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**ON THE FORMULATION AND EVALUATION OF INVESTMENT TAX
INCENTIVES**

City University of New York

Ph.D. 1986

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**ON THE FORMULATION AND EVALUATION OF
INVESTMENT TAX INCENTIVES**

by

LEO C. KAMP

**A dissertation submitted to the Graduate Faculty
in Economics in partial fulfillment of the
requirements for the degree of Doctor of Philosophy,
The City University of New York.**

1986

This manuscript has been read and accepted for the Graduate Faculty in Economics in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

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CHAPTER I

INTRODUCTION

The recent debate over "supply side" economics has focused sharp attention upon the role investment tax incentives play as instruments of fiscal policy. Although such instruments have played a crucial role in the Reagan Administration tax package, they have existed, in one form or another, as part of the tax code for some time. One purpose of this study will be to examine closely certain forms of tax incentives that have existed in the past from a perspective that differs from that of other analysts. The prime purpose of the study will be to develop a way of examining incentive programs (or any change in tax programs) from a different perspective.

The core of this first chapter will concentrate on a review of how incentive programs developed over time, emphasizing legislative intent where appropriate. Once that review is complete the discussion will turn to a brief review and critique of the way certain incentive programs have been evaluated. Finally, an outline will be presented as to how the rest of the discussion in this report will proceed.

A. A Historical Review of the Development of Tax Incentive Programs

1. Depreciation Rules

Since the beginning of the tax code, allowances for wear, tear, and exhaustion have been permitted as legitimate deductions for tax purposes. However, in the earlier years a rule of reasonableness was applicable, resulting in fairly strict control of the claimed estimated useful age of an asset and the wide spread use of the "straight line" method of depreciation (i.e. the deduction of equal proportionate amounts each year for the useful life of the asset). Useful lives were for the most part prescribed by the Internal Revenue Service in their Bulletin F.

The situation however changed in 1954 with the introduction of alternative forms of tax depreciation schemes that have come to be known as "accelerated" forms. These schemes fell into two classes -- declining balance formats and the sum of the years digits method. The former method is based upon a concept of a balance that declines at a certain rate over the estimated useful life of an investment. The formula results in larger depreciation allowances in the early years of asset life than in later years. The rate of decline is stipulated by the IRS for the particular asset being depreciated. For instance, if an asset costs \$1,000 and has a useful life of 10 years,

and the double declining balance system is allowed for depreciation (resulting in 20% of the declining balance being deducted as depreciation each year), the taxpayer holding this asset to produce business income would be able to deduct \$200 for depreciation in the first year (20% of \$1,000) and \$160 in the second year i.e. $20\% \times (\$1,000 - 20\% \times \$1,000)$ and so on. Note that the balance continues to decline (at 20% in our case) each year but never reaches zero by the end of the useful life of the asset. This problem was solved by allowing a switch to straight line at any point during the useful life of the asset to allow the balance to be brought down to zero.

The sum-of-the-years digits (SYD) method works somewhat differently. Again, assume a 10 year useful life and a \$1,000 cost for a capital investment. Then under the SYD system the investor would be entitled to deduct $\frac{10}{55}$ of the \$1,000 cost as depreciation in the first year and $\frac{9}{55}$ of the cost in the second year and so on. Note that the denominator, 55, represents the sum of the year digits (1 through 10) and the numerator represents the year's digit in reverse order (e.g. first year has a numerator of ten). This device allows for relatively high deductions in the earlier years and smaller ones as an asset matures.

Both accelerated systems result in higher after tax internal rates of return for capital investments than are

available under the straight line system. This occurs because the accelerated systems shift the depreciation allowances closer to the beginning of the life of the asset, thereby decreasing tax liability in the earlier less heavily discounted years of the asset's life. The straight line system, still available as an alternative to accelerated schemes, does not do this and hence generates a lower after-tax rate of return.

This higher rate of return potentially induces more investment than would otherwise occur. Such "accelerated" schemes were introduced into the tax code in the hope that capital investment would increase inducing more favorable growth, reduced unemployment and inflation.

The above depreciation rules, the declining balance schemes, the SYD method, and the straight line method, were essentially the only tax depreciation methods that were permitted by the tax code from 1954 to 1981. During 1981, however, the Congress passed the Economic Recovery Tax Act of 1981 (ERTA) which provided for a similar but more generous approach to capital consumption allowances.

Note that each one of the above schemes (including the straight line method) makes no attempt at assessing the way capital truly depreciates but rather is a pro forma method stipulated by the IRS for tax purposes irrespective of the true economic status of the capital item involved.

This situation raises substantial issues of efficiency and equity of such tax provisions.¹

2. Useful Life Times

From the above discussion it is apparent that each depreciation scheme uses useful life, as stipulated by the IRS, in calculating capital consumption (depreciation) allowances. Useful life figures were originally published by the IRS in its Bulletin F. The figures remained basically unchanged until 1962 when they were substantially shortened. The shortening of such useful life figures increase the after tax internal rate of return and thus act as a tax incentive. These shorter lives were introduced in order to stimulate growth and mitigate inflation and unemployment. The passage of the Accelerated Cost Recovery System (ACRS) in ERTA substantially reduces the useful life of most capital items as well as simplifying the complexity of the categories of assets involved. Such changes are likely to substantially affect investment activity as well as generate substantial tax revenue losses to the government.

3. Investment Tax Credits

The above measures, "accelerated" depreciation schemes and variations in stipulated tax lives are two sets of tax instruments chosen by the government to influence capital investment with the hope of affecting the rate of

growth in the U.S. economy. In January, 1962, an additional measure was added, an investment tax credit on capital equipment, which supplemented the other two measures. This scheme allowed for a credit against tax liability based on a percentage (stipulated by the tax code) of new investment expenditures. For instance, if in a given year a firm spent \$1,000 on a new capital item and the investment tax credit rate were 10%, that firm would be allowed to claim \$100 (i.e. 10% of \$1,000) as an offset against its tax liability. Note that the investment tax credit acts as a direct offset against taxes while depreciation schemes and stipulated tax lives operate to reduce taxable income. The investment tax credit was felt to have a more immediate effect upon the propensity to invest than accelerated schemes and tax lives and hence was viewed as better suited as a counter-cyclical tool than the other two methods. Thus, since 1962, the rate of credit has been raised, lowered, or eliminated at certain times as a stabilization measure.

B. Overview of Previous Analyses of Investment Tax Incentive Programs

The passage and implementation of the above incentive programs provided a prime opportunity for economists to examine the impact of such programs upon the economy. In the

following we will discuss briefly the variety of perspectives that have been presented in analyzing these programs, although a more detailed discussion of each kind of analysis will be presented as part of Chapter III.

One class of models examines the impact of depreciation schemes upon discounted income flows and hence capital costs. Accelerated depreciation schemes increase income to firms through reduced tax liability and hence increases the internal rate of return on capital assets. Such increases in the internal rate tend to stimulate the quantity of capital goods demanded. This approach was essentially that presented by E. Cary Brown and Richard Musgrave.² Their analyses were, however, theoretical in nature without any econometric evaluation of size of impact. Their studies indicated that there would be some impact without any estimate of size of impact or what classes of assets would be most affected by the programs.

In an attempt to get some idea of the magnitude of impact, Robert Hall and Dale Jorgenson developed an econometric model through which estimated investment by asset classes due to tax incentives could be ascertained. Their analysis concluded that "accelerated" depreciation schemes greatly stimulated investment in structures while the investment tax credit had a substantial impact upon investment in equipment.³

Other analysts have pursued the analysis of investment tax incentive programs using variants of the Hall-Jorgenson model approach to the problem based upon Jorgenson's neoclassical theory of investment behavior. According to their analysis, accelerated depreciation schemes and investment tax credit affect investment activity through the rental price of capital, for which Hall and Jorgenson developed a formula based on a series of simplifying assumptions concerning profit maximizing firms. Rental prices so developed for different classes of assets were then utilized to estimate an expression for the desired level of each capital asset class. Robert Coen utilized a similar neoclassical approach but incorporated the impact of increased cash flow generated by investment tax incentive programs upon the speed of adjustment of the capital stock to the desired level of capital.⁴ Coen concluded on the basis of his analysis that while the incentive programs did induce some increased investment they were not particularly cost effective since for every dollar of capital expenditure generated by the programs, between two and four dollars of tax revenue were lost. Charles Bischoff also utilized the neoclassical approach but incorporated alternative assumptions concerning the applicable production function and lag distribution.⁵ He concluded that the investment tax credit probably generated more in capital investment than was lost in tax revenues,

while the reverse was probably true for accelerated depreciation. Thus there appears to be a variance of opinion among the above authors concerning the efficiency of the tax incentive programs.

In each of these models, however, the implicit standard utilized to measure the effectiveness of the incentive programs was whether a program stimulated investment to a greater extent than no program. Although some authors take considerable effort to estimate tax revenue lost from incentive programs, no explicit account is taken of the broader impact of programs upon budgetary goals. One broad goal of this study is to refine the evaluation process from the perspective of whether the programs actually passed by the Congress were in some sense optimal.

C. Plan for the Remainder of the Report

As indicated in the previous section, existing models that examine investment tax incentive programs lack any substantive means of evaluating the programs. This study proposes an examination of the programs from the perspective of budgetary goals and optimality. The development of this new perspective will be the main contribution of this report. Chapter II will concentrate on the development of the methodology and will contain a discussion of potential problems underlying the approach. Such problems include a

number of considerations arising from the economic literature such as the time inconsistency of optimal plans and the effectiveness of policy under rational expectations. These problems will be examined to determine their potential impact upon the analysis.

The remainder of the report will be devoted to illustrations of how the approach of Chapter II can be applied. Chapter III, for instance, presents an in-depth review of economic analyses that have been performed in the past and contains a detailed critique of how the analyses presented fail to contain an appropriate measure for evaluation. Chapter IV demonstrates, utilizing a simple model, how the process of Chapter II could be implemented in practice as an evaluative tool. Problems such as the choice of appropriate specifications for an econometric model and an objective function are discussed. An illustration is presented in Chapter IV, namely the retrospective evaluation of a past program. Finally, a summary of conclusions arrived at in the report is presented in Chapter V.

NOTES

CHAPTER I

¹Paul A. Samuelson, "Tax Deductibility of Economic Depreciation to Insure Invariant Valuations," Journal of Political Economy 72 (December 1964) : 604-606.

²E. Cary Brown, "Business Income Taxation and Investment Incentives," in Income, Employment and Public Policy; Essays in Honor of Alvin H. Hansen, ed. Lloyd A. Meltzer (New York: W.W. Norton and Co., 1948), pp. 300-316; Richard A. Musgrave, The Theory of Public Finance (New York: McGraw-Hill Book Co., 1959), pp. 338-346; Leo C. Kamp, "The Impact of Accelerated Depreciation Programs Upon the Investment Decision," M.A. thesis, Hunter College, 1974.

³Robert E. Hall and Dale W. Jorgenson, "Application of the Theory of Optimal Capital Accumulation," in Tax Incentives and Capital Spending, ed. Gary Fromm (Washington: The Brookings Institution, 1971), pp. 9-20.

⁴Robert M. Coen, "The Effect of Cash Flow on the Speed of Adjustment," in Tax Incentives and Capital Spending, ed. Gary Fromm (Washington: The Brookings Institution, 1971), pp. 131-196.

⁵Charles W. Bischoff, "The Effect of Alternative Lag Distributions," in Tax Incentives and Capital Spending, ed. Gary Fromm (Washington: The Brookings Institution, 1971), pp. 61-130.

CHAPTER II

AN APPROACH TO EVALUATING BUDGETARY PROPOSALS

The primary focus of this study is on a means of improving the evaluation of changes in investment tax incentive programs. In order for effective evaluation to proceed, one must first establish a perspective for evaluation. That is the purpose of this chapter. This section will present an approach to evaluation that is inspired by the theory of tax reform and stochastic optimal control theory. The discussion will proceed first from a delineation of the goals of budgetary policy, then present the perspective of optimal tax reform theory, and finally discuss how such a perspective can be implemented into an evaluative tool by means of stochastic optimal control theory.

A. The Goals of Budgetary Policy

The tax system is an integral part of the budget system. Therefore any modification of that system, such as the modification of investment tax incentives, should be evaluated in light of the goals set for the budgetary system. As Musgrave points out, the budget has three major

objectives -- an allocation function, a distribution function, and a stabilization function.¹ The allocation function involves "securing necessary adjustments in the allocation of resources by the market", and the providing for different kinds of social and "merit" wants.² Certain market situations (e.g. situations where externalities exist) provide opportunities for potentially useful government intervention into the market place. Government in such situations can potentially provide (e.g. through tax policy) a more efficient allocation of resources. The allocation function also involves the transfer of resources from the private sector to provide for public goods and services. The determination of the appropriate mixture of public and private goods and services is an essential aspect of the allocation function of the budget. The second major objective of the budget is to "secure adjustments in the distribution of income and wealth."³ The objective is an attempt to modify the existing pattern of income and wealth through taxes, transfers, and expenditures. The objective of establishing a fair distribution is essentially a matter of social philosophy that is reflected in law through the political process. The determination of a "fair" distribution may involve using some concept of equality which may in turn require interpersonal utility comparisons. Since economists generally would rule out such comparisons, the process of determining a "just"

or "fair" distribution of income is essentially a political process.

The final major objective or function of the budget is stabilization. The purpose of stabilization is to maintain "a high level of resource utilization and a stable value of money".⁴ The basic notion of stabilization is to keep certain macroeconomic aggregates from varying excessively, and to provide for high resource utilization and real growth. The ideas of compensatory finance have played a role in implementing this objective. Tax and expenditure policies have been altered over time as an attempt to satisfy this objective. For example, during the Great Depression public works programs were started by the government as a means of increasing resource utilization. This stabilization objective has assumed a major, if not dominant, role in the formulation of budget policy.

Given the above objectives of the budget, one should evaluate tax policies and changes in such policies in light of their impact upon those objectives. Any systematic policy evaluation scheme should account for the impact policy changes have upon budgetary objectives. The following discussion will propose such a scheme for the evaluation of investment tax incentives. The discussion of that scheme will be preceded however by a general discussion of the theory of tax reform.

B. Optimal Tax Reform Considerations

The modification of investment tax incentives involves changes to an existing tax system. In evaluating a change in a tax system, it is important to consider how such changes should be made. Such considerations potentially pinpoint certain aspects that should be incorporated into an evaluative approach.

Although there is no clearcut way of undertaking tax changes optimally, there are certain theoretical as well as practical considerations that should be kept in mind before making such changes. A seminal work in the area of optimal tax reform is Martin Feldstein's paper on the theory of tax reform.⁵ In that paper Feldstein broadly explores various aspects of the tax reform issue and suggests areas that should be investigated in evaluating proposed changes. A brief discussion of his approach to the problem follows.

Feldstein starts by emphasizing the distinction between tax reform and tax design. Tax design concerns itself with the development of de novo tax systems while tax reform concentrates on the modification of an existing tax system. As such tax reform takes as its starting point the existing tax system, which acts to some extent as a constraint on what can be done. Tax reform by its very nature is a piecemeal dynamic process that results in a tax system that differs from one freshly designed from scratch.

According to Feldstein several major factors should be taken into account before undertaking a tax reform -- utilitarian considerations, horizontal equity, and legitimate property rights. Firstly, a utilitarian approach to tax change requires a model to assess the impact of a change. But the dynamic nature of the tax reform process potentially introduces uncertainties about the tax system which in turn can induce a learning process that can alter the parameters of the model being utilized. Also tax changes generate precautionary behavior which can generate inefficiencies as well as revenue losses. The second factor, horizontal equity, is defined by Feldstein as a desire not to disturb the utility ordering of individuals when undertaking tax changes. Thus, for horizontal equity to be maintained, two individuals having the same utility levels prior to a tax change should have the same utility levels after the change. And finally, as part of our legal tradition, Feldstein feels that legitimate property rights should be considered in potentially modifying tax systems.

According to Feldstein, an optimal tax reform should take all these factors into account. Such an effort involves balancing utilitarian considerations with the concept of horizontal equity while at the same time not violating legitimate property rights. In the absence of compensation, such a balancing act is extremely difficult,

but to some extent is facilitated if tax changes are enacted with delayed effective dates. That strategy would tend to mitigate horizontal inequities since prices would adjust over time to reflect the enacted changes. Also postponement allows for no informational surprises thereby allowing individuals to adjust their commitments to new information.

While postponement can mitigate horizontal inequity, such delay can result in a utilitarian loss. Feldstein suggests by way of an example that optimal postponement can be derived by choosing the postponement that maximizes the welfare gain of the tax change. The degree of postponement derived in that fashion takes into account both horizontal equity, and efficiency gains. Since it is an interesting example of how postponement potentially could be utilized to achieve optimal tax reform, the Feldstein approach is described below. However, the derivation presented is this author's to correct for an obvious error in the original Feldstein article.⁶ Consider N individuals having identical utility functions, u , and identical incomes, y . With a tax change, assume n individuals each lose x and $(N-n)$ individuals each gain z . Then for a postponement factor λ , $0 < \lambda < 1$, the welfare gain, G , from the tax change can be expressed as,

$$G = (N-n) [u(y + \lambda z)] + n [u(y - \lambda x) - u(y)].$$

(Note that with no postponement $\lambda = 1$ while with infinite postponement $\lambda = 0$.) For small changes in the tax system, an approximation to the utility function can be used. A second order Taylor series approximation of the utility function implies,

$$G = (N-n)[\lambda zu' + \frac{1}{2} \lambda^2 z^2 u''] - n[\lambda xu' - \frac{1}{2} \lambda^2 x^2 u''] .$$

Defining $R = \frac{-yu''}{u'}$ and deriving the first order condition, one is able to solve for the optimal postponement λ^* :

$$\lambda^* = \frac{1}{R} \frac{z - [n/(N-n)] x}{[(n/(N-n)) \frac{x^2}{y} + \frac{z^2}{y}]} = \frac{z - [n/(N-n)] x}{D}$$

$$\text{where: } D = [(n/(N-n)) \frac{x^2}{y} + \frac{z^2}{y}] .$$

This implies that $\lambda^* = 0$ when $(N-n)z = 0$, or infinite postponement is indicated when aggregate gains just equal aggregate losses. If $u' > 0$ and $u'' < 0$ then $R > 0$ and $D > 0$. If this is true then $\lambda^* > 0$ is indicated when $(N-n)z > nx$, or limited postponement is indicated only when aggregate gains exceed aggregate losses. No postponement is indicated only when $\lambda^* = 1$. One problem with the particular approach by Feldstein is the utility function. In practice it is unlikely that policy makers would know explicitly the true utility of any individual. Public goods by their very nature (i.e. the inapplicability of the exclusion principle) induce persons to not reveal their

true preferences. That is one reason why taxation is necessary to finance public goods. If persons were to reveal preferences then taxes could be based on the benefits received. But since persons are able to consume public goods without paying taxes (i.e. become a free rider) there is an incentive not to accurately reveal their true preferences. In the context of Feldstein's example it is unlikely that policy makers would be able to ascertain the utility functions of losers and beneficiaries of a tax change. Therefore, although utility considerations are important in assessing tax change, the utility function will in practice be that of policy makers who perhaps have some knowledge of the preferences of the public through the political system.

Although the above results are not particularly startling, the approach is useful in examining tax incentive plans as one example of tax reform programs. Essentially what Feldstein is saying is that in order for tax change to be optimal both utilitarian and horizontal equity considerations must be taken into account. He suggests that these objectives can be at least partially fulfilled through the optimal use of postponement of enacted changes. In a dynamic context this suggests that the time path of effective changes to the tax system be determined in such a way that welfare gains are maximized

or welfare losses are minimized. In a stochastic environment this approach suggests that the optimal changes to the tax system over time could be chosen to minimize welfare losses over time. One dynamic stochastic approach to policy formulation and evaluation is stochastic optimal control theory. This approach can incorporate both budgetary goals and timing considerations in the evaluation process. In the next section this approach will be presented along with a suggested way of evaluating policies. A later section of this study will present an example of how that approach can be implemented in practice in evaluating tax incentive programs.

C. The Use of Stochastic Optimal Control Theory as an Evaluative Tool

The previous two sections of this chapter concentrated on certain elements that are appropriate in assessing budgetary programs without any specific delineation of how those elements could enter the total evaluation process. This section will describe the evaluation process that will be utilized in the rest of this report. That description will specifically include references on how both budgetary goals and aspects of optimal tax reform can be incorporated into evaluation. Optimal stochastic control theory is a technique that can be utilized to include both budgetary goals and notions of

optimal tax reforms in an evaluation process. In the following discussion optimal stochastic control theory will first be described with specific mention of how both budgetary goals and optimal tax reform considerations can be accounted for as part of a control problem. Once that has been accomplished the discussion will concentrate on how the solution of a control problem can be utilized to evaluate budgetary programs.

An optimal control problem frequently encountered in the economic literature is the linear-quadratic stochastic dynamic programming problem. Gregory Chow has done extensive work on this problem and also on an approximate solution to the nonlinear-quadratic control problem.⁷ In the linear-quadratic problem, a quadratic loss function is minimized subject to the constraint of a linear stochastic econometric model. Formally the linear-quadratic problem described by Chow is to minimize over T periods expected welfare loss,

$$E_0 W = E_0 \sum_{t=1}^T (y_t - a_t)' K_t (y_t - a_t) \quad (2.1)$$

given a linear econometric model,

$$\begin{aligned} y_t = & A_{1t} y_{t-1} + \dots + A_{mt} y_{t-m} + C_{0t} x_t + \dots \\ & + C_{nt} x_{t-n} + b_t + u_t . \end{aligned} \quad (2.2)$$

In 2.2 y_t is a vector of endogenous variables, x_t is a vector of variables subject to control (control instruments), A_{mt} and C_{nt} are given matrices, b_t is a vector of exogenous variables, and u_t is a vector of error terms with $E(u_t) = 0$ and $E(u_t u_t') =$ a constant covariance matrix V .

Equation 2.2 can be arranged into the following first order form:

$$\begin{bmatrix} y_t \\ y_{t-1} \\ \vdots \\ y_{t-m+1} \\ x_t \\ x_{t-1} \\ \vdots \\ x_{t-n+1} \end{bmatrix} = \begin{bmatrix} A_{1t} & A_{2t} & \cdots & A_{mt} & C_{1t} & \cdots & C_{nt} \\ 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ y_{t-2} \\ \vdots \\ y_{t-m} \\ x_{t-1} \\ x_{t-2} \\ \vdots \\ x_{t-n} \end{bmatrix} + \begin{bmatrix} C_{0t} \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} x_t + \begin{bmatrix} b_t \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} u_t \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (2.3)$$

Equation 2.3 can be redesignated as,

$$y_t = A_t y_{t-1} + C_t x_t + b_t + u_t. \quad (2.4)$$

Note that y_t now contains lagged endogenous variables, and current and lagged control instruments. The purpose of expressing 2.2 in the form 2.4 is to simplify the analytical derivation. The vector y_{t-1} now includes lagged y_t and lagged control instruments.⁸ Equation 2.1 is utilized as a means of incorporating both policy targets at of current endogenous variables (e.g. certain goals of budgetary policy) and the preferences of the measure of welfare loss. K_t are known symmetric positive semidefinite matrices that are used to express the relative weight that a decision-maker places on a deviation of a current endogenous variable y_t from its target a_t . The weighting matrices, K_t , are potentially time varying to allow for the expression of changes in weights by the decision maker over time. Since y_t contains both lagged endogenous and lagged control instruments and usually only a small number of endogenous variables are targeted, the K_t matrix contains many zero elements. Only targeted y_t will be multiplied by nonzero elements in the diagonal of the matrix K_t . E_0 is the expectation operator and expresses that welfare loss is an expected value based upon information available at initial time zero.

Note that the functional form of the expected welfare loss expression is quadratic. This particular functional

form has the disadvantage of weighting equally both positive and negative derivations of variables from their targets. However, according to certain researchers, this particular form has the following points in its favor:

- a. It may be regarded as a second order Taylor series approximation to the "true" objective function of the policy maker;
- b. It gives a simple measure of the weighted deviations of the actual values from their desired trajectories;
- c. The solution of the optimal control problem with a quadratic objective function is well defined and extensively researched.⁹

As stated above, the problem is to find instruments x_t^* that minimize expected welfare loss, 2.1 subject to an econometric model in the form 2.4. This approach, provides a systematic approach for choosing a policy that will approximately satisfy the preferences of decision makers in achieving policy goals over time. This approach facilitates the decision making process in that it provides a decision maker with policies (out of a potentially infinite set of alternatives) that satisfy the objectives of a decision maker. Note the use of "approximate" in the discussion. One must remember that the solution is approximate since both 2.1 and 2.4 are only approximations to the "true" state of affairs.

However, the solution to the above control problem can provide valuable information to decision makers about policies that systematically account for both the preferences and the goals of the decision maker. In terms of the above discussion of budgetary goals and concerns arising from optimal tax reform theory, the optimal control approach is one means of approximating those objectives. The solution to the control problem provides an optimal instrument vector x_t^* over time that accounts for the preferences of decision makers and budgetary objectives through equation 2.1 which at the same time accounting for the estimated behavior of the economy through model 2.4. If the control instrument is an investment tax incentive (e.g. the effective investment tax credit rate on equipment) then the solution to an appropriate control problem provides a potential standard for assessing an actual investment tax incentive. Such an optimal solution would approximately account for factors that theoretically the program should have, and hence is appropriate as an evaluative standard.

The actual solution of the linear-quadratic control problem can be accomplished in a number of ways. One way is to use the technique of Lagrange multipliers.¹⁰ However, a more popular approach among control theorists is the technique of dynamic programming. Chow derives results

using this approach, which is the one which will be presented here.¹¹

The dynamic programming technique is based upon Bellman's Principle of Optimality which states, "An optimal policy has the property that, whatever the initial state and decision (i.e. control) are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision".¹² This solution to the problem is solved backwards optimally from time T to the first period. In that way at time t the solution is optimal since whatever the initial condition all future solutions are optimal. Therefore, the problem for period T is solved first for optimal instrument vector x_T given the initial condition y_{T-1} . With that accomplished the process is repeated for period T-1, so that an optimal x_{T-1} vector is chosen given initial condition y_{T-2} and that x_T is optimal. This one period optimization process is repeated backwards to the first period where finally x_1 is solved given the initial state y_0 . Note that this sequence of solutions satisfies the Principle of Optimality.

Based on dynamic programming Chow derives the following general solution for the linear-quadratic stochastic control program:

$$x_t^* = G_t y_{t-1} + g_t, \text{ where,}$$

$$G_t = -(C_t' H_t C_t)^{-1} C_t' H_t A_t;$$

$$\begin{aligned}
g_t &= -(C_t' H_t C_t)^{-1} C_t' (H_t b_t - h_t), \quad (t = 1, \dots, T); \\
H_{t-1} &= K_{t-1} + (A_t + C_t G_t)' H_t (A_t + C_t G_t), \quad (t = 2, \dots, T); \\
h_{t-1} &= K_{t-1} a_{t-1} + (A_t + C_t G_t)' (h_t - H_t b_t), \quad (t = 2, \dots, T); \\
H_T &= K_T; \\
h_T &= K_T a_T. \quad 13
\end{aligned}$$

This solution is the same whether dynamic programming or the Lagrangian approach is used.

Note that the optimal vector x_t^* is derived according to a rule (i.e. $x_t^* = G_t y_{t-1} + g_t$) that incorporates current information g_t . Thus x_t^* varies over time as current information changes. Such a rule is known as a "closed loop" control rule since x_t^* incorporates current information (feedback) in its solution. Such contingent rules have been shown to be superior to "open loop" rules that do not incorporate current information.¹⁴ Intuitively that makes sense since one is likely to do better when one is able to respond to unforeseen events than when one cannot.

For each period in the solution process there is a minimum expected welfare cost value $V_t(x_t^*)$ associated with optimal policy vector x_t^* . This value shows the welfare cost of following the optimal feedback rule x_t^* in the current period and in the future. Chow uses that value as a basis for evaluating actual policies $\hat{x}_t$. His approach is based upon computing the difference between $V_t(x_t^*)$ and $V_t(\hat{x}_t)$, the expected welfare cost associated with actual

policy $\hat{x}_t$. If for instance a nonoptimal policy $\hat{x}_t$ is followed during the first period the difference $V_1(\hat{x}_1) - V_1(x_1^*)$ would represent the extra welfare cost of following $\hat{x}_t$ rather than x_1^* . If after the first period the optimal rule x_t^* were adopted then $V_1(\hat{x}_1) - V_1(x_1^*)$ would represent the additional welfare cost incurred by using a nonoptimal policy. If however, in the second period a nonoptimal policy $\hat{x}_t$ is adopted, additional welfare cost is incurred, namely, $V_2(\hat{x}_2) - V_2(x_2^*)$. Thus after two periods the additional welfare cost would amount to,

$$[V_1(\hat{x}_1) - V_1(x_1^*)] + [V_2(\hat{x}_2) - V_2(x_2^*)]$$

or, $\sum_{i=1}^2 V_i(\hat{x}_i) - V_i(x_i^*)$. This can be generalized so that after $N < T$ periods the additional welfare cost associated with a nonoptimal policy would be $\sum_{i=1}^N V_i(\hat{x}_i) - V_i(x_i^*)$. That sum represents a means of evaluating how close actual policies come to an optimal policy vector derived from a specified linear-quadratic control problem based upon preferences and the stochastic nature of the economy. From the perspective of optimal tax reform theory optimal stochastic control theory therefore represents a means of incorporating both utilitarian and timing considerations into the selection of policy. Policy selected in that manner could be used potentially as part of an evaluative program outlined above. This approach enables evaluators a

means of assessing whether policy makers could have done better. An example is presented in Chapter IV of how this method can be utilized in evaluating investment tax incentives.¹⁵

D. Some Difficulties with the Above Evaluative Technique

In the prior sections of this chapter the discussion centered around the development of an approach to the evaluation of investment tax incentive programs. That approach used an econometric model to examine an actually promulgated policy compared to a hypothetical one that was generated using stochastic optimal control theory. While that approach may appear fairly reasonable on face, there are certain aspects of that approach that require further explanation. These aspects arise out of the so-called "rational expectations" revolution in econometrics. Proponents of this approach have formulated critiques of the way econometric studies have been conducted in the past.

Therefore, it is imperative that an econometric study at least examine the potential impact of rational expectation critiques upon the results of the study. In the following sections, major critiques of econometric policy evaluation techniques will be examined. The discussion will start with an overview of the rational expectations

approach and then turn to two major areas where this approach has been applied -- assertions about the ineffectiveness of fully anticipated policies and about the time inconsistency of optimal plans. However, the discussion will not only focus on the assertions made by rational expectations proponents but also upon criticisms, both theoretical and pragmatic, of those assertions.

1. The Rational Expectations Approach¹⁶

The fundamental problem confronting econometricians is to estimate from historical data the behavior of economic agents. Agents or collections of agents are observed through time and from these observations econometricians wish to infer, or model, their behavior. In forecasting with a changing environment, the traditional approach has been to assume that the behavior estimated from historical data will not be significantly altered with changes in policy regimes (henceforth referred to as the "invariance" assumption). Rational expectations theorists posit (based on the theory to be presented below) that such an assumption cannot be made if expectations are formed rationally.

a. The Rational Expectations Hypothesis

Consider a single decision maker whose position at any time, t , is described by two variables x_t and z_t . The

"z" variable can be considered as determined by "nature" and expressed over time in the following form,

$$z_{t+1} = f(z_t, \epsilon_t)$$

Therefore z varies over time according to the previous value of z and an "innovation" term, ϵ_t , which is assumed to come from a distribution, ϕ , with zero mean and unit variance. One could conceive of z_t as summarizing the entire history of z up through time t .

The function, f , describes the environment in which a decision maker must operate. The state of the decision maker at t is described by a variable " x ". That state is under the partial control of the decision making agent and assumes the following functional specification:

$$x_{t+1} = g(z_t, x_t, u_t)$$

where: x_t = agent's state at t

u_t = action selected by the agent at
time " t " = $h(z_t, x_t)$.

The above specification posits a situation where a decision making agent's state is a function of that agent's prior state, x_t , the environment (through z_t) and the decision maker's action (u_t) which is also a function of the environment and the prior state of the decision maker. (Note that the function " h " specifies the action that will be taken by the agent at " t " given z_t and x_t .)

The above relations present a way of examining the rational expectations approach.

Typically, estimates of the above functional forms (f , g , and h) have been estimated statistically after observing the movement of z_t , x_t , and u_t through time. However, in estimating such relations there has been an implicit assumption that the functional relations are stable over time. Rational expectation theorists have suggested that there are good reasons why at least some of the functional relations may vary over time. The basic hypothesis underlying their arguments is that economic agents will effectively use information about a changing environment to change the way actions are selected. In terms of the relations established above this translates into $h = T(f)$, or where T relates the way h changes as the environment f changes. Specifically the rational expectations hypothesis asserts that economic behavior will be a function of expectations formulated upon the latest available information, which includes knowledge of the way variables interact. Such behavior is seen as being part of general optimizing behavior on the part of the decision maker.¹⁷ For example, a change from a noninterventionist policy by the federal government to an extremely activist policy would result, according to the rational expectations hypothesis, in a change in the way people select their actions. An example is provided below of how changes in

government policy can affect the environment of private economic agents.¹⁸ Suppose the economy (the private agent's environment) can be represented by the following form with the time invariant coefficients:

$$y_t = A y_{t-1} + C_1 x_{1t} + C_2 x_{2t} + b_t + u_t$$

where: y_t = vector of endogenous variables;

y_{t-1} = vector lagged endogenous variables;

x_{1t} = vector of instruments controlled by private economic agents;

x_{2t} = vector of instruments controlled by the government;

b_t = vector expressing effect of exogenous variables;

u_t = vector of "white" noise terms.

If the government establishes the feedback rule

$x_{2t} = G_2 y_{t-1} + g_2$ to control x_{2t} then,

$$\begin{aligned} y_t &= A y_{t-1} + C_1 x_{1t} + C_2 (G_2 y_{t-1} + g_2) + b_t + u_t \\ &= (A + C_2 G_2) y_{t-1} + C_1 x_{1t} + (b_t + C_2 g_2) + u_t. \end{aligned}$$

Therefore the application of the government rule has affected the environment faced by private agent's. Rational expectations theorists state that the change in environment will be used by optimizing private economic agents in formulating their expectations.

**b. Impact of Rational Expectations Arguments
Upon Econometric Practice**

Based on the rational expectations critique, econometricians are potentially faced with a series of problems. If models are unstable with changes in policy regimes, economic forecasting and policy analysis may become difficult or even impossible. A question arises of how the rational expectations critique shall be implemented as part of econometric practice, if at all.

Implementation of the rational expectations hypothesis is difficult since economics is essentially a nonexperimental science. If it were experimental, it would be possible to trace out the impact of a changing environment upon the decision rules of economic agents by varying the environment holding everything else constant. Unfortunately such a situation does not exist for economists. Rather, what could be done is to make particular assumptions about the nature of the various relations including the objective function of the decision maker. Suppose that the decision maker wishes to choose a decision rule h to maximize the current return, R , where:

$$R = E_0 \sum_{t=0}^{\infty} \beta^t V(z_t, x_t, u_t), \quad 0 < \beta < 1.$$

The symbol E_0 represents the subjective expectation with respect to the distribution of $z_1, z_2, \dots$ conditional upon the latest information (x_0, z_0) . If such a problem is solvable one would be able to infer the decision rule h

from knowledge of V (the return function) given functional forms f and g . But how does one obtain knowledge of V ? Empirically the issue is whether one can first estimate V using observations on f , g , and h . If that could be done one could then obtain some knowledge of the way h varies with changes in f (i.e. $h = T(f)$) by varying f given g and estimated V .

Rational expectations theorists assert that if the above optimizing problem is to be helpful in estimating $T(f)$ then it is essential to equate (or link explicitly) the subjective z - distribution with the actual environment, f , as observed in the data and as altered by hypothetical policy changes. The rational expectations hypothesis essentially equates the subjective z - distribution to the objective f distribution.

The above problem has been rather intractable for most investigators. It has been necessary to restrict V , f , ϕ , and g in such a way that h will be linear. Such restrictions enable one to exploit a property termed "certainty equivalence".¹⁹ If the return function V is quadratic of a particular form and g is linear, the optimal decision $u_t = h(z_t, x_t)$ can be separated into a forecast component, $\tilde{z}_t$, and an optimization component. Let $z_t = (z_t, {}_{t+1}z_t^e, {}_{t+2}z_t^e, \dots)$, where ${}_{t+j}z_t^e$ represents the optimal point forecast of z_{t+j} based upon information at t . Generally, $\tilde{z}_t$ can be expressed as a function of z_t , so that

$\tilde{z}_t = h_2(z_t)$. But h_2 has been shown to depend only upon function f and ϕ . If f is restricted to be linear then $\tilde{z}_t$ are linear least squares forecasts and h_2 is a linear least square predictor. With V quadratic and g linear it is possible to express u_t as a function of z_t and x_t so that $u_t = h_1(z_t, x_t)$. It has been shown that the function h_1 , depends only on V and g . Replacing z_t by $h_2(z_t)$, $h(z_t, x_t) = h_1(h_2(z_t), x_t)$. The property of certainty equivalence enables one to separate h , the optimal decision rule, into two components -- a "forecasting" part h_2 and an optimization part h_1 .

Certainty equivalence also enables one to derive h_1 directly from functions V and g under the assumption that the decision maker knows the future z_t with certainty. If h_2 is a least squares predictor dependent on f , and h_1 can be derived from V and g under certain equivalence, it is possible to derive h from $h(z_t, x_t) = h_1(h_2(z_t), x_t)$.

The point of the above discussion is to emphasize that although much work has concentrated on deriving h_1 from hypotheses about V and g , little attention has been paid to the examination of the dependence of h and h_2 on f . For instance in many formulations of expectations, essentially ad hoc empirical specifications have been utilized without any reference to the dependence of such specifications upon the environment. A major implication of the rational expectations hypothesis is that such procedures

should not be maintained without theoretical justification. Even after utilizing the property of certainty equivalence it is still necessary for econometric investigators to examine whether the "invariance" assumption is valid when conducting studies.

Note however that the above discussion emphasized that one must assess the impact of a change in environment by isolating that component of change from other changes. That isolation process was based on a specification of a particular economic theory and a series of assumptions that permitted such separation. Or to summarize, the separation of the effect of a change in environment (such as a policy change) relied on a priori knowledge. In terms of a conventional econometric model, this process translates into the ability of the econometrician to impose certain a priori restrictions or knowledge so that policy parameters can be isolated from purely structural parameters. If this can be done, it may then be possible to adjust such estimated policy parameters to reflect changes in policy, in evaluation studies and forecasts.²⁰ It is an open question at this time whether such a separation process will ultimately produce superior forecasts and policy evaluations. However, it is abundantly clear that econometric forecasting and evaluation procedures, as conventionally formulated, require substantial change.

From the above it is apparent that the rational expectations hypothesis potentially could have an important impact upon the parameters of econometric models and could result in significant changes in the ways econometric studies are performed. As Thomas Sargent points out, the parameters of time series econometric models are subject to change, at least in part due to effects associated with the rational expectation hypothesis. He concludes that econometric practice must take account of the "laws of motion" that govern economic agents' constraints.²¹ Specifically he demonstrates how an examination of such "laws of motion" can be useful as cross equation restrictions in the identification of model parameters. While not denying the potential theoretical impact of policy changes under rational expectations upon parameter values, other investigators such as Otto Eckstein, doubt whether in practice changes in policy are an important source of parameter change. Eckstein notes, for instance, that a major policy change, the Nixon Administration's wage and price controls, had little impact upon the performance of Data Resources Incorporated's econometric model.²² Nevertheless, the rational expectations hypothesis accentuates the necessity of econometric practitioners to take care in specifying their models and monitoring the performance of model equations (and parameters) over time.

In the following two sections of this chapter, two implications of rational expectations will be discussed. The first relates to what is known as the "policy ineffectiveness" proposition or hypothesis; the second relates to the optimality of optimal plans using dynamic programming with rational expectations. Both topics have bearing upon the evaluative approach discussed earlier in this chapter.

2. The Policy Ineffectiveness Proposition

This proposition is an assertion, that if economic agents form their expectations rationally, systematic policy changes will have no effect upon stabilizing economic variables. Such an assertion, if valid, raises questions whether the formulation of systematic policy, the basis of the above evaluative approach, can affect economic variables. Hence a discussion of this proposition is presented below.

To analyze this proposition further, the following model will be used:²³

$$\text{Aggregate Supply: } y_t - \bar{y} = a(p_t - p_t^*) + u_t \quad (2.5)$$

$$\text{Aggregate Demand: } y_t = -bp_t + cx_t \quad (2.6)$$

$$\text{Rational Expectations: } p_t^* = E(p_t | I_{t-1}) \quad (2.7)$$

where: y_t = aggregate real income at time t ;

$\bar{y}$ = income corresponding to the natural rate of unemployment;

p_t = price level at time t ;

p_t^* = price expectations;

x_t = government policy instrument (e.g. money supply);

I_{t-1} = information available at time $t-1$;

u_t = random error term having $E(u_t) = 0$;

E = expectations operator;

a, b, c = positive parameters.

Solving for the equilibrium price level by equating (2.5) and (2.6) one obtains,

$$p_t = \frac{1}{a+b} (ap_t^* + cx_t - \bar{y} - u_t). \quad (2.8)$$

But $p_t^* = E(p_t | I_{t-1})$ so that after substituting (2.8)

$$\begin{aligned} p_t^* &= E \left[\frac{1}{a+b} (ap_t^* + c E(x_t) - \bar{y} - u_t) \right] \\ &= \frac{1}{a+b} (ap_t^* + c E(x_t) - \bar{y}) \end{aligned} \quad (2.9)$$

since $E(p_t^*) = p_t^*$, $E(\bar{y}) = \bar{y}$, and $E(u_t) = 0$.

Subtracting (2.9) from (2.8) one obtains,

$$p_t - p_t^* = \frac{1}{a+b} (c[x_t - E(x_t)] - u_t). \quad (2.10)$$

Substituting (2.10) into (2.5) one obtains the following :

$$y_t - \bar{y} = \frac{a}{a+b} (c[x_t - E(x_t)] - u_t) + u_t$$

$$\begin{aligned}
&= \frac{ac}{a+b} [x_t - E(x_t)] - \frac{a}{a+b} u_t + u_t \\
&= \frac{ac}{a+b} [x_t - E(x_t)] + \frac{b}{a+b} u_t.
\end{aligned}$$

Therefore for the above model, an equilibrium under rational expectations is a situation where (other than random error) deviations of aggregate income from the level of income associated with the "natural rate" of unemployment results only from unanticipated policy changes (i.e. $x_t - E(x_t)$). Thus with the above model (the "natural rate" level) specification, except for random error, y_t , equals $\bar{y}$ when policy is fully anticipated, and therefore deviates only from the "natural rate level" when there are expectational surprises. In the above model under rational expectations fully anticipated x_t will have no effect upon aggregate income y_t . This is a pretty strong position for it says that fully anticipated policy changes have no impact, and hence such changes cannot be utilized as a means of stabilizing economic variables.

However, that conclusion is actually the result of the particular model employed rather than the rational expectations hypothesis. It is possible to get a different result using a different model specification while still invoking the rational expectations hypothesis. For instance, suppose that instead of 2.5 the following is specified as an aggregate supply function:

$$y_t = ap_t^* + u_t. \quad (2.11)$$

This alternative specification states that aggregate supply is a direct function of expectations of the price level. This specification is a possible alternative to the "natural rate" of unemployment specification of 2.5. Using 2.11 and equating it to 2.6 one can derive the equilibrium price level,

$$p_t = -\frac{a}{b} p_t^* + \frac{c}{b} x_t - \frac{1}{b} u_t.$$

Invoking rational expectations in the form of 2.7 one obtains,

$$p_t^* = \frac{c}{a+b} E(x_t). \quad (2.12)$$

Substituting 2.12 into 2.11 one obtains,

$$y_t = \frac{ac}{a+b} E(x_t) + u_t. \quad (2.13)$$

Equation 2.13 shows that simply changing the aggregate supply specification can have a profound impact. While under the "natural rate" of unemployment specification (i.e. equation 2.5 changes in y_t can come about only through unanticipated policy, with 2.13 y_t will change with changes in expected policy. While in one model only expectational errors can affect y_t , in the other, expectations are the sole determinant (other than the error term u_t) of y_t . Note that since rational expectations was utilized in both cases the difference in results comes solely from the change in model specification. Therefore the policy ineffectiveness proposition rests upon the appropriateness of the "natural rate" of unemployment specification of the aggregate supply function rather than rational expectations

per se. That specification has been attacked as not corresponding to the historical record and when linked with rational expectations, as not explaining the strong historical serial correlation of gross national product and unemployment.²⁴ Furthermore, even if one were to accept the "natural rate" specification with rational expectations there are good reasons for rejecting the policy ineffectiveness proposition including the existence of a difference in private and public opportunity sets, and the existence of revisions in forecasts of future investment values.²⁵ Also, certain empirical tests of the policy ineffectiveness proposition cast some doubt upon its validity.²⁶

There is another aspect, highlighted by the above discussion, that is worth noting. The use of rational expectations requires or assumes that economic agents know what the appropriate economic model is in formulating expectations. That assertion appears rather unlikely given that economists themselves have several alternative models of macroeconomic behavior, each potentially having different forecast implications. Given that there is some difference of opinion among economists as to what is an appropriate macroeconomic model, how can rational expectations theorists assume that economic agents will know what is appropriate? Even if one asserts that economic agents will converge upon the "true" model, one would still have

to model that adjustment process and account for adjustment and information costs. Even if that could be done there is no guarantee of convergence to the "true" model since the nature of the "true" model may be shifting over time due to changes in policy and, more generally, the economic environment.²⁷

3. The Optimality of Optimal Plans Using Dynamic Programming

Up to this point the discussion related to the rational expectations hypothesis has not focused upon the use of optimal control when expectations are formed rationally. In this section we will focus upon that issue. That discussion will in large measure be based upon the work of Kydland and Prescott in this area.²⁸

Kydland and Prescott state that:

... optimal control theory is an appropriate planning device for situations in which current outcomes and the movement of the system's state depend only upon current and past policy decisions and upon the current state. But we argue, this is unlikely to be the case for dynamic economic systems. Current decisions of economic agents depend in part upon their expectations of future policy actions. Only if these expectations were invariant to future policy plans selected would optimal control theory be appropriate. In situations in which the structure is well understood, agents will surely surmise the way policy will be selected in the future.²⁹

Their analysis proceeds on the basis of the following definitions. Let $\Pi = (\pi_1, \pi_2, \dots, \pi_T)$ be a sequence of policies for periods 1 to T (which may be

infinite) and $x = (x_1, x_2, \dots, x_T)$ be the corresponding sequence for economic agents' decisions. Furthermore, an agreed-upon social objective function S , where $S = S(x_1, \dots, x_T, \pi_1, \dots, \pi_T)$ is assumed to exist and is used as a means of selecting policy. A policy π , is defined as consistent if for each time period t , π_t maximizes the agreed upon social objective function, S , taking as given previous decisions of economic agents and that future policy decisions (π_s for $s > t$) are similarly selected. Thus a policy decision selected by dynamic programming would be consistent, according to their definition. However, they show that the consistent policy is not necessarily the optimal policy.

To develop their argument they present the following two period example. Suppose there exists an agreed upon social objective function, S , of the following form:

$$\begin{aligned}
 S &= S(x_1, x_2, \pi_1, \pi_2) \\
 \text{where: } \pi_1 &= \text{policy for period one;} \\
 \pi_2 &= \text{policy for period two;} \\
 x_1 &= \text{economic agents' decisions in} \\
 &\quad \text{period one} \\
 &= x_1(\pi_1, \pi_2); \\
 x_2 &= \text{economic agents' decisions in} \\
 &\quad \text{period two} \\
 &= x_2(x_1, \pi_1, \pi_2).
 \end{aligned}$$

In this situation, the first-order condition for the selection of a consistent policy π_2 would be,

$$\frac{\partial S}{\partial x_2} \frac{\partial x_2}{\partial \pi_2} + \frac{\partial S}{\partial \pi_2} = 0$$

since x_1 is taken as given. The consistent policy π_2 is selected without taking into consideration upon x_1 . More generally an optimal policy π_2^* could be selected that would have the following as its first-order condition:

$$\frac{\partial S}{\partial x_2} \frac{\partial x_2}{\partial \pi_2} + \frac{\partial S}{\partial \pi_2} + \frac{\partial x_1}{\partial \pi_2} \frac{\partial S}{\partial x_1} + \frac{\partial S}{\partial x_2} \frac{\partial x_2}{\partial x_1} = 0 .$$

Only if,

$$\frac{\partial S}{\partial \pi_2} + \frac{\partial x_1}{\partial \pi_2} \frac{\partial S}{\partial x_1} + \frac{\partial S}{\partial x_2} \frac{\partial x_2}{\partial x_1} = 0$$

would π_2 , the consistent policy equal π_2^* , the optimal policy. Therefore consistent policies such as those generated by dynamic programming would not necessarily be optimal policies. However, if,

$$\frac{\partial x_1}{\partial \pi_2} = 0 \quad \text{or,} \quad \frac{\partial S}{\partial x_1} + \frac{\partial S}{\partial x_2} \frac{\partial x_2}{\partial x_1} = 0$$

then the consistent policy would be optimal. Therefore there will be a difference between the optimal policy and the consistent policy if there is an impact of change in π_2 upon x_1 or if there is an impact of change in x_1 upon S . Kydland and Prescott state that with rational expectations there is likely to be a difference between the optimal policy and the consistent policy. They believe this to be the case since economic agents formulate their actions at least partially on the basis of expectations about future

policy. To support their argument they provide an interesting example. Suppose a social objective is that no houses should be built on a particular flood plain but, given that they are there, to take certain costly flood control measures such as the building of levees. If there were never to be any flood control measure built, economic agents would not build on the flood plain. However, in the absence of a law forbidding the construction of homes on the plain, economic agents may very well choose to build on the plain, knowing that once the homes were built that the government (as a result of its policy) would undertake flood control measures. In this case the government's expected policy action had a direct influence on the behavior of economic agents. In this situation the actions of economic agents were dependent upon an expectation of future government policy.

Thus it again appears that the nature of the economic model is important in the presence of rational expectations. Buiter classifies economic models into two classes -- causal and noncausal. Causal models are backward looking so that current endogenous variables are dependent upon lagged endogenous variables and lagged exogenous variables, while noncausal models include expectations concerning future endogenous and exogenous variables. The applicability of dynamic programming to the formulation of policy depends upon whether the economic

environment is causal or noncausal. If economic behavior is causal then dynamic programming is appropriate as a tool in formulating optimal dynamic policy. However, if the economic behavior is noncausal, (i.e., forward looking) then dynamic programming has limited applicability as a tool in formulating optimal policy.³⁰ Therefore the applicability of the dynamic programming approach depends upon one's view of whether economic systems are causal (backward-looking) or noncausal (forward-looking), a question again of the appropriateness of a particular model. Gregory Chow agrees that the Kydland and Prescott critique has validity for noncausal models but he suggests that dynamic programming can be useful in the selection of rules even in those situations. Furthermore, he shows that if the estimated system one is working with is covariance stationary then it may be possible to apply dynamic programming techniques to establish feedback rules in systems having expectations of future variables.³¹

4. The "Lucas Critique" Reexamined

In Section D of this chapter we reviewed some of the difficulties that econometricians must confront as a result of the recent flowering of rational expectations. One particularly troublesome argument (discussed previously in parts a and b of section D) is the possibility that model parameters could change with changes in policy

regimes. That argument has come to be known as the Lucas critique of traditional econometric evaluative procedures where model structure is assumed invariant with presumed policy changes. The purpose of this section is to examine more closely the "Lucas Critique".³²

While the Lucas critique has been valuable in causing a major reexamination of traditional methods, there remain considerable questions as to its applicability to the usual policy making setting. Policy making in the United States takes place in a highly charged environment where policies are the outgrowth of generally evolutionary rather than revolutionary forces, and hence only takes place after considerable debate. Also, given the institutional constraints of frequent national elections and a federal system where states share considerable political power, policies historically have been revised frequently and have been short lived; and changes have been rather small in magnitude. Therefore, it would be a mistake to view typical policy making in the U.S. as radical leaps into the unknown. Radical departures from past policy are rare events. Consequently, it is entirely conceivable that economic agents when formulating decisions heavily discount the future impact of policy change and view any policy changes as transitory shocks to the system or as transitory changes in policy variables rather than permanent changes in policy rules. Given this view, one could consider the

Lucas critique as more of a cautionary note on the possibility of parameter change with changes in policy rather than an argument that parameter change will definitely occur.³³

Also, there is the question of the empirical significance of the Lucas critique. Although the Lucas critique presents a theoretical argument why certain model parameters could change with changes in policy, it contains nothing concerning the magnitude or the significance of parameter change. Testing for such change is a difficult task. There is probably no absolute way in a nonexperimental setting of assessing the significance of parameter change. Econometricians do not operate in a situation where factors affecting parameter values can be isolated from each other and controlled. The best one can do is to carefully examine the evidence under a set of restrictive assumptions in an attempt to assess the impact of policy change. In making such assessments, the final conclusions derived are to a great extent a function of assumptions made in the analysis. Nevertheless, one would expect econometric models to perform poorly during periods of substantial policy change if the effects envisioned by the Lucas critique were significant. Although by no means definitive, there is some empirical evidence that certain econometric models performed rather well during periods of major policy change in the 1970's. This at least raises

some doubt as to the significance of the Lucas
critique.³⁴

NOTES

CHAPTER II

¹Richard A. Musgrave, The Theory of Public Finance (New York: McGraw-Hill Book Company, 1959), pp. 3-27.

²Ibid., pp. 6-17.

³Ibid., pp. 17-22.

⁴Ibid., pp. 22-27.

⁵Martin Feldstein, "On Theory of Tax Reform", Journal of Public Economics 6 (July-August 1976) : 77-104.

⁶Feldstein used an incorrect second order Taylor series approximation to, $u(y + \lambda z) - u(y)$, in the expression for the welfare gain, G. He obtained the incorrect result, $\lambda zu' - \frac{1}{2} \lambda^2 z^2 u''$, rather than the correct expression utilized in this paper.

⁷Gregory C. Chow, Analysis and Control of Dynamic Economic Systems (New York: John Wiley & Sons, 1975).

⁸Ibid., pp. 149-157.

⁹J.H. Wescott et al., "A Control Theory Framework for Policy Analysis," in Optimal Control for Econometric Models, Sean Holly et al., eds. (New York: St. Martin's Press, 1979), p. 11.

¹⁰Chow, Analysis and Control of Dynamic Economic Systems, pp. 163-174.

¹¹Ibid., pp. 176-202.

¹²R. Bellman, Dynamic Programming (Princeton, N.J.: Princeton University Press, 1975), quoted in Willem H. Buiter, "The Superiority of Contingent Rules over Fixed Rules in Models with Rational Expectations," The Economic Journal 91 (September 1981) : 648-649.

¹³Chow, Analysis and Control of Dynamic Economic Systems, pp. 149-202.

¹⁴Willem H. Buiter, "The Superiority of Contingent Rules over Fixed Rules in Models with Rational Expectations," The Economic Journal 91 (September 1981) : 647-670.

¹⁵Gregory C. Chow, Econometric Analysis by Control Methods (New York: John Wiley & Sons, 1981), pp. 123-130.

¹⁶The discussion presented in this section is based (except where indicated) upon Rational Expectations and Econometric Practice, Robert E. Lucas, Jr. and Thomas J. Sargent, editors, (Minneapolis: The University of Minnesota Press, 1981), pp. xi-xl.

¹⁷John F. Muth, "Rational Expectations and the Theory of Price Movements," Econometrica 29 (July 1961) : 315-335.

¹⁸Gregory C. Chow, Econometric Analysis by Control Methods (New York: John Wiley & Sons, 1981), pp. 11-12.

¹⁹Henri Theil, Optimal Decision Rules for Government and Industry (Amsterdam: North Holland, 1964).

²⁰David K. H. Begg, The Rational Expectations Revolution in Macroeconomics (Baltimore: The Johns Hopkins University Press, 1982), pp. 71-128.

²¹Thomas J. Sargent, "Interpreting Economic Times Series," Journal of Political Economy 89 (April 1981) : 213-248.

²²Otto Eckstein, The DRI Model of the U.S. Economy (New York: McGraw-Hill Book Company, 1983), pp. 40-46.

²³This model is taken from Rodney Maddock and Michael Carter, "A Child's Guide to Rational Expectations," Journal of Economic Literature 20 (March 1982) : 50-51.

²⁴Ibid., pp. 44-45.

²⁵Buiter, "Superiority of Contingent Rules over Fixed Rules in Models with Rational Expectations," pp. 653-662.

²⁶Frederick S. Mishkin, "Does Anticipated Monetary Policy Matter," Journal of Political Economy 90 (February 1982) : 22-51.

_____, "Does Anticipated Aggregate Demand Policy Matter," The American Economic Review 72 (September 1982) : 788-802.

²⁷Maddock and Carter, "A Child's Guide to Rational Expectations," pp. 44-45.

²⁸Finn E. Kydland and Edward C. Prescott, "Rules Rather than Discretion: The Inconsistency of Optimal Plans," in Rational Expectations and Econometric Practice, Robert E. Lucas, Jr. and Thomas J. Sargent, editors (Minneapolis: The University of Minnesota Press, 1981), pp. 619-637.

²⁹Ibid., p. 620.

³⁰Willem H. Buiter, "The Role of Economic Policy after the New Classical Macroeconomics," (Cambridge: National Bureau of Economic Research, 1980), pp. 26-27.

³¹Chow, Econometric Analysis by Control Methods, pp. 234-236.

³²Robert E. Lucas, Jr., "Econometric Policy Evaluation: A Critique," in Studies in Business - Cycle Theory (Cambridge: The MIT Press, 1981), pp. 104-130.

³³Christopher A. Sims, "Policy Analysis with Econometric Models," Brookings Papers on Economic Activity no. 1 (1982) : 107-152.

³⁴See for example - Eckstein, The DRI Model of the U.S. Economy; also, Albert Ando, "The 'Failure' of Keynesian Economics and the Phillips Curve in the 1970's: A Fact or a Fiction?," (University of Pennsylvania, September 1981), cited by Stephen M. Goldfeld in comments to Christopher A. Sims, "Policy Analysis with Econometric Analysis," p. 157.

CHAPTER III

MODELS OF THE IMPACT OF INVESTMENT TAX INCENTIVES UPON INVESTMENT BEHAVIOR

This chapter shall present a detailed discussion of qualitative and quantitative models, used in both microeconomic and empirical macroeconomic studies, which have appeared recurrently in the economic literature dealing with the impact of tax incentives upon the investment behavior of the firm. The discussion is not intended to be an exhaustive review of the extensive literature in this area, but is intended to illustrate the scope of analyses undertaken. However, before pursuing the details of the models, a brief introduction to a traditional approach to capital theory will be presented.

A. A Brief Introduction to Capital Theory

Approaches to investment behavior usually involve some notion of alternative choice. For example, in what has come to be known as the rate of return model, the potential investor has a choice at any point in time between lending his funds at a market rate of interest r or purchasing a capital good yielding a rate of return ρ which

is implicitly defined by the relation (in continuous discounting form),

$$p = \int_0^n N(t) e^{-\rho t} dt \quad (3.1)$$

where: p = market price of the capital good;

$N(t)$ = net cash receipt from the capital good
as a function of time t ;

n = economic life of the capital good.

The rational profit maximizing investor will use available funds in such a way as to maximize the yield derived from those funds. Thus, the investor would invest in capital goods when $\rho > r$; and lend funds at the market rate $r > \rho$ provided the risk of investing equals the risk in the market. The investor would be indifferent between lending and investing at the point where $\rho = r$. The point where $\rho = r$ is called the marginal condition; and an investment made at this point is called a marginal investment.¹

B. Qualitative Analyses

1. E. Cary Brown's Model

One of the first models attempting to explain the impact of accelerated depreciation to be introduced into economic literature using the above rate of return theory of investment was one by E. Cary Brown.²

As noted above, a rational profit maximizing investor will invest up to the point where the price of a new asset is equal to the present value of its income stream discounted at the prevailing market rate of interest (i.e., in 3.1 above $\rho = r$ at the point where the investor would stop investing). Using this fact, Brown states that the investor will invest in the absence of uncertainty and risk up to the point where,

$$C = RA$$

where: C = the cost of the asset;
 R = the constant net annual cash receipt;
 A = present value of one dollar per annum discounted at the market rate r .

Introducing a proportional income tax system with full loss offset (i.e., a system that makes tax refunds when losses are incurred as well as taxing profits) into the above prescribed system one obtains,

$$C_T = RA - t(R - \frac{C_T}{d}) \quad A = [R - t(R - \frac{C_T}{d})]A$$

where: t = the proportional tax rate;
 d = a straight line depreciation rate (i.e., constant proportional rate);
 C_T = cost with taxation.

Rearranging the expression results in,

$$C_T(1 - \frac{At}{d}) = (1 - t) RA$$

Noting that if $t = \frac{At}{d}$, which implies that the present value of the stream of tax credits per dollar of asset cost due to depreciation equals the proportional tax rate, one has established a condition for neutrality where the tax rate is irrelevant in its impact upon investment behavior.

From this model, Brown concludes that the tax system distorts investment behavior since the present value of the tax credits per dollar of asset cost due to depreciation (i.e. $\frac{At}{d}$) never equals the tax rate. Thus, the entire discussion hinges on the fact that the present value of the depreciation stream per dollar of asset cost (i.e. $\frac{A}{d}$) is less than one using straight line depreciation. It is also true that accelerated depreciation systems possess a present value of the depreciation allowances per dollar of asset cost that are less than one. Hence, such accelerated systems are also nonneutral.

2. Richard Musgrave's Model

Richard Musgrave in his classic, The Theory of Public Finance, does not use the marginal condition but rather defines the internal rate of interest (i.e., rate of

return), i , as that rate of interest that equates the two sides of the expression,

$$C = RA$$

where: C = the cost of the asset;
 R = the constant net annual cash receipt;
 A = the present value of a unit per annum cash receipt stream, discounted at the internal rate i .³

Introducing a proportional tax rate system with full loss offset one obtains,

$$C = (1 - t) RA_t + \frac{Ct}{d} B_t$$

where: $\frac{Ct}{d} B_t$ = the present value of the tax credit stream resulting from the depreciation allowance.

This new expression would result in a new implicitly defined internal rate of interest i_t under the given tax system. Defining the effective tax, t_{ej} , on alternative investments, neutrality occurs when,

$$t_{e1} = t_{e2} = t_{e3} = \dots = t_{ej} = 1 - \frac{i_{tj}}{i_j}$$

where $j = 1, 2, \dots, n$ indexes alternative investments.

This in turn implies that,

$$\frac{i_1}{i_{t1}} = \frac{i_2}{i_{t2}} = \dots = \frac{i_j}{i_{tj}}.$$

Thus, neutrality would occur if the alternative internal rates were reduced by the same proportion in all cases.

Musgrave then computes alternative tax rates under different depreciation systems and finds that the tax system does not impose the same effective tax rate upon all investments due to the availability of the alternative depreciation systems. Thus, the tax system containing these alternative systems is nonneutral.

3. An Alternative Approach Using the Marginal Condition⁴

To formalize the following discussion the following notational definitions are necessary:

V_i = the present net project value under proportional taxation with full loss offset at rate τ , the i 'th ($i = 1, 2, 3$) tax depreciation write off system, including the investment tax credit;

$D_i(s)$ = the i 'th tax depreciation write-off system's allowance at time s ;

q_i = the price of the asset under the i 'th tax depreciation system (i.e., the maximum amount an investor would be willing to pay under the given conditions);

s = a variable of integration representing time;

a = a constant net cash receipt per continuous unit of time s ;

k = investment tax credit rate;

r = market rate of interest (a given constant);

n = economic life of asset (a given constant).

In addition to the above notation it is assumed that salvage value equals zero; there is an absence of uncertainty and risk; that perfect competition and perfect capital markets prevail; that continuous time and discounting is relevant; and the investor is a profit maximizer.

From the above definitions and assumptions, one can establish the following relation, assuming that the economic life of the asset is identical to the stipulated tax life of the asset:

$$\begin{aligned} V_i &= \int_0^n (a - \tau[a - q_i D_i(s)]) e^{-rs} ds - (1 - k)q_i \\ &= a(1 - \tau) \int_0^n e^{-rs} ds + \tau q_i \int_0^n D_i(s) e^{-rs} ds \\ &\quad - (1 - k)q_i . \end{aligned} \quad (3.2)$$

From the discussion above we establish that an investor would invest rather than lend when $V_i > 0$, or up to the point where, $V_i = 0$, the marginal condition. This is true since when $V_i = 0$, $\rho = r$. Substituting this condition for the marginal investment into 3.2 results in,

$$\begin{aligned} V_i &= a(1 - \tau) \int_0^n e^{-rs} ds + \tau q_i \int_0^n D_i(s) e^{-rs} ds \\ &\quad - (1 - k)q_i = 0 . \end{aligned} \quad (3.3)$$

Solving 3.3 for q_i results in,

$$q_i = \frac{a(1 - \tau) \int_0^n e^{-rs} ds}{1 - k - \tau \int_0^n D_i(s) e^{-rs} ds} = \frac{a(1 - \tau) \int_0^n e^{-rs} ds}{1 - k - \tau z_i} \quad (3.4)$$

with $z_i = \int_0^n D_i(s) e^{-rs} ds$. Note that z_i represents the present value of the depreciation allowance stream. Since all expressions on the right side of 3.4 except k and z_i are constants, q_i will vary with the present value of the depreciation allowances, z_i , and k . Since $0 < z_i < 1$, $0 < k < 1$, and $0 < \tau < 1$, q_i becomes greater as z_i and k become closer to one. Thus, one must first know what relationship exists between the present values of the alternative tax depreciation systems before one can assess the relative impact of these depreciation systems upon the marginal investment decision.

Such a relationship has been established by this author in the earlier paper already cited. However, the following table, which has been derived from formulae in that paper, adequately demonstrates the hierarchy that exists between alternative values of z_i .

TABLE 1

SELECTED PRESENT VALUE FACTORS
FOR ALTERNATIVE DEPRECIATION METHODS

Life- time n (Yrs)	Interest rate r	Straight Line z_1	Double Declining Balance z_2	Sum of The years Digits z_3
5	.06	.863940	.888262	.907067
	.12	.751980	.795041	.826733
10	.06	.751980	.795041	.826733
	.12	.582338	.650637	.696103
20	.06	.582338	.650637	.696103
	.12	.378868	.467874	.517610
30	.06	.463723	.546216	.595855
	.12	.270188	.363624	.405451
40	.06	.378868	.469702	.517610
	.12	.206618	.296946	.330576

With subscripts 1, 2, 3 denoting the historically popular straight line, double declining balance with optimal switchover point equal to $\frac{n}{2}$, and the sum of the year's digits methods of depreciation,

$$0 < z_1 < z_2 < z_3 < 1.$$

What this implies is that under the given assumptions and definitions, an investor would be willing to pay at the margin a higher price for an asset to be depreciated under the sum-of-the-years digits method than under the straight line or double declining balance (with optimal switchover point at $\frac{n}{2}$) methods. Or the investor would be willing to pay a higher price at the margin for an asset to be depreciated under the double declining balance method (with

optimal switchover point at $\frac{n}{2}$) than for an asset depreciated under the straight line method.

Furthermore, if the tax authorities revise tax lives downward (as they did in 1962 and 1981), a further non-neutrality results. In this case they stipulate tax lives, m , that are less than the previously stipulated tax lives, n . This implies that with $m < n$, $0 < z_i < z'_i < 1$ where z'_i represents the present value of the tax depreciation allowances under the new shorter tax live, m . This condition further implies that $q'_i > q_i$ where q'_i represents the price an investor would now be willing to pay for an asset depreciated under a given write off scheme with the new shorter tax life. Thus it is apparent that the shortening of tax lives results in the investor being willing to pay a higher price at the margin for a capital good to be depreciated under the alternative write off schemes.

From the above model a question arises as to why the sum-of-the-years digits method was not utilized more extensively by investors since the time it was first introduced. Besides for restrictions imposed by the Internal Revenue Service as to what depreciation system can be applied to a particular asset, one possible answer is that salvage value has not been included in the above model, and it is this factor that may be decisive in the choice between the sum-of-the-years digits method and the double declining balance method.

The purpose of the discussion was to demonstrate that the introduction of accelerated tax depreciation systems result in a further nonneutrality that distorts the previously existing investment decision under the straight line system.. The discussion further demonstrates that a change to shorter tax lives will result in a further impact upon the investment decision.

4. The Samuelson Theorem on the Necessity of Tax Deductibility of Economic Depreciation to Insure Invariant Valuations

Samuelson establishes the following theorem:

"If and only if, true loss of economic value is permitted as a tax-deductible depreciation expense will the present value of a cash-receipt stream be independent of the tax rate."⁵

The proof of the theorem depends on the following notational definitions:

$V(t)$ = the present value of a cash-receipt stream at time t ;

$N(t)$ = net cash receipts as a function of time;

$r(t)$ = the instantaneous rate of interest at which one can borrow or lend in unlimited amounts.

In addition, it is assumed that $r(t) = r$, a constant, and that interest deductibility is allowed for tax purposes.

From the above definitions and assumptions one can obtain the relation,

$$V(t) = \int_t^n N(x) e^{-r(x-t)} dx \quad (3.5)$$

where x is a variable of integration representing time and the relation is defined in the finite interval, (t, n) .

Explicit differentiation of 3.5 with respect to t results in,

$$\frac{dV}{dt} = V'(t) = r V(t) - N(t) \quad (3.6)$$

which is an expression for the true loss (or gain) in economic value over time.

If a uniform proportional tax $T' = 1 - T$ is introduced, then the interest rate is $Tr < r$ and the cash receipt is $T N(t) < N(t)$. With the introduction of a depreciation allowance $D(t)$, the model, containing modifications due to taxation, becomes,

$$\begin{aligned} V(t) &= \int_t^n [N(x) - (1-T) [N(x) - D(x)]] e^{-Tr(x-t)} dx \\ &= \int_t^n [T N(x) + (1-T) D(x)] e^{-Tr(x-t)} dx . \quad (3.7) \end{aligned}$$

Generalizing 3.7 to allow for the possibility that the depreciation allowance is not only a function of time but also of the tax rate so that $D(x) = D(x, T)$, one obtains,

$$V(t, T) = \int_t^n [T N(x) + (1-T) D(x, T)] e^{-Tr(x-t)} dx . \quad (3.8)$$

Differentiating 3.8 with respect to t results in,

$$\frac{\partial V(t, T)}{\partial t} = TrV(t, T) - [TN(t) + (1-T) D(t, T)] . \quad (3.9)$$

But by hypothesis, the solution is to be independent of T , or $V(t, T) = V(t, 0) = V(t)$, a fact which transforms 3.9 into,

$$\begin{aligned} V'(t) &= TrV(t) - TN(t) - (1-T) D(t, T) \\ &= T [rV(t) - N(t)] - (1-T) D(t, T) . \end{aligned} \quad (3.10)$$

But since $rV(t) - N(t) = V'(t)$ from 3.6 above, we can write 3.10 as,

$$V'(t) = TV'(t) - (1-T) D(t, T), \quad \text{or rearranging as,}$$

$$(1-T)V'(t) = -(1-T) D(t, T),$$

or $D(t, T) = -V'(t)$, which is the correct result according to the Samuelson theorem, since $-V'(t)$ is the true economic loss in value.

If the true economic loss in value is not deducted for tax purposes, the above theorem demonstrates that the cash receipt stream will not be independent of the tax rate. If the cash receipt stream is not independent of the tax rate, then a nonneutrality will exist. Thus when arbitrary tax depreciation systems are employed, as is the case in the U.S. tax laws, nonneutralities must exist. Therefore, Samuelson has demonstrated in general the results of the previous models.

5. A Neoclassical Model of Investment Behavior

In a series of articles, Robert E. Hall and Dale W. Jorgenson have employed a neoclassical model of investment behavior that they used to estimate empirically the impact of the introduction of accelerated depreciation upon investment behavior. In the discussion that follows their model will be presented in detail as an analytical model including tax, investment tax credit, and depreciation variables.⁶

The object of the firm is assumed to be the maximization of,

$$z' = pQ - wL - cK$$

where: p = the price of output;
 Q = the quantity of output;
 w = the price of labor input;
 L = the quantity of labor input;
 c = the rental price of capital;
 K = the quantity of capital.

Note that the product and factor markets are implicitly assumed to be perfectly competitive. The production function is defined as,

$$Q = \phi(L, K) .$$

Substituting this expression for Q into z' , one obtains,

$$z' = p \phi(L, K) - wL - cK .$$

The necessary first order conditions for a maximum are then,

$$\frac{\partial z'}{\partial L} = p \frac{\partial \phi}{\partial L} - w = 0 \text{ and } \frac{\partial z'}{\partial K} = p \frac{\partial \phi}{\partial K} - c = 0$$

$$\text{or, } \frac{w}{p} = \frac{\partial \phi}{\partial L} \text{ and } \frac{c}{p} = \frac{\partial \phi}{\partial K} .$$

These conditions will later be used by the authors to implement their model empirically.

The price $q(t)$ of a new capital good is defined as the present value of future rentals, or in the absence of direct taxation as the expression,

$$q(t) = \int_t^{\infty} e^{-r(s-t)} c(s) e^{-\delta(s-t)} ds$$

where: r = the rate of return (a constant);

c = the rental price of capital input;

$e^{-\delta(s-t)}$ = the quantity of capital input at time s resulting from the purchase of one unit of the capital asset at time t ;

δ = the rate of replacement;

s = a variable of integration representing time.

If prices of new capital remain stationary at q , then c must be expected to remain constant and,

$$q = c \int_t^{\infty} e^{-(r + \delta)(s - t)} ds = \frac{c}{r + \delta}$$

or $q(r + \delta) = c$.

Now if a tax system is imposed that employs a proportional tax rate of u ; tax depreciation allowances $D(s-t)$; investment tax credit rate k ; and interest deductibility then,

$$q(t) = \int_t^{\infty} e^{-(1-u)(s-t)\rho} [e^{-\delta(s-t)}(1-u)c(s) + u q(t) D(s-t)] ds + k q(t)$$

where ρ is the before tax rate of return.

If, as before, the price of new capital goods is expected to remain constant at q , then $c(s) = c$ and,

$$q = (1 - u)c \int_t^{\infty} e^{-[(1-u)\rho + \delta][s - t]} ds + uqz + kq$$

$$\text{where: } z = \int_t^{\infty} e^{-(1-u)\rho(s-t)} D(s-t) ds,$$

the present value of the depreciation allowances.

Performing the integration one obtains,

$$q = \frac{(1-u)c}{(1-u)\rho + \delta} + uqz + kq$$

$$\text{or } c = \frac{q [(1-u)\rho + \delta] [1 - k - uz]}{1 - u} \quad (3.11)$$

If the before tax rate of return is to be held constant and the combined value of depreciation allowances and the investment tax credit is equal to the expression $D(s-t) = \delta e^{-\delta(s-t)}$ the allowance for "economic

depreciation" under the given assumptions, then the present value for "economic depreciation", z^* is,

$$z^* = \int_t^{\infty} e^{-(1-u)\rho(s-t)} \delta e^{-\delta(s-t)} ds$$

$$= \frac{\delta}{(1-u)\rho + \delta}.$$

Provided that $k + uz = uz^*$ then,

$$c = q(\rho + \delta).$$

If the after tax rate of return $r = (1-u)\rho$ is held constant and immediate expensing is employed then $z = 1$ and $k = 0$, so that,

$$c = q(r + \delta).$$

Note however that as the values for z (where $0 < z < 1$) in formula 3.11 become closer to one, the value of c decreases. Thus the adoption of the more accelerated forms of depreciation and higher investment tax credit rates tend to decrease the previously existing implicit rental price and hence increase investment incentives.

The Hall and Jorgenson model is important since it brings together in one model many of the previous results of other authors. In this single model, one can obtain the essential results of any of the previously presented models simply by changing the assumptions.

6. Extensions of the Above Qualitative Analyses

The above works represent major approaches to the qualitative analysis of investment tax incentive programs. In the following we will briefly review some extensions to the above works. Many of these analyses introduce additional features that essentially show that the analysis of investment tax incentives becomes much more complicated once realistic assumptions and constraints are introduced. To conserve space, the discussion below will present a minimum of technical detail but will rather focus on the way analyses have been modified and on the conclusions derived. These extensions have taken a number of directions. We shall concentrate on four areas -- the impact of borrowing restraints, the impact of expected inflation, the effect of the inclusion of adjustment costs, and the influence of personal taxation.

Some investigators have been concerned about the effect of borrowing constraints upon a corporation's incentive to invest. Boadway and Bruce have shown that one needs to introduce a constraint on corporate borrowing before any meaningful analysis of investment incentives can be undertaken. They demonstrate that the corporation's incentives to invest depend upon the form of the borrowing constraint. For instance, they show that if corporate borrowing cannot exceed the firm's accounting stock of capital

and interest deductibility is allowed, the firm has no incentive to alter its investment plans from the no-tax situation when noneconomic tax depreciation schemes (e.g., accelerated schemes) are introduced.⁷ Alan Auerbach, however, suggests an alternative to Boadway's constraint and shows that incentive effects are possible.⁸ Nevertheless, it is apparent that borrowing activities and constraints in the presence of interest deductibility could have an impact upon the effectiveness of investment tax incentives.

In addition, several authors have emphasized that expected inflation should be introduced into the analyses. They point out that nominal depreciation allowances decrease in real value with inflation.⁹ Also, the present value of depreciation allowance stream declines with expected inflation. Furthermore, the presence of expected inflation influences a firm's decision to borrow, which in conjunction with interest deductibility has an influence on the incentive to invest. Thus, once inflation is included one must examine the impact of expected future deductions for interest and depreciation under different borrowing constraints and depreciation systems.¹⁰ Generally, introducing expected inflation results in more complicated analyses.

Some investigators have emphasized that the tax treatment of the adjustment costs associated with investment activities could have an impact upon the incentive to invest. Boadway shows, for instance, that when adjustment costs are not deductible, investment and capital accumulation will be retarded. However, if adjustment costs are deductible he demonstrates that disincentives would tend to be minimized.¹¹ Again the introduction of another area having potential impact upon the effectiveness of investment incentives requires more involved analyses.

Finally, the existence of personal taxation together with corporate taxation is likely to have an influence upon the incentive to invest. Auerbach demonstrates that the introduction of personal taxes into the analysis can result in non-natural results even if the corporate tax structure is neutral.¹² Moreover, the nature of such nonneutralities is complicated by the multiplicity of effective marginal tax rates applying to individuals.

In the above discussion we have focused briefly upon a number of modifications to the fundamental analyses presented previously. One should have noted from that discussion that when fairly realistic considerations (e.g., borrowing constraints and personal taxation) are introduced into analyses the conclusions as to the effectiveness of investment tax incentive programs become obscured. By

focusing, even briefly, upon such considerations one realizes the complexity of realistically modelling investment behavior and the large number of possible models one could construct in attempting to model such behavior. In what follows we will briefly examine how investment tax incentive programs could induce investment. This does not imply, however, that such programs necessarily induce investment.

C. Theoretical Impact of the Adoption of Investment Tax Incentives Upon the Capital Market

From the foregoing discussion it is apparent that the adoption of the investment tax credit and accelerated depreciation have a potential impact upon investment behavior. In the discussion below, this author will demonstrate, by means of a simple model, the implications of this change.

Suppose the economy under discussion has but one type of capital good and we introduce investment tax incentive programs, such as accelerated depreciation and an investment tax credit. From the previous discussion we learned that under these systems an investor may be willing to pay a higher price for a capital good than previously. Thus, assuming that all investors were subject to the same economic conditions, the demand schedule for capital goods would tend to shift to the right. Figures 1 and 2 depict

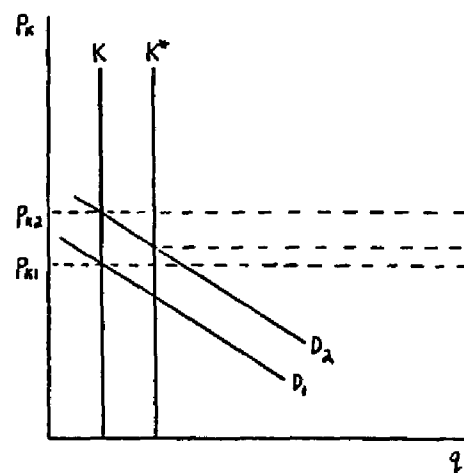


Figure 1
Stock of Capital Goods

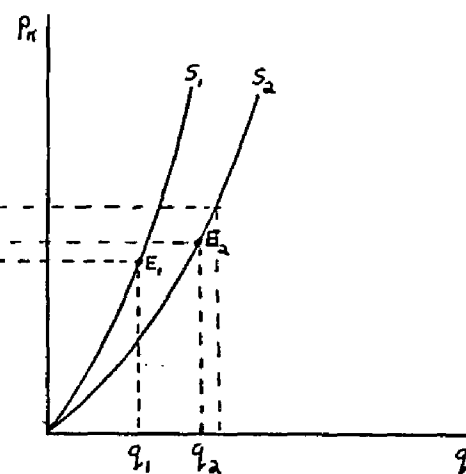


Figure 2
Production of New Capital Goods

the situation under consideration. With the capital stock, K , perfectly inelastic, the price of new capital goods is determined by the intersection of the existing demand curve for capital goods, D_1 , and the existing capital stock, K , resulting in price of p_{k1} . With the introduction of investment tax incentives and the resulting increase in demand, the demand curve shifts from D_1 to D_2 resulting in the price increasing to p_{k2} .

At price p_{k1} the production of new capital goods is just sufficient to maintain the existing capital stock at K . But with the increase of price p_{k2} two things occur. First, the supply curve of all producers of new capital goods shifts to the right due to reservation demand on the part of these producers.¹³ Secondly, the amount of new capital goods being produced exceeds the replacement level q_1 . Since this increased production is above the replacement level, the capital stock shifts to the right resulting in a reduction in price. Note however, that we are now using supply curve S_2 to derive the quantity of capital goods being produced. But the resulting price is still sufficient for the production of new capital goods to be above the replacement level. Thus the capital stock shifts to the right again. In fact, this same process continues until the price is driven down to a level that is just sufficient for production of the same amount of new capital goods necessary to maintain the then existing capital

stock, K^* , the desired level of capital. This new replacement level corresponds to the quantity, q_2 , produced at the new equilibrium E_2 .

Although the preceding discussion is extremely simplistic, much of the existing literature proceeds along essentially similar lines. Net investment results when there is difference between the existing capital stock K_{t-1} and the desired level (stock) of capital, K_t^* . However, net investment resulting from this difference does not take place instantaneously but rather over time. The rate at which this adjustment takes place is called the speed of adjustment. For instance, net investment $I_{t,\text{net}}$ could be denoted at time t by the relation,

$$I_{t,\text{net}} = b(K_t^* - K_{t-1}),$$

where b is the speed of adjustment factor. With the addition of replacement, which is assumed to be a constant portion of the existing capital stock, one obtains, for gross investment, $I_{t,\text{gross}}$,

$$I_{t,\text{gross}} = b(K_t^* - K_{t-1}) + \delta K$$

where δ is the replacement factor. However, this is just one of many possible adjustment schemes.

In light of the above discussion, we can now conclude that the introduction of investment tax incentives could result in a change in the desired level of capital which in turn could result in net investment. It should be noted

however, that this change is a "one shot" affair and that the time path of the adjustment process is not well specified. Once the process has been completed, further net investment is possible only if the desired level of capital changes due to some additional change. In addition, it should be emphasized that the above brief discussion is a partial equilibrium analysis and that to find the total impact of depreciation policy change one should perform a general equilibrium analysis.

The next section will briefly present some of the empirical work which has been done in this subject area. That discussion will focus on the work of Hall and Jorgenson, concentrating on the methodology employed, the data utilized, and the results obtained. The discussion will then turn to other empirical studies in this area.

D. Quantitative Approaches

In the preceding discussion it was qualitatively demonstrated, with a variety of approaches, that the adoption of investment tax incentives by the taxing authorities could result in increased demand for capital goods due to the reduction of the implicit rental price for capital services. The increased demand would in turn result in a new desired level of capital which would result in net investment taking place. The purpose of this section will be

to show what attempts have been made to measure quantitatively the impact of the adoption of investment tax incentives by the taxing authorities.

The following discussion will first focus on the seminal work of Hall and Jorgenson and then turn to a brief discussion of extensions to their work and other approaches.

1. The Hall and Jorgenson Approach¹⁴

Hall and Jorgenson have utilized their neoclassical theory of investment behavior to measure the impact of investment tax incentive programs upon investment behavior. The analytical foundation to their approach was presented above in the discussion on qualitative approaches. To implement their approach they assume that the production function is of the Cobb-Douglas form, $Q = AL^{1-\alpha}K^\alpha$, so that the optimal level of capital K^* is determined by the marginal productivity condition (under competitive assumptions), or

$$\alpha\left(\frac{Q}{K^*}\right) = \frac{c}{p}$$

where α is the elasticity of output with respect to capital input. Or solving for K^* , the optimal or desired level of capital, one obtains, $K^* = \alpha\left(\frac{pQ}{c}\right)$. Thus as $c > 0$ is made smaller, the desired level K^* becomes greater. Assuming lags, a change of this type generates net investment over a

period of time. Thus net investment, $I_{t,\text{net}}$ is a weighted average of projects initiated in prior periods,

$$\begin{aligned} I_{t,\text{net}} &= I_t - \delta K_{t-1} \\ &= \mu_0(K_t^* - K_{t-1}^*) + \mu_1(K_{t-1}^* - K_{t-2}^*) \\ &\quad + \dots, \end{aligned}$$

where I_t is gross investment and δK_{t-1} is replacement investment. Defining the lag operator $s x_t = x_{t-1}$ for any sequence x_t ; and defining $\mu(s) = \mu_0 + \mu_1 s + \dots$, as a power series in the lag operator, then one can write,

$$I_{t,\text{net}} = I_t - \delta K_{t-1} = \mu(s) (K_t^* - K_{t-1}^*).$$

a. The Distributed Lag Functions

To implement the model empirically it is necessary to specify the form of the lag series $\mu(s)$ and to add a random component to the relation.

As for the specification of the lag series, Hall and Jorgenson have considered two approaches - the first being a pure rational form in which $\mu(s) = \frac{\gamma(s)}{w(s)}$ where $\gamma(s)$ and $w(s)$ are polynomials in the lag operator s ; the second being the utilization of the Almon procedure to approximate $\mu(s)$.

We shall first investigate the rational form. This relation is specified by the form,

$$\mu(s) = \frac{\gamma(s)}{w(s)},$$

or the power series $\mu(s)$ is represented by the ratio of two polynomials in the lag operator s . If $\beta(s)$ is a polynomial that is capable of representing the power series $\mu(s)$ then $\beta(s) = \mu(s) = \frac{\gamma(s)}{w(s)}$. In this case the polynomial $\beta(s)$ can be represented by the ratio of the two polynomials $\gamma(s)$ and $w(s)$. Given that,

$$\gamma(s) = \gamma_0 + \gamma_1 s + \gamma_2 s^2 + \dots$$

$$w(s) = 1 + w_1 s + w_2 s^2 + \dots$$

and $\beta(s) = \frac{\gamma(s)}{w(s)} = \mu(s)$ and that we can add an independently and identically distributed random error v_t , we can transform our relation into,

$$I_t - \delta K_{t-1} = \beta(s) (K_t^* - K_{t-1}^*) + v_t$$

$$\text{or, } w(s) (I_t - \delta K_{t-1}) = \gamma(s) (K_t^* - K_{t-1}^*) + w(s) v_t$$

$$\text{or, } w(s) (I_t - \delta K_{t-1}) = \gamma(s) (K_t^* - K_{t-1}^*) + \varepsilon_t$$

where $w(s)v_t = \varepsilon_t$. Thus by assuming the order of $w(s)$ to be one and the number of terms in the polynomial $\beta(s)$ to be five, we obtain,

$$(I_t - \delta K_{t-1}) + w_1(I_{t-1} - \delta K_{t-2}) = \sum_{\tau=0}^5 \gamma_{\tau} (K_{t-\tau}^* - K_{t-(\tau-1)}^*) + \varepsilon_t.$$

Specification of the order and number of terms determines the number of parameters that would have to be estimated. In the above specification seven parameters would have to be estimated namely,

$$w_1, \gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \text{ and } \gamma_5.$$

An alternative approach to the above rational form is to utilize the Almon lag (or polynomial distributed lag) approach where the polynomial in the lag operator, $\beta(s)$ has coefficients generated by a polynomial in the lag, τ , itself.¹⁵ This polynomial of degree k is of the form,

$$\beta_{\tau} = \pi_0 + \pi_1 \tau + \dots + \pi_k \tau^k.$$

Substituting this into our relation results in,

$$I_t - \delta K_{t-1} = \sum_{\tau=0}^4 \beta_{\tau} [K_t^* - \tau - K_{t-(\tau+1)}^*] + v_t.$$

If the order of the polynomial is two and there are five terms in the original polynomial in the lag operator $\beta(s)$, then,

$$\begin{aligned} I_t - \delta K_{t-1} &= \sum_{\tau=0}^4 \beta_{\tau} [K_t^* - \tau - K_{t-(\tau+1)}^*] + v_t \\ &= \sum_{i=0}^2 \pi_i \sum_{\tau=0}^4 \tau^i [K_t^* - \tau - K_{t-(\tau+1)}^*] + v_t. \end{aligned}$$

Note however, that the polynomials are set up so that, $\beta(s) = \frac{\gamma(s)}{w(s)} = \beta_{\tau}$ or $\beta(s)w(s) = \gamma(s) = w(s)\beta_{\tau}$.

Thus if the order of $w(s)$ is unity (i.e. $w(s) = 1 + w_1 s$), our expression is transformed into,

$$\begin{aligned} (I_t - \delta K_{t-1}) + w_1 (I_{t-1} - \delta K_{t-2}) \\ = \sum_{i=0}^2 \pi_i \sum_{\tau=0}^4 \tau^i [K_t^* - \tau - K_{t-(\tau+1)}^*] \\ + w_1 \sum_{i=0}^2 \pi_i \sum_{\tau=0}^4 \tau^i [K_{t-(\tau+1)}^* - K_{t-(\tau+2)}^*] + \varepsilon_t \end{aligned}$$

where $\varepsilon_t = w(s)v_t$. Since a low order Almon approximation can be used to approximate $\beta(s)$, the number of parameters

to be estimated has been reduced. Whereas in our example we had seven parameters to estimate using a rational distributed lag function without the Almon approximation, we have only four (namely π_0 , π_1 , π_2 , and w_1) to estimate using the Almon approximation. This approximation was employed by Hall and Jorgenson in their empirical studies.

The brief discussion above contains the essence of the procedure employed by Hall and Jorgenson in setting up the relations to be estimated. Without presenting the details, which would not add materially to our discussion, it can be stated that regression techniques were employed to effect the estimation of these relations.

b. Sources of Data

The model described above was fitted to data on investment expenditures based on the 1966 capital goods study of the Office of Business Economics (OBE). This data covered structures and equipment separately for manufacturing and nonfarm manufacturing in the United States for the years 1929 through 1965. The price indexes employed had a 1965 base. These indexes were derived from work done in the 1966 capital goods study and from the national income accounts. The capital stock for equipment and structures was derived from the relation,

$$K_t = I_t + (1 - \delta) K_{t-1}.$$

The replacement factor, δ , was assumed to be 2.5 times the inverse of the IRS Bulletin F lifetime.

Since K^* is a function of pQ , the authors used gross value added at factor costs for each sector in current dollars for this value. Since K^* is also a function of c , the authors assumed the following in deriving c :

- a. Through 1953 straight line depreciation was the appropriate system of depreciation.
- b. Since 1953 sum-of-the-years' digits is the appropriate tax depreciation write-off system except that from October 1966 to March 1967 the appropriate system for structures was the one and one-half declining balance system.
- c. The appropriate tax rate was assumed to be the corporate tax rate prevailing during most of the year.
- d. For all years the rate of return before taxes was assumed to be a constant 20 percent.
- e. The asset lifetimes allowable for tax purposes were based on a special Treasury study. These values ranged from 17.5 to 13.1 years for equipment; and from 27.8 to 22.8 years for structures for the period 1929 to 1965.
- f. The appropriate investment tax credit rate is assumed to be the prevailing investment tax credit rate of a given period.

c. The Results

Hall and Jorgenson conclude essentially that accelerated depreciation and investment tax credit systems have had a significant impact upon investment activity in this country. These programs have had an impact on changing the level and timing of investment expenditures, according to these authors. The data in Tables 2 and 3 support their contention of a substantial increase.

2. Coen's Modification to the Above Approach of Hall and Jorgenson¹⁶

Robert Coen develops a major alternative to the quantitative approach implemented by Hall and Jorgenson. However, he essentially follows the approach of Hall and Jorgenson with the following differences:

- a. K^* , the desired level of capital, is assumed to be a function of realized sales, S_r , rather than optimal output to account for the existence of inventories in the real world.
- b. An exponentially decaying or Koyck lag distribution was employed rather than the lag function utilized by Hall and Jorgenson.

TABLE 2

ESTIMATED CHANGES IN GROSS AND NET
INVESTMENT IN EQUIPMENT AND STRUCTURES, AND
IN CAPITAL STOCK RESULTING FROM ACCELERATED
DEPRECIATION, 1954-1964 (IN MILLIONS OF
1965 DOLLARS) *

MANUFACTURING

Year	Equipment			Structures		
	Investment		Capital Stock	Investment		Capital Stock
	Gross	Net		Gross	Net	
1954	209	209	0	79	79	0
1956	862	744	805	432	410	354
1958	621	298	2,199	323	250	1,166
1960	369	-2	2,524	153	59	1,508
1962	487	111	2,557	180	78	1,638
1964	660	245	2,820	165	53	1,771

NONFARM MANUFACTURING

1954	847	847	0	204	204	0
1956	1,756	1,265	2,555	944	886	839
1958	1,662	737	4,808	867	684	2,628
1960	1,294	141	5,998	448	194	3,666
1962	1,573	353	6,344	430	201	4,024
1964	2,090	708	7,188	440	134	4,403

* Adapted from: Robert E. Hall and Dale W. Jorgenson.
"Application of the Theory of Optimum Capital
Accumulation", in Tax Incentives and Capital Spending
(Washington: The Brookings Institution, 1971) p. 48.

TABLE 3

ESTIMATED CHANGES IN GROSS AND
NET INVESTMENT IN EQUIPMENT, AND IN
CAPITAL STOCK RESULTING FROM INVESTMENT
TAX CREDIT, BY INDUSTRIAL SECTOR, 1962-70
(IN MILLIONS OF 1965 DOLLARS) *

Year	Manufacturing equipment			Nonfarm nonmanufacturing equipment		
	Investment			Investment		
	Gross	Net	Capital Stock	Gross	Net	Capital Stock
1962	185	185	0	804	804	0
1963	527	500	185	1,711	1,556	804
1964	992	891	685	2,773	2,319	2,360
1965	1,229	997	1,576	2,726	1,826	4,679
1966	1,015	637	2,573	2,069	818	6,505
1967	724	252	3,210	2,017	609	7,323
1968	635	126	3,462	2,516	991	7,932
1969	723	195	3,588	2,283	567	8,923
1970	884	328	3,783	2,330	505	9,490

* Adapted from: Robert E. Hall and Dale W. Jorgenson, "Application of the Theory of Optimal Capital Accumulation", in Tax Incentives and Capital Spending, (Washington: The Brookings Institution, 1971), p. 52.

c. Using the Koyck lag function, Coen develops an expression for gross investment, I_t , as $I_t = b(K_t^* - K_{t-1}) + \delta K_{t-1}$, where "b" is a speed of adjustment factor with $0 < b < 1$. Coen introduces a "liquidity effect" via the speed of adjustment factor to capture the impact of increased available funds from investment tax incentive programs.

This latter point is developed by Coen by first specifying gross investment, I_t , as,

$$I_t = \sum_{i=0}^n \beta_i (K_{t-i}^* - K_{t-1-i}^*) + \delta K_{t-1}$$

$$= I_{\text{net},t} + \delta K_{t-1}.$$

Utilizing his assumption of a Koyck lag distribution in the form $\beta_i = b(1-b)^i$, ($i = 0, 1, \dots$), so that the β_i 's form an infinitely decaying geometric series, the expression for I_t becomes:

$$I_t = \sum_{i=0}^{\infty} b(1-b)^i (K_{t-i}^* - K_{t-1-i}^*) + \delta K_{t-1}$$

$$= \sum_{i=0}^{\infty} b(1-b)^i K_{t-i}^* - \sum_{i=0}^{\infty} b(1-b)^i K_{t-1-i}^* + \delta K_{t-1}$$

$$= b K_t^* + (1-b) \sum_{i=0}^{\infty} b(1-b)^i K_{t-1-i}^*$$

$$- \sum_{i=0}^{\infty} b(1-b)^i K_{t-1-i}^* + \delta K_{t-1}$$

$$= b[K_t^* - \sum_{i=0}^{\infty} b(1-b)^i K_{t-1-i}^*] + \delta K_{t-1} .$$

Noting that K_t is the sum of all past net investments, or,

$$\begin{aligned} K_t &= \sum_{\tau=0}^{\infty} I_{\text{net}, t-\tau} \\ &= \sum_{\tau=0}^{\infty} \sum_{i=0}^{\infty} b(1-b)^i (K_{t-\tau-i}^* - K_{t-\tau-1-i}^*) \\ &= b(K_t^* - K_{t-1}^*) + b(1-b) (K_{t-1}^* - K_{t-2}^*) \\ &\quad + b(1-b)^2 (K_{t-2}^* - K_{t-3}^*) + \dots \\ &\quad + b(K_{t-1}^* - K_{t-2}^*) + b(1-b) (K_{t-2}^* - K_{t-3}^*) \\ &\quad + \dots + b(K_{t-2}^* - K_{t-3}^*) + \dots \\ &= b K_t^* + b(1-b) K_{t-1}^* + b(1-b)^2 K_{t-2}^* + \dots \\ &= \sum_{i=0}^{\infty} b(1-b)^i K_{t-i}^* . \end{aligned}$$

Therefore,

$$I_t = b(K_t^* - K_{t-1}) + \delta K_{t-1} .$$

Thus utilizing the above Koyck lag distribution results in net investment equalling a fraction "b" (the speed of adjustment factor) of the gap between the desired level of capital K_t^* and the existing capital stock K_{t-1} .

Coen then examines three specifications, including modelling the impact of cash flow, F_t , upon the speed of adjustment factor, b . These specifications follow:

$$I_t = b_t(K_t^* - K_{t-1}) + \delta K_{t-1}$$

where: $b_t = b = \text{a constant};$

$$\text{or, } b_t = b_0 + b_1 \frac{F_{t-1} - \delta K_{t-1}}{K_t^* - K_{t-1}};$$

$$\text{or, } b_t = b_0 + b_1 \frac{F_{t-1}}{K_t^* - (1 - \delta) K_{t-1}}.$$

Following Hall and Jorgenson and replacing optimal output with realized sales, S_R , K_t^* is modelled by Coen as a function of realized sales and the relative price of capital, $\frac{C}{W}$. F_{t-1} is the gross amount of cash flow available from both expansion and replacement, and b_0 and b_1 are parameters to be estimated. After considering the above three specifications for "b", Coen decides that the second specification is the most appropriate, or that b_t should equal,

$$b_0 + b_1 \frac{F_{t-1} - \delta K_{t-1}}{K_t^* - K_{t-1}}$$

Coen then estimates the effectiveness of investment tax incentive programs utilizing the above specification for I_t with b set equal to the second alternative above. In developing his estimates he utilizes data extracted from the Survey of Current Business as well as other sources.

Although he found that his model fit rather well, he estimates that the effectiveness of the investment tax incentive programs was rather disappointing since these policies generated far less in the growth of the capital stock than was lost in tax revenue. For example, in 1954 dollars, only \$2.0 billions in capital stock growth for manufacturing were generated from 1954 to mid-1962 while \$5.1 billions were lost in revenues.

3. Other Quantitative Approaches

Besides the major quantitative studies cited above, other studies have examined the impact of investment tax incentive programs. Two of these studies will be described briefly in the following paragraphs.

Charles Bischoff examined these programs using alternative lag distributions together with the Jorgenson-like neoclassical theory of investment and a CES production function. He undertook this approach to not only assess the magnitude of impact of incentive programs but also the timing of impact. Timing considerations are important in suggesting whether investment incentive programs could be useful as short-term stabilization instruments or as long-term means for promoting the growth of potential output. While Bischoff found that incentive programs do indeed increase investment in equipment they generally do so over a fairly long time period, although the investment

tax credit has a shorter response time than accelerated depreciation. Furthermore, changes in aggregate output appear to have a greater influence in the short run on equipment expenditures than changes in the relative price of capital (which incorporates the impact of tax incentive programs as well as other factors such as interest expense). Also he estimated that the investment tax credit adopted in 1962 probably resulted in more investment spending than revenues lost in the government.¹⁷

Klein and Taubman also analyzed the impact of investment tax incentive programs but in the context of a complete econometric model rather than in the context of the somewhat limited set of relationships utilized by other investigators. After adjusting the investment equations of the Wharton model to account for the estimated impact of investment tax incentive programs, a series of simulation exercises were performed to assess the impact of those programs upon the economy. As a result of this analysis, Klein and Taubman found that the incentive programs increased economic growth somewhat. In addition, alternative investment equations were utilized as replacements for the equations in the Wharton model. These alternative specifications were closer to those of Jorgenson's in their lag structure. Simulations using these alternative equations resulted in higher investment numbers although not as high as those of Hall and Jorgenson. They noted that these

equations would require further testing before they would be willing to stand behind the higher estimates.¹⁸

E. A Critique of the Above Approaches

In the sections above a number of models that have appeared in the economics literature were reviewed. While each of these approaches were beneficial in casting some additional light upon the value of investment tax incentive programs, it is the view of this writer that a major aspect of the evaluation process has been overlooked -- namely the value of the programs as tax reforms in the context of public finance theory.

As described in Chapter II, public finance theory outlines a number of objectives for budgetary programs including stabilization objectives. Furthermore, the theory of tax reform requires that changes to the tax system, such as investment tax incentive programs, should be assessed in light of timing and utilitarian considerations. As explained in Chapter II, stochastic control theory provides a means of both incorporating budgetary objectives (such as stabilization goals) and utilization considerations (via a loss function of the policy maker) into a process to evaluate budgetary programs in a dynamic setting. Essentially, such an analysis can be utilized to establish the opportunity cost of adopting a particular policy. This

estimated opportunity cost can be utilized as a means of evaluating an actual policy undertaken.

In contrast, the analyses presented above are for the most part rather limited in perspective. Rather than focusing upon broad budgetary objectives, these analyses focus upon the more limited question whether tax incentive programs changed investment behavior. While such analyses are beneficial in shedding some light on that rather limited question, they leave unanswered what in my view is a more important question in the context of budgetary theory -- how close the programs approximate optimality. Furthermore, the limited nature of most of the investigations cited above (with the possible exception of the Klein and Taubman study) involved the use of rather limited empirical models -- models of the investment sector alone without the accounting of potential interactions with the rest of the economy. Such interactions are potentially important, particularly the interactions with public sector revenues and the financial sector. The ignoring of such interactions by most of the studies cited above as well as differences in modelling approaches may explain the rather varied set of empirical conclusions arrived at by those studies.

In the following chapter, an illustration will be presented of how the methodology presented in Chapter II could be implemented. Although the illustration is rather

limited, due essentially to the data involved, it illustrates the essential difference in approach of the studies reviewed in this chapter and that outlined in Chapter II. Furthermore, an attempt is made to incorporate in a very general way, various sector interactions that might influence the attainment of budgetary objectives. This illustration is intended not as the last word on the examination of investment tax incentive programs, but rather as an illustration of how those programs could be examined from a budgetary prospective.

NOTES

CHAPTER III

¹For a basic introduction to investment theory refer to R. G. D. Allen, Macro - Economic Theory (New York: St. Martin's Press, 1968), pp. 62-66. For a more advanced discussion with the impact of tax systems included, see Alan J. Auerbach, "Taxation, Corporate Financial Policy, and the Cost of Capital," Journal of Economic Literature 21 (September 1983) : 905-940.

²E. Cary Brown, "Business-Income Taxation and Investment Incentives," in Income, Employment and Public Policy: Essays in Honor of Alvin H. Hansen, Lloyd A. Meltzer, ed. (New York: W.W. Norton and Co., 1948), pp. 300-316.

³Richard Musgrave, The Theory of Public Finance (New York: McGraw-Hill Book Co., 1959), pp. 338-346.

⁴This approach was originally presented without the inclusion of the investment tax credit rate in Leo C. Kamp, "Accelerated Depreciation as a Tax Incentive and its Impact Upon Investment Behavior," M.A. thesis, Hunter College, 1974.

⁵Paul A. Samuelson, "Tax Deductibility of Economic Depreciation to Insure Invariant Valuations," Journal of Political Economy 72 (December 1964) : 604-606.

⁶Dale W. Jorgenson and Robert E. Hall, "Application of the Theory of Optimal Capital Accumulation," in Tax Incentives and Capital Spending, Gary Fromm, ed. (Washington: The Brookings Institution, 1971), pp. 9-20.

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¹² Alan J. Auerbach, "A Note on the Efficient Design of Investment Incentives," pp. 7-8.

¹³ George J. Stigler, The Theory of Price (3rd edition, New York: Macmillan, 1966), pp. 96-98.

¹⁴ Hall and Jorgenson, "Application of the Theory of Optimal Capital Accumulation," pp. 23-60.

¹⁵ Shirley Almon, "The Distributed Lag between Capital Appropriations and Expenditures," Econometrica 33 (January 1965), pp. 178-196.

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¹⁷ Charles W. Bischoff, "The Effect Of Alternative Lag Distributions," in Tax Incentives and Capital Spending, Gary Fromm, ed. (Washington: The Brookings Institution, 1971), pp. 61-130.

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CHAPTER IV

AN ILLUSTRATION OF HOW THE SUGGESTED EVALUATION TECHNIQUE COULD BE IMPLEMENTED

In the previous chapter a number of models that have appeared in the investment tax incentive literature were reviewed and were found to have taken an alternative evaluation approach to the optimal control approach suggested in Chapter II. The purpose of this chapter is to illustrate how the optimal control approach of Chapter II could be utilized as an additional means of evaluation by policy makers. This illustration will focus on the requirements of implementation (including problem areas) as well as a discussion of results obtained.

Since stochastic optimal control theory presumes a model that depicts the stochastic behavior of variables of interest, the discussion will focus first upon a number of data analyses to investigate the behavior of variables utilized, and then will turn to the specification of a stochastic representation of interrelationships among those variables. Once that task has been accomplished, the specified representation will be estimated and utilized within the stochastic control framework of Chapter II to

illustrate the evaluation of historical values of a tax incentive variable. The results of that exercise then will be examined critically, together with certain appropriate qualifications.

A. An Overview to the Suggested Approach to Evaluating Investment Tax Incentive Measures

This chapter will focus upon the implementation of a methodology that could be utilized to evaluate investment tax incentive measures. The methodology involves using a stochastic model together with stochastic control theory to establish a standard against which an actual measure can be compared. The method utilizes the welfare cost of an optimal value of an incentive measure and the welfare cost of an actual value to establish that standard. Optimality is attained by utilizing the stochastic control methodology outlined in Chapter II. That methodology requires representing the stochastic behavior of the variables to be utilized, the specification of target values for variables of interest, and the relative importance of deviations of targeted variables from their targets (i.e. the specification of weights in the policy maker's objective function). Furthermore, due to the computational intensity of this approach, an appropriate computer program is necessary to carry out the required calculations, including the various welfare cost calculations. Each of these required areas in

implementing the approach are discussed in turn in the following sections.

B. Representing the Stochastic Behavior of Variables to Be Utilized

This section will describe the analyses that were undertaken to arrive at a representation of relationships between the variables utilized in this illustration. The first step undertaken was a preliminary examination of the data available relating to investment tax incentive plans to determine its suitability for empirical analysis. Then the incentive data were utilized with selected macroeconomic variables to examine the "exogeneity" of the investment tax incentive variables and to ascertain whether any of the tax incentive variables affected the behavior of the macroeconomic variables. Once that stage was completed, a multiple time series representation of the variables utilized was specified and estimated.

1. Description of Data Utilized

The data utilized in this empirical illustration came from a variety of primary sources. However, all of the investment tax incentive data were obtained from Data Resources, Inc. (DRI), which derived the tax incentive data from U.S. Treasury Department sources. The macroeconomic data, which also were retrieved from the DRI data banks,

had U.S. Government agencies as their primary source. The data were retrieved for the period from the first quarter of 1959 through the final quarter of 1983. This period was chosen to incorporate the introduction of the investment tax credit in 1962 and to encompass the turbulent post 1979 period as an evaluation period. A quarterly frequency was utilized in order to provide a sufficient number of observations for empirical analysis. In addition, the quarterly frequency was chosen to reflect the fact that policy makers probably would be unwilling to make changes to investment tax incentive programs more frequently than once a quarter.

The investment tax incentive data, obtained from DRI, consisted of data on average tax lives and marginal rates of investment tax credit. Specifically, data on average tax lives and marginal rates of investment tax credit were obtained for investments in nonresidential structures by public utilities and by businesses other than public utilities, and for investments in nonresidential producers' durable equipment. However, due to a lack of observations, the marginal rate of investment tax incentive variables for structures (both public utility and other) were excluded from further analysis. The numerical values of the remaining three tax incentive series are contained in Appendix A. Note the sharp reductions in depreciation tax lives starting in the third quarter of 1983 with the passage of ERTA.

The macroeconomic data consisted of only three variables in order to limit the complexity of the investigation. The variables were chosen to detect and estimate interrelationships between the aggregate price level, unemployment, and nominal monetary aggregate, all of which could be of interest to a policy maker. Specifically the variables chosen were the implicit price deflator for gross national product, the civilian unemployment rate, and the nominal stock of money as measured by M1. The source for the first series, the implicit price deflator, is The Survey of Current Business published by the Bureau of Economic Analysis of the U.S. Department of Commerce. The second series, the unemployment rate, has as its source Employment and Earnings, a monthly publication of the U.S. Department of Labor. And finally the third variable, the nominal stock of money (M1 definition), was obtained from the monthly Federal Reserve Bulletin of the Board of Governors of the Federal Reserve System. Series that were reported originally as monthly data, namely the unemployment rate and M1, were converted for this investigation into quarterly averages from the monthly data. All macroeconomic data were seasonally adjusted. The numerical values of the three macroeconomic variables also are displayed in Appendix A. Plots of the macroeconomic variables are presented in Figures 3, 4, and 5. Note the upward

trends evident in M1 and the GNP deflator and the absence of trend in the unemployment rate.

2. Methodology Utilized in Analyzing the Data and in Representing Interrelationships Among the Data

Once the data were obtained, a preliminary investigation was undertaken to ascertain whether any of the tax incentive data was linearly related to the macroeconomic data. As a preliminary approach, vector autoregressions (VAR's) were run, where all variables were presumed endogenous, except for deterministic variables such as linear time trend and a constant term. Specifically, each VAR included all of the macroeconomic variables listed above and a single selected tax incentive variable, resulting in a total of six VAR's, one for each tax incentive variable, each consisting of four regression equations.

As background to the analyses that follow, a brief discussion is presented below of VAR's and certain related tests.¹ One can represent a VAR generally as:

$$A(L)Z_t = U_t \quad (4.1)$$

where: $A(L) = A_0 - A_1L - \dots - A_qL^q$,

a matrix polynomial in the lag operator, L , having a maximum lag, q , and $(n+m) \times (n+m)$ matrices of parameter values (the A 's in the polynomial);

FIGURE 3
NOMINAL STOCK OF MONEY
M1 DEFINITION-BILLIONS OF DOLLARS

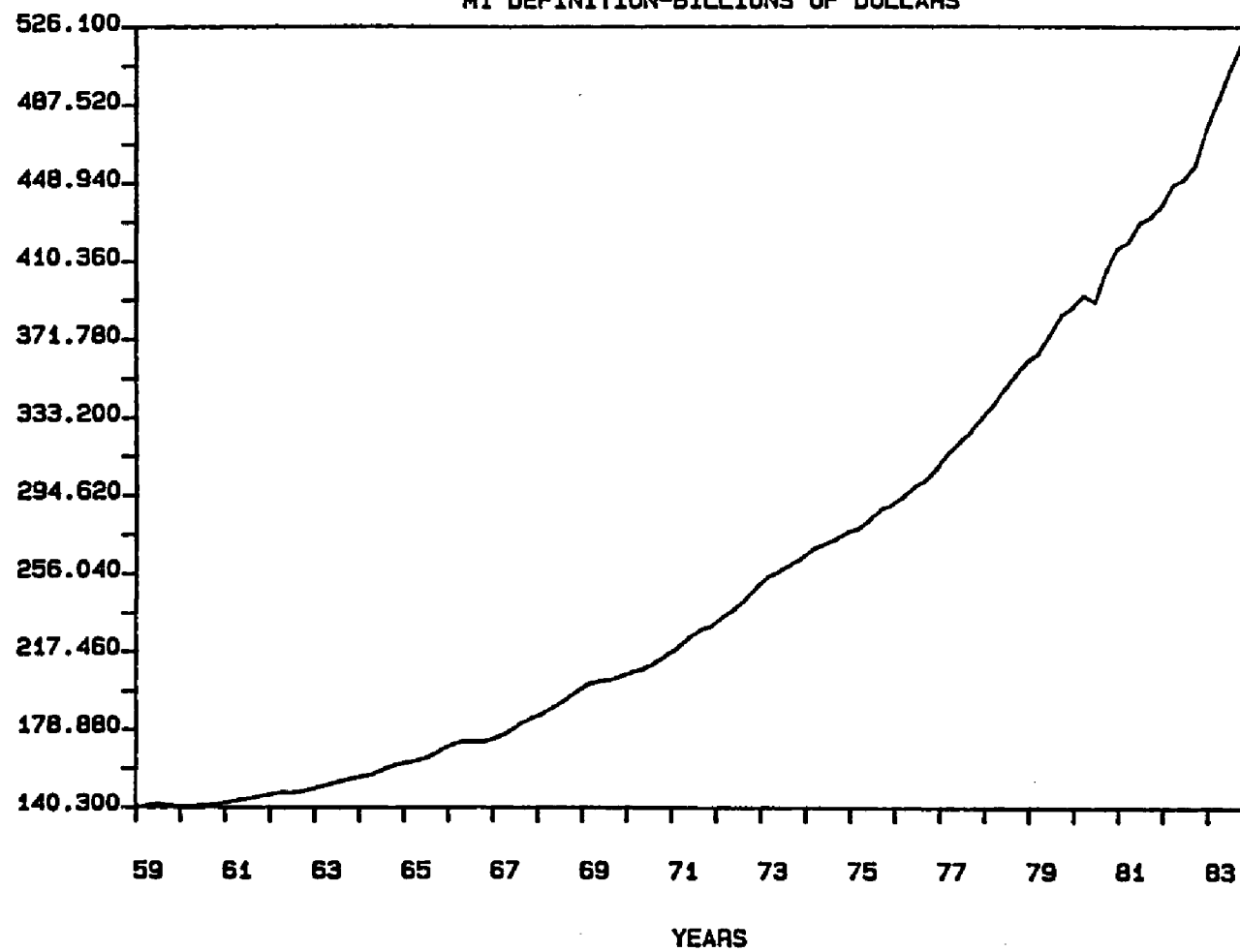


FIGURE 4
CIVILIAN UNEMPLOYMENT RATE
PERCENT

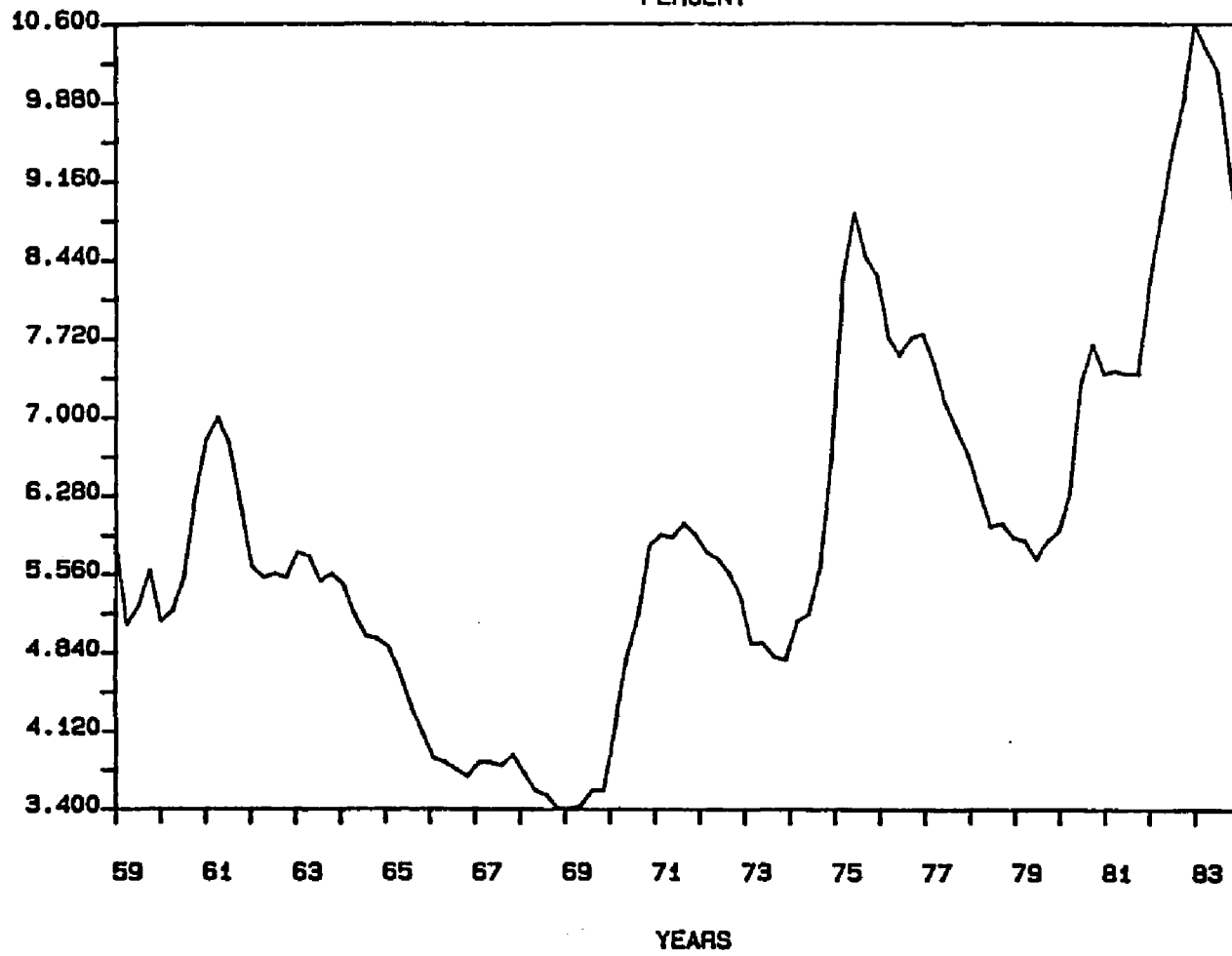
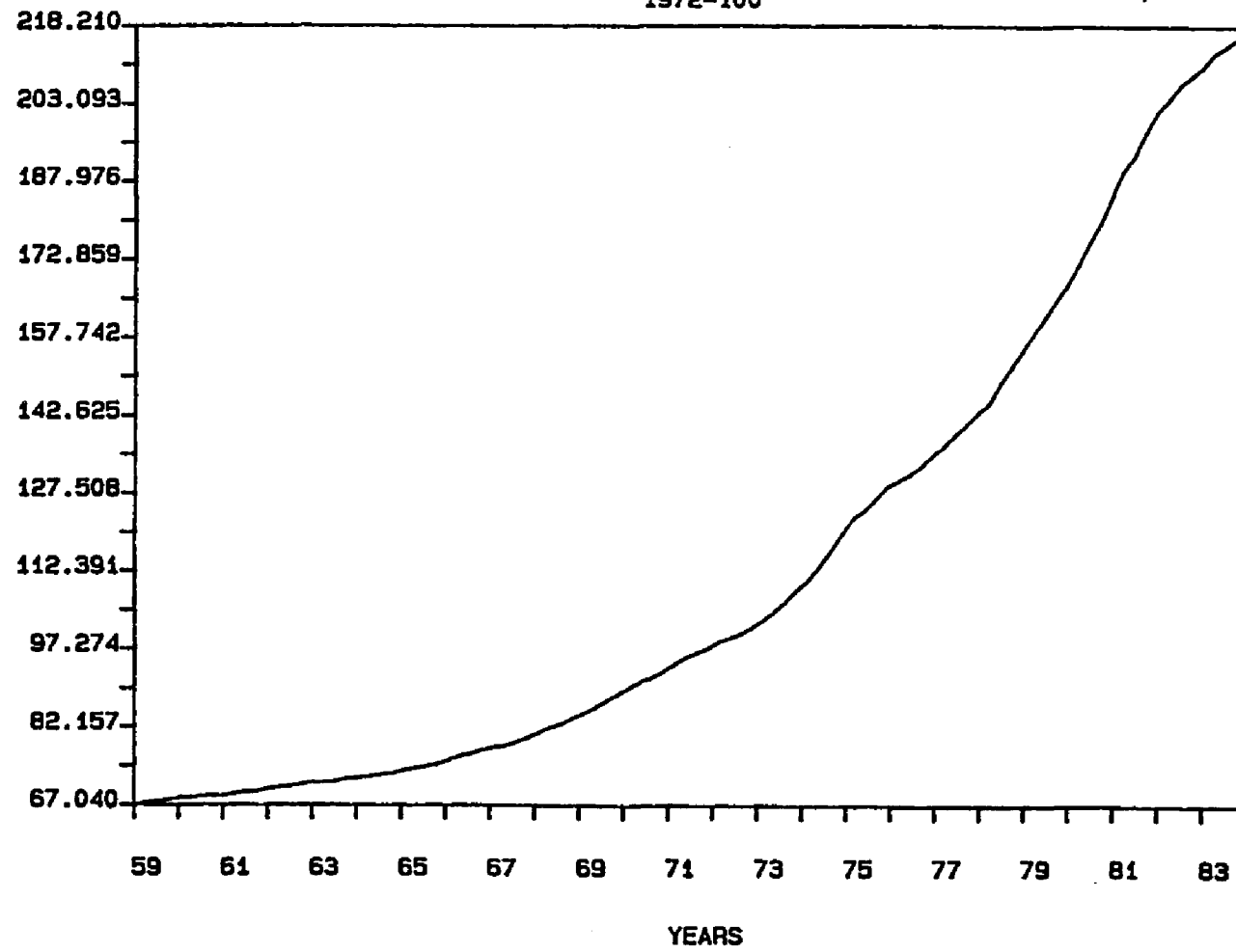


FIGURE 5
IMPLICIT PRICE DEFLATOR-GNP
1972=100



Z_t = a vector of n current endogenous variables, Y_t , and m current exogenous variables, X_t , where deterministic elements, such as constant and linear time trend have been accounted for;

U_t = a $(n+m) \times 1$ vector of error (or innovation) terms.

Note that if A_0 is the identity matrix, the variables in the vector Z_t would be endogenous.

If in estimating VAR's one assumes that Z_t is correlated with past Z_t 's, one could form a best linear predictor of Z_t , $\hat{Z}_t$, that is assumed to be well approximated by lagged values of Z_t , such that the weight on any lagged Z_t is dependent on the lag but not on t . With $\hat{Z}_t$ being the best linear predictor and defining $U_t = Z_t - \hat{Z}_t$, then U_t is a linear combination of past Z_t , and U_t is serially uncorrelated.

One can also represent Z_t as a moving average representation (MAR), that is, as a distributed lag on U_t , so that,

$$Z_t = \sum_{s=0}^{\infty} B_s U_{t-s} \quad (4.2)$$

where: B_s = an $(n+m) \times (n+m)$ matrix of parameters analogous to the "A" matrices in the VAR representation of Z_t .

Note that the B 's in the MAR are also dependent on s but not on t .

The coefficients of the MAR can be interpreted as the response of Z_t to an innovation at lag s . Furthermore the coefficients of the MAR can be more readily interpreted than the coefficients of the VAR representation. In fact, one can derive the MAR from the VAR representation. From expressions 4.1 and 4.2 above one can write,

$$Z_t = [A(L)]^{-1} U_t = \sum_{s=0}^{\infty} B_s U_{t-s} \quad (4.3)$$

or the B 's in the MAR are derived by inverting the matrix polynomial, $A(L)$. The B 's in the MAR are equivalent to simulating the VAR of Z_t , when for a particular equation j in the VAR, $U_{j,t} = 1$ and $U_{i,t+s} = 0$ for i not equal to j and $s = 0, 1, 2, \dots$, with initial condition $Z_{t-r} = 0$ for $r = 0, 1, 2, \dots, q$. One can interpret the i, j 'th component of B_s , b_{ij} , as the response s periods ahead of variable i to an initial shock to the j 'th variable.

The above simulation, however, ignores the possibility of an innovation to one variable in the system affecting other variables in the system contemporaneously. In fact, it may not be possible to directly decompose the

variance of Z_t into components attributable to the innovations of each variable due to possible contemporaneous cross-correlations between the innovations of the variables in the system. To handle such a situation, it is appealing to apply a transformation to U_t , to obtain $V_t = TU_t$, where T is chosen to make the covariance matrix of V_t the identity matrix, so that the variances and covariances of components of V_t are one and zero respectively. This results in the components of V_t being contemporaneously orthogonal to each other, or having no contemporaneous cross-correlation. The MAR then assumes the following form:

$$Z_t = \sum_{s=0}^{\infty} B_s T^{-1} v_{t-s}$$

Although there are several ways one could choose the matrix T^{-1} , a lower triangular matrix derived from a Choleski factorization of the covariance matrix, D , of U_t is frequently used. In particular, if T^{-1} is derived utilizing a Choleski factorization of D , then $D = T^{-1}(T^{-1})'$, where T^{-1} is lower triangular with positive diagonal elements. This transformation now allows us to examine the response of the variables in the system to a one standard deviation innovation to a selected variable in the system (i.e. to determine the impulse responses) and to determine the proportion of the k 'th step ahead forecast error variance of a particular variable accounted for by an innovation to a particular variable in the system (i.e. to determine the

variance decomposition). For both analyses, the results are potentially sensitive to the ordering of D utilized in calculating the Choleski factor, meaning that in empirical work a number of alternative orderings should be examined to ascertain such sensitivities.

Regarding the variance decomposition specifically, utilizing orthogonal innovations, one can write the k-step forecast error variance as,

$$\begin{aligned} Z_{t+k} - E_t Z_{t+k} &= B_0 T^{-1} v_{t+k} + B_1 T^{-1} v_{t+k-1} + \dots + B_{k-1} T^{-1} v_{t+1} \\ &= \sum_{s=0}^{k-1} B_s T^{-1} v_{t+k-1-s}, \end{aligned}$$

where B_0 is the identity matrix and E is the expectation operator.

Once one has estimated a VAR utilizing regression techniques, one can form estimates of the matrices $B_s T^{-1}$, which we denote $\tilde{B}_s$. If one lets the i, j 'th component of $\tilde{B}_s$, $\tilde{b}_{ij}$, be the response of variable i to an orthogonalized innovation of one standard deviation to variable j , then one can write the part of the k-step forecast error variance of variable i accounted for by an innovation to the j 'th variable of Z_t as,

$$\sum_{s=0}^{k-1} \tilde{b}_{ij}^2(s),$$

and the total forecast error variance as,

$$\sum_{j=1}^{m+n} \sum_{s=0}^{k-1} \tilde{b}_{ij}^2(s),$$

since the variances and covariances of the orthogonalized innovations are one and zero respectively by construction. Therefore, the proportion of the k -step forecast error variance of variable i of Z_t accounted for by an orthogonalized one standard deviation innovation to variable j of Z_t can be written as,

$$\frac{\sum_{s=0}^{k-1} \tilde{b}_{ij}^2(s)}{\sum_{j=1}^{m+n} \sum_{s=0}^{k-1} \tilde{b}_{ij}^2(s)}$$

This proportion is known as the variance decomposition of variable i . While the forecast variance does not depend upon the particular ordering of D used in deriving the matrix T^{-1} , the variance decomposition does. Therefore in utilizing variance decompositions, one should examine alternative orderings of D to ascertain the sensitivity of the decomposition to the particular ordering used. In the empirical work that follows, variance decomposition computations will be examined to gain insights into the interrelationships of the variables in the system, especially the question of "exogeneity" of certain variables. Since one would expect the forecast variance of "exogenous" variables to be explained primarily by their own innovations, variance decompositions can be used to shed some light on whether a variable may be treated as "exogenous".

Obviously from the discussion above, the use of VAR's requires some notion of an appropriate size for the VAR.

The size of a VAR is a function of the number of variables included and of the number of lags utilized. Given that one is constrained to a limited number of time-series observations on a given set of variables, some choice must be made of an appropriate maximum lag length to be utilized in an empirical investigation. Christopher Sims has suggested a modified likelihood ratio technique that can be utilized to set an appropriate lag length.² His suggestion relies on the following formula,

$$(T-k)[\log \det D_r - \log \det D_u]$$

where: $\log \det D_r$ = the natural log of the determinant of D_r , the covariance matrix of the residuals for a "restricted" VAR;

$\log \det D_u$ = the natural log of the determinant of D_u , the covariance matrix of the residuals for an restricted" VAR;

T = number of observations utilized;

k = average number of regressors for each of the equations in the "unrestricted" VAR.

The computed statistic (hereafter referred to as the Sims statistic) from the above formula is distributed as chi-squared with degrees of freedom equal to the total number

of restrictions in the "restricted" VAR (i.e. the difference between the total number of regressors in the "unrestricted" VAR and in the "restricted" VAR). This procedure (referred to below as the Sims test) has been tested by another investigator and found to be accurate although it probably results in some overparameterization.³ The Sims test will be utilized in the empirical work below in determining an appropriate lag length for the VAR's utilized, but also in determining whether a block of variables can be considered to have an "exogenous" impact upon the "endogenous" variables of the system. In the case of determining an appropriate lag length, two otherwise identical VAR's are compared using the above test, one with one additional lag, to determine whether increasing the lag length by one adds significantly to the explanatory power of the VAR. For a block "exogeneity" test, two otherwise identical VAR's are utilized, except that one contains a set of "exogenous" variables as part of the deterministic component of the VAR. With that construct, one then uses the above Sims test to ascertain whether the block of "exogenous" variables significantly influences the explanatory power of the other VAR not containing any "exogenous" variables.

In addition to these procedures, partial F tests were

utilized to screen for interrelationships between the variables utilized. In particular the following test statistic was calculated,

$$\frac{(RSS_r - RSS_{ur})/q}{RSS_{ur}/(N - k)}$$

where: RSS_r = residual sum of squares of restricted regression (i.e. a regression with zero restrictions on the coefficients of certain variables;

RSS_{ur} = residual sum of squares of unrestricted regression;

q = number of coefficient restrictions in restricted regression;

N = number of observations;

k = number of regressors (including constant) in unrestricted regression.

This test statistic is distributed as a F distribution with q degrees of freedom in the numerator and $N-k$ degrees of freedom in the denominator.⁴ The test enables one to test exclusion restrictions on a block of variables in regressions (i.e. to test whether the coefficients on the variables being tested are jointly significantly different from zero). This test was useful in determining whether certain variables have influenced other variables, and in determining whether certain variables have behaved

"exogenously" in the sense that "exogenous" variables are not dependent upon other variables in the system.

The techniques described above are computationally intensive. Fortunately, computer packages exist that facilitate the estimation of VAR's and other time-series related work. One such package is the Regression Analysis of Time Series (RATS) program developed by Robert Litterman and Thomas Doan.⁵ RATS contains a series of instructions that enable one to efficiently set up VAR's, to determine associated MAR's, to calculate variance decompositions, and to perform numerous statistical procedures, including the procedures outlined above. All of the empirical work that follows was performed utilizing the microcomputer version of RATS, including the matrix program written by this investigator to make the optimal control calculations.

In the following section, results will be presented of an empirical investigation which utilized the analytical tools presented in this section. That investigation resulted in a representation of the stochastic behavior of the variables utilized in the stochastic control evaluation illustration presented later in this chapter.

3. Empirical Results

This section will focus upon the procedures that were used to formulate a representation of the interrelationships between the variables utilized in illustrating the stochastic control evaluation technique of Chapter II. In the process establishing such a representation, insights concerning such relationships were developed, particularly the linear interactions (or lack of interactions) between the tax incentive variables and the macroeconomic variables. The discussion below will first describe certain data considerations; then will turn to a description of the procedures utilized; and finally will describe the representation that was estimated.

The data utilized in this investigation consisted of the macroeconomic and tax incentive variables described earlier, in the form of levels. Furthermore, the empirical work of this section was based upon data ending the last quarter of 1979. This specification of the observation set used was to allow the remaining data (i.e. the data through the final quarter of 1983) in the overall data set to be utilized as an evaluation period in the optimal control evaluation exercise presented below.

a. Preliminary Data Analysis

The first test undertaken was a test for an appropriate lag length. This test utilized the procedure described above in conjunction with Sims' modified likelihood ratio test. To review, this procedure tests whether a given VAR would be increased in explanatory power by increasing the lag length by some stipulated amount. In the test conducted here, two otherwise identical VAR's were utilized, but with one containing an additional lag. Both however included, as part of their variables, a given investment tax incentive variable having the same lag length as other variables in the particular VAR. The test was extended only to five lags, since greater lag lengths would have resulted in an unacceptable parameter to observation ratio for the given estimation period. The Sims statistic then was calculated and compared to an chi-squared distribution with appropriate degrees of freedom to ascertain significance. The results of that exercise are displayed in Table 4 for a number of lag lengths and tax incentive variables included. The program that generated those results is listed in Appendix B. From Table 4, one can see that extending the lag length to five resulted in no significant improvement in the explanatory power of the VAR utilized. This result is similar to the one obtained

TABLE 4

TESTS FOR APPROPRIATE LAG LENGTH USING A VAR ON
SELECTED MACROECONOMIC VARIABLES AND
THE TAX INCENTIVE VARIABLE INDICATED BELOW

LAG LENGTH	TAX INCENTIVE VARIABLE INCLUDED			
	RITC	LIFENPU	LIFEPU	LIFEEQP
2	S	S	S	S
3	S	S	S	S
4	S	N	N	N
5	N	N	N	N

LEGEND:

S = SIGNIFICANT AT 5% LEVEL

N = NOT SIGNIFICANT AT 5% LEVEL

RITC = MARGINAL RATE OF INVESTMENT TAX INCENTIVE CREDIT ON EQUIPMENT

LIFENPU = DEPRECIATION TAX LIFE ON NON-PUBLIC UTILITY STRUCTURES

LIFEPU = DEPRECIATION TAX LIFE ON PUBLIC UTILITY STRUCTURES

LIFEEQP = DEPRECIATION TAX LIFE ON EQUIPMENT

by Sims in another study that also utilized quarterly data.⁶ Hence a lag length of four was utilized in all VAR's in this study.

The second set of tests used partial F tests to examine interrelationships between variables with particular attention paid to which of the tax incentive variables could be considered "exogenous". An "exogenous" variable would not be influenced by other variables in a VAR, a feature that was tested by means of the partial F tests. Since the particular approach to stochastic control utilized in the illustration below presumes the existence of an "exogenous" variable (known as a control instrument) under the control of the policy maker, it is important to test the "exogeneity" of the variable to be utilized as a control instrument. The results of the partial F tests are shown in Tables 5 through 8. The program that generated those results is listed in Appendix C. Based on the results of those tests, one could conclude that the tax incentive variable behaving most exogenously was the depreciation tax life for non-public utility structures (henceforth denoted TLNPUS), although the marginal rate of investment tax credit for non-residential producers' durable equipment (hereafter referred to as RITC) exhibited "exogeneity" at lag lengths one through three. In addition, there appeared to be only a limited impact of the TLNPUS upon the macroeconomic variables and only at

TABLE 5

TESTS FOR INTERRELATIONSHIPS BETWEEN THE MARGINAL
RATE OF INVESTMENT TAX CREDIT ON PRODUCERS' DURABLE
EQUIPMENT AND MACROECONOMIC VARIABLES

LAG LENGTH	LAGGED ENDOGENOUS VARIABLES	CURRENT ENDOGENOUS VARIABLES			
		M1	UNEMP	PGNP	RITC
1	M1	S	N	S	N
	UNEMP	N	S	N	N
	PGNP	N	N	S	N
	RITC	N	S	N	S
2	M1	S	N	S	N
	UNEMP	N	S	N	N
	PGNP	N	S	S	N
	RITC	N	N	N	S
3	M1	S	S	S	N
	UNEMP	N	S	N	N
	PGNP	S	S	S	N
	RITC	N	N	N	S
4	M1	S	S	S	S
	UNEMP	N	S	N	S
	PGNP	S	S	S	N
	RITC	N	N	N	S

LEGEND: S = SIGNIFICANT AT 5% LEVEL: N = NOT SIGNIFICANT AT 5% LEVEL
M1 = NOMINAL STOCK OF MONEY (M1 DEFINITION)
UNEMP = CIVILIAN UNEMPLOYMENT RATE
PGNP = IMPLICIT PRICE DEFLATOR ON GNP
RITC = MARGINAL RATE OF INVESTMENT TAX CREDIT ON PRODUCERS'
DURABLE EQUIPMENT

TABLE 6

TESTS FOR INTERRELATIONSHIPS BETWEEN THE DEPRECIATION
TAX LIFE ON NON-PUBLIC UTILITY STRUCTURES
AND MACROECONOMIC VARIABLES

LAG LENGTH	LAGGED ENDOGENOUS VARIABLES	CURRENT ENDOGENOUS VARIABLES			
		M1	UNEMP	PGNP	LIFENPU
1	M1	S	S	S	N
	UNEMP	N	S	N	N
	PGNP	N	S	S	N
	LIFENPU	N	S	N	S
2	M1	S	N	S	N
	UNEMP	N	S	N	N
	PGNP	N	S	S	N
	LIFENPU	N	S	N	S
3	M1	S	S	N	N
	UNEMP	N	S	N	N
	PGNP	N	S	S	N
	LIFENPU	N	N	N	S
4	M1	S	S	S	N
	UNEMP	N	S	N	N
	PGNP	S	S	S	N
	LIFENPU	N	N	N	S

LEGEND: S = SIGNIFICANT AT 5% LEVEL; N = NOT SIGNIFICANT AT 5% LEVEL
M1 = NOMINAL STOCK OF MONEY (M1 DEFINITION)
UNEMP = CIVILIAN UNEMPLOYMENT RATE
PGNP = IMPLICIT PRICE DEFLATOR ON GNP
LIFENPU = DEPRECIATION TAX LIFE ON NON-PUBLIC UTILITY STRUCTURES

TABLE 7

TESTS FOR INTERRELATIONSHIPS BETWEEN THE DEPRECIATION
TAX LIFE ON PUBLIC UTILITY STRUCTURES
AND MACROECONOMIC VARIABLES

LAG LENGTH	LAGGED ENDOGENOUS VARIABLES	CURRENT ENDOGENOUS VARIABLES			
		M1	UNEMP	PGNP	LIFEPU
1	M1	S	N	S	N
	UNEMP	N	S	N	S
	PGNP	N	N	S	S
	LIFEPU	N	N	N	S
2	M1	S	N	S	N
	UNEMP	N	S	N	S
	PGNP	N	S	S	N
	LIFEPU	N	N	N	S
3	M1	S	S	N	N
	UNEMP	N	S	N	S
	PGNP	N	S	S	S
	LIFEPU	N	N	N	S
4	M1	S	S	N	N
	UNEMP	N	S	N	S
	PGNP	S	S	S	S
	LIFEPU	N	N	N	S

LEGEND: S = SIGNIFICANT AT 5% LEVEL; N = NOT SIGNIFICANT AT 5% LEVEL
M1 = NOMINAL STOCK OF MONEY (M1 DEFINITION)
UNEMP = CIVILIAN UNEMPLOYMENT RATE
PGNP = IMPLICIT PRICE DEFLATOR ON GNP
LIFEPU = DEPRECIATION TAX LIFE ON PUBLIC UTILITY STRUCTURES

TABLE 8

TESTS FOR INTERRELATIONSHIPS BETWEEN THE DEPRECIATION
TAX LIFE ON PRODUCERS' DURABLE EQUIPMENT
AND MACROECONOMIC VARIABLES

LAG LENGTH	LAGGED ENDOGENOUS VARIABLES	CURRENT ENDOGENOUS VARIABLES			
		M1	UNEMP	PGNP	LIFEEQP
1	M1	S	N	S	N
	UNEMP	N	S	N	S
	PGNP	N	N	S	N
	LIFEEQP	N	N	N	S
2	M1	S	N	S	N
	UNEMP	N	S	N	S
	PGNP	N	S	S	N
	LIFEEQP	N	N	N	S
3	M1	S	S	S	N
	UNEMP	N	S	N	S
	PGNP	N	S	S	N
	LIFEEQP	N	N	N	S
4	M1	S	S	N	N
	UNEMP	N	S	N	S
	PGNP	S	S	S	S
	LIFEEQP	N	N	N	S

LEGEND: S = SIGNIFICANT AT 5% LEVEL; N = NOT SIGNIFICANT AT 5% LEVEL
M1 = NOMINAL STOCK OF MONEY (M1 DEFINITION)
UNEMP = CIVILIAN UNEMPLOYMENT RATE
PGNP = IMPLICIT PRICE DEFLATOR ON GNP
LIFEEQP = DEPRECIATION TAX LIFE ON PRODUCERS' DURABLE EQUIPMENT

short lag lengths. Note however that interrelationships between the macroeconomic variables become more numerous as the lag length was increased to four, affirming the appropriateness of a lag length of four.

However, to further investigate the conclusions concerning the "exogeneity" of the TLNPUS and the RITC, and the limited influence of those tax incentive variables upon macroeconomic variables, an additional set of exercises was performed. Those exercises involved examining variance decompositions of VAR's of lag length four containing either the TLNPUS or the RITC. For each tax incentive variable, two orderings for the covariance matrix of the residuals were examined to ascertain the sensitivity of the results to different orderings. Since an "exogenous" variable would be expected to have its forecast variance almost entirely explained by its own innovations at a given forecast step, variance compositions provide another means of examining "exogeneity". Selected results of these variance decompositions are displayed in Table 9. The program that generated those results is listed in Appendix D. From the results, it is evident that the TLNPUS displays variance decompositions expected of an "exogenous" variable, since a large percentage of its forecast error variance is accounted for by its own innovations, regardless of forecast step or the ordering utilized in

TABLE 9

PERCENTAGES OF FORECAST ERROR k QUARTERS AHEAD
PRODUCED BY EACH INNOVATION *

FORECAST ERROR IN:	ORTHOGONAL INNOVATION IN:				
	k	LIFESNPU	UNEMP	PGNP	M1
LIFESNPU	1	100.0	0.0	0.0	0.0
	4	92.1	1.5	5.4	1.0
	8	86.7	3.9	7.6	1.7
	16	77.3	14.1	5.8	2.8
UNEMP	1	0.0	100.0	0.0	0.0
	4	0.8	79.5	7.2	12.5
	8	0.9	72.7	11.7	14.7
	16	11.3	62.9	12.9	12.9
PGNP	1	2.7	8.5	88.8	0.0
	4	12.1	4.4	78.1	5.4
	8	15.6	2.7	53.8	27.9
	16	8.9	16.5	23.6	51.0
M1	1	1.3	7.4	2.9	88.4
	4	7.9	11.7	1.8	78.7
	8	28.9	21.6	1.8	47.7
	16	24.8	41.8	2.8	30.6

NOTES: * THE DECOMPOSITION UTILIZED IS IN THE ORDER
OF THE VARIABLES LISTED ACROSS THE TOP.

THE SUM OF PERCENTAGES IN A GIVEN ROW MAY NOT
EQUAL 100% DUE TO ROUNDING.

LEGEND: LIFESNPU = DEPRECIATION LIFE FOR INVESTMENTS
IN NON-PUBLIC UTILITY STRUCTURES
RITC = MARGINAL RATE OF INVESTMENT TAX
CREDIT FOR PRODUCERS' DURABLE
EQUIPMENT
M1 = NOMINAL STOCK OF MONEY (M1
DEFINITION)
UNEMP = CIVILIAN UNEMPLOYMENT RATE
PGNP = IMPLICIT PRICE DEFLATOR ON GNP

TABLE 9 - CONTINUED

PERCENTAGES OF FORECAST ERROR k QUARTERS AHEAD
PRODUCED BY EACH INNOVATION *

FORECAST ERROR IN:	k	ORTHOGONAL INNOVATION IN:			
		M1	UNEMP	PGNP	RITC
M1	1	100.0	0.0	0.0	0.0
	4	87.3	0.2	8.6	4.0
	8	78.7	1.3	17.2	2.8
	16	77.9	5.7	13.9	2.5
UNEMP	1	5.0	95.0	0.0	0.0
	4	19.7	69.3	10.8	0.3
	8	26.8	52.1	16.1	5.0
	16	21.2	44.8	28.0	6.0
PGNP	1	0.3	5.3	94.5	0.0
	4	2.0	1.4	95.6	1.0
	8	20.7	1.5	73.8	3.9
	16	61.7	6.1	26.8	5.3
RITC	1	5.3	0.1	0.0	94.5
	4	14.7	0.5	14.7	70.2
	8	14.4	3.2	19.4	63.1
	16	15.9	14.5	19.3	50.3

TABLE 9 - CONTINUED

PERCENTAGES OF FORECAST ERROR k QUARTERS AHEAD
PRODUCED BY EACH INNOVATION *

FORECAST ERROR IN:	k	ORTHOGONAL INNOVATION IN:			
		RITC	UNEMP	PGNP	M1
RITC	1	100.0	0.0	0.0	0.0
	4	79.3	0.6	14.9	5.2
	8	70.1	4.0	18.8	7.1
	16	56.1	17.8	18.3	7.7
UNEMP	1	0.0	99.9	0.0	0.0
	4	1.4	79.9	9.2	9.4
	8	8.9	64.0	13.8	13.2
	16	7.8	52.1	24.5	15.6
PGNP	1	0.1	4.5	95.4	0.0
	4	2.0	1.0	96.6	0.4
	8	9.1	3.7	77.2	10.0
	16	15.7	16.5	27.6	40.3
M1	1	5.3	4.8	0.9	88.9
	4	15.4	4.3	5.3	75.0
	8	11.9	7.2	12.0	68.8
	16	11.1	15.2	10.4	63.2

TABLE 9 - CONTINUED

PERCENTAGES OF FORECAST ERROR k QUARTERS AHEAD
PRODUCED BY EACH INNOVATION *

FORECAST ERROR IN:	k	ORTHOGONAL INNOVATION IN:			
		M1	UNEMP	PGNP	LIFESNPU
M1	1	100.0	0.0	0.0	0.0
	4	90.4	0.9	5.8	2.9
	8	63.1	8.6	8.6	19.7
	16	53.3	23.5	4.2	19.0
UNEMP	1	7.3	92.7	0.0	0.0
	4	23.7	64.7	8.6	2.9
	8	25.0	57.6	13.8	3.6
	16	19.7	52.5	17.6	10.2
PGNP	1	0.4	10.1	89.4	0.0
	4	6.7	7.8	80.1	5.4
	8	28.7	6.1	52.9	12.3
	16	62.2	7.5	22.5	7.8
LIFESNPU	1	1.3	0.2	3.8	94.7
	4	0.8	1.2	15.1	82.9
	8	0.5	3.9	18.2	77.3
	16	0.5	14.4	14.2	70.9

calculating the factorization. While the RITC also exhibited "exogenous" behavior, the variance decompositions for the RITC displayed a somewhat sharper rate of decline with increasing forecast steps, a fact suggesting that the TLNPUS behaved more exogenously than the RITC. Based on these results and the fact that a priori one would expect the TLNPUS to be "exogenous" since it is stipulated by the tax code, the TLNPUS was treated as an exogenous variable in the remainder of this study. Furthermore, since only one control instrument is utilized in the illustration that follows, the TLNPUS was selected as the sole control instrument utilized in that exercise. Note also from the variance decompositions that innovations to the TLNPUS has at most a moderate impact upon the forecast error variances of the macroeconomic variables, meaning that an innovation to the TLNPUS could be expected to have at most moderate impact upon the future movement of the macroeconomic variables. This of course has implications to the ability of the TLNPUS to control macroeconomic behavior in the control exercise below.

To further refine the exogenous treatment of the TLNPUS in the estimation of the regressions utilized, a few other tests were performed utilizing the Sims test described above. Specifically, block exogeneity tests were performed, utilizing a VAR with a lag length of four and three different specifications for the portion of the

deterministic component of the unrestricted VAR containing the TLNPUS -- one with the TLNPUS at lag zero through lag four, another with the TLNPUS at lag one through lag four, and finally one with the TLNPUS at lag zero alone. The results of those tests are displayed in Table 10. Those results suggest a specification for estimation that includes only the TLNPUS as an exogenous variable without any lag.

b. Estimating Relationships

The next step involved initially estimating a VAR of lag length four in the form of three separate regressions, one for each of the macroeconomic variables, each including the TLNPUS at lag zero as an exogenous covariate, a constant term, and a linear time trend term. These regressions were estimated utilizing data through the final quarter of 1979. Once that was done and the estimation results were examined, two exclusion restrictions were tested by means of partial F tests to simplify one of the equations in that VAR. Those restrictions were the exclusion of the constant term and the linear time trend from the regression equation for unemployment. This exclusion was tested since unemployment has no apparent linear time trend during the estimation period when plotted (see Figure 4 above), and since both the constant and trend

TABLE 10

BLOCK EXOGENEITY TESTS
FOR DIFFERENT LAG SPECIFICATIONS OF
THE DEPRECIATION TAX LIFE FOR INVESTMENTS IN
NON-PUBLIC UTILITY STRUCTURES

WITH NO EXCLUSION RESTRICTIONS ON THE UNEMPLOYMENT EQUATION	
LAGS 0 TO 4	NOT SIGNIFICANT AT 5% LEVEL
LAGS 1 TO 4	NOT SIGNIFICANT AT 5% LEVEL
LAG 0 ALONE	SIGNIFICANT AT BETTER THAN 1% LEVEL
WITH EXCLUSION RESTRICTIONS ON THE UNEMPLOYMENT EQUATION	
LAGS 0 TO 4	NOT SIGNIFICANT AT 5% LEVEL
LAGS 1 TO 4	NOT SIGNIFICANT AT 5% LEVEL
LAG 0 ALONE	SIGNIFICANT AT BETTER THAN 1% LEVEL

terms in the initially estimated unemployment equation were not significant. Both the regression results of the initial estimations and the results of the exclusion restrictions are displayed in Appendix E. Once those data consistent exclusion restrictions were made, the three regression equations were reestimated. The regression results of that reestimation for the period ending the last quarter of 1979 are displayed in Appendix F. In addition, the block exogeneity tests were rerun after the exclusion restrictions were made (see Table 10), with the result that the TLNPUS still was significant when included with zero lag. The estimated parameter values exhibited in those equations then were utilized in the stochastic optimal control exercise that follows.

Furthermore to ascertain the response of the macro-economic variables in the estimated representation to a one-time change in the TLNPUS, multipliers were calculated, the results of which are graphically displayed in Figures 6, 7, and 8. These figures generally display a rather small response to a one year increase in the TLNPUS. Although conjectural, the response patterns displayed are not counter-intuitive -- the short lived raise in M1 could be accounted for by increased loan demand as the result of lower corporate cash flows; the initial decrease in the implicit price deflator for GNP could be due to a decrease in the demand for capital goods to be followed by a rise in

FIGURE 6
RESPONSE OF THE NOMINAL STOCK OF MONEY (M1)
TO A ONE-TIME INITIAL INCREASE OF ONE YEAR IN THE TAX
LIFE OF INVESTMENTS IN NON-PUBLIC UTILITY STRUCTURES

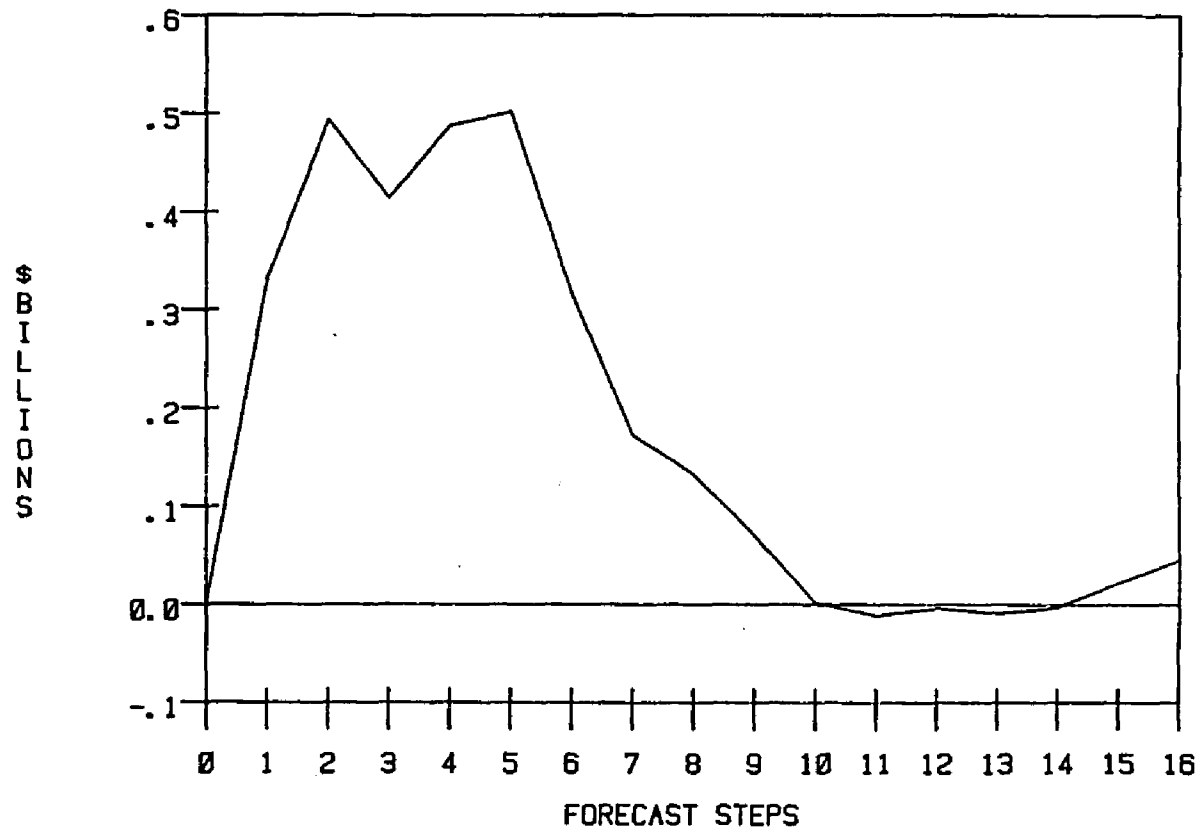


FIGURE 7
RESPONSE OF THE CIVILIAN UNEMPLOYMENT RATE
TO A ONE-TIME INITIAL INCREASE OF ONE YEAR IN THE TAX
LIFE OF INVESTMENTS IN NON-PUBLIC UTILITY STRUCTURES

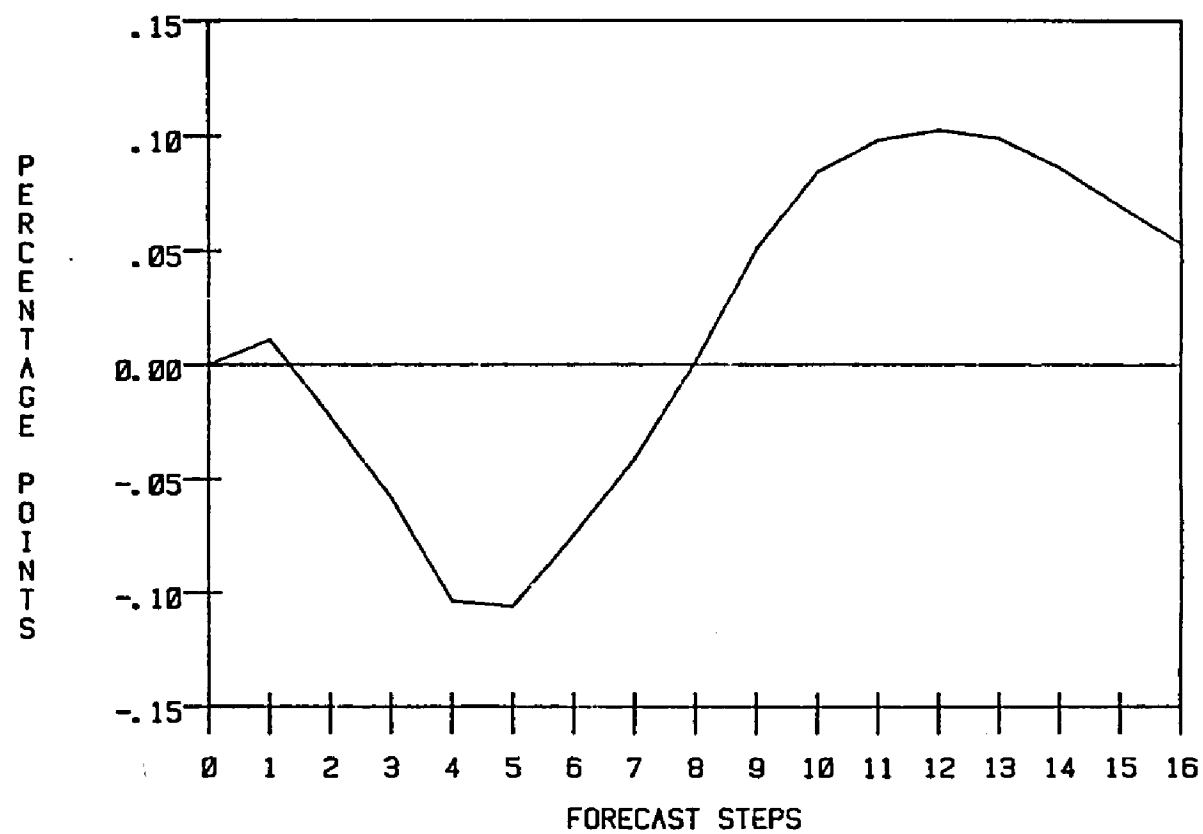
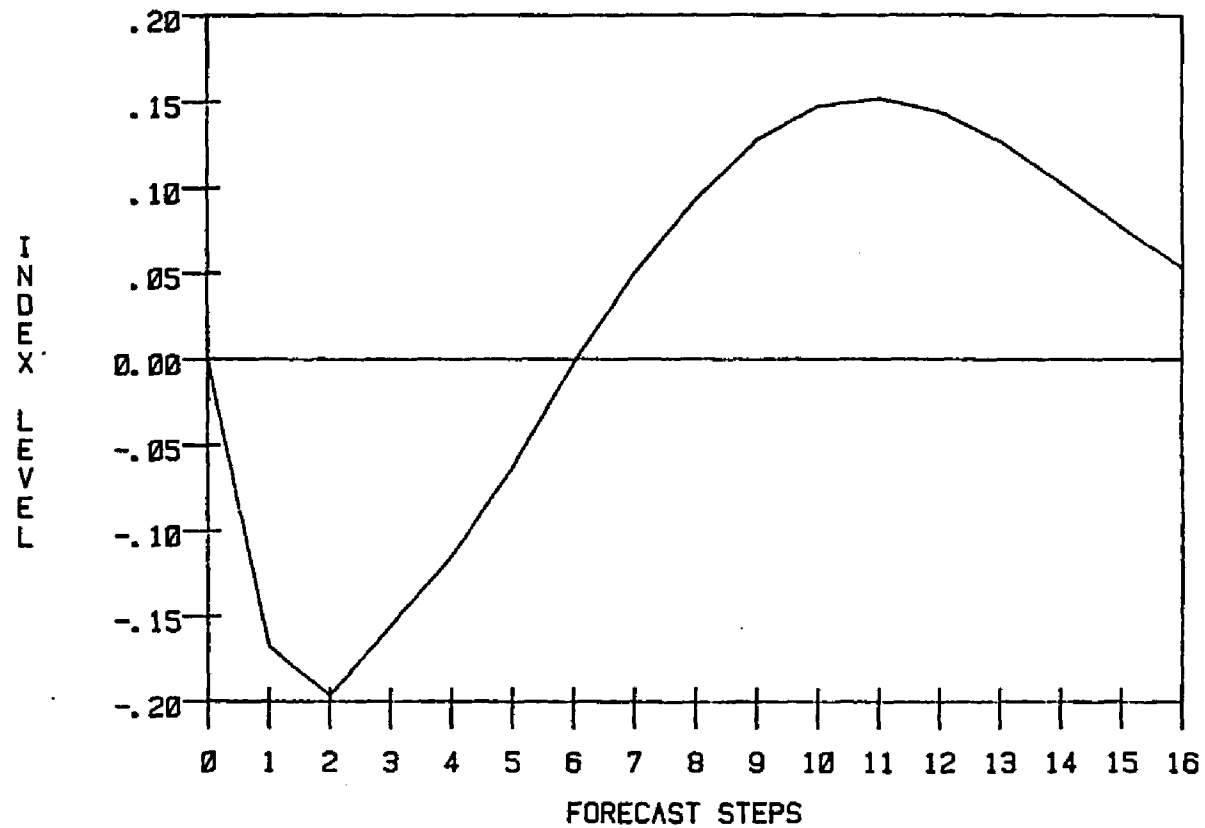


FIGURE 8
RESPONSE OF THE IMPLICIT PRICE DEFLATOR FOR GNP
TO A ONE-TIME INITIAL INCREASE OF ONE YEAR IN THE TAX
LIFE OF INVESTMENTS IN NON-PUBLIC UTILITY STRUCTURES



the price level as productivity wanes and unit labor costs rise; and the initial decrease in the unemployment rate could be due to the substitution of labor for capital to be followed later by increases in the unemployment rate as the money supply tightens.

One should also note, however, that the relatively simple specification employed is one of many representations that could have been utilized to represent the dynamics of the variables. This fairly general specification was utilized since it appears to be reasonably supported by the data and facilitates the control illustration that follows without having to invoke any particular economic theory. A relatively simple representation was utilized, since the primary focus of this chapter is the illustration of the methodology suggested in Chapter II, not the development of a sophisticated representation or model of the behavior of the variables utilized in that illustration. Note also that the estimated representation is backward looking (i.e. "causal" in Buiter's terminology), and hence is not subject to the argument (described in Chapter II) raised by Kydland and Prescott concerning the nonoptimality of plans derived by dynamic programming. Moreover, the suggested control methodology does not preclude the use of alternative representations of the variables' dynamics, including

traditional large scale structural econometric models (e.g. the DRI model).

C. An Exercise to Illustrate the Use of the Stochastic Control Technique in Evaluating a Tax Incentive Measure

In the prior sections of this chapter relationships were investigated between tax incentive variables and selected macroeconomic variables. That investigation resulted in the estimation of a relationship between a tax incentive variable and the macroeconomic variables. The purpose of this section is to utilize that estimated relationship in an optimal control framework to illustrate the use of stochastic control in evaluating historical investment tax incentive measures. The discussion will begin first with a brief overview of this approach to evaluating tax incentive measures and then will turn to the details of implementing that approach and to the empirical results obtained. Finally, a brief critical discussion will be presented of both the advantages and disadvantages of this evaluation technique.

1. An Overview of the Evaluation Technique

The evaluation technique illustrated in this section relies upon the derivation of a feedback control rule for optimally choosing the value of a policy variable, such as a investment tax incentive measure. It is the opportunity

of choosing, at least theoretically, the optimal value of the tax incentive measure over the actual measure that forms a basis for evaluating a historical measure. However, it is important to understand how the optimal value is chosen and how the evaluation measure is derived.⁷

The optimal feedback control rule is derived by means of the optimal control methodology discussed more fully in Chapter II. To briefly review, the feedback control rule for optimally selecting the vector, x_t^* , of policy variables is,

$$x_t^* = G_t y_{t-1} + g_t, \quad (4.4)$$

where G_t and g_t are matrices derived by minimizing,

$$E_0 \sum_{t=1}^T (y_t - a_t)' K_t (y_t - a_t), \quad (4.5)$$

an expected multiperiod welfare loss function, subject to an economic model,

$$y_t = A y_t + C x_t + b_t + u_t, \quad (4.6)$$

where each of the expressions 4.4 through 4.6 are as described in Chapter II. The multiperiod welfare loss function, 4.5 above, is a weighted sum of squared deviations of variables, y_t , from their targets, a_t , with the weighting matrix, K_t , expressing the relative preferences of the policy maker. The linear economic model (4.6 above) depicts a vector of economic variables, y_t , as a function of lagged values of y_t , (y_{t-1}), a vector of "exogenous" variables (x_t) under the control of the policy maker, a

vector of "exogenous" variables not under the control of the policy maker (b_t), and a vector of error terms (u_t). The matrices A and C in 4.6 above contain the estimated parameter values of the model as well as certain identities.

The matrices G_t and g_t in 4.4 are derived using dynamic programming, a method that minimizes the expected multiperiod loss function at time t with respect to x_t given that the expected losses for periods $t+1$ to T (where T is the last period in the planning horizon) have been minimized with respect to the vector x in those periods. Assuming that the expected welfare loss in all future periods is minimized with respect to x , one can express the expected multiperiod loss at time t as a function of x_t , optimal or otherwise. If x_t is chosen to minimize the expected multiperiod at time t (by means of control rule 4.4 above), then one obtains a minimal or optimal value for the expected multiperiod loss, which can be denoted as $V_t(x_t^*)$. If, however, some other value of x_t , $\hat{x}_t$, is chosen then one obtains a non-optimal value for the expected multiperiod loss, denoted as $V_t(\hat{x}_t)$. The difference between the expected two multiperiod welfare losses, $V_t(\hat{x}_t) - V_t(x_t^*)$, represents the additional expected multiperiod loss that would be incurred by choosing $\hat{x}_t$ over x_t^* . If one lets $\hat{x}_t$ represent a historical value, such as the value of an investment tax incentive measure, then $V_t(\hat{x}_t) - V_t(x_t^*)$ would

be the additional expected multiperiod welfare cost associated with the adoption of historical $\hat{x}_t$. Since one had the choice between historical $\hat{x}_t$ and optimal x_t^* over N (where $N < T$) periods in an evaluation period, one could calculate a measure, denoted TDL below, of the total differential loss incurred by adopting historical $\hat{x}_t$ in each of those N periods as,

$$TDL = \sum_{t=1}^N v_t(\hat{x}_t) - v_t(x_t^*). \quad (4.7)$$

$$\begin{aligned} \text{where: } v_t(x_t^*) &= y'_{t-1}(A_t + C_t G_t)' H_t (A_t + C_t G_t) y_{t-1} \\ &\quad + 2y'_{t-1}(A_t + C_t G_t)' (H_t b_t - h_t) \\ &\quad + (b_t + C_t g_t)' H_t (b_t + C_t g_t) \\ &\quad - 2(b_t + C_t g_t)' h_t + c_t + E_{t-1} u_t' H_t u_t \\ v_t(\hat{x}_t) &= x_t' C_t' H_t C_t x_t + 2x_t' C_t' (H_t A_t y_{t-1} + H_t b_t - h_t) \\ &\quad + (A_t y_{t-1} + b_t)' H_t (A_t y_{t-1} + b_t) \\ &\quad + E_{t-1} u_t' H_t u_t - 2(A_t y_{t-1} + b_t)' h_t + c_t. \end{aligned}$$

with the matrices in these expressions defined in Chapter II. Expression 4.7 forms the basis for the evaluation illustration that follows.

2. Implementing an Illustration of This Approach to Evaluation

In order to implement this approach, one must make certain specifications related to the policy makers' objective function. Specifically, one must specify variables to be targeted, the length of the planning horizon of the policy maker, the time path for targeted variables over the

planning horizon of the policy maker, and the weights that would be applied to the deviations of variables from their targets in the objective function. Furthermore, one may wish to apply a discount rate to the weights in the objective function at a particular time period to account for the rate of time preference of the policy maker.

For the purpose of this illustration, it was assumed that the targeted variables were the nominal stock of money (M1 definition), the civilian unemployment rate, and the implicit price deflator on gross national product. Also it was assumed that the targets for the nominal stock of money were derived by increasing the last observed value of that variable at a six percent annual rate for the remainder of the planning horizon. A similar procedure was assumed in deriving the targets for the implicit price deflator, except that a three percent annual rate of increase was assumed. The targets for the civilian unemployment rate for every time period in the planning horizon were set to a value of six percent. The choice of targeted variables is consistent with an assumed stabilization objective for the policy maker. The planning horizon was assumed to extend from the first quarter of 1980 through the final quarter of 1983, resulting in a maximum evaluation period of fifteen quarters extending from the first quarter of 1980 through the third quarter of 1983.

Two alternative sets of weights on the deviations of the unemployment rate and the deflator from their targets were utilized in specifying the objective function -- one with weights one and one respectively on the deviations of the unemployment rate and the deflator from their targets and another with 0.5 and one respectively. Also, since it was assumed that primary emphasis should be placed on the unemployment rate and the deflator, the weight applied to the deviation of the stock of money from its target was at a low value of .05. Furthermore, two values of discount rate were utilized, zero percent and three percent per annum, each in combination with a given set of weights. Altogether, the sets of weights and the discount rates resulted in four different specifications initially for the policy makers' objective function. This strategy was undertaken to examine the sensitivity of results to different weight specifications. Furthermore a penalty variable was included in the objective function to potentially stabilize the period to period movement of the calculated value of the control instrument. In addition, two weight specifications, zero and one hundred, were applied to deviations of that penalty variable from a targeted value in the objective function. The target value of the penalty variable was set to one, approximately one standard deviation of the TLNPUS during the estimation period used in the regressions. These two weight specifications on the penalty

variable together with the other specifications of the objective function resulted in a total of eight specifications of the objective function that were examined -- four with a zero weight on the penalty variable and four with a penalty weight of one hundred.⁷ In addition, the results for the TDL were examined at different lengths for the evaluation period to examine the sensitivity of the results to changes in the length of evaluation period.

To make the required calculations, a computer program was formulated by this investigator, written in RATS. A listing of that program for one specification of the penalty weight in the objective function is contained in Appendix G. This program contains two important RATS procedures, CHOW and BMAT. The procedure CHOW makes the matrix computations required in calculating the TDL of 4.7 above. This procedure was tested for the accuracy of certain crucial matrix calculations against published results. Procedure BMAT builds the required A matrix of 4.6. The rest of the program contains instructions for doing such things as setting the alternative weight specifications applied to the objective function, reinitializing y_{t-1} at each time period, and resetting the targets for new base values.

In performing this illustrative exercise, two crucial simplifying assumptions were made, that the estimated representation of the behavior of the variables utilized is

invariant with time and that certainty equivalence holds. The invariance assumption was made largely to limit complexity of the investigation. However, this assumption, frequently made in econometric work, is subject to the objections raised by Lucas in his famous critique (see Chapter II for further elaboration). Nevertheless, as discussed earlier, the empirical significance of those objections is largely unclear. The latter assumption relating to certainty equivalence is applicable to the illustration conducted in this investigation, since a linear model with additive error terms and a quadratic objective function was used. This assumption permits one to make the feedback control calculations under the assumption that u_t in 4.6 above is set equal to its expected value of zero. This results in the same optimal control equations that one would obtain in a deterministic model with u_t set equal to zero. However, in a stochastic setting with certainty equivalence the solution path for x_t^* can only be calculated numerically after the observation of stochastic y_{t-1} , resulting in a stochastic path for x_t^* . Hence, although certainty equivalence simplifies the computational algorithm, it does not destroy the stochastic nature of x_t^* derived from that algorithm since x_t^* is a function of stochastic y_{t-1} .

3. Review of Empirical Results Obtained

The empirical results for the illustrative optimal exercise for each of the four specifications of the objective function are displayed in Tables 11 and 12 where the weights on the penalty variable in the tables are set to zero and one hundred respectively. Note that in each table, the results are displayed for TDL's for evaluation periods of three different lengths -- five, ten and fifteen periods long. Within each of the length specifications displayed are the results for four specifications of the objective function.

Basically the results in Table 11 show only small TDL's for the given specifications of the objective function and different evaluation period lengths. Although the TDL's are somewhat larger as a percent of the optimal welfare loss at shorter length evaluation periods, they are still small. On the basis of this information one might conclude that the welfare cost associated with the choice of the actual values for the TLNPUS in the 1980 to 1983 period was not substantially different from the welfare cost associated with an optimally chosen TLNPUS. One should note however that such a conclusion would be predicated upon the specifications utilized in making the welfare cost calculations. For instance the result obtained could be reflecting the behavior of the estimated model used in this illustration, a model displaying

TABLE 11

WELFARE COSTS ASSOCIATED WITH THE
OPTIMAL AND ACTUAL VALUES OF INVESTMENT TAX INCENTIVE MEASURE
FOR EVALUATION PERIODS OF THREE DIFFERENT LENGTHS

FIVE PERIOD LENGTH:

WEIGHT SPECIF.	SUM OF VSTAR	SUM OF VHAT	DIFF	%DIFF
1	157,065.7	158,046.5	980.8	0.62
2	146,078.3	146,936.2	857.9	0.59
3	160,502.7	161,334.0	831.3	0.52
4	148,881.3	149,606.5	725.2	0.49

TEN PERIOD LENGTH:

WEIGHT SPECIF.	SUM OF VSTAR	SUM OF VHAT	DIFF	%DIFF
1	317,429.6	318,507.3	1,077.7	0.34
2	293,918.7	294,913.8	995.1	0.34
3	319,412.7	320,369.9	957.2	0.30
4	295,307.5	296,143.2	835.7	0.28

FIFTEEN PERIOD LENGTH:

WEIGHT SPECIF.	SUM OF VSTAR	SUM OF VHAT	DIFF	%DIFF
1	401,729.9	402,865.3	1,135.4	0.28
2	372,080.6	373,133.6	1,053.0	0.28
3	403,061.8	404,082.0	1,020.2	0.25
4	372,901.7	373,800.5	898.8	0.24

LEGEND: VSTAR = WELFARE COST ASSOCIATED WITH OPTIMAL
VALUE OF TAX INCENTIVE MEASURE

VSTAR = WELFARE COST ASSOCIATED WITH ACTUAL
VALUE OF TAX INCENTIVE MEASURE

DIFF = SUM OF VHAT MINUS SUM OF VSTAR = TDL

%DIFF = PERCENT DIFFERENCE BETWEEN SUM OF VHAT
AND SUM OF VSTAR

WEIGHT SPECIFICATIONS IN THE OBJECTIVE FUNCTION FOR DEVIATIONS
OF VARIABLES LISTED FROM THEIR TARGETS USING TWO DISCOUNT RATES:

1 = .05 ON M1, .5 ON UNEMP, 1.0 ON PGNP, 0 ON PENALTY
(0% DISCOUNT RATE)

2 = .05 ON M1, .5 ON UNEMP, 1.0 ON PGNP, 0 ON PENALTY
(3% DISCOUNT RATE)

3 = .05 ON M1, 1.0 ON UNEMP, 1.0 ON PGNP, 0 ON PENALTY
(0% DISCOUNT RATE)

4 = .05 ON M1, 1.0 ON UNEMP, 1.0 ON PGNP, 0 ON PENALTY
(3% DISCOUNT RATE)

TABLE 12

WELFARE COSTS ASSOCIATED WITH THE
OPTIMAL AND ACTUAL VALUES OF INVESTMENT TAX INCENTIVE MEASURE
FOR EVALUATION PERIODS OF THREE DIFFERENT LENGTHS

FIVE PERIOD LENGTH:

WEIGHT SPECIF.	SUM OF VSTAR	SUM OF VHAT	DIFF	%DIFF
1	127,574.3	127,881.6	307.3	0.24
2	118,214.2	118,546.5	332.3	0.28
3	127,666.3	127,969.2	302.9	0.24
4	118,299.3	118,627.0	327.7	0.28

TEN PERIOD LENGTH:

WEIGHT SPECIF.	SUM OF VSTAR	SUM OF VHAT	DIFF	%DIFF
1	190,479.8	240,198.6	49,718.9	26.10
2	177,424.6	226,720.5	49,295.9	27.78
3	190,740.1	240,464.4	49,724.3	26.07
4	177,666.7	226,968.4	49,301.7	27.75

FIFTEEN PERIOD LENGTH:

WEIGHT SPECIF.	SUM OF VSTAR	SUM OF VHAT	DIFF	%DIFF
1	201,896.3	252,419.6	50,523.2	25.02
2	188,520.0	238,606.1	50,086.1	26.57
3	202,165.9	252,702.7	50,536.8	25.00
4	188,771.5	238,871.2	50,099.7	26.54

LEGEND: VSTAR = WELFARE COST ASSOCIATED WITH OPTIMAL
VALUE OF TAX INCENTIVE MEASURE

VSTAR = WELFARE COST ASSOCIATED WITH ACTUAL
VALUE OF TAX INCENTIVE MEASURE

DIFF = SUM OF VHAT MINUS SUM OF VSTAR = TDL

%DIFF = PERCENT DIFFERENCE BETWEEN SUM OF VHAT
AND SUM OF VSTAR

WEIGHT SPECIFICATIONS IN THE OBJECTIVE FUNCTION FOR DEVIATIONS
OF VARIABLES LISTED FROM THEIR TARGETS USING TWO DISCOUNT RATES:

1 = .05 ON M1, .5 ON UNEMP, 1.0 ON PGNP, 100 ON PENALTY
(0% DISCOUNT RATE)

2 = .05 ON M1, .5 ON UNEMP, 1.0 ON PGNP, 100 ON PENALTY
(3% DISCOUNT RATE)

3 = .05 ON M1, 1.0 ON UNEMP, 1.0 ON PGNP, 100 ON PENALTY
(0% DISCOUNT RATE)

4 = .05 ON M1, 1.0 ON UNEMP, 1.0 ON PGNP, 100 ON PENALTY
(3% DISCOUNT RATE)

relatively small responses of the macroeconomic variables to a change in the TLNPUS. In conducting this approach to evaluation one is implicitly assuming that the evaluator is in a position to make such specifications.

In addition to the TDL calculation, one could also look at the optimal path of the TLNPUS versus the actual path to gain some insights. Figure 9 compares the optimal path of the TLNPUS to the actual path for fifteen periods beyond the final quarter of 1979 for one specification of the objective function, containing a zero weight on the penalty variable. It is evident from that figure that the optimal TLNPUS displays an unrealistic path, since it contains negative values and high variability compared to the actual values. Nevertheless, such a comparison is potentially valuable since it shows that even large differences in the two paths result in small TDL's. These observations may lead one to conclude, within the context of this approach to evaluation, that the choice of the TLNPUS is not particularly crucial to the stipulated policy objective. Such information potentially is useful to a policy maker's decision making process (e.g. the current debate on lengthening tax lives in proposed tax reform legislation).

Table 12 and Figure 10, results where the weight on the penalty variable was one hundred, reflect somewhat different results. While the time path of the optimal control instrument in Figure 10 is positive, smoother and

FIGURE 9
OPTIMAL VALUE (LINE) VS. ACTUAL VALUE (DOT)
OF THE DEPRECIATION TAX LIFE FOR NON-PUBLIC UTILITY STRUCTURES
(WEIGHT OF ZERO ON PENALTY VARIABLE)

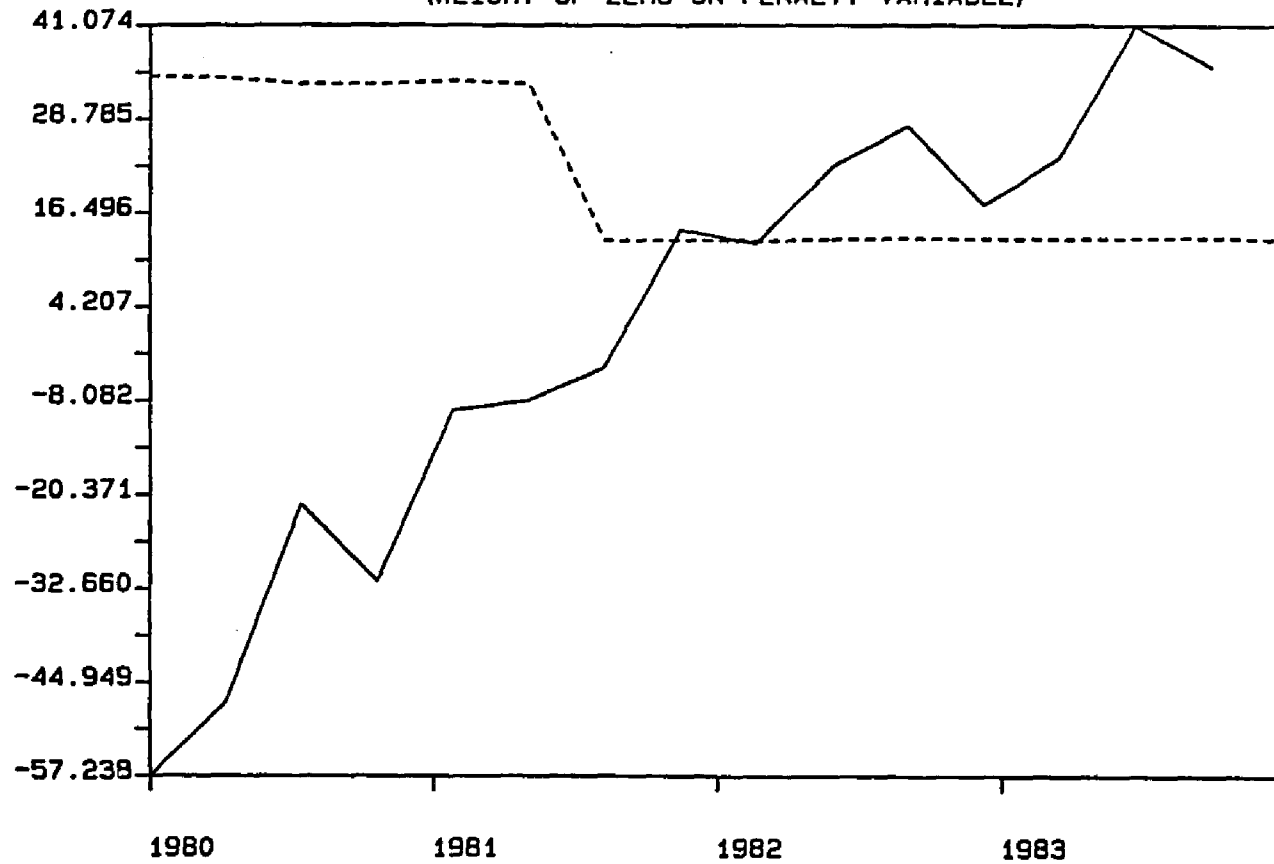
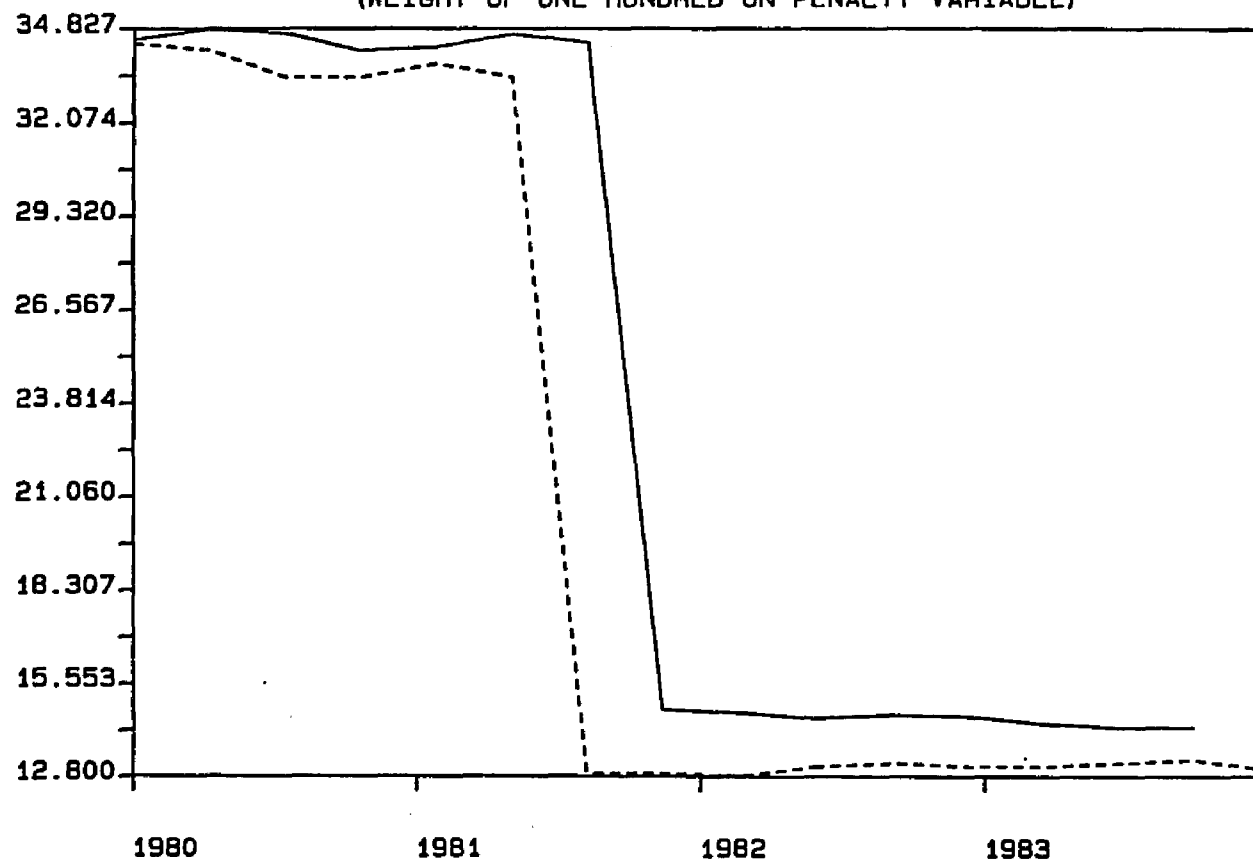


FIGURE 10
OPTIMAL VALUE (LINE) VS. ACTUAL VALUE (DOT)
OF THE DEPRECIATION TAX LIFE FOR NON-PUBLIC UTILITY STRUCTURES
(WEIGHT OF ONE HUNDRED ON PENALTY VARIABLE)



closer to the actual path, due to the imposition of the weight on the penalty variable, the values for the TDL's in Table 12 are larger at longer length evaluation periods than those seen in Table 11. This is due to the large change in the TLNPUS that occurred with the enactment of ERTA in 1981. One can see in Figure 10 a large gap between the calculated optimal value of the policy instrument and the actual value occurring in the third quarter of 1981. This resulted, via the penalty variable in the objective function, in a large difference in welfare costs being incurred for that period, resulting in large TDL's being generated for longer length evaluation periods. Excluding that period of extraordinary change, one finds that the TDL's as a percent of optimal welfare cost are substantially lower (see Table 13). Note however that even when that exclusion is made there appears to be an increase in the TDL as a percent of optimal welfare cost as the evaluation period lengthens. Nevertheless, with the exclusion, the TDL's are relatively small. However, it is not readily apparent whether such an exclusion should be made. Without the exclusion and if the specification of the objective function were considered appropriate (including the penalty weight specification), one might conclude that the shortening of the TLNPUS under ERTA was not desirable due to the high TDL incurred. But since the specification used in this illustration was simply assumed to illustrate a

TABLE 13

WELFARE COSTS ASSOCIATED WITH THE
OPTIMAL AND ACTUAL VALUES OF INVESTMENT TAX INCENTIVE MEASURE
FOR EVALUATION PERIODS OF THREE DIFFERENT LENGTHS
(EXCLUDING PERIOD ASSOCIATED WITH THIRD QUARTER OF 1981)

FIVE PERIOD LENGTH:

WEIGHT SPECIF.	SUM OF VSTAR	SUM OF VHAT	DIFF	%DIFF
1	127,574.3	127,881.6	307.3	0.24
2	118,214.2	118,546.5	332.3	0.28
3	127,666.3	127,969.2	302.9	0.24
4	118,299.3	118,627.0	327.7	0.28

NINE PERIOD LENGTH:

WEIGHT SPECIF.	SUM OF VSTAR	SUM OF VHAT	DIFF	%DIFF
1	177,655.6	178,743.1	1,087.5	0.61
2	165,350.6	166,445.5	1,094.9	0.66
3	177,884.6	178,974.2	1,089.6	0.61
4	165,564.3	166,660.6	1,096.4	0.66

FOURTEEN PERIOD LENGTH:

WEIGHT SPECIF.	SUM OF VSTAR	SUM OF VHAT	DIFF	%DIFF
1	189,072.2	190,964.0	1,891.8	1.00
2	176,446.0	178,331.1	1,885.1	1.07
3	189,310.4	191,212.6	1,902.1	1.00
4	176,669.1	178,563.5	1,894.4	1.07

LEGEND: VSTAR = WELFARE COST ASSOCIATED WITH OPTIMAL
VALUE OF TAX INCENTIVE MEASURE

VSTAR = WELFARE COST ASSOCIATED WITH ACTUAL
VALUE OF TAX INCENTIVE MEASURE

DIFF = SUM OF VHAT MINUS SUM OF VSTAR = TDL

%DIFF = PERCENT DIFFERENCE BETWEEN SUM OF VHAT
AND SUM OF VSTAR

WEIGHT SPECIFICATIONS IN THE OBJECTIVE FUNCTION FOR DEVIATIONS
OF VARIABLES LISTED FROM THEIR TARGETS USING TWO DISCOUNT RATES:

1 = .05 ON M1, .5 ON UNEMP, 1.0 ON PGNP, 100 ON PENALTY
(0% DISCOUNT RATE)

2 = .05 ON M1, .5 ON UNEMP, 1.0 ON PGNP, 100 ON PENALTY
(3% DISCOUNT RATE)

3 = .05 ON M1, 1.0 ON UNEMP, 1.0 ON PGNP, 100 ON PENALTY
(0% DISCOUNT RATE)

4 = .05 ON M1, 1.0 ON UNEMP, 1.0 ON PGNP, 100 ON PENALTY
(3% DISCOUNT RATE)

methodology, there is no particular reason for deeming the specification appropriate.

These results again highlight the fact that any conclusions derived from analyses of this kind rely heavily upon appropriate specifications of the various inputs into the analysis, including appropriate specifications for a representation of economic interrelationships and for the policy maker's objective function. In the illustrative results presented here the specification of the objective function was simply assumed since this investigator, not being a policy maker, has no knowledge of an appropriate specification. However, considerably more attention was paid to the development of an appropriate representation of the stochastic behavior of the variables used in that illustration, although ultimately the TLNPUS did not display substantial linear impacts upon the various macroeconomic variables contained in the objective function used in this illustration. Therefore one should view the the results contained as not definitive but as illustrative of the kinds of information a policy maker could obtain from this evaluation methodology. In this illustration, one major conclusion that a policy maker might have obtained from this analysis, assuming that the various specifications in that particular analysis were appropriate, is that the choice of the TLNPUS was not crucial during one historical

period. If accurate, this would not be unimportant information to policy makers' since it may serve as information to policy makers in deciding how to allocate their resources. For instance, such a conclusion may enable policy makers to concentrate their resources more effectively (e.g. political resources) on areas having greater significance to their objectives.

D. Advantages of and Problems with the Suggested Evaluation Technique

From the above exercise it is apparent that the suggested technique for evaluating investment tax incentive measures has certain advantages over other evaluation techniques (such as those reviewed in Chapter III), but at the same time has certain problems.

The main advantage of this evaluation technique is the possibility of including budgetary goals and policy maker preferences directly in the evaluation process, something that has been totally absent in other approaches to evaluation in the investment tax incentive area. Furthermore, the technique allows one to establish a standard, based on those preferences and goals, against which the actual policy may be assessed. Also, the technique incorporates timing considerations in the evaluation process through the feed-back control derivation of the optimal path and optimal welfare cost values. All of these

aspects are consistent with the evaluation perspective presented in Chapter II and represent improvements to evaluative procedures undertaken in prior studies.

Nevertheless, the suggested technique presents a number of problems. First, the suggested technique requires more of the evaluator. In using this procedure, an evaluator has a substantially greater informational burden than would be necessary with more traditional approaches. For instance, the evaluator must specify the objective function to be utilized -- including the time paths of targets, the weights in the objective function, and a discount rate if deemed appropriate. Such a specification, as alluded to earlier, could be difficult particularly if it is difficult to precisely identify the policy maker. (In the U.S. there is no one policy maker for fiscal matters with both houses of Congress and the Executive branch all playing significant roles in policy making.) This is in addition to constructing a representation of relations among the variables utilized in the evaluative exercise, a task that is potentially more difficult in this approach to evaluation, since this approach focuses on the effects of a change in a tax incentive measure upon budgetary policy objectives rather than the more narrow impact upon investment alone. Furthermore, the greater complexity to this approach potentially has a larger number of sources of error. And finally, if the invariance assumption is utilized, as it

was in this illustration and as it has been in many other econometric studies, the analysis is subject to the Lucas critique of traditional econometric practice, although the empirical significance of that critique is unclear.

However even with these potential problems, the approach suggested here represents an alternative to approaches taken in the past, and as such augments the battery of analytical tools that economists could use in evaluating investment tax incentive measures. With careful application this tool could yield information that could be useful as input into the decision making process related to investment tax incentive measures.

NOTES

CHAPTER IV

¹See the following as references on VAR's and related topics:

Zvi Eckstein, T. Paul Schultz, and Kenneth I. Wolpin, "An Analysis of Swedish Fertility And Mortality from 1790-1955," working paper, Yale University, December, 1981.

Robert B. Litterman, "Techniques of Forecasting Using Vector Autoregressions," working paper 115, Federal Reserve Bank of Minneapolis, November, 1979.

Thomas J. Sargent, Macroeconomic Theory (New York: Academic Press, 1979), pp. 256-297.

Christopher Sims, "Small Econometric Models of the U.S. and West Germany without Prior Restriction," discussion paper, University of Minnesota, December, 1978.

²Christopher A. Sims, "Macroeconomics and Reality," Econometrica 48 (January 1980) : 17-18.

³Gerald Nickelburg, "Small-Sample Properties of Dimensionality Statistics for Fitting VAR Models to Aggregate Economic Data," Journal of Econometrics 28 (May 1985) : 183-192.

⁴Robert S. Pindyck and Daniel L. Rubinfeld, Econometric Models and Economic Forecasts (New York: McGraw-Hill Book Company, 1981), pp. 117-119.

⁵Thomas A. Doan and Robert B. Litterman, User's Manual-RATS (Minneapolis: VAR Econometrics, 1984).

⁶Christopher A. Sims, "Macroeconomics and Reality," p. 18.

⁷The specifications of the objective function have a critical impact upon the welfare cost calculations, which are highly dependent upon the eigenvalues of the H matrix. In this illustration, the specifications, particularly the weight specification on the penalty variable, were chosen so that welfare cost values were always positive. Certain specifications can result in negative welfare cost values.

CHAPTER V

SUMMARY AND CONCLUSIONS

The primary focus of this study has been the presentation and development of an alternative methodology to the ways investment tax incentive programs have been evaluated. The purpose of this chapter is to summarize that alternative perspective, emphasizing the conclusions that were reached in developing that approach to evaluation.

The study began with Chapter I presenting a historical overview of investment tax incentive programs as well as some of the basic economic justifications given for those programs. Once that was accomplished, a brief review and critique was presented of some of the major economic studies that evaluated those programs.

With that background, the discussion then turned in Chapter II to the development of an alternative evaluation perspective to the ones conventionally employed in evaluating investment tax incentive programs. Specifically, it concentrated on the incorporation of budgetary objectives, timing considerations, and utilitarian considerations explicitly into the evaluation process. This is in contrast to more conventional evaluation analyses in this area that focused on the more narrow question of whether investment

tax incentive programs increased investment. While those analyses may be adequate within the context of that limited objective, both public finance theory and the theory of tax reform suggest that such evaluative analyses should proceed on a different basis. More specifically it was argued in Chapter II that since investment tax incentive programs are in fact budgetary programs (a fact so vividly apparent after the introduction of ACRS in 1981) any evaluation scheme of those programs should take budgetary objectives into account. Furthermore, since changes to these programs represent alterations to the tax system, the theory of tax reform suggests that utilitarian and timing considerations should be incorporated into the evaluation process. Stochastic control theory provides a means of potentially incorporating those important aspects into an evaluation process. Specifically stochastic control theory allows one to develop a measure of welfare cost based on those objectives so that the welfare cost associated with the value of an actual investment tax incentive measure can be compared to the welfare cost of an optimal value for an investment tax incentive measure. The difference between the two welfare cost measures serves as the basis for the alternative evaluation scheme developed here.

While the suggested scheme could be useful as an alternative to schemes conventionally employed in evaluating investment tax incentive measures, there are a number of

problems related to the rational expectations hypothesis that potentially limit the usefulness of this alternative approach. In particular, the suggested approach, as with the more conventional analyses, is subject to the the now famous "Lucas" critique of traditional econometric evaluation. That critique argues that changes in policy represent changes in the economic environment for economic actors who potentially could change their behavior with a perceived change in their environment. Such a change in behavior potentially could result in changes in parameter values of the models used to estimate that behavior. Thus traditional econometric evaluations, where parameter values are assumed invariant under policy changes, are potentially flawed. However, as explained in Chapter II, the way that policy changes are perceived by economic actors is unclear and hence the empirical significance of the Lucas critique is unclear. Therefore, the Lucas critique can be viewed more as cautionary note rather than an outright prohibition of the use of the parameter invariance assumption. Additionally, the stochastic control approach suggested in this study potentially suffers from another problem. If expectations of future policy variables affect the current endogenous variables of the objective function, plans generated on the basis of that objective function utilizing dynamic programming would not necessarily be optimal.

However, that critique only applies if expectations of future policy variables are relevant to modelling current endogenous variables. If instead a backward looking (or "causal" model) is relevant, this particular critique does not apply.

After presenting an alternative approach to evaluation in Chapter II, a number of more conventional approaches to evaluation were reviewed and critiqued in Chapter III. That chapter concentrated on a review of both analytical and empirical approaches. Based upon that survey, it was concluded that those approaches focused on the more narrow question of whether investment tax incentive measures increased investment activity, without explicitly incorporating budgetary objectives, timing considerations, or utilitarian considerations into the analysis. Furthermore, even within the narrow context of those analyses, there was a variance of conclusions concerning the effectiveness of the investment tax incentive programs.

Chapter IV presented an empirical illustration of the suggested methodology. The purpose of this chapter was to demonstrate that the methodology is indeed feasible, and to determine the nature of the problems inherent in implementing the methodology. The primary focus of Chapter IV was on methodology, rather than the particular results obtained from the exercise. To implement this illustration

a representation was necessary of interrelationships between the various variables used in the illustration. After a fairly extensive preliminary analysis of the data, a representation of interrelationships was estimated utilizing three macroeconomic variables and a single investment tax incentive measure. That representation was then utilized, together with a computer program written by this investigator and assumed specifications for an objective function, to make the required welfare cost calculations. One possible empirical conclusion arising from the illustration was that the choice of the value for the tax incentive measure was not particularly crucial, since even wide deviations of calculated "optimal" values resulted in relatively small differences in welfare costs. While not definitive, that conclusion illustrated the kind of information that could be available utilizing this technique. Although that conclusion is highly dependent upon the various specifications made in the analysis, the kind of conclusion available to the policy maker from that kind of analysis is much different from the kind available from other evaluation studies. This methodology focuses on the impact of a change upon budgetary objectives, while other approaches usually have focused on the impact upon investment. Therefore, if the appropriate specifications can be made, results derived from this approach to evaluation

could yield alternative, and potentially useful, information to policy makers in evaluating investment tax incentive programs. However, the major drawback of this approach is that much more is required of the evaluator. Not only must an appropriate, and possibly more complicated, model be established but numerous specifications related to the objective function also must be made. (This is perhaps why stochastic control theory has not been widely employed by investigators.) Since this approach is dependent upon such specifications, the feasibility of the approach is dependent the ability of the evaluator to make informed assessments of the required inputs. That task is complicated by the fact that there no single fiscal policy maker in the United States, with the responsibility for fiscal affairs being shared by both houses of Congress and the Executive branch. Nevertheless, this approach to evaluating investment tax incentive programs provides information from a perspective that is different than more conventional analyses, and hence has the potential for capturing information not obtainable from those analyses.

APPENDIX A

LISTING OF DATA UTILIZED IN CHAPTER IV

QTR	M1	UNEMP	PGNP	RITC	LIFENPU	LIFEPU	LIFEEQP
59:1	140.300	5.830	67.040	0.000	35.100	28.700	11.400
59:2	141.400	5.100	67.550	0.000	35.300	28.700	11.200
59:3	142.500	5.270	67.810	0.000	35.300	28.700	11.100
59:4	141.300	5.600	68.000	0.000	35.100	28.700	11.400
60:1	140.900	5.130	68.440	0.000	35.400	28.800	11.300
60:2	140.700	5.230	68.560	0.000	35.400	28.900	11.600
60:3	142.000	5.530	68.860	0.000	35.300	28.900	11.500
60:4	141.900	6.270	68.960	0.000	35.700	28.900	11.600
61:1	142.700	6.800	68.880	0.000	35.800	28.800	11.500
61:2	143.800	7.000	69.220	0.000	35.900	28.800	11.400
61:3	144.700	6.770	69.540	0.000	36.100	28.800	11.300
61:4	146.000	6.200	69.650	0.000	36.000	28.700	11.400
62:1	147.000	5.630	70.230	0.000	35.800	28.700	11.400
62:2	148.000	5.530	70.480	0.000	35.900	28.600	11.400
62:3	147.800	5.570	70.620	0.055	36.000	26.300	9.900
62:4	148.700	5.530	71.080	0.055	35.600	26.300	9.800
63:1	150.200	5.770	71.410	0.056	35.400	26.400	9.800
63:2	151.600	5.730	71.460	0.057	35.500	26.400	10.000
63:3	153.200	5.500	71.660	0.057	35.200	26.400	10.000
63:4	154.600	5.570	72.170	0.056	35.000	26.500	9.900
64:1	155.700	5.470	72.360	0.057	35.000	26.500	10.000
64:2	156.800	5.200	72.570	0.058	35.100	26.500	9.900
64:3	159.400	5.000	72.970	0.056	35.400	26.400	9.900
64:4	161.400	4.970	73.160	0.058	35.300	26.400	10.100
65:1	162.600	4.900	73.770	0.056	35.500	26.400	9.900
65:2	163.600	4.670	74.130	0.056	35.100	26.400	9.900
65:3	165.600	4.370	74.560	0.057	35.400	26.400	10.000
65:4	168.500	4.100	74.960	0.056	35.600	26.400	9.900
66:1	171.400	3.870	75.710	0.056	35.200	26.400	9.900
66:2	173.300	3.830	76.580	0.057	34.900	26.400	10.000
66:3	172.800	3.770	76.990	0.057	35.000	26.400	10.000
66:4	173.200	3.700	77.760	0.000	35.300	26.400	10.000
67:1	175.000	3.830	78.250	0.000	35.300	26.400	10.200
67:2	177.500	3.830	78.530	0.057	35.200	26.400	10.000
67:3	181.400	3.800	79.280	0.057	35.100	26.400	10.000
67:4	184.300	3.900	80.130	0.058	35.400	26.400	10.000
68:1	186.800	3.730	81.150	0.056	35.800	26.300	9.800
68:2	190.000	3.670	82.140	0.056	36.100	26.300	9.800
68:3	193.700	3.530	82.840	0.056	36.000	26.300	9.700
68:4	197.800	3.400	83.990	0.055	35.900	26.300	9.700
69:1	201.400	3.400	84.970	0.055	36.100	26.400	9.700
69:2	203.000	3.430	86.100	0.000	36.000	26.500	9.900
69:3	203.900	3.570	87.490	0.000	36.100	26.500	9.900
69:4	205.600	3.570	88.620	0.000	36.100	26.500	10.000
70:1	207.600	4.170	89.890	0.000	36.300	26.500	10.000
70:2	209.200	4.770	91.070	0.000	36.100	26.500	10.100
70:3	212.000	5.170	91.790	0.000	36.100	26.500	10.100
70:4	215.800	5.830	93.030	0.000	36.000	26.500	10.400
71:1	219.500	5.930	94.400	0.000	36.300	26.600	10.100
71:2	224.200	5.900	95.700	0.063	37.200	26.600	10.000
71:3	228.100	6.030	96.520	0.062	37.600	21.300	8.100
71:4	230.200	5.930	97.390	0.062	37.500	21.300	8.000
72:1	234.700	5.770	98.720	0.062	37.600	21.300	8.000
72:2	238.600	5.700	99.420	0.062	37.700	21.300	8.000
72:3	243.500	5.570	100.260	0.062	37.700	21.300	8.100
72:4	249.600	5.370	101.540	0.062	37.600	21.300	8.000
73:1	254.600	4.930	102.950	0.061	37.700	21.300	7.900

APPENDIX A - CONTINUED

73:2	257.600	4.930	104.750	0.062	37.200	21.300	8.000
73:3	260.700	4.800	106.530	0.062	37.100	21.300	8.100
73:4	264.000	4.770	108.740	0.063	36.700	21.300	8.200
74:1	268.400	5.130	110.720	0.063	36.500	21.300	8.200
74:2	270.800	5.200	113.480	0.063	36.300	21.200	8.200
74:3	273.300	5.630	116.420	0.063	36.000	21.200	8.100
74:4	276.500	6.600	119.790	0.063	35.500	21.200	8.200
75:1	278.500	8.270	122.880	0.091	35.000	21.100	8.300
75:2	282.800	8.870	124.440	0.090	33.900	21.100	8.200
75:3	288.000	8.470	126.680	0.089	33.500	21.000	8.100
75:4	290.400	8.300	128.990	0.089	33.500	21.000	8.100
76:1	294.300	7.730	130.120	0.089	33.800	20.900	8.000
76:2	299.100	7.570	131.300	0.089	34.000	20.900	8.000
76:3	302.400	7.730	132.890	0.088	34.000	20.900	8.000
76:4	308.300	7.770	134.990	0.089	34.300	21.000	8.000
77:1	315.400	7.500	136.800	0.087	33.900	21.200	7.900
77:2	320.900	7.130	139.010	0.088	33.600	21.300	7.900
77:3	326.400	6.900	141.030	0.088	33.900	21.300	7.900
77:4	333.300	6.670	143.240	0.088	33.600	21.400	7.900
78:1	339.600	6.330	145.120	0.088	33.800	21.400	7.900
78:2	347.300	6.000	148.890	0.088	33.700	21.500	7.900
78:3	354.400	6.030	152.020	0.089	33.600	21.500	8.000
78:4	360.700	5.900	155.380	0.089	33.800	21.500	7.900
79:1	365.000	5.870	158.600	0.089	33.300	21.400	8.000
79:2	374.000	5.700	161.850	0.090	34.200	21.400	8.000
79:3	383.700	5.870	165.120	0.090	34.300	21.400	8.100
79:4	387.800	5.970	168.050	0.091	34.300	21.400	8.200
80:1	393.600	6.300	171.940	0.092	34.400	21.400	8.300
80:2	390.100	7.330	176.460	0.093	34.200	21.300	8.400
80:3	405.600	7.670	180.240	0.092	33.400	21.300	8.300
80:4	416.800	7.400	185.130	0.092	33.400	21.300	8.300
81:1	420.200	7.430	190.010	0.092	33.800	21.300	8.200
81:2	429.400	7.400	193.030	0.092	33.400	21.300	8.300
81:3	432.600	7.400	197.700	0.097	12.900	13.800	4.300
81:4	438.200	8.230	201.690	0.097	12.900	13.900	4.400
82:1	447.900	8.830	203.980	0.097	12.800	13.900	4.400
82:2	451.100	9.430	206.770	0.097	13.100	13.900	4.300
82:3	457.900	9.900	208.530	0.097	13.200	13.900	4.300
82:4	476.500	10.600	210.270	0.096	13.100	13.900	4.300
83:1	490.100	10.370	212.870	0.096	13.100	13.900	4.300
83:2	504.900	10.170	214.250	0.096	13.200	13.800	4.300
83:3	517.800	9.330	215.890	0.096	13.300	13.800	4.300
83:4	526.100	8.470	218.210	0.096	13.000	13.800	4.300

LEGEND: QTR = YEAR AND QUARTER
 M1 = NOMINAL STOCK OF MONEY (M1 DEFINITION)
 UNEMP = CIVILIAN UNEMPLOYMENT RATE
 PGNP = IMPLICIT PRICE DEFLATOR FOR GNP
 RITC = MARGINAL RATE OF INVESTMENT TAX CREDIT ON PRODUCERS'
 DURABLE EQUIPMENT
 LIFENPU = DEPRECIATION TAX LIFE ON INVESTMENTS IN NON-PUBLIC
 UTILITY STRUCTURES
 LIFEPU = DEPRECIATION TAX LIFE ON INVESTMENTS IN PUBLIC UTILITY
 LIFEQ = DEPRECIATION TAX LIFE ON INVESTMENTS IN PRODUCERS'

APPENDIX B

RATS PROGRAM UTILIZED IN ESTIMATING AN APPROPRIATE LAG LENGTH UTILIZING SIMS' TEST

```

OPEN DATA TAXINFO.DAT
BMA LOC 120 EXP 200 MAT 250 COM 3500 GLO 225
CAL 59 1 4
ALL 30 83,4 3 30
DATA(ORG=OBS) 1 100 1 TO 7
EQV 1 TO 7
  M1 UNEMP PGNP RITC LIFESNPU LIFESPU LIFEEOQ
SET TREND 1959,1 1983,4 = T
*
*PERFORM TESTS FOR AN APPROPRIATE LAG LENGTH
*
  *SET EXOGENOUS COVARIATE
  DO EXO=4,7
  *SET LAGS IN EACH SYSTEM
  DO LLAG=1,4
  IEVAL LFPER=(59,1)+LLAG
  IEVAL HLAG=1+LLAG
  IEVAL HFPER=(59,1)+HLAG
  IEVAL VARUR=2+4*HLAG
  *DEFINE AND ESTIMATE SYSTEM WITH ONE ADDITIONAL LAG
  SYSTEM 1 TO 4
  VARIABLES 1 TO 3 EXO
  LAGS 1 TO HLAG
  DET CONSTANT TREND
  END(SYSTEM)
  ESTIMATE(NOPRINT,NOFTTESTS) HFPER 79,4 9
  *DEFINE OTHER SYSTEM
  SYSTEM 5 TO 8
  VARIABLES 1 TO 3 EXO
  LAGS 1 TO LLAG
  DET CONSTANT TREND
  END(SYSTEM)
  ESTIMATE(NOPRINT,NOFTTESTS) HFPER 79,4 13
  NOTE      EXOGENOUS VARIABLE:
  WRITE      LABEL(EXO)
  NOTE      HIGHER LAG LENGTH:
  WRITE      HLAG
  NOTE      LOWER LAG LENGTH:
  WRITE      LLAG
  RATIO(DEGREES=16,MCORR=VARUR,NOPRINT) HFPER 79,4
  # 9 10 11 12
  # 13 14 15 16
  END DO LLAG
  END DO EXO
  END

```

APPENDIX C

RATS PROGRAM UTILIZED IN CALCULATING
THE PARTIAL F TESTS FOR EXAMINING INTERRELATIONSHIPS AMONG
THE TAX INCENTIVE VARIABLES AND THE MACROECONOMIC VARIABLES

```

OPEN DATA TAXINFO.DAT
BMA LOC 120 EXP 200 MAT 250 COM 3500 GLO 225
CAL 59 1 4
ALL 30 83,4 3 30
DATA(ORG=OBS) 1 100 1 TO 7
EQV 1 TO 7
  M1 UNEMP PGNP RITC LIFESNPU LIFESPU LIFEEOGP
SET TREND 1959,1 1983,4 = T
*
*PERFORM TESTS FOR INTERRELATIONSHIPS BETWEEN VARIABLES
*
  *SET TAX INCENTIVE VARIABLE
  DO TIV=4,7
  *SET LAGS IN THE SYSTEM
  DO SLAG=1,5
  IEVAL FPER=(59,1)+SLAG
  NOTE      TAX INCENTIVE VARIABLE:
  WRITE      LABEL(TIV)
  NOTE      LAG LENGTH:
  WRITE      SLAG
  *DEFINE AND ESTIMATE SYSTEM WITH ONE ADDITIONAL LAG
  SYSTEM 1 TO 4
  VARIABLES 1 TO 3 TIV
  LAGS 1 TO SLAG
  DET CONSTANT TREND
  END(SYSTEM)
  ESTIMATE(NOPRINT) FPER 79,4 16
  END DO SLAG
  END DO TIV
  END

```

APPENDIX D

PROGRAMS UTILIZED IN CALCULATING THE VARIANCE DECOMPOSITIONS IN CHAPTER IV

For RITC:

```

OPEN DATA TAXINFO.DAT
BMA LOC 120 EXP 200 MAT 250 COM 3500 GLO 225
CAL 59 1 4
ALL 30 83,4 3 30
DATA(ORG=OBS) 1 100 1 TO 7
EQV 1 TO 7
M1 UNEMP PGNP RITC LIFESNPU LIFESPU LIFEEOQ
SET TREND 1959,1 1983,4 = T
*
do ll = 1,4
ieval fp = (59,1)+ll
note      LAG LENGTH
write     ll
system 1 to 4
variables 1 to 4
lags 1 to ll
det constant trend
end(system)
estimate(noprint) fp 79,4 9
vcv(matrix=covm,nocorr) fp 79,4
# 9 to 12
note      RITC UNEMP PGNP M1
list ieqn = 4 2 3 1
errors 4 16 covm
cards ieqn 0 0 ieqn
note      M1 UNEMP PGNP RITC
list ieqn = 1 2 3 4
errors 4 16 covm
cards ieqn 0 0 ieqn
note      UNEMP PGNP M1 RITC
list ieqn = 2 3 1 4
errors 4 16 covm
cards ieqn 0 0 ieqn
note      RITC PGNP UNEMP M1
list ieqn = 4 3 2 1
errors 4 16 covm
cards ieqn 0 0 ieqn
end do ll
end

```

APPENDIX D - CONTINUED

For LIFESNPU:

```
OPEN DATA TAXINFO.DAT
BMA LOC 120 EXP 200 MAT 250 COM 3500 GLO 225
CAL 59 1 4
ALL 30 83,4 3 30
DATA(ORG=OBS) 1 100 1 TO 7
EQV 1 TO 7
  M1 UNEMP PGNP RITC LIFESNPU LIFESPU LIFEEOQ
SET TREND 1959,1 1983,4 = T
*
do 11 = 1,4
  ieval fp = (59,1)+11
  note      LAG LENGTH
  write     11
  system 1 to 4
  variables 1 to 3 5
  lags 1 to 11
  det constant trend
end(system)
estimate(noprint) fp 79,4 9
vcv(matrix=covm,nocorr) fp 79,4
# 9 to 12
note      LIFESNPU UNEMP PGNP M1
list ieqn = 4 2 3 1
errors 4 16 covm
cards ieqn 0 0 ieqn
note      M1 UNEMP PGNP LIFESNPU
list ieqn = 1 2 3 4
errors 4 16 covm
cards ieqn 0 0 ieqn
note      UNEMP PGNP M1 LIFESNPU
list ieqn = 2 3 1 4
errors 4 16 covm
cards ieqn 0 0 ieqn
note      LIFESNPU PGNP UNEMP M1
list ieqn = 4 3 2 1
errors 4 16 covm
cards ieqn 0 0 ieqn
end do 11
end
```

LEGEND: RITC = MARGINAL RATE OF INVESTMENT TAX CREDIT
ON PRODUCERS' DURABLE EQUIPMENT

LIFESNPU = DEPRECIATION TAX LIFE ON NON-PUBLIC
UTILITY STRUCTURES

APPENDIX E

INITIAL REGRESSION RESULTS AND THE RESULTS OF THE EXCLUSION RESTRICTION TESTS ON THE UNEMPLOYMENT RATE EQUATION

A. ESTIMATED EQUATION FOR THE NOMINAL STOCK OF MONEY :

DEPENDENT VARIABLE		1	M1		
FROM	60- 1	UNTIL	79- 4		
OBSERVATIONS		80	DEGREES OF FREEDOM	65	
R**2	.99981924		RBAR**2	.99978031	
SSR	73.431251		SEE	1.0628789	
DURBIN-WATSON	1.80452846				
Q(24)=	24.7577	SIGNIFICANCE LEVEL	.419020		

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*****	***	***	*****	*****	*****
1	M1	1	1	1.255338	.1321991	9.495813
2	M1	1	2	-.6379092	.2146336	-2.972084
3	M1	1	3	.5727685	.2301624	2.488540
4	M1	1	4	-.2632010	.1497484	-1.757621
5	UNEMP	2	1	-.2034601	.5929043	-.3431584
6	UNEMP	2	2	.6536494	1.030553	.6342702
7	UNEMP	2	3	-.5246637	.9440663	-.5657488
8	UNEMP	2	4	-.2775723	.5138049	-.5402289
9	PGNP	3	1	-.4828590	.4054548	-1.190907
10	PGNP	3	2	.6067685	.5879190	1.032061
11	PGNP	3	3	-.8761369	.5964459	-1.468929
12	PGNP	3	4	1.002151	.3779575	2.651491
13	LIFESNPU	5	0	.3308947	.2236844	1.479293
14	CONSTANT	0	0	-16.17435	8.410998	-1.923000
15	TREND	31	0	.6040780E-01	.3706094E-01	1.629959

APPENDIX E - CONTINUED

B. ESTIMATED EQUATION FOR THE UNEMPLOYMENT RATE :

DEPENDENT VARIABLE		2	UNEMP	
FROM	60- 1	UNTIL	79- 4	
OBSERVATIONS	80		DEGREES OF FREEDOM	65
R**2	.97456000		RBAR**2	.96908062
SSR	3.5482239		SEE	.23364088
DURBIN-WATSON		1.89841286		
Q(24)=	34.3913	SIGNIFICANCE LEVEL .779504E-01		

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*****	***	***	*****	*****	*****
1	M1	1	1	-.7633735E-02	.2905986E-01	-.2626900
2	M1	1	2	-.8375728E-01	.4718052E-01	-1.775251
3	M1	1	3	.1843952E-01	.5059406E-01	.3644601
4	M1	1	4	.6222671E-01	.3291754E-01	1.890382
5	UNEMP	2	1	1.380852	.1303316	10.59492
6	UNEMP	2	2	-.6157533	.2265351	-2.718136
7	UNEMP	2	3	.8414969E-01	.2075236	.4054945
8	UNEMP	2	4	.8445132E-01	.1129440	.7477272
9	PGNP	3	1	.2979376	.8912663E-01	3.342857
10	PGNP	3	2	-.3538885	.1292357	-2.738318
11	PGNP	3	3	.2012526	.1311101	1.534989
12	PGNP	3	4	-.1217068	.8308220E-01	-1.464897
13	LIFESNPU	5	0	.9640922E-01	.4917005E-01	1.960730
14	CONSTANT	0	0	-3.235839	1.848896	-1.750146
15	TREND	31	0	.6517371E-02	.8146695E-02	.8000018

APPENDIX E - CONTINUED

C. TESTS OF EXCLUSION RESTRICTIONS ON THE UNEMPLOYMENT RATE EQUATION:

NULL HYPOTHESIS

THE FOLLOWING COEFFICIENTS ARE ZERO

VAR 0 CONSTANT

F(1 , 65) = 3.063013 SIGNIFICANCE LEVEL .8481207E-01

NULL HYPOTHESIS

THE FOLLOWING COEFFICIENTS ARE ZERO

VAR 31 TREND

F(1 , 65) = .6400029 SIGNIFICANCE LEVEL .4266239

NULL HYPOTHESIS

THE FOLLOWING COEFFICIENTS ARE ZERO

VAR 0 CONSTANT

VAR 31 TREND

F(2 , 65) = 1.566401 SIGNIFICANCE LEVEL .2165737

APPENDIX E - CONTINUED

D. ESTIMATED EQUATION FOR THE IMPLICIT PRICE DEFLATOR FOR GNP :

DEPENDENT VARIABLE	3	PGNP
FROM 60- 1 UNTIL 79- 4		
OBSERVATIONS	80	DEGREES OF FREEDOM 65
R**2	.99988440	RBAR**2 .99985950
SSR	7.7168366	SEE .34455850
DURBIN-WATSON	2.01033241	
Q(24)=	17.7119	SIGNIFICANCE LEVEL .816759

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*****	***	***	*****	*****	*****
1	M1	1	1	.7551899E-02	.4285561E-01	.1762173
2	M1	1	2	.6652952E-01	.6957879E-01	.9561753
3	M1	1	3	-.5750905E-01	.7461285E-01	-.7707659
4	M1	1	4	.5891365E-01	.4854466E-01	1.213597
5	UNEMP	2	1	-.2076536	.1922046	-1.080378
6	UNEMP	2	2	.4681711	.3340794	1.401377
7	UNEMP	2	3	-.2853120	.3060424	-.9322628
8	UNEMP	2	4	-.1766628E-01	.1665626	-.1060639
9	PGNP	3	1	1.175836	.1314382	8.945918
10	PGNP	3	2	-.2407349	.1905885	-1.263114
11	PGNP	3	3	-.3004688E-01	.1933527	-.1553993
12	PGNP	3	4	-.3613296E-01	.1225243	-.2949046
13	LIFESNPU	5	0	-.1674906	.7251283E-01	-2.309806
14	CONSTANT	0	0	5.060197	2.726633	1.855841
15	TREND	31	0	-.3960192E-01	.1201422E-01	-3.296254

APPENDIX F

REGRESSION RESULTS OF EQUATIONS USED IN THE OPTIMAL CONTROL EXERCISE

The following are the regression results for the equations that are used in the optimal control exercise:

A. ESTIMATED EQUATION FOR THE NOMINAL STOCK OF MONEY :

DEPENDENT VARIABLE		1	M1			
FROM	60- 1	UNTIL	79- 4			
OBSERVATIONS			80	DEGREES OF FREEDOM	65	
R**2	.99981924			RBAR**2	.99978031	
SSR	73.431251			SEE	1.0628789	
DURBIN-WATSON	1.80452846					
Q(24)=	24.7577		SIGNIFICANCE LEVEL	.419020		
NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*****	***	***	*****	*****	*****
1	M1	1	1	1.255338	.1321991	9.495813
2	M1	1	2	-.6379092	.2146336	-2.972084
3	M1	1	3	.5727685	.2301624	2.488540
4	M1	1	4	-.2632010	.1497484	-1.757621
5	UNEMP	2	1	-.2034601	.5929043	-.3431584
6	UNEMP	2	2	.6536494	1.030553	.6342702
7	UNEMP	2	3	-.5246637	.9440663	-.5557488
8	UNEMP	2	4	-.2775723	.5138049	-.5402289
9	PGNP	3	1	-.4828590	.4054548	-1.190907
10	PGNP	3	2	.6067685	.5879190	1.032061
11	PGNP	3	3	-.8761369	.5964459	-1.468929
12	PGNP	3	4	1.002151	.3779575	2.651491
13	LIFESNPU	5	0	.3308947	.2236844	1.479293
14	CONSTANT	0	0	-16.17435	8.410998	-1.923000
15	TREND	31	0	.6040780E-01	.3706094E-01	1.629959

APPENDIX F - CONTINUED

B. ESTIMATED EQUATION FOR THE UNEMPLOYMENT RATE :

DEPENDENT VARIABLE		2	UNEMP	
FROM	60- 1	UNTIL	79- 4	
OBSERVATIONS		80	DEGREES OF FREEDOM	67
R**2	.97333387		RBAR**2	.96855785
SSR	3.7192375		SEE	.23560774
DURBIN-WATSON	1.88342911			
Q(24)=	28.2721	SIGNIFICANCE LEVEL .248730		

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*****	***	***	*****	*****	*****
1	M1	1	1	.1011714E-01	.2747941E-01	.3681716
2	M1	1	2	-.9397076E-01	.4720567E-01	-1.990667
3	M1	1	3	.3307636E-01	.5017917E-01	.6591651
4	M1	1	4	.5619419E-01	.3301548E-01	1.702056
5	UNEMP	2	1	1.444601	.1255953	11.50203
6	UNEMP	2	2	-.6934751	.2231521	-3.107634
7	UNEMP	2	3	.1546521	.2020284	.7654966
8	UNEMP	2	4	.4936513E-01	.1068926	.4618197
9	PGNP	3	1	.2464035	.7982323E-01	3.086865
10	PGNP	3	2	-.3371751	.1291228	-2.611274
11	PGNP	3	3	.1990326	.1321656	1.505934
12	PGNP	3	4	-.1213237	.8375332E-01	-1.448584
13	LIFESNPU	5	0	.1011280E-01	.5923227E-02	1.707312

APPENDIX F - CONTINUED

C. ESTIMATED EQUATION FOR IMPLICIT PRICE DEFLATOR FOR GNP :

DEPENDENT VARIABLE	3	PGNP
FROM 60- 1 UNTIL 79- 4		
OBSERVATIONS	80	DEGREES OF FREEDOM 65
R**2	.99988440	RBAR**2 .99985950
SSR	7.7168366	SSE .34455850
DURBIN-WATSON	2.01033241	
Q(24)=	17.7119	SIGNIFICANCE LEVEL .816759

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
***	*****	***	***	*****	*****	*****
1	M1	1	1	.7551899E-02	.4285561E-01	.1762173
2	M1	1	2	.6652952E-01	.6957879E-01	.9561753
3	M1	1	3	-.5750905E-01	.7461285E-01	-.7707659
4	M1	1	4	.5891365E-01	.4854466E-01	1.213597
5	UNEMP	2	1	-.2076536	.1922046	-1.080378
6	UNEMP	2	2	.4681711	.3340794	1.401377
7	UNEMP	2	3	-.2853120	.3060424	-.9322628
8	UNEMP	2	4	-.1766628E-01	.1665626	-.1060639
9	PGNP	3	1	1.175836	.1314382	8.945918
10	PGNP	3	2	-.2407349	.1905885	-1.263114
11	PGNP	3	3	-.3004688E-01	.1933527	-.1553993
12	PGNP	3	4	-.3613296E-01	.1225243	-.2949046
13	LIFESNPU	5	0	-.1674906	.7251283E-01	-2.309806
14	CONSTANT	0	0	5.060197	2.726633	1.855841
15	TREND	31	0	-.3960192E-01	.1201422E-01	-3.296254

APPENDIX G

RATS PROGRAM USED TO MAKE OPTIMAL CONTROL CALCULATIONS

The following RATS program is one of two programs utilized in making the optimal control calculations in Chapter IV. This program was used in making the optimal control calculations where the value of the penalty variable in the objective function equaled zero.

```

OPEN DATA TAXINFO.DAT
BMA LOC 130 EXP 300 MAT 270 COM 3700 GLO 235
CAL 59 1 4
ALL 30 83,4 3 30
DATA(ORG=OBS) 1 100 1 TO 7
EQV 1 TO 7
M1 UNEMP PGNP RITC LIFESNPU LIFESPU LIFEEQP
SET TREND 1959,1 1983,4 = T
*****
procedure bmat nendog mlendog ninst mlinst pen
*****
*****BMAT*****
*****
*
*This routine forms the "A" matrix (Chow,1975) from a system
*of reduced form equations under control
*where:
*  nendog=number of endogenous variables
*  mlendog=maximum lag of any endogenous variable in system
*  ninst=number of variables under control(i.e. number of instruments)
*  mlinst=maximum lag of any instrument in system
*  pen=number of first difference penalty variables of instruments
*  acap=the "A" matrix
*
type integer nendog mlendog ninst mlinst pen
declare rect acap
local integer c0 c1 c2 c3 aa ab ac ad ae cc k kk pp cl dimspe
ieval c0 = nendog*(mlendog-1)
ieval c1 = nendog*mlendog+ninst*mlinst
ieval c2 = c1+ninst
ieval c3 = (mlinst-ninst)*ninst
if mlinst.eq.0
begin

```

APPENDIX G - CONTINUED

```

*
*
*ESTABLISH OPTIMAL CONTROL PROGRAM
*
*
*   proc chow A KCAP C y0 start npd met instr Gal Gbl Gul Disc Xa
*
*This procedure computes an optimal set of instruments given:
*   - a quadratic welfare function containing targets of
*     targeted variables and weights showing relative importance
*
*   - a model in reduced form depicting the stochastic
*     behavior of variables of interest.
*
*****
*****
*****  PROCEDURE CHOW  *****
*****
*****
*
*arguments:
*   al=vector of targets
*   bl=vector of exogenous variables
*   ul=vector of innovations
*   A=coefficients matrix
*   KCAP=diagonal matrix of weights without discount
*   C=matrix of coefficients on concurrent instruments
*   y0=initial state vector at time "start"
*   start=initial time period
*   npd=number of periods in planning horizon
*   met=met of state vector
*   Gal=target matrix
*   Gbl=exogenous variables matrix
*   Gul=innovation matrix
*   Disc=discount rate applied to welfare cost
*
*locals:
*
*   Vatar="optimal" expected welfare value at time "start" for n periods
*   Vhat= expected welfare value at time "start" for n periods
*         associated with actual instrument value
*   G=matrix of feedback control coefficients
*   gl=vector of intercept values for feedback control rule

```

APPENDIX G - CONTINUED

```

*      last=start+npd=last time period in planning horizon
*      ct=current time
*      H="H" matrix in Chow,1975
*      hl="h" matrix in Chow,1975
*      Htmp=temporary "H" matrix
*      hltmp=temporary "hl" matrix
*      bltmp=temporary "bl" matrix
*      GG=storage for G's
*      Ggl=storage matrix for gl's
*      xh=optimal x vector
*      yh=optimal y vector
*      xopt=storage matrix for xh
*      yopt=storage matrix for yh
*
*
*
*Specification of variables and matrices:
*
*
type rectangular A Gal Gbl Gul C
type symmetric KCAP
type real Disc
type vector y0 xa
type integer start npd met instr
local rect G GG Ggl xopt yopt R
local symmetric K H
local vector gl kl hl ultmp al bl ul bltmp xh yh
local vector Vhat cl ybar ubar xbar Vstar
local integer ct last tt si kk nn time mn ins nm cnt xj
dimension al(met) kl(met) bl(met) y0(met) ul(met) A(met,met) $
K(met,met) C(met,instr) Gal(met,(npd+start)) Gbl(met,(npd+start)) $
Vhat(1) G(instr,met) gl(instr) H(met,met) hl(met) $
ultmp(met) bltmp(met) GG(met,instr*(npd+start)) Ggl(instr,(npd+start)) $
yh(met) xopt(instr,(npd+start)) yopt(met,(npd+start)) $
cl(1) xh(instr) Gul(met,(npd+start)) $
xa(instr) Vstar(1) R(met,met) KCAP(met,met)
*
*write(format='(14f9.4)') A
*write(format='(14f9.4)') KCAP
*write(format='(14f9.4)') Gal Gbl Gul
*write(format='(1f9.4)') C y0
*write Disc
*write xa

```

APPENDIX G - CONTINUED

```

*write start npd met instr
*
*start iterations
*
ieval last=start+npd
*
*
*compute H's,hl's,G's,gl's for planning horizon starting with
*last period in conformity with the dynamic programming approach
*
*
matrix xopt=const(0.0)
matrix yopt=const(0.0)
matrix bltmp=const(0.0)
matrix al=const(0.0)
matrix bl=const(0.0)
matrix ul=const(0.0)
matrix cl=const(0.0)
  do tt=0, npd
    ieval ct=last-tt
    matrix bltmp=bl
    matrix K=scale((1+Disc)**(-1*.25*(ct-start+1)))*KCAP
  *   eigen K eigtest
*note THE EIGENVALUES OF K ARE:
*   write eigtest
  *write(format='(14f8.4)') K
  matrix al=const(0.0)
  matrix bl=const(0.0)
  matrix ul=const(0.0)
  do si=1, met
    eval al(si)=Gal(si,ct)
    eval bl(si)=Gbl(si,ct)
    eval ul(si)=Gul(si,ct)
  end do si
* write ct
*write(format='(1f8.4)') al bl ul
if ct.eq.last
  begin last
    matrix H=K
    matrix G=scale(-1)*((inv(tr(C)*H*C))*(tr(C)*H*A))
    matrix hl=kl=K*al
    matrix gl=scale(-1)*inv(tr(C)*H*C)*tr(C)*(H*bl-hl)
    matrix cl=tr(al)*K*al
  
```

APPENDIX G - CONTINUED

```

matrix R=A+C*G
end last
else
begin rest
matrix hl=K*al+(tr(R)*(hl-H*bltmp))
matrix cl=(tr(al)*K*al)+((tr(bltmp+C*gl))*H*(bltmp+C*gl)) $
-scale(2)*tr(bltmp+C*gl)*hl+cl
matrix H=K+tr(R)*H*R
matrix G=scale(-1)*(inv(tr(C)*H*C))*tr(C)*H*A
matrix gl=scale(-1)*inv(tr(C)*H*C)*tr(C)*(H*bl-hl)
matrix R=A+C*G
end rest
*
* store G's and gl's in storage arrays
do kk=1,instr
do nn=1,met
eval GG(nn,(instr*(ct-1)+kk))=G(kk,nn)
eval Ggl(kk,ct)=gl(kk)
end do nn
end do kk
*
* write ct
* write al bl ul cl
* write(format='(1f20.4)') hl gl
* write(format='(14f20.4)') G H
* write(format='(16f20.4)') GG Ggl
*
* end do tt
*
*compute minimized multiperiod welfare cost,Vstar,at
*starting period equal to 'start'
*
*eigen H eigtest2
*note EIGENVALUES OF H USED IN WELFARE COST CALCULATIONS:
*write eigtest2
* write ul
matrix ul=const(0.0)
matrix Vstar=tr(y0)*tr(R)*H*R*y0 $
+scale(2)*tr(y0)*tr(R)*(H*bl-hl)+tr(bl+C*gl)*H*(bl+C*gl) $
-scale(2)*tr(bl+C*gl)*hl+cl+tr(ul)*H*ul
matrix Vhat=tr(xa)*tr(C)*H*C*xa $
+scale(2)*tr(xa)*tr(C)*(H*A*y0+H*bl-hl) $

```

APPENDIX G - CONTINUED

```

+tr(A*y0+b1)*H*(A*y0+b1)-scale(2)*tr(A*y0+b1)*h1 $
+cl+tr(ul)*H*ul
*
*compute x's and y's
*
do time=start,last
  do mn=1,met
    eval al(mn)=Gal(mn,time)
    eval bl(mn)=Gbl(mn,time)
  end do mn
  if time.eq.start
    begin first
      matrix ul=const(0.0)
      matrix xh=G*y0+gl
      matrix yh=A*y0+C*xh+b1+ul
    end first
  else
    begin rest1
*
* retrieve G's and gl's from storage arrays
*
do ins=1,instr
  do nm=1,met
    eval G(ins,nm)=GG(nm,(instr*(time-1)+ins))
    eval gl(ins)=Ggl(ins,time)
  end do ins
end do nm
*
      matrix xh=G*yh+gl
      matrix ul=const(0.0)
      matrix yh=A*yh+C*xh+b1+ul
    end rest1
*
* store xh's and yh's into storage arrays
*
do cnt=1,instr
  do xj=1,met
    eval xopt(cnt,time)=xh(cnt)
    eval yopt(xj,time)=yh(xj)
  end do xj
end do cnt
*
end do time

```

APPENDIX G - CONTINUED

```

*
* write(format='(1f20.4)') a1 b1 u1 h1 g1 c1 Vstar Vhat
note    THE FOLLOWING RESULTS ARE FOR THE FOLLOWING PERIOD:
note    OF THE BASE EVALUATION PERIOD:
write    start
note    OPTIMAL INSTRUMENT VALUES FOR REMAINING PERIODS
write(format='(16f20.4)') xopt
note    VALUES OF Y VECTOR UNDER OPTIMAL CONTROL FOR REMAINING PERIODS
write(format='(16f20.4)') yopt
note    WELFARE COST OF OPTIMAL MEASURE INCURRED IN PERIOD
write(format='(1f20.4)') Vstar
note    WELFARE COST OF ACTUAL MEASURE INCURRED IN PERIOD
write(format='(1f20.4)') Vhat
return
end
*****
*****
declare symm KKCAP(14,14)
declare rect GGal(14,16)
matrix GGal=const(0.0)
* write(format='(16f11.4)') GGal
declare rect Gul(14,16)
matrix Gul=const(0.0)
declare rect GGbl(14,16)
matrix GGbl=const(0.0)
DECLARE VECTOR XXA(1)
declare vector yy0(14)
declare rect CCAP(14,1)
MATRIX CCAP=CONST(0.0)
declare rect WHTS(4,4)
input WHTS
.5 1.0 0.0 0.0
.5 1.0 0.0 3.0
1.0 1.0 0.0 0.0
1.0 1.0 0.0 3.0
*
*ESTIMATE MODEL AND REINITIALIZE FOR EACH PERIOD
do lsht=1,15
do lws=1,4
ieval WSET=lws
eval w0=.05
eval w1=WHTS(WSET,1)
eval w2=WHTS(WSET,2)

```

APPENDIX G - CONTINUED

```

eval w3=WHTS(WSET,3)
eval dr=WHTS(WSET,4)
eval drate=dr/100
note WEIGHTS IN OBJECTIVE FUNCTION:
note          On      M1
write          w0
note          On      UNEMP
write          w1
note          On      PGNP
write          w2
note          On      Instrument Penalty
write          w3
note DISCOUNT RATE
write          dr
ieval sht=lsht
ieval initialt=(79,4)+sht-1
ieval pd=16-sht
do mt=0,pd
  eval GGal(1,(sht+mt))=(M1((initialt)))*(1.06)**(.25*(mt+1))
  eval GGal(2,(sht+mt))=6.0
  eval GGal(3,(sht+mt))=(PGNP((initialt)))*(1.03)**(.25*(mt+1))
  eval GGal(14,(sht+mt))=1.0
end do mt
*write(format='(16f8.4)') GGal
matrix KKCAP=const(0.0)
eval KKCAP(1,1)=w0
eval KKCAP(2,2)=w1
eval KKCAP(3,3)=w2
eval KKCAP(14,14)=w3
OLS(DEFINE=1,NOPRINT) M1 60,1 79,4
# -M1 1 4 -UNEMP 1 4 -PGNP 1 4 LIFESNPU CONSTANT TREND
OLS(DEFINE=2,NOPRINT) UNEMP 60,1 79,4
# -M1 1 4 -UNEMP 1 4 -PGNP 1 4 LIFESNPU
OLS(DEFINE=3,NOPRINT) PGNP 60,1 79,4
# -M1 1 4 -UNEMP 1 4 -PGNP 1 4 LIFESNPU CONSTANT TREND
*
*
*
*
*
*
eval CCAP(1,1)=coeff(13,1)

```

APPENDIX G - CONTINUED

```

eval CCAP(2,1)=coeff(13,2)
eval CCAP(3,1)=coeff(13,3)
eval CCAP(13,1)=1
eval CCAP(14,1)=1
* write(format='(1f11.4)') CCAP
*
*
do ttt=0,pd
  eval GGB1(1,sht+ttt)=coeff(14,1)+(coeff(15,1))*(TREND(((initialt)+ttt+1)))
  eval GGB1(3,sht+ttt)=coeff(14,3)+(coeff(15,3))*(TREND(((initialt)+ttt+1)))
end do ttt
*write(format='(16f11.4)') GGB1
*
*
*
eval yy0(1)=M1((initialt))
eval yy0(2)=UNEMP((initialt))
eval yy0(3)=PGNP((initialt))
eval yy0(4)=M1((initialt-1))
eval yy0(5)=UNEMP((initialt-1))
eval yy0(6)=PGNP((initialt-1))
eval yy0(7)=M1((initialt-2))
eval yy0(8)=UNEMP((initialt-2))
eval yy0(9)=PGNP((initialt-2))
eval yy0(10)=M1((initialt-3))
eval yy0(11)=UNEMP((initialt-3))
eval yy0(12)=PGNP((initialt-3))
eval yy0(13)=LIFESNPU((initialt))
eval yy0(14)=LIFESNPU((initialt))-LIFESNPU((initialt-1))
* write(format='(1f11.4)') yy0
EVAL XKA(1)=LIFESNPU((initialt+1))
* write XKA
execute bmat 3 4 1 0 1
execute chow acap KKCAP CCAP yy0 sht pd 14 1 GGal GGB1 Gul drate XKA
end do lws
end do lsht
end

```

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