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Noncoherent detection of trellis coded continuous-phase multilevel-FM

Ishak, Adel W., Ph.D.

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U·M·I
300 N. Zeeb Rd.
Ann Arbor, MI 48106

NONCOHERENT DETECTION OF TRELLIS CODED CONTINUOUS-PHASE MULTILEVEL-FM

by

ADEL ISHAK

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Date

Chair of Examining Committee

5/19/88

Date

Jacques E. Benveniste

Executive Officer

Prof. R. Bozovic

Prof. J. Barba

Prof. N. Scheinberg

Prof. E. Hibshoosh

Supervisory Committee

The City University of New York

ABSTRACT

NON COHERENT DETECTION OF TRELLIS CODED CONTINUOUS-PHASE MUTILEVEL-FM

by

Adel Ishak

Adviser: Professor D. L. Schilling

A noncoherent detection of a narrow band digital communication system employing channel coding with continuous-phase multilevel frequency modulation and Trellis decoding is analyzed. For the codes under consideration the power spectrum density function and 99% energy bandwidth are given.

In particular we consider rate 1/2 code with symmetric and asymmetric signal constellation. The upper bound on bit error rate is obtained for both cases. The results are compared with noncoherent detection of MSK and Duobinary signaling. It is shown that a symmetric signal constellation combined with a four states Trellis code offers no significant system performance improvement over a MSK signal. In addition, it is shown that an asymmetric signal spacing combined with a four states Trellis code gives a significant improvement over a MSK signal.

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To the memory of Mr. Wahba Ishak, my father, who taught me my first mathematics and launched me on the long and fascinating journey of research in probability and coding.

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1 STATEMENT OF THE PROBLEM

1.1 Introduction

In communication systems where a high spectrum efficiency and immunity to nonlinear distortion is required, it is advisable to use continuous phase frequency modulation. In addition to these requirements, if the transmission channel exhibits a strong, fast fading and multipath propagation, noncoherent detection is preferred.

The communication system we propose is narrow band trellis coded continuous-phase multilevel-FM, with noncoherent detection, employing a limiter-discriminator and an integrator as a post-detection filter. The continuous-phase multilevel FM provides an appropriate spectrum shaping to increase the spectrum efficiency which is of great importance in any multi- user radio communications. The trellis coding of the signal space is to improve error performance of the system without sacrificing data rate or increasing the bandwidth. This study is to investigate n/(n+1) trellis codes and continuous-phase M-ary FM, $M=2^{n+1}$ with symmetric and asymmetric signal constellation. For given n and symmetric signal constellation we shall determine the system bandwidth B, minimum free Euclidean distance d_{free} and upper bound on the bit error rate P_b. The effect of an asymmetric signal constellation on B, d_{free} and P_b will be studied. For a given n we will find the signal constellation which minimizes B and maximizes d_{free}. Theoretical methods will be used as well as computer simulation to obtain the upper bound on the bit error rate P_b for a signal embedded in white Gaussian noise.

In digital radio communications, due to the existing system constraints in power and spectrum economy, continuous-phase frequency modulation (CP-FM) schemes are preferred. In order to meet all the requirements for transmission over radio channels, a number of spectrally efficient CP-FM schemes have been developed [1]-[6]. A (CP-FM) signal received over a radio channel can be demodulated by using coherent detection, differential detection or discriminator detection. The coherent detection requires carrier recovery which becomes extremely difficult in the fast fading environment that characterizes mobile radio communications. For such an environment, because of its immunity against fast fading and carrier frequency drift (due to the Doppler effect), noncoherent detection employing discriminator appears to be the best solution. Trellis coding of the signal set was first introduced by Ungerboeck [9], who has analyzed n/(n+1) trellis codes and symmetric MPSK $M=2^{n+1}$ signals constellation. It is shown that significant system performance improvement is achieved over uncoded MPSK $M=2^{n}$. Appropriate combination of trellis coding of the signal space and continuous-phase multilevel FM with noncoherent detection, employing discriminator, will result in a modulation scheme very suitable for mobile communications. This new modulation scheme will provide high spectrum and power efficiency, low error rate and high immunity against fast fading and doppler shifts.

2 PREVIOUS WORK

The notion of correlative coding to data transmission by FM was introduced by Lender [2]. Lender suggested "a direct combination of digital coding and modulation. Such combinations would improve the data transmission rate in bits/sec/Hz as compared to the conventional Nyquist rate. This technique enhances the spectral efficiency of digital data for the given channel constraints such as bandwidth and the input power." Correlative techniques deliberately introduce a limited amount of ISI (intersymbol interference) over one or more bit duration. The idea of deliberately introducing ISI is to shape the spectrum of the statistically independent and equally likely to occur random binary data. It is required to reduce the power within the sidelobes of a MSK system, which is equivalent to 10 % of the total power, which is considered as a source for channel interference. Therefore by introducing correlative coding to random binary data at the transmitter, it is possible to shape the spectrum and to improve the spectral efficiency. For example G.S Deshpande, and P.H. Wittke [4], succeded to improve the spectral efficency of a MSK system; they succeded to obtain an explicit expression for the power spectrum of a correlative encoded digital FM. Their results confirm that correlative encoding are spectrally more efficient than a MSK system and truly promise a better error performance. For example by using TFM it is possible to reduce the bandwidth to $(0.8/T_s)$ relative to MSK $(1.18/T_s)$ for the same 99% power and same modulation index $(.5\pi)$. The spectral efficiency improved from (.85 bits/sec/Hz) MSK system to (1.25 bits/sec/Hz) TFM system. These improvements are at the expense of high bit error rate.

Ungerboeck [9], "has constructed trellis codes that provide the same noise immunity as is given by increasing the power of uncoded data by factors ranging from two to four." His method is to specify a convolutional code and a mapping rule to map the output of this code to a fixed signal constellation. Trellis codes are attractive and feasible because of their rich structure. By introducing trellis coding which is a composite of two parts, the first one is the encoder through which we introduce deliberately inter-symbol interference; its structure depends on the type of code to be applied, and the required amount of coding gain. The secod part is to assign a mapping rule to map the channel signals in a manner to maximize the minimum Euclidean distance, which it is an essential quantity in determining both the required coding gain and to improve the system error performance.

Ungerboeck utilized a trellis coding technique to improve the error performance without increasing the bandwidth and sacrificing data rate. For example if n bits per modulation interval are required to be transmitted then we use an expanded set of 2^{n+1} discrete channel signals. The encoder which satisfies this requirement is the convolutinal encoder with a rate $R = n/(n+1)$. The resulting (n+1) bits must be mapped into the 2^{n+1} discrete channel signals for transmission; by applying this mapping technique we achieve redundancy for coding. We note that the expanded signal set results in a reduced minimum distance between the sympol points for a given averge power. However because of the dependancy introduced by the convolutional encoder between the successive transmitted symbols, this minimum distance is no longer the relevant measure of error performance; instead, it is the minimum distance between the allowed sequence of symbols that determines this performance.

2.1 A Simple Illustrative Example

If n=2 , then the number of messages to be transmitted without coding is 4 i.e., (00, 01, 10, 11). For these 4 messages to be transmitted we have to have 4 frequencies.

Assign S_0

to the message 00

Assign S_1

to the message 01

Assign S_2

to the message 10

Assign S_3

to the message 11

To achieve redundancy we must have 2^{n+1} frequencies to transmit the same information per modulation interval, i.e., the same 2 bits in our case. The expanded signals set {000, 001, 010, 011, 100, 101, 110, 111}

Assign S_0

to the message 000

Assign S_1

to the message 001

Assign S_2

to the message 010

Assign S_3

to the message 011

Assign S_4

to the message 100

Assign S_5

to the message 101

Assign S_6

to the message 110

Assign S_7

to the message 111

For the uncoded case the message 00 needs only one frequency S_0 to transmit it, but for the expanded case the message 00 required two frequncies S_0 and s_1 to transmit the same information. The same is true for the remaining messages. By expanding the binary data sequences utilizing convolutional encoding with a rate R =n/(n+1), we gained redundancy for the code without introducing redundancy at the transmitter such as parity checks. To maximize the free Euclidean distance, the approach is based on a mapping rule called "mapping by set partitioning". This mapping follows from successive partitioning of a channel signal set into subsets. The main reason for partitioning is to increase the minimum Euclidean distance between the signals within the subset. For our case the expanded signal set has 8 frequencies, which are contained within the specified distance by the maximum frequency deviations as shown in the Fig. 1.

2.2 Set Partitioning Method

The first level of set partitioning is to divide the signal set into two subsets, each subset having an equal number of signals and each signal in one subset having a maximum distance from each signal in the other subset. This is considered the first level of set partitioning. The partitioning process is repeated until the maximum signal separation is equal to or more than the required free distance of the TCM scheme to be designed.

2.2.1 Mapping Rule

The efficient mapping rule is as follows :

1) All M-FSK signals should occur with equal probability and with reasonable amount of symmetry and regularity.

2) Assign signals from either of the two partitions (each contains M/2 signals) generated at the first level of partitioning to transitions diverging from a given state, similarly, assign signals from the second one to transitions re-emerging to a given state. This mapping technique is to assure that at any merging state the signals must have a maximum Euclidean distance between them.

Rule 1) tells us that good codes should have a regular structures.

Rule 2) Assures that the Euclidean distance (ED) associated with the error event d_{free} coded exceeds the d_{free} uncoded.

2.3 The Concept Of Coding Gain :

Robert Calderbank and James E.Mazo [10], combined correlative coding and mapping by set partitioning in one step. Their description is analytic rather than graphical. The coding gain obtained is expressed as follows :

$$coding \quad gain(db) = 10\log_{10}\left\{\frac{(d^2_{min}/p)_{coded}}{(d^2_{min}/p)_{encoded}}\right\} \qquad (1)$$

where d^2_{min} refers to the characteristic squared minimum distance, and the power p is the average signal power at the channel input. In practice the encoder can be represented as a linear-finite state machine. The number of states depends on the past history of the encoder. For example, if the number of the previous bits which determine the history of the encoder is v, then the number of the states should be equal to 2^v , and if the number of bits per modulation interval is equal to n then the number of the transitions to the successor state is equal to 2^n. After completing the state transition diagram, what is left is the assignment of channel signals from an extended set of 2^{n+1} signals to the transitions to achieve a maximum free Euclidean distance, and thereby improve the error performance of the system. The purpose of encoding is to gain noise immunity i.e., by increasing the Euclidean distances of the signals. By drawing this encoding procedure sequentially in time we have the trellis structure shown in Fig.2. Hence the name trellis code. For example consider the trellis code with n =1 , v = 2. If the encoder is in state 10 and the new input is 1 then a transfer from state 10 to state 11 occurs and the encoder output is x(1;10), the new state must be determined by the previous state and the present input. The beauty of the trellis encoder is its ability to generate linear and non linear codes, such as :

$$X = b_2 - 2b_1 b_3 \tag{2}$$

$$X = b_2 - b_1 b_2 b_3 \tag{3}$$

$$X = 8b_2 + 4b_3 + b_4 - 2b_1 b_4 b_7 \tag{4}$$

For the simple trellis code as given in (2) :

$$X = b_2 - 2b_1 b_3$$

b_1 is the present input bit, b_2 is the bit preceeding bit b_1 and b_3 is the bit preceeding bit b_2.

If k = 1 & V=2 therefore for V=2 the number of the states = $2^V = 2^2 = 4$. The number-of transitions from each state is equal to $2^k = 2^1 = 2$.

By changing the signal constellation from symmetric to asymmetric and designing an efficient mapping rule we expect to increase the minimum Euclidean distance and to enhance the system error performance.

Shannon's information theory [17], predicted the existence of the coding scheme with these characteristics three decades ago. Ungerboeck [9], studied the effect of coding in terms of channel capacity i.e., to find the limits to performance gains in signals to noise ratio for coded system relative to uncoded system. Ungerboeck concluded that by expanding the signal set, using set partitioning, or by applying the code rate n/(n+1), resulted in similar coding gain.

2.3.1 The Applications Of Trellis Coding :

It is advisable to choose the signal constellation such that the sequence of signals repreesenting information should have the largest distance among them in the Euclidean signal space. It is the responsibility of the designer of TCM (trellis code modulation) to choose the codes which maximizes the minimum Euclidean distance and to expand the signal set to provide redundancy for coding. One of the major applications of trellis coding is in data modems, interfacing between digital computers. For example, uncoded voice data were usually transmitted at 9.6k bits/sec, over a voice channel. This was the upper limit for data modems. But by using TCM and other improvements in equalization and synchronization. Data modems reached the rate 14.4 k bits/sec, and higher up to 19.6k bits/sec, a lot of research and emphasis on this area to increase the data rate up to 64k bits/sec. Another application of TCM technique is in Satellite & Mobile communication systems, in order to improve the throughput rate or to permit satisfactory operations at lower signal to noise ratio.

2.4 Soft Decision Decoding of Convolutional Code

Ungerboeck [9] codes are convolutional codes. His codes have a large free Euclidean distances and they fit into a bandpass channel. The decoding of convolutional codes could be done by utilizing the soft decision version of the Viterbi algorithm. The Viterbi algorithm finds the path through the trellis that is most closely matched to the received code word (our code word is a sequence of signals). The role of modulation on the performance of a coded system becomes evident when one examines the decoding process. Optimum decoding involves the searching of the most likely path in the trellis based on the sequence of received analog symbols, which are corrupted by noise and other channel impairments. A received symbol sequence $Y = (y_1, y_2, y_3, ----, y_n)$ is decoded into one of the allowed symbol sequences in the set $\{x_i\}$ where $X_i = (x^i_1 , x^i_2, x^i_3, -------, x^i_n)$ based on the optimum decision rule that selects X_i if and only if $P(Y/X_i) > P(Y/X_j)$ for all i not equal to j. In the presence of an additive Gaussian noise channel, this decision rule is equivelent to computing the Euclidean distance squared. If all sequences are equally likely to occur, then sequences which are closest, in the sense of Euclidean distance, will determine the error rate of the coded system. The modulation process must attempt to maximize the Euclidean distance between all possible sequences X_i and X_j , i not equal to j. The larger the distance, the lower is the error rate. At high signal to noise ratios (SNR), the performance is dominated by the error event with the minimum Euclidean distance. Thus, assigning signal points to the encoder output in a manner to maximize the minimum Euclidean distance is essential for optimizing the decoding process. Each branch of any path has assigned to it, its Euclidean distance, for comparing every two paths separated at node i, merged at node j and not having any two nodes in between. We must sum the squares of their Euclidean distance difference, branch by branch, starting from node i up to node j. Thus the concept of Euclidean distances, can be applied to unquantized channel. Since we can not store the analog samples, in practical system we have to convert our analog samples to digits using A/D converter, the number of levels ranges from (8) levels up to (65,536) levels.

Blahut [18], described the Viterbi algorithm as follows :

The Viterbi algorithm is easy to use when the channel noise is stationary and modeled as Gaussian (AWGN).

"Let P(V/C) be the probability that the symbol V is the output of the demodulator given that symbol C is the input to the channel. Suppose the received word V_i has n_o components denoted by :

$$(V_i \ \ for \ \ i = 1, 2, 3, --------, n_0)$$

If you choose any path on the trellis closest to the received code word C_i

$$(C_i \ \ for \ \ i = 1, 2, 3, --------, n_0)$$

then the distance of a code word C from the corresponding code word V is given by the log-likelihood function d(V,C) :

$$d(V,c)=\sum_{i=1}^{n_0}\log P(V_i/C_i). \tag{5}$$

"

The soft decision Viterbi decoder is the same as the hard-decision Viterbi decoder except that it uses the new path metrics in place of Hamming distance metrics. At each node the algorithm determines the surviving path which has the best metrics among the all other paths entering this node, the algorithm retains the surviving path at each node and discards the rest, each surviving path contains the maximum likelihood of having the minimum probability of error, The maximum-likelihood decoder may make a decision at this point with no loss in performance. This is exactly what the Viterbi algorithm does, the retained paths are called "survivors", and the number of the survivors are equal to the number of the states. If the noise is modeled as stationary, additive, independent, and Gaussian then :

$$P(V/C)=\prod_{i=1}^{n_0}\left\{\frac{1}{\sqrt{2\pi\sigma^2}}\exp-\frac{(C_i-V_i)^2}{2\sigma^2}\right\} \tag{6}$$

For a Gaussian channel maximizing the likelihood function is equivalent to minimizing the Euclidean distance.

$$d_e^2(V,C) = \sum_{i=1}^{n_0} (C_i - V_i)^2 \qquad (7)$$

The Euclidean distance often is measured in decibels, defined by

$$d_e(db) \quad = \quad 20\log_{10} d_e \qquad (8)$$

the asymptotic gain is defined as :

$$G \quad = \quad 20\log_{10}\left(\frac{d_e}{d_{ref}}\right) \qquad (9)$$

where d_{free} is the minimum Euclidean distance of an uncoded reference wave has the same average power. The distance $d^2(\epsilon)$ associated with an error event ϵ, is important because the probability P_i a viterbi decoder output an incorrect path starting at state i is upperbounded by

$$P_i \le \sum_{l=l_{min}}^{\infty} \frac{1}{2^{kl}} \sum_{\epsilon \in S(l,i)} \exp\left(\frac{-d^2(\epsilon)}{8\sigma^2}\right) \qquad (10)$$

where S(l,i) is the set of error event of length l that starts at state i. The main attraction of trellis codes are the achievement of significant coding gains over conventional uncoded multilevel modulation without compensating bandwidth efficiency, or sacrificing data rate. TCM exhibits redundancy by expanding the signal set.

2.5 Combined Trellis Coding With Asymmetric MPSK Modulati on

Simon [11], combined MPSK modulation and coding which resulted in increased the power efficiency, without bandwidth expansion, with either symmetric or asymmetric signal constellation. This resulted in further improvement in performance. Simon also designed the constellation to be asymmetric to obtain better performance for the same bandwidth and power constraints of the system. The object of introducing coding is to maximize the minimum free distance d_{-}. The ratio of d_{-}/d_{nf} is an indication of the maximum reduction in the required E_b/N_o that can be achieved for arbitrarly small bit error rate.

2.5.1 System Model

The system model is illustrated in Fig.3. The study of system performance for symmetric and asymmetric signal signal is also shown. The symmetric contains $M = 2^n$ point set, uniformly distributed on a circle. The asymmetric contains $M = 2^{n+1}$ points set on the circle are created by adding together the symmetrical M/2 points set with a rotated version of its self by an angle ϕ. The main goal is to find the optimum value of the angle ϕ, which gives the optimum performance.

Simon concluded that by introducing an appropriate amount of asymmetry into the constellation design of a combined modulation/trellis coding is a cost effective. A better improvement is achieved relative to symmetric and uncoded system, for the low coding case i.e., two states trellis. If the coding complexity increases, no improvement relative to symmetric case could be achieved, but there is a better improvement relative to uncoded system.

3 RESEARCH PERFORMED

The objective of the research performed herein is to obtain a new modulation scheme for satellite, mobile communications, local area networks, computer communications, modems and for all digital integrated networks. The problem is to develop a modulation scheme which will offer high immunity against fading and doppler shifts. Our study will be focused on narrow band continuous-phase M-ary FM combined with trellis coding of the signal space. The research is aimed at increasing the spectrum efficiency i.e., increasing the number of users without increasing the system bandwidth, and at the same time to maintain low error rate.

The goal of the study is to find the optimum signal constellation of a trellis coded continuous-phase M-ary FM scheme, which will maximize the minimum Euclidean distance d_{free}, and minimizes the 99% energy bandwith and have spectrum with small sidelobes.

4 OUR WORK

In order to provide reasonably low error rates with reasonably efficient use of the frequency spectrum, traditional digital radio systems have focused of continuous-phase, frequency modulation (CP-FM) techniques. Unfortunately, these CP-FM techniques often trade spectral efficiency in order to improve bit error rate (BER) performance. While these tradeoff may have been acceptable in the past, there is an increasing need for systems that can provide both high spectral efficiency and low BER. We have identified a modulation-technique that does exactly that. Additionally, a CP-FM signal can be demodulated through coherent detection or discriminator (non-coherent) detection. Most digital radio systems use coherent detection. This requires carrier recovery, which is complex and can be quite difficult in the fast fading environment that characterizes mobile radio (and many other) communications systems. In these systems, carrier frequency drift caused by Doppler effects can be a problem as well. Our approach to CP-FM will be implemented through non-coherent detection techniques, reducing these problem significantly. Minimum shift keying (MSK) was among the earliest of the CP-FM schemes. It is still among the most widely used. Among its advantges : simplicity, cost and reasonable bit error rate efficiency (Typically 10^{-4} at 10 dB) when demodulated noncoherently. Its chief limitation is in its low spectral efficiency (on the order of 0.85 bits/sec/Hz).

As the need for systems providing more efficient use of the frequency spectrum began to merge, the performance limits of MSK were quickly reached. Several digital modulation techniques were developed that achieved the requisite spectral efficiency, but at a price: complexity, cost and lower performance. Tamed FM is typical of these techniques. Its spectral efficiency is on the order of 1.25 bits/sec/hz. However, to achieve a BER of 10-5 requires an SNR of more than 15 db. How critical these limitations of traditional digital modulation techniques are depends on the application. For satellite communications system, for cellular radio systems and for many military applications, they are serious indeed.

The research we propose will investigate the use of trellis coding of the signal space with CP-M-ary FSK, using a non-coherent, FM discriminator based detection system. This innovative coding scheme will be applied for two cases. Our mathematical analysis to date indicates that the first of these cases, using a symmetrical constellation, will have a spectral efficiency exceeding MSK and a BER better than TFM. We believe, based on preliminary investigations, that the second case, using an asymmetrical constellation, will have a spectral efficiency greater than TFM and an expected BER better than MSK. As important : by implementing our modulation technique through a non-coherent detection scheme, we will achieve significant improvements in cost and reliability, far better performance under conditions of fast fading, and lower vulnerability to doppler effects. while all the "components" of our approach already exist (at least in communications theory), they have never been combined in the manner we intend. Our, research project will complete the mathematical formulation, (MSK, TFM, Duobinary FM) and select an optimum signal constellation for a given number of information bits per symbol, details of our approach are provided in the sections that follow. The implications of a successful outcome are enormous: far more efficient communications systems that require less power (for both ground and satellite components) for the same BER ; reduced weight, higher reliability on-board, lower cost component; significantly better recovery from Rician fading; and reduced Doppler shift effects. There are additional advantages in the application of our approach to low probability of intercept (LPI) systems.

4.1 The Objective Of Our Research

The overall objective of the research effort is to develop a modulation scheme which will offer high spectrum and power efficiency, low bit error rate and high immunity against fast fading and Doppler shifts. Our research will focus on narrow-band continuous-phase M-ary FSK combined with trellis coding, noncoherent detection and the Viterbi decoding algorithm. More specifically our investigations will:

(a) determine the 99% energy bandwidth and the upper bound on the probability of error for rate n/(n+1) trellis coded continuous phase M-ary FSK $M=2^{n+1}$ trellis coded continuous phase M-ary FSK $M=2^{n+1}$ with a symmetric signl constellation.

(b) Investigate the same approach, but with an assymetric signal constellation. For a given code rate n/(n+1), we will find a signal constellation which jointly minimizes the 99% energy bandwidth and the probability of error. The tradeoff between spectrum and error rate will be determined.

(c) compare the performance of the proposed system, the probability of error and spectrum efficiency, to noncoherent detection of standard modulation techniques such as MSK, Duobinary and TFM.

(d) For a given code rate n/(n+1) select the best modulation scheme, i.e., the scheme with smallest probability of error and with largest spectrum efficiency.

The communication system we propose is a narrow-band trellis coded continuous-phase M-ary FSK, with noncoherent detection employing a limiter-discriminator and integrator as post-detection filter. The decoding of the trellis signal space is performed by the Viterbi decoding algorithm. Our research will investigate n/(n+1) trellis codes and continuous-phase M-ary FSK $M=2^{n+1}$ with symmetric and asymmetric signal constellations which will jointly minimize the 99% energy B and the bit error probability P_b.

4.2 The Factors Which Determine The Signal Space Design

The system requirements such as information rate , bandwidth, bit error rate, etc., will determine the signal space design, the required symbol rate and number of information bits n per symbol. An uncoded system would require a 2^n-ary FSK modulation scheme. To be able to keep the same information rate (n bits per symbol) and the same bandwidth, the signal space is expanded to 2^{n+1} frequencies, with the same maximum frequency deviation, and then a trellis code of a rate n/(n+1) is applied. Since by this technique the symbol rate is not changed and only the number of transmitted frequencies is increased, the spectrum efficiency of the coded system with the SNR reduced for an amount proportional to the coding gain exhibits the same error rate as an uncoded system. This approach, coding of signal space, was first introduced by Ungerboeck [9], who has analyzed n/(n+1) trellis coded 2^{n+1} PSK signal constellations. The signal assignment of MPSK $M = 2^{n+1}$ to transitions of the trellis code is performed by set partitioning. It has been shown that a significant performance improvement is achieved over uncoded MPSK $M = 2^n$. The disadvantage of the Ungerboeck technique is that it requires coherent detection and therefore perfect carrier recovery at the receiver side. Also the power spectral density function exhibits strong sidelobes which reduces its application in mobile and satellite supported communications.

5 RESEARCH TASKS

We will analyze a symmetrical signal constellation by considering 4, 8, and 16 continuous phase FSK modulation schemes combined with trellis codes having 2, 4, 8, and 16 states. For each case , The 99% energy bandwidth will be obtained by the means of a computer simulation. Also, the upper bound on the bit error rate will be calculated as described in section 5.3.

A computer simulation will be developed to estimate the bit error rate as a function of the signal to noise ratio. The results will be used to confirm the calculated bound and vice-versa. Next, we will analyze 4, 8 and 16 continuous phase FSK with asymmetrical signal constellation combined with trellis codes having 2, 4, 8 and 16 states. for a given number of frequencies and number of states we will determine the optimum asymmetry, i.e., the asymmetry which minimizes the 99% energy bandwidth and maximizes the minimum free Euclidean distance. This task will be performed by the method described in Section 5.3 and by a computer simulation as explained above. The performance of the system we propose will be compared with the standard continuous phase modulation technique (MSK, Duobinary, TFM) employing noncoherent detection. We will show that by our proposed technique it is possible to achieve significant spectrum efficiency improvement bits/sec/Hz over MSK, Duobinary, TFM, and still have an acceptable rate. Finally, for a given information rate we will select the best modulation scheme i.e., the scheme which exhibits the best spectrum efficiency and offers the smallest error rate.

5.1 Procedures

Our objective in this research is to achieve high performance using the less complex non-coherent detection method. Non-coherent systems provide additional benefits relating to better operation under fading conditions and less vulnerability to Doppler effects. Our work confirmed the anticipated performance improvements. The procedures used in this earlier research are described in the following paragraphs.

In our earlier work we investigated a rate 1/2 trellis coded continuous phase FM, and we calculated the upper bound on the probability of error based on a 99% energy bandwidth. For the same code we determined the upper bound on the bit error rate and the power spectrum density function for symmetric and asymmetric signal constellations. The system we considered is shown in Fig.4. The incoming data bits b_n are encoded by a trellis structure shown in Fig.5. To each incoming data bit, based on the past history, one of four possible signals of the signal set S_n(n= -2, -1, 1, 2) is assigned. Signal assignment is performed by set partitioning as shown in Fig.7. And described in [9]. The received signal is embedded in white Gaussian noise and is given as:

$$r(t) = s(t) + n(t) \qquad 0 < t < T_s \qquad (11)$$

where T_s is the signaling period. The receiver consists of an ideal band-pass filter where bandwidth is adjusted to 99% energy bandwidth for a given code. The limiter-discriminator and the integrator are as described in [13]. The receiver consists of an ideal band-pass filter whose bandwidth is adjusted to 99% of the energy bandwidth for a given code. The decoding is performed using the Viterbi algorithm. The output signal of the VCO may be represented as :

$$S_n(t) = a\cos\left(2\pi f_c t + k\int_0^t X_n(t) \quad dt + \theta\right) \qquad (12)$$

where f_c is the carrier frequency, K is the VCO constant, θ is an initial phase and $X_n(t)$ is the input voltge of the VCO corresponding to the n-th transmitted signal S_n(n= -2, -1, 1, 2) where $X_n(t)$ is given as follow :

$$X_n(t) = \begin{cases} X_n & 0 < t < T_s \\ 0 & \text{elsewhere} \end{cases}$$

The value of X_n will determine the deviation frequency of the VCO The maximum frequency deviation from the carrier is chosen to be $(1/4T_s)$. For an arbitrary signal constellation and for a noiseless system the sampled output of the integrator q(t) at $t=T_s$ will be distributed as shown in Fig.6.a. Where the zero value represents the carrier frequency.

The sampled values (S-2, s-1, s1, s2) correspond to the transmitted signals,

S_n (n = -2, -1, 1, 2) respectively. The maximum distance d_3 is determined by the maximum frequency deviation $(1/4T_s)$ and in this case, the distances d_0, d_1 and d_2 depend on the chosen signal constellation and for the symmetric constellation are given as :

$$d_0 = \frac{\pi}{3}, \qquad d_1 = \frac{\pi}{3}, \qquad d_2 = 2\frac{\pi}{3}, \qquad d_3 = \pi. \quad (14)$$

When the received signal consists of a signal plus noise, the output sample of the integrator at t = T_s for a given S_n, is a random variable whose exact probability density function is given in [13]. We limit our analysis to the case of a high signal to noise ratio. Since the system under consideration is a narrow-band system we can neglect the so called "clicks effect" [17].

Next we evaluate the upper bound on the bit error rate for the signal constellation given in Fig.6.a. To evaluate the upper bound on P_b we proceed as described in [12] and utilize the error state diagram shown in Fig.9. Where the initial and final states are all zero error states. First we give the upper bound for a code denoted as code 1. For this code we choose a symmetric signal constellation as follows :

S_n(n = -2, -1, 1, 2) such that the distances d_i (i = 0, 1, 2, 3) from Fig.6a, are given by Eq (14). The power spectrum density function of the code 1 is estimated by a method based on time averaging over short modified periodograms. The estimated spectrum of code 1 is plotted in Fig. 8-a. The 99% energy bandwidth for this code is equal to :

$$B = \left(\frac{1.13}{T_s}\right) \tag{15}$$

using this result, the set of distances of this code and the error state diagram, we find the upper bound on P_b. The upper bound on the bit error rate for code 1 is plotted in Fig.10.

5.2 The Analysis Of Asymmetric Signal Constellation

For an asymmetric signal constellation Fig.6b, we chose the following set of distances.

$$d_0 = \frac{\pi}{4}, \qquad d_1 = \frac{\pi}{2}, \qquad d_2 = \frac{3\pi}{4}, \qquad d_3 = \pi. \tag{16}$$

we denote this scheme as code 2. The power spectrum density function of code 2 with this asymmetric signal constellation is estimated in the same way as for code 1 and is plotted in Fig.8-a. It appears that the 99% energy bandwidth, and the power spectral efficiency for this code is the same as for code 1. The 99% energy bandwidth for both code 1 and code 2 is $(1.18/T_s)$, which is considered relatively worst than TFM $(.8/T_s)$ as shown in Fig.8-b. The upper bound on the bit error rate is calculated in the same way as for code 1, with the utilization of the error state diagram in Fig. 9. The result is given in Fig.10. For the purpose of comparison in the same figure we give the upper bound on the bit error rate for MSK and Duobinary; Fig 8-c . The 99% energy bandwidth for MSK is $(1.18/T_S)$. The 99% energy bandwidth for Duobinary is $(0.92/T_S)$. By comparing the curves in Fig.10. We can see that asymmetric signal constellation offers a significant system performance improvement over non coherent detection of a MSK and Duobinary, and at the same time has better spectrum than a MSK.

5.3 Related Research

The increased demand for digital transmission channels in the radio frequency (RF) band creates a serious problem of spectrum congestion. One way of solving this problem is the use of high spectrally efficient modulation techniques. In order to solve this problem several modulation techniques such as MSK, Duobinary and TFM were developed [1], [2], [3] [14], [15], [16]. The commonality among all these schemes is that the price paid for the improved spectrum efficiency is a poorer BER and increased complexity. The 99% energy bandwidth for MSK, Duobinary and TFM is calculated in [4]. Based on these results,the spectrum efficiency of MSK is shown to be (.85 bits/sec/hz), Duobinary (1.09 bits/sec/hz), and TFM (1.25 bits/sec/hz). It is also shown how much the spectrum efficiency is increased by proper shaping of the base-band data pulses. In the past few years non-coherent detection of the above mentioned schemes has received increased attention. Non-coherent detection of narrow-band Duobinary FM is analyzed in [5]. It is shown that the "click effect", a major problem in conventional digital FM, does not contribute significantly to errors for low frequency deviation systems. In [6], noncoherent detection TFM is analyzed. It is shown that there is a loss of 4.7 dB with respect to the optimal coherent receiver (BER of 10^{-5} requires SNR of 15.2 dB). A multilevel decision technique for band-limited digital FM employing discriminator detection is analyzed in [7]. It is shown that this method significantly improves the BER performance of digital FM using discriminator detection. For coherently detected signals, error-rate improvement can be achieved in a spectrally efficient manner by using several techniques. Aulin and Sundberg [8] make use of the properties of continuous phase modulated signals that provide narrow bandwidth with low sidelobes and allows for trellis decoding in the phase plane. Coherent detection establishes phase memory, which is used by a decoding process that produces the desired improvement in error-rate performance. As an example, the authors cite any 8-ary continuous phase FSK scheme that has a bandwidth comparable to that of the commonly used MSK scheme and a 4.3 dB gain in error rate, over MSK. The disadvantage of Aulin work is that there is no much improvement in bandwith reduction relative to MSK scheme, the signal detection is performed coherently and therefore perfect carrier recovery at the receiverside.

Trellis coding of the signal set was first introduced by Ungerboeck [9], who has analyzed n/(n+1) trellis codes and symmetric 2^{n+1} phase shift keying (PSK) signal constellation. The signal assignment of MPSK $M = 2^{n+1}$ to the transitions of the trellis code is performed by set partitioning. It is shown that compared with uncoded modulation, the same amount of information can be transmitted within the same bandwidth with coding gains in excess of 3 dB using using simple codes with four to eight states. Therefore a significant system performance improvement is achieved over uncoded MPSK $M = 2^n$. Although the mapping rule for "set partitioning" of a symmetric signal set offers significant coding gain, It can be shown [10], [11], that a symmetric signal constellation is not necessarily always optimum. In [10] are given the best codes, i.e., the codes with maximum asymptotic gain, of rate 1/2 and the corresponding derived codes of rate 2/3. It is found that codes with asymmetric signal spacing achieve better coding gain than codes with symmetric signal sets. In [11] an asymmetric MPSK constellation is analyzed and the upper bounds on the bit error rate are given for rates 1/2 and 2/3 trellis coded asymmetric 4 and 8-PSK. Also the, optimum symmetry with respect to the asymptotic coding gain is obtained. Finally,in [12] we have analyzed the noncoherent detection of trellis coded continuous phase 4-FSK. We have shown that a symmetric signal constellation and four state trellis code does not offer significant improvement in the BER over noncoherent detection of MSK. Also it is shown that an asymmetric signal constellation combined with a four state trellis code gives a significant improvement over a MSK.

6 RESULTS

The output signal of the VCO may be represented as :

$$S_n(t) = a\cos\left(2\pi f_c t + k\int_0^t X_n(t)d(t) + \Theta\right) \qquad (17)$$

where f_c is the carrier frequency, k is the VCO constant, θ is an initial phase and $X_n(t)$ is the input voltge of the VCO corresponding to the n-th transmitted signal S_n (n= -2, -1, 1, 2) where $X_n(t)$ is given as follows :

$$X_n(t) = \begin{cases} X_n & 0 < t < T_s \\ 0 & \text{elsewhere} \end{cases}$$

The value of X_n will determine the deviation frequency of the VCO. In this analysis we assume that the maximum frequency deviation from the carrier frequency is $(1/4T_s)$. For an arbitrary signal constellation and for a noiseless system the sampled of the integrator q(t) at $t = T_s$ will be distributed as shown in Fig.6.a. where the zero value corresponds to the carrier frequency.

The sampled values (S_{-2}, S_{-1}, S_1, S_2) correspond to the transmitted signals $S_n(n = -2, -1, 1, 2)$ respectively. The maximum distance d_3 is determined by the maximum frequency deviation ($1/4T_s$) and this case is.

$$\frac{d_3}{2} = \frac{\pi}{2} \tag{19}$$

The distances d_0, d_1, and d_2 depend on the chosen code, and for a symmetric signal constellation they are given as:

$$d_0 = \frac{\pi}{3}; \qquad d_1 = \frac{\pi}{3}; \qquad d_2 = 2\frac{\pi}{3}$$

6.1 Analysis Of A Received Signal Embedded In Gaussian No ise

When the received signal consists of a signal plus noise the output sample of the integrator at $t = T_s$, for a given S_n, is a random variable whose probability density function is given in [5]. Since our analysis is limited to the case of a high signal- to noise ratio, and since the system under consideration is a narrow-band system we neglect the so called "click effect". Therefore the density function of the random samples is given in [5].

$$P_q(q)=\int_{-\infty}^{\infty}P_\phi(\phi)P_\phi(q-\phi)d\phi \qquad (21)$$

where :

$$P_\phi(\phi)=\exp\left(-\frac{\rho}{\pi}\right)+\sqrt{\frac{\rho}{\pi}}\cos\phi\exp(-\rho\sin^2\phi).erf\left(\sqrt{\rho}\cos\phi\right) \qquad (22)$$

$$-\frac{\pi}{2}<\phi<\frac{\pi}{2}$$

with rho we denote the signal to noise ratio defined as :

$$\rho=\frac{a^2}{2\sigma^2} \qquad (23)$$

where $(\sigma)^2$ is the variance of the noise at the output of the band-pass filter Fig.4, and is a function of the 99% energy bandwidth for a given code. For a large signal to noise ratio Eq.(22) can be written as

$$P_\phi(\phi)=\frac{1}{\sqrt{2\pi\sigma_\phi^2}}\exp\left(\frac{-\phi^2}{2\sigma_\phi^2}\right) \qquad (24)$$

which is a Gaussian p.d.f. with zero mean and variance σ_ϕ^2, based on all these assumptions and approximations the p.d.f. of $q = q(T_s)$ is given as:

$$P_q(q) = \frac{1}{\sqrt{2\pi\sigma_q^2}} \exp\left(\frac{-q^2}{2\sigma_q^2}\right) \qquad (25)$$

where :

$$\sigma_q^2 = 2\sigma_\phi^2 = \frac{1}{\rho} \qquad (26)$$

For a given code, i.e., given the distances d_i(i = 0, 1, 2, 3) and the trellis diagram, Fig.5, we calculate the upper bound on the probability of error. To be able to compare the results for different codes (the system bandwidth depends on the code). We redefine the signal to noise ratio as follow :

$$\rho = \frac{E_s/N_0}{T_s B} = \frac{SNR}{T_s B} \qquad (27)$$

where E_s is the signal energy, and (N_0 /2) is the power spectral density of white noise and B is the 99% energy bandwidth.

6.2 Analysis Of Bit Error Rate

If the transmitted message is m, the probability that the Viterbi algorithm will receive an alternate message $(\overline{m})$ [6] is given as :

$$P_i(m \Rightarrow \overline{m}) = Q\sqrt{\frac{\sum_k d_{k,i}^2}{4\sigma_q^2}} \qquad i = 0, 1, 2, 3, \ldots \qquad (28)$$

Using the bound :

$$Q(X) < \exp(-X^2/2) \qquad (29)$$

and averaging over all messages the bit error rate can be bounded as

$$P_b < \sum_{\overline{m}} \frac{w(\overline{m})}{2^{w|\overline{m}|}} \prod_k \exp\left(-\frac{SNR}{8T_s B} d_{k,i}^2\right) \qquad (30)$$

where $w(\overline{m})$ is the number of bits that the path corresponding to $(\overline{m})$ differs from the correct path. $2^{w|\overline{m}|}$ denotes the number of transitions that the path corresponding to $(\overline{m})$.

Before we proceed with the evaluation of P_b we find the time bandwidth product (T_sB), which depends on a given code. From [4] it is found that for MSK signaling (T_sB =1.18), and for Duobinary (T_sB = 0.92). By utilizing these results it is easy to show that the upper bound on

P_b for noncoherent detection of MSK signaling is given by

$$P_b < \exp\left(\frac{-\pi^2}{8T_s B} SNR\right) \tag{31}$$

For Duobinary signaling the bit error rate is bounded by

$$P_b < 4\frac{D^{\frac{\varepsilon^2}{2}}}{\left(1 - D^{\varepsilon^2}\right)^2} \tag{32}$$

where

$$D = \exp\left(-\frac{SNR}{8T_s B}\right) \tag{33}$$

Both bounds are plotted in Fig. 10.

Next, we evaluate the upper bound on the bit error rate for the signal constellation given in Fig. 6a. Using trellis coding, the result is shown in Fig.5. To evaluate the bound on P_b, i.e. to calculate Eq. (29) we utilize the error state diagram shown in Fig.9, where the initial and final states are the all zero error states.

The coefficient are :

$$a = \frac{z}{2}D^{d_0^2} \;;\quad b = \frac{z}{2}D^{d_1^2} \;;\quad c = \frac{z}{2}D^{d_2^2} \;;\quad d = \frac{z}{2}D^{d_3^2}$$

$$e = \frac{1}{2}D^{d_0^2} \;;\quad f = \frac{1}{2}D^{d_1^2} \;;\quad g = \frac{1}{2}D^{d_2^2} \;;\quad h = \frac{1}{2}D^{d_3^2} \;;\quad i = \frac{1}{2} \tag{34}$$

The coefficients having the factor z are assigned to the branches where a bit error is made. The upper bound (29) is obtained by differentiating the generating function T (d_0, d_1, d_2, d_3, z) with respect to z and setting z = 1

$$P_b < \frac{\partial T(d_0, d_1, d_2, d_3, z)}{\partial z}|_{z=1} \qquad (35)$$

By using the above described technique we find the upper bound for a code denoted as code 1. For this code we choose a symmetric signal constellation i.e., we choose S_n as follow, $S_n(n = -2, -1, 1, 2)$ such that the distances $d_i(i = 0, 1, 2, 3)$ from Fig. 6.a are given by Eq.(14) The power spectral density function of the code 1 is estimated by a method based on time averaging over short modified periodigram [7]. The estimated spectrum of the code 1 is plotted in Fig.10, and the 99% energy bandwidth for this code is (B=1.13/Ts). Using this result, the set of distances of this code and the error state diagram we evaluate P_b.

Because of the complexity of the error state diagram the generating function T(d_0, d_1, d_2, d_3, z) and its derivatives is calculated numerically. The upper bound on the bit error rate for code 1 is plotted in Fig.10.

Comparing the bounds on P_b of the Duobinary and code 1 we find that for $P_b < 10^{-5}$, the Duobinary requires more than 2 db greater signal to noise ratio. But comparing code 1 and MSK signaling we see that there is no significant improvement in the bit error rate. This can be explained by the poor minimum distance property of code 1.

Now we evaluate the upper bound on P_b of a code 2 whose signal set has the following distance properties:

$$d_0 = \frac{\pi}{4}, \qquad d_1 = \frac{\pi}{2}; \qquad d_2 = 3\frac{\pi}{4}; \qquad d_3 = \pi$$

The power spectrum density function of code 2 with this asymmetric signal constellation is estimated in a same way as for code 1 and is plotted in Fig. 8-a. The 99% energy bandwidth for code 2 is ($B=1.13/T_s$). By using this result Eq. (29) and the error state diagram we evaluate P_b. The result is given in Fig. 9. Comparing the upper bound on P_b of code 1 and 2, we can see that code 2 with its asymmetric signal constellation offers significant error improvement over MSK as well as over code 1.

High Rate Code 2/2/8

The first digit indicates the number of states, the second digit indicates the number of bits (two frequencies for single transition), and the third digit indicates the total number of frequencies.

In our example the 8 frequencies are (S_{-4}, S_{-3}, S_{-2}, S_{-1}, S_1, S_2, S_3, S_4), since the number of input bits is equal to 2 therefore the total number of transitions is 2^2 =4. We assigned signals (S_{-4}, S_1) from the first level of signal partitioning For the transitions diverging from state 0. We assigned signals (S_{-3}, S_2) from the second set at the first level of set partitioning to transitions merging to state 0. Similarly we assigned the signals (S_{-2}, S_3) from set one to transitions diverging from state 0 to state 1, and assigned signals (S_{-1}, S_4) from the other set at the first level of set partitioning to transitions merging to state 1.

The above technique is to assure maximum Euclidean distaces among the signals in the signal space. The analysis is shown in the appendix A.

CONCLUSION

Introducing an appropriate amount of asymmetry into the constellation design of a combined modulation/trellis coding system, quite a bit of performance improvement is achievable relative to the equivalent symmetric design. Furthermore, the overall improvement of the asymmetric coded system relative to the standard non-coherent detection (MSK, Duobinary, TFM) continues to increase remarkably.

The high rate codes are the worst and have bad error performance, relative to MSK, code 1 and code 2. By comparing the error performance for both Duobinary and the high rate codes, indeed the Duobinary has much better error performance relative to the high rate codes this due to the increase of the information rate of the high codes rate.

We found that doubling the number of channel-signals makes most of the theoritically improvement accessible. We deduce ageneral a rule for multilevel FM encoders as binary convolutional encoders of rate $R = m / (m + 1)$ followed by mapping of coded bits into channel signals by "set partitioning". This mapping allowed better error performance. Improvements in order of 3 db or more require codes with more states and signals (increasing the system complexity). It could be that by expanding channel-signals sets more than twice, codes could be found with better free Euclidean distances for the given constraint length than code 1 and code 2.

As we expected, the high rate codes have the poorest error performance, due to the increased amount of information to be transmitted. Thus, we cannot compare such codes to MSK or the codes 1 and 2. However, by comparing the high rate code with Duobinary, the latter is seem to have better error performance relative to high rate codes. Based on the results obtained by increasing the system complexity, i.e., by increasing the number of states and the number of frequencies, it is seen that we could improve the system error performance.

7 SUPPLEMETARY DERIVATIONS

Signal Constellation For Duobinary :

Fig. 11-a shows the signal constellation for Duobinary, the zero represents the carrier, and S_1 and S_{-1} are symmetrical around the carrier. Their distances from the carrier are given as follows :

$$d_1 = \pi \quad , \quad d_0 = \frac{\pi}{2} \tag{1}$$

Fig. 11-b, shows the two state Trellis diagram for Duobinary.

Fig. 11-c, shows the error state diagram for Duobinary

Fig. 11-d, shows the simplification of the error state diagram.

$$T = \frac{z a^2(d_0)}{1 - \frac{z}{2} a(d_1) + \frac{z}{2} a(0)} \tag{2}$$

$$a(d_0) = \exp\left(-\frac{d_0^2}{8\sigma^2}\right) \tag{3}$$

$$D = \exp\left(\frac{-1}{8\sigma^2}\right) \tag{4}$$

$$T = \frac{z D^{2d_0^2}}{1 - \frac{z}{2} D^{d_1^2} - \frac{z}{2}} \tag{5}$$

$$T = \frac{z D^{\frac{\kappa^2}{2}}}{1 - \frac{z}{2} D^{\kappa^2} - \frac{z}{2}} \tag{6}$$

$$\frac{\partial T}{\partial z} = \frac{D^{\frac{\kappa^2}{2}}\left(1 - \frac{z}{2} D^{\kappa^2} - \frac{z}{2}\right) + z D^{\frac{\kappa^2}{2}}\left(\frac{1}{2} D^{\kappa^2} + \frac{1}{2}\right)}{\left(1 - \frac{z}{2} D^{\kappa^2} - \frac{z}{2}\right)^2} \tag{7}$$

$$\frac{\partial T}{\partial z}\Big|_{z=1} = \frac{4 D^{\frac{\kappa^2}{2}}}{\left(1 - D^{\kappa^2}\right)^2} \tag{8}$$

$$P_b < \frac{4 D^{\frac{\kappa^2}{2}}}{\left(1 - D^{\kappa^2}\right)^2} \tag{9}$$

$$P_b = \frac{4 \exp\left(-\frac{\kappa^2}{16\sigma^2}\right)}{\left(1 - \exp\left(-\frac{\kappa^2}{8\sigma^2}\right)\right)^2} \tag{10}$$

Evaluation Of The Branch Coefficients For High Rate Code 2/2/8 :

Initially The system is in state S_6 (0,0). The first digit indicates the present state, and the second digit indicates the estimated state. In the event, that an error is made, the system may possibly make a transition to S_4 (0,0) or to S_5 (1,1). (The first digit represents the next state and the second digit represents the estimate of the state). During this transition an error could be made on the next state or its estimated one.

The error coefficient "a" is due to an error in the transmitting signal and is determined as follows :
The system is in state (0) and it goes to the correct state (0), but an error could be made in the transmitted and the received signals (0,4), as assigned to the states transitions of the Trellis diagram, Fig. 11.

The probability of this error event is defined as a, where a is given as follows :

$$a = p(0/4)p(4) + p(4/0)p(0)$$

if we assume equal a priori probabilities, the conditional probability is a function of the Euclidean distance between signals in the signal space, and the difference between the transmitted and the received bits. The same technique could be applied to the other branches resulting in:

$$a = \frac{z}{2} D^{\left(\frac{\epsilon_3 + \epsilon_0}{2}\right)}$$

$$b = \frac{z}{2} D^{\left(\frac{\epsilon_1 + \epsilon_2}{2}\right)}$$

$$c = \frac{1}{2}\left\{ D^{\frac{(\epsilon_3 - \epsilon_2)}{2}} + D^{\frac{(\epsilon_1 - \epsilon_0)}{2}} \right\} + \frac{z}{2}\left\{ D^{\frac{(\epsilon_0 + \epsilon_2)}{2}} + D^{\frac{(\epsilon_3 + \epsilon_2)}{2}} \right\}$$

$$d = \frac{z^2}{2}\left\{ D^{\frac{(\epsilon_3 - \epsilon_1)}{2}} + D^{\frac{(\epsilon_2 - \epsilon_0)}{2}} \right\} + \frac{z}{2}\left\{ D^{\frac{(\epsilon_3 + \epsilon_2)}{2}} + D^{\frac{(\epsilon_1 + \epsilon_0)}{2}} \right\}$$

$$e = z^2 D^{\frac{(\epsilon_2 - \epsilon_1)}{2}} + \frac{z}{2} D^{d_1} + \frac{z}{2} D^{d_2}$$

$$f = \frac{1}{2}\left\{ D^{\frac{(\epsilon_1 - \epsilon_0)}{2}} + D^{\frac{(\epsilon_3 - \epsilon_2)}{2}} \right\} + \frac{z}{2}\left\{ D^{\frac{(\epsilon_3 + \epsilon_1)}{2}} + D^{\frac{(\epsilon_2 + \epsilon_0)}{2}} \right\}$$

$$g = \frac{z^2}{2}\left\{ D^{\frac{(\epsilon_2 - \epsilon_0)}{2}} + D^{\frac{(\epsilon_3 - \epsilon_1)}{2}} \right\} + \frac{z}{2}\left\{ D^{\frac{(\epsilon_3 + \epsilon_2)}{2}} + D^{\frac{(\epsilon_1 + \epsilon_0)}{2}} \right\}$$

$$h = \frac{z}{2} D^{\frac{(d_2 + d_1)}{2}}$$

$$x = \frac{z}{2} D^{\frac{(d_3 + d_0)}{2}}$$

$$y = \frac{z^2}{2} D^{\frac{(d_3 - d_0)}{2}} + \frac{z}{2} D^{d_0} + \frac{z}{2} D^{d_3}$$

$$f = S_0 + S_1 \tag{11}$$

$$S_0 = c \cdot S_2 + a \cdot S_4 + a \cdot S_6 + h \cdot S_5 + c \cdot S_3 \tag{12}$$

$$S_1 = f \cdot S_3 + x \cdot S_5 + b \cdot S_6 + b \cdot S_4 + f \cdot S_2 \tag{13}$$

$$S_2 = \left(\frac{1}{2c + 2f + e - y} \right) \cdot \left(eS_3 + gS_5 + dS_6 + dS_4 \right) \tag{14}$$

$$S_3 = \left(\frac{1}{2f + 2c + e - y} \right) \cdot \left(eS_2 + dS_4 + dS_6 + gS_5 \right) \tag{15}$$

$$S_4 = \left(\frac{1}{2b+2d}\right) \cdot \left(cS_2 + cS_3 + hS_5 + aS_6\right) \qquad (16)$$

$$S_5 = \left(\frac{1}{2h+2g}\right) \cdot \left(fS_3 + fS_2 + bS_4 + bS_6\right) \qquad (17)$$

To evaluate the upper bound on P_b, i.e. to calculate eq. (35) given in section 6-2, we utilized the error state diagram shown in Fig. 14. Assuming the initial and the final states are zero error states, the linear equations 11-17, can be solved numerically to evaluate the generating function $T(d_0, d_1, d_2, d_3, z)$. The coefficients having the factor z are assigned to the branches where a bit error is made, while the coefficients having the factor Z^2 are assigned to the branches where two bits error are made. The upper bound on the bit error rate is obtained by differentiating the generating function $T(d_0, d_1, d_2, d_3, z)$ with respect to z and then setting z =1.

To calculate the generating function T(d0, d1, d2, d3, z) we utilized the signal constellation diagram shown in Fig. 12. , its two states Trellis diagram shown in Fig. 13, and the error state diagram shown in Fig. 14, as well as the 99% energy Bandwidth of this code which is $B = (1.05/T_s)$.

The upper bound on the bit error rate for high rate code 2/2/8 and its spectrum are shown respectively in Fig. 15, and Fig. 16.

Fig. 17. Indicates that the Duobinary has a better performance than the high rate code 2/2/8. As an example we can apply the above technique to evaluate the upper bound on the bit error for high rate codes 2/4/8 and 2/4/16.

8 TABLES

TABLE 1.

PERFORMANCE OF RATE N/(N+1) TRELLIS CODED MFSK VERSUS STANDARD NON COHERENT FM

Type	99% energy-bandwidth	(E_b/N_o)error-free
MSK	$(1.18/T_s)$	11.0 db
DUOBINARY	$(0.92/T_s)$	12.8 db
TFM	$(0.80/T_s)$	15.2 db
CODE (1)	$(1.13/T_s)$	11.0 db
CODE (2)	$(1.13/T_s)$	10.0 db
HIGH RATE CODE	$(1.05/T_s)$	13.5 db

9 FIGURES

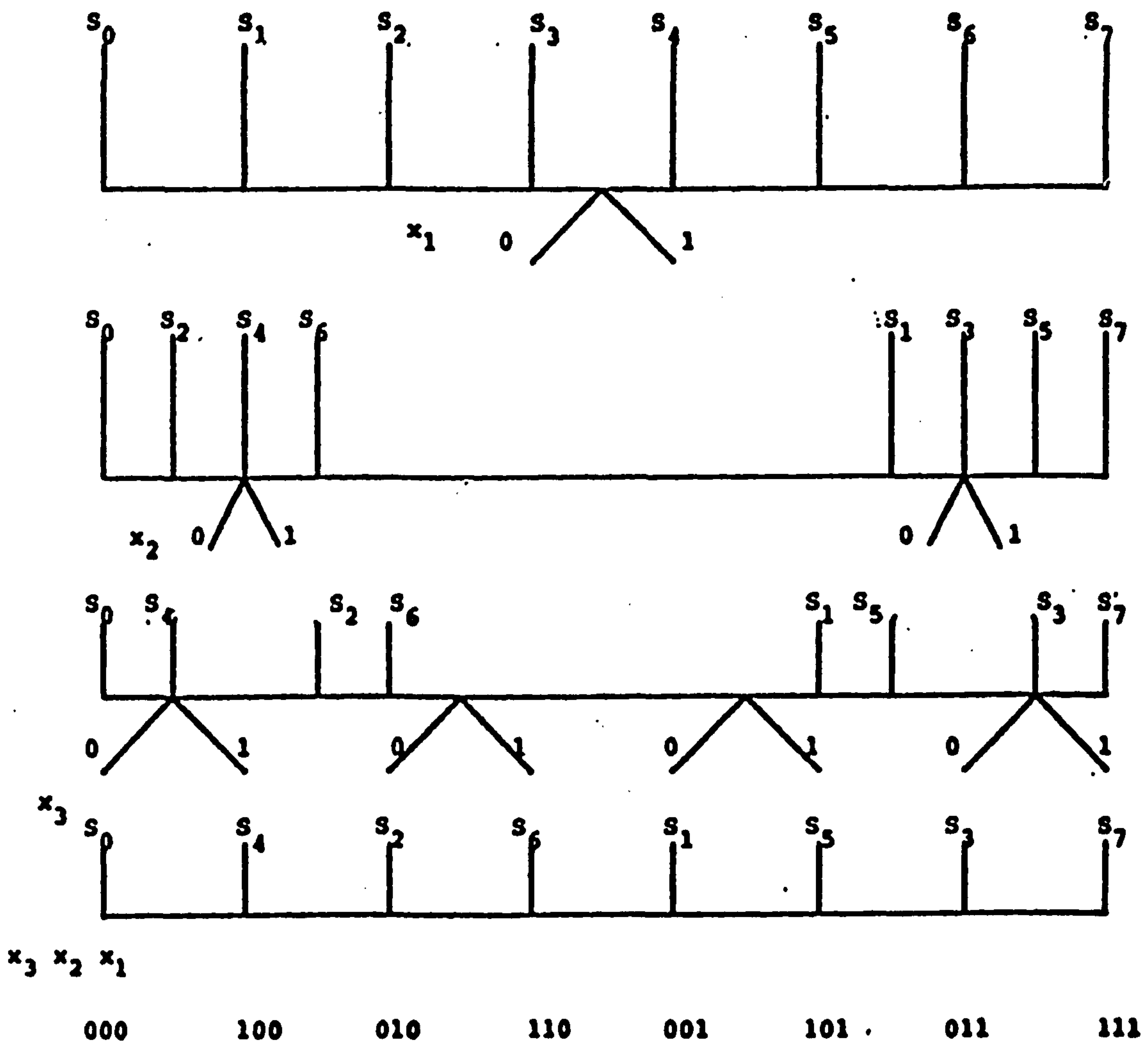


FIg. 1 SET PARTITIONING OF 8 FREQUENCIES

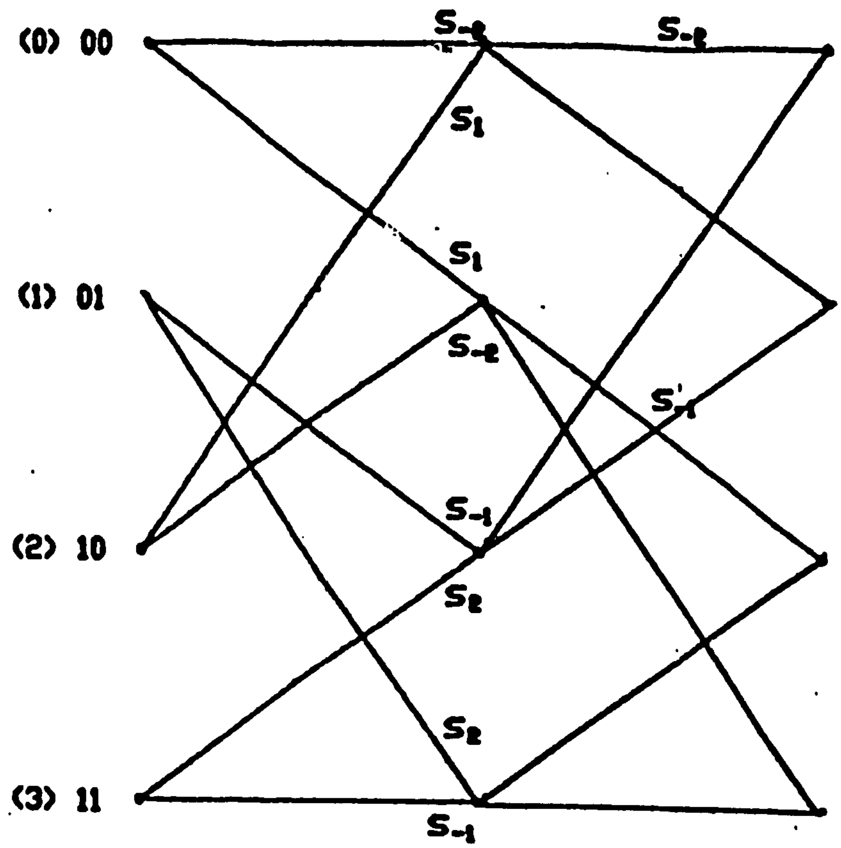


FIG. 2 FOUR STATE TRELLIS DIAGRAM

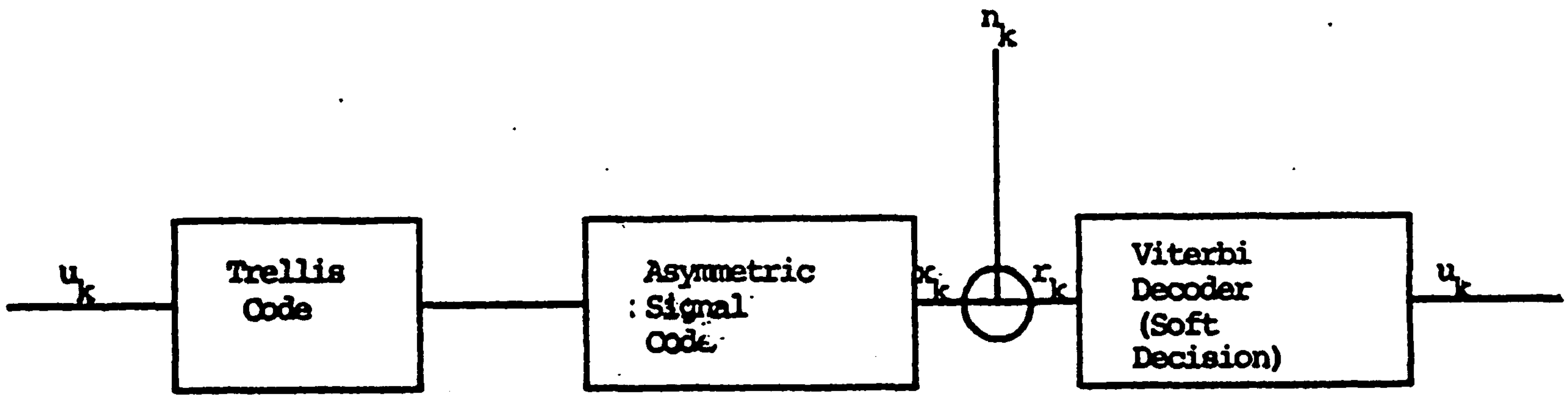


Fig. 3 Simon System Block Diagram

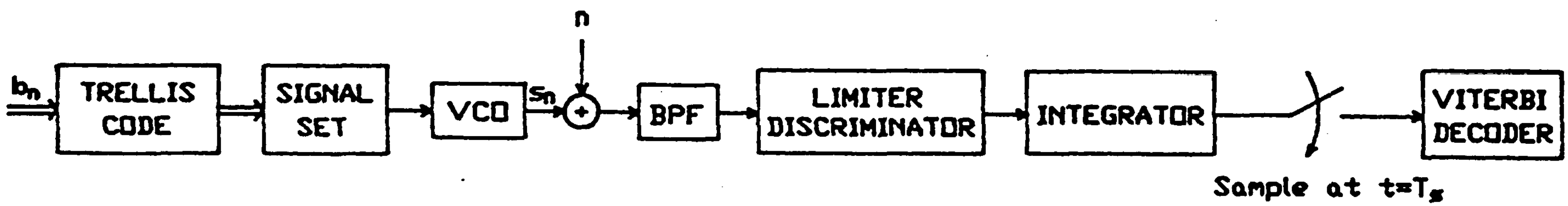


Fig. 4 System Block Diagram

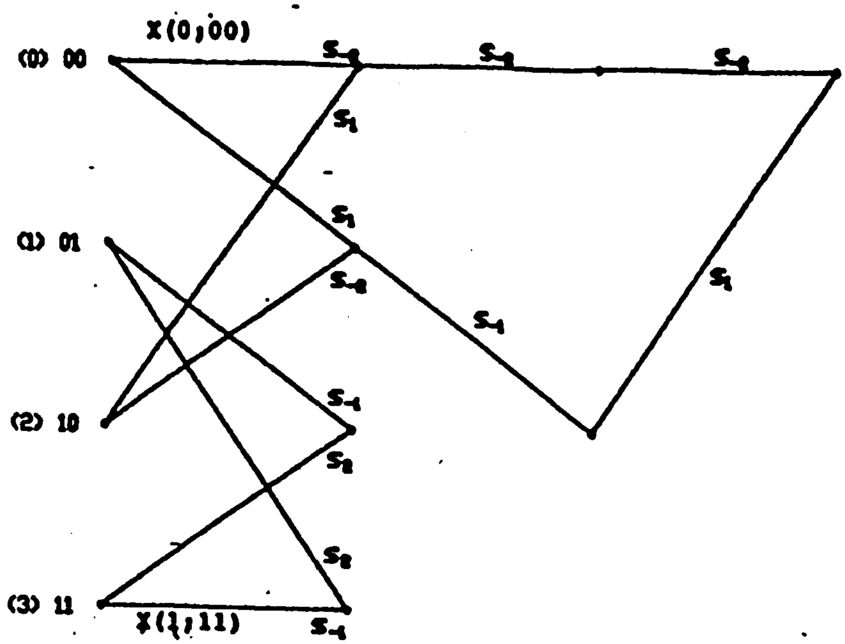


Fig. 5 Four State Trellis Diagram

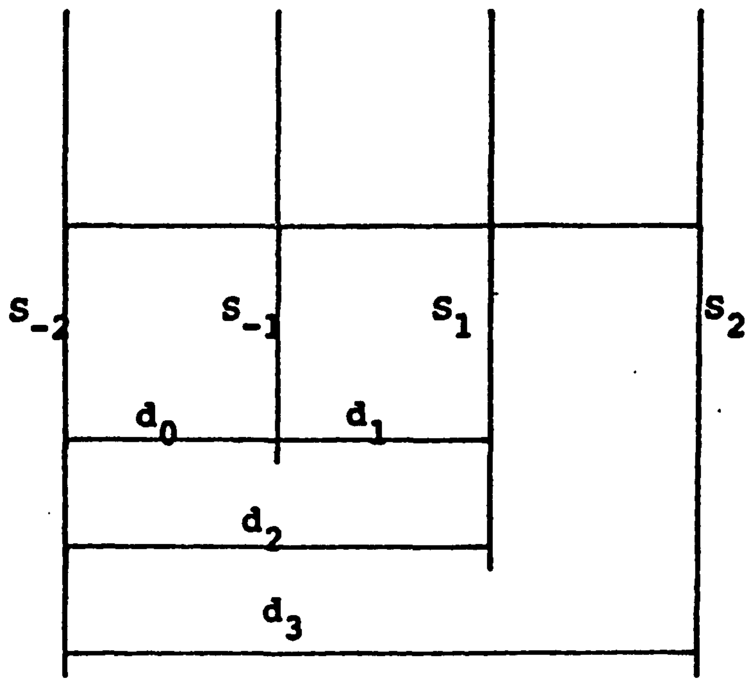
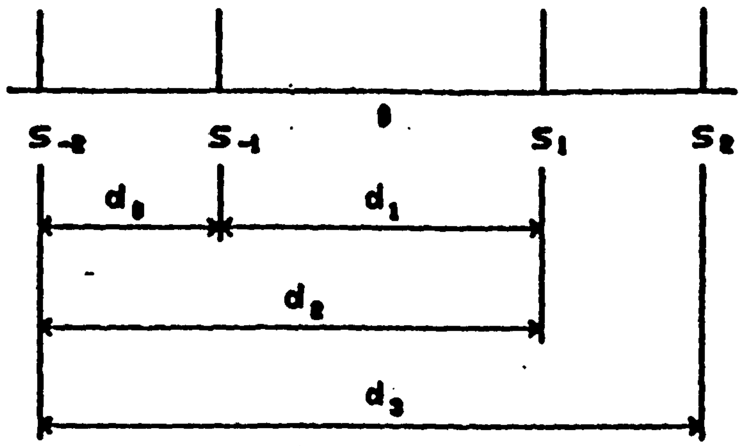


Fig. 6-a Symmetric Signal Constellation



(b)

FIG. 6 b) Asymmetric Signal Constellation

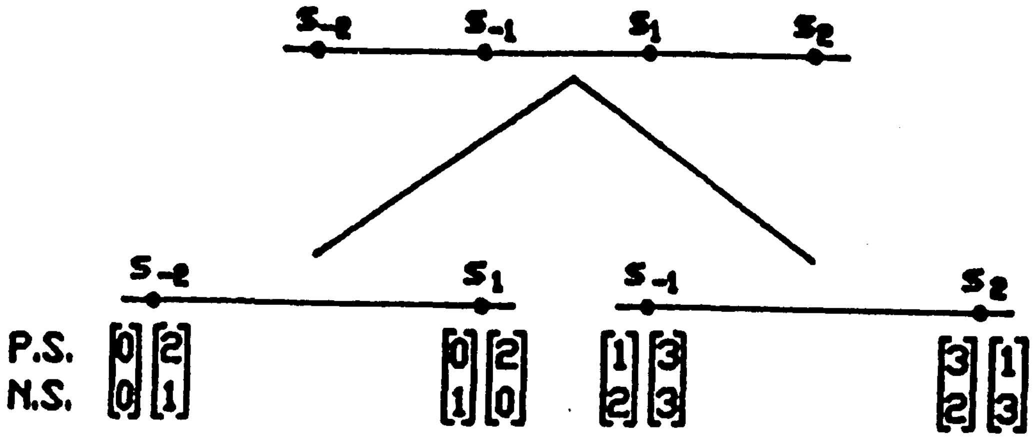


Fig. 7 Signal Assignment by Set Partitioning

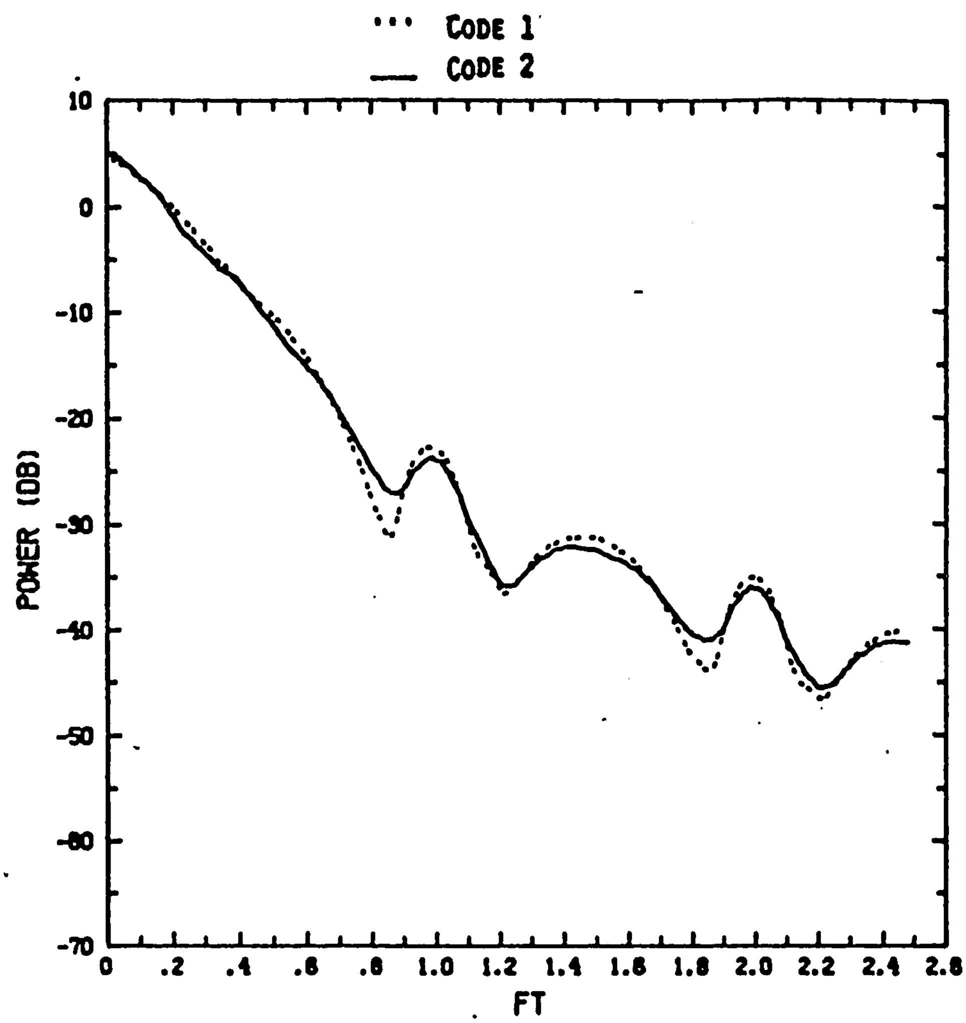


Fig. 8-a SPECTRUM OF CODE 1 AND 2

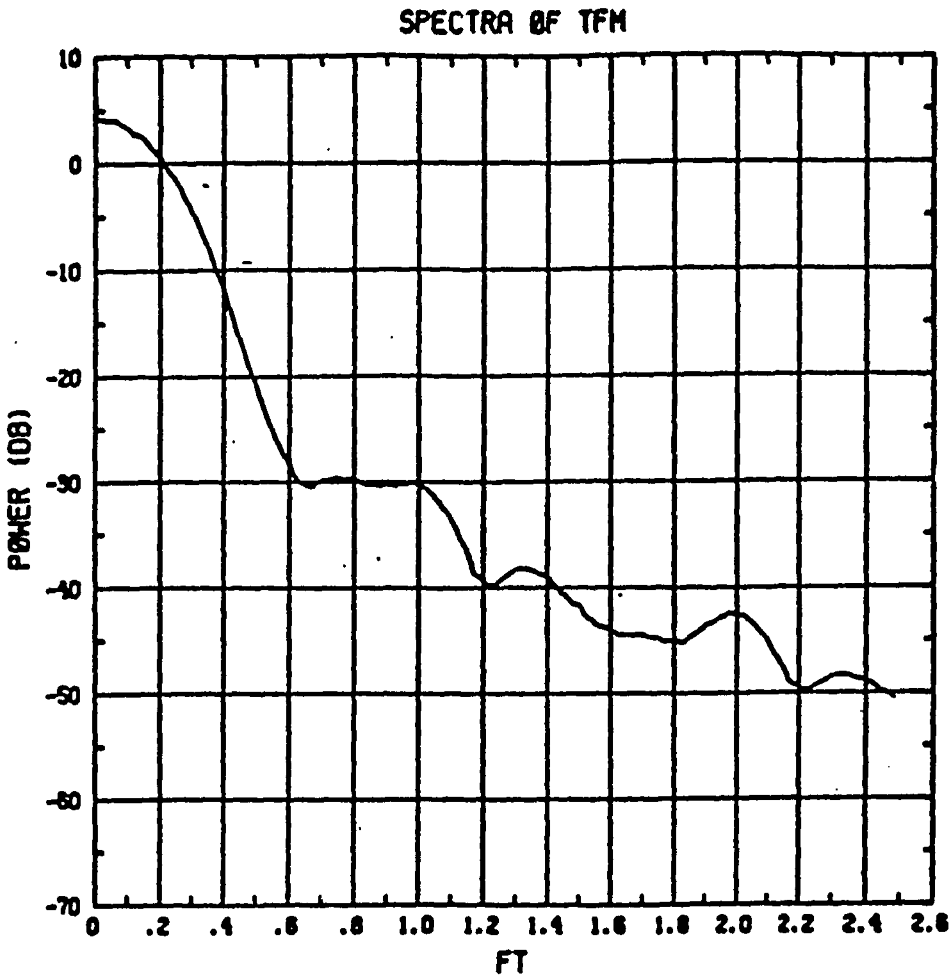


Fig. 8-b SPECTRUM OF TFM

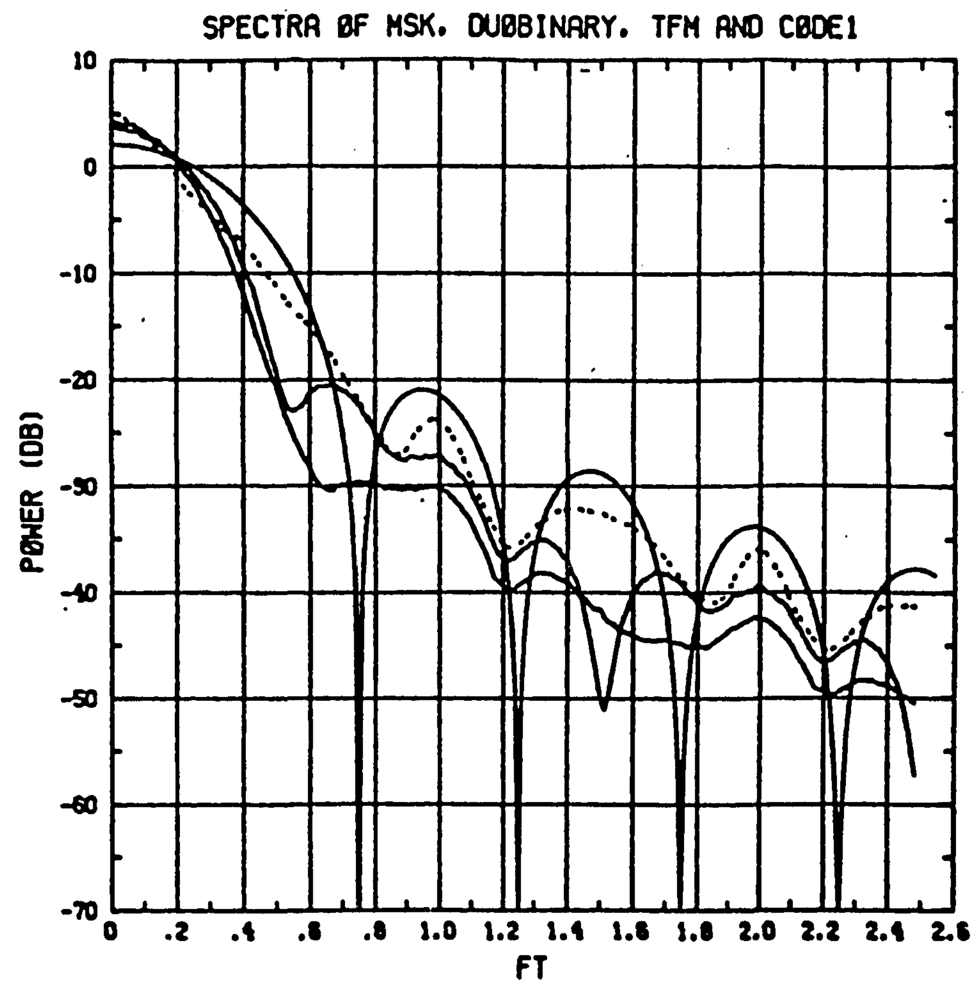


Fig. 8-c SPECTRA OF MSK, DUOBINARY, TFM AND CODE 1

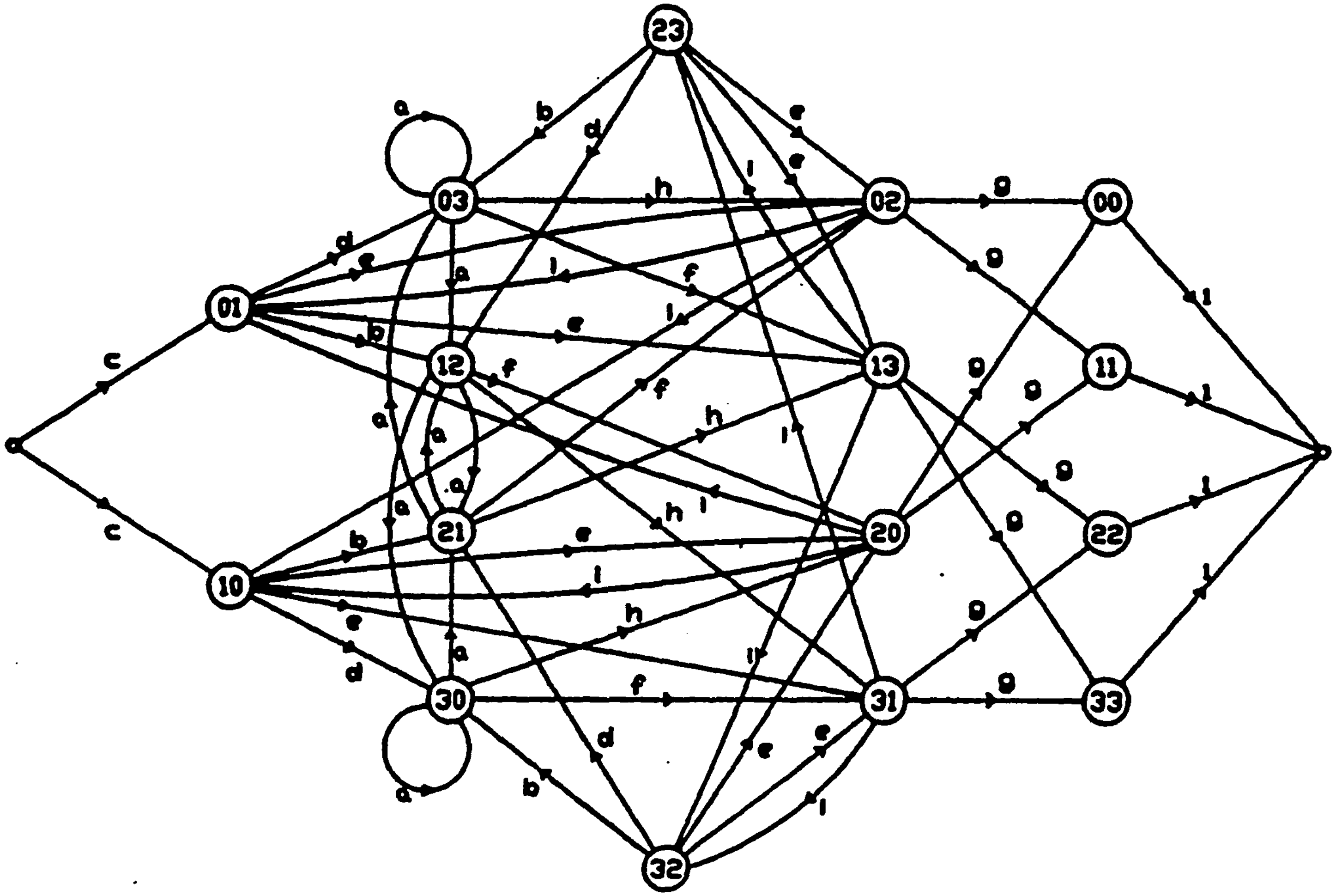


Fig. 9 Error State Diagram

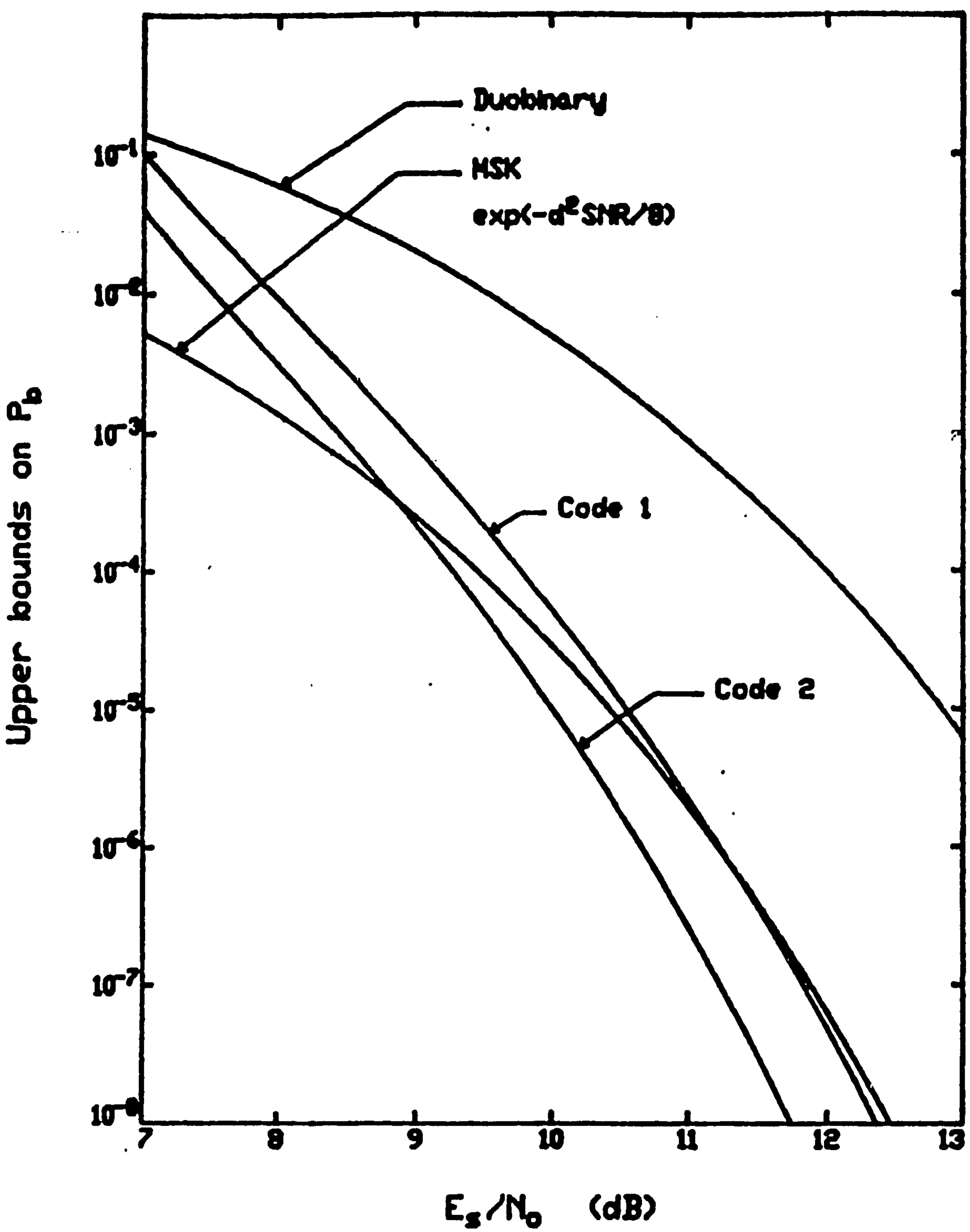


Fig. 10 Upper Bounds on Probability of Error for Noncoherent Detection of Rate 1/2 Trellis Coded Continuous Phase FM

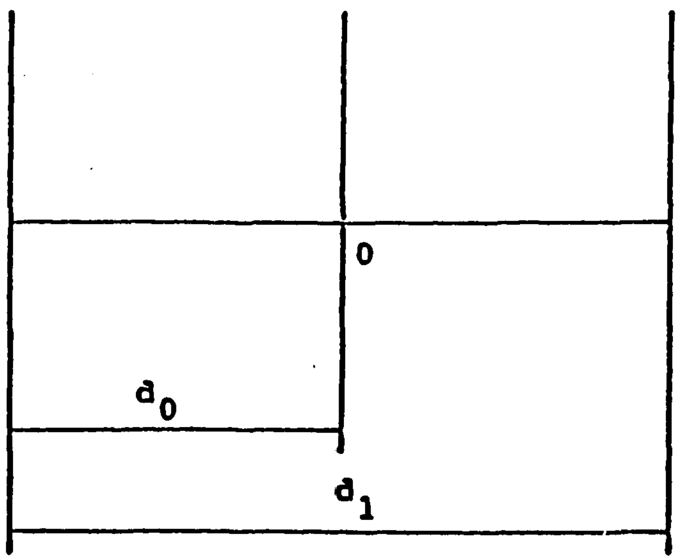
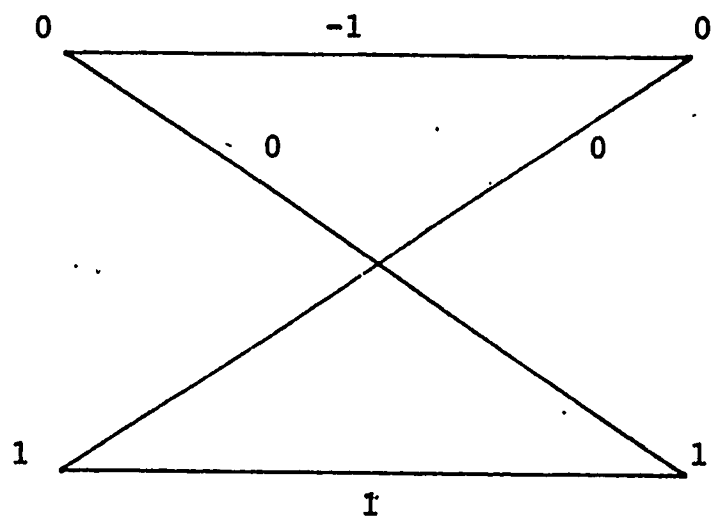


Fig.11-a. Signal Constellation for Duobinary

Fig. 11-b Two State Trellis Diagram for Duobinary.



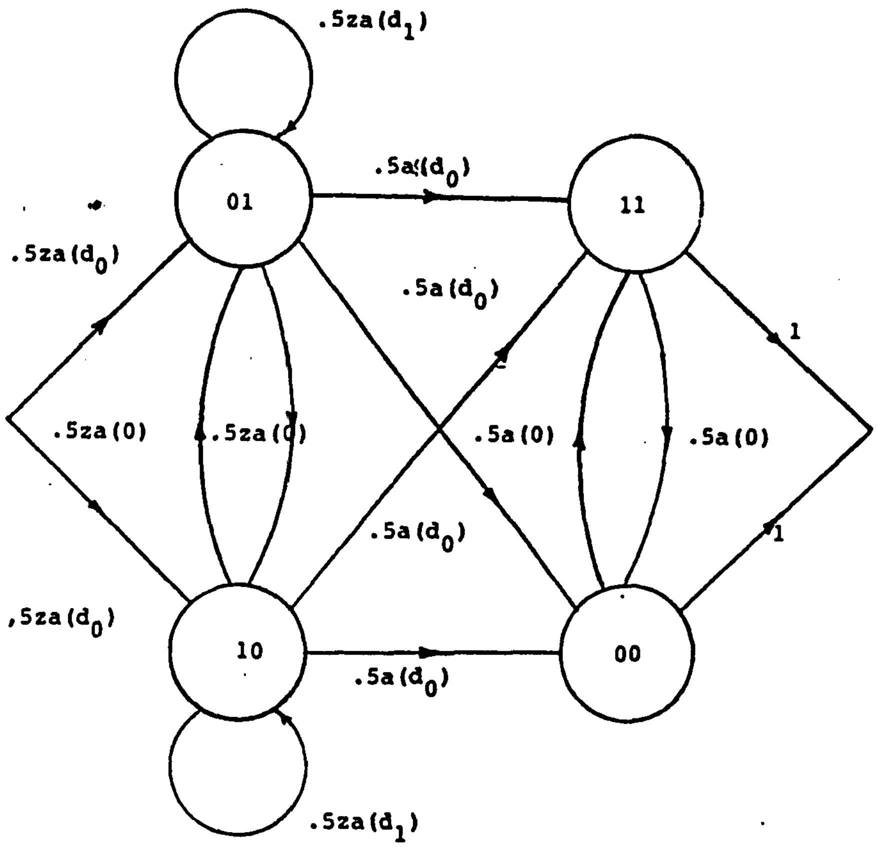


Fig. 11-c ERROR STATE DIAGRAM

FOR DUOBINARY

Simplification of Error State for Duobinary.

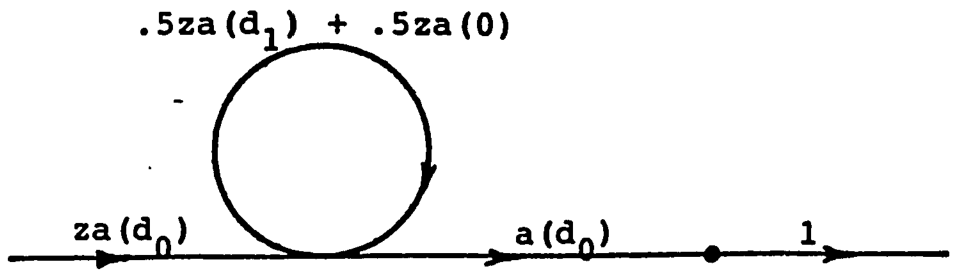


Fig. 11-d.

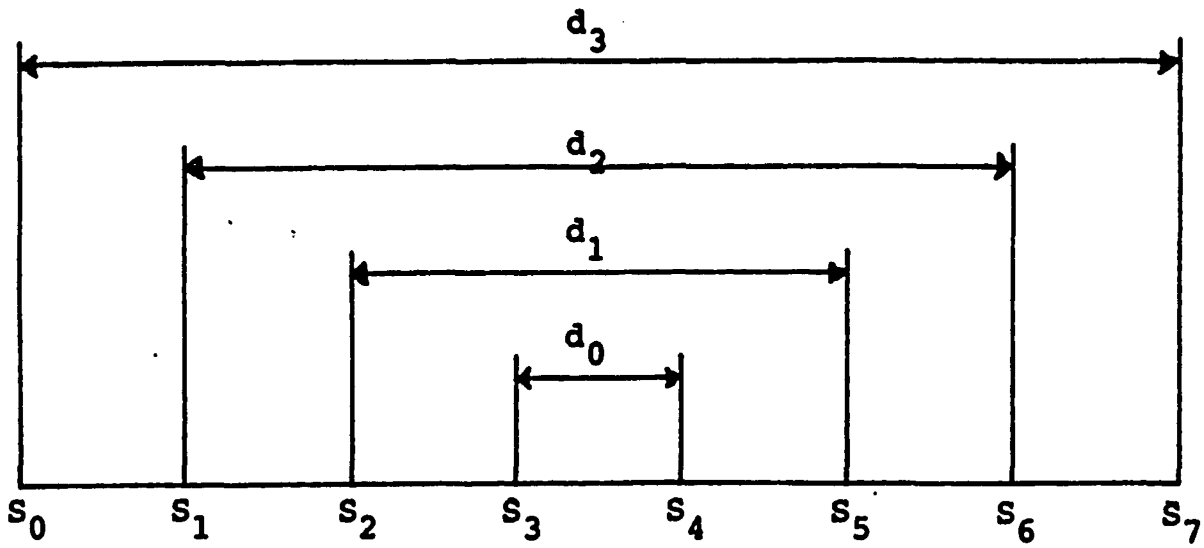


Fig. 12 Signal Constellation for High Rate Code 2/2/8

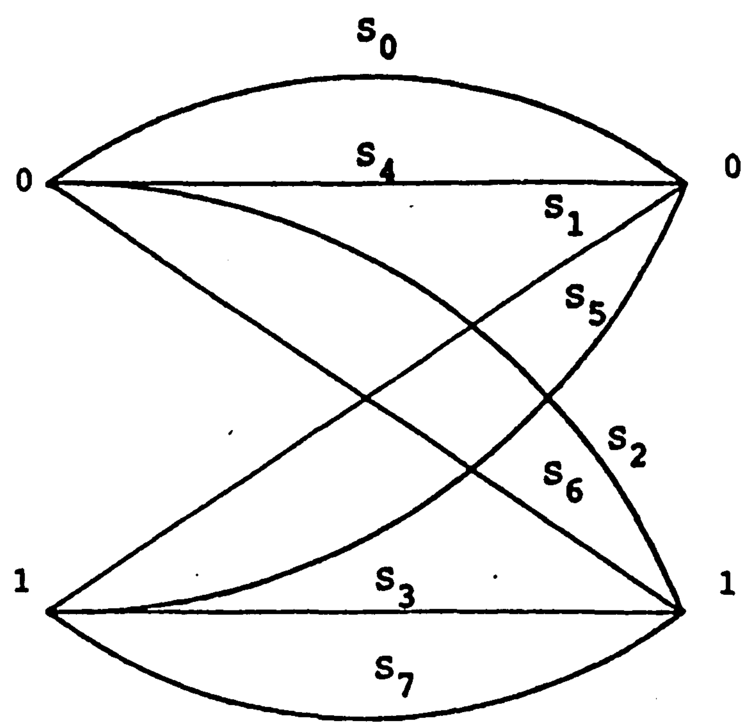


Fig. 13. Two State Trellis Diagram for High Rat: Code - 2/2/8

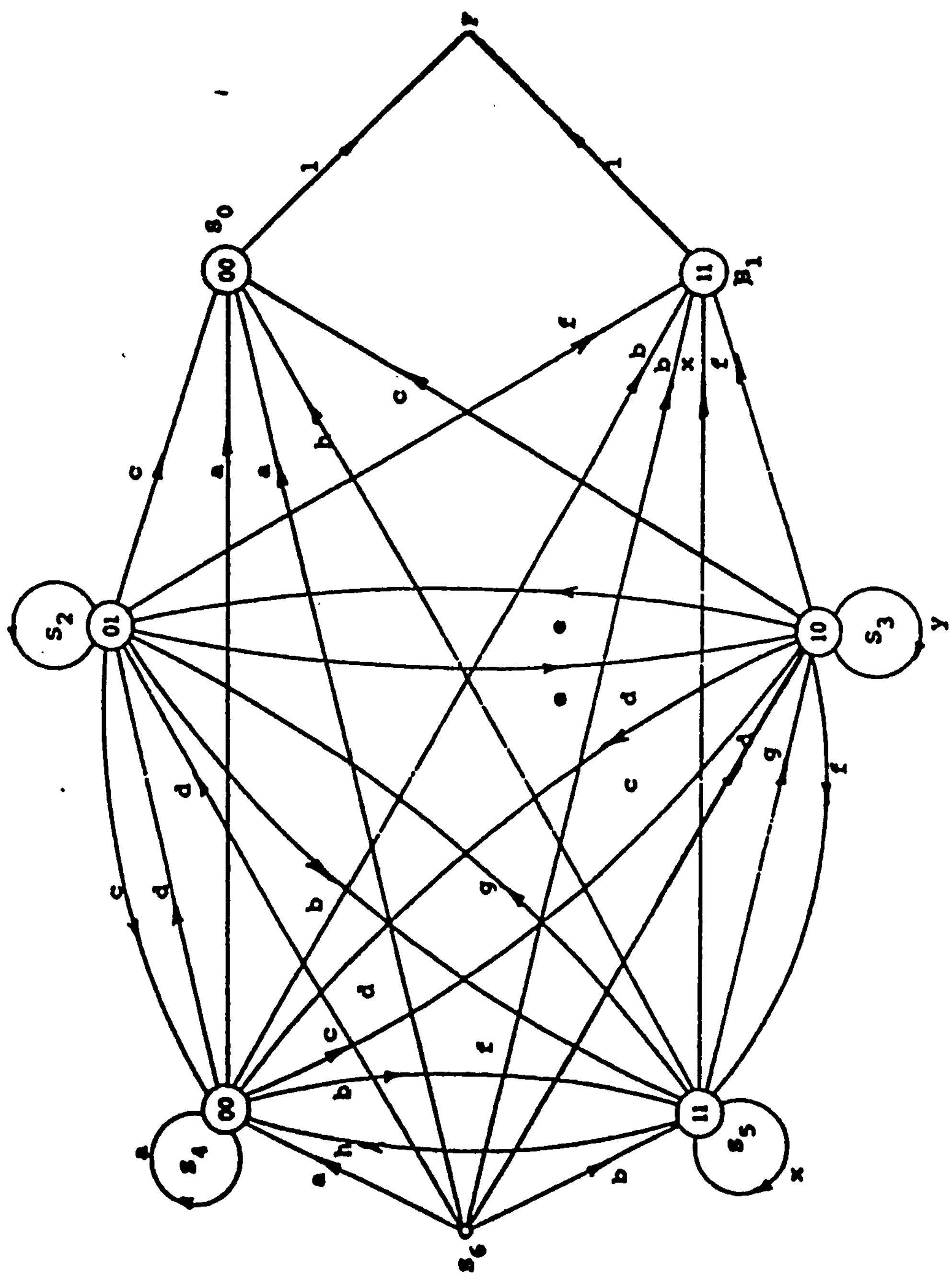
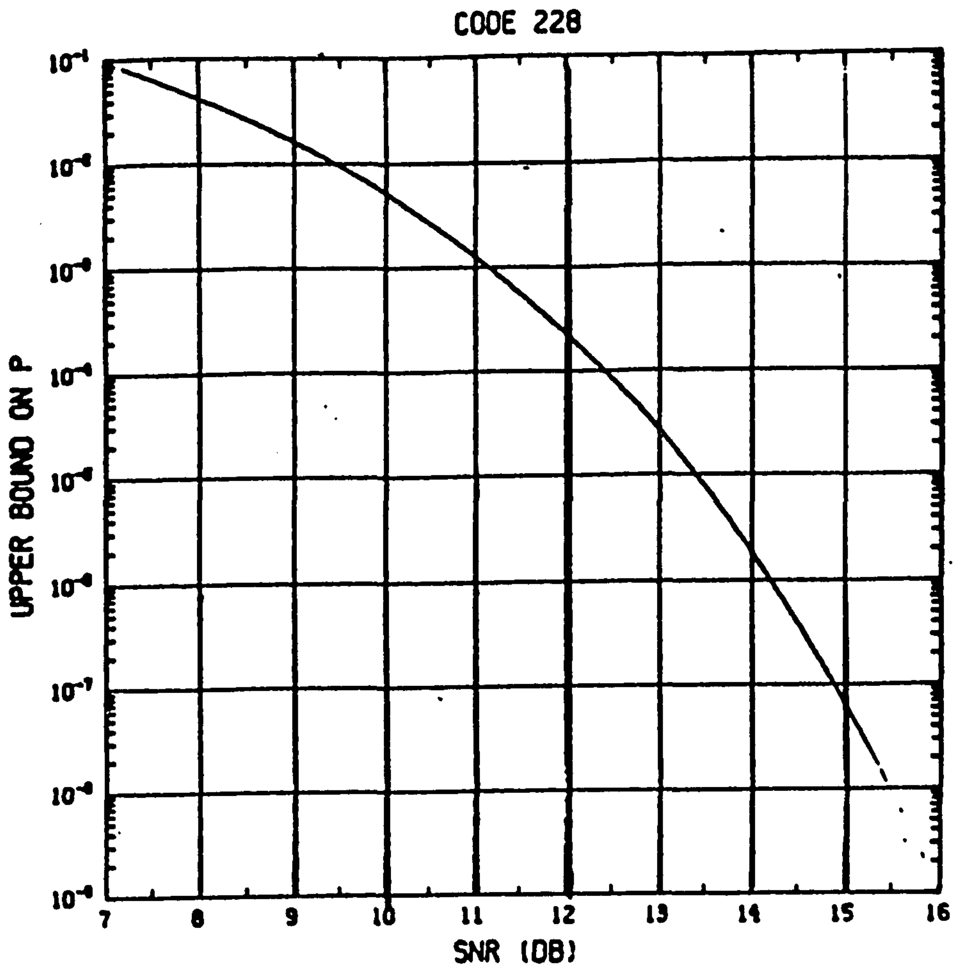


Fig. 16 SPECTRUM OF HIGH RATE CODE 2/2/8

Fig. 15 Upper Bound On Bit Error Rate for High Rate Code 2/2/8



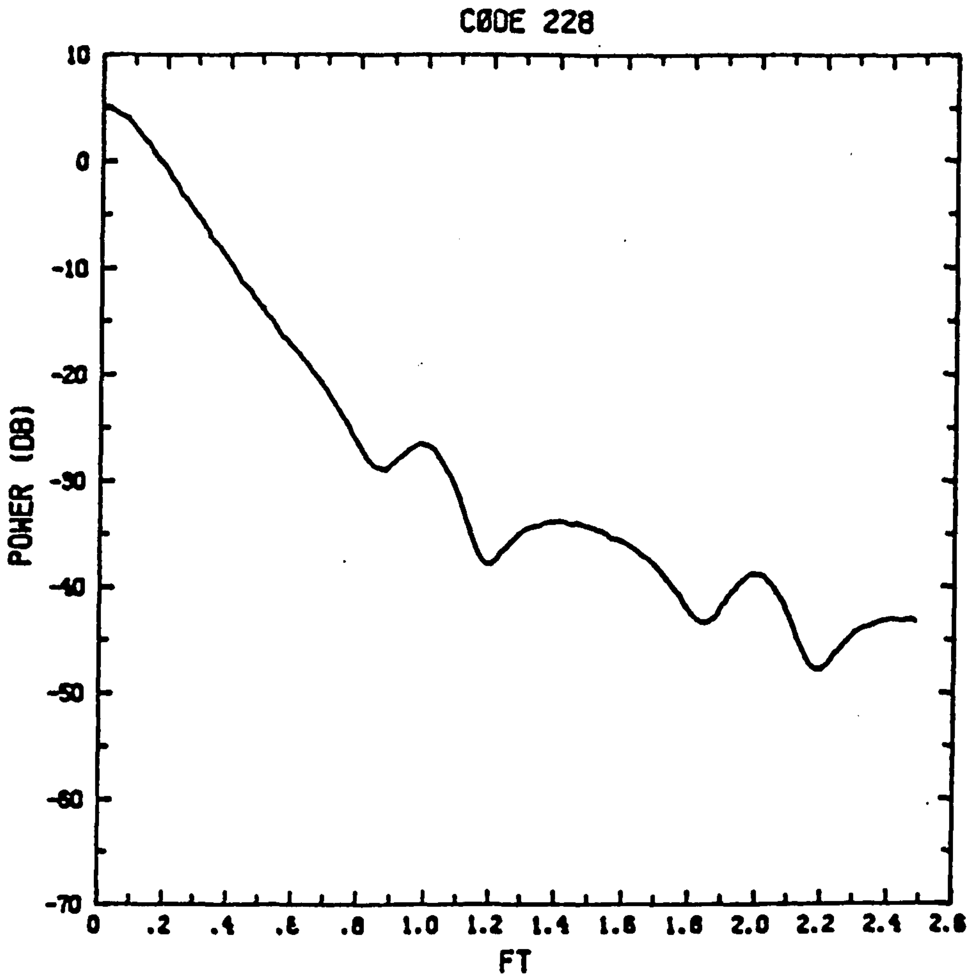


Fig. 16. spectrum of high rate code 2/2/8

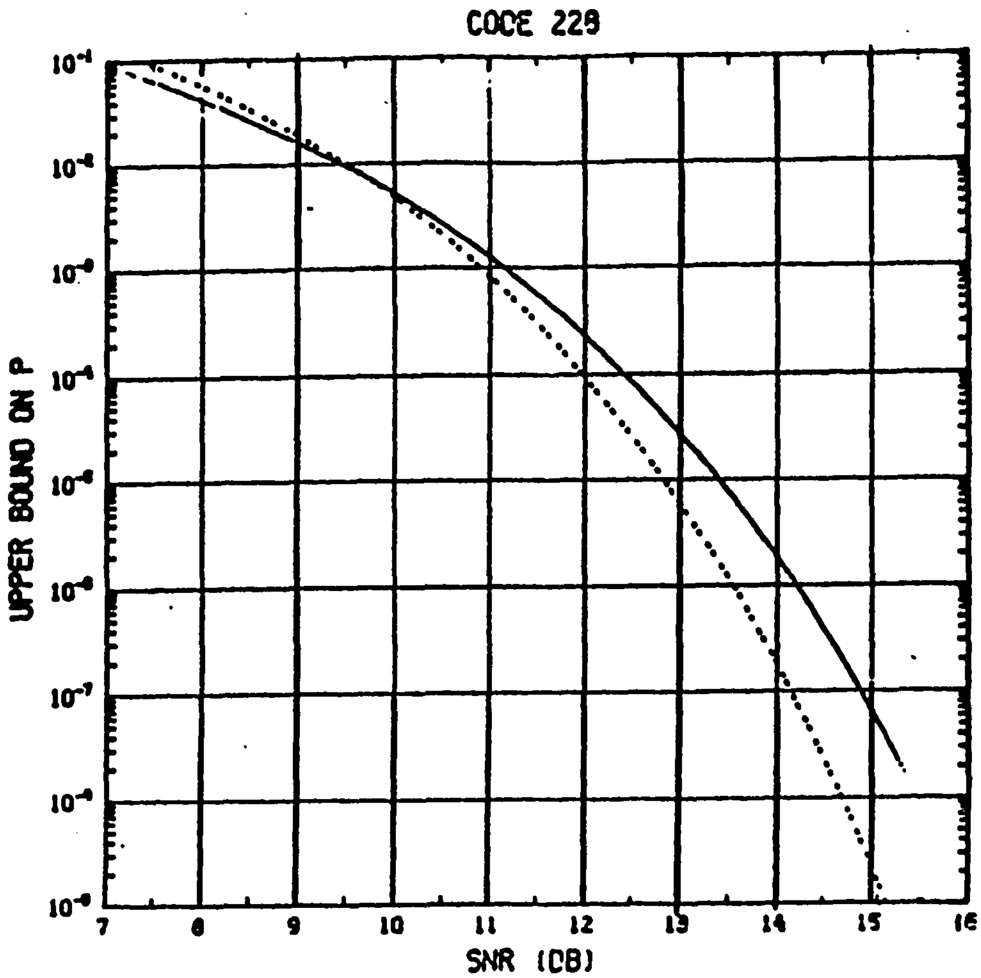


Fig. 17 Upper Bound On Bit Error Rate

__________ Code -2/2/8

---------- Duobinary

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