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VALUE-AT-RISK

BY

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ABSTRACT

Value-at-Risk

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This paper develops a comprehensive framework of the new emerging market risk measurement model known as Value-at-Risk, VaR. It analyses, summarizes and integrates the existing literature which it supplements in two important respects. First, it presents, assesses and compares the three basic approaches to computing VaR - historical, analytical, and Monte Carlo simulation. Then, it explores proposed adjustments and improvements to the currently used models. Finally, it addresses the difficulties of model validity and accuracy testing techniques. Second, parametric and nonparametric forms of VaR models are applied to seventeen years of historical Dow Jones Industrial Average data to appraise the approaches' respective ability to estimate market risk exposures against the history of profits and losses (P&L) incurred during the Great Depression on portfolio of blue chip stocks.

Statistical treatment of the data generally supports the theory. It confirmed that not only the methodologies can provide substantially different estimates but also that these VaRs are dependent on the choice of time horizon, reference period and confidence level. The application

also confirmed that VaR estimates are more accurate in assessing market risk exposures under normal conditions and that they may exhibit greater failure rates than expected from their confidence level. Finally, it corroborated that shorter-time VaRs, parametric VaRs and higher confidence level VaRs are generally more accurate than others. Overall, even in a context of financial markets turmoil and economic depression, the VaR approaches did perform surprisingly well and provided accurate and reliable estimates of potential losses. This VaR application confirmed that the methodology is indeed a valuable market risk measurement technique that would apprise investors of blue chip stocks of their risk exposures and potential losses in a satisfying manner. In the end, the most interesting finding lies in the realization, that, had the VaR framework been discovered seventy years ago, the Great Depression might not have had such a tragic impact on investors in high quality stocks (and even maybe on the US financial industry)!

A ma Maman Chérie:
Merci pour mes ailes,
et au reste de ma Famille

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INTRODUCTION

Over the past two decades, important changes in financial markets have increased the risks incurred when trading and investing in capital markets. In effect, a combination of several factors has significantly modified trading and investment practices in the US and worldwide. Markets have become more complete, interrelated, liquid and thus more volatile and complex[1], but, as the markets became more "esoteric" they also became more "accessible" to all (two conflicting directions which do not facilitate the making of a sound financial system).

To capture the growing demand for private and corporate investment, financial institutions subsequently entered a period of accentuating profitability to the public and underemphasizing risk when marketing investment alternatives. In the mean time, as trading accelerated, a large portion of the financial industry did not keep up with the increasing sophistication and interplay of financial risks. The price to pay was a series of financial corporate disasters caused by improper risk assessment/management and financial market adverse moves.

This surge of dismal events highlighted the need to back up trading operations with adequate capital and to improve risk measurement and control systems and prompted the financial community and regulators to enter a period of focus on financial risks. Credit risk was the subject of scrutiny in the late eighties and then, in the nineties, market risk (the potential damages caused by changes in financial variables such as asset prices, interest and exchange

[1] For a more detailed account of recent financial market changes, see Exhibit I of the Appendix.

rates to financial assets profitability), became in turn the point of focus.

Thus, in recent years, the debate between regulators and financial institutions has been on institutional capital requirements, and the focus for financial experts/economists and institutions has been on the conceptualization and the implementation of a market risk framework (assessment and management). A new market risk measurement concept has emerged and been the point of convergence of the financial community's endeavor, in Europe and in the US: the firm-wide, cross-product, global concept known as Value-at-Risk, (VaR).

A VaR estimate is a probabilistic figure, which indicates, with a certain confidence, the worst loss, an institution stands to lose due to market risk. Statistically, a VaR estimate is a lower tail critical value on a portfolio profit and loss distribution. For instance, if a bank's daily VaR on its trading portfolio is calculated to be, say US$ 20 million, at the 99% confidence level, this means that this loss should not be exceeded more than 1% of the time on average, or on two to three days in a calendar year.

VaR is fast becoming an essential tool for assessing market risk but also for conveying trading risk to senior management and shareholders. The comprehensiveness and simplicity of the concept may explain its fast growing acceptance as a standard of market risk measurement. In effect some of the greatest qualities of the model are that (1) it provides a summary measure of a firm's potential worse loss due to its firmwide exposure to market risk (2) expressed simply as a dollar value, (3) which also indicates the probability that this loss will be exceed due to a more adverse market move.

Available literature on VaR is now abundant. However, it is generally very specific (publications usually focus on one particular aspect of the VaR framework). Only a marginal fraction of current publications is theoretical and addresses more than one aspect of the VaR framework. Available conceptual research deals with the modeling of the basic approaches of VaR and their variants. There is a general convergence toward three basic VaR methodologies, the historical, analytical and stochastic simulation approaches. Allen [1994], Wilson [1994], Leong [1996], Reed [1996] have provided interesting and in-depth comparisons of these methods with different levels of model specification and degrees of technical advancement. This is virtually the most comprehensive type of research available today.

Another point of focus of the research has been the limiting underlying assumptions built in the different VaR models and has led to the proposal of improved variants of VaR. Wilson [1993] proposes to replace the normal distribution by the t-distribution in parametric methods to improve on model forecasting quality. Paul-Choudhury [1996] presents two adjustments to VaR basic models to better cope with optionality in portfolios and to integrate the very damaging effects of market crashes in the VaR metric. Alexander [1993] highlights the dangers of using past returns correlations in VaR computations and present the corrective qualities of Garch models.

Others have explored ways of improving on some VaR tools or of complementing the current VaR models. For example, variants of the Monte Carlo simulation technique are presented in Brocheron-Ratcliffe [1994], Moro [1995] and Papageorgiou and Traub [1996]. Garman 1996] develops a model of marginal contribution to VaR, DelVaR, which provides risk managers using analytical VaRs with a simplified method that can be used to (1) assess, *ex ante*, the effect on a portfolio's VaR of adding changing the investment in a position, and (2) to rank comparable candidate trades based on their marginal effect on total VaR.

The bulk of the literature, though, is not about the VaR conceptual framework, but on VaR applications. The majority of publications on VaR deal with financial institutions' capital requirements, Capital-at-Risk, (CaR); promote the merits/discuss the limits of currently available VaR systems[2], or present a variant of VaR for nonfinancial corporates and relate the experiences of these industrial firms with VaR.

The general conclusions to emerge from all this literature are that: (1) Value-at-Risk is an increasingly popular market risk concept recommended by many regulatory and supervisory organizations, studied by an expanding number of financial economists and adopted at an increasing rate by financial and industrial corporations; however (2) to this day there is still no clear consensus of what VaR approach is best, each method has important restricting specifications or drawbacks which are still not completely understood by a large number of traders/practitioners, and (3) there is limited work published on the assessment of VaR models validity and of VaR measures accuracy and statistical significance, although there is, in this regard, a consensus that it is virtually impossible to test a model's accuracy, even in large samples, and that VaR estimates may be substantially biased by sample errors.

This paper makes two contributions to the existing literature. First, it develops a comprehensive theoretical framework of the Value-at-Risk concept and, second, it applies parametric and nonparamatric VaR methods to historical data and investigates, retrospectively, in a Great Depression context, their abilities to estimate future losses and even forewarn investors in blue chip equities of imminent dangers.

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[2] Different available methodologies are briefly presented in Exhibit I of the Appendix: JP Morgan's RiskMetrics, CF Bankers Trust's RaRoc 2020, Deustche Bank's dB-Analyst, CSBF' s PrimeRisk, Chase's CHARISMA, and C•ATS' CARMA.

Once the conceptual framework of Value-at-Risk has been analyzed, general VaR methodologies are applied to a portfolio of blue chip stocks during a sample of US history that includes the Great Depression. The data consists of statistics of monthly and weekly Dow Jones Industrial averages (DJIA) readjusted for stock changes, dividends and splits sampled from "Technical Analysis Explained, an illustrated guide for the investor"[3]. The reference period chosen for this study is 1920-1936. The empirical results generally support the theory and confirm that (1) VaR estimates are model-parameter dependent and are affected by the choice of holding period (longer-time VaRs do not aggregate well over time) and confidence levels; (2) that the approaches yield different estimates of potential losses; (3) VaR models may underestimate the frequency of unfavorable events. But the application also confirms, that in spite of its acknowledged weaknesses, the VaR methodology is a crucial invention for the financial industry: the empirical application revealed the comforting, yet unprecedented news, that, had VaR been known and implemented in the early twenties it would indeed have forewarned investors in portfolios of blue chip stocks of imminent losses and would well possibly have avoided many financial disasters and/or failures if measures to reduce risk exposures had been taken and/or prudential capital had been allocated following VaR guidelines.

This paper is organized as follows. First, the historical development of VaR is retraced. Then chapter II presents the modeling of VaR basic approaches to VaR and, their assumptions, methodologies and limits are compared in a theoretical framework in chapter III. Then, the fourth chapter, discusses the different corrections and improvements proposed for VaR. Chapter V deals with market risk model validity/accuracy verification testing techniques. The sixth chapter presents the empirical work of this paper in which several years of DJIA averages are

[3] Martin J. Pring, "Technical Analysis Explained, an illustrated guide for the investor", McGraw Hill,

used to implement an application of parametric and nonparametric VaR methodologies on a Great Depression background. Finally, concluding thoughts make up the seventh and last chapter.

1980.

Chapter I – BACKGROUND

I - Financial risks

Any situation that involves the making of decisions whose outcome(s) will take place in the future bears the possibility that these results will differ from what was expected: this variability in outcomes is the essence of risk. At the basis of any risk assessment are computations of changes in quantities or values of interest from what they were expected to be. The two most common points of focus are *risk exposure*, the amount at risk (or the potential loss), and *variability*, the pattern in the changes that indicates the likelihood of the risk.

Firms face a variety of risks which can be classified in three broad categories: *business risk* which comes from the strategic choices made in managing a firm, *strategic risk* which arises from the economic and political environment and *financial risk*.

1.1. The corporate concept of financial risk

In the corporate world, risk is generally synonymous with uncertainties in future earnings, and *percent* returns, such as changes relative to initial assets prices, are frequently used to measure risk exposures.

Financial risk can be divided into five classes: (1) *market risk* represents the hazards associated with being a participant in financial markets, and is generally triggered by the adverse effect of changes in interest rates, exchange rates or underlying asset prices, on security prices, (2) *liquidity risk*, the possible inability of a firm to sell assets in a timely transaction conducted at (close to) market price, (3) *credit risk* in a transaction is a counterparty's potential incapacity to meet its obligations, (4) *operational risk*, comes from internal human error or fraud that impairs a firm's revenues, and (5) *legal risk* which arises when losses occur as the result of a counterparty not having the legal or regulatory authority to engage in a transaction it conducted, or from lawsuits against a firm.

Movements in financial variables such as interest or exchange rates create risks for most institutions. The service industry or industrial corporations derive most of their revenues from the marketing of specific services or real assets, the price of which they generally set (or participate in setting) and, is fairly *predictable*, and their profitability depends on their ability to provide quality goods, assets or services. They are, therefore, affected by financial risk only in their treasury, if they conduct operations in foreign currency, or/and on their financial assets. The story is quite different for financial institutions. They derive most of their profits from selling standardized intangible assets whose prices are generally set through the interaction of millions of market participants, and on which they, thus, have no (or very little) control over. In this context, financial institutions' profitability lies in their ability to choose and manage portfolios of assets whose prices are determined outside the firm, and can be subject to large unpredictable swings. As a result, it is a vital requirement for such institutions to manage financial risk actively and efficiently.

In effect, in a world ever increasing competition and pressure (to satisfy more and more informed and demanding shareholders, and attract scarce capital), it is becoming essential for financial institutions to identify, quantify and control their risk exposures, in order to minimize losses, survive and outperform their competition.

Understanding risk enables financial managers to consciously plan for the consequences of adverse outcomes, to avoid, or at least, minimize losses and set capital provisions for losses, and by so doing, be better prepared to optimize revenues. Risk management, thus, involves the estimation of the frequency and size of the risks (the distribution of risk exposures), the setting of position limits through an asset allocation strategy, and the building of a capital cushion against potential losses, and constant control, to preserve the profitability and survival of the firm.

1.2. The underemphasis of risk

For decades, financial economists and scholars have researched, conceptualized and studied investment theory in terms of both risk and return, and the trade-off that generally exists between the two. This dual framework, however, still remains to be really understood and adopted by individual and corporate investors, as well as at the operational/trading and management/supervision levels of a large number of financial institutions. Traditionally, investors, traders, corporate managers and the media relied primarily on earnings to assess an investment's potential and measure corporate performance. Mutual funds, for example, are typically advertised to savers investors through reports of prestigious historical returns without offering the corresponding warning about potential losses

Although risk is a concern for many investors, perhaps because it is a reminder of potential negative events, notwithstanding that measures of risk are less easy to understand and less readily available than those of return, asset riskiness is often overlooked/neglected in investment performance assessment For instance, traditional measures of financial assets' "quality" or risk, such as security ratings or betas are generally less frequently reported in the media than earnings and less easy to understand than actual or expected returns. But consideration of risk is crucial in investment since it may wipe out the entire profit and even cause losses in even the most attractive and potentially profitable investment strategy.

At the corporate level, even when some risk management practices are implemented, some types or aspects of risks are overlooked. This underemphasis of risk or inappropriate risk assessment are still frequent. In the past years, many institutions have incurred financial losses as the result of insufficient risk consideration, and some of the most spectacular stories which report colossal losses of billions of dollars have been highly publicized in the media[4].

Finally, while for corporates most of the focus on risk has been on estimation of liquidity and credit risks, the lack of an effective standard, market risk measure and the absence of risk reporting requirements until now, have, with the other reasons mentioned above, contributed to maintain an under-emphasis of this dimension of investment analysis and made it easier for firms to elude risk concern.

1.3. - Traditional measures of financial risk

As mentioned earlier, the use of mathematical models to measure financial risk is not new, although today, they are much more sophisticated and ubiquitous. For years, modern fixed-income portfolio managers, for instance, have relied on analytical concepts such as duration, convexity and basis point value to measure their risk exposure to interest rate risk, the possibility that a change in interest rate will depress security prices.

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[4] A short summary of some of these cases are presented in Exhibit II of the Appendix: S&Ls, Metallgesellschaft, Orange County, Daiwa, and Barings.

The well-established basis point value measure, for example, is a deterministic risk measure which quantifies the change in the value of a bond as interest rate moves up and down by one basis point. However, it does not provide the expected value of potential losses, in other words, it cannot, for example, estimate the likelihood for a bond to lose one fourth of its value within a day.

Another problem with the older risk measures is that they tend to look at risk in isolation. Basis point does not incorporate the historical volatilities of interest rates and gives little information about the interactions of assets risks in a portfolio context. Risk, however, is a concept which is more meaningful in a portfolio context. In addition, simple linear measures of risk such as basis point, and duration are not always appropriate because they cannot be aggregated across assets[5]. The absolute riskiness of an asset, its returns variability on a stand alone basis, is of limited interest because seldom will a portfolio be made of only one single type of financial instrument.

Risk, is better assessed in a portfolio context and, of greater interest than stand-alone risk measures, are an asset's relative riskiness in a portfolio or its marginal contribution to a portfolio's riskiness. The latter is a function of the components of that portfolio. For this reason, the total risk of a portfolio is generally not the same as the direct summation of its components' risk, evaluated on a stand-alone basis. To be accurate, a risk measurement system has to account for all possible interactions between all components of a relevant portfolio.

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[5] Duration is the average life of a debt instrument measures as the weighted average of the present values of the payoffs of the assets and it has become more difficult to obtain a measure of market risk now that portfolios have become more complex due to great financial innovation of sophisticated instruments. This McKaulay metric shows that in reality a bond repays itself before maturity because of the coupon payments, (ΔP due to duration) = - (duration)(P)(Δyield), where P is the market price of the bond. Its is also the measure of the sensitivity of a bond price to changes in interest rates which is called convexity, (ΔP due to convexity) = (convexity)(initial price)(.5)(Δ in interest rate).

II - The historical development of VaR

2.1. - A new era of risk consciousness

For financial institutions, however, financial risk management has always been a preoccupation and more advanced than for corporate investors, perhaps because their survival is at stake and/or because their industry is highly regulated. In effect, certain types of risks, such as liquidity and default, have "always" been estimated by banks, but not necessarily really integrated in their investment practices. In recent years, however, the creation of more liquid securities, and breakthroughs in data processing and software sophistication have encourages more frequent trading. A corollary of these changes was the increase in the volatility of asset prices and returns which makes it not only a necessity to frequently revalue portfolios to assess performance ("marking-to-market"), but also to evaluate the risks of potential losses associated with being a participant in financial markets. Therefore, as banks started trading securities more actively[6], they also increased their market risk exposures and many of these in turn begun implementing a more thorough, more systematic and more prudential management of this financial risk.

The recent series of mammoth losses in some of the world's largest financial institutions[7], however, has contributed to turn things around. They have served as a tragic wake-

[6] Banks are more and more dependent on trading revenues, a list of financial institutions are presented with 6 to 51% of their revenues generated by trading. French banks and then Swiss banks lead the way. Banque Indosuez for instance reported FF. 6.458 of trading revenues n 1993, that is 49% of their total revenues. Banque Paribas was the second bank with FF. 13.923, that is 45% of total revenues. The leaders in the US were Bank trust (US$ 1.631 and 36% of revenues and JP Morgan with US$2,059 or 32% of revenues. The leading US brokers were Salomon with US$1.716 and 41% of revenues, Morgan Stanley 1,617 and 39% and Lehman Brothers 1.967 and 38%) Source Standard & Poor's. Chew . "Shock Treatment". *Risk Magazine*. September 1994. pp.63-70

[7] A brief presentation of some of these financial disasters is presented in Exhibit II of the Appendix.

up call and established, beyond doubt, the vital importance of a systematic approach to corporate market risk management, control, reporting and disclosure. In the face of such poor (or lack of) risk management and control, government regulators and oversight agencies -the Bank for International Settlements[8], the Federal Reserve Bank, the Office of the Comptroller of the Currency, and the SEC and others- have drawn up new requirements for corporations and financial institutions to calculate, report, and disclose their market risk exposure, and to build capital charges against them, using the modern techniques and mathematical models recently developed by financial.

Private and corporate investors and financial institutions, are now paying more attention to risk. Until 1991, bank regulators and financial institutions focused on credit risk. Once this type of risk was better regulated and controlled, the attention shifted toward market risk. The objectives were to decide how to (1) measure the hazards inherent in financial markets and security trading, how to (2) incorporate them in the prudential management of an institution and to (3) allocate proper capital charges against it.

2.2. The emergence of VaR

There has been a noticeable convergence of interest on a specific market risk measurement technique developed recently: Value-at-Risk. In the early nineties, following regulatory recommendations, banks started assessing the capital they stood to lose in security

[8] The central banks of the Group of Ten countries reached an agreement in Basle, Switzerland, in July 1988. They cosigned a landmark accord toward greater and systematic prudential protection against credit risk. They defined the Cooke ratio, the 8% capital requirement against risk-adjusted bank assets and off-balance sheet bank trading activities in the derivatives markets, to be fully implemented in 1993 by all insured banks of signatory countries. The agreement also limited large risks, positions of at least 1% of a bank's capital and required greater disclosure of a bank's risky activities.

trading due to adverse moves in financial markets: they called that potential loss their Capital-at-Risk (CaR). It is the probabilistic estimation of the magnitude of the losses a financial institution stands to lose on a daily basis, due to its exposure to adverse market moves and should be covered by capital provisions to absorb actual losses and prevent failure.

In 1993, the Basle Committee on Banking Supervision issued a first proposal referred to as the "standard model" approach recommending to allocate prudential capital calculated on a specific market risk estimation technique called Value-at-Risk. Then, it reviewed and reissued a new proposal, the "internal model" approach allowing some financial institutions to use their own VaR methods to calculate capital requirements, provided that they respect the committee's guidelines (99% confidence, 10 day-holding period, at least one year of data). The Federal Reserve recommended that banks precommit to penalties should their cumulative losses exceed their cumulative VaR on a quarterly basis.

In subsequent years, reporting of additional financial losses maintained risk management concern in the lime light and prompted additional research on the VaR methodology. The result was the emergence, from the exclusively banking industry CaR practice, of a new - more comprehensive yet more refined - version with increased versatility (extension to any financial institution and multiple applications): Value-at-Risk. During that period, JP Morgan and other financial institutions launched their own market risk management systems. The methodology gained attention and popularity and an increasing number of institutions and financial managers began implementing some type of VaR approach. VaR has now become one of the key concepts of modern risk management and is today the acronym of market risk measurement.

2.3. - The VaR framework

As distinct from the conventional financial risk measures, the new approach to market risk measurement, VaR, incorporates what is known about the historical volatilities of asset prices or rates, and the historical correlations between different financial instruments. To avoid the pitfalls of partial analysis, the VaR methodology attempts to measure risk in a comprehensive and integrated manner, aggregating all possible interactions between all relevant assets. This represents the real innovation of VaR: this quantification of firm-wide, cross-product market risk exposure. Two important by-products are the extensive use of modern and sophisticated statistical techniques in the risk measurement process of many corporates, and the forming of probabilistic statements about the likelihood of various degrees of potential loss.

As explained earlier, the VaR concept originated in the banking industry as CaR. VaR is a measure of market-based financial risk which is used to place a probabilistic limit on losses which result in a portfolio as the result of movements in financial prices. VaR measures the worst expected loss of a portfolio, over a given time horizon, under normal market conditions, at a given confidence level. "Risk exposures are quantified in terms of a VaR, a specific critical value of a portfolio's potential one-day profit and loss (P&L) distribution that is large enough so that the probability that the portfolio could post a larger loss is at most some specified value, like 1% or 5%. A VaR measure thus corresponds to a specific left-hand critical value of the portfolio's potential P&L distribution"[9] VaR is an effective tool because it assesses risk in dollar amounts of a common metric notional losses relative to a standard unit likelihood. It can be used to compare and aggregate risks across instruments types and markets

[9] Kupiec, "Techniques for verifying the accuracy of risk measurements models", *The Journal of Derivatives*, pp.73

The concept behind VaR is very simple: a portfolio is valued based on the current value of the market factors that determine the price of its asset components, and then revalued at different periods in the past. Changes in the portfolio values are calculated and sorted in a distribution curve (the profits and losses, P&L, distribution). VaR is a negative change, or loss, such that 99% (or 95% ,etc.) of the rest of the changes in portfolio values are greater or that a loss even greater can occur only 1% of the time. The actual VaR computation is more complex because a relationship has to be established between the market factors and the portfolio, to price it.

There are three basic methods of computing VaR, all three uses the historical values of the market factors over a reference period. The historical method directly prices the portfolio across time and calculate changes in its values, the analytical method assumes that the market factors are normally distributed and aggregates the portfolio values based on chosen pricing models, the simulation method prices the portfolio based on randomly generated sample paths of market factors and calculate the VaR from the distribution of simulated changes.

2.4. VaR applications

The VaR concept has become a popular reporting tool because it is a risk measure which is easy to intuitively understand, even by "non experts" of the field of risk assessment. In addition to its appealing user-friendliness, another reason for the huge success of VaR is the numerous purposes it may serve.

2.4.1. - Internal uses of VaR

Financial institutions use VaR, at different levels of the firm's organization and hierarchy, to (1) measure market risk exposure(s), to (2) allocate resources, to (3) control trading operations and to (4) report to senior management and shareholders.

VaR is used at the *trading desks* or in the trading units of financial institutions to evaluate the riskiness candidate positions. Daily analytical VaRs are generally used to provide real-time estimates of portfolios' market risk exposures as changes occur in market risk factors and/or in the portfolio's composition. At a higher level, financial managers of the trading units calculate the total VaR of their respective portfolios and follow its evolution. These portfolio VaRs can then aggregated to yield the firm-wide VaR (which can also be estimated from the firm's global portfolio) and, in general, more sophisticated methods are used for these portfolios' and institutional VaRs.

VaR is also used to decide among candidate trades, on a daily (or less frequent) basis when changes are going to be made to a portfolio. An *incremental* VaR can be computed to assess the marginal contributions of proposed asset positions. At a higher level of organization, managers also decide how risk capital is deployed across units (or portfolios) on a risk-adjusted return basis, according to the profitability of these units relative to the VaR they exhibit.

In summary, VaR is used to control risk taking of traders, evolution of the riskiness of the portfolios, verify the proper assessment of risk exposures, set positions limits, assess portfolio composition changes, compare the actual riskiness to the objective of the firm, etc. This control is conducted by financial and/or risk managers and has to be consistent with the institution's entire strategy and the most sophisticated and precise VaR techniques are generally used at this level of decision making. These results are then generally aggregated, summarized and simplified to report to senior management and disclose information to shareholders.

2.4.2. **Financial system soundness**

As mentioned earlier, after a series of market adverse moves and an increasing number of financial disasters related to market risk, financial regulators and organizations have rallied on the decision that, to protect the soundness of the international financial system, measures had to be taken, one financial institution at a time, to implement a comprehensive risk management system, and that prudential capital should be allocated to guard against adverse market moves. There has also been convergence toward using VaR to estimate market risk exposures (including those related to holding derivative instruments) and set capital requirements.

The premise is that financial institutions should become more concerned about (market) financial risk and learn to manage it better. An appropriately used VaR should enable management to decrease the required hedging by decreasing risk exposures and the capital charges should be wiped out on a regular basis, for instance, on average twice or three times a year when VaR is calculated at the 99% confidence level, but protect firms for failure. In addition, to make sure that the capital cushion would be enough to withstand unexpected larger adverse events, the capital set aside should be some multiple of the capital at risk directly calculated from VaR.

The European Community in 1993, the Bank of International Settlements in 1996[10], the Group of Thirty Derivatives, the Stock Exchange Commission, the Federal Reserve, to quote

[10] A brief presentation of the EC-CAD and of the Basle Committee's final amendment are presented in Exhibit III of the Appendix.

only a few, have all issued recommendations promoting a greater awareness of market risk that suggested to use VaR to set financial institutions capital requirements and to increase disclosure of market risk exposures.

2.4.3. - **VaR for corporates**

Finally, although VaR is developed in a context of financial (market) risk management an important feature of this methodology is that it provides a firm-wide measure of risk. In that respect, the concept can be extended to the risk management of any business enterprise because centralized risk management is useful to any firm. Indeed, a growing number of industrial firms are now implementing variants to the original VaR for other type of risk, financial or otherwise.

VaRs for nonfinancial corporates, takes a wide variety of forms, because of the innumerable possibilities. Thus, no specific methodology has been developed, so far, but the main principle is the same. In addition, for nonfinancial corporates, there is some unease with traditional VaR methodologies because their future cash flows are not *marked-to-market* and sometimes their assets may not be traded. However, while financial managers are more interested by VaR in a capital adequacy context, corporates maybe more interesting in reducing the volatilities in their treasury, or in their sales revenues.

Dow Chemical Company and Mobil Corporation use the same type of approach derivatives dealers implement to monitor the foreign exchange and interest rate risk exposures embedded in their debt and derivatives portfolios. PepsiCo, for example, uses VaR techniques to evaluate its exposure to credit risk. Other institutions have been using VaR models to evaluate the risk of fluctuations in input costs and/or demand impose on operational revenues, the influence of changes in foreign exchange on their foreign currency-denominated operational cash flows or on their equity investments in their foreign subsidiaries[11].

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[11] Risk exposures are sometimes computed from elasticities, for instance, η=[%Δ of expected future values / %Δ in financial price]. Cash flow at risk, CFaR = Δ in Domestic Sales = [1+ (η¥/$ of domestic sales)x(%Δ¥/$)$_t$ + ($\eta i_{int.\ rates}$ of domestic sales)x(%Δint. rates)$_t$] x (domestic sales)$_{t-1}$, where η denotes an elasticity. They also calculate VaR as the difference between the expected value (present value) of future cash flows and the worst case scenario (which shows the minimum attainable level of revenues, VaR is thus the fluctuating level of cash flows (the difference between what is the guaranteed minimum and the possible achievement).

Chapter II - VaR modeling

I - Introduction

Three basic VaR methodologies and several variants have been developed by financial economists and adopted by banks and securities houses, and until this day, there is a lack of consensus on various topics pertaining to VaR. (1) For some institutions, the VaR metric represents the absolute dollar loss (estimated relative to the portfolio actual initial notional value)[12], while for others it represents the worst loss relative to the expected value of the portfolio. (2) The most appropriate or simpler methods to calculate VaR are a matter of controversy. (3) Finally, opinions also differ on the importance and consequences of some of the simplifying assumptions underpinning VaR computations.

Typically, VaR models rely on the past being a guide to the future. They tend to build around statistical estimates, using probability distributions for the changes in value of a given portfolio over a specific holding period (but there again, is no consensus among market practitioners or regulators as to how to generate these distributions).

There are three basic VaR methodologies but only tow fundamentally different approaches. The first one consists in reading VaR off a distribution, as a critical value. The

[12] It is the VaR value generally used by managers of trading operations and treasurers and favored by regulators. This absolute measure of market risk shows the potential loss in a position (or portfolio) irrespective of the average performance the position The relative measure of market risk, on the other hand, is favored by investment managers interested by portfolios performance. For them, an absolute VaR of say, of $9.6 million still represent an important loss but is smaller compared to an average performance of $2 million in profits (relative VaR of 2 - (-9.6) 11.6), versus an average profit of $5.8 million (relative VaR of 5.8 - (-9.6) 15.4 million) Relative VaRs measure worse potential losses in terms of a given average profit.

general (historical or empirical) distribution approach relies on the actual variations in a portfolio values (P&L) to estimate VaR directly. It can be refined or increased in sophistication in two methods referred to respectively as *historical* and *stochastic (Monte Carlo) simulation*. The former revalues the portfolio under permutations of the empirical path, while the latter uses randomly simulated paths for the revaluations. The other approach is the *statistical* (*parametric* or *model-based*) approach. It involves forecasting risk exposures (potential losses) by estimating the true portfolio's return distribution using known probability distributions and statistical models.

From a survey administered to risk management system suppliers, it appears, generally, that to their clients (the practitioners): (1) the simpler empirical method is mostly used to report to shareholders, (2) the parametric method is used to conduct rapid real-time checks at the trading desk, set capital requirements and to report to back-office controllers, senior management, shareholders and regulators while (3) the more sophisticated simulation approach is used to highlight defects in the parametric approach and is favored in the front office for assessment an control of trading risk exposures where an accurate measurement of riskiness is crucial[13].

In its most general form, VaR can be derived from the probability distribution of the future portfolio value *f(Vp)*, where *f* represents the probability density function of portfolio value Vp. At a given confidence level $(1-\alpha)100\%$, for instance, at 95% confidence α is 5%, the worst possible realization Vp_{MIN} is calculated such that the probability of exceeding this value is:

$$(1-\alpha)100\% \quad (1-\alpha)100\% = \int_{Vp_{MIN}}^{+\infty} f(Vp)\,dv_p.$$

[13] Marshall and Siegel. "Get it into your system". *Risk Magazine*. VaR supplement. June 1996. pp.26-32

or such that the probability of a value lower than Vp_{MIN}, $p = P(Vp \leq Vp_{MIN})$, is $\alpha(100\%)$:

$$\alpha(100\%) = \int_{-\infty}^{Vp_{MIN}} f(Vp)dv_p = p = P(Vp \leq Vp_{MIN}).$$

In other words, the area from $-\infty$ to Vp_{MIN} must sum up to $\alpha(100\%)$, for instance 5%. Vp_{MIN} is the *sample quantile* of the distribution. This relationship has the advantage to be valid for any distribution, discrete or continuous, fat or thin tailed.

II - General methodology

Calculating VaR is not quite as simple as the concept may lead to believe though. Basically, the difficulty arises, not so much from the necessity to obtain a series of listing of past changes in market factors, but from the calculation of the corresponding portfolio values. First, the current portfolio is valued using the current *price list* (or values of the market factors) of all its components[14]. Then, the current portfolio is revalued, using an *alternative price list* of the values of the factors at different points in time, and the changes in the portfolio value (the differences in the value using the current and alternative price lists) are computed. Given this, VaR is specified in terms of confidence levels: the financial manager can calculate the maximum the institution can lose over a specified "time horizon" at a specified probability level. For instance, the maximum loss for a one-day period at a 95% probability can be defined as the loss that should be exceeded on only 5 days out of 100 (if the VaR is calculated on a daily basis).

[14] For example, the market factors that affect the value of a bond denominated in a foreign currency are the term structure of the foreign interest rate and the exchange rate.

2.1. VaR for empirical distributions

The following explains the basic methodology used in the historical and simulation approaches. To compute the VaR of a portfolio, let: Vp_0 be the initial investment, R_t the rate of return from time t-1 to time t, μ the average historical return on that portfolio and σ the actual volatility of the portfolio returns over a given reference period. Then, the portfolio value at the end of any period t, the target horizon is

$$Vp_t = Vp_0\,(1+R_t)$$

Let also the lowest portfolio value at the given confidence level $(1-\alpha)100\%$ be:

$$Vp_{MIN} = Vp_0\,(1+R_{MIN})$$

VaR can be defined in two ways: (1) as the dollar loss relative to the average portfolio value:

$$VaR_M = E(Vp) - Vp_{MIN} = VaR_M = -Vp_0\,(R_{MIN} - \mu);$$

(2) or as the absolute dollar loss (that is, relative to zero or without reference to the mean):

$$VaR_/ = Vp_{MIN} = -Vp_0 R_{MIN}$$

In both cases, finding the VaR is equivalent to identifying the minimum value Vp_{MIN}, or the cutoff return R_{MIN}.

The following is an application of this process for a daily VaR . Let μ = $5.1 million and let the reference period be 255 days. Then, Vp_{MIN} at, say the 95% confidence, must be such that the number of observations to its left is 255x5%=12.75, or approximately 13. If there are say, only 11 observations to the left of -$10 million, and 15 to the left of -$9 million, interpolating these values yield Vp_{MIN} = $9.6 million. The daily VaR of that portfolio relative to its average value is: $VaR_M = E(Vp) - Vp_{MIN}$ = $5.1million - (-$9.6million)=$14.7million.

The VaR in terms of absolute dollars is: Vp_{MIN} = $9.6million.

2.2. VaR for parametric distributions

The VaR computation can be considerably simplified if the distribution can be assumed to be normal, or approximately normal. When this is the case, the VaR figure can be derived directly from the portfolio standard deviation, using a multiplicative factor that depends on the confidence level. This approach is sometimes called *parametric* since it calculates VaR from the estimates of parameters of the portfolio P&L distribution, the mean and standard deviation of returns, instead of just reading it as a quantile, off the empirical distribution.

To compute the VaR of a specific financial asset (or portfolio), the periodic changes or returns are assumed to be identically and independently distributed[15] (*iid*) so that, VaR can then be derived, at say, the 95% confidence level, as 5% left-side *losing tail* critical value of the P&L density function.

First, it is necessary to transform the general P&L distribution into a standard normal distribution, $\Phi(\epsilon)$, where ϵ has mean of zero and standard deviation of unity. Vp_{MIN} is still associated with the cutoff return R_{MIN} such that:

$$Vp_{MIN} = Vp_0(1+R_{MIN}).$$

In VaR calculations, R_{MIN} is negative and so is the standard normal deviate $Z_{\alpha/2}$ associated with the confidence level underpinning R_{MIN}

$Z_{\alpha/2} = [R_{MIN} - \mu]/\sigma$, which is 2.33 for 99% confidence and 1.96 for 95%.

Using the standard deviate is equivalent to setting:

$$\alpha(100\%) = \int_{-\infty}^{Vp_{MIN}} f(Vp)\,dv_p = \int_{-\infty}^{R_{MIN}} f(R)\,dr = \int_{-\infty}^{\alpha} \Phi(\epsilon)\,d(\epsilon),$$

[15] This assumption has empirically been invalidated: existence of volatility cluster that denote heteroscedastisity and of autocorrelation for squared changes in returns.

Thus the problem of finding VaR is equivalent to finding the deviate $Z_{\alpha/2}$ such that the area to the left of it is equal to $\alpha/2$. The cutoff return R_{MIN} and VaR can then be calculated from $Z_{\alpha/2}\,\sigma$. The cutoff return (from the deviate equation) and the VaR are:

$$R_{MIN} = -Z_{\alpha/2}\,\sigma + \mu,$$

$$VaR_M = E(Vp) - Vp_{MIN} = -Vp_0(R_{MIN} - \mu).$$

When VaR is defined as an absolute dollar loss:

$$VaR_Z = Vp_{MIN} = -Vp_0 R_{MIN} = Vp_0\, Z_{\alpha/2}\,\sigma$$

This method generalizes to other probability distributions than the normal, as long as all the uncertainty is contained in σ. Other distributions will entail different values of the critical value $Z_{\alpha/2}$. The normal distribution is just particularly easy to deal with, since it adequately represents many empirical distributions. This is especially true for large, well-diversified portfolios but certainly not for portfolios with heavy option components and/or exposures to a small number of financial risks.

For greater variety in time horizon, the parameters μ and F can be expressed on an annual basis or on a different periodicity. The time interval considered is Δt where t is expressed in years. Replacing this in the original equation yields:

$$VaR_M = -Vp_0(Z_{\alpha/2}\,\sigma\sqrt{\Delta t} - \mu) \text{ and } VaR_Z = Vp_0(Z_{\alpha/2}\,\sigma\sqrt{\Delta t})$$ [16].

III - Choice of models parameters

Measures of VaR depend on two arbitrarily set variables: the selected time horizon and the confidence level which defines α.

[16] This extrapolation is highly debated. The next subsection on time horizon, and later, the problems of the linearity assumption and options discussed in Chapter IV will clarify the problems with this extrapolation.

3.1. - Time horizon

There are two levels of time periods that be must be determined in any VaR methodology, and both are somewhat arbitrary decisions: the reference and the holding periods. The holding period, or *time horizon*, dictates the frequency with which to calculate the changes in value used for the VaR metric. The observation or *reference* period is that from which the observations are extracted. It conditions the level of information captured in the VaR measure. The choice of reference period used to measure the behavior of prices is a balancing act between increasing the sample size to capture a full variety of events and relationships between factors, and selecting a period which is too long. In effect, the events and relationships observed may no longer be representative of what can be expected in the future.

The holding period used to calculate VaR is function of the turnover of the instruments composing a portfolio. It is the longest period necessary to liquidate a portfolio. The speed to modify a position is itself a function of the nature of the instruments (for instance, standardized instruments are more easily traded and more liquid than exotic ones), the size of the position (the notional value) and the market condition (for instance, in times of stress, liquidity of the markets is decreased). In brief, it is a function of liquidity.

Commercial banks, for example, currently report their trading VaR over a daily horizon because of the rapid turnover in their portfolios. In contrast, investment portfolios managers such as pension funds managers, adjust their portfolios only slowly, which is why they generally choose a one-month or even quarterly horizon. In any case, the holding period should be related to the liquidity of the securities, but, risk managers should bear in mind that, during times of market stress, market liquidity evaporates, and there is no or little opportunity for stop-loss trades, for instance, and even if there was, implementing stop-loss strategy may be counterproductive and exacerbate the situation. When thinking ahead and assessing worse case scenarios, this temporary illiquidity must be considered[17].

Many practitioners using daily VaRs have protested the 10-day-holding-period requirement in the Basle amendment advocating that a longer time horizon does not represent more information but is just a fudge factor that unnecessarily inflates the reported VaR and the capital requirement set against it (because the longer the time horizon, the higher the VaR). They argue that a daily VaR is better suited for their trading practices, risk profile, risk management and objectives. However, the committee's stand is that, although daily VaRs are certainly appropriate for trading exposures and model validity assessments purposes, they are not sufficient to ensure the long-term solvency of an institution.

Another point of controversy has been on the allegation that daily VaR could simply be aggregated to longer period VaRs. This only yields an approximation, however, because longer-holding-period VaRs cannot be extrapolated from daily VaR, at least not when the portfolio contains derivatives because of the nonlinearity of option risks[18]. Some users also insist on the fact that the longer-time VaR could well be below the daily VaR scaled by the square root of the time period because of the offsets that may occur in the period. In any case, for longer time horizons, to obtain a more accurate and reliable result, the VaR metric should be entirely recalculated[19]. In addition, the endogeneity of trading risks makes evaluating longer than daily risk exposures complicated because risk exposures are frequently changed by management trading practices and schemes designed to reduce/adjust risk taking. When daily VaRs and risk

[17] For greater details, see Chapter IV, stress testing.
[18] Bond prices are not really linear in their payoffs either but to a much lower extent than options.
[19] Or at least the variance/covariance matrix in the analytical approach, as presented later.

exposures are extrapolated to longer time periods this endogeneity is ignored.

In general, piecemeal measures of risk exposure do not aggregate into a representative measure of a bank's total market risk and, the sum or average of these piecemeal risks is generally greater than total risk, if there are offset among risks. For longer-period VaRs, two methods are frequently used because of their speed: (1) multiply daily VaR by square root of the holding period, or (2) calculate returns using a longer holding interval. To be valid, these methods only require the returns be *iid* and that the weights of the assets returns in linear pricing models are independent of the changes in primitive asset values. And these are not true in reality. And, the longer the holding period the daily VaR is extrapolated to, the more inaccurate this extrapolation.

3.2 . - Confidence interval

Fewer guidelines are available for the choice of confidence level, though. VaR users have generally set different levels of confidence. For instance, Bankers Trust chose 99%, Chemical and Chase 97.5%, BankAmerica and JP Morgan 95%. The confidence interval shows the level of precision and protection the firm is seeking. The rationale when using small tail probabilities, or high confidence levels, to calculate VaR and then allocate capital charges, is that the estimated worse loss is, in theory, unlikely to be exceeded.

Risk aversion as well as the cost of a loss greater than VaR must be taken into consideration, when choosing a confidence level. Higher aversion and higher cost imply a greater amount of capital to cover possible losses, and call for higher levels of confidence and thus higher VaRs. The purpose VaR is used for, also conditions this choice. If VaR is calculated simply to provide a company-wide yardstick to compare risk across different markets, confidence levels are not crucial, and if, in addition, a normal distribution is assumed for returns, it is easy to convert disparate measures of risk exposure (internal VaR versus "Basle VaR, or VaRs from different units/products) into comparable metrics[20].

It is also important to choose a confidence level that allows model validation and enables users to check their VaR estimates on a regular basis, and with high confidence levels, such as 99%, it is virtually impossible to detect an inaccurate VaR model from a good one[21]. Some users think that the higher the confidence, the greater the safety, since VaR would then, be rarely exceeded (the higher VaR metric would be exceeded less often). For instance, a 99% confidence level also means that due to chance only, a portfolio can experience a loss worse than VaR 1 day out of 100, but it also means that a risk manager would have to wait 100 days before he could confirm that the model conforms to reality. Therefore, a higher the confidence level *also* means that it is more difficult to assess model validity.

First, the higher the confidence, the more infrequently will VaR be exceeded, and this rarity implies that it takes a long delay to verify model accuracy, since for instance, as many as 100 periods may have to pass before one failure, a loss greater than VaR, is recorded. Second, when a sufficient sample has accumulated, the higher the confidence, the more difficult it is to assess whether a model is really accurate, or if the firm has just been lucky. In other words, even

[20] It is possible to convert risk measures based on different confidence levels and time horizons into comparable to compare relative/total resin exposures. For instance, the RiskMetrics VaR may be converted into the Basle Committee VaR.. RiskMetrics provides a 95% confidence interval (1.65F) over one day. The Basle proposal defines a 99% confidence interval (2.33F) over 10 days. The adjustment takes the following form: $VaR_{BC} = VaR_{RM} (2.33/1.65)$ o10= $4.45 VaR_{RM}$.This adjustment is valid only when positions are assumed constant and the portfolio does not contain options.

[21] See chapter V for greater details on the verification testing procedures of VaR Models.

if fewer violations than expected have occurred, this may simply be due to chance. As a consequence, a 99% confidence-computed VaR, although it calls for a higher prudential capital charge, cannot be used to verify the accuracy of the model

The choice of confidence level is also a cause of controversy. Regulators require high confidence levels but some institutions insist that higher confidence levels are also a fudge factor yielding higher VaRs and capital requirements which are not justified since they do not protect the firm in case of market turmoil.

There is a conflict of interest here: VaR users generally implement the methodology to conduct frequent, routine market risk measurement assessment and control and are mostly concerned by regular market conditions whereas rare events are those at the center of risk managers' and controllers' concern, since they are more interested by the outlying events which may jeopardize financial institutions' survival and the financial system soundness.

Instead of setting large capital provisions, some users advocate, risk taking can be revised on the basis of VaR and other measures such as stress testing can be used to deal with abnormal situations. Ultimately, though, even if prudential capital allocations based on 99% confidence VaRs will not withstand the blows inflicted in times of important market stress they are more likely to do so than lower confidence VaRs and in addition, afford greater protection to the institution on average[22].

Some VaR users have rejected these arguments since for them, even at 99% confidence, it is impossible to know: (1) the probability, (2) date, (3) frequency, or even (4) magnitude of future market crashes or losses larger than the first or fifth percentile, and since VaR models

[22] The results of the empirical treatment show that, sometimes a 99% VaR can even fully protect an institution under very stressful market conditions.

accuracy can absolutely not be verified at that level, although a very risk adverse user would have probably opted for that confidence level, it seems a better choice to use 95% confidence and conduct scenario analysis and stress testing to evaluate worst case scenarios, that the traditional VaR methods derived for normal market conditions cannot show. There is not really anything to gain by using the 99% confidence level, and accuracy testing abilities to gain by using 95%.

Presumably, the Basle Committee chose a 99% level to reflect the trade-off between the desire of regulators to ensure a safe and sound financial system (as increasing the information to derive VaR) and the adverse effect of capital requirements on bank returns. But, higher confidence levels do not necessarily guarantee safety since, in reality it is probable that VaR will be exceeded and that, even more frequently than expected.[23], and since when markets crash, for instance, VaR estimates may be exceeded dramatically whether calculated at 99% or 95% confidence. On the other hand, higher confidence levels provide better protection for normal market conditions since they imply higher VaR figures, and the purpose of calculating VaR is, after all, to monitor risk exposures under normal conditions.

[23] For more information, see VaR performance and accuracy in Chapter V.

Chapter III - The different VaR approaches

The three basic methods used to calculate VaR are presented in this chapter, in order of the increased complexity of their implementation: (1) the historical or empirical approach and its variant the historical simulation, (2) the analytical approach also known as correlation or variance/covariance method, and (3) the stochastic or Monte Carlo simulation.

I - The historical approach

This method is also called the empirical approach because it uses the actual distribution of the daily (or monthly) P&L of their component assets.

1.1. - General principle

To compute VaR by this approach, all that is needed is a historical record of the past daily (or weekly) P&L of the portfolio assets from which an empirical distribution of the periodic actual portfolio P&L can be reconstructed. This method contains minimal analytical requirements since no valuation model is required to compute the VaR metric[24].

The process starts with the recording of the portfolio's assets values and the calculation of their successive changes for a specific time horizon, over a given historical period. If, for example, a daily VaR is to be obtained using say, the past previous 101 days, each of the assets will have a vector of 100 observed changes. In each time period, and for each asset, a vector of alternative values is created by adding the current value of the asset to each vector of changes.

[24] If market factors and not assets prices are used, then a valuation model must be used to arrive at the portfolio value.

The 101 portfolio values are found as the weighted value of notional positions in the different assets, for each day. Then, changes in the portfolio value, between succeeding days are calculated. Finally, these changes are sorted in ascending order and the VaR is determined based on the desired confidence level. For a one-day, 95% confidence level, the VaR would be the 95th most adverse change in portfolio value (or more directly the fifth largest loss in portfolio value).

A variant to this approach consists in using a pricing model to compute the portfolio values from the portfolio market factors, building a distribution of the aggregated changes in the market factors and then calculating the VaR from the relevant critical value by using the portfolio pricing method, or simply expressing the VaR in terms of percent changes instead of dollar value.

1.2 - Advantages of the historical method

Its most compelling advantages are that it is powerfully simple and the use of the empirical distribution is user-friendly[25]. Another appealing aspect is its lack of theoretical baggage since it makes no assumption about the volatilities of the portfolio' assets or about the shape of the returns' distribution. It is free of model risk unless a pricing model is used to compute portfolio values.

Besides being extremely simple, the historical approach has the additional benefit of being realistic. Since actual daily P&L fluctuations are used, the true volatilities and correlations are used and, the outliers (such as abnormal losses that result from markets' crashes), they may

[25] Many financial institutions, including JP Morgan and Merryll Lynch, have used this simple method to communicate in annual reports the risk profile of the company in their to their shareholders, although they use more sophisticated VaR methodologies in their risk management systems.

contain are captured and not averaged out[26]. As a result, this approach accurately reflects the stress experienced by a portfolio during chaotic periods such as on the Black Monday or during the 1992 breakdown of the Exchange Rate Mechanism, (ERM).

An additional benefit of the historical method is that it does not require any simplification (for instance, no cash flow mapping is necessary). Thus the substantial distortion that these processes may create, especially for portfolios with sizable optionality, is avoided. The historical approach, by using actual P&L, circumvents this problem.

1.3. - Limits of the historical method

This method weights all past observations equally, and by giving the same importance to recent changes as to older ones, it ignores the dynamic ordering of observations. As a result an outlier, because of its size and irrespective of its date of occurrence may dominate other observations in the VaR magnitude, causing "plateau" effects in the VaR as long as it remains in the window of reference and then causes a sharp decrease in VaR when it is dropped from the observation period.

In addition, if too many rare events and crashes occurred during the observation period, they may obscure the day-to-day benefits of VaR by unduly distorting the distributions. This is not practical since a chaotic period is not more likely to be followed by another one. Thus, the resulting VaRs may be artificially and uselessly inflated and call for higher prudential capital charges although the storm may have been weathered. On the other hand, the fact that the method ultimately relies on only a few observations (those around the quantile), it may also

[26]As they might be in the analytical approach, as invoked in the next chapter.

understate true risk exposures by systematically ignoring most of the worse losses.

This method also only allows one sample path of possibilities, one history of evolution of asset prices/market risks, albeit the true history, and therefore predict VaR in a restricted manner. Last but not least, because this approach is based strictly on past data and uses the constitution of the current portfolio, the VaR produced may no longer reflect the current situation. This aging bias exists even if the portfolio mix does not change over time. In effect, over a long time horizon, the shortening of maturities of debt instruments and the closing in of the derivatives' expiry substantially modify the risk profile of the portfolio.

Finally, the empirical approach is not particularly useful for scenario analysis[27] (unless it relies on an analytical valuation model to calculate portfolio values from the market factors). Adverse movements cannot be simulated since only actual assets prices are used and no distribution of the underlying factors has been identified that can be reproduced in different scenarios to prepare a better damage control strategy

1.4. - Historical simulation

This variant is more involved than the simpler original historical method. Instead of relying on a history of portfolio P&L derived from assets prices, a historical simulation relies on a historical record of a set of the basic market factors which determine the portfolio value. Using the current portfolio composition and the historical market data, it is then possible, to simulate, *ex post*, the portfolio value at any point in the past. This simulated history can then be used to construct an "empirical" distribution of the portfolio P&L and to derive the associated

[27] In the analytical approach, for example, volatilities and correlations could be changed to see the effects on VaR, here this is not possible.

VaR: just as in the empirical approach VaR is read directly as a quantile off the portfolio P&L distribution[28], only this time a simulated one.

For each of the market factors, a vector of alternative values is created by adding the current value of the market factor to each of the values in the vector of simulated changes. One approach to obtain simulated changes consists in using the range of the changes in the market factors (or the range of the changes in the assets prices) over the historical period to create a multipoint sensitivity for each factor and shock these sensibilities by different levels of the factor prices[29]. Another method is to permute the different values of the market factors to generate new, simulated, paths. Once this vector of alternative market factors is obtained, the current and alternative values for the portfolio, the changes in the value of the portfolio, and the VaR are calculated by exactly the same method as in the historical approach. The VaR can be determined by listing out all of the outcomes in ascending order and cutting off at the desired confidence level. That cutting off point, or critical value, represents the VaR.

This method is the VaR approach the most commonly used and recommended by regulators[30]. It makes very few assumptions about the market price processes generating the portfolio returns. It simply assumes that the past is representative of the future and that market

[28] Or off the distribution of the VaR, since alternatively, instead of building a portfolio simulated P&L distribution, that of the VaR obtained from each simulated P&L distribution can be sorted in its own distribution, directly.

[29] Simplifications can be used to avoid a full simulation each time, for instance by using bands for the range of changes in price factors instead of shocking the sensitivities (simulating) over the entire range of the changes in the factors. But those simplifications are not recommended when there are not simple derivatives in portfolio. They are suitable for non contingent cash flow decomposable assets and for regulatory purposes but not for risk management purposes since they are approximations.

[30] It is the methodology the Bank of International Settlements based its Basle Committee Capital Accord of August, 1993, on.

price changes in the future are drawn from the same empirical distribution as market price changes generated by the historical data. It is computer intensive, however, since a simulation is required for each path.

The simulation generally uses the actual price functions to calculate portfolio values but it is also possible to rely on specific models for market factor changes (each market risk factor evolution must be explicitly modeled, in which case, model risk is introduced). It may also require large investment in intellectual power (since the risk manager can model his own expectations in the model) and system infrastructure. However, when the modeling is done correctly this method is one of the most robust methods in terms of accuracy and applicability (since it can be used for many diverse product structures, including derivatives). It is probably not cost effective to implement simulation only for risk measurement, but if the firm is already using simulation, for instance to price complex investments, than this method is worth implementing.

This more sophisticated approach retains many of the benefits of the simpler historical approach while it may eliminate its aging bias. In effect, although the “historical” portfolio P&L is generally simulated while keeping the portfolio character constant, its composition can also be altered to reflect any changes in its components and thus provide a more updated and accurate VaR. However, the price to pay for this increased precision may be high. Model risk is introduced because an analytical valuation model is necessary to derive the history of the portfolio value from the history of market factors. In addition, more time is needed to perform the analysis since a much larger number of computations is required. The need to resort to this simulation approach is a function of the contents of the portfolio and of the objectives of the firm. As long as the portfolio mix remains relatively constant and there is no substantial aging effect, the simple historical approach suffices.

II - The analytical approach

This method is also called variance/covariance approach, correlation method, delta-normal method, or delta-gamma-normal method based on the different degrees of simplification it contains. Its prominent characteristic is that it uses the assumption of normally distributed returns to simplify the computation of VaR and some practitioners that it to be the simplest VaR approach.

2.1. - General principle

The analytical method assumes that (1) the returns on the factor prices are normally distributed, (2) the correlations between the factors are constant and (3) the delta, or price sensitivity to changes in the risk factor, of each portfolio component is constant. To implement this method, it is necessary to (1) identify and estimate the risk factors the portfolio is exposed to, (2) to decompose the instruments in the portfolio into cash equivalent positions based on only one the market factors[31]; (3) then, the exact distributions for the market factors are specified; and (4) finally the portfolio variance and VaR are calculated using standard statistical methods of portfolio variance.

In this method, the volatility of each risk factor is extracted from the historical observation period. The potential effect of each market factor on the overall portfolio value is

[31] These procedure is known as cash flow mapping.

thus worked out from each component's delta with respect to a particular risk factor and that risk factor's volatility. These effects are then aggregated across the whole portfolio using the historical correlations between the risk factors[32], to give the overall volatility of the portfolio value. VaR can be obtained by scaling the volatility of the portfolio by the desired confidence interval index.

2.2. - Implementation

The analytical method is based on the premise that the financial instruments in the portfolio can be decomposed, "mapped", into a set of cash flows that are exposed to only one market risk factor (by using a process similar to that used to strip bonds) and are assessed at their present value. For instance an equity is decomposed into its dividend payments or a two-year foreign note is mapped in a set of 2 zero-coupon notes, one for the first coupon and one for the second coupon and the principal, and a foreign currency cash payment equivalent to the present value of these payments. The zero coupons are exposed to 1 and 2 year zero-coupon interest rates and the cash payment to the foreign exchange rate.

Then it is necessary to make assumptions about the shapes of the distributions of the market factors. Generally the underlying distributions are assumed to be normal, and as a result, all the historical information is summarized in the means, variances and covariances of the market factors.

A direct implication of the normality assumption is that the portfolio, as the sum of the individual normally distributed instruments, can also be assumed to be normally distributed.

[32]The volatility and correlation data provided free by JP Morgan can be used for this method.

This means that both the portfolio variance and VaR can be calculated very easily from the factors' volatilities:

$$\sigma_p^2 = \Sigma \omega_i^2 \sigma_i^2 + \Sigma\Sigma \omega_i \omega_j \rho_{i,j} \sigma_i \sigma_j, \text{ and VaR} = k\, \sigma_p{}^{33},$$

where k represents the critical value of the distribution of the returns, $k = Z_{\alpha/2}$, when the distribution of the factors is standard normal and the confidence level is $(1-\alpha)100\%$. This equation shows that the portfolio variance, and thus VaR, only depend on the volatilities of each factor in the portfolio and the correlations between them.

2.3. - Advantages

The attractiveness of this method lies in the simplicity in the VaR computation when the normal distribution is assumed. Because it is based on the variance-covariance matrix of the market factors it allows for easy updating for changes in portfolio composition. All that is needed is to change the weights corresponding to the new notional positions and if necessary to include new volatilities and correlations in the matrix. Just as the method allows for portfolio endogeneity (and solves the aging problem), it allows for scenario analysis conducted in a similar way. The method can be run quickly once the covariance matrix established and thus allows real-time VaR calculations which are very important in a trading environment.

The analytical approach also offers an interesting level of versatility. Historical volatilities and correlations for the factors can be extracted from the actual data, but there are other methods to calculate the relevant risk factor volatilities and correlations. Although, using empirical volatilities (correlations) is the most straightforward approach it creates *plateau* effects

[33] Or, VaR = $\mu - k\,\sigma_p$, when it is calculated with respect to the mean.

when large off market moves occur during the reference period, that can significantly distort volatilities (correlations) over the required forecasting period.

A more sophisticated approach consists in weighting past observations unequally. Exponentially weighted moving averages generally give greater importance to more recent observations so that large jumps in volatility (correlation), are not caused by events that occurred a long time ago. The Generalized AutoRegressive Conditional Heteroskedasticity (Garch) family of models express returns as functions of their previous values and provide *dynamic* volatilities because an element of randomness is included in their computation, but needs to be fine-tuned for each factor time series (which is not required in exponential smoothing).

Finally, the analytical method can also be implemented with various degrees of sophistication and allow for a better treatment of nonlinear instruments such as options. In the delta-normal approach capital charges are allocated as a direct proportion of the portfolio market risk:

$$CaR_\delta = Z_{\alpha 2}\, \sigma_\delta, \text{ where } \sigma_\delta \text{ is the market risk of the portfolio.}$$

This method van be applied to any portfolio, even with options, but only takes into consideration the linear risk factors of the assets in the portfolio. Here, the price functions here are assumed to be reasonably approximated by a first-order Taylor's series expansion on portfolio value, Vp, which only captures delta risk:

$$\Delta Vp = \theta\Delta t + \delta'\Delta S + o(2) \text{ or } \Delta\, Vp \cong \theta\Delta t + \delta'\Delta S,$$

where $\theta = dP(S)/dt$ and $\delta = dP(S)/dS$, and S represents the changes in the market factors (the returns)[34]. This relationship is true if the time horizon is very short, because over such short time

[34] Wilson, "Plugging the gap", *Risk Magazine*, October 1994, pp. 74-80

periods, financial products are approximately linear in their payoff. This method is therefore appropriate for quick and/or daily CaR/VaR calculations and is suitable for intraday risk assessment.

Another more sophisticated version is the delta-gamma method, which integrate convexity, the gamma risk of nonlinear instruments[35]. This method uses a second order Taylor's series expansion of the portfolio values around current market rates instead of first order expansion used in the delta-normal method.

$$\Delta Vp \cong \theta \Delta t + \delta' \Delta S \gamma + \Delta S' \Delta S/2 \text{ and } CaR_{\delta\gamma} = k\, \sigma_{\delta\gamma}$$ [36]

One method used is to *linearize* gamma risk[37], or as presented in RiskMetrics, to calculate the four moments of the distribution of returns, when gamma risk is included, to find a distribution with known properties which has the same moments and to use it to approximate the true distribution of the returns. This approximating distribution is then used to calculate k and VaR.

2.4. - Limits of the method

This method is limited by the simplifying assumptions it relies on. Analytical VaRs are parametric and thus distribution-dependent, and as such, they carry more model risk than the previously described historical approach

First, this approach makes strong simplifying assumptions about the distribution of the market factors and thereby only accounts for some of the risks of derivatives which have nonlinear payoff functions (only delta and sometimes also gamma risks are generally included).

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[35] The normal distribution with mean zero assumption can be justified for most continuous processes using Ito's lemma if the time horizon is sufficiently short.
[36] Ibid.
[37] Ibid.

As a result, this method underestimates the riskiness of portfolios containing derivative instruments. This is the major drawback of this method.

The second greatest disadvantage may be the cash flow decomposition of the components of the portfolio which also undermines the accuracy of the obtained VaR metric because some precision may be lost by the interpolations required in the mapping. In addition, this mapping may be discouraging for firms with a large number and variety of assets.

2.4.1. Normality

This is a choice of simplification made to gain speed because without the normality assumption VaR is not simply some multiple of the standard deviation of the portfolio P&L distribution. This assumption allows to convert the portfolio variance into the VaR figure. However, to the extent that the returns are not really normally distributed the resulting VaR can be grossly biased. It may help to check the validity of the normality assumption[38] on different component assets but also to calculate VaR using other approaches which are not so distribution-dependent and compare the VaR figures thus obtained to the actual losses.

In the real world, the return distributions of financial prices show greater density in their tails than the normal distribution. The distributions of asset prices have fatter tails (corresponding to larger possible changes in security prices) and a greater number of observations in the center (more small changes). The real distributions have fatter tails and thinner waist: they are leptokurtotic. As a result, VaR calculations using confidence loss limits based on a normality assumptions will generally underestimate real market risk exposures.

[38]Several testing procedures have been developed to assess the normality of a distribution, e.g., tests of skewness, kurtosis, Q-Q (quantile-Quantile) plots.

In addition, the critical values derived from the empirical distributions and even the closest fitting normal distribution can be dramatically different, for instance 1.65σ may be found to represent the cutoff value for 91.3% or 93.4% confidence in a specific empirical distribution where as it is assumed to be 95% for the normal distribution: this discrepancy underestimates the size of VaR and the frequency with which a 95% VaR-based capital charge would be consumed (or similarly, the true VaR should have been higher). The normality assumption for returns gives a false sense of security about the rarity of events against which the protection is sought: they can occur more frequently than the suggested 1% of the time. This 1% (for a 99% confidence level) is only an estimation, and gives only and idea of the average frequency of rare events.

2.4.2. - Linear asset pricing models

This is a corollary of the normality assumption. In effect, to justify the use of the normal distribution price functions are assumed to be linear in terms of changes in the underlying market factors. If returns are normally distributed and prices of assets are linear combinations (functions) of such returns than the assets and portfolio are also normally distributed (since a linear combination of normally distributed variables is also normally distributed). Therefore, it is assumed that the portfolio's delta is sufficient to characterize the portfolio's risk profile. This is not true when options are involved because their delta is not linear and because they carry other types of non linear risk. This method is not appropriate when anything but the simplest, linear, derivatives such as forwards and swaps are included in the portfolio. When options are part of a portfolio, the computed VaR will seriously understates the true market risk since it will ignore nonlinear risks embedded in them.

There is no linear relationship between option prices and their underlying assets' prices. In addition, for some options, the largest possible loss may not be generated by a large change in the in the underlying, as it does when risk exposure estimates are linear functions of portfolio returns standard deviations. A small change (and sometimes no change at all) may cause a bigger loss than that caused by a large change in the underlying. Therefore, for VaR purposes the portfolio positions should be revalued at many intermediate points within a range circumscribed, for instance by the 1 and 99% critical values of the underlying asset values.

2.4.3. **Stability**

The assumption of constant correlation is also a by-product of the normality assumption. It is also wrong, though, because returns are not really identically distributed, homoskedastic. In reality, volatility/correlation do change over time, and sometimes dramatically. It may help to do some sensitivity analysis, changing the estimates of the volatilities and correlations to see how much the VaR would change[39].

2.4.4. **Cash flow mapping**

To generate a variance covariance matrix that can be used repeatedly until the portfolio composition changes. Typically, assets income streams are converted into zero-coupon, domestic-currency denominated fixed cash flows, and their present are set among a determined number of grid points on a time line called *vertices*. Thus, only a reduced number of maturities, for instance 3, and 6 months, 1, 2 , 3, 5 years, (and so on), are used. Second, even variable or

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[39] This problem can be solved by using dynamic volatilities as mentioned earlier or by following Wilson [1993]'s advice to replace the normal distribution by the Student's t. (See Chapter IV).

contingent cash flows, such as stock dividends or cash flows resulting from option positions are converted into deterministic ones.

For options, this simplification calls for using cash flows based only on delta risk only which means that the higher-order sensitivities of the instrument are ignored. This standardization of the cash flows lead to adding more imprecision to the model and to weakening the significance of the VaR estimate thus derived.

III - Monte Carlo Simulation

3.1. - General principle

In an attempt to generate a more comprehensive risk profile, some risk managers simulate the random behavior of all the basic market factors which have an impact on portfolio value. This is commonly done using a stochastic sampling methodology, Monte Carlo simulation, in which a mathematical process is used to generate series of *pseudo-random* changes in factors to simulate reality.

This third method, more flexible than the previous two, is correspondingly the most computer intensive. Like the historical simulation, it allows the practitioner to use the historical distributions of the risk factor returns, rather than to assume normal returns. However, when the historical approach quantifies portfolio risk by going through one historical path of market evolution (from which it permutes the actual realizations of market factors to obtain a larger number of scenarios) the stochastic approach generates a series of random paths.

Stochastic simulation only differs from the historical simulation in two respects. First, while the evolution of market factors is simulated through mathematical modeling in the stochastic approach. Second, only one sample path is used in the historical simulation, whereas here, thousands of paths can be generated.

3.2. – Implementation

First, the stochastic processes must be specified for the behavior of the market factors[40], second the statistical parameters for each stochastic process must be estimated[41]. Then, fictitious paths are simulated for each market factor by generating pseudo-random numbers which are fed into the stochastic processes. The complex interplay of the risk factors is captured by generating a large sample of simulated paths[42]. Then the portfolio is valued form the simulated market factors values.

3.3. - Advantages of the stochastic approach

3.3.1. **Comprehensiveness**

Stochastic simulation is much more comprehensive because it allows for a large collection of sample scenarios. It generates a large number of sample paths that can be used to explore a wide range of possibilities and test large variety of scenarios. So many iterations are run that in the end convergence toward the true distributions can be achieved. The complex interplay of the risk factors is captured by the large sample of simulated paths. When, say, 1,000 paths are simulated, thousands of permutations can be obtained and used for the valuation. As

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[40] For example, one can assume that the evolution of interest rates follows a *random walk*, option prices follow a *geometric brownian motion*..

[41] For example, the mean and variances of the random walk of the returns.

[42] There are new techniques used to guarantee more evenly spread random numbers, for more information see Brocheron-Ratcliffe [1994], Moro [1995] orand Papageorgiou and Traub [1996]

the number of simulations increases the portfolio P&L which is obtained converges toward the true distribution. The VaR figure thus obtained represents a market risk measure which is averaged across a large number of potential sample paths generated in the simulation process. In this sense, the VaR number is also more realistic and reliable

3.3.2. **Flexibility**

This approach is more flexible since most of the analytical modeling is done by the risk manager who has the total freedom to specify the evolution of random system over time. It offers virtually boundless flexibility in specifying how the random system evolves over time. For instance, if so desired, a different portfolio mix and/or a different set of statistical parameters, such as volatilities and correlations, can be specified for each future period. The manager can also include his own expectations of future changes of the markets and the portfolio in the VaR valuation. In effect he may even change the stochastic process of some financial assets or the composition or the portfolio from one simulation to the next.

3.3.3 **Precision**

Stochastic simulation is the most efficient approach in terms of precisely capturing the risk profile of portfolios. This is because precise valuation models are used to determine changes in portfolio values as the market environment changes. In contrast with the analytical approach where cash flows are only approximated. or with the historical, where the portfolio composition may already be outdated. no simplifications or approximations are made. Because of its modeling precision, stochastic simulation is the best approach to use when quantifying the effect of optionality, large market movements or deviations from normality (which may be present in each path).

3.4. - Limits of the method

Just like any other approach, Monte Carlo simulation also has its drawbacks. While this method is likely to match reality more closely than the other two, and therefore estimate VaR more accurately, its price is time and intellectual and computer power. These costs may outweigh the benefits of increased accuracy and many interested users will as a result turn to the easier, less accurate methods sufficient to meet their objectives.

3.4.1. **Substantial sophistication requirements**

This method requires a substantial investment in intellectual *know-how*, substantial mathematical sophistication, and hardware infrastructure. Thus it is not worth implementing on a stand alone basis for the sole purpose of VaR evaluation. In addition, although its flexibility can be a valuable advantage, it will really be so only if the user does have the proper financial/econometric expertise. This flexibility will backfire if he does not because, a VaR metric can only be as good as the modeling done to compute it.

3.4.2. **Lack of speed**

Another major drawback is speed (or lack of): stochastic simulation is the most time-consuming way to run risk analysis. Although there is no theoretical limit to the complexity of the model(s) that can be built to implement this approach, there are many practical constraints.

In effect, to generate a large sample of simulated paths, the portfolio has to be revalued many times. Even in this day and age, when computing power is relatively cheap and readily available, the need to perform tens of thousands of portfolio valuations can still be daunting, especially if the portfolio contains a large variety of assets and the simulation involves a large number of sample paths.

This method will generally be used as a strategic risk management/control tool in the back office but not directly at the trading desk since it will generally not produce real time estimates of market risk exposures. However, several large financial institutions with advanced systems, have reported that they are now able to run simulations overnight and obtain Monte Carlo VaRs on a daily basis.

IV - Choosing among the methods

The three methods deliver different VaR measures: the inevitable question is, thus, which method is best, or which one should be chosen? Realistically, in selecting a risk measurement approach, the financial/risk manager has to understand the relative strengths and weaknesses of each alternative and the particular situation of the institution - what is the portfolio composition; what kind of hardware, software and databases are available for the risk management effort, what are the firm's risk aversion and strategic objectives, and so on.

Temptation to opt for method that produces highest VaR is obvious since it would better ensure the solvency of the institution, but this impulse must be tempered by the realization that commercial enterprises have to earn competitive rates of return for their shareholders, and thus are restricted to the amount of prudential capital that keep since these resources remain idle and do not earn a positive return (or even worse, can be lost). In addition, because of disclosure requirements, some institutions will hesitate to use higher VaR yielding methods since they may "scare" shareholders away by reporting larger risk exposures

Ultimately, the choice of method always involves a trade-off between practicality and accuracy, therefore, the method utilized should be determined by the composition of the portfolio, the objectives of the manager/institution, the amount of compromise (a function of risk aversion) the institution is willing to accept.

4.1. - Convenience versus accuracy: Portfolios with options

Discrepancies among VaRs are especially large with portfolios that contains options. Analytical-VaR may be convenient, but because of its normality simplifying assumption, it ignores some of the risk factors which enter the market risk makeup of options. Ignoring the higher-order *Greek relatives* of delta and gamma severely underestimates VaR when there is substantial optionality in the portfolio. The historical and simulation methods, in contrast, fully capture option risk. Simulations can account for potential changes in all the market factors that affect the price of options. and their revaluation process allows the market risk of options to be more accurately measured for large and small estimated changes in the underlying[43].

[43] Jordan and Mackay have computed and compared (daily, weekly and monthly) VaRs of stylized equity and equity and option portfolios using historical. simulation and analytical VaRs: In equity portfolios, parametric VaR was systematically lower but not by much, in mix portfolios, it was substantially lower (by a factor of as much as 2 to 3). Smithson. "Value-at-Risk". *Risk Magazine*, January, 1996, pp.25-27

4.2. - Convenience versus accuracy: Periods of market stress

The normality assumption of parametric VaRs which make their computation practical also cause them to underestimate the frequency and impact of market crashes. They estimate more-than-3-standard-deviation adverse market moves to occur less 1% of the time which is generally not true. Here again, historical and simulation VaR do not make assumptions on outliers frequencies and do not average out the impact of market stress[44]. Because analytical VaRs underestimate risk when there is optionality and market turmoil, regulators and sophisticated risk managers do not use this method for market risk assessment. Since analytical VaR is an "optimistic" metric, it should not be used for capital charges allocation: analytical CaR would be consumed faster and more frequently than expected and this could seriously jeopardize the survival of the institution in case of extremely adverse market moves.

4.3. - Convenience versus accuracy: Real-time VaRs

When time rather than precision is of essence, or when there is no substantial optionality: many traders and risk managers will use analytical VaRs to assess intraday VaR evolution as trading alter the portfolio composition. The analytical approach provides quick answers and a chance to test alternative small trades impact on portfolio VaR in real-time conditions. In effect, since analytical VaR requires no portfolio pricing model (just the weights of the assets) and uses the covariance matrix of market factors, it can be used to provide an estimate quickly, since minimal calculations are needed[45]. For the same reason, scenario analysis can be implemented

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[44] Although as mentioned previuosly, historical VaR may understate market risk by ignoring some of the worse losses of the P&L distribution.
[45] See DelVaR in the next chapter.

quickly, for instance, if markets start showing threatening signals. However, for best results, full valuation and simulation are respectively preferable when considering new trades and doing scenario analysis.

4.4. - Accuracy versus convenience: Exigent requirements of accuracy

The stumbling blocks to using the simulation method is (1) the evidently complex task of doing Monte Carlo simulations and (2) that simulation, quickly grows extremely time-consuming and can be costly in terms of computer-power. The amount of effort required to build a good simulation is much greater than that required for an analytical approach where simplifying assumptions can be used (provided their implications are understood) and even than that of historical simulation. However, there is software that will take care of the simulations.

The choice between the historical and stochastic simulation, comes down to user preference either using actual changes in the market factors (which incorporates market characteristics implicit in the historical data) or generating distributions based on the model designer's expectations of the market behavior. For small users, or in situations where time is of essence, the refinements provided by using simulation techniques are generally outweighed by the difficulty of constructing and running them.

4.5. - Subjective versus objective risk assessment

Subjective estimation implies utilizing user-defined methodologies while an objective approach calls for standardized methods. The advantage of the former is that, if the model designer is highly skilled, the results will exceed in quality those of any standard approach since he may customize the system to the idiosyncrasies of his portfolio and institution. On the other hand, when there is only an average knowledge of econometrics and financial asset pricing theories using a standard method is safer. A disadvantage of standardized methods is its dubious assumptions: parametric VaRs, for instance, are distribution dependent and make simplifying hypotheses, and therefore, as such, they more than others include model risk.

4.6. - Conclusions

The choice of VaR model is not so much a matter of finding an optimal methodology but rather of assessing the level of compromise one is willing to accept: this is what in the end has the greater impact on the VaR metric.

Limitations are a fact of life. It is impossible to find a risk management methodology which does not have some kind of drawbacks. Effective risk management is not so much a matter of finding the perfect method, as a matter of knowing the relative strengths and weaknesses of each alternative approach and possibly implement a few mutually complementary ones.

The three different basic approaches to VaR: the historical (and the historical simulation), the analytical, and the stochastic (simulation) approaches, have each their own strengths and weaknesses and none is absolutely superior to the others. One method may simply be better suited for specific situations or portfolio types and may fit some risk management objectives or the firm's existing resources better than others: intelligent risk managers will seek to understand the limitations of each approach to VaR and find one - or a combination of several - which best fits their organization's constraints and needs. A few comparative studies conducted on VaR computation methods, such as that of Jordan and Mackay, may however, shed some light on how to select a VaR methodology.

In 1994 and 1995, several initiatives taken to assess the diversity in institutions' assessments of the valuation and risk of traded instruments revealed that even the most advanced international financial institutions still experience difficulties in assessing market risk exposures and calculating VaR. The Basle Committee on Banking Supervision has conducted benchmarking exercises twice, with a small group of internationally active banks. It provided the portfolio description and the data and the financial institutions were instructed to calculate VaR with their internal models but within the guidelines of the Basle proposal. There were substantial discrepancies in the outputs. The bank of England also conducted a survey in February, 1995, with forty banks and securities houses to identify the difference in their pricing and risk assessment of derivative products. They too, were given the description of the instruments and left to the assessment. While the results on equity and exchange rate options were fairly consistent and consensual, only a limited number of participating firms were able to price interest rate options and, the variability across results was substantial. These surveys were conducted to verify financial institutions' abilities to calculate to price financial assets since they were soon supposed to do that to calculate VaR and set capital requirements against these estimates of potential losses within the two organizations' capital adequacy framework.

Chapter IV - Variations on a theme

VaR methodologies are financial models which provide a way of encapsulating a firm's market risk into a single metric that can only be used to help management's understanding of its risk position. In effect, reliance on models to handle risk carries its own risk, model risk. Models are, by definition, approximations of reality and to be practical and useful, they have to make certain simplifying assumptions: the user must be aware of them and of their consequences.

No VaR model shows exactly what the exposures to market risk are, they are approximations of risk exposures (of worse potential losses) and must be considered to have error bars. Some researchers have focused their attention on the simplifying assumptions embedded in some VaR models and looked for solutions to release them or circumvent their debilitating effect on the quality of the computed VaR metric.

I - Limits and proposed adjustments to the VaR models

I.I. - Reliance on past data

The main assumption underpinning the VaR concept is also one of its main drawbacks: the distribution of future price changes is similar to that of past price variations. The portfolio loss calculations are worked out using distributions or parameters from historical asset price data collected during the observation period. This assumption may induce missing extreme situations in financial markets, since parametric VaRs even out outliers or the historical method may calculate VaR from a fairly stable observation period.

In addition, any method that relies on historical volatilities, especially analytical and historical methods, keep a shock in the computations for all the reference period and distort the resulting VaR metric, the so-called plateau effect. A solution is to weight changes unequally or to use Garch models.

Even if the day-to-day risk management is not conducted under "normal" conditions. VaR remains a useful tool though, provided that users are aware of its built-in simplification assumptions and limitations. Most users should use stress testing or scenario analysis to determine the risk of their portfolios in such situations, and sensitivity analysis that investigates the impact on VaR of changes in volatilities and correlations.

1.2. - Normality assumption

Assuming normally distributed returns requires them to be *iid.* However, abundant research has proved that returns are not identically distributed. In effect, although they tend to fluctuate around a mean value, they do generally display heteroskedasticity since they exhibit volatility clusters with periods of high and low variability. As for the second assumption of independence, although short-run returns such as daily returns, tend to show no autocorrelation, there variances do[46]. In any case, the *iid* requirement is not met, and assuming returns to be normally distributed distorts the VaR estimates

Returns are often assumed to be at leas approximately normal, though, in order to allow linear computations and substantially simplify calculations. This simplification does not provide highly biased results as long as the portfolios does not contain substantial optionality, and when they do, the assumption of constant delta is made, so that the P&L remains

[46] Existence of correlation can be assessed through correlograms or the Box-Ljung test.

approximately normal: but doing this creates a potential for large errors.

1.2.1 The problem with market crashes and options

The market risk of a portfolio may be quantified as a single VaR number but thorough risk management requires a more careful consideration of the portfolio risk sensitivities than VaR alone can provide. This is particularly true (1) for options, where the *Greek* risk sensitivities can prove extremely significant, and (2) when markets crash. Even a carefully computed VaR figure would do little to predict the likelihood of a market crash occurrence or its consequences if it did. The normality assumptions poses a problem when considering portfolios with options or the impact on markets downfalls.

Outlying negative events, such as market crashes, are more common in the real world than the normal distribution suggests: this calls for the use of stress testing and scenario analysis to investigate the impact of more frequent market falls on the firm's risk exposure. Nor are returns on an options portfolio normally distributed. Any instrument which includes optionality will have a non-linear payout profile. Put more simply, the relationship between a change in the underlying asset price (or rate) and the resultant change in the value of the derivative is not constant. This relationship is quantified as delta, the first of the Greeks which is graphically represented in the slope of an option value curve Delta is not constant (linear), it depends on the value (or change in the value) of the underlying. This dependence creates a variability in delta which defines the gamma risk of options which is also called convexity because graphically it is the curvature of the option value curve.

Delta and gamma are just the first two members of a long series of sensitivities[47] to the

[47] v-risk (vega risk) is the effect of volatility in the price of the underlying on the value of the option. θ-

underlying, the rest of which are generally ignored. The importance of non-linearity increases as the time to expiry increases and also as the option approaches the money. The shape of the volatility curve is more important for options with a longer time remaining until expiration (as measured by vega).

1.2.2. Proposed solutions

Until recently, VaR models have only taken account of delta (the only approximately linear component of option risk) to allow the use of the normal distribution, but for portfolios containing large options large numbers of options and therefore significant exposure to gamma risk, this is not a sufficiently good approximation, and regulators are no longer complacent in that respect. After the wave of reported enormous losses incurred from portfolios encompassing derivatives, January 1996's proposed amendment to the Basle Capital Accord stipulates that banks which write options are required to measure delta, gamma and vega risks when calculating capital requirements to be held against them.

However, using a delta-gamma approach does not necessarily yield a more accurate VaR than a delta-normal model, because with the introduction of gamma the distribution becomes non-normal but the assumption that VaR is a linear function of volatility that has ceased to be valid but sometimes continues to be used[48]. The distribution of returns becomes asymmetric, skewed to the left. Other variables, such as the mean and variance, may also differ from those of the normal distribution, invalidating their use. Just because the parameters of the normal distribution no longer apply, though, does not mean similar but more refined techniques cannot

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risk (tau or theta-risk) is the time decay of option values. it shows the risk sensitivity due to the passing of time and it increases as the time horizon increases.

[48] As shown earlier, this can be circumvented though. See Analytical approach in Chapter. III.

be used on the skewed returns generated by the portfolio.

The portfolio VaR can then be calculated on the basis of this skewed distribution. in one of three ways: (1) the skewed distribution can be approximated to a deformed normal distribution; (2) the 5th and 95th percentiles can be calculated for the skewed distribution. and (3) the skewed distribution can be fitted to a more general family of distributions whose characteristic statistical measures are known[49].

1.3. - Correlations

There are two correlation-related difficulties in VaR techniques: correlations are generally assumed to be (1) known and/or (2) constant. Generally, risk analysis uses constant parameter assumptions (constant volatilities. correlations. deltas and gammas) because that greatly simplifies the models but this is unrealistic and yields contradictions.

1.3.1. **Unknown correlations**

Correlations are important in an investment portfolio context. because the lower the assets correlations the greater the diversification effect and overall risk reduction in the portfolio. However. in periods of market stress not only do the magnitudes in the changes in price (volatilities) of the underlying assets increase[50] but in addition the correlations increase[51] as

[49] Paul-Choudhury. "Optional Extras". *Value-at-Risk. Risk special supplement* .June 1996. pp. 63-65

[50] Longin and Solnik (199) found that correlations of national stock markets typically increase by 27% in periods of high turbulence. The direct impact of this alone. could underestimate VaR by 13%. P. Jorion. Value-at-Risk: the new benchmark for controlling market risk. pp. 178

[51] The breakdown of the European Community's Exchange Rate Mechanism (ERM) in September. 1992. is a vivid example of this increase in volatilites. For instance. from the close on Friday. September 11. 1992. to the opening on the following Monday. the one-month implied volatility of Sterling/Deustchmark options. for instance. which had been relatively stable in the 4-7% range over the previous months. increased to an historical high of 35%. implying that some risk managers those who had relied on implicit

markets tend to move together. As a result, VaR relying on historical volatilities/correlations are underestimated at two levels in periods of market stress.

To make things worse, most VaR methods assume that the covariance matrix is known. This is not true: volatilities/correlations are stochastic and thus unknown when VaR is calculated for the next period and prudential capital is allocated. Extrapolating volatilities and correlations over the next period, based on historical estimates or implicit volatilities curves, is very difficult while using past ones is very dangerous..

Regulators are so concerned about the impact of correlations on risk exposures that they have gone to extreme measures to guard against their stochastic character. The European Community and the BIS, for instance, impose maximum correlations ($\rho_{i,j} = +1$) across risk factors to aggregate the measure of market risk when calculating prudential capital allocation[52].

1.3.2. **Proposed solution: t-distribution**

Wilson [1993] developed a model variant that uses stochastic covariance matrices and relies on simple assumptions. In the most simple case, it calls for replacing the normal distribution with the standard t-distribution since the latter is leptokurtotic and reflects the fact that VaR users have only limited prior information, especially, about the covariance matrix since it is a random variable[53]

Empirical research applied to banks' capital allocation, showed that the student's t CaR

(or historical) volatilities/correlations, may have underallocated risk capital by a factor of five. Wilson, "Infinite wisdom", *Risk Magazine*, June 1993, pp 37-45

[52] See their directives in Exhibit III of the Appendix

[53] The more limited the information the model user has (about the correlations of returns the fewer the observations the VaR calculations are based on), and the smaller the number of degrees of freedom in the t-distribution, that is the fatter its tails are. However, as information increases, (so do the number of degrees of freedom) this distribution converges toward the normal distribution.

constantly allocates more prudential coverage than the normal distribution-based computation does. Capital provisions would therefore, be more conservative if it is recognized that the covariance matrix is unknown when risk exposures are assessed. In addition, the 99%-confidence CaR metric based on the t-distribution was exceeded only 0.7% of the time by actual returns, versus 2.7% for the normal distribution based one, confirming that the former distribution might provide a better prudential capital allocation[54].

These findings are very appealing because, since the properties of the t-distribution are well documented, and this simple modification can be quickly implemented at low cost, even in existing systems. In addition, empirical researchers have found the t-distribution fits the data better than other distributions for a range of markets. Finally, the adjustment is easy to understand: the less certain one is about the interplay between risk factors, the greater the capital that needs to be allocated to guard against the risk exposure of the portfolio.

Furthermore, empirical treatment has shown that when the returns are generated based on the assumption that the covariance matrix is known with certainty (the standardized normal distribution assumption) as opposed as to the assumption that it is not known (the standardized t-distribution with two degrees of freedom), *ceteris paribus*, there is a greater potential for larger losses if the risk manager's information about the covariance matrix is poorer (not only are losses more likely, but greater losses are more likely). Any method of allocation of prudential capital charges for analytical VaRs should therefore, set aside greater capital protection for two reasons when the uncertainty about market volatilities and correlations increases (as it does in times of market stress).

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[54] Ibid

1.3.3. **Stationary correlations**

The problem with the basic historical data is that the obtained correlations (1) no longer include current investors' expectations, (2) do not account for chronology (the dynamic ordering of pairwise observations is ignored: for instance, historical correlations do not capture the fact that two successive returns were high, but simply that two returns were high), (3) it may be a contradiction to try to calculate time-varying series for historical correlations which are assumed to be stationary (Alexander[1994] argues that the differences among them only come from sampling errors), (4) they depend on the length of the reference period (because of the plateau effect)[55].

For all these reasons, using historical correlations may lead to misleading results. A solution may be to use the t-distribution to replace the normal distribution in parametric VaRs, or to use implied correlations: those extracted from option prices since their prices reflect people's expectations about the future. Such correlations can, thus, provide as a proxy of future correlations (however, some option prices are not available, there is therefore not a complete distribution of future correlations available, or calculated implied correlations sometimes turn out to be greater than 1, which is inconsistent).

1.3.4. **Proposed solutions: dynamic correlations**

Another way to account for and integrate time-varying volatilities and correlations is to use a moving window of data to calculate them as time-varying parameters, but it carries its undesirable plateau effects Exponentially weighting moving averages and Garch models[56] of

[55] Alexander," History debunked", *Risk Magazine*, December 1994, pp. 59-63

[56] Arch models are autoregressive conditional heteroskedasticity models which are stochastic processes whose volatility varies over time (heteroskdacticity), in a way that depends on (conditional) past values of

dynamic correlations circumvent that problem.

Garch models provide instantaneous measures of volatilities and correlations. The principle is to use regression analysis to find the best fitting relationship to a series that describes current returns as functions of past returns. Even a Garch model may provide misleading results (for instance if it is based on series which are not jointly stationary, the model will estimate correlations which in reality do not exist) but, these errors are generally much smaller than those of stationary models. In all cases, however, there is a loss of information because, historical, implied, exponentially weighted and Garch correlations are all calculated on stationary return series and contain no information about the trends in the data.

II - VaR supplements

2.1.- Marginal VaR: DelVaR

Once portfolio VaR is assessed, there remains the important question of how the firm will use it to adjust its trading practice and in its market risk management. In particular, the institution must decide what can be done to improve on its risk exposure and reduce potential losses? The choice of trading positions should be based on their VaR/profitability tradeoff. To make such decisions, the non-linear nature of VaR imposes the creation and revaluation of a new portfolio which incorporates the new proposed trades (or does not include the securities that are going to be sold), to arrive at the new portfolio VaR. Since it has been shown that such full valuations, of maybe tens of thousands of assets, can be a very demanding process, Garman

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a stochastic error process (an autoregressive one). Garch, generalized Arch, models characterize processes based on their own past values.

[1996] designed a tool to assess the contribution of a position to analytical portfolio VaR: DelVaR.

The DelVaR approach provides a short-cut to the full valuation process by evaluating the marginal contribution of any securities purchase/sale on portfolio/institutional VaR, without the extensive recalculation of total VaR[57]. It allows, an *ex ante* (before the transaction) rapid real-time evaluation of candidate positions, which is extremely practical in a trading environment. Moreover, by adding one additional feature to DelVaR, "trade normalization", it becomes possible to determine not just whether certain trades will increase/decrease VaR, but indeed which trade will have the greater impact.

Garman develops DelVaR for analytical VaR, since empirical VaR does not allow scenario analysis and simulation is too time-consuming. In such a context, DelVaR is only a function of the current portfolio, it is not a function of the specific candidate trade.

Let VaR = $v = \sqrt{p'Qp}$, where P is the current portfolio, $p = m(P)$ is a column vector of cash flow amounts. Let, $r_i(\epsilon) = p + \epsilon a_i$, be the cash flow vector of the portfolio scaled by a small positive amount (which represents the notional investment in that new purchase and is small relative to the total portfolio value) of the candidate trade Now, VaR can be rewritten as:

[57] The "brute-force" method recalculates VaR of new augmented portfolio $R_i = P + A_i$, where P is the current portfolio of existing trades, A_i the portfolio consisting only of the ith candidate trade. Then, new VaR = $w_i = \sqrt{r_i' Q\, r_i}$, where $r_i = m(R_i)$ is the cash flow map of the augmented portfolio. The test of VaR improvement occurs by examining the difference ($w_i - r_i$) When this quantity is negative, the candidate trade will improve institutional VaR. Garman, "Improving on VaR", *Risk Magazine*, May 1996, pp.31-37

$$VaR = w_i(\epsilon) = \sqrt{r_i'(\epsilon) Q\, r_i(\epsilon)}.$$

When a Taylor series expansion of VaR is taken around $\epsilon = 0$, it appears that

$$w_i(\epsilon) = w_i(0) + \epsilon[\nabla w_i(0).a_i] + o(\epsilon^2) = v + \epsilon[DelVaR.a_i] + o(\epsilon^2),$$

where ∇ is the del (derivative) operator.

If ϵ is sufficiently small, the improvement of VaR is solely dictated by the sign and magnitude of a_i ($=m(A_i)$) the vector of cash flow for the ith candidate trade, since $o(\epsilon^2)$ the higher order term can be ignored provided ϵ is sufficiently small. This incremental VaR calculation is, thus, an approximation since the higher order term is ignored, but it is a fairly good one for most small trade candidates (unless trading volume is quite low or the average trade is held for an extremely short period).

$$DelVaR = \nabla w_i(0) = \nabla v.$$

Thus, DelVaR depends only on the current portfolio, and not on the candidate trade, and needs only be calculated once during those trading intervals over which the institution portfolio is substantially unchanged. The incremental effect of any new trade is then approximated as:

$$\text{Incremental VaR} = DelVaR.\ a_i\,.$$

which requires no recalculation of the new portfolio, and uses the same DelVaR vector for all candidate trades.

These characteristics make DelVaR particularly convenient for establishing VaR-based trading limits over relatively short periods, without significant recomputation of VaR metrics. But so far, this process only indicates the sign of the contribution of a trade to portfolio VaR, (whether its effect will be positive, increase VaR, or not).

To quantify the magnitude of this effect, though, and rank candidate trades in terms of their relative contribution to the portfolio VaR, trades must be standardized, made comparable. Garman suggests several bases for that normalization, cash flows, stand-alone VaR, notional value, price, and so on, which enables to calculate a consistent DelVaR per unit, for each trade.

2.2. - Stress testing

2.2.1. - **The case for stress testing**

No risk measurement model is without implied assumptions or limitations. It is therefore helpful to understand what would happen should some of the assumptions underlying the model break down. There is no standard way to do stress testing. It is just a way to experiment with the limits of a risk model and to "think outside the box" Stress testing is, in fact, the catch-all term for running a series of scenario analyses to investigate the effects of violating some of the basic assumptions underlying the risk model It is also a means to measure the residual risk from tail events which is not effectively captured by the "official" risk model, and thus complement the VaR framework. The 1% of market moves that are excluded in 99%-confidence VaR computations contain events such as the 1987 stock market crash, a 20 standard deviation move, or the 1994 bond market crash In these events not only the correlation between markets suddenly increased well above levels normally assumed in VaR models as all markets went down together, but in addition, market moves were much larger than any VaR system would have accounted for.

2.2.2 - **Maximum loss optimization**

The most crucial question is of the magnitude of the expected losses on these days and what an institution can do to protect itself against these losses that could cause its ruin. VaR become useless in this situation. In particular, assuming normally distributed returns may provide a workable day-to-day approximation for estimating market risk exposure but when market moves are more extreme, relying on such assumptions is positively dangerous, since the resulting estimated loss may be dramatically understated compared to the actual one. VaR metrics provide estimates of the worse loss based on a given P&L distribution and confidence level. It is also crucial for the firm, however, to explore other situations in order to assess *worst* losses, and they need not come from extreme moves since for certain types of options, worst losses occur when the price of the underlying remains unchanged.

Maximum loss optimization is a portfolio value search designed to find the combination(s) of changes in market factors that result(s) in the portfolio greatest loss. The method uses only the ranges for the changes in factor prices and multipoint price sensitivity. It does not rely on past data and correlations to build the sensitivities and is not correlation dependent because it allows risk factors to move freely within their ranges. Given a set of scenarios, the maximum portfolio loss can be worked out, backward. The optimization program finds the market event as identified by the combination of market factors values that minimizes the P&L given that this event belongs to a confidence level and the portfolio is valued to calculate the size of the loss.

In a similar vein, some institutions may be interested in conducting ruin analysis, and estimate the probability of total loss and/or the market event that may lead to it. It is an important additional information that any risk measurement program should consider.

2.2.3. - **Scenario and sensitivity analysis**

The one thing that risk managers can hope to do to identify their true portfolio risks in stress situations is to simulate extreme market moves over a range of differing scenarios. In the Group of Thirty's July 1993's reports, it is recommended that dealers regularly perform simulations to determine how their portfolios would perform under stress conditions. The most popular and flexible approach is to use Monte Carlo simulation. This allows dealers to push the risk factors to greater limits and is not dependent on the assumption of normality or indeed the distribution of returns over the historical observation period. A 99% confidence interval captures the events up to 2.33 standard deviations away from the mean asset or portfolio or such that they are not exceed more than 1% of the time. A risk manager may want to find out what would happen to the trading portfolio if a 10 standard deviation move occurred.

Financial/risk managers may also want to change the correlation assumptions under which they normally work. If markets all move down together, losses are obviously going to be greater than if some markets are offset by other negatively correlated markets. By pushing the bounds of market moves that are covered in the risk management process, financial institutions have a better chance of seeing where losses might occur and therefore a better chance of managing those risks effectively.

Scenario analysis need to be sufficiently rigorous to provide substantial coverage in times of stress, but even in very adverse market circumstances, financial organizations take steps to reduce their risks and conserve capital. Thus, stress testing must include portfolio endogeneity but also the fact that when markets crash liquidity decreases and stop-loss strategies may be impossible or counterproductive. Finally, it should also account for qualitative factors, such as the trading objectives, the cash available, or the goals of the organization.

2.2.4. - **Limits of stress testing**

Sound risk measurement practices go beyond assessing risk exposures under normal conditions, thus while VaR expresses normal market risks, stress testing identifies risk exposures under abnormal circumstances. Stress testing should help identify unrecognized vulnerabilities, often the results of hidden assumptions, and make clear of consequences of being wrong in these assumptions. However, even scenario analysis is not sufficient because it also has limits.

It is costly to stress test and stress simulation of VaR because of its computational complexity, gives results with a delay. Stress testing can generally not be implemented on a daily basis to monitor day-to-day positions and VaRs and use the results the next day. As a consequence, some institutions use the linear simplifications for the VaR computation and scenario analysis with distribution of normal distributions (which may be nonsensical and may beat its own purpose) since low tail events (one of the very subject/reason of the scenario analysis) are understated under the normal distribution.

Finally, stress testing only answers a limited number/range of questions. For instance it does not provide an estimate of the risk that an institution's trader/manager will enter very dangerous transactions which will jeopardize the survival of the firm. After all, this question is of the domain of other types of financial risks (legal or operational risks, which may turn out to firmkillers, whereas it is very unlikely that market risk alone may cause, a financial institution failure, today.

Chapter IV - Accuracy assessment

I - The framework

1.1. - The case for model accuracy assessment

Because VaR plays a crucial role in market risk management and is used to set regulatory capital requirements, it is important to (1) select the best VaR model based on the specific portfolio and goals of the institution (that is, the approach which will yield estimates closer to the actual losses), but also to (2) conduct *ex post* test of its accuracy, that is, to statistically test the significance of the estimates obtained through the implemented methodology.

No one has a good estimate of the error involved in the VaR metric. Despite the obvious importance of accuracy assessment, little research has considered the statistical techniques that would be appropriate for judging the quality of an institution's VaR estimates and there is no commonly accepted standard statistical approach for determining the accuracy of VaR.

RiskMetrics provides an abbreviated discussion of model verification issues. Crnkovic and Drachman [1995] propose a measure of goodness-of-fit for an entire estimated P&L distribution. The Group of Thirty "Derivatives" study [1995] suggests that institutions perform *reality checks* for judging model performance, by backtesting (comparing an institution's VaR estimates against its portfolio's subsequent profit and loss outcomes), but this report does not provide any detail regarding the formal statistics that facilitate this comparison. Similarly, the Basle Supervisors Committee recommends back testing as a means of verifying the accuracy of a bank's risk exposure estimates, but again, the recommendation does not provide the details of the proposed verification test.

Kupiec [1995], however, provides formal techniques to derive alternative statistics as well as their formal statistical properties, that can be used to verify the accuracy of VaR estimates computed by the historical approach within the guidelines of the Basle Committee proposal. His work is the one generally cited in the literature when the discussion covers the accuracy of VaR models. His procedures use historical P&L distributions of the institution's portfolio or historical simulations to verify the accuracy of an institution's VaR estimate of its potential loss exposure both in a capital requirement regulatory context and in internal management framework. The testing procedures described below provide a summary of his research.

Verifying the integrity of VaR models could be done by comparing risk exposure estimates to (1) *ex-ante* measures such as the firm's objective or benchmark or some benchmark of trading risk (but such benchmark does not exist yet, and most people in the industry are worried that even such a standard be created and imposed because it may be very restrictive and may even jeopardized the survival of institutions), in addition, as VaR models are not statistical regression models, there is no ex ante measure of their goodness-of-fit. or (2) *ex-post* against the actual losses.

Presumably, a loss estimate could be considered accurate if an institution's actual losses do not frequently exceed its (*ex ante*) critical value estimates, or if its loss estimates do not exceed the potential losses that would have been generated by the portfolio if it had been held by the firm through some historic period.

1.2. - Testing difficulties

When actual losses exceed risk exposure estimates it is difficult to determine whether an accurate estimate was exceeded because of a low probability event or whether the estimate is biased and set too low. Conversely, lack of such "violations" does not suffice to validate a VaR estimate/model since this non violation may also just be the result of chance and not of an appropriately calculated risk exposure. For these reasons, VaR models do not appear to be really statistically verifiable. Indeed the statistics developed to retroactively validate VaR models, even when based on performance analysis have very low statistical power to detect inaccurate prior tail probability estimate.

The problem is that, since VaR is reported only at a specified confidence level, its value will be exceeded in some instances, for example in 5% of the observations at the 95% confidence level. But exactly 5% excess deviations will not necessarily be observed. A greater percentage could occur due to bad luck, perhaps 6-8%, and similarly, due to chance, the actual loss frequency could be less than predicted by the confidence level. It is only if the frequency of deviations becomes too large or too small that one must conclude that the problem lies with the model, not with bad/good luck, and that a regulator can impose penalties on an institution that willfully understates its VaR, or that the firm reviews his model specifications.

The use of historical performance data to verify potential loss estimates over long holding period is complicated by the endogenous nature of the portfolio's risk. Verification of risk exposure estimates calculated over a long horizon against actual portfolio P&L, compares the market-based risk estimate for a portfolio of fixed composition to a P&L performance generated by a series of portfolios different in composition[58]. Any true long-horizon risk

[58] As mentioned earlier, historical VaR methods cannot circumvent a portfolio aging process. Indeed, even

exposure verification scheme would also have to be based on the long-horizon profits or losses generated by repricing the initial portfolio. Monitoring schemes based on historical performance are internally consistent, only when they compare a portfolio's potential loss estimate with the same portfolio's actual performance.

Finally, because reliable performance-based verification tests require significant amounts of data, the verification process is necessarily time-consuming which generally compels some institutions to work a while with VaRs whose accuracy they have not even yet been able to verify and will not be able to do so until enough data has been recorded. It would take so long[59] to gather a history of required two-week P&L that Kupiec and the Basle Committee recommend to do one-day performance comparisons when backtesting VaR accuracy. Some institutions, as a result, have opted to conduct daily VaR controls using analytical VaRs rather than using historical ones.

II - <u>Performance-based testing procedures</u>

2.1. - <u>Test statistics</u>

Day-to-day P&L determine the outcome of a binomial event: either the firm's loss on trading activities is less than its *ex ante* VaR estimate (a success), or the loss exceeds the VaR estimate (a failure). If daily forecasts are efficient, estimates are independent across days and the performance data are distributed as a series of draws from a Bernoulli distribution.

For a 99% confidence level, the null hypothesis - that the probability of a failure on any

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over a single day, an institution risk profile can be significantly altered by intraday changes in positions. If intraday exposure changes are significant, verification tests should be based on performance calculated by revaluing the original portfolio.

[59] It takes as long as ten years to gather 1,250 two-week returns.

day is 1% - can be tested in a variety of ways. The appropriate test depends on how the institution is being monitored and on the size of the sample available for performance comparison. For internal verification, since the institution is monitored continuously, as soon as a single failure is observed, the financial manager/risk controller can formally test the hypothesis that the firm's true failure rate is the 1% used for its reported VaR: this test is the time until first failure test (TUFF).[60]

An alternative approach is appropriate for supervisors who can only oversee performance at less frequent intervals and test the null hypothesis using the proportion of failures observed in the monitoring period. This test is the proportion of failures test (PFT). These two tests can be used complementarily.

Finally, for external verification of VaR models accuracy by regulators or examiners, it is also possible to compare the reported VaRs against losses the firm could have experienced by simulating a P&L. In effect, external supervision aims to unveil whether the model used, accurately estimates potential losses, therefore, it calls for using a P&L other than the one the firm could provide: since the purpose is to verify the institution/model integrity it would not be appropriate to rely on an internally reported P&L distribution.

Verification tests can also be constructed that use information about the parametric form of a portfolio potential P&L distribution. For example, Crnkovic and Drachman (CD) [1995] use Kuiper's [1962] results to construct a goodness-of-fit measure for an estimate of the entire P&L distribution. Using a symmetric weighting-function, CD specialize their goodness-of-fit measure into a test of the accuracy of a risk management model tail's profitability estimates.

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[60] Kupiec, "Techniques for verifying the accuracy of VaR measurement models, *The Journal of Derivatives*, Winter 1995, pp.73-84

This weighting function places an equal importance on the extreme P&L tail events which implicitly assumes the underlying distribution is symmetric, which may not be appropriate since for financial instruments occurrence of extreme losses is more common than that of extreme gain. This underlying asymmetry may be compounded if there is a concentration of option positions in an institution's portfolio. CD conclude from a Monte Carlo analysis that their testing procedure requires a minimum of 1,000 observations to be reliable. If the asymmetry assumption is discarded, the test requires additional data. Compared to CD's testing procedure, Kupiec's are computationally simpler and may be more accurate in smaller data sets[61].

2.2. - TUFF test statistic

This test is based on the number of observations (or days) until the first failure (until the first time VaR is exceeded by actual losses). The variable is T, p is the probability of a failure on any given day, V is the period in which the first failure is observed. Then:

$$P(T=V^*)=p(1-p)^{V-1},$$

and T follows a geometric distribution with expected value 1/p, the average time until the first failure. Given a specific value of V, Kupiec tests whether the VaR estimate is consistent with H_0, that is, whether it is possible that the VaR corresponds to the stated confidence level given that the first failure occurred on day V and given a specific Type I error rate (probability of incorrectly rejecting the null). A likelihood ratio test is used for the procedure, and the test statistic allows to calculate acceptance regions in terms of $T=V^*$, the given number of days until the first failure.

It is found that the acceptance region grows as the tail probability used for VaR

[61] Ibid.

decreases, that is, the higher the confidence level the more difficult it becomes to reject the null hypothesis. If V is lower than the smaller value prescribed by the acceptance region, the tail-probability is higher than the one tested (the true failure rate is higher and the true confidence level is lower and, the VaR is underreported), if V exceeds the highest possible value, than the true tail-probability is lower than the value tested and the true confidence level is higher than reported and the is overstated

With a Type I error rate of 5% (TUFF(5%)), testing failure rates greater than 5% always yield a failure on the first day and the rejection of H_o. When p is assumed to be greater than 5%, the TUFF(5%) rejects H_o only when the true tail probability is smaller than 5%, in brief, the TUFF(5%) cannot detect models with failure rates greater than 5%. If the Type I error rate exceeds the failure rate, the test can detect failure rates greater then the level tested in the null.

When p is small, the acceptance region is very large, for instance, when H_o : p = .01, for the TUFF(5%), the acceptance region is $6<V<439$, meaning that if a failure occurs on he 7th day, the null is accepted although the maximum likelihood value of the failure rate p is $1/V^*=1/7=14.3\%$, showing that the test cannot distinguish among a wide variety of alternative hypotheses. A failure rate of 1% is accepted even when on average it could be 14 times higher (14%): the TUFF(5%) has very low power

Kupiec also investigates the Type II error rate, the probability of wrongly accepting the null. It is inversely related with the Type I error rate, and is a function of the true value of the tested parameter, here the failure rate or tail probability. The farther the true value of p from its tested value p*, the more easily will the test detect a difference between p and p*, and the smaller will the Type II error rate be. The power of the test is (1- Type II error rate). Kupiec finds that the TUFF has very low power. For instance, it will accept that the VaR is calculated at 99% confidence (that is with a failure rate, or p, of 1%) even when it is really calculated with 95% confidence, in TUFF(5%), H_o : p*=1% will be accepted even when p=2% in reality, and the Type II error will be 86.8% and, there is still as much as a 75.1% chance (Type II error rate) to accept p*=1% when p is really 4%!

The test cannot distinguish between low probabilities such as a 1% versus a 2% failure rates which can ,however, translate in huge differences in critical values (that is, in VaRs). Kupiec takes a rigged example of a t-distributed returns series with 1 degree of freedom in which the 1% critical value is -31.82 whereas the 2% critical value (VaR) is -15.89. In this case reporting a VaR of half the true risk exposure would go undetected. Small probability differences (small failure rate differences) are virtually impossible to detect with the TUFF(5%) even when such small differences can yield substantially different VaR metrics.

The TUFF(5%) Type II error rate can be reduced at the expense of the Type I error: this reduces the acceptance region from both sides but the power remains low, even for Type I error rates as large as 25% because of the, then, small sample size.

2.3. - PF test statistic

Continued monitoring beyond the first failure provides additional information. This test verifies the consistence of the reported failure rate (or correspondingly, the confidence level) with the proportion of failures observed in a given period. Thus, when the TUFF cannot reject H_o, the proportion of failures test (PF) can be conducted.

The procedure involves the following. X is the number of failures, regardless of their order, observed in an n-day period and is a binomial variable:

$$P(X:n)=nCx\ p^x(1-p)^{n-x},$$

where p is, as before, the failure rate for VaR on any one day. Once again the likelihood ratio test is conducted, but this time, Kupiec highlights the maximum sample sizes required to reject the null that p = p*. For instance when 6 failures are observed, it takes at most 240 days to reject the null that p = 1% with a 5% Type I error rate, if there are 6 failures in 241 days you cannot reject the null, or it takes 49 days to reject p = 5% with the same 6 failures.

Kupiec also pegs the sample size n, to investigate the number of failures that must be observed to reject the tested failure rate (or correspondingly, the VaR confidence level). He finds that in one year (n=255 days), it takes at most 6 failures to accept that p = 1% (while its maximum likelihood value goes from 0 to 7/255=2.3%), and from 2 to 10 failures in two years to accept p = 1% with 95% confidence (Type I error rate of 5%), in 4 years it takes from 5 to 16 failures (if there are 4 or fewer failures than p<1%, if there are more than 16 failures p>1%).

The PF(5%) also has low power in small samples: the test shows large acceptance regions (high Type II error rates, high probabilities of accepting the null when it is wrong). And even in large samples, the power of the test for local alternatives (values of p, the failure rate, closed to the tested value, within 110% of p*) is very low. The power rises when the true p is more than 110% away from p*.

2.4. - Results

The results indicate that, unless a relatively long performance history or historical simulation data base is available, there are significant difficulties surrounding verification of

VaR estimates. Even when tests are based on daily performance comparisons, small sample test statistics have extremely poor power for detecting a model or institution that habitually underestimates market risk exposures even when the underestimation is substantial. Reliable performance-based verification techniques require a relatively long comparison period.

III - Simulation-based test

Verification schemes need not be based on historical performance, however. As an alternative to monitoring an institution's actual performance, its loss estimates can be compared periodically to the performance of a simulated distribution, which (it is reminded) would be appropriate for external supervision and VaR model validity assessment. For example, a bank examiner who would not have the continuous monitoring period data (or would not want to use it for integrity reasons) could still evaluate the institution's reported VaR performance by simulating retrospectively the P&L (using historical changes in market factors) the current portfolio could have experienced compute VaR based on these simulations and then compare the reported VaR against the simulated VaRs

3.1. - The dilemma

There are two levels of uncertainty in VaR constructs, (1) the frequency of their failures (which was just assessed), and (2) the accuracy of the obtained VaR values: in other words, are the losses' sizes (or dollar values) well forecasted, should the VaR have been reported to be exactly $15 million or assumed to be in that range within $2 (or 5) millions? No one knows the amount of error in the VaR metric (some practitioners do report VaRs with error bands). But the accuracy of the VaR magnitude is not just one of dollar values of losses, it also affects its probability of being exceeded. In effect, If the VaR value is underestimated (due to sampling errors), it will be violated at a greater frequency than predicted by the confidence level used to construct it. Thus a valid and well implemented VaR methodology could be invalidated due to too frequent failures, not because of its intrinsic (in-)accuracy but simply because of undetectable distortions in the computed VaR caused by unavoidable sampling errors.

From the viewpoint of VaR users (and regulators), testing the statistical significance of the model is important but it is, also crucial, to assess the degree of precision in the magnitude of the reported VaR number. This measure of potential loss, whichever method is used to compute it, is always calculated by estimating actual losses through samples of past data. Once the essential parameters for the measurement of VaR, means, standard deviations, and quantiles from actual or simulated data are estimated they should not be taken for granted. They are affected by estimation error, the natural sampling variability due to the limited sample size. It is important to be aware of this for model validation: for VaR users the model must be useful and predict risk well; regulators need to be able to detect systematic biases in the reporting of VaR. For instance, when a VaR is found to be, say, $10 million, in what range is the actual loss likely to occur? Within $2 millions of VaR? Within $5 millions? The computed VaR is just an estimate of the true VaR but to be useful it should not be substantially different from it.

Kupiec illustrates this dilemma by the case where VaR is obtained from the historical-simulation method, with a reference period of 7 days to assess the riskiness a portfolio exhibited in the past: VaR model accuracy is tested retroactively by comparing reported VaR estimates to the simulated VaR estimates. VaR is calculated based on estimates of the critical values of the true distributions of returns, therefore the simulated VaRs (critical value estimates) have substantial sampling errors (although, ideally, with an infinite number of observations and a perfectly stable system, these estimates converge to the true values). In practice, however, sample sizes are limited, either because some series, like those of emerging markets are relatively recent or because structural changes make it meaningless to go back too far in time. As some estimation error will generally remain, the natural dispersion of estimated values can be measured by the sampling distribution of the estimates (of the true critical values).

3.2. - Verification scheme

In this approach, the critical values of the portfolio P&L distribution are generated by historically simulating the day-to-day P&L the institution's current portfolio would have generated if it were held over some fixed historical period. The only assumption, needed here, is that the statistical process that generates the asset price changes from the market factors is stationary. No assumption is needed about the covariances since they are captured by the historical simulation.

Loss exposure estimates (VaR estimates) can, in theory, be corroborated using the critical value estimates of the historical loss distribution of an institution's current portfolio. However, the simulated VaRs, because they are obtained from estimates of the true distribution contain sampling errors which vary with the length of the observation period (or the size of the sample) and the number of simulations conducted. In the limit, the simulated P&L distributions and resulting critical values will approach the true value but large sample sizes are necessary to reduce the errors substantially.

3.3. - Measurement errors

It has been shown that the variance of an estimate of a critical value is a function of the tail probability and of the sample size[62]. Thus the variability in the VaR (critical value) is based on its confidence level [(1-p)x100%] and the length of the reference period: the higher that confidence level, or the shorter the observation period, the greater the variability in the VaR. For instance, the standard errors of the 1% and 5% critical values of X_p, when X_p has a standard normal distribution , are respectively:

$$SE(1\%)= 3.7989\ \sigma\ n^{-1/2} \text{ and } SE(5\%)= 2.3104\ \sigma\ n^{-1/2},$$

When the length of the observation period increases, the standard error falls. the estimate of the critical value becomes closer to the true theoretical (average value), for instance. X_p approaches 2.33 for 99% confidence, or 1.96 for 95% confidence.

3.4. - Results

Kupiec shows the variability in the critical value (or VaR) estimates of standard normal and t-distributed (with 8 and 2 degrees of freedom) P&L when sample sizes varying from 100 to 2500 observations (from half a year to 10 years of daily data). He conducts comparisons of simulated critical values for normal and t-distributions to show the effects of sample size, type of distribution and sample errors on the magnitude of critical values of the left-hand tail probabilities.

When potential loss distribution are fat-tailed. simulation-based critical value estimates exhibit significant biases and have standard errors of substantial magnitude, even in relatively

[62] Kendall and Stuart [1960] have shown that when X_p is the critical value or *quantile*, then variance(X_p)≅ $p(1-p)/[n\ f(X_p)^2]$. where f is the probability density function and n is the sample size (or length or the reference period). so that the integral from negative infinity to X_p is p%. Ibid.

large samples. Kupiec underscores the average value of the estimate, its standard error, its maximum and minimum values, for each sample size as well as the theoretical average value. He shows that historical simulation-based verification schemes also perform poorly unless historical simulation sample sizes are large.

The drawback of this external verification scheme is that the resulting simulated VaRs against which the reported VaRs are verified, are not reliable. In effect, large sampling errors are associated with the simulation obtained critical values. The historically simulated distribution is only an estimate of the true distribution of returns and is subject to estimation errors associated with in sampling. Very large samples are required to decrease the sampling error (or standard error) of the critical values of low probability events (higher confidence level VaRs).

The results show that because of large sample errors, historical simulation-based verification schemes also perform poorly unless historical simulation sample sizes are large. The ranges for the estimated critical values (and thus, of the obtained benchmarking VaRs) are large, even in large samples. Regulators using that verification scheme would therefore have no way of verifying model accuracy nor of detecting model underreporting since they cannot simulate reliable VaR benchmarks to gauge reported bank VaRs against.

Some financial economists think that using a parametric approach in the simulation is more precise because the estimated standard deviation relies on the entire distribution of the returns whereas the quantile only relies on the two values around it (the quantile is obtained by ranking the observations and selecting the value such that only 1% of the observations are smaller) and they may be significantly different from one sample to the next. However, using the normal distribution (or any other parametric distribution) introduces other biases in the VaR method that an historical (empirical) approach does not generate. Thus the decision of which method is best should be based on a comprehensive comparison.

In the end, whether the VaR is estimated from a parametric or an empirical distribution it is always subject to large sampling errors which are more important the smaller the sample size and the greater the confidence level. As a result risk managers and regulators should always keep in mind the error bands embedded in the obtained VaR constructs and that (whether obtained through historical or analytical approaches) stochastic VaRs are the most reliable estimates.

Chapter V - Empirical Treatment

I - Introduction

1.1. - Objectives of the empirical treatment

"The specter of the great stock market crash of 1929 continues to haunt and to fascinate the public... After over half a century of mulling over the events of the 1930s, there is still no consensus among economists as to what really caused the Great Depression"[63]. The purpose here is not to answer this question but rather to observe what happened in the stock market during this forty-three month contraction of the US economy and to investigate, retrospectively, whether parametric and nonparametric VaR calculations (1) would be consistent with the events and losses of the Great Depression for portfolios of blue chip stocks and, (2) would have allowed investors to protect themselves against losses and even bankruptcies. Could these VaR models, had them been known at that time, have forewarned stock market investors of the imminence of market contractions and signaled how much of an hedge was necessary against possible losses? Could they have apprised investors of their actual risk taking and capital at risk by providing adequate measures of the value at risk in their portfolio of blue chip stocks so that they can set aside provisions to survive financial markets downfalls? Answering such questions is the purpose of this empirical treatment, and the Dow Jones Industrial Average, DJIA, is going to be used as the proxy for the investment portfolio on which VaR measurements are computed.

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[63] "Understanding 1939-1933. *Rochester Studies in Economics and Policy* issues. *Vol. 2, The Great Depression revisited.* 1981

1.2 - The context: the Great Depression

The Great Depression was a forty-three month economic contraction experienced by the US economy in the 1930s. The turning point is, now in retrospect, estimated to have occurred around June of 1929, contrarily to people's common belief that the October stock market crash was the detonator. This recession lasted four years until September of 1933. Although most people believe that it was the financial market crash which initiated this recession most economists now believe that this crash was just a symptom/consequence of this depression just as the activity decline, fall in the general level of prices, and the massive unemployment which characterized this economic crisis.

There was a high level of speculation in the stock market in the late twenties[64] as measured by the high proportion of low-quality stocks traded relative to that of high quality ones[65]. By late 1929, stock prices had been rising almost uninterruptedly for eight years increasing fivefold, and volume at the time of the peak was exceptionally heavy[66]. However, from September, 1929, to April, 1930 stock values fell in real terms over seven months by almost 20% of their values.

For some economists, the explanation of the Great Depression lies in the Keynesian income-expenditure framework: collapse in investment following a period of overinvestment

[64] Investment and banking functions became closely intertwined in the twenties as banks joined the general public in speculating on equities. Some of them even capitalized on the opportunity to speculate with depositors' money by purchasing equities outright and/or underwriting new stock issues. The market collapse of 1929 wiped out numerous banks and their depositors. In 1933, Congress passed the Glass-Steagal Act that separated traditional banking from investment banking activities.

[65] The rationale is that low quality stocks are more volatile and in a bull market and their demand and prices will rise proportionately faster than better-quality stocks because of the attractiveness of their high potential profits. Studies have shown an outstanding speculative peak in 1928-29 with a trough in 1932. [65] Martin J. Pring, "Technical Analysis Explained, an illustrated guide for the investor", McGraw Hill, 1980.

[66] See Graphs of the DJIA monthly and weekly closes in Exhibit V of the Appendix.

which led to the stock market crash and the revision of expectations which through the multiplier effect induced a deep decline in output and employment[67].

Schwartz distinguishes five negative and one positive shocks during that period which serve as benchmarks to divide the depression into five subperiods referred to later in the analysis of the empirical findings:

> (1) August '29 to October '30, the period prior to the first bank panic which includes the stock market crash; (2) the final quarter of 1930, when the first bank panic occurred; (3) the first quarter '31 which is characterized by the second bank crisis in March '30; (4) the last half of '31 during which the Federal Reserve's reaction to Britain's departure from the gold standard worsened the US economy's decline; (5) the second quarter of '32 when the Federal Reserve's open market purchases provided a revival of the economy in the summer and fall of 1932 and; (6) the final six months of contraction when bank problems spread (Bank Holiday of March 1933) when the economy worsened again[68].

Analysis of the empirical treatment and of the VaR measures computed in the process will look into the patterns of the P&L of the DJIA stocks during these periods, the monthly data is separated in several periods: (1) 1920-1928, the years before the depression; (2) 1929-1933: the Great Depression years: (3) 1934-1936: the years immediately after the depression.

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[67] "Understanding 1939-1933. Anna J. Schwartz, *Rochester Studies in Economics and Policy* issues, *Vol. 2, The Great Depression revisited*, 1981, pp. 5-48

[68] Ibid.

Figure 1.a.

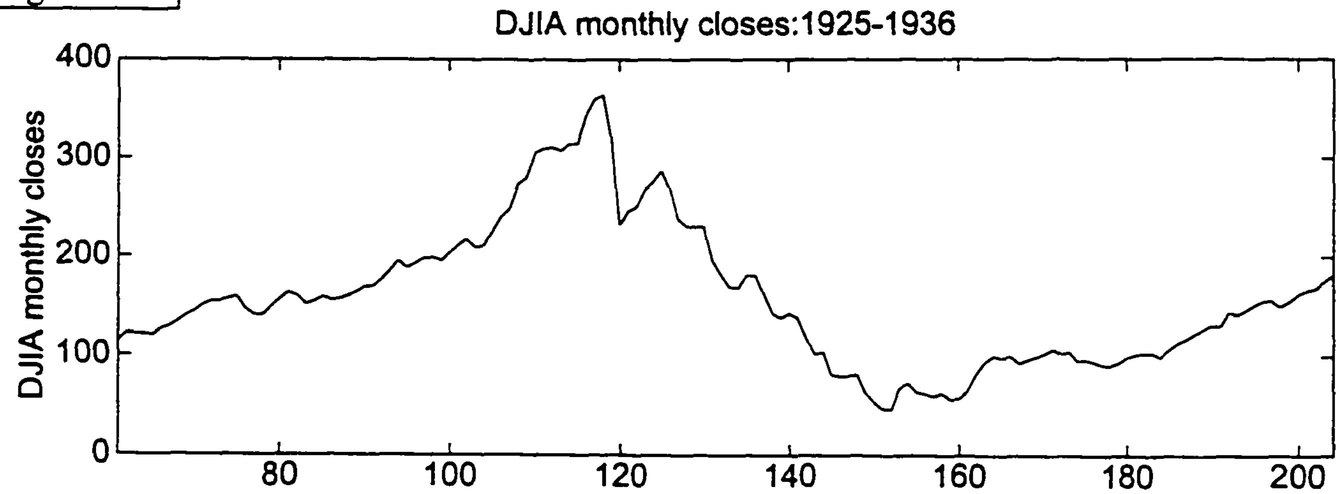


Figure 1.b.

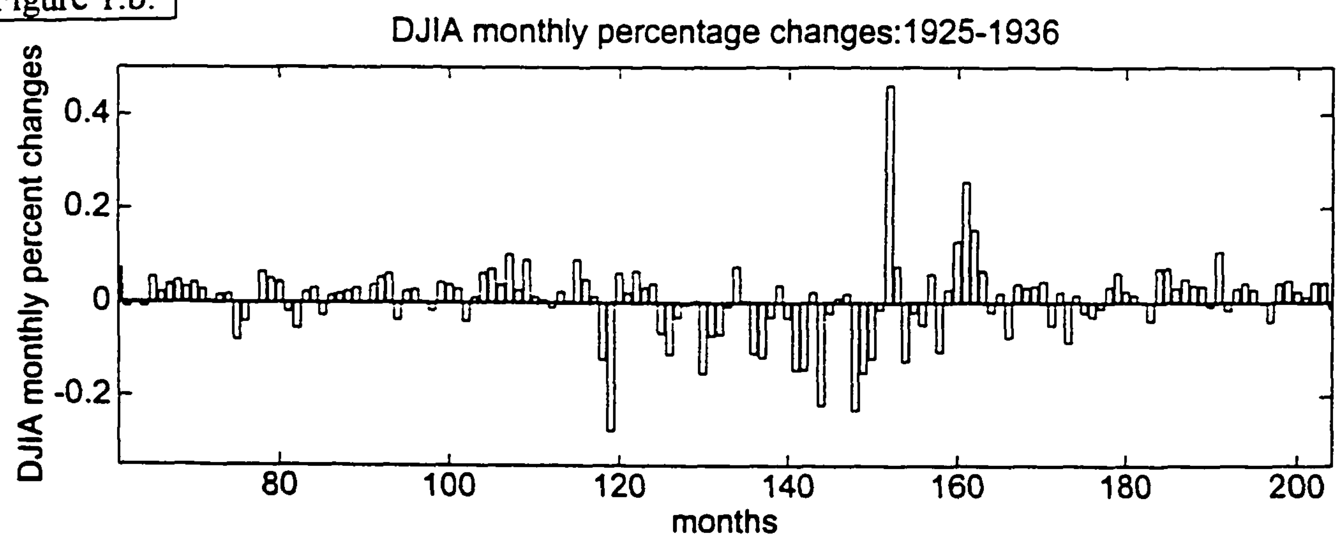


Figure 2.a.

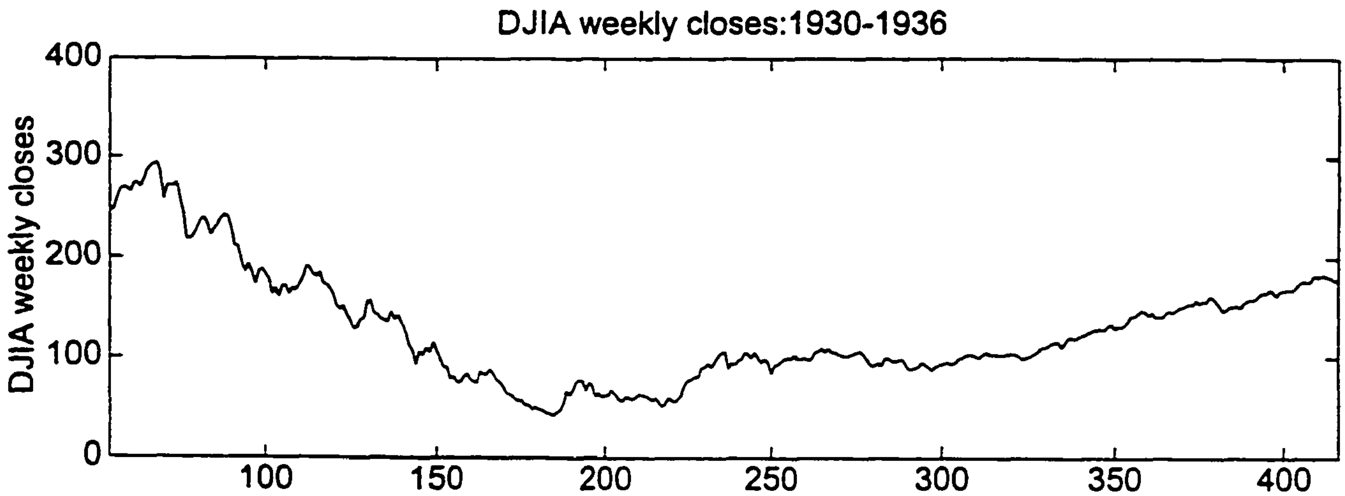
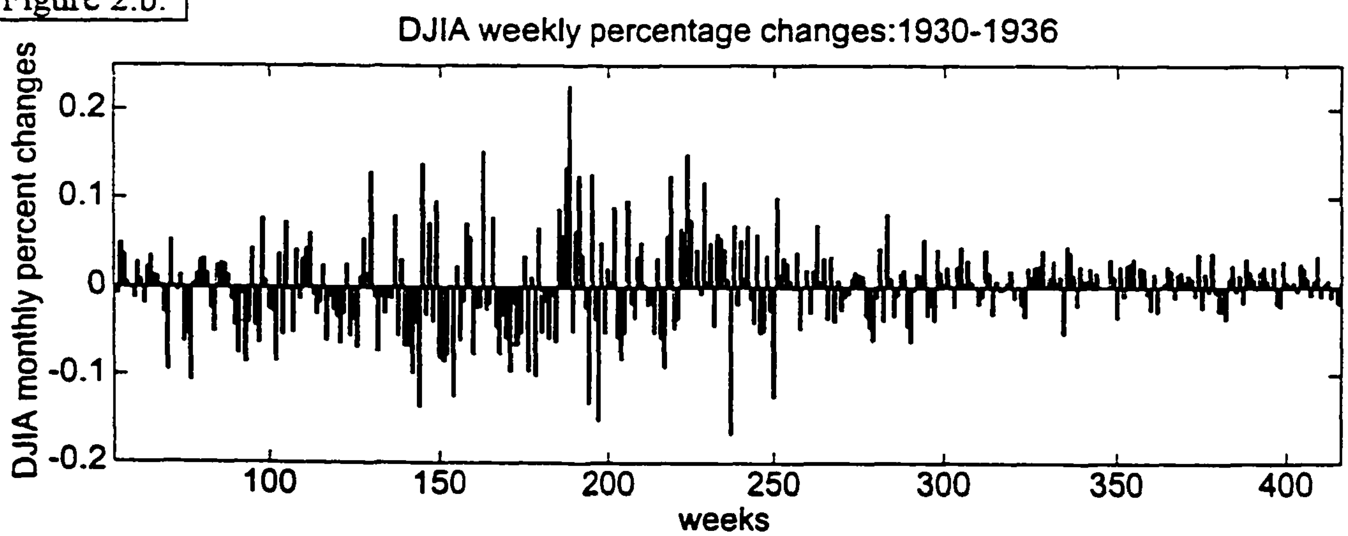


Figure 2.b.



1.3. - Stock market indicators

There are two types of stock market indicators which measure the general level of stock prices: unweighted and weighted ones. Unweighted indicators are market averages calculated by adding together the prices of component stocks and dividing that sum by a specific divisor. Market indexes are weighted indicators that give a greater weight to the larger companies: they take the average of the prices of a number of stocks by weighting them by their capitalization, that is the number and market values of shares of each company, and compare the current level of component securities' prices or market values with a base period value. These latter indicators are thus more representative of changes in value of a nation's portfolios.

The DJIA is the most widely followed stock market indicator in the world. It was created in 1884 with 11 popular stocks, mostly railroads companies[69]. It is a price weighted average, which since 1928, is constructed by summing up the prices of thirty blue chip stocks and scaling the total by a divisor which is changed after stock dividends and splits, and changes in the composition of the average. When a stock price increases and there is no split, its influence on the value of the DJIA becomes substantially higher, especially if other component stocks are growing in price and splitting. The "Dow" is not very representative of the stock market because (1) it only includes a small number of (2) high quality stocks and also because (3) it is distorted over time because of its divisor adjustment procedures which create a built-in bias for greater volatility. However, although it is not a composite index, its capitalization accounts for 25 to 30% of the NYSE market value and has proved to be a thoroughly reliable indicator of the general movements of the market. In effect, over the years, the Dow has acted fairly consistently

[69] The original reason for including a relatively small number of stocks was convenience because the average was calculated by hand.

with many of the more widely capitalized market indicators such as the S&P500.

1.4. Data set description

The data consists of seventeen years of monthly DJIA closes ([1920-1936], 204 observations) and of seven years of weekly DJIA closes ([1929-1936], 419 observations). These DJIA statistics are readjusted for stock changes, dividends and splits sampled from "Technical Analysis Explained, an illustrated guide for the investor"[70]. The period of reference chosen for this study is 1920 to 1936 in order to follow up on the evolution the VaRs before, during and after the Great Depression. This is not a real record of the Dow though, since, from time to time (through mergers or takeovers, for example) new stocks have been substituted. From 1920 to 1928, only twenty stocks composed the average whereas thirty securities were used thereafter. As a result, there are, in fact, two different indexes, spliced together to maintain some continuity. The weekly DJIA unfortunately start only in 1929 and do not directly allow VaR calculations before this period[71].

Finally, it is worth underlining that these DJIA statistics are closes and not period averages, thus for instance, while the highest fall between two consecutive monthly closes, over the reference period occurred October and November of 1929 (-27.45%). This change only represents the relative difference between DJIA levels at the end of those months and not the difference of their average value (even more since the Black Monday occurred in late October, 1929). The closes may not accurately reflect true events and trends and may thus, introduce

[70] Martin J. Pring, "Technical Analysis Explained, an illustrated guide for the investor", McGraw Hill, 1980.

[71] Although similar figures could have been found elsewhere, this choice was not made to preserve the consistency of the data and not mix DJIAs calculated differently, which would have defeated the purpose of completing/improving the data.

some bias in the analysis.

In the same vein, when examining the weekly statistics for that same period, the closes were recorded on October 25 (before the crash) and on November, 1. As a result, although during that week, changes were abnormally high between days, the closes do not reflect this high volatility. Therefore, parallel comparisons, over same time periods when done at the monthly and weekly levels may not exhibit the same patterns nor yield consistent results.

II - Computations

2.1. - Patterns of changes in the DJIA

Throughout the twenties, the long eight-year bull market, interrupted briefly from time to time by *corrective* reactions is fully reflected in the monthly Dow levels which rose steadily from 75.10 in January, 1920 to culminate at 364 on September, 1929. The DJIA values also closely mirror the New York Stock Exchange volume statistics for that period. In effect, at the beginning they of the decade, they fluctuated within a 10-to-20 index range and started accelerating at the end of 1924. For the next three years volume oscillated between 30 and 50 to jump to another level in 1928 when, for the first time, it exceeded 100. It reached 115.40 in September, 1928, a level surpassed only three times in the reference period, in October, 1929 (141.70), and in June and July ,1933 (respectively 125.60 and 120.30). During the depression, volume fell steadily back to its levels prior to the contraction[72]. Figures (1.a.) and (2.a.) and (3.a. and b.) show the evolution of the DJIA during the reference period and the depression[73].

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[72] Volume statistics obtained from Martin J. Pring, Ibid

[73] For a wider view of this expansion (from 1920 to 1936), see Figures (A1.a) and (A1.b) in Exhibit V of the appendix

Figure 3.a.

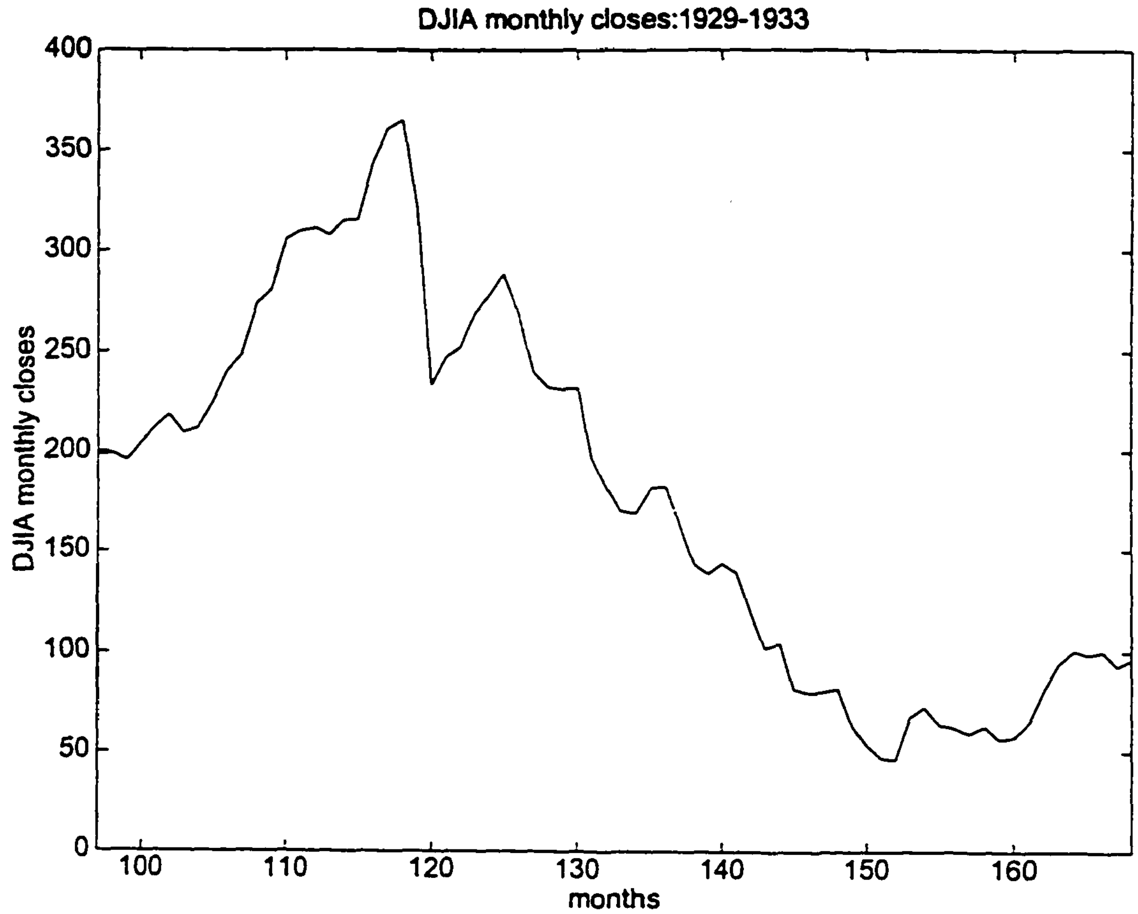


Figure 3.b.

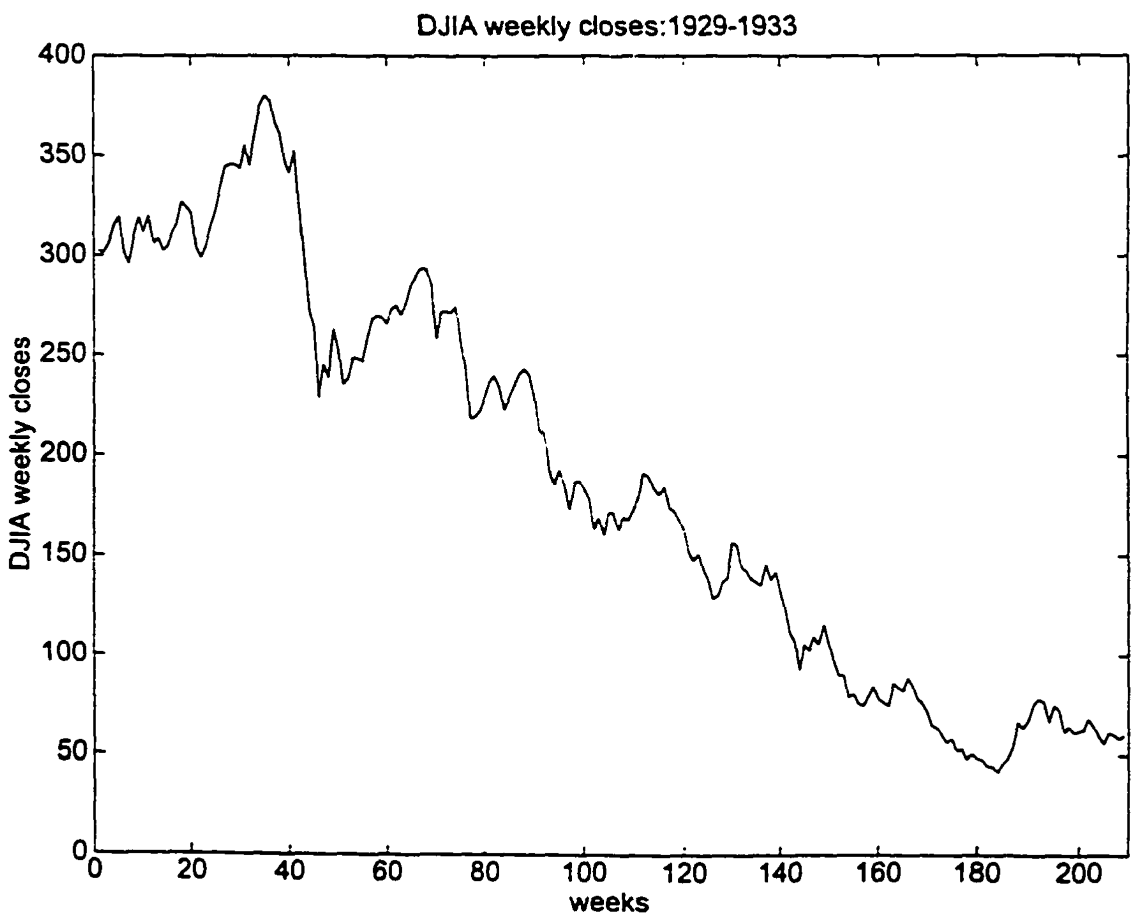


Figure 4 .a.

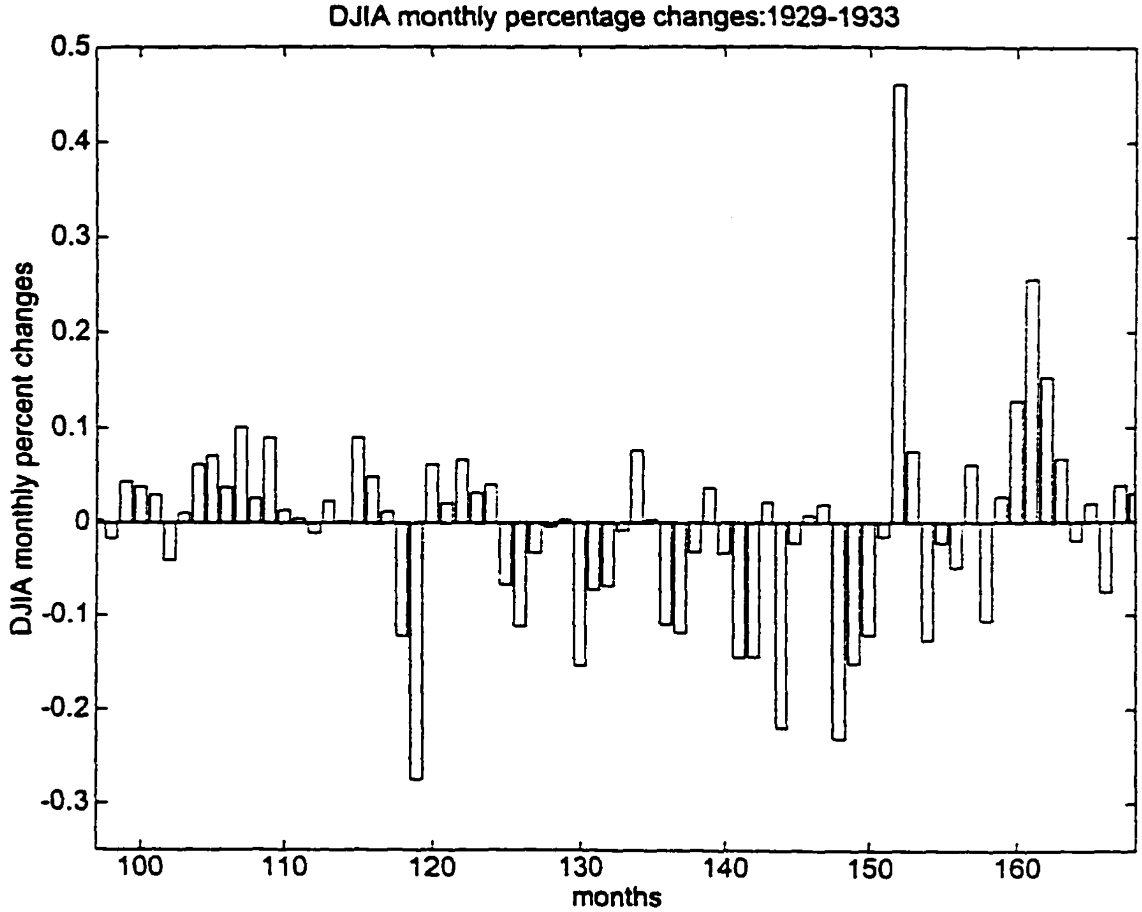
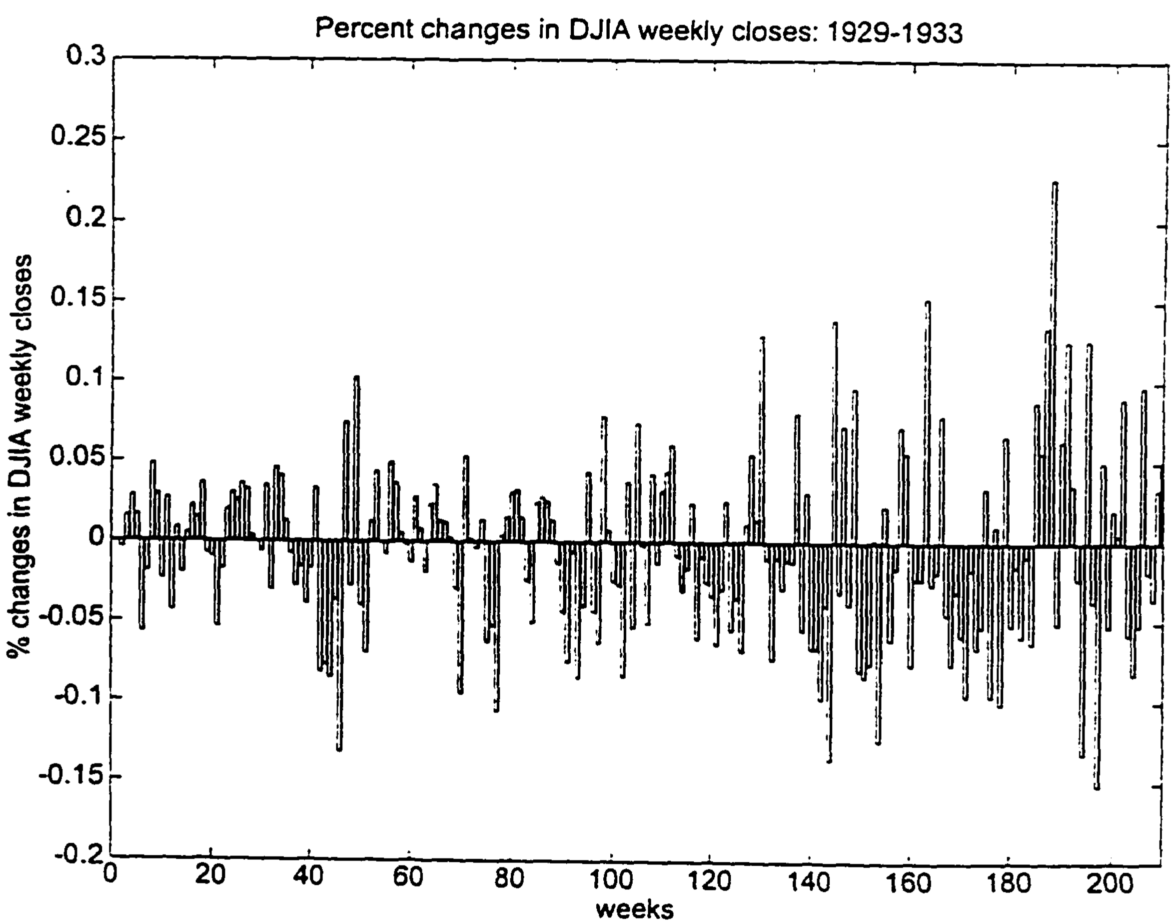


Figure 4 .b.



Returns[74] are computed from the DJIA monthly and weekly variables and are actually relative or percent changes in DJIA consecutive values. The rationale is that a corporate investor which would have invested, say $100,000, in the DJIA stocks would have earned that return would him have held on to its portfolio for a month, or a week. Returns are calculated for one week and one month holding periods since VaR computations are typically calculated on a one-holding-period basis, and mostly because (1) in general, investors will hold on to stocks at least a few days and (2) back in the twenties and thirties trading was certainly not as frequent as today.

Average returns, standard deviations and worst losses, for each data set and each year, are calculated and presented below to provide a greater insight of the DJIA profitability and variability during the reference period.

<u>Table 1: DJIA changes statistics: 1920-1936</u>

	Before the Great Depression									Great Depression					Aftermath		
Years	'20	'21	'22	'23	'24	'25	'26	'27	'28	'29	'30	'31	'32	'33	'34	'35	'36
μ_{month}	-3.0	86	1.83	-.25	1.69	2.6	36	1.89	3.0	-.53	-2.9	-5.6	-1.42	4.83	28	2.89	2.04
σ_{month}	4.85	4.47	2.54	3.35	3.68	2.6	4.44	2.86	3.85	10.1	6.62	8.92	17.2	9.81	4.33	4.18	2.66
WL^1	-9.33	-10.5	-4.58	-5.31	-4.78	-.91	-8.04	-3.68	-4.04	-27.45	-15.33	-21.94	-23.1	-10.5	-8.63	-3.88	-4.17
μ_{week}										-.37	-.67	-1.3	-.16	1.11	.21	62	.43
σ_{week}										4.31	4.18	5.96	7.73	6.01	3.18	2.09	1.77
WL										-13.2	-10.61	-13.6	-15.2	-16.7	-6.24	-5.31	-3.63

μ: annual average return (%). σ annual standard deviation of returns (%). WL: worst annual loss.

[74] Percent changes in the DJIA closes are referred to as *returns* in this chapter since they are the closest proxies of one-period returns

Figures (1.b.) and (2.b.) and (4.a.) and (4.b.) illustrate the pattern in the monthly and weekly percent changes respectively. It is important to mention that there is a substantial difference between monthly and weekly changes: during the depression there is a dominance of negative returns in monthly data whereas in the weekly set, observations exhibit a great number of losses (of smaller size than the monthly losses) but also of numerous positive returns of equivalent sizes which offset them. This is worth noticing since it will have an impact on the VaRs.

Another point worth mentioning is that, during the period, *standard percent changes*, ([%ΔDJIA$_{t-1,t}$ - μ)]/σ), where μ and σ represent the average and volatility of such changes over the period), had a very "thin waist" distribution with a large concentration around zero and awithin two deviations of the mean (See Figure (A2.a.) and (A2.b.) in Exhibit V of the Appendix). This distribution reflects a general small range of variability in the percent changes (whether at the monthly or weekly level) and few outliers (more-than-three-deviation returns four, and three respectively for the monthly and weekly data).

Finally, it is also interesting to examine the pattern of these changes according to the different phases of the Great Depression. Monthly changes do mirror the events of the contraction, as seen in Figure (4.a.), and they do follow the general state of the economic performance since each of the five negative shocks (the stock market crash and the four episodes of bank crisis) as well as the two brief revivals correspond respectively to periods of losses (or decreases in the DJIA) when the shocks are negative and periods of profits (or increases in the DJIA) when they are positive. This correspondence which is striking at the "aggregate" monthly level, is not however, at the weekly level as it can be seen in Figure(4.b.), where random fluctuations in returns cloud out and obscure the trend (or pattern) in the state of the economy (as shorter-term variables are expected to. and usually do).

2.1. - VaR computations

The historical VaR method calls for the use of past observations (prices of securities composing the portfolio studied) to compute VaR estimates. Accordingly, in the nonparametric approach, changes in a portfolio value are calculated over previous periods, say from 53 previous weeks, to obtain a P&L distribution which is then sorted in ascending order. The first observation thus obtained represents the worst loss of the period. Then, first, fifth and tenth percentiles are approximated from the next higher returns (in that ordering) of the historical P&L and serve as VaR estimates for the next period (t+1). Then. at time (t+1), to obtain the next period VaR estimate, that of time (t+2), the oldest observation is dropped from the reference period, that of time (t-51), the second oldest observation becomes the new oldest one. and the most recent one is added (the one which was previously the actual current profit or loss against which the VaR estimate was measured at time (t) This is how new VaR estimates are calculated for successive periods, in a moving motion

The parametric version of VaR also calls for the use of past data. But this time, they serve to compute the expected value and standard deviation of the obtained P&L distribution, which in turn are used to compute the VaR estimates through the same procedure, using a moving window of previous observations VaR is estimated relative to the mean return of the period, at different confidence levels Returns are assumed to be roughly normally distributed, and for instance, at the 99% confidence level, the VaR estimate is the mean return minus 2.33

standard deviations:

$VaR_{Mean} = \mu - 2.33\sigma$

VaR estimates provided by these methods exhibit one-period lags since next period VaRs are computed currently based on previous values.

A five-year reference period is used to calculate monthly VaR estimates (sixty observations) and one year is used (fifty-two values) for the weekly statistics(as a result, the first monthly VaR outputs is in 1925 whereas the weekly output starts in 1930). The choice of observation period is the result of a balancing act between providing enough data for the computations but not using a period too long so that it would no longer reflect the current dynamics of the markets. These choices are satisfying since they correspond to regulatory guidelines and general practices among VaR users.

III - Results

3.1. - General findings

Parametric VaR measures consistently yield greater values at risk than nonparametric estimates (except on a few instances at the 99% level), closer to the actual realizations and, less frequently violated. As such they provide a better basis to calculate capital requirements. 99% level estimates generally forecast potential worse losses better both in terms of both magnitude and schedule of occurrence, and more accurately since they are generally closer to actual realizations[75]. Although, both parametric and nonparametric VaRs tend to be better estimates in

[75] In light of these findings, which support opponents to the Basle Committee requirement to use a scaling factor of 3 to 99% confidence VaRs for prudential capital allocation and investigators' findings that the model approach sufficed to withstand most contemporary market stress periods, it is understandable that so much controversy surrounded that issue. However, these results cannot serve as an additional argument against that proposal since they are obtained from only a brief, simplified application that only addresses a

times of low volatility (when returns are small, above and below zero) and in addition, tend to yield very close loss estimates, parametric VaRs, however are more distorted by and take longer to adjust to shocks than nonparametric ones. But these latter do suffer the expected *plateau* effects due to the presence of abnormal changes in the reference period. These abnormal returns are smoothed out in the parametric approach but remain in the VaR computation for 60 months and contribute to delay its adjustment after a shock has occurred.

In general, parametric VaRs also exhibit a better track record in terms of violations. In effect, comparisons of the two types of VaRs at the same confidence levels show that parametric VaRs are less frequently violated than the nonparametric estimates (See tables (3) and (4)). Finally, calculations at the weekly level provide more accurate predictions of potential worse losses: in general the VaR measure falls closer to the actual realization, is better synchronized (no or almost no time lag) and suffer fewer failures which, in addition, are smaller size violations.

However, in spite of these defects, computed VaR estimates are consistent with the general theory that they apprise investors of the value at risk and could provide a fair basis to calculate a prudential capital provision against potential losses. It remains important though, to bear in mind that these are warning signs which although they are sometimes overstated will, ultimately be exceeded when the market is down

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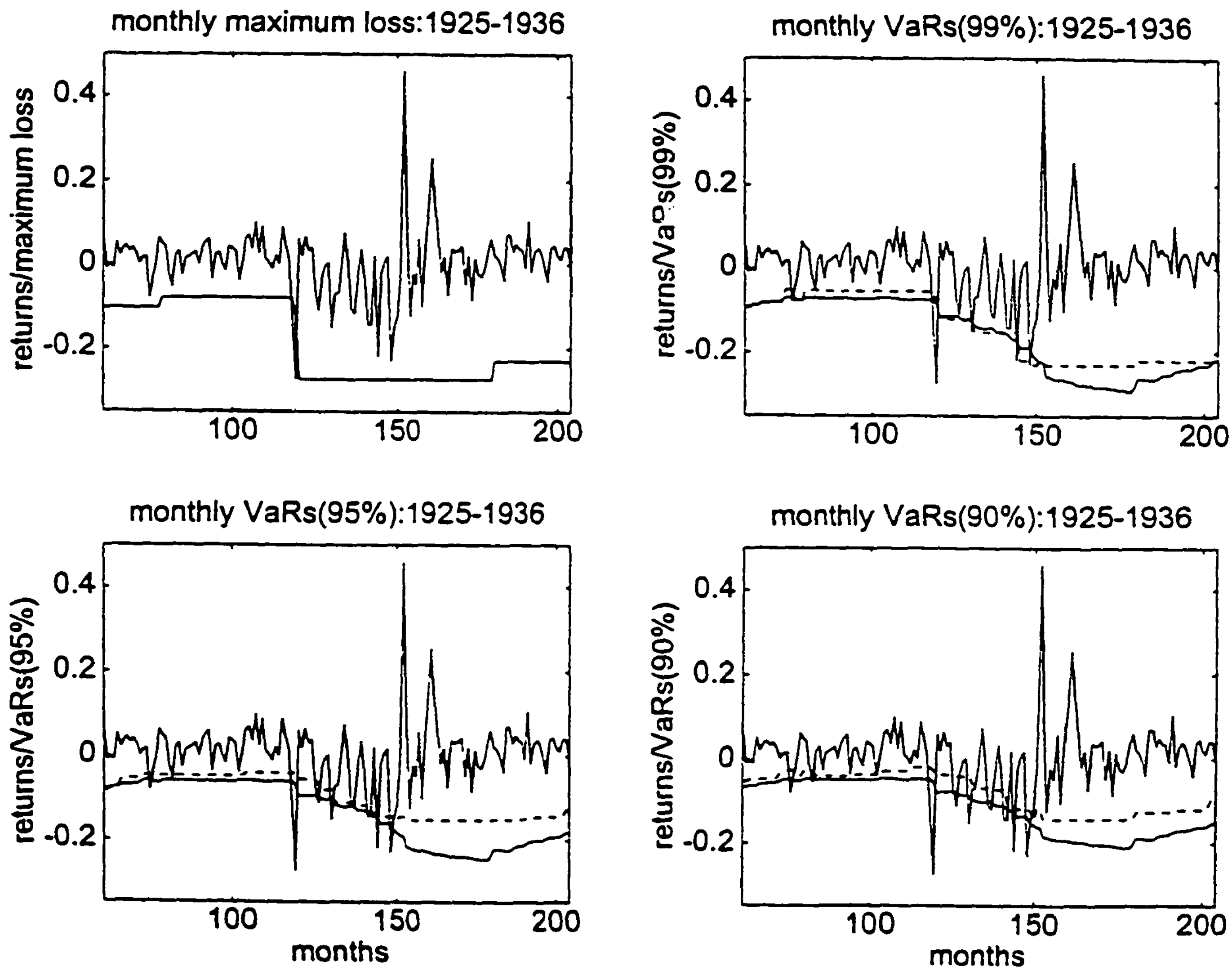
portfolio of equities and does not involve a variety of financial instruments.

Figure 5

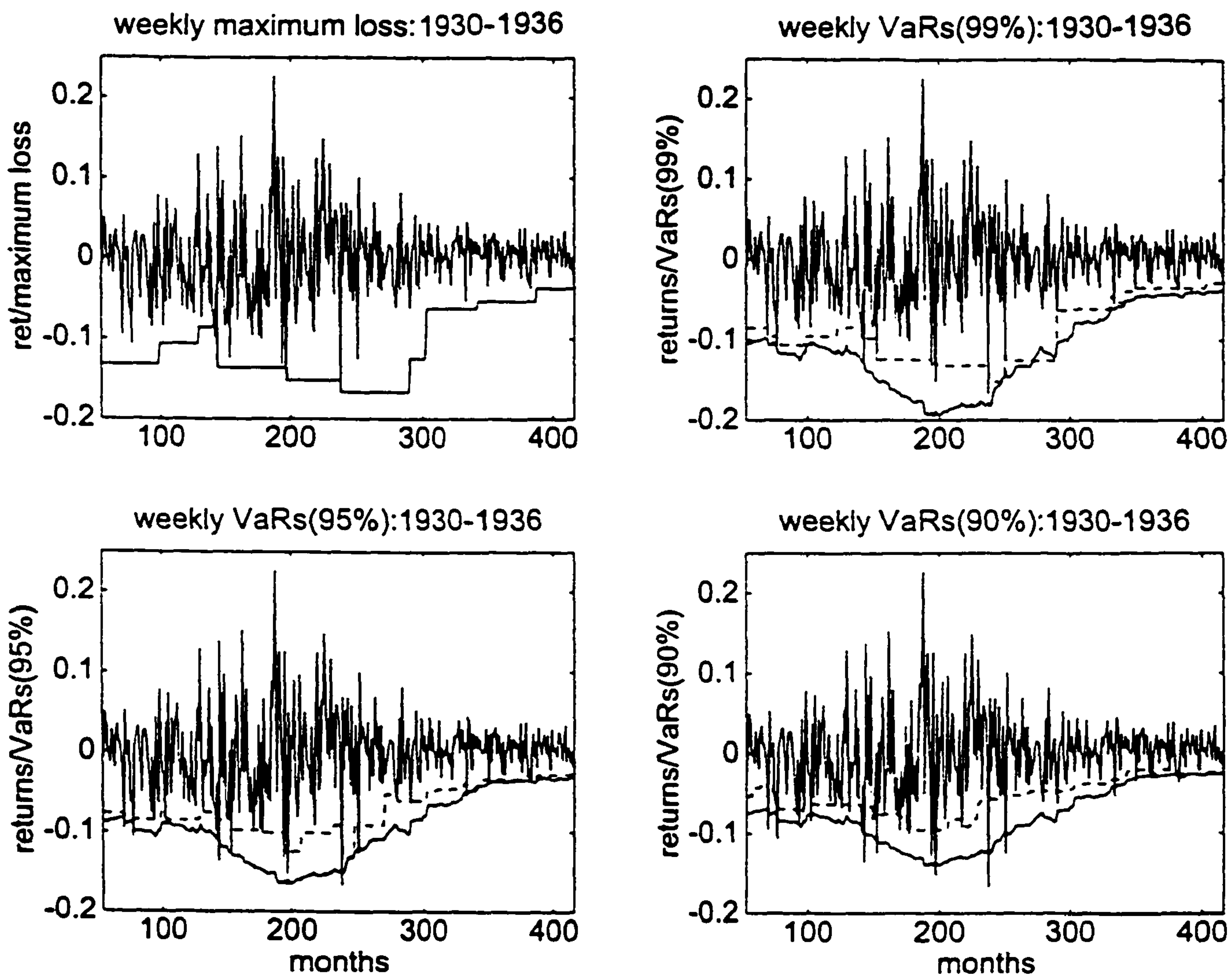
Pairwise comparison of monthly parametric and nonparametric VaRs: 1925-1936



Parametric VaR: ________
Nonparametric VaR. ------

Figure 6

Pairwise comparison of weekly parametric and nonparametric VaRs: 1930-1936



Parametric VaR. ________
Nonparametric VaR. -------

3.2. - Parametric versus nonparametric methodology

Table 2: VaR differences (Monthly versus weekly statistics)

	Monthly data		Weekly data	
VaR differences	Mean	Stand. Dev.	Mean	Stand. Dev.
Par.-Nonp.(99%)	-1.72%	2.09%	-2.21%	1.89%
Par.-Nonp.(95%)	-3.63%	2.95%	-2.24%	1.78%
Par.-Nonp.(90%)	-3.40%	1.96%	-2.53%	1.63%

As expected, the two methods yield different results, although, they are not dramatically different. Parametric VaRs provide more conservative estimates because they include all the observations in the time range, and especially the worst losses, that the nonparametric approach systematically excludes since VaR estimates are critical values (percentiles) read directly off the historical distribution.

When markets are extremely volatile and patterns in returns are not stable, parametric VaRs which are direct functions of volatilities will be inflated whereas, on the other hand, nonparametric VaR estimates tend to underestimate potential losses. In effect, since some of the worst losses are excluded from their calculation this may provide results that underreport the stress experienced in the market. In fact they could eve have been obtained in more favorable times when the excluded returns were not outliers but "close neighbors" of the chosen percentile. This bias which may be negligible when patterns in returns are very stable can be very damaging when estimating potential worse losses during periods of market stress.

Both approaches generate closer estimates when there is a low volatility in the returns and the differences between them are small as exhibited in Figures (5) and (6)[76], where the two methodologies are compared pairwise at the three confidence levels, for both data sets.

In addition, although the two methods exhibit lags (time delays) in reflecting occurrences of abnormal returns (they "announce" greater losses after these have occurred, and do so during a noticeable period), but their lags are different in structure. Parametric VaRs are more distorted by and take longer to adjust to a negative shock[77] than nonparametric VaRs. As predicted by the theory the latter suffer the abrupt fall/rises and plateau effects due to the very nature of their derivation (the same past loss, if large enough will remain say, a 5th percentile, as long as a worse loss replaces it).

Finally, as will be seen below, and for the reasons mentioned above, parametric VaRs systematically outperform nonparametric VaR estimates in terms of failure rates (the difference is substantial for weekly data. (See table 4 on failure rates, below).

3.3. - Choice of aggregation: monthly versus weekly data

This choice is generally dictated by the assets held in the portfolio. The investor will generally choose an holding period commensurate to the liquidation period of its portfolio. Differences in the obtained VaRs are described below

Weekly VaRs, probably because they are based on a greater data set (even if the holding period is shorter) and are calculated from a period of stress, are a much better fit to the actual

[76] Larger individual graphs are provided in Exhibit V of the Appendix.
[77] For instance, the worst loss of the entire period, -27.45% incurred on November, 1929, after the stock market crash, remains in the VaR metric for five years after it has occurred but never enters the nonparametric.

losses. They are closer in size and, more in "sync" (almost no time lag), and the discrepancies (differences between the estimate and the actual loss, the errors) when failures occur are much smaller.

Monthly observations are aggregates and, as a result, are more likely to experience wider ranges of variability. This happens to be the case here (see Table 1. and Figures 7 through 10) and affects the VaR metrics obtained. In effect, monthly VaRs are proportionately larger relative to actual losses. This also contributes to increase the time response/adjustment of monthly parametric VaRs (as reported earlier) since monthly returns have more negative values and larger volatilties during the Great Depression that weekly ones do. Monthly computations exhibit longer recoveries (adjustments) from the occurrence of an outlier or after a period of turbulence. This is probably because (1) an observation will remains in the reference period for as long a time as the length of the observation period and also because, monthly data (2) are based on five-years histories (weekly are based on 2), and (3) they vary within larger ranges (they have greater volatility).

Finally, the choice of reference period also substantially affect the VaR estimates. For instance, monthly VaRs calculated with only two years of data adjust much faster to (negative) shocks (see Figure 11). Although not presented here, using two years of data to calculate nonparametric VaRs also provided longer *plateau* effects and higher VaRs since larger losses were captured as percentiles and kept in the time range for twice as long.

Figure 7

Monthly parametric VaRs: 1925-1936

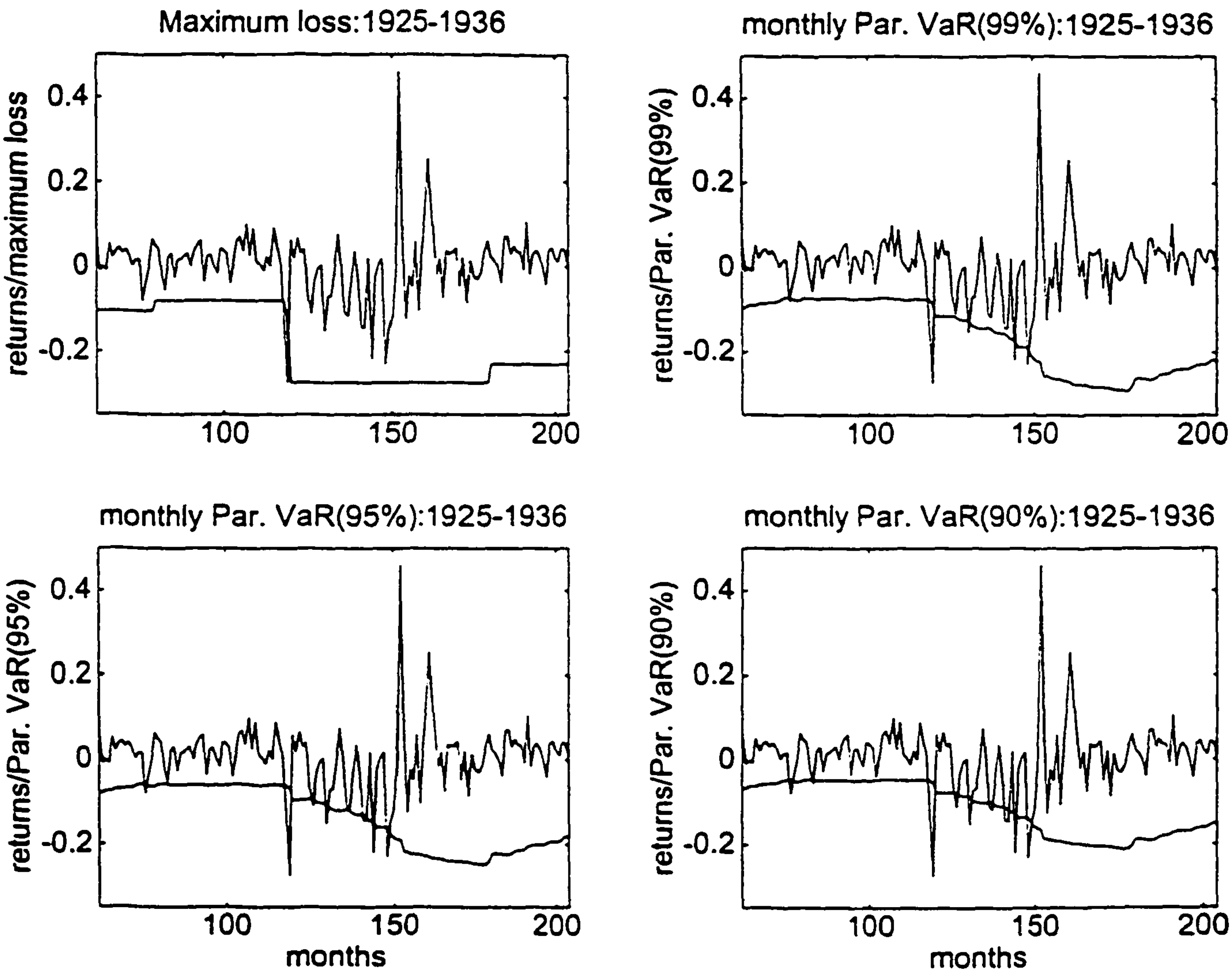


Figure 8

Monthly nonparametric VaRs: 1925-1936

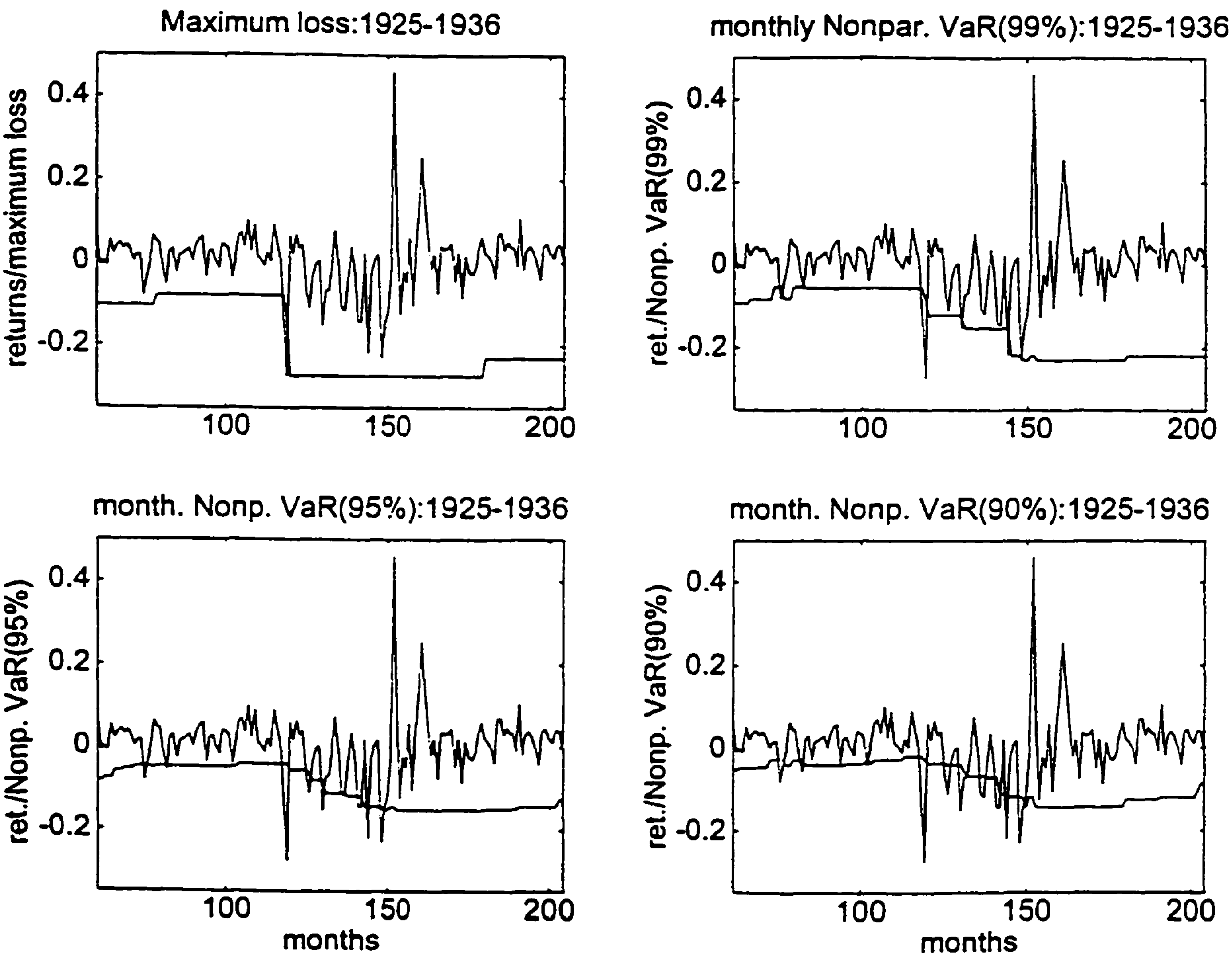


Figure 9

Weeky parametric VaRs: 1930-1936

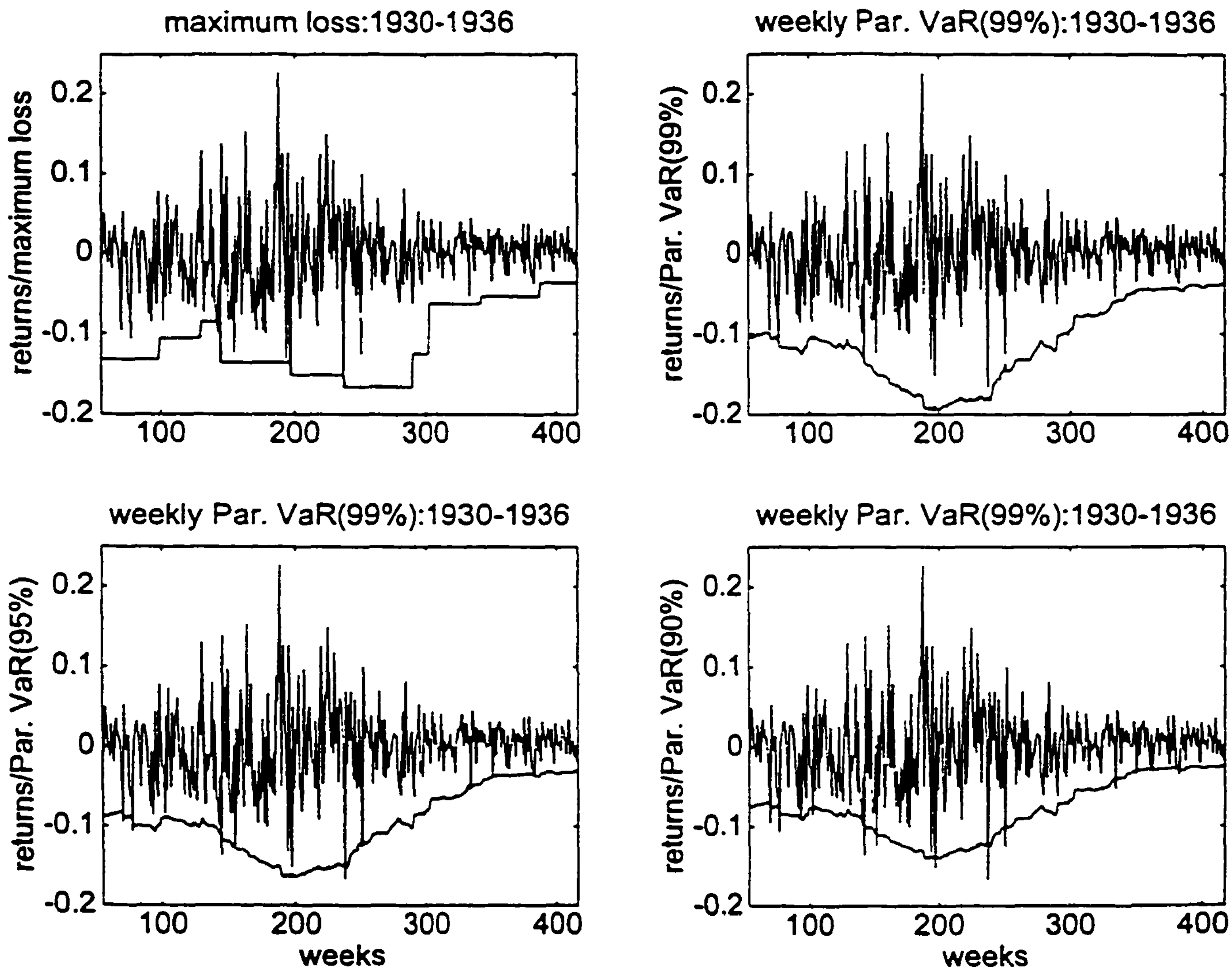


Figure 10

Weekly nonparametric VaRs: 1930-1936

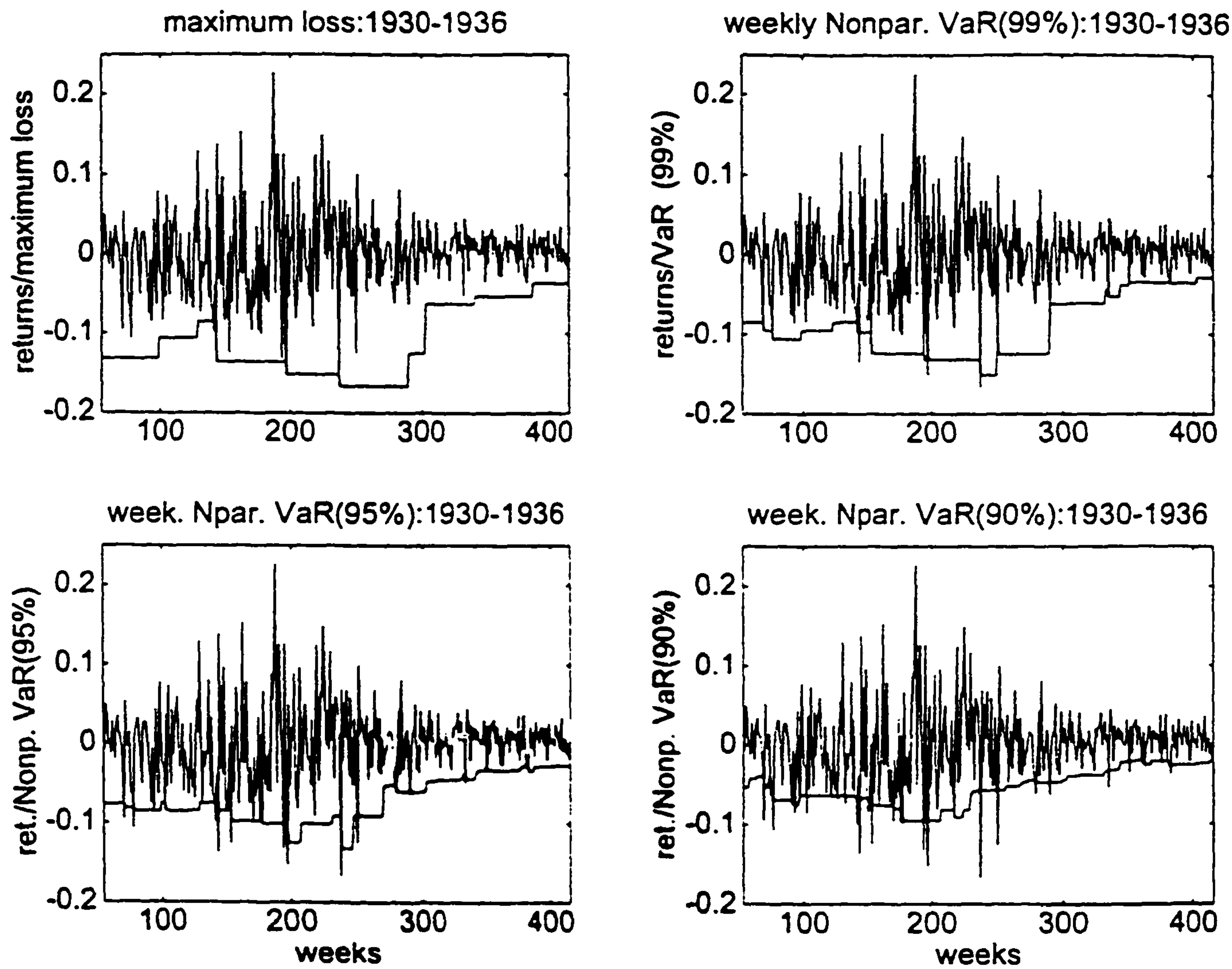
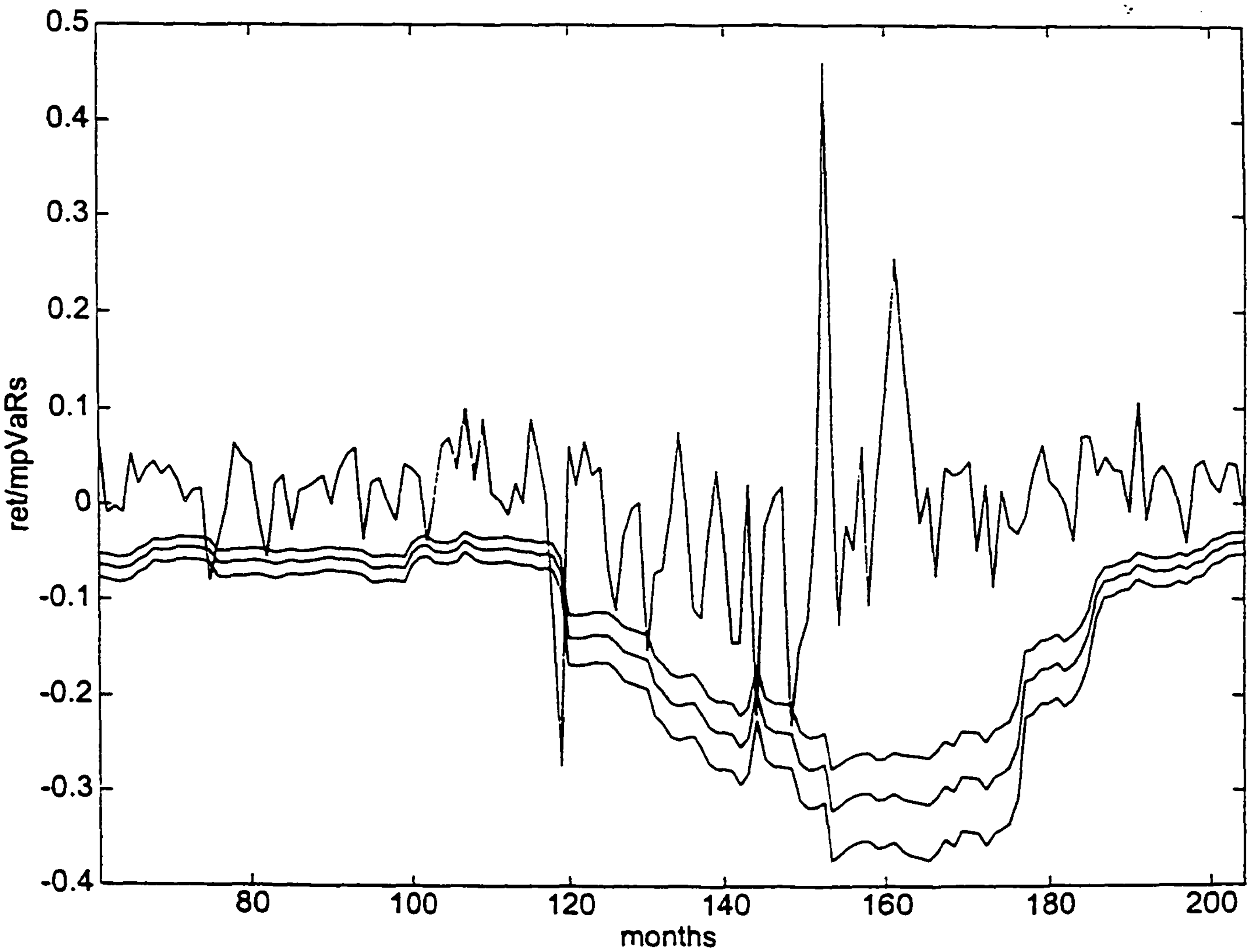


Figure 11

Monthly parametric VaRs (with two years of data): 1925-1936



3.4. - Failure rates

Table 3: Failures in Parametric VaRs

	Failures of Parametric VaRs	
VaRs	Failure rates	Worse violation[1] statistics (%)
Monthly 99%	5/144=3.47%, 4 during GD[2]	By 19.02%; Nov. '29; V[3] -8.43%, R[4] -27.45%
Monthly 95%	9/144=6.25%, 8 during GD	By 20.68%, Nov. '29; V -6.77%, R -27.45%
Monthly 90%	12/144=8.33%,10 during GD	By 22.07%, Nov. '29; V -5.38%, R -27.45%
Weekly 99%	1/363=.28%, during GD	By .87%; Oct. '31; V -12.72%, R -13.59%
Weekly 95%	7/363=1.92%, 5 during GD	By 2.70%; Oct. '31; V -10.89%, R -13.59%
Weekly 90%	12/363=3.30%, 8 during GD	By 4.72%, Jul. '33, V -11.94%, R -16.66%

1. A violation occurs when the loss exceeds the Var. 2. GD: Great Depression. 3. V: VaR estimate. 4. R: Return

Table 4: Failures in Nonparametric VaRs

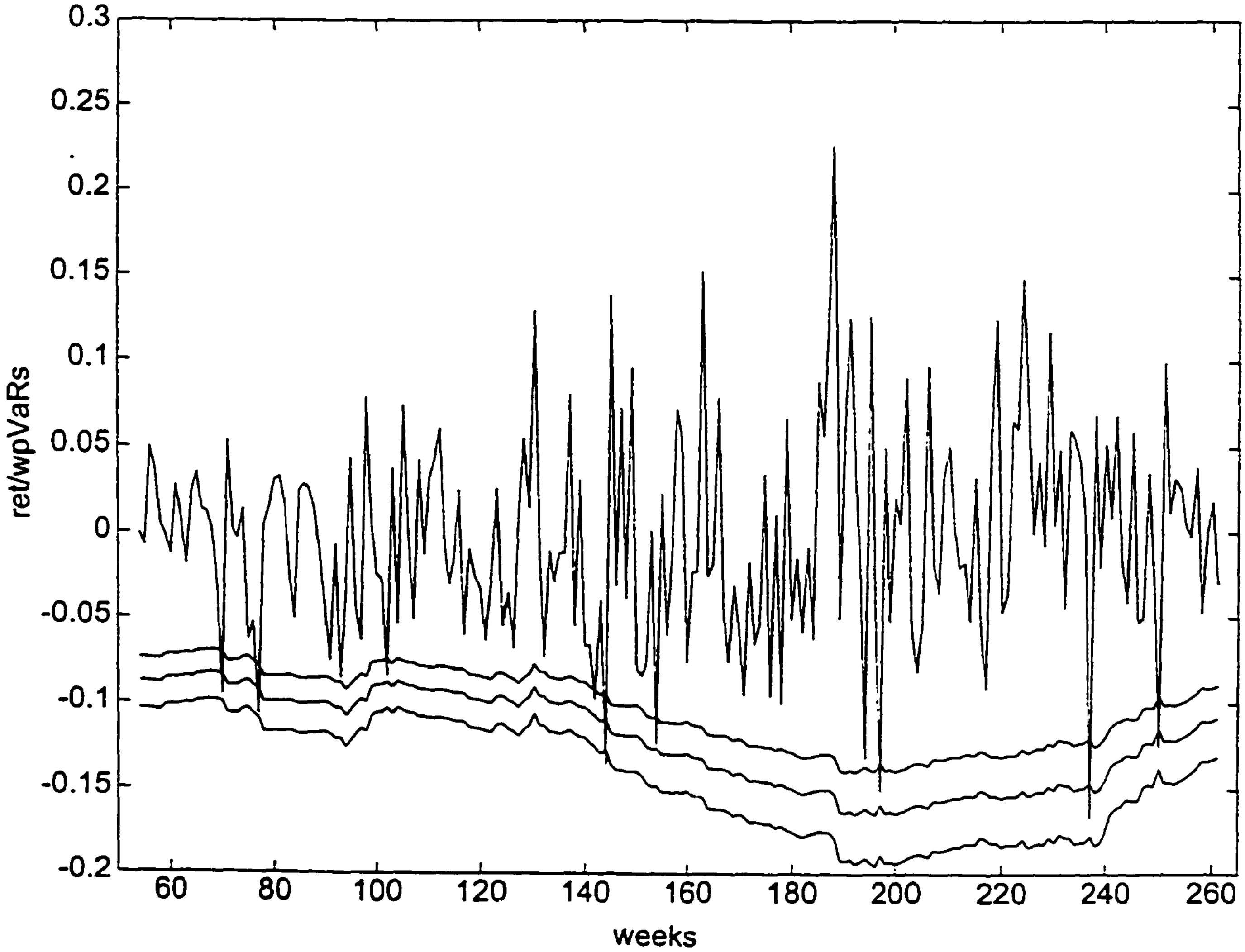
	Failures of Nonparametric VaRs	
	Failure rates	Worse violation statistics(%)
Monthly 99%	7/144=4.86%,5 during GD	By 19.41%, Nov. '29, V -8.04%, R -27.45%
Monthly 95%	12/144=8.33%, 10 during GD	By 23.41%, Nov. '29, V -4.04%, R -27.45%
Monthly 90%	14/144=9.72%, 12 during GD	By 24.75%, Nov. '29, V -2.70%, R -27.45%
Weekly 99%	11/363=3%, 8 during GD	By 5.04%, Oct. '31, V -8.55%, R -13.59%
Weekly 95%	15/363=4.13%, 10 during GD	By 7.55%, Jul. '33, V -9.12%, R -16.66%
Weekly 90%	35/363=9.64%, 25 during GD	By 19.41%, Jul. '33, V -5.70%, R -16.66%

The rates of failure confirm the theory that VaR provides loss estimates under "normal" market conditions, and that, as a result, it may be inefficient during times of market crash. In effect, most of the failures occurred during the Great Depression, and on the greater losses of the effect, most of the failures occurred during the Great Depression, and on the greater losses of the period (and especially on the outliers uncovered on the distribution of the standard returns). However, there are other patterns in these failures which are also worth important: (1) parametric VaRs are more accurate, and (2) weekly calculations outperform monthly statistics. This is probably because as mentioned earlier (1) parametric VaRs include all the observations of the range and especially the worst ones, and (2) weekly computations have smaller spreads and rely on a larger data base (this may be compounded by the historical circumstances since, the weekly data only covers a period of market stress, whereas the monthly data encompasses the eight-year bull market values).

Although the different VaR levels did fail "together" on occasion, it would be a bad idea to allocate prudential capital onnthe basis of the lower VaRs because: (1) these estimates are violated much more often than; (2) although on average the size of the violation is not much greater for lower confidence level VaRs; (3) the earnings that could be made of the released prudential capital would be dwarfed by the losses incurred (from the capital consumed) on the many occasions theses estimates would be exceeded. (On November, 1929, the violation of the 90% reached 22.07%). Figure(12) shows weekly VaRs at different confidence levels and allow a better comparison of their failure rates

Figure 12

Weeky parametric VaRs: 1933-1936



3.5. - Choice of confidence level

The choice of confidence level is a function of the risk aversion of the financial manager/institution, the cost of capital (idled for prudential purposes), the cost of violating the estimate... In brief, it is as much the result of a subjective decision as it is that of an objective decision. However, it is worth noticing that, for monthly data, the 99 and 95% level VaRs give a false sense of security since their true failure rates do exceed what the theory predicts. However, it is now common knowledge that what the theory calls abnormal, rare or outlying returns are more frequent in reality. In light of this reminder, and of the previously calculated statistics on the differences in the VaRs of different confidence level, the decision should be made on a case-by-case basis by investigating the particularities in terms of cost of capital, cost of a failure, etc.

The application confirmed that VaR is inefficient when the market crashes but also that the choice of confidence level is crucial in market risk measurement. In effect, it significantly affects the failure rate of the chosen VaR model, which may be of critical importance if prudential capital is set equal to VaR, and or VaR is used to readjust risk exposures. Not only does the choice of confidence level condition the failure rate and thus the reliability of an estimate but it also influences the magnitude of the size of the loss (the size of the danger) the investor is exposed to when a failure occurs. The application shows that on a given failure, the actual loss can be dramatically different from VaR (the worst failure occurs with the monthly data in November 1929, in the aftermath of the market crash and varies from a 19.02% difference relative to 99% parametric VaR to as much as 24.75% discrepancy with the 90% nonparametric. (See Table 4)

IV - Conclusions

The statistical findings presented in this chapter are generally consistent with the theory. In effect, they confirm that (1) different VaR methodologies yield significantly different estimates of potential losses, and the same approach may even perform differently and be more or less reliable at various level of time aggregation (therefore, a firm must try different VaR approaches and compare their performance before choosing a specific approach which should best fit its profile); (2) the VaR estimate is affected by the choice of time horizon, holding period and confidence levels; (3) since it can capture their risk structure parametric VaR yields appropriate risk exposure estimates for portfolios of equities; (4) nonparametric VaR suffers the predicted impact of *plateau* effects while monthly parametric VaRs do not adjust quickly to negative shocks; (5) longer time VaRs do not aggregate well over time and produce less reliable results than shorter term VaRs (because the range of variability in more frequent measurements is smaller); (6) VaR metrics may provide a false sense of security since they underestimate the frequency of unfavorable events (those that create losses greater than VaR); large samples (or stable volatility ones) are necessary to improve on VaR estimates quality.

It was also very interesting to discover that, although, returns exhibited very thin waist P&L density distributions, the "normal" parametric approach did generate results that did not necessarily underestimate the failure rate (this is especially true for the weekly data set). In addition, although parametric VaR is frequently criticized for excessively evening out outliers, here, the fact that the VaRs included outliers proved valuable since it contributed to this approach yielding more reliable loss estimates than the nonparametric VaR which, by nature, systematically excludes some of the larger losses. Finally, although the parametric approach may be appealing due to its simplicity, when there is significant volatility clusters in the returns, the approach may not be such a good tool since it may provide VaR estimates never "in step" with the actual P&L because of the distortion it contains.

The empirical results support the theory that VaR estimates show the worse loss that will be surpassed only 1, 5 or 10% of the time, at least for parametric weekly VaRs. However, they also revealed that the 99% VaRs do not generally draw a pessimistic picture by overstating losses by large amounts, whereas 90% VaR computations are too "optimistic" and frequently exceeded. However, although they understate actual losses more frequently, as expected, they do so less frequently than the 10% (of the time) expected and do not call for exaggerated coverage against potential losses. Therefore, a cautious, risk-averse investor who might, *a priori* have been tempted to implement 99% VaR computations would have indeed allocated appropriate capital provisions and not uselessly hampered profitability, but would have protected himself more efficiently than a 95 and 90% VaR user would have, (at least for weekly data). This would be true provided that the cost of capital does not exceed the cost of underestimating actual losses.

The results of this empirical treatment must be judged carefully, though, because of the previously described the idiosyncrasies of the data sets and the biases in the implementation, especially those built in nonparametric VaRs). (1) Monthly VaRs are based on longer histories of five years, even if there are fewer data points then for the weekly estimations, they are therefore based on returns reflecting first the long-eight-year-bull markets (when returns were mostly small and positive), then the great depression years (when returns fluctuations were mostly large and negative), and finally the aftermath of the contraction (when spread, volatility of returns receded again and returns became more positive). As a result, monthly VaRs which rely on fewer and more heterogeneous observations (while weekly VaRs are calculated over the Great depression only, and exhibit smaller spreads) provide less accurate results than the weekly ones. (2) The returns are based on DJIA closes which fall on different dates of the months, which also creates some distortion between shorter-time and longer-time VaRs. (3) Finally nonparametric VaR metrics are based on only a few of the worse outcomes of a P&L distribution and systematically excludes some of the worst losses but also the greatest positive changes, (whereas the parametric estimates include all the observations of the reference period). This exclusion has an important impact on the results since the observation period includes a large period of market turbulence).

When this is said, it is necessary, however, to remember that the purpose of this empirical application and of VaR estimation is to verify if it is possible, ahead of time, to assess one's risk taking and exposure through the forecast of the worse potential losses at a given confidence level, in order to set prudential capital provisions to cover actual losses when they occur and/or readjust the risk exposure in light of the values at risk.

Ultimately, even if in some instances the potential losses were generally overstated/understated, the VaR estimate did provide warning of increasing risk and of greater future (potential) losses. The morale of the story is, thus that, VaR computations will not provide exactly accurate measures of potential losses (they are not supposed to, and should not be expected to either), but will serve the purpose they were supposed to, offering a fair warning.

In fact, these estimates turned out to be valuable indicators because, in retrospect they were not exceeded too often and not by too much (except after the Wall Street crash of October, 1929). A final necessary word of caution for the investor, though, is that VaRs are only measures of potential losses and not of potential gains. Substantial positive profits were achievable during the Great Depression to the investor who stuck with market and did not liquidate his positions when the market was at its lowest.

Finally, the results were, overall, very satisfying: the pattern in VaR was appropriate (potential loss estimates rose when market stress increased, failures rates were in concordance with the theory and even of remarkable quality for weekly data). After all, VaR *did* do a valuable job as forewarning of the potential losses during the Great Depression, even if it did so a little too pessimistically after the Wall Street crash.

Thus this empirical research also confirmed that, in spite of its acknowledged weaknesses, the VaR methodology is a crucial invention for the financial industry since the application revealed the very satisfying news, that, had VaR been known and implemented in the early twenties it would indeed have, at least for portfolios of blue chip DJIA stocks, forewarned of imminent losses and would well possibly have avoided many financial disasters and/or failures if measures to reduce risk exposures had been taken or/and prudential capital had been allocated in concordance with VaR guidelines.

Chapter VI - Conclusions

This paper developed a comprehensive framework of the new emerging market risk measurement model known as Value-at-Risk. It supplements current publications in two important respects. First, the same document presented, analyzed and compared the three basic approaches to computing VaR - historical, analytical, and Monte Carlo simulation and also provided a review and assessment of proposed adjustments/improvements to the currently used models. Finally, it discussed model validity and accuracy testing techniques. Thus the present research on VaR was not specific, but integrated.

Second, the empirical contribution of this research was to investigate and measure up the performance of parametric and nonparametric VaR methodologies in a Great Depression context by using historical data of an equity portfolio of Dow Jones Industrial Average stocks. This application confirmed that (1) VaR estimates are not proportionately affected by the choice of holding period (longer time VaRs do not aggregate well over time) and confidence levels; (2) that for portfolios of stocks, parametric VaR models produce more reliable measures of risk exposure; (3) that nonparametric VaR underestimates the probability of adverse market moves (those that create losses worse than the computed VaR metric). But the statistical treatment also confirmed, that in spite of its acknowledged weaknesses, the VaR methodology is a crucial invention for the financial industry since the empirical application revealed the remarkable and comforting news, that, had VaR been known and implemented in the early twenties it would indeed have forewarned of imminent losses (at least on blue chip stocks) and could well possibly have prevented many financial disasters and/or failures if measures to reduce risk exposures had been taken and/or prudential capital had been allocated following VaR guidelines.

While the Value-at-Risk concept is a simple one, its implementation is more involved and controversial that one might, at first, have been led to expect. The increasing number of institutions which decided to set up VaR systems have run into a series of stumbling blocks. For starters, all the firm (worldwide) positions must be normalized (made compatible) and gathered into one central database. This enterprise may be quite a challenge, knowing that some institutions have international branches running not only on different time zones but also on and different, and even incompatible, systems. The next step is to calculate the overall risk by aggregating the risks from the individual positions in the entire portfolio. This requires working out the effect of the moves in the different risk factors across the portfolio, which once again, may be a long, complex and discouraging task since many currencies may be involved and, within each currency, a large number of asset classes will generally exist.

Once the firm-wide, cross product portfolio is valued, changes in its value are calculated to obtain its P&L distribution, institutional VaR can be computed (or aggregated from the different units' VaRs). This process requires a huge amount of data since, for regulatory purposes, the potential move in each risk factor is required to be inferred from an observation period of one to five year(s).

Although the world leading banks and securities houses have now been implementing comprehensive and sophisticated market risk management systems based on VaR, for several years, and many of them, pioneered by U.S. JP Morgan in 1994, have even marketed their own VaR systems, it has been a whole different story at the smaller firms' level. In effect, many firms find it overwhelming to implement the process and/or some just do not have the data, the scientific knowledge and/or the infrastructure necessary to carry out the process, and in the best of cases they hire financial consultants and/or purchase financial risk management system packages.

Although VaR has started to become ubiquitous, difficulties surrounding the concept itself, have also emerged. First, there is a lack of consensus in the financial community on which VaR methodology is more appropriate and there is a lack of understanding of the impact of the simplifying assumptions built in some of VaR models. Second, the limits of the concept's usefulness start to filter down to all practitioners: although VaR is, now indeed, one of the fundamental tools of financial market risk management and reporting, it is clearly not the whole picture.

Its purpose is to provide an estimate of losses under "normal" market conditions, at a given confidence level. It does not show what might happen during market crashes: for that stress testing (scenario analysis) is required. In no circumstances, will VaR forecast what might happen during a market fall and especially the magnitude of the actual losses endured. While VaR expresses normal market risks, stress testing identifies risk exposures under abnormal circumstances and investigates the impact for the firm of the breakdown of some of the models simplifying assumptions. In addition, unless positions are readjusted after VaR calculations: VaR cannot prevent losses it can only avoid bankruptcies by setting the necessary capital cushion to absorb losses.

Furthermore, as any risk management model -or any model for that matter- even if the VaR procedure has been validated, its output, the estimated loss, can only be as good as the data entered into the model. For instance, a rogue user, entering false data or deliberately underreporting VaR can easily and undetectably underestimate this market risk exposure and potential losses and allocate less prudential capital than necessary.

The accuracy of VaR also depends on the use the organization wants to make of it. For some institutions, piecemeal VaR can be used as a "quick" approximate evaluation tool at the trading desk, in real-time, to assess temporary, intraday, risk exposures and adjust positions in a portfolio; but VaR can also be used as an elaborate control mechanism by back office and senior management supervision; VaR can be used as a simple, transparent, communication and disclosure metric to report to top management and shareholders; but VaR may also serve as a guideline in strategic decision-making for the firm, or more aggressively to measure performance and or allocate resources at division levels, or in capital management to calculate the appropriate level of prudential capital requirement (in which case its precision may be crucial).

Although VaR is being adopted at an increasing rate, heavy reliance on VaR metrics can be dangerous because of their sizable measurement errors and the quasi-impossibility to test their true value[78]. However, in this new era of risk consciousness, one has only to hope that VaR will contribute to improve market and financial risk management so that financial catastrophes are limited if not totally avoided in the future, and that, globally, the safety of the financial system durably improves.

In light of these limits, financial institutions cannot rely on a single VaR metric to assess

[78]In both cases, long periods may be required to verify the accuracy of VaR models. Hence the verification process will be time-consuming, even more so if models are frequently altered. With a new model, the data collection process will start over again, and long periods may be required before the new model can be accepted.

their risk exposures but must consider other factors to create a more complete risk assessment. At a minimum. VaR needs to be considered to have (substantial) error bars and to be used in conjunction with stress testing. It is also necessary to keep tabs on other types of financial and business risks such operational and legal risks (the billion dollar losses at Barings and Daiwa. for instance, were not simply due to market risk but also to operational risk, for lack of proper internal control of traders). Therefore. this concept, although it is an extremely interesting innovation of firm-wide. cross-product market risk management tool, which can be used for day-to-day risk management but also for strategic planning. it is no substitute for a wider risk management process. and acute internal and regulatory supervision.

Finally. analyzing risk management models and choosing between methodologies for calculating VaR have indubitably forced managers to consider risk in a greater detail and reconsider some of the questionable assumptions they may have inadvertently made in the past. This in itself, is a substantial contribution of VaR which is very favorable for the safety of the financial system. In the end. the greatest benefit of VaR probably lies in this imposition of a structured methodology for critically thinking about risk: institutions that go through the process of computing their VaR are forced to confront their exposure to financial risk. Thus the process of integrating VaR in a risk management strategy may be at least as important as the computed risk exposure itself.

Although there is no doubt that VaR is here to stay. the concept is still in its infancy. Additional efforts to improve its understanding and implementation remain necessary, as well as supplementary research to improve on the concept and especially its accuracy still needs to be carried out.

An interesting topic for future research could be to retrospectively pass a selection of real portfolios of financial assets through the test of VaR, over time, and especially during episodes of financial markets turmoil, in order to scrupulously investigate "an eventual pattern" in VaR based on how the estimates did perform and use this pattern in the future to prevent the enormous losses that occur too frequently when markets plunge.

APPENDIX

EXHIBIT I : RECENT FINANCIAL MARKET CHANGES

In the past two decades, US financial markets experienced an unprecedented growth, innovation, and often speculative excess. Regulation receded in most markets, the stock market experienced a boom, the junk bond market developed, commercial and consumer loans rose sharply. The financial services industry also experienced an unsustainable growth that was halted in 1987 with the stock market crash. After that event and until the mid-nineties, the financial industry went through a period of adjustment: growth slowed down, numerous financial institutions merged to cut costs, gain economies of scale and become more competitive. Since then, there has been a resurgence in financial markets (and especially capital markets) activities, and corporate lending and investing are rebounding.

Financial markets have become more complete, liquid, volatile, interrelated and complex over the past two decades. At the onset (1) financial product innovation has led to the rapid development of new securities and in particular of hybrid securities with optionlike features (such as convertible or putable bonds or indexed certificates of deposit), and of *derivatives* (created at the origin to provide an hedge against the risks associated with holding certain positions); (2) the invention of *securitization* has allowed to convert illiquid long-term nonstandardized debt instruments into more liquid, safer shorter-term, smaller-denomination, standardized, marketable instruments.

(3)This creativity has contributed to improving the access of capital markets to small investors. Greater participation of small investors was also encouraged by the introduction of user-friendly customer-oriented services but also of more affordable securities such as new stripped (zero coupon) securities and the lowering of minimum denomination of existing instruments (the Fed, for instance, lowered the minimum denomination of T-bills from 10,000 to 5,000 and $1,000). Regulators also encouraged, through deregulation, the financial industry to create structures and offer services more adapted to small savers. For instance, the Stock Exchange Commission abolished NYSE minimum commissions in the seventies, and discount brokers made their appearance to service smaller investors.

The introduction of money market mutual funds (MMMF) was also very instrumental in this process, although mutual funds were in existence since 1940. They allow (small) investors to indirectly afford investments until then prohibitively expensive and yet benefit from portfolio diversification. This type of fund literally exploded in the eighties, its assets were multiplied almost tenfold over that decade alone, because MMMF offered tailored investment opportunities and, in addition, they were able to lure away commercial banks and thrifts depositors when short-term rates rose and banks interest payment on deposits were legally restricted: but MMMF could offer higher-interest paying opportunities and thus, more attractive short-term investment alternatives. Banks were then deregulated to stop their deposits "bleeding" in favor of these mutuals and allowed to offer fundlike services to their depositors. Capital markets were also made increasingly "certificateless" to facilitate trading and enhance liquidity.

Parallely, (4) exchanges and the OTC expanded as shown by the substantial increase they experienced in their capitalization and trading volume. An increasing number of corporations raised funds in financial markets, as individual savers frantically turned to them for increased earnings. The emergence of the junk bond market was a vivid example of the development of (new) markets. Financial markets were able to develop to that extent not solely due to the increased domestic supply and demand of securities but also due to higher international trading.

In effect, (5) security trading intensified with existing foreign markets but also because new financial markets were created abroad. While until then, investors' attention was almost solely on domestic securities, with the opening of securities exchanges and the trading of financial securities overseas, they were, from then on, given an even greater multitude of choices. This globalization of financial markets was also permitted because markets, such as the NYSE and the OTC in the US, were able to inaugurate, in the seventies and eighties, automated quotation and trading systems with increased capacities tenfold those handled until then (capacities are expected to reach a billion transactions daily), which also contributed to abolish international trading frontiers.

This latter change is a corollary of (5) the technological breakthroughs in data processing and telecommunication of information which have allowed substantially faster asset pricing procedures, enhanced trading by allowing deals to be made at a greater speed, and anywhere, in real-time. For instance, market orders are now processed overnight to be filled first thing as the markets open, and generate opening prices for next day exchange's opening; brokers/dealers can process trades for their firm/clients all over the world and match trades sometimes in matters of seconds; asset pricing and portfolio valuations which used to take days to compute can now be done in a few minutes if not seconds

The media (6), in recent years, have also played an important part in the development of financial markets. The number of financial publications and shows has risen and the reporting of financial news has also taken a substantially greater place in the existing news papers and

TV/radio programs. Everyday thousands of statistics are published at the aggregate (volume, securities indexes and so on) level but also through individual listings (opening/closing price, bid/offer spread, yield, volume, etc.) of securities.

Finally, (7) financial research has become increasingly analytical, sophisticated and trading practices in financial institutions have followed suit. Many financial institutions, now, have their own teams of in-house researchers and financial and technical experts who are hired to work hand-in-hand and combine their financial, econometric and computer science knowledge to innovate/adjust/implement pricing methodologies and conceptualize risk management frameworks.

But paradoxically, as markets became more sophisticated in nature they became more popular in access. Entry to the market is no longer a privilege of the "rich, famous, sophisticated and of the financial experts": *John Do* can also play, now, and that makes him feel good about himself. In effect, while "trading on the stock market" has become a common fact of life, (or at least an experience), for numerous corporates and individuals, investment practices have changed substantially and have led to more frequent trading, increased asset/market liquidity but also, a much higher investment in a much wider range of varied instruments, including exotic and complex securities, and thus to increased riskiness of being a financial market participant.

The financial industry has also experienced the dual expansion markets have. Banks and security firms have been so occupied filling trading orders for a fast growing number of clients, that many institutions, and maybe the vast majority, have been unable to ensure that its staff was able to keep up with the sophistication of the financial techniques (at the trading, asset pricing, and risk management levels) although, they did make the adjustments to the changes in trading since they were obligatory to satisfy the clients and participate in the market. This lack of timely theoretical training in asset valuation and risk assessment/management has contributed to increase the riskiness of the industry (by worsening the principal/agent problem). The large leading banks and security houses, were almost exclusively the only ones keeping up with the sophistication of the financial theoretical approach.

To capture their shares of a growing market, most financial institution entered a period of emphasizing profitability and understressing the risk counterpart. As a result of this trading "bulimia" and the overlooking of the complex interplay across markets and of the risk associated with trading, news of financial disasters resulting from lack of risk management started taking the limelight in the media, at a steady and increasing rate.

In addition, financial markets suffered a series of down falls which resulted in another wave of financial losses for corporate and individual investors. The stock market crashed on Black Monday, October 19, 1987 when the DJIA dropped 508 points losing 23% of its value; the European Exchange rate Mechanism blew up in September, 1992; the bond market tumbled in February/March, 1994 (when bonds lost 80 basis points).

The accumulation of these financial dismal news drew concern of regulators and served as a wake-up call for the entire financial industry which entered a new era of financial risk awareness. At first, after the series of banks and thrifts failures, and reported corporate financial catastrophes (in the US and worldwide), related to off-balance sheet transactions in derivatives, focus was placed on credit risk. in the late eighties for instance bank regulators imposed that banks hold capital against credit, by defining the Cooke 8% capital/asset ratio. Then in turn, in light of another series of financial losses due this time to lack of or improper assessment/management of the risk associated with changes in financial variables such as asset prices, interest and exchange rates, the focus shifted to market risk in the early nineties.

EXHIBIT II: REVIEW OF SOME RECENT FINANCIAL DISASTERS

The 1980s wave of S&Ls failures

In the 1980s, the US savings and loans industry experienced tremendous losses which are now estimated to amount to as much as $150 billion. S&Ls were prime lenders in the real estate markets, making long-term loans mostly for residential housing, that were funded by the (shorter-term) savings deposits of their customers When short-term interest rates rose sharply in the 1980s, S&Ls became trapped in a duration gap because they were paying a much higher rate to their depositors than they were receiving from the long-term loans they were locked in. As a result a large number of thrifts became insolvent and were closed.

The Metallgesellschaft's story

Metallgesellschaft (MG), was Germany fourteenth-largest industrial group when it almost went bankrupt after the enormous losses its US subsidiary, MG refining & Marketing (MGRM), suffered. The story begins with MRGM's decision to sell long-term forward contracts in oil supply which where very popular since they allowed their purchasers to lock in oil prices for a long period. The marketing of such contracts was so successful that by 1993, MGRM had committed to supplying 180 million of oil barrels over the next decade, a commitment equivalent to Kuwait's 85-day oil output of that period and many times greater than MGRM's refining capacity. To hedge against acquiring oil price swings, the subsidiary entered short-term futures positions because, at that time, there was no possibility of long-term contracts in futures markets, and built a rolling hedge against its price risk exposure. In 1993, a break in the oil price structure occurred, making short-term prices diverge from the present value of a real long-term futures. Short-term prices fell by $10, losing 40% of their original level. Just as the S&Ls a decade earlier, MGRM found itself caught in a duration squeeze, acquiring its products at a higher price than it was compelled to sell them. The German parent stepped in, replaced the subsidiary's management team and liquidated the contracts, realizing a $1.3 billion loss in the process. The conglomerate was saved from failure by Deustche Bank's injection of $2.4 billion in the group.

Orange County

This case is one of massive loss incurred in a government fund managing the money of schools, cities, districts and of the county. Citron, the county treasurer, in an attempt to maximize the earnings from that fund that he was managing, leveraged $12.5 billion through reverse repurchased agreements (very short-term instruments), which had to be constantly rolled over, and with the $7.5 billion of the fund, acquired a total of $20 billion of agency notes of an average maturity of four years. Citron did achieve good results, at first, because of an upward sloping yield curve which generated earnings exceeding his cost of borrowing.

However, in February 1994, the yield curve reverted and Citron's strategy broke down and backfired. Just as for Leeson a few years later, margin calls to cover positions started coming in. As Citron was unable to post the margin requirements, the money being tied up into longer-term investments, alarmed investors started withdrawing their money out of the fund.

which eventually defaulted on collateral payments. Brokers proceeded to liquidate them and the county went bankrupt (the realized loss amounted to $1.62 billion).

Orange County is also a case of unlimited power allowed by a lack of supervision, control and disclosure requirement that backfired. Because Citron had been a winner in the past, he was entrusted with increased amounts of money and no supervision. But this is also, obviously, a case of deficient risk assessment and management. Citron thought that he had zero risk because he had purchased government debt instruments that he was planning to hold until maturity (no default or maturity risk). The losses were allowed to accumulate and worsen because they were not reported, as it is government practice not to report paper losses or gains (Citron did not report the portfolio notional value and only entered it at cost).

Daiwa

Daiwa's story is also the story of an entire institution's survival single-handedly jeopardized by one of its employees' trading. But although Japan's twelfth largest bank flirted with failure, because it had a very large capital base, it was able to withstand the blow. On September 26, 1995, the bank announced that one of its trader, 44-year-old Toshihide Igushi had accumulated losses of approximately $1.1 billion (as a result, only approximately 15% of the bank's capital was wiped out)

Starting in 1984, and for over eleven years, Igushi hid from the bank's management more than 30,000 trades he had conducted. He was trading US treasury bonds, and when he started making losses he concealed them and even sold customers' securities deposited at the New York branch, to try to make up for his losses

This is obviously the story of a risk taker who was a bad risk assessor and had control of both front and back offices. But it is also the tale of a rogue character who willfully deceived his firm by hiding his losses but also its clients by selling their and falsifying listings to hide his illegal behavior. It is also the story of a bank that purposely lied to regulators to pass inspections and covered for its employee it had lacked looking over, itself. In effect, the Federal Reserve board, following the failure of Bank of Credit and Commerce International had conducted inspections of Daiwa's New York branch in 1992 and 1993 and warned management of the risk inherent in its management structure. Daiwa did not implement the recommended changes and even hid records and temporarily removed brokers to pass the 1992 inspection. The bank ultimately moved Igushi to the back office under regulators' pressure, but he continued his trading there, hiding behind fellow traders The Fed's scrutiny, however, made concealing his losses increasingly difficult and, in July, 1995, Igushi finally informed the bank of the extent of the losses he had contracted other the years

Barings' failure

Late, on February 23, 1996, Nicholas Leeson, chief trader for Barings Futures in Singapore, left his work place as usual, and later faxed his supervisors his apologies for the situation he had put the bank in On February 26, Great Britain and the world was shocked by the news of the failure of the conservative and reputable 233-year-old bank, Barings, PLC.

Leeson, had been accumulating positions in stock index futures on the Nikkei 225 and in the process had acquired up to $7 billion in positions on the Singapore and Osaka exchanges. When the market fell by more than 15% at the beginning of 1995, Barings Futures experienced enormous losses that were worsened because Leeson, as he had expected markets to be stable, had also sold options. Even when losses kept mounting, he continued to increase the notional investments because he was convinced he was right and that the markets would finally evolve in his favor. When he was unable to make the margin requirements dictated by exchanges on derivative trading, he walked out.

Because of Barings strong image of a well-established and conservative bank, its bankruptcy compounded with the fact that it was due to the trading losses of only one (very young) executive, was a shocking wake-up call for the world financial community. The lesson to be learned was not simply one of the danger of market risk exposure to the survival of an institution but also one on how ruinous the lack of control in risk management can be. In effect, although Leeson turned out to be an extremely risk-taking executive, had he not been in charge of both the front and back-offices (trading and supervision) of its department, someone could have unveiled his dangerous trading practices and risk exposures and probably imposed limits on the positions he was engaging the bank in. Barings's fall was a great case for market risk management. Positions must be limited and diversified, market risk must be assessed and portfolio constitution readjusted, back and front office functions must be separated; and finally, a risk management system quality, just as that of any endeavor, is dependent on the personality and skills of the person undertaking it.

EXHIBIT III: PRESENTATION OF REGULATORY CAPITAL REQUIREMENT GUIDELINES

The European Community Capital Adequacy Directive (EC-CAD)

The EC's directive was issued in March 1993 and had to be implemented by January, 1996. It was written in a context of planning for Europe's integration and unique market. It mandates banks and investment firms to set capital aside to cover market risks. The directive relies on VaR computations but computes capital requirements by adding up the capital requirements on positions of different types, in different markets. This method assumes perfect correlations across risk factors and calculates market risk as the square root of the sum of the factors variances: it does not allow for the risk reducing effect of assets diversification. The EC directive approach prohibits the recognition of correlations between risk factors when calculating capital allocation, because of concern about their instability. It also sets a floor for specific risk charges when it has been widely acknowledged that proper portfolio diversification does eliminate specific risk (it is the component of total risk inherent in an instrument because of its issuer and it can be eliminated from a portfolio through diversification which only retains market risk)

While the prudential aspect of the CAD comes from a justified concern to protect the soundness of the financial system, its coming to application has generated great concern in the European financial industry. Many financial firms have protested that some of them (and especially those active in different markets) would be forced to overestimate their market risks and consequently force them to maintain very high (and even excessive and detrimental) capital levels. Some institutions have also expressed concern that as a result of the application of the CAD, they may well be bared from some types of trading activities because the inefficient allocation of capital would too seriously undermine their trading profitability.

The Basle Committee Proposal on Market Risk Capital Requirements

In January, 1996, the Basle Committee on Banking Supervision of the Bank of International Settlements of the Group of Ten Central Banks released its final amendment of its Capital Requirement proposal which must be implemented by the end of 1997. The Committee had issued a preliminary consultative proposal in 1993 that called for the use of a standard model by all banks of member countries of the Group of Ten, to calculate VaR, to guard against market risk. Under this approach, VaR was computed through the building block procedure which computes total VaR as the sum of the piecemeal VaRs calculated with respect to the different relevant market risk factors. This approach generated a heated debate in the international banking industry and especially because the method does not allow for portfolio diversification effect since it uses perfect correlations among factors (just as the EC-CAD).

Since then, important progress has been made since the Basle Committee has realized that some financial institutions have implemented their own proprietary risk management system which are more sophisticated than the standard-VaR model. The new proposal allows these institutions to use their internal models to evaluate their market risk while firms less advanced in terms of risk management can still use the model approach. "Internal"-VaRs, however, would have to be multiplied by a factor of three to obtain the prudential capital charges, where as "standard" VaR could be used without any adjustment. In any case VaRs should be computed at

99% confidence, with a ten-day holding period, using a reference period of five years. Banks using their internal model would in addition have to pay a penalty in terms of holding more capital if backtesting indicates that their VaR is underestimated (the scaling factor of three for capital requirement would then be raised).

Many institutions have protested what they think is a disadvantageous treatment of risk-conscious firms which have moved forward to implement a sophisticated risk management system. They argue that the Basle amendment penalizes them since more capital would have to be allocated for protection under the internal models approach. They think their models are more accurate than the standard approach and, they should be trusted and acknowledged for having, on their own, implemented complex and expensive risk management systems.

The International Swaps and Derivatives Association (ISDA) has conducted a research concluding that VaR based on the core parameters of the Basle Committee proposal provided capital covers of P&L movements for the 1987 stock market crash, the 1990 Gulf War, the 1992 Exchange Rate Mechanism crisis, the 1994 bond market decline[70].

The London Investment Banking Association (LIBA) has also showed that capital charges would be higher under the internal models approach than under the standardized approach, showing, thus, that banks would be penalized for using more accurate risk management systems. In all cases, the models alternative produced capital charges at least 40% higher than the standardized approach, and in some cases as much as three times higher capital charges.

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[70] Elderfield, "Capital Incentives", *Risk Magazine*, September, 1995, pp. 20-21

EXHIBIT IV: A BRIEF TOUR OF SOME VAR SYSTEMS

RiskMetrics

On October 10, 1994, JP Morgan released a risk management systems it had been working on for a year. The firm stated that it was seeking to establish a standard market risk measurement for the industry, promote greater transparency on market risks and enhance its relationship with its clients. It has been frequently reported that since 1990, the firm itself had been using the VaR methodology and that, by 4:15 p.m., the chairman of the investment bank received a daily report of the summary of the firm's market risk exposure for that day.

RiskMetrics key techniques of market risk management and market data are since freely available. Originally, daily and monthly volatitilies and correlations data on 15 markets, in total over 100,000 numbers were available daily on the Internet, CompuServe and Telerate, the data sets were later expanded to cover 450 instruments in 23 markets.

The method assumes changes in prices and yield are normally distributed and relies on analytical VaR, and uses exponentially weighted historical volatility and correlation data to predict how markets are likely to move in the future. The choice of model variables are a daily holding period, a 95% confidence level, and a 75-day rolling observation window, and longer-time VaR is aggregated from Daily Earnings at Risk, (DeaR), as DeaR times the square root of the appropriate liquidation period. This methodology should only be used for linear risks but not for such associated with options (the system treats those separately in Monte Carlo simulations. JP Morgan launched its Fourfifteen spreadsheet package in March 1996, and added an option market risk measurement package by the end of 1996, as a Monte Carlo simulation (partial and full) add-in.

JP Morgan expects users to be mostly corporates. fund managers. commercial and central banks. and also provided the data and methodology to financial consultants and software suppliers which will incorporate RiskMetrics in a range of risk management software packages. Although. the bank wanted to establish a benchmark for market risk, most professionals agree that (1) the idea. in itself. is questionable since financial institutions' risk profiles. risk aversion. resources and infrastructure. and strategic objectives are so heterogeneous: (2) even if a yardstick was created it should not be that of a specific competitor but rather a regulatory (neutral) one: (3) the most likely adopters of RiskMetrics are smaller firms. less advanced in their risk management implementation and that rival financial institutions will never use it for their own risk management (even if they may use it for research or comparison purposes).

RaRoc 2020

Bank Trust issued RaRoc 2020 in June. 1995. by licensing it to the Chrysler pension fund: it is a Risk-Adjusted-Return-on-Capital methodology. The system was developed in the 1980s. for internal use. to measure how effectively traders were allocating the bank's capital on a risk adjusted basis. It produces CaR estimates on a firmwide. but also on a portfolio. asset and manager basis. RaRoc 2020 methodology is based on Monte Carlo simulation. includes data on as many as 400 risk factors. uses a 99% confidence level and one year of data for the computations.

d-B Analyst

It is an end-user analysis tool launched by Deustche Bank, in January, 1995, to help manage cash flow and understand the concepts of risk management. It includes an interest rate module to calculate risk in symmetric and asymmetric interest rate sensitive positions such as loans, bonds, notes, floating rate notes, deposit, forward rate agreements, swaps, and futures. It also provides a system to identify foreign exchange risk, liquidity risk, and credit risk. Finally, it also allows gap, cash flow and sensitivity analysis and provides a scenario analysis framework.

The system is now used by wide number of the bank's clients and was relaunched in an integrated risk management system called db-treasury-network, because it was not the success it was expected to be the first time around.

PrimeRisk

Unlike RiskMetrics, PrimeRisk was not launched in the public domain, by CFS Boston, on March 18, 1996. It is destined only to current and prospective clients and is part of an integrated risk management software package called PrimeClear, a central interface that allows real-time trade reconciliation and confirmation, combined with flexible account structuring, between eight other highly-integrated products launched after, in 1996.

PrimeRisk software package offers customers the flexibility to customize risk analysis systems, some of the clients only use the data but their own model, while others use the whole package. It provides a forecasting model for risk parameters such as volatilities, correlations, prices and zero-coupon yield curves and tools to model VaR concepts. PrimeClear interface, on the other hand, produces the VaR and transaction sensitivity in its consolidation profit center, using its spreadsheet calculator.

PrimeRisk data are accessible to users via Internet, proprietary website, electronic mail, fax and diskettes. VaR, correlation and volatilities are updated internally at CSFB every hour, and data are downloaded to customers two to three times a day. Unlike RiskMetrics, the system uses different volatility models for the different relevant assets markets and fractional exponential weighting (because CSFB found Garch models less accurate than exponential weighting for its settings) where lower weights are applied to recent and more distant days and increased weights are used for intermediate days.

CHARISMA

It is Chase Manhattan's risk management system: Chase Risk Management Analyzer. It uses the historical VaR method and constructs distribution of the probable future price changes using historical data. Portfolios' exposures to specific risks, such as interest rate or foreign exchange volatility are identified then the portfolio is reevaluated as if each change occurred from today, and 100 changes in portfolio value are created. Then VaR is derived from that distribution. The methodology uses a 97.5% confidence level (thus approximately one failure in every 40 days in the 100 day-reference period used for the computations).

CARMA

CARMA is a software system issued by C•ATS that provides fully integrated market and credit risk analysis and management for global portfolios. It relies on the historical simulation method that can be generated in C•atalyst front-back office trading desk management system, while parametric Monte Carlo simulation can be handled in CARMA, and parametric analytical VaR can be derived in its FiCAD application. The firm targets derivative traders and end-users, financial institutions' risk managers but the package can also be used by corporates and money managers.

EXHIBIT V: ADDITIONAL GRAPHS

Reminder: Chronology of the Great Depression

- (1) August '29 to October '30, the period prior to the first bank panic which includes the stock market crash;

- (2) the final quarter of 1930, when the first bank panic occurred;

- (3) the first quarter '31 which is characterized by the second bank crisis in March '30;

- (4) the last half of '31 during which the Federal Reserve's reaction to Britain's departure from the gold standard worsened the US economy's decline;

- (5) the second quarter of '32 when the Federal Reserve's open market purchases provided a revival of the economy in the summer and fall of 1932 and;

- (6) the final six months of contraction when bank problems spread (Bank Holiday of March 1933) when the economy worsened again

Reading guide

Monthly data: **1920**:1-12; **1921**:13-24, **1922** 25-36, **1923**:37-48; **1924**:49-60; **1925**:61-72 **1926**:73-84; **1927**: 85-96, **1928** 97-108, **1929**:109-120; **1930**:121-132; **1931**:133-144;**1932**:145-156;**1933**:157-168;**1934**:169-180;**1935**:181-192; **1936**:193:204

Weekly data: **1929**:1-52;**1930**:53-104;**1931** 105-156;**1932**:157-209;**1933**:210-261; **1934**:262-313;**1935**:314-355; **1936**:356-417

FIGURE A1.a

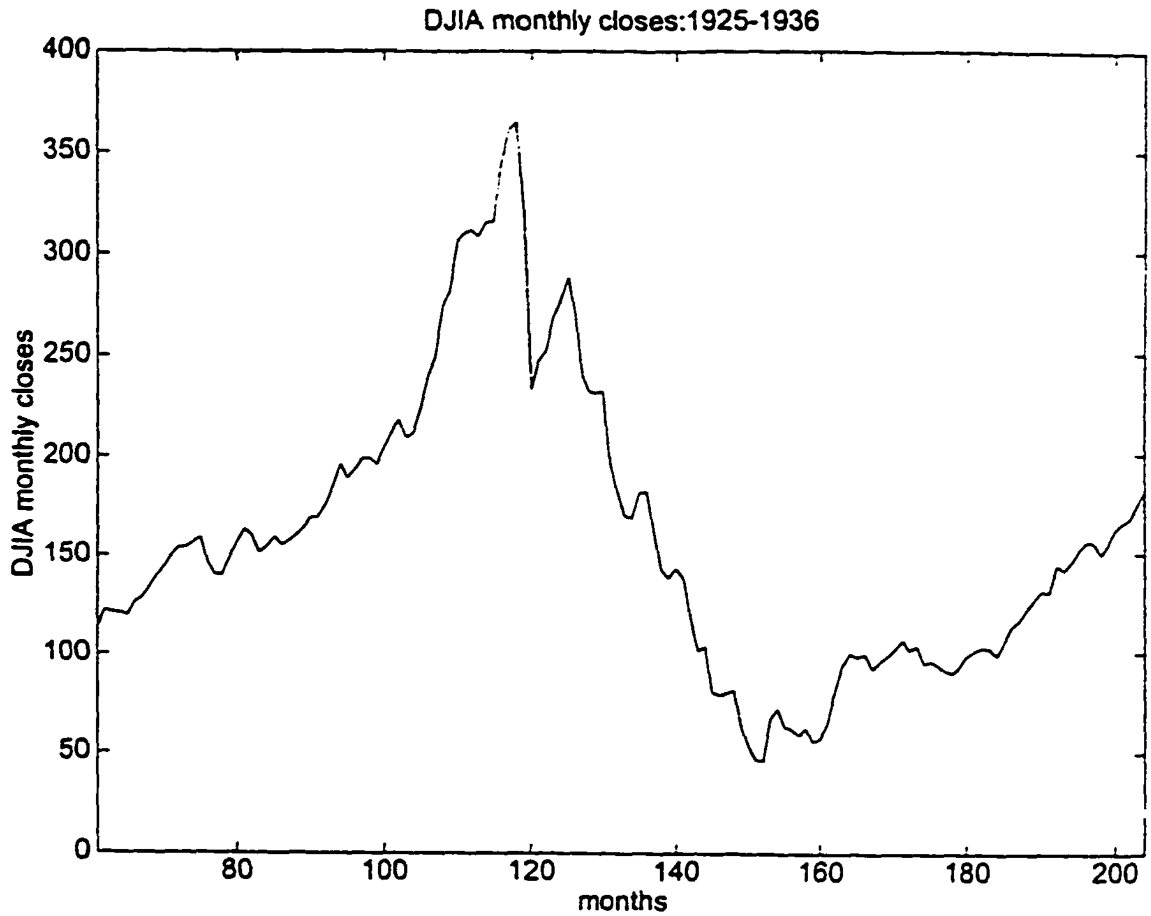


FIGURE A1.b

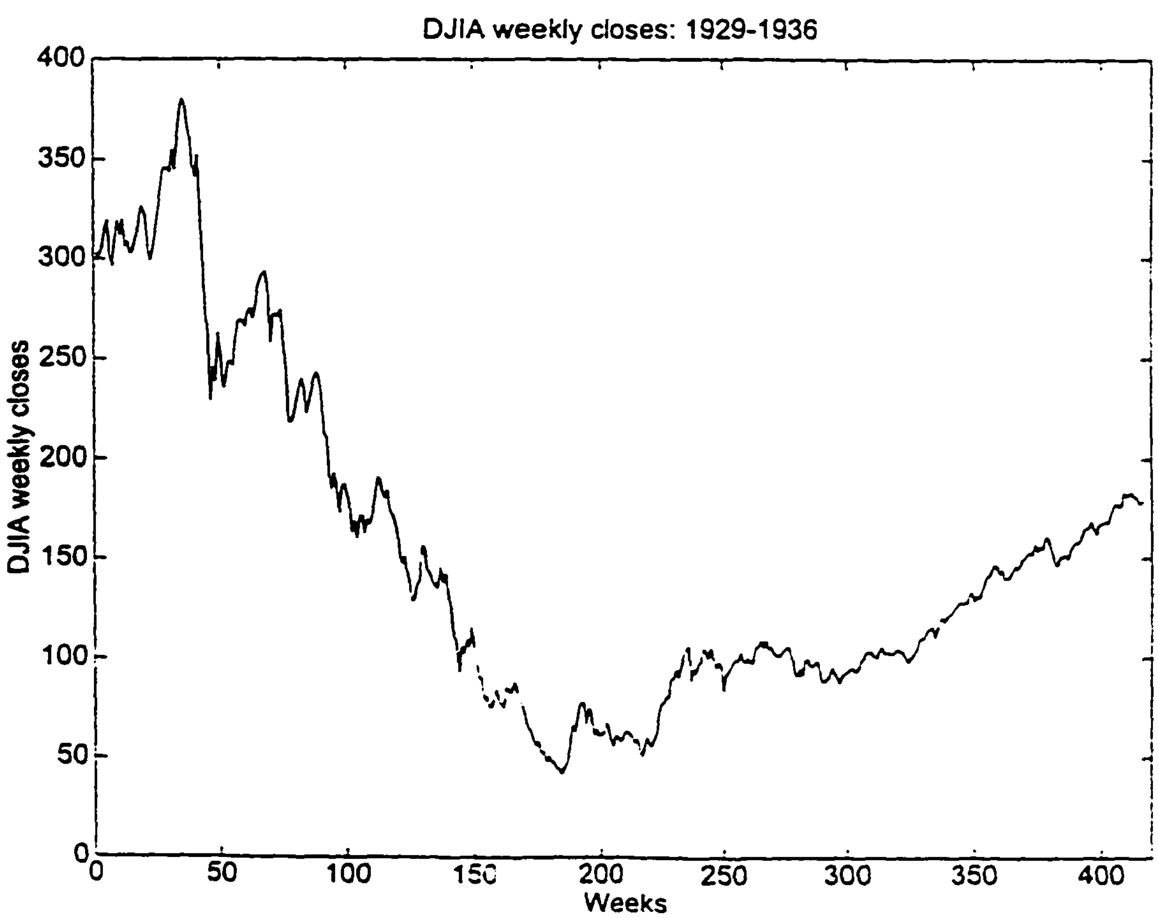


FIGURE A2.a

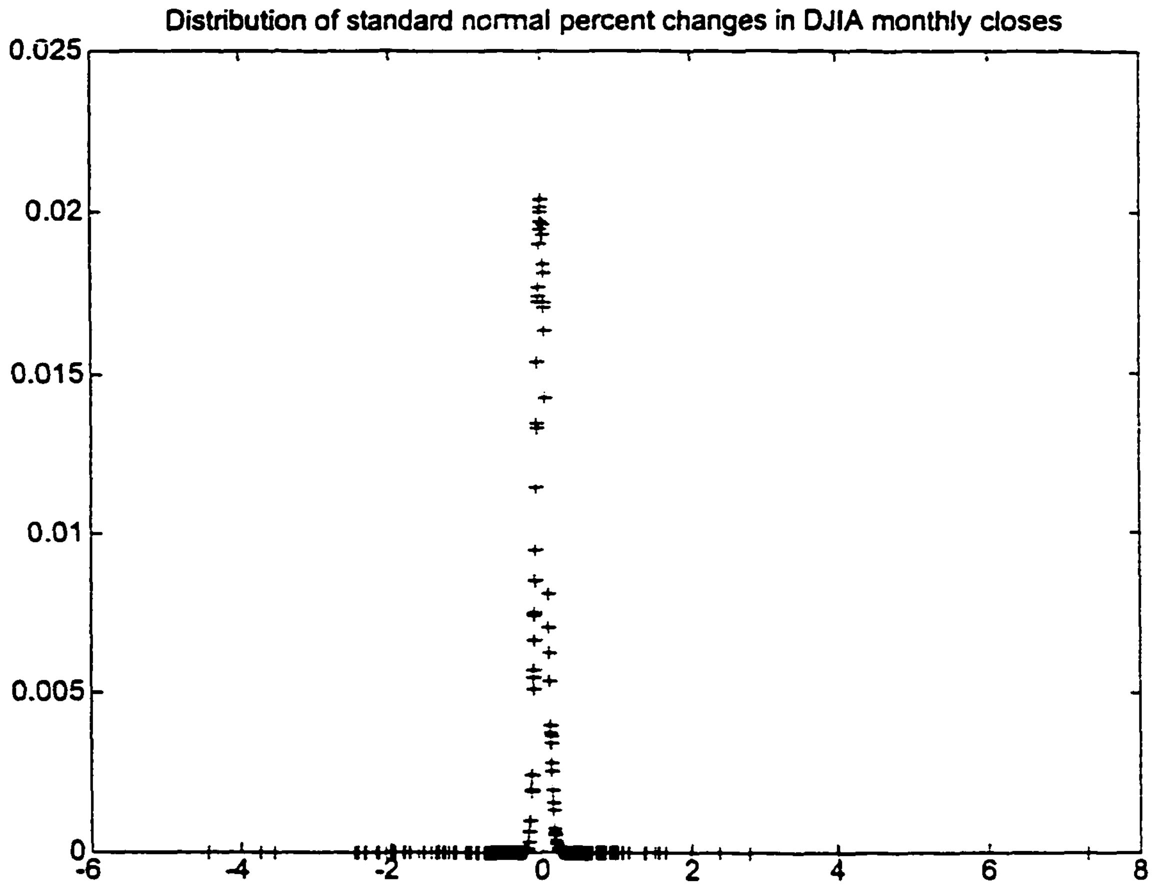


FIGURE A2.a

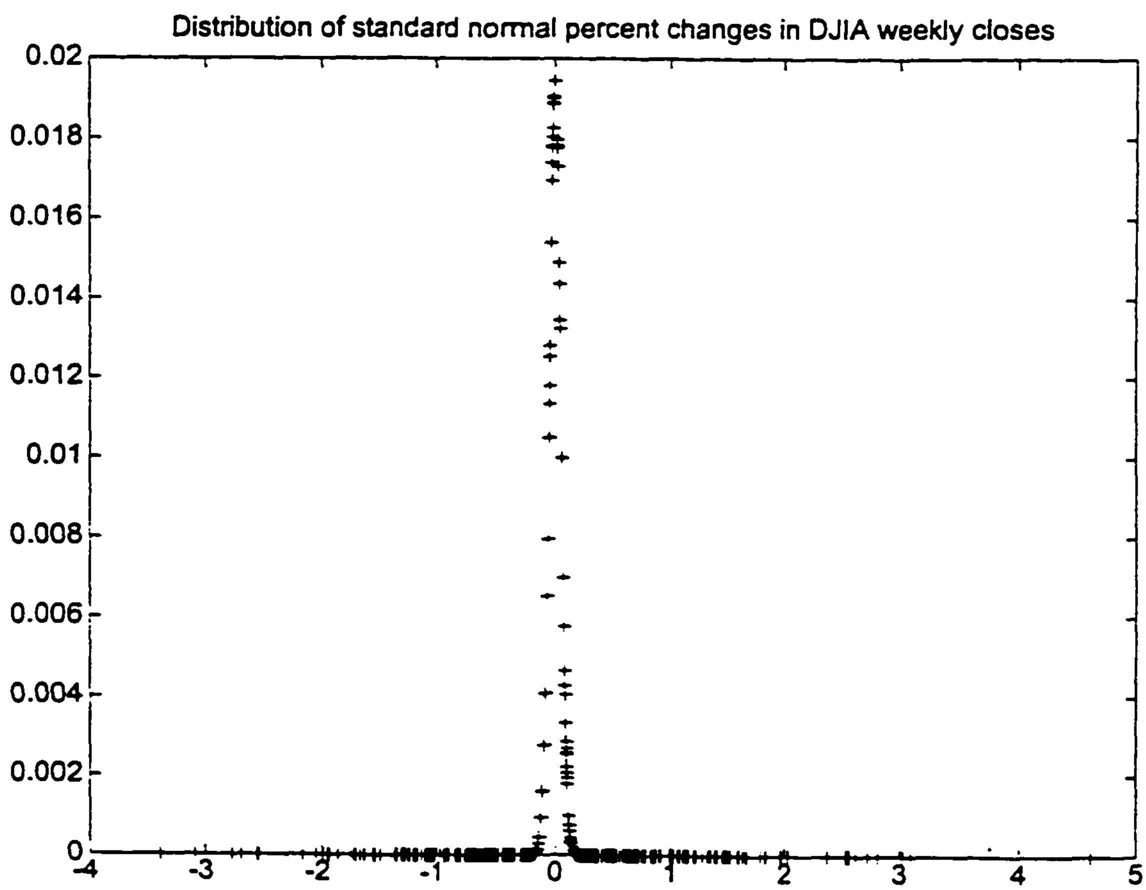


FIGURE A3

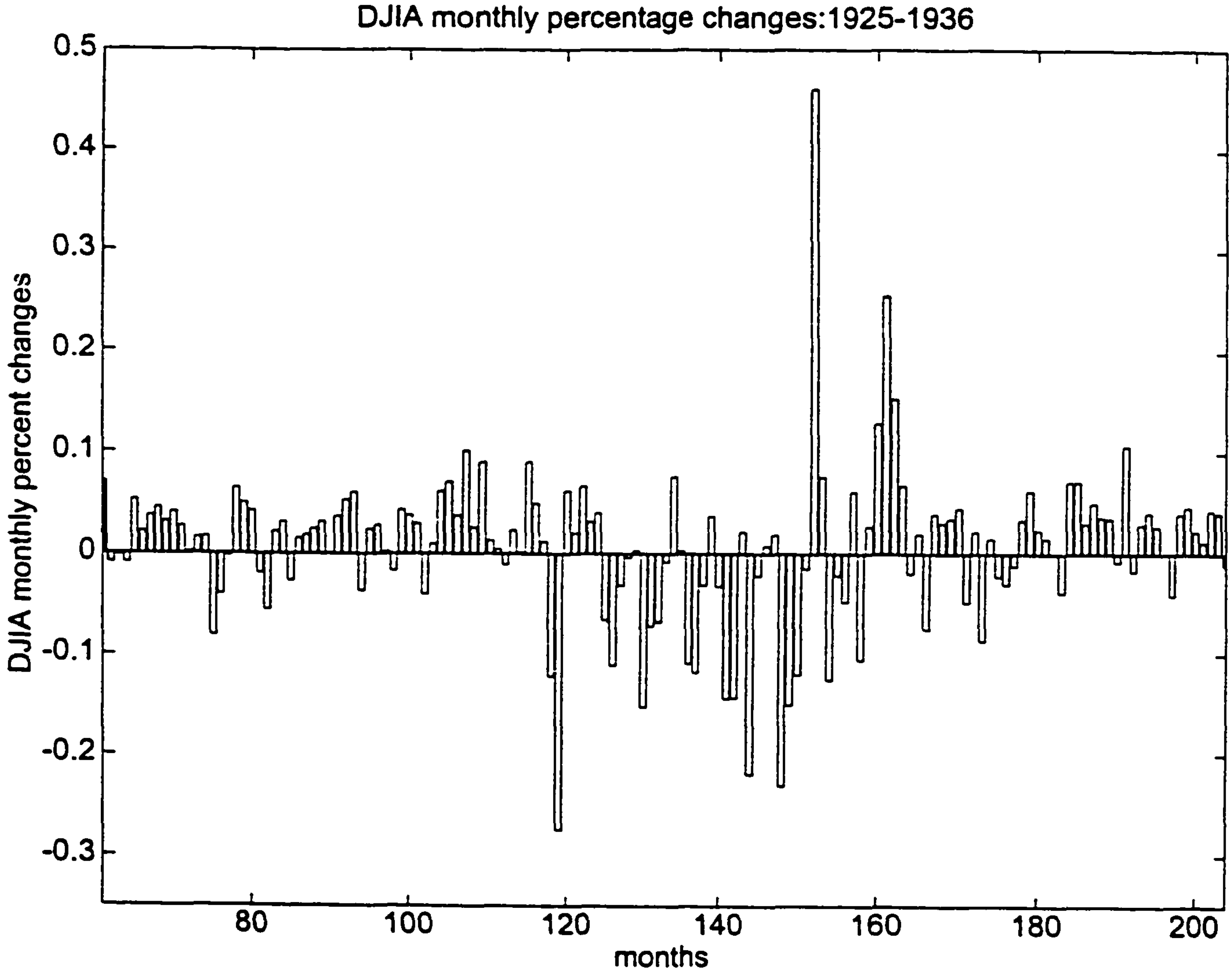

DJIA monthly percentage changes:1925-1936
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
DJIA monthly percent changes
80
100
120
140
160
180
200
months

FIGURE A4

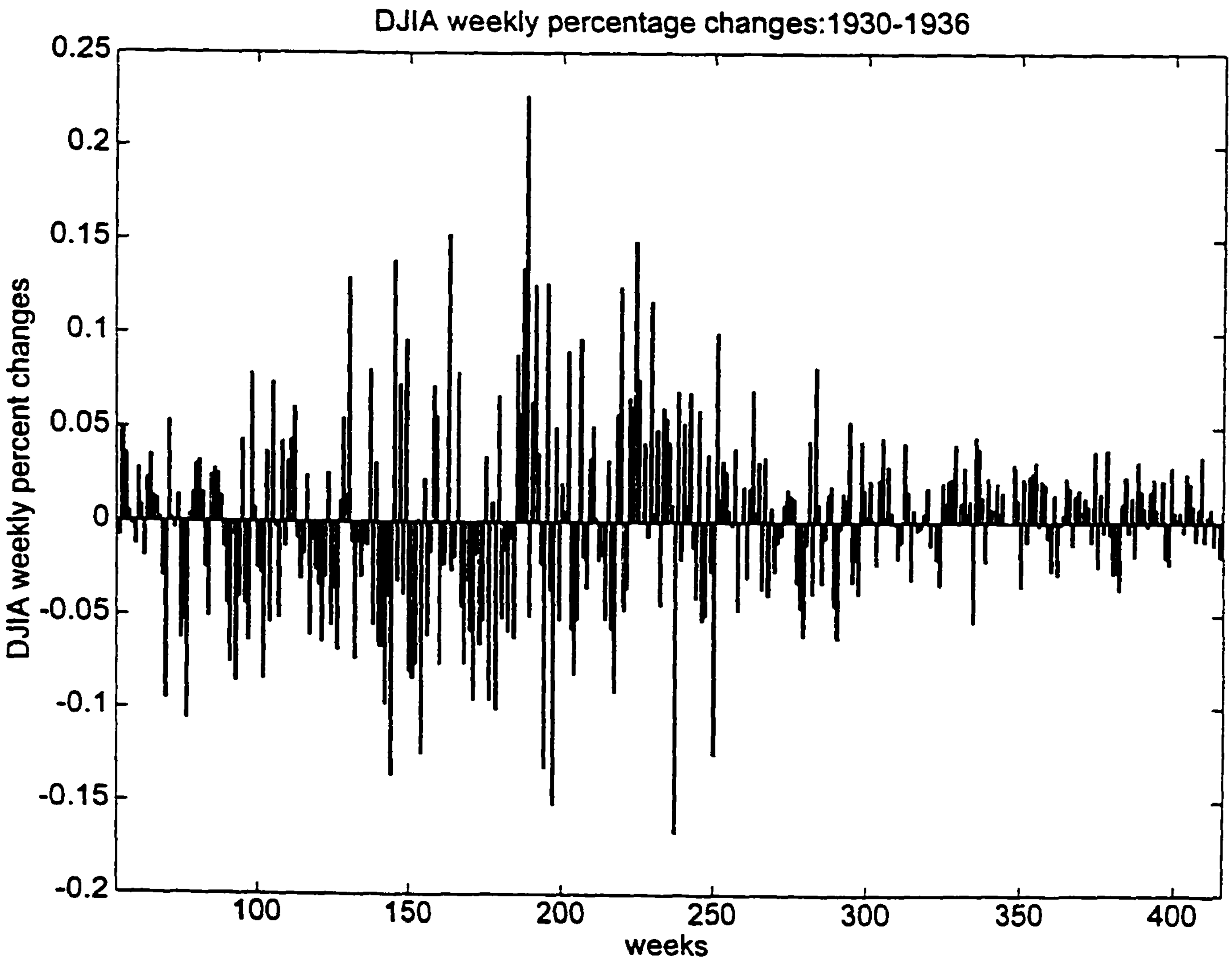

DJIA weekly percentage changes:1930-1936
DJIA weekly percent changes
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
100
150
200
250
300
350
400
weeks

FIGURE A5

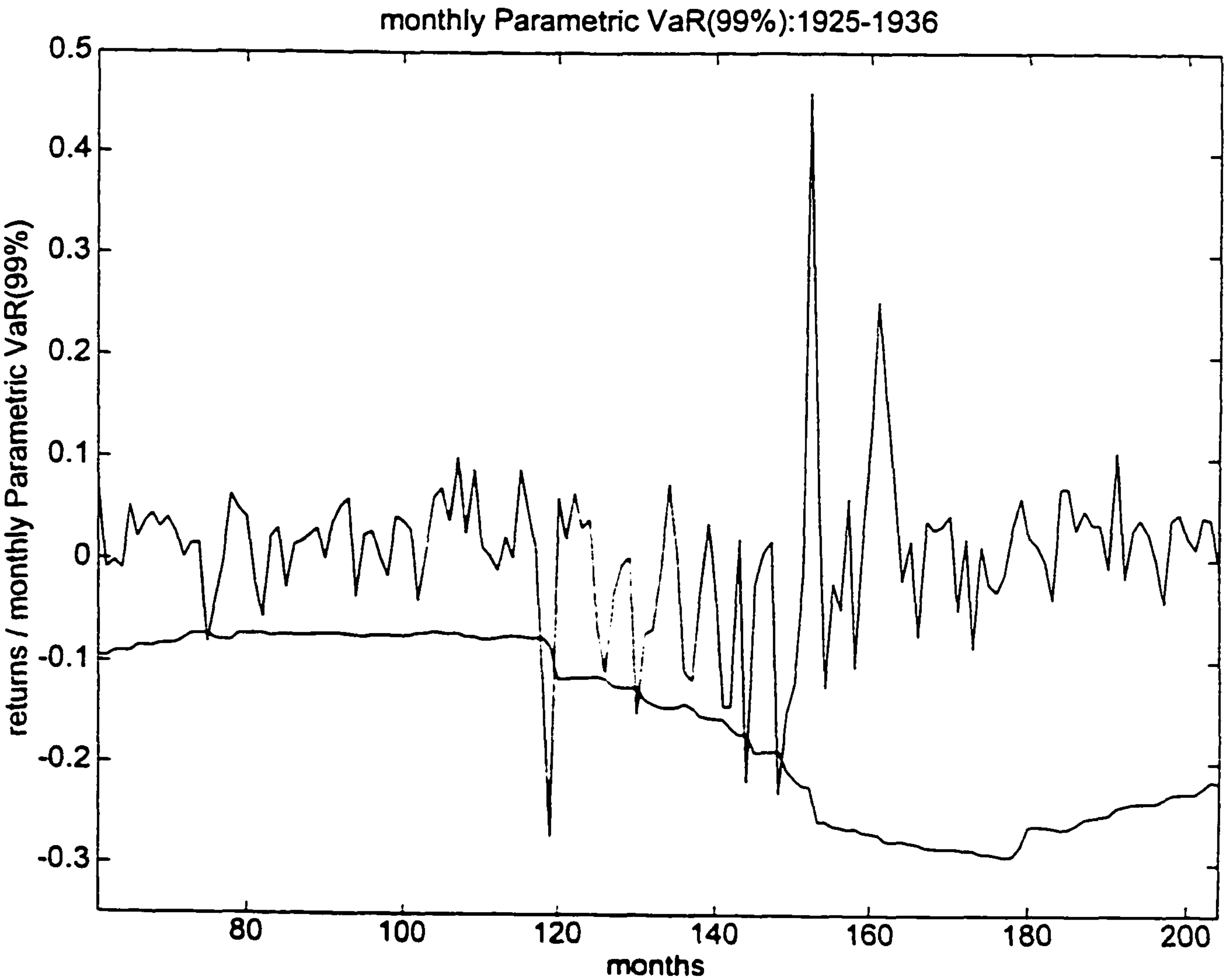

monthly Parametric VaR(99%):1925-1936
returns / monthly Parametric VaR(99%)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
80
100
120
140
160
180
200
months

FIGURE A6

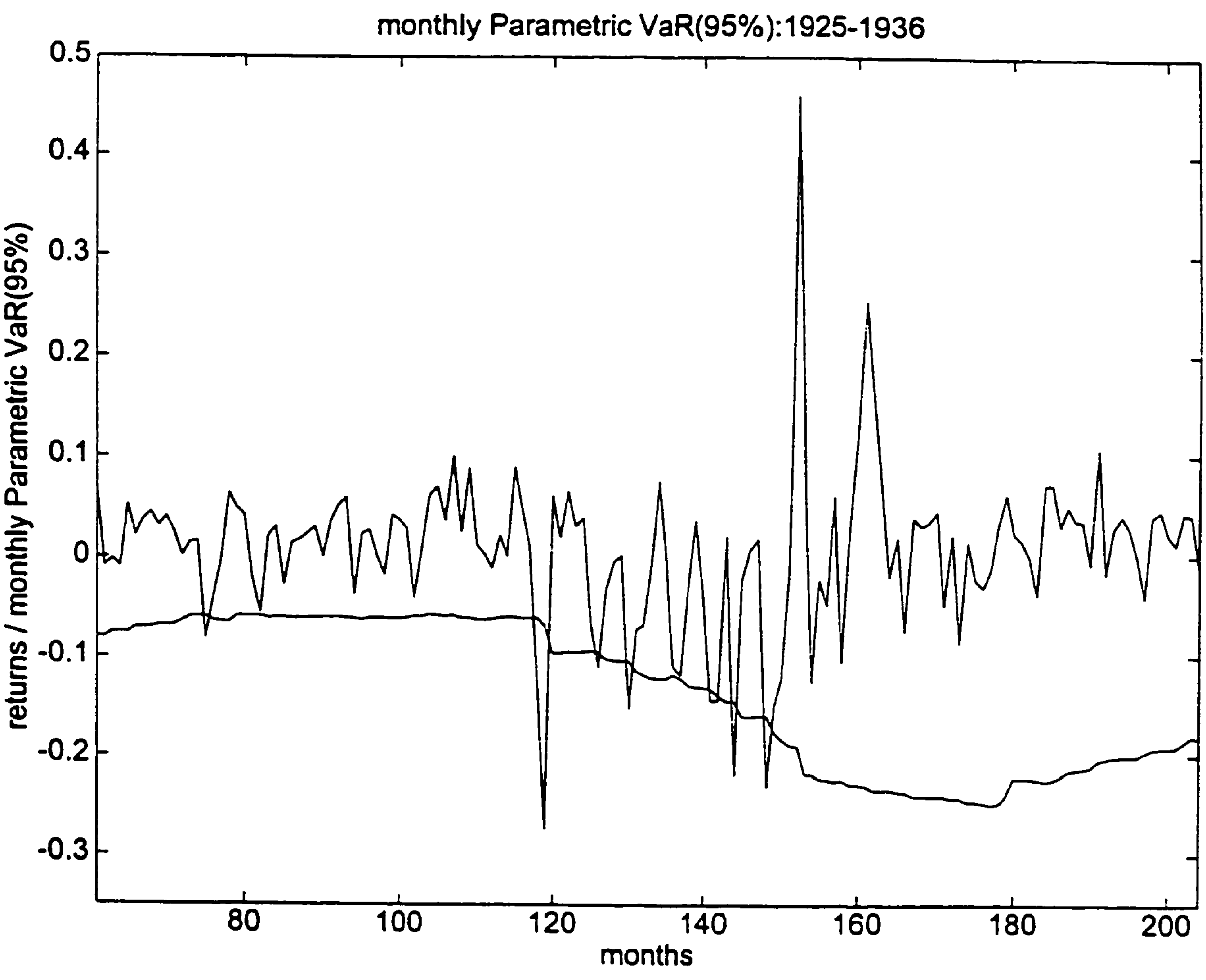

monthly Parametric VaR(95%):1925-1936
returns / monthly Parametric VaR(95%)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
80
100
120
140
160
180
200
months

FIGURE A7

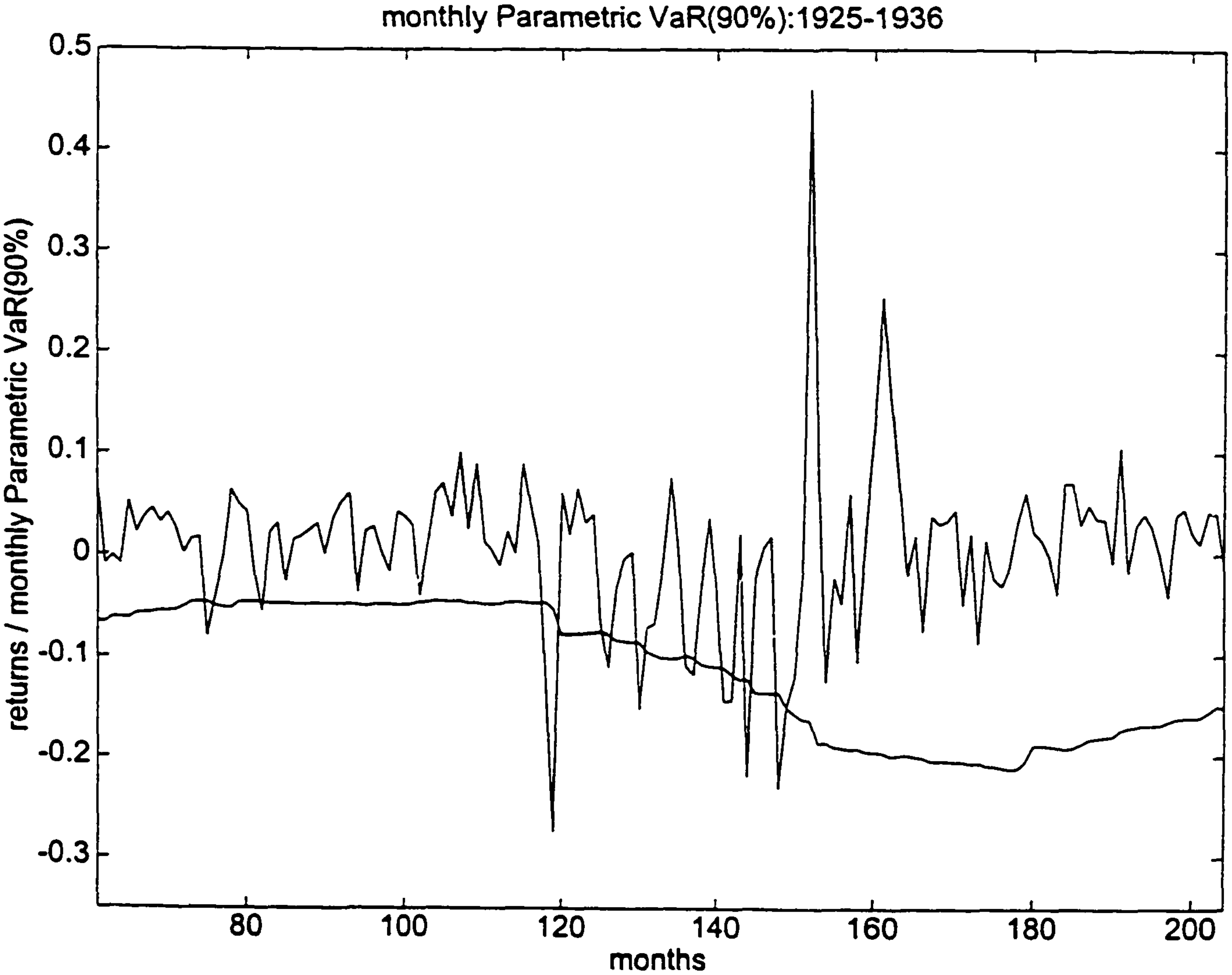

monthly Parametric VaR(90%):1925-1936
returns / monthly Parametric VaR(90%)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
80
100
120
140
160
180
200
months

FIGURE A8

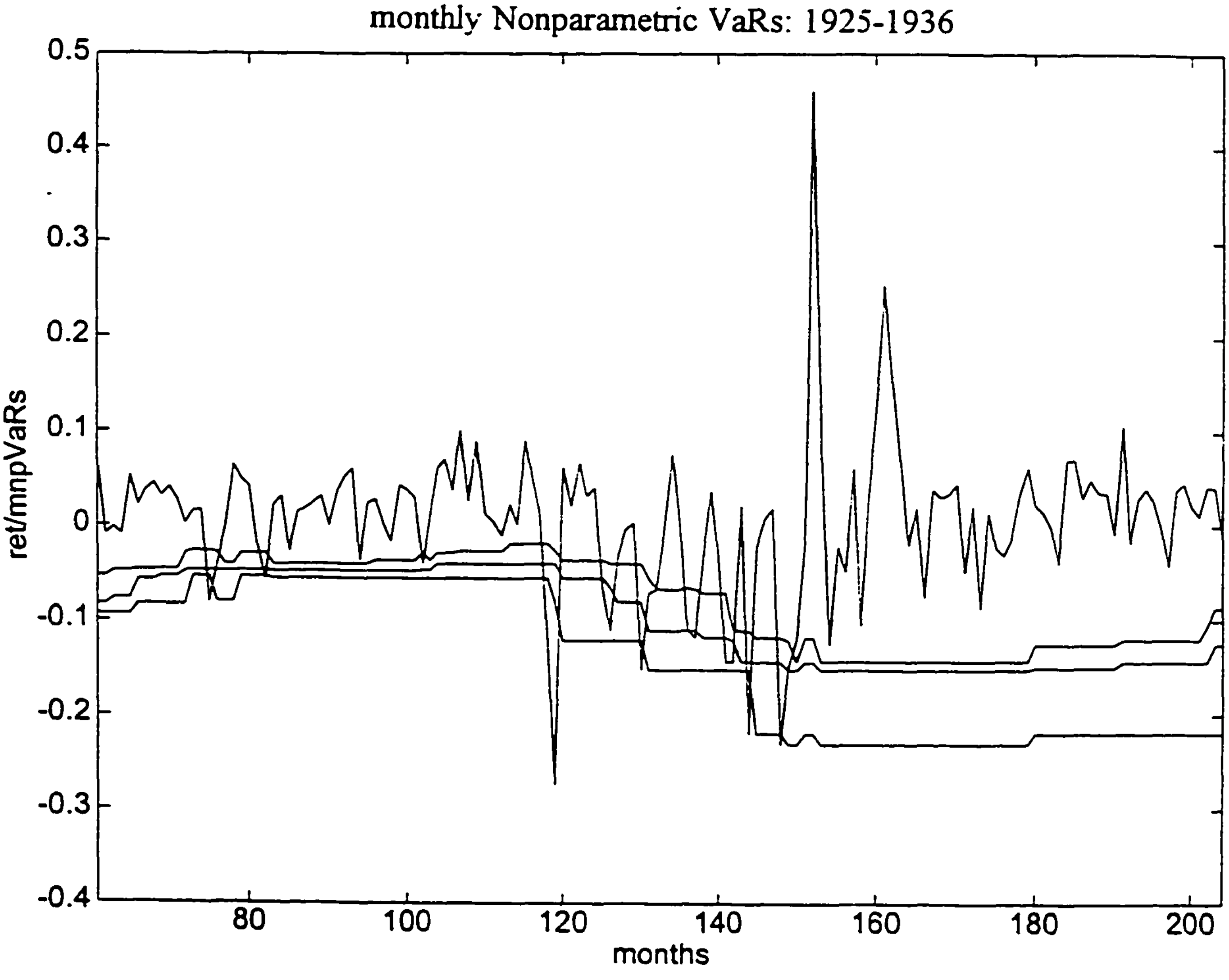
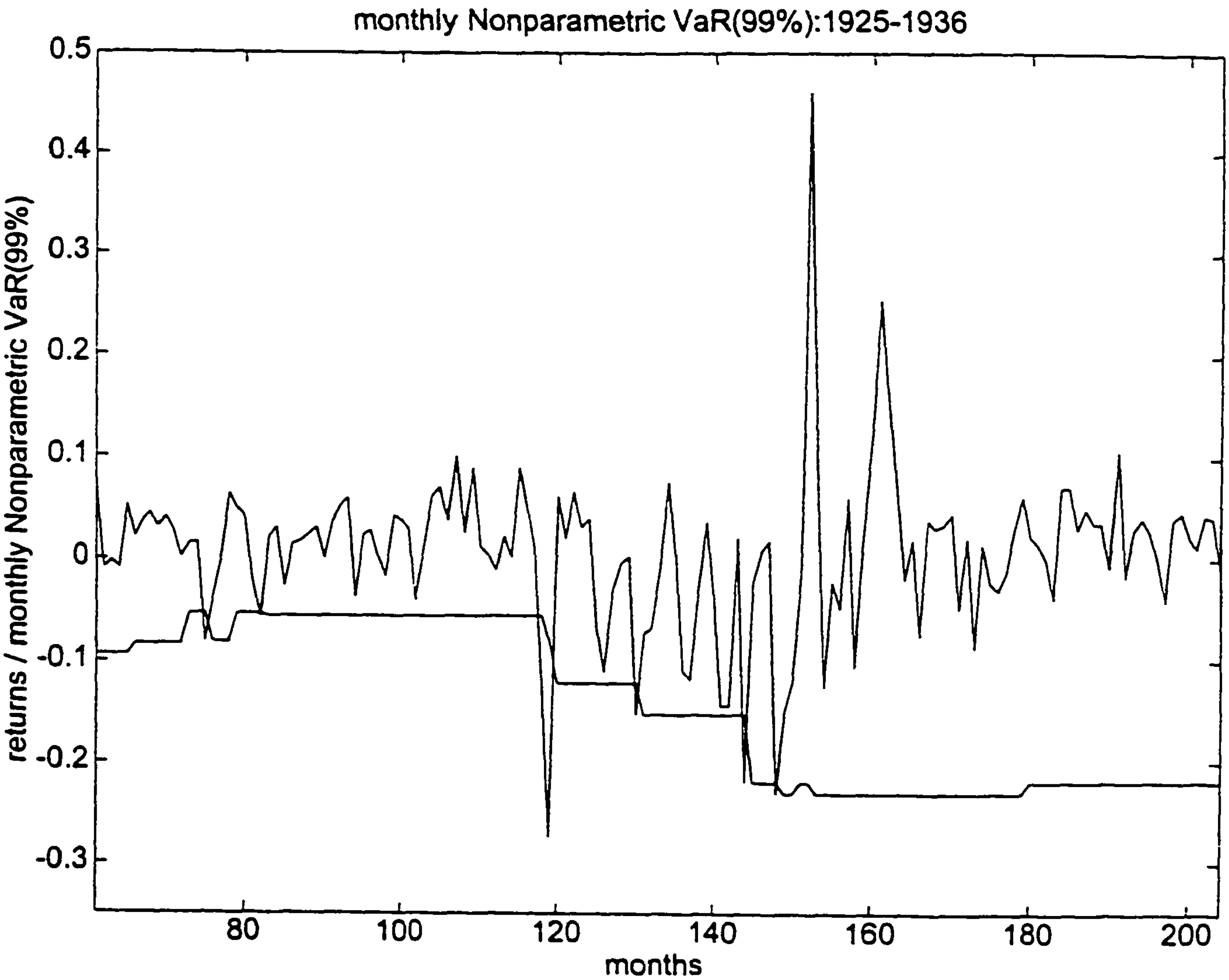

monthly Nonparametric VaRs: 1925-1936
ret/mnpVaRs
months
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
80
100
120
140
160
180
200

FIGURE A9



monthly Nonparametric VaR(99%):1925-1936
returns / monthly Nonparametric VaR(99%)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
80
100
120
140
160
180
200
months

FIGURE A10

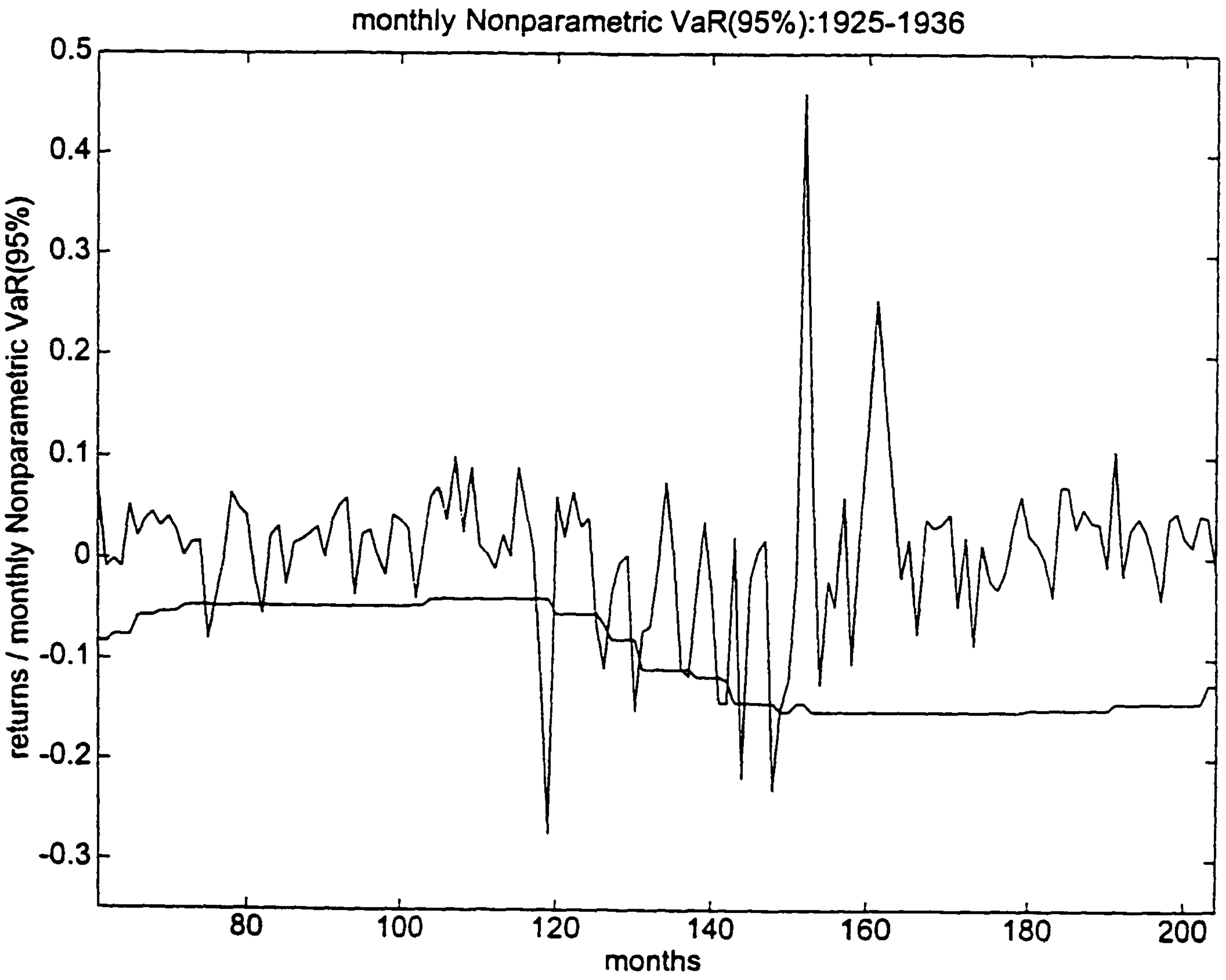

monthly Nonparametric VaR(95%):1925-1936
returns / monthly Nonparametric VaR(95%)
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
80
100
120
140
160
180
200
months

FIGURE A11

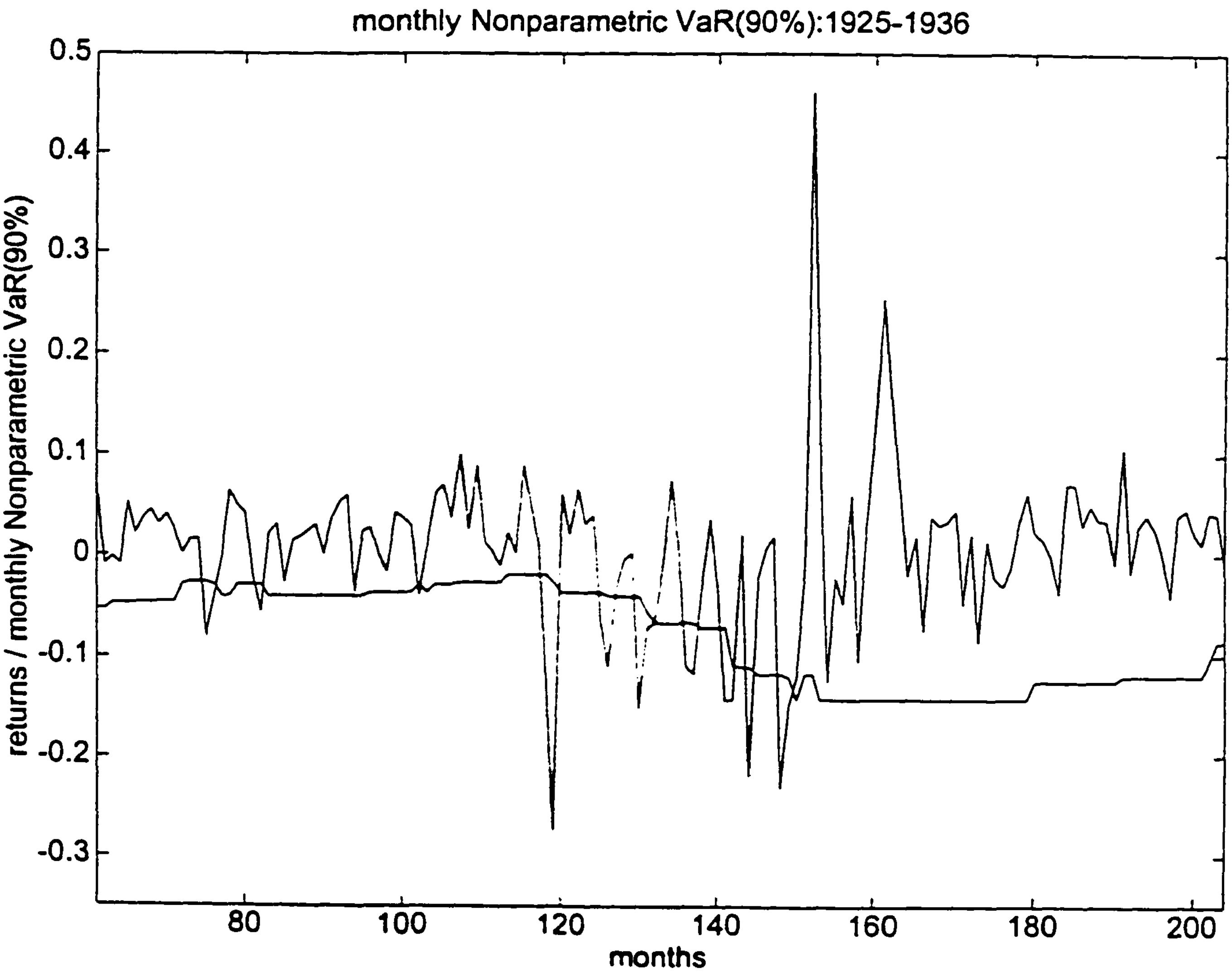
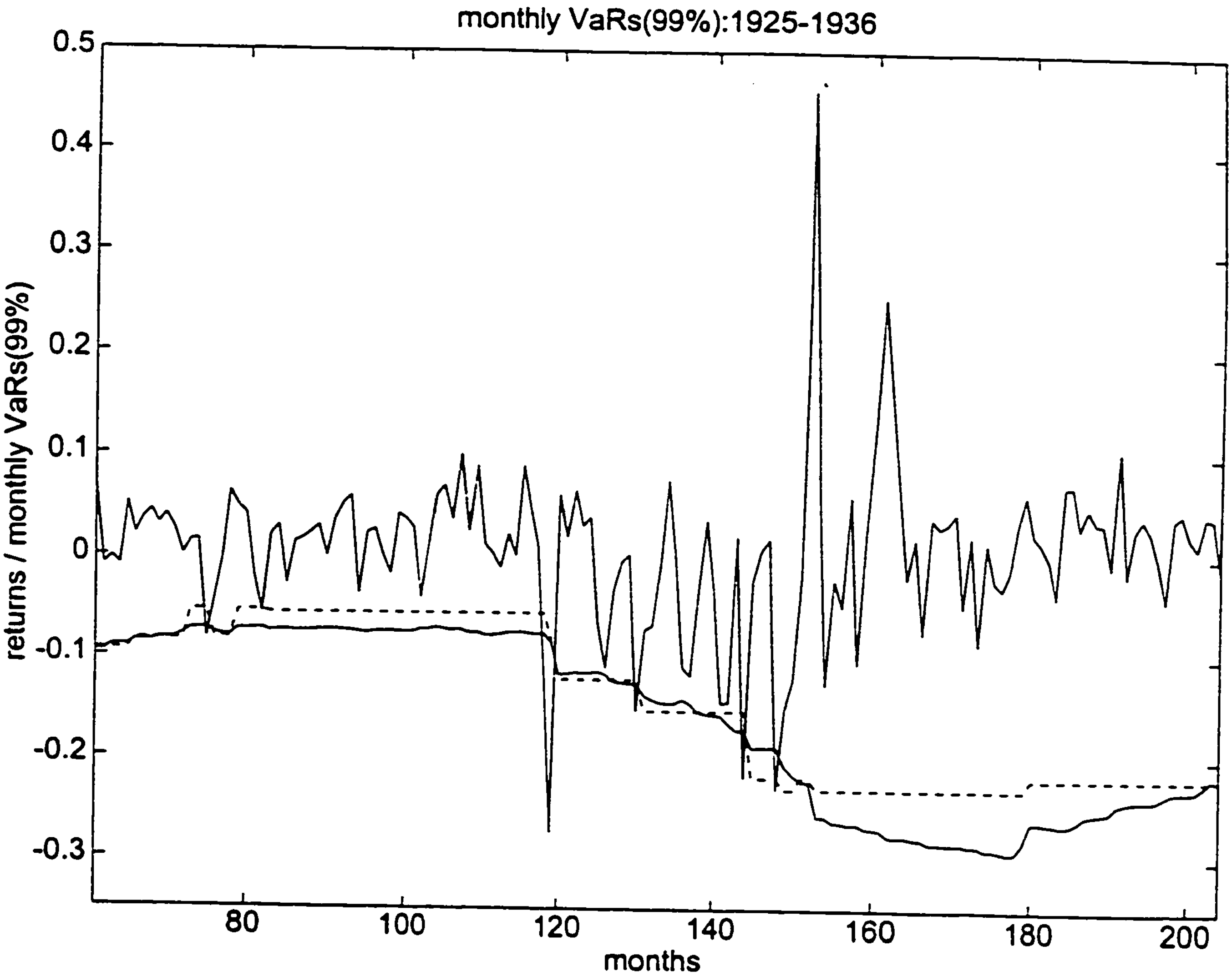

monthly Nonparametric VaR(90%):1925-1936
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
returns / monthly Nonparametric VaR(90%)
80
100
120
140
160
180
200
months

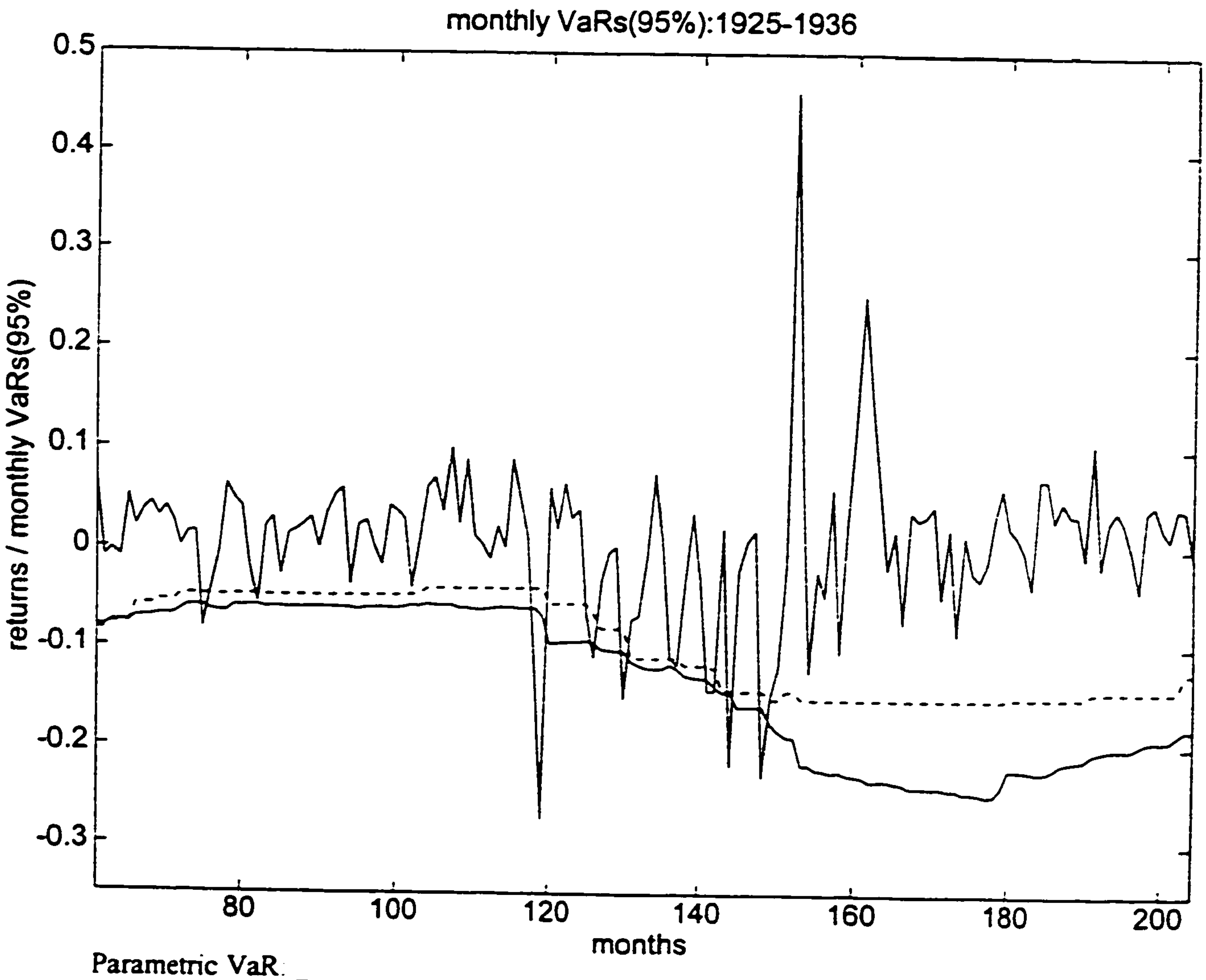
FIGURE A12



Parametric VaR: ________

Nonparametric VaR: ------

FIGURE A13



Parametric VaR: ________
Nonparametric VaR: ------

FIGURE A14

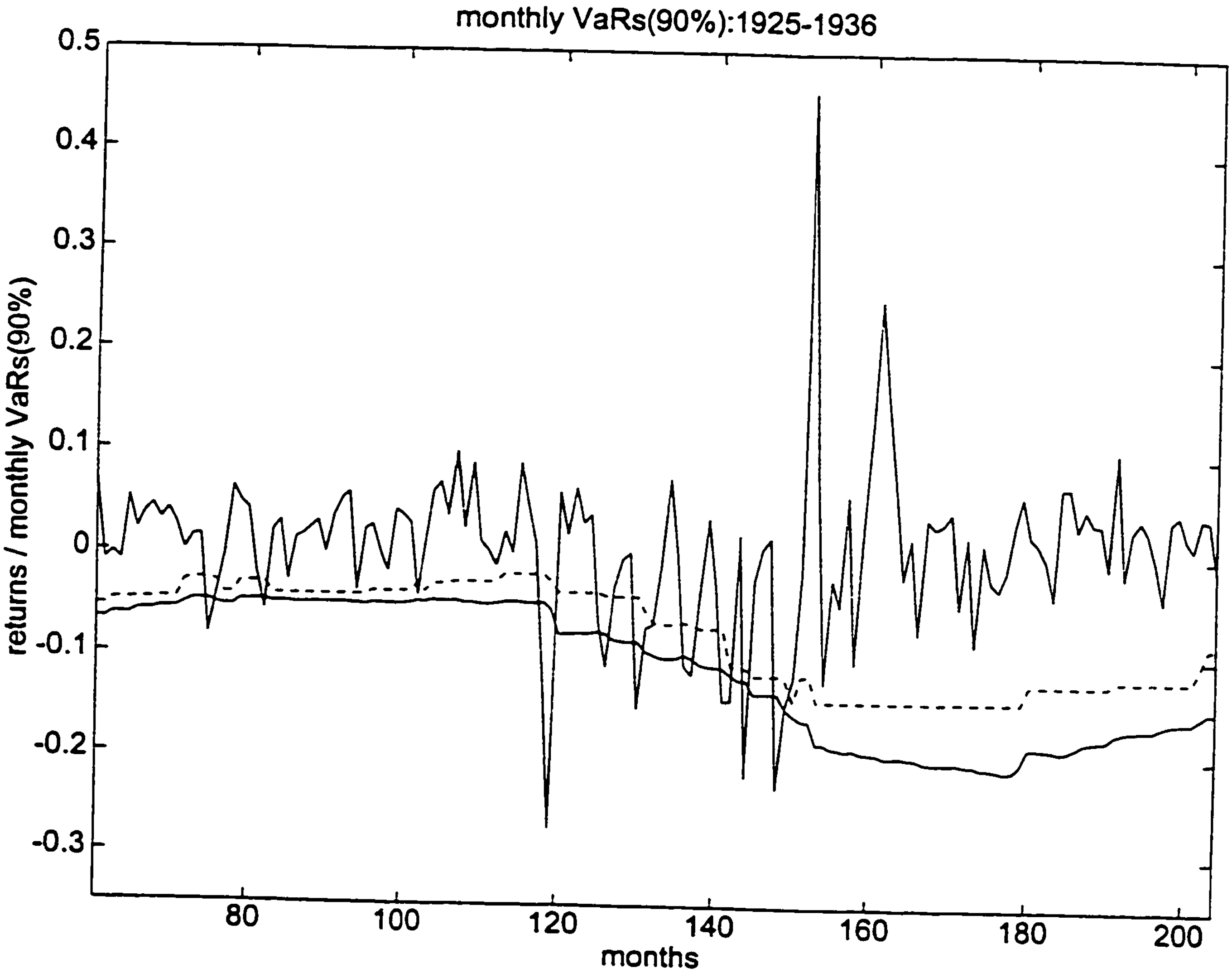

monthly VaRs(90%):1925-1936
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
returns / monthly VaRs(90%)
80
100
120
140
160
180
200
months

FIGURE A15

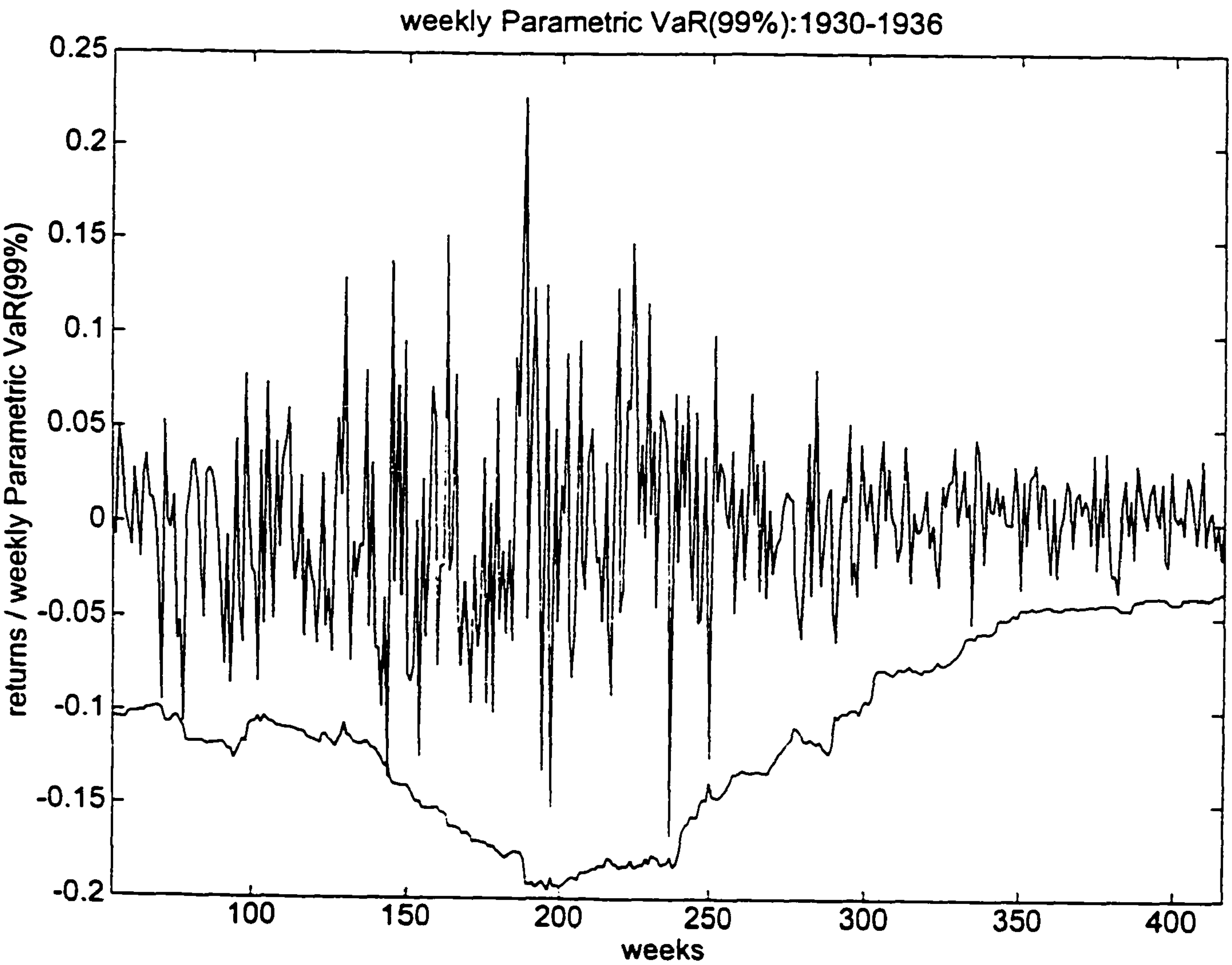

weekly Parametric VaR(99%):1930-1936
returns / weekly Parametric VaR(99%)
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
100
150
200
250
300
350
400
weeks

FIGURE A16

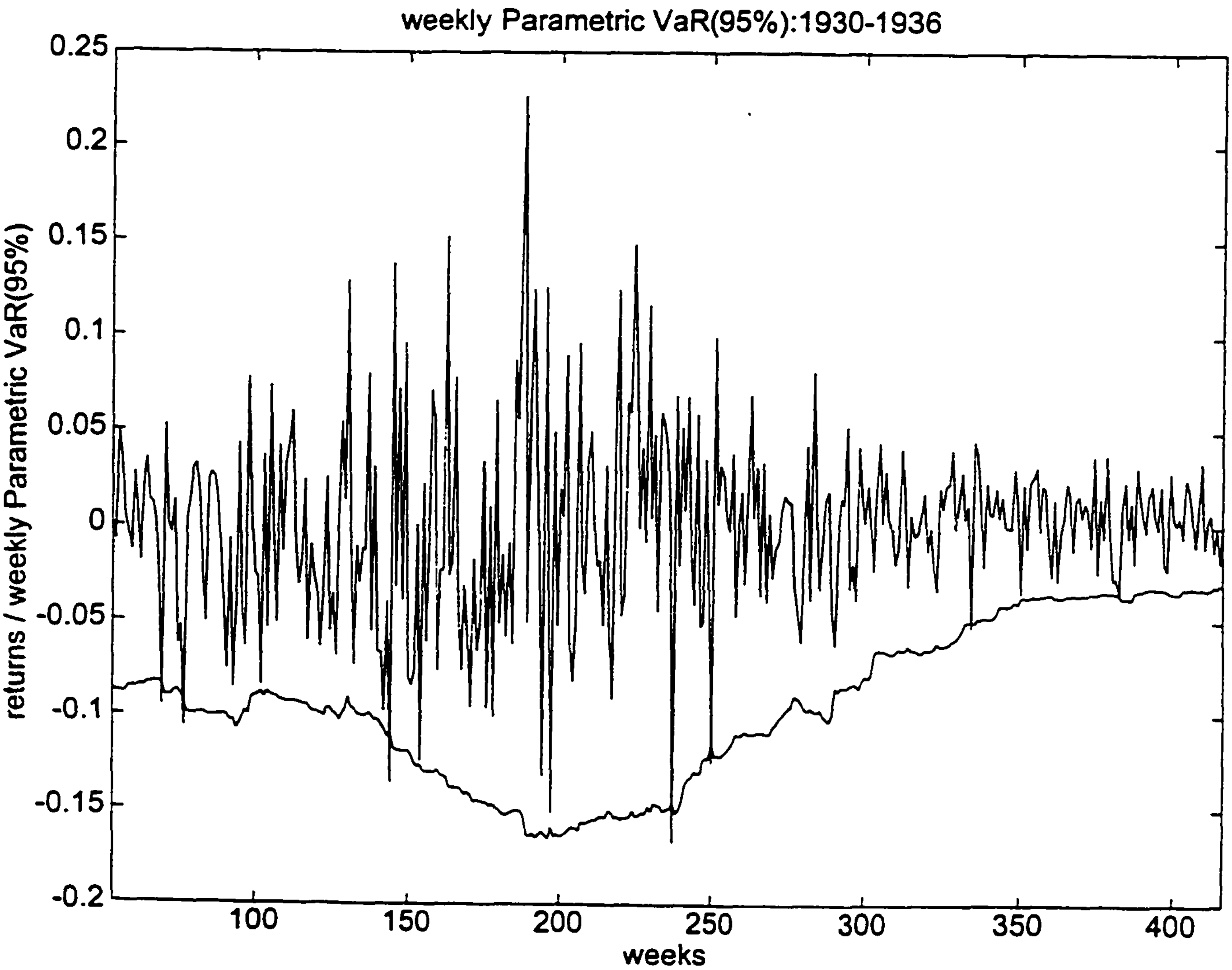

weekly Parametric VaR(95%):1930-1936
returns / weekly Parametric VaR(95%)
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
100
150
200
250
300
350
400
weeks

FIGURE A17

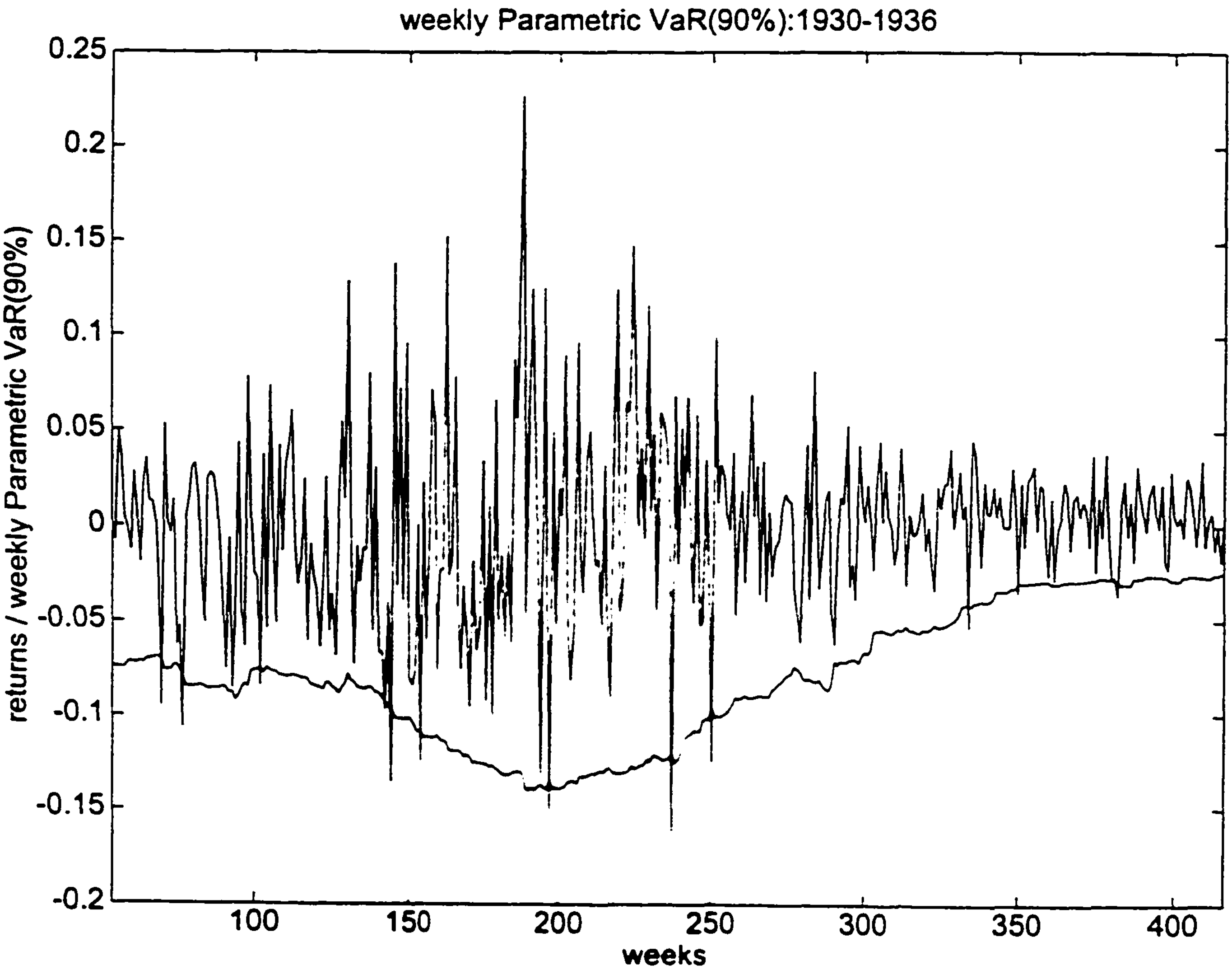

weekly Parametric VaR(90%):1930-1936
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
returns / weekly Parametric VaR(90%)
100
150
200
250
300
350
400
weeks

FIGURE A18

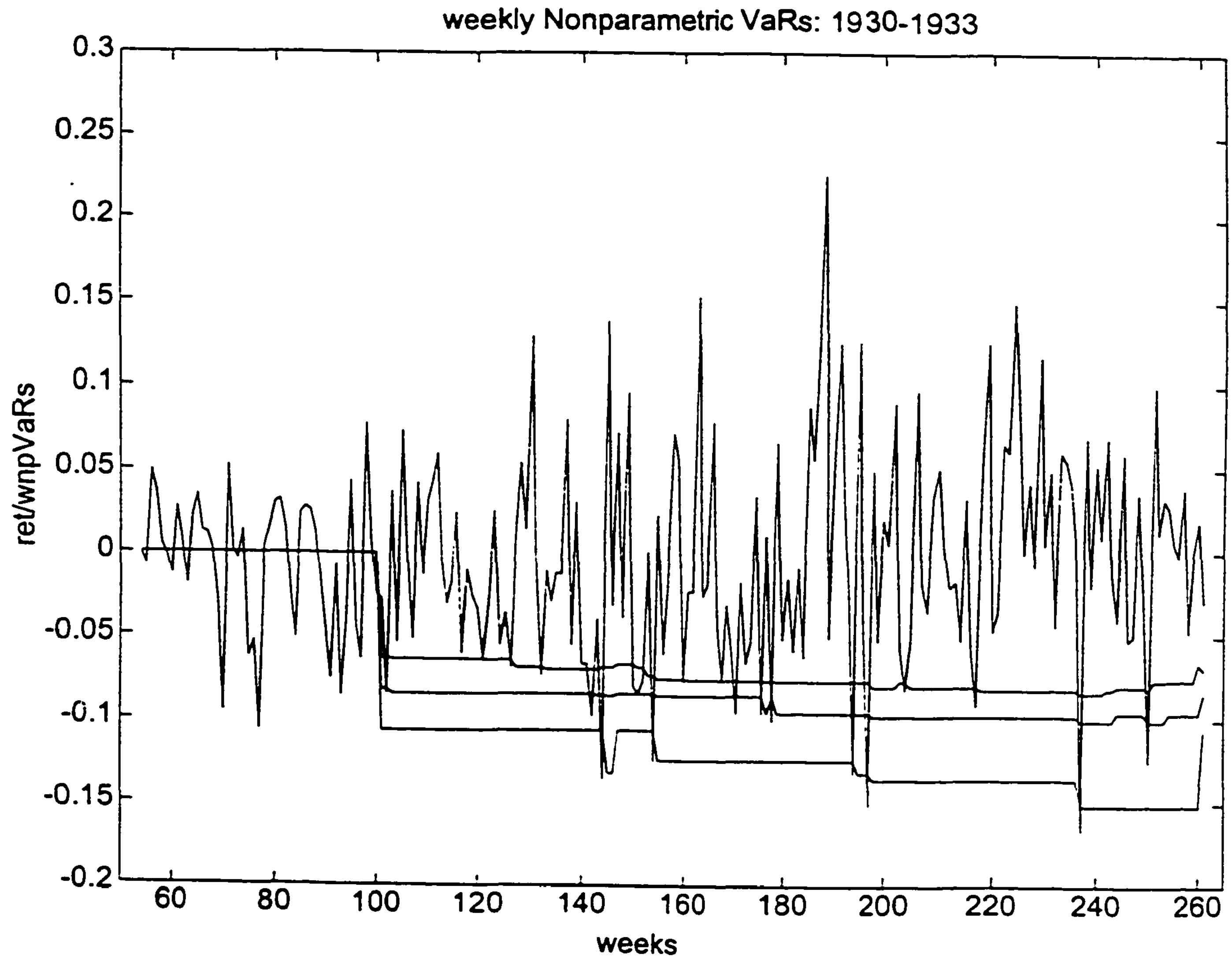

weekly Nonparametric VaRs: 1930-1933
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
ret/wnpVaRs
60
80
100
120
140
160
180
200
220
240
260
weeks

FIGURE A19

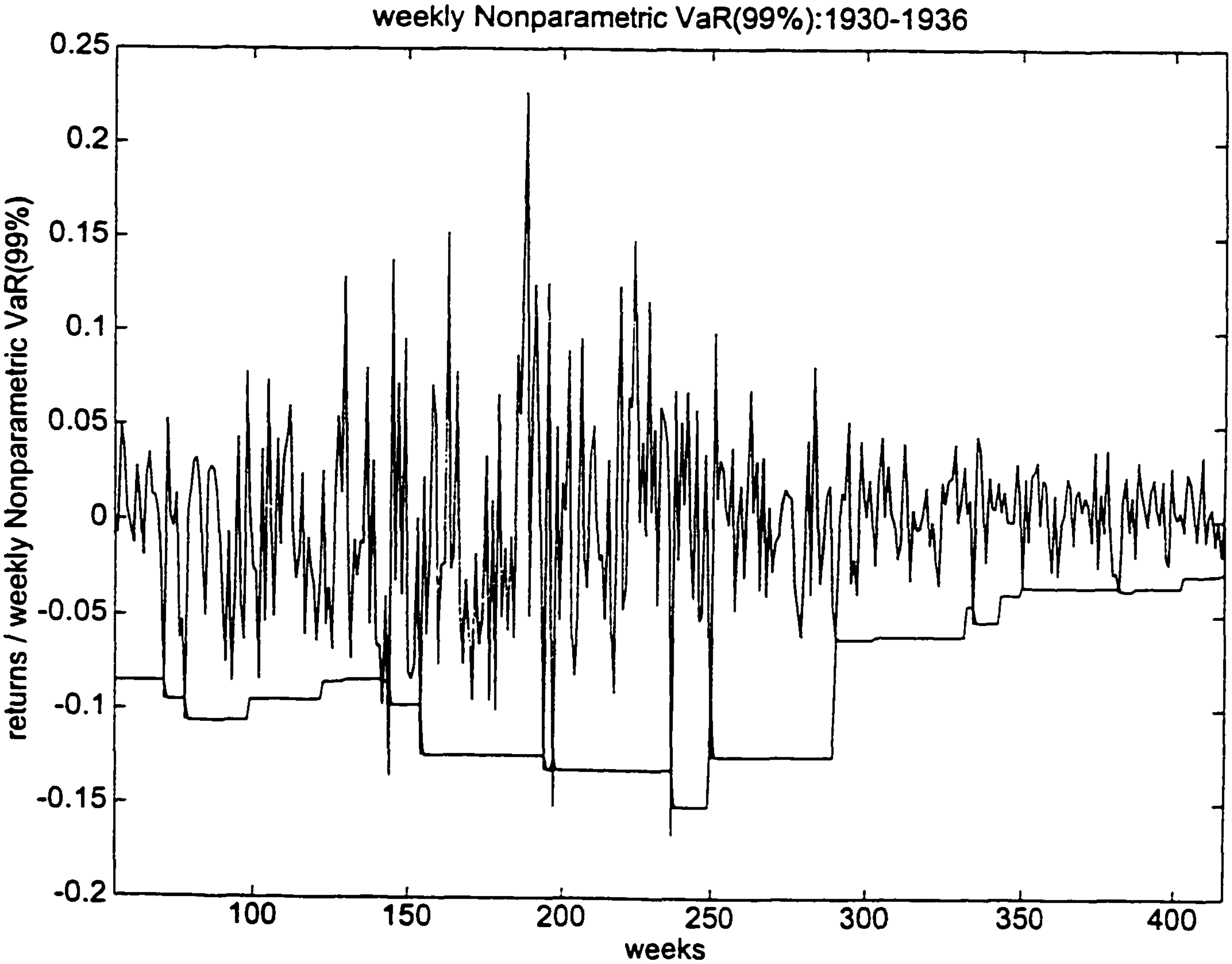

weekly Nonparametric VaR(99%):1930-1936
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
returns / weekly Nonparametric VaR(99%)
100
150
200
250
300
350
400
weeks

FIGURE A20

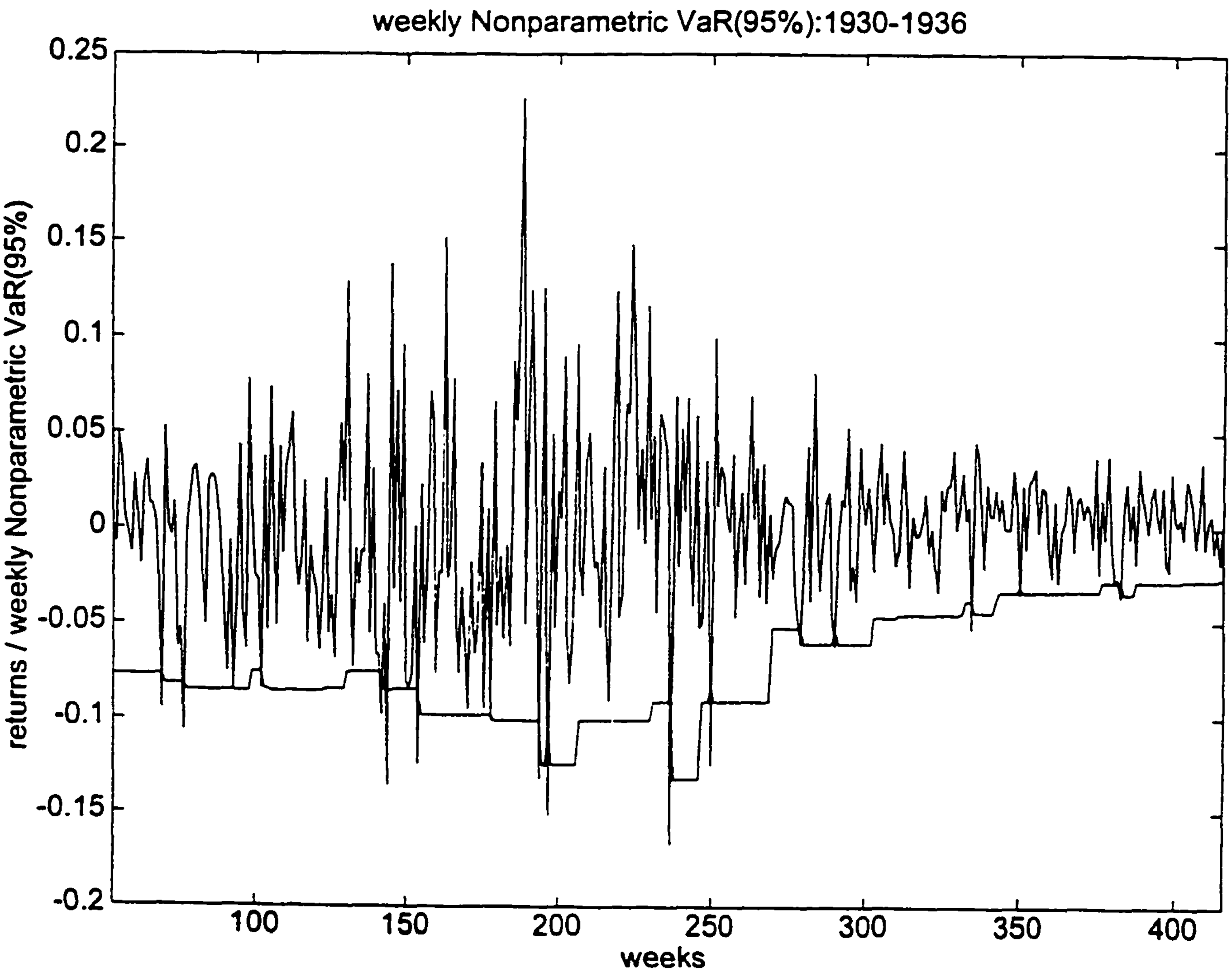

weekly Nonparametric VaR(95%):1930-1936
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
returns / weekly Nonparametric VaR(95%)
100
150
200
250
300
350
400
weeks

FIGURE A21

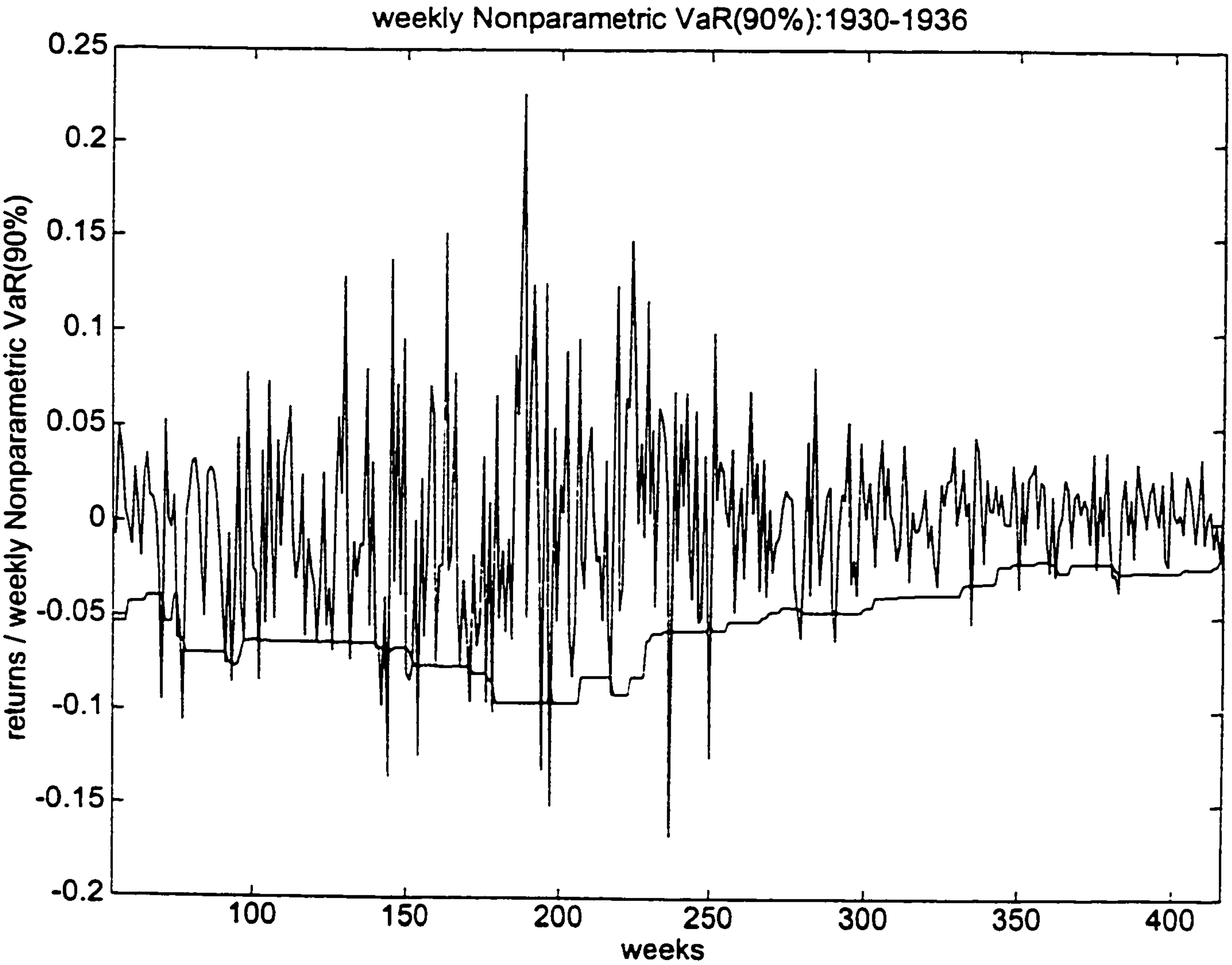
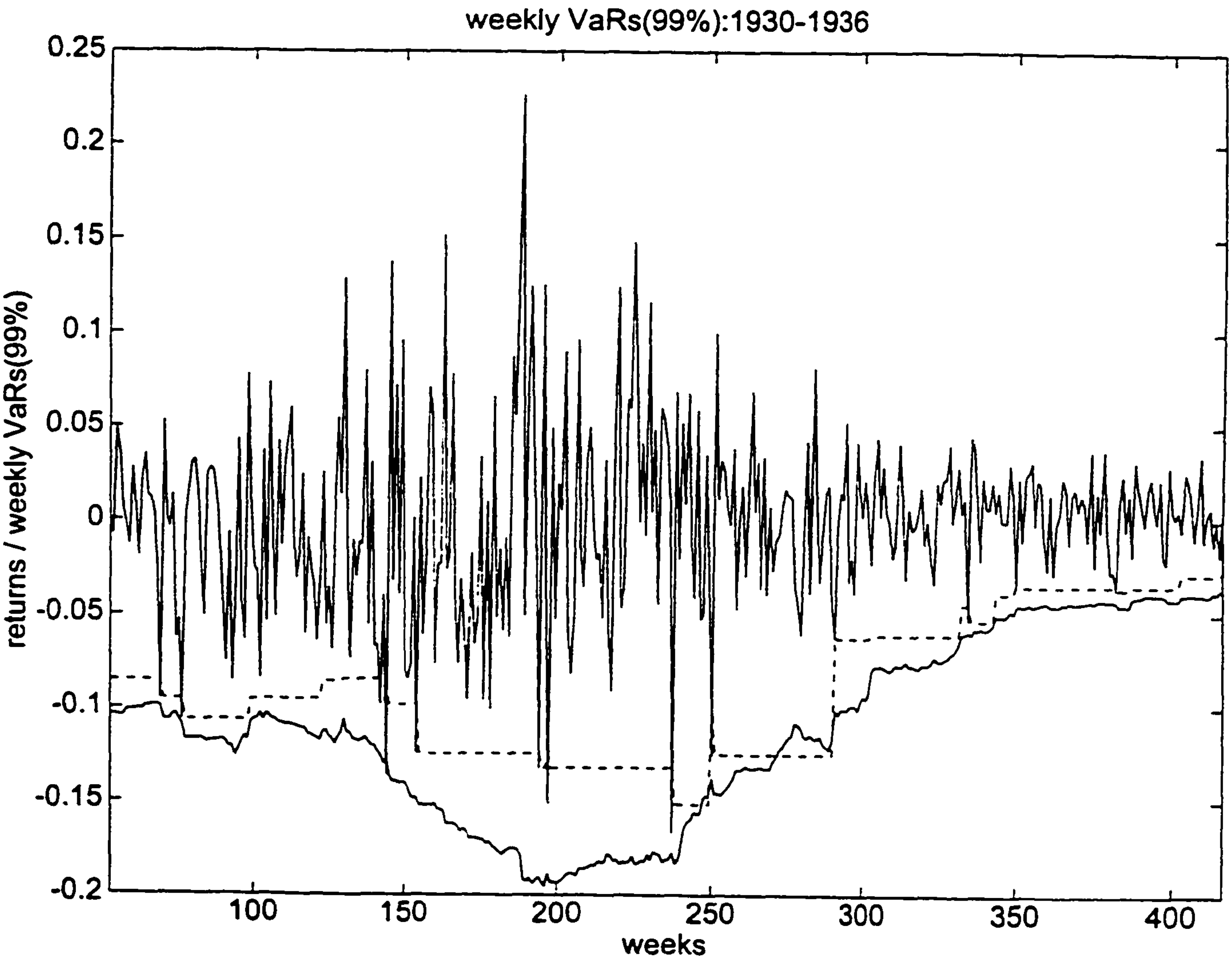
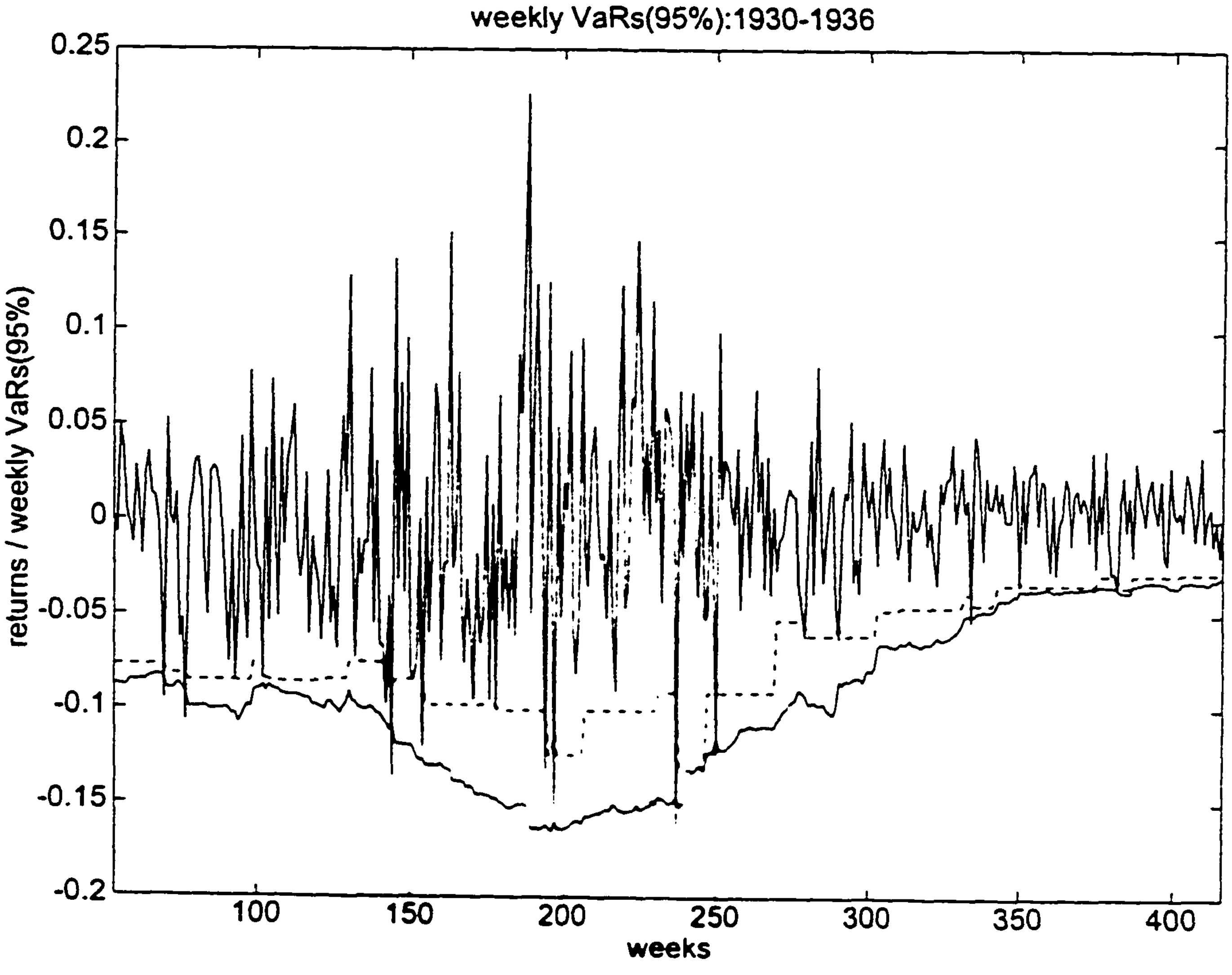

weekly Nonparametric VaR(90%):1930-1936
returns / weekly Nonparametric VaR(90%)
weeks
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
100
150
200
250
300
350
400

FIGURE A22



Parametric VaR: ________
Nonparametric VaR: -------

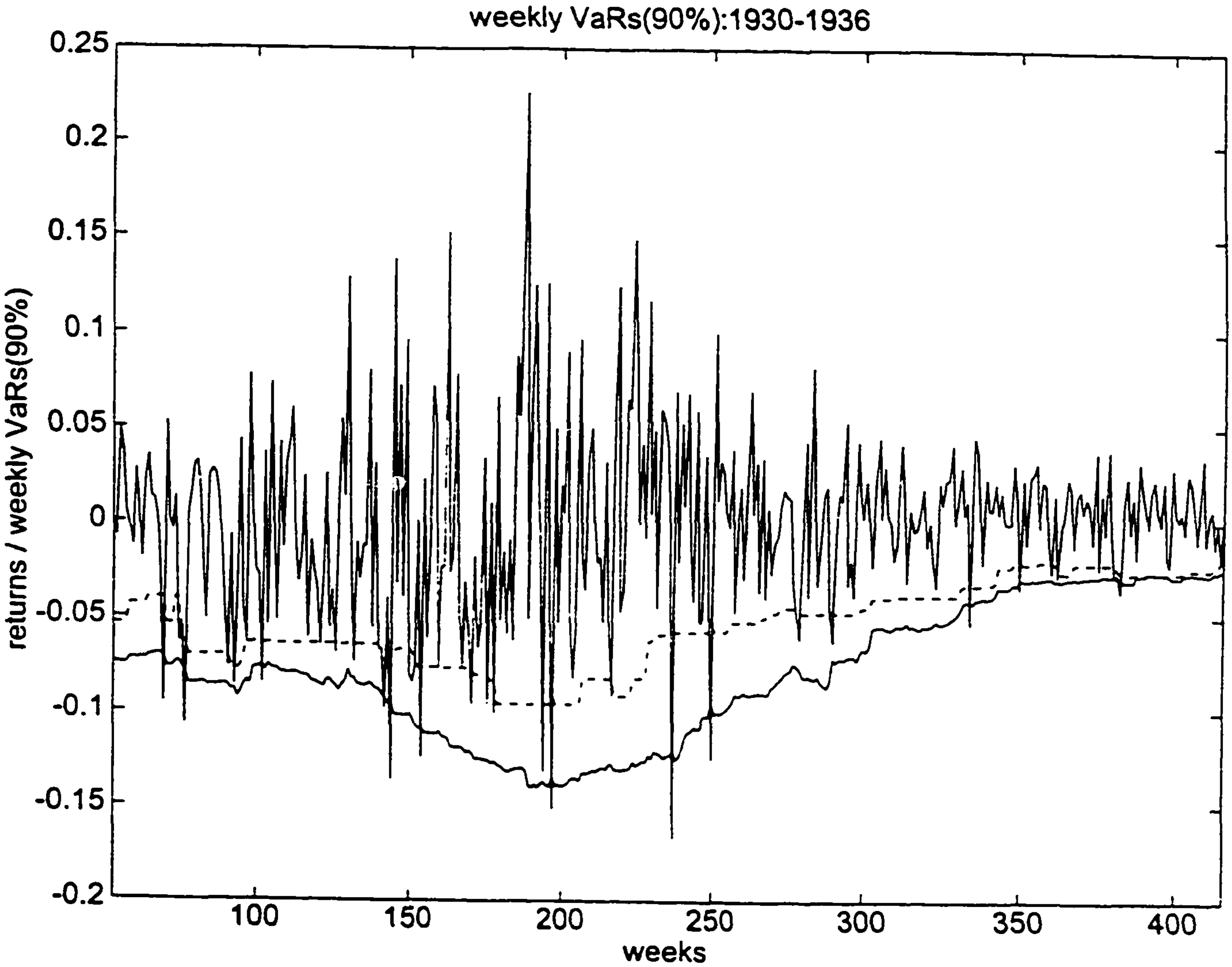
FIGURE A23



Parametric VaR ———

Nonparametric VaR -------

FIGURE A24



Parametric VaR: ______
Nonparametric VaR -------

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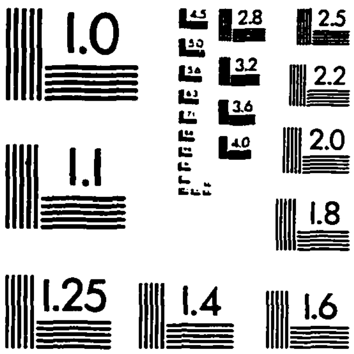
IMAGE EVALUATION TEST TARGET (QA-3)



150mm

6"

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