Twelve-Tone Cartography: Space, Chains, and Intimations of "Tonal" Form in Anton Webern's Twelve-Tone Music

by

Brian Christopher Moseley

A dissertation submitted to the Graduate Faculty in Music in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York

2013

©2013 by Brian Christopher Moseley
All Rights Reserved.

This manuscript has been read and accepted by the Graduate Faculty in Music in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

Shaugn O'Donnell

Date

Chair of the Examining Committee

Norman Carey

Date

Acting Executive Officer

Joseph N. Straus, Advisor

Philip Lambert, First Reader

Julian Hook, Indiana University
Supervisory Committee

Graduate Center, The City University of New York

Abstract

Twelve-Tone Cartography: Space, Chains, and Intimations of "Tonal" Form in Anton Webern's Twelve-Tone Music

by

Brian Christopher Moseley

Advisor: Joseph N. Straus, CUNY Graduate Center
First Reader: Philip Lambert, CUNY Graduate Center and Baruch College

This dissertation proposes a theory and methodology for creating musical spaces, or maps, to model form in Webern's twelve-tone compositions. These spaces are intended to function as "musical grammars," in the sense proposed by Robert Morris. And therefore, significant time is spent discussing the primary syntactic component of Webern's music, the transformation chain, and its interaction with a variety of associational features, including segmental invariance and pitch(-class) symmetry. Throughout the dissertation, these spaces function as an analytical tools in an exploration of this music's deep engagement with classical formal concepts and designs. This study includes analytical discussions of the Piano Variations, Op. 27 and the String Quartet, Op. 28, and extended analytical explorations of the second movement of the Quartet, Op. 22, and two movements from the Cantata I, Op. 29.

Acknowledgements

I am extremely fortunate to have had a wonderful group of friends, family, colleagues, and mentors support me as I completed this project. Writing a dissertation is not a solitary act, and though I take full responsibility for all that follows, I would not have been able to complete this without the encouragement of this community of people.

Joe Straus has been the best kind of advisor. While I was mired in the routine of organizing, writing, and revising, his extraordinary wisdom ensured that I never lost sight of this project's scope and its most important contributions, and his intense pragmatism ensured that I finished. I will forever strive to communicate complexity with the kind of clarity and efficiency that Joe inspires.

Many of my ideas for this dissertation emerged in Phil Lambert's seminar on transformation theory. I thank him for supporting those ideas, challenging some of them at a critical point, and helping me express them more clearly. Jay Hook influenced this project long before I asked him to be a part of it. As an undergraduate, his "Uniform Triadic Transformations" was one of the first music theory articles I ever read (I should have started with something a bit easier!), and his work has shaped my thinking about transformation theory and analysis since. Jay's fastidiousness is surpassed only by his extraordinary knowledge and unfailing generosity. Shaugn O'Donnell was an ideal committee chair— supportive and responsive. I thank him for graciously offering his time and for providing many of the philosophical and metaphorical ideas that informed my approach.

In addition to the members of this committee, many scholars have shared with me ideas that shaped this work greatly. Though I didn't know it at the time, this project really began in Catherine Losada's set theory courses at CCM in 2005 and 2006. In those courses Catherine inspired me to approach music with an attention to detail, both musically and technically, that

has had the greatest impact on my analytical philosophy today. Her support has meant a great deal. I thank Jessica Barnett, Ryan Jones, Rachel Lumsden, Andrew Pau, Chris Segall, Phil Stoecker, and Dmitri Tymoczko for sharing their ideas with me and offering comments on my work. At CCM and at CUNY, Mark Anson-Cartwright, Poundie Burstein, David Carson Berry, Steven Cahn, Miguel Roig-Francoli, Richard Kramer, William Rothstein, and Mark Spicer inculcated great scholarly intuitions and encouraged the critical inquisitiveness I needed to complete this project. I thank my colleagues at Furman University, especially Mark Britt, Mark Kilstofte, and Dan Koppelman, for providing such an encouraging environment in which to write a dissertation.

My family and friends have been *sui generis*— incredible people that have supported and inspired me and made it possible for me to finish this project. My mother and father, Will and Danielle Moseley, motivated me to work harder than I thought I could and gave me more love than I could have ever hoped for. David Moseley always helped me take a break at the right time, and Alex Moseley reminded me what real hard work is. Linda Barnett has been a great champion, and Jim Barnett offered great advice and kept the coffee flowing. My best ideas and most productive days came when Oliver was sitting on my lap.

Most of all, I thank my wife, Jessica Moseley. She has been my greatest advocate and favorite interlocutor. She has always reassured me when I questioned my resolve or my ability and she's challenged me when I needed challenging. Directly and indirectly, she influenced this project in innumerable ways. I'll need the rest of my life to repay her.

TABLE OF CONTENTS

Abstract *iv*
Acknowledgements *v*
Table of Contents *vii*
List of Illustrations *viii*

PART 1: THEORY AND METHODOLOGY

Introduction **1**

Chapter 1: Transformation Chains, Syntax, and Representation **7**

§1.1 Preliminaries: Defining Chains 3
§1.2 Formalities: Chains and Intervals 20
§1.3 Chain Meaning 42
§1.4 Representing Row Class Temporality 50

Chapter 2: Twelve-Tone Cartography **88**

§2.1 Product Groups 87
§2.2 The Efficacy of Chain Groups 104
§2.3 Organizing Spatial Networks: Paradigmatics and Syntagmatics 124
§2.4 Combinatoriality, Row Areas, and Transformational Character 163

PART 2: ANALYTICAL STUDIES

Chapter 3: "Theme," "Key," and "False Recapitulation" in Webern's Quartet, Op. 22, II **207**

Chapter 4: Musical Images of Nature in the Cantata I, Op. 29 **259**

I: Circularity: "Kleiner Flügel Ahornsamen" 259
II: Linearity: "Blitz und Donner" 280

Conclusions *312*
Appendix 1: Chain Possibilities in Webern's Serial Music *319*
Appendix 2: Invariance Measurements *331*
References *344*

ILLUSTRATIONS

TABLES

1.1. Intervallic transformations under the four chains. 29

1.2. *TCH* chains in Webern's music. 32

1.3. *RICH* chains in Webern's music. 43

1.4. Mapping table for all CH_i acting on any capable twelve-tone row. 48

1.5. CH_1 and CH_4 acting on Webern's Op. 28. 50

1.6. Transformation Chains as UTTs. 54

1.7. The homomorphism α maps H to G. 74

2.1. Four cycles in Op. 27, II. 159

FIGURES

1.1. *TCH* and *RICH* in the String Quartet, Op. 5, III. 11

1.2. Comparing $RICH_2$ and $RICH_1$ in the Piano Variations, Op. 27, I. 15

1.3. Duration chains in the Cantata I, Op. 29, I. 18

1.4. Combining chains: $TCH_7 * TCH_7 = TCH_2$ in the Orchestra Variations, Op. 30. 20

1.5. Three mapping tables comparing operations and transformations. 23

1.6. A mapping table for $RECH_3$ as it transforms a time-point series. 25

1.7. Comparing TCH_1 in the Piano Variations and the Orchestra Variations. 27

1.8. *TCH*'s intervallic requirements. 30

1.9. *TCH* chains in Op. 30. 30

1.10. Intervallic equivalencies under ICH_3. 33

1.11. ICH_3 in the Concerto for Nine Instruments, Op. 24. 34

1.12. *ICH* possibilities. 37

1.13. Intervallic equivalencies under $RECH_3$. 39

1.14. Interval limitations on *RECH*. 40

1.15. $RECH_{12}$ and the Symphony, Op. 21. 42

1.16. $RICH_3$'s and $RICH_4$'s intervallic equivalencies. 42

1.17. Adjacent interval series for Opp. 28, 29, and 30. 43

1.18. $RICH_7$ transforming $\mathbf{P_9}$ in the Orchestra Variations, Op. 30. 45

1.19. In Op. 31, $RICH_3$ can transform **P** and **I** forms, but not **R** or **RI** forms. 46

1.20. TCH_1 transforming $\mathbf{I_3}$ in Op. 23 and Op. 28. 49

1.21. In Op. 28, $RICH_4(\mathbf{I_3}) = \mathbf{R_{10}}$. 50

1.22. Two networks representing major triads and pc transposition. 64

1.23. A network showing TCH_4 as it acts on P forms from Webern's String Quartet, Op. 28. 67

1.24. Creating a Cayley diagram for $G = \langle T_7 \rangle$. 68

1.25. In Op. 28, the cyclic group $C_3 = \langle TCH_4 \rangle$. 72

1.26. Homomorphisms, Isomorphisms, and Automorphisms amongst transposition groups and chain groups. 75

1.27. An automorphism between groups generated by T_7 and T_1. 76

1.28. Turning a Caley diagram into a spatial network. 78

1.29. Spatial representation of Table 1.4. 80

1.30. Cayley diagram for $\langle TCH_1 \rangle$, where $y = 1$. 83

1.31. TCH_2 and TCH_4 in Op. 28. 84

1.32. $(TCH_1)^3 \bullet (ICH_1)^3 \bullet (RECH_1)^3 = (RICH_1)^3$. 86

2.1. Manhattan's street grid as a spatial network partitioned by bus lines. 88

2.2. A "path of influence" from the row class to a partitioned spatial network. 90

2.3. Piano Variations, Op. 27, II. 94

2.4. Two transformation diagrams of the Piano Variations, Op. 27, II. 95

2.5. A procedure for generating direct product groups. 97

2.6. Constructing a non-commutative group. 101

2.7. A procedure for generating semidirect product groups. 102

2.8 Constructing the *classical serial group*: $(T_1 \rtimes I) \times R$. 106

2.9. Constructing a *chain group*: $(TCH_1 \rtimes ICH) \rtimes RECH$. 111

2.10. Transformation diagrams of the Piano Variations's second movement, showing path distance. 119

2.11. Piano Variations, Op. 27, I. 122

2.12. Transformation diagrams for the Piano Variations's first movement. 124

2.13. A hybrid group for the Piano Variations. 128

2.14. Analyzing the Piano Variations's second movement in a hybrid group, showing path distance. 129

2.15. Paradigmatic and Syntagmatic relationships in language shown along two orthogonal axes. 135

2.16. Robert Gauldin's "Basic Classification of Diatonic Chords in Functional Harmony". 136

2.17. An event network for Bach's E-major fugue (Rings, "Tonality and Transformation," 340, Figure 3.14). 138

2.18. Rings's event network reconfigured. 129

2.19. Rings's event network modeled in relation to an underlying spatial network. 140

2.20. An event network modeling the first and third entries of Bach's fugue in pc space. 141

2.21. The "gazing network": Cohn, "As Wonderful as Star Clusters," 220, Figure 5. 144

2.22. Reconstructing Cohn's "gazing network." 146

2.23. Cohn's "hyper-hexatonic system": "Maximally Smooth Cycles," 24, Figure 5. 150

2.24. An I_6-organized $\langle TCH_1, ICH_1 \rangle$ network. 153

2.25. A diagram of the Piano Variations's second movement, with imaginary chains. 156

2.26. Transformation cycles in each of the second movement's formal units. 157

2.27. Two event networks for the second movement of the Piano Variations. 160

2.28. Invariance relationships in the Piano Variations's third movement. 163

2.29. Chain-generated spatial network organized by the *WT* relation. 165

2.30. Piano Variations's third movement, "Theme." 166

2.31. Piano Variations's third movement, "Variation 1." 169

2.32. Row areas represent a "paradigmatic" relationship, binding together rows sharing hexachordal content. 173

2.33. A spatial network constructed from Lewin's comments about the Violin Fantasy ("A Study of Hexachord Levels"). 175

2.34. Lewin's event network of the first two sections of Schoenberg's Violin Fantasy with my annotations added. 176

2.35. Many symmetries in the opening A section of the Piano Variations's first movement. 179

2.36. Recapitulation in the Piano Variations's first movement. 180

2.37. Shared discrete hexachords create paradigmatic relationships amongst eight row forms. 182

2.38. An organized, chain-generated space for the Piano Variations's first movement. 183

2.39. An event network for the Piano Variations's first movement. 185

2.40. A reduction of the event network showing an amplification of the opening symmetries onto the larger row-area progression of the movement. 187

2.41. A formal diagram and event network for the String Quartet, Op. 28, I.

2.42. The String Quartet's "theme," segmented into chromatic dyads and BACH tetrachords. 196

2.43. The String Quartet's "second theme" segmented into BACH tetrachords. 198

2.44. A dyadic and tetrachordal row segmentation. 200

2.45. Spatial networks for the String Quartet generated by TCH_4 and TCH_2 and organized by segmental invariance. 200

2.46. Comparing the String Quartet's "main subject" and "reprise." 202

2.47. The "transition" from the "main subject" to the "second theme." 204

3.1. Formal chart for the second movement of Webern's Quartet showing Fennelly's "characteristics" and comparing the movement to Beethoven's Rondo, Op. 14, no. 2, III. 208

3.2. One-note chains and their effect on a row's two chromatic tetrachords (**CTETS**). 214

3.3. Spatial networks showing $\mathbf{A_0}$, a row area generated from one-note chains. 215

3.4. An event network for the first and third refrain. 217

3.5. First refrain, first half (mm. 1–7). 218

3.6. Third refrain, mm. 136-52 (reduced). 219

3.7. First refrain, second half (mm. 20–30). 221

3.8. Rows in intersecting ovals share **CTETS** and inversional potential. 224

3.9. Six row areas ($\mathbf{A_0}$–$\mathbf{A_5}$), generated from one-note chains, shown as they relate to a row's **CTETS** and "inversional potential." 226

3.10. $RICH_2$ as a progressive and static transformation. 228

3.11. A one-note, chain generated space organized by CTETS and inversional potential. 231

3.12. A compositional design for the second movement. 233

3.13. An event network for the second movement with formal annotations above and below. 235

3.14. Comparing the first and second refrains. 240

3.15. Inversional structuring from refrain to episode. 242

3.16. The false and real recapitulations, mm. 122–130. 247

3.17. **CTET**(**S**) and inversional structuring at the false recapitulation. 248

3.18. The central episode, divided into a "pre-core," "core," and "false recapitulation." 250

3.19. Mapping the central episode on the spatial network shows the "leaps" over the central column, which is filled in at the recapitulation. 251

3.20. **CTET** statements in the "pre-core," "core," and "core." 253

3.21. Reduction of the central episode. 254

4.1. Illustration of a "maple key." 266

4.2. Jone's "Kleiner Flügel Ahornsamen" set against the formal structure of Webern's movement. 268

4.3. A variety of symmetrical features in the introduction. 269

4.4. "Kleiner Flügel Ahornsamen," B section, mm. 27–36. 273

4.5. Symmetrical amplification from "Ahornsamen" to the B section. 276

4.6. A transformational derivation of the four-voice canon. 277

4.7. A TCH_4-generated spatial network, organized by the transformation graph in Figure 4.6. 279

4.8. Recursion of inversional structure across three levels of the movement. 281

4.9. Horizontal amplifications of retrograde and near-retrograde relationships. 283

4.10. Tetrachord association at important formal locations in "Kleiner Flügel Ahornsamen." 285

4.11. A formal diagram for "Blitz und Donner." 288

4.12. "Blitz und Donner's" three choral sections, mm. 14-35. 289

4.13. Boundary chord echoes in three choral sections. 292

4.14. "Echoes" in the first two canon sections, mm. 1–13. 297

4.15. "Echoes" in the last two canon sections, mm. 36–47. 299

4.16. Spatial networks modeling four "quartet types" in the canons. 304

4.17. Four types of pc association found in the canons. 305

4.18. Vertical and horizontal invariance in canon 1, mm. 2–5. 308

4.19. Vertical and horizontal invariance in canon 2, mm. 8–12 308

4.20. Vertical and horizontal invariance in canon 3, mm. 37–40. 309

4.21. Vertical and horizontal invariance in canon 4, mm. 43–46. 309

4.22. Spatial ramifications if the "broken chain" had not occurred at m. 14. 311

Reproductions of portions of Webern's Piano Variations's, Op. 27 and String Quartet, Op. 28, found in Chapter 2, were graciously permitted by Universal Edition:

UE 10881: © Copyright 1937 by Universal Edition, Copyright renewed 1965
UE 16845: © Copyright 1937, 1979 by Universal Edition A.G., Wien

PREFACE

Writing about Webern's serial music is difficult, at least in part, because one is forced to reconcile the composer's radical innovations with his musical conservatism. Apart from twelve-tone composition itself, those innovations include Webern's rhythmic writing, his use of *Klangfarbenmelodie*, his originality in the realm of musical gesture, and the intense brevity of his music, but they especially include his novel and obsessive use of polyphony, and his music's reinvention (or near total abolition) of "theme."[1] These latter innovations are most difficult to reconcile with the conservative elements of Webern's music—in particular, his unyielding appreciation for and use of classical formal models. Perhaps sensing this difficulty, but surely aware of its importance to "new music," Webern's writings and recorded lectures contain repeated references to the absolute thematic unity and "inter-penetration" of "horizontal and vertical" elements allowed by the twelve-tone method.[2] Not only is an understanding of this synthesis needed to fully engage with the music, but it is impossible to fully comprehend the richness of Webern's conservatism or radicalism without understanding how the two interface.

---

[1] Whether themes were totally abandoned in Webern's music, or just conceived in a new way has been the subject of some debate, much of which necessarily centers around one's definition of "theme." Herbert Eimert, a musicologist who edited *Die Reihe* with Stockhausen from 1955-62, claimed that beginning with the Symphony, Op. 21, Webern effectively abolished thematicism. By contrast, Webern's student Leopold Spinner notes that Schoenberg's formulation of the twelve-tone method (and hence, Webern's understanding of it) was meant to create a sort of *hyperthematicism*, and therefore, to speak of the abolition of themes is a contradiction of the most egregious sort: "the unity of thematic relations has been established in an absolute degree because all interval relations throughout are based on the basic interval succession of the twelve-tone row of which the theme itself is derived, and from it all structural derivative. To speak of abolition of thematicism in a twelve-tone composition as Dr. Eimert does, referring to Webern's Symphony, is an absolute contradiction, as the primary concept of the method is the realization of complete thematic unity" ("The Abolition of Thematicism and the Structural Meaning of the Method of Twelve-Tone Composition," *Tempo* no. 146 (September 1, 1983): 5).

[2] In the lectures posthumously published as *The Path to the New Music*, these references are constant. For example: "the style Schoenberg and his school are seeking is a new inter-penetration of music's material in the horizontal and the vertical[. ...] It's not a matter of reconquering or reawakening the [polyphony of the] Netherlanders, but of re-filling their forms by way of the classical masters, of linking these two things. Naturally it isn't purely polyphonic thinking; it's both at once" (*The Path to the New Music*, ed. Willi Reich, trans. Leo Black (Bryn Mawr, PA: Theodore Presser, 1963): 35).

In this study I explore how this interaction takes shape, and especially, how it creates musical form. Webern's sensitivity in this regard is quite deep, engaging (as Andrew Mead has shown) three levels of the twelve-tone system: (1) the "primitives" (i.e., "relationships that hold for all possible orderings"); (2) the "potentialities inherent in a row class"; and (3) the way that "[specific rows] are articulated on the musical surface."[3] Keeping these levels in mind, the present study's originality is manifest in two respects.

First, while most previous studies have emphasized the associational features of Webern's music, especially as they influence row combination, in this study I am interested primarily in the forces that guide horizontal connections between rows. Unique among members of Schoenberg's compositional circle, Webern, throughout his compositional career, consistently linked (or "chained") horizontally adjacent row forms by eliding a pitch or pitches at the end of one row with those at the beginning of the next. Many studies of Webern's twelve-tone music mention this particular predilection, but none have explored in detail how the meaning of row chains are influenced by the primitives and potentialities of the twelve-tone system or how they influence the compositional surface, up to and including large-scale formal design.

Second, in this study I explore how row chains constrain and interact with associational features of the music, such as segmental invariance between rows or composition around an axis of symmetry. To do so, I develop a theory and analytical methodology that finds its primary outlet in transformational musical spaces (or maps) that are capable of representing a (pre-)compositional environment in which analytical "performances" can take place. Such spaces have many historical precedents—they somewhat resemble, for example, many recent models of parsimonious voice leading. My intention, however, is that they function as "musical grammars"

[3] Andrew Mead, "Webern, Tradition, and 'Composing with Twelve Tones'," *Music Theory Spectrum* 15, no. 2 (September 1, 1993): 173–4.

in the sense outlined by Robert Morris.[4] That is, these spaces suggest a musical syntax for Webern's serial music (created primarily by row chains) that communicates with associational aspects (suggested by the primitives of the system or the peculiarities of a row class) that are substitutional or combinational in nature.

For cartographers, maps of the real world are representations of socially- and/or environmentally-conditioned arguments. They are not entirely fixed or absolute, but reflect relative degrees of "zoom" and underscore some types of proximity while hiding others. I imagine the spaces constructed in this study in that manner, and liken their production to an act of "musical cartography." Though I propose a general theory and methodology, my maps of Webernian serial syntax are interpretively-created tools designed to allow for interpretations of music. My central contention is that, carefully constructed, these maps imply norms of syntax and row combination, and that these norms capture ways that Webern's serial music interacts with classical conceptions of musical form.

Chapters 1 and 2 propose the core theory and methodology. In Chapter 1 I define and investigate the properties of transformation chains as they interface with the primitives of the twelve-tone system and the peculiarities of particular row classes. Transformation chains are "contextual transformations," and Chapter 1 shows how they derive their meaning from a row class. Because that meaning is the primary determinant of the syntactical properties of a row class, Chapter 1 details how a row class and a particular chain (or collection of chains) imply rudimentary musical grammars that represent a row class's inherent temporality. Chapter 2 continues to explore how groups generated by transformation chains accrue spatial representations in a broader context. In particular, I discuss how such groups differ from "classical serial groups" generated by the operations of transposition, inversion, and retrograde, and I

[4] Robert Morris, "Compositional Spaces and Other Territories," *Perspectives of New Music* 33, no. 1/2 (January 1, 1995): 328–358.

explore the efficacy of chain groups as analytical tools. The final half of Chapter 2 proposes a separation suggested by Saussure's categories of paradigmatic and syntagmatic relationship that allows for an interaction between syntactic transformation chains and other types of relationships. That separation leads to the formulation of a robust musical grammar called an "organized spatial network." Chapter 2 contains a number of analytical vignettes that explore the relevant issues.

The final two chapters are close analytical studies of two works by Webern—one of them instrumental, the second movement of the Quartet, Op. 22, and the other vocal, the first two movements of the Cantata I, Op. 29. Taken together with the analyses in Chapter 2, these more detailed analyses investigate ways that Webern utilized the different levels of the twelve-tone system to create analogies with classical formal models and concepts, and to varying degrees, most of these analyses investigate the concept of "closure" in Webern's serial music. Chapter 3 approaches the concept of recapitulation by searching for representatives of "theme" and "key" in the second movement of the Quartet, Op. 22 (1928-30). Chapter 4 offers analytical studies of two movements from the Cantata I, Op. 29 (1939). Both of the movements studied are ternary forms with obvious reprises, and my analyses show how the closure engendered by the details of these reprises are representatives of natural images in the poetic text of each movement.

Orthographic Matters

In this study *row* and *row form* indicate specific orderings of the twelve pitch classes. A *row class* contains rows related by transposition, inversion, retrograde, and retrograde inversion. In analyses of Webern's works, row forms are often labeled differently. Many analysts label rows only after determining a row that is analogous to some "tonic." Others label rows as transformations of a single prime form **P**, and still others label rows such that $\mathbf{P_0}$ is always equal to an arbitrarily

determined prime form beginning on C.[5] For consistency, and because determinations of "tonic analogues" are not easily made, I follow the latter methodology, but I have made every effort to align my own row designations with those found elsewhere in the scholarly literature. $\mathbf{P_0}$ should not be understood to be more "tonic-like" than any other row form.

As above, row forms are always with bold type ($\mathbf{P_0}$, $\mathbf{I_0}$, and so on). To differentiate between a row form and a transformation acting on a row form, transformations and operations are always indicated with italicized type (T_0, I_0, *RICH*, *TCH*, and so on). Thus, $RI_0(\mathbf{RI_0})$ symbolizes the operation RI_0 transforming the row $\mathbf{RI_0}$. Generally speaking, transformations are understood in left-to-right order. To apply (*TCH*)(*ICH*) to some row form, first transform the row by *TCH*, and then transform the result by *ICH*. Note that this differs from the usual practice, wherein T_xI ($\mathbf{P_0}$) is calculated by first applying I to $\mathbf{P_0}$ and then T_x. On occasion, especially when classical serial operations are used by themselves, I will make use of that orthographic practice, but I will always make note when that is the case.

[5] Julian Hook and Jack Douthett offer an interesting discussion of labeling systems in "Uniform Triadic Transformations and the Twelve-Tone Music of Webern," *Perspectives of New Music* (2008): 101-5.

PART I: THEORY AND METHODOLOGY

CHAPTER 1

TRANSFORMATION CHAINS, SYNTAX, AND REPRESENTATION

As compared with the associational features of Webern's music, syntactical (or, more generically, linear) considerations have been relatively neglected. Nonetheless, a fundamental similarity often links the two types of relation. Pitch-class associations amongst row forms within a particular row class—associations that often take the form of statements like "these rows share this pitch-class segment"—are often byproducts of the particular ordering of twelve pitches within that row class.[1] Similarly, linear relationships (why particular row forms follow others) are frequently determined by properties of a row class. The latter relationship occurs because Webern frequently "chains" a row form to its successor, relying on an overlap in pitch content between linearly adjacent forms. Thus, both types of relation are in some sense predetermined by the constructive principles of the row class itself.

In the chapter that follows, these syntactical considerations are forefront. Four basic classes of chains are defined and studied in relation to one another and to other canonical transformations. The discussion follows in two large sections. In §1.1-2 chains are defined generally and then demonstrated and explored in a variety of contexts. Though we will most often be interested in the way that chains can transform twelve-tone rows, they may in principle act on any ordered series, whether that series contains pitches, durations, dynamics, articulations, and so forth. Section 1.1 shows that chain transformations require only some concept of "interval." In §1.2 the exploration narrows. Chains are considered as they act on twelve-tone rows

[1] Generally, these relationships are forged at the level of the row class or within the space of a particular composition. Andrew Mead encourages twelve-tone analysts to think of compositional possibilities as emanating from three different levels of the twelve-tone system: (1) "primitives of the system" are "relationships that hold for all possible orderings"; (2) "potentials inherent in a row class" include relationships that are dependent on a particular ordering. And finally, (3) "the way [a row class's] members are articulated on the musical surface" involve relationships that emerge only as a result of composition-specific elements—register, rhythm, instrumentation, and so on. See Andrew W. Mead, "Webern, Tradition, and 'Composing with Twelve Tones'," *Music Theory Spectrum* 15, no. 2 (1993): 173–74. Pc invariance belongs to the second level.

and particular attention is paid to the number and variety of chains that can act on the members of a row class. Because this section centers on those aspects of chains that are determined by properties of a specific row class, it also provides an opportunity to think about Webern's row construction from a novel perspective.

The second large part of this chapter begins a pivot towards Chapter 2, which constitutes the core methodology of this study. In §1.3–5 we will explore chains as members of *groups* and detail a spatial framework for representing these groups. Forefront here is the representation of the syntactical component of Webern's music. Unlike other transformational actions that connect row forms (especially members of the "classical serial group"), the "meaning" of a chain (that is, what it *does* to a particular row form) is determined by the row class itself; that is, chains, like other ways of studying twelve-tone rows and their interactions with one another, have a "natural" basis in the row class's structure.

1.1 Preliminaries: Defining Chains

I will approach each chain as it relates to one of four basic chain-classes: *TCH*, *ICH*, *RECH*, and *RICH*.

1.1.1 Definition

The chain family (*CH*) acts on ordered series of elements. Members of this family connect transposed, inverted, retrograded or retrograde-inverted objects by eliding the end of the inputted object with the beginning of its transformation. Corresponding to the four serial transformations of an ordered series, there are four classes of chains: a transpositional chain (TCH_i), inversion chain (ICH_i), retrograde chain ($RECH_i$), and retrograde-inversion chain ($RICH_i$).[2] The length i of a chain refers to the number of overlapped elements linking two objects. Consequently, while two chains may belong to the same class—TCH_1 and TCH_2, for example— they can be distinguished by the length of the segment involved in their overlap—one and two pitches, respectively.

When discussing the family generically, I will occasionally speak of a "*CH*," which stands for any of the chain types. Potentially, each of these four *CH* classes has as many types, as defined

---

[2] *RICH* and *TCH* chains have been famously described and explored in David Lewin's writing as a way to describe music by Wagner, Webern, and Bach. See *Generalized Musical Intervals and Transformations* (New Haven: Yale University Press, 1987), 180-8. Lewin had earlier described *RICH* (the name was not fully formed) in the context of Webern's music as a way to illustrate contextual inversion: "Webern, in his serial practice, frequently elides the last two pc's u and v of one row form together with the first two pc's u and v of the suitably retrograde-inverted form: RI^{UV} of the first form" ("A Label-Free Development for 12-Pitch-Class Systems," *Journal of Music Theory* 21, no. 1 (1977): 35). Joseph Straus described the four classes of chains in the chain family in "Motivic Chains in Bartók's Third String Quartet," *twentieth-century music* 5, no. 1 (2008): 25–44.

In principle, we could imagine other types of chains. *TCH* and *ICH* call to mind the class of forty-eight "canonical operators" that includes pitch-class multiplication. A multiplication chain (*MUCH?*) could be of interest in more recent serial music. (Terminologically, this would pose a problem. Lewin uses *MUCH* to mean something quite different. See *GMIT*, 183-84.) *RECH* and *RICH* invoke the "order operations" that also include rotation, perhaps implying that a rotation chain (*ROCH?*) may have some analytical or compositional application.

by the length *i* of the chain, as there are elements in the ordered series being chained. Given an ordered series $s_0, \ldots, s_n$, it is possible for $CH_1, \ldots, CH_n$ to act on that series. Though, as we will see below, considerations of order generally limit the number of distinct chain types.

Implicitly, this definition of a chain involves other musical elements. Though it is framed such that the "overlapped elements" refers most easily to pitches, those elements could be other objects. It is also easy to imagine the definition be loosened in a variety of ways to apply in different musical contexts. The examples below explore these ideas and are also intended to refine our conception of chains in ways that will be explored later.

1.1.2 Example: Webern's String Quartet, Op. 5

Imagine a space *S* of ordered (015) trichords. Each of these trichords has pitch-class intervals ordered as <1, 4> or any of its serial permutations: inversion <11, 8>, retrograde <8, 11>, or retrograde inversion <4, 1>. *S*, then, has forty-eight members and contains the ordered (015) trichord {G♯, A, C♯} but not {G♯, C♯, A}.
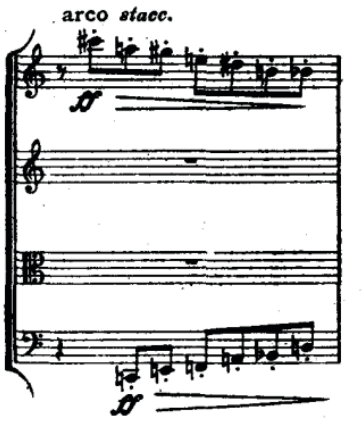
This space nicely models some passages in Webern's Five Movements for String Quartet, Op. 5, iii, one of which is shown in Figure 1.1 There, the violin and cello play a series of (015) trichords. Figure 1.1(b) shows the violin's first trichord {C♯, A, G♯} being transformed into a transposed-form {G♯, E, D♯} through TCH_1, the single-element overlap created through a shared G♯. The $RICH_2$ interpretation in (c) elides two elements in conjoined, *RI*-related trichords. $RICH_2$ requires two additional chain links to complete the measure of music as compared to TCH_1.[3] Both interpretations are "complete." They subsume the entire measure and neither leave nor require any additional pitches to finish their transformational action. By contrast, the interpretation at (d)—transform the initial trichord by RI_0 to produce the next three notes—

[3] Straus's analysis of Webern's Concerto, Op. 24, ii, which shows the *RICH* basis of the melody in mm. 1-28, is strikingly similar to my melodic analysis of the string quartet passage (d) ("Contextual-Inversion Spaces," *Journal of Music Theory* 55, no. 1 (2011): 57-61).

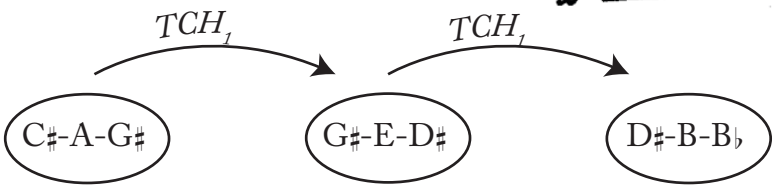
leaves a B♭ "hanging" at the end of the measure. TCH_3({C♯, A, G♯}) is shown at (e). The example is trivial because every ordered series of *n* elements may TCH_n with itself, TCH_n being somewhat similar to T_0.

FIGURE 1.1. *TCH* and *RICH* in the String Quartet, Op. 5, III.

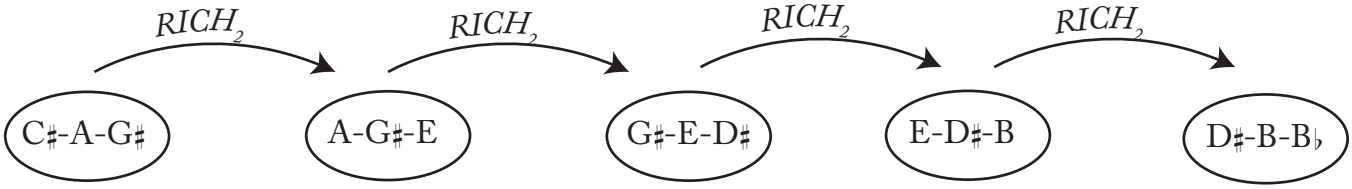
(a)



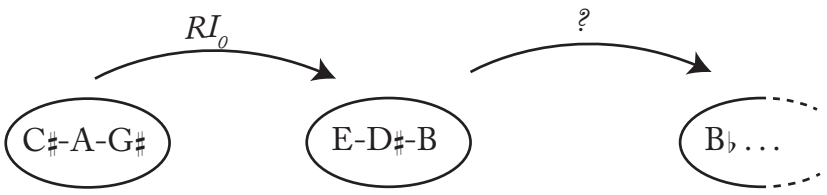
(b)



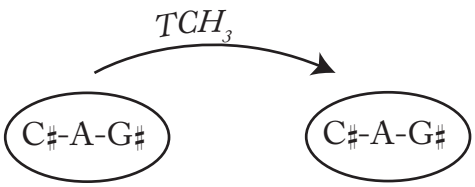
(c)



(d)



(e)



Generally, transformation chains overlap pitches. We might occasionally wish to imagine a more abstract type of chain, one that links pitch classes, or even a type of chain that links unordered series that share a common pitch or pitch class. Linkages of this variety would imply that the common elements are chained only in the abstract, but not at the musical surface.[4] The "promiscuity" of such a chain would necessarily require careful usage, but could yield interesting analytical results.[5] Consider the passage shown below at (f), the six measures preceding the violin/cello passage shown at (a). Above the C♯ pedal, the violins and viola play a collection of (014) trichords. Treating these trichords as unordered sets of pitch classes shows that chronologically adjacent trichords are always related by TCH_1 or ICH_1, following the interpretations given in (g)-(k).
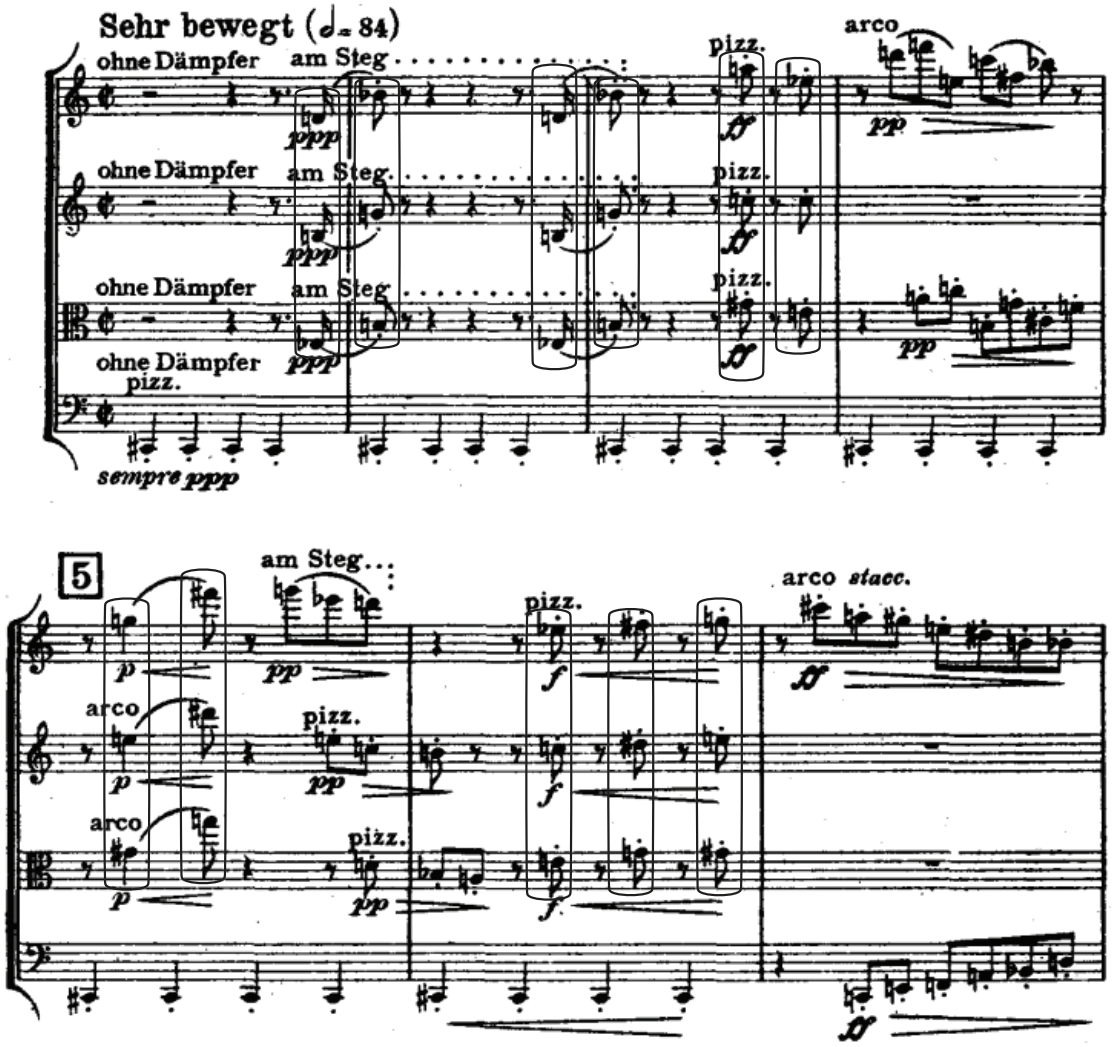
In isolation, this analysis would be a strange way to view these chords. But because it shows that a common logic exists between the verticalities in the passage and the melodic snippets in m. 7, it has some analytical elegance. The chain interpretation shows that adjacent chords are always related in one of only two ways—TCH_1 or ICH_1. Imagining relations between the chords as *transpositions* or *inversions* of one another would involve greater complexity. We should not downplay the significance of those transformations, but the chain-perspective does say

[4] Along these lines, the "neo-Riemannian" transformations *P*, *L*, and *R* can be viewed as transformation chains acting on unordered series. In fact, Joseph Straus generalizes *P*, *L*, and *R* for all trichords, tetrachords, and pentachords in this manner by noting that the neo-Riemannian transformations are akin to retaining two pitch classes while "flipping" the remaining pitch class(es) around the pitch-class axis of symmetry implied by the invariant pcs. Straus connects the transformations to Lewin's *RICH*, though because the objects in question are unordered, they may be better understood as a type of *ICH*. In an unordered context, there is no real way to distinguish between the two transformations, as their is no way to distinguish between *TCH* and *RECH*. See "Contextual-Inversion Spaces," 43–88.

[5] I use the term "promiscuity" here and elsewhere in the spirit of Shaugn O'Donnell's "exclusivity-promiscuity" continuum ("Transformational Voice Leading in Atonal Music" (Ph.D. dissertation, City University of New York, 1997), 7-8.) It is not just that chain transformations could devolve into rebrandings of the four classical transformations. But more generally, understanding chains as acting on unordered series would allow a great deal many more connections than in an ordered context.

something new and fascinating about the passage.[6] It also indicates a possible precedent for twelve-tone chains in Webern's non-serial music.[7]
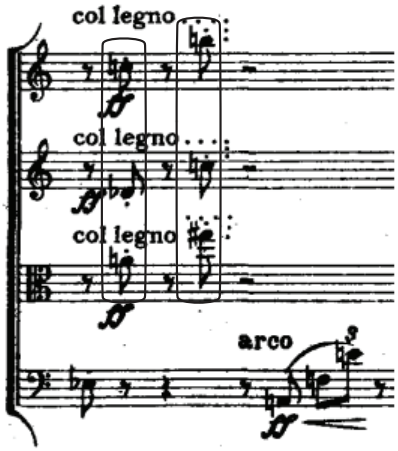
(f) Webern, Five Pieces for String Quartet, Op. 5, III, mm. 1-8



[6] One facet of the interpretation that is quite interesting involves the canonic passages in mm. 4, 5-6, and 7-8. These passages tend to have minimal overlap in pitch-class content, thereby "rubbing against" the chain interpretations of the chords. For example, in mm. 5-6, the three upper strings play a series of (015) trichords that have no pitch-class intersection.

[7] J. S. Bach's keyboard music offers another suggestive precedent. Lewin shows *RICH* and *MUCH* chains in Bach's Two-Part Invention in C major, suggestively positioning his analysis of the Invention just after an analysis of Webern's Piano Variations, Op. 27 (*GMIT*, 183-84). Webern's admiration for Bach's polyphonic writing makes the connection particularly interesting.

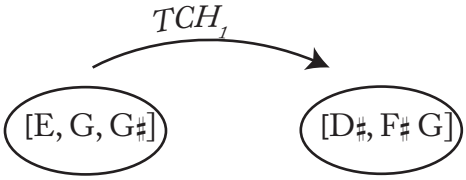
(f) cont.



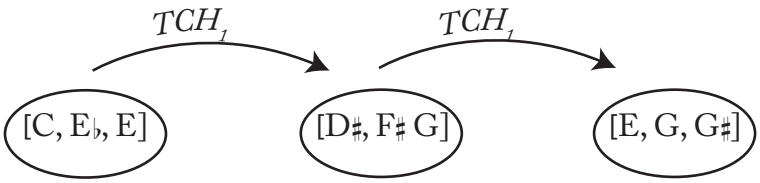
(g) mm. 1-3



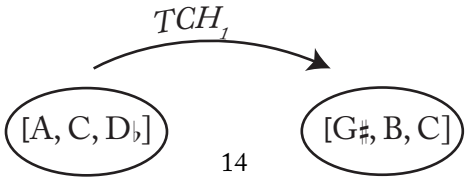
(h) m. 5



(i) m. 6



(k) m. 8



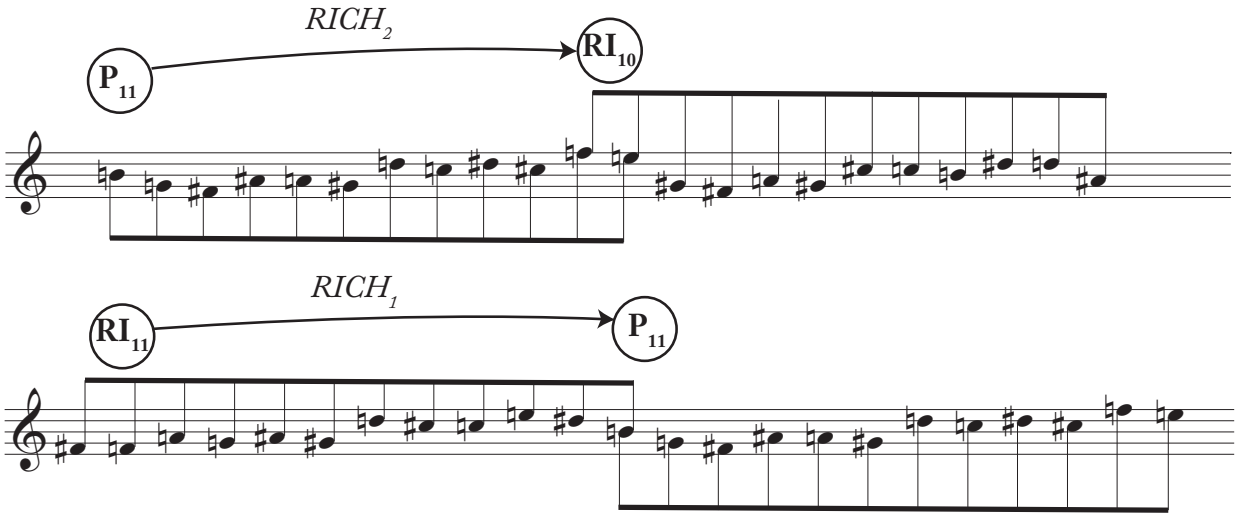
Throughout this study, I will often be interested in distinguishing transformation chains from the group of "classical" serial operations. One of the most basic ways that chains differ from those operations is through their engagement with temporality. Classical serial operations can describe rows that are adjacent "horizontally," "vertically," and rows that are great temporal distances from one another. Chains are generally limited to adjacent, horizontal row connections —one way in which they act as carriers of syntax. Therefore, they are capable of leading to different analytical discoveries.

1.1.3 Example: Webern's Piano Variations, Op. 27

Imagine a space *S* containing members of the row class from Webern's Piano Variations, Op. 27, whose **P**-form is {E♭, B, B♭, D, C♯, C♮, F♯, E, G, F, A, G♯}. Figure 1.2(a) shows two types of chain that are prevalent in the movement. *RICH*$_2$ transforms $\mathbf{P_{11}}$ into $\mathbf{RI_{10}}$ through the shared {F, E} at the end of $\mathbf{P_{11}}$. This gesture is common in the first movement's B section, a passage

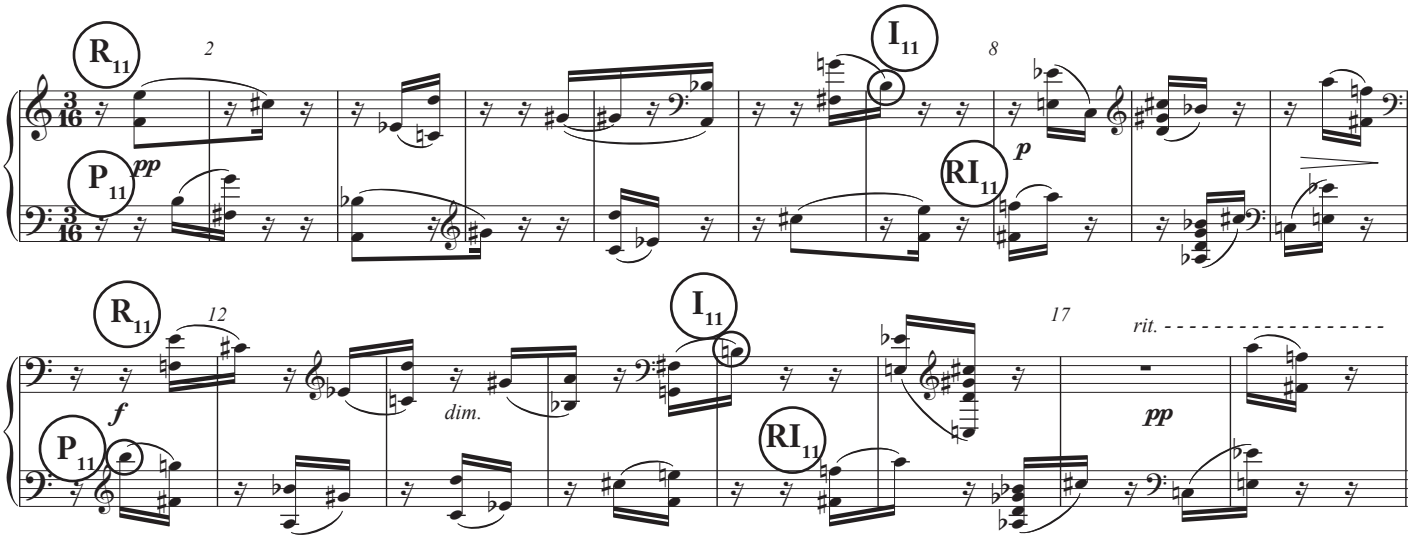
Figure 1.2. Comparing *RICH*$_2$ and *RICH*$_1$ in the Piano Variations, Op. 27, I.

(a) *RICH*$_2$ and *RICH*$_1$



discussed in exactly these terms by David Lewin.[8] In the outer A sections, the *RICH$_1$* connections, like that from **R$_{11}$** to **I$_{11}$**(shown on the first stave of Figure 1.2(b)), have far greater structural power.[9]

(b) Piano Variations, Op. 27, i, mm. 1-18, reduction



RICH$_1$ connections are prevalent in the canon that opens the movement, a reduction of which is given as Figure 1.2(b). This reduction places the *dux* and *comes,* which are rhythmic retrogrades of one another, on different staves. The *dux* plays two row forms, **R$_{11}$** and **I$_{11}$,** and the *comes* plays **P$_{11}$** and **RI$_{11}$**. In the passage shown, **R$_{11}$** initiates the *dux* and links with **I$_{11}$** through a shared B in m. 7, a *RICH$_1$* connection. But, **I$_{11}$** is unable to return to **R$_{11}$** by similar means. The final pitch class of **I$_{11}$** is F♯, and the first pitch of **R$_{11}$** is E. Thus, **I$_{11}$** ends in m. 10 and **R$_{11}$** begins anew in m. 11—no overlap. Notice that the converse attains in the *comes* voice: **P$_{11}$** begins the

[8] Lewin *GMIT*, 182.

[9] This movement is discussed in more detail in Chapter 2. An important part of the analysis there involves determinations of the structural power of these two transformations as they interact with the movement's ABA structure.

comes in m. 1, but a $RICH_1$ connection does *not* exist in m. 7 as it did in the *dux*. Such a connection does, however, in m. 11.[10]

Whether a pair of adjacent rows can chain or not has important canonic consequences. As $\mathbf{RI_{11}}$ $RICH_1$*'s* into $\mathbf{P_{11}}$ at m. 11—at the moment where $\mathbf{I_{11}}$ and $\mathbf{R_{11}}$ become disconnected—the *comes* "jumps ahead" of the *dux*. In the ensuing system (beginning at m. 11), the canonic relationships are reversed. A classical understanding of this passage would note its transformational consistency. Each row is an RI_{10} relation to the next. Consistency is often prized, but here, the *lack* of consistency is analytically interesting because it partly explains how the structure of the canon morphs over the first eighteen measures.

Chains have the potential to act on more than one type of musical object—pitch, rhythm, and so on. To define a chain robustly in different musical domains, the objects under consideration need to be ordered.

1.1.4 Example: Webern's Cantata I, Op. 29

Figure 1.3(a) investigates the rhythmic construction of the canon subject for the first movement of Webern's Cantata I, Op. 29.[11] This subject, labeled as $\mathbf{P_0}$, is treated as part of a double canon that recurs in the movement. In Figure 1.3(a) I treat this subject as an ordered series of time-points (x) and durations (y), and so, below each rhythmic figure in the example, every attack is represented as an ordered pair (x, y). The symbol <x, y> indicates a rest of y quarter notes at time-point x. Imagine a space S containing $\mathbf{P_0}$, all of its serial permutations, and their transpositions.[12] In this space, the quarter note receives a duration of one, and therefore, $\mathbf{P}_x$ will

[10] While $\mathbf{RI_{11}}$ $RICH_1$s with $\mathbf{P_{11}}$ (see Figure 1.3(a) and m. 11 of (b)), the converse is not true: $\mathbf{P_{11}}$ does not $RICH_1$ with $\mathbf{RI_{11}}$: $RICH_1$ ($\mathbf{P_{11}}$) = $\mathbf{RI_9}$, a row form not represented in this passage at all. The same is true for $\mathbf{R_{11}}$ and $\mathbf{I_{11}}$, the two row forms used in the canon's leading voice.

[11] This movement is analyzed in detail in Chapter 4.

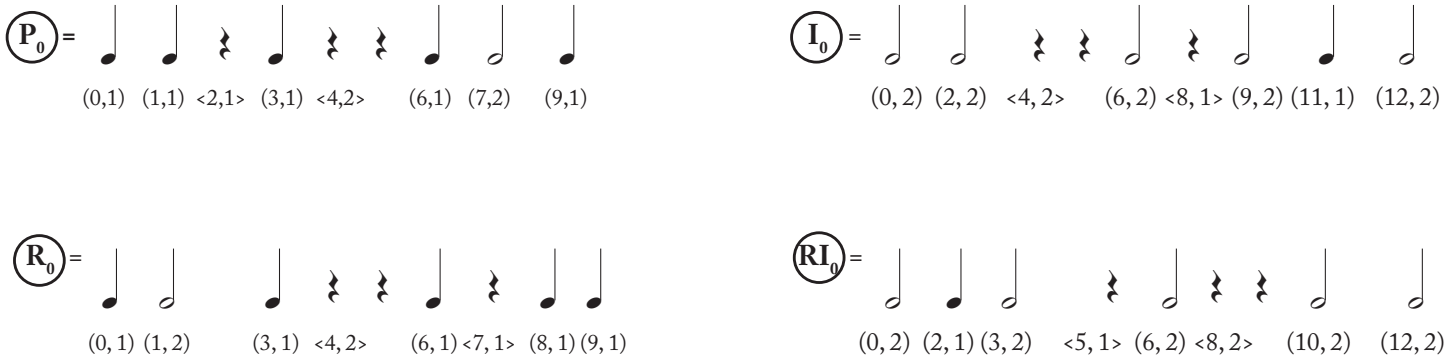
[12] Because the movement is a fixed duration, the space is not infinite, but will have many members.

have the following series of durations: 1 1 <1> 1 <2> 1 2 1.[13] Given x we can calculate the remaining time points using durations as intervals. If $x = 0$, $\mathbf{P_0} = \{(0, 1), (1, 1), <2, 1>, (3, 1), <4, 2>, <6, 1>, (7, 2), (9, 1)\}$, as shown in Figure 1.3(a).

To retrograde the series, read the intervals of **P** backwards.[14] Inverse-related series's project corresponding durations that sum to mod 3: quarter notes in **P** become half notes in **I** and vice versa.[15] We can also transpose a series by adding a constant to each time-point. Transposition maintains the series of durations and moves the series forward or backward in time. For example, $T_3(\mathbf{R_0}) = \{(3, 1), (4, 2), (6, 1), <7, 2>, (9, 1), <10, 1>, (11, 1), (12, 1)\}$.

FIGURE 1.3. Duration chains in the Cantata I, Op. 29, I.

(a) the cantata's canon subject ($\mathbf{P_0}$), its inversion, retrograde, and retrograde inversion



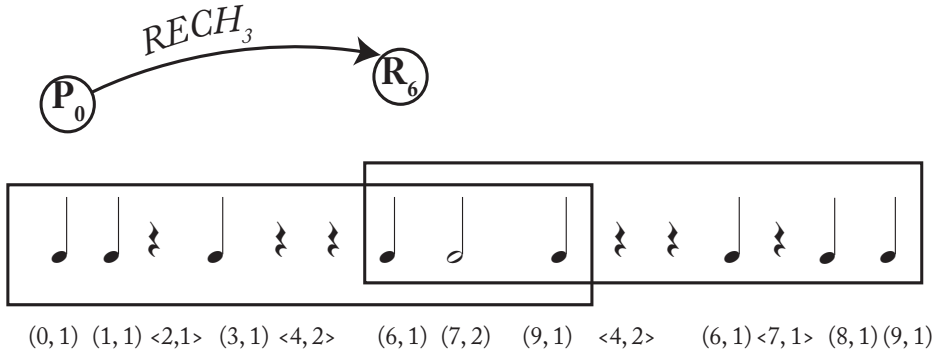
[13] This presentation of a rhythmic series is similar to the algebraic model used by Julian Hook to model rhythmic characters in Messiaen's "Turangalîla Symphony." As there, I am not considering the subject in terms of its metric situation. See "Rhythm in the Music of Messiaen: An Algebraic Study and an Application in the "Turangalîla Symphony," *Music Theory Spectrum* 20 (Spring 1998): 97-120.

[14] This conception of retrograde seems intuitive. But in one important way, it is very different from the understanding of retrograde that is common to twelve-tone theory. Retrograded series of pitches project an intervallic series that is the *retrograde inversion* of the original. Conversely, *retrograde-inverted* series of pitches convey an intervallic series that is the *retrograde* of the original. Our understanding here proposes that the intervals, which we understand to be durations, are *not* retrograde-inverted but simply retrograded. Again, this idea of retrograde seems more in line with intuitions about rhythmic retrograde. We might redefine rhythmic retrograde to conform with the understanding from twelve-tone theory, but that seems undesirable.

[15] Inversion is calculated mod 3 so that quarter notes (1) become half notes (2) and vice versa. Hook 1998 does not discuss inversions of rhythmic characters. My understanding of this owes much to Lewin, who uses durational motives (DM's) to analyze some motivic work in Mozart's Symphony, No. 40, in G minor (*GMIT*, 220-25). Lewin uses *RICH* and *TCH* chains to understand relationships between DMs.

Now imagine *RECH*$_3$ acting on this space. In order for *RECH*$_3$ to transform $\mathbf{P_0}$ we must find a retrograde form whose first three ordered pairs elide with the last three of $\mathbf{P_0}$—(6,1), (7, 2), (9, 1). That retrograde form form begins at time-point 6, and is shown below at (b). Similarly, **I**- and **RI**-forms can *RECH*$_3$, though the relationship is different. More specifically, *RECH*$_3$ ($\mathbf{I_0}$) = $\mathbf{RI_9}$.[16]

(b) *RECH*$_3$ing $\mathbf{P_0}$ to $\mathbf{R_6}$



Apart from showing the potential of various chains to transform rhythmic figures, the example shows that, in an ordered context, some concept of interval is necessary to understand *if* a chain can transform an object and *what* that chain's action means. Though it *RECH*$_3$'s into $\mathbf{R_6}$, $\mathbf{P_0}$ does not *RECH*$_3$ into *any* **R** form. The *intervallic relationship* between $\mathbf{P_0}$ and $\mathbf{R_6}$ is what allows for the chain to act there and not in other situations.

One benefit of exploring chains as syntactic elements involves their simplicity. To chain two series together, a number of elements at the end of one series need only be represented at the beginning of a different series. But as we consider *if* two series can chain and *how* a chain transforms a series, the *intervallic* makeup of that series will become increasingly important. To some extent this seems counterintuitive and complicates a chain's simplicity. The elements

[16] In this discussion, I have intentionally avoided saying that the **P** and **I** were "transformed" by *RECH*$_3$. This avoidance of the word "transformation" has to do with the fact that neither **R** nor **RI** can be *RECH*$_3$-ed at all. Figure 1.3(a) confirms that there is no retrograde of **R** or **RI** whose initial elements link to it by three durations. This interesting distinction between **P**/**I** and **R**/**RI**, as they exist in connection with *RECH*$_3$ will be taken up in greater detail soon.

involved in a chain are pitches, attacks, and so on. But those objects always exist in *reference* to other elements in a series. And understanding an objects referentially requires invoking the concept of interval.

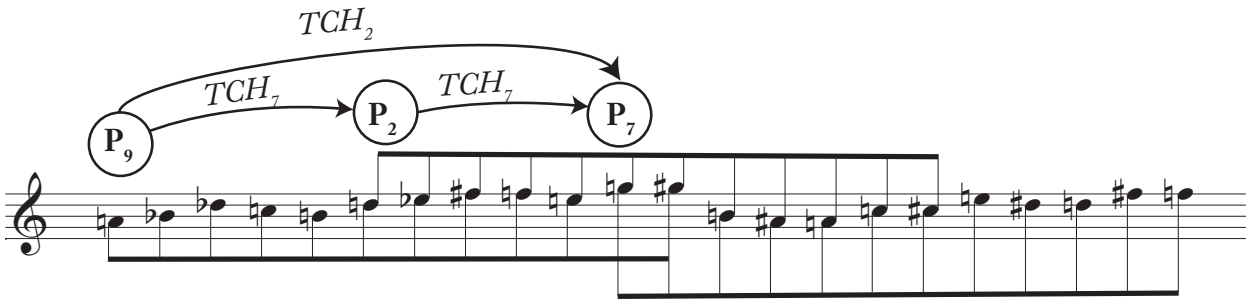
1.1.5 Example: Webern's Variations, Op. 30

In the Variations for Orchestra, Op. 30, TCH_7 works in tandem with TCH_2. Like the two compositions that preceded Op. 30, the row for this piece is *RI*-symmetrical. Because $\mathbf{P}_x = \mathbf{RI}_{x+4}$, $TCH_7 = RICH_7$. The seven-note chain (shown in Figure 1.4(a)) engulfs so much of the original row that when applied twice a third row appears that is TCH_2 of the original. In Figure 1.4(b), from the first variation, both types of chain are evident in the textural changes from solo melody to accompanimental quarter notes. TCH_2 connects $\mathbf{P}_9$ and $\mathbf{P}_7$ at mm. 21 and 29, both passages unfolding an identical rhythmic series that projects ic1-related pitch classes. While TCH_2 relates these passages of solo melody, TCH_7 coincides with the change to accompaniment at m. 24, as well as the change back to solo melody at m. 29.

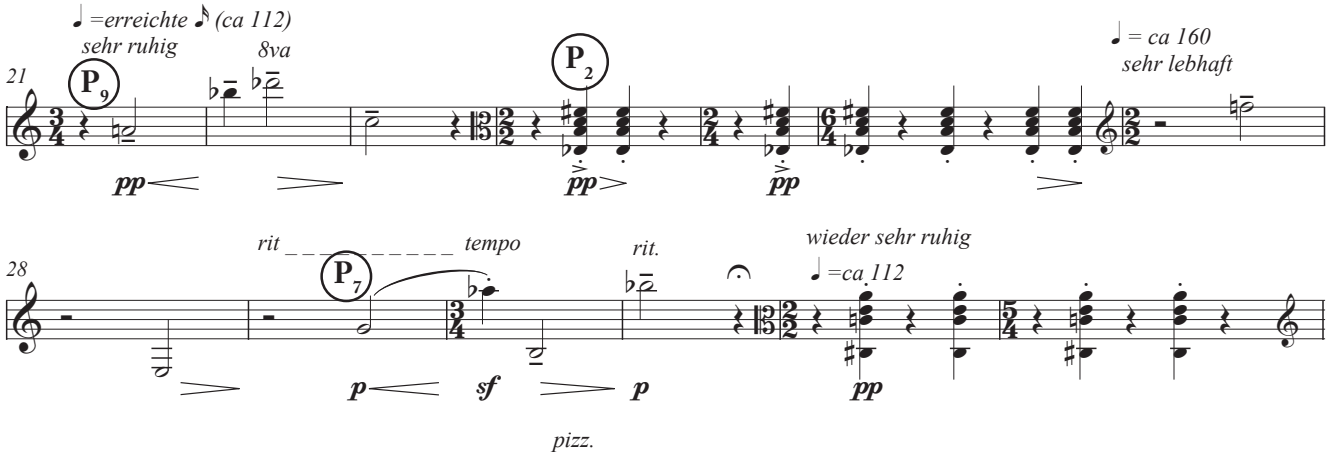
Each of these transformations's "meaning" is determined by its context—in particular, the *intervallic context* of the row. TCH_7 turns $\mathbf{P}_x$ into $\mathbf{P}_{x+5}$ because +5 is the directed interval from the first order position of the row to the sixth. Similarly, $\mathbf{P}_x$ becomes $\mathbf{P}_{x-10}$ when transformed by TCH_7 because +10 is the directed interval from the first order position to the penultimate one.

Figure 1.4. Combining chains: $TCH_7 * TCH_7 = TCH_2$ in the Variations, Op. 30.

(a) $TCH_7 * TCH_7 = TCH_2$



(b) Variations, Op. 30, mm. 21-33, reduction.



These examples show that chains can work in other musical contexts. Because they most often act on ordered objects, chains engage temporality, which is one of their defining features and one of the most important ways that they are distinguished from classical serial operations. Their reliance on context is also a distinguishing characteristic. The intervallic makeup of a series is especially important as a determinant of a chain's ability to transform something as well as that chain's transformational meaning

1.1.6 Function, Transformation, Operation

In the discussion of the musical excerpts above, I often called chains "transformations." It is worth specifying exactly what "transformation" means.[17] A transformation is a type of function, which we can symbolize as *f*. If we have two spaces, *S*, containing elements $\{s_0, s_1, \ldots\}$, and *T*, containing elements $\{t_0, t_1, \ldots\}$, a function *f* is a "rule" that specifies how to send each member *s* of *S* to some member *t* of *T*, often symbolized as $f(s) = t$.[18] In many musical situations, *S* and *T* are

[17] The terminological discussion that follows relates some basics of "transformation theory," though it is not should not be taken as a primer on transformation theory. An accessible introduction can be found in Steven Rings, *Tonality and Transformation* (New York: Oxford University Press, 2011), 9-40.

[18] This symbology calls to mind the grade-school plotting of functions on a Cartesian plane, and that act exactly depicts what a function is. We might imagine the x and *y* axis to contain integers. If $f(x) = y$, then graphing the function will create a line sloped at 45º.

the same, and this was the case in all but one of the examples above. In all of those examples, *f* was a chain.

In the (015) space and the two twelve-tone row spaces, each chain mapped an element in that space to another element *in the same space*. These types of functions are called *transformations*. A function's action can be easily visualized with a "mapping table" that shows the result of transforming any of the space's elements. Such a table is shown in Figure 1.5(a). This space visualizes the action of $RICH_2$ on a space *S* whose elements are the forty-eight row forms for Webern's Piano Variations. To interpret the table, substitute an integer from 0 to 11 for *x*-value (the row form's index number): if $x = 11$, for example, $RICH_2$ ($\mathbf{P_{11}}$) = $\mathbf{RI_{11+11} = RI_{10}}$. Because $\mathbf{RI_{10}}$ is a member of both columns, the action, $RICH_2$, links elements that both belong to the space *S*. And therefore, $RICH_2$ is a *transformation* from *S* onto *S*.

Two additional features of the Figure 1.5(a) are worth noting:

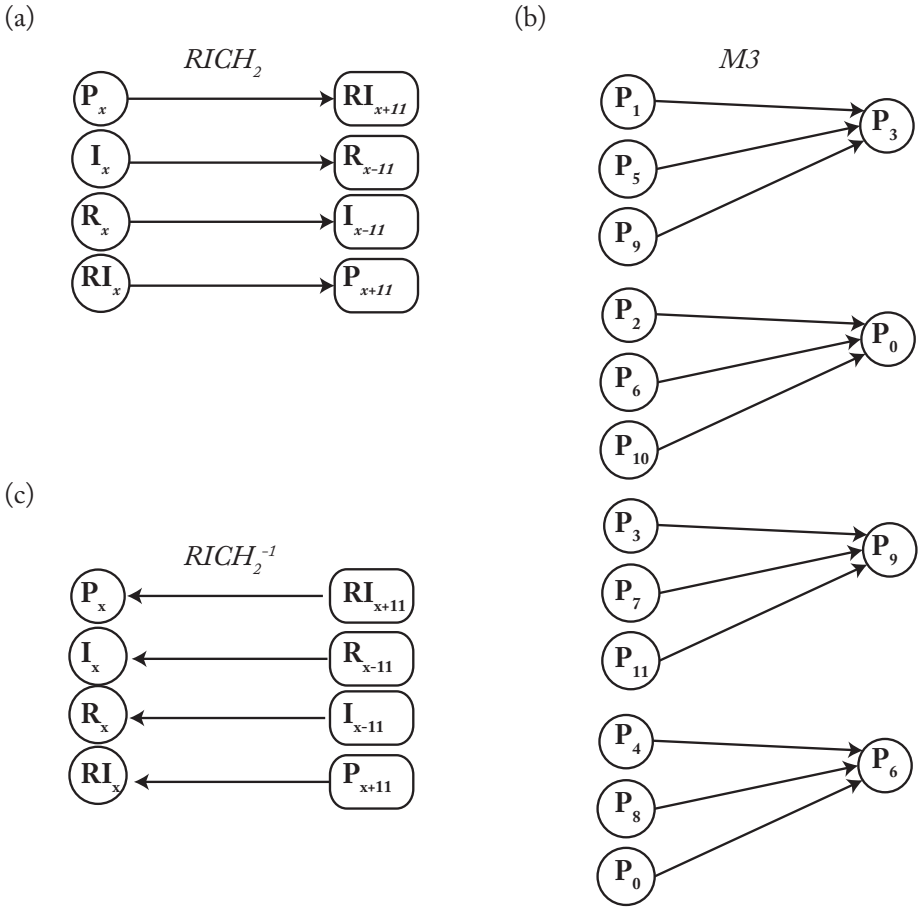
(1) Every member of the the left column has a *unique* target in the right column. That is, given two different rows $\boldsymbol{x}$ and $\boldsymbol{y}$, $RICH_2(\boldsymbol{x})$ is never the same as $RICH_2(\boldsymbol{y})$. This type of function is *one-to-one*.

(2) Every member of *S* is represented in the right column—all forty-eight row forms. Given that $RICH_2$ is a transformation, this situation means that both columns have the same number of elements, and every row form in the right column is a target of $RICH_2$ when applied to some member in the left column. This type of function is *onto*.

Transformations that are both *one-to-one* and *onto* are called *operations*. Thus, the $RICH_2$ chain is both a *transformation* and an *operation*.

Not every transformation is an operation, though many of the most familiar musical transformations are. To contrast the two, imagine a transformation (that is not an operation) acting on a space *S* of twelve rows, {$\mathbf{P_0}$, $\mathbf{P_1}$, […], $\mathbf{P_{11}}$}. This transformation is called *M3*, and it

FIGURE 1.5. Mapping tables showing an *operation* (at (a)), a *transformation* (at (b)) that is not an operation, and the *inverse* of $RICH_2$ at (c).



sends $\mathbf{P}_x$ to another row whose subscript is three times the subscript of the original, $\mathbf{P}_{x \cdot 3}$ (mod 12). The mapping table for *M3* is shown in Figure 1.5(b). This transformation is not *one-to-one* because not every element in the left-hand column has a *unique* target in the right hand column. Row forms whose subscripts are four more or four less than one another map to the same member of the right column. Moreover, *M3* is not *onto*: the four elements in the right-hand column represent only one-third of the totality of *S*, forms of the row whose subscripts are 0, 3, 6, or 9.[19]

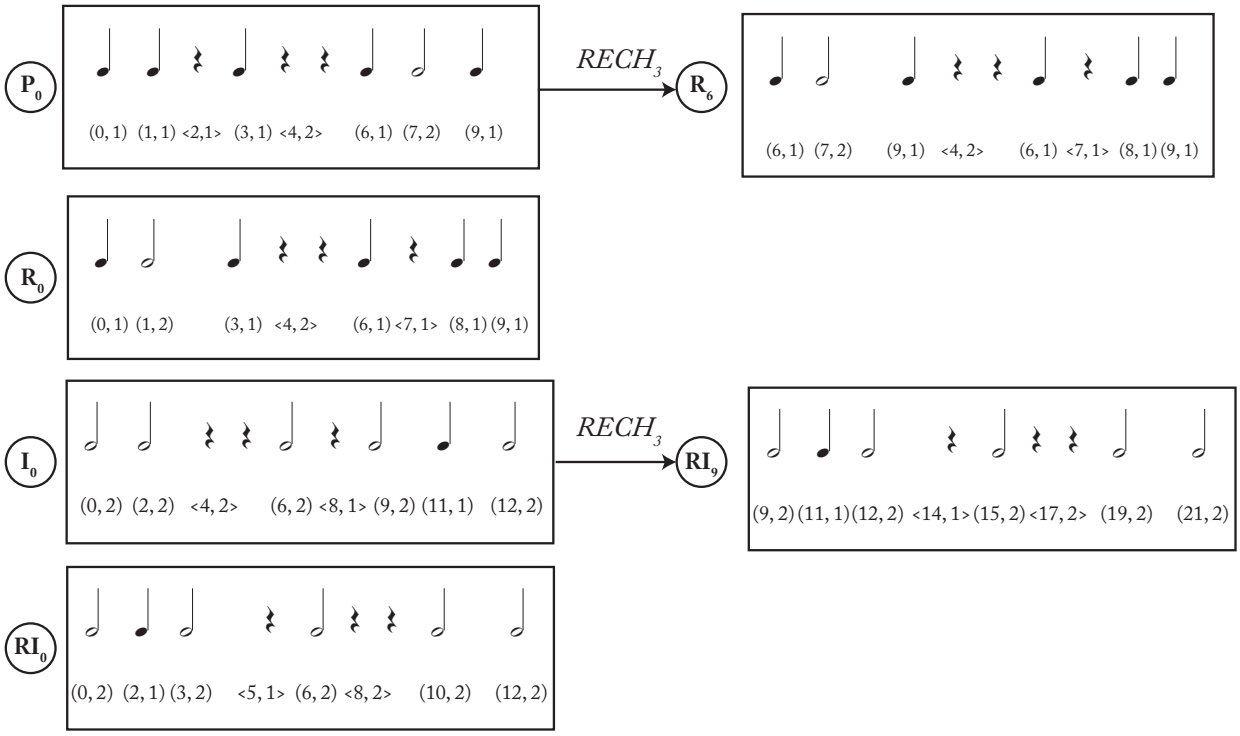
[19] Most commonly used transformations are also operations. Steven Rings has explored interesting tonal transformations that are neither *one-to-one* nor *onto*. For example, "resolving" transformations map members of a scale to particular triads. His "resI" would send a four-note dominant seventh chords onto a three-note tonic triad. This "many-to-few" relationship is common to transformations that are not also operations. See Rings, *Tonality and Transformation*, 125-9.

Because of these requirements, operations conform more intuitively to our concept of moving from one place to another. One particular reason for this intuition is that operations, unlike transformations, always have an *inverse*. The inverse of any operation (typically symbolized with a superscript "-1") reverses the arrows in that operation's mapping table, an action shown in Figure 1.5(c) for the operation $RICH_2$. Applied to $\mathbf{RI}_{x+11}$, the inverse of $RICH_2$—$RICH_2^{-1}$—produces $\mathbf{P}_x$. Transformations that are not operations do not have inverses. The mapping table for *M3* shows why. Imagine transforming $\mathbf{P_3}$ by $M3^{-1}$. Reversing every arrow would mean that a single object in the right column would point to three different members of the left column. There is no way to arbitrate between the three destinations. Operations, then, conform to a common bodily experience: if we know how to get from point *x* to point *y* (whether the points are physical locations in the real world or notes on a staff), we only need to do that backwards to get from point *y* to point *x*.

Most chains are operations. However, some chains are not even transformations. For example, imagine an infinite space *S* that contains serial permutations and transpositions of the time-point series for Webern's Cantata I, which was discussed earlier in §1.1.4. When $RECH_3$ acts on the elements of this space, it creates the mapping table shown in Figure 1.6: $RECH_3(\mathbf{P_0}) = \mathbf{R_6}$ and $RECH_3(\mathbf{I_0}) = \mathbf{RI_9}$. Although those **R**- and **RI**-forms of the series are *targets* of $RECH_3$, $RECH_3$ cannot act on either of them. *S* contains no elements that are the $RECH_3$ of **R** or **RI**.

For $RECH_3$ to be a transformation, it must be able to act on *every* element of the space, but it cannot do so in this case. If $RECH_3$ is not a transformation, then what is it? It is clear in this context that **P** and **I** are different types of elements than **R** and **RI**. The former are capable of being $RECH_3$ed and the latter are not. Therefore, we might imagine two spaces: *S* contains **P** and **I** while *T* contains **R** and **RI**. $RECH_3$ sends elements from *S* onto *T*. Defined in this way, $RECH_3$ is not a transformation, but it is, however, a *function*.

FIGURE 1.6. A mapping table for *RECH*$_3$ as it transforms a time-point series.



In the following section we will encounter a musical example of a chain that is not a transformation, but a function. Because of its broader currency in current music theory, we will often use the word *transformation* to refer to a general category that includes operations in the sense discussed above. *Functions* that are not transformations will be distinguished as such.

1.2 Formalities: Chains and Intervals

The complexity of musical syntax derives, at least in part, from music's diversity. That diversity belies a universal syntax, and therefore, much music theory has sought syntactical principles that have limited and varying degrees applicability—within a particular stylistic oeuvre, genre, or composer's output.[20] Nonetheless, most syntaxes proceed from basic principles that are shared with language. (1) In both music and language, syntax exists alongside a "lexicon," which in language refers to a person's vocabulary of words, and in music, refers to notes, chords, keys, and other musical objects. (2) Syntax is a force that constrains the *ordering* of these lexical objects. (3) Most explanations of musical syntax have shown that syntactical routines are determined to a some extent by the properties of the musical objects involved. Rameau's *corps sonore* generated the major triad and also described the intervals by which the fundamental bass could move. Schenker described music as unfolding of the tonic triad. Neo-Riemannian theory views many chromatic and parsimonious triadic progressions as the byproduct of the internal properties of the consonant triad.

Thus it would seem that inasmuch as transformation chains describe the linear ordering of rows and derive their meaning from them, they have a syntactical role in Webern's music. In the remainder of this chapter, I explore this role. First, I show (in §1.2 and 1.3) how a chain gets its meaning from a row or row class intervallic environment. That meaning directly impacts the temporality inherent in a row class, which I am concerned with in the final portion of this

[20] These studies often proceed from very different theoretical outlooks, most often in keeping with the purview of the repertoire under consideration. Dmitri Tymoczko's description of an "extended common practice" finds syntactical constraints that come about as a result of five general features of tonality. The generality of Tymoczko's approach allows him to study music spanning more than a century. See *A Geometry of Music* (New York: Oxford University Press, 2011. Robert Gjerdingen's study of Galant schemas has broad application, but within a particular stylistic era as its claims rest on a pedagogical tradition and repertoire of social cues that are specific to that time period. See *Music in the Galant Style* (Oxford University Press, 2007). Corpus studies, such as Ian Quinn's study of J.S. Bach's chorales, rely on the relatively small sample sizes and stylistic homogeneity, to find specific syntactical principles. See "Are Pitch-Class Profiles Really 'Key for Key'?," *Zeitschrift Der Gesellschaft Für Musiktheorie* 7, no. 2 (2010), http://www.gmth.de/zeitschrift/artikel/513.aspx.

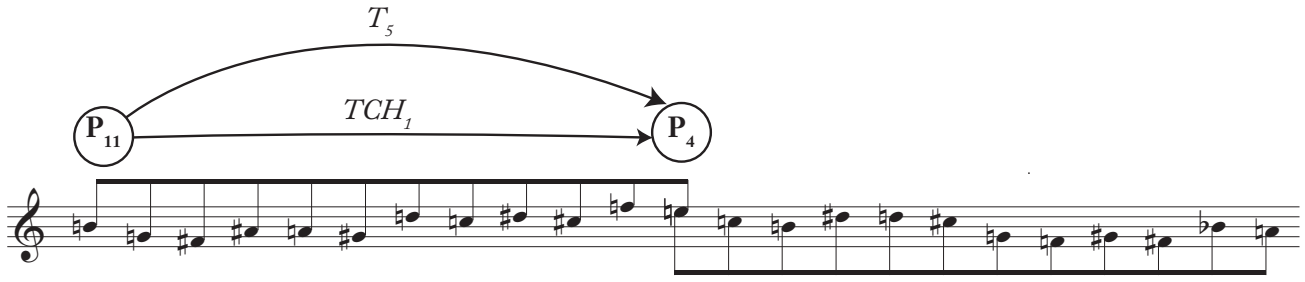
chapter (§1.4). There, I concern myself with spatial representations of compositional environments that capture a row class's temporality. These networks are shaped by the linear ordering imposed by a chain and by that chain's relationship to a row class. As before, part of this exploration is intended to emphasis the differences between chains and the classical serial operations that they superficially resemble. But the larger goal is to show how the special nature of chains bestows them with characteristics of *syntax* that are not enjoyed by the classical serial transformations.

1.2. What's Possible?

In earlier examples, I remarked that the intervallic structure of the row determined the target of a chain. Figure 1.7, given below, shows that the TCH_1 chains in Op. 27 and Op. 30 have very different meanings: at (a), TCH_1 sends the $\mathbf{P_{11}}$ row to $\mathbf{P_4}$; at (b), it sends $\mathbf{P_{11}}$ to $\mathbf{P_{10}}$. [21] They are different because the intervallic structure of each row is different—in the first case, the directed interval from the first pitch class to the last is 5, and in the second case, it is 11. (Note that TCH_1 is not equivalent to any particular transposition.) Apart from the specific meaning of a chain, intervallic structure also determines whether a chain can transform a row at all. In Figure

Figure 1.7. Comparing TCH_1 in the Piano Variations and the Orchestra Variations.

(a) TCH_1 connection in the Piano Variations, Op. 27

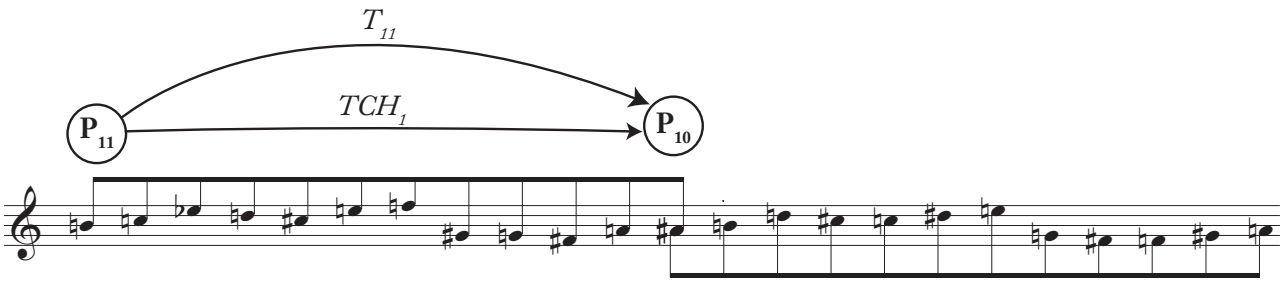


[21] The careful reader may object that the networks above the musical staves in Figure 1.7 are not "path consistent" and are, therefore, not well-formed. If I inserted different objects into the nodes, TCH_1 would not equal T_5. For now I will defer discussion of this important topic until later.

1.4, shown earlier, TCH_7 could transform *every* Op. 30 row. But TCH_7 cannot transform *any* Op. 27 row, or any other twelve-tone row that Webern conceived.

Considerations such as these do not apply to the classical serial operations *T*, *I*, *R*, and *RI* that chains superficially resemble, nor do the contextual considerations that give chains their

(b) TCH_1 connection in the Variations, Op. 30



meaning. Serial operations are universal. They do not belong to a specific compositional environment but are imposed on that environment from the "outside." Transformation chains exist "inside" a row class. While we can imagine the concept of a transformation chain in the abstract, chains have no real meaning outside of a particular row class.

In fact, whether a transformation chain exists or not is entirely dependent upon the intervallic characteristics of row class. A row class, which typically contains forty-eight members, can be defined by the four permutations of an adjacent interval series (AIS), $\{x_0, x_1, \ldots, x_{10}\}$, that I have shown in Table 1.1. Our initial exploration of chains will explore AIS conditions necessary for a particular chain to exist within a row class. Before continuing, the following the terms will be used throughout this exploration:

- A *segment* is a consecutive string of pitches or intervals drawn from an ordered series of pitches or an adjacent interval series.
- A segment that begins a series is called the *initial segment*.

- A segment that ends the series is called the *final segment*.[22]

Note that, in order to avoid confusion, abstract representations of pitch use the designation "*s*" while intervals are signified with "*x*".

TABLE 1.1. Intervallic transformations under the four chains.

	TCH	*ICH*	*RECH*	*RICH*
original series	same	inversion	retrograde inversion	retrograde
$\langle x_0, x_1, \ldots, x_{10} \rangle$	$\langle x_0, x_1, \ldots, x_{10} \rangle$	$\langle -x_0, -x_1, \ldots, -x_{10} \rangle$	$\langle -x_{10}, \ldots, -x_1, -x_0 \rangle$	$\langle x_{10}, \ldots, x_1, x_0 \rangle$

1.2.1 *TCH* CONDITIONS

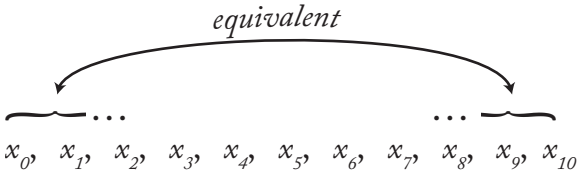
TCH_i can act on any member of its row class if, given any row, the initial and final segments of i - 1 directed pitch-class intervals are equivalent.

Figure 1.8 displays *TCH*'s "necessary intervallic configuration" at (a). Transpositionally related rows have the *same* adjacent interval series, and therefore, when transformed by TCH_i, the target row's adjacent interval series remains invariant. To overlap, then, the final segment of a row must be equal to the same-sized initial segment of the row. For example, a row class capable of TCH_3 requires equivalent segments at its beginning and end containing 2 = [3 - 1] intervals. Figure 1.8(b) illustrates: the interval segment $\langle x_9, x_{10} \rangle$ must be equivalent to the two-interval segment that initiates $T(\boldsymbol{S})$—$\langle x_0, x_1 \rangle$. For this equivalence to occur, the row form must have *equivalent* initial and concluding segments.

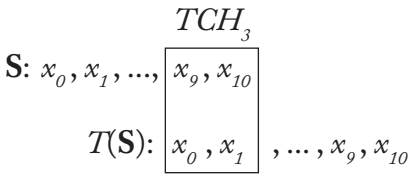
[22] Here, I leave the particular length of an initial or final segment undefined so that the terms may function in a variety of more specific contexts.

FIGURE 1.8. *TCH*'s intervallic requirements.

(a) required intervallic configuration for TCH_i



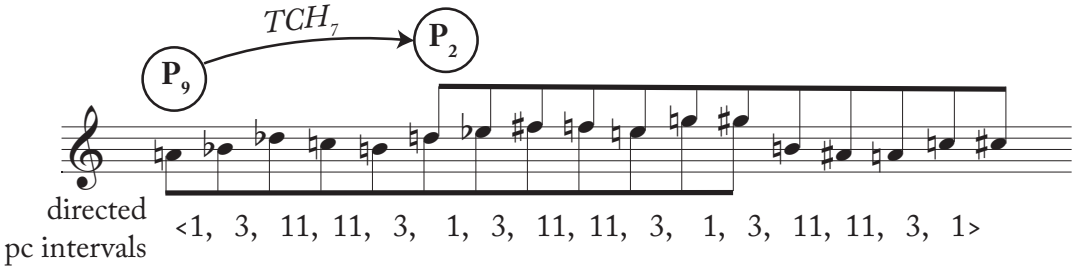
(b) intervallic equivalencies under TCH_3: $x_9 = x_0$; $x_{10} = x_1$



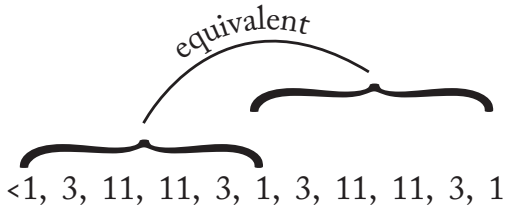
In general, a chain's length is one greater than the length of the interval segment involved in the elision. In Figure 1.8, the equivalent segments contained *two* intervals—hence, the $TCH_{\underline{3}}$ chain. In Op. 30, whose $\mathbf{P_9}$ and $\mathbf{P_2}$ rows are reprinted as Figure 1.9(a), the final *seven-note* segment of $\mathbf{P_9}$ elides with the initial seven notes of $\mathbf{P_2}$, allowing the TCH_7 chain discussed in 1.1.5. That such an elision is available to this row class is a byproduct of the ordered equivalence that exists between its initial and final segments of *six* intervals: <1, 3, 11, 11, 3, 1>, as shown in Figure 1.9(b).

FIGURE 1.9. *TCH* chains in Op. 30.

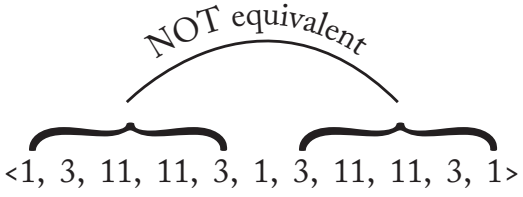
(a) TCH_7 transforming $\mathbf{P_9}$



(b) TCH_7



(c) TCH_6



Given a chain (CH_n), CH_n's ability to transform a series is not a guarantee that CH_{n-1}, CH_{n-2}, and so on can transform that series. For example, TCH_7's ability to transform the Op. 30 series does not transfer to TCH_6, TCH_5, and so on.Figure 1.9(c) shows that, if TCH_6 were to transform a row in the Op. 30 row class, the initial segment of five intervals would not be equivalent to the final segment of five intervals. Though the same unordered set of intervals occur in these segments, they are not ordered equivalently and TCH_6 cannot transform this row.

At its most extreme, an initial segment might be defined as comprising eleven intervals. Those eleven intervals subsume the entire row, and are (trivially) equivalent to the row's final segment. As a result, TCH_{12} can act on every twelve-tone series, and even more generally, given an interval series with n intervals, TCH_n can always transform that series.

Larger *TCH* chains are often correlated with row classes that are "derived" by some set-class type, which is not surprising given the intervallic equivalencies required for *TCH* to be available. Table 1.2 gives the Webern compositions associated with these large chains. Rows for Webern's Opp. 20, 25, 28, 29, and 30 are among the most highly derived in Webern's oeuvre. TCH_2-related pairs in Webern's Symphony, Op. 20 share the same set of discrete dyads: compare the *TCH_2-related* pair $\mathbf{P_0}$ = {C, B, F♯, F, B♭, A, C♯, D, G, A♭, E, F} and $\mathbf{P_4}$ = {E, F, B♭, A, D, C♯, F, F♯, B, C, A♭, G}. The Three Songs, Op. 25 and the String Quartet, Op. 28 have a similar structure. In both cases, the length of the chain (three and four, respectively) is coincident with the row's

derivation: in Op. 25, *TCH_3-related* rows have equivalent trichords,[23] and in Op. 28 *TCH_2-related* rows share discrete dyads and *TCH_4-related rows* share discrete tetrachords.[24]

TABLE 1.2. *TCH* chains in Webern's music.

TCH_2	op. 20, op. 28, op. 29, op. 30
TCH_3	op. 25
TCH_4	op. 28
TCH_7	op. 30

1.2.2. *ICH* CONDITIONS

ICH_i can act on any member of its row class if, given any row, the initial and final segments of i - 1 directed pitch-class intervals are inversionally equivalent.

TCH, as we have seen, creates a row whose intervallic series is the same as the original series. Table 1.1 shows that *ICH* inverts the original object's interval series. For the final segment of a row **S** to elide with the initial segment of **S**'s inversion, *I*(**S**), Figure 1.10 shows that **S**'**s** final segment must be the ordered inversion of its initial segment.

[23] In this piece, Webern never makes use of the TCH_3 chain, which would occur between rows related as $\mathbf{P}_x$ to $\mathbf{P}_{x+5}$. The whole first and last movements use **P**, **I**, **R**, and **RI** forms whose subscript is 0. Interestingly, the middle movement uses only row forms whose **P**, **I**, **R**, and **RI** forms whose subscript is 5. Therefore, in some sense, the middle movement is TCH_3-related to the first.

[24] The String Quartet, Op. 28 is discussed in greater detail at the close of Chapter 2.

FIGURE 1.10. Intervallic equivalencies under ICH_3.

$$\begin{array}{rl} & \overset{ICH_3}{\boxed{\phantom{x_9, x_{10}}}} \\ \mathbf{S}: & x_0, x_1, \ldots, \boxed{x_9, x_{10}} \\ I(\mathbf{S}): & \boxed{-x_0, -x_1}, \ldots, -x_9, -x_{10} \end{array}$$

ICH chains are uncommon in Webern's twelve-tone music. The Concerto for Nine Instruments, Op. 24, which features a highly derived row containing four serial permutations of the (014) trichord, is the only mature serial work to use a large *ICH* chain in its composition.[25]

1.2.3 EXAMPLE: WEBERN'S CONCERTO FOR NINE INSTRUMENTS, OP. 24

Figure 1.11(a) shows that the final trichord of an Op. 24 row is the ordered inverse of the first, which allows for ICH_3. In this case, ICH_3 links rows related as $\mathbf{P}_x$ is to $\mathbf{I}_{x+5}$ (Figure 1.11(b)). That linkage is interesting because rows related in this way share discrete trichords. This chain plays an important role in the final variation of Op. 24's third movement. The passage, reduced in Figure 1.11(c), shows that two $RICH_6$ transformations flank a central ICH_3-created oscillation between $\mathbf{P_9}$ and $\mathbf{I_2}$. [26] Like ICH_3, $RICH_6$ connects rows who share discrete trichords, and therefore, the transformational consistency in the passage is connected to its trichordal consistency.

In whole, the transformational structure of the passage is also symmetrical. And that large-scale symmetry echoes the smaller chordal symmetries bracketed below the piano part in mm. 58-60, mm. 63-65, and mm. 68-70, which together create an even larger symmetry, spanning the length of the passage.

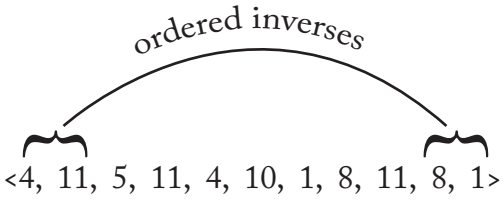
---

[25] Julian Hook and Jack Douthett discuss this movement in terms similar to these in "Uniform Triadic Transformations and the Twelve-Tone Music of Webern," *Perspectives of New Music* (2008): 91–151.

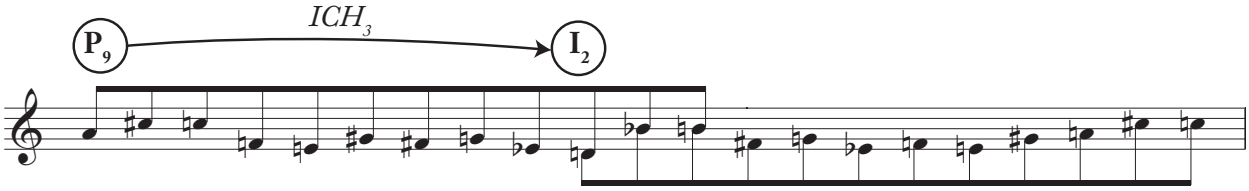
[26] The oscillation between $\mathbf{P_9}$ and $\mathbf{I_2}$ occurs because *ICH* is an *involution*: applied twice, *ICH* will always result in the original series.

FIGURE 1.11. *ICH$_3$* in the Concerto for Nine Instruments, Op. 24.

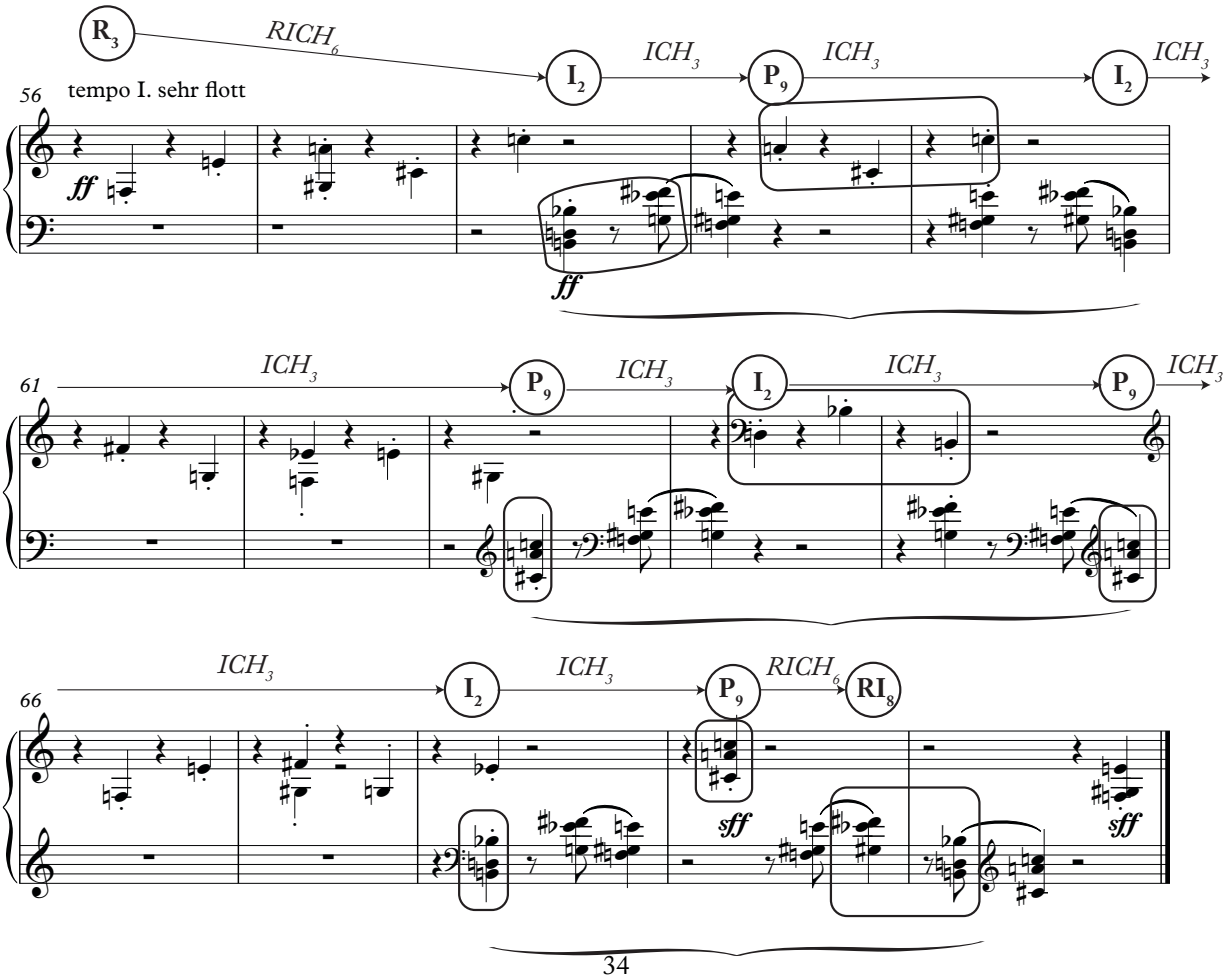
(a) The initial and final segments are ordered inverses of one another.



(b) *ICH$_3$* of $\mathbf{P}_x = \mathbf{I}_{x+5}$



(c) Webern, Concerto for Nine Instruments, Op. 24, III, mm. 56ff (reduction). Elided pitches are circled.



This variation highlights one of the practical reasons for a chain's presence in Webern's music. Chains, especially larger ones, obviate the need for unnecessary pitch repetition. Because each of the rows in this passage is highly similar as regards their discrete trichords, had the four rows followed one another without the elisions, each of the four discrete trichords would have appeared many more times. Avoiding those repetitions clarifies the interesting musical symmetries in the passage. Elided trichords occur melodically in the center each of the bracketed piano chords—mm. 59-60, mm. 64-5, and (harmonically) m. 69. Without the elisions, those trichords would have been repeated. As singularities, their larger symmetry becomes apparent. The {A, C♯, C} trichord in mm 59-60 is echoed in m. 69, separated by the {D, B♭, B} trichord in mm. 64-5.

1.2.4 *ICH* Possibilities

It is interesting to delimit the degree to which certain chain types can even act on a twelve-tone row. While Webern uses *ICH* less than *TCH*, it may simply be that *ICH*-able rows are more difficult to create or that they have less ability to transform twelve-tone rows. Outside of the strictures of twelve-tone composition, large *ICH* chains are easy to contrive. The basic twelve-tone axiom, however, requires that a row contain no duplicate pitches, which has important consequences for a row's interval series, and, therefore, its *ICH* possibilities. How large of an *ICH* chain is possible? To answer that question, we need to look more carefully at the intervallic requirements of an *ICH*-able series.

Imagine the most extreme example, ICH_{12}. In Figure 1.12(a), ICH_{12} requires all eleven intervals of the two AIS series to overlap; therefore, every interval in **S** must be its own inversion: $x_0 = -x_0$; $x_1 = -x_1$, and so on. But only the tritone is its own inversion, and within the bounds of twelve-tone composition, successive tritones are not allowed—they automatically create duplicate

pitches. Because we are studying intervals to determine a row's *ICH* capabilities, we need to know what intervallic rule constrains an eleven-interval AIS. In particular, a well-formed twelve-tone row cannot contain *any* segment of intervals that sums to 0 (mod 12).

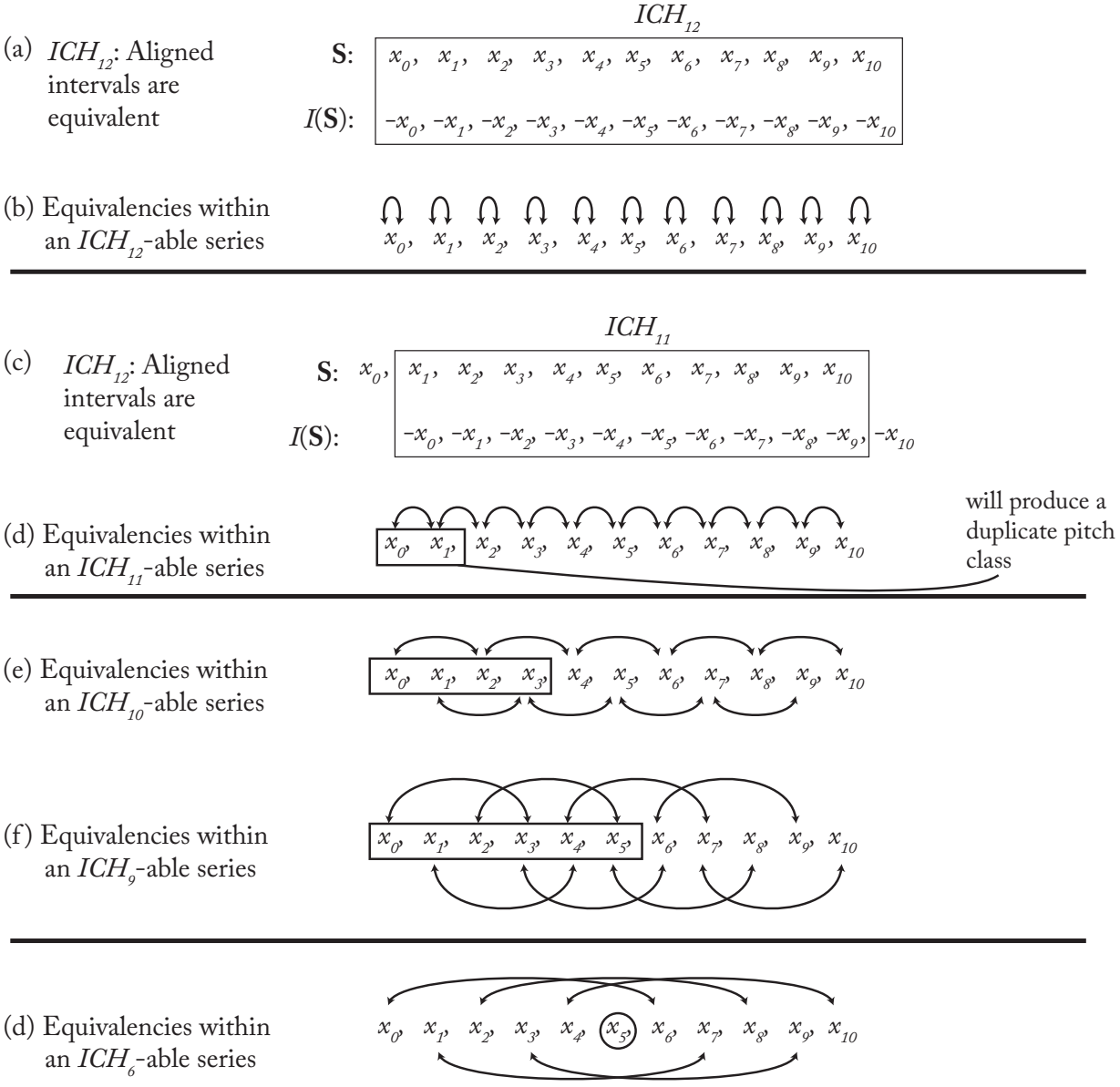
Because ICH_{12} requires the AIS to have all tritones, every adjacent interval series in an ICH_{12}-able row contains its complement. ICH_{11} is attempted at (c), and the required interval configuration is shown at (d). The arrows at (d) show that the intervallic conditions necessary to produce a row that can be ICH_{11}-ed violate the twelve-tone axiom: every adjacent segment of two intervals will sum to 12.

As the length of an *ICH* chain decreases by one, the size of the interval segment that sums to 0 increases by two. For example:

- ICH_{11}: a duplicate pitch occurs after the second interval (x_1).
- ICH_{10}, a duplicate pitch occurs after the fourth interval (x_3).
- ICH_9, a duplicate pitch occurs after the sixth interval (x_5).
- ICH_8, a duplicate pitch occurs after the sixth interval (x_7).
- ICH_7, a duplicate pitch occurs after the sixth interval (x_9).

A series that can be ICH_6-ed would contain a duplicate pitch after twelve intervals. But because a twelve-tone row contains only eleven intervals, ICH_6 *will be able to transform* an appropriately formed row, and therefore, ICH_6 is the largest *ICH* chain that can transform a twelve-tone row. In the ICH_6 configuration given at (g), the central interval (x_5) has no inverse in the intervallic series. As a result, the intervals within a series constructed like this will never sum to 0, and the series will always contain twelve different pitches.

FIGURE 1.12. *ICH* possibilities.



When acting on a row class, *TCH* and *ICH* are always operations—transformations that are *one-to-one* and *onto*. We have seen that the target of *TCH* or *ICH* is determined by the intervallic structure of the row. For example, in Figure 1.7(a), *TCH* transformed $\mathbf{P}_x$ onto $\mathbf{P}_{x+5}$ because the directed interval from the first to last pitch class was 5. Because the intervallic structure of a row class is constant for all rows in that class, given an x or y that is unique, $CH(\mathbf{P}_x)$

is never equivalent to $CH(\mathbf{P}_y)$ when *CH* is *TCH* or *ICH*. Therefore, *TCH* and *ICH* are *one-to-one* transformations. To be receptive to *TCH* or *ICH*, a row must possess an equivalency between the *initial* and *final* segments of a row. Because the members of a row class are *T*, *I*, *R*, or *RI* related, those intervallic equivalencies are necessarily contained in every row, and therefore, *TCH* and *ICH* are *onto*—if one row in a class can be transformed by *TCH* or *ICH*, every row can be transformed by them.

While *RECH* and *RICH* are often operations; they are not necessarily operations.

1.2.5 *RECH* Conditions

$RECH_i$ can act on a twelve-tone *row* if the interval constituents of the segment of $i - 1$ ordered intervals are inversionally symmetrical and i is an even number.

Note that $RECH_i$ is defined only on a row, not a row class. When *RECH*-ed, an interval series is retrograded and inverted (see Table 1.1 above). And therefore, the final segment of a row **S** becomes the initial segment of the target row $R(\mathbf{S})$, backwards and inverted. Figure 1.13(a) shows $RECH_3$ transforming **S**. The new series $R(\mathbf{S})$ can an overlap with **S** only if the inversion of the final interval (x_{10}) is equal to the penultimate interval (x_9)—more plainly, the final two intervals in **S** (x_9 and x_{10}) need to be complements of one another. Figure 1.13(b) makes this more concrete by substituting the intervallic series <3, 9> for x_9 and x_{10}. When $RECH_3$-ed, the directed interval 9 (= x_{10} of **S**) becomes 3 and interval 3 becomes 9, allowing the intervallic overlap needed for $RECH_3$.

Because the final segment of **S** becomes the initial segment of the new row, only the *final segment* needs to be inversionally symmetrical. Therefore, if a row form **S** can be *RECH*-ed, so can $I(\mathbf{S})$—both types of row will have final interval segments that are inversionally symmetrical. But within the same row class, it is not a given that $R(\mathbf{S})$ or $RI(\mathbf{S})$ will be able to be *RECH*-ed

because the final segments of those rows have no necessary relationship to the final segment of **S**. For an entire row class to be capable of $RECH_i$, the initial *and* final segments of intervals must be inversionally symmetrical, and those segments need not be inversionally symmetrical in equivalent ways.

FIGURE 1.13. Intervallic equivalencies under $RECH_3$.

(a) $x_9 = -x_{10}$; $x_{10} = -x_9$.

$$\begin{array}{rl} & \overset{RECH_3}{} \\ \mathbf{S}: & x_0, x_1, \ldots, \boxed{\begin{array}{c} x_9, \; x_{10} \\ -x_{10}, -x_9 \end{array}}, \ldots, -x_1, -x_0 \\ R(\mathbf{S}): & \end{array}$$

(b) Complementary intervals in the final segment lead to overlap when *RECH*-ed.

$$\begin{array}{rl} & \overset{RECH_3}{} \\ \mathbf{S}: & x_0, x_1, \ldots, \boxed{\begin{array}{c} 3, \; 9 \\ 3, \; 9 \end{array}}, \ldots, -x_1, -x_0 \\ R(\mathbf{S}): & \end{array}$$

1.2.6 EXAMPLE: *RECH₂*

Imagine $RECH_2$. Per §1.2.5, the final segment contains one interval, and that interval must must be its own complement. Therefore, any twelve-tone row ending with a tritone can be $RECH_2$-ed. Interestingly, none of Webern's row classes ends or begins with a tritone.

1.2.7 *RECH* POSSIBILITIES

Figure 1.13 constructs interval configurations for $RECH_3$ similar to those I created earlier for *ICH*. The row at (a) satisfies the requirements for $RECH_3$ because the two intervals in the row's final segment are complements in symmetrical positions within the segment. But the pitch-

class series at (b) shows that, in such situations, pitch duplications are assured. As long as the two intervals are complements, $RECH_3$ is not possible.

We saw something similar when discussing *ICH*. The pc duplication in Figure 1.14(a) is the result of the final segment summing to 0, which always occurs when complementary intervals are adjacent. Creating a twelve-tone row (with no pc duplications) that can be transformed by $RECH_3$ requires that the complementary intervals in the final segment be non-adjacent. This is possible only when the length *i* of the chain is even, as Figure 1.14(c) shows for $RECH_4$. In this case, the final segment of three intervals satisfies the requirement of the *RECH* while separating the complementary intervals. In a twelve-tone context, then, only seven types of *RECH* are possible, $RECH_1$ and the six even-length *RECH*'s.

FIGURE 1.14. Interval limitations on *RECH*.

(a) adjacent interval series for $RECH_3$: $\overset{\frown}{\langle x_0,\ x_1, \ldots, 3,\ 9\rangle}$

(b) example pitch-class series: {... , C, E♭, C}

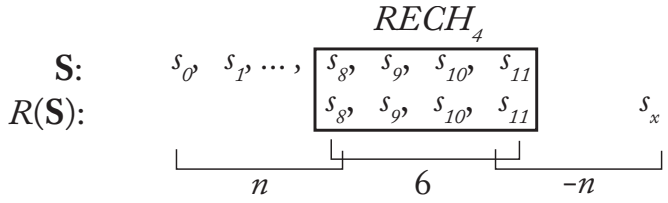
(c) adjacent interval series for $RECH_4$: $\overset{\frown}{\langle x_0,\ x_1, \ldots, 3, 6, 9\rangle}$

(d) example pitch-class series: {... , C, E♭, A, F♯}

Every $RECH_1$ has the equivalent effect of the order operation *R* because the first element of a $RECH_1$-ed row is the last element of the original row. By contrast, every *RECH* that is not $RECH_1$—that is, any of the six large, even-length *RECH* chains—is equivalent to T_6R, as shown at (e). As explained above, the directed interval between the first and last pitches in the elided pitch segment (s_8–s_{11}) must be a tritone because the first and last intervals (from s_8 to s_9 and s_{10} to

s_{11}) are complementary and the central interval is a tritone. When retrograded, the interval n becomes $-n$; and therefore, the final pitch class of $R(\mathbf{S})$, which is also the index number of that row, will always be $T_{n+6-n} = T_6$ away from the first pitch class of **S**.

(e) $RECH_i$, where i is greater than 1, is always equal to T_6R

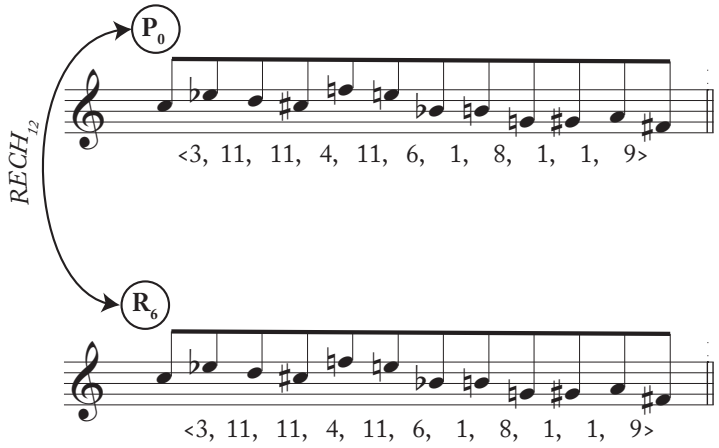


1.2.8 Example: *RECH*$_{12}$ and Webern's Symphony, Op. 21

Webern wrote only one row that could be *RECH*-ed by a chain larger than *RECH*$_1$. And as it happens, it is the largest possible, *RECH*$_{12}$. For *RECH*$_{12}$ to transform a series, the entire adjacent interval series is the *final segment*, and therefore, the entire adjacent interval series must be inversionally symmetrical, with a tritone at its center. Webern's row for the Symphony, Op. 21 has all of these properties. (See Figure 1.15.) When *RECH*-ed, the intervals are retrograded and inverted, as shown at (b), but the inversional symmetry means that the new interval series is identical to the original. The practical implications are that the row has only twenty-four distinct permutations because every $\mathbf{P}_x = \mathbf{R}_{x+6}$ and every $\mathbf{I}_x = \mathbf{RI}_{x+6}$.

Though this property has been oft-noted, the present conception is novel. It locates the smaller number of permutations in the rows's ability to *RECH*$_{12}$, of which the rows's symmetry is a necessary requirement.

FIGURE 1.15. *RECH*$_{12}$ and the Symphony, Op. 21.



1.2.9 *RICH* Conditions

Given a twelve-tone row form, *RICH*$_i$ can act on a row if the final segment of i - 1 intervals is non-retrogradable.

RICH-ing a series of intervals retrogrades the interval series (see Table 1.1). The last interval of the final segment becomes the first interval of the target rows's initial segment, the penultimate interval becomes the second interval, and so on, and therefore, as Figure 1.17 shows for *RICH*$_3$ and *RICH*$_4$, the first interval of the final segment must be equal to the last interval, the penultimate interval equal to the segment's second interval, and so on.

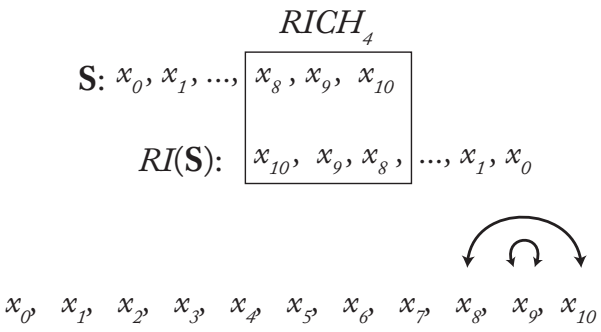
FIGURE 1.16. *RICH*$_3$'s and *RICH*$_4$'s intervallic equivalencies.

(a) $x_9 = x_{10}$; $x_{10} = x_9$.

$$\begin{array}{rl} & \overset{RICH_3}{\boxed{\phantom{x_9,\ x_{10}}}} \\ \mathbf{S}: & x_0, x_1, \ldots, \boxed{x_9,\ x_{10}} \\ RI(\mathbf{S}): & \boxed{x_{10},\ x_9,} \ldots, x_1, x_0 \end{array}$$

$$x_0,\ x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ x_6,\ x_7,\ x_8,\ \overset{\frown}{x_9,\ x_{10}}$$

(b) $x_8 = x_{10}$; $x_9 = x_9$.



For $RICH_2$, the final segment contains only one interval. That one interval is always equivalent to itself, and as a result, any row **S** will always be able to overlap with some row *RI*(**S**). $RICH_2$, then, can act on *any* ordered series of pitch-classes, and it is the only chain, aside from the one-note chains, that can do so. While $RICH_2$ carries the same promiscuity as the one-note chains, the larger *RICH* chains require the large intervallic symmetries noted in §1.2.9. These types of symmetries are characteristic of the rows in Webern's later twelve-tone music, and therefore, the largest *RICH* chains in Webern's music occur in those works (Table 1.3).

TABLE 1.3. *RICH* chains in Webern's music.

$RICH_3$	Op. 31
$RICH_6$	Op. 24
$RICH_4$	Op. 20, Op. 28
$RICH_7$	Op. 30
$RICH_{12}$	Op. 28, Op. 29, Op. 30

1.2.10 Example: *RICH*$_{12}$ and Webern's Opp. 28, 29, and 30

Earlier, we noted that *RECH*$_{12}$ offers a novel perspective on Webern's row for the Symphony, Op. 21. *RICH*$_{12}$ interacts similarly with the rows for Webern's Opp. 28, 29, and 30, the adjacent interval series of which are shown in Figure 1.17. When a row's entire adjacent interval series is non-retrogradable, the entire row satisfies the conditions in §1.2.9, and the whole row may be transformed by *RICH*$_{12}$. The resulting *RI*-related row entirely subsumes the original, and the therefore, every **P**-form is equivalent to an **RI**-form, and every **I**-form is equivalent to some **R**-form.

Figure 1.17. Adjacent interval series for Opp. 28, 29, and 30.
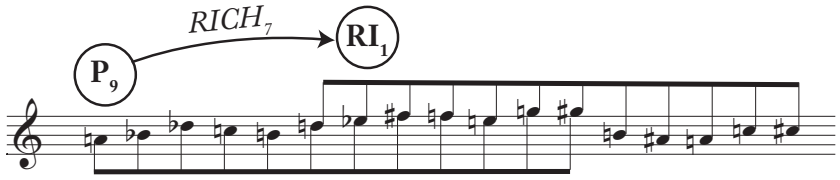
(a) String Quartet, Op. 28	<11, 3, 11, 4, 9, 1, 4, 11, 3, 11>
(b) Cantata I, Op. 29	<8, 3, 11, 4, 11, 3, 11, 4, 11, 3, 8>
(c) Variations, Op. 30	<1, 3, 11, 11, 3, 1, 3, 11, 11, 3, 1>

Rows equivalent under some *R* are *R*-symmetrical and can be transformed by *RECH*$_{12}$, as we saw in Figure 1.16, and those equivalent under some *RI* are *RI*-symmetrical. These symmetries create certain chain equivalencies. Within an *R*-symmetrical row class, *RECH*$_{12}$ = *TCH*$_{12}$. More interestingly, when a row class is *RI*-symmetrical, every *RICH*$_i$ is equivalent to *TCH*$_i$.

1.2.11 Example: Webern's Variations, Op. 30

As we have seen, the final segment of six intervals in the row class of the Variations, Op. 30, is non-retrogradable— <1, 3, 11, 11, 3, 1>. Therefore, *RICH*$_7$ is able to transform the row, as shown in Figure 1.18. This same relationship was shown earlier in Figure 1.9 to be *TCH*$_7$.

FIGURE 1.18. *RICH*$_7$ transforming **P**$_9$ in the Orchestra Variations, Op. 30.



As was true with *RECH*, *RICH* is not always an operation. Given a row class, if a row **S** can be transformed by *RICH*, *I*(**S**) can be as well, but not necessarily *R*(**S**) or *RI*(**S**).

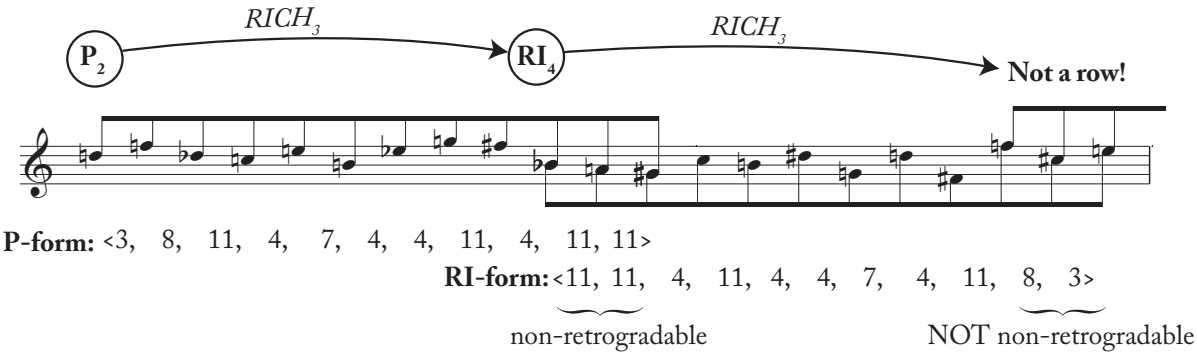
1.2.12 EXAMPLE: WEBERN'S CANTATA, OP. 31

These situations are rare, but one interesting occurrence is found Webern's last completed work, the Cantata II, Op. 31. Shown in Figure 1.19, **P**- and **I**-rows, by virtue of a final interval segment of <11, 11>, can be chained by *RICH*$_3$. In that figure, the final three pitches of **P**$_2$ become the first three of **RI**$_4$. However, the new **RI**-form does *not* end with a non-retrogradable final segment. Thus, *RICH*$_3$ of **RI**$_4$ is not possible; there is no row within the forty-eight members of the row class that begins {F, C♯, E}.

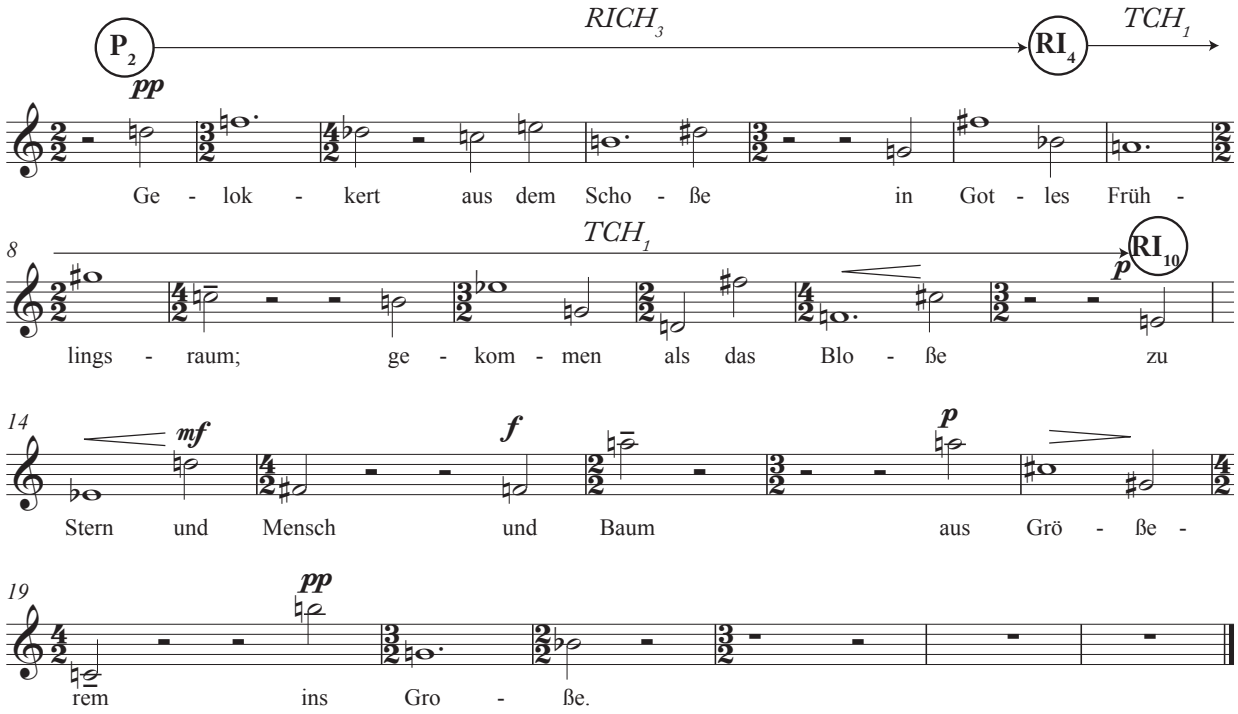
In Webern's setting of Hildegard Jone's poem for the final movement (the entire tenor line is shown in Figure 1.19(b), *RICH*$_3$ is "blocked" in mm. 12–13. In the preceding twelve measures, **P**$_2$ and **RI**$_4$ were chained together to set the twenty syllables that comprise the first three lines of Jone's poem. (The three-note elision allows two row forms—which would typically require twenty-four syllables—to set twenty instead.) Because **RI**$_4$ cannot be *RICH*$_3$-ed, Webern uses *TCH*$_1$ to link **RI**$_4$ to the **RI**$_{10}$ row that begins with the pickup to m. 14. At this point, however, the poem requires thirteen additional syllables (from "zu" to the end) and Webern's row has only twelve notes available. The setting solves this compositional problem by repeating the A5 from "Baum" to "aus," at the climactic moment of the piece.

FIGURE 1.19. In Op. 31, *RICH*$_3$ can transform **P** and **I** forms, but not **R** or **RI** forms.

(a) *RICH*$_3$ (**P**$_2$) = **RI**$_4$, but *RICH*$_3$ (**RI**$_4$) does not produce a row in the row class.



(b) *RICH*$_3$ in Webern's Cantata II, Op. 31 "Gelockert aus dem Schosse".



The example is reminiscent of the rhythmic series discussed in §1.1.4, on which $RICH_3$ could transform only half of the row class. As there, the $RICH_3$ chain in the Cantata II is not a transformation at all but a simple *function* from *S* to *S'*, where *S* contains all of the **P**- and **I**-forms, and *S'* contains **R**- and **RI**-forms.

1.3 Chain Meaning

How does the row class determine the the result of applying a chain? We can view the result of applying any chain to any row using a *mapping table*, such as that given in Table 1.4. The many variables (x, y, z, and q) in the header column and within the body of the table are indicative of its abstract, contextual nature: each of those variables represent intervallic ingredients that are individual to a given row (all addition and subtraction is performed mod 12):

- x = a row's index number.

The following three variables are calculated from the *row class's* prime form:[27]

- y = int(s_0, s_{12-i}), the directed interval between the row's first pitch (s_0) and its first chained pitch (s_{12-i});
- z = int (s_0, s_{11}), the directed interval between the first and last pitch (s_{11}) of the row;
- q = int(s_0, s_{i-1}), the interval between the first pitch and the i-th pitch class minus 1, where i is the length of the chain.

Table 1.4. Mapping table for all CH_i acting on any capable twelve-tone row.

	TCH_i	ICH_i	$RECH_i$	$RICH_i$
$\mathbf{P}_x$	$\mathbf{P}_x \rightarrow \mathbf{P}_{x+y}$	$\mathbf{P}_x \rightarrow \mathbf{I}_{x+y}$	$\mathbf{P}_x \rightarrow \mathbf{R}_{x+(y-z)}$	$\mathbf{P}_x \rightarrow \mathbf{RI}_{x+(y+z)}$
$\mathbf{I}_x$	$\mathbf{I}_x \rightarrow \mathbf{I}_{x-y}$	$\mathbf{I}_x \rightarrow \mathbf{P}_{x-y}$	$\mathbf{I}_x \rightarrow \mathbf{RI}_{x+(y-z)}$	$\mathbf{I}_x \rightarrow \mathbf{R}_{x-(y+z)}$
$\mathbf{R}_x$	$\mathbf{R}_x \rightarrow \mathbf{R}_{x-y}$	$\mathbf{R}_x \rightarrow \mathbf{RI}_{x+y}$	$\mathbf{R}_x \rightarrow \mathbf{P}_{x+(y-z)}$	$\mathbf{R}_x \rightarrow \mathbf{I}_{x+q}$
$\mathbf{RI}_x$	$\mathbf{RI}_x \rightarrow \mathbf{RI}_{x+y}$	$\mathbf{RI}_x \rightarrow \mathbf{R}_{x-y}$	$\mathbf{RI}_x \rightarrow \mathbf{I}_{x+(y-z)}$	$\mathbf{RI}_x \rightarrow \mathbf{P}_{x-q}$

[27] Analysts often disagree about which row should be the "prime" form. We take the view that these distinctions are largely irrelevant. For the sake of the calculations in this table and in all that follows, however, it will not matter which row is the "prime" form of the row.

1.3.1 EXAMPLE: *TCH1* IN WEBERN OP. 23 AND OP. 28

Supposing a one-note chain (i = 1) were acting on one of the forty-eight rows for Webern's *Drei Gesange*, Op. 23, where **P0 =** {C, G, B, G♯, D, B♭, F♯, A, F, E, C♯, E♭}, y, z, and q are calculated as follows:

$$y = \text{int}(\text{C, E}\flat) = 3\ ;$$

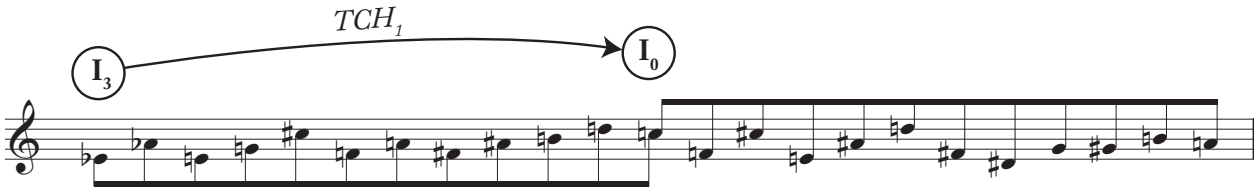
$$\text{z} = \text{int}\ (\text{C, E}\flat) = 3;$$

$$\text{and, } q = (\text{C, C}) = 0.$$

With these variables in hand, the mapping table specifies the precise result of performing any chain transformation. For example, to determine $TCH_1(\mathbf{I_3})$, follow the process indicated in Table 1.4: subtract y (= 3) from the row's index number (x): $TCH_1(\mathbf{I_3}) = \mathbf{I_{3\text{-}3}} = \mathbf{I_0}$. (See Figure 1.20(a)).

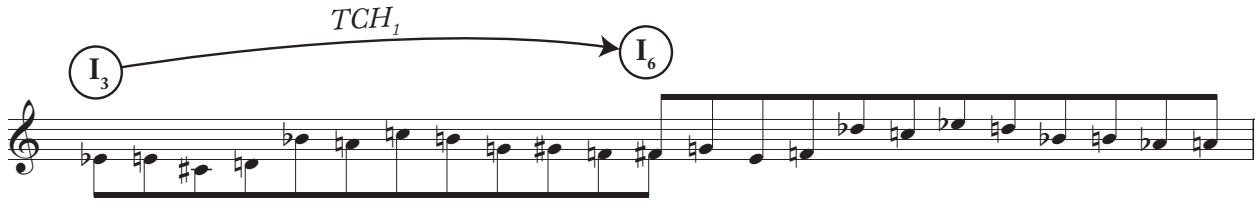
FIGURE 1.20. *TCH1* transforming **I3** in Op. 23 and Op. 28.

(a) Webern's Op. 23: $TCH_1(\mathbf{I_3}) = \mathbf{I_0}$



Of course, if the row class were different, the result of *TCH1* may be as well. Imagine TCH_1 acting on Webern's String Quartet, Op. 28, where **P0 =** {C, B, D, C♯, F, F♯, D♯, E, G♯, G, B♭, A}: Here, y = 9, z = 9 and q = 0. Therefore, $TCH_1(\mathbf{I_3}) = \mathbf{I_{3\text{-}9}} = \mathbf{I_6}$, as shown in Figure 1.20(b).

(b) Webern's Op. 28: $TCH_1(\mathbf{I_3}) = \mathbf{I_6}$



Though simple, the similarities and differences between Figure 1.20(a) and (b) are crucial. Both show the *same* transformation being performed on a row that has the *same* label, though each are from *different* compositions. The internal structure of each row (in particular, their *y* value, the directed interval from the first to last pitch class of the row class's prime form) is entirely responsible for the divergent results of applying the chains. Transpositional interpretations of these row successions would of course be different, and that difference is real and may be meaningful in some context. Here, however, the identical TCH_1 relationship foregrounds the role of row overlap in "driving" *both* relationships between successive rows.

1.3.2 Example. TCH_4 in Webern's Op. 28

The variables *x*, *y*, *z*, and *q* vary with a chain's length. Above, I indicated how the action of one-note chains on the row for Webern's String Quartet, Op. 28 was calculated using $y = 9$, $z = 9$ and $q = 0$. If instead we suppose that a length 4 chain (such as $RICH_4$) were transforming the row, then y = int(C, G♯) = 8; z = int (C, A) = 9; and, q = int(C, C♯) = 1. These differences are shown in Tables 1.5(a) and (b), and applying $RICH_4$ ($\mathbf{I_3}$) (= $\mathbf{R_{3\text{-}(8+9)} = R_{10}}$) is shown in Figure 1.21.

Table 1.5. CH_1 and CH_4 acting on Webern's Op. 28.

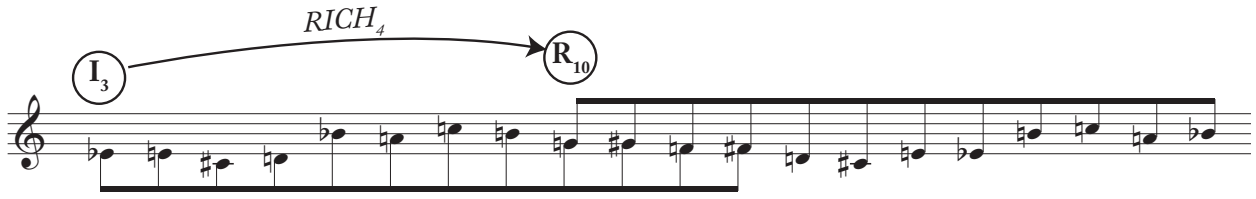
(a) $\boldsymbol{CH_1}$, where $y = 9$, $z = 9$ and $q = 0$ (cf. Table 1.4)

	TCH_1	ICH_1	$RECH_1$	$RICH_1$
$\mathbf{P}_x$	$\mathbf{P}_x \rightarrow \mathbf{P}_{x+9}$	$\mathbf{P}_x \rightarrow \mathbf{I}_{x+9}$	$\mathbf{P}_x \rightarrow \mathbf{R}_{x+(9-9)}$	$\mathbf{P}_x \rightarrow \mathbf{RI}_{x+(9+9)}$
$\mathbf{I}_x$	$\mathbf{I}_x \rightarrow \mathbf{I}_{x-9}$	$\mathbf{I}_x \rightarrow \mathbf{P}_{x-9}$	$\mathbf{I}_x \rightarrow \mathbf{RI}_{x+(9-9)}$	$\mathbf{I}_x \rightarrow \mathbf{R}_{x-(9+9)}$
$\mathbf{R}_x$	$\mathbf{R}_x \rightarrow \mathbf{R}_{x-9}$	$\mathbf{R}_x \rightarrow \mathbf{RI}_{x+9}$	$\mathbf{R}_x \rightarrow \mathbf{P}_{x+(9-9)}$	$\mathbf{R}_x \rightarrow \mathbf{I}_{x+0}$
$\mathbf{RI}_x$	$\mathbf{RI}_x \rightarrow \mathbf{RI}_{x+9}$	$\mathbf{RI}_x \rightarrow \mathbf{R}_{x-9}$	$\mathbf{RI}_x \rightarrow \mathbf{I}_{x+(9-9)}$	$\mathbf{RI}_x \rightarrow \mathbf{P}_{x-0}$

(b) ***CH₄***, where $y = 8$, $z = 9$ and $q = 1$ (cf. Figure 1.21)[28]

	TCH₄	*ICH₄*	*RECH₄*	*RICH₄*
$\mathbf{P}_x$	$\mathbf{P}_x \rightarrow \mathbf{P}_{x+8}$			$\mathbf{P}_x \rightarrow \mathbf{RI}_{x+(8+9)}$
$\mathbf{I}_x$	$\mathbf{I}_x \rightarrow \mathbf{I}_{x-8}$			$\mathbf{I}_x \rightarrow \mathbf{R}_{x-(8+9)}$
$\mathbf{R}_x$	$\mathbf{R}_x \rightarrow \mathbf{R}_{x-8}$			$\mathbf{R}_x \rightarrow \mathbf{I}_{x+1}$
$\mathbf{RI}_x$	$\mathbf{RI}_x \rightarrow \mathbf{RI}_{x+8}$			$\mathbf{RI}_x \rightarrow \mathbf{P}_{x-1}$

FIGURE 1.21. In Op. 28, *RICH₄* **(I_3) = R_{10}**.



1.3.3 CHAINS AS "UNIFORM TRIADIC TRANSFORMATIONS"

Most importantly, the table makes it clear that some chain transformations affect half of the class in equal but opposite ways. For example, *TCH* adds the constant *y* to the index number of **P** and **RI** forms but subtracts it from **I** and **R** forms. *ICH* has a similar property. The *y* value is *added* to the index number of **P** and **R** forms, but *subtracted* from **I** and **RI** forms.

These are important properties of transformation chains that distinguish them from transposition and fixed-axis inversion. A compelling way to specify the difference is by notating

[28] *ICH₄* and *RECH₄* are not available here for the reasons explored in §1.2.2. and §1.2.5. *ICH₄* requires that the initial and final segments of three intervals be inversionally equivalent, but in this case those segments are identical: <11, 3, 11>. *RECH₄* requires the final segment be inversionally symmetrical, which it is not.

each of the transformations as a "Uniform Triadic Transformation" (UTT). This notation will prove useful later as well when we consider the commutative properties of chains.

Julian Hook described the 288 UTTs as transformations acting on *triads*.[29] Hook uses a novel system of notation where each triad has a root corresponding to a pc number (0-11) and a sign (+ or -) that indicates whether the triad is major or minor. C major is represented as 0^{+} and E minor as 4^{-}. Every UTT $\langle \mathbf{\sigma}, t^{+}, t^{-} \rangle$ has two transposition levels (t^{+} and t^{-}) that indicate the how a major (+) or minor (-) triad is transposed. The value **σ** is a sign (+ or -) that indicates whether the UTT is "mode-preserving" or "mode-reversing": a "-" sign changes a major triad into a minor one, and a minor triad into a major one; a "+" sign indicates that the triad will maintain its mode.[30] So if we apply the UTT ⟨+, 7, 7⟩ to C major (0^{+}) the resulting triad is G major (7^{+}). Similarly, applying ⟨+, 7, 7⟩ to C minor (0^{-}) results in G minor (7^{-}). Thus, the UTT ⟨+, 7, 7⟩ describes the pc transposition T_7.

With only slight alterations, the UTT system can be used to describe twelve-tone rows and the transformations on them.[31] Like the triadic representations, each row has a root corresponding to the first pc of the row for **P** and **I** forms, and the last pc of the row for **R** and **RI** forms. To account for the retrograde modes, each row has two signs. The first sign in each pair is the *inversion sign* and the second is the *retrograde sign*. Thus, $\mathbf{P_0} = 0^{++}$, $\mathbf{I_0} = 0^{-+}$, $\mathbf{R_0} = 0^{+-}$, and $\mathbf{RI_0} = 0^{--}$. UTTs act on twelve-tone rows in much the same way that they act on triads. The sign **σ** indicates if the row's *inversion sign* is changed. For example, applying ⟨+, 7, 7⟩ to $\mathbf{P_0}$ (0^{++}) creates

---

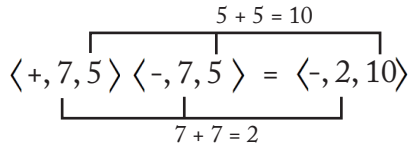
[29] Julian Hook, "Uniform Triadic Transformations," *Journal of Music Theory* 46, no. 1/2 (2002): 57–126. Hook, "Uniform Triadic Transformations," generally describes triads in the form (*r*, **σ**). C major is (0, +), and the formulation 0^{+} is shorthand.

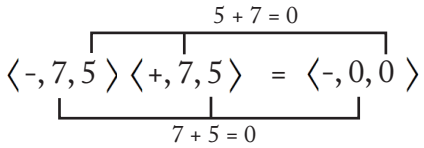
[30] Ibid., 61-2.

[31] See Julian Hook and Jack Douthett, "Uniform Triadic Transformations and the Twelve-Tone Music of Webern," *Perspectives of New Music* (2008): 91–151.

$\mathbf{P_7}$ (7^{++}). But, applying ⟨-, 7, 7⟩ to $\mathbf{P_0}$ (0^{++}) creates $\mathbf{I_7}$ (7^{-+}).Because the retrograde operation (*R*) is commutative, it can be adjoined to the any of the 288 UTTs to create a larger group of 576 transformations. When *R* is appended to a UTT, the *retrograde sign* of the row is changed: when ⟨+, 7, 7⟩*R* acts on $\mathbf{P_0}$ (0^{++}), the result is $\mathbf{R_7}$ (7^{+-}). Thus, ⟨+, 7, 7⟩*R* describes T_7R.

UTTs combine in simple ways. UTTs with like signs—"++" or "--"—produce a mode-preserving transformation (+). Those with unlike signs produce a mode-reversing transformation. The transposition values combine based upon the sign of the first UTT. For example:





UTT language allows us to see similarities amongst transformations that at first may seem very different. For example, Hook notes that a pc transposition T_y always has a UTT representative ⟨+, *y*, *y*⟩. "Riemannian" transformations—such as, but not limited to, *P*, *L*, and *R* have t^+ and t^- values that sum to 0, exhibiting what Hook calls the "Riemannian dualism condition."[32] The Riemannian transformation group contains twenty-four members, twelve *Schritts* and twelve *Wechsels*. *Schritts* are mode-preserving UTTs and *wechsels* are mode-reversing

[32] Hook, "Uniform Triadic Transformations," 74.

TABLE 1.6. Transformation Chains as UTTs

Transformation	UTT	UTT Type	Notes	Ex.: $y = 1, z = 1, q = 0$
TCH (**P** or **I**)	$\langle +, y, -y \rangle$	Riemannian	Equivalent to a Riemannian *Schritt*, S_y	$\mathbf{P_0} \xrightarrow{TCH_1} \mathbf{P_1}$; $\mathbf{I_0} \xrightarrow{TCH_1} \mathbf{I_{11}}$
TCH (**R** or **RI**)	$\langle +, -y, y \rangle$	Riemannian	Equivalent to a Riemannian *Schritt*, S_{-y}	$\mathbf{R_0} \xrightarrow{TCH_1} \mathbf{R_{11}}$; $\mathbf{RI_0} \xrightarrow{TCH_1} \mathbf{RI_1}$
T_y	$\langle +, y, y \rangle$	Pc transposition		$\mathbf{P_0} \xrightarrow{T_1} \mathbf{P_1}$; $\mathbf{I_0} \xrightarrow{T_1} \mathbf{I_1}$
ICH	$\langle -, y, -y \rangle$	Riemannian	Equivalent to a Riemannian *Wechsel*, W_y	$\mathbf{P_0} \xrightarrow{ICH_1} \mathbf{I_1}$; $\mathbf{I_0} \xrightarrow{ICH_1} \mathbf{P_{11}}$
RICH (**P** *or* **RI**)	$\langle -, (y+z), -q \rangle R$			$\mathbf{P_0} \xrightarrow{RICH_1} \mathbf{RI_2}$; $\mathbf{RI_0} \xrightarrow{RICH_1} \mathbf{P_0}$
RICH (**R** *or* **I**)	$\langle -, q, -(y+z) \rangle R$			$\mathbf{I_0} \xrightarrow{RICH_1} \mathbf{R_{10}}$; $\mathbf{R_0} \xrightarrow{RICH_1} \mathbf{I_0}$
$RECH_1$	$\langle +, 0, 0 \rangle R$	Riemannian	Equivalent to the order operation R	$\mathbf{P_0} \xrightarrow{RECH_1} \mathbf{R_0}$; $\mathbf{I_0} \xrightarrow{RECH_1} \mathbf{RI_0}$
$RECH_i$, when $i > 1$	$\langle +, 6, 6 \rangle R$	Riemannian	Equivalent to T_6R	$\mathbf{P_0} \xrightarrow{RECH_6} \mathbf{R_6}$; $\mathbf{I_0} \xrightarrow{RECH_6} \mathbf{RI_6}$

UTTs. Thus, *P*, *L*, and *R* are *wechsels*: $P = \langle -, 0, 0 \rangle$, $L = \langle -, 4, 8 \rangle$, and $R = \langle -, 9, 3 \rangle$.[33]

Table 1.6 shows each of the four chains as a UTT. The table indicates that two of the four chains are Riemannian in character. *TCH* will always be equal to some Riemannian *Schritt*, though the specific *Schritt* varies according to the row's *y* value, and *ICH* is a *Wechsel*.[34] *RECH* is an order operation equal to *R*, except when *RECH*'s length is greater than one. In that case, *RECH* is equal to T_6R (see §1.2.7).

This table also shows that *TCH* and *RICH* stand for two unique UTTs each, depending on the row type being transformed, and those UTTs are always inverse related. Thus, while *TCH* (**P** or **I**) = $\langle +, y, -y \rangle$, *TCH* (**R** or **RI**) = $\langle +, -y, y \rangle$.[35] The *y* -values for each transpositional level sum to 0. It may be less obvious that *RICH* (**P** or **RI**) is the inverse of *RICH* (**I** or **R**), so consider the following compound *RICH* transformation:

$$\underbrace{\langle -, (y+z), -q \rangle R}_{-q + q = 0} \underbrace{\langle -, q, -(y+z) \rangle R}_{(y + z) - (y - z) = 0} = \langle +, 0, 0 \rangle$$

In the expression, note that the two *R*s "cancel themselves," as do the two transposition levels. Therefore, the two types of *RICH* represented in Table 1.6 are always inverses.

---

[33] The twenty-four Riemannian transformations have a variety of names. Following Riemann, Klumpenhouwer named them as *x-Schritt* or *x-Wechsel*, where *x* is the interval between dual roots. A *Quintschritt*, for example, would describe the relationship between an C major chord and a G major chord, a C minor chord and an F minor chord ("Some Remarks on the Use of Riemann Transformations," *Music Theory Online* 9 (1994).) Gollin's system is similar. It labels *Schrits* and *Wechsels* as S_n or W_n, where *n* is the interval between dual roots. S_5 is the same as Klumpenhouwer's *Quintschritt*. See Edward Gollin, "Some Aspects of Three-Dimensional 'Tonnetze'," *Journal of Music Theory* 42, no. 2 (1998): 195–206.

[34] Hook, "Uniform Triadic Transformations," 78-81 helpfully provides a classification of Riemannian UTTs that also indicates which UTTs correspond to terminology used elsewhere. For example, the *wechsel* $\langle -, 5, 7 \rangle$ has elsewhere been called *L', nebenverwandt*, *Seitenwechsel*, or W_0. Hook also proposes an intuitive shorthand terminology (similar to Gollin, "Some Aspects of Three-Dimensional 'Tonnetze'") for each of these transformations. A *Schritt* $\langle +, y, -y \rangle$ is called S_y, and a *wechsel* $\langle -, y, -y \rangle$ is called W_y.

[35] Thus, a *TCH* chain operating on a row whose $y = 1$, is S_1 when transforming **P** or **I** rows and S_{11} when transforming **R** or **RI** rows.

1.4 Representing Row Class Temporality

Reciprocality, as I have shown in §1.2 and §1.3 is, *sine qua non,* the most important descriptor of the chain/row relationship. Specific chain types emerge only in a highly specified environments, and moreover, those environments determine a chain's meaning entirely, not unlike the plants that Webern observed on alpine hikes. In fact, reciprocality of this sort is entirely in keeping with the Webern's natural, organicist compositional aesthetic.[36] About the Variations, Op. 30, which we discussed earlier in §1.1.5 and §1.2.10, Webern wrote:

> Imagine this: 6 notes are given, in a *shape* determined by the sequence and the rhythm, and what follows […] is nothing other than this shape over and over again!!! Naturally in continual 'Metamorphosis' […] but it is the same every time. Goethe says of the "Prime Phenomenon" [*Urphänomen*]:
>
> > 'ideal as the ultimate recognizable thing,
> > real when recognized*,
> > symbolic, since it embraces every case,
> > identical with every case**'.
>
> * in my piece that is what it is, *namely the shape mentioned above!* (The comparison serves only to clarify the *process*.)
> ** Namely in my piece! That is what it does! [37]

[36] Webern, like the other members of the Second Viennese School, was very much influenced by Goethe's explanations of organicism. Their association is well known and has been the object of many studies. Two recent contributions to this line of inquiry are: Gareth Cox, "Blumengruß and Blumenglöckchen: Goethe's Influence on Anton Webern," in *Goethe: Musical Poet, Musical Catalyst: Proceedings of the Conference Hosted by the Department of Music, National University of Ireland, Maynooth, 26 & 27 March 2004*, ed. Lorraine Byne (Dublin: Carysfort Press, 2004), 203–224; and Lorian Meyer-Wendt, "Anton Webern's Musical Realization of Goethe's Urpflanze Concept in Drei Lieder, Op. 18" (M.M. thesis, The Florida State University, 2004). This influenced is discussed in more detail in Chapter 4.

[37] Webern, *Letters to Hildegard Jone and Josef Humplik*, ed. Josef Polnauer, trans. Cornelius Cardew (Bryn Mawr, PA: Theodore Presser, 1967), 44, emphasis is Webern's.

By invoking Goethe's *Urphänomen*, his description of the music's "process," "Metamorphosis," and "what it does" becomes tied organically to the "shape" of the "6 notes" that initiate the row. In other words, the object suggests the behavior.[38]

Within transformation theory, more generally, this interrelationship places chains within the larger world of "contextual transformations."[39] In that they respond to an object, contextual transformations subtly alter the technical separation of musical object and transformation group that is basic to the group theory that rests at the foundation of transformation theory. Separation of object and transformation is basic to transformation theory, and in many ways it has proven to be quite valuable. It allows formal comparisons of relationships between different types of musical objects.[40] And separating object and transformation allows analysts to detail compelling musical recursions.[41]

Nonetheless, transformation theory's separation of object and transformation can be viewed as problematic as it potentially places transformations in an "active" role and musical objects in a "passive" role, perhaps voiding the anthropomorphic roles that analysts often bestow onto music. Daniel Harrison, for example, says that this separation causes "[o]bjects [to be] inert

---

[38] Severine Neff discusses Goethe's *Urphänomen* in the context of Schoenberg's theoretical and analytical terminology. She notes that the *Urphänomen* was "the archetype" ("Schoenberg and Goethe: Organicism and Analysis," *Music Theory and the Exploration of the Past* (1993): 413).

[39] Aside from the neo-Riemannian transformations *P*, *L*, and *R*, *RICH* is perhaps the most discussed contextual transformation. Contextual transformations, according to Philip Lambert," are "transformations that are sensitive to particular aspects of a given musical context" ("On Contextual Transformations," *Perspectives of New Music* 38, no. 1 (2000): 46). I might specify this somewhat precisely: contextual transformations are sensitive to particular aspects of a given musical *object*, though the transformations themselves are often suggested by a musical *context*. Lambert notes that many contextual transformations derive from "invariance patterns," including the chain *RICH* that we have been studying. Generally, such transformations fall into one of two categories, those that related T_n/I_n-equivalent objects (like chains) and those that do not. See Lewin's discussion of Jonathan Bernard's FLIPEND and FLIPSTART as an example of the latter category (*GMIT*, 189). Jonathan W. Bernard, *The Music of Edgard Varèse* (New Haven: Yale University Press, 1987).

[40] For example, in *GMIT* Lewin uses the separation to show *RICH* in a variety of contexts in music by Bach, Wagner, and Webern (180-92).

[41] Lewin's analytical use of Klumpenhouwer networks, for example, often sought recursion between pitch classes and chords. See "Klumpenhouwer Networks and Some Isographies That Involve Them," *Music Theory Spectrum* 12, no. 1 (1990): 83–120; David Lewin, "A Tutorial on Klumpenhouwer Networks, Using the Chorale in Schoenberg's Opus 11, No. 2," *Journal of Music Theory* 38, no. 1 (1994): 79–101.

and without tendency, and all activity and meaning [to be] supplied by transformations applied to them."[42] For Harrison, the separation is problematic because it models musical motion in an illusory way. Musical objects are not moving themselves, according to their own properties and those of their musical environment, but instead, musical motion is accomplished by an external force: "transformational theory appears to model the metaphor of musical motion by constructing a ventriloquist's dummy; it only appears to be alive, but is in fact a construction of lifeless parts that are made to move by some external force."[43]

Harrison's "dummy" assumes that object and transformation are *independent*, and that musical motion is a passive act. It may be that we should not expect musical motion to imitate the natural world, and thus, metaphors like Harrison's—which criticize transformation theory for its inability to do so—are somewhat unfair.[44] But apart from that criticism, contextual transformations generally, and transformation chains specifically, *are dependent* on the objects they transform. In the chain-conditioned twelve-tone context we are exploring, objects are not "inert"

---

[42] Daniel Harrison, "Three Short Essays on Neo-Riemannian Theory," in *The Oxford Handbook of Neo-Riemannian Music Theories*, ed. Edward Gollin and Alexander Rehding (New York: Oxford, 2011), 552.

[43] Ibid.

[44] Some might say that transformation theory invites these criticisms, and that Lewin's invocation of the "transformational attitude" incited the poetic, anthropomorphic metaphors that are common in many transformational analyses (*GMIT,* 159). Indeed, these considerations are at the core of the opening of Harrison's essay (2011, 548-53). A great deal of discussion has centered on "attitude" shift that may or may not have been important in Lewin's description of transformation groups. Henry Klumpenhouwer has argued that a central narrative in *GMIT* is the displacement of the static, intervallic, Cartesian thinking with dynamic, anti-Cartesian, "transformational" thinking. See Henry Klumpenhouwer, "Essay: In Order to Stay Asleep as Observers: The Nature and Origins of Anti-Cartesianism in Lewin's Generalized Musical Intervals and Transformations," *Music Theory Spectrum* 28, no. 2 (2006): 277–289. Julian Hook is skeptical that this type of philosophical reorientation was indeed the motivation for Lewin's invocation of the transformational attitude. He notes that Lewin calls interval-language and transformational-language "two aspects of one phenomenon" (160) and that transformation theory " 'subsumes' GIS theory" ("David Lewin and the Complexity of the Beautiful," *Intégral* 21 (2007): 155–190).

and reliant upon a transformation to give it "activity and meaning," but instead, they participate in the creation of that transformation's meaning and thus give rise to their own activity.[45]

In §1.2 I claimed that syntactical descriptions of music have generally accounted not just for the temporal arrangement of music, but also shown that that *temporality is suggested by the kinds of musical objects involved.* Diatonic tonal syntax and chromatic tonal syntax, for instance, evince different syntactical routines, but those routines are, in both cases, influenced by the structure of tonal elements like triads and seventh chords.[46] Similarly, by virtue of their interrelationship, transformation chains forge a relationship with twelve-tone rows that describes Webern's twelve-tone syntax, a relationship that *does not exist* when between twelve-tone rows and the classical serial operations.

Transformation chains find similarities with the syntactical rules of tonal voice leading and harmonic progression in at least three ways, none of which are shared by the classical serial operations. (1) Chains have "universality" within a row class. Many voice-leading rules, such as the commonly-known axiom that perfect fifths and octaves should not move in parallel motion, are universals that hold outside of a particular tonal context. Amongst the forty-eight serial operations, none has "primacy" in a given twelve-tone context. (2) Chains make row progression more "exclusive." A multitude of requirements influences whether a chain can connect a pair of rows. By contrast, the group of classical serial operations is extremely promiscuous, allowing connections between *any* two row forms in a row class. (3) Chains are "contextual," depending

---

[45] Richard Cohn advances a similar argument in response to Harrison's claim, but in the context of neo-Riemannian operations and their relationships to consonant triads: "[o]ne of the desirable qualities of a theory is the ability to demonstrate a relationship between the *internal properties* of an object and its *function* within a system […]." The "structure of triads, as objects"—their near-evenness—"is intimately related to their function, as participants in hexatonic (and, more broadly, pan-triadic) syntax." See *Audacious Euphony: Chromaticism and the Triad's Second Nature* (New York: Oxford, 2012): 39-40). Cohn's work in this area has consistently, and compelling, shown that the properties of the objects themselves suggest the neo-Riemannian transformations. We can make a similar claim as to the status of transformation chains within a twelve-tone environment.

[46] Cohn has an interesting discussion of "double syntax." The central claim of his study, he notes, is that "two incommensurate ways of measuring triadic distance—*"triadic syntax"*—emerge respectively from two independent *properties of consonant triads*" (*Audacious Euphony*, 195-210, emphasis is mine).

upon the row to tell them what they can do. Harmonic progression, similarly, depends on a triad's tonal position to determine what progressions are normative.

1.4.1 Representation

Musical syntax has long been depicted with graphical representations. Peter Westergaard notes that spatial depictions of pitch are found as far back as Boethius and were suggested, more than two centuries ago, in ancient Greek theorists "spatial reasoning."[47] Spatial diagrams of key relationships were relatively common in the eighteenth and nineteenth centuries, and to be sure, most of those diagrams were conceived as depictions of normative syntax. Heinichen's famous *Musicalischer Circul*, which was imitated and improved on by numerous (mostly German) music theorists, is, above all, a practical, compositional guide to key relationships.[48] In the nineteenth century, these models expanded to include relationships amongst chords—the most famous being the *Tonnetz*.[49] More recently, music theory has witnessed a profusion of spatial diagrams, prompting Joseph Straus to christen the era a "new space age."[50] Many of these representations have origins in Lewin's transformation theory. [51] Others, particularly those describing voice

---

[47] Peter Westergaard, "Geometries of Sounds in Time," *Music Theory Spectrum* 18, no. 1 (1996): 1–21.

[48] Among those following in Heinichen's footsteps are David Kellner, Lorenz Christoph Mizler Kolof, Georg Andreas Sorge, and Gottfried Weber. Heinichen's practical orientation is best contrasted with Weber, whose "table of key relationship" had an explicitly psychological orientation.

[49] Leonhard Euler created the first *Tonnetz* in 1739. It was revived in the latter half of the nineteenth-century by German theorists, including Hugo Riemann. For more on its history, see Michael Kevin Mooney, "The 'Table of Relations' and Music Psychology in Hugo Riemann's Harmonic Theory" (Ph.D. dissertation, Columbia University, 1996); Edward Gollin, "Some Further Notes on the History of the Tonnetz," *Theoria* 13 (2006): 99–111; and Richard Cohn, "Tonal Pitch Space and the (Neo-) Riemannian Tonnetz," in *The Oxford Handbook of Neo-Riemannian Music Theories*, ed. Edward Gollin and Alexander Redhing (New York: Oxford, 2011), 322–48.

[50] Straus, "Contextual-Inversion Spaces," 46. Of course, as we have noted, spaces have been part of music theory for quite some time. This "new space age" is at least as much the product of the importance of mathematics in recent music theory, as well as the explosion of technological means by which to created such diagrams.

[51] Of particular note in this context is David Lewin's *Musical Form and Transformation: Four Analytical Essays* (New Haven: Yale University Press, 1993), particularly Chapter 2 (16-67), which explores the process of making a spatial network for Stockhausen's *Klavierstück III*. Steven Rings, in *Tonality and Transformation*, has used many of Lewin's ideas about network construction to describe tonal phenomenon (9-150).

leading, have the neo-Riemannian brand of transformation theory as important predecessor.[52]

Transformational spaces, of the type pioneered in Lewin' *GMIT* and *Musical Form and Transformation*, will be of great relevance to all that follows. But of no less significance is the concept of "compositional space," which has been explored by Robert Morris.[53] In his "model of the compositional process (329), compositional spaces are "out-of-time" structures" that precede a compositional design.[54] More specifically, "a compositional space is a set of musical objects related and/or connected in at least one specific way."[55] Morris defines two broad categories of compositional space. "Literal" spaces contain actual musical objects, while "abstract spaces assert possible literal, more specific spaces," but in fact contain categories, like a set-class type, for example. Abstract and literal spaces can be un-ordered—a twelve-tone matrix, for example—or ordered.

In the following, and for much of the succeeding chapter, I will discuss transformational spaces containing rows that are ordered by transformation chains. In Morris's terms, these abstract spaces are "musical grammars" because they order musical categories temporally.[56]

---

[52] The "neo-Riemannian" revival of the *Tonnetz* was inspired by Lewin's *GMIT*. The *Tonnetz* has been explored by many authors since. Transformational approaches to voice leading are important predecessors of the more recent spatial explorations of the geometry of voice leading. See, for example, John Roeder, "A Theory of Voice Leading for Atonal Music" (Ph.D dissertation, Yale University, 1984); John Roeder, "A Geometric Representation of Pitch-Class Series," *Perspectives of New Music* 25, no. 1/2 (1987): 362–409; Jack Douthett and Peter Steinbach, "Parsimonious Graphs: A Study in Parsimony, Contextual Transformations, and Modes of Limited Transposition," *Journal of Music Theory* 42, no. 2 (1998): 241–263; Richard Cohn, "A Tetrahedral Graph of Tetrachordal Voice-leading Space," *Music Theory Online* 9, no. 4 (2003); Dmitri Tymoczko, "The Geometry of Musical Chords," *Science* 313, no. 5783 (2006): 72–74; Clifton Callender, Ian Quinn, and Dmitri Tymoczko, "Generalized Voice-Leading Spaces," *Science* 320, no. 5874 (2008): 346–348; Dmitri Tymoczko, *A Geometry of Music.*

[53] Morris has discussed this topic in a number of articles emanating from his book, *Composition With Pitch-Classes: A Theory of Compositional Design* (New Haven: Yale University Press, 1987). "Compositional Spaces and Other Territories" takes up the idea of a "compositional space" in greater detail (*Perspectives of New Music* 33, no. 1/2 (1995): 328–358. And "Voice-Leading Spaces" explores the concept as it relates to neo-Riemannian-like transformations that exploit efficient voice leading (*Music Theory Spectrum* 20, no. 2 (1998): 175–208).

[54] Morris "Compositional Spaces," 329-30.

[55] Ibid., 336.

[56] Ibid., 340.

Furthermore, these spaces are "cyclic"—they have no beginning or ending.[57] As compared with the abstract musical grammars that represent chord progression in tonal music, these spaces will, at first, seem incredibly rudimentary. In Chapter 2, I introduce additional concepts that make the spaces more robust models for Webern's music.

1.4.2 Some Spatial Networks

Chapter 2 of Lewin's *Musical Form and Transformation* is a tutorial on constructing a network. Lewin discusses two types of network, *formal networks* and *figural networks*.[58] Formal networks capture out-of-time, a priori relationships between objects. Figural networks capture chronology. They are "blow-by-blow," left-to-right. I will adopt John Roeder's terminology by calling Lewin's formal networks *spatial networks* and figural networks *event networks*.[59] Lewin's tutorial underscores the significant differences between spatial and event networks, particularly the efficacy of each network type as a representational tool. In this vein, Lewin deals with issues of node/arrow arrangement and with the necessity or lack thereof of a network to represent "'potentialities' rather than 'presences.'"[60] Though Lewin's discussion of the relevant issues of representation are quite rich, many have noted that they leave open other, more practical

---

[57] Many of these spaces are *cyclic* in the group theoretic way as well, though that is not what Morris means by the term.

[58] Lewin, *Musical Form and Transformation*, 16-67. The terms "formal and "figural are borrowed from Jeanne Bamberger, "Cognitive Issues in the Development of Musically Gifted Children," in *Conceptions of Giftedness*, ed. Robert J. Sternberg and Davidson (New York: Cambridge University Press, 1986), 388–413.

[59] John Roeder, "Constructing Transformational Signification: Gesture and Agency in Bartók's Scherzo, Op. 14, No. 2, Measures 1–32," *Music Theory Online* 15, no. 1 (2009), http://www.mtosmt.org/issues/mto.09.15.1/mto.09.15.1.roeder_signification.html.

[60] Lewin, *Musical Form and Transformation*, 35.

questions, including how to select object families, the merits of event networks, and the formalities underlying spatial and event networks.[61]

In the following section, I explore the construction of a spatial network, leaving the details of event networks for Chapter 2.[62] This discussion will involve a brief review of some basic axioms of group theory, particularly the way in which *Cayley diagrams* are visual means for representing the structure of groups.[63] Spatial networks are isomorphic to the Cayley diagram created from a transformation groups's generator. Aside from provide a methodologically consistent way to create a spatial network, Cayley diagrams visually highlight the importance of a generator, which for present purposes, are nearly always a transformation chain. Because those chains are determinants of temporality in Webern's twelve-tone music, these spatial networks become powerful, abstract musical grammars that capture the unique syntax of a particular compositional environment.

Figure 1.22(a) begins this discussion with an exemplary spatial network, the circle of fifths. Like most compositional spaces, the circle-of-fifths emphasizes a privileged theoretical relationship that arranges the twelve major triads in a particular way. In fact, we could imagine the space as a particular type of *transformation network*[64] that allows us to visualize *group* of

---

[61] Edward Gollin study of spatial representations details conceptions of distance on "formal" (or spatial) networks. See "Representations of Space and Conceptions of Distance in Transformational Music Theories" (Ph.D. dissertation, Harvard University, 2000): 1-107. Steven Rings has defined "figural" networks in particularly detailed terms. See his "Tonality and Transformation" (Ph.D. dissertation, Yale University, 2006). Also relevant in this regard is Roeder, "Constructing Transformational Signification."

[62] Much of this discussion is influenced by Gollin, "Representations of Space," 1-107; Nathan Carter, *Visual Group Theory* (Washington D.C.: Mathematical Association of America, 2009); and Julian Hook, *Musical Spaces and Transformations* (forthcoming, n.d.).

[63] This discussion is not a primer on group theory or transformation theory. Those interested in more detail should see Robert Morris, *Class Notes for Advanced Atonal Music Theory* (Lebanon, New Hampshire: Frog Peak Music, 2001); Ramon Satyendra, "An Informal Introduction to Some Formal Concepts from Lewin's Transformational Theory," *Journal of Music Theory* 48, no. 1 (2004): 99–141; Rings, *Tonality and Transformation*; and Hook, *Musical Spaces and Transformation*.

[64] See Lewin, *GMIT*, 196, Definition 9.3.1.

operations acting on a set of *objects*. Underlying a transformation network such as Figure 1.22(a) are three primary components:

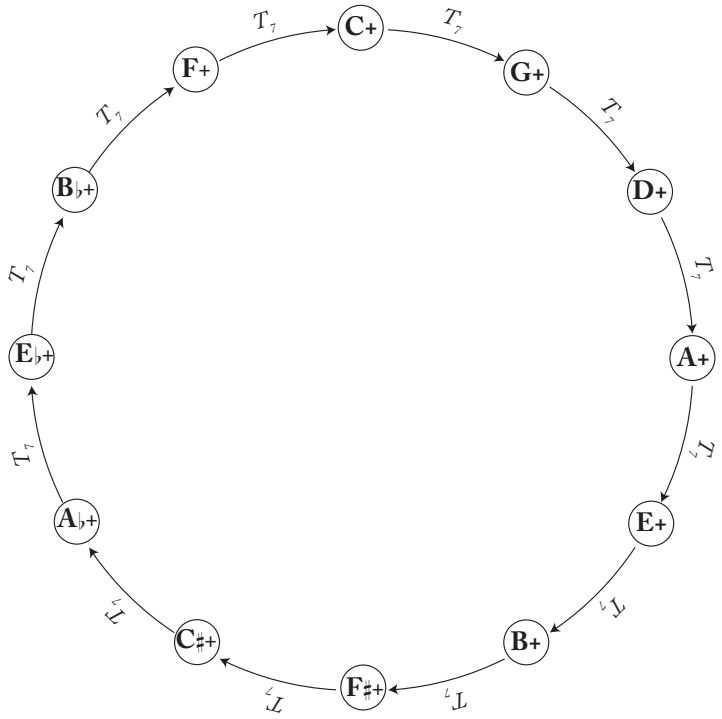
(1) A *node/arrow system*.[65]

(2) A *transformation system* (*S*, *G*). *S* is an unordered set of objects. (Here, *S* = {C+, C♯+ D +, E♭+ E+, F+, F♯+, G+, A♭+, A+, B♭+, B+}.) *G* is a *group* of operations or a *semigroup* of transformation. (Here, *G* is the group of pitch-class transpositions.)

(3) Finally, a pair of functions (Lewin calls them *TRANSIT* and *CONTENTS*) coordinate the *transformation system* with the *node/arrow system*. Elements in *S* are mapped to a network's nodes and arrows are labeled with members of *G*.

FIGURE 1.22. Two networks representing major triads and pc transposition.

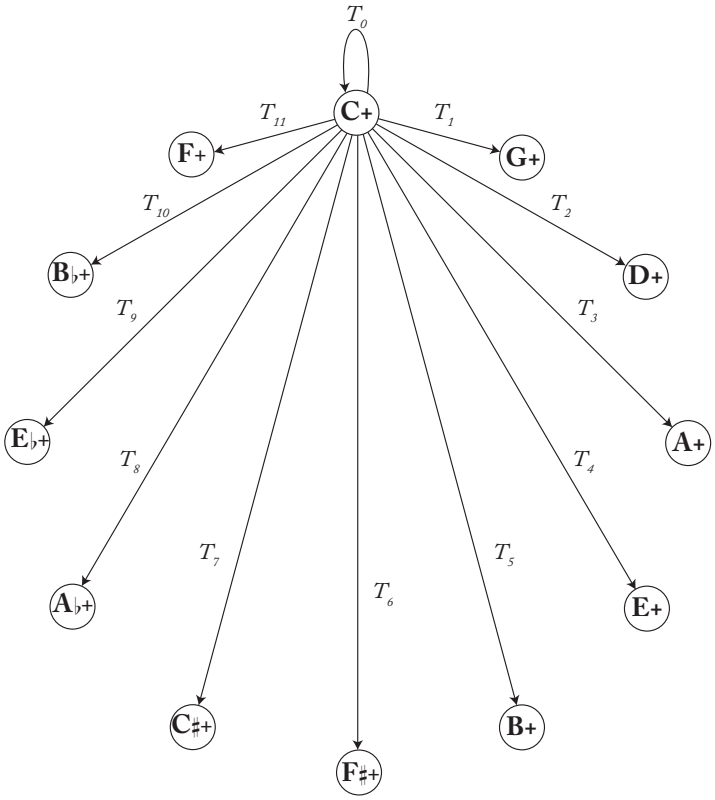
(a) the Circle-of-Fifths: (Major Triads, Pc Transposition)



[65] Lewin, *GMIT*, 193, Definition 9.1.1.

In some sense, then, a transformation network is a way to represent a set of elements and a group acting on those elements. Figure 1.22(a) is just one network representation of this transformation system. Figure 1.22(b) shows another. This network includes arrows for all twelve operations that make up the group *G*. It is visually obvious that these spaces are related, but only the first is a common representation. The most important differences regard the types of transformations represented. At (a) only one transformation (T_7) is shown on the network. At (b) twelve unique transformations are shown.

(b) network showing every member of the pc transposition group.



Often, spatial networks "hide" some operations in a group at the expense of foregrounding just one (or a small set); although the representation in Figure 1.22(a) may seem

to suggest that *G* contains only a single operation (T_7), all twelve transpositions shown in Figure 1.22(b) are *implicit* in that network; there are lots of "hidden arrows" on this space, all of those in Figure 1.22(b) and more. Spatial networks hide certain group elements and emphasize others for good reason. Those transformations shown are generally fundamental in some way to our understanding of the system represented by the space. T_7 is an important relationship between triads, one also contained within them. T_1 describes a less important relationship, and so spaces generally downplay its significance optically by leaving it off. All of this is to say that although the operations in a group are all theoretically equal in status, we tend to interpret music primarily through the lens of just a few operations.

Another, "fundamental" aspect of the spatial network in Figure 1.22(a) concerns the relationship of the one transformation shown (T_7) to all of the the other transformations in the group *G*: by itself, T_7 can generate every other transformation in *G*. This becomes apparent by examining the arrangement of triads in Figure 1.22(b). Arrows there show how all twelve operations in *G* can be understood as combinations of T_7. That is, T_7 is "fundamental" to *G* in large part because we can write every element of the group as some "power of T_7": If $(T_7)^m$ symbolizes *m* iterations of T_7, $T_2 = (T_7)^2$, $T_9 = (T_7)^3$, $T_4 = (T_7)^4$, and so on.[66] Or: imagine Figure 1.22(a) as a clock, the triad at "*m* o'clock" is *m* iterations distant from **C+**. This property of T_7 is only somewhat unique in *G*—T_1, T_{11}, and T_5 can also generate the group. But before proceeding, imagine if all of the arrows on Figure 1.22(a) were replaced by T_2, for example. Not all of the major triads would appear on the space, which means that not every member of *G* would be implicit in its structure.

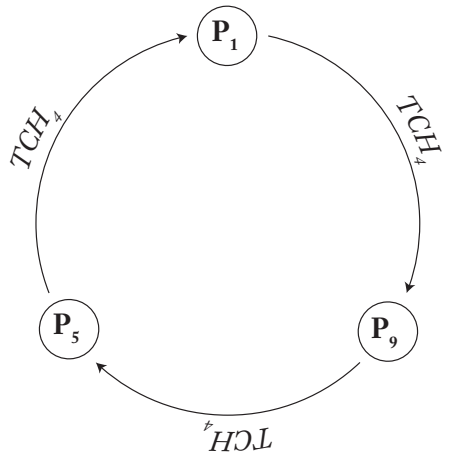
Groups in which every element can be understood as a "power of" one element are called *cyclic groups*. They form the basis of every other type of group and represent some of the most

[66] When I said above that T_7 could "generate every other transformation in the *G*," I was referencing the act of *binary composition*. A group's binary composition indicates how group elements combine. It's given a variety of symbols (for example, the "+" sign is the binary composition that often indicates numerical addition), but is symbolized here with as "•". In the pc transposition group, composition is additive mod 12: $T_7 \bullet T_7 = T_2$.

important musical groups. The special element that can create all others is called the *generator* of the group, often symbolized by ⟨*g*⟩, where *g* is the generator. Thus, the pc transposition group $G = \langle T_7 \rangle$, a cyclic group generated by T_7.

Let us look at another spatial diagram representing a cyclic group, one generated by TCH_4 as it acts on **P** forms from Webern's String Quartet, Op. 28. Figure 1.23 shows three iterations of TCH_4, the number required to arrive back at the originating row form (cf. Table 1.5(b)). In this group, TCH_4 is special in the same way that T_7 was earlier. But unlike T_7, which generated a twelve-element group, TCH_4 generates a group of only three elements: $\{TCH_4, (TCH_4)^2, (TCH_4)^3\}$, where the superscripted number *m* stands for *m* iterations of TCH_4. For example, $(TCH_4)^2 = TCH_4 \bullet TCH_4$.[67]

FIGURE 1.23. A network showing TCH_4 as it acts on P forms from Webern's String Quartet, Op. 28.



[67] $(TCH_1)^2$ is equivalent to $TCH_1 \bullet TCH_1$. As we move forward, the expression $f \bullet g$, where f and g are elements of a group, means that we calculate the result of applying "f-then–g." Thus, $TCH_1 \bullet ICH_1 \bullet RECH_1 \bullet RICH_1 (\mathbf{I}_x) = \mathbf{I}_x \xrightarrow{TCH_1} \mathbf{I}_{x-y} \xrightarrow{ICH_1} \mathrm{P}_{(x-y)-y} \xrightarrow{RECH_1} \mathrm{R}_{x-2y} \xrightarrow{RICH_1} \mathrm{I}_{x-2y}$. Note: unless noted, I will calculate expressions using left-to-right orthography. This differs from the standard practice of right-to-left orthography. In this system the expression T_0I, which often symbolizes inversion around the pitch-class axis of C/F♯, is expressed as IT_0.

1.4.3 CAYLEY DIAGRAMS

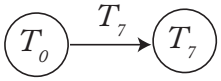
Networks like Figure 1.22(a) and Figure 1.23 are excellent ways to visualize each group. They imply every member of the group, thereby outlining every possible pathway, but in the most optically efficient way. This structure of this type of network, first described in 1878 by Arthur Cayley, is called a *Cayley diagram*.[68] A Cayley diagram on the group G is characterized by a transformation system (S, G) where the set S contains the same elements as the group G, and the diagram is visually organized by the group generator $\langle g \rangle$. Figure 1.24 demonstrates the process of constructing a Cayley diagram on the pc transposition group, $G = \{T_0, T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8, T_9, T_{10}, T_{11}\}$, using the group generator $\langle T_7 \rangle$:

FIGURE 1.24. Creating a Cayley diagram for $G = \langle T_7 \rangle$.

(1) Create a node for the *identity element* in the group (***e***). In the group G, T_0 is the identity element.

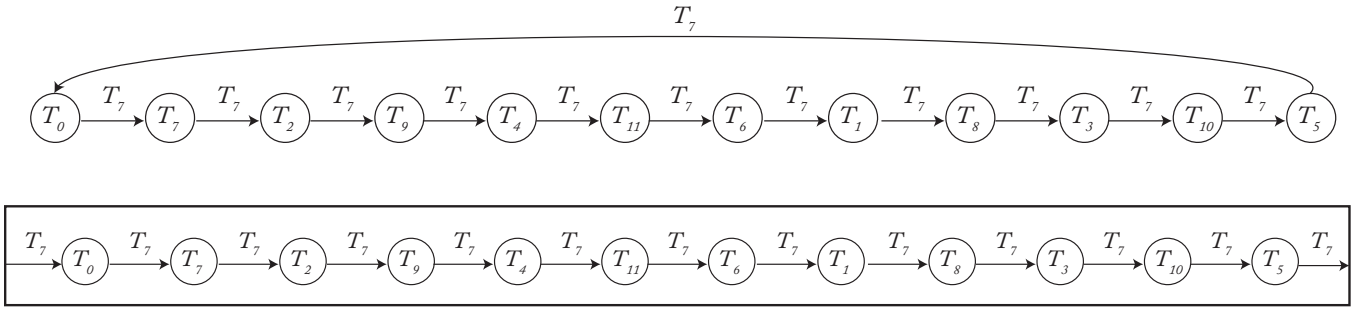


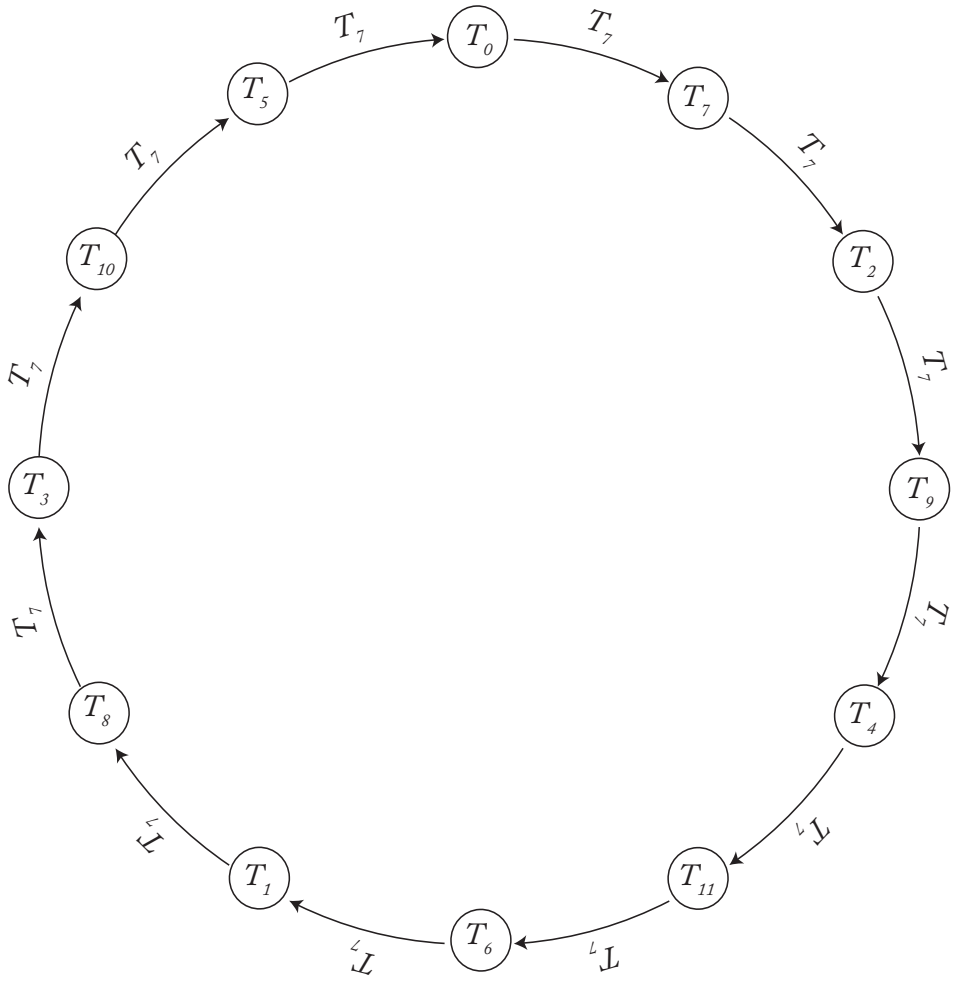
(2) For each generating element g, create an arrow labeled by that element originating at ***e*** and pointing to a node representing the binary composition $e \bullet g$.



[68] Nathan Carter has designed the free software program "Group Explorer" to accompany his *Visual Group Theory*. Among other things, it automatically displays Cayley diagrams for many common groups. See *Group Explorer*, version 2.2 (Waltham, Massachusetts, 2009), http://groupexplorer.sourceforge.net/. Also relevant in this regard is the interesting software package GAP (for Groups, Algorithms, and Programming). The Gap Group, *GAP —Groups, Algorithms, and Programming*, version 4.6.4, 2013, http://www.gap-system.org/. Robert Peck has an interesting tutorial explaining how to use the software: Robert W. Peck, "A GAP Tutorial for Transformational Music Theory," *Music Theory Online* 17, no. 1 (2011), http://www.mtosmt.org/issues/mto.11.17.1/mto.11.17.1.peck.html.

(3) Repeat, taking the end node(s) of step 2 as the originating node(s), until the identity node is regained.





Such spaces can be laid out in many ways. The top two shown above are "linear." In the second of the two, the constraining box indicates that when T_7 "hits the right border" it will emerge on the

left side. The third diagram wraps the space into a circle.[69] The latter reveals a similarity between this Cayley diagram and the circle-of-fifths that I will soon detail.

Cayley diagrams embody four criteria that characterize every group.

(1) Closure: A group is closed if any composition of two members of the group always produces another member of the group. Because every element of a Cayley diagram is generated by the group generator, it can easily symbolize closure by representing each element in a composition as powers of the generator. For example, $T_2 \bullet T_9 = (T_7)^2 \bullet (T_7)^3 = (T_7)^5 = T_{11}$. In general, any $T_{nm} = T_m$. Because of its circularity, a Cayley diagram captures the fact that any number of iterations of a generator will always create an element in the group.

(2) Inverse: Every group element must be able to be reversed so that the composition of that element and its opposite is the equivalent of doing nothing. The opposite of a group element is called its inverse. For instance, on the Cayley diagram above, $T_7 \bullet T_7(\boldsymbol{T_0}) = \boldsymbol{T_2}$—two "clicks" around the circle from $\boldsymbol{T_0}$. To return to $\boldsymbol{T_0}$ *and follow the arrows on the diagram* requires ten more clicks. Therefore, T_{10} is the inverse of T_2.

(3) Identity: Every group contains an element that stands for the action of "doing nothing," often symbolized as *e* and called the identity element. By convention, it is located at the top of the Cayley diagram. The identity element in the Cayley diagram for $\langle T_7 \rangle$ is $\boldsymbol{T_0}$.

(4) Associativity: Elements in a group compose associatively: $x(yz) = (xy)z$. Imagine the following series of operations along the outer edge of the Cayley diagram, beginning at any node (remember, $T_m = T_{nm}$): $T_{10} \bullet T_3 \bullet T_5$. We know from the Cayley diagram that $T_{10} \bullet T_3 = T_1$, thus we can "reduce" that part of the series to make its meaning simpler:

$$(T_{10} \bullet T_3) \bullet T_5 = (T_1) \bullet T_5.$$

[69] The three layouts highlight the topological similarity of a circle and a line. Topology ignores the "wraps" or "bends" necessary to create the circle, concentrating instead only on the structure's local topology. All three layouts of the network are 1-dimensional manifolds, though it requires two dimensions on paper to represent the circle. That topological similarity accounts for the many ways that such networks are often drawn.

Or, we might want to emphasize the fact that $T_3 \bullet T_5 = T_8$, again allowing us to simplify the equation, but in a different way:

$$T_{10} \bullet (T_3 \bullet T_5) = T_{10} \bullet (T_8).$$

Both associations make the meaning of the series or operations simpler. That we can move the parenthesis around within the series, allowing us to associate certain operations without changing the meaning, is called associativity.

Cayley diagrams also make it easy to identify *subgroups* of a group. Given a group G, a subgroup H contains some collection of operations in G and satisfies the four criteria above. For any element in one of the Cayley diagram's nodes, a subgroup can be created by using that element as a generator. To verify this let us create a subgroup $H = \langle T_4 \rangle$ of pc transposition group. $H = \{T_0, T_4, T_8\}$. T_0 is the group's identity and T_8 is the inverse of T_4.

There are twelve subgroups of the pc transposition group, one for every element of the Cayley diagram, including T_0 (called the *trivial subgroup*) and T_7. (Every group is considered a subgroup of itself, called a *non-proper subgroup.*)

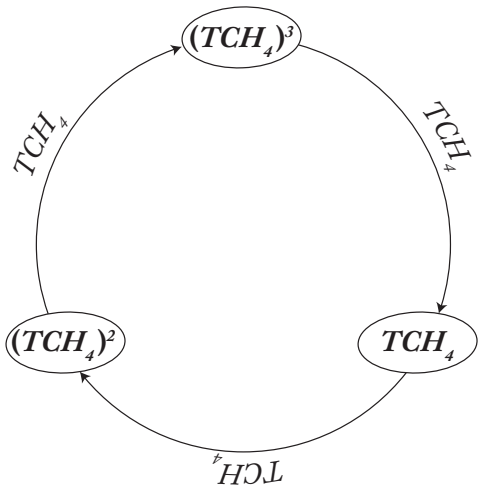
1.4.4 Cyclic Groups C_N

If there is there is a single generator involved in the process of creating a Cayley diagram, it will create a Cayley diagram for a *cyclic group*, which was defined above as a group for which every operation can be understood as a power of one of the group's operations. Cyclic groups form the basis of every all other groups. Within a cyclic group, the number of applications of an operation necessary to generate identity is called that operation's *order*. In the group pc transposition group, the group generator T_7 is an operation of order 12 because—as the Cayley diagram shows—it requires twelve iterations of T_7 to generate identity. The pc transposition group has four operations of order 12 (T_1, T_5, T_7, T_{11}), two operations of order six (T_2, T_{10}), two

operations of order four (T_3, T_9), two operations of order three (T_4, T_8), one operation of order two (T_6), and one operation of order one (T_0).

Furthermore, every cyclic group has an *order* that indicates the size of the group and that is equal to the order of that group's generator. Therefore, the pc transposition group is a cyclic group of order 12, often symbolized as C_{12}, where the subscripted number is the group's order. The symbol C_n is an abstract way of referring to a group that can indicate correspondences between more concrete instantiations. Notice that the TCH_4 operation, when acting on Op. 28, generates a cyclic group of order 3, C_3, whose Cayley diagram is shown in Figure 1.25.

FIGURE 1.25. In Op. 28, the cyclic group $C_3 = \langle TCH_4 \rangle$.



As I mentioned earlier, every group element must have an inverse. The inverse of T_2 was T_{10}, and more generally, for any operation f, the inverse of f^m is f^{-m}. In a cyclic group, every element's inverse is equivalent to some non-negative element, determined by the order of the group. The inverse of T_2 could be represented as T_{-2}, and it is the equivalent of T_{10} because $T_{12-2} = T_{10}$—twelve being the order of the group. Less obviously, in $C_3 = \langle TCH_4 \rangle$, the inverse of every element $(TCH_4)^m = (TCH_4)^{3-m}$. Thus, the inverse of TCH_4 is $(TCH_4)^{3-1} = (TCH_4)^2$.

Three additional properties are common to all cyclic groups:

(1) All operations in the group are commutative. Given a cyclic group G and any two elements f_1 and f_2 in G, $f_1 \bullet f_2 = f_2 \bullet f_1$. In $C_{12} = \langle T_7 \rangle$, for example, $T_1 \bullet T_2 = T_2 \bullet T_1$. More plainly, commutativity means that the order in which the operations are performed does not impact the result of performing those operations.

(2) Every subgroup of a cyclic group is also cyclic. To illustrate, reconsider the subgroup of H of the pc transposition group that was generated by T_4. Group closure required that every group element combined under binary composition; thus, T_4 must combine with the identity element and with itself: $T_4 \bullet T_4 = T_8$. If the group contains T_4 and T_8, those elements must compose as well: $T_8 \bullet T_4 = T_0$, and this new element (T_0) must also compose with T_4 ($T_0 \bullet T_4 = T_0$). Repeating this process indicates that $H = \{T_0, T_4, T_8\}$, and that every element can be generated by T_4.

(3) The inverse of a cyclic group's generator may also generate the group. Within $C_{12} = \langle T_7 \rangle$, the inverse of T_7 is T_5 ($= T_{12-7}$), also an element of order 12. Therefore, like T_7 it can generate the entire group.

1.4.5 Homomorphisms, Isomorphisms, and Automorphisms

We often want to assert similarities and differences between groups. In fact, I said earlier that the circle-of-fifths and the Cayley diagram of the pc transposition group seemed similar. The concepts of *homomorphism*, *isomorphism*, and *automorphism*—all concerned in some respect with the "shape" of a group—allow us to more precisely specify that similarity.

Homomorphisms are functions that manifest as "embeddings" or "quotients" that map one Cayley diagram onto another. Homomorphic mappings find a copy of the original diagram's structure in the new diagram, which is often easy to see visually. Imagine the two groups $G = \langle T_7 \rangle$

and $H = \langle T_4 \rangle$, symbolized in the columns of Table 1.7, a mapping table. There, the homomorphism α specifies that $\alpha(\boldsymbol{T_m}) = \boldsymbol{T_m}$, creating the following map:

TABLE 1.7. The homomorphism α maps *H* to *G*.

	$H = \langle T_4 \rangle$	$G = \langle T_7 \rangle$
α:	$\boldsymbol{T_4} \rightarrow \boldsymbol{T_4}$	
α:	$\boldsymbol{T_8} \rightarrow \boldsymbol{T_8}$	
α:	$\boldsymbol{T_0} \rightarrow \boldsymbol{T_0}$	

The homomorphism can be followed on Figure 1.26, where the dotted arrows represent α.

For such a mapping to be a homomorphism, it must also preserve the binary composition from the originating group: that is, for every arrow *g* in *H* that leads from node s_1 to node s_2, in *G* an arrow $\alpha(g)$ must lead from $\alpha(s_1)$ to $\alpha(s_2)$. The figure shows that this is true for *H* and *G*. The T_4 arrow that connects $\boldsymbol{T_0}$ to $\boldsymbol{T_4}$ in *H* connects $\boldsymbol{T_0}$ to $\boldsymbol{T_4}$ in *G*. This requirement shows that homomorphisms map not only nodes in one Cayley diagram to nodes in another, but they also map arrows in the the original Cayley diagram to arrows in the other. Earlier, I showed that *H* was a subgroup of *G*. And in fact, every subgroup *G* will be related by some homomorphism. Visually, this is easy to see as the structure of the subgroup *H* will be *embedded* in the *G*, and that is why this type of homomorphism is called an embedding.

Homomorphisms do not necessarily involve group/subgroup relationships, however. At the bottom of Figure 1.26, short-dashed arrows show that the group generated by TCH_4, diagrammed earlier in Figure 1.25, is related to *G* by the homomorphism β, which specifies that

$\beta\,(TCH_4^m) = T_{m*4} \pmod 4$. (For example, $\beta(TCH_4) = T_4$). Of course, $\langle TCH_4 \rangle$ is not a subgroup of *G*, one group is generated by a row chain and the other by transposition. But $\langle TCH_4 \rangle$ can be embedded in *G*.

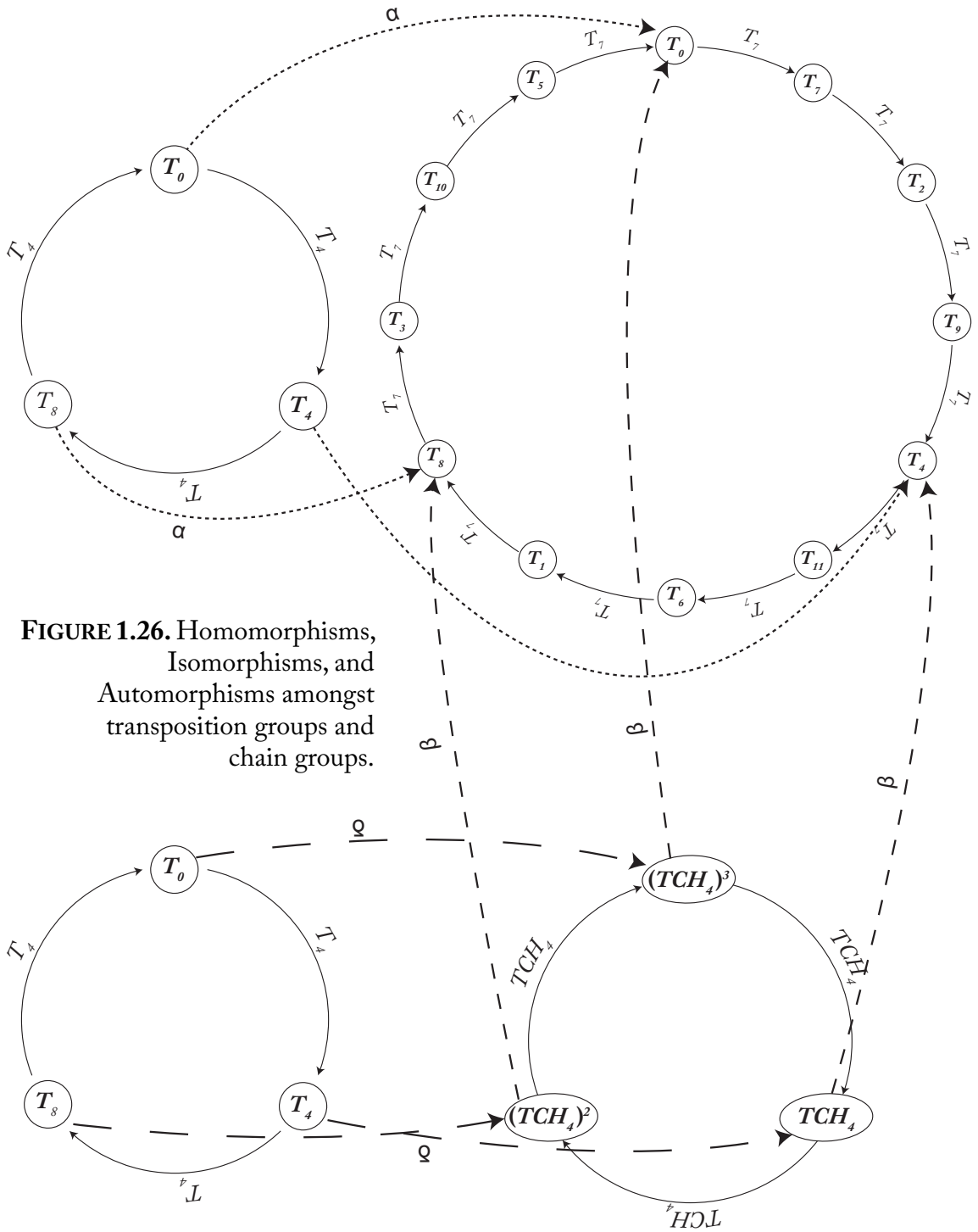
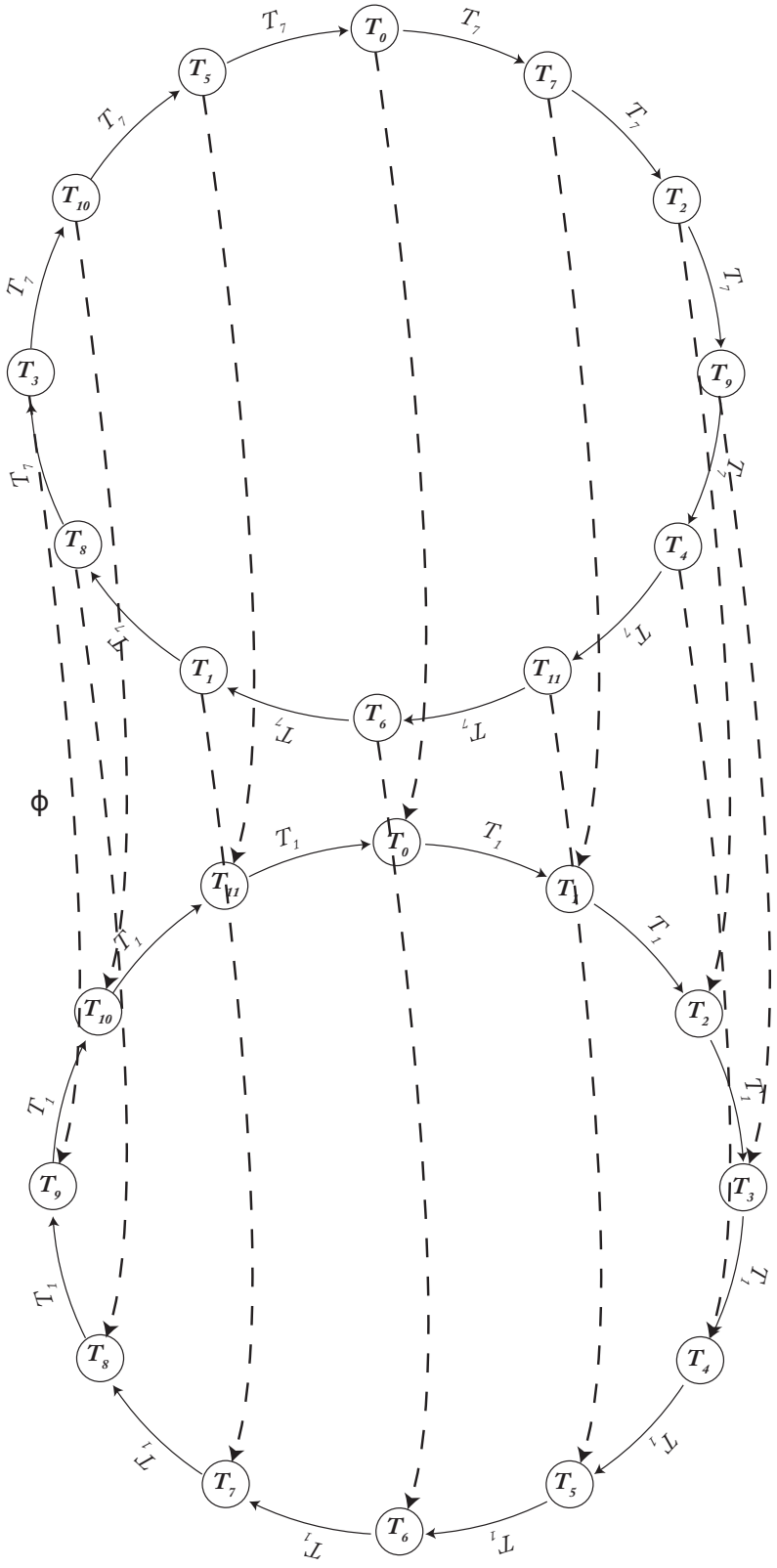


FIGURE 1.26. Homomorphisms, Isomorphisms, and Automorphisms amongst transposition groups and chain groups.

FIGURE 1.27. An automorphism between groups generated by T_7 and T_1.



When a homomorphism maps a group to another that has "the same structure," it is called an *isomorphism*. In these cases, the entirety of one group can be embedded in another, and therefore, both groups are the same size. In this case, the function is *one-to-one* and *onto*. At the bottom of Figure 1.26, the group $H = \langle T_4 \rangle$ is mapped to $\langle TCH_4 \rangle$ through the isomorphism ϙ. Isomorphic groups have the same abstract group structure. Therefore, because $\langle T_4 \rangle$ and $\langle TCH_4 \rangle$ are isomorphic, $C_3 = \langle TCH_4 \rangle = \langle T_4 \rangle$.

Finally, Figure 1.27 shows an *automorphism* ϕ between $\langle T_7 \rangle$ and the group $\langle T_1 \rangle$. Automorphisms are

isomorphisms between a group and itself. As the nodes of Figure 1.27 show, $\langle T_7 \rangle$ and $\langle T_1 \rangle$ have the same group elements, and therefore, represent the same group, the pc transpositions. The mapping between the two groups is one-to-one and onto, sending $\boldsymbol{T_m}$ in $\langle T_7 \rangle$ to $\boldsymbol{T_{m*7}}$ in $\langle T_1 \rangle$. Thus, ϕ transforms $\boldsymbol{T_7}$ into $\boldsymbol{T_1}$, and thus "shuffles" the elements of $\langle T_7 \rangle$ into a new configuration in $\langle T_1 \rangle$.

1.4.6 PARTITIONS

It is likely clear that the Cayley diagrams representing C_{12} and C_3 are related to the spatial networks earlier called "the circle-of-fifths" and the "TCH_4 space for Webern's String Quartet, Op. 28." And in fact, a spatial network and Cayley diagrams generated by the same group are isomorphic. Figure 1.28(a) maps out precisely how the Cayley diagram for $\langle T_7 \rangle$ becomes the circle of fifths through the isomorphism Ω: given any operation g in the Cayley diagram for $\langle T_7 \rangle$, $\Omega(\boldsymbol{g}) = \Omega(\mathbf{REF})$, where **REF** is an arbitrarily chosen member of the major triads.[70]

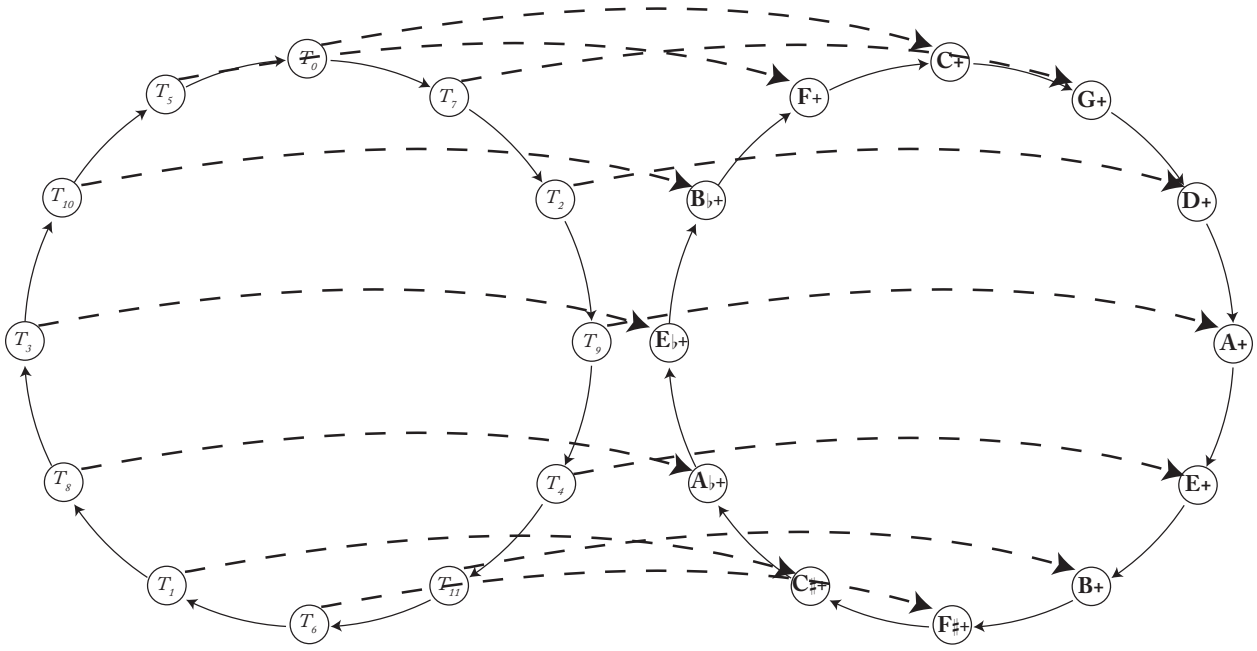
More plainly, the isomorphism Ω chooses a referential major triad (**REF**), and then applies the operation g to that triad. Figure 1.28(b) uses a similar isomorphism to transform the Cayley diagram for $\langle TCH_4 \rangle$ into a spatial network for Webern's String Quartet. The choice of a referential triad in Figure 1.28(a) will have no impact on the triads contained in the resulting space, only their placement on the space.

The same is not true of the network for Op. 28. I *chose* $\mathbf{P_1}$ as **REF**. That choice placed $\mathbf{P_1}$ at the top of that network, which automatically filled in the remaining nodes with $\mathbf{P_9}$ and $\mathbf{P_5}$. Had I *chosen* a different referential row form, say $\mathbf{P_2}$ or $\mathbf{RI_7}$, the contents of the nodes would have

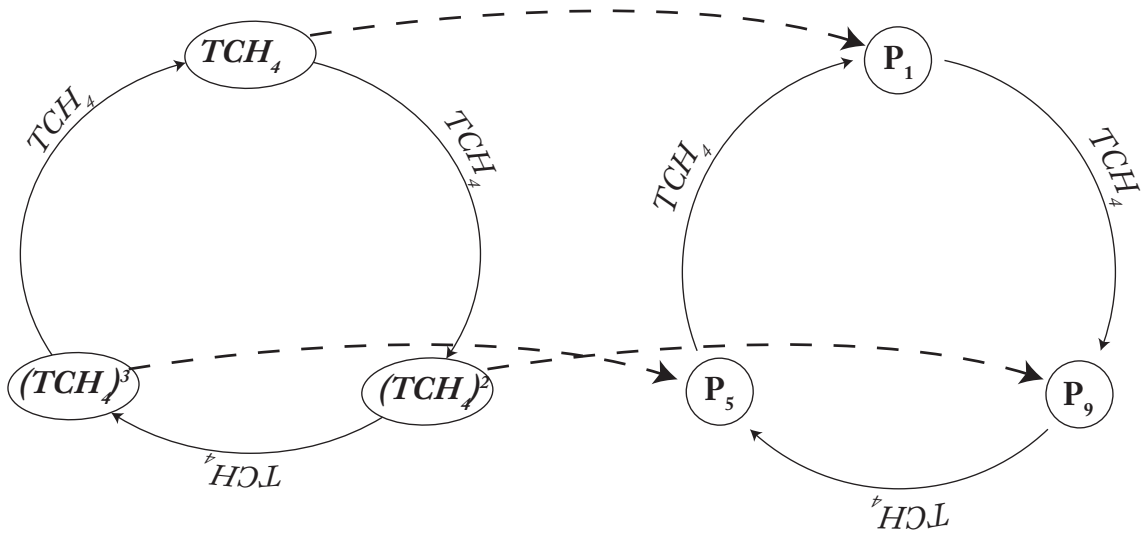
[70] This procedure copies one outlined in Gollin, "Representations of Space," 71-4.

FIGURE 1.28. Turning a Cayley diagram into a spatial network

(a) The isomorphism **Ω** maps the Cayley diagram for ⟨T_7⟩ onto the spatial network known as



(b) Turning the Cayley diagram for ⟨*TCH4*⟩ into a spatial network.



been different. In other words, when acting on rows from Webern's String Quartet, *TCH4* *partitions* the row class. In this case, partition refers to the process of evenly "dividing" the larger

set of rows in the row class into a collection of smaller sets, all of which contain rows that can reach one another through TCH_4.

Given a transformation system (S, G), the number of partitions created by G is related to the *size* of the set S of objects being transformed and the *order* of the group G transforming them. When acting on the twelve major triads, the order 12 group $\langle T_7 \rangle$ divides those triads into 1 (= 12 ÷ 12) partitions.[71] By contrast, the 48 row forms in Webern's String Quartet are partitioned by $\langle TCH_4 \rangle$, a group of order 3, into 16(= 48 ÷ 3) partitions.

I return to the idea of partitions in §1.5 and explore them more fully in the two analytical chapters, but suffice it to say here that the ability of chains to "naturally" partition a set of row forms is the one source of their power to influence a compositional environment. Inasmuch as chains act as voice-leading constraints, moving along the paths implied by a chain naturally partitions the rows in a row class by making available certain connections and prohibiting others.

1.4.7 Spatial Network

By now the characteristics of a spatial network should be coming into focus. All spatial networks have the following two traits:

(1) A spatial network is a transformation network as described in §1.4.2 representing a group G of operations acting on a set S of objects.

(2) A spatial network is isomorphic to the Cayley diagram created from the group's generator $\langle G \rangle$ or generators $\langle G, \ldots \rangle$.[72]

---

[71] If the transformation system (S, G) were the twenty-four major and minor triads, and $G = \langle T_7 \rangle$, G would partition the into 2 (= 24 ÷ 12) partitions.

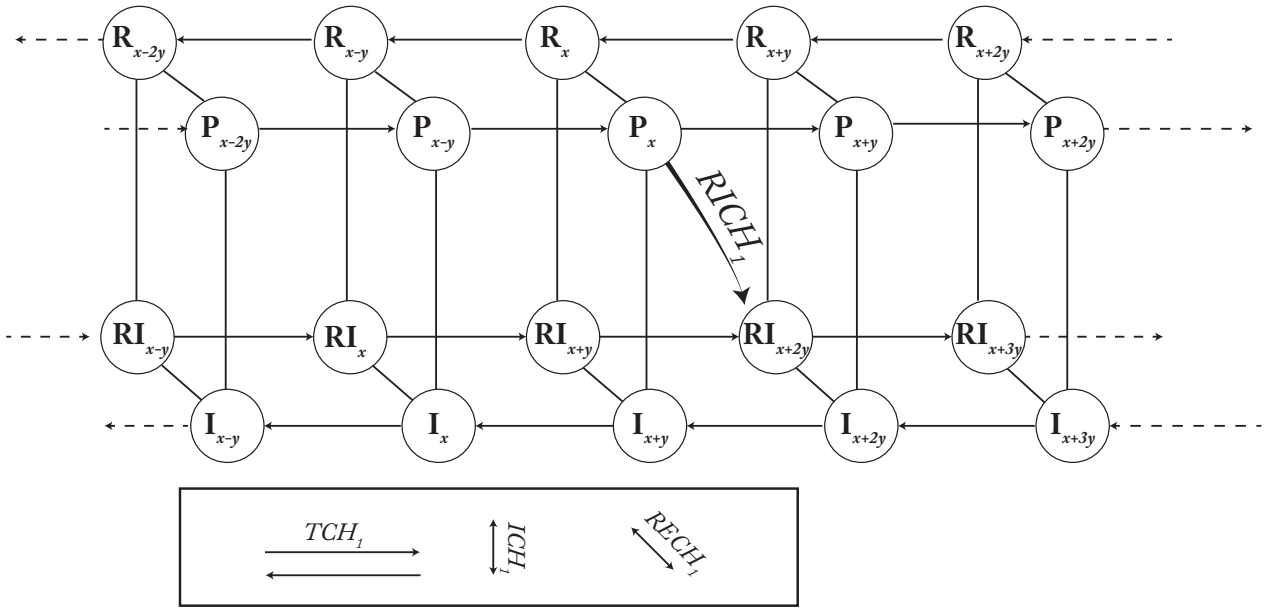
[72] Thus far, we have seen only groups generated by a single generator. Chapter 2 has many examples of groups that have more than one generator.

Though a Cayley diagram has an important influence on the visual arrangement of nodes and arrows, some representational decisions remain. Spatial networks generated by cyclic groups may have a linear orientation or can be "bent" into circles. Depending on the relationship of the two components of the transformation system, a spatial network may be completely connected (that is, every node is connected to every other node by some some group transformation) or disconnected into partitions. I will note that it may seem that these restrictions allow each object in a transformation network to be represented only once. There are, however, analytical situations in which the duplication of an object on a spatial network is suggestive.

1.5. Chain Spaces

Figure 1.29 is a spatial network generated by the TCH_1, ICH_1, and $RECH_1$.[73] Rather than filling the nodes with specific row forms, I have left them under-determined. As represented, the figure's abstract structure makes it a primitive example of an abstract

Figure 1.29. Spatial representation of Table 1.4



[73] Technically, $RICH_1$ is not a generator of the group, but it could be. We will explore this more below.

compositional space—a *musical grammar*.[74] A literal "interpretation" of this grammar would fill in nodes by substituting row forms given a value *y* gleaned from some row class, and an arbitrary value for *x*. Then, we could "surf" along the space using the various pathways to chain into and out of various rows. In essence, Figure 1.29 is a graphic representation of the mapping table shown in Table 1.4.

Depending on the value for *y*, a variety of literal interpretations of the space could be produced. Many of these will be differently sized, while some will be isomorphic. If *y* were 6 for example, the space would be quite small—containing only 4 rows. If *y* were 1, the space would be much larger, encompassing all forty-eight rows in a row class, and it would be isomorphic to a space were *y* = 5, 7, or 11. The size of such a space directly correlates with the order of the generating transformations, which I discussing briefly in the following section.

1.5.1 ⟨*ICH*⟩ AND ⟨*RECH*⟩

Both *ICH* and *RECH* generate cyclic groups isomorphic to C_2—the only group of order two. Irregardless of the chain's length, it is an *involution*—an operation that is its own inverse.[75] To verify follow the following compound ICH_1 chain on Figure 1.29: $\mathbf{R}_x \xrightarrow{ICH_1} \mathbf{RI}_{x+y} \xrightarrow{ICH_1} \mathbf{R}_x$. Involutions are often shown as lines with no arrowheads to indicate that the path can be traversed in either direction, and that the transformations "undo themselves."

1.5.2 ⟨*TCH*⟩

---

[74] Morris "Compositional Spaces," 339-40.

[75] For *ICH*, this is easily enough verified by consulting Table 1.4 and applying *ICH* successively: $ICH \bullet ICH\ (\mathbf{P}_x) = ICH\ (\mathbf{I}_{x+y}) = \mathbf{P}_{(x+y)-y} = \mathbf{P}_x$

To prove that *RECH* is always an involution as well, suppose a compound $RECH_2$ chain transforming $\mathbf{R}_x$: according to the mapping table $\mathbf{R}_x \xrightarrow{RECH_2} \mathbf{P}_{x+(y-z)} \xrightarrow{RECH_2} \mathbf{R}_{x+(y-z)+(y-z)}$. If $RECH_2$ is an involution, $\mathbf{R}_{x+(y-z)+(y-z)}$ must equal $\mathbf{R}_x$. And therefore, $y - z$ must be equal to 6, which will always be true because the sum of intervals in the final segment of a *RECH*-able row is always 6, per §1.2.7.

The cyclic groups (C_n) generated by *TCH* or *RICH* are more directly tied to the specific intervallic properties of a row class. To determine which cyclic group *TCH* creates, we need an answer to the following question: "how many *TCH*s do we need to perform to produce *identity*?" Figure 1.29 shows that $(TCH)^2(\mathbf{P}_x) = \mathbf{P}_x \xrightarrow{TCH} \mathbf{P}_{x+y} \xrightarrow{TCH} \mathbf{P}_{x+2y}$. In that expression, notice that for every *TCH* the interval *y* is increase incrementally by 1. More generally, then, $(TCH_1)^n(\mathbf{P}_x) = \mathbf{P}_{x+ny}$. To reframe the above question in these terms, "how many *TCH*s (n) do we need to perform such that $n(y) = 0$," for in that case, $(TCH_1)^n = identity$. The answer to the question, of course, depends on the value of *y*.

1.5.3 When $G = \langle TCH \rangle$…

The order of *TCH* is *n*, where $n = 12 \div (\text{GCD}(12, y))$. Note: this is *not* solved mod 12.[76] Given the value *y* for a particular row class, $\langle TCH \rangle = C_n$, a cyclic group isomorphic to one of eleven cyclic groups, C_2 - C_{12}. Inverse values for *y* (1 and 11, for example) generate automorphic groups. The *y* value of 6 results in a *TCH* group of order 2, isomorphic to the groups generated by *ICH* and *RECH*. In this group, *TCH* is an involution.

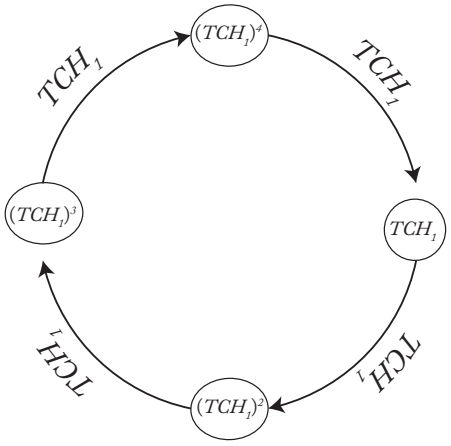
1.5.4 Example: Webern, Op. 23

Remember, *y* represents the directed interval from the first pitch class of a row to the first elided pitch class. For a one-note chain, then, *y* is the interval from the first to last pitch of a **P** form. Transforming the row for Op. 23 (where $y = 3$) TCH_1 invokes the four-element, cyclic group C_4. We know this because when $y = 3$, the order *n* of *TCH* = $12 \div (\text{GCD}(12, 3)) = 12 \div 3 =$

[76] Given that we are seeking the *n* where $ny = 0 \pmod{12}$, it may seem that we could solve for *n* by dividing 0 (or 12) by *y*. Therefore, $n = 0 \div y$. Division, however, is not defined on modular sets because often there will be more than one solution. This definition remedies the problem by dividing 12 by the *greatest common factor* of 12 and *y*. This definition is the same as the equation for the cyclic length of a *simple interval cycle* set forth in Edward Gollin, "Multi-Aggregate Cycles and Multi-Aggregate Serial Techniques in the Music of Béla Bartók," *Music Theory Spectrum* 29, no. 2 (2007): 143–176.

4.) This group contains the four elements (TCH_1, $(TCH_1)^2$, $(TCH_1)^3$, and $(TCH_1)^4$) shown in the Cayley diagram in Figure 1.30. That diagram shows that $(TCH_1)^4$ is the *identity* operation in the group and helps us visualize each element's inverse. $(TCH_1)^{-1} = (TCH_1)^3$, and $(TCH_1)^2$ is an involution. Were we to create a spatial network for Op. 23 where S = the row class's forty-eight rows and $G = \langle TCH \rangle$, G would partition S into twelve disconnected subsets.

FIGURE 1.30. Cayley diagram for $\langle TCH_1 \rangle$, where $y = 3$.



1.5.5 EXAMPLE: WEBERN, OP. 28

Under the right conditions, a multitude of *TCH* chains, each with a different length, may act in a row class. These chains often create unique cyclic groups. As an example, Figure 1.31(a) shows two different *TCH* chains transforming $\mathbf{P_7}$ from Webern's String Quartet, Op. 28. TCH_4 generates the cyclic group C_3, shown in Figure 1.31(b) and TCH_2, generates the cyclic group C_6, shown at (c).

Each type of chain will partition the twenty-four *distinct* rows of Webern's String Quartet into the following *disjoint* subsets of row forms:

TCH_2 = ($\mathbf{P_7}$, $\mathbf{P_5}$, $\mathbf{P_3}$, $\mathbf{P_1}$, $\mathbf{P_{11}}$, $\mathbf{P_9}$), ($\mathbf{P_8}$, $\mathbf{P_6}$, $\mathbf{P_4}$, $\mathbf{P_2}$, $\mathbf{P_0}$, $\mathbf{P_{10}}$), ($\mathbf{R_7}$, $\mathbf{R_9}$, $\mathbf{R_{11}}$, $\mathbf{R_1}$, $\mathbf{R_3}$, $\mathbf{R_5}$),

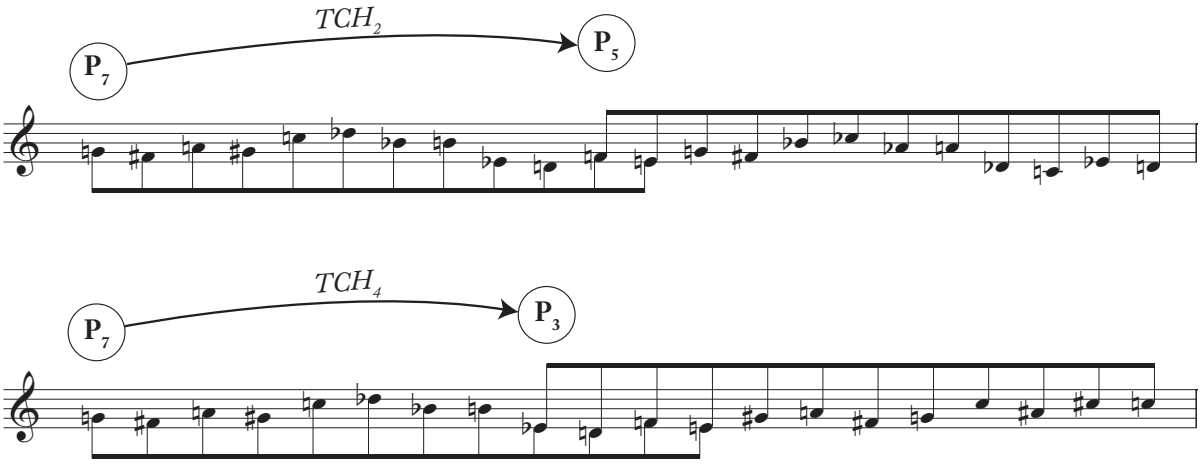
($\mathbf{R_8}$, $\mathbf{R_{10}}$, $\mathbf{R_0}$, $\mathbf{R_2}$, $\mathbf{R_4}$, $\mathbf{R_6}$)

TCH_4 = (**P_7, P_3, P_{11}**), (**P_8, P_4, P_0**), (**P_9, P_5, P_1**), (**P_{10}, P_6, P_2**), (**R_7, R_{11}, R_3**), (**R_8, R_0, R_4**), (**R_9, R_1, R_5**), (**R_{10}, R_2, R_6**),

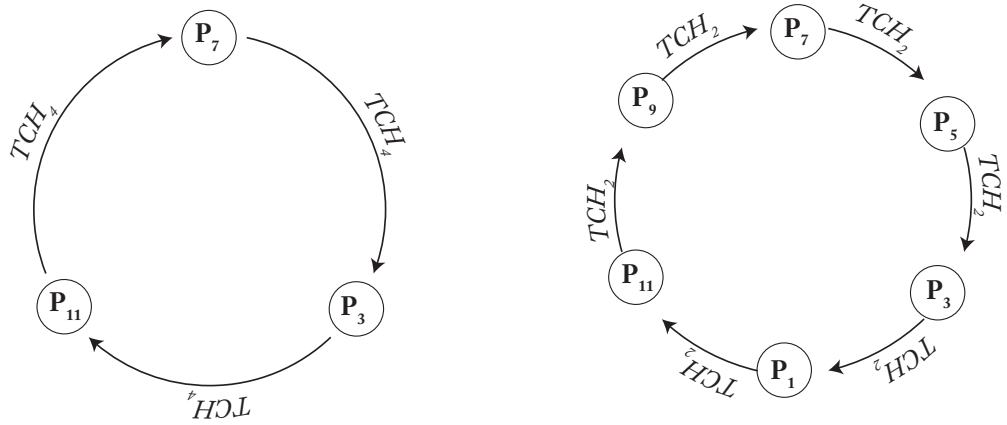
⟨TCH_2⟩ partitions the 24 rows into fewer but larger disjoint subsets than does ⟨TCH_4⟩. Being smaller, the subsets generated by TCH_4 embed homomorphically into those generated by TCH_2, as can be seen at (b) and (c).

FIGURE 1.31. TCH_2 and TCH_4 in Op. 28.

(a) acting on P_7



(b) ⟨TCH_4⟩ (c) ⟨TCH_2⟩



1.5.6 GENERATING *RICH*

One quirk of my exploration of chains in this chapter is that *RICH* has received relatively little attention. *RICH*, after all, is the row chain *par excellence*. Its exploration elsewhere in the scholarly literature is inspiration for the current study in many different ways. But, from the perspective of the current discussion, *RICH* is simply one of the four chain types. And furthermore, *RICH* can be *generated* by the other chains. On Figure 1.29, TCH_1, ICH_1, and $RECH_1$ occupied the *x*-, *y*-, and *z*-axes. The one $RICH_1$ shown moves between antipodally situated row forms ($RICH_1(\mathbf{P_x}) = \mathbf{RI_{x+2y}}$), engaging all three axes: *RICH* could be imagined as a compound operation, $TCH_1 \bullet ICH_1 \bullet RECH_1$.[77]

In fact, compounds of TCH_1 , ICH_1 , and $RECH_1$ can express compounds of *RICH* such that $(TCH_1)^n \bullet (ICH_1)^n \bullet (RECH_1)^n = (RICH_1)^n$. Compare the following expressions, both applied to $\mathbf{P_{x-y}}$: (Each statement can be followed below on Figure 1.32.)

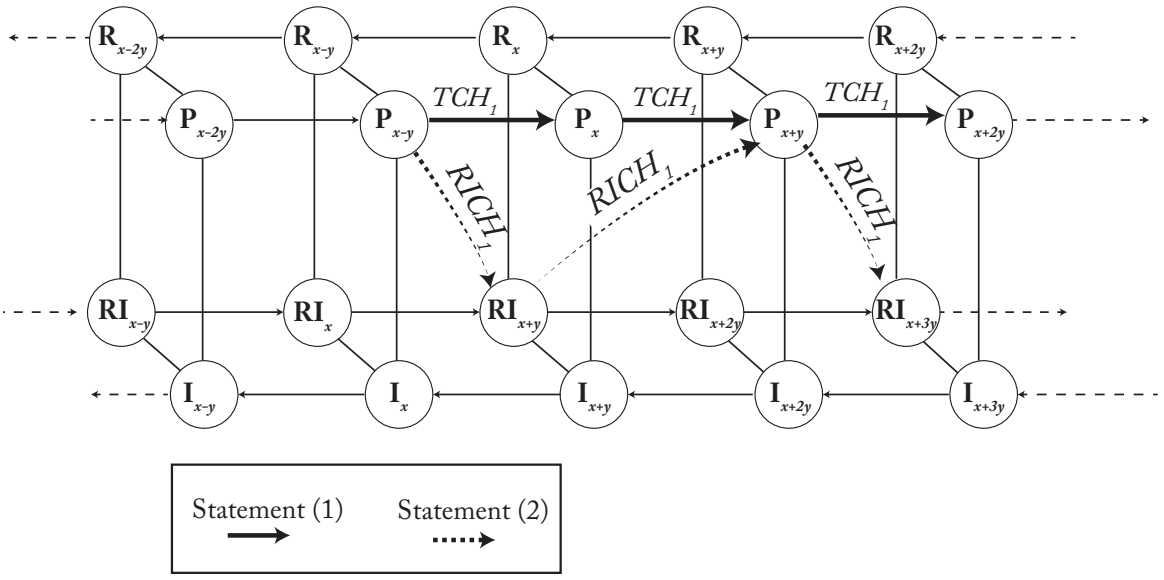
Statement (1)	Statement (2)
$(TCH_1)^3 \bullet (ICH_1)^3 \bullet (RECH_1)^3(\mathbf{P_{x-y}}) = (TCH_1)^3 \bullet ICH_1 \bullet RECH_1(\mathbf{P_{x-y}})$; therefore,	$(RICH_1)^3(\mathbf{P_{x-y}})$,
$\mathbf{P_{x-y}} \xrightarrow{(TCH_1)3} \mathbf{P_{x+2y}} \xrightarrow{ICH_1} \mathbf{I_{x+3y}} \xrightarrow{RECH_1} \mathbf{RI_{x+3y}}$.	$\mathbf{P_{x-y}} \xrightarrow{RICH_1} \mathbf{RI_{x+y}} \xrightarrow{RICH_1} \mathbf{P_{x+y}} \xrightarrow{RICH_1} \mathbf{RI_{x+3y}}$

Though the statements seem very different, they are the same. Figure 1.32 shows that statement (1) moves along the *x-axis* three places to the right before snaking downward through *ICH* and *RECH*. By contrast, statement (2) bounces between **P** and **RI** forms, but it ends up in the same

[77] In actuality, both *ICH* and *RECH* could be generated by *RICH* as well.

location. Therefore, although the paths seem different, $(TCH_1)^3 \bullet (ICH_1)^3 \bullet (RECH_1)^3 = (RICH_1)^3$. Because ICH_1 and $RECH_1$ are involutions, in statement (1) the three iterations of each are equivalent to a single iteration.

FIGURE 1.32. $(TCH_1)^3 \bullet (ICH_1)^3 \bullet (RECH_1)^3 = (RICH_1)^3$.



Continuing along these lines, because $(ICH_1)^n = identity$ and $(RECH_1)^n = identity$ whenever n is an even number, $(TCH_1)^n = (RICH_1)^n$, whenever n is even as well.[78] On Figure 1.32, compare $\mathbf{P}_{x-y} \xrightarrow{TCH_1} \mathbf{P}_x \xrightarrow{TCH_1} \mathbf{P}_{x+y}$ to $\mathbf{P}_{x-y} \xrightarrow{RICH_1} \mathbf{RI}_{x+y} \xrightarrow{RICH_1} \mathbf{P}_{x+y}$. In *GMIT* Lewin makes a similar observation: given an object s "the RICH transform of RICH(s) is a transposed form of s".[79] Lewin calls the compound transformation *TCH*. As we have seen here, any compound *RICH*

[78] $TCH_i{}^n \bullet ICH_i{}^n \bullet RECH_i{}^n = RICH_i{}^n$ holds for all i as long as each chain is capable of transforming the given row. Otherwise, $RICH_i$ is unique in the sense that it cannot be generated by the other chains.

[79] Lewin, *GMIT*, 181. In that context, Lewin was speaking of 2-note *RICH* chains, but the observation is true in any context where i is 1, or in situations where i is greater than 2—as long as the row is capable of being *TCH*-ed to such a degree.

chain is equal to some *TCH* chain when the chain contains an even number of iterations. However, Lewin does not conceive of *TCH* in quite the same manner as in the present study. Lewin's *TCH* is a transpositional chain *generated by RICH*: $TCH = RICH^2$. It does not invoke an overlap in itself. Here, I am showing that *TCH* chains can connect rows by overlap, they can generate groups, and in fact, *TCH* (with *ICH* and *RECH*) can often generate *RICH*.

1.5.7 *RICH* Groups

TCH_i and $RICH_i$ often imply the same abstract cyclic group. Only when, TCH_i*'s* order is odd will they differ, and in these cases, the order of *RICH* is twice the order of *TCH*.

1.5.8 When $G = \langle RICH \rangle$…

The order of *RICH* is n, where $n = 2(12 \div (\text{GCD}(12, 2y)))$. Note: this is *not* solved mod 12.[80] Therefore, given the value y for a particular row class, $\langle RICH_i \rangle = C_n$.

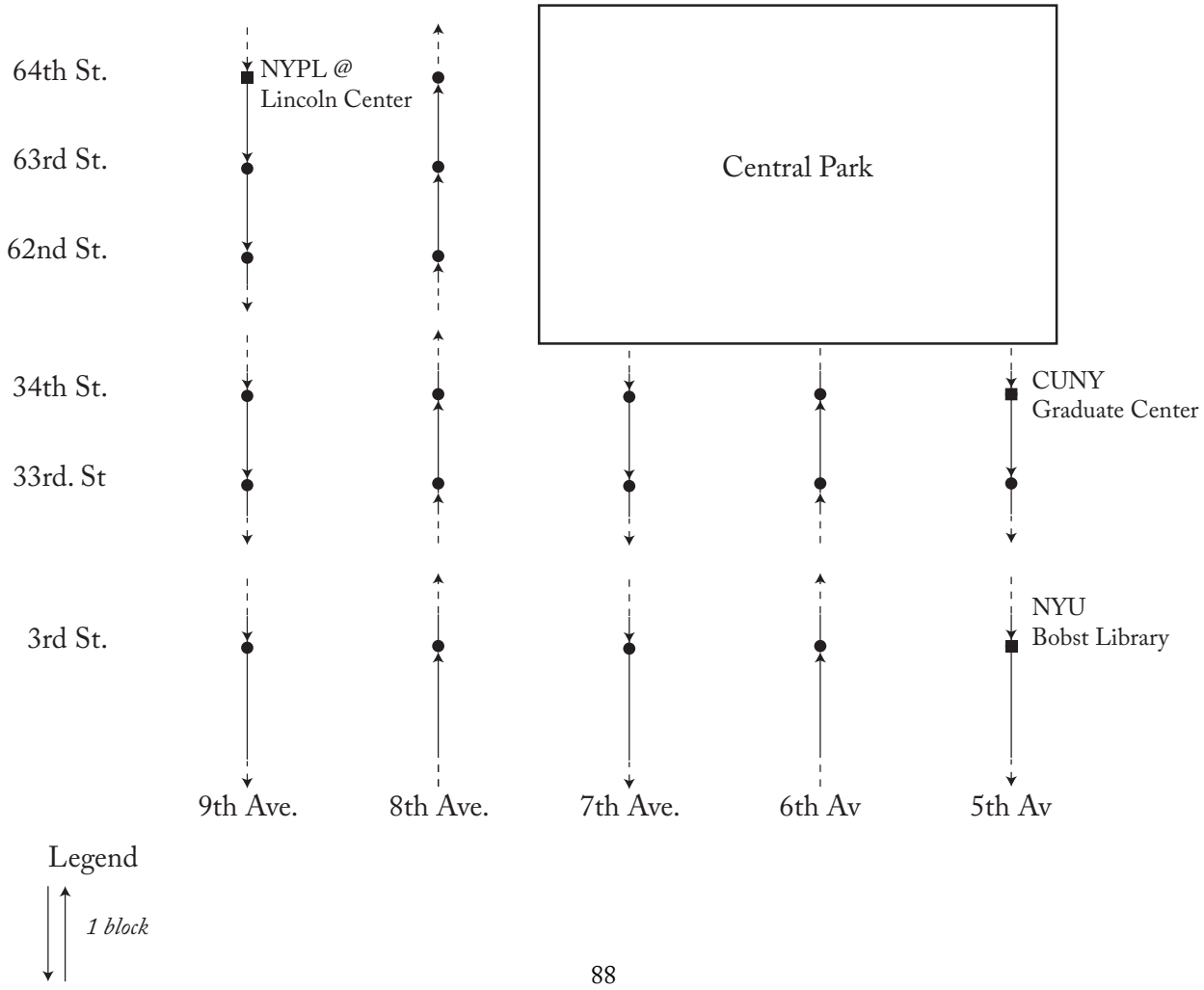
[80] See Gollin, "Multi-Aggregate Cycles," 146 for the origin of this definition. There, Gollin uses the definition to specify the length of a compound interval cycle. See also J. Philip Lambert, "Interval Cycles as Compositional Resources in the Music of Charles Ives," *Music Theory Spectrum* 12, no. 1 (1990): 55 and Straus, "Contextual-Inversion Spaces," 47-52.

CHAPTER 2

TWELVE-TONE CARTOGRAPHY

Spatial networks as defined can capture an idea of *distance* between objects that is conditioned by the group's generator. This conception of musical space is suggestive of *road* maps that calculate distance between locations not "as the crow flies," but on the basis of available routes between those objects. For example, imagine the spatial network of Manhattan's street grid, shown in Figure 2.1. The locations on the space correspond to a set S of locations, where each location s is an ordered pair (x, y) and x designates a "street" and y designates an "avenue."

FIGURE 2.1. Manhattan's street grid as a spatial network partitioned by bus lines.



For example, the location *s* corresponding to the CUNY Graduate Center is (34th St., 5th Ave.), or more colloquially, "34th St. and 5th Ave."

On this map only locations due north or south from one another are connected; that is, as the road map *depicts* this space, we can travel only between two locations s_1 and s_2 if their ordered pair has the same avenue. These "privileged" connections could reflect a number of environmental impediments to getting between avenues: when traveling by car, streets between avenues often contain a lot of parked cars, and you are more likely to hit a red light when moving that direction. But, let us imagine instead that these routes have been created by the Metropolitan Transit Authority (MTA). The MTA has created bus lines that move only north and south, but not east and west (perhaps responding to the environmental impediments mentioned above.) The MTA's bus lines, then, establish a group of operations *G* generated by moving *1 block* north or south, the distance between streets. $G = \langle 1\ block \rangle$ partitions the locations on this map into those that share the same avenue, creating the north/south pathways shown on the map. Acting on the locations, *1 block* (34th St., 5th Ave.) = (33rd St., 5th Ave.) and *1 block* (34th St., 6th Ave.) = (35rd St., 6th Ave.): that is, these are "one-way streets."

We will call the rightmost partition "M5"—perhaps representing the particular bus line traveling that avenue—the one next to it "M6," and so on. The *partitioning* makes certain locations close while others, which may be close by other standards, are not even reachable by bus. Therefore, a student at the CUNY Graduate Center who needs to go to a library may be more likely to choose the NYU Bobst Library at "3rd St. and 5th Ave." over the Lincoln Center Library at "64th St. and Columbus Ave." In the mind of the graduate student, distance is conceived not "as the crow flies," but is judged instead on the basis of available routes created by the bus's only metric, *1 block* north or south.

The Manhattan bus map is a good analogy for the concept of a spatial network, especially one generated by a contextual transformation. Our (or the MTA's) interest in the operation *1*

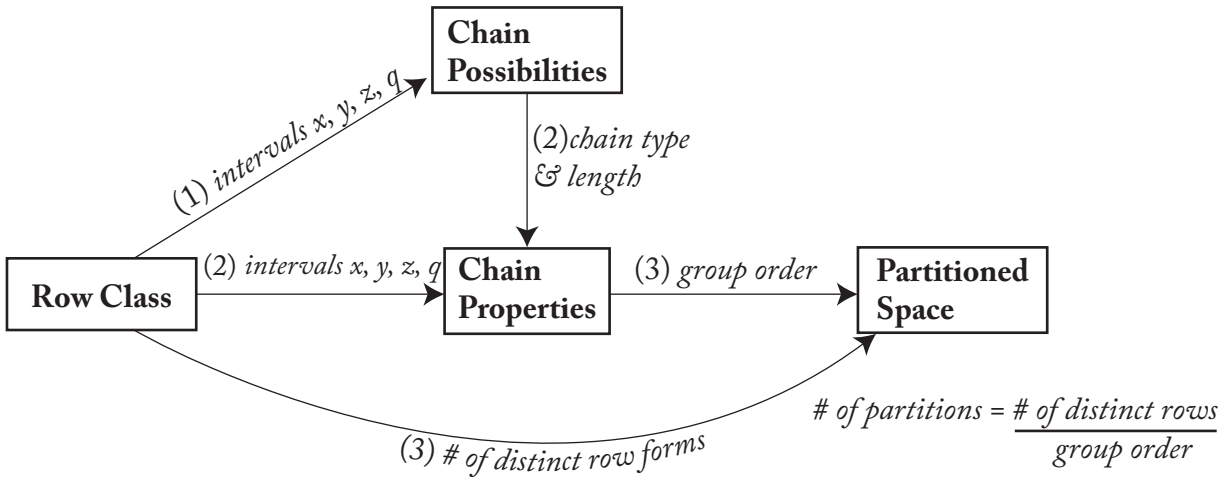
block is influenced by the locations (objects *s*), giving real meaning to the distance represented by *1 block*—the lack of paths from east to west, and so on. By analogy, transformation chains are manifestations of an intervallic environment specific to a particular row class, and a spatial network created by a transformation chain—all of its pathways, and the distances implied by them—is *molded by the row class*. In Figure 2.2 I have sketched a "path of influence" from a row class to a partitioned space, with row chains acting as intermediaries. That sketch summarizes Chapter 1 by showing how features internal to the row class have influence at certain points along this path, as do the transformation chains themselves:

(1) the intervallic properties of the row (symbolized by the values *x*, *y*, *z*, and *q*) determine the chain(s) that can transform that row;

(2) those intervallic properties, along with the type and length of the chain, dictate the chain group's properties;

(3) those properties and the number of distinct rows in a row class partition the space.

FIGURE 2.2. A "path of influence" from the row class to a partitioned spatial network.



At this point, the process may seem rather concrete, and as this chapter is concerned more with analytical methodology, it is worth considering at this point the *decisions* that do go into creating a spatial network. For one, if it represents a specific composition, a spatial network may need to contain more than one type of chain or a collection of some.[1] These are "horizontal" concerns. A spatial network capable of projecting a musical grammar should also communicate "vertical" properties—segmental invariance, inversional axis, and so on.[2] Creating an analytically interesting spatial network, then, is a creative act, perhaps resembling the job of a cartographer who chooses amongst a variety of social and/or environmental factors to create *maps*. "Map," more than "space," reflects the work of an outside influence—a cartographer, music analyst, "music cartographer"—in producing a representation.[3] These are representations that, as the cartographer Denis Wood might say, are socially constructed arguments.[4] Musical maps, then, should place the syntactical properties of objects into more robustly-conceived environments.

---

[1] Roeder, "Constructing Transformational Signification," notes the difficulty of choosing an "expressive object family" for transformational analysis. I would note that it should be similarly difficult to choose the appropriate transformation group, and that those decisions should often be made in conjunction.

[2] Many articles by Milton Babbitt are important in this regard: for example, "Twelve-Tone Invariants as Compositional Determinants" and "Set Structure as a Compositional Determinant," both in *The Collected Essays of Milton Babbitt*, edited by Stephen Dembski, Andrew Mead, and Joseph N Straus (Princeton: Princeton University Press, 2003): 55-69 and 86-108. Other significant studies include Bo Alphonce, "The Invariance Matrix" (Ph.D. dissertation, Yale University, 1974) and David W. Beach, "Segmental Invariance and the Twelve-Tone System," *Journal of Music Theory* 20, no. 2 (October 1, 1976): 157–184.

[3] In music theory and analysis, these terms, it seems, are often conflated, though they have the potential to articulate important conceptual differences. Cohn (*Audacious Euphony*) uses the term "map" when referring to the *creative* analytical act: "It acts as a stage upon which imaginative performances are mounted […] A musical map can illuminate compositional decisions as selections from a finite menu" (14-5). A *space*, by contrast, represents something real and factual, beyond dispute. Cohn (*Audacious Euphony*) again: the Cube Dance space "is a '*true*' model of voice-leading distance between triads" (84). At times in the scholarly literature, these distinctions seem to acquire quality judgements. For example, Dmitri Tymoczko notes that many transformation networks, notably the *Tonnetz*, "*distort* voice-leading relationships" ("Geometrical Methods in Recent Music Theory," *Music Theory Online* 16, no. 1 (2010), http://www.mtosmt.org/issues/mto.10.16.1/mto.10.16.1.tymoczko.html.), emphasis added.

[4] Denis Wood: "*the knowledge of the map is knowledge of the world from which it emerges* […] This of course would be to site [sic] the source of the map in a realm more diffuse than cartography; it would be to insist on a *sociology* of the map. It would force us to admit that the knowledge it embodies was socially *constructed*, not tripped over and no more than … *reproduced*" (Dennis Wood and John Fels, *The Power of Maps* (New York: Guilford Press, 1992): 18).

This chapter explores the theoretical and methodological concerns prompted by the concept of a musical "map." Section §2.1 and 2.2 shows how to construct multi-chain-generated spatial networks through the *product* process and compares the efficacy of *chain groups*, *classical serial groups*, and *hybrid groups* as representations of transformational distance. In §2.3 I return to the concept of chain syntax, arguing that chains form one half of a *paradigmatic/syntagmatic* understanding of Webern's music and showing how the concepts tie together spatial and "event networks." Those concepts lead to a refinement of the spatial network in which *relations* such as invariance and inversional axis act as "molecular bonds" that organize a network. I argue that this kind of "dual understanding" has predecessors in transformational analysis, showing two complementary spatial networks created by Richard Cohn. As much of the chapter is concerned with these relationships between the "horizontal" and the "vertical," the chapter closes (in §2.4) with an exploration of possible intersections with Schoenberg's system of combinatoriality and the nature of transformational "character."

§2.1 Product Groups
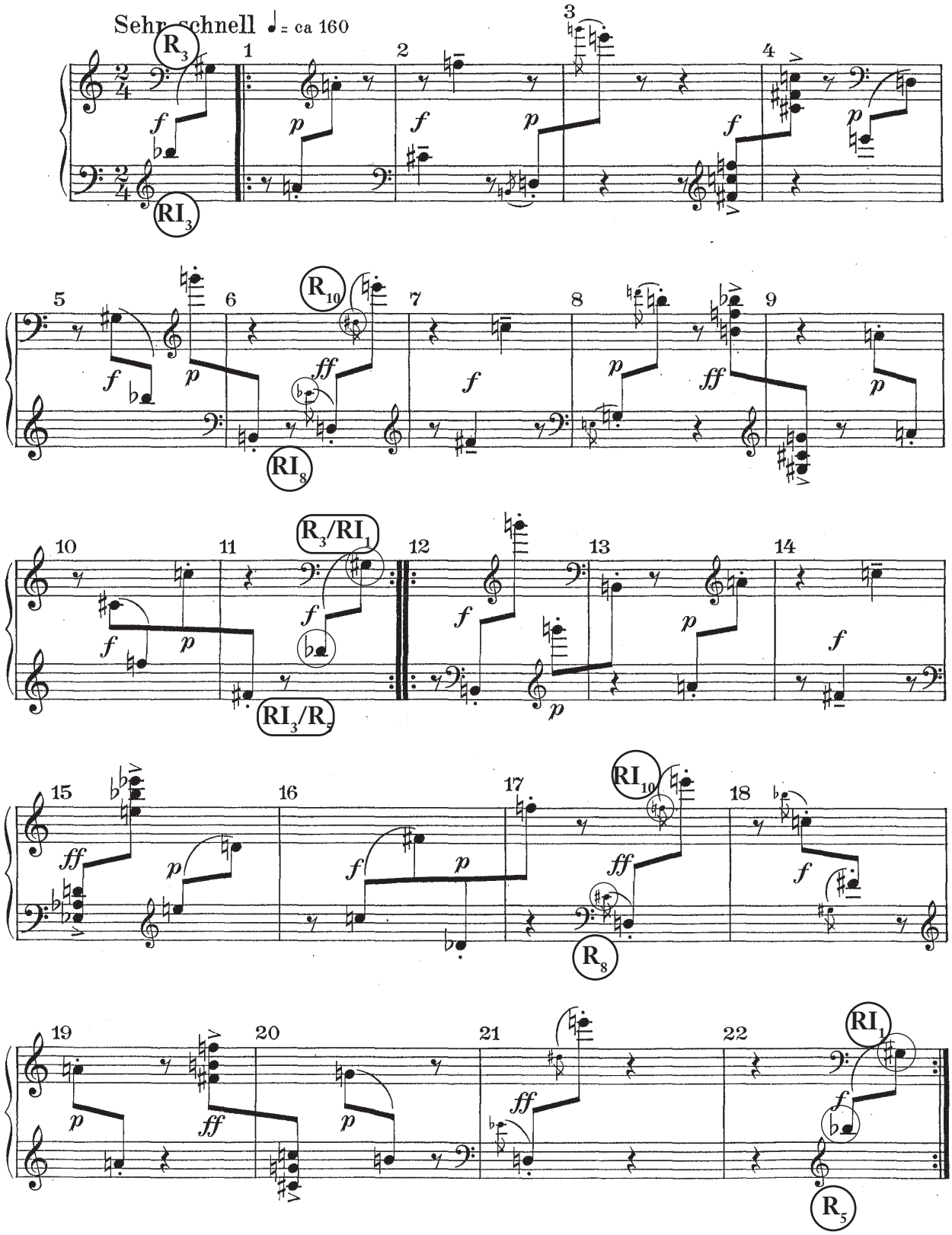
Figure 2.3 gives the score to the second movement of Webern's Piano Variations, Op. 27.[5] The movement is a two-voice canon whose voices are mirrored symmetrically around A4. The score in Figure 2.3 has been marked with row forms, and I have circled those pitches that involved in the transformation chains. Figure 2.4(a) shows an analytical diagram of this piece that reflects some basic transformational actions through the lens of the *classical T/I group* of transpositions and *fixed-axis* inversions.[6] In general, transpositions drive linear connections in the first half, and inversions take over in the second half. Notice the transformational diversity in this passage: two different types of transposition (T_5 and T_7) and five types of inversion (I_1, I_{11}, I_6, I_9, I_3).

By contrast, Figure 2.4(b) surveys the movement as a series of TCH_1s and ICH_1s. These two transformations form a *chain group*. This group includes TCH_1, ICH_1, $RECH_1$, and their compounds. This analytical diagram has some advantages over (a), and some disadvantages. First, there is a great deal more transformational *consistency*. Within the *classical T/I group* analysis at (a), the profusion of inversion operations invite comparisons: what is the relationships between I_1/I_{11} in the first half and I_9/I_3 in the second half? And how are we to understand those

[5] A model of economy, formal clarity, and a distillation of many Webernian characteristics, this movement has attracted considerable analytical attention. Mead, "Webern and Tradition," 179-87 offers an analysis showing how the form of the movement emerges at different "levels" of the system: the primitives of the twelve-tone system, the properties associated with a given row class, and the specific composition of row forms on the musical surface. While Mead's analysis considers serial organization, Catherine Nolan's revisionist analysis "circumvent[s] references to row deployment and transformation" ("Structural Levels and Twelve-Tone Music: A Revisionist Analysis of the Second Movement of Webern's 'Piano Variations' Op. 27," *Journal of Music Theory* 39, no. 1 (1995): 47–76.) In that sense it shares much in common with two important earlier studies by Peter Westergaard and Ray Travis, both of which considered linear aspects of the work apart from its serial organization: Peter Westergaard, "Webern and 'Total Organization': An Analysis of the Second Movement of Piano Variations, Op. 27," *Perspectives of New Music* 1, no. 2 (1963): 107–120; and Roy Travis, "Directed Motion in Schoenberg and Webern," *Perspectives of New Music* 4, no. 2 (1966): 85–89. My understanding of the work borrows somewhat from Hook and Douthett, "UTTs and Webern's Twelve-tone Music," 98-107.

[6] Each inversion $I_x = T_xI$.

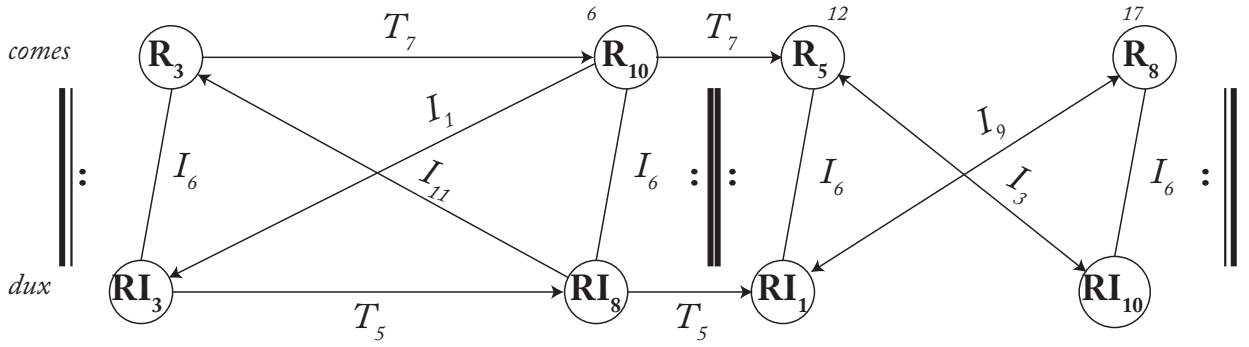
FIGURE 2.3. Piano Variations, Op. 27, II.



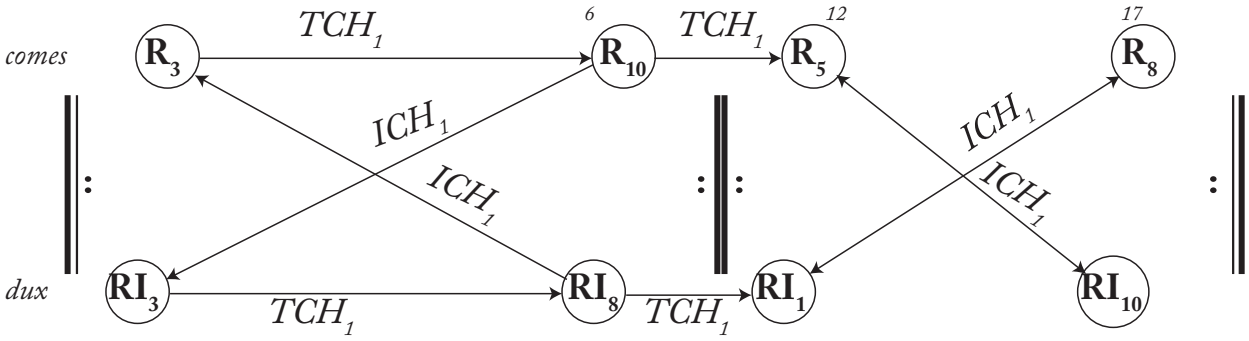
transformations in relation to the consistent I_6 that *binds* the two canonic strands together? These questions are not evident at (b). Instead, a narrative emerges: the *ICHs* that were somewhat subsidiary in the first section are preeminent in the second section. Yet, interpreting the music entirely within the chain group has one distinct disadvantage: it removes the structuring influence of I_6 that is captured at (a). Rows related as **R3** to **RI3** are not chain related, and thus, the vertical bonds go unnoticed at (b).[7]

FIGURE 2.4. Two transformation diagrams of the Piano Variations, Op. 27, II.

(a) *classical T/I group* analysis.



(b) *chain group* analysis.



[7] Actually, rows related as **R3** is to **RI3** are chain related, just not in the intuitive sense we have been pursuing. In particular *TCH1 ICH1* (**R3**) = **RI3**. But that compound operation does not *directly connect* the two rows by pitch elision.

My two analytical diagrams emphasize differences in the transformation groups that underlie these representations. Groups formed by TCH_1 and ICH_1 often produce analytical observations with significant differences as compared to a group formed from T_n and I_n. In what follows, I will foreground some of the reasons for these differences in terms of group construction and the ways in which each group projects distance. In each analysis above, I mentioned aspects of the representation that seemed important—the I_6 consistency at (a) and the chain consistency at (b). I will show that the good parts of these analyses do not need to be mutually exclusive. I do believe, however, that they represent different types of relationship that are often worth *separating*. The I_6 consistency responds to a feature of the music that is "binding" or "vertical." By contrast, the consistency created by the transformation chains represents an aspect of the music that is "syntactic" or "linear." This section closes with some thoughts about hybrid groups that try to represent both categories, an effort that is redoubled in §2.2 where these relationships differences are explored as representations of *paradigmatic* or *syntagmatic* categories of meaning.

2.1.1 Product Groups

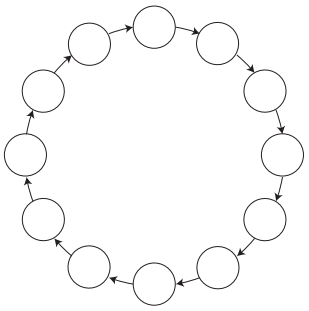
The spatial networks representing these groups can foreground these differences. And it turns out that these differences become most salient in the *process* used to create the networks. Before looking at these larger groups, Figure 2.5 sketches the process of creating the group $T/I = \langle T_1, I \rangle$ as it transforms a collection S of row forms.[8] In this group, both T_1 and I are generating transformations. Transposition should be understood in the normal manner and the inversion I is a contextual inversion—and *not* a *fixed-axis* inversion—that maps a **P**-form to the **I**-form with whom it shares the same first pitch class. Because we are combining features of two groups (transpositions and inversions), we call this a product group.

[8] Here I am following the process set forth in Carter, *Visual Group Theory*, 117-21.

FIGURE 2.5. A procedure for generating direct product groups.

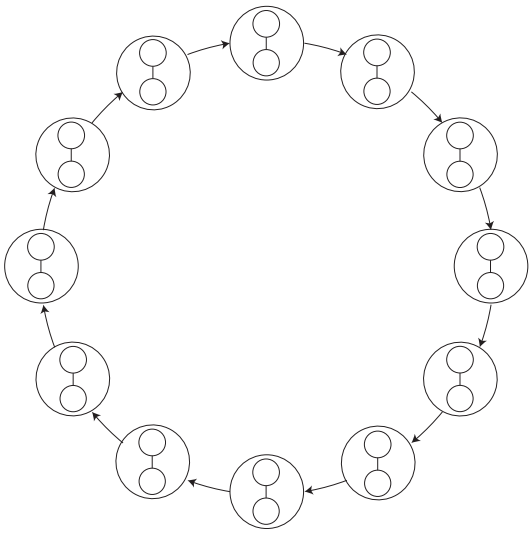
(1) Begin with a Caley diagram for either of the generating transformations.[9]

(a) Caley diagram of C_{12}, representing the generating transformation $\langle T_1 \rangle$.



(2) "Inflate" each noted of C_{12} such that a copy of the Cayley diagram for C_2, representing I, can be placed within.

(b) Inflated C_{12}, each node containing copies of C_2, which represents $\langle I \rangle$.

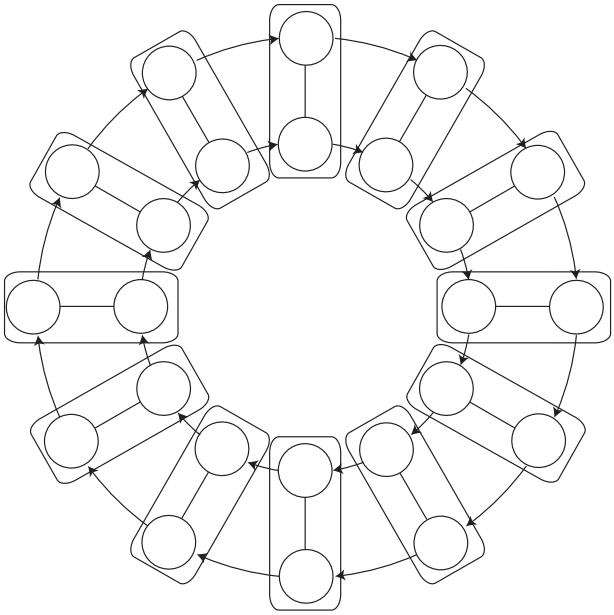


(3) Connect corresponding nodes from inside each larger node. [10]

[9] For clarity, I have omitted the contents of each node in these Cayley diagrams.

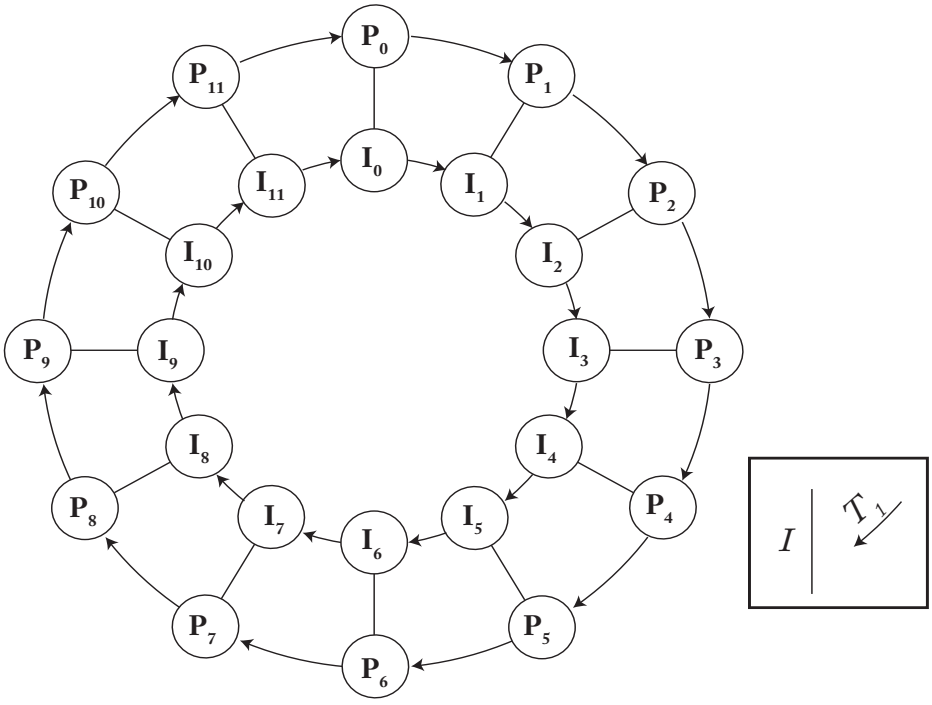
[10] If an additional generator were involved, steps 2 and 3 could be repeated. This shows a direct product group created by two *cyclic* groups. A direct product group may be generated from other types of groups, however.

(c) Caley diagram for $T/I = T_1 \times I$.



(4) Remove the inflated nodes. Insert any object from S into any one of the graph's nodes, automatically filling the remaining nodes.

(d) Spatial network for (S, T/I), where S = twelve-tone row forms and $T/I = T_1 \times I$.



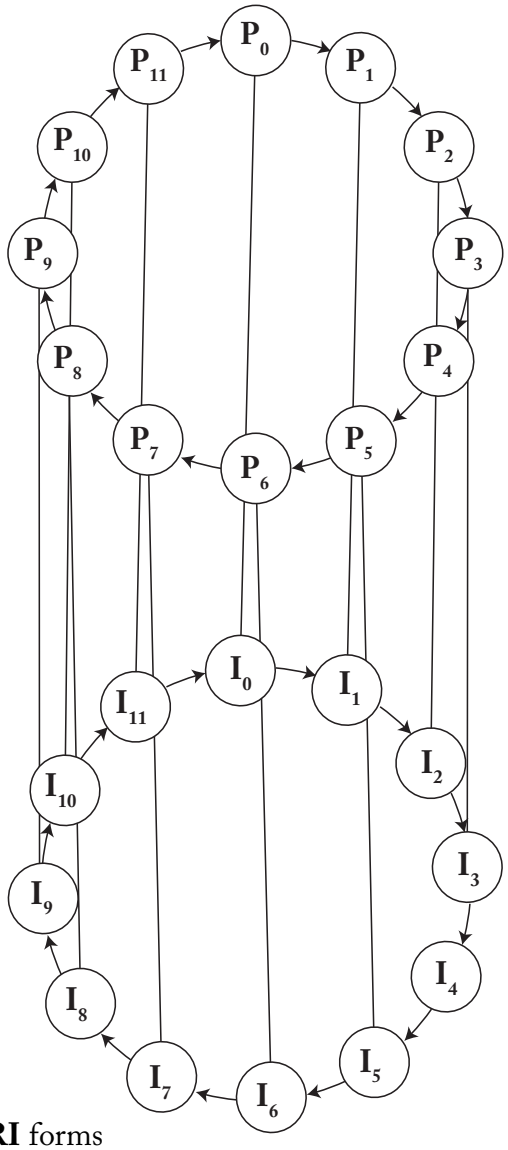
The spatial network at (d) symbolizes the group $T/I = T_1 \times I$ (pronounced "T_1 cross I") as it transforms a collection S of row forms. In this process, the two generating transpositions were responsible for the work of creating the network in steps (1) and (2), and they are called the group's *factors*.

Factors of a group are always subgroups of the group, and T_1 and I are but two of this group's many subgroups. From a representational standpoint, the process described above guarantees that the group's factors, which were also the group's generators, are emphasized in the network's visual presentation.

Factors in a direct product are *commutative*, meaning that the $T/I = T_1 \times I = I \times T_1$. Optically, spaces tend to resemble the first factor in the operation more than the second. Figure 2.5(e) is isomorphic to the spatial network in Figure 2.5(d), but more greatly accentuates the structuring characteristics of I. Both of these spatial networks (in (c) and (d)) are isomorphic to $C_{12} \times C_2$, the abstract groups resembling T_1 and I respectively. Like $C_{12} \times C_2$, the order of $T_1 \times I$ is 24—equal to the product of the order of each of the group's factors: $12 \bullet 2 = 24$.

As was the case with the simple groups containing a single generator, the order of larger groups is indicative of how the group will partition a collection of objects—in this case row forms. Forty-eight **P**, **I**, **R**, and **RI** forms

FIGURE 2.5(e) $T/I = I \times T_1$. The space is isomorphic to (d), but more greatly accentuates the structuring influence of I.



are partitioned by the *T/I group* into two, disconnected networks (48 ÷ 24 = 2). Figure 2.5(d) and (e) show only one of those subsets.

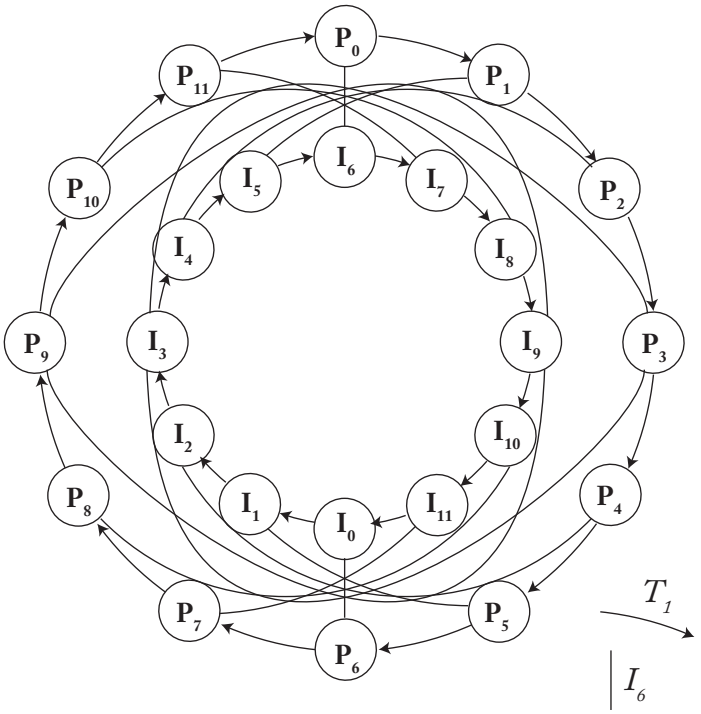
For our present purposes, perhaps the most important feature of the space is its *commutativity*. The order of operations performed in the group (and on the space) is inconsequential: $T_1I = IT_1$. As noted in §1.4.4, commutativity is a feature of all cyclic groups, and this commutativity is inherited in a direct product group.[11] The visual presentation of the space generally makes it is easy to identify whether or not a group is commutative—such groups have *concentric circles of arrows pointing in the same direction*. Note in Figure 2.5(d) and (e) that the two concentric circles of T_1 arrows point the same direction.

Logically, then, non-commutative groups *cannot* be assembled via direct product and they create differently structured spaces. Imagine the group $\langle T_1, I_6 \rangle$ that I used in my earlier analysis of the Piano Variations movement. There, the inversion I_6 was not defined contextually, but is instead *fixed around the A4 axis*. It is a well-known that this type of group, which I will call the *classical T/I group*, is non-commutative. As an illustration of the representational differences between commutative and non-commutative groups, I have attempted in Figure 2.6(a) to create a representation that retains the parallel, concentric circles from the commutative spatial network while using unoriented I_6 arrows to join them. The result is a tangled mess! Untangling the network by placing each row form opposite its I_6-partner (as at (b)) also creates a problem: the transposition arrows on the concentric circles are no longer labeled the same way. Rather, T_1 designates arrows on the outside of the circle, but is replaced by its inverse (T_{11}) in the center. To make this a true spatial network, showing only the group's generators as factors, the inner circle

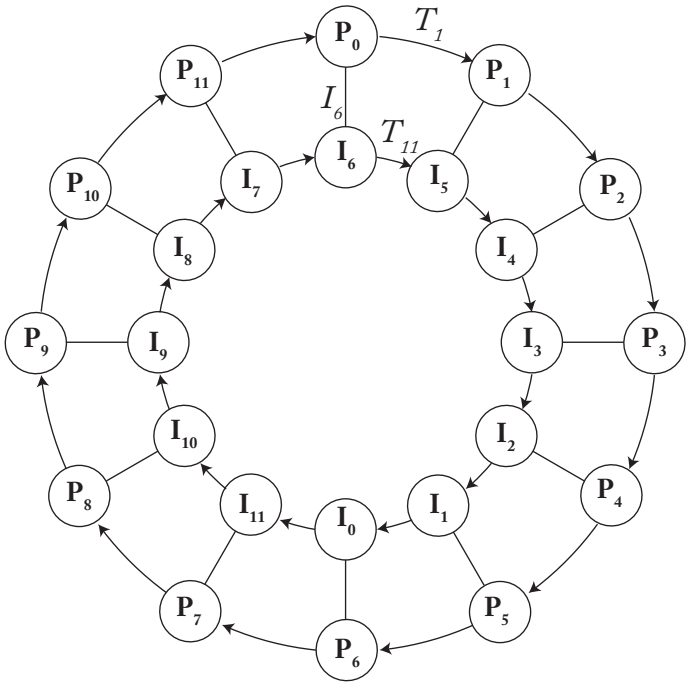
[11] In fact, every commutative (also called abelian) group C_{mn} can be expressed as the direct product of cyclic subgroups C_m and C_n, as long as m and n are co-prime. We saw earlier that the pc transposition group T, isomorphic to C_{12}, could be singly generated by T_1, T_5, T_7, or T_{11}. The group could also be generated as a direct product group $T = T_3 \times T_4$. Doing so makes those subgroups visually prominent in the representation. See Carter, *Visual Group Theory*, 101.

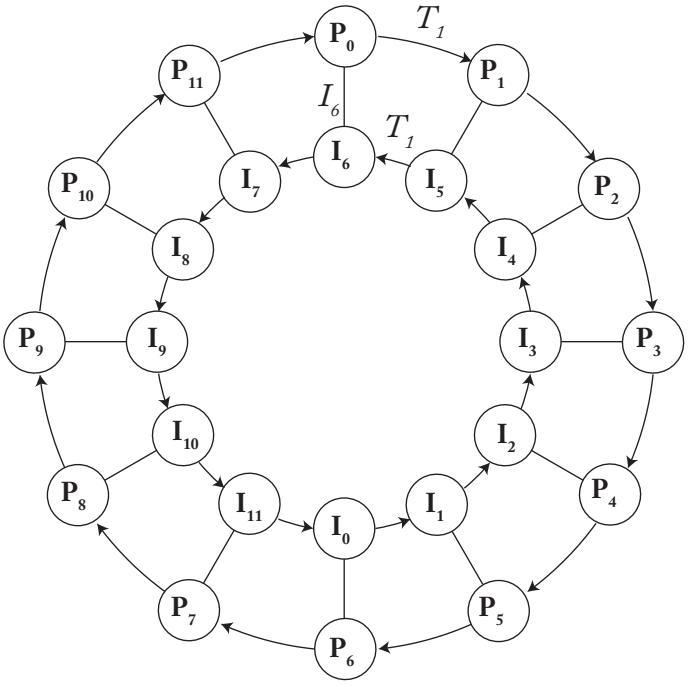
FIGURE 2.6. Constructing a non-commutative group.

(a) Retains the parallel, concentric circles.



(b) Relabels the inner circle arrows as T_{11}.





(c) Non-commutative space with arrows pointing in opposing directions

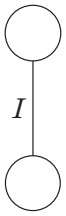
of arrows need be reversed (as is shown at (c)).

Therefore, the mark of a non-commutative group is concentric circles of arrows pointing in *opposite* directions. The space at (c) suggests the following product process (similar to the direct product process above) that leads to a representation of the *classical T/I group*:

FIGURE 2.7. A procedure for generating semi-direct product groups.

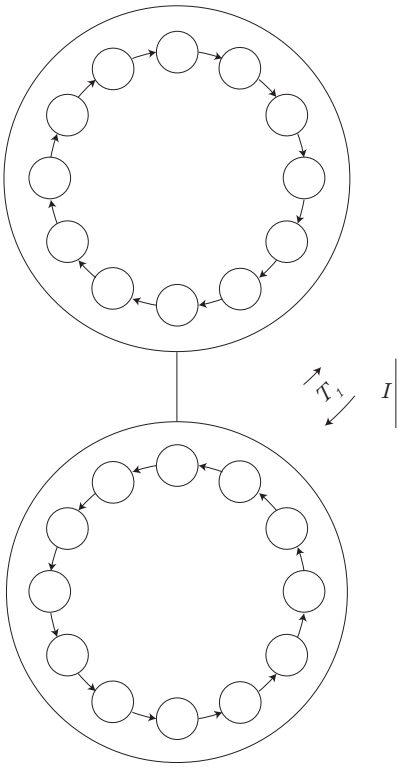
(1) Begin with Cayley diagram on the generator isomorphic to C_2; in this case, I_6.

(a) Caley diagram of C_2, representing the generating transformation $\langle T_1 \rangle$



(2) As before, inflate each nodes and place the Cayley diagram for the other generator within. *Then, reverse the arrows of one of these copies while retaining the transformation label.*

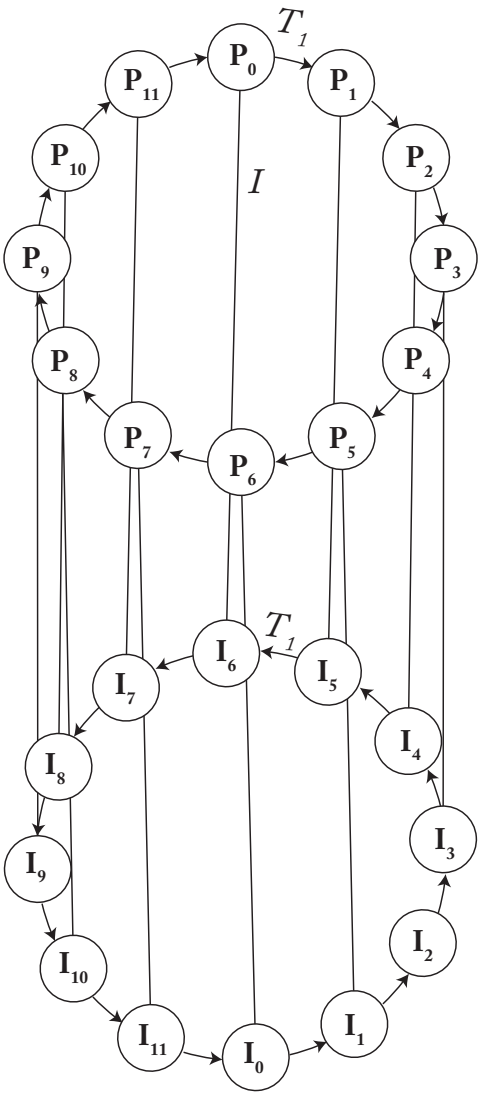
(b) Inflated C_2, each node containing copies of C_{12} representing $\langle T_1 \rangle$ Arrows in one node are reversed.



(3) Connect corresponding nodes from inside each larger node.

(4) Remove the inflated nodes. Insert any object from *S* into any one of the graph's nodes, automatically filling the remaining nodes. (As earlier, this space can be reorganized so that one of the large, T_1 circles sits inside the other. (See Figure 2.6(c), for example.)

(d) Spatial network for (*S*, *T/I*), where *S* = twelve-tone row forms and $T/I = T_1 \rtimes I$



This process bears a great deal of similarity to the direct-product process, the most importance difference being the reversal of one circle's arrows in step two. [12] An additional and deeply-important part of this new process is that it is *not commutative*. Had I begun with $\langle T_1 \rangle$ in step one and inserted copies of $\langle I \rangle$ into each of its nodes in step two, reversing the arrows of $\langle I \rangle$ in step three would have produced a different network. It is essential to begin a

[12] Carter, *Visual Group Theory*, 128-32 and 177-9. See also Hook, "Uniform Triadic Transformations," 103-4.

A more precise way of defining this would be to say that the nodes in C_2 contain the members of the automorphism group of C_{12}.

semi-direct product with the group isomorphic to C_2.[13] This type of product operation is *semi-direct*. It is symbolized with by "⋊," and it underlies some of the most common non-commutative groups studied by music theorists.

Note that the two factors of this group are isomorphic to the cyclic groups C_{12} and C_2, which formed the structural basis of the commutative direct product group $T_1 \times I$ explored earlier. Their difference, which is crucial, lies in the fact that the *classical T/I* group is constructed by semi-direct product, rather than direct product. Semi-direct products of the type $C_n \rtimes C_2$ form *dihedral* groups, symbolized as D_n. Therefore, the non-commutative, *classical T/I group* is isomorphic to the dihedral group D_{12}.[14] The order of a dihedral group (equal to the number of operations in the group, is always $2n$.[15] Therefore, $T_1 \rtimes I$ has 24 (=12 • 2) elements, and as with commutative *T/I group*, $T_1 \times I$, it will partition the forty-eight row forms into two disconnected networks.

Before proceeding, a quick summary:

- A group $G = \langle G, H \rangle$, whose generators commute with one another, can be combined to create a *direct product group* of the form $C_n \times C_m$ *that is also commutative*.[16] The process is

---

[13] In the direct product group $G \times H$, both of the generators are *normal subgroups*. That is, for all g in G, gHg^{-1} is H and for all h in H, hGh^{-1} is G. These are *conjugates* of H and G (Carter, *Visual Group Theory*, 142-7)). By contrast, a semi-direct product group $G \rtimes H$ contains only one *normal subgroup*. In the classical *T/I* group, the subgroup *T* of pc transpositions is normal, but the subgroup *I* is not. We may say more generally, then, when creating the space for a semi-direct product group, always begin the process with the subgroup that is *not normal*.

[14] Note that the group of neo-Riemannian transformations, generated by *L* and *R*, create a dihedral group D_{12} (occasionally called D_{24}) that is isomorphic to the *classical T/I group*. See Satyendra, "An Informal Introduction to Some Formal Concepts from Lewin's Transformational Theory," 118-23, for example.

[15] It is not uncommon for the dihedral group D_n to be called D_{2n}. For example, D_{12}, which symbolizes the neo-Riemannian group, is sometimes called D_{24}. In these cases, the subscript represents the order of the group, as it does with cyclic groups. Cyclic groups describe objects that have rotational symmetry, like propellers. A three-blade propeller can spin in only three ways that will preserve its shape, and can be described by C_3. Dihedral groups describe objects that can spin *and* flip—regular polygons, for example. Therefore, these groups possess twice as many actions that we find in a similarly shaped cyclic group. The dihedral group describing a three-side polygon while therefore be called D_3.

[16] Though the group contains to cyclic groups, it is *not* necessarily a cyclic group itself. Given two cyclic groups *G* and *H*, $G \times H$ is cyclic only when the generators of the group are coprime—that is, divisible by no positive number but 1.

commutative (for example, when I is contextually-defined, $T_1 \times I = I \times T_1$), though the space will often emphasize the first of the factors.

- By contrast, a group $G = \langle G, H \rangle$, where H is isomorphic to C_2 and does not commute with G, combines to form *semi-direct product groups* of the form $C_n \rtimes C_2$. These groups are *non-commutative* and the semi-direct product process is not commutative either. When T_1 and I do not commute, $T_1 \rtimes I \neq I \rtimes T_1$. These groups are also called dihedral groups.

2.1.2 Serial Groups

Larger groups can be generated by a additional "passes" through the two processes. Before constructing these spatial networks, we must determine whether the transformations are commutative, as commutative transformations generate direct-product groups and non-commutative transformations generate semi-direct product groups.

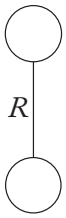
The *classical serial group* combines the non-commutative *classical T/I group* with the order operation R, which retrogrades the pitch classes in an ordered series.[17] (Thus, RI is a compound operation, R then I.) Order operations such as R generally are commutative, and they form two-element groups isomorphic to C_2. As such, the *classical serial group* is the direct product the *classical T/I group* together with R: $(T_1 \rtimes I) \times R$. Figure 2.8. diagrams the process of constructing the group.

This is a very familiar group of transformations. Nonetheless, some facets of the group should deserve emphasis in the present context. First, in Figure 2.8(c) the concentric circles on the top of the space contain arrows pointing in opposite directions, as do the two circles on the bottom. This reflects the non-commutativity of the subgroup $T_1 \rtimes I$. However, comparing the dual pair of circles at the top of the figure (perhaps easiest seen at (b)) with those on the bottom

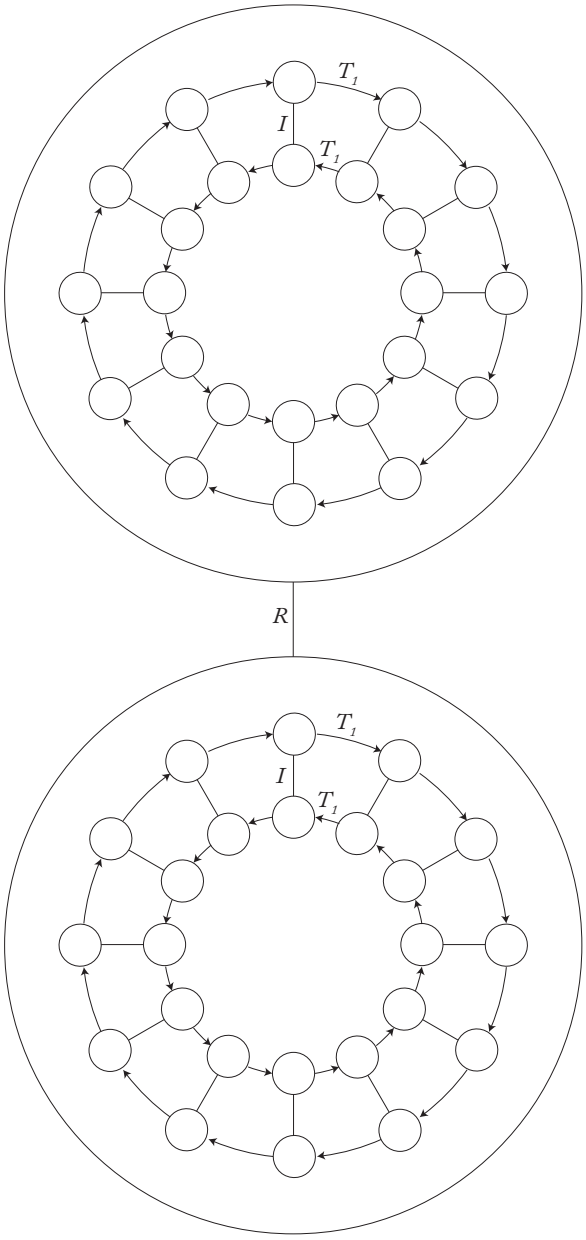
[17] See Morris, *Class Notes for Advanced Atonal Music Theory*, 56. Morris calls this the "serial group". Following Hook, *Musical Spaces and Transformation,* I prefer "classical serial group" to distinguish it from a group that would include operations such as series rotation.

FIGURE 2.8 Constructing the *classical serial group*: $(T_1 \rtimes I) \times R$.

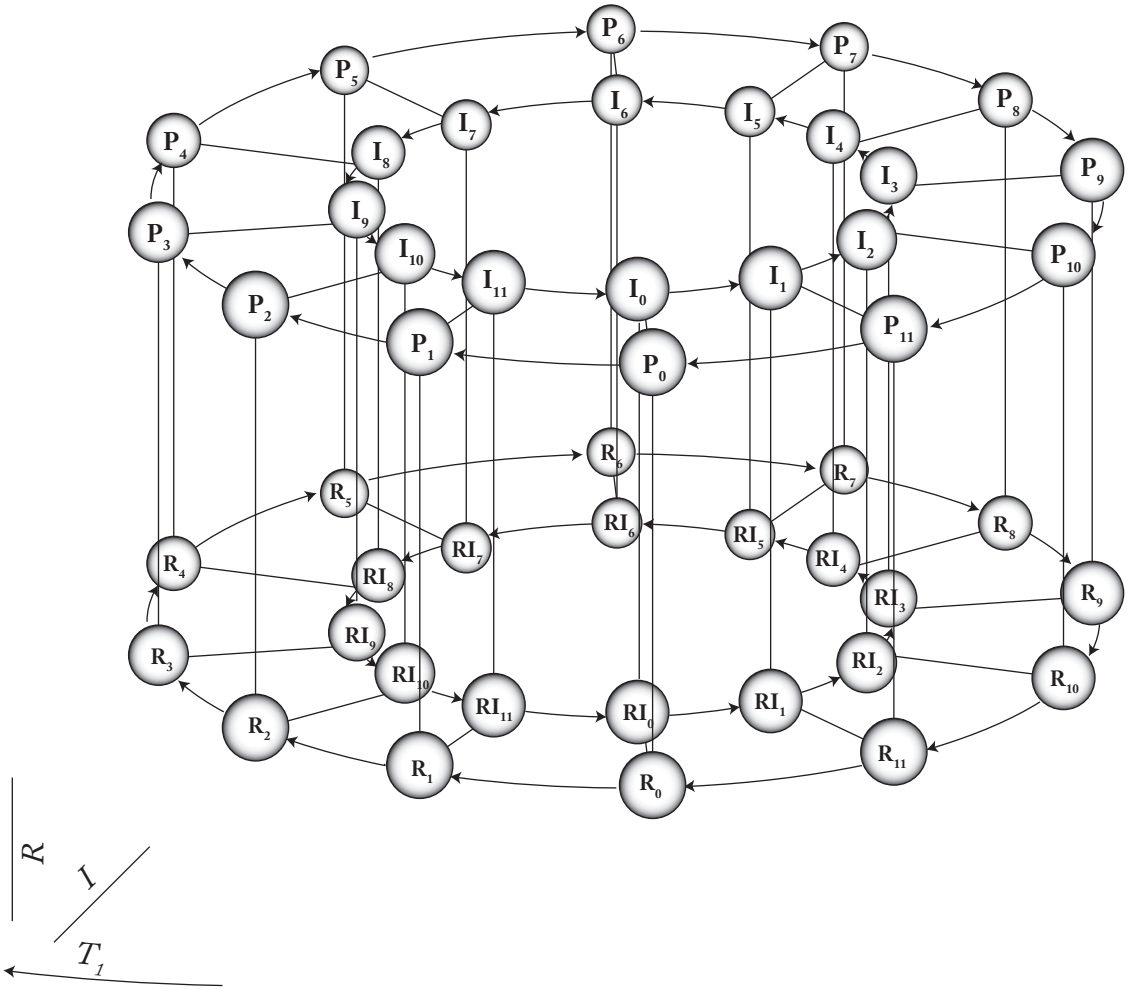
(a) Cayley diagram for R, isomorphic to C_2.



(b) $(T_1 \rtimes I)$ inserted inside each of the nodes from (a). Note the non-commutativity inside each of the nodes, represented by arrows pointing in *opposite* directions, but commutativity between those operations and R, which is emphasized by the identical arrow structure of arrows in the two nodes.



(c) The *classical serial group*: $(T_1 \rtimes I) \times R$
acting on twelve-tone row forms



reveals structural similarity—arrows pointing in the same direction. That parallelism represents the fact that *R*, which connects the circles, *does* commute with T_1 and *I*. Though the space can model all forty-eight classical serial operations (*T*, *I*, *R*, and *RI*), the group is generated by T_1, *I*, and *R* alone. In the group, the transformation *RI* is a "compound operation" equivalent to performing *"R* and then *I*." In most analytical systems, *RI* is no less "basic" than the other serial operations, and so its worth questioning its absence here. I could have included *RI* as a generator,

but its inclusion would have been "redundant," and the simplest presentation of a group always avoids these redundancies.[18]

2.1.3 Chain-Generated Groups

As an analogue to the *classical serial group*, we can imagine *chain groups* containing just chain transformations. To create *chain groups*, we need the following principles of commutativity:

(1) TCH_i and TCH_j commute, for any i or j ;

(2) $RICH_i$ and $RICH_j$ do not generally commute;

(3) TCH generally commutes with *RICH*;

(4) *ICH* and *RECH* commute.

(5) *TCH* and *RICH* do not generally commute with *ICH* or *RECH*.

Our understanding of chains in UTT terms (see §1.3.3, especially Table 1.6) is helpful in understanding the chains's commutative properties. Because two of the four transformation chains (*TCH* and *ICH*) resemble Riemannian *Schritts* and *Wechsels*, the commutative properties of the Riemannian group are applicable here.[19]

Hook notes that mode-preserving UTTs always commute, while mode-reversing UTTs commute only in special situations. Therefore, *TCH* (which is equivalent to a mode-preserving *Schritt*) commutes with itself regardless of *TCH*'s length. *ICH,* however, is equivalent to a mode-reversing *Wechsel.* The only two *Wechsels* that commute are ⟨-, 0, 0⟩, also known as the *P(arallel)*

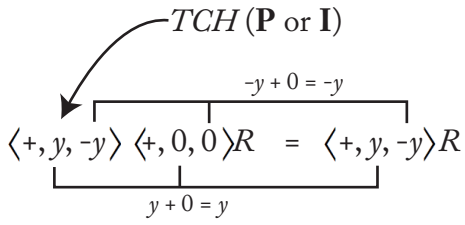
[18] Gollin, "Representations of Space," 3-12 argues that *RI* (as opposed to *IR*) can represent distinct ways of hearing a passage. To emphasize that *RI* can be a "unitary," rather than composite operation, Gollin memorably renames *RI* as "George" (11). We might imagine many different serial groups, depending on which transformations we take to be basic. For example: $(T_1 \rtimes I) \times RI$: In this presentation, *R* is not a generator but is understood as a compound operation equivalent to "*RI* and then *I*." This is an order 48 group. Replacing T_1, with T_5, T_7, or T_{11} would generate a space automorphic to Figure 2.8(c). Substituting other transposition values, for example T_3, decreases the size of the group and would therefore partition a typical row class into disconnected subsets.

[19] Hook provides the following, helpful summary of UTT commutativity: "mode-preserving UTTs always commute; a mode-preserving UTT *U* and a mode-reversing UTT *V* commute if and only if *U* is some transposition T_n; two mode-reversing UTTs ⟨-, *m*, *n*⟩ and ⟨-, *i*, *j*⟩commute if and only if $n - m = j - i$" ("Uniform Triadic Transformations," 69-70).

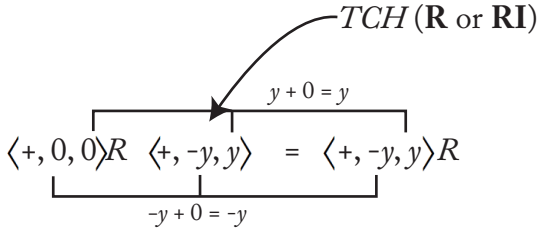
transformation, and ⟨-, 6, 6⟩, the *Gegenleittonwechsel*. While some *ICH* chains may be ⟨-, 6, -6⟩, the twelve-tone system does not allow for chains equivalent to ⟨-, 0, 0⟩, and therefore, *ICH* chains never commute. Further, *Schritts* and *Wechsels* do not commute, so *TCH* and *ICH* do not commute with one another.

Surprisingly, *TCH* does not commute with *RECH*. Because $RECH_1$ is equivalent to *R*, and order operations tend to commute with other transformations, it would seem that $RECH_1$ should commute with *TCH*, but that is not the case. Compare the following two expression:

(a) *TCH* • *RECH*



(b) *RECH* • *TCH*

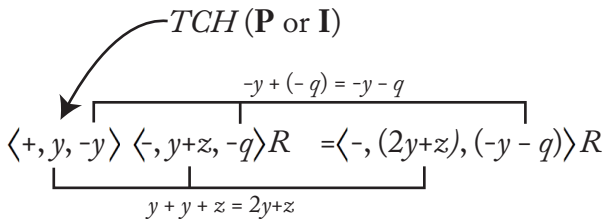


Recall from Table 1.6 that *TCH* stands for two *unique*, but inverse-related UTTs. In swapping the order of *TCH* and *RECH*, these two expressions necessarily require that *TCH* be unique in each case. Thus, whereas $\mathbf{P}_x \xrightarrow{TCH} \mathbf{P}_{x+y} \xrightarrow{RECH} \mathbf{R}_{x+y}$; $\mathbf{P}_x \xrightarrow{RECH} \mathbf{R}_x \xrightarrow{TCH} \mathbf{R}_{x-y}$.

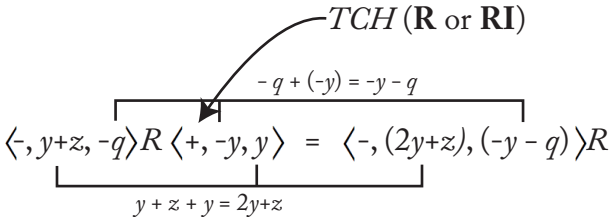
Interestingly, *TCH* does commute with *RICH* for the same reason that it does not commute with *RECH*. This is also somewhat surprising. Hook notes that a mode-preserving transformation (such as *TCH*) will not generally commute with a mode-reversing transformation

(such as *RICH*), unless the mode-preserving operation is some T_n, which *TCH* is not. But when combining with *RICH*, *TCH* becomes *somewhat* like a transposition. Compare the following expressions.

(a) *TCH* • *RICH*



(b) *RICH* • *TCH*



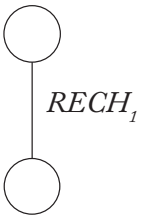
Thus, *TCH* • *RICH* = *RICH* • *TCH*. Note that, like the above combination of *TCH* and *RECH*, both "types" of *TCH* appear here. And because *RICH* is a mode-reversing transformation, the expression at (b) adds "*y*" to "*y* + *z*" and "-*y*" to "-*q*," just as it did at (a).

In both cases, the commutativity or lack thereof comes as the result of *TCH* standing for two, inverse-related UTTs. Therefore, while *TCH* mimics the *classical T/I* group in that it does not commute with *ICH*, it has an exact opposite relationship to the *classical serial group*. There, T_n commutes with *R*, but not with *RI*. Here, *TCH* commutes does not commute with *RECH*, but does commute with *RICH*.

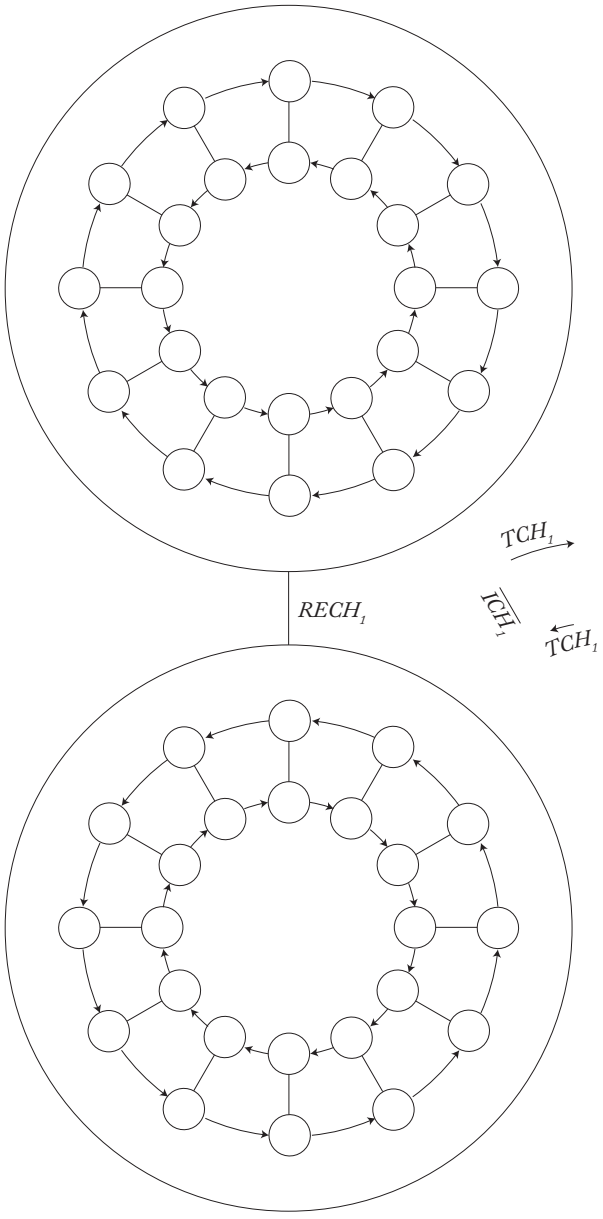
The properties have very interesting analytical repercussions, which we will see shortly. Before exploring those, Figure 2.8 constructs a spatial network for the *chain group* = $\langle TCH_1, ICH, RICH_1 \rangle$, where $y = 5$.

FIGURE 2.9. Constructing a *chain group*: $(TCH_1 \rtimes ICH) \rtimes RECH$.

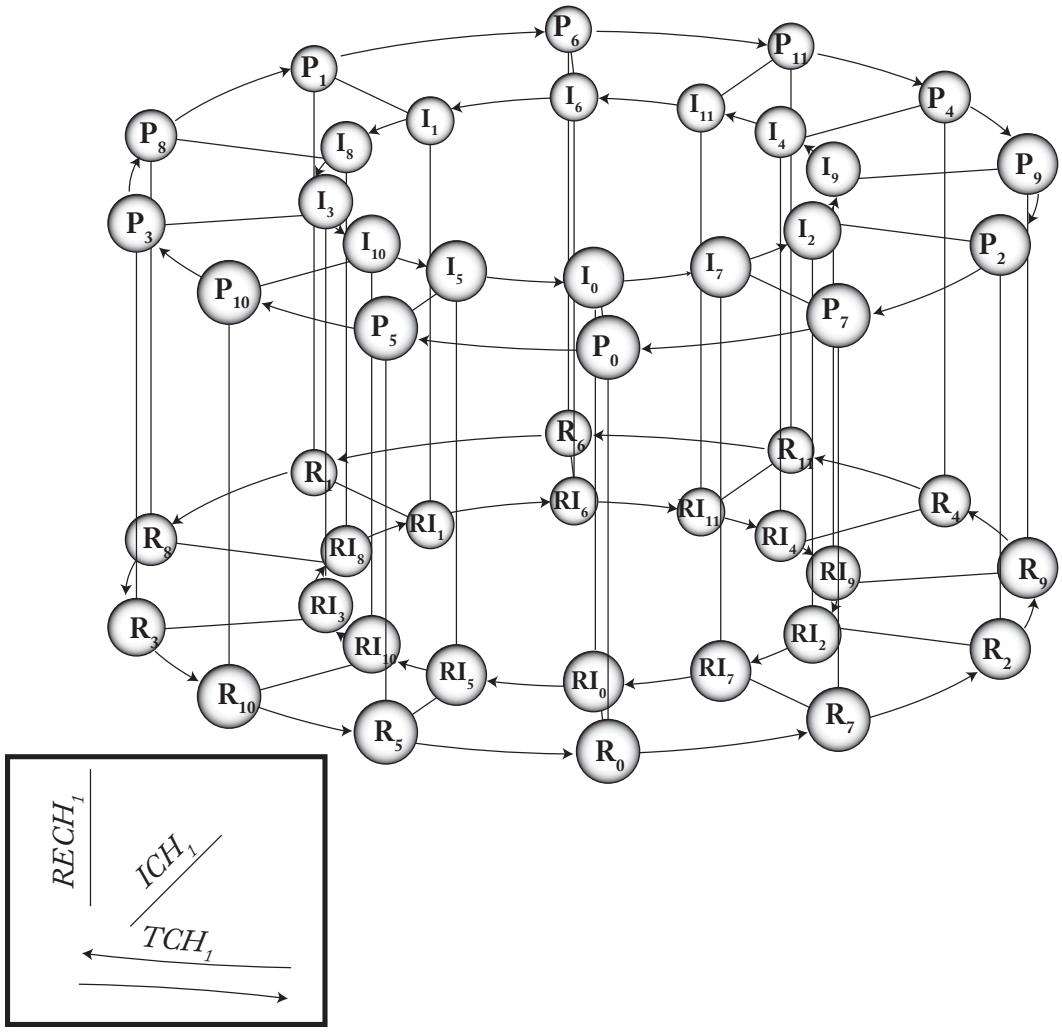
(a) Cayley diagram for R, isomorphic to C_2.



(b) $(TCH_1 \rtimes ICH)$ inserted inside each of the nodes from (a). Note the non-commutativity inside each of the nodes, represented by arrows pointing in *opposite* direction, *and* the non-commutativity between the nodes (cf. Figure 2.7(b)). The lower node's arrows are pointing in opposite directions as compared to the upper node.



(c) The *chain group* = $(TCH_1 \rtimes ICH_1) \rtimes RECH_1$ acting on twelve-tone row forms where $y = 1$.



Because neither *RECH* or *ICH* commute with *TCH*, the *chain group* $\langle TCH_1, ICH_1, RECH_1 \rangle$ is generated by two semi-direct products—$(TCH_1 \rtimes ICH_1) \rtimes RECH_1$. Thus, the "disc" on the bottom of Figure 2.9 is oriented opposite to the disc at the top, and the circles within each disc are oriented opposite as well. As a result, *RI*-related rows follow *TCH* arrows moving in the same direction and *R*-related rows follow *TCH* arrows moving in opposite directions. Compared to the classical serial group (Figure 2.9(c)), this is completely reversed.[20]

[20] Comparing the abstract groups behind these structures, the classical serial group is isomorphic to $D_{12} \times C_2$, while a chain group is isomorphic to $D_{12} \rtimes C_2$.

Before moving on, I will note that chain groups generated by TCH_1, ICH_1 and $RECH_1$ vary in size according the value for y. When y = 1, 5, 7, or 11, the *chain group* = $\langle TCH_1, ICH_1, RECH_1 \rangle$ is an order 48 group that connects *all forty-eight rows* in a typical row class. But if y = 2 or 10, the order of the group is 24; if y = 3 or 9, the order is 16; if y = 4 or 8, the order is 12; and if y = 6, the order is only 8. These specific groups partition a row class into two or more subsets of disconnected rows.

2.2 The Efficacy of Chain Groups

Since §1.4.1, I have been promoting chain-generated spatial networks as models of "musical grammar" for Webern's twelve-tone music. It is my understanding that rows are lexical objects, and transformation chains are syntactical elements. Navigating Figure 2.9 through the arrows on the network, then, is akin to "obeying" normative grammatical rules. Row connections need not follow *only* those paths, but the space gives us room to interpret those kinds of connections as exceptional.

Of course, Figure 2.8, which represented the *classical serial group*, could also be a musical grammar, with its objects and transformations fulfilling the same roles that I ascribed to the *chain group*. And thus, it raises the question as to the efficacy of *chain groups* as compared to *classical serial groups*. I believe there are at least three reasons for that chain-generated analyses often have greater value. The first, which was the primary subject of Chapter 1, is that these transformations are contextual and derive their meaning from the lexical objects (row classes and rows) on which they act. I will add, now that we have seen how multiple chains generate larger spaces, that this reciprocality also shapes the resulting spatial network in a very literal way. A spatial network's size depends upon the intervallic configuration of a row class.

Chains groups are more potent analytical structures for two additional reasons, that I will explore in the following section. First, transformation chains often offer *simpler* transformational interpretations of a passage. As a principle for music analysis, Occam's razor is often deficient. Music is complex. But if a system claims to model syntax, as I believe transformation chains can, simple interpretations should be the normative ones.[21] Finally, in both technical and conceptual ways, transformation chains interact with serial operations in highly suggestive ways. §2.2.3

[21] In §2.2.3 below, I will show an example, in connection with the first movement of Op. 27, where the "simplest" syntactical interpretation is not the most interesting.

shows that the commutative properties of "hybrid groups" of chains and serial operations have considerable analytical potential.

2.2.1 Transformational Distance and Analytical Simplicity

To return to the bus-map analogy that began this chapter: when imagining the distance of a path on the bus map as representative of some transformation in a group, we are thinking metaphorically. This is because in the group $G = \langle 1\ block \rangle$ the group element *1 block* is *not,* in group terms, any less "far" than *3 blocks*, or *30 blocks*. Nor is T_1 "smaller" than T_7 because "one" is a smaller number than "seven." And in fact, in the *chain group* represented in Figure 2.9(c), TCH_1 is not "less distant" than $TCH_1 \bullet TCH_1$.[22] All of this is despite the fact that we often imagine group elements in these terms. But groups are very abstract structures, and do not represent distance by necessity. Dmitri Tymoczko has criticized this aspect of transformation theory, contending that "[transformation theory] simultaneously asserts that intervals represent 'measurements' or 'distances' […] while also proposing a formal group-theoretical model in which magnitudes are not explicitly represented."[23]

But although ascribing primacy to *1 block*, T_1, or TCH_1 is a metaphorical act, it is one that most music theorists are quite comfortable with. Metaphors, after all, lie at the heart of many of music theories and analytical methodologies, and they are even more central to most music

[22] Rings, *Tonality and Transformation*, 13-4, has offered a helpful summary of these issues.

[23] Dmitri Tymoczko, "Generalizing Musical Intervals," *Journal of Music Theory* 53, no. 2 (September 21, 2009): 243. See also Dmitri Tymoczko, "Lewin, Intervals, and Transformations: a Comment on Hook," *Music Theory Spectrum* 30, no. 1 (2008): 164–168.

theorists use of musical spaces.[24] Although we often feel comfortable imparting distance and even direction upon a spatial representation, it is important, as Tymoczko has noted, to support these metaphors rigorously. Tymockzo proposes a simple addition to Lewin's formulation of a transformation group that stipulate each "intervals" size. T_0 may have a size of "zero," T_1 and T_2 are size "one," and so on.[25] Alternatively, Edward Gollin has shown how group structure itself can provide a notion of distance.[26]

Group structure, inasmuch as it is shaped by group generators, has been a central concern to this point, and therefore, Gollin's conception of transformational distance reverberates with the spirit of the present work. For all elements *g* in a group *G,* Gollin establishes their size as the *length* of a *word* representing them. A *word* is expressed in terms of the generator(s) of *G*, which have a size of 1.[27] Consider the pc transposition group $T = \langle T_1 \rangle$. In this group, T_1 is a word of length one, while $T_2 = T_1T_1$ has a length of two, $T_3 = T_1T_1T_1$ has a length of three, and so on. Gollin's formulation of distance therefore relies heavily on *the generation* of a group, which will

---

[24] Research in cognitive linguistics, particularly that by George Lakoff, Mark Johnson, and Mark Turner has developed concepts, such as "conceptual metaphors," that have been used effectively to describe music theories and create music analyses. (See, for example, George Lakoff and Mark Johnson, *Metaphors We Live By* (Chicago: University of Chicago Press, 2008). Music research in this area is too extensive to survey. Representative studies include Candace Brower, "A Cognitive Theory of Musical Meaning," *Journal of Music Theory* 44, no. 2 (2000): 323–379 and "Paradoxes of Pitch Space," *Music Analysis* 27, no. 1 (2008): 51–106. More general studies include Michael Spitzer, *Metaphor and Musical Thought* (Chicago: University of Chicago Press, 2004); Lawrence M. Zbikowski, *Conceptualizing Music: Cognitive Structure, Theory, and Analysis* (Oxford University Press, 2005). Straus (2011, 57-63) has an interesting discussion of Webern's Concerto for Nine Instruments that uses the image schema "SOURCE-PATH-GOAL" as an analytical backdrop.

[25] Tymoczko, "Generalizing Musical Intervals," 245-6.

[26] Gollin, "Representations of Space." Tymoczko, "Generalizing Musical Intervals," 247-8 summarizes these two approaches, noting primarily that his proposal is less restrictive than Gollin's.

[27] This is the source of Tymoczko's differences with Gollin (cf. Tymoczko's "Generalizing Musical Intervals" with Gollin's "Representations of Space). Tymoczko notes that there may be situations in which group generators should be different sizes, pointing especially to the Riemannian *Tonnetz*. For Tymoczko, those determinations most often relate to voice-leading distance as measured in semitone displacements. Thus, if the *Tonnetz* were generated by *L* and *R*, Tymoczko would prefer that *L*, which involves a single semitone of motion, be an element of size 1 and *R,* which requires two semitones of motion, an element of size 2.

At present, it is difficult to see how similar criteria might be created for representing transformation chains acting on twelve-tone rows. Section §2.3 makes some suggestions in this regard by understanding chains acting within a paradigmatically defined environment that is influenced by invariance and inversional axis, for example.

always have a defining affect on its measure of distance.[28] If we imagine the pc transposition group generated differently, for example, as $T' = \langle T_7 \rangle$, each element may have a different size. Though T' is automorphically related to T, containing the same group elements, in T', T_7 is word of length one, and $T_1 = T_7\ T_7\ T_7\ T_7\ T_7\ T_7\ T_7$, is a word of length seven. In terms of the present study, Gollin's formulation of distance is powerful primarily because the generators we are considering are pre-determined by the row class; and therefore, interpretive decisions about a group's generation are kept to a minimum

Spatial networks created from group generators capture distance as paths from one node to another along a *single* arrow. Thus, in Figure 2.9(c), TCH_1 has a size of 1, following a single arrow from one node to another, while $TCH_1 \bullet TCH_1$, requiring two arrows, has a size of 2. Measurements such as these provide a rigorous method for judging the "simplicity" of an analysis as a function of its parsimony, efficiency, and consistency. A transformational pathway that traverses less distance is simpler than one that requires more movements.

Figure 2.10 uses word length to compare my earlier analyses of the second movement of the Piano Variations, which involved two different transformation groups. My analysis within the *classical T/I group* is given at (a), the *chain group* analysis at (c). Determining transformational distance in this passage requires first finding the best *interpretation* of the the *T/I* group.[29] Specifically, we must decide which transformations should be primary and should receive a unit of 1. Studying Figure 2.10(a), the fixed-inversion operation I_6 immediately emerges as a primary transformation because of its vertical consistency throughout the passage.[30] Each row of the *dux* is related by I_6 to its partner in the *comes*. The relative simplicity of that decision does not extend

[28] In addition to formulating these transformational distance, Gollin, "Representations of Space," uses distance to compare "tonal" spaces. His study is significantly more robust and complex than the present discussion may make it seem. In addition to group generators, the concept of a group's "relators" impacts Gollin's understanding of distance.

[29] Ibid., 48-78 frames this in terms of the group's "presentation," which in addition to the group's generators, contains the group's *relators*.

[30] Here, and in the following commentary, $I_n = IT_n$, which is calculated as I followed by T_n.

to the determination of a transposition generator, as T_7 and T_5 have equal representation in the first part of the movement's binary form. My analysis choses, arbitrarily, T_7 instead of T_5, and thus the *T/I* group for the passage = $\langle T_7, I_6 \rangle$

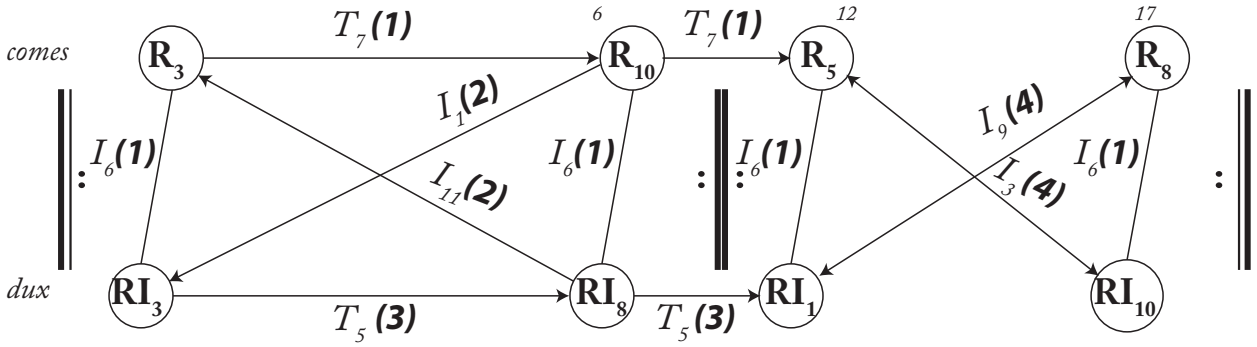
Much less interpretation is need in selecting the appropriate chain group because we can begin with the premise that TCH_1 and ICH_1 are primary. As I have noted, that determination is made not simply by trying to find the best "fit" with the transformations in the movement, but is arrived at primarily by consulting the pre-compositional structure of the row class. When the *chain group* = $\langle TCH_1, ICH_1 \rangle$, each adjacent connection is a single unit in length. TCH_1 drives connections in the first half. ICH_1 takes over in the second half. But in one way, this leaves us with the opposite problems of the serial analysis at (a): the vertical connections are neither consistent nor simple.

"Unfolded" spatial networks at (b) and (d) interpret the distance of the *comes* voice in each diagram, follow the voice as if the spaces were maps and the arrows represented the only possible pathways. The "path distance" of each transformation on this space is shown in bold next to every transformation on the diagrams at (a) and (c).[31] For example, both T_7 and I_6 represent only one unit of distance. But the decision to privilege those transformations has repercussions for the other transformational actions actions in the movement. In the *dux* voice, $T_5 = I_6T_7I_6$ is three units in length, for example. Thus, the *dux* and the *comes* are not doing the same amount of "work" in the first half. Overall, Figure 2.10(a) and (b) show that the total path distance for each canon voice is 17, though each voice traverses only seven row forms.

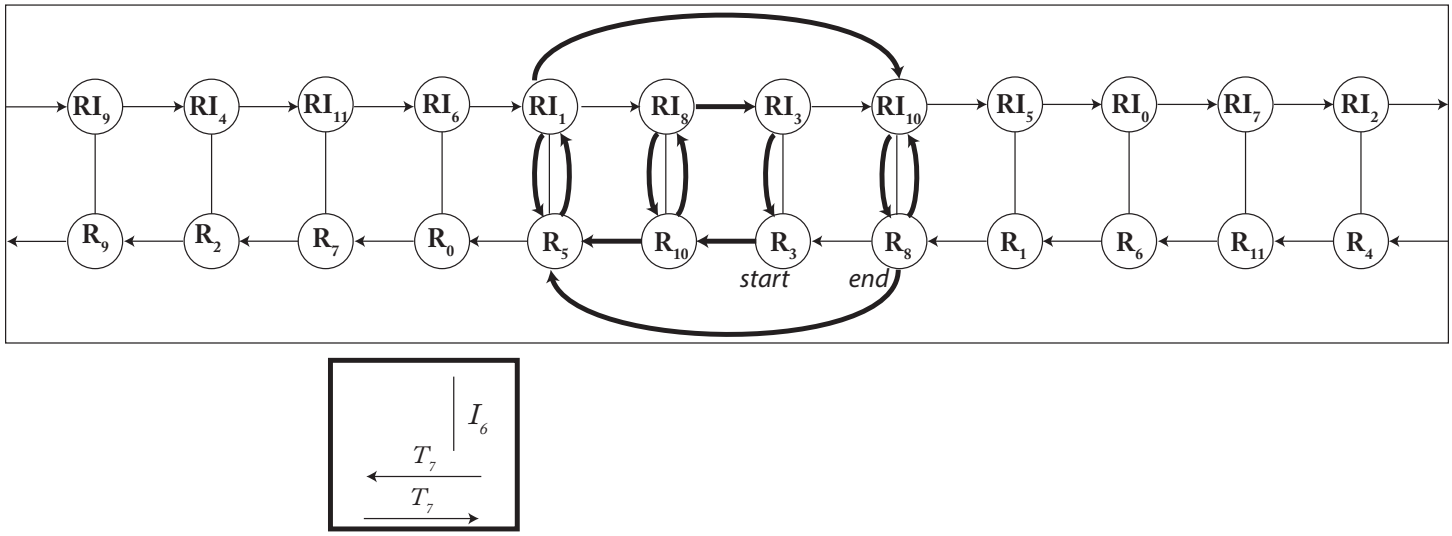
[31] Gollin defines three types of distance: (1) "path distance," (2) "class distance," and (3) "absolute distance" ("Representations of Space," 99-107). Path distance traces every step along a path, even if such a path is quite circuitous. To demonstrate, Gollin uses the analogy of a neighborhood in which an occupant of house A wants to pay a visit to his or her neighbor, the occupant of house B. But rather than directly going to house B, the occupant of house A first travels to the pharmacy, the grocery, and the cleaners. Thus, while the absolute distance from the house A to house B is small, the "path distance" is quite large. Path distance is the most appropriate measurement in the present context because I have been primarily concerned with "local" connections amongst adjacent row forms.

FIGURE 2.10. Transformation diagrams of the Piano Variations's second movement, showing path distance.

(a) Analysis with transpositions and fixed-axis inversions. Bolded numbers represent distant judged in terms of the *classical T/I group* = ⟨T_7, I_6⟩.

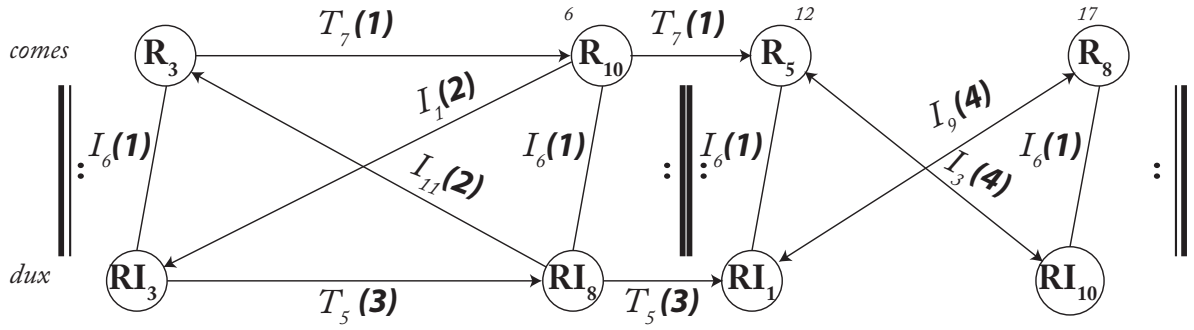


(b) Charting the *comes* voice as a path in the *classical T/I group* = ⟨T_7, I_6⟩.

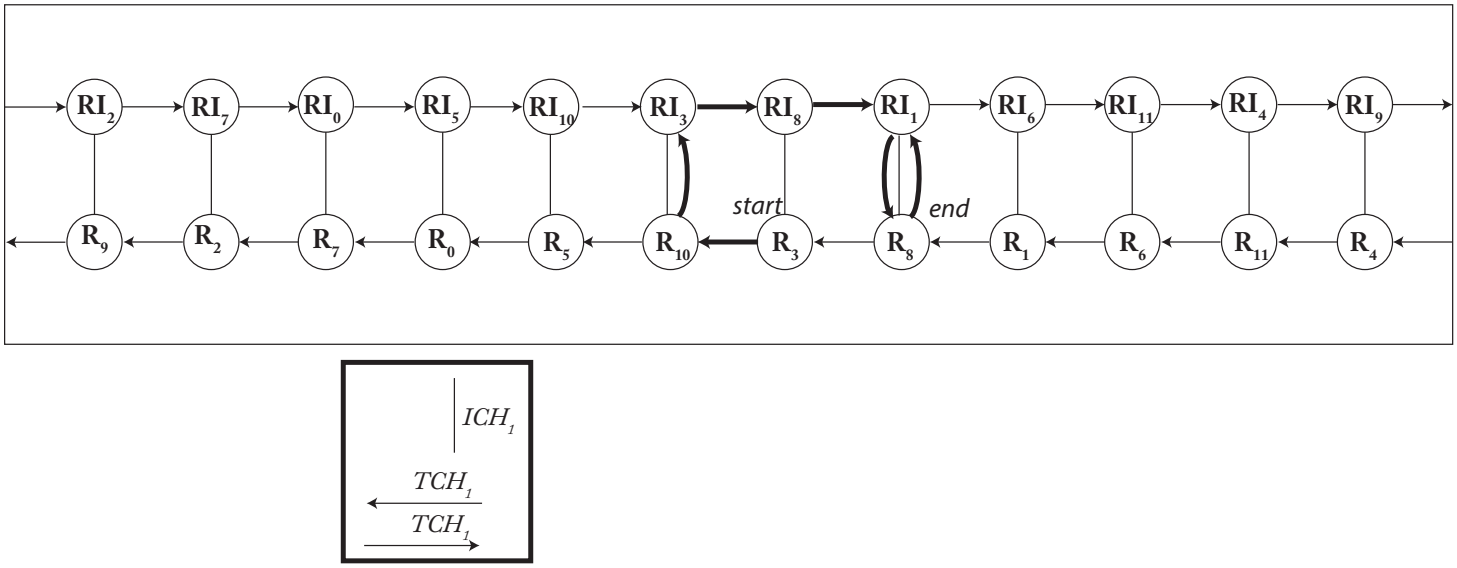


Interpreting the passage in the chain group (shown at (c) and (d) is far simpler. Each chain transformation is a single unit of length, and because both voices travel only via *TCH1* and *ICH1* pathways, the total length of each canon voice is seven units. This is simpler and more intuitive. There are only seven rows in each voice, matching the number of transformations in each voice exactly.

(c) Analysis with TCH_1 and ICH_1. Bolded numbers represent distance judged in terms of the *chain group* = ⟨ TCH_1, ICH_1⟩.



(d) Charting the *comes* voice as a path in the *chain group* = ⟨TCH_1, ICH_1⟩.



2.2.2. Example: Webern, Piano Variations, Op. 27, I

It is tempting to imagine T_5 and T_7 to represent similar (if not the same) transformations. In inversional canons, as in the second movement of the Piano Variations, the two transformations materialize in identical places amongst corresponding canonic voices. We might, therefore, prefer to imagine the serial analysis above in a "redundant" group: ⟨T_7, T_5, I_6⟩.[32] The group is "redundant" because the transformation T_5 is already generated by T_7. Such a group

[32] A redundant transformation (see Gollin, "Representations of Space," 75)) is not needed to generate a group, but is nonetheless felt to be "primary" in the way that the group's generators are.

could be helpful if we wished to equate the distance of T_5 with that of T_7. Had we decided to use a *contextual inversion* instead of a fixed-axis inversion to generate the group, we could have made the serial analysis look even more like the chain analysis.

These are a quick fixes. But there are other instances in which an analysis carried out within a *serial group* is not easily transferable to a *chain group* (and vice versa) without disrupting the meaning of that analysis. For example, because *RECH* does not commute with *TCH* (as we saw in §2.1.3), an analysis carried out in some *T/R* group will often be quite different from one carried out in a *chain group,* ⟨*TCH, RECH*⟩.

Let us consider an example from the first movement of the Piano Variations where these differences are forefront.[33] The first eighteen measures are shown in Figure 2.11(a), along with a rhythmic reduction at (b). Like the second movement, the movement is canonic throughout. There are two canonic voices, but the canons are constructed in *canzicrans*, with the *comes* of each canon echoing the *dux* but in retrograde as I have shown at (b). For the most part, we can consider the rows to be analogous with the canon voices. In mm. 11-18, the four rows heard in mm. 1–10 return, still in canon. However, at this return, the canonic relationship changes: **P** and **RI** rows that acted as *comites* in mm. 1-10 become *duces* in mm. 11-18.

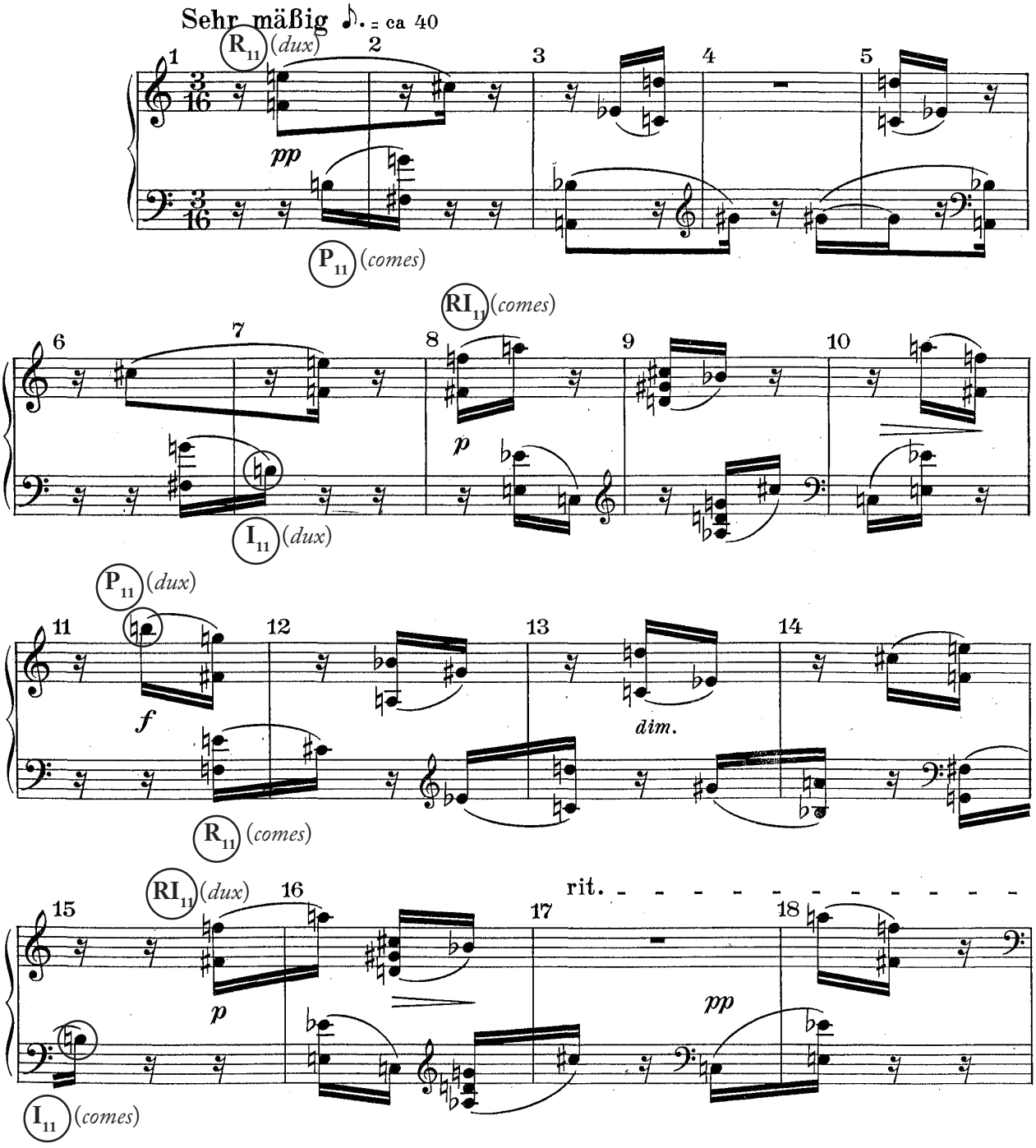
Two transformational networks of this passage in Figure 2.12 diagram these changes. The first (at (a)) is from the perspective of a group $G = \langle R, RI_{10} \rangle$. These generators seems appropriate because canon voices are *R*-related and RI_{10} connects every adjacent row in the canon. [34] Moves along the $\langle R, RI_{10} \rangle$ spatial network show how easily this passage is traversed in this group. Each

[33] Analytical studies of this movement include Robert W. Wason, "Webern's 'Variations for Piano', Op. 27: Musical Structure and the Performance Score," *Intégral* 1 (1987): 95-9; Catherine Nolan, "Hierarchic Linear Structures in Webern's Twelve-tone Music" (Ph.D. dissertation, Yale University, 1989): 258-90; Kathryn Bailey, *The Twelve-Note Music of Anton Webern: Old Forms in a New Language*, Music in the Twentieth Century (New York: Cambridge University Press, 1991): 191-4; and Nicholas Cook, *A Guide to Musical Analysis* (New York: Oxford University Press, 1994): 300-11.

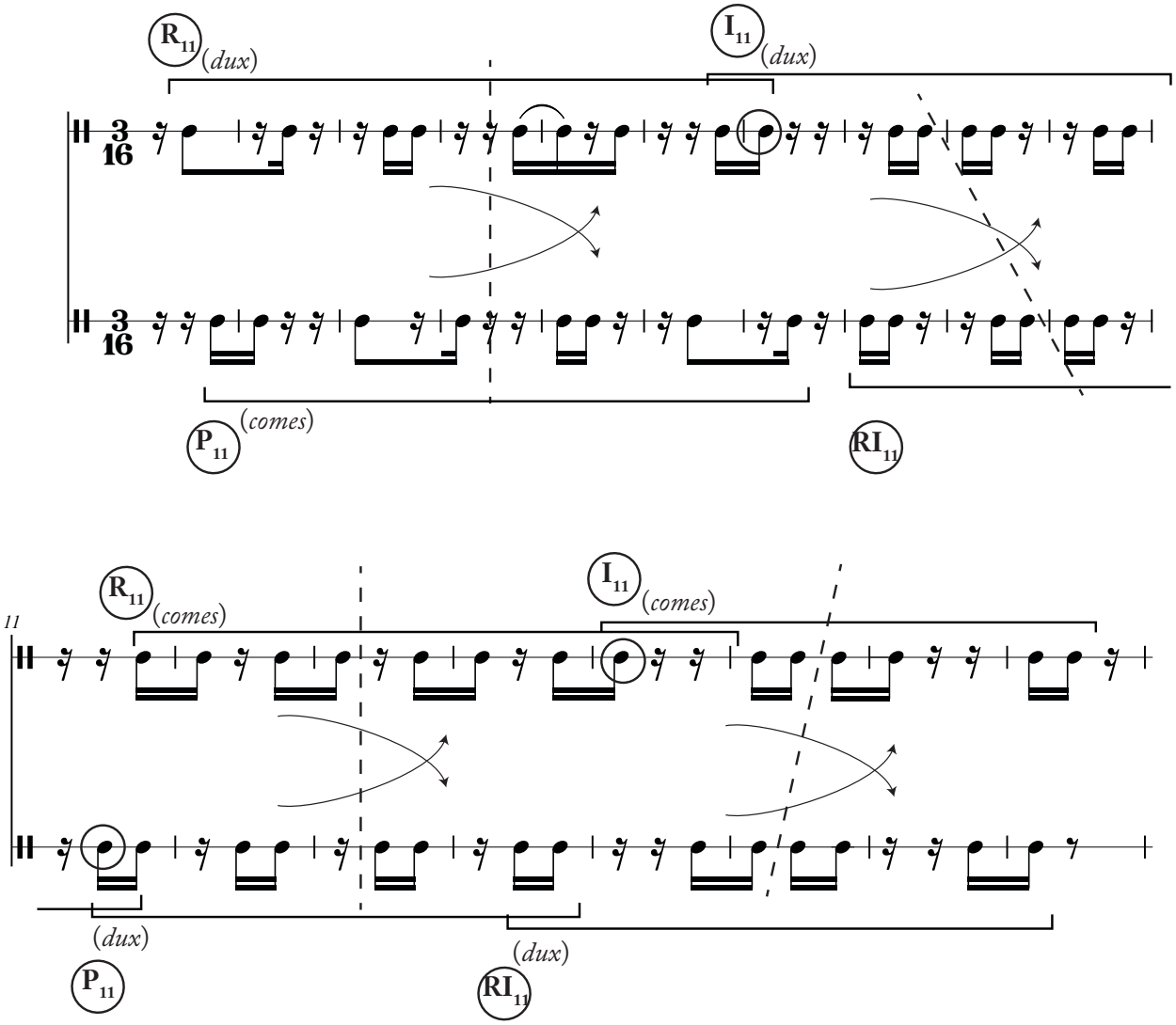
[34] *G* is a four-element group, and thus partitions the forty-eight rows in the row class into twelve, disconnected subsets. If we were analyzing the remainder of the movement, a larger group, including some sort of pc transposition, would be appropriate. But as the opening eighteen measures contain only four row forms, *G* is adequate and has the added benefit of efficiency.

FIGURE 2.11. Piano Variations, Op. 27, I.

(a) mm. 1-18.



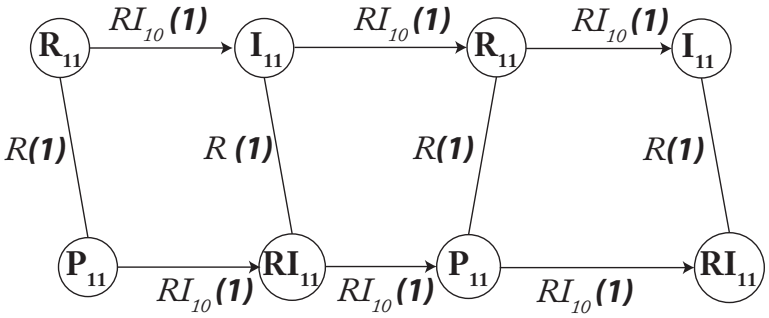
(b) rhythmic reduction, mm. 1-18.



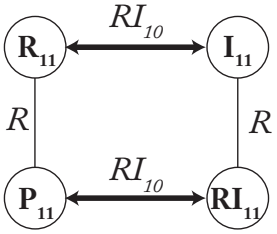
of the RI_{10} successions are accomplished by a motion through one of the horizontal arrows. And because RI_{10} is an involution, every other RI_{10} undoes the action of the first. This nicely accounts for the recapitulation of the opening row configuration at m. 11 in group-structural terms.

Inasmuch as it can label *every* linear row succession the same way, the serial group has the advantage of simplicity over the chain group analysis, shown at (b). This expression of the chain group is generated from $RECH_1$ and $RICH_1$. As was true earlier, this generation is appealing for two reasons. First, $RECH_1$ and $RICH_1$ are present in the passage. Second, those chains are suggested pre-compositionally, by the row class itself. $RECH_1$ and $RICH_1$ generate the space

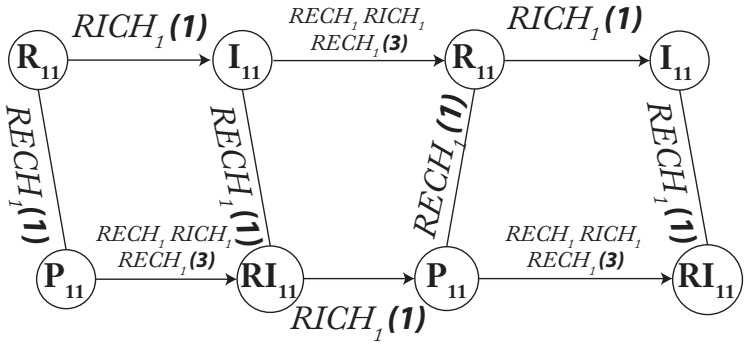
FIGURE 2.12. Transformation diagrams for the Piano Variations, first movement.
(a) Analysis with serial operations. Bolded numbers represent distance judged in terms of the *group* $G = \langle RI_{10}, R \rangle$.



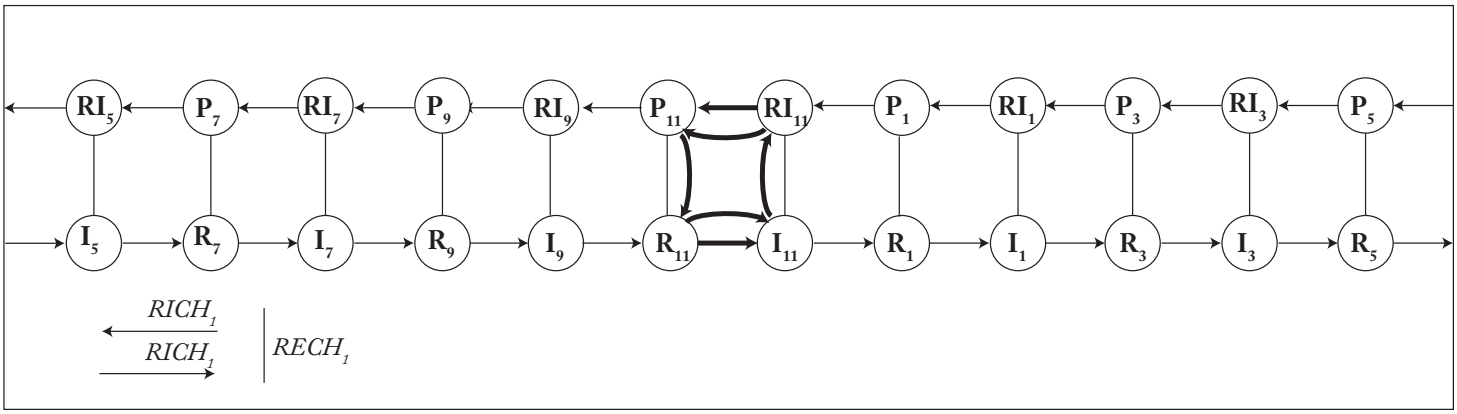
(b) spatial network for $G = \langle RI_{10}, R \rangle$.



(c) Analysis with chains Bolded numbers represent distance judged in terms of the *chain group* = $\langle RECH_1, RICH_1 \rangle$.



(d) spatial network for *chain group* = $\langle RECH_1, RICH_1 \rangle$.



shown at (d), which engages a far larger swath of rows than the spatial network for ⟨*R*, *RI*$_{10}$⟩.[35] In fact, if Occam's razor were invoked as a way to arbitrate between the two analyses, the serial analysis would win. ⟨*R*, *RI*$_{10}$⟩ is efficient. It engages only the four rows in the opening eighteen measures. And in terms of transformational distance, it is the simplest. Every row relationship shown occupies just one path on Figure 2.12(b).

Why does the chain group show less simple distances here? The reason is that not every row successions involve an elision. The score and reduction (in Figure 2.11) shows elisions connecting **R**$_{11}$→**I**$_{11}$ and **RI**$_{11}$→**P**$_{11}$, but the others are not. On Figure 2.12(c), **R**$_{11}$→**I**$_{11}$ and **RI**$_{11}$→**P**$_{11}$ can be labeled as *RICH*$_1$, but **P**$_{11}$→**RI**$_{11}$ and **I**$_{11}$→**R**$_{11}$ involve the more complex, 3-unit path, (*RECH*$_1$)(*RICH*$_1$)(*RECH*$_1$).[36] The ⟨*RECH*$_1$, *RICH*$_1$⟩ space at (d) shows why. Though *RI*$_{10}$ is an involution, *RICH*$_1$ is *not*. This both explains why the ⟨*RECH*$_1$, *RICH*$_1$⟩ space is larger, and also why *RICH*$_1$ arrows point from **R**$_{11}$ to **I**$_{11}$ but not from **I**$_{11}$ to **R**$_{11}$. In mm. 1-10, the *dux* can move from **R**$_{11}$ to **I**$_{11}$ through *RICH*$_1$, but at m. 11, **I**$_{11}$ *cannot follow the same path back*. Thus, the recapitulation at m. 11 requires a "break" in the transformation chain. There really *is* a greater transformational distance from **I**$_{11}$ to **R**$_{11}$ than there was from **R**$_{11}$ to **I**$_{11}$.

Note how the analysis at (c) explains the change in canon relationships at m. 11, where **P**$_{11}$ and **RI**$_{11}$ become *duces* after having been *comites* in mm. 1-10. Though **I**$_{11}$ cannot chain into **R**$_{11}$ at m. 11, **RI**$_{11}$ *can* chain into **P**$_{11}$, as the space at (d) shows. Thus, at m. 11 **P**$_{11}$ becomes the *dux* voice. Comparing the opening with m. 11, notice that the {B, F♯, G} trichord that acts as accompaniment to the {F, E, C♯} trichord in mm. 1-2 initiates the new canon at m. 11, and seems to inspire a new canonic accompaniment. In fact, its increased importance is reflected in a relocation to the right hand, where that {B, F♯, G} trichord is transferred up an octave (and {B} is

[35] Because *RICH*$_1$ is an order 12 operation in Op. 27, the the order of ⟨*RECH*$_1$, *RICH*$_1$⟩ 24.

[36] In the serial group, expressions like RRIR (perhaps the closest correspondent to *RECH*, *RICH*, *RECH*) can always be reduced to a single RI. This is not generally true in the chain group because *RECH* and *RICH* do not commute.

transferred up two octaves!), coinciding with an octave demotion of {F, E, C♯} trichord. This voice "switch" certainly represents some the "variation" in the movement's title, but it also shows how the smaller-scale *canzicrans* is represented on a large scale, as can be seen in the the rhythmic reduction.

These two analyses underscore two important points. First, a simple analysis may not always be the best analysis. Greater transformational distance is spanned in the analysis at (c), but that distance reflects facts at the musical surface: $\mathbf{R_{11}}\rightarrow\mathbf{I_{11}}$ is *not the same* as $\mathbf{I_{11}}\rightarrow\mathbf{R_{11}}$.[37] The inconsistency in the chain analysis has real meaning. It underscores the exceptional characteristics of mm. 10-11, characteristics that are wrapped up nicely with the varied recapitulation. Second, comparing the two analyses shows that we generally cannot swap a serial analysis for a chain analysis. The RI_{10} transformations at (a) are not the same as the $RICH_1$ transformations at (c). Not only does $RICH_1$ not commute with *RECH*, but $RICH_1$ is not generally an involution, while RI_{10} is always an involution.

2.2.3. Hybrid Groups

When labeling *horizontal*, syntactical connections, chains are often simpler; and in those occasions in which they are not, they can reveal interesting transformational "blockages," suggesting a "path not taken," and forcing us to reckon with why not. But as descriptors of *vertical* connections, chains are generally no better than serial operations, and quite often—as in my analysis of the Piano Variations's second movement in §2.2.1—they are worse. Certain transformations are simply better labeled by serial operations: I_6, the fixed inversion around A4 in the second movement, really does have symbolic significance.

---

[37] Compare this with the serial analysis of the second movement in Figure 2.10. There are no musical reasons for T_5 and T_7 to project different distances.

These sorts of situations invite the "mingling"; that is, *hybrid groups* containing serial and chain operations. Hybrid groups have advantages beyond simply letting us "have our cake and eat it too." Most notably, chain transformations often commute with operations in the *classical serial group*, even when they do not commute with themselves. Thus, while TCH_1 does not commute with ICH_1, and T_n does not commute with I, TCH_1 does commute with I and ICH does commute with T_n. This follows from an observation by Hook: though "the schritt/wechsel group […] and the transposition/inversion group […] are non-commutative," the only transformations "that commute with all transpositions and inversions are the schritts and wechsels, and vice versa."[38] Our earlier work showed that TCH and ICH are equivalent to Riemannian *schritts* and *wechsels*, and thus, although they do not commute with themselves, they *do* commute with transpositions and inversions. This fact can have significant analytical advantages.

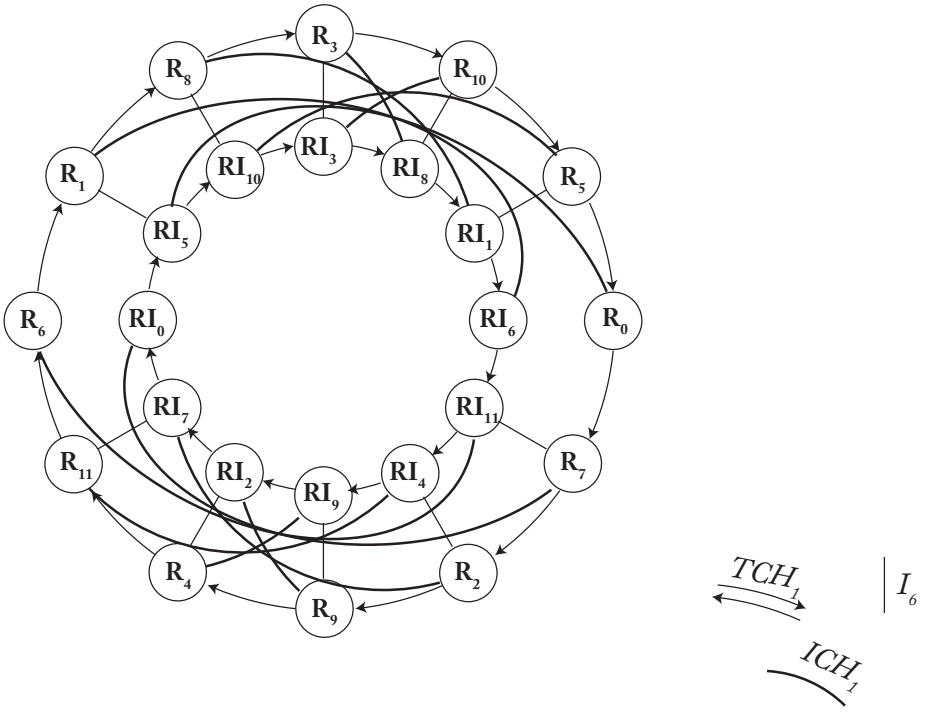
2.2.4. Example: Webern, Piano Variations, Op. 27, II

A hybrid group $G = \langle TCH_1, ICH_1, I_6 \rangle$ is shown in Figure 2.13. TCH_1 is an order 12 operation, and ICH_1 and I_6 are involutions. Therefore, G has forty-eight elements, but as Figure 2.13 shows, connects only twenty-four rows. Quite often, these spatial representations are visually fascinating, but unwieldy. ICH_1 is particularly hard to follow. Often, "unfolding" the space, as I have done in Figure 2.14, remedies those graphical difficulties and reveals the regularity underlying the network. This style of representation, which I will use a great deal in the pages to come, also has the benefit of showing "horizontal" transformations—the chains; those that connect adjacent rows in the movement—moving along the x-axis and "vertical" transformations —those related by I_6—along the y-axis.

---

[38] Hook, "Uniform Triadic Transformations," 111.

FIGURE 2.13. A hybrid group $G = \langle TCH_1, ICH_1, I_6 \rangle$ acting on **R** and **RI** forms in the Piano Variations.



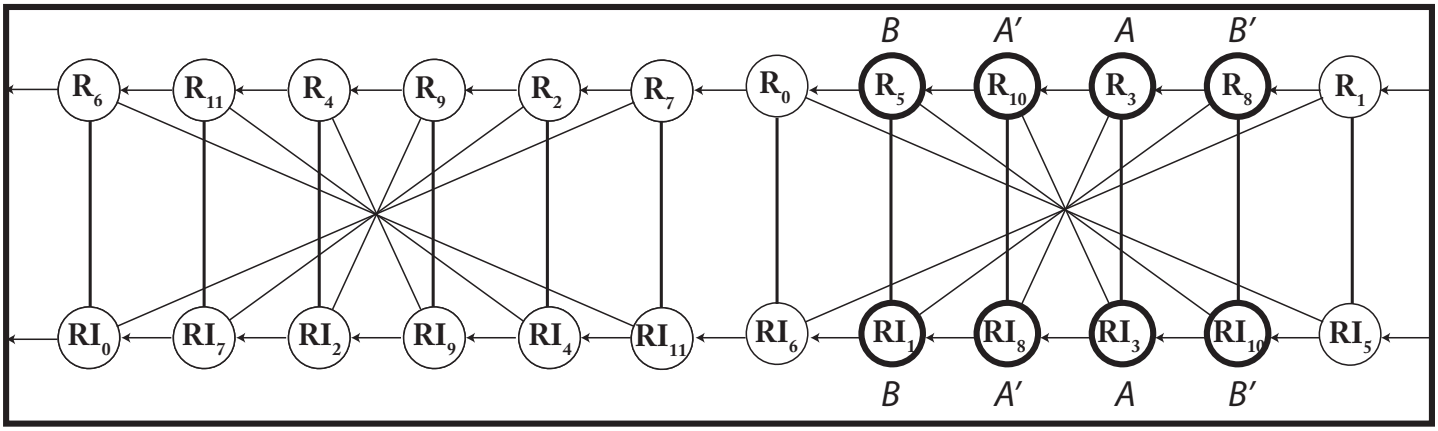
Notably, 2.14 shows clearly that TCH_1 commutes with I_6. The TCH_1 arrows move from right-to-left along the top *and* bottom of the space. This has significant advantages in charting the transformational action of the *dux* and *comes*, which are shown there. In fact, it shows that concurrent TCH_1 or ICH_1 motions in the two voices *automatically* maintain the axis of inversion.[39] This is incredibly suggestive as regards characteristics often associated with Webern's serial music. Plainly, given two canon voices related by I_n, *TCH*, *ICH*, *RECH*, or *RICH* will *always* maintain the I_n relationship between the two voices.[40] The transformational diagram at (b) should be judged in comparison with those show earlier in Figure 2.10. Bolded numbers there

[39] This was noted in regards to this passage, but framed in UTT language, by Hook and Douthett, "UTTs and Webern's Twelve-tone Music," 101. Mead, "Webern and Tradition," 176-7 notes the general principle in terms associated with transformation chains: given a "linkage between successive blocks of rows by overlapped dyads" … "all of the inversional relations are automatically preserved."

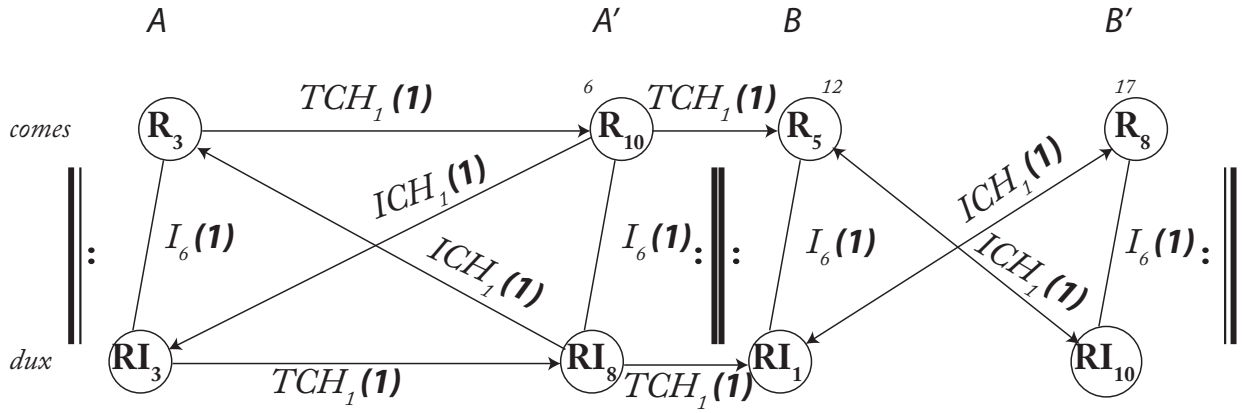
[40] By contrast given two canon voices related by I_n, T_x will never maintain the I_n relationships unless T_x is T_0 or T_6.

FIGURE 2.14. Analyzing the Piano Variations's second movement in the hybrid group $G = \langle TCH_1, ICH_1, I_6 \rangle$.

(a) an "unfolded" versions of Figure 2.13. The labels (**A** and **B)** refer to the two parts of the movement's binary form.



(b) Analysis with TCH_1, ICH_1, and I_6. Bolded numbers represent distant judged in terms of the *hybrid group* = $\langle TCH_1, ICH_1, I_6$.



show that horizontal *and* vertical connections are a single unit of distance apart. And therefore, in distance terms, this is the simplest of the three analyses.

2.2.5. HYBRID GROUPS AND SIMPLE TRANSITIVITY

Taken alone, both the *classical serial group* and the *chain groups* are *simply transitive* groups: choosing any two row forms on a serial space or chain space, there exists one and only one *unique*

transformation connecting those two row forms.[41] For example, within the *classical serial group* represented on Figure 2.8(c), imagine beginning at $\mathbf{P_0}$ and traveling to $\mathbf{P_1}$. There *seem* to be many ways to make this journey. Spatially, the shortest distance path distance is $\mathbf{P_0} \xrightarrow{T_1} \mathbf{P_1}$, but many other paths symbolize equivalent actions: $\mathbf{P_0} \xrightarrow{R} \mathbf{R_0} \xrightarrow{T_1} \mathbf{R_1} \xrightarrow{R} \mathbf{P_1}$ and $\mathbf{P_0} \xrightarrow{I} \mathbf{I_6} \xrightarrow{R} \mathbf{RI_6} \xrightarrow{I} \mathbf{R_0} \xrightarrow{R} \mathbf{P_0} \xrightarrow{T_1} \mathbf{P_1}$ both accomplish the job, as do an infinite number of other paths. We can assert, in other words, that $T_1 = RT_1R = IRIRT_1$, and so on. Furthermore, T_1 is *always* equivalent to those other expressions, and therefore, none of those transformations are unique. Rather, they are all different ways of saying the same thing. This property is evident in the chain-generated space in Figure 2.9(c) as well. A single transformation connects every pair of rows, though there are infinite ways of expressing that transformation.

If a group is simply transitive, the order of the group and the size of the set of objects must be equal. The classical serial group has forty-eight members, and therefore, when acting on the forty-eight row forms in a row class, the group is simply transitive. Lots of musically interesting groups are *not* simply transitive.[42] In *GMIT*, Lewin calls these kinds of groups "non-*intervallic*" transformation groups because they cannot be subsumed within a generalized *interval* system (GIS).[43]

---

[41] The bipartite structure *GMIT*, whose two parts discuss "Generalized Interval Systems (GISs)" and "Transformation Graphs and Networks," hinges on Lewin's discussion of simply transitive groups in §7.1.1 (157). There, Lewin proves that every simply transitive transformation group can be represented by represented by a GIS. Lewin says, "all the work we have done with GIS structures ... can be regarded as a special branch of transformational theory, namely that branch in which we study a space S and a simply transitive group STRANS of operations on S" (158).

[42] For example, the *classical T/I group* transforming pitch classes. The order of the *classical T/I group* is 24 and there are only twelve pitch classes; thus, while $T_1(\mathbf{C}) = \mathbf{C\sharp}$ and $I_1(\mathbf{C}) = \mathbf{C\sharp}$, $T_1 \neq I_1$. By contrast, when the *classical T/I group* transforms major and minor triads, of which there are twenty-four, it is simply transitive.

[43] See Lewin, *GMIT*, 175-92. And in fact, Lewin's introduction of *RICH* occurs as part of explanation of such groups.

In general, when transformation chains combine with serial operations to create a "hybrid group," as in the analysis above, that group will not be simply transitive. In connection with Figure 2.13 above, I noted that the group *G* contained forty-eight transformations, but the space itself only had twenty-four rows. That inequality creates situations in which more than one transformation connects the same two objects. To illustrate, on Figure 2.13, consider moving from $\mathbf{R_3}$ to $\mathbf{RI_8}$. There are a few ways to do this: $\mathbf{I_8}$ and $\mathbf{R_3} \xrightarrow{I \cdot TCH_1} \mathbf{I_8}$. This seems to suggest, however, that ICH_1 and $I \bullet TCH_1$ are the "same" transformation, but the space does not bear that out. Perform both transformations, now beginning at $\mathbf{R_8}$: $\mathbf{R_8} \xrightarrow{ICH_1} \mathbf{RI_1}$, but $\mathbf{R_8} \xrightarrow{I \cdot TCH_1} \mathbf{RI_3}$. The two transformations lead to different places, and therefore, $ICH_1 \neq I \bullet TCH_1$.

Groups that are not simply transitive are no less valuable analytically than simply transitive groups.[44] But in general, two important qualifications apply to analyses carried out in these groups. First, by their nature, non-simply transitive groups admit ambiguity. In the analysis above, I labeled the first connection in the *dux* as $\mathbf{RI_3} \xrightarrow{TCH_1} \mathbf{RI_8}$, but I could have labeled it in another way: $\mathbf{RI_3} \xrightarrow{ICH_1 \cdot I} \mathbf{RI_8}$. Our discussion in this section shows that distance is one way to arbitrate between such choices: TCH_1 is a single unit of length, while $ICH_1 \bullet I$ is two units length. Thus, TCH_1 is the "simpler" analysis. I will propose another framework for this type of decision making in §2.3.

Second, spatial representations of these kinds of analyses often lack *path consistency*. Hook uses the term path consistency to describe a condition that Lewin placed on transformation

[44] When making the pivot toward transformation groups, Lewin does not seem to be making a quality judgement wherein the first, GIS half of *GMIT* is less "good" than the second, transformational half. Lewin says: "more significant than this dichotomy [between intervals and transformations], I believe, is the *generalizing* power of the transformational attitude: It enables us to *subsume* the theory of GIS structure, along with the theory of simply transitive groups, into a broader theory of transformations" (*GMIT*, 159). See also Hook, "David Lewin and the Complexity of the Beautiful," 172-77.

networks.[45] Primarily, path consistency was meant to ensure that a network will be *universally-realizable*, no matter which object in a set *S* of objects is inserted into one of the network's nodes. Non-simply transitive groups often lead to networks that are not *universally-realizable*, but are *realizable* under for some set of objects in *S*. (Figure 1.7, in Chapter 1, is one such example. That network is well-formed for **P** and **RI** forms, but not for **I** or **R** forms, who require a T_7 transposition instead of T_5.) Hook suggests that we loosen Lewin's formulation, and we will follow his lead. In general, transformation graphs need not be path consistent or universally realizable, though they should always be realizable.

[45] Julian Hook, "Cross-Type Transformations and the Path Consistency Condition," *Music Theory Spectrum* 29, no. 1 (2007): 1–40.

2.3 Organizing Spatial Networks: Paradigmatics and Syntagmatics

Simply transitive groups bring clarity to analytical decisions. Any two objects are related by one and only one transformation within a simply transitive group. And yet, there are many analytical situations in which these groups are simply inadequate.[46] We saw two examples in §2.2. Neither the *chain group* nor the *classical serial group* were able to adequately account for *linear and vertical* relationships in the second movement of Op. 27, but together the two groups convincingly accounted for both of those relationships. Overall, §2.2 suggests that the two groups of transformations are often good at accounting for different types of things. Chains nicely described the *syntactical*, *chronological* relationships, and serial operations better accounted for *vertical*, *binding* relationships.

Nonetheless, in relying on hybrid groups, deciding which kinds of transformation should account for a relationship is not always as simple. This section proposes a conceptual separation between transformational relationships that are *syntactical* and those that are *binding*. These ideas are explored first in connection with Saussure's "paradigmatic" and "syntagmatic" relationships. Saussure's ideas are springboards towards more robust musical grammars, symbolized by organized spatial networks that better embody this distinction than do simple chain-generated spaces.

---

[46] Chapter 8 of *GMIT* offers many other examples. Chromatic music, in particular, often benefits from descriptions in terms of *P*, *L*, and *R* transformations in addition to *D(ominant)* and *M(ediant)* ones. Those groups are not simply transitive. One of Lewin's most discussed analyses take place within such a group. His network analyses of the "Tarnhelm" and "Valhalla" passages of *Das Rheingold* in *GMIT* (178-9) were revised in "David Lewin, "Some Notes on Analyzing Wagner: The Ring and Parsifal," *19th-Century Music* 16, no. 1 (Summer 1992): 49–58. Lewin's revision was prompted by the "ill-formed" network in *GMIT*, that Hook, "Cross-Type Transformations and the Path Consistency Condition," considers in terms of path consistency.

§2.3.1. Paradigmatic and Syntagmatic Relationships

Distinctions between members of the *classical serial group* and the *chain group* roughly resemble the distinction between "paradigmatic" and "syntagmatic" relationships described by the linguist Ferdinand de Saussure. Patrick McCreless used Saussure's terms to describe tonal and event hierarchies of the type created by Fred Lerdahl, and Steven Rings has understood the terms in relation to tonal "intention."[47] For Saussure, *paradigmatic* (sometimes called *associative*) relationships occur "out-of-time" and obtain between associated linguistic terms. Paradigms are formed in any number of ways: through meaning (friend, companion, confidante are synonyms); phonetic similarity (friend, friendship, and friendly have the same stem); parts of speech (friend, man, boy, girl, truck are each nouns); and so on. In language, paradigmatic relationships are limited only by our own mind. Saussure says that "the mind creates as many associative [paradigmatic] series as there are diverse relations".[48] Such relationships call to mind a mind's personal lexicon, where linguistic "substitution" is an important manifestation of paradigmatic thinking.

Syntagms, by contrast, are linguistic terms that depend on order and temporality. In the phrase "my friend sings," the word "friend" gathers meaning through its temporal relationship to what came before it ("my") and after it ("sings") While paradigmatic relationships exist *outside* of a given utterance, syntagmatic relationships obtain *within* it.[49] According to Saussure, "whereas a syntagm immediately suggests an order of succession and a fixed number of elements, terms in an

---

[47] Patrick McCreless, "Syntagmatics and Paradigmatics: Some Implications for the Analysis of Chromaticism in Tonal Music," *Music Theory Spectrum* 13, no. 2 (October 1, 1991): 147–178; Rings, "Tonality and Transformation," 115-72.

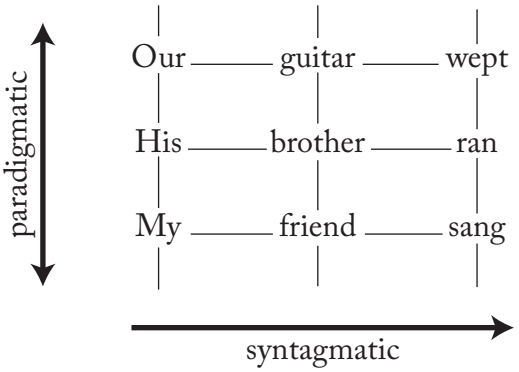
[48] Ferdinand de Saussure, *Course in General Linguistics* (New York: Columbia University Press, 2011): 125.

[49] Rings promotes the idea that paradigmatic relationships correspond to metaphors of "tonic-as-center," where "subordinate [harmonic] elements are arrayed 'around' the central tonic in the system and gain their meaning from it" ("Tonality and Transformation," 137). Syntagmatic relationships intentions are temporal: they "involve our awareness of a tonic already heard ('tonic-as-point-of-departure'), and a tonic we expect to hear again at some point in the future ('tonic-as-goal')". His association of the concepts with tonal "intention" foregrounds a crucial difference between his study and the present one, and tonal music and serial music more generally. Where Rings imagines an abstract system of relationships *oriented* toward a tonic, serial music in general presumes no such hierarchy.

associative [paradigmatic] family occur neither in fixed numbers nor in a definite order."[50] A syntagmatic relationship, then, is dependent upon normative rules of *syntax*.

In Figure 2.15 I have diagramed how paradigmatic and syntagmatic series interface along orthogonal axes. On the bottom row, the forward-pointing syntagmatic arrow indicates the importance of chronology as "friend" is limited by the possessive adjective "my" that precedes it and animated by the verb that follows. But in paradigmatic terms, the word "friend" gathers additional meaning as it extends outward, infinitely, in both directions. This shows not only the substitutional nature of paradigmatic relationships, but also how these kinds of relationships determine a term's meaning in relation to other associated terms.

FIGURE 2.15. Paradigmatic and Syntagmatic relationships in language shown along two orthogonal axes.



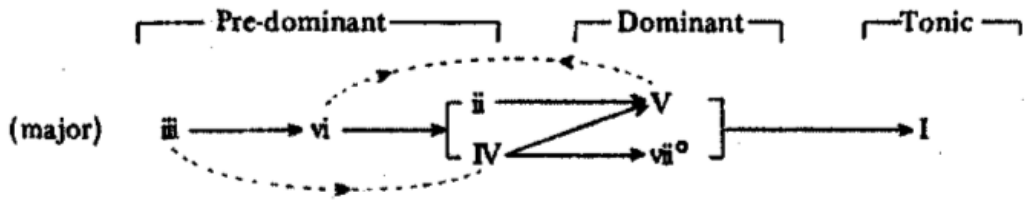
These linguistic concepts nicely capture the relational differentiation I have been pointing towards. Chain transformations are inherently chronological, and thus, they describe row syntax and syntagmatic relationships between row forms. In my analysis of the opening movement of

[50] Saussure, *Course in General Linguistics*, 126, notes that of the two characteristics of a paradigmatic relationship—"indeterminate order and indefinite number"—only the first is always true. This is an important characteristic of music as well. Though a mental lexicon is vast and perhaps infinite, in musical analysis we are often considering a *finite* number of objects, though those objects though are paradigmatically joined in an unordered manner.

Op. 27, the *lack of a RECH connection* at certain points in each canon voice, which accounted for the temporal changes in the passage, is a specific manifestation of unique syntagmatic relationships between rows. As noted there, a transformation group containing *RECH* captures those unique relationships, while one generated by serial operations only does not. By contrast, the binding power of I_6 in the second movement described the consistent, paradigmatic relationship that occupies that movement in a way that chains could not.

My spatial representations of Webern's row classes have been, to this point, very rudimentary music grammars that symbolize syntagmatic relationships conditioned by a chain-generated system of syntax. More robust representations of a musical grammar generally have a distinctly paradigmatic component—rules for "substitution," for example. In Figure 2.16 Robert Gauldin's diagram of harmonic progression is shown.[51] Like my spatial networks, this is a *cyclical* network. As in those spaces, syntax is read by following arrows, generally from left to right, in what may be an infinite loop.[52] Unlike my spatial networks, however, Gauldin's grammar classifies Roman numerals that have equivalent syntactical roles. Vertically-adjacent Roman numerals are paradigms, "syntactic substitutes" in the same way that "Our, His, and My" are paradigms in the sentence diagram above in Figure 2.15. Paradigms in tonal grammar, then, are often imagined as substitutions. Such rules are generally only one component of an elaborate

FIGURE 2.16. Robert Gauldin's "Basic Classification of Diatonic Chords in Functional Harmony."



[51] Robert Gauldin, *Harmonic Practice in Tonal Music*, 2nd ed. (New York: Norton, 2004), Figure 1.1.

[52] The "tonal intentional" character of these spaces is generally captured by locating the tonic on the far left or right as a "root node." See Rings, "Tonality and Transformation," 125-33.

musical grammar. In addition to substitution, Robert Morris notes these grammars might contain "*embedding* (secondary dominants)," *transformation* (relative and parallel minor substitutes); and *realization* (voice leading and chord registration and doubling)".[53] Below, I will explore two ways to represent musical syntax in terms of these paradigmatically defined constraints.

2.3.2 Event Networks, Spatial Networks

Transformational *event networks* are the syntagmatically conditioned, chronological cousins of the out-of-time, spatial networks I have been concerned with. Figure 2.14(a), is for example, an event network.[54] Thus far, I have constructed these networks in an ad hoc manner. When chronicling real musical events rather than abstract spatial organization, some freedom certainly seems justified. But some ad hoc representational decisions have no real meaning when an additional layer of meaning might be advantageous. Why, for example, are $\mathbf{R_3}$ and $\mathbf{R_{10}}$ in Figure 2.14(b) *above* $\mathbf{RI_3}$ and $\mathbf{RI_8}$? Does it mean anything to say that $\mathbf{RI_8}$ is "below" $\mathbf{R_3}$? This section shows how that event networks diagramming tonal music often impose paradigmatic relationships on chronological events, which suggests similar ways to structure representations of row classes.

Figure 2.17 shows a simple event network that Steven Rings uses to model the subject from Bach's E major fugue, from the *Well-Tempered Clavier,* Book II. Rings's event network resembles both the contour of that subject and the chronological placement of its six pitch events. To capture the chronology, Rings creates a detailed formalism that maps pitch events in

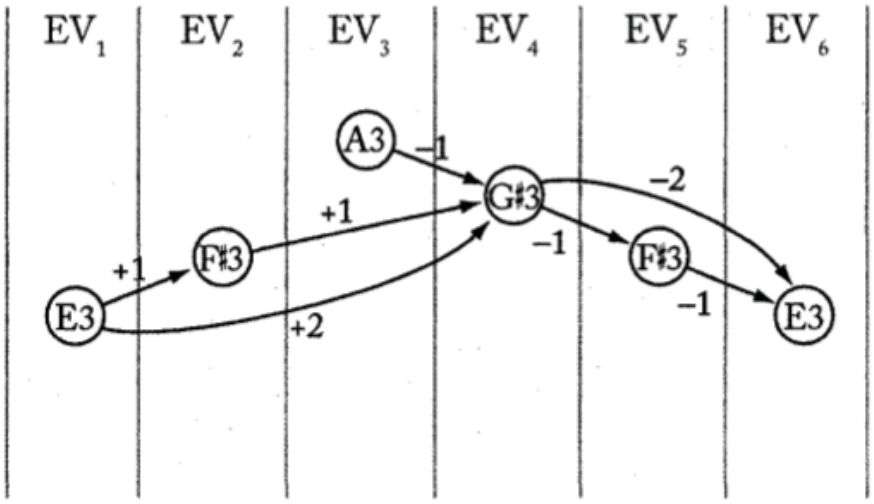
[53] Morris, "Compositional Spaces," 340.

[54] The term "event network" originates in John Roeder, "Constructing Transformational Signification." Lewin, *Musical Form and Transformation*, called these types of networks "figural networks," terminology adopted in Rings, "Tonality and Transformation."

the network to "event categories," EV_1, EV_2, and so on, and transformation arrows onto chronological relations between various events.[55]

To model real music, event networks relax two restrictions that are typically placed on spatial networks. First, event networks allow an object to appear in the network many times, permitting the common musical situation shown in Figure 2.17: E4 both begins and ends the passage, and F♯ appears twice as well.[56] Second, event networks often include "non-normative" transformations. That is, while spatial networks are constrained and organized by a set of generating transformations, usually drawn from a larger group, event networks are not. These generating transformations still structure the musical transformation groups used: Rings's network is organized by a group *G* of integers under addition (Z, +) acting on the (infinite) E major diatonic gamut, and generated by diatonic steps ($G = \langle 1 \rangle$). But on Figure 2.17, Rings has

FIGURE 2.17. An event network: Rings, "Tonality and Transformation," 340, Figure 3.14).



[55] Ibid., 145-56.

[56] Rings frames this in the following way: the function *CONTENTS* does not map NODES onto S 1-to-1.)

labeled many transformations other than diatonic steps, including the *+2* gesture from E3 up to G♯3.

Event networks have the advantage of capturing syntagmatic relationships. That E3 initiates the passage and concludes it captures the temporal syntagmatic intention of "tonic-as-point-of-departure" and "tonic-as-goal." However, event networks also generally capture paradigmatic relationships that are borrowed from an underlying spatial network. I noted that in addition to the left-to-right organization of Rings's event network, it also captures the contour of the fugue subject. Nothing about the formalism of Ring's event network requires it to show the contour. Figure 2.18, an alternative event network for Bach's subject scrubs the vertical dimension of any reference to contour. The network accords with Ring's rules for these types of networks, but with decreased descriptive power. *Rings's network is powerful precisely because it depicts chronology in terms of a normative melodic transformation—the "step"* (+/-1).

FIGURE 2.18. Rings's event network for Bach's fugue subject reconfigured (cf. Figure 2.17).

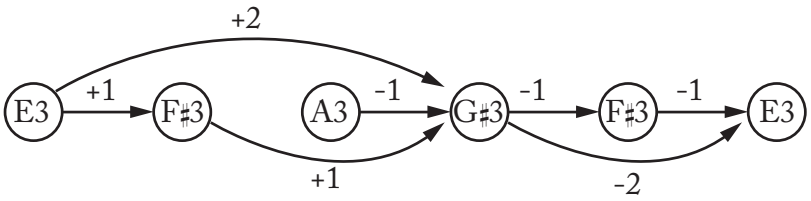
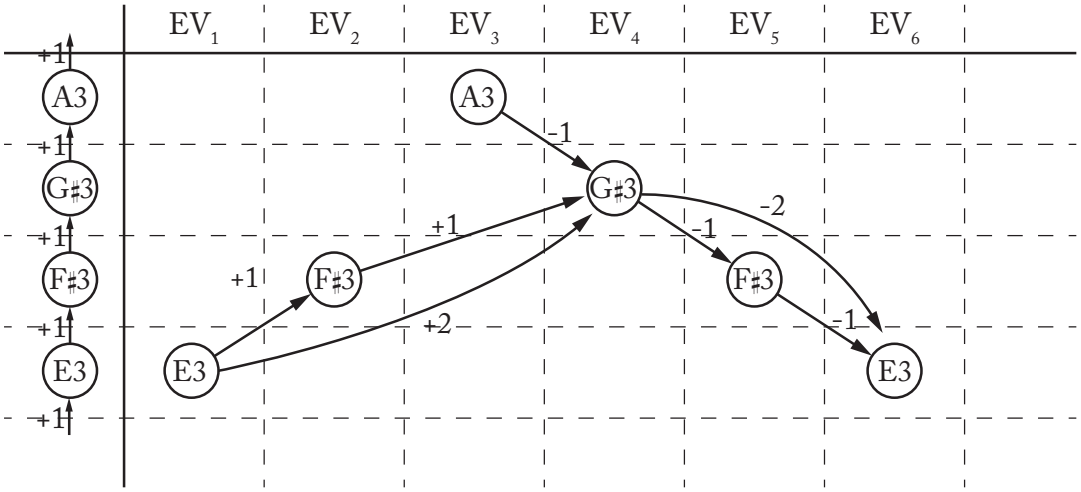


Figure 2.19 shows precisely *how* an event network relates to its underlying spatial network. Running along the left side of the figure is a segment of the infinite spatial network generated by the diatonic step. It shows that, in addition to mapping pitch events chronologically to event categories (EV_1, EV_2, and so on), the network often also maps a pitch event's vertical location to an abstract space structured by the diatonic step.

Therefore, changing the underlying transformation group or its generation will impact the way the network looks and the meaning it communicates. For example, because it represents *pitch space*, Rings network would need to be vertically expanded to accommodate the third entry of Bach's fugal subject (not shown on Figure 2.19), which occurs an octave higher. *Pitch-class* spaces reduce the size of these groups and represent commonly held equivalencies, such as E3 and E4 have the same tonal position in E major, but they involve visual and metaphorical tradeoffs as

FIGURE 2.19. Rings's event network in relation to an underlying spatial network. (Cf. Figure 2.17 and 2.18)

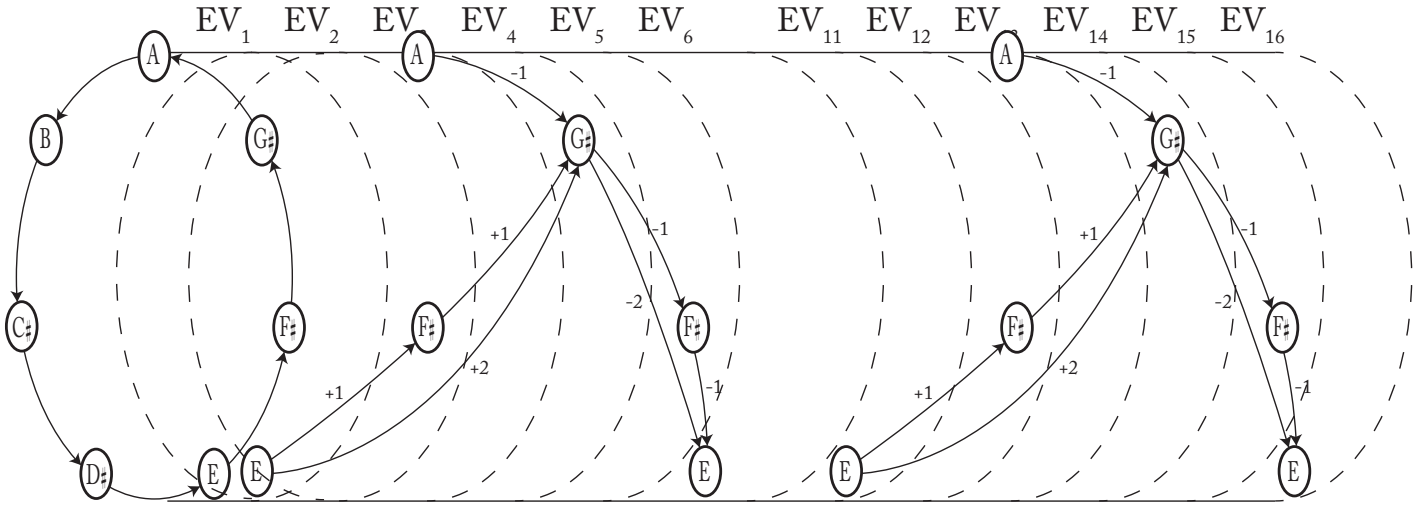


well. Figure 2.20 demonstrates. It models both subject entries in an event network organized by the circular E major *pitch-class* space. The three-dimensional, tube-like representation understands both the first and third fugue statements as "equivalent." The network demonstrates an important differences between infinite and finite spaces. While the idea of "above" and "below" was metaphorical on Rings's network (Figure 2.17), we *could* nonetheless say quite confidently that if F♯4 was "above" E4, then A4 was as well. In the pitch-class space in Figure 2.20, however, how do we know if the pitch-class A is "above" or "below" E?

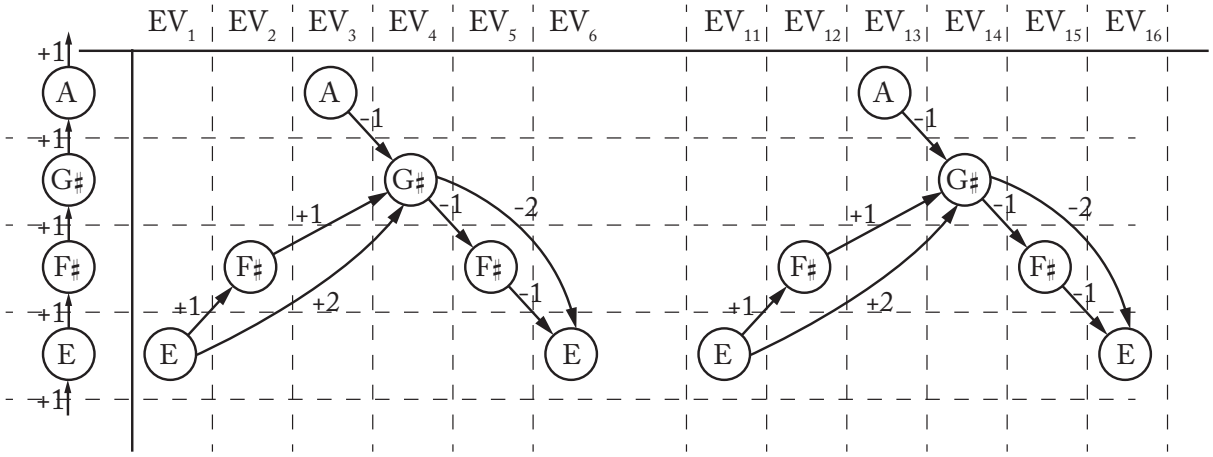
Event spaces representing finite groups (such as pitch-*class* transposition), then, often constrain the metaphors we can use to describe objects and their relationships to one another.

FIGURE 2.20. Modeling the first *and* third entries of Bach's E major fugue in pitch-class space.

(a) A "tube" representation.



(b) A "flattened" representation.



Instead of saying the pitch class A is above the pitch class E, a dubious statement in pitch-class space, we are instead forced to rely on a simpler metaphor, but one that is still driven by the transformations that generate the underlying group: A is not as "close" to E as F♯ is, for example, or that, F♯ and D♯ are spatial "neighbors" of E. Flattening a three-dimensional event network, as in Figure 2.20(b) does makes them easier to work with. But—and this point must be underscored

again—the similarity between Figure 2.20(b) and Rings's network in Figure 2.17 is entirely visual. Both spaces tell us a great deal about proximity in relation to a privileged set of generating transformations. However, only the pitch space accurately represents some events as *above* and others as *below*. As Peter Westergaard says in his discussion of Gottfried Weber's table of key relationships, "The traveler who floats in this space gets no compass. Here, as in that Swedenborgian heaven that Schoenberg quotes from Seraphita, 'no absolute down, or right or left, forward or backward' guides your flight."[57]

2.3.3 Organizing a Spatial Network

I have been suggesting that transformation chains are syntactic, and that they can capture syntagmatic relationships. Other analytical considerations, such as the inversional axis represented by I_6, are paradigmatic. The event network for Bach's E major fugue that I just discussed contained the same distinction: the E3 at the beginning and end of Figure 2.17 has the same paradigmatic meaning—hence, the same vertical location on the page—but the horizontal separation encapsulates the two E3s's differing syntagmatic relationships—"tonic-as-point-of-departure" and "tonic-as-goal." Figure 2.20 took this paradigmatic distinction further. It "divided" the infinite, E major gamut, into a finite space. All Es, for example, were assigned to an equivalence class represent an important paradigmatic relationship. Thus, the space is a organized, or conformed, version of the the earlier pitch space.

These are important distinctions in Webern's serial music as well. Assigning particular types of relationships to "paradigmatic" or "syntagmatic" categories can clarify an analysis carried out within a particular transformation group, thereby allowing an analyst to avoid hybrid groups that can easily conflate different types of relationships. In this section, I will outline a method of

[57] Westergaard, "Geometries of Sounds in Time," 16.

organizing spatial networks. Simply, these networks are syntactically-driven chain spaces that are divided by an important paradigmatic relationship, such as "inversional axis" or "invariance."

Dividing a network by equivalent objects is not new to this study. Many of the most important spatial networks used by music theorists are in some way organized, or conformed. The "enharmonically conformed" *Tonnetz* is one famous example. To produce this space, its unconformed cousin, which is theoretically infinite in size, is divided into equivalence classes that "fold" designations like B♮, C♭, D𝄫, and so on, into a single category (B♮) representing them all. Other, similarly instructive spaces populate music theory and analysis, and before demonstrating how I understand them to apply to Webern's music, I will review two exemplary tonal spaces, both created by Richard Cohn to model maximally smooth voice in nineteenth-century music.

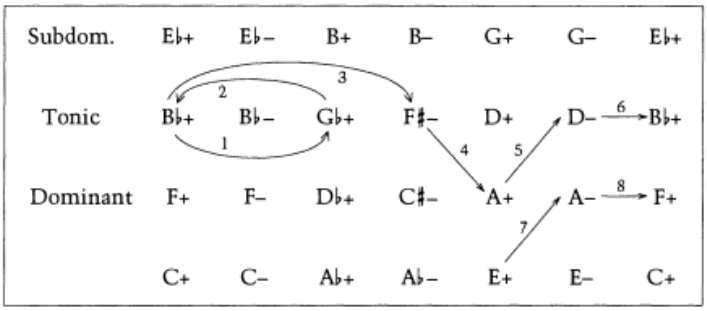
2.3.4 Example: Cohn's "Gazing" Network

Figure 2.21 gives Richard Cohn's analysis of the exposition of Schubert's Piano Sonata in B-flat. Though he calls it a "formal model," in the Lewinian sense, Cohn's figure also somewhat resembles an event network. It models ten tonal events in the exposition, placing them in relation to one another chronologically with numbered arrows, and spatially, with reference to the labeled rows. The figure could easily be translated into a "Rings-ian" event network, but for our purposes it is nice as is because it shows quite clearly the network's underlying structure.

The underlying structure is shaped by two types of transformation. The first category is represented by labels at the head of the three rows ("Subdominant," "Tonic," "Dominant") and by vertical associations of triads in the network: triads that are vertically-adjacent sit next to one another along the circle of fifths. With one foot sitting in the tonic-dominant universe, the network places its other in the world of maximally smooth voice leading: the second category of transformation abuts triads adjoined by one of the two maximally smooth transformations, *L* or

P. Those triads that are capable of being joined by maximally smooth transformations belong to the same "tonic," "subdominant," or "dominant" category.

FIGURE 2.21. The "gazing network": Cohn "As Wonderful as Star Clusters," 220, Figure 5.



Cohn's thesis is that Schubertian harmony resembles the *duality* of a star cluster: "A star cluster evokes a network of elements and relations, none of which hold prior privileged status. These two contrasting images of cosmic organization provide a lens through which to compare two conceptions of tonal organization in Schubert's music."[58] On the one hand, Schubertian harmony responds well to "an approach that de-emphasizes diatonic collections and emphasizes voice-leading efficiency," and on the other hand, Cohn recognizes that maximally smooth voice leading exists alongside "the abiding strength of the tonic-dominant framework."[59]

On this front, Cohn's "gazing" space is a lens through which we can imagine the interaction of *two conceptually different* types of relationship. To emphasize these two modes, Figure 2.22 outlines a reconstruction of Cohn's network:

(1) At (a), I have shown a spatial network containing the twenty-four major and minor triads acted upon by the group $G = \langle D \rangle$, where D sends a major or minor triad to the

[58] Richard L. Cohn, "As Wonderful as Star Clusters: Instruments for Gazing at Tonality in Schubert," *19th-Century Music* 22, no. 3 (1999): 213.

[59] Ibid., 215-17.

triad of which it is the dominant. The space contains two disconnected partitions of major and minor triads, and those networks are unfolded at (b).

(2) Unfolding the partitions allows us to align them *horizontally* at (c) with members of the maximally smooth group $H = \langle P, L \rangle$. H stitches the two partitions together. Greek letters are used to show that D transformations leaving the top of the figure emerge at its bottom, two columns over. Re-folding the space at (d) by sewing the L P cycles together—Cohn's own representation—shows that it forms a torus.

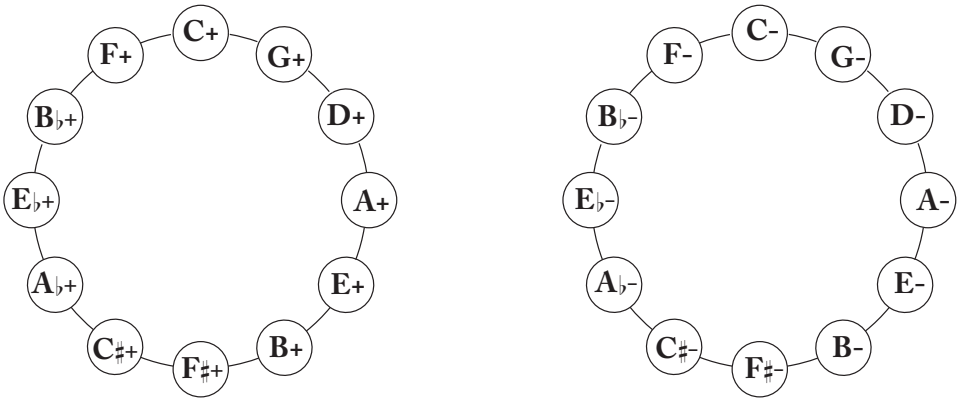
Orthogonal axes at (c) and (d) embody the duality Cohn wishes to emphasize: the tonic-dominant associations structuring the deeper levels of Schubert's sonata are contrasted with the maximally smooth voice leading that operates at the surface. Separating them in this reconstruction implies that the members of either category of transformations ($\langle D \rangle$ or $\langle R, L \rangle$) are interchangeable parts, and Cohn implies as much. Referring to the dominant-based alignment at (d), Cohn says that "this is one of several available alignments; others, such as those that pair triads with their relative major or minor, are more appropriate for some music."[60] Figure 2.29(e) shows precisely such a realignment, substituting the relative transformation R for D. This network is isomorphic to the enharmonically-conformed triadic *Tonnetz*. But—and this is the important point—this *representation*, as opposed to a triadic *Tonnetz*, indicates a *unique* role for R in relation to L and P, perhaps the one suggested by Cohn, where R is a "global," "regional" relationship.

In his analysis of the Schubert sonata, Cohn recognizes that the syntax created by L and P *binds together* chords into harmonic regions, *organizing* the dominant-structured, circle of fifths

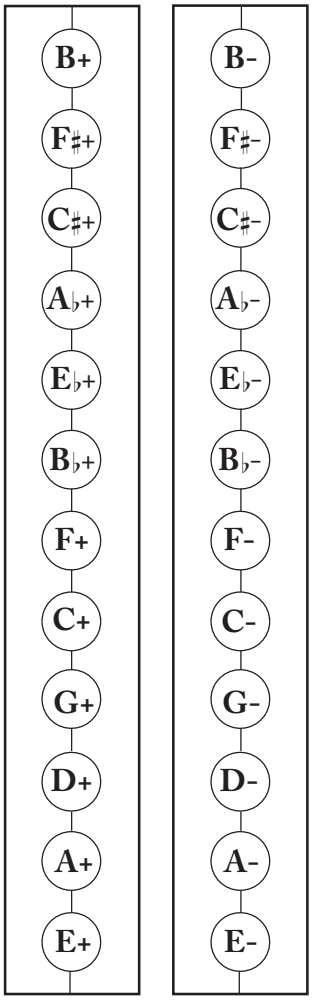
[60] Ibid., 217.

FIGURE 2.22. Reconstructing Cohn's "gazing network."

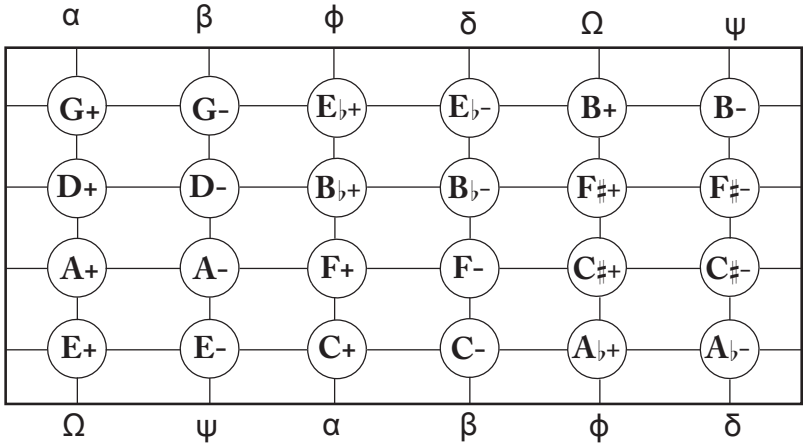
(a) $G = \langle D \rangle$ partitions major and minor triads into two networks.



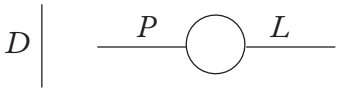
(b) "flattened" version of (a) containing two partitions—major and minor triads.



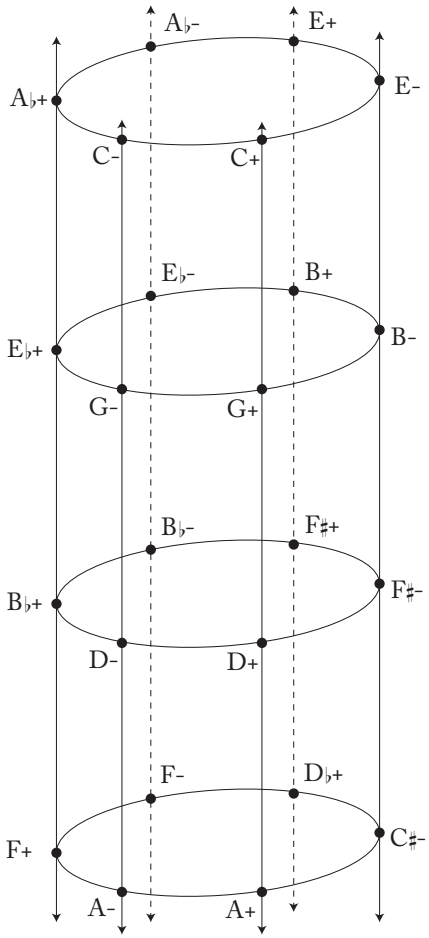
(c) the two partitions horizontally aligned according to *P* and *L*. (Cf. Figure 2.21).



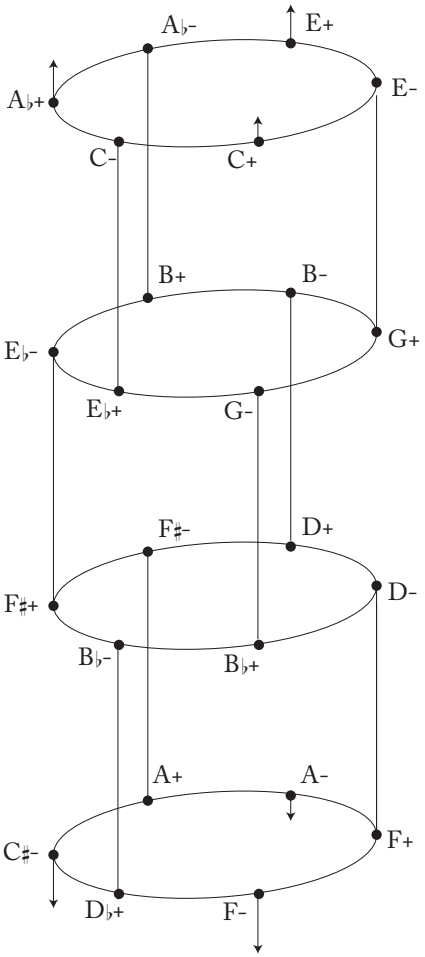
LEGEND



(d) Sewing together (c) produces a torus. Cohn ("As Wonderful as Star Clusters," 216, Figure 2).



(e) An alternate space, with *R* acting as the "paradigmatic" relationship.



by creating *equivalence classes* of triads ("tonic," "dominant," "subdominant") that share maximally smooth connective potential. Cohn's representation avoids conflating these transformations by associating them with unique musical roles: *L* and *P* are local transformations, nonetheless responsible for creating the harmonic regions responsible for large-scale tonal motion.

In this case, we can be somewhat more specific about what exactly ⟨*L, P*⟩ is doing when we say that it creates "regions": given the collection *S* of major and minor triads, ⟨*L, P*⟩ creates four *equivalence class* of triads related by the transformations of ⟨*L, P*⟩—"tonic," "subdominant," and so on. Equivalency is a *relation* (symbolized as ~) among triads in *S*, and in this case the equivalence

relation is $\sim_{\langle L, P\rangle}$. For two triads x and y in S to be equivalent, the $\sim_{\langle L, P\rangle}$ must be *reflexive* ($x \sim x$), *symmetric* ($x \sim y$ and $y \sim x$), and *transitive* (if $x \sim y$ and $y \sim z$, then $x \sim z$). The conditions required to create a group guarantee that $\sim_{\langle L, P\rangle}$ satisfies these properties. For two triads x and y to be *L/P-equivalent*, there must be some transformation f in $\langle L, P\rangle$ such that $f(x) = y$. We can prove that this is a relation as follows:

(1) *L/P* equivalency is reflexive because the group $\langle L, P\rangle$ must contain an identity element, and therefore, $x \sim_{\langle L, P\rangle} x$, for any x ;

(2) *L/P* equivalency is symmetric because of the requirement that the group contain an inverse—if $f(x) = y$, then $f^{-1}(y) = x$;

(3) *L/P* equivalency is transitive due to the group's binary composition—if $f(x) = y$ and $g(y) = z$, then $fg(x) = z$, for any f and g in $\langle L, P\rangle$.

Equivalence classes created by a group action upon a set of objects are called *orbits*. In the "gazing" space, the four orbits contain the six unique triads in the four rows of Figure 2.21. A set of all orbits created by a particular relation is called a *quotient set*; and thus, Cohn's "gazing" network is *quotient space* containing four orbits created by *L/P* equivalency. The term "quotient" here is meant in much the same sense that "product" was meant in §2.1. Whereas a product combines two smaller groups, a quotient "divides" a larger group into a smaller one.[61]

[61] Quotient "maps" are a type of homomorphism. Unlike embeddings, which I discussed in §1.4.5, quotient maps send more than one element in a group to the same element in another group. See Carter, *Visual Group Theory*, 163-7. In order for a subgroup to divide a larger group, that subgroup must be a *normal subgroup*. I will often be interested in creating spatial networks organized by subgroups that are not normal. And therefore, I will generally avoid the term quotient and instead refer to the manner in which subgroups or other types of relations *organize* a spatial networks.

2.3.5 COHN'S "HYPER-HEXATONIC SYSTEM"

Subtle changes in the underpinnings of a quotient space's organization can communicate different meaning. Cohn's "gazing" network was organized by *D*, and therefore, the four *L/P* orbits are named in accordance with the minimal distance, in iterations of ⟨*D*⟩, that a given triad in one orbit is from some triad in another orbit. A hypothetical triad in the "dominant" orbit is one *D* transformation from some triad in the "tonic" orbit," and so on.

Cohn (1996) created a different organization of ⟨*L, P*⟩ orbits that is not constrained by dominant relationships. There, he understood each of the four regions (which he calls "hexatonic systems") as being related by their *total pitch-class content*. Referring to Figure 2.23, a reprint of Cohn's "hyper-hexatonic system," Cohn says:

> The basis for this cyclic arrangement [of orbits] is discovered at the centre of [Figure 2.23], where the twelve pitch-classes are partitioned into the four T_4-cycles (augmented triads). The intersecting ovals in which they are enclosed portray the four hexatonic collections of pitch-classes, labelled H_0(pc) to H_3(pc), each of which includes two T4-cycles. The arrows from centre to periphery show the affiliations between hexatonic collections and hexatonic systems. Neighbouring hexatonic systems (those connected directly) share three pcs, while the pc content of opposite systems is complementary with respect to the twelve-pc aggregate.[62]

Note that, unlike the "gazing network," Cohn does not organize the "hyper-hexatonic system by creating an organizing *transformation*, such as ⟨*D*⟩. Rather, Cohn establishes a set of pitch-class restrictions that characterize triads in the same system in relation to those in adjacent systems. These pitch-class restrictions are *relations* that organize the network in the same way that ⟨*D*⟩ did earlier:

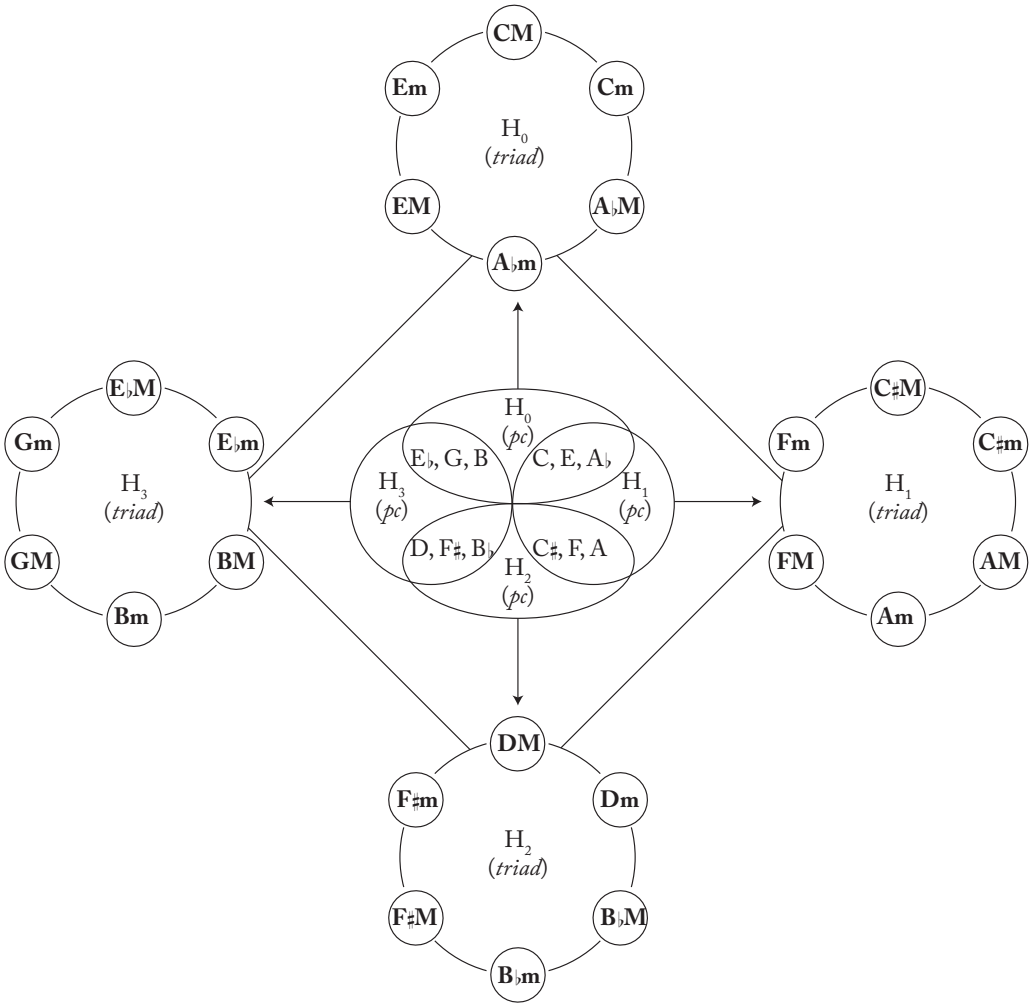
(1) Two triads *x* and *y* are *hexatonically equivalent* ($\sim_{HEX}$) if their pitch-class content belongs to the same hexatonic collection. **C+** $\sim_{HEX}$ **E -** because both belong to the hexatonic orbit $\mathbf{H_0}$.

[62] Richard Cohn, "Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions," *Music Analysis* 15, no. 1 (1996): 23.

(2) Two *orbits* are in the *hexatonic-neighbor* (~HN) relationship if their total pitch-class content overlaps by three pitch classes. **H₀** ~HN **H₁**: the two orbits share the pitch classes C, E, and A♭.

Note that the second relationship occurs between orbits, and not triads, and that it is not an equivalence relationship. Though the *hexatonic-neighbor* relationship is reflexive (given any orbit *x*, *x* ~ *x*) and symmetric (given any two orbits *x* and *y*, if *x* ~ *y*, then *y* ~ *x*), the relation ~HN is not transitive. For example, although **H₀** ~HN **H₁**, and **H₁** ~HN **H₂**, **H₀** is *not* a hexatonic neighbor of

FIGURE 2.23. Cohn's "hyper-hexatonic system": "Maximally Smooth Cycles," 24, Figure 5.



H_2 because they do not have any overlapping pitch content. Relationships that are reflexive and symmetric, but not transitive, are called *similarity* relationships.

2.3.6 Organized Spatial Networks and the Molecular Metaphor

In each of Cohn's spaces, one type of relationship *conformed* triads into orbits, and a second type *organized* those equivalence classes in relationship to one another. In both cases, the *conforming* relationship was the same—the *maximally smooth group* = ⟨*L, P*⟩. However, each spatial representation *organized* the *L/P* orbits differently. While the "gazing" network is organized to capture minimal voice-leading *in relation to a tonic/dominant framework*, the hyper-hexatonic system models triadic music that is tonally indeterminate and *organized by shared-pitch class content*. Cohn's analyses using the hyper-hexatonic system, rather than indicating how regions are related to an established "tonic" region, as was the case in his analysis of Schubert's sonata, show how motion between regions can take advantage of invariant pitch class content.

Turning back to the idea of paradigmatic and syntagmatic relationships, the organization of Cohn's spatial networks specifies the meaning of the orbits whose members are paradigmatically related; that is, in the larger scheme members of an orbit have similar functions. As musical grammars, the visual separation of Cohn's networks show not just syntactical relationships but also substitutional ones. This means of organization is a useful way to represent the dual modes of relationship often in evidence in Webern's music, as well, wherein pitch-class invariance and inversional axes are important paradigmatic relations in his serial music, often *organized* by the syntax of transformation chains.

Recall the Piano Variations's second movement. We have seen that the chain group ⟨TCH_1, ICH_1⟩ acting on **R** and **RI** forms is a useful way to depict linear connections in the movement, while inversional symmetry around A4, symbolized by the transformation I_6, best describes the

bond between canonic voices. Figure 2.24 describes how to create a spatial network that visually depicts this duality.

(1) Figure 2.24(a) and (b) show a spatial network for $\langle TCH_1, ICH_1 \rangle$ acting on **R** and **RI**-forms. At (b), the circular spatial network has simply been unfolded.

(2) Figure 2.24(b) shows two I_6-related row forms. A finished space will place such-related rows in the same vertical "container," which is shown at (c). Because the relation $\sim_{I6}$ is an equivalence relation, the jagged strips on the space divide it into twelve, completely separated regions or "row areas."

The space at (c) is an I_6-organized $\langle TCH_1, ICH_1 \rangle$ network. Spatial organization is meant to reflect the dual relationships embodied in Op. 27. On the space, I have identified equivalence classes ("regions" or "row areas") created by I_6 with the notation $\mathbf{A}_x$, where x is arbitrarily equal to the subscript of the **R** form in that region. Thus, $\mathbf{A}_5$ represents the equivalence class containing $\mathbf{R}_5$ and $\mathbf{RI}_{10}$.

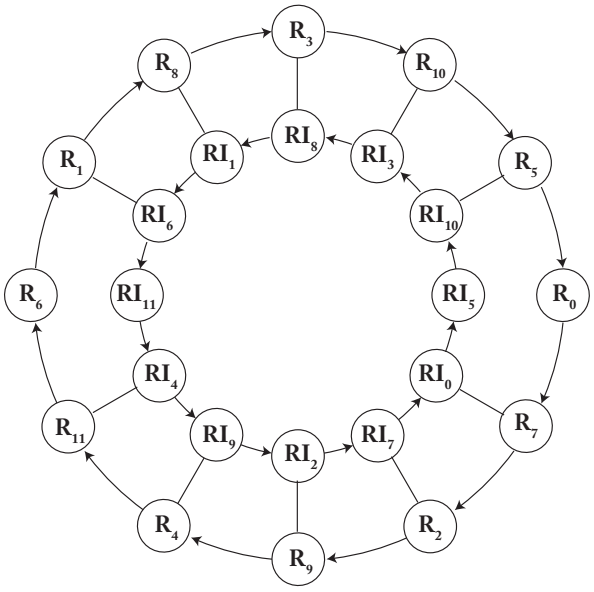
This space strongly resembles the hybrid group representation shown in Figure 2.14, but is conceptually quite different. Most importantly, the row areas, created by I_6, are meant to symbolize cohesive *units*. Their row constituents are not meant to be distant, but are instead dependent upon one another to create the overall meaning of each row area unit. Cohesiveness, in this sense, is suggestive of the bonds that create molecular structure, which Shaugn O'Donnell has used to describe the utility of Klumpenhouwer networks as models of a set's internal structure: "I visualize K-nets as three-dimensional ball-and-stick models with nodes standing in for atoms, and transformations functioning as bonds."[63]

O'Donnell's molecular metaphor is suggestive in the present context because it encourages a separation of transformational relationships that are *internal*, *binding*, or *vertical* with those that are *horizontal* and drive music forward. Klumpenhouwer networks are good models of the

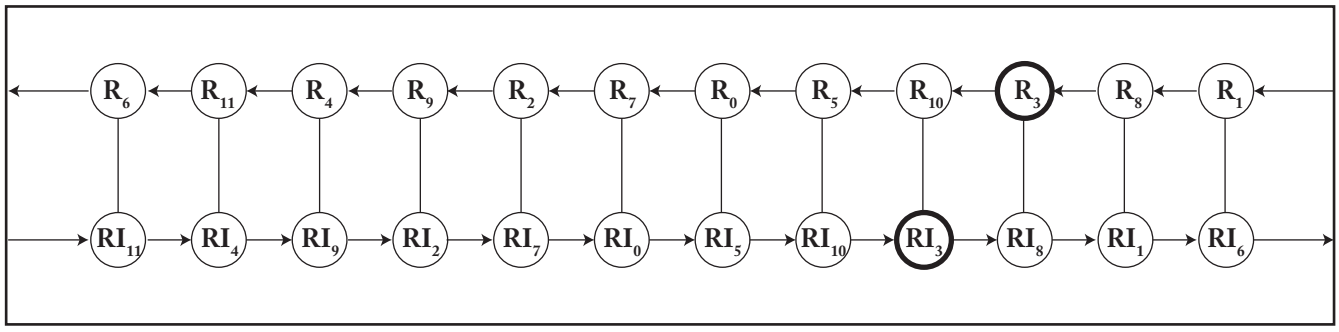
[63] Shaugn O'Donnell, "Klumpenhouwer Networks, Isography, and the Molecular Metaphor," *Intégral* 12 (1998): 74

FIGURE 2.24. An I_6-organized $\langle TCH_1, ICH_1 \rangle$ network.

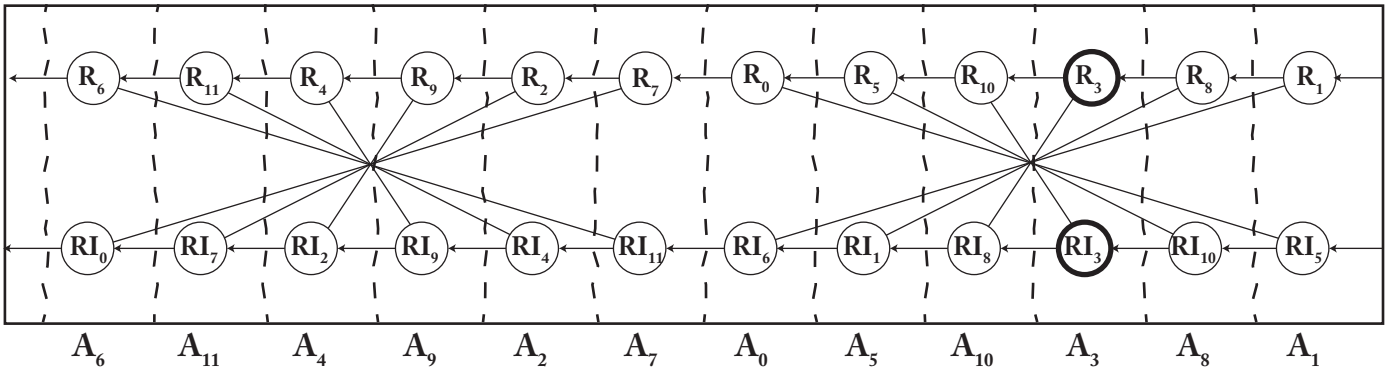
(a) Spatial network for **R**- and **RI**-forms in Op. 27, generated by $\langle TCH_1, ICH_1 \rangle$.



(b) The spatial network at (a), unfolded. $\mathbf{R_3}$ and $\mathbf{RI_3}$ are in the I_6 relation.



(c) An I_6-organized spatial network. Vertically-aligned rows are in the I_6 relation.



"vertical" bonds that create chord structure, while O'Donnell's dual transformations are better models of the horizontal connections between these chords. Binding relationships are akin to paradigmatic relationships, and are compelling ways to organize chain-generated syntactical spaces into paradigmatically organized spatial networks to model Webern's compositional language.

I understand the molecular bonds created by these relationships in two ways, which I will explore in the following examples. First, inversional axes created by coinciding row forms produce bonds that are dependent upon the presence of *both* row forms. Second, invariance relationships create bonds between row forms that share a particular type of invariance. Unlike an inversional axis, these bonds do not require the presence of every row form related as such. Rather, they call to mind an out-of-time universe of rows that are *substitutional* in nature. That is, if a row **S** is related to **T** by a particular invariance relationship, the two rows can substitute for one another in a compositional grammar.

2.3.7 Analytical Vignette: Webern Piano Variations, Op. 27, II

Though Webern titled his Op. 27 "Piano Variations," there have been some questions as the sense in which the first and second movements are variations at all.[64] In that the constant axis of symmetry guarantees the regular circulation of a set of motivic dyads and trichords, the movement is certainly a regular variation of the *order* of these pitch motives. But there are at least

[64] Much of the controversy was initiated by Kathryn Bailey's interpretation of Willi Reich's notes about the movement, which seems to have been guided by an incomplete consideration of historical evidence. See Kathryn Bailey, "Willi Reich's Webern," *Tempo* no. 165 (1988): 18–22. Upon completion of the third movement, Webern wrote to Hildegard Jone and Josef Humplik, "The completed part is a variations movement; the whole will be a kind of 'Suite'" (Webern, *Letters*, 32). According to Kathryn Bailey, *The Twelve-Note Music of Anton Webern,* 190-1, this letter is evidence that the title of the piece refers *only* to the third movement. Regina Busch notes that "this does not mean, as [Bailey] concludes, that the rest of the work is 'a kind of "suite"', and it also does not exclude the first two movements being variations as well" (Regina Busch, "[Letter to the Editor]," *Tempo* no. 166 (1988): 68. Busch goes on to cite three pieces of historical evidence that Bailey failed to consider. Among them are a letter from Webern to

two other, structural means of variation as well, both of which account for the brevity of the movement:

(1) In the binary form scheme, an ICH_1 chain accomplishes each repeat. The two halves of the piece are *varied* transformationally in that, despite the canonic voices maintaining their I_6-relationship, a TCH_1 chains connect rows in the first half and an ICH_1 chains connect rows in the second half. Figure 2.25 shows this variation with bold arrows showing the variation. It suggests an *imaginary* TCH_1 at the close of the movement because, had it occurred, the transformational variation of mm. 1-11 in the second half would have been exact:

(mm. 1–11): $\boldsymbol{TCH_1} \bullet ICH_1 \bullet \boldsymbol{TCH_1} \bullet TCH_1$

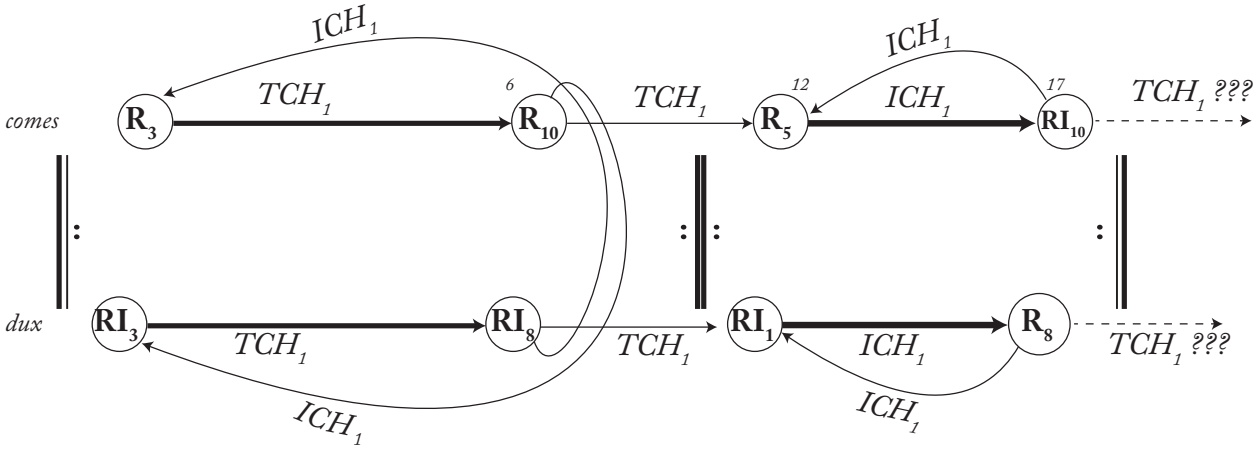
(mm. 12–22): $\boldsymbol{ICH_1} \bullet ICH_1 \bullet \boldsymbol{ICH_1} \bullet (TCH_1)$

(2) Throughout the movement, transformation *cycles* are coincident with formal units, and as I will show below, those cycles are varied.

---

the work's dedicatee Edward Steuermann (printed in Regina Busch, "Aus Dem Briefwechsel Webern-Steuermann," in *Musik-Konzepte, Soderband Anton Weber I*, ed. Heinz-Klaus Metzger and Rainer Riehn (Vienna: Universal Edition, 1983), 32-33) in which Webern says: "I am sending you my Variations […]. As, I believe, I have already indicated to you, they are divided into self-contained movements (three). Also I make the theme by no means expressly prominent (at the top, as it used to be for instance). […] (It is—naturally I shall tell you straight away—the first 11 bars of the third movement.)" This letter makes clear that Webern considered the whole work as variations. See also Neil Boynton, "Some Remarks on Anton Webern's 'Variations, Op. 27'," in *Webern_21*, ed. Dominik Schweiger and Nikolaus Urbanek, Wiener Veröffentlichungen Zur Musikgeschichte 8 (Vienna, Cologne, Weimar: Böhlau, 2009), 199–220.

FIGURE 2.25. A diagram of the Piano Variations's second movement. Bolded arrows, occurring in the body of each half of the movement, are transformational "variations" of one another. Chains that occur at repeats are not varied, which suggests an imaginary *TCH₁* at the close of the movement that corresponds to the *TCH₁* from m. 11 to m. 12.

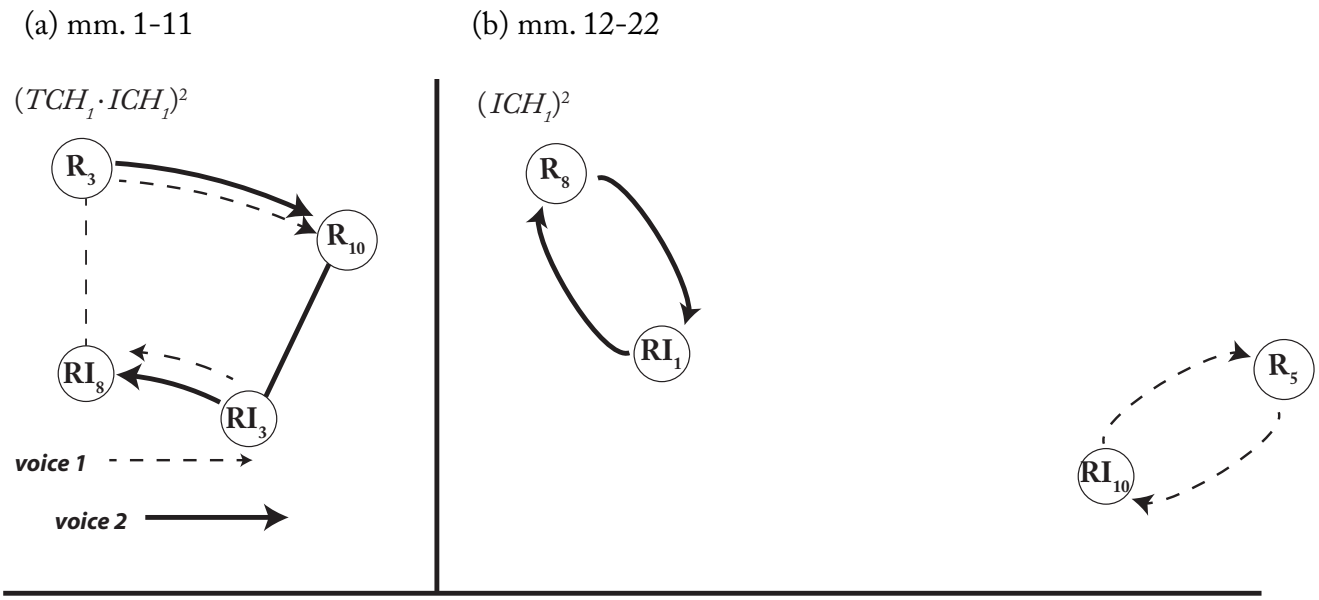


In Figure 2.26 I have provided four "snapshots" of the movement showing cycles on the chain-generated spatial network from 2.24(a). Each of the four cycles describe a formal unit in the piece.[66] Each cycle occupies part of a small, 8-row section of Figure 2.24(a), but—in keeping with the spirit of variation—none of the cycles are the same. In the first half (shown at (a)), "voice 1" is echoed by "voice 2," and *together* they complete a cycle (*TCH₁* • *ICH₁* • *TCH₁* • *ICH₁*) engaging four unique row forms. A single voice completing this cycle would require four transformations, but (as Figure 2.25 showed) only three transformations occur in each voice in the first half; both realize the transformational path *TCH₁* • *ICH₁* • *TCH₁*. The spatial network shows that the two voices are superimposed in such a way that the passage projects the complete, four-transformation cycle in only three transformations.

As if varying that cycle, the second half of the piece (mm. 12–22, shown at (b)) projects a complete cycle as well, with each voice performing *ICH₁* • *ICH₁* • *ICH₁*. At the end of the second half, the two voices are relatively "far apart" spatially, positioned at **R5** and **RI1**. However, both

[66] I am understanding a cycle to be a "closed" loop on the spatial network.

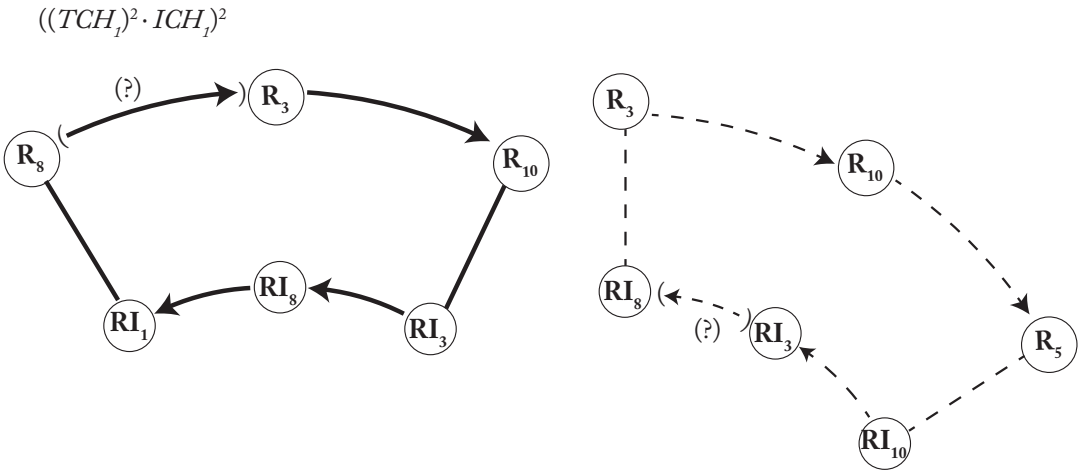
FIGURE 2.26. Transformation cycles in each of the second movement's formal units.



voices at the end (at **R8** and **RI10**) are a single move away from their position at the beginning of the piece. The sense of this is given at (c), which shows both voices as they move through the entirety of the piece. (Notice the overlap in node content between the two, which occurs entirely in the first half of the movement.) This extraction shows that in the course of the movement each voice traces a nearly complete cycle of $(TCH_1 \bullet ICH_1 \bullet TCH_1)^2$. In both cases, the cycle is cut one chain short: to complete the cycle, an imaginary TCH_1 that I hypothesized earlier is missing and shown there with a question mark. Had that chain occurred, the piece would be be in prime position for a recap of the opening— in "da Capo" fashion, perhaps.

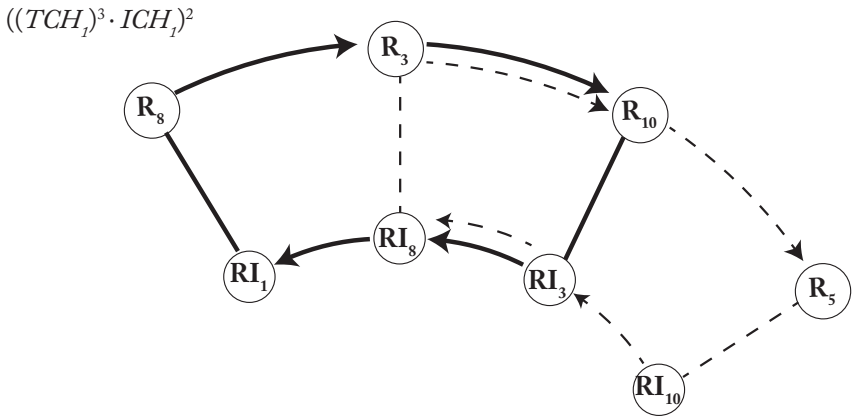
I first proposed this imaginary TCH_1 above as reinforcement of the sense of transformational variation in the second half. On this space, we can now see why that chain does not occur: in the case of both voices, a TCH_1 chain following the final rows would have led to the collection of rows that began the piece. That the piece begins with the same {B♭5, G♯3} dyad that it ends with allows the listener to imagine this situation, maybe even supplying the missing TCH_1 chain. Note that the idea of a larger cycle, encapsulating the cyclic character of each of the

(c) Complete piece, mm. 1-22, voices separated



two variations, indicates that the piece as a whole might be a sort of "meta-variation." At (d) a final network superimposes the two voice paths that were separated at (c). This graph shows two interesting facets of the movement's transformational action. First, it indicates that the music exhausts every transformational path in this eight-row corner of the ⟨*TCH1*, *ICH1*⟩ space.

(d) Complete piece, mm. 1-22, voices combined



Second, it shows that the superimposed voices together create an even larger cycle, $((TCH_1)^3 \bullet ICH_1)^2$.[67]

These paths on the $\langle TCH_1, ICH_1 \rangle$ network indicate how the idea of *cycle* characterizes every formal unit in the piece, a fact underscored in Table 2.1:

TABLE 2.1. Four cycles in Op. 27, II

	Cycle
First half (mm. 1-11)	$TCH_1 \bullet ICH_1 \bullet TCH_1$
Second half (mm. 12-22)	$ICH_1 \bullet ICH_1 \bullet ICH_1$
Full movement, individual voices	$(TCH_1 \bullet ICH_1 \bullet TCH_1)^2$
Full movement, combined voices	$((TCH_1)^3 \bullet ICH_1)^2$

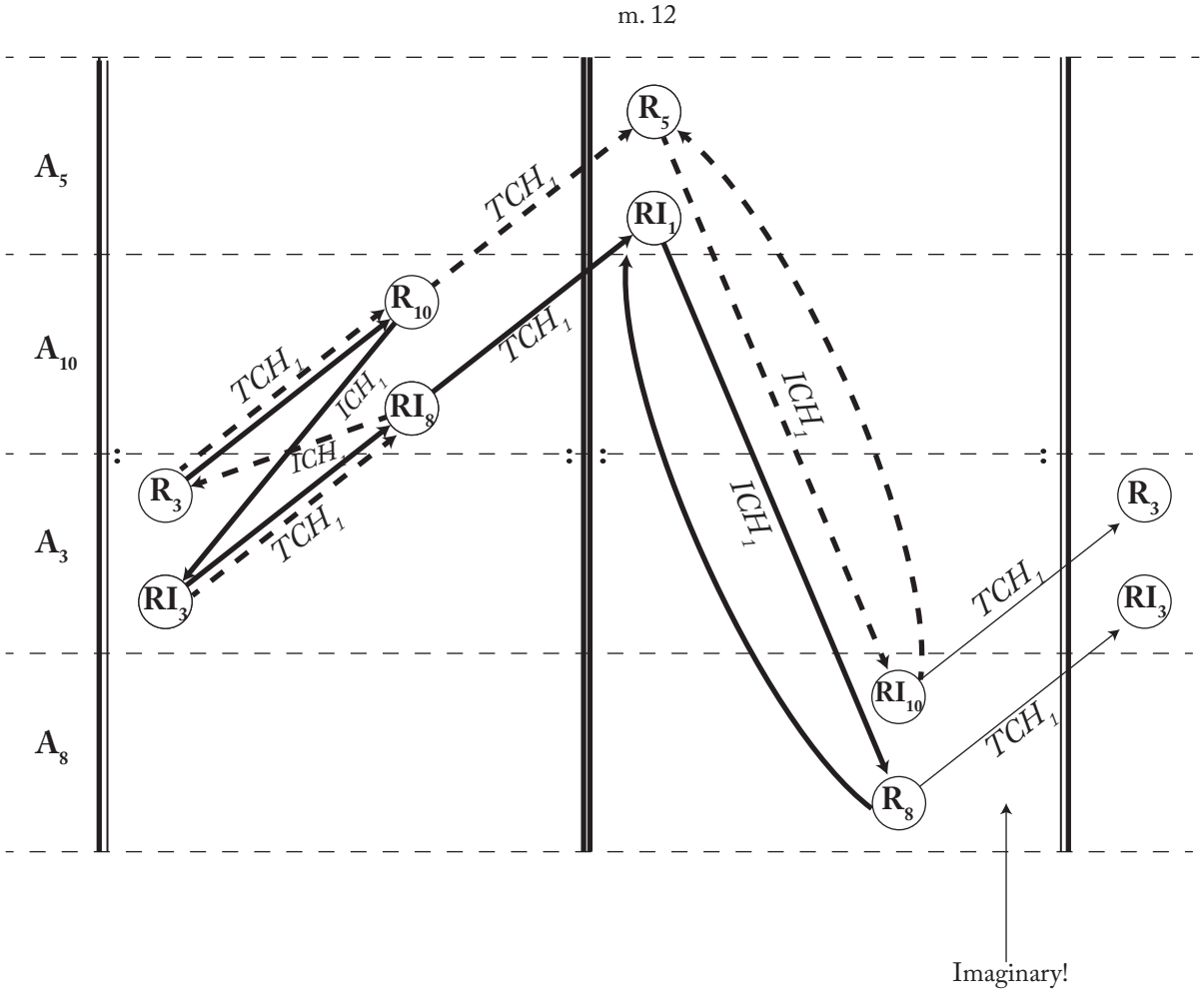
Now it is worth considering how the axis of symmetry that organized the network in Figure 2.24(c) is involved. Of course, the axis of symmetry is an element of constancy in the course of the variations. But furthermore, the I_6-constrained row areas *limit* the spatial locations on the network where the transformational paths discussed above will create cycles. To demonstrate, Figure 2.27 shows two event networks charting transformation chains in the space in reference to the rows areas created in Figure 2.24(c). The first is an actual event network for the piece, while the second (at (b)) transposes the *dux* by T_7, but retains the transformation chains and the I_6 axis.

The event network at (a) shows how the second half of the piece spatially surrounds the two row areas characterizing the first half: the row area $\mathbf{A_5}$ is adjacent to $\mathbf{A_{10}}$, and $\mathbf{A_8}$ (at the close of

[67] We might also think of this hypothetical TCH_1 in relation to the next movement. Does the opening row of that movement complete this cycle? Not exactly. Completing the cycle would have led to $\mathbf{R_3}$/$\mathbf{RI_3}$. The third movement of Op. 27 begins with a single row, $\mathbf{P_3}$. $\mathbf{P_3}$ is, of course, the retrograde of $\mathbf{R_3}$, and the retrograde of $\mathbf{RI_3}$ follows shortly after. Even more than this, however, the final movement is initiated and ended with a 3-family of rows (including $\mathbf{P_3}$, $\mathbf{I_3}$, $\mathbf{R_3}$, and $\mathbf{RI_3}$) that act as representatives of a sort of "tonic" family of row forms.

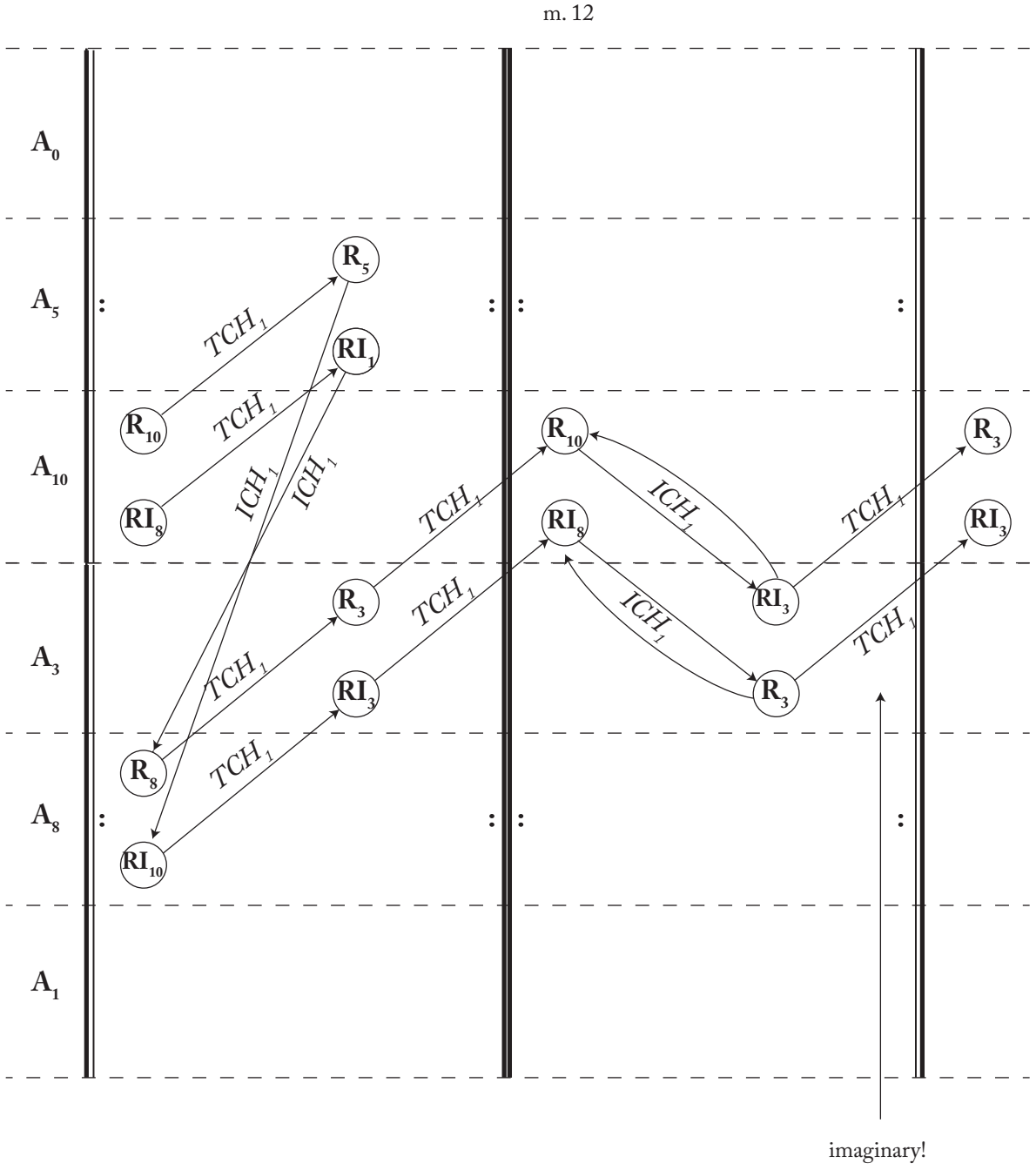
FIGURE 2.27. Two event networks for the second movement of the Piano Variations.

(a) The whole movement (cf. Figure 2.25).



the second half), is one "imaginary" *TCH1* removed from **A3**. That sense of symmetricality is voided in the hypothetical, transposed network at (b). There, following the initial *TCH1*, an *ICH1* moves both rows from **A5** to **A8**. And therefore, unlike the network at (a), mm. 1–12 are not repeated exactly. Instead, the opening traverses four unique row areas. Instead of the second symmetrically surrounding the first half, it recapitulates two row areas heard there: mm. 12, rather than leading off with a unique row area, returns to the area heard in the opening.

(b) A hypothetical event network imagining that the *dux* was transposed by T_7 but the I_6 axis was retained.



The organized space in Figure 2.24(c) shows that the row area that Webern chose to begin the piece ($\mathbf{A_3}$) exists at a "nodal" point. This nodal point is (in addition to its tritone transposition) the only portion of the space that would allow this symmetrical surrounding in so few

transformations. Beginning at any other location in the space and perform the same four transformations would *not* result in the same symmetry. Thus, the paradigmatic, I_6 relation influences the results of the syntactic transformation chains that create the movement's cyclic variations.[68]

2.3.8 Analytical Vignette: Webern, Piano Variations, Op. 27, III

Like inversional axes, pitch-class associations between row forms may be important markers of row affiliation that imply the same sorts of "molecular" bonds that created the inversionally defined row areas I just discussed. Shared pitch-class segments between unique row forms (often termed *invariant pc segments*) are particularly Webernian, and such associations generally establish equivalence or similarity relationships that are analytically interesting in terms of their ability to organize chain-generated spatial representations.[69]

Consider the passage shown at in Example 2.28(a), from the "theme" of the Piano Variations's third movement. Dynamics, articulation, and durational patterns associate certain

---

[68] In some ways, my analysis of the movement here is a "variation" on an analysis of the movement found in Mead (1993, 179-87).

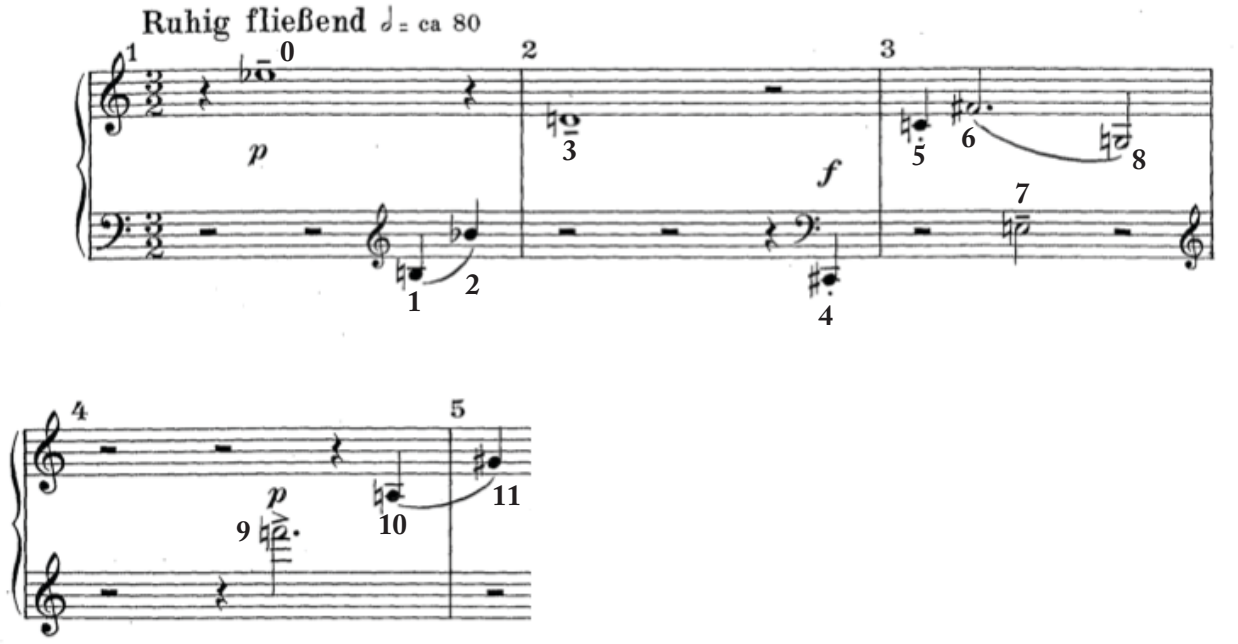
[69] Invariance is often a very generic term indicating many different types of pitch-class association. Robert Morris, *Class Notes for Advanced Atonal Music Theory*, 158-62, has catalogued pc associations according to five "types" that indicate the degree of "closeness" or "remoteness" of such relationships. His five "types" track whether associated pc segments are ordered and whether they occupy adjacent order positions. The closest of these relationships (Morris's *Type-1* relation) associates pc segments that occur in adjacent order positions and whose pcs are in the same order. The most remote relationship (*Type 5*) associates segments whose pcs occur in non-adjacent order positions and are unordered.

An invariance relationship, let us call it ~INVAR., is an *equivalence relation* if and only if the associated segments occur in the same order positions. For example, imagine a segment *x* in a row **S**. This segment has the same pc content, and perhaps the same internal ordering, as a segment *y* in the row **T**. Similarly, the segment *y* has the same pc content, and perhaps the same internal ordering, as a segment *z* in the row **U.** The relationship ~INVAR is of course reflexive(**S** ~INVAR **S**), and it is symmetric (**S** ~INVAR **T** and **T** ~INVAR **S**). But the invariance relation is guaranteed to be associative only if *x*, *y*, and *z* occur *in the same order position*. More commonly, invariance relationships are simply similarity relations, which need not be associative, and describe *mappings* between segments at different order positions.See Morris, *Class Notes for Advanced Atonal Music Theory*, 164-5; and David W. Beach, "Segmental Invariance."

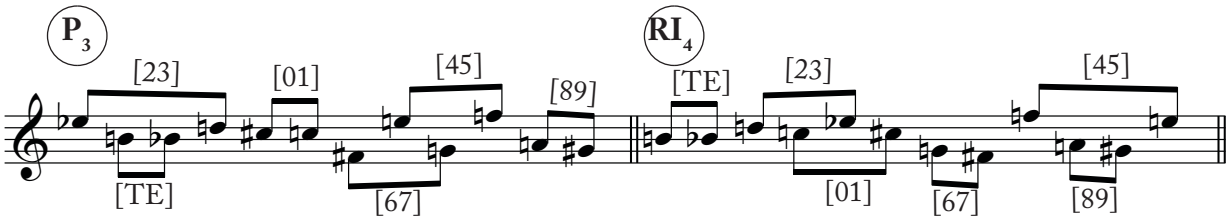
semi-tonal dyads, particularly those at order positions **{03}**, **{12}**, **{45}**, **{68}**, **{79}**, and **{te}**.[70] On that figure, for example, *tenuto* markings and duration associate E♭5 and D4 in mm. 1-2, although they occur at non-adjacent order number positions **{03}** within the row. Similar associations

FIGURE 2.28. Invariance relationships in the Piano Variations, third movement.

(a) Associated semi-tonal dyads at **{03}, {12}, {45}, {68}, {79},** and **{te}**.



(b) **P3** and **RI4**, when partitioned as shown, create the same *mosaic*, whose *parts* are semi-tonal dyads whose "roots" belong to the "even" whole-tone collection. The two partitions are related by the order operation I_e, retrograde in traditional terms.



[70] To distinguish order numbers from pitch-class numbers, order numbers are indicated with bold face. See Andrew Mead, "Some Implications of the Pitch Class/Order Number Isomorphism Inherent in the Twelve-Tone System: Part One," *Perspectives of New Music* 26, no. 2 (1988): 96–163.

relate non-adjacent pitches at order positions **{68}** and **{79}**, found in mm. 3-4. Those non-adjacent pitches have longer durations in the context of the passage.[71]

At (b) I have highlighted this partitioning of $\mathbf{P_3}$. Because it envelops the entire aggregate, the partitioning creates a *mosaic*, whose subsets (called *parts* in "mosaic theory") are a catalog of semi-tonal dyads: [{01}, {2,3}, {4,5}, {6,7}, {8,9}, {10,11}]. The six semi-tonal dyads have "roots"—the lowest pitch-class of the dyad in normal form—that belong to the "even" whole-tone collection. Six **P** forms and six **I** forms have the same mosaic when partitioned as $\mathbf{P_3}$ is in Figure 2.28(b), and the remaining **P** and **I** forms have a mosaic whose parts belong the "odd" whole-tone collection

Figure 2.28(b) also shows a partitioning of $\mathbf{RI_4}$ that creates the same mosaic. This partitioning is related to that of $\mathbf{P_3}$ by the order operation $\boldsymbol{I_{11}}$, which is equivalent to retrograde in traditional terms.[72] I will say that rows sharing the same mosaic, at these two partitions, are *WT*-related because their parts are semi-tonal dyads whose roots belong to the same whole-tone collection. Because there are only two whole-tone collections, the relation $\sim_{WT}$ is very coarse; it divides a row class into two large collections, which I will call $\mathbf{A_0}$ and $\mathbf{A_1}$. A row is in $\mathbf{A_0}$ if the "root" of its parts belong to the even whole-tone collection. Thus, measures 1–5, which began the opening theme of the movement, exemplify the row area $\mathbf{A_0}$: each of the associated dyads are "even."

Figure 2.29 organizes a chain-generated spatial network by assigning its rows to $\mathbf{A_0}$ or $\mathbf{A_1}$. That organization shows the "function" each transformation chain in reference to the *WT*-

---

[71] This particular partitioning was first noted by Peter Westergaard, "Some Problems in Rhythmic Theory and Analysis," *Perspectives of New Music* 1, no. 1 (October 1, 1962): 180–191. It has been the basis for many discussions of meter in the opening theme of this movement. Robert Wason makes an interesting case, buttressed by Webern's annotations in Robert Stadlen's performance score of the piece, that Webern may himself have heard the piece this way ("Webern's '"Variations for Piano,' Op. 27, 75-9).

[72] See Mead (1988, 99). If the order-number aspect of a row is <**0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11**>, its retrograde is <**11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0**>. Corresponding order numbers in the two retrograde-related rows sum to 11; therefore, retrograde can be described as the order operation $\boldsymbol{I_{11}}$.

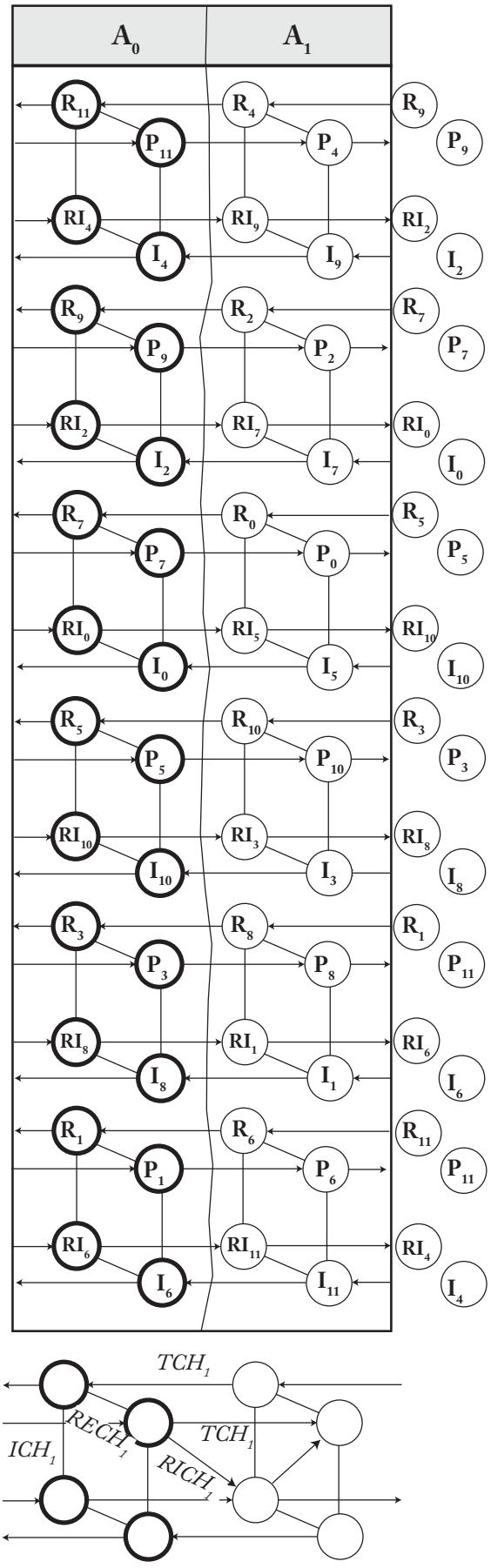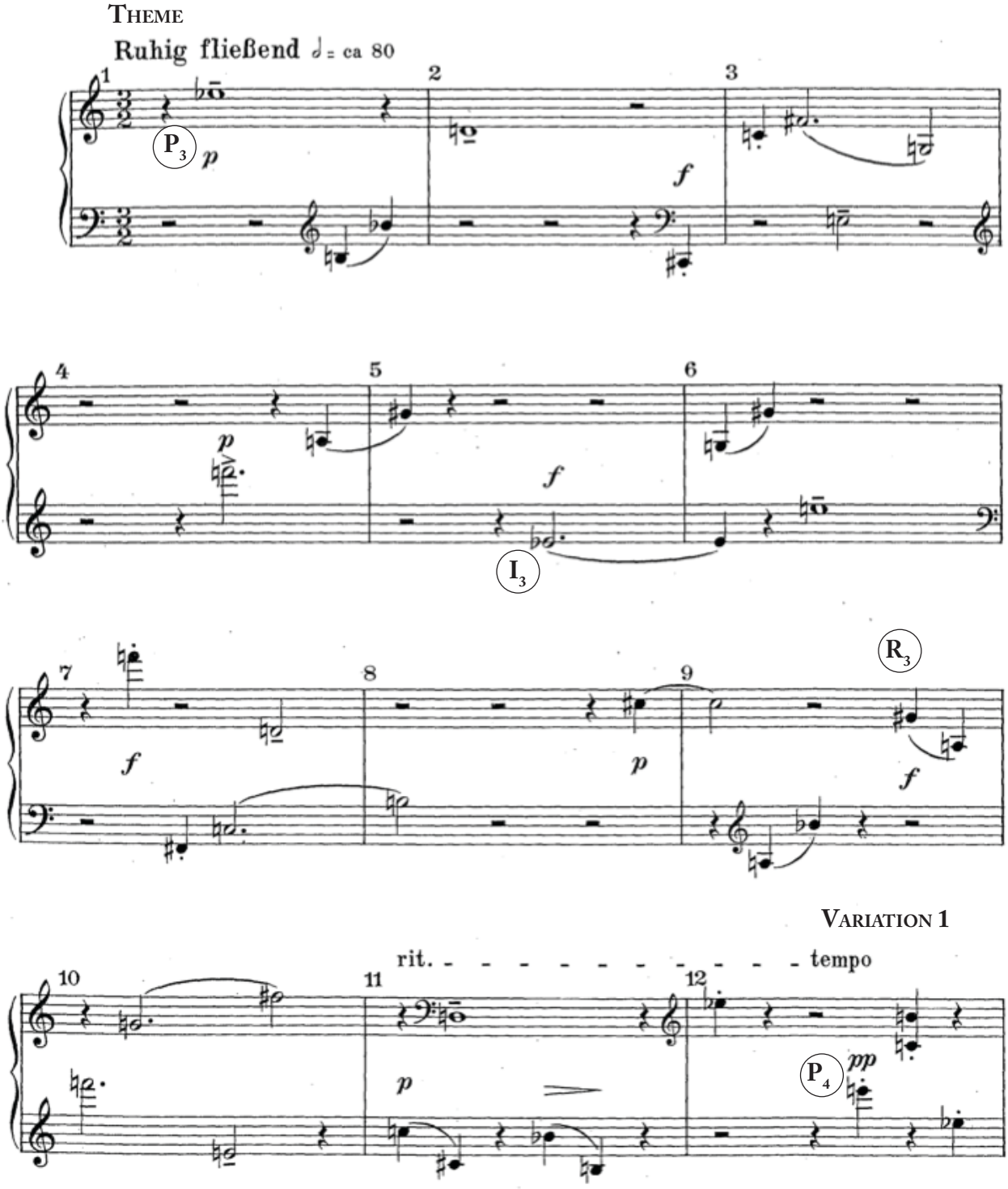


FIGURE 2.29. Chain-generated spatial network organized by the *WT* relation. Rows within the same "area" have the same catalogue of semi-tonal dyads.

relation. In these terms, there are generally two types of transformation. *TCH1* is *always* coincident with movement *away* form an area, while *ICH1*, and *RECH1 always maintain* an area. *RICH1* has a dual function. When applied to a **P** or **I** form, it results in a movement away from an area, while *RICH*ing an **R** or **RI** form has the opposite effect.

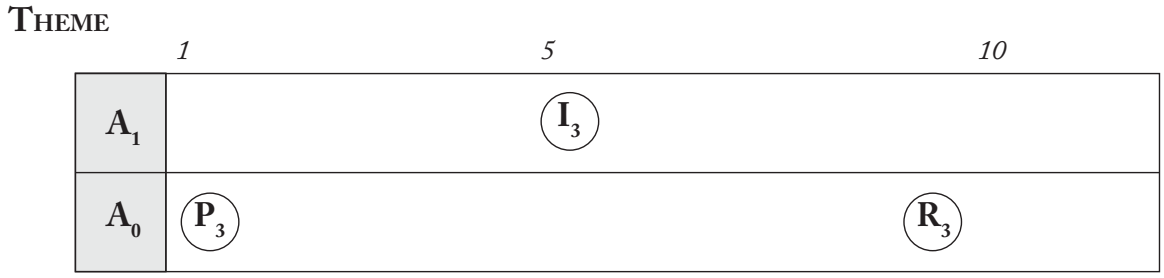
This space is capable of tracking "tonal" motion from $\mathbf{A_0}$ to $\mathbf{A_1}$ in the theme and five variations according to the partition scheme above. I have shown the entire theme in Figure 2.30, along with an event network at (b) tracking row motion within the spatial network's two areas. (Interestingly, the theme is the only section of the piece not to make extensive use of transformation chains.) Essentially, the theme involves a departure from $\mathbf{A_0}$ and a return. $\mathbf{I_3}$ follows $\mathbf{P_3}$ in m. 5, and that change is coincident with a prominent change in the catalogue of semi-tonal dyads, as $\mathbf{I_3}$ is a member of $\mathbf{A_1}$ and $\mathbf{P_3}$ is a member of $\mathbf{A_0}$. A compensatory motion occurs with the theme's close on $\mathbf{R_3}$, a member of the same $\mathbf{A_0}$ row area that began the theme.

FIGURE 2.30. Piano Variations's third movement, "Theme."

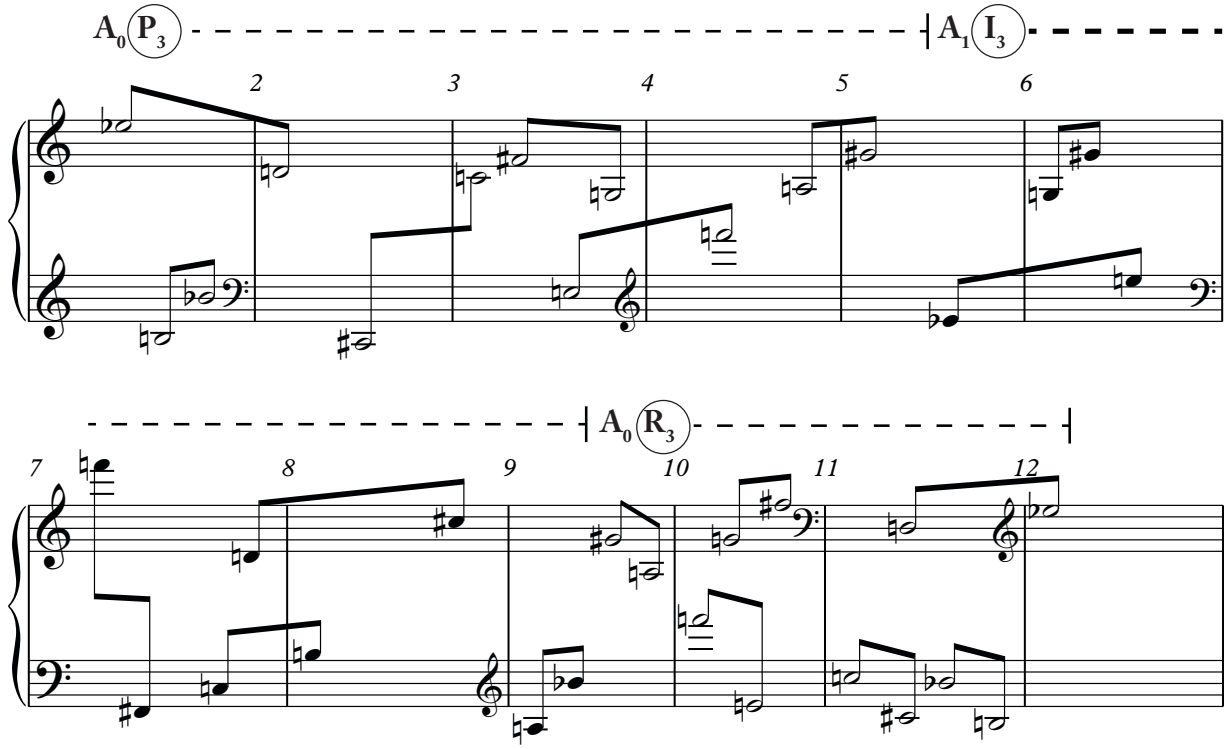
(a) Measures 1-12.



(b) Row area analysis shows "departure" and "return" from **A_0**.



(c) Score reduction. Beamed notes are associated by timbre, dynamics, and duration and represent the partitioning shown in Figure 2.29(b). Open note heads indicate "even" dyads, which belong to **A_0**; Closed note heads indicate "odd" dyads, which belong to **A_1**.



At (c), I have shown a reduction of the passage to make the "tonal change" from **A_0** to **A_1** and back more concrete. The reduction beams together the dyads associated by duration and dynamics—the very dyads that were associated in my original analysis of the theme's opening in Figure 2.28(a). Dyads that belong to **A_0** are shown with open note heads (they have "even" roots); dyads that belong to **A_1** are shown with closed note heads (they have "odd" roots). For example,

track find the three occurrences of E♭ in the passage. In mm. 1-2, E♭5 is associated with D4. The same E♭5 resurfaces at the end of the passage, also associated with D and indicative of the return to $\mathbf{A_0}$. In the departure to $\mathbf{A_1}$ that occurs in m. 5, E♭ is sounded, but there it is associated with E, creating an "odd" dyad.

Similar registral associations occur between nearly every dyad in the passage—for example, when the "even" dyad [G♯4, A3] gives way to the "odd" dyad [G♮3, G♯4] as a change in area occurs over mm. 4-6, and when the {B3, B♭4} dyad in m. 1 becomes {B3, C2} in mm. 7-8. Perhaps the most salient of these associations involves F6, the highest note in the "theme." F6 sounds three times, once each as part of the three rows in the passage. In the first and last instances (mm. 3-4 and m. 10), F6 is associated with its "even" root E. During the departure to $\mathbf{A_1}$, F6 is dramatically juxtaposed with F♯2, the widest registral span in the theme.

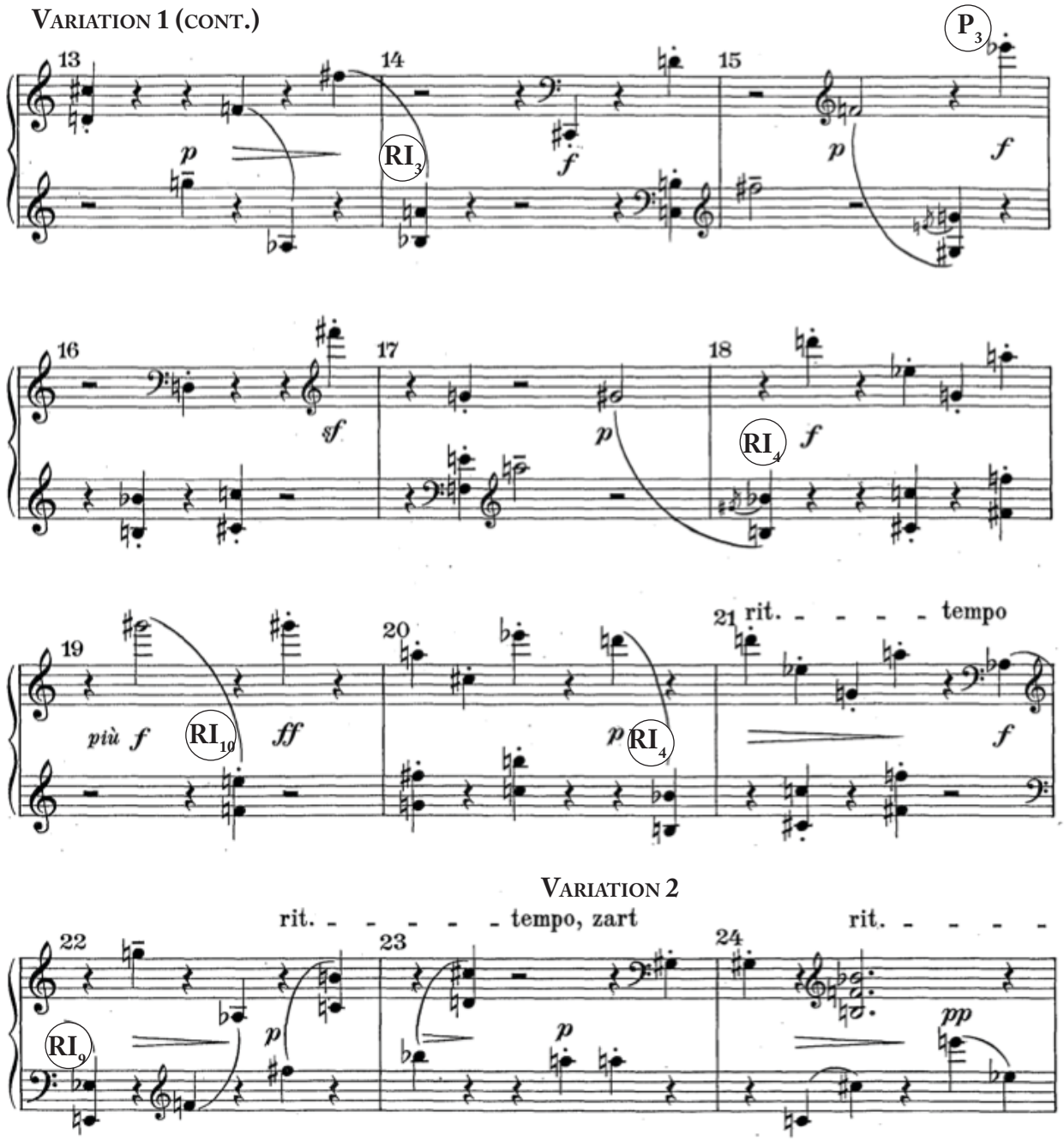
The tonal motion of the "theme" is replicated in the first and third variations. The first variation is shown in Figure 2.31. This variation largely follows the same partition scheme as the theme. Though more row forms are used here (and only one from the theme is heard), the event space at (b) indicates that the tonal motion follows the same scheme of departure and return as the theme, though it accomplishes this in a reciprocal manner—beginning and ending at $\mathbf{A_1}$.[73] In this scheme, the $RICH_1$ chain in m. 15 is answered by TCH_1 in m. 21, the two chains having the same function as regards the spatial organization—each moves a row into the *other* row area.

A reduction is given for this variation at (c). Like the earlier reduction, this one beams together dyads (usually major sevenths or minor ninths) that are associated musically. Those associations follow the same partition scheme as found in the theme; and thus, F4 and F♯5 are beamed in m. 13 though the occupy non-adjacent order positions. After beginning in $\mathbf{A_1}$, shown

[73] In Willi Reich's outline of the piece (printed in Friedhelm Döhl, "Weberns Beitrag Zur Stilwende Der Neuen Musik," in *Berliner Musikwissenschaftliche Arbeiten*, ed. Carl Dahlhaus and Rudolf Stefan, vol. 12 (Munich and Salzburg: Musikverlag Emil Katzbichler, 1976)), which he claimed to have gotten from Webern, the first variation is also a "transition" in the third movement's "sonatina" scheme. If $\mathbf{A_0}$ is considered the "tonic" area, it's notable that the first variation ends with $\mathbf{A_1}$.

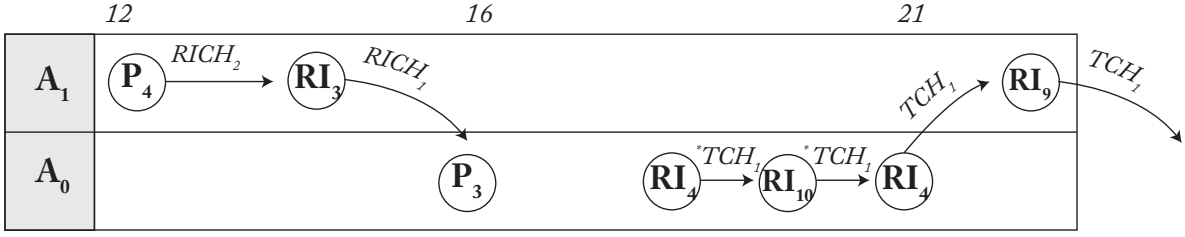
FIGURE 2.31. Piano Variations's third movement, "Variation 1."

(a) Variation 1, mm. 12-22. (Measure 12 shown in Figure 2.30).

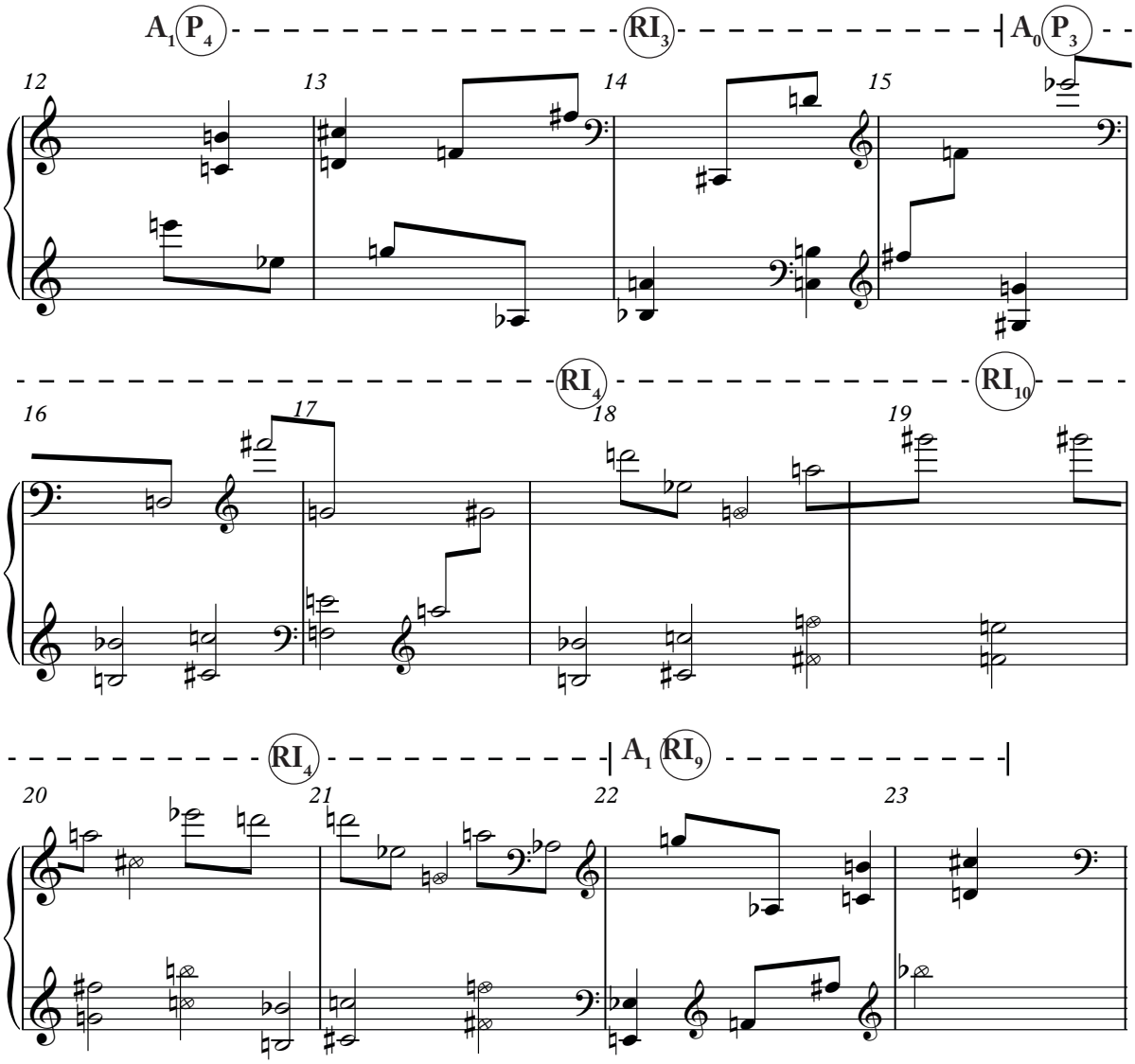


(b) Row area analysis shows “departure” and “return” (cf. Figure 2.30(b)).

VARIATION 1



(c) Reduction of Variation 1, mm. 12-23.



with closed note heads, a new presentation scheme associated with staccato quarter notes (as chordal major sevenths in the left hand; see m. 16) begins the $\mathbf{A_0}$ passage. Nearly every dyad over the course of mm. 16-21 belongs to $\mathbf{A_1}$. (Brief changes in the partition scheme occur in m. 8 and mm. 21-22, shown with crossed-out note heads.) The passage closes with a return to $\mathbf{A_1}$. This is heralded by a resurgence of motives that were heard in the passage that opened the variation. At (a), for example, compare mm. 13-14 and mm. 22-23.

Note the difference in the types of paradigmatic relationships established in the second and third movements. While each bond in the second movement required the presence of *both* rows in the inversional relationship, the bonds in the third movement are of a substitutional nature. On Figure 2.31(b), for example, note that $\mathbf{P_4}$, $\mathbf{RI_3}$, and $\mathbf{RI_9}$ can each substitute for one another in the paradigmatically understood invariance relationship that creates row area $\mathbf{A_1}$.

§2.4 Combinatoriality, Row Areas, and Transformational Character

Webern did not write "combinatorial music," at least not in the sense that Schoenberg did. But imagining Webern's serial music as comprising two distinct but interactive types of relationship—one paradigmatic and one syntagmatic—resonates a great deal with the dual organization that affects musical form in Schoenberg's serial music. The idea of a "row area" describes a *paradigmatic*, molecular relationship between *IH*-combinatorial rows—rows that are bound together "vertically" by shared hexachords.[74] These *IH*-related row areas are also *organized* syntactically. That is, there are relationships that establish typical ways to *order* combinatorially-defined row areas.[75]

In Figure 2.32-34, I have deconstructed Lewin's analysis of Schoenberg's Violin Fantasy to accentuate each of these components and underscore the systemic similarity between Schoenbergian combinatoriality and Webern's compositional practice. Figure 2.32 shows the twelve row areas described in Lewin's analysis of Schoenberg's Violin Fantasy. Lewin's use of the the term "row area," symbolized there with $\mathbf{A_0}$, $\mathbf{A_1}$, and so on, to refer to harmonic "regions" containing similarly-constructed rows. Though he does not use the term paradigmatic, it is clear that the term "area" has something of this meaning for Lewin. Framing the idea of a "row area" in historical terms, Lewin says: "[Liszt] organizes his material into 'areas,' often diatonic; […]

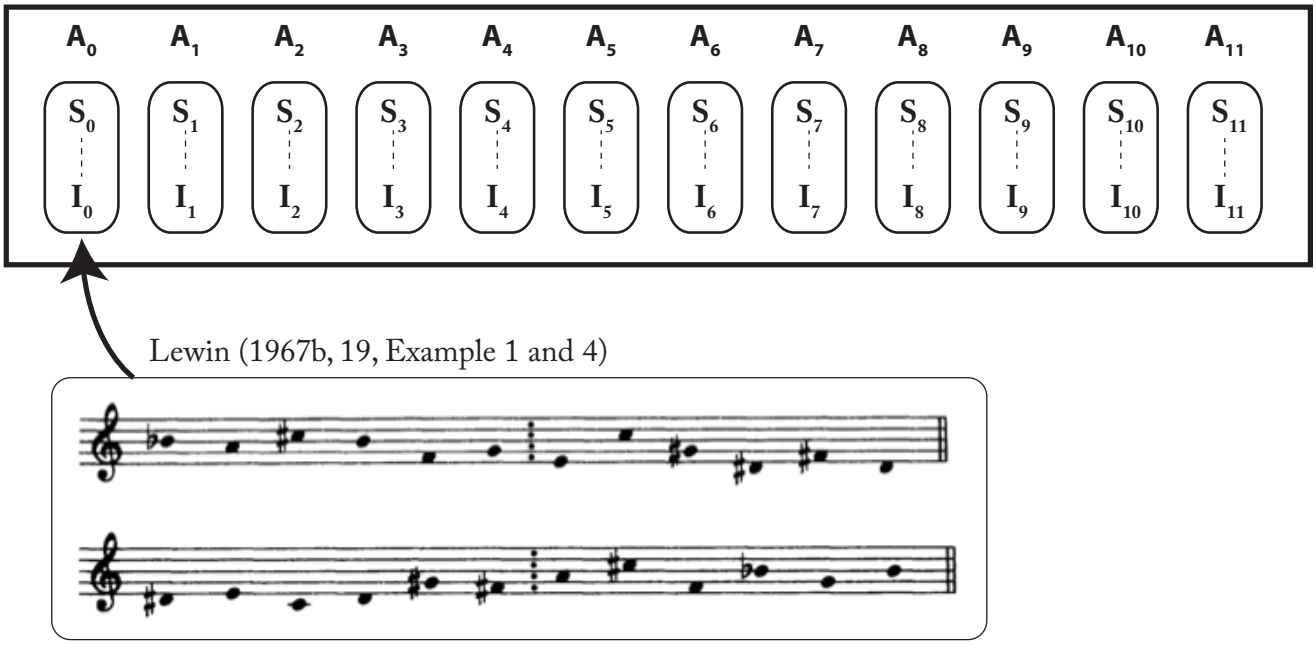
---

[74] David Lewin originated the concept of twelve-tone areas to study Schoenberg's twelve-tone music in three articles, each of which show that the concept of row-area organization was quite fluid. "A Study of Hexachord Levels in Schoenberg's Violin Fantasy," *Perspectives of New Music* 6, no. 1 (October 1, 1967): 18–32; "*Moses Und Aron*: Some General Remarks, and Analytic Notes for Act I, Scene 1," *Perspectives of New Music* 6, no. 1 (October 1, 1967): 1–17; "Inversional Balance as an Organizing Force in Schoenberg's Music and Thought," *Perspectives of New Music* 6, no. 2 (1968): 1–21. See also Andrew Mead, "Large-Scale Strategy in Arnold Schoenberg's Twelve-Tone Music," *Perspectives of New Music* 24, no. 1 (October 1, 1985): 120–157.

[75] Schoenberg does not seem to have had a consistent practice in this regard. Analytical studies—many of which are cited in the preceding note—have shown various methods by which Schoenberg organized row areas to create "inversional balance," or to imitate tonal forms by establishing a row area (or set of row areas) that act as a quasi-tonic.

Liszt's procedures, in this respect, were adapted by (among others) both Wagner and Debussy to their own idioms. […] Wagner used them dialectically, […] Debussy experimented with extending them to work with less 'tonal'-sounding 'areas' than those of Liszt. […] [S]choenberg's practice merely amounts to extending the same methods to 'areas' determined by hexachords."[76] In each of these precedents, Lewin emphasizes that these areas have a local kinship because the objects contained within these areas sound similar; they belong to the same diatonic or modal collection, for example.

FIGURE 2.32. Row areas represent a "paradigmatic" relationship, binding together rows sharing hexachordal content.



But the structure of a composition is largely determined by the *order* in which those areas occur—how they are *organized* in relation to one another. "[T]he structure of [Liszt's] pieces is largely determined by the way in which he transposes one of these 'areas' into another. […]

[76] Lewin, "A Study of Hexachord Levels," 25.

Schoenberg's practice merely amounts to extending the same methods to 'areas' determined by hexachords."[77] In the Violin Fantasy, Lewin finds that the row-area order that gives rise to the structure of the piece is related to serial considerations within the row areas themselves.

Figure 2.32 shows what I mean, using Lewin's comments to organize Figure 2.31 into a spatial network. (This network is *my* interpretation of Lewin's analytical comments and is not found in Lewin's article.) Lewin locates two primary syntactical relationships, called the *9-relation* and the *5-relation*. These relationships emerge from the musical snippets I have shown at (a) and (b). About the *9-relation*, Lewin says, "[t]he 'modulation' from **A$_0$** to **A$_9$** is effected by enlarging the 3-note group [G,B♭,B] […] to the 4-note group [F♯, G, B♭, B] […], and then extracting from this 4-note group the 3-note group [F ♯,G,B♭] […]. Note its [G,B♭,B] preparation through mm. 15-16 (still within **A$_0$**). Then, at the moment of change of area (m. 21 1/2), it 'bridges' the two areas."[78] This 'bridge' is created by shared pc-content between specific locations in the rows that make up the row area. Similarly, the *5-relation* shown at (b) emerges from the shared trichord [E♭, F♯, D].

In Lewin's analysis, these two methods of row area organization create the composition's "structure."[79] Figure 2.34 shows Lewin's structural diagrams, which I have annotated to show how the spatial diagram in Figure 2.33 underlies Lewin's representational decisions. (The figure is essentially an event or, in Lewin's later terminology, figural network.) Lewin notes that a key part of the compositional "plan is to 'move through the diminished seventh chord of row areas'—referring to the motion from **A$_0$** through **A$_9$**, **A$_6$** and **A$_3$** in the first section.[80] The return to **A$_0$** at m. 143 prepares the third section of the piece, which begins with **A$_0$**, and it is similarly prepared by a movement through the diminished seventh of row areas containing **A$_0$**. The *9-relation* drives

[77] Ibid.

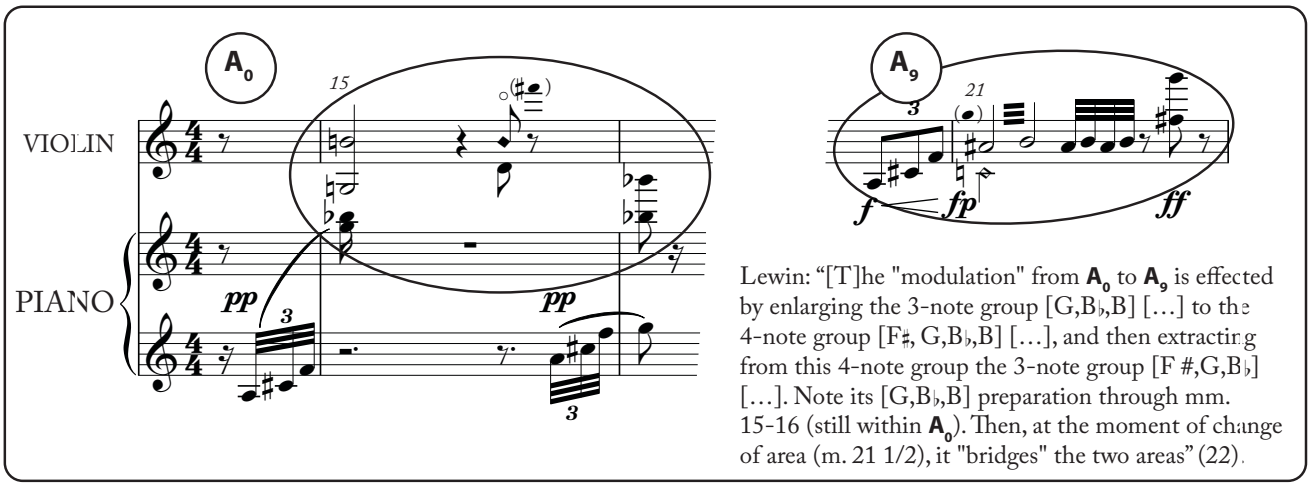
[78] Ibid., 22.

[79] Lewin is careful to indicate that the "structure" is not the same as the "form" of the composition.

[80] Ibid., 24.

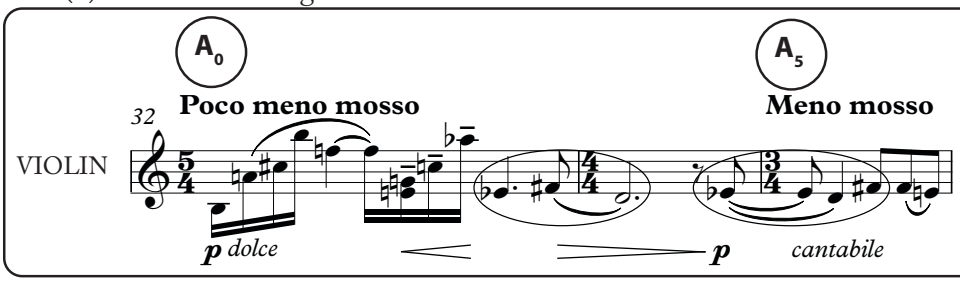
FIGURE 2.33. A spatial network constructed from Lewin's comments about the Violin Fantasy ("A Study of Hexachord Levels").

(a) Lewin describing a "*9-relation*"



Lewin: "[T]he "modulation" from $\mathbf{A_0}$ to $\mathbf{A_9}$ is effected by enlarging the 3-note group [G,B♭,B] […] to the 4-note group [F♯, G,B♭,B] […], and then extracting from this 4-note group the 3-note group [F #,G,B♭] […]. Note its [G,B♭,B] preparation through mm. 15-16 (still within $\mathbf{A_0}$). Then, at the moment of change of area (m. 21 1/2), it "bridges" the two areas" (22).

(b) Lewin describing a "5-relation"



Lewin: "[A]t m. 34 the change from $\mathbf{A_0}$ to $\mathbf{A_5}$ via the 3-note motive [E♭,F♯,D] is made very explicit" (26).

(c) spatial network re-constructed following comments above

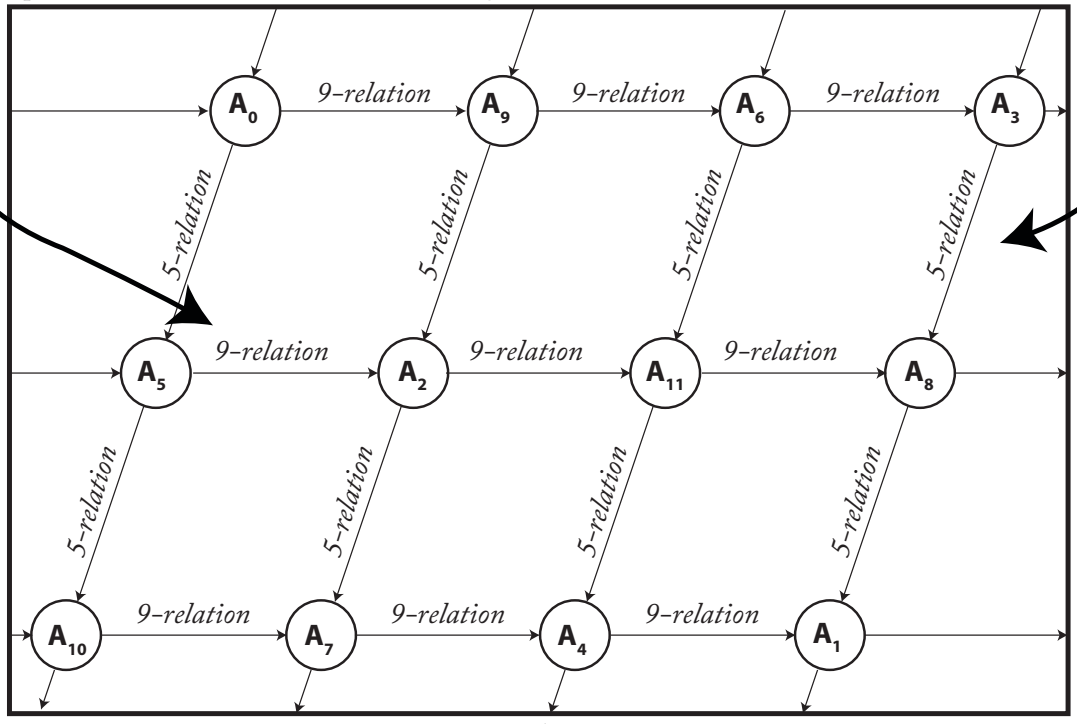
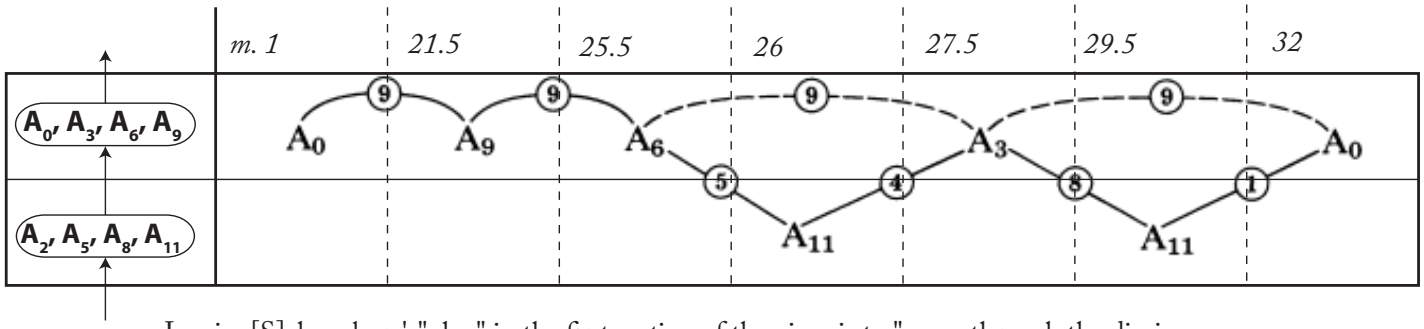
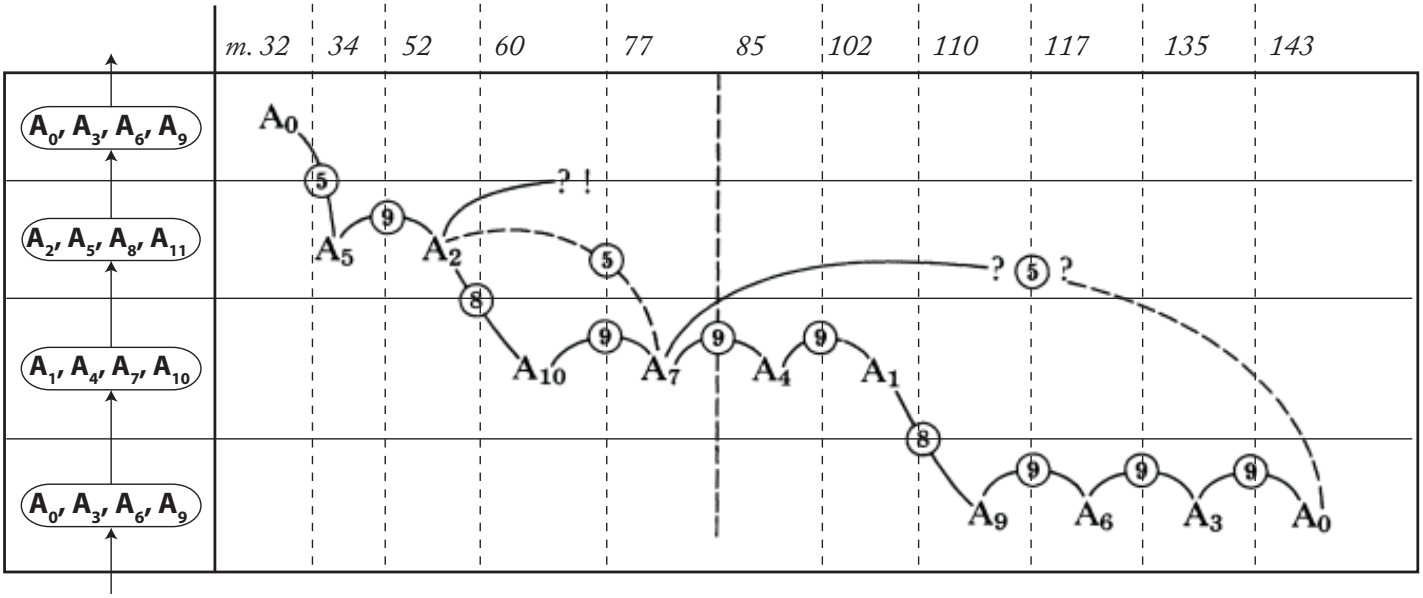


FIGURE 2.34. Lewin's event network of the first two sections of Schoenberg's Violin Fantasy with *my* annotations added.



Lewin: [S]choenberg's"plan" in the first section of the piece is to "move through the diminished seventh chord: $A_0 \rightarrow A_9 \rightarrow A_6 \rightarrow A_3 \rightarrow A_0$"(24).



these fundamental structural motions, and the *5-relation* is variously associated with "interruptions" (at m. 26, for example) and "modulations" between sections of the composition (at m. 34, for example).

Figures 2.32-34 do a good job of accentuating the different types of relationship involved in Schoenbergian combinatoriality. Row areas, themselves containing row forms, are abstract containers that stand for a particular type of relationship—like that shown in Figure 2.32. That relationship, a paradigmatic one, is fundamentally different from the syntactic relationships that

structure the space in Figure 2.33, and which form the basis for the structural diagram in Figure 2.34. Complicating things: however separate these relationships are, in practice they interact. Lewin shows, for example, that the syntactical relationships used in the Violin Fantasy have their basis in serial considerations; pc relationship contained with the row areas themselves are "bridges" to other row areas.

While the concept of a paradigmatic "row area" has near universal applicability over the span of Schoenberg's mature serial music, there is no sense of universal syntax. The situation in Webern's twelve-tone music is nearly opposite. While Webern explored a variety of paradigmatic arrangements—created by particular types of invariance, inversional axes, and so on—the syntax created by transformation chains is quite consistent throughout his serial music. Like Schoenberg's music, these syntactical routines are invariably related to those paradigmatic relationships. The following example explores how these relationships are connected in the first movement of the Piano Variations, and how the syntactical transformations are involved in the formal structure of the composition.

2.4.1. Example: Webern, Piano Variations, Op. 27, I

Earlier, in §2.2.2, I explored the A section (mm. 1–18) of the first movement of the Piano Variations. In that analysis I remarked upon the temporal structure of the section, showing precisely how the availability (or lack thereof) of a $RICH_1$ chain influenced the changing nature of the *dux* and *comes* in the four crab canons that make up the section. Owing to its crab canon structure, much of that passage's foundation is driven by pitch and temporal symmetries. Here, I am going to show how an organized spatial network can capture syntax in terms of invariance-driven paradigmatics. Thus, the analysis is meant to reveal some ways in which the composition reflects principles similar to those in Schoenberg's combinatorial pieces, especially the way in which the progression of row forms is related to the form of the movement. The brief analysis

shows that the row progression is an "amplification" of the temporal and pitch symmetries heard in the opening—an amplification that is, in spirit, like the recursive cyclic variations we saw in our earlier examination of the second movement.

From the perspective of melodic design, the movement is a clear ternary (ABA'), though there are significant problems in understanding *how* the final A section is a "recapitulation" of the first in a "tonal" sense.[81] Shown in Figures 2.35 and 2.36 the two A sections have the same canonic and rhythmic designs. Surrounding a B section that markedly quicker in tempo and rhythm, the final A section is readily identifiable as a "return" to the opening material. But, in a "tonal" sense this recapitulation is strange because it takes place at a new tonal level. While the opening of the piece begins at $\mathbf{P_{11}/R_{11}}$, the recapitulation is initiated from $\mathbf{P_3/R_3}$, and makes use of none of the row forms from the first A section. Further complicating matters, the final eighteen measures are not a simple transpositional adjustment of the opening, as one might expect in a recapitulation. Rather, Figure 2.36 shows that unlike the opening eighteen measures, which utilize only four row forms, the last eighteen measures use eight. Whereas the third canon of the first A section (m. 11 on Figure 2.35) returns to the pitch level of the opening—varying only the canonic and rhythmic aspects of mm. 1–10—the third canon of the final A section (m. 47 on Figure 2.36) begins at an entirely new pitch level. That underscores a final, puzzling dissimilarity. The first A section uses three $RICH_1$ chains to connect the four canons. The final A section, as Figure 2.36 shows, inserts a TCH_1 chain between the second and third canon.

The key to understanding how the final A section is a recapitulation is found in the many symmetries created by the canons. Dotted lines in Figure 2.35 show that there are two types of symmetry in the first A section. *Temporal symmetry* is created when the *dux* and *comes* exchange

[81] These issues are discussed in Bailey, *The Twelve-Note Music of Anton Webern*, 191-4. Boynton has used sketch evidence to probe the relationship between "variation" and sonata in this movement ("Some Remarks on Anton Webern's 'Variations, Op. 27'").

FIGURE 2.35. Many symmetries in the opening A section of the Piano Variations's first movement.

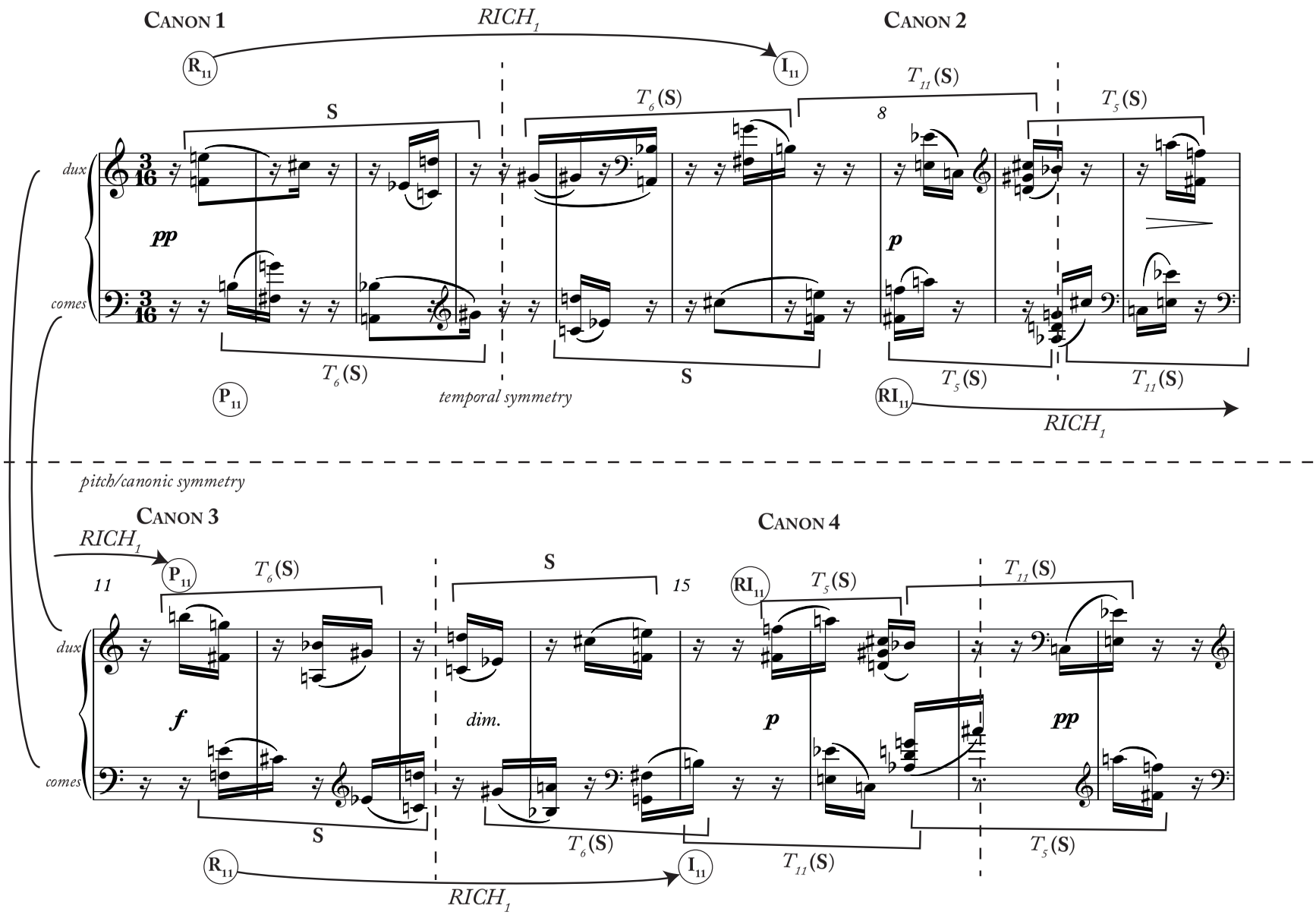
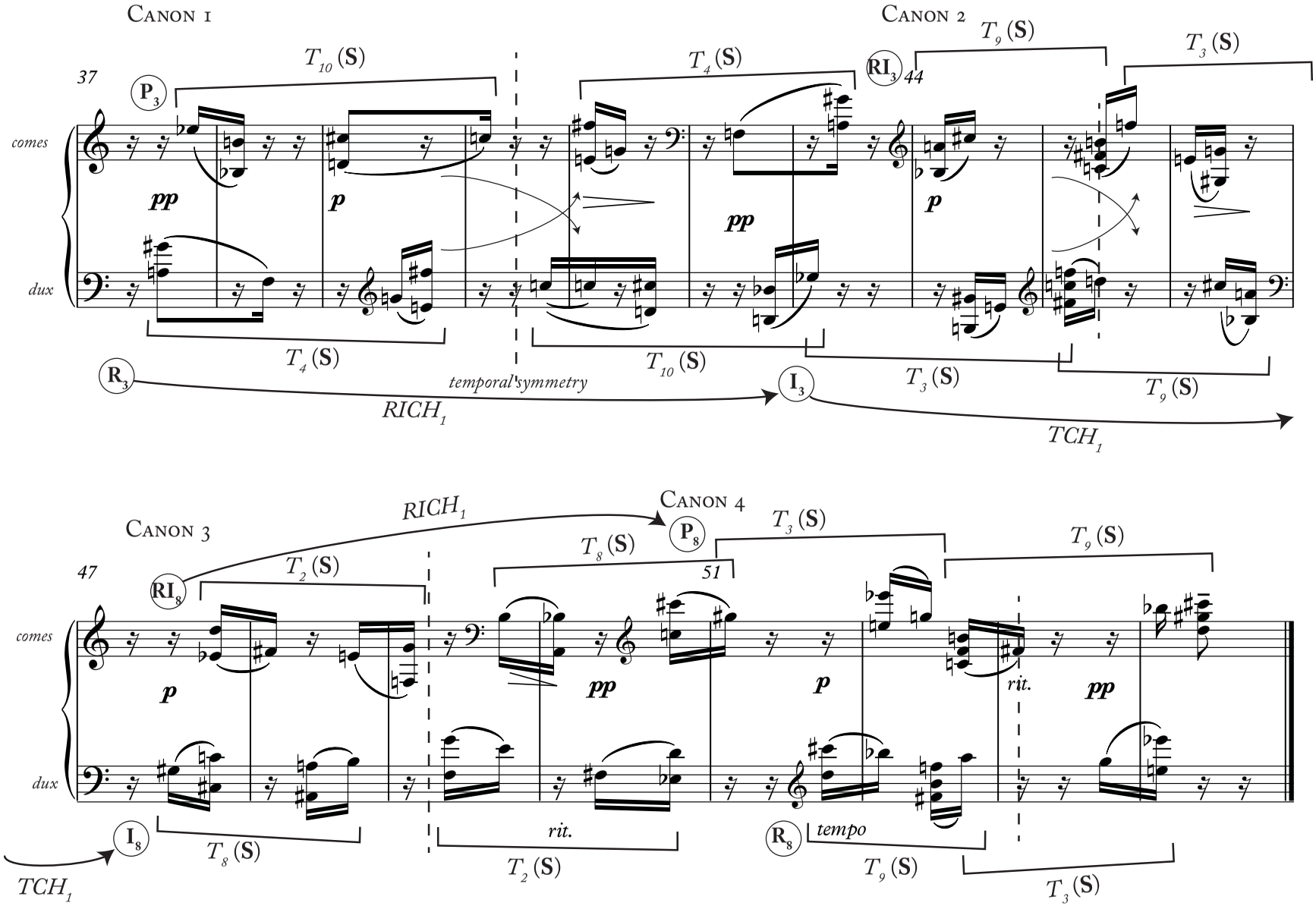


FIGURE 2.36. Recapitulation in the Piano Variations's first movement. Notice that though the canonic and rhythmic design is the same, the tonal level has changed as have the transformational actions.



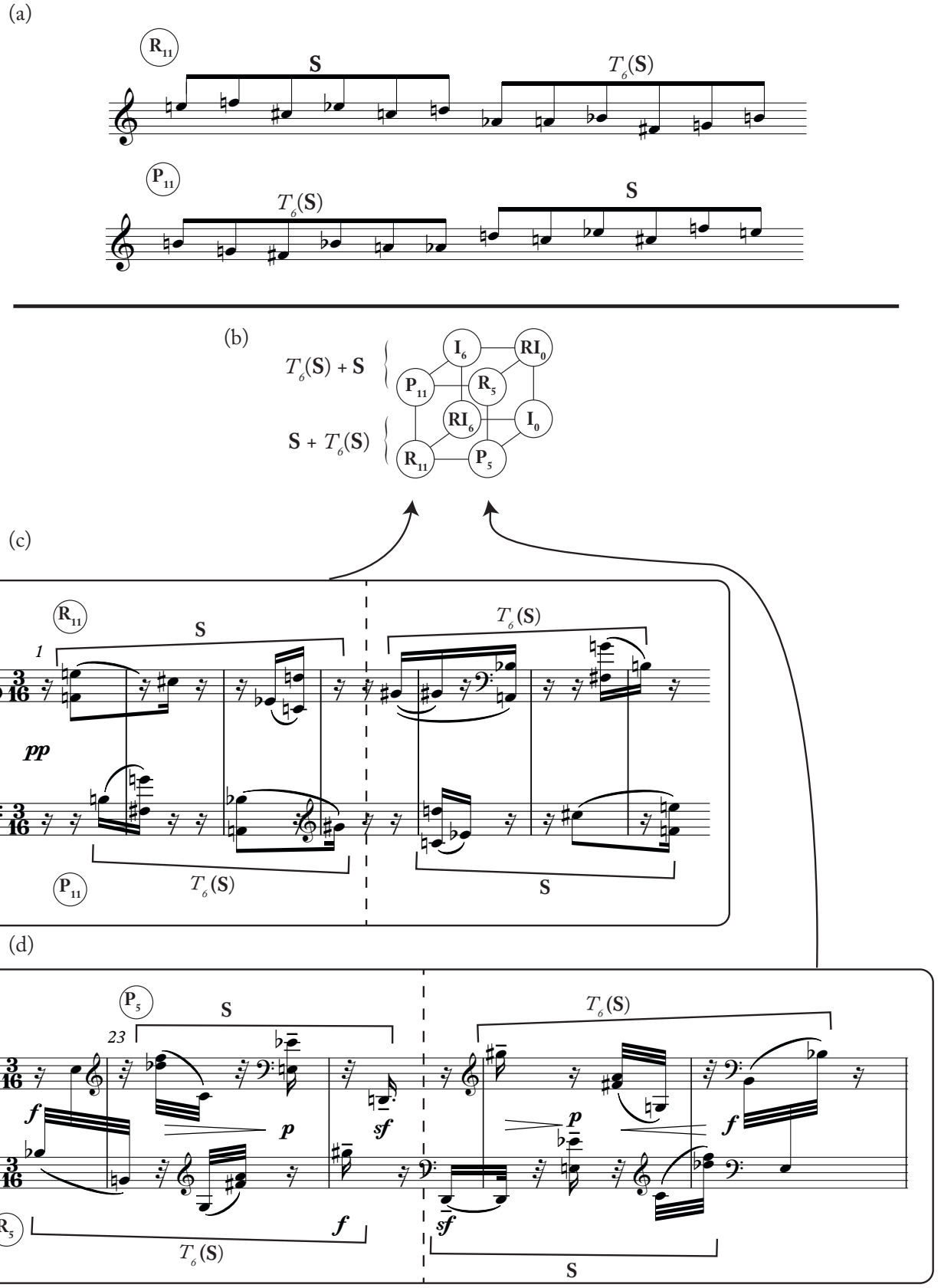
pitch and rhythmic ideas on either side of each of the midpoint of each of the four canons—for example, when, in canon 1, the two voices exchange music around the hinge of m. 4, beat 3. That temporal hinge is, in the first and third canon of each section, coincident with pitch symmetry. But in the second and fourth canons, the pitch aspect becomes slightly "unglued" from the rhythmic aspect of the canon—partly as a result of the transformation chains. Larger-scale *pitch* and *canonic* symmetry is created when the third and fourth canons recapitulate the first and second, but swap the voices canonic role. Figure 2.35 shows this with a horizontal line, indicating how at m. 11 $\mathbf{P_{11}}$ becomes the *dux* when it was the *comes* in m. 1. That change in canonic role is accompanied by balanced registral shifts in the two voices. At m. 11, $\mathbf{P_{11}}$ begins two octaves higher than in m. 1 while $\mathbf{R_{11}}$ begins one octave lower.[82]

It is the concurrent, retrograde-related rows that allow for the temporal symmetry that is involved in each of the crab canons. That relationship acts as the primary paradigmatic relationship throughout the movement, and suggests a broader way to understand row relationships. Figure 2.37 explores this. At (a), I have shown $\mathbf{R_{11}}$, the row of the first canon's *dux* voice, spliced into its two discrete hexachords. Because these are all-combinatorial, fully-chromatic hexachords, each maps to itself at multiple transformational values. The collection of row forms at (b), then, shows that eight row forms share the same discrete hexachords, and that those eight can be divided into two groups that play those hexachords in the same order. Thus, the paradigmatic relationship emblematic of the canons can be broadened: for example, $\mathbf{R_5}$ and $\mathbf{P_5}$ (shown on the right column of (b)) can be understood as representative of precisely the same paradigmatic relationship as $\mathbf{R_{11}}$ and $\mathbf{P_{11}}$.

Two passages at (c) and (d) demonstrate. The first, from the opening canon (cf. Figure 2.35), clearly demarcates the two T_6-related hexachords comprising the concurrent rows. The temporal hinge created by the crab canon helps us hear the hexachordal partitioning as those

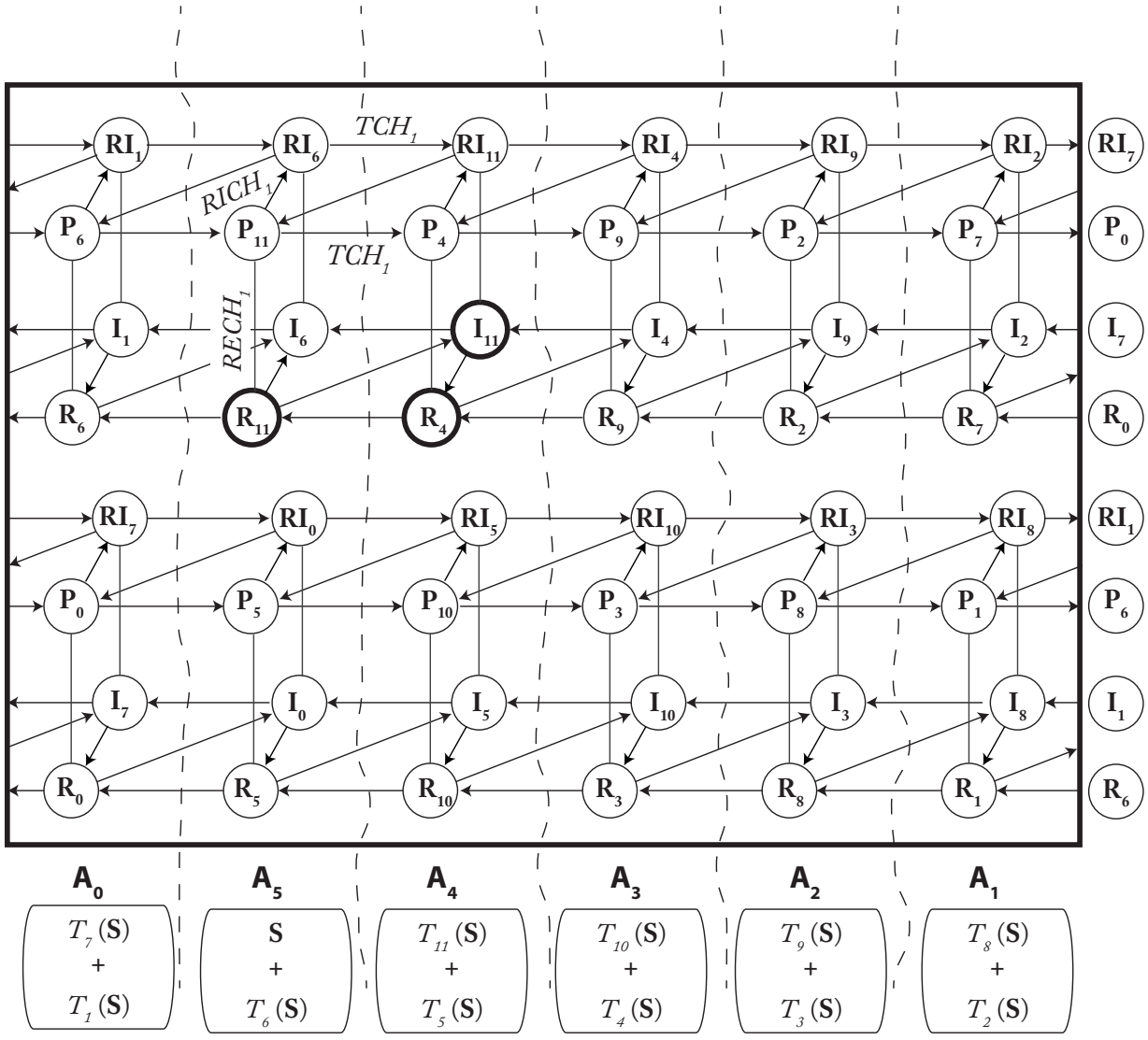
[82] There are other symmetries here, too. Dynamics in the two A sections are also palindromic, for example.

FIGURE 2.37. Shared discrete hexachords create paradigmatic relationships amongst eight row forms.



hexachords pivot around the dotted line on the figure. The second passage, shown at (d), from the contrasting B section, also contains two concurrent, retrograde-related rows. Like the opening canon, those rows pivot around the center of the passage. Most importantly, though these row forms are *not the same* as those at (c), the hexachords on either side of the temporal hinge are *the same hexachords*. Thus, in this sense, we can understand such related rows forms as paradigmatic substitutes for one another.

FIGURE 2.38. Chain-generated space organized by the paradigmatic invariance explored in Figure 2.37.



On Figure 2.38, I have used this paradigmatic relationship to organize a chain space generated by TCH_1, $RICH_1$, and $RECH_1$. This space shows that TCH_1's function is to move a row into an adjacent area. When applied to a **P** or **RI** form, TCH_1 moves a prevailing row area $\mathbf{A}_x$ to $\mathbf{A}_{x-1}$ (mod 6), and it does the opposite when applied to **I** or **R** forms. $RICH_1$ has the *same function* when applied to an **R** or **RI** form. But, when $RICH_1$ acts on a **P** or **I** form belonging to $\mathbf{A}_x$, $\mathbf{A}_x$ is maintained.[83]

Figure 2.39 uses this spatial network as the basis for an event network tracking the movement's syntactical motions in terms of this paradigmatic relationship. Foremost, this network clearly represents the tonal differences between the first A section and its recapitulation at m. 36 that I mentioned earlier. Not only do none of the rows in the recapitulation overlap with those in the first A section, but there is no row area overlap either. Moreover, the space shows the very different local trajectories of each passage in terms of the row area structure: while the opening A section moves down from $\mathbf{A_5}$ to $\mathbf{A_4}$, back to $\mathbf{A_5}$ and then down to $\mathbf{A_4}$ once again, the recapitulation begins at $\mathbf{A_3}$ and moves twice in the same direction, before a *compensatory movement* back to $\mathbf{A_2}$ ends the movement.

I have "reduced" the network at (b) to show how important this compensatory movement is in terms of the recapitulation. While on a local level, each of the A sections has a different tonal trajectory, the reduction shows that on a global level each of the passages does precisely the same thing: move "downwards" from $\mathbf{A}_x$ to $\mathbf{A}_{x-1}$.[84] This reduction also shows the overall tonal function of the contrasting middle section. Unlike the outer A sections, this passage begins and ends in

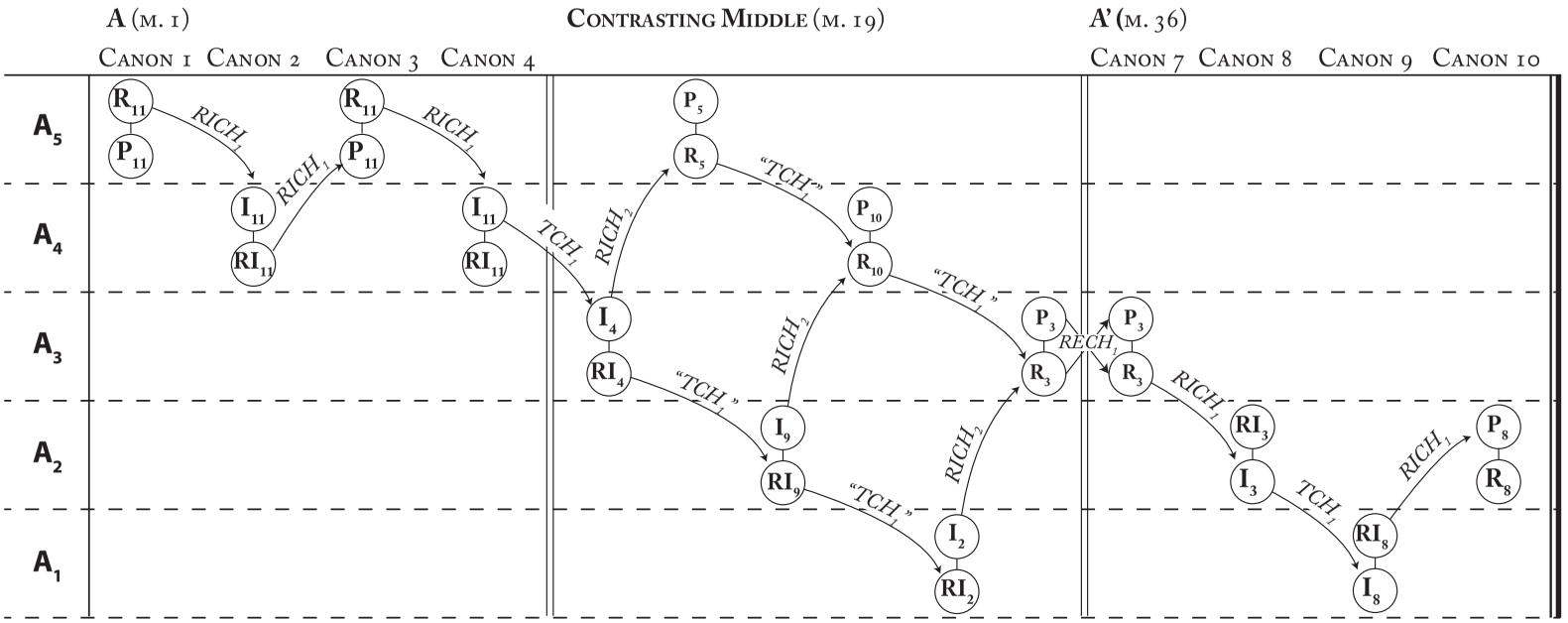
---

[83] This latter fact offers another reason that $RICH_1$ chains were "blocked" in the opening section. Notice on Figure 2.35 that, at m. 7, $\mathbf{R_{11}}$ $RICH_1$'s into $\mathbf{I_{11}}$ at the onset of the second canon. But as I have noted, $\mathbf{I_{11}}$ does not $RICH_1$ into the next row to begin canon 3. Figure 2.38 shows that while $\mathbf{R_{11}}$ coincides with an area movement that transposes the discrete hexachords by T_{11}, $RICH_1$ing $\mathbf{I_{11}}$ would have *maintained* that area. Thus, while the initial $RICH_1$ creates "tonal" motion, $RICH_1$ing $\mathbf{I_{11}}$ would have created "tonal" stasis.

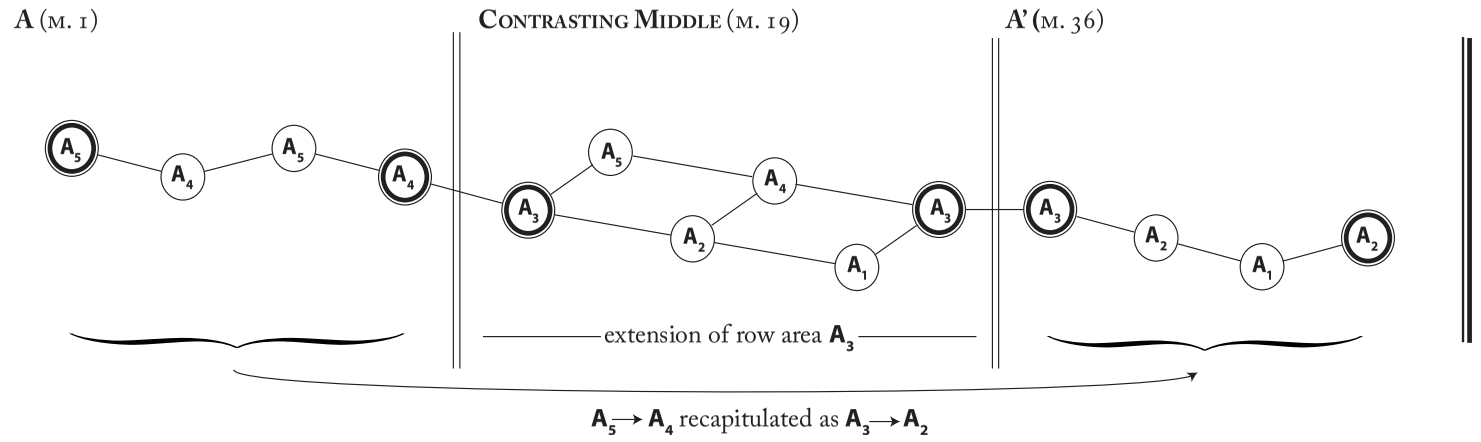
[84] Note that this network also captures the sense in which the recapitulation is a tonal variation of the first A section. Both have the same collection of spatial trajectories: two movement's "down" and one "up." But each of the passages deploys that collection of trajectories in unique ways. This seems related to the composition's title, which has been the subject of much debate.

FIGURE 2.39. An event network showing syntactical actions conditioned by the paradigmatic structuring of Figure 2.38.

(a)



(b) Row Area Reduction of (a)



the same location: row area **A3**. That extension is present despite the fact that the rows that open the contrasting middle are different from those that end it. In the sense of the paradigmatic relationship that links rows's discrete hexachords, however, those rows are "the same."

While it illuminates how the final A section recapitulates the "tonal motion" of the first A section, this figure still does not seem to explain why the final A section takes place at a different tonal level. Figure 2.40 fills in the important gaps. There, notice the means by which the contrasting middle section extends **A3**. The rows in the midst of this expansion have symmetrical partners along the temporal axis of the section. That is, **A5** is echoed by **A1** and **A2** by **A4**, all around the midpoint of the contrasting middle.[85] The symmetrical echoes are the key to understanding how the final A section recapitulates the first. On Figure 2.40(a), notice that the compensating motion from **A1** to **A2**, which closes the piece, is the symmetrical counterpart to the **A5**-**A4** motion that ended the first A section. Even more, it is symmetrical around precisely the same row area (**A3**) as was the contrasting middle.[86]

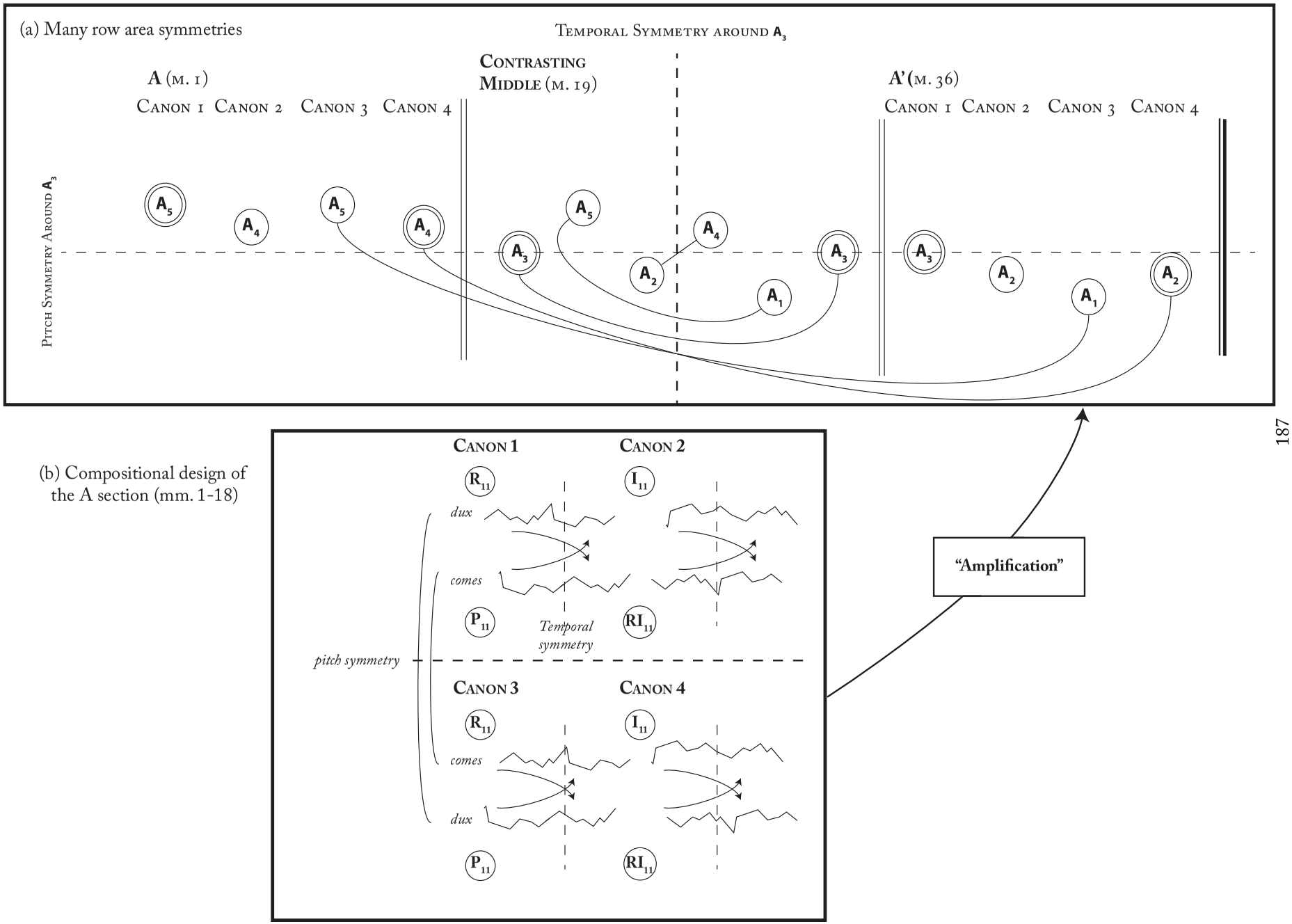
Figure 2.40(b) uses a compositional design to relate this to the many symmetries we saw in the A sections. It shows how the temporal symmetry in the movement's canons are "amplified" onto the movement's form. In this amplification, the center of the movement acts as a hinge around which the row areas pivot, just as the hinges in each of the canons functioned as temporal locations around which the rhythmic motives (and hexachordal content of the rows) pivoted. Figure 2.40(b) also suggests that the pitch symmetry, which was created in m. 11 when **P11** and **R11** swapped registral locations, is a smaller-scale echo of the large formal symmetry around **A3**.

---

[85] The contrasting middle has two sections, and the motion to **A4** occurs just as the final section of the contrasting middle begins at m. 30.

[86] I will not explore this further here, but **A3** contains row forms that many believe to be the "tonic" of the piece as a whole. The members of **A3** include row forms that function as "tonics" in the second and third movement. In the second movement, the I_6 relation is, at the beginning of the piece, presented between **R3** and **RI3**, and the final "imaginary" TCH_1 chain would have led to those rows again. **R3**, is, of course a member of **A3** in the first movement. And finally, in the third movement, the opening theme begins and ends with **P3** and **R3**, two members of **A3** that are subsequently recapitulated at the movement's close.

FIGURE 2.40. A reduction of the event network showing an amplification of the opening symmetries onto the larger row-area progression of the movement.



2.4.2 Transformational Character

One byproduct of understanding transformation chains in the context of some paradigmatically defined relationship is that it shows that different transformations may nonetheless have similar functions. We saw this on Figure 2.39, for example, where TCH_1 and $RICH_1$ often had the same spatial signature, though they obviously transformed rows into different forms.[87]

In general, in an organized space a transformation will have one of two broad functions. Given some row in row area $\mathbf{A}_x$, a *static* transformation chain transforms the row into a row still within $\mathbf{A}_x$, thereby maintaining the spatial "status quo." Transformations that guide a row form *away* from a row area are *progressive*. Thus, a *static* chain makes a row into something much like itself, at least in terms of the paradigmatic relationships that organize the space; it has the character of stasis or prolongation. *Progressive* chains make a row into something very different than itself; they have the character of modulation.

Lewin's analysis of the Violin Fantasy, referred to at the opening of this section, relied on these distinctions in his discussion of *9-* and *5-relations*.[88] And Lewin has elsewhere used the terms "internal" and "progressive" to describe the character of a transformation—specifically in his analysis of right and left hand chords in Schoenberg's Piano Piece, Op. 19, No. 6. Lewin says " 'internal' transformations make a thing […] very like itself; […] 'progressive' transformations make an earlier thing […] very like a later, different thing."[89] He later used the terms "internal,"

---

[87] Organized spaces, therefore, remedy a perceptual difficulty posed by transformation chains—namely, they are not distinctly audible phenomena. Incorporated into an organized space, however, a chain *may* become tied to a prominent musical signature. The effect is could be likened to the introduction of chromaticism into an otherwise diatonic passage. Though we may not have heard a complete change in diatonic collection (e.g., all of the new notes in the collection), the chromaticism is a *signal* for such a change.

[88] Lewin notes the "modulatory" effect of *9-relations* in opposition to the *5-relation*, wherein hexachords have "five notes in common," and are "of maximally closeness to each other in sonority ("A Study of Hexachord Levels," 26-7).

[89] David Lewin, "Transformational Techniques in Atonal and Other Music Theories," *Perspectives of New Music* 21, no. 1/2 (October 1, 1982): 343.

which I call "static," and "progressive" in a more formal sense in connection with the "Injection Function" (INJ). In *GMIT*, Lewin defines the function as follows: "[g]iven sets X and Y, given a transformation f on S, then the injection number of X into Y for f, denoted INJ(X, Y)(f), is the number of elements s in X such that f(s) is a member of Y."[90] Put most simply, the higher the injection number, the more *progressive* a transformation. These terms describe, quite wonderfully, the *character* of a given transformation in a particular musical context, and thus describe something of its function. Lewin says: applied to an object X "an [internal] transformation tends to extend/elaborate/ develop/prolong X in the music, while a progressive transformation tends to urge X onwards, to become something else."[91]

Organized spatial networks show the relative progressivity of a particular transformation in these terms. Returning to my analysis of the Piano Variations's first movement, the event network in Figure 2.39(a), which was organized by hexachordal commonality, indicates that the *only* static transformation occurs at the end of the contrasting middle, where $\mathbf{A_3}$ is extended into the recapitulation, as preparation for it. The scarcity of static transformations is noteworthy. The spatial network in Figure 2.38 shows that static chains are not rare. In fact, $RICH_1$ applied to a **P** or **R** form maintains that rows's area. Numerous $RICH_1$ chains are found in the movement, but Webern completely avoids using $RICH_1$ of a **P** or **R** forms, and therefore, $RICH_1$ is *never static*. More generally, Webern's avoidance of static chains is indicative both of the general tendency to avoid stasis in the movement. The preparation of $\mathbf{A_3}$ at the end of the B section, then, underscores its role as the central row area in the movement.

These character distinctions are interesting ways to view the three sections of the Piano Variations movement just discussed. The two A sections, while differing somewhat in their transformational details, each project progressive transformations that have the same spatial

---

[90] Lewin, *GMIT*, 124, Definition 6.2.1.

[91] Ibid., 142.

trajectory: every transformation in each of these sections moves the music "downward"adjacent row area. Note that the use of *TCH₁* midway through the recapitulation nonetheless creates the *same* transformational character as the the *RICH₁* chains surrounding it. Moreover, the *larger* character of both sections is the same. Both are tonally progressive to the same degree: the first A section moves from $\mathbf{A_5}$ to $\mathbf{A_4}$ and the final A section from $\mathbf{A_3}$ to $\mathbf{A_2}$.

In these terms, the two A sections contrast with the contrasting middle. Musically, this section features a dramatic change in character, and that change is reflected tonally. The three *RICH₂* transformations that characterize the passage are, in relation to the outer A sections, highly progressive. While the A sections are confined to relatively small portions of the space, the B section occupies five of the space's six row areas. Paradoxically, this highly progressive passage is, as a whole, completely static. The three *RICH₂* chains, which are initiated from successively "lower" row areas, cancel one another out. Thus despite its general progressivity, the music in this section is static overall, beginning and ending with row area $\mathbf{A_3}$.

2.4.2 The 'Horizontal' and 'Vertical'

To a degree, these differences in character help to capture the richness of Webern's engagement with "the horizontal and vertical." These terms occur often in Webern's writings and in the writings of those in the compositional circle around him.[92] In the *Path to the New Music*, Webern often refers to the terms in conjunction with his discussions of the "presentation of musical ideas." He generally distinguishes between two modes of presentation. Polyphony, which reached its zenith in the music of the "Netherlanders," demonstrates complete unity in the the

[92] This interesting topic is the subject of two articles by Regina Busch. These studies explore the idea of "musical space," its relationship to Webern's usage of the terms "horizontal" and "vertical," and the ways that Webern's understanding of those terms were conditioned by others in Schoenberg's circle. See Regina Busch, "On the Horizontal and Vertical Presentation of Musical Ideas and on Musical Space (I)," *Tempo* no. 154 (1985): 2–10; Regina Busch and Michael Graubart, "On the Horizontal and Vertical Presentation of Musical Ideas and on Musical Space (II)," *Tempo* no. 156 (1986): 7–15.

horizontal dimension through its use of canon and imitation.[93] Homophony, the other mode of presentation, has origins in Monteverdi's music, opera, and most of all, with popular dance forms. Webern says that this mode of presentation was characterized by a melody and accompaniment. In early homophony, the accompaniment was hierarchically subsidiary.

In Webern's telling, in the nineteenth-century—and beginning with Beethoven— "the function of the accompaniment struck out along a new path":

> The accompaniment's supplement to the sing-line main part became steadily more important, there was a transformation, quite gradual and without any important divisions, stemming from the urge to discover ever more unity in the accompaniment to the main idea—that is, to achieve ever firmer and closer unifying links between the principal melody and the accompaniment.[94]

In Webern's telling, it is the urge to create unity between melody and accompaniment that resulted in a return to polyphony: the two methods of presentation "inter-penetrated to an ever increasing degree." And, as Webern says, "the final result of these tendencies is the music of our time."[95]

Throughout the *Path* and elsewhere, Webern relates this "inter-penetration" to Schoenberg's method of twelve-tone composition. In an analysis of his own String Quartet, Op. 28, Webern says that the "work must be the 'crowning fulfillment,' so to speak, of the '*synthesis*' of the '*horizontal*' and *vertical*' construction (Schoenberg!)."[96] Apart from his invocation of Schoenberg's name, he goes on to say: "as is known, the classical cyclic forms—sonata, symphony, and so forth—evolved on the basis of the [the vertical mode], while '*polyphony*' and its associated practices (canon, fugue, and so on) derived from the [horizontal mode]. And now, here I have attempted not only to comply with the principles of both styles in general, but also specifically to

---

[93] Webern: "That's the strongest unity is when everybody does the same, as with the Netherlanders" (*The Path to the New Music*, ed. Willi Reich, trans. Leo Black (Bryn Maw, PA: Theodore Presser, 1963): 35.

[94] Ibid., 21.

[95] Ibid.

[96] Hans Moldenhauer and Rosaleen Moldenhauer, *Anton von Webern, a Chronicle of His Life and Work* (New York: Knopf, 1979): 753.

combine the forms themselves."[97] Webern's expressed goal—which also functions as an historical precedent for "the new music"—is to integrate the horizontal mode associated with imitative polyphony and the vertical mode associated with sonata and symphonies. Thus, the existence of music that has one foot in the world of classical form, another in the world of polyphony, but both working towards a similar goal—unity.

From the perspective we are exploring, polyphony, then, has two roles. First, it creates horizontal unity. But second, and most interestingly, the polyphonic web itself creates a *vertical* mode of presentation—"tonal blocks of polyphony" that are capable of interacting with form in the same way that "tonal areas" do in classical sonatas and symphony. The richness of Webern's integration comes in the way that blocks of polyphony are connected via transformation chains that have the static character of tonal stasis or of progressive modulation.

2.4.5 Analytical Vignette: Webern, String Quartet, Op. 28, I

In the Piano Variations, chains were primarily associated with tonal motion, In the first movement of the String Quartet, Op. 28—and the second, for that matter—chains are linked primarily with stasis, with non-chain based motions responsible for most of the tonal motion in the piece.

Webern wrote a self analysis of the opening movement his String Quartet, Op. 28 in which he identified a formal combination of "variations" and "adagio-form":

> The first movement is a *variation movement*; however, the fact that the variations also constitute an *adagio form* is of *primary* significance. That is to say, it is the basis of the movement's formal structure, and the variations have come into being *in accordance* with it. Thus, the shaping of an adagio form on the basis of variations.[98]

[97] Ibid.

[98] Ibid., 752.

Webern's indication of the movement's "adagio form" is a *vertical* concern.[99] *Horizontally,* the movement is also two-voice canon, as the analytical diagram in Figure 2.41 illustrates.[100] In most of these variations, each of the two canon voices generally completes a *cycle* of row forms, such that cyclic completion becomes a marker of formal completion on the smaller levels of the movement's form.[101]

Formal combination interested Webern greatly. The Piano Variations, Op. 27, composed just prior to the Sting Quartet, are a kind of suite whose constituent movements (a three-part "andante form," a two-part "scherzo") are also variations. Neal Boynton has discussed how the Orchestral Variations, Op. 30, composed just after the String Quartet, are a combination of variations and adagio form.[102] Variations are ideal for such combination because, as Boynton notes in his study of Op. 27, "[t]he conditions for a variation set do not presuppose a particular shape for the work or movement as a whole, variations are, so to speak, the formless form."[103] The amorphousness of variations, then, invite the superposition of another structure, an "adagio-form," for example, as we find in the first movement of the String Quartet. Boynton says that in the context of a variation, such superpositions "set the boundaries of the whole series of variations, to indicate the closure of the set, or at least to offer something that contributes to the closure of the set, more than simply stopping at the end of the last variation."[104]

---

[99] For Webern, "adagio-form" would signify a three-part form with a contrasting middle section. Webern also referred to three-part structures as "andante-forms."

[100] Many of my findings are also found in Hook and Douthett, "UTTs and Webern's Twelve-tone Music," 110-19. Hook and Douthett, though, take a somewhat different approach to generating the spatial and event networks that structure their analysis.

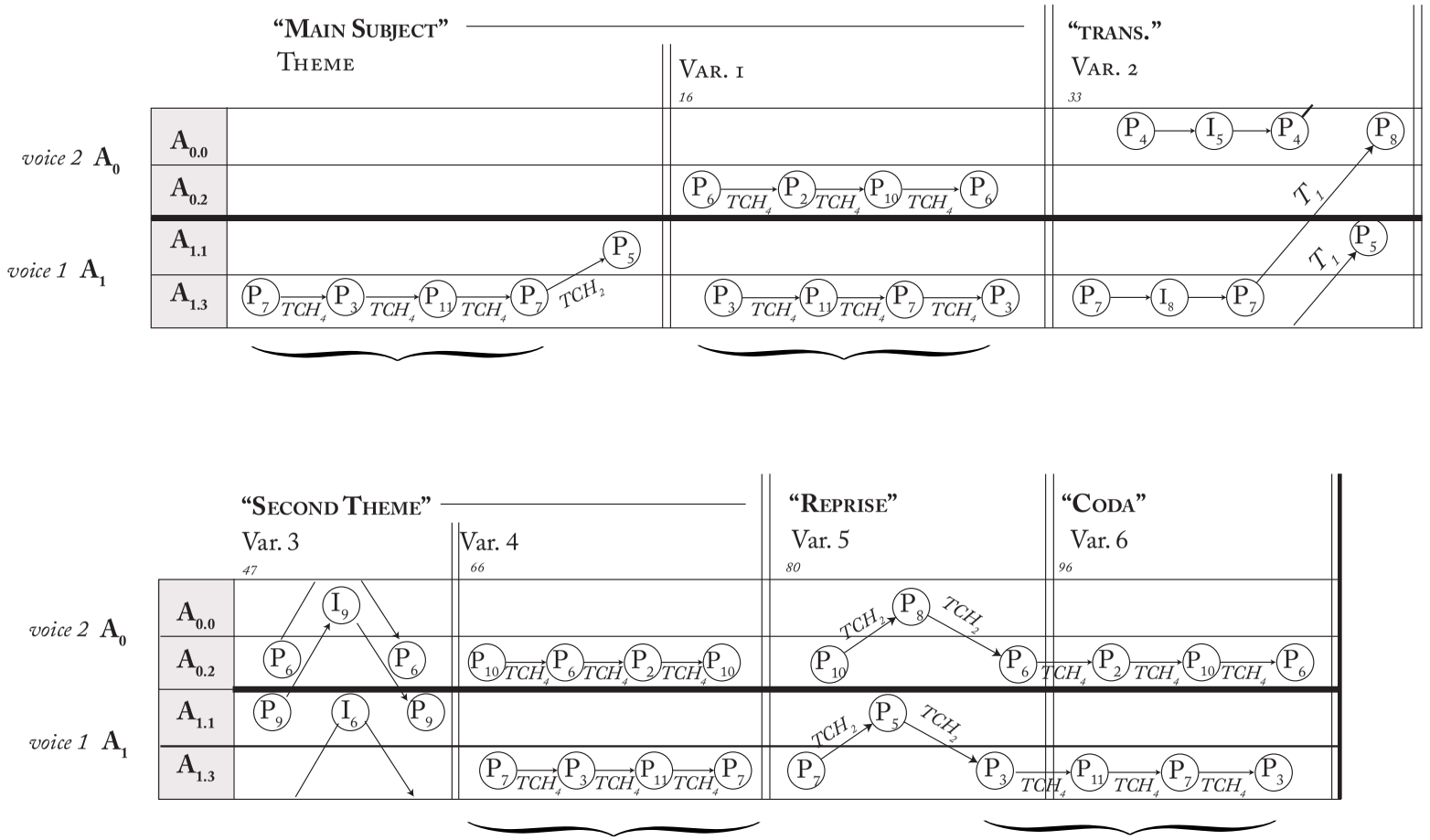
[101] "Cyclic composition" of this sort appears in the String Quartet for the first time, and also occupies Webern's next work, the Cantata I, which I discuss in Chapter 5. It is interesting to consider the degree to which such cycles are also nascent in the second movement of Op. 27, which I discussed §2.3.7, which Webern composed immediately before the String Quartet.

[102] Neil Boynton, "Formal Combination in Webern's Variations Op. 30," *Music Analysis* 14, no. 2/3 (1995): 193–220.

[103] Boynton, Some Remarks on Anton Webern's 'Variations, Op. 27,' 201.

[104] Ibid., 201-2.

FIGURE 2.41. A formal diagram and event network for the String Quartet, Op. 28, I.



In this movement, closure occurs locally, through the completion of transformation cycles, and globally, when a set of row forms return at the head of a section.[105] Returns of the latter type create paradigmatic associations that impose vertical, formal structure onto the horizontal unfolding of the canon, which is coincident with the smaller-scale chain cycles. In Figure 2.41 I have superimposed Webern's "adagio" analysis above the six variations to show how they interact.[106] Observe, for example, that the main subject's reprise in variations five and six returns to the row progression that characterized it in variations one and two (the "main theme") and that the contrasting second theme clearly involves a new set of rows.

Figure 2.41 is an event network derived from a spatial network generated by TCH_4 and TCH_2 (the two primary syntactic transformations in this movement) and organized by two important invariance relationships, one stemming from chromatic dyadic structure and the other from "BACH" tetrachords. The theme (shown in Figure 2.42) contains one complete TCH_4-cycle followed by a single TCH_2. In its presentation of melodic material, the theme is quite clear—semi-tonal dyads predominate, with those dyads combining instrumentally and registrally to form tetrachords. Dyads are distinguished primarily by instrument: over mm. 1-2, for example, two dyads—{G3, F♯5} and {A4, G♯5}—are sounded in the viola and first violin that together create the tetrachord {G3, F♯5, A4, G♯5}. Each of the tetrachords in the passage are ordered transformations of the "BACH" tetrachord, {B♭, A, C, B♮}. Our association of dyads—those that create BACH tetrachords—is helped by contour. Both {G3, F♯5} and {A4, G♯5}, for example, are ascending dyads that contrast with the descending dyads that follow in the other instruments.

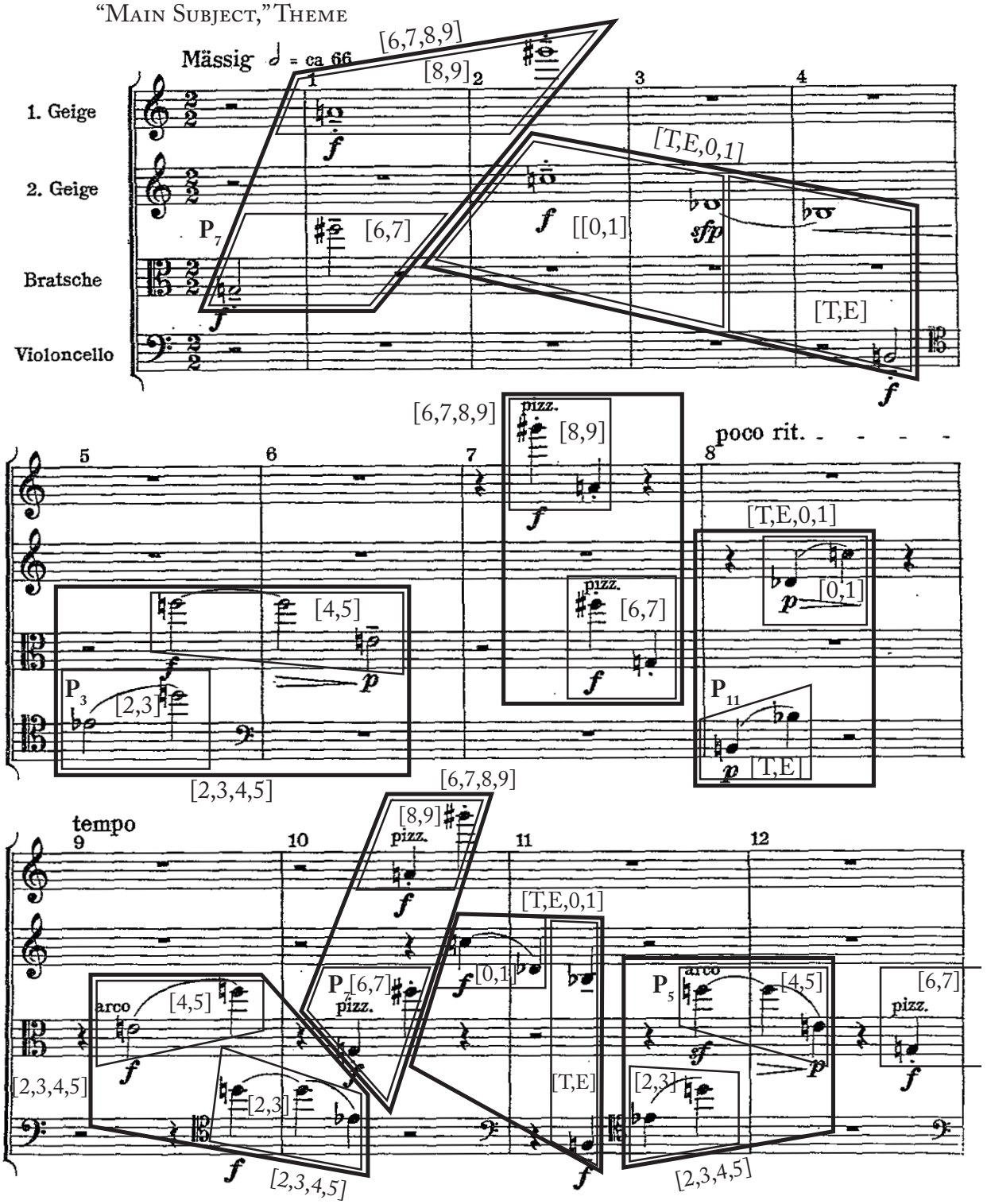
Throughout the entirety of the theme, only six unique semi-tonal dyads are sounded. In fact, the situation is strikingly reminiscent of the partition scheme that predominate in the final

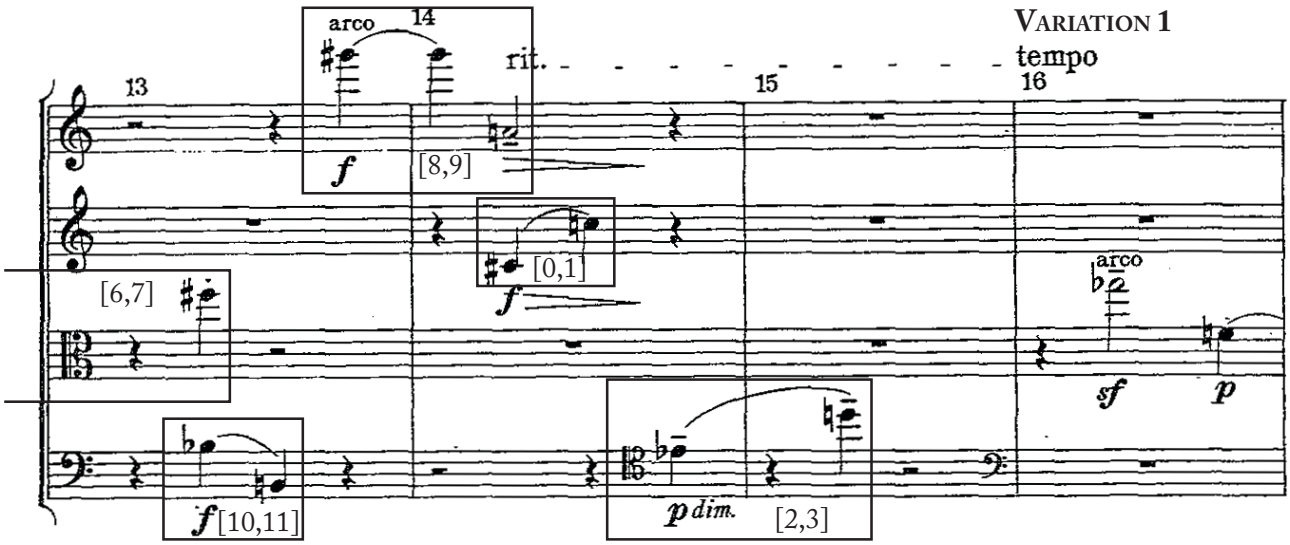
---

[105] In this context, Webern's comment in a 1932 lecture is pertinent: "The original form and pitch of the row occupy a position akin to the 'main key' in earlier music; the recapitulation will naturally return to it" (Webern, *The Path to the New Music*, 54).

[106] Webern's analysis of this work was written in 1939. It was translated by Zoltan Roman and published in Moldenhauer and Moldenhauer, *Webern, a Chronicle*, 751-6.

Figure 2.42. The String Quartet's "theme," segmented into chromatic dyads and BACH tetrachords.





movement of Piano Variations, Op. 27 that we studied in §2.3.8. The six semi-tonal dyads in the theme all share the same whole-tone "root": [0,1], [2,3], [4,5], [6,7], [8,9], and [T,E]. These, and only these, dyads sound throughout the remainder of the theme—even after the first TCH_4 cycle's completion at m. 11. Their salience is largely the result of register and instrumentation. Each of the six dyads recur in the same register and (mostly) in the same instrument. As the primary durational values double at m. 7 (and as the four members of the quartet interject more frequently from m. 10ff), tetrachordal segments recede in importance and dyadic segments increase in importance.[107] Entirely tetrachordal segmentation does not return again until m. 47—at the onset of the more lyrical passage that Webern called the "second theme."

In Figure 2.44 I have shown three row forms segmented into discrete dyads and tetrachords, following the primary partitioning in the main subject and second them. All of the rows shown there share the same set of six dyads when divided evenly into their six adjacent order positions. Because this row class has only twenty-four unique members, this relationship, which I will call *WT*, partitions the row class into two equivalence classes of twelve rows each. Like we saw in Op. 27, members of this class have semi-tonal dyads sharing the same whole-tone

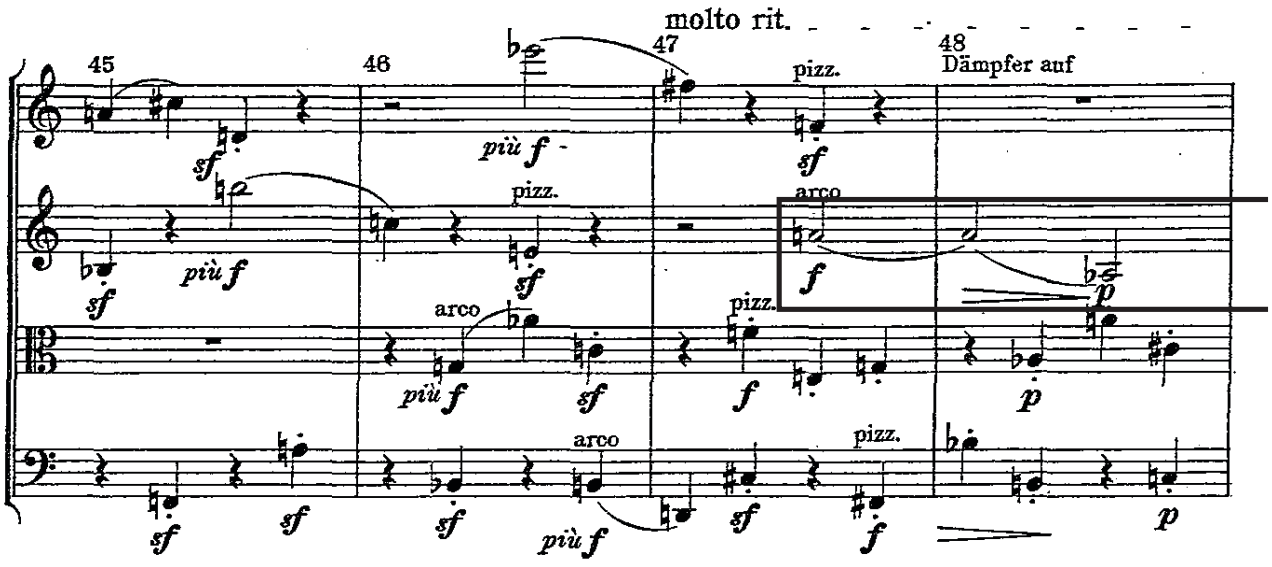
[107] I emphasize the importance of *dyads* here, rather than tetrachords, primarily because most analyses of the quartet's first movement begin from a *tetrachordal* perspective. For, example: Bailey, *The Twelve-Note Music of Anton Webern,* 215-22, and Hook and Douthett, "UTTs and Webern's Twelve-tone Music," 110-19.

root. The theme alone cycles through five members of this class. A second, coarser relationship, called *BACH*, is created by the association of BACH tetrachords. Figure 2.44 shows that $\mathbf{P_7}$ and $\mathbf{P_3}$ share BACH tetrachords, but $\mathbf{P_5}$ does not. The *BACH-relation* creates four equivalence classes of six rows each. Row forms that are in the *BACH-relation* are always in the *WT-relation*, but the converse is not always true.

My formal diagram in Figure 2.41 revealed the pervasiveness of TCH_4 and TCH_2 as syntactic transformations in the movement. Figure 2.45(a) shows how the group $\langle TCH_4, TCH_2 \rangle$ partitions the row class into four collections containing six rows each. (In this group, TCH_4 is a "redundant" generator because $(TCH_2)^2 = TCH_4$.)[108] In Figure 2.45(b) I have organized the space by the two invariance relationships *WT* and *BACH*. As a finer relation, *WT* creates the two large row areas that I call $\mathbf{A_0}$ and $\mathbf{A_1}$. If a row is in $\mathbf{A_0}$ its adjacent dyads have "even" whole-tone roots. Because rows in the *BACH* relation are also in the *WT* relation, Figure 2.45(b) names four

FIGURE 2.43. The String Quartet's "second theme," beginning with the second violin in m. 47, segmented into BACH tetrachords.

"SECOND THEME" (3RD VARIATION)



[108] Figure 2.45(a) shows the same four orbits as in Hook and Douthett, "UTTs and Webern's Twelve-tone Music," 115, Example 11(c). In their analysis, my TCH_4 is a $UTT\ U = \langle -, 4, 4 \rangle$ and TCH_2 is a "schritt" $S_2 = (+, 2, 10)$.
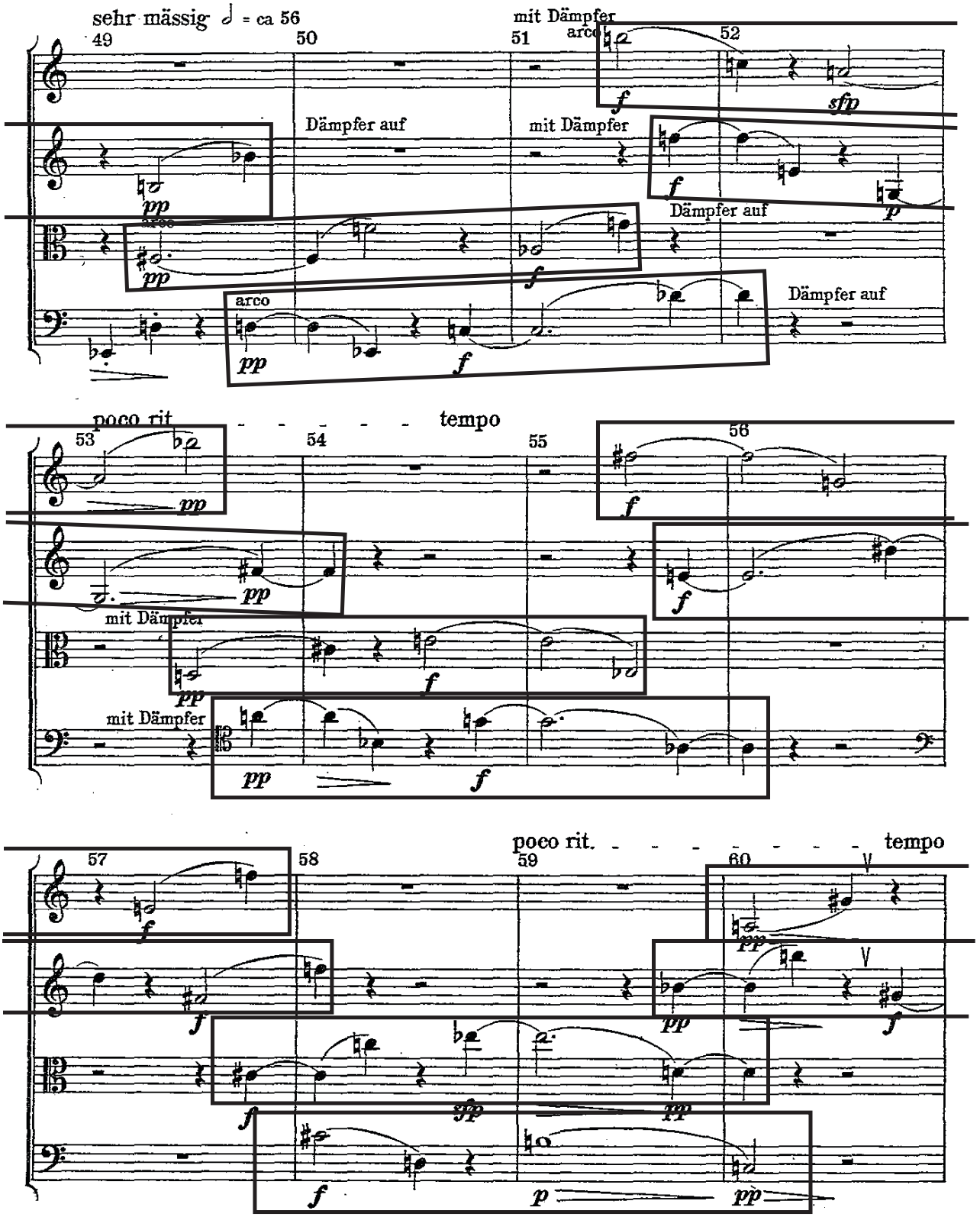

sehr mässig ♩ = ca 56
49
50
51
52
mit Dämpfer
arco
Dämpfer auf
mit Dämpfer
Dämpfer auf
arco
Dämpfer auf
poco rit
tempo
53
54
55
56
mit Dämpfer
mit Dämpfer
poco rit.
tempo
57
58
59
60

FIGURE 2.44. A dyadic and tetrachordal row segmentation.

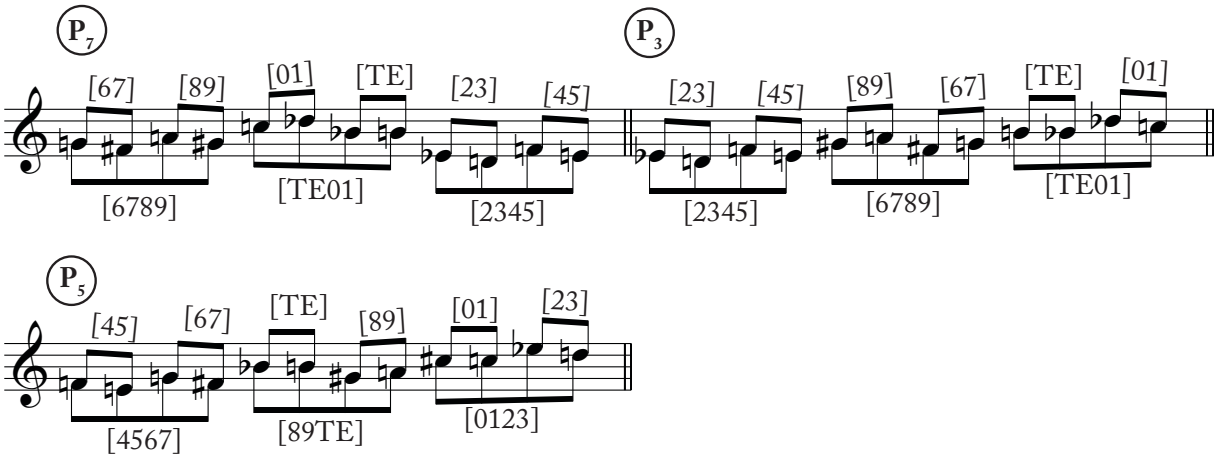
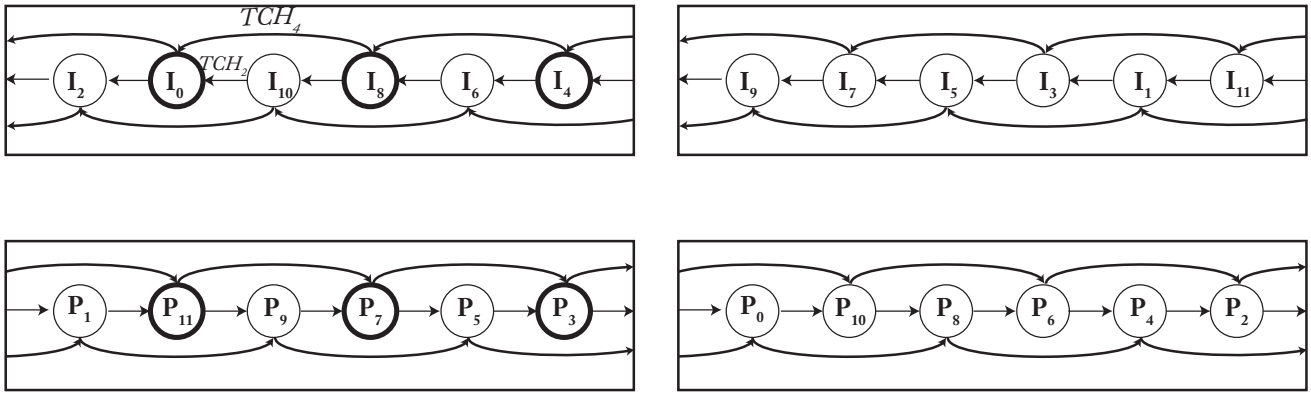
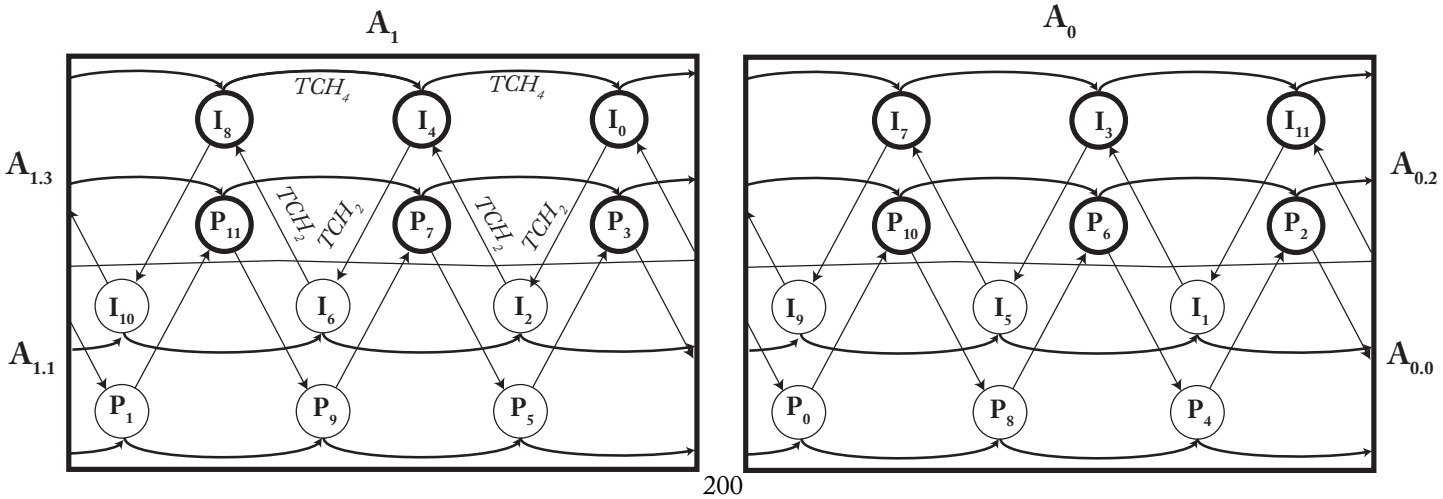


FIGURE 2.45. Spatial networks for the String Quartet generated by *TCH4* and *TCH2* and organized by segmental invariance.

(a) The two chains partition the twenty-four unique rows into four groups.



(b) The chain-generated networks at (a) organized by the two invariance relationships.



less inclusive row areas $\mathbf{A_{0.0}}$, $\mathbf{A_{0.2}}$, $\mathbf{A_{1.1}}$, and $\mathbf{A_{1.3}}$.

Essentially, Figure 2.45(b) "pulls-apart" the four groups in Figure 2.45(a) to reveal how the two transformation chains interact with dyadic and tetrachordal invariance. TCH_4 is a *static* chain that prolongs one of the *BACH*-defined areas. In *BACHian* terms the chain TCH_2 is more *progressive*. However, in terms of the more inclusive *WT* relationship, TCH_2 is static, as is TCH_4. Following TCH_2 chains along any one of the paths in Figure 2.45(b) shows that two progressive TCH_2 chains are, overall, static.

This is precisely the sense in which the two chains function in the opening theme, as shown in the event network in Figure 2.41. The three TCH_4 chains that begin the movement "prolong" both the *BACH* and *WT* basis of the initial row, $\mathbf{P_7}$—thereby allowing for the recurrence of the semit-tonal dyads and BACH tetrachords in m. 6 and m. 10 that I pointed to above. Upon completing the TCH_4-cycle in m. 10, $\mathbf{P_7}$ returns, but with increased surface rhythm and decreased emphasis on tetrachordal segmentation. Interestingly, the decreased importance of tetrachords is coincident with the TCH_2 transformation, which moves the music *away* from the *BACH*-defined $\mathbf{A_{1.3}}$ region that begin the piece and into $\mathbf{A_{1.1}}$. The sense in which TCH_2 is *both* prolongational *and* modulatory is on full display here. The score excerpt in Figure 2.42 shows that over the course of mm. 12–15, the dyadic correspondence with mm. 1-11 remains; that is, the music remains firmly entrenched in $\mathbf{A_0}$, using the same six dyads. However, as the boxed tetrachords on the score indicate, a subtle shift has taken place: the BACH tetrachords that defined mm. 1-11 have changed.[109]

Most importantly, the event network in Figure 2.41 shows that the two large areas ($\mathbf{A_0}$ and $\mathbf{A_1}$) are present throughout the piece, and at every point in the piece, *both* "whole-tone" sets of semi-tonal dyads are being sounded constantly. The polyphonic combination of the two canon voices, then, never overlaps in either *BACH* or *WT* terms—perhaps a means of differentiating the

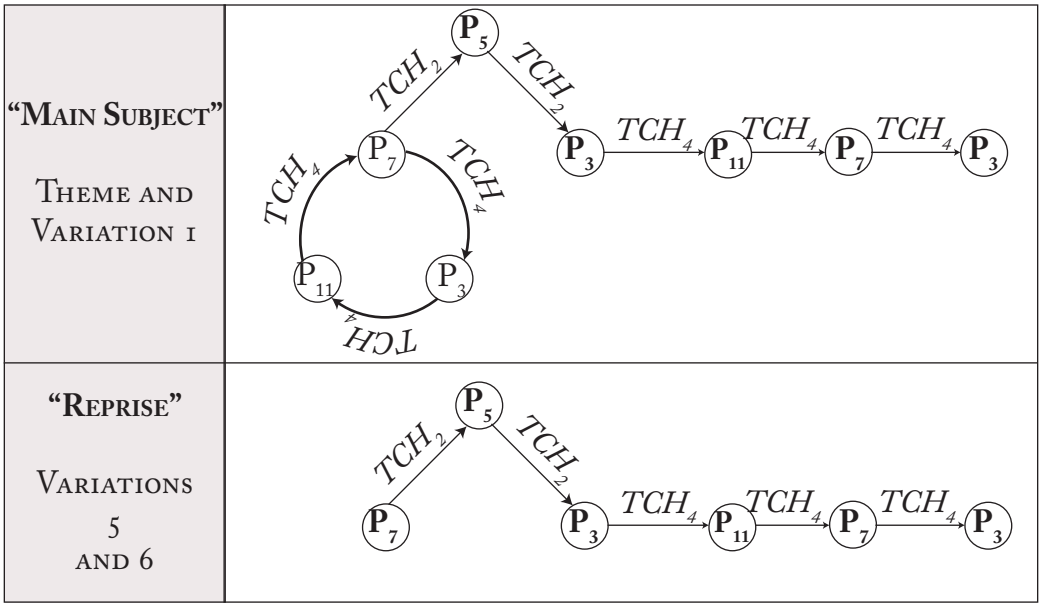
[109] Hook and Douthett, "UTTs and Webern's Twelve-tone Music," are primarily interested in tetrachordal segmentation, and thus, they note the same change in BACH tetrachords at m. 11.

two canon voices. Figure 2.45 shows this primarily through the transformational segregation of the two areas. No chain transformation exits either of the areas, and thus, as long as rows are being chained, the music remains "static" in either *WT* or *BACH* terms. Even when one voice crosses into the others territory (as in the transition or second theme) the alternate voice reciprocates.

To best understand how these variations "constitute an adagio form" we must consider the importance of cycles as carriers of syntagmatic meaning. The row forms at the beginning of each of the formal sections are "initiators," and they are directed syntagmatically towards the same rows, which at the end of each variation, act as "concluders"—row-forms-as-goals. In each of the movement's seven sections, those most associated with "tonal stability" are completely cyclic; those most associated with tonal instability are not.

This is perhaps most keenly felt in the main subject and its reprise. In both passages, TCH_4 cycles are prominent. Figure 2.46 superimposes the two passages (comprising the theme and variation 1, and variations 5 and 6) and shows how the reprise is an abbreviated version of the

FIGURE 2.46. Comparing the String Quartet's "main subject" and "reprise."



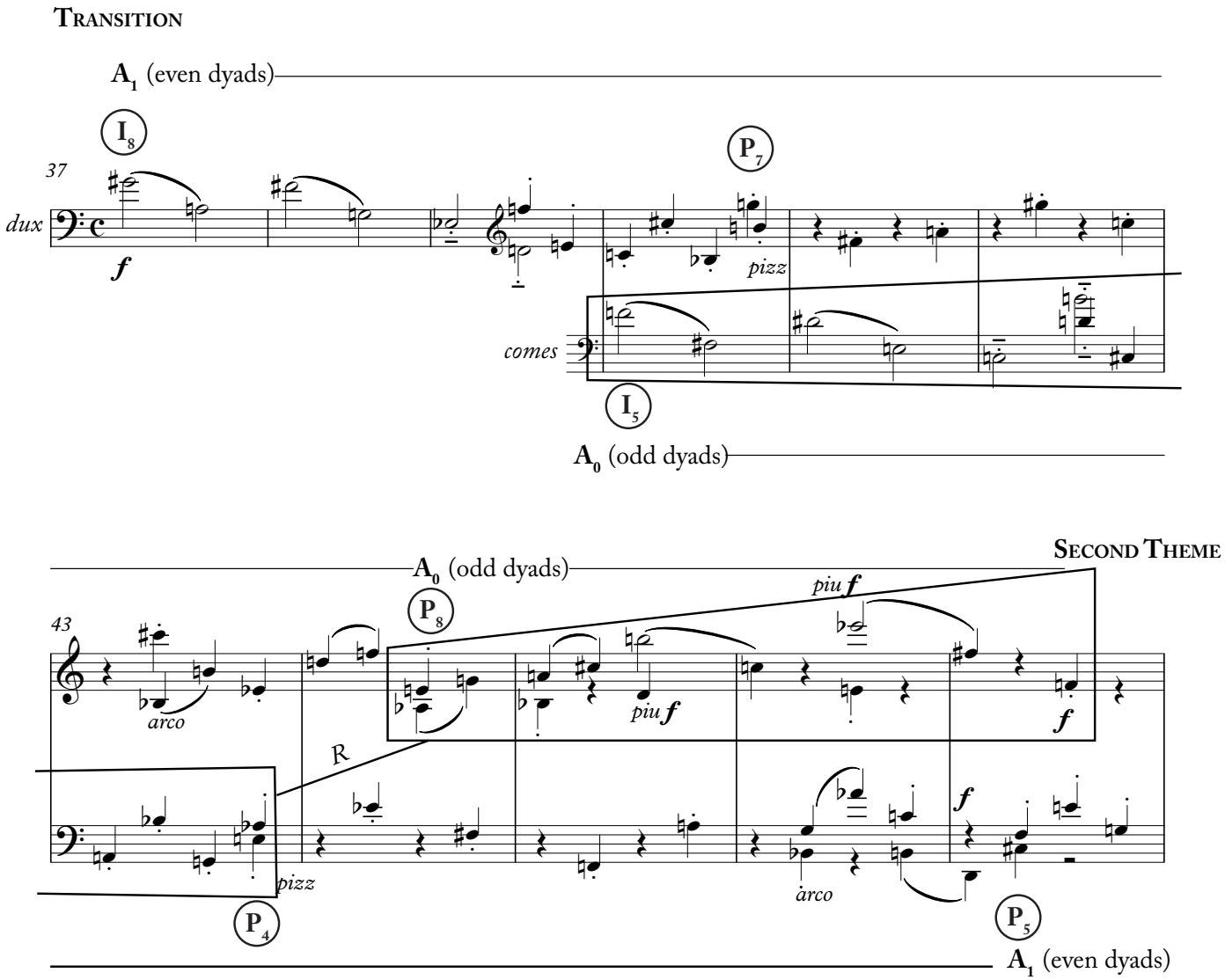
main subject, missing only the *TCH4* cycle that occurred over the course of the theme. **P3/P6**'s location at the end of variation 5—prior to the "coda"—suggests that that row pairing may be the primary "tonic" for the piece, functioning as goals. That **P3/P6** chains *into* variation 6—something that occurs nowhere else in the movement—also suggests the dependence of the coda on variation 5.[110] The **P3/P6** row forms that close variation 5 thus simultaneously take on a concluding and initiating role.

The second theme differentiates itself in both segmental, cyclic, and "tonal" or "spatial" terms. Figure 2.43 showed the second theme's preoccupation with tetrachordal segments of the row. The preeminence of tetrachords here is appropriate in tonal terms; Figure 2.41 indicates how the second theme establishes a novel pairing of *dux* and *comes*. Comparing the tonal location of the rows of the second theme (variation 3) with those at the beginning of variation 2 is illustrative. While variation 2 is initiated by rows in the **A0.0** and **A1.3** areas, the second theme launches from a complementary location (**A0.2** and **A1.1**.) Thus, the tonal contrast is one that occurs primarily in terms of the BACH tetrachords even as it projects the same *dyadic* structure as the previous variation; thus, BACH tetrachords are predominant. In cyclic terms, the second theme is the only variation that is cyclic but not in terms of *TCH4*. Within the second theme both voices pass through two progressive transformations that ultimately cancel one another.

The transition (variation 3) that prepares the second theme is "transitional" in two primary ways. First, the transition is *not* cyclic. And second, Figure 2.41 shows that a final motion, non-chain-based motion carries each canon voice into a new row area. Voice 1 (the *dux*) moves from **A1** to **A0** and voice 2 does the opposite. That final motion results in this "transition" variation being the only one in the piece to start and end in a different row area. These transitional features are highlighted by both the rhythmic and row structure of the passage, which is reduced to two voices in Figure 2.47. At this variation, the canonic interval separating *dux* and *comes* has reached

[110] This is noted by Hook and Douthett, "UTTs and Webern," 118.

FIGURE 2.47. The "transition" from the "main subject" to the "second theme."



its apex. Three whole measures separate the *comes*'s entry at m. 40 from the *dux.* The *dux* moves from **A1** to **A0** halfway through m. 44, manifesting the aforementioned transition into new "tonal" territory, and that movement is accompanied by a change from odd dyads to even ones. The *comes* does the opposite, moving from **A0** to **A1** (even to odd) but three measures later. This three-measure separation is important because it highlights the sense in which the *dux* has morphed into a new area. The boxes I have shown on Figure 2.47 show that the *dux*, halfway through m. 44, begins to play the *comes* voice from mm. 43—only backwards. That the *dux* is able to play the *comes* backwards is the result of the *RI-symmetry* of the row class. **P8** in m. 44 is the

retrograde of **I**$_5$. That retrograde relationship, and the fact of the *dux*'s changed tonal position, is underscored precisely because the canonic interval is sufficiently large that the last row of the *dux* comes just on the heels of the first row of the *comes*.

PART II: ANALYTICAL STUDIES

CHAPTER 3

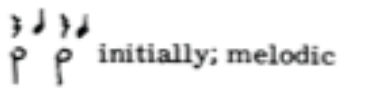
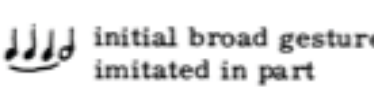
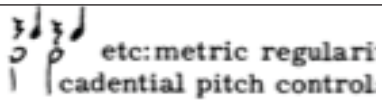
"THEME," "KEY," AND "FALSE RECAPITULATION" IN WEBERN'S QUARTET, OP. 22, II

Due to its "amorphous qualities," the second movement of Webern's Quartet for Violin, Clarinet, Saxophone and Piano has engendered a degree of analytical surrender. Writing in 1966, Brian Fennelly sums up a feeling still present in more recent analyses: "while the nature and limits of the compositional process could be isolated and defined in movement I, movement II abounds in perplexing situations. The intuitive freedom allowed by the *absence of highly restrictive pre-compositional postulations* is mirrored in the spirit of the music: in comparison to I, an elegant, carefully wrought, precision organism, II is unrestrained." [1] Fennelly's description is accurate in many ways, but defining the second movement only in relation to the compositional tidiness of the first movement perhaps unfairly marks this movement as abnormal.[2] An "unrestrained" musical surface does not, of course, belie an unorganized substructure. This dichotomy encourages

[1] Brian Fennelly, "Structure and Process in Webern's Opus 22," *Journal of Music Theory* 10, no. 2 (1966): 315, emphasis added. Leland Smith follows Fennelly's lead: "The [second] movement of this quartet marks an abrupt shift away from such (in reference to the first movement) a multitude of precompositional procedures" ("Composition and Precomposition in the Music of Webern," in *Anton von Webern Perspectives*, ed. Hans Moldenhauer and Demar Irvine (Seattle: University of Washington Press, 1966), 86–101). George Perle postulates that "in the Quartet Webern turn[ed] away from the rigorous logic and austere economy that he had pushed to a "point of no return"in the unfinished third movement of the Symphony [Op. 21]. The contrast between the strictly symmetrical row of the Symphony and the nonsymmetrical row of the Quartet is reflected in the difference in the character of the two works" ("Webern's Twelve-Tone Sketches," *The Musical Quarterly* 57, no. 1 (1971): 15). Kathryn Bailey, *The Twelve-Note Music of Anton Webern*, 242 suggests that the second movement of Op. 22 is indicative of a period during which Webern was less concerned with symmetry and other, more easily specifiable, characteristics. According to Bailey, this period contains this movement of Op. 22—the first movement is not included—and *Das Augenlicht,* Op. 26, which was composed five years later.

[2] In relation to the second movement, studies of the first movement occupy a disproportionate percentage of the analytical literature, no doubt because of that movement's relatively lucid structure. For remarks on the first movement, see Milton Babbitt, "Twelve-Tone Invariants as Compositional Determinants," *The Music Quarterly 46* (1960): 246–59; reprinted in *The Collected Essays of Milton Babbitt*, ed. Stephen Dembski, Andrew Mead, and Joseph N. Straus (Princeton: Princeton University Press, 2003): 55-69; Christopher F. Hasty, "Composition and Context in Twelve-Note Music of Anton Webern," *Music Analysis* 7, no. 3 (October 1, 1988): 285-93; Mead, "Webern and Tradition," 187–96; Observations about both movements, but particularly the first are found in Dora A. Hanninen, "The Variety of Order Relations in Webern's Music: Studies of Passages from the Quartet Op. 22 and the Variations Op. 30," *Theory and Practice* 20 (1995): 31–56.

FIGURE 3.1. Formal chart for the second movement of Webern's Quartet showing Fennelly's "characteristics" and comparing Webern's movement to Beethoven's Rondo, Op. 14, no. 2, III.

Webern's Rondo: Op. 22, II	measure number	"Characteristics," from Fennelly, "Structure and Process," 316, Table 3.	Row Area, cf. Figure 3.11	Beethoven's Rondo: Op. 14, no.2, III
Refrain (A1)	**1**	 initially; melodic imitative figures, canonic	A_0	Refrain (1-22): G major (**I**)
Episode 1 (B1)	33	 initial broad gestur imitated in part imitative, canonic	A_3	Episode (23-42): (**VI—V)**
Refrain (A2)	69	 etc: metric regulari cadential pitch control	A_0	Refrain (42-72) with transition: (**I**)
Episode 2 (C1)	85	cl, sax dyads:  ; "sehr ruhig" pitch controls (esp. piano) increasing tension to mm. 128-131, high point		Episode (73-124): (**IV)** False Reprise (124-138): (**IV)**
Refrain (A3)	132	tension abated, piano pitch control, other registral, textural control	A_0	Refrain (138-188) with transition: (**I**)
Episode 3 (B2)	153	 Pno. initial gesture; complex canonic imitation increase in tension	A_0	Episode (189-237) with transition: (**I**)
Refrain (A4)/Coda	180	dramatic flourishes, cadence.	A_0	Refrain/Coda (237-53): (**I**)

an emphasis on the radical elements of the work (an "absence of highly restrictive pre-compositional postulations") at the expense of musical features that are not.[3]

[3] The second movement of this piece was composed *first*. Because of the compositional order, perhaps it is better to view the first movement as a reaction to the second, inhabiting compositional territory suggested by the second movement.

Despite the music's unrestrained qualities, most analyses note its classical formal model—the rondo. Building off a diagram Fennelly himself creates, the diagram I have shown in Figure 3.1 displays the movement's seven-part formal organization.[4] Kathryn Bailey also notes the movement's rondo design, agreeing in most part with Fennelly's analysis, but makes only a small attempt at understanding how the surface structure is integrated into the larger design. Instead, Bailey identifies a "looseness" that causes the structure to be "elusive and difficult to define" (242-44).[5]

In Bailey's opinion, the relative formlessness of the movement is made more perplexing because of references Webern, himself, made to the structure of the piece in a pre-compositional sketch and in remarks to his student, Willi Reich.[6] The latter reference is found in the postscript to the *Path to the New Music*. There, Reich recounts Webern comparing this movement with the scherzo of Beethoven's Op. 14, no. 2: "He said of the latter [Op. 22], when we were analyzing the Scherzo of Beethoven's Piano Sonata Op. 14, No. 2, that *during the analysis* he had in fact realized that the second movement of his quartet was formally an exact analogy with the Beethoven Scherzo" (emphasis added).[7]

Bailey's specifies her dissatisfaction with the analogy by locating at least six aspects of Webern's movement for which there exists no exact correlate in Beethoven's movement:

(1) The Op. 22 rondo has a much more significant B section than the Beethoven scherzo.

---

[4] Fennelly "Structure and Process," 316.

[5] I will show at numerous points in this chapter that Bailey's understanding of the movement is conditioned to a great extent by two factors: (1) the start and stop of row forms; and (2) changes in "musical character"—tempo, articulation, dynamics. Her identification of the "loose" and "elusive" formal structure is in part due to her unwillingness to accept that row forms do not necessarily begin and end with the change of musical section. Bailey, *The Twelve-Note Music of Anton Webern,* 249: "sections have been delineated on the basis of two things - row structure and changes in treatment or material - and specific bars where these sections begin and end have been suggested. In all cases, however, there is a discrepancy between the row structure and the musical structure […] where the music seems not to reinforce the row structure in any way."

[6] According to Bailey, *The Twelve-Note Music of Anton Webern*, 440, n.9, the plan appears in Sketchbook 1, on p. 53. The sketchbook is housed in New York at the Pierpont Morgan Library.

[7] Webern, *The Path to the New Music*, 57.

(2) The Op. 22 rondo lacks, for all practical purposes, a return of the refrain between episodes B and C, though the row structure would indicate the existence of one. The A section is exactly repeated at this point in the Beethoven, and a return is indicated as well in the Webern outline.
(3) The Op. 22 rondo has an analogue, although again theoretical rather than aural, to the false reprise in the Beethoven. This is not indicated in the projected outline.
(4) The third refrain, following the central episode, is varied in Op. 22/ii and resembles the earlier (hypothetical) return more than it does the original A. This return is exact in the Beethoven.
(5) The final episode in Op. 22/ii is a variation of the first one; in Webern's projected structure it should refer to the first episode, but also to the central one as well; in the Beethoven, the final episode is new and forms a part of the coda.
(6) The final A section functions as a coda in all three of the structures in question, though in the Beethoven the coda has begun much earlier.[8]

Bailey's objections are quite specific, referring both to the proportional features of Beethoven's rondo and the degree to which later refrains are variations of earlier ones, and this specificity is part of what makes her objections jarring. Reich's short, offhand remark is second hand, and given the lack of detail in his account (in the quotation, it is unclear precisely what aspect of the formal structure of Beethoven's rondo Webern was referring to), Bailey seems to be expecting too close a fit between the movements.[9] More important I think, in his recounting Reich clearly states that Webern realized the correspondence *during the analysis*, which was carried out following the composition of the movement. Bailey seems to be suggesting that Webern used the Beethoven as a model (in the context of a different argument, she says: "the Beethoven scherzo said to have been identified by Webern as a model …" (248)), but this is not indicated in the quotation from Reich.

---

[8] Bailey, *The Twelve-Note Music of Anton Webern*, 248.

[9] The first and sixth of these objections (referring respectively to the length of the B section and relative proportional position of Webern's final refrain/coda) represent matters of degree. And in fact, it's not clear that Bailey is entirely correct in her analysis here. Beethoven's refrain (22 measures) and B section (19 measures) are nearly the same length, as are Webern's refrain (30 measures) and B section (33 measures). Further, as Beethoven's rondo is not particularly exceptional in these regards (compare Beethoven's scherzo from Op. 14, no. 2 with the more grandly conceived sonata rondo from Op. 14, no. 1, for example), it seems unlikely that Webern would analogize the proportional and variation qualities of the movements. Her second and fourth objections contradict themselves. It is unclear how the third refrain (following the central episode) would resemble the second refrain if, as her objection two suggests, such a refrain does not exist. In other cases (objections two, three, five, and six), Bailey seems to conflate Webern's finished movement with the outline of an earlier sketch.

Despite these problems, by scrutinizing the relationship in such detail Bailey invites a closer reading of the piece in these terms—especially a reading that attempts to reconcile the amorphous musical surface with any formal logic that lay beneath. In my opinion, which I will explore more rigorously below, the key to the analogy likely lies primarily in two characteristics of the Beethoven rondo that are bound up with interaction of "theme" and "key." First: in Beethoven's rondo the final episode of contrasting thematic material occurs in the tonic key, as I have shown in Figure 3.1.[10] This is an important structural feature of the rondo, and begs the question of how Webern's movement could operate similarly. Second: in her fourth comparison, Bailey locates the "theoretical" potential for a false recapitulation in Webern's rondo that would mirror that found in Beethoven's. Such an idiosyncratic feature would be likely be suggestive to Webern as he analyzed the movement with Reich, and may even override any dissimilarities between the pieces, of which there are some. Rondo reprises require the thematic *and* tonal return of the initial refrain. False recapitulations (or reprises, or refrains) rely on a mismatch between the two; generally, following the second episode, the rondo refrain returns in the wrong key, as is the case in Beethoven's rondo, which as Figure 3.1 indicates, occurs in the subdominant just measures before the "real" recapitulation.[11]

Because false recapitulations rely on the notion of tonality, Bailey's criticism of this shared aspect of the movements reflects, in part, her deeper-seated pessimism towards the idea that the

[10] A tonic-based final episode is common in a sonata-rondo, of which Beethoven's movement is not.

[11] False reprises are found elsewhere in Beethoven's rondos: see, for example, Symphony No. 8 in F, Op. 93, iv, 151—a sonata rondo. Also, on a smaller scale: Rondo for Piano in C, Op. 51, no. 1 (five-part rondo), Violin Sonata in G, Op. 30, no. 3, iii (seven-part rondo), and Cello Sonata in F, Op. 5/1, ii, 60 (sonata rondo).

twelve-tone system may establish systemic corollaries with tonality.[12] And in this vein, the idea of false recapitulation along with the "tonal resolution" of the final episode prompts important questions about the twelve-tone system's (as practiced by Webern) relationship to tonality. If Webern does not use the row in a manner analogous to "theme," how is false recapitulation possible?[13] Is there any corollary to "key"? These are questions that I will explore below.

I will show that understanding Webern's movement in these terms is revelatory in at least two ways: first, it shows one way the composer reconciled the twelve-tone technique with sophisticated formal practices of the classical era. To be sure, Webern mimicked classical form in other ways, some of which I discussed in Chapter 2, and others of which we will explore in Chapter 4. This movement is interesting because, more than these other pieces, it does reveal quite a close relationship between the aspects of the classical form that are most widely discussed. Second, it reveals how the "unrestrained" surface structure interacts with certain "precompositional postulations" associated with the underlying form. In particular, the movement has a clear compositional "design," emerging from basic properties of the row and their connections to one another, on top of which, the rather wild surface structure often operates like a guided improvisation.

---

[12] Mead, "Webern and Tradition," grapples with the sophisticated means through which Webern engaged the formal constructs associated with common-practice tonality in the second movement of Op. 27, the first movement of Op. 22, and the String Trio, Op. 20. That study's springboard, in fact, is Bailey's and George Perle's skepticism that the twelve-tone system (as conceived by Schoenberg, an especially in regard to Op. 20) is capable of engaging form in more than a superficial way. See Perle, "Webern's Twelve-Tone Sketches"; and Bailey, *The Twelve-Note Music of Anton Webern*. Mead's review of Bailey, *The Twelve-Note Music of Anton Webern*, also offers incisive commentary on Bailey's understanding of form and "tonality" as it operates within the precepts of the twelve-tone system: Andrew Mead, "Review of *The Twelve-Note Music of Anton Webern: Old Forms in a New Language*, by Kathryn Bailey," *Intégral* 6 (1992): 107–35.

[13] Webern reportedly stated that "The twelve-note row is, as a rule, not a 'theme'" (Moldenhauer and Moldenhauer, *Webern, A Chronicle*, 667, n. 11).

I: One-Note Chains, Row Invariance, and Inversion

A particularly close relationship exists between a group of four one-note chains plus $RICH_2$, a set of pitch-class invariants, and the "inversional potential" possessed by rows related by the group. Figure 3.2(a) shows $\mathbf{P_6}$, which along with $\mathbf{RI_6}$ is the first row of the piece, and indicates two prominent subsets: two T_6-related chromatic tetrachords (**CTET**s) occupy eight, contiguous pitch classes in the row's center. This row's boundary pitches are also T_6-related, and as a result, when the row is TCH_1-ed (as shown at (b)), the chromatic tetrachords and the boundary pitches swap places, as do the two pitch classes forming the tritone {C♯, G}.[14] Of course, $RECH_1$-related forms also preserve these invariants. While TCH_1 and $RECH_1$ retain the precise pitch-class identity of the **CTETS**, the other half of the one-note chain group—ICH_1 and $RICH_1$—shift the **CTETS** down one half step, as shown at (c). (To make reading easier, I will refer to CTETS throughout this chapter. When CTETS is bolded, as in "ICH_1 shifts $\mathbf{P_6}$'s **CTETS** down one half step," I am referencing the specific pitch classes shown in Figure 3.2(a). When CTETS is not bolded, as in "the CTETS structure of the passage as a whole is static," I am referring not to a specific set of pitch classes, but to the fully chromatic set class [01234567] that is found in the center of Figure 3.2(a).)

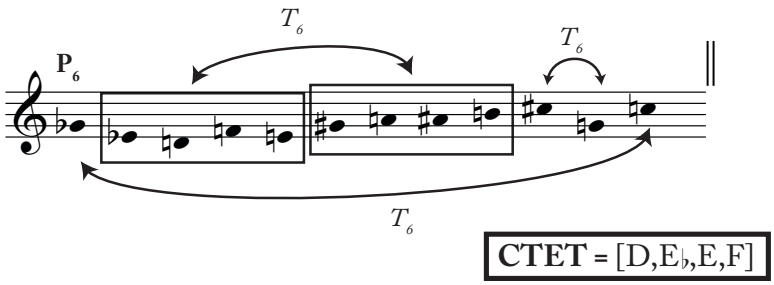
Each of these transformations are involutions, and thus, the one-note chain group $\langle TCH_1, ICH_1, RECH_1, RICH_1 \rangle$ joins eight rows into a single chain-connected space.[15] I have shown this in Figure 3.3(a), a row area called $\mathbf{A_0}$ because it possesses $\mathbf{P_0}$. (As the boundary interval of the row is a tritone, the eight rows in such a space have subscripts that differ by 6.) When unfolded at (b), **P** and **R** forms are on the right side, forming a "**P**-side," and **I** and **RI** forms find a home

[14] Hanninen, "Order Relations in Webern's Music," 43, identifies the same **CTET** in the development, m. 96. Mead, "Webern and Tradition," 188 identifies the prevalence of a property ("property 2") important to formal procedures in the first movement that is similar, but not quite the same as show here.

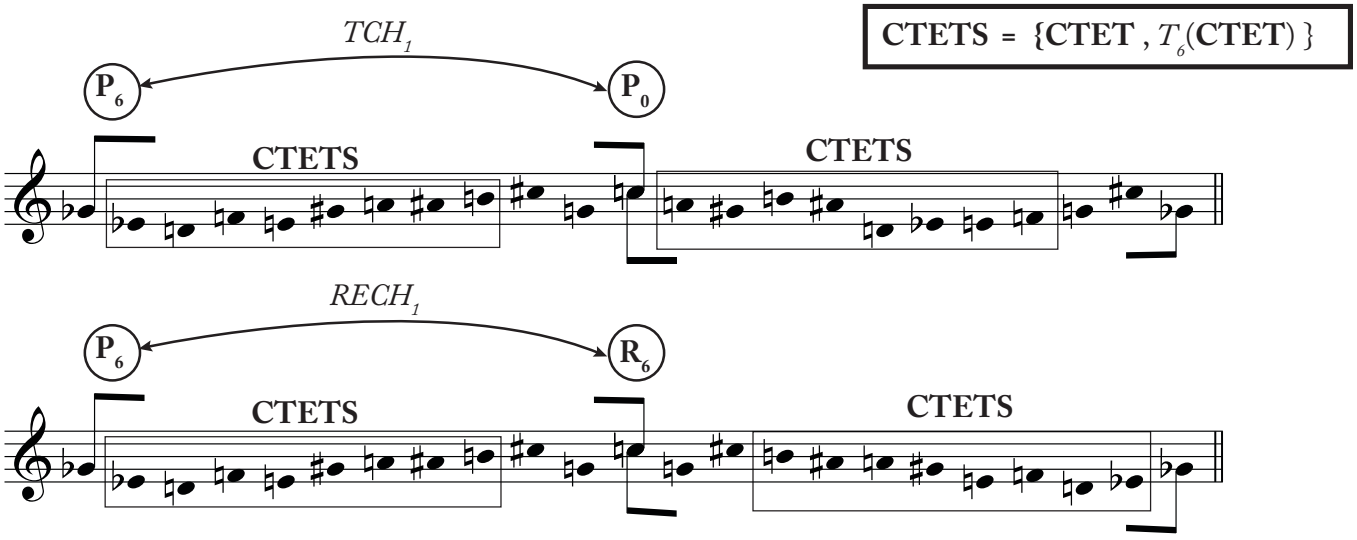
[15] The group's order equals 8, not 16, because $RICH_1$ is a redundant operation equal to $TCH_1 \cdot ICH_1 \cdot RECH_1$.

FIGURE 3.2. One-note chains and their effect on a row's two chromatic tetrachords (**CTETS**).

(a) The prevalence of T_6 within the row.



(b) TCH_1 and $RECH_1$ chains preserve **CTETS**.



(c) ICH_1 and $RICH_1$ chains shift **CTETS** by $T_{+/-1}$.

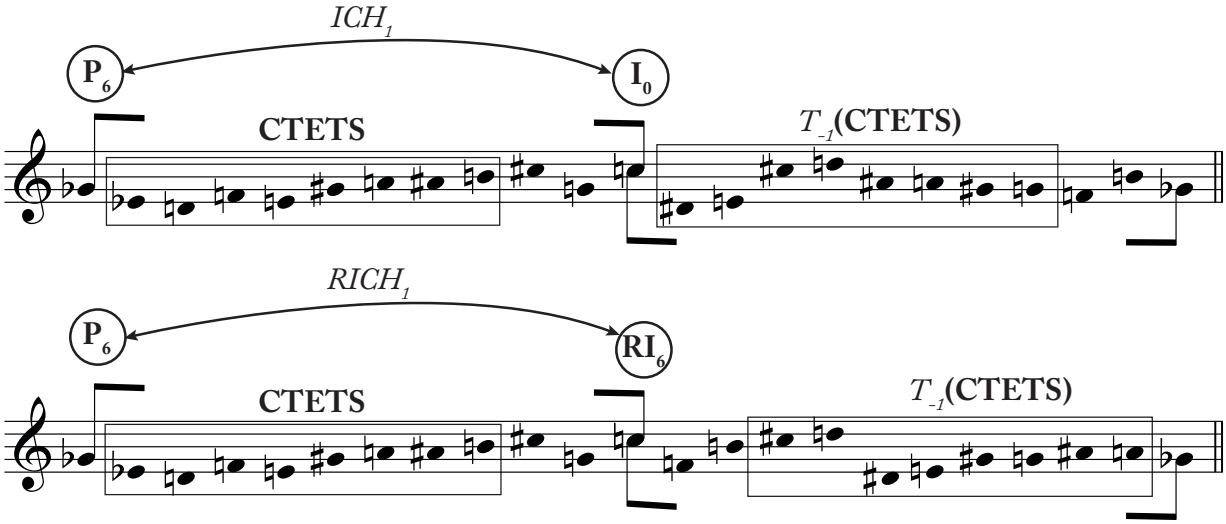
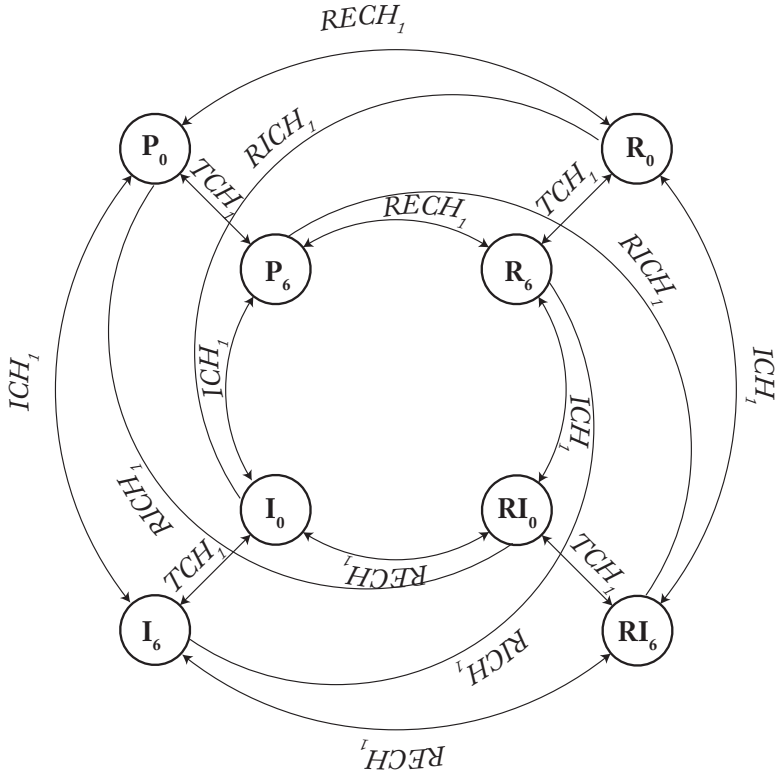
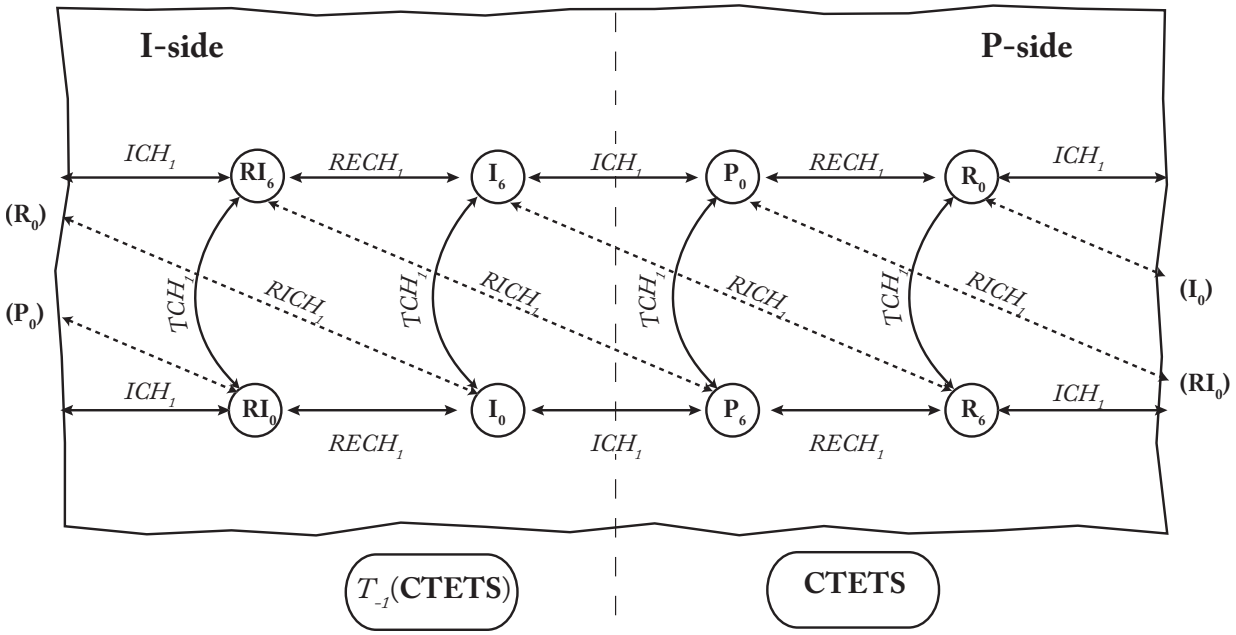


FIGURE 3.3. Spatial networks showing $\mathbf{A_0}$, a row area generated from one-note chains.

(a) A "double-circle" space.



(b) A "flattened" version of (a), with **CTETS** for each "side" shown along the bottom.



on the left side of the figure, the "**I**-side." This unfolding reveals how the CTETS organization exists alongside the chain paths: rows within a side are reachable by TCH_1 and $RECH_1$ only, while ICH_1 and $RECH_1$ are the only paths from one side to the other.[16] TCH_1 and $RECH_1$ are static with regard to CTETS, while moving from the **P**-side to the **I**-side through ICH_1 or $RICH_1$ shifts all CTETS up one half-step, a slightly more progressive transformation.

Throughout the movement, $\mathbf{A_0}$ is the primary "thematic" zone for the four refrains in the movement's rondo form, as Figure 3.1 shows. In the larger formal scheme, the first and third of these refrains carry the greatest weight—the first for obvious reasons, the third because it initiates the large second half of the rondo plan, following the developmental central episode.[17] Figure 3.4(a) and (b) diagram the row structure the two refrains, both of which are divided into two halves. This shows that the two refrains are organized similarly: in each, the first half of the section is divided by a $RICH_1$ chain that carries two row forms from the I-side to the P-side. As if compensating, this $RICH_1$ move is met by the appearance of a new row form ($\mathbf{RI_6}$) in both cases, just before the sectional divide. The second half of both passages begin with $RECH_1$ chains initiated from $\mathbf{P_0/I_0}$.

The organization of the refrains is in fact somewhat more rigorous. If we wish to imagine the movement's surface unfolding a bit improvisationally, we may frame this organization according to two "rules":

(1) Both the P-side and the I-side of $\mathbf{A_0}$ must be present at all times. I noted for example that as $RICH_1$ carries $\mathbf{RI_6/I_6}$ to the P-side in the first half of each refrain, a new row form ($\mathbf{RI_6}$) fills the spatial "hole" left by this change;

---

[16] Rows in the same area are never more than two chain transformations away. Moving from $\mathbf{P_6}$ to $\mathbf{I_6}$ would require $TCH_1(ICH_1)\mathbf{P_6}$. Any three moves are equivalent to one single move. $\mathbf{P_6}$ goes to $\mathbf{RI_6}$ via $TCH_1(ICH_1)(RECH_1)\mathbf{P_6}$ or simply $RICH_1(\mathbf{P_6})$

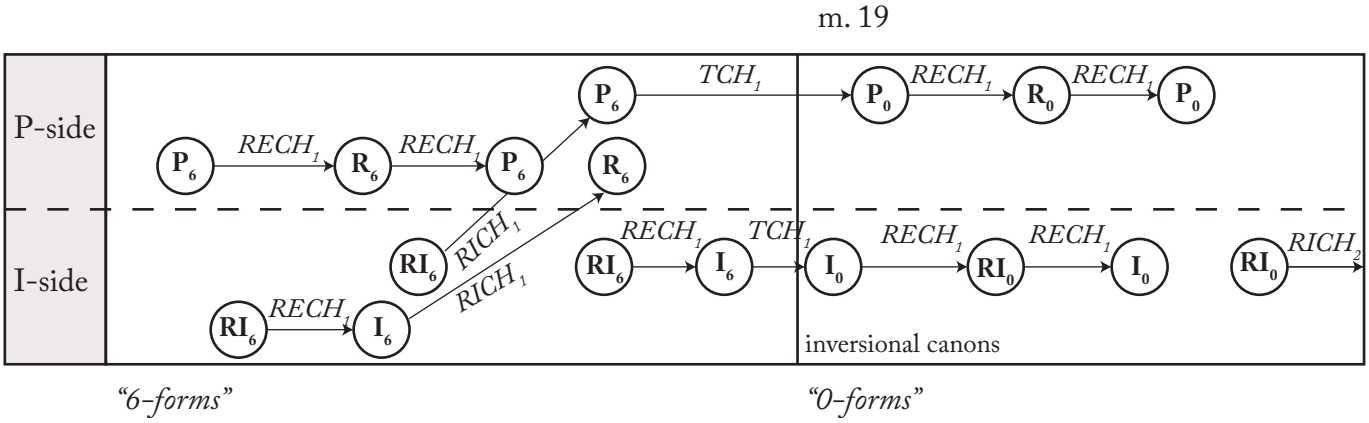
[17] One aspect of the Webern's rondo that is quite different from Beethoven's involves these refrains. In Beethoven's piece, the four refrain are the same. In Webern's movement, the second and final rondos are abbreviated versions of the other two.

(2) Only row forms with the same subscript are played together. In both passages, the first half of the passage uses 6-forms, and the second half uses 0-forms.[18]

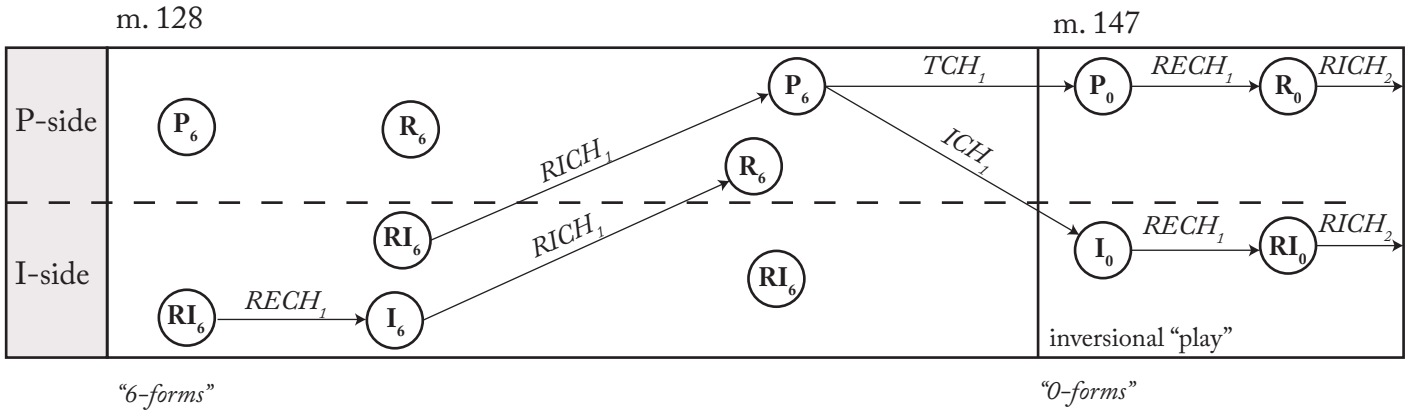
How do these rules affect the surface of the music? In relation to rule one: because both sides of the $\mathbf{A_0}$ are always sounding, both sets of CTETS—that is **CTETS** and T_{-1}(**CTETS**) on Figure 3.3(b)—are consistently juxtaposed. Figure 3.5(a) shows how this juxtaposition works in the opening seven measures of the movement. The two sides of $\mathbf{A_0}$ are contrasted timbrally and

FIGURE 3.4. An event network for the first and third refrain.

(a) Refrain 1: mm. 1–32



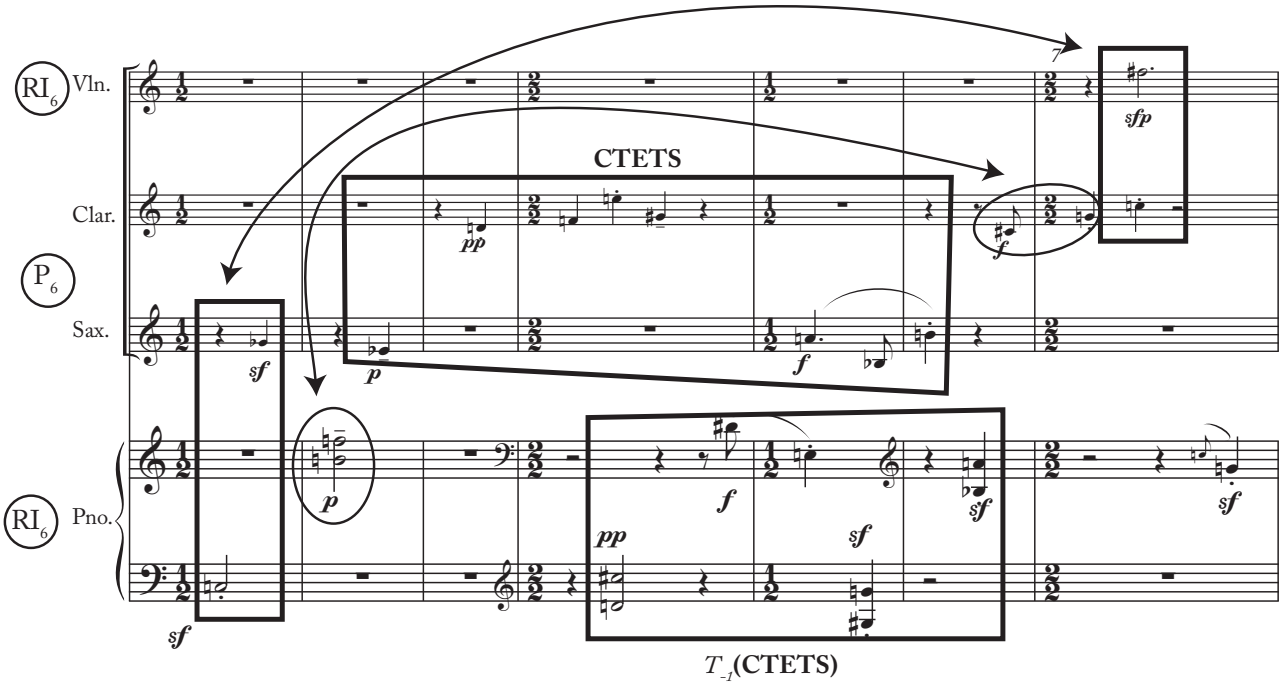
(b) Refrain 3: mm. 128-52



[18] We will see later that both of these rules are in some way operative in all of the refrains, though the "rules" are followed with particular "strictness" here. In particular, the idea that 6-forms initiate refrains proves to be a valuable formal observation.

texturally. While the winds play **CTETS** melodically, the piano separates most of *(T-1)***CTETS** into harmonic major sevenths and minor ninths.[19] Notice also that the passage has a tinge of retrograde structure that involves boundary tritones. The initial {C3, G♭4} that occurs between the two row strands in m. 1 is answered in m. 7 as {C5, F♯5}. On the inner halves of these pitch-class pairs, Webern places each row's lone tritonal adjacency: **RI$_6$**'s vertically realized {B4,F5} is answered over m. 6 and 7 by **P$_6$**'s horizontally realized {C♯4, G4}.

FIGURE 3.5. First refrain, first half (mm. 1–7). **CTETS** are heard melodically, *T-1*(**CTETS**) harmonically (cf. Figure 3.5(a))



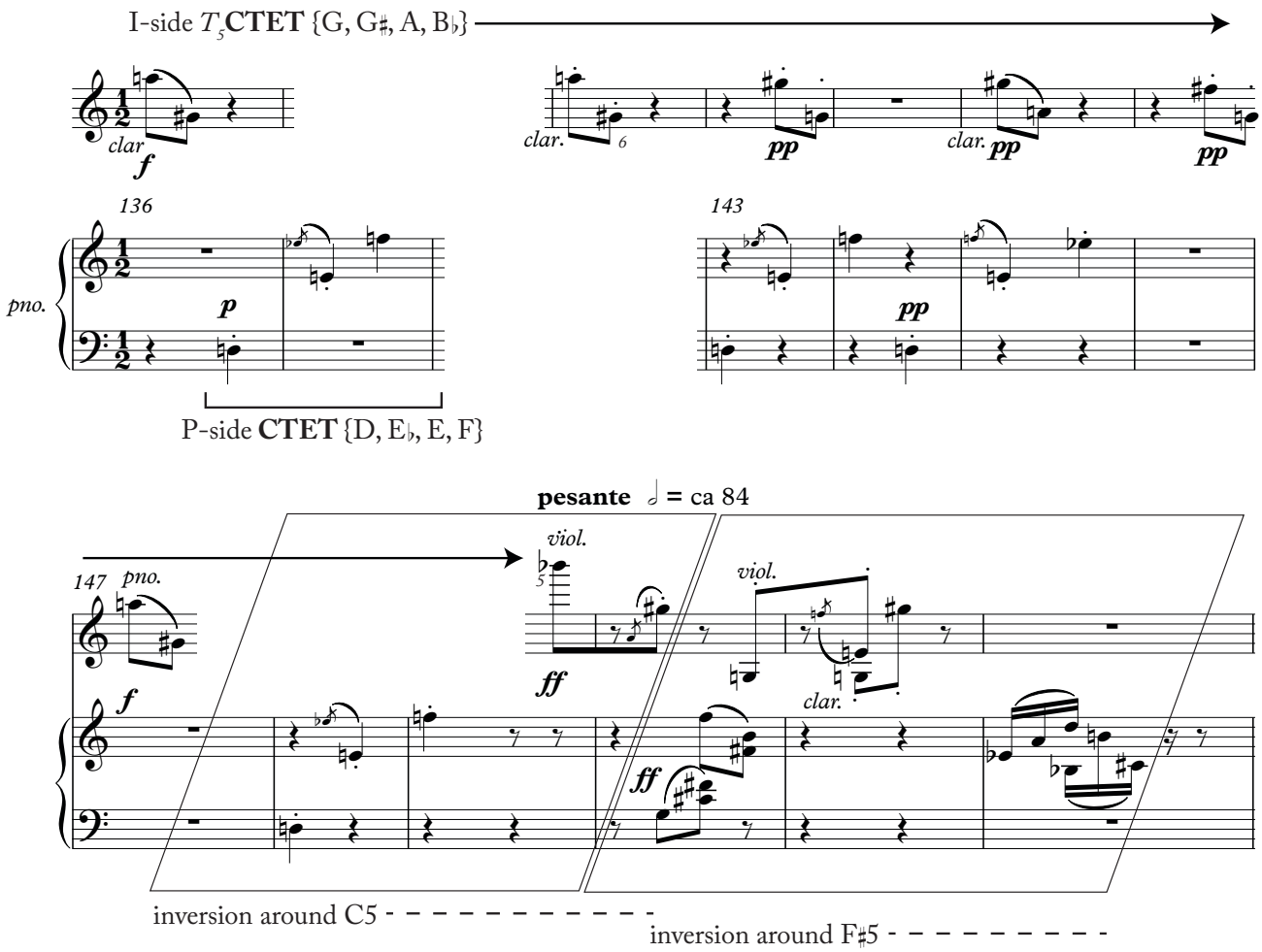
Like the first refrain, CTETS structuring is particularly prominent in the first half of the third refrain. On Figure 3.6(a) I have shown how the P-side **CTET** {D, E♭, E♮, F} is repeated as an ostinato (beginning at m. 136) four times over the course of the first half of the third refrain. (This reduction omits some events from the musical surface. The passage contains a staggering

[19] This relationship between row strands calls to mind the opening movement of Webern's String Quartet that was considered at the close of Chapter 2. There, the two canonic voices were contrasted by projecting *exactly opposite sets of semi-tonal dyads*—a relationship that obtained throughout the movement. Here, two voices are juxtaposed structurally by projecting *different sets of chromatic tetrachords*.

variety of order relationships that obtain between the two or three rows sounding throughout the passage, and are very difficult to convey graphically with crowding the score. Nonetheless, many of these order relationships emphasize points I wish to convey.[20] Each of these repetitions occurs in the piano, they are all in the same register, and each ascends. A compensating gesture is heard throughout the passage in the clarinet, also in a fixed register. Beginning at m. 136, minor ninths {A, G♯}, {G♯, G}, and in m. 146 {F♯, G}, echo the piano but invert its direction. When at m. 149 the violin reaches B♭6, a long range I-side **CTET** {G, G♯, A, B♭} is completed.

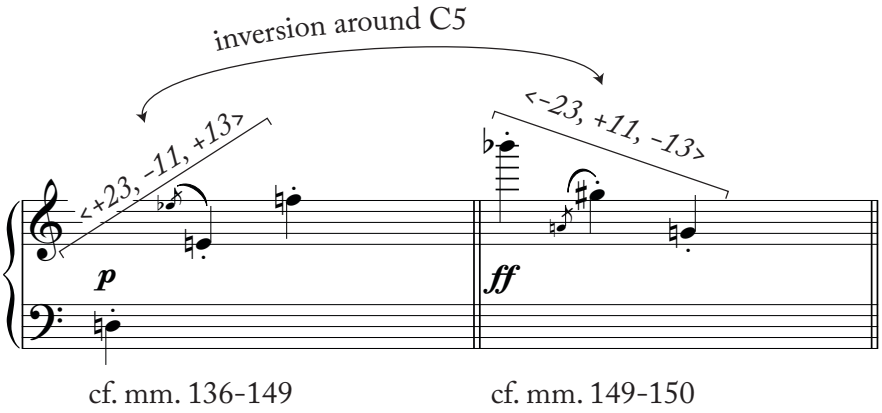
FIGURE 3.6. Third refrain, mm. 136-52 (reduced). **CTETS** ostinatos are fixed in register.

(a) mm. 136-52



[20] See Hanninen, "Order Relations in Webern's Music," for other interesting relationships like this.

(b) "Inversional play" (cf. Figure 3.6(a)).



The second half of these refrains explores the implications of the second rule above, that "only row forms of the same subscript are played together." These implications are both more abstract and, I will show, farther reaching. As row forms within each side of $\mathbf{A_0}$ differ by six, a single *pair* of inversional axes lurks within the structure of the space. For example, given any **P**-form and **I**-form in $\mathbf{A_0}$, their combination can yield one of two axes: I_0 or I_6. I call this the area's "inversional potential." The two potentialities always differ by six.

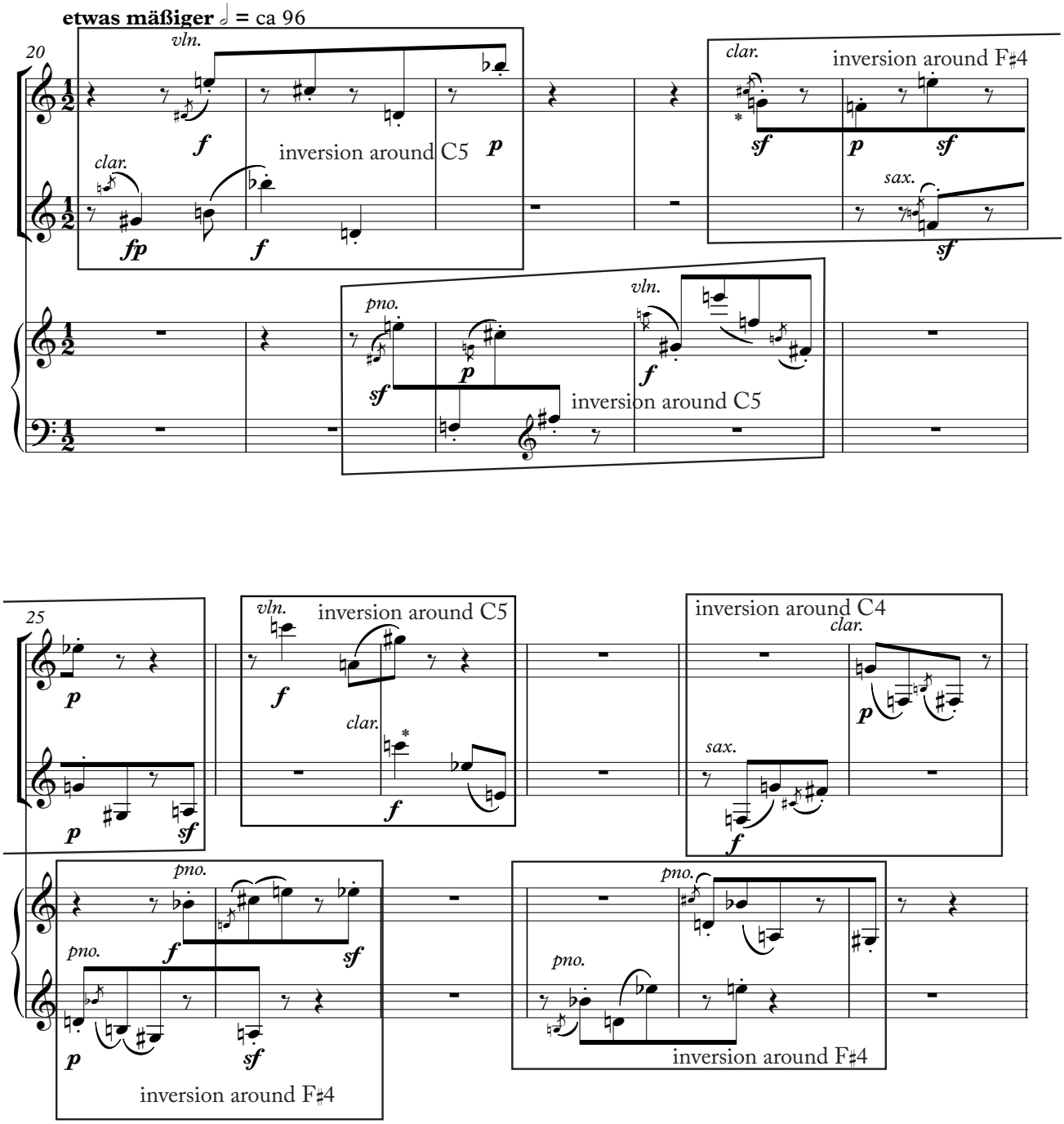
And throughout the first half of these refrains, the potential for an inversional axis to act as a structuring agent lies dormant. In the second half, inversional structuring is primary. Figure 3.7 shows the second half of the opening refrain, where seven short canonic gestures—containing three or four pitches—unleash the nascent inversional potential.[21] Each of these gestures projects the I_0 axis in pitch space.[22] Though inversion around C5 initiates the passage and seems to be the primary center, many of the canonic gestures shift this axis, often depending on what pair of instruments is involved. Webern's handling of the passage conceals CTETS through grace notes that begin (or fall in the middle of) each gesture. Thus, while the inversional structuring of was dormant in the first half of the refrain, CTETS structuring becomes dormant in the second half.

[21] Perle, "Webern's Twelve-Tone Sketches," 15-16, discusses this canon. While imitative the canon is disguised somewhat as it relates a *dux* and *comes* by inverting both pitch-class order and contour.

[22] This is except for the C♯5 in m. 23, and the C6 in m. 27.

By contrast, the inversional play represented in the third refrain (shown in Figure 3.6) *emerges with* the CTETS completions mentioned above. As Figure 3.6(b) shows, the **CTET**—{D3, E♭5, E♮4, F5}—projects a pitch-space intervallic scheme spanning 27 semitones. Repeated

FIGURE 3.7. First refrain, second half (mm. 20–30).



four times over mm. 136-149, the gesture includes D3, which is the lowest pitch in the passage. Using the first refrain's inversion around C5 as a model, an inversional "answer" to the **CTET** would mimic the model shown in the second half of Figure 3.6(b).[23]

When that answer occurs at m. 149 (Figure 3.6(a)), the dramatic B♭6 in the violin, two compositional processes that have unfolded in the preceding measures. First, it completes the I-side CTET, which has acted as a compliment to the P-side CTET sounded in the piano. And second, it satisfies the expectation of inversional structuring that was suggested by the piano/clarinet interaction over mm. 136-147. The greater pitch content of piano and clarinet action over the passage in Figure 3.6(a)—a fully-chromatic, nine-note collection—is symmetrical precisely around C5.[24]

This B♭6 is also the passage's climax, its high register answering the low register of the **CTET**'s D3. In m. 150, the **CTET**'s grace-note gesture {E♭5, E4} is answered by a grace-note gesture {A4, G♯5}. Only as the final inversional partner is expected {the **CTET**'s F5 "needs" a G4} do things begin to change. Just prior to the expected G4 in m. 150, the piano begins a new canonic gesture. (See the lower staves of Figure 3.6(a).) Perhaps replicating the piano's primary axis in the opening refrain (mm. 25-26 and 28-29 in Figure 3.7), this gesture is symmetrical around F♯4. Thus, the expected G4 *does not arrive.* Instead, Webern substitutes G3 in the violin in m. 150, echoing the G4 just heard in the piano. Over the remaining measures of the section, the piano continues to play around F♯4, as Figure 3.6(a) shows.

---

[23] Among other things, the {E♭, E♮} dyad that straddles the middle of the **CTET** is repeatedly answered in the clarinet with its I_0 counterpart, {A, G♯}.

[24] More generally, a lot of the pitch content in the passage is symmetrical around C5, though the clarinet and piano realize this most concretely, and offer a compelling reason for the climax at m. 149.

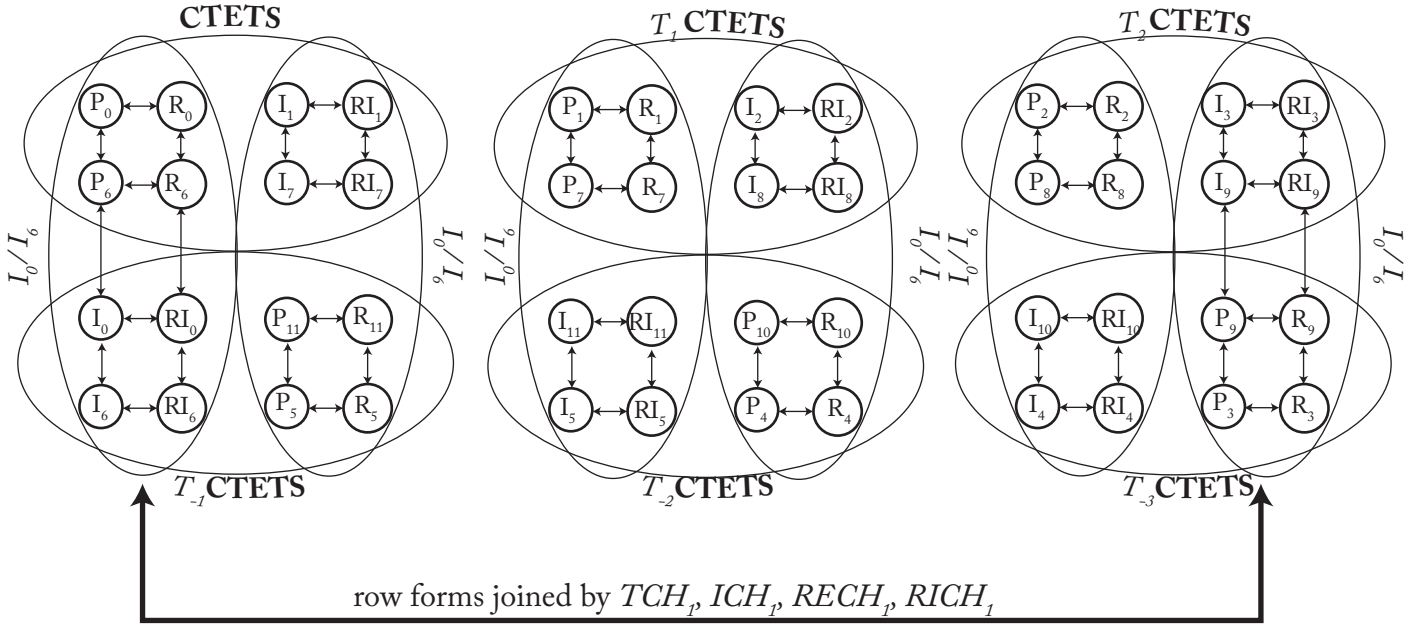
II: A Compositional Space and Compositional Design

In the first and third refrains, I have demonstrated how the spatial network generated by the one-note chain group (*TCH*$_1$, *ICH*$_1$, *RECH*$_1$, and *RICH*$_1$) interfaces with pitch-class invariance and inversional axis. Thus, we have seen three types of "structure" in the refrains (chains, CTETS, inversional axis), each worth exploring further as they relate to one another. Some interesting observations are borne of this exploration. For example, I will illustrate that each of the properties are suggested by one another. "Improvising" with anyone of the properties might lead quite naturally to either of the others. But more interesting in terms of larger goals of this chapter, the three properties that structure the refrain turn out to have an enormous bearing on the movement as a whole—its rondo structure and details of the interaction between refrains and episodes. The refrain principles are, in fact, "amplified" onto the structure of the movement, which I will show through a compositional design.

Figure 3.8 arranges all forty-eight row forms in cross-hatch ovals. Horizontally oriented ovals share CTETS, and vertically oriented ovals project the same inversional potential. Inversional potential, as I am conceiving of it in this movement, exists amongst a collection of eight rows—two each of **P**-forms, **I**-forms, **R**-forms, and **RI**-forms. In the refrains, I_0 was the inversional axis *sine qua non*, but as I mentioned in that exposition, the same collection of eight row forms could also have been presented in such a way as to emphasize I_6. Such a presentation would alter rule (2) above: instead of requiring that "only row forms with the same subscript are played together," the formulation would read "only row forms with different subscripts are played together." The configuration in Figure 3.8 shows all forty-rows organized according to the I_0 / I_6 inversional potential: given a row form in one oval, its I_0 or I_6 partner exists in the same oval, on the opposite side. (The collection of rows in $\mathbf{A_0}$ exist on this space on the far left side.)

The CTETS invariance shown by the horizontally oriented ovals indicate that the interaction of I_0 / I_6 inversional potential with CTETS invariance is coincident with *minimal differentiation* of CTETS. In fact, this figure indicates that CTETS structure and I_0 / I_6 structuring are *mutually exclusive*: two I_0 / I_6-related row forms *cannot* share **CTETS**. And even more generally, *no even axis of inversion can relate row forms that share CTETS*. We may frame this more positively: choosing to compose with with I_0 / I_6 as an axis of inversion, or even more generally, choosing to compose with an even axis of inversion, immediately suggests the sort of differentiation of CTETS that we saw in each of the refrains.[25]

FIGURE 3.8. Rows in intersecting ovals share CTETS and inversional potential.



Though $\mathbf{A_0}$ has a presence on this space, the five other ovals are note quite the same as $\mathbf{A_0}$. Most importantly, not all of them can be joined by the group of one-note chains. The figure shows that, in addition to $\mathbf{A_0}$, only on other collection of rows in the far right oval is similarly connected by chains. (This collection will comprise $\mathbf{A_3}$, and I will soon show that, not

[25] *Minimal* differentiation, however, is a compositional choice. Figure 3.8 indicates that even with an even axis of inversion, CTETS could differ by a greater degree.

surprisingly, it is $\mathbf{A_0}$'s primary partner in the larger structure of the piece.) Thus, as an even inversional axis necessarily involves CTETS differentiation, a particular even axis also limits the collections of row forms that can fulfill that axis *and* be joined by one-note chains.

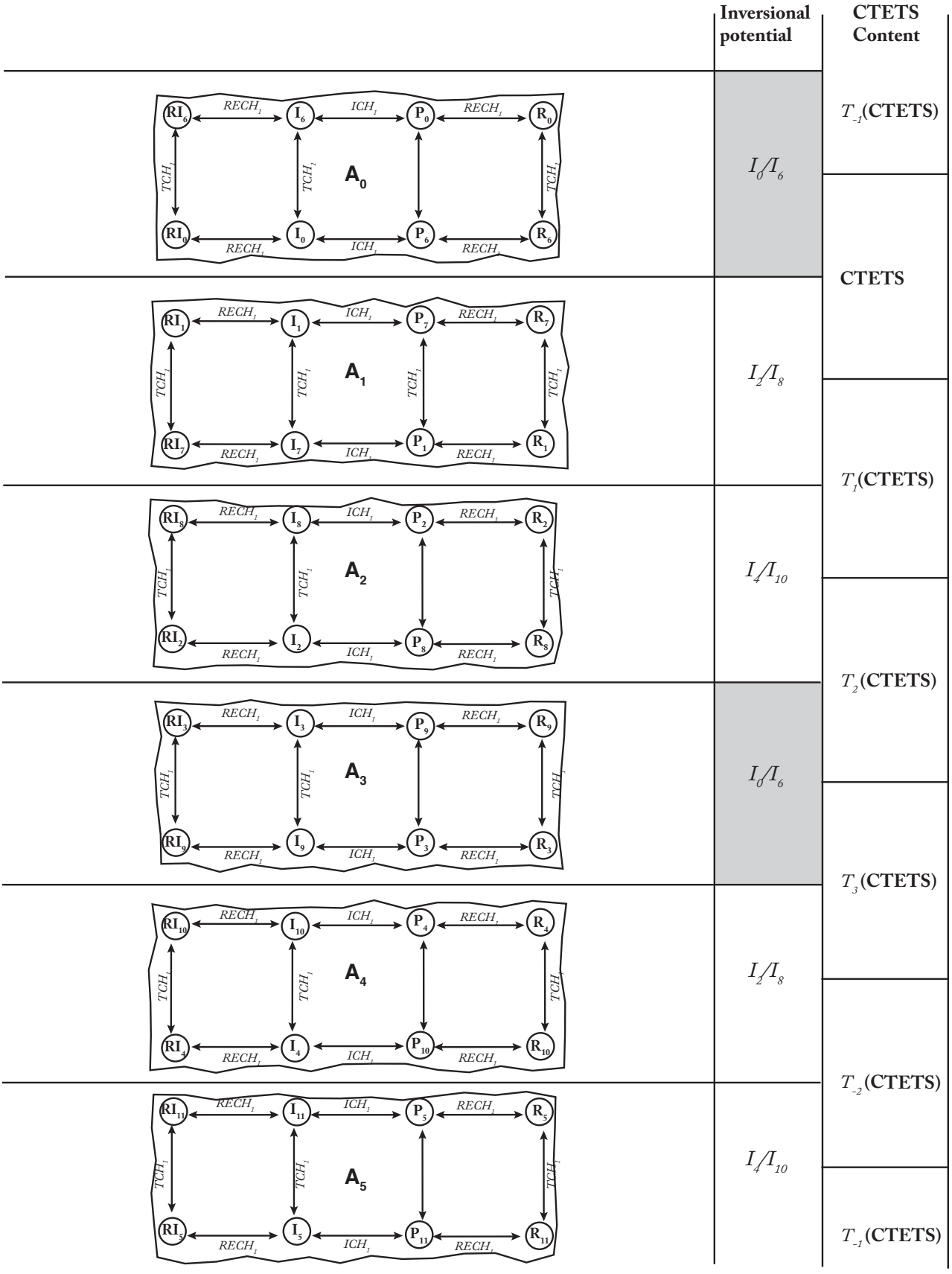
Whereas Figure 3.8 assumed inversion and invariance a priori, Figure 3.9 is generated from the one-note chain group $\langle TCH_1, ICH_1, RECH_1, RICH_1 \rangle$. (As an order 8 group, $\langle TCH_1, ICH_1, RECH_1, RICH_1 \rangle$ partitions the forty-eight rows in 6 (= 48 ÷ 8) areas.) The earlier figure indicated how CTETS and inversional axis suggest particular chain relationships; this figure shows the converse. Unlike that figure, each of the six areas shown here has the same transformational structure, and are thus labeled from $\mathbf{A_0}$ to $\mathbf{A_5}$.[26] Along the right side, each of the six areas are interpreted according to their inversional potential and CTETS content. In this light, certain correspondences appear that were not shown on Figure 8. In particular, every row area whose subscripts differ by 3 (such as, but not exclusive to, $\mathbf{A_0}$ and $\mathbf{A_3}$) share the same inversional potential. Simultaneously, every row area is endowed with two sets of CTETS one half step apart, as we saw in the refrain. Only six distinct sets of CTETS are present (labeled as T_{-2}(**CTETS**) through T_3(**CTETS**)), and the example shows that CTETS content overlaps

[26] The figure is interesting as regards the quartet's larger structure. First, it indicates that the inversional potential of a chain-generated group must be even. (This follows from the the tritone that bounds the first and last pitches of a row.) Thus, if Webern had chosen to compose around an even inversional axis, the chains would have naturally emerged, and vice versa. And once a particular referential row had been chosen, the *specific* even inversional axis would have presented itself. (In fact, this is precisely the character of the refrain. As we saw above, the chain connections predate the inversional canons.)

The structure of Figure 3.8 and Figure 3.9 indicate differences and intersections between the second and first movement. Numerous analyses have noted that the first movement has a inversional canon structure oriented around I_0, which is reflected through vertical ovals on Figure 3.8. That movement, unlike the second, does not make use of chains. Thus, Webern has a different sort freedom as regards row relationships in the first movement: $\mathbf{P_1}$ and $\mathbf{I_{11}}$, for example, initiate the opening of the movement and find their homes across from one another in one of the two central ovals. Those two are in different areas in Figure 3.9, because they cannot be linked by the one-note chain group.

It is worth noting as well that Mead, "Webern and Tradition," finds three properties operant in the first movement, two of which are also visible on Figure 3.9. Property 1, which holds invariant the second-order, all-combinatorial hexachord occurs amongst those rows in the cross-section of two ovals, and the larger collection of rows in the two central ovals. Property 2 is similar to the **CTETS** property, and as we've seen, relates rows in the same vertical ovals.

FIGURE 3.9. Six row areas (**A_0–A_5**), generated from one-note chains, shown as they relate to a row's CTETS and "inversional potential."



between areas whose subscripts differ by 1.[27] (Compare **A_0** /**A_1**, and **A_0** / **A_5**.)

Note that those row areas who share the same inversional potential (**A_0** and **A_3**, for example), have *maximally different CTETS*. This relationship between CTETS and inversion is quite far reaching, impacting the structure of the movement at its deepest and most shallow. To recapitulate, then: given any one of the three principles we have been discussing—chain based-composition, composition around an even inversional axis, or composition with **CTETS**—the other two principles somewhat naturally suggest themselves in the following ways:

(1) Row areas generated by *one-note chains* have the *same inversional potential* amongst their eight constituent row forms.

(2) Row areas that have *same inversional potential* always have *maximally differentiated* ***CTETS***.

(3) Stated another way, row areas that have *maximally differentiated* ***CTETS*** always have the *same inversional potential*.

The whole of the one-note chain group is relatively *static*. The six areas that the group creates have individual CTETS's "flavors," limited inversional potential, and each of the one-note chains ensures maintenance of the spatial status quo, as we saw in the refrain. One-note chains are, of course, unable to link the six areas shown in Figure 3.9. Rather, in the movement a *progressive* chain—*$RICH_2$*—assumes a connective role. As one-note chains are characteristic of the stability associated with refrains, *$RICH_2$* is a "transitional" transformation that often functions as a formal connector.

$RICH_2$ affects row forms differently:

(1) *$RICH_2$* of an **R** or **RI** form leads to a row form belonging to the *area* that has the same inversional potential as the originating row form, but maximally different CTETS.

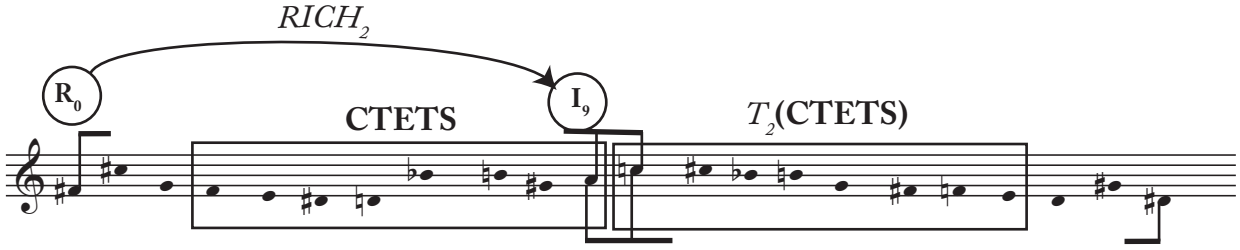
[27] Note that T_{-2}(**CTETS**) and T_3(**CTETS**) are different only by T_1. That is T_3(**CTETS**) = T_{-3}(**CTETS**).

(2) *RICH*$_2$ of a **P** or **I** form leads to a row form that has the same CTETS, but a different inversional potential as considered with the rows in its area.

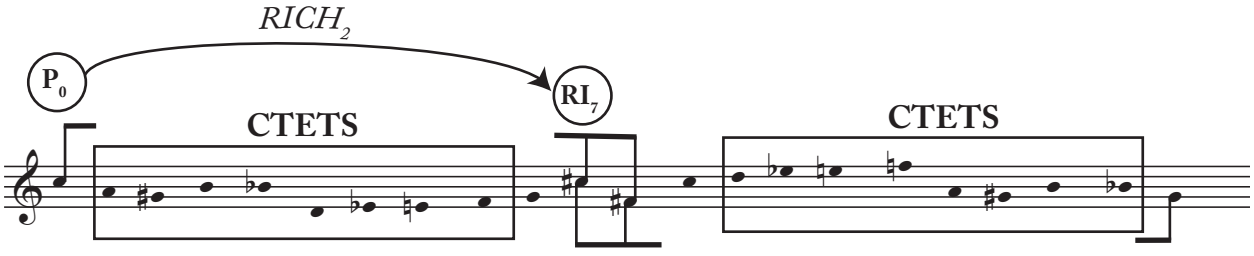
Thus, depending on context, each of the two "varieties" of *RICH*$_2$ could be viewed as quite progressive or very static. Figure 3.10 demonstrates, first showing *RICH*$_2$ acting on $\mathbf{R_0}$ and producing $\mathbf{I_9}$ at (a) and *RICH*$_2$ acting on $\mathbf{P_0}$ and producing $\mathbf{RI_7}$ at (b). At (b) the two CTET's maintain not only overall content, but also their order—a *Type-1* invariance that is stronger than amongst rows in the same area. However, the *row area* containing $\mathbf{RI_7}$ ($\mathbf{A_1}$) will not have the same inversional potential as $\mathbf{P_0}$. As we imagine $\mathbf{RI_7}$ acting in the piece, it cannot completely "substitute" for a row in $\mathbf{A_0}$, perhaps fitting into one of the refrains, simply because it is *not capable* of fulfilling all three of $\mathbf{A_0}$'s membership requirements. Transforming $\mathbf{R_0}$ into $\mathbf{I_9}$ at (a) results in a

FIGURE 3.10. *RICH*$_2$ as a progressive and static transformation.

(a) *RICH*$_2$ as a progressive transformation.



(b) *RICH*$_2$ as a static transformation.



general dispersal of the CTETS, but—as Figure 3.9 confirms—$\mathbf{R_0}$ and $\mathbf{I_9}$ belong to areas sharing the same inversional potential. It is static, then, in an exactly opposite manner.

A more comprehensive space in Figure 3.11 is generated by the one-note chain group plus $RICH_2$.[28] Each of the six row areas from Figure 3.9 has a home on the space, and each of those areas is connected entirely by one-note chains. $RICH_2$ creates the vertical and horizontal connections that *adjoin* the six row areas, showing that certain varieties of $RICH_2$ maintain CTETS and others inversional potential.

This space's geometry has many twists and folds. Each of the six areas are circular strips (flattened here, of course). Running into one of the jagged edges in any one of these areas will cause you to emerge in the same place on the opposite edge.[29] The large space itself is not a strip, but a four-dimensional torus. Exiting the left side of the space will result in reentry in the same horizontal row on the right side, arrow's cardinal direction reverses: when $RICH_2$ ($\mathbf{RI_0}$) leaves the left side, it re-enters in the same horizontal row on the right side, but instead of moving in a "north-westerly" direction, the arrow is now oriented towards the "southeast." That reorientation indicates a toroidal twist as the left side of the larger space is "glued" to the right. As chains move

---

[28] Unlike the one-note chains, $RICH_2$ is not an involution. Further, while the one-note chains are commutative with one another, $RICH_2$ is not commutative with the one-note chains. Therefore, the larger group structure invoked here is $(TCH_1 \times ICH_1 \times RECH) \rtimes RICH_2$. As an order 24 transformation, $RICH_2$ is able to completely connect the row forms that were previously disconnected.

[29] It is interesting that this twelve-tone space, along with that in Figure 13, indicate the potential for a subtype of Schoenberg's inversional combinatoriality. Inversional hexachordal combinatoriality establishes row areas on the basis of pitch-class invariance and inversional level. Pitch-class invariance in Schoenberg's combinatorial music takes on the form of invariance between entire hexachords. In other words, for two rows to be deemed combinatorial, their two, unordered hexachords must map onto one another through some inversion. So, inversionally combinatorial row areas are distinguished on the basis of a consistent inversional axis and pitch-class invariance. Similarly, the two twelve-tone spaces shown in Figures 3.13 and 3.14 has both inversion and pitch-class invariance at its core. There are two primary differences. One, unlike Schoenberg's combinatoriality, this space preserves only the two chromatic tetrachords. In other words, it is "tetrachordal combinatoriality" rather than hexachordal combinatoriality. Second, there is not an exact, one-to-one relationship between inversional level and pitch-class invariance. Maintained inversional levels in Schoenberg's combinatorial music insure maintained hexachordal invariance. In the case of the space in Figure 3.14, only those members of the diagonal threads will maintain both inversional level and pitch-class invariance.

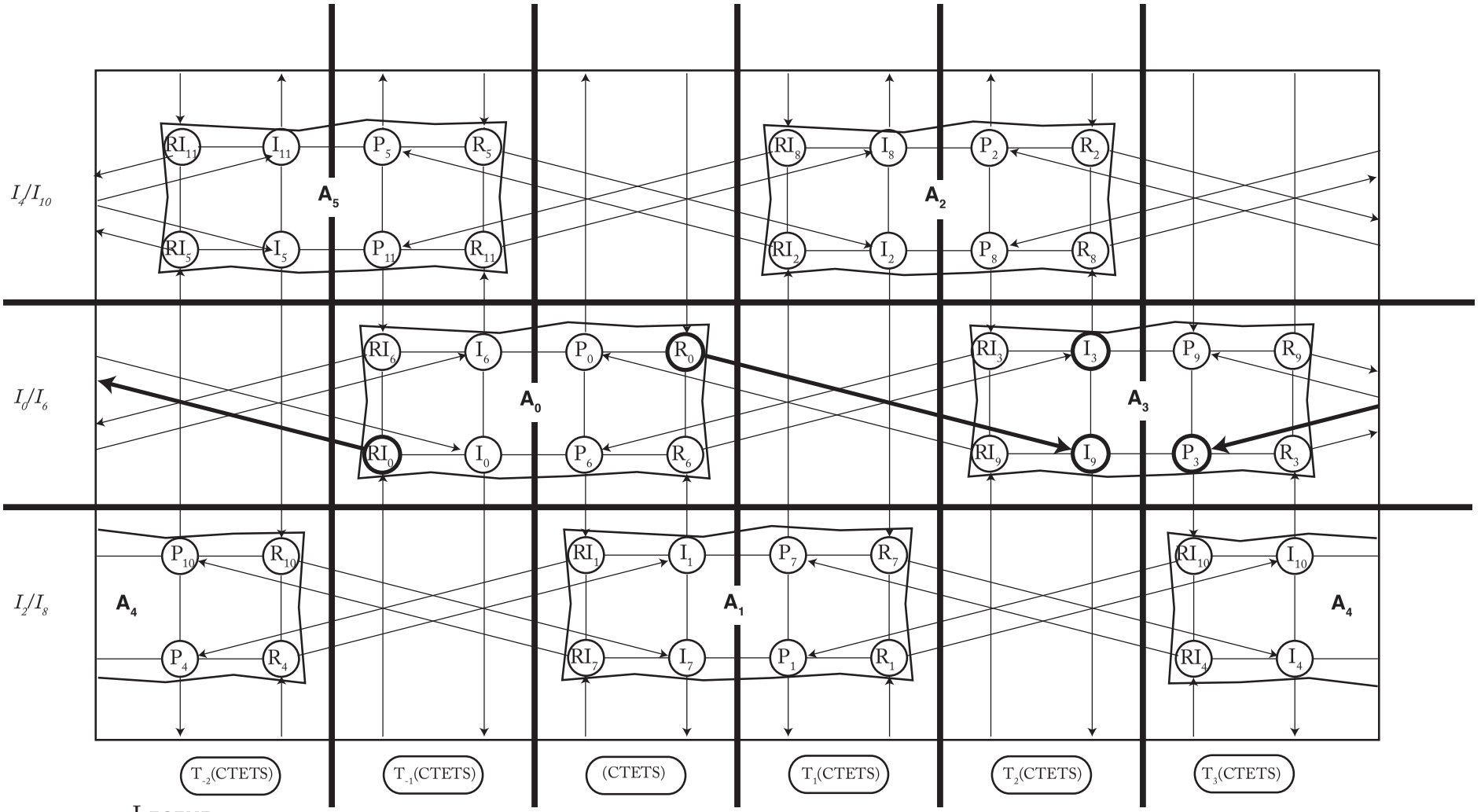
vertically, running into the top or bottom of the space simply causes reentry in the same column on the opposite edge.

The properties of invariance and inversional potential that were tracked in Figure 3.9 are shown here along the horizontal and vertical axis. *RICH*$_2$ arrows that are oriented vertically initiate from **P** or **I** forms *only,* and as the CTETS below the space indicate, are associated with maintaining the initiating row's CTETS. Thus, the $\mathbf{P_0} \rightarrow \mathbf{RI_7}$ motion from Figure 10 is found in one of the two central columns. The arrow leaving $\mathbf{P_0}$ disappears into the top edge, and after reappearing on the bottom, runs into $\mathbf{RI_7}$. The entire collection of row forms in that column have the same CTETS—$\mathbf{P_0}$ and $\mathbf{RI_7}$ included . By contrast, horizontally oriented *RICH*$_2$ arrows begin only at **R** or **RI** forms; they are are coincident with a maintenance of the row area's inversional potential along with a concurrent dispersal of a row's **CTETS**. Imagine, for example, that you are at $\mathbf{RI_0}$ / $\mathbf{R_0}$—sitting on opposite ends of $\mathbf{A_0}$. At this point, you are a valid row pair in one of the movement's refrains. Both sets of CTETS are represented, the I_0 inversional potential is present. If, as the space shows with bold arrows, you both *RICH*$_2$, the arrival point ($\mathbf{I_9}$ / $\mathbf{P_3}$) will still be capable of acting around I_0, though the CTETS will have completely changed.

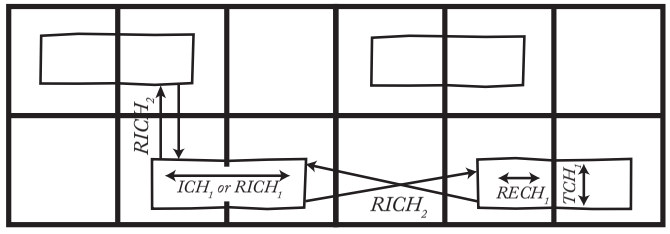
In light of these properties, some general features of the movement's rondo design emerge as amplifications of the structural premises set forth in the refrain.[30] In Figure 3.12(a) I have indicated three principles of row area construction that will be recognized from above: an even inversional axis invokes minimal CTETS differentiation amongst its concurrent, inversionally related row strands. The chain basis of the majority of row connections ensures the status quo; every row in $\mathbf{A}_x$ can fulfill the requirements of the refrain.

[30] What follows in this section imagines the music unfolding much like the process outlined in Robert Morris, "Compositional Space and Other Territories," 329. There, Morris shows a model that proceeds from basic ideas—such as those we have discussed in reference to the refrain—advances throughout a compositional space such like Figure 3.11, and in interaction with a compositional design and improvisation, results in a draft, score, and performance. In Morris's model, the compositional space provides crucial "feedback" from stage to stage.

FIGURE 3.11. A one-note, chain generated space organized by CTETS and inversional potential.



LEGEND



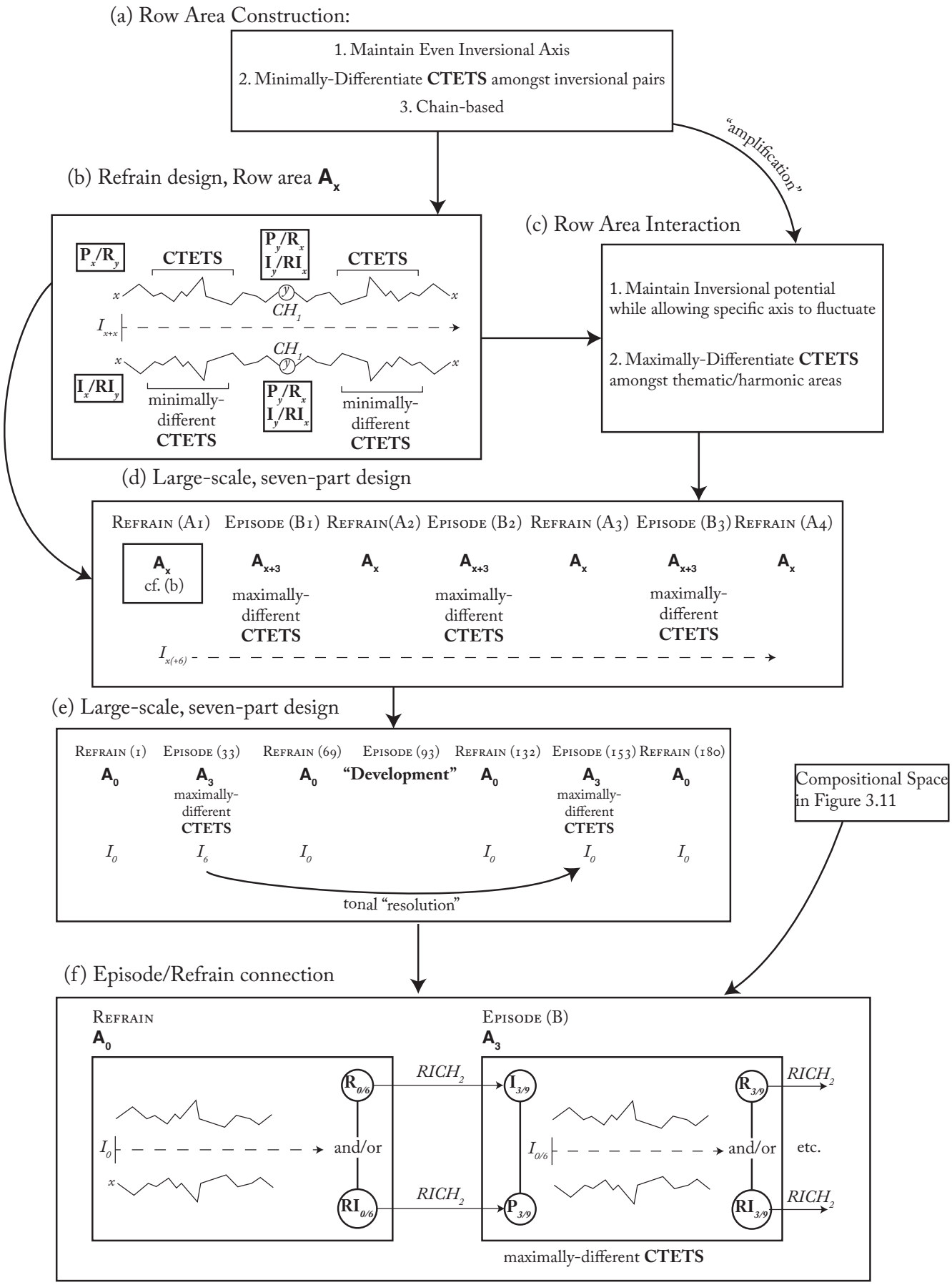
Those constructive principles are shown "in action" as a compositional design in Figure 3.12(b). The design does not capture the subtle increase in the prominence of the inversional axis that we saw in the first and third refrains. Rather, it imagines those features of the refrain as concrete manifestations of the more general axioms shown there. At this stage in the design, the refrain row forms could be drawn from any of the six areas in Figure 3.11, as the compositional space shows that all of them embody the CTETS/inversional potential/chain-construction principles required by the design.

From here, the refrain's constructive principles are "amplified" at (c) into axioms that govern *row area interaction* between refrains and episodes. While CTETS differentiation and axis structure are maintained, as row areas interact, CTETS are *maximally differentiated,* and though the inversional potential is constant, the specific axis of symmetry is allowed to fluctuate. It is possible to imagine row areas interacting that share the same CTETS, but if that were the case, Figure 3.11 shows that it would not be possible for each of those areas to possess the same inversional potential. As noted earlier, CTETS and inversional axis are in some ways mutually exclusive. The choice to vary CTETS in the design excludes a constant axis of symmetry.

That the movement follows the path at (c) is suggestive in two ways: first, it indicates how the larger structure of the piece mimic the refrains, as amplifications of their basic principles; and second, it begins to suggest some analogies to "theme" and "key" that are "baked into" the compositional space itself. In this scheme, certain pitch-structural components change, while others do not, and the potential for change in this respect, and moreover, change that occurs independently of other structural principles will be suggestive at a later point.

The seven-part design at (d) reflects the amplification from (c), simultaneously plugging the compositional design from (b) into the four refrains. The row areas ($\mathbf{A}_x$ and $\mathbf{A}_{x+3}$) involved in this large plan are suggested by the amplification itself, and Figure 3.11 shows why. Given a particular set of CTETS, the collection of CTETS maximally-different will be three columns to

FIGURE 3.12. Compositional design for the second movement.



the left or right. If the refrain is based on rows from $\mathbf{A}_x$, then, Figure 3.11 shows that only one other row area is three columns to the left or right—$\mathbf{A}_{x+3}$.[31] Coincidentally, as we saw above, that row area also shares $\mathbf{A}_x$'s inversional potential, so the row area interaction has the ability to echo the row strands's relationship in the refrain.

Our earlier analysis showed how $\mathbf{A_0}$ acted as a structural background for the refrains. Inserting that detail into the refrain variable at (d) fills out the diagram as I have done at (e)—a fair representation of Webern's movement. $\mathbf{A_0}$'s partner in the larger formal organization is $\mathbf{A_3}$. The properties embodied in the space from Figure 3.11 have been explicitly guiding all this process, and at this point, they also constrain structural features of the connections between refrain and episode. Figure 3.12(f) shows the refrain design again, abutting it against an episode. The design indicates that the connection between the two is guided by the ideals embedded in the large formal plan and the *realities* of the Figure 3.11 space. That space shows that if chains are involved in the process of connecting refrains and episodes, the final row or row forms of the refrain must be an **R** or **RI** form (and the initial row or row forms of the episode must be a **P** or **I** form). This limitation occurs because **R** and **RI** forms are the *only* row forms who, when *RICH*$_2$*-ed*, will connect $\mathbf{A_0}$ to $\mathbf{A_3}$ and vice versa.

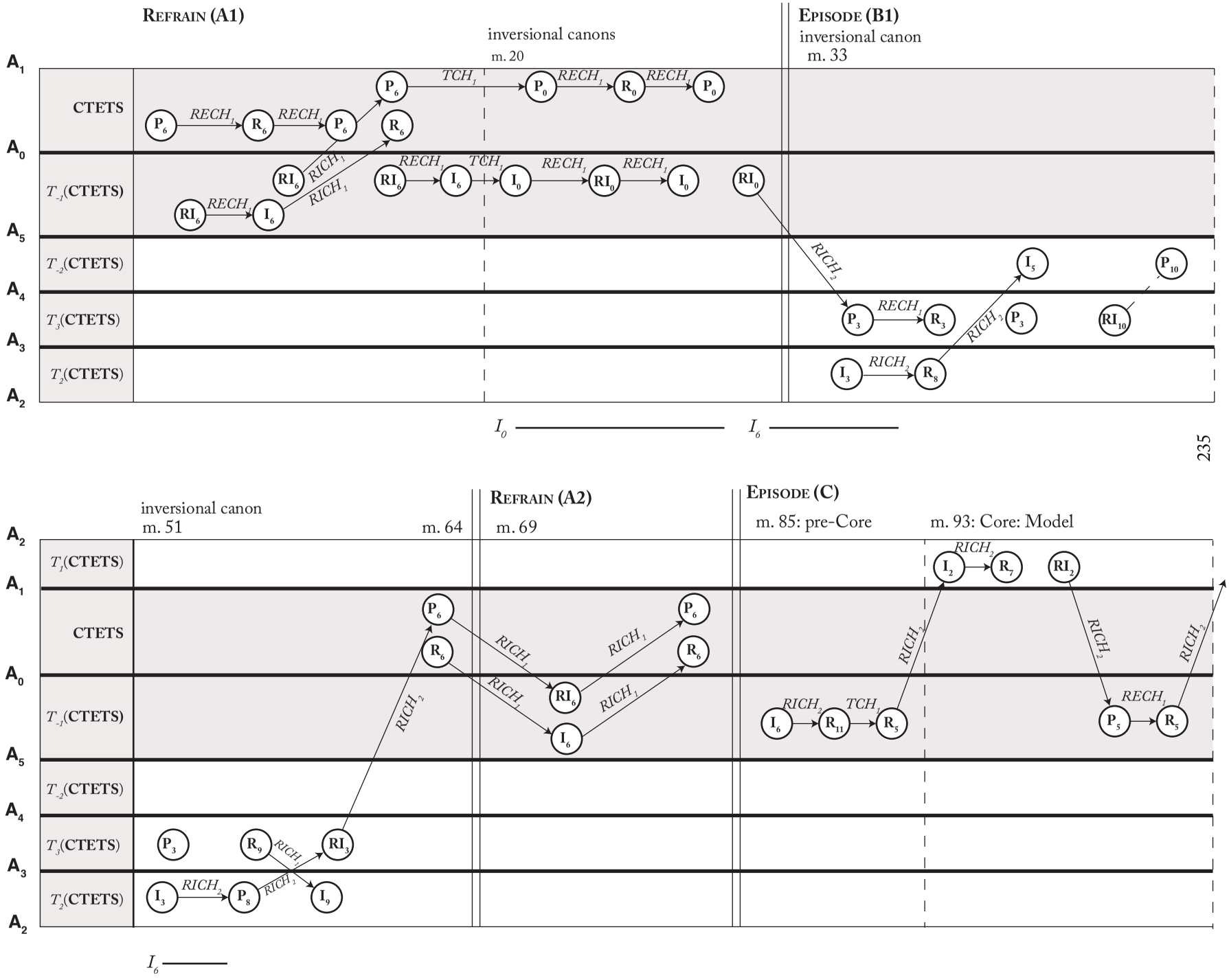
III: Webern Contra Beethoven

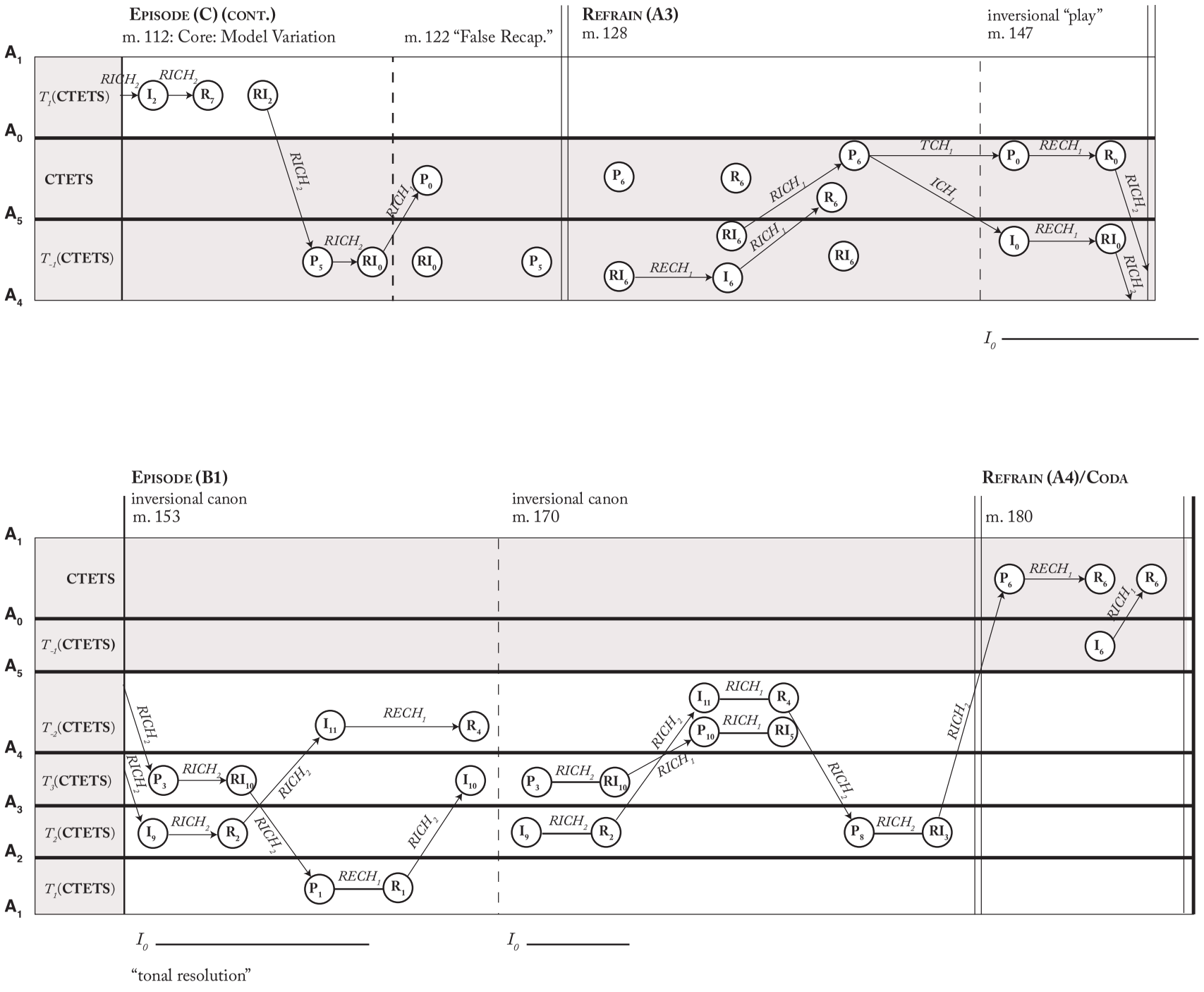
Now that the formal shape of the movement has begun to crystallize as a manifestation of properties of the row class, I can begin to explore precisely how those properties are involved in the movement's interaction with the classical rondo, and Beethoven's rondo in particular.

---

[31] Strictly speaking, two different row areas are found three columns to the left or right. But, only one of those areas is three columns away from the *whole* of $\mathbf{A_x}$.

FIGURE 3.13. An event network for the second movement with formal annotations above and below.




EPISODE (C) (CONT.)
m. 112: Core: Model Variation
m. 122 "False Recap."
REFRAIN (A3)
m. 128
inversional "play"
m. 147
A1
A0
A5
A4
T1(CTETS)
CTETS
T-1(CTETS)
RICH2
RECH1
TCH1
ICH1
I0
EPISODE (B1)
inversional canon
m. 153
inversional canon
m. 170
REFRAIN (A4)/CODA
m. 180
A1
A0
T-1(CTETS)
A5
T-2(CTETS)
A4
T3(CTETS)
A3
T2(CTETS)
A2
T1(CTETS)
CTETS
RICH1
I0
"tonal resolution"

Above, I suggested that Kathryn Bailey's critique of the movement's relationship to that piece reveals an underlying pessimism as to the ways in which the twelve-tone system may reflect the principles of tonality, especially as it interacts with musical form. As a basis for exploring ways that I believe Webern's movement *does* reflect these principles, I will explore some traits of the Beethoven movement that Bailey mentions in terms I have developed. I realize that such this connection can only go so far. Webern noted a "formal analogy" to Beethoven's movement, but did not *model* his composition on the movement, and I do not wish to fall into the trap of expecting Webern's movement to be an exact copy of Beethoven's. Yet, Beethoven's rondo has a number of characteristic features—some idiosyncratic—that involve the interaction between form and tonality, the following four of which will be interesting case studies as I scrutinize Webern's rondo's relationship to tonal form:

(1) In Beethoven's rondo, each of the refrains is tonally "closed." That is, all of the refrains begin and end in the same key.

(2) The first episode is tonally "open." After beginning in a contrasting key, the episode modulates back to the tonic key, preparing the beginning of the second refrain.

(3) Unlike the other two episodes, the final episode is in the tonic key

(4) A "false recapitulation" occurs at the end of the second episode.[32]

To study these traits in relation to Webern's movement, Figure 3.13 unfolds the large space from Figure 3.11 to map the progression of row forms in relation to their underlying CTETS. (In actuality, this figure would be best rendered as a "tube" that extends to the end of the piece. Because this is unwieldy, Figure 13 freely "rotates" the tube to make it easier to see some of the relationships I wish to highlight. In order to maintain a sense of perspective, the rows containing

[32] All of these claims should be uncontroversial and easily identified by consulting the score. As a refinement, I will mention only that, although the refrains are tonally closed, the second and third refrains are followed by "dissolving transitions." By "dissolving," I mean that the transitions are based on thematic material taken from the refrains. In fact, these transitions have counterparts in Webern's movement, but exploring them in detail is somewhat beyond the scope of this chapter.

A_0 are greyed-in.) Below the figure, inversional axes are indicated as appropriate. For example, in both the first and third refrains, the I_0 axis is shown below the second half of the passage.

"Each of the refrains is tonally 'closed.'"

On Figure 3.13, each of the four refrains is confined to **A_0**. I had noted this implicitly in relation to the first and third refrains, and Figure 3.13 confirms that—unlike the three episodes—the four refrains are "landlocked" by the one-note chain pathways that inhibit passage into the surrounding areas. [33]

I have discussed the first and third refrains, in some detail, noting their projection of two compositional "rules." Bailey is skeptical that a second refrain occurs at all: "the return beginning in *bar 64* is particularly difficult to apprehend, as the tonal section defined by untransposed rows [**P_6**, **I_6**, **R_6**, and **RI_6**] does not correspond to the structure outlined by the musical content" (245, emphasis is mine). Bailey, as she often is the case, is seeking an absolute correspondence between row structure and "musical content." After locating a return to "untransposed rows" in m. 64—equivalent to one half of what I have called **A_0**—and noticing that there does not seem to be a corresponding musical return, Bailey deems the formal outline blurry, calling into question the very existence of a second refrain at all. (More on this "mismatch" soon.)

[33] Bailey, *The Twelve-Note Music of Anton Webern*, 245, analyzes only half of **A_0** as the "tonic region," specifically those rows with 6 as a subscript. Invariance relationships suggest, however, that this "tonic region" should be expanded. (**P_0** and **P_6** are very similar from this perspective, for instance.) Bailey's analysis of the smaller tonic region may be due in part to Bailey's tendency to privilege the first note of a row and the first row statement of a piece. But that analysis would imply that the refrain begins and ends in a "different key," and it is not clear that a strong musical basis for that distinction exists.

Figure 3.14(b) shows the passage in question, and at (a), the first two bars of the opening refrain. Rhythmic, dynamic, and articulation correspond, which certainly seems to indicate that the second refrain begins at *m. 69* and not in *m. 64*, where Bailey quote above claims.[34] While shorter, this refrain is built from the rhythmic figure heard in mm. 1-2, which juxtaposes a quarter-note voice (mostly sounding on the second beat) with a half-note voice (articulating the downbeat. My alignment of the two systems of the refrain at (b) and arrangement of the row strands show that both lines are present here and that this refrain's two halves are rhythmically identical: mm. 69-78 are the same as mm. 79-88, though the pitch content varies. Taken as a whole, the quarter-note voice of both halves is rhythmically symmetrical (around m. 74 and m. 84), which also calls to mind the retrograde structuring of the opening seven measures (Figure 3.5).
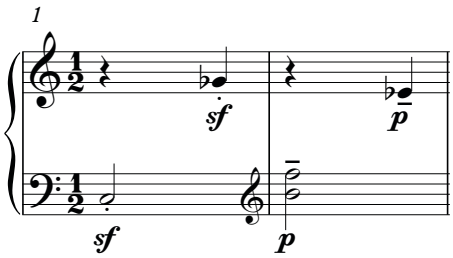
Unlike the first and third refrains, this refrain does not follow the first "rule." That is, as Figure 3.13 can confirm, the P-side and I-side are not "present at all times." Despite this, considered in whole, both "sides" of $\mathbf{A_0}$ are played in the passage: the I-side on the first system and the P-side on the second. In m. 69-78 the two chromatic tetrachords associated with the I-side—T_{-1}(**CTETS**)—sound prominently in the quarter-note voice, and as mm. 79-88 rhythmically recapitulate those measures, one of the chromatic tetrachords {G♯, A, B♭, B} associated with the P-side **CTETS** plays in the quarter note voice and is echoed in the half note voice. As the second of the tetrachords {D, E♭, E, F} is set to complete in m. 84—{E, F} sound in

[34] Bailey's interpretation (*The Twelve-Note Music of Anton Webern*, p. 245, note 14. is shown to differ from both Fennelly, "Structure and Process," and Smith, "Composition and Precomposition," both of whom hear the return at m. 69. Bailey's choice of m. 64 is certainly indicative of her tendency throughout her analysis to hear only one half of $\mathbf{A_0}$—[$\mathbf{P_6}$, $\mathbf{I_6}$, $\mathbf{R_6}$, and $\mathbf{RI_6}$]—as capable of carrying "tonic" function, rather than the larger group containing 0-forms as well.

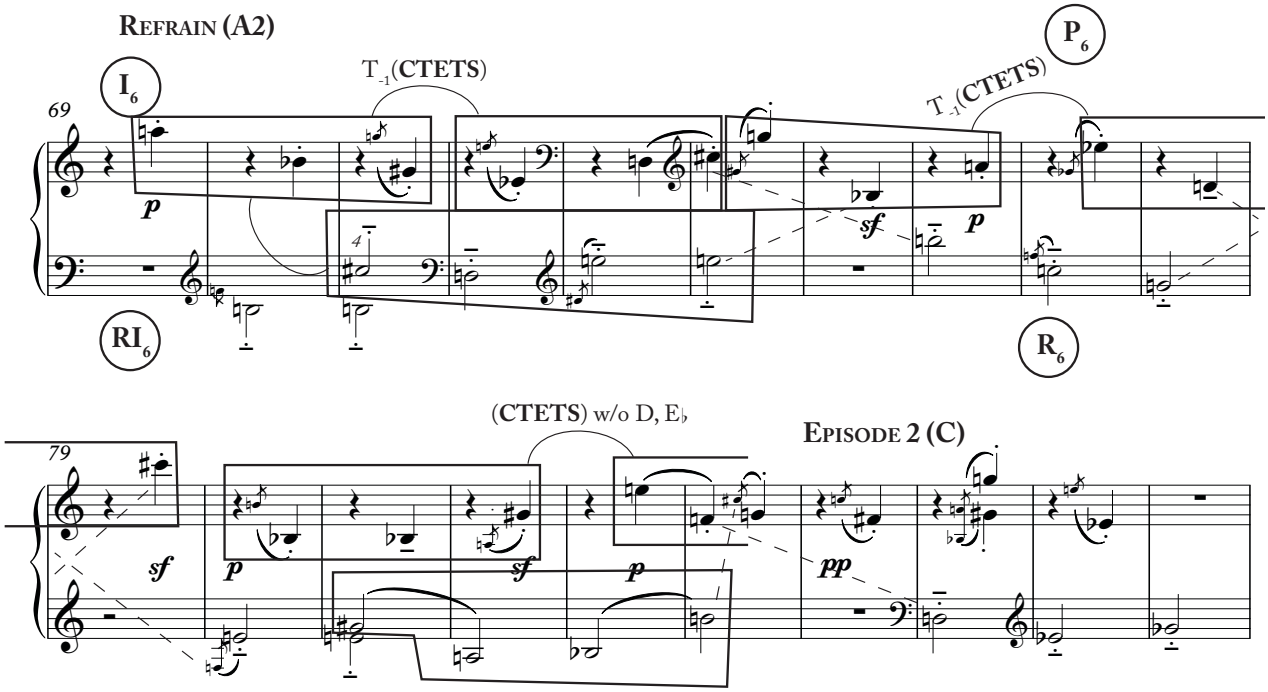
The refrain has a great deal more in common with the first half of the first refrain than the second half. Bailey, *The Twelve-Note Music of Anton Webern*, 245 mentions something similar to this. She notes in relation to the latter halves of the first and third refrains that "in a tonal sense … these sections built on tritone transpositions of the row seem to be simply extensions of the preceding tonic areas."

FIGURE 3.14. Comparing the first and second refrains.

(a) Refrain 1, mm. 1–2.



(b) Second Refrain, mm. 69ff.



m. 83-84—Webern moves the $\mathbf{P_6}$ row strand into the half note voice. This movement destroys the **CTETS** completion, but at an apt formal point: mm. 85 begins the second episode.

"The first episode is tonally 'open.' After beginning in a contrasting key, the episode modulates back to the tonic key, preparing the beginning of the second refrain."

Classical form is created primarily through thematic and tonal differentiation. I just noted that the refrains *always* occur within the confines of $\mathbf{A_0}$. Conversely, the episodic material is based in $\mathbf{A_3}$, venturing into $\mathbf{A_0}$ at only two points. One of these moments occurs at the end of the first episode, in m. 64—the point at which Bailey identified the second refrain. This anticipation of $\mathbf{A_0}$ mimics a similar moment in Beethoven's rondo: there, the first episode is tonally open, beginning in E minor and closing in the tonic, G major. Figure 3.13 shows how the second episode is also "tonally open," its last-minute move into $\mathbf{A_0}$ preparing the return of the second refrain.

"The final episode is in the tonic key."

Both Webern's and Beethoven's rondos mimic the "sonata-rondo," where the second half recapitulates the first, the final episode acting as a "tonal resolution" by stating episodic material in the tonic key. [35] Recapitulation is much more a part of Webern's piece than Beethoven's, [36] and manifests itself in two ways, both owing to details of the row structure diagramed in Figure 3.13: First, the third refrain (beginning at m. 128) has a nearly identical structure to the first refrain. And second, the third episode (at m. 153) "recapitulates" the first (at m. 33), adjusting its "key."

The "tonal resolution" happens in a number of ways. Notice first on Figure 3.13 that the third episode begins in the same spatial territory as the first ($\mathbf{A_3}$), and thus, will project the same abstract **CTETS** as was heard in the first episode. (They share common rhythm and articulation as well.) Unlike the refrains, each of these episodes has the same "loose-knit" spatial construction: after beginning with a canon entrenched in $\mathbf{A_3}$, both venture away to spatial neighbors before

---

[35] Webern's movement seems more like a sonata rondo than Beethoven's. In Beethoven's rondo, the first episode is not in the dominant key, which is typical of a sonata rondo whose first refrain and episode are akin to a sonata exposition; and, the second episode is not a development, but a relatively tight-knit interior theme. We could make a case that Webern's movement has both of those things.

[36] Caplin, *Classical Form*, 235-41, defines the sonata aspects of a sonata-rondo in three terms: (1) the initial refrain and episode constitute a sonata exposition and the third refrain and episode a recapitulation; (2) the second episode is organized as a development; and (3) a coda—including the final refrain—is a required element.

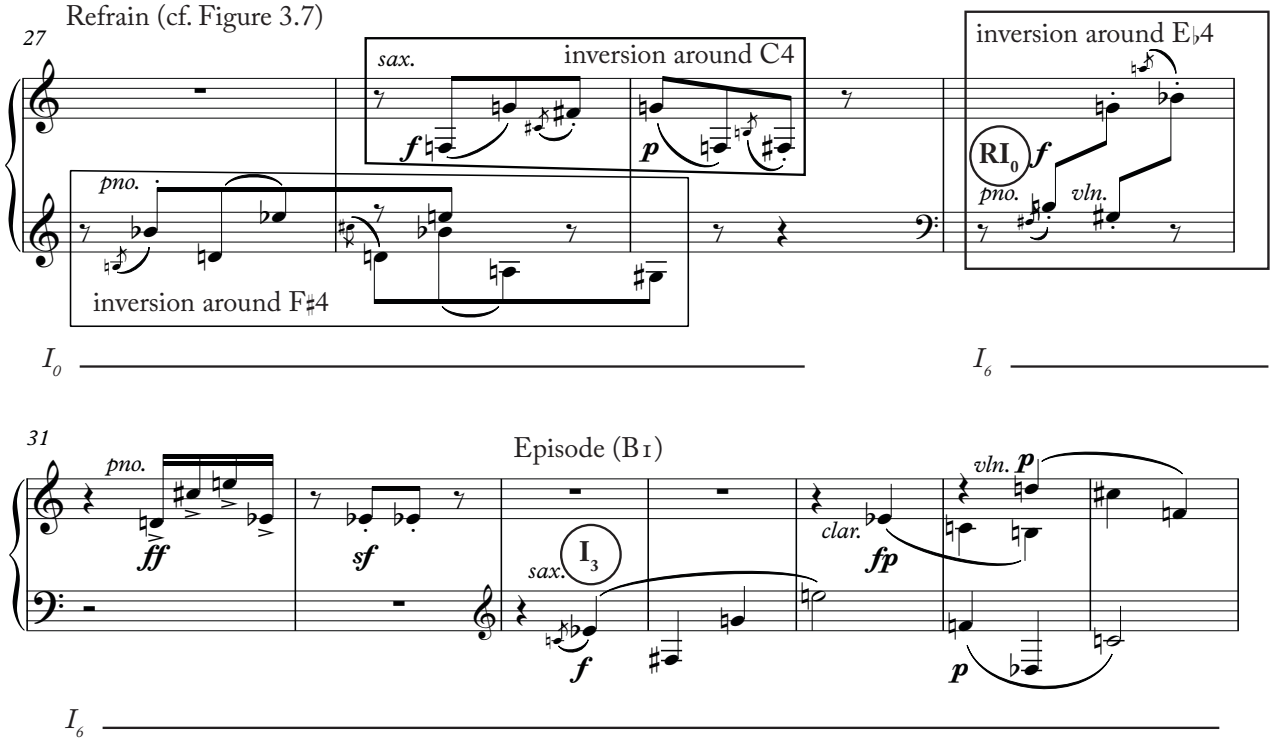
In addition to containing a "recapitulation" like that of the sonata rondo, Webern's movement also has a coda as its final refrain.

returning to **A3** to begin a second canon (at m. 51 and m. 170). Figure 3.13 shows that the "tonal resolution" occurs in the third episode when the I_6 inversional axis associated with the first episode is adjusted to I_0, matching the inversional axis of the refrains.

Figures 3.15(a) and (b) study this resolution in detail. The close of the first refrain is shown in Figure 3.15(a). We saw earlier (Figure 3.7) that the latter half of this refrain presents a series of short, three- and four-note gestures that are inversional around C5, F♯4, and C4—each representative of the abstract I_0 inversional axis. The last of these gestures sounds in m. 29, and is followed by a three-measure transition phrase that introduces a characteristic rhythmic idea. As that figure shows, the pitches of **RI0** (a member of **A0**!) are placed registrally in m. 30 such that a novel pitch axis emerges around E♭4—or more abstractly, I_6. Over the course of mm. 31-33, E♭ is articulated four(!) times as the end of **RI0** *RICH2s* into the beginning of **I0**—the initial row of the first episode. That row sets off a series of gestures that confirm I_6 as the new axis of inversion and

FIGURE 3.15. Inversional structuring from refrain to episode.
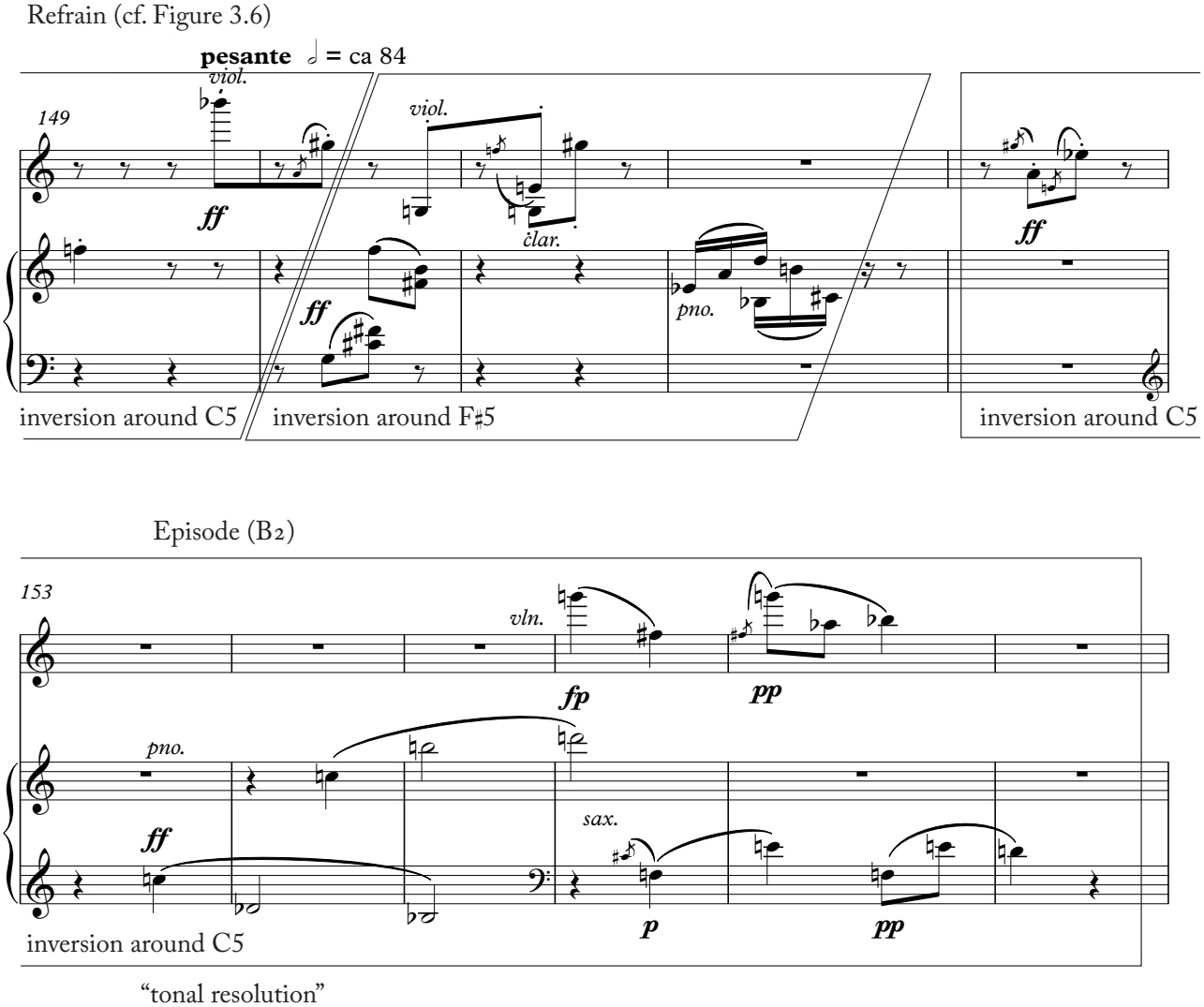
(a) I_0 yields to I_6 in the transition from the first refrain to the second episode.



A3 as the new row area. These new, three- and four-note gestures (clearly analogous to the three- and four-note gestures that ended the refrain) are more lyrical and project this new axis in pitch-*class* space. The imitation continues (somewhat sporadically, and often just rhythmically) for the remainder of the episode.

In this light, the predominance of inversional structuring at the *ends* of each refrain makes a great deal of compositional sense. In the final nine measures of the first refrain, I_0 as a structuring agent is quite powerful, as we saw in earlier. When at m. 30 I_0 yields to its inversional

(b) Inversional structuring of I_0 remains *constant* from the third refrain into the final episode, creating a tonal resolution.



partner I_6, the effect is quite transparent in part because Webern had emphasized I_0 repeatedly at the end of the refrain, and also because of subtle pitch repetitions: I_0 is represented by the repeated C6 in the violin and clarinet in m. 26 and m. 27, which becomes associated with the repeated E♭4, now representing I_6, in mm. 31-32.[37]

The same procedure occurs at the transition into the third refrain, but the inversional axis is "adjusted" to remain in I_0, thereby "resolving" the contrasting key of the the first episode. Figure 3.15(b) shows this passage. Earlier (in Figure 3.6), I showed how the close of this refrain also involves inversional structuring around I_0. At the "pesante" marking, for example, the pitch motive {D3, E♭5, E♮4, F5} finds its I_0-inversional partner beginning at the attainment of the climatic B♭6. As in the first refrain, that axis yields to inversion around F♯4, and in the final measures, a transition passage (with a characteristic rhythm taken from the first refrain) anticipates the episode beginning at m. 153. Like the first episode, this episode begins canonically: two voices unfold symmetrically around C5, the inversional axis represented at the climax of the preceding refrain (m. 149).[38] *Thus, we find music in the same spatial location (**A_3**), but making use of the other inversional axis* (I_0)*, the one found in the refrains.* Once again, the axis is set up by a transition gesture: in m. 153 a staccato figure in the clarinet is disposed symmetrically around C5, making the connection between the refrain and episode possible.

Figure 3.13 calls this moment a "tonal resolution." To understand the degree to which this third episode "resolves" the first, compare the row structure of the the two episodes. The first (at m. 33) operates within $\mathbf{A_3}$, the specific row forms ($\mathbf{I_3}$ and $\mathbf{P_3}$) combining around to create the I_6 axis. The third refrain, in the rondo's recapitulation, also operates within $\mathbf{A_3}$, but the chosen row forms ($\mathbf{P_3}$ and $\mathbf{I_9}$) now combine to create an I_0 axis—the same axis that structures the refrain. The

[37] Interestingly, the second of the C6s is one of only two pitches in the second half of the refrain that are not in the "correct" register, according to the surrounding canonic structure.

[38] C5 was also the most prominent axis in the opening refrain. One nice detail: Webern's graces (along with the particular disposition of row forms) allow G6 and F3 to be the highest and lowest notes of the passage, each heard twice in m. 156 and 157. Those were also the highest and lowest pitches in the second half of the first refrain.

RICH$_2$ connections bolded on the larger space in Figure 3.11 show the similarities and differences between the two connections. Both involve *RICH*$_2$(**RI**$_0$), which leaves from the left side and emerges on the right as it heads to **P**$_3$. In the first episode the arrival at **P**$_3$ is met with the emergence of **I**$_3$, creating the I_6 axis.[39] This is a *singular* connection: **I**$_3$ is *not joined* to the previous section but emerges as from thin air.

The corresponding passage leading to the tonal resolution at m. 153 shows why. Here, the same *RICH*$_2$(**RI**$_0$) leads to **P**$_3$. *But, this chain connection occurs concurrently with a RICH*$_2$ *from* **R**$_0$ *into* **I**$_9$. Because both chains occur simultaneously (and not, as in the first episode, singularly), the arrival point (at m. 153)*maintains* the I_0 axis. In retrospect, Webern's decision to thin the two canonic rows to one at the end of the opening refrain was a necessity. Emerging from a passage based on I_0, dual *RICH*$_2$ chains cannot lead to an episode based on I_6.

IV: False Recapitulation, Theme, and Key

"The primary task of analysis is to show the functions of the individual sections: the thematic side is secondary"[40]

The last of the case studies I proposed above involved the claim, questioned by Kathryn Bailey, that Webern's movement may contain a false recapitulation. In its classic form, the effect requires a discrepancy between theme and key. Typically, the primary theme returns at some point in the development, but in the wrong key.[41] Throughout this chapter, I purposely glossed

[39] The use of the same "subscripted" form is analogous to the practice throughout the refrain.

[40] Webern, *The Path to the New Music*, 57.

[41] False recapitulations have been investigated by many authors, most noting the effect of surprise created when a primary theme returns in a non-tonic key. See Mark Evan Bonds, "Haydn's False Recapitulations and the Perception of Sonata Form in the Eighteenth Century" (Ph.D. dissertation, Harvard University, 1988; Charles Rosen, *Sonata Forms* (New York: Norton, 1988) 282; Caplin, *Classical Form*, 159; James Webster and Laura Macy, "Sonata Form," *Grove Music Online*, accessed 2 August 2013, http://www.grovemusic.com. James Hepokoski and Warren Darcy, *Elements of Sonata Theory: Norms, Types, and Deformations in the Late-Eighteenth-Century Sonata* (New York: Oxford University Press, 2006): 221-226) have questioned whether such effects would have struck pre-1800 listeners as surprising at all.

over a distinction that is often central to classical form (and the false recapitulation effect)—that of "theme" and "key." Scholars often disagree as to their degree of importance, but most theories of form have some place for these components, whether they act independently or not. What is analogous to "theme" and "key" in this movement?

The "tonal resolution" of the first episode in the third offers a hint.[42] There, the CTETS content of the third episode remained the *same* as compared to the first episode (they both occupied $\mathbf{A_3}$) while the inversional axis *changed*, *adjusting itself to match the inversional axis of the refrains*. That adjustment (in which the I_6 "dominant" is resolved to an I_0 "tonic") suggests that inversional axis is acting like a "key" that organizes row strands. Conversely, CTETS *equivalence* between the first and third episodes, as well as the differentiation of CTETS amongst the refrains and episodes throughout, is associated with "theme." [43]
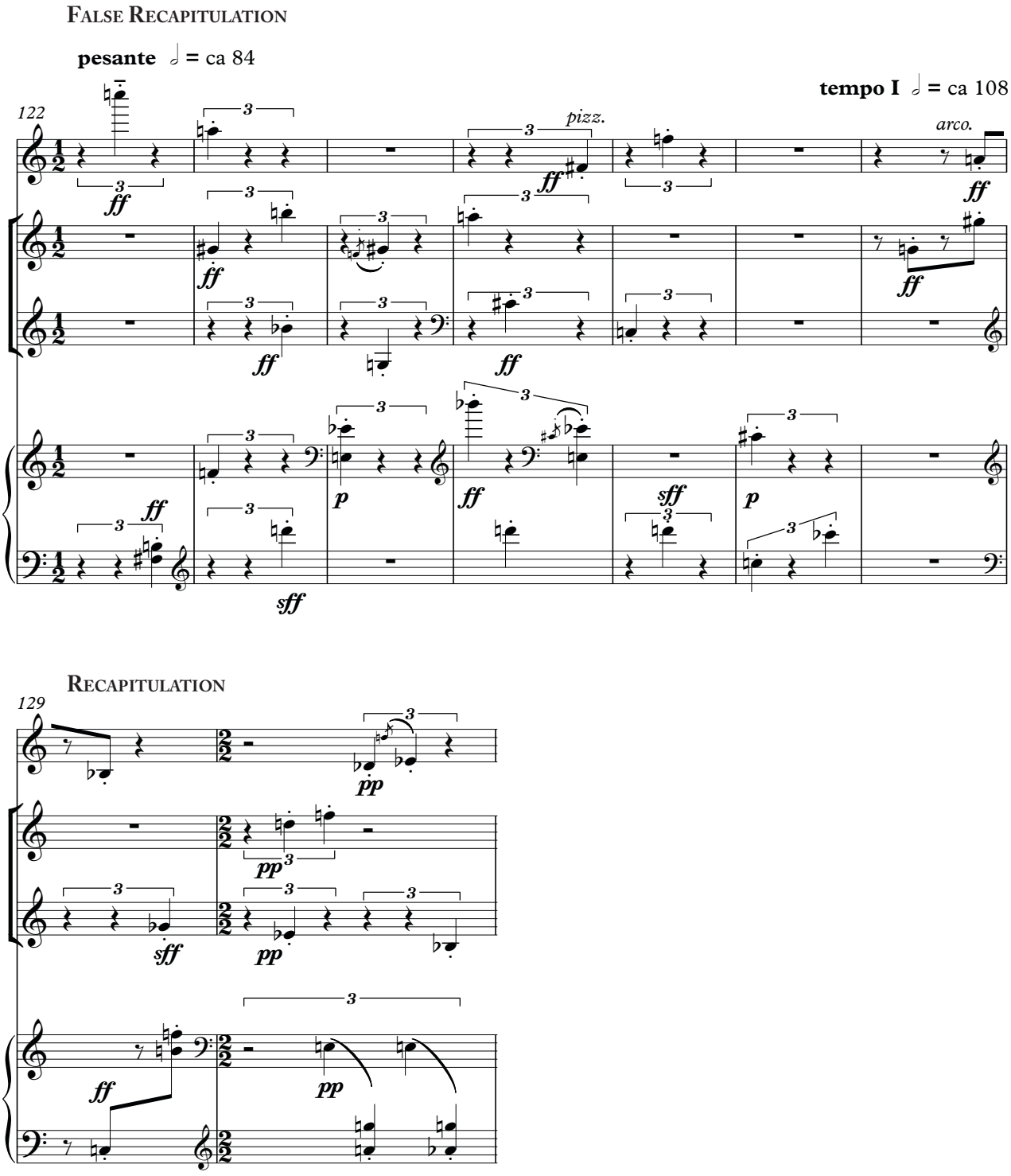
Such distinctions are forefront in the potentially "false recapitulation" at m. 122. Figure 3.16(a) shows both the passage, which ends with the real recapitulation at m. 129. Comparing m. 129 with the opening refrain (Figure 3.5) shows clear motivic and registral correspondences; there is little doubt that this is *the* recapitulation. Some questions remain, nonetheless; most notably, why doesn't the recapitulation correspond with the tempo change at m. 128? Is the four-note motive divided between the violin and clarinet {G, G♯, A, B♭} related to the recapitulation? That figure also shows how m. 122 could be conceived as a recapitulation as well. Though rhythmically and dynamically the passage is still very much a part of the episode that preceded it, m. 122 contains a dramatic C7 in the first violin that is juxtaposed with F♯3 in the piano—

[42] In fact, in later music, such as the opening movement of Op. 28, that lacks clear inversional structuring, this seems to be precisely the distinction used to create formal areas.

[43] In all of Webern's serial music, themes are clearly "underdetermined," often more motivic than thematic. And rather than seeking a strict correlation, it may be better to imagine these two types of structure (pitch(-class) motives and inversional axis) as dual agents in the creation of musical form. Like "theme" and "key," they may generally be better understood as they relate to one another, and not as exact analogies to "theme" and "key." This more holistic viewpoint allows us to imagine the "function" of a passage without assigning a specific, classical form-conditioned meaning to the components that determine that function.

recalling the C3 and G♭4 that began the piece. After m. 129, the incredible increase in rhythmic activity, heightened dynamic, full instrumental texture, and wide registral scope help identify the

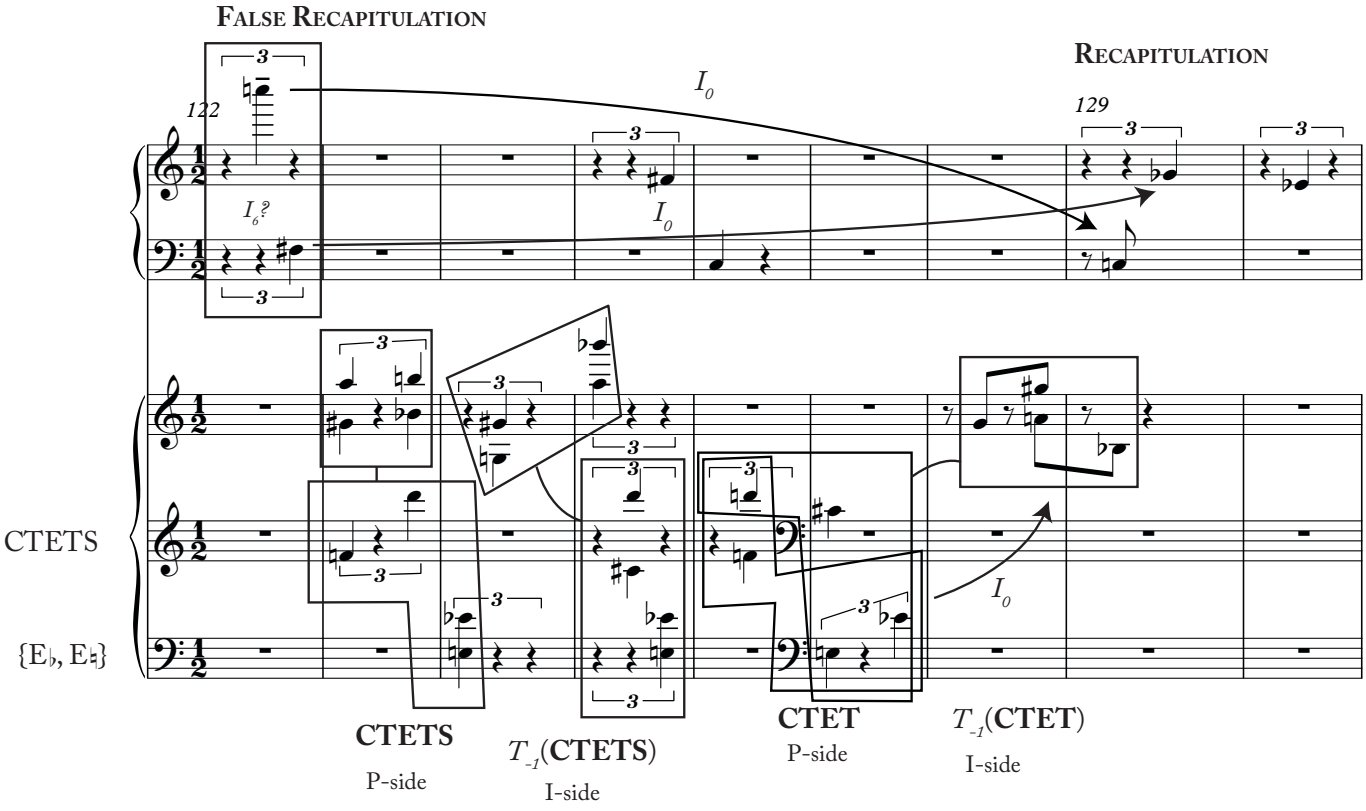
FIGURE 3.16. The false and real recapitulations, mm. 122–130.



music as the culmination of the developmental music that preceded it, and at m. 129, the retrospective reinterpretation of m. 122 as a deceptive early entry of the refrain is more or less clear.

The false recapitulation is more than superficial. Figure 3.17 shows how it involves a carefully controlled return of the "thematic area" associated with the refrain, $\mathbf{A_0}$, without the concomitant I_0 tonal area, which develops only at the moment of recapitulation and involves the violin/clarinet motive questioned earlier. On the bottom three staves of that reduction, I have boxed in the CTETS structure of the passage. As is know becoming expected in refrains, the P-side **CTETS** and I-side T_{-1}(**CTETS**) make an appearance, alternating over the course of mm. 123-127. Notably, the dyad {E♭4 E♮3} emphasizes the connection between the two sides. Its three-fold appearance over the course of the passage, always in the piano and always in the same

FIGURE 3.17. CTET(S) and inversional structuring at the false recapitulation.



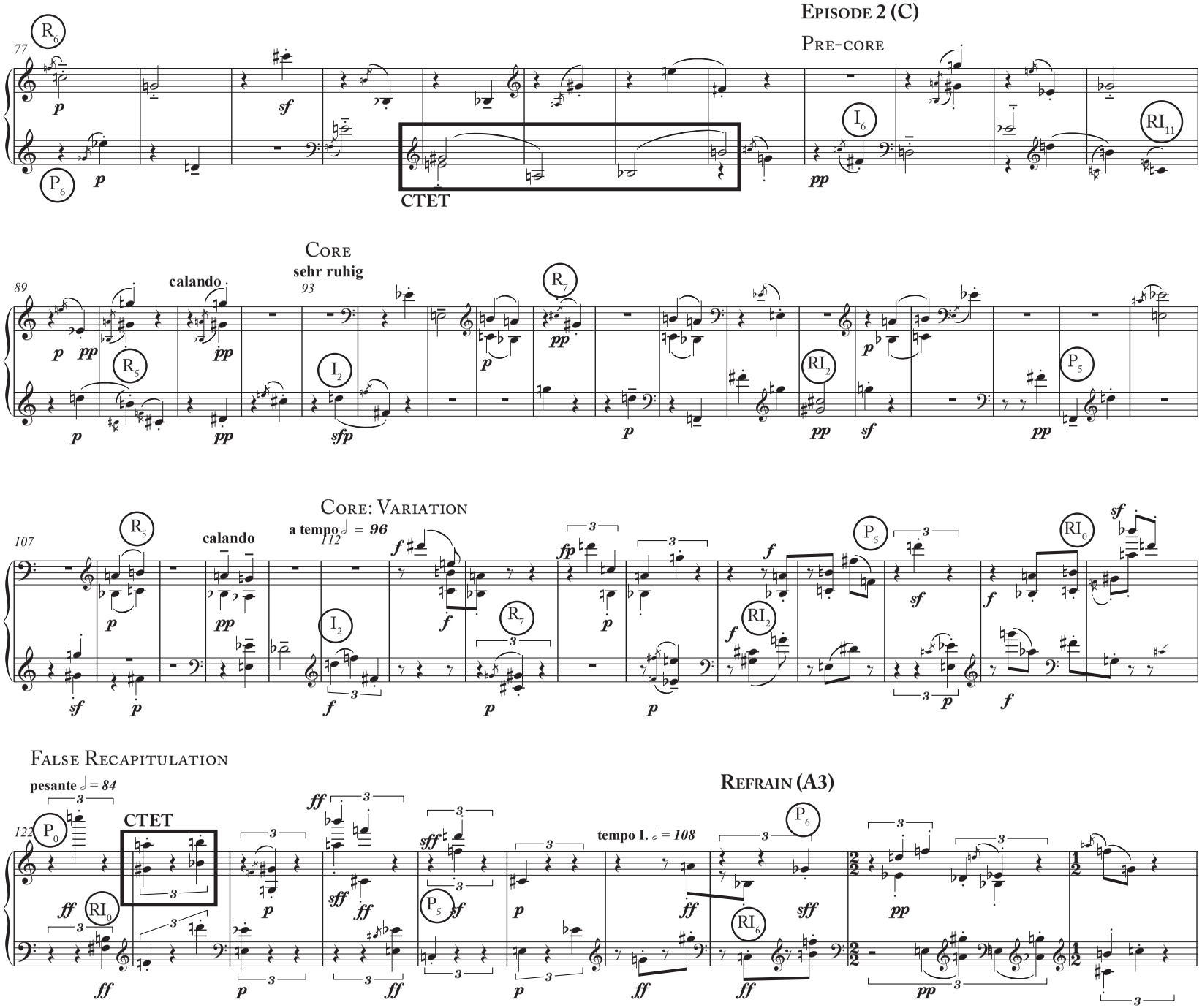
register, implicates it in both sides of the **A$_0$**. As a member of the P-side, it fills in the gap between pitch-class gap separating D and F, which are sounded in close proximity in m. 123 and m. 126. As a member of the I-side, it completes the the chromatic tetrachord {C♯, D, D♯, E} in m. 125 and m. 127.

Despite the return of the "thematic elements" associated with **A$_0$** in m. 122, the I_0 structuring associated with the refrain is missing. Figure 3.17 indicates that, in m. 122, the climactic C7 in the first violin is answered by a F♯3. If we take such registral juxtapositions seriously, and we have seen throughout this chapter that they play important roles elsewhere, this moment implies not I_0 as an axis, but I_6—the "dominant" axis associated with the first episode. On Figure 6, I have shown how the music leading to the refrain "resolves" that axis, coinciding with the real recapitulation at m. 129. First, in the course of mm. 125-126 a C3 and F♯4 are heard in the violin and saxophone. Those pitches are buried in the CTETS action occurring at that moment, but they anticipate m. 129. There, C3 sounds in the piano, accompanied by G♭4 in the saxophone. That arrangement exactly recapitulates the opening refrain, and furthermore, "resolves" the axis. The violin's C7 in m. 122—the highest pitch in the piece—is answered by the piano's C3—the lowest in the near vicinity—completing the I_0 axis. Simultaneously, the piano's F♯3 in m. 122 is answered by F♯4, again completing an I_0 relationship. This completing is echoed in the CTETS structure of the false recapitulation. Figure 3.17 shows how the final CTET in m. 127 finds its inversional partner around I_0 in m. 128, launching the recapitulation that immediately follows by implicating the thematic CTETS structure in the "tonal" resolution.

V: The Central Episode

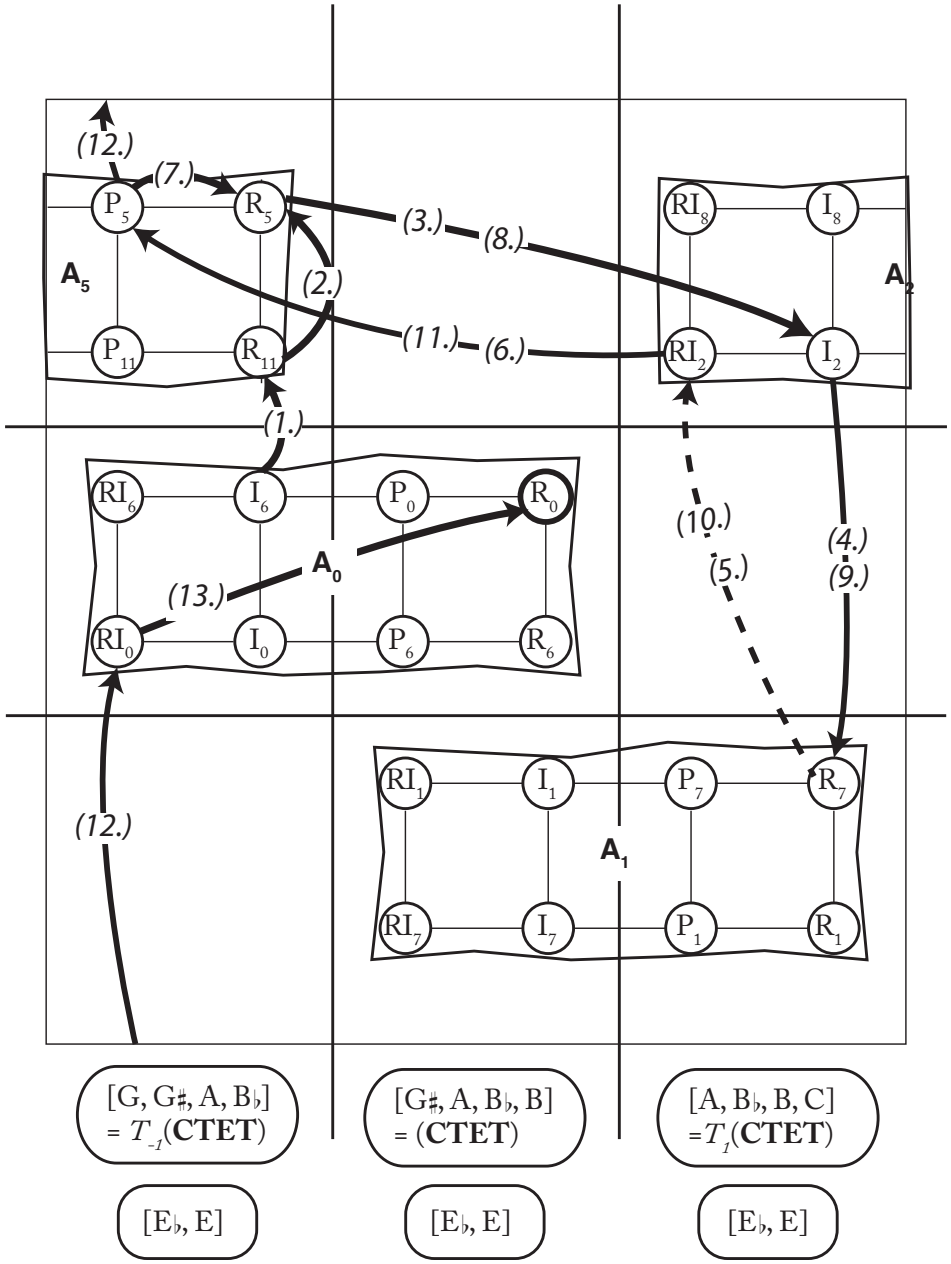
As a final study, I will briefly examine how the episode that precedes the false recapitulation prepares it. This episode is much more developmental than the second episode of Beethoven's rondo, the compositional processes at work here much more integrated into the

FIGURE 3.18. The central episode, divided into a "pre-core," "core," and "false recapitulation."



larger narrative of Webern's movement. In the course of this discussion, I will be concerned with a spatial "gap" in the episode, a gap that is filled at the moment of false recapitulation. This gap has many concrete manifestations on the musical surface, all of which are related to register.

FIGURE 3.19. Mapping the central episode on the spatial network shows the "leaps" over the central column, which is filled in at the recapitulation.

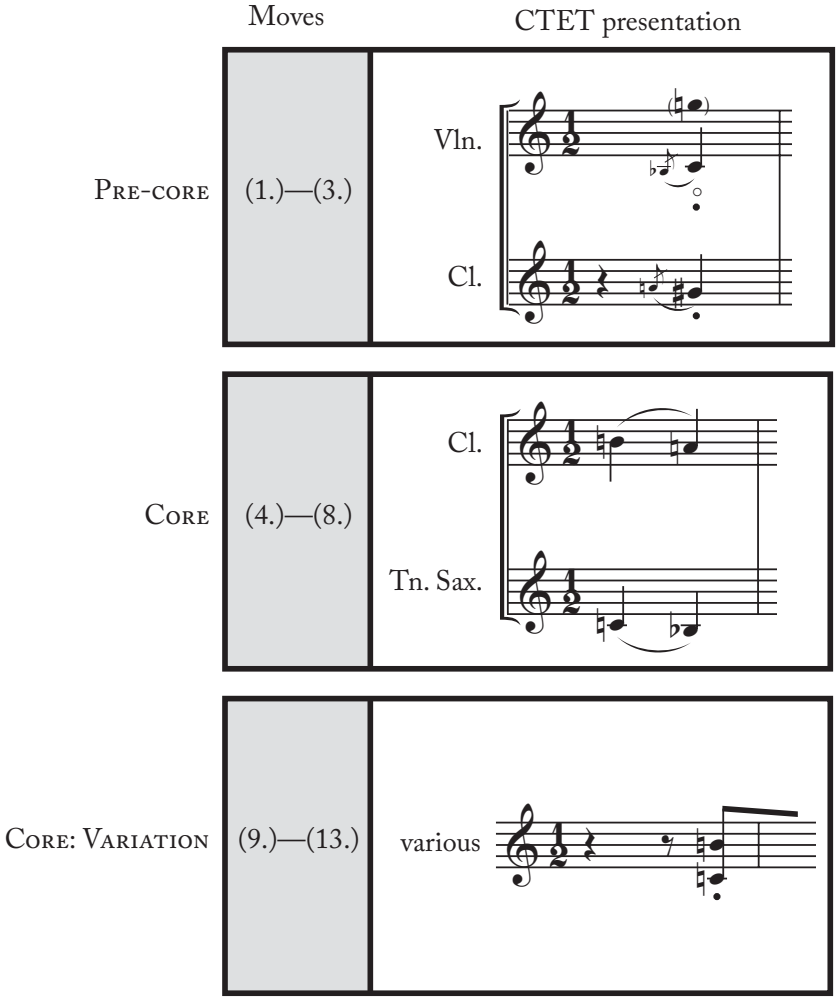


This whole of this passage is condensed in Figure 3.18. The entire passage is structured by "monophonic" row statements connected via mostly *RICH*$_2$ chains. $\mathbf{A_0}$ initiates the passage at m. 85 and, as we have seen, ends it. In relation to the rest of the movement, the texture in this developmental section is very sparse, with slower rhythmic motives and a narrow registral compass. My analysis on Figure 3.18 (and in the formal diagram shown in Figure 3.13) is organized into four sections, borrowing development terminology from William Caplin. After a pre-core, a core-like passage begins at m. 93 and is varied at m. 112. This variation of the core has the same progression of row forms, but as preparation for the climactic false recapitulation, accomplishes them in half the time, veering off into new territory at its end. The final zone is the false recapitulation.

Our formal diagram in Figure 3.13 shows the first three zones of this episode creating a spatial "gap" around the **CTETS** area. *RICH*$_2$ transformations into m. 93 and 112 create are responsible for the gaps. Figure 3.19 uses the large space shown earlier to track the row progression in each of these sections as a series of transformational moves, and makes this gap even more clear. Moves (1.), (2.) and (3.) make up the "pre-core," and comprise three row forms on the leftmost vertical column. Because they are in the same column, all of these row forms will have the same CTETS content. This leg's final move takes the music to the eastern side of the space by using *RICH*$_2$($\mathbf{R_5}$) to skip to $\mathbf{I_2}$. Both the "core" (moves (4)-(8)) and its variation (moves (9)-(13)) begin at this eastern point, circle around the eastern column, and skip over the central column through *RICH*$_2$($\mathbf{RI_2}$) to **P5**. At move (8.), the second leg makes its way back to $\mathbf{I_2}$ to begin the variation. The variation traverses the same path as the core, but after it arrives at $\mathbf{P_5}$—following the skip over the central column—it moves to the top of the space through *RICH*$_2$($\mathbf{P_5}$) for the return to $\mathbf{A_0}$ and the false recapitulation.

Below this space, nodes are filled with two subsets of each row area's CTETS: a chromatic tetrachord and a dyad. These two pitch-class sets characterize two prominent motivic

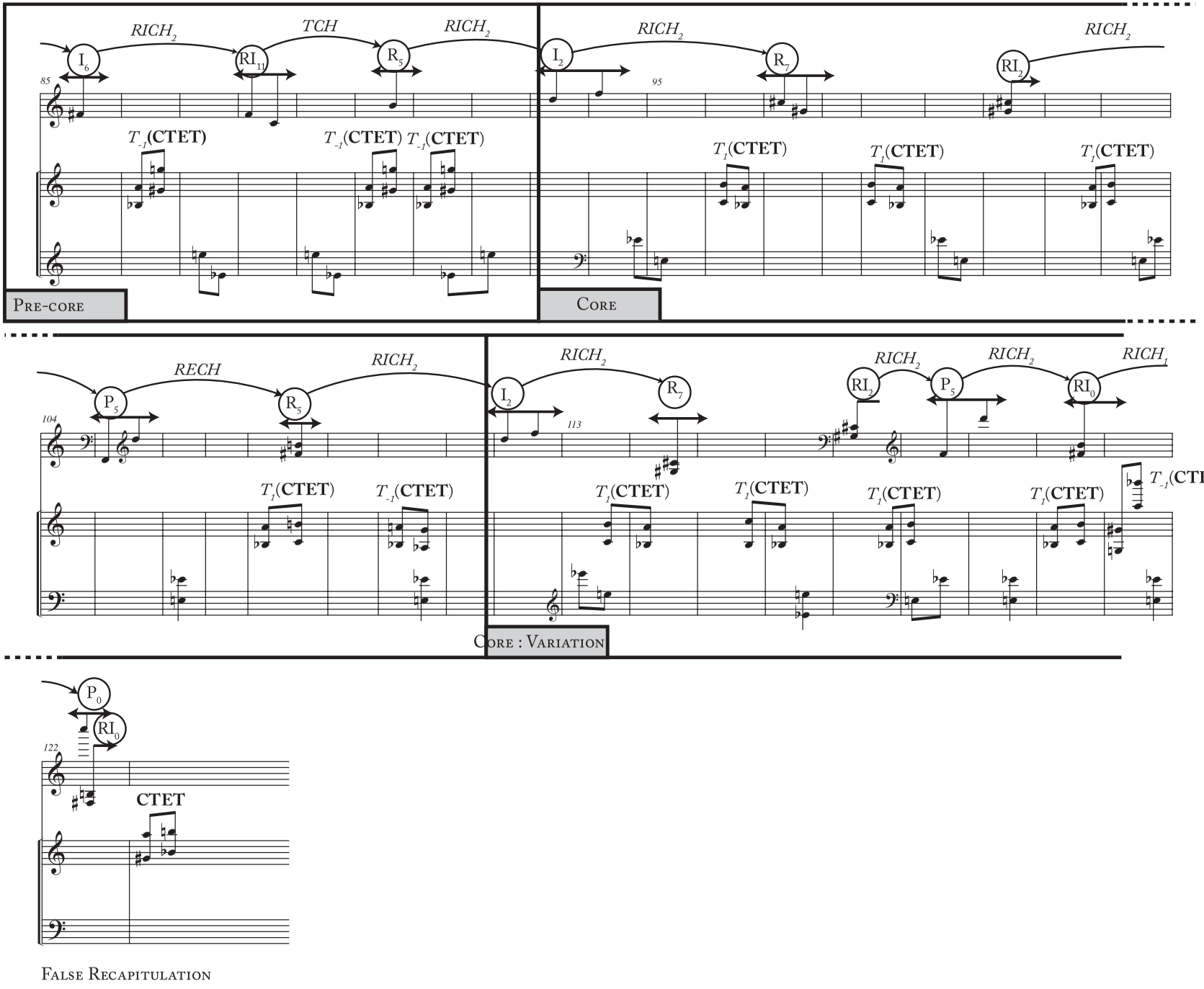
FIGURE 3.20. CTET statements in the "pre-core," "core," and "core."



features of this episode: a fully chromatic tetrachord (CTET) and an {E♭, E♮} dyad. Because this dyad is common to all three columns of the space—it is a member of each columns CTETS—used in the episode section, it acts as a pedal throughout.[44] We know from our discussion of the false recapitulation that these elements play an important role there: CTETS indicating the complete return of $\mathbf{A_0}$, and {E♭, E♮} uniting $\mathbf{A_0}$'s two sides. In the first three zones, the CTET tetrachords shown underneath the space are each leg's most salient motive. Figure 3.20 shows

[44] This dyad would not have been available as a row adjacency had the music ventured any further east or west. Its presence uniquely signifies this portion of the space, and in the passage, Webern uses the dyad as a regularly recurring motive.

FIGURE 3.21. Reduction of the central episode.



that each of the first three zones of the episode has a unique way of presenting its CTET. In the pre-core, every CTET is played in the violin and clarinet and is a single, staccato verticality preceded by quick, grace notes. In the core every CTET is played by the saxophone and clarinet, comprising two quarter-note, semi-tone verticalities played legato. The core's variation at m. 112 disperses the CTET among the various instruments, though it always characterized by two quick, staccato eighth notes.

The section associates each of these CTET and dyadic motives with a particular registral space. Anticipating the important role register plays at the recapitulation, registral movement plays is an essential component of a zone's character. Changes in register occur at the beginnings of a zone. A reduction of the entire passage is given in Figure 3.21 that shows both of these motives on the lower two staves.[45] (For convenience, the top staff labels the passage's row forms and the chain transformations that connect them. CTET tetrachords occupy the middle staff, and the {E♭, E♮} motivic dyad is shown on the bottom.) As mentioned, the dyad acts as a pedal. It is always heard as a major seventh or a minor ninth, and it occupies a single registral space for a long stretch of measures. Changes to this registral space are conspicuous and correspond with with the beginning of a new section. For instance, at the onset of the core, the E♮ from the dyad —which had previously been heard only as E♮5 in the pre-core—leaps down two octaves to E♮3. That E♮3 in m. 95 is particularly prominent because it is the lowest pitch in the episode to this point. Similarly, at the onset of the third leg, the {E♭, E♮} dyad performs another registral move: both members leap upward so that E♭6— the highest pitch in the episode to this point—is heard at the beginning of the core's variation. As the core variation progresses, the music becomes much more active, losing the placid character of the previous music through increased rhythmic action and a much expanded registral compass. As the variation prepares the false recapitulation, the

[45] This reduction does not account for every pitch of the passage. It shows only the two motives and the pitches involved in the chaining.

{E♭, E♮} dyad excitedly cycles through all three prior registers. Along with the registral play of the {E♭, E♮} dyad, the four pitches of the CTET tetrachords occupy a circumscribed registral space. Throughout the passage, Figure 3.21 shows that the motive never never leaves circumscribed space from B♭3 to G5. Only in m. 121, just before the false recapitulation, does the tetrachord leave that space.

The spatial gap shown on Figure 3.18 is more than abstract, but manifest through interactions between CTET tetrachords throughout the passage. Just before the episode began (see Figure 3.18), $\mathbf{P_6}$ sounds the $\mathbf{A_0}$ associated CTET {G♯, A, B♭, B} in the saxophone and piano parts from mm. 81–84, where it is stated as four, legato half notes. *This **CTET** belongs to the central column in the space of Figure 3.18, where $\boldsymbol{P_6}$ is located. Because of the aforementioned transformational "skips" over this column, that **CTET** is not heard again until the false recapitulation at m. 122. Its attainment there signifies the "return" to the $\boldsymbol{A_0}$.* This is most clear in the central staff of the Figure 3.20 reduction. The reduction shows only two CTET tetrachords are heard in the three passages that constitute the pre-core and core of the development: T_{-1}(**CTET**) and T_1(**CTET**) are repeated consistently, with the **CTET** [G♯, A, B♭, B] remaining conspicuously absent.

Thus, when the **CTET** {G♯, A, B♭, B} sounds in m. 123—at the moment of false recapitulation—it fills in the spatial gap prepared in the development, indicating a complete return to the thematic area associated with $\mathbf{A_0}$. An interesting way of hearing the absence of this **CTET** involves focusing on the dyad {A, B♭}. As part of T_{-1}(**CTET**) and T_1(**CTET**), {A, B♭} is a boundary dyad. In the episode reduced in Figure 3.21, the pitches of that dyad are always sounded *together* as a major seventh and in a specific register, as B♭3 and A4. The dyad, however, is not at the *boundary* of the **CTET** [G♯, A, B♭, B]; it sits in the center. *So, when the central column is final filled in conjunction with the false recapitulation at m. 122, that {A, B♭} is excluded as a verticality for the first time in the passage*. In other words, {A, B♭} is an aural marker indicative of the eastern

and western columns of the space in Figure 3.18. Its exclusion as part of the **CTET** is representative of the return to $\mathbf{A_0}$ at m. 122.

The dramatic crux at m. 122 brings together a number of strands that have been woven throughout this paper and involved in nearly every passage of music: the structuring influence of pitch-class invariance, of inversional axes, the importance of register, and the metaphorical power of spatial representations to capture these things. The moment certainly seems associated with the "recapitulation" in the first movement, where Mead has noted multiple processes involved in "imbu[ing] the beginning of the reprise with the kind of multiple significance one associates with analogous recapitulatory moments in tonal music."[46] Given Webern's attention to detail and his deliberate compositional process, this is not so surprising.

Given this, I suspect that the correspondences that Bailey is seeking between Webern's and Beethoven's rondos would have seemed trivial and superficial to someone like Webern, who in his writings and his music repeatedly demonstrates that he is seeking a more sophisticated method of imitating polyphony and integrating it with classical form. As Mead has noted, "the similarities in [Webern's] music to tonal forms are not simply the result of superficial modeling, but spring from a deeper level, one at which the relational properties of the two grammars allow similar narrative patterns to grow."[47]

This chapter sought some correspondences between these two grammars, specifically between "theme" and "key." What the analysis shows is that, despite a seemingly amorphous surface structure, the underlying form is robustly structured. And that structure is defined to a degree that allows rather interesting, sometimes improvisatory, but often carefully controlled

---

[46] Mead, "Webern and Tradition," 193.

[47] Ibid., 204.

music that takes advantage of some of the most sophisticated techniques associated with classical form.

CHAPTER 4

MUSICAL IMAGES OF NATURE IN THE CANTATA I, OP. 29

Webern's friendship with the poet and painter Hildegard Jone and her husband, the sculptor, Josef Humplik, began in 1926 and grew in intensity through the remainder of his life.[1] His twelve-tone vocal music is associated nearly exclusively with Jone.[2] In 1930, four years after meeting Jone and at the end of the composition of Op. 22, Webern asked her for a text for a cantata or stage work.[3] Though he did not complete a large-scale vocal work for five more years (*Das Augenlicht*, Op. 26), he did set six poems from her *Viae inviae* cycle during the years of 1933-34 (the *Drei Gesänge*, Op. 23, and the *Drei Gesänge*, Op. 25) and subsequently set text by no other author. In the last ten years of his life, Webern wrote three cantatas (Opp. 26, 29, and 31) on texts by Jone.

---

[1] Lauriejean Reinhardt, more than any author to date, has offered an extensive account of Jone and Webern's collaboration (see her "From Poet's Voice to Composer's Muse: Text and Music in Webern's Jone Settings" (Ph.D. dissertation, University of North Carolina, Chapel Hill, 1995).

[2] Webern's initial experimentation with twelve-tone composition occurred at the end of a decade (1914-24) during which he composed vocal music nearly exclusively. During those experimental years, Webern turned away from the avant-garde, modernist poetry that he set in his early music and toward liturgical and "religious" folk texts. Webern did not engage with a living poet again until 1933, when he began setting Jone's poetry. Anne Schreffler makes a compelling argument that those early vocal works were essential in forming Webern's conception of twelve-tone technique: "His earliest rows grew out of concrete melodic gestures, a conception that remained potent for a long time. Later he approached the notion of an abstract row as he sought to realize the essence of the religious and folk poems that attracted him. […] [T]he mere presence of a twelve-tone row could provide a subconscious unity for the whole piece. Musical gestures could then be freed from their previous role of ensuring surface comprehensibility" (" 'Mein Weg Geht Jetzt Vorüber': The Vocal Origins of Webern's Twelve-Tone Composition," *Journal of the American Musicological Society* 47, no. 2 (1994): 280).

[3] In a letter to Jone, dated 17 January 1930, Webern mentions the possibility of an opera libretto. Only months later (see the letter dated 8 September 1930),Webern specifically asks for something from her *Farbenlehre* to serve as the text for a cantata. "Ever since I have known your writings the idea has never left me of setting something to music. That was why I suggested that time the idea of a libretto, or better a dramatic text. Now I have the following idea. […] [I] am very occupied with the idea of writing a cantata" (Webern, *Letters,* 15–16).

Because of Jone's relative obscurity after Webern's death, their relationship has provoked a good deal of discussion.[4] Boulez, for example, believed Jone's poetry was lacking in quality and questioned Webern's judgement. Reinhardt points out, though, that although Boulez condemned her work, he believed that Webern had successfully "parlayed the liability represented by Jone's poetry into an asset"—the supposedly inferior quality of the poetry functioned as a sort of blank slate onto which Webern was able to impose his ideas.[5] Boulez says:

> Webern no longer depends on the text to give him his form, but integrates the text into the form: a very different approach, in which the musician recovers confidence in his own individual powers and imposes his will on the poem. It should be added that the poems themselves help in this, being markedly inferior in literary quality to those selected by Webern when he was younger—one could hardly set Hildegard Jone beside Georg Trakl or even Stefan George.[6]

That Webern used Jone's work in such a way is largely unlikely. Letters Webern wrote to Jone and her husband, published first in 1959, make it clear that Webern viewed his settings of Jone's texts as *collaborations* and that he viewed Jone as an artistic equal. Following the first London performance of *Das Augenlicht*, Webern wrote to Jone: "I am especially pleased because

---

[4] Reinhardt explores all of these issues in fascinating detail ("From Poet's Voice to Composer's Muse"). As for the criticism of Jone, Reinhardt notes the following general themes: (1) "Jone was a dilettante whose poetry was derivative and", according to Paul Griffiths, "of 'no great literary quality'"; (2) "the style and substance of her poems hold little in common with the style and substance of Webern's twelve-tone compositions"; (3) "Webern was a poor judge of literary quality, and he was drawn to Jone's poetry because of her reverence for Goethe"; and (4) "Jone's poetry was somehow an inappropriate choice, given Webern's calibre as a composer and the type of twelve-tone music he composed" ("From Poet's Voice to Composer's Muse," 10-11). For the quotation from Griffiths, see "Anton Webern," *The New Grove Second Viennese School: Schoenberg, Webern, Berg* (New York: Norton, 1983), 114. Her answers to these charges occupy much of her study but can be summarized as follows: About the supposed dilettantism, Reinhardt references "epochal song cycles like Schuberts *Die schone Müllerin* and *Winterreise* based on the poems of Wilhelm Müller, and Beethoven's *An die ferne Geliebte* based on the poems of Alois Jeitteles," which show that the "merit of a texted work often lies less with the status of the poem than with the cogency and ingenuity with which the poem has been absorbed into the essence of the composition" ("From Poet's Voice to Composer's Muse,"12). She also notes the positive response of Webern's contemporaries (especially Schoenberg and Berg) to his decision to set Jone's works (13). According to Reinhardt, Jone was a private person and wished to keep much of her work to herself. She shared much of her poetry with Webern only, who encouraged her to make her work more publicly available. And as to charges that Webern "was a poor judge of literary quality," she counters that Webern's early literary tastes were of contemporary, avant-garde poets (including Richard Dehmel, Stefan George, Rainer Maria Rilke, Peter Altenberg and Karl Kraus) that were "not likely due exclusively to popular taste or the influence of others" (14-15). Webern was the first to set Trakl's poems.

[5] Ibid., 3-4.

[6] Pierre Boulez, "Anton von Webern," *Stocktakings from an Apprenticeship*, ed. Paule Thévenin, trans. Stephen Walsh (Oxford: Clarendon Press, 1991), 300.

you too were part of what was heard—it is *our* "Augenlicht," after all."[7] In May 1941, Webern wrote an almost apologetic letter to Jone that addressed both his thoughts about her poetry and his process of finding text for a song project:

> At last I had the chance to make some acquaintance with your other works. How your thoughts move me is difficult for me to express in a letter, but perhaps my music may do it on occasion to some extent. Please understand me correctly: I have *never* gone out looking (as it were) for a "text," with the intention—indeed *I could never have such an intention*—of writing something vocal (a song, a choral piece, etc.). It was never thus; *the text was always provided first!* Given a text, then of course "something vocal" should be the result. […] So when I say that I can't wait to see your new work, that is purely *for the sake of your work* and for no other reason.[8]

Webern was clearly inspired by Jone's poetry. Much of that inspiration no doubt has a root in their shared aesthetic sensibilities—most importantly, their belief in a profound symbiotic relationship between art and nature.[9] Both admired Goethe's reverence for nature and organicism, and his thoughts about color.[10] In explanations of his compositional techniques, Webern often uses Goethian botanical concepts to describe the twelve-tone method. Webern saw Goethe's *Urpflanze* (primeval plant), in particular, as a model for principles that govern the

---

[7] Webern *Letters*, 36 (dated 20 July 1938), emphasis is Webern's.

[8] Webern *Letters*, 43 (dated 3 May 1941), emphasis is Webern's.

[9] Reinhardt sees three intersections: "(1) a renewed faith in the communicative power and lawful nature of art; (2) a clear vision of the future course of modern art based on the cumulative achievements of the Western classical tradition; and (3) a firm conviction of the spiritual and metaphysical nature of art, drawing together ideas from both a Christian essentialist and existentialist point of view" ("From Poet's Voice to Composer's Muse," 463-4).

[10] See, for example, Cox, "Blumengruß and Blumenglöckchen," 203–224. After receiving a copy of Goethe's Zur Farbenlehre in 1929, he annotated it heavily before sending handwritten extracts to Jone. It was not long after that Webern asked Jone for a cantata text based on her own *Farbenlehre*, which dates from the early 1920s.

Goethe's organicism was a strong influence on the Second Viennese School more generally. Severine Neff, for example, has noted that Schoenberg's theoretical writings and analytical method draw heavily from Goethe's *Metamorphose der Pflanzen.* ("Schoenberg and Goethe: Organicism and Analysis," in *Music Theory and the Exploration of the Past*, ed. Christopher Hatch and David N. Bernstein (Chicago: University of Chicago Press, 1993), 409–34).

natural world and find reverberations in art: "Goethe sees art as a product of nature in general […] there is no essential contrast between a product of nature and a product of art."[11]

In addition to believing that art is a manifestation of nature, both Webern and Jone believed that spirituality was manifest in art. They were both Judeo-Christian and espoused a pantheistic, Christian mysticism that was closely connected to the natural world. Reinhardt notes that for the two artists "all of the arts were seen to reflect certain absolute values or spiritual 'laws,' which were manifested likewise in God's own creative handiwork; i.e., in nature."[12] Their belief in a spiritual resonance between the divine and natural worlds was immensely important to both artists and formed the core of their collaboration.[13]

As Goethe was a model for Webern's understanding of art and nature, an historical precedent for Webern's spiritual beliefs can be located in the writings of Emanuel Swedenborg, an influential early-eighteenth-century philosopher and theologian. Swedenborg's influence in the nineteenth-and twentieth centuries was large, and many of his ideas resonate with Goethe's organicism.[14] His theory of "correspondence" was particularly influential in Webern's spiritual

---

[11] Webern, *The Path to the New Music*, 10-1. Goethe's *Urpflanze* also provides a link between common-practice music and twelve-tone composition. In a later lecture Webern says: "[V]ariation form is a forerunner of twelve-note composition. An example: Beethoven's Ninth Symphony, finale—theme in unison; all that follows is derived from this idea, which is the archetypal form. Unheard-of things happen, and yet it is constantly the same thing! You'll already have seen where I am leading you. Goethe's *Urpflanze*; the root is in fact no different from the stalk, the stalk no different from the leaf, and the leaf no different from the flower: variations of the same idea" (*The Path to the New Music*, 52-3).

[12] Reinhardt, "From Poet's Voice to Composer's Muse," 486

[13] Kathryn Bailey frames it thusly: "Jone's strange mystical/Christian poetry with its rapturous metaphors and allusions to nature found a kindred spirit in the naïve but intense composer who customarily outlined the movements of projected works in his sketchbooks by making associations with favorite alpine flowers and mountain retreats" (*The Twelve-Note Music of Anton Webern*, 265).

[14] Goethe, Balzac, and August Strindberg all cite Swedenborg's work, and John Covach makes the case that Schoenberg's philosophical and aesthetic ideals owe a great deal to him, though Schoenberg might never have read his work directly (he likely learned of them through reading Balzac's Séraphita). See John Covach, "The Sources of Schoenberg's 'Aesthetic Theology'," *19th-Century Music* 19, no. 3 (1996): 252–262; "Schoenberg and The Occult: Some Reflections on the Musical Idea," *Theory and Practice* 17 (1992): 103–118.

Though Schoenberg may never have read Swedenborg directly, we know that Webern did. He wrote to Schoenberg on 30 October 1913: "I am now reading Swedenborg. It takes my breath away. It is incredible. I had expected something colossal, but it is even more" (Moldenhauer and Moldenhauer, *Webern, A Chronicle*, 199).

understanding of the natural world. Correspondence theory asserts that every detail of nature corresponds to a spiritual reality.[15]

Abstraction, which is at the essence of Goethe's organicism and Swedenborg's correspondence theory, was influential across Webern's musical output. It finds representation primarily in Webern's love of symbolism and metaphor, and likely influenced his understanding of the twelve-tone method. [16] And it certainly seems to have contributed to Webern's love of Jone's poetry, which often describe natural images as metaphors for the spiritual and the divine. This is certainly the case in his Cantata I, Op. 29. The poems used in the first two movements, "Blitz und Donner" and "Kleiner Flügel Ahornsamen," describe natural phenomena, a lightning strike and a maple key, as spiritual analogues of life and death.

In this chapter, I offer an interpretation of these two settings that locate musical images corresponding to the universal metaphors proposed in the poems. Taken together, my analyses capture ways in which the two movements exemplify distinct conceptions of these ideas—one is "circular," the other, "linear." Spread across its many formal layers and involving multiple musical domains, "Kleiner Flügel Ahornsamen" contains manifestations of a collection of musical transformations organized recursively. The structure of the movement resonates with Goethe's *Urpflanze* but its interaction with Jone's poem shows that the recursion has spiritually transcendent overtones. "Blitz und Donner" has the same ABA formal scheme as "Kleiner Flügel

---

[15] Julian Johnson has noted that Webern's understanding of correspondences is well illustrated in *Tot*, a stage play Webern wrote in October 1913: "The plot is minimal, hinging on the gradual consolation that a mother and father find through nature following the death of their young child. There is very little action, and the six scenes are more like tableaux in which nature is encountered in different ways as the threshold of a spiritual presence" (*Webern and the Transformation of Nature* (Cambridge: Cambridge University Press, 2000), 34).

[16] Reinhardt, "From Poet's Voice to Composer's Muse," points out that "Webern himself acknowledged the difficulty he found in realizing a mode of expression comprised strictly of essence and divorced from symbolic or imitative representation […] Friedrich Deutsch, who attended a conducting class Webern taught at the Schwarzwald School in the early 1920s, later recalled that the composer "had a preference for symbol and metaphor" (493-94). For the Deutsch quote, see Moldenhauer and Moldenhauer, *Webern, A Chronicle*, 466.

Ahornsamen" but uses that scheme to portray life's linearity through a progression that involves subtle changes in the A section's recapitulation.[17]

[17] Ternary form seems to have been a particular preoccupation of Webern where, as Julian Johnson aptly notes, "the reflection of the first section in the third is transformed by the process to something richer" (*Webern and the Transformation of Nature*, 179).

CIRCULARITY: CANTATA I, OP. 29, "KLEINER FLÜGEL AHORNSAMEN"

> [H]owever freely it seems to float around—possibly music has never before known anything so loose—it is the product of a *regular procedure more strict*, possibly, than anything that has formed the basis of a musical conception before (the *"little wings*", "they bear within themselves"—but really, not just figuratively—the "whole" … form. Just as your words have it!).
>
> Webern, *Letters*, 37

Webern's description of the cantata's second movement is preoccupied with a paradox—a piece of music that is simultaneously the most "loose" and "strict" ever to exist. Jone's poem is the inspiration for this paradox. It describes a maple key, the boomerang-shaped seed of a maple tree that is illustrated in Figure 4.1.[18] In the poem (shown below) Jone seized upon the multiple paradoxes inherent within the object's two parts: the papery wings create its characteristic, *chaotic* flutter, and are associated with the key's *fall*; the seed, contained between the wings initiates a new tree' s predetermined, *controlled rise*.[19] Jone conveys the universality of these natural oppositions in part by describing other forms of the idea. She contrasts darkness and daylight in the first stanza, but most especially, the terrestrial and the divine in the central stanza.

This latter association is the key to the poem's larger metaphorical context, wherein earthly, natural phenomena associated with the cycle of life have divine correspondences —a conceit that has Swedenborgian reverberations.[20] This transcendence is captured not only through this central metaphor, but also in the *abstract structure* of the poem. As an image, the maple key represents life's circularity ("Wieder wirst aus dir du kleine Flügel senden" [*Again* there will be sent from you, you little wing]," which in the context of the central stanza comes to have a

---

[18] The poem is from Jone's *Fons Hortorum*, an unpublished journal dating from early 1934 that Jone loaned to Webern that spring. Webern kept the manuscript copy of the journal until 1937. Reinhardt notes that Webern was quite inspired by the *Fons Hortorum*. He note upon its return: "Thanks so much that you allowed me to keep it so long. So long and yet far too short" ("From Poet's Voice to Composer's Muse," 130-31).

[19] Graham Phipps pointed to this opposition as well. See "Tonality in Webern's Cantata I," *Music Analysis* 3, no. 2 (1984): 141.

[20] Themes of transcendence and transformation were apparently common in Jone's writing, as noted by Reinhardt, "From Poet's Voice to Composer's Muse," 98.

broader meaning. Structurally, that circularity is manifest as a symmetry that embeds the paradox that Webern referenced above. In the poem's tripartite form, the image of the maple key's flutter and fall is recapitulated in the final stanza (cf. "du kleine Flügel" in the final stanza with "Kleiner Flügel" in the first), while the controlled rise occupies the poem's center.[21]

FIGURE 4.1. Illustration of a "maple key"(Ivy Livingstone, *Acer*).



"Kleiner Flügel Ahornsamen," from *Fons Hortorum*

Kleiner Flügel Ahornsamen schwebst im Winde! Mußt doch in der Erde Dunkel sinken. Aber du wirst auferstehn dem Tage, all den Düften und der Frühlingszeit;	Little wing, maple seed, you float in the wind! You must yet sink in the dark earth. But you will arise to the day, all the fragrances and the springtime;
wirst aus Wurzeln in das Helle steigen, bald im Himmel auch verwurzelt sein.	you will arise from roots into the brightness, also soon become rooted in heaven.
Wieder wirst aus dir du kleine Flügel senden, die in sich schon tragen deine ganze schweigend Leben sagende Gestalt.	Again there will be sent from you, you little wing, that which already carries your entire silent life speaking form.

[21] Webern believed this movement possessed a certain "centricity" within the cantata. It was the first movement of the cantata that he composed, though during its composition he believed it would be part of a symphonic cycle. Webern always intended it to be the central movement. Moldenhauer and Moldenhauer, *Webern, A Chronicle*, 561 show that even when Webern was planning a five-movement,"Second Symphony, Op. 29," "Kleiner Flügel" was the middle movement. In a letter to Jone during its composition, Webern states that "it is to become the key to a sizable symphonic cycle." See Webern, *Letters*, 36.

SEED/TREE RELATIONSHIPS, PART 1: INTRODUCTION AND B SECTION

Abstractly, we might view this as "recursion," or "self-replication," the primary relationship between a "seed" and a "tree." The poem's structure stands for the many different manifestations of the seed/tree relatinoship, which has a tiny, natural expression in the maple key, and much larger spiritual connotations as well. In different, though related ways, all of these ideas find expression in Webern's setting, though it is the idea of self-replication that most seems to inform the larger compositional strategy. Webern indicates as much in the quotation at the head of this section, which quotes liberally from Jone's poem: "the 'little wings,' 'they bear within themselves'—but really, not just figuratively—the 'whole' … form.'"

Figure 4.2 is a simple diagram showing the form of the movement, which echoes the circularity of Jone's poem. In its ABA structure, the recurrence of the maple key's "wings" in the final stanza aligns with a varied recapitulation of the first section of canons, both of which are polyphonic, pointillistic, and sound quite free.[22] Jone's image of the tree's rise, and the turn toward's spiritual transcendence is set apart texturally in the homophonic B section.[23]

Musically, recursion is manifest through the presence of a variety of equivalent symmetries that are—like the symbols in Jone's poem—projected across multiple formal spans The "seed," so to speak, lies in various symmetries that are latent in the instrumental introduction, which I have reduced to two strands (clarinet/voice and orchestral accompaniment) in Figure 4.3(a). Accompanying the clarinet ($\mathbf{I_2}$), the orchestra plays a series of chordal, (016) trichords—

---

[22] For a variety of reasons, some of which will be outlined below, the A sections, which describe the *fall* of the maple key, sound very free. And by contrast, the B section, describing the tree's growth, sounds very controlled.

[23] The movement's form has been analyzed differently by others. Phipps, "Tonality in Webern's Cantata I," hears a large AB structure, the second half beginning at m. 31 midway through the passage I have called B. Bailey, *The Twelve-Note Music of Anton Webern*, 288 sees the movement as a four-part ABAB form. Her four sections align with the four that I have shown in Figure 4.2, though she does not hear the first A as an introduction.

It may be worth noting that Webern, himself, saw the movement in three parts. See Moldenhauer and Moldenhauer, *Webern, A Chronicle*, 561. He notes, also, the presence of an introduction (575).

FIGURE 4.2. Jone's "Kleiner Flügel Ahornsamen" set against the formal structure of Webern's movement. Note the circularity in both.

INTRODUCTION (M. 1)

A (M. 6)	Kleiner Flügel, Ahornsamen, schwebst im Winde!
• *polyphonic*	Mußt doch in der Erde Dunkel sinken.
• *pointillistic*	Aber du wirst auferstehn dem Tage,
	all den Düften und der Frühlingszeit;
B (M. 27)	wirst aus Wurzeln in das Helle steigen,
• *homophonic*	bald im Himmel auch verwurzelt sein.
A' (M. 36)	Wieder wirst aus dir du kleine Flügel senden,
• *polyphonic*	die in sich schon tragen deine ganze
• *pointillistic*	schweigend Leben sagende Gestalt.

formed from three concurrently stated rows—that echo the clarinetist canonically at the distance of a sixteenth note. Within each part, the rhythmic pattern is symmetrical in terms of attack and duration. Both contain three four-note gestures, each separated by a sixteenth note.[24] That rhythmic symmetry manifests itself in the vocal part, as well, which is also in canon with the clarinet, but at a much larger canonic interval.

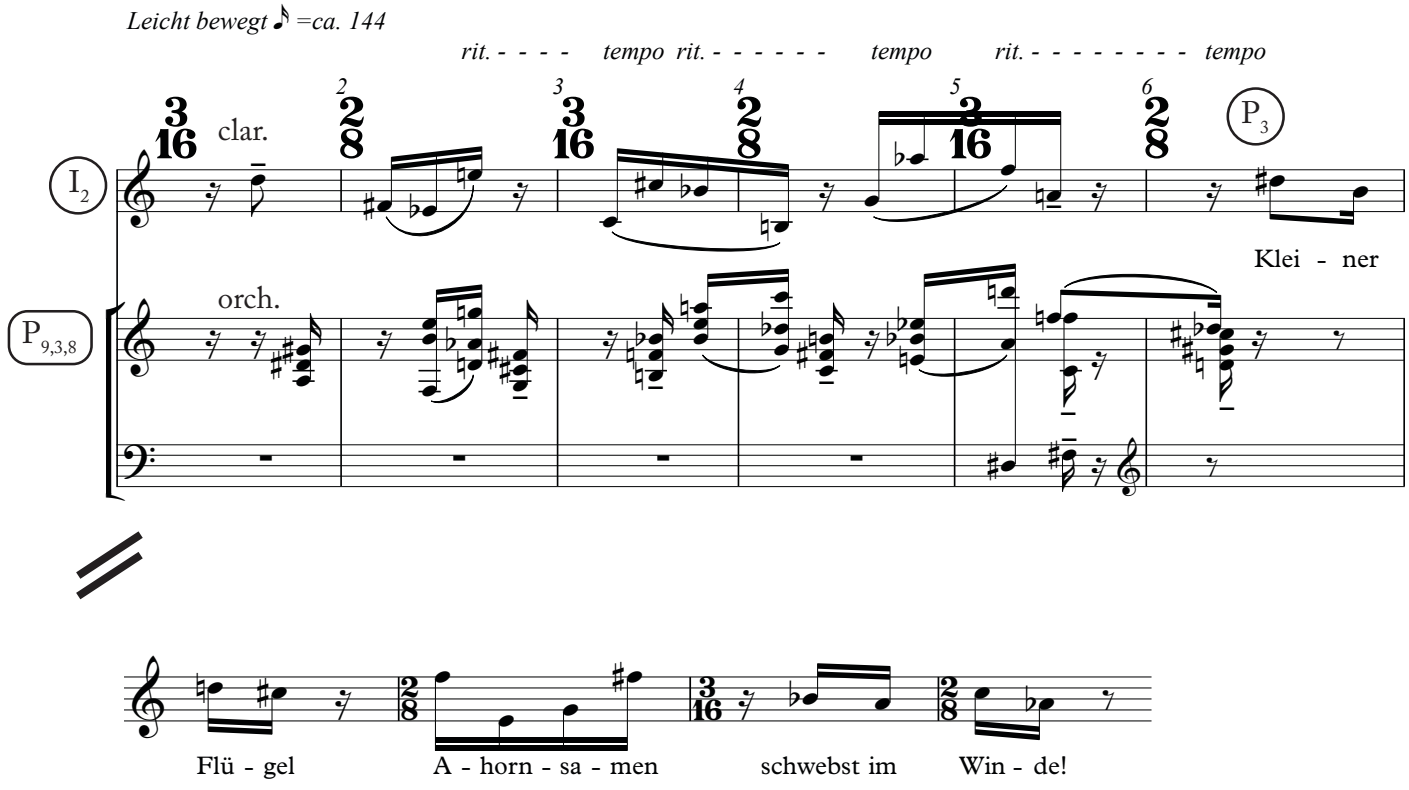
The row's construction resonates with this rhythmic symmetry.[25] Figure 4.3(b) shows the row's *pitch-class* intervallic symmetry (or "*RI*-symmetry"), which is also found in the instrumental

[24] That this rhythmic symmetry has *three* parts is certainly suggestive as regards the structure of the poem, which also has three parts.

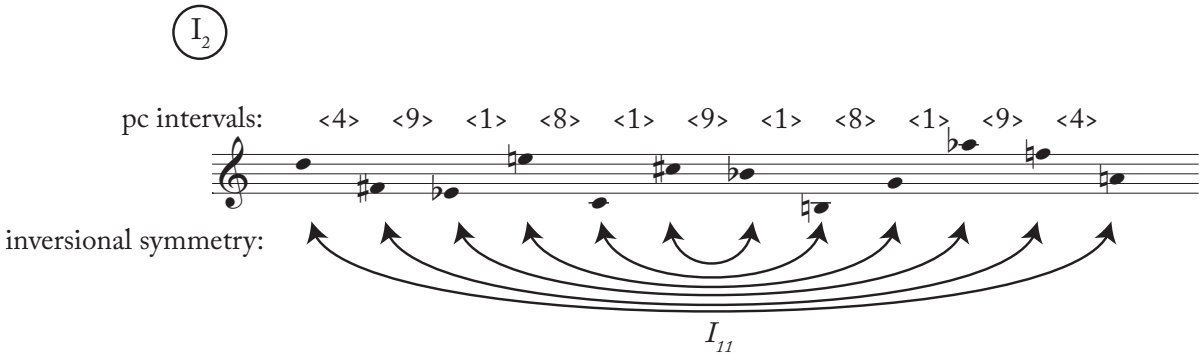
[25] Phipps sees a similar poetic resonance in the row's construction: "suggestions of the physical opposites in nature's growth cycle (growth upward from roots in the earth's soil as opposed to the 'rooting' in the sky and once more sending seed to earth); the propagation of the species; the bearing of new life—all must have had a direct influence upon the construction of Webern's tone row" ("Tonality in Webern's Cantata I," 141).

FIGURE 4.3. A variety of symmetrical features in the introduction.

(a) Introduction, mm. 1–10.



(b) Intervallic and inversional symmetry in the row (cf. clarinet at (a))



works that preceded and followed the cantata's composition.[26] Writing to Jone, Webern noted the recursive, seed-like role of the first six notes and tied it to the poetic content of the three poems in the cantata: "the 12 notes […] [have] the peculiarity that the second set of six notes is, in its intervals, the *backwards inversion* of the first set, so that everything that occurs can be traced back to a sequence of *6 notes*. *Ever the same*: whether it's the "blissful strings", the "charm of mercy", the "little wings", the "lightning of life" or the "thunder of the heartbeat." Surely it is evident from this how well the text can be built into the said sequence And musically it is just the same. And yet, each time something quite different!"[27]

As a result of the row's intervallic symmetry, pcs in complementary order positions in the row's two discrete hexachords are related by the same inversional value—fixed around the same pc axis; that is, the pc in order position **0** is inversionally related to the pc in order position **11** at the same fixed-inversion value as pcs in order positions **1**-**10**, **2**-**9**, **3**–**8**, and so on—as illustrated in Figure 4.3(b). I will call this a row's "internal" inversional symmetry in order to distinguish from "external" inversional relationships between rows and other objects. Within $\mathbf{I_2}$, for example, which comprises the clarinet's melody, the internal inversional symmetry is I_{11}. Interestingly, the discrete hexachords of $\mathbf{I_2}$, $\mathbf{P_3}$, and $\mathbf{P_9}$ all have internal inversional symmetry of I_{11}, while $\mathbf{P_8}$ alone

[26] Webern's Symphony, Op. 21 also has an intervallically symmetrical row, but of a different type. There, corresponding intervals in the row are complements. In this case, $\mathbf{P}_x = \mathbf{RI}_{x+5}$. The intervallic symmetry of the row means that the row class will have only twenty-four distinct members. That is, if we retain the **P**-labels for the orchestra, the clarinet in Figure 4.3 could have been labeled as $\mathbf{I_2}$ *or* $\mathbf{R}_9$. There is no extramusical means for deciding how to label these row forms. Within the movement, we will see that there are times in which it seems appropriate to emphasize the inversional relationship between rows and times in which retrograde relationships seem more suitable. And in fact, this relationship between inversional- and retrograde-thinking is an important part of the larger narrative. Unfortunately, we must make a single decision typographically. Because of the emphasis on inversional structuring here in the introduction and also because the retrograde relationship is not emphasized (cf. $\mathbf{I_2}$ and $\mathbf{P_9}$ in Figure 4.3(a)), we will label all row forms in the piece with a **P** or **I** label.

[27] Webern, *Letters*, 39 (dated 2 October 1939).

is different—its internal inversional symmetry is I_9.[28] The importance of these two specific inversional values, and the "near-consistency" of the introduction's four row in this regard, is a characteristic of the introduction to which we will return to later.

The row's intervallic symmetry halves the size of the row class—there are only twenty-four distinct rows. That is, if we retain the **P**-labels for the orchestra, the clarinet in Figure 4.3 could have been labeled as $\mathbf{I_2}$ *or* $\mathbf{R_9}$. This means that the transformational labels are often ambiguous as well. $\mathbf{I_2}$ and $\mathbf{P_9}$ are I_{11} transforms of one another, *but they are also* R *transforms of one another*. (Cf. the clarinet melody on Figure 4.3(a) with the $\mathbf{P_9}$ voice in the orchestral reduction.) It should be noted, however, that I_{11} *is not the same as R*. Two row forms (**P** and **I**) are also retrogrades only when those rows are $\mathbf{P}_x$ and $\mathbf{I}_{x+5}$.[29] There are no *extramusical* means for deciding how to label row forms or transformations between them.[30] But there are *musical* reasons here for emphasizing the inversional structuring over the retrograde structuring; namely, the fact that I_{11} is a prominent transformation *within* three of the four introduction rows encourages a similar understanding *between* rows. Nonetheless, the fact that I_{11} can in special situations also be *R* will prove to be valuable at certain points in our analysis, and we will point those out as we proceed.

In the introduction these symmetries are "latent" or "germinal"—not quite fully formed, abstract. (Think, again, of a seed.) For example, although the row is symmetrical in terms of ordered pitch-*class* intervals, none of the voices in the introduction plays this symmetry in terms of *pitch intervals*. (In a related sense, though each of the rows are inversionally symmetrical, they

---

[28] Given any **P** form, $\mathbf{P}_x$, its internal inversional symmetry can be described as $2x + 5$ (mod 12), while $\mathbf{I}_x$'s internal inversional symmetry is $2x + 7$(mod 12). Because $2x = 2(x + 6)$ when calculated mod 12, every **P** and **I** form has the same internal inversional symmetry as its T_6 transposition. Thus, $\mathbf{P_3}$ and $\mathbf{P_9}$ share that facet of internal structure. Moreover, a **P** form, $\mathbf{P}_x$, shares internal inversional symmetry with the **I** form whose index number is five higher, $\mathbf{I}_{x+5}$: $\mathbf{P_9}$ and $\mathbf{I_2}$ in the introduction, for example. Note that it is impossible for three distinct **P** forms (as is found in the orchestra in mm. 1–6) to share the same internal inversional structuring. Thus the row forms in the introduction overlap in their internal inversional structuring to the maximum degree possible.

[29] For example: $\mathbf{I_2}$ and $\mathbf{P_9}$ are I_{11} transforms of one another *and R* transforms of one another, but $\mathbf{I_9}$ and $\mathbf{P_2}$ are only I_{11}-related
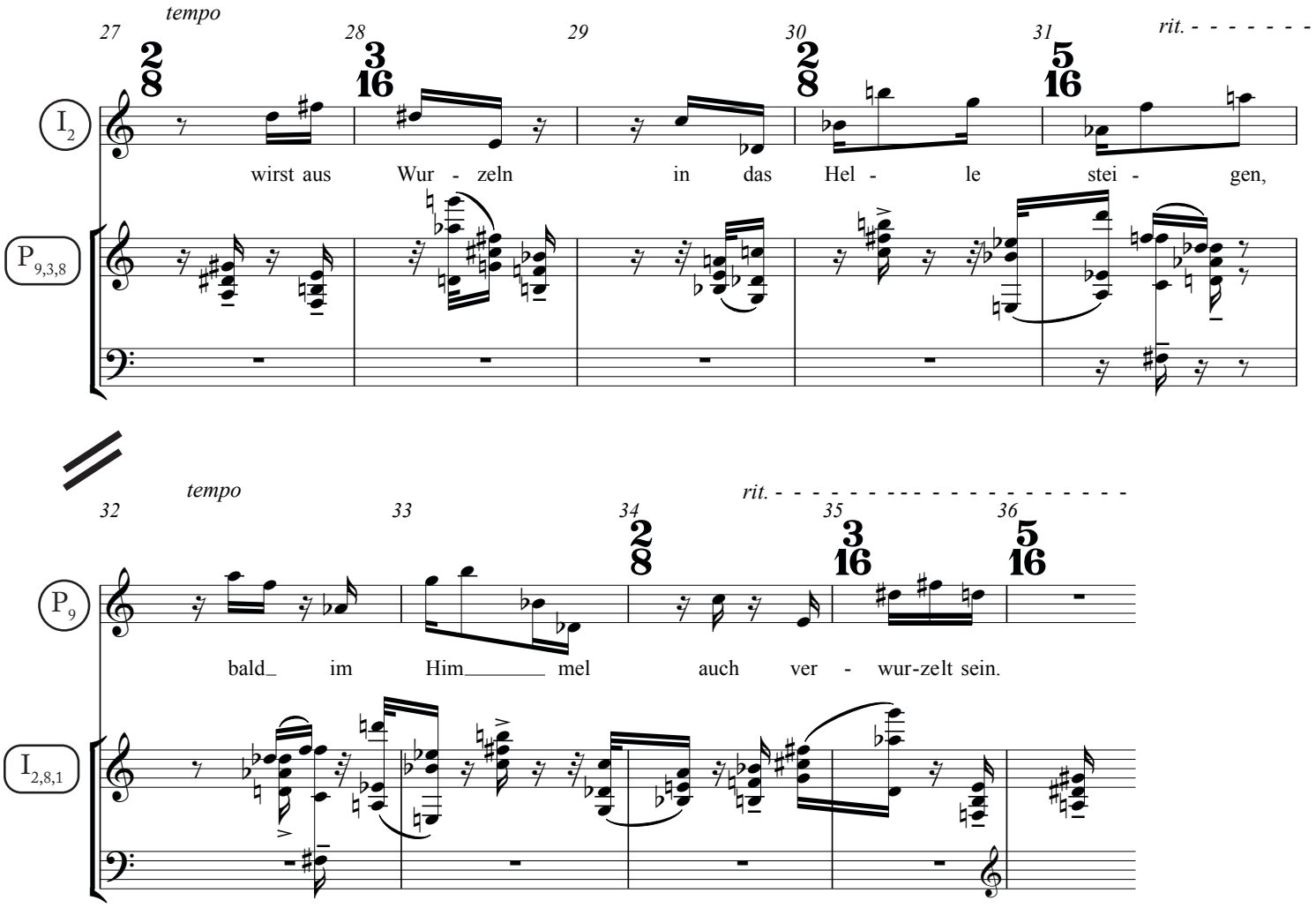
[30] See also Mead, "Webern and Tradition," 174–8.

do not articulate that axis around a particular *pitch*.) Similarly, although the strands separately play symmetrical rhythmic patterns, together, they create a composite rhythm that is not symmetrical.

Understanding those symmetries as "germinal" is appropriate because of the passage's relationship to the B section—which sits at the center of the movement (Figure 4.4(a); mm. 27-36). There, the introduction's collection of row forms return, but the music is expanded and reconfigured. This expansion is the most prominent aspect of recursive structure in the movement, and coincides with *explicit* symmetry in nearly every musical domain. Beginning at m. 27 the voice, accompanied by the orchestra, sings the words found in the center strophe of Jone's poem, the passage associated with the seed, its roots, and their spiritual correspondences. After five measures (mm. 27-31), each part reverses, playing mm. 27-31 in retrograde over mm. 32-36. Aside from the obvious pitch symmetry, that retrograde coincides with an explicit realization of the introduction's latent symmetries. First, both the voice and orchestra are now symmetrical in terms of *pitch* (not just *pitch-class*) intervals. Second, as each part reverses at m. 32, the rhythmic relationships between the parts change as well. And as a consequence, not only are each of the two parts rhythmically symmetrical as individuals, but *together*, the two parts create the composite rhythmic symmetry that was missing in the introduction.

The B section's explicit realization of the introduction's abstract symmetries are poetically suggestive in two ways: first, as representative of recursion—of the Goethian *Urpflanze*—the self-replicating, seed/tree relationship; and second, as an embodiment of the passage's Swedenborgian metaphorical transcendence. Each of these ideas has a unique "music transformational" association, outlined in Figure 4.5. Arrows on the figure trace a narrative of recursion that has its origins in the vocal phrase that sets the first line of text. Like the clarinet and orchestra parts that preceded it (cf. Figure 4.3(a)), this phrase is rhythmically symmetrical. That symmetry places the poem's most important word, "Ahornsamen" [maple seed], at its very center, where the four

FIGURE 4.4. "Kleiner Flügel Ahornsamen," B section, mm. 27–36. Note the melodic and rhythmic retrograde that begins at m. 32.



pitches that set the four-syllable word highlight the I_{11} symmetry that $\mathbf{P_3}$ contains within it. That *I*-structuring is the *same* as found in the instrumental introduction, which—as the arrow from the introduction to the B section shows—is the *same* as the *I*-structuring in the B section. The recursion, here, involves three levels: (1) the *four pitch classes* that set "Ahornsamen," (2) the *discrete hexachords* in the instrumental introduction, and (3) the *row forms* in the B section. This amplification, as we have seen, is associated with other types of symmetry, which grow more

explicit at higher levels. The rhythmic symmetry of the "Ahornsamen" is expanded in the introduction (see Figure 4.3(a)), which is subsequently expanded even more in the B section.

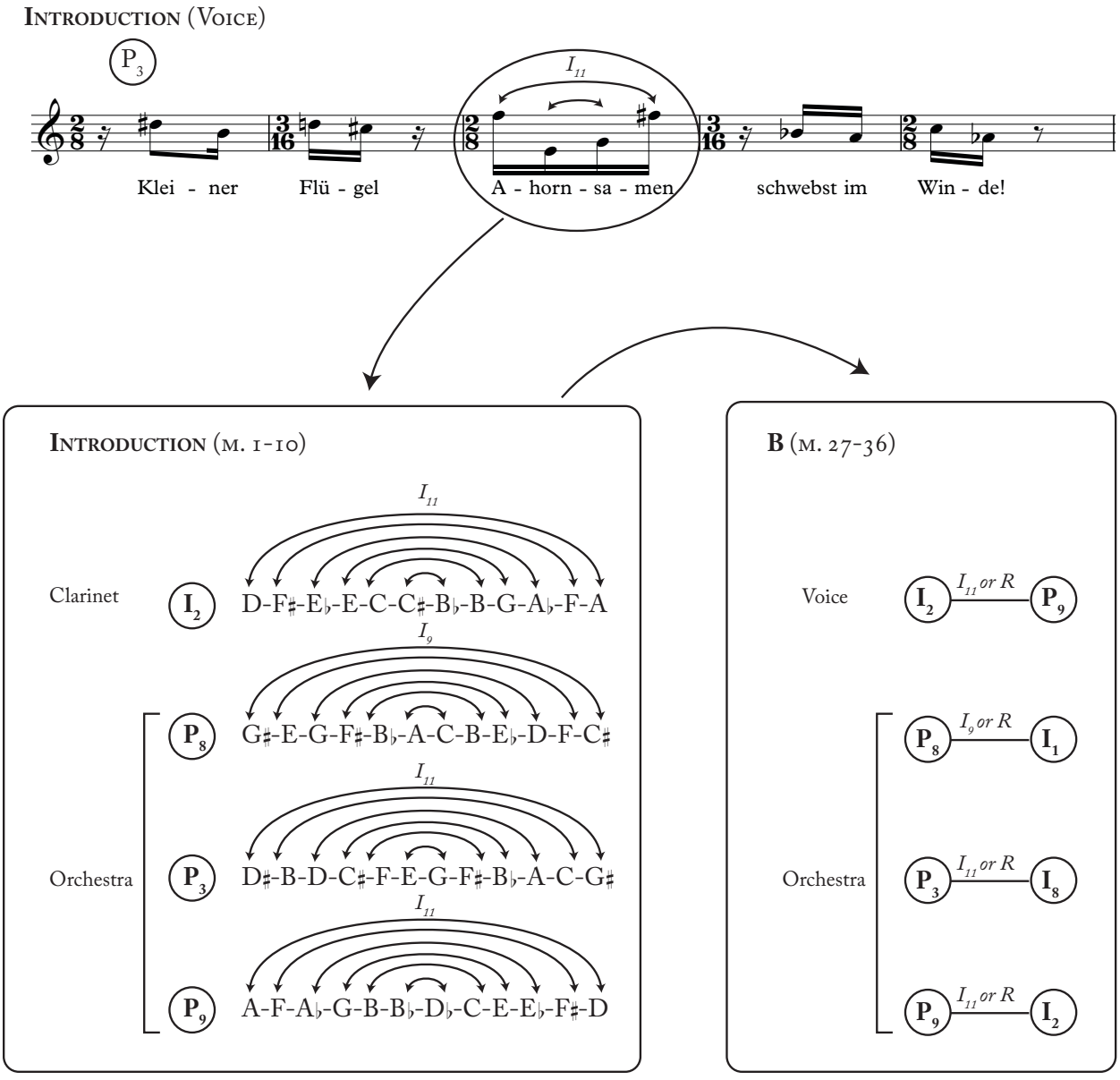
This narrative of recursion points toward the B section, as it should. That passage sets the text that is the key to the poem's metaphorical enlargement. As part of the musical representation of this idea, an important "music transformational" change happens in the B section that takes advantage of the transformational ambiguity afforded by the row class. On Figure 4.5, the transformational labels in the B section are shown as I_{11} *or* R (or I_9 *or* R). The emergence of R in the B section is of paramount poetic importance and is directly tied to lexical associations within the poem. Though the two lines of the central strophe are not of the same length (they have ten and nine syllables, respectively), the phonetic associations between the strophe's most important words *are* symmetrical: "Helle" [light] becomes "Himmel" [Heaven] while "Wurzeln" [roots] becomes "verwurzelt sein" [to be rooted].

The passage's R structuring is a direct response to the strophe's symmetry.[31] The pitch and rhythmic retrograde allows Webern to locate each of these words within nearly identical pitch and rhythmic contexts, as can be seen on the score excerpt in Figure 4.4. Whereas at first appearance "Helle" (m. 30) has an earthly connotation, representing the light towards which the maple tree reaches, its motivic association with "Himmel" at m. 33 widens its lexical scope. Associated with "Himmel," "Helle" becomes an earthly metaphor for heaven. Similarly, "Wurzeln" at m. 28 has a limited lexical meaning associated with nature—tree roots—but the retrograde underscores its broadened scope: "verwurzelt sein" (to be rooted) occurs in a heavenly context.

It is interesting that these two musical representations of the poem's themes are not mutually exclusive. From a technical standpoint, the I structure in the B section did not have to

[31] In fact, a strong argument could be made for analyzing the rows beginning at m. 32 with **R** and **RI** labels. This might better capture the sense in which the second half of the B section manifests I becoming R. I have decided to retain the **P** and **R** labels for reasons that become apparent in Figure 4.8.

FIGURE 4.5. Symmetrical Amplification from "Ahornsamen" to the B section.



B (M. 27-36)

wirst aus Wurzeln in das Helle steigen, bald im Himmel auch verwurzelt sein.

coincide with *R* structure because, as mentioned earlier, I_{11} and I_9—the *I*-transformation that recur in the B section—have the same action as *R* only in particular circumstances. Webern could have chosen eight different rows for the B section that would have had the same I_{11}/I_9 structure, but would not have been able to produce the metaphorically important *R* structure.

SEED/TREE RELATIONSHIPS, PART 2: THE CANONS

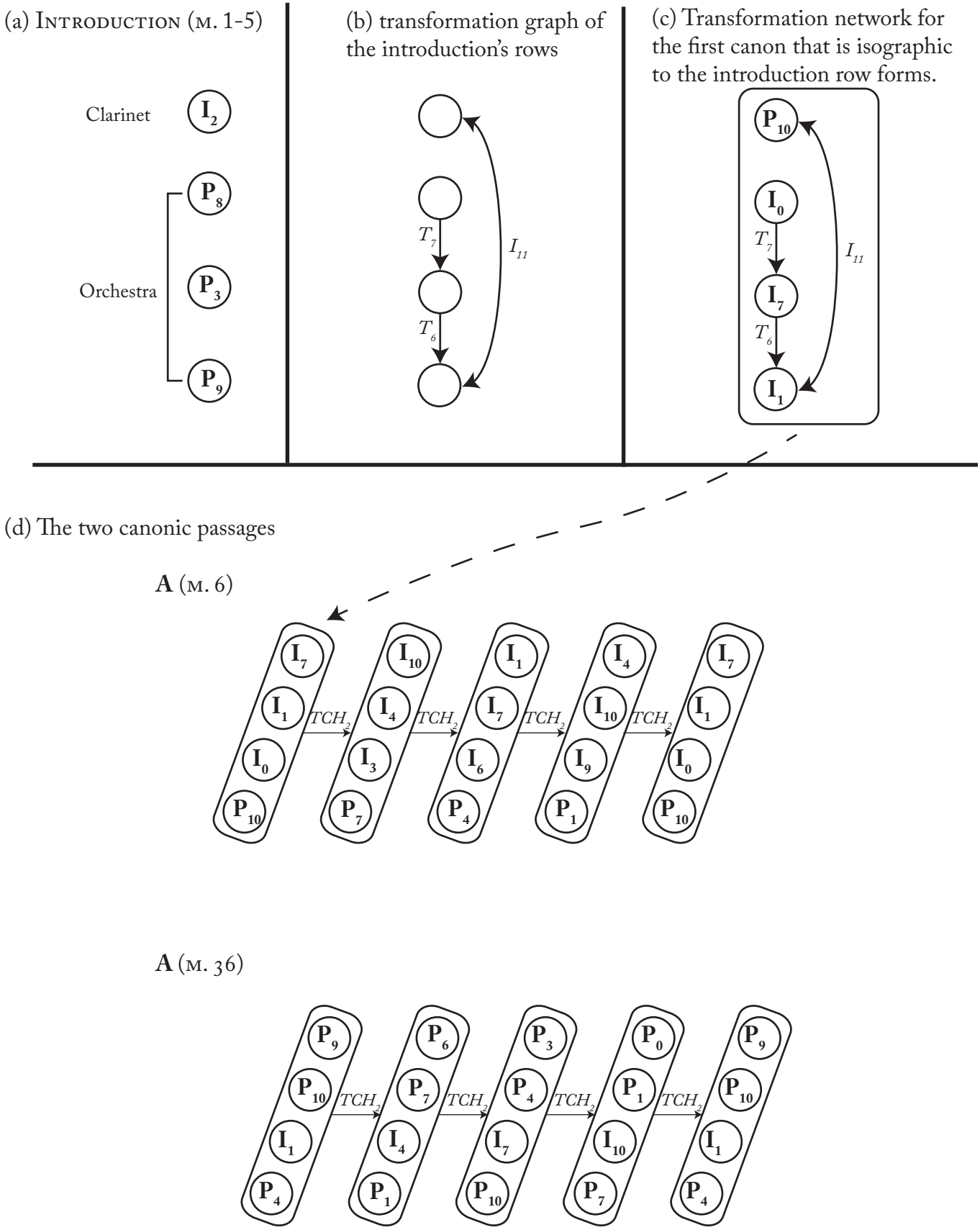
"Loose," four voice canons surround the B section and contain the text from the outer strophes of Jone's poem.[32] The relationship of these A sections to one another, and to the introduction and B section, mimics the relationship between those passages as large-scale projections of the inversional structuring that we saw there. Before showing that relationship, Figure 4.6 shows that the rows corresponding to the four canon voices are derived from the row structure of the introduction. The transformation graph at (b) models the transformational relationships of those rows—three transpositionally related row forms, and one row related to another by the inversion I_{11}—and that graph forms the basis for each of the larger nodes in the two canonic passages, as can be seen at (c) and (d).[33] At (d) the networks modeling each canonic passage show that each traverses a complete, 5-row, TCH_2 cycle. Notably, the row contents of the cycles are not the same. When the A section is recapitulated (beginning at m. 36) an important "reversal" occurs. Whereas at m. 6 the four voices in the canon projected *three **I**-rows and one **P**-row*, at m. 36—following the B section—the four voices in the canon are comprised of *three **P**-rows and one **I**-row*.

---

[32] Bailey (*The Twelve-Note Music of Anton Webern*, 286-92) outlines the canonic structure in detail. The pitch and rhythmic structure of the passages are controlled by the requirements of the canon, but as Bailey shows, that structure is loosened rhythmically, and the pitch content of the subjects is divided amongst the orchestra. The result is "music […] never before so loose."

[33] This is just one of many ways transformational interpretations of the introduction. For example, one could label all of the inversional relationships present, which would linking each of the **"P**-forms" to the "**I**-form." I have avoided that construction for interpretive reasons. Recalling that I_{11} related discrete hexachords *within* three of the four rows in the introduction, it seems more interesting to highlight the fact that the same relationship exists *between* rows as well. The four rows in each large node at (d) are positively isographic, though the nodes are not transpositionally related.

FIGURE 4.6. A transformational derivation of the four-voice canon.



An organized spatial network in Figure 4.7 illustrates the relationship between the two A sections more precisely. Rows on the space are partitioned by TCH_2 and conformed by the transformation graph shown in Figure 4.7(b).[34] These constraints partition the space into six disconnected networks, each containing sixteen rows; and therefore, the space has ninety-six (= 16 * 6) row forms. There are ninety-six rows on this space and not twenty-four (the number of distinct rows in the row class) because every row has a duplicate in two other partitions.

The six partitions are arranged to show that the same "seed/tree" relationships that links the introduction to the B section also connects the canons in the two A sections to one another. Each of the six partitions sits adjacent to its I_{11}/I_9 transformation—the transformation that connected both the discrete hexachords in the movement's introduction and the row forms in the B section.[35] For example, at the top of the space, two groups of four nodes have been bolded. The group of rows on the left—{$\mathbf{I_2}$, $\mathbf{P_3}$, $\mathbf{P_9}$, $\mathbf{P_8}$}—represents the introduction and the opening five measures of the B section. The group of rows on the right is the final five measures of the B section (cf. Figure 4.5). Collectively, those two groups of rows are related by I_{11}/I_9 ($\mathbf{I_2} \xrightarrow{I_{11}} \mathbf{P_9}$, $\mathbf{P_3} \xrightarrow{I_{11}} \mathbf{I_8}$, $\mathbf{P_9} \xrightarrow{I_{11}} \mathbf{I_2}$, $\mathbf{P_8} \xrightarrow{I_9} \mathbf{I_1}$), as we saw earlier.

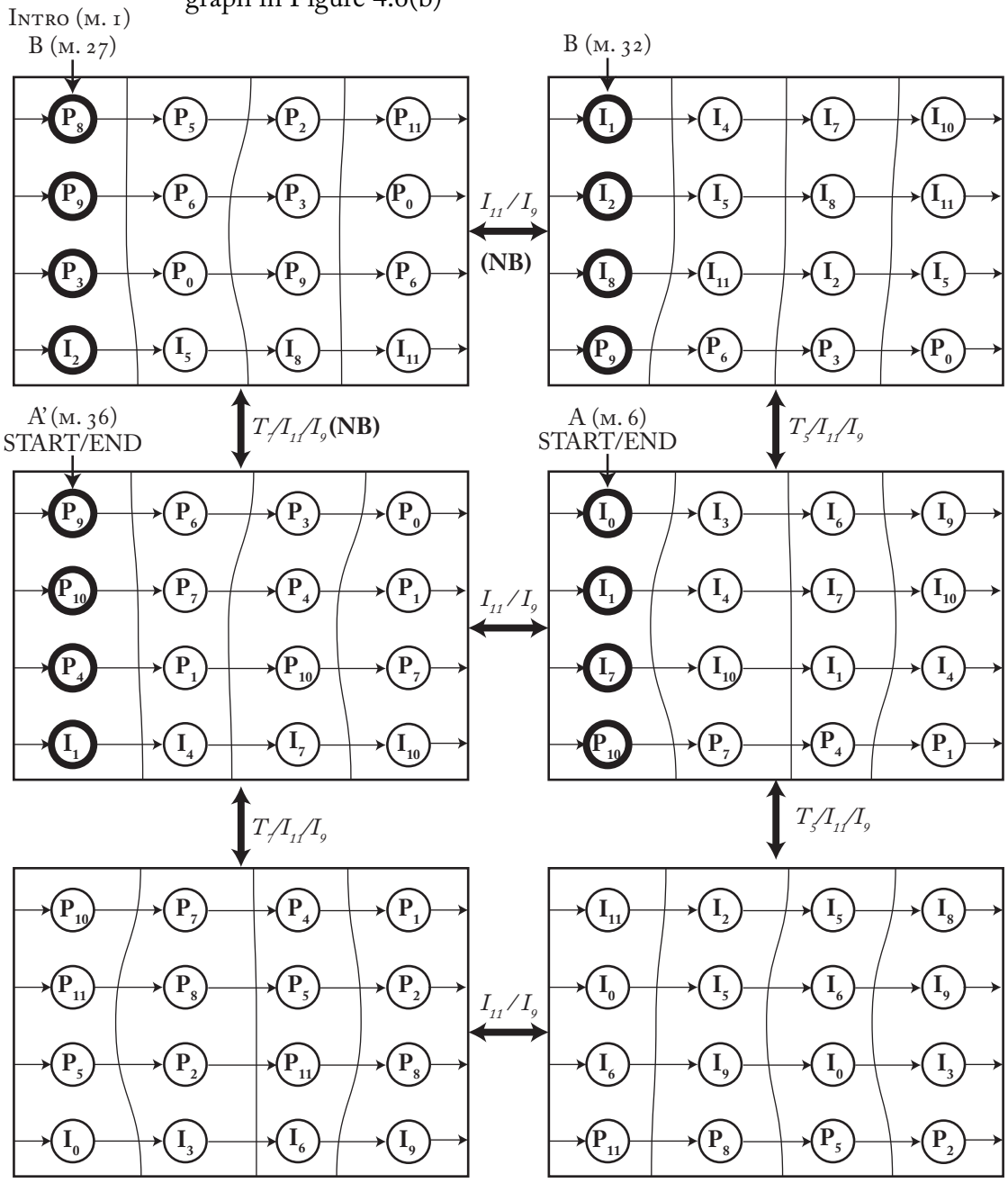
Compared to those passages, note that the two A sections belong to different partitions, both in the center of the space. Each canon completes a TCH_2 cycle, and therefore, the row quartets that begin each canon also end that canon. Because the A sections's partitions are adjacent on the space, the transformational relationship between the two canonic passages is the same as the relationship between the pairs of rows in the B section, which as we saw earlier, is the same as the relationship between the discrete hexachords in the introduction, which is the same as the relationship between the "Ahornsamen" pitches in the vocal line.

---

[34] We will discuss the pc invariance endemic to the graph soon.

[35] Formally speaking, the relationships between partitions that I have shown here—such as I_{11}/I_9—are "split transformations." See, for example, Shaugn J. O'Donnell, "Transformational Voice Leading in Atonal Music" (Ph.D. dissertation, City University of New York, 1997).

FIGURE 4.7. A *TCH4* generated spatial network, organized by the transformation graph in Figure 4.6(b)



NB: The *I* transformations are (at only these places) also *R*.

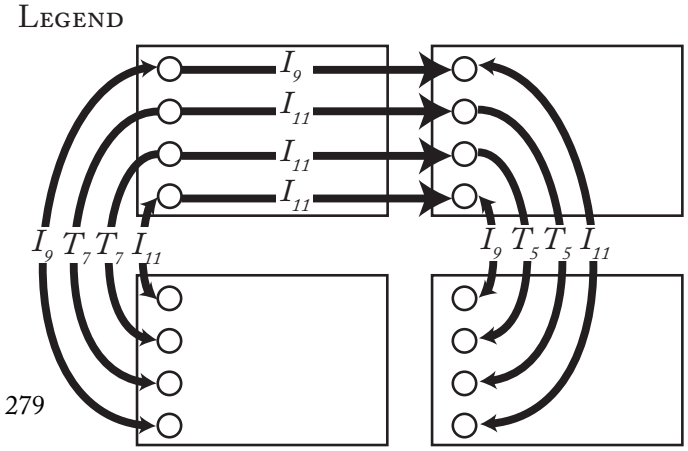


Figure 4.8 summarizes this musical *Urpflanze* by showing how these recursions are manifest across three levels of the movement, joining I_{11}/I_9 related hexachords in the introduction, rows in the B section, and row partitions amongst the two A sections. Appropriately, the B section sits directly in the center of this large-scale symmetry. And, only that passage projects both I_{11}/I_9 *and* R; while the row partitions that set the canons are I_{11}/I_9 transforms, they are not R-related. The uniqueness of the B section's dual relationships is easily seen on the spatial network Figure 4.7. That space shows that only the top two partitions are capable of joining all four row forms in each quartet by I_{11}/I_9 *and* R.

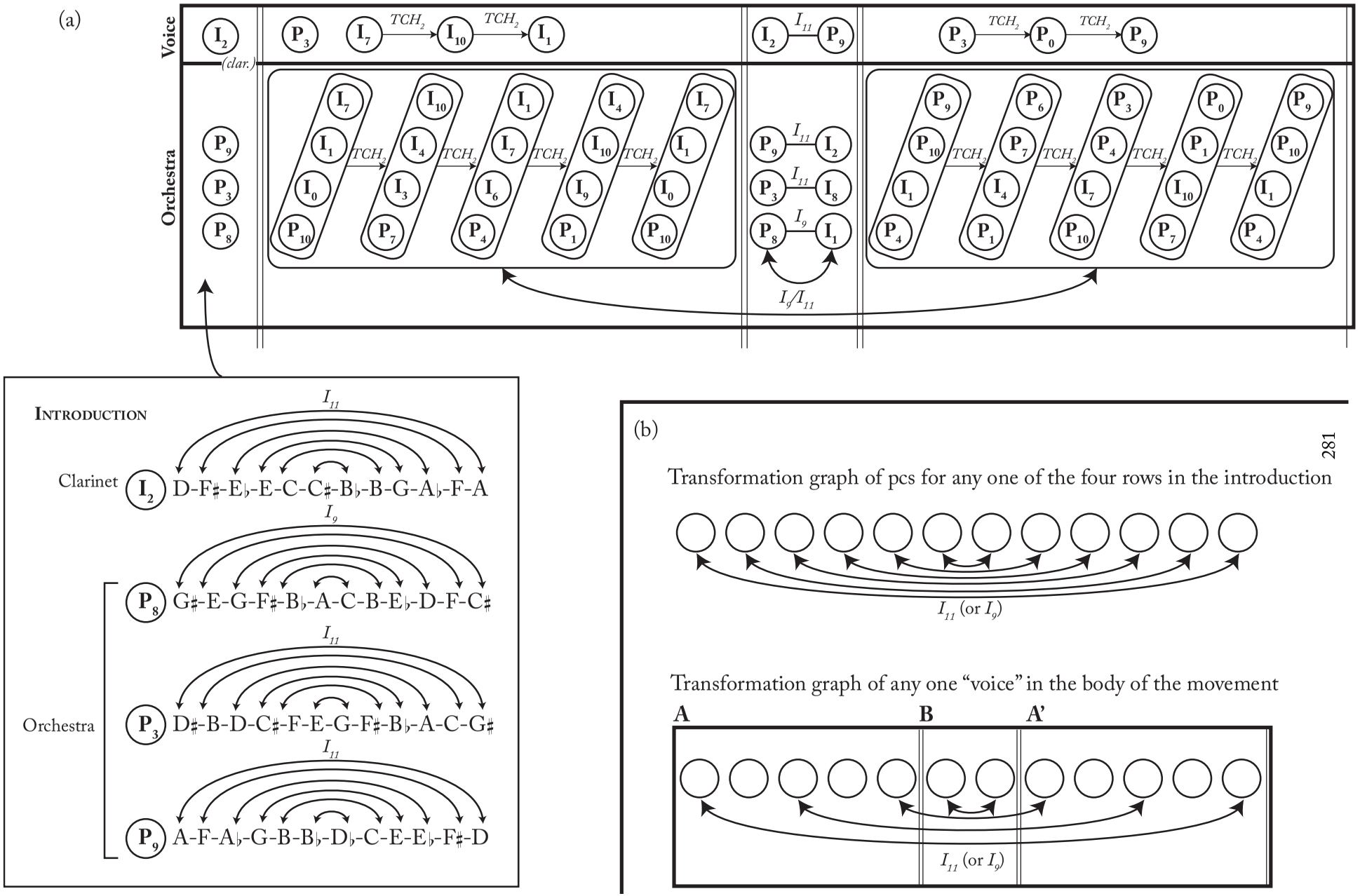
This certainly has poetic significance. Remember that the B section set the text that broadened the metaphorical scope of the poem to engage natural/spiritual "correspondences." The retrograde relationship allowed Webern to project that musically through rather simple word painting that associated important words ("Himmel" and "Helle," "Wurzeln" and "verwurzelt sein") through shared pitch classes. The I_{11}/I_9 (but not R) relationship between the A sections is representative of not only the larger metaphorical scope suggested by the B section, which sits in their center, but also the increasingly *abstract* nature of that metaphor.

The increasing abstraction has another representative. Figure 4.8(b) shows that the pitch class succession in the introduction's row forms is very nearly isographic to the row succession in the A and B sections. The transformation graph at the top symmetrically connects the twelve pitch classes in any of the four rows found in the introduction by I_{11} or I_9. That same transformation graph very nearly applies to the succession of *twelve* rows that comprise the A and B sections, as shown at the bottom of (b).

More Seeds: the 'Vertical' and the 'Horizontal'

Figure 4.8's transformational understanding of the movement's plan resonates quite strikingly with comments that Webern made to Willi Reich. Discussing the introduction and

FIGURE 4.8. Recursion of inversional structure across three levels of the movement.



opening vocal phrase, Webern understands its relation to the rest of the movement as follows:

> Formally it [mm. 1–10] is like an introduction, a recitative! But this section is constructed in a way that perhaps none of the "Netherlanders" ever thought of; it was probably the most difficult task I have ever had to fulfill! […] The melody […] may be the *law* (Nomos) for all that follows! In the sense of the "primeval plant" of Goethe: "With this model, and the key to it, one can proceed to invent plants ad infinitum … The same law can be applied to everything else that lives!"[36]

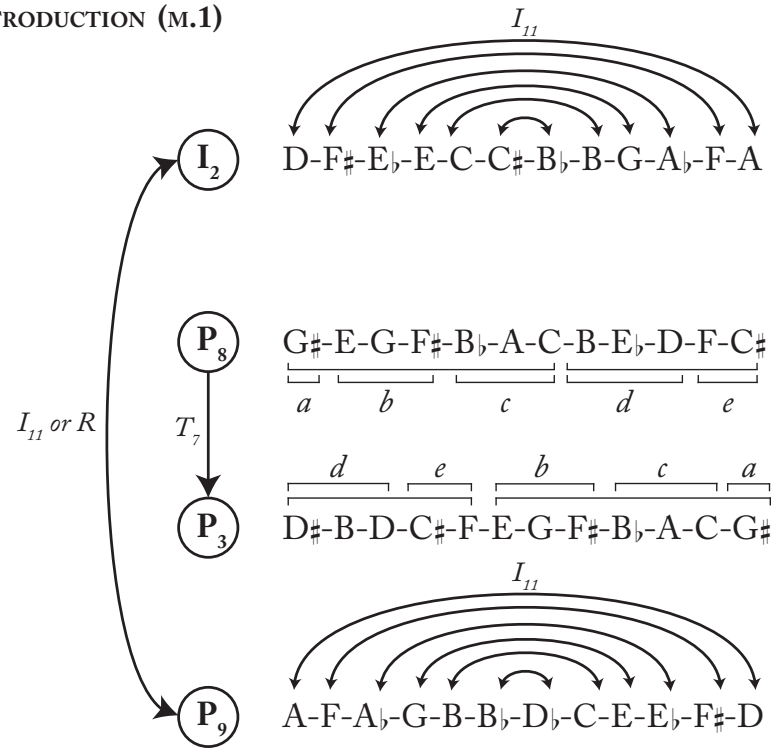
Up to this point, we have shown how this "law" is applied across multiple musical objects (pitch, row, row partition), but in only one musical dimension—*horizontally*. That is, the relationships between pcs within the introduction's rows, between rows in the B section, and between row partitions all engaged chronology.

Our concern with these horizontal relationships came—to some extent—at the expense of concerns with pc relationships between rows. Figure 4.9 studies the transformational relationships *between* row forms in the introduction, and shows one interesting way in which these *vertical* relationships are projected onto the *horizontal* plan of the movement. One central way in which the "horizontal" is manifest in the "vertical" is through the projection of a row's internal inversional structuring onto its vertical relationship to other rows. Figure 4.9 shows that the horizontal relationship of pcs *within* both $\mathbf{I_2}$ and $\mathbf{P_9}$ also describes the vertical relationship between the rows. That correspondence between the horizontal and vertical dimensions is what allows us to describe the relationship simultaneously as retrograde. The remaining row forms at (a) are T_7-related. Rows related by T_7 are very similar, and the distribution of their invariant pc segments strongly resembles that of retrograde related rows. The initial seven pcs of $\mathbf{P_8}$ are the final seven pcs of $\mathbf{P_3}$, and moreover, the five labeled pc segments of $\mathbf{P_8}$ (*a*–*e*) occur in $\mathbf{P_3}$ in near-retrograde order. The vertical relationships in the introduction are both retrograde and T_7, or "near-retrograde."

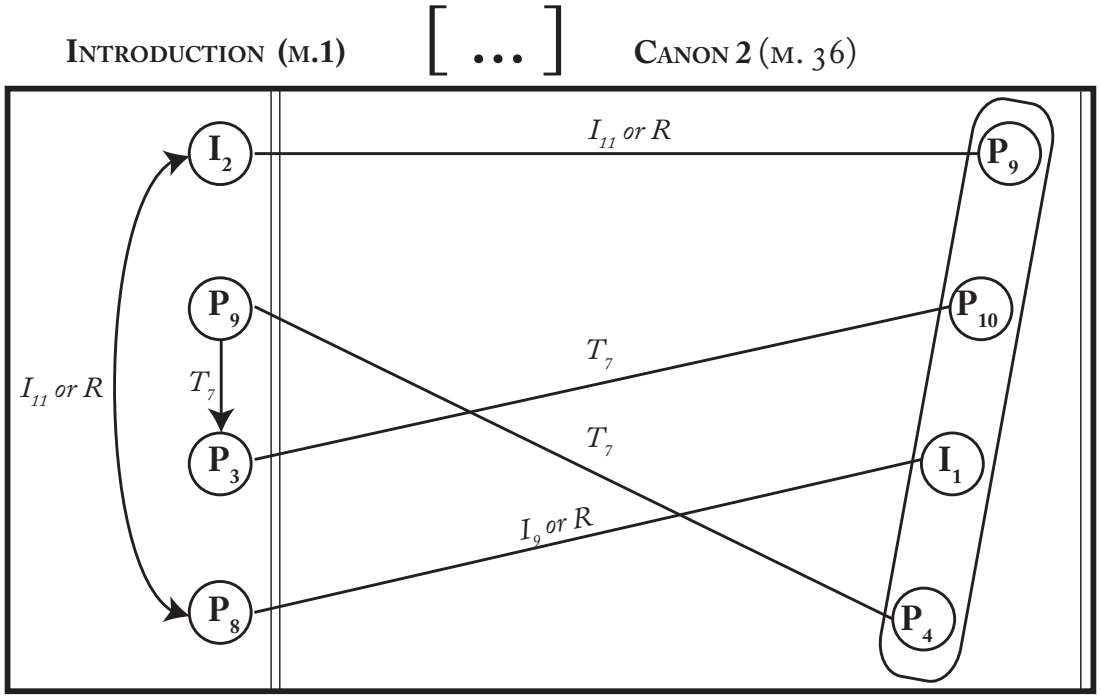
[36] Moldenhauer and Moldenhauer, *Webern, A Chronicle*, 575.

Both of these play an important part in the very largest-scale assocations in the piece, between the introduction and final canon. I've shown this in Figure 4.9(b). There, we can see that the single retrograde relationship that *vertically* related rows in the introduction is converted into two retrograde relationships that occur *horizontally* between the first and last pairs of row forms. And importantly, both of those retrogrades can also be described as I_{11} or I_9, the two prominent I transformations in the movement. Simultaneously, the single T_7 relationship (a "near-retrograde") in the introduction is converted into two T_7 relationships between the first and last pairs of row forms. As a result, the large-scale relationship between the introduction and final canon is an amplification of the vertical relationships amongst row forms within the introduction, another example of the type of musical recursion that echoes the central metaphor in Jone's poem.

FIGURE 4.9. Horizontal amplifications of retrograde and near-retrograde relationships. (a) Retrograde and near-retrograde relationships in the introduction.



(b) Horizontal amplifications of the introduction's vertical construction.



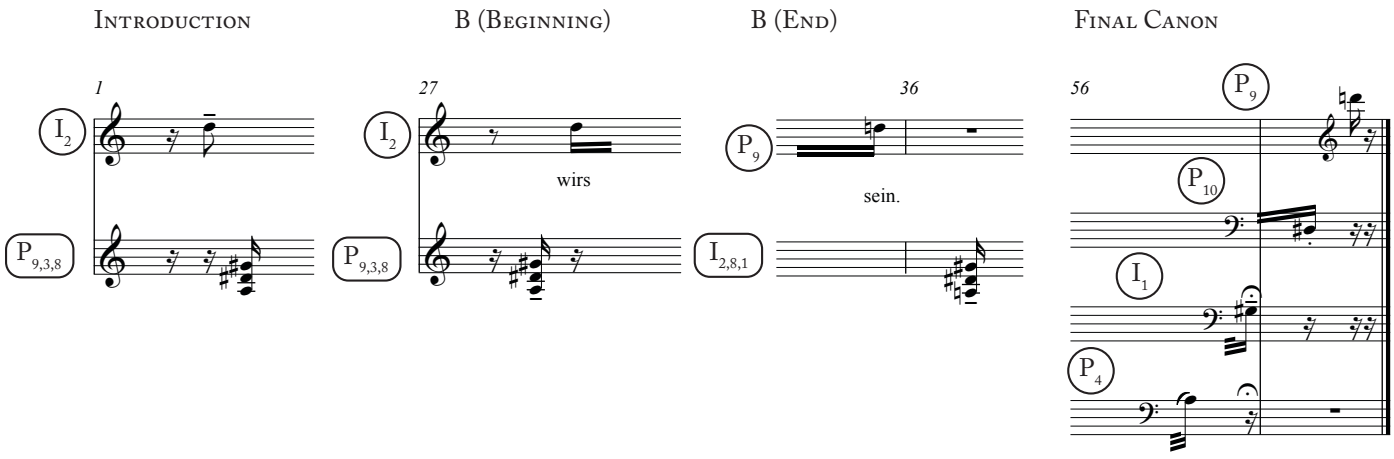
Our space in Figure 4.7 shows that—like the *R* relationship in the B section—this relationship is similarly unique. Vertically aligned partitions on the space are $T_7/I_{11}/I_9$ related in the manner shown by the legend. The I_{11}/I_9 component of the partition's relationships are suggested by the I_{11}/I_9 relationships that occurred at many points earlier in our analysis. The space indicates that only the vertically aligned two partitions on the upper left have I_{11}/I_9 relationships that are also *R*.

The projection of the retrograde and near-retrograde relationships creates a pc association that encapsulates the many other symmetries in the movement and creates the largest recursive relationships in the movement. Figure 4.10 illustrates. As a result of those relationships the first and last tetrachords of the movement [D, D♯, G♯, A] are the same, relating the introduction to the final canon.[37] The figure shows that this large symmetry, like so many other symmetries in the

[37] This was observed, in a different context, by Phipps, "Tonality in Webern's Cantata I," 147. Phipps sees the opening and closing sonorities as "dominants" of "tonics" found in the first movement.

movement, echoes the B section, whose first and last tetrachords are also [D, D♯, G♯, A]. These three moments comprise the temporal boundaries of the movement's important formal sections. Their relationships to one another show how the introduction acts like a seed for the rest of movement in both horizontal and vertical terms. While the B section's symmetry is a projection of horizontal relationships within the introduction (*I_{11}/I_9 between* hexachords), the larger symmetry between the introduction and final canon is a horizontal projection of *vertical* relationships in the introduction.

Figure 4.10. Tetrachord association at important formal locations in "Kleiner Flügel Ahornsamen."



LINEARITY: CANTATA I, OP. 29, "BLITZ UND DONNER" [38]

Like "Kleiner Flügel," the poetic structure of Jone's "Blitz und Donner" is parallel to the central image of the poem.[39] Here, that image—a lightning strike and its sonic echo—is likewise imbued with larger metaphorical significance, and once again, the operant metaphor relates to life:

"Blitz und Donner," from *Der Monkopf*

Zündender Lichtblitz des Lebens schlug ein aus der Wolke des Wortes.	Lightning, the kindler of Being, struck, flashed from the word in the storm cloud.
Donner der Herzschlag folgt nach,	Thunder, the heartbeat, follows,
bis er in Frieden verebbt.	at last dissolving in peace.

More than "Kleiner Flügel," whose structure and content represent's life's circularity, "Blitz und Donner" communicates a *linear* process. The opening image of a lightning strike as a kindler "of Being" ("des Lebens") conveys the immediacy of life's inception, which echoes (like a heartbeat

---

[38] Analytical remarks on "Blitz und Donner" have appeared more often than for other movements. See George Rochberg, "Webern's Search for Harmonic Identity," *Journal of Music Theory* 6, no. 1 (1962): 109–122; David H. Saturen, "Symmetrical Relationships in Webern's First Cantata," *Perspectives of New Music* 7, no. 1 (1967): 142–43; Jonathan D. Kramer, "The Row as Structural Background and Audible Foreground: The First Movement of Webern's First Cantata," *Journal of Music Theory* 15, no. 1–2 (1971): 158–81; Robin Hartwell, "Duration and Mental Arithmetic: The First Movement of Webern's First Cantata," *Perspectives of New Music* 23, no. 1 (1984): 348–359; Phipps, "Tonality in Webern's Cantata I," 130-41; Bailey, *Twelve-Note Music*, 272-85; Mead, "Webern and Tradition," 174–78.

[39] We do not know of Webern's original source for "Blitz und Donner." Reinhardt indicates that Jone included it in her *Iris* collection, but that it appears to have been from *Der Mohnkopf*, an earlier manuscript that is now lost ("From Poet's Voice to Composer's Muse," 541).

("Herzschlag")) into the future and finally dissipates into peace ("Frieden").[40] Suggestively, while "Herzschlag" is usually translated as "hearbeat," it can also mean "heart failure."[41]

Jone's poem communicates this metaphor with less subtlety than in "Kleiner Flügel," perhaps because the analogy is less obvious. But the *structure* of the poem is no less compelling as a representative of Jone's larger ideas. Dissolution over time is an important metaphor, and Jone's poem captures this through stanzas that become shorter and filled with less detail. The lightning strike is described with precision—note the echo of alliterative echo of "Lichtblitz des Lebens" in "Wolke des Wortes." But the thundering heartbeat and its dissolution are recounted sparsely.

The Choral Music

Webern's setting of the poem, sung by a choir, places the text in the very center of the movement (see Figure 4.11), surrounded by instrumental canons. As was true in "Kleiner Flügel," much of the analytical challenge here lies in understanding the instrumental canons in terms of the text—as a musical image of the text's central metaphor. Unlike that movement, which was obsessed with organicism as manifest through structural recursion, "Blitz und Donner" is linear, and appropriately, Webern's setting communicates a *progression* that reverberates sympathetically with that poetic idea. A lightning strike's dissolution occurs sonically. And therefore, it seems appropriate that the musical image of this dissolution takes place in the four-voice instrumental

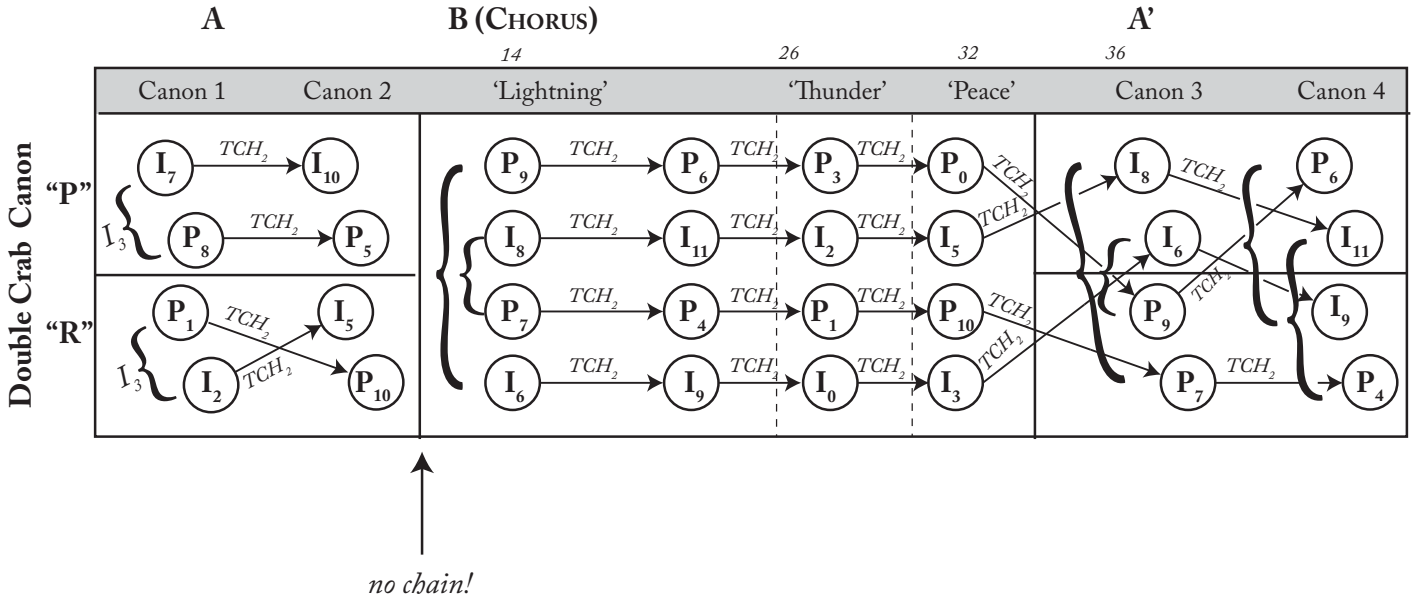
---

[40] Bailey views the poem similarly: "Hildegard Jone's poem refers to lightning as the moment of life's inception, thunder as the moment of its cessation, and the eventual quiet following the thunder as the peace and tranquillity of death" (*The Twelve-Note Music of Anton Webern,* 272). She also notes the ambiguity of "Herzschlag" (442, note 1).

[41] Like "Kleiner Flügel," the poem certainly has spiritual connections, though they are less obvious here. In particular, Jone's reference to "der Wolke des Wortes" alludes to the Christian existentialist thought of Ferdinand Ebner. Ebner's theology of language, based on a notion called *Ich-Du*, placed unequivocal faith in the "word" as a way for the human *Ich* to address the only true *Thou*, that is, to God. Reinhardt ("From Poet's Voice to Composer's Muse") notes that Ebner's philosophy greatly influenced Jone. The two were good friends in the final years of Ebner's life. Webern's first Jone setting, the songs in Op. 23, were taken from Jone's *Viae inviae*, an elegy to Ebner published in the Catholic literary journal, *Der Brenner*. Reinhardt has used sketch materials to show how "the qualities of Jone's verse that Ebner valued so highly are […] mirrored in Webern's music" (3781). See "«ICH UND DU UND ALLE»: Hildegard Jone, Ferdinand Ebner, and Anton Webern's 'Drei Gesänge' Op. 23," *Revista de Musicología* 16, no. 6 (1993): 3766–3782.

canons that surround the choral music, which occur in the movement's two A sections. Primarily, the dissolution occurs through a constantly morphing conception of a *voice exchange* that is heard melodically and rhythmically. Within this progression the abstract details of the canons—their rhythmic subject, tempo, melodic and harmonic relationships—grow "fuzzy," like an echoing thunder clap that dissolves into silence. Thus, dissolution as a poetic idea is often represented musically as "misremembrance"—the imperfect recollection of a musical object (pitch, rhythm, and so on) that came before.

FIGURE 4.11. A formal diagram for "Blitz und Donner."

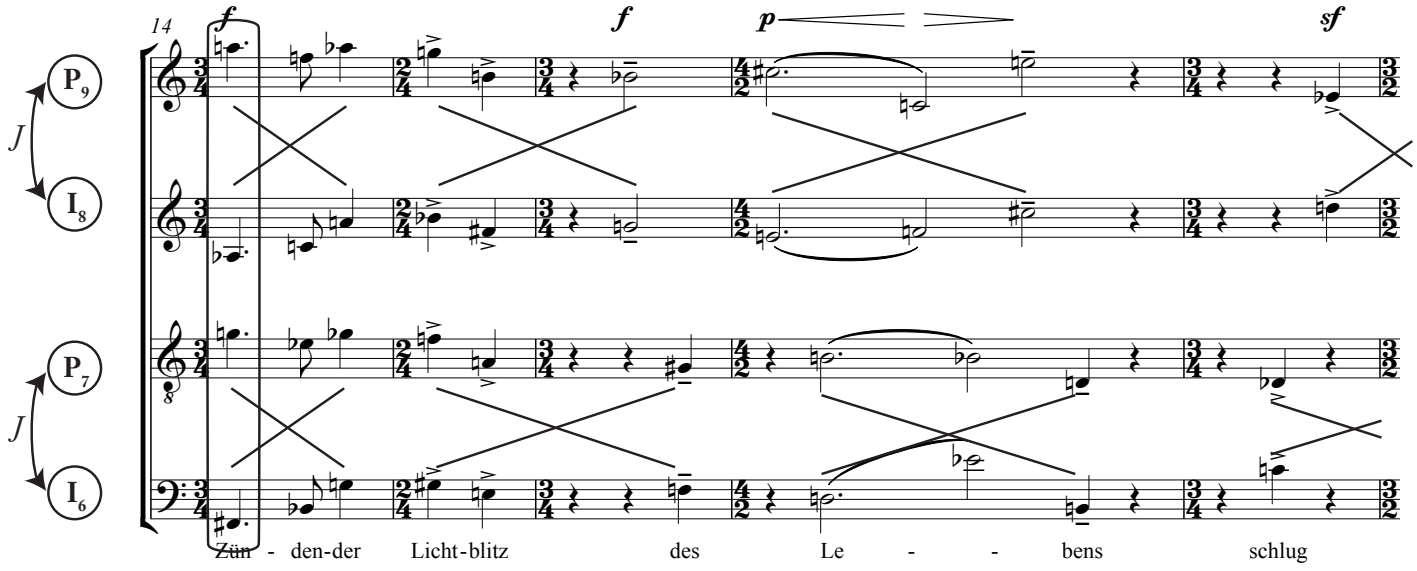


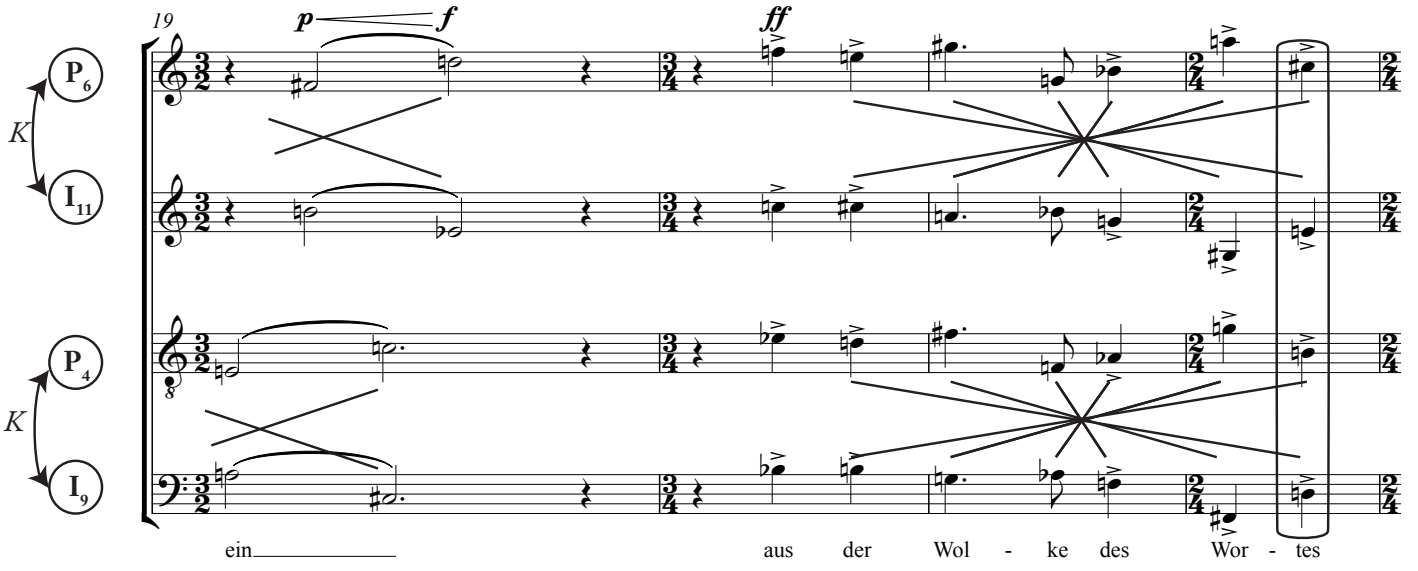
Though the choral music (mm. 14–36; see Figure 4.11) that sits in the center of the movement stands outside of this progression, the instrumental canons involved in the progression and the concepts of dissolution are best understood in terms this music, which contain the only setting of the poem in the movement. Within this section the three stanzas of the poem (shown in Figure 4.12, and referred to there as "Lightning," "Thunder," and "Peace") are separated by interjections from the orchestra. Unlike the canon passages, the choral music is entirely homophonic. In it, Webern captures the thunder echoes in the poem in two imagistic ways—one

involving voice exchanges, and the other involving pc relationships between the boundary sonorities of the three stanzas.

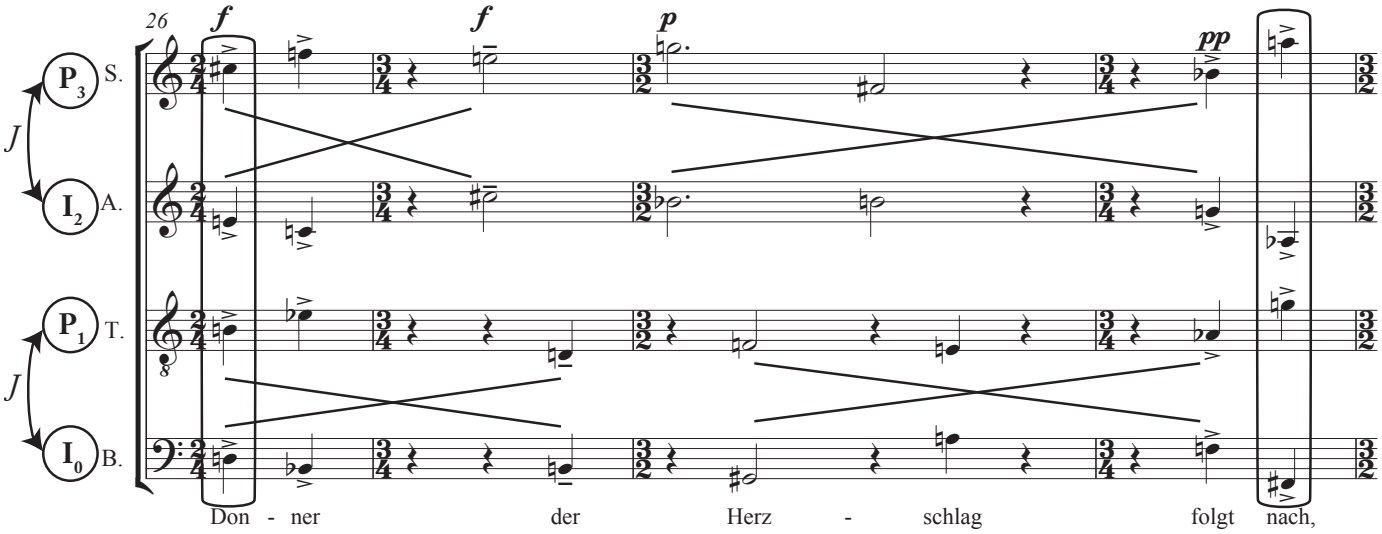
FIGURE 4.12. "Blitz und Donner's" three choral sections, mm. 14-35.

(a) Choral Section 1: Lightning

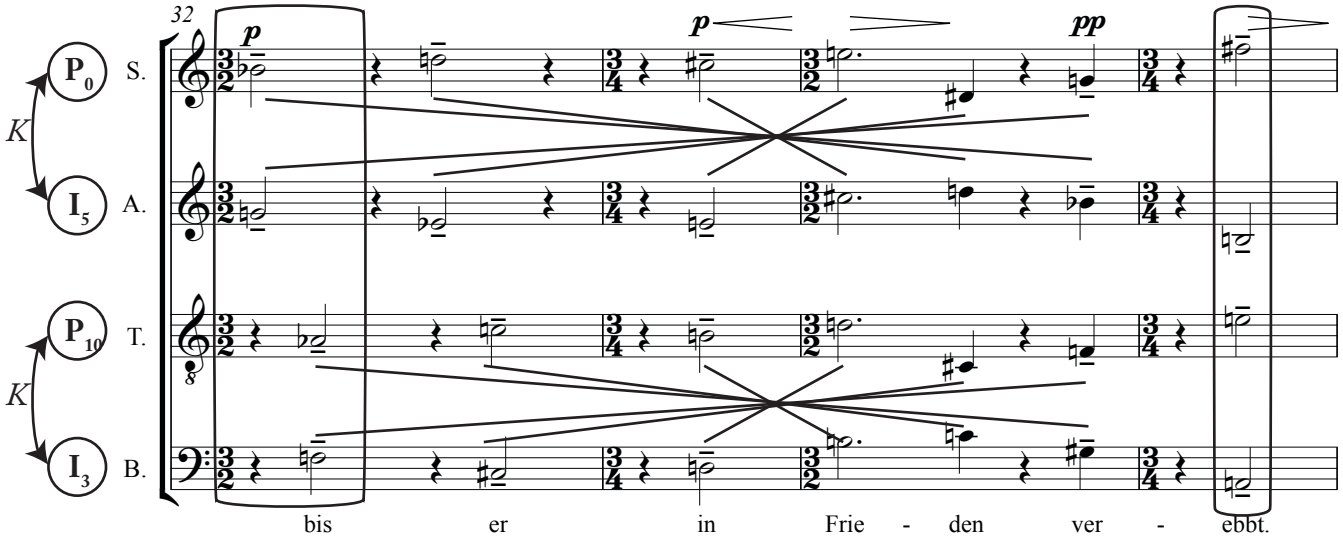




(b) Choral Section 2: Thunder (the first chorus note in each part is row order number 4)



(c)Choral Section 3: Peace (the first chorus note in each part is row order number 4)



The Lightning music (at (a)) divides musically and grammatically into two parts, the first of which contains four simple "voice exchanges" between the soprano/alto and tenor/bass voice

pairs.[42] At m. 20 the voice exchange becomes more "elaborate": the last six syllables of the stanza —"der Wolke des Wortes"—enlarge the voice exchange into a complete retrograde between the voices in both pairs.[43] Both the Thunder and the Peace music (at (b) and (c)) contain these pc echoes as well, but in the spirit of of Jone's poem—where each stage of the lightning strike contained less detail than the prior stage—these passages "misremember" the Lightning music, or at least remember its details with less precision. The Thunder music contains a partial setting of the voice exchange that began 'Lightning,' but not the more elaborate retrograde, and the Peace music sets only the retrograde.

Thus, the types of pc relationship that characterize this music embody two images from the poem. The sonic echo of the thunder is mimicked by through the pc echoes in the voice exchanges, and the thematic dissolution is represented by the different types of voice exchange that set each stanza. A similar dissolution involves the three passages's boundary sonorities, which are circled in Figure 4.12 and diagrammed in Figure 4.13. Together, the six boundary chords in the three passages contain four distinct sonorities that belong to three set-class types. An [F♯, G, A♭, A] chromatic tetrachord begins the Lightning passage and ends the Thunder passage, while a [B, C♯, D, E] tetrachord ends Lightning and begins Thunder (see the arrows on the top of Figure 4.13). Thus, while the Thunder music echoes the Lightning that occurred just before it, the music jumbles the order of its boundary sonorities—the resulting symmetry reflective of the smaller-

---

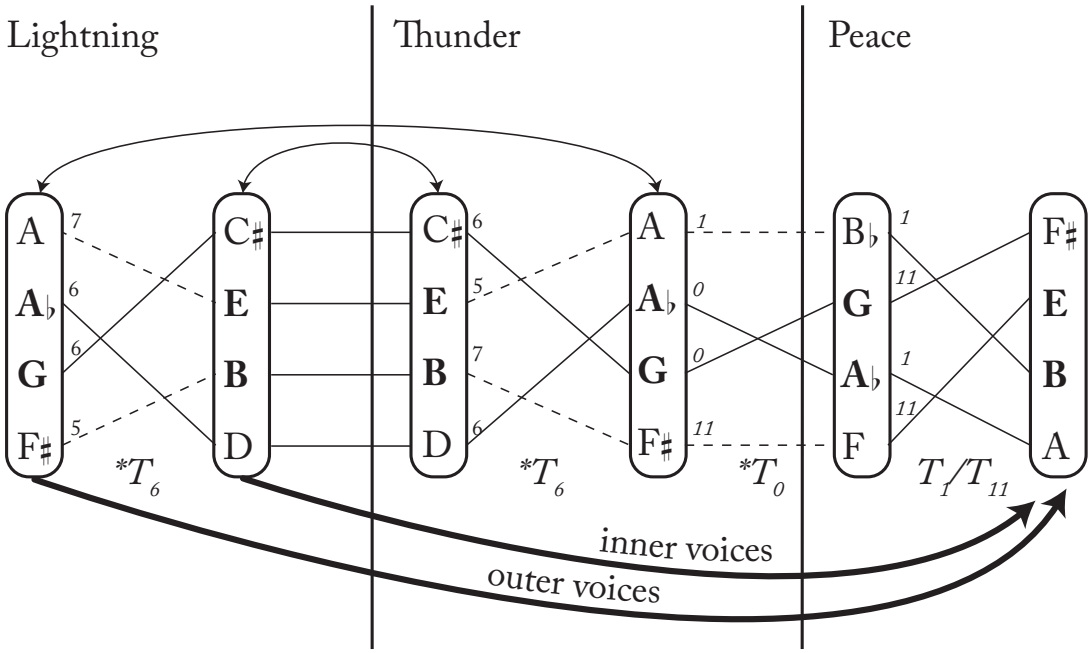
[42] This property of the particular row combinations in the choral music has been noted by many others: see Rochberg "Webern's Search," 117; Saturen, "Symmetrical Relationships"; Kramer, "The Row as Structural Background and Audible Foreground," 168; Phipps, "Tonality in Webern's Cantata I," 137-38; and Bailey, *Twelve-Note Music*, 281. There are many characteristics of this passage—resulting from the properties of the row combinations—that I am glossing over for now. For example, in the passages with simple voice exchanges (mm. 14-19, for instance) the four-voice chordal sonorities are also echoed every other beat—cf. [F♯, G, A♭, A] in m. 14, b. 1 and [F♯, G, A♭, A] in the same measure, on beat 3. Mead, "Webern and Tradition," 174-78 has the most complete discussion of those sonorities, much of which echoes comments found first in Kramer, but reframed.

[43] In a serial context, we often simply label characterize passages like this (mm. 21-22) as projecting "retrograde." While retrograde is undoubtedly present here, I prefer to understand it as an enlarged voice exchange because of its context; the prior music carried out a similar musical act, to which this seems related. In tonal music these kinds of passages are often understood in this way. Consider, for example, the omnibus, which often prolongs a harmony through a chromatic voice exchange spanning a number of chords.
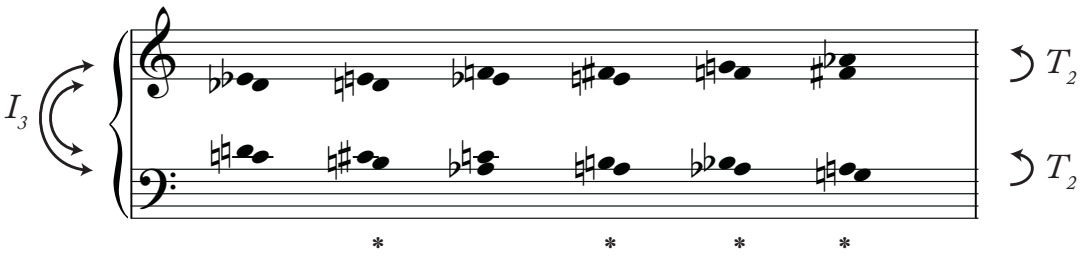
scale voice exchanges present between voices in shown in Figure 4.12. In the final passage of Peace music, those sonorities are entirely "forgotten"—its boundary chords, [F, G, A♭, B♭] and [E, F♯, A, B], are new.

FIGURE 4.13. Boundary chord echoes in three choral sections.

(a) Boundary chords for the three choral sections, mm. 14-35 (cf. Figure 4.12)



(b) "General repertoire of tetrachords formed by relations" in the choral music (after Mead, "Webern and Tradition," 175). Asterisked chords are boundary chords in the choral music as shown at (a).



And yet, the novelty of these chords—and even the relationship between the boundary chords of Lightning and Thunder—also seems to represent the idea of echo and dissolution.

Figure 4.13(a) shows that the vertical tetrachord that *ends* Lightning and *begins* Thunder is [B, C♯, D, E]. This is as a "misremembered," "fuzzy" transpositional echo of the fully-chromatic [F♯, G, A♭, A] that *began* 'Lightning.' Two voices articulate the T_6 motion, while the remaining voices slightly misremember that motion, missing by a semitone. The initial sonority of the Peace passage, [F, G, A♭, B♭], is a similarly misremembered, "fuzzy" transposition of the [F♯, G, A♭, A] tetrachord that ended 'Thunder.' Echoing the earlier fuzzy transposition, the outer two voices are offset by a semitone from the T_0 heard in the inner voices. And the final chord of the choral section [E, F♯, A, B] is completely novel (0257). The chord is created as a balanced split transposition of the prior chord.[44]

An important part of the chordal dissolution involves the central dyads of these sonorities. Figure 4.15 shows that as the three passages's boundary chords morph, the central dyads of the chords remain mostly invariant. That is, each chord—even the two completely new chords found in Peace—contain either [G, A♭] or [B, E] as the dyad in the chord's registral center. In this sense, the final chord of Peace is a misremembered echo of the Lightning music:

[44] Fuzzy transpositions and inversions have been explored in many publications by Joseph N. Straus as a way to represent atonal voice leading. See, in particular, "Voice Leading in Atonal Music," in *Music Theory in Concept and Practice*, ed. James M. Baker, David W. Beach, and Jonathan W. Bernard (Rochester: University of Rochester Press, 1997), 237–74; and "Uniformity, Balance, and Smoothness in Atonal Voice Leading," *Music Theory Spectrum* 25, no. 2 (October 1, 2003): 305–352. Straus's work has many forbearers, in particular, Henry James Klumpenhouwer, "A Generalized Model of Voice Leading for Atonal Music," (Ph.D. dissertation Harvard University, 1991); O'Donnell, "Transformational Voice Leading in Atonal Music"; and David Lewin, "Some Ideas About Voice-Leading Between PCSets," *Journal of Music Theory* 42, no. 1 (1998): 15–72.

the outer voices are taken from the first chord of Lightning and the inner voices from the last chord of 'Lightning.'[45]

In part, this chordal dissolution is inherent within the compositional "system," as a byproduct of the two types of transformational relationship relating voices in the music. Andrew Mead has shown the "general repertoire of chordal tetrachords" for this passage (see Figure 4.13(b)) as a byproduct of the fixed axis inversional structuring of I_3 between voice parts (S/A and T/B) and those voice parts's T_2 relationship. The "general repertoire" is small—only six distinct sonorities, four of which appear in the choral music. Mead notes that this general repertoire is a representative of "relations that depend on the fundamental properties of the twelve-tone system," as opposed to those that derive from "particular orderings [row forms] used in a composition" (176).[46] That is, many different row forms could have been used in this passage and created the same verticalities. The particular row forms used here, and Webern's placement of those row forms in relation to the text (note, for example, that both the Thunder and Peace music begin at order number 4), are interesting in part because of the boundary sonorities that emerge. Of the six chordal tetrachords in the "general repertoire," Figure 4.13(b) shows that two have [G,

[45] Mead, "Webern and Tradition," 174-8, shows the "general repertoire of chordal tetrachords" for this passage. That general repertoire is a byproduct of the fixed axis I_3 inversional structuring between voice parts (S/A and T/B) and those voice parts T_2 relationship. The "general repertoire" is small—only six distinct sonorities, four of which appear here. Mead shows this general repertoire because as a representative of "relations that depend on the fundamental properties of the twelve-tone system," as opposed to those that derive from "particular orderings [row forms] used in a composition" (176).

The particular row forms used here, and Webern's placement of those row forms in relation to the text (note that both the Thunder and Peace music begin at order number 4), are interesting in part because of the boundary sonorities that emerge. Of the six chordal tetrachords in the "general repertoire," two have [G,A♭] as a dyadic constituent and two have [B, E] as a constituent. That Webern's composition of the passage placed only *those tetrachords* at these prominent junctures indicates the potential importance of these echoes and the tetrachordal dissolution on the large-scale structure of the choral music.

[46] Mead's discussion of the passage is, in part, a response to the many previous authors (Rochberg, "Webern's Search," Saturen, "Symmetrical Relationships", and Kramer, "The Row as Structural Background and Audible Foreground") who viewed the "R relationship as the serial source of the various chords" in the passage. As we noted earlier, the *RI*-symmetry of the row class allows for *I* or *R* understandings of the transformational relationships between rows. Mead notes, and we adopt this view, that "[t]he trouble with basing the description of the passage on the *R* relation is that it tends to obscure the deeper *I* relations at work, and to imply that aspects of order have something to do with the nature of the chords themselves" ("Webern and Tradition," 175-6).

A♭] as a dyadic constituent and two have [B, E] as a dyadic constituent. Webern used *only* row forms and combinations capable of placing those sonorities at the boundaries of each passage.

This choral music, then, communicates two musical images that resonate with the poetic ideas echo and dissolution found in Jone's poem. First, echoes are heard locally, manifest as two types of voice exchange—a smaller version at the beginning of Lightning and a more elaborate (retrograde) version at its end. On a larger scale, the boundary chords of the three passages are echoes of one another in terms of the dyads in the center of those chords. And second, dissolution is conveyed through subtle musical misremembrances. In the voice exchanges, the pitch echoes are inexact, pitch classes are freely transferred from one octave to another. And in the boundary chord succession, echoes of the central dyads are accompanied by small changes in the outer voices. Each novel sonority is related to the first in a "fuzzy," slightly misremembered way.

The Canons

Because these passages are homophonic and the textural contrast with the surrounding, polyphonic canons is so vivid, the choral music is a concentrated musical representation of the text's ideals. Reconciling the movement's four canons with those ideals requires approaching them in a more diluted, abstract form; many of the same relationships obtain, but because of the polyphony, those relationships appear in less concrete ways. In the larger scheme, the greater abstractness of the canon music (it is for orchestra alone) hints at the universal metaphor communicated in the poem.

As mentioned at the head of this section, the poem communicates a *linear progression of dissolution*. This progression is manifest in many musical domains and involves canonic structure, rhythmic structure, instrumentation, pc invariance, and so on. In its most abstract conception, the idea of canon is already symbolic of the poem's idea of echo. Webern's construction of the canons

even more greatly suggests sonic echo along with the progressive dissolution we noted in the choral music.

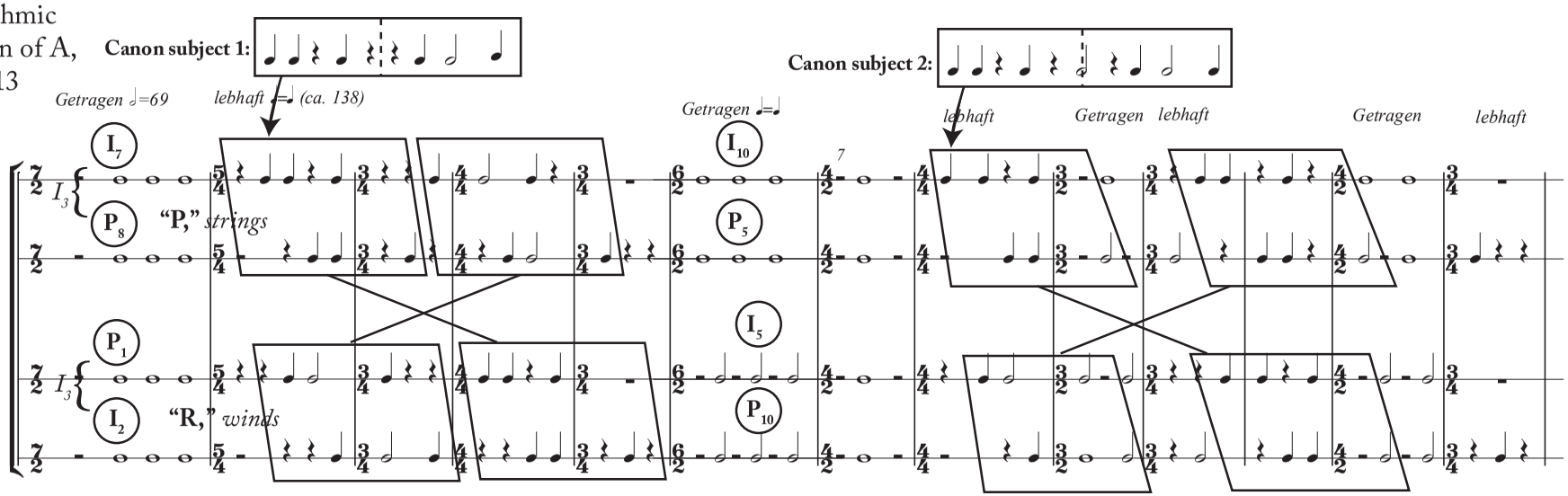
Each of the two large canon passages (see Figure 4.11, labeled as A and A') contains two short canons with brief homophonic interjections separating them. Figure 4.14 reduces them rhythmically. Every canon has a similar rhythmic subject, shown in boxes above each canon, that is comprised of two rhythmic patterns (see the box above Figure 4.14(a)), which gives the complete subject for the first canon): (1) a four-beat pattern (♩♩𝄽♩) containing three attacks and (2) a simple variation on that idea that replaces the (♩𝄽) with a half note— (♩𝅗𝅥♩).[47] The variation has the same number of attacks and replicates the the original's durational structure. Both are symmetrical. Taken together, the patterns divide the canon subject into two halves (around the dotted lines in Figure 4.14) which also foregrounds the whole subject's durational symmetry. This durational symmetry resonates nicely with the intervallic symmetry of the row class. All four canons are crab canons in which two voices play the canon subject in prograde (**"P"** on Figure 4.11 and 5.14) and two voices play the subject in retrograde (**"R"**). And in general, **"P**" and **"R**" are reinforced by instrumentation, strings play one of the parts and winds the other.

The crab characteristic of the movement's canonic structure is important in a poetic sense as it creates a rhythmic version of the "voice exchange" echo from winds to strings. This is most apparent in Figure 4.14(a)—which models the first canon—but has been annotated in all four canons. There, notice that when the strings are playing the basic rhythmic pattern (♩♩𝄽♩), the winds play the variation (♩𝅗𝅥♩), and *vice versa*. As the crossed lines on the figure show, this creates a "rhythm exchange"—very much analogous to the voice exchanges we noted in the choral music.

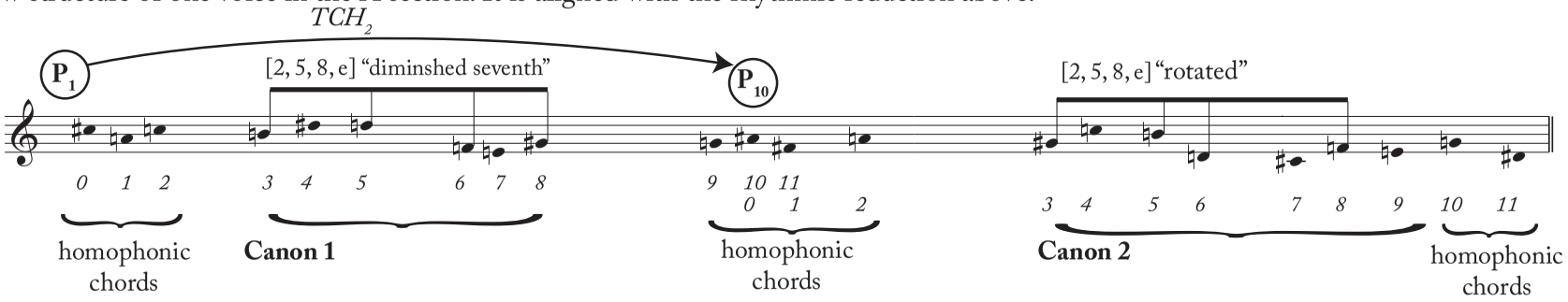
[47] According to Bailey, *The Twelve-Note Music of Anton Webern*, 442, n. 4, Hartwell, "Duration and mental arithmetic," was the note the rhythmic structure of the canon, along with the tempo changes and that create durational augmentation.

FIGURE 4.14. The first two canon sections.

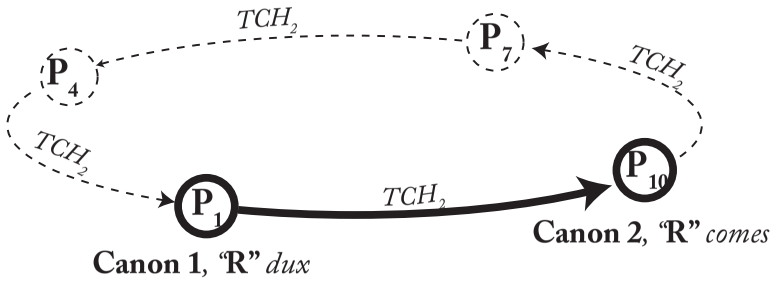
(a) Rhythmic reduction of A, mm. 1–13



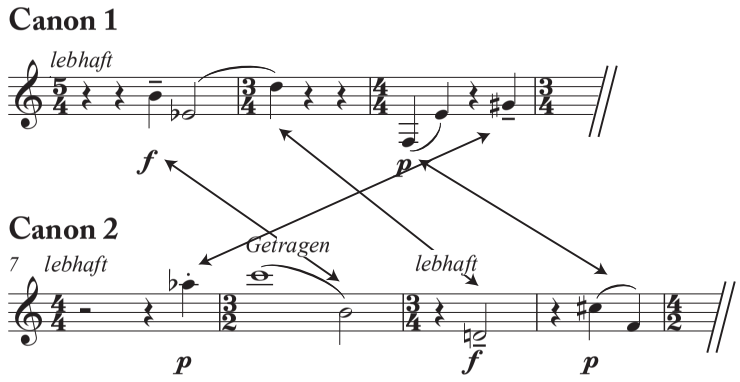
(b) Row structure of one voice in the A section. It is aligned with the rhythmic reduction above.



(c) Each canonic voice traverses a half cycle



(d) Realization of one canonic voice (cf. (a) and (b)



All four canons are constructed in the same way, as Figure 4.14 shows, and therefore, the very basics of canonic structure in the movement communicate the concept of "voice-exchange" echo.

As part of the progression towards (canonic) dissolution, the opening canon is the only one that not only embodies these abstract principles but also makes them salient.[48] In keeping with the ideal of misremembrance that we saw in the choral music's boundary chords, the final three canon passages operate with nearly the same canon subject and the same abstract, crab-canonic structure as the first, but introduce tactus changes that slowly dissolve the canonic structure's salience over the course of the four canons. On Figure 4.14(a), note that the entirety of the first canon occurs within the tempo marking of *Lebhaft*. All of the other canons (at m. 8 on Figure 4.14(a) and at m. 37 and m. 43 on Figure 4.15) contain "tempo" changes from *Lebhaft* to *Getragen* within themselves that correspond with changes in tactus; notice the denominator on each time signature changes from "4" to "2" as the tempo does.[49] These are not tempo changes in the most obvious sense, because the quarter note value remains consistent, but in *Getragen* measures the tactus is half as fast as in *lebhaft* measures.

Most importantly for the perceptual salience of the canon, *Getragen* measures also correspond with augmentations of the rhythmic values within the portion of the canon subject being played during that bar; in particular, note values from the canon subject are doubled when they occur in *Getragen* measures. As an example, Figure 4.14(a) shows the second canon beginning at m. 8. This canon is also a double crab canon and contains a nearly identical canon subject as the first, but with a half note inserted as the fourth rhythmic value between the two rhythmic ideas from the first canon. Follow $\mathbf{I_{10}}$—the *dux* voice of the **"P"** canon: the first three

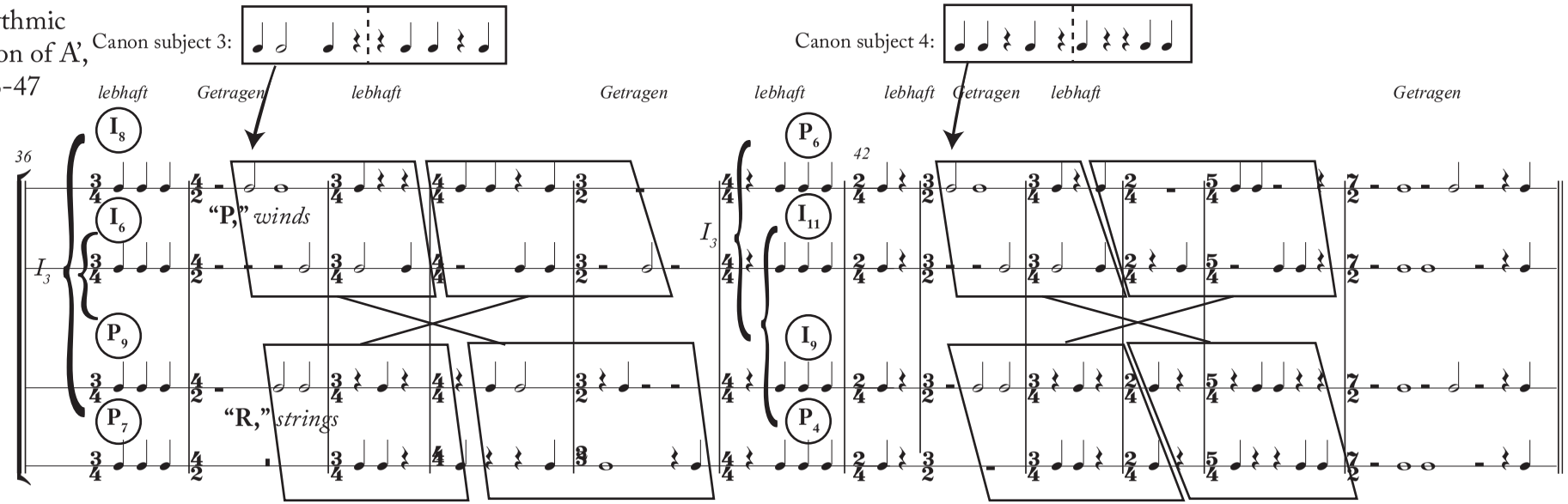
---

[48] Details of this dissolution are noted in Hartwell, "Duration and mental arithmetic," 353 and Bailey, *The Twelve-Note Music of Anton Webern*, 276–278. Neither tie this more generally to the poetic content of the poem.

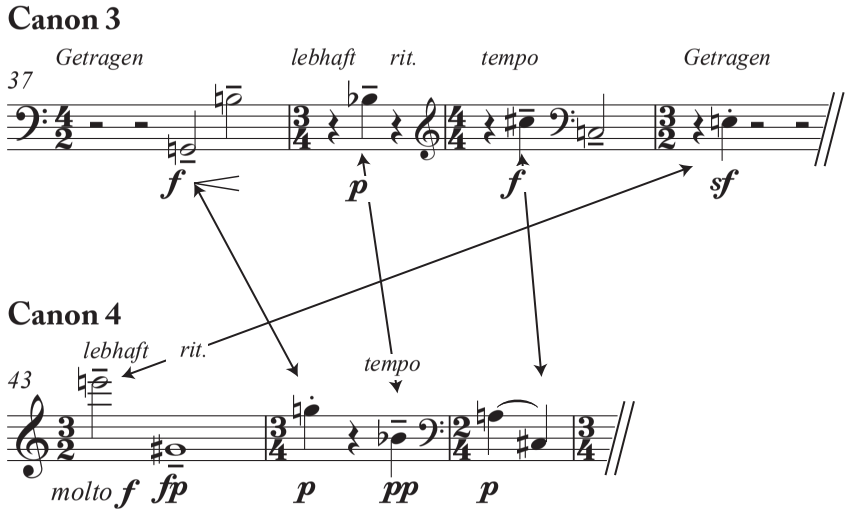
[49] Julian Johnson has found that the tempo direction *Getragen* is rare in Webern, "and always associated with the funereal" (*Webern and the Transformation of Nature*, 175). His discussion of *Getragen* is in specific reference to the first movement of Op. 23, a setting of Jone's "Das dunkle Herz." Johnson notes that the poem, which "begins 'in the dark realm of roots which reaches to the dead'", is "concerned with the perception of 'spring' in this inner darkness" (175). That description is certainly suggestive as regards the poetic content of "Blitz und Donner."

FIGURE 4.15. The last two canon sections.

(a) Rhythmic reduction of A', mm. 36-47



(b) Realization of one canonic voice



notes values track the canon subject in the box, but the fourth—a whole note—is double the duration of the canon subject's half note because it occurs during a *Getragen* measure. Note that because each canon begins at a different point, these rhythmic augmentations do not occur at the same place in each canon. The whole note in m. 9 of $\mathbf{I_{10}}$ corresponds canonically to the half-note in m. 10 of $\mathbf{P_5}$, which occurs during a *Lebhaft* measure.

As a result of these tempo alternations, the saliency of canonic structure dissolves; the final three canons sound less and less like canons. It seems an essential part of the musical representation of progressive dissolution that the *abstract* structure of the canons remain intact throughout the movement (each canon's basis in the first canon can be derived as I described above) though in every successive canonic passage, that structure becomes perceptually *fuzzy*. This dissolution is especially apparent in the last two canons (see Figure 4.15): the boxed-in rhythmic patterns—which diagram the abstract, echo-inspired voice-exchange structure of the canons—hardly correspond at all.

These are rhythmic processes that do not depend on pc relationships. Nonetheless, two types of pc invariance are similarly implicated in this process, and interact with the canonic dissolution: (1) invariance *within* a canon (that is, amongst the four voices in a single canon) and (2) *between* canons. We will consider this latter type of invariance first, which is tied to the chain-based connections between canons in the two larger canon sections. To illustrate, Figure 4.14(c) extracts a single "voice" from the first canon passage (the "**R**" component's *dux* ($\mathbf{P_1}$), which becomes its *comes* ($\mathbf{P_{10}}$)). It shows that in the course of the A section, a single voice traverses half of a TCH_2 cycle. Since TCH_2 is an order 4 operation, every successive row in a cycle is three semitones above or below the prior row.

Figure 4.14 (b) aligns this extracted voice rows below the rhythmic reduction to show how order positions of each row are distributed in relation to the canons and the homophonic chords that interject between them. The beamed pitch classes in $\mathbf{P_1}$ comprise a "fully diminished"

seventh chord [2, 5, 8, 11], and because *TCH₂* leads to a transposed a row three semitones away, that fully diminished seventh remains invariant at those order positions: {11, 2, 5, 8} becomes {8, 11, 2, 5}. This is significant because, as Figure 4.14(b) shows, each of the two canons begin at the same order position (*3*) and are similar in length—six and seven notes respectively. And therefore, the melodic content of the canonic portions of **P₁** will be echoed in **P₁₀**.

More precisely, four of the six notes of **P₁**'s canonic subject will be echoed in **P₁₀**'s. And furthermore, the distribution of those invariant pitch classes interacts interestingly with the symmetry of the canon subject's rhythm. Figure 4.14 (d) shows this most clearly by aligning the two canons. There, it becomes clear that the invariance takes the form of a pitch class rotation that occurs between canon 1 and canon 2, resembling a "voice exchange" heard over time: echoing the first canon, the boundary pitches of the second canon's rhythmic patterns remain invariant, but are shifted, "misremembering" the order in which those pitch classes occurred. Figure 4.14(d) shows only one voice in the canon, but every voice realizes the same invariance pattern because each of the voices in the two canon sections are joined by *TCH₂*. Thus, four rotated echoes occur as the music progresses from canon 1 to canon 2, and from canon 3 to canon 4.

TCH₂ structures horizontal connections in the movement. On the formal diagram in Figure 4.11, note that both A and A' traverse half cycles. Only the choral music in the B sections travels through a full *TCH₂* cycle, which seems appropriate given that it is the only portion of the piece that contains text. The fixed axis inversion *I₃* is its vertical counterpart. Figures 4.14 and 4.15 show that each canon is a "quartet" of rows containing two **P**-forms and two **I**-forms. Every quartet is divisible into two, two-voice pairs, whose constituent rows are related by *I₃*.[50] Braces on those figures show the *I₃* relations. *I₃* is similarly immanent in the choral music, a connection we explored earlier in relation to Figure 4.13(b). *I₃*'s structuring influence throughout the movement

[50] Other inversional relations are present too, but *I₃* is the only one present throughout the movement.

is certainly in keeping with the organic imagery of the poem. Musically, it has two distinct functions. First, in the choral section, I_3 is present as an actual axis of symmetry, generally heard around C♯4/D5.[51] (It is not generally an axis of symmetry in the canon passages.) Second, I_3 ensures a "general repertoire" of dyadic verticalities between such related rows, as we saw earlier.[52]

As it is partly responsible for vertical relationships, I_3, then, seems a natural place to begin exploring relationships *within* canons.[53] On Figure 4.16(a), I have shown two unique spatial networks, both generated by TCH_2 and I_3. I_3 "strands" occupy the top and bottom half of each network, which wraps around when TCH_2 is initiated from either of the network's eastern boundaries. These networks are unique in the way that these I_3 strands are aligned. As we saw earlier, the choral music contains I_3 strands related by T_2, and Figure 4.16(a) shows that the same relationship obtains in the final A section. In fact, the end of the choral music TCH_2's into the A section, as can be seen on Figure 4.11. A different network is required to model the first A section: the network on the left aligns I_3 strands that are T_5 related.[54]

Though it is not an event network, the network can be read chronologically by moving from left to right. The first two canons begin in the left partition, move through a half cycle, and after a "broken chain" at m. 14, enter the right partition to begin the choral music. Notably, that broken chain—as the network shows—was necessitated by only one I_3 strand. Though $\mathbf{I_5/P_{10}}$ could have chained into the choral music, $\mathbf{I_{10}/P_5}$ could not. The earlier formal diagram in Figure

---

[51] This was noted by Kramer, "The Row as Structural Background and Audible Foreground," 172. Kramer also notes that the axis is occasionally transposed by T_6, probably out of concern for the vocal registers.

[52] This is noted by Mead, "Webern and Tradition," 176. That general repertoire includes {C♯, D}, {C, E♭}, {B, E}, {B♭, F}, {A, F♯}, and {A♭, G}

[53] About the divisions: These I_3-created divisions are not necessarily related to those voices contrapuntal position as a *dux* or *comes* in the components of the double canon. In keeping with the idea of progressive dissolution, I_3 divides each canon in a different way: Canon 1 is simplest as I_3 relates *dux* and *comes* within each of the **"P"** and **"R"** components. The second canon is divided similarly, but with the **I**-forms acting as the two *dux* voices. Canon 3, shown in Figure 4.18, places transpositionally related forms within each of the two parts of the double canon, such that I_3 relates the *dux* of one canonic part to the *comes* of the other. And finally, the fourth canon shows I_3 dividing the canonic texture into corresponding voices in the two canonic parts.

[54] Both of these networks are partial.

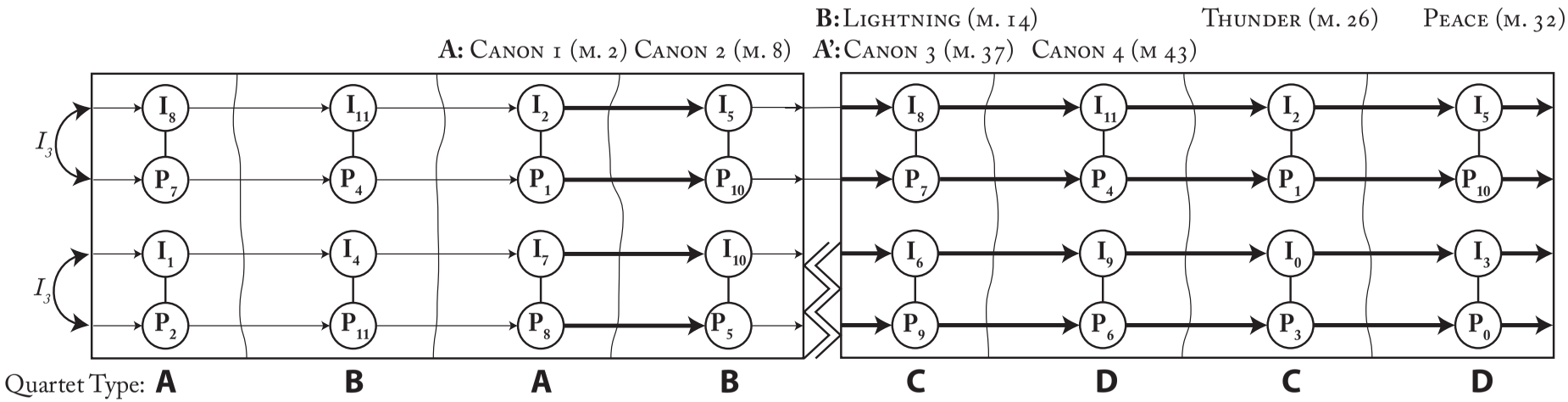
4.11 indicates that this is the *only* portion of the movement where a chain is absent. Following the broken chain, the choral music moves through a complete cycle on the right partition before the final two canons recapitulate the canonic texture from the first A, but in a novel spatial location.

Studying this space allows us to consider the invariance potential of each row quartet, which has some bearing on a large progression of dissolution as well as on the proportions of the movement. Row quartets within a single partition are isographic, though because each quartet is structured by a *fixed-axis* inversion (I_3), the pc invariances implied by each quartet are *not the same*.[55] That is, although the transformation I_3 has some explanatory power in the movement—it provides organic unity and limits the collection of dyads possible between I_3-related rows—because it is a fixed-axis inversion I_3 does not describe particular types of invariance or pc associations associated with a particular *ordering* of the row.

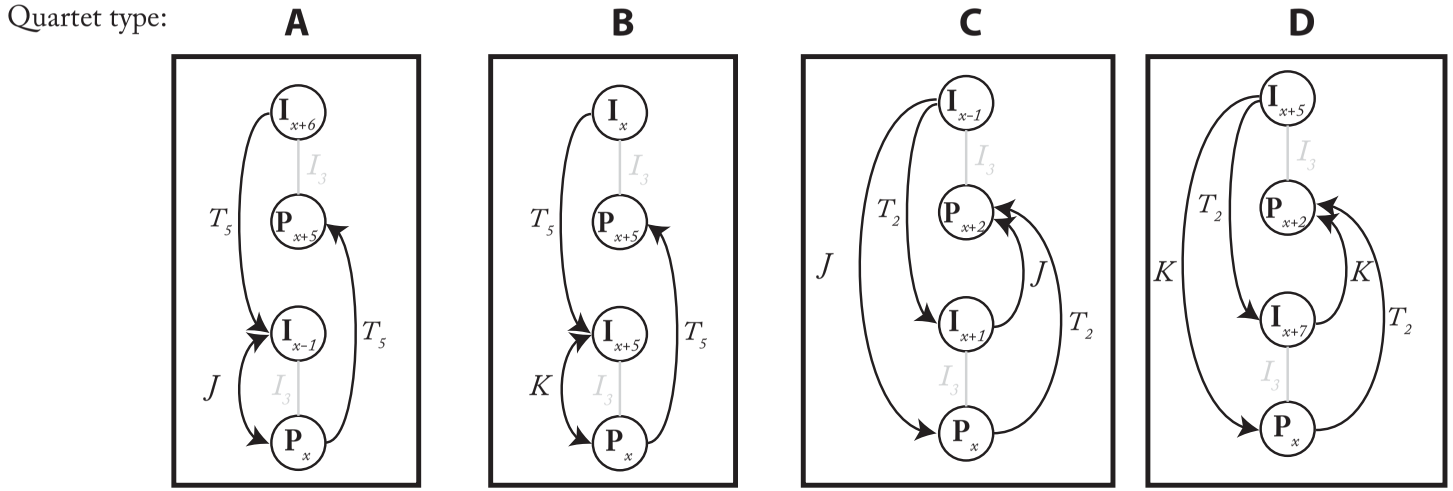
Though fixed-axis inversions cannot describe pc invariance, contextual inversions can. Figure 4.17 shows four of these relationships, two associated with contextual inversions called *J* and *K*. The rows shown there have been annotated with dotted lines to show the portions of the rows that will appear as part of the canons. Remember, as Figure 4.14(b) illustrated, only order positions *3-8* of a row are generally associated with the canon; the remainder of the row sets the homophonic chords. At (a) and (b), two contextual inversions are shown that were heard in the choral music. The contextual inversion *J* creates the simple voice exchange at the opening of the Lightning music. *J* occurs between a **P**-form and the **I**-form whose index number is a semitone lower. Figure 4.12 shows how *J* operates over the first half of Lightning (Figure 4.12(a)), as well as in Thunder (Figure 4.12(b)). In that passage, the row pairs $\mathbf{P_8/I_7}$ are *J*-related as are $\mathbf{P_7/I_6}$.

[55] To begin pivoting back toward the progression associated with the canon sections, although I_3 structure is present throughout the movement, I_3 does *not* create the voice exchanges that we noticed in the choral music and which resonated so strongly with the idea of sonic echo. That is, though I_3 influences the general repertoire of verticalities—it does not figure in the order in which those verticalities present themselves. Figure 4.15 shows that I_3 relates the two inner voices, along with the two outer voices. The voice exchanges, however, occurred amongst the soprano/alto and tenor/bass voice pairs. Therefore, those pc invariances were *not* produced by I_3.

Figure 4.16 (a) Two unique spatial networks that model the row quartet structure of the movement. Both networks are structure by TCH_2 and the fixed axis inversion I_3. The network on the left aligns I_3 related by T_5. The network on the righ aligns I_3 related by T_2.



(b) Networks describing the row quartet types **A/B** are isographic, as are those describing **C/D**. However, each of them projects a unique set of invariants associated with the contextual inversions created by *J* and *K*.



(Note that the fixed-axis inversion for each pair is not equivalent.) *J's* more elaborate partner, the contextual inversion *K*, relates a **P**-form to the **I**-form whose index number is five semitones above. *K* is associated with the retrograde in the second half of Lightning (Figure 4.12(a)) and in Peace (Figure 4.15(c)).

FIGURE 4.17. Four types of pc association found in the canons.

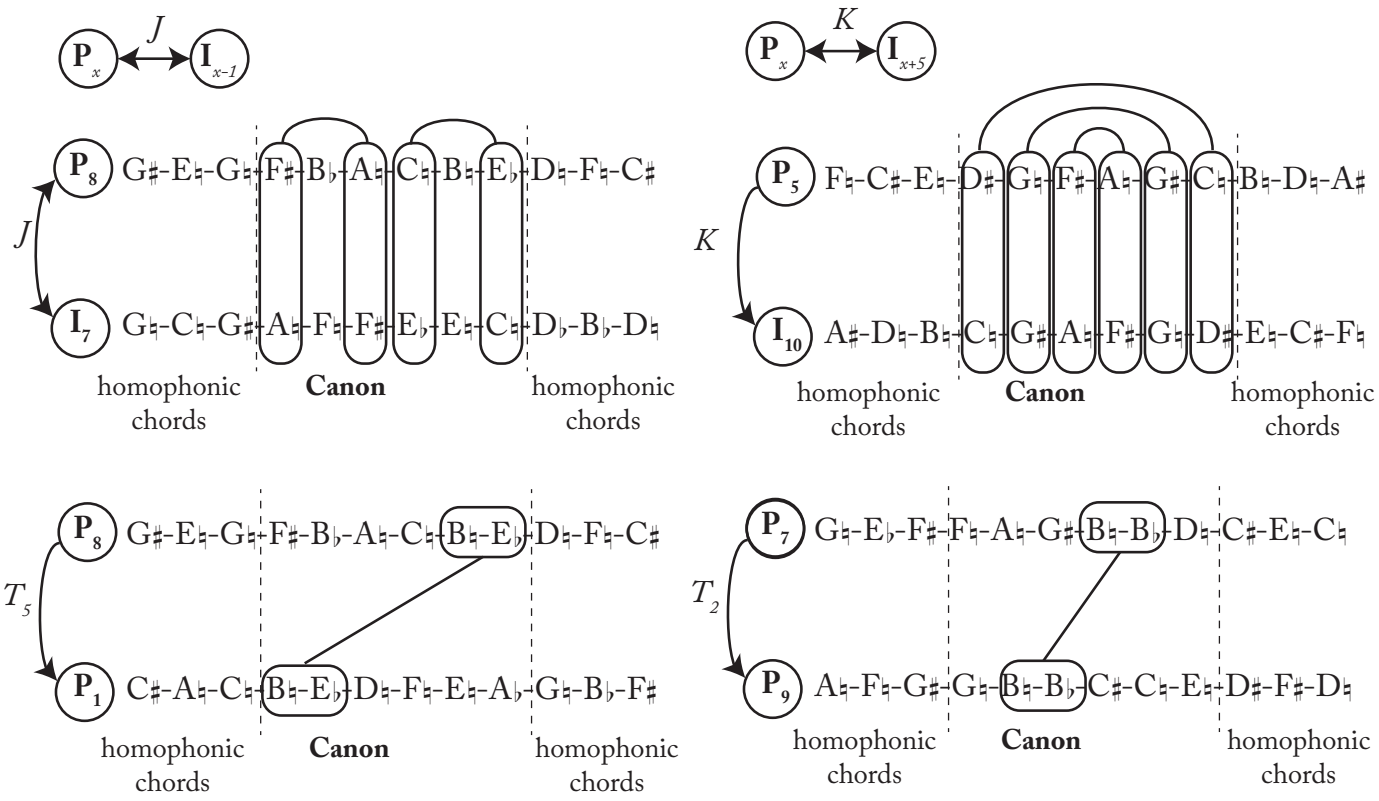


Figure 4.17 also diagrams two pc invariances associated with T_5 and T_2. These are the two transpositional relationships that related I_3 strands in the spatial network shown in Figure 4.16(a). T_5 creates a dyadic invariance that pivots symmetrically around the center of the row; within its six-note canon section, the first two notes of one canon, which form ic3 become the last two notes of the other. T_2 creates a similar invariance involving ic1.

Extracts of the row quartets from the spatial network are shown in Figure 4.16(b). These indicate how each of these four types of invariance are represented in the row quartets. Though isographic as defined by the fixed-axis inversion I_3, the row quartets in the two networks are different in terms of the invariances they project. To distinguish amongst them, the quartets have been labeled as **A** through **D**. Quartet types **A** and **B** occur within the left network, and contain invariances associated with T_5. Quartet **A** possesses *J* invariance between $\mathbf{P}_x$ and $\mathbf{I}_{x-1}$, while Quartet **B** contains *K* invariance between $\mathbf{P}_x$ and $\mathbf{I}_{x+5}$. Quartet types **C** and **D** are best understood in relation to types **A** and **B**. In those terms, these quartet types exchange T_2 for T_5 and possess twice the number of *J*- and *K*-created invariance relationships. In these terms, Quartet **C** is an altered, amplified quartet **A**, while Quartet **D** bears a similar relationship to Quartet **B**. It is useful to think of Quartets **C** and **D** in relation to **A** and **B**, respectively, because they relate corresponding canons in the first A section and its recapitulation. The spatial network in Figure 4.16(a) shows that all four canons project a unique quartet type. Quartet **C** echoes Quartet **A** as the initiator of the recapitulation, and both Quartets **B** and **D** end their respective sections.

These associations also highlight a progression involving pc invariance that corresponds with the various types of echo and dissolution that we noted earlier. To make these abstract observations somewhat more concrete, Figure 4.18-21 displays each of the four canons, aligned at (a), and instrumentally reduced at (b). The alignments at (a) illustrate the structuring influence of the contextual inversions *J* and *K*. Canon 1 is structured by Quartet **A** (Figure 4.18(a)). This quartet possesses one *J*-relationship, which creates the single voice exchange in the strings. Once again, note that Webern's rhythmic motive is well calibrated to highlight this relationship. In particular, the vertical invariance reinforces the durational symmetry of each of the three note rhythms comprising the subject: the {A5, G♭4} dyad that begins the string canon is echoed three beats later as {A4, G♭4}, associating the two symmetrically related quarter notes, and the {C,E♭} dyad similarly bounds the rhythmic pattern in the subject's second half. Canon 2 (Figure 4.19(a))

is structured by Quartet **B**. This quartet type contains a single instance of the contextual inversion *K*; thus, the second canon expands the pitch symmetry of the first. Notice that this pitch symmetry still occurs only in the strings.

The final two canons are echoes of the first, but as part of the dissolution process, each misremembers features of the first two canons. These canons are associated with Quartets **C** and **D**, which amplify the contextual inversion characteristics of the Quartets **A** and **B**. Thus, like canon 1, canon 3 (Figure 4.20(a)) contains the simple, *J*-associated voice exchange, but it now encompasses the strings *and* winds. Canon 4, echoing canon 2, invokes the *K*-associated retrograde, but expands it to include the full orchestra.

The alignments at (a) are helpful ways to imagine these passages, but they are abstractions to some degree because these passages are not homophonic. In fact, the realizations at (b) show that although the symmetries at (a) are latent, they are not as neatly manifest in the canonic music as they are in the choral music shown earlier.[56] And in fact, as the canonic structure dissolves over the progress of the movement, these symmetries become less apparent. The score reductions at (b) show how a series of voice exchanges, created by T_5- and T_2-associated invariance, do have greater perceptual salience, are tied to the rhythmic construction of each subject, and also interact with the ideas of echo and dissolution. Figure 4.18(b) shows that T_5 is responsible for a dyadic exchange of *ic* 3*s* across the central division of the canon. Like the symmetries at (a), this figure shows that the crab canon structure allows the invariances to occur at corresponding portions of the subject. For example, the 𝅗𝅥. 𝅗𝅥 setting {B4, E♭4} in the winds is echoed by a 𝅗𝅥 𝅗𝅥. setting {B3, E♭4} in the strings. T_5-created invariance is similarly apparent in the second canon, in Figure 4.19(b). Webern's insertion of a half note in the middle of the canon subject (cf. Figure 4.14(a)) allows the invariance heard in the first canon to be expanded. Rather than a simple *ic* 3 *echo,* the entire trichord that begins each of the subjects in the winds is heard at

[56] This observation is also found in Kramer, "The Row as Structural Background and Audible Foreground."

Figure 4.18

(a) Canon 1, mm. 2-5, voices aligned to show vertical invariance (b) Canon 1, mm. 2-5, horizontal invariance

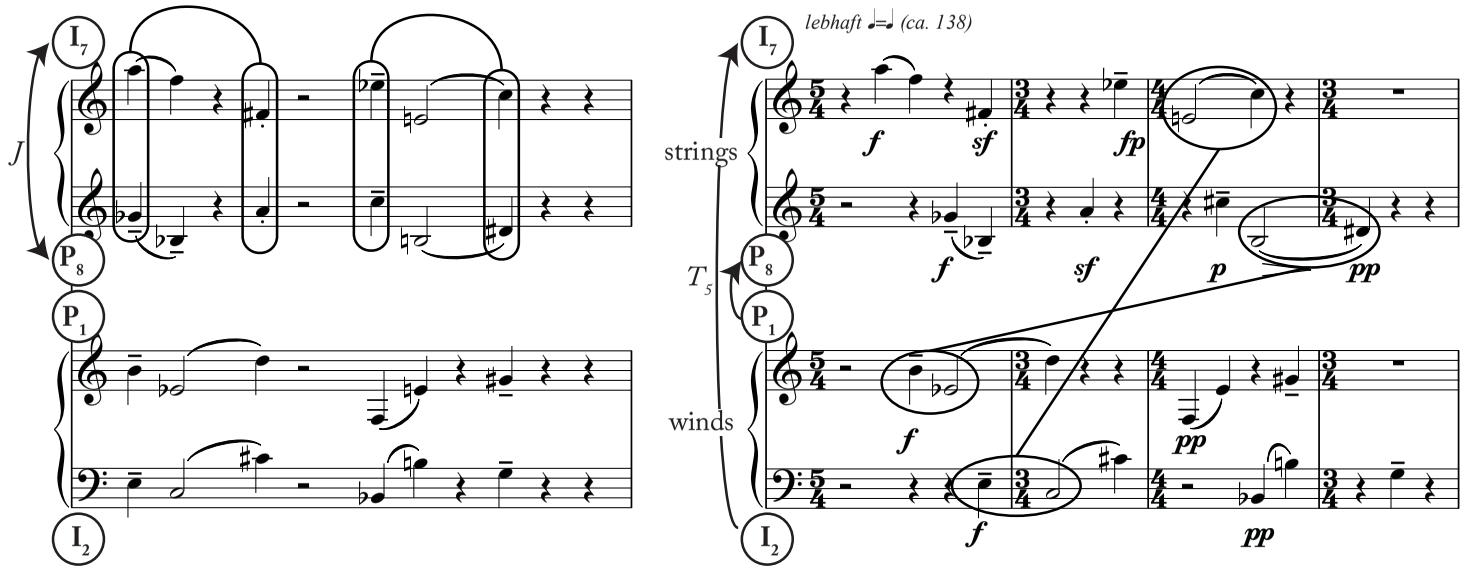


Figure 4.19

(a) Canon 2, mm. 8-12, voices aligned to show vertical invariance (b) Canon 1, mm. 8-12, horizontal invariance

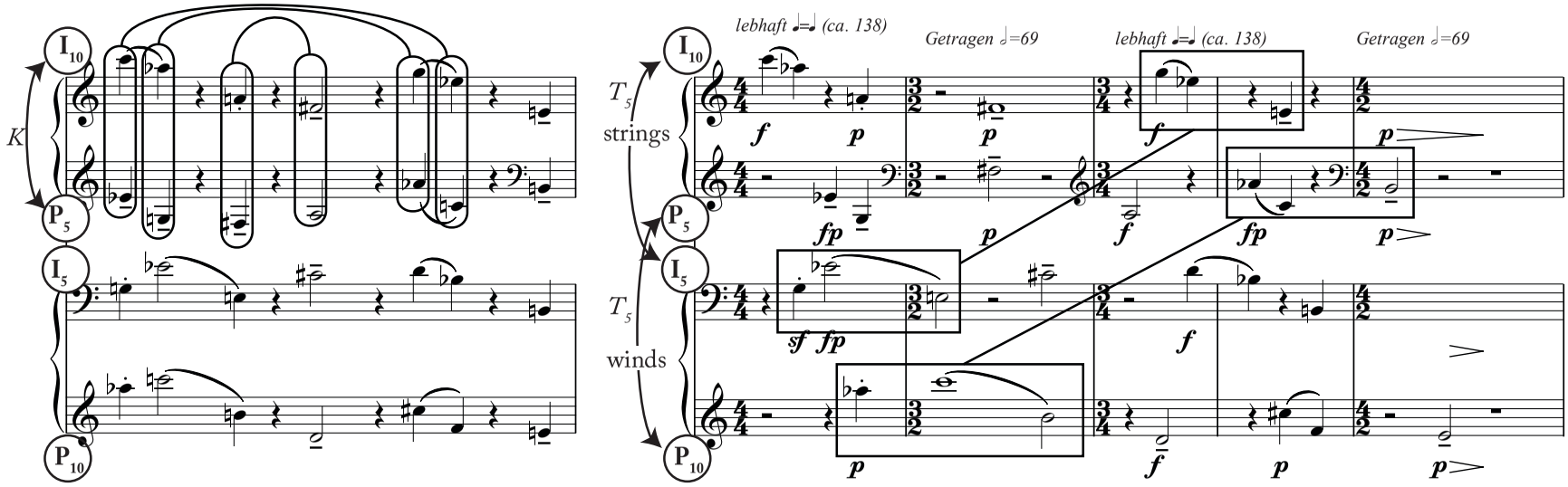


Figure 4.20

(a) Canon 3, mm. 37-40, voices aligned to show vertical invariance (b) Canon 3, mm. 37-40, horizontal invariance

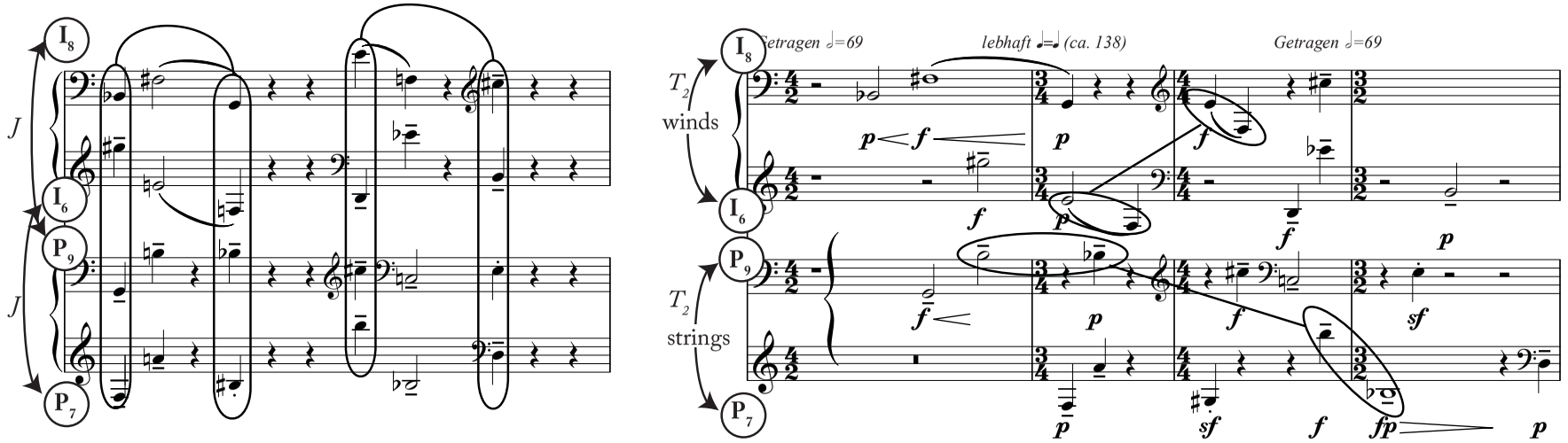
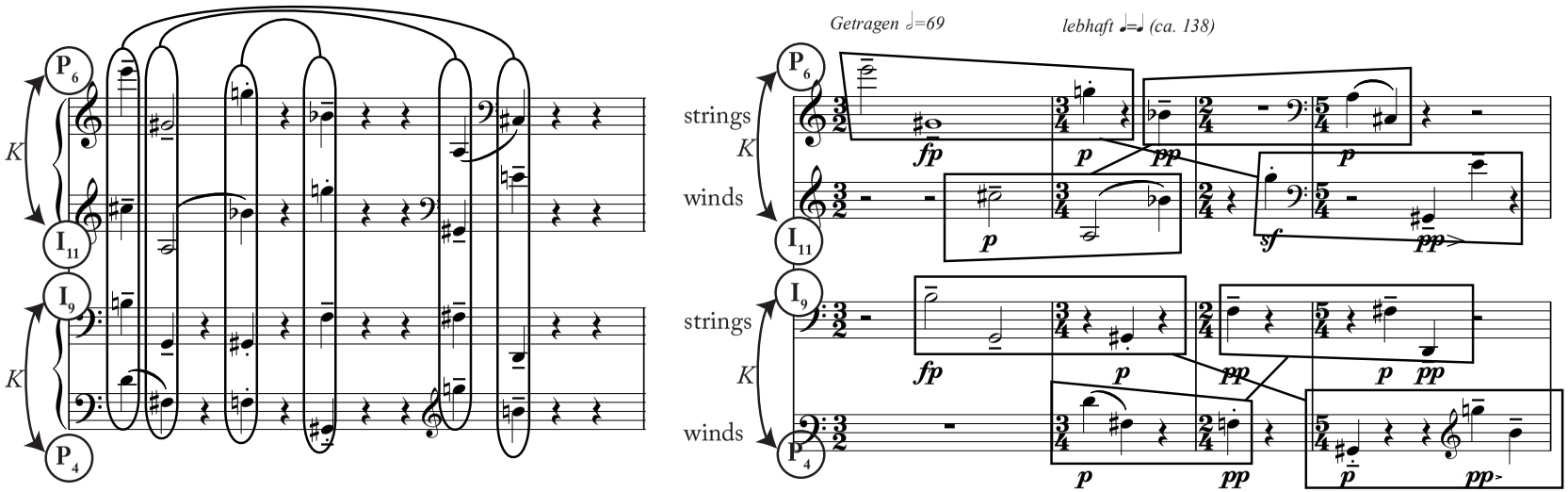


Figure 4.21

(a) Canon 4, mm. 43-46, voices aligned to show vertical invariance (b) Canon 4, mm. 43-46, horizontal invariance



the end of the string canons.

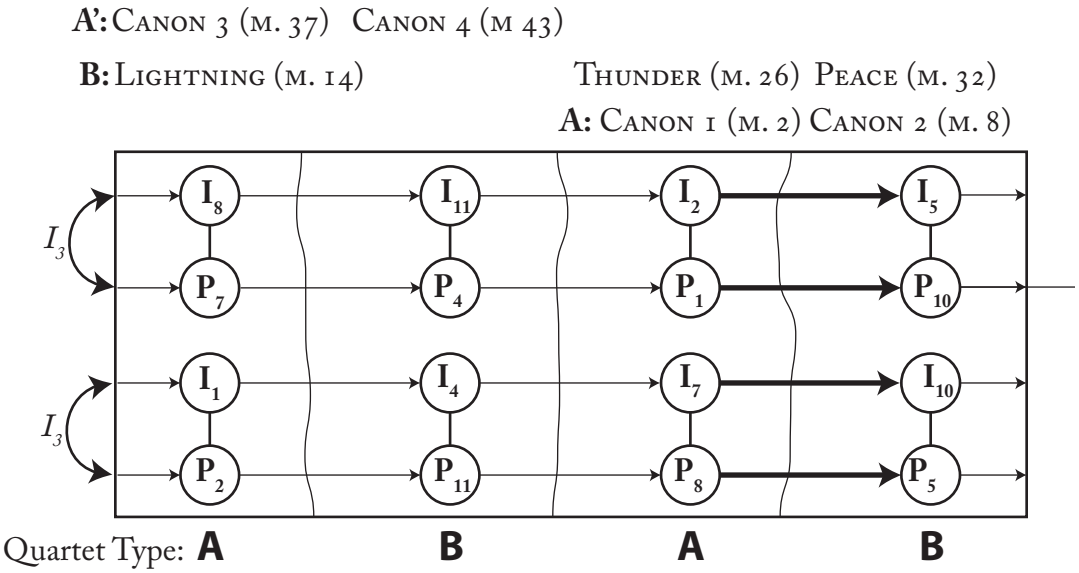
These invariances are continued in the third and fourth canons, but slightly misremembered. In Figure 4.20(b), the third canon is shown to echo the first. Here, *ic* 1 dyads—instead of *ic* 3—created as a result of T_2 relations amongst rows echo within each part of the canon. Again, the rhythmic symmetry of the motive enhances the relationship. Each of the *ic* 1 dyads occurs in symmetrically-equivalent positions of the two rhythmic patterns comprising the canon subject. And finally, echoing the second canon, the final canon also invokes trichordal invariance, but here it is expanded. Corresponding with the canonic division in two parts, the first and last trichords within each canonic pair are swapped.

Thus, the canonic echoes and dissolution have a number of representatives in terms of pitch and rhythmic structure, between and within each of the four canons. Much of the commentary on this movement has questioned why Webern changed the canonic structure at m. 14, which as the spatial network in Figure 4.16(a) shows, necessitated the broken TCH_2 chain just prior to the choral music. That spatial network indicates how concerns for *unique types* of pc invariance, in particular, might have motivated adjustment. The tripartite constraints on the spatial networks—of TCH_2, I_3, and T_5 or T_2—produce four row quartets per network. But, each of those network contain only *two row quartet types*. Within the left network, for example, which contains the first two canons, the two row quartet types are **A** and **B**.

Thus, avoidance of redundancy between the four canons may be the simple explanation for the adjustment and broken chain after the second canon. An alternative diagram in Figure 4.22 considers what the spatial ramifications would have been had this break not occurred. Following the first canon section, the choral music would have began with the westernmost row quartet, a quartet of type **A**. Assuming the rest of the movement played out similarly, the third canon section would have begun in the not in the same spatial location as the first canon, but it

would have projected the same quartet type. Thus, the types of pc invariance heard in the third canon would have been the same as those heard in the first.

Figure 4.22. Spatial ramifications if the "broken chain" had not occurred at m. 14.



Note that in this alternative, the third quartet would have an entirely unique collection of rows. But the particular pitch classes involved matter less here than the *types of invariance* that heard. Those types of pc invariance, after all, are the most direct musical image of the poem's sonic echoes. Webern's image of the poetic dissolution is convincing only if the four canons are related but slightly changed. Altering the canonic structure at m. 14 allows for the echoes associated with pc invariance to subtly change, as the canonic structure did, dissolving the sonic image of the first canon slowly, just as thunder grows fuzzy as it trails away.

Conclusions

In the Preface to this study, I quoted Andrew Mead, who in "Webern, Tradition, and 'Composing with Twelve Tones'" noted that "[Webern's] works show an extraordinary sensitivity to the possibilities of the twelve-tone system for embodying the formal strategies of earlier music—possibilities that range from the primitives of the system, through the potentialities inherent in a row class, to the way its members are articulated on the musical surface."[1] In the intervening pages, I have tried to approach each of these levels in novel ways, and in the analyses throughout, I have attempted to use that information to shed some new light on the many ways that Webern's "radical" compositional language interacts with the traditions of the past. Principally, this dissertation demonstrates how Webern's engagement with classical form takes place in two dimensions, where vertical, associational relationships work in tandem with transformation chains, whose primary formal role is syntactic. The paradigmatically organized spaces I developed in Part I capture this interrelationship, and the analyses in Part II demonstrate the sensitivity, as Mead notes above, with which Webern approached the possibilities inherent in the system and within a composition's unique environment.

As a matter of methodology, this dissertation's most original contribution is its concern both for the horizontal dimension engaged by transformation chains and the reciprocal relationship between transformation chains and row structure. Transformation chains are deeply constrained by the intervallic restrictions of the twelve-tone system, as I demonstrated in §1.2. Notably, at the most primitive level of the twelve-tone system, intervallic restrictions establish a small collection of transformation chains (the one-note chains and $RICH_2$) as potentially available to every row class—certainly accounting for their prevalence in Webern's music. Other transformation chains require particular types of row class structure. Row derivation, for example, is often coincident with large *TCH* and *ICH* chains due to those chains's need for an equivalence

[1] Mead, "Webern and Tradition," 173.

between initial and final interval segments of the row. *RICH* and *RECH*, by contrast, are associated with Webern's most symmetrical rows. In §1.2.8 and §1.2.10 I reframed the concepts of *R*- and *RI*-symmetry, structural features of four of Webern's row classes, as instantiations of $RECH_{12}$ and $RICH_{12}$.

I believe that the importance of this reciprocality can hardly be overstated. As I noted in §1.4 the idea that the "object suggests the behavior"—that is, that a row or row class suggests particular transformational routines—resonates in many directions, both within and outside of Webern's music. The interrelationship of row and transformation chain is suggestive, for example, in a broader music-theoretical context. Multiple authors have shown how the "pan-triadic syntax" common in the nineteenth-century may have emerged as a byproduct of the voice-leading properties of triads. This was exactly my argument in Chapter 1 (especially §1.4), only substituting "triad" for row and "voice-leading" for transformation chain. There may seem to be a rather large gulf between nineteenth-century triadic syntax and Webern's serial practice, but given Webern's conservative tendencies and the overlapping ideological views of many of the main players, that gulf may not be as large as it seems.

Perhaps most importantly, this reciprocality has justification in Webern's interest in organicism, which seems to have grown in intensity throughout his life.[2] It is notable, for example, that his final four compositions, the String Quartet Op. 28, Cantata I, Op. 29, Variations, Op. 30, and Cantata II, Op. 31, contain the most thoroughgoing relationship between row and transformation. Cyclic composition—which, as we saw in §2.3.7, is already nascent in the Variations, Op. 27—is predominant in his final four works, as my analyses of Op. 28 (in §2.4.5) and Op. 29 (in Chapter 4) demonstrated.

Because they emerge "naturally" from the intervallic properties of a row class, studying transformation chains allows us to understand the temporality inherent in a row class. My

[2] My discussion of his relationship with Hildegard Jone in Chapter 4, for example, suggests that his primary collaborator late in life was as enthusiastic about these ideas as Webern was.

approach to representing that temporality was guided by a concern for highlighting a row class's natural transformational pathways while hiding others. Chapters 1 and 2 argued that, inasmuch as these pathways represent temporal "norms," musical space's generated by transformation chains are a simple cyclic music grammar, in the sense discussed by Robert Morris.[3]

In this grammar, transformation chains are carriers of syntax, and in Chapters 1 and 2, I argued that this is one of the most significant reasons to prefer groups generated by transformation chains over those generated by classical serial operations. When studying horizontal connections between rows in Webern's music, chains often provide a "simpler" analysis, which I measured (in §2.2) through Edward Gollin's concept of "path distance."[4] Their simplicity is partly the result of their unique structural properties; chains are not generally equivalent to a classical serial operation, but (as §1.3 demonstrated) more closely resemble neo-Riemannian *Schritts* and *Wechsels*. Their inherent "dualism," then, is advantageous in analytical situations that evince a similar duality—such as the second movement of the Piano Variations, Op. 27 (discussed in §2.2.1).

Moreover, transformation chains interact with classical serial operations in compelling ways. Generally speaking, transformation chains do not commute with one another, nor do classical serial operations commute amongst themselves. But, transformation chains do generally commute with classical serial operations.[5] This proves beneficial when combinations of rows exist around an axis-of-symmetry (as in Op. 27, II) or are *R*-related (as in Op. 27, I). My analyses in Chapter 4, which showed the importance of cycles of transformation chains as formal determinants, relied heavily on the ability of chains to commute with inversion operations.

---

[3] Morris, "Compositional Spaces."

[4] Gollin, "Representations of Space."

[5] I noted there that this commutativity was earlier proven by Hook, "Uniform Triadic Transformations" and noted in an explicitly Webernian context in Hook and Douthett, "UTTs and Webern."

Nevertheless, conceptually separating these types of transformations is important. For one, because combining chains and classical operations results in non-simply-transitive transformation groups, every relationship between rows can be expressed in two, non-equivalent ways. Therefore, it is possible to conflate transformational relationships. However, because they best describe different types of relationships—while chains are elements of horizontal syntax, serial operations often better describe vertical relationships of row combination—I suggested in §2.3 a separation based on Saussure's paradigmatic and syntagmatic categories of relationship. There, and in §2.4, I suggested that this separation is an effective way to organize the two dimensions of a spatial network.

These *organized spatial networks* have historical precedents. I suggested a commonality with two of Cohn's spatial diagrams of the maximally smooth group in §2.3.4 and §2.3.5, but there are many others as well.[6] My intent is that these spatial networks function as robust musical grammars whose horizontal and vertical dimensions capture the syntactic and associational forces at work in a given compositional environment. Most importantly, the networks are capable of showing how syntax interacts with those associational features, thus giving some sense of a composition's "tonal motion" as the product of both paradigmatic and syntagmatic relationships.

As an analytical study, this dissertation has demonstrated that a prescriptive approach to form in Webern's serial music should be undertaken with caution. Though nearly all of Webern's serial works interact with traditions of classical form in one manner or another, my analyses show that Webern engaged those formal concepts and designs in different ways throughout his compositional career. Therefore, I suggest that although Webern occasionally hinted at certain principles underlying his work's interaction with classical form (he said, for example, that "[t]he original form and pitch of the row occupy a position akin to the 'main key' in earlier music; the

[6] An instructive example is found in Lewin's "formal" network (i.e. *spatial network*) for Stockhausen's *Klavierstück III*. See *Musical Form and Transformation*, 34.

recapitulation will naturally return to it"), any absolutes gleaned from those statements should be examined carefully.[7]

For example, the short analytical vignettes in Chapter 2 and the extended analyses in Chapters 3 and 4 show that Webern often created "tonal closure" in different ways. In the opening movement of the Piano Variations, Op. 27, the recapitulation's sense of closure stems from a large-scale symmetrical completion of the underlying row area progression. In the second movement of the Quartet, Op. 22, which was the subject of Chapter 3, the closure created in the sonata-rondo's recapitulation is both more traditional in conception—involving the return of the primary row area—and at the same time more complicated due to that row area's ability to project classical analogues of "theme" and "key." Closure in the String Quartet, Op. 28 and the Cantata I, Op. 29, which I analyzed in Chapter 4, are of an entirely different type—both involve cyclic completion.

Though these differences caution against prescriptive analysis, certain formal strategies do recur in many of these pieces. In many cases, I demonstrated that the large-scale form was an amplification of structural principles found at much smaller levels of the form. Such was the case in the first movement of Op. 27, where the temporal, rhythmic, and pitch symmetry of the opening were telescoped onto the large-scale row area progression. The second movement also displayed a similar amplification, wherein the cyclicity of each formal part found an expression at every higher level of the form. All of the extended analyses found in Chapters 3 and 4 involved a an amplification of some smaller formal unit onto larger ones. In Chapter 4 I suggested an intellectual context for the general principle in Webern's love for Goethe's organicism and Emmanuel Swendeborg's concept of "correspondence."

---

[7] Webern, *The Path to the New Music*, 54. Those absolutes are, for example, the basis of Kathryn Bailey's study of form in Webern's music (*The Twelve-Note Music of Anton Webern*), which I critiqued in Chapter 3.

The findings of this dissertation suggest many future avenues for exploration. Of course, because the present work promotes a general theory and methodology, much more analytical work should be done to confirm and refine its primary findings. Much of the work done here has attempted to uncover compositional, or even pre-compositional, environments; therefore, the general approach would provide a novel perspective on the many historical documents that contain evidence of Webern's compositional process. There are many avenues for such an exploration. In its most simple form, the musical grammars created here could be refined based on evidence from sketch material, and the results could be used to provide new analyses of Webern's works.

To a different end, these spatial networks could also be used to study the compositional process itself. One of the central questions raised by this study is the degree to which "horizontal" concerns influenced Webern's pre-compositional construction of row classes, as the creation of derived rows and symmetrical rows certainly did. Evidence from the music itself is mixed. Webern's earlier works, such as the Quartet, Op. 22, make use of a profusion of chains, most of them $RICH_2$ or one-note chains. On the one hand this indicates that compositional usage of the chains was an afterthought—or at least not a significant part of the pre-compositional process—because those chains require no special intervallic requirements on the part of the row. On the other hand, those chains seem calibrated to have a specific meaning—one that interacts with the associational features of the music in ways that impact the small and large levels of musical form. As mentioned earlier, Webern's later works (for example, the first two movements of the String Quartet, Op. 28, studied in §2.4.5) use one or two types of chains, many of them large chains, in an almost single-minded manner. This suggests that determining the meaning (or "transformational character" as I called it in §2.4) of the primary chains was likely an important pre-compositional activity.

This study also suggests that the reason for transformation chains's commonality in Webern's output needs to be questioned further. In Chapter 1 I proposed that chains would have appealed to Webern because they were actions suggested by the objects on which they were acting. Studying sketch documents, among other historical documents, may shed some light on this conjecture. As I mentioned above, Webern's works do suggest that transformation chains became more important structural principles in the course of his career. It is certainly worth studying that progression. Is it fair to say that the cycles found in Webern's later works were nascent in earlier, non-cyclic compositions? Or is that conjecture simply a byproduct of the present approach, in which cyclic groups have played such a prominent role?

Finally, it is worth examining possible precursors as well as searching for manifestations of these procedures in the second generation of serial music. Though he does not suggest any explicit path of influence, Lewin's juxtaposition in *GMIT* of Webern's serial chains with motivic chains in music by Bach and Wagner (and later, Mozart and Bartók) urges further investigation of the commonalities and differences in these composer's use of similar procedures. Because in some form chains provide another layer of rules on top of the basic axioms of serial composition, this study also invites an exploration of their influence on total serial procedures.

APPENDIX 1

CHAIN POSSIBILITIES IN WEBERN'S SERIAL MUSIC

This appendix contains a listing of chain possibilities in Webern's mature serial works (Op. 20ff), in addition to a "T-matrix" for each composition. Blacked out squares are "impossible chains," as discussed in §1.2. It should be underscored that these are *possibilities* that in a given work Webern may or may not make use of. In the charts of chain possibilities, the length (*i*) for a given chain type is shown in addition to the order of that chain in the body of the chart. For example, in the Symphony, Op. 21, the chain $RICH_2$ is an order 2 operation (an involution) that joins the minimum number of rows possible, while in the Concerto for Nine Instruments, Op. 24, $RICH_2$ is an order 24 operation, joining the maximum possible.

STRING TRIO, OP. 20

0	11	6	5	10	9	1	2	7	8	4	3
1	0	7	6	11	10	2	3	8	9	5	4
6	5	0	11	4	3	7	8	1	2	10	9
7	6	1	0	5	4	8	9	2	3	11	10
2	1	8	7	0	11	3	4	9	10	6	5
3	2	9	8	1	0	4	5	10	11	7	6
11	10	5	4	9	8	0	1	6	7	3	2
10	9	4	3	8	7	11	0	5	6	2	1
5	4	11	10	3	2	6	7	0	1	9	8
4	3	10	9	2	1	5	6	11	0	8	7
8	7	2	1	6	5	9	10	3	4	0	11
9	8	3	2	7	6	10	11	4	5	1	0

CHAIN POSSIBILITIES

i =	1	2	3	4	5	6	7	8	9	10	11	12
TCH_i	4											
ICH_i	2						■	■	■	■	■	■
$RECH_i$	2		■		■		■		■		■	
$RICH_i$	4	6										

SYMPHONY, OP. 21

0	3	2	1	5	4	10	11	7	8	9	6
9	0	11	10	2	1	7	8	4	5	6	3
10	1	0	11	3	2	8	9	5	6	7	4
11	2	1	0	4	3	9	10	6	7	8	5
7	10	9	8	0	11	5	6	2	3	4	1
8	11	10	9	1	0	6	7	3	4	5	2
2	5	4	3	7	6	0	1	9	10	11	8
1	4	3	2	6	5	11	0	8	9	10	7
5	8	7	6	10	9	3	4	0	1	2	11
4	7	6	5	9	8	2	3	11	0	1	10
3	6	5	4	8	7	1	2	10	11	0	9
6	9	8	7	11	10	4	5	1	2	3	0

CHAIN POSSIBILITIES

i =	1	2	3	4	5	6	7	8	9	10	11	12
TCH_i	2											1
ICH_i	2						■	■	■	■	■	■
$RECH_i$	2		■		■		■		■		■	1
$RICH_i$	2	2										

QUARTET, OP. 22

0	9	8	11	10	2	3	4	5	7	1	6
3	0	11	2	1	5	6	7	8	10	4	9
4	1	0	3	2	6	7	8	9	11	5	10
1	10	9	0	11	3	4	5	6	8	2	7
2	11	10	1	0	4	5	6	7	9	3	8
10	7	6	9	8	0	1	2	3	5	11	4
9	6	5	8	7	11	0	1	2	4	10	3
8	5	4	7	6	10	11	0	1	3	9	2
7	4	3	6	5	9	10	11	0	2	8	1
5	2	1	4	3	7	8	9	10	0	6	11
11	8	7	10	9	1	2	3	4	6	0	5
6	3	2	5	4	8	9	10	11	1	7	0

CHAIN POSSIBILITIES

i =	1	2	3	4	5	6	7	8	9	10	11	12
TCH_i	2											1
ICH_i	2											
$RECH_i$	2											
$RICH_i$	2	12										

THREE SONGS, OP. 23

0	7	11	8	2	10	6	9	5	4	1	3
5	0	4	1	7	3	11	2	10	9	6	8
1	8	0	9	3	11	7	10	6	5	2	4
4	11	3	0	6	2	10	1	9	8	5	7
10	5	9	6	0	8	4	7	3	2	11	1
2	9	1	10	4	0	8	11	7	6	3	5
6	1	5	2	8	4	0	3	11	10	7	9
3	10	2	11	5	1	9	0	8	7	4	6
7	2	6	3	9	5	1	4	0	11	8	10
8	3	7	4	10	6	2	5	1	0	9	11
11	6	10	7	1	9	5	8	4	3	0	2
9	4	8	5	11	7	3	6	2	1	10	0

CHAIN POSSIBILITIES

i =	1	2	3	4	5	6	7	8	9	10	11	12
TCH_i	4											
ICH_i	2						■	■	■	■	■	■
$RECH_i$	2		■		■		■		■		■	
$RICH_i$	4	8										

CONCERTO FOR NINE INSTRUMENTS, OP. 24

0	4	3	8	7	11	9	10	6	5	1	2
8	0	11	4	3	7	5	6	2	1	9	10
9	1	0	5	4	8	6	7	3	2	10	11
4	8	7	0	11	3	1	2	10	9	5	6
5	9	8	1	0	4	2	3	11	10	6	7
1	5	4	9	8	0	10	11	7	6	2	3
3	7	6	11	10	2	0	1	9	8	4	5
2	6	5	10	9	1	11	0	8	7	3	4
6	10	9	2	1	5	3	4	0	11	7	8
7	11	10	3	2	6	4	5	1	0	8	9
11	3	2	7	6	10	8	9	5	4	0	1
10	2	1	6	5	9	7	8	4	3	11	0

CHAIN POSSIBILITIES

i =	1	2	3	4	5	6	7	8	9	10	11	12
TCH_i	4											
ICH_i	2		2									
$RECH_i$	2											
$RICH_i$	4	24				2						

THREE SONGS, OP. 25

0	9	8	11	6	10	7	4	3	5	2	1
3	0	11	2	9	1	10	7	6	8	5	4
4	1	0	3	10	2	11	8	7	9	6	5
1	10	9	0	7	11	8	5	4	6	3	2
6	3	2	5	0	4	1	10	9	11	8	7
2	11	10	1	8	0	9	6	5	7	4	3
5	2	1	4	11	3	0	9	8	10	7	6
8	5	4	7	2	6	3	0	11	1	10	9
9	6	5	8	3	7	4	1	0	2	11	10
7	4	3	6	1	5	2	11	10	0	9	8
10	7	6	9	4	8	5	2	1	3	0	11
11	8	7	10	5	9	6	3	2	4	1	0

CHAIN POSSIBILITIES

i =	1	2	3	4	5	6	7	8	9	10	11	12
TCH_i	12		12									1
ICH_i	2											
$RECH_i$	2											
$RICH_i$	12	4										

DAS AUGENLICHT, OP. 26

0	2	1	4	3	7	8	5	9	6	10	11
10	0	11	2	1	5	6	3	7	4	8	9
11	1	0	3	2	6	7	4	8	5	9	10
8	10	9	0	11	3	4	1	5	2	6	7
9	11	10	1	0	4	5	2	6	3	7	8
5	7	6	9	8	0	1	10	2	11	3	4
4	6	5	8	7	11	0	9	1	10	2	3
7	9	8	11	10	2	3	0	4	1	5	6
3	5	4	7	6	10	11	8	0	9	1	2
6	8	7	10	9	1	2	11	3	0	4	5
2	4	3	6	5	9	10	7	11	8	0	1
1	3	2	5	4	8	9	6	10	7	11	0

CHAIN POSSIBILITIES

i =	1	2	3	4	5	6	7	8	9	10	11	12
TCH_i	12											1
ICH_i	2											
$RECH_i$	2											
$RICH_i$	12	24										

PIANO VARIATIONS, OP. 27

0	8	7	11	10	9	3	1	4	2	6	5
4	0	11	3	2	1	7	5	8	6	10	9
5	1	0	4	3	2	8	6	9	7	11	10
1	9	8	0	11	10	4	2	5	3	7	6
2	10	9	1	0	11	5	3	6	4	8	7
3	11	10	2	1	0	6	4	7	5	9	8
9	5	4	8	7	6	0	10	1	11	3	2
11	7	6	10	9	8	2	0	3	1	5	4
8	4	3	7	6	5	11	9	0	10	2	1
10	6	5	9	8	7	1	11	2	0	4	3
6	2	1	5	4	3	9	7	10	8	0	11
7	3	2	6	5	4	10	8	11	9	1	0

CHAIN POSSIBILITIES

i =	1	2	3	4	5	6	7	8	9	10	11	12
TCH_i	12											1
ICH_i	2						■	■	■	■	■	■
$RECH_i$	2		■		■		■		■		■	
$RICH_i$	12	8										

STRING QUARTET, OP. 28

0	11	2	1	5	6	3	4	8	7	10	9
1	0	3	2	6	7	4	5	9	8	11	10
10	9	0	11	3	4	1	2	6	5	8	7
11	10	1	0	4	5	2	3	7	6	9	8
7	6	9	8	0	1	10	11	3	2	5	4
6	5	8	7	11	0	9	10	2	1	4	3
9	8	11	10	2	3	0	1	5	4	7	6
8	7	10	9	1	2	11	0	4	3	6	5
4	3	6	5	9	10	7	8	0	11	2	1
5	4	7	6	10	11	8	9	1	0	3	2
2	1	4	3	7	8	5	6	10	9	0	11
3	2	5	4	8	9	6	7	11	10	1	0

CHAIN POSSIBILITIES

i =	1	2	3	4	5	6	7	8	9	10	11	12
TCH_i	4	6		3								1
ICH_i	2						■	■	■	■	■	■
$RECH_i$	2		■		■		■		■		■	
$RICH_i$	4	6		3								1

CANTATA I, OP. 29

0	8	11	10	2	1	4	3	7	6	9	5
4	0	3	2	6	5	8	7	11	10	1	9
1	9	0	11	3	2	5	4	8	7	10	6
2	10	1	0	4	3	6	5	9	8	11	7
10	6	9	8	0	11	2	1	5	4	7	3
11	7	10	9	1	0	3	2	6	5	8	4
8	4	7	6	10	9	0	11	3	2	5	1
9	5	8	7	11	10	1	0	4	3	6	2
5	1	4	3	7	6	9	8	0	11	2	10
6	2	5	4	8	7	10	9	1	0	3	11
3	11	2	1	5	4	7	6	10	9	0	8
7	3	6	5	9	8	11	10	2	1	4	0

CHAIN POSSIBILITIES

i =	1	2	3	4	5	6	7	8	9	10	11	12
TCH_i	12	4										1
ICH_i	2						■	■	■	■	■	■
$RECH_i$	2		■		■		■		■		■	
$RICH_i$	12	4										1

VARIATIONS, OP. 30

0	1	4	3	2	5	6	9	8	7	10	11
11	0	3	2	1	4	5	8	7	6	9	10
8	9	0	11	10	1	2	5	4	3	6	7
9	10	1	0	11	2	3	6	5	4	7	8
10	11	2	1	0	3	4	7	6	5	8	9
7	8	11	10	9	0	1	4	3	2	5	6
6	7	10	9	8	11	0	3	2	1	4	5
3	4	7	6	5	8	9	0	11	10	1	2
4	5	8	7	6	9	10	1	0	11	2	3
5	6	9	8	7	10	11	2	1	0	3	4
2	3	6	5	4	7	8	11	10	9	0	1
1	2	5	4	3	6	7	10	9	8	11	0

CHAIN POSSIBILITIES

i =	1	2	3	4	5	6	7	8	9	10	11	12
TCH_i	12	6					12					1
ICH_i	2											
$RECH_i$	2											
$RICH_i$	12	6					12					1

CANTATA II, OP. 31

0	3	11	10	2	9	1	5	4	8	7	6
9	0	8	7	11	6	10	2	1	5	4	3
1	4	0	11	3	10	2	6	5	9	8	7
2	5	1	0	4	11	3	7	6	10	9	8
10	1	9	8	0	7	11	3	2	6	5	4
3	6	2	1	5	0	4	8	7	11	10	9
11	2	10	9	1	8	0	4	3	7	6	5
7	10	6	5	9	4	8	0	11	3	2	1
8	11	7	6	10	5	9	1	0	4	3	2
4	7	3	2	6	1	5	9	8	0	11	10
5	8	4	3	7	2	6	10	9	1	0	11
6	9	5	4	8	3	7	11	10	2	1	0

CHAIN POSSIBILITIES

i =	1	2	3	4	5	6	7	8	9	10	11	12
TCH_i	2											1
ICH_i	2						■	■	■	■	■	■
$RECH_i$	2		■		■		■		■		■	
$RICH_i$	2	12	1									

APPENDIX 2

MEASUREMENTS OF SEGMENTAL INVARIANCE IN WEBERN'S SERIAL WORKS

The following pages show tables with measurements of segmental invariance, calculated for each of Webern's mature serial works. Each work contains two tables. The first calculates invariance under transposition T_n, and the second calculates invariance under the contextual inversion I_n. I_n sends a row $\mathbf{S}_x$ to its inversion that whose first pitch is n semitones above x. Thus, I_7 sends $\mathbf{P_0}$ to $\mathbf{I_7}$.

In the body of the table, invariance is measured according to the length of the discrete segment. For example, the two columns below "3-note" contain measurements of segmental invariance of trichords under the indicated transformations. The left-most number in each column displays the number of invariant trichords (disregarding order) and the right-most number displays triadic invariants as a percentage of the total possible. For example, the table for the Cantata I, Op. 29, indicates that under T_5, a row will retain six of its segmental trichords, which is 60 percent of the maximum possible.

On the far left side of the table, a total "invariance number" is shown that indicates the total number of invariants of all sizes maintained under the given transformation. The "invariance percentage" displays those invariants as a percentage of the total possible number of segmental invariants. For example, in the Cantata I, Op. 29, the rows *RI*-symmetry means that a **P** form is equivalent to $\mathbf{RI}_{x+5}$. Therefore, in the I_n column of the invariance table, the entry for "5" has sixty-six as its invariance number, one-hundred percent of the total possible.

STRING TRIO, OP. 20

T_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
0	66	*100%*	11	*100*	10	*100*	9	*100*	8	*100*	7	*100*	6	100	5	*100*	4	*100*	3	*100*	2	*100*	1	*100*
1	2	*10%*	1	*9*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
2	10	*17%*	6	*55*	1	*10*	2	*22*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
3	3	*14%*	1	*9*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	1	*50*	1	*100*
4	18	*28%*	7	*64*	3	*30*	2	*22*	1	*13*	1	*14*	1	17	1	*20*	0	*0*	1	*33*	0	*0*	1	*100*
5	2	*10%*	1	*9*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
6	14	*23%*	6	*55*	0	*0*	4	*44*	0	*0*	2	*29*	0	0	1	*20*	0	*0*	0	*0*	0	*0*	1	*100*
7	2	*10%*	1	*9*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
8	18	*28%*	7	*64*	3	*30*	2	*22*	1	*13*	1	*14*	1	17	1	*20*	0	*0*	1	*33*	0	*0*	1	*100*
9	3	*14%*	1	*9*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	1	*50*	1	*100*
10	10	*17%*	6	*55*	1	*10*	2	*22*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
11	2	*10%*	1	*9*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*

I_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
8	1	*9%*	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
7	21	*33%*	6	*55*	2	*20*	4	*44*	2	*25*	3	*43*	0	0	2	*40*	0	0	1	*33*	0	*0*	1	*100*
6	2	*14%*	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	1	*50*	1	*100*
5	20	*27%*	9	*82*	6	*60*	3	*33*	0	*0*	0	*0*	0	0	1	*20*	0	0	0	*0*	0	*0*	1	*100*
4	4	*12%*	2	*18*	0	*0*	1	*11*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
3	21	*45%*	7	*64*	2	*20*	2	*22*	0	*0*	1	*14*	0	0	1	*20*	2	50	3	*100*	2	*100*	1	*100*
2	1	*9%*	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
1	23	*30%*	8	*73*	6	*60*	4	*44*	2	*25*	2	*29*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
0	5	*16%*	3	*27*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	1	*50*	1	*100*
11	14	*25%*	6	*55*	0	*0*	3	*33*	0	*0*	1	*14*	0	0	2	*40*	0	0	1	*33*	0	*0*	1	*100*
10	2	*10%*	1	*9*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
9	11	*18%*	7	*64*	2	*20*	1	*11*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*

SYMPHONY, OP. 21

T_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
0	**66**	***100%***	11	*100*	10	*100*	9	*100*	8	*100*	7	*100*	6	100	5	*100*	4	*100*	3	*100*	2	*100*	1	*100*
1	**3**	***11%***	2	*18*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
2	**3**	***11%***	2	*18*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
3	**6**	***14%***	4	*36*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	1	*20*	0	*0*	0	*0*	0	*0*	1	*100*
4	**3**	***11%***	2	*18*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
5	**3**	***11%***	2	*18*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
6	**66**	***100%***	11	*100*	10	*100*	9	*100*	8	*100*	7	*100*	6	100	5	*100*	4	*100*	3	*100*	2	*100*	1	*100*
7	**3**	***11%***	2	*18*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
8	**3**	***11%***	2	*18*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
9	**6**	***14%***	4	*36*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	1	*20*	0	*0*	0	*0*	0	*0*	1	*100*
10	**3**	***11%***	2	*18*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
11	**3**	***11%***	2	*18*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*

I_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
2	**2**	***10%***	1	*9*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
1	**5**	***12%***	4	*36*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
0	**21**	***40%***	6	*55*	4	*40*	2	*22*	2	*25*	0	*0*	0	0	1	*20*	2	50	1	*33*	2	*100*	1	*100*
11	**5**	***13%***	2	*18*	0	*0*	0	*0*	0	*0*	2	*29*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
10	**7**	***14%***	4	*36*	2	*20*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
9	**17**	***31%***	6	*55*	0	*0*	3	*33*	0	*0*	2	*29*	0	0	3	*60*	0	0	2	*67*	0	*0*	1	*100*
8	**2**	***10%***	1	*9*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
7	**5**	***12%***	4	*36*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
6	**21**	***40%***	6	*55*	4	*40*	2	*22*	2	*25*	0	*0*	0	0	1	*20*	2	50	1	*33*	2	*100*	1	*100*
5	**5**	***13%***	2	*18*	0	*0*	0	*0*	0	*0*	2	*29*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
4	**7**	***14%***	4	*36*	2	*20*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
3	**17**	***31%***	6	*55*	0	*0*	3	*33*	0	*0*	2	*29*	0	0	3	*60*	0	0	2	*67*	0	*0*	1	*100*

QUARTET, OP. 22

T_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
0	66	*100%*	11	*100*	10	*100*	9	*100*	8	*100*	7	*100*	6	100	5	*100*	4	*100*	3	*100*	2	*100*	1	*100*
1	5	*12%*	3	*27*	1	*10*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
2	4	*12%*	2	*18*	1	*10*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
3	2	*10%*	0	*0*	0	*0*	1	*11*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
4	4	*12%*	2	*18*	1	*10*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
5	4	*12%*	2	*18*	0	*0*	0	*0*	0	*0*	1	*14*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
6	12	*29%*	5	*45*	0	*0*	2	*22*	0	*0*	0	*0*	0	0	1	*20*	0	*0*	1	*33*	2	*100*	1	*100*
7	4	*12%*	2	*18*	0	*0*	0	*0*	0	*0*	1	*14*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
8	4	*12%*	2	*18*	1	*10*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
9	2	*10%*	0	*0*	0	*0*	1	*11*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
10	4	*12%*	2	*18*	1	*10*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
11	5	*12%*	3	*27*	1	*10*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*

I_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
0	8	*25%*	4	*36*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	1	*33*	2	*100*	1	*100*
11	3	*11%*	2	*18*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
10	1	*9%*	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
9	5	*15%*	3	*27*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	1	*33*	0	*0*	1	*100*
8	14	*23%*	5	*45*	1	*10*	1	*11*	2	*25*	2	*29*	1	17	1	*20*	0	0	0	*0*	0	*0*	1	*100*
7	18	*30%*	7	*64*	4	*40*	2	*22*	0	*0*	0	*0*	0	0	1	*20*	2	50	1	*33*	0	*0*	1	*100*
6	8	*26%*	2	*18*	1	*10*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	1	25	1	*33*	2	*100*	1	*100*
5	3	*11%*	2	*18*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
4	1	*9%*	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
3	11	*19%*	2	*18*	2	*20*	2	*22*	2	*25*	2	*29*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
2	4	*12%*	3	*27*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
1	11	*20%*	4	*36*	0	*0*	3	*33*	0	*0*	2	*29*	0	0	1	*20*	0	0	0	*0*	0	*0*	1	*100*

THREE SONGS, OP. 23

T_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
0	66	100%	11	100	10	100	9	100	8	100	7	100	6	100	5	100	4	100	3	100	2	100	1	100
1	3	11%	2	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
2	5	13%	2	18	1	10	1	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
3	3	14%	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	50	1	100
4	3	11%	1	9	1	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
5	8	16%	3	27	1	10	1	11	1	13	1	14	0	0	0	0	0	0	0	0	0	0	1	100
6	2	10%	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
7	8	16%	3	27	1	10	1	11	1	13	1	14	0	0	0	0	0	0	0	0	0	0	1	100
8	3	11%	1	9	1	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
9	3	14%	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	50	1	100
10	5	13%	2	18	1	10	1	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
11	3	11%	2	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100

I_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
8	4	12%	2	18	1	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
7	8	18%	4	36	0	0	1	11	0	0	0	0	0	0	1	20	0	0	1	33	0	0	1	100
6	3	14%	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	50	1	100
5	10	17%	5	45	2	20	2	22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
4	18	32%	5	45	3	30	1	11	0	0	2	29	2	33	1	20	2	50	1	33	0	0	1	100
3	10	27%	3	27	2	20	1	11	0	0	0	0	0	0	0	0	0	0	1	33	2	100	1	100
2	2	10%	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
1	1	9%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
0	8	19%	3	27	3	30	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	50	1	100
11	1	9%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
10	8	15%	3	27	3	30	0	0	1	13	0	0	0	0	0	0	0	0	0	0	0	0	1	100
9	4	12%	3	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100

CONCERTO FOR NINE INSTRUMENTS, OP. 24

T_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
0	**66**	***100%***	11	*100*	10	*100*	9	*100*	8	*100*	7	*100*	6	100	5	*100*	4	*100*	3	*100*	2	*100*	1	*100*
1	**3**	***11%***	2	*18*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
2	**16**	***28%***	4	*36*	2	*20*	2	*22*	2	*25*	3	*43*	1	17	0	*0*	0	*0*	0	*0*	1	*50*	1	*100*
3	**1**	***9%***	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
4	**8**	***16%***	4	*36*	0	*0*	1	*11*	0	*0*	2	*29*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
5	**3**	***11%***	2	*18*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
6	**15**	***22%***	8	*73*	4	*40*	0	*0*	0	*0*	2	*29*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
7	**3**	***11%***	2	*18*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
8	**8**	***16%***	4	*36*	0	*0*	1	*11*	0	*0*	2	*29*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
9	**1**	***9%***	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
10	**16**	***28%***	4	*36*	2	*20*	2	*22*	2	*25*	3	*43*	1	17	0	*0*	0	*0*	0	*0*	1	*50*	1	*100*
11	**3**	***11%***	2	*18*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*

I_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
10	**3**	***11%***	2	*18*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
9	**13**	***21%***	4	*36*	2	*20*	2	*22*	2	*25*	2	*29*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
8	**3**	***11%***	1	*9*	1	*10*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
7	**9**	***18%***	4	*36*	0	*0*	1	*11*	0	*0*	2	*29*	0	0	1	*20*	0	0	0	*0*	0	*0*	1	*100*
6	**8**	***16%***	2	*18*	1	*10*	2	*22*	1	*13*	1	*14*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
5	**18**	***28%***	8	*73*	4	*40*	0	*0*	0	*0*	3	*43*	0	0	0	*0*	2	50	0	*0*	0	*0*	1	*100*
4	**10**	***27%***	2	*18*	0	*0*	0	*0*	0	*0*	1	*14*	1	17	2	*40*	1	25	1	*33*	1	*50*	1	*100*
3	**9**	***19%***	4	*36*	0	*0*	1	*11*	0	*0*	2	*29*	0	0	0	*0*	0	0	1	*33*	0	*0*	1	*100*
2	**5**	***23%***	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	1	25	1	*33*	2	*100*	1	*100*
1	**13**	***21%***	4	*36*	2	*20*	2	*22*	2	*25*	2	*29*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
0	**4**	***15%***	2	*18*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	1	*50*	1	*100*
11	**31**	***38%***	10	*91*	8	*80*	6	*67*	4	*50*	2	*29*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*

THREE SONGS, OP. 25

T_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
0	66	100%	11	100	10	100	9	100	8	100	7	100	6	100	5	100	4	100	3	100	2	100	1	100
1	5	18%	2	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	33	1	50	1	100
2	7	14%	3	27	2	20	1	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
3	3	11%	1	9	1	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
4	2	10%	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
5	10	18%	5	45	3	30	0	0	0	0	0	0	0	0	0	0	1	25	0	0	0	0	1	100
6	5	13%	2	18	2	20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
7	10	18%	5	45	3	30	0	0	0	0	0	0	0	0	0	0	1	25	0	0	0	0	1	100
8	2	10%	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
9	3	11%	1	9	1	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
10	7	14%	3	27	2	20	1	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
11	5	18%	2	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	33	1	50	1	100

I_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
10	3	11%	2	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
9	8	21%	3	27	0	0	0	0	0	0	0	0	0	0	1	20	2	50	1	33	0	0	1	100
8	7	15%	3	27	1	10	1	11	0	0	0	0	0	0	1	20	0	0	0	0	0	0	1	100
7	7	14%	5	45	0	0	1	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
6	5	13%	2	18	0	0	0	0	1	13	0	0	1	17	0	0	0	0	0	0	0	0	1	100
5	9	17%	5	45	0	0	2	22	0	0	1	14	0	0	0	0	0	0	0	0	0	0	1	100
4	4	12%	3	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
3	5	15%	3	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	33	0	0	1	100
2	15	36%	4	36	1	10	0	0	0	0	0	0	1	17	2	40	3	75	2	67	1	50	1	100
1	10	28%	2	18	0	0	0	0	2	25	2	29	0	0	0	0	0	0	1	33	2	100	1	100
0	6	17%	4	36	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	50	1	100
11	2	10%	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100

DAS AUGENLICHT, OP. 26

T_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
0	66	100%	11	100	10	100	9	100	8	100	7	100	6	100	5	100	4	100	3	100	2	100	1	100
1	6	19%	2	18	1	10	0	0	0	0	0	0	0	0	0	0	0	0	1	33	1	50	1	100
2	8	18%	2	18	1	10	1	11	0	0	0	0	1	17	1	20	1	25	0	0	0	0	1	100
3	5	12%	3	27	1	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
4	4	12%	2	18	1	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
5	4	12%	2	18	0	0	0	0	1	13	0	0	0	0	0	0	0	0	0	0	0	0	1	100
6	3	11%	2	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
7	4	12%	2	18	0	0	0	0	1	13	0	0	0	0	0	0	0	0	0	0	0	0	1	100
8	4	12%	2	18	1	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
9	5	12%	3	27	1	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
10	8	18%	2	18	1	10	1	11	0	0	0	0	1	17	1	20	1	25	0	0	0	0	1	100
11	6	19%	2	18	1	10	0	0	0	0	0	0	0	0	0	0	0	0	1	33	1	50	1	100

In	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
8	1	9%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
7	2	10%	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
6	3	11%	2	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
5	7	14%	3	27	2	20	1	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
4	4	13%	1	9	0	0	0	0	1	13	0	0	1	17	0	0	0	0	0	0	0	0	1	100
3	10	17%	5	45	2	20	1	11	0	0	1	14	0	0	0	0	0	0	0	0	0	0	1	100
2	16	27%	6	55	3	30	1	11	1	13	0	0	2	33	0	0	1	25	1	33	0	0	1	100
1	11	19%	3	27	2	20	2	22	2	25	0	0	0	0	1	20	0	0	0	0	0	0	1	100
0	12	26%	4	36	2	20	0	0	0	0	1	14	1	17	0	0	2	50	0	0	1	50	1	100
11	7	24%	2	18	0	0	1	11	0	0	0	0	0	0	0	0	0	0	1	33	2	100	1	100
10	8	24%	3	27	0	0	0	0	0	0	0	0	0	0	0	0	1	25	2	67	1	50	1	100
9	13	22%	5	45	2	20	2	22	2	25	0	0	0	0	0	0	0	0	1	33	0	0	1	100

T_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
0	66	100%	11	100	10	100	9	100	8	100	7	100	6	100	5	100	4	100	3	100	2	100	1	100
1	4	12%	3	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
2	3	11%	2	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
3	2	10%	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
4	2	10%	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
5	6	17%	2	18	1	10	1	11	0	0	0	0	0	0	0	0	0	0	0	0	1	50	1	100
6	10	19%	3	27	0	0	0	0	2	25	2	29	2	33	0	0	0	0	0	0	0	0	1	100
7	6	17%	2	18	1	10	1	11	0	0	0	0	0	0	0	0	0	0	0	0	1	50	1	100
8	2	10%	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
9	2	10%	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
10	3	11%	2	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
11	4	12%	3	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100

I_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
6	11	18%	5	45	2	20	1	11	1	13	1	14	0	0	0	0	0	0	0	0	0	0	1	100
5	12	28%	5	45	2	20	1	11	0	0	0	0	0	0	0	0	0	0	1	33	2	100	1	100
4	4	12%	3	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
3	4	12%	3	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
2	3	11%	2	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
1	9	17%	4	36	2	20	0	0	0	0	2	29	0	0	0	0	0	0	0	0	0	0	1	100
0	3	14%	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	50	1	100
11	3	13%	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	33	0	0	1	100
10	2	14%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	50	1	100
9	2	10%	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
8	8	17%	4	36	2	20	0	0	0	0	0	0	0	0	0	0	0	0	1	33	0	0	1	100
7	8	17%	3	27	0	0	1	11	0	0	2	29	0	0	1	20	0	0	0	0	0	0	1	100

T_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
0	66	*100%*	11	*100*	10	*100*	9	*100*	8	*100*	7	*100*	6	100	5	*100*	4	*100*	3	*100*	2	*100*	1	*100*
1	1	*9%*	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
2	17	*32%*	6	*55*	0	*0*	1	*11*	0	*0*	1	*14*	2	33	3	*60*	2	*50*	1	*33*	0	*0*	1	*100*
3	3	*14%*	1	*9*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	1	*50*	1	*100*
4	20	*27%*	9	*82*	6	*60*	3	*33*	0	*0*	0	*0*	0	0	1	*20*	0	*0*	0	*0*	0	*0*	1	*100*
5	1	*9%*	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
6	7	*14%*	6	*55*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
7	1	*9%*	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
8	20	*27%*	9	*82*	6	*60*	3	*33*	0	*0*	0	*0*	0	0	1	*20*	0	*0*	0	*0*	0	*0*	1	*100*
9	3	*14%*	1	*9*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	1	*50*	1	*100*
10	17	*32%*	6	*55*	0	*0*	1	*11*	0	*0*	1	*14*	2	33	3	*60*	2	*50*	1	*33*	0	*0*	1	*100*
11	1	*9%*	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*

I_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
10	1	*9%*	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
9	66	*100%*	11	*100*	10	*100*	9	*100*	8	*100*	7	*100*	6	100	5	*100*	4	100	3	*100*	2	*100*	1	*100*
8	1	*9%*	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
7	17	*32%*	6	*55*	0	*0*	1	*11*	0	*0*	1	*14*	2	33	3	*60*	2	50	1	*33*	0	*0*	1	*100*
6	3	*14%*	1	*9*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	1	*50*	1	*100*
5	20	*27%*	9	*82*	6	*60*	3	*33*	0	*0*	0	*0*	0	0	1	*20*	0	0	0	*0*	0	*0*	1	*100*
4	1	*9%*	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
3	7	*14%*	6	*55*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
2	1	*9%*	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
1	20	*27%*	9	*82*	6	*60*	3	*33*	0	*0*	0	*0*	0	0	1	*20*	0	0	0	*0*	0	*0*	1	*100*
0	3	*14%*	1	*9*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	1	*50*	1	*100*
11	17	*32%*	6	*55*	0	*0*	1	*11*	0	*0*	1	*14*	2	33	3	*60*	2	50	1	*33*	0	*0*	1	*100*

CANTATA I, OP. 29

T_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
0	66	100%	11	100	10	100	9	100	8	100	7	100	6	100	5	100	4	100	3	100	2	100	1	100
1	1	9%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
2	7	14%	4	36	2	20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
3	5	15%	3	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	33	0	0	1	100
4	2	10%	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
5	23	34%	7	64	6	60	3	33	3	38	1	14	1	17	0	0	0	0	0	0	1	50	1	100
6	1	9%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
7	23	34%	7	64	6	60	3	33	3	38	1	14	1	17	0	0	0	0	0	0	1	50	1	100
8	2	10%	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
9	5	15%	3	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	33	0	0	1	100
10	7	14%	4	36	2	20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
11	1	9%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100

I_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
6	1	9%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
5	66	100%	11	100	10	100	9	100	8	100	7	100	6	100	5	100	4	100	3	100	2	100	1	100
4	1	9%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
3	7	14%	4	36	2	20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
2	5	15%	3	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	33	0	0	1	100
1	2	10%	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
0	23	34%	7	64	6	60	3	33	3	38	1	14	1	17	0	0	0	0	0	0	1	50	1	100
11	1	9%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
10	23	34%	7	64	6	60	3	33	3	38	1	14	1	17	0	0	0	0	0	0	1	50	1	100
9	2	10%	1	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100
8	5	15%	3	27	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	33	0	0	1	100
7	7	14%	4	36	2	20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	100

T_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
0	66	100%	11	100	10	100	9	100	8	100	7	100	6	100	5	100	4	100	3	100	2	100	1	100
1	17	34%	4	36	0	0	2	22	4	50	2	29	0	0	0	0	1	25	2	67	1	50	1	100
2	11	21%	5	45	0	0	2	22	2	25	0	0	0	0	0	0	0	0	1	33	0	0	1	100
3	7	15%	4	36	0	0	1	11	1	13	0	0	0	0	0	0	0	0	0	0	0	0	1	100
4	9	17%	4	36	0	0	1	11	2	25	1	14	0	0	0	0	0	0	0	0	0	0	1	100
5	27	36%	8	73	5	50	5	56	4	50	2	29	1	17	1	20	0	0	0	0	0	0	1	100
6	13	22%	4	36	0	0	2	22	4	50	2	29	0	0	0	0	0	0	0	0	0	0	1	100
7	27	36%	8	73	5	50	5	56	4	50	2	29	1	17	1	20	0	0	0	0	0	0	1	100
8	9	17%	4	36	0	0	1	11	2	25	1	14	0	0	0	0	0	0	0	0	0	0	1	100
9	7	15%	4	36	0	0	1	11	1	13	0	0	0	0	0	0	0	0	0	0	0	0	1	100
10	11	21%	5	45	0	0	2	22	2	25	0	0	0	0	0	0	0	0	1	33	0	0	1	100
11	17	34%	4	36	0	0	2	22	4	50	2	29	0	0	0	0	1	25	2	67	1	50	1	100

I_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
6	27	36%	8	73	5	50	5	56	4	50	2	29	1	17	1	20	0	0	0	0	0	0	1	100
5	13	22%	4	36	0	0	2	22	4	50	2	29	0	0	0	0	0	0	0	0	0	0	1	100
4	27	36%	8	73	5	50	5	56	4	50	2	29	1	17	1	20	0	0	0	0	0	0	1	100
3	9	17%	4	36	0	0	1	11	2	25	1	14	0	0	0	0	0	0	0	0	0	0	1	100
2	7	15%	4	36	0	0	1	11	1	13	0	0	0	0	0	0	0	0	0	0	0	0	1	100
1	11	21%	5	45	0	0	2	22	2	25	0	0	0	0	0	0	0	0	1	33	0	0	1	100
0	17	34%	4	36	0	0	2	22	4	50	2	29	0	0	0	0	1	25	2	67	1	50	1	100
11	66	100%	11	100	10	100	9	100	8	100	7	100	6	100	5	100	4	100	3	100	2	100	1	100
10	17	34%	4	36	0	0	2	22	4	50	2	29	0	0	0	0	1	25	2	67	1	50	1	100
9	11	21%	5	45	0	0	2	22	2	25	0	0	0	0	0	0	0	0	1	33	0	0	1	100
8	7	15%	4	36	0	0	1	11	1	13	0	0	0	0	0	0	0	0	0	0	0	0	1	100
7	9	17%	4	36	0	0	1	11	2	25	1	14	0	0	0	0	0	0	0	0	0	0	1	100

CANTATA, OP. 31

T_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
0	**66**	***100%***	11	*100*	10	*100*	9	*100*	8	*100*	7	*100*	6	100	5	*100*	4	*100*	3	*100*	2	*100*	1	*100*
1	**9**	***17%***	3	*27*	2	*20*	2	*22*	1	*13*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
2	**4**	***12%***	3	*27*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
3	**6**	***13%***	4	*36*	1	*10*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
4	**4**	***12%***	2	*18*	1	*10*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
5	**4**	***12%***	2	*18*	1	*10*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
6	**10**	***26%***	4	*36*	2	*20*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	1	*33*	2	*100*	1	*100*
7	**4**	***12%***	2	*18*	1	*10*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
8	**4**	***12%***	2	*18*	1	*10*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
9	**6**	***13%***	4	*36*	1	*10*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
10	**4**	***12%***	3	*27*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*
11	**9**	***17%***	3	*27*	2	*20*	2	*22*	1	*13*	0	*0*	0	0	0	*0*	0	*0*	0	*0*	0	*0*	1	*100*

I_n	Inv. #	Inv. %	2-note		3-note		4-note		5-note		6-note		7-note		8-note		9-note		10-note		11-note		12-note	
0	**20**	***37%***	6	*55*	4	*40*	3	*33*	1	*13*	1	*14*	1	17	0	*0*	0	0	1	*33*	2	*100*	1	*100*
11	**9**	***16%***	5	*45*	2	*20*	1	*11*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
10	**3**	***11%***	1	*9*	1	*10*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
9	**8**	***15%***	4	*36*	2	*20*	1	*11*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
8	**1**	***9%***	0	*0*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
7	**5**	***13%***	2	*18*	2	*20*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
6	**27**	***49%***	5	*45*	3	*30*	2	*22*	3	*38*	4	*57*	3	50	2	*40*	1	25	1	*33*	2	*100*	1	*100*
5	**5**	***12%***	4	*36*	0	*0*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
4	**5**	***13%***	2	*18*	2	*20*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	0	*0*	0	*0*	1	*100*
3	**10**	***19%***	6	*55*	2	*20*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	0	0	1	*33*	0	*0*	1	*100*
2	**9**	***17%***	5	*45*	2	*20*	0	*0*	0	*0*	0	*0*	0	0	0	*0*	1	25	0	*0*	0	*0*	1	*100*
1	**6**	***16%***	3	*27*	0	*0*	1	*11*	0	*0*	0	*0*	0	0	0	*0*	0	0	1	*33*	0	*0*	1	*100*

References

Adorno, Theodor W. *Klangfiguren. Musikalische Schriften, I.* Suhrkamp Verlag, 1959.

Alphonce, Bo. "The Invariance Matrix." Ph.D. dissertation, Yale University, 1974.

Babbitt, Milton. "Set Structure as a Compositional Determinant." In *The Collected Essays of Milton Babbitt*, edited by Joseph N. Straus, Andrew Mead, and Stephen Dembski, 86-108. Princeton: Princeton University Press, 2003.

———. "Twelve-Tone Invariants as Compositional Determinants." In *The Collected Essays of Milton Babbitt*, edited by Joseph N. Straus, Andrew Mead, and Stephen Dembski, 55–69. Princeton: Princeton University Press, 2003.

Bailey, Kathryn. *The Twelve-Note Music of Anton Webern: Old Forms in a New Language.* Music in the Twentieth Century. New York: Cambridge University Press, 1991.

———. "Willi Reich's Webern." *Tempo* no. 165 (June 1, 1988): 18–22.

Bamberger, Jeanne. "Cognitive Issues in the Development of Musically Gifted Children." In *Conceptions of Giftedness*, edited by Robert J. Sternberg and Davidson, 388–413. New York: Cambridge University Press, 1986.

Beach, David W. "Segmental Invariance and the Twelve-Tone System." *Journal of Music Theory* 20, no. 2 (1976): 157–184.

Bernard, Jonathan W. *The Music of Edgard Varèse*. New Haven: Yale University Press, 1987.

Bonds, Mark Evan. "Haydn's False Recapitulations and the Perception of Sonata Form in the Eighteenth Century." Ph.D. dissertation, Harvard University, 1988.

Boulez, Pierre, and Paule Thévenin. *Stocktakings from an Apprenticeship*. Translated by Stephen Walsh. Oxford: Clarendon Press, 1991.

Boynton, Neil. "Formal Combination in Webern's Variations Op. 30." *Music Analysis* 14, no. 2/3 (July 1, 1995): 193–220.

———. "Some Remarks on Anton Webern's 'Variations, Op. 27'." In *Webern_21*, edited by Dominik Schweiger and Nikolaus Urbanek, 199–220. Wiener Veröffentlichungen Zur Musikgeschichte 8. Vienna, Cologne, Weimar: Böhlau, 2009.

Brower, Candace. "Pathway, Blockage, and Containment in 'Density 21.5'." *Theory and Practice* 22 (1997): 35–54.

———. "Paradoxes of Pitch Space." *Music Analysis* 27, no. 1 (2008): 51–106.

———. "A Cognitive Theory of Musical Meaning." *Journal of Music Theory* 44, no. 2 (2000): 323–379.

Busch, Regina. "Aus Dem Briefwechsel Webern-Steuermann." In *Musik-Konzepte, Soderband Anton Webern I*, edited by Heinz-Klaus Metzger and Rainer Riehn, 23–51. Universal Edition. Vienna, 1983.

———. "On the Horizontal and Vertical Presentation of Musical Ideas and on Musical Space (I)." *Tempo* no. 154 (1985): 2–10.

———. Busch, Regina, and Michael Graubart. "On the Horizontal and Vertical Presentation of Musical Ideas and on Musical Space (II)." *Tempo* 156 (1986): 7–15.

———. "[Letter from Regina Busch]." *Tempo* 166 (September 1, 1988): 67–69.

Callender, Clifton, Ian Quinn, and Dmitri Tymoczko. "Generalized Voice-Leading Spaces." *Science* 320, no. 5874 (2008): 346–348.

Caplin, William E. *Classical Form: A Theory of Formal Functions for the Instrumental Music of Haydn, Mozart, and Beethoven*. Oxford University Press, 1998.

Carter, Nathan. *Group Explorer* (version 2.2). Waltham, Massachusetts, 2009. http://groupexplorer.sourceforge.net/.

———. *Visual Group Theory*. Washington D.C.: Mathematical Association of America, 2009.

Cherlin, Michael. "The Formal and Dramatic Organization of Schoenberg's 'Moses Und Aron'." Ph.D. dissertation, Yale University, 1983.

Cohn, Richard. "Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions." *Music Analysis* 15, no. 1 (March 1, 1996): 9–40.

———. "As Wonderful as Star Clusters: Instruments for Gazing at Tonality in Schubert." *19th-Century Music* 22, no. 3 (April 1, 1999): 213–232.

———. "A Tetrahedral Graph of Tetrachordal Voice-leading Space." *Music Theory Online* 9, no. 4 (2003).

———. "Tonal Pitch Space and the (Neo-) Riemannian Tonnetz." In *The Oxford Handbook of Neo-Riemannian Music Theories*, edited by Edward Gollin and Alexander Redhing, 322–48. New York: Oxford, 2011.

———. *Audacious Euphony: Chromaticism and the Triad's Second Nature*. New York: Oxford, 2012.

Cook, Nicholas. *A Guide to Musical Analysis*. New York: Oxford University Press, 1994.

Covach, John. "Schoenberg and the Occult: Some Reflections on the Musical Idea." *Theory and Practice* 17 (1992): 103–118.

———."The Sources of Schoenberg's' Aesthetic Theology'." *19th-Century Music* 19, no. 3 (1996): 252–262.

Cox, Gareth. "Blumengruß and Blumenglöckchen: Goethe's Influence on Anton Webern." In *Goethe: Musical Poet, Musical Catalyst: Proceedings of the Conference Hosted by the Department of Music, National University of Ireland, Maynooth, 26 & 27 March 2004*, edited by Lorraine Byne, 203–224. Dublin: Carysfort Press, 2004.

Döhl, Friedhelm. "Weberns Beitrag Zur Stilwende Der Neuen Musik." In *Berliner Musikwissenschaftliche Arbeiten*, edited by Carl Dahlhaus and Rudolf Stefan. Vol. 12. Munich and Salzburg: Musikverlag Emil Katzbichler, 1976.

Douthett, Jack, and Peter Steinbach. "Parsimonious Graphs: A Study in Parsimony, Contextual Transformations, and Modes of Limited Transposition." *Journal of Music Theory* 42, no. 2 (1998): 241–263.

Fennelly, Brian. "Structure and Process in Webern's Opus 22." *Journal of Music Theory* 10 (1966): 300–328.

Fuller, Sarah. "Tendencies and Resolutions: The Directed Progression in 'Ars Nova' Music." *Journal of Music Theory* 36, no. 2 (1992): 229.

Gap Group, The. *GAP—Groups, Algorithms, and Programming* (version 4.6.4), 2013. http://www.gap-system.org/.

Gauldin, Robert. *Harmonic Practice in Tonal Music*. 2nd ed. New York: Norton, 2004.

Gollin, Edward. "Some Aspects of Three-Dimensional 'Tonnetze'." *Journal of Music Theory* 42, no. 2 (1998): 195–206.

———. "Representations of Space and Conceptions of Distance in Transformational Music Theories." Ph.D. dissertation, Harvard University, 2000.

———. "Some Further Notes on the History of the *Tonnetz*." *Theoria* 13 (2006): 99–111.

———. "Multi-Aggregate Cycles and Multi-Aggregate Serial Techniques in the Music of Béla Bartók." *Music Theory Spectrum* 29, no. 2 (2007): 143–176.

Gjerdingen, Robert. *Music in the Galant Style*. New York: Oxford University Press, 2007.

Griffiths, Paul. "Anton Webern." *The New Grove Second Viennese School: Schoenberg, Webern, Berg*. New York: Norton, 1983.

Haimo, Ethan, Paul Johnson, and Schoenberg. "Isomorphic Partitioning and Schoenberg's 'Fourth String Quartet'." *Journal of Music Theory* 28 (1984): 47–72.

Hanninen, Dora A. "The Variety of Order Relations in Webern's Music: Studies of Passages from the Quartet Op. 22 and the Variations Op. 30." *Theory and Practice* 20 (1995): 31–56.

Harrison, Daniel. "Three Short Essays on Neo-Riemannian Theory." In *The Oxford Handbook of Neo-Riemannian Music Theories*, edited by Edward Gollin and Alexander Rehding, 548–78. New York: Oxford University Press, 2011.

Hartwell, Robin. "Duration and Mental Arithmetic: The First Movement of Webern's First Cantata." *Perspectives of New Music* 23, no. 1 (1984): 348–59.

Hasty, Christopher F. "Composition and Context in Twelve-Note Music of Anton Webern." *Music Analysis* 7, no. 3 (1988): 281–312.

Heinichen, Johann David. *Der General-Baß in Der Composition*. Dresden, 1728.

Hepokoski, James, and Warren Darcy. *Elements of Sonata Theory: Norms, Types, and Deformations in the Late-Eighteenth-Century Sonata*. New York: Oxford University Press, 2006.

Hook, Julian. "Rhythm in the Music of Messiaen: An Algebraic Study and an Application in the 'Turangalîla Symphony'." *Music Theory Spectrum* 20, no. 1 (1998): 97–120.

———. "Uniform Triadic Transformations." *Journal of Music Theory* 46, no. 1/2 (2002): 57–126.

———. "David Lewin and the Complexity of the Beautiful." *Intégral* 21 (2007): 155–190.

———. "Cross-Type Transformations and the Path Consistency Condition." *Music Theory Spectrum* 29, no. 1 (2007): 1–40.

Hook, Julian, and Jack Douthett. "Uniform Triadic Transformations and the Twelve-Tone Music of Webern." *Perspectives of New Music* (2008): 91–151.

Johnson, Julian. *Webern and the Transformation of Nature*. Cambridge: Cambridge University Press, 2000.

Klumpenhouwer, Henry. "A Generalized Model of Voice Leading for Atonal Music." Ph.D. dissertation, Harvard University, 1991.

———. "Some Remarks on the Use of Riemann Transformations." *Music Theory Online* 9 (1994).

———. "Essay: In Order to Stay Asleep as Observers: The Nature and Origins of Anti-Cartesianism in Lewin's Generalized Musical Intervals and Transformations." *Music Theory Spectrum* 28, no. 2 (2006): 277–289.

Kramer, Jonathan. "The Row as Structural Background and Audible Foreground: The First Movement of Webern's First Cantata." *Journal of Music Theory* 15, no. 1/2 (1971): 158–181.

Lakoff, George, and Mark Johnson. *Metaphors We Live By*. Chicago: University of Chicago Press, 2008.

Lambert, Philip. "Interval Cycles as Compositional Resources in the Music of Charles Ives." *Music Theory Spectrum* 12, no. 1 (1990): 43–82.

———. "On Contextual Transformations." *Perspectives of New Music* 38, no. 1 (2000): 45–76.

Lerdahl, Fred, and Ray S. Jackendoff. *A Generative Theory of Tonal Music*. Boston: MIT Press, 1983.

Lewin, David. "A Label-Free Development for 12-Pitch-Class Systems." *Journal of Music Theory* 21, no. 1 (1977): 29–48.

———. "A Metrical Problem in Webern's Op. 27." *Music Analysis* 12, no. 3 (1993): 343–354.

———. "A Study of Hexachord Levels in Schoenberg's Violin Fantasy." *Perspectives of New Music* 6, no. 1 (1967): 18–32.

———. "A Tutorial on Klumpenhouwer Networks, Using the Chorale in Schoenberg's Opus 11, No. 2." *Journal of Music Theory* 38, no. 1 (1994): 79–101.

———. *Generalized Musical Intervals and Transformations*. New Haven: Yale University Press, 1987.

———. "Inversional Balance as an Organizing Force in Schoenberg's Music and Thought." *Perspectives of New Music* 6, no. 2 (1968): 1–21.

———. "Klumpenhouwer Networks and Some Isographies That Involve Them." *Music Theory Spectrum* 12, no. 1 (1990): 83–120.

———. "Moses und Aron: Some General Remarks, and Analytic Notes for Act I, Scene 1." *Perspectives of New Music* 6, no. 1 (1967): 1–17.

———. *Musical Form and Transformation: 4 Analytical Essays*. New Haven: Yale University Press, 1993.

———. "Some Ideas About Voice-Leading Between PCSets." *Journal of Music Theory* 42, no. 1 (1998): 15–72.

———. "Some Notes on Analyzing Wagner: The Ring and Parsifal." *19th-Century Music* 16, no. 1 (1992): 49–58.

———. "Transformational Techniques in Atonal and Other Music Theories." *Perspectives of New Music* 21, no. 1/2 (1982): 312–371.

Livingstone, Ivy. *Acer*. Accessed June 6, 2013. http://biodidac.bio.uottawa.ca/thumbnails/filedet.htm?File_name=ACER001B&File_type=GIF.

McCreless, Patrick. "Syntagmatics and Paradigmatics: Some Implications for the Analysis of Chromaticism in Tonal Music." *Music Theory Spectrum* 13, no. 2 (1991): 147–178.

Mead, Andrew. "Large-Scale Strategy in Arnold Schoenberg's Twelve-Tone Music." *Perspectives of New Music* 24, no. 1 (1985): 120–157.

———. "Some Implications of the Pitch Class/Order Number Isomorphism Inherent in the Twelve-Tone System: Part One." *Perspectives of New Music* 26, no. 2 (1988): 96–163.

———. "Review of The Twelve-note Music of Anton Webern: Old Forms in a New Language, by Kathryn Bailey." *Intégral* 6 (1992): 107–35.

———. "Webern, Tradition, and 'Composing with Twelve Tones'." *Music Theory Spectrum* 15, no. 2 (1993): 173–204.

Meyer-Wendt, Lorian. "Anton Webern's Musical Realization of Goethe's Urpflanze Concept in Drei Lieder, Op. 18." M.M. thesis, The Florida State University, 2004.

Moldenhauer, Hans, and Rosaleen Moldenhauer. *Anton von Webern, a Chronicle of His Life and Work*. New York: Knopf, 1979.

Morris, Robert. *Composition With Pitch-Classes: A Theory of Compositional Design*. New Haven: Yale University Press, 1987.

———. *Class Notes for Atonal Theory*. Lebanon, NH: Frog Peak Music, 1991.

———. "Compositional Spaces and Other Territories." *Perspectives of New Music* 33, no. 1/2 (1995): 328–358.

———. *Class Notes for Advanced Atonal Music Theory*. Lebanon, NH: Frog Peak Music, 2001.

———. "Voice-Leading Spaces." *Music Theory Spectrum* 20, no. 2 (October 1, 1998): 175–208.

Moseley, Brian. "Getting RICH: Chain Transformations and Row Areas in Webern's Op. 22, ii." Annual meeting of the Society for Music Theory, Montréal, 2009 and the Music Theory Society of New York State, Hempstead, NY, 2009.

———."A Spatial Representation of Webern's 'Synthesis' and an Analysis of 'Blitz und Donner,' from the Cantata I." Annual meeting of Music Theory Midwest, Ann Arbor, MI, 2012.

Neff, Severine. "Schoenberg and Goethe: Organicism and Analysis." In *Music Theory and the Exploration of the Past*, edited by Christopher Hatch and David N. Bernstein, 409–34. Chicago: University of Chicago Press, 1993.

Nolan, Catherine. "Hierarchic Linear Structures in Webern's Twelve-tone Music." Ph.D. dissertation, Yale University, 1989.

———. "Structural Levels and Twelve-Tone Music: A Revisionist Analysis of the Second Movement of Webern's 'Piano Variations' Op. 27." *Journal of Music Theory* 39, no. 1 (1995): 47–76.

———."Webern's Twelve-Tone Rows through the Medium of Klumpenhouwer Networks." In *Mathematics and Computation in Music*, edited by Timour Klouche and Thomas Noll, 375–385. Berlin and Heidelberg: Springer, 2009.

O'Donnell, Shaugn. "Transformational Voice Leading in Atonal Music." Ph.D. dissertation, City University of New York, 1997.

———. "Klumpenhouwer Networks, Isography, and the Molecular Metaphor." *Intégral* 12 (1998): 53–80.

Peck, Robert W. "A GAP Tutorial for Transformational Music Theory." *Music Theory Online* 17, no. 1 (2011). http://www.mtosmt.org/issues/mto.11.17.1/mto.11.17.1.peck.html.

———. "Wreath Products in Transformational Music Theory." *Perspectives of New Music* 47, no. 1 (2009): 193–210.

Perle, George. "Webern's Twelve-Tone Sketches." *The Musical Quarterly* 57, no. 1 (January 1, 1971): 1–25.

Phipps, Graham H. "Tonality in Webern's Cantata I. Winner of the Elisabeth Lutyens Essay Prize, 1984." *Music Analysis* 3, no. 2 (1984): 124–158.

Quinn, Ian. "Are Pitch-Class Profiles Really 'Key for Key'?" *Zeitschrift Der Gesellschaft Für Musiktheorie* 7, no. 2 (2010). http://www.gmth.de/zeitschrift/artikel/513.aspx.

Reinhardt, Lauriejean. "«ICH UND DU UND ALLE»: Hildegard Jone, Ferdinand Ebner, and Anton Webern's 'Drei Gesänge' Op. 23." *Revista de Musicología* 16, no. 6 (1993): 3766–3782.

———. "From Poet's Voice to Composer's Muse: Text and Music in Webern's Jone Settings." Ph.D. dissertation, University of North Carolina, 1995.

Riemann, Hugo. *Musikalische Syntaxis: Grundriss einer harmonischen Satzbildungslehre*. Leipzig: Breitkopf und Härtel, 1877.

Rings, Steven. "Tonality and Transformation." Ph.D. dissertation, Yale University, 2006.

———. *Tonality and Transformation*. New York: Oxford University Press, 2011.

Rochberg, George. "Webern's Search for Harmonic Identity." *Journal of Music Theory* 6, no. 1 (1962): 109–122.

Roeder, John. "Constructing Transformational Signification: Gesture and Agency in Bartók's Scherzo, Op. 14, No. 2, Measures 1–32." *Music Theory Online* 15, no. 1 (March 2009).

Rosen, Charles. *Sonata Forms*. New York: Norton, 1988.

Saslaw, Janna. "Forces, Containers, and Paths: The Role of Body-Derived Image Schemas in the Conceptualization of Music." *Journal of Music Theory* 40, no. 2 (1996): 217–243.

Saturen, David. "Symmetrical Relationships in Webern's First Cantata." *Perspectives of New Music* 6, no. 1 (1967): 142–143.

Satyendra, Ramon. "An Informal Introduction to Some Formal Concepts from Lewin's Transformational Theory." *Journal of Music Theory* 48, no. 1 (April 1, 2004): 99–141.

Saussure, Ferdinand de. *Course in General Linguistics*. New York: Columbia University Press, 2011.

Shaffer, Kristen (Kris) P. "'Neither Tonal nor Atonal'?: Harmony and Harmonic Syntax in György Ligeti's Late Triadic Works." Ph.D. dissertation, Yale University, 2011.

Shreffler, Anne. "Anton Webern." In *Schoenberg, Berg, and Webern: A Companion to the Second Viennese School*, 251–314. Westport, Conn.: Greenwood, 1999.

———. "'Mein Weg Geht Jetzt Vorüber': The Vocal Origins of Webern's Twelve-Tone Composition." *Journal of the American Musicological Society* 47, no. 2 (1994): 275–339.

Smith, Leland. "Composition and Precomposition in the Music of Webern." In *Anton von Webern Perspectives*, edited by Hans Moldenhauer and Demar Irvine, 86–101. Seattle: University of Washington Press, 1966.

Spitzer, Michael. *Metaphor and Musical Thought*. Chicago: University of Chicago Press, 2004.

Spinner, Leopold. "The Abolition of Thematicism and the Structural Meaning of the Method of Twelve-Tone Composition." *Tempo* 146 (1983): 2–9.

Straus, Joseph N. "Voice Leading in Atonal Music." In *Music Theory in Concept and Practice*, edited by James M. Baker, David W. Beach, and Jonathan W. Bernard, 237–74. Rochester, NY: University of Rochester Press, 1997.

———."Uniformity, Balance, and Smoothness in Atonal Voice Leading." *Music Theory Spectrum* 25, no. 2 (2003): 305–352.

———. *Introduction to Post Tonal Theory*. 3rd ed. Upper Saddle River, NJ: Pearson Prentice Hall, 2005.

———. "Voice Leading in Set-Class Space." *Journal of Music Theory* 49, no. 1 (2005): 45–108.

———. "Motivic Chains in Bartók's Third String Quartet." *Twentieth-century Music* 5, no. 1 (2008): 25–44.

———. "Contextual-Inversion Spaces." *Journal of Music Theory* 55, no. 1 (2011): 43–88.

Travis, Roy. "Directed Motion in Schoenberg and Webern." *Perspectives of New Music* 4, no. 2 (1966): 85–89.

Tymoczko, Dmitri. "The Geometry of Musical Chords." *Science* 313, no. 5783 (2006): 72–74.

———. "Lewin, Intervals, and Transformations: a Comment on Hook." *Music Theory Spectrum* 30, no. 1 (2008): 164–168.

———."Generalizing Musical Intervals." *Journal of Music Theory* 53, no. 2 (2009): 227–254.

———. "Geometrical Methods in Recent Music Theory." *Music Theory Online* 16, no. 1 (2010). http://www.mtosmt.org/issues/mto.10.16.1/mto.10.16.1.tymoczko.html.

———. *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice: Harmony and Counterpoint in the Extended Common Practice*. New York: Oxford University Press, 2011.

Wason, Robert W. "Webern's 'Variations for Piano', Op. 27: Musical Structure and the Performance Score." *Intégral* 1 (1987): 57–103.

Webern, Anton. *The Path to the New Music*. Edited by Willi Reich. Translated by Leo Black. Bryn Mawr, PA: Theodore Presser, 1963.

———. *Letters to Hildegard Jone and Josef Humplik*. Edited by Josef Polnauer. Translated by Cornelius Cardew. Bryn Mawr, PA: Theodore Presser, 1967.

Webster, James. "Sonata Form." Edited by Laura Macy. *Grove Music Online*. Accessed August 2, 2013. http://www.grovemusic.com.

Westergaard, Peter. "Some Problems in Rhythmic Theory and Analysis." *Perspectives of New Music* 1, no. 1 (1962): 180–191.

———. "Webern and 'Total Organization': An Analysis of the Second Movement of Piano Variations, Op. 27." *Perspectives of New Music* 1, no. 2 (1963): 107–120.

———. "Geometries of Sounds in Time." *Music Theory Spectrum* 18, no. 1 (1996): 1–21.

Wood, Dennis, and John Fels. *The Power of Maps*. New York: Guilford Press, 1992.

Zbikowski, Lawrence M. *Conceptualizing Music: Cognitive Structure, Theory, and Analysis*. New York: Oxford University Press, 2005.