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Soft-photon analysis of pion-proton bremsstrahlung and the magnetic moment of Delta(1232)

Lin, Dahang, Ph.D.

City University of New York, 1992

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SOFT-PHOTON ANALYSIS OF PION-PROTON BREMSSTRAHLUNG AND THE MAGNETIC MOMENT OF DELTA(1232)

by

Dahang Lin

A dissertation submitted to the Graduate Faculty in Physics in
partial fulfillment of the requirements for the degree of Doctor of
Philosophy, The City University of New York

1992

This manuscript has been read and accepted for the Graduate Faculty in Physics in satisfaction of the dissertation requirment for the degree of Doctor of Philosophy.

May 15, 1992
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Abstract

Soft-photon Analysis of Pion-proton Bremsstrahlung and the Magnetic Moment of $\Delta(1232)$

by

Dahang Lin

Adviser : Professor MingKung Liou

We have rigorously derived a special two-energy-two-angle amplitude for the $\pi^{\pm}p\gamma$ process near the $\Delta(1232)$ resonance. In order to take into account bremsstrahlung emission from an internal Δ line with both charge and the anomalous magnetic moment $\lambda(\lambda_{\Delta}$ for the Δ^{++} and λ_{Δ}' for the Δ^{0}), we have applied a radiation decomposition identity to modify Low's standard prescription for constructing a soft-photon amplitude. We have also used the special two-energy-two-angle amplitude to calculate all $\pi^{\pm}p\gamma$ cross sections which can be compared with the experimental data. Treading λ as a free parameter in these calculations, the magnetic moments of the Δ, μ_{Δ} for the Δ^{++} and $\mu_{\Delta}'$for the Δ^{0} , have been extracted from the UCLA data and the SIN data. The average value of μ_{Δ} determined from the experimental data is $4.35e/(2m_p)$ (m_p is the proton mass), which is in good agreement with the value $4.25\ e/(2m_p)$ predicted by a modified SU(6) model. The average value of μ_Δ' determined from the experimental data is 0.5 $e/(2m_p)$, which is in accord with the value predicted by the SU(6) model. Finally, we show that the overall agreement between theory and experiment is excellent if the special two-energy-two-angle amplitude is used in the calculation with the extracted μ_Δ (or μ_Δ') as a input.

ACKNOWLEDGMENTS

Without support of many friends and colleagues, I could never have finished this work. I have this opportunity to express my deepest gratitude to those people, who gave me support and encouragement. First, there is Professor Ming-Kung Liou, my mentor, who always give me constant encouragement and valuable guidance. Then there are Professor Louis Celenza and Professor Micheal I. Sobel, who knew what kind of help I needed and when I needed it. There are Dr. B. Gibson and Prof. R. Brown, who gave me useful suggestions and discussions. There is Qixian Zhang, my wife, who gave me strong support, spiritual, and practical, throughout all my life.

I wish to acknowledge with gratitude the Department of Physics, Brooklyn College and the Research Foundation of the CUNY for the financial support during my graduate work.

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I. Introduction

Historically, the most important reason for studying the radiation accompanying a scattering process has been to investigate the off-shell behavior of the process; information which is not available in the process itself. In 1963, for example, Sobel and Cromer[1] suggested that nucleon-nucleon bremsstrahlung would be an ideal (the simplest and the most direct) process for investigating the off-shell behavior of the two-nucleon interaction. They introduced a nonrelativistic potential model calculation in which the nuclear interaction is treated exactly by using a phenomenological potential and the electromagnetic interaction is treated only to first order as a perturbation. Since various phenomenological potentials, which are equivalent on the energy shell, can be used to generate the half-off-shell T-matrix elements, different potentials can be distinguished by bremsstrahlung measurements. When theoretical predictions are compared with the experimental measurements, the best potential may be selected[2].

In addition to the study of the off-shell effects, there are several areas in which one hopes to use the radiation accompanying certain processes to obtain specific information about these processes. The use of bremsstrahlung emission as a tool for investigating nuclear reactions is a well-known example. This idea was first proposed by Eisberg, Yennie and Wilkinson in 1960[3]. Their classical treatment was extended later to a quantum mechanical treatment by Feshbach and Yennie[4]. The theory behind this idea is very simple. Briefly, the amplitude which represents the photon emission before nuclear scattering and the amplitude which represents the photon emission after scattering add coherently. Since these two amplitudes differ in phase by $\omega\tau$ (ω is the radiation frequency and τ is the time delay), the bremsstrahlung cross section evaluated from these two amplitudes (and an internal amplitude obtained through the gauge invariant condition) will contain an interference term which depends upon the time delay τ. For small values of $\omega\tau$, one obtains a typical, smooth bremsstrahlung spectrum with $1/k$ dependence (k is the photon energy). As $\omega\tau$ increases, the interference between the two amplitudes is altered, causing a change in the bremsstrahlung spectrum. In other words, the bremsstrahlung spectrum will show structure when a long-lived resonant state is formed. A quantitative measurement of the bremsstrahlung cross section can then provide a measure of the time delay. This information about the time delay can be used to distinguish unambiguously between a direct nuclear reaction and a compound nuclear reaction. A serious attempt to measure the proton-carbon bremsstrahlung ($p^{12}C\gamma$) cross sections near the 1.7- and 0.5-MeV resonances and to extract useful information about the time delay was made by the Bologna group[5], the Brooklyn group[6,7] and the Tokyo group[8]. Each group has clearly observed the resonant structure, and a delay time of the order of 10^{-20} second has also been extracted from the measured resonant structure[9].

Another important application of bremsstrahlung processes is to study the electromagnetic properties of resonances. A typical example is the study of pion-proton bremsstrahlung ($\pi^{\pm}p\gamma$) in the region of the $\Delta(1232)$ resonance which was originally suggested for investigating the electromagnetic multipole moments of the Δ(1232) resonance.

The idea of using the $\pi^{\pm}p\gamma$ processes to determine the electromagnetic multipole moments of the Δ resonance was first proposed by Kondratyuk and Ponomarev[10] in 1968. It is well-known that the magnetic moment of the Δ(1232) cannot be measured by the conventional spin precession or atomic x-ray methods because of the short life time of the Δ. The bremsstrahlung method works as follows: One measures the $\pi^{\pm}p\gamma$ cross section as a function of the photon energy k at an incident kinetic energy about the resonance. If this incident kinetic energy is very far from the resonance, one obtains a typical bremsstrahlung spectrum with a characteristic 1/k dependence since the contribution from the resonance effect is small. In the vicinity of the resonance, however, such effects become significant and one expects a resonant structure to appear in the bremsstrahlung spectrum, which can be used to extract the magnetic multipole moments of the Δ resonance. Hoping to see such structure due to the resonance, an experimental group at UCLA[11] used 19 photon counters at many different angles to measure the $\pi^{\pm}p\gamma$ spectrum at three bombarding energies for each of the π^{+} and π^{-} beams. As a result, 108 spectra were obtained, but, surprisingly, no resonant structure was clearly observed in these spectra. Except for some cases with bad statistical accuracy, most of them exhibit a simple 1/k dependence, implying very little contribution from the resonance.

It was these unexpected experimental results that ruled out most of the theoretical calculations. The large discrepancy between the UCLA data and most of the theoretical predictions has encouraged two other experimental groups, the SIN group[12] and the CERN group[13], to obtain new experimental results which again confirm the observations of the UCLA experiment. It has also encouraged theorists to search for a fundamental theory which can be used not only t describe the experimental observations but also to extract the magnetic moment of the Δ from the $\pi^{\pm}p\gamma$ data.

Among various theoretical models and approximations proposed during the past three decades for bremsstrahlung calculations, the most important and commonly used approximations are the soft-photon approximation (or the model independent approximations). The soft-photon approximations are based upon a fundamental theorem, known as the soft-photon theorem or the low-energy theorem for photons. It was first derived by Low[14] and was extended later by Adler and Dothan[15]. This theorem states that the first two terms in the series expansion of the bremsstrahlung amplitude (or differential cross section) in powers of the photon energy k may be calculated exactly in terms of the corresponding elastic amplitude and the electromagnetic constants of the participating particles. Thus, the theorem provides a method for constructing an approximate bremsstrahlung amplitude, which can be used to calculate the bremsstrahlung cross section in terms of the corresponding elastic amplitude.

However, the soft-photon theorem states nothing about the energy and the scattering angle at which the elastic amplitude should be evaluated. Since there are two different energies (the initial energy $\sqrt{s_i}$ and the final energy $\sqrt{s_f}$) and two different scattering angles (determined from $t_p$and t_q, quantities s_i, s_f, t_p, and t_q will be defined in section II.) which can be defined for any bremsstrahlung process, the elastic amplitude can be evaluated at any linear combination of s_i and s_f $[s_{\alpha\beta}=(\alpha s_i+\beta s_f)/(\alpha+\beta)]$ and any linear combination of t_p and t_q $[t_{\alpha'\beta'}=(\alpha' t_p+\beta' t_q)/(\alpha'+\beta')]$[16]. This is the theoretical ambiguity involved in using this theorem, and this ambiguity implies that the prescription used to construct an approximate bremsstrahlung amplitude is by no means unique.

Various soft-photon amplitudes, which are consistent with the soft-photon theorem, have been constructed by using Low's prescription. Low's prescription involves the following steps : (A) Obtain the external amplitude, M^E_μ, from the four external emission diagrams and expand M^E_μ in powers of k. (B) Impose the gauge invariant condition, $M^I_\mu k^\mu = -M^E_\mu k^\mu$, to obtain the leading term (order k^0) of the internal amplitude, M^I_μ . (C) Combine M^E_μ and M^I_μ to obtain the total bremsstrahlung amplitude, M_μ. The first two terms of the expansion of M_μ, which are independent of the off-shell effects, define a soft-photon amplitude. Depending upon how many energies and scattering angles are involved, soft-photon amplitudes have been divided into the following classes:[16] (i) the one-energy-one-angle (OEOA) approximation, which includes Low's original soft-photon approximation[14], the external-emission-dominance (EED) approximation of Nefkens and Sober[17], and the modified soft-photon approximation of Nutt, Liu and Liou[18], (ii) the one-energy-two-angle (OETA) approximation (iii) the two-energy-one-angle (TEOA) approximation, which includes the Feshbach-Yennie approximation (FYA)[4,19], (iv) the two-energy-two-angle (TETA) approximation, which includes the

Fischer-Minkowski approximation (FMA)[20], Heller's approximation[21], and Ding-Lin-Liou approximation[22], and (v) other approximations.

Recent studies[16,22] show that the OEOA and OETA approximations have failed to adequately describe the $\pi^{\pm}p\gamma$ and $p^{12}C\gamma$ data. The combined $\pi^{\pm}p\gamma$ and $p^{12}C\gamma$ data can only be described by special two-energy amplitudes (i.e., those amplitudes which depend upon two special energies, the initial energy $\sqrt{s_i}$ and the final energy $\sqrt{s_f}$). Moreover, TETAS amplitudes (i.e., the special two-energy-two-angle amplitudes which depend upon two special energies and two special scattering angles) are found to give the best fit to the combined data.

The TETAS amplitudes have been investigated by Fischer and Minkowski[20], by Heller[21], and most recently by Ding, Lin and Liou[22]. The amplitude obtained by Heller has ignored the contributions from the magnetic moment of proton and the magnetic moment of the $\Delta(1232)$, while the amplitude of Ding-Lin-Liou includes these contributions generated from the magnetic moment of proton. Both amplitudes can be successfully applied to describe the combined $\pi^{\pm}p\gamma$ and $p^{12}C\gamma$ data, but neither of them can be used to determine the magnetic moment of the Δ from the $\pi^{\pm}p\gamma$ data. Let us explain this point more precisely. As we know, bremsstrahlung emissions from the internal Δ^{++} line, for example, involve two sources : one contribution comes from the charge of the Δ^{++} and another contribution is due to the magnetic moment of the Δ^{++}. Low's prescription can be applied to find the expression for the charge contribution. (The expressions for the charge contribution obtained in Refs. 20, 21 and 22 are all identical even though the expressions are written in different forms.) But it is very difficult to obtain the expression for the magnetic contribution by using Low's prescription. This is because the magnetic contribution involves an important term which depends upon the anomalous magnetic moment of the Δ^{++}, λ_Δ, and this λ_Δ-dependent term is separately gauge invariant[23]. (If M^μ_λ is the λ_Δ-dependent term which is separately gauge invariant, then we have $M^\mu_\lambda K_\mu=0$. In that case, M^μ_λ cannot be derived from the external amplitude by imposing the gauge invariant condition. Imposing the gauge invariant condition to determine the leading term of the internal amplitude is the most important step in Low's prescription.) This explains why a soft-photon amplitude which takes into account photon emission from the Δ^{++} (including both the charge contribution and the magnetic contribution) has never before been constructed. Since the amplitudes obtained before do not have the λ_Δ-dependent term, these amplitudes cannot be used to extract λ_Δ or the magnetic moment of the Δ^{++} from the $\pi^+p\gamma$ data.

In this thesis, we focus our study on the $\pi^\pm p\gamma$ processes near the $\Delta(1232)$ resonance. The major contribution of our study (some of our results have already been published[23]) can be summarized as follows:

(i) A soft-photon bremsstrahlung amplitude in the special two-energy-two-angle (TETAS) approximation has been derived for the $\pi^\pm p\gamma$ processes near the $\Delta(1232)$ resonance. The amplitude includes both the external amplitude, which takes into account photon emissions from the pions and protons (with charge and magnetic moment), and the internal amplitude, which represents photon emissions from the internal Δ-line. Since Low's original prescription cannot be used to obtain an internal contribution which is separately gauge invariant (the λ_Δ-dependent term), we have developed a modified procedure to construct the TETAS amplitude which includes the λ_Δ-dependent term[23]. The first step in this new procedure is exactly the same as Low's original prescription, i.e., to obtain the external amplitude M^E_μ and to expand it in powers of photon energy k. The second step is to obtain an internal contribution M^Δ_μ, which represents photon emission from the internal Δ-line, and to split M^Δ_μ into four quasiexternal amplitudes by using a generalized Brodsky-Brown identity.[24] The third step is to obtain an additional gauge invariant term M^G_μ by imposing the gauge invariant condition, $M^G_\mu k^\mu = - M^{E\Delta}_\mu k^\mu$. Here, $M^{E\Delta}_\mu = M^E_\mu + M^\Delta_\mu$. The last step is to obtain the total amplitude M_μ by combining $M^{E\Delta}_\mu$ with M^G_μ : $M_\mu = M^{E\Delta}_\mu + M^G_\mu$. The first two terms of the expansion of M_μ, which can be written in terms of the complete elastic T-matrix, define the TETAS amplitude.

(ii) Using the TETAS amplitudes obtained in (i), we have determined the magnetic moments of the Δ^{++}, μ_Δ, and the Δ^0, μ_Δ', from the $\pi^+ p\gamma$ and the $\pi^- p\gamma$ data, respectively. Our work represents the first successful attempt to extract μ_Δ and μ_Δ' by fitting to more than 85% of the available experimental $\pi^\pm p\gamma$ data. We have also demonstrated that the overall agreement between the experimental data and the theoretical calculations (based upon the TETAS amplitude with the extracted value of the magnetic moment of the Δ as an input) is excellent. To the best of our knowledge, such an agreement has never before been obtained.

The plan of this thesis is as follows: In section II, we derive the TETAS amplitude for the $\pi^{+}p\gamma$ process near the Δ^{++} resonance. In section III, we determine the magnetic moment of the Δ^{++} from the UCLA data[11] and the SIN data[12] by using the TETAS amplitude derived in section II. The extracted values of μ_{Δ} are in good agreement with the value predicted by the modified SU(6) model. In section IV, we show that the overall agreement between the $\pi^{+}p\gamma$ data and the theoretical predictions is excellent if the extracted values of μ_{Δ} are used as an input in the TETAS amplitude for calculations. In section V, we first obtain another TETAS amplitude for the $\pi^{-}p\gamma$ process near the Δ^{0} resonance. We then use the obtained amplitude to extract the magnetic moment of the Δ^{0}, μ_{Δ}' , from the UCLA data. Finally, we present a comparison between the experimental $\pi^{-}p\gamma$ spectra and the calculated spectra. Section VI is devoted to further studies and discussions. Our conclusion is given in the last section. There are two Appendices. Appendix A gives the detailed expressions for the elastic T-matrix and the four half-off-shell T-matrices at the tree-level approximation. The explicit expressions for some off-shell terms are shown in Appendix B.

II. Bremsstrahlung Amplitude for the $\pi^+p\gamma$ Process

This section is divided into three parts. In part (A), we discuss the π^+p elastic scattering process (Fig. 1a). We define the general form for the π^+p elastic T-matrix, T, which is an important input for bremsstrahlung calculations. In the energy region of the Δ^{++} resonance, a tree diagram given by Fig. 1b, $\pi^+p \longrightarrow \Delta^{++} \longrightarrow \pi^+p$, becomes the dominant elastic diagram. We derive the explicit expression for the T-matrix corresponding to Fig. 1b, $\tilde{T}$, which will be used to define a TETAS amplitude $\tilde{M}_\mu^{TETAS}$ for the $\pi^+p\gamma$ process at the tree level. In part (B), we treat Fig. 1b as a source graph to generate π^+p bremsstrahlung diagrams at the tree level (Figs. 2a-2e). By using a generalized Brodsky-Brown identity[24] for photon emission from the internal Δ^{++} line (Fig. 2e), we derive the expression for $\tilde{M}_\mu^{TETAS}$ in terms of $\tilde{T}$ and the electromagnetic constants of π^+, p and Δ^{++}. The amplitude $\tilde{M}_\mu^{TETAS}$ plays a vital role in our derivation of a more general TETAS amplitude, M_μ^{TETAS}, for the $\pi^+p\gamma$ process. In part (C), we use the modified Low procedure to derive the amplitude M_μ^{TETAS} which can be written in terms of the general form of the elastic T-matrix T and the electromagnetic constants of π^+, p, and Δ^{++}. In deriving M_μ^{TETAS}, we have imposed a condition that M_μ^{TETAS} reduces to $\tilde{M}_\mu^{TETAS}$ in the energy region of the Δ^{++} resonance.

(A) π^+p elastic scattering T-matrix :

We consider the $\pi^+p\gamma$ process,

$$\pi^+(q_i^\mu) + P(p_i^\mu) \longrightarrow \pi^+(q_f^\mu) + P(p_f^\mu) + \gamma(k^\mu) , \qquad (1)$$

where $q_i^\mu(q_f^\mu)$ and $p_i^\mu(p_f^\mu)$ are the initial (final) four-momenta of the pion and proton, respectively, and k^μ is the four-momentum of the emitted photon. These four-momenta satisfy energy-momentum conservation:

$$q_i^{\mu} + p_i^{\mu} = q_f^{\mu} + p_f^{\mu} + k^{\mu}. \tag{2}$$

In the limit when k approaches zero, the $\pi^+ p\gamma$ process reduces to the corresponding $\pi^+ p$ elastic scattering process,

$$\pi^+(q_i^{\mu}) + P(p_i^{\mu}) \longrightarrow \pi^+(\bar{q}_f^{\ \mu}) + P(\bar{p}_f^{\ \mu}) \ , \tag{3}$$

where

$$\bar{p}_f^{\ \mu} = \lim_{k\to 0} p_f^{\mu}$$

$$\bar{q}_f^{\ \mu} = \lim_{k\to 0} q_f^{\mu} \ .$$

The energy-momentum conservation becomes

$$q_i^{\mu} + p_i^{\mu} = \bar{q}_f^{\ \mu} + \bar{p}_f^{\ \mu} \ . \tag{4}$$

A diagram which represents the $\pi^+ p$ elastic scattering process is shown in Fig. 1a. In this diagram, T represents the $\pi^+ p$ elastic scattering T-matrix. Although we are interested in the TETAS amplitude which depends only on the elastic (on-shell) T-matrix, the exact bremsstrahlung amplitude without the soft-photon approximation involves off-shell T-matrices. Thus, we have to show how a TETAS amplitude which is independent of the off-shell effects can be derived. All T-matrices, on-shell or off-shell, can be written in terms of six Lorentz invariants as

$$T(s, t, p_i^2, q_i^2, p_f^2, q_f^2). \tag{5}$$

Here, s is the total energy squared and t is the momentum transfer squared. For the $\pi^+ p$ elastic scattering process, the elastic T-matrix depends only on two independent variables, s and t, since all four external lines (legs) are on their mass shells, i.e., the on-mass-shell conditions,

$$p_i^2 = p_f^2 = m_p^2 \tag{6}$$

and

$$q_i^2 = q_f^2 = m_\pi^2 ,$$

are satisfied. Here m_p and m_π are the masses of proton and pion, respectively. A half-off-shell T-matrix is defined if one of the external lines is off its mass shell. For example, if $q_i^2 \neq m_\pi^2$, then we have a half-off-shell T-matrix which can be written as

$$T (s , t , m_p^2 , q_i^2 , m_p^2 , m_\pi^2). \tag{7}$$

We can write the π^+p elastic T-matrix in the standard form

$$T(s , t) \equiv T(s , t , m_p^2 , m_\pi^2 , m_p^2 , m_\pi^2)$$

$$= A(s , t) + \frac{1}{2}(\not{q}_i + \not{\bar{q}}_f) B(s , t) \tag{8}$$

where

$$s = (p_i + q_i)^2 = (\bar{p}_f + \bar{q}_f)^2$$

and

$$t = (\bar{p}_f - p_i)^2 = (\bar{q}_f - q_i)^2 .$$

If s and t are given (or if the incident energy and the scattering angle are known), the amplitudes A(s , t) and B(s , t) can be calculated in terms of π^+p phase shifts and inelasticities, determined by the π^+p elastic scattering experiments. The experimentally determined T-matrix has been used as an input for all bremsstrahlung calculations using soft-photon amplitudes.

In the energy region of the $\Delta^{++}(1232)$ resonance, the Feynman diagram given by Fig. 1b is the dominant contribuion to the π^+p

elastic process and the photon emission from the intermediate Δ^{++} line becomes significant in that region. This diagram, which will be treated as a source graph to generate photon emission diagrams at the tree level, is important in our derivation of the TETAS amplitude. The elastic T-matrix corresponding to Fig. 1b has the form:

$$\tilde{T} = [\ g\ \bar{q}_f^{\ \rho}\ ]\ G_{\rho\alpha}(p)\ [\ g\ q_i^{\ \alpha}\ ] \tag{9}$$

where g is the $\pi^+ p\Delta^{++}$ vertex, $p^\mu = p_i^{\ \mu} + q_i^{\ \mu}$,

$$G_{\rho\alpha}(p) = \frac{i\ d_{\rho\alpha}(p)}{p^2 - M_\Delta^2 + i\varepsilon}\quad , \tag{10a}$$

$$d_{\rho\alpha}(p) = (\ \not{p} + M_\Delta\ )[\ g_{\rho\alpha} - \frac{1}{3}\ \gamma_\rho\gamma_\alpha - \frac{1}{3M_\Delta}(\ \gamma_\rho p_\alpha - \gamma_\alpha p_\rho) - \frac{2}{3M_\Delta^2} p_\rho p_\alpha\ ]$$

$$+ \frac{2}{3M_\Delta^2}(\ p^2 - M_\Delta^2)[\ \gamma_\rho p_\alpha - \gamma_\alpha p_\rho + (\ \not{p} + M_\Delta\ )\ \gamma_\rho\gamma_\alpha\ ] \tag{10b}$$

and M_Δ is the mass of the Δ^{++}. In terms of $s, t, p_i^2, q_i^2, \bar{p}_f^2$ and $\bar{q}_f^2$, $\tilde{T}$ can be written as

$$\tilde{T} \equiv \tilde{T}\ (\ s,\ t,\ p_i^2,\ q_i^2,\ \bar{p}_f^2,\ \bar{q}_f^2\ )$$

$$= \tilde{A}\ (s,t,p_i^2,q_i^2,\bar{p}_f^2,\bar{q}_f^2\ ) + \frac{1}{2}(\ \not{q}_i + \bar{\not{q}}_f\ )\ \tilde{B}\ (s,t,p_i^2,q_i^2,\bar{p}_f^2,\bar{q}_f^2) \tag{11}$$

where

$$\tilde{A}\ (\ s,\ t,\ p_i^2,\ q_i^2,\ \bar{p}_f^2,\ \bar{q}_f^2\ )$$

$$= \frac{ig^2}{s - M_\Delta^2 + i\varepsilon}\left\{\frac{1}{2}(M_\Delta + m_p)\left[-t + q_i^2 + \bar{q}_f^2 - \frac{1}{3M_\Delta^2}(s - \bar{p}_f^2 + \bar{q}_f^2\ )(s - p_i^2 + q_i^2\ )\right]\right.$$

$$\left. - \frac{1}{6M_\Delta}\ \bar{q}_f^2\ (s - p_i^2 + q_i^2\ ) + (s - p_i^2\ )\left[\frac{1}{6M_\Delta}(s - \bar{p}_f^2 + \bar{q}_f^2\ ) + \frac{2s - 3M_\Delta^2}{3M_\Delta^2}(M_\Delta + m_p)\right]\right\} \tag{12a}$$

$$= \frac{ig^2}{s - M_\Delta^2 + i\varepsilon} \left\{ \frac{1}{2}(M_\Delta + m_p)\left[-t + q_i^2 + \bar{q}_f^2 - \frac{1}{3M_\Delta^2}(s - \bar{p}_f^2 + \bar{q}_f^2)(s - p_i^2 + q_i^2) \right] \right.$$

$$\left. - \frac{1}{6M_\Delta} q_i^2 (s - \bar{p}_f^2 + \bar{q}_f^2) + (s - p_i^2)\left[\frac{1}{6M_\Delta}(s - p_i^2 + q_i^2) + \frac{2s - 3M_\Delta^2}{3M_\Delta^2}(M_\Delta + m_p) \right] \right\} \tag{12b}$$

and

$$\tilde{B}(s, t, p_i^2, q_i^2, \bar{p}_f^2, \bar{q}_f^2)$$

$$= \frac{ig^2}{s - M_\Delta^2 + i\varepsilon} \left\{ \frac{1}{2}\left[-t + q_i^2 + \bar{q}_f^2 - \frac{1}{3M_\Delta^2}(s - \bar{p}_f^2 + \bar{q}_f^2)(s - p_i^2 + q_i^2) \right] \right.$$

$$+ \frac{1}{2}\left(\frac{2s}{3M_\Delta^2} - 1 - \frac{m_p}{3M_\Delta} \right)(\bar{p}_f^2 - p_i^2 + q_i^2 - \bar{q}_f^2) + \left(\frac{2s - 3M_\Delta^2}{3M_\Delta^2} \right)\bar{q}_f^2 \tag{12c}$$

$$\left. - 2m_p \left[\frac{1}{6M_\Delta}(s - \bar{p}_f^2 + \bar{q}_f^2) + \frac{2s - 3M_\Delta^2}{3M_\Delta^2}(M_\Delta + m_p) \right] \right\}$$

$$= \frac{ig^2}{s - M_\Delta^2 + i\varepsilon} \left\{ \frac{1}{2}\left[-t + q_i^2 + \bar{q}_f^2 - \frac{1}{3M_\Delta^2}(s - \bar{p}_f^2 + \bar{q}_f^2)(s - p_i^2 + q_i^2) \right] \right.$$

$$+ \frac{1}{2}\left(\frac{2s}{3M_\Delta^2} - 1 - \frac{m_p}{3M_\Delta} \right)(p_i^2 - \bar{p}_f^2 + \bar{q}_f^2 - q_i^2) + \left(\frac{2s - 3M_\Delta^2}{3M_\Delta^2} \right) q_i^2 \tag{12d}$$

$$\left. - 2m_p \left[\frac{1}{6M_\Delta}(s - p_i^2 + q_i^2) + \frac{2s - 3M_\Delta^2}{3M_\Delta^2}(M_\Delta + m_p) \right] \right\} .$$

In Eqs. (12a)-(12d), p_i^2, q_i^2, $\bar{p}_f^2$ and $\bar{q}_f^2$ satisfy the on-mass-shell conditions given by Eq. (6). Without imposing these on-mass-shell conditions explicitly, the expressions for $\tilde{A}$ and $\tilde{B}$ can be extended to define the half-off-shell T-matrix later.

(B) TETAS amplitude for the $\pi^+ p\gamma$ process at the tree level:

As we have already mentioned , Fig. 1b will be used as a source graph to generate photon emission diagrams at the tree level. Five diagrams generated by Fig. 1b are shown in Fig. 2. (We shall impose the gauge invariant condition later to take care of the remaining contributions.) The first four Feynman diagrams (2a-2d) represent the photon emissions from external pion lines and external proton lines and the last Feynman diagram (2e) represents photon emission from the internal Δ^{++} line.

From the four external emission diagrams (2a-2d), we can define the following half-off-shell T-matrices for $\pi^+ p$ interactions ($\pi^+ p \to \Delta^{++} \to \pi^+ p$):

$$\tilde{T}_a = [g\,(q_f+k)^\rho]\; G_{\rho\alpha}(p)\; [g\, q_i^{\;\alpha}], \tag{13a}$$

$$\tilde{T}_b = [g\, q_f^{\;\rho}]\; G_{\rho\alpha}(p')\; [g\,(q_i-k)^\alpha\;], \tag{13b}$$

$$\tilde{T}_c = [g\, q_f^{\;\rho}]\; G_{\rho\alpha}(p)\; [g\, q_i^{\;\alpha}\;]\;, \tag{13c}$$

and

$$\tilde{T}_d = [g\, q_f^{\;\rho}]\; G_{\rho\alpha}(p')\; [g\, q_i^{\;\alpha}]\;, \tag{13d}$$

Here, $p^\mu = (q_i + p_i\;)^\mu$, $p'^\mu = (q_i + p_i - k\;)^\mu = (q_f + p_f\;)^\mu$,and $G_{\rho\alpha}(p)$ are defined by Eq. (10a). It is easy to show that $\bar{u}(p_f,\nu_f)\;\tilde{T}_a\; u(p_i,\nu_i)$ and $\bar{u}(p_f,\nu_f)\;\tilde{T}_b\; u(p_i,\nu_i)$ can be written in terms of $\tilde{A}$ and $\tilde{B}$ [given by Eqs. (12a) and (12c), respectively] as

$$\bar{u}\;\tilde{T}_a\; u = \bar{u}\left\{ \tilde{A}\,[\; s_i\;, t_p,\; p_i^2,\; q_i^2,\; p_f^2,\; \Delta_a] \right.$$
$$\left. + \frac{1}{2}(\not{q}_i + \not{q}_f + \not{k}\;)\;\tilde{B}\,[\; s_i,\; t_p,\; p_i^2,\; q_i^2,\; p_f^2,\; \Delta_a] \right\} u \tag{14a}$$

and

$$\bar{u}\,\tilde{T}_b\,u = \bar{u}\left\{ \tilde{A}\,[\,s_f\,,t_p,\ p_i^2,\ \Delta_b,\ p_f^2,\ q_f^2] + \frac{1}{2}(\not{q}_i + \not{q}_f - \not{k}\,)\ \tilde{B}\,[\,s_f,\ t_p,\ p_i^2,\ \Delta_b,\ p_f^2, q_f^2\,]\right\} u \qquad (14b)$$

where

$$s_i = (\,p_i + q_i\,)^2 = (\,p_f + q_f + k\,)^2\ ,$$

$$s_f = (\,p_f + q_f\,)^2 = (\,p_i + q_i - k\,)^2\ ,$$

$$t_p = (\,p_f - p_i\,)^2$$

$$\Delta_a = (\,q_f + k\,)^2 = m_\pi^2 + 2q_f\cdot k\ ,$$

and

$$\Delta_b = (\,q_i - k\,)^2 = m_\pi^2 - 2q_i\cdot k\ .$$

However, the expressions for $\tilde{T}_c u(p_i,\nu_i)$ and $\bar{u}(p_f,\nu_f)\ \tilde{T}_d$ involve extra off-shell amplitudes:

$$\tilde{T}_c\ u = \left\{ \tilde{A}\ [s_i,t_q,\ p_i^2,\ q_i^2,\ \ \Delta_c,\ q_f^2\,] + \frac{1}{2}(\,\not{q}_i+\not{q}_f\,)\ \tilde{B}\ [s_i,\ t_q,\ p_i^2,\ q_i^2,\ \ \Delta_c,\ q_f^2\,] + \frac{1}{2}(\,\not{p}_f+\not{k}\ -m_p)\ \tilde{C}\ [s_i,\ t_q,\ p_i^2,\ q_i^2,\ \ \Delta_c,\ q_f^2\,]\right\} u \qquad (15a)$$

and

$$\bar{u}\,\tilde{T}_d = \bar{u}\left\{ \tilde{A}\ [s_f,t_q,\ \ \Delta_d,\ q_i^2,\ p_f^2,\ q_f^2\,] + \frac{1}{2}(\,\not{q}_i + \not{q}_f\,)\ \tilde{B}\ [s_f,t_q,\ \ \Delta_d,\ q_i^2,\ p_f^2,\ q_f^2\,] + \frac{1}{2}\ \tilde{C}\ [s_f,t_q,\ \ \Delta_d,\ q_i^2,\ p_f^2,\ q_f^2\,](\,\not{p}_i - \not{k}\ -m_p\,)\right\} \qquad (15b)$$

where

$$t_q = (\,q_f - q_i\,)^2\ ,$$

$$\Delta_c = (\,p_f + k\,)^2 = m_p^2 + 2p_f\cdot k\ ,$$

$$\Delta_d = (\,p_i - k\,)^2 = m_p^2 - 2p_i\cdot k\ ,$$

and the extra off-shell amplitude $\tilde{C}$ has the form

$$\tilde{C}\,[s,t,\,p_i^2,\,q_i^2,\,p_f^2,\,q_f^2\,]$$

$$= \frac{ig^2}{s-M_\Delta^2+i\varepsilon}\left\{\frac{1}{2}\left[-t+q_i^2+q_f^2-\frac{1}{3M_\Delta^2}(s-p_f^2+q_f^2)(s-p_i^2+q_i^2)\right]\right.$$

$$+\frac{1}{2}\left(\frac{2s}{3M_\Delta^2}-1-\frac{m_p}{3M_\Delta}\right)(2s-p_i^2-p_f^2+q_i^2+q_f^2)-\left(\frac{2s-3M_\Delta^2}{3M_\Delta^2}\right)m_\pi^2$$

$$-2m_p\left[\frac{2s-3M_\Delta^2}{3M_\Delta^2}(m_p+M_\Delta)+\frac{1}{6M_\Delta}(s-m_p^2+m_\pi^2)\right] \qquad (16)$$

$$\left.-2\not{p}_i\left[\frac{2s-3M_\Delta^2}{3M_\Delta^2}(m_p+M_\Delta)+\frac{1}{6M_\Delta}(s-m_p^2+m_\pi^2)\right]\right\}\;.$$

The expressions for $\tilde{A}$ and $\tilde{B}$ in Eq. (15b) are given by Eqs. (12a) and (12c), respectively, but the expressions for $\tilde{A}$ and $\tilde{B}$ in Eq. (15a) are given by Eqs. (12b) and (12d), respectively. Again, p_i^2, q_i^2, p_f^2 and q_f^2 in Eqs. (14) and (15) satisfy the on-mass-shell conditions given by Eq.(6). Since we are interested in the soft-photon approximation, the extra off-shell amplitude involving $\tilde{C}$ will be neglected later in our derivation of the TETAS amplitude. (Justification for neglecting the extra off-shell amplitude will be discussed again in the next section.)

From the expressions for T_x (x=a,b,c,d) given by Eqs.(14) and (15),we can see that T_x depends on the square of the invariant mass Δ_x (x=a,b,c,d) of the off-mass-shell leg on which the photon emission occurs. As k approaches zero, Δ_x reduces to $(\text{mass})^2$ and T_x reduces to on-shell (elastic) T-matrix. Since the TETAS amplitude which we wish to derive depends only on the on-shell T-matrix [evaluated at four different sets : $(s_i,\ t_p)$, (s_f, t_p), $(s_i,\ t_q)$ and $(s_f,\ t_q)$], we must expand T_x in powers of k. Keeping only terms to order k, we obtain

$$\bar{u}\,\tilde{T}_a\, u = \bar{u}\ [\ \tilde{T}\ (s_i, t_p) + 2q_f\cdot k\ (\frac{\partial\ \tilde{T}_a}{\partial\ \Delta_a}) + \ldots\ldots]\ u\ , \tag{17a}$$

$$\bar{u}\,\tilde{T}_b\, u = \bar{u}\ [\ \tilde{T}\ (s_f, t_p) - 2q_i\cdot k\ (\frac{\partial\ \tilde{T}_b}{\partial\ \Delta_b}) + \ldots\ldots]\ u\ , \tag{17b}$$

$$\tilde{T}_c\ u = [\ \tilde{T}(s_i, t_q) + 2p_f\cdot k\ (\frac{\partial\ \tilde{T}'_c}{\partial\ \Delta_c}) + \tilde{T}_{cc}(s_i, t_q) + \ldots\ldots]\ u\ , \tag{17c}$$

and

$$\bar{u}\,\tilde{T}_d = \bar{u}\ [\ \tilde{T}(s_f, t_q) - 2p_i\cdot k\ (\frac{\partial\ \tilde{T}'_d}{\partial\ \Delta_d}) + \tilde{T}_{dc}(s_f, t_q) + \ldots\ldots]\ , \tag{17d}$$

where

$$\tilde{T}(s_i, t_p) = \tilde{A}\ (s_i, t_p,\ m_p^2,\ m_\pi^2,\ m_p^2,\ m_\pi^2\ ) + \frac{1}{2}(\ \not{q}_i + \not{q}_f + \not{k}\ )\ \tilde{B}\ (s_i, t_p, m_p^2, m_\pi^2, m_p^2, m_\pi^2) \tag{18a}$$

$$\tilde{T}(s_f, t_p) = \tilde{A}\ (s_f, t_p,\ m_p^2,\ m_\pi^2,\ m_p^2,\ m_\pi^2\ ) + \frac{1}{2}(\ \not{q}_i + \not{q}_f - \not{k}\ )\ \tilde{B}\ (s_f, t_p, m_p^2, m_\pi^2, m_p^2, m_\pi^2) \tag{18b}$$

$$\tilde{T}(s_i, t_q) = \tilde{A}\ (s_i, t_q,\ m_p^2,\ m_\pi^2,\ m_p^2,\ m_\pi^2\ ) + \frac{1}{2}(\ \not{q}_i + \not{q}_f\ )\ \tilde{B}\ (s_i, t_q, m_p^2, m_\pi^2, m_p^2, m_\pi^2) \tag{18c}$$

$$\tilde{T}(s_f,t_q) = \tilde{A}\,(s_f,t_q,\,m_p^2,\,m_\pi^2,\,m_p^2,\,m_\pi^2\,)$$

$$+\,\frac{1}{2}(\,\not{q}_i + \not{q}_f\,)\,\tilde{B}\,(s_f,t_q,m_p^2,m_\pi^2,m_p^2,m_\pi^2) \tag{18d}$$

$$\tilde{T}_{cc}(s_i,t_q) = \frac{1}{2}(\,\not{p}_f + \not{k} - m_p)\,\tilde{C}\,(s_i,t_q,m_p^2,m_\pi^2,m_p^2,m_\pi^2)$$

$$+\,(p_f\cdot k)(\,\not{p}_f + \not{k} - m_p)\left(\frac{\partial\,\tilde{C}}{\partial\Delta_c}\right)\,, \tag{18e}$$

$$\tilde{T}_{dc}(s_f,t_q) = \frac{1}{2}\,\tilde{C}\,(s_f,t_q,m_p^2,m_\pi^2,m_p^2,m_\pi^2)(\,\not{p}_i - \not{k} - m_p)$$

$$-\,(p_i\cdot k)\left(\frac{\partial\,\tilde{C}}{\partial\Delta_d}\right)(\,\not{p}_i - \not{k} - m_p), \tag{18f}$$

and $\tilde{T}'_c$ and $\tilde{T}'_d$ are defined by Eqs. (15a) and (15b), respectively, but without those terms involving $\tilde{C}$.

The external scattering amplitude corresponding to the four external diagrams (2a-2d) at the tree level can be written in terms of the half-off-shell T-matrices $\tilde{T}_a$, $\tilde{T}_b$, $\tilde{T}_c$ and $\tilde{T}_d$ as

$$\tilde{M}^{(\varepsilon)}_{\mu} = \bar{u}(p_f,\nu_f)\left[\,\frac{Q_a q_{f\mu}}{q_f\cdot k}\,\tilde{T}_a - \tilde{T}_b\,\frac{Q_b q_{i\mu}}{q_i\cdot k}\right.$$

$$\left.+\,\frac{Q_c(p_f+R_f)_\mu}{p_f\cdot k}\,\tilde{T}_c - \tilde{T}_d\,\frac{Q_d(p_i+R_i)_\mu}{p_i\cdot k}\,\right]u(p_i,\nu_i), \tag{19}$$

where Q_a=Q_b represents the charge of pion, Q_c=Q_d represents the charge of proton, and $R_{i\mu}$ and $R_{f\mu}$ have the form :

$$\varepsilon^\mu R_{i\mu} = \frac{1}{4}[\,\not{\varepsilon}\,,\,\not{k}\,] + \frac{\lambda_p}{8m_p}\left\{\,[\,\not{\varepsilon}\,,\,\not{k}\,],\,\not{p}_i\,\right\}, \tag{20a}$$

and

$$\varepsilon^\mu R_{f\mu} = \frac{1}{4}[\,\not{\varepsilon}\,,\,\not{k}\,] + \frac{\lambda_p}{8m_p}\left\{\,[\,\not{\varepsilon}\,,\,\not{k}\,],\,\not{p}_f\,\right\}. \tag{20b}$$

In Eqs. (20a) and (20b), ε^{μ} is the photon polarization, λ_p is the anomalous magnetic moment of proton, and we have used $[X,Y] \equiv XY-YX$ and $\{X,Y\} \equiv XY+YX$. [Note that $R_{i\mu}$, $R_{f\mu}$ and R_{μ} (Eq.(43)) defined in this paper are slightly different from those defined in Ref.29. There is a sign difference between the two definitions since $[\not p , \not k]=-[\not k, \not p]$.] The factors $[Q_c(p_f+R_f)_{\mu} \,/\, p_f \cdot k]$ and $[-Q_d(p_i+R_i)_{\mu} \,/\, p_i \cdot k]$ in Eq. (19) are obtained from the following relations :

$$\bar{u}(p_f,\nu_f)[-iQ_c\Gamma_{\mu}][i/(\not p_f + \not k - m_p)]=\bar{u}(p_f,\nu_f)[Q_c(p_f+R_f)_{\mu}/p_f \cdot k], \tag{21}$$

which describes photon emission by an outgoing proton line with charge Q_c, anomalous magnetic moment λ_p and momentum p_f^{μ} (Fig.3a), and

$$[i/(\not p_i - \not k - m_p)](-iQ_d\Gamma_{\mu})u(p_i,\nu_i)=[-Q_d(p_i+R_i)_{\mu}/p_i \cdot k]u(p_i,\nu_i), \tag{22}$$

which describes photon emission by an incoming proton line with charge Q_d , anomalous magnetic moment λ_p and momentum p_i^{μ} (Fig.3b). Here, Γ_{μ} is the (on-shell) electromagnetic vertex,

$$\Gamma_{\mu} = \gamma_{\mu} - i\lambda_p\sigma_{\mu\nu}k^{\nu}/(2m_p) \tag{23}$$

with

$$\sigma_{\mu\nu} = i\,[\,\gamma_{\mu}\,,\,\gamma_{\nu}\,]/2\,.$$

It is easy to show that $R_{i\mu}$ and $R_{f\mu}$ are separately gauge-invariant, i.e., they satisfy

$$R_i \cdot k = R_f \cdot k = 0\,. \tag{24}$$

The internal amplitude (at the tree level) $\tilde{M}_{\mu}^{(\Delta)}$ corresponding to Fig. 2e has the form

$$\varepsilon^{\mu}\,\tilde{M}_{\mu}^{(\Delta)} = \bar{u}(p_f,\nu_f)[gq_f^{\ \rho}]G_{\rho\sigma}(p')[-i(Q_b+Q_d)\Gamma_{\mu}^{\sigma\beta}\varepsilon^{\mu}]G_{\beta\alpha}(p)[gq_i^{\ \alpha}]u(p_i,\nu_i), \tag{25}$$

where $p^{\mu} = q_i^{\mu} + p_i^{\mu}$, $p'^{\mu} = p^{\mu} - k^{\mu} = q_f^{\mu} + p_f^{\mu}$, $G_{\rho\sigma}(p')$ is the propagator for the Δ^{++} given by Eq.(10a) and $\Gamma^{\sigma\beta}_{\mu}\varepsilon^{\mu}$ is the electromagnetic vertex for the Δ^{++} (in the Rarita-Schwinger formalism but neglecting the contribution from the electric quadrupole and magnetic octupole moment of the Δ^{++}),

$$\Gamma^{\sigma\beta}_{\mu}\varepsilon^{\mu} = (\not\varepsilon + \frac{\lambda_{\Delta}}{2M_{\Delta}} \not\varepsilon \not k)g^{\sigma\beta} - \frac{1}{3} \not\varepsilon \gamma^{\sigma}\gamma^{\beta} - \frac{1}{3}(\gamma^{\sigma}\varepsilon^{\beta}-\gamma^{\beta}\varepsilon^{\sigma}) \tag{26}$$

In Eq.(26), λ_{Δ} is the anomalous magnetic moment of the Δ^{++}. The amplitude $\tilde{M}^{(\Delta)}_{\mu}$ can be decomposed into four quasi-external amplitudes by using a generalized Brodsky-Brown decomposition identity.[24] To do this, we introduce an operator $\Lambda^{\sigma\beta}(p)$,

$$\Lambda^{\sigma\beta}(p) = (\not p -m_{\Delta}) g^{\sigma\beta} - \frac{1}{3}(\gamma^{\sigma}p^{\beta}+\gamma^{\beta}p^{\sigma}) + \frac{1}{3} \gamma^{\sigma}(\not p + M_{\Delta})\gamma^{\beta} , \tag{27}$$

which satisfies the condition

$$d_{\rho\sigma}(p) \Lambda^{\sigma\beta}(p) = (p^{2}- M_{\Delta}^{2}) g_{\rho}^{\ \beta} , \tag{28}$$

where $d_{\rho\sigma}(p)$ is defined by Eq.(10b). It is easy to prove the following useful relations :

$$\frac{d_{\rho\sigma}(p')}{p'^{2}- M_{\Delta}^{2}} \Gamma^{\sigma\beta}_{\mu}\varepsilon^{\mu} d_{\beta\alpha}(p) = \frac{d_{\rho\sigma}(p')}{p'^{2}- M_{\Delta}^{2}} \left[\Gamma^{\sigma\beta}_{\mu}\varepsilon^{\mu} d_{\beta\alpha}(p) + \Lambda^{\sigma\beta}(p') \Gamma_{\mu,\beta\alpha}\varepsilon^{\mu}\right]$$
$$- g_{\rho}^{\ \beta} \Gamma_{\mu,\beta\alpha}\varepsilon^{\mu} , \tag{29}$$

$$d_{\rho\sigma}(p') \Gamma^{\sigma\beta}_{\mu}\varepsilon^{\mu} \frac{d_{\beta\alpha}(p)}{p^{2}-M_{\Delta}^{2}} = \left[d_{\rho\sigma}(p') \Gamma^{\sigma\beta}_{\mu}\varepsilon^{\mu} + \Gamma_{\mu,\rho\sigma}\varepsilon^{\mu} \Lambda^{\sigma\beta}(p)\right] \frac{d_{\beta\alpha}(p)}{p^{2}-M_{\Delta}^{2}}$$
$$- \Gamma_{\mu,\rho\sigma}\varepsilon^{\mu} g^{\sigma}_{\ \alpha} , \tag{30}$$

and

$$\frac{d_{\rho\sigma}(p')}{p'^{2}- M_{\Delta}^{2}} \Gamma^{\sigma\beta}_{\mu}\varepsilon^{\mu} d_{\beta\alpha}(p) - d_{\rho\sigma}(p') \Gamma^{\sigma\beta}_{\mu}\varepsilon^{\mu} \frac{d_{\beta\alpha}(p)}{p^{2}- M_{\Delta}^{2}}$$
$$= 2(p'\cdot k)\frac{d_{\rho\sigma}(p')}{p'^{2}- M_{\Delta}^{2}} \Gamma^{\sigma\beta}_{\mu}\varepsilon^{\mu} \frac{d_{\beta\alpha}(p)}{p^{2}- M_{\Delta}^{2}} \tag{31}$$

Combining Eqs.(29), (30) and (31), we find

$$\frac{d_{\rho\sigma}(p')}{p'^2-M_\Delta^2}\,\Gamma^{\sigma\beta}_{\mu}\varepsilon^{\mu}\,\frac{d_{\beta\alpha}(p)}{p^2-M_\Delta^2}=\frac{1}{2p'\cdot k}\,\frac{d_{\rho\sigma}(p')}{p'^2-M_\Delta^2}\left[\Gamma^{\sigma\beta}_{\mu}\varepsilon^{\mu}\,d_{\beta\alpha}(p)+\Lambda^{\sigma\beta}(p')\Gamma_{\mu,\beta\alpha}\varepsilon^{\mu}\right]$$

$$-\frac{1}{2p\cdot k}\left[d_{\rho\sigma}(p')\,\Gamma^{\sigma\beta}_{\mu}\varepsilon^{\mu}+\Gamma_{\mu,\rho\sigma}\varepsilon^{\mu}\,\Lambda^{\sigma\beta}(p)\right]\frac{d_{\beta\alpha}(p)}{p^2-M_\Delta^2} \qquad (32)$$

which gives the following decomposition identity

$$G_{\rho\sigma}(p')[Q_\Delta\Gamma^{\sigma\beta}_{\mu}\varepsilon^{\mu}]G_{\beta\alpha}(p) = iG_{\rho\sigma}(p')\left[\frac{Q_\Delta O^{\sigma}{}_{\alpha}}{2p'\cdot k}\right]+\left[-\frac{Q_\Delta O'^{\beta}_{\rho}}{2p\cdot k}\right]iG_{\beta\alpha}(p)\ , \qquad (33)$$

where

$$O^{\sigma}{}_{\alpha} = \Gamma^{\sigma\beta}_{\mu}\varepsilon^{\mu}\,d_{\beta\alpha}(p) + \Lambda^{\sigma\beta}(p')\,\Gamma_{\mu,\beta\alpha}\varepsilon^{\mu}\ , \qquad (34)$$

$$O'^{\beta}_{\rho} = d_{\rho\sigma}(p')\,\Gamma^{\sigma\beta}_{\mu}\varepsilon^{\mu} + \Gamma_{\mu,\rho\sigma}\varepsilon^{\mu}\,\Lambda^{\sigma\beta}(p)\ , \qquad (35)$$

and

$$Q_\Delta = Q_b + Q_d = Q_a + Q_c. \qquad (36)$$

It should be pointed out that the factor $[Q_\Delta O^{\sigma}{}_{\alpha} / (2p'\cdot k)]$ in Eq.(33) can also be obtained from the following expression,

$$\bar{u}^{(\Delta)}_{\sigma}(p',\lambda)\left[-iQ_\Delta(\Gamma^{\sigma\beta}_{\mu}\varepsilon^{\mu})\right]\frac{id_{\beta\alpha}(p)}{p^2-M_\Delta^2} = \bar{u}^{(\Delta)}_{\sigma}(p',\lambda)\left[\frac{Q_\Delta O^{\sigma}{}_{\alpha}}{2p'\cdot k}\right], \qquad (37)$$

which describes photon emission by an outgoing Δ line with charge Q_Δ, anomalous magnetic moment λ_Δ and momentum $p^{\mu}=(p'+k)^{\mu}$. (See Fig. 3c). Here the vector-spinors $\bar{u}^{(\Delta)}_{\sigma}$ satisfy the following conditions :

$$\begin{aligned}
&\bar{u}^{(\Delta)}_{\sigma}(p',\lambda)(\not{p}' - M_\Delta) = 0\ ,\\
&\bar{u}^{(\Delta)}_{\sigma}(p',\lambda)(p'^2 - M_\Delta^2) = 0\ ,\\
&\bar{u}^{(\Delta)}_{\sigma}(p',\lambda)\,\gamma^{\sigma} = 0\ ,\\
&\bar{u}^{(\Delta)}_{\sigma}(p',\lambda)\,p'^{\sigma} = 0\ ,
\end{aligned} \qquad (38)$$

and

$$\bar{u}^{(\Delta)}_{\sigma}(p',\lambda)\ \Lambda^{\sigma\beta}(p') = 0\ .$$

The proof is very simple. The left hand side of Eq.(37) can be rewritten in the form

$$\bar{u}^{(\Delta)}_{\sigma}(p',\lambda)Q_{\Delta}\left[\Gamma^{\sigma\beta}_{\mu}\varepsilon^{\mu}\, d_{\beta\alpha}(p)+\Lambda^{\sigma\beta}(p')\,\Gamma_{\mu,\beta\alpha}\varepsilon^{\mu}-\Lambda^{\sigma\beta}(p')\,\Gamma_{\mu,\beta\alpha}\varepsilon^{\mu}\right]\frac{1}{p^{2}-M^{2}_{\Delta}}$$

$$= \bar{u}^{(\Delta)}_{\sigma}(p',\lambda)Q_{\Delta}\left[O^{\sigma}{}_{\alpha}-\Lambda^{\sigma\beta}(p')\,\Gamma_{\mu,\beta\alpha}\varepsilon^{\mu}\right]\frac{1}{p^{2}-M^{2}_{\Delta}} \quad . \tag{39}$$

Since $p^{2}=(p'+k)^{2}=M^{2}_{\Delta}+2p'\cdot k$ and $\bar{u}^{(\Delta)}_{\sigma}\Lambda^{\sigma\beta}=0$, we obtain Eq.(37). Similarly, we can also show that

$$\frac{id_{\rho\sigma}(p')}{p'^{2}-M^{2}_{\Delta}}\left[-iQ_{\Delta}\,\Gamma^{\sigma\beta}_{\mu}\varepsilon^{\mu}\right]u^{(\Delta)}_{\beta}(p_{i},\lambda) = \left[-\frac{Q_{\Delta}O'^{\beta}_{\rho}}{2p\cdot k}\right]u^{(\Delta)}_{\beta}(p_{i},\lambda) \tag{40}$$

which describes photon emission by an incoming Δ line with charge Q_{Δ}, anomalous magnetic moment λ_{Δ} and momentum p^{μ}. (See Fig.3d).

If we substitute the expression for $\Gamma^{\sigma\beta}_{\mu}\varepsilon^{\mu}$ given by Eq. (26) into Eqs. (34) and (35), we find

$$O^{\sigma}{}_{\alpha} = [\,2p'\cdot\varepsilon + 2R\cdot\varepsilon\,]\, g^{\sigma}{}_{\alpha} + E^{\sigma}{}_{\alpha} \tag{41}$$

and

$$O'^{\beta}_{\rho} = [\,2p\cdot\varepsilon + 2R\cdot\varepsilon\,]\, g_{\rho}{}^{\beta} + E'^{\beta}_{\rho} \tag{42}$$

where the expressions for $E^{\sigma}{}_{\alpha}$ and E'^{β}_{ρ} are given in Appendix B and

$$R\cdot\varepsilon = \frac{1}{4}[\,\not\varepsilon\,,\not k\,] + \frac{\lambda_{\Delta}}{8M_{\Delta}}\left\{[\,\not\varepsilon\,,\not k\,]\,,\not p\right\} . \tag{43}$$

In Eq. (43), $p^{\mu}=(p_{i}+q_{i})^{\mu}$ and we have used $[X,\ Y]\equiv XY-YX$ and $\{X,\ Y\}\equiv XY+YX$. Again, it is easy to verify that R^{μ} is separately gauge-invariant, $R\cdot k=0$. If we compare Eq.(43) with Eqs.(20a) and (20b), we find that $R_{i}\cdot\varepsilon$, $R_{f}\cdot\varepsilon$ and $R\cdot\varepsilon$ can be written in the same form. Inserting Eq.(33) (with the expressions for $O^{\sigma}{}_{\alpha}$ and O'^{β}_{ρ} given by Eqs.(41)and (42), respectively) into Eq.(25) and remembering that charge is conserved (Eq.36) , we obtain

$$\varepsilon^{\mu}\,\tilde{M}_{\mu}^{(\Delta)} = -\bar{u}(p_f,\nu_f)\Big[Q_a\Big(\frac{p'\cdot\varepsilon + R\cdot\varepsilon}{p'\cdot k}\Big)\,\tilde{T}_a - Q_b\,\tilde{T}_b\Big(\frac{p\cdot\varepsilon + R\cdot\varepsilon}{p\cdot k}\Big)$$

$$+ Q_c\Big(\frac{p'\cdot\varepsilon + R\cdot\varepsilon}{p'\cdot k}\Big)\,\tilde{T}_c - Q_d\tilde{T}_d\,\Big(\frac{p\cdot\varepsilon + R\cdot\varepsilon}{p\cdot k}\Big)\Big]u(p_i,\nu_i)$$

$$+ \bar{u}(p_f,\nu_f)[\ \varepsilon\cdot\tilde{D}\ ]u(p_i,\nu_i) \tag{44}$$

where

$$\varepsilon\cdot\tilde{D} = Q_a\Big(\frac{p'\cdot\varepsilon + R\cdot\varepsilon}{p'\cdot k}\Big)[g\ k^{\rho}]G_{\rho\alpha}(p)[g\ q_i{}^{\alpha}]$$

$$+ Q_b\Big[g\ q_f{}^{\rho}\Big]G_{\rho\alpha}(p')\Big[g\ k^{\alpha}\Big]\Big(\frac{p\cdot\varepsilon + R\cdot\varepsilon}{p\cdot k}\Big) \tag{45}$$

$$+(Q_b+Q_d)\Big[g\ q_f{}^{\rho}\Big]\Big[\frac{G_{\rho\sigma}(p')E^{\sigma}{}_{\alpha}}{2p'\cdot k} - \frac{E'_{\rho}{}^{\beta}G_{\beta\alpha}(p)}{2p\cdot k}\Big]\ \Big[g\ q_i{}^{\alpha}\Big]$$

Since $Q_a=Q_b$ and $Q_c=Q_d$, it is easy to show that the leading term in Eq. (44)(i.e.,the amplitude $\tilde{M}_{\mu}^{(\Delta)}$ without those terms involving $\varepsilon\cdot R$ and $\varepsilon\cdot\tilde{D}$) is of order k^0 and is independent of k^{μ} when $k\to 0$. Thus $\tilde{M}_{\mu}^{(\Delta)}$ has no kinematic singularity at k=0.

Now let us add the internal amplitude $\tilde{M}_{\mu}^{(\Delta)}$ given by Eq.(44) to the external amplitude $\tilde{M}_{\mu}^{(E)}$given by Eq.(19). We find

$$\varepsilon^{\mu}\,\tilde{M}_{\mu}^{(E\Delta)} = \varepsilon^{\mu}\,\tilde{M}_{\mu}^{(E)} + \varepsilon^{\mu}\,\tilde{M}_{\mu}^{(\Delta)} \tag{46}$$

$$= \varepsilon^{\mu}\,\tilde{M}_{\mu}^{TETA} + \bar{u}(p_f,\nu_f)[\ \varepsilon^{\mu}\cdot\tilde{D}_{\mu}\ ]u(p_i,\nu_i)$$

where

$$\begin{aligned}\tilde{M}^{TETA}_{\mu} = \bar{u}(p_f,\nu_f)\Bigg\{ & Q_a\left[\frac{q_{f\mu}}{q_f\cdot k} - \frac{(p_f+q_f+R)_{\mu}}{(p_f+q_f)\cdot k}\right]\tilde{T}_a \\ & - Q_b\,\tilde{T}_b\left[\frac{q_{i\mu}}{q_i\cdot k} - \frac{(p_i+q_i+R)_{\mu}}{(p_i+q_i)\cdot k}\right] \\ & + Q_c\left[\frac{(p_f+R_f)_{\mu}}{p_f\cdot k} - \frac{(p_f+q_f+R)_{\mu}}{(p_f+q_f)\cdot k}\right]\tilde{T}_c \qquad (47) \\ & - Q_d\tilde{T}_d\left[\frac{(p_i+R_i)_{\mu}}{p_i\cdot k} - \frac{(p_i+q_i+R)_{\mu}}{(p_i+q_i)\cdot k}\right]\Bigg\}u(p_i,\nu_i)\end{aligned}$$

which is defined in terms of four half-off-shell T-matrices, $\tilde{T}_a$, $\tilde{T}_b$, $\tilde{T}_c$ and $\tilde{T}_d$. Although $\tilde{M}^{TETA}$ is gauge invariant,

$$\tilde{M}^{TETA}_{\mu}\,k^{\mu} = 0\ , \qquad (48)$$

the amplitude $\tilde{M}^{(E\Delta)}_{\mu}$ does not satisfy the gauge invariant condition since

$$\tilde{D}_{\mu}\,k^{\mu} \neq 0\ .$$

That $\tilde{M}^{(E\Delta)}_{\mu}$ is not gauge invariant is not surprising because there are other Feynman diagrams for $\pi^{+}p\gamma$ process which are not shown in Fig. 2. To obtain the total bremsstrahlung amplitude $\tilde{M}_{\mu}$ which is completely gauge invariant, we have to impose the gauge invariant condition :

$$\tilde{M}_{\mu} = \tilde{M}^{(E\Delta)}_{\mu} + \tilde{M}^{(G)}_{\mu}\ , \qquad (49)$$

$$\tilde{M}_{\mu}\,k^{\mu} = (\ \tilde{M}^{(E\Delta)}_{\mu} + \tilde{M}^{(G)}_{\mu})\ k^{\mu} = 0\ . \qquad (50)$$

The additional gauge term $\tilde{M}^{(G)}_{\mu}$, which is required to make the total amplitude $\tilde{M}_{\mu}$ gauge invariant, can be determined from the gauge invariant condition, Eq.(50). Such calculations are very lengthy and the final expression for $\tilde{M}_{\mu}$ can be written as

$$\varepsilon^{\mu}\,\tilde{M}_{\mu} = \varepsilon^{\mu}\,\tilde{M}^{TETA}_{\mu} + \varepsilon^{\mu}\,\tilde{M}^{X}_{\mu}\ , \qquad (51)$$

where

$$
\begin{aligned}
\varepsilon^{\mu}\,\tilde{M}_{\mu}^{\;X} = \bar{u}(p_f,\nu_f)\;\Bigg\{ & Q_a\, g^2\left[\frac{(p_f+q_f)^{\rho}(\varepsilon_{\rho}k^{\beta}-\varepsilon^{\beta}k_{\rho})+R\cdot\varepsilon\; k^{\beta}}{(p_f+q_f)\cdot k}\right]G_{\beta\alpha}(p_i+q_i)\; q_i^{\;\alpha} \\
& + Q_b g^2 q_f^{\;\rho}\; G_{\rho\sigma}(p_f+q_f)\left[\frac{(k^{\sigma}\varepsilon_{\alpha}-k_{\alpha}\varepsilon^{\sigma})(p_i+q_i)^{\alpha} + k^{\sigma}\; R\cdot\varepsilon}{(p_i+q_i)\cdot k}\right] \qquad (52) \\
& +(Q_b+Q_d)g^2 q_f^{\;\rho}\left[\frac{G_{\rho\sigma}(p_f+q_f)\;\tilde{C}^{\sigma}_{\alpha\mu}\varepsilon^{\mu}}{2(p_f+q_f)\cdot k} - \frac{\varepsilon^{\mu}\tilde{C}^{\prime\,\beta}_{\rho\mu}\;G_{\beta\alpha}(p_i+q_i)}{2(p_i+q_i)\cdot k}\right]q_i^{\;\alpha}\Bigg\}u(p_i,\nu_i),
\end{aligned}
$$

and the expressions for $\tilde{C}^{\sigma}_{\alpha\mu}$ and $\tilde{C}^{\prime\,\beta}_{\rho\mu}$ are given in Appendix B.

It is clear that the amplitude $\tilde{M}_{\mu}^{\;X}$ is gauge invariant, but it cannot be written in terms of the $\pi^{+}p$ elastic T-matrix. This amplitude will be ignored in the soft-photon approximation mainly because it cannot be calculated if the $\pi^{+}p$ elastic T-matrix and the electromagnetic constants of p,π^{+} and Δ^{++} are the only input for the $\pi^{+}p\gamma$ calculation. In order to estimate the contribution from $\tilde{M}_{\mu}^{\;X}$, we have used two amplitudes, $\tilde{M}_{\mu}$ and $\tilde{M}_{\mu}^{\;TETA}$, to calculate the $\pi^{+}p\gamma$ cross sections. The average cross sections over G1-G18 at 298 MeV have been calculated. When two results are compared, the difference between the two calculations is within 11%. If $\tilde{M}_{\mu}^{\;X}$ is ignored, then the total amplitude $\tilde{M}_{\mu}$ reduces to $\tilde{M}_{\mu}^{TETA}$. As we have already mentioned, the amplitude $\tilde{M}_{\mu}^{TETA}$ involves the half-off-shell T-matrices. It can be calculated if we have a dynamical model from which the half-off-shell T-matrix elements can be determined. However, since we are interested in the on-shell soft-photon approximation in this work, the possibility of developing a new approximation based on the off-shell amplitude $\tilde{M}_{\mu}^{TETA}$ will not be discussed here.

With the help of the expansions given by Eqs.(17a)-(17d), $\tilde{M}_{\mu}^{TETA}$ can be expanded as follows:

$$\tilde{M}_\mu^{TETA} = \tilde{M}_\mu^{TETAS} + \tilde{M}_\mu^{off}, \qquad (53)$$

where

$$\begin{aligned}
\tilde{M}_\mu^{TETAS} = \bar{u}(p_f,\nu_f) \Bigg\{ & Q_a \left[\frac{q_{f\mu}}{q_f \cdot k} - \frac{(p_f+q_f+R)_\mu}{(p_f+q_f)\cdot k} \right] \tilde{T}(s_i,t_p) \\
& - Q_b \tilde{T}(s_f,t_p) \left[\frac{q_{i\mu}}{q_i \cdot k} - \frac{(p_i+q_i+R)_\mu}{(p_i+q_i)\cdot k} \right] \\
& + Q_c \left[\frac{(p_f+R_f)_\mu}{p_f \cdot k} - \frac{(p_f+q_f+R)_\mu}{(p_f+q_f)\cdot k} \right] \tilde{T}(s_i,t_q) \qquad (54) \\
& - Q_d \tilde{T}(s_f,t_q) \left[\frac{(p_i+R_i)_\mu}{p_i \cdot k} - \frac{(p_i+q_i+R)_\mu}{(p_i+q_i)\cdot k} \right] \Bigg\} u(p_i,\nu_i)
\end{aligned}$$

and

$$\begin{aligned}
\tilde{M}_\mu^{off} = \bar{u}(p_f,\nu_f) \Bigg\{ & 2Q_a \left[q_{f\mu} - (q_f \cdot k) \frac{(p_f+q_f)_\mu}{(p_f+q_f)\cdot k} \right] \left(\frac{\partial \tilde{T}_a}{\partial \Delta_a} \right) \\
& + 2Q_b \left(\frac{\partial \tilde{T}_b}{\partial \Delta_b} \right) \left[q_{i\mu} - (q_i \cdot k) \frac{(p_i+q_i)_\mu}{(p_i+q_i)\cdot k} \right] \\
& + 2Q_c \left[p_{f\mu} - (p_f \cdot k) \frac{(p_f+q_f)_\mu}{(p_f+q_f)\cdot k} \right] \left(\frac{\partial \tilde{T}'_c}{\partial \Delta_c} \right) \\
& + 2Q_d \left(\frac{\partial \tilde{T}'_d}{\partial \Delta_d} \right) \left[p_{i\mu} - (p_i \cdot k) \frac{(p_i+q_i)_\mu}{(p_i+q_i)\cdot k} \right] \qquad (55) \\
& + Q_c \left[\frac{(p_f+R_f)_\mu}{p_f \cdot k} - \frac{(p_f+q_f+R)_\mu}{(p_f+q_f)\cdot k} \right] \tilde{T}_{cc}(s_i,t_q) \\
& - Q_d \tilde{T}_{dc}(s_f,t_q) \left[\frac{(p_i+R_i)_\mu}{p_i \cdot k} - \frac{(p_i+q_i+R)_\mu}{(p_i+q_i)\cdot k} \right] + \cdots\cdots \Bigg\} u(p_i,\nu_i).
\end{aligned}$$

$\tilde{M}_\mu^{TETAS}$ defined by Eq. (54) is the special (on-shell) TETA amplitudefor the $\pi^+ p\gamma$ at the tree level. It is gauge invariant,

$$\tilde{M}_{\mu}^{TETAS} \, k^{\mu} = 0 \, , \tag{56}$$

and it depends only upon the elastic T-matrix, evaluated at (s_i, t_p), (s_f, t_p), (s_i, t_q) and (s_f, t_q). Moreover, it is easy to show that $\tilde{M}_{\mu}^{TETAS}$ has no kinematic singularity at k=0. The amplitude $\tilde{M}_{\mu}^{off}$ given by Eq. (55), on the other hand, is an off-shell amplitude. It depends upon the off-shell derivatives and also upon the extra off-shell amplitudes involving $\tilde{C}$. Thus, if we ignore those terms which cannot be expressed in terms of the elastic T-matrix (i.e., the amplitude $\tilde{M}_{\mu}^{X}$) and those terms which involve off-shell effects (i.e., the amplitude $\tilde{M}_{\mu}^{off}$), then we obtain the TETAS amplitude $\tilde{M}_{\mu}^{TETAS}$,

$$\tilde{M}_{\mu} \approx \tilde{M}_{\mu}^{TETAS} , \tag{57}$$

which can be evaluated exactly in terms of the $\pi^+ p$ elastic T-matrix and the electromagnetic constants of p, π^+ and Δ^{++}.

For the purpose of comparison between the modified procedure and Low's standard (original) procedure for deriving a soft-photon amplitude, let us derive a different version of the TETAS amplitude by using the standard procedure. We first expand $\tilde{M}_{\mu}^{(E)}$ in powers of k. Substituting Eqs. (17a)-(17d) into Eq. (19), we obtain

$$\begin{aligned} \tilde{M}_{\mu}^{(E)} = \bar{u}(p_f, \nu_f) \Big[& Q_a \frac{q_{f\mu}}{q_f \cdot k} \tilde{T}(s_i, t_p) - \tilde{T}(s_f, t_p) \, Q_b \frac{q_{i\mu}}{q_i \cdot k} \\ & + Q_c \frac{(p_f + R_f)_{\mu}}{p_f \cdot k} \tilde{T}(s_i, t_q) - \tilde{T}(s_f, t_q) \, Q_d \frac{(p_i + R_i)_{\mu}}{p_i \cdot k} \\ & + 2Q_a \, q_{f\mu} \Big(\frac{\partial \tilde{T}_a}{\partial \Delta_a} \Big) + 2Q_b \, q_{i\mu} \Big(\frac{\partial \tilde{T}_b}{\partial \Delta_b} \Big) + 2Q_c p_{f\mu} \Big(\frac{\partial \tilde{T}'_c}{\partial \Delta_c} \Big) + 2Q_d p_{i\mu} \Big(\frac{\partial \tilde{T}'_d}{\partial \Delta_d} \Big) \\ & + (\text{ terms involving } \tilde{C} \,) + \cdots\cdots \Big] \, u(p_i, \nu_i) \, . \end{aligned} \tag{58}$$

Since those terms involving $\tilde{C}$ will be completely ignored later, we shall neglect them in the rest of our derivation. The second step is to obtain the leading term of the internal amplitude $\tilde{M}^{(I)}_{\mu}$ by imposing the gauge invariant condition :

$$k^{\mu}\,\tilde{M}^{(I)}_{\mu} = -\,k^{\mu}\,\tilde{M}^{(E)}_{\mu}$$

$$= -\,\bar{u}(p_f,\nu_f)\Big[\,\tilde{T}^{(EK)} + 2Q_a\,q_f\cdot k\,\Big(\frac{\partial\tilde{T}_a}{\partial\Delta_a}\Big) + 2Q_b\,q_i\cdot k\,\Big(\frac{\partial\tilde{T}_b}{\partial\Delta_b}\Big) \qquad (59)$$

$$+\,2Q_c p_f\cdot k\,\Big(\frac{\partial\tilde{T}'_c}{\partial\Delta_c}\Big) + 2Q_d p_i\cdot k\,\Big(\frac{\partial\tilde{T}'_d}{\partial\Delta_d}\Big) + \cdots\cdots\Big]u(p_i,\nu_i)\;,$$

where

$$\tilde{T}^{(EK)} = Q_a\,\tilde{T}(s_i,t_p) - Q_b\,\tilde{T}(s_f,t_p) + Q_c\,\tilde{T}(s_i,t_q) - Q_d\,\tilde{T}(s_f,t_q) \qquad (60)$$

and we have $Q_a=Q_b$ and $Q_c=Q_d$ for the $\pi^+p\gamma$ process. The amplitude $\tilde{T}^{(EK)}$ has been studied by Fischer and Minkowski[13] and also by Heller.[23] For example , Heller has used the mean value theorem for derivatives,

$$T(s_i,t_p) - T(s_f,t_p) = (s_i-s_f)\frac{\partial T(s_0,t_p)}{\partial s} = 2(q_i+p_i)\cdot k\,\frac{\partial T(s_0,t_p)}{\partial s}\;, \qquad s_f \le s_0 \le s_i \qquad (61a)$$

and

$$T(s_i,t_q) - T(s_f,t_q) = (s_i-s_f)\frac{\partial T(s'_0,t_q)}{\partial s} = 2(q_i+p_i)\cdot k\,\frac{\partial T(s'_0,t_q)}{\partial s}\;, \qquad s_f \le s'_0 \le s_i\;, \qquad (61b)$$

to obtain

$$\tilde{T}^{(EK)} = 2Q_a(q_i+p_i)\cdot k\,\frac{\partial T(s_0,t_p)}{\partial s} + 2Q_c(q_i+p_i)\cdot k\,\frac{\partial T(s'_0,t_q)}{\partial s}\;. \qquad (62)$$

Inserting Eq. (62) into Eq. (59), we find

$$\tilde{M}^{(I)}_{\mu} = \bar{u}(p_f,\nu_f)[-2Q_a(q_i+p_i)_{\mu}\,\frac{\partial T(s_0,t_p)}{\partial s} - 2Q_c(q_i+p_i)_{\mu}\,\frac{\partial T(s'_0,t_q)}{\partial s}$$

$$- 2Q_a\, q_{f\mu}\left(\frac{\partial\tilde{T}_a}{\partial\Delta_a}\right) - 2Q_b\, q_{i\mu}\left(\frac{\partial\tilde{T}_b}{\partial\Delta_b}\right) - 2Q_c p_{f\mu}\left(\frac{\partial\tilde{T}'_c}{\partial\Delta_c}\right) \qquad (63)$$

$$- 2Q_d p_{i\mu}\left(\frac{\partial\tilde{T}'_d}{\partial\Delta_d}\right) + \cdots\cdots\Big]u(p_i,\nu_i) \quad .$$

If we apply Eqs.(61a) and (61b) again, we can rewrite $\tilde{M}^{(I)}$ in the form

$$\begin{aligned}\tilde{M}^{(I)}_\mu = \bar{u}(p_f,\nu_f)\Bigg[&-Q_a\,\frac{(p_f+q_f)_\mu}{(p_f+q_f)\cdot k}\,\tilde{T}\,(s_i,t_p) + Q_b\,\tilde{T}\,(s_f,t_p)\frac{(p_i+q_i)_\mu}{(p_i+q_i)\cdot k}\\ &-Q_c\,\frac{(p_f+q_f)_\mu}{(p_f+q_f)\cdot k}\,\tilde{T}\,(s_i,t_q) + Q_d\,\tilde{T}\,(s_f,t_q)\frac{(p_i+q_i)_\mu}{(p_i+q_i)\cdot k}\\ &- 2Q_a\, q_{f\mu}\left(\frac{\partial\tilde{T}_a}{\partial\Delta_a}\right) - 2Q_b\, q_{i\mu}\left(\frac{\partial\tilde{T}_b}{\partial\Delta_b}\right) - 2Q_c p_{f\mu}\left(\frac{\partial\tilde{T}'_c}{\partial\Delta_c}\right)\\ &- 2Q_d p_{i\mu}\left(\frac{\partial\tilde{T}'_d}{\partial\Delta_d}\right) + \cdots\cdots\Bigg]u(p_i,\nu_i)\end{aligned} \qquad (64)$$

Here, we have used the fact that $(q_i+p_i)\cdot k = (q_f+p_f)\cdot k$, $(q_i+p_i)\cdot\varepsilon = (q_f+p_f)\cdot\varepsilon$, $Q_a=Q_b$ and $Q_c=Q_d$. Finally, we add $\tilde{M}^{(E)}_\mu$ (Eq.(58)) and $\tilde{M}^{(I)}_\mu$ (Eq.(64)) to obtain the total amplitude $\tilde{M}'_\mu$:

$$\begin{aligned}\tilde{M}'_\mu = \bar{u}(p_f,\nu_f)\Bigg\{&Q_a\left[\frac{q_{f\mu}}{q_f\cdot k} - \frac{(p_f+q_f)_\mu}{(p_f+q_f)\cdot k}\right]\tilde{T}\,(s_i,t_p)\\ &- Q_b\,\tilde{T}\,(s_f,t_p)\left[\frac{q_{i\mu}}{q_i\cdot k} - \frac{(p_i+q_i)_\mu}{(p_i+q_i)\cdot k}\right]\\ &+ Q_c\left[\frac{(p_f+R_f)_\mu}{p_f\cdot k} - \frac{(p_f+q_f)_\mu}{(p_f+q_f)\cdot k}\right]\tilde{T}\,(s_i,t_q)\\ &- Q_d\tilde{T}\,(s_f,t_q)\left[\frac{(p_i+R_i)_\mu}{p_i\cdot k} - \frac{(p_i+q_i)_\mu}{(p_i+q_i)\cdot k}\right]+\cdots\cdots\Bigg\}u(p_i,\nu_i)\end{aligned} \qquad (65)$$

which is to be compared with the amplitude $\tilde{M}_\mu$ obtained by using the modified procedure,

$$\tilde{M}_\mu \approx \tilde{M}_\mu^{TETAS} + \tilde{M}_\mu^{off} , \qquad (66)$$

where $\tilde{M}_\mu^{TETAS}$ and $\tilde{M}_\mu^{off}$ are given by Eqs. (54) and (55), respectively. (But we neglect those terms involving $\tilde{T}_{cc}$ and $\tilde{T}_{dc}$ in Eq. (55) in this comparison.) Two substantial differences can be observed : (i) Eq. (65) shows that the first two terms in the series expansion of the amplitude $\tilde{M}'_\mu$ in powers of k are independent of off-shell derivatives. Eq. (66),on the other hand, shows that the amplitude $\tilde{M}_\mu$ does depend upon off-shell derivatives of order k^0. (ii) The amplitude $\tilde{M}_\mu$ has extra terms involving R_μ. These terms represent photon emission from the internal Δ^{++} line with spin-$\frac{3}{2}$ and the anomalous magnetic moment λ_Δ. The amplitude $\tilde{M}'_\mu$ does not include any R_μ term which is separately gauge invariant, $R_\mu k^\mu = 0$. This is because the standard procedure (or the gauge invariant condition, Eq. (59)) cannot be used to determine an internal term which is separately gauge invariant. (Note that both $\tilde{M}_\mu$ and $\tilde{M}'_\mu$ include internal terms which are proportional to either $(q_i+p_i)_\mu$ or $(q_f+p_f)_\mu$. These internal terms represent photon emission from the charge of the internal Δ^{++} line.)

(c) General TETAS amplitude and modified Low procedure:

The result obtained in the last section will be used as an important guide to develop a modified procedure for constructing a more general TETA amplitude for the $\pi^+ p\gamma$ process. The first step in the modified procedure is exactly the same as the one used in Low's standard procedure. We obtain the external amplitude $M_\mu^{(E)}$ from four external emission diagrams, Fig. 4a-4d, which are generated from the source diagram shown in Fig. 1a. We find

$$M_\mu^{(E)} = \bar{u}(p_f, \nu_f) \left[\frac{Q_a q_{f\mu}}{q_f \cdot k} T_a - T_b \frac{Q_b q_{i\mu}}{q_i \cdot k} + \frac{Q_c (p_f + R_f)_\mu}{p_f \cdot k} T_c - T_d \frac{Q_d (p_i + R_i)_\mu}{p_i \cdot k} \right] u(p_i, \nu_i) \qquad (67)$$

where $R_{i\mu}$ and $R_{f\mu}$ are defined by Eqs. (20a) and (20b), respectively, and T_x ($x = a, b, c, d$) which are the half-off-shell T-matrices can be written in the form

$$\bar{u}\, T_a u \equiv \bar{u}\, T\, [s_i,\ t_p,\ p_i^2 = m_p^2,\ q_i^2 = m_\pi^2,\ p_f^2 = m_p^2,\ \Delta_a = (q_f + k)^2]\ u$$

$$= \bar{u} \Big\{ A\,(s_i,\ t_p,\ m_p^2,\ m_\pi^2,\ m_p^2,\ \Delta_a)$$

$$+ \tfrac{1}{2}(\not{q}_i + \not{q}_f + \not{k})\ B\,(s_i,\ t_p,\ m_p^2,\ m_\pi^2,\ m_p^2,\ \Delta_a) \Big\}\, u\ , \qquad (68a)$$

$$\bar{u}\, T_b u \equiv \bar{u}\, T\, [s_f,\ t_p,\ p_i^2 = m_p^2,\ \Delta_b = (q_i - k)^2,\ p_f^2 = m_p^2,\ q_f^2 = m_\pi^2\ ]\ u$$

$$= \bar{u} \Big\{ A\,(s_f,\ t_p,\ m_p^2,\ \Delta_b,\ m_p^2,\ m_\pi^2\ )$$

$$+ \tfrac{1}{2}(\not{q}_i + \not{q}_f - \not{k})\ B\,(s_f,\ t_p,\ m_p^2,\ \Delta_b,\ m_p^2,\ m_\pi^2\ ) \Big\}\, u\ , \qquad (68b)$$

$$T_c u \equiv\ T\, [s_i,\ t_q,\ p_i^2 = m_p^2,\ q_i^2 = m_\pi^2,\ \Delta_c = (p_f + k)^2,\ q_f^2 = m_\pi^2\ ]\ u$$

$$= \Big\{ A\,(s_i,\ t_q,\ m_p^2,\ m_\pi^2,\ \Delta_c,\ m_\pi^2\ )$$

$$+ \tfrac{1}{2}(\not{q}_i + \not{q}_f\ )\ B\,(s_i,\ t_q,\ m_p^2,\ m_\pi^2,\ \Delta_c,\ m_\pi^2\ )$$

$$+ \tfrac{1}{2}(\not{p}_f + \not{k} - m_p)\ C\,(s_i,\ t_q,\ m_p^2,\ m_\pi^2,\ \Delta_c,\ m_\pi^2\ ) \Big\}\, u\ , \qquad (68c)$$

and

$$\bar{u}\, T_d \equiv \bar{u}\, T\, [s_f,\ t_q,\ \Delta_d = (p_i - k)^2, q_i^2 = m_\pi^2,\quad p_f^2 = m_p^2,\ q_f^2 = m_\pi^2\ ]$$

$$= \bar{u} \Big\{ A\,(s_f,\ t_q,\ \Delta_d,\ m_\pi^2,\ m_p^2,\ m_\pi^2\ )$$

$$+ \frac{1}{2}(\not{q}_i + \not{q}_f)\, B\, (s_f,\ t_q,\ \Delta_d,\ m_\pi^2,\ m_p^2,\ m_\pi^2\ )$$

$$+ \frac{1}{2}\, C'\ (s_f,\ t_q,\ \Delta_d,\ m_\pi^2,\ m_p^2,\ m_\pi^2\ )(\not{p}_i - \not{k} - m_p) \Big\} , \qquad (68d)$$

In Eq. (69c) and (69d), the expressions for T_c and T_d have extra off-shell terms involving amplitude C or C'. These extra off-shell terms vanish on the mass-shell. The expressions for off-shell amplitudes A, B, C and C' are much more complicated than those for off-shell amplitudes $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ at the tree level. (See Eqs. (12) and (15c).) However, since the Feynman diagrams given by Figs. 2a-2e are the dominant contribution to the $\pi^+ p\gamma$ process in the energy region of the Δ^{++}(1232) resonance, we expect that A reduces to $\tilde{A}$, B reduces to $\tilde{B}$, and C and C' reduce to $\tilde{C}$ when Figs. 4a-4e reduce to Figs. 2a-2e, respectively. The on-shell values of the amplitudes A, B, C and C' are defined by

$$A(\ s_\alpha,\ t_\beta) \equiv A(\ s_\alpha,\ t_\beta,\ m_p^2,\ m_\pi^2,\ m_p^2,\ m_\pi^2\ )\ , \qquad (69a)$$

$$B(\ s_\alpha,\ t_\beta) \equiv B(\ s_\alpha,\ t_\beta,\ m_p^2,\ m_\pi^2,\ m_p^2,\ m_\pi^2\ )\ , \qquad (69b)$$

$$C(\ s_i,\ t_q) \equiv C(\ s_i,\ t_q,\ m_p^2,\ m_\pi^2,\ m_p^2,\ m_\pi^2\ )\ , \qquad (69c)$$

and

$$C'(\ s_f,\ t_q) \equiv C'(\ s_f,\ t_q,\ m_p^2,\ m_\pi^2,\ m_p^2,\ m_\pi^2\ )\ , \qquad (69d)$$

with α = i or f and β = p or q. The on-shell amplitudes $A(s_\alpha, t_\beta)$ and $B(s_\alpha, t_\beta)$, which determine the on-shell (elastic) $\pi^+ p$ T-matrix $T(s_\alpha, t_\beta)$, can be calculated in terms of $\pi^+ p$ phase shifts and inelasticities, determined by the $\pi^+ p$ elastic scattering experiments. Since the expressions for $C(s_i, t_q)$ and $C'(s_f, t_q)$ are not known, all extra terms involving C and C' have been completely ignored in the on-shell soft-photon approximation.

The second step is to find an internal amplitude $M_\mu^{(\Delta)}$ which represents photon emission from the intermediate Δ^{++} line. The idea is to write $M_\mu^{(\Delta)}$ as a linear combination of T_x(x=a,b,c,d) and D :

$$\varepsilon^\mu M_\mu^{(\Delta)} = \bar{u}(p_f,\nu_f) \, [\, Y_a T_a + T_b Y_b + Y_c T_c$$
$$+ T_d Y_d + \varepsilon\cdot D \,] \, u(p_i,\nu_i) \, , \qquad (70)$$

where Y_x (x=a,b,c,d) are the coefficients to be determined and $\varepsilon\cdot D$ represents the remainder of other terms which cannot be written in terms of T_x. To determine Y_x (x=a,b,c,d), we demand that $M_\mu^{(\Delta)}$ reduce to $\tilde{M}_\mu^{(\Delta)}$ given by Eq. (44) when Fig. 4e reduces to Fig. 2e. Since T_x(x=a,b,c,d) reduce to $\tilde{T}_x$ and D reduces to $\tilde{D}$, we find

$$Y_a = - Q_a \frac{\varepsilon\cdot(p_f+q_f) + \varepsilon\cdot R}{(p_f+q_f)\cdot k} \, ,$$
$$Y_b = Q_b \frac{\varepsilon\cdot(p_i+q_i) + \varepsilon\cdot R}{(p_i+q_i)\cdot k} \, , \qquad (71)$$
$$Y_c = - Q_c \frac{\varepsilon\cdot(p_f+q_f) + \varepsilon\cdot R}{(p_f+q_f)\cdot k} \, ,$$

and

$$Y_d = Q_d \frac{\varepsilon\cdot(p_i+q_i) + \varepsilon\cdot R}{(p_i+q_i)\cdot k} \, .$$

Combining $M_\mu^{(E)}$ with $M_\mu^{(\Delta)}$, we obtain

$$\varepsilon^\mu M_\mu^{(E\Delta)} = \varepsilon^\mu M_\mu^{TETA} + \bar{u}(p_f,\nu_f)[\, \varepsilon^\mu D_\mu \,]u(p_i,\nu_i) \, , \qquad (71a)$$

where

$$M_\mu^{TETA} = \bar{u}(p_f,\nu_f) \left\{ Q_a \left[\frac{q_{f\mu}}{q_f\cdot k} - \frac{(p_f+q_f+R)_\mu}{(p_f+q_f)\cdot k} \right] T_a \right.$$
$$- Q_b T_b \left[\frac{q_{i\mu}}{q_i\cdot k} - \frac{(p_i+q_i+R)_\mu}{(p_i+q_i)\cdot k} \right]$$
$$+ Q_c \left[\frac{(p_f+R_f)_\mu}{p_f\cdot k} - \frac{(p_f+q_f+R)_\mu}{(p_f+q_f)\cdot k} \right] T_c \qquad (71b)$$

$$- Q_d \, T_d \left[\frac{(p_i+R_i)_\mu}{p_i \cdot k} - \frac{(p_i+q_i+R)_\mu}{(p_i+q_i)\cdot k} \right] \Bigg\} \, u(p_i, \nu_i) \ .$$

Since D_μ cannot be expressed in terms of T_x, it has been ignored in the soft-photon approximation.

The third step is to impose the gauge invariant condition,

$$k^\mu (\, M_\mu^{(E\Delta)} + M_\mu^{(G)}) = 0 \ , \tag{72a}$$

in order to obtain an additional gauge term $M_\mu^{(G)}$ so that the total amplitude, $M_\mu = M_M^{(E\Delta)} + M_\mu^{(G)}$, will be completely gauge invariant. Since

$$k^\mu \, M_\mu^{TETA} = 0,$$

the condition (72a) gives

$$k^\mu \, M_\mu^{(G)} = -k^\mu D_\mu \tag{72b}$$

which shows that $M_\mu^{(G)}$ can be determined if the detailed expression for D_μ is known. However, since D_μ has been ignored in the soft-photon approximation, $M_\mu^{(G)}$ will also be ignored in our derivation. Thus, the total amplitude has the form

$$M_\mu = M_\mu^{TETA} \tag{73}$$

if D_μ and $M_\mu^{(G)}$ are neglected. Eq. (73) shows that the off-shell TETA amplitude M_μ^{TETA} is an approximate amplitude which can be rigorously derived for the $\pi^+ p\gamma$ process.

Finally, to obtain a special on-shell TETAS amplitude M_μ^{TETAS}, we have to expand T_x (x=a, b, c, d). We obtain

$$M_\mu^{TETA} = M_\mu^{TETAS} + M_\mu^{off} \ , \tag{74}$$

where

$$M_\mu^{TETAS} = \bar{u}(p_f, \nu_f) \Bigg\{ Q_a \left[\frac{q_{f\mu}}{q_f \cdot k} - \frac{(p_f+q_f+R)_\mu}{(p_f+q_f)\cdot k} \right] T \, (s_i, t_p)$$

$$
\begin{aligned}
&- Q_b\, T\,(s_f, t_p) \left[\frac{q_{i\mu}}{q_i \cdot k} - \frac{(p_i + q_i + R)_\mu}{(p_i + q_i)\cdot k} \right] \\
&+ Q_c \left[\frac{(p_f + R_f)_\mu}{p_f \cdot k} - \frac{(p_f + q_f + R)_\mu}{(p_f + q_f)\cdot k} \right] T\,(s_i, t_q) \qquad\qquad (75) \\
&- Q_d T\,(s_f, t_q) \left[\frac{(p_i + R_i)_\mu}{p_i \cdot k} - \frac{(p_i + q_i + R)_\mu}{(p_i + q_i)\cdot k} \right] \Bigg\}\, u(p_i, \nu_i)
\end{aligned}
$$

and M_μ^{Off} represents the rest of the other terms which include off-shell derivatives of the amplitudes A and B and extra off-shell terms involving amplitudes C and C' . The amplitude M_μ^{TETAS} , Eq.(75), is identical to the one given by Eq.(1) in Ref.29. Because of different definitions for $R_{i\mu}$, $R_{f\mu}$ and R_μ used in this work, there is a sign difference for those terms involving $R_{i\mu}$, $R_{f\mu}$ and R_μ in the expression for M_μ^{TETAS}. In Eq. (75), $T(s_i,t_p)$, $T(s_f,t_p)$, $T(s_i,t_q)$ and $T(s_f,t_q)$ are the elastic π^+p T-matrices, evaluated at (s_i,t_p), (s_f,t_p), (s_i,t_q) and (s_f,t_q), respectively. (See Ref.28 for a discussion on the calculation of these T-matrices.) . We have shown that if we neglect those terms which cannot be expressed in terms of the T-matrix (i.e.,$\varepsilon\cdot D$) and ignore all off-shell terms (i.e.,$\varepsilon\cdot M^{off}$), then we obtain the amplitude M_μ^{TETAS} which can be calculated exactly in terms of the π^+p elastic T-matrix and the electromagnetic constants of p, π^+ and Δ^{++} .

(III) The "Experimental" Magnetic Moment of $\Delta^{++}(1232)$ Extracted from Experimental Data :

We have used the special two-energy-two-angle amplitude, M_μ^{TETAS} given by Eq. (75), to calculate $\pi^+ p\gamma$ cross sections as a function of photon energy k,

$$\frac{d^3\sigma}{d\Omega_\pi\, d\Omega_\gamma\, dk} = \frac{J}{(2\pi)^5} \int \delta^4(p_i+q_i-p_f-q_f-k)$$

$$\left\{\frac{1}{2} \sum_{pol,spin} (M_\mu^{TETAS}\varepsilon^\mu)^+ (M_\nu^{TETAS}\varepsilon^\nu)\right\} d^4F \, , \qquad (76)$$

where

$$J = e^2 m_p^2 \, / \, [(p_i \cdot q_i)^2 - m_\pi^2 m_p^2]^{1/2} \, ,$$

$$d^4F = [q_f^2 \, dq_f \, /(2E_\pi)] \, [d^3\vec{p}_f/(2E_p)] \, [k^2/(2k)] \, ,$$

$$E_\pi = (m_\pi^2 + \vec{q}_f^{\,2})^{1/2} \, ,$$

and

$$E_p = (m_p^2 + \vec{p}_f^{\,2})^{1/2} \, .$$

In these calculations, the anomalous magnetic moment of the Δ^{++}, λ_Δ, has been treated as a free parameter and it is to be determined from the UCLA data[11] at three bombarding energies, 269, 298 and 324 MeV, and the SIN data[13] at 299 MeV.

The UCLA group has used 19 photon counters, G_i (i=1-19), to measure $\pi^+ p\gamma$ differential cross sections. As a result, 18 sets of cross sections have been obtained for each bombarding energy. (Cross sections for the photon counter G16 have not been determined.) In each set, the UCLA data are given at the following photon energies : k_1= 22.5 MeV, k_2= 40 MeV, k_3= 60 MeV, k_4= 80 MeV, k_5= 100 MeV, k_6= 120

MeV and k_7= 140 MeV. We shell use the "spectrum G_i " to label the set of cross sections obtained from the photon counter G_i. Thus, if we define E_1= 269 MeV, E_2= 298 MeV and E_3= 324 MeV, then the UCLA data can be denoted by $\sigma^{UCLA}(E_i,G_j,k_l)$, which represents cross section at the bombarding energy E_i (i=1,2,3) and the photon energy k_l(l=1, ⋯, 7) for the spectrum G_j(j=1,⋯, 19). The corresponding theoretical cross section, calculated using Eq. (76), will be denoted by $\sigma^{TH}(E_i,G_j,k_l)$.

Using the experimental cross sections $\sigma^{UCLA}(E_i,G_j,k_l)$ and the theoretical cross sections $\sigma^{TH}(E_i,G_j,k_l)$, we first calculated the following average cross sections :

$$\sigma^{UCLA}_{1-10}(E_i,k_l) = \sum_{j=1}^{10} \sigma^{UCLA}(E_i,G_j.k_l)/10, \tag{77a}$$

$$\sigma^{TH}_{1-10}(E_i,k_l) = \sum_{j=1}^{10} \sigma^{TH}(E_i,G_j,k_l)/10 \quad , \tag{77b}$$

$$\sigma^{UCLA}_{11-15}(E_i,k_l) = \sum_{j=11}^{15} \sigma^{UCLA}(E_i,G_j.k_l)/5 \ , \tag{77c}$$

$$\sigma^{TH}_{11-15}(E_i,k_l) = \sum_{j=11}^{15} \sigma^{TH}(E_i,G_j,k_l)/5 \quad , \tag{77d}$$

$$\sigma^{UCLA}_{1-15}(E_i,k_l) = \sum_{j=1}^{15} \sigma^{UCLA}(E_i,G_j.k_l)/15 \ , \tag{77e}$$

and

$$\sigma^{TH}_{1-15}(E_i,k_l) = \sum_{j=1}^{15} \sigma^{TH}(E_i,G_j,k_l)/15 \quad , \tag{77f}$$

for all photon energies k_l (l=1,⋯,7) at three bombarding energies, E_i (i=1,2,3). The values of $\sigma^{UCLA}_{1-10}(E_i,k_l)$, $\sigma^{UCLA}_{11-15}(E_i,k_l)$ and $\sigma^{UCLA}_{1-15}(E_i,k_l)$, without including the experimental errors, are shown in Table 1. We then use these cross sections to define the following average deviations :

$$D_{1-10}\,(E_i,\ \lambda_\Delta) = \sum_l \frac{|\sigma^{UCLA}_{1-10}(E_i,k_l) - \sigma^{TH}_{1-10}(E_i,k_l)|}{\sigma^{UCLA}_{1-10}(E_i,k_l)}\ , \tag{78a}$$

$$D_{11-15}(E_i,\ \lambda_\Delta) = \sum_l \frac{|\sigma^{UCLA}_{11-15}(E_i,k_l) - \sigma^{TH}_{11-15}(E_i,k_l)|}{\sigma^{UCLA}_{11-15}(E_i,k_l)}\ , \tag{78b}$$

and

$$D_{1-15}\,(E_i,\ \lambda_\Delta) = \sum_l \frac{|\sigma^{UCLA}_{1-15}(E_i,k_l) - \sigma^{TH}_{1-15}(E_i,k_l)|}{\sigma^{UCLA}_{1-15}(E_i,k_l)}\ , \tag{78c}$$

Since there are three bombarding energies, we obtain nine deviation functions, which are all functions of λ_Δ. Varying the value of λ_Δ, we find nine deviation curves. As shown in Figs. 5a,5b and 5c, each of these nine deviation curves clearly exhibits a minimum point. The values of λ_Δ at these minimum points are as follows : $\lambda_\Delta^{1-10}(E_1) = 1.53$ $\lambda_\Delta^{1-10}(E_2) = 1.47$,$\lambda_\Delta^{1-10}(E_3) = 1.20$, $\lambda_\Delta^{11-15}(E_1) = 1.90$, $\lambda_\Delta^{11-15}(E_2) = 2.00$, $\lambda_\Delta^{11-15}(E_3) = 1.44$, $\lambda_\Delta^{1-15}(E_1) = 1.86$, $\lambda_\Delta^{1-15}(E_2) = 1.58$, and $\lambda_\Delta^{1-15}(E_3) = 1.36$. Taking an average, we have

$$\lambda_\Delta = \sum_{i=1}^{3} \lambda_\Delta^{1-10}\,(E_i)/3 = 1.4 \tag{79a}$$

for spectra G1 - G10 ,

$$\lambda_\Delta = \sum_{i=1}^{3} \lambda_\Delta^{11-15}(E_i)/3 = 1.8 \tag{79b}$$

for spectra G11 - G15 , and

$$\lambda_\Delta = \sum_{i=1}^{3} \lambda_\Delta^{1-15}\,(E_i)/3 = 1.6 \tag{79c}$$

for spectra G1 - G15 . Using these results for λ_Δ, the value of the "experimental" magnetic moment of the $\Delta^{++}(1232)$, μ_Δ, can be calculated. We find

$$\mu_\Delta = 2\ (1 + \lambda_\Delta)\ \frac{e}{2\ M_\Delta} = 2\ (1 + \lambda_\Delta)\ \frac{m_p}{M_\Delta}\ \left(\frac{e}{2\ m_p} \right)$$

$$= \begin{cases} 3.7\ \dfrac{e}{2m_p} & \text{for spectra G1 - G10} \\ 4.0\ \dfrac{e}{2m_p} & \text{for spectra G1 - G15} \\ 4.2\ \dfrac{e}{2m_p} & \text{for spectra G11 - G15} \end{cases} \tag{80}$$

This gives us a range of the value of μ_Δ which can be extracted from the UCLA data.

Of course, the UCLA data sets can be analyzed together to yield a single value of μ_Δ . This has been done and the value is 3.8 $e/(2m_p)$. It should be pointed out that if the χ^2 as a function of λ_Δ were used to determine the value of λ_Δ, then we would also obtain about the same result as that obtained by using the deviation function defined by Eq.(78). For spectra G1-G10 at 298 MeV, for example, the χ^2 fit gives $\lambda_\Delta = 1.4$ while the method based on Eq.(78) gives $\lambda_\Delta \equiv \lambda_\Delta^{1-10}(E_2) = 1.47$. In Fig.5e, we show the χ^2 curve with a clear minimum point at λ_Δ = 1.4. This curve is to be compared with the solid curve exhibited in Fig. 5a.

We have used the term "experimental" magnetic moment to describe the result obtained in this work for the following reason: The magnetic moment of the Δ^{++} obtained in this work is based upon the TETAS amplitude. In deriving this amplitude, we have ignored the emission from the internal pion-proton loop. In a recent study using a nonrelativistic dynamical model, Heller et al.[30] have reported that an "effective" magnetic moment of the Δ^{++} can be defined if the contribution from the loop diagrams is involved. This effective moment, which is different from the "bare" moment predicted by the SU(6) or the quark model, is a complex and energy-dependent quantity. Although these authors have found that the imaginary part of the effective moment is not negligible, the problem of defining and calculating the effective moment for an off-shell unstable Δ^{++} particle remains unsolved mainly because they were unable to demonstrate that their model could be used to describe most of the $\pi^{+}p\gamma$ data. It is obvious that the effective moment cannot be calculated to arbitrary precision in any model-independent calculations since it is difficult to take into account the loop contribution in the soft-photon approximation. This is why the magnetic moment of the Δ^{++} extracted from the $\pi^{+}p\gamma$ data by using the TETAS amplitude is an approximation with theoretical errors to the effective moment. Since it is also difficult to identify our magnetic moment with the "bare" moment, we have therefore used the "experimental" magnetic moment to describe the result obtained by us.

We have also extracted the value of μ_{Δ} from the SIN data. The SIN group has measured the $\pi^{+}p\gamma$ cross sections at 299 MeV. Depending upon the angular regions for the outgoing pions, the group has obtained three sets of cross sections. We shall call the set for $55^{\circ} < \theta_{\pi} < 95^{\circ}$ as the first set, the set for $55^{\circ} < \theta_{\pi} < 75^{\circ}$ as the second set, and the set for $75^{\circ} < \theta_{\pi} < 95^{\circ}$ as the third set. In each set, the SIN data are given at the following photon energies : $k_1' = 27.5$ MeV,

$k_2' = 42.5$ MeV, $k_3' = 57.5$ MeV, $k_4' = 72.5$ MeV, $k_5' = 87.5$ MeV, $k_6' = 102.5$ MeV, and $k_7' = 117.5$ MeV. Thus, the SIN data will be denoted by $\sigma_i^{SIN}(k_j')$ and the corresponding theoretical cross section by $\sigma_i^{TH}(k_j')$. Here, $k_j'(j = 1,\cdots,7)$ are photon energies and the subscript i indicates the set number $(i = 1,2,3)$. The values of $\sigma_i^{SIN}(k_j')$, without including the experimental errors, are shown in Table 1. Using $\sigma_i^{SIN}(k_j')$ and $\sigma_i^{TH}(k_j')$, we define three deviation functions :

$$D_1(\lambda_\Delta) = \sum_j \frac{|\sigma_1^{SIN}(k_j') - \sigma_1^{TH}(k_j')|}{\sigma_1^{SIN}(k_j')} , \qquad (81a)$$

$$D_2(\lambda_\Delta) = \sum_j \frac{|\sigma_2^{SIN}(k_j') - \sigma_2^{TH}(k_j')|}{\sigma_2^{SIN}(k_j')} , \qquad (81b)$$

and

$$D_3(\lambda_\Delta) = \sum_j \frac{|\sigma_3^{SIN}(k_j') - \sigma_3^{TH}(k_j')|}{\sigma_3^{SIN}(k_j')} , \qquad (81c)$$

which are all functions of λ_Δ. Varying the value of λ_Δ, we obtain three deviation curves. As shown in Fig. 5d, each curve has a minimum point. The values of λ_Δ at these minimum points are

$$\lambda_\Delta = 2.1$$

for the first set, $55^\circ < \theta_\pi < 95^\circ$,

$$\lambda_\Delta = 2.2$$

for the second set, $55^\circ < \theta_\pi < 75^\circ$, and

$$\lambda_\Delta = 2.0$$

for the third set, $75^\circ < \theta_\pi < 95^\circ$. The values of μ_Δ calculated from λ_Δ are as follows :

$$\mu_\Delta = 2\ (1 + \lambda_\Delta)\ \frac{m_p}{M_\Delta}\ \left(\frac{e}{2\ m_p}\right)$$

$$= \begin{cases} 4.7\ \dfrac{e}{2m_p} & \text{for} \quad 55^\circ < \theta_\pi < 95^\circ \\ 4.9\ \dfrac{e}{2m_p} & \text{for} \quad 55^\circ < \theta_\pi < 75^\circ \\ 4.6\ \dfrac{e}{2m_p} & \text{for} \quad 75^\circ < \theta_\pi < 95^\circ \end{cases} \tag{82}$$

The range of μ_Δ determined by the SIN data is therefore $4.6\ e/(2m_p) \le \mu_\Delta \le 4.9\ e/(2m_p)$. If, on the other hand, all the SIN data sets are analyzed together to yield a single value of μ_Δ, we find $\mu_\Delta = 4.6\ e/(2m_p)$.

It is clear that the values of μ_Δ extracted from either the UCLA data or the SIN data are smaller than the "bare" magnetic moment, μ_Δ=5.58 $e/(2m_p)$, predicted by the SU(6) model[26] and the quark model. However, as pointed out by the UCLA group, a modified SU(6) model (with mass corrections) suggested by Beg and Pais[25] predicts $\mu_\Delta = \left(\frac{m_p}{M_\Delta}\right)\times 5.58\ e/(2m_p) = 4.25\ e/(2m_p)$. Moreover, Meyer et al. have also pointed out that bag-model corrections to the quark model[27] give μ_Δ= 4.41~ 4.89 $e/(2m_p)$. Thus the values of μ_Δ extracted from the data (the average value of μ_Δ determined from both the UCLA and the SIN data is 4.35 $e/(2m_p)$) are in much better agreement with the value predicted by the modified SU(6) model or the quark model with corrections. The values of μ_Δ previously obtained by other authors were 3.6 ± 2.0 by Musakhanov,[28] 5.6 ± 2.1 by Pascual and Tarrach,[29] 7.0 ~ 9.8 by Heller et al.[30], and 5.58 ~ 7.53 by Wittman[31] in units of $e/(2m_p)$. All of these results were extracted from the UCLA data using quite different approximations and methods. Most recently, by fitting the asymmetry data to predictions calculated in the MIT model, Bosshard et al. have found $\mu_\Delta = 4.58 \pm 0.33\ e/(2m_p)$.[32] A comparison of these results is shown in Table II. For other theoretical predictions, we refer to an article published recently by Krivoruchenko et al.[33].

IV Theoretical $\pi^+p\gamma$ Cross Sections and Comparison to Experimental Data:

Using the values of μ_Δ extracted from the experimental data [Eqs.(80) and (82)] as input, we have applied the amplitude M_μ^{TETAS} to calculate all $\pi^+p\gamma$ cross sections which can be compared with the experimental data at the five bombarding energies, 165, 269, 298, 299,and 324 MeV. Some of these calculations are shown in Figs. 6-12. In these figures, the calculated cross sections at 269, 298, and 324 MeV are compared with the UCLA data of Nefkens et al. and the calculated cross sections at 165 MeV and 299 MeV are compared with the UCLA data of Smith et al.[3] and the SIN data ,respectively.

For G11 - G17, the calculated cross sections are insensitive to the variation of μ_Δ between 3.7 $e/(2m_p)$ and 4.2 $e/(2m_p)$. As shown in Figs.8,9 and 10, the calculations using μ_Δ = 3.7,4.0 and 4.2 $e/(2m_p)$ give almost identical spectra. Although different values of μ_Δ(3.7 $e/(2m_p) \le \mu_\Delta \le 4.2\ e/(2m_p)$) predict spectra which are slightly different at k > 70 MeV for G1 - G10, Figs.6, 7 and 8 show that the difference is smaller than the experimental errors. The overall agreement between theory and the UCLA data is excellent. This fact can also be seen from the following χ^2 values. We have calculated the χ^2 values for those UCLA cross sections shown in Figs.6 ~ 9 using μ_Δ= 4.0 $e/(2m_p)$ as an input for all theoretical predictions. The χ^2 values are 1.6(0.9), 0.4, 4.1, 5.3(2.1), 0.8(0.3), 0.5, 0.6(0.5), 0.5, 1.0, 0.6, 2.8, 8.1(1.8), 3.6(1.6), 0.7 and 1.0 for G1 at 298 MeV (Fig.6a), G2 at 298 MeV (Fig.6b), G3 at 269 MeV (Fig.6c), G4 at 269 MeV (Fig.6d), G5 at 324 MeV (Fig.7a), G6 at 298 MeV (Fig.7b), G7 at 324 MeV (Fig.7c), G8 at 298 MeV (Fig.7d), G9 at 298 MeV (Fig.8a), G10 at

269 MeV (Fig.8b), G11 at 298 MeV (Fig.8c), G12 at 269 MeV (Fig.8d), G13 at 269 MeV (Fig.9a), G14 at 269 MeV (Fig.9b), G15 at 269 MeV (Fig.9c), respectively. The χ^2 values in parentheses are obtained from the calculation which does not include the last datum with zero cross section. Thus, the TETAS amplitude with values of μ_Δ in the range from 3.7 to 4.2 $e/(2m_p)$ can be used to describe all $\pi^+p\gamma$ data obtained by the UCLA group except for the measurements obtained for G18. Here , we must point out that our predicted cross sections for G18 at 269, 298 and 324 MeV are quite different from the UCLA data . (These three spectra for G18 are the only exceptions. The rest of other spectra at 165, 269, 298 and 324 MeV for G_i ($i = 1, \cdots, 19$ but $i \neq 18$) are in excellent agreement with the UCLA data.) However, the agreement is much better for G18 at 165 MeV. This comparison is shown in Fig.10b.

In Fig. 11, we present the results of our calculation using μ_Δ = 4.6, 4.7 and 4.9 $e/(2m_p)$ at 299 MeV. These results are compared with the SIN data. Using μ_Δ = 4.6 and 4.9 $e/(2m_p)$ as input for theoretical predictions, we have calculated the χ^2 values for the three sets of cross sections shown in Fig.11. The χ^2 values [corresponding to μ_Δ = (4.6, 4.9) $e/(2m_p)$] are (4.3, 3.4), (2.6, 2.8) and (2.2, 2.1) for the second set (Fig.11a), the third set (Fig.11b) and the first set (Fig.11c), respectively. We therefore conclude that the SIN data can be described by the TETAS amplitude with the value of μ_Δ between 4.6 $e/(2m_p)$ and 4.9 $e/(2m_p)$.

Finally, we have also used μ_Δ= 5.58 $e/(2m_p)$, the value predicted by SU(6), SU(3) and the naive quark model, to calculate $\pi^+p\gamma$ spectra at

298 MeV for G7, G14, G15 and G 1-10 (the average cross section over the ten photon counters G1 to G10). As shown in Fig. 12, these results are compared with the calculations using μ_Δ = 3.7, 4.0 and 4.2 $e/(2m_p)$ and also with the calculations using an approximate TETAS amplitude [M_μ(TETAS)] obtained in Ref.22. (The expression for M_μ(TETAS) is given by Eq.(83) in the next section.) This comparison reveals three important facts : (i) For G14 and G15 , all calculations give similar results which are in excellent agreement with the UCLA data. This confirms our statement that the calculated cross sections for G11-G19 are insensitive to the variation of μ_Δ. (ii) The data for G7 and G1-10 can be used to differentiate between the calculation using μ_Δ= 5.58 $e/(2m_p)$ and the calculation using μ_Δ=3.7 ~ 4.2 $e/(2m_p)$. The latter is in better agreement with the data. For k>70 MeV, all calculations with μ_Δ=5.58 $e/(2m_p)$ are in complete disagreement with the data. (iii) The spectra calculated using an approximate TETAS amplitude obtained in Ref.22 are very close to those calculated using the amplitude M_μ^{TETAS} given by Eq. (75) with the value of μ_Δ between 3.7 $e/(2m_p)$ and 4.2 $e/(2m_p)$.

V. The TETAS Amplitude for the $\pi^-p\gamma$ Process and the Magnetic Moment of the Δ^0 (1232)

We have already shown a TETAS amplitude can be rigorously derived for the $\pi^+p\gamma$ process near the Δ^{++}(1232) resonance and we have also discussed how the magnetic dipole moment of the Δ^{++} can be extracted from the $\pi^+p\gamma$ data. The fact that an excellent result has been obtained for the $\pi^+p\gamma$ process encourages us to apply the same method to construct a TETAS amplitude for the $\pi^-p\gamma$ process near the Δ^0(1232) resonance and to extract the magnetic dipole moment of the Δ^0 from the UCLA data[11]. We have found that the amplitude for the $\pi^-p\gamma$ process has same form as that for the $\pi^+p\gamma$ process, except that there is no charge contribution from the internal Δ^0 line. Thus, both amplitudes have very similar features. Since the magnetic moment of the Δ^0 has never been determined from the $\pi^-p\gamma$ data by any group, it is interesting to investigate (i) whether the extracted magnetic moment of the Δ^0 is about zero as predicted by the SU(6) model and (ii) whether the $\pi^-p\gamma$ cross sections, calculated using the TETAS amplitude and the extracted magnetic moment of the Δ^0 as the input, agree with the UCLA data and the CERN data[13].

(A) The TETAS Amplitude for the $\pi^-p\gamma$ Process

At the tree level, the amplitude $\bar{M}_\mu^{(\Delta)}$ which represents photon emission from the internal Δ^0 line can be written as

$$\varepsilon^\mu \bar{M}_\mu^{(\Delta)} = \bar{u}(p_f,\nu_f)[g q_f{}^\rho]\, G_{\rho\sigma}(p')\, [-iQ^0\Gamma_\mu^{\sigma\beta}\,\varepsilon^\mu]\, G_{\beta\alpha}(p)\, [g q_i{}^\alpha] u(p_i,\nu_i). \tag{83}$$

Here, $[-iQ^0\Gamma_\mu^{\sigma\beta}\varepsilon^\mu]$ is the electromagnetic vertex for the Δ^0 with $Q^0 = 1e$

(one unit of proton charge). Except for different electromagnetic vertex, this expression for $\bar{M}_{\mu}^{(\Delta)}$ is identical to the expression for $\tilde{M}_{\mu}^{(\Delta)}$ given by Eq. (25). Since Δ^0 has only the anomalous magnetic moment λ_{Δ}', $\Gamma_{\mu}^{\sigma\beta}$ has the form

$$\Gamma_{\mu}^{\sigma\beta} = g^{\sigma\beta} \frac{\lambda_{\Delta}'}{4M_{\Delta}'} (\gamma_{\mu}\gamma_{\nu} - \gamma_{\nu}\gamma_{\mu}) \, k^{\nu} \, , \tag{84}$$

where M_{Δ}' is the mass of the Δ^0. Let Q_{π^-} be the charge of π^-, then we can write

$$Q^0 = \frac{1}{2} Q_p - \frac{1}{2} Q_{\pi^-} \, . \tag{85}$$

Applying the radiation decomposition identity to decompose $\bar{M}_{\mu}^{(\Delta)}$, we find

$$G_{\rho\sigma}(p')[Q^0\Gamma_{\mu}^{\sigma\beta}\varepsilon^{\mu}]G_{\beta\alpha}(p) = iG_{\rho\sigma}(p')\left[\frac{Q^0\bar{O}^{\sigma}{}_{\alpha}}{2p'\cdot k}\right]+\left[-\frac{Q^0\bar{O}'_{\rho}{}^{\beta}}{2p\cdot k}\right]iG_{\beta\alpha}(p) \, , \tag{86}$$

where

$$\bar{O}^{\sigma}{}_{\alpha} = 4R'\cdot\varepsilon \;\; g^{\sigma}{}_{\alpha} + \bar{E}^{\sigma}{}_{\alpha} \, , \tag{87a}$$

$$\bar{O}'_{\rho}{}^{\beta} = 4R'\cdot\varepsilon \;\; g_{\rho}{}^{\beta} + \bar{E}'_{\rho}{}^{\beta} \, , \tag{87b}$$

and

$$R'\cdot\varepsilon = \frac{\lambda_{\Delta}'}{16M_{\Delta}'} \left\{ [\, \not{\varepsilon} \, , \not{k} \,] \, , \not{p} \right\} . \tag{87c}$$

The expressions for $\bar{E}^{\sigma}{}_{\alpha}$ and $\bar{E}'_{\rho}{}^{\beta}$ can be obtained from the expressions for $E^{\sigma}{}_{\alpha}$ and $E'_{\rho}{}^{\beta}$ given in Appendix B. Changing λ_{Δ} to λ_{Δ}' and M_{Δ} to M_{Δ}' in the expressions for $E^{\sigma}{}_{\alpha}$ and $E'_{\rho}{}^{\beta}$ and then dropping those terms which are independent of λ_{Δ}', we obtain the expressions for $\bar{E}^{\sigma}{}_{\alpha}$ and $\bar{E}'_{\rho}{}^{\beta}$, respectively. Inserting Eqs. (85) and (86) into Eq. (83) gives

$$\varepsilon^{\mu} \, \bar{M}_{\mu}^{(\Delta)} = \bar{u}(p_f, \nu_f)\Big[(Q_{\pi^-}) \, \frac{R'\cdot\varepsilon}{p'\cdot k} \, \tilde{T}_a - (Q_{\pi^-}) \, \tilde{T}_b \frac{R'\cdot\varepsilon}{p\cdot k}$$

$$- Q_p \frac{R'\cdot\varepsilon}{p'\cdot k} \tilde{T}_c + Q_p \tilde{T}_d \frac{R'\cdot\varepsilon}{p\cdot k} \Big] u(p_i,\nu_i)$$

$$+ \bar{u}(p_f,\nu_f)[\ \varepsilon\cdot\bar{D}\ ]u(p_i,\nu_i)\ , \tag{88}$$

where $\tilde{T}_a$, $\tilde{T}_b$, $\tilde{T}_c$, and $\tilde{T}_d$ are defined by Eqs.(13) and $\bar{D}$ can be obtained from Eq.(45). If we replace both Q_a and Q_b by $(-\ Q_{\pi^-})$ and (Q_a+Q_b) by Q^0, and drop those terms involving $p'\cdot\varepsilon$ and $p\cdot\varepsilon$, we get $\varepsilon\cdot\bar{D}$. The external amplitude $\bar{M}^{(E)}_\mu$ can be obtained from Eq.(19) by changing Q_a, Q_b, Q_c, and Q_d to Q_{π^-}, Q_{π^-} , Q_p, and Q_p, respectively. Combining $\varepsilon^\mu \bar{M}^{(E)}_\mu$ with $\varepsilon^\mu \bar{M}^{(\Delta)}_\mu$, we find

$$\begin{aligned}
\varepsilon^\mu M^{(E\Delta)}_\mu = \bar{u}(p_f,\nu_f) \Bigg\{ & (Q_{\pi^-}) \left[\frac{q_f\cdot\varepsilon}{q_f\cdot k} + \frac{R'\cdot\varepsilon}{p'\cdot k} \right] \tilde{T}_a \\
& - (Q_{\pi^-})\ \tilde{T}_b \left[\frac{q_i\cdot\varepsilon}{q_i\cdot k} + \frac{R'\cdot\varepsilon}{p'\cdot k} \right] \\
& + Q_p \left[\frac{(p_f+R_f)\cdot\varepsilon}{p_f\cdot k} - \frac{R'\cdot\varepsilon}{p'\cdot k} \right] \tilde{T}_c \\
& - Q_p\ \tilde{T}_d \left[\frac{(p_i+R_i)\cdot\varepsilon}{p_i\cdot k} - \frac{R'\cdot\varepsilon}{p\cdot k} \right] \Bigg\} u(p_i,\nu_i) \\
& + \bar{u}(p_f,\nu_f)\ (\varepsilon\cdot\bar{D})\ u(p_i,\nu_i)\ .
\end{aligned} \tag{89}$$

Obviously, the amplitude $M^{(E\Delta)}_\mu$ is not gauge invariant, $K^\mu M^{(E\Delta)}_\mu \neq 0$, even though $K^\mu R'_\mu = 0$. To obtain the total amplitude $\bar{M}_\mu$ which is completely gauge invariant, we have to impose the gauge invariant condition:

$$\begin{aligned}
&\bar{M}_\mu = \bar{M}^{(E\Delta)}_\mu + \bar{M}^{(G)}_\mu\ , \\
&\bar{M}_\mu\ k^\mu = (\ \bar{M}^{(E\Delta)}_\mu + \bar{M}^{(G)}_\mu)\ k^\mu = 0\ .
\end{aligned} \tag{90}$$

Without repeating the detailed calculations, which are similar to those given by Eq.(50) for the $\pi^+ p\gamma$ case, the final expression for $\varepsilon^\mu \overline{M}_\mu$ can be written as

$$\varepsilon^\mu \overline{M}_\mu = \varepsilon^\mu \overline{M}^{TETA}_\mu + \varepsilon^\mu \overline{M}_\mu^{\,X} , \tag{91a}$$

$$= \varepsilon^\mu \overline{M}^{TETAS}_\mu + \varepsilon^\mu \overline{M}^{off}_\mu + \varepsilon^\mu \overline{M}_\mu^{\,X} , \tag{91b}$$

where

$$\begin{aligned} \overline{M}^{TETAS}_\mu = \bar{u}(p_f, \nu_f) \Bigg\{ & (Q_{\pi^-}) \left[\frac{q_{f\mu}}{q_f \cdot k} - \frac{(p_f+q_f-R')_\mu}{(p_f+q_f)\cdot k} \right] \tilde{T}(s_i, t_p) \\ & - (Q_{\pi^-}) \, \tilde{T}\,(s_f, t_p) \left[\frac{q_{i\mu}}{q_i \cdot k} - \frac{(p_i+q_i-R')_\mu}{(p_i+q_i)\cdot k} \right] \\ & + Q_p \left[\frac{(p_f+R_f)_\mu}{p_f \cdot k} - \frac{(p_f+q_f+R')_\mu}{(p_f+q_f)\cdot k} \right] \tilde{T}\,(s_i, t_q) \\ & - Q_p \tilde{T}\,(s_f, t_q) \left[\frac{(p_i+R_i)_\mu}{p_i \cdot k} - \frac{(p_i+q_i+R')_\mu}{(p_i+q_i)\cdot k} \right] \Bigg\} u(p_i, \nu_i) \end{aligned} \tag{92}$$

and the expression for $\overline{M}^{TETA}_\mu$ is obtained if $\tilde{T}(s_i,\ t_p)$, $\tilde{T}(s_f,\ t_p)$, $\tilde{T}(s_i,\ t_q)$, and $\tilde{T}(s_f,\ t_q)$ in Eq.(92) are replaced by $\tilde{T}_a$, $\tilde{T}_b$, $\tilde{T}_c$, and $\tilde{T}_d$, respectively. Although the expressions for $\overline{M}^X_\mu$ and $\overline{M}^{off}_\mu$ can be obtained by slightly modifying the expressions for $\tilde{M}^X_\mu$ and $\tilde{M}^{off}_\mu$ given by Eqs.(52) and (55), respectively, we omit them here mainly because they will be ignored in our numerical calculations.

We have shown how the TETAS amplitude at the tree level, $\overline{M}^{TETAS}_\mu$, can be derived for the $\pi^- p\gamma$ process near the $\Delta^0(1232)$ resonance. The result obtained here can be used as a guide to construct a more general TETAS amplitude, $M^{TETAS}_\mu(\pi^-)$, by using a modified procedure for constructing the soft-photon amplitude. It turns out that $M^{TETAS}_\mu(\pi^-)$ has the same form as the expression for $\overline{M}^{TETAS}_\mu$ except that all T-matrices [$\tilde{T}(s_i, t_p)$, $\tilde{T}(s_f, t_p)$, $\tilde{T}(s_i, t_q)$, and $\tilde{T}(s_f, t_q)$] at the tree level are replaced by the realistic experimentally determined elastic T-matrices [$T(s_i, t_p)$, $T(s_f, t_p)$, $T(s_i, t_q)$, and $T(s_f, t_q)$] :

$$M_\mu^{TETAS}(\pi^-) = \overline{M}_\mu^{TETAS}(\ \tilde{T} \longrightarrow T\ ) \qquad (93)$$

where $\overline{M}_\mu^{TETAS}$ is given by Eq.(92).

(B) The Magnetic Moment of the $\Delta^0(1232)$

Using Eq.(76) with the amplitude $M_\mu^{TETAS}(\pi^-)$ given by Eq.(93), we have calculated $\pi^- p\gamma$ cross sections as a function of photon energy k. In these calculations, the anomalous magnetic moment of the Δ^0, λ_Δ', has been treated as a free parameter and it is to be determined from the UCLA data[11] at three bombarding energies, 263, 298, and 330 MeV.

As we have already discussed in section III, the UCLA data can be denoted by $\sigma^{UCLA}(E_i,\ G_j,\ k_l)$, which represents cross section at the bombarding energy E_i $(i=1,2,3)$ and the photon energy k_l $(l=1,\ 2,\ \cdots,\ 7)$ for the spectrum G_j $(j=1,\ 2,\ \cdots,\ 19)$. The definitions for k_l and G_j are the same for both the $\pi^+ p\gamma$ case and the $\pi^- p\gamma$ case. The only difference between the two cases is E_i. For the $\pi^- p\gamma$ case, we define E_1= 263 MeV, E_2= 298 MeV and E_3= 330 MeV. Again, the corresponding theoretical cross section, calculated using the amplitude $M_\mu^{TETAS}(\pi^-)$ given by Eq.(93), will also be denoted by $\sigma^{TH}(E_i,\ G_j,\ k_l)$.

Using the experimental cross sections $\sigma^{UCLA}(E_i,\ G_j,\ k_l)$ and the theoretical cross sections $\sigma^{TH}(E_i,\ G_j,\ K_l)$, we first calculate $\sigma^{UCLA}_{1-10}(E_i,\ k_l)$, $\sigma^{TH}_{1-10}(E_i,\ k_l)$, $\sigma^{UCLA}_{1-15}(E_i,\ k_l)$, and $\sigma^{TH}_{1-15}(E_i,\ k_l)$ using Eqs.(77a), (77b), (77e) and (77f), respectively. The values of $\sigma^{UCLA}_{1-10}(E_i,k_l)$ and $\sigma^{UCLA}_{1-15}(E_i,\ k_l)$ are shown in Table III. We then use these cross sections to calculate average deviations $D_{1-10}(E_i,\lambda_\Delta')$ and $D_{1-15}(E_i,\lambda_\Delta')$, which are defined by Eqs.(78a) and (78c), respectively. Varying the value of λ_Δ', we obtain two deviation curves for each bombarding energy. As shown in Figs. 14a, 14b, and 14c, each curve gives a minimum value for λ_Δ'. We find the following minimum values for λ_Δ' from six curves: $\lambda_\Delta'^{(1-10)}(E_1)= 0.001$, $\lambda_\Delta'^{(1-10)}(E_2)= 1.0$, $\lambda_\Delta'^{(1-10)}(E_3)= 0.001$, $\lambda_\Delta'^{(1-15)}(E_1)= 1.0$, $\lambda_\Delta'^{(1-15)}(E_2)= 1.0$, $\lambda_\Delta'^{(1-15)}(E_3)= 1.0$. Note that we have chosen $\lambda_\Delta'^{(1-10)}(E_2)= 1.0$ mainly because the deviation curve (for E_2= 298 MeV and spectra G1-G10) reaches its minimum point at $\lambda_\Delta'= 2.0$ and the curve becomes flat in the region $0\le \lambda_\Delta' \le 2.0$. Taking an average, we have $\lambda_\Delta'= \sum_{i=1}^{3} \lambda_\Delta'^{(1-10)}(E_i)/3 = 0.33$ for spectra G1-G10 and $\lambda_\Delta'= \sum_{i=1}^{3} \lambda_\Delta'^{(1-15)}(E_i)/3 = 1.0$ for spectra G1-G15. If we take an average again, we find $\lambda_\Delta' = (0.33 + 1.0)/2 = 0.67$. Thus, the range of λ_Δ' extracted from the $\pi^- p\gamma$ data of the UCLA is $0 \le \lambda_\Delta' \le 1.0$ and the average value for λ_Δ' is 0.67. Now, the magnetic moment of the Δ^0 can be calculated from the following formula

$$\mu_\Delta' = \lambda_\Delta' \left(\frac{m_p}{M_\Delta}\right) \left(\frac{e}{2m_p}\right).$$

Therefore, the average μ_Δ', calculated from $\lambda_\Delta'= 0.67$, is 0.5 $e/(2m_p)$ and the range of μ_Δ', calculated from $0 \le \lambda_\Delta' \le 1.0$, is $0 \le \mu_\Delta' \le 0.76$ $e/(2m_p)$. The value of μ_Δ' in this range is in accord with the value $\mu_\Delta'=0$ predicted by the SU(6) model. This agreement is also supported by the following two facts: (i) All those deviation curves which give $\lambda_\Delta'=1.0$ are flat and insensitive to the variation of λ_Δ' in the range $0 \le \lambda_\Delta' \le 2$. (ii) The calculated $\pi^- p\gamma$ cross sections, as will be shown in next subsection, are very insensitive to the variation of μ_Δ' in the region $0 \le \mu_\Delta' \le 0.5$ $e/(2m_p)$. Since μ_Δ' has never before been directly measured or determined from the $\pi^- p\gamma$ data, no other result can be used to compare with ours.

(C) Theoretical $\pi^- p\gamma$ Cross Sections and Comparison to Experimental Data:

We have applied the amplitude $M_\mu^{TETAS}(\pi^-)$ to calculate all $\pi^- p\gamma$ cross sections which can be compared with the UCLA data at 263, 298, and 330 MeV and the CERN data at 192 MeV. The input in these calculations is the magnetic moment μ_Δ' which is chosen to be 0 and 0.5 $e/(2m_p)$. Some of these calculations are shown in Figs. 15-20.

As shown in Figs. 15-18, the calculations using $\mu_\Delta' = 0$ and 0.5 $e/(2m_p)$ give almost identical spectra which are in excellent agreement with the UCLA data. Thus, the calculated $\pi^- p\gamma$ cross sections are completely insensitive to the variation of μ_Δ' in the range between 0 and 0.5 $e/(2m_p)$. Such finding is expected because it is consistent with the behavior of the deviation curves in the region $0 \le \lambda_\Delta' \le 0.67$ (see Figs. 14a, 14b, and 14c). Since the extracted magnetic moment of the Δ^0, in the range $0 \le \mu_\Delta' \le 0.5\ e/(2m_p)$, gives essentially the same $\pi^- p\gamma$ cross sections, we conclude that our result is in agreement with the value ($\mu_\Delta'=0$) predicted by the SU(6) model.

Using $\mu_\Delta'=0$ (more precisely, we use $\lambda_\Delta' = 0.001$), we have also calculated the $\pi^- p\gamma$ cross sections at 192 MeV. The result is compared with the CERN data[13]. As shown in Figs. 19 and 20, the agreement between theory and experiment is also good but not as good as we would like to see. This may not be surprising since the agreement between the data and the EED calculations[17] is even worse. Further studies are required in order to understand some discrepancy between the CERN data and our calculations.

V. Discussion

It has been reported in Ref.22 that almost all of the $\pi^{\pm}p\gamma$ cross sections obtained by the UCLA group can be described by a TETAS amplitude of the form

$$
\begin{aligned}
M_{\mu}(\mathrm{TETAS}) = \bar{u}(p_f,\nu_f)\{Q_a[\frac{q_{f\mu}}{q_f\cdot k} - \frac{(q_f+p_f+R_f)_{\mu}}{(q_f+p_f)\cdot k}]\ T(s_i,t_p) \\
- Q_b\ T(s_f,t_p)\ [\frac{q_{i\mu}}{q_i\cdot k} - \frac{(q_i+p_i+R_i)_{\mu}}{(q_i+p_i)\cdot k}] \qquad (94) \\
+ Q_c\ [\frac{p_{f\mu}+R_{f\mu}}{p_f\cdot k} - \frac{(q_f+p_f+R_f)_{\mu}}{(q_f+p_f)\cdot k}]\ T(s_i,t_q) \\
- Q_d\ T(s_f,t_q)\ [\frac{p_{i\mu}+R_{i\mu}}{p_i\cdot k} - \frac{(q_i+p_i+R_i)_{\mu}}{(q_i+p_i)\cdot k}]\}u(p_i,\nu_i)\ ,
\end{aligned}
$$

where $R_{i\mu}$ and $R_{f\mu}$ are defined by Eqs.(20a) and (20b), respectively. This amplitude, which cannot be rigorously derived, is slightly different from the amplitude M_{μ}^{TETAS} given by Eq. (75). In section IV, as shown in Fig. 12, we have found that the $\pi^{+}p\gamma$ cross sections calculated with the amplitude $M_{\mu}(\mathrm{TETAS})$ are very close to the cross sections predicted by the amplitude M_{μ}^{TETAS} if the value of μ_{Δ} used in M_{μ}^{TETAS} is about $4\ e/(2m_p)$. To understand why the amplitude $M_{\mu}(\mathrm{TETAS})$ works so well and why the two amplitudes, $M_{\mu}(\mathrm{TETAS})$ and M_{μ}^{TETAS}, can give similar results, let us compare these two amplitudes carefully. From Eqs.(75) and (94), we can see that both amplitudes have the same form for the external contribution but they differ from one another in the expression for the internal contribution. They can predict about the same cross sections only under the following condition:

$$R_{i\mu} \approx R_{\mu} \approx R_{f\mu} \ . \tag{95}$$

To study this condition without any approximation is very difficult. Fortunately, a good approximation can be found. If we replace $\not{p}_i$, $\not{p}_f$ and $\not{p}$ in the expressions for $R_{i\mu}$, $R_{f\mu}$ and R_{μ} (Eqs.(20a), (20b) and (43)) by m_p, m_p and M_Δ, respectively, then we find

$$R_{i\mu} = R_{f\mu} = R_{\mu} \tag{96}$$

provided that

$$\lambda_\Delta = \lambda_p \ . \tag{97}$$

This implies that the two amplitudes can produce about the same result if the magnetic moment of the Δ^{++} (treated as a parameter in the amplitude M_μ^{TETAS}) is

$$\begin{aligned}\mu_\Delta &= 2(1+\lambda_\Delta) \ \ e/(2M_\Delta) \\ &= 2(1+\lambda_p) \ \frac{m_p}{M_\Delta} \ \frac{e}{2m_p} \\ &= 2(1+1.79) \ \frac{938}{1232} \ \frac{e}{2m_p} \\ &= 4.25 \ e/(2m_p)\end{aligned}$$

which is exactly the value predicted by the modified SU(6) model of Beg and Pais.[25] This value also agrees very well with the average value of μ_Δ , 4.35 $e/(2m_p)$, extracted from both the UCLA data and the SIN data. Thus, the fact that the UCLA data can be described by the amplitude M_μ(TETAS) suggests that the value of μ_Δ is about 4 $e/(2m_p)$.

It is obvious that the modified Low procedure can be applied to obtain TETAS amplitudes for other bremsstrahlung processes near a scattering resonance. For example, the TETAS amplitude for the $p^{12}C\gamma$ process has the same expression as the amplitude M_μ^{TETAS} given by Eq. (75) but without those terms involving $R_{i\mu}$, $R_{f\mu}$ and R_μ.[22] This is because the contribution from $R_{i\mu}$, $R_{f\mu}$ and R_{μ} terms is negligible for the low energy $p^{12}C\gamma$ process near either the 1.7-MeV resonance[33-35] or the 0.5-MeV resonance.[7] As shown in Ref.22, those gauge terms involving $(p_i + q_i)_\mu$ or $(p_f + q_f)_\mu$ represent photon emissions from the charge of the intermediate $^{13}N^*$ resonance.

As we have already mentioned, the effective moment which is a complex quantity has been studied by Heller et al.[30] We cannot calculate this moment to arbitrary precision in this work since it is difficult to take into account the loop contribution in the soft-photon approximation. Nevertheless, we have done a numerical study by treating λ_Δ in Eq.(75) as a complex quantity, $\lambda_\Delta = \lambda_R + i\lambda_I$, in order to estimate the contribution from the imaginary part λ_I. We have chosen λ_Δ to be $1.47+i\lambda_I$, $1.6+i\lambda_I$ and $2.4+i\lambda_I$. By varying λ_I from -1.0 to 1.0 in each case, we have used the UCLA data (at 298 MeV for counters G1-G10) to calculate average deviations as a function of λ_I . As shown in Fig.13, we have obtained three deviation curves which have the same interesting feature. The value of the average deviation decreases rapidly as λ_I increases from -1.0 to zero and then it increases rapidly as λ_I increases from zero to 1.0. Thus, the minimum points for all three average deviation curves are around $\lambda_I = 0$, independent of the choice of λ_R , indicating that the best fit to the UCLA data (at 298 MeV for counters G1-G10) can be obtained by choosing λ_Δ to be a real quantity, as we have done in this work. This result implies that the dynamical contribution (photon emissions from the π^+p loop) to the imaginary part λ_I is very small. If there is no dynamical contribution to λ_I , we also expect very little dynamical contribution to the real part λ_R . We may therefore conclude that the whole dynamical contribution would be small and hence the "experimental" magnetic moment should be very close to the effective moment.

To understand why the best fit to the UCLA data can be obtained only if λ_Δ is chosen to be a real quantity, we have performed another study. Our numerical investigation of the amplitude M_μ^{TETAS} reveals that the best agreement between theory and experiment is obtained when the contribution from the R_μ-dependent terms cancels the total contribution from those terms involving $R_{i\mu}$ and $R_{f\mu}$ in Eq. (75). This cancellation occurs when μ_Δ is around 4 $e/(2m_p)$. However, no cancelation is possible if λ_Δ is chosen to be a complex quantity with a large imaginary part since the anomalous magnetic moment of proton λ_p is a real quantity ($\lambda_p = 1.79$). This explains why the minimum point is always found around $\lambda_I = 0$, independent of the choice of λ_R, if the average deviation is plotted as a function of λ_I. As we have already pointed out, our numerical study also indicates that the spectra calculated by using Eq.(75) agree very well with those spectra predicted by Eq.(94) (which is identical to Eq.(16) of Ref.22) if μ_Δ used in Eq.(75) is about 4 $e/(2m_p)$. Both results are in excellent agreement with the experimental data.

Now let us discuss what would happen if those terms involving $R_{i\mu}$, $R_{f\mu}$ and R_μ are canceled out precisely. We would obtain an amplitude M_μ^{TETAS} with $R_{i\mu} = R_{f\mu} = R_\mu = 0$. Such an amplitude was first proposed by Heller[21] and it was discussed in great details in Ref.22 (Heller's amplitude is identical to Eq.(3) of Ref.22). It is a well known fact that Heller's amplitude can be successfully applied to describe both the $\pi^{\pm}p\gamma$ data and the $p^{12}C\gamma$ data. This fact may have two possible implications that are consistent with our findings. (i) The cancelation between the contribution from the magnetic moment of the Δ^{++}(including all possible loop corrections) and the contribution from the magnetic moment of proton exists. (ii) The imaginary part of the effective magnetic moment of the Δ^{++} is small and the real part is 3.7 ~ 4.9 $e/(2m_p)$. In short, the data seem to suggest that dynamical corrections from the loop diagrams are small. In other words, our best fit implies that the effective magnetic moment of the Δ^{++} should be nearly equal to not only the "experimental" moment obtained in this work but also the bare moment given by the modified SU(6) model or the quark model with corrections.The problem requires further careful studies.

VII. Conclusion

We conclude the following.

(i) We have derived a radiation decomposition identity for bremsstrahlung emission from an internal Δ line, which includes the Δ^{++} line with an anomalous magnetic moment λ_Δ and the Δ^0 line with an anomalous magnetic moment λ_Δ'. We show how this identity can be applied to modify Low's standard prescription for constructing soft-photon amplitudes.

(ii) Using the modified Low procedure, we have derived the TETAS amplitudes for the $\pi^\pm p\gamma$ processes near the $\Delta(1232)$ resonance. The M_μ^{TETAS} amplitude for the $\pi^+p\gamma$ process is given by Eq. (75) while the $M_\mu^{TETAS}(\pi^-)$ amplitude for the $\pi^-p\gamma$ process is given by Eq.(93). These TETAS amplitudes have many interesting features : (1) They are relativistic, gauge invariant and consistent with the soft-photon theorem. (2) They depend only on the elastic T-matrix, evaluated at four different sets of (s, t): (s_i, t_p), (s_i , t_q), (s_f , t_p) and (s_f , t_q), but they are free of any derivative of T with respect to s or t. (3) They take into account bremsstrahlung emissions from (a) the incoming pion and the outgoing pion, (b) the incoming proton and the outgoing proton (with charge +e and the anomalous magnetic moment λ_p), (c) the internal Δ^{++} line (with charge +2e and the anomalous magnetic moment λ_Δ) for the $\pi^+p\gamma$ case and the internal Δ^0 line (with the anomalous magnetic moment λ_Δ') for the $\pi^-p\gamma$ case, and (d) other sources by imposing the gauge invariant condition.

(iii) We have used the amplitude M_μ^{TETAS} to calculate $\pi^+p\gamma$ cross sections as a function of photon energy K, $d^3\sigma/d\Omega_\pi d\Omega_\gamma dK$, at five bombarding energies, 165, 269, 298, 299 and 324 MeV. Treating λ_Δ as a free parameter in these calculations, the "experimental" magnetic moment of the Δ^{++}, μ_Δ, has been extracted from 45 sets of the UCLA data and 3 sets of the SIN data. The extracted values of μ_Δ are

$$\mu_\Delta = \begin{cases} 3.7\ e/(2m_p) & \text{for photon counters G1-G10} \\ 4.0\ e/(2m_p) & \text{for photon counters G1-G15} \\ 4.2\ e/(2m_p) & \text{for photon counters G11-G15} \end{cases}$$

from the UCLA data and

$$\mu_\Delta = \begin{cases} 4.6\ e/(2m_p) & \text{for } 75^\circ < \vartheta_\pi < 95^\circ \\ 4.7\ e/(2m_p) & \text{for } 55^\circ < \vartheta_\pi < 95^\circ \\ 4.9\ e/(2m_p) & \text{for } 55^\circ < \vartheta_\pi < 75^\circ \end{cases}$$

from the SIN data. These extracted values of μ_Δ (the average is 4.35 $e/(2m_p)$), are smaller than the value 5.58 $e/(2m_p)$, the "bare" magnetic moment predicted by the SU(6) model or the quark model, but they are close to the value 4.25 $e/(2m_p)$ predicted by the modified SU(6) model of Beg and Pais and also in accord with the value 4.41~4.89 $e/(2m_p)$ obtained by Brown, Rho and Vento. Using the amplitude M_μ^{TETAS} and the values of μ_Δ extracted from the experimental data, we have calculated all $\pi^+p\gamma$ cross sections which can be compared with the UCLA data and the SIN data. In general, the agreement between the theoretical predictions and the experimental measurements is excellent. This agreement demonstrate that the amplitude M_μ^{TETAS} is valid and it can be used to describe almost all the available $\pi^+p\gamma$ data near the $\Delta(1232)$ resonance.

(iv) We have also treated λ_Δ as a complex quantity, $\lambda_\Delta = \lambda_R + i\lambda_I$, in order to estimate the contribution from the imaginary part λ_I. The best fit to the data gives $\lambda_I = 0$, independent of the choice of λ_R. This finding suggests that further dynamical corrections to the amplitude M_μ^{TETAS} from the open pion-proton channel are small and hence the "effective" moment, the "experimental" moment, and the "bare" moment predicted by the modified SU(6) model of Beg and Pais should have about the same value.

(v) We have shown that the approximate amplitude given by Eq. (94), an amplitude used in Ref. 22, is theoretically justified. This explains why the amplitude (used in Ref.22) works remarkably well for the $\pi^+ p\gamma$ process. We have also explained why the amplitude M_μ^{TETAS} given by Eq.(75) can be used to describe $p{}^{12}C\gamma$ cross sections near either the 1.7-MeV resonance or the 0.5-MeV resonance.

(vi) We have used the amplitude $M_\mu^{TETAS}(\pi^-)$ to calculate $\pi^- p\gamma$ cross sections as a function of photon energy k at three bombarding energies, 263, 298, and 330 MeV. Treating λ_Δ' as a free parameter in these calculations, the magnetic moment of the Δ^0, μ_Δ', has been extracted from the UCLA data. The range of the extracted value of μ_Δ' is $0 \le \mu_\Delta' \le 0.76\ e/(2m_p)$ (with an average value about $0.5\ e/(2m_p)$). Since the calculated $\pi^- p\gamma$ cross sections are very insensitive to the variation of μ_Δ' in this range, we conclude that the extracted value of μ_Δ' is in accord with the value $\mu_\Delta' = 0$, predicted by the SU(6) model. Using $\mu_\Delta'=0$ in the amplitude $M_\mu^{TETAS}(\pi^-)$, we have calculated those $\pi^- p\gamma$ cross sections which can be compared with the UCLA data and the CERN data. The agreement between the theory and the UCLA data is excellent. Although the agreement between the theory and the CERN data is mixed (with some good agreement and some fair agreement), we can still conclude that the amplitude $M_{\mu}^{TETAS}(\pi^-)$ can be used to describe the $\pi^- p\gamma$ data near the $\Delta^0(1232)$ resonance.

TABLE I

The UCLA data and the SIN data : the values of $\sigma^{UCLA}_{1-10}(E_i,\ k_l)$, $\sigma^{UCLA}_{11-15}(E_i,\ k_l)$, $\sigma^{UCLA}_{1-15}(E_i,\ k_l)$, and $\sigma^{SIN}_{i}(\ k'_j\ )$

photon energy(MeV)		22.5	40.0	60.0	80.0	100.0	120.0	140.0
269 (UCLA)	average G1-10	2.53 ±0.41	1.65 ±0.27	1.25 ±0.21	0.79 ±0.15	0.28 ±0.08		
	average G11-15	31.8	15.1	8.8	5.8	2.5	1.2	
	average G1-15	12.3	6.11	3.77	2.44	1.03	0.55	
298 (UCLA)	average G1-10	1.78 ±0.28	1.03 ±0.17	1.03 ±0.15	0.74 ±0.13	0.44 ±0.09	0.44 ±0.12	
	average G11-15	25.0	14.1	7.62	5.68	3.38	1.14	
	average G1-15	9.50	5.41	3.21	2.39	1.44	0.61	
324 (UCLA)	average G1-10	1.05 ±0.31	1.04 ±0.22	0.66 ±0.17	0.77 ±0.18	0.30 ±0.11	0.10 ±0.06	
	average G11-15	22.6	8.30	4.80	2.22	1.64	0.58	0.38
	average G1-15	8.24	3.45	2.05	1.25	0.74	0.43	

photon energy(MeV)		27.5	42.5	57.5	72.5	87.5	102.5	117.5
299 (SIN)	average 55^0-95^0	1.57 ±0.23	1.24 ±0.18	1.40 ±0.16	1.21 ±0.14	1.12 ±0.13	0.85 ±0.10	0.71 ±0.09
	average 55^0-75^0	1.23 ±0.28	1.31 ±0.21	1.40 ±0.18	1.20 ±0.16	1.27 ±0.17	0.83 ±0.13	0.82 ±0.11
	average 75^0-95^0	1.91 ±0.30	1.17 ±0.24	1.40 ±0.20	1.22 ±0.18	0.90 ±0.16	0.90 ±0.13	

Table II

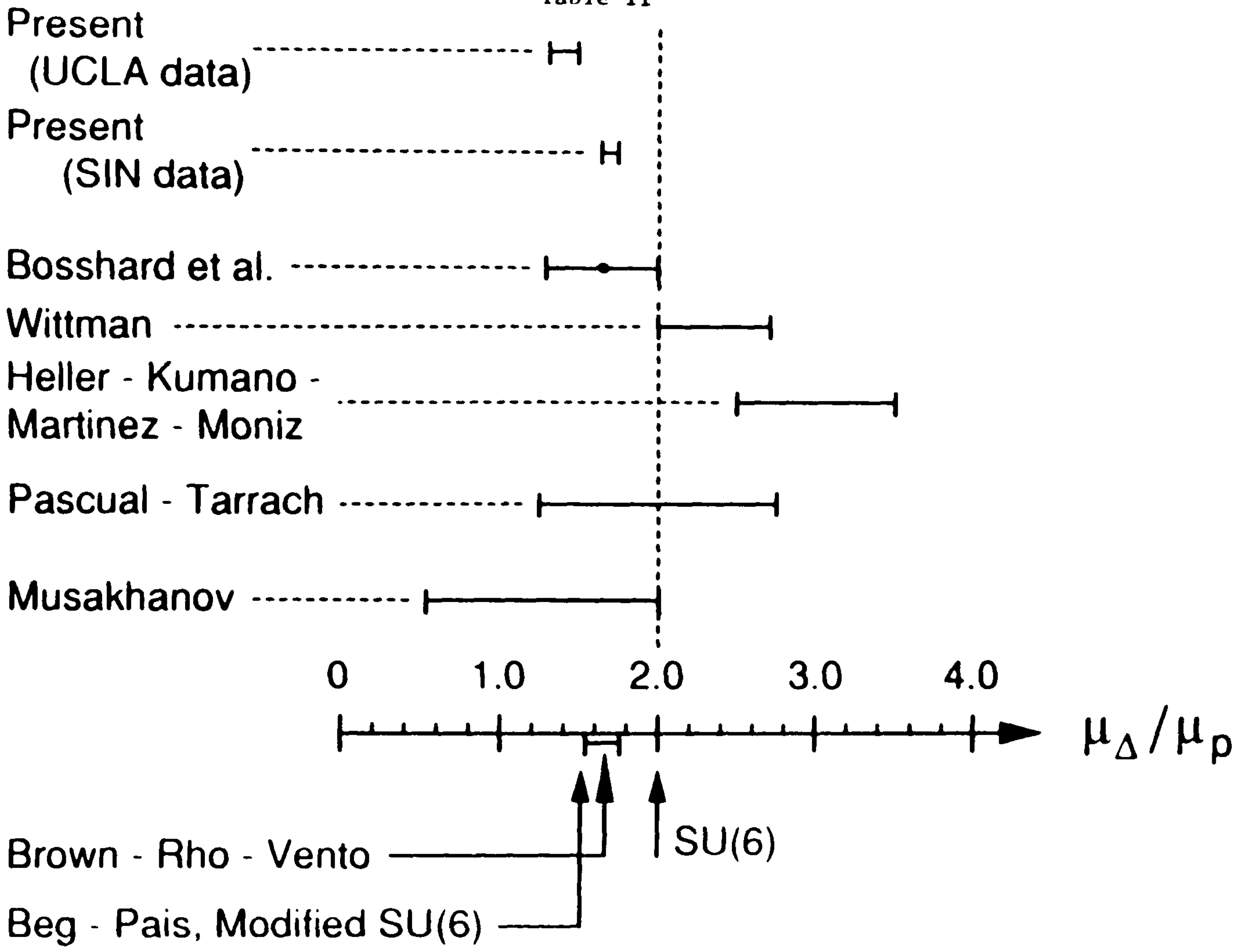
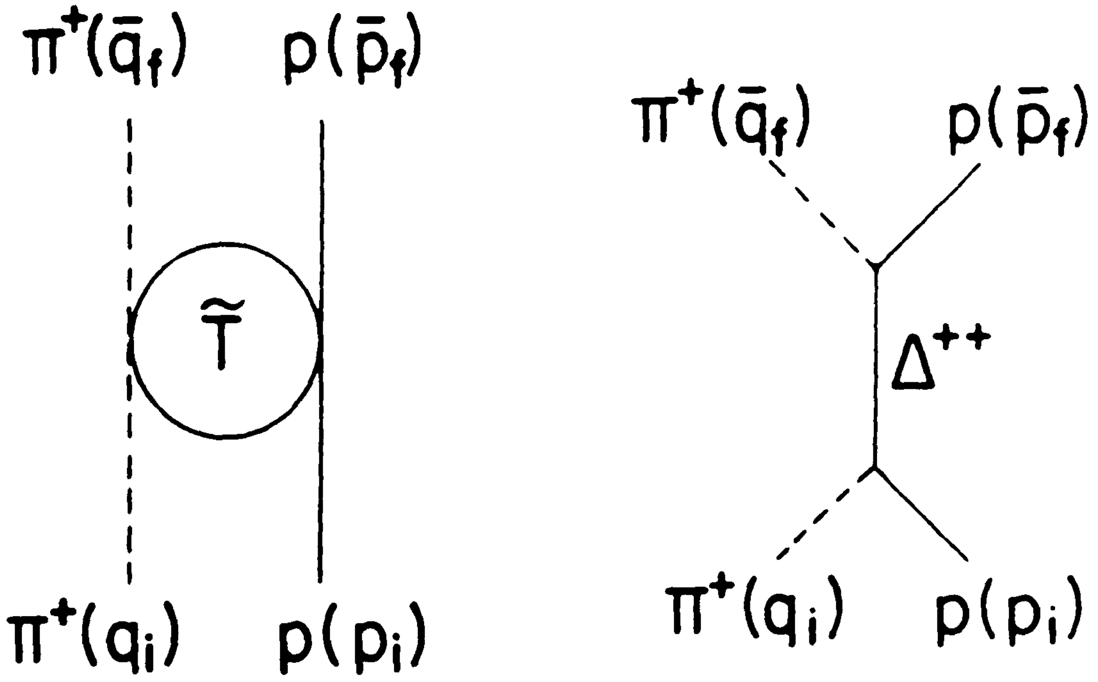

Present
(UCLA data)
Present
(SIN data)
Bosshard et al.
Wittman
Heller - Kumano -
Martinez - Moniz
Pascual - Tarrach
Musakhanov
0
1.0
2.0
3.0
4.0
μ_Δ/μ_p
Brown - Rho - Vento
SU(6)
Beg - Pais, Modified SU(6)

TABLE III

The UCLA data and the CERN data for $\pi^- p \gamma$ process

photon energy(MeV)		22.5	40.0	60.0	80.0	100.0	120.0	140.0
263 (UCLA)	average G1-10	4.56 ±0.88	3.48 ±0.65	1.77 ±0.42	1.28 ±0.35	0.50 ±0.21		
	average G11-15							
	average G1-15	4.63	3.34	2.21	1.78	0.51		
298 (UCLA)	average G1-10	5.7 ±1.1	4.7 ±0.9	2.2 ±0.5	2.6 ±0.6	0.94 ±0.34	0.61 ±0.31	
	average G11-15							
	average G1-15	5.9	4.7	2.4	2.3	0.91	0.71	
330 (UCLA)	average G1-10	2.9 ±0.9	2.6 ±0.7	1.8 ±0.6	1.5 ±0.5	1.5 ±0.5	0.4 ±0.3	
	average G11-15							
	average G1-15	3.5	3.3	1.8	1.7	1.0	0.27	

photon energy(MeV)		50	70	90	110	130		
192 (CERN)	average 90°-120°	5.4 ±3.3	3.7 ±2.0	2.7 ±0.8	2.2 ±0.3	1.07 ±0.19		
	average 120°-150°	3.7 ±1.6	2.7 ±0.6	1.3 ±0.2	0.95 ±0.12	0.58 ±0.09		
	average 150°-180	2.8 ±0.9	1.5 ±0.4	1.11 ±0.24	0.69 ±0.17	0.20 ±0.13		


$\pi^+(\bar{q}_f)$
$p(\bar{p}_f)$
$\tilde{T}$
$\pi^+(q_i)$
$p(p_i)$
$\pi^+(\bar{q}_f)$
$p(\bar{p}_f)$
Δ^{++}
$\pi^+(q_i)$
$p(p_i)$

(a) (b)

Fig. 1

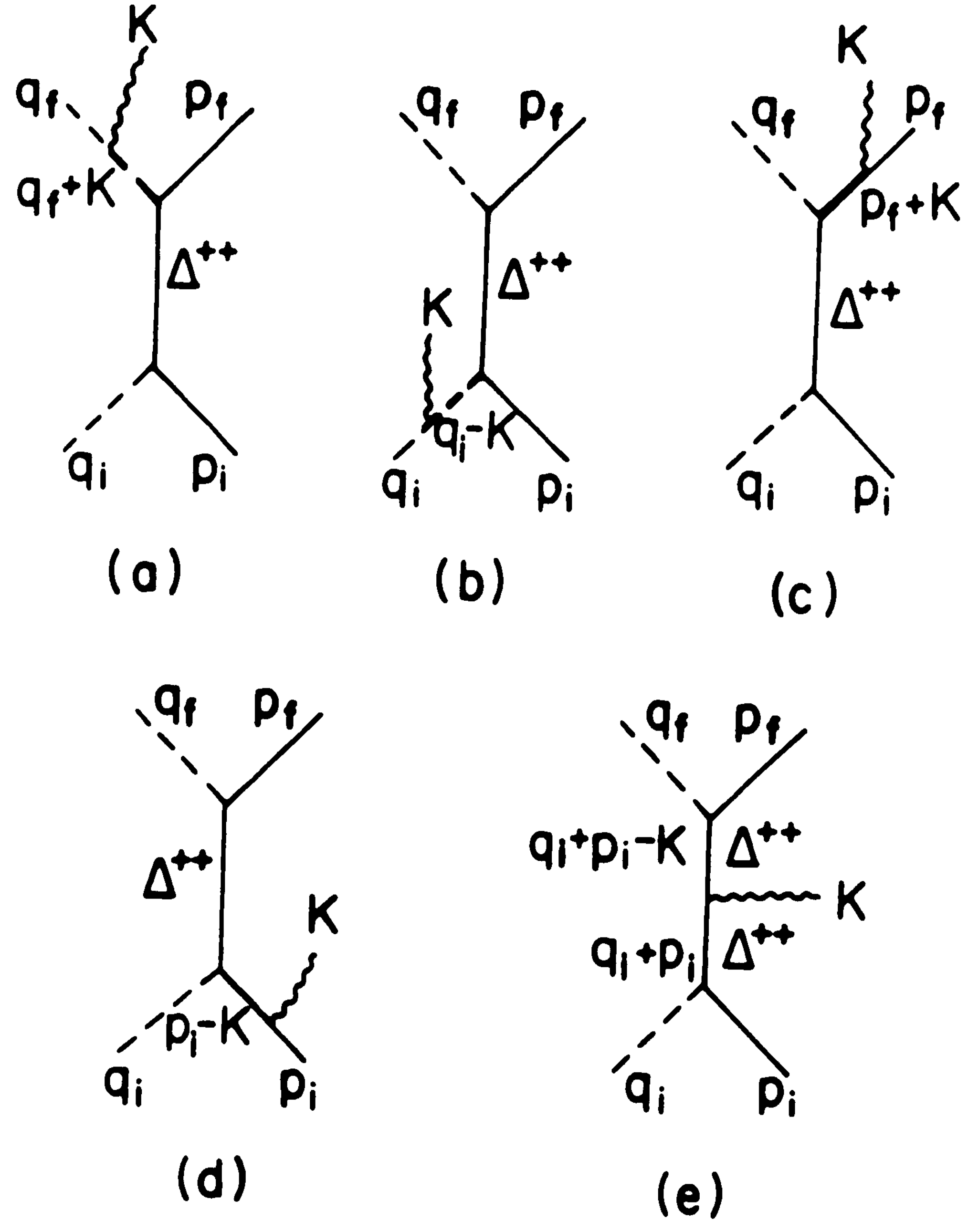

K
q_f
p_f
q_f+K
Δ++
q_i
p_i
(a)
q_f
p_f
Δ++
K
q_i-K
q_i
p_i
(b)
K
q_f
p_f
p_f+K
Δ++
q_i
p_i
(c)
q_f
p_f
Δ++
K
p_i-K
q_i
p_i
(d)
q_f
p_f
q_i+p_i-K
Δ++
K
q_i+p_i
Δ++
q_i
p_i
(e)

Fig. 2

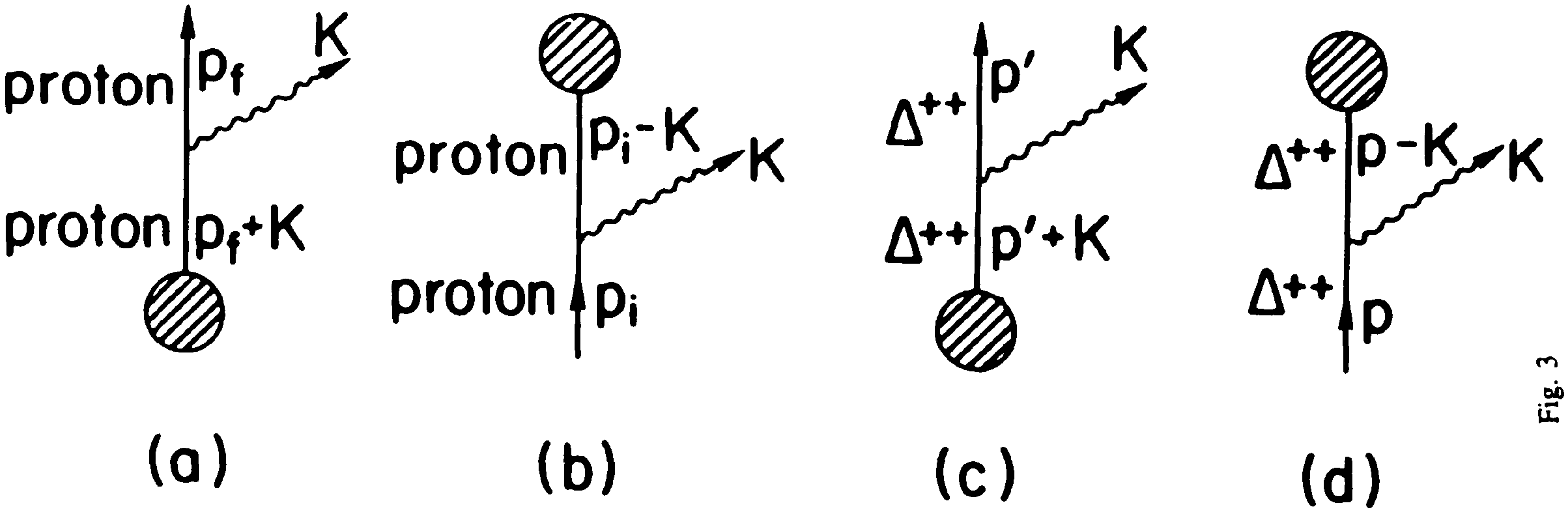
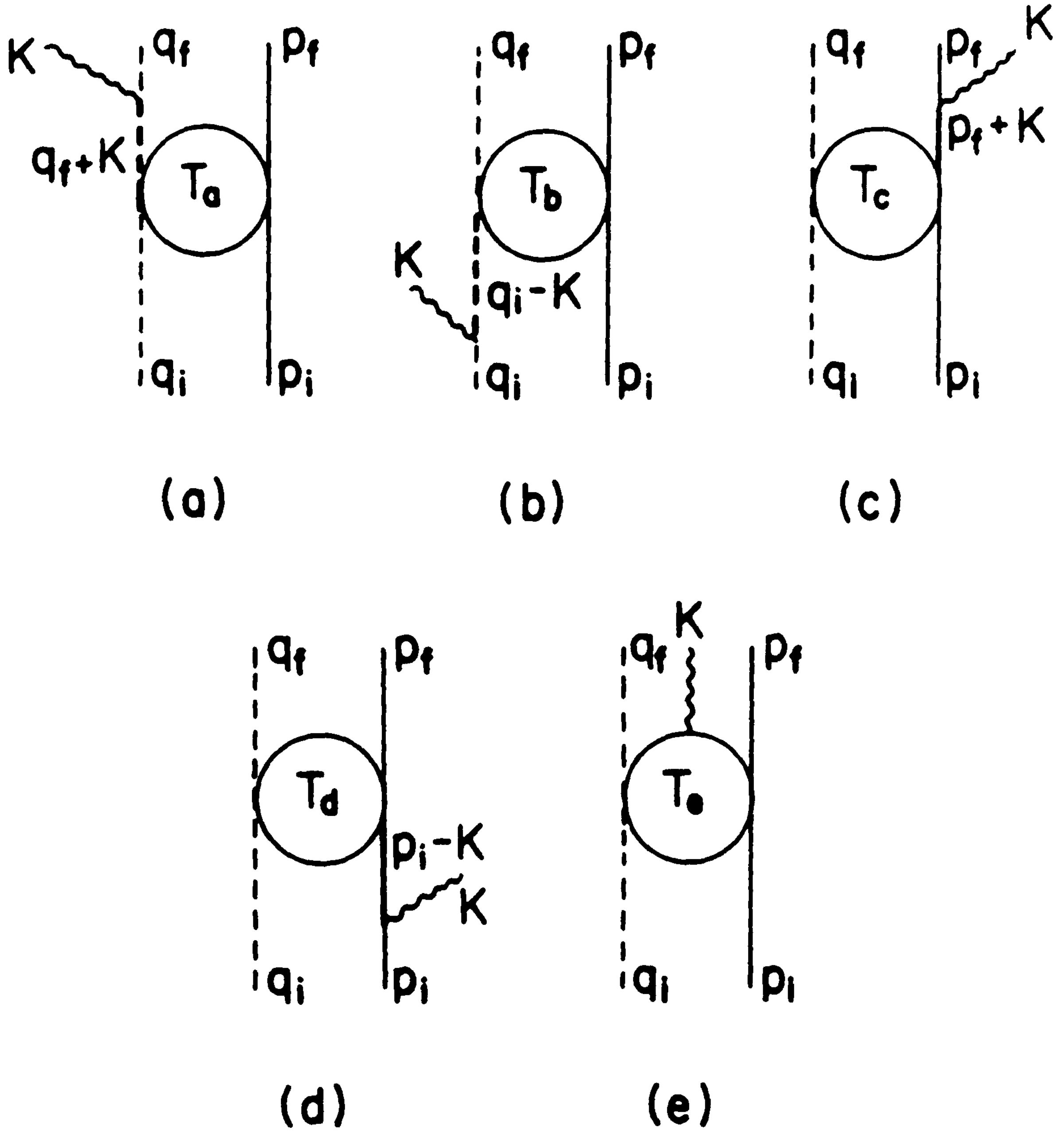

proton
p_f
K
proton
$p_f + K$
(a)
proton
$p_i - K$
K
proton
p_i
(b)
Δ^{++}
p'
K
Δ^{++}
$p' + K$
(c)
Δ^{++}
$p - K$
K
Δ^{++}
p
(d)

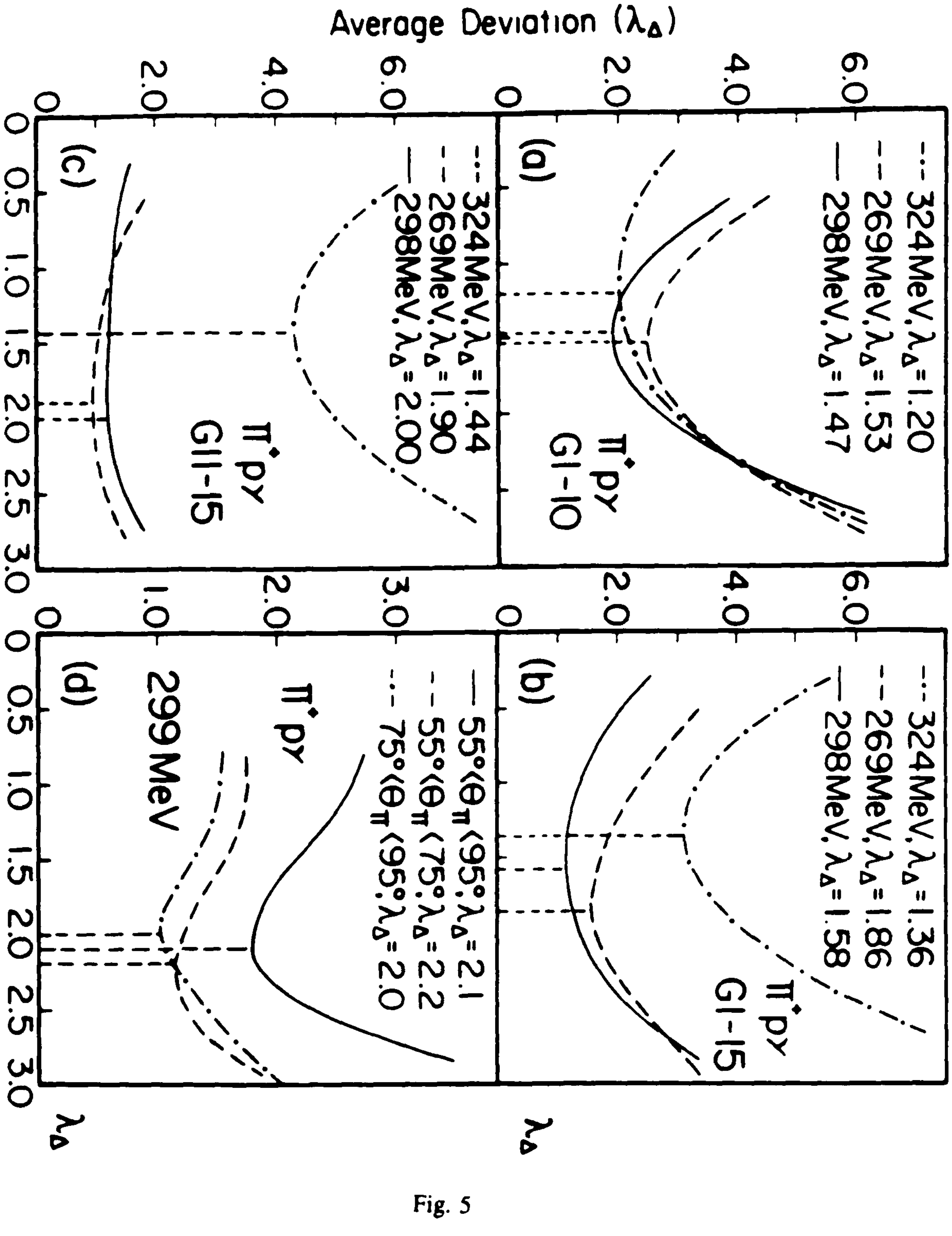
Fig. 3


K
q_f
p_f
q_f+K
T_a
q_i
p_i
q_f
p_f
T_b
K
q_i-K
q_i
p_i
q_f
p_f
K
p_f+K
T_c
q_i
p_i
q_f
p_f
T_d
p_i-K
K
q_i
p_i
K
q_f
p_f
T_e
q_i
p_i

(a) (b) (c)

(d) (e)

Fig. 4



Average Deviation (λ_Δ)
(a)
324MeV, λ_Δ=1.20
269MeV, λ_Δ=1.53
298MeV, λ_Δ=1.47
$\pi^+p\gamma$
GI-10
(b)
324MeV, λ_Δ=1.36
269MeV, λ_Δ=1.86
298MeV, λ_Δ=1.58
$\pi^+p\gamma$
GI-15
(c)
324MeV, λ_Δ=1.44
269MeV, λ_Δ=1.90
298MeV, λ_Δ=2.00
$\pi^+p\gamma$
GII-15
(d)
55°<θ_π<95°, λ_Δ=2.1
55°<θ_π<75°, λ_Δ=2.2
75°<θ_π<95°, λ_Δ=2.0
$\pi^+p\gamma$
299MeV
0
2.0
4.0
6.0
1.0
3.0
0.5
1.5
2.5
λ_Δ

Fig. 5

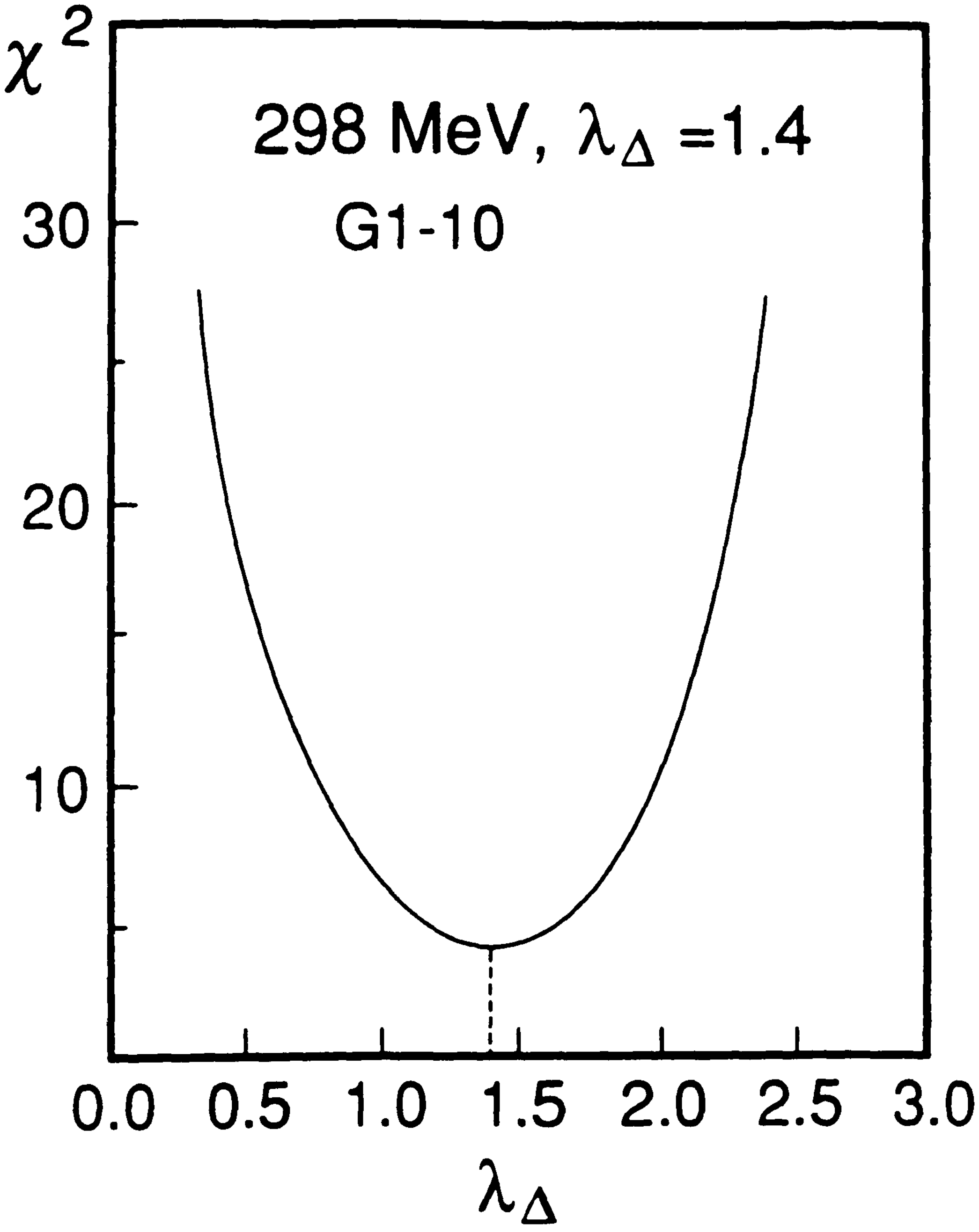

χ^2
298 MeV, λ_Δ =1.4
G1-10
30
20
10
0.0
0.5
1.0
1.5
2.0
2.5
3.0
λ_Δ

Fig. 5(e)

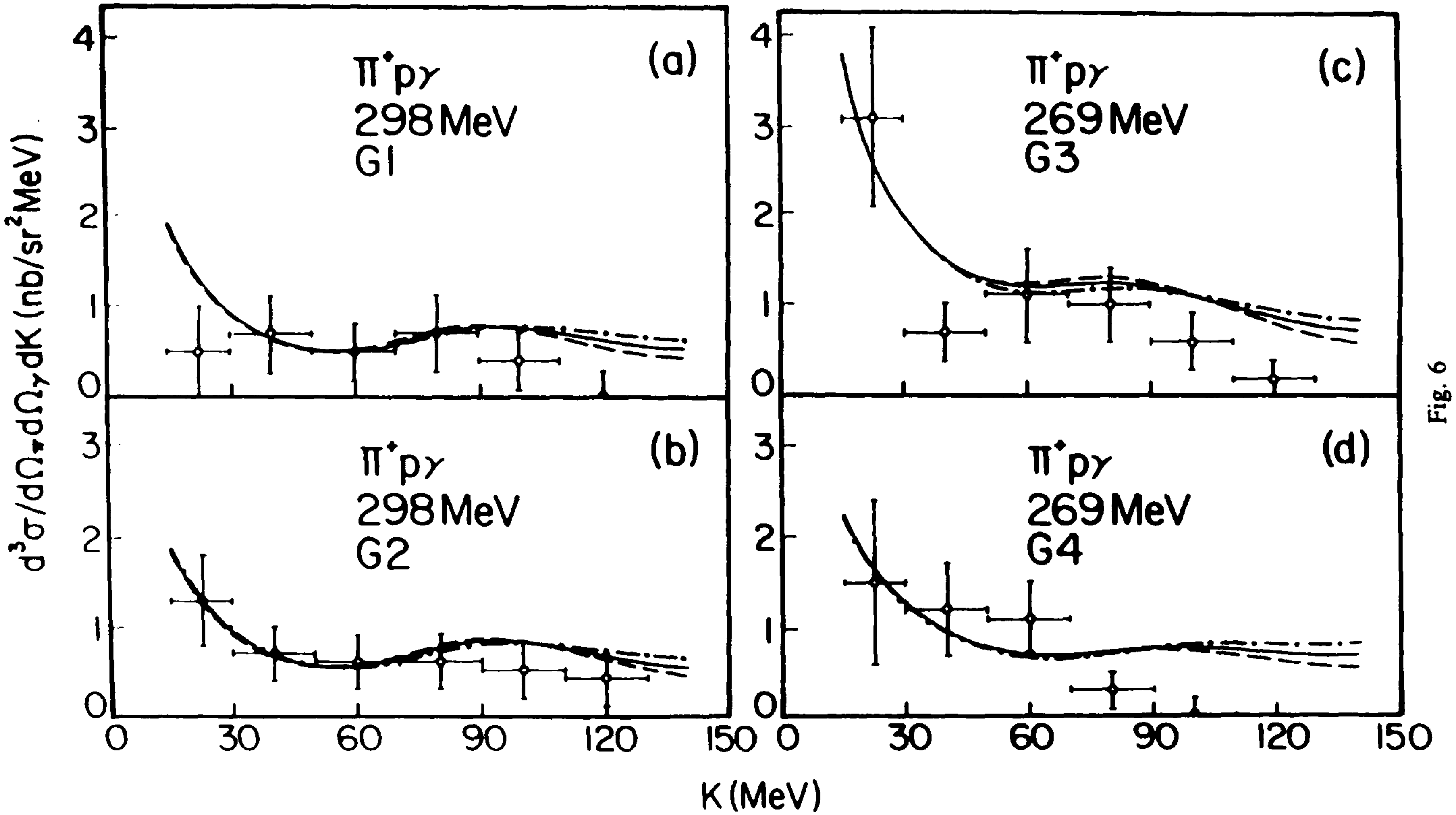

(a)
π⁺pγ
298 MeV
G1
(b)
π⁺pγ
298 MeV
G2
(c)
π⁺pγ
269 MeV
G3
(d)
π⁺pγ
269 MeV
G4
$d^3\sigma/d\Omega_\pi d\Omega_\gamma dK$ (nb/sr^{2}MeV)
K (MeV)
0
1
2
3
4
0
30
60
90
120
150

Fig. 6

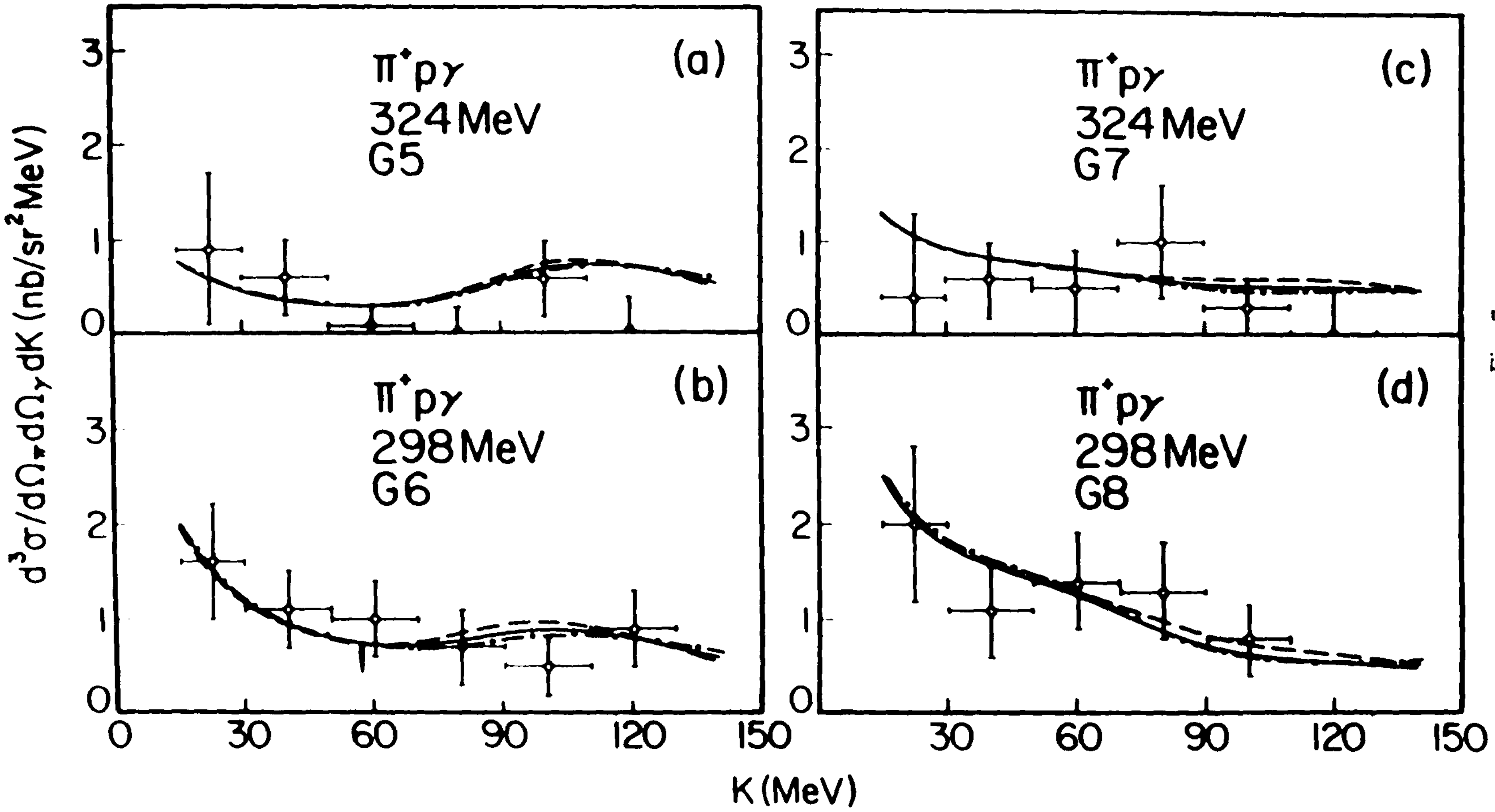

(a)
$\pi^+ p\gamma$
324 MeV
G5
(b)
$\pi^+ p\gamma$
298 MeV
G6
(c)
$\pi^+ p\gamma$
324 MeV
G7
(d)
$\pi^+ p\gamma$
298 MeV
G8
$d^3\sigma/d\Omega_\pi d\Omega_\gamma dK$ (nb/sr^{2}MeV)
0
1
2
3
0
30
60
90
120
150
K (MeV)

Fig. 7

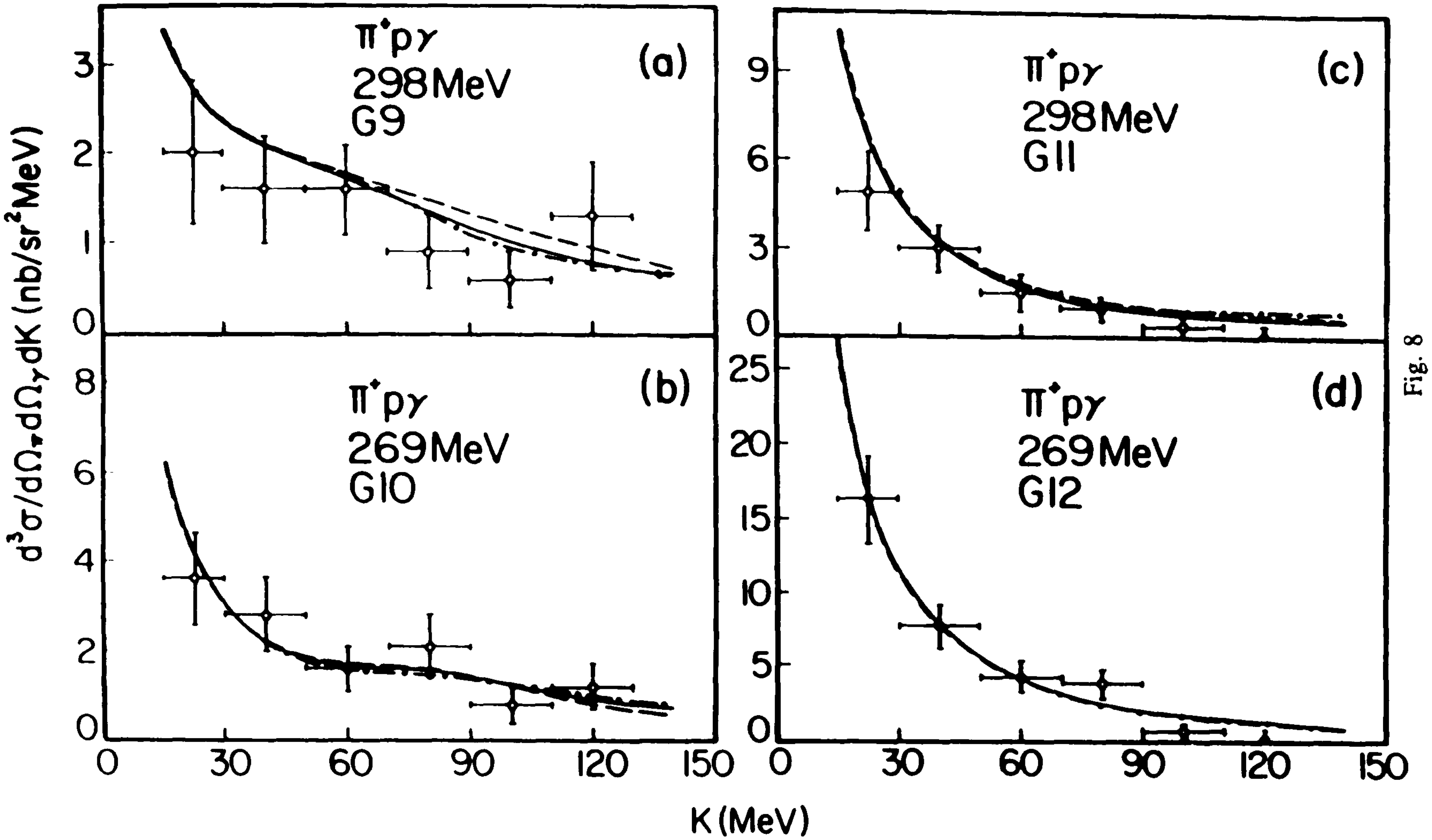

$\pi^+p\gamma$
298 MeV
G9
(a)
$\pi^+p\gamma$
298 MeV
G11
(c)
$\pi^+p\gamma$
269 MeV
G10
(b)
$\pi^+p\gamma$
269 MeV
G12
(d)
$d^3\sigma/d\Omega_\pi d\Omega_\gamma dK$ (nb/sr^{2}MeV)
K (MeV)
0
30
60
90
120
150
0
1
2
3
0
2
4
6
8
0
3
6
9
0
5
10
15
20
25

Fig. 8

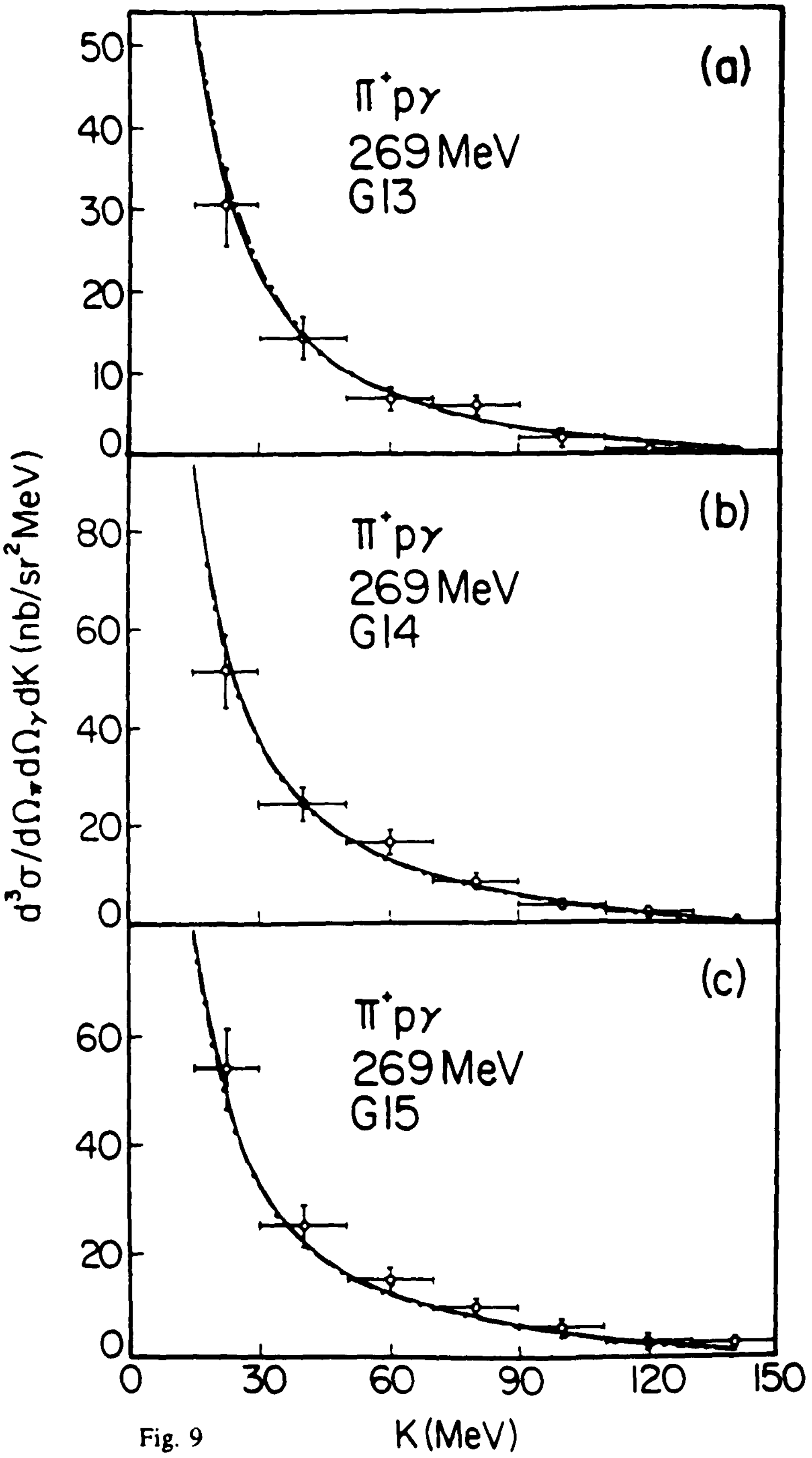

(a)
π⁺pγ
269 MeV
G13
(b)
π⁺pγ
269 MeV
G14
(c)
π⁺pγ
269 MeV
G15
d³σ/dΩπ dΩγ dK (nb/sr² MeV)
0
10
20
30
40
50
0
20
40
60
80
0
20
40
60
0
30
60
90
120
150
K (MeV)

Fig. 9

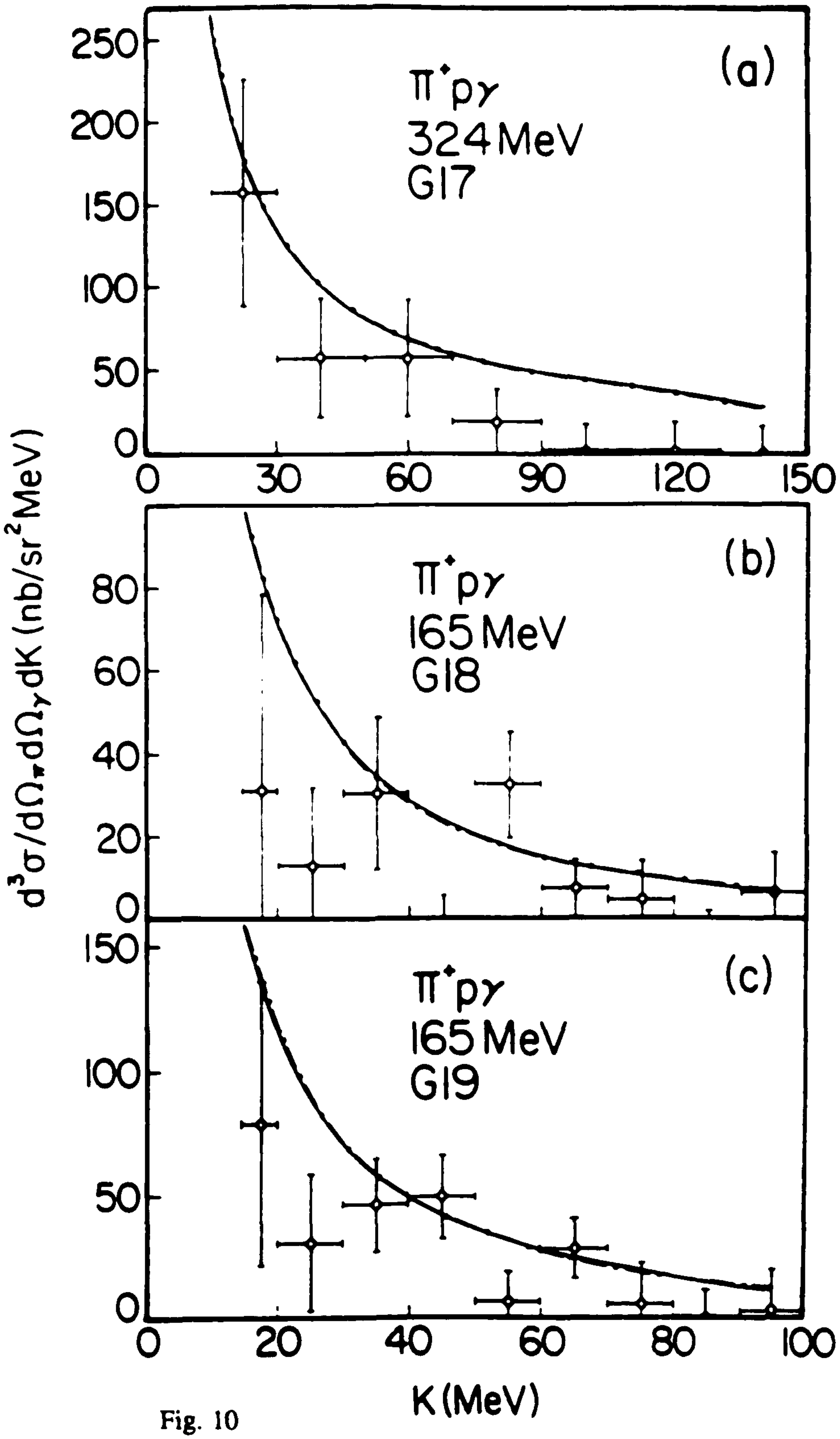

(a)
$\pi^+ p\gamma$
324 MeV
G17
(b)
$\pi^+ p\gamma$
165 MeV
G18
(c)
$\pi^+ p\gamma$
165 MeV
G19
$d^3\sigma/d\Omega_\pi d\Omega_\gamma dK$ (nb/sr^{2}MeV)
K (MeV)
0
50
100
150
200
250
0
30
60
90
120
150
0
20
40
60
80
0
50
100
150
0
20
40
60
80
100

Fig. 10

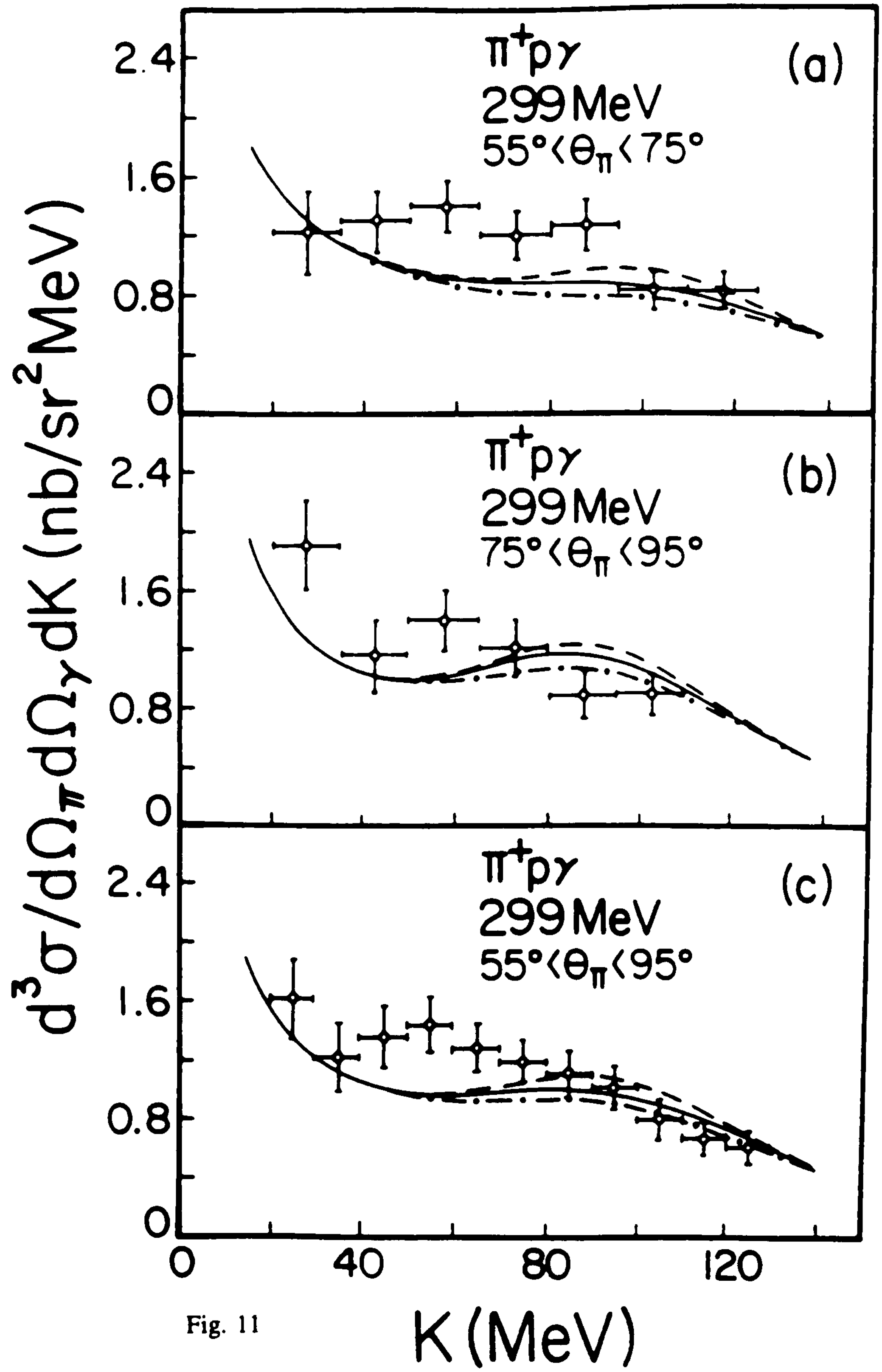

$\pi^+p\gamma$
299 MeV
55°<θ_π<75°
(a)
$\pi^+p\gamma$
299 MeV
75°<θ_π<95°
(b)
$\pi^+p\gamma$
299 MeV
55°<θ_π<95°
(c)
2.4
1.6
0.8
0
0
40
80
120
$d^3\sigma/d\Omega_\pi d\Omega_\gamma dK$ (nb/sr^{2}MeV)
K (MeV)

Fig. 11

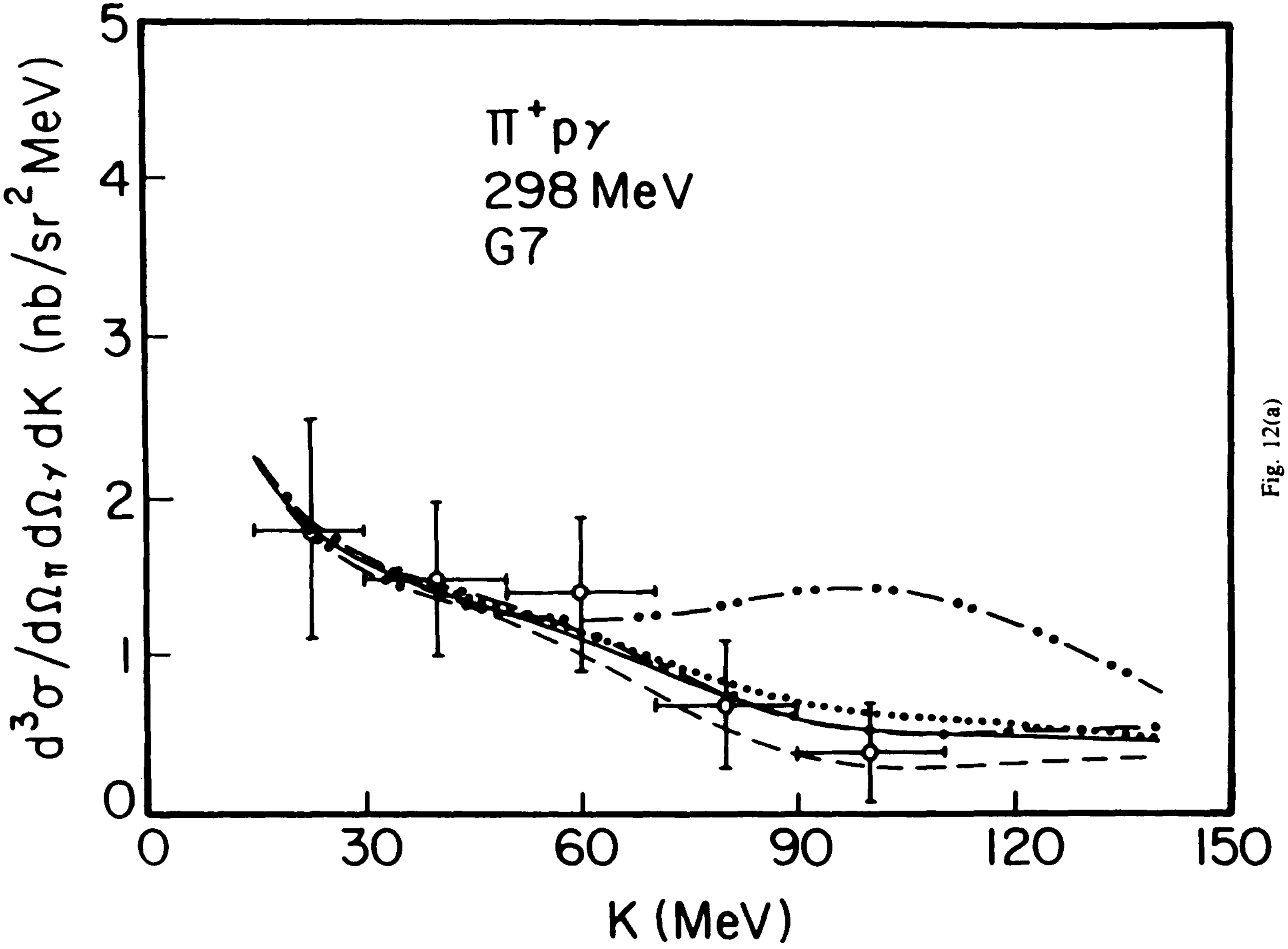

π⁺pγ
298 MeV
G7
d³σ/dΩ_π dΩ_γ dK (nb/sr² MeV)
0
1
2
3
4
5
0
30
60
90
120
150
K (MeV)

Fig. 12(a)

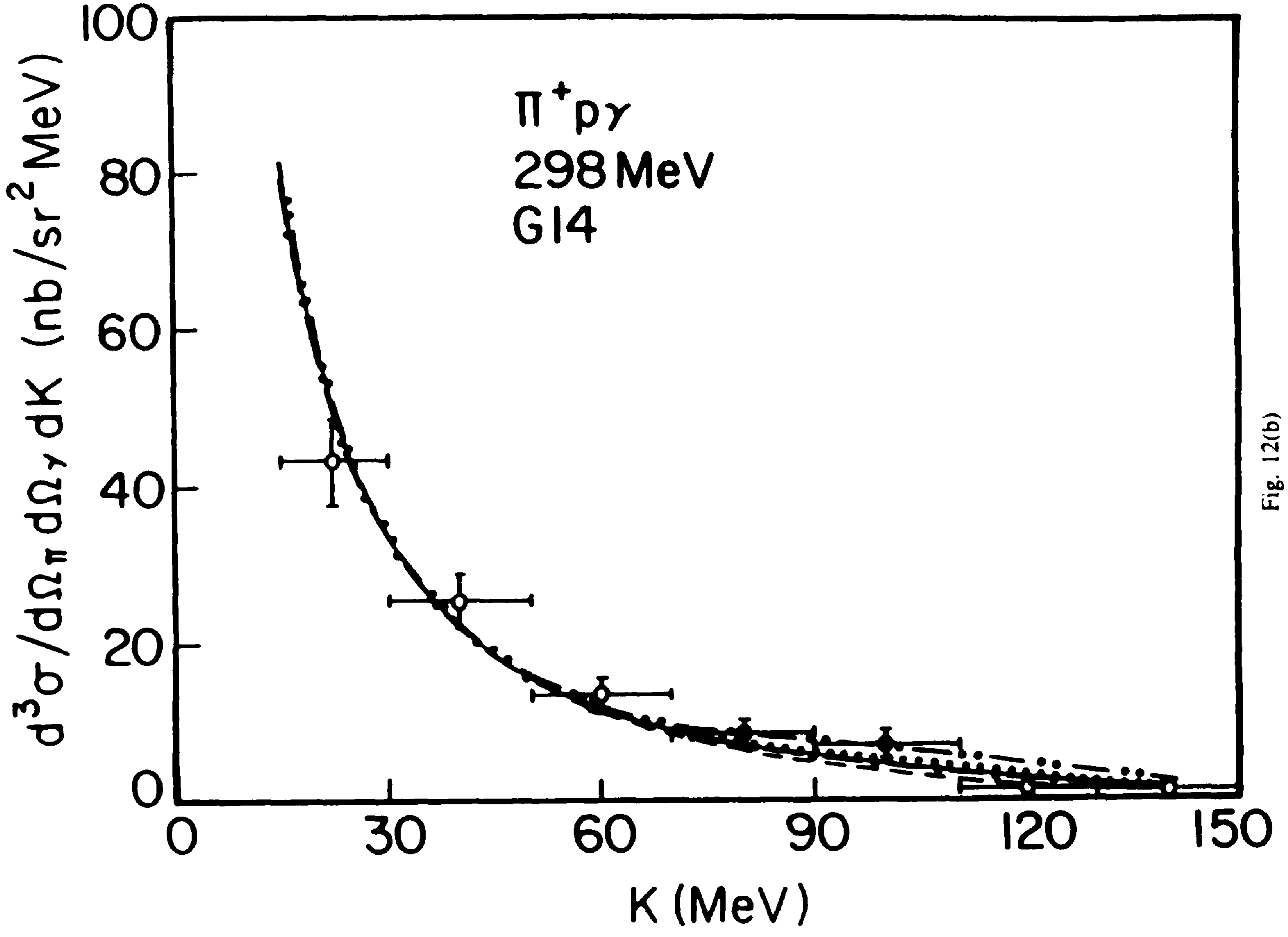

$\pi^+ p\gamma$
298 MeV
G14
$d^3\sigma/d\Omega_\pi d\Omega_\gamma dK$ (nb/sr^{2}MeV)
100
80
60
40
20
0
0
30
60
90
120
150
K (MeV)

Fig. 12(b)

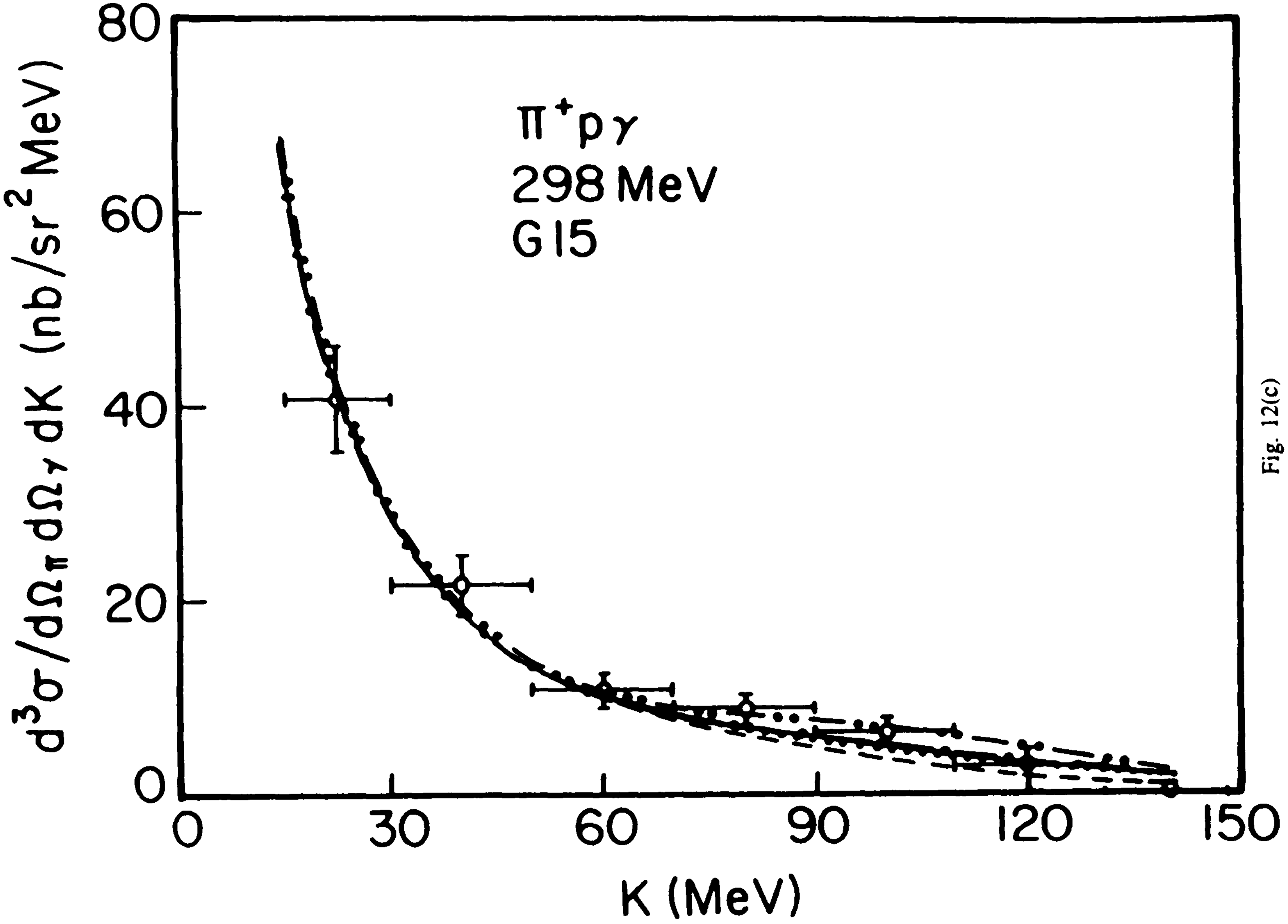

π⁺pγ
298 MeV
G15
d³σ/dΩπ dΩγ dK (nb/sr² MeV)
K (MeV)
0
20
40
60
80
0
30
60
90
120
150

Fig. 12(c)

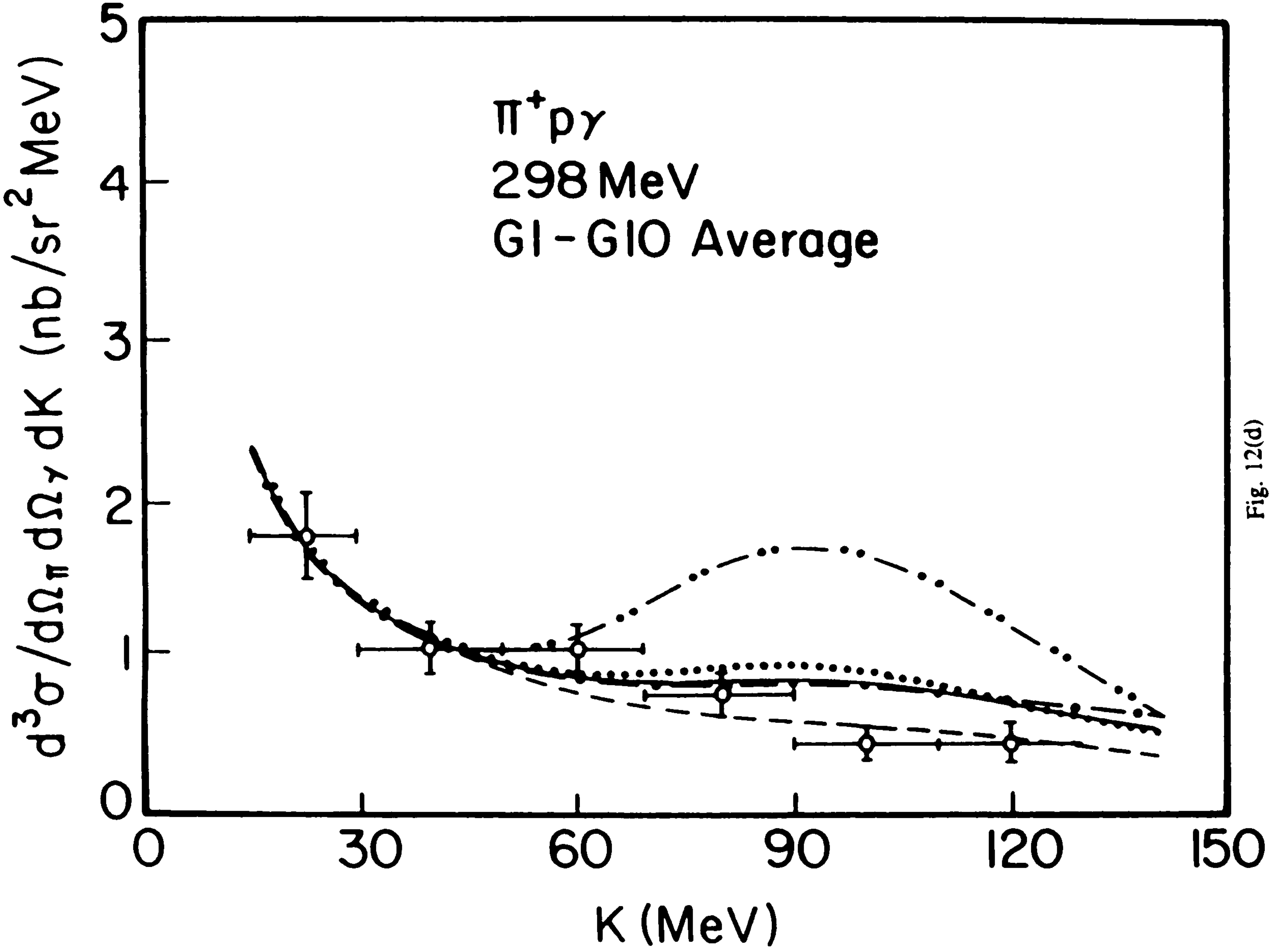

$\pi^+ p \gamma$
298 MeV
G1 - G10 Average
$d^3\sigma/d\Omega_\pi d\Omega_\gamma dK$ (nb/sr^2 MeV)
0
1
2
3
4
5
K (MeV)
0
30
60
90
120
150

Fig. 12(d)

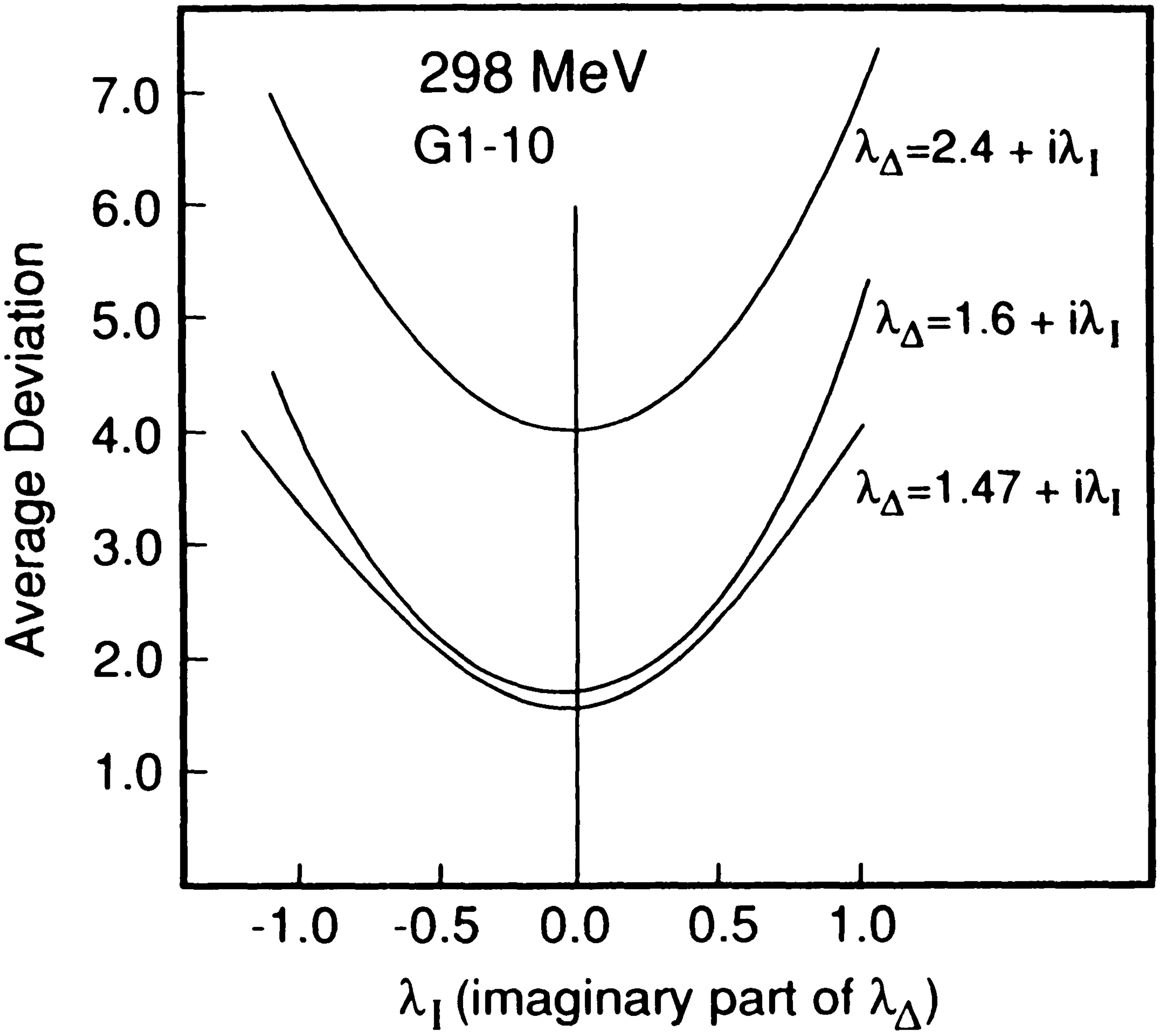
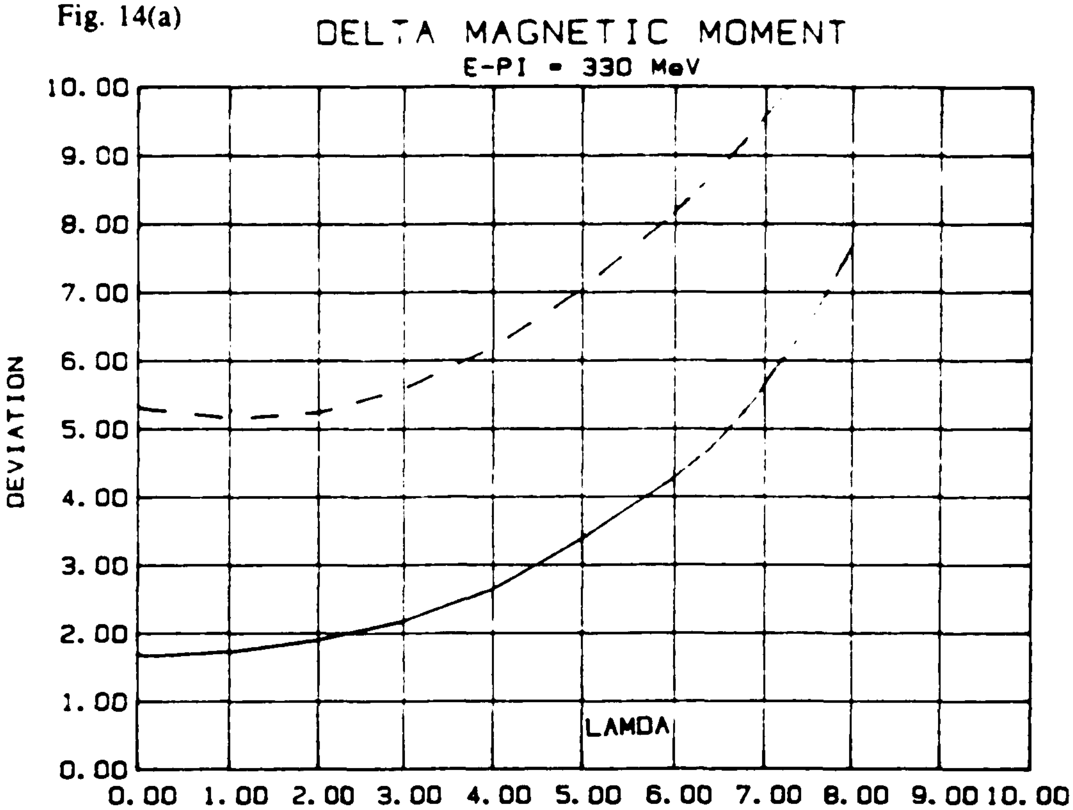

298 MeV
G1-10
$\lambda_\Delta = 2.4 + i\lambda_I$
$\lambda_\Delta = 1.6 + i\lambda_I$
$\lambda_\Delta = 1.47 + i\lambda_I$
Average Deviation
7.0
6.0
5.0
4.0
3.0
2.0
1.0
-1.0
-0.5
0.0
0.5
1.0
λ_I (imaginary part of λ_Δ)

Fig. 13

Fig. 14(a)



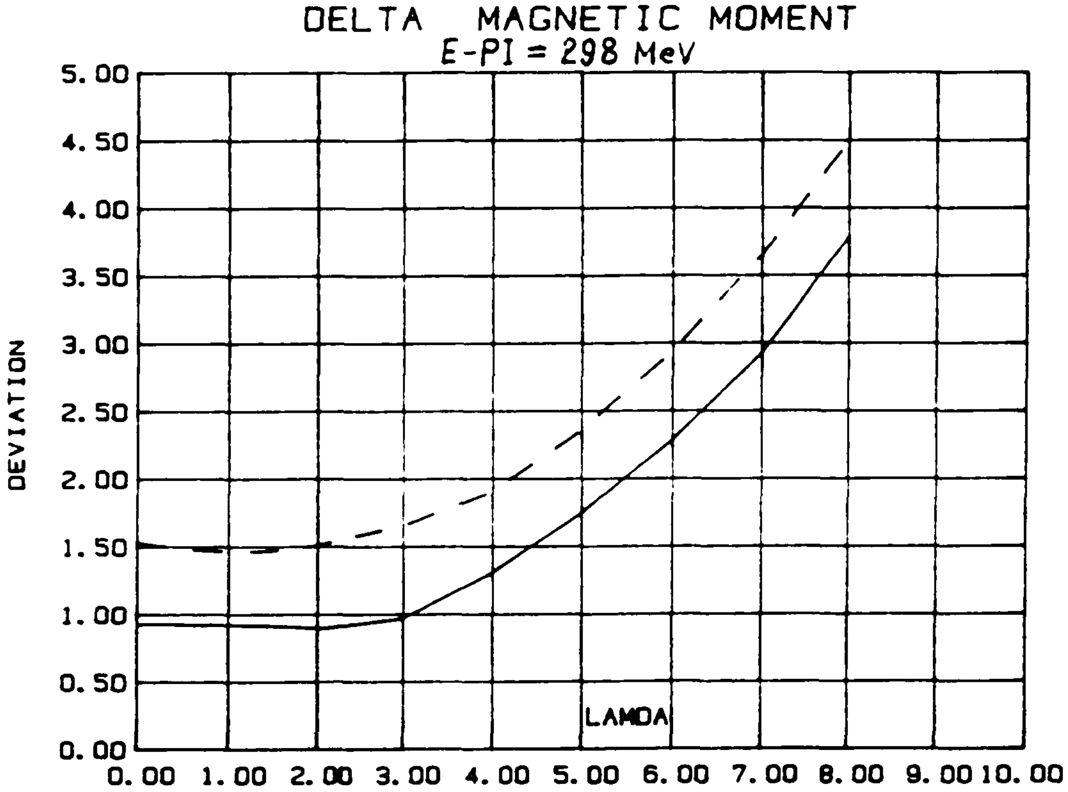


Fig. 14(b)

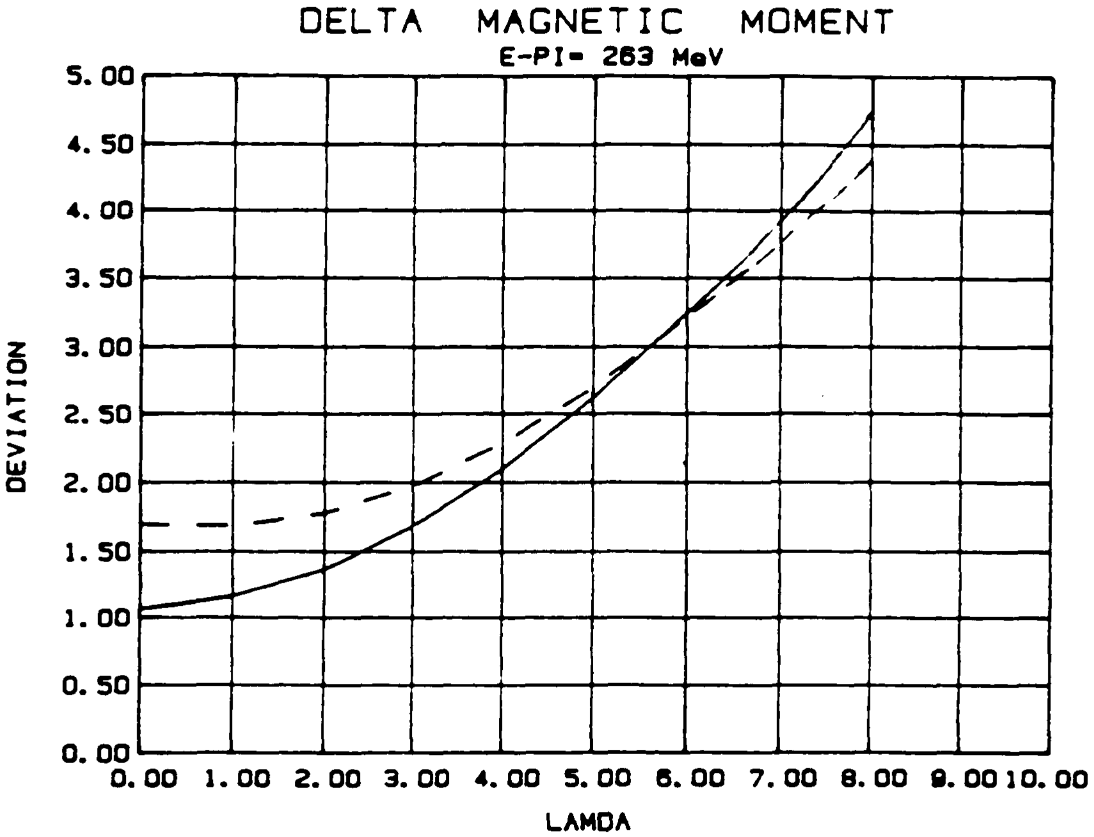

DELTA MAGNETIC MOMENT
E-PI= 263 MeV
DEVIATION
LAMDA
5.00
4.50
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
0.00
1.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
9.00
10.00

Fig. 14(c)

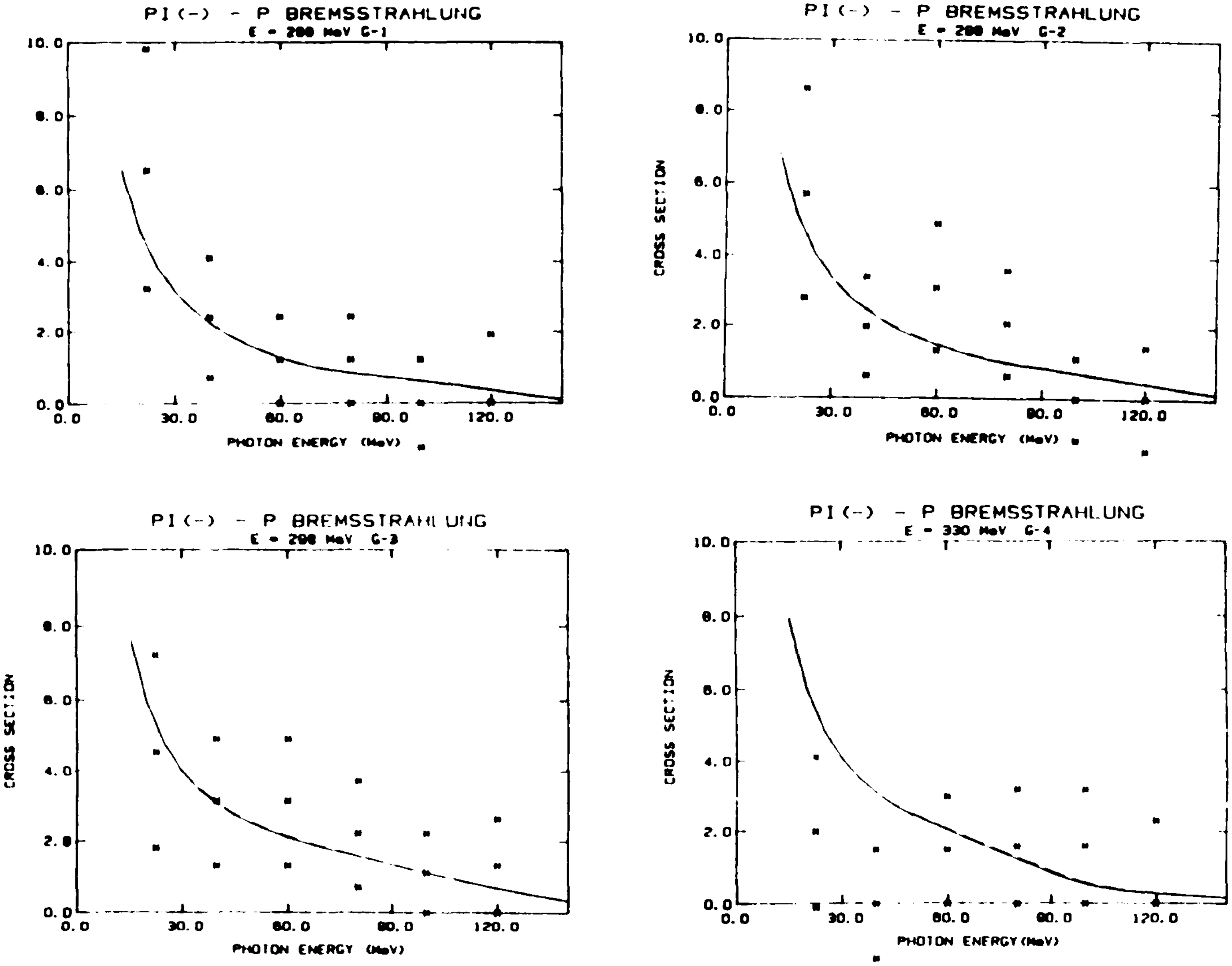

PI(-) - P BREMSSTRAHLUNG
E = 200 MeV G-1
10.0
8.0
6.0
4.0
2.0
0.0
0.0
30.0
60.0
90.0
120.0
PHOTON ENERGY (MeV)
PI(-) - P BREMSSTRAHLUNG
E = 200 MeV G-2
CROSS SECTION
10.0
8.0
6.0
4.0
2.0
0.0
0.0
30.0
60.0
90.0
120.0
PHOTON ENERGY (MeV)
PI(-) - P BREMSSTRAHLUNG
E = 200 MeV G-3
CROSS SECTION
10.0
8.0
6.0
4.0
2.0
0.0
0.0
30.0
60.0
90.0
120.0
PHOTON ENERGY (MeV)
PI(-) - P BREMSSTRAHLUNG
E = 330 MeV G-4
CROSS SECTION
10.0
8.0
6.0
4.0
2.0
0.0
0.0
30.0
60.0
90.0
120.0
PHOTON ENERGY (MeV)

Fig. 15

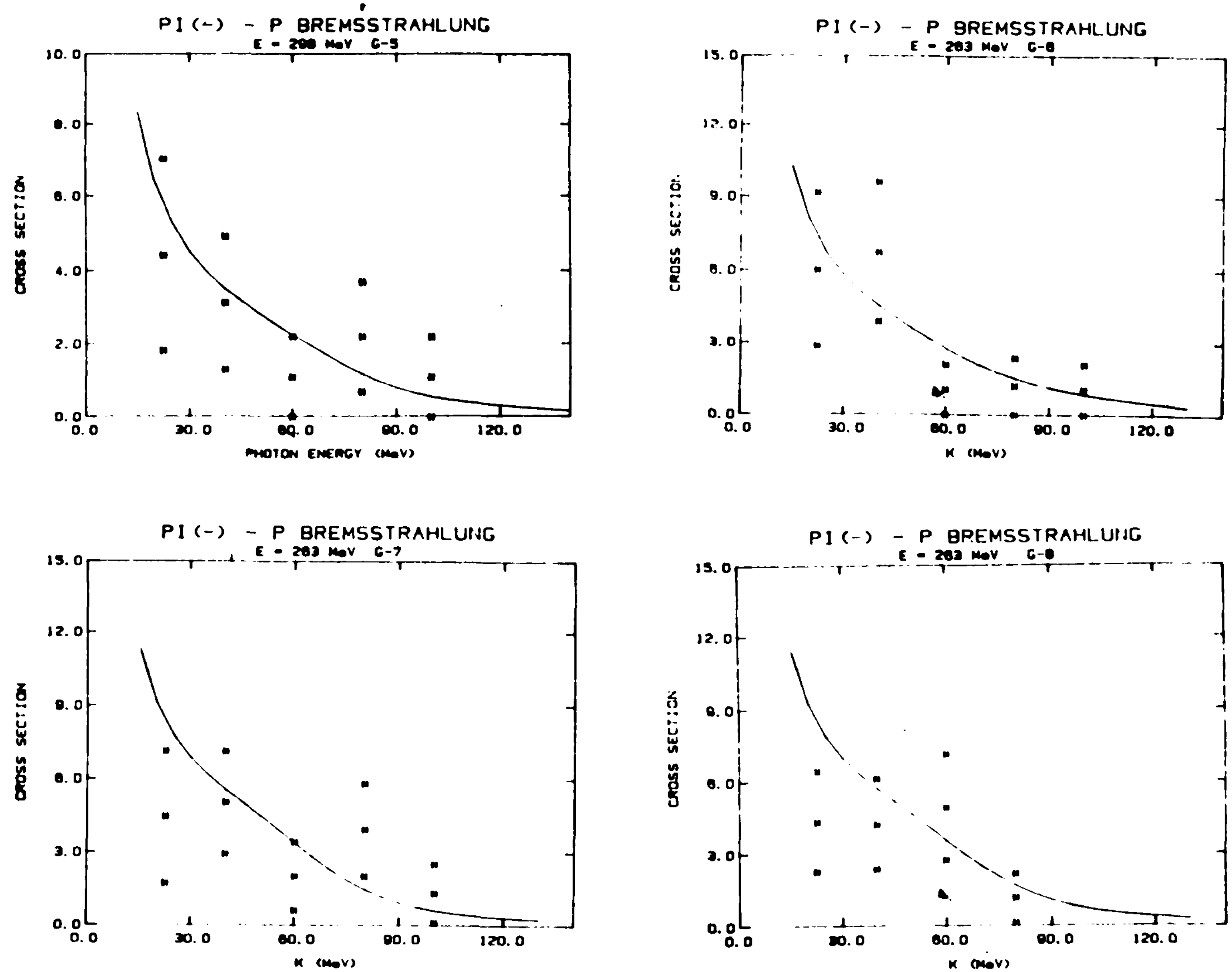

PI (-) - P BREMSSTRAHLUNG
E = 208 MeV G-5
CROSS SECTION
PHOTON ENERGY (MeV)
0.0
2.0
4.0
6.0
8.0
10.0
30.0
60.0
90.0
120.0
PI (-) - P BREMSSTRAHLUNG
E = 283 MeV G-6
CROSS SECTION
K (MeV)
0.0
3.0
6.0
9.0
12.0
15.0
30.0
60.0
90.0
120.0
PI (-) - P BREMSSTRAHLUNG
E = 283 MeV G-7
CROSS SECTION
K (MeV)
0.0
3.0
6.0
9.0
12.0
15.0
30.0
60.0
90.0
120.0
PI (-) - P BREMSSTRAHLUNG
E = 283 MeV G-8
CROSS SECTION
K (MeV)
0.0
3.0
6.0
9.0
12.0
15.0
30.0
60.0
90.0
120.0

Fig. 16

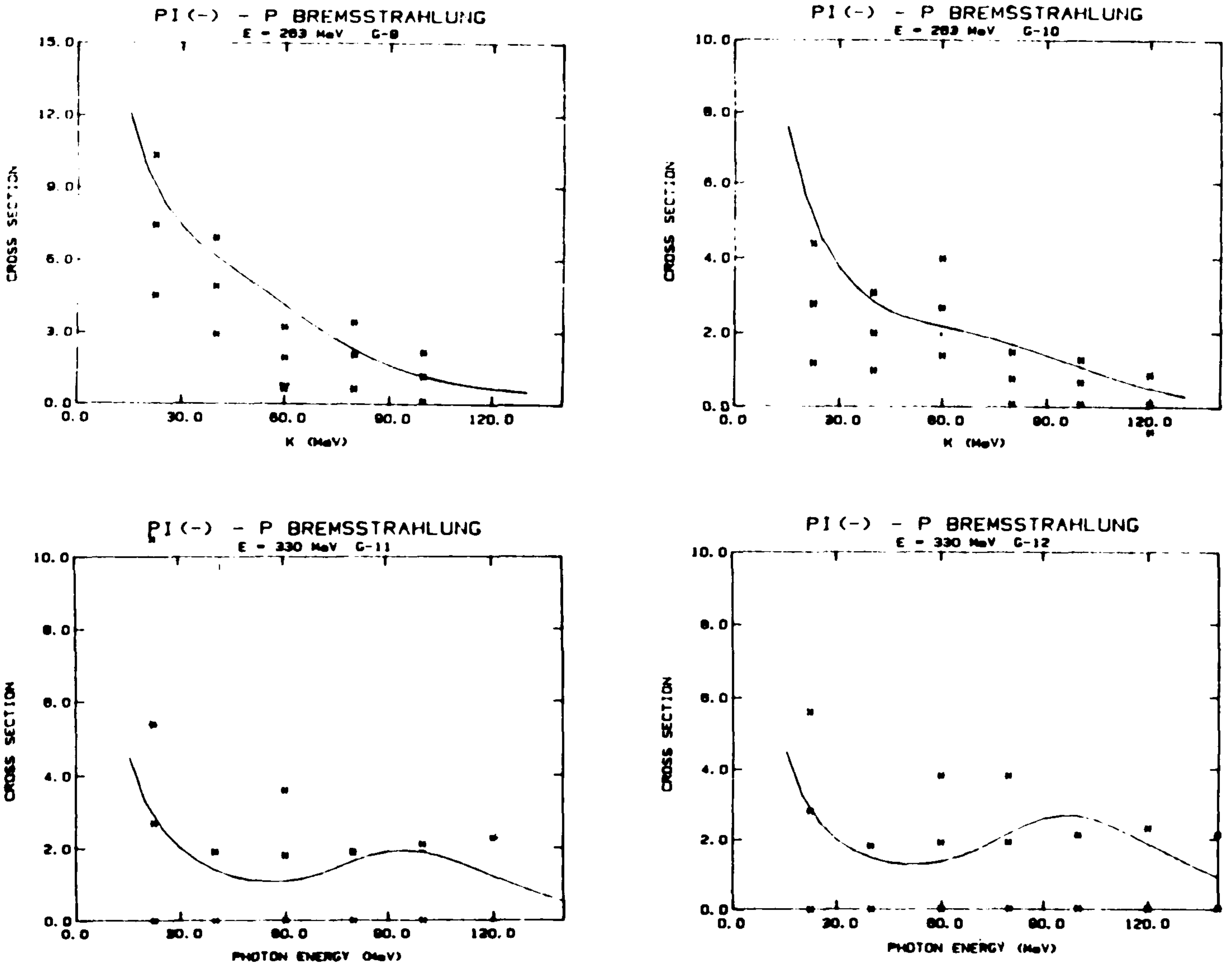
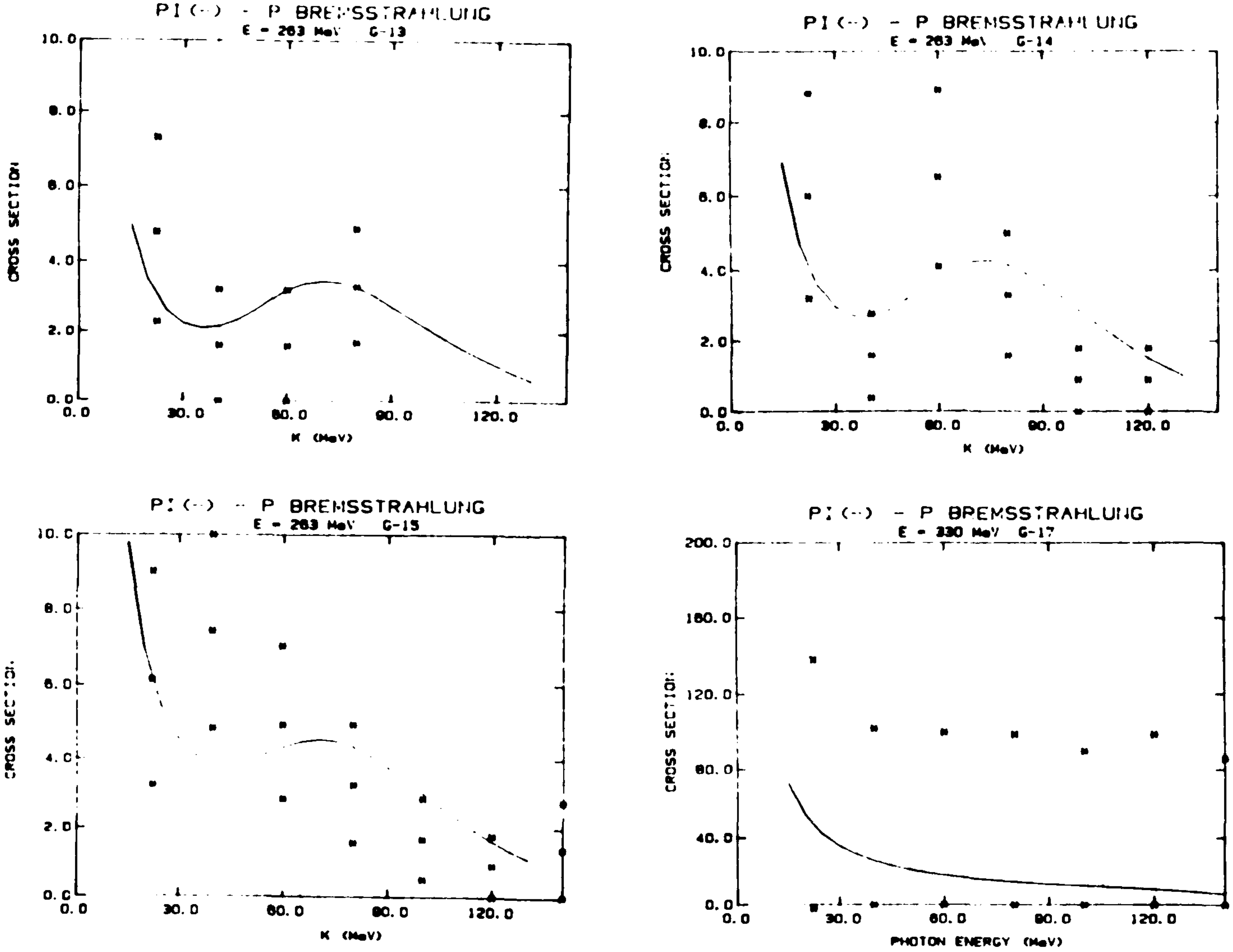

PI (-) - P BREMSSTRAHLUNG
E = 283 MeV G-9
CROSS SECTION
15.0
12.0
9.0
6.0
3.0
0.0
0.0
30.0
60.0
90.0
120.0
K (MeV)
PI (-) - P BREMSSTRAHLUNG
E = 283 MeV G-10
CROSS SECTION
10.0
8.0
6.0
4.0
2.0
0.0
0.0
30.0
60.0
90.0
120.0
K (MeV)
PI (-) - P BREMSSTRAHLUNG
E = 330 MeV G-11
CROSS SECTION
10.0
8.0
6.0
4.0
2.0
0.0
0.0
30.0
60.0
90.0
120.0
PHOTON ENERGY (MeV)
PI (-) - P BREMSSTRAHLUNG
E = 330 MeV G-12
CROSS SECTION
10.0
8.0
6.0
4.0
2.0
0.0
0.0
30.0
60.0
90.0
120.0
PHOTON ENERGY (MeV)

Fig. 17



PI (-) - P BREMSSTRAHLUNG
E = 283 MeV G-13
PI (-) - P BREMSSTRAHLUNG
E = 283 MeV G-14
PI (-) - P BREMSSTRAHLUNG
E = 283 MeV G-15
PI (-) - P BREMSSTRAHLUNG
E = 330 MeV G-17
CROSS SECTION
K (MeV)
PHOTON ENERGY (MeV)
0.0
2.0
4.0
6.0
8.0
10.0
30.0
60.0
90.0
120.0
40.0
80.0
120.0
160.0
200.0

Fig. 18

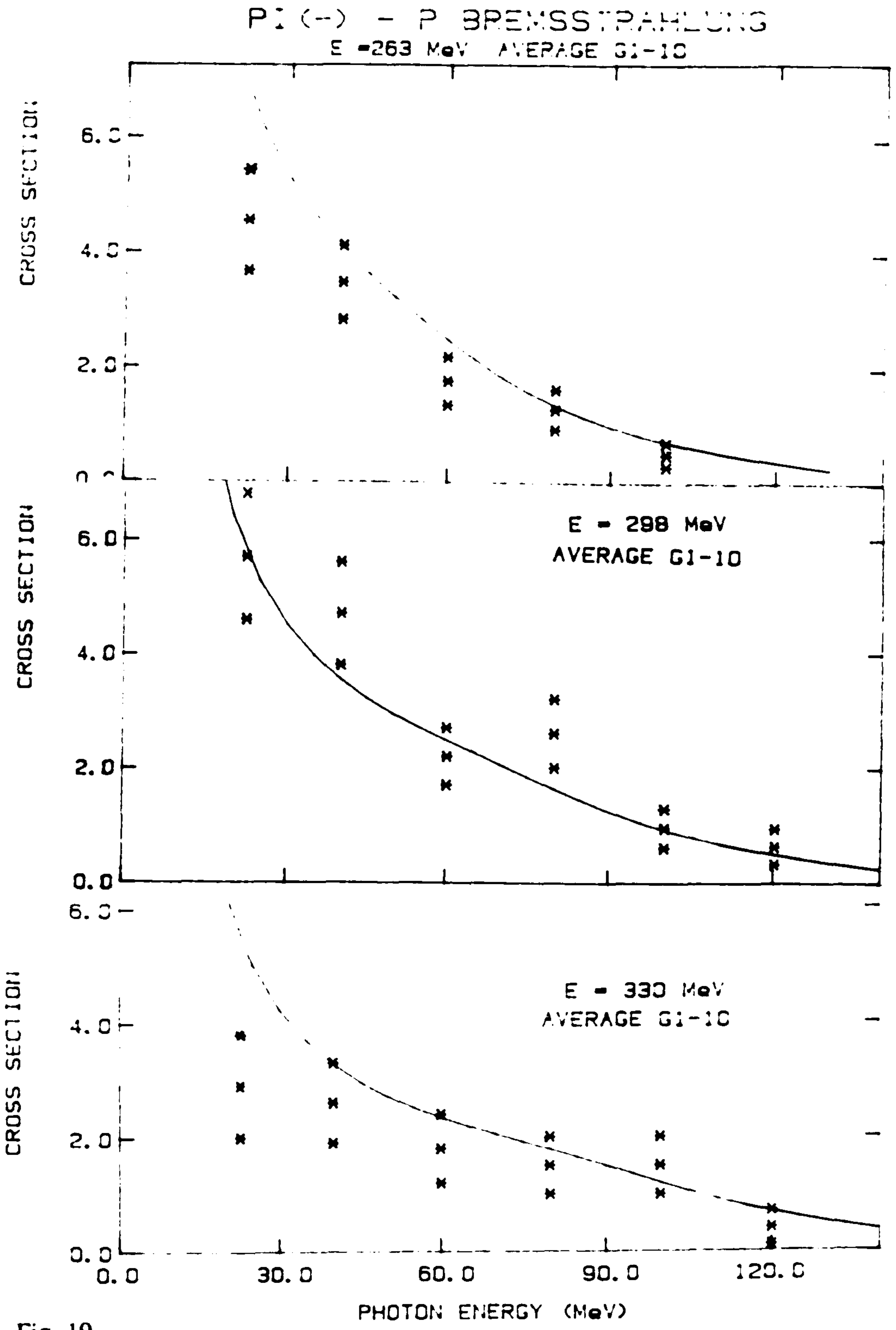

PI(-) - P BREMSSTRAHLUNG
E =263 MeV AVERAGE G1-10
E = 298 MeV
AVERAGE G1-10
E = 330 MeV
AVERAGE G1-10
CROSS SECTION
0.0
2.0
4.0
6.0
0.0
30.0
60.0
90.0
120.0
PHOTON ENERGY (MeV)

Fig. 19

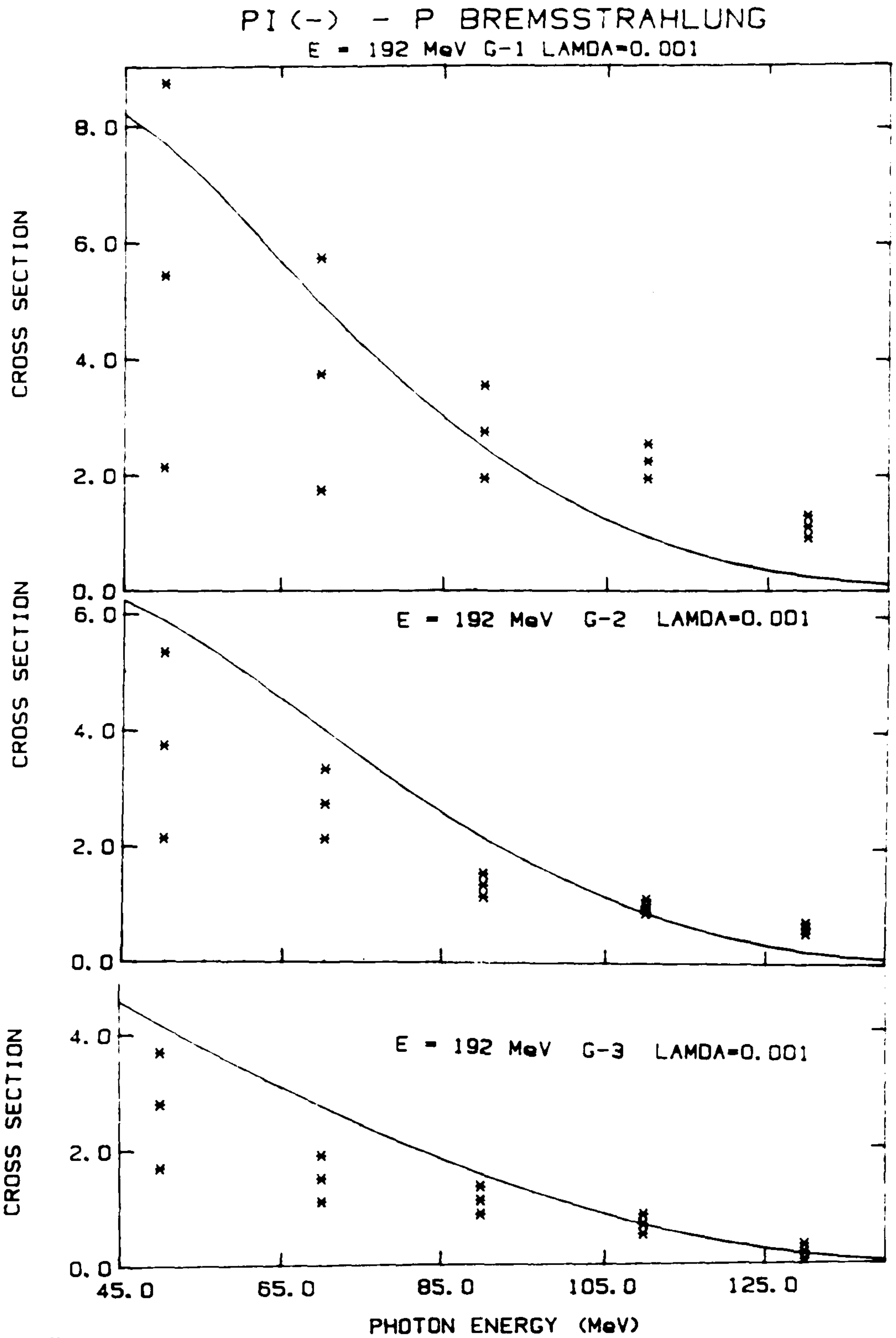

PI(-) - P BREMSSTRAHLUNG
E = 192 MeV G-1 LAMDA=0.001
E = 192 MeV G-2 LAMDA=0.001
E = 192 MeV G-3 LAMDA=0.001
CROSS SECTION
CROSS SECTION
CROSS SECTION
8.0
6.0
4.0
2.0
0.0
6.0
4.0
2.0
0.0
4.0
2.0
0.0
45.0
65.0
85.0
105.0
125.0
PHOTON ENERGY (MeV)

Fig. 20

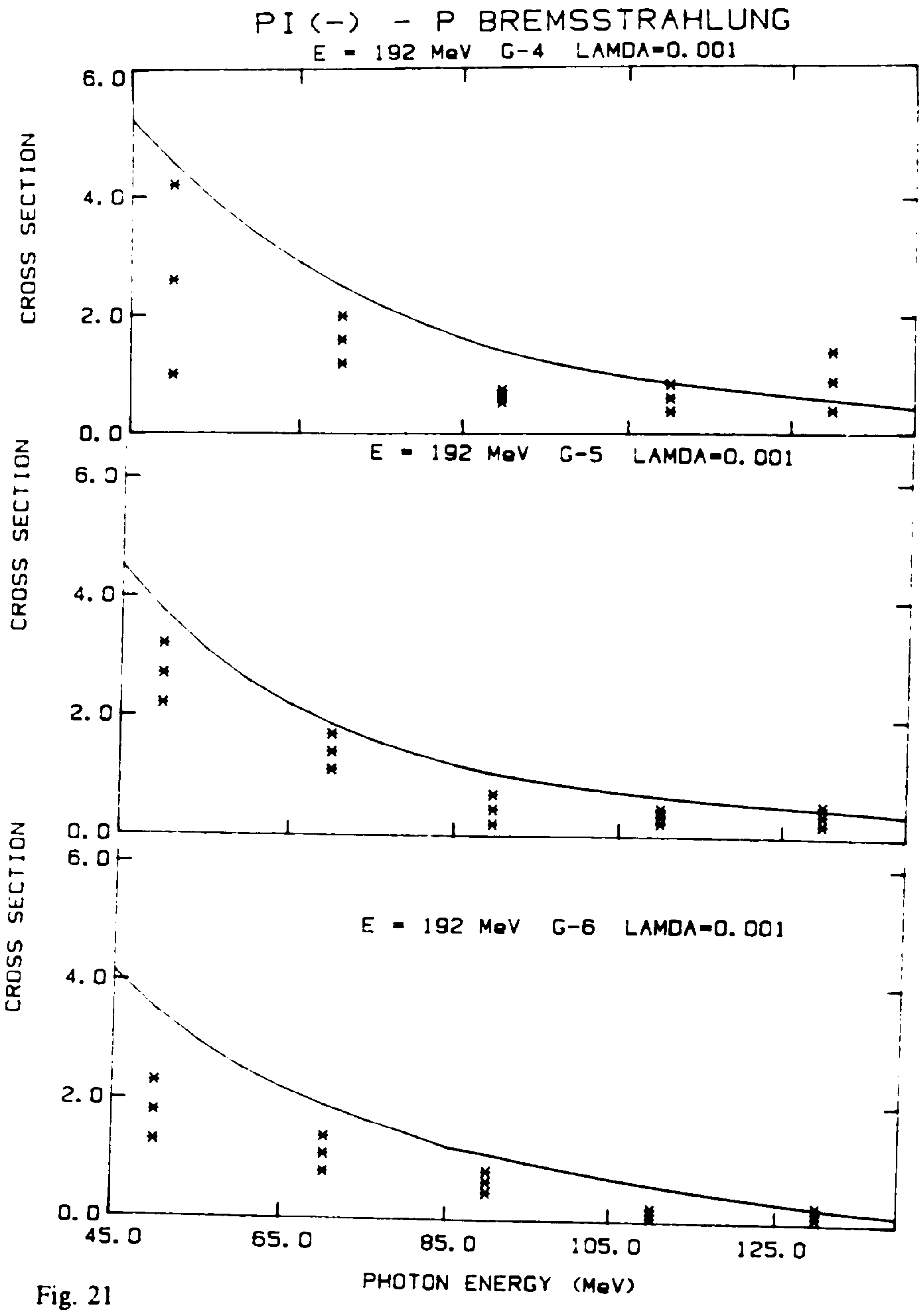

PI (-) - P BREMSSTRAHLUNG
E = 192 MeV G-4 LAMDA=0. 001
E = 192 MeV G-5 LAMDA=0. 001
E = 192 MeV G-6 LAMDA=0. 001
CROSS SECTION
CROSS SECTION
CROSS SECTION
6. 0
4. 0
2. 0
0. 0
6. 0
4. 0
2. 0
0. 0
6. 0
4. 0
2. 0
0. 0
45. 0
65. 0
85. 0
105. 0
125. 0
PHOTON ENERGY (MeV)

Fig. 21

Appendix A

Derivation of T-matrices $\tilde{T}$, $\tilde{T}_a$, $\tilde{T}_b$, $\tilde{T}_c$, and $\tilde{T}_d$

(i) The elastic T-matrix $\tilde{T}$ is defined by the following expression:

$$\tilde{T} = [g\bar{q}_f^{\rho}]\ G_{\rho\alpha}(p)\ [\ g{q_i}^{\alpha}] = \frac{iq^2}{p^2-M_\Delta^2+i\epsilon}\ \tilde{T}_{el}$$

where

$$\tilde{T}_{el}=(\not{p} + M_\Delta)[\ \bar{q}_f{\cdot}q_i - \frac{1}{3}\bar{\not{q}}_f \not{q}_i - \frac{1}{3M_\Delta}(\ \bar{\not{q}}_f\ p{\bullet}q_i - \not{q}_i\ p{\bullet}\bar{q}_f) - \frac{2}{3M_\Delta^2} p{\bullet}\bar{q}_f\ p{\bullet}q_i]$$

$$+ \frac{2}{3M_\Delta^2}(p^2-M_\Delta^2)\ [\ \bar{\not{q}}_f\ p{\bullet}q_i - \not{q}_i\ p{\bullet}\bar{q}_f +(\not{p} + M_\Delta)\,\bar{\not{q}}_f\,\not{q}_i]$$

Using

$$p = q_i+p_i = \bar{q}_f+\bar{p}_f\ ,\ s = p^2,$$

$$\bar{u}(\bar{p}_f)\ \not{q}_i\ u(p_i) = \bar{u}(\bar{p}_f)\ \bar{\not{q}}_f\ u(p_i)$$

$$= \bar{u}(\bar{p}_f)\ \frac{1}{2}\ (\ \bar{\not{q}}_f + \not{q}_i\ )\ u(p_i)\ ,$$

$$\bar{u}(\bar{p}_f)\ \bar{\not{q}}_f\,\not{q}_i\ u(p_i) = \bar{u}(\bar{p}_f)\ [\ \not{q}_i + \not{p}_i - \bar{\not{p}}_f\ ]\ \not{q}_i\ u(p_i)$$

$$= \bar{u}(\bar{p}_f)\ [q_i^2 + 2p_i{\bullet}q_i - 2M_p\ q_i\ ]\ u(p_i),$$

$$t = (q_i- \bar{q}_f)^2 = {\bar{q}_f}^{\,2} + {q_i}^2 - 2\bar{q}_f{\bullet}q_i, \qquad \bar{q}_f{\bullet}q_i = -\frac{t}{2} + \frac{\bar{q}_f^{\,2}}{2} + \frac{q_i^2}{2}\ ,$$

$$s = (q_i+ p_i)^2 = {p_i}^2 + {q_i}^2 + 2p_i{\bullet}q_i, \qquad p_i{\bullet}q_i = \frac{s}{2} - \frac{p_i^2}{2} - \frac{q_i^2}{2}\ ,$$

$$p\bullet q_i = (q_i + p_i)\bullet q_i = \frac{s}{2} - \frac{p_i^2}{2} + \frac{q_i^2}{2},$$

and

$$p\bullet \bar{q}_f = (\bar{q}_f + \bar{p}_f)\bullet \bar{q}_f = \frac{s}{2} - \frac{\bar{p}_f^2}{2} + \frac{\bar{q}_f^2}{2},$$

We obtain

$$\tilde{T} = \left(\frac{ig^2}{s - M_\Delta^2 + i\epsilon}\right)\left[\, A(s,t) + \frac{1}{2}\left(\bar{\not{q}}_f + \not{q}_i\right) B(s,t)\right]$$

where

$$A(s,t) = (M_p + M_\Delta)\left[-\frac{t}{2} + \frac{\bar{q}_f^2}{2} + \frac{q_i^2}{2} - \frac{2}{3M_\Delta^2}\left(\frac{s}{2} - \frac{\bar{p}_f^2}{2} + \frac{\bar{q}_f^2}{2}\right)\left(\frac{s}{2} - \frac{p_i^2}{2} + \frac{q_i^2}{2}\right)\right]$$

$$- \frac{1}{3M_\Delta}\bar{q}_f^2\left(\frac{s}{2} - \frac{p_i^2}{2} + \frac{q_i^2}{2}\right) + (s - p_i^2)\left[\frac{1}{3M_\Delta}\left(\frac{s}{2} - \frac{\bar{p}_f^2}{2} + \frac{\bar{q}_f^2}{2}\right)\left(\frac{2s - 3M_\Delta^2}{3M_\Delta^2}(M_p + M_\Delta)\right]$$

$$B(s,t) = -\frac{t}{2} + \frac{\bar{q}_f^2}{2} + \frac{q_i^2}{2} - \frac{2}{3M_\Delta^2}\left(\frac{s}{2} - \frac{\bar{p}_f^2}{2} + \frac{\bar{q}_f^2}{2}\right)\left(\frac{s}{2} - \frac{p_i^2}{2} + \frac{q_i^2}{2}\right)$$

$$+\left(\frac{2s}{3M_\Delta^2} - 1 - \frac{M_P}{3M_\Delta}\right)\left(\frac{\bar{p}_f^2}{2} - \frac{p_i^2}{2} + \frac{q_i^2}{2} - \frac{\bar{q}_f^2}{2}\right) + \frac{2s - 3M_\Delta^2}{3M_\Delta^2}\, q_f^2$$

$$- 2M_p\left[\frac{1}{3M_\Delta}\left(\frac{s}{2} - \frac{\bar{p}_f^2}{2} + \frac{\bar{q}_f^2}{2}\right) + \frac{2s - 3M_\Delta^2}{3M_\Delta^2}(M_p + M_\Delta)\right].$$

These expressions for A(s,t) and B(s,t) are not unique; they can be written in other forms. If we use the following relations,

$$\not{p}\,\bar{\not{q}}_f = 2p\bullet\bar{q}_f - \bar{\not{q}}_f\,\not{p}, \qquad \not{p}\,\not{q}_i = 2p\bullet q_i - \not{q}_i\,\not{p},$$

$$\not{p}\,\bar{\not{q}}_f\,\not{q}_i = 2p\bullet\bar{q}_f\,\not{q}_i - 2p\bullet q_i\,\bar{\not{q}}_f + \bar{\not{q}}_f\,\not{q}_i\,\not{p}\ ,$$

and

$$\bar{u}\,\bar{\not{q}}_f\,\not{q}_i\,u = \bar{u}\,(\,\not{q}_i + \not{p}_i - \bar{\not{p}}_f\,)\,\not{q}_i\,u = \bar{u}\,(\,q_i^2 + 2p_i\bullet q_i - 2M_p\,\not{q}_i\,)\,u\ ,$$

then we find

$$A(s,t) = (M_p+M_\Delta)[-\frac{t}{2} + \frac{\bar{q}_f^2}{2} + \frac{q_i^2}{2} - \frac{2}{3M_\Delta^2}(\frac{s}{2} - \frac{\bar{p}_f^2}{2} + \frac{\bar{q}_f^2}{2})(\frac{s}{2} - \frac{p_i^2}{2} + \frac{q_i^2}{2})]$$

$$- \frac{1}{6M_\Delta}[\,q_i^2\,(\frac{s}{2} - \frac{\bar{p}_f^2}{2} + \frac{\bar{q}_f^2}{2}) + \bar{q}_f^2\,(\frac{s}{2} - \frac{p_i^2}{2} + \frac{q_i^2}{2})]$$

$$- (s - p_i^2)[\frac{1}{6M_\Delta}(\,s - \frac{p_i^2}{2} + \frac{q_i^2}{2} - \frac{\bar{p}_f^2}{2} + \frac{\bar{q}_f^2}{2}) + \frac{2s-3M_\Delta^2}{3M_\Delta^2}(M_p+M_\Delta)]$$

and

$$B(s,t) = -\frac{t}{2} + \frac{\bar{q}_f^2}{2} + \frac{q_i^2}{2} - \frac{2}{3M_\Delta^2}\,(\frac{s}{2} - \frac{\bar{p}_f^2}{2} + \frac{\bar{q}_f^2}{2})(\frac{s}{2} - \frac{p_i^2}{2} + \frac{q_i^2}{2})$$

$$+(\frac{2s}{3M_\Delta^2} - 1 - \frac{M_P}{3M_\Delta})\,(\,0\,) + \frac{2s-3M_\Delta^2}{3M_\Delta^2}\,(\,\bar{q}_f^2 + q_i^2\,)$$

$$- 2M_p[\frac{1}{6M_\Delta}(\,s - \frac{\bar{p}_f^2}{2} + \frac{\bar{q}_f^2}{2} - \frac{p_i^2}{2} + \frac{q_i^2}{2}) + \frac{2s-3M_\Delta^2}{3M_\Delta^2}(M_p+M_\Delta)]\,.$$

(ii) The half-off-shell T-matrix $\tilde{T}_a$ is defined by the following expression:

$$\tilde{T}_a = [g(q_f+K)^{\rho}]\;G_{\rho\alpha}(p)\;[\,gq_i{}^{\alpha}] = \frac{iq^2}{p^2-M_\Delta^2+i\epsilon}\,\tilde{T}'_a$$

where

$$\tilde{T}'_a = (\not{p} + M_\Delta)\{(q_f+k)\bullet q_i - \frac{1}{3}(\not{q}_f + \not{k})\not{q}_i - \frac{1}{3M_\Delta}[(\not{q}_f + \not{k})\, p\bullet q_i - \not{q}_i\, p\bullet(q_f+k)]$$

$$- \frac{2}{3M_\Delta^2} p\bullet(q_f+k)\; p\bullet q_i]$$

$$+ \frac{2}{3M_\Delta^2}(p^2 - M_\Delta^2)\; [(\not{q}_f + \not{k})\, p\bullet q_i - \not{q}_i\, p\bullet(q_f+k) + (\not{p} + M_\Delta)(\not{q}_f + \not{k})\not{q}_i\}$$

Using

$$p = q_i + p_i = q_f + K + p_f \,,$$

$$\bar{u}(p_f)\not{p}_f = M_p\, \bar{u}(p_f)\,,$$

$$\bar{u}(p_f)(\not{q}_f + \not{k})\not{q}_i\, u(p_i) = \bar{u}(p_f)(q_i^2 + 2p_i\bullet q_i - 2M_p \not{q}_i)\, u(p_i),$$

$$\bar{u}(p_f)\not{q}_i\, u(p_i) = \bar{u}(p_f)(\not{q}_f + \not{k})\, u(p_i)$$

$$= \bar{u}(p_f)\frac{1}{2}(\not{q}_f + \not{q}_i + \not{k})\, u(p_i)\,,$$

$$t_p = (p_i - p_f)^2 = (q_i - q_f - k)^2 \,, \quad (q_f+k)\bullet q_i = -\frac{t_p}{2} + \frac{(q_f+k)^2}{2} + \frac{q_i^2}{2}\,,$$

$$s_i = (q_i + p_i)^2 \,, \qquad p_i\bullet q_i = \frac{s_i}{2} - \frac{p_i^2}{2} - \frac{q_i^2}{2}\,,$$

$$p\bullet q_i = (q_i + p_i)\bullet q_i = \frac{s_i}{2} - \frac{p_i^2}{2} + \frac{q_i^2}{2}\,,$$

$$p\bullet(q_f+k) = (q_f + k + p_f)\bullet(q_f+k) = \frac{s_i}{2} - \frac{p_f^2}{2} + \frac{(q_f+k)^2}{2}\,,$$

$\tilde{T}_a$ can be written as

$$\bar{u}\,\tilde{T}_a\, u = \bar{u}\,\left(\frac{ig^2}{s_i - M_\Delta^2 + i\epsilon}\right)[\, A(s_i,t_p) + \frac{1}{2}(\not{q}_f + \not{q}_i + \not{k})\, B(s_i,t_p)]\, u$$

where

$$A(s_i,t_p) = (M_p+M_\Delta)\Big[-\frac{t_p}{2}+\frac{(q_f+k)^2}{2}+\frac{q_i^2}{2}-\frac{2}{3M_\Delta^2}\Big(\frac{s_i}{2}-\frac{p_f^2}{2}+\frac{(q_f+k)^2}{2}\Big)\Big(\frac{s_i}{2}-\frac{p_i^2}{2}+\frac{q_i^2}{2}\Big)\Big]$$

$$-\frac{1}{3M_\Delta}(q_f+k)^2\Big(\frac{s_i}{2}-\frac{p_i^2}{2}+\frac{q_i^2}{2}\Big)+(s_i-p_i^2)\Big[\frac{1}{3M_\Delta}\Big(\frac{s_i}{2}-\frac{p_f^2}{2}+\frac{(q_f+k)^2}{2}\Big)+\frac{2s_i-3M_\Delta^2}{3M_\Delta^2}(M_p+M_\Delta)\Big]$$

and

$$B(s_i,t_p) = -\frac{t_p}{2}+\frac{(q_f+k)^2}{2}+\frac{q_i^2}{2}-\frac{2}{3M_\Delta^2}\Big(\frac{s_i}{2}-\frac{p_f^2}{2}+\frac{(q_f+k)^2}{2}\Big)\Big(\frac{s_i}{2}-\frac{p_i^2}{2}+\frac{q_i^2}{2}\Big)$$

$$+\Big(\frac{2s_i}{3M_\Delta^2}-1-\frac{M_P}{3M_\Delta}\Big)\Big(\frac{p_f^2}{2}-\frac{p_i^2}{2}+\frac{q_i^2}{2}-\frac{(q_f+k)^2}{2}\Big)+\frac{2s_i-3M_\Delta^2}{3M_\Delta^2}(q_f+k)^2$$

$$-2M_p\Big[\frac{1}{3M_\Delta}\Big(\frac{s_i}{2}-\frac{p_f^2}{2}+\frac{(q_f+k)^2}{2}\Big)+\frac{2s_i-3M_\Delta^2}{3M_\Delta^2}(M_p+M_\Delta)\Big].$$

or

$$A(s_i,t_p)=(M_p+M_\Delta)\Big[-\frac{t_p}{2}+\frac{(q_f+k)^2}{2}+\frac{q_i^2}{2}-\frac{2}{3M_\Delta^2}\Big(\frac{s_i}{2}-\frac{p_f^2}{2}+\frac{(q_f+k)^2}{2}\Big)\Big(\frac{s_i}{2}-\frac{p_i^2}{2}+\frac{q_i^2}{2}\Big)\Big]$$

$$-\frac{1}{3M_\Delta^2}\,q_i^{\,2}\Big(\frac{s_i}{2}-\frac{p_f^2}{2}+\frac{(q_f+k)^2}{2}\Big)+(s_i-p_i^2)\Big[\frac{1}{3M_\Delta}\Big(\frac{s_i}{2}-\frac{p_i^2}{2}+\frac{q_i^{\,2}}{2}\Big)+\frac{2s_i-3M_\Delta^2}{3M_\Delta^2}(M_p+M_\Delta)\Big]$$

and

$$B(s_i,t_p) = -\frac{t_p}{2}+\frac{(q_f+k)^2}{2}+\frac{q_i^2}{2}-\frac{2}{3M_\Delta^2}\Big(\frac{s_i}{2}-\frac{p_f^2}{2}+\frac{(q_f+k)^2}{2}\Big)\Big(\frac{s_i}{2}-\frac{p_i^2}{2}+\frac{q_i^2}{2}\Big)$$

$$+\Big(\frac{2s_i}{3M_\Delta^2}-1-\frac{M_P}{3M_\Delta}\Big)\Big(-\frac{p_f^2}{2}+\frac{p_i^2}{2}-\frac{q_i^2}{2}+\frac{(q_f+k)^2}{2}\Big)+\frac{2s_i-3M_\Delta^2}{3M_\Delta^2}\,q_i^{\,2}$$

$$-2M_p\Big[\frac{1}{3M_\Delta}\Big(\frac{s_i}{2}-\frac{p_i^2}{2}+\frac{q_i^{\,2}}{2}\Big)+\frac{2s_i-3M_\Delta^2}{3M_\Delta^2}(M_p+M_\Delta)\Big].$$

(iii) The half-off-shell T-matrix $\tilde{T}_b$ is defined by the following expression:

$$\tilde{T}_b = [g\, q_f{}^{\rho}]\, G_{\rho\alpha}(p)\, [\, g(q_i-K)^{\alpha}] = \frac{iq^2}{p^2-M_\Delta^2+i\epsilon}\, \tilde{T}'_b$$

where

$$\tilde{T}'_b=(\not{p} + M_\Delta)\{q_f\bullet(q_i-k)- \frac{1}{3} \not{q}_f\, (\not{q}_i- \not{k})- \frac{1}{3M_\Delta}[\, \not{q}_f\, p\bullet(q_i-k) - (\not{q}_i - \not{k}\,)\, p\bullet q_f]$$

$$- \frac{2}{3M_\Delta^2} p\bullet q_f\; p\bullet(q_i-k)]$$

$$+ \frac{2}{3M_\Delta^2}(p^2-M_\Delta^2)\, [\, \not{q}_f\, p\bullet(q_i-k) - (\, \not{q}_i -\not{k}\,)\, p\bullet q_f +(\not{p} + M_\Delta)\, \not{q}_f\, (\not{q}_i-\not{k})\}$$

Using

$$p = q_f+p_f = q_i- k +p_i\ ,$$

$$\bar{u}(p_f)\, \not{p}_f\ =\ M_p\, \bar{u}(p_f)\ ,$$

$$\bar{u}(p_f)\, \not{q}_f(\, \not{q}_i-\not{k}\,)\, u(p_i) = \bar{u}(p_f)\, [q_i^2+ 2p_i\bullet(q_i-k) - 2M_p(\, \not{q}_i - \not{k}\,)]u(p_i),$$

$$\bar{u}(p_f)\, (\, \not{q}_i - \not{k}\,)\, u(p_i) = \bar{u}(p_f)\, \not{q}_f\, u(p_i)$$

$$= \bar{u}(p_f)\, \frac{1}{2}\, (\, \not{q}_f + \not{q}_i - \not{k}\,)\, u(p_i)\ ,$$

$$t_p= (p_i- p_f)^2 = (q_i - q_f - k)^2\ , \quad q_f\bullet(q_i-k) = - \frac{t_p}{2} + \frac{q_f{}^2}{2} + \frac{(q_i-k)^2}{2}\ ,$$

$$s_f= (q_f+ p_f)^2\ , \qquad p_i\bullet(q_i-k) = \frac{s_f}{2} - \frac{p_i^2}{2} - \frac{(q_i-k)^2}{2},$$

$$p\bullet(q_i-k) = (q_i-k + p_i)\bullet(q_i-k) = \frac{s_f}{2} - \frac{p_i^2}{2} + \frac{(q_i-k)^2}{2}\ ,$$

$$p\bullet q_f = (\, q_f+ p_f)\bullet q_f = \frac{s_f}{2} - \frac{p_f^2}{2} + \frac{q_f{}^2}{2}\ ,$$

We can express $\tilde{T}_b$ in terms of s_f and t_p as

$$\bar{u}\,\tilde{T}_b\,u = \bar{u}\left(\frac{ig^2}{s_f - M_\Delta^2 + i\epsilon}\right)\left[\, A(s_f, t_p) + \frac{1}{2}\left(\not{q}_f + \not{q}_i - \not{k}\right) B(s_f, t_p)\right] u$$

where

$$A(s_f, t_p) = (M_p + M_\Delta)\left[-\frac{t_p}{2} + \frac{q_f^{\,2}}{2} + \frac{(q_i - k)^2}{2} - \frac{2}{3M_\Delta^2}\left(\frac{s_f}{2} - \frac{p_f^2}{2} + \frac{q_f^{\,2}}{2}\right)\left(-\frac{s_f}{2} - \frac{p_i^2}{2} + \frac{(q_i - k)^2}{2}\right)\right]$$

$$-\frac{1}{3M_\Delta} q_f^{\,2}\left(\frac{s_f}{2} + \frac{p_i^2}{2} + \frac{(q_i - k)^2}{2}\right) + (s_i - p_i^2)\left[\frac{1}{3M_\Delta}\left(\frac{s_f}{2} - \frac{p_f^2}{2} + \frac{q_f^2}{2}\right) \frac{2s_i - 3M_\Delta^2}{3M_\Delta^2}(M_p + M_\Delta)\right]$$

and

$$B(s_f, t_p) = -\frac{t_p}{2} + \frac{q_f^2}{2} + \frac{(q_i - k)^2}{2} - \frac{2}{3M_\Delta^2}\left(\frac{s_f}{2} - \frac{p_f^2}{2} + \frac{q_f^2}{2}\right)\left(\frac{s_f}{2} - \frac{p_i^2}{2} + \frac{(q_i - k)^2}{2}\right)$$

$$+\left(\frac{2s_f}{3M_\Delta^2} - 1 - \frac{M_p}{3M_\Delta}\right)\left(\frac{p_f^2}{2} - \frac{p_i^2}{2} - \frac{q_f^2}{2} + \frac{(q_i - k)^2}{2}\right) + \frac{2s_f - 3M_\Delta^2}{3M_\Delta^2}\, q_f^{\,2}$$

$$-\,2M_p\left[\frac{1}{3M_\Delta}\left(\frac{s_f}{2} - \frac{p_f^2}{2} + \frac{q_f^2}{2}\right) + \frac{2s_f - 3M_\Delta^2}{3M_\Delta^2}(M_p + M_\Delta)\right].$$

(iv) The half-off-shell T-matrix $\tilde{T}_d$ is defined by the following expression:

$$\tilde{T}_d = [g\, q_f^{\,\rho}]\; G_{\rho\alpha}(p')\; [\, g\, q_i^{\,\alpha}] = \frac{iq^2}{p'^{\,2} - M_\Delta^2 + i\epsilon}\, \tilde{T}'_d$$

where

$$\tilde{T}'_d = (\not{p}' + M_\Delta)\left[q_f \bullet q_i - \frac{1}{3}\not{q}_f\,\not{q}_i - \frac{1}{3M_\Delta}\left(\not{q}_f\; p'\bullet q_i - \not{q}_i\; p'\bullet q_f\right) - \frac{2}{3M_\Delta^2} p'\bullet q_f\; p'\bullet q_i\right]$$

$$+\,\frac{2}{3M_\Delta^2}(p'^{\,2} - M_\Delta^2)\left[\,\not{q}_f\; p'\bullet q_i - \not{q}_i\; p'\bullet q_f + (\not{p}' + M_\Delta)\,\not{q}_f\,\not{q}_i\,\right].$$

Using

$$p' = q_f + p_f = q_i + p_i - K ,$$

$$\bar{u}(p_f)\, \not{p}_f = M_p\, \bar{u}(p_f) ,$$

$$\bar{u}(p_f)\, \not{q}_f\, \not{q}_i = \bar{u}(p_f)\, [q_i^2 + 2(p_i - k)\bullet q_i - 2M_p \not{q}_i - \not{q}_i\, (\not{p}_i - \not{K} - M_p)],$$

$$\bar{u}(p_f)\quad \not{q}_f = \bar{u}(p_f)\, [\frac{1}{2}(\not{q}_f + \not{q}_i) + \frac{1}{2}(\not{p}_i - \not{K} - M_p)] ,$$

$$\bar{u}(p_f)\quad \not{q}_i = \bar{u}(p_f)\, [\frac{1}{2}(\not{q}_f + \not{q}_i) - \frac{1}{2}(\not{p}_i - \not{K} - M_p)] ,$$

$$t_q = (q_i - q_f)^2 = (p_i - p_f - k)^2 , \qquad q_f \bullet q_i = -\frac{t_q}{2} + \frac{{q_f}^2}{2} + \frac{q_i^2}{2} ,$$

$$s_f = (q_f + p_f)^2 , \qquad (p_i - K)\bullet q_i = \frac{s_f}{2} - \frac{q_i^2}{2} - \frac{(p_i - k)^2}{2},$$

$$p'\bullet q_i = \frac{s_f}{2} - \frac{q_i^2}{2} + \frac{(p_i - k)^2}{2} ,$$

and

$$p'\bullet q_f = \frac{s_f}{2} - \frac{p_f^2}{2} + \frac{{q_f}^2}{2} ,$$

$\tilde{T}_d$ can be written in terms of s_f and t_q as

$$\bar{u}\, \tilde{T}_d\, u = \bar{u}(\frac{ig^2}{s_f - M_\Delta^2 + i\epsilon})[A(s_f, t_q) + \frac{1}{2}(\not{q}_f + \not{q}_i) B(s_f, t_p) + \frac{1}{2} C(s_f, t_q, \not{q}_i)(p_i - k - M_p)]u$$

where

$$A(s_f, t_q) = (M_p + M_\Delta)[-\frac{t_q}{2} + \frac{{q_f}^2}{2} + \frac{{q_i}^2}{2} - \frac{2}{3M_\Delta^2}(\frac{s_f}{2} - \frac{p_f^2}{2} + \frac{{q_f}^2}{2})(\frac{s_f}{2} - \frac{p_i^2}{2} + \frac{q_i^2}{2})]$$

$$-\frac{1}{3M_\Delta}m_\pi^{\,2}\left(\frac{s_f}{2}+\frac{m_\pi^2}{2}-\frac{(p_i-k)^2}{2}\right)+[s_f-(p_i-k)^2]\left[\frac{1}{3M_\Delta}\left(\frac{s_f}{2}-\frac{M_p^2}{2}+\frac{m_\pi^2}{2}\right)+\frac{2s_f-3M_\Delta^2}{3M_\Delta^2}(M_p+M_\Delta)\right]$$

$$B(s_f,t_q) = -\frac{t_q}{2}+\frac{q_f^2}{2}+\frac{q_i^2}{2}-\frac{2}{3M_\Delta^2}\left(\frac{s_f}{2}-\frac{p_f^2}{2}+\frac{q_f^2}{2}\right)\left(\frac{s_f}{2}+\frac{q_i^2}{2}-\frac{(p_i-k)^2}{2}\right)$$

$$+\left(\frac{2s_f}{3M_\Delta^2}-1-\frac{M_P}{3M_\Delta}\right)\left(\frac{M_p^2}{2}-\frac{(p_i-K)^2}{2}\right)+\frac{2s_f-3M_\Delta^2}{3M_\Delta^2}\,m_\pi^{\,2}$$

$$-2M_p\left[\frac{1}{3M_\Delta}\left(\frac{s_f}{2}-\frac{M_p^2}{2}+\frac{m_\pi^2}{2}\right)+\frac{2s_f-3M_\Delta^2}{3M_\Delta^2}(M_p+M_\Delta)\right].$$

and

$$C(s_f,t_q,\not{q}_i) = -\frac{t_q}{2}+\frac{q_f^2}{2}+\frac{q_i^2}{2}-\frac{2}{3M_\Delta^2}\left(\frac{s_f}{2}-\frac{p_f^2}{2}+\frac{q_f^2}{2}\right)\left(\frac{s_f}{2}+\frac{q_i^2}{2}-\frac{(p_i-k)^2}{2}\right)$$

$$+\left(\frac{2s_f}{3M_\Delta^2}-1-\frac{M_P}{3M_\Delta}\right)\left(s_f-\frac{p_f^2}{2}-\frac{(p_i-K)^2}{2}+\frac{q_f^2}{2}+\frac{q_i^2}{2}\right)-\frac{2s_f-3M_\Delta^2}{3M_\Delta^2}\,m_\pi^{\,2}$$

$$-2M_p\left[\frac{1}{3M_\Delta}\left(\frac{s_f}{2}-\frac{M_p^2}{2}+\frac{m_\pi^2}{2}\right)+\frac{2s_f-3M_\Delta^2}{3M_\Delta^2}(M_p+M_\Delta)\right].$$

$$-2\not{q}_i\left[\frac{1}{3M_\Delta}\left(\frac{s_f}{2}-\frac{M_p^2}{2}+\frac{m_\pi^2}{2}\right)+\frac{2s_f-3M_\Delta^2}{3M_\Delta^2}(M_p+M_\Delta)\right].$$

(v) The half-off-shell T-matrix $\tilde{T}_c$ is defined by the following expression:

$$\tilde{T}_c = [g\, q_f^{\,\rho}]\, G_{\rho\alpha}(p)\, [g\, q_i^{\,\alpha}] = \frac{iq^2}{p^2-M_\Delta^2+i\epsilon}\,\tilde{T}'_c$$

where

$$\tilde{T}'_c = [q_f \bullet q_i - \frac{2}{3M_\Delta^2}p\bullet q_f\ p\bullet q_i](\not{p}+M_\Delta) + \frac{2}{3M_\Delta^2}(p^2-M_\Delta^2)[\not{q}_f\ p\bullet q_i - \not{q}_i\ p\bullet q_f]$$

$$+ \frac{2p^2-3M_\Delta^2}{3M_\Delta^2} \not{q}_f \not{q}_i (\not{p} + M_\Delta) + \frac{2p^2-3M_\Delta^2}{3M_\Delta^2}(2p\bullet q_f \not{q}_i - 2p\bullet q_i \not{q}_f)$$

$$- \frac{1}{3M_\Delta}(\not{q}_f \, p\bullet q_i - \not{q}_i \, p\bullet q_f)(- \not{p} +M_\Delta)$$

Using

$$p = q_f+p_f = q_i+p_i-K ,$$

$$(\not{p} + M_\Delta) u(p_i) = (\not{q}_i + M_p + M_\Delta)u(p_i) ,$$

$$\not{q}_i \; u(p_i) = [\frac{1}{2}(\not{q}_f + \not{q}_i) + \frac{1}{2}(\not{p}_f + \not{K} -M_p)]u(p_i) ,$$

$$\not{q}_f \; u(p_i) = [\frac{1}{2}(\not{q}_f + \not{q}_i) - \frac{1}{2}(\not{p}_f + \not{K} -M_p)]u(p_i) ,$$

$$\not{q}_f \not{q}_i \; u(p_i) = [q_i^2 + 2p_i\bullet q_i-2M_p \not{q}_i - (\not{p}_f + \not{K} -M_p) \not{q}_i]u(p_i) ,$$

$$t_q= (q_i- q_f)^2 = (p_i - p_f - K)^2 , \qquad q_f\bullet q_i = - \frac{t_q}{2} + \frac{{q_f}^2}{2} + \frac{q_i^2}{2} ,$$

$$s_i= (q_i+ p_i)^2 , \qquad p_i\bullet q_i = \frac{s_i}{2} - \frac{q_i^2}{2} - \frac{p_i^2}{2} ,$$

$$p\bullet q_i = \frac{s_i}{2} - \frac{p_i^2}{2} + \frac{q_i^2}{2} ,$$

and

$$p\bullet q_f = \frac{s_i}{2} - \frac{(p_f+k)^2}{2} + \frac{{q_f}^2}{2} ,$$

$\tilde{T}_c$ can be written in terms of s_i and t_q as

$$\bar{u} \, \tilde{T}_c \, u = \bar{u}(\frac{ig^2}{s_f-M_\Delta^2+i\epsilon})[A(s_i,t_q)+\frac{1}{2}(\not{q}_f+\not{q}_i)B(s_i,t_p)+\frac{1}{2}(\not{p}_f+\not{K}-M_p)C(s_i,t_q,\not{q}_i)]u$$

where

$$A(s_i,t_q) = (M_p+M_\Delta)[-\frac{t_q}{2}+\frac{q_f^2}{2}+\frac{q_i^2}{2}-\frac{2}{3M_\Delta^2}(\frac{s_i}{2}-\frac{(p_f+k)^2}{2}+\frac{q_f^2}{2})(\frac{s_i}{2}-\frac{p_i^2}{2}+\frac{q_i^2}{2})]$$

$$-\frac{1}{3M_\Delta}m_\pi^2(\frac{s_i}{2}+\frac{m_\pi^2}{2}-\frac{(p_f+k)^2}{2})+[s_i-p_i^2][\frac{1}{3M_\Delta}(\frac{s_i}{2}-\frac{M_p^2}{2}+\frac{m_\pi^2}{2})+\frac{2s_i-3M_\Delta^2}{3M_\Delta^2}(M_p+M_\Delta)]$$

and

$$B(s_i,t_q) = -\frac{t_q}{2}+\frac{q_f^2}{2}+\frac{q_i^2}{2}-\frac{2}{3M_\Delta^2}(\frac{s_i}{2}-\frac{(p_f+k)^2}{2}+\frac{q_f^2}{2})(\frac{s_i}{2}+\frac{q_i^2}{2}-\frac{p_i^2}{2})$$

$$+(\frac{2s_i}{3M_\Delta^2}-1-\frac{M_P}{3M_\Delta})\ (\frac{M_p^2}{2}-\frac{(p_f+k)^2}{2})+\frac{2s_i-3M_\Delta^2}{3M_\Delta^2}\ m_\pi^2$$

$$-2M_p[\frac{1}{3M_\Delta}(\frac{s_i}{2}-\frac{M_p^2}{2}+\frac{m_\pi^2}{2})+\frac{2s_i-3M_\Delta^2}{3M_\Delta^2}(M_p+M_\Delta)].$$

$$C(s_f,t_q,\not{q}_i) = -\frac{t_q}{2}+\frac{q_f^2}{2}+\frac{q_i^2}{2}-\frac{2}{3M_\Delta^2}(\frac{s_i}{2}-\frac{(p_f+k)^2}{2}+\frac{q_f^2}{2})(\frac{s_i}{2}+\frac{q_i^2}{2}-\frac{p_i^2}{2})$$

$$+(\frac{2s_i}{3M_\Delta^2}-1-\frac{M_P}{3M_\Delta})\ (s_i-\frac{p_i^2}{2}-\frac{(p_f+k)^2}{2}+\frac{q_f^2}{2}+\frac{q_i^2}{2})-\frac{2s_i-3M_\Delta^2}{3M_\Delta^2}\ m_\pi^2$$

$$-2M_p[\frac{1}{3M_\Delta}(\frac{s_i}{2}-\frac{M_p^2}{2}+\frac{m_\pi^2}{2})+\frac{2s_i-3M_\Delta^2}{3M_\Delta^2}(M_p+M_\Delta)].$$

$$-2\not{q}_i\ [\frac{1}{3M_\Delta}(\frac{s_i}{2}-\frac{M_p^2}{2}+\frac{m_\pi^2}{2})+\frac{2s_i-3M_\Delta^2}{3M_\Delta^2}(M_p+M_\Delta)].$$

Appendix B

The expressions for $E^{\sigma}{}_{\alpha}$, E'^{β}_{ρ} , $\tilde{C}^{\sigma}_{\alpha\mu}$ and $\tilde{C}'^{\beta}_{\rho\mu}$.

$$E^{\sigma}{}_{\alpha}=\gamma^{\sigma}\gamma_{\alpha}[-2\not{\epsilon}\not{p}'/9-4p'\cdot\epsilon/9-2\not{\epsilon}\not{k}/3+p'\cdot\epsilon(\not{p}'+\not{k})/(9M_{\Delta})$$

$$-(p'^{2}+2p'\cdot k)\not{\epsilon}/(9M_{\Delta})+2(p'^{2}+2p'\cdot k)(-p'\cdot\epsilon+\not{\epsilon}\not{p}'+\not{\epsilon}\not{k})/(9M_{\Delta}^{2})]$$

$$+\epsilon^{\sigma}\gamma_{\alpha}[-\not{p}'-4\not{k}/3-2M_{\Delta}/3+(p'^{2}+2p'\cdot k)/(3M_{\Delta})+2(p'^{2}+2p'\cdot k)(\not{p}'+\not{k})/(3M_{\Delta}^{2})]$$

$$+\gamma^{\sigma}\epsilon_{\alpha}[7\not{p}'/9+\not{k}+2M_{\Delta}/9+2(p'^{2}+2p'\cdot k)/(9M_{\Delta})-4(p'^{2}+2p'\cdot k)(\not{p}'+\not{k})/(9M_{\Delta}^{2})]$$

$$+\epsilon^{\sigma}(p'+k)_{\alpha}[2/3+2(\not{p}'+\not{k})/(3M_{\Delta})-2(p'^{2}+2p'\cdot k)/(3M_{\Delta}^{2})]+2\epsilon^{\sigma}k_{\alpha}/3$$

$$+(p'+k)^{\sigma}\epsilon_{\alpha}[-14/9-2(\not{p}'+\not{k})/(3M_{\Delta})+4(p'^{2}+2p'\cdot k)/(3M_{\Delta}^{2})]-4k^{\sigma}\epsilon_{\alpha}/9$$

$$+\gamma^{\sigma}(p'+k)_{\alpha}[-\not{\epsilon}/9-4\not{\epsilon}(\not{p}'+\not{k})/(9M_{\Delta})+2p'\cdot\epsilon/(9M_{\Delta})$$

$$+2(p'\cdot\epsilon\not{p}'+p'\cdot\epsilon\not{k}+p'^{2}\not{\epsilon}+2p'\cdot k\not{\epsilon})/(9M_{\Delta}^{2})]-5\gamma^{\sigma}k_{\alpha}\not{\epsilon}/9$$

$$+(p'+k)^{\sigma}\gamma_{\alpha}\not{\epsilon}[1/3+(\not{p}'+\not{k})/(3M_{\Delta})-2(p'^{2}+2p'\cdot k)/(3M_{\Delta}^{2})]+2k^{\sigma}\gamma_{\alpha}\not{\epsilon}/3$$

$$-(p'+k)^{\sigma}(p'+k)_{\alpha}\,2\not{\epsilon}(\not{p}'+\not{k}+M_{\Delta})/(3M_{\Delta}^{2})$$

$$+\lambda_{\Delta}/(2M_{\Delta})\{\gamma^{\sigma}\gamma_{\alpha}[-(\not{\epsilon}\not{k}\not{p}'+\not{p}'\not{\epsilon}\not{k})/3+2\not{\epsilon}\not{k}(\not{p}'+M_{\Delta})(p'^{2}-M_{\Delta}^{2})/(3M_{\Delta}^{2})]$$

$$+p'^{\sigma}\gamma_{\alpha}\not{\epsilon}\not{k}(2p'^{2}-M_{\Delta}\not{p}'-4M_{\Delta}^{2})/(3M_{\Delta}^{2})-\gamma^{\sigma}p'_{\alpha}\not{\epsilon}\not{k}(2p'^{2}-M_{\Delta}\not{p}'-4M_{\Delta}^{2})/(3M_{\Delta}^{2})$$

$$+(4k^{\sigma}\epsilon_{\alpha}-4\epsilon^{\sigma}k_{\alpha}+2\epsilon^{\sigma}\gamma_{\alpha}\not{k}-2\gamma^{\sigma}\epsilon_{\alpha}\not{k}+2\gamma^{\sigma}k_{\alpha}\not{\epsilon}-2k^{\sigma}\gamma_{\alpha}\not{\epsilon})(\not{p}'+M_{\Delta})(2p'^{2}-3M_{\Delta}^{2})/(3M_{\Delta}^{2})$$

$$+[2\not{k}(\epsilon^{\sigma}p'_{\alpha}-p'^{\sigma}\epsilon_{\alpha})+2\not{\epsilon}(p'^{\sigma}k_{\alpha}-k^{\sigma}p'_{\alpha})](2\not{p}'-3M_{\Delta})(\not{p}'+M_{\Delta})/(3M_{\Delta}^{2})$$

$$-2p'^{\sigma}p'_{\alpha}\not{\epsilon}\not{k}(\not{p}'+M_{\Delta})/(3M_{\Delta}^{2})\}$$

$$E'^{\beta}_{\rho}=[-2\not{p}\,\not{\epsilon}/9-4p\cdot\epsilon/9-2\,\not{\epsilon}\,\not{k}/3+p\cdot\epsilon(\not{p}-\not{k})/(9M_\Delta)-(p^2-2p\cdot k)\,\not{\epsilon}/(9M_\Delta)$$

$$+2(p^2-2p\cdot k)(-p\cdot\epsilon+\not{p}\,\not{\epsilon}-\not{k}\,\not{\epsilon})/(9M_\Delta^2)]\,\gamma_\rho\gamma^\beta$$

$$+[-\not{p}+4\not{k}/3-2M_\Delta/3+(p^2-2p\cdot k)/(3M_\Delta)+2(p^2-2p\cdot k)(\not{p}-\not{k})/(3M_\Delta^2)]\,\gamma_\rho\epsilon^\beta$$

$$+[7\not{p}/9-\not{k}+2M_\Delta/9+2(p^2-2p\cdot k)/(9M_\Delta)-4(p^2-2p\cdot k)(\not{p}-\not{k})/(9M_\Delta^2)]\,\epsilon_\rho\gamma^\beta$$

$$+[2/3+2(\not{p}-\not{k})/(3M_\Delta)-2(p^2-2p\cdot k)/(3M_\Delta^2)](p-k)_\rho\epsilon^\beta\;-\;2k_\rho\epsilon^\beta/3$$

$$+[-14/9-2(\not{p}-\not{k})/(3M_\Delta)\;+\;4(p^2-2p\cdot k)/(3M_\Delta^2)]\epsilon_\rho(p-k)^\beta\;+\;4\epsilon_\rho k^\beta/9$$

$$+[-\not{\epsilon}/9-4(\not{p}-\not{k})\,\not{\epsilon}/(9M_\Delta)+2p\cdot\epsilon/(9M_\Delta)$$

$$+2(p\cdot\epsilon\;p-p\cdot\epsilon\,\not{k}+p^2\,\not{\epsilon}-2p\cdot k\,\not{\epsilon})/(9M_\Delta^2)](p-k)_\rho\gamma^\beta\;+\;5\,\not{\epsilon}\,k_\rho\gamma^\beta/9$$

$$+[1/3+(\not{p}-\not{k})/(3M_\Delta)-2(p^2-2p\cdot k)/(3M_\Delta^2)]\,\not{\epsilon}\,\gamma_\rho(p-k)^\beta\;-\;2\,\not{\epsilon}\,\gamma_\rho k^\beta/3$$

$$-2(\not{p}-\not{k}+M_\Delta)\,\not{\epsilon}\,(p-k)_\rho(p-k)^\beta/(3M_\Delta^2)$$

$$+\lambda_\Delta/(2M_\Delta)\{[-(\not{\epsilon}\,\not{k}\,\not{p}+\not{p}\,\not{\epsilon}\,\not{k})/3\;+2(\not{p}+M_\Delta)\,\not{\epsilon}\,\not{k}\,(p^2-M_\Delta^2)/(3M_\Delta^2)]\,\gamma_\rho\gamma^\beta$$

$$+(2p^2-M_\Delta\,\not{p}-4M_\Delta^2)\,\not{\epsilon}\,\not{k}\,\gamma_\rho p^\beta/(3M_\Delta^2)\;-\;(2p^2-M_\Delta\,\not{p}-4M_\Delta^2)\,\not{\epsilon}\,\not{k}\,p_\rho\gamma^\beta/(3M_\Delta^2)$$

$$+(2p^2-3M_\Delta^2)(\not{p}+M_\Delta)(4k_\rho\epsilon^\beta-4\epsilon_\rho k^\beta+2\not{k}\,\epsilon_\rho\gamma^\beta-2\not{k}\,\gamma_\rho\epsilon^\beta+2\,\not{\epsilon}\,\gamma_\rho k^\beta-2\not{\epsilon}\,k_\rho\gamma^\beta)/(3M_\Delta^2)$$

$$+(2\not{p}-3M_\Delta)(\not{p}+M_\Delta)[2\not{k}\,(\epsilon_\rho p^\beta-p_\rho\epsilon^\beta)+2\,\not{\epsilon}\,(p_\rho k^\beta-k_\rho p^\beta)]/(3M_\Delta^2)$$

$$-2(\not{p}+M_\Delta)\,\not{\epsilon}\,\not{k}\,p_\rho p^\beta/(3M_\Delta^2)\}$$

$$\tilde{C}^{\sigma}_{\alpha\mu}(p',k) = \gamma^{\sigma}\gamma_{\alpha}[\,2(p'^{2}-3M_{\Delta}^{2})\gamma_{\mu}\not{k}/(9M_{\Delta}^{2}) + (p'_{\mu}\not{k} - \gamma_{\mu}p'\bullet k)/(9M_{\Delta})]$$

$$+\gamma_{\alpha}(g_{\mu}{}^{\sigma}\not{k} - k^{\sigma}\gamma_{\mu})(-7M_{\Delta}^{2} - M_{\Delta}\not{p}' + 4p'^{2})/(6M_{\Delta}^{2})$$

$$+\gamma^{\sigma}(g_{\alpha\mu}\not{k} - k_{\alpha}\gamma_{\mu})(15M_{\Delta}^{2} + 4M_{\Delta}\not{p}' - 6p'^{2})/(18M_{\Delta}^{2})$$

$$+\gamma^{\sigma}(p'_{\mu}k_{\alpha} - p'\bullet k\, g_{\alpha\mu})\,2(M_{\Delta} + \not{p}')/(9M_{\Delta}^{2})$$

$$+p'_{\alpha}(g_{\mu}{}^{\sigma}\not{k} - k^{\sigma}\gamma_{\mu})(2M_{\Delta} + \not{p}')/(3M_{\Delta}^{2})$$

$$+(g_{\mu}{}^{\sigma}k_{\alpha} - k^{\sigma}g_{\alpha\mu})(5M_{\Delta}^{2} + 2M_{\Delta}\not{p}' - 3p'^{2})/(3M_{\Delta}^{2})$$

$$+p'^{\sigma}(g_{\alpha\mu}\not{k} - k_{\alpha}\gamma_{\mu})\not{p}'/(3M_{\Delta}^{2}) + p'^{\sigma}\gamma_{\alpha}\gamma_{\mu}\not{k}/(3M_{\Delta})$$

$$+\gamma^{\sigma}p'_{\alpha}[-4M_{\Delta}\gamma_{\mu}\not{k} + 2(p'_{\mu}\not{k} - \gamma_{\mu}p'\bullet k)]/(9M_{\Delta}^{2}) - 2p'^{\sigma}p'_{\alpha}\gamma_{\mu}\not{k}/(3M_{\Delta}^{2})$$

$$+\lambda_{\Delta}/(2M_{\Delta})\{\gamma^{\sigma}\gamma_{\alpha}[-(\not{p}'\gamma_{\mu}\not{k} + \gamma_{\mu}\not{k}\not{p}')/3 + 2(p'^{2}-M_{\Delta}^{2})\gamma_{\mu}\not{k}(\not{p}'+M_{\Delta})/(3M_{\Delta}^{2})]$$

$$+p'^{\sigma}\gamma_{\alpha}\gamma_{\mu}\not{k}(2p'^{2} - M_{\Delta}\not{p}' - 4M_{\Delta}^{2})/(3M_{\Delta}^{2}) - \gamma^{\sigma}p'_{\alpha}\gamma_{\mu}\not{k}(2p'^{2} - M_{\Delta}\not{p}' - 4M_{\Delta}^{2})/(3M_{\Delta}^{2})$$

$$+(2p'^{2}-3M_{\Delta}^{2})[4(k^{\sigma}g_{\alpha\mu} - g_{\mu}^{\sigma}k_{\alpha}) + 2\gamma_{\alpha}(g_{\mu}^{\sigma}\not{k} - k^{\sigma}\gamma_{\mu}) - 2\gamma^{\sigma}(g_{\alpha\mu}\not{k} - k_{\alpha}\gamma_{\mu})](\not{p}'+M_{\Delta})/(3M_{\Delta}^{2})$$

$$+[2(g_{\mu}{}^{\sigma}\not{k} - k^{\sigma}\gamma_{\mu})p'_{\alpha} - 2p'^{\sigma}(g_{\alpha\mu}\not{k} - k_{\alpha}\gamma_{\mu})](2\not{p}' - 3M_{\Delta})(\not{p}'+M_{\Delta})/(3M_{\Delta}^{2})$$

$$-2p'^{\sigma}p'_{\alpha}\gamma_{\mu}\not{k}(\not{p}' + M_{\Delta})/(3M_{\Delta}^{2})\}$$

$$\tilde{C}'^{\beta}_{\rho\mu}(p,k)= [2(p^2-3M_\Delta^2)\gamma_\mu \not{k}\,/(9M_\Delta^2) - (p_\mu \not{k} - p\bullet k\,\gamma_\mu)/(9M_\Delta)]\,\gamma_\rho\gamma^\beta$$

$$+(7M_\Delta^2+M_\Delta \not{p} -4p^2)(\not{k}\,g^\beta{}_\mu - \gamma_\mu k^\beta)\,\gamma_\rho/(6M_\Delta^2)$$

$$+(-15M_\Delta^2 -4M_\Delta \not{p} +6p^2)(\not{k}\,g_{\rho\mu} - \gamma_\mu k_\rho)\,\gamma^\beta/(18M_\Delta^2)$$

$$+2(-M_\Delta- \not{p})(p_\mu k_\rho-p\bullet k\,g_{\rho\mu})\,\gamma^\beta/(9M_\Delta^2)$$

$$+(-2M_\Delta- \not{p})(\not{k}\,g^\beta{}_\mu-\gamma_\mu k^\beta)p_\rho\,/(3M_\Delta^2)$$

$$+(-5M_\Delta^2-2M_\Delta \not{p} +3p^2)(k_\rho g^\beta{}_\mu - g_{\rho\mu}k^\beta)/(3M_\Delta^2)$$

$$- \not{p}(\not{k}\,g_{\rho\mu}-k_\rho\gamma_\mu)\,p^\beta/(3M_\Delta^2) + \gamma_\mu \not{k}\,\gamma_\rho p^\beta/(3M_\Delta)$$

$$+[-4M_\Delta\,\gamma_\mu \not{k} - 2(p_\mu \not{k} - \gamma_\mu\,p\bullet k)]p_\rho\gamma^\beta/(9M_\Delta^2)\ -2\gamma_\mu \not{k}\,p_\rho p^\beta/(3M_\Delta^2)$$

$$+\lambda_\Delta/(2M_\Delta)\{[-(\not{p}\,\gamma_\mu \not{k} + \gamma_\mu \not{k}\,\not{p})/3+2(p^2-M_\Delta^2)(\not{p} +M_\Delta)\gamma_\mu \not{k}\,/(3M_\Delta^2)]\gamma_\rho\gamma^\beta$$

$$+(2p^2-M_\Delta \not{p} -4M_\Delta^2)\gamma_\mu \not{k}\,\gamma_\rho p^\beta/(3M_\Delta^2)- (2p^2-M_\Delta \not{p} -4M_\Delta^2)\gamma_\mu \not{k}\,p_\rho\gamma^\beta/(3M_\Delta^2)$$

$$+(2p^2-3M_\Delta^2)(\not{p} +M_\Delta)[4(k_\rho g^\beta{}_\mu-g_{\rho\mu}k^\beta)+2(\not{k}\,g_{\rho\mu}-\gamma_\mu k_\rho)\gamma^\beta+2(\gamma_\mu k^\beta-\not{k}\,g^\beta{}_\mu)\gamma_\rho]/(3M_\Delta^2)$$

$$+(2\not{p}-3M_\Delta)(\not{p}+M_\Delta)[2(\not{k}\,g_{\rho\mu}-\gamma_\mu k_\rho)p^\beta+2(\gamma_\mu k^\beta- \not{k}\,g^\beta{}_\mu)p_\rho]/(3M_\Delta^2)$$

$$-2(\not{p} + M_\Delta)\,\gamma_\mu \not{k}\,p_\rho p^\beta/(3M_\Delta^2)\}$$

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