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**Quality control and maintenance float models in unreliable
flexible manufacturing systems**

Lin, Chinho, Ph.D.

City University of New York, 1992

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QUALITY CONTROL AND MAINTENANCE FLOAT MODELS
IN UNRELIABLE FLEXIBLE MANUFACTURING SYSTEMS

By

Chinho Lin

A Dissertation submitted to the Graduate Faculty
in Business in partial fulfillment of the
requirements for the degree of Doctor of
Philosophy, The City University of New York

1992

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ABSTRACT

**QUALITY CONTROL AND MAINTENANCE FLOAT MODELS
IN UNRELIABLE FLEXIBLE MANUFACTURING SYSTEMS**

by

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Advisers: Professors Georghios Sphicas, William Chien, and
Lie-Fern Hsu

ABSTRACT

This dissertation investigates two reliability issues pertaining to flexible manufacturing systems (FMS): the reliability of the manufactured products and the reliability of the equipment.

In studying the product reliability, we model the system as an open queueing network and derive the relationships between inspection rates and operational characteristics of the FMS. This study deals with the working, reworking, partial scrapping, and in-process inspection issues. Three separate cases are examined to illustrate the effect of the quality control function on system characteristics:

- (1) The FMS is completely reliable;
- (2) The FMS is unreliable and there is no quality control;
- and
- (3) The FMS is unreliable and has quality control.

Through the examination of these three cases, we show that some different effects exist. Thus, a nonlinear mathematical program is formulated to capture the revenue and cost factors of the system to find the optimal quality control policy. We develop an approximate procedure to obtain near-optimal in-process inspection rates for the nonlinear program. Based on the results of our study, we have found that incorporating an appropriate in-process quality control policy can improve the quality level and enhance the revenue of the FMS. Therefore, we conclude that building a quality control system within the FMS is a viable alternative to the much more expensive zero-defect technology to ensure product reliability.

For the equipment reliability, a model is proposed to deal with the maintenance float of the key flexible manufacturing cell (FMC) and the repair system of the FMS. The entire FMS that we study can be divided into two subsystems, a key FMC and the rest of the system. In the early stage of this research, an analytical approach was conducted for some simplified versions of this problem. We found that the analytical approach became intractable as a means of solving the problem. Therefore, a systematic approach is proposed, which combines the simulation, Taguchi design, regression and classical queueing models. This approach is useful in determining the expression of the fractional utilization of the system. These models are then used to determine the optimal (or near-optimal) number of floats of the

module in a key flexible manufacturing cell and the optimal (or near-optimal) capacity of repair stations in the FMS. The benefits of the approach are:

- (1) Flexibility of simulation by not requiring some of the restrictive assumptions made in analytical models;
- (2) Reduction of the amount of work needed in order to obtain a reliable and applicable model; and
- (3) Savings in time and cost by not requiring to run full-scale simulation experiments, as a result of adopting the Taguchi experiment design.

Furthermore, this approach provides a guide and reference source for operations managers and industrial practitioners involved in design and planning problems of maintenance float systems. It also serves as an alternative to dealing with the intricate issues existing in complex stochastic models.

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CHAPTER ONE

INTRODUCTION

Due to intensified domestic and international competition, and rapid technological changes, the life cycles of products have been greatly shortened. Frequently, manufacturers are confronted with the need to replace their production systems in order to adapt successfully to dynamic changes in product designs. The replacement of an existing technology with a new one is also necessary in order to compete effectively. Therefore, an increasing number of companies have been adopting flexible manufacturing systems to fulfill these needs.

A Flexible Manufacturing System (FMS) is a computer-controlled production system that, in general, includes the following main components (Van Looveren et al. 1986, Dupont-Gatelmand 1982, Hutchinson and Wynne 1973):

- 1) several computer numerically controlled (CNC) machines, including tools and a tool delivery system,
- 2) an automated material handling system (MHS), including a storage facility, and
- 3) an on-line computer control system, which manages the entire FMS.

Through a careful combination of computer control, manufacturing process, operational control, and related equipment, FMS can enable an organization to process different types of parts

simultaneously so that it can respond economically and rapidly to current market requirements. FMS is becoming widely accepted now (Luggen 1991), thanks to advanced technologies that help to change the traditional perception that an FMS is only applicable to a relatively small portion of manufacturing operations.

From the managerial point of view, major issues in the design and operation of FMS can be classified into the following categories (Lin 1989a):

- 1) System models,
- 2) Optimal configuration models,
- 3) Loading models,
- 4) Part routing models,
- 5) Part selection models,
- 6) Releasing and scheduling models,
- 7) Unreliable system models, and
- 8) Inventory models.

Research interest in studying these FMS related issues has been steadily increasing over the years. Different types of mathematical and computational procedures, such as mathematical programming, queueing network theory, static allocation models, simulation, petri nets, and perturbation analysis (Suri 1985), have been developed to solve these problems. In particular, queueing network theory (QNT) has been widely adopted in modeling FMS so as to capture the dynamics, interactions, and uncertainties (i.e., probability property of events) in an aggregate form (Yao

and Buzacott 1986a and 1986b, Lin 1989a).

In this research, we have focused on the construction and analysis of unreliable system models that belong to the seventh category of the above classification scheme. Lin (1989a) points out that there are two types of unreliable issues in the FMS (or, in any manufacturing system). These are associated with the product and with the equipment. In practice, the product reliability can be improved by implementing appropriate quality control policies, while maintenance of equipment is necessary to achieve a specified level of reliability for a manufacturing system. In this study, we have applied the classical queueing, open queueing network and simulation approaches to construct and analyze quality control models and maintenance float models for both a flexible manufacturing cell (FMC) and an FMS.

The quality control model deals with the reworking, partial scrapping, and in-process inspection issues in an unreliable FMS. We have formulated a nonlinear mathematical program that captures the revenue and cost factors of the system. It incorporates the results obtained by an open queueing network analysis. An approximate procedure is then developed to obtain near-optimal in-process inspection rates for the nonlinear program.

A model dealing with the maintenance float of the key FMC and repair system for the entire FMS is established by using the simulation approach which is combined with the Taguchi experimental design and regression metamodel. We divide the

entire FMS into two subsystems, a key FMC and the rest of the system in this study. The key FMC has a great impact on the utilization of the FMS when its module fails. The module could be a tool magazine (Luggen 1991) or some expensive parts used in the FMS. We apply a maintenance float policy to this key FMC for its module to enhance the fractional utilization of the system. Before doing this research, we conducted an analytical approach for this problem in Appendix A. We found that the analytical approach was difficult as a means of solving our problem. Therefore, a systematic approach combining the simulation, Taguchi design, regression, and classical queueing models (Kuei et al. 1990, Chanin et al. 1990, Lin 1989a) is developed to derive the expression of the fractional utilization of the system. The process of the approach has been clearly defined. The benefit of this approach is that we can obtain a simple expression of the fractional utilization of the system in terms of some important independent variables. Therefore, a decision model can be easily developed and solved.

CHAPTER TWO

LITERATURE REVIEW AND METHODOLOGY

In this chapter, we review some relevant literature and discuss the methodology used in this study. Queueing network theory, quality control models for an FMS, maintenance float models, and maintenance models for an FMS are respectively reviewed in Sections 1, 2, 3, and 4. The methodology will be discussed in Section 5.

1. Queueing Network Theory (QNT)

Two types of models exist in the literature in the area of queueing network theory.

- 1) Exact models--In this case, all the models are strictly derived using stochastic theory assumptions.
- 2) Approximate models--In this case, approximate models or algorithms are presented for the various stochastic processes.

For the exact models, there are two distinct approaches:

- 1) Infinite buffer models (IBMs)--These assume that the waiting capacity in each station (i.e., the local buffer) is infinite.
 - 2) Finite buffer models (FBMs)--These assume that the waiting capacity in each station is limited by a finite number.
- For the FBMs, the models for performance measures are

derived using reversibility theory.

The steady state probabilities of these two types of models can be expressed either by a product-form or a closed-form analytical solution. The results obtained from the exact models are based on the assumption of Poisson arrivals and departures. Therefore, various approximate models have been developed to address more general cases. Five different approaches are followed in the approximate models (Bitran and Tirupate 1988): 1) Approximate Decomposition Method (ADM), 2) Approximate Mean Value Analysis (AMVA), 3) Operational Analysis (OA), 4) Exponentialization Approximation (EA), and 5) Diffusion Approximation (DA).

INFINITE BUFFER MODELS (IBMs)

Jackson (1957 and 1963), a pioneer in queueing network study, developed a queueing network representation to capture the stochastic aspects of job shops. Solberg (1977) originally modeled and analyzed an FMS based on closed queueing network (CQN) theory developed by Gordon and Newell (1967).

Three methods, Convolution Algorithm (CA) (Buzen 1973), Mean Value Analysis (MVA) (Reiser and Lavenberg 1980) and Z-Transform Algorithm (Malone et al. 1986), can be used to obtain performance measures such as throughput, length of waiting time, etc. Solberg (1977) also developed a software package-CAN-Q, which employs a Convolution Algorithm to solve for the performance measures of an

FMS. Co and Wysk (1986) examined and confirmed the robustness of CAN-Q.

Moore (1972) derived the performance formulas for classical queueing networks using the generating function method. Malone et al. (1986) extended this model to incorporate multiple part types with a single server, and solved the model using the Z-transform method.

Based on the algorithm developed by Baskett et al. (1975), Solot and Bastos (1988) proposed an exact multiple part types (i.e. for several pallet types) model called MULTIQ.

FINITE BUFFER MODELS (FBMs)

According to Hatvany (1983), in practice, the storage capacity at each station (i.e., the size of each local buffer) is very small. For the finite buffer case, it is only necessary to show a system possessing a reversible process (Kelly 1979) to ensure a steady-state product-form solution (Melamed 1982). Yao (1983) and Yao and Buzacott (1985a and 1987) showed that a system should have a reversible process if any of the following routing rules are used 1) symmetric routing (i.e., all stations are equally visited), 2) jobs are routed to a central-server network, and 3) dynamic routing (i.e., the routing rate depends on the number of jobs at the dispatching and receiving stations).

For the zero local buffer case, Yao and Buzacott (1987) showed that a product-form solution exists even with general

service time distributions. Dallery (1986) used the "global reversible routing" to model an FMS with a limited global storage capacity (i.e., the total number of jobs at a given set of stations is limited by a fixed value), and derived a product-form solution for the model.

APPROXIMATE DECOMPOSITION METHOD (ADM)

The basic idea of ADM is to decompose the network and analyze each station or subset of stations separately. To use this method, two conditions must be satisfied: 1) the nodes must be treated as being independent, and 2) reasonable results for the mean and variance must be obtained.

This method was first developed by Reiser and Kobayashi (1974). There are several other researchers applying this method to derive the models for the system performance of a queueing network. In general, the ADM involves the following three steps:

- 1) analyze the interaction between stations,
- 2) decompose the network into subsystems (each contains a station or a set of stations) and analyze each subsystem separately, and
- 3) combine the results obtained in step 2 to obtain the formulas for the performance of the entire system.

Whitt (1983a and 1983b) also proposed an approximate algorithm for an open queueing network(OQN). Marchal (1985) proposed two approximate mean delay models. Using ADM, the mean

delay and departure coefficients of variation of each station (cell) can be evaluated, and hence, the mean time a job takes to pass through the system can be obtained. Bitran and Tirupate (1988) modified the above algorithm and proposed three approximate algorithms to capture the interference effect of multiple part types using the Poisson and Erlang approximations for the aggregate product. Shanthikumar and Gocmen (1983) also developed an algorithm based on a CQN, which differs from the above algorithms. The previous models used the traditional ADM approach which involves the aggregation of results of the GI/G/1 queueing systems directly. While, Shanthikumar and Gocmen (1983) used an iterative algorithm to find the steady-state probabilities and the utilization rate at each station.

Yao and Buzacott (1985a) developed a decomposition algorithm for the finite local buffer case. Their model assumed 1) an OQN model for an FMS with a centralized MHS (material handling system), 2) finite local buffers, 3) multiple parallel servers at each station, and 4) general service time distributions. The algorithm essentially involves the following two steps: 1) find the relationships between the blocking probabilities of all stations and the inter-arrival time distribution at each station. 2) decompose the system into a set of GI/G/C/S queues and evaluate the performance of each station using the algorithm proposed by Yao and Buzacott (1985b).

APPROXIMATE MEAN VALUE ANALYSIS (AMVA)

This is a heuristical approach for the exact MVA. Schweitzer (1979) first introduced AMVA and considered a single server, single job type case. Neuse and Chandy (1981) extended it to a multiple servers model. Krzesinske and Grcyling (1984) independently developed an improved algorithm for the same problem. Recently, a more powerful algorithm was introduced by Akyildiz and Bolch (1988).

OPERATIONAL ANALYSIS (OA)

OA was first proposed by Buzen (1976) and extended by Denning and Buzen (1978). Four assumptions of OA (Denning and Buzen, 1978) are: one step behavior, flow balance, routing homogeneity, and homogeneous service time. The basic idea is to get measurable quantities directly from a set of data. The data are obtained by observing the on-line system or through simulation. The data is then used to predict the performance of the network based on the results obtained from a closed exponential network model.

EXPONENTIALIZATION APPROXIMATION (EA)

This approach was first used to model the computer system (Shum and Buzen 1977, Marie 1979). Yao and Buzacott (1986a) adopt it to analyze a closed FMS with a classical queueing network. It can also be extended to networks with limited local buffers and multiple-server stations.

DIFFUSION APPROXIMATION (DA)

The diffusion approximation technique is based on the assumptions that the number of events in a given time interval is approximately normally distributed and that queues are almost always nonempty. We can then apply the central limit theorem to capture the variations in queue lengths, and approximate the discrete-value queueing process. A diffusion equation is used to describe the probability distribution of the continuous process $X(t)$. $X(t)=0$ is imposed as a reflecting boundary so that no probability mass can be collected at $X(t)=0$. Therefore, the model is useful only in heavy traffic conditions and becomes less adequate as the queue lessens. Gelenbe (1975 and 1979) introduced additional models to represent the behavior of the empty queues. Reiser and Kobayashi (1974) showed that in most cases the accuracy of the diffusion approximation is quite adequate and is much higher than that obtained by assuming exponential service times. Several researchers (Cox and Miller 1965, Gaver 1968, Newell 1980, Gaver and Shedler 1973a and 1973b, Kobayashi 1974, Reiser and Kobayashi 1974, Gelenbe 1975, Gelenbe and Pujolle 1976, Halachim and Franta 1977, Newell 1980, Pujolle and Al 1986) have used DA to evaluate the system performance of queueing networks.

2. Quality Control Models for an FMS

Almost all papers in the area of FMS assume that the FMS is a zero-defect technology. The more practical approach that incorporates the quality control function into the FMS has not been explored until recently. Tapiero and Hsu (1988) considered a non-zero-defect and unreliable FMS, and proposed an algorithm to compute the average outgoing quality (AOQ) of the system for a queueing network with GI/G/S queues based on Whitt's approximation formulas (Whitt 1983a and 1983b). They derive the relationships between the inspection effort and the quality characteristics under a random Bernoulli (Duncan, 1974) inspection plan. Unfortunately, similar results cannot be obtained for a CSP-1 (Duncan 1974) inspection plan. Furthermore, no optimal inspection rules are provided for the FMS environment.

Recently, Hsu and Tapiero (1989) modified the above model. In this paper, the authors:

- 1) introduced a general procedure for measuring and managing the in-process quality control of an FMS described by an OQN,
- 2) provided some managerial insights regarding the role, position and distribution of the quality control effort in an FMS, and
- 3) formulated the relationships between the operating characteristics and the control of the FMS.

They also constructed a nonlinear program to determine an

optimal quality control plan. The objective function of the model includes the following:

- 1) the inspection cost of each part type at each processing step,
- 2) the waiting cost per unit time,
- 3) the post sales failure cost of each part type,
- 4) the revenue of each part type, and
- 5) the scrapping cost of each part type.

The constraints of the model are the prespecified inspection rates for the parts. Using ADM, they computed the throughput and AOQ of the system. The objective function value can then be obtained for a fixed inspection rate for each part type. However, no complete algorithm is provided to solve the nonlinear program although some sensitivity analysis on the relationships between the profit, inspection rates and AOQ is performed.

3. Maintenance Float Models

A maintenance float policy is typically used in both the manufacturing and service sectors to keep the operating equipment at a high level of availability. The float is made up of standby units that can be used to replace the key operational units immediately after a failure occurs. Failed units are sent to the repair shop for repairing, and are placed in the standby mode after the repair is done.

One of the earliest attempts in determining the required number of standby units is by Levine (1965). He presented some analytical results of the maintenance float factors under an exponential failure-time distribution. A maintenance float factor is the fraction of an operating center's population that will fail in the long run. Using this float factor, the optimal number of standby units can be calculated. This model is subsequently extended to the cases of Weibull (Lowe and Lewis 1983), gamma (Madu 1985), and triangular (Georgantzas 1987) failure-time distributions. This problem was also modeled using the queueing theory technique. Koenigsberg (1958 and 1960) used a two-stage cyclic queue model to study the effect of standby units on the efficiency of the system. Madu (1988a, 1988b and 1990) used the closed queueing network theory to develop maintenance float models for a one repair center case and for two repair centers, load independent servers case. The author points out that "the expected number of units required to be in float on the average in

order to maximize the availability of the operating system is the expected number of failed parts (machines or modules) either awaiting or undergoing repair in the stations" (Madu 1988a, 1988b and 1990). Some cost functions were also used in these papers to determine the optimal number of machines (parts) and the optimal number of repair servers in the system. Other researchers (Sahu and Sharam 1970) used simulation to study this problem. Rueda and Miller (1985) studied the fleet size of a transit bus system using simulations. Madu and Georgantzas (1988) compared the float levels obtained from analytical models to those obtained from experimental simulations. Madu et al. (1990) used experimental simulations to study the effects of the coefficient of variation on maintenance float policy. Madu and Chanin (1989) used the fractional factorial design to simulate the maintenance float system. Kuei et al. (1990) and Chanin et al. (1990) adopted a new approach (Lin 1989a)--a combination of simulation modeling, Taguchi experimental design, and metamodels to analyze the maintenance float problem.

However, all the papers reviewed above focus on the maintenance float problem for single-station systems.

4. Maintenance Models for an FMS

Several papers (Gilbert 1985, Hora 1987, Kennedy 1987, Claire 1988) have discussed the impact of a maintenance policy on the operations of an FMS. However, all the above papers focus only on the guidelines for and the importance of the maintenance policies and no mathematical models are provided in these papers. Recently, Singhal et al. (1987) discussed the issues pertaining to the maintenance policies in an FMS environment. They are:

- (1) How much self-diagnostic equipment is justified for the FMS?
- (2) How much preventive maintenance should be performed?
- (3) How should downtime costs be calculated?
- (4) How much maintenance should be done in-house, and how much by outside contractors?
- (5) What spare parts should be stocked, and in what quantities?

There are only a few papers that deal with modeling in an FMS.

Buzacott and Shanthikumar (1980) demonstrated the effect of machine breakdowns on the production capacity but do not provide optimal rules.

A general model for an FMS with unreliable machines is proposed by Vinod and Solberg (1984). In this model, each work station has a pure delay station which is visited by preemptive failure parts of machines. Assuming an ample supply of repairmen,

the authors analyzed the problem using a CQN and proposed two approximate methods--MCA (multiple class approximation method), which is modified from MVA, and APF (approximate product-form method), which is based on the concept of product-form solutions to calculate the throughput, waiting time and utilization of the system. Simulation was used to validate the following characteristics of MCA and APF:

- 1) MCA always underestimates the actual throughput.
- 2) In MCA, the relative throughput error decreases with the total jobs (N) in the system. However, the APF does not possess this property.
- 3) Both MCA and APF estimate the mean queue length fairly accurately. The error, though small, frequently increases with N , but the rate of increase decreases with N .

Vinod and Altiook (1986) modelled the above problem using an exact equivalent network in which the service time is represented by a two-stage Coxian distribution. The model is approximated using the CAN-Q model and is validated by performing simulations with a wide range of parameters. The results show that the approximate formulas are fairly robust.

Vinod and John (1986) studied a failure-prone FMS with a two-stage repair facility. The FMS in their model has M stations that perform distinct operations, and each station has one or more identical machines. Each station represents a distinct input source to the repair system. Hence, there are $M+2$ stations

in the system and the stationary probability distribution can be obtained using the formulas for a CQN with multiple job types. A mathematical model is constructed to determine the optimal capacities for repair facilities. They prove the monotonicity property of the steady-state behavior of the model. That is, when the number of repair channels at either one or both repair facilities increases by one, the steady-state number of operational machines at each station increases stochastically. The model can then be solved by the traditional discrete optimization procedures. An algorithm developed by Lawler and Bell (1966) is proposed to solve the problem.

Vinod and Sabbagh (1986) examined a tool availability model, which can be used to determine the optimal number of spares for each type of tool and the optimal capacity level of the repair facility. In this paper, the authors present an approximate method that incorporates failure time into the total service time. Three extreme cases are tested to validate the approximate method. The results show that the approximate method is quite accurate for practical applications. The model is then formulated as a nonlinear integer program that embeds a closed queueing network with multiple job types. The program is difficult to solve using general mathematical programming algorithms, and therefore, a modified version of the lexicographic partial enumeration algorithm (Sabbagh 1983) is proposed to solve the nonlinear program.

5. Methodology

Based on the above review of the literature, we conclude that quality control and maintenance problems in an FMS environment have not been fully addressed. Therefore, our study will focus on these issues.

In an unreliable FMS, defective products may be produced. To improve the quality of products, an appropriate quality control policy must be in place. According to Taguchi and Wu (1980), two types of quality control policies may be adopted to improve product quality--the off-process and the in-process quality control policies. The off-process quality control policies seek to improve the quality level of products before they are produced, which is highly recommended by Taguchi and has been popularly adopted in industry (Taguchi and Wu 1980). The in-process quality control policies concentrate on the quality control issues during the time that products are being processed. The classical statistic quality control techniques, such as control chart, inspection-sample planning etc. belong to the in-process quality control. The in-process quality control is required in the manufacturing process because we want to maintain the quality of our products at a certain level.

The approach followed in this study is the in-process quality management. The basic idea of the in-process quality management is that product inspection is carried out first and then

statistical methods are applied to analyze the results of the inspection. This helps to characterize the production process, so proper action for improvement can be undertaken based on the feedback obtained. As to the sampling method used in the inspection, although the CSP (continuous sampling plan) would be used in practice for automated equipment, it is difficult to analyze the resulting FMS with analytical models (Tapiero and Hsu 1988). Therefore, we adopt the random Bernoulli inspection plan (Duncan 1974) in this study.

The study here is similar to the works of Tapiero and Hsu (1988 and 1989). But three issues in our study are different from the above researches (Tapiero and Hsu 1988, 1989). They are

- (1) Mathematical approach--We first analyze the characteristics of the FMC, then use these results to describe the performance measurements of the FMS. Tapiero and Hsu directly derive the performance measurements of the FMS.
- (2) Assumptions--The FMS in our case permits defective parts to be sent back for repairing and reworking if the defects in the part are made in the manufacturing process of the current cell.
- (3) Practical issue--The FMS we study is an assembly-oriented system. Most of the characteristics of flexible machining systems can also be found in flexible assembly systems (Kusiak 1990). However, the process planning and

design of products and parts in the flexible assembly systems are different from those in the flexible machining systems (Kusiak 1990, Boothroyd et al. 1982, Laszcz 1985).

We model the FMS as an open queueing network and evaluate the system characteristics for a given set of inspection rates. Several factors such as output rate, working, reworking, scrapping, and inspection, are considered in our model, which, together with the waiting time information, will be used to construct a mathematical program for obtaining near-optimal inspection rates.

Equipment failure, on the other hand, is handled in this study using a maintenance policy. We separate the sources of failure of the FMS into two types. One is the module used in the key FMC of the FMS. The other is the component used in the whole FMS excluding the key FMC. The module is a very important and expensive unit in the key FMC. It could be a group of components. We group these components which are subject to failure into a portable unit (i.e., module) for speeding the replacement when it fails. This type of module should enhance the utilization of the FMS.

Before dealing with the above complex problem, we use classical queueing theory and formulate mathematical programs to model the maintenance float issues (see Appendix A). In this prior study, we found that the approach we present in Appendix A

is difficult to use in solving the complex case (see Case 4 in Appendix A). Therefore, we developed a novel approach to solve this complex problem. This new approach is modified from Lin (1989a), Kuei et al. (1990), and Chanin et al. (1990). It uses simulation, Taguchi experiment design, regression, and classical queueing models to determine the main performance measurement of the system (i.e., fractional utilization in our case) and then a decision model is constructed and solved. Three different points from the above three papers are proposed in the new approach:

- (1) We propose a method to determine the experimental range of the independent factor rather than using a given range in experiment and argue that the given range is determined from the technical information (see step 3 of systematic approach in Section 2 of Chapter 4).
- (2) The process of choosing linear graph--in addition to considering the requirements of experimental factors, we also need to be concerned with the effects of the confounding of high order interaction factors on the experimental results (see step 4 of systematic approach in Section 2 of Chapter 4).
- (3) In the re-design, we also consider separating the experimental design into two experimental designs (i.e., two regression equations will be obtained) rather than only increasing the experimental level of the independent

factors whose F values are very high compared to others in the analysis of variance (ANOVA) table (full explanation is given in Section 3.2 of Chapter 4).

The detailed procedure of the new approach will be discussed in Chapter 4.

CHAPTER THREE

A QUALITY CONTROL MODEL WITH REWORKING AND SCRAPPING

1. Introduction

In this chapter we propose an analytically-based queueing network model for an unreliable FMS with inspection, reworking and scrapping. The present FMS consists of several manufacturing cells. The main components in each cell are assembly machines, robots, and an inspection station. In the assembly-oriented manufacturing cell, the defective parts which are detected by the inspection station can be dis-assembled (i.e., "reworked") if the defects are made in the manufacturing process of the current cell. Then they can be processed just like the original input materials from the immediately preceding cell. On the other hand, if defects of parts are caused by the processing from one of the preceding cells, the parts will be scrapped. In practice, the defective parts made in the current cell, even after being reworked, may have to be scrapped because they are impossible to use anyway. For example, in the process of cutting tubes, we found it too long after the first cut, but after recutting, it may be too short and thus it has to be scrapped. However, we exclude this possible situation by assuming that the probability that an unrepairable defect is made by the reworking process is negligible.

The main assumptions are as follows:

1. After going through a particular cell for processing, the part will not return to that cell again.
2. The local buffer is of infinite size.
3. The processing capacity of each cell is large enough to handle the input , i.e., the aggregate processing time (including the working time, reworking time and inspection time for all part types) multiplied by the aggregate arrival rate of all part types should be less than one.
4. The working time, reworking time and inspection time of parts are mutually independent.
5. The working time and reworking time belong to a general distribution.
6. For each part type, the visitation sequence in the FMS is fixed and given.
7. A part may go through the working area, reworking area and inspection area many times, but the processing time in these three processes does not change in each visit.

Assumptions 1 and 6 are practical situations in the assembly-oriented system because the works of assembly rarely go back to the stations which were visited before unless they need to be reworked, and also because the routing of an assembly system for a specified job is difficult to change. The assumption of infinite local buffer size is adopted by many researchers

(Buzacott and Yao 1986a, 1986b) to simplify the solution procedure. To ensure that we reach the steady state of the queueing system, assumption 3 is needed. Assumptions 4 and 7 are required to aggregate the working time, reworking time and inspection time.

The remainder of the chapter is organized as follows. In Section 2 we derive preliminary results for an unreliable manufacturing cell with in-process inspection, reworking and scrapping. These results will be used in Section 3 to examine the performance of an unreliable FMS under the effect of quality control. A queueing network model is developed for this purpose. In Section 4 we formulate a profit-maximization, nonlinear mathematical program that incorporates the results obtained in Section 3. We then use a numerical example to illustrate the effect of the quality control process on the system performance and implement an enumerative procedure to obtain a near-optimal quality control policy for the FMS in Section 5. Finally, Section 6 concludes this chapter.

2. An Unreliable Manufacturing Cell With Inspection, Reworking and Scrapping

In this section, we define the functions of an unreliable manufacturing cell and the movements of parts in the cell, and derive preliminary results to characterize the entire FMS.

The manufacturing cell that we consider consists of a working area, a reworking area, and an inspection station. A part will leave the cell if it passes or skips inspection. If defects are detected, two possibilities arise: if the defect was due to the processing in the current cell, the unit is routed to the reworking area and then recirculated back to the working area for processing. Otherwise, it will be scrapped. This process is shown in Figure 3-1. Inspection is assumed to be 100% reliable and all defective items can be examined to determine if they are from the current cell or from one of the preceding cells. The operational parameters of the manufacturing cell are defined as follows:

- r : the inspection percentage at the manufacturing cell, $0 \leq r \leq 1$,
- D : the defective percentage of parts, including the defects that are made in the current cell and in the preceding cells ($0 \leq D \leq 1$) before inspection,
- AOQ_{-1} : the average output quality of the immediately preceding cell, i.e., the defective percentage of the parts entering the current cell,

q : probability that the work in the current cell is defective, $0 \leq q \leq 1$,

τ, V : the mean and variance of the working time,

T : the inspection time (a constant).

In order to avoid unreliable processing of parts and inspection, a typical part may need to pass through the working area, inspection station, and reworking area several times before it leaves the cell. Let

N : the number of times a part passes through the working area,

$K(N), \tau_N, V_N$: the p.d.f., mean and variance of N , respectively,

θ : the actual total time (i.e., working time, reworking time and inspection time) a part spends in the cell, and

τ_θ, V_θ : the mean and variance of θ , respectively.

The probability distribution ($K(N)$), mean (τ_N) and variance, (V_N) of N can be expressed in operational terms. These results are summarized in Propositions 1 and 2 below.

Proposition 1:

For the first visitation cell of all types of parts, N is geometrically distributed with

$$K(N) = p(rq)^{N-1}, \quad N=1,2,3,\dots, \text{ and} \quad (1)$$

$$\tau_N = 1/p, \quad V_N = rq/p^2, \quad \text{where } p=1-rq. \quad (2)$$

■

Proof:

In the first visitation cell, the probability that a part will be reworked is rq , and the probability that a part will pass the cell with one visit to the working area is $1-rq$. Therefore, from the definition of the geometric distribution, we obtain equations (1) and (2). Q.E.D.

Proposition 2:

For the i th visitation cell of a part ($i \geq 2$), N follows a quasi-geometric distribution with

$$K(1) = 1 - r(D-AOQ_{-1}), \quad (3)$$

$$K(N) = r(D-AOQ_{-1})(1-rq)(rq)^{N-2}, \quad N=2,3,\dots, \quad (4)$$

$$\tau_N = 1 + [r(D-AOQ_{-1}) \left(\frac{1}{1-rq} \right)], \text{ and} \quad (5)$$

$$V_N = r(D-AOQ_{-1}) \left[\frac{1 + rqAOQ_{-1}}{(1-rq)^2} \right] \quad (6)$$

■

Proof:

After the first visitation cell, the probability that a part will be reworked after the first pass through the working area is $r(D-AOQ_{-1})$. This implies that the probability a part will visit the working area only once before it leaves the cell (to the next visitation cell or to be scrapped) can be expressed by equation (3).

Furthermore, after the first reworking, the probability that a part will leave the cell is $(1-rq)$. The probability that it

will be reworked again is rq . Therefore, we obtain equation (4).

τ_N and V_N are the expected value and variance, respectively, and they can be obtained directly from definition.

Equation (5) is derived as follows:

$$\begin{aligned}
 \tau_N &= [1-r(D-AOQ_{-1})] + \sum_{N=2}^{\infty} Nr(D-AOQ_{-1})(1-rq)(rq)^{N-2} \\
 &= [1-r(D-AOQ_{-1})] + r(D-AOQ_{-1})(1-rq) \sum_{N=2}^{\infty} N(rq)^{N-2} \\
 &\stackrel{\Delta}{=} [1-r(D-AOQ_{-1})] + r(D-AOQ_{-1})(1-rq) B, \text{ where} \\
 B &= \frac{1}{rq} \left[\sum_{i=0}^{\infty} \binom{i+1}{1} (rq)^i - 1 \right] \\
 &= \frac{1}{rq} \left[\frac{1}{(1-rq)^2} - 1 \right] \\
 &= \frac{2-rq}{(1-rq)^2}.
 \end{aligned}$$

Thus,

$$\begin{aligned}
 \tau_N &= 1-r(D-AOQ_{-1}) + r(D-AOQ_{-1})(1-rq) \left(\frac{2-rq}{(1-rq)^2} \right) \\
 &= 1 + \left\{ r(D-AOQ_{-1}) \left(\frac{1}{1-rq} \right) \right\}.
 \end{aligned}$$

Similarly, equation (6) is derived as follows:

$$\begin{aligned}
 V_N &= \sum_{N=0}^{\infty} N^2 K(N) - \tau_N^2. \\
 \sum_{N=0}^{\infty} N^2 K(N) &= [1-r(D-AOQ_{-1})] + r(D-AOQ_{-1})(1-rq) \sum_{N=2}^{\infty} N^2 (rq)^{N-2} \\
 &\stackrel{\Delta}{=} [1-r(D-AOQ_{-1})] + r(D-AOQ_{-1})(1-rq) A, \text{ where} \\
 A &= \sum_{N=2}^{\infty} \left[2 \binom{N}{2} + N \right] (rq)^{N-2} = 2 \sum_{N=2}^{\infty} \binom{N}{2} (rq)^{N-2} + \sum_{N=2}^{\infty} N (rq)^{N-2}
 \end{aligned}$$

$$\begin{aligned}
&= 2 \sum_{i=0}^{\infty} \binom{i+2}{2} (rq)^i + \frac{2-rq}{(1-rq)^2} \\
&= \frac{2}{(1-rq)^3} + \frac{2-rq}{(1-rq)^2} = \frac{4-3rq+(rq)^2}{(1-rq)^3}.
\end{aligned}$$

Thus,

$$V_N = r(D-AOQ_{-1}) \left[\frac{1+rqAOQ_{-1}}{(1-rq)^2} \right]. \quad \text{Q. E. D.}$$

It is easy to see that if it is assumed that the part passes the cell without reworking, then N follows a geometric distribution with a passing rate equal to $r(D-AOQ_{-1})(1-rq)$, and a feedback rate equal to $r(D-AOQ_{-1})(rq)$. That is, the right tail of this distribution is purely geometrically distributed.

Using the results of Propositions 1 and 2, we can then derive the mean and variance of the total processing time. We derive the expressions for the i th ($i \geq 2$) visitation cell in Proposition 3 first, and then use them to obtain the expressions for the first visitation cell in Proposition 4.

Proposition 3:

For the i th visitation cell ($i \geq 2$), the mean and variance of the actual total time a part spends in the cell are, respectively,

$$\tau_{\theta} = (\tau_M + rT)\tau_N + \tau_R(\tau_N - 1) \quad (7)$$

$$V_{\theta} = (\tau_N - 1)(V_M + V_R) + V_M + (\tau_M + \tau_R + rT)^2 V_N. \quad (8)$$

■

Proof:

The total mean processing time for the working and inspection areas equals $\tau_M + rT$. The expected number of times a part will be reworked equals $\tau_M - 1$. Therefore, using the compounded stochastic process (Ross 1985), and by the definition of τ_θ , we obtain equation (7).

The total mean processing time for the working, reworking, and inspection equals $\tau_M + \tau_R + rT$. The total variance for the working and reworking equals $V_M + V_R$. Since the working time and reworking time are assumed to be independent, V_θ can be expressed by equation (8).

Q.E.D.

Proposition 4:

For the first cell in the visitation sequence, the mean and variance of the actual total time a part spends in the cell are, respectively,

$$\tau_\theta = (\tau_M + rT + rq\tau_R)/p, \text{ and} \quad (9)$$

$$V_\theta = (V_M + rqV_R)/p + rq(\tau_M + rT + \tau_R)^2/p^2, \quad (10)$$

where $p = 1 - rq$.

■

Proof:

Substituting equation (2) into equations (7) and (8), we obtain equations (9) and (10). Q.E.D.

3. The Flexible Manufacturing System

The flexible manufacturing system (FMS) that we consider consists of several manufacturing cells with processing, inspection and reworking functions in each cell. Parts of different types will flow through the FMS following prespecified routes that visit various cells in the system. As described earlier, a part leaves a cell if it passes or skips inspection. On the other hand, when a defective part is detected, the part will be reworked and recirculated back to the working area of the cell if the defects are made in the current cell. However, if the defects are found to be made in one of the preceding cells, the part will be scrapped from the current cell without reworking. This section is further divided into two subsections. In Section 3.1 we obtain, for each part type, the expected number of parts that will be reworked and scrapped in each of their visitation cells, the expected number of undetected defective parts, the total flow to the next cell in the visitation sequence, and finally, the average outgoing quality of each part type and of the FMS as a whole. In Section 3.2 we examine the waiting time spent by each part in the system due to working, reworking and inspection.

3.1 System Performance Measurements

We first define the operational parameters of the FMS and the variables that measure the performance of the FMS.

Operational Parameters

R : the number of part types to be processed by the FMS,

M : the number of manufacturing cells in the FMS,

λ_k : the arrival rate of type- k parts,

$V_k = [V_k(1), V_k(2), \dots, V_k(z_k)]$, the prespecified visitation sequence of type- k parts, where $V_k(1)$ denotes the cell visited by type- k parts at their 1th processing step,

$q_{k, V_k(\ell)}$: the probability of manufacturing a defective type- k part in the $V_k(\ell)$ visitation cell, and

$r_{k, V_k(\ell)}$: the inspection percentage for type- k parts in the $V_k(\ell)$ visitation cell.

Performance Variables

$(ER_k)_{V_k(\ell)}$: the expected number of type- k parts that will be reworked in the ℓ th visitation cell per unit time,

$(ES_k)_{V_k(\ell)}$: the expected number of type- k parts that will be scrapped in the ℓ th visitation cell per unit time,

$(ED_k)_{V_k(\ell)}$: the expected number of undetected defective type- k parts exiting from the ℓ th visitation cell per unit time,

$(EF_k)_{V_k(\ell)}$: the expected flow of type- k parts exiting from the ℓ th visitation cell, (and entering the next cell in the visitation sequence) per unit time,

$(D_k)_{v_k(\ell)}$: the probability that a type-k part is defective after processing and prior to inspection in its ℓ th visitation cell, and

$(AOQ_k)_{v_k(\ell)}$: the average outgoing quality of type-k parts from their ℓ th visitation cell.

Furthermore, we define a function as follow:

$$1\{V_k(1)=j\} = \begin{cases} 1, & \text{if cell } j \text{ is the } 1\text{th visitation cell of type-}k \text{ part;} \\ 0, & \text{otherwise.} \end{cases}$$

In Propositions 5 and 6 we express, for each part type, the performance variables in terms of the operational parameters for the first and the subsequent visitation cells, respectively.

Proposition 5:

For the first visitation cell of type-k parts ($k=1, \dots, R$), we have

$$(ER_k)_{v_k(1)} = \lambda_k (r_{k,v_k(1)} q_{k,v_k(1)}) / (1 - r_{k,v_k(1)} q_{k,v_k(1)}), \quad (11)$$

$$(ED_k)_{v_k(1)} = (\lambda_k + (ER_k)_{v_k(1)}) [1 - r_{k,v_k(1)}] q_{k,v_k(1)}, \quad (12)$$

$$(ES_k)_{v_k(1)} = 0, \quad (13)$$

$$(EF_k)_{v_k(1)} = \lambda_k, \quad (14)$$

$$(D_k)_{v_k(1)} = q_{k,v_k(1)}, \text{ and} \quad (15)$$

$$(AOQ_k)_{v_k(1)} = (ED_k)_{v_k(1)} / \lambda_k. \quad (16)$$

■

Proof (dropping all indices of notations):

Since this is the first visitation cell of the parts, all detected defects are made in the current cell, and all the detected defective parts will be reworked and recirculated back to the working area. Therefore, there will be no scrapped parts and the total flow of parts exiting from this cell should equal the arrival rate of the parts. This proves equations (13) and (14).

Equation (11) can be obtained from the steady-state balance equation (i.e., input rate=output rate) for the expected number of reworked parts: $(\lambda+ER)r_q=ER$.

Equation (12) is obtained from the fact that the total number of parts entering the working area is $\lambda+ER$ and the probability that a defective part is not inspected in this cell is $(1-r)q$.

Finally, equations (15) and (16) are obtained by definition.

Q.E.D.

Proposition 6:

For the l th visitation cell ($l=2, \dots, z_k$) of type- k parts ($k=1, \dots, R$), we have

$$(D_k)_{v_k(l)} = (AOQ_k)_{v_k(l-1)} + q_{k,v_k(l)} - (AOQ_k)_{v_k(l-1)} q_{k,v_k(l)}, \quad (17)$$

$$(ES_k)_{v_k(l)} = (ED_k)_{v_k(l-1)} r_{k,v_k(l)}, \quad (18)$$

$$(EF_k)_{v_k(l)} = (EF_k)_{v_k(l-1)} - (ES_k)_{v_k(l)}. \quad (19)$$

$$(ER_k)_{v_k(l)} = (EF_{k,v_k(l-1)} (1 - (AOQ_k)_{v_k(l-1)}) r_{k,v_k(l)} q_{k,v_k(l)}) / [1 - r_{k,v_k(l)} q_{k,v_k(l)}], \quad (20)$$

$$(ED_k)_{v_k(l)} = ((EF_k)_{v_k(l)} + (ER_k)_{v_k(l)}) [1 - r_{k,v_k(l)}] (D_k)_{v_k(l)}, \quad (21)$$

$$(AOQ_k)_{v_k(l)} = (ED_k)_{v_k(l)} / (EF_k)_{v_k(l)}. \quad (22)$$

■

Proof (dropping all indices of notations):

To obtain (17), observe that the rate of defective parts entering from the preceding cell is $(AOQ)_{-1}$. During processing at the current cell, the probability of a defective work is q . Therefore, the joint rate of defective parts before inspection is:

$$(AOQ)_{-1} + (1 - AOQ_{-1})q = (AOQ)_{-1} + q - (AOQ)_{-1}q.$$

Since the parts are scrapped if the detected defects are made in one of the preceding cells in the visitation sequence, and the outflow of parts equals the inflow minus the number of parts that are scrapped in that cell, we obtain equations (18) and (19).

Equation (20) is proved as follows. The number of parts that will be reworked once in a cell is

$$ER' = EF_{-1} rD - EF_{-1} rAOQ_{-1} = rq [EF_{-1} (1 - AOQ_{-1})],$$

Thus,

$$\begin{aligned} ER &= ER' + ER'rq + ER'r^2q^2 + \dots \\ &= [(EF_{-1} (1 - AOQ_{-1}))rq] / (1 - rq). \end{aligned}$$

Equation (20) can also be obtained directly from equation (5) as $(\tau_N - 1)EF_{-1}$.

Equation (21) is obtained from equation (12) by replacing λ and q with EF and D , respectively.

Finally, equation (22) is obtained by definition. Q.E.D.

Using the results presented in Propositions 5 and 6, we can then express the average outgoing quality of type- k parts by $(AOQ_k)_{v_k(z_k)}$, i.e., the average outgoing quality of type- k parts from the last visitation cell. Furthermore, the average outgoing quality of all types of parts from the FMS can be represented by the ratio of the total number of undetected defective parts (of all types) and the total outflow of parts (of all types) from the FMS. That is,

$$\text{Overall AOQ} = \left(\sum_{k=1}^R (ED_k)_{v_k(z_k)} \right) / \left(\sum_{k=1}^R (EF_k)_{v_k(z_k)} \right).$$

3.2. Evaluating Part Waiting Time

In this section we derive the expected waiting time for each type of part and express the expected total time a type- k part spends in each of its visitation cells and in the entire system. We first derive the aggregate arrival rate and the aggregate time spent by all types of parts in each cell of the FMS. Then, we use the approximate decomposition method to evaluate the approximate waiting time spent by an aggregate part in each cell. This approach is widely used in queueing network studies to deal with

situations where general processing time distributions are assumed.

Let $\tau_{W,V_k(\ell)}$, $\tau_{R,V_k(\ell)}$, $V_{W,V_k(\ell)}$, and $V_{R,V_k(\ell)}$ denote the means and variances of the working and reworking times of type- k parts in their ℓ th visitation cell, respectively. We define additional variables that will be used in this section as follows.

λ_{o1} : the aggregate arrival rate of all types of parts entering cell 1,

λ_{ij} : the aggregate arrival rate of parts to cell j from cell i ,

λ_{i0} : the aggregate completion rate of parts exiting the FMS from cell i ,

TR_{ij} : the transition probability of an aggregate part (i.e., disregarding part types) from cell i to cell j , and

$\tau_{V_k(\ell)}^\theta$, $V_{V_k(\ell)}^\theta$: the mean and variance of the "total" processing time for a type- k part in its ℓ th visitation cell (i.e., θ is used to represent the sum of the work, rework, and inspection time).

We now express these variables in Propositions 7 and 8 below.

Proposition 7:

The aggregate arrival and completion rates, and the transition probabilities under the effect of quality control can be expressed in terms of individual arrival rates and expected flows as follows.

$$\lambda_{01} = \sum_{k=1}^R \lambda_k 1\{V_k(1)=1\}, \quad i=1, \dots, M, \quad (23)$$

$$\lambda_{1j} = \sum_{k=1}^R \sum_{\ell=1}^{z_k} (EF_k)_{v_k(\ell)} 1\{V_k(\ell)=1\} 1\{V_k(\ell+1)=j\}, \quad i, j=1, \dots, M, \quad (24)$$

$$\lambda_{10} = \sum_{k=1}^R (EF_k)_{v_k(z_k)} 1\{V_k(z_k)=1\}, \quad i=1, \dots, M, \quad \text{and} \quad (25)$$

$$TR_{1j} = \lambda_{1j} / \sum_{\ell=1}^M \lambda_{1\ell}, \quad i, j=0, 1, \dots, M. \quad (26)$$

■

Proposition 8:

The mean and variance of the total processing time of type- k part in its first visitation cell are, respectively,

$$\tau_{k, v_k(1)}^\theta = [\tau_{H, v_k(1)} + r_{k, v_k(1)} T_{k, v_k(1)} + r_{k, v_k(1)} q_{k, v_k(1)} \tau_{R, v_k(1)}] / (1 - r_{k, v_k(1)} q_{k, v_k(1)}), \quad \text{and} \quad (27)$$

$$\begin{aligned} v_{k, v_k(1)}^\theta = & \{ [V_{H, v_k(1)} + r_{k, v_k(1)} q_{k, v_k(1)} V_{R, v_k(1)}] / \\ & (1 - r_{k, v_k(1)} q_{k, v_k(1)}) \} + \{ [(\tau_{H, v_k(1)} + \tau_{R, v_k(1)} \\ & + r_{k, v_k(1)} T_{k, v_k(1)})^2 r_{k, v_k(1)} q_{k, v_k(1)}] / \\ & (1 - r_{k, v_k(1)} q_{k, v_k(1)})^2 \} \end{aligned} \quad (28)$$

For the subsequent visitation cells, we have

$$\begin{aligned} \tau_{k,v_k}^{\theta} = & (\tau_{n,v_k}(\ell) + r_{k,v_k}(\ell) T_{k,v_k}(\ell)) \tau_{n,v_k}(\ell) \\ & + \tau_{R,v_k}(\ell) (\tau_{n,v_k}(\ell) - 1), \end{aligned}$$

(29)

$$\begin{aligned} v_{k,v_k}^{\theta} = & (\tau_{n,v_k}(\ell) - 1) (V_{n,v_k}(\ell) + V_{R,v_k}(\ell)) + V_{n,v_k}(\ell) \\ & + (\tau_{n,v_k}(\ell) + \tau_{R,v_k}(\ell) + r_{k,v_k}(\ell) T_{k,v_k}(\ell))^2 V_{n,v_k}(\ell). \end{aligned}$$

(30)

■

Proof:

By adding the indices for part types and visitation sequence to (7), (8) (9), and (10), Proposition 8 is obtained. Q.E.D.

In order to simplify our model and meet the requirements of the approximate formulation for solving the waiting time problem (Whitt 1983a), we assume that there is no dependent relationship between the reworking process and working process. Thus, we can aggregate the total process time of all part types together.

Proposition 9:

The mean and variance of the total processing time for an aggregate part in cell j are, respectively,

$$\tau_j = \frac{\sum_{k=1}^R \sum_{\ell=1}^{Z_k} (EF_k)_{v_k(\ell-1)} 1\{V_k(\ell)=j\} \tau_{k,v_k}^{\theta}}{\sum_{k=1}^R \sum_{\ell=1}^{Z_k} (EF_k)_{v_k(\ell-1)} 1\{V_k(\ell)=j\}}, \quad j=1, \dots, M, \text{ and} \quad (31)$$

$$V_j = \frac{\sum_{k=1}^R \sum_{\ell=1}^{Z_k} (EF_k)_{v_k(\ell-1)} \left[v_{k, v_k(\ell)}^\theta + (\tau_{k, v_k(\ell)}^\theta)^2 \right] 1\{V_k(\ell)=j\}}{\sum_{k=1}^R \sum_{\ell=1}^{Z_k} (EF_k)_{v_k(\ell-1)} 1\{V_k(\ell)=j\}} - \tau_j^2, \quad j=1, \dots, M, \quad (32)$$

where,

$$(EF_k)_{v_k(0)} = \lambda_k, \quad k=1, \dots, R.$$

■

Proof:

Equation (31) can be directly derived by the method of aggregation of means. The result of equation (32) is based on the property that the second moment of a mixture of distributions is the mixture of the second moments of the individual distributions. Q.E.D.

Proposition 10:

The expected waiting time of a part in cell j can be approximated by

$$W_{qj} = \tau_j \rho_j (ca_j + cs_j) g_j / 2(1-\rho_j), \quad (33)$$

where,

$$g_j = \begin{cases} \exp \left[-\frac{2(1-\rho_j)}{3\rho_j} - \frac{(1-ca_j)^2}{ca_j+cs_j} \right], & \text{if } ca_j < 1; \\ 1, & \text{if } ca_j \geq 1, \end{cases}$$

$\rho_j = (\sum_{i=0}^M \lambda_{ij}) \tau_j$, the traffic intensity at cell j , $0 \leq \rho_j < 1$,

$cs_j = V_j/\tau_j^2$, the squared coefficient of variation of the processing time at cell j , and

ca_j = the square coefficient of variation of the arrival rates at cell j , which can be determined by solving the following system of linear equations:

$$\alpha_i = \sum_{j=0}^M \lambda_{ji}, \quad i=1, \dots, M, \quad (34)$$

$$\alpha_i ca_i - \sum_{j=1}^M \{ \alpha_j (1-\rho_j^2) TR_{ji}^2 ca_j \} = \lambda TR_{oi} + \sum_{j=1}^M \{ \alpha_j TR_{ji} (\rho_j^2 TR_{ji} cs_j + 1 - TR_{ji}) \},$$

$$i=1, \dots, M, \quad (35)$$

$$\text{where } \lambda = \sum_{i=1}^M \lambda_i$$

Proof:

Equation (33) is obtained from the results presented by Whitt (1983a). g_j is the Kraemer and Langenbach-Belz approximation when $ca_j < 1$. The Kraemer and Langenbach-Belz refinement is not useful and also it is not necessary when $ca_j \geq 1$ (Whitt 1983a). Equation (33) is an exact formula for an $M/G/1$ system when $ca_j = 1$. Equations (34) and (35) were derived by Shanthikumar and Buzacott (1981). Q.E.D.

Using Propositions 8 and 10, we can now express the expected total time a type-k part spends in its ℓ th visitation cell ($W_{k, v_k(\ell)}$) and the expected total time a type-k part spends in the FMS (WE_k) as follows.

$$W_{k, v_k(\ell)} = W_{q, v_k(\ell)} + \tau_{k, v_k(\ell)}^{\theta}, \text{ and} \quad (36)$$

$$WE_k = \sum_{\ell=1}^z (W_{q, v_k(\ell)} + \tau_{k, v_k(\ell)}^{\theta}) . \quad (37)$$

4. A Quality Control Model for an Unreliable FMS

Although incorporating a quality control process into an unreliable FMS can improve the quality of the outputs, the trade-off is the inevitable decrease in the throughput of the FMS due to the presence of the quality control process. One of the reasons is that the quality control process can increase the total processing time of parts and the units of scrapping. Since the in-process quality control system will dynamically affect the performance measures we discussed in the previous section, we have to use a systematic approach to analyzing these effects. Therefore, in this section, we construct a profit-maximization model that seeks a near-optimal (i.e., only a heuristic solution algorithm is proposed here) quality control policy for an unreliable FMS. Relevant revenue and cost parameters are defined as follows.

REV_k : the revenue for a type-k part,

CI_{kl} : the cost of inspecting a type-k part in its l th visitation cell,

CW_k : the waiting cost of a type-k part per unit time,

CF_k : the post-sales failure cost of a type-k part (as defined in Lee and Tapiero (1986)),

CS_{kl} : the cost of scrapping a type-k part in its l th visitation cell, with $CS_{kl} < CS_{k,l+1}$, $l = 1, 2, \dots, z_k - 1$,

CR_{kl} : the cost of reworking a type-k part in its l th

visitation cell, and

$CP_{k\ell}$: the cost of processing a type-k part in its ℓ th visitation cell.

Let r_{kj} (the inspection rate for type-k parts in cell j) be decision variables, we now present a profit-maximizing model that incorporates the quality control into an unreliable FMS.

$$\text{Maximize } \sum_{k=1}^R J_k \quad (38)$$

$$\begin{aligned} \text{where: } J_k = & (REV_k)(EF_k)_{v_k(z_k)} - CI_{k1} [\lambda_k + (ER_k)_{v_k(1)}] r_{k,v_k(1)} \\ & - \sum_{\ell=2}^{z_k} CI_{k\ell} [(EF_k)_{v_k(\ell-1)} + (ER_k)_{v_k(\ell)}] r_{k,v_k(\ell)} \\ & - CW_k [\lambda_k + (ER_k)_{v_k(1)}] W_{k,v_k(1)} \\ & - CW_k \sum_{\ell=2}^{z_k} [(EF_k)_{v_k(\ell-1)} + (ER_k)_{v_k(\ell)}] W_{k,v_k(\ell)} \\ & - (CF_k)(AOQ_k)(EF_k)_{v_k(z_k)} \\ & - \sum_{\ell=1}^{z_k} (CS_{k\ell})(ES_k)_{v_k(\ell)} - CP_{k1} [\lambda_k + (ER_k)_{v_k(1)}] \\ & - \sum_{\ell=2}^{z_k} CP_{k\ell} [(EF_k)_{v_k(\ell-1)} + (ER_k)_{v_k(\ell)}] - \sum_{\ell=1}^{z_k} CR_{k\ell} (ER_k)_{v_k(\ell)}, \end{aligned}$$

$$\text{and} \quad (39)$$

$$\rho_j < 1, 0 \leq r_{kj} \leq 1, \text{ for all } k \text{ and } j. \quad (40)$$

In this model, the profit is accounted for on a per unit time basis and J_k represents the total return of type-k parts produced

by the FMS. Equation (39) computes the the profit generated by type-k parts, which is the total revenue (the 1st term) minus inspection costs (the 2nd and 3rd terms), waiting costs (the 4th and 5th terms), post-sales failure costs (the 6th term), scrapping costs (the 7th term), processing costs (the 8th and 9th terms), and reworking costs (the last term).

The above model is a complex nonlinear optimization problem which becomes more cumbersome when we attempt to incorporate the results of the queueing network model into it (Lin 1989a). Thus a two-step and two-digit enumerative search procedure is proposed as follows.

Step 1: Search for the optimal inspection rates in the range of 0 and 1 with increments of 0.1. Denote the solutions by r_{kj}^* .

Step 2: Search for the optimal inspection rates in the range of $r_{kj}^* - 0.05$ and $r_{kj}^* + 0.05$ with increments of 0.01.

In the next section, we use a numerical example to demonstrate the effect of the quality control policy on system characteristics, and implement the above enumerative procedure to obtain approximate solutions to the optimal inspection rates.

5. A Numerical Example

Consider an FMS with three cells where each cell contains two operating areas, one for working (which also contains an inspection station) and the other for reworking. Two types of parts are processed in the FMS. The visitation sequence for part types 1 and 2 are given by $V_1=(1,2,3)$ and $V_2=(1,3)$, respectively. Other relevant data are given in Table 3-1.

Three separate cases are examined to illustrate the effect of the quality control function on system characteristics: (1) the FMS is completely reliable, (2) the FMS is unreliable and there is no quality control, and (3) the FMS is unreliable and has quality control; 20 percent of the processed parts are inspected.

For each case and each part type, we computed the average number of parts scrapped (ES's), the average number of parts reworked (ER's), the average flow rates (EF's), the average outgoing quality (AOQ's), and the part waiting time at the three cells and in the entire system. The results are summarized in Table 3-2.

For the reliable case (column 2 in Table 3-2), the AOQ's of the outgoing parts are all zeros and the waiting times in the system for both part types are 0.8507 and 0.8440 time units, respectively. On the other hand, when the FMS is unreliable and there is no quality control (column 3 in Table 3-2), the AOQ's for both part types increase to 0.2217 and 0.1628, respectively. Note

that although a zero-defect technology is available to reduce the AOQ to zero, it may be too expensive to adopt such an FMS. Therefore, an alternative strategy is to incorporate the quality control process into the manufacturing system as in case three. In this case (see column 4 in Table 3-2), the AOQ's of both part types go down to 0.1190 and 0.1443, respectively. This reduction in AOQ is at the expense of the inspection, reworking, and increased part waiting time. For instance, the waiting times of both part types in the system increase to 0.9903 and 0.9827 time units, respectively; the average number of reworked parts of both part types increase to 1.373 and 1.336, respectively. Based on the above observation, one may expect that incorporating an appropriate quality control policy can improve the quality level and the productivity of an FMS. It is also a viable alternative to the implementation of a much more expensive zero-defect technology.

Other results of the FMS operating characteristics based on different levels of the quality control effort are shown in Figures 3-2 to 3-16. For example, in Figure 3-2, the part waiting time in cell 1 can be greatly reduced if we adopt a lower inspection rate at cell 1. In Figure 3-3, we see that the waiting time in cell 2 depends on the inspection rate at cells 1 and 2, but it is more sensitive to the inspection rate at cell 2 than to the inspection rate at cell 1. As to the waiting time in cell 3, described in Figures 3-4 through 3-9, it changes dynamically with

the inspection rates. However, we observe that it is highly sensitive to the inspection rate at cell 3, and not as sensitive to the inspection rate at cell 1. It is worth noting that the waiting time in cell 3 decreases when we increase the inspection rate at cell 2. That is, it seems that the increase in the congestion level at cell 2, which is caused by the increase in the inspection rate at cell 2, helps to reduce the congestion level at cell 3. However, this phenomenon cannot be clearly observed for cell 1. Since the part waiting time in each cell is very sensitive to the inspection rate adopted at that cell, the above observation seems to suggest that we can increase the system throughput rate by speeding up the inspection process at the most congested cell. Therefore, one can conjecture that reducing the inspection time may be a key factor in reaching a higher level of productivity for a manufacturing system. Finally, although it is obvious that the AOQ can be reduced by increasing the inspection rates, Figures 3-10 to 3-16 indicate that the relationships between the AOQ and the inspection rate at each cell are somewhat dynamic and, therefore, no simple conclusion can be obtained here.

To obtain approximate solutions to the optimal inspection rates, we implement the two-step and two-digit enumerative search procedure. The computer time is about 30 seconds on an IBM PC-486 compatible machine using a BASIC interpreter. Since the total profit is insensitive when the variation of inspection rate is smaller than 0.01, the solutions we obtained by this algorithm

were almost equal to the optimal solutions which we obtained from a one-step and five-digit search procedure.

Using this algorithm, we further conduct a sensitivity analysis to examine the effects of cost parameters on the objective value (i.e., the expected profit).

For the post-sales failure cost, (see Table 3-3), we found that (1) no inspection is needed when $CF_1 \leq 220$ and $CF_2 \leq 170$; (2) r_2 increases faster than r_3 when the value of CF_1 is increased. The values of r_1^* , r_2^* , and r_3^* are .15, .57, and 1, respectively, when $CF_1 \geq 330$; (3) An increase in the value of CF_2 , will lead to an increase in r_3 . The near-optimal inspection rates are $r_1^* = 0.15$, $r_2^* = 0.57$, $r_3^* = 1$ when $CF_2 \geq 240$. (4) When $CF_1 = CF_2 = 400$, the near-optimal inspection rates are $r_1^* = 0.15$, $r_2^* = 0.57$, and $r_3^* = 1$. The above observations (2) and (3) are reasonable since after r^* reaches 1.0, i.e., we perform complete inspection in the last station of both parts, any further increase in CF_1 or CF_2 will not affect the total costs and thus no change in optimal inspection rates is observed.

For the inspection cost, (see Table 3-4), we found that increasing it will decrease the optimal inspection rate of its corresponding station.

Tables 3-5 showed that (1) increasing CS_{12} will lead to an increase in r_1^* and a decrease in r_2^* and r_3^* ; (2) r_2^* will decrease when we increase CS_{13} ; (3) increasing CS_{22} will lead to an increase the r_1^* and a decrease in r_2^* and r_3^* . In general,

increasing in scrapping cost of a station will lead to the increase of the prior station's optimal inspection rate. The reasons of (3) are: to compensate the increasing cost of CS_{22} , one has to reduce the number of the defective type-2 products scrapped at station 3, which were made at station 1 and found at station 3. Thus, one has to increase the inspection rate at station 1, while decrease the inspection rate at station 3 (note that $V_2 = [1,3]$). The increase of inspection rate at station 1 will result in the decrease of the defective rate of the type-1 products. So the inspection rate at station 2 might be decreased.

The results on the sensitivity analysis of repair costs are showed in Table 3-6. But it is difficult to obtain conclusions from this observation. It shows that the effects on the optimal inspection rates of each station due to the variation of repair cost may be dynamic in the system. In Table 3-7, it is observed that an increase in the processing cost of a station will lead to a decrease in the optimal inspection rate of that station and may also increase the optimal inspection rate of other stations. These results can be directly observed by evaluating the changes in the optimal inspection rate when CP_{12} or CP_{13} increases.

According to the results of the sensitivity analysis of waiting time cost shown in Table 3-8, increasing the waiting cost of part type k will decrease the optimal inspection rate of all stations through which part type k has to visit (note the changes when CW_1 increases), and seems to increase the optimal inspection

rate of all stations where no type-k part is processed. This later case is observed when CW_2 is increased.

In Appendix C, two numerical examples--one has five stations with three part types, the other has five stations with two part types are solved by the enumerative algorithm. We found that it took about three hours to execute the first step of the enumerative algorithm using a IBM PC-486 compatible. Therefore, a new enumerative algorithm is proposed to reduce the computing time (a full description is given in Appendix C).

6. Conclusion

In this chapter we obtained the operational characteristics of an assembly-oriented FMS with an in-process quality control policy using an open queueing network approach. A nonlinear profit-maximization formulation was presented, and an enumerative algorithm to solve this problem was provided. For small-scale problems, the enumerative procedure can be used to obtain efficient approximate solutions. However, for large-scale problems, the proposed search procedure will require more computing time because of the increased number of alternative policies that have to be evaluated (Appendix C). Some aspects of an FMS, such as part transportation time, finite local buffer size, and other types of quality control methods (such as CSP) are not incorporated in our model. Therefore, there are four issues for future research.

- (1) We can extend our model to consider the transportation time by adding a new station, and imposing an upper limit on the size of the local buffer at each station. The extended model of the finite local buffer case can then be solved using an iterative algorithm developed by Yao and Buzacott (1986b).
- (2) It will also be interesting and useful to use simulation, together with appropriate experimental designs and regression analyses, to investigate the relationships

between the in-process quality control policy and the operational characteristics of an actual manufacturing system. A cost or profit objective function may then be defined to determine the optimal inspection rates to be used in the system.

- (3) It is worthwhile to investigate the other types of in-process quality control methods (such as CSP) in the FMS. The approach proposed in (2) can be used in this study.
- (4) Developing a better heuristic method to solve the nonlinear program for large scale problems is worth studying.

CHAPTER FOUR

MAINTENANCE FLOAT MODELS FOR A FLEXIBLE MANUFACTURING SYSTEM

1. Introduction

In this chapter, we propose a simulation approach to determine the satisfactory maintenance float policy for a key flexible manufacturing cell (FMC), and the satisfactory repair capacities for the entire flexible manufacturing system to ensure a high level of system utilization. The FMS that we consider consists of the key FMC, the rest of the system, and two repair stations.

Before conducting this research, we analyzed some simple cases of this problem using analytical approaches described in Appendix A. We found that it is possible to derive the solution procedures for the optimal maintenance float and repair capacities for some simplified cases (see cases 1 and 2 in Appendix A). However, the derivations become intractable analytically for more complex cases. For example, in case 3 of Appendix A, we can not guarantee that the solution obtained by the procedure is optimal. Furthermore, for case 4 of Appendix A, no solution procedure can be derived. Thus, only an enumerative approach is suggested because it is too complex to derive the properties of the fractional utilization of the system.

Cases 1 and 2 in Appendix A describe a situation where the FMC contains the principal production line of the FMS, and the fractional utilization of the FMS is totally affected by this cell. Since the major module in the FMC is subject to failure (and when this happens, the operations of the FMC stop and the entire system will be forced to be idle), a maintenance float policy needs to be employed to achieve a high fractional utilization of the system. With a maintenance float policy, several units of the key module are kept in the FMC (one is engaged in operation while the remaining are standbys). When the module in operation fails, it is immediately replaced by one of the standby modules and the failed module is sent to the repair shop for repair. After the repair is done, the repaired module will be sent back to the FMC to serve as one of the standbys. The difference between these two cases is that in the first case a single-server queue is assumed, while in the second case a multiple-server queue is used. The results obtained for these simplified cases are useful when the repair cost of the rest of system is negligible compared with the repair cost of the FMC, and when the "reward" of the system depends almost entirely on the fractional utilization of the FMC. Two profit-maximization models are proposed to determine the optimal maintenance policies in both cases, respectively. The solution procedures of these two models are formally derived, which exploit the properties of the fractional utilization of the system.

Case 3 in Appendix A considers two repair stations (one used for repairing modules and the other used to repair the components sent from the rest of system). The idle time of the system is caused not only by the FMC but also by the rest of the system. A two-dimension queue is proposed to capture the characteristics of the fractional utilization in this case. A profit maximization problem is considered to determine the near-optimal maintenance policies based on the properties of the fractional utilization of the system.

Case 4 is an extension of Case 3. In this case, modules could be sent to either one of the repair stations. However, in this shift repair process, the service priority of the component is higher than that of the module and the priority discipline is preemptive repeat-identical (Jaiswal 1968). However, the properties of the fractional utilization of the system are difficult to derive. Therefore, we only propose an enumerative approach to solve a profit-maximization model.

The situation that we consider in this chapter excludes the higher service priority of the components to be repaired at Station 2 of Case 4 in Appendix A. The repair flow of this problem is showed in Figure 4-1. The main assumptions of the problem are:

- 1) There are F modules in the key FMC, one is engaged in the operation and the remaining $F-1$ modules are standbys;
- 2) The changeover time for replacing the failed module with a

standby module is negligible;

- 3) Failures of modules and components occur according to independent Poisson distributions;
- 4) Repair times of servers are independently exponentially distributed;
- 5) Modules will be repaired at Station 1 but when Station 2 is empty, the first waiting module can be sent to Station 2 to repair, and the components will be repaired only at Station 2; and
- 6) The system will be idle when all F modules are in the repair Station 1, or there are at least M units of components in repair Station 2.

The module is a very important unit in the key FMC. The characteristics of a module are that 1) it is very expensive, and 2) it is easy to fail. In practice, it may be a tool magazine (Luggen 1991) or a portable unit (used in the key FMC) which contains many very unreliable components. To speed up the replacing process when a failure occurs, we can put those unreliable components in a portable unit (i.e. module). Thereby, a firm only have to prepare a number of standby modules in the operation site.

In this study, although our focus is on the float (standby) system of the module to maintain the system reliability, we do not omit the failure events due to components. We assume that if the

total number of failure components is larger than a given level, the system will break down even there is an operative module available in the key FMC.

According to the above discussion, we develop a systematic approach to obtain some approximate formula to simplify the solution procedure for more complex, yet more practical system settings. This systematic approach incorporates simulation, Taguchi experimental design, regression analysis, and classical queuing models to determine the functional form of the fractional utilization of the system. This approach provides an efficient alternative for constructing a metamodel and solving a decision model. The metamodel is defined as "models of simulation models - which express the input-output relationship in the form of a regression equation" (Gardenier 1990). This approach is an extension of Kuei, et al. (1990), Chanin, et al. (1990), and Lin (1989b). The remainder of this chapter is organized as follows. In Section 2 we discuss the systematic approach. The model analysis is proposed in Section 3. The conclusions are presented in Section 4.

2. Systematic Approach

The framework of this approach is shown in Figure 4-2. There are eight steps in this approach. First, the problem should be defined clearly. Secondly, a simulation model needs to be developed based on the problem definition. Third, we need to conduct a preliminary study to determine the feasible ranges for control (independent) variables. Fourth, an appropriate Taguchi experimental design needs to be chosen to determine the experimental design points. Fifth, data are collected for each design point. Sixth, the Analysis of Variance (ANOVA) is used to examine the relationships between the dependent variable and independent variables. Seventh, a regression metamodel is developed to determine the approximate relationships between the independent variables (which are significant in the ANOVA) and the dependent variable. Finally, a mathematical program is formulated and solved to determine the optimal levels of the independent variables by introducing the relevant revenue and cost factors. The detail descriptions of the procedures of our approach is as follows.

STEP 1: Define the Problem

In this step, we have to answer the following questions:

- What is the problem?
- Which performance measurements are to be used?
- Which factors are involved in the problem?

-- Which factors are controllable (independent)?

-- Which factor is the dependent variable?

Through a detailed consideration of the above questions, we can construct the simulation model for our problem and determine the factors we need to include in the Taguchi experimental design.

STEP 2: Develop the Simulation Model

Three problems are involved in this step:

-- Which language will be adopted?

-- How to verify the correctness of the model?

-- How long should the running time be to guarantee that the system reaches the steady state.

There is no special simulator software available for our simulation model. Hence, there are only two alternatives in selecting an appropriate language for coding our simulation program. One is to write the program in a general purpose language, such as Fortran, C,... etc. The other is using a simulation language, such as GPSS, SIMSCRIPT,... etc. For model verification, we need to examine the computer code to insure that the program represents the proper concept of our model.

To determine the running time required to reach the steady state of the system, we can conduct a very long simulation run and examine the output values of time interval sets. Comparing these values, we may find that the output values are almost all equal to or approach a value occurring after some time interval set. The

running time corresponding to the above interval set is then the time required to reach the steady state of the system. This heuristic approach is similar to the concepts proposed by Solomon (1983).

STEP 3: Decide the Feasible Ranges of Independent Variables

There are three ways to determine the feasible ranges of the independent variables. The first is that they can be directly determined by the relevant technological information. The second is to use the analytic approaches to determine the approximate feasible ranges first. Then, simulation is conducted to determine the feasible ranges based on the approximate feasible ranges. The third way that we can adopt when no technological information and no analytic approaches are available is to conduct the simulation run under the worst or the best cases of some of the independent variables in which we are not interested to determine their bound values. In this simulation run, we observe the bound value of the dependent variable and the corresponding bound value of our concerned independent variable. From this observation we can determine the bound values of the some of the independent variables.

Step 4: Build the Taguchi Experimental Design

Problems in this step:

--How many levels should be set for each independent variable?

--Which orthogonal array needs to be adopted?

--Which linear graph is appropriate?

If there is not enough information to determine levels of factors, we usually adopt the two-level design first. However, after Step 6, if the results of regression analysis cannot represent the points between these two levels, the number of levels will be increased (see Step 6 for a more detailed discussion).

Ignoring the high order effects (Taguchi 1977), an appropriate orthogonal array (Taguchi 1977) can be selected based on the number of independent variables (i.e., factors) and their corresponding levels.

For each orthogonal array there are many linear graphs to correspond to it (Taguchi 1977). It means that for a specific set of independent variables and their corresponding levels there are many ways to assign the factors and their interactions to the columns in the orthogonal array. However, we must consider the problem of confounding (Box et al. 1978, Kuei et al. 1990) because the effects of high order interactions would confound with the effects of factors or two-factor interactions. Therefore, we have to select an appropriate linear graph to avoid the effects of confounding of high order interactions in our experiment.

STEP 5: Run Simulation

A single run for each design point is not enough to represent the simulation results because they are random events. Therefore,

we have to use different random number seeds to simulate each design point. We use ten different random seeds to replicate the simulation runs for each design point. The average of these simulation results is considered as the output of each design point.

STEP 6: Implement ANOVA of Taguchi Method

In this process, we use ANOVA to determine which of the factors and which of the two-factor-interactions are significant. The ANOVA is used to represent the total variation of observations into accountable sources such as factors effects and the experimental error. Other than the calculations of mean square errors in the ANOVA of the Taguchi approach, we also need to determine which of the columns in the orthogonal array would be used as the sources of error messages (i.e., the experimental error). Taguchi and Wu (1980) suggest that the total columns of error message have to be about half of the total columns in the orthogonal array. The sources of error messages would be:

- (1) The unassigned columns in the orthogonal array;
- (2) The smaller column effects relative to other columns (Taguchi and Wu 1980, Wu 1976, Ross 1988).

STEP 7: Implement Regression Analysis and Metamodel Validation

As noted by Hicks (1982), the regression analysis will be

conducted only if significant correlations between the independent factors and the system performance is found in the ANOVA and further investigations are needed to see how system performance varies with these significant independent factors. In other words, we are interested in finding a functional relationship between system performance and significant independent variables (factors).

In metamodel validation, we choose some points which are located between the levels of the factors (independent variables) to conduct the additional simulation runs. We compare the results of the simulation runs of these additional points with the values of regression output in this validation process. More clearly, we denote

S_i : the result of i th additional simulation run of some factor;

O_i : the output of regression of i th additional design point of some factor; and let

$$MAD = \sum_{i=1}^M |S_i - O_i| / M$$

where M is the number of additional design points,

MAD is the mean absolute derivation of the regression metamodel.

The lower the value of MAD , the better the representation of the system response by the regression equation. Therefore, we have to redesign the Taguchi design when the MAD value is large. The unsolved problem here is how to decide the threshold value of MAD .

In practice, the verification of the model is an interactive process between the model builder and the decision maker (manager). If the MAD is agreed upon by the manager then the metamodel is accepted. Otherwise, we can further refine the experimental design by focusing on some highly significant (in terms of F-values in ANOVA) independent variables and derive a more detailed and sophisticated metamodel.

STEP 8: Formulate and Solve Decision Models

After we validate the regression metamodel, the function of system response in terms of the significant factors and two-factor interactions would be used to predict the system performance. A profit or cost decision model is constructed based on this function and their relevant factors, such as the revenue function, the cost function, and the range of factors. This approach is suggested by Hoover and Perry (1989) and Shanthikumar and Sargent (1983). They argue that one of the problem solving techniques for a complex system is to combine the simulation with analytic modeling.

3. Model Analysis

3.1 Define the Problem

The problem in the model (see Figure 4-1) is similar to that of Case 4 in Appendix A, except that the assumption that the service priority of the component is higher than that of the module is waived. The performance measure in the model is the fractional utilization of the system. It is related to the reward of the system. We will use it as an independent variable of the revenue function in the decision model. However, the fractional utilization of the system depends on the following factors:

- The mean repair time per server in station 1 (denoted by R_1).
- The mean repair time per server in station 2 (denoted by R_2).
- The total number of modules (denoted by F ; 1 in operation and $F-1$ standbys).
- The number of servers in station 1 (denoted by S_1).
- The number of servers in station 2 (denoted by S_2).

For the simulation model, the above five factors are the independent variables with respect to the fractional utilization of the system. However, these five independent variables are the dependent variables of the cost functions in the decision model. The characteristics of the fractional utilization of the system in terms of concavity or convexity are difficult to derive with respect to its five independent variables (see Case 4 in Appendix A) in the model. Thus, no exact solution procedure could be developed. Therefore, we adopt the simulation approach and

regression metamodel to construct the simple expression function of the fractional utilization of the system in terms of the five independent variables (i.e., R_1 , R_2 , S_1 , S_2 , and F). In our model, we assume that the mean times between failures of components and modules are 30 and 15 time units, respectively, and the system will be idle when there are at least 9 units of components in the repair system.

3.2 Develop the Simulation Model

Since no special software is available for our simulation model, we used GPSS-H language for simulation coding. The block storages of GPSS-H were used to represent an FMC with F-1 standby units, the rest of the system with M standby units, repair station one with S_1 servers, and repair station two with S_2 servers. The transactions were generated from the beginning of the major segment. The detailed flow of simulation is shown in Figure 4-3.

To verify the simulation program, rather than examining the actual computer code, we did simulation runs under different values of failure rates of module and component as well as repair rates of repair stations to observe if the following two rules were true in the results of these simulation runs:

- (1) The function of the fractional utilization of the system is non-increasing in terms of the mean time to repair.

- (2) The function of the fractional utilization of the system is non-decreasing in terms of the mean time between failures.

We conducted the simulation run for one million time units. The outputs (i.e., fractional utilization) of this simulation run were observed at every 0.1 million interval time. We found that 0.2 million time units simulation run was long enough to guarantee the system reaching the steady state.

3.3 Decide the Feasible Ranges of Independent Variables

Based on technological information, we knew that the upper and lower bounds of the mean repair time of a server at station 1 were equal to 11 and 4, respectively, and those bounds of a server at station 2 were 14 and 6, respectively.

To determine the upper bound of the units of float of modules, we used the second approach--analytical approach combined with simulation run. Equation (A3) of Case 1 in Appendix A was used to determine the fractional utilization of worst cases of this model in terms of different number of float of modules when we let the values of R_1 , R_2 , S_1 and S_2 be 11, 14, 1, and 1, respectively (i.e., the worst case of R_1 , R_2 , S_1 and S_2). Based on equation (A3), increasing the value of F , we found that the value of the fractional utilization increased and approached 1 (i.e., the value was 0.99999981) when F was 15. After we obtained this upper bound we used the simulation approach to evaluate the

fractional utilization beginning from this upper bound (i.e., 15) to search the real upper bound of the model. In this process, we find that all values of the fractional utilization are one when the F value was equal to 8 through 15. Therefore, the upper bound of F of the model was 8.

To decide the upper bound of S_1 , we used the same procedure as we did for F . Equation (A16) was used to evaluate the fractional utilization. We found that the fractional utilization approached 1 (i.e., 0.9999999922) when the value of S_1 was 6. With the aid of the simulation run, we determined the upper bound of S_1 of the model which was equal to 4.

The third approach was adopted to determine the upper bound of S_2 because we could not use the first two approaches in this case. Let $R_1=11$, $R_2=14$, $S_1=1$, $F=1$, we found that the upper bound of S_2 was 5 but its corresponding fractional utilization of the system was less than one (i.e., equal to 0.816). We had to consider better cases in terms of the values of S_1 and F . The upper bound of these revised cases (let $(1, k)$ denote the case with the values of S_1 and F equal to 1 and k , respectively) $(1,2)$, $(1,3)$, $(1,4)$, $(2,4)$, $(3,4)$, $(1,5)$, $(2,5)$ was 5 (see Table 4-1). According to the above simulation runs we concluded that the upper bound of S_2 was 5 because the fractional utilization of the system of case $(2,5)$ with $S_2=5$ was equal to one. We also found that the upper bound of S_2 was 5 by performing the above analysis for different combination values of F , S_1 , R_1 , and R_2 .

Because we need at least one unit of module to operate the system, the lower bound of F is one. Furthermore, since there are two repair stations (one for modules and the other for components) in the repair system, we conclude that the lower bounds of S_1 and S_2 are all equal to one.

3.4 Build the Taguchi Experimental Design

Five controllable factors (independent variables) F , R_1 , R_2 , S_1 , S_2 were considered in the model. We initially adopted the two-level design for each factor and investigated all main effects of controllable factors and all possible combinations of the two-factor interactions. In this investigation, the significant controllable factors and/or two-factor interactions could be identified. Therefore, $L_{16}(2^{15})$ orthogonal array (see Table 4-2) was chosen as our experimental design array which is the minimum number of experiments in the orthogonal arrays we can use to accomplish the above requirements (Taguchi and Wu 1980; Wu 1976).

Three linear graphs satisfy our requirements in orthogonal array $L_{16}(2^{15})$ (Taguchi and Wu 1980). Because there is no information on which can be used to design the required confounding pattern, we can choose any one of them. Here, we used the first linear graph of these three linear graphs (Taguchi and Wu 1980). The corresponding experimental design of this linear graph is shown in Table 4-3. The Table explains the number of assigning column for each factor.

3.5 Run Simulation

We used ten different random seeds to replicate the simulation runs for each design point. The average of these simulation results was considered as the output of each design point. The results of the fractional utilization of all simulation runs of the experimental design are shown in Table 4-4.

3.6 Implement ANOVA of Taguchi Method

Since the values of the sum of square error (SSE) of S_1 , $F*S_1$, $F*S_2$, S_1*S_2 , R_1*R_2 , S_1*R_1 , S_2*R_1 , S_1*R_2 , $F*R_2$ were quite low compared with other factors (see Table 4-5), the joint effects of these variables yielded error messages (or the experimental error) (Ross 1988). Therefore, nine "insignificant" messages are shown in Table 4-5. The ANOVA Table (see Table 4-5) shows that F , R_1 , $F*R_1$ are significant at $\alpha=0.01$, while S_2 , S_2*R_2 and R_2 are significant at $\alpha=0.05$.

3.7 Implement Regression Analysis and Metamodel Validation

Based on the ANOVA results, the following regression metamodel is proposed.

$$FU = a_0 + a_1 F + a_2 R_1 + a_3 (F*R_1) + a_4 S_2 + a_5 (S_2*R_2) + a_6 R_2 \quad (4-1)$$

Based on Table 4-6, we obtain the following regression of the fractional utilization:

$$\begin{aligned} FU = & 1.109708227 - 0.006755102F - 0.010703125S_2 - 0.024545918R_1 - 0.009 \\ & 64837R_2 + 0.003045918FR_1 + 0.001929688S_2R_2 \end{aligned} \quad (4-2)$$

From the ANOVA table (see Table 4-5), the mean square error of F and R_1 is quite high compared to those of other factors. According to the experimental results of previous research (Chanin et al. 1990, Kuei et al. 1990, Lin 1989b), one can improve the model by increasing the experimental levels of these two factors. Moreover, Table 4-7 shows that the MAD is 0.0652722 (Table 4-7). Accordingly, the manager (decision maker) would not be satisfied with it. Thus, further analysis is recommended. Further analysis was conducted by using $L_{16}(2^{15})$ design but was separated into two independent experimental designs based on the two ranges of F (i.e., [1, 4] and [5, 8]) because its F -distribution value in the ANOVA Table (see Table 4-5) is much higher than those of the others. We adopted a four-level design of R_1 since its F -distribution value is the second highest in the ANOVA. The factor $S_2 * R_2$ was ignored in the design because its F -distribution is the lowest in the ANOVA and there is no available $L_{16}(2^{15})$ orthogonal table which can include all significant factors in the ANOVA (i.e., Table 4-5). The experimental designs are shown in Table 4-8.

We separated the range of F into two parts, (i.e., [1,4] and [5,8]) and performed two independent experiments. In these two experiments, we adopted a four level design of R_1 (i.e., 4, 6, 9,

11) by using three columns (i.e., columns 2,4,6) of the orthogonal array (see Table 4-8) to represent it. The levels of factors and their corresponding simulation results are shown in Table 4-9. The ANOVA table of these two experiments are shown in Table 4-10 and Table 4-11.

The ANOVA table of the first experiment (see Table 4-10) shows that $F, R_2, S_2, R_1, R_1 \times F$ were significant at $\alpha=0.01$. But in the second experiment (see Table 4-11) the sums of the square errors of F and $R_1 \times F$ were too small. Thus we let factor F and $R_1 \times F$ be two error factors in the second experiment. The ANOVA table of the second experiment (see Table 4-11) shows that R_2, S_2 , and R_1 were significant at $\alpha=0.001$.

For the first experiment, we suggest the following regression model based on the results of the ANOVA (see Table 4-10):

$$FU = a_0 + a_1 F + a_2 R_2 + a_3 S_2 + a_4 R_1 + a_5 FR_1 \quad (4-3)$$

The analysis of variance of (4-3) regression model is shown in Table 4-12. It shows that the model seems to fit very well because its R^2 value is quite high (i.e., 0.986194).

Based on Table 4-12, we obtain the regression line as follows:

$$FU = 1.094806034 - 0.00737931F - 0.00559375R_2 + 0.0129375S_2 - 0.032474138R_1 + 0.005517241FR_1 \quad (4-4)$$

Table 4-13 shows that the second model fit very well (i.e., $R^2=0.9478$, and all values of significant level are quite small). Thus the regression line of the model is:

$$FU=1.060672414-0.00565625R_2+0.011437S_2-0.008189655R_1 \quad (4-5)$$

In order to evaluate and test these three equations, twenty simulation runs were conducted and the average errors were calculated. The results are shown in Table 4-14 and Table 4-15. We find that the average error of one-regression-line is higher than that of two-regression-lines. Therefore, the manager (decision maker) decides to adopt the two-regression-lines in the decision model.

3.8 Decision Model and Numerical Example

The decision model for a profit-maximization problem can be stated as following:

$$\text{Max. } Z(F, S_1, S_2, R_1, R_2) = R(FU) - k(F) - g_1(S_1) - g_2(S_2) - h_1(R_1) - h_2(R_2)$$

s. t.

the ranges of F, S_1, S_2, R_1 , and R_2

where,

$R(FU)$: revenue function of the fractional utilization,

$k(F)$: cost function for keeping F modules in the FMC,

$g_1(S_1)$: cost function of having S_1 servers in repair station 1,

$g_2(S_2)$: cost function of having S_2 servers in repair station 2,

$h_1(R_1)$: cost function for maintaining a mean repair time of R_1 time units for modules, and

$h_2(R_2)$: cost function for maintaining a mean repair time of R_2 time units for components.

Suppose we face the following numerical decision problem,

$$\text{Max. } Z(F, R_1, R_2, S_1, S_2) = 30000FU - 300F - 900 - \frac{1}{R_1} - 400\frac{1}{R_2} - 400S_1 - 200S_2$$

We can let $S_1 = 1$ because S_1 is insignificant in ANOVA. (i.e. use the lower bound). The problem can be separated into two subproblems based on equations (4-4) and (4-5) in terms of the ranges of F value. Furthermore, the first subproblems using equation (4-4) can be separated into two subproblems as follows:

Subproblem 1-1:

$$\text{Max. } Z^1(F, R_1) = 30000(-0.0073793F - 0.032474138R_1 + 0.005517241FR_1)$$

$$-300F - 900 - \frac{1}{R_1}$$

s. t.

$$1 \leq F \leq 4, \quad F \in \text{integer}$$

$$4 \leq R_1 \leq 11$$

Subproblem 1-2:

$$\text{Max. } Z^2(S_2, R_2) = 30000(-0.00559375R_2 + 0.0129375S_2) - 400\frac{1}{R_2} - 200S_2$$

s.t.

$$1 \leq S_2 \leq 5, \quad S_2 \in \text{integer}$$

$$6 \leq R_2 \leq 14$$

In solving Subproblem 1-1, we fix the value of F (i.e. F would be 1, 2, 3, and 4) and then determine the optimal R_1 for each F value. We find that the optimal values of R_1 for all the above cases are smaller than the lower bound of R_1 . Thus $R_1^* = 4$ and the optimal value of F equals to 4.

For Subproblem 1-2, we let $S_2 = 1, 2, 3, 4$, and 5 and determine the optimal R_2 . Since R_2 is independent to S_2 , we can determine the optimal value of R_2 by the following procedure:

$$\text{let } Z(R_2) = 30000(-0.00559375R_2) - 400\frac{1}{R_2}, \text{ and}$$

$$\text{let } \frac{dZ}{dR_2} = 0, \text{ we obtain } R_2^* = 4.3668,$$

since $\frac{d^2Z}{dR_2^2} < 0$ and $R_2 \geq 6$, the optimal value of R_2 is equal to 6.

Because Z^2 is increasing with respect to S_2 , the optimal values of R_2 and S_2 are 6 and 5 respectively.

Based on the solutions of Subproblems 1-1 and 1-2, we obtain the optimal solution of the problem for the first range (i.e., [1, 4]) of F is $F^* = 4$, $R_1^* = 4.3668$, $R_2^* = 6$, $S_1^* = 1$, $S_2^* = 5$, and its corresponding objective value is 28808.33.

Subproblem 2 can be described as follows:

$$\text{Max. } Z^3(S_2, R_1, R_2) = 30000(-0.00565625R_2 + 0.0114375S_2 - 0.008189655R_1)$$

$$-900\frac{1}{R_2} - 400\frac{1}{R_2} - 200S_2$$

s. t.

$$1 \leq S_2 \leq 5, \quad S_2 \in \text{integer}$$

$$4 \leq R_1 \leq 11$$

$$6 \leq R_2 \leq 14$$

In Subproblem 2, R_1 , R_2 and S_2 are mutually independent, thus we can solve for the optimal values of them individually. Because Z^3 is increasing in terms of S_2 , the optimal value of S_2 is equal to 5. To solve for the optimal value of R_1 , we let

$$Z(R_1) = 30000(-0.008189655R_1) - 900\frac{1}{R_1},$$

$$\text{let } \frac{dZ}{dR_1} = 0, \text{ we obtain } R_1^* = 5.4134,$$

$$\text{since } \frac{d^2Z}{dR_1^2} < 0, \text{ and } R_1 \geq 4, \text{ the optimal value of } R_1 \text{ is}$$

equal to 5.4134. Similarly, we obtain the optimal value of R_2 to be 6. Therefore, for the $5 \leq F \leq 8$ range, the optimal point of the problem is $F^* = 5$, $S_1^* = 1$, $S_2^* = 5$, $R_1^* = 5.4134$, $R_2^* = 6$. Its corresponding objective value is 26867.07.

Based on the above results, the optimal solution of the problem is $F^* = 4$, $R_1^* = 4.3668$, $R_2^* = 6$, $S_1^* = 1$, $S_2^* = 5$, and its corresponding objective value is equal to 28808.33.

To verify the above optimal solution, we conduct further

simulation sets which include the neighborhood points of each independent variable (see the first column of Table 4-16). The values of $Z(F, R_1, R_2, S_1, S_2)$ for each simulation set are evaluated (shown in Table 4-16). Comparing these results with the objective value of the above optimal solution, we found that the above optimal solution is still better than any one of the simulation sets.

In the solution procedure to determine the optimal solution, the above case seems relatively easy because its corresponding function of fractional utilization can be separated into two equations. However, for the cases with complex functional form of the fractional utilization, such as the following equation (i.e. the most complex case)

$$\begin{aligned} FU = & a_0 + a_1 F + a_2 R_1 + a_3 R_2 + a_4 S_1 + a_5 S_2 + a_6 FR_1 + a_7 FR_2 \\ & + a_8 FS_1 + a_9 FS_2 + a_{10} R_1 R_2 + a_{11} R_1 S_1 + a_{12} R_1 S_2 + a_{13} R_2 S_1 \\ & + a_{14} R_2 S_2 + a_{15} S_1 S_2, \end{aligned}$$

it can still be solved by a traditional calculus algorithm if a maximum point exists (i.e., we assume that all variables are continuous). But for cases without an existing maximum point we can use a direct grid search algorithm (Wang 1986) to determine the optimal solution under feasible ranges of decision variables.

4. Conclusion

In Appendix A, we proposed four cases for determining the optimal float of modules used in the key FMC and the optimal repair capacity for the repair system of the FMS. The first two cases focused only on the FMC and characteristics of the fractional utilization were derived to solve the problems presented. These two cases can be extended to handle the case where the operations of the FMC requires $K > 1$ modules be used simultaneously. In this case, the operations of the FMC will stop and the FMS will be forced to be idle if there are $F-k+1$, $F-k+2, \dots$, or F modules in the repair shop. Assuming module failures are independent events, the fractional utilization of the system is still a function of F and μ (or F and S), and the properties of the fractional utilization of the system in terms of the decision variables remain valid for this more general case. In the other two cases (Cases 3 and 4) we extended the repair system to include the repair station for the rest of system and the idleness can also be caused by the rest of the system. The proprieties of the fractional utilization of the system are not easy to derive in these cases. Although we derived some of the required properties of the fractional utilization in Case 3, we could not guarantee that the solution is optimal. In Case 4, we extended Case 3 in order to obtain a higher fractional utilization of the system. But we could not derive the concave and increasing properties of the function of the fractional utilization of the

system. According to the analysis of these four cases, we found that although in the more complex cases we captured more practical operational considerations, the solution procedure for the profit-maximization problem of more complex cases was difficult to derive. Therefore, we proposed a new approach which combined the simulation with the Taguchi design technique, regression metamodel, and classical queueing models. Using this approach, we obtained a useful expression of the fractional utilization of the system for solving the decision model. According to this result, we find that the new approach is very useful in dealing with complex decision models although it is an approximate method.

CHAPTER FIVE

CONCLUSIONS AND FUTURE RESEARCH DIRECTIONS

A study of the unreliable FMS has been presented. Two unreliability issues of an FMS were addressed--the issue of unreliable products and the issue of unreliable equipment.

For the issue of unreliable product, we first examined the effects of an in-process inspection policy on the operational characteristics of an FMC, then combined the results into a network system (FMS) based on the open queueing network theory. We also studied three different types of operational systems: a zero-defect FMS, an unreliable system without quality control, and an unreliable system with quality control. The differences in operational characteristics of these three systems were described. We also showed that these differences influence the expected number of scrapped parts and reworked parts, throughputs, AOQ's, and the waiting time. It is concluded that an in-process quality control policy needs to be implemented in order to improve the productivity of a system when a zero-defect technology is not available or is too expensive to incorporate. Finally, we constructed a mathematical program and proposed an enumerative solution procedure to find the near-optimal inspection rates, which can be used for small to medium-sized systems. A numerical example was illustrated and used to conduct sensitivity analysis

on all cost factors. In practice, building a quality control system in the FMS is required because the zero-defect system may not be available from the technical point of view or its cost may be too high to implement. Based on these considerations, this study proposed an approach to build a quality control system in the unreliable FMS to improve the reliability of the products.

For the unreliable issue of equipment, five models were proposed for the related maintenance float problems. In the first four models, we applied classical queueing theory to derive the fractional utilization of the system, and proposed solution procedures to determine the maintenance float and repair capacity for the FMS. These results are useful when there is a key FMC in the FMS and the efficiency of the FMS is totally affected by the key FMC or is partially affected by the rest of the system. However, this approach cannot successfully handle the more complex case (see Case 4 in Appendix A). Thus, in Chapter 4, we proposed a new approach which incorporates the simulation, Taguchi design, regression analysis, and classical queueing models to derive the functional form of the fractional utilization of the system. With this approximate function, a decision model was then developed and solved. This approach benefits from the flexibility of simulation by not requiring some of the restricting assumptions made by the analytical model. Thus, it provides a useful and applicable procedure to deal with the complex issues of stochastic models. Furthermore, we found that:

- (1) it reduces the amount of work needed to obtain a reliable model for the maintenance float systems in the FMS,
- (2) it enables us to conduct "what if" type of analyses, such as sensitivity analyses for model parameters, and
- (3) it provides a guide and reference source for operations managers and industrial practitioners involved in design and planning problems of maintenance float systems in an FMS.

For future research, we have suggested several possible directions at the end of Chapter three. In addition, it is important to combine the quality control and maintenance management together because there exist interrelationships between them, and to examine their joint effects on the operational characteristics of an FMS. Two approaches can be taken to study these problems.

The first approach is to use the new approach that we presented in Chapter 4. It can be applied to determine the relationships between the maintenance policy, inspection rates and the operational characteristics of the FMS via regression equations, using the inspection rate and maintenance policy as the independent variables and the operational characteristics of the FMS as dependent variables. A profit-maximization mathematical program can then be constructed to determine the optimal inspection rates and maintenance policy. The solution procedure for the mathematical program could be less complex than the one we

proposed in Chapter three. This approach can also incorporate the continuous inspection plans (CSP) in addition to the random inspection plans.

As to the second approach, we can first study a simplified FMC using the classical queueing theory, and then extend the results to an FMS using queueing network theory. However, this model may become very complex if we cannot find a simple expression to describe the inter-dependence between the quality control policy and the maintenance policy.

Table 3-1 Data of the numerical example

station	product 1			product 2	
	1	2	3	1	3
τ_M	0.01	0.005	0.01	0.015	0.006
V_M	0.005	0.006	0.006	0.008	0.009
τ_R	0.005	0.001	0.009	0.005	0.008
V_R	0.005	0.001	0.004	0.006	0.001
q	0.1	0.08	0.06	0.09	0.08
CI	16	16	15	10	8
CS	-	20	30	-	16
OR	15	15	10	10	12
OP	12	14	12	12	15
T	0.0005	0.0006	0.0003	0.0005	0.0003
λ	30			40	
CF	220			230	
REV	180			200	
CW	1.5			2	

Table 3-2 Operational characteristics of the FMS

	zero-defect system	unreliable system with	
		no quality control	quality control
Cell 1			
(ES ₁) ₁ /(ES ₂) ₁	0/0	0/0	0/0
(ER ₁) ₁ /(ER ₂) ₁	0/0	0/0	.0612/.7330
(EF ₁) ₁ /(EF ₂) ₁	30/40	30/40	30/40
(AOQ ₁) ₁ /(AOQ ₂) ₁	0/0	0/0	.0816/.0733
W _{q1}	.81500	.81500	.93438
Cell 2			
(ES ₁) ₂	0	0	.48980
(ER ₁) ₂	0	0	.44800
(EF ₁) ₂	30	30	29.51021
(AOQ ₁) ₂	0	.1720	.12600
W _{q2}	.00648	.006648	.00753
Cell 3			
(ES ₁) ₃ /(ES ₂) ₃	0/0	0/0	.7435/.5866
(ER ₁) ₃ /(ER ₂) ₃	0/0	0/0	.3130/.6030
(EF ₁) ₃ /(EF ₂) ₃	30/40	30/40	29.767/39.413
(AOQ ₁) ₃ /(AOQ ₂) ₃	0/0	.2217/.1628	.1190/.1443
W _{q3}	.02901	.02901	.9903/.9827
Entire system			
ER ₁ /ER ₂	0/0	0/0	1.3730/1.3360
EF ₁ /EF ₂	30/40	30/40	29.630/39.410
AOQ ₁ /AOQ ₂	0/0	.2217/.1628	.1190/.1443
W ₁ /W ₂	.8507/.8440	.8507/.8440	.9903/.9827

Table 3-3 Sensitivity analysis of post sales failure cost

CF_1	CF_2	r_1^*	r_2^*	r_3^*	optimal objective value
≤ 220	≤ 170	0	0	0	8901.267
230	170	0	0	0.11	8837.818
240	170	0	0.05	0.15	8782.805
250	170	0	0.95	0	8750.896
260	170	0	1.0	0	8733.611
290	170	0	1.0	0.05	8685.510
310	170	0	1.0	0.38	8660.779
320	170	0	1.0	0.52	8651.618
≥ 330	170	0.15	0.57	1.0	8650.144
220	180	0	0	0.09	8838.442
220	190	0	0	0.22	8783.466
220	200	0	0	0.34	8736.539
220	210	0	0	0.46	8697.592
220	220	0	0	0.58	8666.500
220	≥ 240	0.15	0.57	1.0	8650.049
400	400	0.15	0.57	1.0	7432.010

Table 3-4 Sensitivity analysis of inspection cost

CI_{11}	CI_{12}	CI_{13}	CI_{21}	CI_{22}	r_1^*	r_2^*	r_3^*
8, 10	16	15	10	8	1	0	1
13, 14	16	15	10	8	.84	0	1
15	16	15	10	8	.68	0	1
16	16	15	10	8	.15	.82	1
$\geq 17, \leq 100$	16	15	10	8	0	.62	1
16	$\geq 8, \leq 15$	15	10	8	.15	1	1
16	16	15	10	8	.15	.57	1
16	$\geq 17, \leq 100$	15	10	8	.47	0	.89
16	16	$\geq 8, \leq 12$	10	8	.57	0	1
16	16	13	10	8	.28	.45	1
16	16	14	10	8	.21	.52	1
16	16	15	10	8	.15	.57	1
16	16	16	10	8	.24	0	.69
16	16	19	10	8	.1	0	.46
16	16	21	10	8	.39	0	.24
16	16	$\geq 25, \leq 100$	10	8	.62	0	0
16	16	15	8	8	.81	0	1
16	16	15	10	8	.15	.57	1
16	16	15	$\geq 12, \leq 100$	8	0	.62	1
16	16	15	10	6	.15	.58	1
16	16	15	10	8	.15	.57	1
16	16	15	10	10	.06	0	.54
16	16	15	10	12	.27	0	.33
16	16	15	10	$\geq 13, \leq 100$	.62	0	0

Table 3-5 Sensitivity analysis of scrapping cost

CS_{12}	CS_{13}	CS_{22}	r_1^*	r_2^*	r_3^*
10	30	16	0	.83	1
15	30	16	0	.72	1
20	30	16	.15	.57	1
$\geq 25, \leq 100$	30	16	.47	0	.89
20	10	16	.27	0	1
20	20	16	.47	0	1
20	30	16	.15	.57	1
20	40	16	.35	1	1
20	50	16	.15	1	1
20	30	5	0	.62	1
20	30	14	0	.62	1
20	30	18	.25	.57	1
20	30	20	.55	0	.15
20	30	$\geq 25, \leq 30$	.65	0	.15

Table 3-6 Sensitivity analysis of repair cost

CR_{11}	CR_{12}	CR_{13}	CR_{21}	CR_{22}	r_1^*	r_2^*	r_3^*
$\geq 10, \leq 12$	15	10	10	12	.5	0	.98
15	15	10	10	12	.15	.57	1
$\geq 18, \leq 100$	30	10	10	12	.47	0	.89
15	12	10	10	12	.12	.65	1
15	15	10	10	12	.15	.57	1
15	$\geq 18, \leq 100$	10	10	12	0	.47	1
15	15	$\geq 7, \leq 16$	10	12	.15	.57	1
15	15	19	10	12	.35	0	.77
15	15	$\geq 22, \leq 30$	10	12	.25	0	.72
15	15	10	7	12	.59	0	.98
15	15	10	10	12	.15	.57	1
15	15	10	$\geq 12, \leq 100$	12	0	.62	1
15	15	10	10	9	.25	.52	1
15	15	10	10	$\geq 12, \leq 15$	.15	.57	1
15	15	10	10	$\geq 18, \leq 40$	.21	0	.69

Table 3-7 Sensitivity analysis of processing cost

CP_{11}	CP_{12}	CP_{13}	CP_{21}	CP_{22}	r_1^*	r_2^*	r_3^*
$\geq 6, \leq 9$	14	12	12	15	.59	0	.98
12	14	12	12	15	.15	.57	1
$\geq 15, \leq 50$	14	12	12	15	0	.55	1
12	$\geq 9, \leq 11$	12	12	15	.12	.65	1
12	14	12	12	15	.15	.57	1
12	$\geq 17, \leq 30$	12	12	15	.47	0	.89
12	14	6	12	15	.57	0	1
12	14	9	12	15	.53	0	1
12	14	12	12	15	.15	.57	.95
12	14	$\geq 15, \leq 18$	12	15	0	.74	.95
12	14	12	6	15	.65	0	1
12	14	12	9	15	.61	0	1
12	14	12	$\geq 15, \leq 30$	15	0	.62	1
12	14	12	12	$\geq 9, \leq 12$	.2	.55	1
12	14	12	12	15	.15	.57	1
12	14	12	12	18	.14	.58	1
12	14	12	12	$\geq 21, \leq 23$	.15	.95	.75
12	14	12	12	30	.07	.95	.65

Table 3-8 Sensitivity analysis of waiting cost

CW_1	CW_2	r_1^*	r_2^*	r_3^*
1.5	2	.15	0.57	1
3	2	0	0.62	1
6	2	0	0	.66
12	2	0	0	.61
1.5	2	.15	.57	1
1.5	4	0	.63	1
1.5	6	0	.63	.65
1.5	12	0	.63	.58

Table 4-1 Upper bounds of S_1 , S_2 , F and their corresponding fractional utilization values

S_1	S_2	F	Fractional Utilization
1	1	1	.735
1	3	1	.812
1	≥ 5	1	.916
1	≥ 5	2	.996
1	≥ 5	3	.997
2	≥ 5	4	.997
3	≥ 5	4	.997
1	≥ 5	5	.999
2	≥ 5	5	.9999*

* The FU value in this observation was very close to 1

Table 4-2 $L_{16}(2^{15})$ Orthogonal array

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2
3	1	1	1	2	2	2	2	1	1	1	1	2	2	2	2
4	1	1	1	2	2	2	2	2	2	2	2	1	1	1	1
5	1	2	2	1	1	2	2	1	1	2	2	1	1	2	2
6	1	2	2	1	1	2	2	2	2	1	1	2	2	1	1
7	1	2	2	2	2	1	1	1	1	2	2	2	2	1	1
8	1	2	2	2	2	1	1	2	2	1	1	2	2	1	2
9	2	1	2	1	2	1	2	1	2	1	2	1	2	1	2
10	2	1	2	1	2	1	2	2	1	2	1	2	1	2	1
11	2	1	2	2	1	2	1	1	2	1	2	2	1	2	1
12	2	1	2	2	1	2	1	2	1	2	1	1	2	1	2
13	2	2	1	1	2	2	1	1	2	2	1	1	2	2	1
14	2	2	1	1	2	2	1	2	1	1	2	2	1	1	2
15	2	2	1	2	1	1	2	1	2	2	1	2	1	1	2
16	2	2	1	2	1	1	2	2	1	1	2	1	2	2	1

Table 4-3 Experimental design

Column number	Factor
1	F
2	S_1
3	$F \times S_1$
4	S_2
5	$F \times S_2$
6	$S_1 \times S_2$
7	$R_1 \times R_2$
8	R_1
9	$F \times R_1$
10	$S_1 \times R_1$
11	$S_2 \times R_2$
12	$S_2 \times R_1$
13	$S_1 \times R_2$
14	$F \times R_2$
15	R_2

Table 4-4 Systematic assignment of the levels to the 5 factors
and simulation results

OBS	F	S ₁	S ₂	R ₁	R ₂	UR
1	1	1	1	4	6	0.953
2	1	1	1	11	14	0.735
3	1	1	5	4	14	0.962
4	1	1	5	11	6	0.816
5	1	4	1	4	14	0.908
6	1	4	1	11	6	0.816
7	1	4	5	4	6	0.962
8	1	4	5	11	14	0.816
9	8	1	1	4	14	0.937
10	8	1	1	11	6	0.995
11	8	1	5	4	6	0.9999*
12	8	1	5	11	14	0.9999*
13	8	4	1	4	6	0.9999*
14	8	4	1	11	14	0.937
15	8	4	5	4	14	0.9999*
16	8	4	5	11	6	0.9999*

* The FU values in these observations were very close to 1

Table 4-5 ANOVA table

Factor	Level 1	Level 2	S.S.	D.F.	M.S.	F-value
F	6.968	7.868	0.050625	1	0.050625	107.7892 ^{**}
S ₁	7.398	7.438	0.000100	0	insignificant	
F×S ₁	7.402	7.434	0.000064	0	insignificant	
S ₂	7.280	7.556	0.004761	1	0.004761	10.13697 [*]
F×S ₂	7.412	7.424	0.000009	0	insignificant	
S ₁ ×S ₂	7.398	7.438	0.000100	0	insignificant	
R ₁ ×R ₂	7.402	7.434	0.000064	0	insignificant	
R ₁	7.722	7.114	0.023104	1	0.023104	49.19233 ^{**}
F×R ₁	7.716	7.120	0.022201	1	0.022201	47.26969 ^{**}
S ₁ ×R ₁	7.420	7.416	0.000001	0	insignificant	
S ₂ ×R ₂	7.542	7.294	0.003844	1	0.003844	8.184528 [*]
S ₂ ×R ₁	7.430	7.406	0.000036	0	insignificant	
S ₁ ×R ₂	7.424	7.412	0.000009	0	insignificant	
F×R ₂	7.420	7.416	0.000001	0	insignificant	
R ₂	7.542	7.294	0.003844	1	0.003844	8.184528 [*]

^{**} $F_{1,9,0.99} = 10.56$ ^{*} $F_{1,9,0.95} = 5.12$

Table 4-6 Analysis of variance

Source	D.F.	S.S.	M.S.	F-value	Significance level
Regression	6	0.10839387	0.01806565	419.52	0.0001
error	9	0.00038756	0.00004306		
$R^2=0.996437$					
Estimated	Coefficient		t	significant level	
Intercept	1.109708227		110.61	0.0001	
F	-0.006755102		-6.09	0.0002	
S_2	-0.010703125		-4.85	0.0009	
R_1	-0.024545918		-32.15	0.0001	
R_2	-0.00964837		-13.05	0.0001	
$F \cdot R_1$	0.003045918		22.74	0.0001	
$S_2 \cdot R_2$	0.001929688		9.41	0.0001	

Table 4-7 Metamodel validation of (4-2)

F	S_1	S_2	R_1	R_2	FU	Sim.	Error
5	2	2	10	7	0.920839	0.999	0.078238
2	1	2	9	13	0.833447	0.955	0.121553
3	1	4	7	10	0.919476	0.997	0.077756
1	4	1	8	12	0.827625	0.883	0.055375
3	4	2	9	6	0.894628	0.993	0.098372
3	2	3	1	14	0.987894	0.985	0.002938
5	2	1	1	11	0.971007	0.98	0.009176
1	3	2	6	13	0.877289	0.934	0.056711
1	1	1	10	12	0.784625	0.702	0.082625
2	2	3	5	13	0.921646	0.991	0.069983

Average: 0.0652722

Table 4-8 Improved experimental design

	R_1				*	F	$R_1 \times F$			R_2	e	S_2	e	e	e	e	e
	2	4	6			1	3	5	7	8	9	10	11	12	13	14	15
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
3	1	2	2	2	1	1	2	2	1	1	1	1	1	2	2	2	2
4	1	2	2	2	1	1	2	2	2	2	2	2	2	1	1	1	1
5	2	1	2	3	1	2	1	2	1	1	2	2	2	1	1	2	2
6	2	1	2	3	1	2	1	2	2	2	1	1	1	2	2	1	1
7	2	2	1	4	1	2	2	1	1	1	2	2	2	2	2	1	1
8	2	2	1	4	1	2	2	1	2	2	1	1	1	1	1	2	2
9	1	1	1	1	2	2	2	2	1	2	1	2	1	2	1	2	2
10	1	1	1	1	2	2	2	2	2	1	2	1	2	1	2	1	2
11	1	2	2	2	2	2	1	1	1	2	1	2	2	2	1	2	1
12	1	2	2	2	2	2	1	1	2	1	2	1	1	1	2	1	2
13	2	1	2	3	2	1	2	1	1	2	2	1	1	1	2	2	1
14	2	1	2	3	2	1	2	1	2	1	1	2	2	2	1	1	2
15	2	2	1	4	2	1	1	2	1	2	2	1	2	1	1	1	2
16	2	2	1	4	2	1	1	2	2	1	1	2	1	2	2	2	1

*This column is used to represent the 4-level design of R_1 .

Table 4-9 Levels of factors and simulation results

	F	S ₂	R ₁	R ₂	FU
1	1	1	4	6	.953
2	1	5	4	14	.962
3	1	1	6	6	.911
4	1	5	6	14	.925
5	1	5	9	6	.861
6	1	1	9	14	.782
7	1	5	11	6	.816
8	1	1	11	14	.735
9	4	1	4	6	.999
10	4	5	4	14	.9999*
11	4	1	6	6	.996
12	4	5	6	14	.9999*
13	4	5	9	6	.999
14	4	1	9	14	.898
15	4	5	11	6	.997
16	4	1	11	14	.872

(a) First experiment

	F	S ₂	R ₁	R ₂	FU
1	5	1	4	6	.9999*
2	5	5	4	14	.9999*
3	5	1	6	6	.998
4	5	5	6	14	.9999*
5	5	5	9	6	.9999*
6	5	1	9	14	.912
7	5	5	11	6	.999
8	5	1	11	14	.885
9	8	1	4	6	.9999*
10	8	5	4	14	.9999*
11	8	1	6	6	.9999*
12	8	5	6	14	.9999*
13	8	5	9	6	.9999*
14	8	1	9	14	.924
15	8	5	11	6	.9999*
16	8	1	11	14	.914

(b) Second experiment

* The FU values in these observations were very close to 1.

Table 4-10 ANOVA table of the first experiment

Factor	S. S.	M. S.	F-value
F	0.041616	0.041616	338.1123**
R ₂	0.008010	0.008010	65.07989**
e	0.000036	-----	-----
e	0.000072	-----	-----
e	0.000025	-----	-----
S ₂	0.010712	0.010712	87.03249**
e	0.000020	-----	-----
R ₁	0.013750	0.013750	111.7203**
R ₁ *F	0.002652	0.002652	21.55179**
e	0.000441	-----	-----
e	0.000144	-----	-----

$$**F_{1,8,0.99} = 13.74$$

Table 4-11 ANOVA table of the second experiment

Factor	S. S.	M. S.	F-value
F	0.000121	-----	-----
R ₂	0.008190	0.008190	124.5665**
e	0.000012	-----	-----
e	0.000090	-----	-----
e	0.000020	-----	-----
S ₂	0.008372	0.008372	129.3346**
e	0.000072	-----	-----
R ₁	0.002820	0.002820	42.89987**
R ₁ *F	0.000047	-----	-----
e	0.000090	-----	-----
e	0.000110	-----	-----

$$**F_{1,8,0.999} = 25.42$$

Table 4-12 Analysis of variance of the first model

Source	DF	SS	MS	F-value	Significance level
Regression	5	0.10876513	0.02175303	142.87	0.0001
error	10	0.00152262	0.00015226		
$R^2=0.986194$					
Estimated	coefficient		t	significance level	
Intercept	1.094806034		55.03	0.0001	
F	-0.007379310		-1.21	0.0253	
R_2	-0.005593750		-7.25	0.0001	
S_2	0.012937500		8.39	0.0001	
R_1	-0.032474138		-14.58	0.0001	
$F \cdot R_1$	0.005517241		7.22	0.0001	

Table 4-13 Analysis of variance of the second model

Source	DF	SS	MS	F-value	Significance level
Regression	3	0.02434267	0.00811422	72.70	0.0001
error	12	0.00133933	0.00011161		
$R^2=0.94785$					
Estimated	coefficient		t	significance level	
INTERCEPT	1.060672414		96.67	0.0001	
R_2	-0.005656250		-8.57	0.0001	
S_2	0.011437500		8.66	0.0001	
R_1	-0.008189655		-8.35	0.0001	

Table 4-14 Evaluation of one-regression-line

F	S ₂	R ₁	R ₂	FU	Sim.	Error
2	2	9	13	0.833447	0.94	0.118045
3	4	7	10	0.919476	0.997	0.07776
1	1	8	12	0.82762	0.872	0.050888
3	3	11	14	0.833812	0.985	0.153489
1	2	6	13	0.877289	0.924	0.050552
1	1	10	12	0.784625	0.778	0.008515
2	3	5	13	0.921646	0.991	0.069983
3	2	9	6	0.894628	0.992	0.098156
4	4	5	7	0.964556	0.9999*	0.035443
2	1	11	8	0.8207550	0.933	0.120310
Average: 0.078314						
5	2	10	7	0.920839	0.999	0.078238
7	3	6	8	0.980091	.9999*	0.019908
5	1	11	11	0.877843	0.98	0.104241
8	3	11	14	0.967562	.9999*	0.032437
7	4	11	10	0.964843	.9999*	0.035156
6	1	5	9	0.957653	0.997	0.039464
6	2	7	12	0.934409	0.999	0.064654
4	3	8	13	0.901508	0.998	0.096684
7	4	10	9	0.969997	.9999*	0.030002
8	5	7	12	1.000000#	.9999*	0.000000
Average: 0.050079						

* The FU values in these observations were very close to 1.

The FU value is 1.000901 estimated by the regression equation.

Table 4-15 Evaluation of two-regression-line

F	S ₂	R ₁	R ₂	FU	Sim.	Error
2	2	9	13	0.8402468	0.945	0.1047532
3	4	7	10	0.950238	0.997	0.0399762
1	1	8	12	0.8175841	0.872	0.0544159
3	3	11	14	0.8580216	0.985	0.0126978
1	2	6	13	0.8788416	0.924	0.0451584
1	1	10	12	0.7636703	0.778	0.0143297
2	3	5	13	0.9389429	0.991	0.0520571
3	2	9	6	0.9216789	0.992	0.0703210
4	4	5	7	1.0000000#	0.9999*	0.0258567
2	1	11	8	0.8123988	0.933	0.1206013
Average: 0.0540167						
5	2	10	7	0.962057	0.999	0.036979
7	3	6	8	1.0000000@	0.9999*	0.000596
5	1	11	11	0.919804	0.98	0.061423
8	3	11	14	0.925711	0.9999*	0.074288
7	4	11	10	0.959773	0.9999*	0.040226
6	1	5	9	0.980255	0.997	0.016794
6	2	7	12	0.958344	0.999	0.040695
4	3	8	13	0.955936	0.998	0.042147
7	4	10	9	0.973619	0.9999*	0.026380
8	5	7	12	0.992657	0.9999*	0.007342
Average: 0.034687						

* The FU values in these observations were very close to 1.

@ The FU value is 1.0258570 estimated by the regression equation.

The FU value is 1.000596 estimated by the regression equation.

Table 4-16 Further simulation sets for optimal
solution

No. of Set	F	R_1	R_2	S_1	S_2	FU	$Z(F, R_1, R_2, S_1, S_2)$
1	3	4	7	2	3	.9998	27412
2	3	4	7	2	4	.9999	27215
3	3	5	7	2	3	.9998	27457
4	3	5	7	2	4	.9999	27260
5	5	4	7	2	3	.9999	26815
6	5	4	7	2	4	.9999	26615
7	5	5	7	2	3	.9998	26857
8	5	5	7	2	4	.9999	26660

Figure 3-1 Flow of parts in the FMS

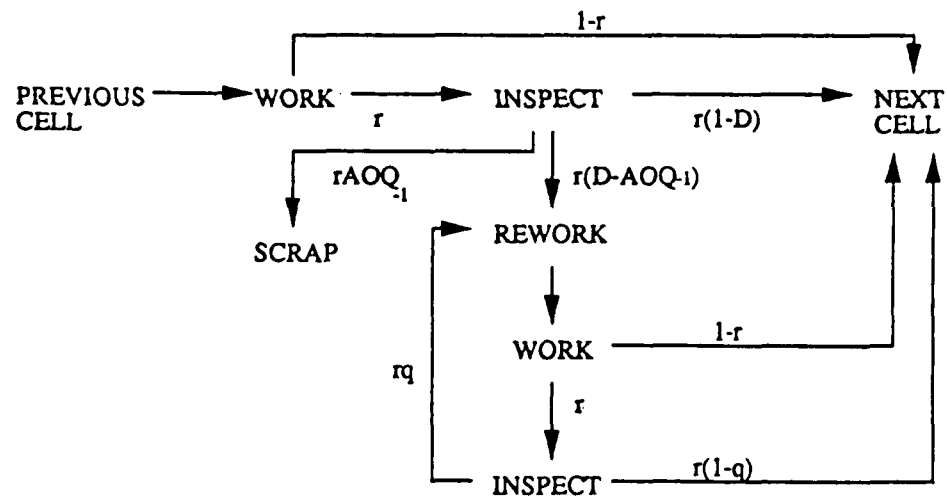


Figure 3-2 Waiting time in cell 1

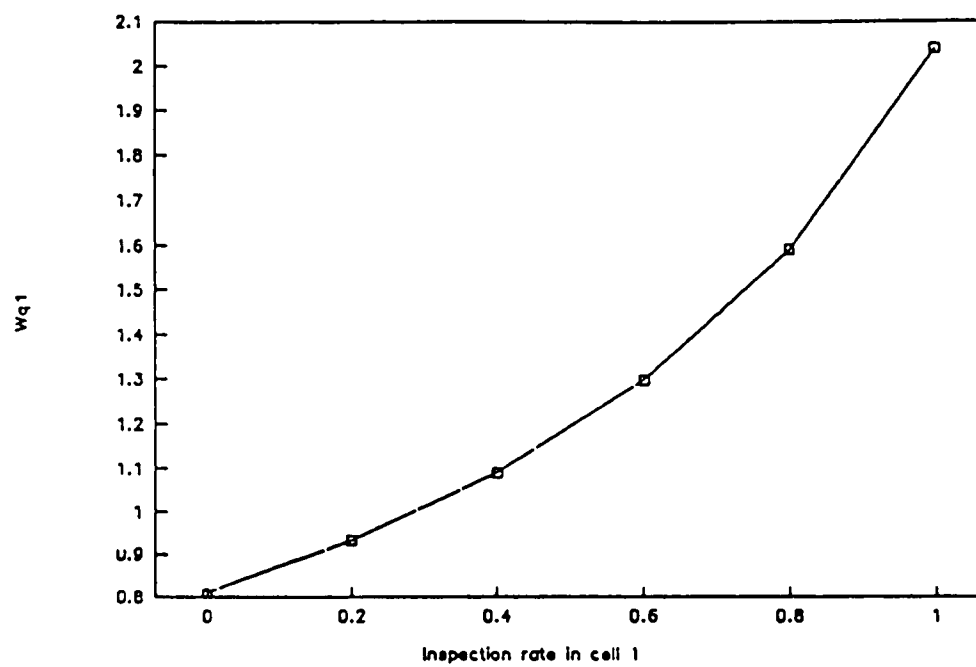


Figure 3-3 Waiting time in cell 2

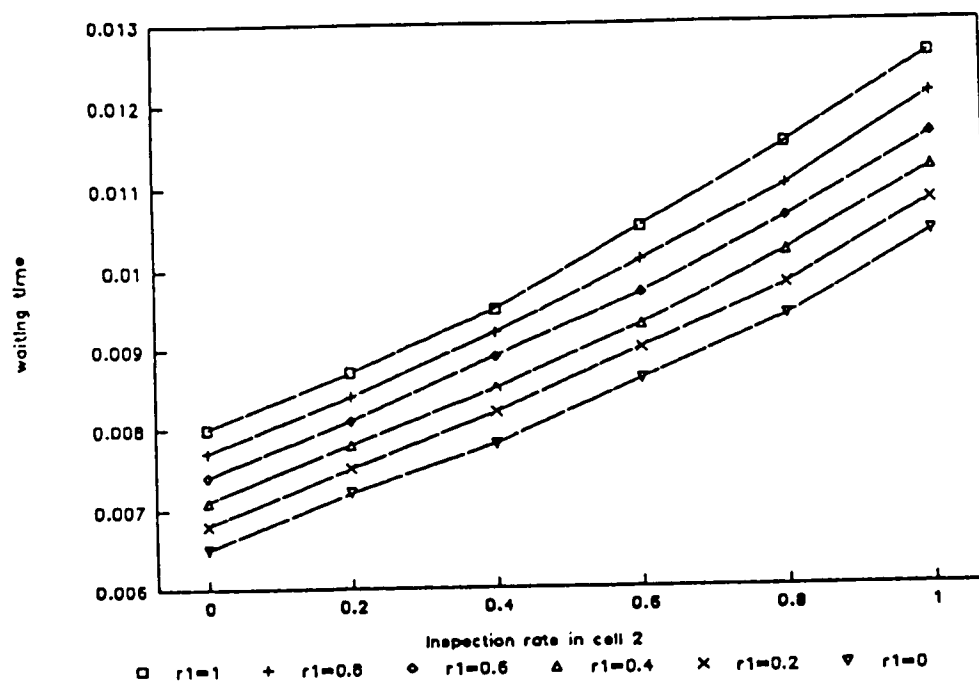


Figure 3-4 Waiting time in cell 3-(a)

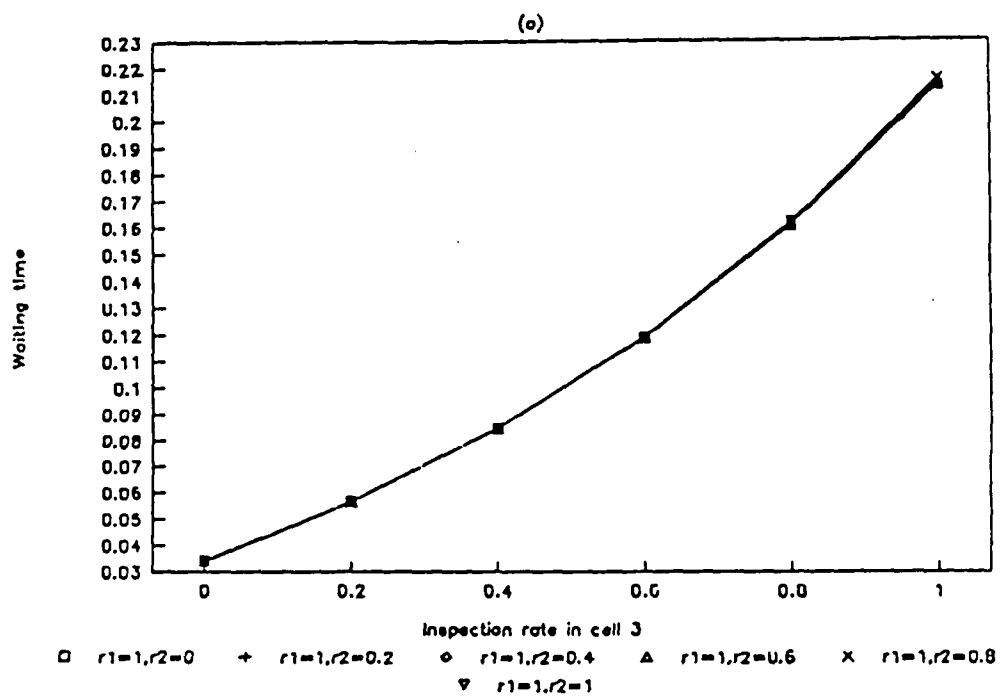


Figure 3-5 Waiting time in cell 3-(b)

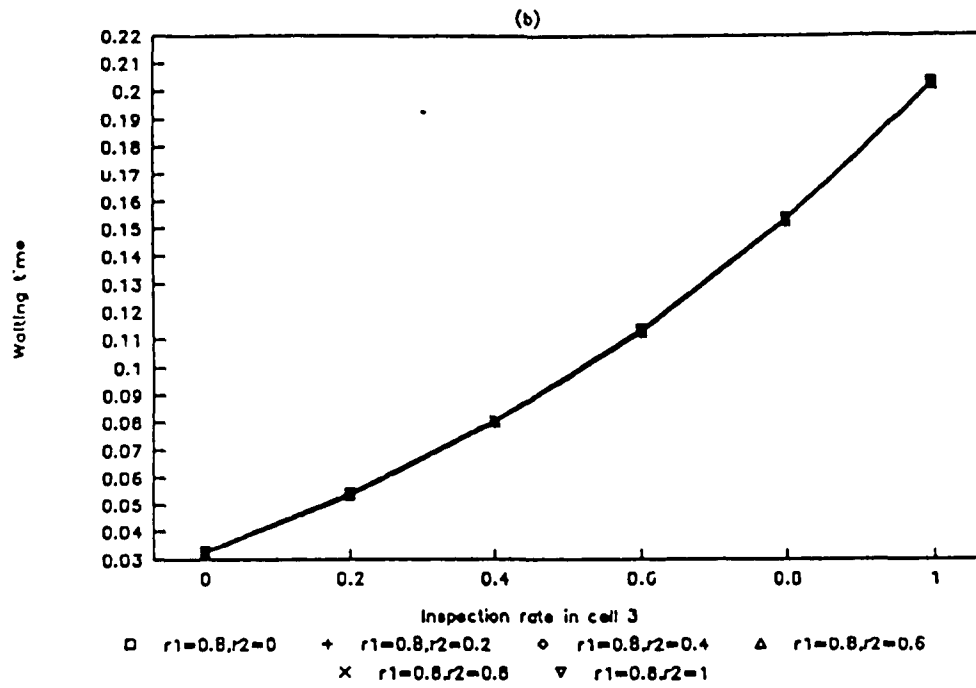


Figure 3-6 Waiting time in cell 3-(c)

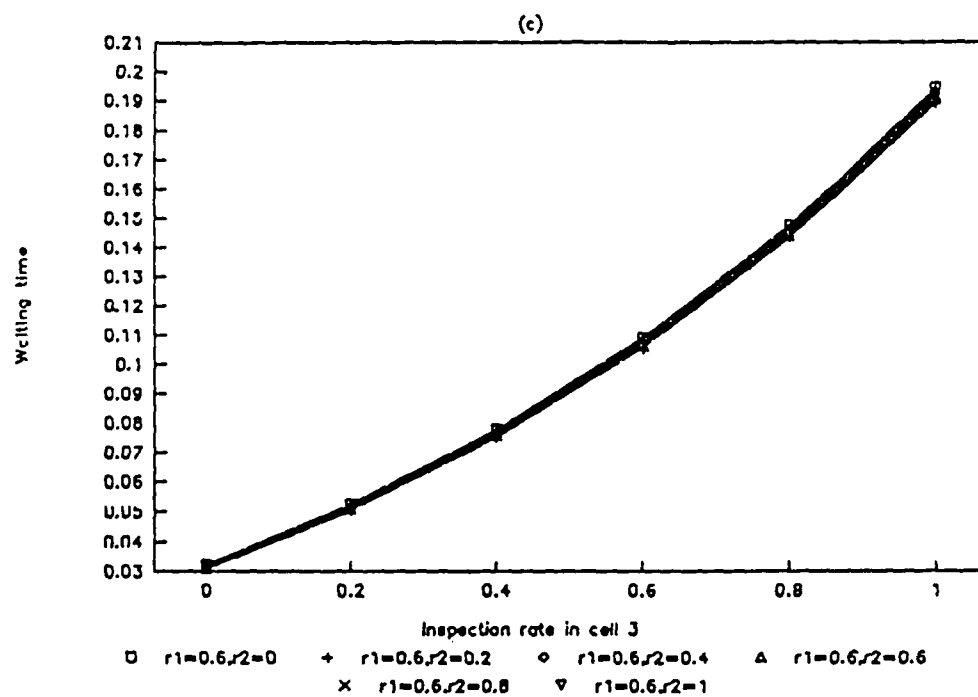


Figure 3-7 Waiting time in cell 3-(d)

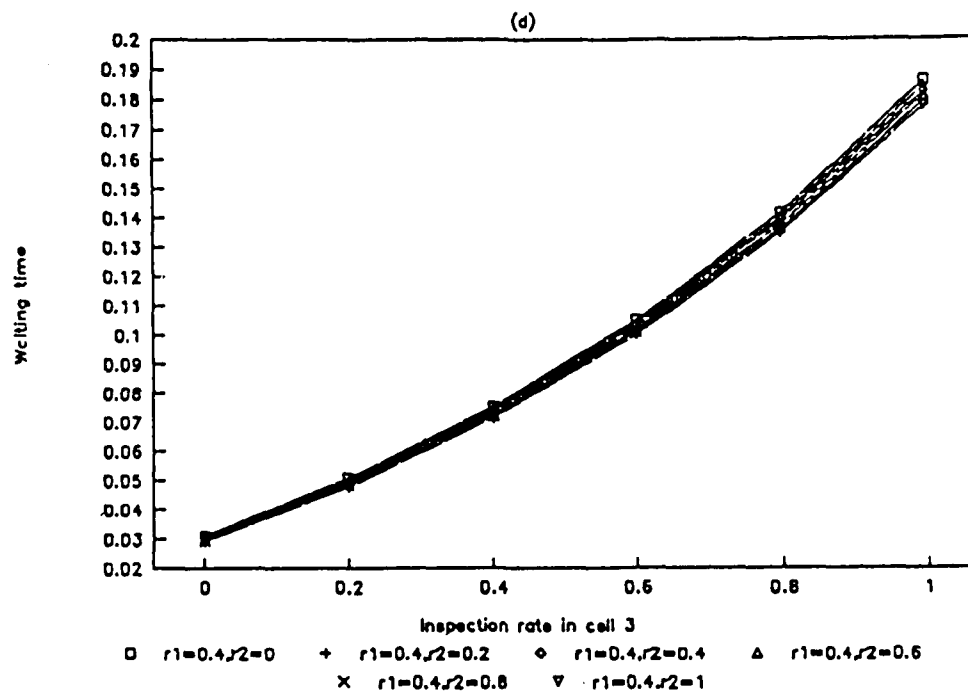


Figure 3-8 Waiting time in cell 3-(e)

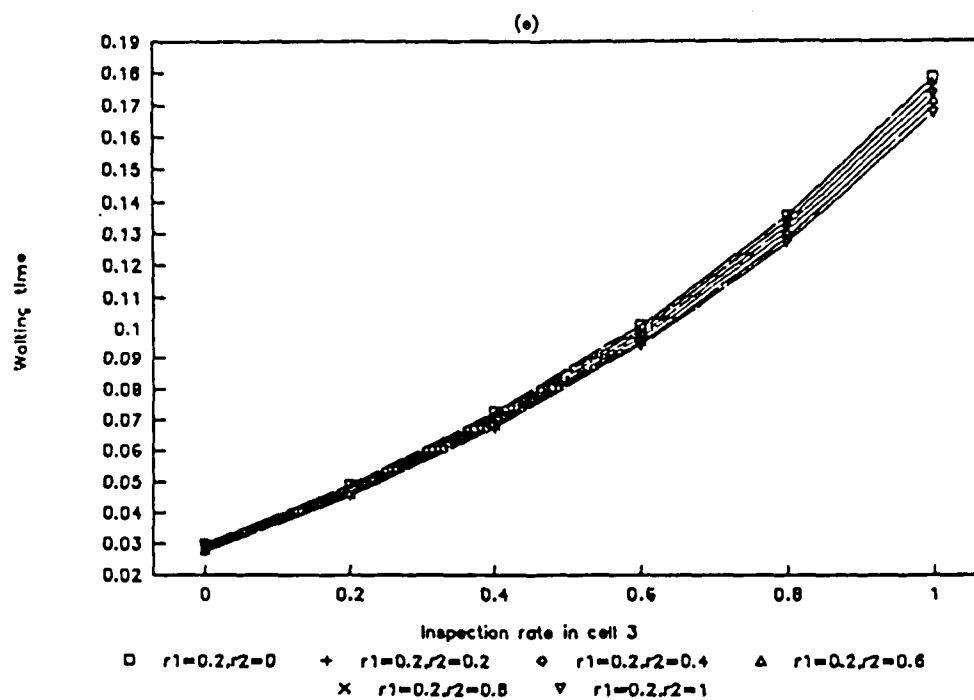


Figure 3-9 Waiting time in cell 3-(f)

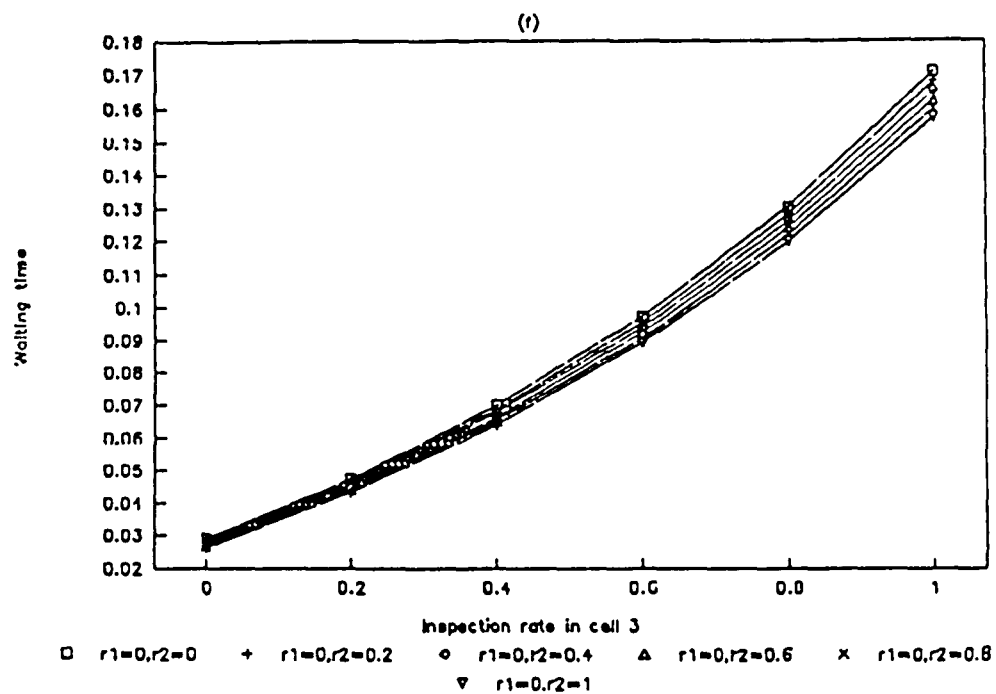


Figure 3-12 Average outgoing quality of part type 1-(b)

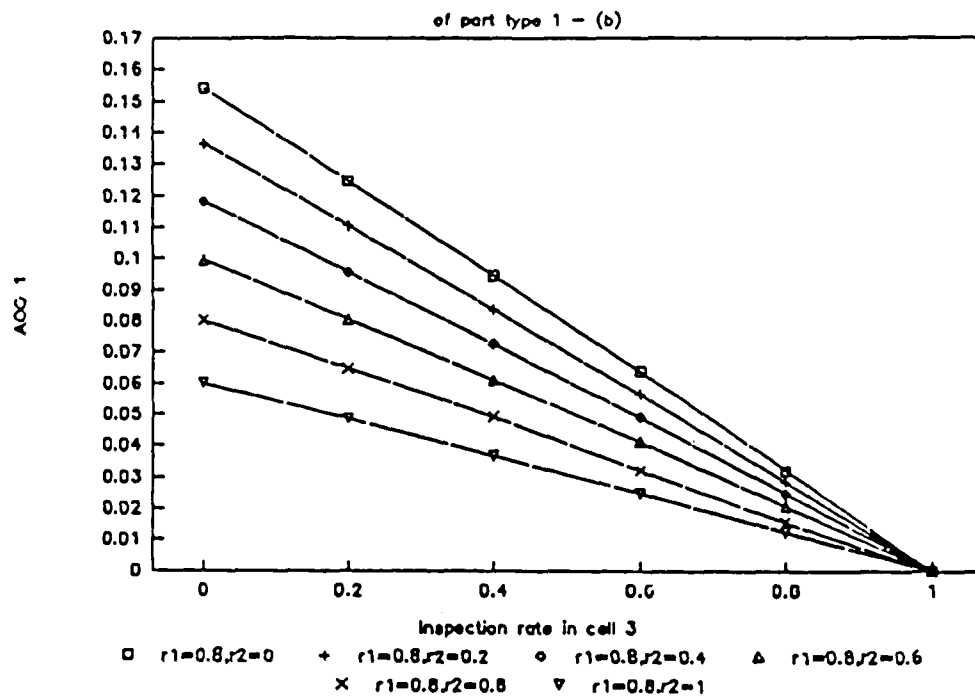


Figure 3-14 Average outgoing quality of part type 1-(d)

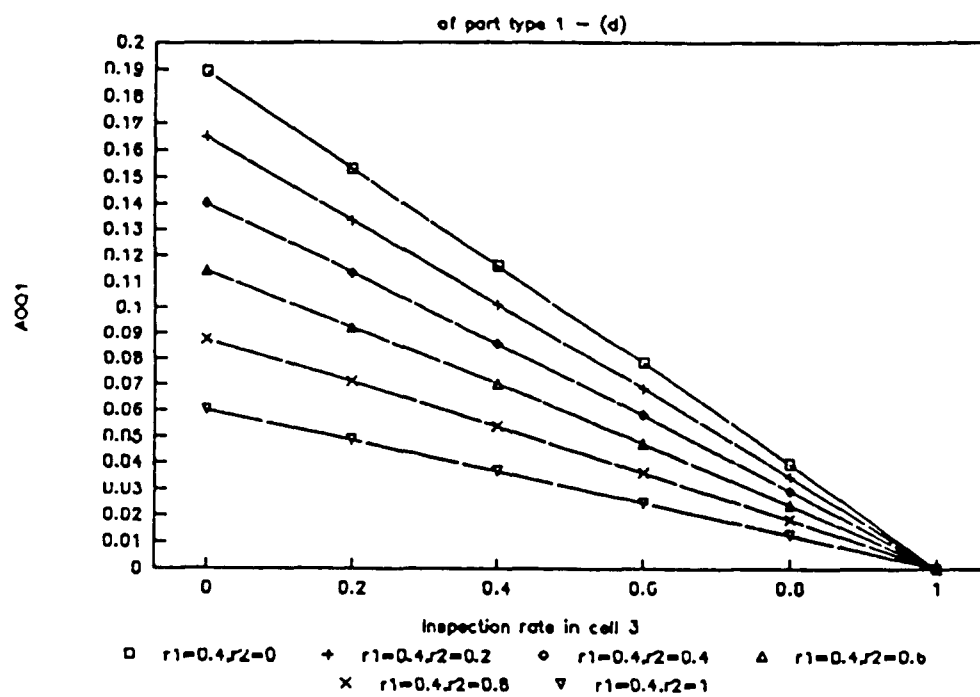


Figure 3-15 Average outgoing quality of part type 1-(e)

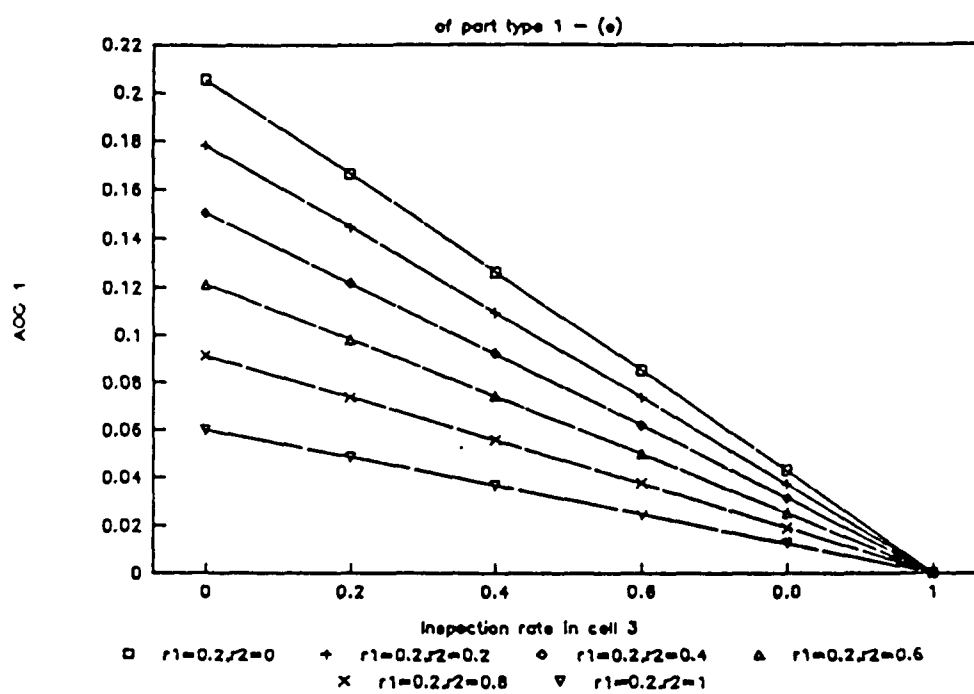


Figure 3-16 Average outgoing quality of part type 1-(f)

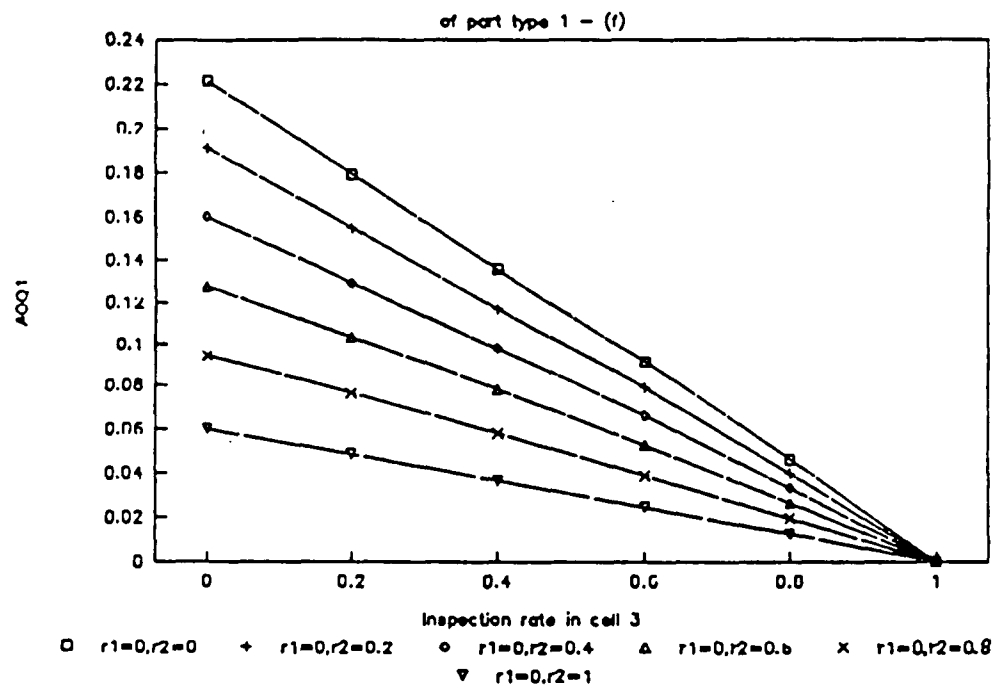


Figure 4-1 Maintenance system of the FMS

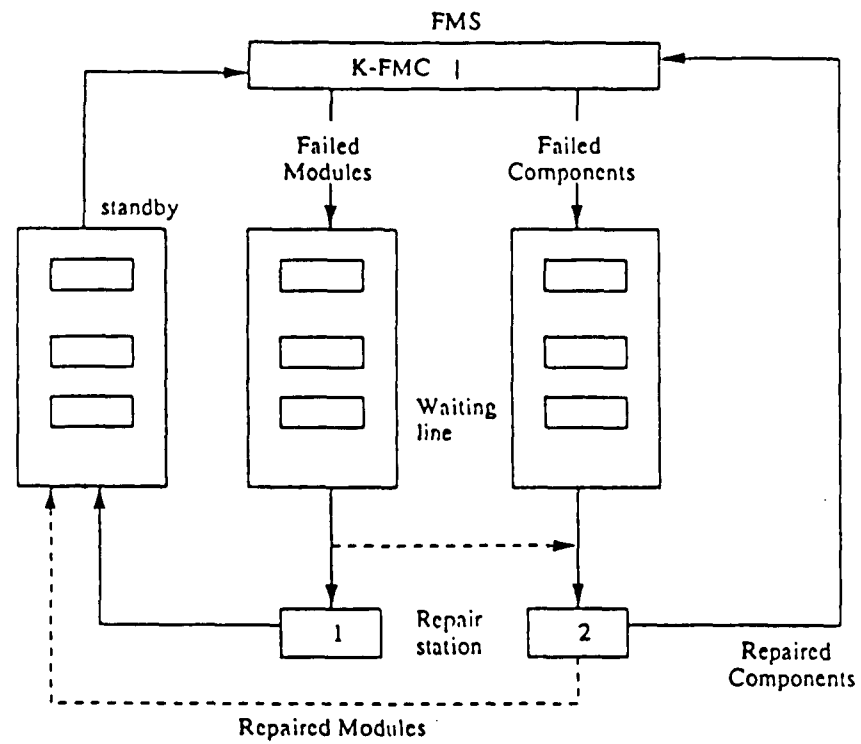


Figure 4-2 Framework of approach

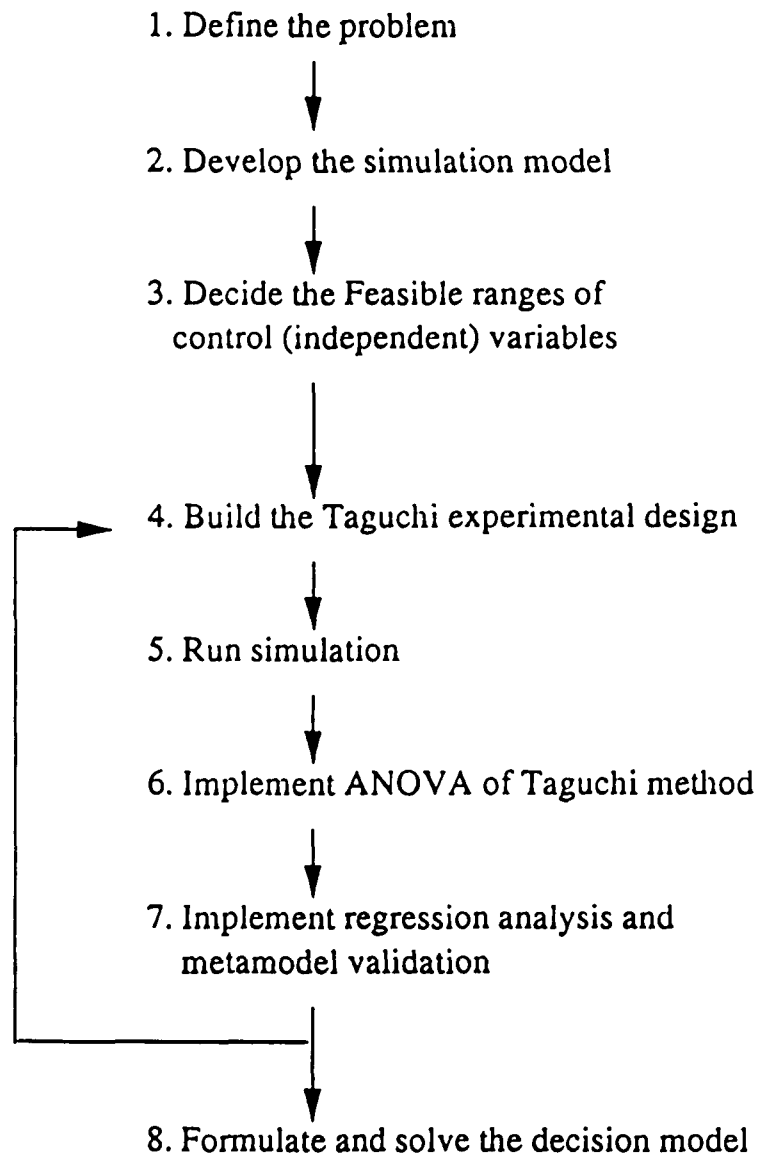
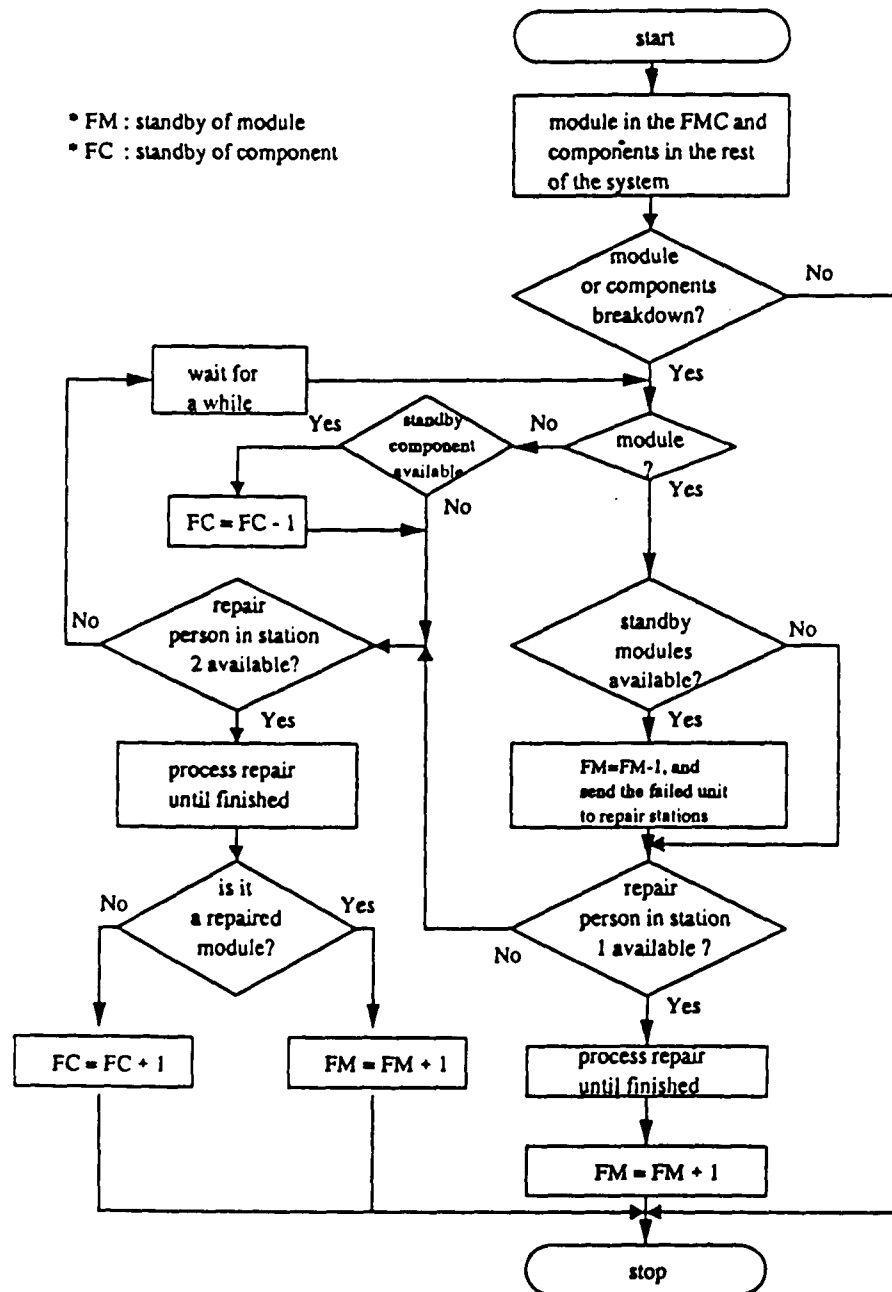


Figure 4-3 Flow diagram of simulation



APPENDIX A: ANALYTICAL MODELS OF MAINTENANCE FLOAT

Case 1: The Single Repair Server Case Without Consideration Of The Rest Of The System

1.1. Queuing System Model

We first specify the assumptions and define the notations for the model.

- 1) The module in the FMC is repairable;
- 2) There are F modules in the FMC, one is engaged in operation and the remaining $F-1$ modules are standbys;
- 3) The changeover time for replacing the failed module with a standby module is negligible;
- 4) The failure rate of the modules (in the FMC) follows a Poisson distribution with parameter λ ; and
- 5) The repair rate of the single server at the repair station follows a Poisson distribution with parameter μ , where $\lambda < \mu$ (i.e., we assume that the mean repair rate is greater than the failure rate of the modules).

Proposition 1:

The repair system of the FMC is an M/M/1/F queue with

$$P_n = \left(\frac{1-\rho}{1-\rho^{n+1}} \right) \rho^n, \quad n=0,1,2,\dots,F \quad (A1)$$

where $\rho = \frac{\lambda}{\mu}$.

Proof:

(A1) can be derived directly from the results for an M/M/1/F queue (see, for example, Kleinrock, Vol. 1, 1974, pp. 90-94).

Q.E.D.

For a maintenance float policy that specifies a total of F modules (1 in operation and $F-1$ standbys), the FMC stops when all F modules are in the repair shop. Since the entire FMS will be forced to idle when the FMC stops, the idle rate of the system is a function of F and μ , and can be expressed by:

$IR(F, \mu)$ = the idle rate of the system

$$P_F = \left(\frac{1-\rho}{1-\rho^{F+1}} \right) \rho^F \quad (A2)$$

and the fractional utilization of the system is, thus,

$$\begin{aligned} FU(F, \mu) &= \text{the fractional utilization of the system} \\ &= 1 - IR(F, \mu) \end{aligned} \quad (A3)$$

1.2. A Profit-Maximization Model for Determining the Optimum Maintenance Float Policy

Suppose we are facing the following revenue and cost functions:

$f(FU(F, \mu))$: an increasing and concave revenue function of the fractional utilization of the system,

$g(F)$: a convex cost function for keeping F modules in the FMC, and

$h(\mu)$: a convex cost function for maintaining an average repair rate of μ at the repair station.

The optimum maintenance float policy can then be determined by solving the following mathematical program:

$$\text{Maximize } Z(F, \mu) = f(FU(F, \mu)) - g(F) - h(\mu) \quad (A4)$$

Subject to:

$$F \geq 1 \text{ integer, and} \quad (A5)$$

$$\mu > \lambda, \quad (A6)$$

where, $FU(F, \mu)$ is the fractional utilization of the system and is determined by (A3).

Proposition 2:

The fractional utilization $FU(F, \mu)$ is increasing in F for a given μ . ■

Proof:

Since $\rho < 1$ implies $\rho^{F+1} < \rho^F$ and $(1-\rho^{F+1}) < (1-\rho^{F+2})$,

$$\text{Thus, } \left(\frac{1-\rho}{1-\rho^{F+2}} \right) \rho^{F+1} < \left(\frac{1-\rho}{1-\rho^{F+1}} \right) \rho^F.$$

That is, $IR(F, \mu)$ is decreasing in F for a given μ , and therefore, $FU(F, \mu)$ is increasing in F for a given μ . Q.E.D.

Proposition 3:

The fractional utilization $FU(s, \mu)$ is concave in F for a given μ . ■

Proof:

We have to prove that the idle rate $IR(F, \mu)$ is convex in F for a given μ , that is, $P_{F+1} + P_{F-1} > 2P_F$, and it suffices to show that

$$\begin{aligned}
 & \rho^2 \left(\frac{1-\rho}{1-\rho^{F+2}} \right) + \left(\frac{1-\rho}{1-\rho^F} \right) > 2\rho \left(\frac{1-\rho}{1-\rho^{F+1}} \right). \\
 & \rho^2 \left(\frac{1-\rho}{1-\rho^{F+2}} \right) + \left(\frac{1-\rho}{1-\rho^F} \right) - 2\rho \left(\frac{1-\rho}{1-\rho^{F+1}} \right) \\
 & = \frac{(1-\rho)}{(1-\rho^{F+2})(1-\rho^F)(1-\rho^{F+1})} [\rho^2(1-\rho^F)(1-\rho^{F+1}) + (1-\rho^{F+2})(1-\rho^{F+1}) \\
 & \quad - 2\rho(1-\rho^{F+2})(1-\rho^F)] \\
 & = \frac{(1-\rho)}{(1-\rho^{F+2})(1-\rho^F)(1-\rho^{F+1})} [(1-\rho)^2(1+\rho^{F+1})] > 0, \text{ since } 0 < \rho < 1.
 \end{aligned}$$

Q.E.D.

Proposition 4:

The fractional utilization $FU(F, \mu)$ is increasing in μ for a given F . ■

Proof:

For $F \geq 1$, we have

$$\begin{aligned}
 IR(F, \mu) &= \frac{(1-\rho)\rho^F}{1-\rho^{F+1}} = \frac{(\mu-\lambda)(\lambda^F)}{\mu^{F+1}-\lambda^{F+1}} \quad (A7) \\
 &= \lambda^F / \left(\sum_{k=0}^F \mu^{F-k} \lambda^k \right),
 \end{aligned}$$

where $IR(F, \mu)$ is decreasing in μ for a given F . Therefore,

$FU(F, \mu)$ is increasing in μ .

Q.E.D.

Proposition 5:

The fractional utilization $FU(F, \mu)$ is concave in μ for a given F . ■

Proof:

The Proposition will be proved by showing that $IR(F, \mu)$ is convex in μ for a given F . From (A7) we have $IR(F, \mu) = (\mu - \lambda)(\lambda^F) / (\mu^{F+1} - \lambda^{F+1})$. For a given F , let $n = F + 1$ and define

$$f(\mu) = \frac{\mu - \lambda}{\mu^n - \lambda^n}.$$

It then suffices to show that $f''(\mu) > 0$ for every feasible μ .

$$f'(\mu) = - \frac{(n-1)\mu^n - n\lambda\mu^{n-1} + \lambda^n}{(\mu^n - \lambda^n)^2}$$

$$f''(\mu) = \frac{1}{(\mu^n - \lambda^n)^4} \{ 2(\mu^n - \lambda^n)n\mu^{n-1}[(n-1)\mu^n - n\lambda\mu^{n-1} + \lambda^n] - n(n-1)(\mu^{n-1} - \lambda\mu^{n-2})(\mu^n - \lambda^n)^2 \} = \frac{n\mu^{n-2}}{(\mu^n - \lambda^n)^3} g(\mu), \text{ where}$$

$$g(\mu) = (n-1)\mu^{n+1} - (n+1)\lambda\mu^n + (n+1)\lambda^n\mu - (n-1)\lambda^{n+1}.$$

Now, consider the first and second derivatives of $g(\mu)$:

$$g'(\mu) = (n-1)(n+1)\mu^n - n(n+1)\lambda\mu^{n-1} + (n+1)\lambda^n, \text{ and}$$

$$g''(\mu) = (n-1)(n)(n+1)(\mu^{n-1} - \lambda\mu^{n-2}).$$

Clearly, $g''(\mu) > 0$, for $\mu > \lambda$.

→ $g'(\mu)$ is increasing in μ and $g'(\mu) > g'(\lambda) = 0$, for $\mu > \lambda$.

→ $g(\mu)$ is increasing in μ and $g(\mu) > g(\lambda) = 0$, for $\mu > \lambda$.

Since the repair rate μ is restricted to be greater than λ by constraint (A6), we have $g(\mu) > 0$, and hence, $f''(\mu) > 0$. That is, for

a given F , $IR(F, \mu)$ is convex in μ within the feasible range of μ . Therefore, for a given F , the fractional utilization $FU(F, \mu)$ is concave in μ within the feasible range of μ . Q.E.D.

It is difficult to prove that $Z(F, \mu)$ is concave with respect to F and μ simultaneously. Hence, to maximize $Z(F, \mu)$ we introduce the concept and the relationships between the superlinearity (sublinearity) and convexity (concavity). Following the notations and results in Shanthikumar and Yao (1987), we obtain the following Proposition.

Proposition 6:

- (i) A function $\phi(n)$ ($n=0,1,\dots$) is superlinear (sublinear) if for all n , $n\phi(n+1) \geq (\leq) (n+1)\phi(n)$.
- (ii) IF $\phi(0)=0$, then $\phi(n)$ is convex (concave) implies that $\phi(n)$ is superlinear (sublinear).
- (iii) $\phi(n)$ is convex (concave) if and only if $(j-1)[\phi(k)-\phi(j)] \geq (\leq) (k-j)[\phi(j)-\phi(1)]$ for all $1 \leq j \leq k$.
- (iv) $\phi(n)$ is superlinear (sublinear) if and only if $\phi(k)-\phi(j) \geq (\leq) (k/j-1)\phi(j)$ for all $j \leq k$.

For solving $\text{Max. } Z(F, \mu)$, we propose the following two Propositions.

Proposition 7:

Define $H(F, \mu) = f(FU(F, \mu)) - h(\mu)$, then $H(F, \mu)$ is concave in F for a given μ , and $H(F, \mu)$ is concave in μ for a given F .

Proof:

To prove the concavity of $H(F, \mu)$ in F we only need to show that $f(FU(F, \mu))$ is concave in F for a given μ .

Let $f_\mu(FU(F)) \triangleq f(FU(F, \mu))$, we have

$$\begin{aligned} f_\mu(\alpha F_1 + (1-\alpha)F_2) &\geq f_\mu(\alpha FU(F_1) + (1-\alpha)FU(F_2)) \\ &\geq \alpha f_\mu(FU(F_1)) + (1-\alpha)f_\mu(FU(F_2)) \end{aligned}$$

The first inequality holds because FU is concave in F for a given μ and f_μ is increasing; the second inequality holds because f_μ is assumed to be concave. Similarly, we can prove that $H(F, \mu)$ is concave in μ . Q.E.D.

$$\text{Let } H(F, \mu_F) = \max_{\mu} H(F, \mu). \quad (\text{A8})$$

$$\text{then } \max_{F, \mu} Z(F, \mu) = \max_F \{H(F, \mu_F) - g(F)\}. \quad (\text{A9})$$

Given these notations and Propositions 6 and 7, we derive the following proposition.

Proposition 8:

The optimal solution F^* is the optimal value that solves

$$\text{Maximize } \{H(F, \mu_F) - g(F)\}, \text{ for } 1 \leq F \leq F_0, \quad (\text{A10})$$

where F_0 is the smallest integer that satisfies

$$H(F_0, \mu_{F_0}) / F_0 \leq g(F_0) - g(F_0 - 1) \quad (\text{A11})$$

■

Proof:

From A(10) and A(11), F^* is the optimal maintenance float for $F \leq F_0$. For $F > F_0$, we have

$H(F, \mu_F) - H(F_0, \mu_{F_0}) \leq H(F, \mu_F) - H(F_0, \mu_F)$ (from the definition of $H(F, \mu_F)$).

$\leq \frac{F - F_0}{F_0} H(F_0, \mu_F)$ (from Proposition 6-(ii), (iv), and Proposition 7).

$$\leq \frac{F - F_0}{F_0} H(F_0, \mu_{F_0}).$$

$$\leq (F - F_0) [g(F_0) - g(F_0 - 1)] \text{ (from (A11))}.$$

$$\leq g(F) - g(F_0) \text{ (From Proposition 6-(iii))}.$$

Thus, for $F > F_0$

$$H(F, \mu_F) - g(F) \leq H(F_0, \mu_{F_0}) - g(F_0) \leq H(F^*, \mu_{F^*}) - g(F^*).$$

Therefore, (F^*, μ_{F^*}) is optimal.

Q.E.D.

1.3. A Solution Procedure and a Numerical Example

Based on Propositions 7 and 8, the solution procedure starts with $F=1$ (F is increased by 1 in subsequent iterations). μ_F is solved using the concavity property of $H(F, \mu)$. Once this is done, condition (A11) is checked. The solution procedure is now formally presented below.

Step 1: Initialization.

Set $F \leftarrow 1$, $\mu \leftarrow 0$, $H^{\circ}(F, \mu) \leftarrow -\infty$.

Step 2: Solve $\partial H(F, \mu) / \partial \mu = 0$ for μ_F , and evaluate $H(F, \mu_F)$.

Step 3: If $H(F, \mu_F) > H^{\circ}(F, \mu)$, go to Step 4; otherwise, go to Step 5.

Step 4: Set $\mu \leftarrow \mu_F$ and $H^{\circ}(F, \mu) \leftarrow H(F, \mu_F)$.

Step 5: If F satisfies condition (A11), stop, (F, μ) is optimal with the maximum profit equal to $Z(F, \mu)$; otherwise, set $F \leftarrow F+1$, and go to Step 2.

Suppose the failure rate is 3, and we are faced with the maximization of $Z(F, \mu)$, where

$$Z(F, \mu) = 9000FU(F, \mu) - 200F - 150\mu, \text{ and}$$

the mean repair rate must be greater than the mean failure rate.

The optimal solution is as follows: $F = 4$, $\mu = 6$, and $Z^{\circ}(F, \mu) = \$7009.678$. Notice that this is a special case since all terms in the objective function are linear.

Case 2. The Multiple Repair Servers Case Without Consideration Of The Rest Of The System

Here we discuss the case where there are S identical servers at the repair station. The repair rate of each server for the modules sent from the FMC follows a Poisson distribution with parameter μ . The first four assumptions made for the single-server case will be retained here. We also require that

the total repair rate of the servers be greater than the failure rate of the modules.

For the multiple repair servers case, we are interested in obtaining the optimal number of servers (S) at the repair station, under a fixed repair rate μ , and the optimal number of floats so as to maximize profit.

2.1. Modeling and Analyzing the Multiple-Server Maintenance Flow Problem

Suppose we are facing the following revenue and cost functions:

$f(F, S)$: an increasing and concave revenue function,

where S is the number of servers,

$g(F)$: a convex cost function of having F units of float in the system, and

$h(S)$: a convex cost function of having S servers at the repair station.

The optimal maintenance float policy can then be determined by solving the following mathematical program:

$$\text{Maximize } Z(F, S) = f(F, S) - g(F) - h(S) \quad (A12)$$

Subject to:

$$F \geq 1, \text{ integer, and} \quad (A13)$$

$$S\mu > \lambda. \quad (A14)$$

For solving the above mathematical program, we derive the following Propositions.

Proposition 9:

Given that $\frac{\lambda}{S\mu} < 1$, we have

$$IR(F, S) = P_F = \frac{\lambda^F}{S! \mu^F S^{(F-S)}} P_0, \text{ for } F \geq S \quad (A15)$$

$$FU(F, S) = 1 - IR(F, S) = 1 - P_F \quad (A16)$$

where,

$$P_0 = \left[1 + \sum_{\ell=1}^S \frac{\lambda^\ell}{\mu^\ell \ell!} + \sum_{\ell=S+1}^F \frac{\lambda^\ell}{S! \mu^\ell S^{(\ell-S)}} \right]^{-1}. \quad (A17)$$

■

Proof:

Equations (A15) and (A17) can be derived directly from the results of an M/M/S/F queue. (See, for example, Kleinrock, Vol. 1, 1974, pp. 90-94.) Equation (A16) is obtained by definition.

Proposition 10:

The fractional utilization $FU(F, S)$ is increasing in F for a given S .

■

Proof:

Let $P_0^{(F)}$ and $P_0^{(F+1)}$ be the values of P_0 evaluated using (A17) with the upper limit for ℓ in the second summation equal to F and $F+1$, respectively. Clearly, $P_0^{(F)} > P_0^{(F+1)}$. Using (A15), we have

$$\frac{IR(F,S)}{IR(F+1,S)} = \frac{P_F}{P_{F+1}} = \left(\frac{S\mu}{\lambda} \right) \left(\frac{P_0^{(F)}}{P_0^{(F+1)}} \right) > 1,$$

because $S\mu > \lambda$ and $P_0^{(F)} > P_0^{(F+1)}$. Thus, $IR(F,S) > IR(F+1,S)$. Therefore, $FU(F,S) < FU(F+1,S)$. Q.E.D.

Proposition 11:

For a given F , the fractional utilization $FU(F,S)$ is increasing in S , for $S=1,2,\dots,F-1$. ■

Proof:

Let $P_0^{(n)}$ and $P_0^{(n+1)}$ be the values P_0 evaluated using (A17) with S equal to n and $n+1$, respectively, where $1 \leq S < F$. Then,

$$P_0^{(n)} = \left[A + \sum_{\ell=n+1}^F \frac{\lambda^\ell}{n! \mu^\ell n^{(\ell-n)}} \right]^{-1}, \text{ and}$$

$$P_0^{(n+1)} = \left[A + \sum_{\ell=n+1}^F \frac{\lambda^\ell}{n! \mu^\ell (n+1)^{(\ell-n)}} \right]^{-1},$$

where,

$$A = 1 + \sum_{\ell=1}^n \frac{\lambda^\ell}{\mu^\ell \ell!}.$$

Therefore,

$$\begin{aligned}
 \frac{IR(F,n)}{IR(F,n+1)} &= \frac{\frac{\lambda^F}{n! \mu^F n^{(F-n)}} P_0^{(n)}}{\frac{\lambda^F}{(n+1)! \mu^F (n+1)^{(F-n-1)}} P_0^{(n+1)}} \\
 &= \left(\frac{n+1}{n}\right)^{(F-n)} \left(\frac{P_0^{(n)}}{P_0^{(n+1)}}\right) \\
 &= \frac{A(n+1)^{(F-n)} + \sum_{\ell=n+1}^F \frac{\lambda^\ell (n+1)^{F-\ell}}{n! \mu^\ell}}{A(n)^{(F-n)} + \sum_{\ell=n+1}^F \frac{\lambda^\ell (n)^{F-\ell}}{n! \mu^\ell}} > 1,
 \end{aligned}$$

which implies $IR(F,n) > IR(F,n+1)$. Therefore, $FU(F,n) < FU(F,n+1)$.

Q. E. D.

Proposition 12:

The fractional utilization $FU(F,S)$ is concave in F for a given S . ■

Proof:

To prove that $FU(F,S)$ is concave in F for a given S , we need to show that $IR(F,S)$ is convex in F for a given S , which means

that $P_{F-1} + P_{F+1} - 2P_F$ must be greater than zero.

$P_{F-1} + P_{F+1} - 2P_F = K [P_0^{(F+1)} X^2 - 2P_0^{(F)} X + P_0^{(F-1)}]$, where

$K = \frac{S^5 \lambda^{F-1}}{S! \mu^{F-1} S^{F-1}}$, $X = \frac{\lambda}{\mu S}$, and $P_0^{(1)}$ is the value of P_0 evaluated

using (A17) with the upper limit on ℓ equal to 1.

Since $K > 0$ and $P_0^{(F+1)} > 0$, to show that $P_{F-1} + P_{F+1} - 2P_F > 0$ we need to show that the determinant of $P_0^{(F+1)} X^2 - 2P_0^{(F)} X + P_0^{(F-1)}$, Δ , is less than zero.

Let a_i be the $(i+1)$ th term in (A17), then $a_{F+1} = \frac{\lambda}{\mu S} a_F$, and

$$\begin{aligned} \Delta &= (P_0^{(F)})^2 - P_0^{(F+1)} P_0^{(F-1)} \\ &= \frac{a_F (P_0^{(F)})^3}{(1 + a_{F+1} P_0^{(F)}) (1 - a_F P_0^{(F)})} \left(-1 + \frac{\lambda}{\mu S} - \frac{\lambda}{\mu S} a_F P_0^{(F)} \right) < 0, \end{aligned}$$

since $\frac{\lambda}{\mu S} < 1$. Thus, $P_{F+1} + P_{F-1} - 2P_F > 0$ and $IR(F, S)$ is convex in F for a given S . Therefore, $FU(F, S)$ is concave in F for a given S . Q.E.D.

Let $H(F, S) = f(FU(F, S)) - h(S)$, then $H(F, S)$ is concave in F (same as the result we proved in Proposition 7). Define $H(F, S) = H(F, F)$ if $S > F$,

$$H(F, S_F) = \text{Max}_S H(F, S), \text{ and} \quad (\text{A18})$$

$$\text{Max}_F Z(F, S) = \text{Max}_{S, F} \{H(F, S) - g(F)\} \quad (\text{A19})$$

$$\text{then equation (A19) reduces to } \text{Max}_F [H(F, S_F) - g(F)] \quad (\text{A20})$$

Proposition 13:

The optimal solution (F^*, S_{F^*}) satisfies

$$H(F^*, S_{F^*}) - g(F^*) = \max_{1 \leq F \leq F_0} \{H(F, S_F) - g(F)\}$$

Where F_0 is the smallest integer that satisfies

$$H(F_0, S_{F_0}) / F_0 \leq g(F_0) - g(F_0 - 1) \quad (A21)$$

Proof:

For $F \leq F_0$, $H(F, S_F) - g(F) \leq H(F^*, S_{F^*}) - g(F^*)$. For $F > F_0$, we have.

$$\begin{aligned} & H(F, S_F) - H(F_0, S_{F_0}) \\ & \leq H(F, S_F) - H(F_0, S_F) \quad (\text{from the definition of } H(F, S_F)) \\ & \leq \frac{F - F_0}{F_0} H(F_0, S_F) \quad (\text{from Proposition 6-(ii), (iv) and} \\ & \quad \text{Proposition 7}) \\ & \leq \frac{F - F_0}{F_0} H(F_0, S_{F_0}) \\ & \leq (F - F_0)[g(F_0) - g(F_0 - 1)] \quad (\text{From (A21)}) \\ & \leq g(F) - g(F_0) \quad (\text{from Proposition 6-(iii)}) \end{aligned}$$

Hence, (F^*, S_{F^*}) is optimal.

Q.E.D.

2.3 A Solution Procedure and a Numerical Example

The solution procedure is as follows:

Step 1: $F^* \leftarrow 1$, $S_{F^*} \leftarrow 1$, $F \leftarrow 1$, $S_F \leftarrow 1$.

Step 2: Evaluate $Z(F^*, S_{F^*}) = \{H(F^*, S_{F^*}) - g(F^*)\}$, and compute F_0 .

Step 3: IF $F > F_0$, stop; otherwise go to Step 4.

Step 4: $F \leftarrow F+1$.

Step 5: Find S_F by evaluating $H(F,S)$ for $S=1,\dots,F$, and let

$$S_F = \text{Max} \left\{ S_F, \left\lceil \frac{\lambda}{\mu} \right\rceil \right\}.$$

Step 6: Evaluate $Z(F, S_F)$.

Step 7: IF $Z(F, S_F) \leq Z(F^*, S_F^*)$, go to Step 3; otherwise go to

Step 8.

Step 8: $F^* \leftarrow F$, $S_F^* \leftarrow S_F$, $Z(F^*, S_F^*) \leftarrow Z(F, S_F)$, and go to Step 3.

Suppose the failure rate of modules is 4, and we face the following problem (all functions are linear):

$$\text{Maximize } Z(F,S) = 9500FU(F,S) - 200F - 300S$$

Solving it by the above procedure, we obtain the optimal value of float, repair sever, and objective function as 3, 6, and 6874, respectively.

The results obtained for the above two cases are useful when the repair cost of the FMC dominates and the "reward" of the system depends almost entirely on the fractional utilization of the system. In practice, the fractional utilization of the system could also be affected by the breakdown of the rest of the system. Therefore, it is worth expanding our system to include the effects of the overall system breakdown on the fractional utilization of the system and the repair system. This issue will be discussed in the next two cases.

Case 3. The Single Repair Server Case with Consideration Of The Rest Of The System.

3.1 Queuing System Model

In this section, we consider the case where system idleness will occur if either there is no available float in the FMC or there are at least M units of components sent from the rest of the system to the repair shop. Assumptions 1 through 3 in Case 1 are also adopted here. Other relevant assumptions are listed below.

- 1) The failure rates of the module (in the FMC) and of the components (in the rest of the system) follow the Poisson probability distributions with parameters λ_1 and λ_2 , respectively.
- 2) There are two single-server repair stations (denoted as 1 and 2) in the repair shop. The repair rates at stations 1 and 2 follow the Poisson distribution with parameters μ_1 and μ_2 , respectively. Furthermore, $\mu_1 > \lambda_1$ and $\mu_2 > \lambda_2$.
- 3) First-come-first-serve discipline is adopted in both repair stations.
- 4) Modules will be repaired only at station 1 and the components will be repaired only at station 2.
- 5) The system will be idle when there are F units of modules in repair station 1, or there are at least M units of components in repair station 2

The system can now be modeled using a two-dimensional birth-and-death process with states (i, j) , where i and j represent the number of modules (sent from the FMC) and the number of failed components (sent from the rest of the system) in the repair shop respectively. We note here that although the system can also be modelled by two independent M/M/1 queues, we elect to use the two-dimensional birth-and-death process to describe the idle rate of the system more clearly. We first derive the transitional probabilities $P(i, j)$, for $i=0, 1, \dots, F$ and $j=0, 1, \dots, M$.

Based on the theory of the birth-and-death process, we obtain the following equations. For

$$\begin{aligned} \text{Node } (i, j) : & \lambda_1 P(i-1, j) + \lambda_2 P(i, j-1) + \mu_2 P(i, j+1) + \mu_1 P(i+1, j) \\ & = (\mu_1 + \mu_2 + \lambda_1 + \lambda_2) P(i, j), \quad i=1, \dots, F-1; j=1, \dots, M-1, \quad (\text{A22}) \end{aligned}$$

$$\text{Node } (0, 0) : \mu_2 P(0, 1) + \mu_1 P(1, 0) = (\lambda_1 + \lambda_2) P(0, 0), \quad (\text{A23})$$

$$\begin{aligned} \text{Node } (i, 0) : & \lambda_1 P(i-1, 0) + \mu_2 P(i, 1) + \mu_1 P(i+1, 0) \\ & = (\mu_1 + \lambda_1 + \lambda_2) P(i, 0), \quad i=1, \dots, F-1, \quad (\text{A24}) \end{aligned}$$

$$\begin{aligned} \text{Node } (0, j) : & \lambda_2 P(0, j-1) + \mu_2 P(0, j+1) + \mu_1 P(1, j) \\ & = (\mu_2 + \lambda_1 + \lambda_2) P(0, j), \quad j=1, \dots, M-1, \quad (\text{A25}) \end{aligned}$$

$$\text{Node } (F, 0) : \lambda_1 P(F-1, 0) + \mu_2 P(F, 1) = (\mu_1 + \lambda_2) P(F, 0), \quad (\text{A26})$$

$$\begin{aligned} \text{Node } (F, j) : & \lambda_1 P(F-1, j) + \lambda_2 P(F, j-1) + \mu_2 P(F, j+1) \\ & = (\mu_1 + \mu_2 + \lambda_2) P(F, j), \quad j=1, \dots, M-1, \quad (\text{A27}) \end{aligned}$$

$$\begin{aligned} \text{Node } (i, M) : & \lambda_1 P(i-1, M) + \lambda_2 P(i, M-1) + \mu_1 P(i+1, M) \\ & = (\mu_1 + \mu_2 + \lambda_1) P(i, M), \quad i=1, \dots, F-1, \quad (\text{A28}) \end{aligned}$$

$$\text{Node } (0, M): \lambda_2 P(0, M-1) + \mu_1 P(1, M) = (\mu_2 + \lambda_1) P(0, M), \text{ and} \quad (\text{A29})$$

$$\text{Node } (F, M): \lambda_1 P(F-1, M) + \lambda_2 P(F, M-1) = (\mu_1 + \mu_2) P(F, M). \quad (\text{A30})$$

Proposition 13:

$$P(i, j) = P(0, 0) \left(\frac{\lambda_1}{\mu_1} \right)^i \left(\frac{\lambda_2}{\mu_2} \right)^j, \quad i=0, 1, \dots, F; j=0, 1, \dots, M. \quad (\text{A31})$$

■

Proof:

Proposition 13 can be verified by substituting (A31) into (A22)-(A30) as follows.

For equation (22):

$$\begin{aligned} \text{Left-hand-side} &= \lambda_1 P(0, 0) \left(\frac{\lambda_1}{\mu_1} \right)^{i-1} \left(\frac{\lambda_2}{\mu_2} \right)^j \\ &+ \lambda_2 P(0, 0) \left(\frac{\lambda_1}{\mu_1} \right)^i \left(\frac{\lambda_2}{\mu_2} \right)^{j-1} \\ &+ \mu_2 P(0, 0) \left(\frac{\lambda_1}{\mu_1} \right)^i \left(\frac{\lambda_2}{\mu_2} \right)^{j+1} \\ &+ \mu_1 P(0, 0) \left(\frac{\lambda_1}{\mu_1} \right)^{i+1} \left(\frac{\lambda_2}{\mu_2} \right)^j \\ &= (\mu_1 + \mu_2 + \lambda_2 + \lambda_1) P(0, 0) \left(\frac{\lambda_1}{\mu_1} \right)^i \left(\frac{\lambda_2}{\mu_2} \right)^j \\ &= \text{Right-hand-side.} \end{aligned}$$

We can verify equation (A23) through equation (A30) in a similar way. Q.E.D.

Since the transitional probabilities should sum up to 1, we obtain $P(0, 0)$ as follows.

$$\begin{aligned}
\sum_{i=0}^F \sum_{j=0}^M P(i,j) &= 1 \\
\Rightarrow \sum_{i=0}^F \sum_{j=0}^M P(0,0) (\lambda_1/\mu_1)^i (\lambda_2/\mu_2)^j &= 1 \\
\Rightarrow P(0,0) \left[\left(\frac{1-\rho_1^{F+1}}{1-\rho_1} \right) \left(\frac{1-\rho_2^{M+1}}{1-\rho_2} \right) \right] &= 1 \\
\Rightarrow P(0,0) &= \frac{(1-\rho_1)(1-\rho_2)}{(1-\rho_1^{F+1})(1-\rho_2^{M+1})} \quad (A32) \\
\text{where } \rho_1 &= \frac{\lambda_1}{\mu_1}, \quad \rho_2 = \frac{\lambda_2}{\mu_2}.
\end{aligned}$$

Based on these derivations, the idle rate of the system can be expressed as:

$$\begin{aligned}
\text{IR}(F, \mu_1, \mu_2) &= \text{the idle rate of the system} \\
= \sum_{j=0}^M P(F, j) + \sum_{i=0}^{F-1} P(i, M) &= \frac{\rho_1^F (1-\rho_1) + \rho_2^M (1-\rho_2) - \rho_1^F \rho_2^M (1-\rho_1 \rho_2)}{(1-\rho_1^{F+1})(1-\rho_2^{M+1})} \quad (A33)
\end{aligned}$$

and the fractional utilization of the system is, thus,

$$\text{FU}(F, \mu_1, \mu_2) = 1 - \text{IR}(F, \mu_1, \mu_2) \quad (A34)$$

3.2. A Profit-Maximization Model for Determining the Best Maintenance Float Policy

Suppose we adopt the cost function $g(F)$ (as defined in Section 2.2) and define the following notations:

$h_1(\mu_1)$: a convex cost function for maintaining a repair rate of μ_1
for the modules,

$h_2(\mu_2)$: a convex cost function for maintaining a repair rate of μ_2
for the components,

$f(FU(F, \mu_1, \mu_2))$: an increasing and concave revenue function.

The maintenance policy can then be determined by solving the
following mathematical program:

$$\text{Maximize } Z(F, \mu) = f(FU(F, \mu)) - g(F) - h_1(\mu_1) - h_2(\mu_2) \quad (A35)$$

Subject to:

$$F \geq 1 \text{ integer}, \quad (A36)$$

$$\mu_1 > \lambda_1, \quad (A37a)$$

$$\mu_2 > \lambda_2, \quad (A37b)$$

where $\mu = [\mu_1, \mu_2]$.

In order to solve the above mathematical program, we propose
the following Propositions.

Proposition 14:

The fractional utilization $FU(F, \mu)$ is concave in F for a
given μ . ■

Proof:

Let $W(F) = IR(F, \mu)$. By rewriting equation (A33), we obtain

$$\begin{aligned}
W(F) &= \frac{(1-\rho_1)\rho_1^F}{1-\rho_1^{F+1}} + \frac{(1-\rho_2)\rho_2^M}{1-\rho_2^{M+1}} - \frac{\rho_1^F \rho_2^M (1-\rho_1)(1-\rho_2)}{(1-\rho_1^{F+1})(1-\rho_2^{M+1})} \\
&= A + B \frac{\rho_1^F}{1-\rho_1^{F+1}} \quad (A38) \\
\text{where } A &= \frac{(1-\rho_2)\rho_2^M}{1-\rho_2^{M+1}}, \text{ and } B = 1-\rho_1 - \frac{\rho_2^M(1-\rho_1)(1-\rho_2)}{1-\rho_2^{M+1}} > 0.
\end{aligned}$$

Thus,

$$\begin{aligned}
W(F+1) + W(F-1) - 2W(F) &= B \left(\frac{\rho_1^{F+1}}{1-\rho_1^{F+2}} + \frac{\rho_1^{F-1}}{1-\rho_1^F} - \frac{2\rho_1^F}{1-\rho_1^{F+1}} \right) \\
&= B \rho_1^{F-1} q(F), \text{ where } q(F) = \frac{\rho_1^2}{1-\rho_1^{F+2}} + \frac{1}{1-\rho_1^F} - \frac{2\rho_1}{1-\rho_1^{F+1}}.
\end{aligned}$$

Now, since

$$\begin{aligned}
(1-\rho_1^F)(1-\rho_1^{F+1})(1-\rho_1^{F+2})q(F) &= \rho_1^2(1-\rho_1^F-\rho_1^{F+1}+\rho_1^{2F+1}) + (1-\rho_1^{F+1}-\rho_1^{F+2}+\rho_1^{2F+3}) \\
&\quad - 2\rho_1(1-\rho_1^F-\rho_1^{F+2}+\rho_1^{2F+2}) \\
&= 1+\rho_1^2-2\rho_1^{F+2}+\rho_1^{F+3}-2\rho_1+\rho_1^{F+1} \\
&= (1-\rho_1)^2 + \rho_1^{F+1}(1-\rho_1)^2 \\
&= (1-\rho_1)^2(1+\rho_1^{F+1}) > 0,
\end{aligned}$$

$IR(F, \mu)$ is convex in F , and hence, $FU(F, \mu)$ is concave in F for a given μ . Q.E.D.

Proposition 15:

The fractional utilization $FU(F, \mu)$ is increasing in F for a given μ . ■

Proof:

From equation (A38) we have,

$$IR(F+1, \mu_1, \mu_2) - IR(F, \mu_1, \mu_2) = B \left(\frac{\rho_1^{F+1}(1-\rho_1^{F+1}) - \rho_1^F(1-\rho_1^{F+2})}{(1-\rho_1^{F+2})(1-\rho_1^{F+1})} \right), \text{ and}$$

the numerator of the above equation is less than 0. Thus, IR is decreasing with respect to F , and FU is increasing in F for a given μ . Q.E.D.

Proposition 16:

The fractional utilization $FU(F, \mu)$ is increasing in μ_1 for a given F and a given μ_2 . ■

Proof:

$$\text{Let } IR(\rho_1) = \frac{1}{1-\rho_2^{N+1}} \left[\frac{\rho_1^F(1-\rho_1) + \rho_2^N(1-\rho_2) - \rho_1^F \rho_2^N(1-\rho_1 \rho_2)}{1-\rho_1^{F+1}} \right].$$

We will prove Proposition 16 by showing that $IR(\rho_1)$ increases with ρ_1 .

$$\begin{aligned} \frac{d}{d\rho_1} IR(\rho_1) &= \frac{(1-\rho_2^N) \rho_1^{F-1} [F - \rho_1 - F\rho_1 + \rho_1^{F+1}]}{(1-\rho_2^{N+1}) (1-\rho_1^{F+1})^2} \\ &= \frac{(1-\rho_2^N) \rho_1^{F-1} (1-\rho_1)}{(1-\rho_2^{N+1}) (1-\rho_1^{F+1})^2} \left[F - \frac{\rho_1 - \rho_1^{F+1}}{1-\rho_1} \right]. \end{aligned}$$

$$\text{Since } \frac{\rho_1 - \rho_1^{F+1}}{1-\rho_1} = \sum_{i=1}^F \rho_1^i, \text{ and } \rho_1 < 1 \text{ implies } \rho_1^i < 1 \text{ for } i=1, \dots, F.$$

It is, therefore, easy to see that

$$\frac{\rho_1 - \rho_1^{F+1}}{1 - \rho_1} = \sum_{i=1}^F \rho_1^i < \sum_{i=1}^F (1) = F. \text{ Thus, } \frac{d}{d\rho_1} IR(\rho_1) > 0.$$

That is, $IR(\rho_1)$ increases with ρ_1 and decreases with μ_1 . Hence, $FU(F, \mu)$ is increasing in μ_1 , for a given F and a given μ_2 . Q.E.D.

By symmetry, we have the following Proposition.

Proposition 17:

The fractional utilization $FU(F, \mu)$ is increasing in μ_2 for a given F and a given μ_1 .

3.3 A Heuristical Solution Procedure and A Numerical Example

Following the definitions of the H function and μ_F and replacing μ by μ in Case 1, we obtain the same conclusions that we derived in Proposition 8. Thus, the solution procedure here is similar to that for Case 1. The only thing that needs to be changed is the procedure to determine the optimal value of μ . Only the properties of Propositions 16 and 17 and the upper bound on the fractional utilization are used here. The solution procedure is as follows:

Step 0: $F \leftarrow 1$, $H^*(F, \mu_F) \leftarrow -\infty$, $\mu_F^* \leftarrow 0$, $F^* \leftarrow 1$.

Step 1: Solve for μ_F and evaluate $H(F, \mu_F)$. The solution procedure for μ_F is as follows:

(a) $\mu_1 \leftarrow \lambda_1 + \Delta\mu_1$.

- (b) $\mu_2 \leftarrow \lambda_2 + \Delta\mu_2$, $H'(F, \underline{\mu}_F) \leftarrow -\infty$, $\underline{\mu}_F = (\mu_1, \mu_2)$.
- (c) Evaluate the objective function $H(F, \underline{\mu}_F)$.
- (d) If $H(F, \underline{\mu}_F) > H'(F, \underline{\mu}_F)$, then $H'(F, \underline{\mu}_F) \leftarrow H(F, \underline{\mu}_F)$, $\underline{\mu}_F' = \underline{\mu}_F$, go to (e); otherwise, go to (e).
- (e) If $|\text{new FU}(F, \underline{\mu}_F) - \text{old FU}(F, \underline{\mu}_F)| < \epsilon$ (ϵ is a very small number), go to (g).
- (f) $\mu_2 \leftarrow \mu_2 + \Delta\mu_2$, go to (c).
- (g) $\mu_1 \leftarrow \mu_1 + \Delta\mu_1$.
- (h) If $|\text{new FU}(F, \underline{\mu}_F) - \text{old FU}(F, \underline{\mu}_F)| < \epsilon$, stop; otherwise, go to (b).

Step 2: If $\text{new } H(F, \underline{\mu}_F) > \text{old } H(F, \underline{\mu}_F)$, then $H^*(F, \underline{\mu}_F) = \text{new } H(F, \underline{\mu}_F)$, $\underline{\mu}_F^* \leftarrow \text{new } \underline{\mu}_F$, $F^* \leftarrow \text{new } F$, go to Step 3; otherwise, go to Step 3.

Step 3: If F satisfies the optimal condition described in Proposition 8, then stop; otherwise, $F \leftarrow F+1$, go to Step 1.

Suppose we face the following problem: Maximize $Z(F, \underline{\mu}) = 15000\text{FU}(F, \underline{\mu}) - 300F - 450\mu_1 - 150\mu_2$, and assume that the repair rates at repair stations 1 and 2 are greater than the respective failure rates. Using the above approach we obtain the optimal solution: $F^* = 5$, $\mu_1^* = 3.3$, $\mu_2^* = 10.4$, $Z^* = 9733.592$. In this case, we let $\Delta\mu_1 = \Delta\mu_2 = 0.1$.

In real life, the function of the modules could be more important than that of the components. Therefore, it is essential

that the modules are repaired in a timely fashion. For this reason, in the next model, we consider the case where the idle capacity of the repair station, which is normally used to repair the components, is used to repair the modules in order to minimize the waiting time of the modules.

Case 4. Two Stations with Single Server, Shift Property, and Preemptive Repeat-identical Priority.

4.1 Queueing System Model

Suppose again that there are two single-server stations, denoted as Stations 1 and 2. Station 1 repairs the modules, and Station 2 can repair the components and the modules. The queue disciplines are as follows:

- (1) Modules wait at Station 1 but when Station 2 is empty, the first module in the queue advances to Station 2 to be repaired.
- (2) Components are repaired at Station 2 on a first-come-first-serve basis.
- (3) At Station 2, the service priority of the components is higher than that of the modules. The priority discipline is preemptive and repeat-identical.

The other assumptions and the required notations are as following:

- (1) The repair rate of the server at Station 1 follows a

Poisson distribution with parameter μ_1 :

- (2) The repair rates of the server at Station 2 for the modules and components follow Poisson distributions with parameters μ_1 and μ_2 , respectively. Note that the same repair rate for the modules is assumed at both Stations;
- (3) The failure rates of modules and components follow Poisson distributions with parameters λ_1 and λ_2 , respectively;
- (4) There are F modules in the FMC, one is used in operation, the others ($F-1$ units) are standbys; and
- (5) The system breaks down when there are F modules or M failed components at the repair stations.

Based on a two-dimensional birth-and-death process we obtain the following equations:

$$\text{Node } (0,0): (\lambda_1 + \lambda_2)P(0,0) = \mu_1 P(1,0) + \mu_2 P(0,1), \quad (\text{A39})$$

$$\text{Node } (1,0): (\lambda_1 + \mu_1 + \lambda_2)P(1,0) = \lambda_1 P(0,0) + \mu_2 P(1,1) + 2\mu_1 P(2,0), \quad (\text{A40})$$

$$\begin{aligned} \text{Node } (i,0): (2\mu_1 + \lambda_1 + \lambda_2)P(i,0) &= \lambda_1 P(i-1,0) + \mu_2 P(i,1) + 2\mu_1 P(i+1,0) \\ i &= 2, 3, \dots, F-1, \end{aligned} \quad (\text{A41})$$

$$\text{Node } (F,0): (2\mu_1 + \lambda_2)P(F,0) = \lambda_1 P(F-1,0) + \mu_2 P(F,1), \quad (\text{A42})$$

$$\begin{aligned} \text{Node } (0,j): (\mu_2 + \lambda_1 + \lambda_2)P(0,j) \\ = \lambda_2 P(0,j-1) + \mu_2 P(0,j+1) + \mu_1 P(1,j), \end{aligned} \quad (\text{A43})$$

$$j = 1, 2, \dots, M-1,$$

$$\begin{aligned}
\text{Node } (F, j): & (\mu_1 + \lambda_2 + \lambda_2)P(F, j) \\
& = \lambda_2 P(F, j-1) + \lambda_1 P(F-1, j) + \mu_2 P(F, j+1), \quad (A44) \\
& j=1, 2, \dots, M-1,
\end{aligned}$$

$$\begin{aligned}
\text{Node } (1, j): & (\mu_1 + \lambda_2 + \lambda_1 + \mu_2)P(1, j) \\
& = \lambda_1 P(1-1, j) + \mu_2 P(1, j+1) + \mu_1 P(1+1, j) + \lambda_2 P(1, j-1), \quad (A45) \\
& i=1, 2, \dots, F-1, \quad j=1, 2, \dots, M-1,
\end{aligned}$$

$$\text{Node } (0, M): (\mu_2 + \lambda_1)P(0, M) = \lambda_2 P(0, M-1) + \mu_1 P(1-M), \quad (A46)$$

$$\begin{aligned}
\text{Node } (1, M): & (\mu_1 + \mu_2 + \lambda_1)P(1, M) = \lambda_1 P(1-1, M) + \lambda_2 P(1, M-1) + \mu_1 P(1+1, M), \\
& i=1, 2, \dots, F-1, \text{ and} \quad (A47)
\end{aligned}$$

$$\text{Node } (F, M): (\mu_1 + \mu_2)P(F, M) = \lambda_1 P(F-1, M) + \lambda_2 P(F, M-1). \quad (A48)$$

Proposition 18:

The steady state probability can be expressed by $P(1, j) = \left(\frac{\lambda_1}{\mu_1}\right)^1 \left(\frac{\lambda_2}{\mu_2}\right)^j P(0, 0)$ when $(1, j) \neq (2, 0), (3, 0), (4, 0), \dots, (F-1, 0)$.

Proof:

For (A48)

$$\begin{aligned}
\text{RHS} &= \lambda_1 \left(\frac{\lambda_1}{\mu_1}\right)^{F-1} \left(\frac{\lambda_2}{\mu_2}\right)^M + \lambda_2 \left(\frac{\lambda_1}{\mu_1}\right)^F \left(\frac{\lambda_2}{\mu_2}\right)^{M-1} \\
&= \mu_1 P(F, M) + \mu_2 P(F, M) = \text{LHS}.
\end{aligned}$$

Similarly, we can verify that equations (A39), (A40), (A42)-(A47) are all satisfied by the solution

$$P(1, j) = \left(\frac{\lambda_1}{\mu_1}\right)^1 \left(\frac{\lambda_2}{\mu_2}\right)^j P(0, 0). \quad \text{Q.E.D.}$$

Proposition 19:

The steady state probability of states $(2,0)$, $(3,0)$, ..., and $(F-1,0)$ can be expressed by

$$P(n,0) = \left[\frac{(2\beta - \rho_1 - r)}{4(\beta - \alpha)(\rho_1 - \alpha)} \alpha^{n-1} \rho_1^2 + \frac{2\alpha - \rho_1 - r}{4(\beta - \alpha)(\beta - \rho_1)} \beta^{n-1} \rho_1^2 + \frac{r}{2(\rho_1 - \alpha)(\rho_1 - \beta)} \right]$$

$$\rho_1^{n+1} P(0,0), \quad n=2,3,\dots,F-1,$$

$$\text{where } \rho_1 = \frac{\lambda_1}{\mu_1}, \quad r = \frac{\lambda_2}{\mu_1}, \quad \text{and}$$

$$\alpha, \beta = \frac{2+r+\rho_1 \pm \sqrt{(2+r+\rho_1)^2 - 8\rho_1}}{4}.$$

■

Proof:

Let $P(i-1,0) = f_n P(0,0)$. The equation for node $(1,0)$ can then be expressed by

$$2f_{n+2} - (2+r+\rho_1)f_{n+1} + \rho_1 f_n = -\rho_1^{n+1} r.$$

Recall the shift operator E : $Ef_n = f_{n+1}$. Denote

$$Q(E) = 2E^2 - (2+r+\rho_1)E + \rho_1 = 2(E-\alpha)(E-\beta),$$

where α and β are the two roots of the polynomial $Q(E)$,

$$\alpha, \beta = \frac{2+r+\rho_1 \pm \sqrt{(2+r+\rho_1)^2 - 8\rho_1}}{4}.$$

Then the equation of the node $(1,0)$ becomes

$$Q(E)f_n = -\rho_1^{n+1} r.$$

Since $(E-\rho)(-r\rho^{n+1}) = -r(\rho^{n+2} - \rho^{n+2}) = 0$, and $Q(E)f_n = -\rho_1^{n+1} r$

implies

$$(E-\alpha)(E-\beta)(E-\rho_1)f_n=0,$$

we have $f_n = a\alpha^n + b\beta^n + c\rho_1^n$ (see, Jordan, 1950, pp.541-549).

Let $n=1,2,3$, we obtain following relationships,

$$\begin{bmatrix} \alpha & \beta & \rho_1 \\ \alpha^2 & \beta^2 & \rho_1^2 \\ \alpha^3 & \beta^3 & \rho_1^3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} \rho_1 \\ \rho_1^2/2 \\ \rho_1^3/4 - \rho_1^2 r/4 \end{bmatrix}$$

Since

$$\begin{bmatrix} \beta\rho_1 - (\beta+\rho_1) & 1 \\ \alpha\rho_1 - (\alpha+\rho_1) & 1 \\ \alpha\beta - (\alpha+\beta) & 1 \end{bmatrix} \begin{bmatrix} \alpha & \beta & \rho_1 \\ \alpha^2 & \beta^2 & \rho_1^2 \\ \alpha^3 & \beta^3 & \rho_1^3 \end{bmatrix} = \begin{bmatrix} \alpha(\alpha-\beta)(\alpha-\rho_1) & 0 & 0 \\ 0 & \beta(\beta-\alpha)(\beta-\rho_1) & 0 \\ 0 & 0 & \rho_1(\rho_1-\alpha)(\rho_1-\beta) \end{bmatrix}$$

we obtain

$$\alpha(\alpha-\beta)(\alpha-\rho_1)a = \beta\rho_1^2 - (\beta+\rho_1)\rho_1^2/2 + \rho_1^3/4 - \rho_1^2 r/4,$$

$$\beta(\beta-\alpha)(\beta-\rho_1)b = \alpha\rho_1^2 - (\alpha+\rho_1)\rho_1^2/2 + \rho_1^3/4 - \rho_1^2 r/4,$$

$$\rho_1(\rho_1-\alpha)(\rho_1-\beta)c = \alpha\beta\rho_1 - (\alpha+\beta)\rho_1^2/2 + \rho_1^3/4 - \rho_1^2 r/4.$$

Thus,

$$a\alpha^n = \frac{2\beta - \rho_1 - r}{4(\beta-\alpha)(\rho_1-\alpha)} \alpha^{n-1} \rho_1^2,$$

$$b\beta^n = \frac{2\alpha - \rho_1 - r}{4(\beta-\alpha)(\beta-\rho_1)} \beta^{n-1} \rho_1^2,$$

$$c\rho_1^n = \frac{r}{2(\rho_1-\alpha)(\rho_1-\beta)} \rho_1^{n+1}.$$

Q. E. D.

According to Proposition 19 and equation (A42), we can express $P(F,0)$ as follows:

$$P(F,0) = \frac{1}{2\mu_1 + \lambda_2} (\lambda_1 f_{F-1} + \mu_1 \lambda_1^F \lambda_2) P(0,0).$$

The fractional utilization can be then expressed as

$$\begin{aligned} FU(F, \rho_1, \rho_2) &= 1 - \left[\sum_{j=1}^M \rho_1^F \rho_2^j P(0,0) + \sum_{i=0}^{F-1} \rho_1^i \rho_2^M P(0,0) \right. \\ &\quad \left. + \left(\frac{1}{2\mu_1 + \lambda_2} \right) (\lambda_1 f_{F-1} + \mu_1 \rho_1^F \rho_2) P(0,0) \right] \\ &= 1 - P(0,0) \left[\frac{\rho_1^F \rho_2 (1 - \rho_2^M)}{1 - \rho_2} + \frac{(1 - \rho_1^F) \rho_2^M}{1 - \rho_1} \right. \\ &\quad \left. + \frac{1}{2\mu_1 + \lambda_2} (\lambda_1 f_{F-1} + \mu_1 \lambda_1^F \lambda_2) \right], \text{ where} \end{aligned}$$

$$\begin{aligned} P(0,0) &= \left[\sum_{i=0}^F \sum_{j=1}^M \rho_1^i \rho_2^j + 1 + \rho_1 + \sum_{n=2}^{F-1} \left(\frac{(2\beta - \rho_1 - r)}{4(\beta - \alpha)(\rho_1 - \alpha)} \alpha^{n-1} \rho_1^2 \right. \right. \\ &\quad \left. \left. + \frac{2\alpha - \rho_1 - r}{4(\beta - \alpha)(\beta - \rho_1)} \beta^{n-1} \rho_1^2 + \frac{r}{2(\rho_1 - \alpha)(\beta - \rho_1)} \rho_1^{n+1} \right) \right. \\ &\quad \left. + \frac{1}{2\mu_1 + \lambda_2} (\lambda_1 f_{F-1} + \mu_1 \rho_1^F \rho_2) \right] \end{aligned}$$

$$\begin{aligned} \text{where } r &= \frac{\lambda_2}{\mu_1}, \text{ and} \\ \alpha, \beta &= \frac{2 + r + \rho_1 \pm \sqrt{(2 + \rho_1 + r)^2 - 8\rho_1}}{4}. \end{aligned}$$

4.2 A Numerical Example

In this model, properties of utilization rate would be used to solve the profit-maximization problems which we proposed in Sections 1,2, and 3. We evaluate the value of $Z(F, \mu)$ as

$$Z(F, \mu) = 15000FU(F, \mu) - 300F - 450\mu_1 - 150\mu_2.$$

Where $FU(F, \mu)$ is the fractional utilization of the system with F units float and repair rates μ_1 and μ_2 in repair stations one and two, respectively.

The enumerative search method is adopted to solve this problem. We also set the ranges of μ_1 , μ_2 , and F be (2.1, 3.1), (4.1, 11.0), and (1, 5), respectively, and the increments for μ_1 and μ_2 are both set at 0.1. Under these settings, the optimal solution for the decision variables were $\mu_1=2.4$, $\mu_2=10.1$, $F=2$, and the $Z(F, \mu)=11646.09$ ($FU=0.989406$).

5. Conclusions

In the above analytical approaches, we found that it is possible to derive the solution procedures for the optimal maintenance float and repair capacities for some simplified cases (see Cases 1 and 2). However, the derivations become intractable analytically for more complex cases. For example, in Case 3, we can not guarantee that the solution obtained by the procedure is optimal. Furthermore, for Case 4, no solution procedure can be derived. Therefore, we proposed an approximate approach to simplify the solution procedure in Chapter 4.

APPENDIX B: DEFINITIONS OF NOTATIONS IN CHAPTER 3

AOQ_{-1} : the average outgoing quality of the immediately preceding cell, i.e., the defective percentage of the parts entering the current cell.

$(AOQ)_k^{v_k(\ell)}$: the average outgoing quality of type-k parts from their ℓ th visitation cell.

ca_j : the square coefficient of variation of the arrival rates at cell j .

CF_k : the post-sales failure cost of a type-k part (as defined in Lee and Tapiero 1986).

$CI_{k\ell}$: the cost of inspecting a type-k part in its ℓ th visitation cell.

$CP_{k\ell}$: the cost of processing a type-k part in its ℓ th visitation cell.

$CR_{k\ell}$: the cost of reworking a type-k part in its ℓ th visitation cell.

$CS_{k\ell}$: the cost of scrapping a type-k part in its ℓ th visitation cell, with $CS_{k\ell} < CS_{k,\ell+1}$, $\ell = 1, 2, \dots, z_k - 1$.

cs_j : the squared coefficient of variation of the processing time at cell j .

CW_k : the waiting cost of a type-k part per unit time.

D : the defective percentage of parts, including the defects that are made in the current cell and in the preceding cells ($0 \leq D \leq 1$) before inspection.

$(D_k)_{v_k(l)}$: the probability that a type-k part is defective after processing and prior to inspection in its l th visitation cell.

$(ED_k)_{v_k(l)}$: the expected number of undetected defective type-k parts exiting from the l th visitation cell per unit time.

$(EF_k)_{v_k(l)}$: the expected flow of type-k parts exiting from the l th visitation cell, (and entering the next cell in the visitation sequence) per unit time.

$(ER_k)_{v_k(l)}$: the expected number of type-k parts that will be reworked in the l th visitation cell per unit time.

$(ES_k)_{v_k(l)}$: the expected number of type-k parts that will be scrapped in the l th visitation cell per unit time.

$K(N)$: the p.d.f. of N .

M : the number of manufacturing cells in the FMS.

N : the number of times a part passes through the working area.

q : probability that the work in the current cell is defective, $0 \leq q \leq 1$.

$q_{k,v_k(l)}$: the probability of manufacturing a defective type-k part in the $V_k(l)$ visitation cell.

R : the number of part types to be processed by the FMS.

r : the inspection percentage at the manufacturing cell, $0 \leq r \leq 1$.

REV_k : the revenue for a type-k part.

r_{kj} : the inspection rate for type-k parts in cell j .

$r_{k,v_k(\ell)}$: the inspection percentage for type-k parts in the $v_k(\ell)$ visitation cell.

T : the inspection time (a constant).

TR_{ij} : the transition probability of an aggregate part (i.e., disregarding part types) from cell i to cell j .

V : the variance of the working time.

$V_k = [V_k(1), V_k(2), \dots, V_k(z_k)]$, the prespecified visitation sequence of type-k parts, where $V_k(i)$ denotes the cell visited by type-k parts at their i th processing step.

$V_{N,v_k(\ell)}$: the variance of the working times of type-k parts in their ℓ th visitation to a cell.

V_N : the variance of N .

$V_{R,v_k(\ell)}$: the variance of the reworking times of type-k parts in their ℓ th visitation cell.

V_θ : the variance of θ .

$V_{v_k(\ell)}^\theta$: the variance of the "total" processing time for a type-k part in its ℓ th visitation cell.

$W_{k,v_k(\ell)}$: the expected total time a type-k part spends in its ℓ th visitation.

WE_k : the expected total time a type-k part spends in the FMS.

θ : the actual total time (i.e., working time, reworking time and inspection time) a part spends in the cell.

λ : total arrival rate of the system.

λ_k : the arrival rate of type-k parts.

λ_{01} : the aggregate arrival rate of all types of parts entering into cell 1.

λ_{ij} : the aggregate arrival rate of parts to cell j from cell i.

λ_{i0} : the aggregate completion rate of parts exiting the FMS from cell i.

τ : the mean of the working time.

τ_N : the mean of N.

τ_θ : the mean of θ .

$\tau_{N,v_k(\ell)}$: the means of the working times of type-k parts in their ℓ th visitation cell.

$\tau_{R,v_k(\ell)}$: the means of the reworking times of type-k parts in their ℓ th visitation cell.

$\tau_{v_k(\ell)}^\theta$: the mean of the "total" processing time for a type-k part in its ℓ th visitation cell.

ρ_j : the traffic intensity at cell j, $0 \leq \rho_j < 1$.

APPENDIX C: TWO COMPLEX EXAMPLES OF QUALITY CONTROL MODEL

The first example we considered is an FMS with five cells and three types of parts. The visitation sequences for part types 1, 2, and 3 are given by $V_1=(1, 2, 3, 5)$, $V_2=(1, 3, 5, 4)$, and $V_3=(1, 2, 4)$, respectively (see Figure A-1). Other relevant data are given in Table A-1. Using an IBM PC-486 compatible to solve this problem, the computing time is about 210 minutes for step 1 and 10 minutes for step 2 by the heuristic search method. The optimal solution of the inspection rates are 0, 1, 0, 0, and 0 for cell 1, 2, 3, 4, and 5 respectively. The objective value is 5548.838.

Example 2 is another FMS with five cells and two types of parts. The visitation sequences for part types 1 and 2 are given by $V_1=(1, 2, 4, 5)$ and $V_2=(1, 2, 3, 4)$ respectively (see Figure A-2). Other relevant data are given in Table A-2. Using an IBM PC-486 compatible to solve this problem, the computing time is about 190 minutes for step 1 and 17 minutes for step 2 by the heuristic search method. The optimal solution of inspection rates are 0, 0, .8, 0, and 0 for cell 1, 2, 3, 4, and 5 respectively. The objective value is 6073.556.

In these two numerical examples, stations 2 and 3 seems to be the critical stations for example 1 and 2, respectively, according to the data of cost and time we show in Table A-1 and A-2. However, the optimal inspection rate in example 1 is (0,0,0,0.9,0) when $CF_1=340$, $CF_2=280$, and $CF_3=390$. It is also found that the optimal inspection rate at each station of example 1 is equal to 1

in the following two cases: 1) $CF_1=340$, $CF_2=280$, and $CF_3=390$; 11) $CF_1=340$, $CF_2=280$, and $CF_3=390$.

These two examples showed that an increase in the number of stations or part-types will lead to an increase in computing time. It is also found that using the heuristic search method to solve them spent too much computing time. Thus, we modify the value of the increment of the first step of this heuristic method to 0.2. Then the computing time of step 1 of this new method is 11 minutes and 9 minutes for example 1 and 2 respectively. The optimal solution obtained by the new method is the same as the one obtained from the original method. The new algorithm cannot take care of the cases when the optimal inspection rate is not a multiple of 0.2. Therefore, the first step of the new algorithm should be modified. The modified procedure is illustrated as follows:

Step 1-a:

- (a) Let initial value and incremental value of inspection rate be 0 and 0.2, respectively.
- (b) Compute the objective value until inspection rate equals 1.
- (c) Find the optimal objective value.

Step 1-b:

- (a) Let initial value and incremental value of inspection rate be 0.1 and 0.2, respectively.
- (b) Compute the objective value until inspection rate equals 0.9.

(c) Find the optimal objective value.

Step 1-c:

Compare the results of Step 1-a and Step 1-b and choose the better one as a basis in Step 2 of the old algorithm.

Table A-1 Data of numerical example 1

	product 1				product 2				product 3		
station	1	2	3	5	1	3	5	4	1	2	4
T_M	0.01	0.005	0.01	0.01	0.015	0.006	0.005	0.01	0.01	0.003	0.007
V_M	0.005	0.006	0.006	0.005	0.008	0.009	0.006	0.007	0.0008	0.0005	0.0008
T_R	0.005	0.001	0.009	0.004	0.005	0.008	0.004	0.002	0.002	0.001	0.002
V_R	0.005	0.001	0.004	0.001	0.006	0.001	0.0004	0.0002	0.0005	0.0003	0.0008
q	0.1	0.12	0.06	0.05	0.09	0.1	0.06	0.09	0.01	0.05	0.06
CI	16	16	15	14	10	8	10	9	12	10	11
CS	-	30	50	70	-	35	50	70	-	15	16
CR	15	15	10	12	10	12	18	20	25	20	18
CP	12	14	12	16	12	15	14	15	16	18	20
T	0.0005	0.0006	0.0003	0.0004	0.0005	0.0003	0.0004	0.0005	0.0005	0.0006	0.0005
λ	20				30				20		
CF	220				230				450		
REV	280				200				350		
CW	1.5				2				2.5		

Table A-2 Data of numerical example 2

station	product 1				product 2			
	1	2	4	5	1	2	3	4
T_M	0.01	0.005	0.01	0.01	0.015	0.006	0.005	0.01
V_M	0.005	0.006	0.006	0.005	0.008	0.009	0.006	0.007
T_R	0.005	0.001	0.009	0.004	0.005	0.008	0.002	0.002
V_R	0.005	0.001	0.004	0.0001	0.006	0.001	0.0004	0.0002
q	0.1	0.18	0.06	0.05	0.09	0.15	0.06	0.09
CI	16	16	15	14	10	8	10	9
CS	-	50	100	125	-	50	80	100
OR	15	15	30	12	20	22	28	30
CP	12	14	12	16	12	15	14	15
T	0.0004	0.0005	0.0004	0.0004	0.0005	0.0003	0.0004	0.0005
λ	20				30			
OF	350				320			
REV	280				200			
CW	1.5				2			

Figure A-1 Example 1

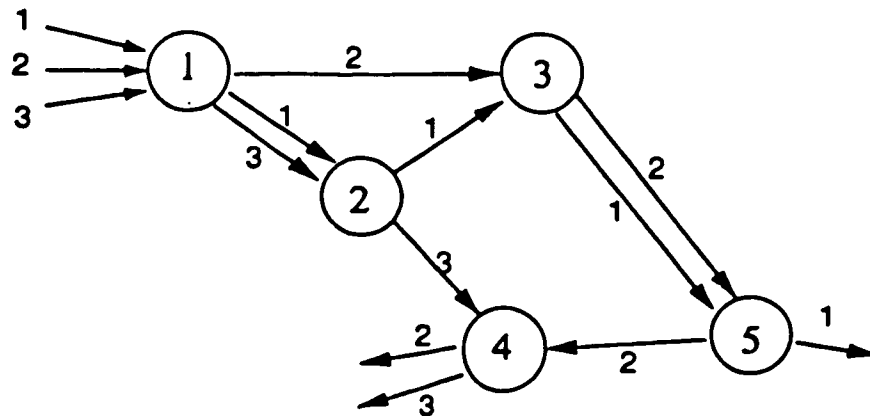
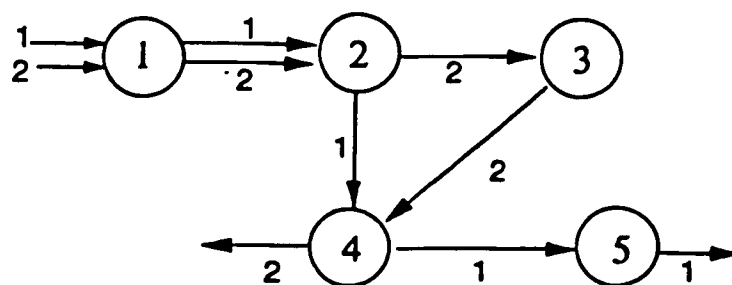


Figure A-2 Example 2



APPENDIX D: COMPUTER CODE OF EXAMPLES 1 & 2

```

1 REM Computer Code of Example 1
5 CLS
9 REM Input Process Time
10 T (1,1) = .0005: T (1,2) = 6.000001E-04: T (1,3) = .0003: T (1,4) = .0004: T
(2,1) = .0005: T (2,2) = .0003: T (2,3) = .0004: T (2,4) = .0005: T (3,1) = .000
5: T (3,2) = 6.000001E-04: T (3,3) = .0005
20 UM(1,1) = .01: UM(1,2) = .005: UM(1,3) = .01: UM(1,4) = .01: UM(2,1) = .015: U
M(2,2) = .006: UM(2,3) = .005: UM(2,4) = .01: UM(3,1) = .01: UM(3,2) = .003: UM(
3,3) = .007
30 VM(1,1) = .005: VM(1,2) = .006: VM(1,3) = .006: VM(1,4) = .005: VM(2,1) = 8.000001E-03
40 VM(2,2) = 8.999999E-03: VM(2,3) = .006: VM(2,4) = .007: VM(3,1) = .0008: VM(3,2) = .0005:
VM(3,3) = .0008
50 UR(1,1) = .005: UR(1,2) = .001: UR(1,3) = 8.999999E-03: UR(1,4) = .004: UR(2,3) = .002: U
R(2,4) = .001: UR(2,1) = .005: UR(2,2) = 8.000001E-03: UR(3,1) = .002: UR(3,2) = .001: UR(3
,3) = .002
60 VR(1,1) = .005: VR(1,2) = .001: VR(1,3) = .004: VR(1,4) = .001: VR(2,1) = .006: VR(2,2) =
.001: VR(2,3) = .0004: VR(2,4) = .0002: VR(3,1) = .0005: VR(3,2) = .0003: VR(3,3) = .0008
69 REM Input Defective Rate
70 Q(1,1) = .1: Q(1,2) = .12: Q(1,3) = .06: Q(1,4) = .05: Q(2,1) = 9.000001E-02: Q(2,2) = .1
: Q(2,3) = .06: Q(2,4) = 9.000001E-02: Q(3,1) = .1: Q(3,2) = .05: Q(3,3) = .06
79 REM Input Cost
80 CI(1,1) = 16: CI(1,2) = 16: CI(1,3) = 15: CI(1,4) = 14: CI(2,1) = 10: CI(2,2) = 8: CI(2,3
) = 10: CI(2,4) = 9: CI(3,1) = 12: CI(3,2) = 10: CI(3,3) = 11
90 CF(1) = 350: CF(2) = 290: CF(3) = 400
100 CS(1,2) = 30: CS(1,3) = 50: CS(1,4) = 70: CS(2,2) = 35: CS(2,3) = 50: CS(2,4) = 70: CS(3,1) =
12: CS(3,2) = 15: CS(3,3) = 16
110 CR(1,1) = 15: CR(1,2) = 15: CR(1,3) = 10: CR(1,4) = 12: CR(2,1) = 10: CR(2,2) = 12: CR(2
,3) = 18: CR(2,4) = 20: CR(3,1) = 25: CR(3,2) = 20: CR(3,3) = 18
120 CP(1,1) = 12: CP(1,2) = 14: CP(1,3) = 12: CP(1,4) = 16: CP(2,1) = 12: CP(2,2) = 15: CP(2
,3) = 14: CP(2,4) = 15: CP(3,1) = 16: CP(3,2) = 18: CP(3,3) = 20
129 REM Input Arrival Rate
130 AR(1) = 20: AR(2) = 30: AR(3) = 20: P(2) = 350: P(1) = 400: P(3) = 450
140 TEST = 0
144 REM Print Start Time
145 LOCATE 1,70
146 PRINT TIME$
149 REM Input Inspection Rate
150 FOR I=1 TO 11
160 R(1,1) = (I-1)/10
170 FOR J=1 TO 11
180 R(1,2) = (J-1)/10
190 FOR S=1 TO 11
200 R(1,3) = (S-1)/10
202 FOR K=1 TO 11
203 R(2,4) = (K-1)/10
204 FOR L=1 TO 11
205 R(1,4) = (L-1)/10
210 R(2,1) = R(1,1): R(2,2) = R(1,3): R(2,3) = R(1,4): R(3,1) = R(1,1): R(3,2) = R(1,2): R
(3,3) = R(2,4)
219 REM Evaluate Measurements Without Considering Waiting Time
220 ER(1,1) = AR(1)*Q(1,1)*R(1,1)/(1-Q(1,1)*R(1,1))
230 ER(2,1) = AR(2)*Q(2,1)*R(2,1)/(1-Q(2,1)*R(2,1))
240 ER(3,1) = AR(3)*Q(3,1)*R(3,1)/(1-Q(3,1)*R(3,1))
250 ED(1,1) = (AR(1)+ER(1,1))*(1-R(1,1))*Q(1,1)
260 ED(2,1) = (AR(2)+ER(2,1))*(1-R(2,1))*Q(2,1)
261 ED(3,1) = (AR(3)+ER(3,1))*(1-R(3,1))*Q(3,1)
270 OQ(1,1) = ED(1,1)/AR(1): OQ(2,1) = ED(2,1)/AR(2): OQ(3,1) = ED(3,1)/AR(3)
280 ES(1,1) = 0: ES(2,1) = 0: ES(3,1) = 0
290 OQ(1,1) = Q(1,1): OQ(2,1) = Q(2,1): OQ(3,1) = Q(3,1)
300 OQ(1,2) = OQ(1,1)+Q(1,2)-OQ(1,1)*Q(1,2)

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310 QQ(2,2)=OQ(2,1)+Q(2,2)-OQ(2,1)*Q(2,2)
311 QQ(3,2)=OQ(3,1)+Q(3,2)-OQ(3,1)*Q(3,2)
320 ES(1,2)=ED(1,1)*R(1,2)
330 ES(2,2)=ED(2,1)*R(2,2)
331 ES(3,2)=ED(3,1)*R(3,2)
340 EF(1,2)=AR(1)-ES(1,2)
350 EF(2,2)=AR(2)-ES(2,2)
351 EF(3,2)=AR(3)-ES(3,2)
360 ER(1,2)=AR(1)*(1-OQ(1,1))*Q(1,2)*R(1,2)/(1-Q(1,2)*R(1,2))
370 ER(2,2)=AR(2)*(1-OQ(2,1))*Q(2,2)*R(2,2)/(1-Q(2,2)*R(2,2))
371 ER(3,2)=AR(3)*(1-OQ(3,1))*Q(3,2)*R(3,2)/(1-Q(3,2)*R(3,2))
380 IF ER(1,2)<0 THEN ER(1,2)=0
390 IF ER(2,2)<0 THEN ER(2,2)=0
391 IF ER(3,2)<0 THEN ER(3,2)=0
400 ED(1,2)=(EF(1,2)+ER(1,2))*(1-R(1,2))*OQ(1,2)
410 ED(2,2)=(EF(2,2)+ER(2,2))*(1-R(2,2))*OQ(2,2)
411 ED(3,2)=(EF(3,2)+ER(3,2))*(1-R(3,2))*OQ(3,2)
420 OQ(1,2)=ED(1,2)/EF(1,2)
430 OQ(2,2)=ED(2,2)/EF(2,2)
431 OQ(3,2)=ED(3,2)/EF(3,2)
432 QQ(1,3)=OQ(1,2)+Q(1,3)-OQ(1,2)*Q(1,3)
433 ES(1,3)=ED(1,2)*R(1,3)
434 EF(1,3)=EF(1,2)-ES(1,3)
435 ER(1,3)=EF(1,2)*(1-OQ(1,2))*Q(1,3)*R(1,3)/(1-Q(1,3)*R(1,3))
436 ED(1,3)=(EF(1,3)+ER(1,3))*(1-R(1,3))*OQ(1,3)
440 QQ(2,3)=OQ(2,2)+Q(2,3)-OQ(2,2)*Q(2,3)
441 ES(2,3)=ED(2,2)*R(2,3)
442 EF(2,3)=EF(2,2)-ES(2,3)
443 ER(2,3)=EF(2,2)*(1-OQ(2,2))*Q(2,3)*R(2,3)/(1-Q(2,3)*R(2,3))
444 ED(2,3)=(EF(2,3)+ER(2,3))*(1-R(2,3))*OQ(2,3)
445 OQ(2,3)=ED(2,3)/EF(2,3)
446 QQ(3,3)=OQ(3,2)+Q(3,3)-OQ(3,2)*Q(3,3)
447 ES(3,3)=ED(3,2)*R(3,3)
448 EF(3,3)=EF(3,2)-ES(3,3)
449 ER(3,3)=EF(3,2)*(1-OQ(3,2))*Q(3,3)*R(3,3)/(1-Q(3,3)*R(3,3))
450 ED(3,3)=(EF(3,3)+ER(3,3))*(1-R(3,3))*OQ(3,3)
451 OQ(3,3)=ED(3,3)/EF(3,3)
452 QQ(1,4)=OQ(1,3)+Q(1,4)-OQ(1,3)*Q(1,4)
453 ES(1,4)=ED(1,3)*R(1,4)
454 EF(1,4)=EF(1,3)-ES(1,4)
455 ER(1,4)=EF(1,3)*(1-OQ(1,3))*Q(1,4)*R(1,4)/(1-Q(1,4)*R(1,4))
456 ED(1,4)=(EF(1,4)+ER(1,4))*(1-R(1,4))*OQ(1,4)
457 OQ(1,4)=ED(1,4)/EF(1,4)
458 QQ(2,4)=OQ(2,3)+Q(2,4)-OQ(2,3)*Q(2,4)
459 ES(2,4)=ED(2,3)*R(2,4)
460 EF(2,4)=EF(2,3)-ES(2,4)
461 ER(2,4)=EF(2,3)*(1-OQ(2,3))*Q(2,4)*R(2,4)/(1-Q(2,4)*R(2,4))
462 ED(2,4)=(EF(2,4)+ER(2,4))*(1-R(2,4))*OQ(2,4)
463 OQ(2,4)=ED(2,4)/EF(2,4)
499 REM Evaluate Reward Without Considering Waiting Cost
500 IN=CI(1,1)*(ER(1,1)+AR(1))*R(1,1)+CI(1,2)*(ER(1,2)+AR(1))*R(1,2)
510 ISS=IN+CI(1,3)*(ER(1,3)+EF(1,2))*R(1,3)+CI(1,4)*(ER(1,4)+EF(1,3))*R(1,4)+CI(
2,1)*(ER(2,1)+AR(2))*R(2,1)
520 ISR=ISS+CI(2,2)*(ER(2,2)+AR(2))*R(2,2)
521 ISE=ISR+CI(2,3)*(ER(2,3)+EF(2,2))*R(2,3)+CI(2,4)*(ER(2,4)+EF(2,3))*R(2,4)+CI
(3,1)*(ER(3,1)+AR(3))*R(3,1)+CI(3,2)*(ER(3,2)+AR(3))*R(3,2)+CI(3,3)*(ER(3,3)+EF
(3,2))*R(3,3)
530 CF=CF(1)*OQ(1,4)*EF(1,4)+CF(2)*OQ(2,4)*EF(2,4)+CF(3)*OQ(3,3)*EF(3,3)
540 CS=CS(1,2)*ES(1,2)+CS(1,3)*ES(1,3)+CS(1,4)*ES(1,4)+CS(2,2)*ES(2,2)+CS(2,3)*E
S(2,3)+CS(2,4)*ES(2,4)+CS(3,2)*ES(3,2)+CS(3,3)*ES(3,3)

```



```

O(1,2))/RF(2)
1050 CA(3)=(RO(1,3)^2*RF(1)*(1-UT(1)^2)+RF(2)*(1-UT(2)^2)*CA(2)*(RO(2,3))^2+RF(1)
)*RO(1,3)*(UT(1)^2*RO(1,3)*KS(1)+1-RO(1,3))*RF(2)*RO(2,3)*(UT(2)^2*RO(2,3)*KS(2)
+1-RO(2,3))/RF(3)
1051 CA(5)=(RF(3)*(1-UT(3)^2)*RO(3,5)*CA(3)+RF(3)*RO(3,5)*(UT(3)^2*RO(3,5)*KS(3)
+1-RO(3,5))/RF(5)
1052 CA(4)=(RO(2,4)^2*RF(2)*(1-UT(2)^2)*CA(2)+RF(5)*(1-UT(5)^2)*CA(5)*(RO(5,4))^
2*RF(2)*RO(2,4)*(UT(2)^2*RO(2,4)*KS(2)+1-RO(2,4))*RF(5)*RO(5,4)*(UT(5)^2*RO(5,4)
*KS(5)+1-RO(5,4))/RF(4)
1060 W(1)=1.5: W(2)=2: W(3)=2.5
1070 FOR N=1 TO 5
1080 IF CA(N)<1 THEN 1110
1090 G(N)=1
1100 GOTO 1120
1110 G(N)=EXP(-(2*(1-UT(N))*(1-CA(N))^2/(3*UT(N)*(CA(N)+KS(N))))))
1120 PZ(N)=U(N)*UT(N)*(CA(N)+KS(N))*G(N)/(2*(1-UT(N)))
1130 IF PZ(N)<0 THEN PZ(N)=0
1140 NEXT N
1150 WA(1,1)=PZ(1)+UO(1,1)
1160 WA(1,2)=PZ(2)+UO(1,2)
1170 WA(1,3)=PZ(3)+UO(1,3)
1171 WA(1,4)=PZ(5)+UO(1,4)
1180 WA(2,1)=PZ(1)+UO(2,1)
1190 WA(2,2)=PZ(3)+UO(2,2)
1191 WA(2,3)=PZ(5)+UO(2,3)
1192 WA(2,4)=PZ(4)+UO(2,4)
1193 WA(3,1)=PZ(1)+UO(3,1)
1194 WA(3,2)=PZ(2)+UO(3,2)
1195 WA(3,3)=PZ(4)+UO(3,3)
1199 REM Evaluate Total Profit
1200 TWW=W(1)*((AR(1)+ER(1,1))*WA(1,1)+(AR(1)+ER(1,2))*WA(1,2)+(EF(1,2)+ER(1,3))
)*WA(1,3)+(EF(1,3)+ER(1,4))*WA(1,4))
1202 TW1=TWW+W(2)*((AR(2)+ER(2,1))*WA(2,1)+(AR(2)+ER(2,2))*WA(2,2)+(EF(2,2)+ER(2
,3))*WA(2,3)+(EF(2,3)+ER(2,4))*WA(2,4))
1203 TW=TW1+W(3)*((AR(3)+ER(3,1))*WA(3,1)+(AR(3)+ER(3,2))*WA(3,2)+(EF(3,2)+ER(3,
3))*WA(3,3))
1210 TT=TI-TW
1220 IF TT<TEST THEN 1250
1230 TEST=TT: AF1=R(1,1): AF2=R(1,2): AF3=R(1,3): AF4=R(2,4): AF5=R(1,4)
1250 NEXT L
1260 NEXT K
1270 NEXT S
1271 NEXT J
1272 NEXT I
1280 PRINT TEST,AF1,AF2,AF3,AF4,AF5: GOTO 1304
1290 PRINT "NO SOLUTION"
1304 LOCATE 3,70
1305 PRINT TIMES$
1306 STOP

```

```

1 REM Computer Code of Example 2
10 CLS
19 REM Input Process Time
20 T(1,1)=.0004: T(1,2)=5.000001E-04: T(1,3)=.0004: T(1,4)=.0004: T(2,1)=.0005:
T(2,2)=.0003: T(2,3)=.0004: T(2,4)=.0005
30 UM(1,1)=.01: UM(1,2)=.005: UM(1,3)=.01: UM(1,4)=.01: UM(2,1)=.015: UM(2,2)=.00
6: UM(2,3)=.005: UM(2,4)=.01
40 VM(1,1)=.005: VM(1,2)=.006: VM(1,3)=.006: VM(1,4)=.005: VM(2,1)=8.000001E-03
50 VM(2,2)=8.999999E-03: VM(2,3)=.006: VM(2,4)=.007
60 UR(1,1)=.005: UR(1,2)=.001: UR(1,3)=8.999999E-03: UR(1,4)=.004: UR(2,3)=.002: U
R(2,4)=.001: UR(2,1)=.005: UR(2,2)=8.000001E-03
70 VR(1,1)=.005: VR(1,2)=.001: VR(1,3)=.004: VR(1,4)=.0001: VR(2,1)=.006: VR(2,2)
=.001: VR(2,3)=.0004: VR(2,4)=.0002
79 REM Input Defective Rate
80 Q(1,1)=.1: Q(1,2)=.18: Q(1,3)=.06: Q(1,4)=.05: Q(2,1)=9.000001E-02: Q(2,2)=.1
5: Q(2,3)=.06: Q(2,4)=9.000001E-02
89 REM Input Cost
90 CI(1,1)=16: CI(1,2)=16: CI(1,3)=15: CI(1,4)=14: CI(2,1)=10: CI(2,2)=8: CI(2,3
)=10: CI(2,4)=9
100 CF(1)=350: CF(2)=320
110 CS(1,2)=50: CS(1,3)=100: CS(1,4)=125: CS(2,2)=50: CS(2,3)=80: CS(2,4)=100
120 CR(1,1)=15: CR(1,2)=15: CR(1,3)=30: CR(1,4)=12: CR(2,1)=20: CR(2,2)=22: CR(2
,3)=28: CR(2,4)=30
130 CP(1,1)=12: CP(1,2)=14: CP(1,3)=12: CP(1,4)=16: CP(2,1)=12: CP(2,2)=15: CP(2
,3)=14: CP(2,4)=15
139 REM Input Arrival Rate
140 AR(1)=20: AR(2)=30: P(2)=220: P(1)=280
150 TEST=0
159 REM Print Starting Time
160 LOCATE 1,70
170 PRINT TIMES
179 REM Input Inspetion Rate
180 FOR I=1 TO 11
190 R(1,1)=(I-1)/10
200 FOR J=1 TO 11
210 R(1,2)=(J-1)/10
220 FOR S=1 TO 11
230 R(2,3)=(S-1)/10
240 FOR K=1 TO 11
250 R(2,4)=(K-1)/10
260 FOR L=1 TO 11
270 R(1,4)=(L-1)/10
280 R(2,1)=R(1,1): R(2,2)=R(1,2): R(1,3)=R(2,4)
299 REM Evaluate Measurements Without Considering Waiting Time
300 ER(1,1)=AR(1)*Q(1,1)*R(1,1)/(1-Q(1,1)*R(1,1))
310 ER(2,1)=AR(2)*Q(2,1)*R(2,1)/(1-Q(2,1)*R(2,1))
320 ED(1,1)=(AR(1)+ER(1,1))*(1-R(1,1))*Q(1,1)
330 ED(2,1)=(AR(2)+ER(2,1))*(1-R(2,1))*Q(2,1)
340 OQ(1,1)=ED(1,1)/AR(1): OQ(2,1)=ED(2,1)/AR(2)
350 ES(1,1)=0: ES(2,1)=0
360 QQ(1,1)=Q(1,1): QQ(2,1)=Q(2,1)
370 QQ(1,2)=OQ(1,1)+Q(1,2)-OQ(1,1)*Q(1,2)
380 QQ(2,2)=OQ(2,1)+Q(2,2)-OQ(2,1)*Q(2,2)
390 ES(1,2)=ED(1,1)*R(1,2)
400 ES(2,2)=ED(2,1)*R(2,2)
410 EF(1,2)=AR(1)-ES(1,2)
420 EF(2,2)=AR(2)-ES(2,2)
430 ER(1,2)=AR(1)*(1-OQ(1,1))*Q(1,2)*R(1,2)/(1-Q(1,2)*R(1,2))
440 ER(2,2)=AR(2)*(1-OQ(2,1))*Q(2,2)*R(2,2)/(1-Q(2,2)*R(2,2))
450 ED(1,2)=(EF(1,2)+ER(1,2))*(1-R(1,2))*QQ(1,2)

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460 ED(2,2)=(EF(2,2)+ER(2,2))*(1-R(2,2))*QQ(2,2)
470 QQ(1,2)=ED(1,2)/EF(1,2)
480 QQ(2,2)=ED(2,2)/EF(2,2)
490 QQ(1,3)=QQ(1,2)+Q(1,3)-QQ(1,2)*Q(1,3)
500 ES(1,3)=ED(1,2)*R(1,3)
510 EF(1,3)=EF(1,2)-ES(1,3)
520 ER(1,3)=EF(1,2)*(1-QQ(1,2))*Q(1,3)*R(1,3)/(1-Q(1,3)*R(1,3))
530 ED(1,3)=(EF(1,3)+ER(1,3))*(1-R(1,3))*QQ(1,3)
540 QQ(2,3)=QQ(2,2)+Q(2,3)-QQ(2,2)*Q(2,3)
550 ES(2,3)=ED(2,2)*R(2,3)
560 EF(2,3)=EF(2,2)-ES(2,3)
570 ER(2,3)=EF(2,2)*(1-QQ(2,2))*Q(2,3)*R(2,3)/(1-Q(2,3)*R(2,3))
580 ED(2,3)=(EF(2,3)+ER(2,3))*(1-R(2,3))*QQ(2,3)
590 QQ(2,3)=ED(2,3)/EF(2,3)
600 QQ(1,4)=QQ(1,2)+Q(1,4)-QQ(1,2)*Q(1,4)
610 ES(1,4)=ED(1,2)*R(1,4)
620 EF(1,4)=EF(1,2)-ES(1,4)
630 ER(1,4)=EF(1,2)*(1-QQ(1,2))*Q(1,4)*R(1,4)/(1-Q(1,4)*R(1,4))
640 ED(1,4)=(EF(1,4)+ER(1,4))*Q(1,4)
650 QQ(1,4)=ED(1,4)/EF(1,4)
660 QQ(2,4)=QQ(2,2)+Q(2,4)-QQ(2,2)*Q(2,4)
670 ES(2,4)=ED(2,2)*R(2,4)
680 EF(2,4)=EF(2,2)-ES(2,4)
690 ER(2,4)=EF(2,2)*(1-QQ(2,2))*Q(2,4)*R(2,4)/(1-Q(2,4)*R(2,4))
700 ED(2,4)=(EF(2,4)+ER(2,4))*(1-R(2,4))*QQ(2,4)
710 QQ(2,4)=ED(2,4)/EF(2,4)
719 REM Evaluate Reward Without Considering Waiting Cost
720 IN=CI(1,1)*(ER(1,1)+AR(1,1))*R(1,1)*CI(1,2)*(ER(1,2)+AR(1,1))*R(1,2)
730 IS=IN+CI(1,3)*(ER(1,3)+EF(1,3))*R(1,3)+CI(1,4)*(ER(1,4)+EF(1,3))*R(1,4)+CI(
2,1)*(ER(2,1)+AR(2,1))*R(2,1)
740 ISR=IS+CI(2,2)*(ER(2,2)+AR(2,2))*R(2,2)
750 ISE=IS+CI(2,3)*(ER(2,3)+EF(2,2))*R(2,3)+CI(2,4)*(ER(2,4)+EF(2,2))*R(2,4)
760 CP=CP(1)*QQ(1,4)+CP(1,4)+CP(2)*QQ(2,4)+CP(3)*QQ(3,3)+EF(3,3)
770 CS=CS(1,2)*ES(1,2)+CS(1,3)*ES(1,4)+CS(1,4)+CS(2,2)*ES(2,2)+CS(2,3)*E
S(2,3)+CS(2,4)*ES(2,4)+CS(3,2)*ES(3,3)+CS(3,3)*ES(3,3)
780 CR=CR(1,1)*ER(1,1)+CR(1,2)*ER(1,2)+CR(1,3)*ER(1,4)+CR(1,4)*ER(2,1)*
ER(2,1)+CR(2,2)*ER(2,2)+CR(2,3)*ER(2,3)+CR(2,4)*ER(2,4)
790 PP= CP(1,1)*(ER(1,1)+AR(1,1))*CP(1,2)*(ER(1,2)+AR(1,1))+CP(1,3)*(ER(1,3)+EF(1,3)
)
800 PK=PP+CP(1,4)*(ER(1,4)+EF(1,3))*CP(2,1)*CP(2,1)+AR(2,1)+CP(2,2)*(ER(2,2)+AR(
2,1))+CP(2,3)*(ER(2,3)+EF(2,2))+CP(2,4)*(ER(2,4)+EF(2,3))
810 NP=P(1)*EF(1,4)+P(2)*EF(2,4)+P(3)*EF(3,4)
820 TI=RX-ISE-CF-CS-CR-PK
829 REM Evaluate Waiting Time
830 LA(1,2)=AR(1)+AR(2)
840 LA(2,3)=EF(2,2)
850 LA(2,4)=EF(1,2)
860 LA(4,5)=EF(1,4)
870 RO(1,2)=1: RO(2,3)=LA(2,3)/(LA(2,3)+LA(2,4))
880 RO(4,5)=LA(4,5)/(LA(4,5)+EF(1,4))
890 RO(3,4)=1
900 RF(1)=AR(1)+AR(2)
910 RF(2)=AR(1)+AR(2)
920 RF(3)=EF(2,2)
930 RF(4)=EF(1,3)+EF(2,3)
940 RF(5)=EF(1,4)
950 ME(1,1)=1/(1-R(1,1))*Q(1,1)
960 ME(2,1)=1/(1-R(2,1))*Q(2,1)
970 ME(1,2)=1*(R(1,2))*QQ(1,2)-QQ(1,2)/(1-R(1,2))*Q(1,2))
980 ME(1,3)=1*(R(1,3))*QQ(1,3)-QQ(1,3)/(1-R(1,3))*Q(1,3))

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990 ME(1,4)=1+(R(1,4)*(QQ(1,4)-QQ(1,2)))/(1-R(1,4)*Q(1,4))
1000 ME(2,2)=1+(R(2,2)*(QQ(2,2)-QQ(2,1)))/(1-R(2,2)*Q(2,2))
1010 ME(2,3)=1+(R(2,3)*(QQ(2,3)-QQ(2,2)))/(1-R(2,3)*Q(2,3))
1020 ME(2,4)=1+(R(2,4)*(QQ(2,4)-QQ(2,3)))/(1-R(2,4)*Q(2,4))
1030 VN(1,1)=R(1,1)*Q(1,1)/(1-R(1,1)*Q(1,1))-2
1040 VN(2,1)=R(2,1)*Q(2,1)/(1-R(2,1)*Q(2,1))-2
1050 VN(2,2)=R(2,2)*(QQ(2,2)-QQ(2,1))*(1+R(2,2)*Q(2,2)+QQ(2,2)*Q(2,1))/(1-R(2,2)*Q(2,2))
1060 VN(2,3)=R(2,3)*(QQ(2,3)-QQ(2,2))*(1+R(2,3)*Q(2,3)+QQ(2,3)*Q(2,2))/(1-R(2,3)*Q(2,3))
1070 VN(2,4)=R(2,4)*(QQ(2,4)-QQ(2,3))*(1+R(2,4)*Q(2,4)+QQ(2,4)*Q(2,3))/(1-R(2,4)*Q(2,4))
1080 VN(1,2)=R(1,2)*(QQ(1,2)-QQ(1,1))*(1+R(1,2)*Q(1,2)+QQ(1,2)*Q(1,1))/(1-R(1,2)*Q(1,2))
1090 VN(1,3)=R(1,3)*(QQ(1,3)-QQ(1,2))*(1+R(1,3)*Q(1,3)+QQ(1,3)*Q(1,2))/(1-R(1,3)*Q(1,3))
1100 VN(1,4)=R(1,4)*(QQ(1,4)-QQ(1,3))*(1+R(1,4)*Q(1,4)+QQ(1,4)*Q(1,3))/(1-R(1,4)*Q(1,4))
1110 UO(1,1)=UM(1,1)*ME(1,1)+UR(1,1)*(ME(1,1)-1)+HE(1,1)*R(1,1)*OT(1,1)
1120 UO(1,2)=UM(1,2)*ME(1,2)+UR(1,2)*(ME(1,2)-1)+HE(1,2)*R(1,2)*OT(1,2)
1130 UO(1,3)=UM(1,3)*ME(1,3)+UR(1,3)*(ME(1,3)-1)+HE(1,3)*R(1,3)*OT(1,3)
1140 UO(1,4)=UM(1,4)*ME(1,4)+UR(1,4)*(ME(1,4)-1)+HE(1,4)*R(1,4)*OT(1,4)
1150 UO(2,1)=UM(2,1)*ME(2,1)+UR(2,1)*(ME(2,1)-1)+HE(2,1)*R(2,1)*OT(2,1)
1160 UO(2,2)=UM(2,2)*ME(2,2)+UR(2,2)*(ME(2,2)-1)+HE(2,2)*R(2,2)*OT(2,2)
1170 UO(2,3)=UM(2,3)*ME(2,3)+UR(2,3)*(ME(2,3)-1)+HE(2,3)*R(2,3)*OT(2,3)
1180 UO(2,4)=UM(2,4)*ME(2,4)+UR(2,4)*(ME(2,4)-1)+HE(2,4)*R(2,4)*OT(2,4)
1190 VO(1,1)=(ME(1,1)-1)*(VM(1,1)+VR(1,1))+VM(1,1)*(UM(1,1)+UR(1,1)*R(1,1)*OT(1,1))
1200 VO(1,2)=(ME(1,2)-1)*(VM(1,2)+VR(1,2))+VM(1,2)*(UM(1,2)+UR(1,2)*R(1,2)*OT(1,2))
1210 VO(1,3)=(ME(1,3)-1)*(VM(1,3)+VR(1,3))+VM(1,3)*(UM(1,3)+UR(1,3)*R(1,3)*OT(1,3))
1220 VO(1,4)=(ME(1,4)-1)*(VM(1,4)+VR(1,4))+VM(1,4)*(UM(1,4)+UR(1,4)*R(1,4)*OT(1,4))
1230 VO(2,1)=(ME(2,1)-1)*(VM(2,1)+VR(2,1))+VM(2,1)*(UM(2,1)+UR(2,1)*R(2,1)*OT(2,1))
1240 VO(2,2)=(ME(2,2)-1)*(VM(2,2)+VR(2,2))+VM(2,2)*(UM(2,2)+UR(2,2)*R(2,2)*OT(2,2))
1250 VO(2,3)=(ME(2,3)-1)*(VM(2,3)+VR(2,3))+VM(2,3)*(UM(2,3)+UR(2,3)*R(2,3)*OT(2,3))
1260 VO(2,4)=(ME(2,4)-1)*(VM(2,4)+VR(2,4))+VM(2,4)*(UM(2,4)+UR(2,4)*R(2,4)*OT(2,4))
1270 U(1)=(AR(1)*UO(1,1)+AR(2)*UO(2,1))/(AR(1)+AR(2))
1280 U(2)=(AR(1)*UO(1,2)+AR(2)*UO(2,2))/(AR(1)+AR(2))
1290 U(3)=(EF(2,2)*UO(2,3))/EF(2,2)
1300 U(4)=(EF(2,3)*UO(2,4)+EF(1,2)*UO(1,3))/(EF(2,3)+EF(1,2))
1310 U(5)=(EF(1,4)*UO(1,4))/EF(1,4)
1320 V(1)=(AR(1)*(VO(1,1)+(UO(1,1))-2)+AR(2)*(VO(2,1)+(UO(2,1))-2))/(AR(1)+AR(2))
1330 V(2)=(AR(1)*(VO(1,2)+(UO(1,2))-2)+AR(2)*(VO(2,2)+(UO(2,2))-2))/(AR(1)+AR(2))
1340 V(3)=(EF(2,2)*(VO(2,3)+(UO(2,3))-2))/EF(1,2)
1350 V(4)=(EF(1,2)*(VO(1,3)+(UO(1,3))-2)+EF(2,3)*(VO(2,4)+(UO(2,4))-2))/(EF(1,2)+EF(2,3))
1360 V(5)=(EF(1,3)*(VO(1,4)+(UO(1,4))-2))/EF(1,3)
1370 UT(1)=RF(1)*U(1)
1380 UT(2)=RF(2)*U(2)
1390 UT(3)=RF(3)*U(3)
1400 UT(4)=RF(4)*U(4)
1410 UT(5)=RF(5)*U(5)

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1420 IF UT(1)>1 THEN 1850
1430 IF UT(2)>1 THEN 1850
1440 IF UT(3)>1 THEN 1850
1450 IF UT(4)>1 THEN 1850
1460 IF UT(5)>1 THEN 1850
1470 KS(1)=V(1)/((1/U(1))^-2)
1480 KS(2)=V(2)/((1/U(2))^-2)
1490 KS(3)=V(3)/((1/U(3))^-2)
1500 KS(5)=V(5)/((1/U(5))^-2)
1510 KS(4)=V(4)/((1/U(4))^-2)
1520 CA(1)=1
1530 CA(2)=(RO(1,2)^2*RF(1)*(1-UT(1))^-2+RO(1,2)*RF(1)*(UT(1)^2*RO(1,2)*KS(1)+1-RO(1,2))/RF(2)
1540 CA(3)=(RF(2)*(1-UT(2)^2)*CA(2)*(RO(2,3))^-2+RF(2)*RO(2,3)*(UT(2)^2*RO(2,3)*KS(2)+1-RO(2,3))/RF(3)
1550 CA(4)=(RO(2,4)^2*RF(2)*(1-UT(2)^2)*CA(2)+RF(3)*(1-UT(3)^2)*CA(3)*(RO(3,4))^-2+RF(2)*RO(2,4)*UT(2)^2*RO(2,4)*KS(2)+1-RO(2,4)+RF(3)*RO(3,4)*UT(3)^2*RO(3,4)*KS(3)+1-RO(3,4))/RF(4)
1560 CA(5)=(RF(4)*(1-UT(4)^2)*RO(4,5)*CA(4)+RF(4)*RO(4,5)*(UT(4)^2*RO(4,5)*KS(4)+1-RO(4,5))/RF(5)
1570 W(1)=1.5: W(2)=2
1580 FOR N=1 TO 5
1590 IF CA(N)<1 THEN 1620
1600 G(N)=1
1610 GOTO 1630
1620 G(N)=EXP(-(2*(1-UT(N))*(1-CA(N))^-2/(3*UT(N)*(CA(N)+KS(N))))
1630 PZ(N)=U(N)*UT(N)*(CA(N)+KS(N))*G(N)/(2*(1-UT(N)))
1640 IF PZ(N)<0 THEN PZ(N)=0
1650 NEXT N
1660 WA(1,1)=PZ(1)+UO(1,1)
1670 WA(1,2)=PZ(2)+UO(1,2)
1680 WA(1,3)=PZ(3)+UO(1,3)
1690 WA(1,4)=PZ(5)+UO(1,4)
1700 WA(2,1)=PZ(1)+UO(2,1)
1710 WA(2,2)=PZ(3)+UO(2,2)
1720 WA(2,3)=PZ(5)+UO(2,3)
1730 WA(2,4)=PZ(4)+UO(2,4)
1739 REM Evaluate Total Profit
1740 TW=WA(1)*(AR(1)+ER(1,1))*WA(1,1)+(AR(1)+ER(1,2))*WA(1,2)+(EF(1,2)+ER(1,3))*WA(1,3)+(EF(1,3)+ER(1,4))*WA(1,4)
1750 TW=TW+W(2)*((AR(2)+ER(2,1))*WA(2,1)+(AR(2)+ER(2,2))*WA(2,2)+(EF(2,2)+ER(2,3))*WA(2,3)+(EF(2,3)+ER(2,4))*WA(2,4))
1760 TT=TI-TW
1770 IF TT<TEST THEN 1790
1780 TEST=TT: AF1=R(1,1): AF2=R(1,2): AF3=R(2,3): AF4=R(1,3): AF5=R(1,4)
1790 NEXT L
1800 NEXT K
1810 NEXT S
1820 NEXT J
1830 NEXT I
1840 PRINT TEST,AF1,AF2,AF3,AF4,AF5: GOTO 1860
1850 PRINT "NO SOLUTION"
1860 LOCATE 3,70
1870 PRINT TIME$
1880 STOP

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APPENDIX E: COMPUTER CODE OF CHAPTERS 3 & 4

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1 REM Computer Code of Chapter 3
5 REM Input Process Time
10 T(1,1)=.0005: T(1,2)=6.000001E-04: T(1,3)=.0003: T(2,1)=.0005: T(2,2)=.0003
20 UM(1,1)=.01: UM(1,2)=.005: UM(1,3)=.01: UM(2,1)=.015: UM(2,2)=.006
30 VM(1,1)=.005: VM(1,2)=.006: VM(1,3)=.006: VM(2,1)=8.000001E-03
40 VU(2,2)=8.999999E-03
50 UR(1,1)=.005: UR(1,2)=.001: UR(1,3)=8.999999E-03: UR(2,1)=.005: UR(2,2)=8.000
001E-03
60 VR(1,1)=.005: VR(1,2)=.001: VR(1,3)=.004: VR(2,1)=.006: VR(2,2)=.001
69 REM Input Defective Rate
70 Q(1,1)=.1: Q(1,2)=.08: Q(1,3)=.06: Q(2,1)=9.000001E-02: Q(2,2)=.08
79 REM Input Cost
80 CI(1,1)=16: CI(1,2)=16: CI(1,3)=15: CI(2,1)=10: CI(2,2)=8
90 CF(1)=220: CF(2)=230
100 CS(1,2)=20: CS(1,3)=30: CS(2,2)=16
110 CR(1,1)=15: CR(1,2)=15: CR(1,3)=10: CR(2,1)=10: CR(2,2)=12
120 CP(1,1)=12: CP(1,2)=14: CP(1,3)=12: CP(2,1)=12: CP(2,2)=15
129 REM Input Arrival Rate
130 AR(1)=30: AR(2)=40: P(2)=180: P(1)=200
140 TEST=0
149 REM Input Inspection Rate
150 FOR I=1 TO 11
160 R(1,1)=(I-1)/10
170 FOR J=1 TO 11
180 R(1,2)=(J-1)/10
190 FOR S=1 TO 11
200 R(1,3)=(S-1)/10
210 R(2,1)=R(1,1): R(2,2)=R(1,3)
219 REM Evaluate Measurements Without Considering Waiting Time
220 ER(1,1)=AR(1)*Q(1,1)*R(1,1)/(1-Q(1,1)*R(1,1))
230 ER(2,1)=AR(2)*Q(2,1)*R(2,1)/(1-Q(2,1)*R(2,1))
240 OQ(1,1)=ED(1,1)/AR(1): OQ(2,1)=ED(2,1)/AR(2)
250 ED(1,1)=(AR(1)+ER(1,1))*(1-R(1,1))*Q(1,1)
260 ED(2,1)=(AR(2)+ER(2,1))*(1-R(2,1))*Q(2,1)
270 OQ(1,1)=ED(1,1)/AR(1): OQ(2,1)=ED(2,1)/AR(2)
280 ES(1,1)=0: ES(2,1)=0
290 OQ(1,1)=Q(1,1): OQ(2,1)=Q(2,1)
300 OQ(1,2)=OQ(1,1)+Q(1,2)-OQ(1,1)*Q(1,2)
310 OQ(2,2)=OQ(2,1)+Q(2,2)-OQ(2,1)*Q(2,2)
320 ES(1,2)=ED(1,1)*R(1,2)
330 ES(2,2)=ED(2,1)*R(2,2)
340 EF(1,2)=AR(1)-ES(1,2)
350 EF(2,2)=AR(2)-ES(2,2)
360 ER(1,2)=AR(1)*(1-OQ(1,1))*Q(1,2)*R(1,2)/(1-Q(1,2)*R(1,2))
370 ER(2,2)=AR(2)*(1-OQ(2,1))*Q(2,2)*R(2,2)/(1-Q(2,2)*R(2,2))
380 IF ER(1,2)<0 THEN ER(1,2)=0
390 IF ER(2,2)<0 THEN ER(2,2)=0
400 ED(1,2)=(EF(1,2)+ER(1,2))*(1-R(1,2))*OQ(1,2)
410 ED(2,2)=(EF(2,2)+ER(2,2))*(1-R(2,2))*OQ(2,2)
420 OQ(1,2)=ED(1,2)/EF(1,2)
430 OQ(2,2)=ED(2,2)/EF(2,2)
440 OQ(1,3)=OQ(1,2)+Q(1,3)-OQ(1,2)*Q(1,3)
450 ES(1,3)=ED(1,2)*R(1,3)
460 EF(1,3)=EF(1,2)-ES(1,3)
470 ER(1,3)=EF(1,2)*(1-OQ(1,2))*Q(1,3)*R(1,3)/(1-Q(1,3)*R(1,3))
480 ED(1,3)=(EF(1,3)+ER(1,3))*(1-R(1,3))*OQ(1,3)
490 OQ(1,3)=ED(1,3)/EF(1,3)
499 REM Evaluate Reward Without Considering Waiting Cost
500 IN=CI(1,1)*(ER(1,1)+AR(1))*R(1,1)+CI(1,2)*(ER(1,2)+AR(1))*R(1,2)
510 IS=IN+CI(1,3)*(ER(1,3)+EF(1,2))*R(1,3)+CI(2,1)*(ER(2,1)+AR(2))*R(2,1)

```

```

520 ISB-ISB+CI(2,2)*(ER(2,2)+AR(2))*R(2,2)
530 CP-CF(1,1)*OQ(1,2)*EF(1,2)+CF(2,2)*OQ(2,2)*EF(2,2)
540 CS-CS(1,2)*ES(1,2)+CS(1,2)*ES(1,2)+CS(2,2)*ES(2,2)
550 PR-CR(1,1)*ER(1,1)+CR(1,2)*ER(1,2)+CR(1,2)*ER(1,2)+CR(2,1)*ER(2,1)
560 CRN-PR+CR(2,2)*ER(2,2)
570 PK-CP(1,1)*(ER(1,1)+AR(1))+CP(1,2)*ER(1,2)+AR(1,2)+CP(1,2)*ER(1,2)+EF(1,2)
)+CP(2,1)*ER(2,1)+AR(2))+CP(2,2)*ER(2,2)+EF(2,1))
580 RM-P(1)*EF(1,2)+P(2)*EF(2,2)
590 TIK-ISE-CF-CS-CRN-PK
599 REM Evaluate Melting Time
600 LA(1,2)=AR(1)
610 LA(1,2)=AR(2)
620 LA(2,2)=EF(1,2)
630 RO(1,2)=LA(1,2)/(LA(1,2)+LA(1,2)): RO(1,2)=LA(1,2)/(LA(1,2)+LA(1,2))
640 RO(2,2)=1
650 RF(1)=AR(1)+AR(2)
660 RF(2)=AR(1)
670 RF(2)=AR(2)+EF(1,2)
680 ME(1,1)=1/(1-R(1,1)*Q(1,1))
690 ME(2,1)=1/(1-R(2,1)*Q(2,1))
700 ME(1,2)=1/(R(1,2)*(OQ(1,2)-OQ(1,1)))/(1-R(1,2)*Q(1,2))
710 ME(1,2)=1/(R(1,2)*(OQ(1,2)-OQ(1,1)))/(1-R(1,2)*Q(1,2))
720 ME(2,2)=1/(R(2,2)*(OQ(2,2)-OQ(2,1)))/(1-R(2,2)*Q(2,2))
730 VM(1,1)=R(1,1)*Q(1,1)/(1-R(1,1)*Q(1,1))^-2
740 VM(2,1)=R(2,1)*Q(2,1)/(1-R(2,1)*Q(2,1))^-2
750 VM(2,2)=R(2,2)*Q(2,2)/(1-R(2,2)*Q(2,2))
760 VM(1,2)=R(1,2)*Q(1,2)/(1-R(1,2)*Q(1,2))
770 VM(1,2)=R(1,2)*Q(1,2)/(1-R(1,2)*Q(1,2))
770 VM(1,2)=R(1,2)*Q(1,2)/(1-R(1,2)*Q(1,2))
780 UO(1,1)=UM(1,1)*ME(1,1)+UR(1,2)*(ME(1,2)-1)+ME(1,1)*R(1,1)*OT(1,1)
790 UO(1,2)=UM(1,2)*ME(1,2)+UR(1,2)*(ME(1,2)-1)+ME(1,2)*R(1,2)*OT(1,2)
800 UO(1,2)=UM(1,2)*ME(1,2)+UR(1,2)*(ME(1,2)-1)+ME(1,2)*R(1,2)*OT(1,2)
810 UO(2,1)=UM(2,1)*ME(2,1)+UR(2,2)*(ME(2,1)-1)+ME(2,1)*R(2,1)*OT(2,1)
820 UO(2,2)=UM(2,2)*ME(2,2)+UR(2,2)*(ME(2,2)-1)+ME(2,2)*R(2,2)*OT(2,2)
830 VO(1,1)=ME(1,1)+VM(1,1)+VR(1,1)+UM(1,1)+UR(1,1)+R(1,1)*OT(1,1)
840 VO(1,2)=ME(1,2)+VM(1,2)+VR(1,2)+UM(1,2)+UR(1,2)+R(1,2)*OT(1,2)
850 VO(1,2)=ME(1,2)+VM(1,2)+VR(1,2)+UM(1,2)+UR(1,2)+R(1,2)*OT(1,2)
860 VO(1,2)=ME(1,2)+VM(1,2)+VR(1,2)+UM(1,2)+UR(1,2)+R(1,2)*OT(1,2)
870 VO(2,2)=ME(2,2)+VM(2,2)+VR(2,2)+UM(2,2)+UR(2,2)+R(2,2)*OT(2,2)
880 U(1)=AR(1)*UO(1,1)+EF(2,1)*UO(2,1)/(AR(1)+EF(2,1))
890 U(2)=UO(1,2)
900 U(2)=(EF(1,2)*UO(1,2)+AR(2)*UO(2,2))/(EF(1,2)+AR(2))
910 V(1)=AR(1)*(VO(1,1)+UO(1,1))^2+AR(2)*(VO(2,1)+UO(2,1))^2)/(AR(1)+AR(2))
920 V(2)=VO(1,2)
930 V(2)=(EF(1,2)*(VO(1,2)+UO(1,2))^2+AR(2)*(VO(2,2)+UO(2,2))^2)/(EF(1,2)+AR(2))
940 UT(1)=RF(1)*U(1)
950 UT(2)=RF(2)*U(2)
960 UT(2)=RF(2)*U(2)
970 IF UT(1)>1 THEN 1290
980 IF UT(2)>1 THEN 1290
990 IF UT(2)>1 THEN 1290
1000 KS(1)=V(1)*(1/U(1))^2

```

```

1010 KS(2)=V(2)/((1/U(2))-2)
1020 KS(3)=V(3)/((1/U(3))-2)
1030 CA(1)=1
1040 CA(2)=(RO(1,2)-2*RF(1)*(1-UT(1))-2*RO(1,2)*RF(1)*(UT(1)-2*RO(1,2)*KS(1)+1-R
O(1,2))/RF(2)
1050 CA(3)=(RO(1,3)-2*RF(1)*(1-UT(1))-2*RF(2)*(1-UT(2))-2*CA(2)+R(1,3)*RF(1)*(UT
(1)-2*RO(1,3)*KS(1)+1-RO(1,3))*RF(2)*UT(2)-2*KS(2))/RF(3)
1060 W(1)=1.5; W(2)=2
1070 FOR K=1 TO 3
1080 IF CA(K)<1 THEN 1110
1090 G(K)=1
1100 GOTO 1120
1110 G(K)=EXP(-(2*(1-UT(K))*(1-CA(K))-2/(3*UT(K)*(CA(K)+KS(K))))))
1120 PZ(K)=U(K)*UT(K)*(CA(K)+KS(K))*G(K)/(2*(1-UT(K)))
1130 IF PZ(K)<0 THEN PZ(K)=0
1140 NEXT K
1150 WA(1,1)=PZ(1)+UO(1,1)
1160 WA(1,2)=PZ(2)+UO(1,2)
1170 WA(1,3)=PZ(3)+UO(1,3)
1180 WA(2,1)=PZ(1)+UO(2,1)
1190 WA(2,2)=PZ(2)+UO(2,2)
1199 REM Evaluate Total Profit
1200 TW=W(1)*(AR(1)+ER(1,1))*WA(1,1)+(AR(1)+ER(1,2))*WA(1,2)+(EF(1,2)+ER(1,2))*
WA(1,3)+W(2)*(AR(2)+ER(2,1))*WA(2,1)+(AR(2)+ER(2,2))*WA(2,2)
1210 TT=TI-TV
1220 IF TT<TEST THEN 1240
1230 TEST=TT: AF1=R(1,1): AF2=R(1,2): AF3=R(1,3)
1240 PRINT TT
1250 NEXT S
1260 NEXT J
1270 NEXT I
1280 PRINT TEST,AF1,AF2,AF3: GOTO 1300
1290 PRINT "NO SOLUTION"
1300 STOP

```

SIMULATION CODE OF CHAPTER 4

```

.
.   Define Aspervariables
.
      INTEGER      &I
      REAL          &X(10), &Y(10), &Z(10), &AVGX, &AVGY, &AVGZ
      REAL          &TOTX, &TOTY, &TOTZ
.
.
.   AB MODEL
.
.   Define output file
.
OUT   FILE          'A: AB.OUT'
.
      INITIAL       XH1,3
      INITIAL       XH2,9
.
      STORAGE       S(REP1),4/S(REP2),2
.
NEW   FUNCTION       PH3,L2
4,2/2,1
.
1     FVARIABLE      RVEXPO(1,12)
2     FVARIABLE      RVEXPO(1,15)
.
      GENERATE       ...,1,..4PH
      ASSIGN         1,V1,PH
      ASSIGN         2,V2,PH
BACK  SELECT MIN     3PH,1,2,..PH
      SEIZE          SYS
      ADVANCE        PH*PH3
      RELEASE        SYS
      ASSIGN         FN(NEW)-,PH*PH3,PH
      SPLIT          1,MTTR
      TEST C         XH*PH3,0
      SAVEVALUE      PH3-,1,XH
      ASSIGN         PH3,V*PH3,PH
      TRANSFER       ,BACK
.
MTTR  TEST E         PH3,1,OTHER
      ASSIGN         4,RVEXPO(1,11),PH
      TRANSFER       ,TRY1
OTHER  ASSIGN         4,RVEXPO(1,12),PH
      TRANSFER       ,REP2
.
TRY1  GATE SNF       REP1,TRY2
      QUEUE          REP1
      ENTER          REP1
      DEPART         REP1
      ADVANCE        PH4
      LEAVE          REP1
      SAVEVALUE      1+,1,XH
      BUFFER
      TERMINATE
.
.
TRY2  GATE SNF       REP2, WAIT

```

```

TRANSFER      .REP2
ADVANCE        0.5
TRANSFER       .TRY1

REP2
  QUEUE        REP2
  ENTER        REP2
  DEPART       REP2
  ADVANCE      PH4
  LEAVE        REP2
  SAVEVALUE    PH3+.1,XH
  BUFFER
  TERMINATE

.
GENERATE
  BLET         ..200000
  BLET         LX(L1)=FR(SYS)/1000
  BLET         LTOTX=LTOTX+LX(L1)
  BLET         LY(L1)=SR(REP1)/1000
  BLET         LTOTY=LTOTY+LY(L1)
  BLET         LZ(L1)=SR(REP2)/1000
  BLET         LTOTZ=LTOTZ+LZ(L1)
  BLET
  TERMINATE    1

.
START          1
RMULT          37487
CLEAR
INITIAL        XH1.3
INITIAL        XH2.9
START          1
RMULT          59412
CLEAR
INITIAL        XH1.3
INITIAL        XH2.9
START          1
RMULT          10203
CLEAR
INITIAL        XH1.3
INITIAL        XH2.9
START          1
RMULT          72444
CLEAR
INITIAL        XH1.3
INITIAL        XH2.9
START          1
RMULT          12487
CLEAR
INITIAL        XH1.3
INITIAL        XH2.9
START          1
RMULT          59500
CLEAR
INITIAL        XH1.3
INITIAL        XH2.9
START          1
RMULT          70203
CLEAR
INITIAL        XH1.3
INITIAL        XH2.9
START          1

```

```

RMULT      33334
CLEAR
INITIAL    XH1,3
INITIAL    XH2,9
START      1
RMULT      11257
CLEAR
INITIAL    XH1,3
INITIAL    XH2,9
START      1
LET        &AVCX=&TOTX/&I
LET        &AVGY=&TOTY/&I
LET        &AVGZ=&TOTZ/&I
PUTPIC     FILE=OUT,LINES=4

```

AVG EU AVG SU1 AVG SU2

```

-----
PUTPIC     FILE=OUT,(&AVCX,&AVGY,&AVGZ)
          .,...  .,...  ...,.
END

```

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