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Asset expected returns and risk

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ASSET EXPECTED RETURNS AND RISK

by

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Abstract

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All major equilibrium models describing the behavior of prices of financial assets are stated in terms of expected returns and expected payoffs. As a result, empirical verification of those models depends heavily on the ability to calculate those expected values, which are required for each model specifically, using observed data.

In this paper I propose a method for deriving expected returns on individual assets and the market as a whole by simply assuming that markets are efficient, in the sense that prices fully reflect all currently available information, (the semi-strong form of market efficiency). No particular equilibrium model is assumed since the Efficient Markets Hypothesis, hence EMH, is much broader and should be consistent with any single equilibrium model. In fact all equilibrium models utilize some form of the EMH and any test of those models is indirectly a test of the EMH.

The advantage of the method proposed is that it can be applied to single securities, as well as the market index, and it allows for the expected returns to be time dependent (random).

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INTRODUCTION

Most research so far, with some exceptions, has been geared towards explaining cross-sectional variations of expected returns among securities on the assumption that individual security and market expected returns were constant over time. This assumption, even though not required by theory, greatly simplified empirical testing of the major asset pricing models. An increasing number of authors[1], however, are suggesting that the constancy assumption may be untenable. Economic theory itself suggests that relationships may change over time because of changes in technology, tastes etc. In the case of expected returns on financial assets, changes may be due to the changing marginal rate of substitution of present for future consumption, or possibly to changes in fiscal and monetary policy.

It is then possible that the inconclusive results of the tests of the various asset models, may in fact be due to the misspecification of the expected returns. In addition to the problems asociated with the testing of these models, the general notion of efficient markets is in doubt, given the volatility of the prices of financial assets. Shiller (1981) using the variance bound test, suggests that the only way to save the notion of efficient markets would be to attribute the movement in stock prices to changes in expected real discount factors. Putting it differently, if the Efficient Markets Hypothesis is correct

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1. See Merton Robert C. (1980),Grossman J.S. and Shiller R. J. 1981, Shiller R.J. (1981), Litzenberger R. H. and Ronn E.I. (1986), French K. R., Schwert G. W. and Stambaugh R. F. (1987) , Conrad J. and Kaul G. (1988)

then expected discount factors cannot be constant. Litzenberger and Ronn (1986) suggest that discount factors are endogenous and therefore they change substantially over time. Despite the growing number of authors suggesting that discount factors, or what amounts to the same thing, expected returns, are time dependent, empirical work towards this direction has been limited. The reason, as Merton R. C.(1980) suggests, may be due to the fact that "estimating expected returns from time series of realized stock return data is very difficult".

This paper consists basically of two parts. In the first part, I assume that capital markets are efficient and I propose an operational interpretation of the EMH, which allows the estimation of expected returns over time via the use of the State Space Model, commonly known as the Kalman Filter Model. The advantage of this method is that it can be used for portfolios of securities, as well as individual securities. The empirical work that has been done so far along similar lines has been restricted to the estimation of expected returns of portfolios on the grounds that individual security returns are to noisy to allow the application of the Kalman Filter Model[2]. This approach, however, excludes the possibility of using estimated expected returns to test the asset pricing models because all of those models are stated in terms of individual security expected returns.

By allowing expected returns to change over time we gain three things: a) We are able to characterize the stochastic nature of expected returns on individual securities and the market. b) We can estimate security betas using expected rather than actual returns. c)

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2. See Conrad Jennifer and Gautman Kaul (1988)

We can retest the Capital Asset Pricing Model using ex-ante betas (i.e betas based on expected returns). In addition, it is possible to test whether the market factor becomes more important in explaining cross-section variations over time. That line of research can be pursued if in addition to time varying expected returns we assume that betas also vary over time.

In the second part of this paper I attempt to retest the Capital Asset Pricing Model utilizing estimated expected returns over time rather than actual returns. For the purpose of compatibility with the traditional tests, a time series analysis will be used to estimate the beta coefficient for each security, on the assumption that beta is constant, and also a cross sectional analysis examining variations of expected returns among securities, and testing whether the beta coefficient is significant in explaining cross-sectional variations. The results will be compared with the results of the traditional tests for the same sample of securities.

A Summary of What Follows

Chapter 1 is a review of the relevant literature whith emphasis on research that has been done on the estimation of expected returns as random coefficients.

Chapter 2 proposes an operational interpretation of the EMH based on the assumptions of the EMH. A regression equation is derived which in principle would allow estimation of expected returns, on the assumption that these are constant. Estimates for 60 securities are obtained and the constancy hypothesis is tested. The test utilized is the Langrange Multiplier Test.

In Chapter 3, I analyze the Kalman Filter model using recursive projections and assuming that expected returns follow a driftless random walk. I derive estimates over time for the 60 security sample and the market using the S&P 500 as a proxy.

In Chapter 4, assuming that the CAPM holds, I get beta estimates using expected returns. A cross-section analysis is included and the results are compared to the traditional test of the CAPM using the same sample of 60 securities.

Finally, in Chapter 5, I conclude with a discussion of the overall results, and an evaluation of the advantages and limitations of this analysis.

CHAPTER 1

Review of the Literature

In early empirical tests of the Capital Asset pricing Model, individual security and market expected returns were assumed constant[1], and as a result historical averages were used to get the corresponding estimates. The same assumption is still utilized in the calculation of stock betas which are widely published by investment firms.

Merton R.C. (1980) outlines a model attempting to characterize explicitly market expected returns through time. In the model changes in market expected returns are due to changes in market risk. Notationally, (a-r)=Yg(var(a)), where a-r is the expected excess return over and above the risk-free rate, var(a) is the variance rate on the market and Y is the "reward to risk ratio" which could be equal to the representative investor's relative risk aversion.

In the analysis it is assumed that the function g is known and that var(a) can be observed. If the variance rate on the market were constant, then expected excess returns would also be constant, or (a-r)=Y. However, the hypothesis that the variance rate is constant can be rejected at almost any confidence interval[2].

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1. Black, Jensen and Scholes (1972), Fama and MacBeth (1974), Friend and Blume (1970)

2. See Black (1976) and Rosenberg (1972a)

In the estimation part, Merton is assuming that the variance rate is constant over subperiods of four years and he obtains estimates for Y and Var(a) for the time period, 8/26 to 7/78, (13 subperiods).

Briefly, in this model market expected returns vary over time, because the variance rate on the market is not constant. So "for capital market and corporate finance applications the estimate of Y times the estimate of the current variance will provide better estimates of expected excess returns than the historical average"[3].

A similar methodology was adopted by French Schwert and Stambaugh (1987). In this paper, it is also assumed that expected stock returns are positively related to stock volatility. The statistical methodology adopted is the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model, proposed by Engle (1982). Evidence is found that the expected risk premium on common stocks are positively related to the predictable level of market volatility.

The advantage of the two models is that, they explicitly characterize market expected returns which take into account changes in the level of market risk. The limitation is that, the models are fully specialized for the market, and they cannot be used for the estimation of expected returns of individual securities, even though varying expected returns for the market would certainly imply varying expected returns for individual assets.

For example using the CAPM and assuming that it holds intertemporally we have that, $ER_{it}=(1-b)R_t+bER_{mt}$, where ER_{it} and ER_{mt} are expected return on the individual security and the market

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3. See Merton R.C. (1980)

respectively at time t, b is the beta of the security and R_f the risk-free rate. We can see that even if the beta and the risk-free rate are constant, ER_{it} will change over time because of changes in ER_{mt}, i.e. i.e. $d(ER_{it}) = b\ d(ER_{mt})$.

In a more recent article, Jennifer Conrad and Gautam Kaul (1988) test the hypothesis that expected returns are constant against the alternative that expected returns are random. The return process is assumed to follow,

$$R_t = E_{t-1}(R_t) + e_t$$

which is their observation equation in Kalman Filter terminology and the coefficient process is assumed to follow the first-order autoregressive scheme,

$$E_{t-1}(R_t) = G\ E_{t-2}(R_{t-1}) + u_{t-1}$$

which is the system equation. R_t are realized returns and $E_{t-1}(R_t)$ expected returns as of t-1, and $G < 1$. The problem with the system equation is that it implies that over time expected returns become smaller and smaller since $G < 1$. The AR1 process is appropriate only if it is stated in deviation from some unconditional mean so that the restriction $G < 1$ would imply that expected returns converge to their long-term unconditional mean.

The model in principle could be used to get estimates on expected returns, on portfolios as well as individual securities. The authors use weekly returns on 10-size based portfolios over the 1921-85 period. The 10-size portfolio is used because, "it is much more difficult to extract the signal from noisy returns of a single security".[4]

The estimation techniques is the Kalman filter model, and the findings are: a) the constancy of expected returns is strongly rejected for all 10 portfolios; b) the variation of expected returns over time is a relatively large fraction of ex-post return variance; c) although the information used is only the past returns, the estimated expected returns subsume the information in other predictor variables.

The merit of this model is that it can be used for any portfolio including the market, so it is more general than Merton's even though the authors choose not to apply it on individual securities. In addition no specific test is mentioned as to the constancy of the coefficients. Furthermore, no attempt is made to tie the model to any asset-pricing model, despite the fact that all major asset pricing models require as their input, expected returns among other things. In other words, there is no hint as to how estimated expected returns over time could be used.

In examining stock price movements, and the determinants of those movements, Shiller[5] concluded on the basis of the variance bound

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4. See Jennifer Conrad and Gautam Kaul (1988), Campbell (1987), Ferson, Kandel, and Stambaugh (1987).

5. See Grossman J. S., and Shiller, R. J. , May (1981) and also Shiller J. R., June (1981)

tests[6] that, subsequent changes in dividends, could not justify price variability. As a result, the general notion of efficient markets could be maintained, only if movement in prices could be attributed to changes in expected discount factors.

In general, a small but growing body of research points to the direction of utilizing random coefficient models for the estimation of expected returns, because the assumption of stationary return generating processes cannot be maintained.

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6. Shiller's Variance bound test can be outlined as follows:
The perfect foresight stock price, P^*_t, is defined as $P^*_t = \sum_{i=1}^{n} k^i d_{t+i}$ (1)
k=discount factor (assumed constant), d_t=dividends paid. The actual stock price according to the fundamendal model is, $P_t = \sum_{i=1}^{n} k^i D^e_{t+1}$ (2)
where, d^e_t=expected dividend for period t. Assuming a constant discound factor, P_t is a forecast of P^*_t so that, $P^*_t = P_t + e_t$ (3) where e_t=forcast error. Since P_t is the optimal forecast of P_t^*, then according to the efficient markets hypothesis e_t should be uncorrelated with Pt. Taking variances on both sides of equation (3) we get:
$Var(P^*_t) = Var(P_t) + Var(e_t)$ or $Var(P^*_t) > Var(P_t)$. The P_t series fails the variance test when discount factors are assumed constant. According to Shiller: " Once we permit the discount factor to vary, the inequality has a much greater chance of being true".

CHAPTER 2

1. Efficient Markets Hypothesis (EMH)

In general terms the EMH states that current asset prices "fully reflect" available information.[1] The various forms of efficiency were then defined and tested in terms of the contents of the information set.

In its weak-form, efficiency implies that the information set includes all past information, and investors and analysts using past information could not earn returns above normal. The semi-strong form implies that in addition to past information, current prices reflect all current publicly available information, relevant for price formation (e.g., earnings announcements, stock splits, etc). The strong-form of efficiency maintains that current prices reflect all relevant information whether publicly available or not. Notationally whatever the form of efficiency, the EMH can be described as,

$$E(Y_{it+1}|I_t) = [1+E(R_{it+1}|I_t)]\ P_{it} \qquad (1)$$

where E is the expected value operator, Y_{it+1} is equal to the price plus the dividend at t+1 (or, what amounts to the same thing, price at t+1 with reinvestment of any intermediate cash income), R_{it+1} the return at t+1, and P_t, current price.

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1. See Fama, E. (1970)

The assumption that the conditions of market equilibrium can be stated in terms of expected returns, and that equilibrium expected returns are formed on the basis of I_t has the implication that expected profits in excess of equilibrium expected profits should be zero. Thus, let the conditional forecast error,

$$u_{it+1} = Y_{it+1} - E(Y_{it+1} | I_t) \qquad (2)$$

Then, $$E(u_{it+1} | I_t) = 0 \qquad (3)$$

Relation (3) says that the sequence $\{u_t\}$ is a "fair" game with respect to the information sequence $\{I_t\}$, and it is basically a version of rational expectations. It states that forecasts about next period's payoffs are unbiased, and the possibility of expected excess profits is ruled out. This interpretation of $\{u_t\}$ is also consistent with Fama (1976) who identified the notion of market efficiency with the assumption that the market has rational expectations and that "it" knows all the relevant data.

In a multiperiod setting, though, an additional assumption is needed namely that forecast errors are not serially correlated. That is a direct implication of efficient markets and unbiased expectations, when we consider more than two periods.

Thus, $$E(u_{it}, u_{it+1}) = 0 \qquad (4)$$

For if, $u_{it}=au_{t-1}+e_t$ where $E(e_t)=0$ then individuals could have improved their forecasts by taking into account the autoregressive structure of their previous forecast errors. But market efficiency implies that all information currently available, (and that includes the structure of previous forecast errors) is fully and optimally utilized.

2. An Operational Interpretation of the EMH

Equations (1), (2), (3) and (4) form the basis for an operational interpretation of the EMH. Substituting (1) into (2) we get,

$$Y_{it+1} = k_{it+1} P_{it} + u_{it+1} \qquad (5)$$

Where, $k_{it+1} = [1+E(R_{it+1}|I_t)$, $E(u_{it+1})=0$, $E(u_{it},u_{it+1})=0$, $var(u_{it+1})=constant$ and the price at t is uncorrelated with the one period ahead forecast error, so that $E(P_{it},u_{it+1}) = 0$. On the market level the EMH states that,

$$E(Y_{mt+1}|I_t) = [1+E(1+R_{mt+1}|I_t)]\ P_{mt} \qquad (6)$$

where, Y_{mt+1} is the price of the market portfolio plus the dividend at t+1, P_{mt} is the current price, and R_{mt+1} is the rate of return at t+1. Again the conditional forecast error will be,

$$u_{mt+1} = Y_{mt+1} - E(Y_{mt+1}) \qquad (7)$$

The expected as of time t forecast error will be zero, that is

$$E(u_{mt+1} | I_t) = 0 \tag{8}$$

and also forecast errors will be serially uncorrelated,

$$E(u_{mt}, u_{mt+1}) = 0 \tag{9}$$

Substituting (7) into (6) we get,

$$Y_{mt+1} = k_{mt+1} P_{mt} + u_{mt+1} \tag{10}$$

Where, $k_{mt+1} = [1+E(R_{mt+1} | I_t]$, $E(u_{mt+1})=0$, $E(u_{mt}, u_{mt+1})=0$, $Var(u_{mt+1})$=constant and $E(P_{mt+1}, u_{mt})=0$.

Equations (5) and (10) are in standard regression form and the required returns on the individual securities and the market are coefficients that can be estimated by using linear least squares projections (regressions).

An interesting implication of this analysis, is that the current stock prices are sufficient statistics[2], or to put it differently, the information set P_{it} is equivalent to the information set I_t. For example, projecting P_{it} on both sides of the equation (5) we get,

$$\hat{E}[Y_{it+1} | P_{it}] = k_{it+1} P_{it} \tag{11}$$

2. See Grossman, S. and Stiglitz J., May 1976

where $\hat{E}$, is the linear projections operator. Equations (1) and (11) imply that $E(Y_{it+1}|I_t)=\hat{E}(Y_{it+1}|P_{it})$. Of course we cannot conclude that I_t is identical to P_{it}, but we can certainly say that aspects of I_t not included in the current price can be treated as noise.

If expected returns are constant, then we can apply standard OLS to equations (5) and (10) to get estimates of k_i and k_m, since the error process $\{u_t\}$ satisfies all the assumptions of the standard linear model. The sample estimate for k_i would be,

$$\hat{k}_i = \sum_{t=2}^{n} Y_{it} P_{it-1} \Big/ \sum_{t=2}^{n} (P_{it-1})^2 \qquad (12)$$

According to the Gauss-Markov theorem the least squares estimate of k_i will be best linear unbiased estimates of the gross return on the individual security. Examining (12) further we see that the estimate is a weighted average of past gross returns. By dividing and multiplying each term in the numerator of equation (12) by P_{it-1} for t= 1, 2, ... we get that,

$$\hat{k}_i = \sum_{t=2}^{n} W_{it}(1+R_{it})$$

where $W_{it} = P_{it-1}{}^2 / \sum_{t=2}^{n} P_{it1}{}^2$ and $\sum_{t=2}^{n} W_{it} = 1$

The weights will in general be unequal and if stock prices have the tendency to increase over time (which is a reasonable assumption, since for most stocks part of the return will come in the form of capital gain), then our estimates would have the characteristic that more recent information will have a higher weight in the calculation of the expected rate of return. The simple arithmetic mean of past returns which has been used in the past assigns to all data points an equal weight.

In the event that expected returns are random, ordinary least squares is inapplicable and the estimate given by (12) will not be BLUE. If, for example, the correct specification for the expected returns is that they follow a random walk, (i.e. $k_t = k_{t-1} + v_t$), then we can write equation (5) as follows,

$$\begin{aligned} Y_t &= (k_{t-1} - k + k + v_t)\ P_{t-1} + u_t \\ &= k\ P_{t-1} + e_t \end{aligned} \tag{13}$$

where, $e_t = a_t\ P_{t-1} + u_t$, $a_t = k_{t-1} - k + v_t$ and k the constant coefficient associated with equation (5).

Hence, (13) can be viewed as a standard regression model with fixed parameter k and a zero-mean error term e_t. However, if the parameter k_t is stochastic (i.e. not identically equal to its mean k), the error process $\{e_t\}$, will not have a constant variance. In addition, if the stochastic parameter k_t is autocorrelated, then so will be the $\{e_t\}$, as can be seen below:

$$\mathrm{Var}(e_t) = \mathrm{Var}(a_t)\ P_{t-1}^2 + \mathrm{Var}(u)$$

$$E(e_t, e_{t-1}) = E(a_t, a_{t-1})\ P_{t-1}\ P_{t-2} \neq 0$$

The presence of both, heteroscedasticity and autocorrelation would suggest the applicability of GLS, if the parameters of the error process were known. However, a more convenient framework for analyzing such models, is the state space model or Kalman filtering.[3]

3. Test of the Fixed Parameter Model Against the Random Walk Model

Before we proceed with the estimation of the model suggested by equation (5), using stochastic parameter techniques, it would be useful to test whether such techniques are appropriate. In other words, we would have to test the null hypothesis that the coefficient in equation (5) is fixed, against the alternative that it follows a random walk.

The most widely used hypothesis testing procedure in practice is the likelihood ratio test (LR). In order to implement the LR test, one has to estimate the model under the null hypothesis, which could take the form of K restrictions imposed on the parameters, and under the alternative hypothesis. If we denote by L_0 and L_1, the maximum value of the likelihood function under the null and alternative hypothesis respectively, then the LR test is given by,

$$LR = 2(\text{Log } L_1 - \text{Log } L_0)$$

The statistic follows, under the null hypothesis, a chi-square

3. See Harrison, P. J. and Stevens C. F. (1976)

distribution with K degrees of freedom. The null hypothesis is rejected for large values of the statistic.

An alternative test, equivalent to LR, is the Lagrange Multiplier[4] (LM) test, which has been used to test, heteroscedasticity and model specification.[5] The LM test is based on the derivative of the log likelihood function, evaluated under the null hypothesis. In large samples this derivative under the null hypothesis is distributed normally with zero mean. The equivalency of LR and the LM test, stems from the fact that, the higher in absolute value the derivative under the null hypothesis the higher the LR statistic will be, and both statistics will tend to reject the null hypothesis. Both tests share the same optimality properties, in large samples. However, the LM test is much simpler to implement because it only requires estimation of the model under the null hypothesis. In contrast, the LR test requires that the model be estimated under both, the null and the alternative hypothesis.

Using equation (13) and assuming normal distribution, the likelihood function for a sample of size N can be written as,

$$L = (2\pi)^{-n/2} \prod_{t=1}^{N} (\sigma_u^2 + P_{t-1}^2 \sigma_a^2)^{-1/2}$$

$$\exp \left[- \sum_{t=1}^{N} \frac{(Y_t - k P_{t-1})^2}{2(\sigma_u^2 + P_{t-1}^2 \sigma_a^2)} \right]$$

4. See Rao C. R. (1948), and Silvey, D. S. (1959)

5. See Breusch, T. S. and Pagan, A. R. (1979), (1980)

The log likelihood can be written as,

$$\log L = -\frac{N}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{N}\log(\sigma_u^2+P_{t-1}^2\sigma_a^2)$$

$$-\frac{1}{2}\sum_{t=1}^{N}\frac{(Y_t-k\,P_{t-1})^2}{(\sigma_u^2+P_{t-1}^2\sigma_a^2)}$$

Differentiating with respect to σ_a^2, we obtain the slope of the log likelihood,

$$\frac{d\log L}{d\,\sigma_a^2} = -(1/2)\left[\sum_{t=1}^{N}\frac{P_{t-1}^2}{(\sigma_u^2+P_{t-1}^2\,\sigma_a^2)}\right]$$

$$-(1/2)\left[\sum_{t=1}^{N}\frac{P_{t-1}^2\,(Y_t-k\,P_{t-1})^2}{(\sigma_u^2+P_{t-1}^2\,\sigma_a^2)^2}\right]$$

We can now evaluate the slope under the null hypothesis of a fixed parameter model, i.e. we assume that $Y_t = k\,P_{t-1} + u_t$ holds. Then we can estimate the model by using ordinary least squares. Let c_t denote the residuals from the OLS regression. The maximum likelihood estimate of the variance of u_t will be equal to $s_u^2 = \Sigma\, c_t^2/N$. Evaluating the slope using those estimates and also the fact that σ_a^2 will be zero under the null hypothesis we get,

$$\frac{d\log L}{d\,\sigma_a^2} = -(1/2)\left[\sum_{t=1}^{N}\frac{P_{t-1}^2}{s_u^2} + \sum_{t=1}^{N}\frac{P_{t-1}^2\,c_t^2}{s_u^2}\right]$$

The Langrange multiplier statistic is obtained by dividing the quantity above by an estimate of its standard deviation under the null hypothesis. This yields according to Breusch and Pagan (1979) the following test statistic:

$$S = \left[\frac{1}{2} \sum_{t=1}^{N} P_{t-1}^{2} \left(\frac{c_t^2}{s_u^2} - 1 \right) \right]$$

divided by, $$\left[\frac{1}{2} \sum_{t=1}^{N} P_{t-1}^{4} - \frac{1}{2N} \left(\sum_{t=1}^{N} P_{t-1}^{2} \right)^2 \right]^{1/2}$$

where c_t, denotes residuals and s_u^2 the variance estimate obtained from the OLS. Under the null hypothesis, the statistic S has for large samples, a distribution that is well approximated by the standard normal. The hypothesis for random walk coefficients will be preferred to the null for large positive values of the S statistic.

Thus, in order to implement the test, we estimate by OLS equation (13) obtaining the residuals c_t and the estimated error variance s_u^2. We then compute the statistic S, and we reject the null hypothesis, at significance level α if S is greater than $Z\alpha$[6].

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6. See Newbold P. and Bos T. (1985)

4. The Data Sample and Transformations

The sample used in this study consists of 60 companies randomly selected from the CRSP (Center for Research in Security Prices). The series for each company are prices and dividends. The data are monthly and the monthly dividend reported is the latest known annual cash dividend. In order to get a monthly series I divided the reported dividend by 12, assuming the dividends are paid monthly rather than quarterly. The payoff at the end of each period is the sum of price plus dividend (variable Y_t, in equation 5).

In this study the Standard and Poor 500 index is used as a proxy for the market portfolio. The series are again price and dividend, the frequency is monthly, and the data were taken from the Economic Report of the President. Table 1, on page 23 presents the information on each of the 60 companies, i.e. market in which the stock is traded, the Standard Industrial Code, and the time period over which data are available, for each security.

5. Estimation and Results of the Constant Coefficient Model

The results of the estimation of the constant coefficient model are reported in Table 2, page 26. The regression equation is $Y_t = k P_{t-1} + u_t$ where Y_t is current price plus divident, P_{t-1} is last period's price and k is the monthly gross expected return. If k is constant and the error process $\{u_t\}$ is normal with zero mean and constant variance then the ordinary least squares procedure is applicable.

The results reveal that current payoffs in terms of price and dividend, can largely be explained by last period's price; in almost all regressions the coefficient of determination is above 0.90. This seems to support the notion that share prices, when adjusted for dividends and discounting, follow a first-order univariate Markov process and that no other variables Granger cause share prices.[7]

The results on the estimated S[8] statistic, which tests whether the coefficient is constant, reveal that, at almost any confindence interval the null hypothesis that k is constant can be rejected for 57 out of 60 stocks. Also for the market (S&P 500) the constancy assumption can be rejected. In fact the S statistic value for the index is among the highest (tenth in descending order).

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7. Samuelson (1965), first pointed out that anticipated changes in prices follow a random walk.
According to Granger (1969), a random process Z_t is said not to cause a random process X_t if $E(X_{t+1}|X_t,..X_0, Z_t,..Z_0) = E(X_{t+1}|X_t,..X_0)$.

8. op. cit. pp 17-19.

It is interesting to note that the estimated coefficient for the market is almost identical to the historical average. The same cannot be said for the individual securities. For 50% of the securities the estimated monthly gross expected return is less than one. The historical average is less than one for 15% of the securities.

The results are consistent with the findings of other authors with respect to the variabilty of expected returns.[9] The difference is that in their works the results are either for the market, or for large portfolios, whereas in this study, I test the stability of expected returns of individual securities.

9. See Fama and Schwert (1977), Fama (1981) , Keim and Stambaugh (1986), Fama and French (1988) , Merton R.C. (1980), Kaul (1987), Conrad J. and Kaul G. (1988)

TABLE 1

LIST OF COMPANIES AND CLASSIFICATION

COMPANY	SIC	NYSE	ASE	OTC	S&P 400	PERIOD
American Greeting Corp.	2771			+		62:2-88:12
American Maize-Product	2046		+			63:2-88:12
American Ship Building	3730		+			65:2-88:12
Avx Corp.	3679	+				73:5-88:12
British Petroleum	2911	+				66:5-88:12
Casablanca Inds Inc	3630	+				65:2-88:12
Clabir Corp.	2480	+				76:2-88:12
Coleman Co Inc.	3940	+			+	66:2-88:12
Combustion Engineering	3560	+			+	62:2-88:12
Commercial Metals Co.	5050	+				66:2-88:12
Conrac Corp.	3600	+				66:5-88:12
Crystal Brands	2300	+				66:5-88:12
Dallas Corp.	3442	+				66:2-88:12
Dillard Dep. Stores 566311	5311		+		+	69:5-88:12
Domtar Inc.	2600		+			66:5-88:12
Eagle-Picher Inds	3714	+				62:2-88:12
Eeco Inc.	3679		+			66:5-88:12
Genuine Parts Co.	5012	+			+	62:2-88:12
Giant Foods Inc.	5411		+			66:2-88:12
Giant Group LTD	3241	+				66:2-88:12
Great Atl. & Pac. Tea	5411	+			+	62:2-88:12
Harris Corp.	3663	+			+	62:2-88:12
Hartmax Corp.	2300	+			+	62:2-88:12

TABLE 1 Continued

COMPANY	SIC	NYSE	ASE	OTC	S&P 400	PERIOD
Hipotronics Inc.	3825		+			61:2-88:12
ICN Pharmaceuticals	2730	+				69:5-88:12
Imperial Oil LTD.	2911		+			66:2-88:12
Intl. Rectifier Corp.	3674	+				62:2-88:12
Ipco Corp.	2649	+				66:5-88:12
Kellog Co.	2000	+			+	62:2-88:12
Kerr Glass Mfg.	3221	+				76:1-88:12
Kinark Corp.	2890		+			66:5-88:12
Lee Pharmaceuticals	2844		+			73:5-88:12
M/A Com. Inc.	3674	+			+	62:2-88:12
Manville Corp.	3290	+				62:2-88:12
Martin Marietta Corp.	3760	+			+	62:2-88:12
Maytag Corp.	3630	+			+	62:2-88:12
Monarch Machine Tools	3540	+			+	65:2-88:12
Morton Thiokol Inc.	2800	+			+	65:2-88:12
Noxell Corp.	2844			+	+	68:2-88:12
Oakite Products	2841	+				66:5-88:12
Owens-Corning Inc.	3290	+			+	62:2-88:12
Penzoil Co.	2911	+			+	68:2-88:12
Penril Corp.	3825		+			73:5-88:12
Phillips-Van Heusen	2300	+				66:2-88:12
Procter & Gamble	2841	+			+	62:2-88:12
Quebecor Inc.	2711		+			73:5-88:12
Reynolds Metals Co.	3330	+			+	62:2-88:12

TABLE 1 Continued

COMPANY	SIC	NYSE	ASE	OTC	S&P 400	PERIOD
Rymer Co.	2010	+				62:2-88:12
Seagram Co. Ltd.	2085	+			+	62:2-88:12
Shell Transport & Trade	2911	+				73:5-88:12
Standex Intl. Corp.	3580	+				66:5-88:12
Stonebridge Resources	2030	+				72:2-88:12
Sun City Inds.	5140		+			72:2-88:12
Tootsie Roll Inds. Inc.	2065	+				62:2-88:12
Tyler Corp.	5065	+				67:2-88:12
USX Corp.	2911	+			+	62:2-88:12
Varity Corp.	3520	+			+	62:2-88:12
Wean United Inc.	3540	+				67:2-88:12
Weiman Co Inc.	5040		+			66:2-88:12
Wiener Enterprises Inc.	5600		+			71:2-88:12
Standard and Poor 500 Index						61:1-88:12

SIC: Standard Industrial code. 2000 or 3000 level indicate industrial companies, 5000 levels indicate retailing companies.

NYSE: The New York Stock Exchange.
ASE: The American Stock Exchange.
OTC: The over-the counter Market.
S&P 400: The Standard and Poor 400 Index
PERIOD: The time period over which data were obtained for each company.

TABLE 2

RESULTS OF THE CONSTANT COEFFICIENT MODEL

(Regression equation: $Y_t = k P_{t-1} + u_t$)

	COEFF[1]	TSTAT	RBAR	S^2	DW	MEAN[3]
Standard & Poor 500	1.00860	469	.99	18.0	1.25	1.00822
American Greeting Corp.	1.00052	209	.98	12.5	1.94	1.00776
American Maize-Product	.99841	149	.90	12.6	2.00	1.00738
American Ship Building	.99923	164	.91	2.68	2.09	1.00602
Avx Corp.	.99277	88	.91	7.83	2.26	1.00839
British Petroleum	1.00799	178	.97	17.3	1.77	1.01060
Casablanca Inds Inc	.99627	127	.93	9.28	1.79	1.00343
Clabir Corp.	.99999	108	.95	1.87	2.17	1.00535
Coleman Co Inc.	1.00232	168	.94	5.43	2.04	1.00991
Combustion Engineering	1.00111	193	.96	9.43	1.78	1.00868
Commercial Metals Co.	1.01070	161	.97	13.7	1.97	1.01352
Conrac Corp.	.99946	143	.92	6.18	2.05	1.00652
Crystal Brands	.99946	143	.92	6.18	2.05	1.00465
Dallas Corp.	1.00440	172	.89	1.35	2.06	1.00670
Dillard Dep. Stores	1.00535	133	.98	20.0	1.76	1.01511
Domtar Inc.	1.00492	173	.95	13.6	2.03	1.00833
Eagle-Picher Inds	.99841	168	.95	14.6	1.84	1.00687
Eeco Inc.	.99711	144	.96	9.80	1.60	1.00522
Genuine Parts Co.	1.00826	260	.98	14.4	2.07	1.01136
Giant Foods Inc.	1.02162	211	.99	24.9	1.79	1.01827
Giant Group LTD	.99797	179	.93	5.41	1.92	1.00308
Great Atl. & Pac. Tea	.99719	235	.97	8.16	2.05	1.00121

TABLE 2 Continued

	COEFF[1]	TSTAT	RBAR	S^2	DW	MEAN[3]
Harris Corp.	.99910	184	.96	11.0	1.99	1.00603
Hartmax Corp.	1.00382	208	.97	13.4	2.09	1.01062
Hipotronics Inc.	1.00263	124	.95	6.59	2.05	1.00889
ICN Pharmaceuticals	.98293	97	.92	9.47	2.31	.99568
Imperial Oil LTD.	1.01820	185	.93	6.34	2.14	1.00748
Intl. Rectifier Corp.	.99136	134	.94	9.13	1.94	.99995
Ipco Corp.	.99266	136	.95	8.94	1.96	.99997
Kellog Co.	1.01494	273	.99	22.7	2.09	1.01177
Kerr Glass Mfg.	1.00106	208	.95	5.09	2.13	1.00706
Kinark Corp.	.99158	123	.90	11.8	1.89	.99905
Lee Pharmaceuticals	.98080	75	.92	12.2	1.89	.99735
M/A Com. Inc.	.99371	143	.97	16.9	2.01	1.00376
Manville Corp.	.99904	239	.97	2.70	2.00	.99607
Martin Marietta Corp.	1.00516	199	.98	21.9	2.08	1.00925
Maytag Corp.	1.00471	201	.97	18.5	2.02	1.00936
Monarch Machine Tools	1.00091	150	.95	11.3	2.02	1.00914
Morton Thiokol Inc.	1.00456	205	.98	19.5	2.26	1.00862
Noxell Corp.	1.00538	186	.98	16.0	1.82	1.01242
Oakite Products	1.00887	221	.95	3.65	1.97	1.00934
Owens-Corning Inc.	.98657	99	.75	55.1	1.99	1.00228
Penzoil Co.	1.004456	153	.95	6.95	2.19	1.00847
Penril Corp.	.994333	114	.94	3.46	2.29	1.00252
Phillips-Van Heusen	1.002293	157	.97	31.8	1.63	1.00772
Procter & Gamble	1.006498	311	.98	11.6	2.13	1.00717

TABLE 2 Continued

	COEFF[1]	TSTAT	RBAR	S^2	DW	MEAN[3]
Quebecor Inc.	1.013358	144	.98	12.7	1.61	1.01009
Reynolds Metals Co.	1.004807	149	.98	25.7	2.59	1.00631
Rymer Co.	.995227	165	.93	5.23	1.91	1.00053
Seagram Co. Ltd.	1.007651	240	.98	22.3	2.05	1.01074
Shell Transport & Trade	1.010426	178	.97	9.39	1.95	1.01209
Standex Intl. Corp.	1.008643	200	.97	10.3	2.01	1.01168
Stonebridge Resources	.991298	115	.79	0.75	1.99	1.00057
Sun City Inds.	.995314	116	.94	8.11	2.19	1.00429
Tootsie Roll Inds. Inc.	1.010386	177	.98	33.3	2.30	1.01204
Tyler Corp.	.996514	138	.95	11.9	2.09	1.00773
USX Corp.	.998773	234	.91	2.45	2.09	1.00299
Varity Corp.	.997508	196	.97	8.08	1.83	.99720
Wean United Inc.	.989330	151	.96	4.51	2.09	.99348
Weiman Co Inc.	.983958	126	.92	9.76	2.29	.99619
Wiener Enterprises Inc.	.999388	120	.93	3.78	1.88	1.00828

NOTES

[1] The coefficient is the gross expected rate of return (one plus the expected rate of return on a monthly basis).

[2] S is the Langrange Multiplier statistic (pp. 17-19)

[3] The mean is the historic average gross rate of return on a monthly basis.

CHAPTER 3

1. Stochastic Coefficient Models

Stochastic coefficient models have been used extensively in the literature of economics and finance, following the successful employment of those methods in the physical sciences. Instability in economic relationships has long been recognized by econometricians.[1] One source of instability cited was model misspecification. A misspecified relationship can lead to parameter variation over time. Time variation in the parameters, may also be due to the fact that many economic relationships change over time. Lucas (1981) argues that econometric models assuming constant structural parameters are inappropriate tools for long-term policy, because the parameters will be revised by economic agents in order to incorporate the new policy. In other words, the parameters will change over time, as policy changes. Finally, economic theory suggests, that relationships may change over time due to changes in technology, tastes, etc. The assumption of constant coefficients in many instances, is dictated by convenience rather than adherence to economic theory.

Once the assumption that the coefficients of a regression model are fixed is questioned, the next step is to model the pattern of change. Most of the time the pattern of change is not provided by theory. In general two models have been used in the empirical

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1. See Dusenberry and Klein (1965)

literature:

a) the random coefficient model[2], which states that the parameter k_t at any period t will be equal to its long-term stationary mean k plus a white noise, or $k_t = k + v_t$.

b) the first order autoregressive process[3], where it is postulated that the coefficient follows an AR1 process, or $k_t - k = G(k_{t-1} - k) + v_t$. Notice that in the case that G is equal to zero, the process reduces to the random coefficient model, which is important in the study of cross section data. If G=1 then the process reduces to the random walk model with zero drift. The random walk process is first-order homogeneous nonstationary, meaning that the process itself is nonstationary but first-differences follow a stationary process. In other words, expected changes in k_t have zero mean and constant variance. As noted by Rosenberg (1972), "the leading case is that where, the increments to the parameter vector, rather than the parameter vector itself, follow a stationary process."

In this study, the parameter (expected returns) is assumed to follow the random walk process. Two considerations make this process appealing. First, it is reasonable to assume that expected returns change over time but changes are smooth, so that the value for next period will be strongly correlated with the value of the current period. Second, and most important, revisions in the estimate of expected returns, will be possible only in the light of new surprise information. For example, let us define,

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2. See Rosenberg (1972b), and Burnette, T. and Guthrie, D. (1972)

3. See Newbold, P. and Bos, T. (1985)

$$\hat{k}_t = \hat{E}\,[\,k_t \mid Y_t,\ Y_{t-1},\ \ldots,\ Y_0] \qquad (1)$$

where, k_t denotes gross expected returns, $\hat{E}$ is the linear projections operator, and Y_t is the payoff at period t (price plus dividend). If, no new information is revealed, then the j period ahead forecast would be[4],

$$\hat{k}_{t+j} = \hat{E}\,[\,k_{t+j} \mid Y_t,\ Y_{t-1}, \ldots\ Y_0] = \hat{k}_t \qquad (2)$$

(1) and (2) imply that k_t follows a random walk. If, on the contrary, the coefficient followed an AR1 process, then forecasts would be revised, simply because of changes in the forecasting period, even though the information set does not include any new surprise information.[5]

In the case of the random coefficient model, the current estimate as well as the j periods ahead forecast will be equal to the mean of the process which is k.[6] Therefore, new information does not lead to a revision of the estimate, but it is treated instead as random noise.

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4. If $k_t = k_{t-1} + v_t$, then $\hat{E}[k_{t+j} | Y_t, ..Y_0] = P[k_t + \sum_{j=1}^{n} v_{t+j} | Y_t, .Y_0] = \hat{k}_t$, assuming that v_t is orthogonal to Y_t.
5. If, $k_t - k = G(k_{t-1} - k) + v_t$, then $\hat{E}[k_{t+j} | Y_t, .Y_0] = k + G^j (k_t - k)$
6. If $k_t = k + v_t$ then, $\hat{E}[k + v_t | Y_t, ..Y_0] = k$ and $\hat{E}[k_{t+j} | Y_t, ..Y_0] = k$

2. The Kalman Filter Model

Let Y_t, Y_{t-1},......Y_1 denote the observed values of the variable of our interest (the payoffs on an individual security or the market) at times t, t-1, ... 1. We assume that Y_t depends on an unobservable quantity k_t (expected gross return), known as the state of nature. The relationship between Y_t and k_t is linear and is specified by the observation equation,

$$Y_t = k_t P_{t-1} + u_t \qquad (3)$$

which is basically equation (5) that we got earlier by assuming market efficiency and rational expectations. The difference between the Kalman filter and the conventional linear model is that in the former, the state of nature, (the regression coefficient of the latter) is not assumed constant but may vary over time. This dynamic feature is incorporated via the system equation,

$$k_t = k_{t-1} + v_t \qquad (4)$$

The error terms u_t and v_t are zero mean, constant variance and uncorrelated. Equation (4) states that expected changes in the coefficient as of time t are zero i.e. $E(k_t - k_{t-1} | I_t) = 0$.

The problem now is to get estimates of the expected returns k_t over time as new information becomes available. For that purpose I will use the linear projections technique. Let us define,

$$\hat{k}_t = \hat{E}[k_t | Y_t, \ldots Y_0],$$

where $\hat{E}$ is the linear projections operator, $Y_t \ldots Y_0$ is the history of payoffs (prices plus dividends) up to t and $\hat{k}_t$ the estimate for expected returns given information up to t. Using recursive projections we get,[7]

$$\hat{k}_t = \hat{E}[k_t | Y_{t-1}, \ldots Y_0] + \hat{E}[(k_t - \hat{E}[k_t | Y_{t-1}, \ldots Y_0])$$

$$| (Y_t - \hat{E}[Yt | Y_{t-1}, \ldots Y_0])] \tag{5}$$

Substituting (3) and (4) in the equation above and assuming that,

$\hat{E}[v_t | Y_{t-1} \ldots Y_0] = 0, \quad \hat{E}[u_t | v_t] = 0,$ we get,

$$\hat{k}_t = \hat{E}[k_{t-1} + v_t | Y_{t-1}, \ldots Y_0] + \hat{E}[(k_{t-1} + v_t - \hat{E}[k_{t-1} + v_t | Y_{t-1}, \ldots Y_0])$$

$$| (P_{t-1}(k_{t-1} + v_t) + u_t - \hat{E}[P_{t-1}(k_{t-1} + v_t) + u_t | Y_{t-1}, \ldots Y_0]))]$$

$$= \hat{k}_{t-1} + \hat{E}[\{(k_{t-1} - \hat{k}_{t-1}) + v_t\} | \{P_{t-1}(k_{t-1} - \hat{k}_{t-1}) + P_{t-1} v_t + u_t\}]$$

Simplifying the expression above we get that,

$$\hat{k}_t = \hat{k}_{t-1} + A_t [P_{t-1}(k_{t-1} - \hat{k}_{t-1}) + P_{t-1} v_t + u_t] \tag{6}$$

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7. If the regression equation is $Y = aX_1 + bX_2 + u_t$, then according to the linear projections theory, $Y = \hat{E}[Y | X_1, X_2] + u_t$ and, $\hat{E}[Y | X_1, X_2] = \hat{E}[Y | X_1] + \hat{E}[(Y - \hat{E}[Y | X_1]) | (X_2 - \hat{E}[X_2 / X_1])]$, See Sargent (1987)

Where $A_t = \dfrac{E\{(k_{t-1}-\hat{k}_{t-1})+v_t\}\ \{P_{t-1}(k_{t-1}-\hat{k}_{t-1})+P_{t-1}v_t+u_t\}}{E\{P_{t-1}(k_{t-1}-\hat{k}_{t-1}) + P_{t-1}v_t+u_t\}^2}$

We define $\Sigma_{t-1} = E(k_{t-1}-\hat{k}_{t-1})^2$ and the expression for A_t simplifies to the following:

$$A_t = \frac{P_{t-1}(\Sigma_{t-1} + \sigma_v^2)}{P_{t-1}^2(\Sigma_{t-1} + \sigma_v^2) + \sigma_u^2} \qquad (7)$$

where σ_u^2 and σ_v^2 are the variance of the observation error and the system error respectively.

The next step now is to derive an expression for the variance of k_t. Using (6) and (4) we get that,

$$k_t-\hat{k}_t = (1-A_tP_{t-1})[(k_{t-1}-\hat{k}_{t-1})+v_t]-A_tu_t$$

Squaring both sides and taking expectations we get that,

$$Var(k_t) = \Sigma_t = (1-A_tP_{t-1})^2(\Sigma_{t-1}+\sigma_v^2)+A_t^2\sigma_u^2 \qquad (8)$$

Expressions (7) and (8) can be simplified to the following equivalent expressions:

$$\hat{k}_t=\hat{k}_{t-1}+A_t\hat{u}_t \qquad (\text{where } \hat{u}_t=Y_t-\hat{k}_{t-1}P_{t-1}) \qquad (7')$$

$$\Sigma_t=S_t-A_t(P_{t-1}S_t) \qquad (\text{where } S_t=\Sigma_{t-1}+\sigma_v^2) \qquad (8')$$

Summarizing the procedure; at time t-1 before observing Y_t our best choice about next period's expected returns k_t is governed by the system equation $k_t = k_{t-1} + v_t$ so that $\hat{E}[k_t | Y_{t-1} \ldots Y_0] = k_{t-1}$ and the variance as of time t-1 is given by $S_t = \Sigma_{t-1} + \sigma_v^2$.

We can also state that before observing Y_t, our prior for k_t has mean $\hat{k}_{t-1}$ and variance S_t.

On observing Y_t and the one step ahead forecast error $\hat{u}_t = Y_t - \hat{k}_{t-1} P_{t-1}$ we form our posterior for k_t with mean and variance given by (7′) and (8′) respectively. It turns out that A_t is the coefficient of the least squares regression of k_t on u_t (conditional on Y_{t-1})[8]. So the Kalman filter can be viewed as an updating procedure that consists of forming a prior about the state of nature (in our case the expected returns) and then adding a correction to this prior, the correction being determined by how well the prior has performed in predicting the next observation.

The mean of the distribution of Y_t, given information up to time t-1 is obtained by taking projections on both sides of equation (3), so that

$$\hat{E}[Y_t | Y_{t-1}, \ldots Y_0] = \hat{E}[k_t P_{t-1} + u_t | Y_{t-1}, \ldots Y_0] = \hat{k}_{t-1} P_{t-1} \qquad (9)$$

and the conditional variance of Y_t is given by,

$$\mathrm{Var}[Y_t | Y_{t-1}, \ldots Y_0] = P_{t-1}^2 (\Sigma_{t-1} + \sigma_v^2) + \sigma_u^2 \qquad (10)$$

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8. See Meinhold R.J. and Singpurwalla (1983)

Equations (6), (7), (8), (9) and (10) provide the basis for the Kalman filter algorithm. The computations are started off by picking initial values for the coefficient and its variance given no information, i.e. k_0 and Σ_0. The initial values in many applications are chosen rather arbitrarily. However, their influence diminishes as the computations proceed. By setting t=1,2,3.. we can iteratively calculate k_1, Σ_2, k_2, Σ_2 etc. The quantities required for the calculation at t are the results found at t-1.

3. Estimation and Results of the Kalman Filter Model

The Kalman filter model involves two constants, namely the variance of the observation error, σ_u^2 and the variance of the system error, σ_v^2. Estimates of those constants have to be used as inputs for the Kalman Filter algorithm along with the initial values k_0 and Σ_0. An especially difficult task, is the efficient estimation of σ_v^2. Doan T., Litterman R., and Sims C., (1983) assume that the random change in the parameter is drawn from a distribution with zero mean and variance proportional to Σ_0. The factor of proportionality determines the amount of time variation allowed in the parameter. In this study I use a technique similar in spirit to the jackknife approach[9]. The essence of the jackknife approach is to partition out the impact of a particular subset of the data on an estimate derived from the total samle. Suppose, for example, that a random samle of size N is

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9. See Hair,Jr J.,F., Anderson R.,E., and Tatham, R.,L.,(1987)

considered. Let the sample be partioned into L subsets of size M (i.e ML=N), so that a new random sample can be created by deleting one of the subsets from the original sample. Let b be defined as an estimator using the entire sample, and b_i an estimator defined over those values of the sample which remain after the i_{th} subset has been deleted. Then, the jackknife statistic is simply the average of the b_i values. These values can be treated as idependent identically distributed random variables, and they can be used to obtain confindence intervals.

In our particular setting the approach is the following: Equation (3) is estimated over the whole sample and an estimate of the parameter is obtained. Then, by dropping one observation at a time (with replacement) a series of changes in the parameter is obtained. In this case the number of subsets is equal to the number of observations N. The variance of the series is then used as an estimate for the unknown σ_v^2.

I believe that this approach is better than deciding in advance on the amount of time variation allowed in the parameter. Using the technique described above, one allows the data sample to decide in a way, as to what is the appropriate amount of time variation.

The second problem in initializing the Kalman Filter, is the estimate of the variance of the observation error. Ordinary Least Squares estimates will not be appropriate because in the presence of time varying papameter the OLS residuals wil be heterescedastic and, or autocorrelated. Therefore, the Generalized Least Squares estimate of σ_u^2, is appropriate.

The choice of the initial values for k_0 and Σ_0 is not very important. Their influence typically diminishes after five or six iterations. Given the variance estimates for the observation error, the system error and the initial values tha algorithm starts at the beginning of the data and by adding one more data point it revises the estimate for k_t and Σ_t according to the equations (6), (7) and (8).

Since we know that the distribution of Y_t given $Y_{t-1}...Y_0$ is normal with mean and variance given by (9) and (10) respectively, it is possible to write the likelihood function as,

$$L = (2\pi)^{-N/2} \prod_{t=1}^{N} Var(Y_t)^{-1/2}$$

$$\exp\left[-\sum_{t=1}^{N} (Y_t - k_{t-1} P_{t-1})^2 / 2Var(Y_t) \right] \tag{11}$$

Taking logarithms we obtain,

$$\log L = -(N/2)\log(2\pi) - (1/2)\left[\sum_{t=1}^{N} \log[Var(Y_t)] \right.$$

$$\left. + \sum_{t=1}^{N} (Y_t - k_{t-1}P_{t-1})^2 / Var(Y_t) \right] \tag{12}$$

It should be noted that the log likelihood function given by equation (12) does not form the basis for the algorithm, at least in this study. Nontheless, values for the term in the brackets of equation (12) are provided in retrospect (i.e. after the algorithm has computed k_t and Σ_t over the sample period). Table 3 (next page) provides the results of the last iteration for k_t, the average value of the estimated series k_t (i.e $\Sigma\ k_t/N$), and the estimates of σ_v and σ_u, which were obtained earlier (i.e. they are not the result of the Kalman algorithm). In addition there are two values reported in Table 3 labeled L_0 and L_1. The first one is the value for the term in brackets of equation (12) and it is calculated under the assumption that there is a single unobserved parameter k and that more information allows us to get a better estimate, or to put it differently there is no time variation. The second is the value for the same quantity calculated under the assumption that the pameter process is a random walk, or the variance of the system error is not zero. As can be observed from Table 3, L_1 is in all cases smaller than L_0 meaning that the log likelihood has a greater value under the assumption that the parameter is time dependent.

TABLE 3

RESULTS OF THE KALMAN FILTER MODEL

(Observation equation: $Y_t = k_t P_{t-1}$, System equation: $k_t = k_{t-1} + v_t$)

	k_t	$\Sigma k_t/N$	σ_u	σ_v	L_0	L_1
Standard and Poor 500	1.01004	1.0077	.4048	.0020	31.67	-269.4
American Greeting Corp.	1.00318	1.0069	.3134	.0050	17.86	-424.3
American Maize-Product	1.00374	1.0058	.3641	.0063	22.34	-321.8
American Ship Building	.99399	1.0058	.3175	.0065	12.80	-367.8
Avx Corp.	1.00238	1.0085	.5221	.0110	43.75	-53.99
British Petroleum	1.01274	1.0108	.4372	.0056	31.70	-174.8
Casablanca Inds Inc	1.01569	1.0025	.2568	.0079	14.85	-489.5
Clabir Corp.	1.00625	1.0111	.3009	.0110	12.76	-215.7
Coleman Co Inc.	1.00905	1.0065	.4809	.0060	27.95	-124.4
Combustion Engineering	1.00094	1.0081	.4193	.0051	28.59	-234.6
Commercial Metals Co.	1.01792	1.0132	.2511	.0063	14.09	-480.0
Conrac Corp.	1.00700	1.0045	.3942	.0072	20.59	-230.7
Crystal Brands	1.00709	1.0049	.3890	.0068	18.64	-253.2
Dallas Corp.	1.00518	1.0055	.3632	.0060	13.41	-217.0
Dillard Dep. Stores	1.01068	1.0176	.3709	.0074	32.38	-230.3
Domtar Inc.	1.00979	1.0089	.2166	.0055	8.737	-555.0
Eagle-Picher Inds	1.00529	1.0078	.3930	.0054	35.98	-277.3
Eeco Inc.	1.00564	1.0066	.2971	.0077	12.94	-384.6
Genuine Parts Co.	1.00959	1.0116	.2592	.0042	11.10	-543.9
Giant Foods Inc.	1.02162	1.0075	.1636	.0050	7.920	-715.5
Giant Group LTD	1.00248	1.0034	.3828	.0057	15.19	-334.0
Great Atl. & Pac. Tea	1.01663	.9988	.3813	.0045	18.22	-299.6

TABLE 3 Continued

	k_t	$\Sigma k_t/N$	σ_u	σ_v	L_0	L_1
Harris Corp.	1.00499	1.0061	.4283	.0053	38.10	-221.2
Hartmax Corp.	1.00591	1.0080	.2964	.0048	17.41	-459.1
Hipotronics Inc.	1.01438	1.0092	.3121	.0086	15.05	-282.0
ICN Pharmaceuticals	.99401	.9936	.5149	.0104	26.10	-75.08
Imperial Oil LTD.	1.00574	1.0063	.4599	.0052	33.15	-148.8
Intl. Rectifier Corp.	.99159	1.0019	.3226	.0079	18.25	-404.0
Ipco Corp.	.99815	.9976	.4201	.0075	15.01	-197.3
Kellog Co.	1.01765	1.0095	.2558	.0037	15.17	-555.7
Kerr-McGee Corp.	1.00569	1.0060	.4334	.0047	28.95	-212.9
Kinark Corp.	1.00256	.9990	.2314	.0078	5.184	-519.8
Lee Pharmaceuticals	.99377	.9966	.3379	.0133	12.86	-218.0
M/A Com. Inc.	1.00289	1.0057	.3644	.0071	33.18	-327.7
Manville Corp.	.99624	.9999	.4512	.0053	35.01	-187.8
Martin Marietta Corp.	1.00617	1.0107	.3404	.0050	29.75	-369.2
Maytag Corp.	1.00590	1.0096	.2461	.0045	13.95	-578.4
Monarch Machine Tools	1.00623	1.0084	.3659	.0066	26.68	-286.4
Morton Thiokol Inc.	1.00528	1.0084	.3204	.0048	19.72	-363.0
Noxell Corp.	1.00544	1.0115	.2359	.0056	11.05	-471.7
Oakite Products	1.01649	1.0073	.3475	.0046	14.42	-299.5
Owens-Corning Inc.	1.00233	1.0036	.6455	.0072	103.3	43.69
Penzoil Co.	1.00968	1.0076	.6032	.0066	61.76	00.13
Penril Corp.	1.00162	1.0041	.4283	.0095	11.79	-238.8
Phillips-Van Heusen	1.00944	1.0071	.2370	.0060	12.35	-602.8
Procter & Gamble	1.00940	1.0074	.3563	.0030	22.41	-339.7

TABLE 3 Continued

	k_t	$\Sigma k_t/N$	σ_u	σ_v	L_0	L_1
Quebecor Inc.	1.01826	1.0165	.2183	.0074	9.410	-380.3
Reynolds Metals Co.	1.02027	1.0064	.4572	.0059	35.77	-180.1
Rymer Co.	1.00084	1.0029	.4128	.0063	24.11	-244.9
Seagram Co. Ltd.	1.00901	1.0112	.3537	.0046	23.10	-344.6
Shell Transport & Trade	1.01270	1.0138	.3293	.0059	15.40	-226.3
Standex Intl. Corp.	1.01214	1.0080	.2477	.0054	10.52	-488.1
Stonebridge Resources	.99862	1.0001	.3470	.0090	13.51	-223.0
Sun City Inds.	1.00381	1.0059	.2335	.0088	8.440	-383.7
Tootsie Roll Inds. Inc.	1.01496	1.0126	.2372	.0055	13.88	-603.2
Tyler Corp.	1.00473	1.0080	.3060	.0071	19.07	-356.6
USX Corp.	1.00511	1.0029	.4184	.0043	23.91	-236.2
Varity Corp.	.99220	.9999	.3566	.0054	7.381	-340.7
Wean United Inc.	.98989	.9938	.3265	.0077	11.95	-322.1
Weiman Co Inc.	.99832	.9953	.3028	.0080	9.310	-378.2
Wiener Enterprises Inc.	1.00443	1.0074	.3337	.0095	12.48	-253.3

CHAPTER 4

1. The CAPM and Early Tests

The Capital Asset pricing model is founded on the early works of Markowitz (1959) and Tobin (1958). Markowitz developed a normative analysis of portfolio management using the mean-variance paradigm. The extention of the normative mean-variance analysis into a positive equilibrium framework in the capital markets , was done by Sharpe (1964) and Lintner (1965a). Under the usual assumptions of homogeneity of beliefs, existence of a risk-free asset which could be bought and sold by all economic agents, and either quadratic utility functions or normal distribution of returns, it was found that the security risk premium would be proportional to the market risk premium. The factor of proportionality was the beta of the security. Assuming that the model holds intertemporally[1] we have that,

$$E(R_{it})-R_{ft} = b_i \, [E(R_{mt})-R_{ft}] \qquad (1)$$

where $E(R_{it})$ is the expected return on security i at time t, $E(R_{mt})$ the expected return on the market portfolio, R_f the risk-free rate and b_i the the systematic risk of security i over the market risk.

In order to get an ex-post representation of (1) two additional assumptions are needed, namely: $R_{it}=E(R_{it})+n_t$ and $R_{mt}=E(R_{mt})+m_t$, where n_t and u_t are white noises. Substituting those two relations into (1)

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1. See Merton R.C. (1973)

we get,

$$R_{it}=(1-b_i)R_{ft}+b_iR_{mt}+u_t \quad (u_t=n_t-bm_t) \qquad (2)$$

Equation (2) is the market model which is used to get ex-post beta estimates. Notice that the composite error term u_t is not orthogonal to R_{mt}, so that OLS estimates of beta will be biased.

The traditional empirical testing of the CAPM involves two steps. In the first step (first-pass regression), OLS is applied to the market model for each of N securities, and betas are obtained. In the second step (second-pass regression), a cross-section regression is applied for the N securities, where the dependent variable is the arithmetic mean of the i^{th} security in the sample, and the independent variable, the beta of the i^{th} security obtained in the first-pass regression. The estimated slope in the second regression would be an estimate for the market premium and the intercept an estimate of the risk-free rate. Lintner (1965b), tested the CAMP for the years 1954-1963, employing annual rates of return for 301 stocks. He found that, as predicted by the CAPM expected returns are positively related to the betas, but the estimate of the slope was a downward biased estimate of the market premium. In other words the empirical line was flatter than the CAMP line meaning that low beta securities earn more than the CAPM would predict and high beta securities earn less. A surpising result was the fact that the residual variance from the first-pass regression, when used as an independent variable in the second-pass , had a positive and significant coefficient, indicating that expected returns were a function of both, systematic risk (beta) as well as unsystematic (residual variance). Miller and Scholes (1972) run the same test, for the same period, but with a larger sample. The results were basically the same. The authors suggested that the possible source of bias is the fact that ex-post betas are used in place of the true ex-ante betas in the second-pass regresssion.

Table 4 reproduces the results of Miller and Scholes. The residual variance obtained in the first-pass regression has a positive and significant coefficient. The estimate of the market risk-premium (.071) is less than the average observed premium. The coefficient of determination R^2 is .19 with b_i as the only explanatory variable, .28 with the residual variance as the only explanatory variable and .33 with both variables. If a choice were to be made between systematic and unsystematic risk, as the main determinant of cross-section variation of expected returns, the latter would be chosen on the basis of the R^2.

TABLE 4

Summary Results of the Miller Scholes Study

(Second-pass regression)

$\bar{R}_i$ =	C_0	+	$C1\ b_i$	+	$C_3\ S_e^2$	(R^2)
	0.122		0.071			0.19
	(.007)*		(.006)			
	0.163				0.393	0.28
	(.004)				(.025)	
	0.127		0.042		0.310	0.33
	(.006)		(.006)		(.026)	

* Numbers is parentheses are standard errors.

Source: M. Miller and M. Scholes, "Rates of Return in Relation to Risk: a Reexamination of some Recenr Studies" in M. Jensen, Ed., Studies in the Theory of Capital Markets, Preager, 1972.

Black,Jensen and Scholes (1972), tested the CAPM by grouping securities into portfolios in an attempt to minimize the error of measuring individual security betas. Then they applied the first and second-pass regressions for the portfolios. They found an almost perfect fit between expected excees returns and betas (R^2 was .98). It should be remembered, though, that the CAPM is an equilibrium model for portfolios as well as individuals securities. Therefore, we cannot empirically validate the model on the basis of portfolio results.

2. Testing the CAPM using ex-ante Betas

In this section, I attempt to test the CAPM using as inputs , the results I derived earlier with the Kalman Filter method. Specifically, I run the first-pass regression for the 60 securities, using the estimated expected returns rather than the actual. The estimated betas should be closer to the true ex-ante betas, since estimated expected returns are used in the regression. Table 7, on page 51 presents the results of the first-pass regression using both estimated expected returns, and historic returns for the purpose of direct comparison.

The results of the regression which uses expected returns ($k_{it}=a+bk_{mt}+u_t$) are corrected for first-degree autocorrelation. The reason that the residuals are autocorrelated is that expected returns were estimated under the assumption that they follow a random walk. Thus, k_t is strongly correlated with k_{t-1} and failure to include k_{t-1} in the explanatory variables of the regression forces the error term to follow an AR1 process. The results indicate that the autocorrelation coefficient is above 0.9 for all securities and strongly significant.

An alternative procedure to correct for autocorrelation would be to take first differences on k_{it} and k_{mt}. The results, not reported, show that the estimated betas and their corresponding standard errors are almost identical, whether one uses first-differencing or AR1 correction.

The estimated ex-ante beta coefficients are in general different than the ex-post with few exceptions. Table 5 provides information on the sample values of first-pass regressions under both methods, which provide indirect evidence as to the randomness of the sample of the 60 securities used in this study. The cross section mean of the sample should be close to the mean of the index and the average beta should be close to one. For the period under consideration the arithmetic mean of the index is .0082 and the average Kalman filter estimate for the index is .0077. Table 6 (next page) reports the results for the traditional cross-section regression, using ex-post betas along with historic cross-section averages, and for the cross-section using ex-ante betas along with Kalman Filter cross-section averages.

TABLE 5

Summary Statistics of the Cross-Section Analysis

	mean(k_i)	mean(b_i'ex-ante)	mean(R_i)	mean(b_i'ex-post)
Cross section avg. value	.0062[2]	1.056	.0057	1.15
Cross section Stand. deviation	.00499	.256	.0075	.256

2. It should be noted that returns are monthly. The effective annual return would be: $R_a = (1+R_m)^{12} - 1$.

The unexpected result in the traditional cross-section is that the estimated risk premium is negative. When I include the squared residuals, the coefficient for the risk-premium becomes insignificant and the coefficient for the residuals becomes significant with a negative sign. The results from the cross-section which uses ex-ante betas validate at least the basic aspect of the CAPM since the coefficient for the market risk premium is positive and significant. The coefficient for the squared residuals is insignificant meaning that non-systematic risk does not explain any variation in expected returns, which is also very important for the CAPM. In both regressions the fit as measured by R^2 is very poor.

TABLE 6

Cross-Section Analysis

(Traditional second-pass regression)

$\bar{R}_i$	=	C_0	+	$C_1\ b_i$	+	$C_2\ S_e^2$	(R^2)
		0.0142		-.007357			.049
		(.0043)*		(.00364)			
		0.0182		-.000075		-0.120	.15
		(.00429)		(.00424)		(.0413)	

(Cross-section using ex-ante betas)

$\bar{k}_i$	=	C_0	+	$C_1\ b_i$	+	$C_2\ S_e^2$	(R^2)
		0.00016		0.0057			.067
		(.0026)		(.0024)			
		0.00046		0.0060		-.321	.060
		(.00271)		(.0025)		(.516)	

* Numbers in parentheses are standard errors

In search of an answer for the unexpected results of the traditional cross-section, I ran both regressions ommiting nine securities for which the expected returns were negative and the corresponding estimated betas were above one. The results reported in table 6A indicate that the previous findings remain basically unchanged, even when the outliers are removed. The fit of the ex-ante regression improves considerably and the estimated monthly risk premium increases from .0057 to .0062.

TABLE 6A
Cross-Section Analysis Omitting Outliers

(Traditional second-pass regression)

$\bar{R}_i$	=	C_0	+	$C_1\ b_i$	+	$C_2\ S_e^2$	(R^2)
		0.0122		-.003052			.019
		(.0024)		(.00215)			
		0.0129		.000564		-0.058	.11
		(.00244)		(.00249)		(.0231)	

(Cross-section using ex-ante betas)

$\bar{k}_i$	=	C_0	+	$C_1\ b_i$	+	$C_2\ S_e^2$	(R^2)
		0.00098		0.0062			.168
		(.0020)		(.0018)			
		0.00097		0.0062		.0044	.151
		(.00211)		(.0019)		(.366)	

By comparison, the results obtained using ex-ante betas are closer to the theoretical results of the CAPM than the traditional test would indicate. This is in support of Miller and Scholes' (1972) finding that the bias in the traditional testing is probably due to the fact that ex-post betas are used. That notion is reinforced if one regresses ex-ante betas on ex-post betas. The estimated intercept and the slope are 0.389 and 0.576 respectively. Both are significant, and the R^2 is 0.337. These results indicate that securities with high historical betas (higher than .917) tend to have lower ex-ante betas and vice versa, which may explain the fact that the estimated empirical line in earlier studies, was flatter than the true security market line.

TABLE 7

Time-Series Analysis Results
(regressions: $k_{it} = a + b\,k_{mt} + u_t$, $R_{it} = a + bR_{mt} + u_t$)

	BETA	DW	RBAR	RHO -	BETA	DW	RBAR
American Greeting Corp.	.979 (.127)	2.23	.97	.98 (.009)	1.381 (.125)	2.15	.27
American Maize-Product	1.313 (.149)	2.12	.87	.92 (.021)	1.24 (.157)	2.14	.16
American Ship Building	.848 (.152)	2.21	.91	.96 (.016)	1.28 (.127)	2.36	.15
Avx Corp.	1.457 (.317)	2.29	.86	.95 (.021)	1.426 (.261)	2.22	.12
British Petroleum	1.012 (.140)	1.94	.91	.92 (.022)	.87 (.146)	2.06	.11
Casablanca Inds Inc	1.291 (.236)	1.65	.91	.92 (.022)	1.129 (.194)	1.93	.10
Clabir Corp.	1.004 (.320)	2.31	.92	.97 (.022)	1.345 (.348)	2.61	.08
Coleman Co Inc.	1.180 (.140)	2.25	.93	.94 (.018)	1.278 (.149)	2.31	.20
Combustion Engineering	1.050 (.129)	2.00	.89	.96 (.015)	1.103 (.131)	2.34	.17
Commercial Metals Co.	1.264 (.170)	1.99	.81	.92 (.023)	1.097 (.157)	2.56	.14
Conrac Corp.	.967 (.160)	2.07	.89	.89 (.027)	1.317 (.177)	2.34	.16
Crystal Brands	.975 (.142)	2.11	.91	.93 (.021)	1.252 (.186)	2.15	.14
Dallas Corp.	.918 (.136)	2.29	.89	.95 (.018)	1.239 (.152)	2.39	.19
Dillard Dep. Stores	1.574 (.221)	1.98	.93	.94 (.021)	1.341 (.148)	1.78	.25
Domtar Inc.	1.231 (.126)	2.36	.90	.92 (.023)	1.053 (.127)	2.37	.19
Eagle-Picher Inds	1.491 (.123)	2.16	.89	.97 (.016)	1.407 (.124)	2.09	.28
Eeco Inc.	1.401 (.178)	2.15	.92	.95 (.018)	1.522 (.226)	2.02	.14
Genuine Parts Co.	.946 (.106)	2.41	.90	.94 (.017)	.936 (.115)	1.82	.16
Giant Foods Inc.	.953 (.145)	2.00	.92	.92 (.023)	.905 (.138)	1.97	.13
Giant Group LTD	.971 (.132)	2.01	.92	.93 (.022)	1.018 (.146)	2.33	.14
Great Atl. & Pac. Tea	.816 (.110)	2.32	.96	.97 (.013)	.778 (.137)	2.21	.08
Harris Corp.	1.294 (.128)	2.30	.94	.97 (.011)	1.482 (.122)	2.35	.31

TABLE 7 Continued

Time-Series Analysis Results

(regressions: $k_{it} = a + b\,k_{mt} + u_t$, $R_{it} = a + bR_{mt} + u_t$)

	BETA	DW	RBAR	RHO -	BETA	DW	RBAR
Hartmax Corp.	1.064	2.38	.96	.96	1.023	2.38	.17
	(.124)			(.015)	(.122)		
Hipotronics Inc.	1.038	1.99	.89	.95	1.272	2.23	.11
	(.237)			(.019)	(.243)		
ICN Pharmaceuticals	1.277	2.35	.92	.91	1.518	2.40	.11
	(.249)			(.026)	(.269)		
Imperial Oil LTD.	1.031	2.29	.91	.95	.928	2.29	.16
	(.131)			(.018)	(.126)		
Intl. Rectifier Corp.	1.335	1.99	.92	.96	1.619	2.14	.16
	(.200)			(.015)	(.200)		
Ipco Corp.	1.060	2.08	.95	.96	1.425	2.19	.17
	(.166)			(.016)	(.189)		
Kellog Co.	.757	2.25	.94	.95	.603	2.34	.10
	(.109)			(.017)	(.097)		
Kerr-McGee Corp.	.721	2.18	.94	.95	.802	2.27	.11
	(.120)			(.015)	(.121)		
Kinark Corp.	1.128	2.07	.92	.95	1.499	2.09	.18
	(.170)			(.018)	(.187)		
Lee Pharmaceuticals	1.276	1.60	.93	.95	1.775	2.14	.17
	(.390)			(.021)	(.279)		
M/A Com. Inc.	1.293	2.17	.94	.97	1.540	2.27	.16
	(.210)			(.011)	(.192)		
Manville Corp.	.454	2.12	.94	.97	.740	2.23	.03
	(.111)			(.011)	(.209)		
Martin Marietta Corp.	1.285	2.34	.92	.93	1.047	2.38	.19
	(.120)			(.020)	(.119)		
Maytag Corp.	1.185	2.33	.93	.91	.873	2.32	.18
	(.108)			(.024)	(.101)		
Monarch Machine Tools	1.049	2.08	.91	.95	1.227	2.30	.15
	(.178)			(.016)	(.165)		
Morton Thiokol Inc.	1.272	2.52	.96	.94	1.154	2.37	.25
	(.109)			(.019)	(.117)		
Noxell Corp.	1.189	2.29	.95	.95	1.116	2.10	.17
	(.154)			(.020)	(.153)		
Oakite Products	.636	2.04	.90	.93	.846	2.18	.15
	(.106)			(.022)	(.117)		
Owens-Corning Inc.	.640	1.89	.71	.85	1.311	2.18	.09
	(.187)			(.029)	(216)		
Penzoil Co.	.795	2.03	.86	.88	.994	2.17	.12
	(.160)			(.029)	(.163)		
Penril Corp.	.724	2.42	.92	.97	.896	2.28	.04
	(.263)			(.017)	(.275)		
Phillips-Van Heusen	1.278	2.00	.94	.93	1.354	2.30	.19
	(.152)			(.020)	(.153)		
Procter & Gamble	.608	2.25	.92	.95	.525	2.27	.12
	(.077)			(.016)	(.076)		

TABLE 7 Continued

Time-Series Analysis Results

(regressions: $k_{it} = a + b\,k_{mt} + u_t$, $R_{it} = a + bR_{mt} + u_t$)

	BETA	DW	RBAR	RHO	BETA	DW	RBAR
Quebecor Inc.	1.607 (.182)	1.84	.91	.91 (.029)	1.032 (.188)	1.89	.13
Reynolds Metals Co.	1.189 (.096)	2.57	.91	.92 (.023)	1.151 (.140)	2.44	.17
Rymer Co.	1.279 (.147)	2.04	.93	.94 (.018)	1.435 (.164)	2.25	.19
Seagram Co. Ltd.	1.026 (.114)	2.54	.91	.90 (.024)	1.011 (.118)	2.67	.11
Shell Transport & Trade	.951 (.151)	2.12	.88	.93 (.025)	.864 (.154)	2.15	.14
Standex Intl. Corp.	.941 (.121)	2.22	.89	.91 (.024)	1.126 (.143)	2.28	.18
Stonebridge Resources	.804 (.201)	1.84	.81	.89 (.030)	1.023 (.234)	2.12	.08
Sun City Inds.	.857 (.224)	2.07	.88	.90 (.031)	1.113 (.213)	2.70	.11
Tootsie Roll Inds. Inc.	1.165 (.159)	2.28	.95	.95 (.017)	1.160 (.137)	2.57	.17
Tyler Corp.	1.306 (.178)	2.29	.86	.96 (.018)	1.359 (.189)	2.27	.16
USX Corp.	.601 (.104)	2.24	.91	.96 (.014)	.779 (.112)	2.28	.12
Varity Corp.	.568 (.123)	1.97	.96	.98 (.008)	1.101 (.165)	2.15	.11
Wean United Inc.	.918 (.176)	2.03	.89	.94 (.020)	1.339 (.209)	2.18	.13
Weiman Co Inc.	1.042 (.180)	2.33	.92	.91 (.024)	1.233 (.206)	2.48	.11
Wiener Enterprises Inc.	1.082 (.232)	1.91	.91	.94 (.021)	1.333 (.262)	2.06	.10

CHAPTER 5

Summary and Conclusions

The findings of this study can be organically decomposed in two parts. In the first part, the constancy of expected returns of securities and the market was tested, using a regression equation which was an operational interpretation of the Efficient Markets Hypothesis. Using the Langrange Multiplier statistic, it was found that for 57 out of the 60 stocks used in this study, the assumption that expected returns are constant, can be rejected at almost any confindence interval. The results for the Standard and Poor 500 index which is used as a proxy for the market, also indicate that market expected returns vary over time.

These results are conditional on the assumption that markets are efficient and as a result, currently observed prices are sufficient statistics of the total information set facing individual traders. Taking that into account we can interpret the results in a slightly different manner. If the EMH holds true, then expected returns cannot be constant; that holds for the market as well, as for individual securities. Therefore, the variance bound tests used by Shiller (1981), and LeRoy and Porter (1981), assuming constant discount factors, could be interpreted as rejecting the constancy of the discount factors rather than the EMH.

Once the assumption of constant expected returns was rejected, I used the Kalman Filter model to get estimates of expected returns over time for the 60 securities and the index. The amount of time variation allowed for the expected returns was determined by a technique similar to the jackknife. The results on the value of the likelihood function L_0 and L_1, in table 3 provide indirect support of the time dependency of expected returns.

The results of the second part are related to tests of the Capital Asset Pricing Model. The traditional testing of the CAPM involved pooling time-series and cross-section data. Specifically, time series on historic returns on individual securities and some index were used to get beta estimates (first-pass regression) , and then security historic averages were regressed against betas (second-pass regression) to get an estimate of the risk premium. Replicating the same analysis for the sample of the 60 securities, I derived an unexpected negative risk premium. The results did not change when I omitted 9 securities for which the betas were more than one and the corresponding mean returns negative. Miller and Scholes (1972) pointed out that the source of bias is the estimated ex-post beta, which is used in place of the true ex-ante beta. They estimated that the estimated pemium was about 64% of the true premium. It seems that for my sample the bias was sufficient to produce a negative estimated premium.

Following the same methodology, and incorporating the results of the first part I ran the first-pass regression using estimated expected returns, so that the bias caused by historic betas could be eliminated. The results showed that for securities with high ex-post betas (above one) the corresponding ex-ante beta was smaller and vice versa, which may explain why high beta securities earn less than the CAPM would predict and vice versa. In the second-pass regression, where ex-ante betas, and average ex-ante returns were used, two basic aspects of the CAPM were validated; namely the estimated risk-premium was positive and unsystematic risk, as measured by the squared residuals of the first-pass, was insignificant. The fit of the regression though, was very poor. When the 9 securities were dropped the fit improved considerable.

With respect to the size of the estimated premium, if we take the historic average premium to be the true premium, then the second-pass regression estimate seems to be an upward biased estimate. For the period 1961 to 1988 the historic monthly premium is .31% and the estimated premium is .57%. If, on the other hand, we take the ex-ante premium to be the true premium, then taking the Kalman Filter estimated expected return on the index at the end of the sample (table 3) and subtracting the average T-Bill rate for the same period we obtain a premium of .51% which is much closer to the estimated risk-premium.

Evaluating the results, I would say that, with respect to expected returns, time variation is significant, and treating them as constant could produce misleading results. I mentioned earlier the case of the EMH tests as a case in point.

With respect to the test of the CAPM, some biases associated with the tradional tests were at least reduced by using ex-ante betas instead of historic betas. Still, I cannot say on the basis of the results that the CAPM holds because the fit of the second-pass regression is very poor. As compared to other studies, the sample utilised in this study is rather small. A much larger sample would allow more general conclusion to be drawn. The problem with utilising a sample of say 1,000 securities using the Kalman Filter is that the costs associated with computation and time may be prohibitively high.

Another problem which may be important is the fact that the ex-ante beta was assumed constant. It is possible that both expected returns and perceived risk, as measured by ex-ante betas, are time dependent. Some authors[1] have suggested that historic betas are not stable over time. That could also be the case with ex-ante betas. An analysis therefore, treating beta as a time dependent coefficient could be more meaningfull. Of course in such an analysis the pooling of time-series and cross-section data would not be appropriate. Rather, the analysis would take the form of obtaining expected returns and betas for a sample of securities over time, and the cross-section regression would be applied over the number of securities, for that particular point in time. In other words no use would be made of information contained in the sample beyond that point.

This analysis would allow us to examine an additional aspect of the CAPM which I think has been neglected, namely the possibility that the CAPM becomes more significant over time in terms of explaining cross-section variations of expected returns. One of the reasons cited for the inability of the CAPM to adequately describe reality, was the fact that investors did not seem to hold diversified portfolios . For example the Federal Reserve Board's 1967 survey of the Financial Characteristics of Consumers found that the average number of

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1. Fabozzi, F. J. and Francis, J. C. (1978) and also Alexander, G. J. and Benson, G. P. (1982)

securities held in a portfolio was about 3.4.[2] Over the years the situation has changed as individual investors holding largely diversified portfolios, become more important players in the financial markets. For those investors, the costs of diversification are minimal and they are more likely to implement modern portfolio managent techniques. On the other hand, individual investors are more diversified, through their pension plans, holdings of shares of mutual funds and so on. So that, over time it is very likely that securities will be valued according to their systematic risk, or to put it differently the market factor will become more important in terms of explaining individual security returns.

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2. See Blume, M. E. and Friend, I. (1975)

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