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DESIGN OF PROCESS CONTROLLERS

by

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TO MY PARENTS, MIRI, INBAR AND HELITH

ABSTRACT

DESIGN OF PROCESS CONTROLLERS

by

Zalman J. Palmor

Adviser: Professor Reuel Shinnar

An advanced design method for simple sampled data control algorithms as well as an interactive design and tuning method for analog processes controllers are presented.

The real goals of a design and the requirements of a sampled data process controller algorithm are stated. Based on the latter, a thorough evaluation of modern stochastic design methods is performed. It is found that although modern methods serve as useful tools in interactive design and provide usable suggestions for improved designs they are not useful as straightforward design procedures as they do not guarantee successful controllers.

It is shown that actual measurement and study of process disturbances has limited value in the actual design of control algorithms for process control. The major reason for that is that in process control stability constraints are usually dominating.

The control problems associated with relatively infrequent samplings are discussed and treated.

The results derived for sampled data controllers are extended to continuous control. An interactive tuning procedure for analog controllers for processes with relatively large dead times is suggested. The method is based on sensitivity analysis and a modified version of the mini-max procedure.

The special stability problems associated with dead time compensation and inversion of transfer function are pointed out. It is shown that while classical methods are not suitable for such cases the suggested procedure handles effectively these situations. It is demonstrated that in the presence of dead time the dead time compensator when properly tuned improves considerably the performance of the conventional analog controllers.

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PRESENTATION

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Parts of this research are being prepared for publication.

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NOMENCLATURE

a	coefficient of dead time compensator (5.9)
a_t	stationary white noise sequence
B	backward shift operator (3.3)
c	residue of the ratio θ/T ($\theta/T = k+c$)
$C(B)$	transfer function of lead-lag compensators
d_i	coefficients of compensator in forward loop
e_t	error sequence
$E\{\cdot\}$	expected value of $\{\cdot\}$, $(\langle\cdot\rangle)$
g_i	coefficients of elements of dead time compensator (5.25)
G_p	process transfer function
G_{po}	process transfer function without delay
G^*	overall transfer function (2.1)
G_c	controller transfer function
$[G_c{}^n]$	space of controllers associated with the noise n_t
$[G_c]^*$	set of optimal controllers fulfilling all criteria
G_n	noise generator
k	integral value of θ/T
K_c	controller gain
K_D	derivative gain
K_I	integral gain
K_p	process gain
n_t	noise sequence

s	Laplace transform variable
s_i	coefficients of the polynomial $\delta(B)$ (denominator of $G_p(B)$)
t	time
T	sampling interval
X_c	transfer function of main controller
y	system output
u_t	controller signal
α	relative perturbation in process parameters
β	relative perturbation in process time delay
η	damping ratio of the characteristic equation of the closed loop (Appendix D)
$\gamma(B)$	polynomial resulting from factorization (Appendix A)
γ_i	coefficients of γ (B)
$\delta(B)$	denominator of $G_p(B)$
δ_i	$=e^{-T/\tau_i}$
θ	process time delay
$\lambda(B)$	numerator polynomial of noise filter (4.1)
λ_i	parameter of λ (B)
λ_a	design value of λ
λ_r	actual value of λ
λ_{base}	value of λ at the smallest sampling interval
ν	weight on control effort (5.23)
ξ	damping ratio of second order transfer function

σ_a^2	variance of a_t
σ_n^2	variance of n_t
σ_m^2	variance of measurement noise
τ_D	derivative time
τ_I	reset rate
τ_i	process time constants
$\phi(B)$	the autoregressive part in noise model (4.1)
ϕ_i	parameters of $\phi(B)$
ϕ_a	design value of ϕ
ϕ_r	actual value of ϕ
$\psi_i(B)$	defined in (5.11a)
$\omega(B)$	numerator of $G_p(B)$
ω_i	coefficients of (B)
ω	frequency
ω_ℓ	comparison frequency in the low frequencies

CHAPTER 1: Evaluation of Modern Design Methods of Sampled Data Control Algorithms for Process Control

1. Introduction

In the process industries it is quite common to control continuous process on the basis of laboratory tests based on samples taken from the process. This is especially common in the paper, polymers, and metallurgical industries. The final properties of the material are effected by process conditions, but are not measurable at this stage of the process, at least in an instantaneous way. For example, polymers for spinning must be tested in an actual spinning machine for spinability strength and dyeability. Metals from a rolling mill have to be tested for strength, hardness, etc.

This normally involves a considerable time delay and also limits the frequency at which we can sample. To compensate for this we often introduce secondary continuous feedback loops measuring related variables. In the case of the polymer for spinning we use molecular weight or viscosity as the measured variable adjusting catalyst flow or process temperature in the continuous loop.

The operator then adjusts the set point of these secondary loops according to the result of the laboratory tests.

This chapter is concerned with the design of simple control strategies for such cases. Several methods have been suggested in recent years and we will try to compare them and suggest an alternative advanced design algorithm.

We are going to show that some of the optimal design algorithms proposed lead to significant insights in controller design, though they do not guarantee a usable control strategy. Furthermore, once we have understood their implications we can get the same results using classical control methods. We shall also show that the latter method has advantages for the designer.

While we concentrate on the control problems associated with relatively infrequent samplings, the results should be of wider interest as they have some important implications to continuous control, and the use of stochastic predictors in process control.

2. The Requirements of a Sampled Data Controller Algorithm

Before going into any detailed discussion we want to first outline our approach to the design of a control algorithm. In a paper by Kestenbaum et al. (1976) the specifications of a continuous analog controller were discussed. We will restate and modify them here in a form suitable for sampled data control. The general configuration of the control system is given in fig. 1.

1. The controller must be able to maintain the desired output variable at a given set point. In the type of problem that one often deals with in polymers or other complex materials this is often difficult as varying only one manipulated variable is not always sufficient to achieve this. For simplicity we will assume that it is possible to control the state of the measured variable by adjusting just one set point. The approach can be generalized to several set points.

Maintaining the output variable at the desired set point is the most important function of the control and one should always remember that we don't know a priori the exact steady state control setting that will lead to the desired steady state output.

2. Set point changes should be fast and smooth. However, minimum time is not a sufficient criterion. We

can't afford large overshoots as the operator does not know the final steady state and would have to step in.

3. <u>Asymptotic stability and satisfactory performance for different types of disturbances that could arise</u>. The algorithm must lead to stable overall control and converge fast on the desired steady state.

While we can study the nature of the disturbances for the system and try to optimize our controller accordingly, the controller must be reasonably able to handle any unforeseen disturbance that might arise later. In a sampled data system we have an additional requirement. Such systems normally involve some uncorrelated measurement errors in individual samples. If we act on these measurements we will introduce perturbations into the system and the controller must be designed such that these introduced perturbations are small.

4. <u>The controller should be designable with a minimum information with respect to the nature of the inputs and the structure of the system</u>. The second requirement is the more stringent one. Any design method must be related to a modelling and identification method. As the models which result from such methods are almost always crude approximations and simpler than the real system, our total design method must be able to handle a complex system

with a simple model, the parameters of which can be experimentally determined. This is especially important in optimal control as unconstrained optimal control algorithms are often very model sensitive.

5. The controller must be reasonably insensitive to changes in system parameters and must be stable over a reasonable range of system parameters.

6. Excessive control actions should be avoided. In process control the cost of the control effort (steam, catalyst, etc.) is determined mainly by the amount used to keep the system at its steady state. Dynamic variations of the control effort have little effect on costs. However, limiting the control effort is a mathematical convenience in design as the real controller has a limited range. In most optimization schemes it is useful in reducing the sensitivity of the controller.

In our specific case we can add a seventh criterion.

7. Simple control schemes are preferable. As we deal here with sampled data on a relatively infrequent basis there is an inherent advantage in simple transparent control schemes. Cost-wise, today, there is no problem to use the complete history of the unit on a computer, printing out the instructions or directly upgrading the valves after the operator punches in the last result. One should,

however, convince oneself that there is a real advantage to that. If a simple scheme gives almost the same result it is preferable as it is easier to override it in special situations, and helps the operator to understand the unit.

We can look at these criteria quantitatively by investigating the closed loop frequency response of G*(B) defined as follows (see fig. 1).

$$G^*(B) = \frac{1}{1 + G_p(B)G_c(B)} \tag{2.1}$$

and we can rewrite our initial criteria just by writing them in terms of $G^*(e^{-j\omega T})$. Normally it has a form as given in fig. 1a. Criterion 1 means that the value of $G^*(e^{-j\omega T})$ must be zero for low frequencies. Criterion 3 means that G* (B) can't have any poles inside the unit circle in the complex B plane. Furthermore, $G^*(e^{-j\omega T})$ must not have a high peak in the resonance region. Criterion 5 means that G* (B) should not change too much for reasonable perturbations in Gp(B). This can also be related to criterion 4. Criterion 4 is best investigated in the time domain by plotting the continuous time response of the overall system to step inputs. Criterion 6 can be evaluated by plotting $G_c(e^{-j\omega T}) \cdot G^*(e^{-j\omega T})$.

8. <u>Satisfactory response of the system to the actual disturbances specific to the process</u>. It is also common to

define an eighth criterion for the controller design, that the controller should be specifically designed to deal with the specific types of disturbances common to this process.

In classic design we just look if the plant has any frequencies in the resonance region or in the high frequency range and adjust G* accordingly. In stochastic optimal control we look at the problem in a somewhat different way. Most disturbances are not completely uncorrelated, but have a pattern. If we study the time behavior of the disturbance n_t then, from the past and present states of the system, we can predict n_t for the future. We can consequently design a control system as in fig. 2.

Any control action will only affect the system in the future, t+k. If we know G_p we can predict the state of the system at t+k as a result of the control action previous to t. We can then use our prediction to n_{t+k} made at time t, $(\hat{n}_{t+k/t})$, together with our knowledge of the previous control action to find a u_t that will exactly cancel $\hat{n}_{t+k/t}$. As we really don't know $G_p(B)$ exactly we are matching here two predictions. It can be easily shown that fig. 2 and fig. 1 are identical as the predictor is really based on measuring past values of e_t.

What we hope to gain by writing the controller in the form of fig. 2 is some insight as to how the feedback controller should look.

We can look at the criterion that the estimated value of e_t should be small for a given stochastically known n_t as our 8th criterion and we normally define it in a stochastic way such that $\langle e_t^2 \rangle$ should be minimized given an estimate of n_t in a suitable probability definition.

We should remember that this is one of eight criteria; at least the first five have equal importance and one and three are overriding.

While the eighth criterion can be nicely defined mathematically, the same cannot be said of the other seven. Interestingly, we can get formulations for any one of these or pairs, but to formulate all of them together in a rigorous way is rather difficult. On the other hand, given a controller G_c, we can find by simulation quite easily if it fulfills all of our criteria.

Let us assume that there is a set of different controllers $[G_c]^*$ in which all G_c fulfill our criteria; not necessarily equally well, but sufficiently well. Some of them will be better with one criteria, some with another.

Talking about any real optimum here is not very meaningful as we don't know a priori how to define it. For some of these criteria we can get the limits that an optimal controller optimized for these criteria alone could achieve.

If we can find a controller optimum for criterion (8) that fulfills all of our other conditions we really have an optimum. There are, however, two questions we have to ask ourselves:

1. n_t is hard to measure. Is it worth the effort? We first have to be convinced that the properties of the structure of n_t are sufficiently time independent. But this is not the whole story. Assume we could measure n_t. To get a workable design we have to give n_t a reasonable mathematical formulation which given the inaccuracies of such methods, is not unique. We have to guess a form of n_t, which is equivalent to defining a mathematical model for the disturbance n_t and estimate the parameters of this noise model from measurements. In most cases it will be hard to estimate more than two parameters, so we need simple models. We can now look at the whole reasonable space of these two parameters which is associated with a space of $[G_c{}^n]$ and look what part of it is a congruent with the permissible model space of $[G_c]^*$ based on the first

conditions. If only a small part of $[G_c^n]$ falls into the space $[G_c]^*$ than measuring n_t might be a questionable exercise. We could still only modify G_c for any measured n_t by some mechanism (as quadratic criterion, for example) such that it falls into $[G_c]^*$, but then we have to see if the final result is really sufficiently better than that for a reasonable controller obtained by either guessing n_t or G_c.

2. Can we find a structure of n_t such that it gives us a good first guess for G_c? There are two schools on stochastic controls. One believes in actually studying n_t and using the information in controller design. The other considers n_t solely a mathematical design tool leading to a better controller design similar, but more efficient than using frequency response methods. Finding a model for n_t is not a unique process, as we can get reasonable fits with different models. As the controller is really defined once we write down n_t, we have to have an a priori knowledge as to what forms of n_t give useful controllers. One of the goals of the present work is to answer those questions for the specific type of system outlined in the introduction.

We can now outline the rest of Chapter I. In section 3 we will deal with the choice and identification of process models for our purpose. In section 4 we will discuss noise models suitable for the identification of process noise. In section 5 we will discuss the structure of the controllers obtained by different design methods and in section 6 we will test these controllers in terms of our criteria. In the last section we will propose a design method based on these results, and show how the results of optimal stochastic control theory lead to better designs that can be interpreted in terms of classical controller design.

3. Identification of the Process Model

There are basically two common approaches for identifying a process model suitable for our purpose. The first is to set up a theoretical model of the process, building a set of differential equations describing the known physics of the process, and then trying to identify the parameters of the model by separate measurements. Considerable research is normally needed for this approach (Weekman, 1975). To be useful for control purposes, we normally, in the end, have to simplify this process model into a simplified linearized version; but the complete model is useful to study the behavior of the controller.

Another approach is direct identification, preferably by a step response. In our case we mainly require a model that predicts the effect of the manipulated variable on the measured variable.

We are, in this paper, concerned only with stable systems, as unstable systems need a more frequent control for stabilization. We are also looking for a linear transfer function giving the output at the time of sampling. The safest way to get such a model is to follow several step inputs in the manipulated variable using steps in both directions. In an existing process a lot can be learned from studying cross correlations in the closed loop but in the experience of the authors it is risky to rely solely on such measurements unless they contain a sufficient number of significant intentional step changes.

There have been several claims that in the process industries second order models coupled with delays are sufficient for most purposes. One can approximately describe the continuous form of the desired transfer function by

$$G_p(s) = \frac{Y(s)}{U(s)} = \frac{K_p(\tau_1 s + 1)e^{-\theta s}}{(\tau_2 s + 1)(\tau_3 s + 1)} \tag{3.1}$$

where τ_2 and τ_3 might be complex.

This allows one to describe both overdamped and underdamped stable processes as well as processes with an inverse initial response, see fig. 3. In our case this claim is even more justified. We can approximate any linear response at fixed time intervals by the transfer function

$$G_p(B) = \frac{\omega(B)}{\delta(B)} B^{k+1} = G_{po}(B) B^{k} \tag{3.2}$$

where $\omega(B)$ and $\delta(B)$ are polynomials in B.

B here is the backward shift operator defined by

$$B^k y_t = y_{t-k} \tag{3.3}$$

In table I we show the discrete equivalents to the four types of transfer functions represented by (3.1), see fig. 3, and the relations between the discrete polynomials and the parameters of the underlying continuous process.

One can show that for any given stable transfer function of interest to us, the higher terms in the polynomials $\omega(B)$ and $\delta(B)$ decrease exponentially with increasing sample interval T (see Appendix B). An example of this dependence is given in table II.

In practice, therefore, a second order polynomial for $\omega(B)$ and $\delta(B)$ are probably the highest order that can be justified, and in most cases a first order plus delay is sufficient.

$$y_t = \frac{\omega_0 - \omega_1 B}{1 - \delta B} u_{t-k} \tag{3.4}$$

We should note that the transfer function G_p is not what is normally considered the open loop transfer function of the process. In most cases of control based on sampled data the process will have several continuous feedback loops, and $G_p(s)$ is the transfer function for adjusting the set point of one of these control loops. $G_p(s)$ itself can often be modified, as it depends on the adjustment of the continuous feedback loop.

If this loop is adjusted in the normal way $G_p(s)$ will have an overshoot and therefore be underdamped. If T is larger than the settling time of the process then $G_p(B)$ is mainly a delay. If the settling time is long it might make sense to adjust $G_p(s)$ such that the overshoot is minimal. Here we might look at $G_p(s)$ and the desired control strategy in an integrated way.

For the present we can summarize that a process model of the form

$$G_p(B) = \frac{\omega_0 - \omega_1 B - \omega_2 B^2}{1 - \delta_1 B - \delta_2 B^2} B^{k+1} \tag{3.5}$$

is sufficient for all practical cases of interest in sampled data control based on laboratory measurements, and when sampling intervals are reasonably large, then in most cases the model in equation (3.4) is sufficient. Furthermore, $G_p(B)$ not only depends on the interval T, but also on the timings of the continuous process controllers and therefore can be adjusted.

4. Noise Models

In early control applications, disturbances were modelled by means of their power spectra or autocorrelation functions (Weiner (1942), Newton et al (1957)). Lately, due to the extensive development of state space representation of linear systems and linear filtering (Kalman (1960)) state variable models are usually used to characterize the process as well as the disturbances. The noise in this representation may be thought of as an output of a dynamical system driven by a stochastic process which usually is assumed to follow a white Gaussian noise with known mean and variance. (Or, in the multivariable case, a vector of white noise variables, with known mean and covariance matrix.)

A similar representation, though in terms of transfer function, (therefore, suitable for representing linear processes) was developed from statistical point of view, (Box and Jenkins (1968, 1970) and Astrom (1970)). They used statistical time series models; called Autoregressive-Integrated Moving Average (ARIMA) models to describe the stochastic noises for the system and developed extensively procedures for identifying their structure and estimating their parameters.

The general ARIMA model of order (p,d,q) is written as:

$$n_t = G_n(B)a_t = \frac{\lambda_q(B)}{\nabla^d \phi_p(B)} a_t = \frac{(1-\lambda_1 B - \lambda_2 B^2 - \ldots - \lambda_q B^q)}{(1-B)^d (1-\phi_1 B - \ldots - \phi_p B^p)} a_t \qquad (4.1)$$

where a_t is a white noise sequence with zero mean and given variance. The roots of $\lambda(B)$ and $\phi(B)$ are assumed to be outside the unit circle. Whenever $d \neq 0$ the process n_t is nonstationary due to the poles on the unit circle. In this description the noise is seen to be the result of a stationary white noise passing through a filter $G_n(B)$. Specifying G_n (B) as well as the variance of a_t will determine the disturbance completely.

The disturbance sequence, n_t, is considered to represent the total effect at the output of all unobserved disturbances. Thus, the uncontrolled open loop system may be represented by: (see fig. 1).

$$y_t = G_p(B)u_t + n_t = G_p(B)u_t + G_n(B)a_t \qquad (4.2)$$

Knowledge of G_n(B) allows one to predict the future values of n_t based on its history. Thus,

$$\hat{n}_{t+k/t} = \frac{\psi_2(B)}{G_n(B)} a_t \qquad (4.3)$$

where $\psi_2(B)$ is found from the following expansion of $G_n(B)$:

$$G_n(B) = \psi_1(B) + B^{k+1}\psi_2(B) \qquad (4.3a)$$

and therefore contains present and past values of the a_t sequence.

To describe a disturbance in a specific system we need to guess a simplified form of (4.1). Several simplifications are common. The simplest is given by

$$G_n(B) = \frac{1}{1-\phi B} \qquad -1 < \phi < 1 \qquad (4.4)$$

For $0<\phi<1$ this is equivalent to a simple negative exponential correlation function for n_t. Here, $G_n(B)$ is an exponential filter.

Another one parameter model promoted especially by Box and Jenkins (1970) is given by

$$G_n(B) = \frac{1-\lambda B}{1-B} \qquad (4.5)$$

It is nonstationary as it has one pole on the unit circle. Also the variance of n_t is infinite. It is still a very useful tool in controller design as we will see later.

For more complex cases we can go to second order polynomials in $\lambda(B)$ and $\phi(B)$. In our discussion we will suffice ourselves with the case

$$G_n(B) = \frac{1-\lambda B}{(1-B)(1-\phi B)} \tag{4.6}$$

One of the reasons for choosing (4.6) over other possible second order forms is that stationary forms of $G_n(B)$, as for example

$$G_n(B) = \frac{1}{(1-\phi_1 B)(1-\phi_2 B)} \tag{4.7}$$

do not lead to useful controller designs as we will show later. Eqn. (4.6) can be written as

$$G_n(B) = \frac{(1-\lambda)/(1-\phi)}{1-B} + \frac{(\lambda-\phi)/(1-\phi)}{1-\phi B} \tag{4.8}$$

and can be considered the sum of two first order disturbances. Higher order forms of $G_n(B)$ are of limited interest. It is very hard to measure n_t experimentally and to determine a model with more than two parameters is not only difficult, but could lead to large measurement errors.

There is another problem that should be mentioned here. Consider a series of measurements taken at time interval T_o. If we now increase the time interval to $T = m\ T_o$ we get a new time series n_t^*. If we know $G_n(B)$ we can compute a new $G_n^*(B)$ to describe n_t^*. One of the properties of the G_n in eqn. (4.5) is that G_n^* stays invariant with change in sampling rate, (see Box and Jenkins (1970))., i.e.,

$$G_n^*(B) = \frac{1-\lambda^*(m)B}{1-B} \tag{4.9}$$

$$\lambda^*(m) = (-b \mp \sqrt{b^2-4})/2 \qquad (-\text{ for } \lambda > 0)$$

$$\text{and} \quad -b = (m(1-\lambda)^2/\lambda + 2) \qquad (+\text{ for } \lambda < 0) \tag{4.9a}$$

At last, we should point out that any $G_n(B)$ may also be represented in state space description and vice versa (see Macregor (1973)). The results of our work should therefore apply to the latter case.

5. Controller Design

In this chapter we will list and compare the results of different design methods and explain their interrelation, and we will start with classical design.

The simplest sampled data controller used in industry by operators for long times can be written as

$$\nabla u_t = M_c e_t \tag{5.1}$$

The operator makes an adjustment proportional to the deviation. In our notation we will assume that u and e are suitably normalized, such that exact compensation will require an M_c of unity. In good industrial practice this simple algorithm is used with a simple nonlinear filter of the type

$$\nabla u = 0 \qquad -2\sigma < e < 2\sigma \tag{5.2}$$

where σ is the standard deviation of the measurement error (or the inherent uncontrollable error of the system determined experimentally).

If we forget one moment about the nonlinear filter and look at (5.1) as a transfer function using the backward shift operator B, ($Bu_t=u_{t-1}$)

$$u_t = \frac{M_c}{1-B} e_t = M_c(1+B+B^2+\cdots)e_t = M_c\left(e_t + \sum_{i=1}^{t} e_{t-i}\right) \quad (5.3)$$

which is a proportional integral controller.[a]

An integral controller has the form

$$K_I \cdot \sum_{i=1}^{t} e_{t-i} \cdot T$$

and therefore the integral constant K_I is equal to M_c/T.

This immediately illustrates one of the common problems in operator control or any control based on control charts applied to processes with delays. Intuitively we might choose $M_c=1$ or, if we are aware of stability problems, $M_c= 0.5$. Very small values of M_c are hard to use as an operator cannot make small adjustments. If T is large in terms of the process time constants this is fine. If T is small, then M_c/T will be too large and can easily introduce instabilities and limiting cycles.

If T is small then in proper design M_c is small

a. Footnote: or an integral controller dependent on convection. We prefer here the notation used by Aiken (1974).

too and the controller becomes a pure integral controller.

In a standard proportional integral controller the ratio between proportional and integral control is adjustable

$$u_t = K_c\,(e_t + \frac{1}{T_I}\sum_{i=1}^{t} e_{t-i}) \qquad (5.4)$$

which can be written as

$$\nabla u_t = K_c\,(e_t - (1 - \frac{1}{T_I})e_{t-1}) \qquad (5.5)$$

or in terms of the controller transfer function

$$G_c\,(B) = K_c \;\frac{1-(1-1/T_I)B}{1-B} \qquad (5.6)$$

Tuning the controller involves choosing T_I and K_c such that it gives a satisfactory response to set point changes or to step changes in the process input and is still stable (Lopez (1969), Rovira (1969), Moore (1968)).

In classical controller design for such cases we almost always start with (5.6), and in most cases stop here or we can add a more complex lead lag compensator (C(B))

$$G_c(B) = K_c \frac{1-(1-1/T_I)B}{1-B} C(B)$$
$$C(B) = \frac{P(B)}{Q(B)} \quad (5.7)$$

where P (B) and Q(B) are polynomes in B.

A simple derivative controller which is a lead compensator has the form P(B) = 1-aB; (a< 1), but usually more complex forms are used. Again, we choose P(B) and Q(B) such as to fulfill the conditions outlined before (see, as example (Rosenbrock (1974) and Ragazzini (1968) as to how one can find suitable expression for C(B)).

Our initial design criteria are investigated by looking at the properties of the overall closed loop transfer function

$$G^*(B) = 1/(1+G_p(B)G_c(B)) \quad (5.8)$$

either directly or through the properties of the open loop transfer function $G_p(B)G_c(B)$.

To insure proper set point control, $G^*(B) \xrightarrow[B \to 0]{} 0$ (or $G^*(B)G_c(B)G_p(B) \xrightarrow[B \to 1]{} 1$) , or $G_pG_c(B) \xrightarrow[B \to 1]{} \infty$. Stability is insured by looking at the poles of G*(B), which must be well outside the unit circle and must stay there for reasonable perturbations of all the parameters in both G_p and G_c. Furthermore, in sampled data we have to prevent amplification of measurement noise. We will discuss this later in detail.

We note immediately that unlike the continuous case we have here an additional design parameter, namely the sampling time T, by which we can modify the process transfer function $G_p(B)$ and we have, therefore, to look at the overall performance not only in terms of $G^*(B)$, but also in terms of the sampling time T.

There is one type of very effective lead lag compensator which deserves special mention. Consider a transfer function of the form

$$G_p(B) = G_{po}(B) B^{k}$$

which is the discrete equivalent of

$$G_p(s) = G_{po}(s) e^{-s\theta} \quad \text{b}$$

b. Footnote: Note that $G_{po}(B)$ is not necessarily the equivalent of $G_{po}(s)$. This only is the case if θ/T is an integer.

and we would like to design a compensator to reduce the effect of the time lags. For our case we can construct such a compensator by the following argument.

Assume that at the n^{th} sample we make a correction ∇u. If the sampling interval is smaller than the inherent time lag (process delay + measurement delay), then our correction has no effect whatsoever on the next sample. If nothing happened in the process, we would still measure the same deviation e. If we make another correction we over-correct. We have two sensible choices. Either we reduce the gain K_c and apportion part of the adjustment to each of the samples occurring during the time lag, or we reduce the control action by taking into account all the control actions already done which we as yet cannot observe due to the time delay. The first option is automatically obtained if we choose K_c by a proper stability analysis. Long delays will reduce K_c. The second option can be written as

$$\nabla u_t = K_c \left[e_t - \left(1 - \frac{1}{\tau_I}\right) e_{t-1}\right] - a \sum_{i=1}^{k} \nabla u_{t-i} \qquad (5.9)$$

If we want to be consistent with our argument we actually should choose a = 1 as $\sum_{i=1}^{k} \nabla u_{t-i}$ is the total change in ∇u during the period of the delay. For stability reasons, however, a should be smaller. As we now reduce the danger of over-correction during the delay we can choose K_c larger than for the case without the compensator, which gives significant advantages, as will be discussed in the next section.

The compensator in (5.9) was actually first derived from optimal control algorithms. It can be presented purely in terms of classical concepts. It can be written as

$$C(B) = \frac{1}{1 + a(B + B^2 + \cdots + B^k)} \tag{5.10}$$

and we note that if k = 1 and a is small this is a simple lead compensator 1/(1+aB). The properties of the full compensator can be deduced by looking at its Nyquist diagram (fig. 4).

Let us now look at the controllers one gets from optimal stochastic algorithms.

If we have a process transfer function $G_p(B) = G_{po}(B)B^k$ and noise sequence n_t which is generated by an ARIMA filter (eqn. (4.1)), an unconstrained optimal controller that minimizes the output variance ($\mathrm{Var}(y_t)$) is given by the following expression

$$G_c(B) = B\,G_{po}(B)^{-1}\,\psi_2(B)\,\psi_1(B)^{-1} \tag{5.11}$$

where $\psi_2(B)$ and $\psi_1(B)$ are related to $G_n(B)$ in the following way:

We may expand $G_n(B)$ as power series in B

$$G_n(B) = 1 + \sum_{i=1}^{\infty} \psi_i B^i = \psi_1(B) + B^{k+1}\psi_2(B) \tag{5.11a}$$

where

$$\psi_1(B) = 1 + \sum_{i=1}^{k} \psi_i B^i$$

and

$$\psi_2(B) = \sum_{i=1}^{\infty} \psi_{i+k} B^{i-1}$$

The representation of the optimal control laws in the form of (5.11) enables one to get good understanding of the structure and properties of the optimal controllers. However, in order to get a more systematic insight into the optimal design machinery, it is convenient to use a modified representation. As optimal stochastic control laws for systems having delays are always combinations of components one of which is a predictor, it is possible to decompose (5.11) in the form shown in fig. 5 which includes a compensator $X_c(B)$ and a feedback loop known as a Smith predictor (or Smith dead time compensator) (Smith (1959)) and investigate the properties of the resulted forward compensator $X_c(B)$. It can be shown that the $X_c(B)$ which corresponds to (5.11) is

$$X_c(B) = B\,G_{po}(B)^{-1}\,\psi_2(B)[G_n(B) - B\,\psi_2(B)]^{-1} \tag{5.12}$$

Eqn. (5.11) and (5.12) are only correct if $G_{po}(B)$ is invertible. If $G_{po}(B)$ is noninvertible (i.e., it has zeroes inside the unit circle in the complex B plane, $G_{po}(B)$) in (5.11) and (5.12) must be replaced by $G'_{po}(B)$ in which we replace the numerator of $G_{po}(B)$, $\omega(B)$, by another polynomial $\gamma(B)$ which fulfills:

$$\gamma(B)\gamma(B^{-1}) = \omega(B)\omega(B^{-1}) \tag{5.13}$$

If we invert the original $G_{po}(B)$ the resulted controller is practically unstable (Appendix C). Unless $\omega(B)$ has a root on the unit circle we can always do that. For more rigorous treatment of the factorization (5.13) see Wilson (1969) and Appendix A.

We can get non-invertible transfer functions in sampled data systems even when the underlying continuous model is invertible. For first order + delay process, for example (see table I), this situation arises wherever the $|\omega_0/\omega_1| < 1$ or the residue c is larger than $1 - \ln(\frac{1+\delta}{2})/\ln\delta$

Let us now look at the unconstrained optimal controllers in terms of their $X_c(B)$ and $G_c(B)$, and first consider a simple first order transfer function:

$$G_p(B) = \frac{\omega_0}{1-\delta B} B^{k+1} \tag{5.14}$$

For simplicity we choose $k = (\theta/T)$ as an integer such that $\omega_1 = 0$.

For the relations between δ, ω_0 , and the parameters of the continuous process, see table I. If n_t is given by eqn. (4.4), (stationary first order disturbance), eqn. (5.11) yields

$$G_c(B) = \frac{(\phi^{k+1}/\omega_0)(1-\delta B)}{1-(\phi B)^{k+1}} \tag{5.15}$$

which can be written in terms of the control action u_t

$$u_t = \phi^{k+1} u_{t-1} + \frac{\phi^{k+1}}{\omega_0}(e_t - \delta e_{t-1}) \tag{5.15a}$$

This is essentially a proportional + derivative controller with a dead time compensator. For this case, eqn. (5.12) becomes

$$X_c(B) = \frac{\phi^{k+1}}{\omega_0}(1-\delta B)\frac{1}{1-\phi^{k+1}B} \tag{5.16}$$

which shows that the optimal forward loop controller (for $\phi > 0$) is simply a PD controller with a compensator. If ϕ is less than zero the form of (5.15) depends on k. For even numbers of k, X_c is a positive feedback controller. The overall response of the total closed loop system is:

$$G^*(B) = 1 - \phi^{k+1}B^{k+1} \tag{5.17}$$

which indicates that for step input the offset will be

$$\lim_{B \to 1} G^*(B) = 1 - \phi^{k+1}$$

For this case the deterministic unconstrained controller which minimizes $\sum_{i=1}^{\infty} e_i^2$ for a given initial state is obtained by substituting δ for ϕ. Its offset is therefore

$$\lim_{B \to 1} G^*(B) = 1 - \delta^{k+1}$$

This design does not lead to useful controllers as condition 1 is not fulfilled (unless δ or ϕ are close to unity). We, therefore, need to look for noise models for which

$$G^{*}(B) \rightarrow 0 \quad as \quad B \rightarrow 1 \tag{5.18}$$

This requirement will automatically be achieved for any noise model for which

$$\chi_c(B) \underset{B \rightarrow 1}{\longrightarrow} \infty \tag{5.19}$$

or

$$\frac{\psi_2(B)}{\psi_1(B)} \rightarrow \infty \quad as \quad B \rightarrow 1 \tag{5.19a}$$

This condition may be translated into an equivalent constraint on the state space representation but this is outside our scope here.

The type of noise models described by eqn. (4.1) fulfill the condition in (5.19) whenever $d \neq 0$, i.e., when they exhibit non-stationarity. As stated in sections 2 and 4 most real disturbances may be represented by several models and eqn. (5.19) allows us to choose models which at least have the potential of leading to satisfactory designs.

One of the simplest form of $G_n(B)$ that fulfills (5.19) is eqn. (4.5). In this case the controllers for first order + delay process are given by eqns. (III.3) and (IV.1) in tables III and IV respectively. When θ/T is an integer $(\omega_1=0)$, $X_c(B)$ is simply a PI controller of the form of (5.5) and $G_c(B)$ takes the form of (5.9) with $1/\tau_I = 1-\delta$, $K_c = (1-\lambda)/\omega_0$ and $a=1-\lambda$. The overall closed loop transfer function $G^*(B)$ is

$$G^*(B) = 1 - \frac{(1-\lambda)B^{k+1}}{1-\lambda B} \tag{5.20}$$

which obviously goes to 0 for $B \to 1$

We note that $G^*(B)$ does not contain $G_{po}(B)$. This is due to the fact that $X_c(B)$ contains $G_{po}^{-1}(B)$. If $G_{po}(B)$ is different from the assumed one, as is the case in reality, then $G^*(B)$ will depend on it. This will be discussed later.

Eqn. (5.20) has been suggested by Dahlin (1968) as a basis for designing sampled data controllers. He considered

λ a tuning parameter ranging from 0 to 1, whereas in the original optimal formulation it is an experimentally deterministic property of the inputs and can vary from -1 to 1.

We shall show later that with proper choice of λ the controller (III.3) has an excellent performance. We also note that for our simple example this controller is the same as given in (5.9).

In table V we give the resulting forms of the controllers for second order + delay processes (and ARIMA (0,1,1)) in terms of $G_c(B)$ and the control action u_t. These formulae are also suitable for nonminimum phase second order processes for which the $\omega(B)$ polynomial in their discrete transfer function, is invertible. (This is possible under suitable choice of the sampling interval.) It is seen that the controllers in these cases may be thought of as a PID controller and a dead time compensator. Although the dead time compensators are somewhat more complex than the one in (5.9) the net effect is similar. The representation of these controllers in terms of $X_c(B)$ is easily derived using equation (III.1) and, for θ/T integers, are given by

$$X_c(B) = \frac{1-\lambda}{\omega_0} \cdot \frac{1-S_1 B - S_2 B^2}{1-B} \cdot \frac{1}{1-(\omega_1/\omega_0)B} \qquad (5.21)$$

which is a PID controller (see table V) plus a filter resulting from the inversion of $\omega(B)$. We note that the only difference between the optimal $X_c(B)$'s for first order and second order processes is that in the latter a PID control law replaces the PI controller (see also Philipson (1975)) in the first.

It is of interest to study the effect of more complicated noise models on the resulting optimal controllers. We give for this purpose in table III, equation (III.4), the resulting optimal forward controller, $X_c(B)$, for the ARIMA (1,1,1) which has been presented in section 4, eqn. (4.6). We observe that the net effect of the inclusion of ϕ the autoregressive parameter is to add a "classical" lead lag compensator having the following transfer function

$$C(B) = \frac{1-\eta_1(1-B)}{1-\eta_2(1-B)} \tag{5.22}$$

The properties of C(B), obviously, depend on the relative values of λ and ϕ . That is to say, on the nature of the noise. In addition, the gain of the forward compensator as well as that of the overall controller is altered from $(1-\lambda)/\omega_0$ in the ARIMA (0,1,1) case to $(1-\lambda)/[(1+m_2)\omega_0]$ in the ARIMA (1,1,1). Furthermore, C(B) is invariant to the order and structure of the process.

This conclusion may also be reached by noting that $X_c(B)$ in (5.12) is a product of two factors; one depends solely on the process and the second on the noise.

The real significance of the noise in the optimal design is that it acts as a constraint constraining the control effort. Thus, both λ and ϕ are essentially constraints. If we, however, consider them as properties of the inputs, then we have to look for another way to introduce a constraint into the controller design.

As the noises chosen have unstable poles the controller output, u_t, is a nonstationary process as is n_t. As ∇u_t is stationary we may choose to minimize the expected value of

$$E\left\{y_t^2 + \nu(\nabla u_t)^2\right\} \tag{5.23}$$

The optimal forward controller $X_c(B)$ for these cases is given by eqn. (III.5) (for definitions see Appendix A) and it is readily observed that (III.5) can't be factorized to process factors and noise factors and it is, therefore, impossible to get these controllers from simple arguments as Dahlin (1968) or Philipson (1975). The various $X_c(B)$ for first order process + the ARIMA models are presented in table III (eqns. III.6 to III.8). It can be seen that in all these cases constraining doesn't change the basic unconstrained structure of the controller. Its main result is in a) modifying the lead lag compensator in $X_c(B)$ and b) reducing the gain. The exact nature of these modifications depends on the noise properties, but could be constructed by classical methods as will be demonstrated in the next section. In the ARIMA (0,1,1) case, for example, this compensator is given by

$$C(B) = \frac{1}{1 + \alpha_1 B + \alpha_2 B^2} \tag{5.24}$$

which, for positive values of λ is a lag compensator, while for highly correlated noises (negative λ's close to -1) becomes a high pass filter.

In table IV we give the corresponding expression for $G_c(B)$ and u_t. Again, we see that the controller is a PI + dead time compensator. The constraint reduces the gain and the coefficients of the dead time compensator which generally is given (for θ/T integers) by

$$\frac{1}{1 + g_1 B + g_2 B^2 + \cdots\cdots g_k B^k} \tag{5.25}$$

The relation between (5.24) and (5.25) (for this case) are

$$g_1 = d_1 + (1-\lambda) w_0/\gamma_0$$
$$g_2 = d_2 + (1-\lambda) w_0/\gamma_0$$
$$g_3 = g_4 = \cdots = g_k = (1-\lambda) w_0/\gamma_0$$

To get better insight into the constraint action we have plotted in fig. 6 the above optimal trajectories, g_1 and g_2, for a specific example ($\theta/\tau = .5$; $\theta/T = 2$) and for different values of λ.

For strongly negative values of λ the constraints reduce g_2 keeping g_1 constant. For λ between -0.5 and 0.2 the effect of the constraint is practically to reduce λ. If the constraint is applied to a controller with a λ between 0.5 and 1 (approximately) it will reduce g_1 (the coefficient of ∇u_{t-1}) until it becomes negative.

Inputs characteristic of chemical plants have normally positive λ. It is therefore interesting that for a wide range of λ (between 0 and some maximum value) the constraint design is very insensitive to the observed λ used as a basis for the design. If we constrain $\langle \nabla u_t^2 \rangle$ we will end up with approximately the same controller design regardless of what the λ for the unconstrained design is. This indicates that the controller will have a good performance in a wide range of frequencies.

6. Evaluation of the Optimal Controller Design

In the previous section we described the different controllers that result from various optimal design procedures based on stochastic disturbances. To evaluate these designs in terms of the criteria described in section 2 we will use some specific examples.

As a first example we will use a case based on the continuous transfer function:

$$G_p(s) = \frac{e^{-.5s}}{s+1} \tag{6.1}$$

Using a sampling time of T = .25 units of time, we get the sampled data process transfer function:

$$G_p(B) = \frac{.22B^3}{1-.78B} \tag{6.2}$$

and consider the controller given in equation (IV.1) which is based on a disturbance described by eqn. (4.5), having one measurable parameter λ . The gain of the controller is a direct function of λ and we will present its performance in terms of the λ . As λ changes from -1 to +1, the gain changes from 9.04 to zero.

The first problem we have to deal with in any optimal controller is stability. Optimal controllers contain the inverse of the process transfer function and are therefore sensitive to any small change in the real transfer function. The dead time compensator is the equivalent of the inversion of B^k and therefore introduces stability problems in the following sense.

In fig. 7 we plot the stability limits of a PI controller

$$G_c(B) = K_c + K_I \frac{B}{1-B} \tag{6.3}$$

for the process (6.1) as a function of K_c and K_I (curve a). In curve b we give the stability limits of the same closed loop system for the controller given by

$$G_c(B) = \frac{K_c + K_I B/(1-B)}{1+[K_c + K_I B/(1-B)][.22B(1-B^2)/(1-.78B)]} \tag{6.4}$$

which is analogous to the controller given in eqn. (IV.1). The only difference is that we allow K_c and K_I to vary independently. We note that we can use a much larger gain. In fact, the stability limits of (6.4) are identical to those of a process with the transfer function $G_{po}(B)$

controlled by the controller in eqn. (6.3). The dead time compensator seemingly removes the delay from stability consideration.

This is partially an illusion. Let us consider that we either made a mistake in the transfer function or the condition changed and the real continuous transfer function is

$$G_p^*(s) = \frac{e^{-.35s}}{.7s+1} \tag{6.5}$$

which is equivalent to

$$G_p^*(B) = \frac{.193+.107B}{1-.7B}B^2 \tag{6.5a}$$

In curves c and d the stability limits for the closed loop are plotted for the controllers given by (6.3) and (6.4), respectively. In curve e we do the same for the controller (6.3) operating on $G_{po}^*(B)$.

We note that c and e are only slightly different from a and b. The difference between b and d, however, is tremendous. There is no question that the dead time compensator has a stabilizing effect. Curve d allows larger gains than curve a. The difference is much smaller than indicated by curve b. There is a strong difference between curve d and e. In real systems the dead time compensator is not able to eliminate the dead time. This is a general feature of using an inverse of $G_{po}(B)$ in the controller. While this suggests a good structure for the controller it introduces sensitivity problems (see Rosenbrock (1974)).

This means that the conventional methods of classical controller design are useless here. A set of different transfer functions and their stability limits for the simple PI controller is given in fig. 8 to illustrate this. Using half the permitted gain normally assures that the controller is not too sensitive to the transfer function, that the controller will operate well for reasonable changes in the parameters of $G_p(B)$ and also for a wide range of different transfer functions. This does not mean we should not use in the controller terms that cancel zeroes or poles in the transfer function. We just need different methods to determine the maximum allowable gain.

One method which we find useful is based on intentional perturbation of the process parameters. We use either of two methods with equal success.

In the first, all the time constants of $G_p(s)$ are multiplied by $(1+\alpha)$ and we determine the stability limits for a given controller in terms of the limits of α for which the closed loop system would be stable.

In our method this is done on the continuous transfer function $G_p(s)$. This changes the form of $G_p(B)$. For example, for $\alpha = -0.3$ $G_p(B)$ becomes $G_p{*}(B)$ in (6.5a).

In the second method, we multiply the delay in $G_p(B)$ with the factor $(1+\beta)$. Both methods give reasonable results. For actual design it is enough to fix a priori bounds on either α or β and just limit the gain of the controller such that it is stable within these bounds.

In reality we require the controller to not only be insensitive to perturbation in the parameters of $G_p(s)$, but also to the order and form of $G_p(s)$ itself. The $G_p(s)$ used for design is normally of lower order than $G_p(s)$ itself. We found that if the bounds of either α or β are reasonably wide then the controller is also insensitive to slight changes in $G_p(s)$ itself (as long as G_p is stable and the response to a step input remains similar).

We can now look at the controller in (IV.1). In fig. 9 we plot the stability limits of α and β versus λ.

We note that for all values of λ less than 0.5 the controller is very sensitive to changes in α and β. There is an interesting difference here between this controller and a PI controller. A PI controller is only sensitive to changes involving positive values of α or β. Reducing α (which is equivalent to increasing flow rate through the system) increases the stability. The same is true for reducing the magnitude of the delay. When a dead time compensator is used reducing α or β reduces the maximum permissible gain.

To guarantee good stability margins we require a λ larger than 0.5. The stability limits for $\lambda = 0.5$ are

$$-.5 < \alpha < 2.82 \quad ; \quad -1 < \beta < 2.23$$

That means that the controller will operate reasonably well within half these bounds or for

$$-.25 < \alpha < 1.41 \quad ; \quad -.5 < \beta < 1.11$$

If we know G_p very well and can keep it constant we can decrease λ (or increase the gain), but not very much. If we measured λ and its value was less than 0.5 we could not use the unconstrained controller and would have to employ some constraint. Let us first check the controller itself in terms of our five criteria. With $\lambda = 0.5$ the controller can be written

$$\nabla u_t = -.5(\nabla u_{t-1} + \nabla u_{t-2}) + 2.26(e_t - .78 e_{t-1}) \qquad (6.6)$$

It has an integral action and, therefore, fulfills criterion 1. Its response to a change in set point is fast and smooth (fig. 10). In fact, it is much faster than a PI controller (tuned for set point changes, Rovira (1969)) and has no overshoot. It gives a good response even if the parameters are perturbed (fig. 11).

In fig. 10 we also give the response of a continuous PI controller. The advantage of the dead time compensator is strikingly illustrated here. Despite the infrequent sampling, its response is faster and smoother than a continuous conventional controller.

In fig. 12 we give the frequency response of the controller. The amplification in the resonance range is below 1.68, and compares well with the PI controller. The frequency response, again, is not too sensitive to changes in α. If, however, we increase the gain (or decrease λ) then both the response to a set point change and frequency response will become very sensitive to changes in α or β.

The controller in (6.6) is also reasonably insensitive to the structure of $G_p(s)$. We tested it for the following transfer functions

$$G_p(s) = 1/(.5s+1)^3$$

$$G_p(s) = 1/(.375s+1)^4$$

and we found excellent stability limits for α (stable for at least $-.65 < \alpha < 1$. For all these cases the system is unstable for negative λ's

In terms of our first seven criteria, the controller in eqn. (6.6) is a very good controller. Let us now look at criterion 8.

To evaluate the performance of the controller for a simple stationary stochastic input (disturbance described by eqn. (4.4)), we plot in fig. 13, the variance of the output as compared to the uncontrolled case for this controller (curve b). We can look at such a plot as the equivalent of the frequency response in the ϕ dimension. As a comparison, we also give a plot of the best obtainable response at each ϕ_r from an unconstrained optimal controller designed specifically for each ϕ_r (curve a). The region of ϕ close to unity corresponds to the response close to $\omega \to$ zero in the frequency domain. For a good controller the output variance must approach zero as $\phi \rightarrow 1$. The region of $\phi = 0$ corresponds to a stationary white noise. $\phi = 0$ is characteristic of a disturbance due to measurement errors. The optimal control policy here would be to have no control.

Curve a indicates that between $-0.75 < \phi < .75$ there is very little a controller can do. In any practical case we will need a controller to keep the system at the proper steady state. If there is such a disturbance present, it means that we have to constrain our control action in order not to amplify it.

For this whole range the best we can do is to minimize the amplification while still maintaining a reasonable performance with respect to our other criteria. Only for values of ϕ very close to +1 or -1 can we reduce the variance. In a paper by Kestenbaum et al (1976), it was shown that any controller design can be considered as a minimax problem. One tries to minimize the maximum amplification, in the resonance region, while achieving good control in the low frequency region. There the method was described in terms of the frequency response. One can achieve the same objective looking at curves such as fig.

13, which are easier to construct. In our example, for values of $\phi = 0.75$ to $\phi = 1$ the variance can be reduced and we can minimize the difference between the variance obtained by the controller to the optimum obtainable. We have to evaluate this difference in comparison to the amplification of the controller with a ϕ between 0.75 to zero.

Large measurement errors really have a different implication from other random disturbances in the output and must be given a stronger weight in such considerations.

If there is a real disturbance with a variance σ_n^2 and a $\lambda = 1$ (in (4.5) or $\phi = 0$ in (4.4)), then a controller with an amplification of 1.3 in this range, amplifies it by 30%. If, on the other hand, there is a measurement error σ_m^2 then an amplification of 1.3 introduces a disturbance of $0.3\,\sigma_m$, where none was present in the system. We need to know σ_m^2 as well as the allowed specification to decide what amplification is permissible here. (All controllers will amplify measurement noise.) If we cannot reduce this amplification we have to increase λ_a. For example, a controller designed according to (IV.1) with $\lambda_a = 0.8$ (curve c) reduces the amplification at $\phi = 0$ to 1.11. We will discuss this later.

In fig. 14, we also give the performance of two optimum controllers designed according to eqn. (5.15) for two values of ϕ, as well as that of a PI controller. We note that the overall performance for different ϕ is far worse than of our controller.

In fig. 13 we also note that our basic controller (6.6) amplifies strongly in the region close to $\phi = -1.0$. This is of no importance in process control. A negative ϕ implies a white noise passed through a derivative element, which is not what one finds in process control. The optimal controller for this case was given in (5.15), it is a positive feedback controller if k is even. The disturbance has some strongly cyclic properties. If we choose k to be odd we could take care of this, see fig. 14a, but in practice, there is very little reason for doing so.

We can also plot a similar expression in the λ domain. Here the uncontrolled variance is infinity and can, therefore, not serve as a yardstick. We, therefore, use as a basis, the variance of the system having a disturbance of eqn. (4.5) and controlled by an optimal unconstrained controller designed specifically for each value of λ_r

We then compute the ratio of the variance obtained for a disturbance with λ_r operating with a given controller, to the variance obtainable with the unconstrained optimal controller for this λ_r. The λ chosen for the design is designated λ_a. This ratio can be computed and is given by

$$\frac{Var(y_t)_{\lambda_a}}{Var(y_t)_{\lambda_a=\lambda_r}} = 1 + \frac{(\lambda_a - \lambda_r)^2}{(1-\lambda_a^2)[1+k(1-\lambda_r)^2]} \tag{6.7}$$

and is plotted for different values of λ_r in fig. 15.

The ratio given by (6.7) is independent of G_p and is a function of λ and k only. Fig. 15 would, therefore, be correct for any system with a sampling time interval equal to one half the delay. However, the lowest value of λ_a we can choose for an unconstrained controller would depend on $G_p(B)$.

For our case, $\lambda_a = 0.5$ is permissible and we note that this controller performs better than a conventional PI controller. For values of λ between -0.5 and +0.8 it is very close to the optimum performance achievable. For $\lambda_r = 1$ which is exactly the same disturbance as $\phi = 0$, the amplification is 1.33. This means that a measurement noise will be amplified by a factor of 0.33. If this is too high then we can reduce the gain, or constrain the controller by other means. On the other hand, if the measured disturbance has a characteristic λ less than -.5, there is very little we can do to reduce it below the reduction achieved by our basic controller.

For example, for λ_r = -0.8, the variance is 30% larger than for an optimal controller. However, an unconstrained optimal controller is unstable in a practical sense (see fig. 9) and we would have to use a constraint. Increasing λ is one way of constraining it. Another is to constrain $\langle \nabla u^2 \rangle$ (eqn. 5.23)).

If we use eqn. (5.23) for design we need a way to find the Lagrangian multiplier ν. As this is a different controller from eqn. (6.6), it is impossible to get exactly the same stability margins for α and β. However, being the worst point for our basic controller, we choose an $\alpha_{crit.}$ = -.5 as a criterion for comparison of all controllers. (In actual design one might look at the equivalent of fig. 9 for each final design.)

We can then compute the minimum variance obtainable for each λ_r for such a controller designed for this λ_a and use this variance as a yardstick for our controller. In fig. 16 we give the equivalent ratio. We note that there the potential improvement is less than 7% for λ_a= -.8, almost negligible.

Let us look at the controller designed for $\lambda_a = -0.8$. We plot in fig. 16 its performance for other values of λ_r We note that its amplification for $\lambda_r = 1$ is 1.57. The amplification of measurement errors is 0.57 or almost twice as large as for the controller with $\lambda_a = 0.5$. This is due to the fact that the constrained $\lambda_a = -0.8$ controller contains a lead compensator. Its step response is also less smooth (fig. 17). If the transfer function is not exact this difference in the step response decreases, but the $\lambda_a = 0.5$ controller is still better.

Assume that we actually measured the properties of the disturbances and found it would be filled by eqn. (4.5), with $\lambda = -0.8$. If we then designed a controller constraining the variance of ∇u^2 we would not get a better performance than for a controller designed without any knowledge of the disturbance using $\lambda_a = 0.5$ based on stability considerations. The constrained controller for $\lambda_a = -.8$ is a good controller. It is better than a conventional PI controller. It is inferior to our case as it amplifies any measurement noise without compensating for it by better performance. We received no payoff for measuring the actual properties of the disturbance.

This is a rather general result we get for all cases studied by us. If the transfer function is such that the optimal controller has significant stability constraints in the sense mentioned in the beginning of this section, measuring the disturbance does not help us very much. We only need to know the importance of the measurement error, and the existence of disturbances in the resonance region.

This is not surprising as constraining a controller by increasing λ is not that much different from constraining its action by limiting $<\nabla u^2>$. We noted in the previous section that over a wide range of λ's (see fig. 6) the resultant controllers are almost identical.

If the measurement error is important we can constrain our base case in several ways. One is to increase λ_a, the second is to use a quadratic constraint as in (5.23). In figs. 18 and 19 we compare the performance of three controllers all having equal amplification of the measurement noise.

All of them are fairly equal. In our opinion the unconstrained controller with $\lambda_a = .77$ is the best compromise.

Second order disturbances.

Let us now look at second order disturbances as

$$G_n(B) = \frac{1 - \lambda B}{(1-B)(1-\phi B)} \tag{4.6}$$

which is a two parameter disturbance. Let us first look at stability. In fig. 20 we give the stability limits of the unconstrained controller in terms of λ and ϕ. Again, we choose for a rough comparison a $\alpha_{crit.} = -0.5$ as the basis.

We note that positive ϕ always restrict the region that can be chosen. There is a region of negative ϕ where lower values of λ are permitted. Here the value of ϕ itself acts to constrain the controller effort.

In fig. 21 we give the performance of our base case $(\phi_a = 0, \lambda_a = 0.5)$, for different disturbances. Again, the yardstick is the ratio of the variance of the controller operating on an input given by (4.6) to the variance obtained with an unconstrained optimal controller designed for this specific combination of λ_r and ϕ_r.

We noted before that negative values of ϕ are of no interest. The same applies here, as we can always decompose eqn. (4.6) into the sum of a simple ARIMA (0,1,1) disturbance with $\lambda^* = 0$, and a stationary disturbance with $\phi^* = \phi$ (see eqn.(4.8)).

If we look at $\phi = +0.25$ then the performance is the same as for the simpler case ($\phi = 0$, see fig. 15). The performance is excellent for all λ_r's. If we look at the two other cases plotted, $\phi_r = .5$, $\phi_r = .75$, the difference between the optimum performance and our base controller is large for small and negative values of λ_r. If we compare these curves to the stability limits in fig. 20, we note that for all λ values for which the unconstrained controller is stable the basic controller is excellent. The large differences all refer to disturbances for which the optimal controller is too sensitive. If we now take any of these points, for example, $\phi_a = .75$, $\lambda_a = -.8$ design a controller and constrain it to equal stability with our base case, the difference in the output variance becomes negligible.

This is true for all cases studied in the same sense as for the first order disturbances. For large measurement errors, or high frequency disturbances, the base case may need to be constrained and λ_a increased. For all cases where the optimal controller is unstable (in our sense), the base controller compares favorably to a controller designed for the specific disturbance.

We can gain very little here by getting an empirical measured model of the actual disturbance. A disturbance model is useful in searching for better controller configurations. For this purpose we don't need to study it, we can just consider the effect of different disturbances on the designs. What is, however, important is good information about the accuracy of the measurement and the sampling process itself, as well as any information about the disturbance frequency close to the time scale of the process. This is much easier to get than a model of the noise.

One might ask what happens if the optimal controller has very weak stability constraints. One should be so lucky. It seldom happens in process control, especially in cases where the control is based on actual sampling. Should it happen $(\tau \gg \theta)$, then control is so easy, and disturbances are so strongly filtered that control is normally no problem. Again, we can find a controller with a good performance over the entire spectrum of disturbances. Here we can afford a larger ratio between the actual and optimal variance of the system to the optimum achievable as our reduction is going to be very large.

Second order transfer function

The results in figs. 13 and 15 apply independent of $G_p(B)$ as long as the transfer function is exact. However, the more complex the system becomes, the more severe become stability constraints.

We will discuss just one example; an underdamped second order system with a delay

$$G_p(s) = \frac{e^{-.5s}}{.25s^2 + .5s + 1} \tag{6.8}$$

and for T = .25

$$G_p(B) = \frac{.104 + .088B}{1 - 1.414B + .607B^2} B^3 \tag{6.9}$$

Based on our experience with the previous case we choose as our design disturbance eqn. (4.5).

The controller has the form

$$G_c(B) = 9.62(1-\lambda)\frac{1 - 1.414B + .607B^2}{(1-B)(1-.846B)(1+(1-\lambda)B+(1-\lambda)B^2)} \tag{6.10}$$

which is a PID controller with a dead time compensator (see Table V).

The stability, in terms of α , is given by fig. 22. Through this stability analysis all time constants in eqn. (6.8) are multiplied by $(1+\alpha)$. Again, one should note that in (6.9) this does not just change the constants, but changes the form.

Only values of $\lambda > .83$ result in a stable controller. Let us choose $\lambda = .85$ as our base case. The controller will then be

$$\nabla u_t = 1.44(e_t - 1.414\, e_{t-1} + .607 e_{t-2}) - \\ -.996 \nabla u_{t-1} - .277 \nabla u_{t-2} - .127 \nabla u_{t-3} \qquad (6.10a)$$

In figs. 23 and 24 we give the response of this controller to a change in set point and the performance for different inputs in comparison with a conventional PID[c] controller both for exact parameters and for $\alpha = -.3$

It is a much better controller and not very sensitive. However, it is strongly stability constrained and for values of λ less than .85 we again face the same problem as before; namely, that there is no guarantee that a controller designed, for example, for $\lambda_a = 0$ with a constraint on $<\nabla u^2>$ is going to be better than one designed for $\lambda_a = .85$

<u>Conventional operator control</u>

We mentioned in chapter 5 that a standard practice in industry, in processes based on infrequent sampling, is to adjust the ∇u_t proportionally to e_t (eqn. (5.1)). Let us shortly evaluate the performance of this control strategy

c. Footnote: For tuning the discrete PID controller by continuous tuning methods we used a dead time equivalent of T/2 for the sample and hold (Moore, 1969) for our case.

The maximum value of M_c for our example (eqn. (6.1)) is .56. That means that the maximum permissible value is .3. In figs. 25 and 26, we plot the performance of this controller for a change in set point and in the λ domain compared with our base controller. It has a much poorer performance. The nonlinear filter normally used (quality control chart) will avoid amplifications of measurement noise but will not otherwise improve the performance. It has another serious drawback which is immediately evident. Most operators would choose an M_c too close to unity. Even an M_c of .5 would lead to amplification of any disturbances, with amplitudes larger than the limits of the control chart.

Choice of sampling interval

Till now we were mainly concerned with the effect of the disturbance itself and we looked at the design for a given $G_p(s)$. $G_p(B)$ is, however, a function of k or the sampling interval chosen. To study the effect of varying k we go back to our first example (6.1) and look at a controller designed according to eqn. (IV.1). λ is chosen such as to assure good stability (system has good stability limits when $-.5<\alpha<1$.

If we go to small intervals it is more convenient to write the controller in terms of u_t. It becomes

$$u_t = \frac{1-\lambda}{w_o}\left[e_t + (1-\delta)\sum_{i=1}^{t} e_{t-i}\right] - (1-\lambda)\sum_{i=1}^{k} u_{t-i} \tag{6.11}$$

To be stable the gain $(1-\lambda)/w_o$ which is designated K_c cannot exceed a maximum allowable value $(K_c)_{max}$ which, in our case, is approximately 3.25. This value is the limit of

$$(K_c)_{max} = \lim_{k\to\infty} \frac{1-\lambda}{1-e^{-T/\tau}} \to 3.25$$

As T = θ/k, $T/\tau = \theta/k\tau$, therefore, λ_{min} for stable design approaches

$$\lambda_{min} \to 1 - \frac{(K_c)_{max}\,\theta}{k\tau} \tag{6.12}$$

Thus, when k increases the controller in (6.11) becomes

$$u_t = K_c\left[e_t + \frac{\theta}{k\tau}\sum_{i=1}^{t} e_{t-i}\right] + \frac{K_c\theta}{k\tau}\sum_{i=1}^{k} u_{t-i} \tag{6.13}$$

For completeness we give the continuous analog of (6.13)

$$u(t) = (K_c)_{max} \left[e(t) + 1/\tau \int_0^t e(t)dt - 1/\tau \int_{t-\theta}^{t} u(t)\,dt \right] \qquad (6.13a)$$

The difference between $(K_c)_{max}$ and $[(1-\lambda)/\omega_0]_{max}$ is one indication as to how closely the controller approaches a continuous controller.

A second way of looking at the problem is to use the ratio given by eqn. (6.7) and look at the effect of k in reducing the variance.

Here we have to be careful. When k changes, the noise model changes too (unless we deal with measurement noise). For measurement noise $(\lambda_r = 1)$, the variance is constant. The amplification ratio in (6.7) becomes simply

$$\frac{Var[y(t)]_{\lambda_a = \lambda_{a_{min}}}}{Var[y(t)]_{no\ control}} \xrightarrow{\lambda_r = 1} 1 + \frac{[(K_c)_{max}\,\theta/\tau]}{2k} \qquad (k \to \infty)$$

The variance of disturbance introduced by the measurement noise is, therefore, simply $\{1 + [(K_c)_{max}\theta/2k\tau]\}\sigma_m^2$ and decreases as k increases. This is one of the main benefits of more frequent samplings.

In table VII we give a set of tuning parameters for different θ/τ and for two values of k. All of them were chosen based on our stability criteria.

In figs. 27 and 28 we plot the performance of four controllers for two values of θ/τ and four different k's, in the λ domain. To allow proper comparison the λ in the abcissa denoted λ_{base}, refer to the case k=10. For all the other k's, the λ's and the variances of the driving white noises, σ_a^2, are properly scaled such as to refer to the same disturbance to the real plant. We note that there is practically no difference between k=5 and k=10, (k=10 is already a high sampling rate for either case) and that in both cases k=5 approaches the continuous case.

The required k by the last two criteria is almost independent of τ/θ. This implies that the sampling rate is a function of the delay.

This conclusion seemingly contradicts the conventional assumption that T should be chosen on the basis of the cut-off frequency of the continuous transfer function which in this case is a function of τ only (see Kalman (1959), Koppel (1968), Box (1970)). If we accept $T \sim \tau/4$ then k is 2 for the case of θ/τ = .5 and 16 for $\theta/\tau = 4$

For high values of τ/θ the contradiction is only an apparent one. If we choose as our criteria how close we approach an unconstrained optimal controller, then T is a function of θ only. However, if τ/θ is large the closeness to the optimum is no longer a necessary yard-stick for good performance as then the controller becomes more efficient due to the higher value of the permissible gain.

We may see that in several ways. Let us take a numerical example with fixed sampling interval of unit time (T=1), such that the meaning of λ and ϕ doesn't change. We look at two systems

(a) $\theta = 2 \;;\; \tau = 4$

(b) $\theta = 5 \;;\; \tau = 1.25$

System (a) is the same as our base case (eqn. (6.2)). k is 2 for (a) and 5 for (b). In fig. 29 we plot the ratio between the output variance of system (a) to the output variance of system (b) at $\lambda = \lambda_r$

For each of the above cases the controller is given by eqn. (IV.1) and λ_a is chosen on the basis of equal stability (see table VII).

We note that case (a), despite its lower k, has a better overall performance. We could still decrease the variance of system (a) by increasing k to 5, but it might not be necessary.

For high values of τ/θ, the classical criterion $T \sim \tau/4$ is, therefore, reasonable. For low values of τ/θ it leads to too high values of k. When $\tau/\theta \to 0$ (pure delay), this criterion becomes rather useless. The criterion $k \sim 5$ is, here, a reasonable guideline.

7. Recommendation for Design and Summary

In the previous two chapters we discussed and evaluated some of the optimal design methods and we can summarize our results as follows:

1. Some rather simple algorithms, which are definitely realizable even with human operators, have the potential for far better and efficient control than some of the standard methods now in practice.

2. None of the proposed design algorithms are useful as a straightforward design method. They do not guarantee a successful controller. Nor do they provide a way of checking the quality of the overall performance of the controller which must be done by stability analysis and simulation. They provide usable suggestions for improved design and our recommendations are based on suggestions derived from them.

   In that sense the fears that Rosenbrock (1975) expressed about the direction of optimal control are rather exaggerated. Competent use of these methods is in no way different from the interactive trial and error design methods proposed by him.

3. The actual measurement and study of process disturbances has only a limited value in the design of control algorithms for process control. Such study will however provide valuable information about the plant and may lead to removal of these disturbances at their sources. The reason for this conclusion is that in process control there are strong constraints on the design due to stability considerations. Unconstrained designs are in most cases too sensitive to inaccuracies in the transfer function.

   If we look at constrained designs the differences become rather small and in most cases we are able, by simulation, to find controllers which perform equally well over a wide spectrum of inputs.

   What is important is a knowledge of the measurement error, and sometimes a knowledge of the high frequency content of the disturbance is also helpful.

4. When optimal design strategies are used, the gain margin is not a sufficient criterion. Stability has to be tested by varying either parameters of the transfer function or its form.

The actual design that we are proposing is similar to that proposed by Box and Jenkins (1970) or Dahlin (1968) only that our tuning method is completely different.

The basic controller which we found to be generally useful is of the form:

$$G_c(B) = \frac{B}{G_{po}(B)} \cdot \frac{1-\lambda_a}{1-\lambda_a B-(1-\lambda_a) B^{k+1}} \tag{7.1}$$

This contains a tuning parameter λ_a, and the gain of the controller is proportional to $1-\lambda_a$. Box and Jenkins proposed to measure λ_a experimentally. We propose to obtain it by stability analysis in the way outlined in section 6. A plot similar to fig. 9 can readily be prepared, utilizing ready made computer programs, giving for each λ the bound of α and β for which the controller is stable. The form of the plot normally suggests a value of λ_a. Reasonable bounds for α are $-.5<\alpha<1$ and similar bounds for β. The sensitivity of the final controller can be examined by checking its stability with some higher order transfer function of similar form.

This design should be checked by simulation for its response to set point changes, as well as to its response to different inputs. It is a matter of personal choice and experience if one uses for this a frequency response plot or plots of the form of fig. 15 or fig. 13. In the last, it is recommended to plot ϕ from 0 to 1 only.

The stochastic descriptions (figs. 13 and 15) are more useful in sampled data designs as the amplification of measurement noise is directly accessible.

If the measurement error in sampling is high or if one wants to maintain good performance, one either has to reduce the gain or one can increase the number of samples taken. Several samples can be taken at the same time or the sampling interval can be decreased.

Our results showed both have the same effect for small sampling intervals. In many cases we would still recommend to take two samples at the same time. First, this will eliminate totally wrong samples. Second, it is hard for the operator to make small adjustments.

We also have to specify the sampling interval. In many cases this is simply fixed by the economics and constraints of sampling. If not, we can get a reasonable optimum from looking at the transfer function. For human operators, k=2 (or $T=\theta/2$) will result in a simple controller with only four "numbers" used for the setting. This simple controller will be superior to standard conventional schemes and in most cases sufficient. If θ is small as compared to the other time scales of G_p, k=2 may be too frequent and this can be relaxed.

If θ is large k=3÷5 might be a reasonable goal to shoot for.

The most important problem in the design is to measure G_p itself. It can be obtained either by actually modelling the process and comparing the results to the dynamic response of the system to controlled inputs, or get an empirical model by fitting the results of a step response. In both cases we recommend intentional controlled perturbations, preferably several of them measured at different times.

We hate to give specific recommendations for tuning, but we realize that we do not always have the time to go through that procedure. Thus, for processes that can be described by first order + delay models we give recommended controller settings in table VII.

We want to finish with one comment. We restricted the validity of our general results to process controllers with relatively infrequent sampling. Actually, our results should apply to a much broader range of process controllers. The reason for this restriction is that we could not rigorously prove one of our results; namely, that measuring the properties of the input does not lead to sufficient improvements in the design to justify the effort. For simple systems we could demonstrate it over the range of parameters encountered. For any systems for which second order transfer functions are reasonable approximations of the real systems, our results should apply.

CHAPTER 2: Interactive Design and Tuning Method for Analog Controllers and Dead Time Compensators

8. Introduction

Dead times or transportation lags are common phenomena in the process industries and cause considerable difficulties in controlling such processes.

Numerous articles and textbooks dealing with procedures for tuning the two mode (PI) or three mode (PID) conventional controllers have been published in the last three decades. Ziegler-Nichols (1942), Cohen-Coon (1953), C.L. Smith and Murrill (1966), Lopez et al. (1967), Rovira (1969), C.L. Smith et al (1975) all have suggested tuning based on various criteria, especially for processes having first order and delay transfer functions. Among these procedures the quarter decay ratio of Ziegler-Nichols (Z-N) seems to have gained acceptance in process control. These algorithms, however, are inefficient in the presence of large time delays as we show later.

Much more effective control may be achieved if a specific component, called the dead-time compensator (DTC), aimed to deal directly with the time delay is added to the conventional controllers. O.J. Smith (1959) was probably the first to suggest to add the DTC (or better known as Smith Linear Predictor) to the conventional PI or PID controllers. The inclusion of such a DTC in the controller seemingly removes the time delay from stability consideration and may lead to the risky conclusion that the designer is allowed to tune the controller as though no dead time exists in the process.

We intend to show that although the DTC considerably improves the performance of the controllers and leads to far better and efficient control in the presence of large time delays none of the conventional methods is suitable in tuning these controllers and that different methods are needed in these cases.

In chapter I we have shown that minimum variance control algorithms for sampled data control of processes having time delay include DTC. If simple nonstationary noises are used the resulted controllers are PI or PID controllers with DTC depending on the process transfer function.

We also showed that optimal algorithms do not guarantee a good overall controller design, but only suggest a structure. The gain, as well as the final tunings, have to be done by evaluating the overall performance of the controller, especially the sensitivity to changes and inaccuracies in the process transfer function.

Here we deal with the analogous problem of designing

DTC for continuous control.

We propose here a systematic interactive tuning procedure for such controllers. This procedure is based on sensitivity analysis combined with the mini-max procedure suggested by Kestenbaum (1976).

Though the examples dealt with in this chapter are limited to processes that can be modelled by first and second order transfer functions followed by a delay, the method may be easily extended to more complicated processes as well as to other types of process controllers.

9. Dead Time Compensator (DTC)

The DTC suggested by Smith (1959) is described schematically in fig. 30. A minor loop is put around the main controller $X_c(s)$. This loop has the transfer function $G_{po}(s)\ (1-e^{-\theta s})$ where $G_{po}(s)$ is the transfer function of the process without delay.

If the process model $G_p(s)$ is written as

$$G_p(s) = G_{po}(s)\, e^{-\theta s} \tag{9.1}$$

the overall controller transfer function ($G_c(s)$) is given by:

$$G_c(s) = \frac{X_c(s)}{1 + X_c(s)\, G_{po}(s)(1 - e^{-s\theta})} \tag{9.2}$$

where $X_c(s)$ is the transfer function of the main controller. Under this control and assuming that one has an accurate model the characteristic equation of the closed loop (see figure 30) is

$$1 + G_c(s) G_p(s) = \frac{1 + X_c(s)\, G_{po}(s)}{1 + X_c(s)\, G_{po}(s)(1 - e^{-\theta s})} \tag{9.3}$$

and it is evident that the time delay was removed from the characteristic equation and that (9.3) is identical to the characteristic equation that one gets for a process $G_{po}(s)$ controlled by a controller $X_c(s)$.

The basic controller is usually taken to be two mode (eq.(9.4)) of three mode (eq.(9.5)) controllers

$$X_c(s) = K_c\left(1 + \frac{1}{\tau_I s}\right) \tag{9.4}$$

$$X_c(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right) \tag{9.5}$$

For first order and second order + delay processes the transfer function $G_p(s)$ is given by (9.6) and (9.7), respectively

$$G_p(s) = \frac{1}{\tau s + 1} e^{-\theta s} \tag{9.6}$$

$$G_p(s) = \frac{1}{\tau^2 s^2 + 2\tau\xi s + 1} e^{-\theta s} \tag{9.7}$$

Let us first look at the type of optimal controller discussed in the previous chapter which, in that case, was obtained by searching for the unconstrained optimal controller that minimizes the output variance for the total system when the disturbance is an (0,1,1) ARIMA noise. For the continuous case this controller would be equivalent to the controller described by equation (9.2) with the following $X_c(s)$

$$X_c(s) = K_c\left(1 + \frac{1}{\tau s}\right) \tag{9.8}$$

for the first order system (eq. (9.6)) and

$$X_c(s) = K_c\left(1 + \frac{1}{2\tau\xi s} + \frac{\tau}{2\xi} s\right) \tag{9.9}$$

for the second order system (eq. (9.7)).

Equations (9.9) and (9.10) may also be derived as the limit of the corresponding discrete cases (see chapter I).

Comparison between equations (9.4) and (9.5) to equations (9.8) and (9.9) shows that the optimal stochastic design determines completely the reset rate τ_I (for the first order case) and both τ_I and the derivative time τ_D for the second order case. The only parameter left for tuning is the controller gain K_c. (In pure stochastic design the gain in determined by some measurable noise parameter. We have shown, however, in chapter I that measuring noise properties is usually not justified as stability constrained are dominating.) Hence, if we accept the 'optimal' tuning we still have to determine K_c. On the other hand, these tunings represent only limited part of the space of controller settings and one might look at other settings for $X_c(s)$.

When (9.8) is inserted into (9.2) the overall controller's transfer function is somewhat simplified. In this case we get

$$G_c(s) = \frac{K_c(1+\frac{1}{\tau s})}{1+\frac{K_c}{\tau}\cdot\frac{1-e^{-\theta s}}{s}} \tag{9.10}$$

or if we write (9.10) in the time domain we get

$$u(t) = K_c\left[e(t) + \frac{1}{\tau}\int_0^t e(t)dt\right] - (K_c/\tau)\int_{t-\theta}^{t} u(t)dt \tag{9.11}$$

In this form the function of the DTC may be explained in classical terms. Any control action u(t) made at time t has no effect on the output till θ units of time later. Therefore, any control action made during the time interval (t, t+θ) is simply overcorrection. This compensator tells us that instead of decreasing the gain to compensate for the overcorrection we may maintain the higher gain and compensate for the delay by subtracting from the new control action the total actions already done which has not yet been observed due to the dead time.

10. Stability of Controllers with Dead Time Compensators

The basic problem in tuning any optimal controller is that we have no a priori idea about stability. Most unconstrained optimal control algorithms involve some form of inversion and cancel out poles and zeroes. In reality this cancellation is at best imperfect and, therefore, a real system might be unstable with a controller which has a large stability margin for the exact transfer function.

We noted before that the DTC completely eliminates the term $e^{-\theta s}$ from the characteristic equation. What remains in it is $G_{po}(s)$. This means that we can choose K_c on the basis of $G_{po}(s)$ only. In the two cases discussed here this would allow an infinite gain which obviously is nonsense. Early examples employing DTC (see Lupfer and Oglesby (1961)) used processes with $G_{po}(s)$ that results in finite gains but doing so just clouds the issue. We have to find a way to determine reasonable stability limits.

Consider a simple process having the following transfer functions

$$G_p(s) = \frac{e^{-.5s}}{s+1} \tag{10.1}$$

$$G_{po}(s) = \frac{1}{s+1} \tag{10.1a}$$

and assume that $X_c(s)$ is a conventional PI controller. Due to the cancellation of the dead time, the stability limits for this controller in the K_c-K_I plane are infinite, exactly as those for a PI controller and the process in eq. (10.1a). On the other hand, the stability limits of a PI controller with no DTC for the same process are rather limited (see figure 31, curve a). However, if we perturb the process parameters by -30% the stability limits for a PI controller are slightly increased (curve b) while those for a PI+DTC are dramatically decreased (curve c). On the other hand, if we really had a first order without delay process the stability limits would remain infinite, independent of the size of the perturbation.

We see here the central problem of such designs. We have no idea what stability margin to take. In a conventional design taking half the maximum permissible gain is normally sufficient to take care of all the inaccuracies of our identification and design methods. Here, the required factor might be in some occasions 2, in others 100.

Whenever the minimum phase part of the process model, $G_{po}(s)$, consists of first or second order transfer functions, the accepted method of Z-N doesn't provide any tuning as $G_{po}(s)$, under these circumstances, is always stable and the maximum gain is infinity. The C-C method is irrelevant for such design as the dead time is removed. Furthermore, large portions of the controller settings space (as we show later) lead to overdamped step responses. (This applies particularly to optimal controllers.) Consequently, tuning methods based on prespecified overshoots or decay ratio are useless for tuning DTC in such cases. The conventional tuning methods may work in some situations when the allowable gains for the process $G_p(s)$ and those for the minimum phase part $G_{po}(s)$ are close enough. However, when the time delays are relatively large, and this is what compensation is all about, different methods are needed for treating such cases.

In practice we don't know the transfer functions exactly. It is normally more complex and may change. We need controllers which are reasonably insensitive to this and yet giving good performance. This means that the design method should enable the designer to determine the permitted gains and provide an evaluation of the performance in order to assure reasonable insensitivity.

11. The Interactive Tuning Method

The method proposed here guarantees reasonable insensitivity of the controller. It is based on intentional perturbation of process parameters. For first order + delay process, for example, there are two possibilities. One is to multiply all characteristic times of the transfer function $G_p(s)$ by a factor $(1+\alpha)$ and to determine the stability for the given controller in terms of the limits of α. The second possibility in doing so is to multiply only the dead time by $(1+\beta)$. It was found that when the bounds of either α or β are reasonably wide the controller is apt to be insensitive to moderate changes, not only in the parameters, but also to the structure of the process model (as long as the model is stable and exhibits similar step response). Clearly, in cases of higher order transfer function there are more possibilities in perturbing the parameters. Our investigation indicates that a PI+DTC controller with first or second order processes is relatively more sensitive to negative α's. Therefore, an acceptable yardstick for such cases is $-.5 < \alpha < .7$. For more complicated processes one can similarly find adequate bounds on the relative perturbations.

The first step in the procedure is, thus, to construct for a given controller a constant sensitivity line (CSL) in the plane of controller settings (see Appendix E). A typical example of such a line is given in figure 31 (curve d). Once the CSL is constructed a search for the 'good' settings on this line may be performed. This step may be accomplished by the mini-max procedure (Kestenbaum (1976)), or we might start with the settings suggested by (9.8) and (9.9), and determine the maximum permissible value of K_c from the stability requirements such that the system is stable in the range $-.5 < \alpha < .7$.

The mini-max procedure suggested by Kestenbaum translates the basic requirements of a process controller to constraints on the frequency response of the closed loop system.

It is based on the simple principle that the main purpose of a process controller is to keep a system at the steady state. It, therefore, has to fulfill the requirement

$$|G^{*}(\omega)| = 0 \text{ for } \omega = 0 \tag{11.1}$$

where $|G^{*}(\omega)|$ denotes the magnitude of the frequency response of the system $G^{*}(s)$.

Furthermore, no controller can really deal with high frequencies. However, almost all controllers will have a region of amplification or 'resonance region'. To avoid amplification of unforseen disturbances the maximum amplification has to be kept below specified maximum. It is also preferable to push this peak to high frequencies. (For a discussion as to how these requirements correspond to the actual physical requirements of controller design see the original reference.)

We can evaluate this by first looking at the frequency response of different controllers. To again avoid problems of pole cancellation we might look at the frequency response of slightly perturbed transfer functions. A rigorous mathematical formulation is more difficult. Kestenbaum et al. suggested a simple criterion convenient for mathematical search which approximates the above criterion.

The quality of the performance of the controller below the resonance frequency which is the only region in which we can effectively control is approximated by the derivative of $G^*(\omega)$ at $\omega \to 0$.

$$\left.\frac{d|G^*(\omega)|}{d\omega}\right|_{\omega \to 0} \leqslant \epsilon \tag{11.2}$$

For a given required value of ϵ the best controller is that for which the maximum value of $|G^*(\omega)|$ is minimized, or

$$\min \left\{ |G^*(\omega)|_{max} \right\} \quad \text{given } \epsilon \tag{11.3}$$

The first requirement, eq. (11.1) is automatically satisfied for any controller having I mode. Fulfillment of the second requirement (eq. (11.2)), low amplification at low frequencies, ensures fast response. The third requirement, eq. (11.3), states that the resonance amplification should be kept as low as possible. The last two requirements are usually contradicting; this implies that a compromise is needed and that the 'best' setting is a compromise which depends on our knowledge about the specific control requirements of the process.

However, for a given permissible max $|G^*(\omega)|$ some controllers will perform better in the low frequency range than others. We perform this comparison just by looking at $d|(G^*(\omega))|/d\omega$ for $\omega \rightarrow 0$.

We note that this statement of the mini-max principle is an oversimplification for convenience. We are not really interested in ϵ but rather that $|G^*(\omega)|$ should stay low for as high frequencies as possible. We also neglect the criterion that the resonance frequency should be high.

On the other hand, we normally compare controllers, the structures of which are highly constrained. In these cases the above criterion will, in most situations, automatically choose the controller which fulfills our desired criteria, but it is always worthwhile and easy to check this by inspection.

If we do the evaluation graphically, criterion (11.2) is not very convenient. We choose an alternative criterion which is equivalent or might even have some advantages. We choose a comparison frequency, ω_ℓ , in the low frequency range far from the resonance and use $|G^*(\omega)|_{\omega=\omega_\ell}$ as a yardstick for comparison.

We should point out that this is only one of the criteria needed for a good controller which was stated in the previous chapter. Another important criterion is minimizing the overshoot of a step response. This is important, especially for operators. This requirement sometimes contradicts the mini-max procedure and one has to find an additional compromise.

The other requirements mentioned in chapter I are automatically fulfilled here by our way of performing the stability analysis.

The search for good settings on CSL is thus reduced to comparing a limited number of points along the CSL (usually five to eight points) in the mini-max sense.

This is accomplished by finding $|G_i^*(\omega_\ell)|$ and the corresponding $|G_i^*(\omega)|_{max}$ for each of the n points. A plot of the type shown in figure 32 may then be drawn. This usually suffices in indicating the desired settings.

The incooperation of the sensitivity and the simplified mini-max procedures reduce considerably the amount of effort required for finding suitable settings. Furthermore, for simple cases as first or second order + dead time processes, it is possible to specify regions in the controller tuning parameters plane for which the step responses are going to be oscillatory or nonoscillatory.

The information acquired by the first two steps usually supplies the designer with clear recognition of the region of 'good' settings and the effects of modification in the setting. Final choice should be made on comparative evaluation and depends on the specific case and requirements.

To gain clear understanding of the proposed procedure, we shall demonstrate it on the first order + delay and second order + delay processes. It has been claimed that a large fraction of real transfer functions can be approximated by such transfer functions.

12. First Order + Delay Process

Consider a process having the following transfer function

$$G_p(s) = e^{-\theta s}/(\tau s+1) \tag{12.1}$$

and a PI + DTC controller

$$G_c(s) = \frac{K_c\left(1+\frac{1}{\tau_I s}\right)}{1+K_c\left(1+\frac{1}{\tau_I s}\right)\frac{1-e^{-s\theta}}{\tau s+1}} \tag{12.2}$$

We show in appendix D that for this case the setting plane (K_c-K_I plane) may be divided into two regions (see figure 33) indicating whether the response of the system corresponding to a pair (K_c,K_I) is oscillatory or not. Any pair located above the line denoted by η = 1 lead to oscillatory responses while the whole region below this line results in nonoscillatory responses. It is further shown that inside the nonoscillatory zone there is a region of settings which result in some overshoot in the step response though being nonoscillatory (this region is called "overshoot region" in figure 33).

We stated in section 2 that for the process given in eq.(12.1) the minimum variance controllers of the type shown in section 9. is a PI + DTC controller where the main controller was given in eq. (9.8). It is interesting to note that the optimal tunings stay on the straight line (see figure 33) denoted 'optimal line' or O-L. This means that the minimum output variance in these cases is achieved by holding the reset rate $1/\tau_I$ equal to $1/\tau$ regardless of the magnitude of the time delay. Conventional methods vary τ_I with θ. Z-N method, for example, decreases $1/\tau_I$ as θ increases (see figure 34). The tunings located on the O-L are always included in the nonoscillatory region as can be deduced from figure 33.

We turn back now to demonstrate the proposed interactive tuning method for the process under consideration. We aim to tune the PI + DTC controller, i.e., to find suitable K_c and K_I.

Example 1. Large time delay (θ/τ = 8)

As a first example, consider a process with large time delay, (θ/τ = 8). The first step is to construct the CSL in the K_c-K_I plane. As mentioned previously, α = -.5 is an adequate yet quite stringent choice for this process. This line is shown in figure 35a, curve a. For comparison the stability limits for a conventional PI controller were also drawn (curve b). We note that our stability requirements allow larger values for K_c for a PI + DTC for some τ_I's as compared to a simple PI controller. However, there is no rule of thumb for this difference that can be generally applied.

The second step implies choosing several points along the $\alpha = -.5$ line (CSL) and plotting the mini-max curve. For this purpose we have arbitrarily chosen seven points on the CSL (figure 35a) designated by the numbers 1 - 7. The mini-max plot for this case is given in figure 35b. The typical low frequency ω_ℓ in this case was chosen as .01. To assure that if $|G_i^*(\omega_\ell)| < |G_j^*(\omega_\ell)|$ (where i and j represent two compared points on the CSL) implies that $|G_i^*(\omega)| < |G_j^*(\omega)|$ in the whole range $0 < \omega < \omega_\ell$ we have plotted $|G^*(\omega)|$ for four different settings in the low frequencies (figure 35c). From figure 35b it is evident that in this case there exists an optimal region in the mini-max sense, namely the flat region around point 4. As we move from point 3 toward point 1 the resonance amplification increases while the low frequencies behavior deteriorates indicating that going in this direction leads to worse controllers. In addition, as we approach point 1 we cross the $\eta = 1$ line and get settings with oscillatory responses. On the other hand, lower resonance amplification is achieved on the lower part of the mini-max curve though it is traded off by worse performance in the low frequencies. Point 3 on the SCL lies on the optimal line (see figure 35a). The mini-max curve (figure 35b) indicates that for this case the 'optimal tuning' (given in eq. (9.8)) on the CSL also results in a satisfactory mini-max design.

Actually, all information needed for proper tunings is supplied in this case by figures 35a and 35b. In more complicated cases an evaluation by simulation is suggested. This may be done by plotting the step and the complete frequency responses for exact and perturbed parameters. We have plotted in figure 35d the step responses of the controllers represented by points 3 and 5 (see figure 35a) for exact parameters. To see the striking effect of the DTC we added to the plot the responses of conventional PI controllers tuned by Z-N and C-C methods. (In this case these two methods lead to considerably different tunings. The location of these tunings in the K_c-K_I plane are shown in figure 35a.) In figure 35e the corresponding frequency responses are shown. The performance of the PI + DTC controller is far better than that of the conventional PI through the gain of the controller given by point 3 is smaller than the Z-N gain. The difference between the controller tuned according to points 3 and 5 are minor as is also illustrated by figures 35f-35i. This implies that the final choice should be based upon the specific case and requirements or individual preference.

Example 2. Small dead times ($\theta/\tau = .2$)

As a second example we choose a case where the time delay is relatively small. The first step is represented by figure 36a. Again we show, for comparison, the stability limits for a PI controller, the border ($\eta = 1$) line between oscillatory and nonoscillatory responses and the optimal line. Choosing eight points on the CSL and plotting the mini-max curve (figure 36b) constitute the second step. Figure 36b indicates that no further improvement in the mini-max sense is possible beyond 5 or 6 as in this direction the resonance amplification is growing rapidly while the amplification in low frequencies remains the same. Note that settings located above the $\eta = 1$ line result in oscillatory responses. The further from this line the settings are the larger the overshoots become. A good indication of the size of the overshoot may be supplied by calculating the damping ratio η for the specific settings according to eq. (12.3).

$$\eta = \frac{1+K_c}{2\sqrt{K_I \tau}} \tag{12.3}$$

The O-L and the CSL (α = -.5) intersects at point 2 (see figure 36a) which is quite far from the better settings in the min-max sense (as can be seen in figure 36b), indicating unsatisfactory performance of the 'optimal' controller (point 2) at low frequencies. Note that all settings between the O-L and the $\eta = 1$ line (for $K_c > 1$) result in some overshoot. An example of this is shown in figure 36c where the step response of the PI + DTC controller tuned according to point 3 is shown. On this CSL the best mini-max points lead to overshoot. If small oscillations are acceptable a suitable compromise may be achieved by using the setting 5.

The step and frequency responses (for exact parameters) are shown in figures 36d and 36e respectively. In these figures the performances of the controllers corresponding to points 2 and 5 are compared as well as that of a conventional PI tuned by Z-N method. One can see that the setting 2 sacrifices the performance at low frequencies to achieve excellent step response. This is improved by tuning the controller according to setting 5.

Figure 36e (curve c) indicates that the PI controller tuned by Z-N performs slightly better in the low frequencies. This is achieved, however, by sacrificing the resonance region, the step response, and sensitivity (see figures 36f-36i).

We could get a similar performance at low frequencies by the PI + DTC by relaxing somewhat the sensitivity constraint. We could design for $\alpha = -.3$, for example. This would have resulted in good performance at low frequencies but with higher amplification in the resonance frequencies. The step response, however, could still remain overdamped (as long as the settings are below the $\eta = 1$ line) and would be faster than that for the controllers designed for $\alpha = -.5$. In our opinion, however, this is not advisable, and we prefer the design based on $\alpha = -.5$.

The foregoing discussion indicates that one can get better control for the process (12.1) by the DTC even in the presence of small delays.

The application of our method to three mode controllers is rather complicated as one deals in such cases with three dimensional setting space. This means that step 1 in the procedure results in constant sensitivity surface (CSS) instead of the CSL in the case of the two mode controllers. Consequently, this necessitates a three dimensional mini-max search algorithm, which is possible to construct if a suitable search procedure is combined with the mini-max criteria. We do not attempt to treat the three dimensional problem here as the two dimensional procedure for the two mode controller may be extended to include this case too. This is to say that we use the same two dimensional plots of the CSL's in the K_c-K_I plane treating the third setting, τ_D , as a parameter. Thus, we draw several CSL's each for different τ_D. The second step then is to search for the best settings in the mini-max sense on each of the several CSL's. Comparison between the 'best' points lead to overall suitable settings.

13. Tuning PID + DTC Controller via Sensitivity for 2nd Order Processes

As the procedure for tuning PID or PID + DTC controllers is similar to the one demonstrated in examples 1 and 2 for tuning a PI + DTC, we devote this section to tuning PID and PID + DTC controllers via sensitivity analysis for 2nd order + delay processes. This will give us better insight to some sensitivity problems directly related to our tuning method.

Our basic controllers are those resulting from the optimal stochastic procedure mentioned in section 9. We stated there that this design leads, in the case of 2nd order processes, to PID as the main controller (eq. (9.9)) and that both the reset rate, $1/\tau_I$, and the derivative time, τ_D, are determined by this design. The only free parameter is the gain K_c, which has to be determined by sensitivity analysis. These settings can then be used as a starting point for an interactive design procedure searching for better overall compromises. In many cases it will, however, give rather satisfactory performance. We will try to illustrate this by two examples.

Example 3. Underdamped Second Order + Delay Process

Consider the process having the following transfer function

$$G_p(s) = \frac{e^{-\theta s}}{\tau^2 s^2 + 2\tau\xi s + 1} \tag{13.1}$$

with $\tau = 1$, $\theta = 4$, $\xi = .35$. These processes are typical of many chemical processes especially if the controlled process has some internal control loop.

The main controller for this process is given by eq. (9.9) which is a PID with

$$\tau_I = 2\tau\xi = .7 \qquad \tau_D = \tau/2\xi = 1.43 \tag{13.2}$$

In figure 37 we plot the maximum gain $(K_c)_{max}$ for the PID + DTC controller vs. α and β defined in section 11. It is evident again that the DTC is more sensitive to negative α's and that by assuring reasonable sensitivity with respect to negative α's the controller is practically safe for parameter variation. However, this figure illustrates an important phenomenon that should be taken into consideration by the designer.

In the previous examples we just plotted the value of $(K_c)_{max}$ for $\alpha = -0.5$ for various settings. In those cases it was sufficient as $(K_c)_{max}$ changes monotically with

α. This is not always so and requires careful checking. This can be done best by plotting $(K_c)_{max}$ as a function of α and β for given values of τ_I and τ_D.

In the present case the relation is more complex as can be seen from figure 37. We require a K_c which is safely stable over the whole range of α values and we can only insure this by checking the whole range of α and β for a given setting (or a set of settings). With proper computer programs this is a fast procedure.

Here we choose a value of $K_c = 0.25$ and the step and frequency responses of this controller are given in figures 39a (curve d) and 39b (curve d) respectively.

If we look carefully at figure 37, a way of improving this controller suggests itself immediately. We note that positive values of α are more stable and that the main constraints are negative values of α. We can simply move our settings such that the stability limits are more symmetrical.

This is the equivalent of underestimating all time constants of the system by constant factor, and then basing the design on them.

A modified controller with the settings:

$$\tau_I = .6 \quad ; \quad \tau_D = 1.21 \tag{13.3}$$

will have stability limits as given by curve c in figure 37. We can now simply choose a K_c of 0.3 with the same stability margin for parameter variations.

The overall performance of the controller is also given by figures 39-41 and is better than the base case.

Our optimal case is here again just a starting point that can be improved upon depending on what our design goals are.

Let us compare the performance of the controller to other potential designs. One comparison case chosen is a simple PID controller with settings as suggested by the Z-N method.

$$K_c = .45 \; ; \quad \tau_I = 5 \quad ; \quad \tau_D = 1.244 \tag{13.4}$$

The second controller is also a PID controller in which the values of τ_I and τ_D are the same as in (13.2). This is similar to a suggestion made by Smith (1975), though Smith does not recommend this method for underdamped systems.

First let us look at the stability of these controllers. Figure 38 gives the maximum gain as a function of α and β which is a good indication of stability limits.

While the PID controller tuned according to Z-N is more sensitive to negative α the opposite is true for the PID tuned according to (13.2). Furthermore, one notes that the gain suggested by Z-N (13.4) is too close to the stability limits and, hence, unacceptable.

For the PID controller with values of τ_I and τ_D given by (13.2) the small slope of curve a in the positive α region doesn't allow to increase the gain to more than $K_c = .1$. Due to the relatively narrow region of α that permits higher gains it is hard to gain stability by changing the τ_I and τ_D .

The PID tunings according to (13.2) without the DTC are thus

$$K_c = .1 \quad ; \quad \tau_I = 2\tau\zeta = .7 \quad ; \tau_D = \tau/2\zeta = 1.43 \qquad (13.5)$$

We turn now to investigate the performance of the PID with and without DTC. In figure 39a the step responses of the PID + DTC (13.3) and the PID in (13.5) to step changes in setpoint are shown for exact parameters. Results for the Z-N settings are also given. As expected, the Z-N tunings for this case are completely unacceptable and will not be discussed any more. The advantage of the DTC is apparent. It speeds up the time response without introducing overshoot. Its amplification is smaller than that of the PID in all frequencies up to the resonance frequency. The latter is also larger than that of the PID. Any increase in the PID gain will obviously result in larger sensitivity, higher overshoots and resonance amplification. On the other hand, smaller gains will slow down the response and further deteriorate its low frequency performance.

Let us now look at the sensitivity. In figures 40a, b and 41a, b the step and frequency responses for varied process parameters are shown. We note that a DTC, when properly tuned, considerably improves the quality of the control achieved by the PID alone. This holds even though the real process might be considerably different from the one for which the design was made provided we used a proper sensitivity analysis in our design.

Example 4. Overdamped 2nd Order Process + Delay

In this example the process is characterized once again by the second order transfer function (eq. (13.1)) but is overdamped having the following parameters:

$$\tau = 1 \quad ; \quad \theta = 3 \quad ; \quad \xi = 5/3 \tag{13.6}$$

Note that in this example the time delay θ is close to but smaller than $(\tau_1 + \tau_2)$ (for overdamped second order transfer functions one may factorize the denominator in (12.1) to two first order filters with τ_1 and τ_2 their time constants.)

For both the base controller (of the PID + DTC) and the conventional PID controller we choose as a first guess τ_I and τ_D according to eq. (9.9). In figures 42 and 43 the maximum gain curves as functions of α and β for the PID + DTC and the PID are shown. Again, the negative α region is dominating in the determination of the gain for the PID + DTC. Note that here the α curve is monotonically increasing as α varies from $\alpha = -.5$ to $\alpha = 0$. However, its asymmetry suggests again to underestimate τ by 10%-20%. An underestimation of τ by 16% moves the α curve to left such that at $\alpha = -.5$, $K_c = 2$. Thus, suitable tunings for the main controller are as follows:

$$X_c(s) = 2\left(1 + 1/(2.8s) + .252s\right) \tag{13.7}$$

For the PID controller without DTC the positive β region is dominating. Due to the small slope of this curve in the positive β region an appropriate K_c is .75. The PID controller is thus given by:

$$G_c(s) = .75\left(1 + 1/(3.33s) + .3s\right) \tag{13.8}$$

If we adopt the method suggested by Smith et al. (1975) we get the same controller as in (13.8) but with a $K_c = .56$. In light of figure 14, $K_c = .56$[d] is too conservative.

In figures 44-46 we compare the performance of these two controllers (the PID + DTC in (13.7) and the PID in (13.8)). It is apparent that the DTC considerably improves the performance of the PID controller.

d. Footnote: K_c here and K_c in Smith et al. are defined differently due to the different description of the PID. The transfer function they use for the PID is $K_c(1 + 1/\tau_I s)\ (1 + \tau_D s)$. Comparison to (9.9) shows that the K_c in (9.9) is equal to their K_c multiplied by $(1 + \tau_2/\tau_1)$.

14. Simple Approximation of the DTC

The major difficulty in implementing the DTC with analog equipment is due to the DT (its implementation on digital equipment is quite simple). It is possible, however, to approximate the DT using Pade approximation yet retaining the attractive properties of the DTC for at least simple cases as the first order + delay processes.

The denominator of $G_c(s)$ in (9.10) includes the term $((1 - e^{-s\theta})/s)$. This term will appear in all DTC's for which the main controller has I mode. If we use a first order Pade approximation for the DT this term is replaced by the following simple first order filter:

$$\frac{1-e^{-s\theta}}{s} \xrightarrow[\text{Pade approximation}]{\text{first order}} \frac{\theta}{(\theta/2)s+1} \tag{14.1}$$

The step and frequency responses of the PI + DTC with the filter in (14.1) replacing the exact term are compared to those of the exact PI + DTC for Example 1 in figures

47a and 47b, respectively. One can see that this simple approximation is completely satisfactory for the first order + delay process even though the delay is significantly large.

15. Summary and Discussion

In this chapter we try to provide an interactive design procedure for analog controllers acting on processes with relatively large dead time. The controllers proposed are based on known elements such as proportional integral and derivative control (or lead lag compensators) and dead time compensators. The emphasis here is on the methodology and the basic outlook of the approach to tuning and it is in this aspect alone in which we can, hopefully, claim to have made some contribution.

Our examples demonstrate that dead time compensators can make significant improvements in the operation of such controllers.

Till now, many engineers hesitated to employ them widely due to several reasons.

a. They were claimed to be hard to implement (O.J.M. Smith (1959), D.E. Lupfer and H. W. Oglesby (1961), C.L. Smith et al (1975)).

b. The tuning methods originally proposed did not always lead to successful controllers which make engineers hesitant to use them.

The first claim really does not hold today. With cheap microprocessors it is easy to build them in any form desired. Even in cases where all we have available are simple analog circuits one can approximate their performances by a simple filter with very good results as we showed in the last section.

The second claim requires somewhat more attention as it has plagued not just dead time compensators, but other areas of modern control theory where valuable results are often unused as they were not translated into the formulation and goals where an engineer can use them.

The methods that have been proposed, such as basing the tunings on the transfer function without the delay, or amount of overshoot do not work well in the presence of dead time compensators or actually in all cases where one inverts the transfer function. Even more modern methods such as the inverse Nyquist plot suffers from similar problems if applied to controllers based on inverted transfer function. A DTC is basically an excellent inverse of the delay.

The basic approach has been formulated and demonstrated in the paper by Kestenbaum (1976), and in the first chapter and needs no further elaboration. Here we tried to show that it works well for a PID controller with a dead time compensator.

Our basic design approach can be summarized as follows:

Controllers have to fulfill a variety of functions (see Kestenbaum (1977)) that in different situations will have to be given different weight. Even if we try to formulate an exact optimization formula it will never lead to an exact forward algorithm. The only way we can give meaningful definition to the different weighting coefficients is to look by simulation at the actual results.

This simulation is the case of our method. Controllers based on inversion will give results which can be misleading as the sensitivity of the controller is not apparent (see section 10). We, therefore, suggest the following procedure:

A good start is a controller based on the formula given in equation (9.8) or (9.9). We thus determine its sensitivity to process parameters by making a plot such as fig. 37. This allows us to choose a gain that assumes sufficient stability margins. It also gives us, in some cases, clues as to how to modify our basic controller structure to allow higher gains while maintaining the same stability margins.

Starting with this controller we can now modify its parameters and look at the overall performance while maintaining the same stability margins. For different processes we might choose different compromises, but our method at least assures that the controller chosen is a rasonable compromise.

We find the mini-max procedure based on the closed loop frequency response as very satisfactory. One minimizes the $|G^*(\omega)|$ in the low frequencies while minimizing the maximum height of the resonance peak and trying to push it as far as possible to high frequencies.

Others might prefer other approaches and we make no claims here.

Our examples are based on first and second order systems with delays. For these PI or PID controllers with DTC are sufficient.

We make no claim that one should always limit oneself to process models of such simple form. In many cases there is much more information about the structure of the system available and it could be used (see Rosenbrock (1974)). Even in those cases one might want to establish a good base case to compare different controllers.

In a considerable number of cases the transfer function is, however, inaccurately known. In complex cases using a more complex model does not necessarily lead to a more accurate description of the system. There are basically two types of physical systems: (a) Those for which we can at least formulate a mathematical description which, though complex, accurately describe the physics. Many problems in fluid flow or heat transfer are of these nature; (b) Those such as chemical reactors for which the accurate real model is inaccessible with any reasonable amount of work. Here complex models are valuable to elucidate the potential behavior of the system but they provide understanding more than accurate prediction. It also changes as operating conditions change. What we need here is a model that describes the essential features of the process for control purposes. Does it undershoot or overshoot? Is the response oscilatory or not? What are the principal time lags, etc.?

We can look at it in a spatial way. Consider a set of transfer functions $[G_p]_1$ that contains the subset of the real transfer function $[G_p^o]$ that describe the system at different operating conditions. What simulation can hopefully give us is an idea of the probable properties of this set? If we can now find a wider set of simpler transfer functions $[G_p]_2$ that have a similar overall dynamic behavior, a controller that can operate on the whole set has a good chance of giving satisfactory performance on the $[G_p^o]$. In some cases, if strong changes occur in the process, frequently we might want to test the controller each time. A lot of work has gone into designing just such a controller for a given $[G_p]$ but not enough attention has been paid to the relation between the simplified $[G_p]_z$ used for design and the real form. E.H. Bristol recommends to try each design by simulation on a nonlinear transfer function of higher complexity and order than the one used for design. However, there is again no exact formulation. Here, the judgment of the engineer has to come in but one should not forget that the most complex nonlinear models used in simulation are nothing more than their name implies, namely models.

We overcame this problem here by using a relatively wide set of $[G_p]_z$ for which the controller has to perform satisfactorily. We recommend to choose the simplest process model that contains the essential elements of the response. If the process is nonlinear one has to estimate the level of disturbances for which one requires satisfactory control and include a linear approximation of it in the set of $[G_p]_z$ for which the controller has to perform.

Proper choice of the set of $[G_p]_2$ is a question of judgment. We dealt here with those cases for which equations (9.6) and (9.7) can be justified as reasonable approximations. We deal with the wider problem by simply specifying ranges of the parameters of G_p in these equations for which the controller has to be satisfactory. Specifying the range depends on how much we know about the system. We gave new reasonable guidelines. If the controller is stable for values of α between -0.5 and +0.7 and similar ranges of β it will perform well within half this range and this will give good performance over a wide range of transfer functions having similar base properties as the base case chosen. We simulated, in each case, quite a variety of such transfer functions and found satisfactory performance in each case.

The examples we looked at convinced us that dead time compensators, properly designed, will give a significant improvement to analog controllers operating in processes with significant delays and that our method could be a good starting point for designing such controllers.

Appendix A

In deriving the forms of the constrained controller we used here the procedure suggested by Wilson (1969). In this appendix we shall outline this procedure and the notations used.

Consider a process with the discrete transfer function $G_p(B)$ given in (3.2) subjected to a disturbance n_t given by (4.1). Thus, the uncontrolled output y_t is (see fig. 1):

$$y_t = G_p(B)\,u_t + G_n(B)\,a_t \qquad (4.2)$$

We aim to find the control law that minimizes the weighted loss function

$$E\left\{ y^2_{t+k} + \nu\,(\nabla^d u_t)^2 \right\} \qquad (A.1)$$

This control law is given by

$$u_t = \frac{-\delta'(B)\,Q_1(B)}{\omega(B)\,Q_1(B)\,B^{k+1} + \gamma(B)\,\lambda(B)}\,e_t \qquad (A.2)$$

To construct $Q_1(B)$ and $\gamma(B)$ Wilson suggested the following steps

a) Expand $G_n(B)$ in the following way

$$G_n(B) = \frac{\lambda(B)}{\nabla^d \phi(B)} = \frac{\lambda(B)}{\varphi(B)} = \psi_1(B) + \psi_2(B) B^{k+1} = \psi_1(B) + \frac{T(B)}{\varphi(B)} B^{k+1} \tag{A.3}$$

b) Solve the factorization equation (A.4) for $\gamma(B)$

$$\omega(B)\omega(B^{-1}) + \nu \delta'(B) \delta'(B^{-1}) = \gamma(B)\gamma(B^{-1}) \tag{A.4}$$

where

$$\delta'(B) = \delta(B) \nabla^d$$

c) Express

$$\frac{T(B)\,\omega(B^{-1})}{\varphi(B)\,\gamma(B^{-1})} = \frac{Q_1(B)}{\varphi(B)} + \frac{Q_2(B^{-1})}{\gamma(B^{-1})} \tag{A.5}$$

to obtain $Q_1(B)$

Substitution of $Q_1(B)$ from (A.5) and $\gamma(B)$ from (A.4) into (A.2) results in the optimal control law (A.2).

Appendix B

Consider an n^{th} order + delay transfer function which is given by:

$$G_p(s) = \frac{K_p e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)\ldots\ldots(\tau_n s+1)} \tag{B.1}$$

where the τ_i's may be real or complex conjugates. It is easy to verify that the discrete equivalent of (B.1) for stepped input is given by:

$$G_p(B) = \frac{\omega_0 - \omega_1 B - \ldots\ldots\ldots - \omega_n B^n}{(1-\delta_1 B)(1-\delta_2 B)\cdots\cdots(1-\delta_n B)} B^{k+1} \tag{B.2}$$

where:

$$\delta_i = e^{-T/\tau_i}$$

$$\theta/T = k + c$$

For simplicity but without loss of generality assume that $n=5$ and that τ_1, τ_2, τ_3 are real while τ_4, τ_5 are complex conjugates such that

$$\tau_4 = \eta e^{i\xi} \qquad \xi \neq \pm \frac{n\pi}{2}$$

$$\tau_5 = \eta e^{-i\xi}$$

We may express the product $\prod_{i=1}^{3}(1-\delta_i)$ as follows

$$\prod_{i=1}^{3}(1-\delta_i) = 1-p_1B-p_2B^2-p_3B^3 \qquad \text{(B.3)}$$

where

$$p_1 = \delta_1+\delta_2+\delta_3$$
$$p_2 = -(\delta_1\delta_2+\delta_2\delta_3+\delta_1\delta_3)$$
$$p_3 = \delta_1\delta_2\delta_3$$

It is evident that for large T's p_1 behaves like $e^{-T/\tau}$, p_2 like $(e^{-T/\tau})^2$ and p_3 as $(e^{-T/\tau})^3$ (where τ is the order of magnitude of the τ_i's). Similarly we may write:

$$(1-\delta_4B)(1-\delta_5B) = 1-q_1B-q_2B^2 \qquad \text{(B.4)}$$

where it can be shown that

$$q_1 = 2e^{-T\cos\xi/\eta}\cos(T\sin\xi/\eta)$$
$$q_2 = -(e^{-T\cos\xi/\eta})^2$$

and it is apparent that q_1 and q_2 follow $e^{-T/\eta}$ and $(e^{-T/\eta})^2$ respectively.

Combining eqn. (B.3) and eqn. (B.4) according to (B.2) we get:

$$\prod_{i=1}^{5}(1-\delta_i B) = 1 - \sum_{i=1}^{5} s_i B^i \tag{B.5}$$

where it is easily deduced that the s_i's in (B.5) follow $(e^{-T/\tau})^i$. Thus we may conclude that for large sampling intervals the coefficients s_i of the higher order terms in (B.2) are likely to be negligible as compared to s_1 and s_2

Similar arguments can be applied to show that the w_i's behave in the same manner as the s_i's. Hence, when the sytem is sampled infrequently the higher order terms in the denominator as well as in the numerator drop out and in the reasonable range of the T's the significant coefficients are at most those of B^2.

Appendix C

We show in this appendix that the constrained design procedure for non-invertible systems leads practically to unstable control systems.

Consider a process having a transfer function represented by:

$$G_p(B) = \frac{\omega(B)}{\delta(B)} B^{k+1} \tag{3.2}$$

where $\omega(B)$ contains at least one root inside the unit circle.

The unconstrained optimal controller for the process (3.2) and a disturbance n_t characterized by (4.1) has the following transfer function

$$G_c(B) = \frac{\delta(B)\,T(B)}{\omega(B)[\lambda(B) - T(B)B^{k+1}]} \tag{C.1}$$

where T(B) results from the expansion:

$$n_t = \frac{\lambda(B)}{\varphi(B)} = \psi_1(B) + B^{k+1}\,T(B)/\varphi(B) \tag{C.2}$$

and $\psi_1(B)$ is polynomial of order k.

Thus, the characteristic equation of the closed loop for exact parameters is given by

$$1+G_p(B)G_c(B) = 1+\frac{\omega(B)}{\delta(B)}B^{k+1}\frac{\delta(B)\,T(B)}{\omega(B)[\lambda(B)-T(B)B^{k+1}]} = \frac{N(B)}{D(B)} \quad (C.3)$$

whereupon cancellation of ω(B) and $\delta(B)$ the resultant N(B) polynomial is:

$$N(B) = \lambda(B) \quad (C.4)$$

and as it is assumed that $\lambda(B)$ has all its roots outside the unit circle, it is concluded that for exact parameters the overall optimal control system is stable.

Assume, now, that the process parameters vary slightly. For simplicity, assume that the variation affects only ω(B) (this happens for example when the time delay, θ , varies). Hence, the process transfer function will be:

$$G_p(B) = \frac{\omega_0(B)}{\delta(B)}B^{k+1} \quad (C.5)$$

where ω_0(B) may be expressed as

$$\omega_0(B) = \omega(B) + \Delta\omega(B) \quad (C.6)$$

and $\Delta\omega(B)$ is the deviation of $\omega(B)$ from its nominal value. Under these circumstances the characteristic equation becomes:

$$N(B) = \omega(B)\lambda(B) + \Delta\omega(B)T(B)B^{k+1} \qquad \text{(C.7)}$$

It is evident that the unstable root of $\omega(B)$ cannot be cancelled out as $\Delta\omega(B) \neq 0$. Hence, for at least small variations, the poles of N(B) are close to those of $\omega(B)$ and therefore N(B) is, under these circumstances, unstable.

The factorization procedure given in eqn. (5.13) eliminates the instability of the controller in the unconstrained case $(\nu = 0)$, except when $\omega(B)$ contains roots on the unit circle. This stabilization, however, is obviously accompanied by an increase in the output variance.

Appendix D

We intend to show in this appendix that when a PI + DTC controller is used to control a process described by a first order + delay transfer function, the settings plane (K_c-K_I) may be divided into two regions. The step responses corresponding to the settings in one region are oscillatory while those resulting from the settings in the second region are nonoscillatory.

Consider the process given by eq. (12.1) and the controller (12.2). The closed loop transfer function G(s) from r(s) to y(s) (see figure 30) is given by:

$$G(s) = \frac{y(s)}{r(s)} = \frac{(K_c s+K_I)e^{-\theta s}}{\tau s^2+(1+K_c)s + K_I} = \tag{D.1}$$

$$= \frac{(\tau_I s+1)e^{-\theta s}}{\frac{\tau}{K_I} s^2 + \frac{1+K_c}{K_I} s+1}$$

where $K_I = K_c/\tau_I$.

The characteristic equation is thus a second order equation in s of the following form

$$T^2 s^2 + 2T\eta s+1 \tag{D.2}$$

Equating the coefficients of the denominator in (D.1) to to those of (D.2) leads to:

$$\eta = \frac{1+K_c}{2\sqrt{K_I \cdot \tau}} \tag{D.3}$$

Applying the condition for nonoscillatory response $(\eta \geq 1)$ to (D.3) leads to

$$(K_I \cdot \tau) \leq (1+K_c)^2/4 \tag{D.4}$$

Any setting pair (K_c, K_I) which fulfills (D.4) leads to nonoscillatory step responses of the closed loop. The borderline denoted by $\eta = 1$ is shown in figure 33. All settings under this line produce nonoscillatory responses.

Nonoscillatory responses do not imply that some overshoot may not exist. Inspection of G(s) in (D.1) indicates that the overall response to step change in r(t) is a weighted sum of delayed step and impulse responses of a second order system. We may easily find under what conditions an overshoot will result.

Without the delay G(s) in (D.1) may be expressed as:

$$\frac{\tau_I s+1}{T^2 s^2+2T\eta s+1} = \frac{\tau_I s}{T^2 s^2+2T\eta s+1} + \frac{1}{T^2 s^2+2T\eta s+1} \tag{D.5}$$

and it is easily recognized that when a step is introduced the right hand side in (D.5) is a combination of an impulse and step responses.

Further investigation of this response leads to the following conditions for an overshoot to exist

$$\tau K_I > K_c \tag{D.6}$$

$$\tau K_I < K_c\,(1+K_c)/2 \tag{D.7}$$

The above inequalities combined with the condition for nonoscillatory responses (D.4) indicate that there exist a limited region in the setting plane which results in an overshoot even though the response is nonoscillatory. This region is denoted by 'overshoot region' in figure 33. Note that this may occur only for $K_c > 1$. An example of this type of response is shown in figure 36c.

Appendix E

The stability curves in figures 35a, 36a, 37, 38, 42 and 43 were calculated numerically and displayed graphically by a suitable computer program.

To get these curves by using stability criteria suitable for continuous transfer functions is rather comlicated as one deals here with transcendental functions. By approximating, however, the continuous transfer function by equivalent sampled data transfer functions with small sampling intervals (as compared to the dominant characteristic times) this difficulty is eliminated as simple stability criteria suitable for discrete systems may then be employed. Examination of these curves by continuous methods (like Nyquist criterion) indicates that the resulting error is $\pm 5\%$ at most depending on the specific cases.

Underlying Continuous Process	Discrete Equivalent Transfer Function of Process + Zero Data Hold	Relation Between Parameters
1st order + delay $G_p(s) = \frac{K_p e^{-\theta s}}{\tau s + 1}$	$G_p(B) = \frac{W_0 - W_1 B}{1 - \delta B} B^{\ell+1}$	$\delta = e^{-T/\tau}$ $W_0 = K_p(1-\delta^{1-c})$ $W_1 = K_p(\delta - \delta^{1-c})$
2nd order + delay overdamped $G_p(s) = \frac{K_p e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$		$\delta_1 = e^{-T/\tau_1}$ $S_1 = \delta_1 + \delta_2$ $W_0 = K_p[A_1(1-\delta_1^{1-c}) - A_2(1-\delta_2^{1-c})]$ $\delta_2 = e^{-T/\tau_2}$ $S_2 = -\delta_1\delta_2$ $W_1 = K_p[S_1 + A_2\delta_2^{1-c}(1+\delta_1) - A_1\delta_1^{1-c}(1+\delta_2)]$ $A_i = \tau_i/(\tau_1 - \tau_2)$ $W_2 = -K_p S_2[A_2(1-\delta_2^{-c}) - A_1(1-\delta_1^{-c})]$
2nd order + delay underdamped $G_p(s) = \frac{K_p e^{-\theta s}}{\tau^2 s^2 + 2\tau\xi s + 1}$	$G_p(B) = \frac{W_0 - W_1 B - W_2 B^2}{1 - S_1 B - S_2 B^2} B^{\ell+1}$	$\eta = \cos^{-1}(\xi)$ $W_0 = K_p\{1 - \frac{1}{\sin\eta}[e^{-\frac{T(1-c)\xi}{\tau}} \sin(\frac{T(1-c)\sin\eta}{\tau} + \eta)]\}$ $S_1 = 2e^{-\frac{T\xi}{\tau}} \cos\frac{T\sin\eta}{\tau}$ $W_1 = W_0 - W_2 - K_p(1 - S_1 - S_2)$ $S_2 = e^{-2T\xi/\tau}$ $W_2 = K_p S_2\{1 - \frac{1}{\sin\eta}[e^{\frac{Tc\xi}{\tau}} \sin(-\frac{Tc\sin\eta}{\tau} + \eta)]\}$
2nd order, non-minimum phase + delay $G_p(s) = \frac{K_p(\tau_0 s+1)e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)}$ $\tau_0 < 0$		$\delta_i = e^{-T/\tau_i}$ $W_0 = K_p[1 - A_1\delta_1^{1-c} + A_2\delta_2^{1-c}]$ $S_1 = \delta_1 + \delta_2$ $W_1 = K_p[S_1 - A_1\delta_1^{1-c}(1+\delta_2) + A_2\delta_2^{1-c}(1+\delta_1)]$ $S_2 = -\delta_1\delta_2$ $A_i = \frac{\tau_i - \tau_0}{\tau_1 - \tau_2}$ $W_2 = K_p S_2[1 - A_1\delta_1^{-c} + A_2\delta_2^{-c}]$

Table I. Relation Between the Discrete Equivalent and the Underlying Continuous Transfer Functions

Underlying Continuous Process	Discrete Equivalent Transfer Function of Process + Zero Data Hold	
3rd order + delay $G_p(s) = \frac{K_p e^{-\theta s}}{(\tau_1 s+1)(\tau_2 s+1)(\tau_3 s+1)}$	$G_p(B) = \frac{W_0 - W_1 B - W_2 B^2 - W_3 B^3}{1 - s_1 B - s_2 B^2 - s_3 B^3} B^{k+1}$	$W_0 = K_p[1 - A_1\delta_1^{1-c} + A_2\delta_2^{1-c} - A_3\delta_3^{1-c}]$ $W_1 = K_p[s_1 - A_1(1+\delta_2+\delta_3)\delta_1^{1-c} + A_2(1+\delta_1+\delta_3)\delta_2^{1-c} -$ $- A_3(1+\delta_1+\delta_2)\delta_3^{1-c}]$ $W_2 = K_p[s_2 + A_1(\delta_2+\delta_2\delta_3+\delta_3)\delta_1^{1-c} - A_2(\delta_1+\delta_1\delta_3+\delta_3)\delta_2^{1-c} +$ $+ A_3(\delta_1+\delta_1\delta_2+\delta_2)\delta_3^{1-c}]$ $W_3 = K_p s_3[1 - A_1\delta_1^{-c} + A_2\delta_2^{-c} - A_3\delta_3^{-c}]$ $\delta_i = e^{-T/\tau_i}$ $A_0 = (\tau_1-\tau_2)(\tau_2-\tau_3)(\tau_1-\tau_3)$ $s_1 = \delta_1+\delta_2+\delta_3$ $s_2 = -(\delta_1\delta_2+\delta_2\delta_3+\delta_1\delta_3)$ $A_1 = \tau_1^2(\tau_2-\tau_3)/A_0$ $A_2 = \tau_2^2(\tau_1-\tau_3)/A_0$ $s_3 = \delta_1\delta_2\delta_3$ $A_3 = \tau_3^2(\tau_1-\tau_2)/A_0$

Table Ia. Relations Between the Discrete Equivalent and the Underlying Continuous Transfer Function for 3rd Order + Delay Process

T/τ_1	k	ω_1/ω_0	ω_2/ω_0	ω_3/ω_0	δ_2/δ_1	δ_3/δ_1
.1	20	-3.6	-.80	0	- .87	.25
.5	4	-2.36	-.39	0	-0.49	.08
1	2	-1.42	-.114	0	- .23	.017
2	1	- .547	-.013	0	- .06	7.73×10^{-4}
5	0	- .3	$-.74\times10^{-3}$	1.93×10^{-8}	1.11×10^{-3}	4.88×10^{-8}

Table II. An Example of the Dependence of the Coefficients of the Discrete Equivalent Process on the Sampling Interval T. (The underlying continuous process has the following transfer function $e^{-\theta s}/(\tau_1 s+1)(\tau_2 s+1)(\tau_3 s+1)$ with $\tau_1:\tau_2:\tau_3:\theta = 1:.75:.5:2.$. For the definition of the δ's and ω's see Table Ia).

Types of Processes and Noises	Eqn. No.	Unconstrained Controller	Eqn. No.	Constrained Controller
general	III.1	$\frac{B}{G_{po}(B)} \cdot \frac{\psi_2(B)}{G_n(B) - B\psi_2(B)}$	III.5	$\frac{B}{G_{po}(B)} \cdot \frac{-\omega(B)\ell_1(B)}{\gamma(B)\lambda(B) + B\,\omega(B)\ell_1(B)}$
1st + delay ARIMA (1,0,0)	III.2	$\frac{\phi^{k+1}}{\omega_0}(1-\delta)\frac{1}{[1-(\omega_1/\omega_0)B](1-\phi^{k+1}B)}$	III.6	$\frac{m(1-\delta B)/\gamma_0}{1+(\gamma_1/\gamma_0 - \frac{\omega_0 m}{\gamma_0})B + \frac{\omega_1 m}{\gamma_0}B^2}$ $m = \phi^{k+1}(\omega_0-\omega_1\phi)/(\gamma_0+\gamma_1\phi)$
1st + delay ARIMA (0,1,1)	III.3	$\frac{1-\lambda}{\omega_0} \cdot \frac{1-\delta B}{1-B} \cdot \frac{1}{1-(\omega_1/\omega_0)B}$	III.7	$\frac{1-\lambda}{\gamma_0} \cdot \frac{1-\delta B}{1-B} \cdot \frac{1}{1+d_1B+d_2B^2}$ $d_1 = (1-\lambda)(1-\omega_0/\gamma_0) + \gamma_1/\gamma_0$ $d_2 = \lambda\gamma_2/\gamma_0$
1st + delay ARIMA (1,1,1)	III.4	$\frac{\eta_2/\eta_1}{\omega_0} \cdot \frac{1-\delta B}{1-B} \cdot \frac{1}{1-(\omega_1/\omega_0)B} \cdot \frac{1-\eta_1(1-B)}{1-\eta_2(1-B)}$ $\eta_1 = m_2/(1-\lambda)$ $\quad \eta_2/\eta_1 = \frac{1-\lambda}{1+m_2}$ $\eta_2 = m_2/(1+m_2)$ $m_2 = \frac{(\phi-\lambda)\phi^{k+1} - \phi(1-\lambda)}{1-B}$	III.8	$\frac{g_0}{\gamma_0} \cdot \frac{1-\delta B}{1-B} \cdot \frac{1+(g_1/g_0)B}{1+d_1B+d_2B^2}$ $d_1 = 1-\lambda+\gamma_1/\gamma_0 + \omega_0 g_0/\gamma_0$ $d_2 = \omega_1 g_1/\gamma_0 + \lambda\gamma_1/\gamma_0$ $g_0 = \frac{-m_1\omega_0 + m_2\omega_1 - m_2\omega_0(1+\phi^{-1}) + (1-\lambda)(\gamma_1+\gamma_0(1+\phi^{-1}))}{(\gamma_0+\gamma_1\phi+\gamma_2\phi^2)\phi^{-1}}$ $g_1 = 1-\lambda-g_0$ $\quad m_1+m_2 = 1-\lambda$

Table III. The Optimal Transfer Function, $X_c(B)$, of the Forward Controller for Different Noises and Processes

θ/T	Eqn. No.	Controller Equation	
	IV.1	$G_c(B) = \frac{u(B)}{e(B)} = \frac{[(1-\lambda)/w_0](1-\delta B)}{[1-(w_1/w_0)B](1-\lambda B+(\lambda-1)B^{k+1})}$	
$0 \leq \theta/T < 1$	IV.2	$u_t = (w_1/w_0)\, u_{t-1} + PI$	PI = $\frac{1-\lambda}{w_0} \cdot \left[e_t + (1-\delta) \cdot \sum_{i=1}^{t} e_{t-i}\right]$
$1 \leq \theta/T < 2$	IV.3	$u_t = -(1-\lambda-w_1/w_0)\, u_{t-1} + (1-\lambda)(w_1/w_0)\, u_{t-2} + PI$	
$\theta/T \geq 2$	IV.4	$u_t = -(1-\lambda-w_1/w_0)\, u_{t-1} - (1-\lambda)(1-w_1/w_0)\sum_{i=2}^{k} u_{t-i} + (1-\lambda)(w_1/w_0)\, u_{t-k-1} + PI$	
$0 \leq \theta/T < 1$	IV.5	$\nabla u_t = (w_1/w_0)\, \nabla u_{t-1} + [(1-\lambda)/w_0](e_t - \delta e_{t-1})$	
$1 \leq \theta/T < 2$	IV.6	$\nabla u_t = -(1-\lambda-w_1/w_0)\, \nabla u_{t-1} - (1-\lambda)(w_1/w_0)\, \nabla u_{t-2} + (1-\lambda)/w_0[e_t - \delta e_{t-1}]$	
$\theta/T \geq 2$	IV.7	$\nabla u_t = -(1-\lambda-w_1/w_0)\, \nabla u_{t-1} - (1-\lambda)(1-w_1/w_0)\sum_{i=2}^{k} \nabla u_{t-i} + (1-\lambda)(w_1/w_0)\, \nabla u_{t-k-1} + \frac{1-\lambda}{w_0}(e_t - \delta e_{t-1})$	

Table IV. Unconstrained Minimal Variance Control Algorithms for First Order + Delay Process and ARIMA (0,1,1) Noise.

θ/T	Eqn. No.	Controller Equation
	V.1	$G_c(B) = \frac{u(B)}{e(B)} = \frac{[(1-\lambda)/w_0](1 - s_1 B - s_2 B^2)}{1 - (w_1/w_0 + \lambda)B - (w_2/w_0 - \lambda w_1/w_0)B^2 + \lambda(w_2/w_0)B^3 - (1-\lambda)B^{k+1}(1 - (w_1/w_0)B - (w_2/w_0)B^2)}$
$0 \le \theta/T < 1$	V.2	$u_t = (w_1/w_0)u_{t-1} + (w_2/w_0)u_{t-2} + PID$
$1 \le \theta/T < 2$	V.3	$u_t = -(1-\lambda-w_1/w_0)u_{t-1} - [(1-\lambda)(w_1/w_0) + (w_2/w_0)]u_{t-2} + (1-\lambda)(w_2/w_0)u_{t-3} + PID$
$2 \le \theta/T < 3$	V.4	$u_t = -(1-\lambda-w_1/w_0)u_{t-1} - [(1-\lambda)(1-w_1/w_0) - w_2/w_0]u_{t-2} + [(1-\lambda)(w_1+w_2)/w_0]u_{t-3} + (1-\lambda)(w_2/w_0)u_{t-4} + PID$
$3 \le \theta/T$	V.5	$u_t = -(1-\lambda-w_1/w_0)u_{t-1} - [(1-\lambda)(1-w_1/w_0) - w_2/w_0]u_{t-2} - (1-\lambda)(1-w_1/w_0-w_2/w_0)\sum_{i=3}^{k} u_{t-i} + [(1-\lambda)(w_1+w_2)/w_0]u_{t-k-1} - (1-\lambda)(w_2/w_0)u_{t-k-2} + PID$
		To get eqn. V.2-V.5 in terms of ∇u_t replace each u_i by ∇u_i and PID by $[(1-\lambda)/w_0](e_t - s_1 e_{t-1} - s_2 e_{t-2})$
		$PID = \frac{1-\lambda}{w_0}(1+s_2)\left[e_t + \frac{1-s_1-s_2}{1+s_2}\sum_{i=1}^{t} e_{t-i} + \frac{-s_2}{1+s_2}(e_t - e_{t-1})\right]$

Table V. Unconstrained Minimal Variance Control Algorithms for 2nd Order + Delay Process and ARIMA (0,1,1) Noise

Eqn.	θ/T	Controller equation	
VI.1		$G_c(B) = \dfrac{[(1-\lambda)/\gamma_0](1-\delta B)}{1+(\gamma_1/\gamma_0-\lambda)B+(\gamma_2/\gamma_0-\lambda\gamma_1/\gamma_0)B^2-\lambda(\gamma_2/\gamma_3)B^3+(\lambda-1)(\omega_0/\gamma_0)B^{k+1}-(\omega_1/\gamma_0)B^{k+2}}$	
VI.2	$0 \le \theta/T < 1$	$u_t = -[(1-\lambda)(1-\omega_0/\gamma_0)+\gamma_1/\gamma_0]u_{t-1} - \lambda\gamma_2/\gamma_0 u_{t-2} + PI$	$PI = \frac{1-\lambda}{\gamma_0}[e_t + (1-\delta)\cdot\sum_{i=1}^{t} e_{t-i}]$
VI.3	$1 \le \theta/T < 2$	$u_t = -(1-\lambda+\gamma_1/\gamma_0)u_{t-1} - [(1-\lambda)(1+\gamma_1/\gamma_0-\omega_0/\gamma_0)+\gamma_2/\gamma_0]u_{t-2} + PI$	
VI.4	$2 \le \theta/T < 3$	$u_t = -(1-\lambda+\gamma_1/\gamma_0)u_{t-1} - [(1-\lambda)(1+\gamma_1/\gamma_0)+\gamma_2/\gamma_0]u_{t-2} + (1-\lambda)(\omega_1/\gamma_0)u_{t-3} + PI$	
VI.5	$\theta/T \ge 3$	$u_t = -(1-\lambda+\gamma_1/\gamma_0)u_{t-1} - [(1-\lambda)(1+\gamma_1/\gamma_0)+\gamma_2/\gamma_0]u_{t-2} + (1-\lambda)\frac{\omega_0-\omega_1}{\gamma_0}\sum_{i=3}^{k} u_{t-i} + (1-\lambda)\omega_1/\gamma_0 u_{t-k-1} + PI$	
	To get equations VI.2-VI.5 in terms of ∇u_t replace each u by ∇u and PI by $[(1-\lambda)/\gamma_0](e_t - \delta e_{t-1})$		

Table VI. Constrained Minimal Variance Algorithms for 1st Order + Delay Process and ARIMA (0,1,1) Noise.

θ/τ \ k	2			5		
	K_c	a	δ	K_c	a	δ
.2	5.25	.5	.905	7.65	.3	.961
.5	2.26	.5	.779	3.15	.3	.905
1	1.27	.5	.607	1.65	.3	.819
2	.89	.56	.368	1.0	.33	.670
4	.74	.64	.135	.64	.35	.450
	.65	.65	0	.35	.35	0

Table VII. Tuning Parameters of the Discrete Controller for First Order + Delay Process Model.

(Controller equation:

$$\nabla u_t = -a \sum_{i=1}^{k} \nabla u_{t-i} + K_c (e_t - \delta e_{t-1}))$$

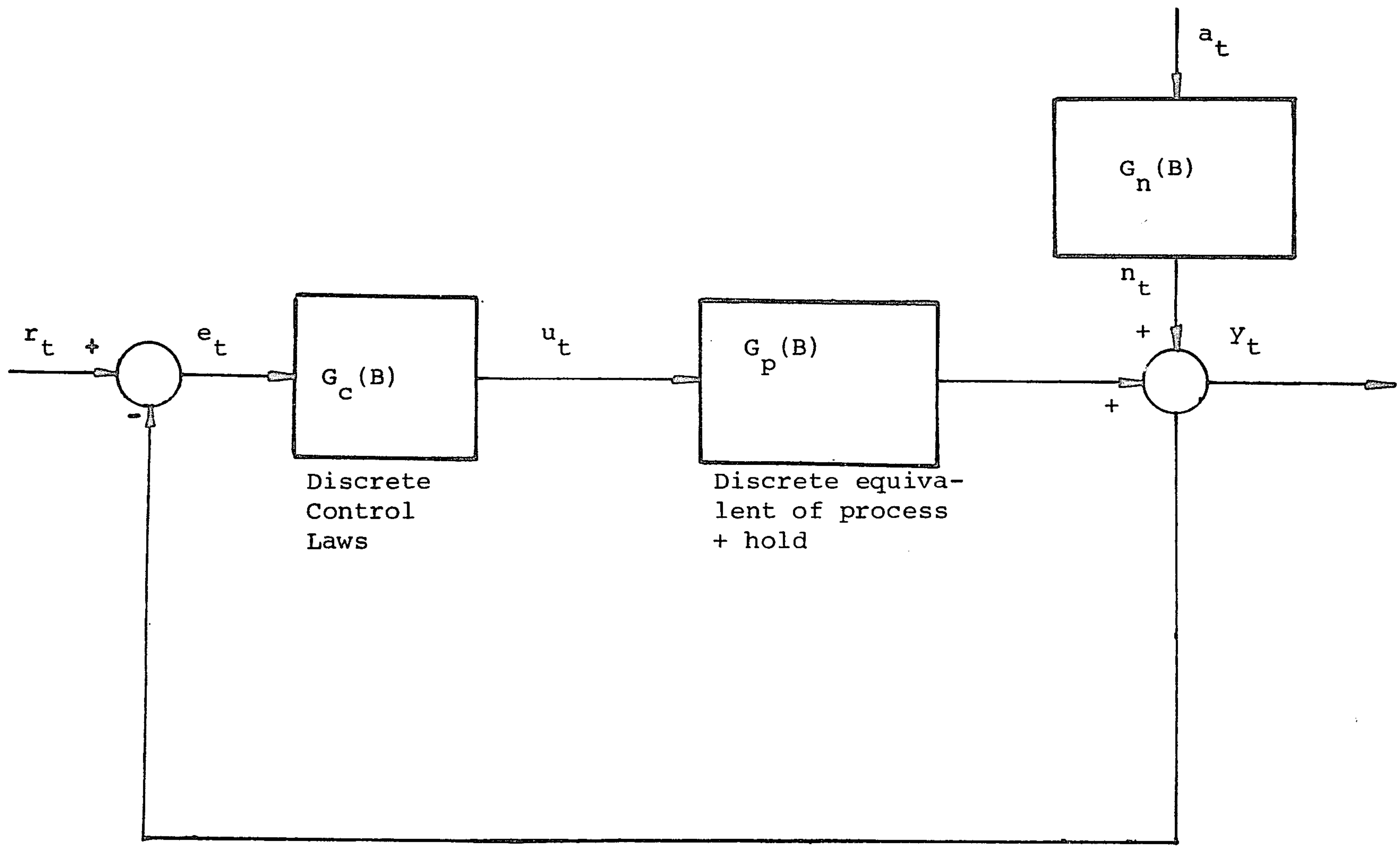


Figure 1. Process-Disturbance Configuration

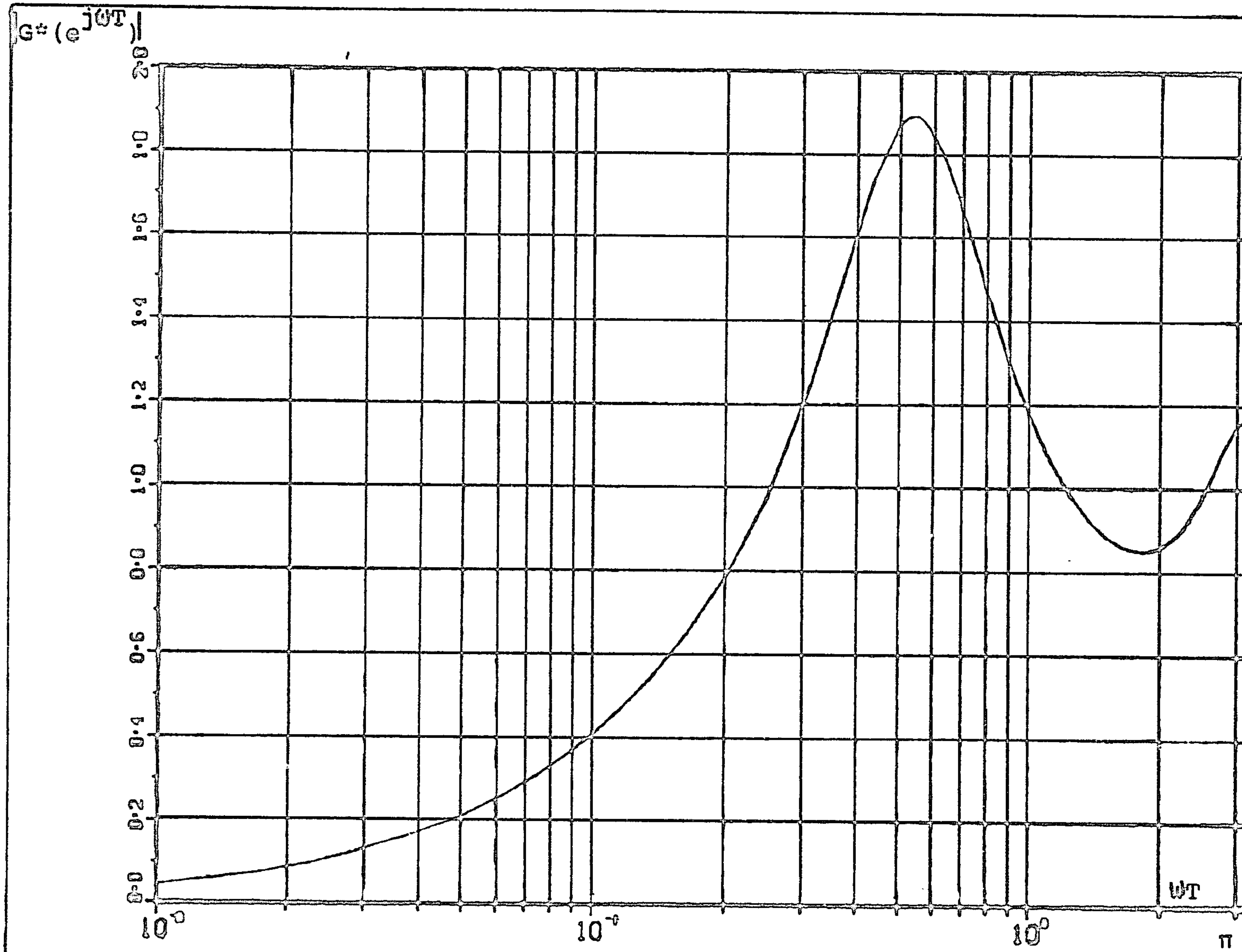
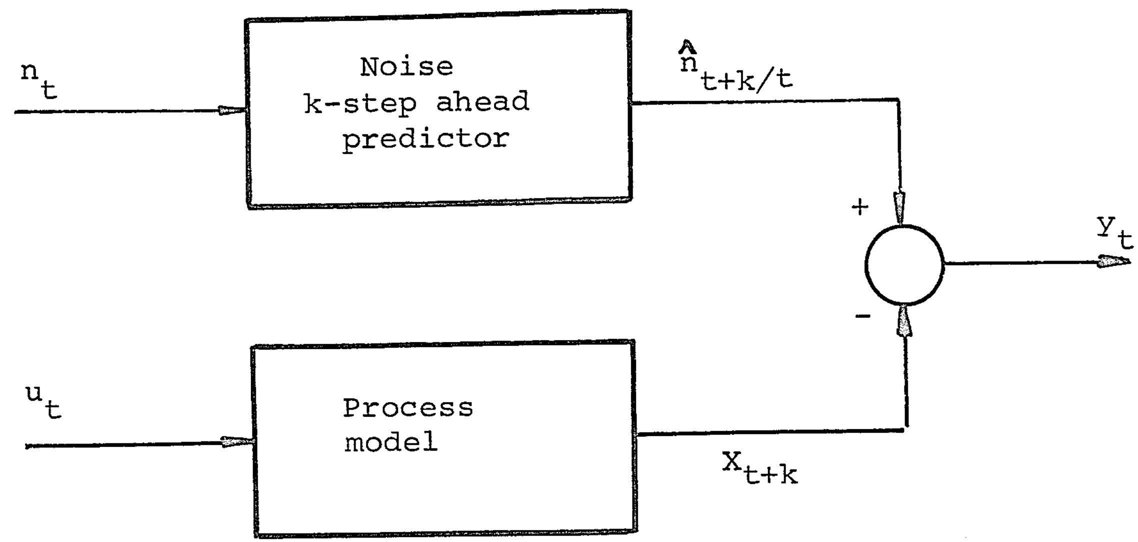


Fig. 1a. A Typical Closed Loop Frequency Response



$\hat{n}_{t+k/t}$ - k-step ahead prediction of n_{t+k} (usually in the mean square sense)

X_{t+k} - Expected process output at time t+k to control input u_t

Fig. 2. Interpretation of the Control Problem as Matching Two Predictions

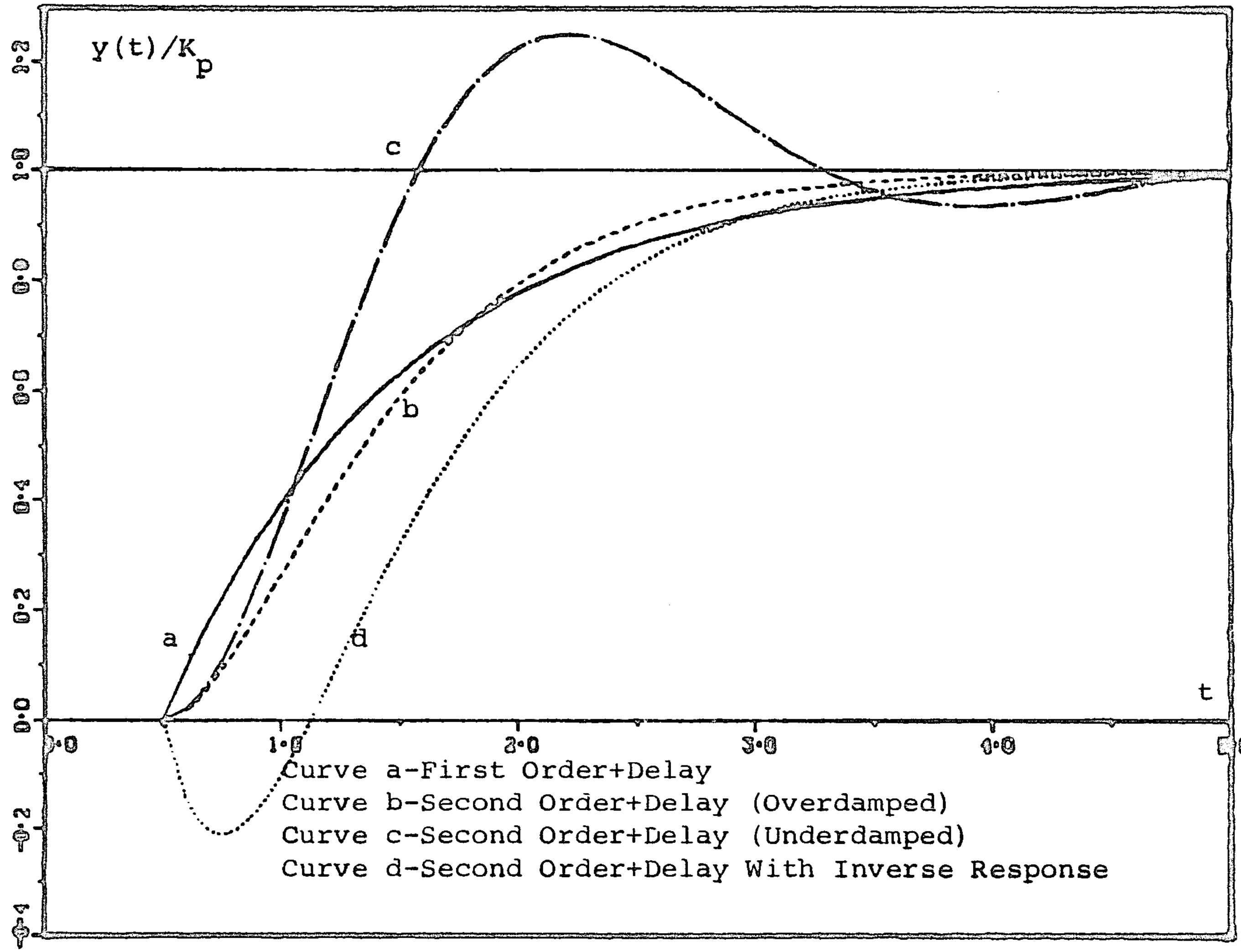


Fig. 3. Responses to Step Change in Set Point of Various Processes Given by Eqn. (3.1)

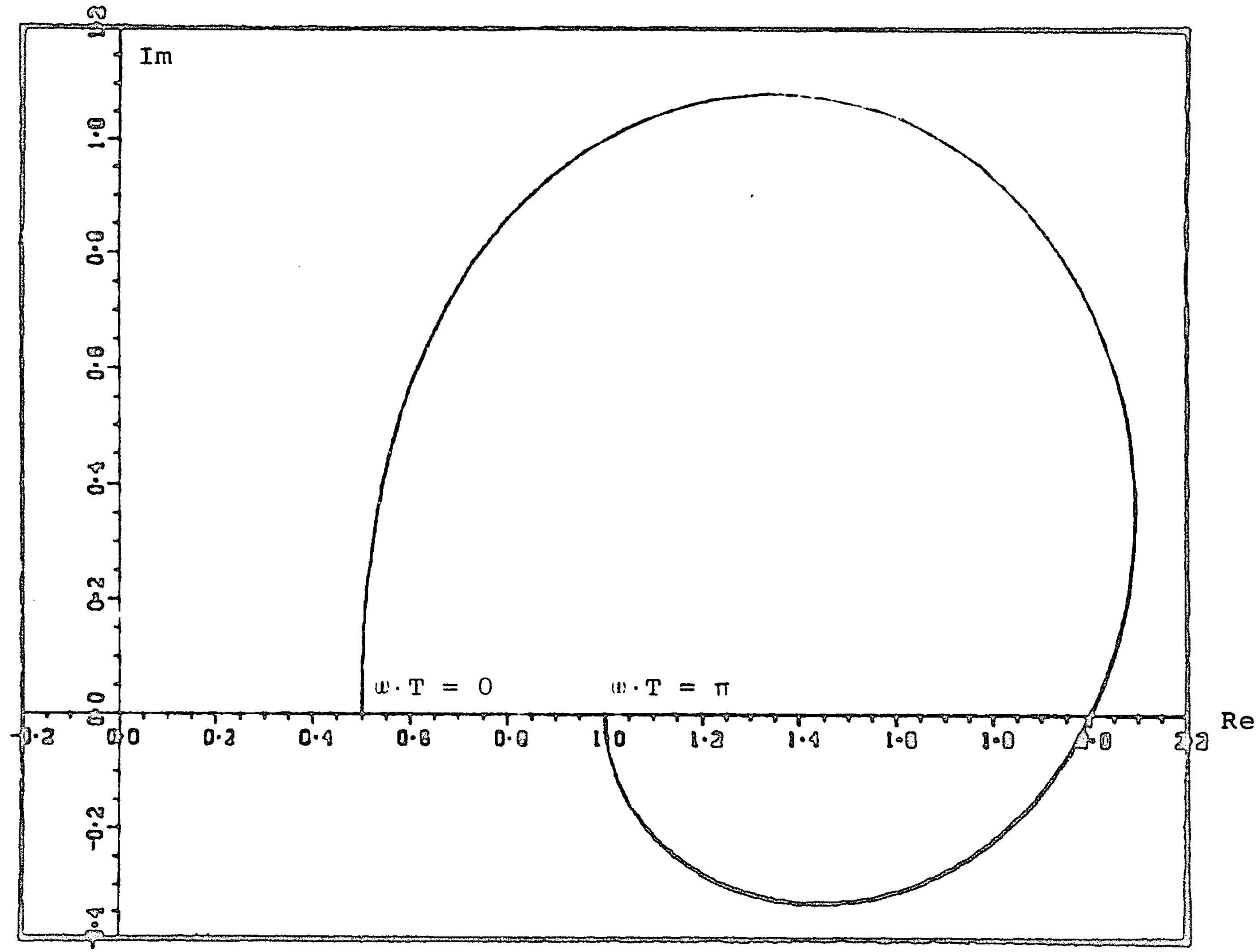


Fig. 4. Polar Plot for the Compensator (5.10) with k = 2 and a = .5

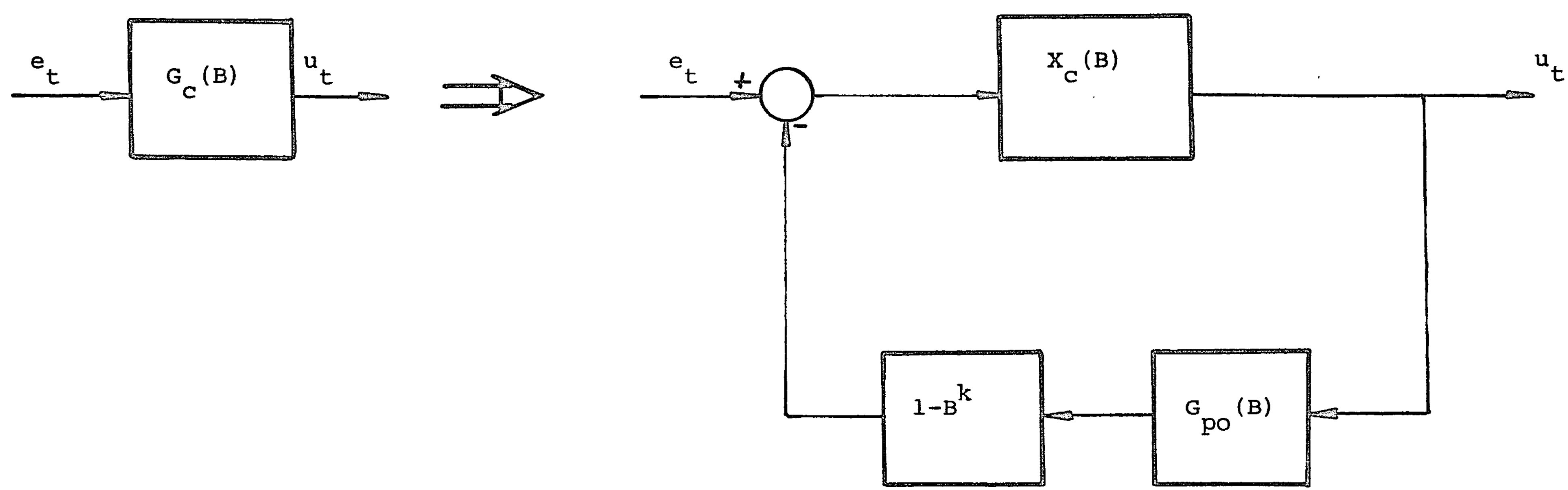


Fig. 5. Decomposition of the Overall Optimal Controller to a Feedback System with Constant Predictor in the Feedback Path

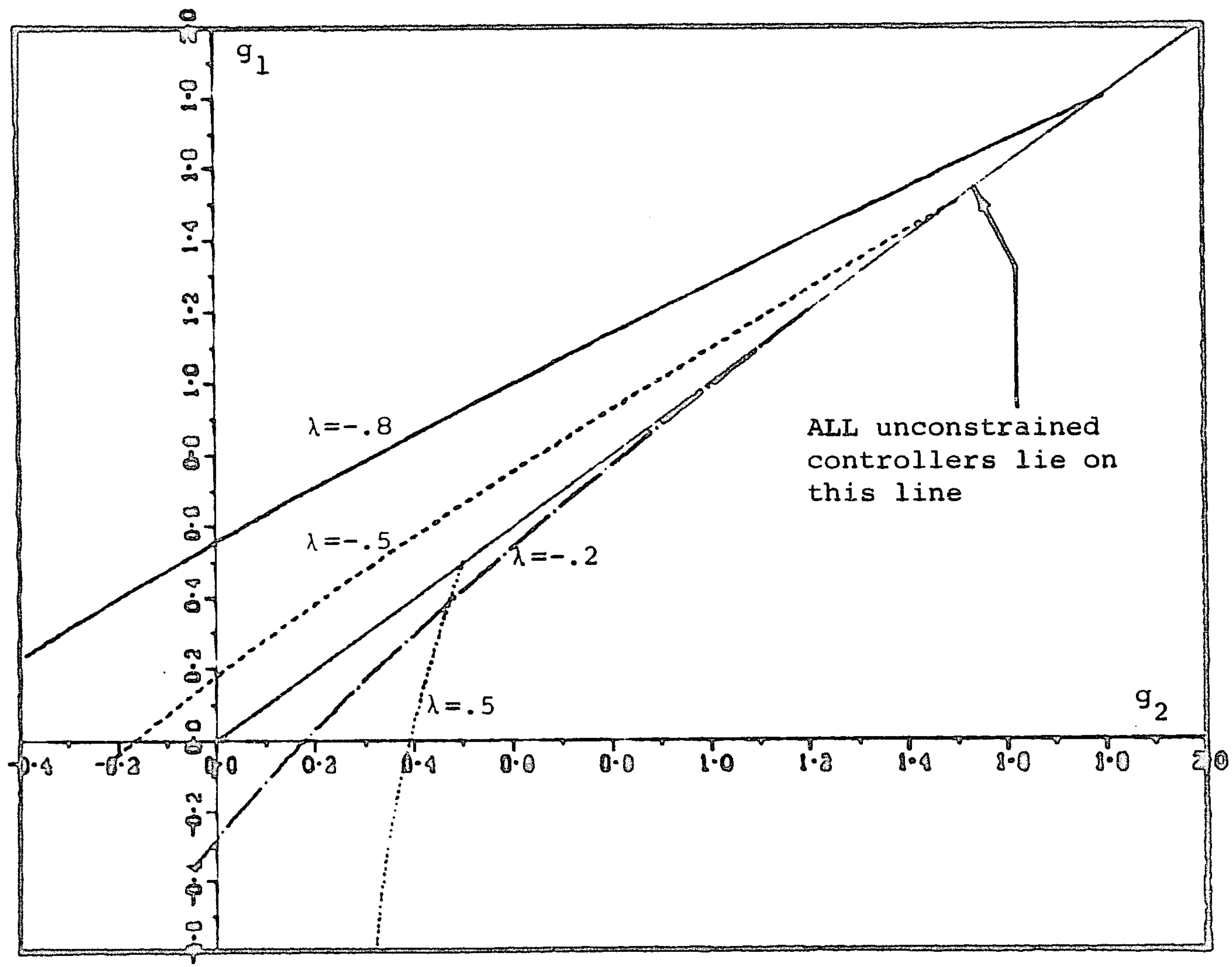


Fig. 6. Optimal Trajectories for the Coefficients of the Dead Time Compensator (Eqn. (5.25)) for Different λ's (θ/T=2, θ/τ=.5)

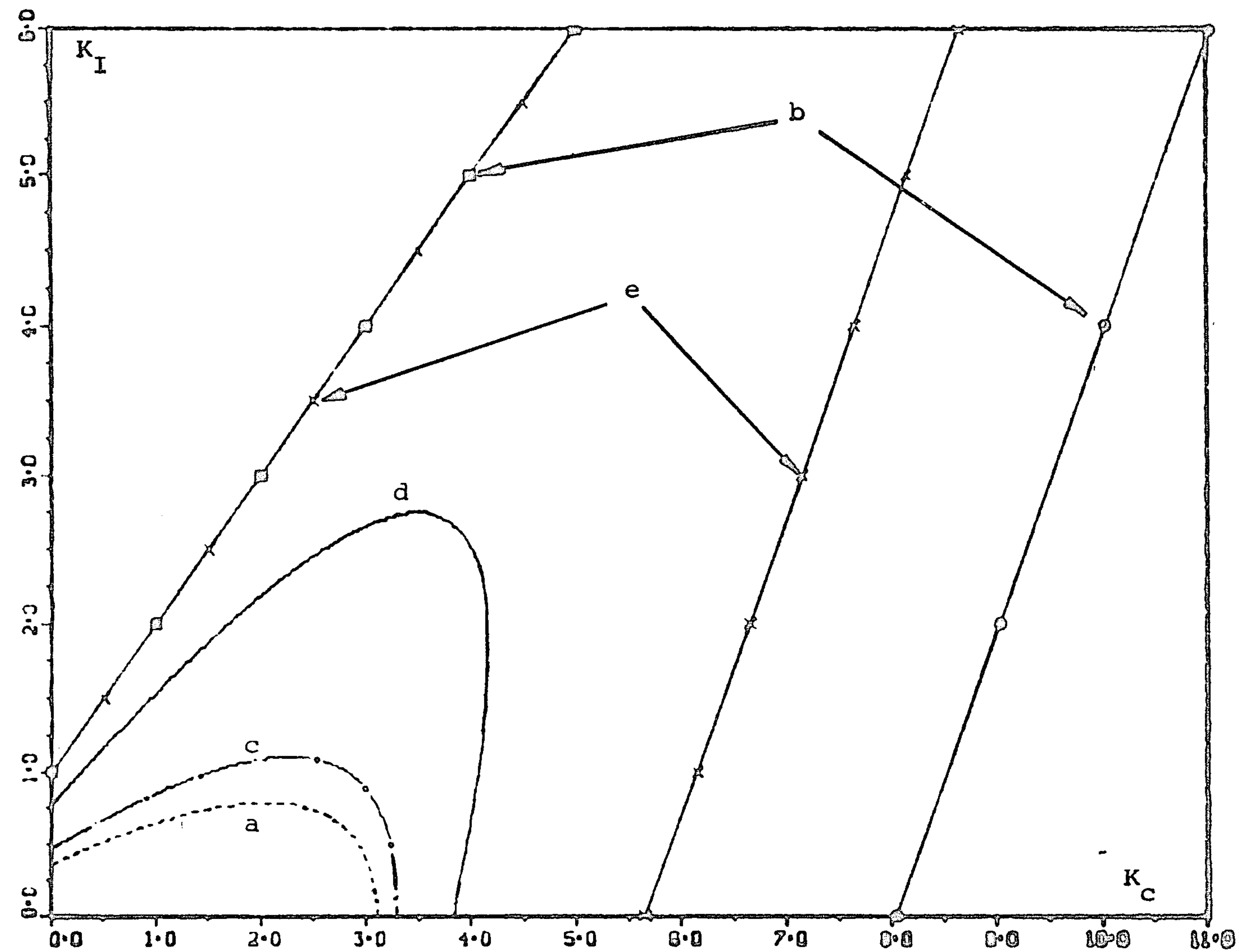


Fig. 7. Stability Limits for a PI Controller in the Forward Loop With and Without Dead Time Compensator (Eqns. (6.3) and (6.4))

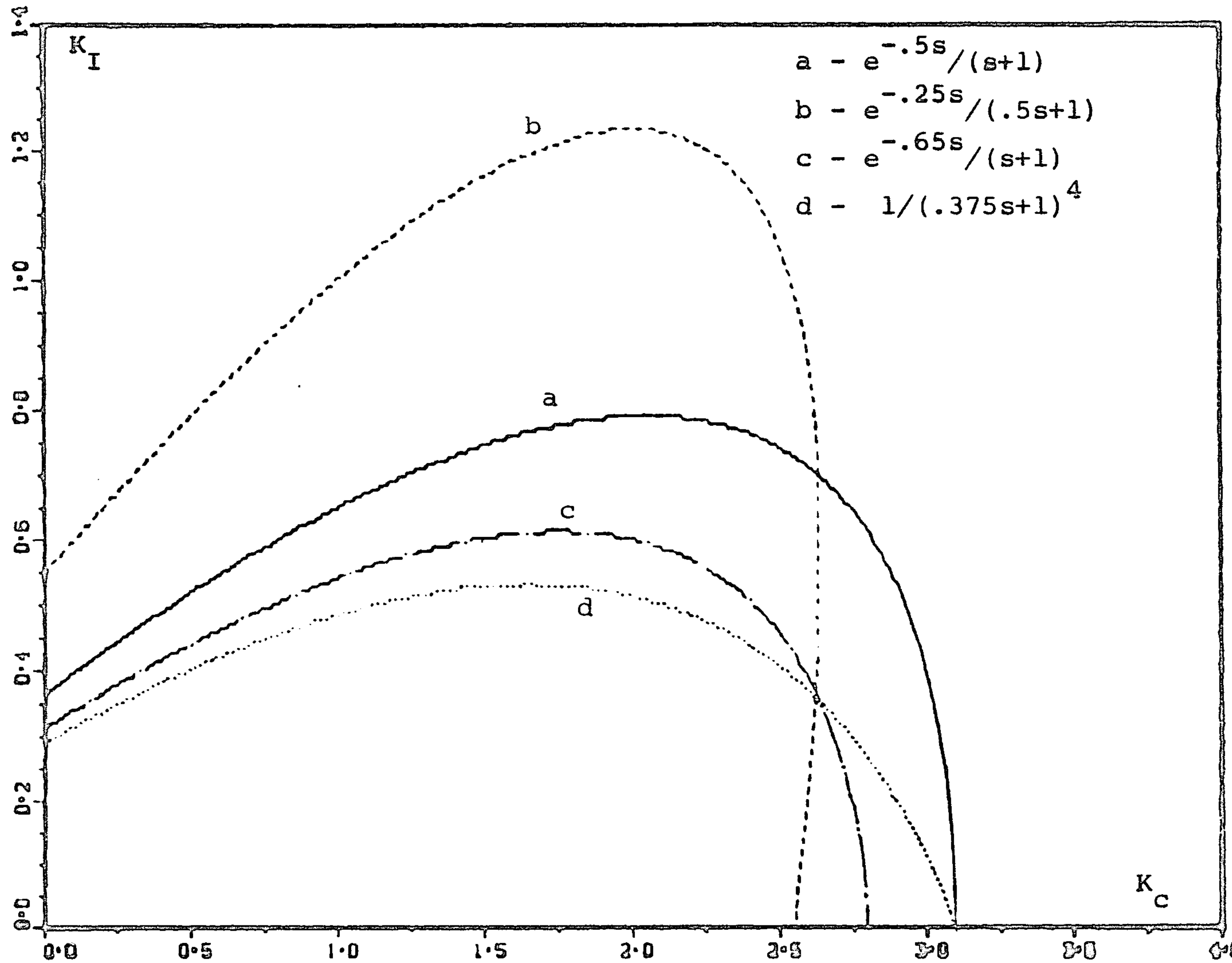


Fig. 8. Stability Limits for a Discrete PI Controller (Eqn. (6.3)) for Various Transfer Functions

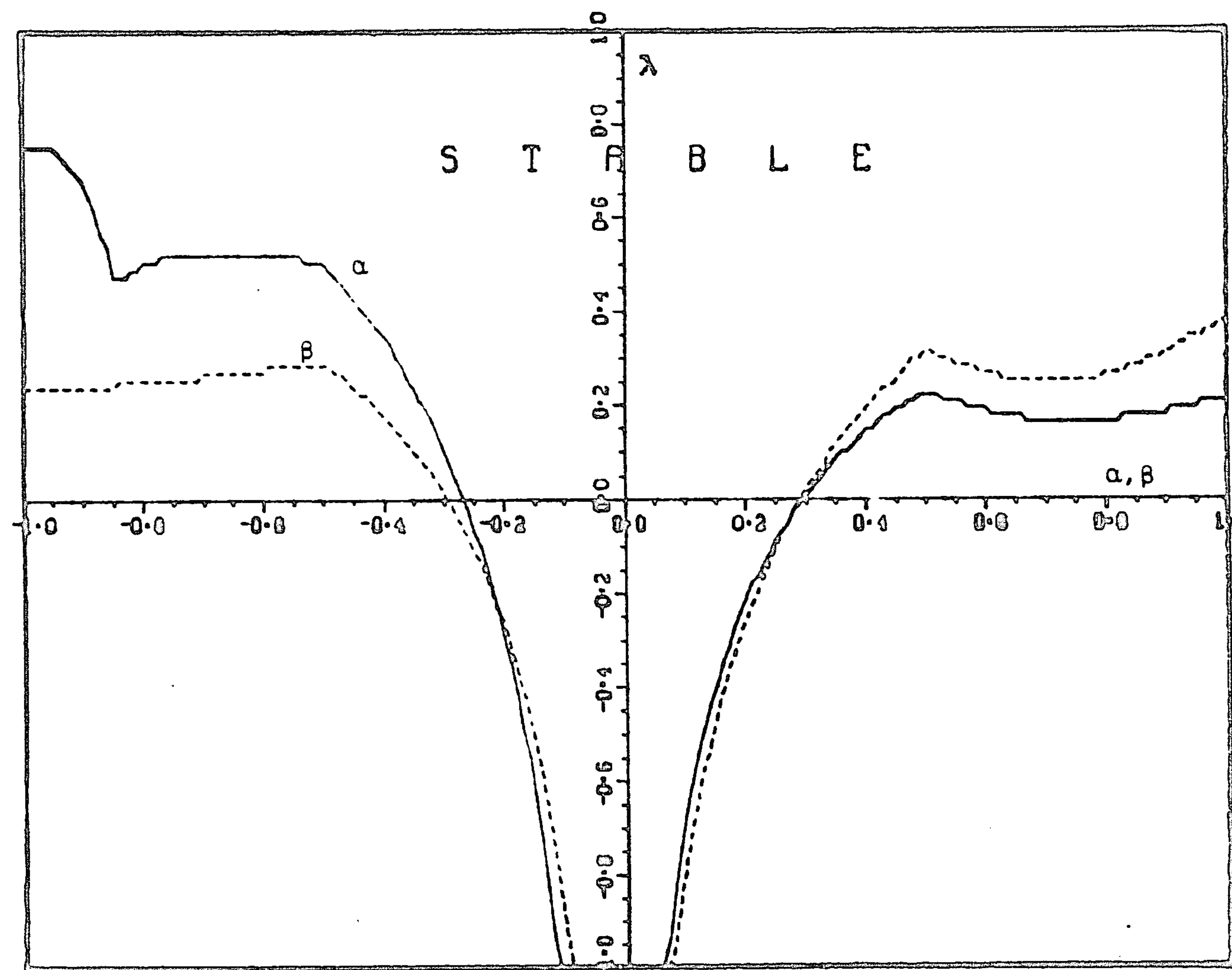


Fig. 9. Stability Limits for the Optimal Controllers (Eqn.(IV.1))as a Function of α and β (Process Given in Eqn. (6.1))

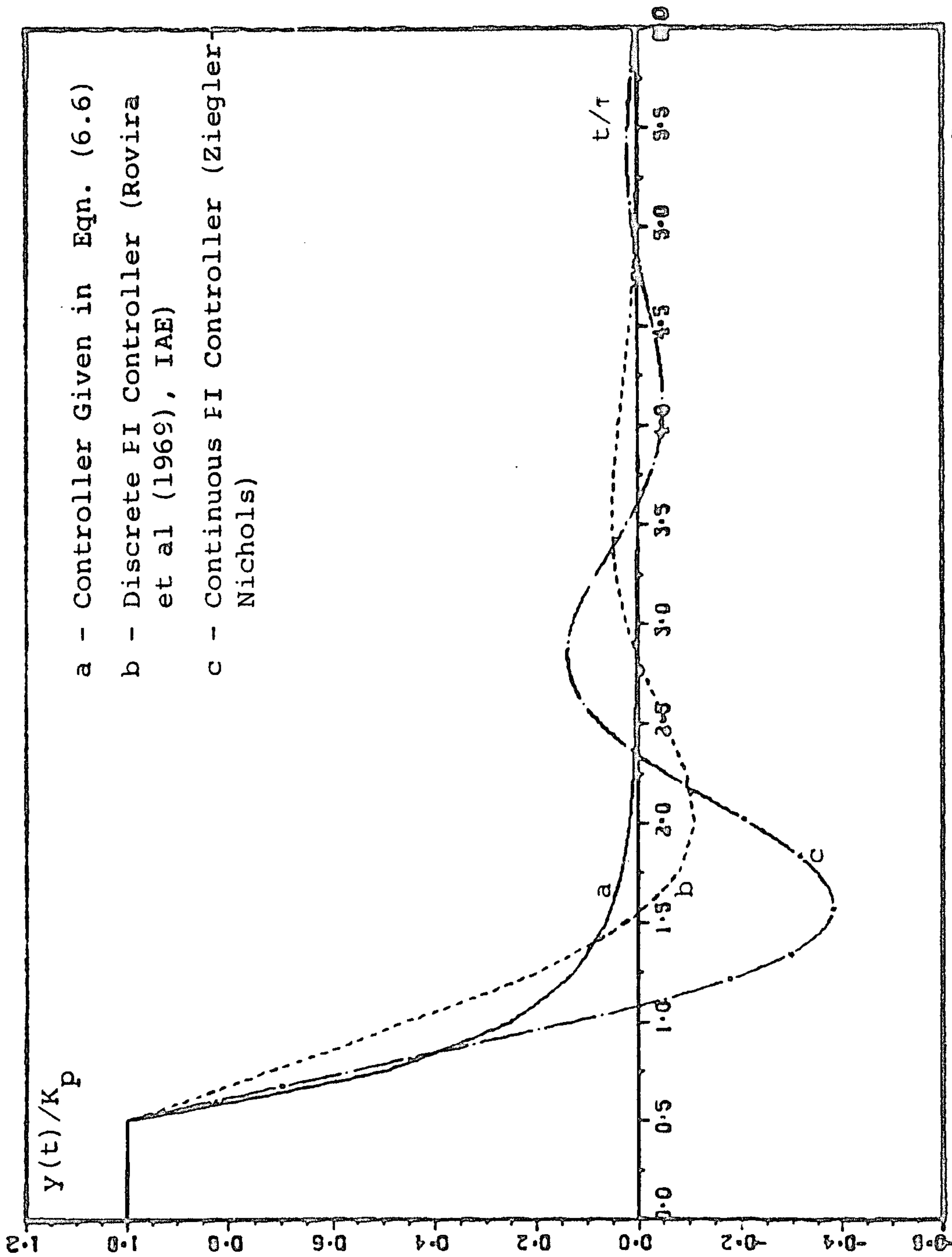


Fig. 10. Response to Unit Step Change in Set Point ($G_p(s)=e^{-.5s}/(s+1)$)

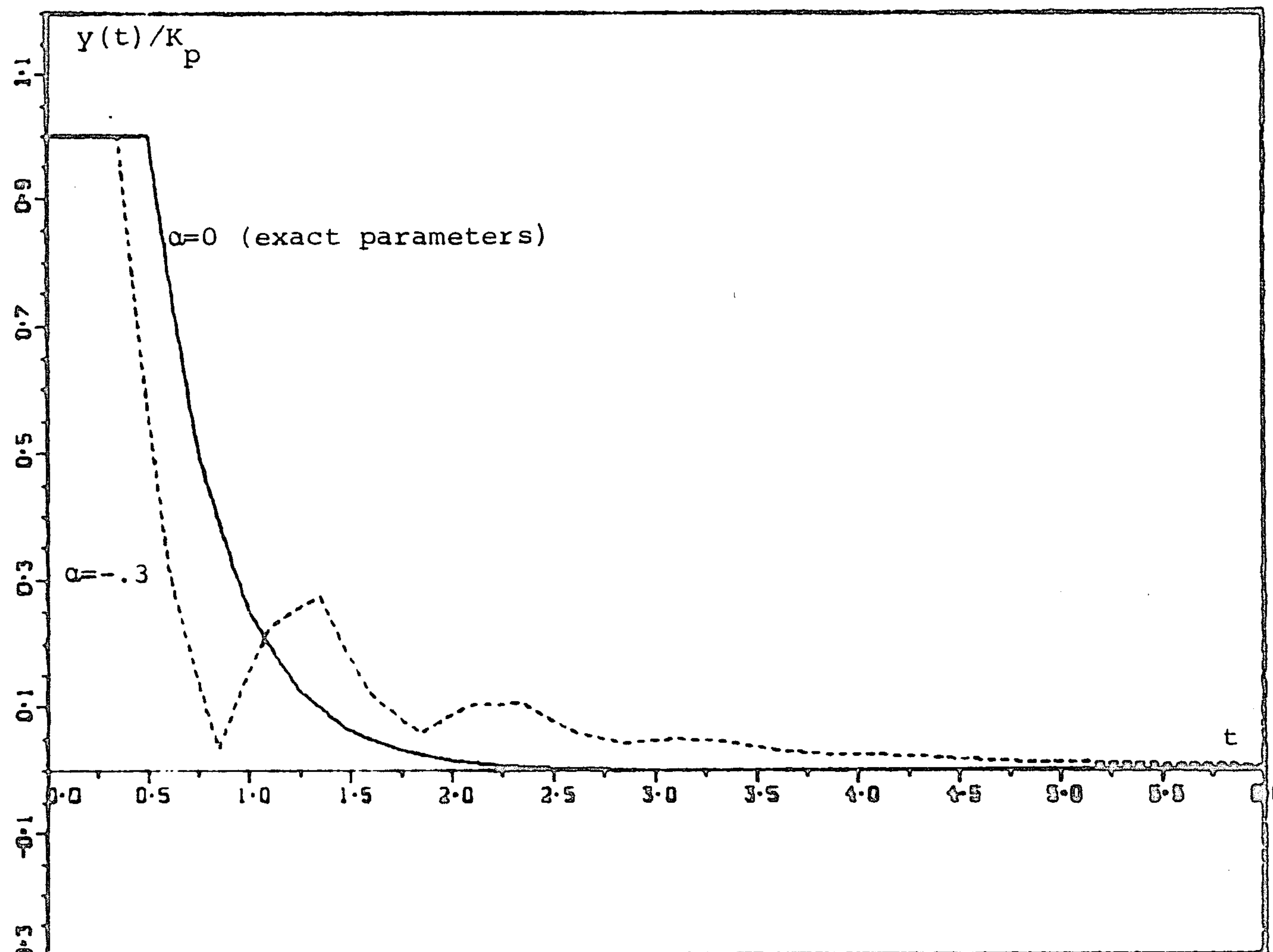


Fig. 11. Response of the Controller (6.6) to Unit Change in Set Point for Exact and Perturbed Parameters ($G_p(s)=e^{-.5s}/(s+1)$)

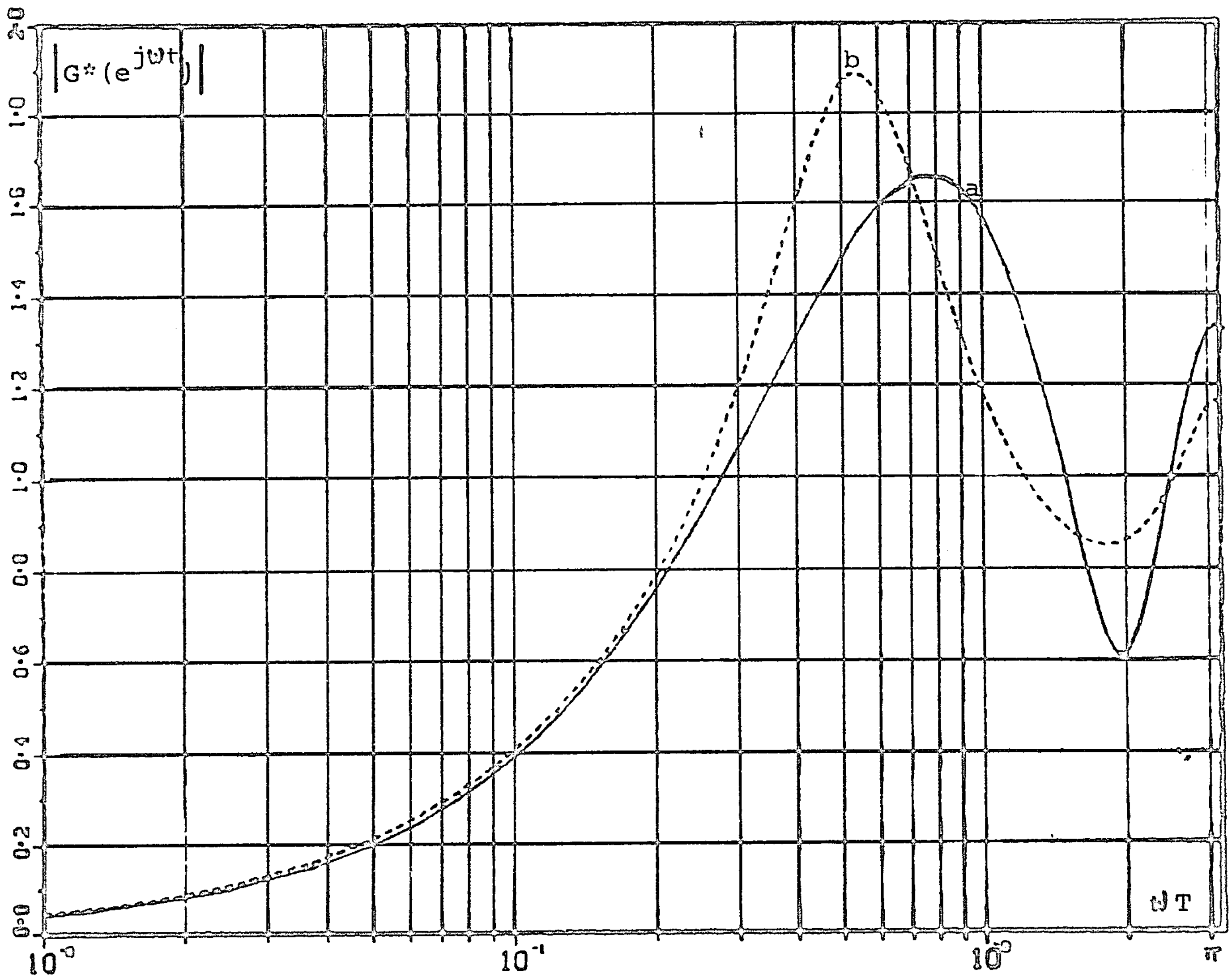


Fig. 12. Closed Loop Frequency Response for a) The Controller in (6.6), b) Discrete PI Controller (Rovira)

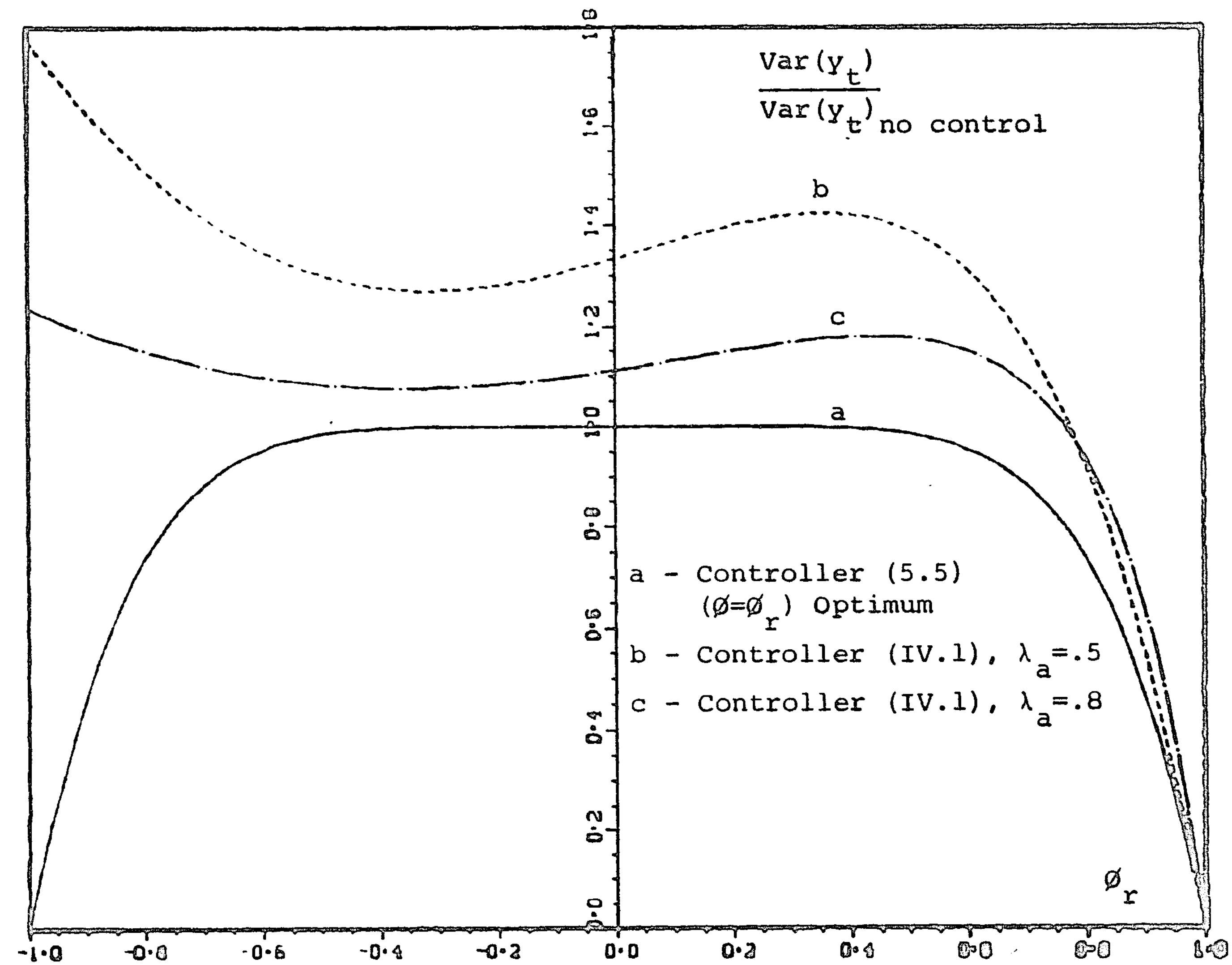


Fig. 13. Ratio Between the Output Variance of the Controlled System to the Output Variance of the Uncontrolled System for Stationary Input (Eqn. (4.4))

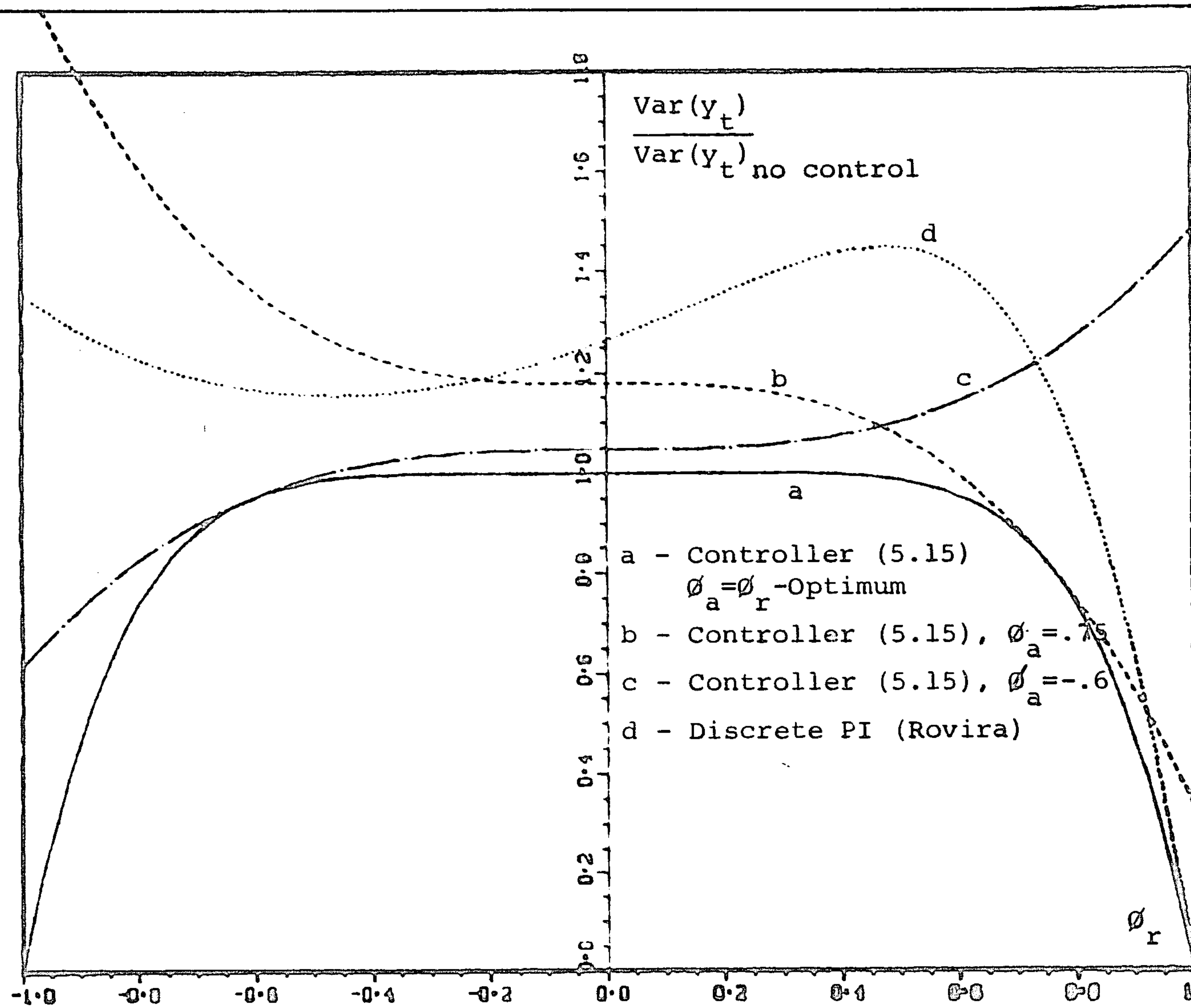


Fig. 14. Ratio Between the Output Variance with Control to the Output Variance with No Control for Stationary Disturbances (Eqn. (4.4))

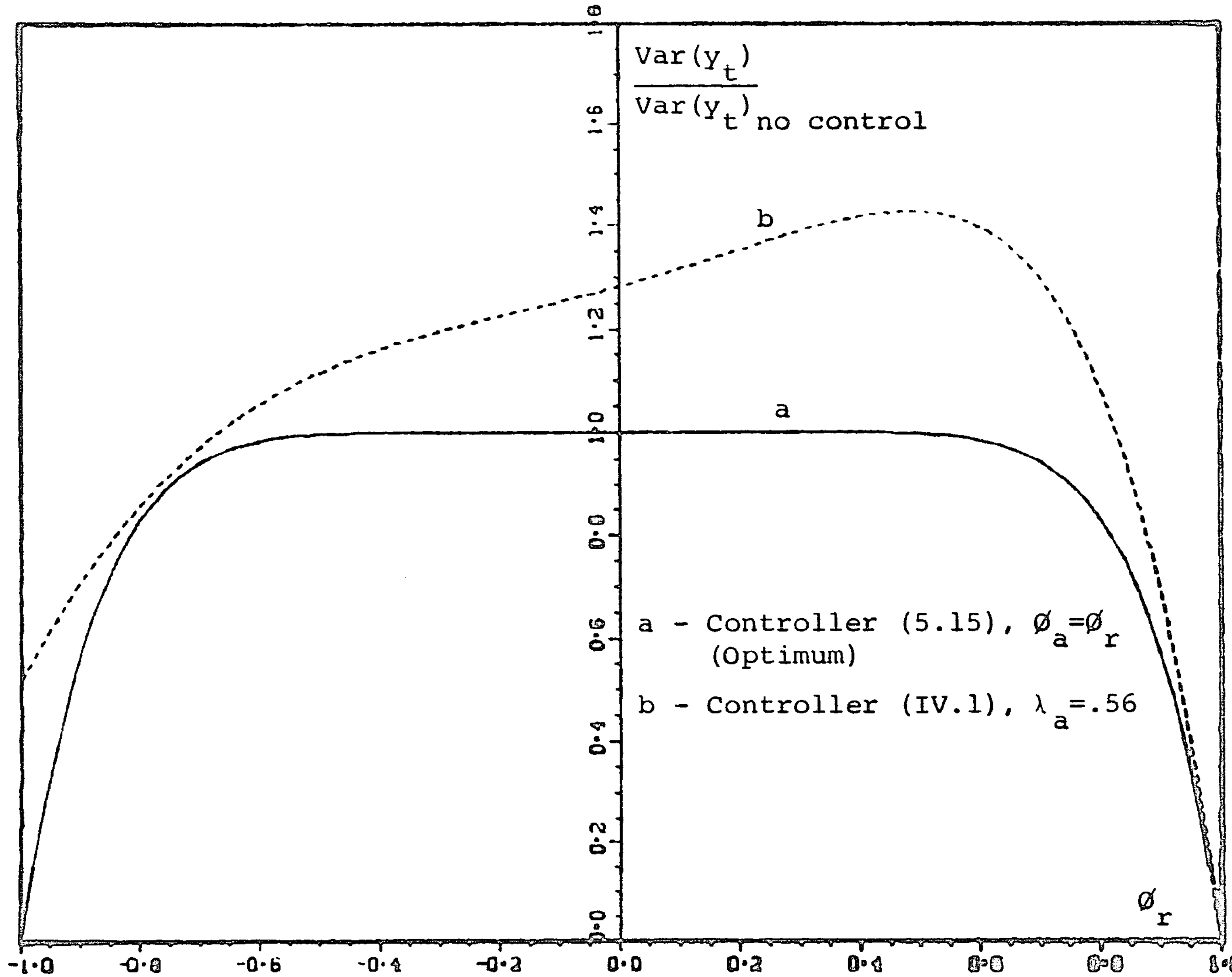


Fig. 14a. Ratio Between the Output Variance with Control to the Output Variance with No Controller For Stationary Disturbances ($\theta/T=k=3$)

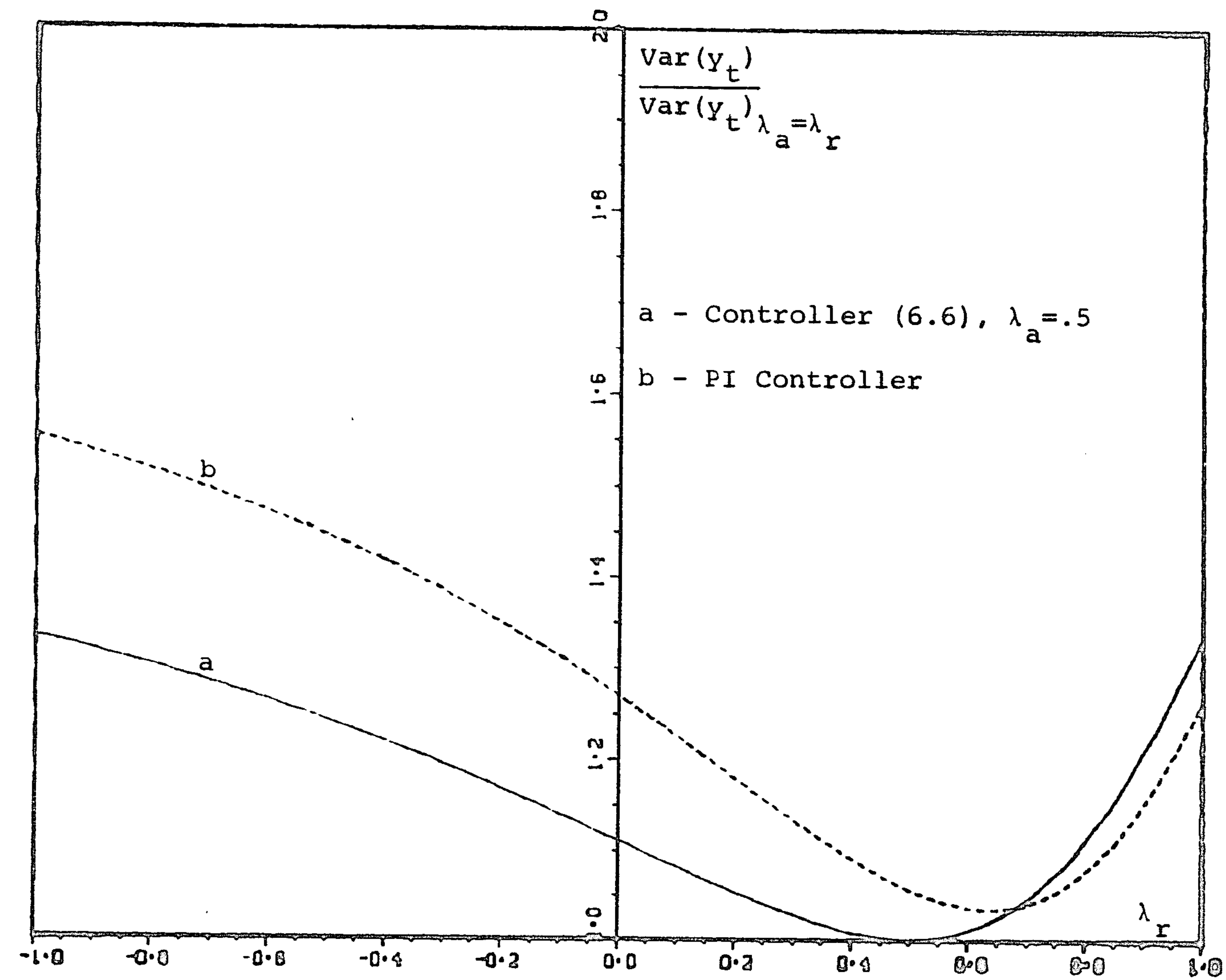


Fig. 15. The Output Variance with the Various Controllers to the Output Variance with the Optimal Unconstrained Controller ($\lambda_a=\lambda_r$) for Disturbances Characterized by (4.5)

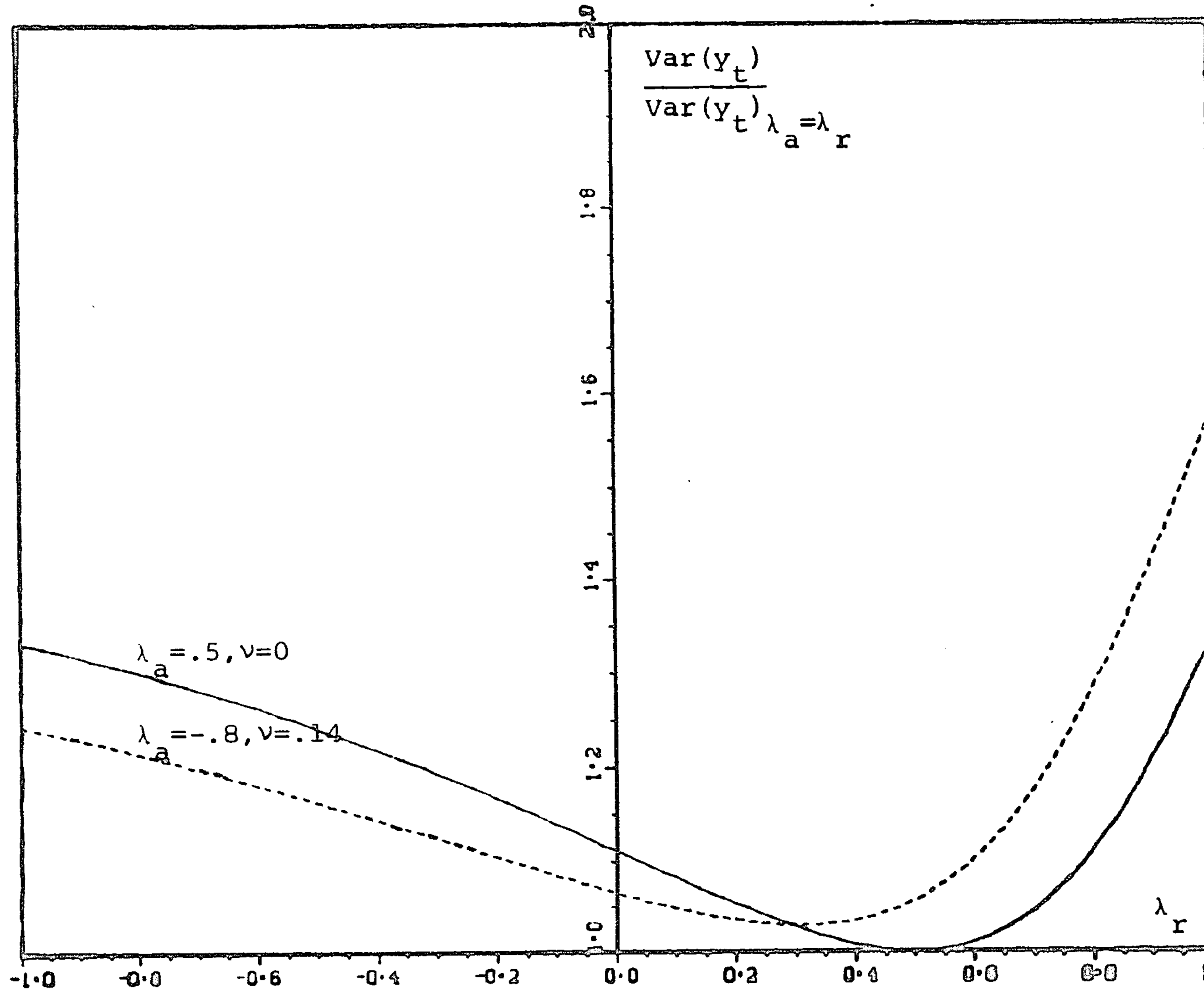


Fig. 16. Comparison Between the Performances of Two Optimal Controllers (Unconstrained (λ_a = .5) and Constrained (λ_a=-.8) with Equal Sensitivity) for Nonstationary Disturbances (Eqn. (4.5))

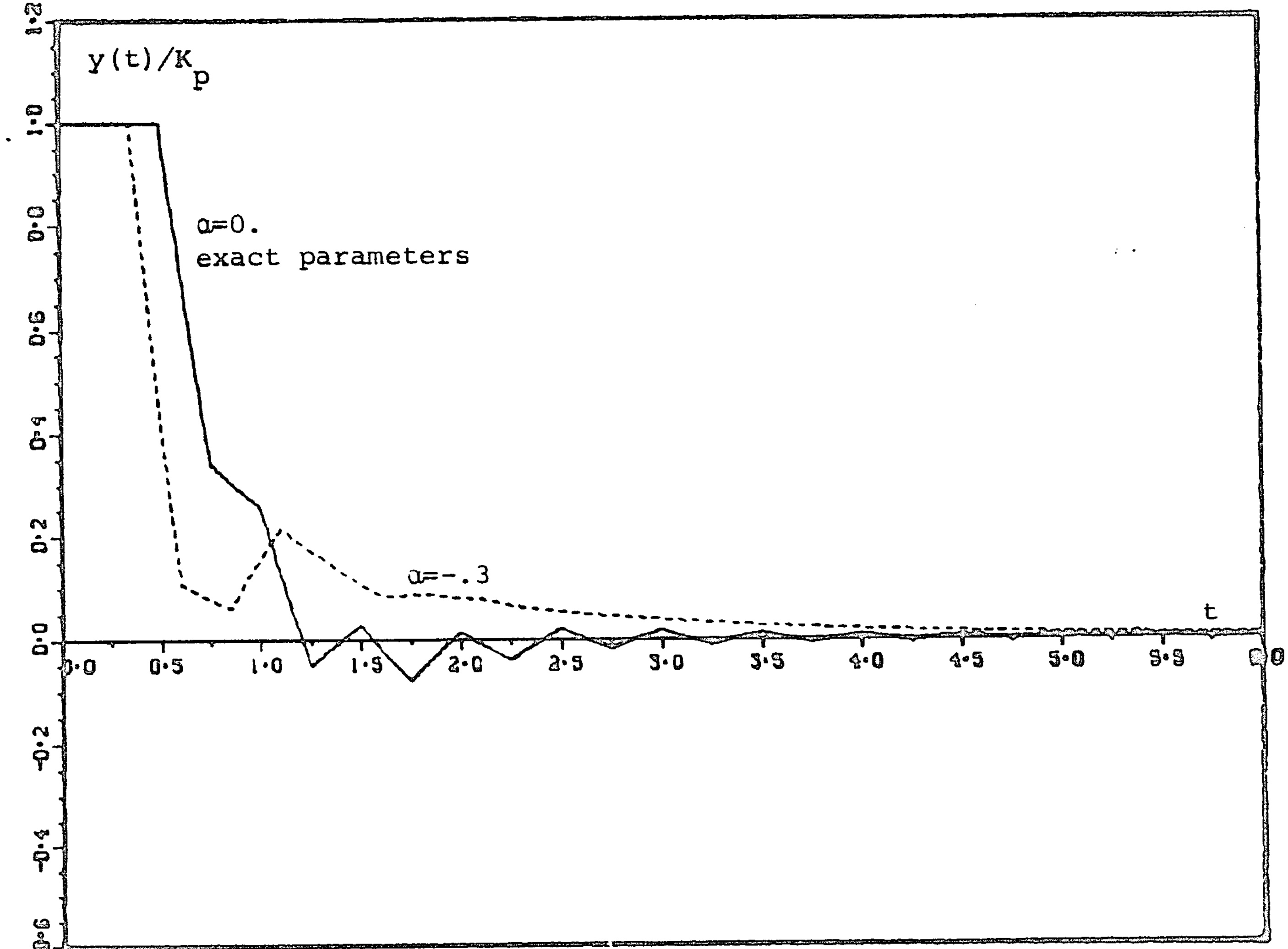


Fig. 17. Response of the Optimal Controller (Eqn. (IV.1), $\lambda_a=-.8$, $\nu=.14$) to Unit Change in Set Point for Exact and Disturbed Parameters

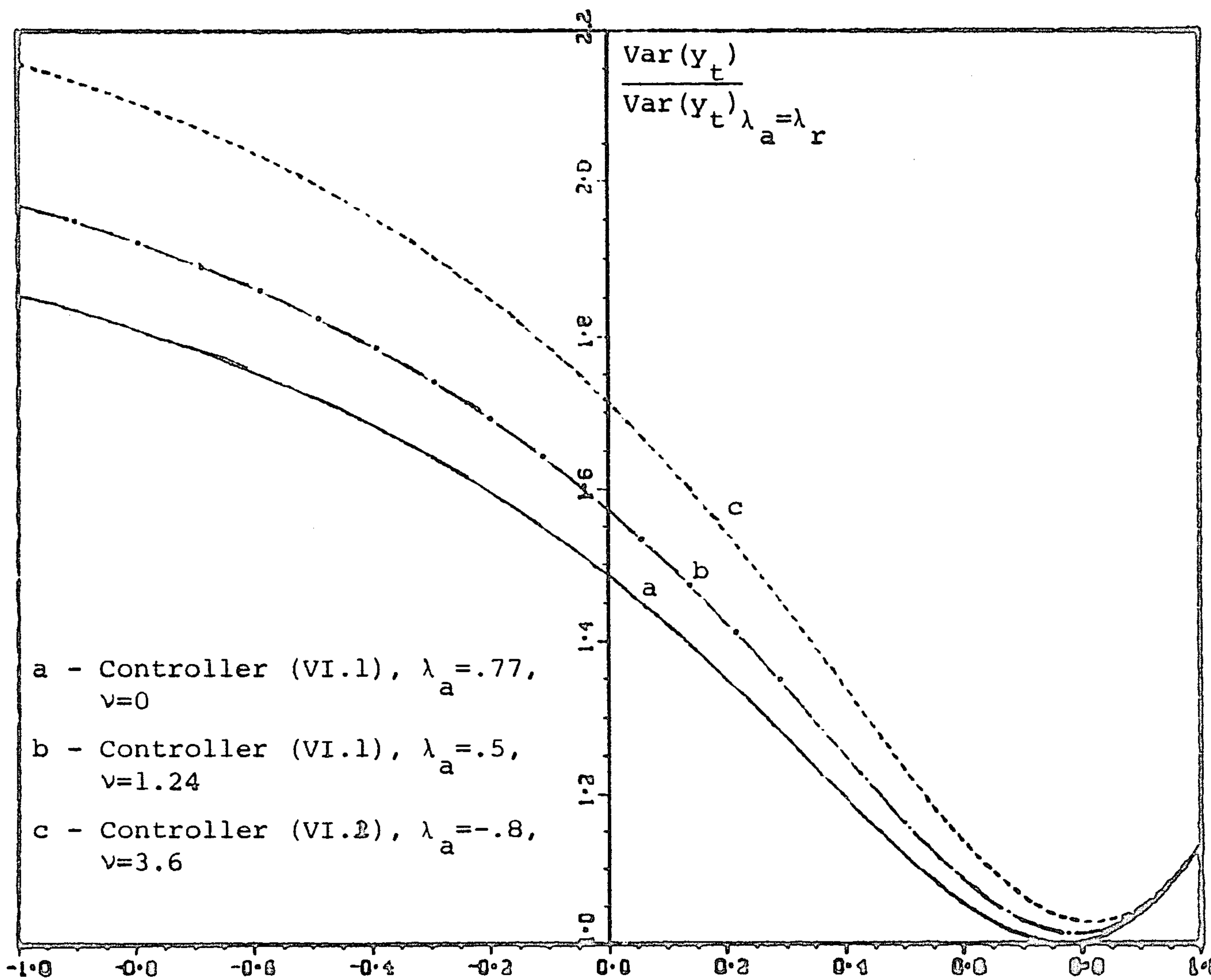


Fig. 18. Output Variance for the Three Controllers Having the Same Amplification of Measurement Noise to the Minimum Achievable Output Variance

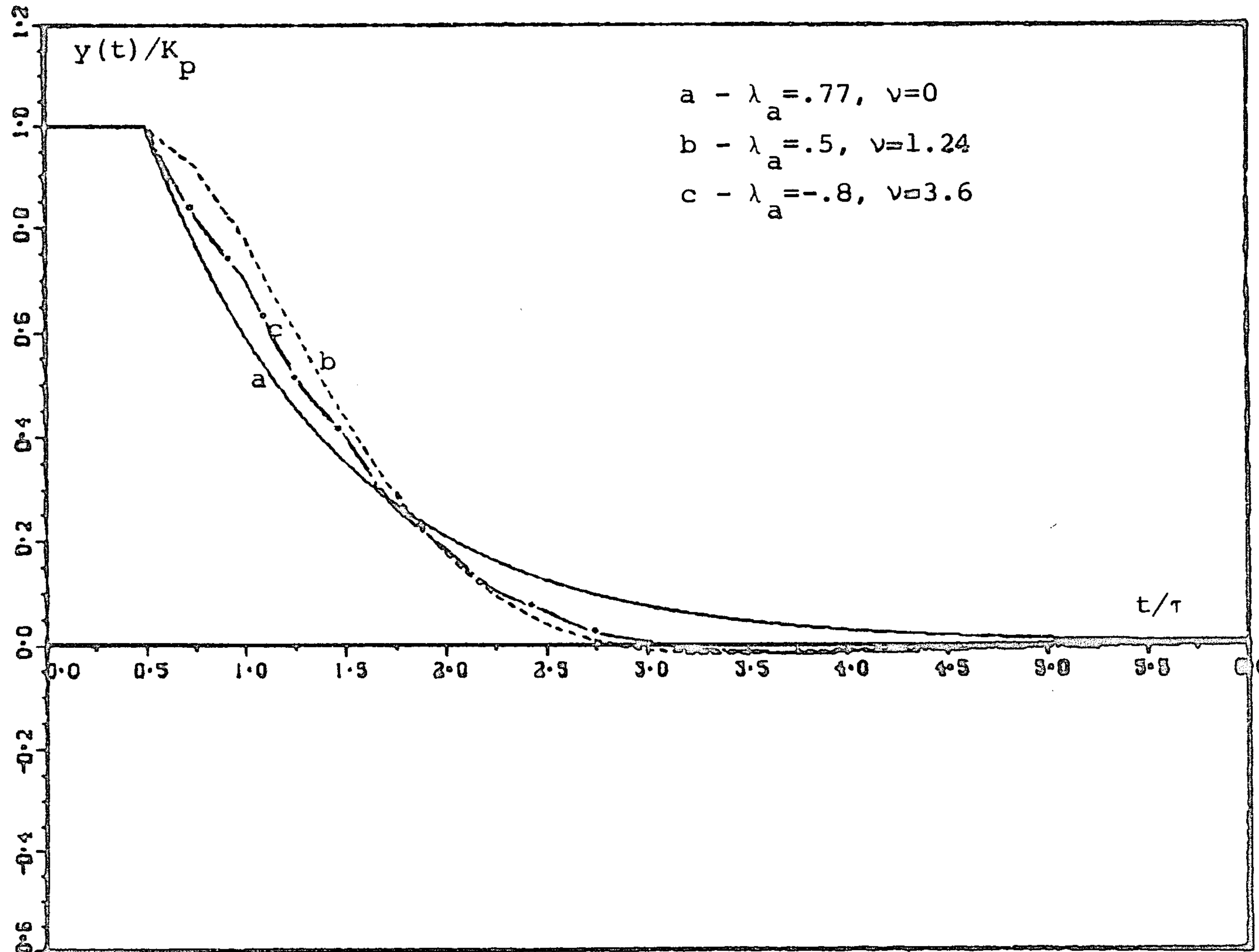


Fig. 19. Response to Unit Change in Set Point of the Closed Loop System Controlled by the Three Optimal Controls (Eqn. (IV.1)) with Equal Amplification of Measurement Noise

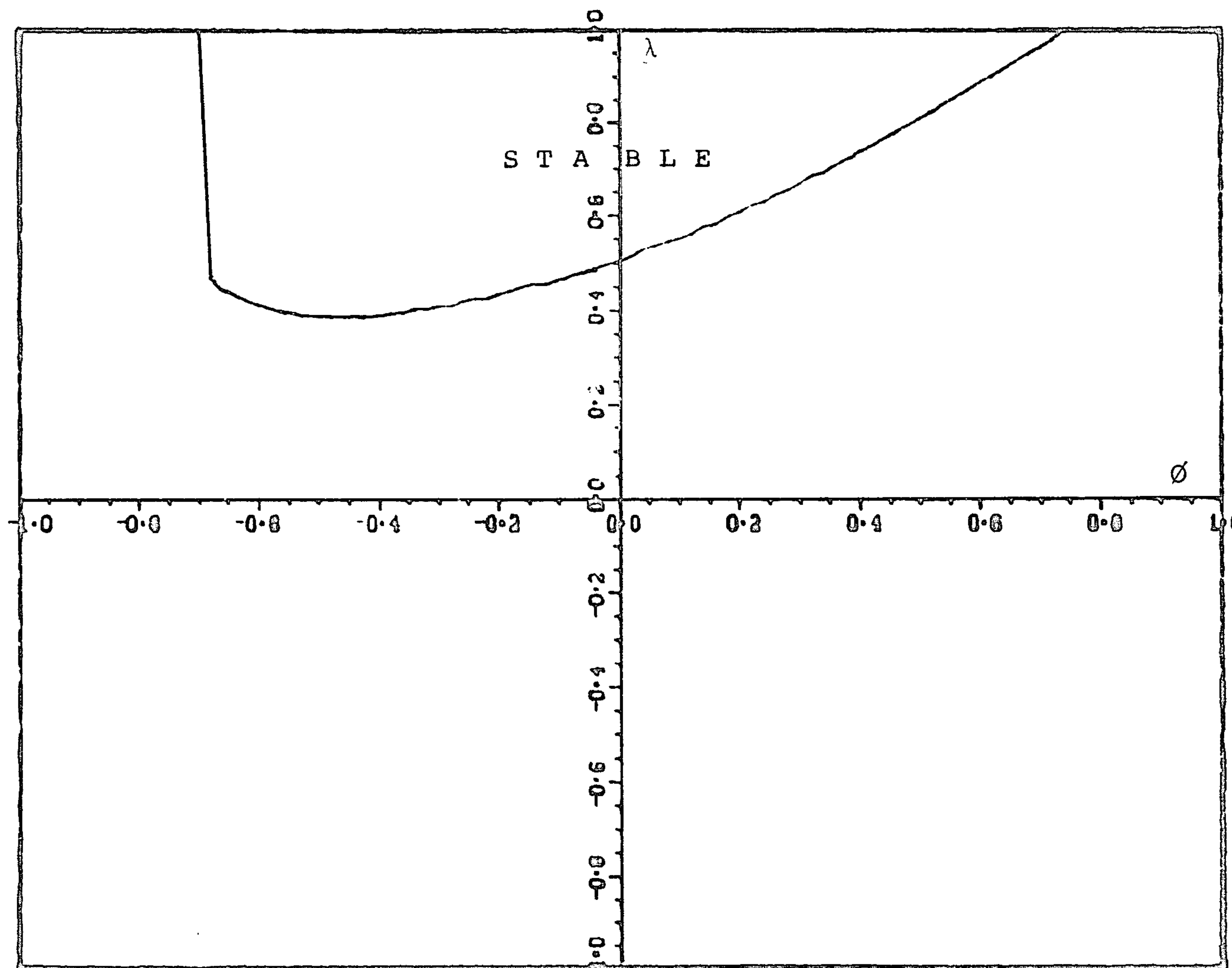


Fig. 20. Stability Limits for an Optimal Unconstrained Controller Designed for an ARIMA (1,1,1) Noise and Sensitivity of $\alpha=-.5$ $(G_p(s) = e^{-.5s}/(s+1))$

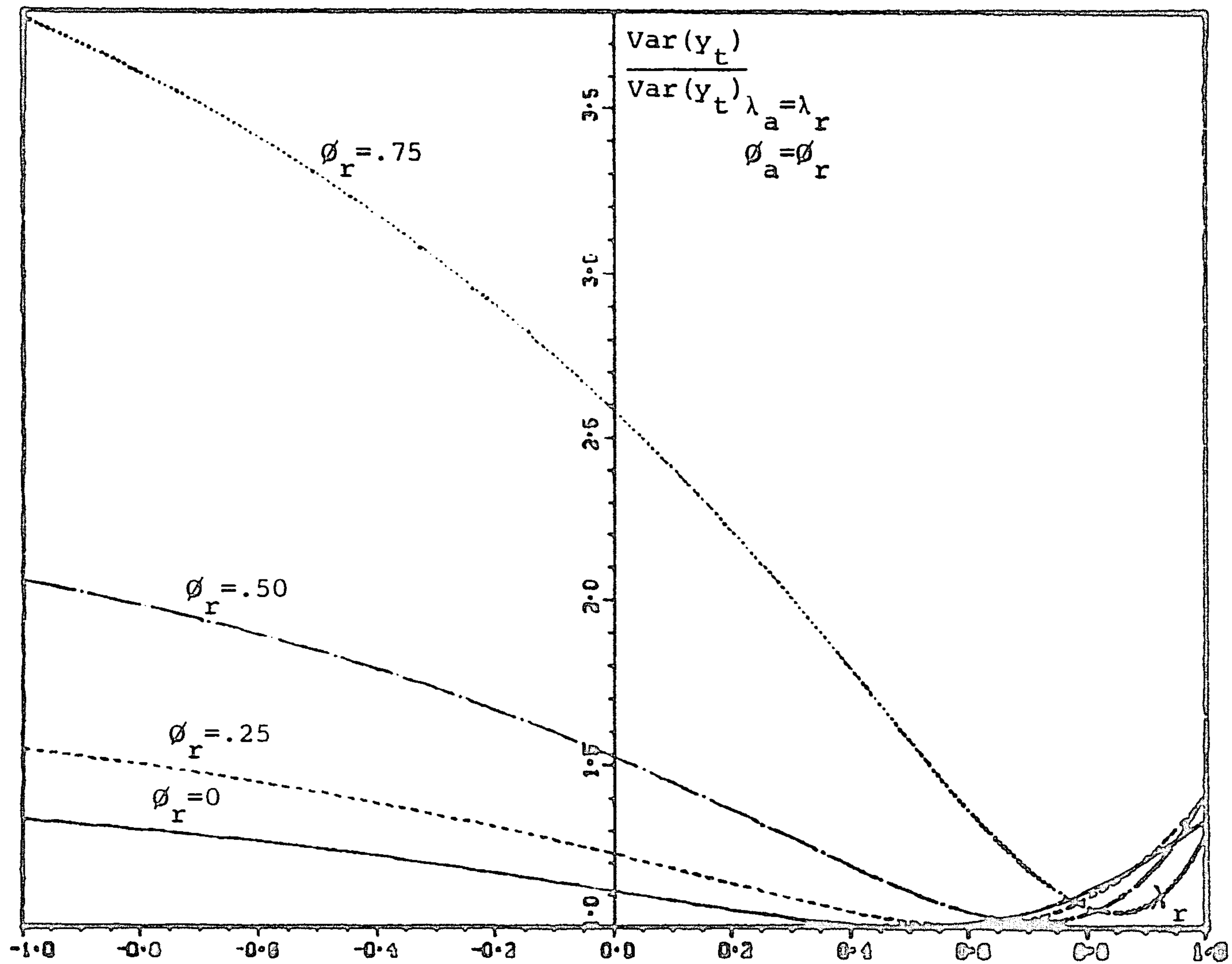


Fig. 21. Output Variance of the Controller (6.6) to the Minimum Achievable Variance for Disturbances Characterized by Eqn. (4.6)

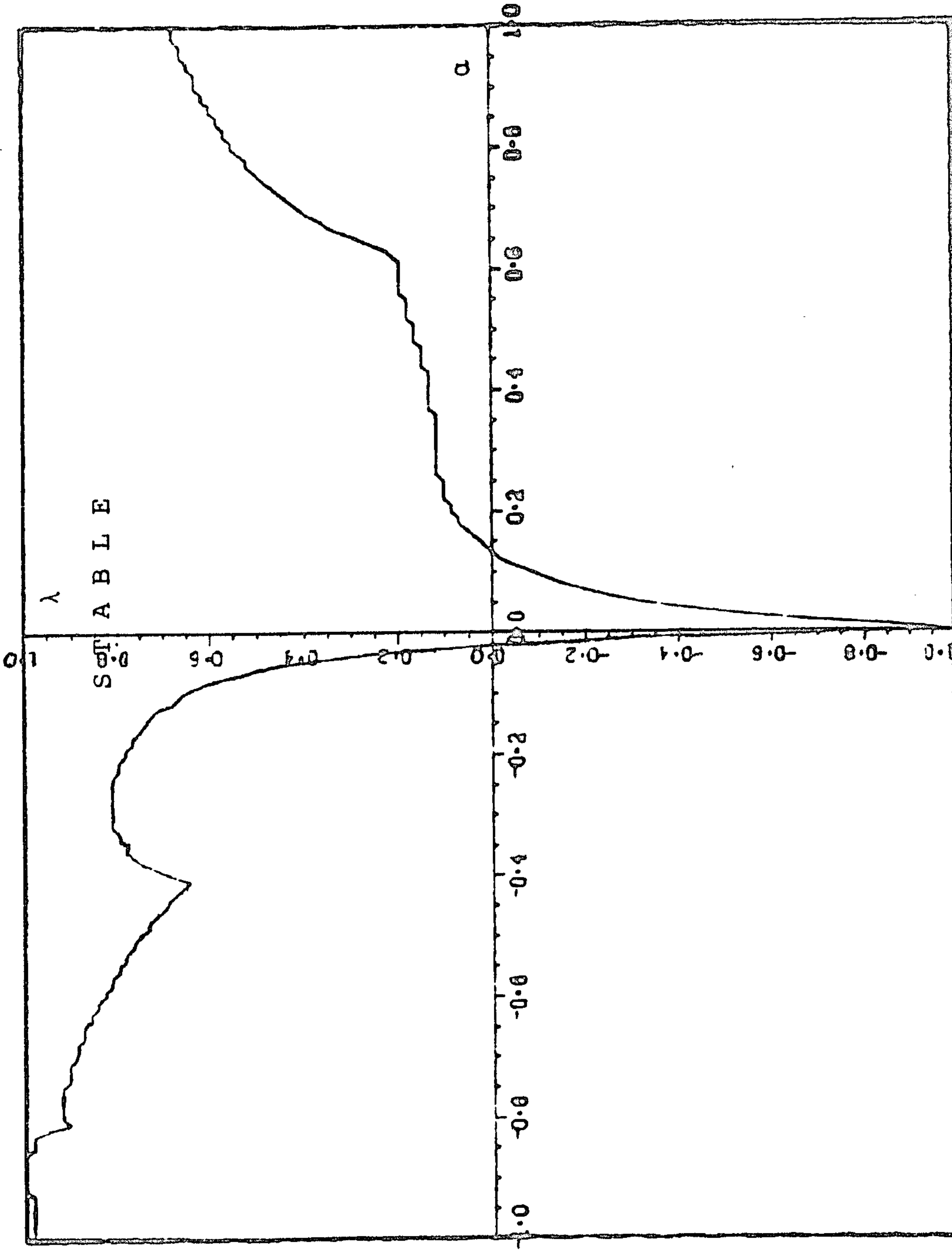


Fig. 22. Stability Limits for the Optimal Controllers (Eqn. (V.1)) as Function of α (Process Given by Eqn. (6.8))

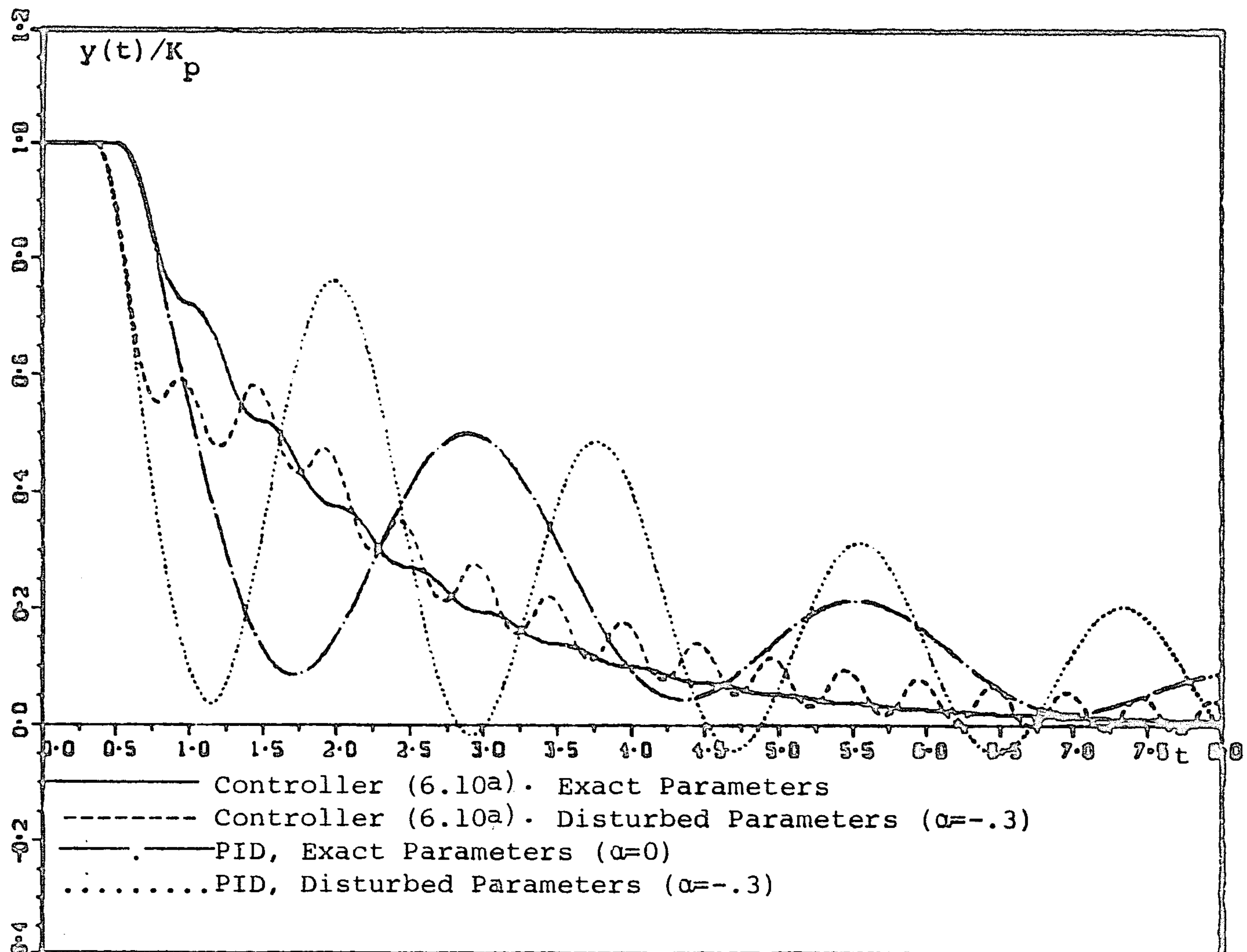


Fig. 23. Response of the Closed Loop System to Unit Change in Set Point (Process Given in (Eqn. (6.8)) Controlled by the Optimal Controller (Eqn. (6.10a))and a Conventional PID Controller

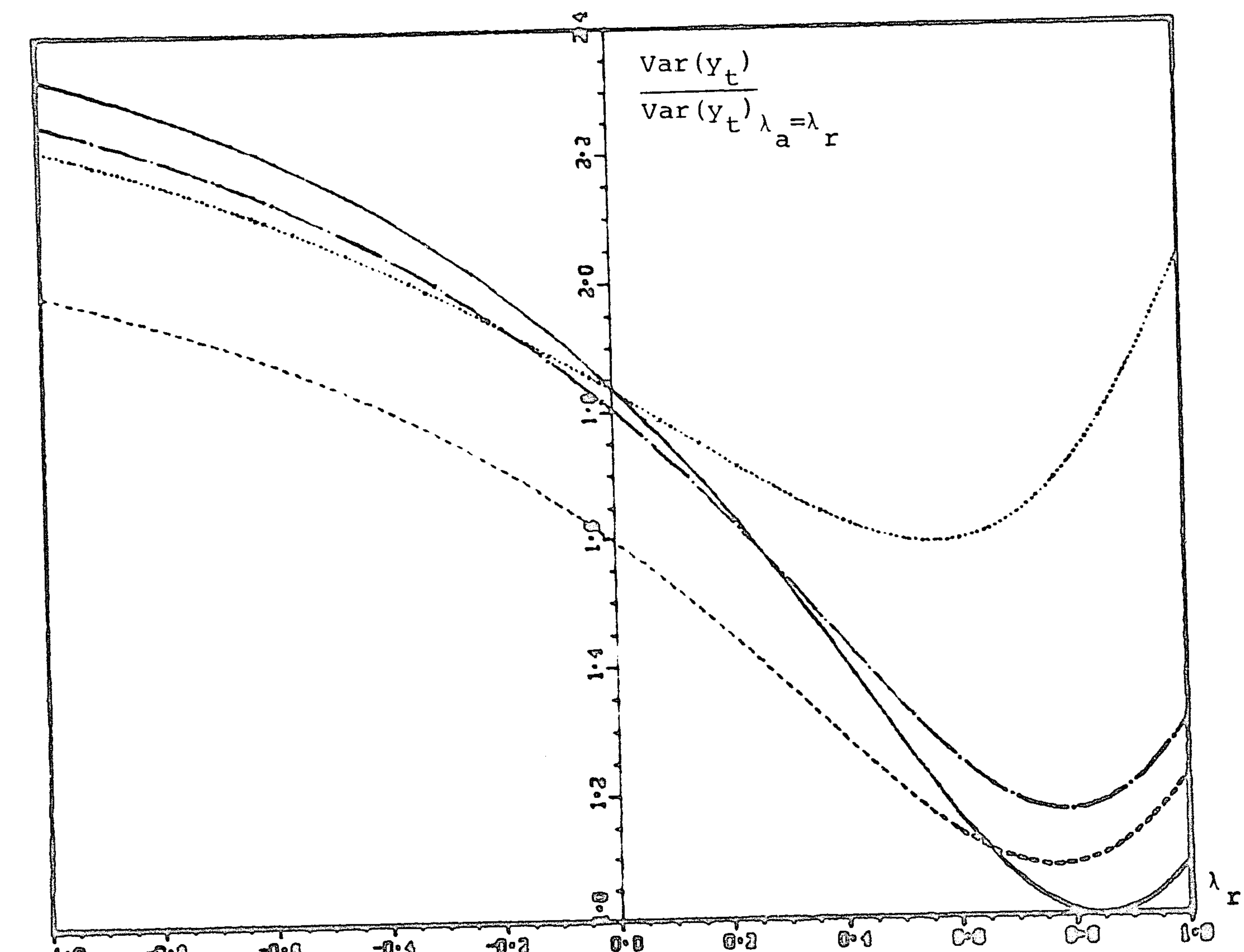


Fig. 24. Output Variance for the Various Controllers to the Minimum Possible Output Variance (For Notation, See Fig. 23) as Function of λ_r

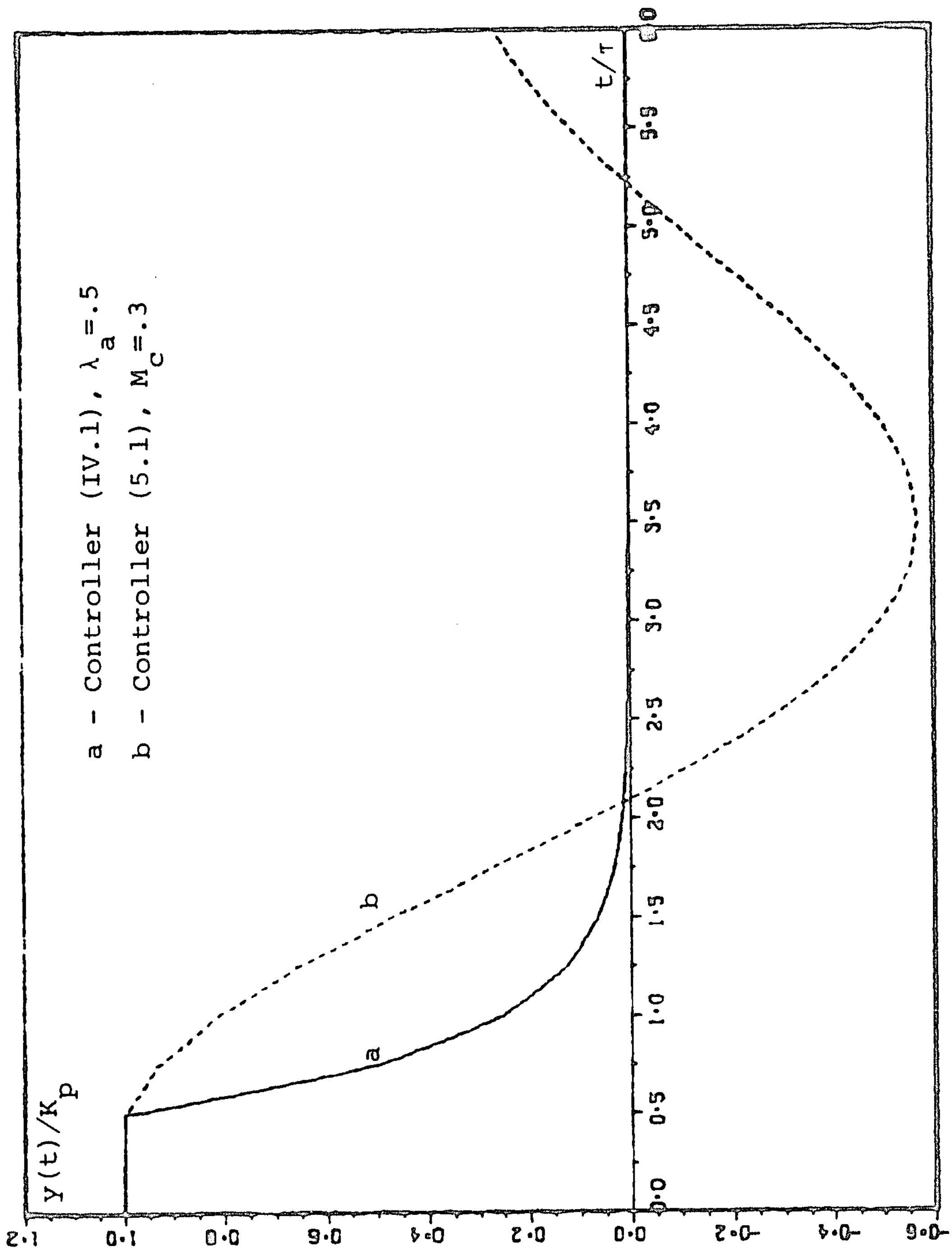


Fig. 25. Response of the Closed Loop System Controlled by a) Controller (6.6), b) Controller (5.1), to Unit Change in Set Point

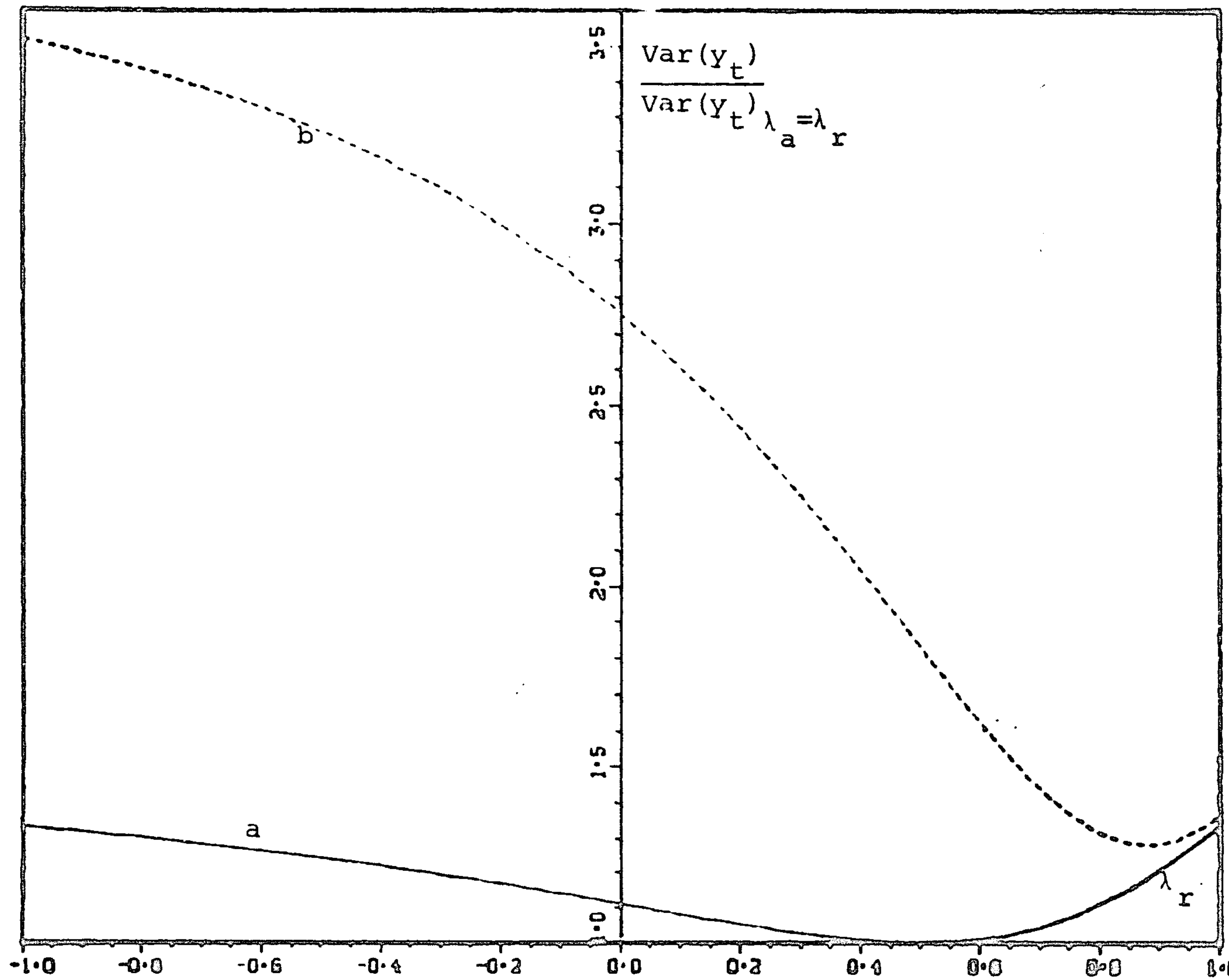


Fig. 26. Output Variances for the Controllers Given by Eqn. (6.6) and (5.1) to the Minimum Possible Output Variance as Functions of λ_r (For Notations see Fig. 25).

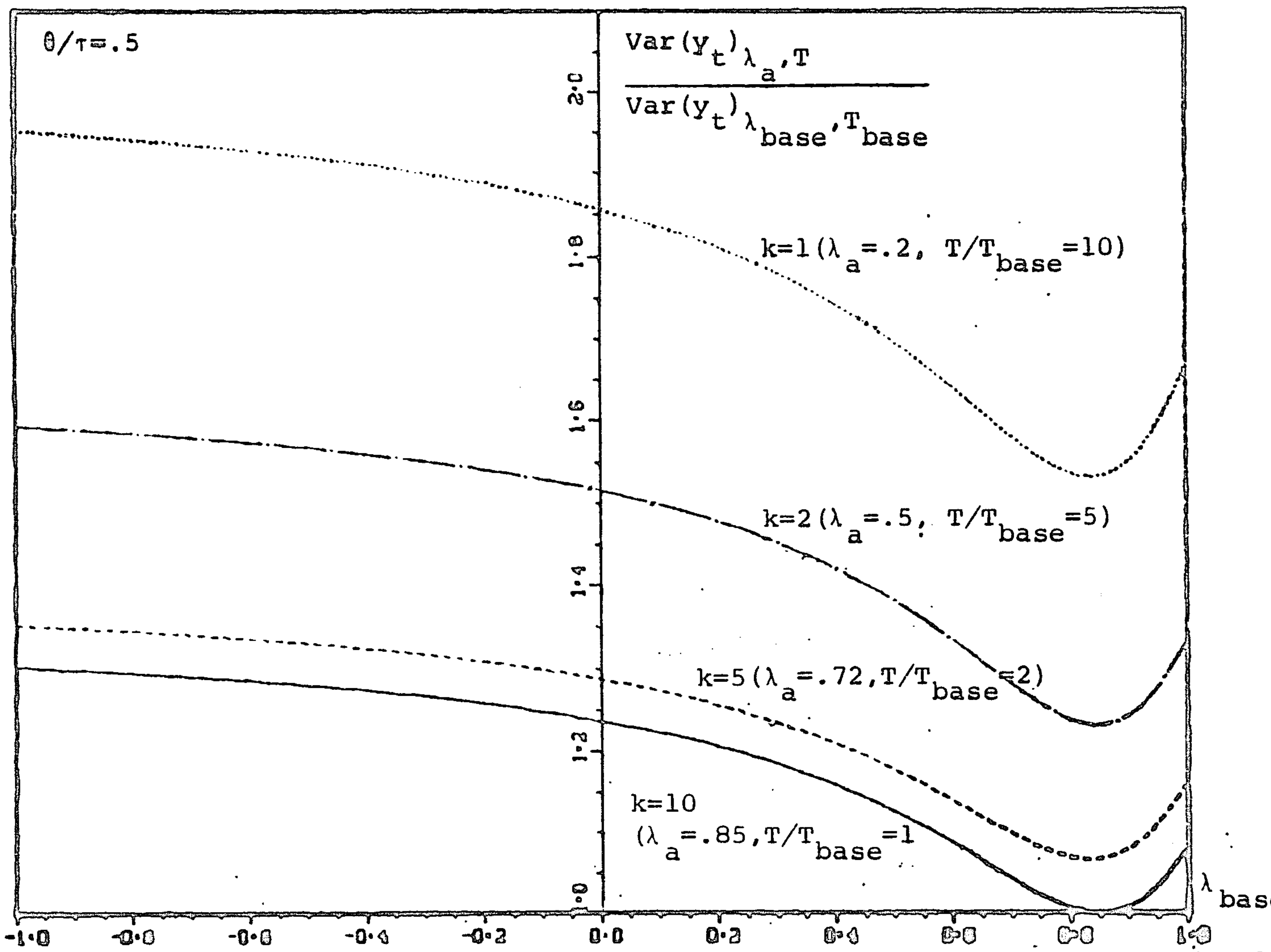


Fig. 27. Output Variance, Scaled to T_{base} for the Controllers Given by (IV.1) (with the Proper λ_a) to the Minimum Possible Output Variance at T_{base} ~~for Various k's, as Function~~ of λ_{base} (θ/τ=.5)

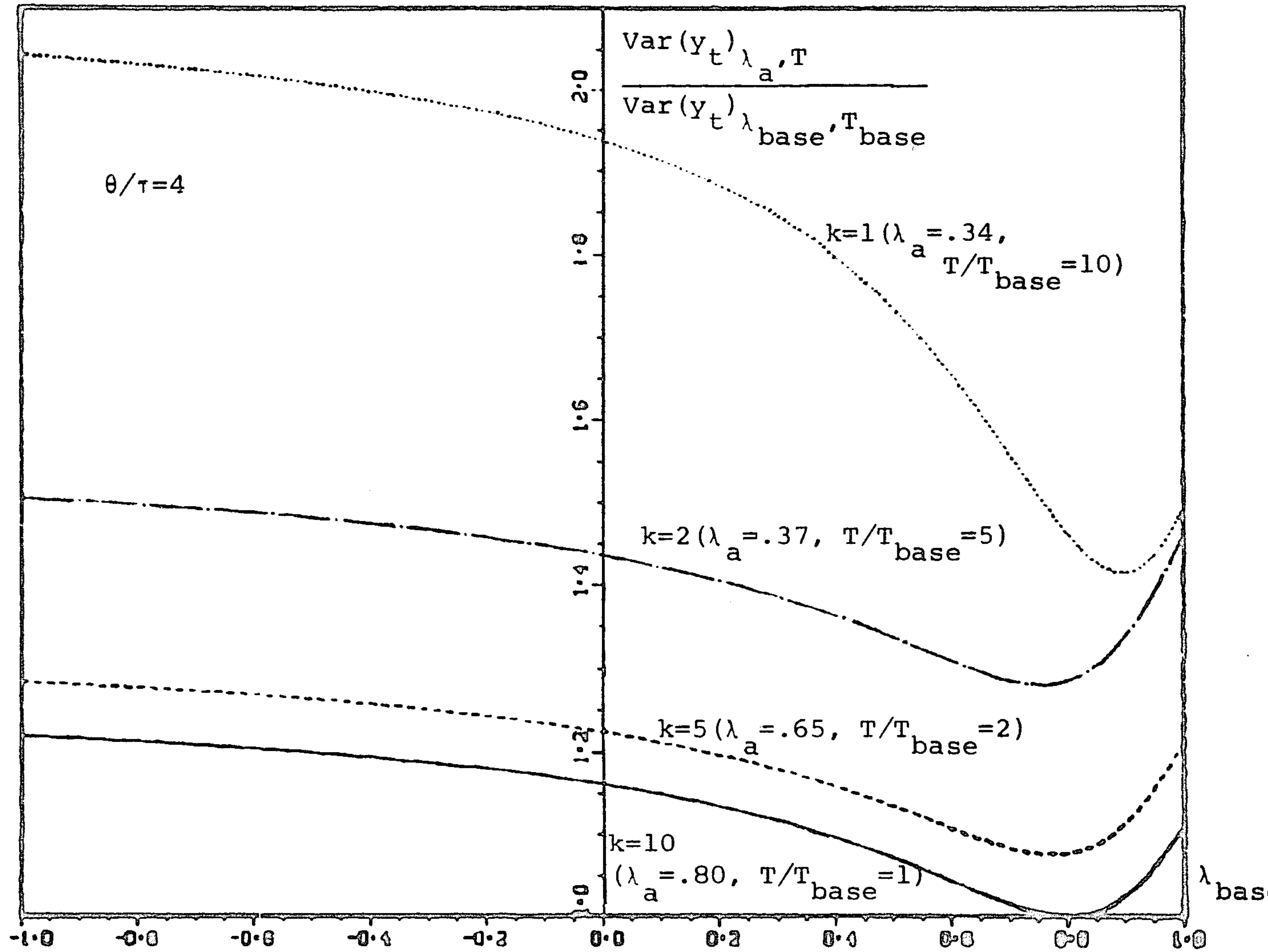


Fig. 28. Output Variance, Properly Scaled to T_{base}, for the Controllers Given by (IV.1) (with the Proper λ_a) to the Minimum Possible Output Variance at T_{base} for various k's as a Function of λ_{base} ($\theta/\tau=4$)

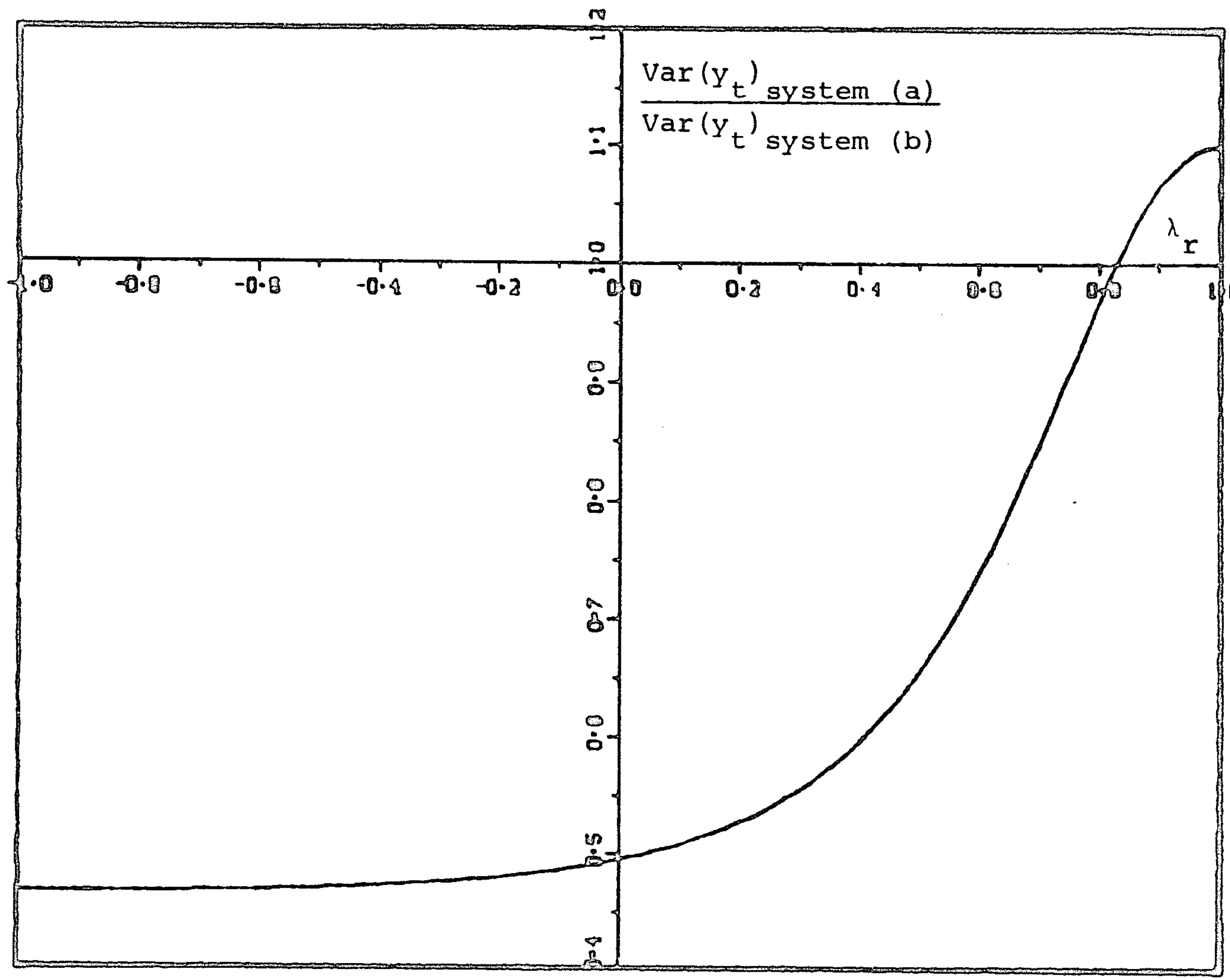


Fig. 29. The Ratio Between the Output Variance of System (a) and the Output Variance of System (b) as a Function of λ_r

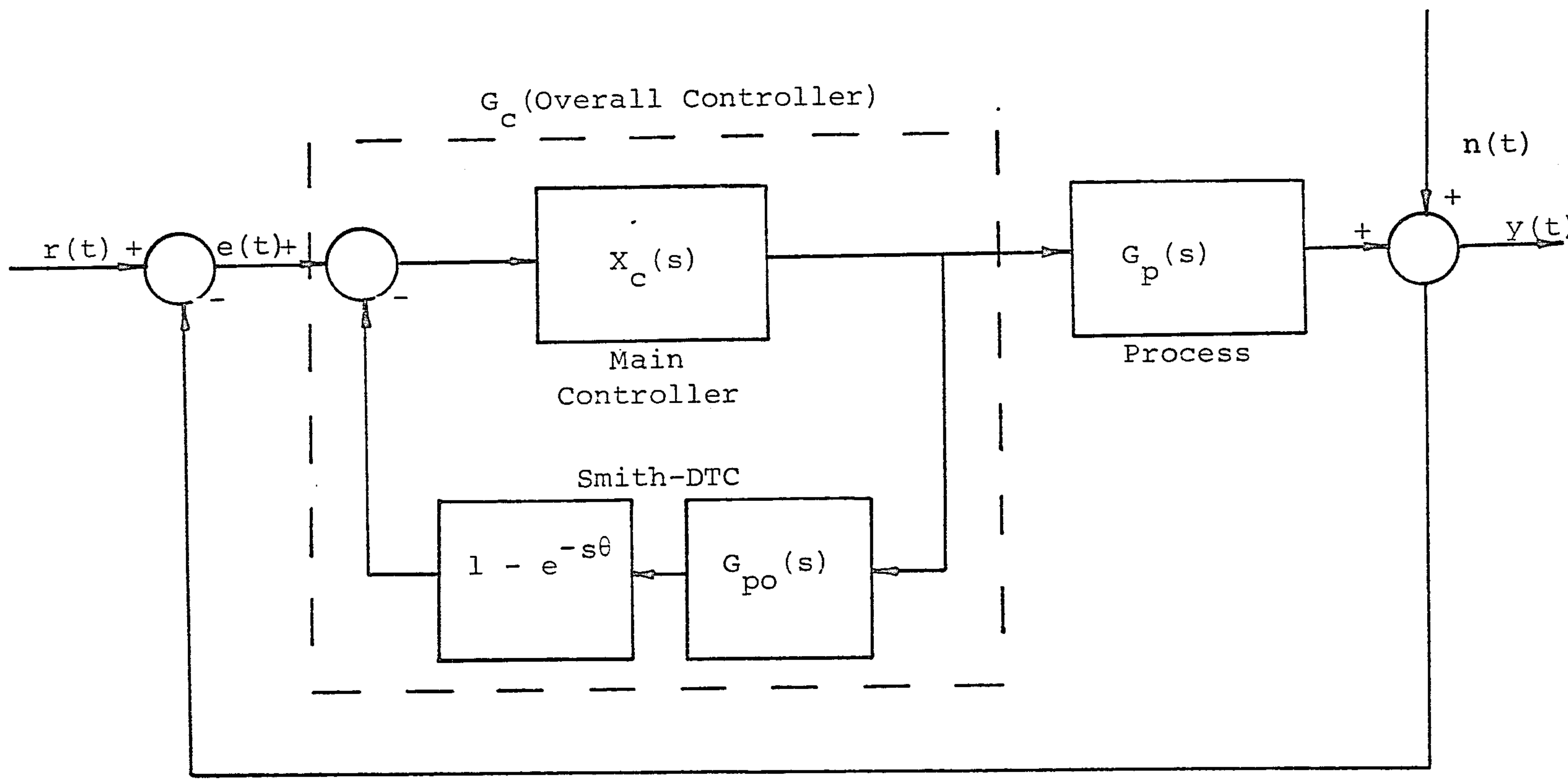


Figure 30. The Closed-Loop System with Smith Dead Time Compensator.

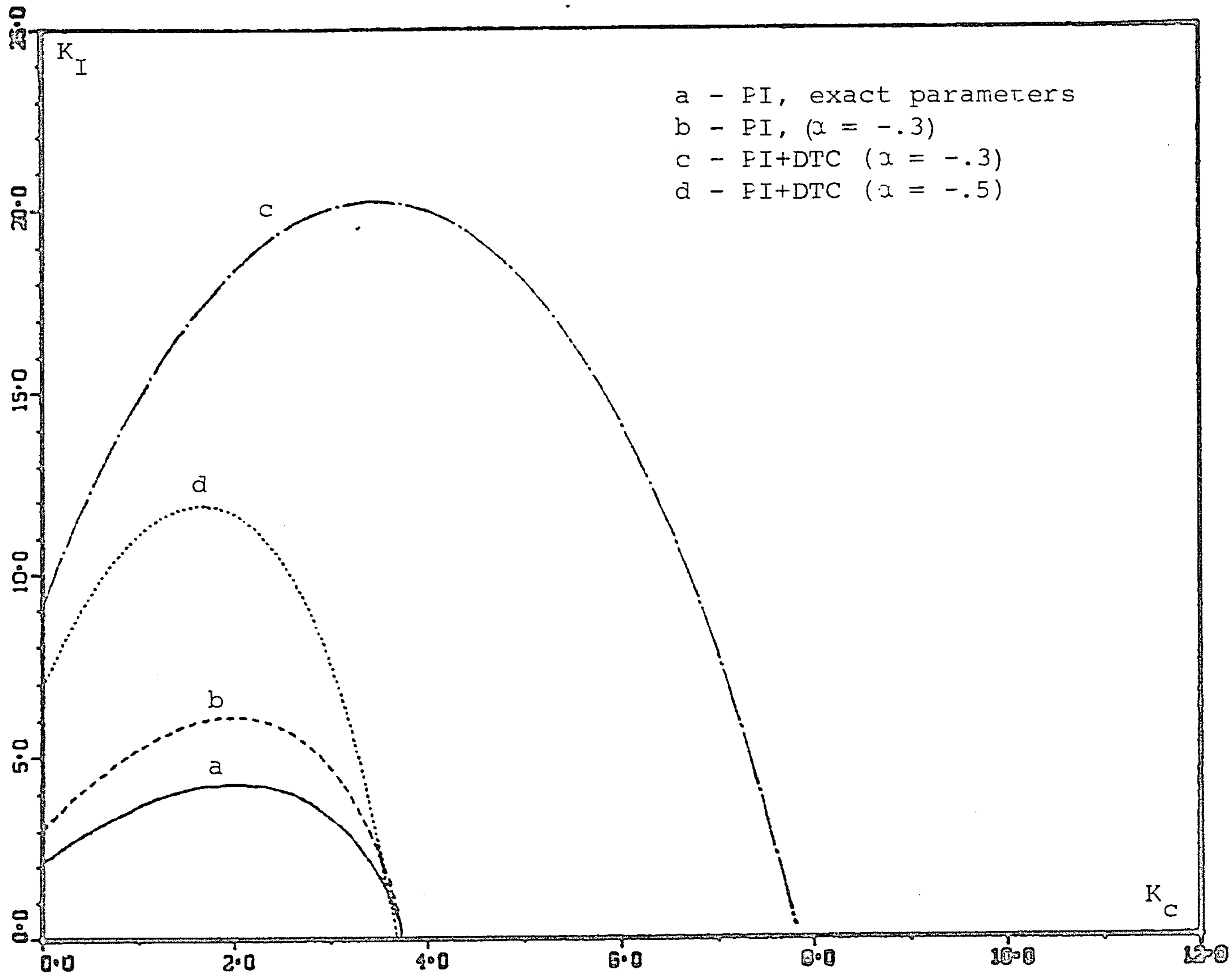


Figure 31. Stability Limits for PI and PI+DTC Controllers (Process Given in Eqn. (10.1).

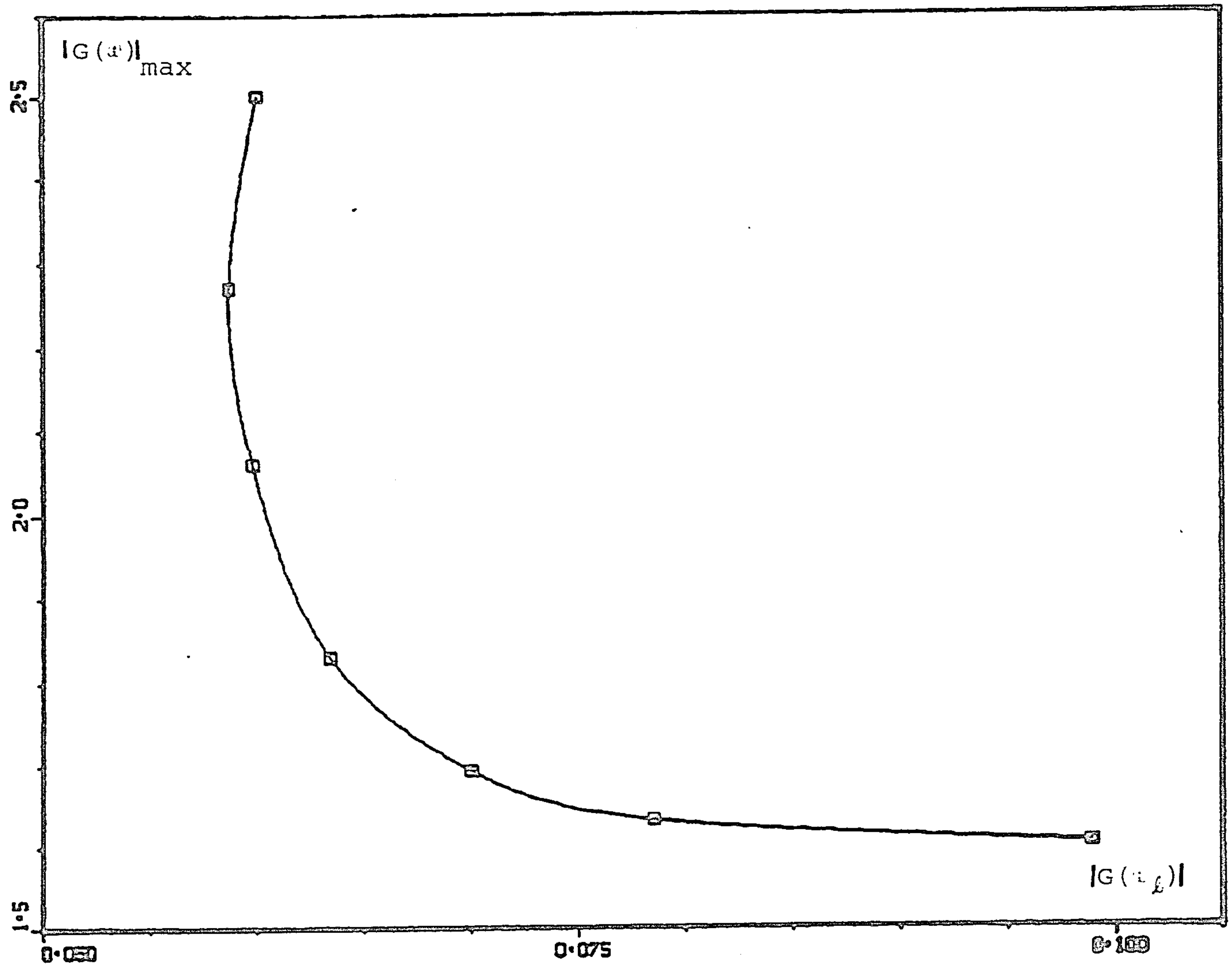


Figure 32. A Typical Mini-Max Plot.

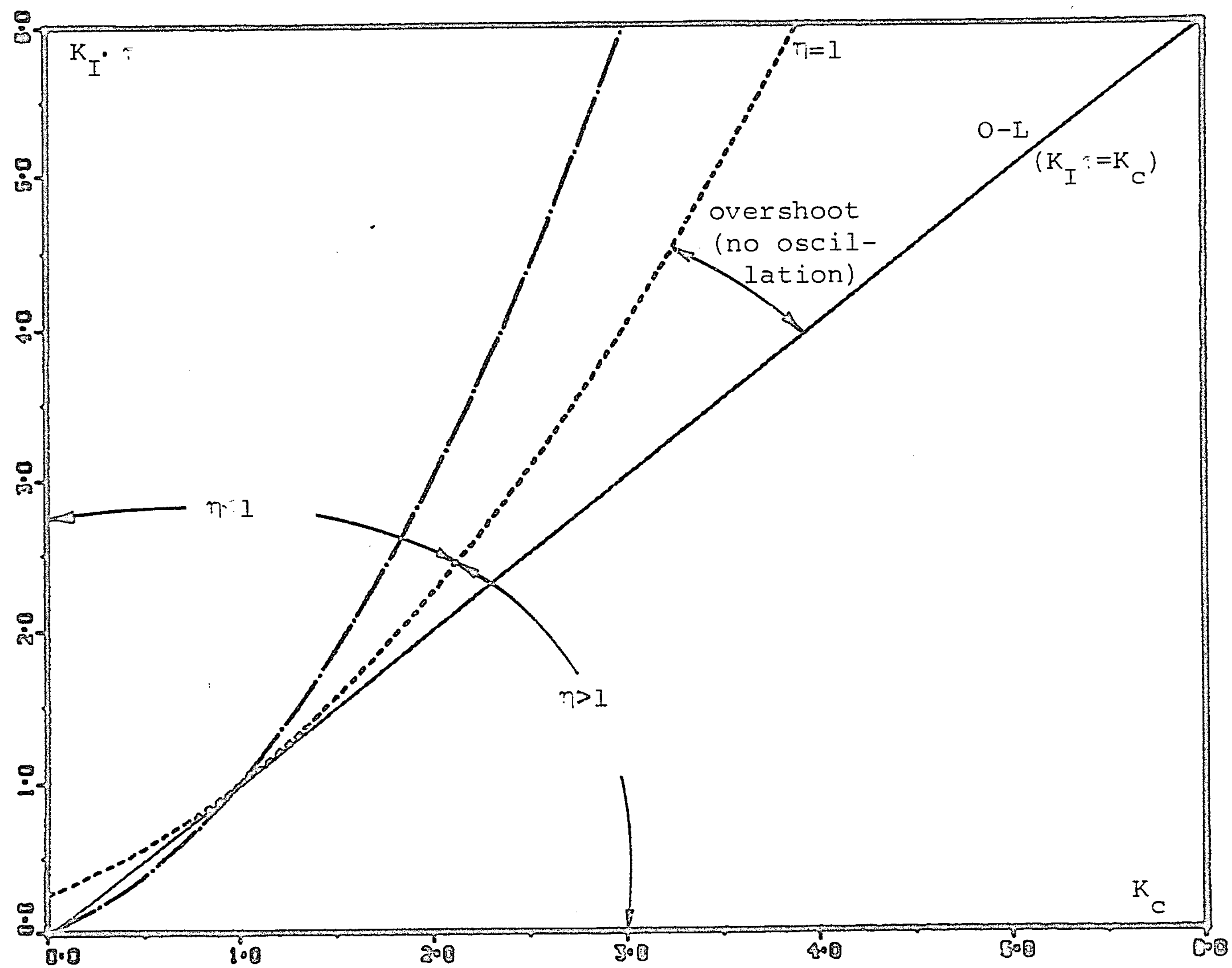


Figure 33. Oscillatory, Nonoscillatory, and Nonoscillatory+Overshoot Regions for PI+DTC Controllers and First Order+Delay Processes.

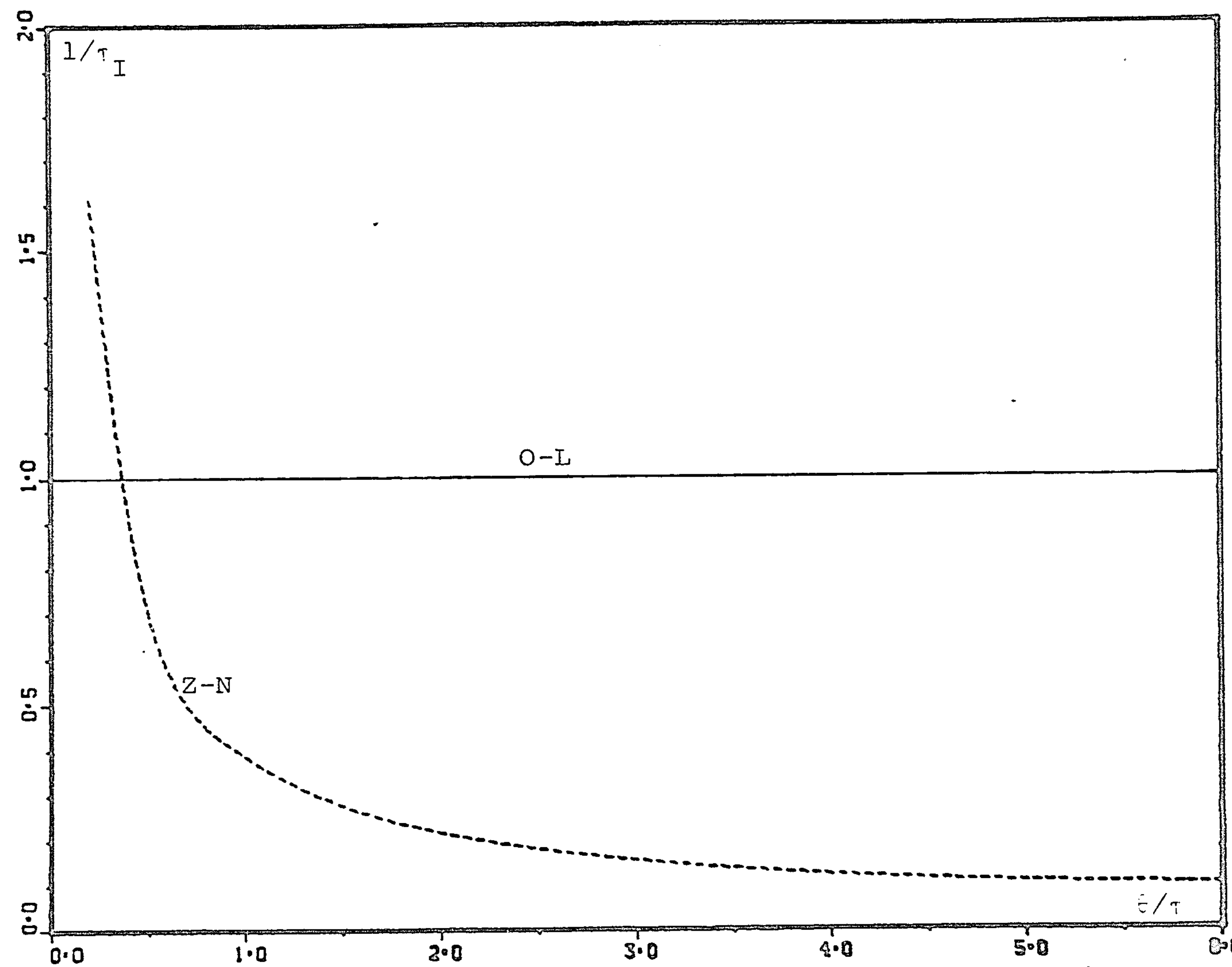


Figure 34. τ_I as Function of θ/τ for Tunings Lying on the O-L (PI+DTC) and for Z-N Tunings for PI.

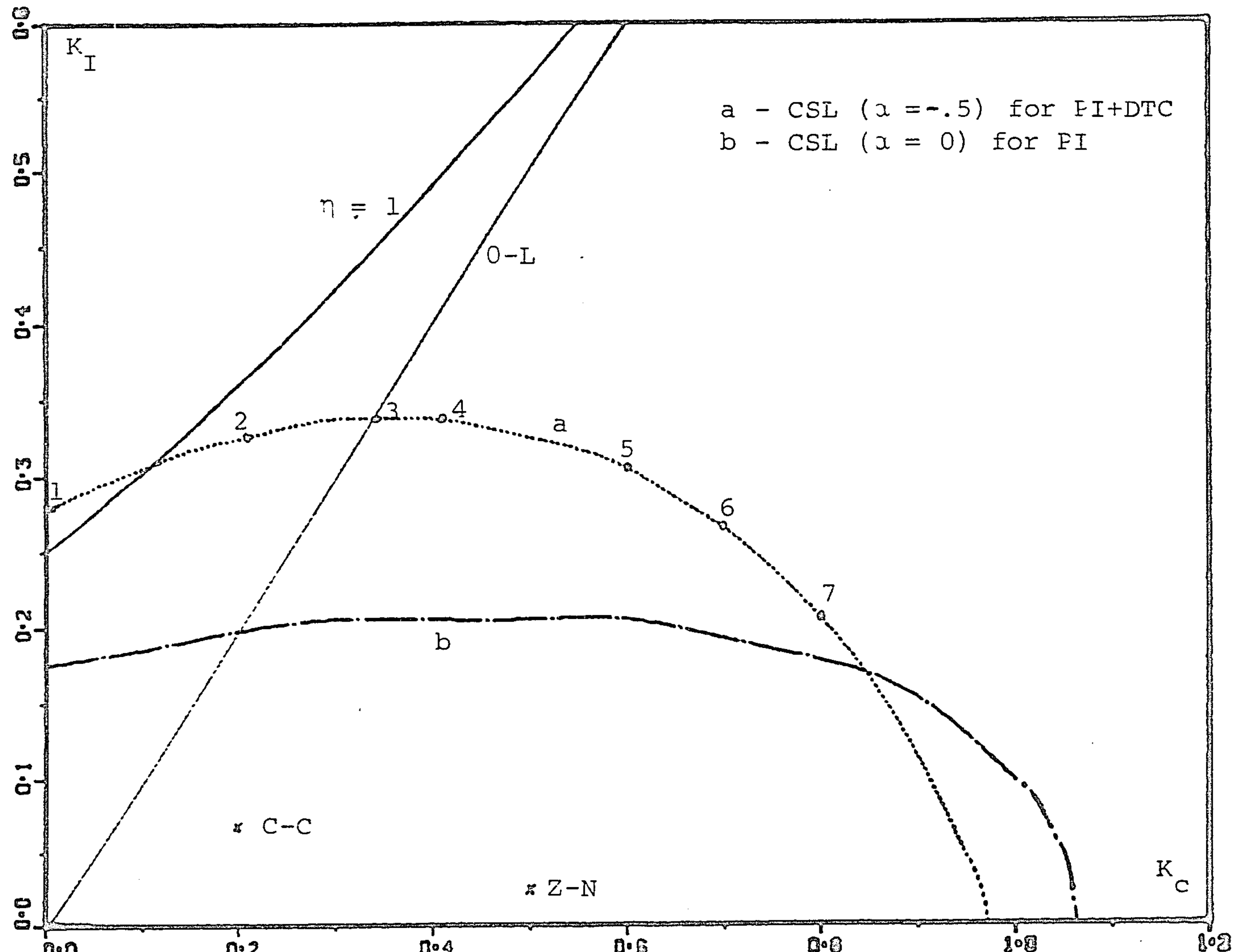


Figure 35a. CSL ($\alpha = -.5$) for PI+DTC and $G_p(s) = e^{-8s}/(s+1)$

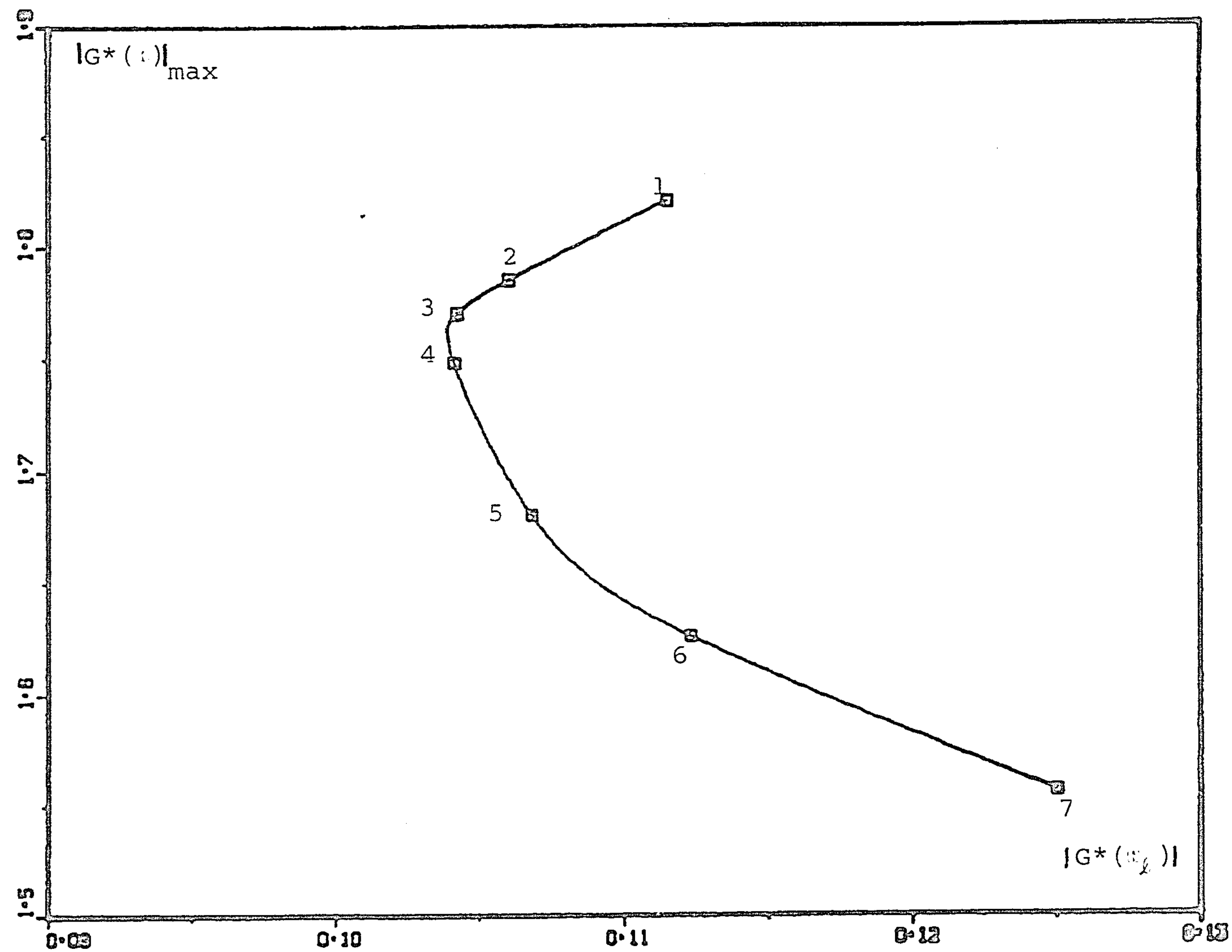


Figure 35b. The Mini-Max Plot for Settings on the CSL (α = -.5) ($G_p(s) = e^{-8s}/(s+1)$).

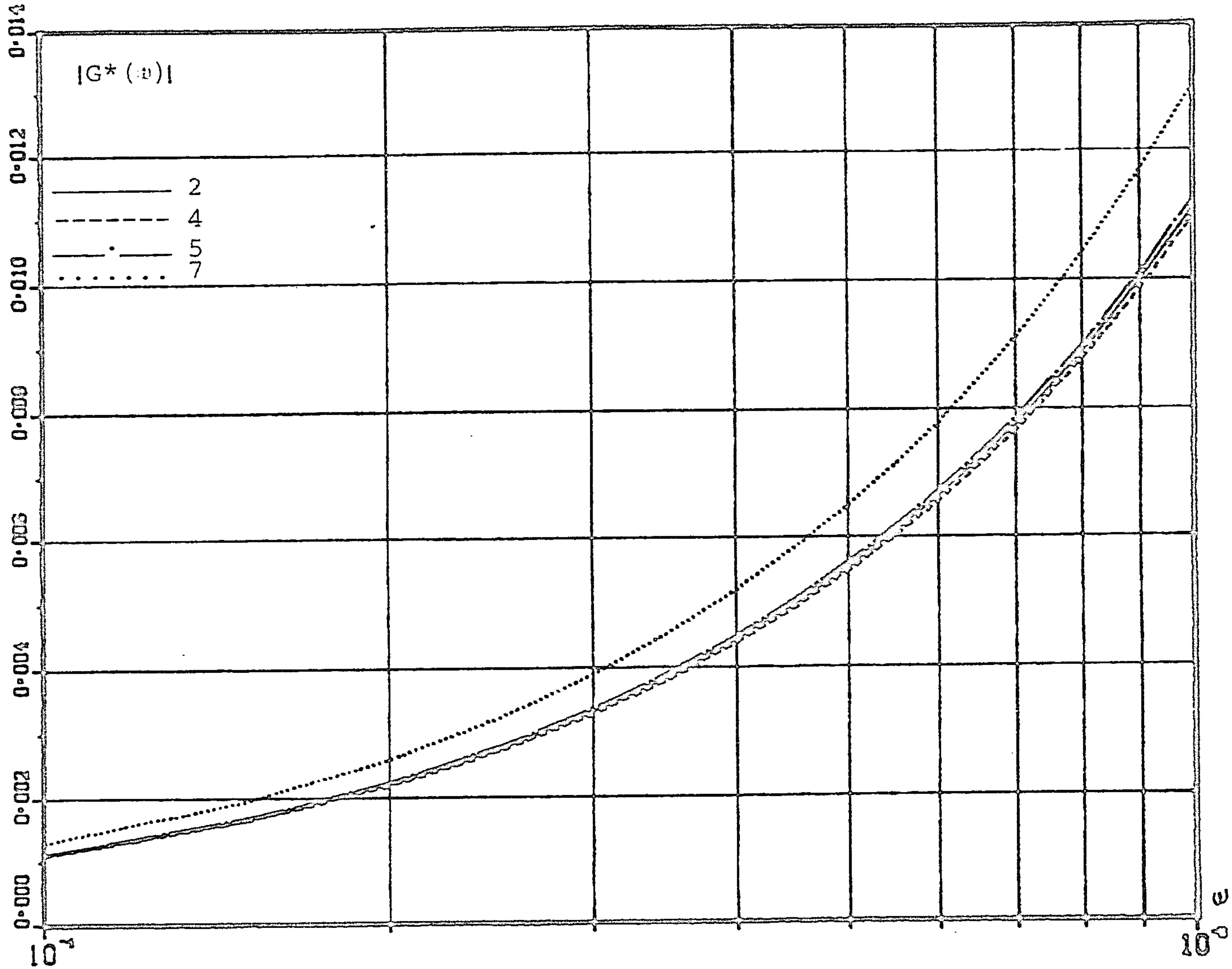


Figure 35c. Closed Loop Frequency Response of Four PI+DTC Controllers on the CSL (α = -.5). The Different Numbers Correspond to Settings on the CSL in Figure 35a.

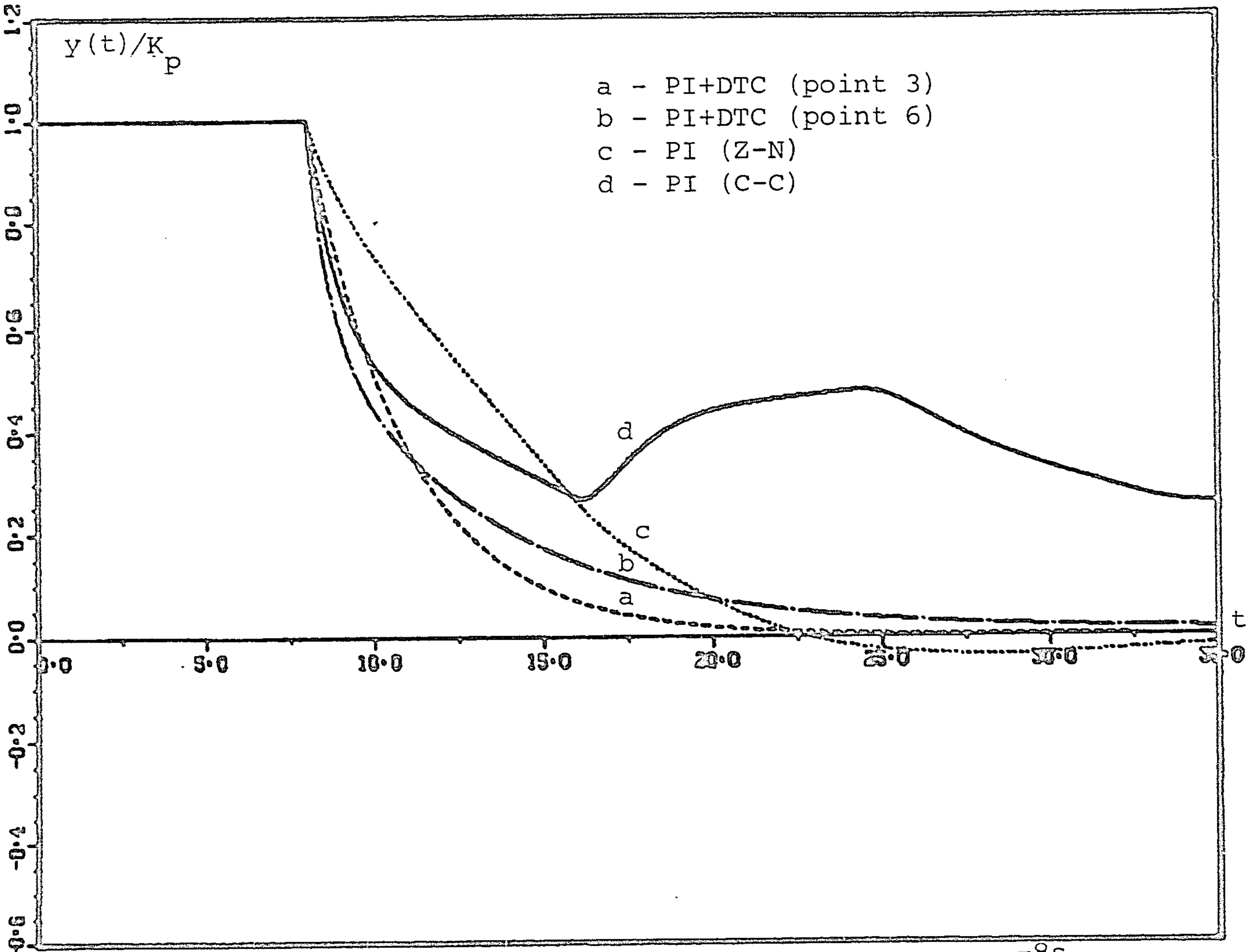


Figure 35d. Responses to Unit Step Change in Set-Point ($G_p(s) = e^{-8s}/(s+1)$) (Exact Parameters).

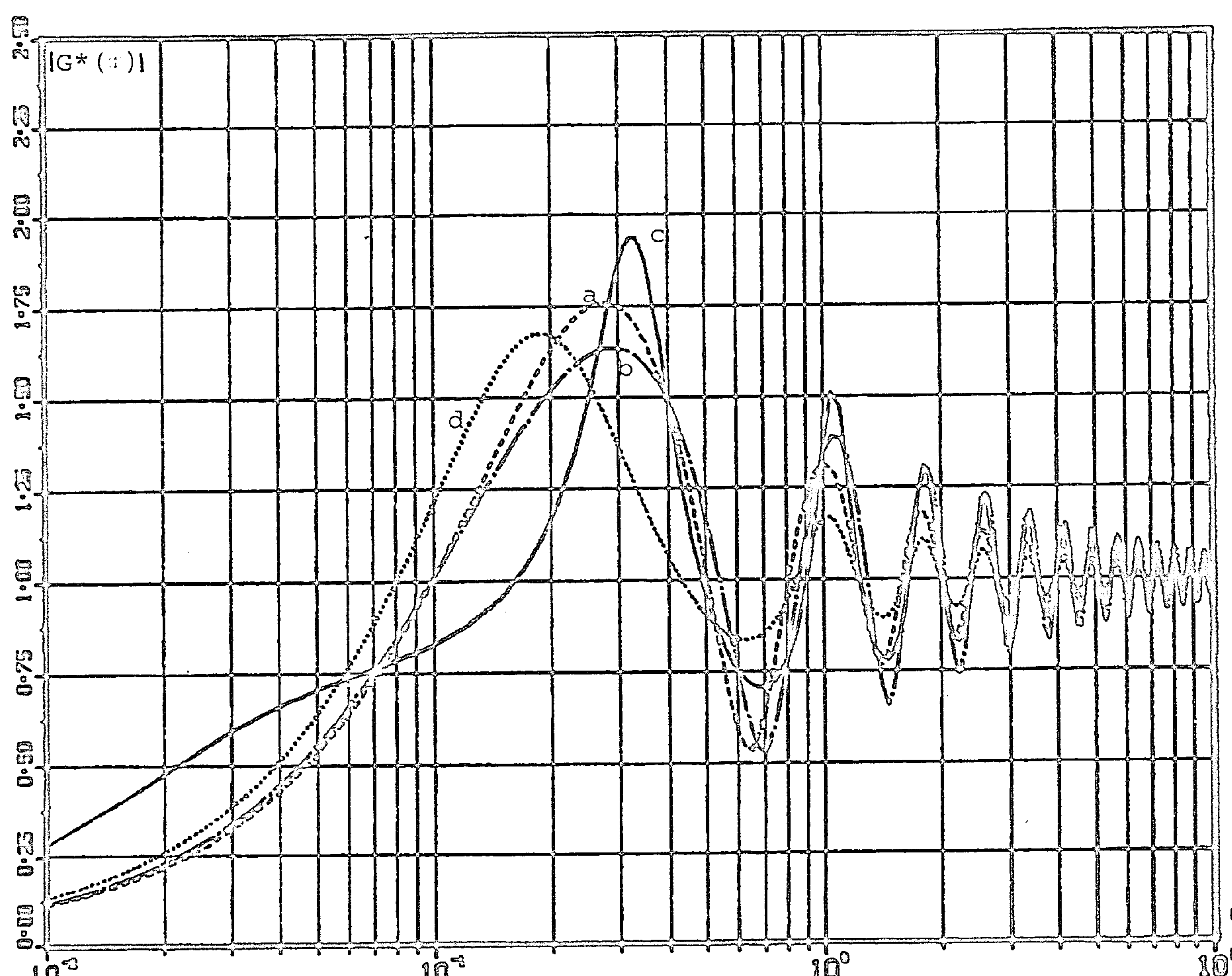


Figure 35e. Closed Loop Frequency Response, Exact Parameters (Example 1). Notation as in Figure 35d.

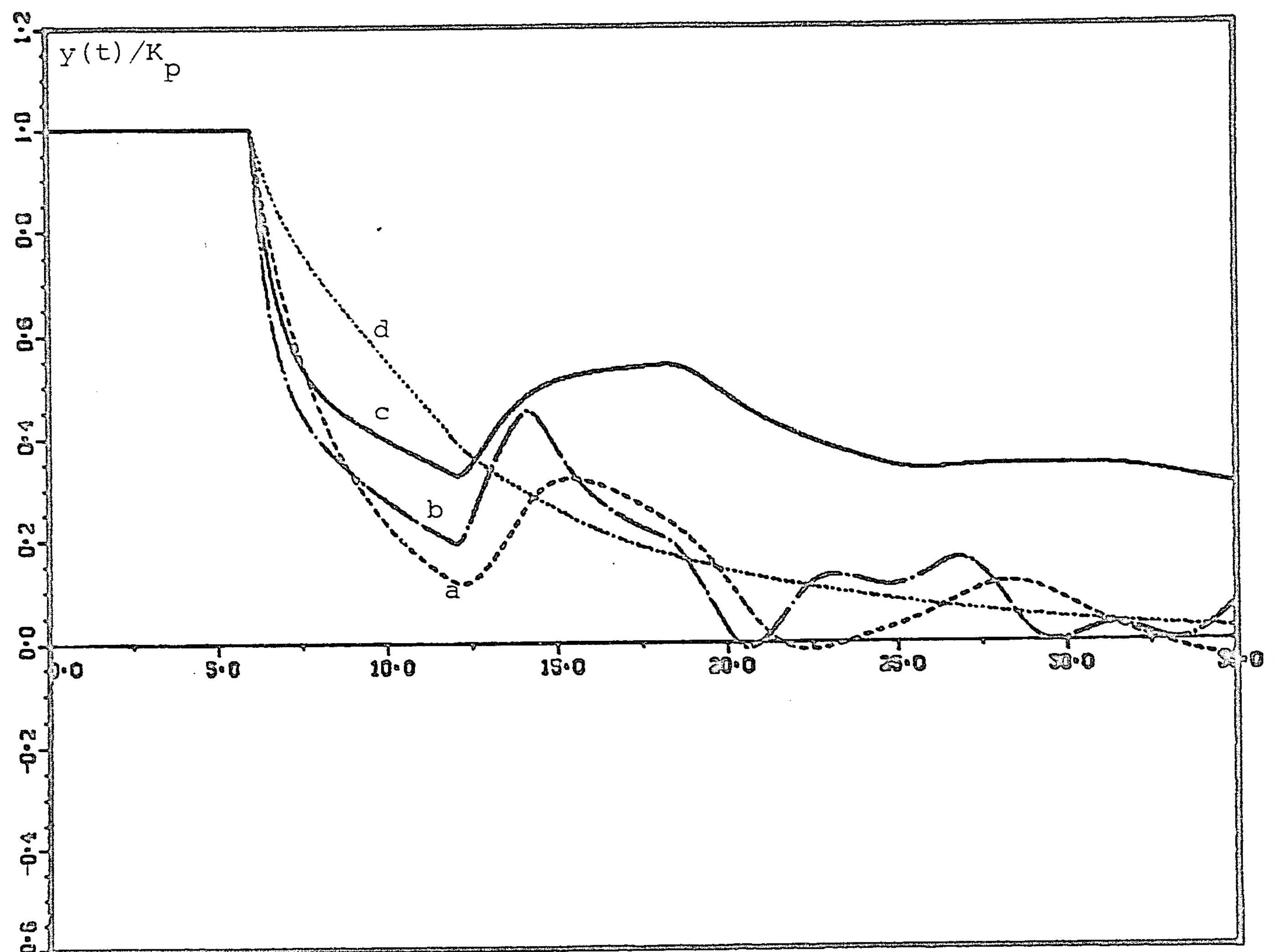


Figure 35f. Response to Unit Step Change in Set-Point, Disturbed Parameters ($\alpha = -.25$), Example 1.

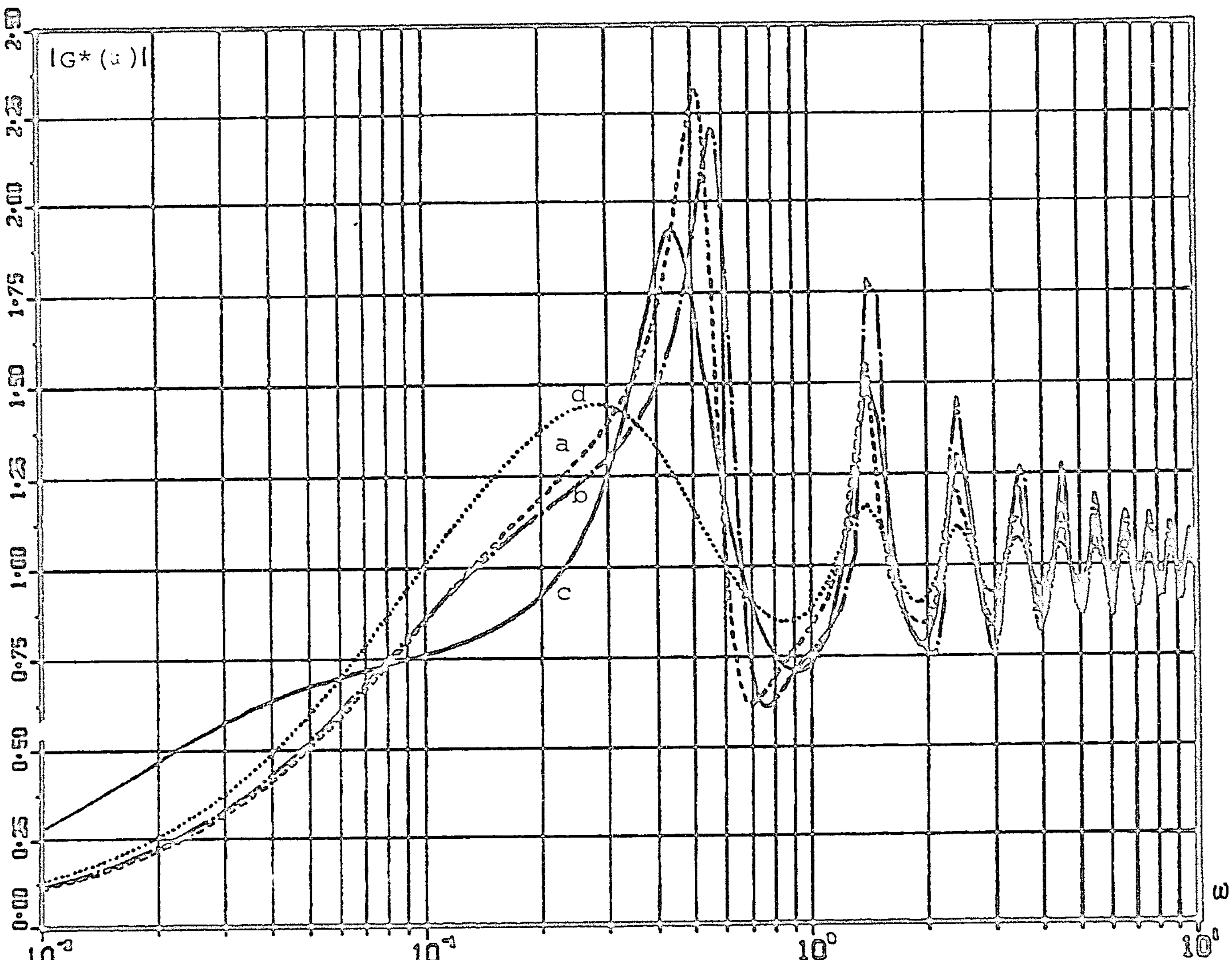


Figure 35g. Closed Loop Frequency Response. Disturbed Parameters (α = -.25), Notation as in Figure 35d.

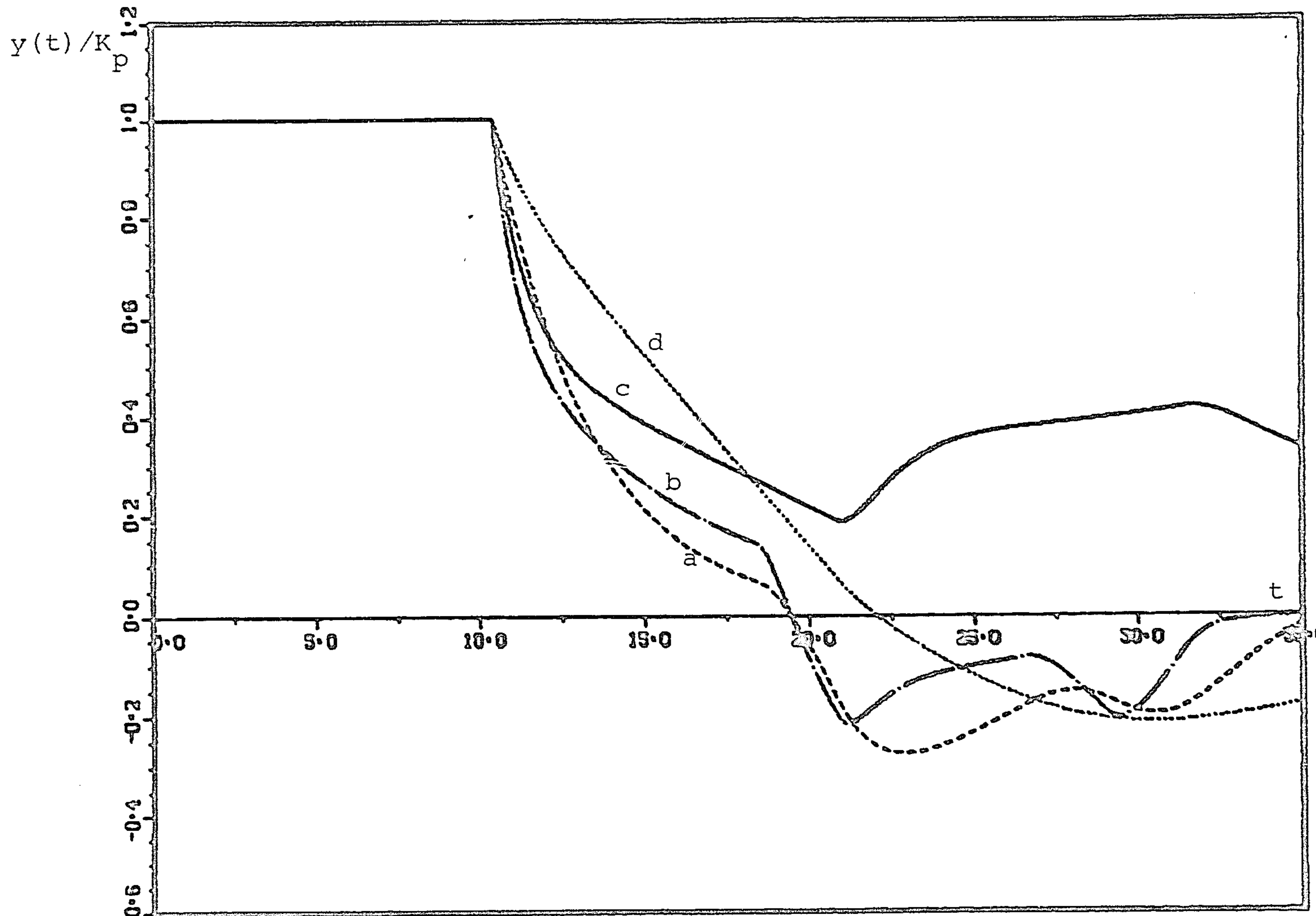


Figure 35h. Response to Unit Step in Set-Point. Disturbed Parameters (β = .3), Example 1.

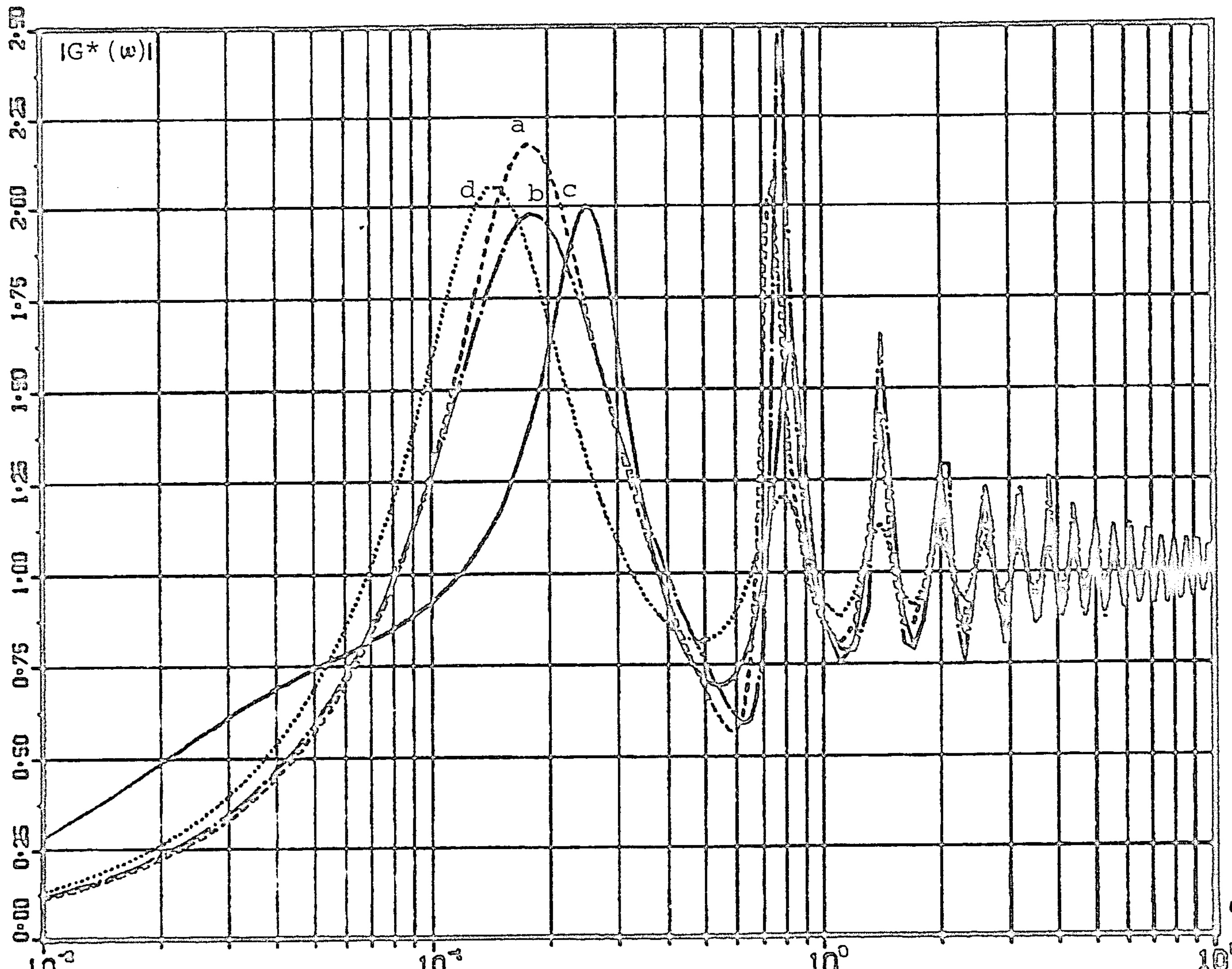


Figure 35i. Closed Loop Frequency Response. Disturbed Parameters (β = .3), Example 1.

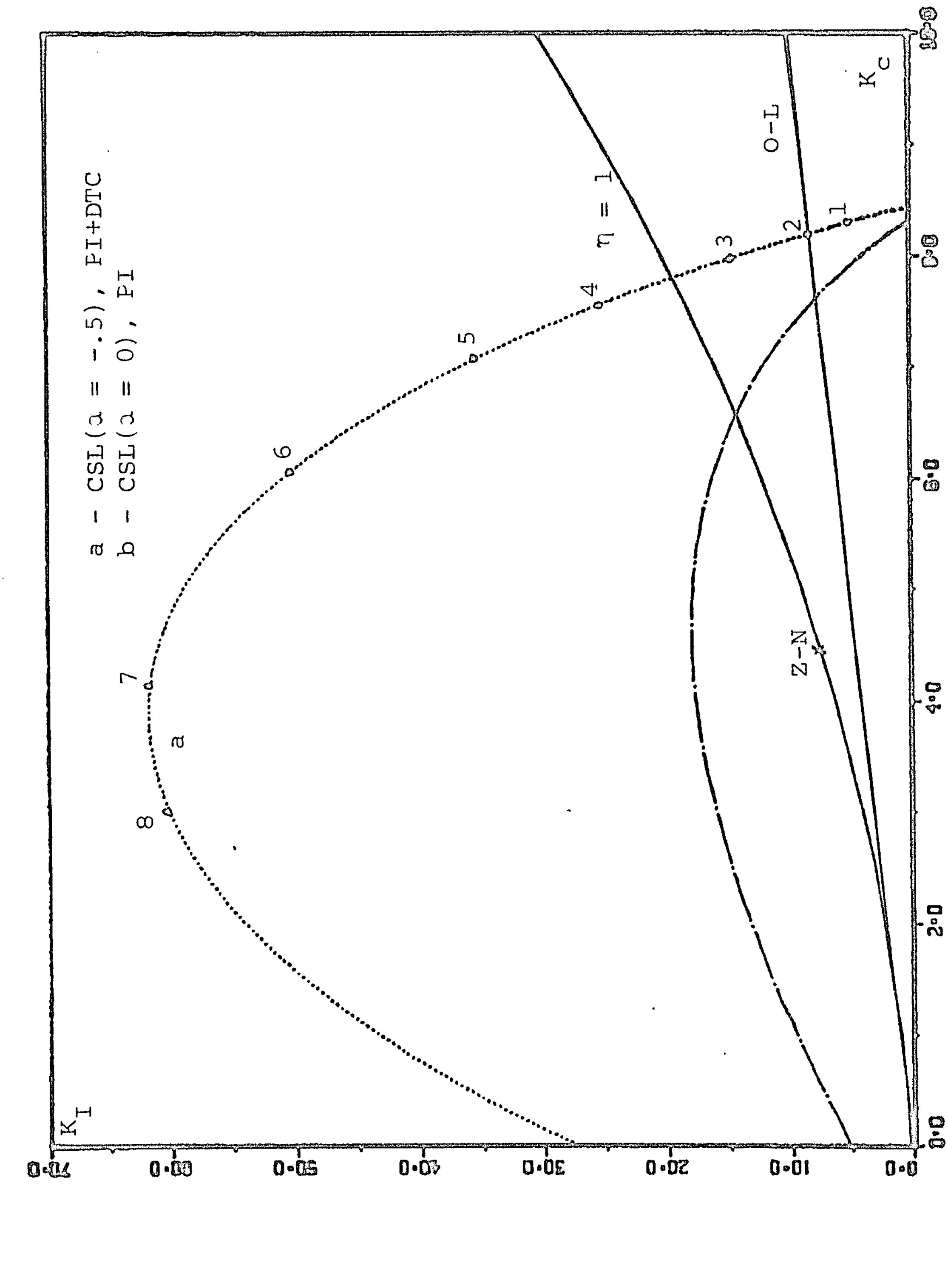


Figure 36a. CSL(α = -.5) for FI+DTC and $G_p(s) = e^{-.2s}/(s+1)$ (Example 2).

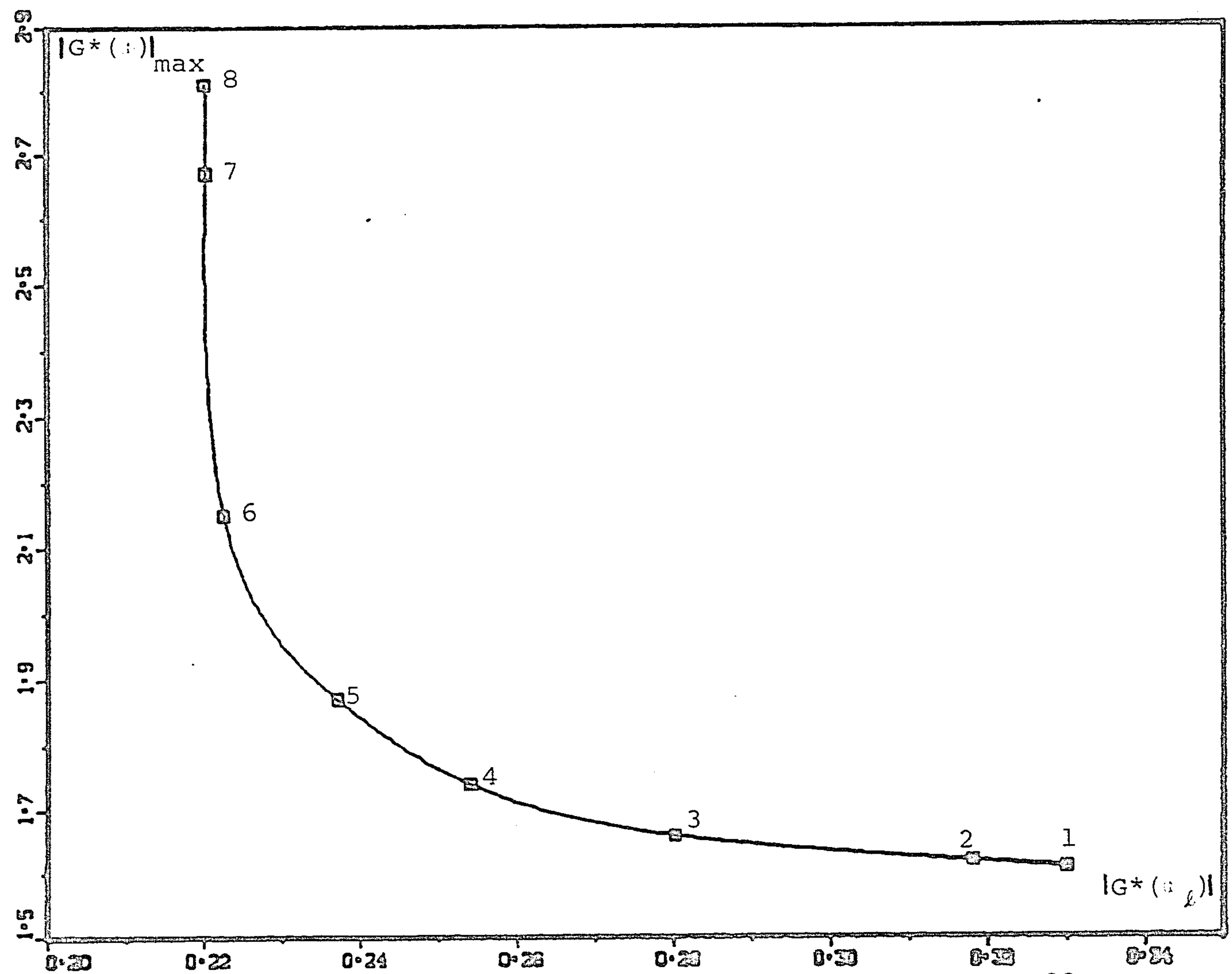


Figure 36b. The Mini-Max Plot for Settings on the CSL ($G_p(s) = e^{-.23}/(s+1)$, Example 2).

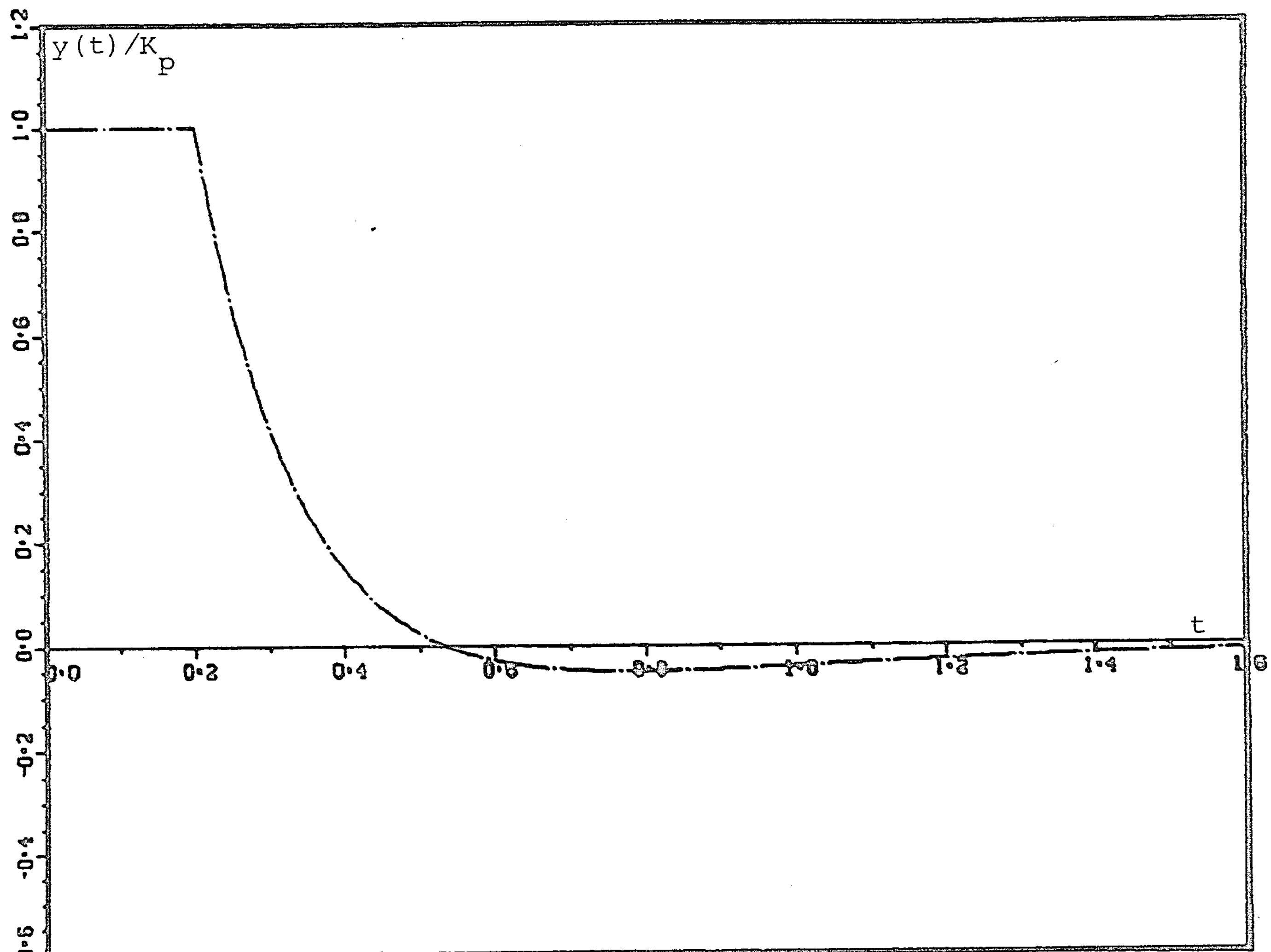


Figure 36c. Response to Unit Step Change in Set-Point of the Controller Represented by Point 3 on the CSL (Figure 36a).

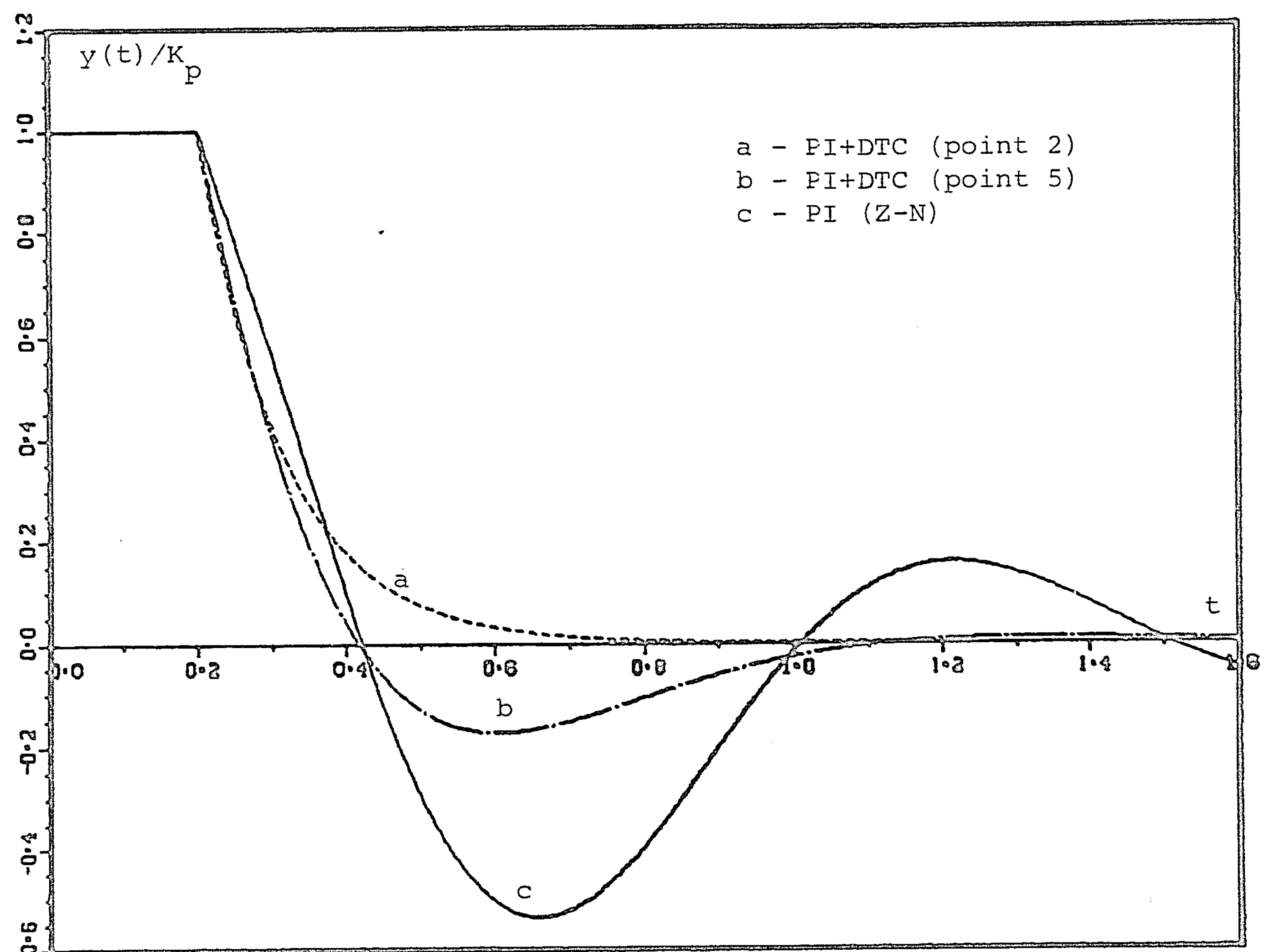


Figure 36d. Response to Unit Step Change in Set-Point, Exact Parameters
$G_p(s) = e^{-.2s}/(s+1)$, Example 2.

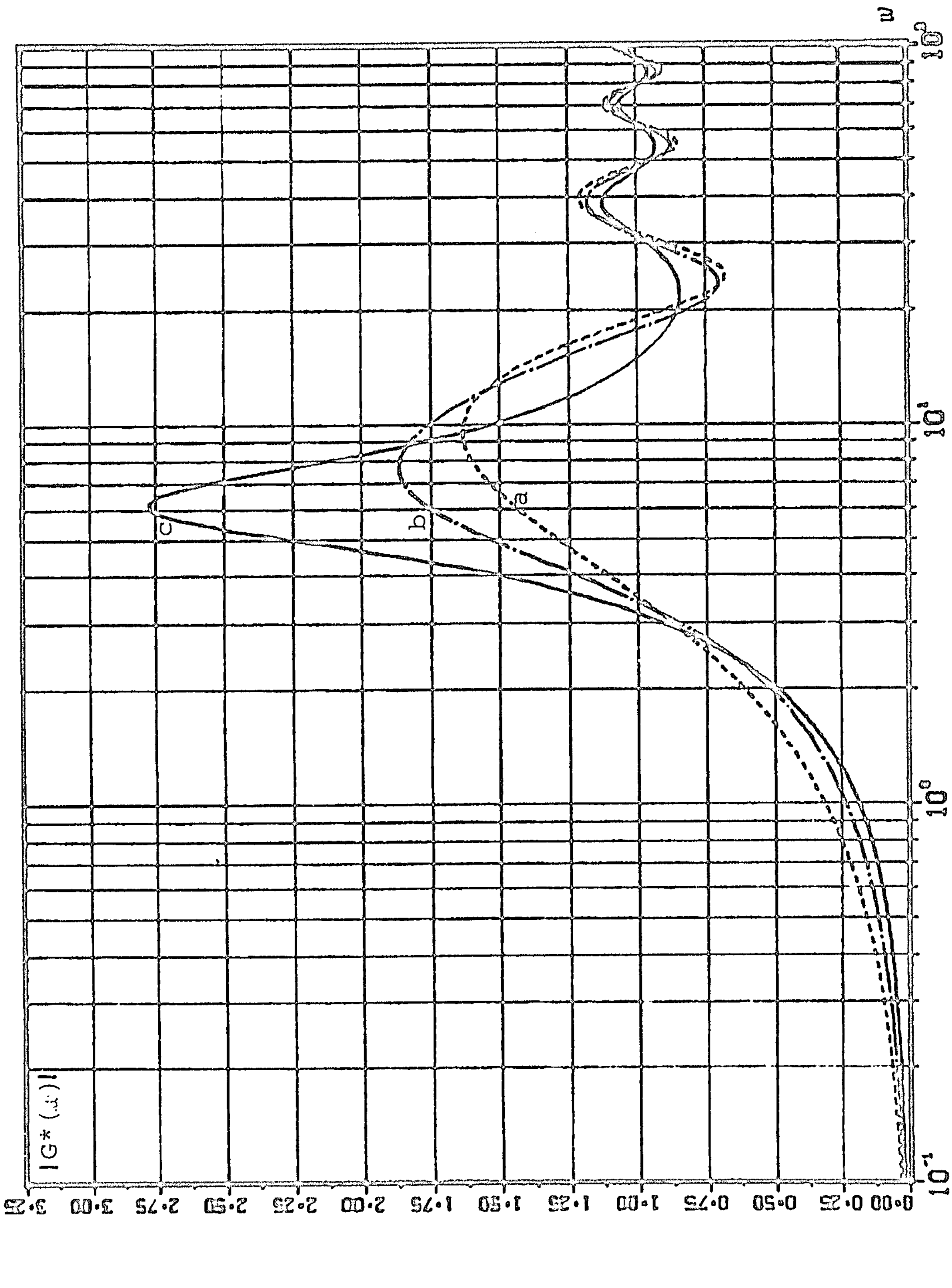


Figure 36e. Closed Loop Frequency Response. Exact Parameters, Example 2.

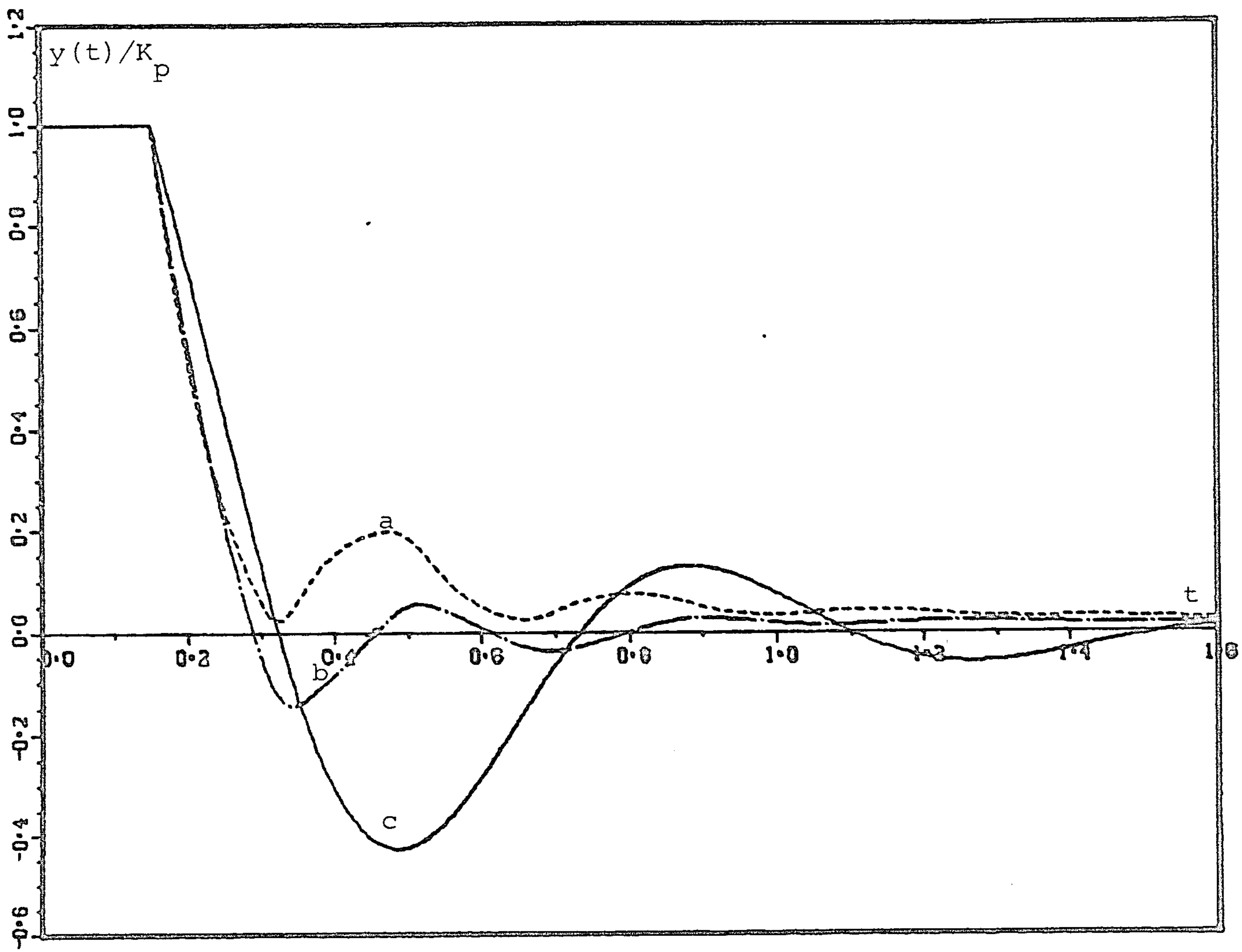


Figure 36f. Response to Unit Change in Set-Point. Disturbed Parameters ($\alpha = -.25$). Example 2. Notation as in Figure 36d.

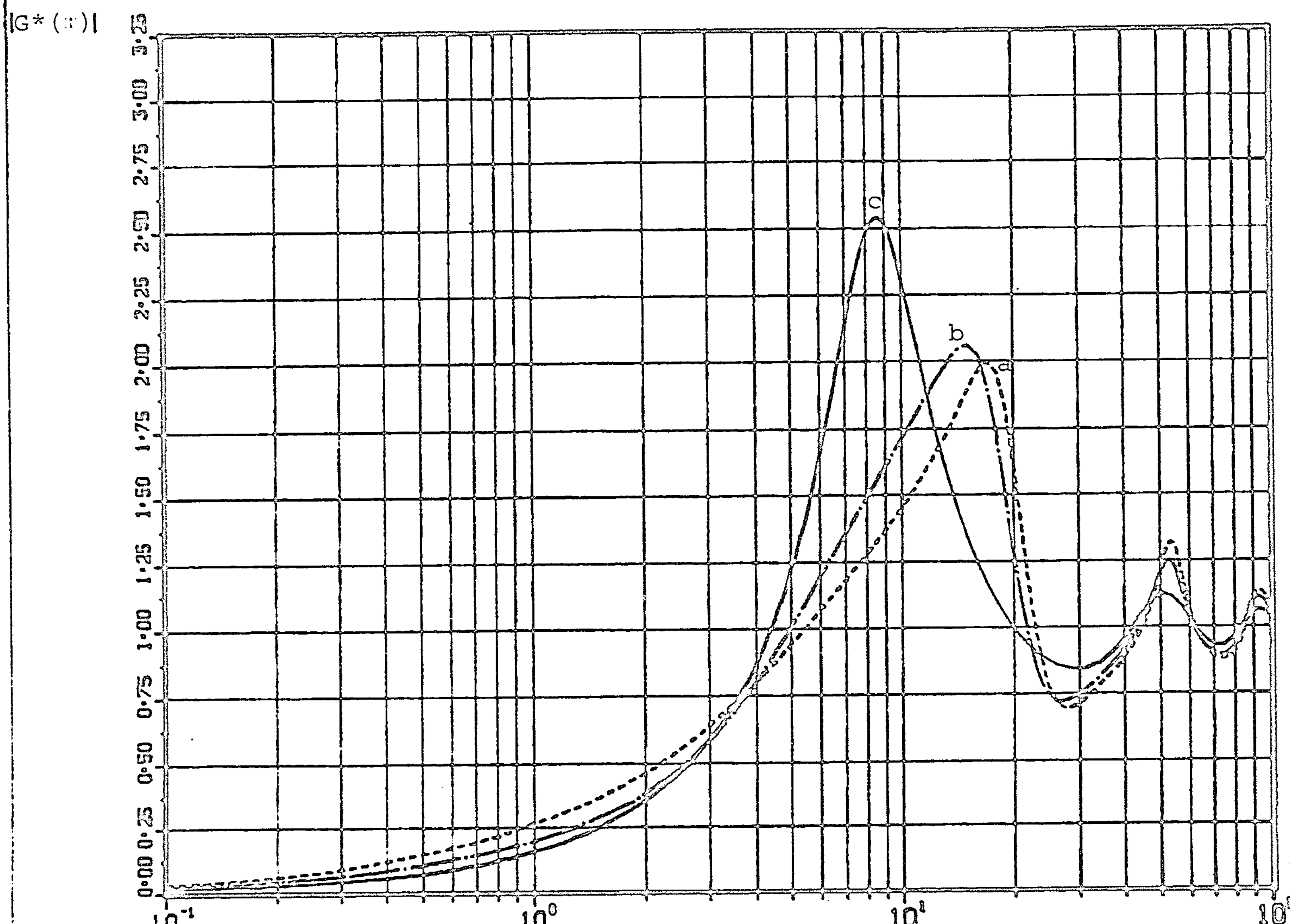


Figure 36g. Closed Loop Frequency Response. Disturbed Parameters (α = -.25). Example 2.

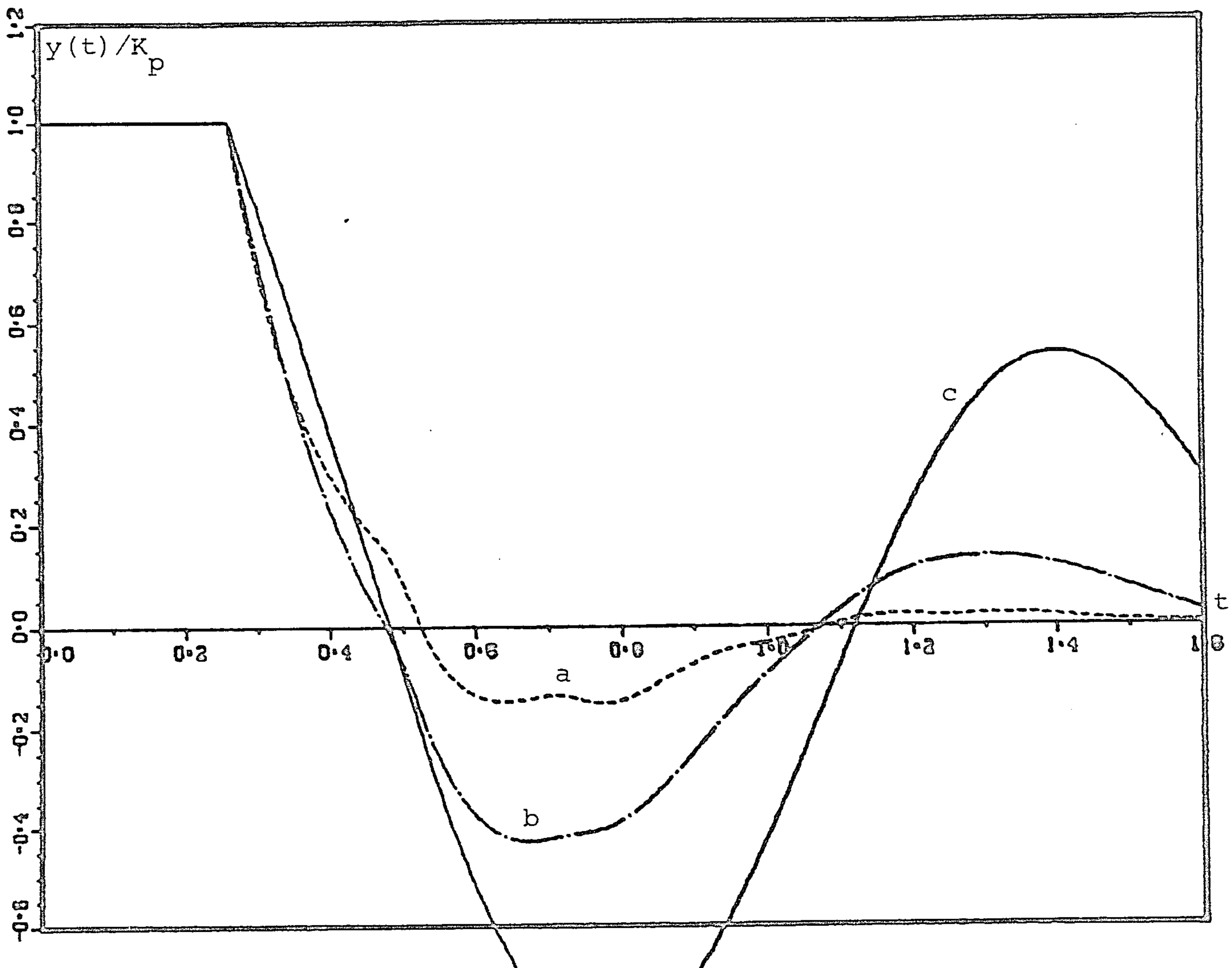


Figure 36h. Response to Unit Step Change in Set-Point. Disturbed Parameters (δ = .3). Example 2.

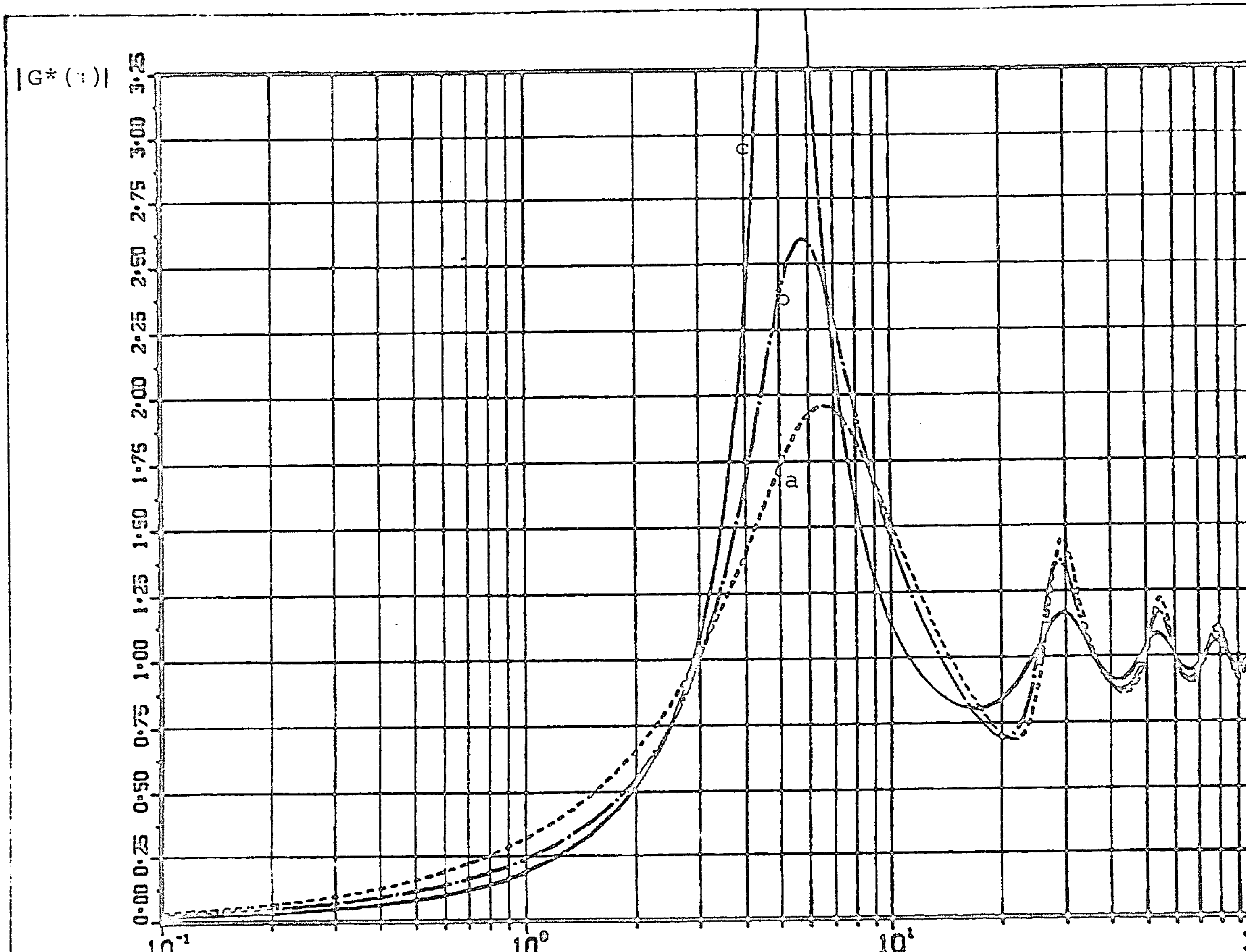


Figure 36i. Closed Loop Frequency Response. Disturbed Parameters (β = .3). Example 2.

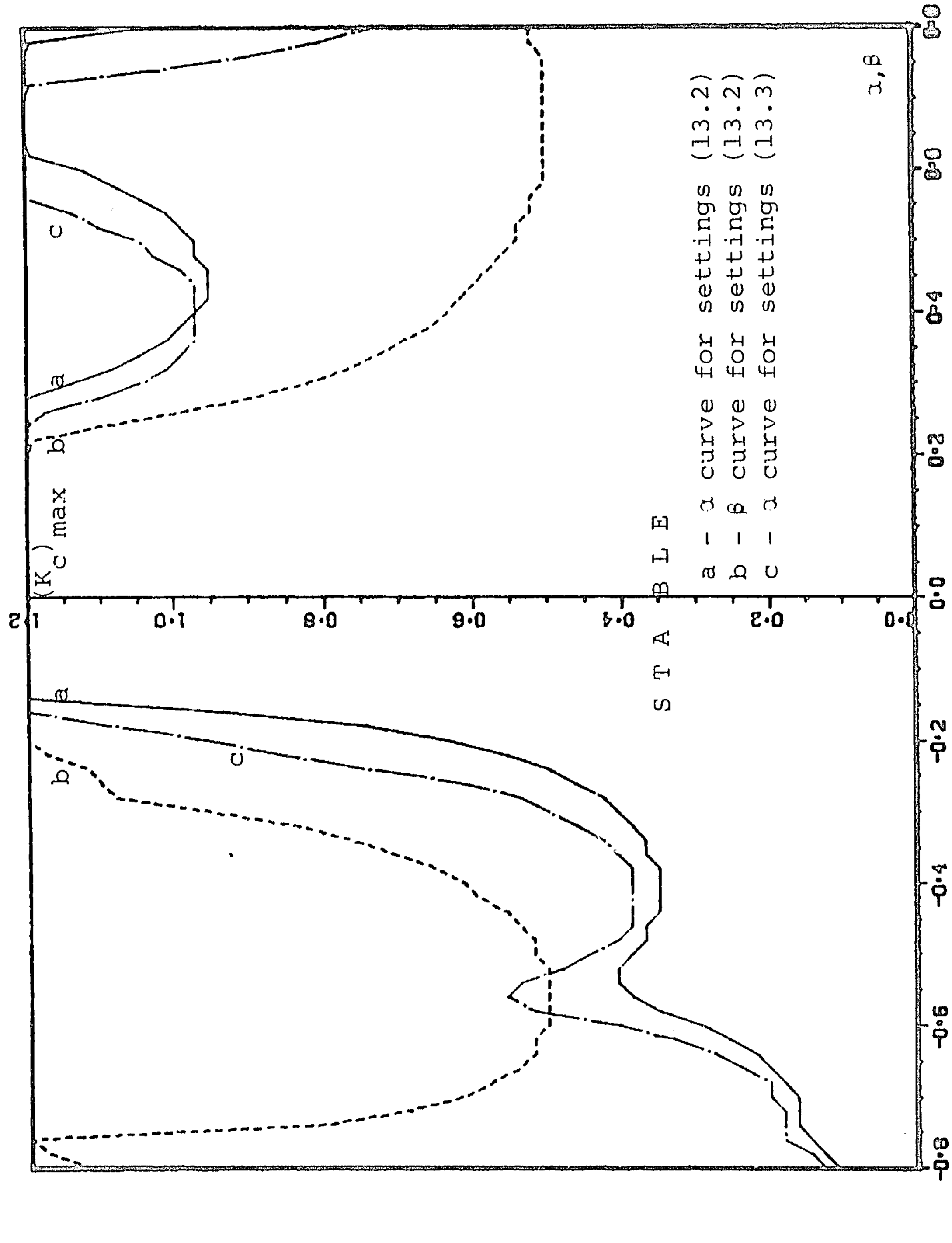


Figure 37. Maximum Gain for FID+DTC as Function of Parameter Variation.

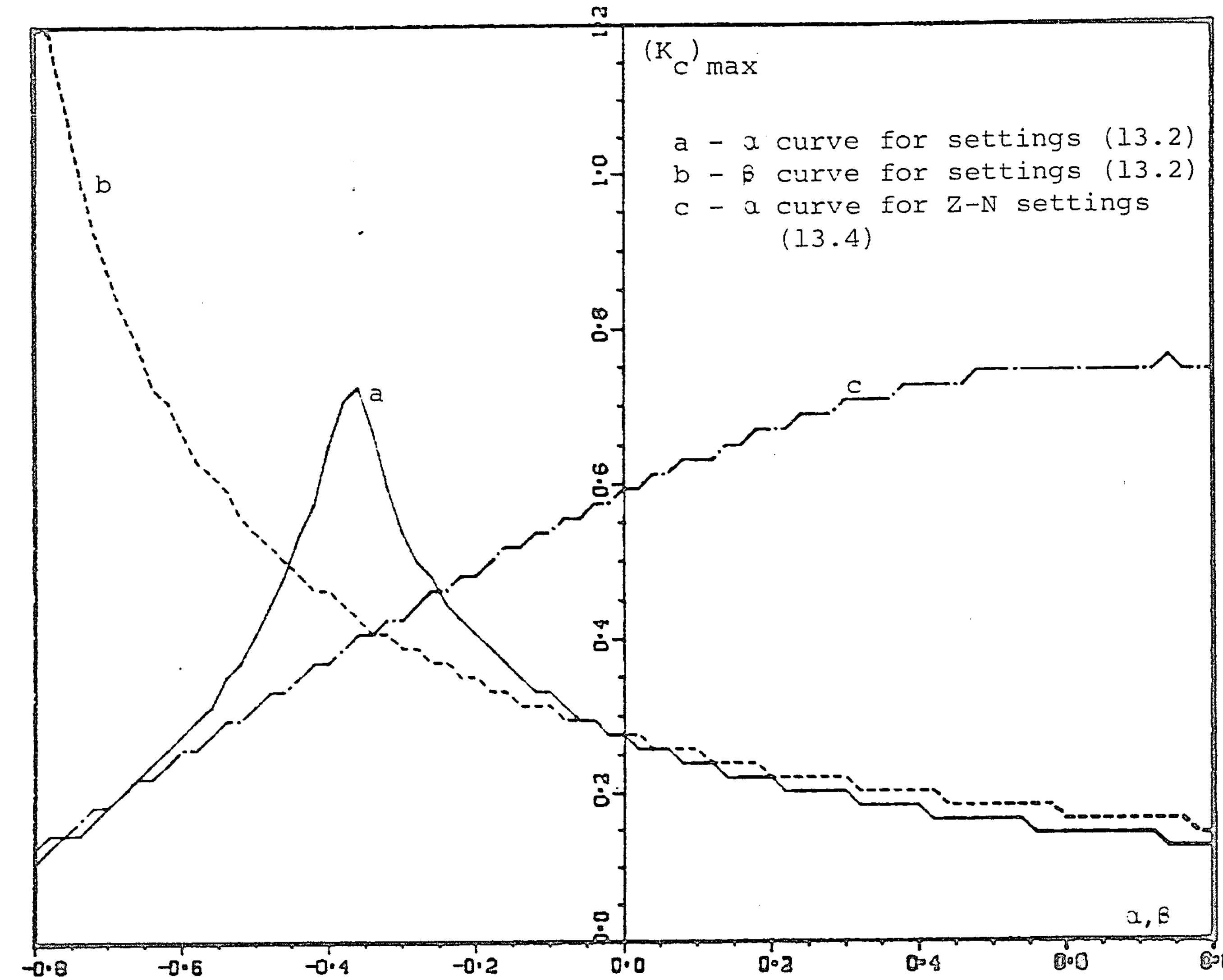


Figure 38. Maximum Gain for PID Controllers as Function of Parameter Variations.

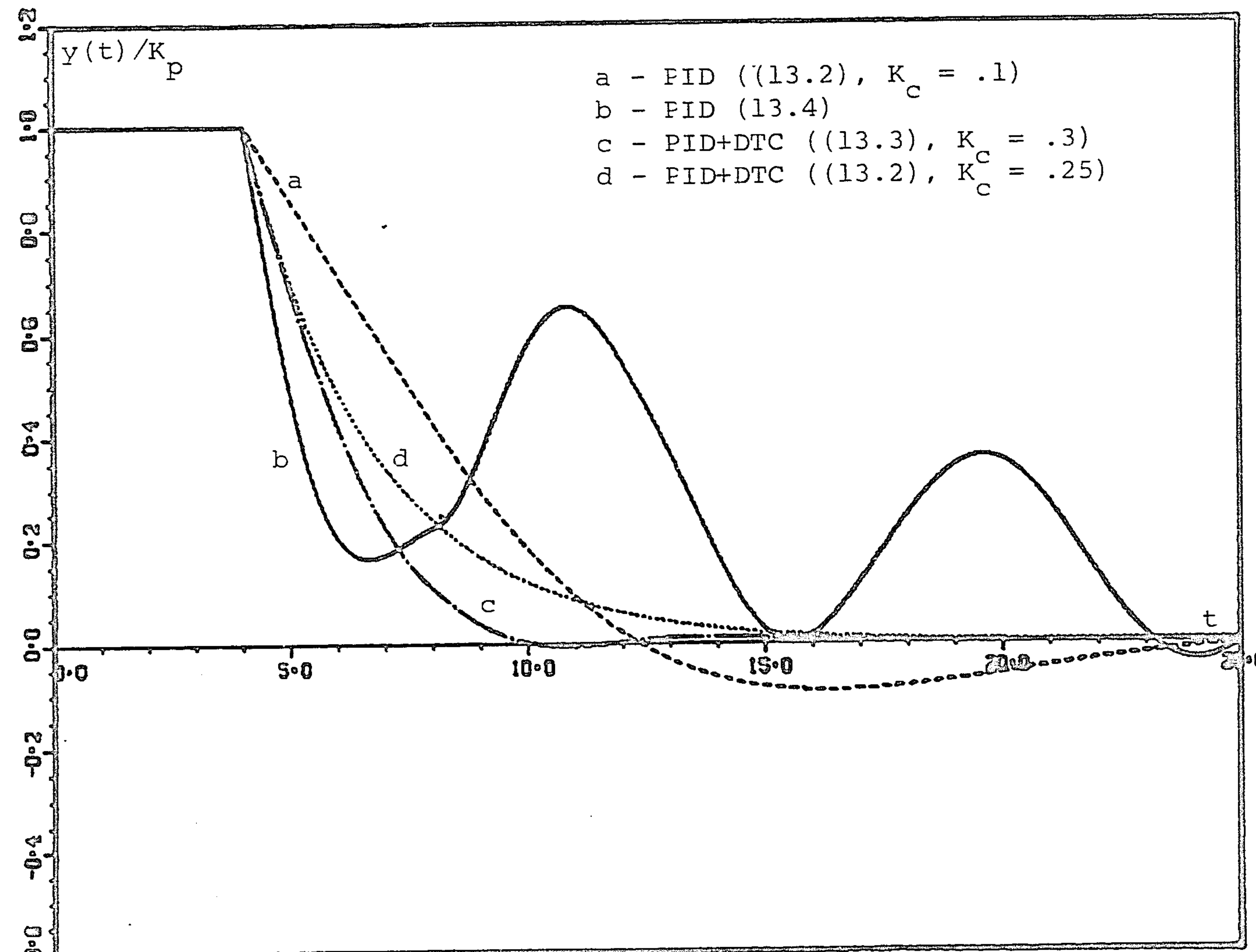


Figure 39a. Time Responses to Step Changes in Set-Point. Exact Parameters (Example 3).

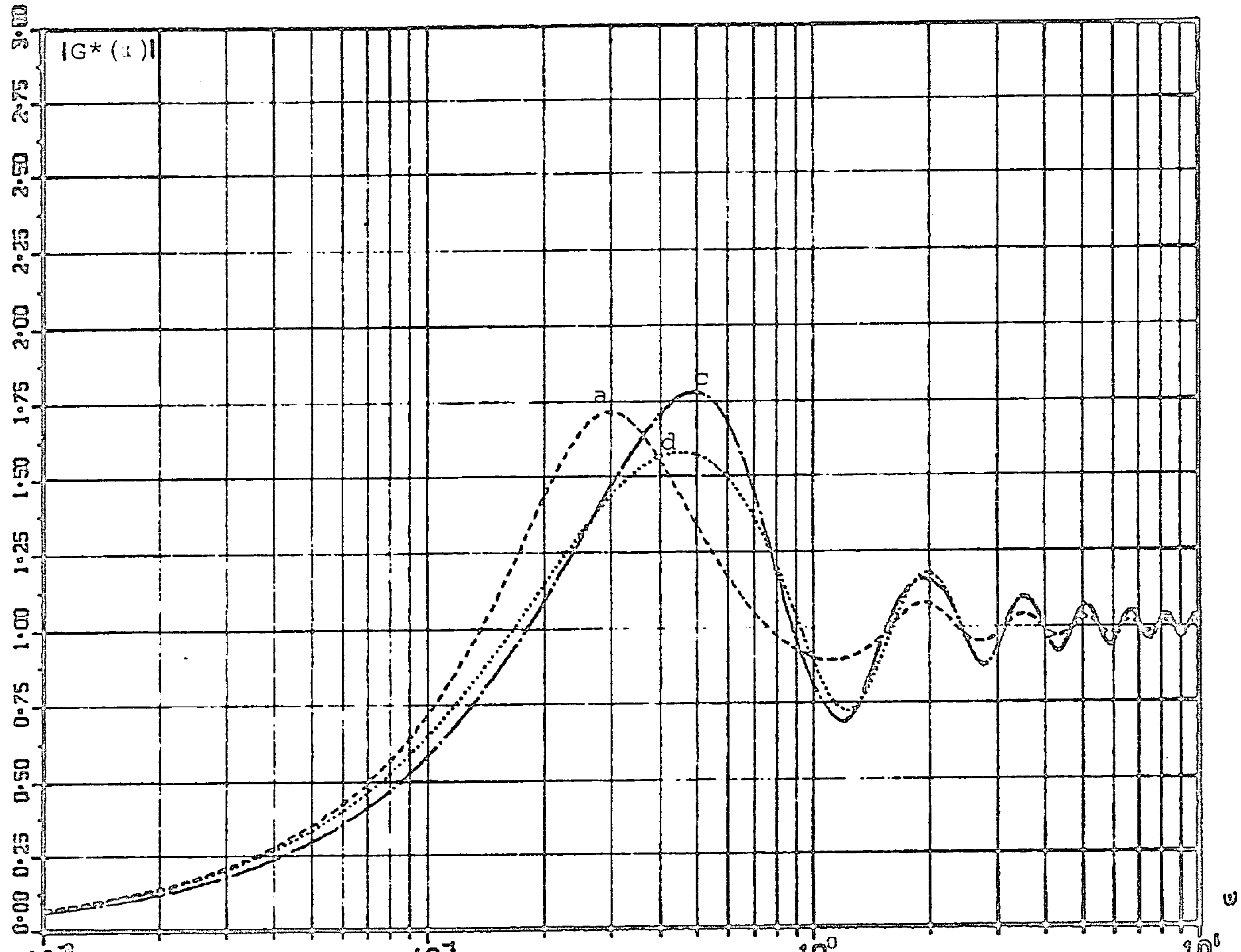


Figure 39b. Closed Loop Frequency Response. Exact Parameters.
(Process Given in Example 3.) For Notation see Figure 39a.

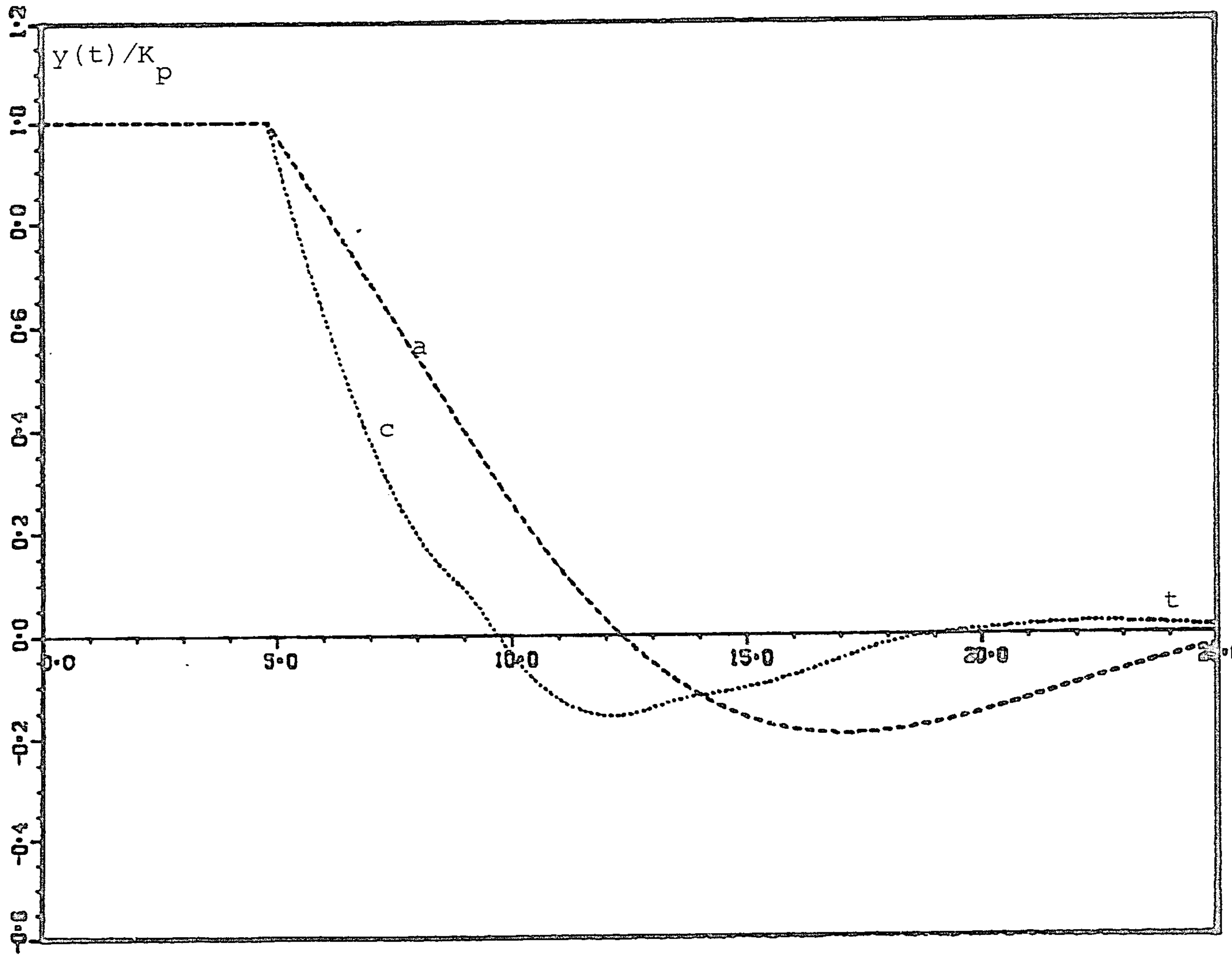


Figure 40a. Response to Unit Step Change in Set-Point. Disturbed Parameters (β = .2). For Notation see Figure 39a.

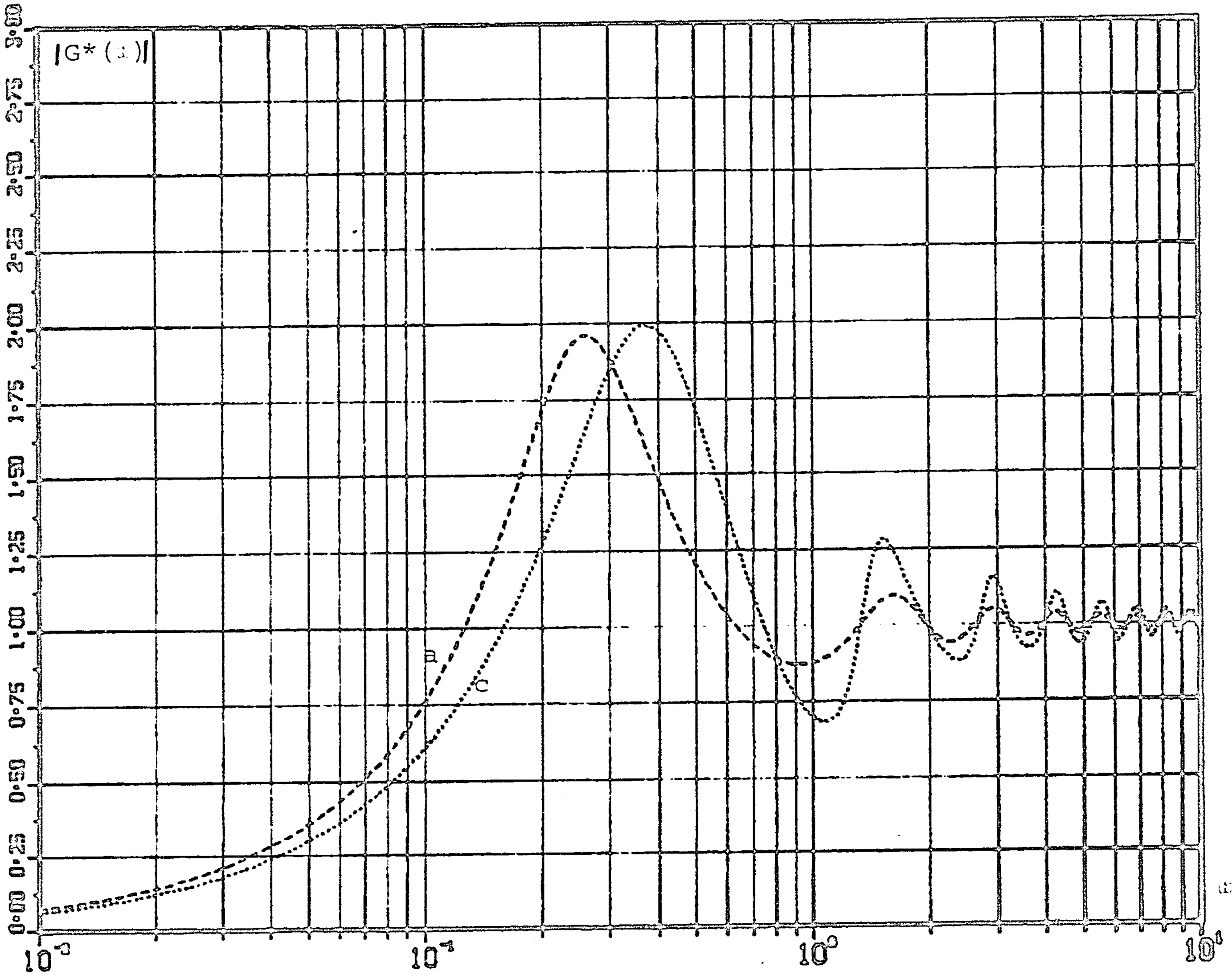


Figure 40b. Closed Loop Frequency Response. Disturbed Parameters (δ = .2).
(For Notation see Figure 39a.)

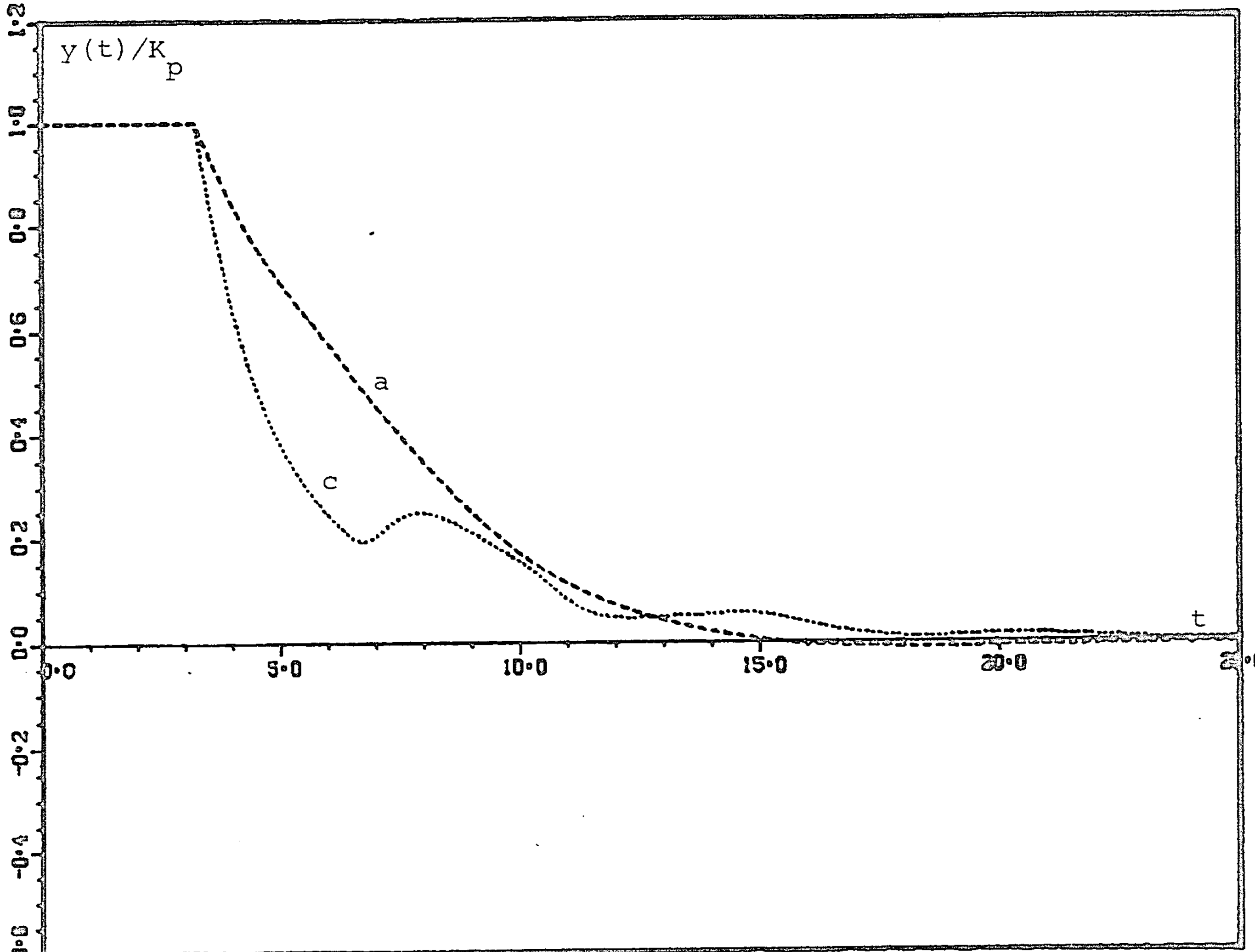


Figure 41a. Response to Unit Step Change in Set-Point. Disturbed Parameters (α = -.2). For Notation see Figure 39a.

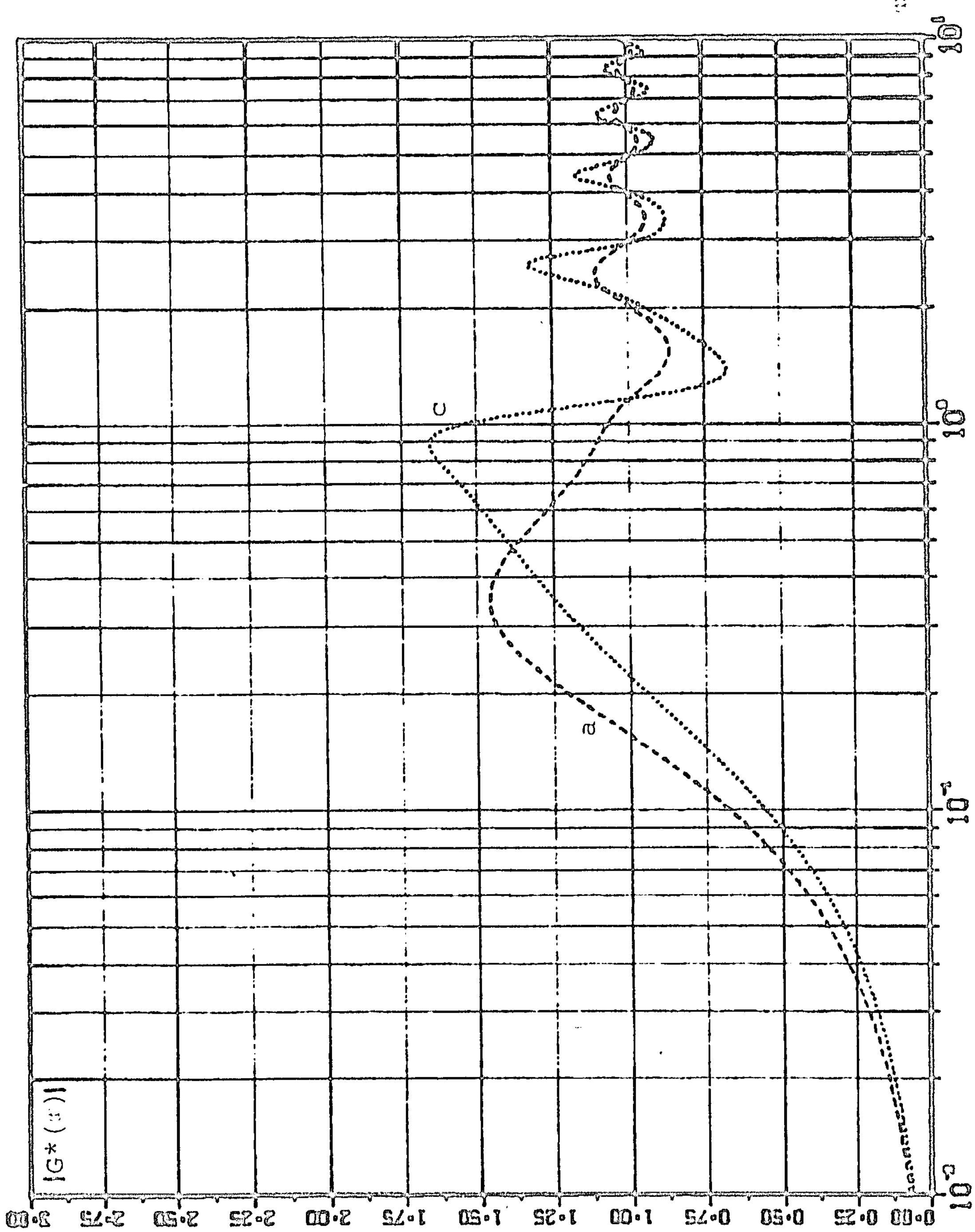


Figure 41b. Closed Loop Frequency Response. Disturbed Parameters (α = -.2)

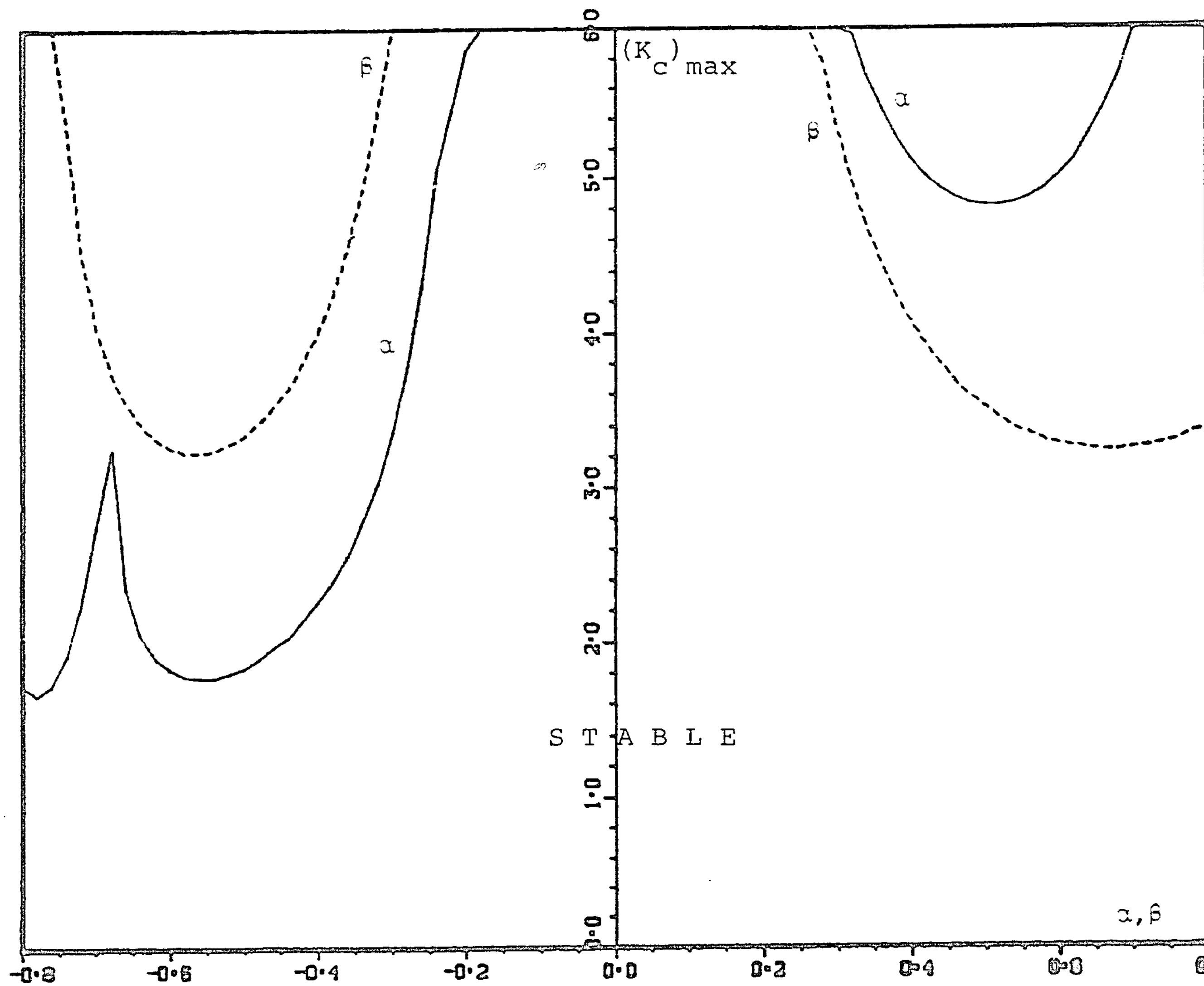


Figure 42. Stability Limits for PID+DTC in Example 4. (τ_I = 3.33, τ_D = .3)

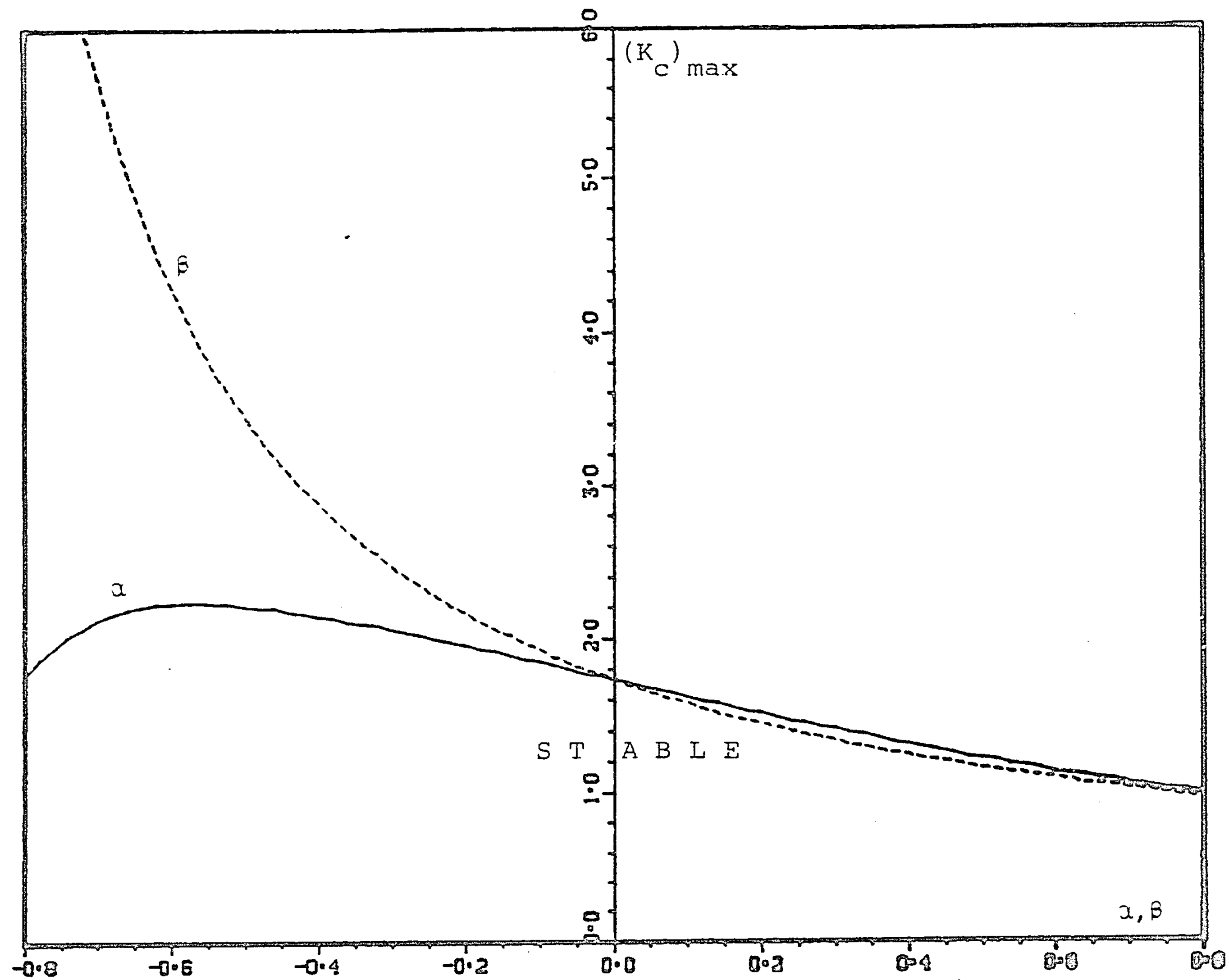


Figure 43. Stability Limits for the FID Controller in Example 4.
(τ_I = 3.333, τ_D = .3)

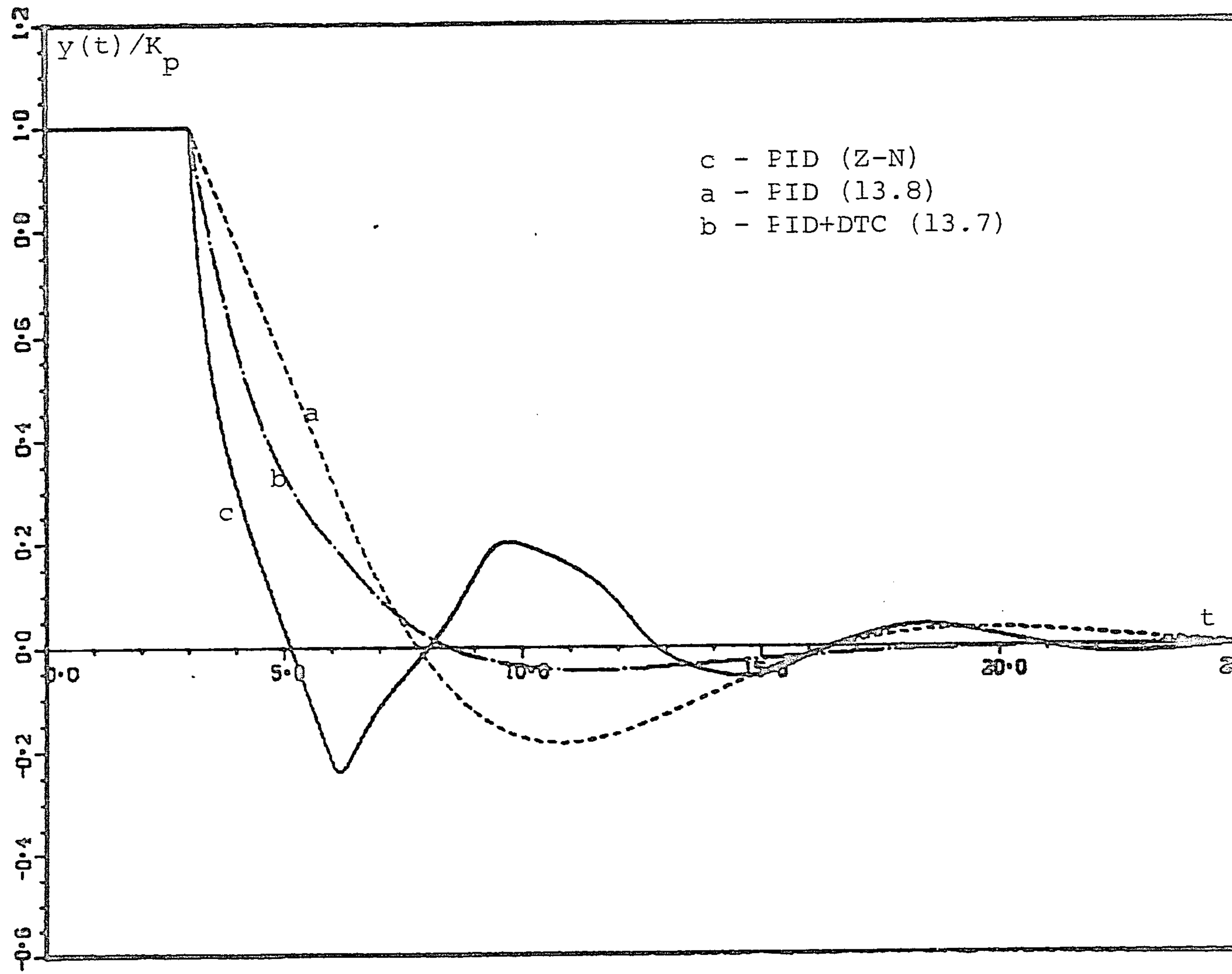


Figure 44a. Response to Unit Step Change at the Set-Point for Example 4 (Exact Parameters).

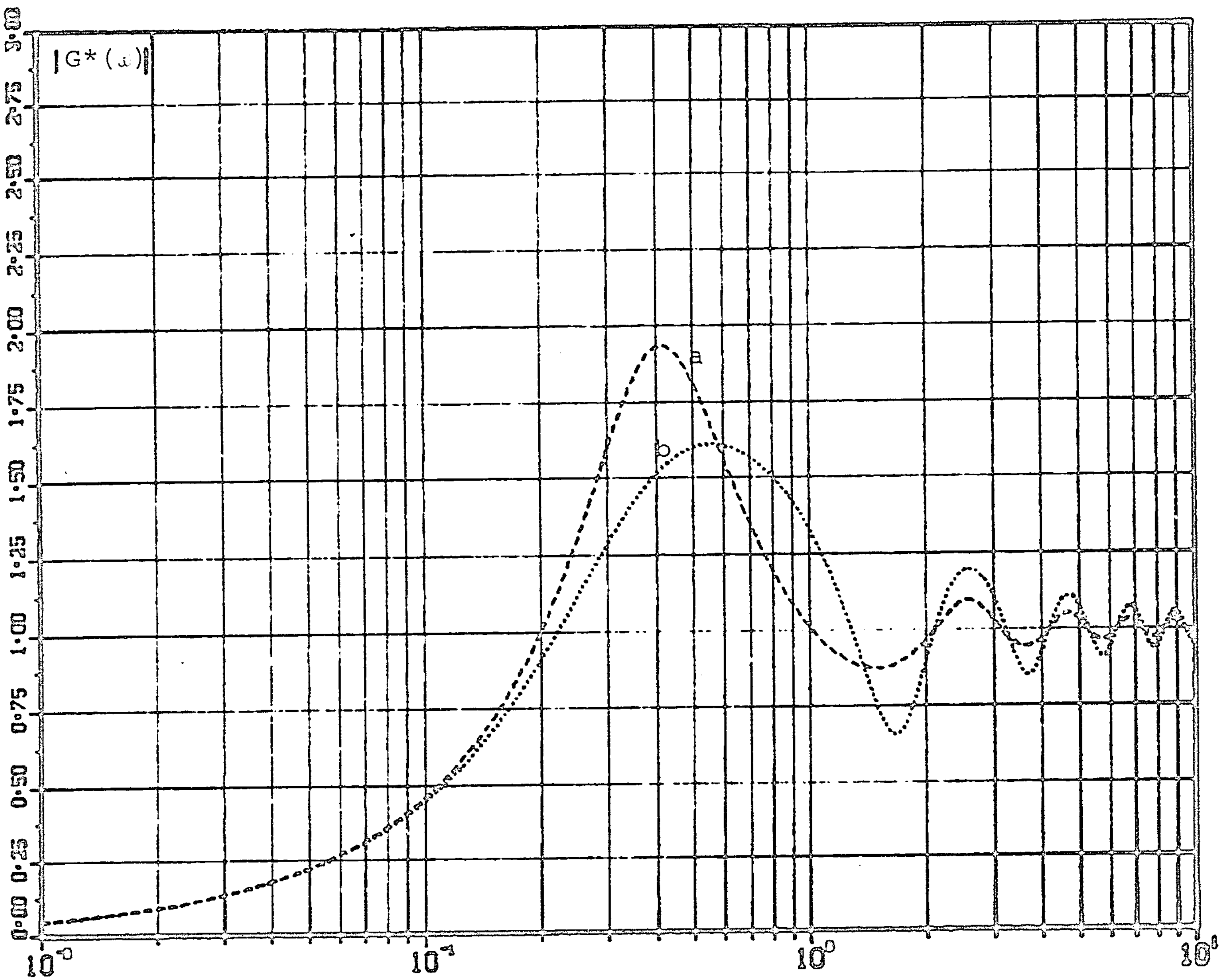


Figure 44b. Frequency Response of the Closed Loops for Example 4 (Exact Paremeters)(Notation as in Figure 44a).

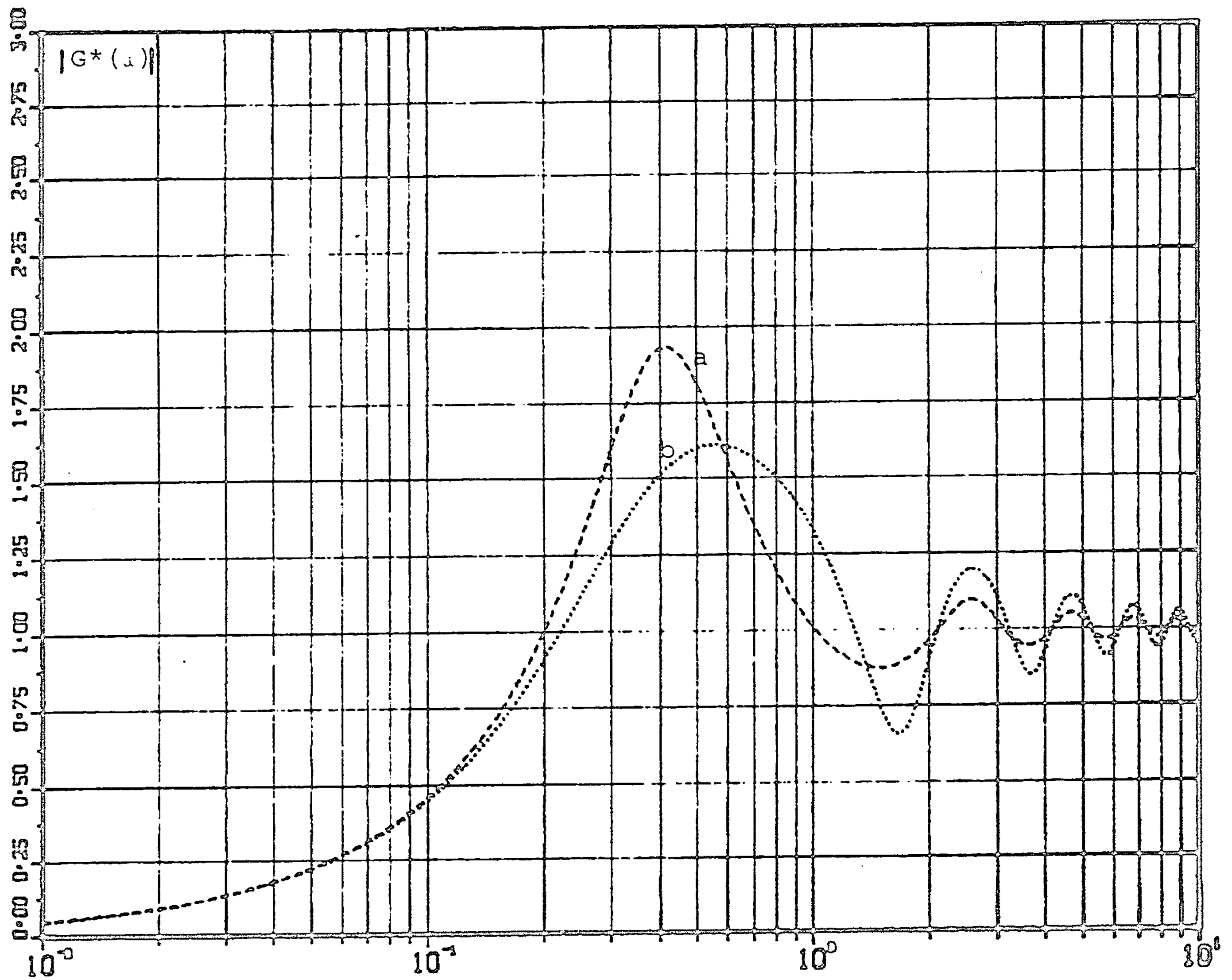


Figure 44b. Frequency Response of the Closed Loops for Example 4 (Exact Paremeters) (Notation as in Figure 44a).

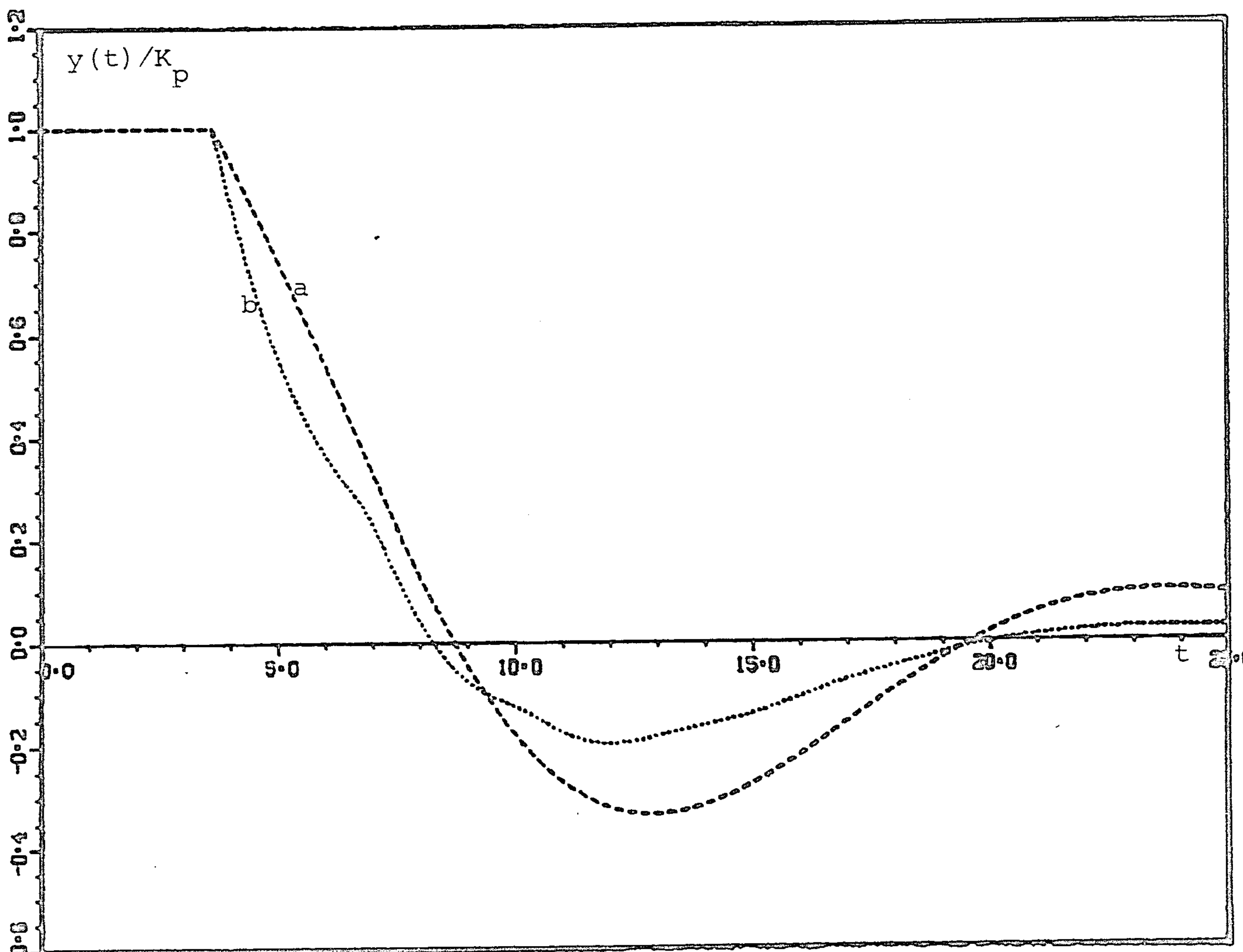


Figure 45a. Responses to Unit Change in Set-Point. Example 4, Disturbed Parameter (α = .2). (Notation as in Figure 44a.)

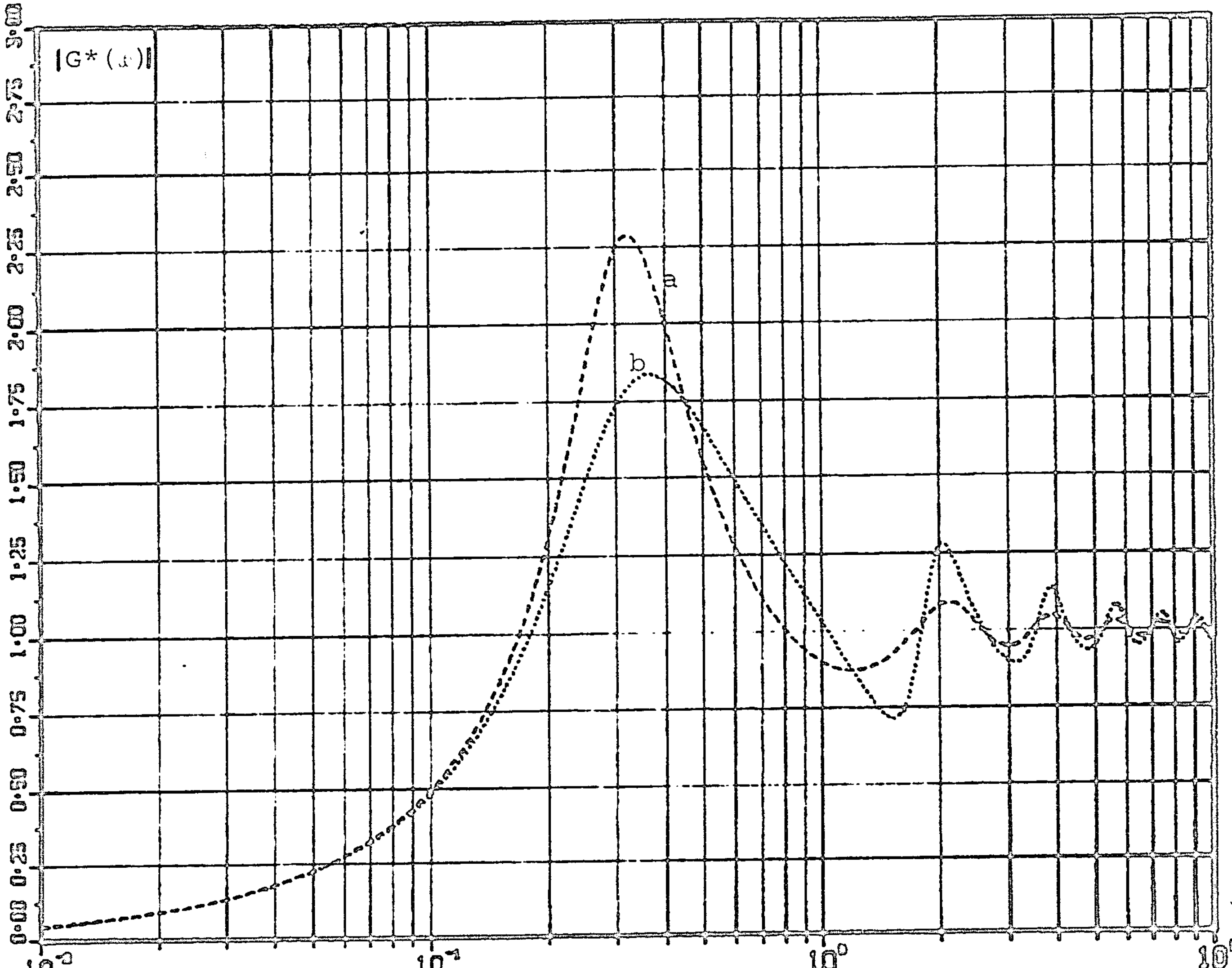


Figure 45b. Frequency Response of the Closed Loop. Example 4, Disturbed Parameters (α = .2). (Notation as in Figure 44a.)

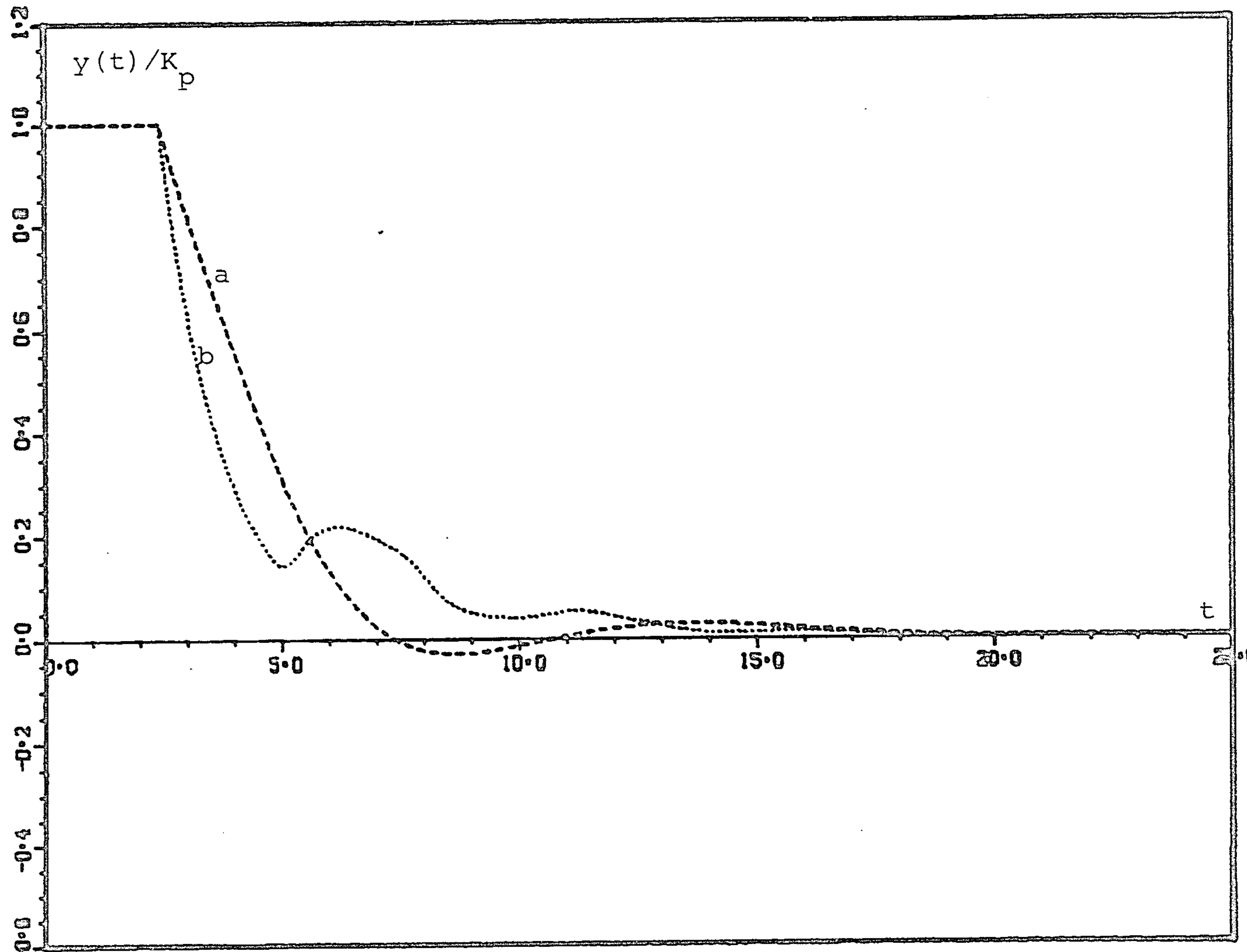


Figure 46a. Response to Unit Step Change in Set-Point. Example 4, Disturbed Parameters ($\alpha = -.2$). (Notation as in Figure 44a.)

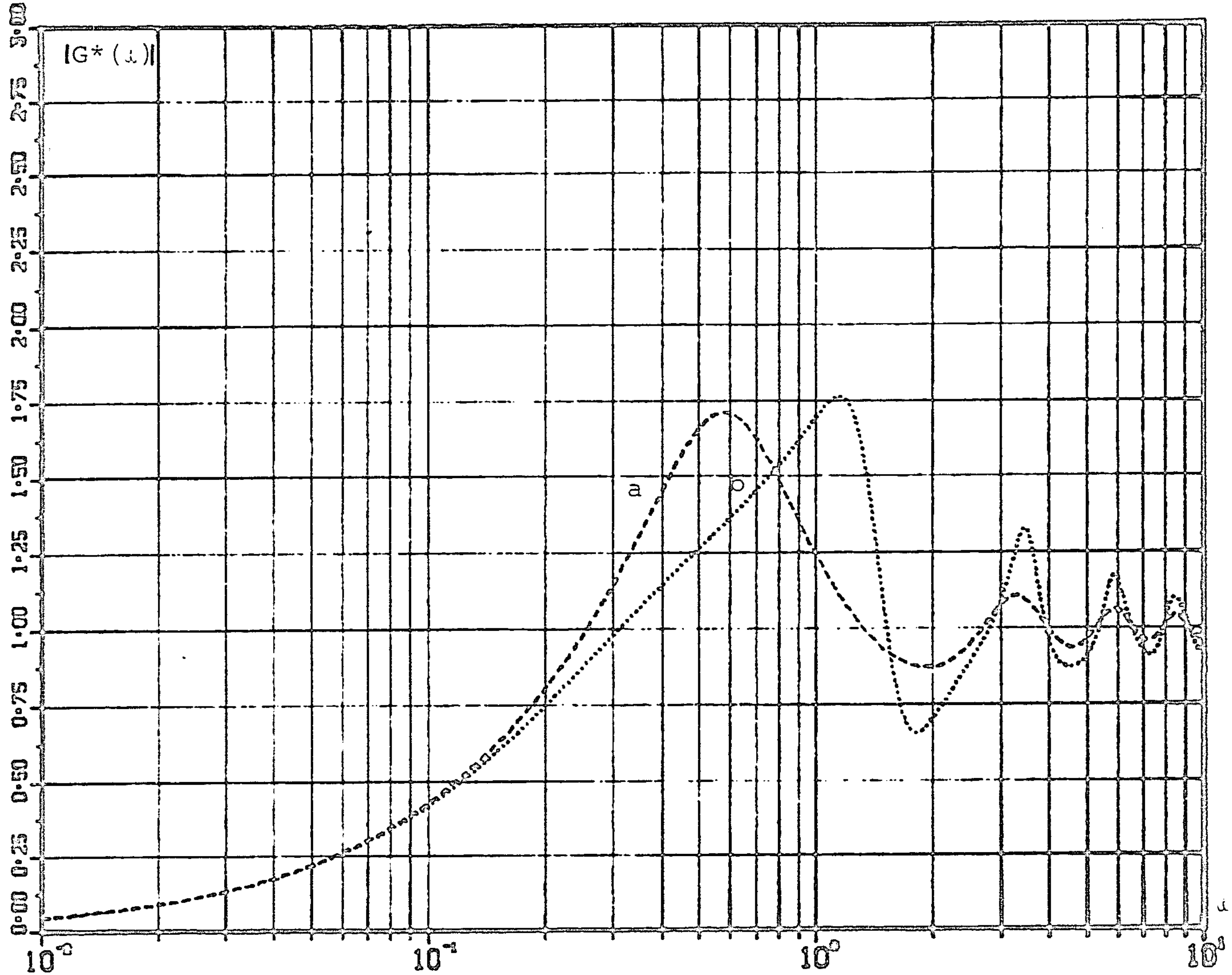


Figure 46b. Frequency Response of the Closed Loop. Example 4, Disturbed Parameters (α = -.2). (Notation as in Figure 44a.)

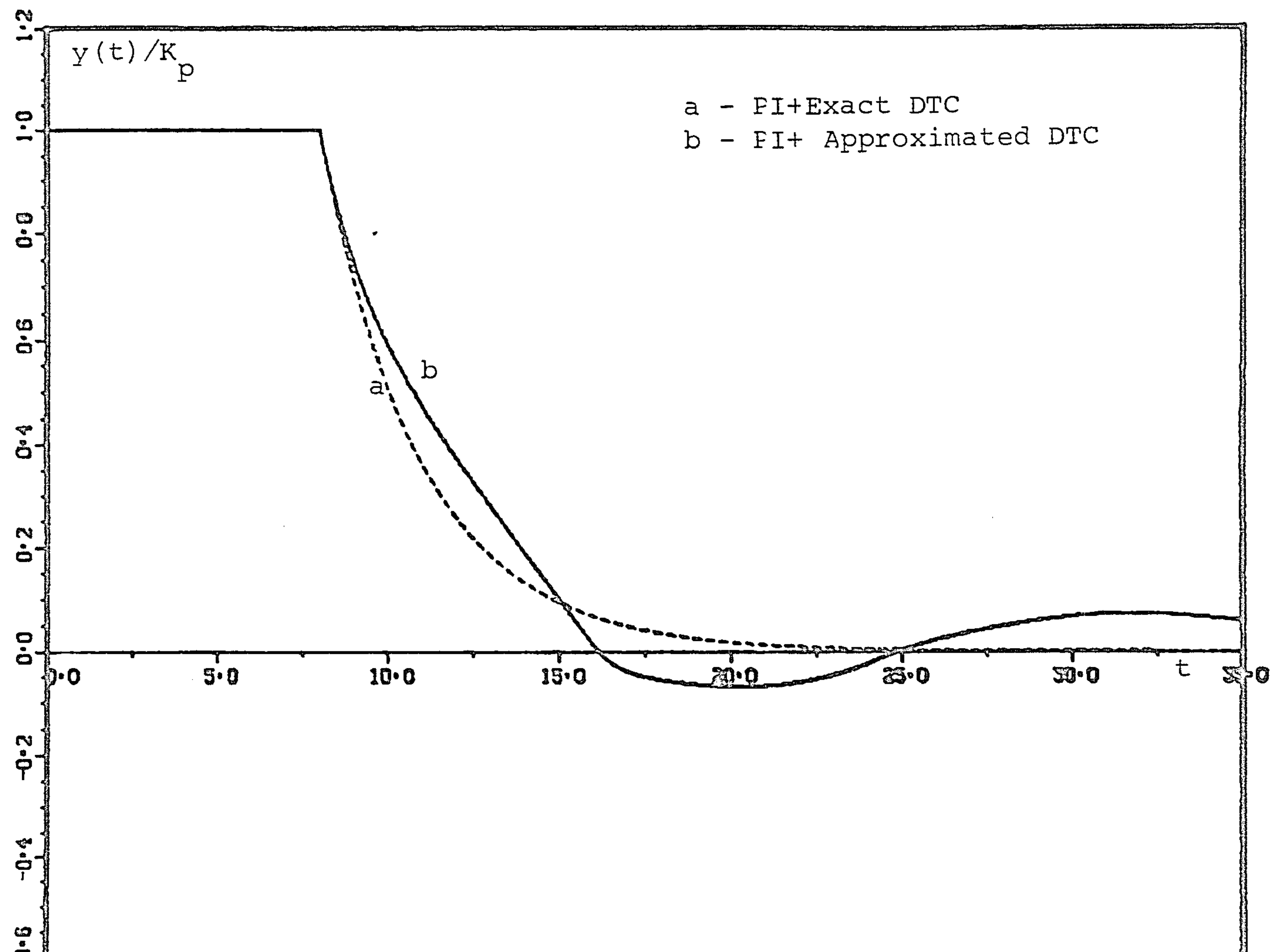


Figure 47a. Responses of the PI+DTC Controllers with Exact DTC and the Approximated DTC to Unit Changes in Set-Point (Example 1). Both are tuned According to Point 3 (Figure 35a).

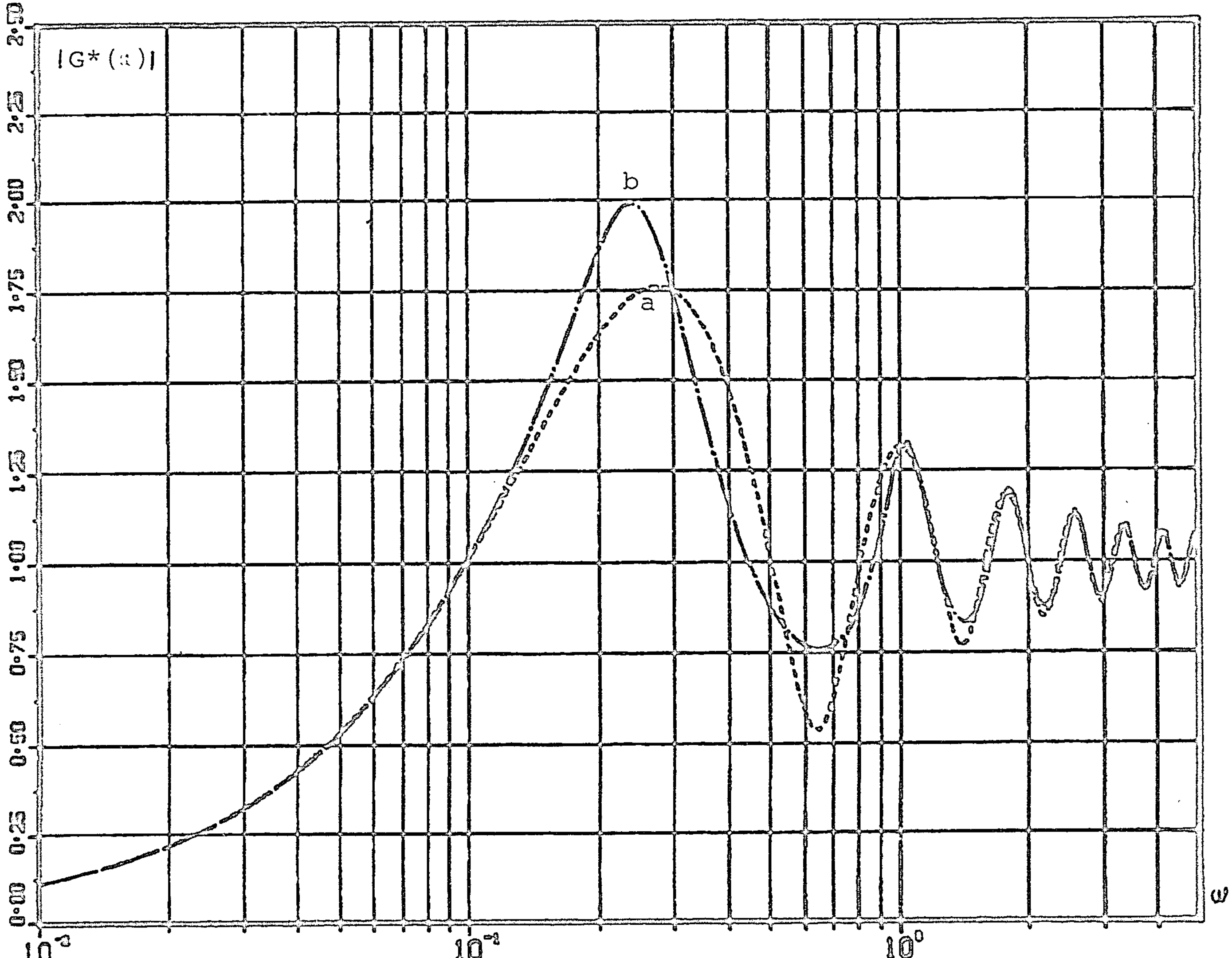


Figure 47b. Frequency Response of the Closed Loop with Exact DTC and Approximated DTC. Example 1. (Notation as in Figure 47a.)

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VITA

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The author is married and has two daughters.