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**Portfolio selection of bonds with interest rate swaps—a  
mean-variance approach**

**Fang, Zhenmin, Ph.D.**

**City University of New York, 1990**

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A

**PORTFOLIO SELECTION OF BONDS WITH INTEREST RATE SWAPS**

**--A MEAN-VARIANCE APPROACH**

**by**

**ZHENMIN FANG**

**A dissertation submitted to the Graduate Faculty in  
Business in partial fulfillment of the requirements  
for the degree of Doctor of Philosophy, The City  
University of New York.**

**1990**

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This manuscript has been read and accepted for the Graduate Faculty in Business in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

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## **Abstract**

### **PORTFOLIO SELECTION OF BONDS WITH INTEREST RATE SWAPS**

#### **--A MEAN-VARIANCE APPROACH**

**by**

**Zhenmin Fang**

**Adviser: Professor Harry Markowitz**

Following the Markowitz general portfolio selection model in 1987, portfolio selection models of bonds without and with interest rate swaps are developed separately. A two factor model is used to obtain the variance-covariance matrix of portfolio returns as a whole without and with interest rate swaps. In both cases, the variance-covariance matrices of portfolio returns as a whole are assumed to be diagonal. Since interest rate swaps are designed to exchange a series of interest payments without exchanging the underlying principal, the proportionate holdings of nonmarketable assets and liabilities can be changed as if they are traded in the market. Thus optimal portfolios can always be achieved.

Using data from the CRSP U.S. Government Bond Tape, the two factor model is used to obtain the variance-covariance matrix of bond portfolio returns as a whole without interest

rate swaps. Empirical evidence shows that the two factor model explains most of the variations of bond portfolio returns and performs better than the single factor model. Using the critical line algorithm developed by Markowitz, the optimal portfolios of bonds without interest rate swaps are determined consistent with the mean-variance theory. With nonmarketable assets in the bond portfolios, the optimal portfolios will be less efficient.

Using data from the "International Financing Review", returns of swaps and swap generated-notes are obtained and the variance-covariance matrix of returns with swaps is estimated. Using the critical line algorithm again, the optimal portfolios of bonds with interest rate swaps are calculated. Empirical evidence shows that: 1. Interest rate swaps can be used to reallocate the nonmarketable bonds in the portfolio selection process; 2. The efficient frontier with interest rate swaps will shift up significantly if the underlying bond portfolios of real principal have longer maturities and higher risk exposures; 3. If the underlying bond portfolios of real principal have shorter maturities and lower risk exposures, the shifting of efficient frontier with interest rate swaps will be less significant.

Duration of 19 bond categories are estimated with a single factor model based on Sharpe (1990). We find that duration of bonds estimated by regressions on different interest rates can be very different. Except bonds with very

short maturities, durations estimated by regressions of bond return on percentage changes of long-term yield explain most of the variation of long-term bond returns. The traditional approach of using the percentage changes of short-term rates to estimate duration is not appropriate.

Using yield curves estimated by Hessel, Kim, and Markowitz, the ex ante returns of 19 bond categories are calculated. Our results show: 1. The values of ex ante returns vary in a very narrow range; 2. For each bond category, the ex post variance is equal to the ex ante variance plus a variable term which increases almost proportionally with the bond maturity. 3. The two factor model also explains most of the variations for the differences between the ex ante and ex post returns.

## **ACKNOWLEDGEMENTS**

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THE BASIC IDEAS

The Markowitz mean-variance portfolio selection model is widely used to select stock portfolios. The model can also be used to select the assets and liabilities of financial institutions such as insurance companies, commercial banks etc.. One problem related to the selection of assets and liabilities is that some assets and liabilities are not allowed to trade during their lifetime. For instance, corporate private placements bypass the SEC registration, and the entire issues are sold to a few knowledgeable investors and therefore, can not be easily traded in the marketplace. As mentioned by Brealey and Myers (1988,p341), corporate private placements outnumbered public issues from year 1976 to 1980, although the public issues were generally large. As to the government securities, the U.S. government saving bonds are issued for the saving purpose to the investors. These issues are also not allowed to trade during their lifetime. Since the U.S. government saving bonds account for about one quarter of the total government bond issues and corporate private placements account for about 35% to 40% of the total corporate bond issues, therefore, the nonmarketable properties of assets and liabilities cannot be ignored in the bond portfolio selection.

Brown (1988) argued that with nonmarketable assets, the consumption opportunity set of an individual who owns a nonmarketable asset will differ from the set available to that individual if the asset were tradable. And the rate of consumption of the individual will be lower if he owns a lot of nonmarketable assets.

Brown's argument may provide some explanations for the past test results of bank portfolio selections (for example, see Fried 1970). Since banks hold a lot of nonmarketable assets and liabilities, the investment opportunity set of banks may be quite different from the set available to them if the assets and liabilities were tradable. Banks may not just use simple "rule of thumb" to determine the portfolio allocation process. They know that the optimal conditions for portfolio allocation may change over time. But they may also have difficulty in adjusting their portfolios, because some assets and liabilities which are not optimal any more in the portfolios are not allowed to be traded easily in the market.

To deal with the nonmarketable property of assets and liabilities in portfolio selection, we try to introduce interest rate swaps. We argue that by introducing interest rate swaps into the mean-variance framework, the optimal allocation of assets and liabilities of the financial institutions can always be achieved whether the assets and liabilities can be bought or sold in the market or not.

To see this, consider a bond portfolio of an insurance

company in which the holdings of assets and liabilities are not optimal. This portfolio can be adjusted to the optimal level easily if all the assets and liabilities are tradable in the market. By simply buying or selling the required assets or liabilities in the market, the proportionate holding can be adjusted to the optimal level which can be obtained from a portfolio selection model. The adjustment of proportionate holding would be difficult, however, if buying and selling of certain types of assets and liabilities are not allowed. Since the proportion of these assets and liabilities would remain at a fixed level no matter what is optimal.

With the development of interest rate swaps, we can solve this problem by introducing a series of interest rate swaps into the bond portfolio selection model. Through simultaneous paying (receiving) and receiving (paying) of a fixed and floating rate of interest based on a notional principal, the proportionate holding of assets and liabilities can be altered as if the assets and liabilities are bought and sold in the market. Thus, the optimal position can always be achieved.

Instead of using full variance-covariance matrix of portfolio returns, the two factor model is used to generate the variance-covariance matrix of portfolio returns. The matrix is diagonal under the two factor model and the optimal portfolios obtained under the diagonal matrix are consistent with the traditional mean-variance theory.

## INTRODUCTION TO INTEREST RATE SWAPS

The significant increase in volatilities of interest rates since the late 70's has resulted in the development of new financial instruments, such as futures and options written on short and long term interest rate, and interest rate swaps.

### 2.1 What Is the Interest Rate Swap

An interest rate swap is an agreement under which two parties exchange a series of interest payments without exchanging the underlying debt for a fixed period of time. One of the simplest types of interest rate swaps is a fixed/floating rate swap. In this swap, the first party agrees to pay the second at designated intervals a fixed rate of interest and receive a floating rate of interest, and the second party agrees to pay the first a floating rate and receive a fixed rate based on the notional principal<sup>1</sup>. The cash flows in an interest rate swap are shown in Figure 1, where  $R_t$  are the fixed interest payments and  $R_t$  are the floating interest payments.

It is shown by Smith, Smithson and Wakeman (1986) that

---

<sup>1</sup> The nominal or principal amount of a swap as measured in a base currency, usually US dollars.

the swap cash flows in Figure 1 are equivalent to a pair of loan contracts. By decomposing the swap into a portfolio of two loan contracts: borrowing at a fixed-rate and lending at a floating-rate, the cash flows of the swap and the portfolio are exactly matched. Figure 2 illustrates the cash flows of the equivalent loan portfolio, where  $R_t$ ,  $R_t$  are the same as that in Figure 1, and  $P$  is the principal.

Typical floating rates used in an interest rate swap are LIBOR (London Interbank Offer Rate), Treasury-bill rate and Prime rate. At every settlement date, no principal changes hand and interest payments are calculated on a net basis (see Figure 1). As an example, consider a five-year interest rate swap in which the first party agrees to pay the second a fixed rate of 8.75% and simultaneously receives a floating rate which is equal to LIBOR-10 basis points; while the second party agrees to pay the first a floating rate of LIBOR-10 basis points and receives a fixed rate of 8.75% at each settlement date. Assume the notional principal of this five-year contract is \$100 million and the interest payments are settled every six months, then the payments or receipts of one party to the another at each settlement is simply:

$$\$100 \text{ million} \times (8.75\% - (\text{LIBOR} - 10 \text{ basis points})).$$

As a numerical example, assume the spot LIBOR rate at time 0 for the five-year interest rate swap is 8.10%. Six month later, the spot LIBOR rate becomes 7.50%. Then, the payment of the first party to the second (the receipt of the second

party from the first) for the settlement will be :

$$\$100 \text{ million} \times (8.75\% - (7.50\% - 10 \text{ basis points}))$$
$$= \$100 \text{ million} \times 1.35\%$$
$$= \$1.35 \text{ million}$$

It is important to notice that the value of any interest rate swap at the time of initiation is equal to zero. Otherwise, financial arbitrage can be easily generated.

## 2.2 Applications of Interest Rate Swaps

The interest rate swap market has grown very fast in recent years. The total notional principal amount arranged in the swap market has reached \$889 billion in 1987 from virtually nothing in 1981.

There are many uses of swaps. Two important functions of swap, as mentioned by Arnold (1984), D.Smith (1983), Kubarych (1983), Bicksler and Chen (1986), Smith, Smithson and Wakeman (1986), and Antl (1987) are:

1. it can be used to manage the maturity gaps of assets and liabilities;
2. it can be used to create a cheap investment through cost saving associated with it.

Smith, Smithson and Wakeman (1986) identify four sources for the explanation of cost saving of swaps: financial arbitrage, tax and regulatory arbitrage, exposure management and completing markets. Early papers related to the interest rate swap (D.Smith (1983), Arnold (1984), and Bicksler and Chen

(1986)) all gave similar examples for the illustration of financial arbitrages. As observed in the credit market, the spread due to the credit difference of two firms with different credit risk is greater in the fixed rate market than in the floating rate market. Through the interest rate swap, the weak credit firm who wants to enter the fixed rate market issues the floating rate debt, while the strong credit firm who wants to borrow at floating rate market issues the fixed rate debt. Thus the comparative advantages can be achieved for both firms. As pointed out by Smith, Smithson and Wakeman (1986), and Turnbull (1987), however, it seems difficult to attribute the financial arbitrage as the only economic basis for the fast growth of swaps. Because the swap markets are becoming increasingly competitive and such opportunities would be eliminated by very process of exploiting it. Turnbull also shows that a difference in quality spreads per se does not provide an explanation for the existence of the interest rate swap.

Wall and Pringle (1989) illustrate four other explanations to demonstrate that the quality spread differentials are not exploitable via an interest rate swap. Thus the arguments about the use of interest rate swaps for financial arbitrage are not convincing.

Unlike the financial arbitrage, tax and regulatory arbitrage will not easily disappear unless there is a change in the existing tax and regulatory codes.

For a firm concerned with exposures to interest rates and currency prices, the swap provides a cheap financial tool to manage these exposures, since the transaction costs associated with swaps are normally lower than that of other financial instruments.

Compared with forward contracts, the swap has a longer time to maturity. As we know, many swaps can extend over a five-to-seven year period. Such swaps can not be duplicated by the existing financial instruments such as futures and options. Thus, swaps provide a way to complete the financial market.

### **2.3 Pricing of Interest Rate Swaps**

There are mainly two approaches to pricing swaps. One approach is to decompose swaps into a series of forward contracts. Thus the valuation of swaps - whether it be the interest rate swap, the currency swap, or the commodity swap - may seem similar to the valuation of a portfolio of forward contracts. This pricing approach of swaps is not completely correct, however, because the default risk is not taken into the consideration. If one party defaults, the entire sequence of forward contracts will be voided rather than one. Therefore, the credit risk of the counterparty must be considered in the valuation of swaps. Ritchken (1987) suggests that swaps must be evaluated in terms of a party's total financial structure, rather than as an isolated

transaction. When interest rate swaps are arranged for two firms with very different yield curves, it can be viewed as an exchange of interest rate risks for credit and basis risks.

The second approach to pricing swaps is to treat the swap as an option. Whittaker provides an option pricing approach to price swaps. The premise of option pricing approach, as Whittaker argued, is that a swap can be viewed as an option and valued as such. According to Whittaker's point of view, a swap can be viewed as the right to buy a bond and sell a floating rate note simultaneously at the prevailing prices. These options will be exercised by the default of the end-user<sup>2</sup> and will occur only if the swap is in the money. For a fixed rate payer, the swap/option will be in the money only when interest rate goes down and bond price goes up. In this case, default/exercise may occur for the end-user by jointly exercising a call option to purchase a bond and a put option to sell a floating rate note. For a fixed rate receiver, the condition will just be reversed. Using the binomial model, Whittaker develops a formula for the pricing of the interest rate swaps, which is similar to the original formula for pricing options.

The big advantage of the option pricing approach for

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<sup>2</sup> A party entering into a swap transaction not as an intermediary or trader rather as a user of the swap to either obtain funding or hedge foreign exchange or interest rate risk.

interest rate swaps is its wide applicability. It can price not only the swaps with the same payment structures but also swaps with payment mismatches.<sup>3</sup>

---

<sup>3</sup> It means that fixed rate and floating rate payment frequencies are not matched. For example, one party of an interest rate swap may receive floating rate payments quarterly and pay the fixed rate interests semi-annually.

**AN OVERVIEW OF PORTFOLIO SELECTION AND ITS APPLICATIONS**

Markowitz developed the portfolio selection theory in the 1950's. His seminal paper "Portfolio Selection" (1952) was a breakthrough in the modern theory of finance. In that paper, Markowitz explicitly quantified the risk in terms of variance. He also introduced the notion of E-V efficient portfolios which are either minimum V for given E or maximum E for given V. Finally, he provided a preliminary approach for solving the portfolio selection model.

After developing an algorithm for solving a more general class of the portfolio selection model, his book (1959) was concentrated on the mean and variance of portfolio returns when selecting securities.

**3.1 The Markowitz Model in the 1950's**

Markowitz mean-variance portfolio selection approach can be described as follows. Suppose that an investor has a given sum of money to invest at the beginning of the period. After making an investment decision and holding for a given length of time, the investor sells the securities he bought at the beginning of the period and uses the proceeds for consumptions and reinvestment at the end of the period. Here the key question is: how can the investor make an optimal investment

decision at the beginning of the period. In other words, how does the investor select an optimal portfolio from a set of possible portfolios? That is the reason why the optimal decision problem is always referred to as the "portfolio selection problem". Since people may have different views about an optimal investment decision, therefore, a reasonable and acceptable common criterion must be developed. The basic contribution of Markowitz's 1959 book is that a simple and testable mean-variance portfolio selection model was presented and it can be used for the practical purpose. In his 1959 book, Markowitz recognized that any investment decision concentrated on only one aspect - the expected return of securities, is unwise. This is because the typical investor not only wants returns to be high, but also wants returns to be as certain as possible. In reality, however, one can hardly find such portfolios which give both maximum expected returns and minimum uncertainty (represented by variance or standard deviation of returns), since they are two conflicting objectives. Nevertheless, Markowitz mean-variance portfolio selection model presented in 1959 book gives full consideration to both these objectives and makes a good balance against each other. Markowitz found that one way to deal these two conflict objectives is to diversify the portfolio holding by purchasing not just one security but several.

### **3.2 The Applications**

The Markowitz model was widely used for the portfolio selection of commercial banks. Fried (1970) used a chance-constrained portfolio selection model to test whether bank portfolio selection is E-V efficient. His test result was not confirmed between actual and generated portfolios. Klein (1970) divided bank securities into three groups - cash, government securities and private loans for the bank portfolio selection. He pointed that portfolio selection theory can be used to explain the bank portfolio allocation process. Vora (1974) argued that the optimal levels of different components of a bank's balance sheet can not be achieved without first determining the optimal set of customers. He developed a portfolio selection model for the customer selection in banking. Francis (1978) used the Markowitz portfolio selection model to select bank's assets and liabilities. He found that small banks performed more efficiently than large banks. Kane and Buser (1979) focused on the diversification at commercial banks. Their empirical results showed that bank diversification was related to the bank size.

### **3.3 Gap Management**

Interest rate risk arises whenever there are mismatches or gaps in the maturity structure of a firm's assets and liabilities. To manage the maturity gaps of assets and liabilities of financial institutions, Goodman and Langer

(1983) divided the financial institutions into two groups: "liability sensitive" or "asset sensitive". Liability sensitive institutions have assets of a longer repricing period than the repricing period of liabilities. They are helped by falling interest rates and hurt by rising interest rates. Asset sensitive institutions have a longer repricing period of liabilities than that of assets. They are helped by rising interest rates and hurt by falling interest rates. Goodman and Langer showed that if the futures contract has a positive expected profit, then asset sensitive institutions will opt for a long position in the futures contract. Because it will increase the expected profitability and reduce the variability of economic profit. If the futures contract has a negative expected profit, the liability sensitive institutions will opt for a short position in financial futures to increase the profit and reduce the risk. Goodman and Langer also pointed out that it is important that the gap management must be a true hedge or a "macro hedge" instead of matching a single asset position with a single liability position, namely a "micro hedge".

Besides futures contracts, interest rate swaps can also be used for hedging purposes by the financial institutions. An important role of interest rate swaps, as we mentioned early, is that it can be used to manage the interest rate exposure arising from the maturity gaps. For example (Bicksler and Chen), the Student Loan Marketing Association (Sallie Mae)

has used interest rate swaps to hedge against its loan positions.

### 3.4 The Markowitz Model of 1987

The Markowitz mean-variance portfolio selection model developed in the 1950's is generalized in his recent book of 1987. Assuming that all portfolio selection problems have the same objective function (to seek the mean-variance efficiency), Markowitz shows that there are three forms of general portfolio selection model, i.e., there are three forms of constraint set for the choice variables. "Form 0" consists of  $n \geq 1$  variables subject to zero, one or more constraints of the following types

$$X_i \geq L_i \quad \text{for some } i \quad (3.1)$$

$$X_i \leq U_i \quad \text{for some } i \quad (3.2)$$

$$\alpha'X \geq C \quad (3.3)$$

$$\alpha'X \leq C \quad (3.4)$$

$$\alpha'X = C \quad (3.5)$$

where  $\alpha$  is a constant vector;  $L_i$ ,  $U_i$  and  $C$  are constant scalar and  $X$  is a vector (portfolio) which includes  $n$  choice variables,  $X' = (X_1, X_2, \dots, X_n)$ .

The general portfolio selection model, "form 2"<sup>4</sup> can be

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<sup>4</sup> The general portfolio selection model, "form 1" can be written as  $AX \geq b$ , where  $A$  is a  $m$  by  $n$  matrix,  $X$  is a  $n$  by 1 vector and  $b$  is a  $m$  by 1 vector. "Form 1" is not related to this paper.

written as

$$AX = b \quad (3.6)$$

$$X \geq 0 \quad (3.7)$$

where  $A$  is any  $m$  by  $n$  matrix and  $b$  is a  $m$  by 1 vector.

Markowitz has proved that these two forms of general portfolio selection model are strictly equivalent.

If the portfolio selection model is feasible, that is, the portfolio  $X$  satisfies the constraints represented by "form 0" or "form 2", then a complete, nonredundant and piecewise linear set of efficient portfolios can be obtained. The method used to find the set of efficient portfolios, in Markowitz' terms, is called critical line algorithm. The elegance of the critical line method is that it can be used to solve the practical problems comparative easily. In next chapter, we are going to develop a portfolio selection model which follows the general portfolio selection model mentioned above to select the assets and liabilities of the financial institutions. We will show how can theory be put into practice.

MODEL BUILDING

4.1 Portfolio Selection Model Without Interest Rate Swaps

To use the portfolio selection model for the management of risk exposure of a firm's assets and liabilities, we define:

- $x_{1L}$  : unit amount (\$100 million) of floating rate assets maturing within one period in the bond portfolio;  
.....
- $x_{nL}$  : unit amount (\$100 million) of floating rate assets maturing within n periods in the bond portfolio;
- $x_{n+1,L}$  : unit amount (\$100 million) of fixed rate assets maturing within one period in the bond portfolio;  
.....
- $x_{2n,L}$  : unit amount (\$100 million) of fixed rate assets maturing within n periods in the bond portfolio;
- $x_{1S}$  : unit amount (\$100 million) of floating rate liabilities maturing within one period in the bond portfolio;  
.....
- $x_{nS}$  : unit amount (\$100 million) of floating rate liabilities maturing within n periods in the bond portfolio;
- $x_{n+1,S}$  : unit amount (\$100 million) of fixed rate liabilities maturing within one period in the bond portfolio;

.....  
 $x_{2n,s}$  : unit amount (\$100 million) of fixed rate liabilities  
 maturing within  $n$  periods in the bond portfolio.

To be a feasible portfolio selection model, the unit amount of  $2n$  bonds  $x_{1L}, x_{2L}, \dots, x_{2n,L}$  as assets and  $x_{1S}, x_{2S}, \dots, x_{2n,S}$  as liabilities in the bond portfolio must satisfy the following constraints:

$$\sum_{i=1}^{2n} x_{iL} - \sum_{i=1}^{2n} x_{iS} \geq 0 \quad (4.1a)$$

$$\sum_{i=1}^{2n} x_{iS} \leq U \quad (4.1b)$$

$$x_{iL} \geq 0 \quad i=1, 2, \dots, 2n \quad (4.1c)$$

$$x_{iS} \geq 0 \quad i=1, 2, \dots, 2n \quad (4.1d)$$

Equation (4.1a) is the no bankruptcy condition of the corporation. That is, the total value of assets in the bond portfolio must be greater than or equal to the total value of liabilities. Otherwise, interest rate swaps are not allowed to be used by the portfolio managers. Equation (4.1b) represents the upper limit of liabilities for a corporation. In other words, a corporation does not have unlimited access to the bond markets. The determination of upper bound  $U$  of a corporation depends on the corporation's credit quality, the corporation's size, the industry, etc..

Constraint (4.1) can be brought into a standard form 0 defined by Markowitz (1987) by writing them as:

$$\sum_{i=1}^{2n} x_i - \sum_{i=2n+1}^{4n} x_i \geq 0 \quad (4.2a)$$

$$\sum_{i=2n+1}^{4n} x_i \leq U \quad (4.2b)$$

$$x_i \geq 0 \quad i=1, 2, \dots, 4n \quad (4.2c)$$

where we write  $x_i$  ( $i \leq 2n$ ) for  $x_{il}$  and  $x_i$  ( $2n+1 \leq i \leq 4n$ ) for  $x_{i-2n,s}$ .

To use the mean-variance criterion for the portfolio selection of assets and liabilities, one must obtain  $E$  and  $V$  of the bond portfolio. Suppose that the returns on floating rate assets or liabilities and fixed rate assets or liabilities at certain periods  $r_1, r_2, \dots, r_n, r_{n+1}, \dots, r_{2n}$  are jointly distributed random variables and the return on the bond portfolio without interest rate swaps is

$$\begin{aligned} R_M &= \sum_{i=1}^{2n} (x_{il} - x_{is}) r_i \\ &= \sum_{i=1}^{2n} (x_i - x_{2n+i}) r_i, \end{aligned} \quad (4.3)$$

then the expected return on the bond portfolio as a whole without swaps is

$$E_N = \sum_{i=1}^{2n} (x_i - x_{2n+i}) \mu_i, \quad (4.4)$$

$$\text{where } \mu_i = E(r_i) \quad i=1, 2, \dots, 2n. \quad (4.5)$$

Notice that in (4.3), returns on bond as assets are assumed to be the same as that of bond as liabilities. That is, both the assets and liabilities are assumed to be in the same risk class.<sup>5</sup> And in (4.4),  $E_N$  is used as the expected value of the return on the bond portfolio, i.e.,  $E_N = E(R_N)$ .

The variance of return on the bond portfolio as a whole without interest rate swaps is given by:

$$V_N = \sum_{i=1}^{2n} \sum_{j=1}^{2n} (x_i - x_{2n+i}) (x_j - x_{2n+j}) \sigma_{ij}, \quad (4.6)$$

where

$$\sigma_{ij} = E[(r_i - \mu_i)(r_j - \mu_j)] \quad i, j = 1, 2, \dots, 2n \quad (4.7)$$

is the covariance between  $r_i$  and  $r_j$ . If  $i=j$ , then

$$\sigma_{ii} = E(r_i - \mu_i)^2 = V(r_i) \quad (4.8)$$

is the variance of  $r_i$ . The use of  $V_N$  in (4.6) is the same as that of  $E_N$  in (4.4), i.e.,  $V_N = V(R_N)$ .

#### 4.2 Portfolio Selection Model With Interest Rate Swaps

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<sup>5</sup> The same risk class here means that the floating rate assets and liabilities with the same maturity time periods  $n$  will have the same returns  $r_1, r_2, \dots, r_n$ ; and the fixed rate assets and liabilities with the same maturity time periods  $n$  will have the same returns  $r_{n+1}, r_{n+2}, \dots, r_{2n}$ .

In case that buying and selling of certain type of assets and liabilities are not allowed in the bond portfolio, a series of interest rate swaps can be introduced to reallocate the assets and liabilities. To incorporate the interest rate swaps with the mean-variance criterion for the portfolio selection of the assets and liabilities, we define:

$x_{2n+1,L}$  : unit notional principal amount (\$100 million) of long (interest rate) swap maturing within one period;

.....

$x_{3n,L}$  : unit notional principal amount (\$100 million) of long (interest rate) swap maturing within  $n$  periods;

$x_{2n+1,S}$  : unit notional principal amount (\$100 million) of short (interest rate) swap maturing within one period;

.....

$x_{3n,S}$  : unit notional principal amount (\$100 million) of short (interest rate) swap maturing within  $n$  periods.

In a long swap  $x_{2n+i,L}$ ,  $i=1,\dots,n$ , the bond portfolio manager pays the floating rate of interests and simultaneously receives the fixed rate based on the notional principal  $x_{2n+i,L}$ . It is equivalent to buying  $x_{2n+i,L}$  fixed rate bonds with  $i$  periods to maturity and simultaneously selling the same amount of floating rate bonds with  $i$  periods to maturity. In a short swap  $x_{2n+i,S}$ , the opposite position would occur. The portfolio

manager would pay the fixed rate (selling fixed rate bond with  $i$  periods) and simultaneously receive the floating rate (buying floating rate bond with  $i$  periods) based on the notional principal  $x_{2n+1,s}$ .

Though the primary market for interest rate swaps is well-developed, the secondary market emerged not very long and trading activities are still relative thin. Therefore, almost all swaps are used as hedge instruments rather than for speculation. For a naked five year interest rate swap with notional principal equal to \$100 million, the initial interest rate exposure is equivalent to a \$100 million five-year treasury security, be it short or long. If there is a 100 basis point movement in treasury securities, it would produce an unrealized loss (or gain) of around 4 percent, or \$4 million. As the risk of pure speculation with interest rate swaps is so large, no one wants to participate in the swaps until market liquidity improves, especially for the long-term swaps.

Taking the above points into the consideration in the portfolio selection of assets and liabilities, the swap portfolio  $x_{2n+1,L}, \dots, x_{3n,L}, x_{2n+1,S}, \dots, x_{3n,S}$  used to hedge against the interest rate risk in the bond portfolio must satisfy the following constraints:

$$\sum_{i=2n+1}^{3n} x_{i,L} \leq \sum_{i=n+1}^{2n} x_{i,S} \quad (4.9a)$$

$$\sum_{i=2n+1}^{3n} x_{i,s} \leq \sum_{i=1}^n x_{i,s} \quad (4.9b)$$

$$x_{iL} = a_i \quad \text{for some } i \quad (4.9c)$$

$$x_{iS} = b_i \quad \text{for some } i \quad (4.9d)$$

$$x_{iL} \geq 0 \quad i=2n+1, \dots, 3n \quad (4.9e)$$

$$x_{iS} \geq 0 \quad i=2n+1, \dots, 3n \quad (4.9f)$$

(4.9a) means that the total notional principal of long swaps

$\sum_{i=2n+1}^{3n} x_{iL}$ , which is used to hedge against the fixed rate

liabilities in the bond portfolio, would not exceed the total

amount of fixed rate liabilities  $\sum_{i=n+1}^{2n} x_{iS}$ . Similarly, (4.9b) means

that the total notional principal of short swaps  $\sum_{i=2n+1}^{3n} x_{iS}$ , which

is used to hedge against the floating rate liabilities in the bond portfolio, would not exceed the total amount of floating

rate liabilities  $\sum_{i=1}^n x_{iS}$ . (4.9c) and (4.9d) mean that some assets

and liabilities in the bond portfolio are constants, which are equal to  $a_i$  and  $b_i$  respectively and are not free to trade.

Constraint (4.9) can be written as the standard form 0 as follows:

$$\sum_{i=4n+1}^{5n} x_i \leq \sum_{i=3n+1}^{4n} x_i \quad (4.10a)$$

$$\sum_{i=5n+1}^{6n} x_i \leq \sum_{i=2n+1}^{3n} x_i \quad (4.10b)$$

$$x_i = c_i \quad \text{for some } i \quad (4.10c)$$

$$x_i \geq 0 \quad i=4n+1, \dots, 6n \quad (4.10d)$$

where  $x_i$  ( $4n+1 \leq i \leq 5n$ ) is used for  $x_{i-2n,L}$  and  $x_i$  ( $5n+1 \leq i \leq 6n$ ) is used for  $x_{i-3n,S}$ ;  $c_i$  is a constant,

$$c_i = \begin{cases} a_i & \text{for some } i \quad (1 \leq i \leq 2n) \\ b_i & \text{for some } i \quad (2n+1 \leq i \leq 4n) \end{cases}.$$

To obtain  $E$  and  $V$  of the bond portfolio with interest rate swaps, we assume:

- The transaction costs of both bonds and swaps are equal to zero;
- Bonds are not callable.
- There are no income taxes on both swaps and bonds.

Under these assumptions, the return on the bond portfolio including interest rate swaps can be obtained:

$$\begin{aligned} R &= \sum_{i=1}^{2n} x_i r_i - \sum_{i=1}^{2n} x_{2n+i} r_i + \sum_{i=1}^{2n} x_{4n+i} r_{4n+i} \\ &= \sum_{i=1}^{2n} (x_i - x_{2n+i}) r_i + \sum_{i=1}^{2n} x_{4n+i} r_{4n+i} \end{aligned} \quad (4.11)$$

where the first term is the return on the net worth and the second term is the return on swaps.

By means of (4.11), the expected return on the bond

portfolio as a whole including interest rate swaps is

$$E = \sum_{i=1}^{2n} (x_i - x_{2n+i}) \mu_i + \sum_{i=1}^{2n} x_{4n+i} \mu_{4n+i} \quad (4.12)$$

$$\text{where } \mu_i = E(r_i) \quad \mu_{4n+i} = E(r_{4n+i}) \quad i=1, 2, \dots, 2n. \quad (4.13)$$

The variance of return on the bond portfolio including interest rate swaps is given by

$$V = \sum_{i=1}^{2n} \sum_{j=1}^{2n} (x_i - x_{2n+i}) (x_j - x_{2n+j}) \sigma_{ij} + \sum_{i=1}^{2n} \sum_{j=1}^{2n} x_{4n+i} x_{4n+j} \sigma_{4n+i, 4n+j} + \quad (4.14)$$

$$+ 2 \sum_{i=1}^{2n} \sum_{j=1}^{2n} (x_i - x_{2n+i}) x_{4n+j} \sigma_{i, 4n+j}$$

where  $\sigma_{ij}$  is the covariance between  $r_i$  and  $r_j$  as defined in (4.7).

Comparing  $E$  and  $V$  of (4.12) and (4.14) with  $E_N$  and  $V_N$  of (4.4) and (4.6), we find that there are more terms in  $E$  and  $V$  than that in  $E_N$  and  $V_N$  because of the introduction of interest rate swaps. In fact,  $E_N$  and  $V_N$  are special cases of  $E$  and  $V$ . If the notional principal of all swaps equal zero, i.e.,  $x_{4n+i}=0$ ,  $i=1, \dots, 2n$ , then  $E$  and  $V$  collapse to  $E_N$  and  $V_N$ .

If the returns on bonds and returns on swaps  $r_i$  and  $r_{4n+i}$  are assumed to be uncorrelated, i.e.,  $\text{cov}(r_i, r_{4n+i})=0$ ,  $i=1, \dots, 2n$ , then we have

$$V = \sum_{i=1}^{2n} \sum_{j=1}^{2n} (x_i - x_{2n+1}) (x_j - x_{2n+1}) \sigma_{ij} + \sum_{i=1}^{2n} \sum_{j=1}^{2n} x_{4n+i} x_{4n+j} \sigma_{4n+i, 4n+j} \quad (4.15)$$

which is a special case of equation (4.14).

### 4.3 The Two Factor Model

To use the mean-variance analysis for the portfolio selection of assets and liabilities, bond portfolio managers must obtain matrix  $V$ , the variance-covariance matrix of the portfolio. In the real world, the matrix  $V$  of the bond portfolio is not easy to obtain, because there are hundreds of different bonds in the market. For 200 bonds, it may be reasonable to expect a bond portfolio management team to prepare 200 carefully considered estimates of expected return and 200 carefully considered estimates of standard deviation of return, but it is not reasonable to expect the team to prepare  $199 \times 200 / 2 = 19,900$  carefully considered estimates of covariance. To solve this difficulty, bond portfolio management team can use one or another model of covariance suggested by Markowitz (1987).

Suppose that the return on each bond and each swap were generated by the following two factor model:

$$r_i = \alpha_i + \beta_{i1} I_1 + \beta_{i2} I_2 + u_i, \quad (4.16)$$

where  $u_i$  is a random variable with  $Eu_i = 0$  for  $i = 1, \dots, 2n$ ;  
 $\text{cov}(u_i, u_j) = 0$  for  $i \neq j$ ;  $\text{cov}(u_i, I_1) = 0$ ,  $\text{cov}(u_i, I_2) = 0$  for

$i=1, \dots, 2n$ ;  $\alpha_i$ ,  $\beta_{i1}$  and  $\beta_{i2}$  are constants;  $I_1$  is the percentage change of one plus the short-term interest rate and  $I_2$  is the percentage change of one plus the long-term interest rate.

One way to incorporate (4.16) into a portfolio selection model is to compute  $V(r_i)$  and  $\text{Cov}(r_i, r_j)$  explicitly. By means of the definition of variance and covariance to the random variables  $r_i$  and  $r_j$  and the assumptions listed following (4.16), we obtain

$$V(r_i) = \beta_{i1}^2 V(I_1) + \beta_{i2}^2 V(I_2) + V(u_i) \quad (4.17)$$

$$\sigma_{ij} = \beta_{i1}\beta_{j1} V(I_1) + \beta_{i2}\beta_{j2} V(I_2) + 2\beta_{i1}\beta_{j2}\text{cov}(I_1, I_2) \quad (4.18)$$

for  $i \neq j$ .

for the return of each bond and each swap.

If the percentage change of two one plus the interest rates  $I_1$  and  $I_2$  are assumed to be uncorrelated, i.e.,  $\text{cov}(I_1, I_2)=0$ , then the covariances between  $r_i$  and  $r_j$  are

$$\sigma_{ij} = \beta_{i1}\beta_{j1} V(I_1) + \beta_{i2}\beta_{j2} V(I_2) \quad \text{for } i \neq j, \quad (4.19)$$

which is a special case of equation (4.18).

The assumption of independence of percentage change of two one plus the interest rates,  $\text{cov}(I_1, I_2)=0$ , is not reasonable. Using Wall Street Journal's 91 day T-bill rate at the end of the month and yield average of 10 bonds with the longest maturities at the end of the month, we find that the correlation coefficient between the percentage change of these

two rates is 0.5897 during the period from January 1985 to December 1987. Obviously, the correlation between these two rates is not low enough to convince people that the assumption of independence of these two rates is reasonable.

With any  $\mu$  and any positive semidefinite variance-covariance matrix  $C$ , in particular, the variances and covariances computed by (4.17) and (4.18), we can use the critical line algorithm provided by Markowitz (1987) to find out the efficient set of bond portfolio including swaps subjected to the following constraint set:

$$\sum_{i=1}^{2n} x_i - \sum_{i=2n+1}^{4n} x_i \geq 0 \quad (4.20a)$$

$$\sum_{i=2n+1}^{4n} x_i \leq U \quad (4.20b)$$

$$\sum_{i=4n+1}^{5n} x_i \leq \sum_{i=3n+1}^{4n} x_i \quad (4.20c)$$

$$\sum_{i=5n+1}^{6n} x_i \leq \sum_{i=2n+1}^{3n} x_i \quad (4.20d)$$

$$x_i = c_i \quad \text{for some } i \quad (4.20e)$$

$$x_i \geq 0 \quad i=1,2,\dots,6n. \quad (4.20f)$$

A more economic approach to obtain the variance and covariance is to consider the return  $R$  on the bond portfolio

as a whole instead of considering the return  $r_i$  on each bond. For the two factor model, the return  $R_M$  on the bond portfolio as a whole without interest rate swaps is:

$$R_M = \sum_{i=1}^{2n} (x_i - x_{2n+i}) r_i \quad (4.21)$$

$$= \sum_{i=1}^{2n} (x_i - x_{2n+i}) \alpha_i + \left[ \sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i1} \right] I_1 + \\ + \left[ \sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i2} \right] I_2 + \sum_{i=1}^{2n} (x_i - x_{2n+i}) u_i ,$$

where  $\sum_{i=1}^{2n} (x_i - x_{2n+i}) \alpha_i$ ,  $\sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i1}$  and  $\sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i2}$  are

constants and  $\sum_{i=1}^{2n} (x_i - x_{2n+i}) u_i$  is a random variable with

$$E \left[ \sum_{i=1}^{2n} (x_i - x_{2n+i}) u_i \right] = 0 \text{ and } \text{cov} \left[ \sum_{i=1}^{2n} (x_i - x_{2n+i}) u_i, I_j \right] = 0 \quad j=1,2.$$

The variance of the return  $R_M$  on the bond portfolio without swaps is

$$V(R_M) = \left[ \sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i1} \right]^2 V(I_1) + \left[ \sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i2} \right]^2 V(I_2) + \\ + 2 \left[ \sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i1} \right] \left[ \sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i2} \right] \text{cov}(I_1, I_2) +$$

$$+ V[\sum_{i=1}^{2n} (x_i - x_{2n+i}) u_i] \quad (4.22)$$

$$= \sum_{i=1}^{2n} [\sum (x_i - x_{2n+i}) \beta_{i1}]^2 V(I_1) + \sum_{i=1}^{2n} [\sum (x_i - x_{2n+i}) \beta_{i2}]^2 V(I_2) +$$

$$+ 2 \sum_{i=1}^{2n} [\sum (x_i - x_{2n+i}) \beta_{i1}] [\sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i2}] \text{cov}(I_1, I_2) +$$

$$+ \sum_{i=1}^{2n} (x_i - x_{2n+i})^2 V(u_i)$$

The last equality holds because  $\text{cov}(u_i, u_j) = 0$ , for  $i \neq j$ .

By means of (4.22), the portfolio selection model of any bond without swaps can be written as follows:

$$\sum_{i=1}^{2n} (x_i - x_{2n+i}) \geq 0 \quad (4.23a)$$

$$\sum_{i=1}^{2n} x_{2n+i} \leq U \quad (4.23b)$$

$$x_i \geq 0 \quad i=1, 2, \dots, 4n \quad (4.23c)$$

$$\sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i1} - b_1 = 0 \quad (4.23d)$$

$$\sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i2} - b_2 = 0 \quad (4.23e)$$

$$E_N = \sum_{i=1}^{2n} (x_i - x_{2n+i}) \mu_i \quad (4.23f)$$

$$V_H = \sum_{i=1}^{2n} (x_i - x_{2n+i})^2 V(u_i) + V(I_1) b_1^2 + V(I_2) b_2^2 + 2b_1 b_2 \text{cov}(I_1, I_2) \quad (4.23g)$$

where  $b_1$  in (4.23d) is constrained to equal  $\sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i1}$  and

$b_2$  in (4.23e) is constrained to equal  $\sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i2}$ , so  $b_1$  and

$b_2$  are indeed the  $\beta_1$  and  $\beta_2$  of the bond portfolio

$(x_1, x_2, \dots, x_{4n})$ .

For the two factor model, the return of bond portfolio as a whole with interest rate swaps is

$$R = \sum_{i=1}^{2n} (x_i - x_{2n+i}) r_i + \sum_{i=1}^{2n} x_{4n+i} r_{4n+i} \quad (4.24)$$

$$= \sum_{i=1}^{2n} (x_i - x_{2n+i}) \alpha_i + \left[ \sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i1} \right] I_1 + \left[ \sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i2} \right] I_2 +$$

$$+ \sum_{i=1}^{2n} (x_i - x_{2n+i}) u_i + \sum_{i=1}^{2n} x_{4n+i} \alpha_{4n+i} + \sum_{i=1}^{2n} [x_{4n+i} \beta_{4n+i,1}] I_1 +$$

$$+ \sum_{i=1}^{2n} [x_{4n+i} \beta_{4n+i,2}] I_2 + \sum_{i=1}^{2n} x_{4n+i} u_{4n+i} ,$$

$$= \sum_{i=1}^{2n} (x_i - x_{2n+i}) \alpha_i + \sum_{i=1}^{2n} x_{4n+i} \alpha_{4n+i} + \left[ \sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i1} + \sum_{i=1}^{2n} x_{4n+i} \beta_{4n+i,1} \right] I_1 +$$

$$+ \sum_{i=1}^{2n} (\alpha_i - \alpha_{2n+i}) \beta_{i2} + x_{4n+i} \beta_{4n+i,2} \big] I_2 + \sum_{i=1}^{2n} (\alpha_i - \alpha_{2n+i}) u_i + \sum_{i=1}^{2n} x_{4n+i} u_{4n+i} ,$$

where  $\sum_{i=1}^{2n} (\alpha_i - \alpha_{2n+i}) \alpha_i$ ,  $\sum_{i=1}^{2n} x_{4n+i} \alpha_{4n+i}$ ,  $\sum_{i=1}^{2n} [(\alpha_i - \alpha_{2n+i}) \beta_{i1} + x_{4n+i} \beta_{4n+i,1}]$  and

$\sum_{i=1}^{2n} [(\alpha_i - \alpha_{2n+i}) \beta_{i2} + x_{4n+i} \beta_{4n+i,2}]$  are constants, and

$\sum_{i=1}^{2n} (\alpha_i - \alpha_{2n+i}) u_i$  and  $\sum_{i=1}^{2n} x_{4n+i} u_{4n+i}$  are random variables with

$$E\left(\sum_{i=1}^{2n} (\alpha_i - \alpha_{2n+i}) u_i\right) = 0, \quad E\left(\sum_{i=1}^{2n} x_{4n+i} u_{4n+i}\right) = 0 \quad \text{and}$$

$$\text{Cov}\left(\sum_{i=1}^{2n} (\alpha_i - \alpha_{2n+i}) u_i, I_j\right) = 0, \quad \text{Cov}\left(\sum_{i=1}^{2n} x_{4n+i} u_{4n+i}, I_j\right) = 0 \quad j=1, 2.$$

The variance of the return on the bond portfolio with interest rate swaps is

$$\begin{aligned} V(R) = & \sum_{i=1}^{2n} [(\alpha_i - \alpha_{2n+i}) \beta_{i1} + x_{4n+i} \beta_{4n+i,1}]^2 V(I_1) + \sum_{i=1}^{2n} [(\alpha_i - \alpha_{2n+i}) \beta_{i2} + x_{4n+i} \beta_{4n+i,2}]^2 V(I_2) + \\ & + 2 \sum_{i=1}^{2n} [(\alpha_i - \alpha_{2n+i}) \beta_{i1} + x_{4n+i} \beta_{4n+i,1}] \sum_{i=1}^{2n} [(\alpha_i - \alpha_{2n+i}) \beta_{i2} + x_{4n+i} \beta_{4n+i,2}] \text{Cov}(I_1, I_2) + \\ & + \sum_{i=1}^{2n} (\alpha_i - \alpha_{2n+i})^2 V(u_i) + \sum_{i=1}^{2n} x_{4n+i}^2 V(u_{4n+i}) \end{aligned} \quad (4.25)$$

based on the assumption that  $\text{cov}(u_i, u_j) = 0$ , for  $i \neq j$ .

By means of (4.25), the portfolio selection model of any bond with interest rate swaps can be written as follows:

$$\sum_{i=1}^{2n} (x_i - x_{2n+i}) \geq 0 \quad (4.26a)$$

$$\sum_{i=2n+1}^{4n} x_i \leq U \quad (4.26b)$$

$$\sum_{i=4n+1}^{5n} x_i - \sum_{i=3n+1}^{4n} x_i \leq 0 \quad (4.26c)$$

$$\sum_{i=5n+1}^{6n} x_i - \sum_{i=2n+1}^{3n} x_i \leq 0 \quad (4.26d)$$

$$x_i \geq 0 \quad i=1, 2, \dots, 6n \quad (4.26e)$$

$$x_i = c_i \quad \text{for some } i \quad (4.26f)$$

$$\sum_{i=1}^{2n} [(x_i - x_{2n+i}) \beta_{i1} + x_{4n+i} \beta_{4n+i,1}] - B_1 = 0 \quad (4.26g)$$

$$\sum_{i=1}^{2n} [(x_i - x_{2n+i}) \beta_{i2} + x_{4n+i} \beta_{4n+i,2}] - B_2 = 0 \quad (4.26h)$$

$$E = \sum_{i=1}^{2n} (x_i - x_{2n+i}) \mu_i + \sum_{i=1}^{2n} x_{4n+i} \mu_{4n+i} \quad (4.26i)$$

$$V = \sum_{i=1}^{2n} (x_i - x_{2n+i})^2 V(u_i) + \sum_{i=1}^{2n} x_{4n+i}^2 V(u_{4n+i}) + V(I_1) B_1^2 + V(I_2) B_2^2 +$$

$$+ 2B_1B_2\text{cov}(I_1, I_2) \quad (4.26j)$$

where  $B_1$  in (4.26g) is constrained to equal

$$\sum_{i=1}^{2n} [(x_i x_{2n+i}) \beta_{i1} + x_{4n+i} \beta_{4n+i,1}],$$

and  $B_2$  in (4.26h) is constrained to equal

$$\sum_{i=1}^{2n} [(x_i x_{2n+i}) \beta_{i2} + x_{4n+i} \beta_{4n+i,2}],$$

so  $B_1$  and  $B_2$  are indeed the  $\beta_1$  and  $\beta_2$  of the bond portfolio with interest rate swaps  $(x_1, x_2, \dots, x_{6n})$ .

There are two advantages to formulating the bond model as in (4.26a) through (4.26j) rather than computing the  $\sigma_{ij}$  explicitly. First, it saves the computations of large amount of covariances. Second, the covariance matrix in (4.26j) is a diagonal matrix with  $6n+2$  dimension. Thus, the computation of the efficient set can be much speeded, since most matrix entries are zero.

#### 4.4 Optimal Conditions Without Interest Rate Swaps

To find the mean-variance efficient set of bond portfolio for the management of assets and liabilities without interest rate swaps, we try to

$$\begin{aligned} \text{Minimize } V_N = & \sum_{i=1}^{2n} (x_i - x_{2n+i})^2 V(u_i) + b_1^2 V(I_1) + b_2^2 V(I_2) \quad (4.27a) \\ & + 2b_1 b_2 \text{cov}(I_1, I_2) \end{aligned}$$

subject to

$$\sum_{i=1}^{2n} (x_i - x_{2n+i}) \geq 0 \quad (4.27b)$$

$$\sum_{i=2n+1}^{4n} x_i \leq U \quad (4.27c)$$

$$x_i \geq 0 \quad i=1, 2, \dots, 4n \quad (4.27d)$$

$$\sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i1} - b_1 = 0 \quad (4.27e)$$

$$\sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i2} - b_2 = 0 \quad (4.27f)$$

$$E = \sum_{i=1}^{2n} (x_i - x_{2n+i}) \mu_i \quad (4.27g)$$

Then the portfolio selection problem becomes a linear programming problem. To find the optimal solution of (4.27), we construct the following Lagrangian function:

$$\begin{aligned} L = & \frac{1}{2} \left[ \sum_{i=1}^{2n} (x_i - x_{2n+i})^2 V(u_i) + V(I_1) b_1^2 + V(I_2) b_2^2 + 2b_1 b_2 \text{cov}(I_1, I_2) \right] + \\ & + \theta_1 \sum_{i=1}^{2n} (x_i - x_{2n+i}) + \theta_2 \sum_{i=2n+1}^{4n} x_i + \theta_3 \left[ \sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i1} - b_1 \right] \end{aligned} \quad (4.28)$$

$$+ \theta_4 \left[ \sum_{i=1}^{2n} (x_i - x_{2n+1}) \beta_{i2} - b_2 \right] - \theta_E \sum_{i=1}^{2n} (x_i - x_{2n+1}) \mu_i .$$

For given constants  $\theta_1, \theta_2, \theta_3, \theta_4$  and  $\theta_E$ , a necessary condition for  $L$  to be minimized is

$$\begin{aligned} \delta L / \delta x_i &= (x_i - x_{2n+1}) V(u_i) + \theta_1 + \theta_3 \beta_{i1} + \theta_4 \beta_{i2} - \theta_E \mu_i \\ &= 0 \end{aligned} \quad i=1, \dots, 2n \quad (4.29a)$$

$$\begin{aligned} \delta L / \delta x_{2n+1} &= -(x_i - x_{2n+1}) V(u_i) - \theta_1 + \theta_2 - \theta_3 \beta_{i1} - \theta_4 \beta_{i2} + \theta_E \mu_i \\ &= 0 \end{aligned} \quad i=1, \dots, 2n \quad (4.29b)$$

$$\delta L / \delta b_1 = b_1 V(I_1) + b_2 \text{cov}(I_1, I_2) - \theta_3 = 0 \quad (4.29c)$$

$$\delta L / \delta b_2 = b_2 V(I_2) + b_1 \text{cov}(I_1, I_2) - \theta_4 = 0 \quad (4.29d)$$

As Markowitz (1987) mentioned in his theorem: any  $X^a$  which satisfies (4.29) minimize  $L$  in (4.28). In other words, (4.29) is a sufficient condition, as well as necessary one for minimizing  $L$  for the given fixed  $\theta$ 's among all  $X$ .

Equation (4.29) can be written as the following matrix form:

$$CX + A'\theta - \theta_E \mu = 0 \quad (4.30a)$$

where

$$C_{(4n+2)} = \begin{bmatrix} V(u_1) & & & & & & & \\ & \ddots & & & & & & \\ & & V(u_{2n}) & & & & & \\ & & & V(u_1) & & & & \\ & & & & \ddots & & & \\ & & & & & V(u_{2n}) & & \\ & & & & & & V(I_1) & \text{cov}(I_1, I_2) \\ & & & & & & \text{cov}(I_1, I_2) & V(I_2) \end{bmatrix} \quad (4.30b)$$

$$X = \begin{bmatrix} x_1 - x_{2n+1} \\ x_2 - x_{2n+2} \\ \vdots \\ x_{2n} - x_{4n} \\ -(x_1 - x_{2n+1}) \\ -(x_2 - x_{2n+2}) \\ \vdots \\ -(x_{2n} - x_{4n}) \\ b_1 \\ b_2 \end{bmatrix} \quad \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_{2n} \\ -\mu_1 \\ -\mu_2 \\ \vdots \\ -\mu_{2n} \\ 0 \\ 0 \end{bmatrix} \quad (4.30c)$$

$$A = \begin{bmatrix} 1 & 1 & \dots & 1 & -1 & -1 & \dots & -1 & 0 & 0 \\ 0 & 0 & \dots & 0 & 1 & 1 & \dots & 1 & 0 & 0 \\ \beta_{11} & \beta_{21} & \dots & \beta_{2n,1} & -\beta_{11} & -\beta_{21} & \dots & -\beta_{2n,1} & -1 & 0 \\ \beta_{12} & \beta_{22} & \dots & \beta_{2n,2} & -\beta_{12} & -\beta_{22} & \dots & -\beta_{2n,2} & 0 & -1 \end{bmatrix} \quad \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} \quad (4.30d)$$

We know that any  $X$  which satisfies (4.30) minimizes  $L$  in (4.28). Using the critical line method developed by Markowitz (1987), we can find a piecewise, linear complete, nonredundant set of bond portfolios by means of equation

(4.30).<sup>6</sup>

#### 4.5 Optimal Conditions With Interest Rate Swaps

To find the mean-variance efficient set of bond portfolios for the management of assets and liabilities with interest rate swaps, we try to

$$\begin{aligned} \text{Minimize } V = & \sum_{i=1}^{2n} (x_i - x_{2n+1})^2 V(u_i) + \sum_{i=1}^{2n} x_{4n+1}^2 V(u_{4n+1}) + V(I_1) B_1^2 + \\ & + V(I_2) B_2^2 + 2B_1 B_2 \text{cov}(I_1, I_2) \end{aligned} \quad (4.31)$$

subject to

$$\sum_{i=1}^{2n} (x_i - x_{2n+1}) \geq 0 \quad (4.32a)$$

$$\sum_{i=2n+1}^{4n} x_i \leq U \quad (4.32b)$$

$$\sum_{i=4n+1}^{5n} x_i - \sum_{i=3n+1}^{4n} x_i \leq 0 \quad (4.32c)$$

$$\sum_{i=5n+1}^{6n} x_i - \sum_{i=2n+1}^{3n} x_i \leq 0 \quad (4.32d)$$

$$x_i \geq 0 \quad i=1, 2, \dots, 6n \quad (4.32e)$$

$$x_i = c_i \quad \text{for some } i \quad (4.32f)$$

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<sup>6</sup> The details can be found in Markowitz's recent book (1987).

$$\sum_{i=1}^{2n} [(x_i - x_{2n+i})\beta_{i1} + x_{4n+i}\beta_{4n+i,1}] - B_1 = 0 \quad (4.32g)$$

$$\sum_{i=1}^{2n} [(x_i - x_{2n+i})\beta_{i2} + x_{4n+i}\beta_{4n+i,2}] - B_2 = 0 \quad (4.32h)$$

$$E = \sum_{i=1}^{2n} (x_i - x_{2n+i})\mu_i + \sum_{i=1}^{2n} x_{4n+i}\mu_{4n+i} \quad (4.32i)$$

To find the optimal solution of (4.32), we construct the following Lagrangian function:

$$\begin{aligned} L = & 1/2 \left[ \sum_{i=1}^{2n} (x_i - x_{2n+i})^2 V(u_i) + \sum_{i=1}^{2n} x_{4n+i}^2 V(u_{4n+i}) + V(I_1)B_1^2 + V(I_2)B_2^2 + \right. \\ & \left. + 2B_1B_2\text{Cov}(I_1, I_2) \right] + \theta_1 \sum_{i=1}^{2n} (x_i - x_{2n+i}) + \theta_2 \sum_{i=2n+1}^{4n} x_i + \theta_3 \left[ \sum_{i=4n+1}^{5n} x_i - \sum_{i=3n+1}^{4n} x_i \right] + \\ & (4.33) \end{aligned}$$

$$\begin{aligned} & + \theta_4 \left[ \sum_{i=5n+1}^{6n} x_i - \sum_{i=2n+1}^{3n} x_i \right] + \theta_5 \left\{ \sum_{i=1}^{2n} [(x_i - x_{2n+i})\beta_{i1} + x_{4n+i}\beta_{4n+i,1}] - B_1 \right\} + \\ & + \theta_6 \left\{ \sum_{i=1}^{2n} [(x_i - x_{2n+i})\beta_{i2} + x_{4n+i}\beta_{4n+i,2}] - B_2 \right\} - \theta_E \left[ \sum_{i=1}^{2n} (x_i - x_{2n+i})\mu_i + \sum_{i=1}^{2n} x_{4n+i}\mu_{4n+i} \right] \end{aligned}$$

For given constants  $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6$  and  $\theta_E$ , the

necessary and sufficient condition for L to be minimized is

$$\begin{aligned}\delta L / \delta x_i &= (x_i - x_{2n+i}) V(u_i) + \theta_1 + \theta_5 \beta_{i1} + \theta_6 \beta_{i2} - \theta_E \mu_i \\ &= 0 \quad i=1, \dots, 2n\end{aligned} \quad (4.34a)$$

$$\begin{aligned}\delta L / \delta x_{2n+i} &= -(x_i - x_{2n+i}) V(u_i) - \theta_1 + \theta_2 - \theta_4 - \theta_5 \beta_{i1} - \theta_6 \beta_{i2} + \theta_E \mu_i \\ &= 0 \quad i=1, 2, \dots, n\end{aligned} \quad (4.34b)$$

$$\begin{aligned}\delta L / \delta x_{3n+i} &= -(x_{n+i} - x_{3n+i}) V(u_{n+i}) - \theta_1 + \theta_2 - \theta_3 - \theta_5 \beta_{n+i,1} - \theta_6 \beta_{n+i,2} + \theta_E \mu_{n+i} \\ &= 0 \quad i=1, 2, \dots, n\end{aligned} \quad (4.34c)$$

$$\begin{aligned}\delta L / \delta x_{4n+i} &= x_{4n+i} V(u_{4n+i}) + \theta_3 + \theta_5 \beta_{4n+i,1} + \theta_6 \beta_{4n+i,2} - \theta_E \mu_{4n+i} \\ &= 0 \quad i=1, 2, \dots, n\end{aligned} \quad (4.34d)$$

$$\begin{aligned}\delta L / \delta x_{5n+i} &= x_{5n+i} V(u_{5n+i}) + \theta_4 + \theta_5 \beta_{5n+i,1} + \theta_6 \beta_{5n+i,2} - \theta_E \mu_{5n+i} \\ &= 0 \quad i=1, 2, \dots, n\end{aligned} \quad (4.34e)$$

$$\delta L / \delta B_1 = B_1 V(I_1) + B_2 \text{cov}(I_1, I_2) - \theta_5 = 0 \quad (4.34f)$$

$$\delta L / \delta B_2 = B_2 V(I_2) + B_1 \text{cov}(I_1, I_2) - \theta_6 = 0 \quad (4.34g)$$

Similar to (4.29), equation (4.34) is a necessary and sufficient condition for minimize L in (4.33) for the given  $\theta$ 's among all X.

Equation (4.34) can be written as the matrix form:

$$C^*X^* + A^*{}^T\theta^* - \theta_E^*\mu^* = 0 \tag{4.35a}$$

where  $C^*_{(6n+2)} =$

$$\begin{bmatrix} V(u_1) & & & & & & & & & \\ & \ddots & & & & & & & & \\ & & \dot{V}(u_{2n}) & & & & & & & \\ & & & V(u_1) & & & & & & \\ & & & & \ddots & & & & & \\ & & & & & \dot{V}(u_{2n}) & & & & \\ & & & & & & V(u_{4n+1}) & & & \\ & & & & & & & \ddots & & \\ & & & & & & & & \dot{V}(u_{5n}) & \\ & & & & & & & & & V(u_{5n+1}) \\ & & & & & & & & & & \ddots \\ & & & & & & & & & & & \dot{V}(u_{6n}) \\ & & & & & & & & & & & & V(I_1) & \sigma_{11,12} \\ & & & & & & & & & & & & \sigma_{11,12} & V(I_2) \end{bmatrix} \tag{4.35b}$$

$$X = \begin{bmatrix} x_1 - x_{2n+1} \\ x_2 - x_{2n+2} \\ \vdots \\ \vdots \\ x_{2n} - x_{4n} \\ -(x_1 - x_{2n+1}) \\ -(x_2 - x_{2n+2}) \\ \vdots \\ \vdots \\ -(x_{2n} - x_{4n}) \\ x_{4n+1} \\ \vdots \\ \vdots \\ x_{5n} \\ x_{5n+1} \\ \vdots \\ \vdots \\ x_{6n} \\ B_1 \\ B_2 \end{bmatrix} \qquad \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \vdots \\ \mu_{2n} \\ -\mu_1 \\ -\mu_2 \\ \vdots \\ \vdots \\ -\mu_{2n} \\ \mu_{4n+1} \\ \vdots \\ \vdots \\ \mu_{5n} \\ \mu_{5n+1} \\ \vdots \\ \vdots \\ \mu_{6n} \\ 0 \\ 0 \end{bmatrix} \tag{4.35c}$$

$$A^* = \begin{bmatrix} 1 & \dots & 1 & -1 & \dots & -1 & -1 & \dots & -1 & 0 & \dots & 0 \\ 0 & \dots & 0 & 1 & \dots & 1 & 1 & \dots & 1 & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & \dots & 0 & -1 & \dots & -1 & 1 & \dots & 1 \\ 0 & \dots & 0 & -1 & \dots & -1 & 0 & \dots & 0 & 0 & \dots & 0 \\ \beta_{11} & \dots & \beta_{2n,1} & -\beta_{11} & \dots & -\beta_{n1} & -\beta_{n+1,1} & \dots & -\beta_{2n,1} & \beta_{4n+1,1} & \dots & \beta_{5n,1} \\ \beta_{12} & \dots & \beta_{2n,2} & -\beta_{12} & \dots & -\beta_{n2} & -\beta_{n+1,2} & \dots & -\beta_{2n,2} & \beta_{4n+1,2} & \dots & \beta_{5n,2} \\ 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ 1 & \dots & 1 & 0 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \beta_{5n+1,1} & \dots & \beta_{6n} & -1 & \dots & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \beta_{5n+1,2} & \dots & \beta_{6n,2} & 0 & \dots & -1 & 0 & \dots & 0 & 0 & \dots & 0 \end{bmatrix} \quad (4.35d)$$

$$\theta^* = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix} \quad (4.35e)$$

Using the critical line method mentioned above, we can find an efficient set of bond portfolios with interest rate swaps by means of equation (4.35).

#### 4.6 Negative Portfolio Betas .

If the regression coefficients  $\beta_{i1}$  and  $\beta_{i2}$  of the two factor model are negative, i.e., the portfolio  $\beta$ s without swaps  $b_1$  in (4.27e) and  $b_2$  in (4.27f) have negative values, four positive variables need to be introduced such that<sup>7</sup>

$$b_1 = x_{4n+1} - x_{4n+3} \quad (4.36a)$$

$$b_2 = x_{4n+2} - x_{4n+4} \quad (4.36b)$$

$$x_{4n+i} \geq 0 \quad i=1,2,3,4. \quad (4.36c)$$

Then the constraints for portfolio  $\beta$ s (4.27e) and (4.27f) become

$$\sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i1} - x_{4n+1} + x_{4n+3} = 0 \quad (4.37a)$$

$$\sum_{i=1}^{2n} (x_i - x_{2n+i}) \beta_{i2} - x_{4n+2} + x_{4n+4} = 0 \quad (4.37b)$$

The variance-covariance between two factors  $I_1$  and  $I_2$  in (4.27a)

$$V(I_1)b_1^2 + V(I_2)b_2^2 + 2b_1b_2\text{cov}(I_1, I_2) \quad (4.38)$$

becomes

$$\begin{aligned} & (x_{4n+1} - x_{4n+3})^2 V(I_1) + (x_{4n+2} - x_{4n+4})^2 V(I_2) + \\ & + 2(x_{4n+1} - x_{4n+3})(x_{4n+2} - x_{4n+4}) \text{cov}(I_1, I_2) \\ & = x_{4n+1}^2 V(I_1) + x_{4n+2}^2 V(I_2) + x_{4n+3}^2 V(I_1) + x_{4n+4}^2 V(I_2) - \end{aligned} \quad (4.39)$$

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<sup>7</sup> The methodology for this transformation can be found at page 26 in Markowitz's book (1987).

$$\begin{aligned}
& - 2x_{4n+1}x_{4n+3}V(I_1) - 2x_{4n+2}x_{4n+4}V(I_2) + 2x_{4n+1}x_{4n+2}\text{COV}(I_1, I_2) - \\
& - 2x_{4n+1}x_{4n+4}\text{COV}(I_1, I_2) - 2x_{4n+2}x_{4n+3}\text{COV}(I_1, I_2) + \\
& + 2x_{4n+3}x_{4n+4}\text{COV}(I_1, I_2)
\end{aligned}$$

The original variance-covariance matrix between  $I_1$  and  $I_2$  in (4.30b)

$$\begin{bmatrix} V(I_1) & \text{cov}(I_1, I_2) \\ \text{cov}(I_1, I_2) & V(I_2) \end{bmatrix} \quad (4.40)$$

becomes

$$\begin{bmatrix} V(I_1) & \text{cov}(I_1, I_2) & -V(I_1) & -\text{cov}(I_1, I_2) \\ \text{cov}(I_1, I_2) & V(I_2) & -\text{cov}(I_1, I_2) & -V(I_2) \\ -V(I_1) & -\text{cov}(I_1, I_2) & V(I_1) & \text{cov}(I_1, I_2) \\ -\text{cov}(I_1, I_2) & -V(I_2) & \text{cov}(I_1, I_2) & V(I_2) \end{bmatrix} \quad (4.41)$$

The last two elements of  $X$  vector and  $\mu$  vector in (4.30c) become

$$\begin{array}{ccc}
x_{4n+1} & 0 & \\
x_{4n+2} & 0 & \\
x_{4n+3} & 0 & \\
x_{4n+4} & 0 & ,
\end{array} \quad (4.42)$$

and the last two columns of  $A$  matrix in (3.30d) become

$$\begin{array}{cccc}
-1 & 0 & 1 & 0 \\
0 & -1 & 0 & 1
\end{array} \quad (4.43)$$

Similar procedures should be used if the portfolio  $\beta$ s with interest rate swaps  $B_1$  in (4.32g) and  $B_2$  in (4.32h) have negative values.

#### 4.7 Why Interest Rate Swaps

Now, we try to explain how interest rate swaps can be used to reallocate the nonmarketable assets and liabilities in the bond portfolio selection model. Remember that the optimal allocations of assets and liabilities can be achieved by solving

$$CX + A'\theta - \theta_e\mu = 0 \quad (4.30a)$$

for the bond portfolio selection without interest rate swaps. After introducing interest rate swaps into the bond portfolio selection model, the optimal allocations of assets and liabilities can be obtained by solving

$$C^*X^* + A^*\theta^* - \theta_e^*\mu^* = 0 \quad (4.35a)$$

Without interest rate swaps, nonmarketable assets and liabilities can not be reallocated in the bond portfolio selection process. This can be seen by looking at the elements in  $C$  and  $\mu$ , the variance and covariance matrix of residual returns and the vector of expected returns, respectively. As we can see from (4.30b),  $C$  is a diagonal matrix with diagonal elements equal to  $V(u_i)$ , the variance of regression residual of  $i^{\text{th}}$  bond portfolio return;  $\mu$  in (4.30c) is a vector with each element equal to  $\mu_i$ , the expected return of  $i^{\text{th}}$  bond portfolio. In obtaining the optimal allocations of assets and liabilities in the bond portfolios, all the elements in the variance-covariance matrix  $C$  and the expected return vector

$\mu$  must be used to determine the optimal allocations in the bond portfolios. This process implicitly assumes that all assets and liabilities in the bond portfolios are free to trade, since they can be adjusted to the required level without restriction. If there are nonmarketable assets or liabilities, they are treated as tradable in the portfolio selection process.

By introducing interest rate swaps into the mean-variance portfolio selection model, we can see that things can be quite different. As we mentioned at the beginning of this chapter, by introducing interest rate swaps into the portfolio selection model, the proportionate holding of assets and liabilities can always be changed whether these assets and liabilities are tradable or not. To see this, compare  $C^*$  and  $\mu^*$  with  $C$  and  $\mu$  respectively. As seen from (4.35b), besides the same elements included in the matrix  $C$ , matrix  $C^*$  has  $2n$  more lines that are the variances of regression residual of swap returns,  $V(u_{4n+i})$   $i=1, \dots, 2n$ . As to  $\mu^*$ , it also includes  $2n$  more elements,  $\mu_{4n+i}$   $i=1, \dots, 2n$ , that are the expected returns of swaps.

The  $2n$  more lines in matrix  $C^*$  and  $2n$  more elements in vector  $\mu^*$  can be used to explain why nonmarketable assets and liabilities can in effect become tradable in the bond portfolio selection process. Notice that to obtain the optimal allocations of assets and liabilities in the bond portfolios with interest rate swaps, the matrix  $C^*$  and the vector  $\mu^*$  are

used to determine the optimal allocation process. That also implicitly assumes that all assets and liabilities in the bond portfolios are tradable, since all  $V(u_i)$  in matrix  $C^*$  and  $\mu_i$  in vector  $\mu^*$  are used to adjust the portfolio to the optimal positions. The contribution of each asset or liability to the adjustment process, however, may be not caused by that asset or liability itself. That is the key point that why nonmarketable assets or liabilities can be treated as if they were tradable.

From (4.30b), we can see that each  $V(u_i)$  in matrix  $C$  and each  $\mu_i$  in vector  $\mu$  has to be used in the portfolio selection process, i.e., the  $i^{\text{th}}$  asset or liability has to be used (treated as being tradable) itself in the determinations of optimal portfolios.

By introducing interest rate swaps (both long swaps and short swaps) into the bond portfolio selection model, the structure of matrix  $C^*$  and vector  $\mu^*$  are not the same as that of  $C$  and  $\mu$ . As we can see from (4.35b), besides the same  $V(u_i)$  and  $\mu_i$  in matrix  $C$  and vector  $\mu$ , there are  $2n$  more  $V(u_{4n+i})$  in matrix  $C^*$  and  $2n$  more  $\mu_{4n+i}$  in vector  $\mu^*$ . Thus, the nonmarketable asset or liability itself does not need to be used in the bond portfolio selection process, since the values of these assets or liabilities can be altered through the arrangement of a series of interest rate swaps ( $V(u_{4n+i})$  and  $\mu_{4n+i}$ ). This argument can be proved much easily when the empirical results are examined in the next chapter.

## EMPIRICAL RESULTS OF BOND SELECTIONS

In this chapter, we are going to use the general portfolio selection model developed in chapter 4 to select bond portfolios. Since there are no similar empirical results available in the literature, no comparison can be made of our results to others.

### 5.1 Data

The data used for the application of model developed in the previous chapter are from the CRSP U.S. Government Bond Tape. Four monthly data are available for each bond. They are:

1. the calendar month;
2. the closing price at the end of the month;
3. the time left to maturity of each bond;
4. the coupon rate of each bond.

These four values are available from January 1954 to December 1987 on the CRSP Government Bond Tape. Table 1 shows part of the data from this tape.

The four values appearing on the tape are in the following fashion. January 1954 corresponds to serial number 338. Assume that the face value of each bond is 100, the bond prices for each month are based on face value. For the treasury-bills, the prices listed on the tape are always less

than 100, since they are sold at a discount. The maturity of bonds vary from close to zero to thirty years. For the bonds with maturity longer than one year, only coupon bonds are available on the tape. Not surprisingly, the coupons on long-term bonds are comparatively low in the early period like 1950's and 1960's. They increase rapidly in the later years, especially in the late 1970's and early 1980's.

In our model, we are going to use the data from January 1985 to December 1987. The total number of observations for each of these four values during the three year period is 7617. There are large number of observations for bonds with short maturities and relatively few observations for bonds with long maturities. In particular, bonds with certain maturities are missing from the data set, therefore, a bond classification scheme is necessary. For instance, there are only 27 observations for bonds maturing within 10 to 11 years, 20 observations for bonds maturing within 11 to 12 years and 35 observations for bonds with maturity from 20 to 21 years during this three year period. In other words, there is less than one observation each month, on average, for these bonds. To guarantee that bonds are available each month for portfolio selection, we divide these bonds into 19 categories. Table 2 shows the bond portfolio categories, its corresponding maturity, and number of observations in each category.

## **5.2 Ex post Monthly Holding Period Returns**

The formula used to calculate the ex post monthly holding period return of each bond is:

$$r_{ti} = [(P_{ti} - P_{t-1,i}) + C_{ti} \times (M_{ti} - M_{t-1,i})] \times 100 / (P_{t-1,i} + A_{t-1,i}) \quad (5.1)$$

$$i=1,2,\dots,7617, \quad t=01/85,\dots,12/87,$$

where  $r_{ti}$  is the percentage monthly holding period return of bond  $i$  at time  $t$ ,  $P_{ti}$  is the closing price at the end of month for bond  $i$  at time  $t$ ,  $C_{ti}$  is the coupon rate of bond  $i$  which is fixed during the life of each bond,  $M_{ti}$  is the time left to maturity for bond  $i$  at time  $t$ , and  $A_{t-1,i}$  is the accrued interest for bond  $i$  at time  $t-1$ .

It is important to note that the "monthly" maturity difference for bond  $i$  between time  $t$  and  $t-1$ ,  $M_{ti} - M_{t-1,i}$ , is not equal to  $1/12$  or  $0.083333$  year. Simply using  $0.083333$  for  $M_{ti} - M_{t-1,i}$  in (5.1) will be wrong. In fact, six values of  $M_{ti} - M_{t-1,i}$  are found in the three year period from January 1985 to December 1987. They are  $0.076660$ ,  $0.079398$ ,  $0.082136$ ,  $0.084873$ ,  $0.087611$  and  $0.090349$ . These values are due to the different number of calendar days between the last trading days in two consecutive months.

The monthly holding period return of each bond  $r_{ti}$  calculated in (5.1) can be used to obtain  $R_{ti}$ , the ex post monthly average holding period return of each bond category defined in Table 2. Table 3 shows the values of  $R_{ti}$ ,  $t=01/85,\dots,12/87$ ,  $i=1,2,\dots,19$ .

It should be noted that 7355 instead of original 7617

individual returns  $r_{it}$  are used to obtain 684 (19X36)  $R_{it}$ , the monthly holding period returns by category. As summed in Table 2, 7355 individual returns do not include  $r_{it}$  for new bonds. In obtaining category returns  $R_{it}$ , the undefined return of newly issued bonds must be excluded. Otherwise, the calculation of monthly holding period return of each bond category will be inaccurate.

Table 4 shows the summary statistics for each category return  $R_{it}$ . As seen from Table 4, the monthly holding period returns are greater than the monthly average coupon rates for bonds with maturity longer than 3 years (category 6) during the sample period. This phenomenon becomes more evident for bonds with maturity longer than 6 years (category 9). This is because the average yield of long-term bonds declined from 11.51% in December 1984 to 8.98% in December 1987. In the long run, the monthly holding period return should be close to the monthly average coupon rate for each bond category.

Except for the short-term bonds (category 1 and 2), monthly holding period returns of bonds can be negative. For long-term bonds (10 years and over), the highest monthly holding period return is more than 10 percent. These returns occurred in February 1986 when long-term yields dropped more than one percent during the month because of the collapse of oil price. Though coupon interest accrues each month, the minimum monthly holding period returns for long-term bonds (10 years and over) are negative 5 percent or less. Most of these

large negative returns occur in February 1985, May 1986 and April 1987 when long-term yields climbed sharply.

As the time left to maturity of bonds increases, the variance of bonds increase proportionally. However, the proportional increase of variance with maturity is not significant for bonds with time left to maturity longer than 20 years.

### 5.3 The Enlarged Variance-Covariance Matrix Without Swaps

In order to use two factor model (4.16) to obtain the variance-covariance matrix C (4.30b) and constraint matrix A (4.30d) and the enlarged variance-covariance matrix<sup>8</sup>, we need to calculate the percentage change of short-term interest rate  $I_1$  and the percentage change of long-term interest rate  $I_2$  at first. The formulas used to obtain  $I_1$  and  $I_2$  in (4.16) are given by

$$I_{t1} = [(1+i_{t1}) - (1+i_{t-1,1})] / (1+i_{t-1,1}) \quad t=01/85, \dots, 12/87 \quad (5.2a)$$

$$I_{t2} = [(1+i_{t2}) - (1+i_{t-1,2})] / (1+i_{t-1,2}) \quad t=01/85, \dots, 12/87 \quad (5.2b)$$

where  $i_{t1}$  is the short-term interest rate and  $i_{t2}$  is the long-term interest rate at time  $t$ , respectively. In our calculation, the Wall Street Journal 91-day T-bill rate at the end of the month is used as the short-term interest rate  $i_{t1}$ ,

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<sup>8</sup> The example of enlarged variance-covariance matrix without interest rate swaps can be found in Chapter 13 of Markowitz (1987).

and the yield average of 10 bonds with the longest maturities at the end of the month is used as the long-term interest rate  $i_{t2}$ . Table 5 shows the time series of  $I_{t1}$  and  $I_{t2}$  from January 1985 to December 1987.

By means of the time series  $R_{ti}$  in Table 3 and  $I_{t1}$  and  $I_{t2}$  in Table 5, we can use two factor model (4.16) to estimate the coefficients  $\beta_{i1}$  and  $\beta_{i2}$  and the residual variances  $V(u_i)$ . As shown in (4.30b) and (4.30d), these  $V(u_i)$  and  $\beta$  values are the critical components in matrix C and matrix A, respectively. Table 6 shows the regression results of the two factor model (4.16).

Two forms of two factor model (4.16) are used to obtain  $V(u_i)$  and  $\beta$  values. Table 6a shows the results obtained by directly regressing  $R_{ti}$  on  $I_1$  and  $I_2$ . Table 6b shows the results obtained by regressing  $R_{ti}$  on  $(I_2 - I_1)$  and  $I_2$ . The difference between the changes in long-term and short-term interest rate,  $I_2 - I_1$ , reflects the change in the shape of the yield curve. It is positive when the yield curve is upward sloping, and negative when the yield curve is downward sloping. The yield curve is flat when  $I_2 - I_1 = 0$ .

As seen from Table 6a, the t-values of  $\beta_{i2}$  (coefficients of percentage change of long-term interest rate) are statistically significant at 5% level for all bond categories except category one (very short maturities). The t-values of  $\beta_{i1}$  (coefficients of percentage change of short-term interest rate), however, are not statistically significant for some

long-term bond categories. From Table 6b, we find that all the t-values of  $\beta_{12}$  are improved such that the coefficient for bond category one also becomes statistical significant.

Not surprisingly, two regression models generate the same  $\alpha_i$  coefficients, the same  $R_i^2$ , the same  $V(u_i)$ , the same absolute values of  $\beta_{11}$  and t-values of  $\beta_{11}$  for the given category, since the change of the form does not provide additional explanatory power for  $I_1$  and  $I_2$ . However, the significance of  $\beta_{12}$ s do improve in form (b) as the corresponding t-values of  $\beta_{12}$  are higher in Table 6b than that in Table 6a. In view of this,  $V(u_i)$  and  $\beta$  values in Table 6b will be used in our estimation of the enlarged variance-covariance matrix.

By means of Table 5, it is not difficult to obtain the following variance-covariance matrix for  $I_1$ ,  $I_2$  and  $I_2-I_1$ .

|           | $I_1$   | $I_2$  | $I_2-I_1$ |       |
|-----------|---------|--------|-----------|-------|
| $I_1$     | 0.1561  | 0.0922 | -0.0640   |       |
| $I_2$     | 0.0922  | 0.1565 | 0.0643    |       |
| $I_2-I_1$ | -0.0640 | 0.0643 | 0.1283    | (5.3) |

Combining the values (5.3) together with the  $\beta$  values and  $V(u_i)$  obtained from Table 6b, we get the enlarged variance-covariance matrix without interest rate swaps. Table 7 shows the structure of that matrix.

As seen from Table 7, the first 23X23 submatrix is the C matrix mentioned in (4.30b), the following 1X23 vector is the  $\mu$  vector of bonds in (4.30c), the next three constraints

are budget constraint, constraints for the first portfolio beta and the second portfolio beta in (4.30d), respectively.

#### 5.4 The Efficient Frontiers Without Interest Rate Swaps

By means of the enlarged variance-covariance matrix estimated in the previous section, we can use critical line algorithm<sup>9</sup> developed by Markowitz to find the optimal bond portfolios. Table 8 shows the solution of these optimal portfolios.

As we can see from Table 8. The first corner portfolio<sup>10</sup> has the highest mean of 1.4948 and the highest standard deviation of 4.1393. It contains 100 percent of bond category 17<sup>11</sup> defined in Table 2. The mean and standard deviation of the last corner portfolio goes all the way down to 0.5825 and 0.1027, respectively. This phenomenon is perfectly consistent with the mean-variance criterion for portfolio selection. It simply reflects the risk-return tradeoff for different bond combinations. Figure 3 plots the mean-standard deviation combinations for all corner portfolios in Table 8, i.e., the efficient frontier for the bonds without interest rate swaps.

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<sup>9</sup> The program of the algorithm written in SIMSCRIPT can be found in Chapter 13 of Markowitz (1987).

<sup>10</sup> A corner portfolio is an efficient portfolio. Any combination of two adjacent corner portfolios will generate an efficient portfolio that lies between the two corner portfolios.

<sup>11</sup> During the sample period from January 1985 to December 1987, bond category 17 has the highest monthly mean return of 1.4948 percent among 19 categories.

To show that nonmarketable assets and liabilities in the bond portfolios can make optimal portfolios less efficient, i.e., shift the efficient frontier down to the right, we force a certain investment in four categories of bonds defined in Table 2. Table 9, 11, 13, 15 and 17 show the enlarged variance-covariance matrix with 20, 40, 60, 80 and 100 percent of nontraded bonds, respectively. And Table 10, 12, 14, 16 and 18 show the corner portfolios with 20, 40, 60, 80 and 100 percent of nontraded bonds, respectively.

Comparing the enlarged variance-covariance matrix in Table 7, we add four 5 percent nontraded bonds in Table 9, namely, we add four constraints  $x_6=0.05$ ,  $x_8=0.05$ ,  $x_{10}=0.05$ , and  $x_{12}=0.05$  in Table 9. In other words, the proportions of these four bonds are not allowed to change in the bond portfolios when the efficient portfolios are determined. The reason to select bond categories 6, 8, 10 and 12 as fixed is that the corresponding swap data are available. Using critical line algorithm, we can obtain the optimal portfolios of bonds with 20 percent nontraded assets. Table 10 shows the results of these optimal portfolios.

Comparing the corner portfolios in Table 10 with those in Table 8, we find that the optimal portfolios with nontraded assets have lower means and larger variances. Due to the existence of nontraded assets in the bond portfolios, not only the mean returns of the optimal portfolios will be reduced, but also the risk of the portfolios will be increased. The

shift down of the efficient frontier can be seen through the comparison of corner portfolios in Table 8 and Table 10. For example, the 11<sup>th</sup> corner portfolio in Table 10 has the mean of 0.8154 and standard deviation of 0.7722. It is dominated in the sense of mean-variance by the 12<sup>th</sup> corner portfolio in Table 8, which has both the higher mean of 0.8271 and the lower standard deviation of 0.7421. Figure 4 plots the efficient frontier of portfolios with 20 percent nontraded assets.

As the percentage of nontraded assets increase in the bond portfolios, the corner portfolios obtained by the critical line algorithm become less efficient, i.e., the efficient frontiers shift down further to the right. This can be shown through the comparison among Table 10, 12, 14, 16 and 18. For instance, the fifth corner portfolio in Table 10 (20 percent nontraded assets) dominates the second corner portfolio in Table 12 (40 percent nontraded assets); The fifth corner portfolio in Table 12 dominates the first corner portfolio in Table 14 (60 percent nontraded assets) etc.. If we have 100 percent nontraded assets in the bond portfolios, all the corner portfolios will have the same mean and standard deviation, i.e., the efficient frontier contains only one point which is dominated by the seventh corner portfolio in Table 8 (without nontraded assets), and fifth corner portfolio in Table 16 (80 percent nontraded assets). These results are perfectly consistent with the basic idea of this paper. That

is, with the existence of nonmarketable assets in the bond portfolio, not only the investment opportunity set will differ, but also the optimal allocation of assets will differ from the set available if the assets are tradable. Figure 5, 6, 7 and 8 plot the efficient frontiers with 40, 60, 80 and 100 percent of nontraded assets.

### **5.5 Portfolio Selection With Interest Rate Swaps**

To find out the optimal bond portfolios with interest rate swaps, we first have to obtain the variance-covariance matrix  $C^*$  and the constraint matrix  $A^*$  in (4.35). This can be done by the procedures described in the previous section. The only difference is that we need to consider the variances-covariances of interest rate swaps in the portfolio selection.

#### **5.5a Data**

The data used to obtain monthly swap returns are from the "International Financing Review" published in London. Weekly data are available from January 1985 to December 1987, which is consistent with the data selected from the CRSP bond tape. Four categories of swap data are used in our portfolio selections: they are 3-year, 5-year, 7-year and 10-year interest rate swaps. Table 19 shows the swap quotations from IFR on July 27, 1985 and August 24, 1985 issues, respectively.

#### **5.5b Ex post Returns of Interest Rate Swaps**

Since interest rate swaps can be decomposed into a portfolio of fixed rate bonds and floating rate notes, we first need to calculate returns of floating rate notes in order to obtain swap returns. The formula used to calculate the ex post monthly holding period return for a  $i$ -year swap adjustable note<sup>12</sup> is given by

$$R_{t,i}^* = (I_{t-1,i} - I_{t,i}) / I_{t,i} * [1 - 1 / (1 + I_{t,i} / 2)^{2i}] * 100 + 100 * I_{t-1,i} / 12 \quad (5.4)$$

$i=3, 5, 7, 10, t=01/85, \dots, 12/87$

where  $R_{t,i}^*$  is the monthly percentage holding period return for a long position of  $i$ -year swap adjustable note,  $I_{t,i}$  is the fixed rate of interest a  $i$ -year long swapper has to pay at time  $t$  and  $I_{t-1,i}$  is the fixed rate of interest a  $i$ -year long swapper can receive at time  $t-1$ . In fact, the return on a  $i$ -year swap adjustable note  $R_{t,i}^*$  is the present value of interest changes for a  $i$ -year floating rate note (based on LIBOR) between time  $t$  and  $t-1$  plus monthly accrued floating interest.

In our calculation of monthly percentage returns on swap adjustable notes (5.4), we implicitly assume that

1. there is a one month roll-over for each swap contract;
2. interest payments are settled every six months;
3. there is no transaction cost for swap roll-overs.

Table 20 shows the returns on 3-, 5-, 7-, and 10-year swap adjustable notes calculated by (5.4) and the means and

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<sup>12</sup> A  $i$ -year swap adjustable note is a floating rate note generated by a  $i$ -year interest rate swap.

standard deviations of the returns during the sample period from January 1985 to December 1987. As we can see from Table 20, the means and standard deviations of returns on swap adjustable notes increase as maturity increases. This reflects the risk-return tradeoff for swap adjustable notes. And most large positive and negative returns on swap adjustable notes occurred at the same months as the corresponding U.S. government bonds. This gives further evidence that swaps can be used to reallocate the bonds that are not tradable in the market.

By means of (5.4), it is not difficult to obtain the ex post monthly percentage return for an interest rate swap. According to the definition of interest rate swaps mentioned at the beginning of the section, we obtain

$$S_{ti} = R_{ti} - R_{ti}^* \quad (5.5)$$

$$i=3,5,7,10, \quad t=01/85, \dots, 12/87.$$

where  $S_{ti}$  is the monthly percentage return on a  $i$ -year swap at time  $t$ ,  $R_{ti}$  is the monthly percentage holding period return on a  $i$ -year bond calculated in Table 3, and  $R_{ti}^*$  is calculated by (5.4). Table 21 gives monthly percentage holding period returns on 3-, 5-, 7-, and 10-year interest rate swaps and the means and standard deviations of the swap returns during the sample period.

As seen from Table 21, the mean returns of all swaps are very close to zero during the sample period, since interest

rate swaps are zero-investment instruments. The standard deviations of swap returns, however, are not close to zero, since variation of returns for fixed rate bonds and floating rate notes are not the same when market interest rate changes.

### 5.5c The Enlarged Variance-Covariance Matrix With Swaps

By means of the swap returns  $S_{it}$  in Table 21 and  $I_1$  and  $I_2$  in Table 5, we can use the same two factor model (4.16) to estimate the coefficients  $\beta_{i1}$ ,  $\beta_{i2}$  and the residual variances  $V(u_i)$  for the interest rate swaps. Table 22 shows the two factor regression results for all swaps .

As seen from Table 22, the  $R^2$  of these four regressions are very small. In particular, the t-values of  $\beta_{i1}$  and  $\beta_{i2}$  for all four regressions are not statistically significant at 5 percent level. These results can be easily explained by equation (5.5), the calculation of swap returns. Since swap returns are return differences between a bond and a note and both returns tend to move in the same direction as interest rate changes, therefore, average swap returns should be insensitive to interest rate changes.

Combining the regression results in Table 22 and Table 6b, we can obtain the  $C^*$  matrix and  $A^*$  matrix in (4.35) and further the enlarged variance-covariance matrix. Table 23 shows the enlarged variance-covariance matrix with interest rate swaps.

Besides four more swap variables, there is one more

constraint in the matrix of Table 23 than that in the matrix of Table 7. The additional constraint is for interest rate swaps. It means that total notional principal of interest rate swaps can not exceed the underlying real principal used in bond selections.

Using the critical line algorithm again, we obtain the optimal portfolios of bonds with interest rate swaps. Table 24 shows the results of corner portfolios with interest rate swaps.

To show that interest rate swaps can be used to reallocate nonmarketable assets, we add the same swap variables and swap constraint mentioned above to the enlarged variance-covariance matrices with nontraded assets. Table 25, 27, 29, 31 and 33 show the enlarged variance-covariance matrices with interest rate swaps and 20, 40, 60, 80 and 100 percent of nontraded assets, respectively. And Table 26, 28, 30, 32 and 34 show the corner portfolios with interest rate swaps and 20, 40, 60, 80 and 100 percent of nontraded assets, respectively.

## **5.6 The Efficient Frontiers With Interest Rate Swaps**

Comparing the optimal portfolios in Table 24 (with interest rate swaps) with that in Table 8 (without interest rate swaps), we find that the optimal portfolios with interest rate swaps are more efficient than those without interest rate swaps, i.e., the efficient frontier with interest rate swaps

will shift up compared to the original one without swaps. As we can see from Table 24, an interest rate swap can be entered on a maturity that is different from the maturity of the bonds that make up real principals. When swaps are entered on bond portfolios with longer maturities and higher risk exposure, the gaps between efficient frontiers without swaps and with swaps become wider, i.e., the efficient frontier with swaps will shift up significantly. When swaps are entered on bond portfolios with shorter maturities and lower risk exposure, the gaps between two frontiers will be narrowed, i.e., the shifting up of the efficient frontier with swaps becomes less significant. For example, the fourth corner portfolio in Table 24 consists of bond portfolios on category 14 and 17 (long maturities), it has a mean of 1.5334 and standard deviation of 4.0314, which significantly dominates, in the sense of mean-variance, the corresponding one (first corner portfolio) in Table 8 with a mean of 1.4948 and standard deviation of 4.1393. And the 13<sup>th</sup> corner portfolio in Table 24 consists of bond portfolios on category 6 and 8 (relatively short maturities), it has a mean of 1.1016 and standard deviation of 1.8044, which does not significantly dominates, in the sense of mean-variance, the corresponding one (eighth corner portfolio) in Table 8 with a mean of 1.0692 and standard deviation of 1.7091. Figure 9 gives the comparison between the efficient frontiers without and with interest rate swaps.

Though two frontiers coincide when the corresponding

corner portfolios have very low risk-return combinations, the efficient frontier with swaps extends farther to the right when the corresponding corner portfolios have high  $E-\sigma$  combinations, since swaps provide additional mean-standard deviation combinations for these corner portfolios.

For optimal portfolios with swaps and nontraded assets, similar results are obtained. As seen from Table 26 (swaps with 20 percent nontraded assets) and Table 10 (20 percent nontraded assets without swaps), the efficient frontier with swaps shifts up significantly when swaps are entered on bond portfolios with longer maturities and higher risk exposure. It does not shift up significantly when swaps are entered on bond portfolios with shorter maturities and lower risk exposure. For example, the fourth corner portfolio in Table 26 consists of bond portfolios with categories 14 and 17 (long maturities) besides 5 percent nontraded bond each on categories 6, 8, 10 and 12. It has a mean of 1.4611 and the standard deviation of 3.6674, which dominates the first corner portfolio in Table 10 with a mean of 1.4175 and standard deviation of 3.7123. And the 10<sup>th</sup> corner portfolio in Table 26 consists of two relatively short term bonds on category 5 and 7 besides four 5 percent nontraded bonds. It has a mean of 0.9821 and standard deviation of 1.3846, which does not dominate the corresponding one (sixth corner portfolio) in Table 10 with a mean of 0.9881 and standard deviation of 1.4497. Figure 10 shows the efficient frontiers with and

without interest rate swaps and 20 percent nontraded assets.

Similar results are obtained when swaps are entered on bond portfolios with 40, 60, 80 and 100 percent of nontraded assets. Figures 11, 12, 13 and 14 plot the efficient frontiers with and without interest rate swaps and 40, 60, 80 and 100 percent of nontraded assets, respectively.

As can be seen from Figures 10, 11, 12, 13 and 14, the efficient frontiers with interest rate swaps will always shift up whether there are nontraded assets in the bond portfolios or not. Moreover, at a given risk ( $\sigma$ ), the gaps between two frontiers increase as the percentage of nontraded assets increases. The reallocation of nonmarketable assets with interest rate swaps can be demonstrated easily by means of Table 18 and Figure 14.

As seen from Table 18 and Figure 14, without interest rate swaps, the efficient frontier with 100 percent nontraded assets contains only one point with  $E=1.1085$  and  $\sigma=2.0815$ . By introducing interest rate swaps into the portfolio selections, the single point efficient frontier becomes a curve with percentage holdings of the nontraded bonds unchanged. In fact, it is the interest rate swaps that play the role of reallocating the nontraded bonds in the portfolio selections. As we mentioned in Chapter 2, interest rate swaps are designed to exchange a series of interest payments without exchanging the underlying principals. Thus, by introducing interest rate swaps into the mean-variance framework, the nonmarketable

property of some assets can always be changed. It is as if the assets are bought and sold in the markets and the optimal allocation of bond portfolios can always be achieved.

## BOND DURATIONS AND EX ANTE BOND RETURNS

## 6.1 The Single Factor Model

In chapter 4, we have used a two factor model

$$r_i = \alpha_i + \beta_{i1}I_1 + \beta_{i2}I_2 + u_i , \quad (4.16)$$

to obtain the variance-covariance matrix C. The single factor model

$$r_i = \alpha_i + \beta_i I + u_i , \quad (6.1)$$

is not used in our estimation, because the results of single factor model (6.1) can be totally different if the different interest rates are used. For the return of stock portfolios, the single factor model may be as good as more complex models in obtaining variance-covariance of portfolio returns.

However, it is not the case in obtaining the variance-covariance of bond portfolio returns. Since bonds have some properties that are quite different from stocks. In particular, bonds can have different maturities which stocks do not have. Table 35a shows the regression results of  $r_i$  on  $I_1$  (the percentage change of short-term rate); and Table 35b shows the regression results of  $r_i$  on  $I_2$  (the percentage change of long-term rate). Both  $I_1$  and  $I_2$  have the same definition as

that in the previous section.

Comparing the results of Tables 35a and 35b with that of Tables 6a or 6b, it is obvious that the regression results of the two factor model are much better than that of the single factor model if the short-term interest rate  $I_1$  is used. Not only the  $R^2$  of each regression is much higher in two factor model than that of the corresponding single factor model, but also that most of the coefficients of  $\beta_{11}$  and  $\beta_{12}$  are statistically significant. Though the results of two models for the long-term bonds are not significantly different if the long-term interest rate is used for the single factor model, the results of two factor model still statistically significantly dominate that of the single factor model for bonds with maturity less than 6 years (category 1-8). This simply means that the percentage change of only one rate is not enough to explain the behavior of bond portfolio returns. In other words, two factors are needed to describe the bond return changes. The conclusion may provide some explanations why duration approach can cause the controversy for the management of bond portfolios.

## **6.2 Bond Durations With the Single Factor Model**

The regression results illustrated in Table 35a and 35b give the estimation of durations for 19 bond categories. Sharpe (1985) explained the duration as the regression coefficient in a single factor model of bond returns. In the

following single factor model:

$$r_i = \alpha_i + \beta_i I + u_i, \quad (6.1)$$

Sharpe pointed out that

$\beta_i = -D_i$  (the duration of bond  $i$ ) and

$I$  = the percentage change of one-year interest rate.

Since duration generally overstates the magnitude of the reaction of a bond's price to changes in short-term rates, Sharpe suggested that a better approach would specify a response proportional to duration:

$$\beta_i = -kD_i \quad (6.2)$$

where  $k$  might be approximately 0.5. In his recent book (1990), Sharpe revised his definition of duration in the following fashion:

$$\begin{array}{l} \text{Percentage change} \\ \text{in bond price} \end{array} \approx -D \times \begin{array}{l} \text{percentage change in} \\ \text{(one plus the bond's yield)} \end{array} \quad (6.3)$$

As seen from Tables 3 and 5, our calculation of bond returns  $R_{ti}$  and percentage change in short-term and long-term rates,  $I_1$  and  $I_2$ , are consistent with Sharpe's definition of duration.

Our empirical results of single factor model in Tables 35a and 35b show some interesting points that have not been mentioned in the duration literature. As shown in Tables 35a and 35b, the values of duration ( $\beta_i$  coefficients) estimated by regressions of  $R_{ti}$  on  $I_1$  can be totally different than that

estimated by regressions of  $R_{ti}$  on  $I_2$ . For bonds with very short maturities (in our case, categories 1, 2 and 3), duration estimated by regressions of  $R_{ti}$  on  $I_1$  are dominant in the sense of higher  $R^2$  and higher t-values for  $\beta_1$ . Besides that, the values of duration estimated by regressions of  $R_{ti}$  on  $I_1$  are slightly higher than the corresponding ones estimated by regressions of  $R_{ti}$  on  $I_2$ . For other 16 bond categories, the results with regressions of  $R_{ti}$  on  $I_2$  are dominant. As seen from Table 35b,  $R^2$ s in regressions of  $R_{ti}$  on  $I_2$  exceed 0.9 for most of the long-term bonds. In particular, the regression coefficients of  $R_{ti}$  on  $I_2$  are not statistically significant different from that obtained by the two factor model for most of the long-term bond categories (see Tables 6a and 6b). This provides further evidence that durations estimated by regressions of  $R_{ti}$  on  $I_2$  explain most of the variation of long-term bond returns. It should be noted that the values of duration estimated by regressions of  $R_{ti}$  on  $I_2$  are much larger for long-term bonds than that estimated by regressions of  $R_{ti}$  on  $I_1$ . Since duration estimated with different interest rates can be very different, therefore, it should be very cautious to use the duration approximation in formula (6.3). Whether percentage change of short-term rate  $I_1$  or percentage change of long-term rate  $I_2$  is used, our estimates show that the response proportional  $k$  is not approximately equal to 0.5 as Professor Sharpe had suggested.

### 6.3 Ex ante Holding Period Returns

Using data from the CRSP bond tape, Hessel, Kim and Markowitz have estimated the monthly yield curve. The model used to estimate the monthly term structure can be written as follows:

$$Y_t = b_{1t} + b_{2t} \text{EXP}(-b_{3t}t) + u_t \quad (6.4)$$

where  $Y_t$  is the estimated yield for bonds with  $t$  years left to maturity;  $b_{1t}$ ,  $b_{2t}$  and  $b_{3t}$  are regression coefficients;  $t$  is the time left to maturity for the bond; and  $u_t$  is the error term. Table 36 shows part of their estimates for  $b_{1t}$ ,  $b_{2t}$  and  $b_{3t}$ .

By means of the estimated yield curve (6.4), we can calculate the approximate present value of bonds, i.e., the approximate price of bonds:

$$P_t = 100/(1+Y_t/2)^{2t} + C_t X 50/(1+Y_t/2)^{2t} X \{ (1-(1+Y_t/2)^{[2t]+1}) / (Y_t/2) \} - C_t X 100 X (([2t]+1)/2-t) \quad (6.5)^{13}$$

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<sup>13</sup> For any bond with  $t$  years left to maturity, the present value of that bond is

$$\begin{aligned} P_t &= 100/(1+Y/2)^{2t} + 50C/(1+Y/2)^{2t} + 50C/(1+Y/2)^{2(t-0.5)} + 50C/(1+Y/2)^{2(t1)} + \\ &+ \dots + 50C/(1+Y/2)^{2(t-([2t]/2-0.5))} + 50C/(1+Y/2)^{2(t-[2t]/2)} - \\ &- 100C/((([2t]+1)/2-t)) \\ &= 100/(1+Y/2)^{2t} + 50C/(1+Y/2)^{2t} * \{ (1-(1+Y/2)^{[2t]+1}) / (1-(1+Y/2)) \} - \\ &- 100C/((([2t]+1)/2-t)) \\ &= 100/(1+Y/2)^{2t} + 50C/(1+Y/2)^{2t} * \{ (1-(1+Y/2)^{[2t]+1}) / (-Y/2) \} - \\ &- 100C/((([2t]+1)/2-t)) \end{aligned}$$

where  $P_t$  is the present value of bond with  $t$  years left to maturity if the yield curve follows (6.4);  $C_t$  is the corresponding coupon rate for the bond  $P_t$  and  $[2t]$  is the whole number part of  $2t$ . In fact, the first term in (6.5) is the present value of principal for bond  $P_t$ ; the second term is the present value of coupon interest payments for bond  $P_t$ ; and the last term is the accrued interest payment for bond  $P_t$ . We assume that coupon interests are paid semi-annually.

It is important to note that the price (present value) of any bond with  $t$  years left to maturity can be obtained by equation (6.5) as long as the yield to maturity of that bond is known as  $Y_t$ . By means of (6.5), it is not difficult to obtain the ex ante monthly percentage holding period return of bond  $P_t$ :

$$r_{ti}^E = \{ [(P_{ti}^{t-1} - P_{t-1,i}^{t-1}) + C_{ti}X(M_{ti} - M_{t-1,i})]X100\} / (P_{t-1,i}^{t-1} + A_{t-1,i})$$

$$i=1,2,\dots,7617, \quad t=01/85,\dots,12/87, \quad (6.6)$$

where  $r_{ti}^E$  is the ex ante monthly percentage holding period return of bond  $i$  at month  $t$ ,  $P_{ti}^{t-1}$  is the ex ante price of bond  $i$  at month  $t$  with the assumption that the yield curve of month  $t$  is the same as that of month  $t-1$ ,  $P_{t-1,i}^{t-1}$  is the calculated price of bond  $i$  at month  $t-1$  given the yield curve at  $t-1$ ,  $C_{ti}$

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Assume that coupon interests are paid semi-annually and annual discount rate is  $Y$ .

is the coupon rate and  $M_{ti}$  is time to maturity and  $A_{t-1,i}$  is accrued interest for bond  $i$ .

Comparing (6.6) with (5.1), we find that both equations have the same formula. The only difference is that in (5.1), we use the actual (realized) prices  $P_{ti}$  and  $P_{t-1,i}$  to obtain the ex post returns  $r_{ti}$ ; while in (6.6), we use estimated prices  $P_{ti}^{t-1}$  and  $P_{t-1,i}^{t-1}$  to obtain the ex ante returns  $r_{ti}^t$ . It is extremely important that the estimation of ex ante price for bond  $i$  at time  $t$   $P_{ti}^{t-1}$  is based on the assumption that the yield curve at month  $t$  is exactly the same as that in the previous month  $t-1$ . Otherwise, no ex ante prices can be obtained.

With the available ex ante monthly percentage holding period returns  $r_{ti}^E$ , we can obtain  $R_{ti}^E$ , the ex ante monthly percentage holding period return of each bond category defined in Table 2. Table 37 shows the values of  $R_{ti}^E$ ,  $t=01/85, \dots, 12/87$ ,  $i=1, \dots, 19$ . Table 38 shows the values of  $R_{ti}^E - r_{ti}$ , the difference between the ex ante and ex post returns for 19 bond categories during the period [01/85, 12/87].

As we can see from Table 37, all the ex ante monthly holding period returns  $R_{ti}^E$  have positive values, since these returns reflect the returns before the fact. The returns before the fact (ex ante) might not be equal to the returns after the fact (ex post) in an efficient market. That is the reason why we have some large values for  $DR_{ti}^E$ , the difference between the ex ante and ex post returns, as can be seen in

Table 38.

#### 6.4 Ex ante Mean and Ex ante Variance

Table 39 shows the summary statistics of  $R_{ti}^E$  and  $DR_{ti}^E$ , respectively. As seen from the first two columns, the mean values of ex ante returns  $R_{ti}^E$  are consistently lower than that of the corresponding ex post returns  $R_{ti}$  except for bond category 1. This is reflected by the negative mean values of  $DR_{ti}^E$ . The differences are very large for the long-term bonds, since the long-term yield dropped from 11.51% on December 31, 1984 to 8.98% on December 31, 1987 during our sample period. The values of ex ante returns  $R_{ti}^E$  vary at a very narrow range [0.3031%, 1.1035%] for all bond categories compared with the range [-7.2256%, +12.5476%] for the ex post returns  $R_{ti}$ . Similar comparison can also be made between the variances of ex ante and ex post returns. As we can see from column 5 in Table 39 and column 3 in Table 4, the variances of ex ante returns rise very slowly as the maturity of bonds increase; while the variances of ex post returns go up quickly as the maturity of bonds increase. In fact, the variance of ex post returns is equal to that of the corresponding ex ante returns plus a variable term for each bond category, i.e.,

$$\text{Var}(R_i) = \text{Var}(R_i^E) + V_i \quad i=1, \dots, 19 \quad (6.7)$$

where  $V_i$  increase almost proportionally as the maturity of bonds increase.

Table 40a shows the regression results obtained by directly regressing  $DR_{ti}$  on  $I_1$  and  $I_2$

$$DR_i = \alpha_i + \beta_{i1}I_1 + \beta_{i2}I_2 + u_i \quad (6.8a)$$

Table 40b shows the regression results obtained by regressing  $DR_{ti}$  on  $I_2 - I_1$  and  $I_2$

$$DR_i = \alpha_i + \beta_{i1}(I_2 - I_1) + \beta_{i2}I_2 + u_i \quad (6.8b)$$

Generally speaking, the two factors  $I_1$  and  $I_2$  ( $I_2 - I_1$  and  $I_2$ ) explain quite well the variations of  $DR_{ti}$  in the sense of higher  $R_i^2$  and significant t-values for  $\beta$  coefficients.

Comparing the regression results in Table 40 with the corresponding results in Table 6, we find that  $R_i^2$  in Table 40 improve significantly for bonds with short maturities. The improvements of  $R_i^2$  for the long term bonds in Table 40 are not very significant, since the corresponding regressions in Table 6 already have the higher  $R_i^2$ . Similar to the regression results in Table 6, the percentage change of long-term rate  $I_2$  still plays a major role for the regressions of long-term returns. Not surprisingly, most regression coefficients for the intercepts are not statistically significant for the two factor model in equations 6.8a and 6.8b, since  $DR_{ti}^E$  is the difference between two returns.

CONCLUSIONS

Following the Markowitz general portfolio selection model of 1987, portfolio selection models of assets and liabilities without and with interest rate swaps are developed. By means of the mean-variance analysis, the optimal bond portfolios can be obtained for the management of risk exposure of assets and liabilities. In case that some assets and liabilities in the bond portfolios are not tradable, a series of interest rate swaps can be introduced to reallocate the assets and liabilities. Through simultaneous paying (receiving) and receiving (paying) the fixed and floating rate of interests based on the notional principals, the proportionate holdings of underlying assets and liabilities in the bond portfolios can be changed as if the bonds are bought and sold in the market. Thus, the optimal allocation of assets and liabilities in the bond portfolios can always be achieved.

A two factor model is used to obtain the variance-covariance matrix of bond portfolio returns as a whole. Without interest rate swaps, the variance-covariance matrix of bond portfolio returns as a whole is diagonal with  $4n+2$  dimensions. By introducing interest rate swaps into the bond portfolio selection model, we find that the variance-

covariance matrix of bond portfolio returns as a whole is still diagonal but with  $6n+2$  dimensions. We have shown that why the  $2n$  more swap variables in the variance-covariance matrix with interest rate swaps can be used to reallocate the nonmarketable assets and liabilities in the bond portfolio selections.

Using data from the CRSP U.S. Government Bond Tape, we divide bonds into 19 categories according to the time left to maturity. We calculate the monthly holding period return for each of the 19 bond categories during the sample period from January 1985 to December 1987. We also calculate the mean, variance, maximum, and minimum values for these 19 bond categories. A two factor model is used to obtain the variance-covariance matrix of bond portfolio returns as a whole without interest rate swaps. Empirical evidence shows that the two factor model explains most of the variations of bond portfolio returns and performs better than the single factor model. Using critical line algorithm developed by Markowitz, the optimal portfolios of bonds without interest rate swaps are calculated and the results are consistent with the mean-variance theory.

With the nonmarketable assets in the bond portfolios, not only the mean returns of the optimal portfolios will be reduced, but also the risk of the portfolios will be increased. That is, the optimal portfolios will be less efficient and the efficient frontiers will shift down.

Using data from the "International Financing Review", returns of swaps and swap adjustable notes were obtained. The two factor model is used again to obtain the variance-covariance matrix of returns of swap adjustable notes and the enlarged variance-covariance matrix is estimated. Using critical line algorithm again, the optimal portfolios of bonds with interest rate swaps are calculated. Empirical evidence shows that: 1. Interest rate swaps can be used to reallocate the nonmarketable bonds in the portfolio selection process; 2. The efficient frontier with interest rate swaps will shift up if the underlying bond portfolios of real principal have longer time to maturity and higher risk exposures; 3. If the underlying bond portfolios of real principal have shorter maturities and lower risk exposures, the efficient frontier with interest rate swaps will shift up less significantly.

Durations of 19 bond categories are estimated with single factor model. We find that the durations of bonds estimated by regressions on different interest rates can be very different. Except bonds with very short maturities, durations estimated by regression of bond returns on percentage changes of long-term yields explain most of the variation of long-term bond returns. The traditional approach to use the percentage changes of short-term rates to estimate durations is not appropriate. Whether percentage changes of short-term rates or percentage change of long-term rates are used, our estimates show that the response proportional  $k$

mentioned by Sharpe is not approximately equal to 0.5.

Using yield curves estimated by Hessel, Kim, and Markowitz, the ex ante monthly holding period returns of 19 bond categories are calculated. We also calculate the differences between the ex ante and ex post monthly holding period returns for the sample period. Our results show: 1. The values of ex ante monthly holding period returns vary at a very narrow range during the sample period; 2. For each bond category, the ex post variance is equal to the ex ante variance plus a variable term which increase almost proportionally with the bond maturity. 3. The two factor model also explains most of the variations for the differences between the ex ante and ex post returns.

**TABLE 1. Data on CRSP Bond Tape  
(Partial Listing)**

| Time Code | Price      | Maturity  | Coupon rate |
|-----------|------------|-----------|-------------|
| 709*      | 99.831940  | 0.025736  | 0.00000     |
| 709       | 99.640640  | 0.044901  | 0.00000     |
| 709       | 99.481339  | 0.064066  | 0.00000     |
| 709       | 100.062500 | 0.083231  | 0.09250     |
| 709       | 99.353302  | 0.083231  | 0.00000     |
| 709       | 99.026245  | 0.119964  | 0.00000     |
| 709       | 89.937500  | 24.871532 | 0.10375     |
| 709       | 100.781250 | 25.122017 | 0.11750     |
| 709       | 87.156250  | 25.371846 | 0.10000     |
| 709       | 108.468750 | 25.871504 | 0.12750     |
| 709       | 117.500000 | 26.371819 | 0.13875     |
| 709       | 118.750000 | 26.871477 | 0.14000     |
| 709       | 89.906250  | 27.871450 | 0.10375     |
| 709       | 102.718750 | 28.621593 | 0.12000     |
| 710       | 99.861557  | 0.015925  | 0.00000     |
| 710       | 99.968750  | 0.037828  | 0.08000     |
| 710       | 100.218750 | 0.037828  | 0.14625     |
| 710       | 99.547913  | 0.054255  | 0.00000     |
| 710       | 100.062500 | 0.073420  | 0.09625     |
| 710       | 99.422882  | 0.073420  | 0.00000     |
| 710       | 99.265976  | 0.099202  | 0.00000     |
| 710       | 92.281250  | 24.786658 | 0.10375     |
| 710       | 103.625000 | 25.037144 | 0.11750     |
| 710       | 89.375000  | 25.286973 | 0.10000     |
| 710       | 111.218750 | 25.786631 | 0.12750     |
| 710       | 120.437500 | 26.286945 | 0.13875     |
| 710       | 121.656250 | 26.786604 | 0.14000     |
| 710       | 92.218750  | 27.786576 | 0.10375     |
| 710       | 105.625000 | 28.536720 | 0.12000     |
| 710       | 116.156250 | 29.286863 | 0.13250     |
| 710       | 110.000000 | 29.536692 | 0.12500     |
| 710       | 104.625000 | 29.786522 | 0.11750     |
| 711       | 99.866608  | 0.022542  | 0.00000     |
| 711       | 99.732834  | 0.041707  | 0.00000     |
| 711       | 99.552002  | 0.060872  | 0.00000     |
| 711       | 99.441559  | 0.080037  | 0.00000     |
| 711       | 100.000000 | 0.088250  | 0.09625     |
| 711       | 100.281250 | 0.088250  | 0.13375     |
| 711       | 99.259171  | 0.097605  | 0.00000     |
| 711       | 99.089996  | 0.116770  | 0.00000     |

\* The time code 709 is equivalent to December 1984 , 710 is equivalent to January 1985 and so on.

**TABLE 2****Bond Portfolio Categories**

| Category | Maturity in Years | Number of Observations |
|----------|-------------------|------------------------|
| 1        | (0 - 0.25]        | 655                    |
| 2        | (0.25 - 0.50]     | 607                    |
| 3        | (0.50 - 1.00]     | 692                    |
| 4        | (1.00 - 2.00]     | 1018                   |
| 5        | (2.00 - 3.00]     | 624                    |
| 6        | (3.00 - 4.00]     | 513                    |
| 7        | (4.00 - 5.00]     | 400                    |
| 8        | (5.00 - 6.00]     | 321                    |
| 9        | (6.00 - 7.00]     | 340                    |
| 10       | (7.00 - 8.00]     | 257                    |
| 11       | (8.00 - 9.00]     | 237                    |
| 12       | (9.00 -10.00]     | 176                    |
| 13       | (10.00 -13.00]    | 87                     |
| 14       | (13.00 -16.00]    | 290                    |
| 15       | (16.00 -19.00]    | 337                    |
| 16       | (19.00 -22.00]    | 175                    |
| 17       | (22.00 -25.00]    | 243                    |
| 18       | (25.00 -28.00]    | 185                    |
| 19       | (28.00 -30.00]    | 198                    |

**Total 7355**

TABLE 3

Monthly Holding Period Returns By Category  
(Partial Listing, Value in Percentages)

| T     | R <sub>1</sub> | R <sub>2</sub> | R <sub>3</sub> | R <sub>4</sub> | R <sub>5</sub> |
|-------|----------------|----------------|----------------|----------------|----------------|
| 01/85 | 0.6878         | 0.7202         | 0.9481         | 1.1716         | 1.3740         |
| 02/85 | 0.5908         | 0.4481         | 0.1712         | -0.1842        | -0.6787        |
| 03/85 | 0.7326         | 0.8556         | 0.9803         | 1.1315         | 1.3673         |
| 04/85 | 0.8229         | 0.9902         | 1.2307         | 1.5682         | 1.8597         |
| 05/85 | 0.8030         | 1.0686         | 1.5876         | 2.3066         | 3.3295         |
| 06/85 | 0.6110         | 0.6759         | 0.8094         | 1.1289         | 1.1655         |
| 07/85 | 0.6255         | 0.5606         | 0.4601         | 0.3858         | 0.1933         |
| 08/85 | 0.6433         | 0.7415         | 0.8721         | 1.0492         | 1.2592         |
| 09/85 | 0.6269         | 0.6688         | 0.7076         | 0.7886         | 0.9482         |
| 10/85 | 0.6184         | 0.6383         | 0.7543         | 0.9716         | 1.3059         |
| 11/85 | 0.6253         | 0.6338         | 0.7517         | 1.0593         | 1.4099         |
| 12/85 | 0.6983         | 0.7186         | 0.8213         | 1.2455         | 1.8237         |
| 01/86 | 0.6155         | 0.6474         | 0.6740         | 0.6677         | 0.6856         |
| 02/86 | 0.5592         | 0.5727         | 0.7211         | 0.9958         | 1.3922         |
| 03/86 | 0.6936         | 0.8713         | 1.0794         | 1.5519         | 2.2749         |
| 04/86 | 0.5988         | 0.6326         | 0.7457         | 0.7878         | 0.6298         |
| 05/86 | 0.4877         | 0.4508         | 0.3813         | 0.0213         | -0.5625        |
| 06/86 | 0.5989         | 0.6660         | 0.8507         | 1.2574         | 1.8548         |
| 07/86 | 0.5744         | 0.6109         | 0.7164         | 0.9717         | 1.2165         |
| 08/86 | 0.5703         | 0.7663         | 1.0677         | 1.5185         | 2.1034         |
| 09/86 | 0.4599         | 0.4172         | 0.3290         | 0.0497         | -0.4318        |
| 10/86 | 0.4736         | 0.5088         | 0.5748         | 0.7939         | 1.1367         |
| 11/86 | 0.4057         | 0.3640         | 0.4342         | 0.5670         | 0.7672         |
| 12/86 | 0.4716         | 0.4118         | 0.3384         | 0.2222         | 0.1700         |
| 01/87 | 0.4963         | 0.5085         | 0.5830         | 0.7411         | 0.7979         |
| 02/87 | 0.4658         | 0.4727         | 0.4321         | 0.4031         | 0.4880         |
| 03/87 | 0.5137         | 0.4490         | 0.4052         | 0.2505         | 0.0849         |
| 04/87 | 0.5089         | 0.4923         | 0.1736         | -0.4384        | -1.2609        |
| 05/87 | 0.4195         | 0.4532         | 0.4058         | 0.1688         | -0.0300        |
| 06/87 | 0.5431         | 0.6329         | 0.7969         | 1.1086         | 1.2358         |
| 07/87 | 0.4971         | 0.5178         | 0.6068         | 0.6139         | 0.4077         |
| 08/87 | 0.5126         | 0.4737         | 0.4207         | 0.2055         | -0.0548        |
| 09/87 | 0.4904         | 0.4081         | 0.2155         | -0.1841        | -0.7346        |
| 10/87 | 0.7556         | 0.9927         | 1.2934         | 1.8828         | 2.6495         |
| 11/87 | 0.5344         | 0.5640         | 0.5940         | 0.6058         | 0.7612         |
| 12/87 | 0.5042         | 0.5906         | 0.5560         | 0.6409         | 0.7606         |

TABLE 3 -- Continued

| T     | R <sub>6</sub> | R <sub>7</sub> | R <sub>8</sub> | R <sub>9</sub> | R <sub>10</sub> |
|-------|----------------|----------------|----------------|----------------|-----------------|
| 01/85 | 1.7305         | 2.0878         | 2.4622         | 2.8577         | 3.2084          |
| 02/85 | -1.2017        | -1.6983        | -1.8270        | -2.4884        | -3.2108         |
| 03/85 | 1.6092         | 1.6416         | 1.7773         | 1.9032         | 2.4692          |
| 04/85 | 2.1045         | 2.3586         | 2.3570         | 2.4037         | 2.3997          |
| 05/85 | 4.2024         | 4.8589         | 5.5492         | 6.1358         | 7.4794          |
| 06/85 | 1.1347         | 1.1269         | 0.8302         | 0.6999         | 0.6019          |
| 07/85 | -0.0057        | -0.0693        | -0.2713        | -0.3667        | -0.6101         |
| 08/85 | 1.4976         | 1.6446         | 2.0326         | 2.1915         | 2.3434          |
| 09/85 | 1.0479         | 0.8947         | 1.0845         | 0.8914         | 0.5312          |
| 10/85 | 1.6550         | 1.8744         | 1.9765         | 2.1450         | 2.4011          |
| 11/85 | 1.7018         | 2.0269         | 2.4472         | 2.6665         | 3.0634          |
| 12/85 | 2.2956         | 2.8529         | 3.3795         | 3.9078         | 4.6554          |
| 01/86 | 0.7804         | 0.6217         | 0.5657         | 0.4066         | -0.0540         |
| 02/86 | 2.1380         | 2.7540         | 3.6125         | 4.8489         | 5.4976          |
| 03/86 | 2.5282         | 3.0400         | 3.5384         | 4.3323         | 4.5059          |
| 04/86 | 0.7181         | 0.6457         | 0.6227         | 0.3149         | 0.2252          |
| 05/86 | -1.4269        | -1.9903        | -2.2433        | -2.8467        | -2.9830         |
| 06/86 | 2.3696         | 2.7729         | 3.1645         | 3.7193         | 3.7176          |
| 07/86 | 1.4333         | 1.4112         | 1.3572         | 1.0985         | 1.0394          |
| 08/86 | 2.6052         | 2.8870         | 3.1003         | 3.3263         | 3.5975          |
| 09/86 | -0.9743        | -1.3458        | -1.4527        | -2.0555        | -2.1416         |
| 10/86 | 1.5118         | 1.7041         | 1.7448         | 1.5621         | 1.8749          |
| 11/86 | 0.9860         | 1.1774         | 1.3811         | 1.4543         | 1.4339          |
| 12/86 | 0.1324         | 0.2645         | 0.2834         | 0.2280         | 0.3561          |
| 01/87 | 0.9328         | 0.9365         | 1.0828         | 1.1775         | 1.2412          |
| 02/87 | 0.5094         | 0.4726         | 0.6142         | 0.6357         | 0.7457          |
| 03/87 | -0.1519        | -0.2575        | -0.5764        | -0.8846        | -0.9690         |
| 04/87 | -1.8066        | -2.3373        | -2.7343        | -3.0377        | -3.2959         |
| 05/87 | -0.2311        | -0.4400        | -0.3136        | -0.6076        | -0.6652         |
| 06/87 | 1.2643         | 1.2849         | 1.2407         | 1.2506         | 1.2119          |
| 07/87 | 0.3201         | 0.1825         | -0.1188        | -0.2642        | -0.2777         |
| 08/87 | -0.2728        | -0.3876        | -0.5321        | -0.7985        | -1.0334         |
| 09/87 | -1.2152        | -1.7753        | -2.2123        | -2.4637        | -2.5026         |
| 10/87 | 3.1256         | 3.5016         | 3.9530         | 4.1422         | 4.1834          |
| 11/87 | 0.7401         | 0.4935         | 0.6382         | 0.5116         | 0.6043          |
| 12/87 | 0.9975         | 1.1444         | 1.3377         | 1.6749         | 1.6160          |

TABLE 3 -- Continued

| T     | R <sub>11</sub> | R <sub>12</sub> | R <sub>13</sub> | R <sub>14</sub> | R <sub>15</sub> |
|-------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 01/85 | 3.2448          | 3.1580          | 3.3593          | 3.8316          | 3.5757          |
| 02/85 | -3.2920         | -3.4592         | -3.4181         | -4.5975         | -4.5028         |
| 03/85 | 2.4328          | 2.3333          | 2.1279          | 2.7303          | 2.6755          |
| 04/85 | 2.0121          | 2.1733          | 2.6531          | 2.2746          | 2.2109          |
| 05/85 | 7.7856          | 8.0479          | 8.3553          | 9.2447          | 8.8850          |
| 06/85 | 0.7978          | 0.8914          | 1.3059          | 0.9829          | 1.0865          |
| 07/85 | -0.7298         | -0.9410         | -2.0771         | -1.3550         | -1.5224         |
| 08/85 | 2.6517          | 2.6056          | 2.6161          | 2.7576          | 2.7055          |
| 09/85 | 0.8599          | 1.8940          | 0.6381          | 0.4481          | 0.1664          |
| 10/85 | 2.6595          | 1.4882          | 3.0074          | 3.2426          | 3.5203          |
| 11/85 | 2.9451          | 2.8975          | 3.3685          | 4.2743          | 4.4082          |
| 12/85 | 4.8229          | 4.5526          | 5.9452          | 5.3640          | 5.4107          |
| 01/86 | -0.0562         | 0.2469          | 0.4051          | 0.4262          | 0.0718          |
| 02/86 | 6.3583          | 6.5487          | 8.3458          | 9.8866          | 10.2421         |
| 03/86 | 4.6604          | 5.2405          | 6.6808          | 6.2907          | 8.4576          |
| 04/86 | 0.3239          | 0.4733          | 1.2730          | -0.0415         | -0.5534         |
| 05/86 | -3.3642         | -3.5967         | -4.3134         | -5.0680         | -5.6282         |
| 06/86 | 4.1168          | 4.6412          | 5.8589          | 5.6484          | 6.0158          |
| 07/86 | 0.8379          | 0.6898          | -0.1631         | -0.3036         | -0.3913         |
| 08/86 | 3.8035          | 3.6294          | 3.8810          | 4.2668          | 4.5576          |
| 09/86 | -2.3829         | -2.6945         | -2.1839         | -3.4096         | -4.1223         |
| 10/86 | 1.9530          | 1.8680          | 2.2640          | 2.3818          | 2.5839          |
| 11/86 | 1.5614          | 1.3628          | 1.2973          | 2.0118          | 2.0134          |
| 12/86 | 0.2748          | 0.1669          | -0.0264         | 0.3116          | 0.1959          |
| 01/87 | 1.2406          | 1.0505          | 0.8770          | 1.4992          | 1.5780          |
| 02/87 | 0.6949          | 0.4940          | 0.9116          | 1.3075          | 1.3286          |
| 03/87 | -1.0412         | -1.4272         | -1.4694         | -1.2811         | -1.5782         |
| 04/87 | -3.5754         | -3.8736         | -4.0005         | -4.3779         | -4.8464         |
| 05/87 | -1.1196         | -1.4517         | -1.5565         | -1.1237         | -0.9202         |
| 06/87 | 1.2702          | 1.3828          | 1.1901          | 1.3148          | 1.1269          |
| 07/87 | -0.5940         | -1.0352         | -1.1201         | -1.1939         | -1.5971         |
| 08/87 | -1.1697         | -1.5967         | -1.5962         | -1.2595         | -1.4773         |
| 09/87 | -2.8855         | -3.3226         | -3.7348         | -3.7053         | -4.1623         |
| 10/87 | 4.9050          | 5.6560          | 6.5286          | 6.4034          | 7.1741          |
| 11/87 | 0.4140          | -0.2307         | -0.4725         | 0.0326          | -0.1367         |
| 12/87 | 1.6526          | 1.8637          | 2.3308          | 1.9684          | 2.3710          |

TABLE 3 -- Continued

| T     | R <sub>16</sub> | R <sub>17</sub> | R <sub>18</sub> | R <sub>19</sub> |
|-------|-----------------|-----------------|-----------------|-----------------|
| 01/85 | 3.6994          | 3.6994          | 3.5609          | 3.8097          |
| 02/85 | -5.0027         | -5.5835         | -5.1454         | -4.9590         |
| 03/85 | 2.8052          | 3.1249          | 2.7641          | 2.9458          |
| 04/85 | 1.8951          | 1.9968          | 1.9319          | 1.7784          |
| 05/85 | 9.4684          | 10.3856         | 9.6599          | 9.1941          |
| 06/85 | 1.5566          | 1.5988          | 1.2652          | 1.4907          |
| 07/85 | -1.6653         | -1.6250         | -1.6748         | -1.6139         |
| 08/85 | 2.8968          | 3.2101          | 2.8412          | 2.5029          |
| 09/85 | 0.0571          | -0.2455         | -0.3323         | 0.0325          |
| 10/85 | 3.6555          | 3.7444          | 3.8373          | 3.7350          |
| 11/85 | 4.7224          | 4.7331          | 4.3323          | 4.2424          |
| 12/85 | 6.0551          | 5.9475          | 5.8332          | 6.1883          |
| 01/86 | 0.0135          | -0.0682         | 0.0363          | 0.0828          |
| 02/86 | 11.4233         | 11.4932         | 11.1247         | 11.4225         |
| 03/86 | 8.5396          | 7.6313          | 8.7732          | 9.2385          |
| 04/86 | -0.6250         | -0.8245         | -0.8554         | -0.2831         |
| 05/86 | -6.4324         | -5.4148         | -5.5861         | -5.5101         |
| 06/86 | 6.6328          | 6.4371          | 6.3733          | 5.9424          |
| 07/86 | -0.5147         | -1.0708         | -1.1448         | -1.0511         |
| 08/86 | 4.8080          | 4.9476          | 5.1096          | 4.6534          |
| 09/86 | -3.9455         | -4.3324         | -4.5108         | -4.1535         |
| 10/86 | 2.1883          | 2.3039          | 2.2896          | 2.2485          |
| 11/86 | 1.9806          | 2.3182          | 2.1877          | 1.6503          |
| 12/86 | -0.0280         | -0.2291         | 0.0506          | -0.3099         |
| 01/87 | 1.5588          | 1.6325          | 1.6632          | 1.7722          |
| 02/87 | 1.6034          | 1.8988          | 1.6634          | 0.9249          |
| 03/87 | -2.1281         | -1.8023         | -1.7523         | -2.5639         |
| 04/87 | -4.9948         | -4.4978         | -4.9378         | -5.8315         |
| 05/87 | -1.5152         | -1.1378         | -1.1071         | -1.3149         |
| 06/87 | 0.6832          | 0.9822          | 1.1337          | 1.6130          |
| 07/87 | -1.8599         | -1.5590         | -1.6938         | -2.5968         |
| 08/87 | -1.5793         | -1.4737         | -1.7418         | -2.1046         |
| 09/87 | -4.8639         | -4.0288         | -4.3537         | -5.2942         |
| 10/87 | 7.8430          | 7.0095          | 7.5054          | 8.2881          |
| 11/87 | -0.1547         | -0.0797         | -0.1391         | 0.2351          |
| 12/87 | 2.5158          | 2.6898          | 2.6093          | 2.5197          |

TABLE 4

Summary Statistics of Bond Returns by Category\*  
(Value in Percentages)

| Category | Mean of $R_{t,i}$<br>(monthly) | Maximum<br>(Minimum)<br>of $R_{t,i}$ | Standard<br>Deviation<br>(Variance) | Average<br>Maturity<br>(years) | Average<br>Coupon<br>Rate |
|----------|--------------------------------|--------------------------------------|-------------------------------------|--------------------------------|---------------------------|
| 1        | 0.5788                         | 0.9505<br>(0.3036)                   | 0.1038<br>(0.0108)                  | 0.1304                         | 3.41**                    |
| 2        | 0.6165                         | 1.2595<br>(0.2319)                   | 0.1753<br>(0.0307)                  | 0.3545                         | 4.55**                    |
| 3        | 0.6803                         | 1.9582<br>(-0.1015)                  | 0.3210<br>(0.1031)                  | 0.7317                         | 7.61**                    |
| 4        | 0.7785                         | 2.8012<br>(-0.8184)                  | 0.6027<br>(0.3632)                  | 1.4834                         | 10.12                     |
| 5        | 0.8806                         | 3.8038<br>(-1.6356)                  | 0.9910<br>(0.9821)                  | 2.4959                         | 10.71                     |
| 6        | 0.9663                         | 5.1758<br>(-2.1478)                  | 1.3403<br>(1.7965)                  | 3.4922                         | 10.79                     |
| 7        | 1.0100                         | 5.4275<br>(-2.6088)                  | 1.6417<br>(2.6951)                  | 4.5065                         | 11.05                     |
| 8        | 1.1070                         | 5.7719<br>(-3.0353)                  | 1.8982<br>(3.6032)                  | 5.5006                         | 10.96                     |
| 9        | 1.1298                         | 6.8050<br>(-3.7928)                  | 2.2359<br>(4.9994)                  | 6.4916                         | 10.12                     |
| 10       | 1.2017                         | 8.2335<br>(-3.8945)                  | 2.5131<br>(6.3157)                  | 7.5170                         | 10.21                     |
| 11       | 1.2241                         | 8.4256<br>(-4.0512)                  | 2.7224<br>(7.4117)                  | 8.4758                         | 10.42                     |
| 12       | 1.1591                         | 10.8601<br>(-6.9547)                 | 2.9075<br>(8.4538)                  | 9.4596                         | 10.21                     |
| 13       | 1.3636                         | 9.5452<br>(-4.6085)                  | 3.3454<br>(11.1920)                 | 11.6583                        | 8.51                      |
| 14       | 1.4218                         | 11.2998<br>(-6.0143)                 | 3.5996<br>(12.9574)                 | 14.6854                        | 11.23                     |
| 15       | 1.4145                         | 10.3908<br>(-6.0247)                 | 3.8869<br>(15.1079)                 | 17.4607                        | 11.63                     |
| 16       | 1.4248                         | 12.5476<br>(-7.2257)                 | 4.2231<br>(17.8348)                 | 20.3687                        | 9.20                      |
| 17       | 1.4948                         | 11.9871<br>(-5.8300)                 | 4.1387<br>(17.1289)                 | 23.4842                        | 10.71                     |
| 18       | 1.4325                         | 11.8226<br>(-5.7168)                 | 4.1565<br>(17.2762)                 | 26.6211                        | 12.15                     |
| 19       | 1.3590                         | 11.7753<br>(-6.4259)                 | 4.2862<br>(18.3714)                 | 28.9791                        | 10.39                     |

\* The number of observations for each category are in Table 1.  
 \*\* These three categories include zero-coupon T-bills.

TABLE 5

Percentage Changes in Short-Term\*  
and Long-Term Interest Rates\*\*

| T     | $I_{1t}$ | $I_{2t}$ |
|-------|----------|----------|
| 01/85 | 0.0557   | -0.2601  |
| 02/85 | 0.5656   | 0.6923   |
| 03/85 | -0.2489  | -0.1072  |
| 04/85 | -0.2773  | -0.1698  |
| 05/85 | -0.6673  | -0.8059  |
| 06/85 | -0.2613  | -0.0632  |
| 07/85 | 0.3649   | 0.2710   |
| 08/85 | -0.2330  | -0.3693  |
| 09/85 | 0.0187   | 0.2080   |
| 10/85 | 0.1401   | -0.3068  |
| 11/85 | -0.0280  | -0.3258  |
| 12/85 | -0.2053  | -0.5630  |
| 01/86 | 0.0935   | 0.0913   |
| 02/86 | -0.0093  | -0.9762  |
| 03/86 | -0.6446  | -0.7923  |
| 04/86 | -0.2257  | 0.1300   |
| 05/86 | 0.1979   | 0.3803   |
| 06/86 | -0.2634  | -0.3141  |
| 07/86 | -0.1603  | 0.2039   |
| 08/86 | -0.5384  | -0.4162  |
| 09/86 | -0.0950  | 0.4923   |
| 10/86 | 0.0000   | -0.2311  |
| 11/86 | 0.1996   | -0.1482  |
| 12/86 | 0.2846   | 0.0371   |
| 01/87 | -0.2081  | -0.1113  |
| 02/87 | -0.0190  | 0.0000   |
| 03/87 | 0.1517   | 0.2321   |
| 04/87 | 0.0947   | 0.6392   |
| 05/87 | -0.0757  | 0.1473   |
| 06/87 | 0.1230   | -0.1471  |
| 07/87 | 0.0851   | 0.2853   |
| 08/87 | 0.2550   | 0.2937   |
| 09/87 | 0.5370   | 0.4393   |
| 10/87 | -1.5744  | -0.5285  |
| 11/87 | 0.6094   | 0.0733   |
| 12/87 | 0.0568   | -0.2471  |

\* 91-day T-bill rate at the end of the month from the Wall Street Journal.

\*\* Yield average of 10 bonds with the longest maturities at the end of the month from the Wall Street Journal.

TABLE 6a

Regression Results of Two Factor Model (a)

$$R_i = \alpha_i + \beta_{i1}I_1 + \beta_{i2}I_2 + u_i$$

| Category | $\alpha_i$<br>(t-value) | $\beta_{i1}$<br>(t-value) | $\beta_{i2}$<br>(t-value) | $R_i^2$ | $V(u_i)$ |
|----------|-------------------------|---------------------------|---------------------------|---------|----------|
| 1        | 0.5691<br>(38.826)      | -0.1044<br>(-2.274)       | -0.0666<br>(-1.452)**     | 0.3417  | 0.0071   |
| 2        | 0.5948<br>(31.297)      | -0.2476<br>(-4.158)       | -0.1374<br>(-2.309)       | 0.6119  | 0.0119   |
| 3        | 0.6355<br>(21.583)      | -0.3878<br>(-4.203)       | -0.3864<br>(-4.192)       | 0.7224  | 0.0286   |
| 4        | 0.6894<br>(14.332)      | -0.5912<br>(-3.922)       | -0.9190<br>(-6.103)       | 0.7898  | 0.0764   |
| 5        | 0.7303<br>(10.202)      | -0.7839<br>(-3.495)       | -1.7282<br>(-7.712)       | 0.8278  | 0.1691   |
| 6        | 0.7621<br>(8.202)       | -0.8242<br>(-2.831)       | -2.5508<br>(-8.769)       | 0.8414  | 0.2849   |
| 7        | 0.7570<br>(7.294)       | -0.8448<br>(-2.598)       | -3.3088<br>(-10.184)      | 0.8681  | 0.3555   |
| 8        | 0.8116<br>(7.485)       | -0.8282<br>(-2.437)       | -3.9958<br>(-11.772)      | 0.8923  | 0.3880   |
| 9        | 0.7800<br>(7.058)       | -0.6423<br>(-1.855)*      | -5.0170<br>(-14.503)      | 0.9194  | 0.4030   |
| 10       | 0.8096<br>(6.656)       | -0.5592<br>(-1.467)**     | -5.7568<br>(-15.118)      | 0.9227  | 0.4883   |
| 11       | 0.7970<br>(6.358)       | -0.6753<br>(-1.719)*      | -6.2174<br>(-15.844)      | 0.9300  | 0.5185   |
| 12       | 0.7011<br>(4.811)       | -1.2035<br>(-2.636)       | -6.2631<br>(-13.729)      | 0.9171  | 0.7008   |
| 13       | 0.8325<br>(5.647)       | -1.2649<br>(-2.738)       | -7.3732<br>(-15.976)      | 0.9359  | 0.7173   |
| 14       | 0.8575<br>(5.729)       | -0.5402<br>(-1.152)**     | -8.5080<br>(-18.157)      | 0.9429  | 0.7394   |
| 15       | 0.8000<br>(5.417)       | -0.7547<br>(-1.631)**     | -9.1257<br>(-19.739)      | 0.9524  | 0.7198   |
| 16       | 0.7594<br>(4.288)       | -0.9325<br>(-1.681)**     | -9.7856<br>(-17.653)      | 0.9420  | 1.0348   |
| 17       | 0.8444<br>(5.361)       | -0.5108<br>(-1.035)**     | -9.9009<br>(-20.081)      | 0.9522  | 0.8187   |
| 18       | 0.7746<br>(5.275)       | -0.7033<br>(-1.529)**     | -9.8595<br>(-21.449)      | 0.9588  | 0.7115   |
| 19       | 0.6783<br>(4.373)       | -0.9635<br>(-1.983)*      | -10.0032<br>(-20.604)     | 0.9568  | 0.7938   |

\* Not statistically significant at 5% level.

\*\* Not statistically significant at 10% level.

TABLE 6b

Regression Results of Two Factor Model (b)

$$R_i = \alpha_i + \beta_{i1}(I_2 - I_1) + \beta_{i2}I_2 + u_i$$

| Category | $\alpha_i$<br>(t-value) | $\beta_{i1}$<br>(t-value) | $\beta_{i2}$<br>(t-value) | $R_i^2$ | $V(u_i)$ |
|----------|-------------------------|---------------------------|---------------------------|---------|----------|
| 1        | 0.5691<br>(38.826)      | 0.1044<br>(2.274)         | -0.1711<br>(-4.114)       | 0.3417  | 0.0071   |
| 2        | 0.5948<br>(31.297)      | 0.2476<br>(4.158)         | -0.3850<br>(-7.140)       | 0.6119  | 0.0119   |
| 3        | 0.6355<br>(21.583)      | 0.3878<br>(4.203)         | -0.7742<br>(-9.268)       | 0.7224  | 0.0286   |
| 4        | 0.6894<br>(14.332)      | 0.5912<br>(3.922)         | -1.5102<br>(-11.066)      | 0.7898  | 0.0764   |
| 5        | 0.7303<br>(10.202)      | 0.7839<br>(3.495)         | -2.5121<br>(-12.369)      | 0.8278  | 0.1691   |
| 6        | 0.7621<br>(8.202)       | 0.8242<br>(2.831)         | -3.3749<br>(-12.802)      | 0.8414  | 0.2849   |
| 7        | 0.7570<br>(7.294)       | 0.8448<br>(2.598)         | -4.1536<br>(-14.106)      | 0.8681  | 0.3555   |
| 8        | 0.8116<br>(7.485)       | 0.8282<br>(2.437)         | -4.8239<br>(-15.680)      | 0.8923  | 0.3880   |
| 9        | 0.7800<br>(7.058)       | 0.6423<br>(1.855)*        | -5.6592<br>(-18.050)      | 0.9194  | 0.4030   |
| 10       | 0.8096<br>(6.656)       | 0.5592<br>(1.467)**       | -6.3160<br>(-18.301)      | 0.9227  | 0.4883   |
| 11       | 0.7970<br>(6.358)       | 0.6753<br>(1.719)*        | -6.8927<br>(-19.381)      | 0.9300  | 0.5185   |
| 12       | 0.7011<br>(4.811)       | 1.2035<br>(2.636)         | -7.4666<br>(-18.059)      | 0.9171  | 0.7008   |
| 13       | 0.8325<br>(5.647)       | 1.2649<br>(2.738)         | -8.6381<br>(-20.652)      | 0.9359  | 0.7173   |
| 14       | 0.8575<br>(5.729)       | 0.5402<br>(1.152)**       | -9.0482<br>(-21.306)      | 0.9429  | 0.7394   |
| 15       | 0.8000<br>(5.417)       | 0.7547<br>(1.631)**       | -9.8805<br>(-23.581)      | 0.9524  | 0.7198   |
| 16       | 0.7594<br>(4.288)       | 0.9325<br>(1.681)**       | -10.7181<br>(-21.334)     | 0.9420  | 1.0348   |
| 17       | 0.8444<br>(5.361)       | 0.5108<br>(1.035)**       | -10.4117<br>(-23.299)     | 0.9522  | 0.8187   |
| 18       | 0.7746<br>(5.275)       | 0.7033<br>(1.529)**       | -10.5628<br>(-25.355)     | 0.9588  | 0.7115   |
| 19       | 0.6783<br>(4.373)       | 0.9635<br>(1.983)*        | -10.9668<br>(-24.924)     | 0.9568  | 0.7938   |

\* Not statistically significant at 5% level.

\*\* Not statistically significant at 10% level.





**TABLE 8. Corner Portfolios Without Interest Rate Swaps**

| Category         | Corner Portfolios-Category Weights |                |                |                |                |                |
|------------------|------------------------------------|----------------|----------------|----------------|----------------|----------------|
|                  | P <sub>1</sub>                     | P <sub>2</sub> | P <sub>3</sub> | P <sub>4</sub> | P <sub>5</sub> | P <sub>6</sub> |
| 1                |                                    |                |                |                |                |                |
| 2                |                                    |                |                |                |                |                |
| 3                |                                    |                |                |                |                |                |
| 4                |                                    |                |                |                |                |                |
| 5                |                                    |                |                |                |                |                |
| 6                |                                    |                |                |                |                |                |
| 7                |                                    |                |                |                |                |                |
| 8                |                                    |                |                | 0.275          | 0.732          | 0.793          |
| 9                |                                    |                |                |                |                |                |
| 10               |                                    |                |                |                |                | 0.038          |
| 11               |                                    |                |                |                |                |                |
| 12               |                                    |                |                |                |                |                |
| 13               |                                    |                | 0.281          | 0.221          | 0.058          | 0.020          |
| 14               |                                    | 0.635          | 0.634          | 0.504          | 0.210          | 0.149          |
| 15               |                                    |                |                |                |                |                |
| 16               |                                    |                |                |                |                |                |
| 17               | 1.000                              | 0.365          | 0.086          |                |                |                |
| 18               |                                    |                |                |                |                |                |
| 19               |                                    |                |                |                |                |                |
| *b <sub>11</sub> | 0.511                              | 0.529          | 0.741          | 0.780          | 0.793          | 0.784          |
| *b <sub>12</sub> |                                    |                |                |                |                |                |
| #b <sub>21</sub> | 10.412                             | 9.546          | 9.050          | 7.797          | 5.934          | 5.586          |
| #b <sub>22</sub> |                                    |                |                |                |                |                |
| E <sup>a</sup>   | 1.4948                             | 1.4484         | 1.4117         | 1.3224         | 1.1881         | 1.1626         |
| σ <sup>a</sup>   | 4.1393                             | 3.7489         | 3.5192         | 3.0104         | 2.2866         | 2.1588         |

\* Value of portfolio's beta-one without interest rate swaps.  
# Value of portfolio's beta-two without interest rate swaps.  
<sup>a</sup> E and σ values are plotted in Figure 3.

**TABLE 8 -- Continued**

|                 | Corner Portfolios-Category Weights |                |                |                 |                 |                 |
|-----------------|------------------------------------|----------------|----------------|-----------------|-----------------|-----------------|
| Category        | P <sub>7</sub>                     | P <sub>8</sub> | P <sub>9</sub> | P <sub>10</sub> | P <sub>11</sub> | P <sub>12</sub> |
| 1               |                                    |                |                |                 |                 |                 |
| 2               |                                    |                |                |                 |                 |                 |
| 3               |                                    |                |                |                 |                 |                 |
| 4               |                                    |                |                |                 |                 | 0.619           |
| 5               |                                    |                |                |                 | 0.378           | 0.267           |
| 6               | 0.069                              | 0.285          | 0.331          | 0.367           | 0.324           | 0.114           |
| 7               |                                    |                |                |                 |                 |                 |
| 8               | 0.770                              | 0.690          | 0.669          | 0.633           | 0.298           |                 |
| 9               |                                    |                |                |                 |                 |                 |
| 10              | 0.041                              | 0.025          |                |                 |                 |                 |
| 11              |                                    |                |                |                 |                 |                 |
| 12              |                                    |                |                |                 |                 |                 |
| 13              |                                    |                |                |                 |                 |                 |
| 14              | 0.120                              |                |                |                 |                 |                 |
| 15              |                                    |                |                |                 |                 |                 |
| 16              |                                    |                |                |                 |                 |                 |
| 17              |                                    |                |                |                 |                 |                 |
| 18              |                                    |                |                |                 |                 |                 |
| 19              |                                    |                |                |                 |                 |                 |
| b <sub>11</sub> | 0.783                              | 0.820          | 0.827          | 0.827           | 0.810           | 0.669           |
| b <sub>12</sub> |                                    |                |                |                 |                 |                 |
| b <sub>21</sub> | 5.290                              | 4.447          | 4.345          | 4.292           | 3.480           | 1.989           |
| b <sub>22</sub> |                                    |                |                |                 |                 |                 |
| E               | 1.1388                             | 1.0692         | 1.0605         | 1.0554          | 0.9758          | 0.8271          |
| σ               | 2.0416                             | 1.7091         | 1.6688         | 1.6457          | 1.3059          | 0.7421          |

TABLE 8 -- Continued

|                 | Corner Portfolios-Category Weights |                 |                 |                 |                 |                 |
|-----------------|------------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Category        | P <sub>13</sub>                    | P <sub>14</sub> | P <sub>15</sub> | P <sub>16</sub> | P <sub>17</sub> | P <sub>18</sub> |
| 1               |                                    |                 |                 |                 |                 | 0.482           |
| 2               |                                    |                 |                 | 0.151           | 0.447           | 0.352           |
| 3               |                                    | 0.416           | 0.629           | 0.562           | 0.418           | 0.166           |
| 4               | 0.666                              | 0.469           | 0.351           | 0.287           | 0.135           |                 |
| 5               | 0.249                              | 0.115           | 0.020           |                 |                 |                 |
| 6               | 0.085                              |                 |                 |                 |                 |                 |
| 7               |                                    |                 |                 |                 |                 |                 |
| 8               |                                    |                 |                 |                 |                 |                 |
| 9               |                                    |                 |                 |                 |                 |                 |
| 10              |                                    |                 |                 |                 |                 |                 |
| 11              |                                    |                 |                 |                 |                 |                 |
| 12              |                                    |                 |                 |                 |                 |                 |
| 13              |                                    |                 |                 |                 |                 |                 |
| 14              |                                    |                 |                 |                 |                 |                 |
| 15              |                                    |                 |                 |                 |                 |                 |
| 16              |                                    |                 |                 |                 |                 |                 |
| 17              |                                    |                 |                 |                 |                 |                 |
| 18              |                                    |                 |                 |                 |                 |                 |
| 19              |                                    |                 |                 |                 |                 |                 |
| b <sub>11</sub> | 0.659                              | 0.529           | 0.467           | 0.425           | 0.353           | 0.202           |
| b <sub>12</sub> |                                    |                 |                 |                 |                 |                 |
| b <sub>21</sub> | 1.918                              | 1.320           | 1.067           | 0.926           | 0.700           | 0.347           |
| b <sub>22</sub> |                                    |                 |                 |                 |                 |                 |
| E               | 0.8199                             | 0.7494          | 0.7188          | 0.6988          | 0.6651          | 0.6089          |
| σ               | 0.7180                             | 0.4926          | 0.4037          | 0.3499          | 0.2639          | 0.1376          |

TABLE 8 -- Continued

|                 | Corner Portfolios-Category Weights |                 |
|-----------------|------------------------------------|-----------------|
| Category        | P <sub>19</sub>                    | P <sub>20</sub> |
| 1               | 0.719                              | 0.901           |
| 2               | 0.281                              | 0.099           |
| 3               |                                    |                 |
| 4               |                                    |                 |
| 5               |                                    |                 |
| 6               |                                    |                 |
| 7               |                                    |                 |
| 8               |                                    |                 |
| 9               |                                    |                 |
| 10              |                                    |                 |
| 11              |                                    |                 |
| 12              |                                    |                 |
| 13              |                                    |                 |
| 14              |                                    |                 |
| 15              |                                    |                 |
| 16              |                                    |                 |
| 17              |                                    |                 |
| 18              |                                    |                 |
| 19              |                                    |                 |
| b <sub>11</sub> | 0.145                              | 0.119           |
| b <sub>12</sub> |                                    |                 |
| b <sub>21</sub> | 0.231                              | 0.192           |
| b <sub>22</sub> |                                    |                 |
| E               | 0.5894                             | 0.5825          |
| σ               | 0.1066                             | 0.1027          |

**TABLE 9. The Enlarged Variance-Covariance Matrix With 20% Nontraded Assets**

[illegible]

**TABLE 9 -- Continued**

```

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.1283 0.0643 -0.1283
-0.0643
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.0643 0.1565 -0.0643
-0.1565
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.1283 -0.0643 0.1283
0.0643
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.0643 -0.1565 0.0643
0.1565

0.5788 0.6165 0.6803 0.7785 0.8806 0.9663 1.0100 1.1070 1.1298
1.2017 1.2241 1.1591 1.3636 1.4218 1.4145 1.4248 1.4948 1.4325
1.3590 0 0 0 0 (a)

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 1 (b)
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.05
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.05 (c)
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0.05
0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0.05

0.1044 0.2476 0.3878 0.5912 0.7839 0.8242 0.8448 0.8282 0.6423
0.5592 0.6753 1.2035 1.2649 0.5402 0.7547 0.9325 0.5108 0.7033
0.9635 -1 0 1 0 0 (d)
-0.1711 -0.3850 -0.7742 -1.5102 -2.5121 -3.3749 -4.1536
-4.8239 -5.6592 -6.3160 -6.8927 -7.4666 -8.6381 -9.0482
-9.8805 -10.7181 -10.4117 -10.5628 -10.9668 0 -1 0 1 0 (e)

```

- (a): The expected return vector.
- (b): The budget constraint.
- (c): Constraints for nontraded assets.
- (d): Constraint for portfolio's beta-one.
- (e): Constraint for portfolio's beta-two.

**TABLE 10. Corner Portfolios With 20% Nontraded Assets**

| Category         | Corner Portfolios-Category Weights |                |                |                |                |                |
|------------------|------------------------------------|----------------|----------------|----------------|----------------|----------------|
|                  | P <sub>1</sub>                     | P <sub>2</sub> | P <sub>3</sub> | P <sub>4</sub> | P <sub>5</sub> | P <sub>6</sub> |
| 1                |                                    |                |                |                |                |                |
| 2                |                                    |                |                |                |                |                |
| 3                |                                    |                |                |                |                |                |
| 4                |                                    |                |                |                |                |                |
| 5                |                                    |                |                |                |                | 0.559          |
| 6                | 0.050                              | 0.050          | 0.050          | 0.050          | 0.050          | 0.050          |
| 7                |                                    |                |                |                | 0.083          | 0.166          |
| 8                | 0.050                              | 0.050          | 0.050          | 0.050          | 0.050          | 0.050          |
| 9                |                                    |                |                |                |                |                |
| 10               | 0.050                              | 0.050          | 0.050          | 0.050          | 0.050          | 0.050          |
| 11               |                                    |                |                |                |                |                |
| 12               | 0.050                              | 0.050          | 0.050          | 0.050          | 0.050          | 0.050          |
| 13               |                                    |                | 0.257          | 0.331          | 0.295          |                |
| 14               |                                    | 0.544          | 0.543          | 0.469          | 0.422          | 0.075          |
| 15               |                                    |                |                |                |                |                |
| 16               |                                    |                |                |                |                |                |
| 17               | 0.800                              | 0.256          |                |                |                |                |
| 18               |                                    |                |                |                |                |                |
| 19               |                                    |                |                |                |                |                |
| *b <sub>11</sub> | 0.579                              | 0.595          | 0.789          | 0.843          | 0.842          | 0.790          |
| *b <sub>12</sub> |                                    |                |                |                |                |                |
| #b <sub>21</sub> |                                    |                |                |                |                |                |
| #b <sub>22</sub> | 9.428                              | 8.686          | 8.232          | 8.202          | 7.809          | 3.869          |
| E <sup>a</sup>   | 1.4175                             | 1.3778         | 1.3442         | 1.3399         | 1.3077         | 0.9881         |
| σ <sup>a</sup>   | 3.7123                             | 3.3862         | 3.1813         | 3.1585         | 2.9985         | 1.4497         |

\* Value of portfolio's beta-one without interest rate swaps and 20% nontraded assets.

# Value of portfolio's beta-two without interest rate swaps and 20% nontraded assets.

@ E and σ values are plotted in Figure 4.

TABLE 10 -- Continued

| Category        | Corner Portfolios-Category Weights |                |                |                 |                 |                 |
|-----------------|------------------------------------|----------------|----------------|-----------------|-----------------|-----------------|
|                 | P <sub>7</sub>                     | P <sub>8</sub> | P <sub>9</sub> | P <sub>10</sub> | P <sub>11</sub> | P <sub>12</sub> |
| 1               |                                    |                |                |                 |                 |                 |
| 2               |                                    |                |                |                 |                 |                 |
| 3               |                                    |                |                |                 | 0.297           | 0.648           |
| 4               |                                    |                | 0.303          | 0.667           | 0.503           | 0.152           |
| 5               | 0.637                              | 0.651          | 0.497          | 0.133           |                 |                 |
| 6               | 0.050                              | 0.050          | 0.050          | 0.050           | 0.050           | 0.050           |
| 7               | 0.163                              | 0.149          |                |                 |                 |                 |
| 8               | 0.050                              | 0.050          | 0.050          | 0.050           | 0.050           | 0.050           |
| 9               |                                    |                |                |                 |                 |                 |
| 10              | 0.050                              | 0.050          | 0.050          | 0.050           | 0.050           | 0.050           |
| 11              |                                    |                |                |                 |                 |                 |
| 12              | 0.050                              | 0.050          | 0.050          | 0.050           | 0.050           | 0.050           |
| 13              |                                    |                |                |                 |                 |                 |
| 14              |                                    |                |                |                 |                 |                 |
| 15              |                                    |                |                |                 |                 |                 |
| 16              |                                    |                |                |                 |                 |                 |
| 17              |                                    |                |                |                 |                 |                 |
| 18              |                                    |                |                |                 |                 |                 |
| 19              |                                    |                |                |                 |                 |                 |
| b <sub>11</sub> | 0.808                              | 0.807          | 0.739          | 0.669           | 0.583           | 0.512           |
| b <sub>12</sub> |                                    |                |                |                 |                 |                 |
| b <sub>21</sub> |                                    |                |                |                 |                 |                 |
| b <sub>22</sub> | 3.376                              | 3.354          | 2.805          | 2.440           | 2.089           | 1.831           |
| E               | 0.9473                             | 0.9455         | 0.8952         | 0.8581          | 0.8154          | 0.7809          |
| σ               | 1.2648                             | 1.2570         | 1.0432         | 0.9061          | 0.7722          | 0.6753          |

**TABLE 10 -- Continued**

|                 | Corner Portfolios-Category Weights |                 |                 |                 |
|-----------------|------------------------------------|-----------------|-----------------|-----------------|
| Category        | P <sub>13</sub>                    | P <sub>14</sub> | P <sub>15</sub> | P <sub>16</sub> |
| 1               |                                    |                 | 0.362           | 0.800           |
| 2               | 0.297                              | 0.547           | 0.438           |                 |
| 3               | 0.503                              | 0.253           |                 |                 |
| 4               |                                    |                 |                 |                 |
| 5               |                                    |                 |                 |                 |
| 6               | 0.050                              | 0.050           | 0.050           | 0.050           |
| 7               |                                    |                 |                 |                 |
| 8               | 0.050                              | 0.050           | 0.050           | 0.050           |
| 9               |                                    |                 |                 |                 |
| 10              | 0.050                              | 0.050           | 0.050           | 0.050           |
| 11              |                                    |                 |                 |                 |
| 12              | 0.050                              | 0.050           | 0.050           | 0.050           |
| 13              |                                    |                 |                 |                 |
| 14              |                                    |                 |                 |                 |
| 15              |                                    |                 |                 |                 |
| 16              |                                    |                 |                 |                 |
| 17              |                                    |                 |                 |                 |
| 18              |                                    |                 |                 |                 |
| 19              |                                    |                 |                 |                 |
| b <sub>11</sub> | 0.439                              | 0.404           | 0.317           | 0.254           |
| b <sub>12</sub> |                                    |                 |                 |                 |
| b <sub>21</sub> |                                    |                 |                 |                 |
| b <sub>22</sub> | 1.603                              | 1.506           | 1.330           | 1.236           |
| E               | 0.7470                             | 0.7310          | 0.7013          | 0.6847          |
| σ               | 0.5909                             | 0.5549          | 0.4932          | 0.4649          |

**TABLE 11. The Enlarged Variance-Covariance Matrix With 40% Nontraded Assets**

[illegible]

**TABLE 11 -- Continued**

```

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.1283 0.0643 -0.1283
-0.0643
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.0643 0.1565 -0.0643
-0.1565
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.1283 -0.0643 0.1283
0.0643
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.0643 -0.1565 0.0643
0.1565

0.5788 0.6165 0.6803 0.7785 0.8806 0.9663 1.0100 1.1070 1.1298
1.2017 1.2241 1.1591 1.3636 1.4218 1.4145 1.4248 1.4948 1.4325
1.3590 0 0 0 0 (a)

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 1 (b)
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.10
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.10 (c)
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0.10
0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0.10

0.1044 0.2476 0.3878 0.5912 0.7839 0.8242 0.8448 0.8282 0.6423
0.5592 0.6753 1.2035 1.2649 0.5402 0.7547 0.9325 0.5108 0.7033
0.9635 -1 0 1 0 0 (d)
-0.1711 -0.3850 -0.7742 -1.5102 -2.5121 -3.3749 -4.1536
-4.8239 -5.6592 -6.3160 -6.8927 -7.4666 -8.6381 -9.0482
-9.8805 -10.7181 -10.4117 -10.5628 -10.9668 0 -1 0 1 0 (e)

```

- (a): The expected return vector.
- (b): The budget constraint.
- (c): Constraints for nontraded assets.
- (d): Constraint for portfolio's beta-one.
- (e): Constraint for portfolio's beta-two.

**TABLE 12. Corner Portfolios With 40% Nontraded Assets**

|                  | Corner Portfolios-Category Weights |                |                |                |                |                |
|------------------|------------------------------------|----------------|----------------|----------------|----------------|----------------|
| Category         | P <sub>1</sub>                     | P <sub>2</sub> | P <sub>3</sub> | P <sub>4</sub> | P <sub>5</sub> | P <sub>6</sub> |
| 1                |                                    |                |                |                |                |                |
| 2                |                                    |                |                |                |                |                |
| 3                |                                    |                |                |                |                |                |
| 4                |                                    |                |                |                |                |                |
| 5                |                                    |                |                |                |                | 0.411          |
| 6                | 0.100                              | 0.100          | 0.100          | 0.100          | 0.100          | 0.100          |
| 7                |                                    |                |                |                | 0.047          | 0.108          |
| 8                | 0.100                              | 0.100          | 0.100          | 0.100          | 0.100          | 0.100          |
| 9                |                                    |                |                |                |                |                |
| 10               | 0.100                              | 0.100          | 0.100          | 0.100          | 0.100          | 0.100          |
| 11               |                                    |                |                |                |                |                |
| 12               | 0.100                              | 0.100          | 0.100          | 0.100          | 0.100          | 0.100          |
| 13               |                                    |                | 0.147          | 0.237          | 0.217          |                |
| 14               |                                    | 0.453          | 0.453          | 0.363          | 0.336          | 0.080          |
| 15               |                                    |                |                |                |                |                |
| 16               |                                    |                |                |                |                |                |
| 17               | 0.600                              | 0.147          |                |                |                |                |
| 18               |                                    |                |                |                |                |                |
| 19               |                                    |                |                |                |                |                |
| *b <sub>11</sub> | 0.648                              | 0.661          | 0.772          | 0.838          | 0.837          | 0.799          |
| *b <sub>12</sub> |                                    |                |                |                |                |                |
| *b <sub>21</sub> |                                    |                |                |                |                |                |
| *b <sub>22</sub> | 8.445                              | 7.827          | 7.567          | 7.530          | 7.307          | 4.409          |
| E <sup>a</sup>   | 1.3402                             | 1.3072         | 1.2879         | 1.2827         | 1.2644         | 1.0293         |
| σ <sup>a</sup>   | 3.2902                             | 3.0275         | 2.9106         | 2.8824         | 2.7922         | 1.6513         |

\* Value of portfolio's beta-one without interest rate swaps and 40% nontraded assets.

\* Value of portfolio's beta-two without interest rate swaps and 40% nontraded assets.

@ E and σ values are plotted in Figure 5.

**TABLE 12 -- Continued**

|                 | Corner Portfolios-Category Weights |                |                |                 |                 |                 |
|-----------------|------------------------------------|----------------|----------------|-----------------|-----------------|-----------------|
| Category        | P <sub>7</sub>                     | P <sub>8</sub> | P <sub>9</sub> | P <sub>10</sub> | P <sub>11</sub> | P <sub>12</sub> |
| 1               |                                    |                |                |                 |                 |                 |
| 2               |                                    |                |                |                 |                 | 0.597           |
| 3               |                                    |                |                |                 | 0.600           | 0.003           |
| 4               |                                    |                | 0.022          | 0.600           |                 |                 |
| 5               | 0.495                              | 0.589          | 0.578          |                 |                 |                 |
| 6               | 0.100                              | 0.100          | 0.100          | 0.100           | 0.100           | 0.100           |
| 7               | 0.105                              | 0.011          |                |                 |                 |                 |
| 8               | 0.100                              | 0.100          | 0.100          | 0.100           | 0.100           | 0.100           |
| 9               |                                    |                |                |                 |                 |                 |
| 10              | 0.100                              | 0.100          | 0.100          | 0.100           | 0.100           | 0.100           |
| 11              |                                    |                |                |                 |                 |                 |
| 12              | 0.100                              | 0.100          | 0.100          | 0.100           | 0.100           | 0.100           |
| 13              |                                    |                |                |                 |                 |                 |
| 14              |                                    |                |                |                 |                 |                 |
| 15              |                                    |                |                |                 |                 |                 |
| 16              |                                    |                |                |                 |                 |                 |
| 17              |                                    |                |                |                 |                 |                 |
| 18              |                                    |                |                |                 |                 |                 |
| 19              |                                    |                |                |                 |                 |                 |
| b <sub>11</sub> | 0.818                              | 0.813          | 0.808          | 0.696           | 0.574           | 0.491           |
| b <sub>12</sub> |                                    |                |                |                 |                 |                 |
| b <sub>21</sub> | 3.878                              | 3.723          | 3.683          | 3.104           | 2.663           | 2.430           |
| b <sub>22</sub> |                                    |                |                |                 |                 |                 |
| E               | 0.9853                             | 0.9732         | 0.9695         | 0.9105          | 0.8516          | 0.8135          |
| σ               | 1.4474                             | 1.3938         | 1.3781         | 1.1569          | 0.9920          | 0.9082          |

**TABLE 12 -- Continued**

| Category | Corner Portfolios-Category Weights |          |
|----------|------------------------------------|----------|
|          | $P_{13}$                           | $P_{14}$ |
| 1        | 0.005                              | 0.600    |
| 2        | 0.595                              |          |
| 3        |                                    |          |
| 4        |                                    |          |
| 5        |                                    |          |
| 6        | 0.100                              | 0.100    |
| 7        |                                    |          |
| 8        | 0.100                              | 0.100    |
| 9        |                                    |          |
| 10       | 0.100                              | 0.100    |
| 11       |                                    |          |
| 12       | 0.100                              | 0.100    |
| 13       |                                    |          |
| 14       |                                    |          |
| 15       |                                    |          |
| 16       |                                    |          |
| 17       |                                    |          |
| 18       |                                    |          |
| 19       |                                    |          |
| $b_{11}$ | 0.489                              | 0.404    |
| $b_{12}$ |                                    |          |
| $b_{21}$ | 2.428                              | 2.301    |
| $b_{22}$ |                                    |          |
| E        | 0.8131                             | 0.7907   |
| $\sigma$ | 0.9074                             | 0.8666   |

**TABLE 13. The Enlarged Variance-Covariance Matrix With 60% Nontraded Assets**

[illegible]

TABLE 13 -- Continued

|         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |          |          |          |          |    |    |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|----------|----------|----------|----------|----|----|---------|---------|---------|---------|
| 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0        | 0        | 0        | 0        | 0  | 0  | 0.1283  | 0.0643  | -0.1283 | -0.0643 |
| 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0        | 0        | 0        | 0        | 0  | 0  | 0.0643  | 0.1565  | -0.0643 | -0.1565 |
| 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0        | 0        | 0        | 0        | 0  | 0  | -0.1283 | -0.0643 | 0.1283  | 0.0643  |
| 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0        | 0        | 0        | 0        | 0  | 0  | -0.0643 | -0.1565 | 0.0643  | 0.1565  |
| 0.5788  | 0.6165  | 0.6803  | 0.7785  | 0.8806  | 0.9663  | 1.0100  | 1.1070  | 1.1298  | 1.2017  | 1.2241  | 1.1591  | 1.3636  | 1.4218  | 1.4145  | 1.4248   | 1.4948   | 1.4325   | 1.3590   | 0  | 0  | 0       | 0       | (a)     |         |
| 1       | 1       | 1       | 1       | 1       | 1       | 1       | 1       | 1       | 1       | 1       | 1       | 1       | 1       | 1       | 1        | 1        | 1        | 0        | 0  | 0  | 0       | 1       | (b)     |         |
| 0       | 0       | 0       | 0       | 0       | 1       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0        | 0        | 0        | 0        | 0  | 0  | 0       | 0       | 0.15    |         |
| 0       | 0       | 0       | 0       | 0       | 0       | 0       | 1       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0        | 0        | 0        | 0        | 0  | 0  | 0       | 0       | 0.15    | (c)     |
| 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 1       | 0       | 0       | 0       | 0       | 0       | 0        | 0        | 0        | 0        | 0  | 0  | 0       | 0       | 0.15    |         |
| 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 0       | 1       | 0       | 0       | 0       | 0        | 0        | 0        | 0        | 0  | 0  | 0       | 0       | 0.15    |         |
| 0.1044  | 0.2476  | 0.3878  | 0.5912  | 0.7839  | 0.8242  | 0.8448  | 0.8282  | 0.6423  | 0.5592  | 0.6753  | 1.2035  | 1.2649  | 0.5402  | 0.7547  | 0.9325   | 0.5108   | 0.7033   | 0.9635   | -1 | 0  | 1       | 0       | 0       | (d)     |
| -0.1711 | -0.3850 | -0.7742 | -1.5102 | -2.5121 | -3.3749 | -4.1536 | -4.8239 | -5.6592 | -6.3160 | -6.8927 | -7.4666 | -8.6381 | -9.0482 | -9.8805 | -10.7181 | -10.4117 | -10.5628 | -10.9668 | 0  | -1 | 0       | 1       | 0       | (e)     |

- (a): The expected return vector.  
(b): The budget constraint.  
(c): Constraints for nontraded assets.  
(d): Constraint for portfolio's beta-one.  
(e): Constraint for portfolio's beta-two.

**TABLE 14. Corner Portfolios With 60% Nontraded Assets**

| Category         | Corner Portfolios-Category Weights |                |                |                |                |                |
|------------------|------------------------------------|----------------|----------------|----------------|----------------|----------------|
|                  | P <sub>1</sub>                     | P <sub>2</sub> | P <sub>3</sub> | P <sub>4</sub> | P <sub>5</sub> | P <sub>6</sub> |
| 1                |                                    |                |                |                |                |                |
| 2                |                                    |                |                |                |                |                |
| 3                |                                    |                |                |                |                |                |
| 4                |                                    |                |                |                |                |                |
| 5                |                                    |                |                |                |                | 0.264          |
| 6                | 0.150                              | 0.150          | 0.150          | 0.150          | 0.150          | 0.150          |
| 7                |                                    |                |                |                | 0.011          | 0.050          |
| 8                | 0.150                              | 0.150          | 0.150          | 0.150          | 0.150          | 0.150          |
| 9                |                                    |                |                |                |                |                |
| 10               | 0.150                              | 0.150          | 0.150          | 0.150          | 0.150          | 0.150          |
| 11               |                                    |                |                |                |                |                |
| 12               | 0.150                              | 0.150          | 0.150          | 0.150          | 0.150          | 0.150          |
| 13               |                                    |                | 0.037          | 0.144          | 0.139          |                |
| 14               |                                    | 0.363          | 0.363          | 0.256          | 0.250          | 0.086          |
| 15               |                                    |                |                |                |                |                |
| 16               |                                    |                |                |                |                |                |
| 17               | 0.400                              | 0.037          |                |                |                |                |
| 18               |                                    |                |                |                |                |                |
| 19               |                                    |                |                |                |                |                |
| *b <sub>11</sub> | 0.717                              | 0.727          | 0.755          | 0.833          | 0.833          | 0.808          |
| *b <sub>12</sub> |                                    |                |                |                |                |                |
| *b <sub>21</sub> |                                    |                |                |                |                |                |
| *b <sub>22</sub> | 7.462                              | 6.967          | 6.901          | 6.854          | 6.805          | 4.948          |
| E <sup>a</sup>   | 1.2630                             | 1.2366         | 1.2317         | 1.2255         | 1.2212         | 1.0705         |
| σ <sup>a</sup>   | 2.8749                             | 2.6746         | 2.6451         | 2.6114         | 2.5903         | 1.8604         |

\* Value of portfolio's beta-one without interest rate swaps and 60% nontraded assets.

# Value of portfolio's beta-two without interest rate swaps and 60% nontraded assets.

@ E and σ values are plotted in Figure 6.

TABLE 14 -- Continued

|                 | Corner Portfolios-Category Weights |                |                |                 |                 |                 |
|-----------------|------------------------------------|----------------|----------------|-----------------|-----------------|-----------------|
| Category        | P <sub>7</sub>                     | P <sub>8</sub> | P <sub>9</sub> | P <sub>10</sub> | P <sub>11</sub> | P <sub>12</sub> |
| 1               |                                    |                |                |                 |                 | 0.400           |
| 2               |                                    |                |                |                 | 0.400           |                 |
| 3               |                                    |                |                | 0.400           |                 |                 |
| 4               |                                    |                | 0.400          |                 |                 |                 |
| 5               | 0.353                              | 0.400          |                |                 |                 |                 |
| 6               | 0.150                              | 0.150          | 0.150          | 0.150           | 0.150           | 0.150           |
| 7               | 0.047                              |                |                |                 |                 |                 |
| 8               | 0.150                              | 0.150          | 0.150          | 0.150           | 0.150           | 0.150           |
| 9               |                                    |                |                |                 |                 |                 |
| 10              | 0.150                              | 0.150          | 0.150          | 0.150           | 0.150           | 0.150           |
| 11              |                                    |                |                |                 |                 |                 |
| 12              | 0.150                              | 0.150          | 0.150          | 0.150           | 0.150           | 0.150           |
| 13              |                                    |                |                |                 |                 |                 |
| 14              |                                    |                |                |                 |                 |                 |
| 15              |                                    |                |                |                 |                 |                 |
| 16              |                                    |                |                |                 |                 |                 |
| 17              |                                    |                |                |                 |                 |                 |
| 18              |                                    |                |                |                 |                 |                 |
| 19              |                                    |                |                |                 |                 |                 |
| b <sub>11</sub> | 0.829                              | 0.826          | 0.749          | 0.667           | 0.611           | 0.554           |
| b <sub>12</sub> |                                    |                |                |                 |                 |                 |
| b <sub>21</sub> |                                    |                |                |                 |                 |                 |
| b <sub>22</sub> | 4.379                              | 4.302          | 3.901          | 3.607           | 3.451           | 3.366           |
| E               | 1.0234                             | 1.0174         | 0.9765         | 0.9372          | 0.9117          | 0.8966          |
| σ               | 1.6388                             | 1.6112         | 1.4603         | 1.3528          | 1.2979          | 1.2710          |

**TABLE 15. The Enlarged Variance-Covariance Matrix With 80% Nontraded Assets**

[illegible]

**TABLE 15 -- Continued**

```

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.1283 0.0643 -0.1283
-0.0643
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.0643 0.1565 -0.0643
-0.1565
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.1283 -0.0643 0.1283
0.0643
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.0643 -0.1565 0.0643
0.1565

0.5788 0.6165 0.6803 0.7785 0.8806 0.9663 1.0100 1.1070 1.1298
1.2017 1.2241 1.1591 1.3636 1.4218 1.4145 1.4248 1.4948 1.4325
1.3590 0 0 0 0 (a)

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 1 (b)
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.20
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.20 (c)
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0.20
0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0.20

0.1044 0.2476 0.3878 0.5912 0.7839 0.8242 0.8448 0.8282 0.6423
0.5592 0.6753 1.2035 1.2649 0.5402 0.7547 0.9325 0.5108 0.7033
0.9635 -1 0 1 0 0 (d)
-0.1711 -0.3850 -0.7742 -1.5102 -2.5121 -3.3749 -4.1536
-4.8239
-5.6592 -6.3160 -6.8927 -7.4666 -8.6381 -9.0482 -9.8805
-10.7181
-10.4117 -10.5628 -10.9668 0 -1 0 1 0 (e)

```

- (a): The expected return vector.
- (b): The budget constraint.
- (c): Constraints for nontraded assets.
- (d): Constraint for portfolio's beta-one.
- (e): Constraint for portfolio's beta-two.

**TABLE 16. Corner Portfolios With 80% Nontraded Assets**

| Category             | Corner Portfolios-Category Weights |                |                |                |                |                |
|----------------------|------------------------------------|----------------|----------------|----------------|----------------|----------------|
|                      | P <sub>1</sub>                     | P <sub>2</sub> | P <sub>3</sub> | P <sub>4</sub> | P <sub>5</sub> | P <sub>6</sub> |
| 1                    |                                    |                |                |                |                |                |
| 2                    |                                    |                |                |                |                |                |
| 3                    |                                    |                |                |                |                |                |
| 4                    |                                    |                |                |                |                | 0.200          |
| 5                    |                                    |                |                | 0.108          | 0.200          |                |
| 6                    | 0.200                              | 0.200          | 0.200          | 0.200          | 0.200          | 0.200          |
| 7                    |                                    |                |                |                |                |                |
| 8                    | 0.200                              | 0.200          | 0.200          | 0.200          | 0.200          | 0.200          |
| 9                    |                                    |                |                |                |                |                |
| 10                   | 0.200                              | 0.200          | 0.200          | 0.200          | 0.200          | 0.200          |
| 11                   |                                    |                |                |                |                |                |
| 12                   | 0.200                              | 0.200          | 0.200          | 0.200          | 0.200          | 0.200          |
| 13                   |                                    |                | 0.049          |                |                |                |
| 14                   |                                    | 0.200          | 0.151          | 0.092          |                |                |
| 15                   |                                    |                |                |                |                |                |
| 16                   |                                    |                |                |                |                |                |
| 17                   | 0.200                              |                |                |                |                |                |
| 18                   |                                    |                |                |                |                |                |
| 19                   |                                    |                |                |                |                |                |
| *<br>b <sub>11</sub> | 0.785                              | 0.791          | 0.827          | 0.817          | 0.840          | 0.801          |
| *<br>b <sub>12</sub> |                                    |                |                |                |                |                |
| *<br>b <sub>21</sub> | 6.479                              | 6.206          | 6.186          | 5.502          | 4.899          | 4.698          |
| *<br>b <sub>22</sub> |                                    |                |                |                |                |                |
| E <sup>a</sup>       | 1.1858                             | 1.1712         | 1.1683         | 1.1129         | 1.0629         | 1.0425         |
| σ <sup>a</sup>       | 2.4700                             | 2.3623         | 2.3476         | 2.0803         | 1.8434         | 1.7693         |

\* Value of portfolio's beta-one without interest rate swaps and 80% nontraded assets.  
# Value of portfolio's beta-two without interest rate swaps and 80% nontraded assets.  
@ E and σ values are plotted in Figure 7.

**TABLE 16 -- Continued**

|                 | Corner Portfolios-Category Weights |                |                |
|-----------------|------------------------------------|----------------|----------------|
| Category        | P <sub>7</sub>                     | P <sub>8</sub> | P <sub>9</sub> |
| 1               |                                    |                | 0.200          |
| 2               |                                    | 0.200          |                |
| 3               | 0.200                              |                |                |
| 4               |                                    |                |                |
| 5               |                                    |                |                |
| 6               | 0.200                              | 0.200          | 0.200          |
| 7               |                                    |                |                |
| 8               | 0.200                              | 0.200          | 0.200          |
| 9               |                                    |                |                |
| 10              | 0.200                              | 0.200          | 0.200          |
| 11              |                                    |                |                |
| 12              | 0.200                              | 0.200          | 0.200          |
| 13              |                                    |                |                |
| 14              |                                    |                |                |
| 15              |                                    |                |                |
| 16              |                                    |                |                |
| 17              |                                    |                |                |
| 18              |                                    |                |                |
| 19              |                                    |                |                |
| b <sub>11</sub> | 0.761                              | 0.733          | 0.704          |
| b <sub>12</sub> |                                    |                |                |
| b <sub>21</sub> |                                    |                |                |
| b <sub>22</sub> | 4.551                              | 4.473          | 4.430          |
| E               | 1.0229                             | 1.0101         | 1.0026         |
| σ               | 1.7165                             | 1.6894         | 1.6761         |

**TABLE 17. The Enlarged Variance-Covariance Matrix With 100% Nontraded Assets**

[illegible]

**TABLE 17 -- Continued**

```

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.1283 0.0643 -0.1283
-0.0643
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.0643 0.1565 -0.0643
-0.1565
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.1283 -0.0643 0.1283
0.0643
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.0643 -0.1565 0.0643
0.1565

0.5788 0.6165 0.6803 0.7785 0.8806 0.9663 1.0100 1.1070 1.1298
1.2017 1.2241 1.1591 1.3636 1.4218 1.4145 1.4248 1.4948 1.4325
1.3590 0 0 0 0 (a)

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 1 (b)
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.249999
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.249999 (c)
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0.249999
0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0.249999

0.1044 0.2476 0.3878 0.5912 0.7839 0.8242 0.8448 0.8282 0.6423
0.5592 0.6753 1.2035 1.2649 0.5402 0.7547 0.9325 0.5108 0.7033
0.9635 -1 0 1 0 0 (d)
-0.1711 -0.3850 -0.7742 -1.5102 -2.5121 -3.3749 -4.1536
-4.8239 -5.6592 -6.3160 -6.8927 -7.4666 -8.6381 -9.0482
-9.8805 -10.7181 -10.4117 -10.5628 -10.9668 0 -1 0 1 0 (e)

```

- (a): The expected return vector.
- (b): The budget constraint.
- (c): Constraints for nontraded assets.
- (d): Constraint for portfolio's beta-one.
- (e): Constraint for portfolio's beta-two.

TABLE 18. Corner Portfolios With 100% Nontraded Assets

|            | Corner Portfolios-Category Weights |
|------------|------------------------------------|
| Category   | $P_1$                              |
| 1          |                                    |
| 2          |                                    |
| 3          |                                    |
| 4          |                                    |
| 5          |                                    |
| 6          | 0.250                              |
| 7          |                                    |
| 8          | 0.250                              |
| 9          |                                    |
| 10         | 0.250                              |
| 11         |                                    |
| 12         | 0.250                              |
| 13         |                                    |
| 14         |                                    |
| 15         |                                    |
| 16         |                                    |
| 17         |                                    |
| 18         |                                    |
| 19         |                                    |
| $b_{11}$   | 0.845                              |
| $b_{12}$   |                                    |
| $b_{21}$   |                                    |
| $b_{22}$   | 5.495                              |
| $E^a$      | 1.1085                             |
| $\sigma^a$ | 2.0815                             |

\* Value of portfolio's beta-one without interest rate swaps and 100% nontraded assets.

# Value of portfolio's beta-two without interest rate swaps and 100% nontraded assets.

@ E and  $\sigma$  values are plotted in Figure 8.

**TABLE 19. Interest Rate Swap Quotations From  
"International Financing Review"**

**Issue 581 July 27, 1985\***

| <b>Amount</b> | <b>Period</b> | <b>Semi-annual rate against LIBOR**</b> |
|---------------|---------------|-----------------------------------------|
| US\$25m+      | 3 years       | 10.01***                                |
| US\$25m+      | 5 years       | 10.63                                   |
| US\$25m+      | 7 years       | 10.86                                   |
| US\$25m+      | 10 years      | 11.05                                   |

**Issue 585 August 24, 1985**

| <b>Amount</b> | <b>Period</b> | <b>Semi-annual rate against LIBOR</b> |
|---------------|---------------|---------------------------------------|
| US\$25m+      | 3 years       | 9.88                                  |
| US\$25m+      | 5 years       | 10.38                                 |
| US\$25m+      | 7 years       | 10.62                                 |
| US\$25m+      | 10 years      | 10.65                                 |

\* The samples listed here are weekly quotations.

\*\* All fixed rates are against 6 months US\$ LIBOR: Amounts between \$25m-\$50m and are applicable to bank counterparties of a minimum credit rating AA or equivalent.

\*\*\* Annual fixed rate,  $(1 + \text{semi-annual rate})^2 = 1 + \text{annual fixed rate}$ .

**TABLE 20. Monthly Holding Period Returns of Swap Adjustable Notes (Value in Percentages)**

| T        | $R_3^*$ | $R_5^*$ | $R_7^*$ | $R_{10}^*$ |
|----------|---------|---------|---------|------------|
| 01/85    | 0.8705  | 2.5165  | 3.2160  | 3.3901     |
| 02/85    | -0.2843 | -1.8303 | -2.8151 | -3.1518    |
| 03/85    | 1.8562  | 2.0861  | 2.3732  | 2.2277     |
| 04/85    | 2.0827  | 2.5193  | 2.5979  | 2.3945     |
| 05/85    | 3.4060  | 4.7727  | 5.8161  | 7.2994     |
| 06/85    | 1.2728  | 0.8750  | 0.3120  | 0.6669     |
| 07/85    | 0.5983  | 0.3807  | 0.6600  | 0.1348     |
| 08/85    | 0.7327  | 1.4583  | 1.9231  | 2.5490     |
| 09/85    | 1.5251  | 1.5632  | 1.5203  | 0.7177     |
| 10/85    | 0.7895  | 0.9734  | 1.0716  | 1.3236     |
| 11/85    | 2.0217  | 2.8364  | 3.2490  | 4.6751     |
| 12/85    | 1.9136  | 3.1378  | 3.7065  | 4.0738     |
| 01/86    | 0.6367  | 0.4496  | 0.6348  | 0.8650     |
| 02/86    | 1.2095  | 2.1190  | 3.0358  | 3.9713     |
| 03/86    | 3.0135  | 3.8955  | 5.1618  | 6.0505     |
| 04/86    | 0.4984  | 0.0333  | -0.4651 | -0.1715    |
| 05/86    | 1.5845  | 1.7852  | 1.2754  | 3.4158     |
| 06/86    | -0.3650 | -0.7381 | -0.4518 | -2.3833    |
| 07/86    | 1.1632  | 1.3835  | 0.8651  | 0.8429     |
| 08/86    | 2.7259  | 3.1826  | 3.2954  | 2.6607     |
| 09/86    | -1.2094 | -1.7097 | -2.1879 | -2.3509    |
| 10/86    | 1.0354  | 1.4110  | 1.4938  | 0.8546     |
| 11/86    | 1.2099  | 1.1949  | 2.0135  | 2.1877     |
| 12/86    | 1.5148  | 1.9677  | 2.2176  | 2.7293     |
| 01/87    | 0.4468  | 0.6283  | 0.7604  | 0.6775     |
| 02/87    | 0.2121  | 0.2191  | 0.2816  | 0.2060     |
| 03/87    | 0.3835  | 0.3507  | 0.2883  | 0.2803     |
| 04/87    | -2.9027 | -4.2693 | -5.7283 | -6.9026    |
| 05/87    | -1.0821 | -1.4422 | -1.8742 | -2.0284    |
| 06/87    | 2.2764  | 2.8102  | 3.2446  | 3.8257     |
| 07/87    | 0.8028  | 0.3160  | -0.1832 | -0.7933    |
| 08/87    | -0.3591 | -0.8810 | -1.2478 | -1.7383    |
| 09/87    | -0.8989 | -1.2871 | -1.9097 | -2.3765    |
| 10/87    | 2.5897  | 3.4640  | 4.3634  | 5.5195     |
| 11/87    | 0.5483  | -0.1901 | -0.2293 | -0.9096    |
| 12/87    | 0.8381  | 1.1945  | 1.3265  | 1.4682     |
| E        | 0.9072  | 1.0327  | 1.1003  | 1.1723     |
| $\sigma$ | 1.2783  | 1.8494  | 2.3450  | 2.8791     |

**TABLE 21. Monthly Returns of Interest Rate Swaps  
(Value in Percentages)**

| T        | S <sub>3</sub> | S <sub>5</sub> | S <sub>7</sub> | S <sub>10</sub> |
|----------|----------------|----------------|----------------|-----------------|
| 01/85    | 0.8600         | -0.0543        | -0.0076        | -0.2321         |
| 02/85    | -0.9174        | 0.0033         | -0.3957        | -0.3074         |
| 03/85    | -0.2470        | -0.3088        | 0.0960         | 0.1056          |
| 04/85    | 0.0218         | -0.1623        | -0.1982        | -0.2212         |
| 05/85    | 0.7964         | 0.7765         | 1.6633         | 0.7485          |
| 06/85    | -0.1381        | -0.0448        | 0.2899         | 0.2245          |
| 07/85    | -0.6040        | -0.6520        | -1.2701        | -1.0758         |
| 08/85    | 0.7649         | 0.5743         | 0.4203         | 0.0566          |
| 09/85    | -0.4772        | -0.4787        | -0.9891        | 1.1763          |
| 10/85    | 0.8655         | 1.0031         | 1.3295         | 0.1646          |
| 11/85    | -0.3199        | -0.3892        | -0.1856        | -1.7776         |
| 12/85    | 0.3820         | 0.2417         | 0.9489         | 0.4788          |
| 01/86    | 0.1437         | 0.1161         | -0.6888        | -0.6181         |
| 02/86    | 0.9285         | 1.4935         | 2.4618         | 2.5774          |
| 03/86    | -0.4853        | -0.3571        | -0.6559        | -0.8100         |
| 04/86    | 0.2197         | 0.5894         | 0.6903         | 0.6448          |
| 05/86    | -3.0114        | -4.0285        | -4.2584        | -7.0125         |
| 06/86    | 2.7346         | 3.9026         | 4.1694         | 7.0245          |
| 07/86    | 0.2701         | -0.0263        | 0.1743         | -0.1531         |
| 08/86    | -0.1207        | -0.0823        | 0.3021         | 0.9687          |
| 09/86    | 0.2351         | 0.2570         | 0.0463         | -0.3436         |
| 10/86    | 0.4764         | 0.3338         | 0.3811         | 1.0134          |
| 11/86    | -0.2239        | 0.1862         | -0.5796        | -0.8249         |
| 12/86    | -1.3824        | -1.6843        | -1.8615        | -2.5624         |
| 01/87    | 0.4860         | 0.4545         | 0.4808         | 0.3730          |
| 02/87    | 0.2973         | 0.3951         | 0.4641         | 0.2880          |
| 03/87    | -0.5354        | -0.9271        | -1.2573        | -1.7075         |
| 04/87    | 1.0961         | 1.5350         | 2.4324         | 3.0290          |
| 05/87    | 0.8510         | 1.1286         | 1.2090         | 0.5767          |
| 06/87    | -1.0121        | -1.5695        | -2.0327        | -2.4429         |
| 07/87    | -0.4827        | -0.4348        | -0.0945        | -0.2419         |
| 08/87    | 0.0863         | 0.3489         | 0.2144         | 0.1416          |
| 09/87    | -0.3163        | -0.9252        | -0.5929        | -0.9461         |
| 10/87    | 0.5359         | 0.4890         | -0.1800        | 0.1365          |
| 11/87    | 0.1918         | 0.8283         | 0.8336         | 0.6789          |
| 12/87    | 0.1594         | 0.1432         | 0.2895         | 0.3955          |
| E        | 0.0591         | 0.0743         | 0.1014         | -0.0132         |
| $\sigma$ | 0.9046         | 1.1880         | 1.4136         | 2.0211          |

TABLE 22

Regression Results of Two Factor Model for Interest Rate Swaps

$$R_i = \alpha_i + \beta_{i1}(I_2 - I_1) + \beta_{i2}I_2 + u_i$$

| Maturity<br>(years) | $\alpha_i$<br>(t-value)* | $\beta_{i1}$<br>(t-value)* | $\beta_{i2}$<br>(t-value)* | $R_i^2$ | $V(u_i)$ |
|---------------------|--------------------------|----------------------------|----------------------------|---------|----------|
| 3                   | 0.0283<br>(0.183)        | 0.1480<br>(0.301)          | -0.5100<br>(-1.135)        | 0.0417  | 0.7841   |
| 5                   | 0.0434<br>(0.213)        | 0.1565<br>(0.240)          | -0.5128<br>(-0.861)        | 0.0242  | 1.3770   |
| 7                   | 0.0619<br>(0.255)        | 0.1427<br>(0.185)          | -0.6394<br>(-0.904)        | 0.0282  | 1.9420   |
| 10                  | -0.0636<br>(-0.183)      | 0.6253<br>(0.565)          | -0.9365<br>(-0.925)        | 0.0254  | 3.9809   |

\* All the t-values in this table are not statistically significant at 10% level.

**TABLE 23. The Enlarged Variance-Covariance Matrix With Swaps**

[illegible]

**TABLE 23 -- Continued**[illegible]

- (a): The expected return vector.
- (b): The budget constraint.
- (c): Constraint for the notional principal of interest rate swaps.
- (d): Constraint for portfolio's beta-one.
- (e): Constraint for portfolio's beta-two.

**TABLE 24. Corner Portfolios With Interest Rate Swaps**

|                   | Corner Portfolios-Category Weights |                |                |                |                |                |
|-------------------|------------------------------------|----------------|----------------|----------------|----------------|----------------|
| Category          | P <sub>1</sub>                     | P <sub>2</sub> | P <sub>3</sub> | P <sub>4</sub> | P <sub>5</sub> | P <sub>6</sub> |
| 1                 |                                    |                |                |                |                |                |
| 2                 |                                    |                |                |                |                |                |
| 3                 |                                    |                |                |                |                |                |
| 4                 |                                    |                |                |                |                |                |
| 5                 |                                    |                |                |                |                |                |
| 6                 |                                    |                |                |                |                |                |
| 7                 |                                    |                |                |                |                |                |
| 8                 |                                    |                |                |                |                |                |
| 9                 |                                    |                |                |                |                | 0.233          |
| 10                |                                    |                |                |                |                |                |
| 11                |                                    |                |                |                |                |                |
| 12                |                                    |                |                |                |                |                |
| 13                |                                    |                |                |                | 0.274          | 0.223          |
| 14                |                                    |                | 0.357          | 0.655          | 0.654          | 0.544          |
| 15                |                                    |                |                |                |                |                |
| 16                |                                    |                |                |                |                |                |
| 17                | 1.000                              | 1.000          | 0.643          | 0.345          | 0.072          |                |
| 18                |                                    |                |                |                |                |                |
| 19                |                                    |                |                |                |                |                |
| S <sub>3</sub>    |                                    |                |                | 0.124          | 0.200          | 0.242          |
| S <sub>5</sub>    |                                    | 0.302          | 0.373          | 0.360          | 0.350          | 0.343          |
| S <sub>7</sub>    | 1.000                              | 0.698          | 0.627          | 0.516          | 0.449          | 0.415          |
| S <sub>10</sub>   |                                    |                |                |                |                |                |
| S <sub>LA</sub>   |                                    |                |                |                |                |                |
|                   |                                    |                |                |                |                |                |
| **B <sub>11</sub> | 0.654                              | 0.658          | 0.669          | 0.678          | 0.885          | 0.918          |
| **B <sub>12</sub> |                                    |                |                |                |                |                |
| ##B <sub>21</sub> |                                    |                |                |                |                |                |
| ##B <sub>22</sub> | 11.051                             | 11.013         | 10.516         | 10.096         | 9.604          | 8.537          |
|                   |                                    |                |                |                |                |                |
| E <sup>a</sup>    | 1.5962                             | 1.5880         | 1.5600         | 1.5334         | 1.4946         | 1.4173         |
| σ <sup>a</sup>    | 4.5825                             | 4.4717         | 4.2247         | 4.0314         | 3.7960         | 3.3649         |

\* Slack variable for the notional principal of interest rate swaps.

\*\* Value of portfolio's beta-one with interest rate swaps.

## Value of portfolio's beta-two with interest rate swaps.

@ E and σ values are plotted in Figure 9.

TABLE 24 -- Continued

|                 | Corner Portfolios-Category Weights |                |                |                 |                 |                 |
|-----------------|------------------------------------|----------------|----------------|-----------------|-----------------|-----------------|
| Category        | P <sub>7</sub>                     | P <sub>8</sub> | P <sub>9</sub> | P <sub>10</sub> | P <sub>11</sub> | P <sub>12</sub> |
| 1               |                                    |                |                |                 |                 |                 |
| 2               |                                    |                |                |                 |                 |                 |
| 3               |                                    |                |                |                 |                 |                 |
| 4               |                                    |                |                |                 |                 |                 |
| 5               |                                    |                |                |                 |                 |                 |
| 6               |                                    |                |                |                 |                 |                 |
| 7               |                                    |                |                |                 | 0.261           | 0.288           |
| 8               | 0.788                              | 0.818          | 0.828          | 0.832           | 0.726           | 0.712           |
| 9               |                                    |                |                |                 |                 |                 |
| 10              |                                    | 0.019          | 0.026          | 0.028           | 0.014           |                 |
| 11              |                                    |                |                |                 |                 |                 |
| 12              |                                    |                |                |                 |                 |                 |
| 13              | 0.025                              | 0.006          |                |                 |                 |                 |
| 14              | 0.187                              | 0.157          | 0.146          | 0.140           |                 |                 |
| 15              |                                    |                |                |                 |                 |                 |
| 16              |                                    |                |                |                 |                 |                 |
| 17              |                                    |                |                |                 |                 |                 |
| 18              |                                    |                |                |                 |                 |                 |
| 19              |                                    |                |                |                 |                 |                 |
| S <sub>3</sub>  | 0.337                              | 0.344          | 0.337          | 0.333           | 0.235           | 0.226           |
| S <sub>5</sub>  | 0.325                              | 0.324          | 0.318          | 0.315           | 0.229           | 0.222           |
| S <sub>7</sub>  | 0.337                              | 0.331          | 0.325          | 0.322           | 0.237           | 0.229           |
| S <sub>10</sub> |                                    |                |                |                 |                 |                 |
| S <sub>1A</sub> |                                    |                | 0.021          | 0.030           | 0.299           | 0.323           |
| B <sub>11</sub> | 0.934                              | 0.930          | 0.925          | 0.925           | 0.928           | 0.928           |
| B <sub>12</sub> |                                    |                |                |                 |                 |                 |
| B <sub>21</sub> |                                    |                |                |                 |                 |                 |
| B <sub>22</sub> | 6.265                              | 6.092          | 6.020          | 5.993           | 4.855           | 4.783           |
| E               | 1.2507                             | 1.2378         | 1.2317         | 1.2294          | 1.1265          | 1.1196          |
| σ               | 2.4951                             | 2.4329         | 2.4042         | 2.3930          | 1.9154          | 1.8841          |

TABLE 24 -- Continued

|                 | Corner Portfolios-Category Weights |                 |                 |                 |                 |                 |
|-----------------|------------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Category        | P <sub>13</sub>                    | P <sub>14</sub> | P <sub>15</sub> | P <sub>16</sub> | P <sub>17</sub> | P <sub>18</sub> |
| 1               |                                    |                 |                 |                 |                 |                 |
| 2               |                                    |                 |                 |                 |                 |                 |
| 3               |                                    |                 |                 |                 |                 |                 |
| 4               |                                    |                 | 0.616           | 0.673           | 0.404           | 0.514           |
| 5               |                                    | 0.382           | 0.269           | 0.247           | 0.478           | 0.416           |
| 6               | 0.369                              | 0.325           | 0.115           | 0.080           | 0.118           | 0.070           |
| 7               |                                    |                 |                 |                 |                 |                 |
| 8               | 0.631                              | 0.293           |                 |                 |                 |                 |
| 9               |                                    |                 |                 |                 |                 |                 |
| 10              |                                    |                 |                 |                 |                 |                 |
| 11              |                                    |                 |                 |                 |                 |                 |
| 12              |                                    |                 |                 |                 |                 |                 |
| 13              |                                    |                 |                 |                 |                 |                 |
| 14              |                                    |                 |                 |                 |                 |                 |
| 15              |                                    |                 |                 |                 |                 |                 |
| 16              |                                    |                 |                 |                 |                 |                 |
| 17              |                                    |                 |                 |                 |                 |                 |
| 18              |                                    |                 |                 |                 |                 |                 |
| 19              |                                    |                 |                 |                 |                 |                 |
| S <sub>3</sub>  | 0.192                              | 0.100           | 0.026           | 0.018           | 0.005           |                 |
| S <sub>5</sub>  | 0.195                              | 0.117           | 0.043           | 0.037           | 0.019           | 0.014           |
| S <sub>7</sub>  | 0.203                              | 0.125           | 0.049           | 0.041           | 0.024           | 0.018           |
| S <sub>10</sub> |                                    |                 |                 |                 |                 |                 |
| S <sub>LA</sub> | 0.409                              | 0.659           | 0.882           | 0.903           | 0.952           | 0.967           |
| B <sub>11</sub> | 0.915                              | 0.861           | 0.687           | 0.672           | 0.539           | 0.505           |
| B <sub>12</sub> |                                    |                 |                 |                 |                 |                 |
| B <sub>21</sub> |                                    |                 |                 |                 |                 |                 |
| B <sub>22</sub> | 4.617                              | 3.660           | 2.061           | 1.962           | 1.358           | 1.221           |
| E               | 1.1056                             | 1.0019          | 0.8373          | 0.8269          | 0.7550          | 0.7381          |
| σ               | 1.8044                             | 1.3896          | 0.7720          | 0.7371          | 0.5080          | 0.4574          |

TABLE 24 -- Continued

| Category        | Corner Portfolios-Category Weights |                 |                 |                 |                 |                 |
|-----------------|------------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 | P <sub>19</sub>                    | P <sub>20</sub> | P <sub>21</sub> | P <sub>22</sub> | P <sub>23</sub> | P <sub>24</sub> |
| 1               |                                    |                 |                 | 0.283           | 0.481           | 0.494           |
| 2               |                                    | 0.137           | 0.447           | 0.391           | 0.352           | 0.349           |
| 3               | 0.631                              | 0.570           | 0.419           | 0.270           | 0.166           | 0.157           |
| 4               | 0.351                              | 0.292           | 0.134           | 0.055           |                 |                 |
| 5               | 0.018                              |                 |                 |                 |                 |                 |
| 6               |                                    |                 |                 |                 |                 |                 |
| 7               |                                    |                 |                 |                 |                 |                 |
| 8               |                                    |                 |                 |                 |                 |                 |
| 9               |                                    |                 |                 |                 |                 |                 |
| 10              |                                    |                 |                 |                 |                 |                 |
| 11              |                                    |                 |                 |                 |                 |                 |
| 12              |                                    |                 |                 |                 |                 |                 |
| 13              |                                    |                 |                 |                 |                 |                 |
| 14              |                                    |                 |                 |                 |                 |                 |
| 15              |                                    |                 |                 |                 |                 |                 |
| 16              |                                    |                 |                 |                 |                 |                 |
| 17              |                                    |                 |                 |                 |                 |                 |
| 18              |                                    |                 |                 |                 |                 |                 |
| 19              |                                    |                 |                 |                 |                 |                 |
| S <sub>3</sub>  |                                    |                 |                 |                 |                 |                 |
| S <sub>5</sub>  | 0.009                              | 0.007           | 0.002           |                 |                 |                 |
| S <sub>7</sub>  | 0.013                              | 0.010           | 0.005           | 0.002           |                 |                 |
| S <sub>10</sub> |                                    |                 |                 |                 |                 |                 |
| S <sub>LA</sub> | 0.978                              | 0.982           | 0.994           | 0.998           | 1.000           | 1.000           |
| B <sub>11</sub> | 0.470                              | 0.431           | 0.353           | 0.264           | 0.202           | 0.199           |
| B <sub>12</sub> |                                    |                 |                 |                 |                 |                 |
| B <sub>21</sub> |                                    |                 |                 |                 |                 |                 |
| B <sub>22</sub> | 1.076                              | 0.946           | 0.703           | 0.493           | 0.347           | 0.340           |
| E               | 0.7203                             | 0.7018          | 0.6655          | 0.6322          | 0.6090          | 0.6079          |
| $\sigma$        | 0.4073                             | 0.3576          | 0.2649          | 0.1869          | 0.1377          | 0.1356          |

TABLE 24 -- Continued

|                 | Corner Portfolios-Category Weights |                 |
|-----------------|------------------------------------|-----------------|
| Category        | P <sub>25</sub>                    | P <sub>26</sub> |
| 1               | 0.719                              | 0.901           |
| 2               | 0.281                              | 0.099           |
| 3               |                                    |                 |
| 4               |                                    |                 |
| 5               |                                    |                 |
| 6               |                                    |                 |
| 7               |                                    |                 |
| 8               |                                    |                 |
| 9               |                                    |                 |
| 10              |                                    |                 |
| 11              |                                    |                 |
| 12              |                                    |                 |
| 13              |                                    |                 |
| 14              |                                    |                 |
| 15              |                                    |                 |
| 16              |                                    |                 |
| 17              |                                    |                 |
| 18              |                                    |                 |
| 19              |                                    |                 |
| S <sub>3</sub>  |                                    |                 |
| S <sub>5</sub>  |                                    |                 |
| S <sub>7</sub>  |                                    |                 |
| S <sub>10</sub> |                                    |                 |
| S <sub>LA</sub> | 1.000                              | 1.000           |
| B <sub>11</sub> | 0.145                              | 0.119           |
| B <sub>12</sub> |                                    |                 |
| B <sub>21</sub> |                                    |                 |
| B <sub>22</sub> | 0.231                              | 0.192           |
| E               | 0.5894                             | 0.5825          |
| σ               | 0.1066                             | 0.1027          |



**TABLE 25 -- Continued**[illegible]

- (a): The expected return vector.
- (b): The budget constraint.
- (c): Constraint for the notional principal of interest rate swaps.
- (d): Constraint for nontraded assets.
- (e): Constraint for portfolio's beta-one.
- (f): Constraint for portfolio's beta-two.

**TABLE 26. Corner Portfolios With Swaps and 20% Nontraded Assets**

| Category          | Corner Portfolios-Category Weights |                |                |                |                |                |
|-------------------|------------------------------------|----------------|----------------|----------------|----------------|----------------|
|                   | P <sub>1</sub>                     | P <sub>2</sub> | P <sub>3</sub> | P <sub>4</sub> | P <sub>5</sub> | P <sub>6</sub> |
| 1                 |                                    |                |                |                |                |                |
| 2                 |                                    |                |                |                |                |                |
| 3                 |                                    |                |                |                |                |                |
| 4                 |                                    |                |                |                |                |                |
| 5                 |                                    |                |                |                |                |                |
| 6                 | 0.050                              | 0.050          | 0.050          | 0.050          | 0.050          | 0.050          |
| 7                 |                                    |                |                |                |                |                |
| 8                 | 0.050                              | 0.050          | 0.050          | 0.050          | 0.050          | 0.050          |
| 9                 |                                    |                |                |                |                |                |
| 10                | 0.050                              | 0.050          | 0.050          | 0.050          | 0.050          | 0.050          |
| 11                |                                    |                |                |                |                |                |
| 12                | 0.050                              | 0.050          | 0.050          | 0.050          | 0.050          | 0.050          |
| 13                |                                    |                |                |                |                |                |
| 14                |                                    |                | 0.161          | 0.564          | 0.563          | 0.477          |
| 15                |                                    |                |                |                |                |                |
| 16                |                                    |                |                |                |                |                |
| 17                | 0.800                              | 0.800          | 0.639          | 0.236          |                |                |
| 18                |                                    |                |                |                |                |                |
| 19                |                                    |                |                |                |                |                |
| S <sub>3</sub>    |                                    |                |                | 0.168          | 0.234          | 0.273          |
| S <sub>5</sub>    |                                    | 0.338          | 0.370          | 0.352          | 0.344          | 0.340          |
| S <sub>7</sub>    | 1.000                              | 0.662          | 0.630          | 0.479          | 0.422          | 0.386          |
| S <sub>10</sub>   |                                    |                |                |                |                |                |
| *S <sub>LA</sub>  |                                    |                |                |                |                |                |
|                   |                                    |                |                |                |                |                |
| **B <sub>11</sub> | 0.722                              | 0.727          | 0.732          | 0.744          | 0.923          | 0.986          |
| **B <sub>12</sub> |                                    |                |                |                |                |                |
| ##B <sub>21</sub> |                                    |                |                |                |                |                |
| ##B <sub>22</sub> | 10.068                             | 10.025         | 9.801          | 9.232          | 8.806          | 8.766          |
|                   |                                    |                |                |                |                |                |
| E <sup>a</sup>    | 1.5189                             | 1.5098         | 1.4972         | 1.4611         | 1.4275         | 1.4209         |
| σ <sup>a</sup>    | 4.1792                             | 4.0492         | 3.9367         | 3.6674         | 3.4686         | 3.4343         |

\* Slack variable for the notional principal of interest rate swaps.

\*\* Value of portfolio's beta-one with interest rate swaps.

## Value of portfolio's beta-two with interest rate swaps.

@ E and σ values are plotted in Figure 10.

TABLE 26 -- Continued

| Category        | Corner Portfolios-Category Weights |                |                |                 |                 |                 |
|-----------------|------------------------------------|----------------|----------------|-----------------|-----------------|-----------------|
|                 | P <sub>7</sub>                     | P <sub>8</sub> | P <sub>9</sub> | P <sub>10</sub> | P <sub>11</sub> | P <sub>12</sub> |
| 1               |                                    |                |                |                 |                 |                 |
| 2               |                                    |                |                |                 |                 |                 |
| 3               |                                    |                |                |                 |                 |                 |
| 4               |                                    |                |                |                 |                 |                 |
| 5               |                                    | 0.261          | 0.554          | 0.641           | 0.666           | 0.280           |
| 6               | 0.050                              | 0.050          | 0.050          | 0.050           | 0.050           | 0.520           |
| 7               | 0.077                              | 0.116          | 0.162          | 0.159           | 0.134           | 0.050           |
| 8               | 0.050                              | 0.050          | 0.050          | 0.050           | 0.050           |                 |
| 9               |                                    |                |                |                 |                 | 0.050           |
| 10              | 0.050                              | 0.050          | 0.050          | 0.050           | 0.050           |                 |
| 11              |                                    |                |                |                 |                 | 0.050           |
| 12              | 0.050                              | 0.050          | 0.050          | 0.050           | 0.050           |                 |
| 13              | 0.290                              | 0.153          |                |                 |                 | 0.050           |
| 14              | 0.433                              | 0.271          | 0.084          |                 |                 |                 |
| 15              |                                    |                |                |                 |                 |                 |
| 16              |                                    |                |                |                 |                 |                 |
| 17              |                                    |                |                |                 |                 |                 |
| 18              |                                    |                |                |                 |                 |                 |
| 19              |                                    |                |                |                 |                 |                 |
| S <sub>3</sub>  | 0.285                              | 0.339          | 0.189          | 0.144           | 0.133           | 0.091           |
| S <sub>5</sub>  | 0.338                              | 0.326          | 0.189          | 0.148           | 0.140           | 0.102           |
| S <sub>7</sub>  | 0.377                              | 0.335          | 0.196          | 0.155           | 0.147           | 0.108           |
| S <sub>10</sub> |                                    |                |                |                 |                 |                 |
| S <sub>LA</sub> |                                    |                | 0.426          | 0.552           | 0.580           | 0.699           |
| B <sub>11</sub> | 0.986                              | 0.961          | 0.873          | 0.874           | 0.869           | 0.789           |
| B <sub>12</sub> |                                    |                |                |                 |                 |                 |
| B <sub>21</sub> |                                    |                |                |                 |                 |                 |
| B <sub>22</sub> | 8.402                              | 6.560          | 4.243          | 3.619           | 3.561           | 2.966           |
| E               | 1.3908                             | 1.2399         | 1.0377         | 0.9821          | 0.9766          | 0.9215          |
| σ               | 3.2880                             | 2.5670         | 1.6315         | 1.3846          | 1.3611          | 1.1315          |

TABLE 26 -- Continued

| Category        | Corner Portfolios-Category Weights |                 |                 |                 |                 |                 |
|-----------------|------------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 | P <sub>13</sub>                    | P <sub>14</sub> | P <sub>15</sub> | P <sub>16</sub> | P <sub>17</sub> | P <sub>18</sub> |
| 1               |                                    |                 |                 |                 |                 |                 |
| 2               |                                    |                 |                 |                 |                 |                 |
| 3               |                                    | 0.289           | 0.294           | 0.649           | 0.257           | 0.297           |
| 4               | 0.672                              | 0.511           | 0.506           | 0.151           | 0.523           | 0.503           |
| 5               | 0.128                              |                 |                 |                 |                 |                 |
| 6               | 0.050                              | 0.050           | 0.050           | 0.050           | 0.050           | 0.050           |
| 7               |                                    |                 |                 |                 |                 |                 |
| 8               | 0.050                              | 0.050           | 0.050           | 0.050           | 0.050           | 0.050           |
| 9               |                                    |                 |                 |                 |                 |                 |
| 10              | 0.050                              | 0.050           | 0.050           | 0.050           | 0.050           | 0.050           |
| 11              |                                    |                 |                 |                 |                 |                 |
| 12              | 0.050                              | 0.050           | 0.050           | 0.050           | 0.050           | 0.050           |
| 13              |                                    |                 |                 |                 |                 |                 |
| 14              |                                    |                 |                 |                 |                 |                 |
| 15              |                                    |                 |                 |                 |                 |                 |
| 16              |                                    |                 |                 |                 |                 |                 |
| 17              |                                    |                 |                 |                 |                 |                 |
| 18              |                                    |                 |                 |                 |                 |                 |
| 19              |                                    |                 |                 |                 |                 |                 |
| S <sub>3</sub>  | 0.012                              |                 |                 |                 |                 |                 |
| S <sub>5</sub>  | 0.039                              | 0.026           | 0.026           | 0.004           |                 |                 |
| S <sub>7</sub>  | 0.046                              | 0.033           | 0.032           | 0.010           | 0.005           | 0.005           |
| S <sub>10</sub> |                                    |                 |                 |                 |                 |                 |
| S <sub>1A</sub> | 0.902                              | 0.941           | 0.942           | 0.985           | 0.995           | 0.995           |
| B <sub>11</sub> | 0.683                              | 0.594           | 0.593           | 0.514           | 0.450           | 0.440           |
| B <sub>12</sub> |                                    |                 |                 |                 |                 |                 |
| B <sub>21</sub> |                                    |                 |                 |                 |                 |                 |
| B <sub>22</sub> | 2.491                              | 2.129           | 2.125           | 1.839           | 1.637           | 1.606           |
| E               | 0.8659                             | 0.8214          | 0.8208          | 0.7822          | 0.7521          | 0.7475          |
| σ               | 0.9277                             | 0.7886          | 0.7869          | 0.6783          | 0.6032          | 0.5921          |

TABLE 26 -- Continued

|                 | Corner Portfolios-Category Weights |                 |                 |                 |
|-----------------|------------------------------------|-----------------|-----------------|-----------------|
| Category        | P <sub>19</sub>                    | P <sub>20</sub> | P <sub>21</sub> | P <sub>22</sub> |
| 1               |                                    |                 | 0.362           | 0.800           |
| 2               | 0.438                              | 0.547           | 0.438           |                 |
| 3               | 0.362                              | 0.253           |                 |                 |
| 4               |                                    |                 |                 |                 |
| 5               |                                    |                 |                 |                 |
| 6               | 0.050                              | 0.050           | 0.050           | 0.050           |
| 7               |                                    |                 |                 |                 |
| 8               | 0.050                              | 0.050           | 0.050           | 0.050           |
| 9               |                                    |                 |                 |                 |
| 10              | 0.050                              | 0.050           | 0.050           | 0.050           |
| 11              |                                    |                 |                 |                 |
| 12              | 0.050                              | 0.050           | 0.050           | 0.050           |
| 13              |                                    |                 |                 |                 |
| 14              |                                    |                 |                 |                 |
| 15              |                                    |                 |                 |                 |
| 16              |                                    |                 |                 |                 |
| 17              |                                    |                 |                 |                 |
| 18              |                                    |                 |                 |                 |
| 19              |                                    |                 |                 |                 |
| S <sub>3</sub>  |                                    |                 |                 |                 |
| S <sub>5</sub>  |                                    |                 |                 |                 |
| S <sub>7</sub>  |                                    |                 |                 |                 |
| S <sub>10</sub> |                                    |                 |                 |                 |
| S <sub>LA</sub> | 1.000                              | 1.000           | 1.000           | 1.000           |
| B <sub>11</sub> | 0.420                              | 0.404           | 0.317           | 0.254           |
| B <sub>12</sub> |                                    |                 |                 |                 |
| B <sub>21</sub> |                                    |                 |                 |                 |
| B <sub>22</sub> | 1.548                              | 1.506           | 1.330           | 1.236           |
| E               | 0.7380                             | 0.7310          | 0.7013          | 0.6847          |
| σ               | 0.5698                             | 0.5545          | 0.4932          | 0.4649          |

**TABLE 27. The Enlarged Variance-Covariance Matrix With Swaps and 40% Nontraded Assets**

[illegible]

**TABLE 27 -- Continued**[illegible]

- (a): The expected return vector.
- (b): The budget constraint.
- (c): Constraint for the notional principal of interest rate swaps.
- (d): Constraint for nontraded assets.
- (e): Constraint for portfolio's beta-one.
- (f): Constraint for portfolio's beta-two.

**TABLE 29. Corner Portfolios With Swaps and 40% Nontraded Assets**

| Category          | Corner Portfolios-Category Weights |                |                |                |                |                |
|-------------------|------------------------------------|----------------|----------------|----------------|----------------|----------------|
|                   | P <sub>1</sub>                     | P <sub>2</sub> | P <sub>3</sub> | P <sub>4</sub> | P <sub>5</sub> | P <sub>6</sub> |
| 1                 |                                    |                |                |                |                |                |
| 2                 |                                    |                |                |                |                |                |
| 3                 |                                    |                |                |                |                |                |
| 4                 |                                    |                |                |                |                |                |
| 5                 |                                    |                |                |                |                |                |
| 6                 | 0.100                              | 0.100          | 0.100          | 0.100          | 0.100          | 0.100          |
| 7                 |                                    |                |                |                |                |                |
| 8                 | 0.100                              | 0.100          | 0.100          | 0.100          | 0.100          | 0.100          |
| 9                 |                                    |                |                |                |                |                |
| 10                | 0.100                              | 0.100          | 0.100          | 0.100          | 0.100          | 0.100          |
| 11                |                                    |                |                |                |                |                |
| 12                | 0.100                              | 0.100          | 0.100          | 0.100          | 0.100          | 0.100          |
| 13                |                                    |                |                |                | 0.127          | 0.230          |
| 14                |                                    |                |                | 0.474          | 0.473          | 0.370          |
| 15                |                                    |                |                |                |                |                |
| 16                |                                    |                |                |                |                |                |
| 17                | 0.600                              | 0.600          | 0.600          | 0.126          |                |                |
| 18                |                                    |                |                |                |                |                |
| 19                |                                    |                |                |                |                |                |
| S <sub>3</sub>    |                                    |                | 0.015          | 0.212          | 0.248          | 0.294          |
| S <sub>5</sub>    |                                    | 0.367          | 0.366          | 0.345          | 0.341          | 0.336          |
| S <sub>7</sub>    | 1.000                              | 0.633          | 0.619          | 0.443          | 0.412          | 0.370          |
| *S <sub>10</sub>  |                                    |                |                |                |                |                |
| *S <sub>LA</sub>  |                                    |                |                |                |                |                |
|                   |                                    |                |                |                |                |                |
| **B <sub>11</sub> | 0.791                              | 0.796          | 0.796          | 0.811          | 0.906          | 0.981          |
| **B <sub>12</sub> |                                    |                |                |                |                |                |
| ##B <sub>21</sub> |                                    |                |                |                |                |                |
| ##B <sub>22</sub> | 9.085                              | 9.038          | 9.036          | 8.368          | 8.139          | 8.092          |
|                   |                                    |                |                |                |                |                |
| E <sup>a</sup>    | 1.4417                             | 1.4317         | 1.4311         | 1.3888         | 1.3708         | 1.3630         |
| σ <sup>a</sup>    | 3.7852                             | 3.6355         | 3.6301         | 3.3086         | 3.2026         | 3.1616         |

\* Slack variable for the notional principal of interest rate swaps.

\*\* Value of portfolio's beta-one with interest rate swaps.

## Value of portfolio's beta-two with interest rate swaps.

@ E and σ values are plotted in Figure 11.

TABLE 28 -- Continued

| Category        | Corner Portfolios-Category Weights |                |                |                 |                 |                 |
|-----------------|------------------------------------|----------------|----------------|-----------------|-----------------|-----------------|
|                 | P <sub>7</sub>                     | P <sub>8</sub> | P <sub>9</sub> | P <sub>10</sub> | P <sub>11</sub> | P <sub>12</sub> |
| 1               |                                    |                |                |                 |                 |                 |
| 2               |                                    |                |                |                 |                 |                 |
| 3               |                                    |                |                |                 |                 |                 |
| 4               |                                    |                |                |                 |                 |                 |
| 5               |                                    | 0.184          | 0.405          | 0.500           | 0.600           | 0.600           |
| 6               | 0.100                              | 0.100          | 0.100          | 0.100           | 0.100           | 0.100           |
| 7               | 0.041                              | 0.068          | 0.103          | 0.100           |                 |                 |
| 8               | 0.100                              | 0.100          | 0.100          | 0.100           | 0.100           | 0.100           |
| 9               |                                    |                |                |                 |                 |                 |
| 10              | 0.100                              | 0.100          | 0.100          | 0.100           | 0.100           | 0.100           |
| 11              |                                    |                |                |                 |                 |                 |
| 12              | 0.100                              | 0.100          | 0.100          | 0.100           | 0.100           | 0.100           |
| 13              | 0.212                              | 0.115          |                |                 |                 |                 |
| 14              | 0.347                              | 0.232          | 0.092          |                 |                 |                 |
| 15              |                                    |                |                |                 |                 |                 |
| 16              |                                    |                |                |                 |                 |                 |
| 17              |                                    |                |                |                 |                 |                 |
| 18              |                                    |                |                |                 |                 |                 |
| 19              |                                    |                |                |                 |                 |                 |
| S <sub>3</sub>  | 0.301                              | 0.339          | 0.226          | 0.177           | 0.134           | 0.131           |
| S <sub>5</sub>  | 0.335                              | 0.326          | 0.223          | 0.179           | 0.145           | 0.143           |
| S <sub>7</sub>  | 0.365                              | 0.335          | 0.230          | 0.186           | 0.153           | 0.151           |
| S <sub>10</sub> |                                    |                |                |                 |                 |                 |
| S <sub>LA</sub> |                                    |                | 0.321          | 0.458           | 0.568           | 0.574           |
| B <sub>11</sub> | 0.981                              | 0.964          | 0.897          | 0.899           | 0.876           | 0.875           |
| B <sub>12</sub> |                                    |                |                |                 |                 |                 |
| B <sub>21</sub> | 7.899                              | 6.595          | 4.851          | 4.171           | 3.946           | 3.943           |
| B <sub>22</sub> |                                    |                |                |                 |                 |                 |
| E               | 1.3470                             | 1.2401         | 1.0880         | 1.0274          | 1.0060          | 1.0055          |
| σ               | 3.0846                             | 2.5765         | 1.8693         | 1.5951          | 1.5030          | 1.5009          |

TABLE 28 -- Continued

| Category        | Corner Portfolios-Category Weights |                 |                 |                 |                 |                 |
|-----------------|------------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 | P <sub>13</sub>                    | P <sub>14</sub> | P <sub>15</sub> | P <sub>16</sub> | P <sub>17</sub> | P <sub>18</sub> |
| 1               |                                    |                 |                 |                 |                 |                 |
| 2               |                                    |                 |                 |                 |                 |                 |
| 3               |                                    |                 | 0.007           | 0.600           | 0.600           | 0.600           |
| 4               | 0.600                              | 0.600           | 0.593           |                 |                 |                 |
| 5               |                                    |                 |                 |                 |                 |                 |
| 6               | 0.100                              | 0.100           | 0.100           | 0.100           | 0.100           | 0.100           |
| 7               |                                    |                 |                 |                 |                 |                 |
| 8               | 0.100                              | 0.100           | 0.100           | 0.100           | 0.100           | 0.100           |
| 9               |                                    |                 |                 |                 |                 |                 |
| 10              | 0.100                              | 0.100           | 0.100           | 0.100           | 0.100           | 0.100           |
| 11              |                                    |                 |                 |                 |                 |                 |
| 12              | 0.100                              | 0.100           | 0.100           | 0.100           | 0.100           | 0.100           |
| 13              |                                    |                 |                 |                 |                 |                 |
| 14              |                                    |                 |                 |                 |                 |                 |
| 15              |                                    |                 |                 |                 |                 |                 |
| 16              |                                    |                 |                 |                 |                 |                 |
| 17              |                                    |                 |                 |                 |                 |                 |
| 18              |                                    |                 |                 |                 |                 |                 |
| 19              |                                    |                 |                 |                 |                 |                 |
| S <sub>3</sub>  | 0.012                              | 0.001           |                 |                 |                 |                 |
| S <sub>5</sub>  | 0.048                              | 0.040           | 0.039           | 0.003           |                 |                 |
| S <sub>7</sub>  | 0.056                              | 0.048           | 0.048           | 0.011           | 0.008           | 0.006           |
| S <sub>10</sub> |                                    |                 |                 |                 |                 |                 |
| S <sub>LA</sub> | 0.884                              | 0.911           | 0.913           | 0.986           | 0.992           | 0.994           |
| B <sub>11</sub> | 0.713                              | 0.709           | 0.708           | 0.576           | 0.575           | 0.575           |
| B <sub>12</sub> |                                    |                 |                 |                 |                 |                 |
| B <sub>21</sub> |                                    |                 |                 |                 |                 |                 |
| B <sub>22</sub> | 3.171                              | 3.156           | 3.150           | 2.671           | 2.668           | 2.666           |
| E               | 0.9204                             | 0.9184          | 0.9176          | 0.8529          | 0.8524          | 0.8522          |
| σ               | 1.1846                             | 1.1782          | 1.1757          | 0.9953          | 0.9940          | 0.9934          |

**TABLE 28 -- Continued**

| Category | Corner Portfolios-Category Weights |          |          |          |
|----------|------------------------------------|----------|----------|----------|
|          | $P_{19}$                           | $P_{20}$ | $P_{21}$ | $P_{22}$ |
| 1        |                                    |          | 0.005    | 0.600    |
| 2        | 0.184                              | 0.597    | 0.595    |          |
| 3        | 0.416                              | 0.003    |          |          |
| 4        |                                    |          |          |          |
| 5        |                                    |          |          |          |
| 6        | 0.100                              | 0.100    | 0.100    |          |
| 7        |                                    |          |          |          |
| 8        | 0.100                              | 0.100    | 0.100    |          |
| 9        |                                    |          |          |          |
| 10       | 0.100                              | 0.100    | 0.100    |          |
| 11       |                                    |          |          |          |
| 12       | 0.100                              | 0.100    | 0.100    |          |
| 13       |                                    |          |          |          |
| 14       |                                    |          |          |          |
| 15       |                                    |          |          |          |
| 16       |                                    |          |          |          |
| 17       |                                    |          |          |          |
| 18       |                                    |          |          |          |
| 19       |                                    |          |          |          |
| $S_3$    |                                    |          |          |          |
| $S_5$    |                                    |          |          |          |
| $S_7$    |                                    |          |          |          |
| $S_{10}$ | 1.000                              | 1.000    | 1.000    | 1.000    |
| $S_{LA}$ |                                    |          |          |          |
| $B_{11}$ | 0.548                              | 0.491    | 0.489    | 0.404    |
| $B_{12}$ |                                    |          |          |          |
| $B_{21}$ | 2.591                              | 2.430    | 2.428    | 2.301    |
| $B_{22}$ |                                    |          |          |          |
| E        | 0.8398                             | 0.8135   | 0.8131   | 0.7907   |
| $\sigma$ | 0.9646                             | 0.9082   | 0.9074   | 0.8666   |

**TABLE 29. The Enlarged Variance-Covariance Matrix With Swaps and 60% Nontraded Assets**

|        |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
|--------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 27   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |
| 0.0071 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0      | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0 |   |
| 0.0119 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0      | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0 |   |
| 0.0286 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0      | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0 |   |
| 0.0764 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0      | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0 |   |
| 0.1691 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0      | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0 |   |
| 0.2849 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0      | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0 |   |
| 0.3555 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0      | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0 |   |
| 0.3880 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0      | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0 |   |
| 0.4030 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0      | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0 |   |
| 0.4883 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0      | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0 |   |
| 0.5185 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0      | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0 |   |
| 0.7008 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0      | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0 |   |
| 0.7173 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0      | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0 |   |
| 0.7394 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0      | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0 |   |
| 0.7198 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0      | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0 |   |
| 1.0348 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0      | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0 |   |
| 0.8187 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0      | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0 |   |
| 0.7115 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0      | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0 |   |
| 0.7938 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0      | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0 |   |
| 0.7841 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0      | 0 |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   | 0 |   |



TABLE 30. Corner Portfolios With Swaps and 60% Nontraded Assets

| Category        | Corner Portfolios-Category Weights |                |                |                |                |                |
|-----------------|------------------------------------|----------------|----------------|----------------|----------------|----------------|
|                 | P <sub>1</sub>                     | P <sub>2</sub> | P <sub>3</sub> | P <sub>4</sub> | P <sub>5</sub> | P <sub>6</sub> |
| 1               |                                    |                |                |                |                |                |
| 2               |                                    |                |                |                |                |                |
| 3               |                                    |                |                |                |                |                |
| 4               |                                    |                |                |                |                |                |
| 5               |                                    |                |                |                |                |                |
| 6               | 0.150                              | 0.150          | 0.150          | 0.150          | 0.150          | 0.150          |
| 7               |                                    |                |                |                |                |                |
| 8               | 0.150                              | 0.150          | 0.150          | 0.150          | 0.150          | 0.150          |
| 9               |                                    |                |                |                |                |                |
| 10              | 0.150                              | 0.150          | 0.150          | 0.150          | 0.150          | 0.150          |
| 11              |                                    |                |                |                |                |                |
| 12              | 0.150                              | 0.150          | 0.150          | 0.150          | 0.150          | 0.150          |
| 13              |                                    |                |                |                | 0.017          | 0.136          |
| 14              |                                    |                |                | 0.383          | 0.383          | 0.264          |
| 15              |                                    |                |                |                |                |                |
| 16              |                                    |                |                |                |                |                |
| 17              | 0.400                              | 0.400          | 0.400          | 0.017          |                |                |
| 18              |                                    |                |                |                |                |                |
| 19              |                                    |                |                |                |                |                |
| S <sub>3</sub>  |                                    |                | 0.097          | 0.257          | 0.261          | 0.315          |
| S <sub>5</sub>  |                                    | 0.364          | 0.355          | 0.338          | 0.337          | 0.332          |
| S <sub>7</sub>  | 1.000                              | 0.636          | 0.549          | 0.406          | 0.402          | 0.353          |
| S <sub>10</sub> |                                    |                |                |                |                |                |
| S <sub>LA</sub> |                                    |                |                |                |                |                |
| B <sub>11</sub> | 0.859                              | 0.864          | 0.865          | 0.877          | 0.890          | 0.976          |
| B <sub>12</sub> |                                    |                |                |                |                |                |
| B <sub>21</sub> |                                    |                |                |                |                |                |
| B <sub>22</sub> | 8.101                              | 8.055          | 8.044          | 7.504          | 7.472          | 7.417          |
| E <sup>a</sup>  | 1.3644                             | 1.3546         | 1.3507         | 1.3165         | 1.3140         | 1.3050         |
| σ <sup>a</sup>  | 3.4038                             | 3.2398         | 3.2040         | 2.9569         | 2.9425         | 2.8947         |

\* Slack variable for the notional principal of interest rate swaps.

\*\* Value of portfolio's beta-one with interest rate swaps.

## Value of portfolio's beta-two with interest rate swaps.

@ E and σ values are plotted in Figure 12.

**TABLE 30 -- Continued**

| Category        | Corner Portfolios-Category Weights |                |                |                 |                 |                 |
|-----------------|------------------------------------|----------------|----------------|-----------------|-----------------|-----------------|
|                 | P <sub>7</sub>                     | P <sub>8</sub> | P <sub>9</sub> | P <sub>10</sub> | P <sub>11</sub> | P <sub>12</sub> |
| 1               |                                    |                |                |                 |                 |                 |
| 2               |                                    |                |                |                 |                 |                 |
| 3               |                                    |                |                |                 |                 |                 |
| 4               |                                    |                |                |                 |                 |                 |
| 5               |                                    | 0.108          | 0.257          | 0.359           | 0.400           | 0.400           |
| 6               | 0.150                              | 0.150          | 0.150          | 0.150           | 0.150           | 0.150           |
| 7               | 0.005                              | 0.021          | 0.044          | 0.041           |                 |                 |
| 8               | 0.150                              | 0.150          | 0.150          | 0.150           | 0.150           | 0.150           |
| 9               |                                    |                |                |                 |                 |                 |
| 10              | 0.150                              | 0.150          | 0.150          | 0.150           | 0.150           | 0.150           |
| 11              |                                    |                |                |                 |                 |                 |
| 12              | 0.150                              | 0.150          | 0.150          | 0.150           | 0.150           | 0.150           |
| 13              | 0.134                              | 0.077          |                |                 |                 |                 |
| 14              | 0.261                              | 0.194          | 0.099          |                 |                 |                 |
| 15              |                                    |                |                |                 |                 |                 |
| 16              |                                    |                |                |                 |                 |                 |
| 17              |                                    |                |                |                 |                 |                 |
| 18              |                                    |                |                |                 |                 |                 |
| 19              |                                    |                |                |                 |                 |                 |
| S <sub>3</sub>  | 0.316                              | 0.338          | 0.263          | 0.209           | 0.192           | 0.116           |
| S <sub>5</sub>  | 0.331                              | 0.327          | 0.257          | 0.209           | 0.196           | 0.140           |
| S <sub>7</sub>  | 0.353                              | 0.335          | 0.265          | 0.217           | 0.203           | 0.149           |
| S <sub>10</sub> |                                    |                |                |                 |                 |                 |
| S <sub>LA</sub> |                                    |                | 0.215          | 0.364           | 0.409           | 0.594           |
| B <sub>11</sub> | 0.976                              | 0.966          | 0.921          | 0.923           | 0.914           | 0.886           |
| B <sub>12</sub> |                                    |                |                |                 |                 |                 |
| B <sub>21</sub> |                                    |                |                |                 |                 |                 |
| B <sub>22</sub> | 7.395                              | 6.629          | 5.459          | 4.723           | 4.630           | 4.529           |
| E               | 1.3031                             | 1.2404         | 1.1383         | 1.0726          | 1.0638          | 1.0498          |
| σ               | 2.8858                             | 2.5894         | 2.1138         | 1.8138          | 1.7745          | 1.7160          |

TABLE 30 -- Continued

| Category        | Corner Portfolios-Category Weights |                 |                 |                 |                 |                 |
|-----------------|------------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 | P <sub>13</sub>                    | P <sub>14</sub> | P <sub>15</sub> | P <sub>16</sub> | P <sub>17</sub> | P <sub>18</sub> |
| 1               |                                    |                 |                 |                 |                 |                 |
| 2               |                                    |                 |                 |                 |                 |                 |
| 3               |                                    |                 |                 | 0.400           | 0.400           | 0.400           |
| 4               | 0.400                              | 0.400           | 0.400           |                 |                 |                 |
| 5               |                                    |                 |                 |                 |                 |                 |
| 6               | 0.150                              | 0.150           | 0.150           | 0.150           | 0.150           | 0.150           |
| 7               |                                    |                 |                 |                 |                 |                 |
| 8               | 0.150                              | 0.150           | 0.150           | 0.150           | 0.150           | 0.150           |
| 9               |                                    |                 |                 |                 |                 |                 |
| 10              | 0.150                              | 0.150           | 0.150           | 0.150           | 0.150           | 0.150           |
| 11              |                                    |                 |                 |                 |                 |                 |
| 12              | 0.150                              | 0.150           | 0.150           | 0.150           | 0.150           | 0.150           |
| 13              |                                    |                 |                 |                 |                 |                 |
| 14              |                                    |                 |                 |                 |                 |                 |
| 15              |                                    |                 |                 |                 |                 |                 |
| 16              |                                    |                 |                 |                 |                 |                 |
| 17              |                                    |                 |                 |                 |                 |                 |
| 18              |                                    |                 |                 |                 |                 |                 |
| 19              |                                    |                 |                 |                 |                 |                 |
| S <sub>3</sub>  | 0.036                              |                 |                 |                 |                 |                 |
| S <sub>5</sub>  | 0.077                              | 0.050           | 0.035           | 0.011           |                 |                 |
| S <sub>7</sub>  | 0.086                              | 0.060           | 0.046           | 0.021           | 0.011           |                 |
| S <sub>10</sub> |                                    |                 |                 |                 |                 |                 |
| S <sub>LA</sub> | 0.801                              | 0.890           | 0.919           | 0.968           | 0.989           | 1.000           |
| B <sub>11</sub> | 0.778                              | 0.765           | 0.761           | 0.672           | 0.669           | 0.667           |
| B <sub>12</sub> |                                    |                 |                 |                 |                 |                 |
| B <sub>21</sub> |                                    |                 |                 |                 |                 |                 |
| B <sub>22</sub> | 4.014                              | 3.965           | 3.949           | 3.626           | 3.614           | 3.607           |
| E               | 0.9931                             | 0.9863          | 0.9838          | 0.9402          | 0.9383          | 0.9372          |
| σ               | 1.5085                             | 1.4868          | 1.4793          | 1.3601          | 1.3554          | 1.3528          |

TABLE 30 -- Continued

|                 | Corner Portfolios-Category Weights |                 |                 |
|-----------------|------------------------------------|-----------------|-----------------|
| Category        | P <sub>19</sub>                    | P <sub>20</sub> | P <sub>21</sub> |
| 1               |                                    |                 | 0.400           |
| 2               |                                    | 0.400           |                 |
| 3               | 0.400                              |                 |                 |
| 4               |                                    |                 |                 |
| 5               |                                    |                 |                 |
| 6               | 0.150                              | 0.150           | 0.150           |
| 7               |                                    |                 |                 |
| 8               | 0.150                              | 0.150           | 0.150           |
| 9               |                                    |                 |                 |
| 10              | 0.150                              | 0.150           | 0.150           |
| 11              |                                    |                 |                 |
| 12              | 0.150                              | 0.150           | 0.150           |
| 13              |                                    |                 |                 |
| 14              |                                    |                 |                 |
| 15              |                                    |                 |                 |
| 16              |                                    |                 |                 |
| 17              |                                    |                 |                 |
| 18              |                                    |                 |                 |
| 19              |                                    |                 |                 |
| S <sub>3</sub>  |                                    |                 |                 |
| S <sub>5</sub>  |                                    |                 |                 |
| S <sub>7</sub>  |                                    |                 |                 |
| S <sub>10</sub> |                                    |                 |                 |
| S <sub>LA</sub> | 1.000                              | 1.000           | 1.000           |
| B <sub>11</sub> | 0.667                              | 0.611           | 0.554           |
| B <sub>12</sub> |                                    |                 |                 |
| B <sub>21</sub> |                                    |                 |                 |
| B <sub>22</sub> | 3.607                              | 3.451           | 3.366           |
| E               | 0.9372                             | 0.9117          | 0.8966          |
| σ               | 1.3528                             | 1.2979          | 1.2710          |

**TABLE 31. The Enlarged Variance-Covariance Matrix With Swaps  
and 80% Nontraded Assets**

|        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |   |   |   |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|---|---|---|
| 8      | 27     |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |   |   |   |
| 0.0071 | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0 | 0 | 0 |
| 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0 | 0 | 0 |
| 0      | 0.0119 | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0 | 0 | 0 |
| 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0 | 0 | 0 |
| 0      | 0      | 0.0286 | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0 | 0 | 0 |
| 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0 | 0 | 0 |
| 0      | 0      | 0      | 0.0764 | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0 | 0 | 0 |
| 0      | 0      | 0      | 0      | 0.1691 | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0 | 0 | 0 |
| 0      | 0      | 0      | 0      | 0      | 0.2849 | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0 | 0 | 0 |
| 0      | 0      | 0      | 0      | 0      | 0      | 0.3555 | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0 | 0 | 0 |
| 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0.3880 | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0 | 0 | 0 |
| 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0.4030 | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0 | 0 | 0 |
| 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0.4883 | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0 | 0 | 0 |
| 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0.5185 | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0 | 0 | 0 |
| 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0.7008 | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0 | 0 | 0 |
| 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0.7173 | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0 | 0 | 0 |
| 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0.7394 | 0      | 0      | 0      | 0      | 0      | 0      | 0 | 0 | 0 |
| 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0.7198 | 0      | 0      | 0      | 0      | 0      | 0 | 0 | 0 |
| 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 1.0348 | 0      | 0      | 0      | 0      | 0 | 0 | 0 |
| 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0.8187 | 0      | 0      | 0      | 0 | 0 | 0 |
| 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0.7115 | 0      | 0      | 0 | 0 | 0 |
| 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0.7938 | 0      | 0 | 0 | 0 |
| 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0.7841 | 0 | 0 | 0 |
| 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0 | 0 | 0 |

**TABLE 31 -- Continued**

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1.3770 0 0 0      0  
    0          0  
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1.9420 0 0      0  
    0          0  
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 3.9809 0        0  
    0          0  
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.1283 0.0643  
-0.1283 -0.0643  
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.0643 0.1565  
-0.0643 -0.1565  
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.1283 -0.0643  
0.1283 0.0643  
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.0643 -0.1565  
0.0643 0.1565  
  
0.5788 0.6165 0.6803 0.7785 0.8806 0.9663 1.0100 1.1070 1.1298  
1.2017 1.2241 1.1591 1.3636 1.4218 1.4145 1.4248 1.4948 1.4325  
1.3590 0.0591 0.0743 0.1014 -0.0132 0 0 0 0 {a)  
  
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 1 (b)  
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 1 (c)  
  
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.20  
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.20  
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.20  
0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0.20  
                                                                (d)  
  
0.1044 0.2476 0.3878 0.5912 0.7839 0.8242 0.8448 0.8282 0.6423  
0.5592 0.6753 1.2035 1.2649 0.5402 0.7547 0.9325 0.5108 0.7033  
0.9635 0.1480 0.1565 0.1427 0.6253 -1 0 1 0     0 (e)  
  
-0.1711 -0.3850 -0.7742 -1.5102 -2.5121 -3.3749 -4.1536  
-4.8239 -5.6592 -6.3160 -6.8927 -7.4666 -8.6381 -9.0482  
-9.8805 -10.7181 -10.4117 -10.5628 -10.9668 -0.5100 -0.5128  
-0.6394 -0.9365 0 -1 0 1   0 (f)
```

- (a): The expected return vector.
- (b): The budget constraint.
- (c): Constraint for the notional principal of interest rate swaps.
- (d): Constraint for nontraded assets.
- (e): Constraint for portfolio's beta-one.
- (f): Constraint for portfolio's beta-two.

**TABLE 32. Corner Portfolios With Swaps and 80% Nontraded Assets**

| Category          | Corner Portfolios-Category Weights |                |                |                |                |                |
|-------------------|------------------------------------|----------------|----------------|----------------|----------------|----------------|
|                   | P <sub>1</sub>                     | P <sub>2</sub> | P <sub>3</sub> | P <sub>4</sub> | P <sub>5</sub> | P <sub>6</sub> |
| 1                 |                                    |                |                |                |                |                |
| 2                 |                                    |                |                |                |                |                |
| 3                 |                                    |                |                |                |                |                |
| 4                 |                                    |                |                |                |                |                |
| 5                 |                                    |                |                |                |                |                |
| 6                 | 0.200                              | 0.200          | 0.200          | 0.200          | 0.200          | 0.200          |
| 7                 |                                    |                |                |                |                |                |
| 8                 | 0.200                              | 0.200          | 0.200          | 0.200          | 0.200          | 0.200          |
| 9                 |                                    |                |                |                |                |                |
| 10                | 0.200                              | 0.200          | 0.200          | 0.200          | 0.200          | 0.200          |
| 11                |                                    |                |                |                |                |                |
| 12                | 0.200                              | 0.200          | 0.200          | 0.200          | 0.200          | 0.200          |
| 13                |                                    |                |                |                |                | 0.042          |
| 14                |                                    |                |                | 0.200          | 0.200          | 0.158          |
| 15                |                                    |                |                |                |                |                |
| 16                |                                    |                |                |                |                |                |
| 17                | 0.200                              | 0.200          | 0.200          |                |                |                |
| 18                |                                    |                |                |                |                |                |
| 19                |                                    |                |                |                |                |                |
| S <sub>3</sub>    |                                    |                | 0.179          | 0.262          | 0.317          | 0.335          |
| S <sub>5</sub>    |                                    | 0.360          | 0.343          | 0.334          | 0.329          | 0.327          |
| S <sub>7</sub>    | 1.000                              | 0.640          | 0.478          | 0.404          | 0.354          | 0.337          |
| S <sub>10</sub>   |                                    |                |                |                |                |                |
| S <sub>LA</sub>   |                                    |                |                |                |                |                |
|                   |                                    |                |                |                |                |                |
| **B <sub>11</sub> | 0.928                              | 0.933          | 0.934          | 0.940          | 0.940          | 0.970          |
| **B <sub>12</sub> |                                    |                |                |                |                |                |
| **B <sub>21</sub> |                                    |                |                |                |                |                |
| **B <sub>22</sub> | 7.118                              | 7.072          | 7.051          | 6.769          | 6.763          | 6.743          |
|                   |                                    |                |                |                |                |                |
| E                 | 1.2872                             | 1.2774         | 1.2703         | 1.2524         | 1.2503         | 1.2471         |
| σ                 | 3.0397                             | 2.8587         | 2.7902         | 2.6635         | 2.6512         | 2.6357         |

\* Slack variable for the notional principal of interest rate swaps.

\*\* Value of portfolio's beta-one with interest rate swaps.

\*\* Value of portfolio's beta-two with interest rate swaps.

@ E and σ values are plotted in Figure 13.

TABLE 32 -- Continued

|                 | Corner Portfolios-Category Weights |                |                |                 |                 |                 |
|-----------------|------------------------------------|----------------|----------------|-----------------|-----------------|-----------------|
| Category        | P <sub>7</sub>                     | P <sub>8</sub> | P <sub>9</sub> | P <sub>10</sub> | P <sub>11</sub> | P <sub>12</sub> |
| 1               |                                    |                |                |                 |                 |                 |
| 2               |                                    |                |                |                 |                 |                 |
| 3               |                                    |                |                |                 |                 |                 |
| 4               |                                    |                |                |                 | 0.200           | 0.200           |
| 5               | 0.013                              | 0.093          | 0.200          | 0.200           |                 |                 |
| 6               | 0.200                              | 0.200          | 0.200          | 0.200           | 0.200           | 0.200           |
| 7               |                                    |                |                |                 |                 |                 |
| 8               | 0.200                              | 0.200          | 0.200          | 0.200           | 0.200           | 0.200           |
| 9               |                                    |                |                |                 |                 |                 |
| 10              | 0.200                              | 0.200          | 0.200          | 0.200           | 0.200           | 0.200           |
| 11              |                                    |                |                |                 |                 |                 |
| 12              | 0.200                              | 0.200          | 0.200          | 0.200           | 0.200           | 0.200           |
| 13              | 0.036                              |                |                |                 |                 |                 |
| 14              | 0.152                              | 0.107          |                |                 |                 |                 |
| 15              |                                    |                |                |                 |                 |                 |
| 16              |                                    |                |                |                 |                 |                 |
| 17              |                                    |                |                |                 |                 |                 |
| 18              |                                    |                |                |                 |                 |                 |
| 19              |                                    |                |                |                 |                 |                 |
| S <sub>3</sub>  | 0.338                              | 0.301          | 0.244          | 0.101           | 0.061           |                 |
| S <sub>5</sub>  | 0.327                              | 0.293          | 0.242          | 0.137           | 0.105           | 0.060           |
| S <sub>7</sub>  | 0.336                              | 0.302          | 0.250          | 0.148           | 0.116           | 0.073           |
| S <sub>10</sub> |                                    |                |                |                 |                 |                 |
| S <sub>LA</sub> |                                    | 0.104          | 0.264          | 0.614           | 0.717           | 0.867           |
| B <sub>11</sub> | 0.969                              | 0.947          | 0.949          | 0.897           | 0.843           | 0.821           |
| B <sub>12</sub> |                                    |                |                |                 |                 |                 |
| B <sub>21</sub> |                                    |                |                |                 |                 |                 |
| B <sub>22</sub> | 6.663                              | 6.098          | 5.307          | 5.115           | 4.858           | 4.776           |
| E               | 1.2405                             | 1.1913         | 1.1207         | 1.0941          | 1.0658          | 1.0544          |
| σ               | 2.6047                             | 2.3753         | 2.0506         | 1.9405          | 1.8384          | 1.8013          |

TABLE 32 -- Continued

|                 | Corner Portfolios-Category Weights |                 |                 |                 |                 |                 |
|-----------------|------------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Category        | P <sub>13</sub>                    | P <sub>14</sub> | P <sub>15</sub> | P <sub>16</sub> | P <sub>17</sub> | P <sub>18</sub> |
| 1               |                                    |                 |                 |                 |                 | 0.200           |
| 2               |                                    |                 |                 |                 | 0.200           |                 |
| 3               |                                    | 0.200           | 0.200           | 0.200           |                 |                 |
| 4               | 0.200                              |                 |                 |                 |                 |                 |
| 5               |                                    |                 |                 |                 |                 |                 |
| 6               | 0.200                              | 0.200           | 0.200           | 0.200           | 0.200           | 0.200           |
| 7               |                                    |                 |                 |                 |                 |                 |
| 8               | 0.200                              | 0.200           | 0.200           | 0.200           | 0.200           | 0.200           |
| 9               |                                    |                 |                 |                 |                 |                 |
| 10              | 0.200                              | 0.200           | 0.200           | 0.200           | 0.200           | 0.200           |
| 11              |                                    |                 |                 |                 |                 |                 |
| 12              | 0.200                              | 0.200           | 0.200           | 0.200           | 0.200           | 0.200           |
| 13              |                                    |                 |                 |                 |                 |                 |
| 14              |                                    |                 |                 |                 |                 |                 |
| 15              |                                    |                 |                 |                 |                 |                 |
| 16              |                                    |                 |                 |                 |                 |                 |
| 17              |                                    |                 |                 |                 |                 |                 |
| 18              |                                    |                 |                 |                 |                 |                 |
| 19              |                                    |                 |                 |                 |                 |                 |
| S <sub>3</sub>  |                                    |                 |                 |                 |                 |                 |
| S <sub>5</sub>  | 0.031                              | 0.019           |                 |                 |                 |                 |
| S <sub>7</sub>  | 0.044                              | 0.032           | 0.014           |                 |                 |                 |
| S <sub>10</sub> |                                    |                 |                 |                 |                 |                 |
| S <sub>LA</sub> | 0.925                              | 0.950           | 0.986           | 1.000           | 1.000           | 1.000           |
| B <sub>11</sub> | 0.812                              | 0.768           | 0.763           | 0.761           | 0.733           | 0.704           |
| B <sub>12</sub> |                                    |                 |                 |                 |                 |                 |
| B <sub>21</sub> |                                    |                 |                 |                 |                 |                 |
| B <sub>22</sub> | 4.742                              | 4.581           | 4.560           | 4.551           | 4.473           | 4.431           |
| E               | 1.0493                             | 1.0275          | 1.0243          | 1.0229          | 1.0101          | 1.0026          |
| σ               | 1.7865                             | 1.7278          | 1.7197          | 1.7165          | 1.6894          | 1.6761          |



**TABLE 33 -- Continued**

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1.3770 0 0 0      0  
    0          0  
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1.9420 0 0      0  
    0          0  
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 3.9809 0        0  
    0          0  
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.1283 0.0643  
-0.1283 -0.0643  
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.0643 0.1565  
-0.0643 -0.1565  
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.1283 -0.0643  
0.1283 0.0643  
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -0.0643 -0.1565  
0.0643 0.1565  
  
0.5788 0.6165 0.6803 0.7785 0.8806 0.9663 1.0100 1.1070 1.1298  
1.2017 1.2241 1.1591 1.3636 1.4218 1.4145 1.4248 1.4948 1.4325  
1.3590 0.0591 0.0743 0.1014 -0.0132 0 0 0 0   (a)  
  
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0 1 (b)  
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0 1 (c)  
  
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.249999  
  
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.249999  
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0.249999  
0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0.249999  
                                     (d)  
  
0.1044 0.2476 0.3878 0.5912 0.7839 0.8242 0.8448 0.8282 0.6423  
0.5592 0.6753 1.2035 1.2649 0.5402 0.7547 0.9325 0.5108 0.7033  
0.9635 0.1480 0.1565 0.1427 0.6253 -1 0 1 0     0 (e)  
  
-0.1711 -0.3850 -0.7742 -1.5102 -2.5121 -3.3749 -4.1536  
-4.8239 -5.6592 -6.3160 -6.8927 -7.4666 -8.6381 -9.0482  
-9.8805 -10.7181 -10.4117 -10.5628 -10.9668 -0.5100 -0.5128  
-0.6394 -0.9365 0 -1 0 1     0 (f)
```

- (a): The expected return vector.
- (b): The budget constraint.
- (c): Constraint for the notional principal of interest rate swaps.
- (d): Constraint for nontraded assets.
- (e): Constraint for portfolio's beta-one.
- (f): Constraint for portfolio's beta-two.

**TABLE 34. Corner Portfolios With Swaps and 100% Nontraded Assets**

| Category          | Corner Portfolios-Category Weights |                |                |                |                |                |
|-------------------|------------------------------------|----------------|----------------|----------------|----------------|----------------|
|                   | P <sub>1</sub>                     | P <sub>2</sub> | P <sub>3</sub> | P <sub>4</sub> | P <sub>5</sub> | P <sub>6</sub> |
| 1                 |                                    |                |                |                |                |                |
| 2                 |                                    |                |                |                |                |                |
| 3                 |                                    |                |                |                |                |                |
| 4                 |                                    |                |                |                |                |                |
| 5                 |                                    |                |                |                |                |                |
| 6                 | 0.250                              | 0.250          | 0.250          | 0.250          | 0.250          | 0.250          |
| 7                 |                                    |                |                |                |                |                |
| 8                 | 0.250                              | 0.250          | 0.250          | 0.250          | 0.250          | 0.250          |
| 9                 |                                    |                |                |                |                |                |
| 10                | 0.250                              | 0.250          | 0.250          | 0.250          | 0.250          | 0.250          |
| 11                |                                    |                |                |                |                |                |
| 12                | 0.250                              | 0.250          | 0.250          | 0.250          | 0.250          | 0.250          |
| 13                |                                    |                |                |                |                |                |
| 14                |                                    |                |                |                |                |                |
| 15                |                                    |                |                |                |                |                |
| 16                |                                    |                |                |                |                |                |
| 17                |                                    |                |                |                |                |                |
| 18                |                                    |                |                |                |                |                |
| 19                |                                    |                |                |                |                |                |
| S <sub>3</sub>    |                                    |                | 0.261          | 0.345          | 0.284          | 0.086          |
| S <sub>5</sub>    |                                    | 0.356          | 0.332          | 0.324          | 0.279          | 0.134          |
| S <sub>7</sub>    | 1.000                              | 0.644          | 0.408          | 0.331          | 0.287          | 0.146          |
| S <sub>10</sub>   |                                    |                |                |                |                |                |
| S <sub>LA</sub>   |                                    |                |                |                | 0.150          | 0.634          |
| **B <sub>11</sub> | 0.996                              | 1.001          | 1.002          | 1.003          | 0.980          | 0.908          |
| **B <sub>12</sub> |                                    |                |                |                |                |                |
| ##B <sub>21</sub> |                                    |                |                |                |                |                |
| ##B <sub>22</sub> | 6.135                              | 6.090          | 6.059          | 6.049          | 5.967          | 5.702          |
| E                 | 1.2099                             | 1.2003         | 1.1899         | 1.1866         | 1.1751         | 1.1384         |
| σ                 | 2.6999                             | 2.4988         | 2.3952         | 2.3754         | 2.3216         | 2.1716         |

\* Slack variable for the notional principal of interest rate swaps.

\*\* Value of portfolio's beta-one with interest rate swaps.

## Value of portfolio's beta-two with interest rate swaps.

@ E and σ values are plotted in Figure 13.

TABLE 34 -- Continued

|                 | Corner Portfolios-Category Weights |                |                |                 |
|-----------------|------------------------------------|----------------|----------------|-----------------|
| Category        | P <sub>7</sub>                     | P <sub>8</sub> | P <sub>9</sub> | P <sub>10</sub> |
| 1               |                                    |                |                |                 |
| 2               |                                    |                |                |                 |
| 3               |                                    |                |                |                 |
| 4               |                                    |                |                |                 |
| 5               |                                    |                |                |                 |
| 6               | 0.250                              | 0.250          | 0.250          | 0.250           |
| 7               |                                    |                |                |                 |
| 8               | 0.250                              | 0.250          | 0.250          | 0.250           |
| 9               |                                    |                |                |                 |
| 10              | 0.250                              | 0.250          | 0.250          | 0.250           |
| 11              |                                    |                |                |                 |
| 12              | 0.250                              | 0.250          | 0.250          | 0.250           |
| 13              |                                    |                |                |                 |
| 14              |                                    |                |                |                 |
| 15              |                                    |                |                |                 |
| 16              |                                    |                |                |                 |
| 17              |                                    |                |                |                 |
| 18              |                                    |                |                |                 |
| 19              |                                    |                |                |                 |
| S <sub>3</sub>  |                                    |                |                |                 |
| S <sub>5</sub>  | 0.071                              | 0.026          |                |                 |
| S <sub>7</sub>  | 0.085                              | 0.042          | 0.016          |                 |
| S <sub>10</sub> |                                    |                |                |                 |
| S <sub>LA</sub> | 0.844                              | 0.932          | 0.984          | 1.000           |
| B <sub>11</sub> | 0.877                              | 0.864          | 0.856          | 0.854           |
| B <sub>12</sub> |                                    |                |                |                 |
| B <sub>21</sub> |                                    |                |                |                 |
| B <sub>22</sub> | 5.586                              | 5.536          | 5.506          | 5.495           |
| E               | 1.1224                             | 1.1147         | 1.1102         | 1.1085          |
| σ               | 2.1189                             | 2.0969         | 2.0854         | 2.0815          |

TABLE 35a

Regression Results of Single Factor Model (a)

$$R_i = \alpha_i + \beta_{i1}I_i + u_i$$

| Category | $\alpha_i$<br>(t-value) | $\beta_{i1}$<br>(t-value) | $R_i^2$ | $V(u_i)$ |
|----------|-------------------------|---------------------------|---------|----------|
| 1        | 0.5712<br>(38.541)      | -0.1438<br>(-3.814)       | 0.2997  | 0.0075   |
| 2        | 0.5992<br>(29.840)      | -0.3287<br>(-6.437)       | 0.5493  | 0.0138   |
| 3        | 0.6477<br>(18.128)      | -0.6159<br>(-6.777)       | 0.5746  | 0.0438   |
| 4        | 0.7186<br>(10.445)      | -1.1337<br>(-6.479)       | 0.5525  | 0.1625   |
| 5        | 0.7853<br>(6.684)       | -1.8040<br>(-6.038)       | 0.5174  | 0.4739   |
| 6        | 0.8432<br>(5.072)       | -2.3299<br>(-5.511)       | 0.4718  | 0.9489   |
| 7        | 0.8622<br>(4.163)       | -2.7979<br>(-5.312)       | 0.4536  | 1.4727   |
| 8        | 0.9386<br>(3.873)       | -3.1868<br>(-5.170)       | 0.4401  | 2.0174   |
| 9        | 0.9394<br>(3.194)       | -3.6037<br>(-4.817)       | 0.4056  | 2.9716   |
| 10       | 0.9926<br>(2.957)       | -3.9574<br>(-4.635)       | 0.3872  | 3.8702   |
| 11       | 0.9946<br>(2.759)       | -4.3454<br>(-4.739)       | 0.3979  | 4.4633   |
| 12       | 0.9002<br>(2.432)       | -4.9006<br>(-5.206)       | 0.4436  | 4.7039   |
| 13       | 1.0668<br>(2.498)       | -5.6172<br>(-5.171)       | 0.4402  | 6.2651   |
| 14       | 1.1279<br>(2.319)       | -5.5624<br>(-4.496)       | 0.3728  | 8.1263   |
| 15       | 1.0901<br>(2.104)       | -6.1416<br>(-4.661)       | 0.3898  | 9.2183   |
| 16       | 1.0704<br>(1.908)*      | -6.7089<br>(-4.702)       | 0.3941  | 10.8069  |
| 17       | 1.1591<br>(2.065)       | -6.3552<br>(-4.451)       | 0.3682  | 10.8224  |
| 18       | 1.0879<br>(1.955)*      | -6.5233<br>(-4.610)       | 0.3846  | 10.6317  |
| 19       | 0.9962<br>(1.760)*      | -6.8684<br>(-4.770)       | 0.4010  | 11.0054  |

\* Not statistically significant at 5% level.

TABLE 35b

Regression Results of Single Factor Model (b)

$$R_i = \alpha_i + \beta_{i2}I_2 + u_i$$

| Category | $\alpha_i$<br>(t-value)* | $\beta_{i2}$<br>(t-value)* | $R_i^2$ | $V(u_i)$ |
|----------|--------------------------|----------------------------|---------|----------|
| 1        | 0.5707<br>(36.793)       | -0.1282<br>(-3.264)        | 0.2386  | 0.0082   |
| 2        | 0.5987<br>(25.934)       | -0.2832<br>(-4.847)        | 0.4086  | 0.0182   |
| 3        | 0.6416<br>(17.872)       | -0.6148<br>(-6.766)        | 0.5738  | 0.0439   |
| 4        | 0.6987<br>(12.191)       | -1.2673<br>(-8.735)        | 0.6918  | 0.1120   |
| 5        | 0.7427<br>(9.007)        | -2.1900<br>(-10.494)       | 0.7641  | 0.2317   |
| 6        | 0.7751<br>(7.604)        | -3.0363<br>(-11.768)       | 0.8029  | 0.3541   |
| 7        | 0.7703<br>(6.873)        | -3.8065<br>(-13.417)       | 0.8411  | 0.4281   |
| 8        | 0.8247<br>(7.115)        | -4.4837<br>(-15.282)       | 0.8729  | 0.4579   |
| 9        | 0.7901<br>(6.914)        | -5.3953<br>(-18.654)       | 0.9110  | 0.4450   |
| 10       | 0.8184<br>(6.625)        | -6.0862<br>(-19.464)       | 0.9171  | 0.5201   |
| 11       | 0.8076<br>(6.273)        | -6.6152<br>(-20.298)       | 0.9238  | 0.5650   |
| 12       | 0.7201<br>(4.564)        | -6.9721<br>(-17.459)       | 0.8997  | 0.8483   |
| 13       | 0.8524<br>(5.304)        | -8.1184<br>(-19.958)       | 0.9214  | 0.8802   |
| 14       | 0.8660<br>(5.765)        | -8.8262<br>(-23.212)       | 0.9406  | 0.7691   |
| 15       | 0.8119<br>(5.375)        | -9.5703<br>(-25.028)       | 0.9485  | 0.7778   |
| 16       | 0.7740<br>(4.264)        | -10.3350<br>(-22.490)      | 0.9370  | 1.1234   |
| 17       | 0.8524<br>(5.413)        | -10.2018<br>(-25.593)      | 0.9507  | 0.8453   |
| 18       | 0.7856<br>(5.254)        | -10.2738<br>(-27.147)      | 0.9559  | 0.7619   |
| 19       | 0.6934<br>(4.295)        | -10.5709<br>(-25.868)      | 0.9516  | 0.8883   |

\* All the t-values in this table are statistically significant at 1% level.

TABLE 36

Coefficients for the Estimation of Yield Curve\*  
(Partial Listing)

$$Y_t = b_{1t} + b_{2t} \text{EXP}(-b_{3t}t) + u_t$$

| T     | $b_{1t}$ | $b_{2t}$  | $b_{3t}$ | $R^2$ |
|-------|----------|-----------|----------|-------|
| 11/84 | 0.114710 | -0.030783 | 0.223191 | 0.993 |
| 12/84 | 0.114956 | -0.036659 | 0.213803 | 0.997 |
| 01/85 | 0.111707 | -0.030964 | 0.190584 | 0.997 |
| 02/85 | 0.119001 | -0.031441 | 0.188117 | 0.997 |
| 03/85 | 0.116630 | -0.030753 | 0.191026 | 0.996 |
| 04/85 | 0.116575 | -0.033175 | 0.146263 | 0.996 |
| 05/85 | 0.108297 | -0.029725 | 0.101350 | 0.994 |
| 06/85 | 0.106817 | -0.035059 | 0.134331 | 0.993 |
| 07/85 | 0.110035 | -0.034279 | 0.129301 | 0.992 |
| 08/85 | 0.108047 | -0.031995 | 0.115010 | 0.991 |
| 09/85 | 0.110436 | -0.033397 | 0.095969 | 0.991 |
| 10/85 | 0.107014 | -0.030768 | 0.093998 | 0.992 |
| 11/85 | 0.102657 | -0.027440 | 0.089060 | 0.992 |
| 12/85 | 0.099644 | -0.025322 | 0.057634 | 0.992 |
| 01/86 | 0.099792 | -0.026229 | 0.065493 | 0.992 |
| 02/86 | 0.091232 | -0.015880 | 0.026842 | 0.994 |
| 03/86 | 0.083016 | -0.013701 | 0.025495 | 0.995 |
| 04/86 | 0.083791 | -0.015877 | 0.033765 | 0.994 |
| 05/86 | 0.085811 | -0.016459 | 0.095516 | 0.989 |
| 06/86 | 0.081556 | -0.015034 | 0.064218 | 0.992 |
| 07/86 | 0.085424 | -0.023360 | 0.058383 | 0.990 |
| 08/86 | 0.082288 | -0.028114 | 0.059376 | 0.993 |
| 09/86 | 0.086577 | -0.029028 | 0.068341 | 0.990 |
| 10/86 | 0.085894 | -0.030231 | 0.061630 | 0.991 |
| 11/86 | 0.084534 | -0.029485 | 0.058386 | 0.992 |
| 12/86 | 0.086531 | -0.028352 | 0.048418 | 0.992 |
| 01/87 | 0.085360 | -0.028454 | 0.049590 | 0.993 |
| 02/87 | 0.083799 | -0.025870 | 0.049169 | 0.994 |
| 03/87 | 0.085966 | -0.025667 | 0.051602 | 0.995 |
| 04/87 | 0.089299 | -0.021194 | 0.065436 | 0.995 |
| 05/87 | 0.090041 | -0.019403 | 0.077054 | 0.995 |
| 06/87 | 0.089921 | -0.022775 | 0.082018 | 0.995 |
| 07/87 | 0.093195 | -0.026058 | 0.081486 | 0.996 |
| 08/87 | 0.095114 | -0.025033 | 0.089173 | 0.996 |
| 09/87 | 0.100195 | -0.024223 | 0.101900 | 0.996 |
| 10/87 | 0.092880 | -0.027049 | 0.118572 | 0.996 |
| 11/87 | 0.094164 | -0.028260 | 0.113487 | 0.995 |
| 12/87 | 0.092188 | -0.024516 | 0.106888 | 0.996 |

\* Values from Hessel, Kim, and Markowitz.

TABLE 37

Ex ante Monthly Holding Period Returns By Category  
(Partial Listing, Value in Percentages)

| T     | $R_1^E$ | $R_2^E$ | $R_3^E$ | $R_4^E$ | $R_5^E$ |
|-------|---------|---------|---------|---------|---------|
| 01/85 | 0.6933  | 0.7193  | 0.7420  | 0.8383  | 0.8848  |
| 02/85 | 0.6419  | 0.6556  | 0.6468  | 0.7212  | 0.7742  |
| 03/85 | 0.7153  | 0.7340  | 0.7547  | 0.8271  | 0.8744  |
| 04/85 | 0.7794  | 0.7951  | 0.7868  | 0.8880  | 0.9148  |
| 05/85 | 0.7284  | 0.7447  | 0.7147  | 0.7978  | 0.8501  |
| 06/85 | 0.6136  | 0.6245  | 0.6208  | 0.6748  | 0.7099  |
| 07/85 | 0.6715  | 0.6850  | 0.6799  | 0.7685  | 0.7984  |
| 08/85 | 0.6402  | 0.6565  | 0.6389  | 0.7115  | 0.7636  |
| 09/85 | 0.6579  | 0.6741  | 0.6617  | 0.7391  | 0.7734  |
| 10/85 | 0.6687  | 0.6822  | 0.6914  | 0.7459  | 0.7677  |
| 11/85 | 0.6203  | 0.6269  | 0.5879  | 0.6563  | 0.6927  |
| 12/85 | 0.6703  | 0.6849  | 0.6645  | 0.7283  | 0.7551  |
| 01/86 | 0.6437  | 0.6482  | 0.6411  | 0.6760  | 0.6899  |
| 02/86 | 0.5751  | 0.5791  | 0.5528  | 0.6019  | 0.6254  |
| 03/86 | 0.6461  | 0.6488  | 0.6452  | 0.6632  | 0.6772  |
| 04/86 | 0.5801  | 0.5770  | 0.5583  | 0.5771  | 0.5799  |
| 05/86 | 0.5656  | 0.5684  | 0.5273  | 0.5652  | 0.5763  |
| 06/86 | 0.5964  | 0.6071  | 0.5994  | 0.6400  | 0.6607  |
| 07/86 | 0.5741  | 0.5790  | 0.5612  | 0.5851  | 0.5970  |
| 08/86 | 0.5052  | 0.5064  | 0.4809  | 0.5227  | 0.5442  |
| 09/86 | 0.4883  | 0.4915  | 0.4825  | 0.5244  | 0.5472  |
| 10/86 | 0.5032  | 0.5086  | 0.5139  | 0.5425  | 0.5665  |
| 11/86 | 0.4382  | 0.4422  | 0.4219  | 0.4628  | 0.4845  |
| 12/86 | 0.5088  | 0.5172  | 0.5131  | 0.5548  | 0.5798  |
| 01/87 | 0.4897  | 0.4926  | 0.4737  | 0.5085  | 0.5271  |
| 02/87 | 0.4471  | 0.4477  | 0.4340  | 0.4702  | 0.4887  |
| 03/87 | 0.5185  | 0.5209  | 0.5185  | 0.5534  | 0.5725  |
| 04/87 | 0.5074  | 0.5083  | 0.4904  | 0.5244  | 0.5435  |
| 05/87 | 0.5489  | 0.5530  | 0.5299  | 0.5659  | 0.5845  |
| 06/87 | 0.6274  | 0.6337  | 0.6310  | 0.6705  | 0.6900  |
| 07/87 | 0.5811  | 0.5887  | 0.5654  | 0.6098  | 0.6338  |
| 08/87 | 0.5826  | 0.5877  | 0.5736  | 0.6174  | 0.6410  |
| 09/87 | 0.5861  | 0.5939  | 0.5957  | 0.6343  | 0.6647  |
| 10/87 | 0.6382  | 0.6448  | 0.6215  | 0.6729  | 0.7010  |
| 11/87 | 0.5747  | 0.5835  | 0.5713  | 0.6274  | 0.6578  |
| 12/87 | 0.5731  | 0.5865  | 0.5906  | 0.6392  | 0.6783  |

TABLE 37 -- Continued

| T     | $R_6^E$ | $R_7^E$ | $R_8^E$ | $R_9^E$ | $R_{10}^E$ |
|-------|---------|---------|---------|---------|------------|
| 01/85 | 0.9193  | 0.9447  | 0.9517  | 0.9969  | 1.0114     |
| 02/85 | 0.8036  | 0.8186  | 0.8559  | 0.8308  | 0.7518     |
| 03/85 | 0.9100  | 0.9333  | 0.9498  | 0.9600  | 0.9510     |
| 04/85 | 0.9635  | 0.9872  | 0.9968  | 1.0422  | 1.0378     |
| 05/85 | 0.8815  | 0.8949  | 0.9194  | 0.9395  | 0.9359     |
| 06/85 | 0.7381  | 0.7566  | 0.7742  | 0.7888  | 0.7925     |
| 07/85 | 0.8488  | 0.8767  | 0.8937  | 0.9410  | 0.9504     |
| 08/85 | 0.7948  | 0.7992  | 0.8204  | 0.8521  | 0.8034     |
| 09/85 | 0.8182  | 0.8411  | 0.8622  | 0.8807  | 0.8844     |
| 10/85 | 0.7976  | 0.8173  | 0.8310  | 0.8784  | 0.8862     |
| 11/85 | 0.7099  | 0.7208  | 0.7420  | 0.7669  | 0.7573     |
| 12/85 | 0.7897  | 0.8068  | 0.8256  | 0.8425  | 0.8503     |
| 01/86 | 0.7054  | 0.7140  | 0.7229  | 0.7595  | 0.7677     |
| 02/86 | 0.6402  | 0.6503  | 0.6534  | 0.6399  | 0.6734     |
| 03/86 | 0.6837  | 0.6853  | 0.6922  | 0.6885  | 0.6946     |
| 04/86 | 0.5843  | 0.5938  | 0.5971  | 0.6104  | 0.6185     |
| 05/86 | 0.5790  | 0.5771  | 0.5906  | 0.5970  | 0.5987     |
| 06/86 | 0.6753  | 0.6862  | 0.6989  | 0.7016  | 0.7138     |
| 07/86 | 0.6056  | 0.6209  | 0.6301  | 0.6483  | 0.6622     |
| 08/86 | 0.5483  | 0.5536  | 0.5777  | 0.5657  | 0.5619     |
| 09/86 | 0.5705  | 0.5878  | 0.6047  | 0.6141  | 0.6264     |
| 10/86 | 0.5853  | 0.6011  | 0.6178  | 0.6432  | 0.6578     |
| 11/86 | 0.4993  | 0.5116  | 0.5339  | 0.5405  | 0.5254     |
| 12/86 | 0.5985  | 0.6169  | 0.6357  | 0.6467  | 0.6563     |
| 01/87 | 0.5387  | 0.5580  | 0.5719  | 0.5952  | 0.6033     |
| 02/87 | 0.5020  | 0.5036  | 0.5017  | 0.5343  | 0.5173     |
| 03/87 | 0.5866  | 0.6009  | 0.6102  | 0.6220  | 0.6304     |
| 04/87 | 0.5543  | 0.5744  | 0.5854  | 0.6078  | 0.6168     |
| 05/87 | 0.5935  | 0.6032  | 0.6200  | 0.6324  | 0.5998     |
| 06/87 | 0.7059  | 0.7204  | 0.7284  | 0.7423  | 0.7452     |
| 07/87 | 0.6473  | 0.6738  | 0.6871  | 0.7146  | 0.7201     |
| 08/87 | 0.6576  | 0.6762  | 0.6743  | 0.6928  | 0.7161     |
| 09/87 | 0.6864  | 0.7048  | 0.7162  | 0.7301  | 0.7419     |
| 10/87 | 0.7285  | 0.7467  | 0.7630  | 0.7886  | 0.8026     |
| 11/87 | 0.6832  | 0.7011  | 0.7173  | 0.7196  | 0.7267     |
| 12/87 | 0.7077  | 0.7314  | 0.7478  | 0.7602  | 0.7755     |

TABLE 37 -- Continued

| T     | $R_{11}^E$ | $R_{12}^E$ | $R_{13}^E$ | $R_{14}^E$ | $R_{15}^E$ |
|-------|------------|------------|------------|------------|------------|
| 01/85 | 1.0123     | 1.0067     | 1.0217     | 1.0236     | 1.0114     |
| 02/85 | 0.8387     | 0.8324     | 0.8919     | 0.7740     | 0.8191     |
| 03/85 | 0.9663     | 0.9635     | 0.9921     | 0.9718     | 0.9702     |
| 04/85 | 1.0540     | 1.0523     | 1.0810     | 1.0592     | 1.0593     |
| 05/85 | 0.9592     | 0.9294     | 0.8751     | 0.8873     | 0.9559     |
| 06/85 | 0.8115     | 0.8089     | 0.8151     | 0.8259     | 0.8438     |
| 07/85 | 0.9729     | 0.9700     | 0.9737     | 0.9815     | 0.9988     |
| 08/85 | 0.8186     | 0.9036     | 0.9176     | 0.8015     | 0.8654     |
| 09/85 | 0.8981     | 0.9093     | 0.9286     | 0.9159     | 0.9371     |
| 10/85 | 0.9031     | 0.9154     | 0.9381     | 0.9309     | 0.9555     |
| 11/85 | 0.7381     | 0.7942     | 0.8445     | 0.8164     | 0.7710     |
| 12/85 | 0.8550     | 0.8715     | 0.8951     | 0.8984     | 0.9004     |
| 01/86 | 0.7730     | 0.7880     | 0.8128     | 0.8195     | 0.8240     |
| 02/86 | 0.6515     | 0.7197     | 0.7473     | 0.6759     | 0.7247     |
| 03/86 | 0.6953     | 0.7066     | 0.7271     | 0.7155     | 0.7255     |
| 04/86 | 0.6191     | 0.6292     | 0.6440     | 0.6356     | 0.6436     |
| 05/86 | 0.5721     | 0.6283     | 0.5011     | 0.6242     | 0.5928     |
| 06/86 | 0.7127     | 0.7212     | 0.7083     | 0.7371     | 0.7347     |
| 07/86 | 0.6609     | 0.6690     | 0.6606     | 0.6889     | 0.6890     |
| 08/86 | 0.5983     | 0.6271     | 0.6301     | 0.5935     | 0.6150     |
| 09/86 | 0.6380     | 0.6539     | 0.6619     | 0.6771     | 0.6908     |
| 10/86 | 0.6698     | 0.6860     | 0.6920     | 0.7077     | 0.7202     |
| 11/86 | 0.5503     | 0.5970     | 0.6123     | 0.5895     | 0.6020     |
| 12/86 | 0.6722     | 0.6830     | 0.7067     | 0.7169     | 0.7284     |
| 01/87 | 0.6179     | 0.6261     | 0.6460     | 0.6567     | 0.6682     |
| 02/87 | 0.5489     | 0.5730     | 0.5557     | 0.5532     | 0.6036     |
| 03/87 | 0.6450     | 0.6537     | 0.6717     | 0.6762     | 0.6990     |
| 04/87 | 0.6305     | 0.6381     | 0.6559     | 0.6601     | 0.6817     |
| 05/87 | 0.6363     | 0.6700     | 0.5774     | 0.6562     | 0.6455     |
| 06/87 | 0.7564     | 0.7531     | 0.7554     | 0.7825     | 0.7886     |
| 07/87 | 0.7331     | 0.7316     | 0.7352     | 0.7619     | 0.7689     |
| 08/87 | 0.7532     | 0.7577     | 0.7218     | 0.7260     | 0.7389     |
| 09/87 | 0.7587     | 0.7534     | 0.7619     | 0.7769     | 0.7827     |
| 10/87 | 0.8202     | 0.8107     | 0.8175     | 0.8339     | 0.8378     |
| 11/87 | 0.7296     | 0.7790     | 0.7954     | 0.7449     | 0.7737     |
| 12/87 | 0.7851     | 0.7835     | 0.8088     | 0.8075     | 0.8140     |

TABLE 37 -- Continued

| T     | $R_{16}^E$ | $R_{17}^E$ | $R_{18}^E$ | $R_{19}^E$ |
|-------|------------|------------|------------|------------|
| 01/85 | 1.0044     | 0.9948     | 0.9996     | 0.9938     |
| 02/85 | 0.8265     | 0.8213     | 0.8758     | 0.8809     |
| 03/85 | 0.9775     | 0.9645     | 0.9759     | 0.9711     |
| 04/85 | 1.0661     | 1.0504     | 1.0650     | 1.0596     |
| 05/85 | 0.8889     | 0.9241     | 0.9346     | 0.9573     |
| 06/85 | 0.8359     | 0.8432     | 0.8474     | 0.8530     |
| 07/85 | 0.9891     | 0.9929     | 0.9949     | 1.0024     |
| 08/85 | 0.8662     | 0.8905     | 0.8778     | 0.8529     |
| 09/85 | 0.9324     | 0.9448     | 0.9491     | 0.9310     |
| 10/85 | 0.9545     | 0.9685     | 0.9749     | 0.9559     |
| 11/85 | 0.8105     | 0.7507     | 0.7244     | 0.8094     |
| 12/85 | 0.9102     | 0.9046     | 0.8975     | 0.9069     |
| 01/86 | 0.8360     | 0.8363     | 0.8329     | 0.8446     |
| 02/86 | 0.6909     | 0.7308     | 0.7642     | 0.7291     |
| 03/86 | 0.7279     | 0.7353     | 0.7435     | 0.7391     |
| 04/86 | 0.6465     | 0.6515     | 0.6579     | 0.6551     |
| 05/86 | 0.6543     | 0.6056     | 0.6320     | 0.6695     |
| 06/86 | 0.7421     | 0.7362     | 0.7457     | 0.7438     |
| 07/86 | 0.6979     | 0.6952     | 0.7056     | 0.7045     |
| 08/86 | 0.6311     | 0.6718     | 0.6126     | 0.6533     |
| 09/86 | 0.7053     | 0.7154     | 0.7126     | 0.7220     |
| 10/86 | 0.7342     | 0.7431     | 0.7387     | 0.7474     |
| 11/86 | 0.5882     | 0.5829     | 0.5839     | 0.6276     |
| 12/86 | 0.7419     | 0.7451     | 0.7454     | 0.7579     |
| 01/87 | 0.6798     | 0.6864     | 0.6869     | 0.6999     |
| 02/86 | 0.5889     | 0.6118     | 0.6115     | 0.6107     |
| 03/86 | 0.6978     | 0.7127     | 0.7177     | 0.7189     |
| 04/87 | 0.6813     | 0.6939     | 0.6978     | 0.7006     |
| 05/87 | 0.6724     | 0.6467     | 0.6848     | 0.6789     |
| 06/87 | 0.7945     | 0.7899     | 0.8024     | 0.7959     |
| 07/87 | 0.7749     | 0.7706     | 0.7837     | 0.7762     |
| 08/87 | 0.7624     | 0.8044     | 0.7170     | 0.8050     |
| 09/87 | 0.7941     | 0.8040     | 0.7899     | 0.7973     |
| 10/87 | 0.8478     | 0.8577     | 0.8414     | 0.8492     |
| 11/87 | 0.7050     | 0.7040     | 0.7417     | 0.7657     |
| 12/87 | 0.8062     | 0.8095     | 0.8083     | 0.8166     |

TABLE 38

The Differences Between Ex ante and Ex post Monthly  
Holding Period Returns By Category  
(Partial Listing, Value in Percentages)

| T     | DR <sub>1</sub> <sup>E</sup> | DR <sub>2</sub> <sup>E</sup> | DR <sub>3</sub> <sup>E</sup> | DR <sub>4</sub> <sup>E</sup> | DR <sub>5</sub> <sup>E</sup> |
|-------|------------------------------|------------------------------|------------------------------|------------------------------|------------------------------|
| 01/85 | 0.0055                       | -0.0009                      | -0.2061                      | -0.3333                      | -0.4892                      |
| 02/85 | 0.0511                       | 0.2075                       | 0.4756                       | 0.9054                       | 1.4529                       |
| 03/86 | -0.0173                      | -0.1216                      | -0.2255                      | -0.3045                      | -0.4928                      |
| 04/85 | -0.0435                      | -0.1951                      | -0.4439                      | -0.6803                      | -0.9449                      |
| 05/85 | -0.0746                      | -0.3239                      | -0.8728                      | -1.5088                      | -2.4794                      |
| 06/85 | 0.0026                       | -0.0514                      | -0.1886                      | -0.4541                      | -0.4557                      |
| 07/85 | 0.0460                       | 0.1244                       | 0.2198                       | 0.3826                       | 0.6051                       |
| 08/85 | -0.0031                      | -0.0851                      | -0.2332                      | -0.3377                      | -0.4956                      |
| 09/85 | 0.0309                       | 0.0054                       | -0.0459                      | -0.0495                      | -0.1748                      |
| 10/85 | 0.0503                       | 0.0438                       | -0.0629                      | -0.2257                      | -0.5382                      |
| 11/85 | -0.0050                      | -0.0069                      | -0.1638                      | -0.4029                      | -0.7171                      |
| 12/85 | -0.0280                      | -0.0336                      | -0.1567                      | -0.5172                      | -1.0687                      |
| 01/86 | 0.0282                       | 0.0009                       | -0.0329                      | 0.0083                       | 0.0043                       |
| 02/86 | 0.0159                       | 0.0065                       | -0.1682                      | -0.3939                      | -0.7669                      |
| 03/86 | -0.0475                      | -0.2225                      | -0.4341                      | -0.8887                      | -1.5977                      |
| 04/86 | -0.0186                      | -0.0556                      | -0.1874                      | -0.2107                      | -0.0499                      |
| 05/86 | 0.0779                       | 0.1175                       | 0.1460                       | 0.5439                       | 1.1388                       |
| 06/86 | -0.0025                      | -0.0589                      | -0.2513                      | -0.6174                      | -1.1941                      |
| 07/86 | -0.0003                      | -0.0320                      | -0.1551                      | -0.3866                      | -0.6195                      |
| 08/86 | -0.0651                      | -0.2599                      | -0.5868                      | -0.9958                      | -1.5591                      |
| 09/86 | 0.0284                       | 0.0743                       | 0.1535                       | 0.4747                       | 0.9790                       |
| 10/86 | 0.0296                       | -0.0003                      | -0.0608                      | -0.2514                      | -0.5702                      |
| 11/86 | 0.0325                       | 0.0782                       | -0.0123                      | -0.1042                      | -0.2827                      |
| 12/86 | 0.0372                       | 0.1055                       | 0.1747                       | 0.3327                       | 0.4098                       |
| 01/87 | -0.0067                      | -0.0158                      | -0.1092                      | -0.2326                      | -0.2708                      |
| 02/87 | -0.0188                      | -0.0250                      | 0.0019                       | 0.0670                       | 0.0007                       |
| 03/87 | 0.0048                       | 0.0720                       | 0.1134                       | 0.3029                       | 0.4876                       |
| 04/87 | -0.0016                      | 0.0160                       | 0.3168                       | 0.9627                       | 1.8044                       |
| 05/87 | 0.1294                       | 0.0999                       | 0.1241                       | 0.3971                       | 0.6145                       |
| 06/87 | 0.0843                       | 0.0008                       | -0.1659                      | -0.4382                      | -0.5458                      |
| 07/87 | 0.0840                       | 0.0709                       | -0.0414                      | -0.0041                      | 0.2262                       |
| 08/87 | 0.0700                       | 0.1140                       | 0.1528                       | 0.4120                       | 0.6957                       |
| 09/87 | 0.0957                       | 0.1858                       | 0.3802                       | 0.8184                       | 1.3992                       |
| 10/87 | -0.1174                      | -0.3479                      | -0.6719                      | -1.2099                      | -1.9484                      |
| 11/87 | 0.0403                       | 0.0195                       | -0.0227                      | 0.0216                       | -0.1034                      |
| 12/87 | 0.0690                       | -0.0042                      | 0.0346                       | -0.0018                      | -0.0823                      |

TABLE 38 -- Continued

| T     | DR <sub>6</sub> <sup>E</sup> | DR <sub>7</sub> <sup>E</sup> | DR <sub>8</sub> <sup>E</sup> | DR <sub>9</sub> <sup>E</sup> | DR <sub>10</sub> <sup>E</sup> |
|-------|------------------------------|------------------------------|------------------------------|------------------------------|-------------------------------|
| 01/85 | -0.8112                      | -1.1432                      | -1.5105                      | -1.8608                      | -2.1970                       |
| 02/85 | 2.0053                       | 2.5169                       | 2.6829                       | 3.3192                       | 3.9627                        |
| 03/85 | -0.6992                      | -0.7083                      | -0.8276                      | -0.9432                      | -1.5182                       |
| 04/85 | -1.1410                      | -1.3714                      | -1.3602                      | -1.3615                      | -1.3619                       |
| 05/85 | -3.3210                      | -3.9641                      | -4.6298                      | -5.1963                      | -6.5435                       |
| 06/85 | -0.3966                      | -0.3702                      | -0.0560                      | 0.0889                       | 0.1907                        |
| 07/85 | 0.8544                       | 0.9460                       | 1.1650                       | 1.3077                       | 1.5604                        |
| 08/85 | -0.7028                      | -0.8454                      | -1.2122                      | -1.3393                      | -1.5401                       |
| 09/85 | -0.2297                      | -0.0536                      | -0.2222                      | -0.0107                      | 0.3531                        |
| 10/85 | -0.8575                      | -1.0572                      | -1.1456                      | -1.2665                      | -1.5148                       |
| 11/85 | -0.9919                      | -1.3062                      | -1.7052                      | -1.8996                      | -2.3060                       |
| 12/85 | -1.5058                      | -2.0461                      | -2.5539                      | -3.0653                      | -3.8052                       |
| 01/86 | -0.0750                      | 0.0923                       | 0.1572                       | 0.3529                       | 0.8217                        |
| 02/86 | -1.4977                      | -2.1037                      | -2.9591                      | -4.2090                      | -4.8243                       |
| 03/86 | -1.8446                      | -2.3547                      | -2.8462                      | -3.6438                      | -3.8112                       |
| 04/86 | -0.1338                      | -0.0519                      | -0.0255                      | 0.2955                       | 0.3933                        |
| 05/86 | 2.0060                       | 2.5674                       | 2.8339                       | 3.4437                       | 3.5816                        |
| 06/86 | -1.6943                      | -2.0867                      | -2.4656                      | -3.0176                      | -3.0038                       |
| 07/86 | -0.8277                      | -0.7903                      | -0.7271                      | -0.4502                      | -0.3772                       |
| 08/86 | -2.0569                      | -2.3334                      | -2.5226                      | -2.7606                      | -3.0356                       |
| 09/86 | 1.5448                       | 1.9336                       | 2.0574                       | 2.6696                       | 2.7680                        |
| 10/86 | -0.9265                      | -1.1029                      | -1.1269                      | -0.9189                      | -1.2172                       |
| 11/86 | -0.4867                      | -0.6658                      | -0.8472                      | -0.9138                      | -0.9085                       |
| 12/86 | 0.4661                       | 0.3524                       | 0.3523                       | 0.4187                       | 0.3002                        |
| 01/87 | -0.3940                      | -0.3784                      | -0.5109                      | -0.5824                      | -0.6379                       |
| 02/87 | -0.0074                      | 0.0310                       | -0.1126                      | -0.1014                      | -0.2284                       |
| 03/87 | 0.7385                       | 0.8584                       | 1.1867                       | 1.5066                       | 1.5994                        |
| 04/87 | 2.3609                       | 2.9117                       | 3.3197                       | 3.6456                       | 3.9126                        |
| 05/87 | 0.8246                       | 1.0432                       | 0.9336                       | 1.2401                       | 1.2650                        |
| 06/87 | -0.5584                      | -0.5645                      | -0.5123                      | -0.5083                      | -0.4667                       |
| 07/87 | 0.3272                       | 0.4913                       | 0.8058                       | 0.9787                       | 0.9977                        |
| 08/87 | 0.9304                       | 1.0638                       | 1.2063                       | 1.4913                       | 1.7495                        |
| 09/87 | 1.9016                       | 2.4801                       | 2.9285                       | 3.1938                       | 3.2445                        |
| 10/87 | -2.3971                      | -2.7549                      | -3.1900                      | -3.3536                      | -3.3808                       |
| 11/87 | -0.0568                      | 0.2076                       | 0.0790                       | 0.2080                       | 0.1224                        |
| 12/87 | -0.2898                      | -0.4130                      | -0.5899                      | -0.9147                      | -0.8404                       |

TABLE 38 -- Continued

| T     | DR <sub>11</sub> <sup>E</sup> | DR <sub>12</sub> <sup>E</sup> | DR <sub>13</sub> <sup>E</sup> | DR <sub>14</sub> <sup>E</sup> | DR <sub>15</sub> <sup>E</sup> |
|-------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| 01/85 | -2.2325                       | -2.1513                       | -2.3376                       | -2.8080                       | -2.5643                       |
| 02/85 | 4.1307                        | 4.2917                        | 4.3099                        | 5.3715                        | 5.3219                        |
| 03/85 | -1.4666                       | -1.3698                       | -1.1358                       | -1.7584                       | -1.7053                       |
| 04/85 | -0.9582                       | -1.1211                       | -1.5722                       | -1.2154                       | -1.1516                       |
| 05/85 | -6.8264                       | -7.1185                       | -7.4802                       | -8.3574                       | -7.9292                       |
| 06/85 | 0.0137                        | -0.0825                       | -0.4908                       | -0.1570                       | -0.2427                       |
| 07/85 | 1.7027                        | 1.9111                        | 3.0509                        | 2.3365                        | 2.5212                        |
| 08/85 | -1.8332                       | -1.7020                       | -1.6985                       | -1.9562                       | -1.8401                       |
| 09/85 | 0.0382                        | -0.9847                       | 0.2904                        | 0.4678                        | 0.7707                        |
| 10/85 | -1.7564                       | -0.5728                       | -2.0693                       | -2.3116                       | -2.5648                       |
| 11/85 | -2.2071                       | -2.1033                       | -2.5240                       | -3.4579                       | -3.6372                       |
| 12/85 | -3.9680                       | -3.6812                       | -5.0501                       | -4.4656                       | -4.5103                       |
| 01/86 | 0.8292                        | 0.5411                        | 0.4077                        | 0.3933                        | 0.7523                        |
| 02/86 | -5.7067                       | -5.8290                       | -7.5986                       | -9.2107                       | -9.5174                       |
| 03/86 | -3.9651                       | -4.5340                       | -5.9537                       | -5.5753                       | -7.7321                       |
| 04/86 | 0.2952                        | 0.1559                        | -0.6290                       | 0.6771                        | 1.1970                        |
| 05/86 | 3.9363                        | 4.2250                        | 4.8145                        | 5.6922                        | 6.2210                        |
| 06/86 | -3.4041                       | -3.9200                       | -5.1506                       | -4.9114                       | -5.2811                       |
| 07/86 | -0.1770                       | -0.0208                       | 0.8237                        | 0.9925                        | 1.0803                        |
| 08/86 | -3.2053                       | -3.0023                       | -3.2509                       | -3.6734                       | -3.9426                       |
| 09/86 | 3.0208                        | 3.3484                        | 2.8458                        | 4.0867                        | 4.8130                        |
| 10/86 | -1.2832                       | -1.1820                       | -1.5719                       | -1.6741                       | -1.8637                       |
| 11/86 | -1.0111                       | -0.7658                       | -0.6851                       | -1.4223                       | -1.4115                       |
| 12/86 | 0.3974                        | 0.5161                        | 0.7331                        | 0.4054                        | 0.5326                        |
| 01/87 | -0.6228                       | -0.4244                       | -0.2310                       | -0.8426                       | -0.9099                       |
| 02/87 | -0.1460                       | 0.0790                        | -0.3560                       | -0.7544                       | -0.7250                       |
| 03/87 | 1.6862                        | 2.0810                        | 2.1411                        | 1.9573                        | 2.2772                        |
| 04/87 | 4.2059                        | 4.5116                        | 4.6564                        | 5.0379                        | 5.5282                        |
| 05/87 | 1.7559                        | 2.1216                        | 2.1339                        | 1.7800                        | 1.5657                        |
| 06/87 | -0.5138                       | -0.6298                       | -0.4347                       | -0.5323                       | -0.3383                       |
| 07/87 | 1.3271                        | 1.7668                        | 1.8553                        | 1.9558                        | 2.3660                        |
| 08/87 | 1.9229                        | 2.3544                        | 2.3180                        | 1.9855                        | 2.2161                        |
| 09/87 | 3.6442                        | 4.0760                        | 4.4967                        | 4.4822                        | 4.9450                        |
| 10/87 | -4.0848                       | -4.8453                       | -5.7111                       | -5.5695                       | -6.3363                       |
| 11/87 | 0.3156                        | 1.0097                        | 1.2678                        | 0.7123                        | 0.9104                        |
| 12/87 | -0.8676                       | -1.0802                       | -1.5221                       | -1.1609                       | -1.5570                       |

TABLE 3C -- Continued

| T     | DR <sub>16</sub> <sup>E</sup> | DR <sub>17</sub> <sup>E</sup> | DR <sub>18</sub> <sup>E</sup> | DR <sub>19</sub> <sup>E</sup> |
|-------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| 01/85 | -2.6950                       | -2.7046                       | -2.5613                       | -2.8159                       |
| 02/85 | 5.8292                        | 6.4048                        | 6.0212                        | 5.8399                        |
| 03/85 | -1.8277                       | -2.1604                       | -1.7882                       | -1.9747                       |
| 04/85 | -0.8290                       | -0.9464                       | -0.8669                       | -0.7188                       |
| 05/85 | -8.5795                       | -9.4608                       | -8.7253                       | -8.2368                       |
| 06/85 | -0.7207                       | -0.7556                       | -0.4178                       | -0.6377                       |
| 07/85 | 2.6544                        | 2.6179                        | 2.6697                        | 2.6163                        |
| 08/85 | -2.0306                       | -2.3197                       | -1.9634                       | -1.6500                       |
| 09/85 | 0.8753                        | 1.1903                        | 1.2814                        | 0.8985                        |
| 10/85 | -2.7010                       | -2.7759                       | -2.8624                       | -2.7791                       |
| 11/85 | -3.9119                       | -3.9824                       | -3.6079                       | -3.4330                       |
| 12/85 | -5.1449                       | -5.0429                       | -4.9357                       | -5.2814                       |
| 01/86 | 0.8225                        | 0.9042                        | 0.7966                        | 0.7618                        |
| 02/86 | -10.7324                      | -10.7624                      | -10.3605                      | -10.6934                      |
| 03/86 | -7.8117                       | -6.8960                       | -8.0297                       | -8.4994                       |
| 04/86 | 1.2715                        | 1.4760                        | 1.5133                        | 0.9382                        |
| 05/86 | 7.0867                        | 6.0198                        | 6.2181                        | 6.1796                        |
| 06/86 | -5.8907                       | -5.7009                       | -5.6276                       | -5.1991                       |
| 07/86 | 1.2126                        | 1.7660                        | 1.8504                        | 1.7556                        |
| 08/86 | -4.1769                       | -4.2758                       | -4.4970                       | -4.0001                       |
| 09/86 | 4.6508                        | 5.0478                        | 5.2234                        | 4.8755                        |
| 10/86 | -1.4541                       | -1.5608                       | -1.5509                       | -1.5011                       |
| 11/86 | -1.3924                       | -1.7353                       | -1.6038                       | -1.0227                       |
| 12/86 | 0.7699                        | 0.9742                        | 0.6948                        | 1.0678                        |
| 01/87 | -0.8790                       | -0.9461                       | -0.9763                       | -1.0723                       |
| 02/86 | -1.0145                       | -1.2870                       | -1.0519                       | -0.3142                       |
| 03/87 | 2.8259                        | 2.5150                        | 2.4700                        | 3.2828                        |
| 04/87 | 5.6761                        | 5.1917                        | 5.6356                        | 6.5321                        |
| 05/87 | 2.1876                        | 1.7845                        | 1.7919                        | 1.9938                        |
| 06/87 | 0.1113                        | -0.1923                       | -0.3313                       | -0.8171                       |
| 07/87 | 2.6348                        | 2.3296                        | 2.4775                        | 3.3730                        |
| 08/87 | 2.3416                        | 2.2781                        | 2.4588                        | 2.9096                        |
| 09/87 | 5.6580                        | 4.8328                        | 5.1436                        | 6.0915                        |
| 10/87 | -6.9952                       | -6.1518                       | -6.6640                       | -7.4389                       |
| 11/87 | 0.8597                        | 0.7837                        | 0.8808                        | 0.5306                        |
| 12/87 | -1.7096                       | -1.8803                       | -1.8010                       | -1.7031                       |

TABLE 39

Summary Statistics of Ex ante Returns and the Differences  
Between Ex ante and Ex post Returns (Value in Percentage)

| Category | Mean<br>of<br>$R_{ti}^E$ | Mean<br>of<br>$DR_{ti}^E$ | Maximum<br>(Minimum)<br>of $R_{ti}^E$ | Maximum<br>(Minimum)<br>of $DR_{ti}^E$ | STD<br>(VAR) of<br>$R_{ti}^E$ | STD<br>(VAR) of<br>$DR_{ti}^E$ |
|----------|--------------------------|---------------------------|---------------------------------------|----------------------------------------|-------------------------------|--------------------------------|
| 1        | 0.5945                   | 0.0157                    | 0.8386<br>(0.4233)                    | 0.2856<br>(-0.3252)                    | 0.0796<br>(0.0063)            | 0.0512<br>(0.0026)             |
| 2        | 0.6027                   | -0.0138                   | 0.8401<br>(0.4303)                    | 0.3496<br>(-0.5222)                    | 0.0843<br>(0.0071)            | 0.1280<br>(0.0164)             |
| 3        | 0.5912                   | -0.0891                   | 0.8285<br>(0.3032)                    | 0.6677<br>(-1.4356)                    | 0.0889<br>(0.0079)            | 0.2800<br>(0.0784)             |
| 4        | 0.6418                   | -0.1367                   | 0.9200<br>(0.3373)                    | 0.3128<br>(-2.2319)                    | 0.1051<br>(0.0110)            | 0.5603<br>(0.3140)             |
| 5        | 0.6686                   | -0.2119                   | 0.9785<br>(0.3507)                    | 2.0668<br>(-3.1404)                    | 0.1127<br>(0.0127)            | 0.9498<br>(0.9021)             |
| 6        | 0.6901                   | -0.2762                   | 1.0100<br>(0.3676)                    | 2.6614<br>(-4.4802)                    | 0.1228<br>(0.0151)            | 1.2988<br>(1.6869)             |
| 7        | 0.7053                   | -0.3047                   | 1.0415<br>(0.3645)                    | 3.1877<br>(-4.7005)                    | 0.1263<br>(0.0159)            | 1.6015<br>(2.5648)             |
| 8        | 0.7195                   | -0.3875                   | 1.0604<br>(0.4017)                    | 3.5176<br>(-4.9675)                    | 0.1283<br>(0.0164)            | 1.8589<br>(3.4555)             |
| 9        | 0.7365                   | -0.3933                   | 1.0828<br>(0.4182)                    | 4.3963<br>(-5.8618)                    | 0.1348<br>(0.0182)            | 2.1972<br>(4.8276)             |
| 10       | 0.7379                   | -0.4638                   | 1.0894<br>(0.4183)                    | 4.5465<br>(-7.2638)                    | 0.1337<br>(0.0179)            | 2.4704<br>(6.1028)             |
| 11       | 0.7515                   | -0.4726                   | 1.0878<br>(0.4388)                    | 4.6918<br>(-7.4115)                    | 0.1349<br>(0.0182)            | 2.6865<br>(7.2173)             |
| 12       | 0.7665                   | -0.3925                   | 1.0989<br>(0.5670)                    | 7.8939<br>(-9.9264)                    | 0.1259<br>(0.0159)            | 2.8705<br>(8.2397)             |
| 13       | 0.7717                   | -0.5919                   | 1.1011<br>(0.4675)                    | 5.2429<br>(-8.8032)                    | 0.1393<br>(0.0194)            | 3.3074<br>(10.9389)            |
| 14       | 0.7695                   | -0.6522                   | 1.0968<br>(0.4514)                    | 6.6467<br>(-10.5529)                   | 0.1264<br>(0.0160)            | 3.5718<br>(12.7575)            |
| 15       | 0.7828                   | -0.6317                   | 1.1035<br>(0.4782)                    | 6.5224<br>(-9.6476)                    | 0.1263<br>(0.0160)            | 3.8574<br>(14.8799)            |
| 16       | 0.7851                   | -0.6397                   | 1.1017<br>(0.4879)                    | 7.8695<br>(-11.8538)                   | 0.1219<br>(0.0149)            | 4.2025<br>(17.6609)            |
| 17       | 0.7886                   | -0.7061                   | 1.0971<br>(0.4786)                    | 6.6466<br>(-11.2242)                   | 0.1214<br>(0.0147)            | 4.1130<br>(16.9171)            |
| 18       | 0.7910                   | -0.6416                   | 1.1014<br>(0.4836)                    | 6.4648<br>(-11.0699)                   | 0.1230<br>(0.0151)            | 4.1343<br>(17.0925)            |
| 19       | 0.7995                   | -0.5595                   | 1.0998<br>(0.4886)                    | 7.0826<br>(-11.0174)                   | 0.1158<br>(0.0134)            | 4.2647<br>(18.1874)            |

TABLE 40a

Regression Results of Two Factor Model (c)

$$DR_i = \alpha_i + \beta_{i1}I_1 + \beta_{i2}I_2 + u_i$$

| Category | $\alpha_i$<br>(t-value) | $\beta_{i1}$<br>(t-value) | $\beta_{i2}$<br>(t-value) | $R_i^2$ | $V(u_i)$ |
|----------|-------------------------|---------------------------|---------------------------|---------|----------|
| 1        | 0.0215<br>(3.907)       | 0.0907<br>(5.256)         | 0.0171<br>(0.989)*        | 0.6174  | 0.0000   |
| 2        | 0.0039<br>(0.419)*      | 0.2376<br>(8.089)         | 0.0825<br>(2.813)         | 0.8231  | 0.0029   |
| 3        | -0.0479<br>(-2.275)     | 0.3872<br>(5.865)         | 0.3284<br>(4.978)         | 0.8131  | 0.0147   |
| 4        | -0.0516<br>(-1.305)*    | 0.5990<br>(4.836)         | 0.8487<br>(6.860)         | 0.8358  | 0.0515   |
| 5        | -0.0659<br>(-1.032)*    | 0.7964<br>(3.980)         | 1.6506<br>(8.258)         | 0.8509  | 0.1345   |
| 6        | -0.0764<br>(-0.895)*    | 0.8400<br>(3.140)         | 2.4691<br>(9.239)         | 0.8574  | 0.2405   |
| 7        | -0.0558<br>(-0.582)*    | 0.8618<br>(2.872)         | 3.2306<br>(10.777)        | 0.8820  | 0.3026   |
| 8        | -0.0962<br>(-0.960)*    | 0.8444<br>(2.689)         | 3.9189<br>(12.495)        | 0.9041  | 0.3313   |
| 9        | -0.0474<br>(-0.466)*    | 0.6544<br>(2.053)         | 4.9431<br>(15.522)        | 0.9293  | 0.3415   |
| 10       | -0.0764<br>(-0.677)*    | 0.5719<br>(1.617)*        | 5.6720<br>(16.056)        | 0.9311  | 0.4203   |
| 11       | -0.0492<br>(-0.422)*    | 0.6818<br>(1.865)*        | 6.1525<br>(16.849)        | 0.9378  | 0.4490   |
| 12       | 0.0614<br>(0.444)*      | 1.2347<br>(2.846)         | 6.1740<br>(14.246)        | 0.9232  | 0.6325   |
| 13       | -0.0651<br>(-0.462)*    | 1.3148<br>(2.980)         | 7.2634<br>(16.480)        | 0.9402  | 0.6542   |
| 14       | -0.0914<br>(-0.628)*    | 0.5562<br>(1.219)*        | 8.4399<br>(18.522)        | 0.9452  | 0.6992   |
| 15       | -0.0211<br>(-0.146)*    | 0.7783<br>(1.717)*        | 9.0441<br>(19.975)        | 0.9536  | 0.6904   |
| 16       | 0.0228<br>(0.130)*      | 0.9371<br>(1.704)*        | 9.7342<br>(17.713)        | 0.9424  | 1.0170   |
| 17       | -0.0595<br>(-0.382)*    | 0.5124<br>(1.051)*        | 9.8398<br>(20.210)        | 0.9528  | 0.7983   |
| 18       | 0.0133<br>(0.091)*      | 0.7201<br>(1.579)*        | 9.7953<br>(21.494)        | 0.9591  | 0.6994   |
| 19       | 0.1187<br>(0.782)*      | 0.9829<br>(2.068)         | 9.9453<br>(20.942)        | 0.9582  | 0.7595   |

\* Not statistically significant at 5% level.

TABLE 40b

Regression Results of Two Factor Model (d)

$$DR_i = \alpha_i + \beta_{i1}(I_2 - I_1) + \beta_{i2}I_2 + u_i$$

| Category | $\alpha_i$<br>(t-value) | $\beta_{i1}$<br>(t-value) | $\beta_{i2}$<br>(t-value) | $R_i^2$ | $V(u_i)$ |
|----------|-------------------------|---------------------------|---------------------------|---------|----------|
| 1        | 0.0215<br>(3.907)       | -0.0907<br>(-5.256)       | 0.1078<br>(6.896)         | 0.6174  | 0.0000   |
| 2        | 0.0039<br>(0.419)*      | -0.2376<br>(-8.089)       | 0.3201<br>(12.038)        | 0.8231  | 0.0029   |
| 3        | -0.0479<br>(-2.275)     | -0.3872<br>(-5.865)       | 0.7156<br>(11.970)        | 0.8131  | 0.0147   |
| 4        | -0.0516<br>(-1.305)*    | -0.5990<br>(-4.836)       | 1.4476<br>(12.910)        | 0.8358  | 0.0515   |
| 5        | -0.0659<br>(-1.032)*    | -0.7964<br>(-3.980)       | 2.4469<br>(13.508)        | 0.8509  | 0.1345   |
| 6        | -0.0764<br>(-0.895)*    | -0.8400<br>(-3.140)       | 3.3091<br>(13.662)        | 0.8574  | 0.2405   |
| 7        | -0.0558<br>(-0.582)*    | -0.8618<br>(-2.872)       | 4.0924<br>(15.026)        | 0.8820  | 0.3026   |
| 8        | -0.0962<br>(-0.960)*    | -0.8444<br>(-2.689)       | 4.7633<br>(16.756)        | 0.9041  | 0.3313   |
| 9        | -0.0474<br>(-0.466)*    | -0.6544<br>(-2.053)       | 5.5975<br>(19.393)        | 0.9293  | 0.3415   |
| 10       | -0.0764<br>(-0.677)*    | -0.5719<br>(-1.617)*      | 6.2439<br>(19.501)        | 0.9311  | 0.4203   |
| 11       | -0.0492<br>(-0.422)*    | -0.6818<br>(-1.865)*      | 6.8343<br>(20.651)        | 0.9378  | 0.4490   |
| 12       | 0.0614<br>(0.444)*      | -1.2347<br>(-2.846)       | 7.4087<br>(18.862)        | 0.9232  | 0.6325   |
| 13       | -0.0651<br>(-0.462)*    | -1.3148<br>(-2.980)       | 8.5782<br>(21.475)        | 0.9402  | 0.6542   |
| 14       | -0.0914<br>(-0.628)*    | -0.5562<br>(-1.219)*      | 8.9960<br>(21.783)        | 0.9452  | 0.6992   |
| 15       | -0.0211<br>(-0.146)*    | -0.7783<br>(-1.717)*      | 9.8223<br>(23.936)        | 0.9536  | 0.6904   |
| 16       | 0.0228<br>(0.130)*      | -0.9371<br>(-1.704)*      | 10.6713<br>(21.425)       | 0.9424  | 1.0170   |
| 17       | -0.0595<br>(-0.382)*    | -0.5124<br>(-1.051)*      | 10.3522<br>(23.460)       | 0.9528  | 0.7983   |
| 18       | 0.0133<br>(0.091)*      | -0.7201<br>(-1.579)*      | 10.5154<br>(25.460)       | 0.9591  | 0.6994   |
| 19       | 0.1187<br>(0.782)*      | -0.9829<br>(-2.068)       | 10.9282<br>(25.390)       | 0.9582  | 0.7595   |

\* Not statistically significant at 5% level.

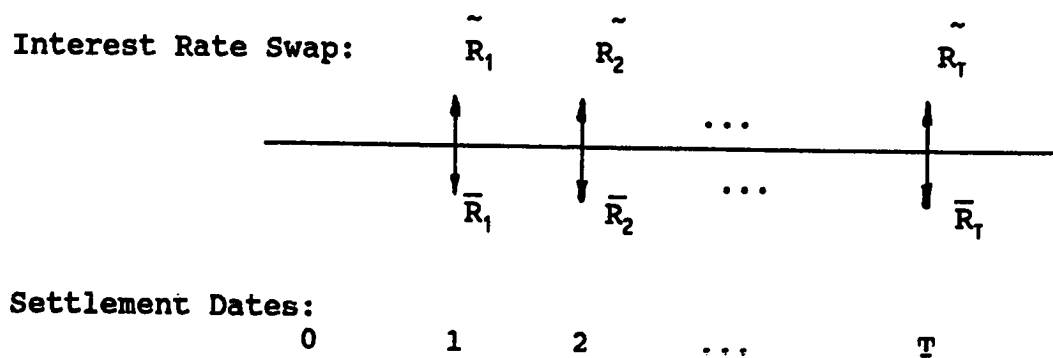


**FIGURE 1.** Cash Flows in An Interest Rate Swap

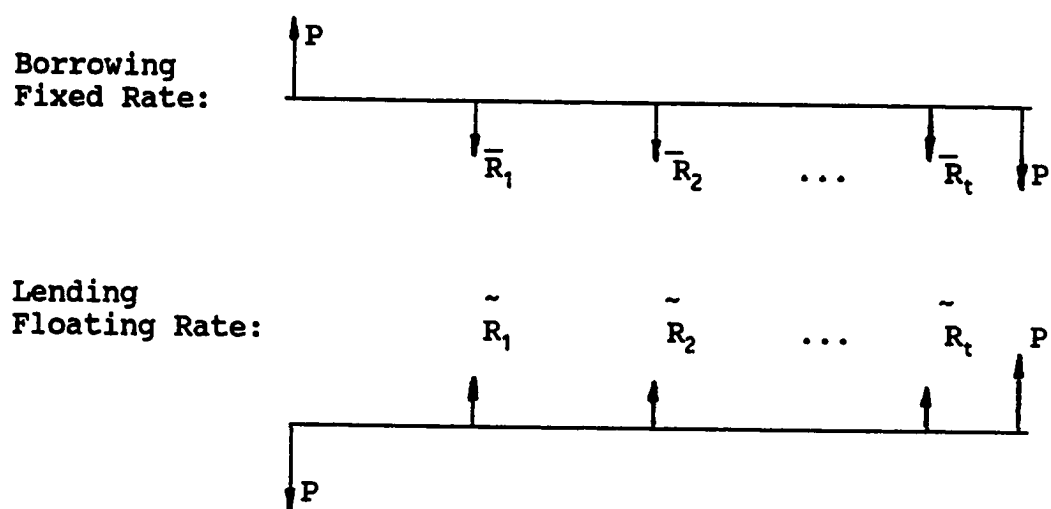


**FIGURE 2.** Cash Flows From A Comparable Portfolio of Loans

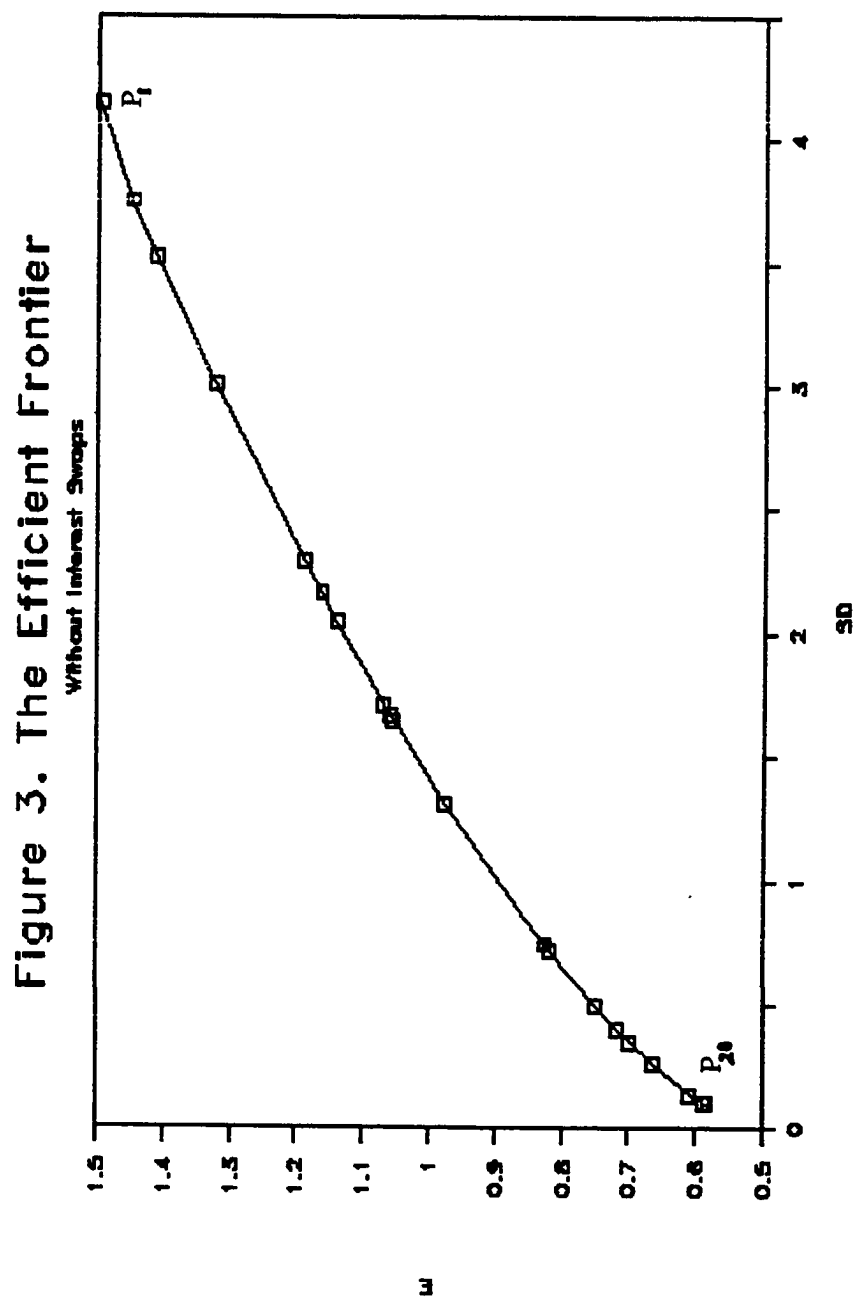


Figure 4. The Efficient Frontier

With 20 Percent Nontraded Assets

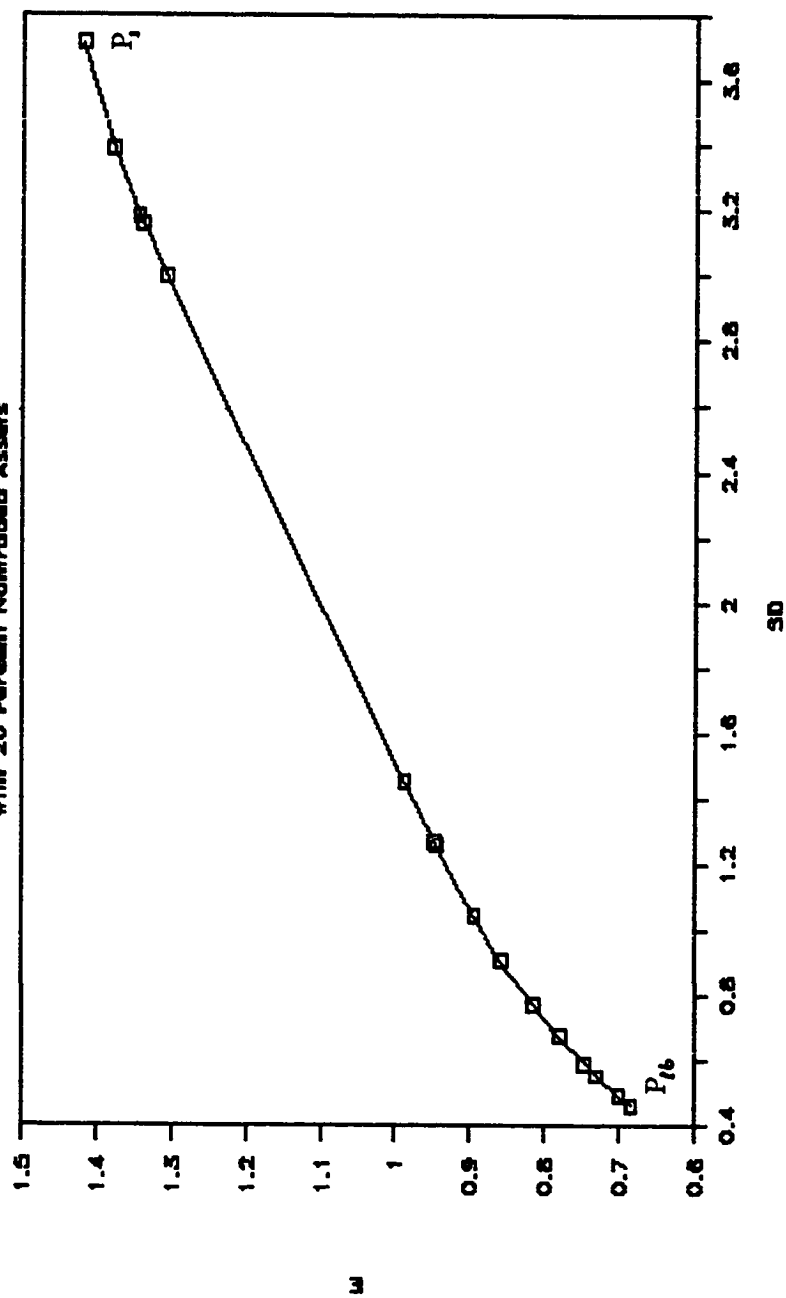


Figure 5. The Efficient Frontier

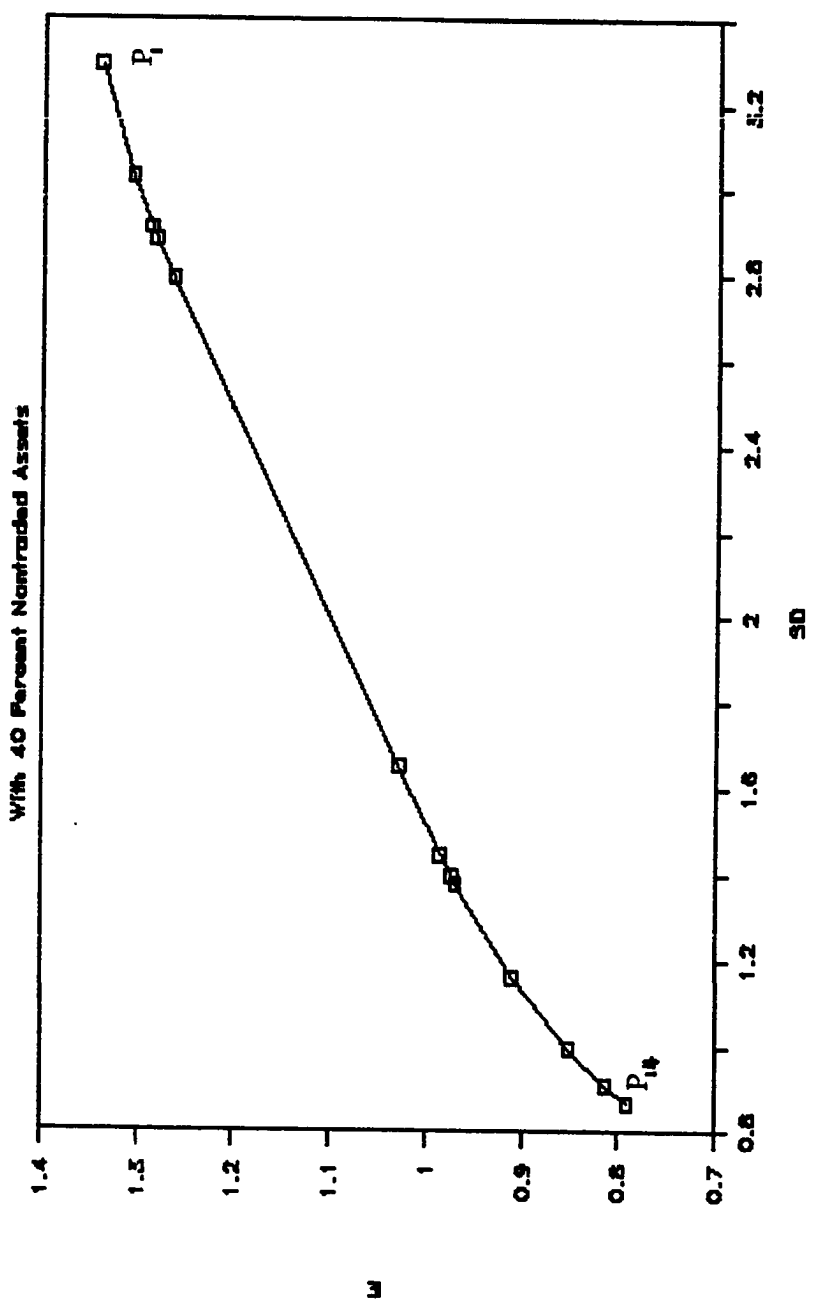


Figure 6. The Efficient Frontier

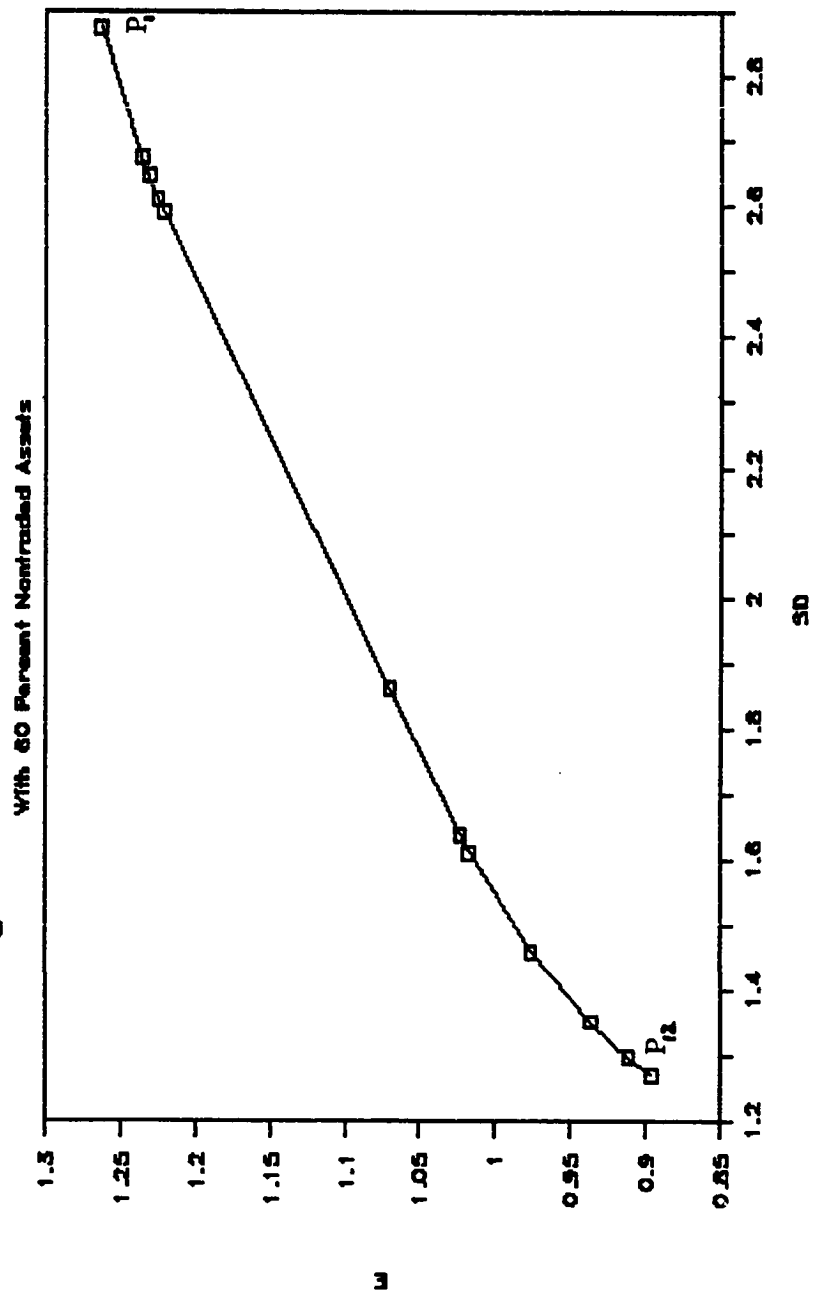


Figure 7. The Efficient Frontier

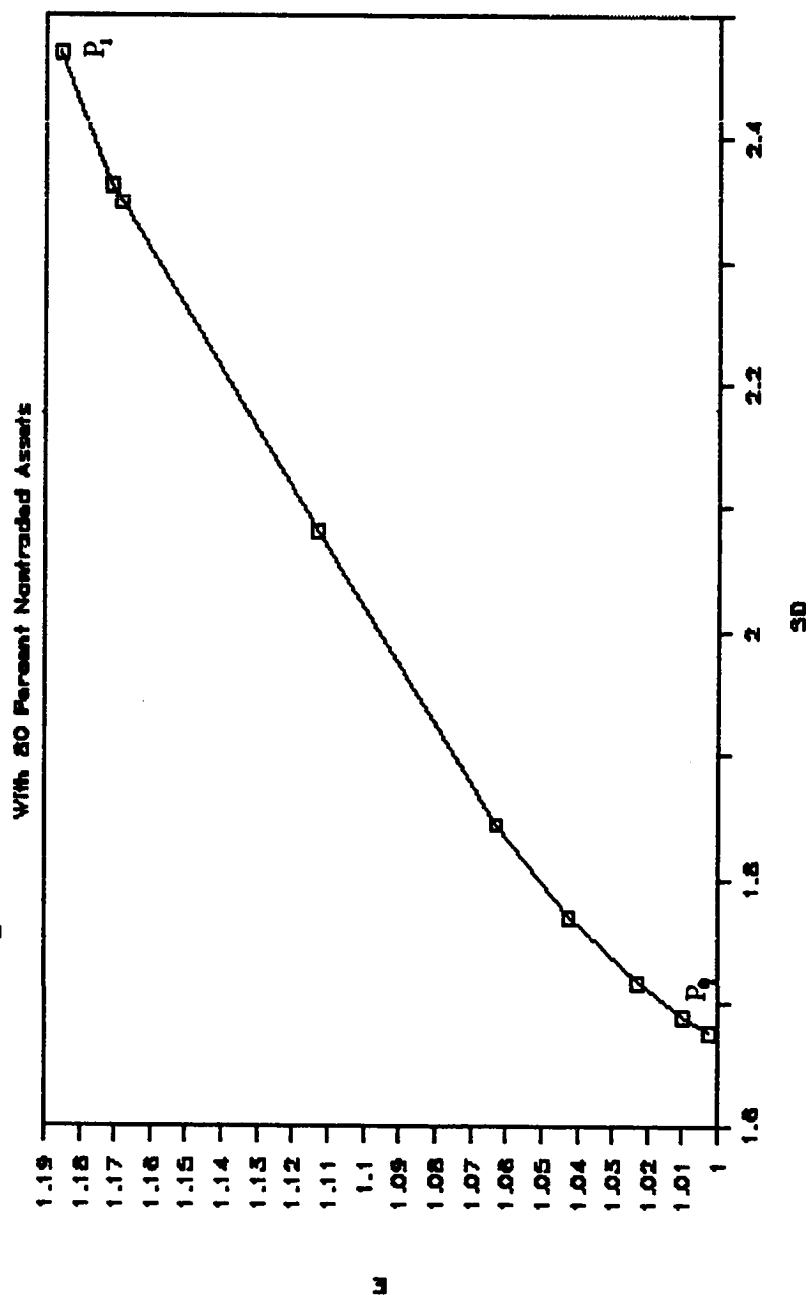


Figure 8. The Efficient Frontier

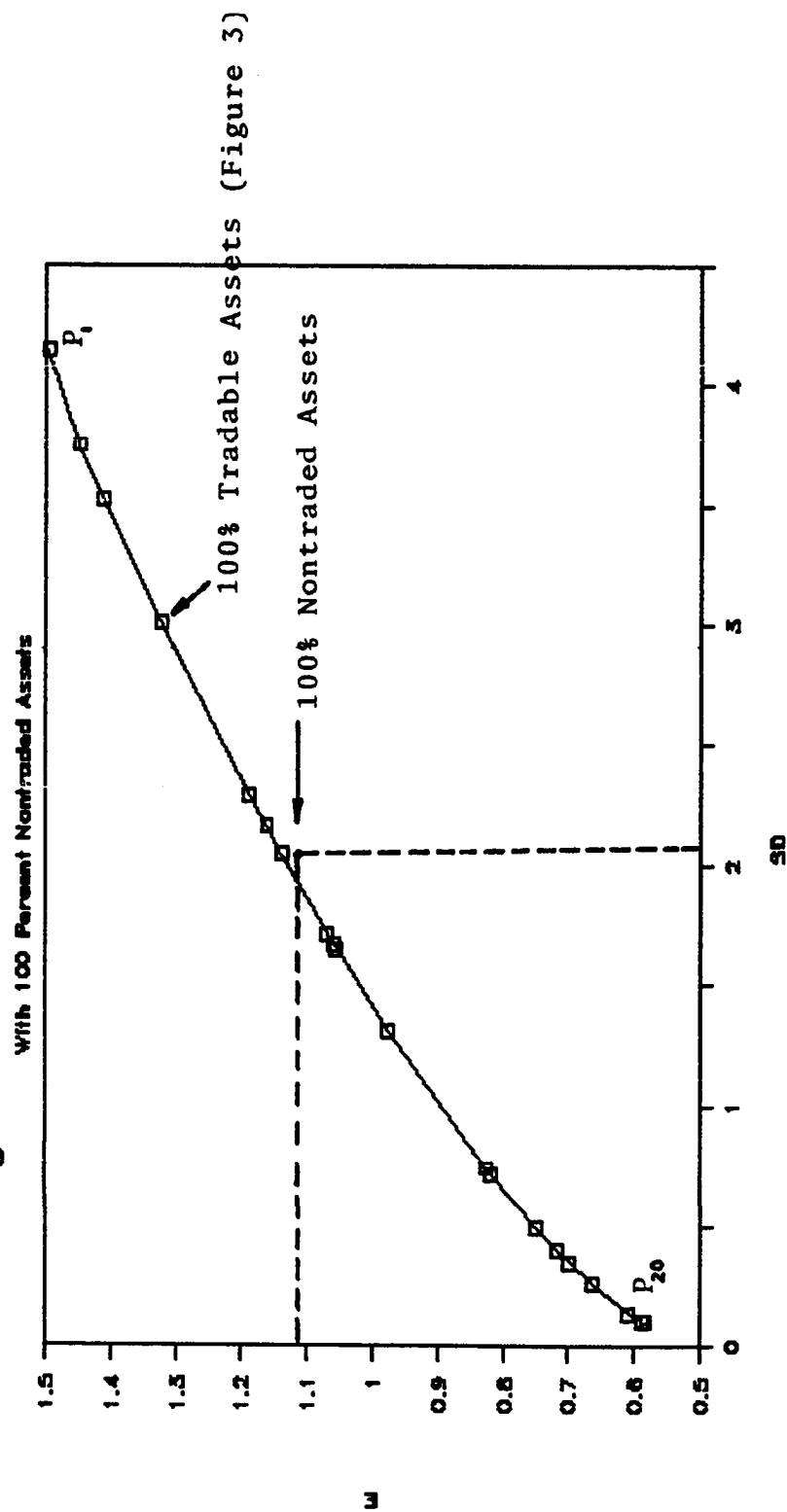


Figure 9. The Efficient Frontier

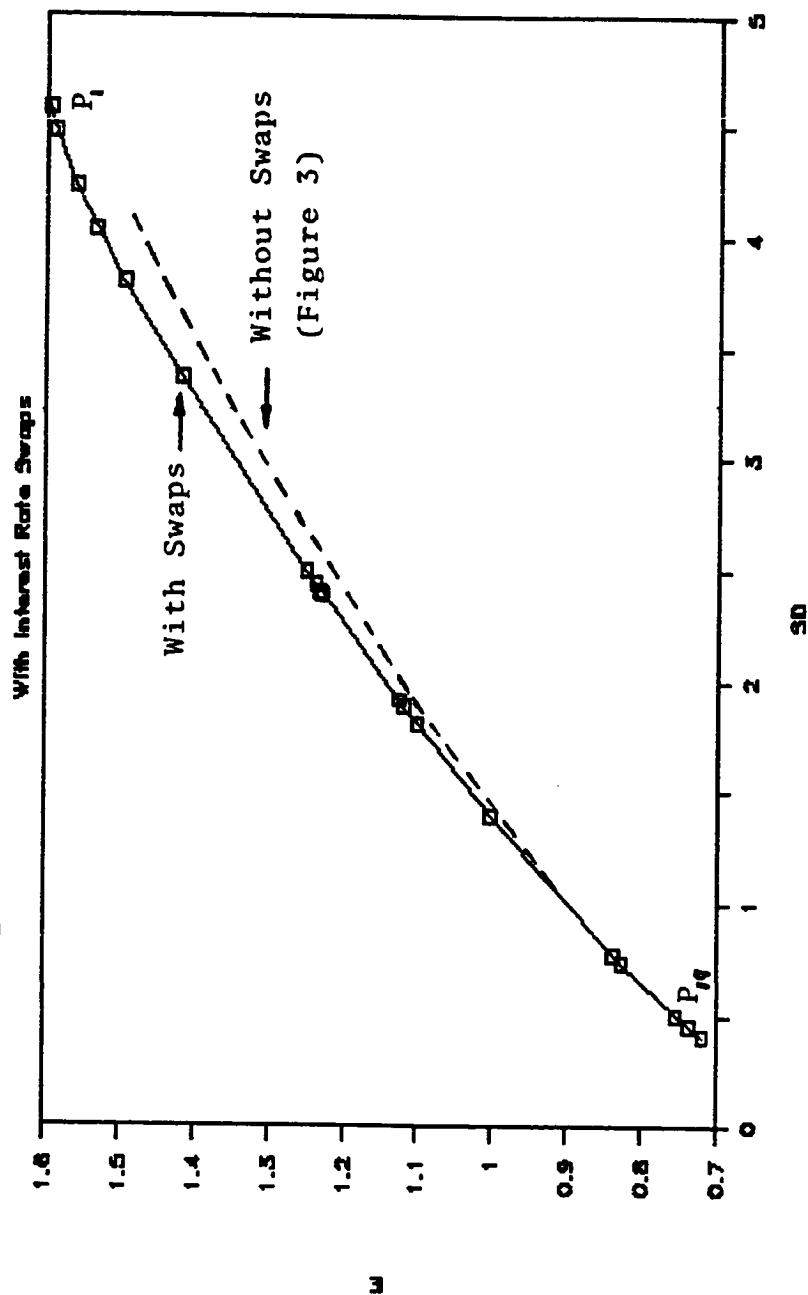


Figure 10. The Efficient Frontier

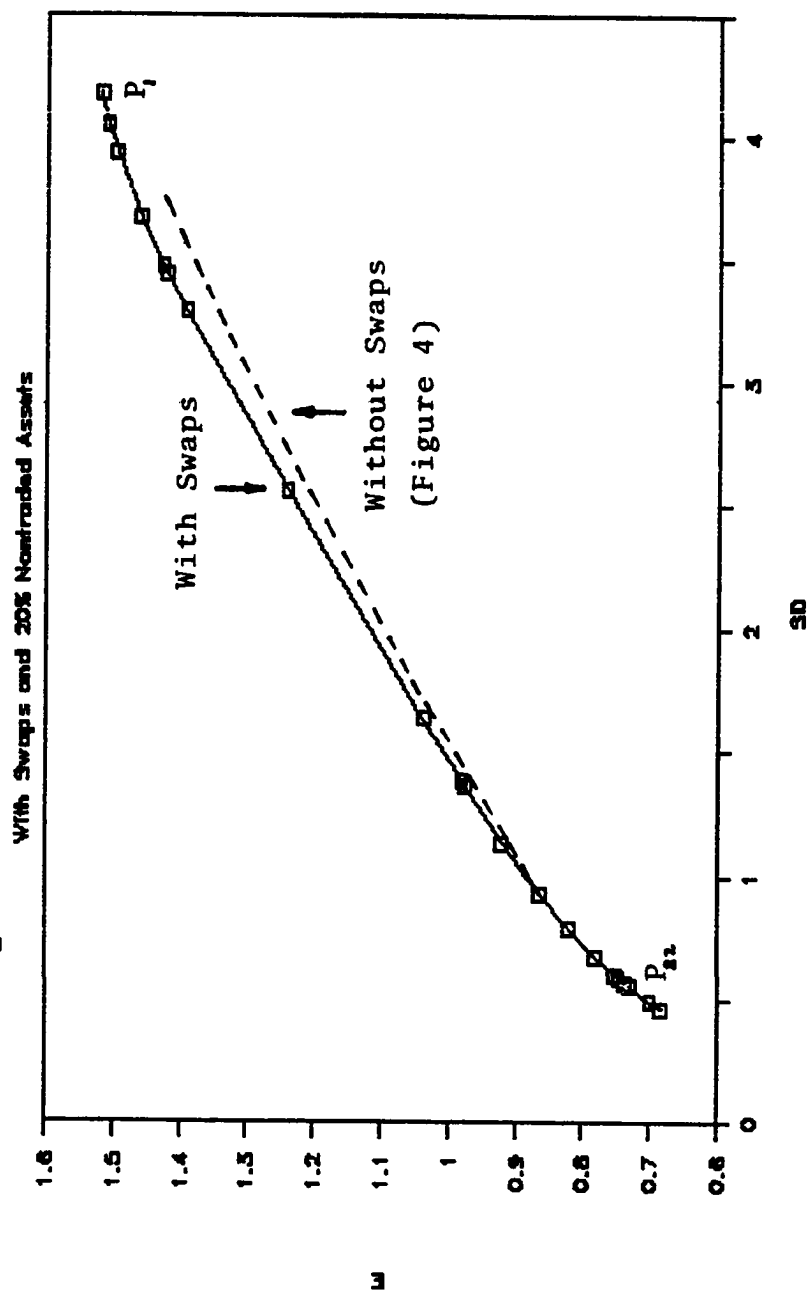


Figure 11. The Efficient Frontier

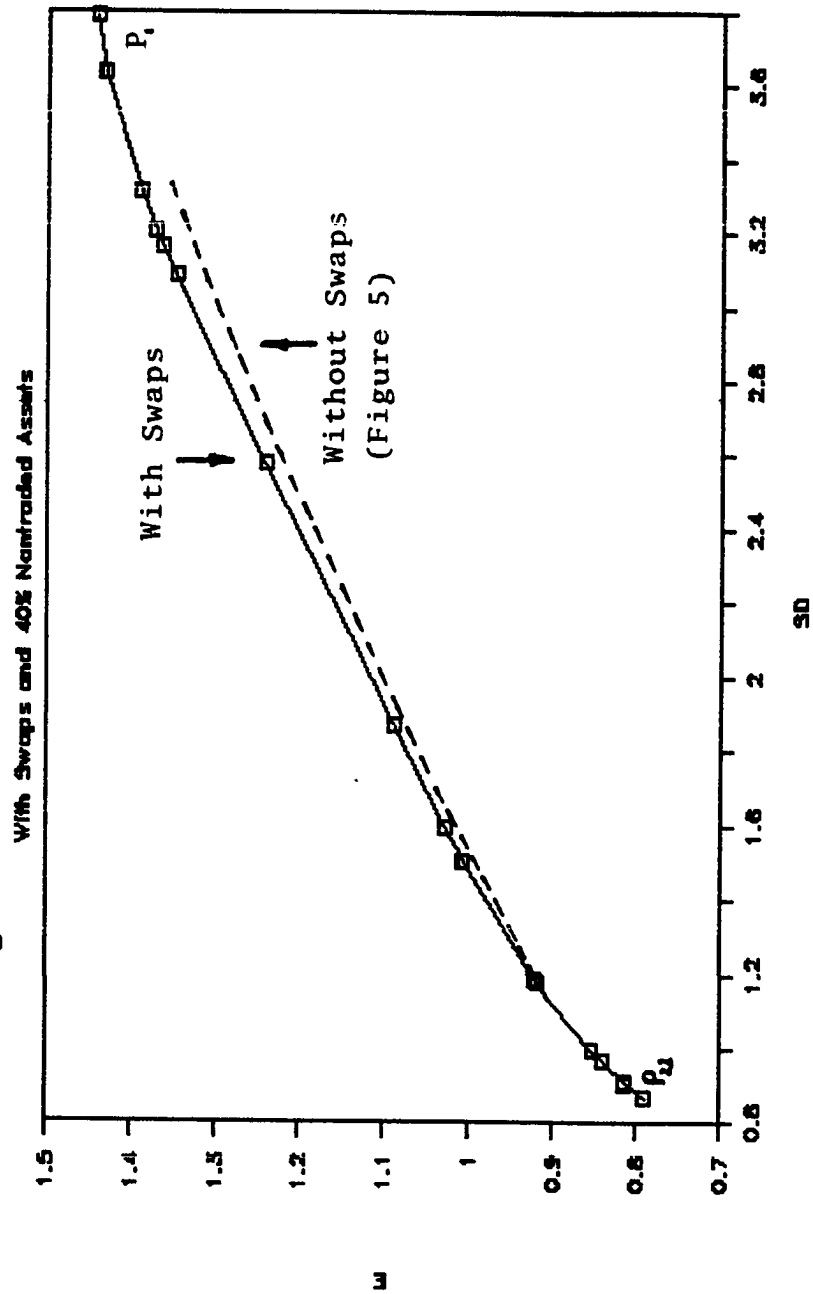


Figure 12. The Efficient Frontier

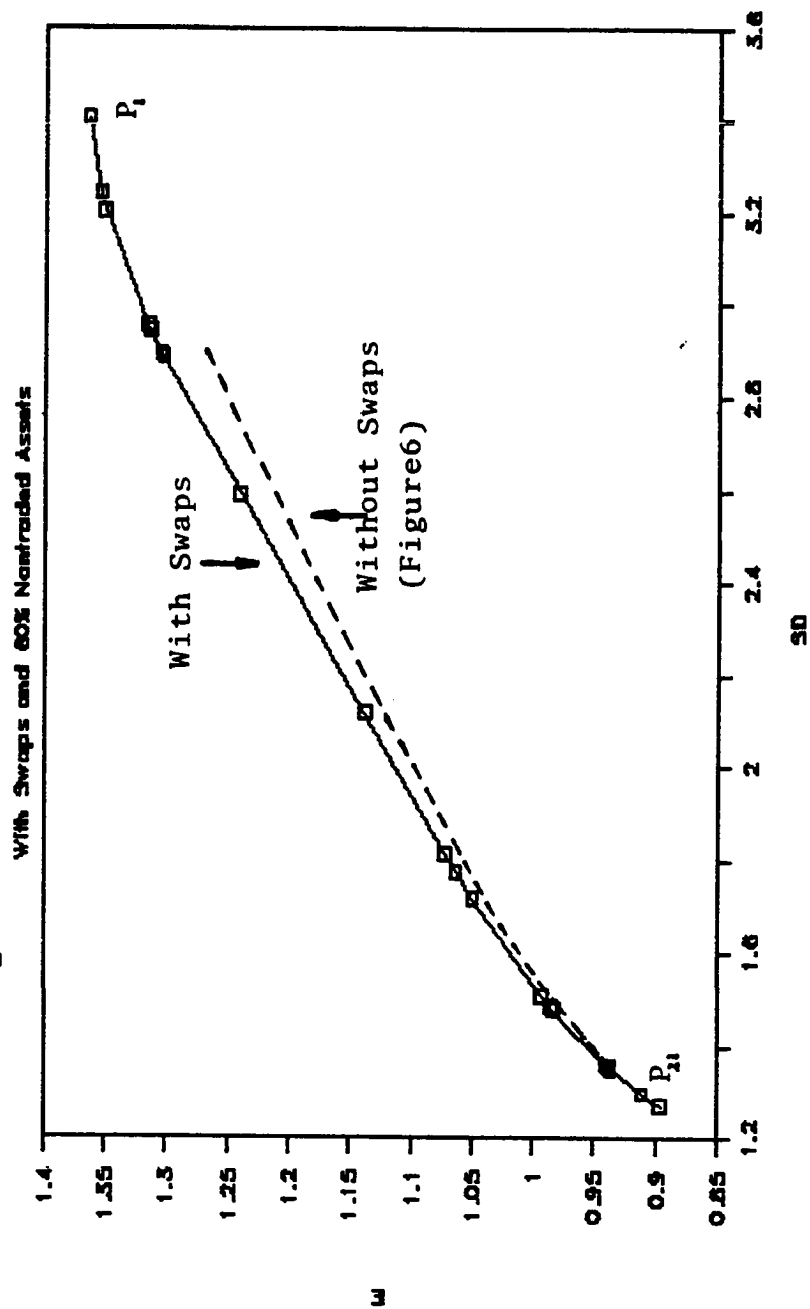


Figure 13. The Efficient Frontier

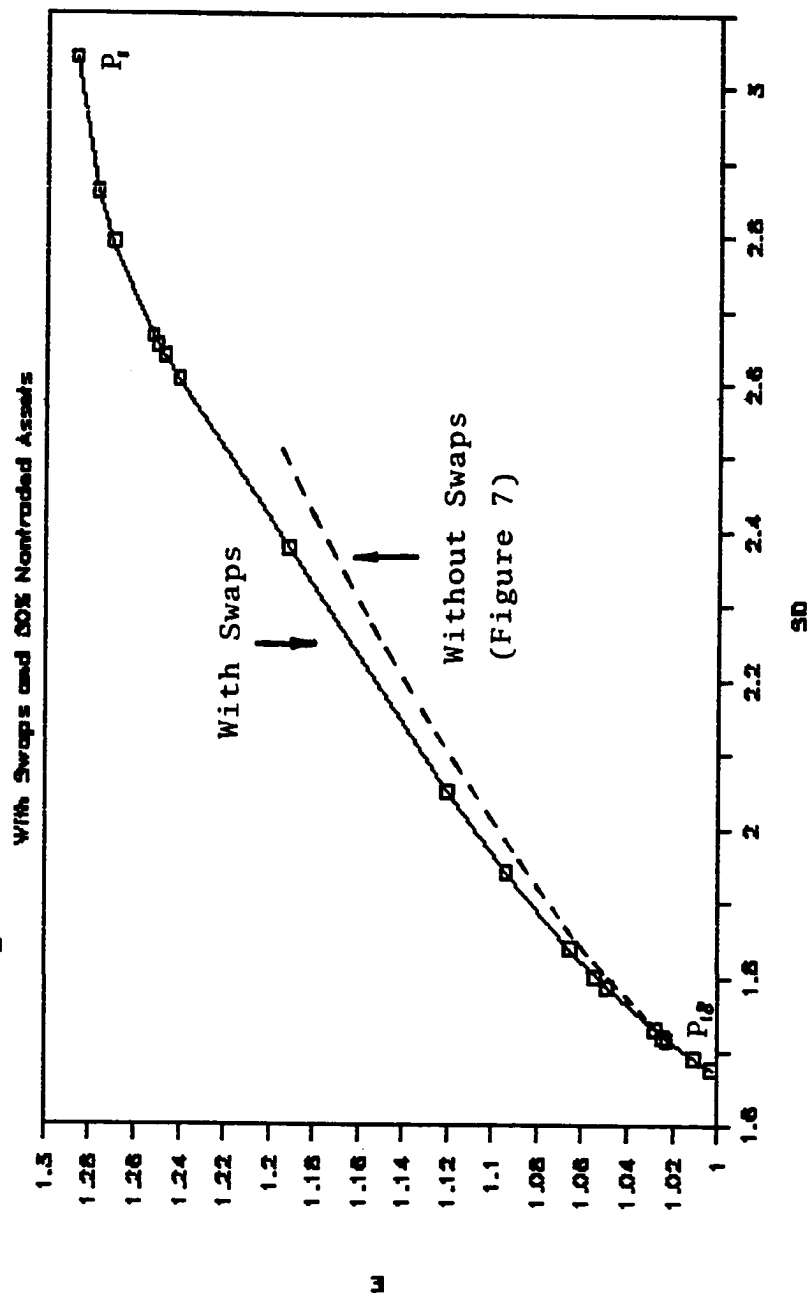
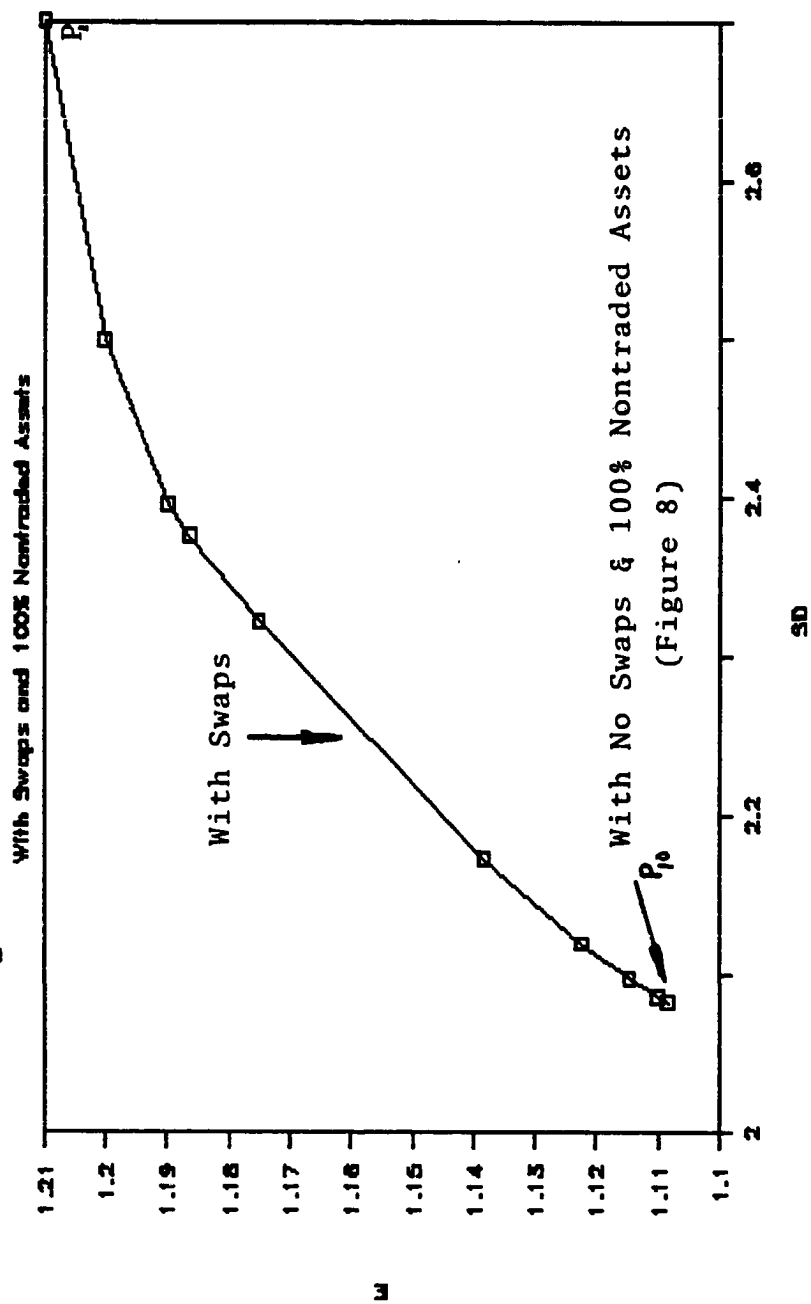


Figure 14. The Efficient Frontier



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