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Halpern, Marc

OPTIONS ON STOCK INDEXES: A COMPARISON OF THE BLACK-SCHOLES OPTION PRICING FORMULA AND AN AUTOREGRESSIVE MODEL

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OPTIONS ON STOCK INDEXES:

A COMPARISON OF THE BLACK-SCHOLES

OPTION PRICING FORMULA

AND AN AUTOREGRESSIVE MODEL

by

MARC HALPERN

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Dr. Salih Neftci
Chairman of Examining Committee

9/5/85
date

Dr. Michael Grossman
Executive Officer

Supervisory Committee
Dr. Salih Neftci
Dr. Ronald Anderson
Dr. Michael Grossman

The Graduate Center
The City University of New York

DEDICATION

This disertation is dedicated to my wife, Myroon, in appreciation for all her help and encouragement; to my children, Sahra, Candy, Levine, and Roger; to my parents, Ruth and Jesse; to my sister Andrea; and to my brother Joel.

INTRODUCTION

The recent introduction of stock index options has been greeted with great enthusiasm by investors and traders. The options on the S and P 100 stock index are the most heavily traded of all options.

The Black-Scholes call option formulal has long been a popular yardstick among traders and has figured in many trading strategies. The Black-Scholes call option pricing formula assumes the underlying security's price follows a random walk through time, the rate of return on the underlying security is a stochastic variable posessing a lognormal disttribution, and markets are efficient. This paper measures the forecasting performance of the Black-Scholes formula when it is applied to the S and P 100 stock index call option traded on the Chicago Board of Exchange. A comparison is made between the option prices produced by the Black-Scholes formula and option prices generated by a AR model. The information content of the Black-Scholes model is evaluated by including them in the AR model. An option strategy is examined that is based upon forecasts produced by the AR model.

A data bank containing daily closing prices of the S and P 100 stock index, closing prices of options on the S and P 100 stock index, daily trading volume of options on the S and P 100 stock index, and daily three month T-bill bond equivalent yields was generously supplied by Data

Resources Incorporated.

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I. EXCHANGE TRADED OPTION CONTRACTS

An option is a contract between two parties, the option writer and the option buyer. The option writer sells the contract to the option buyer. The option writer guarantees the option buyer that he will either buy or sell an item described in the contract at a prearranged price. The contract is in force until the expiration date specified in the contract. The contract is called an option because the option buyer may either require the option writer to perform according to the terms of the contract or allow the contract to expire unfulfilled.

The two types of options traded on financial markets are calls and puts. A call is an option to buy. A put is an option to sell.

The buyer of a call option is guaranteed the right to buy a particular item for the price specified in the option contract. The buyer of a put option is guaranteed the right to sell a particular item for the price specified in the option contract. After the option's expiration date the guarantee is no longer in effect.

Most people are familiar with call options. A person renting a house often has an option to buy the house at a certain price written into the rental contract. At the end of the contract period he may buy the house for the agreed upon price or elect to continue to rent.

Call options on stocks guarantee the buyer of the option that he will be able to obtain the number of shares of stock specified in the contract at a fixed price regardless of fluctuations in the market during the time the option is in effect.

People are less familiar with put options. The holder of a put option on a share of stock may elect to "put"(sell) a share of stock to the option writer in the contract and receive the price specified in the option contract. When the owner of 100 shares of IBM buys a put contract guaranteeing him the right to sell 100 shares of IBM for $110 each, he acquires the right to deliver his 100 shares to the put writer and receive $110 per share at any time during the life of the contract. By buying the contract he protects himself from realizing a loss when the price of IBM declines below $110.

Option contracts that are traded in financial markets are created when an option writer sells an option to an option buyer. This transaction takes place at a public market such as the Chicago Board Options Exchange. As with a stock transaction neither buyer nor seller need know each other. After the transaction is completed the names of the option writer and option buyer are entered onto the books of the exchange. The option writer and option seller are separated and the Option Clearing Corporation becomes the buyer of the contract from the option writer and the seller of the contract to the option buyer. The option writer is said to be short the contract and the option buyer is said to be long the contract.

Exchange traded contracts are standardized to facilitate ease of trading. Because contracts are standardized, there will be many contracts in existence that have the same specifications. The standardization of exchange traded option contracts combined with the services of the Option Clearing Corporation assures the liquidity of exchange traded options. Once an option contract is created, buyer and seller do not have to seek each other out if one wants to close out his position or if the option buyer wants the option writer to fulfill the terms of the contract.

If an option writer wishes to close out his position he simply purchases an option with the same specifications as the one he sold. This new transaction will cancel out his position on the books of the Option Clearing Corporation. Similarly an option buyer can close out his position by selling his option.

The option buyer does not have to concern himself with the ability of the option writer to fulfill the terms of the contract. If the option buyer wants the terms of the contract fulfilled, the Options Clearing Corporation will fulfill the terms of the contract. In turn the Options Clearing Corporation will demand that an option writer fulfill the terms of the contract. Margin requirements imposed upon option writers protect the Options Clearing Corporation from the possibile refusal of an option writer to fulfill his obligation.

An option series is defined by its exercise price and its expiration month. A December 160 call option is an option that expires in December and has an exercise price of 160.

The expiration date of an option contract is the last day the holder of the contract may demand that the writer of the contract fulfill the contract's terms.

The contract's exercise price is the sum of money specified in the contract that the specified item (a house, stock, etc.) can be bought or sold for under the terms of the contract.

An option contract is exercised when an option holder demands that the terms of the contract be fulfilled.

An "American option" is an option that may be exercised before the expiration date or on the expiration date. An "European option", on the other hand, is an option that may be exercised only on the expiration date. Options traded on United States exchanges are all of the American type.

II. OPTIONS ON STOCK INDEXES

Options on stock indexes belong to the special class of options called cash options. Cash options are contracts on the value of an index such as a stock index. Cash options are not settled by the delivery of the item that the contract specifies. One could not deliver a stock index. Cash options are settled by the delivery of the cash value of the item specified in the contract.

A stock index option is an option on the cash value of a stock index. A stock index call contract guarantees the option holder the right to receive the current value of a stock index upon payment of the exercise price to the option writer. A stock index put contract guarantees the option holder the right to receive the exercise price upon payment of the current value of the stock index to the option writer.

The current value of the stock index is determined by the closing value of the stock index on the day it is exercised. For a call option on a stock index to generate a positive cash flow to the option holder, the exercise price must be less than the closing value of the stock index. Because the closing value of the stock index is unknown up until the close of the trading day, the only time that the option can be exercised without risk is at the close of the trading day.

Example 1: The holder of a call option on a stock index with an exercise price of $160 exercises the option on the expiration date. The closing value of the stock index is $165. The option holder pays $160, the exercise price of the option, and receives $165, the closing value of the stock index. There is a positive cash flow of $5 to the option holder.

III. OPTIONS ON THE S and P 100

Options on the S and P 100 index are traded on the Chicago Board Option Exchange. The S and P 100 was originally known as the CBOE 100. It is a market weighted index of 100 stocks listed for options trading on the CBOE. The index is constructed with the following formula:

$$\text{S and P 100 Index} = \frac{\Sigma S_t N_t}{\Sigma S_b N_b} \times 100$$

where

S_t = the current market price of stock i.

N_t = the current number of shares outstanding of stock i.

S_b = the price of stock i on the base date.

N_b = the number of shares outstanding of stock i on the base date.

Trading in index options on the S and P 100 began at the Chicago Board Options Exchange in March 1983. Expiration dates were set for the Saturday following the third Friday in March, June, September, and December. Index options expire on the Saturday following the third Friday of the expiration month. Trading for each expiration date began nine months previous to expiration. Although the options proved to be popular, trading was active for only the three months prior to expiration. In 1984 expiration dates were set for every month, with trading commencing three months prior to expiration.

Options on the S and P 100 index are traded in multiples of 100. If a call option on the S and P 100 with an exercise price of $160 is selling for $2, then one may buy the option for $200 ($2 x $100). If the option is exercised when the closing value of the stock index is $165, then the option writer will pay $16,000 ($160 x 100) and receive $16,500 (165 x 100).

IV. THE PRICING OF CALL OPTIONS

Let t = the current date.

Let P_t = the closing value of the stock index call option at time t.

Let I_t = the closing value of the stock index at time t.

Let X = the exercise price of the option.

Let t' = the number of days until the option expires.

At the close of any day previous to expiration the value of the American call will be greater than or equal to $\max(0, I_t - X)$. The holder of the option will not exercise the option unless he receives a positive cash flow. If he exercises the option he will pay X and receive I_t (he will receive the closing value of the index on that day and pay the exercise price of the option). At the end of any trading day the option holder has the choice of either exercising his option or holding it. The value of the option to the option holder will not fall below zero because if $I_t - X < 0$ than the option holder will not exercise the option. Given a viable market speculators will be willing to pay a price higher than $\max(0, I_t - X)$ as long as there is the possiblity that the value of the index will increase causing an increase in the value of the option.

The value of a call on a stock index, P_t, is a function of the current price of the underlying stock index, the time left to expiration, the exercise price of the contract, and expectations of the value of exercising the option at the close of trading on all the days prior to expiration. Recall that if exercised the value of a stock index option is I_t - X and that the value of I_t is set by the closing value of I_t on the date of exercise. In the literature the value of exercising an option at any point in time is commonly called its intrinsic value. The actual value of the option minus its intrinsic value is referred to as its time value. This time value may be expressed as expectations of future intrinsic values up to and including the time of expiration.

As an example of how expectations of the future path of the stock index will effect the current price of the call at time t consider the following:

Assume all investors have the same expectation of the value of the stock index at time t+4. They are $Prob(I_{t+4} = 174) = .2$, $Prob(I_{t+4} = 175) = .3$, $Prob(I_{t+4} = 176) = .3$, and $Prob(I_{t+4} = 177) = .2$. If I_{t+4} were to equal 174, exercising the option at t+4 would yield a negative cash flow ($I_t - X = 174 - 175$). If I_{t+4} were to equal 175, exercising the option would yield a cash flow of zero. Exercizing the option would only yield a positive cash flow if $I_{t+4} > 175$. Since a rational investor would only exercise an option that produces a positive cash flow, the expected intrinsic value of the option at t + 4 is then simply .2 x (177 - 175) + .3 x (176 - 175) + .5 x 0 = .7. By discounting to the present all future expected intrinsic values an investor would be able to set a current value on a particular call option. The risk neutral investor would simply price the call option at the highest discounted value of all the possible future expected intrinsic values.

If the option is of the European type the investor will only be concerned with the probability distribution of stock prices on the expiration date. Given a particular stochastic process underlying the evolution of stock prices that allows us to derive the probability distribution of stock prices at expiration, the above discussion would allow us to derive the current value of a European option for a risk neutral investor. This insight will be employed later when a risk neutrality argument is introduced to derive the Black-Scholes model.

Merton has observed that the right to early exercise will be nonnegative and that the value of a European call and American call will be the same if the underlying security pays no dividends over the life of the option.[1] For the risk neutral case above allowances will have to be made for subtracting out known dividend payable before expiration when evaluating the evolution of stock prices and the resulting distribution of stock prices at expiration.

The current value of the call option can then be written: $P_t = f(E(I_{t+t'} - X \mid I_{t+t'} > X), prob(I_{t+t'} > X), t', r)$.

Note that the distribution of $I_{t+t'}$ includes such factors as uncertainty about the economy and other factors that effect stock market value. r is a interest rate and t' is the time left to expiration.

Because changes in expectations about the value of the stock index at t' will be reflected in changes in the current value of the stock index:

$$\partial P/\partial I > 0$$

As the value of the stock index increases the value of the option increases. In an efficient market a rise in the value of the stock index signals that expectations of future values of the component stocks are higher.

As the time to expiration decreases (all other things held equal including the value of the index), the value of the option decreases. With a shorter time frame within which the value of the index may increase, the value of the option will fall:

$$\partial P/\partial t' > 0$$

IV. FOOTNOTE

1. Merton, R. C. Theory of Rational Option Pricing. Bell J. Spring 1973, 4(1), p. 144

V. THE BLACK-SCHOLES OPTION PRICING MODEL

A number of call option pricing models have been proposed that build upon hypothesis about the nature of the underlying stochastic process of the evolution of stock prices. The first of these and perhaps the most influential because of its ease of calculation is the model proposed by Fisher Black and Myron Scholes in an important paper in 1972.[1] Fisher Black and Myron Scholes derived a formula for the price of a call option on a security such as a stock when certain conditions are fulfilled. Black and Scholes describe these conditions as "ideal." The conditions are:[2]

1) An short term interest rate that is known and constant for the life of the option contract.

2) The stock's price follows a random walk with drift through time. Changes in the stock's price at time t+1 are independent of changes in the stock's price at time t. Changes in the stock price over time occur randomly, and the stock price is said to follow a random walk through time.

At the end of any finite interval the distribution of possible stock prices is lognormal. The distribution of the rate of return of the stock is normal and the stock possesses a variance rate of return that is constant.

3) The stock pays no dividends. Shareholders do not receive other types of distributions such as

warrants or additional securities.

4) The option can only be exercised at maturity. Merton has explored this restriction and has found that if condition 3 holds, than this condition is unnecessary.

5) There are no transaction costs.

6) There is no limit on borrowing at the short term interest rate.

7) There are no added costs to short selling. Moral hazard does not exist. Short sellers are not required to maintain margin accounts. Option writers are not bound by margin requirements.

Using the above assumptions a riskless hedge is formed that is continuously adjusted through time. For each share of stock in the portfolio, $(\partial P/\partial S)^{-1}$ options must be written. P_t is the price of the option. S_t is the price of the stock.

The hedge is riskless because the value of a portfolio of one share of stock and $(\partial P/\partial S)^{-1}$ shares of stock is $S-P(\partial P/\partial S)^{-1}$. For small changes in the value of the stock the value of the hedge changes by $\partial S - \partial P(\partial P/\partial S)-1 = 0$. Thus the hedge can be said to be riskless for small changes in the price of the underlying stock. Because the hedge is riskless it should produce the same rate of return over time as a riskless bond such as a treasury bill.

Assumption 2 implies that the stochastic process of the evolution of the stock's price is an Ito process such that:

$$\log(S_t+\Delta S/S_t) = a\Delta t + (V\Delta t)^{1/2}Z$$

where Z is a random variable with a mean of zero and a variance equal to 1 and V is the variance of the stocks rate of return.[3]

The above formulation can be rewritten $S+\Delta S_t\Delta = S_t \exp(a\Delta t + (V\Delta t)^{1/2}Z)$. This says that changes in the value of the stocks price over time are due to both constant ($a\Delta t$) and stochastic (Z) elements. The greater Δt the greater the magnitude of the stochastic element.

Given the above, the value of the price of a European call is shown to be:

$$P_t = S_tN(b) - Xe^{-rt'}N(b-(V\ t')^{1/2})$$

$$b = (\ln(S_t/X) + rt' + (Vt'/2))/(Vt')^{1/2}$$

P_t is the Black-Scholes price of the call option.

S_t is the current stock price.

V is the variance of the stocks rate of return.

X is the exercise price of the option.

N( ) is the cumulative normal density function.

$N(b)\int= \ \trianglerighteq\ (2\pi)^{-1/2}\exp(-x^2/2)dx$

t' is the number of days left until expiration.

r is the risk free short term interest rate.

By altering the assumption about the distribution of the stocks rate of return in condition 2, other authors have derived alternate option pricing models. R. C. Merton derived a formula using a Poisson process to describe stock returns that jump from one value to another as new information is received.[4] J.C. Cox and S.A. Ross derived call pricing formulas for the general cases where the variance is proportional to the stock price and where the variance is independent of the stock price.[5]

Other authors have explored option models that allow for dividends. R. Roll extended the Black-Scholes formula to allow for dividends.[6] Geske[7] and Whaley[8] made contributions in their discussions of Roll's paper.

V. FOOTNOTES

1. Black, F. and Scholes, M., The Pricing of Options and Corporate Liabilities, J. Polit. Econ., May-June 1973, 81(3), pp. 637-54.

2. Black, F. and Scholes, M., p.640.

3. Jarrow, R. and Rudd, A. Option Pricing, Dow Jones-Irwin, 1983, p. 88.

4. Merton, R. C. Option Pricing When Underlying Stock Returns Are Discontinuous. J. Finan. Econ., January/March 1976, 3(1/2), pp. 125-44.

5. Cox, J. C. and Ross, S. A. The Valuation of Options for Alternative Stochastic Processes. J. Finan. Econ., January/March 1976, 3(1/2), pp. 145-166.

6. Roll, R. An Analytical Valuation Formula for Unprotected American Call Options on Stocks with Known Dividends. J. Finan. Econ., November 1977, 5(2), pp. 251-58.

7. Geske, R. A Note on an Analytical Valuation Formula for Unprotected American Call Options on Stocks with Known Dividends. J. Finan. Econ., December 1979, 7(4), pp. 375-380.

8. Whaley, R. E. On the Valuation of American Call Options on Stocks with Known Dividends (An Analytical Valuation Formula for Unprotected American Call Options on Stock with Known Dividends). J. Finan. Econ., June 1981, 9(2), pp. 207-11.

VI. THE BLACK-SCHOLES FORMULA AND STOCK INDEX OPTIONS

Unlike a call option on a security, a call option on an index does not carry the right to purchase a particular underlying security. A single security whose behaviour exactly mimics the behaviour of the stock index does not exist. Difficulties arise when constructing a riskless hedge between the option and the basket of stocks in the portfolio. If a call is exercised the option writer will have to deliver the index's end of day value on the day the call is exercised. To liquidate his hedged position he will have to sell his stocks and deliver the cash value of the index. Additional transaction costs will be generated by selling odd lots. Last minute fluctuations in the market may lead to discrepencies between the value of the index and the amount of money realized by the sale of the stocks in the portfolio. However the Black-Scholes model and other models derived using the same arbitrage condition are used extensively by financial institutions to obtain implied stock index option prices. It could be argued that options on indexes should not only be thought of as a means to bet on the markets direction, but as a means to hedge a portfolio against undiversifiable risk. This argument gives some legitimacy to the application of the Black-Scholes model to these options.

The third condition of the Black-Scholes formulation (see chapter VI), the underlying security does not pay dividends, is violated in the sense that the stocks held in the hedged portfolio pay dividends. A method for dealing with this difficulty would be to subtract out the present value of known dividends payable before the options expiration date. One could then calculate a stock index reflecting the risky part of its component stocks value. The value of the risky index would replace the value of the index in the Black-Scholes formula. Using this method a better valuation of the call index option would be obtained. To the extent that the value of dividend payments payable before expiration are included in the value of the stock index, application of the Black-Scholes formula to the reported stock index will produce option prices that are too high. Because of data limitations the present paper does not calculate option prices using only the "risky" component of the index. However, the data should be readily available and examination of Black-Scholes prices utilizing calculations of the "risky" index would be a worthwhile project.

The fourth condition (the option can only be exercised at maturity) is fulfilled in the sense that as long as the price of the option is greater than the value of the stock index minus the exercise price the option will be sold and not exercised. An examination of the closing data has shown this to be the case.

The fifth condition (no transaction cost) is assumed to be fulfilled because of the existence of arbitragers on the floor of the exchange. However, even parties with no commission costs will still have costs of doing business.

The sixth and seventh conditions are also violated (no limit on borrowing and no margin costs), but again these violations are no worse than in the case where the model is applied to individual stocks.

VII. DERIVING THE BLACK-SCHOLES OPTION PRICING MODEL

The following exposition follows Black's and Scholes' derivation with the help of material developed in OPTION PRICING by Robert Jarrow and Andrew Rudd[1] and in SECURITIES MARKETS by Kenneth Garbade.[2]

In this section we will drop the time subscript. P will be understood to equal P_t, S will be understood to equal S_t, and so on. ΔP will be understood to equal $P_{t+\Delta t} - P_t$, and so on.

Let P be the current price of the call option

Let S be the current stock price.

Let V be the variance of the stocks rate of return.

Let X be the exercise price of the option.

Let H be the value of a hedged portfolio at time t.

Let t be the current time.

Recall Black and Scholes second assumption: The path of the stock price through time can be described as a random walk. The variance rate of the stock price is proportional to the square of the stock price. Therefore, at the end of any finite interval the distribution of possible stock prices is log normal. The stock possesses a variance rate of return that is constant. The implication is that the distribution of stock prices will be such that

$$\log((S+\Delta S)/S) = a\Delta t+(V\Delta t)^{1/2}Z \qquad (1)$$

where

$$\Delta S = S_{t+\Delta t} - S_t$$

Z is the standard normal variable with mean 0 and variance 1.

a is the mean logrithmic stock return per unit of time.

V is the variance of the logrithmic stock return per unit of time.

Expanding (1):

$$(S+\Delta S)/S = \exp(a\Delta t+(V\Delta t)^{1/2}TZ)$$

$$1 + \Delta S/S = 1 + (a\Delta t+(V\Delta t)^{1/2}Z) + (a\Delta t+(V\Delta t)^{1/2}Z)^2/2 + (a\Delta t+(V\Delta t)^{1/2}Z)^3/3 + \ldots$$

Here we employ Ito's lemma to ignore all terms in the expansion containing Δt to a power greater than one. Then:

$$\Delta S/S = a\Delta t+(V\Delta t)^{1/2}Z + V\Delta tZ^2/2$$

$$\Delta S/S = (a + VZ^2/2)\Delta t + (V\Delta t)^{1/2}Z$$

Taking expectations and recalling that $E[Z] = 0$ and that $E[Z^2] = 1$:

$$E[\Delta S/S] = a\Delta t + V\Delta t/2$$

$$Var[\Delta S/S] = E[(a+VZ^2/2)\Delta t+(V\Delta t)^{1/2}Z)^2] - (a\Delta t+V\Delta t/2)^2$$

$$Var[\Delta S/S] = V\Delta t$$

The mean and variance define the evolution of $\Delta S/S$ to be:

$$\Delta S/S = (a + V/2)\Delta t + (V\Delta t)^{1/2}Z \qquad (1a)$$

$$\Delta S = (a + V/2)S\Delta t + (V\Delta t)^{1/2}SZ$$

Now if we form a hedged portfolio by buying a share of stock and selling $(\partial P/\partial S)^{-1}$ calls, our position will be riskless for small changes in S.

$$H = S - (\partial P/\partial S)^{-1} P$$

$$\Delta H = \Delta S - (\partial P/\partial S)^{-1} \Delta P \qquad (2)$$

For small changes in time the value of the calls will deteriorate and the value of the hedge will appreciate. To avoid arbitrage the change in the hedge should return the riskless rate.

$$r = (\Delta H/H) / \Delta t \qquad (3)$$

Using a Taylor expansion to expand ΔP around S and t and dropping all powers of Δt greater than 1 (Ito's Lemma), we find:

$$\Delta P = \partial P/\partial S \Delta S + \partial P/\partial S \Delta t + 1/2\ \partial^2 P/\partial S^2 (\Delta S)^2$$
$$+ \partial^2 P/\partial S \partial P\ \Delta S \Delta t \qquad (4)$$

Examining $\Delta S \Delta t$ in the fourth term on the right hand side of (4) and recalling that $\Delta S = (a + V/2) S \Delta t + (V \Delta t)^{1/2} Z$ (1a):

$$\Delta S \Delta t = (a + V/2) S \Delta t^2 + (V^{1/2} \Delta t^{3/2} Z)$$

and so we may drop the third term (Ito's Lemma).

Examining ΔS^2 in the third term of (4) and using (1) and Ito's Lemma to evaluate ΔS^2:

$$(\Delta S)^2 = V \Delta t S^2 Z^2$$

$$(\Delta S)^2 = V \Delta t S^2 \qquad E[Z^2] = 1$$

Substituting back into the third term on the right hand side of (4):

$$1/2\ \partial^2 P/\partial S^2 \Delta S^2 = 1/2\ \partial^2 P/\partial S^2\ V S^2 \Delta t$$

Equation (4) can now be rewritten as:

$$\Delta P = (\partial P/\partial S)^{-1}\Delta S + \partial P/\partial t \Delta t + (1/2)\, \partial^2 P/\partial S^2\, V^2 S^2 \Delta t \quad (5)$$

Substituting (5) into (2):

$$\Delta H = \Delta S - (\partial P/\partial S)^{-1}((\partial P/\partial S)\Delta S + \partial P/\partial t \Delta t + (1/2)\partial^2 P/\partial S^2 V^2 S^2 \Delta t)$$

$$= (\partial P/\partial S)^{-1}(\partial P/\partial t\, \Delta t + (1/2)\partial^2 P/\partial S^2\, V^2 S^2 \Delta t) \quad (6)$$

Substituting (6) into the formula for the risk free rate (3):

$$r = (\partial P/\partial S)^{-1}\Delta t(\partial P/\partial t + (1/2)\partial^2 P/\partial S^2 V^2 S^2 \Delta t) \,/\, ((S - (\partial P/\partial S)^{-1}P)\Delta t)$$

$$r = (\partial P/\partial S)^{-1}\, (\partial P/\partial t + (1/2)\partial^2 P/\partial S^2\, V^2 S^2 \Delta t) \,/\, ((S - (\partial P/\partial S)^{-1}\, P))$$

Multiplying both numerator and denominator of the right hand side by $(\partial P/\partial S)$:

$$r = (\partial P/\partial t + (1/2)\, \partial^2 P/\partial S^2\, VS^2) \,/\, (S\, \partial P/\partial S - P)$$

$$rS\partial P/\partial S - rP = \partial P/\partial t + (1/2)\, \partial^2 P/\partial S^2\, VS^2$$

and we obtain the partial differential equation:

$$0 = P + (1/2)\, \partial^2 P/\partial S^2 VS^2) - rS\, \partial P/\partial S + r\, P$$

The solution of this equation is the Black-Scholes call option formula.

The Black-Scholes model can be intuitively derived by discounting to the present the probability distribution of option prices at the moment of expiration.[3] The reasoning behind this method is that a security that returns a risk free rate of return will be valued the same by a risk neutral as by a risk adverse society. Assuming risk neutrality simplifies the problem. The risk neutral hedge will return the risk free rate in both societies. In a risk free society the stock will return the risk free rate. The value of a neutral hedge would be the same in both societies. Then in a risk free society the call will also return the risk free rate if the value of the hedge is risk free. Evaluating the call under risk free conditions produces the value of the call for all societies because of the constraint of the neutral hedge.

Let t' be the amount of time to expiration at time t.

Let S' denote the price of the stock at expiration.

Let P the value of the call option at expiration discounted to the present.

If the call is exercised at expiration the person long the call will pay X and receive S'. He will only exercise the option if S' > X. At expiration the call is then worth Max(0,S'-X).

If p=Prob(S'>X) and 1-p=Prob(S'<X), then the discounted value of the call is

$$P = e^{-rt'} p\, E[S'-X|S'>X] + (1-p)(0).$$

$$= e^{-rt'} p\, E[S'|\, S'>X] - e^{-rt}\, p\, X,$$

$$\text{since } E[X]=X \qquad (7)$$

We evaluate the first term with help from our assumptions about the distribution of stock prices.

From the assumption that the distribution of stock prices over a finite period is lognormal, we write:

$\log(S'/S) = at' + V(t')^{1/2}Z$

Exponentiating the above, we obtain:

$S' = S \exp(at' + V(t')^{1/2}Z)$

Then

$p = \text{prob}(S \exp(at' + (Vt')^{1/2}Z) > X)$

$= \text{prob}(\log S + at' + (Vt')^{1/2}Z > \log X)$

$= \text{prob}((Vt')^{1/2}Z > \log X - \log S - at')$

$= \text{prob}(Z > (\log X - \log S - at') / (Vt')^{1/2})$

$= \text{prob}(Z > -(\log S - \log X + at') / (Vt')^{1/2})$

Because prob $(Z>X)$ = prob $(Z<-X)$:

$p = \text{prob}(Z < (\log(S/X) + at') / (Vt')^{1/2})$

$= N((\log(S/X) + at') / (Vt')^{1/2})$ (8)

where N( ) = the cumulative normal density function

$E[S' | S'>X]\, p$ = the expected value of S' given that S' > X.

$= \int_{x}^{\infty} S \exp(at' + (Vt')^{1/2}x - x^2/2)(2\ )^{-1/2}\, dx$

$= \int_{x}^{\infty} S \exp(at' + (Vt'/2) - (Vt'/2) + (Vt')^{1/2}x - x^2/2)\, (2\pi)^{-1/2}\, dx$

$= S \exp((a + V/2)t') \int_{x}^{\infty} \exp(-((Vt')^{1/2} - x)^2/2)\, (2\pi)^{-1/2}\, dx$

Let $y = (Vt')^{1/2} - x$

Then the integral can be rewritten:

$E[S' | S'>X]\, p = S \exp((a + V/2)t')$

$$\int_c exp(-y/2)(2\ )^{-1/2}\ dx$$

where $c = (Vt')^{1/2} - X$

Now $S' > X$ implies:

$S\ exp(at' + (Vt')^{1/2}Z) > X$

$(\log S)\ (at' + (Vt')^{1/2}Z > \log X$

$(Vt')^{1/2}Z > (\log X / \log S) - at'$

$Z > -((\log S/X) + at') / (Vt')^{1/2}$

$y > ((\log S/X) + at') / (Vt')^{1/2} - (Vt')^{1/2}$

$y < ((\log S/X) + at') / (Vt')^{1/2} + (Vt')^{1/2}$

$y < ((\log S/X) + at' + (Vt')) / (Vt')^{1/2}$

then

$E[S'|S'>X]\ p = S\ exp((a + V/2)t')$

$$\int_{-\infty}^{b} exp(-y^2/2)(2\pi)^{-1/2}\ dy \quad (9)$$

where $b = (\log(S/X) + at' + Vt') / (Vt')^{1/2}$ (10)

Making the assumption that the expected return on the stock is the risk free rate and recalling that E[S/S] = $a\Delta t + V\Delta t/2$, then $r = a + V/2$. (11)

Substituting (11) into (10)

$b = (\log(S/X) + rt' + (Vt')/2)/(Vt')^{1/2}$

(9) can be rewritten:

$E[S'|S'>X]\ p = S\ e^{-rt}N(b)$ (12)

(8) can be rewritten with the help of (11):

$p = N((\log (S/X) + at' + Vt') / (Vt')^{1/2}) -$
$(Vt')^{1/2}$

$p = N((\log (S/X) + rt' + Vt'/2) / (Vt')^{1/2}) -$

$(Vt')^{1/2}$ (13)

Substituting (11) into (13):

$$p = N(b - (Vt')^{1/2}) \quad (14)$$

Then substituting (12) and (14) back into (7):

$$P = e^{rt'}e^{-rt'} S\, N(b) - e^{-rt'} \; N(b-(Vt')^{1/2})$$

We obtain the Black Scholes formula!

$$P = S\, N(b) - e^{-rt}N(b-(Vt')^{1/2})$$

VII. FOOTNOTES

1. Jarrow, R. and Rudd, A. Option Pricing, Dow Jones-Irwin, 1983, pp. 94-103.

2. Garbade, Kenneth D., Securities Markets. McGraw-Hill. 1983. pp. 394 - 396.

3. Jarrow, R. and Rudd, A. pp. 90 - 93.

VIII. THE DATA

Because closing option prices are reported for days with no volume and because closing prices reported on days when there is light volume may not be true closing prices, the accuracy of estimated prices is explored only over periods of time when volume on an option series is over 1,000 contracts per day for almost all days in the period. This study focuses on options expiring in June, September, and December 1983. The data was supplied by Data Resources Incorporated.

Table 1 lists the call options on the S and P 100 that are examined in this paper. FIRST DAY is the number of days before expiratrion of the first observation, LAST DAY is the number of days before expiration of the last observation.

IX. ESTIMATING THE BLACKR-SCHOLES MODEL

To calculate the Black-Scholes price the following information is needed: the exercise price of the option, days remaining to expiration, an estimate of the variance of the rate of return of the stock index, and an estimate of the risk free interest rate.

The value of the current stock price and the days remaining to expiration are straightforward enough. Both the interest rate and the variance of the rate of return may contain stochastic elements that are not accounted for in the assumptions.

An examination of the first derivitive of the Black-Scholes formula with respect to the interest rate and with respect to the variance of the rate of return indicates that the formula is not very sensitive to changes in the rate of interest but very sensitive to changes in the variance of the rate of return.[1]

Estimating the variance of the rate of return presents a number of problems. Because of the closing of markets on weekdays and holidays, one is not in possession of equally spaced daily data to estimate the variance of the rate of return. Although the formula assumes the variance of the rate of return is constant, one cannot be sure that this is the actual case. How far back should data be used to estimate the variance of the rate of return? The further back one reaches for data, the likelier that some new factor has entered the picture to change the variance of the rate of return.

In calculating an estimate of the variance of the rate of return for this paper the following strategies are employed:

The problem of weekends and holidays is solved by omitting these intervals from the calculation of the variance of the rate of return. The variance of the rate of return is reestimated for each day. This is a compromise with the assumption that the variance of the rate of return is constant.

The variance of the rate of return on the stock index is estimated by taking all one day log differences of the closing value of the S and P 100 index for the 360 days previous to the day the Black-Scholes option value is to be calculated for. Only log differences for Friday-Thursday, Thursday-Wednesday, Wednesday-Tuesday, and Tuesday-Monday intervals are used. Weekend and holiday intervals are omitted. The variance is then estimated by taking the variance of the one day log differences. The option formula is very sensitive to changes in the variance.

The one day rate of return on the stock index is simply $\log(I_t / I_{t-1})$.

The estimate of the variance of the rate of return is calculated as follows:

$V_t = \Sigma(L_t - \bar{L})^2/n-1$

$L_t = \log(I_t / I_{t-1})$, for all t when closing prices are available for both t and t-1.

$L_t = \bar{L}$, for all t when closing prices are not available for both t and t-1.

where V_t is an estimate of the one day rate of return

where $\bar{L}$ is the arithmetic mean for all L_t when closing prices are available for both t and t+1.

where n is the number of days when closing prices are available for both t and t-1.

For each day that a Black-Scholes price is calculated the variance of the rate of return on the stock index is estimated using data from the past 360 days.

The current three month T-bill bond equivalent yield is used to estimate the risk free interest rate.

IX. FOOTNOTES

1. Jarrow, R. and Rudd, A. Option Pricing, Dow Jones-Irwin, 1983, p. 120 for a numerical example.

X. AUTOREGRESSIVE PRICING MODEL

Stock index option prices are dependent on expectations of the future path of the stock index. If markets are efficient, changes in the value of the stock index reflect a change in expectations of the future path of the stock index. A model that focuses on the effect of changes in the stock index on the value of the option could have the general form:

$$P_t = f(P_{t-1}, I_t, I_{t-1}, t')$$

Investors price the option based on last period's option price, last period's index price, and this period's index price.

Autoregressive models are best linear forecasting models in the sense that they minimize the mean square error of the forecast. A linear autoregressive model employing these parameters would take the form:

$$P_t = aP_{t-1} + bI_t + cI_{t-1} + dt' + C + u$$

where:

t = the current date.

P_t = the option price at time t.

I_t = the value of the stock index at time t.

t' = days left to expiration.

C = a constant.

a,b,c,d = parameters to be estimated.

u = an error term.

In this model the signs of a and b should be positive and the signs of c and d should be negative. The sign of a should be positive because the higher the value of last period's option, holding all other factors constant, the higher one would expect the value of this period's option. The sign of b should be positive because the value of a call option varies directly with the value of the index. The sign of c should be negative because if the past period's index is less than this period's index, then any increase in last period's index reduces the amount of positive change in the index and thus reduces the change in the value of the option. If the past period's index is greater than this period's index then any positive increase in the difference between this period's and last period's index produces a greater decline in the value of the index and so one would expect a greater decline in the value of the option. The sign of d should be positive because the greater the time left for the index to increase, the greater the speculative value of the call option.

As with the Black-Scholes model, difficulties arise when applying this model to the data. The lack of closing data for holidays and weekends leaves discontinuities in the data set. When estimating an autoregressive model we require equally spaced data, assuming that the passage of time effects the variables under study proportionally to the amount of time elapsed. When studying financial data this assumption implies that the stream of information effecting prices is proportional to the amount of time elapsed.

If we assume that trading time is the important component of time passing we ignore this difficulty, and use the subscript i to indicate today's closing price, the subscript i-1 to indicate the closing price on the previous day the market was open, and the subscript i+1 to indicate the closing price on the next day the market will be open. Thus if observation i is on a Monday, then observation i-1 is on Friday and observation i+1 is on Tuesday. To still allow for the effect of the actual passage of time the variable t', days to expiration, has been retained.

The model may now be written:

$P_i = aP_{i-1}+bI_i+cI_{i-1}+dt'+C+u$

where:

i = the current date.

i-1 = the date previous to i that markets were open on.

P_i = the option price at time i.

I_i = the value of the stock index at time i.

t' = days left to expiration at time i.

C = a constant.

a,b,c,d = parameters to be estimated.

u = an error term.

A test of the hypothesis that the call prices follow a random walk would involve a test of the null hypothesis that a=1, b=0, c=0, and d=0. This test is performed later.

To compare the effectiveness of this model with the effectiveness of the Black-Scholes, conditional forecasts dependent on I_{i+1} are obtained for the twelfth to the last day of each option series. The model is estimated using closing data from the first eleven days of each series. Then the model is used to estimate the closing price of the option on the twelfth day. Next the model is estimated using closing data from the first twelve days, and the closing option price is estimated for the thirteenth day. The model is updated each day and then used to obtain a forecast for the next trading day's closing option price, conditional upon the closing value of the stock index.

$$\dot{P}_{i+1} = aP_i + bI_{i+1} + cI_i + dt' + C$$

where $\dot{P}_i$ = the forecast of the option price at time i+1 conditional upon the value of the stock index at time i+1.

Unconditional forecasts are also obtained with I_{i+1} set equal to I_i. This suggests a further test of the random walk hypothesis, a rejection of b=-c would be a more general rejection of the random walk constraint on the coefficients of b and c.

XI. AUTOREGRESSIVE MODEL INCORPORATING BLACK-SCHOLES PRICES

To test whether the information in the Black-Scholes model can be utilized to improve the autoregressive model, current Black-scholes prices are included in the autoregressive model. In this model investors price this period's option based on last period's option price, last period's index price, the current index price and the current Black-Scholes price:

$$P_i = aP_{i-1}+bI_i+cI_{i-1}+dB_i+et'+C+u$$

where:

i = the current date.

i-1 = the date previous to i that markets were open on.

P_i = the option price at time i.

I_i = the value of the stock index at time i.

B_i = the Black-Scholes option price at time i.

t' = days left to expiration at time i.

C = a constant.

a,b,c,d = parameters to be estimated.

u = an error term.

The model was estimated for each day i, starting with the eleventh day of the series. The estimated model was then used to obtain conditional forecasts for the next trading day i+1.

$$\dot{P}_{i+1} =aP_i+bI_{i+1}+cI_i+dB_{i+1}+et'+C$$

XII. RESULTS OF ESTIMATING THE AUTOREGRESSIVE MODEL

Table 2 contains the coefficient estimates of the autoregressive model when the model is estimated over the whole data set contained in Table 1. Table 3 contains the coefficient estimates of the autoregessive model that includes the Black-Scholes model.

We first examine the coefficients of the parameters for the autoregressive model without Black-Scholes prices (Table 2). The coefficient of P_{t-1} is significantly different than zero with a confidence level of 95 percent in seven out of thirteen of the estimated series in Table 2. The coefficient of I_t is significantly different than zero in twelve of the estimated series. The coefficient of I_{t-1} is significantly different than zero in eleven of the estimated series. The coefficient of t' is significantly different from zero in ten of the estimated series. The coefficient of C is significant in 11 of the estimated series.

The signs of the coefficients for the autoregressive model are as we expected for all but three cases. The sign of I_{t-1} is positive for the September 160 series, the September 170 series, and the September 175 series. We would expect the sign of I_{t-1} to be negative (see chapter X).

To test the random walk hypothesis additional tests of the hypothesis that a=1 and the hypothisis that b=-c were made. The t statistic for the hypothesis that a=1 is a-1 divided by the standard error of a. The F(1,n-k) statistic for the hypothesis that a=-b is $(b + c)^2$ divided by Var(a) + Var(b) + 2Cov(a,b). The following results were obtained when a rejection region of .05 was selected:

	H: a=1	H: b=-c
June 150	reject	reject
June 155	accept	accept
June 160	accept	reject
June 165	reject	reject
June 170	reject	accept
sept. 160	reject	reject
sept. 165	accept	reject
Sept. 170	reject	reject
Sept. 175	reject	reject
Dec. 160	reject	reject
Dec. 165	reject	reject
Dec. 170	reject	reject
Dec. 175	reject	reject

We next examine the coefficients of the parameters for the autoregressive model that includes Black-Scholes prices (Table 3). For the JUNE 150 series none of the parameters have coefficients that can be said to be different from zero on a 95 percent confidence level. The coefficients fare better for the remaining series. The coefficient of P_{t-1} is significantly different than zero in seven of the estimated series. The coefficient of I_t is significantly different from zero in seven of the estimated series. The coefficient of I_{t-1} is significantly different from zero in nine of the estimated series. The coefficient of the current Black-Scholes price, B_t, is significantly different from zero in nine of the estimated series. The coefficient of the trend term t' is significantly different from zero in six of the estimated series. The constant term, C is significantly different from zero in six of the estimated series.

When comparing the two models it is interesting to note that sum of the squared residuals are lower for the autoregressive model that includes Black-Scholes prices than for the autoregressive model that doesn't include Black-Scholes prices. The standard error of estimate for series including Black-Scholes are lower than or equal to the standard error of estimate for the same series without Black-Scholes prices.

XIII. DESCRIPTION OF FORECAST STATISTICS

The relationship between the actual value of the call at time t and the forecasted value of the call at time t can be expressed as: the reported closing price of the option at time t equals the forecasted closing price of the option at time t plus the forecast error at time t. A positive forecast error is caused by a too low forecast, a negative forecast error is caused by a too high forecast. To compare the three models the following statistics are gathered and analyzed:

mean forecast error = $\Sigma(P_t - \dot{P}_t)/n$

mean absolute forecast error = $\Sigma(|P_t - \dot{P}_t|)/n$

root mean square forecast error = $(\Sigma(P_t - \dot{P}_t)^2/n)^{1/2}$

Theil u = $(\Sigma(P_t - \dot{P}_t)^2)^{1/2} / (\Sigma(P_t - P_{t-1})^2)^{1/2}$

$\dot{P}_t$ = the forecast of the closing price of the option at time t.

P_t = the reported closing price of the option at time t.

The mean forecast error, mean absolute forecast error, root mean square forecast error, and Theil u statistics are all measures of the forecasting ability of the model.

The mean forecast error measures the average forecast error. It is not a very good measure because errors in opposite directions will cancel each other. A model that exactly predicts will have the same mean error as a model having large positive and negative forecasting errors exactly balancing each other out.

The mean absolute forecast error corrects the defect of the mean error by ignoring the signs of forecast errors. If the magnitude of the mean absolute forecast error is close to the magnitude of the mean forecast error, it indicates that the errors are largely of one sign and the model has a tendency to over or underpredict the true value. As a quick example of the use of these two statistics suppose that forecast errors of -3, -10, -18, -8, 3, 2, -10 and -12 are generated by a particular model. The mean forecast error is (-3 -10 -18 -8 +3 +2 -10 -12) / 8 = -7. The mean absolute forecast error is (3 +10 +18 +8 +3 +2 +10 +12) / 8 = 66 / 8 = 8.25. Both the mean and absolute forecast error have approximately the same magnitude. The sign of the mean forecast error is negative, indicating a tendency of the model to produce negative forecast errors, overpredicting the true value.

The root mean square forecast error emphasizes the effects of large errors and minimizes the effects of small errors. An error of -100 and 5 will yield an absolute mean error of 52.5 and a root mean square forecast error of approximately 70.2.

The Theil U statistic compares the forecasting ability of the model with the forecast obtained by simply using the previous period's value as the forecast.

XIV. COMPARING THE FORECASTING PERFORMANCE OF THE THREE MODELS

Black-Scholes prices are calculated for the thirteen S and P 100 call option series described in the chapter IX using the methods described in chapter X. Black-Scholes closing prices are forecasts of closing option prices when the current value of the stock index is known and the closing price of the option is unknown. Traders know the current price of the stock index but must bargain with other traders to set the price of the option. The Black-Scholes price is a forecast of the actual option price, conditional upon the current value of the stock index.

Forecast statistics are gathered for each series and reported in table 4. Autocorrelations of the forecast errors are reported in table 5. Forecast statistics are generated using forecasts beginning with the twelfth day of each data set. The first forecast of the autoregressive model is produced with a model estimated using data from the first eleven days of the data set.

The forecast statistics are reported to eight decimal places (table 4). The tables of forecast errors (tables 6 to 18) are rounded to two decimal places (the nearest penny.) Because the options themselves are priced in sixteenths of a dollar, a further refinement would be to round all forecasts to the nearest sixteenth of a dollar.

In the following discussion the forecasts statistics are all rounded to two decimal places.

Model prices for the June 150 option are computed from 80 days before expiration to 65 days before expiration. The forecast statistics for the Black Scholes model of the June 150 option indicates that all forecast errors are negative. The Black-Scholes model produces a mean forecast error of -1.17 and a mean absolute forecast error of 1.17. The Black-Scholes model overestimates all the closing prices for the June 150 option. A Theil u statistic greater than one for this series indicates that simply using the past value of the option would do a better job at predicting option prices than the Black-Scholes model. The root mean square forecast error of the Black-Scholes model for this series is 1.22.

Both the autoregressive model and the autoregressive model incorporating Black-Scholes prices do markedly better than the Black-Scholes model for the June 150 option. The autoregressive model producess a root mean square forecast error of .41. The autoregressive model incorporating Black-Scholes prices produces a root mean square forecast error of .37. The Black-Scholes prices improve the autoregressive forecast, despite the fact that the Black-Scholes prices in this case are poor estimates of the actual option prices!

Model prices for the June 155 option are computed from 70 days before expiration to 42 days before expiration.

Again we find that the Black-Scholes model produces forecast errors that are all negative (the mean forecast error is negative and equal to the mean absolute forecast error). Both the root mean square forecast error and the Theil u statistic of the Black-Scholes model are more than double the root mean square forecast error and Theil u statistic produced by the two autoregressive models. The addition of Black-Scholes prices to the autoregressive model improves the forecast statistics of the autoregressive model.

Model prices for the June 160 option are computed from 70 days before expiration to 1 day before expiration. Although the Black-Scholes model's negative mean forecast error indicates a tendency to overpredict, the tendency is not as overwhelming as in the June 150 and June 155 options. In fact, the large negative forecast errors for the dates farthest from expiration are largely responsible for the relatively high negative mean error (see table 8 for errors of the Black-Scholes model).

The root mean square forecast error of the Black-Scholes model for the June 160 series is .65, for the autoregressive model it is .53, and for the autoregressive model incorporating Black-Scholes prices it is .42.

Model prices for the June 165 option are computed from 45 days before expiration to 1 day before expiration. The root mean square forecast errors for the three models are all within .03 of each other. The root mean square forecast error of the Black-Scholes model is .31, the root mean square forecast error of the autoregressive model without Black-Scholes prices is .30, and the root mean square forecast error of the autoregressive model incorporating Black-Scholes prices is .28.

Model prices for the June 170 option are computed from 35 days before expiration until 1 day before expiration. In contrast to the forecast performance of the three models for series containing observations further from expiration, the Black-Scholes model produces the lowest root mean square forecast error, .16. The root mean square forecast error of the autoregressive model without Black-Scholes prices is .23 and root mean square forecast error of the autoregressive model incorporating Black-Scholes prices is .20.

For the June options the forecast statistics of the autoregressisve models are the best (lowest root mean square forecast errors) when the series under consideration includes observations far from the options expiration date. The addition of Black-Scholes prices to the autoregressive model improves the forecasting performance of the autoregressive model. When the Black-Scholes model produces better forecasts than the autoregressive model, the inclusion of Black-Scholes prices in the autoregression does not improve the forecasts of the autoregression enough to surpass the Black-Scholes model.

The forecast statistics for the September series exhibit the same pattern as the forecast statistics for the June series. For series containing only observations from the month before expiration, the Black-Scholes model provides better forecasts. For series containing observations from dates further away from expiration the autoregressive models provide better forecasts. Again, the incorporation of Black-Scholes prices into the auroregressive model lowers the root mean square forecast error.

Model prices for the September 160 option are computed from 31 days before expiration to 1 day before expiration. The root mean square forecast error of the Black-Scholes model is .44, the root mean square forecast error of the autoregressive model without Black-Scholes prices is .86, and the root mean square forecast error of the autoregressive model incorporating Black-Scholes prices is .49.

Model prices for the September 165 option are computed from 21 days before expiration until 1 day before expiration. The root mean square forecast error of the Black-Scholes model is .35, the root mean square forecast error of the autoregressive model without Black-Scholes prices is .50, and the root mean square forecast error of the autoregressive model when Black Scholes prices are included is .49. For the Black-Scholes model of the

September 160 option the closeness in magnitude of the mean error and the mean absolute error, and the negative sign of the mean error indicate a tendency of the errors to be negative.

Model prices for the September 170 option are computed from 67 days before expiration to 2 days before expiration. Model prices for the September 175 option are computed from 70 days before expiration until 22 days before expiration. The root mean square forecast errors of the autoregressive models are both lower than the root mean square forecast errors of the Black-Scholes model for these two series.

For the September 170 option the root mean square forecast error for the Black-Scholes model is .52, the root mean square forecast error for the autoregressive model without Black-Scholes prices is .38, and the root mean square forecast error for the autoregressive model incorporating Black-Scholes prices is .37.

For the September 175 option the root mean square forecast error for the Black-Scholes model is .42, the root mean square forecast error for the autoregressive model without Black-Scholes prices is .27, and the root mean square forecast error for the autoregressive model incorporating Black-Scholes prices is .26.

Again for the December series the same pattern is observed. The Black-Scholes model does better when the series under study contains data only from the month and a half prior to expiration.

Model prices are calculated for the December 160 option from 37 days before expiration to 1 day before expiration. As expected the Black-Scholes model produces the lowest root mean square forecast error, .30. The autoregressive model without Black-Scholes prices produces a root mean square forecast error of .32 and the autoregressive model incorporating Black-Scholes prices produces a root mean square forecast error of .33.

Model prices are calculated for the December 165 option from 84 days before expiration to 1 day before expiration. For the Black-Scholes model the closeness in magnitude of the mean forecast error and the mean absolute forecast error, and the negative sign of the mean forecast error indicate a tendency of this model to produce negative errors. The Black-Scholes model's high Theil u statistic indicates that a better forecast could be obtained by simply observing last period's price.

For the December 165 option the autoregressive models both produce root mean square forecast errors of .28, while the Black-Scholes model produces a root mean square forecast error of .99.

Model prices are calculated for the December 170 option from 93 days before expiration until 3 days before expiration. Model prices are calculated for the December

175 option from 87 days before expiration until 14 days before expiration. In both cases the mean forecast error and mean absolute forecast error of the Black-Scholes models indicate that they are producing all negative errors. In both cases the Black-Scholes model produces very high Theil u statistics.

For the December 170 option the autoregressive models both produce root mean square forecast errors of .27, while the Black-Scholes model produces a root mean square forecast error of 1.01.

For the December 175 option the autoregressive models both produce root mean square forecast errors of .18, while the Black-Scholes model produces a root mean square forecast error of .98.

The Black-Scholes model produces larger forecast errors for dates further from observation. This may be attributed to the equation itself. Any estimation error in the variance will be magnified by high values of t'. The autoregressive models provide better forecasts than the Black-Scholes model for series containing observations further away from expiration. The forecasting ability of the Black-Scholes model improves as the expiration date approaches. The forecasting ability of the autoregressive model is not as effected by the amount of time until expiration.

Another possible explanation for the poor performance of the Black-Scholes model is that as the time to expiration increases, the number of dividends that will be paid out between the observed date and the expiration date increases. As pointed out in chapter VII, failure to account for the effect of dividends should produce negative forecast errors. The June and December Black-Scholes series seem to confirm this.

An examination of the actual forecast errors in tables 6 to 18 dramatically confirms the observation that the Black-Scholes prices improve as the expiration date approaches. The autoregressive forecast errors in these tables do not exhibit the obvious serial correlation that the Black-Scholes errors exhibit.

Tables 19 through 21 list the closing values of the S and P 100 index and the forecast errors for the Black-Scholes model for each of the three expiration dates. From these tables it appears that the Black-Scholes model produces better forecasts the farther out of the money (the greater the value of the exercise price minus the index) the option series is.

XV. AN OPTION STRATEGY BASED ON THE AUTOREGRESSIVE MODEL

An option strategy based on the autoregressive model would use the autoregressive model to obtain forecasts for next period's option price and institute a trading strategy emplying the information contained in the forecast. From the autoregressive model we have:

$$\dot{P}_{t+1} = aP_t + bI_{t+1} + cI_t + dt' + C$$

To obtain an unconditional forecast of P_{t+1} we need a forecast of I_{t+1}. Assuming that the index follows a random walk we will use I_t as a forecast for I_{t+1}. Forecast statistics for these forecasts can be found in table 22. A Theil statistic of close to one for these forecasts indicates their poor quality. However it may be that these forecasts indicate the markets direction. To test the usefulness of the forecasts in setting up an option strategy the following methodolody is adopted:

Because index options are priced in sixteenths of a dollar we will institute our strategy when the forecasted option price for the next period differs from the actual option price for the current period by more than $.07. Two option strategies will be employed.

Naked strategy: If the forecast for next period's option price is greater (less) than the current option price, than buy (sell) the option this period and sell (buy) the option next period.

Hedged strategy: If the forecast for next period's option price is greater (less) than the current option price, simultaneously buy (sell) b^{-1} options and sell (buy) the index portfolio this period. Next period sell (buy) b^{-1} options and buy (sell) the portfolio.

These two strategies are compared with the strategy of simply buying the index portfolio this period and selling it next period. Mean daily returns for the three strategies are compared in table 23.

A high hedge ratio renders the hedged strategy useless for the September 165, September 170, and September 175 series. Low t-statistics indicate that none of the observed mean returns are significantly different from zero. The comparatively high variance of the naked option strategy when compared with the variances of the hedged and index strategies is to be expected given the high degree of risk associated with naked option strategies.

XVI. CONCLUSION

We can conveniently divide our conditional forecast results into two classes. The first class contains those series where the Black-Scholes model produces lower root mean square forecast errors than the autoregressive model without Black- Scholes prices. For these series adding Black-Scholes prices to the autoregressive model lowers the autoregressive model's forecast statistics. However, when the three models are compared the Black-Scholes model still produces the lowest root mean square forecast error.

The second class of conditional forecast results contains those series where the Black-Scholes model produces higher root mean square forecast errors than the autoretgressive model without Black-Scholes prices. For these series adding Black-Scholes prices to the autoregressive model also lowers the autoregressive model's forecast statistics. Although the improvement is only a small one, it is striking that the combination of the two models produces lower forecast statistics for this class.

It is also observed that for dates far away from expiration the autoregressive model provides better forecasts than the Black-Scholes model. The forecasts of the Black-Scholes model improves as the expiration date approaches. The errors produced by the autoregressive model do not exhibit the systematic pattern of mispricing produced by the Black-Scholes model (see table 5).

An option strategy based upon unconditional forecasts using the autoregressive model do not appear very promising.

TABLE 1
S AND P 100 CALL OPTIONS EXAMINED IN THIS PAPER

EXPIRATION MONTH	EXERCISE PRICE	FIRST DAY	LAST DAY
June 1983	150	80	65
June 1983	155	70	42
June 1983	160	70	1
June 1983	165	45	1
June 1983	170	35	1
Sept. 1983	160	31	1
Sept. 1983	165	21	1
Sept. 1983	170	67	2
Sept. 1983	175	70	22
Dec. 1983	160	37	1
Dec. 1983	165	84	1
Dec. 1983	170	93	1
Dec. 1983	175	87	14

First day refers to the first day of the series. It is the number of days before expiration of the first day of the series. Last day refers to the last day of the series. It is the number of days before expiration of the last day of the series.

TABLE 2
PARAMETER ESTIMATES FOR THE AUTOREGRESSIVE MODEL WITHOUT BLACK-SCHOLES PRICES

JUNE 150

OBSERVATIONS 20 DEGREES OF FREEDOM 15
R SQUARED 0.93 RBAR SQUARED .91
SSR 1.43 SEE .31
DURBIN WATSON 1.86
Q(10)= 7.16 SIGNIFICANCE LEVEL .71

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.18	.28	.63	.54
I_t	.69	.62E-1	11.1	.15E-7
I_{t-1}	-.15	.20	-.73	.48
t'	.55E-1	.21E-1	2.56	.22E-01
C	-81.08	28.76	-2.81	.13E-1

JUNE 155

OBSERVATIONS 30 DEGREES OF FREEDOM 25
R SQUARED .99 RBAR SQUARED .99
SSR 2.86 SEE .34
DURBIN WATSON 1.70
Q(15)=18.94 SIGNIFICANCE LEVEL .22

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.93	.97E-1	9.52	.46E-8
I_t	.66	.50E-1	13.39	.37E-8
I_{t-1}	-.58	.83E-1	-7.04	.22E-6
t'	.69E-2	.15E-1	.45	.66
C	-13.04	11.44	-1.14	.27

JUNE 160

OBSERVATIONS 58 DEGREES OF FREEDOM 53
R SQUARED .97 RBAR SQUARED .97
SSR 10.23 SEE .44
DURBIN WATSON 1.83
Q(21)= 11.10 SIGNIFICANCE LEVEL .96

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.83	.86E-1	9.63	.37E-8
I_t	.66	.42E-1	15.59	.37E-8
I_{t-1}	-.53	.74E-1	-7.12	.66E-8
t'	.11E-1	.63E-2	1.7	.96E-1
C	-20.57	9.51	-2.16	.35E-1

JUNE 165

OBSERVATIONS	40	DEGREES OF FREEDOM	35
R SQUARED	.98	RBAR SQUARED	.98
SSR	1.86	SEE	.23
DURBIN WATSON	1.95		
Q(18)= 1.95		SIGNIFICANCE LEVEL	.89

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{i-1}	.57	.90E-1	6.37	.25E-6
I_i	.49	.26E-1	18.77	.37E-8
I_{i-1}	-.19	.61E-1	-3.12	.36E-2
t'	.29E-1	.63E-2	.47	.45E-4
C	-49.46	9.69	-5.11	.12E-4

JUNE 170

OBSERVATIONS	32	DEGREES OF FREEDOM	27
R SQUARED	.97	RBAR SQUARED	.97
SSR	.84	SEE	.18
DURBIN WATSON	1.46		
Q(15)=11.85		SIGNIFICANCE LEVEL	.69

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{i-1}	.70	.13	5.58	.64E-5
I_i	.15	.25E-1	6.05	.19
I_{i-1}	-.89	.38E-1	2.33	.27E-1
t'	.16E-1	.76E-2	2.14	.42E-1
C	-10.45	5.24	-1.99	.56E-1

SEPTEMBER 160

OBSERVATIONS	31	DEGREES OF FREEDOM	26
R SQUARED	.85	RBAR SQUARED	.82
SSR	9.39	SEE	.60
DURBIN WATSON	2.00		
Q(15)= 7.81		SIGNIFICANCE LEVEL	.93

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{i-1}	.23	.18	1.32	.20
I_i	.60	.89E-1	6.82	.31E-6
I_{i-1}	.46E-	.15	.30	.77
t'	.53E-1	.16E-1	3.24	.32E-2
C	103.39	25.12	-4.12	.35E-2

SEPTEMBER 165

OBSERVATIONS	23	DEGREES OF FREEDOM	18
R SQUARED	.94	RBAR SQUARED	.93
SSR	1.69	SEE	.31
DURBIN WATSON	1.43		
Q(11)=6.98		SIGNIFICANCE LEVEL	.80

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.60	.20	3.07	.66E-2
I_t	.58	.52E-1	11.21	.15E-8
I_{t-1}	-.29	.13	-2.22	.40E-1
t'	.38E-1	.17E-1	2.24	.38E-1
C	-47.30	20.74	-2.28	.35E-1

SEPTEMBER 170

OBSERVATIONS	55	DEGREES OF FREEDOM	50
R SQUARED	.97	RBAR SQUARED	.97
SSR	5.28	SEE	.32
DURBIN WATSON	1.53		
Q(21)=19.32		SIGNIFICANCE LEVEL	.56

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.79E-1	.92E-1	.86	.39
I_t	.80E-1	.293-1	2.74	.84E-2
I_{t-1}	.21	.39E-1	5.31	.26E-5
t'	.61E-1	.67E-1	9.07	.39E-11
C	-47.96	5.04	-9.52	.81E-12

SEPTEMBER 175

OBSERVATIONS	43	DEGREES OF FREEDOM	38
R SQUARED	.97	RBAR SQUARED	.96
SSR	1.87	SEE	.22
DURBIN WATSON	1.54		
Q(18)=11.93		SIGNIFICANCE LEVEL	.85

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.16	.90E-1	1.73	.91E-1
I_t	.53E-2	.23E-1	.23	.82
I_{t-1}	.17	.27E-1	6.40	.16E-6
t'	.35E-1	.54E-2	6.58	.93E-7
C	-29.78	3.53	-8.43	.31E-9

DECEMBER 160

OBSERVATIONS	34	DEGREES OF FREEDOM	29
R SQUARED	.98	RBAR SQUARED	.97
SSR	1.56	SEE	.23
DURBIN WATSON	1.89		
Q(15)=	14.18	SIGNIFICANCE LEVEL	.51

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.13	.19	.66	.51
I_t	.87	.36E-1	24.55	.37E-8
I_{t-1}	-.16	.16	-.98	.33
t'	.55E-1	.12E-1	4.55	.88E-4
C	-113.86	24.98	-4.56	.86E-4

DECEMBER 165

OBSERVATIONS	67	DEGREES OF FREEDOM	62
R SQUARED	.99	RBAR SQUARED	.99
SSR	3.34	SEE	.23
DURBIN WATSON	1.98		
Q(24)=	23.58	SIGNIFICANCE LEVEL	.49

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.67	.89E-1	7.54	.25E-9
I_t	.58	.23E-1	24.92	.00
I	-.41	.54	-7.56	.23E-9
t'	.19E-1	.54E-2	3.60	.63E-3
C	-27.86	8.18	-3.41	.12E-2

DECEMBER 170

OBSERVATIONS	72	DEGREES OF FREEDOM	67
R SQUARED	.99	RBAR SQUARED	.99
SSR	3.88	SEE	.24
DURBIN WATSON	2.21		
Q(24)=	20.82	SIGNIFICANCE LEVEL	.65

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.66	.79E-	8.36	.37E-8
I_t	.35	.23E-1	15.13	.37E-8
I_{t-1}	-.22	.40E-1	-5.43	.84E-6
t'	.17E-1	.39E-2	4.31	.54E-4
C	-22.80	5.51	-4.13	.10E-3

DECEMBER 175

OBSERVATIONS 61 DEGREES OF FREEDOM 56
R SQUARED .98 RBAR SQUARED .98
SSR 1.39 SEE .16
DURBIN WATSON 1.87
Q(21)= 15.47 SIGNIFICANCE LEVEL .79

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.65	.11	6.10	.10E-2
I_t	.21	.16E-1	12.87	.00
I_{t-1}	-.14	.27E-1	-5.13	.38E-5
t'	.13E-1	.40E-2	3.16	.25E-2
C	-11.89	3.92	-3.03	.36E-2

TABLE 3
PARAMETER ESTIMATES FOR THE AUTOREGRESSIVE MODEL WITH BLACK-SCHOLES PRICES

JUNE 150

OBSERVATIONS 20	DEGREES OF FREEDOM		14
R SQUARED .94	RBAR SQUARED .92		
SSR 1.15	SEE .29		
DURBIN WATSON 1.15			
Q(10)= 4.66	SIGNIFICANCE LEVEL		.91

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.17	.26	.64	.53
I_t	.35	.19	1.78	.98E-1
I_{t-1}	-.18	.19	-.98	.34
B_t	.49	.27	1.82	.90E-1
t'	.24E-1	.26	.95	.36
C	-24.84	40.84	-.61	.55

JUNE 155

OBSERVATIONS 30	DEGREES OF FREEDOM		24
R SQUARED .99	RBAR SQUARED .99		
SSR 1.95	SEE .29		
DURBIN WATSON 2.29			
Q(15)= 13.32	SIGNIFICANCE LEVEL		.58

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.71	.11	6.69	.64E-6
I_t	.26	.13	1.99	.58E-1
I_{t-1}	-.44	.81E-1	-5.46	.13E-4
B_t	.54	.16	3.35	.27E-2
t'	-.22E-1	.16E-1	-1.42	.17
C	28.63	15.73	1.82	.81E-1

JUNE 160

OBSERVATIONS 58	DEGREES OF FREEDOM		52
R SQUARED .98	RBAR SQUARED .98		
SSR 5.82	SEE .33		
DURBIN WATSON 2.37			
Q(21)= 12.25	SIGNIFICANCE LEVEL		.93

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.52	.82E-1	.64	.52E-7
I_t	.15	.87E-1	1.77	.83E-1
I_{t-1}	-.28	.69E-1	-4.03	.19E-3
B_t	.73	.12	6.27	.71E-7
t'	.19E-1	.67E-2	2.79	.73E-2
C	19.48	9.66	2.02	.49E-1

JUNE 165

OBSERVATIONS 40 DEGREES OF FREEDOM 34
R SQUARED .98 RBAR SQUARED .98
SSR 1.56 SEE .21
DURBIN WATSON 2.27
Q(18)= 9.62 SIGNIFICANCE LEVEL .94

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.52	.87E-1	5.95	.10E-5
I_t	.33	.68E-1	4.84	.27E-4
I_{t-1}	-.15	.59E-1	-2.60	.14E-1
B_t	.32	.13	2.56	.15E-1
t'	.10E-1	.94E-2	1.11	.28
C	-29.15	12.00	-2.43	.21E-1

JUNE 170

OBSERVATIONS 32 DEGREES OF FREEDOM 26
R SQUARED .98 RBAR SQUARED .97
SSR .60 SEE .15
DURBIN WATSON 2.30
Q(15)= 12.65 SIGNIFICANCE LEVEL .63

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.50	.12	4.03	.43E-3
I_t	.77E-1	.31E-1	2.47	.20E-1
I_{t-1}	.48E-1	.35E-1	-1.37	.18
B_t	.47	.15	3.26	.31E-2
t'	.94E-3	.81E-2	.12	.91
C	-4.88	4.81	-1.01	.32

SEPTEMBER 160

OBSERVATIONS 31 DEGREES OF FREEDOM 25
R SQUARED .95 RBAR SQUARED .94
SSR 3.24 SEE .36
DURBIN WATSON 1.96
Q(15)= 6.58 SIGNIFICANCE LEVEL .97

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.16	.11	1.52	.14
I_t	-1.31	.28	-4.64	.95E-4
I_{t-1}	-.57E-1	.93E-1	-.60	.55
B_t	2.36	.34	6.90	.32E-6
t'	-.12	.26E-1	-4.40	.18E-3
C	217.76	48.94	4.45	.16E-3

SEPTEMBER 165

OBSERVATIONS 23 DEGREES OF FREEDOM 17

R SQUARED .98 RBAR SQUARED .97

SSR .66 SEE .20

DURBIN WATSON 1.49

Q(11)= 14.72 SIGNIFICANCE LEVEL .20

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.43	.13	3.27	.45E-2
I_t	.52E-1	.11	.48	.63
I_{t-1}	-.19	.87E-1	-2.22	.40E-1
B_t	.83	.16	5.13	.84E-4
t'	-.28E-1	.17E-1	-1.7	.11
C	22.81	19.13	1.19	.25

SEPTEMBER 170

OBSERVATIONS 55 DEGREES OF FREEDOM 49

R SQUARED .97 RBAR SQUARED .97

SSR 4.78 SEE .31

DURBIN WATSON 1.95

Q(21)= 6.77 SIGNIFICANCE LEVEL .99

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.84E-1	.89E-1	.95	.34
I_t	-.32E-1	.57E-1	-.56	.58
I_{t-1}	.21	.37E-1	5.66	.78E-6
B_t	.30	.13	2.24	.29E-1
t'	.42E-1	.11E-1	3.94	.26E-3
C	-30.36	9.22	-3.29	.19E-2

SEPTEMBER 175

OBSERVATIONS 43 DEGREES OF FREEDOM 37

R SQUARED .97 RBAR SQUARED .97

SSR 1.74 SEE .22

DURBIN WATSON 1.88

Q(18)= 4.64 SIGNIFICANCE LEVEL .79

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.14	.88	1.61	.12
I_t	-.49E-1	.39E-1	-1.25	.22
I_{t-1}	.17	.26E-1	6.62	.94E-7
B_t	.22	.13	1.67	.10
t'	.28E-1	.67E-2	4.22	.15E-3
C	-21.02	6.27	-3.35	.18E-2

DECEMBER 160

OBSERVATIONS 34 DEGREES OF FREEDOM 28

R SQUARED .98 RBAR SQUARED .97

SSR 1.45 SEE .23

DURBIN WATSON 1.96

Q(15)= 14.55 SIGNIFICANCE LEVEL .48

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.11	.19	.58	.57
I_t	.69	.13	5.23	.15E-4
I_{t-1}	-.15	.16	-.98	.34
B_t	.22	.15	1.41	.17
t'	.41E-1	.15E-1	2.71	.11E-1
C	-85.62	31.66	-2.70	.12E-1

DECEMBER 165

OBSERVATIONS 67 DEGREES OF FREEDOM 61

R SQUARED .99 RBAR SQUARED .99

SSR 3.15 SEE .23

DURBIN WATSON 2.06

Q(24)= 22.28 SIGNIFICANCE LEVEL .56

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.60	.96	6.19	.59E-7
I_t	.47	.63E-1	7.46	.41E-8
I_{t-1}	-.38	.56E-1	-6.71	.11E-7
B_t	.18	.96E-1	1.89	.64E-1
t'	.11E-1	.68E-2	1.70	.94E-1
C	-15.25	10.43	-1.46	.15

DECEMBER 170

OBSERVATIONS 72 DEGREES OF FREEDOM 66

R SQUARED .99 RBAR SQUARED .99

SSR 3.48 SEE .23

DURBIN WATSON 2.25

Q(24)= 18.88 SIGNIFICANCE LEVEL .76

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.54	.88E-1	6.14	.52E-7
I_t	.22	.53E-1	4.16	.93E-4
I_{t-1}	-.19	.40E-1	-4.69	.14E-4
B_t	.29	.11	2.73	.81E-2
t'	.56E-2	.56E-2	.99	.33
C	-5.94	8.11	-.73	.47

DECEMBER 175

OBSERVATIONS	61	DEGREES OF FREEDOM		55
R SQUARED	.99	RBAR SQUARED	.98	
SSR 1.25		SEE .15		
DURBIN WATSON 1.99				
Q(21)= 13.10		SIGNIFICANCE LEVEL .91		

INDEPENDENT VARIABLE	COEFFICIENT	STANDARD ERROR	T-STATISTIC	SIGNIFICANCE LEVEL
P_{t-1}	.54	.11	4.93	.79E-5
I_t	.14	.32E-1	4.49	.37E-4
I_{t-1}	-.13	.27E-1	-4.70	.18E-4
B_a	.22	.90E-1	2.47	.17E-1
t'	.62E-2	.46E-2	1.34	.19
C	-2.88	5.23	-.55	.58

TABLE 4
CONDITIONAL FORECAST STATISTICS

1st line Black-scholes
2nd line Autoregressive model without Black-Scholes prices
3rd line Autoregressive model with Black-Scholes prices

June 150 from 80 days before expiration to 65 days before expiration. 11 observations.

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
-1.1733061	1.1733061	1.2247784	1.3378821
-0.19766049	0.30657346	0.41378814	0.45199992
-0.19986928	0.23891092	0.37497954	0.41067799

June 155 from 70 days before expiration to 42 days before expiration. 21 observations.

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
-1.0040916	1.0040916	1.1261387	0.94989485
0.28272167	0.44386049	0.52860181	0.44587415
0.24839188	0.39874342	0.48137816	0.40604114

June 160 from 70 days before expiration to 1 day before expiration. 49 observations.

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
-0.30575828	0.49400813	0.65474517	0.58814037
0.14425936	0.42727418	0.53840752	0.48363732
0.070146682	0.36411721	0.42338695	0.38031737

June 165 from 45 days before expiration to 1 day before expiration. 31 observations.

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
0.069199795	0.23941513	0.30622463	0.41551140
-0.023609827	0.22847179	0.28973272	0.39313378
-0.036590544	0.22094130	0.28046489	0.38055841

June 170 from 35 days before expiration to 1 day before expiration. 23 observations.

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
-0.031409946	0.11018699	0.15651522	0.62691323
0.036281135	0.18741942	0.23076829	0.92432991
0.044758202	0.16123138	0.20318002	0.81382658

September 160 from 31 days before expiration to 1 day before expiration. 22 observations.

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
-0.31372491	0.36449011	0.44196975	0.39711897
0.20594131	0.65451057	0.86036131	0.77305245
0.084676013	0.40372904	0.49478774	0.44457703

September 165 from 21 days before expiration to 1 day before expiration. 14 observations.

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
-0.16116458	0.26684056	0.35737258	0.38625805
-0.71546140	0.35458726	0.50437974	0.54514741
0.053249663	0.28449665	0.42763869	0.46220359

September 170 from 67 days before expiration to 2 days before expiration. 46 observations.

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
.059291381	0.38067795	0.52054988	1.0321976
-0.089712544	0.28589755	0.37918513	0.75188562
-0.061654842	0.27875933	0.36848599	0.73067033

September 175 from 70 days before expiration to 22 days before expiration. 34 observations.

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
0.0098331713	0.31095387	0.41560585	1.2514269
0.058110417	0.21707968	0.26780987	0.80639880
0.056327453	0.21149083	0.26228694	0.78976974

December 160 from 37 days before expiration to 1 day before expiration. 25 observations.

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
-0.10390351	0.22621566	0.2953496	0.26362230
-0.051697317	0.25909324	0.32006151	0.28568310
-0.036094557	0.25927815	0.32502817	0.29011628

December 165 from 84 days before expiration to 1 day before expiration. 58 observations.

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
-0.78914172	0.80506053	0.99023621	1.2097150
-0.047559092	0.21330583	0.27805446	0.33968325
-0.046321619	0.21632735	0.27840523	0.34011177

December 170 from 93 days before expiration to 3 days before expiration. 64 observations.

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
-0.85468383	0.85468383	1.0149509	1.9886526
-0.076307163	0.22032279	0.27388775	0.53665530
-0.065037056	0.21017202	0.26762374	0.52437087

December 175 from 87 days before expiration to 14 days before expiration. 52 observations.

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
-0.80404456	0.80404456	0.98111300	2.9987016
0.013815205	0.15004199	0.18305042	0.55948051
0.019543281	0.14629403	0.17992387	0.54992441

TABLE 5
AUTOCORELLATIONS OF THE FIRST FIVE CONDITIONAL FORECAST ERRORS

1st line Black-Scholes
2nd line Autoregressive model without Black-Scholes prices
3rd line Autoregressive model with Black-Scholes prices

June 150 from 80 days before expiration to 65 days before expiration

-.09962	.05600	.09978	-.05254	.2202
-.01030	-.2807	-.3402	-.04928	.4530
-.02454	-.1154	-.1096	-.1681	.3752

June 155 from 70 days before expiration to 42 days before expiration

.5378	.5469	.4258	.3010	.03931
.09327	-.1132	-.2358	.1806	.002509
.1167	-.2850	.08329	.2009	-.006086

June 160 calls from 70 days before expiration to 1 day before expiration

.6471	.6663	.5694	.4894	.3705
.03922	.04338	-.1284	-.1629	.1505
.003022	-.09178	-.1231	-.1041	.1548

June 165 from 45 days before expiration to 1 day before expiration

.3937	.3191	.2103	.06369	.03547
.1293	-.1154	-.002407	-.1084	.2092
.1338	-.02151	.05943	-.1178	.1418

June 170 from 35 days before expiration to 1 day before expiration

.1249	.07094	-.007337	-.1006	-.05781
.4500	-.1017	-.3865	-.4434	-.1327
.3783	-.1864	-.4229	-.4214	-.06957

September 160 from 31 days before expiration to 1 day before expiration

.6700	.4451	.2521	.03815	.05543
.5564	.1999	.1312	.05482	-.007771
.3316	.04776	.1622	.1674	.006216

September 165 from 21 days before expiration to 1 day before expiration

.6780	.2827	.03558	-.2017	-.2294
.3220	-.1697	-.2369	-.1175	.08839
-.02064	-.3036	-.3925	-.008582	.2913

September 170 from 67 days before expiration to 2 days before expiration

.3631	.05081	.1275	.2035	-.1095
.3577	.2158	.06408	.02712	-.01645
.3067	.1469	.03107	.02523	.02190

September 175 from 70 days before expiration to 22 days before expiration

.1807	-.03821	.09582	.08812	-.2164
.3494	.1025	-.04764	.04849	.007428
.2960	.08072	-.06081	.05971	.05984

December 160 from 37 days before expiration to 1 day before expiration

-.01827	.07232	-.2816	-.04991	.1551
-.01350	.08258	-.1697	.005246	.3449
-.1568	.09765	-.1131	-.007652	.3820

December 165 from 84 days before expiration to 1 day before expiration

.6400	.5380	.5834	.5035	.5391
.1478	.1380	.04120	.02130	.3540
.1774	.1641	.01965	.03570	.3461

December 170 from 93 days before expiration to 3 days before expiration

.7607	.7003	.7049	.6276	.6149
.07044	.1710	.02440	.1698	.2023
.1571	.09366	.1575	.1674	.2501

December 175 from 87 days before expiration to 14 days before expiration

.8319	.7739	.7799	.6825	.6490
.1241	.1971	-.1207	.0530	.1803
.1439	.1874	-.1491	.09155	.2133

TABLE 6
JUNE 150 FORECAST ERRORS

DAYS BEFORE EXPIRATION	BLACK SCHOLES	AUTOREGRESSIVE	AUTOREGRESSIVE BLACK-SCHOLES
80	- .95	.05	- .09
79	- .83	- .35	- .20
78	-1.63	- .99	- .96
74	- .84	.03	.09
73	-1.22	- .26	- .32
72	-1.05	.20	.03
71	- .51	- .26	- .01
70	-1.44	- .68	- .65
67	-1.27	- .24	- .18
66	-1.50	.04	.00
65	-1.65	.28	.10

TABLE 7

JUNE 155 FORECAST ERRORS

DAYS BEFORE EXPIRATION	BLACK SCHOLES	AUTOREGRESSIVE	AUTOREGRESSIVE BLACK-SCHOLES
70	-1.08	- .35	- .32
67	- .80	.78	.96
66	-1.28	1.03	.98
65	-1.42	.72	.46
64	-1.59	.83	.46
63	-1.20	.07	.59
60	-2.00	.20	- .17
59	-1.22	- .12	.39
58	-1.37	.88	.61
57	-1.22	.22	.24
56	-1.47	.14	.06
53	-1.03	- .19	.02
52	-1.60	.66	.14
51	- .51	.59	.71
50	- .57	.63	.47
49	-1.16	- .51	- .64
46	- .27	- .07	.14
45	- .11	.35	.37
44	- .17	.14	.09
43	- .47	- .44	- .39
42	- .53	.40	.16

TABLE 8
JUNE 160 FORECAST ERRORS

DAYS BEFORE EXPIRATION	BLACK SCHOLES	AUTOREGRESSIVE	AUTOREGRESSIVE BLACK-SCHOLES
70	- .79	- .16	- .13
67	- .81	.31	- .41
66	-1.16	.60	- .61
65	-1.38	.67	.49
64	-1.47	.90	- .47
63	- .90	- .36	.35
60	-1.91	.08	- .41
59	- .76	- .16	- .32
58	-1.07	.86	- .64
57	- .81	.04	- .02
56	-1.15	- .05	.27
53	- .58	- .36	- .16
52	-1.36	.64	.19
51	.13	.57	.75
50	- .25	.42	.29
49	- .61	- .17	- .37
46	.33	- .56	- .32
45	.24	.16	.17
44	.19	.27	.16
43	.11	- .45	- .36
42	.03	.55	.31
39	.67	.04	.03
38	.52	- .31	- .28
37	.20	- .74	- .66
36	.07	- .10	- .18
35	.49	.30	.36
32	- .10	- .57	- .58
31	.33	.27	.36
30	.22	- .01	.07
29	- .56	- .50	- .54
28	- .73	.54	.27
25	- .34	.81	.60
24	- .18	.97	.57
23	.08	.31	.10
22	.21	- .09	- .13
21	.25	- .09	- .05
17	- .40	-1.13	- .88
16	- .14	.65	.53
15	- .15	.23	.20
14	- .05	.27	.16
11	- .33	- .11	- .26
10	- .53	- .81	- .62
9	- .24	.25	.35
8	- .24	.23	.33
7	- .28	- .04	.15

DAYS BEFORE EXPIRATION	BLACK SCHOLES	AUTOREGRESSIVE	AUTOREGRESSIVE BLACK-SCHOLES
4	.31	1.32	.96
3	- .21	- .16	- .42
2	.25	1.24	.42
1	.09	.52	- .56

TABLE 9
JUNE 165 FORECAST ERRORS

DAYS BEFORE EXPIRATION	BLACK SCHOLES	AUTOREGRESSIVE	AUTOREGRESSIVE BLACK-SCHOLES
44	.24	.14	.14
43	.49	- .39	- .35
42	.04	- .34	- .54
39	.78	- .10	- .12
38	.70	- .18	- .19
37	.28	- .60	- .60
36	.05	.00	.03
35	.52	.09	.04
32	- .18	- .28	- .24
31	.37	.25	.17
30	.13	.05	.04
29	- .32	.13	.20
28	- .49	.32	.37
25	- .32	.00	- .01
24	- .41	.07	.10
23	.27	.00	- .01
22	.22	- .32	- .32
21	.06	- .15	- .15
17	- .06	- .11	- .10
16	- .16	.18	.18
15	.11	- .09	- .12
14	- .12	- .30	- .30
11	- .13	- .37	- .37
10	- .13	- .05	- .06
9	- .08	.59	.58
8	- .12	.22	.22
7	- .02	- .14	- .08
4	.21	.08	.14
3	.05	- .50	- .43
2	.25	.62	.54
1	- .10	.42	.09

TABLE 10
JUNE 170 FORECAST ERRORS

DAYS BEFORE EXPIRATION	BLACK SCHOLES	AUTOREGRESSIVE	AUTOREGRESSIVE BLACK-SCHOLES
32	- .24	- .13	- .12
31	.17	.07	.04
30	.04	.10	.09
29	- .13	.28	.31
28	- .26	.28	.29
25	- .13	- .05	- .07
24	- .53	- .31	- .34
23	.11	.03	.03
22	.10	- .04	- .04
21	- .03	.04	.04
17	- .03	.29	.28
16	- .13	.19	.19
15	- .04	- .15	- .13
14	- .08	- .07	- .06
11	- .04	- .16	- .14
10	.05	.37	.32
9	.03	.60	.49
8	.02	.29	.22
7	.05	.04	.06
4	.07	- .22	- .10
3	.05	- .16	- .09
2	.18	- .14	- .02
1	.03	- .31	- .23

TABLE 11
SEPTEMBER 160 FORECAST ERRORS

DAYS BEFORE EXPIRATION	BLACK SCHOLES	AUTOREGRESSIVE	AUTOREGRESSIVE BLACK-SCHOLES
31	- .64	- .27	- .27
30	- .75	.70	.71
29	- .62	- .68	- .72
28	- .65	- .34	.30
25	- .41	.29	.33
24	- .54	- .42	- .43
23	- .48	-1.24	-1.19
22	- .20	- .96	- .60
21	- .37	- .15	.09
18	- .18	.35	.46
17	- .21	.36	.39
16	- .70	.83	.36
15	- .51	.72	.35
14	- .73	.62	.22
10	- .43	2.38	.65
9	.19	2.01	.87
8	.07	.43	.09
7	.00	- .17	.05
4	- .02	- .40	.11
3	.22	.27	.47
2	.03	.48	.12
1	.05	- .31	.09

TABLE 12
SEPTEMBER 165 FORECAST ERRORS

DAYS BEFORE EXPIRATION	BLACK SCHOLES	AUTOREGRESSIVE	AUTOREGRESSIVE BLACK-SCHOLES
21	- .22	- .24	- .23
18	- .09	.11	.10
17	- .20	- .03	- .03
16	- .72	- .09	- .19
15	- .66	.05	.04
14	- .67	- .07	- .06
10	- .40	1.39	1.31
9	.23	.44	- .24
8	.17	- .60	- .52
7	- .04	- .69	- .34
4	.10	- .52	.19
3	.18	- .11	.47
2	.02	- .19	.19
1	.04	- .45	.06

TABLE 13
SEPTEMBER 170 FORECAST ERRORS

DAYS BEFORE EXPIRATION	BLACK SCHOLES	AUTOREGRESSIVE	AUTOREGRESSIVE BLACK-SCHOLES
66	1.79	.12	.06
65	.30	.05	.01
64	- .02	.15	.18
63	.79	- .02	.04
60	.47	.26	.26
59	- .37	- .04	- .30
58	-1.34	.44	.40
57	- .05	- .23	- .19
56	.30	.51	.55
53	- .15	.46	.44
52	- .71	.05	.11
51	.95	- .04	- .03
50	1.00	- .58	- .67
49	.84	- .46	- .45
46	.41	.14	.14
45	.47	.01	.01
44	- .37	- .06	- .07
43	.39	- .18	- .20
42	.38	.07	.07
39	.66	.19	.17
38	.54	.82	.82
37	.21	.16	.18
36	.05	- .35	- .36
35	- .26	- .27	- .27
32	- .31	- .19	- .17
31	- .35	- .79	- .77
30	- .18	- .23	- .16
29	- .07	-1.17	-1.13
28	- .37	- .36	- .32
25	- .15	- .39	- .28
24	- .23	- .58	- .59
23	- .08	- .15	- .28
22	- .02	.22	.14
21	- .09	.35	.37
18	- .07	.01	.05
17	- .12	- .09	- .06
16	- .48	- .04	.07
15	- .46	- .61	- .53
14	- .34	- .36	- .19
10	- .34	.30	.52
9	- .04	- .36	- .21
8	- .28	- .54	- .42
7	- .13	- .56	- .44
4	.48	.11	.17
3	.07	.02	.05
2	.02	.08	.19

TABLE 14
SEPTEMBER 175 FORECAST ERRORS

DAYS BEFORE EXPIRATION	BLACK SCHOLES	AUTOREGRESSIVE	AUTOREGRESSIVE BLACK-SCHOLES
67	- .79	.19	.21
66	1.17	- .01	- .14
65	.24	.17	.17
64	- .12	.22	.22
63	.35	.00	.00
60	.17	.23	.23
59	- .41	.09	.09
58	-1.14	.10	.10
57	- .24	- .33	- .33
56	.08	.29	.30
53	- .10	.34	.34
52	- .56	.11	.13
51	.44	- .18	- .17
50	.75	- .29	- .31
49	.44	- .19	- .19
46	.24	.39	.39
45	.33	.33	.33
44	- .20	.23	.23
43	.21	.05	.04
42	.24	.20	.20
39	.38	.27	.23
38	.30	.68	.67
37	.10	.25	.26
36	- .07	- .15	- .16
35	- .22	- .05	- .07
32	- .22	.02	.05
31	- .32	- .44	- .44
30	- .19	- .13	- .59
29	- .06	- .59	- .57
28	- .20	- .07	- .05
25	- .10	- .11	- .08
24	- .11	- .16	- .19
23	- .07	.09	.06
22	.01	.44	.36

TABLE 15
DECEMBER 160 FORECAST ERRORS

DAYS BEFORE EXPIRATION	BLACK SCHOLES	AUTOREGRESSIVE	AUTOREGRESSIVE BLACK-SCHOLES
36	- .35	.19	.19
35	.75	.37	.30
32	- .07	.43	.61
31	- .06	- .11	- .10
30	- .46	- .34	- .31
29	- .22	.12	.18
28	.16	.17	.16
25	.35	.48	.50
24	- .24	- .51	- .53
23	- .28	- .11	- .10
21	- .28	- .09	- .08
18	- .54	.59	- .59
17	- .14	.37	.40
16	- .57	- .62	- .61
15	- .08	.04	.03
14	- .35	- .35	- .33
11	.07	.15	.14
10	.06	.09	.11
9	- .08	- .03	- .04
8	.02	- .04	- .01
7	- .26	- .34	- .33
4	- .12	.06	.04
3	.05	.13	.14
2	.08	- .19	- .14
1	- .03	- .57	- .50

TABLE 16
DECEMBER 165 FORECAST ERRORS

DAYS BEFORE EXPIRATION	BLACK SCHOLES	AUTOREGRESSIVE	AUTOREGRESSIVE BLACK-SCHOLES
81	-1.89	- .59	- .67
80	-1.49	- .40	- .42
79	-1.76	- .18	- .07
78	-1.63	.04	.08
77	-1.19	.23	.20
74	-1.16	.22	.21
73	-1.47	- .26	- .30
72	-1.40	.16	.11
71	-2.24	.02	.12
70	-2.10	.21	.26
67	- .09	.56	.36
66	-1.96	- .14	.08
65	-1.91	- .04	.03
64	-1.68	.24	.30
63	-1.64	.27	.33
60	-1.39	.36	.39
59	-1.17	- .05	- .14
58	-1.16	- .05	- .10
57	- .92	.25	.23
56	- .82	- .08	- .13
53	- .42	.57	.58
52	- .40	.02	.00
51	- .59	- .03	- .02
50	- .51	.04	.05
49	- .47	.13	.16
46	- .30	- .03	- .01
45	- .44	- .10	- .09
44	- .72	- .48	- .51
43	- .82	- .16	- .16
42	- .62	.08	.10
39	- .33	.18	.22
38	.33	.00	- .05
37	- .72	- .45	- .44
36	- .70	- .29	- .32
35	.09	- .02	- .14
32	- .44	.06	.04
31	- .58	- .20	- .22
30	- .74	- .22	- .23
29	- .64	- .03	- .03
28	- .29	.05	.03
25	- .03	.32	.32
24	- .47	- .40	- .42
23	- .66	- .19	- .20
21	- .56	- .08	- .08

DAYS BEFORE EXPIRATION	BLACK SCHOLES	AUTOREGRESSIVE	AUTOREGRESSIVE BLACK-SCHOLES
18	- .71	- .52	- .53
17	- .57	.13	.13
16	- .68	- .60	- .60
15	- .50	- .01	.00
14	- .57	- .31	- .30
11	- .28	- .06	- .05
10	- .41	- .32	- .30
9	- .36	- .10	- .09
8	- .27	- .23	- .19
7	- .61	- .48	- .46
4	- .25	- .04	- .01
3	- .25	- .31	- .25
2	- .18	- .10	.06
1	.04	.67	.60

TABLE 17
DECEMBER 170 FORECAST ERRORS

DAYS BEFORE EXPIRATION	BLACK SCHOLES	AUTOREGRESSIVE	AUTOREGRESSIVE BLACK-SCHOLES
92	- .74	- .72	- .64
91	-1.17	.50	.27
88	- .96	- .24	- .13
87	-1.68	- .39	- .68
86	-1.47	- .77	- .80
85	-1.39	- .01	- .01
84	-1.35	- .40	- .34
81	-1.76	- .26	- .35
80	-1.30	- .50	- .42
79	-1.69	- .29	- .36
78	-1.65	- .15	- .19
77	-1.27	.06	.08
74	-1.12	.28	.27
73	-1.34	- .16	- .13
72	-1.22	.22	.28
71	-2.22	- .10	- .21
70	-2.16	- .11	- .16
67	- .67	.11	.34
66	-1.95	- .41	- .41
65	-1.84	- .14	- .14
64	-1.61	.16	.16
63	-1.54	.19	.19
60	-1.25	.23	.23
59	-1.09	- .24	- .24
58	-1.12	- .07	- .06
57	- .93	.25	.26
56	- .73	.03	.04
53	- .42	.56	.56
52	- .49	- .08	- .07
51	- .59	.08	.08
50	- .51	.12	.12
49	- .33	.36	.35
46	- .15	.14	.13
45	- .48	- .17	- .18
44	- .53	- .25	- .24
43	- .55	.08	.06
42	- .42	.19	.16
39	- .39	.08	.03
38	- .03	.04	.04
37	- .70	- .42	- .42
36	- .56	- .17	- .14
35	- .42	- .42	- .32
32	- .65	- .12	- .06

DAYS BEFORE EXPIRATION	BLACK SCHOLES	AUTOREGRESSIVE	AUTOREGRESSIVE BLACK-SCHOLES
31	- .57	- .05	- .02
30	- .79	- .31	- .28
29	- .66	- .01	.00
28	- .27	.15	.17
25	- .50	- .25	- .20
24	- .51	- .24	- .16
23	- .84	- .35	- .31
21	- .85	- .28	- .23
18	- .64	- .15	- .10
17	- .66	- .11	- .05
16	- .42	- .03	.03
15	- .44	- .01	.01
14	- .39	.07	.05
11	- .48	- .36	- .29
10	- .38	.05	.06
9	- .42	- .19	- .13
8	- .17	.28	.26
7	- .26	- .11	- .07
4	- .11	- .29	- .14
3	.00	.31	.32

Table 18

DECEMBER 175 FORECAST ERRORS

DAYS BEFORE EXPIRATION	BLACK SCHOLES	AUTOREGRESSIVE	AUTOREGRESSIVE BLACK-SCHOLES
86	-1.31	- .10	- .13
85	-1.21	.07	.09
84	-1.22	- .11	- .10
81	-1.71	- .19	- .22
80	-1.26	- .27	- .27
79	-1.52	- .13	- .14
78	-1.39	.00	.00
77	-1.16	- .07	- .07
74	-1.20	- .02	- .02
73	-1.31	- .13	- .13
72	-1.21	.15	.17
71	-2.16	- .08	- .06
70	-2.07	.14	.15
67	- .89	.28	.24
66	-1.82	- .10	- .05
65	-1.74	.05	- .04
64	-1.47	.21	.20
63	-1.61	.10	.12
60	-1.32	.20	.19
59	-1.06	.07	.03
58	-1.07	- .03	- .04
57	- .94	.18	.18
56	- .68	.12	.09
53	- .58	.30	.33
52	- .52	- .05	- .06
51	- .55	.17	.21
50	- .48	.07	.10
49	- .28	.38	.44
46	- .15	- .01	.00
45	- .36	- .20	- .21
44	- .34	- .17	- .18
43	- .38	.17	.18
42	- .21	.29	.29
39	- .18	.08	.06
38	- .04	- .01	- .02
37	- .44	- .46	- .45
36	- .42	- .14	- .12
35	- .37	- .26	- .21
32	- .43	.02	.05
31	- .44	.07	.08
30	- .50	- .11	- .10
29	- .53	- .04	- .03
28	- .29	.18	.18
25	- .42	- .23	- .20

DAYS BEFORE EXPIRATION	BLACK SCHOLES	AUTOREGRESSIVE	AUTOREGRESSIVE BLACK-SCHOLES
24	- .42	- .13	- .09
23	- .53	- .08	- .06
21	- .53	- .12	- .09
18	- .31	.19	.19
17	- .43	- .26	- .20
16	- .13	.41	.39
15	- .16	.07	.08
14	- .06	.32	.27

TABLE 19

VALUES OF THE INDEX AND BLACK-SCHOLES ERRORS FOR THE JUNE OPTION

DAYS BEFORE EXPIRATION	INDEX VALUE	JUNE 150	JUNE 155	JUNE 160	JUNE 165	JUNE 170
80	152.95	- .95				
79	155.06	- .83				
78	153.76	-1.63				
74	153.77	- .84				
73	152.31	-1.22				
72	151.81	-1.05				
71	152.61	- .51				
70	153.72	-1.44	-1.08	- .79		
67	156.10	-1.27	- .80	- .81		
66	156.62	-1.50	-1.28	-1.16		
65	158.01	-1.62	-1.42	-1.38		
64	159.43		-1.59	-1.47		
63	160.17		-1.20	- .90		
60	161.58		-2.00	-1.91		
59	160.30		-1.22	- .76		
58	162.85		-1.37	-1.07		
57	161.62		-1.22	- .81		
56	162.12		-1.47	-1.14		
53	160.18		-1.03	- .58		
52	164.01		-1.60	-1.36		
51	161.30		- .51	.13		
50	165.33		- .57	- .25		
49	166.63		-1.16	- .61		
46	164.14		- .27	.33		
45	164.62		- .11	.24		
44	165.13		- .17	.19		
43	166.05		- .47	.11	.49	
42	167.70		- .53	.03	.04	
38	167.06			.67	.78	
37	165.59			.20	.28	
36	164.82			.07	.05	
35	165.36			.49	.52	
32	163.88			- .10	- .18	- .24
31	163.84			.33	.37	.17
30	162.78			.22	.13	.04
29	161.34			- .56	- .32	- .13
2	161.64			- .73	- .49	- .26
25	163.55			- .34	- .32	- .13
24	165.62			- .18	- .41	- .53
23	166.41			.08	.27	.11
22	165.42			.21	.22	.10
21	164.26			.25	.06	- .03

DAYS BEFORE EXPIRATION	INDEX VALUE	JUNE 150	JUNE 155	JUNE 160	JUNE 165	JUNE 170
17	162.20			- .40	- .06	- .03
16	162.86			- .14	- .16	- .13
15	164.53			- .14	.11	- .04
14	164.73			- .05	- .12	- .08
11	165.42			- .33	- .13	- .04
10	162.99			- .53	- .13	.05
9	161.33			- .24	- .08	.03
8	161.82			- .24	- .12	.02
7	162.69			- .28	- .02	.05
4	165.00			.31	.21	.07
3	165.59			- .21	.05	.05
2	167.42			.25	.25	.18
1	169.31			.09	- .10	.03

TABLE 20
VALUES OF THE INDEX AND BLACK-SCHOLES ERRORS FOR THE SEPTEMBER OPTION

DAYS BEFORE EXPIRATION	INDEX VALUE	SEPT 160	SEPT 165	SEPT 170	SEPT 175
67	168.25				- .79
66	165.43			1.79	1.17
65	165.29			.30	.24
64	166.02			- .02	- .12
63	164.13			.79	.35
60	163.86			.47	.17
59	165.03			- .37	- .41
58	169.83			-1.34	-1.14
57	169.47			- .05	- .24
56	168.69			.30	.08
53	169.88			- .15	- .10
52	170.80			- .71	- .56
51	167.95			.95	.44
50	165.54			1.00	.75
49	163.13			.84	.44
46	163.38			.41	.24
45	163.13			.47	.33
44	164.66			- .37	- .20
43	163.13			.39	.21
42	163.23			.38	.24
39	160.66			.66	.38
38	161.87			.54	.30
37	163.19			.21	.10
36	162.72			.05	- .07
35	163.20			- .26	- .22
32	164.97			- .31	- .22
31	164.53	- .64		- .35	- .32
30	166.44	- .75		- .18	- .19
29	164.37	- .62		- .07	- .06
28	164.92	- .65		- .37	- .20
25	165.22	- .41		- .15	- .10
24	163.89	- .54		- .23	- .11
23	162.13	- .47		- .08	- .07
22	161.85	- .20		- .02	.01
21	163.35	- .37	- .24	- .09	
18	163.72	- .18	- .09	- .07	
17	163.83	- .21	- .20	- .12	
16	165.98	- .70	- .72	- .48	
15	165.53	- .51	- .66	- .46	
14	166.38	- .73	- .67	- .34	
10	169.51	- .41	- .40	- .34	
9	169.31	.19	.23	- .04	
8	168.96	.07	.17	- .28	
7	167.82	.00	- .04	- .13	

DAYS BEFORE EXPIRATION	INDEX VALUE	SEPT 160	SEPT 165	SEPT 170	SEPT 175
4	166.22	- .02	.10	.48	
3	165.78	.22	.18	.07	
2	166.64	.03	.02	.02	
1	169.31	.05	.04		

TABLE 21
VALUES OF THE INDEX AND BLACK-SCHOLES ERRORS FOR THE DECEMBER OPTION

DAYS BEFORE EXPIRATION	INDEX VALUE	DEC 160	DEC 165	DEC 170	DEC 175
92	165.28			- .74	
91	167.45			-1.17	
88	168.93			- .96	
87	170.40			-1.68	
86	169.06			-1.47	-1.31
85	170.77			-1.39	-1.21
84	170.49			-1.35	-1.22
81	171.11		-1.89	-1.76	-1.71
80	168.40		-1.49	-1.30	-1.26
79	168.99		-1.79	-1.69	-1.52
78	168.36		-1.63	-1.65	-1.39
77	167.06		-1.19	-1.27	-1.16
74	167.14		-1.16	-1.12	-1.20
73	167.63		-1.47	-1.34	-1.31
72	169.45		-1.40	-1.22	-1.21
71	172.26		-2.24	-2.22	-2.16
70	172.47		-2.10	-2.16	-2.07
67	174.65		- .09	- .67	- .89
66	171.92		-1.96	-1.95	-1.82
65	171.35		-1.91	-1.84	-1.74
64	171.90		-1.68	-1.61	-1.47
63	171.57		-1.64	-1.54	-1.61
60	172.33		-1.39	-1.21	-1.32
59	169.03		-1.17	-1.09	-1.06
58	168.08		-1.16	-1.12	-1.07
57	167.78		- .92	- .93	- .94
56	166.87		- .82	- .73	- .68
53	167.15		- .42	- .42	- .58
52	167.30		- .40	- .49	- .52
51	165.90		- .59	- .59	- .55
50	165.91		- .59	- .59	- .48
49	164.11		- .47	- .33	- .28
46	164.32		- .30	- .15	- .15
45	165.02		- .44	- .48	- .36
44	166.10		- .72	- .53	- .34
43	164.41		- .82	- .55	- .38
42	163.23		- .62	- .42	- .21
39	162.76		- .33	- .39	- .18
38	162.82		.33	- .03	- .04
37	165.54		- .72	- .70	- .44
36	165.87	- .35	- .70	- .56	- .42
35	167.68	.75	.09	- .42	- .37
32	167.58	- .07	- .44	- .65	- .43
31	166.25	- .06	- .58	- .57	- .44

DAYS BEFORE EXPIRATION	INDEX VALUE	DEC 160	DEC 165	DEC 170	DEC 175
30	166.80	- .46	- .74	- .79	- .50
29	166.71	- .22	- .64	- .66	- .53
28	165.69	.16	- .29	- .27	- .29
25	167.24	.35	- .03	- .50	- .42
24	167.61	- .24	- .47	- .51	- .42
23	167.66	- .28	- .66	- .84	- .53
21	167.94	- .28	- .56	- .85	- .53
18	166.91	- .54	- .71	- .64	- .31
17	168.58	- .14	- .57	- .66	- .43
16	166.45	- .57	- .68	- .42	- .13
15	166.62	- .08	- .50	- .44	- .16
14	165.60	- .35	- .57	- .39	- .06
11	166.60	.07	- .28	- .48	
10	166.02	.06	- .41	- .38	
9	166.76	- .08	- .36	- .42	
8	165.56	.02	- .27	- .17	
7	165.90	- .26	- .61	- .26	
4	167.08	- .12	- .25	- .11	
3	165.95	.05	- .25	.00	
2	164.20	.08	- .18		
1	162.53	- .03	.04		

TABLE 22
UNCONDITIONAL FORECAST STATISTICS FOR THE
AUTOREGRESSIVE MODEL WITHOUT BLACK-SCHOLES PRICES

June 150
from 80 days before expiration to 65 days before expiration
11 observations

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
0.10771796	0.69251273	0.91933371	1.0042307

June 155
from 70 days before expiration to 42 days before expiration
21 observations

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
0.64821034	1.0601633	1.2981252	1.0949650

June 160
from 70 days before expiration to 1 day before expiration
49 observations

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
0.28111602	0.94793386	1.1724974	1.0532236

June 165
from 45 days before expiration to 1 day before expiration
31 observations

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
0.046918470	0.61262989	0.73226289	0.9935960

June 170
from 35 days before expiration to 1 day before expiration
23 observations

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
0.060043805	0.21487756	0.25431833	1.0186583

September 160
from 31 days before expiration to 1 day before expiration
22 observations

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
0.20194456	0.99480231	1.2909276	1.1599252

September 165
from 21 days before expiration to 1 day before expiration
14 observations

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
0.00063035746	0.73506494	1.0069957	1.0883885

September 170
from 67 days before expiration to 2 days before expiration
46 observations

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
-0.092344669	0.30485491	0.41178554	0.816542874

September 175
from 70 days before expiration to 22 days before expiration
34 observations

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
0.053942223	0.21249516	0.26451268	0.79647166

December 160
from 37 days before expiration to 1 day before expiration
25 observations

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
-0.16050948	0.85612206	1.0816899	0.96550358

December 165
from 84 days before expiration to 1 day before expiration
58 observations

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
-0.13248671	0.62637542	0.81407338	0.99450693

December 170
from 93 days before expiration to 3 days before expiration
63 observations

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
-0.084361436	0.39100386	0.53708951	1.0523509

December 175
from 87 days before expiration to 14 days before expiration
52 observations

MEAN ERROR	MEAN ABS. ERR	RMS ERROR	THEIL
-0.012867603	0.21685893	0.32429804	0.99119372

TABLE 23
AUTOREGRESSIVE OPTION STRATEGY

June 150 6 OBSERVATIONS

	MEAN	STD. DEV.	VARIANCE	T-STATISTIC
NAKED	.00106	.14944	.02233	.00710
HEDGED	-.00010	.00489	.00002	-.01942
INDEX	.00193	.00865	.00007	.22297

June 155 16 OBSERVATIONS

	MEAN	STD. DEV.	VARIANCE	T-STATISTIC
NAKED	-.06828	.14881	.02215	-.45880
HEDGED	-.00221	.00433	.00002	-.51057
INDEX	.00361	.01066	.00011	.33826

June 160 38 OBSERVATIONS

	MEAN	STD. DEV.	VARIANCE	T-STATISTIC
NAKED	-.03288	.24273	.05891	-.13546
HEDGED	.00003	.00619	.00004	.00504
INDEX	.00262	.00902	.00008	.28993

June 165 25 OBSERVATIONS

	MEAN	STD. DEV.	VARIANCE	T-STATISTIC
NAKED	.02061	.45092	.20333	.04571
HEDGED	.00172	.00369	.00001	.46631
INDEX	.00025	.00842	.00007	.02922

June 170 15 OBSERVATIONS

	MEAN	STD. DEV.	VARIANCE	T-STATISTIC
NAKED	.17090	.45128	.20366	.37871
HEDGED	.00318	.00481	.00002	.66238
INDEX	.00004	.00861	.00007	.00476

SEPTELMBER 160 20 OBSERVATIONS

	MEAN	STD. DEV.	VARIANCE	T-STATISTIC
NAKED	.01148	.17798	.03168	.06450
HEDGED	.00154	.01054	.00011	.14584
INDEX	-.00035	.00808	.00007	-.04330

SEPTEMBER 165 7 OBSERVATIONS

	MEAN	STD. DEV.	VARIANCE	T-STATISTIC
NAKED	-.15350	.28323	.08022	-.54197
INDEX	.00499	.00802	.00006	.62218

SEPTEMBER 170 42 OBSERVATIONS

	MEAN	STD. DEV.	VARIANCE	T-STATISTIC
NAKED	.11369	.37254	.13879	.30518
INDEX	-.00029	.00970	.00009	-.02998

SEPTEMBER 175 27 OBSERVATIONS

	MEAN	STD. DEV.	VARIANCE	T-STATISTIC
NAKED	.13281	.26695	.07126	.49751
INDEX	-.00112	.01082	.00117	-.10375

DECEMBER 160 17 OBSERVATIONS

	MEAN	STD. DEV.	VARIANCE	T-STATISTIC
NAKED	.03763	.20753	.04307	.18131
HEDGED	.00137	.00278	.00001	.49182
INDEX	-.00036	.00732	.00005	-.04873

DECEMBER 165 37 OBSERVATIONS

	MEAN	STD. DEV.	VARIANCE	T-STATISTIC
NAKED	-.05750	.39609	.15689	-.14517
HEDGED	.00047	.00287	.00001	.16468
INDEX	-.00063	.00787	.00006	-.08061

DECEMBER 170 39 OBSERVATIONS

	MEAN	STD. DEV.	VARIANCE	T-STATISTIC
NAKED	.00984	.20521	.04211	.04797
HEDGED	-.00016	.00442	.00002	-.03658
INDEX	-.00007	.00767	.00006	-.00881

DECEMBER 175 21 OBSERVATIONS

	MEAN	STD. DEV.	VARIANCE	T-STATISTIC
NAKED	.03739	.19911	.03965	.18777
HEDGED	.00064	.00337	.00001	.18884
INDEX	-.00066	.00857	.00007	-.07697

GLOSSARY

CALL an option to buy.

EXERCISCE when an option holder uses his right to buy or sell, he is said to exercise his option.

EXPIRATION DATE the last day that the option contract is valid.

OPTION CONTRACT a contract that imparts the right to buy or sell a particaular item for a certain pricce during a specified time period.

OPTION HOLDER the party in an option contract who has the right to buy or sell the item specified in the contract.

OPTION WRITER the party in an option contract who stands ready to fulfill the terms of the option contract, if the option holder so demands.

PUT an option to sell.

STRIKE PRICE the price that the option contract specifies.

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