

INFORMATION TO USERS

This material was produced from a microfilm copy of the original document. While the most advanced technological means to photograph and reproduce this document have been used, the quality is heavily dependent upon the quality of the original submitted.

The following explanation of techniques is provided to help you understand markings or patterns which may appear on this reproduction.

1. The sign or "target" for pages apparently lacking from the document photographed is "Missing Page(s)". If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting thru an image and duplicating adjacent pages to insure you complete continuity.
2. When an image on the film is obliterated with a large round black mark, it is an indication that the photographer suspected that the copy may have moved during exposure and thus cause a blurred image. You will find a good image of the page in the adjacent frame.
3. When a map, drawing or chart, etc., was part of the material being photographed the photographer followed a definite method in "sectioning" the material. It is customary to begin photoing at the upper left hand corner of a large sheet and to continue photoing from left to right in equal sections with a small overlap. If necessary, sectioning is continued again — beginning below the first row and continuing on until complete.
4. The majority of users indicate that the textual content is of greatest value, however, a somewhat higher quality reproduction could be made from "photographs" if essential to the understanding of the dissertation. Silver prints of "photographs" may be ordered at additional charge by writing the Order Department, giving the catalog number, title, author and specific pages you wish reproduced.
5. PLEASE NOTE: Some pages may have indistinct print. Filmed as received.

Xerox University Microfilms

300 North Zeeb Road
Ann Arbor, Michigan 48106

75-20,521

MANDELMAN, Jack Allan, 1946-
MICROSTRIP-OXIDE-DISTRIBUTED P_N DEVICES.

The City University of New York, Ph.D., 1975
Engineering, electrical

Xerox University Microfilms, Ann Arbor, Michigan 48106

MICROSTRIP-OXIDE-DISTRIBUTED P/N DEVICES

by

Jack Allan Mandelman

**A dissertation submitted to the
Graduate Faculty in Engineering
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy, The City
University of New York.**

1975

This manuscript has been read and accepted for the Graduate Faculty in Engineering in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

MAY 12, 1975
date

Joseph J. Nadan
Chairman of Examining Committee

May 12, 1975
date

Jacques E. Benveniste
Executive Officer

Prof. Morris Ettenberg

Prof. Paul Karmel

Prof. Joseph S. Nadan

Chairman, Supervisory Committee

The City University of New York

Abstract

MICROSTRIP-OXIDE-DISTRIBUTED P_{VN} DEVICES

by

Jack Allan Mandelman

Advisor: Professor Joseph S. Nadeau

The dc and microwave properties of microstrip-oxide-distributed p_{VN} devices are studied through the use of distributed one-dimensional dc and small signal rf models and measurements on devices designed and fabricated in the semiconductor device laboratory.

The distributed dc model is obtained from systematic modeling theory with the use of approximations concerning the current density and potential distributions within the structure for the materials used. DC surface voltage profiles on the device are obtained from the numerical solution of the surface voltage differential equation derived from the distributed dc model. Transfer characteristics, derived from the voltage profiles, are used to demonstrate the feasibility of the device for use as an MOS field effect transistor gate-oxide protection network. The dc surface voltage profiles are used to establish the quiescent operating conditions for the rf analysis.

The distributed rf model is developed from a cascade of short five-layer, lossy, parallel-plate waveguide seg-

ments, each having uniform dc surface voltage and propagation characteristics for a dominant TM mode. Wave propagation on the structure is studied as a function of the physical dimensions and electronic parameters of each layer, excitation frequency and dc biasing voltages. The usefulness of the device as a semiconductor analog of a microwave stub tuner and a phase shifter is shown to be limited by the losses of the materials available.

Measurements at dc and microwave frequencies are used to demonstrate both the adequacy and limitations of the models in predicting the electronic behavior of the device.

ACKNOWLEDGEMENTS

I wish to express my thanks to my mentor Professor Joseph S. Nadan for his technical and personal guidance during the course of my graduate studies.

The assistance of Mr. Andrés G. Fortino with the preparation of the photographs in this dissertation is gratefully acknowledged. I extend my thanks to Mr. Abraham Glasser for his help in maintaining the semiconductor device fabrication facilities in top condition.

I am indebted to the National Science Foundation for the Summer Traineeship awarded to me in 1971 and to the City University of New York for the Research Assistantship of the Summer of 1974.

My appreciation is due The General Instrument Corporation (my employer) for allowing me to use their Xerox facilities.

Last but definitely not least I thank Antoinette, my wife, for her patience and understanding in living with this doctoral student.

TABLE OF CONTENTS

Chapter		Page
1	INTRODUCTION	1
2	DC CHARACTERISTICS OF THE DISTRIBUTED P/N STRUCTURE	11
	2.1 A Distributed Model Simulation for the DC Operation of the Distributed P/N Structure	11
	2.2 Investigation of the Behavior of the Solution of the Surface Voltage Differential Equation	21
	2.3 The Numerical Method of Solution of the Surface Voltage Differential Equation	25
	2.4 Computer Generated Voltage Profiles	31
	2.5 The Input Volt-Ampere Characteristics	40
	2.6 DC Measurements	51
3	RF ANALYSIS OF THE M-O-D P/N DEVICE	74
	3.1 Electromagnetic Wave Propagation on a Microstrip-Oxide-Distributed P/N Structure	74
	3.2 Parallel-Plate Waveguide Model of Segment	82
	3.3 Electromagnetic Properties of a Microstrip-Oxide-Distributed P/N Segment	92
	3.4 Measurement of the Microwave Operation of the Microstrip-Oxide-Distributed P/N Device	102
4	SUMMARY AND CONCLUSIONS	132

TABLE OF CONTENTS

Chapter	Page
APPENDIX I - STATIC CURRENT-VOLTAGE CHARACTERISTIC OF PYN DIODES	136
APPENDIX II- EFFECTIVE TRANSVERSE COMPLEX PERMITTIVITY OF THE PV SPACE CHARGE REGION	151
APPENDIX III- DERIVATION OF MULTIPLE-LAYER EIGEN-VALUE EQUATION USING ELECTROMAGNETIC BOUNDARY CONDITIONS AND TRANSVERSE RESONANCE METHODS	157
APPENDIX IV- EVALUATION OF THE CHARACTERISTIC IMPEDANCE	164
APPENDIX V - LISTING OF COMPUTER PROGRAMS	166
APPENDIX VI- COMPUTER PROGRAM FLOW CHARTS	177
APPENDIX VII- DEVICE FABRICATION PROCEDURE	185
REFERENCES	189
AUTOBIOGRAPHICAL STATEMENT	193

LIST OF FIGURES

<u>No.</u>	<u>Caption</u>	<u>Page</u>
1-1	Driving-Point Impedance of a Shorted Transmission Line	2
1-2	Adjustable Shorted Stub Tuner	3
1-3	The Microstrip-Oxide-Distributed PYN Device	5
1-4	Current Flow and the Plane of Strong Reflection in the Microstrip-Oxide-Distributed PYN Device	7
1-5	Five Layer Parallel-Plate Waveguide RF Model	10
2.1-1	Subregion Boundaries and Currents	16
2.1-2	The General Subregion Network	19
2.1-3	The Device Subregion Network	19
2.1-4	The DC Model of the Device	19
2.2-1	Piecewise J_0 -Y Characteristic	22
2.3-1	Convergence to a Solution	29
2.3-2	Convergence to a Solution	30
2.4-1	Computer Generated Voltage Profiles, $Y(z_1) = Y(o)$	32
2.4-2	Computer Generated Voltage Profiles, $Y(z_1) \neq Y(o)$	33
2.4-3	Computer Generated Voltage Profiles, $Y(z_1) \neq Y(o)$	34
2.4-4	Zero Voltage Point vs $Y(o)$ with $Y(z_1)$ as a Parameter, $Y(o) > 0$ $Y(z_1) < 0$	37
2.4-5	Zero Voltage Point vs Y with $Y(z_1)$ as a Parameter, $Y(o) < 0$ $Y(z_1) > 0$	38
2.4-6	The Effect of Voltage Profile Curvature on the Sensitivity of z_v to Changes in $Y(o)$	39

LIST OF FIGURES

<u>No.</u>	<u>Caption</u>	<u>Page</u>
2.5-1	Sketch of Potential Variation in the Vicinity of a Biasing Contact	41
2.5-2	Evaluating a Biasing Contact Current	42
2.5-3	Biasing Contact Current vs Number of Sections in Model	45
2.5-4	Input Volt-Ampere Characteristic	47
2.5-5	The Distributed PYN Diode as a Protection Network	48
2.5-6	Obtaining the Transfer Characteristics	49
2.5-7	Normalized Transfer Characteristics	50
2.6-1	PYN Test Diode	53
2.6-2	Measured J_0 -V Characteristic	54
2.6-3	Obtaining τ_{PH}	58
2.6-4	Voltage Profile Measuring Contacts	60
2.6-5	Measured Voltage Profiles, $Y(x) = Y(o)$	62
2.6-6	Measured Voltage Profiles, $Y(x) \neq Y(o)$	64
2.6-7	Current Crowding	66
2.6-8	Alternate Biasing Contact Geometry with Voltage Profile Measuring Contacts	67
2.6-9	Displayed Input Volt-Ampere Characteristics	69
2.6-10	Measured Input Volt-Ampere Characteristics	70
2.6-11	Effect of Bottom Contact Resistance	71
2.6-12	Measured Transfer Characteristics	73
3.1-1	Voltages and Currents of Interest	75
3.1-2	N Cascaded Segment Model	75
3.1-3	Cascade of Two-Port Sections	78

LIST OF FIGURES

<u>No.</u>	<u>Caption</u>	<u>Page</u>
3.1-4	The i^{th} Two-Port Section	78
3.1-5	Signal Flow Graph of i^{th} Section	78
3.2-1	Parallel-Plate Waveguide Segment	83
3.2-2	Finding the Transmission Line Current	90
3.3-1	Attenuation vs ν Layer Parameters	94
3.3-2	Z_0 vs Epi Layer Thickness	95
3.3-3	Attenuation vs Surface Voltage	97
3.3-4	Normalized Wavelength vs Surface Voltage	98
3.3-5	Attenuation vs Frequency	100
3.3-6	Z_0 vs Surface Voltage	101
3.4-1	Microwave Network Analyzer Test Setup	103
3.4-2	Device with RF Measurement Jig	104
3.4-3	Device and Test Diode	106
3.4-4	Measured $1/C^2$ vs V Curve	108
3.4-5	Experimentally Determined Impurity Profile	109
3.4-6	Effect of Minority Carrier Lifetime on Calculated Transmission Magnitude	111
3.4-7	Effect of Minority Carrier Lifetime on Calculated Transmission Phase	112
3.4-8	Transmission Magnitude, $Y(\omega) = Y(\omega)$, 2, 3 and 4 GHz	116
3.4-9	Transmission Phase, $Y(\omega) = Y(\omega)$, 2, 3 and 4 GHz	117
3.4-10	Transmission Magnitude, $Y(\omega) = Y(\omega)$, 8, 10 and 12 GHz	118
3.4-11	Transmission Phase, $Y(\omega) = Y(\omega)$, 8, 10 and 12 GHz	119

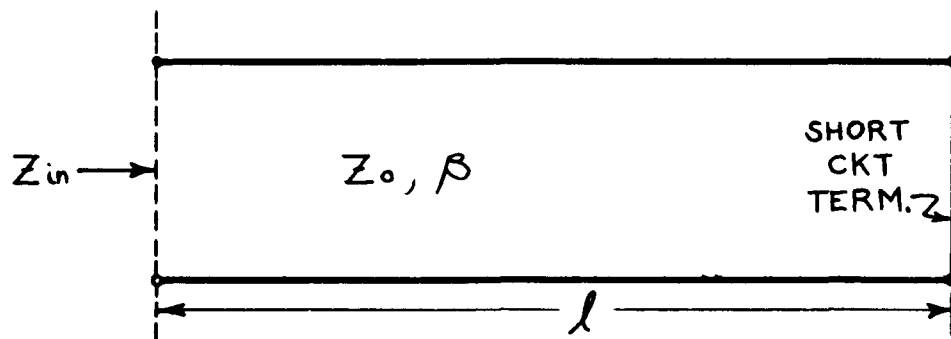
LIST OF FIGURES

<u>No.</u>	<u>Caption</u>	<u>Page</u>
3.4-12	Reflection Magnitude, $Y(z_L) = Y(o)$, 2, 3 and 4 GHz	120
3.4-13	Reflection Phase, $Y(z_L) = Y(o)$, 2, 3 and 4 GHz	121
3.4-14	Reflection Magnitude, $Y(z_L) = Y(o)$, 8 and 12 GHz	122
3.4-15	Reflection Phase, $Y(z_L) = Y(o)$, 8 and 12 GHz	123
3.4-16	Transmission Magnitude, $Y(z_L) = 34$, 2, 3 and 4 GHz	124
3.4-17	Transmission Phase, $Y(z_L) = 34$, 2, 3 and 4 GHz	125
3.4-18	Transmission Magnitude, $Y(z_L) = 34$, 8, 10 and 12 GHz	126
3.4-19	Transmission Phase, $Y(z_L) = 34$, 8, 10 and 12 GHz	127
3.4-20	Reflection Magnitude, $Y(z_L) = 34$, 2, 3 and 4 GHz	128
3.4-21	Reflection Phase, $Y(z_L) = 34$, 2 and 4 GHz	129
3.4-22	Reflection Magnitude, $Y(z_L) = 34$, 8 and 12 GHz	130
3.4-23	Reflection Phase, $Y(z_L) = 34$, 8 and 12 GHz	131
I-1	Structure for the Evaluation of the J_0 -V Characteristic	137
III-1	Transverse Transmission Line	161
VI-1	Main Program	178
VI-2	Main Program (cont. 1)	179
VI-3	Main Program (cont. 2)	180
VI-4	Main Program (cont. 3)	181
VI-5	Subroutine SIGN	182
VI-6	Subroutine ITER	183
VI-7	Subroutine RF	184

TO
ANTOINETTE

CHAPTER 1 - INTRODUCTION

A length of transmission line terminated in a short circuit is a widely utilized and convenient means of obtaining a reactive element at microwave frequencies. The driving point reactance of the transmission line is a function of the electrical distance of the short circuit from the input terminals and the characteristic impedance of the transmission line, as schematically illustrated in Fig. 1-1. By building a mechanical device which allows the position of the short circuit termination to be varied, any arbitrary reactance may be obtained. In practice coaxial type stub tuners are employed, as shown in Fig. 1-2. However, the mechanical versions presently in use are awkward to tune remotely and cannot be incorporated into a microwave monolithic integrated circuit format. The study of the microstrip-oxide-distributed pyn device is therefore motivated by a desire to develop a solid state analog of the microwave stub tuner. The microstrip-oxide-distributed pyn device is fabricated as part of a monolithic integrated circuit, is voltage tuneable at electronic rates, and offers a significant size reduction over the mechanical stub tuner. Electromagnetic wave propagation on such a structure is studied to determine the feasibility of using the device as a stub tuner. As of the beginning date of research on the microstrip-oxide- distributed pyn device, February 1, 1971, no known work had been done on this structure.



$$\frac{Z_{in}}{Z_o} = j \tan(\beta l)$$

FIGURE 1-1 DRIVING-POINT IMPEDANCE OF A SHORTED TRANSMISSION LINE

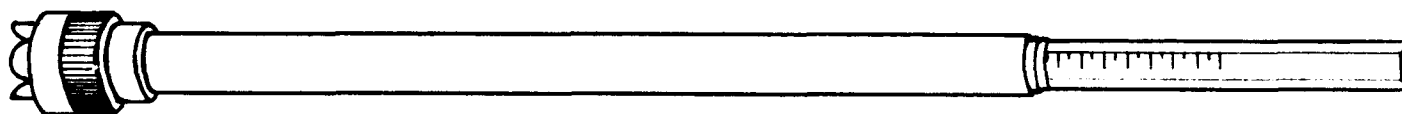


FIGURE 1-2 ADJUSTABLE SHORTED STUB TUNER

The microstrip-oxide-distributed pyn device, shown in Fig. 1-3, consists of a p layer diffused into a ν type epitaxial layer on an n type silicon substrate. Ohmic biasing contacts are provided at both ends of the p layer (in addition to the groundplane at the bottom of the substrate); a microstrip transmission line is rf coupled to the p layer through an oxide, approximately 500 Å to 1000 Å thick.

When a positive voltage is applied to one contact at $z=0$, and a negative voltage to the other, at $z=z_L$, a voltage gradient is established in the p layer causing the voltage on the surface with respect to the groundplane to go through zero at some value of z ($z=z_x$) in the p layer. The distributed diode is, therefore, forward biased in one region, $0 \leq z < z_x$, and reverse biased in its remaining portion, $z_x < z \leq z_L$. The total current from one contact may be thought of as consisting of two components; a longitudinal component through the p layer parallel to the surface and a transverse component across the p ν junction. An electromagnetic wave propagating on the microstrip line over the reverse biased region, in the z direction, encounters a large gradient in the characteristic impedance in the vicinity of the zero voltage point z_x , referred to as the plane of strong reflection (more rigorously defined as the value of z at which $|\nabla Z_0|$ is a maximum), and excites a reflected wave. The microwave source generator is therefore terminated in an impedance that is, ideally, dependent on the electrical distance of the plane of strong reflection from the generator. By varying the bias

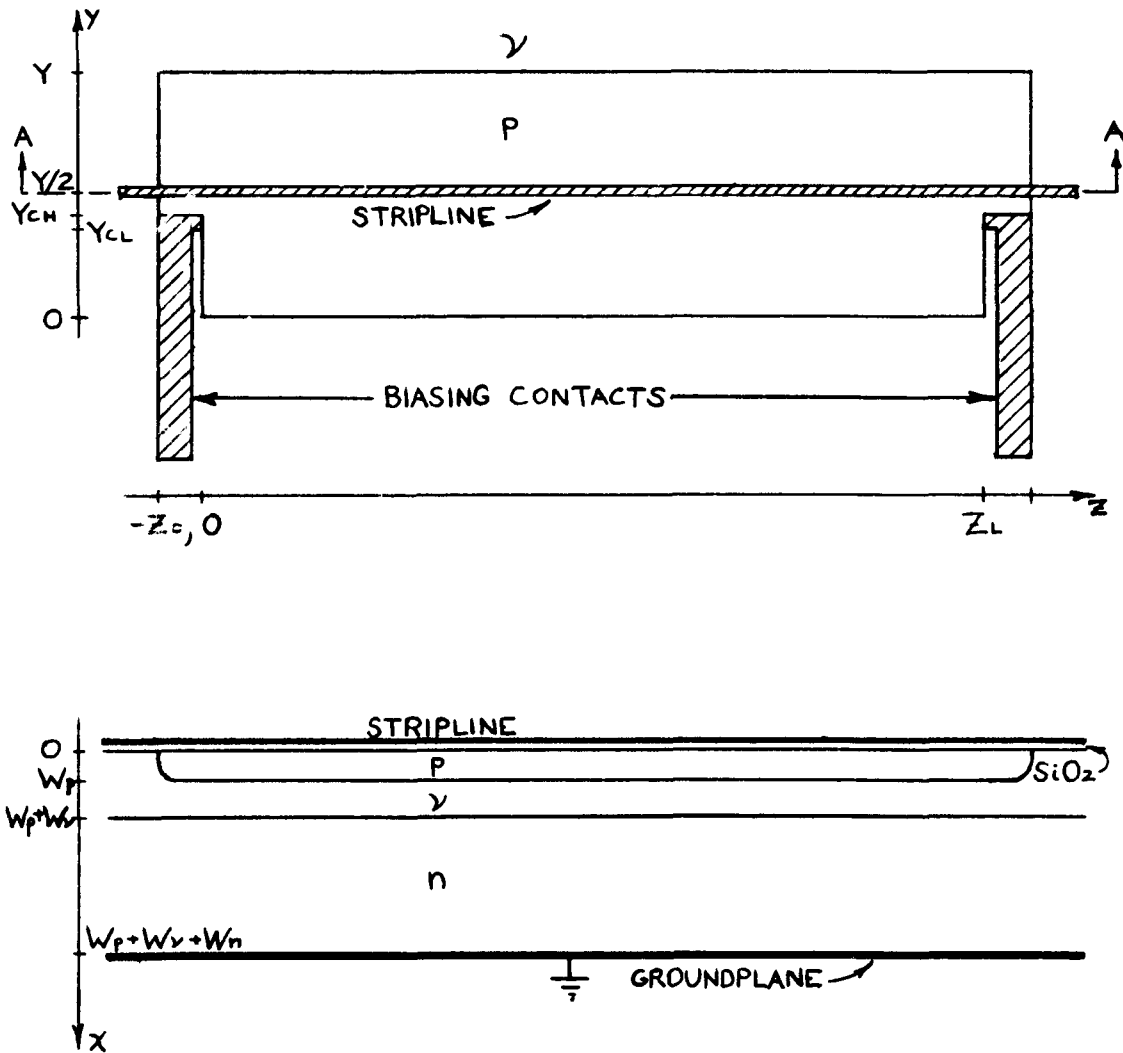
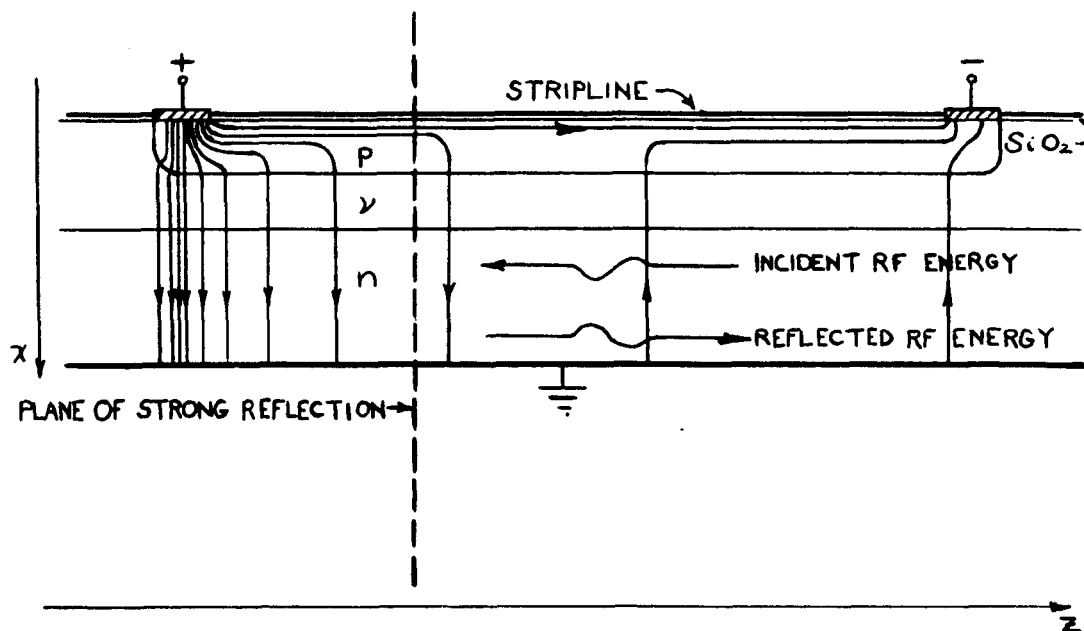


FIGURE 1-3 THE MICROSTRIP-OXIDE-DISTRIBUTED P-V-N DEVICE

to the diode, the plane of strong reflection is shifted in the z direction along the p layer. Increasing the magnitude of one or both of the biasing contact voltages increases the surface voltage gradient in the z direction in the p layer making the transition region (the region around the plane of strong reflection where the diode goes from forward to reverse bias) more abrupt resulting in a larger $|\nabla Z_s|$ at the plane of strong reflection. Ideally, a variable reactance will be seen at the input to the microstrip line coupled to the p - n substrate; in practice the driving point impedance is not pure imaginary due to losses in the device. The currents and the plane of strong reflection are illustrated in Fig. 1-4.

The analysis of the microstrip-oxide-distributed p - n device consists of solving a dc problem, i.e. the surface voltage profile in the z direction, and a small signal rf problem, i.e. electromagnetic wave propagation on the dc biased structure.

The solution of the dc problem uses systematic semiconductor device modeling theory to develop a one dimensional dc model of what is essentially a three dimensional device. The dc solution yields the surface voltage profile on the p layer, when the biasing contact voltages are specified. Alternately, specifying one biasing contact voltage and the location of the zero voltage point ($z=z_1$) yields the voltage profile and the other required biasing contact



NOTE: dc CURRENT REVERSAL MAY
NOT OCCUR AT PLANE OF
STRONG REFLECTION

FIGURE 1-4 CURRENT FLOW AND PLANE OF STRONG
REFLECTION IN THE MICROSTRIP-OXIDE-
DISTRIBUTED P-V-N DEVICE

voltage.

The input dc I-V characteristics with the output biasing contact voltage as a parameter allows one to study the usefulness of the distributed pvn structure as a field effect transistor gate-oxide protection network. The voltage developed from gate to ground, when the gate is connected to one biasing contact, is calculated as a function of the voltage at the other biasing contact from the distributed dc model. Although a diffused resistor structure is currently being used as a gate-oxide protection network¹, no known distributed analysis of the structure has been undertaken. The feasibility of the distributed pvn structure for use as a gate-oxide protection network is investigated in Sections 2.5 and 2.6.

The solution of the rf problem reveals the nature of electromagnetic wave propagation on the device subject to dc biasing conditions. The rf problem is reduced to one of investigating electromagnetic wave propagation on a nonuniform, stratified, lossy, parallel-plate waveguide system. The characteristic impedance and propagation constant of the structure at any point z is found allowing the driving point impedance and transmission coefficient through the device to be calculated.

Several authors have investigated electromagnetic wave propagation on microstrip on a SiO_2 -Si substrate. Hasegawa, et. al.², and Guckel, et. al.³, treat the problem as a two

layered, uniform parallel-plate waveguide problem. Although the solution of the two layered problem provides insight into methods for the solution of the rf problem of the microstrip-oxide-distributed p_vn structure, the problem being studied is greatly complicated by the anisotropy of the p_v space charge layer, the extension of a two layer device to five layers, and the dc surface voltage gradient that makes the transmission line nonuniform. The five layers are; the n substrate, the ν epitaxial layer, the p_v space charge region, the diffused p layer and the thin oxide film, as shown in Fig. 1-5.

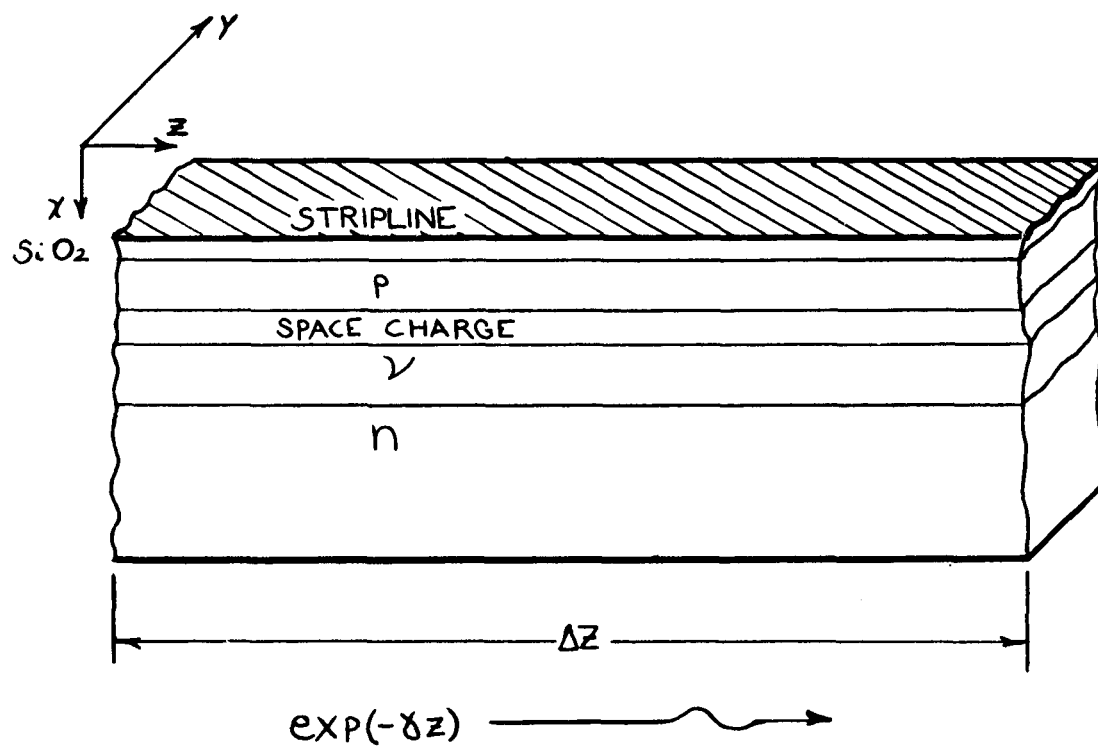


FIGURE 1-5 FIVE LAYER PARALLEL-PLATE WAVEGUIDE
RF MODEL

CHAPTER 2 - DC CHARACTERISTICS OF THE DISTRIBUTED P/N STRUCTURE

The purpose of this chapter is the development of a one dimensional model describing the dc operation of the device, establishing the quiescent operating conditions for the rf analysis of the microstrip-oxide-distributed p/n device in Chapter 3. Numerical results from the model are compared with experiments on fabricated devices.

2.1 A Distributed Model Simulation for the DC Operation of the Distributed P/N Structure

In this section a distributed circuit model is established to describe the dc operation of the distributed p/n structure. Of interest, in developing the model, is the surface voltage, $V(z)$, measured with respect to the ground-plane at any point z in the p layer in the plane $y=Y/2$. The analysis results in a differential equation with $V(z)$ as the dependent variable and z as the independent variable. A distributed circuit model is then deduced from this differential equation. A number of simplifying assumptions are made, their conditions for validity being discussed next. These assumptions are:

1. The static electric field is derived from a scalar potential, $\phi(x,y,z)$, as $\vec{E} = -\vec{\nabla}\phi(x,y,z)$, since all time variations are zero.

2. The total variation of surface potential in the y direction, $\int_0^Y \frac{\partial \phi(0,y,z)}{\partial y} dy$, for a

constant value of z is small compared to $V(z)$. It follows from this assumption that the current density across the pn junction is approximately independent of y . A potential at a point on the surface of the p layer gives rise to a current density across the pn junction supplied by current in the y and z directions in the p layer. In practice current in the y direction gives rise to non-zero $\frac{\partial \phi(x,y,z)}{\partial y}$. The total potential variation in the y direction at the surface for a constant z may be minimized by making the dimension Y as small as possible while still maintaining the desired device characteristics. The range of applied bias voltages for which the approximation

$$\int_0^Y \frac{\partial \phi(0,y,z)}{\partial y} dy \ll V(z) \quad \text{is valid is}$$

established by comparing the analytically calculated surface voltage profile with experiments and is discussed in Sec. 2.6.

3. The resistivity of each layer is uniform.
4. The electric field in the p layer is z directed. Equivalently, x directed current in the p layer is a diffusion current only. This assumption is valid when high level injection does not occur in the p layer; i.e. excess carrier concentrations do not approach the equilibrium hole concentration in the p layer. P and n layer impurity concentrations of

the same order of magnitude result in high level injection occurring at approximately the same forward voltage in both layers. Since the p layer thickness is much less than the n layer thickness $\left(\frac{W_p}{W_n} \approx \frac{1}{1000}\right)$, the voltage drop is much greater in the n layer than in the p layer and the effects of the x directed electric field in the p layer are neglected.

5. E_x is independent of x in the p layer. This, along with Assumption 3, leads to the z directed drift current density in the p layer being independent of x, to be used in the development of the model.
6. In the n substrate, $\frac{\partial \phi(x, y, z)}{\partial z} = 0$ and $\frac{\partial \phi(x, y, z)}{\partial y} = 0$ for any value of x. The validity of this assumption is justified since the n region, fabricated using very low resistivity silicon (approximately $10^{-2} \Omega\text{-cm}$), is in direct contact with the groundplane over the entire y-z range of the device. As a consequence of this assumption the z and y directed drift current in the n region is zero. The validity of the assumption becomes questionable under high level injection in the n substrate.
7. The z directed drift current is confined to the p layer. Since the device being studied is fabricated with the resistivity of the v layer much greater than

the resistivity of the p layer, the z directed drift current in the ν layer is negligible compared with the z directed drift current in the p layer.

8. The diffusion current in all layers in the y and z directions is neglected. Since the total variation of surface potential in the y direction is neglected the excess carrier gradient and hence the diffusion current in the y direction is neglected. The current in the z direction is assumed to consist of the drift current in the p layer. The excess carrier gradient in the z direction for any value of x or y is assumed to contribute negligibly to the total z directed current.
9. The biasing contacts are ideally ohmic. The two biasing contacts to the p layer are formed by alloying aluminum into the contact areas of the p layer, which are highly doped with boron ($N_A \approx 1 \times 10^{20} \text{ cm}^{-3}$). Similarly, aluminum is alloyed into the groundplane region of the arsenic doped n substrate whose surface is degenerately doped with phosphorous ($N_{Dn} \approx 1 \times 10^{21} \text{ cm}^{-3}$). Contacts of this type exhibit negligible impedance and have a linear I-V characteristic as long as the excess carrier densities in the silicon at the contacts are well below the equilibrium majority values, which is true over the range of operation being studied for the device ^{4, 5}.

The device is divided into a number of subregions ⁶, as illustrated in Fig. 2.1-1. The model of each subregion is found; the model for the entire device being obtained by cascading the subregion models. This is justified since the currents are continuous across the subregion boundaries and the voltage, $V(z)$, is a continuous function of z .

It is to be emphasized that the subregion boundaries do not cross into the groundplane, making it possible to evaluate the current into the groundplane from the subregion. Were the boundary extended into the metal groundplane, the current in the groundplane would be an additional unknown, complicating the derivation of the distributed model. Since the z directed current is assumed to be only in the p layer, a current $I(z_1)$ enters the subregion through the p layer and $I(z_2)$ exits the other side of the subregion in the p layer. $I_0(z)$ is the current crossing the subregion boundary into the groundplane. Consistent with the previously made assumptions $I_0(z)$ is the current crossing the $p-v$ junction and is evaluated from the I - V characteristic of the $p-n$ diode (Appendix I). The recombination current at the surface of the p layer is neglected since it is shown in Appendix I that the boundary condition at the surface of the p layer does not significantly effect the I - V characteristic of the diode.

The continuity equation is applied to the subregion to obtain

$$I(z_1) - I(z_2) = I_D(z) \quad . \quad 2.1$$

Equation 2.1 is the distributed analog of Kirchhoff's current law and is referred to as the distributed Kirchhoff equation or the d-K equation for the subregion $?$. $I(z_1)$ and $I(z_2)$ are further expressed as

$$I(z_1) = J(z_1)W_pY = E_z(z_1) \frac{W_pY}{\rho_p} = \left(- \frac{\partial V(z)}{\partial z} \Big|_{z=z_1} \right) \frac{W_pY}{\rho_p} \quad 2.2$$

and similarly,

$$I(z_2) = \left(- \frac{\partial V(z)}{\partial z} \Big|_{z=z_2} \right) \frac{W_pY}{\rho_p} \quad . \quad 2.3$$

The diffused depth of the p layer, W_p , is an approximation to the undepleted depth of the p layer, more accurately written as $W_p - x_{dp}$. The width of the depletion region in the p layer is much less than W_p ; typical values, for the doping densities used, being $W_p \approx .5\mu$ and $x_{dp} < 1 \text{ \AA}$. Furthermore,

$$I_D(z) = Y \int_{z_1}^{z_2} J_D(z) dz$$

allowing the d-K equation to be rewritten as

$$\frac{W_pY}{\rho_p} \left(\frac{\partial V(z)}{\partial z} \Big|_{z=z_2} - \frac{\partial V(z)}{\partial z} \Big|_{z=z_1} \right) = Y \int_{z_1}^{z_2} J_D(z) dz \quad . \quad 2.4$$

Using the mathematical identity

$$\frac{\partial V(z)}{\partial z} \Big|_{z=z_2} - \frac{\partial V(z)}{\partial z} \Big|_{z=z_1} = \int_{z_1}^{z_2} \frac{\partial^2 V(z)}{\partial z^2} dz \quad 2.5$$

results in

$$\frac{W_P Y}{\rho_P} \int_{z_1}^{z_2} \frac{\partial^2 V(z)}{\partial z^2} dz = Y \int_{z_1}^{z_2} J_D(z) dz \quad 2.6$$

or

$$\int_{z_1}^{z_2} \left(\frac{W_P}{\rho_P} \frac{\partial^2 V(z)}{\partial z^2} - J_D(z) \right) dz = 0. \quad 2.7$$

Since the subregion has non-zero volume, $z_1 \neq z_2$ and

$$\frac{W_P}{\rho_P} \frac{\partial^2 V(z)}{\partial z^2} - J_D(z) = 0. \quad 2.8$$

Equation 2.8 has been derived from the d-K equation and is, therefore, called the d-K partial differential equation⁸.

The parameters of Eq. 2.8 may be related to the elements of the resistive distributed network section shown in Fig. 2.1-2. The length of the subregion represented by this network is $\Delta z = z_2 - z_1$. Applying Kirchhoff's current law at the internal node results in the relationship

$$I_1 - I_2 = V_1 G(z) Y \Delta z - I_1 \frac{R(z)}{2} G(z) (\Delta z)^2. \quad 2.9$$

Furthermore,

$$V_2 - V_1 = - \frac{R(z) \Delta z}{2 Y} (I_1 + I_2). \quad 2.10$$

In the limit as $\Delta z \rightarrow 0$, $V_1 \rightarrow V_2 \rightarrow V(z)$, $I_1 \rightarrow I_2 \rightarrow I(z)$ and Eqs. 2.9 and 2.10 reduce to

$$\frac{\partial I(z)}{\partial z} = - Y G(z) V(z) \quad 2.11$$

and

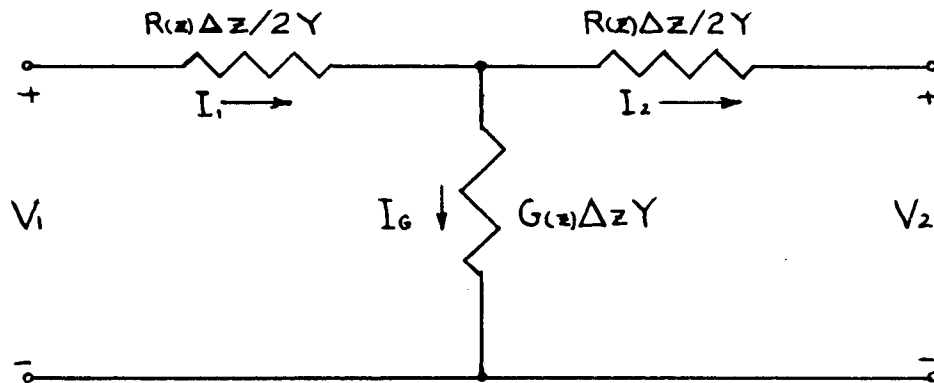


FIGURE 2.1-2 THE GENERAL SUBREGION NETWORK

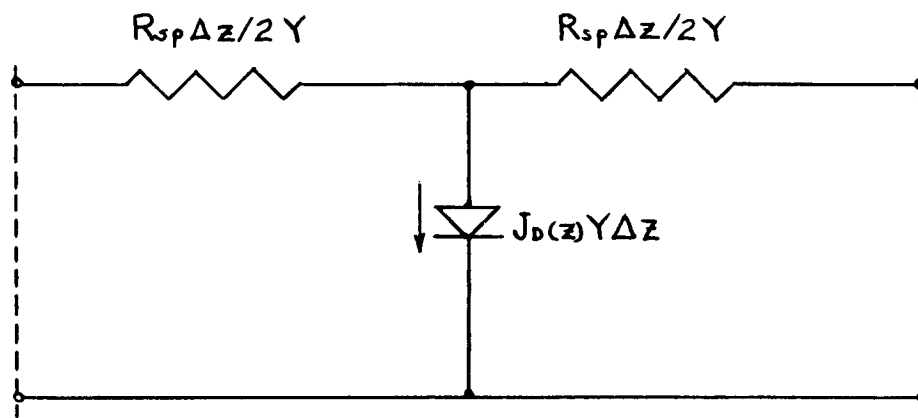


FIGURE 2.1-3 THE DEVICE SUBREGION NETWORK

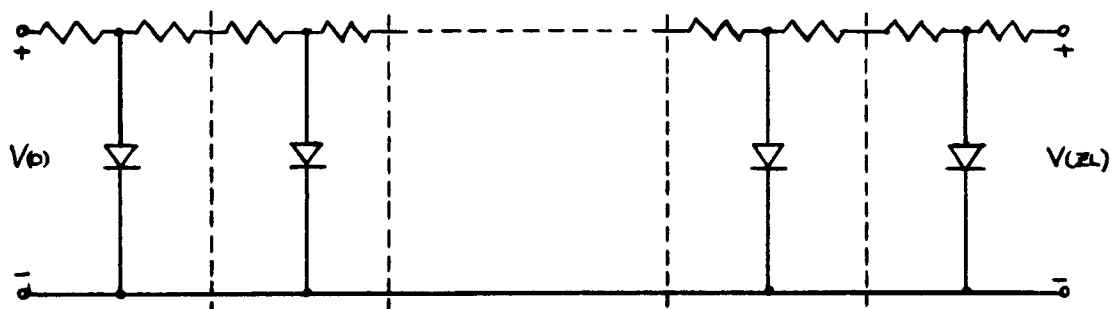


FIGURE 2.1-4 THE DC MODEL OF THE DEVICE

$$\frac{\partial V(z)}{\partial z} = - \frac{R(z) I(z)}{Y} \quad . \quad 2.12$$

Differentiation of Eq. 2.12 with respect to z and substitution of Eqs. 2.11 and 2.12 into the result yields

$$\frac{1}{R(z)} \frac{\partial^2 V(z)}{\partial z^2} - \frac{1}{R(z)} \frac{\partial R(z)}{\partial z} \frac{\partial V(z)}{\partial z} - G(z) V(z) = 0 \quad . \quad 2.13$$

If $R(z)$ is chosen to be the sheet resistance of the p layer, and $G(z) V(z) \equiv J_D(z)$, then $\frac{1}{R(z)} \frac{\partial R(z)}{\partial z} \frac{\partial V(z)}{\partial z}$ is interpreted as the dc surface voltage modulation of the sheet resistance of the p layer. The nature of the approximation that the variation in undepleted p layer width may be neglected, used in Eq. 2.8, implies that

$$\left| \frac{1}{R(z)} \frac{\partial R(z)}{\partial z} \frac{\partial V(z)}{\partial z} \right| \ll \left| J_D(z) \right| \quad . \quad 2.14$$

Although this approximation is invalid for $V(z) \lesssim 0$, neglecting $\frac{1}{R(z)} \frac{\partial R(z)}{\partial z} \frac{\partial V(z)}{\partial z}$ produces an error in $\frac{\partial^2 V(z)}{\partial z^2}$ which is too small to produce an observable change in the voltage profile for the device dimensions used (determined from parameters and voltage profiles in Sec. 2.4). By defining the p layer sheet resistance, $R(z) = R_{sp} = \frac{\rho_p}{W_p}$, Eq. 2.13 is equivalent to Eq. 2.8. The symbolic model for the subregion is now redrawn as shown in Fig. 2.1-3. Figure 2.1-4 shows how the symbolic models for the subregions are cascaded to give the symbolic model for the device. Equation 2.8, which is further discussed in Secs. 2.2 and 2.3, is re-expressed

in the simplified form

$$\frac{\partial^2 V(z)}{\partial z^2} - J_D(z) R_{sp} = 0. \quad 2.15$$

2.2 Investigation of the Behavior of the Solution of the Surface Voltage Differential Equation

The dc surface voltage differential equation, Eq. 2.15, is studied here to gain insight into its solution. Since z is the only independent variable being considered, the partial derivative in Eq. 2.15 is replaced with the total derivative. $V(z)$ is normalized with respect to V_T to define the normalized variable $Y(z) = V(z)/V_T$. Equation 2.15 is now rewritten as

$$\frac{d^2 Y(z)}{dz^2} - \frac{J_D(z) R_{sp}}{V_T} = 0 \quad 2.16$$

The diode current density, $J_D(z)$, derived as a function of $V(z)$ in Appendix I is illustrated in Fig. 2.2-1. A general analytic solution has not been developed for Eq. 2.16, however, a numerical solution on the digital computer has yielded the result of $Y(z)$ vs. z . Before the numerical solution is discussed, the analytic solution to Eq. 2.16 is investigated in several regions of interest.

When the normalized applied voltage at the biasing contact at $z=0$ is large negative with respect to the ground-

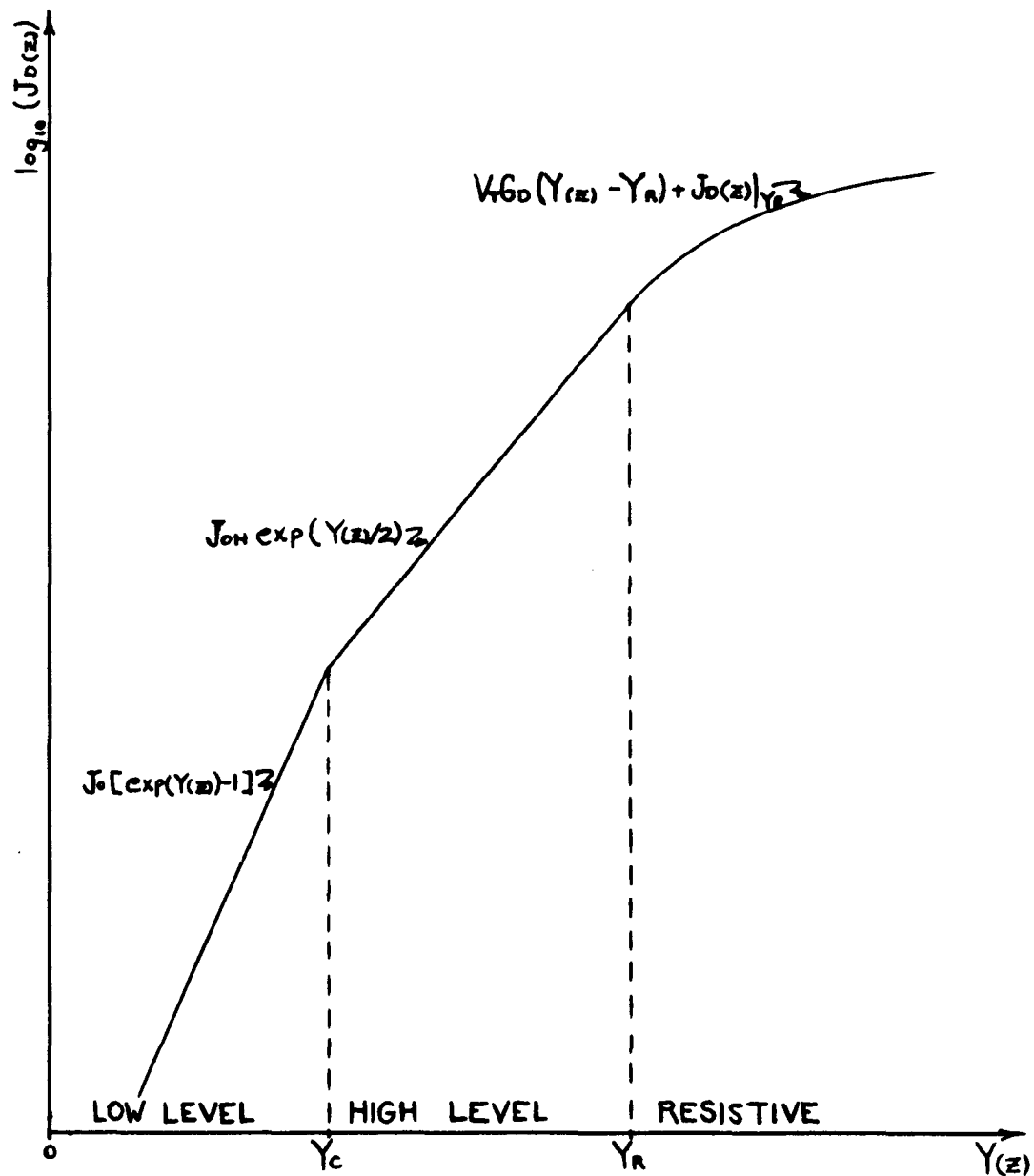


FIGURE 2.2-1- PIECEWISE J_0 - Y CHARACTERISTIC

plane, $J_0(z) = J_0 [\exp(Y(z)) - 1] \approx J_0$ in the region $0 < z < z_L$ and Eq. 2.16 reduces to $\frac{d^2 Y(z)}{dz^2} + \frac{J_0 R_{sp}}{V_T} = 0$ having the solution

$$Y(z) = -\frac{J_0 R_{sp}}{V_T} z^2 + \left. \frac{dY(z)}{dz} \right|_{z=0} z + Y(0)$$

for the strongly reverse biased region. The solution in this region is nearly linear with z when

$\frac{J_0 R_{sp}}{V_T} z \ll \left| \left. \frac{dY(z)}{dz} \right|_{z=0} \right|$, as expected, since the reverse biased p layer is physically a diffused resistor.

Worst case for this inequality occurs at $z = z_L$ when $z_L = z_L$

requiring that $\left| \left. \frac{dY(z)}{dz} \right|_{z=0} \right| \gg \frac{J_0 R_{sp}}{V_T} z_L$.

The least negative value of $Y(0)$ needed to satisfy this

inequality is estimated by using the approximation that

$$\left. \frac{dY(z)}{dz} \right|_{z=0} \approx \frac{Y(0)}{z_L} \quad \text{resulting in} \quad |Y(0)| \gg \frac{J_0 R_{sp}}{V_T} z_L^2$$

For typical diodes fabricated, $J_0 \approx 10^{-9} \text{ A/cm}^2$, $z_L \approx .25 \text{ cm}$ and

$R_{sp} \approx 10 \Omega/\square$ resulting in $Y(0) \ll -2.5 \times 10^{-8}$, placing no practical

restriction on the value of $Y(0)$, for $Y(z)$ to vary

linearly with z in the reverse biased region for $0 < z < z_L$.

In a very heavily forward biased region where

$J_0(z) = J_{0H} \exp(Y(z)/2)$, Eq. 2.16 reduces to

$$\frac{d^2 Y(z)}{dz^2} - \frac{J_{0H} R_{sp}}{V_T} \exp(Y(z)/2) = 0 \quad . \quad \text{An equation of}$$

this form has been studied by Ghosh⁹ in connection with a

distributed model of the bipolar junction transistor having

the implicit analytic solution $z = \int_{W|z=0}^{W|z} \frac{dW}{W(C_1 + 2K_1 W)^{1/2}}$

where $W = \exp(Y(z)/2)$ and $K_1 = J_{0H} R_{sp}/V_T$. The form of solution

of W depends on the sign of the constant C_1 which is

determined from the boundary conditions $\left. \frac{dY(z)}{dz} \right|_{z=z_L}$ and $Y(0)$.

Evaluation of C_1 for any specified endpoint voltages, as in the case of the distributed pyn diode, is cumbersome. This method of solution is not used to find the voltage profile on the distributed pyn structure since the condition that $Y_c < Y(z) < Y_R$ ($Y_c = V_c/V_T$, $Y_R = V_R/V_T$, V_c and V_R are defined in Appendix I and illustrated in Fig. 2.2-1) must be satisfied for all z , placing severe restrictions on the biasing voltages. The numerical solution is more general since any combination of biasing voltages may be specified.

About the point of zero voltage, z_z , $Y(z) \ll 1$ and Eq. 2.16 is approximated by $\frac{d^2 Y(z)}{dz^2} = \frac{J_0 R_{sp}}{V_T} Y(z)$. It is seen from this equation that a minimum in $\frac{dY(z)}{dz}$ occurs at $Y(z) \Big|_{z=z_z} = 0$. Operation of the device as a dc voltage controllable stub tuner requires that the gradient of the characteristic impedance be as large as possible to produce the strongest possible reflection. The minimum in $\frac{dY(z)}{dz}$ at $z=z_z$, while not desirable, is not expected to be detrimental to device operation as a stub tuner since $|\nabla Z_0|_{max}$ is expected to occur at $Y(z) > 0$ and not at $Y(z) = 0$.

The approximations used in each of the regions are not sufficient to obtain a general voltage vs. position solution of Eq. 2.16 for all values of z and biasing contact voltages. Therefore, a numerical solution technique is developed and discussed in the next section.

2.3 The Numerical Method of Solution of the Surface Voltage Differential Equation

It is desired to obtain the dc surface voltage vs. z profile on the device for the specification of $Y(0)$ and $Y(z_L)$ or for the specification of $Y(0)$ and z_z , the location of the zero voltage point. The interval from $0 \leq z \leq z_L$ is discretized into N equal segments. There are, therefore, $N+1$ equally spaced points from $0 \leq z \leq z_L$, located at the junctions of adjacent segments and at the endpoints. The dc surface voltage at each of the $N-1$ points is to be found.

One method of solution, the matrix inversion method, involves the generation of a system of difference equations which are solved for the voltage at each point of interest. The non-linear dependence of the current density term, $J_0(z)$, on $V(z)$ in Eq. 2.16 results in a system of non-linear equations. To use the matrix inversion technique, the system of equations must be linearized¹⁰. An initial guess is made of the voltage profile to generate the value of $J_0(z)$ at each of the $N+1$ points. The resulting set of equations no longer has the non-linear dependence on $V(z)$, but appears linear since $J_0(z)$ is known, and is treated as a constant at each point. A matrix inversion is used to solve for the new value of voltage at each point. The new values of voltage are used to generate a new value of $J_0(z)$ at each point and, again, the system of equations is solved for the new voltages. The process continues until the voltage profile

converges ¹¹ to within the desired degree of accuracy. If the boundary conditions $Y(0)$ and $Y(z_L)$ are specified, $N+3$ difference equations are generated by writing Eq. 2.16 in difference equation form (by expressing the second derivative in finite difference form) at the $N+1$ points and by using the two boundary conditions. If $Y(0)$ and z_x are specified, the difference equations are generated by considering the boundary value problem in the region $0 \leq z \leq z_x$ with $Y(0)$ and $Y(z_x) = 0$ as the boundary conditions. The solution of the voltage at the points in the region $0 \leq z \leq z_x$ allow $\left. \frac{dY(z)}{dz} \right|_{z=z_x}$ to be calculated. The conditions $Y(z_x) = 0$ and $\left. \frac{dY(z)}{dz} \right|_{z=z_x}$ allow a system of equations to be set up and solved for the voltage at the remaining points in the interval $z_x < z \leq z_L$. Although the matrix inversion is not difficult, since a tri-diagonal matrix is generated, the solution is cumbersome due to the choice of input conditions that are to be accommodated. A less awkward method that gives more physical insight to the solution is, therefore, used.

Equation 2.16, using the result of Appendix I, is integrated once by expressing $\frac{d^2 Y(z)}{dz^2}$ as $\frac{d}{dz} \left(\frac{dY(z)}{dz} \right)$ and separating variables to obtain

$$\frac{dY(z)}{dz} = \begin{cases} \pm \sqrt{\frac{2J_0 R_{sp}}{V_T} [\exp(Y(z)) - Y(z)] + C} & Y(z) \leq Y_c \\ \pm \sqrt{\frac{4J_0 R_{sp}}{V_T} \exp(Y(z)/2) + C_c} & Y_c < Y(z) \leq Y_R \quad 2.17 \\ \pm \sqrt{R_{sp} G_D (Y(z) - Y_R)^2 + \frac{2R_{sp} J_D}{V_T} Y(z) + C_R} & Y_R < Y(z) \end{cases}$$

where

$$C_c = \frac{2J_o R_{sp}}{V_T} [\exp(Y_c) - Y_c] - \frac{4J_o R_{sp}}{V_T} \exp(Y_c/2) + C \quad 2.18$$

and

$$C_R = \frac{4J_o R_{sp}}{V_T} \exp(Y_R/2) - \frac{2R_{sp}}{V_T} J_D \bigg|_{Y=Y_R}^{Y_R + C_c} \quad 2.19$$

The algorithm for a fourth-order Runge-Kutta method ¹² is used to generate the values of $Y(z)$ at the N points from $0 < z \leq z_L$, subject to the values of $Y(0)$ and C . A first value of $-2J_o R_{sp}/V_T$ is assigned to C to calculate the first trajectory, i.e. the value of voltage at each point in the interval $0 < z \leq z_L$. This is the most negative possible value for C while still assuring that $\frac{dY(z)}{dz}$, calculated from Eq. 2.17, is real for all values of $Y(z)$. A second value of C is calculated by adding a positive increment to the previous value and a new trajectory is generated. The value of $Y(z)$ calculated at $z=z_L$ is then compared to the boundary condition of $Y(z_L)$. Or, if the location of the zero voltage point is specified, the value of $Y(z)$ at $z=z_x$ is compared to zero. If the value of the voltage specified at either z_x or z_L does not lie somewhere between the current and previous trajectories, a new value of C is calculated by adding a positive increment to the previous value and the next trajectory is generated. This is continued until the voltage specified at either z_x or z_L lies between the latest and previous trajectories. Then it is known that the value of

C corresponding to the desired boundary conditions lies somewhere between the two previous values used. Interval halving is then used to converge to a solution within a specified accuracy. A simple case is illustrated in Fig. 2.3-1. If the value of voltage specified at $z=z_1$ is close to the value specified at $z=0$, the initial value of C may be larger than the value needed for the correct trajectory. For this case, the value of C is decreased by adding negative increments, as long as the argument of the square root of Eq. 2.17 remains positive. This is illustrated in Fig. 2.3-2.

A program listing and flow chart showing the numerical solution in detail, for $Y(0)$ and $Y(z_1)$ specified, are in Appendices V and VI. Subroutine ITER contains the fourth-order Runge-Kutta algorithm and is used to generate the trajectory for a given $Y(0)$ and C . Subroutine SIGN selects the sign of the square root in Eq. 2.17 at $z=0$.

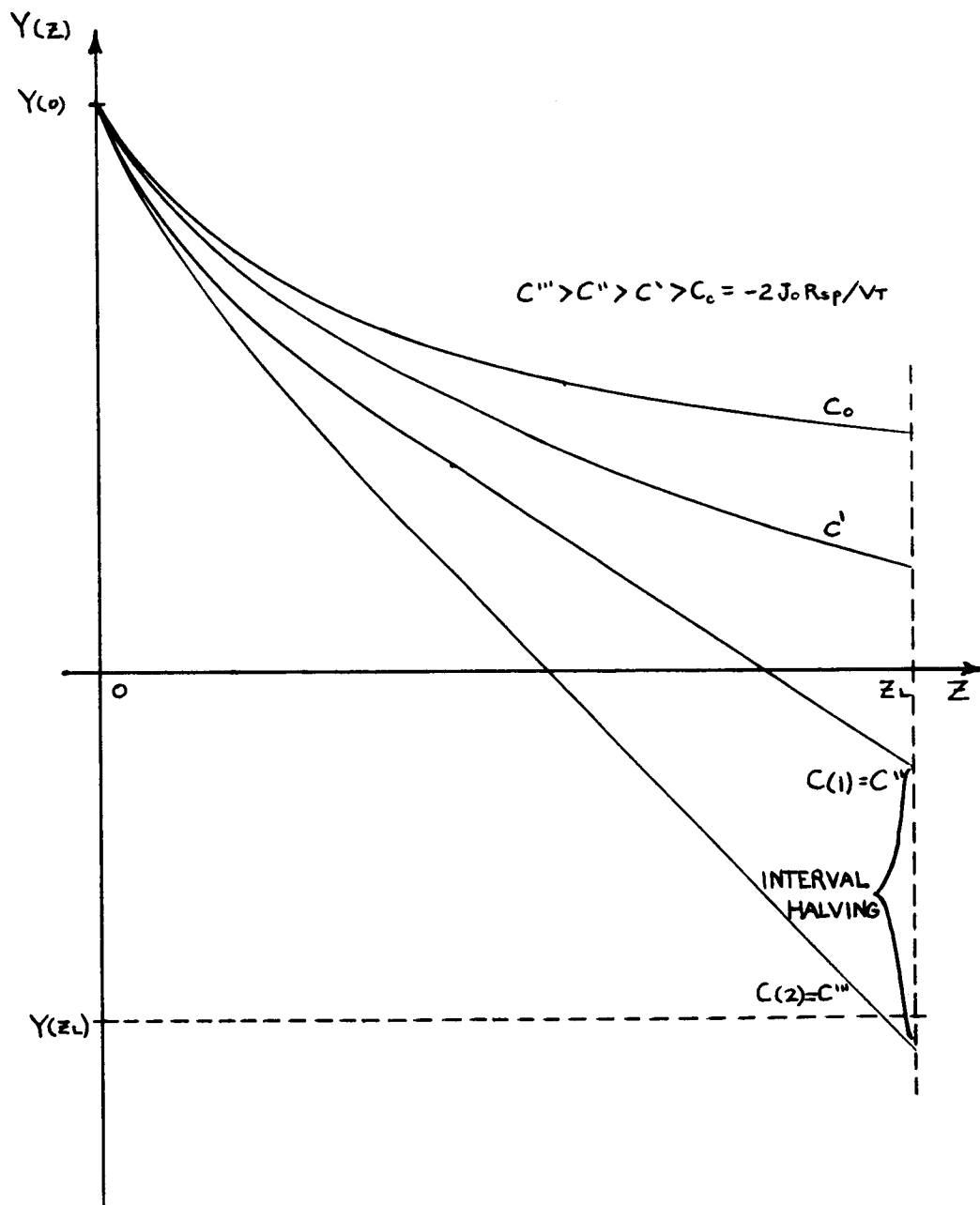


FIGURE 2.3-1 CONVERGENCE TO A SOLUTION

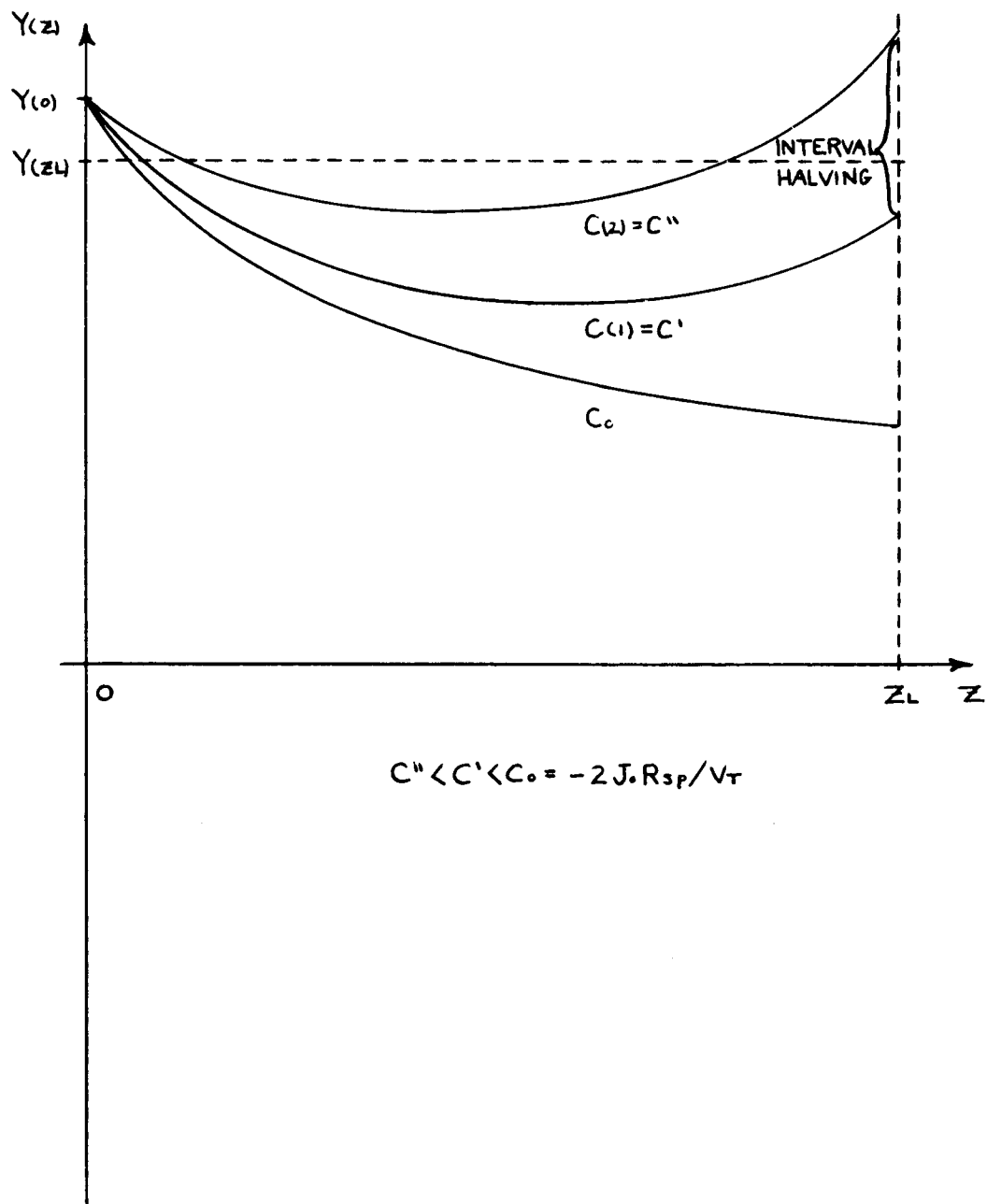


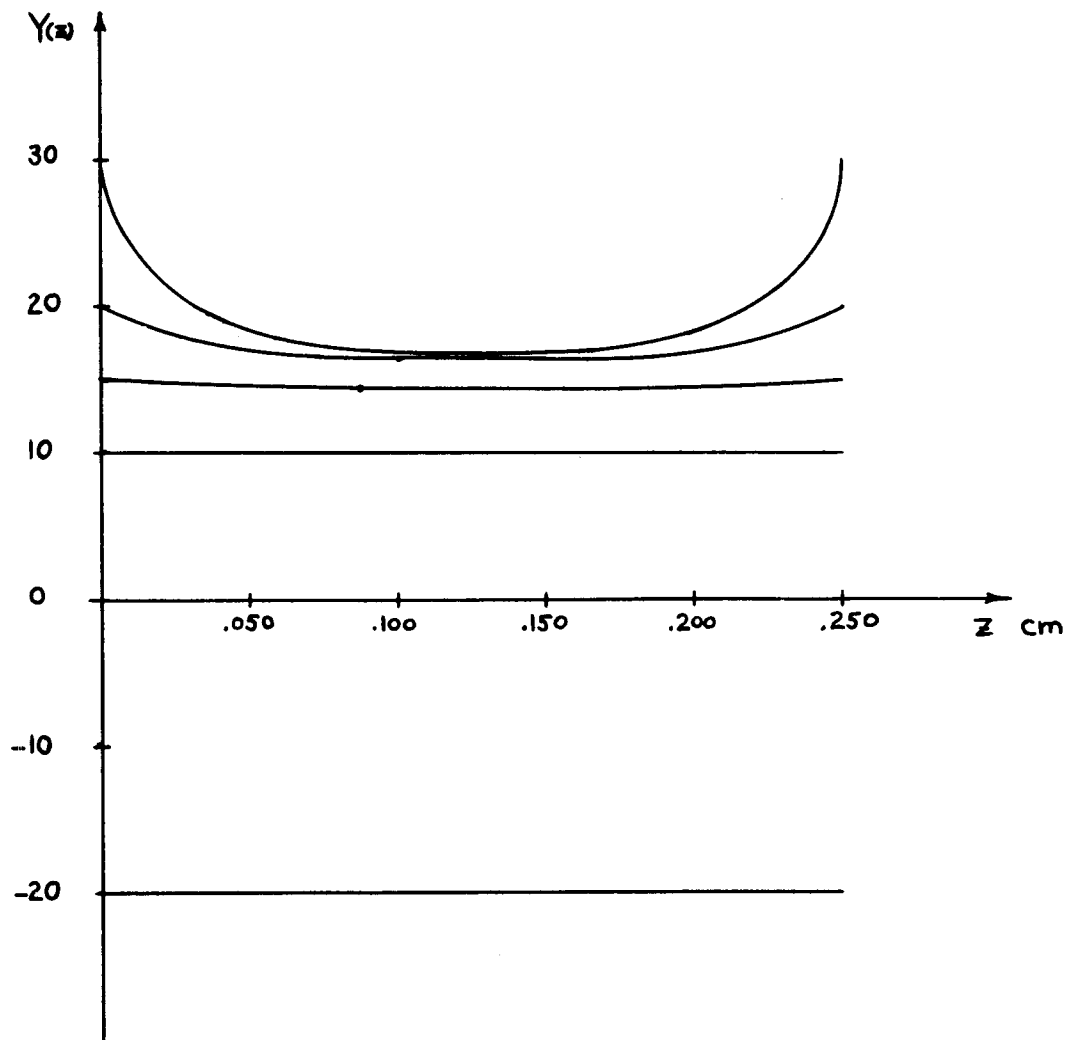
FIGURE 2.3-2 CONVERGENCE TO A SOLUTION

2.4 Computer Generated Voltage Profiles

Computer generated voltage profiles which are obtained from the numerical solution of Eq. 2.17 are illustrated and discussed in this section.

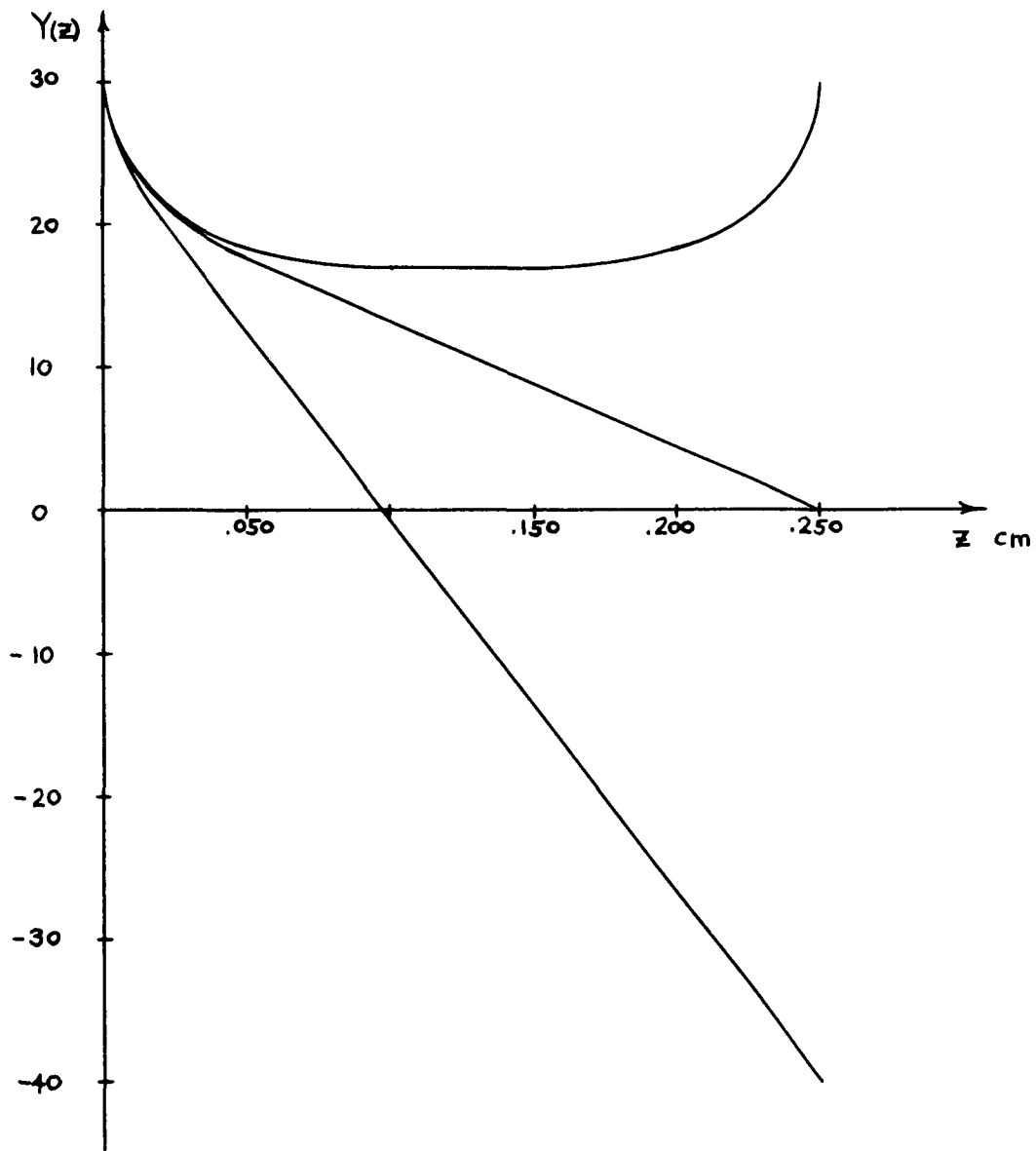
Although increasing the number of sections used in the model beyond $N=25$ produces no noticeable change in the computed voltage profile for the endpoint voltages being considered, a value of $N=100$ is used throughout the numerical solutions to accurately compute the biasing contact currents (discussed in Sec. 2.5) and rf device parameters (discussed in Sec. 3.4).

Curves in this section show the effect of different endpoint voltages on the voltage profile and on the location of a possible zero voltage point. Figure 2.4-1 is a family of voltage profile curves satisfying the condition that $Y(o) = Y(z_L)$. The effect of the junction current density, $J_b(z)$, on the voltage profile is illustrated by this graph. For $Y(o)$ and $Y(z_L) \lesssim 17$ the profiles are nearly linear, corresponding to the negligible effect that $J_b(z)$ has on the profile. This can be thought of as an extension of the negligible effect that $J_b(z)$ has on the curvature of the profile for reverse bias, discussed in Sec. 2.2; even for small forward biases the junction current density has an insignificant effect on the curvature. Larger endpoint voltages give rise to larger values of $J_b(z)$, producing noticeable curvature of the voltage profiles. Figures 2.4-2 and 2.4-3 illustrate the effect



PARAMETERS
 $J_0 = 2.67 \times 10^{-9} \text{ A/cm}^2$
 $J_{0H} = 1.78 \times 10^{-5} \text{ A/cm}^2$
 $R_{sp} = 200 \text{ } \Omega/\square$

FIGURE 2A-1 COMPUTER GENERATED VOLTAGE PROFILES, $Y(z_L) = Y(0)$



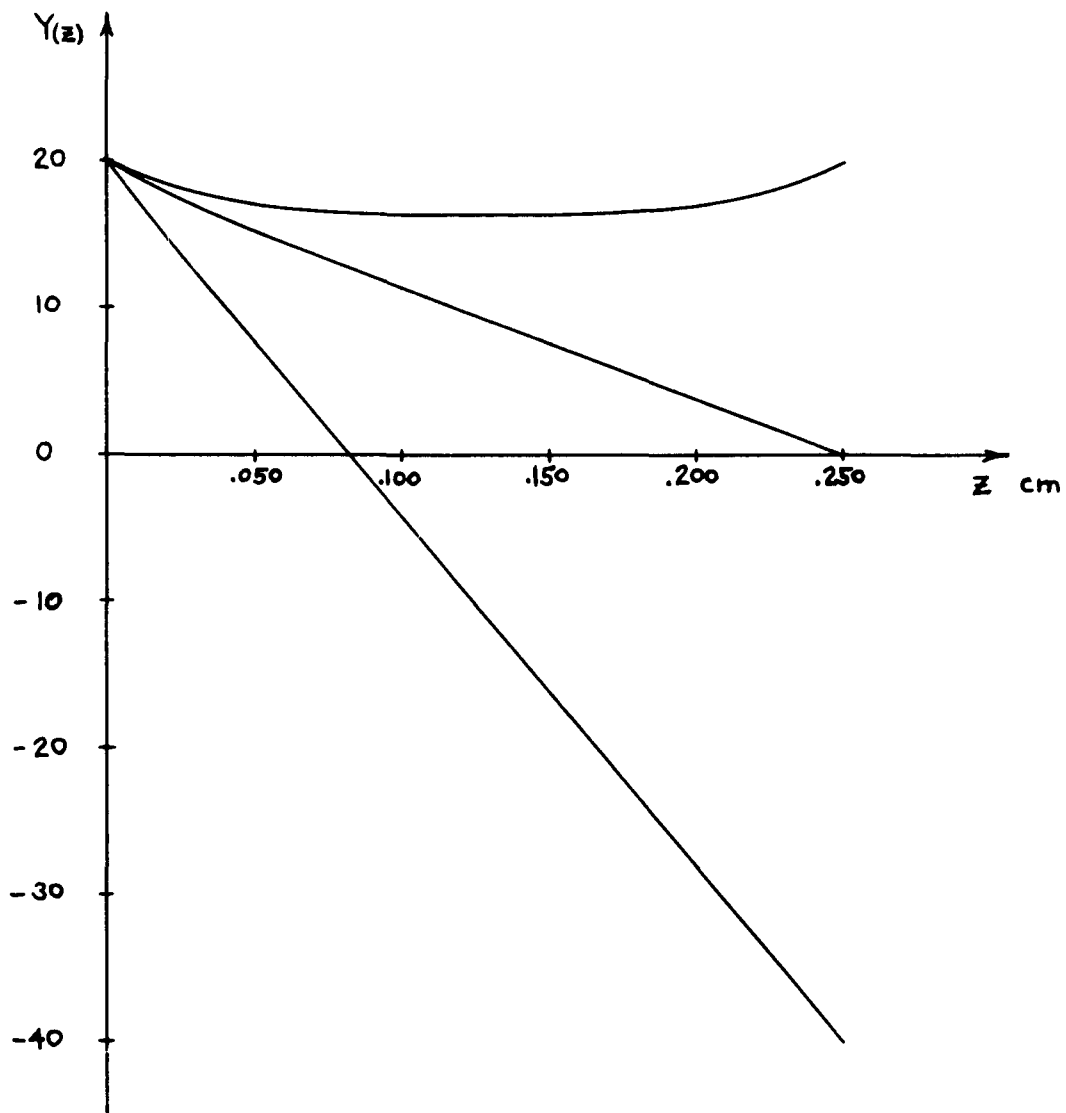
PARAMETERS

$$J_0 = 2.67 \times 10^{-9} \text{ A/cm}^2$$

$$J_{0H} = 1.78 \times 10^{-5} \text{ A/cm}^2$$

$$R_{sp} = 200 \text{ } \Omega/\square$$

FIGURE 2.4-2 COMPUTER GENERATED VOLTAGE PROFILES



PARAMETERS

$$J_0 = 2.67 \times 10^{-9} \text{ A/cm}^2$$

$$J_{\text{ex}} = 1.78 \times 10^{-5} \text{ A/cm}^2$$

$$R_{\text{sp}} = 200 \text{ } \Omega/\square$$

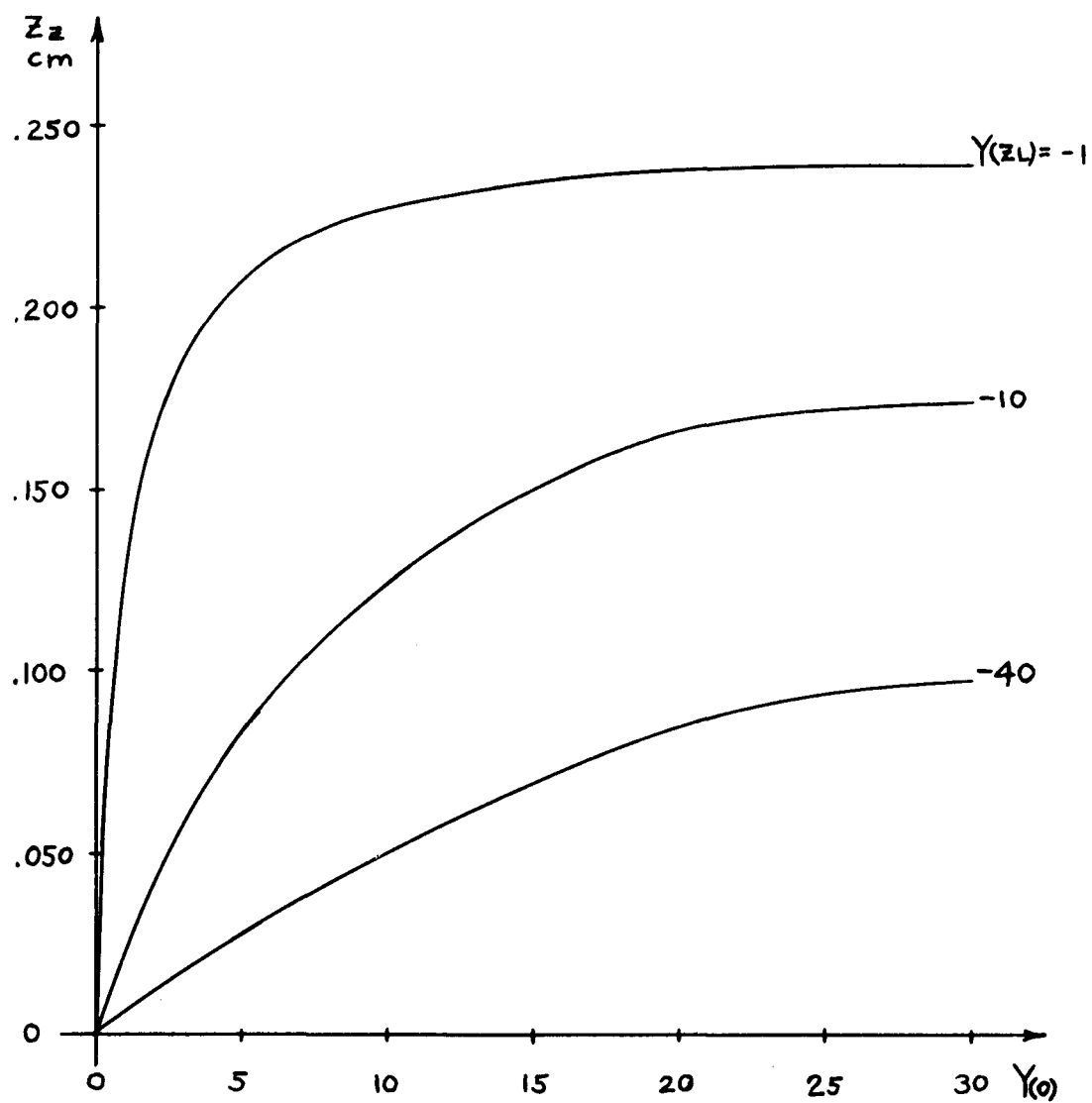
FIGURE 2.4-3 COMPUTER GENERATED VOLTAGE PROFILES

of varying the voltage on the negatively biased contact while holding the voltage on the positively biased contact constant. As the difference between $Y(0)$ and $Y(z_L)$ is increased the voltage profiles become more linear with respect to z . Increasing the difference between $Y(0)$ and $Y(z_L)$ increases the z directed current density in the p layer reducing the effect of $J_D(z)$ on the curvature of the voltage profile. Therefore, for a given $Y(0) > 0$, and not allowing $Y(z_L)$ to exceed $Y(0)$, the largest departure from a linear voltage profile occurs when $Y(z_L) = Y(0)$. For $Y(z_L) = Y(0)$ the effect of $J_D(z)$ on the voltage profile is most noticeable since the z directed current in the p layer at $z = z_L/2$ is zero. Furthermore, the total current from the contact at $z=0$, subject to the conditions that $Y(0) > 0$ and $Y(z_L) \leq Y(0)$, is a minimum for $Y(z_L) = Y(0)$. The evaluation of the currents from each biasing contact is discussed in the next section.

For the doping densities being considered, the onset of high level injection in the v region occurs at approximately $Y(z) = Y_c = 17$. Since the voltage profile is very nearly linear for the largest normalized biasing contact voltage less than or approximately equal to 17 the effect of $J_D(z)$ on the voltage profile is very small and the exact form of $J_D(z)$ for $Y(z) \leq 17$ is unnecessary. This is noteworthy since the actual low level injection volt-ampere characteristic, $J_D(z)$ vs. $V(z)$, departs from the ideal since $J_D(z)$ is not precisely proportional to $\exp(V(z)/V_T)$. The experimentally

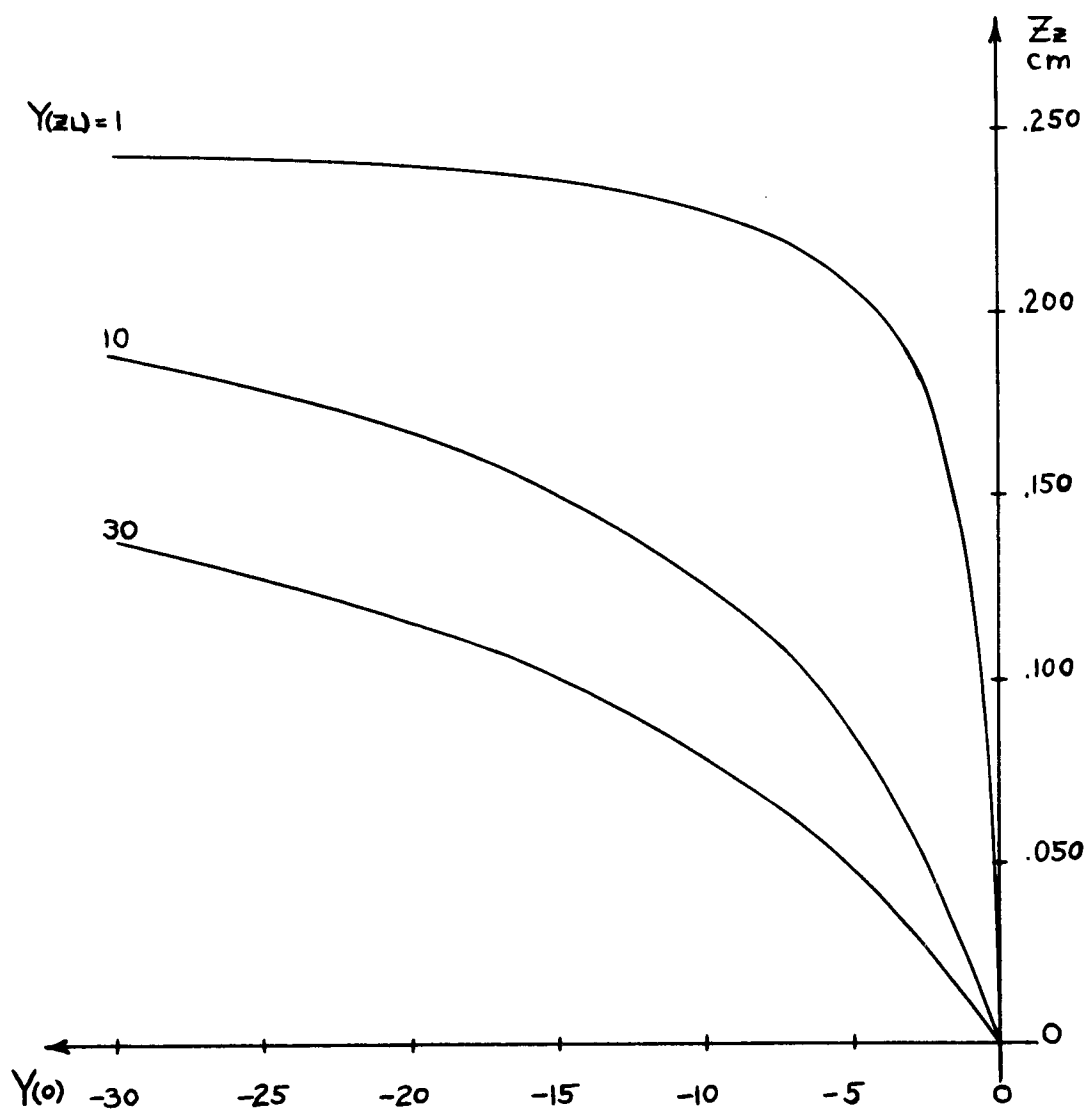
obtained $J_0(z)$ vs. $V(z)$ characteristic is discussed in Sec. 2.6.

It should be emphasized that for $Y_0 \cdot Y(z_L) < 0$ the location of the zero voltage point, $z=z_x$, is in general more sensitive to a change in voltage at the negative biasing contact than at the positive biasing contact, as illustrated in Figs. 2.4-4 and 2.4-5. This is a direct consequence of the curvature of the voltage profile in the vicinity of the positive biasing contact, as shown in Fig. 2.4-6. When using the microstrip-oxide-distributed pyn device as a voltage controllable solid state stub tuner it is desirable to make the rf driving point impedance as sensitive to changes in the biasing contact voltages as possible. Therefore, it appears to be desirable to vary the voltage on the negative biasing contact while holding the voltage on the positive biasing contact constant, producing the largest change in the location of the plane of strong reflection per unit change in biasing contact voltage. It is desired that the voltage on the positive biasing contact, which is limited by power dissipation, be as large as possible, maximizing the gradient in the characteristic impedance. As seen in Fig. 2.4-4, z_x changes by approximately .150 cm when $Y(z_L)$ is varied from -1 to -40 and Y_0 is held at 30. This corresponds to a change of approximately one tenth of a wavelength at 3 GHz, in the reverse biased portion of the device (see Sec. 3.3).



PARAMETERS	
$J_0 = 2.67 \times 10^{-9} \text{ A/cm}^2$	
$J_{0H} = 1.78 \times 10^{-5} \text{ A/cm}^2$	
$R_{sp} = 200 \text{ } \Omega/\square$	
$Z_L = .250 \text{ cm}$	

FIGURE 2.4-4 ZERO VOLTAGE POINT VS. $Y(0)$ WITH $Y(z_L)$ AS A PARAMETER, $Y(0) > 0$ $Y(z_L) < 0$



PARAMETERS

$$J_0 = 2.67 \times 10^{-9} \text{ A/cm}^2$$

$$J_{0H} = 1.78 \times 10^{-5} \text{ A/cm}^2$$

$$R_{sp} = 200 \text{ } \Omega/\square$$

$$Z_L = .250 \text{ cm}$$

FIGURE 2.4-5 ZERO VOLTAGE POINT VS. $Y(0)$ WITH $Y(z_L)$ AS A PARAMETER, $Y(0) < 0$ $Y(z_L) > 0$

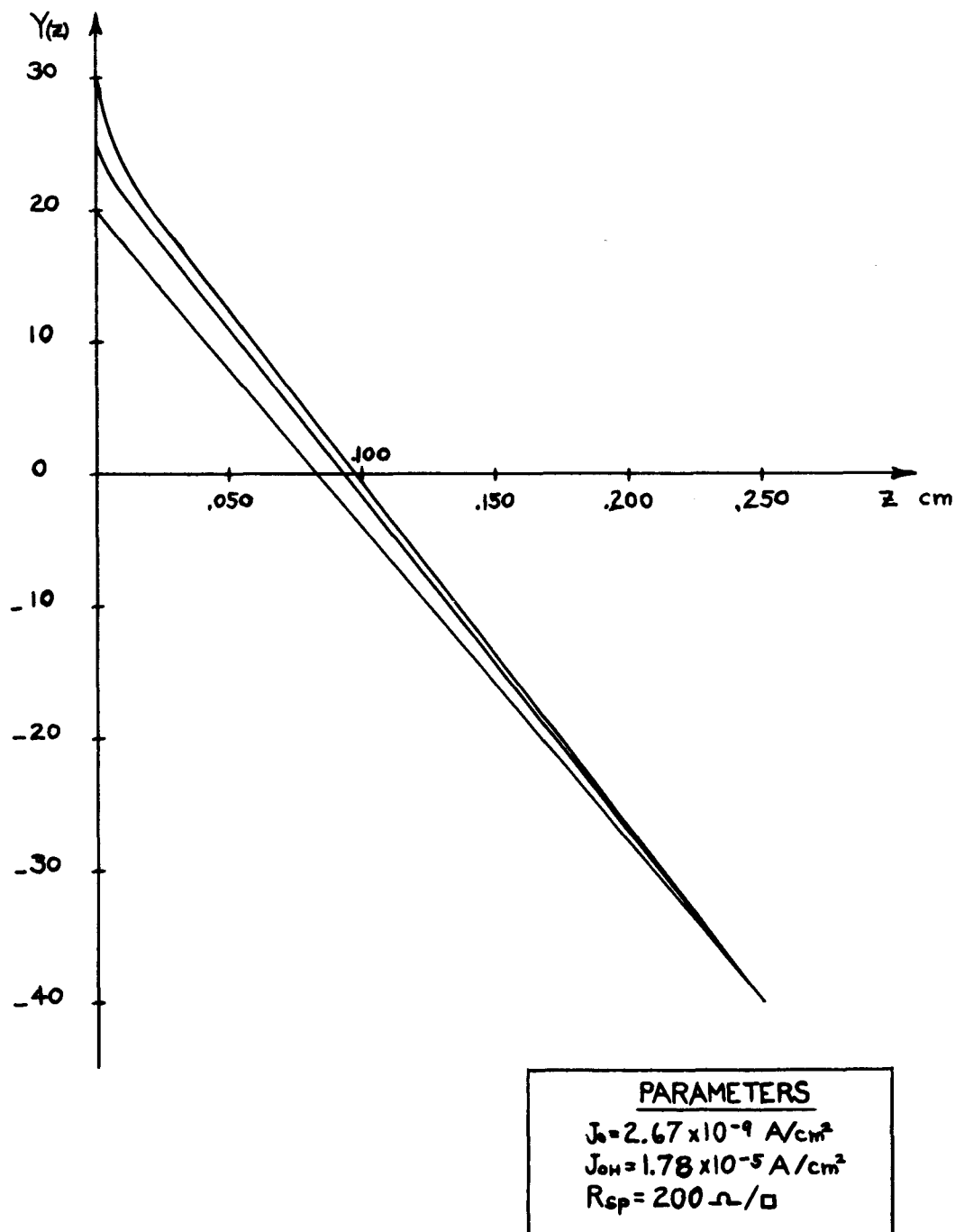


FIGURE 2.4-6 THE EFFECT OF VOLTAGE PROFILE CURVATURE ON THE SENSITIVITY OF z_2 TO CHANGES IN $Y(0)$

2.5 The Input Volt-Ampere Characteristics

In this section knowledge of the dc surface voltage profile is used to derive the dc input volt-ampere characteristic at one biasing contact for different values of dc voltage at the other biasing contact. Use of the device as a field effect transistor gate-oxide protection network is also investigated.

In the vicinity of the biasing contacts equipotential curves are not straight lines and do not necessarily extend from $y=0$ to $y=Y$. This transition region for $0 < z < \xi$ is illustrated in Fig. 2.5-1, in which the microstrip transmission line is omitted for clarity. At $z=\xi$ the equipotential lines are nearly straight and extend across the full width of the p layer, as assumed in the development of the distributed model in Sec. 2.1 (Assumption 2). The current from the dc source into the biasing contact at $z=0$, I_0 , as a function of the biasing contact voltages is found by isolating a region of the device in the vicinity of the contact and evaluating the currents normal to the boundaries of the region, as illustrated in Fig. 2.5-2. Consistent with the assumptions made in Sec. 2.1, y directed current is zero and z directed current is confined to the p layer, with zero z directed current crossing the p layer to the y background. Therefore, only the current density normal to the plane $z=\xi$ and the junction current density, $J_b(x)$, contribute to I_0 . The current crossing the surface $z=\xi$, I_1 , is evaluated from

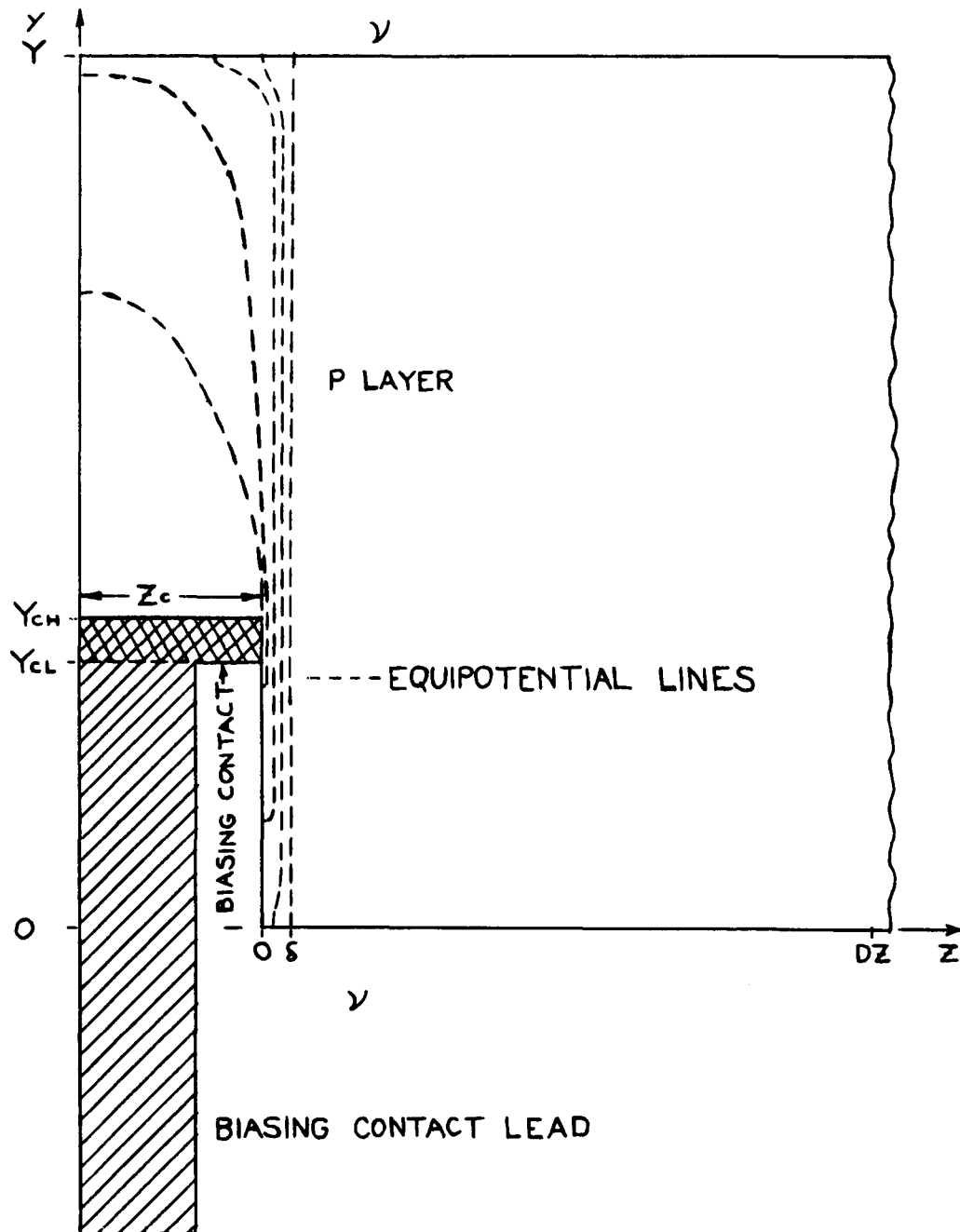


FIGURE 2.5-1 SKETCH OF
POTENTIAL VARIATION IN THE VICINITY
OF A BIASING CONTACT

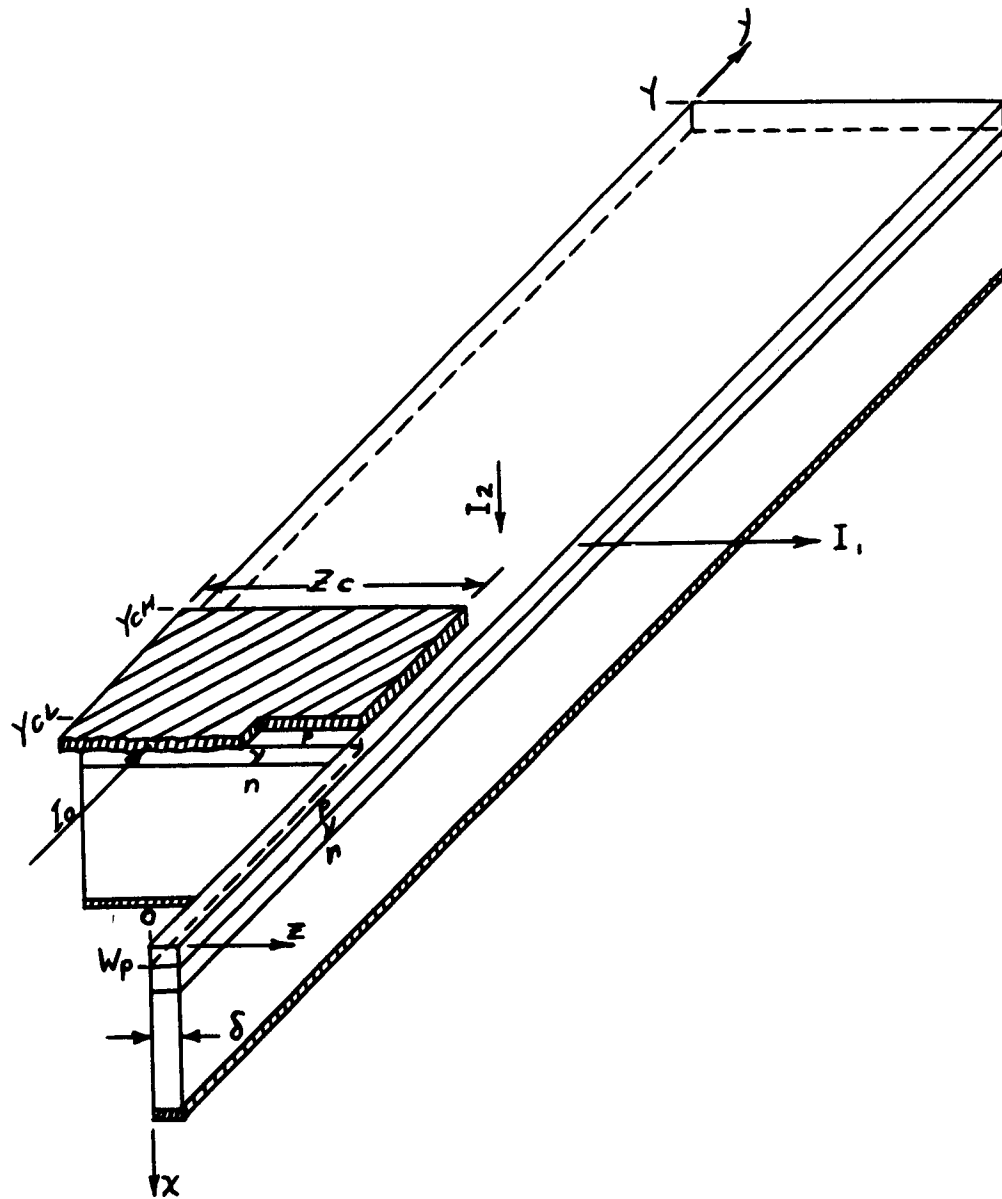


FIGURE 2.5-2 EVALUATING A BIASING CONTACT CURRENT

$$I_1 = - \frac{V_T}{\rho_P} \int_0^{W_P} \int_0^Y \left. \frac{dY(z)}{dz} \right|_{z=\delta} dy dx \quad 2.20$$

(see Assumption 8, Sec. 2.1). Integration of Eq. 2.20 with use of Assumptions 2 and 5 of Sec. 2.1 yields

$$I_1 = - \frac{W_P Y V_T}{\rho_P} \left. \frac{dY(z)}{dz} \right|_{z=\delta} \quad 2.21$$

It is assumed that $Y(\delta) \approx Y(0)$, $\delta \ll DZ$ and $\left. \frac{dY(z)}{dz} \right|_{z=\delta} \approx \frac{Y(DZ) - Y(0)}{DZ}$, where DZ is the distance between grid points used in the numerical solution, permitting Eq. 2.21 to be expressed as

$$I_1 = \frac{Y V_T}{R_{SP} DZ} (Y(0) - Y(DZ)), \quad 2.22$$

where $R_{SP} = \rho_P / W_P$. The junction current in the region defined in Fig. 2.52, I_2 , is written as

$$I_2 = \int_{-z_c}^0 \int_{Y_{CL}}^Y J_0(z) dy dz + \int_0^{\delta} \int_0^Y J_0(z) dy dz \quad 2.23$$

Assuming that the transition region is very small compared to z_c , $\delta \ll z_c$, the contribution from the second integral in Eq. 2.23 is neglected and Eq. 2.23 simplifies to

$$I_2 = \int_{-z_c}^0 \int_{Y_{CL}}^Y J_0(z) dy dz \quad 2.24$$

The surface potential is nearly independent of y and $Y(z) \approx Y(0)$ for $-z_c < z < 0$ allowing Eq. 2.24 to be evaluated as

$$I_2 = z_c (Y - Y_{CL}) J_0(z) \Big|_{Y(z)=Y(0)} \quad 2.25$$

Use of the relationship $I_0 = I_1 + I_2$ with Eqs. 2.22 and 2.25 result in

$$I_0 = \frac{Y V_T}{R_{sp} DZ} (Y(0) - Y(DZ)) + Z_c (Y - Y_{cl}) J_0(z) \Big|_{Y(z)=Y(0)}, \quad 2.26$$

which is used in the numerical evaluation of I_0 . Similarly, the current into the biasing contact at $z=z_L$ is

$$I_L = \frac{Y V_T}{R_{sp} DZ} (Y(z_L) - Y(z_L - DZ)) + Z_c (Y - Y_{cl}) J_0(z) \Big|_{Y(z)=Y(z_L)}. \quad 2.27$$

The currents I_0 and I_L are evaluated by first calculating the voltage profile to obtain the values of $Y(0Z)$, $Y(z_L - DZ)$ and, if necessary, $Y(z_L)$, which are substituted Eqs. 2.26 and 2.27.

The calculated values of I_0 and I_L are functions of the number of sections, N , used in the model. Although the calculated voltage profiles, typically, do not appear to change when N is increased above 25, the calculated biasing contact currents continue to change well beyond $N=25$, as a result of slight changes in the calculated values of $\frac{dY(z)}{dz}$ at the biasing contacts. Figure 2.5-3 illustrates the variation of I_L (the biasing contact current most sensitive to changes in N , for this typical example) with N , for biasing contact voltages of $Y(0) = -30$ and $Y(z_L) = 30$. As N increases above 100, I_L appears to asymptotically approach approximately 26 mA. In this example, $N=100$ results in I_L which is within 6% of the value produced by $N \rightarrow \infty$. For the biasing contact voltages considered, $N=100$ sections is used in the model and results in

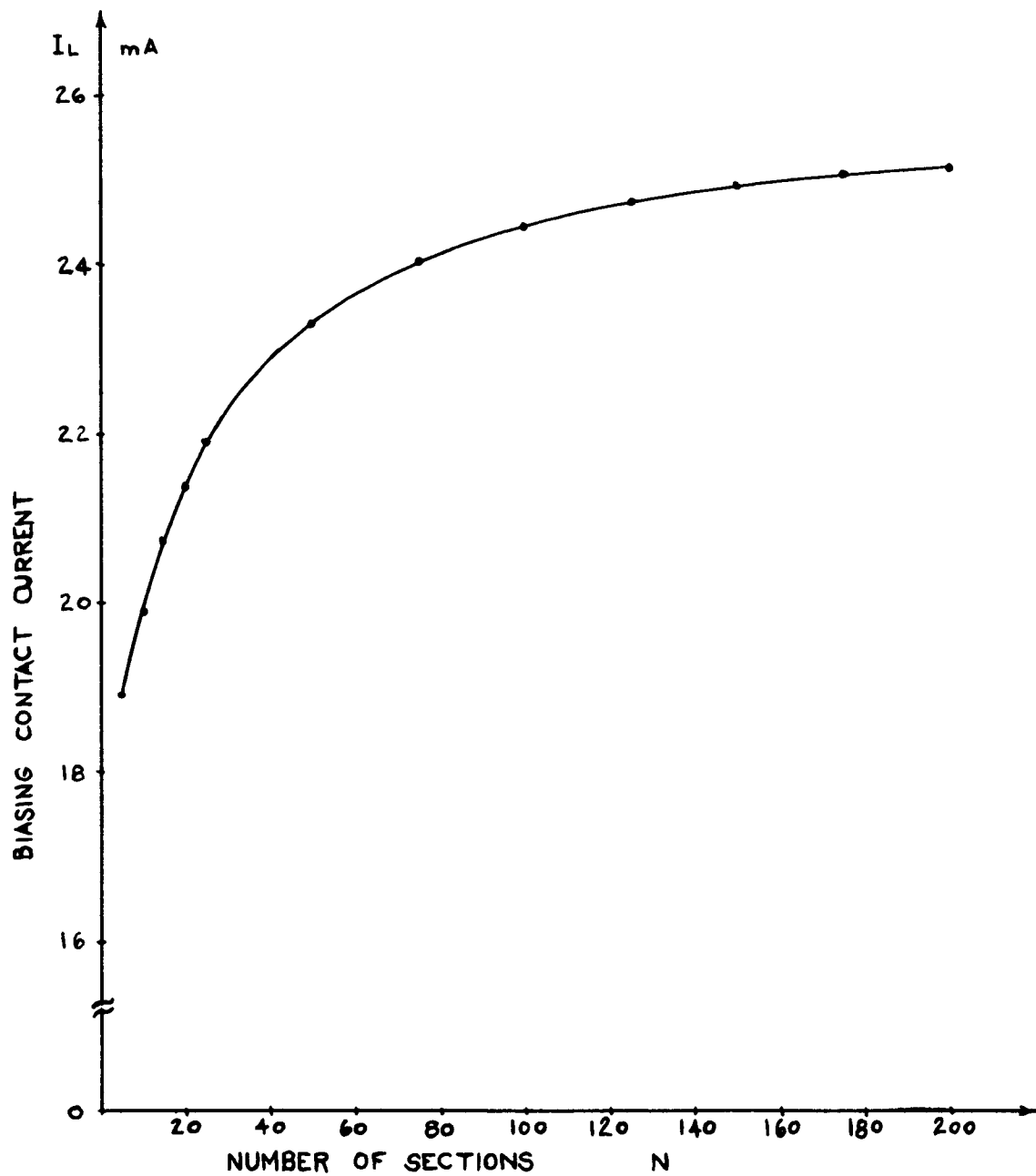


FIGURE 2.5-3 BIASING CONTACT CURRENT VS. NUMBER OF SECTIONS IN MODEL

excellent convergence of the calculated quantities to their asymptotic values.

Calculated input volt-ampere characteristics are shown in Fig. 2.5-4. The input current, I_0 , increases rapidly with $Y(0)$ for $Y(0) \gtrsim 25$ due to the large component of junction current. This limiting type of characteristic makes the device useful as a protection network.

When a load resistance of value R_L is connected from the biasing contact at $z=z_L$ to ground and a voltage $Y(0)$ is applied to the biasing contact at $z=0$ a voltage $Y(z_L)$ is developed across R_L , as illustrated schematically in Fig. 2.5-5. The load line equation, $Y(z_L) = -\frac{R_L}{V_T} I_L$, is plotted for $R_L = 10\Omega$, $1k\Omega$, and ∞ on the characteristics in Fig. 2.5-6. The load line for $R_L = \infty$ coincides with the $Y(z_L)$ axis. The transfer characteristics, shown in Fig. 2.5-7, are obtained from the intersection of the appropriate load line with the volt-ampere characteristics. For $R_L = \infty$, the contact at $z=z_L$ open circuited, $Y(z_L) \approx Y(0)$ for $Y(0) \lesssim 12$ since the current drawn across the junction is not large enough to drop the voltage from $z=0$ to $z=z_L$ noticeably. For $Y(0) > 12$ the magnitude of the junction current results in observable curvature of the transfer characteristic. As $Y(0)$ is increased beyond approximately 20, the voltage in the vicinity of $z=0$ drops very sharply with z resulting in reduced sensitivity of $Y(z_L)$ to changes in $Y(0)$. As a result $Y(z_L)$ is nearly constant at 15.9 for $Y(0) \gtrsim 22$, as seen in Fig. 2.5-6. For a load resistance,

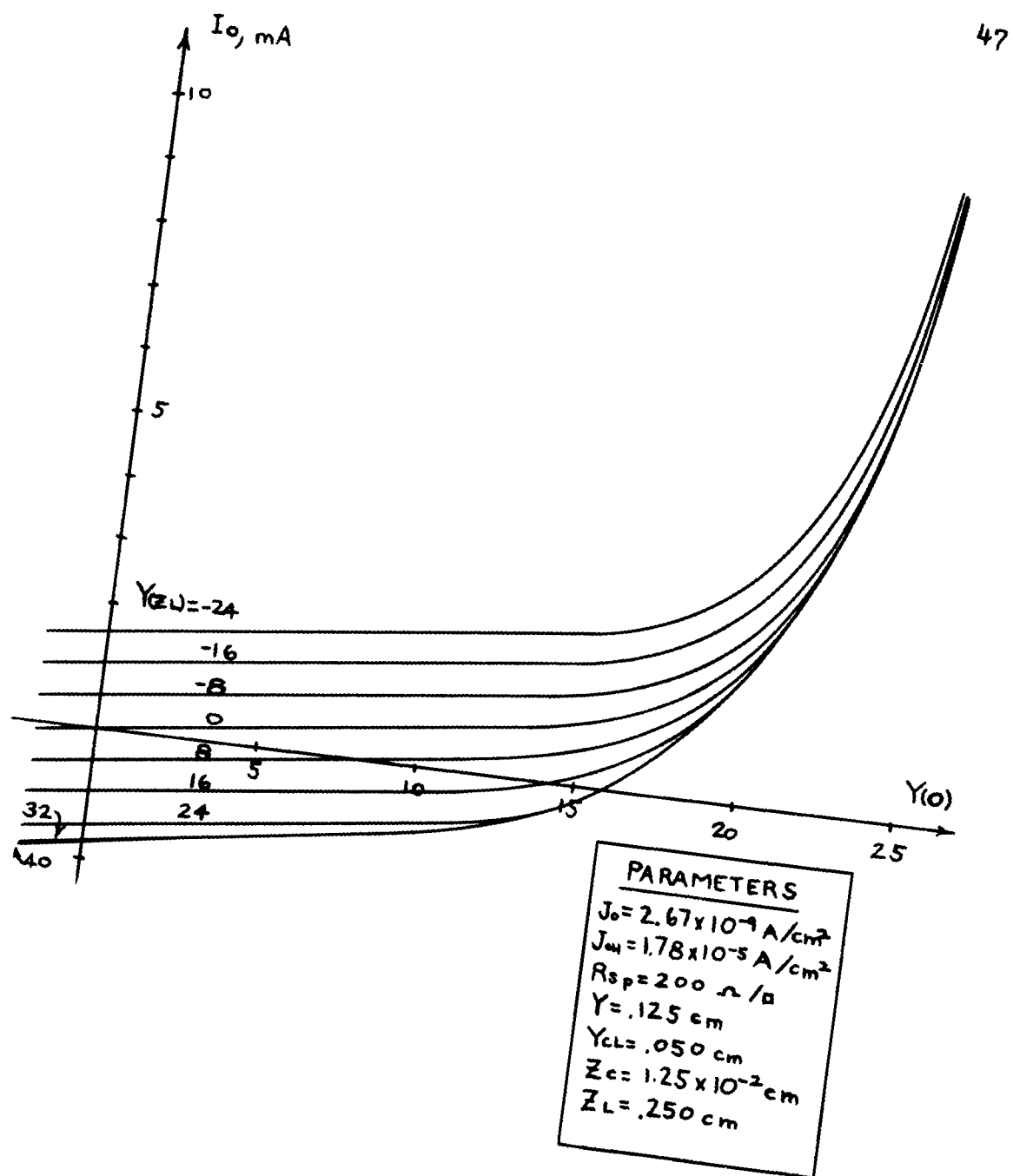


FIGURE 2.5-4 INPUT VOLT-AMPERE CHARACTERISTIC

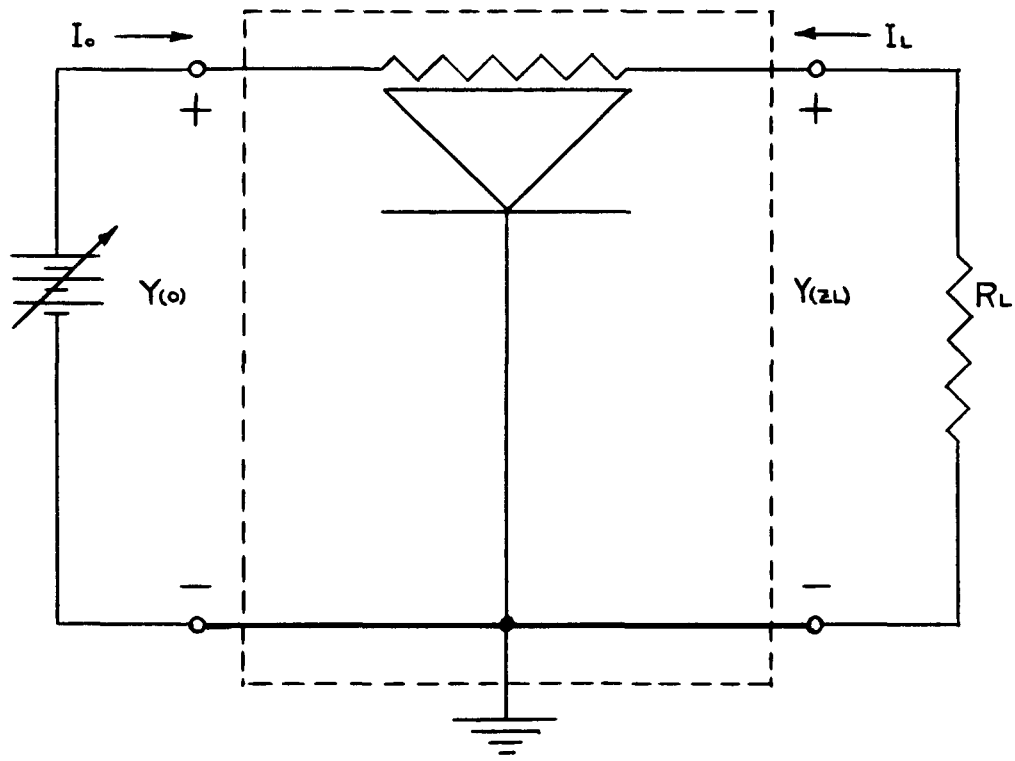


FIGURE 2.5-5 THE DISTRIBUTED P-N DIODE AS A PROTECTION NETWORK

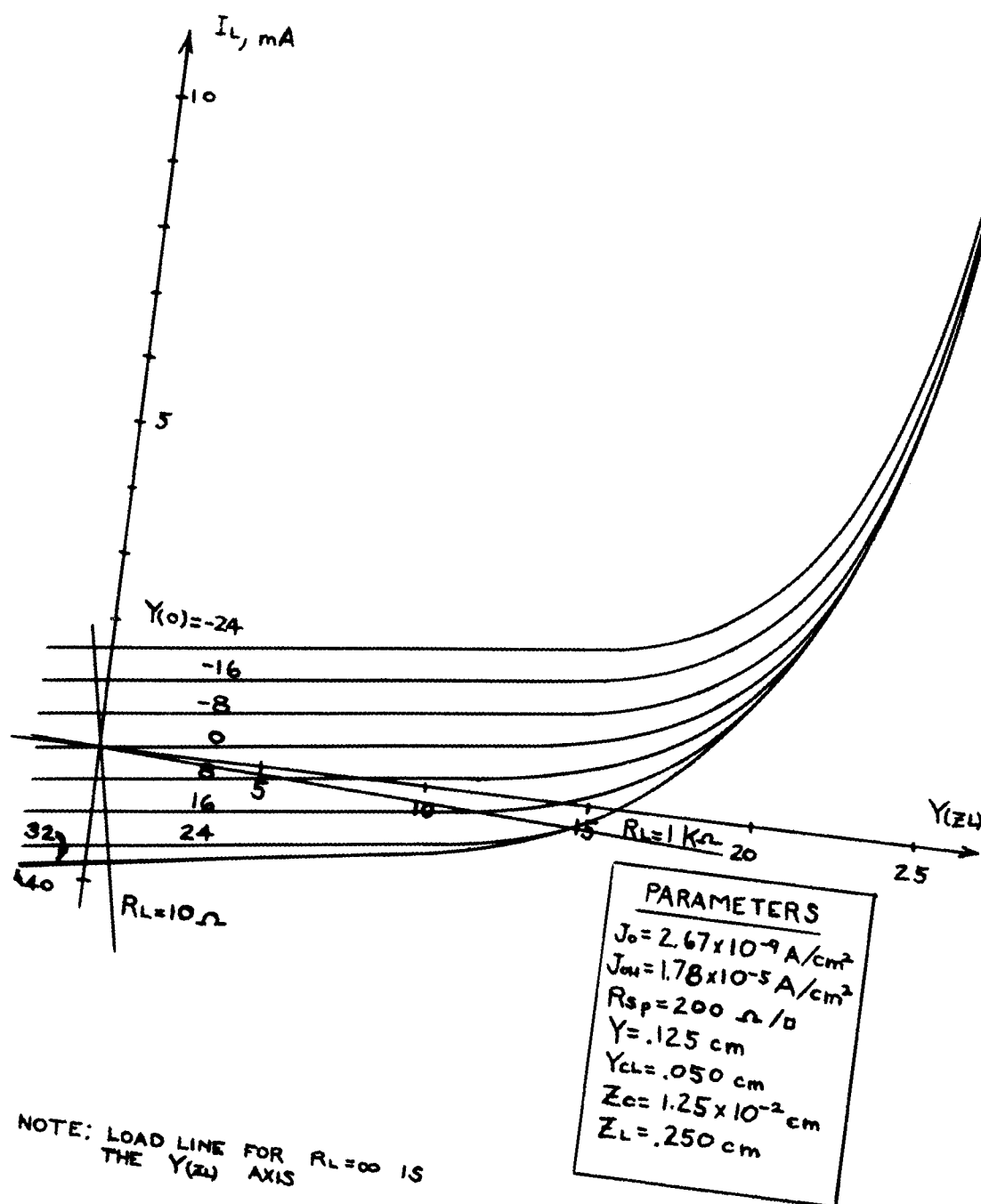


FIGURE 2.5-6 OBTAINING THE TRANSFER CHARACTERISTICS

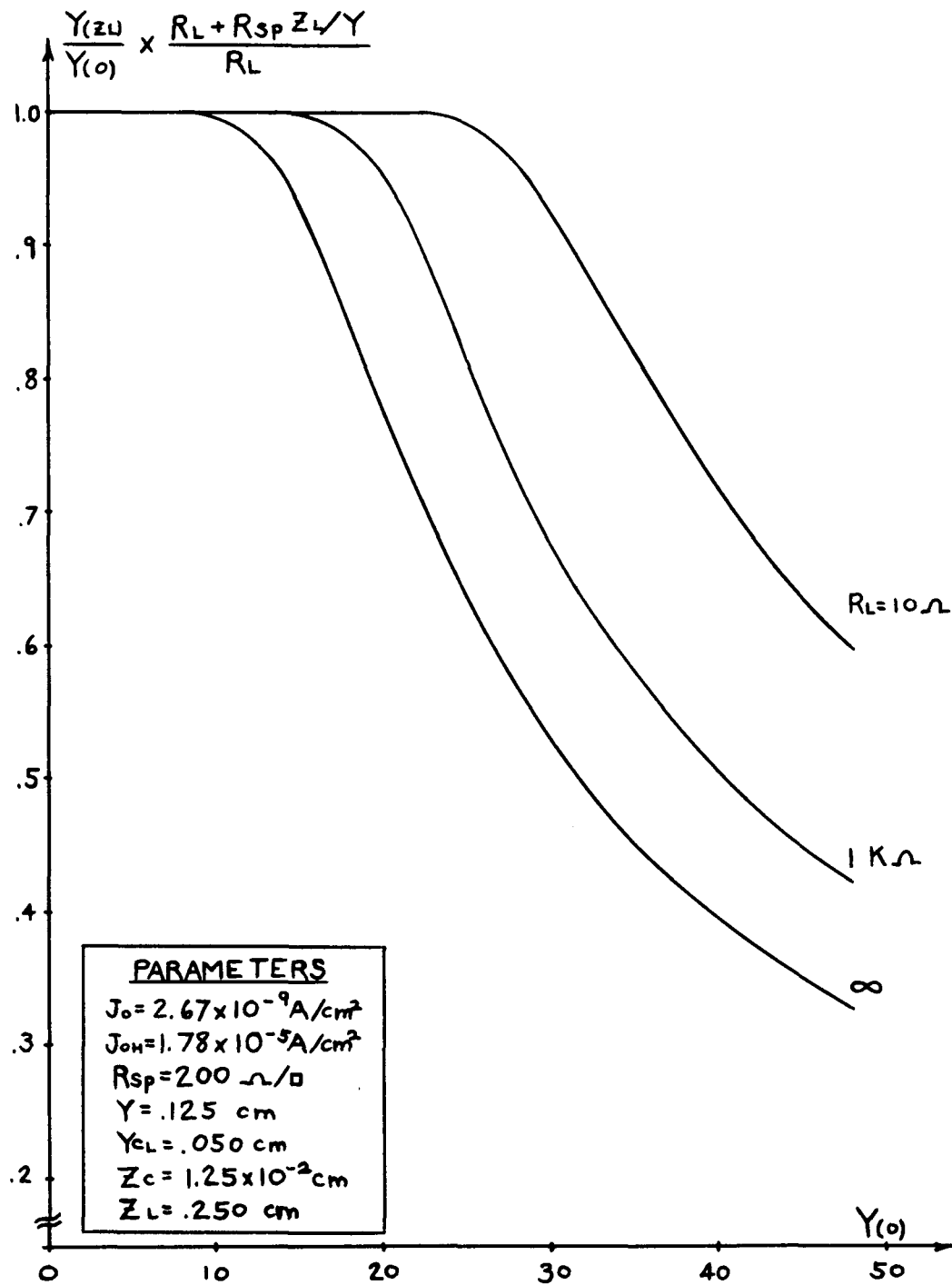


FIGURE 2.5-7 NORMALIZED TRANSFER CHARACTERISTICS

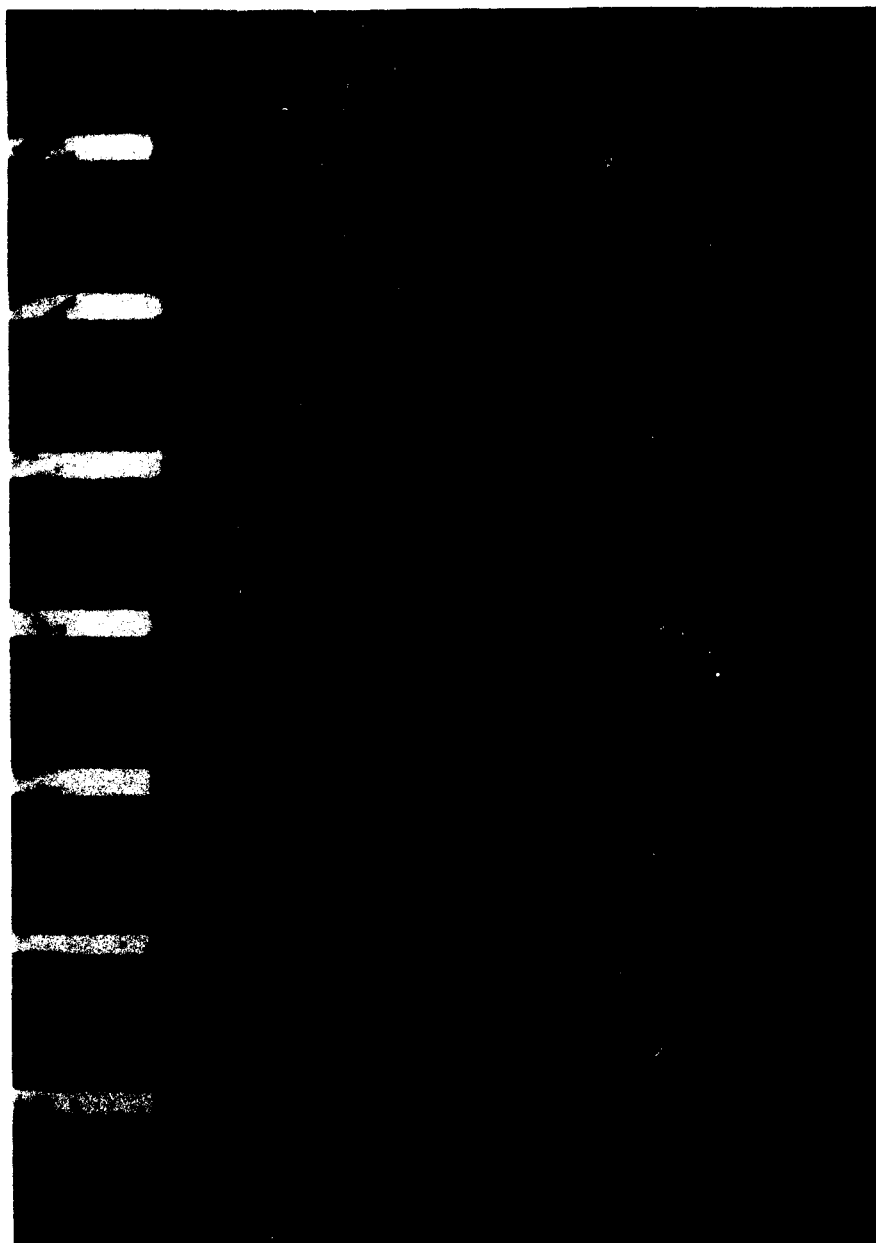
$R_L = 10\Omega$, a value much smaller than the 400Ω resistance of the p layer, the curvature of the transfer characteristic is not as pronounced as for $R_L = \infty$ since the increased z directed p layer current masks the limiting effect of the junction. However, the case for which $R_L = \infty$ is of particular interest since the infinite load resistance statically approximates the gate of a MOSFET. The capacitance associated with the distributed pyn structure and the input of the MOSFET along with the resistance of the p layer integrates and limits transient voltages to non-destructive levels in addition to the dc limiting characteristic discussed previously. Structures similar to the distributed pyn geometry are currently fabricated as part of MOS integrated circuits to serve as gate-oxide protection networks, making use of both forward limiting and reverse breakdown characteristics, to prevent oxide breakdown ¹².

2.6 DC Measurements

The purpose of this section is to compare the theoretically obtained results with measurements made on devices fabricated in the laboratory: J_b -V characteristics, voltage profiles, input volt-ampere characteristics and transfer characteristics.

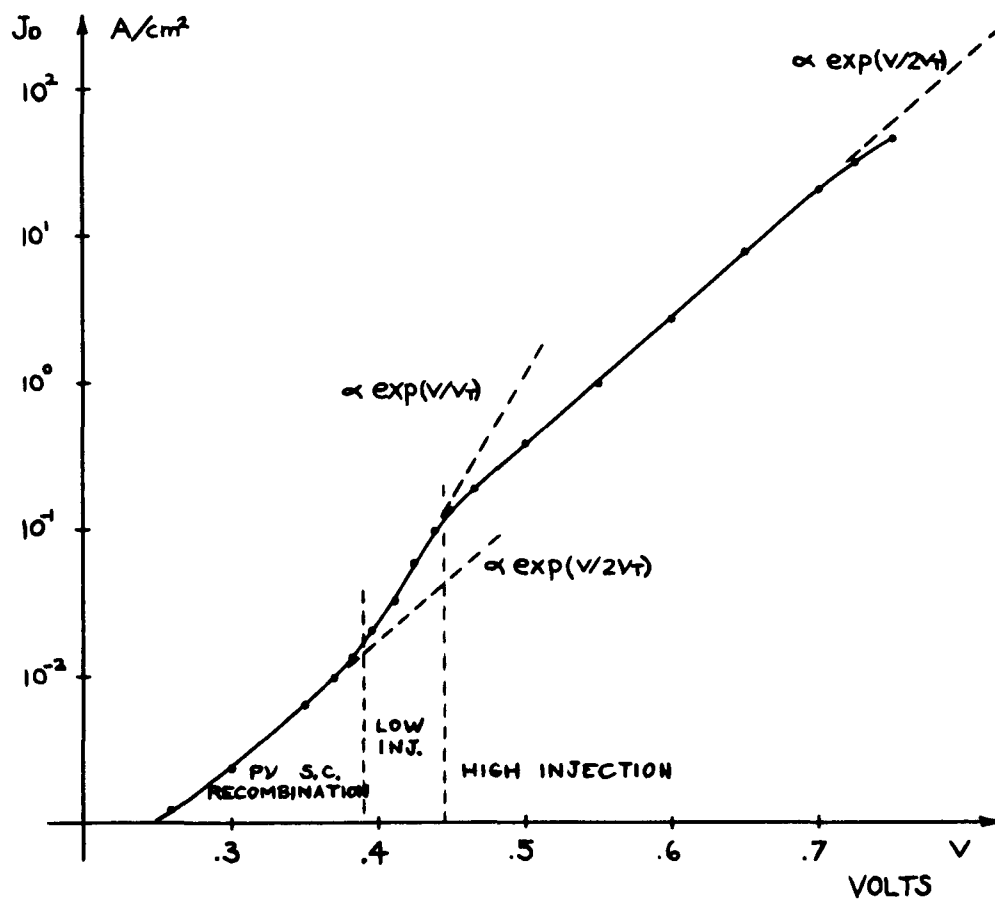
The J_b -V characteristic of the pyn structure is experimentally obtained by measuring the I-V characteristic of a 10 mil x 10 mil test diode fabricated in close proximity

to the distributed pyn structure (so that the test diode has approximately the same impurity profile as the distributed pyn diode in the x direction), illustrated in Fig. 2.6-1. The p region contact is used to apply a specified voltage between the surface of the p region and the groundplane of the test diode. The test diode theoretically has an I-V characteristic that is identical to the I-surface voltage characteristic of a pyn diode of the same impurity profile but without a top contact, as discussed in Appendix I. The I-V characteristic of test diodes were measured using a Tektronix type 575 transistor curve-tracer oscilloscope, permitting measurements at larger values of voltage, without heating and destroying the diode, than if a dc source were used. The J_D -V characteristic obtained by averaging the I-V characteristics of three test diodes on the same wafer is shown in Fig. 2.6-2. Applied forward voltages less than approximately .39 volts result in $J_D \propto \exp(V/2V_T)$ due to the recombination current in the p ν space charge region dominating over the recombination current throughout the rest of the diode¹³. This type of J_D -V characteristic is not predicted by the low level injection model of Appendix I since p ν space charge recombination is not considered. This discrepancy is tolerable since the voltage profiles are nearly linear for $Y(x) \lesssim 17$ and $J_D(x)$ is small enough to make the actual form of the J_D -V characteristic unimportant. The curvatures produced in the voltage profiles for these values of $J_D(x)$ are too



10 mils

FIGURE 2.6-1 P-N TEST DIODE (1 inch $\approx$ 11.76 mils)



PARAMETERS	
$N_A = 4 \times 10^{20} \text{ cm}^{-3}$	
$N_{Dv} = 1 \times 10^{14} \text{ cm}^{-3}$	
$N_{Dn} = 2 \times 10^{21} \text{ cm}^{-3}$	
$W_p = .1 \mu$	
$W_v = 8 \mu$	
$W_n = 250 \mu$	

FIGURE 2.6-2 MEASURED J_0 -V CHARACTERISTIC

small to be easily observed. The current due to recombination outside the $p\nu$ space charge region rapidly becomes larger than the $p\nu$ space charge region recombination current and J_0 becomes proportional to $\exp(V/V_T)$ as V is made larger than approximately .39 volts. This type of dependence is predicted in Appendix I. The low level injection leakage current density, J_0 , is found by solving Eq. I-37 for J_0 and evaluating the resulting expression at a point on the J_0 - V characteristic within the range where $J_0 \propto \exp(V/V_T)$. For the devices being discussed, Batch 773E10, J_0 was found to be 2.67 nA/cm^2 . At approximately $V = .44$ volts J_0 rapidly becomes proportional to $\exp(V/2V_T)$ with increasing V , due to the onset of high level injection in the ν region. The critical voltage, V_c , is theoretically approximated by considering the value of V at which the ν region hole concentration is approximately equal to $N_{D\nu}$. For low level injection in the ν region

$$p_\nu(w_p + x_{d\nu}) = \frac{n_i^2}{N_{D\nu}} \exp(V/V_T) \quad 2.28$$

(see Eq. I-7), which is approximately valid on the low side of V_c . The value of V_c obtained from measurement, .44 volts, when used in Eq. 2.28 gives the result that $p_\nu(w_p + x_{d\nu}) = 10^{14} \text{ cm}^{-3}$ which is equal to the value of $N_{D\nu}$ used in Batch 773E10. This result shows that the measured value of V_c is in excellent agreement with theory.

At forward voltages greater than .44 volts the hole

concentration in the ν region increases into the high level injection regime, part of V is dropped across the νn space charge region and J_0 becomes proportional to $\exp(V/2V_T)$. The $J_0 \propto \exp(V/2V_T)$ dependence is clearly observable in Fig. 2.6-2 up to $V \approx .7$ volts. At approximately $V = .7$ volts J_0 increases less rapidly with V since the ohmic drops in the p and n regions are no longer negligible¹⁴. The voltage drops in the p and n regions are approximately proportional to J_0 since they are predominantly due to ohmic effects of the material; whereas the voltage drop in the ν region, V_ν , is independent of J_0 since it is due to both ohmic and carrier gradient mechanisms¹⁵. The high level injection leakage current density, J_{0H} , is found by solving Eq. I-37 for J_{0H} and evaluating the resulting expression at a point on the J_0 - V characteristic within the range $.44 \lesssim V \lesssim .7$ volts resulting in $J_{0H} = 1.78 \times 10^{-5} \text{ A/cm}^2$.

A very important corollary of the measurement of J_{0H} is the determination of the high level injection carrier lifetime in the ν region, $\tau_{\nu H}$. Carrier lifetime is dependent on the concentration of recombination-generation centers and the values of their capture cross sections. Uncertainty in the types and concentrations of extraneous impurities introduced into the device during fabrication makes it impossible to evaluate the carrier lifetime by applying the well known recombination rate expression¹⁶

$$U = \frac{\sigma_p \sigma_n v_{th} N_t [pn - n_i^2]}{\sigma_n [n + n_i \exp[(E_t - E_i)/kT]] + \sigma_p [p + n_i \exp[(E_i - E_t)/kT]]} \quad 2.29$$

where σ_p and σ_n are the capture crosssections for holes and electrons, v_{th} is the thermal velocity of the carriers and N_t and E_t are the concentration and energy level of the recombination-generation centers. As a result, τ_{yn} is evaluated indirectly using the results of Appendix I. Equation I-31 is rewritten as

$$\Phi_b = \ln \left[\frac{en_i 2 D_{ny} D_{py}}{W_b J_{0H} (D_{ny} + D_{py})} \right] + \ln \left[\frac{W_b}{L_y^*} \right] \quad 2.30$$

where D_{ny} , D_{py} and W_b are specified quantities for the material used and J_{0H} is obtained from measurements on test diodes. Equations 2.30 and I-33 are plotted as a function of $\frac{W_b}{L_y^*}$ in Fig. 2.6-3. Their intersection reveals that $\frac{W_b}{L_y^*} = .63$. Since, for large forward bias, the py and the yn space charge region widths approach zero allowing the approximation that $W_b \approx W_y = 8 \mu$, one obtains $L_y^* = \frac{W_b}{.63} = 12.7 \mu$. The high level injection lifetime is found to be $\tau_{yn} = 1 \times 10^{-7}$ sec from $\tau_{yn} = \frac{L_y^{*2}}{D_y^*}$, where $D_y^* = \frac{2D_{ny}D_{py}}{D_{ny} + D_{py}}$ is the ambipolar diffusion constant. The value of high level injection lifetime obtained is three orders of magnitude smaller than typical values expected from high quality, high resistivity silicon¹⁷. Equation 2.29 for high level injection ($p \approx n$, $pn \gg n_i^2$, $n \gg n_i \exp[(E_t - E_i)/kT]$, $p \gg n_i \exp[(E_i - E_t)/kT]$) reduces to

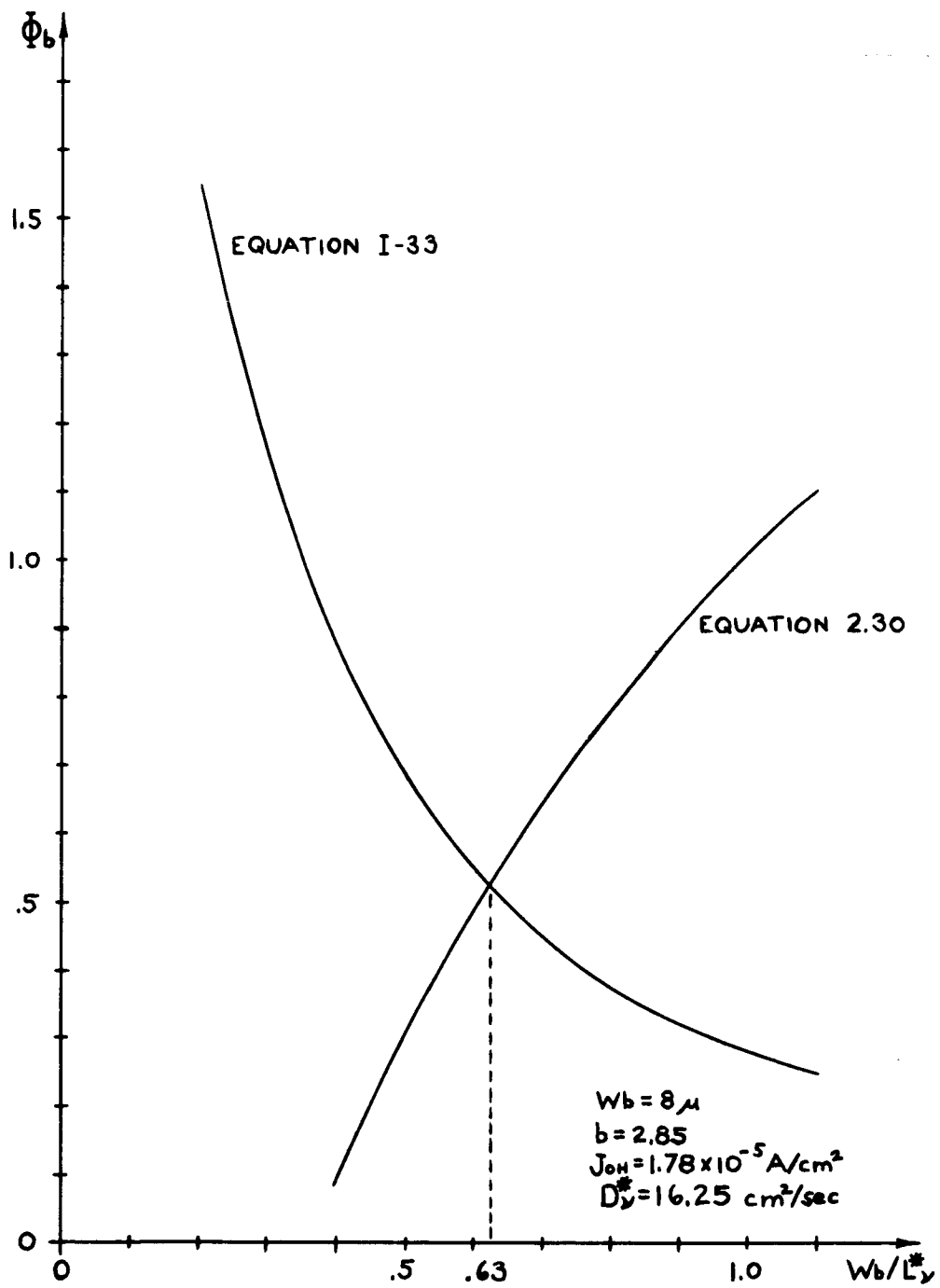


FIGURE 2.6-3 OBTAINING τ_{yH}

$$U = p \sqrt{\frac{\sigma_p + \sigma_n}{\sigma_p \sigma_n V_{th} N_t}} \quad 2.31$$

revealing that

$$\tau_{vn} = \frac{\sigma_p + \sigma_n}{\sigma_p \sigma_n V_{th} N_t} \quad 2.32$$

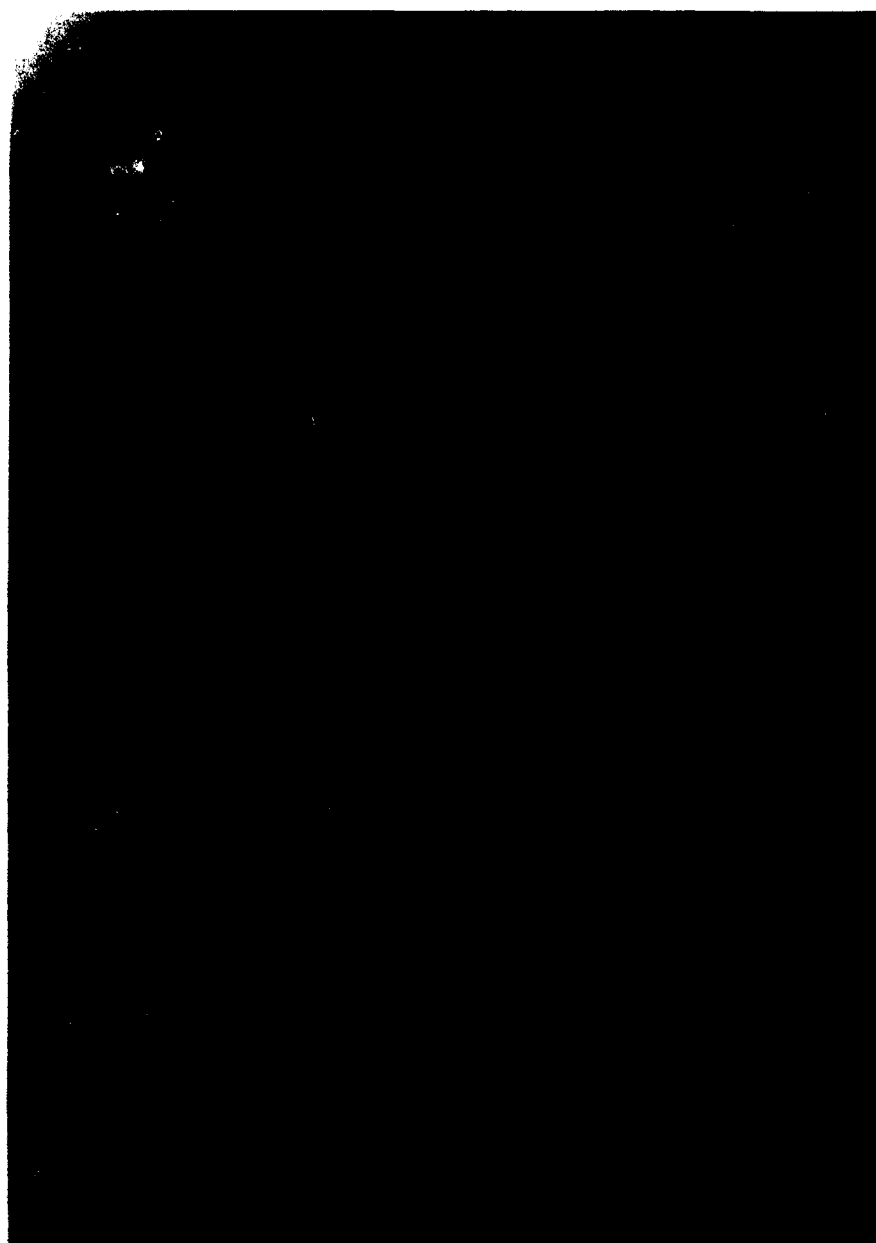
indicating that the short carrier lifetime is due to impurities or crystal structure defects resulting in a high density of recombination-generation centers, N_t . Lack of knowledge of the types and concentrations of the recombination-generation centers makes the evaluation of the low level injection excess carrier lifetime, using Eq. 2.29, impossible. It is therefore assumed that $\tau_{pv} = \tau_{vl} = \tau_{vn} = 1 \times 10^{-7}$ sec, resulting in $L_{pv} = (D_{pv} \tau_{vl})^{1/2} = 10.5 \times 10^{-4}$ cm. Solving Eq. I-21 for α one obtains

$$\alpha = \frac{1}{\beta - 1} \left[\frac{\sinh^2(W_b/L_{pv}) - \beta \cosh^2(W_b/L_{pv})}{\sinh(W_b/L_{pv}) \cosh(W_b/L_{pv})} \right] \approx .407$$

for $W_b = 4.7 \times 10^{-4}$ cm and $\beta = .297$, corresponding to a highly blocking pn junction (see Appendix I).

The reverse I-V characteristics of the test diodes were found to have current densities of less than $1 \mu A/cm^2$ with reverse breakdown voltages of approximately 100 volts.

Surface voltage profiles of the p layer were measured on distributed pn diodes specially designed and fabricated with voltage measuring contacts on the surface of the p layer from $0 < z < z_L$, as shown in Fig. 2.6-4. Each of the eleven



(1 inch $\approx$ 20.83 mils) $\overbrace{\hspace{1.5cm}}^{20 \text{ mils}}$

FIGURE 2.6-4 VOLTAGE PROFILE MEASURING CONTACTS

voltage measuring contacts have an area of 1 mil^2 contacting the surface of the p layer at $y=Y/2$ and are brought out to a contact pad at the edge of the chip via a metalized strip insulated from the surface of the silicon by an SiO_2 layer. The relatively small area of contact to the p layer ensures that the measured voltage profile is perturbed minimally from the true voltage profile that exists on a device without the voltage measuring contacts. Potentiometric voltage measurements were made at each of the voltage measurement contact pads while holding the biasing voltages constant, to obtain the surface voltage profiles. Normalized biasing voltages, $Y(0)$ and $Y(1)$, were not permitted to exceed 24 to minimize thermal drift. Results of the measurements obtained by maintaining equal biasing voltages are compared with the numerical solutions in Fig. 2.6-5. The results show a discrepancy between the measured profile and the theoretical profile which increases with increasing biasing voltage due to the breakdown of Assumption 2 of Sec. 2.1. Sufficiently large values of biasing voltage lead to values of $J_b(x)$ which produce a significant total variation in surface potential in the y direction. The variation of $J_b(x)$ in the y direction associated with the variation in surface potential in the y direction results in smaller values of $\int_0^Y J_b(y, x) dy$ than those predicted from the assumption of uniform surface potential and junction current density in the y direction, leading to the measurement of smaller values of $\frac{d^2 Y(x)}{dx^2}$ and

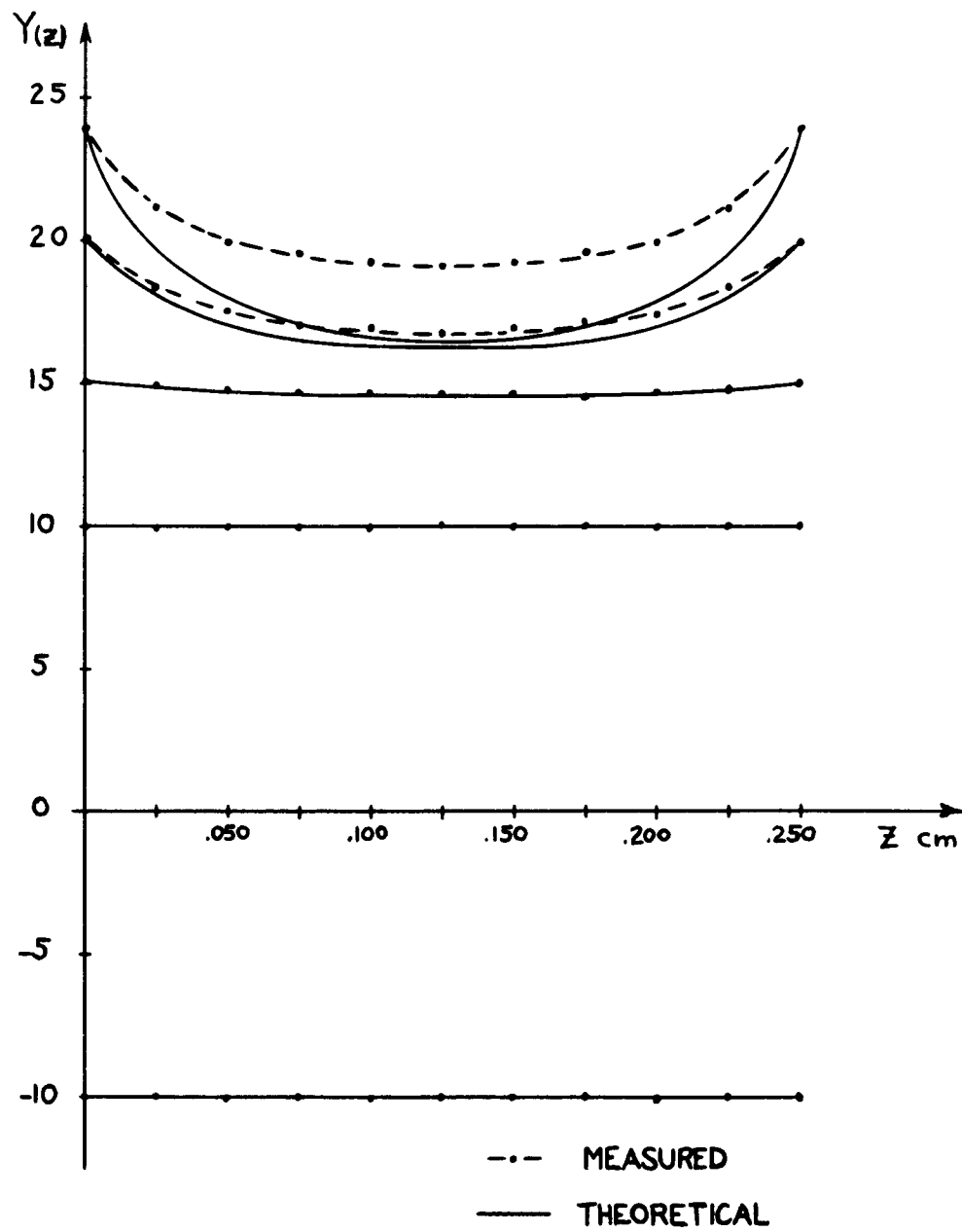


FIGURE 2.6-5 MEASURED VOLTAGE PROFILES, $Y(z) = Y(0)$

$\frac{dY(z)}{dz}$ than those predicted from the model developed in Sec. 2.1. This discrepancy is shown for the worst case in Fig. 2.6-5 since for a specified Y_0 and $Y(z) = Y_0$ the z directed p layer current minimally masks the effect of the junction current density resulting in a greater discrepancy than if $Y(z) \neq Y_0$ and $Y(z) < Y_0$.

A discrepancy, due to another effect, that results in a measured departure from a linear voltage profile was observed for the case of $Y(z) \neq Y_0$, shown in Fig. 2.6-6. For surface voltage profiles that are theoretically nearly linear with z , curvature of the profile was measured in the vicinity of each biasing contact as long as $Y(z) \neq Y_0$, regardless of the polarity of the biasing voltages. This discrepancy results because the edge of the biasing contacts at $y=Y_c$ is not at the same value of y at which the voltage profile is measured. The z directed p layer current crowds in the vicinity of the biasing contacts producing equipotential lines at the surface of the p layer which are not necessarily constant in z from $y=0$ to $y=Y$. As a result, at a specified value of z , the measured voltage in the vicinity of the biasing contacts is lower than the voltage at $y=Y/2$. Increasingly larger values of $|Y_0 - Y(z)|$ result in larger values of z directed p layer current producing increasingly larger voltage discrepancies in the vicinity of the biasing contacts. The curvature in the measured surface voltage profile, due to current crowding near the biasing contacts, is

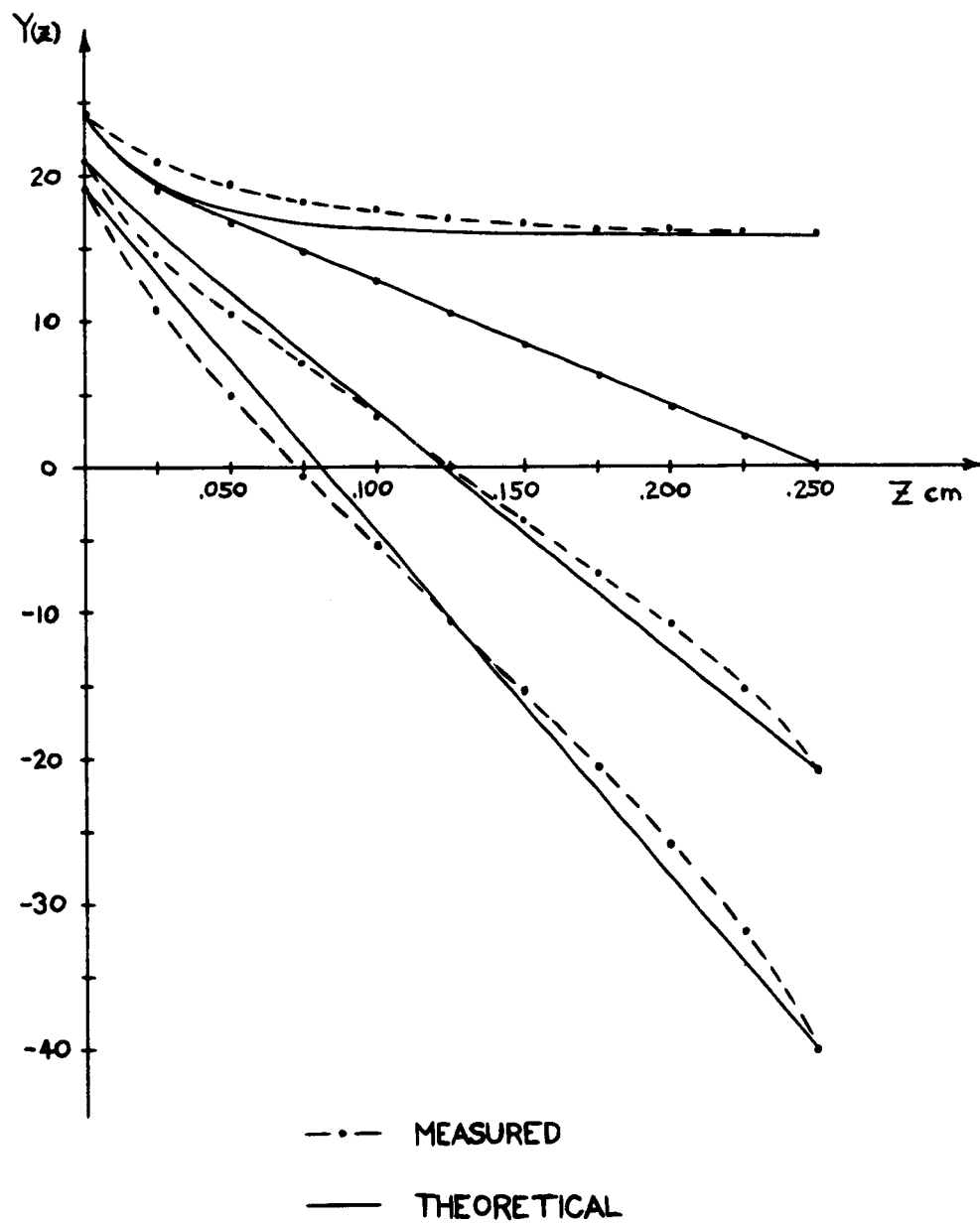
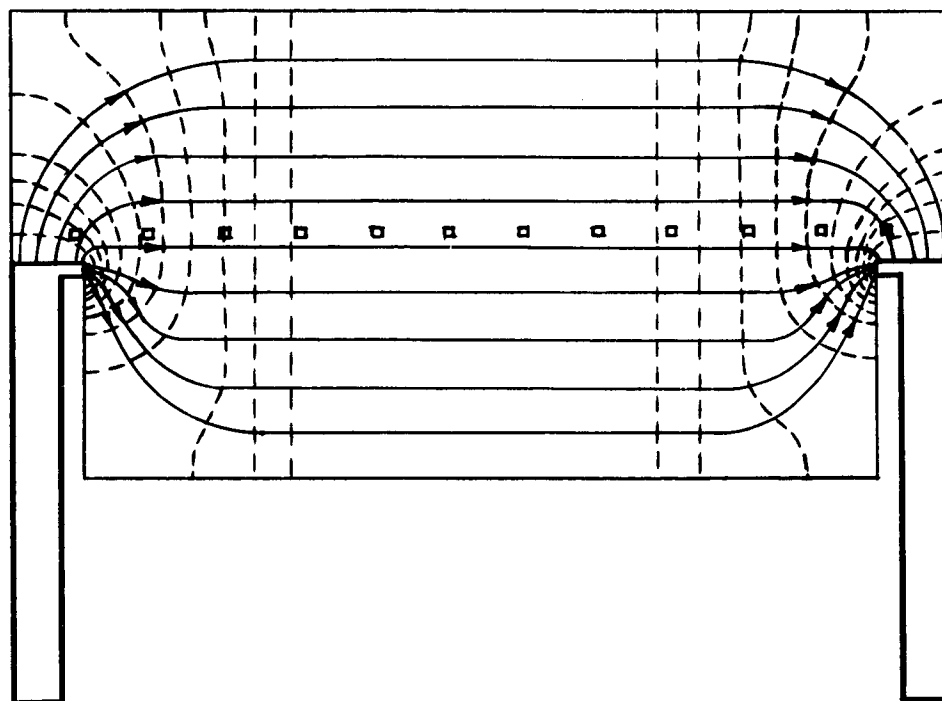


FIGURE 2.6-6 MEASURED VOLTAGE PROFILES, $Y(z) \neq Y(0)$

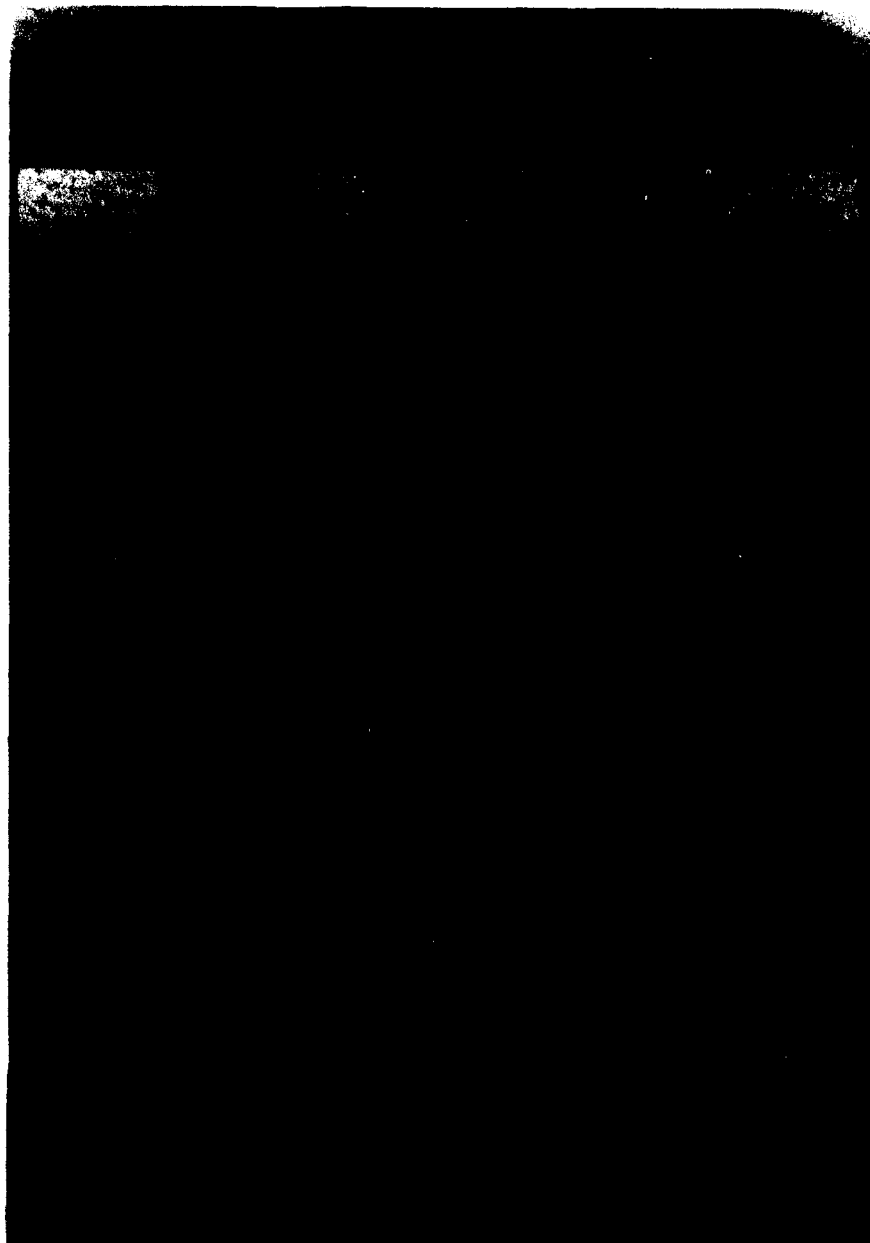
observable regardless of the polarities of the biasing voltages as long as $|Y(0) - Y(z_s)|$ produces a z directed p layer current large enough to mask the curvature producing effect of $J_D(z)$, discussed earlier. As the value of z is removed from the vicinity of the biasing contacts the discrepancy vanishes since the equipotential lines become more constant in z from $y=0$ to $y=Y$. This effect is illustrated and exaggerated qualitatively in Fig. 2.6-7.

Both the discrepancy caused by nonuniform $J_D(z)$ in the y direction and current crowding near the biasing contacts may be reduced by modifying the biasing contact geometry. By designing biasing contacts extending from $y=0$ to $y=Y$ more uniform z directed current density and surface potential, with respect to y , are established. However, this type of geometry is not desirable since it precludes the use of a microstrip transmission line. Separation of each biasing contact allows a microstrip transmission line to extend over the p layer, although both halves of each biasing contact must be contacted and maintained at the same voltage to force the desired potential variation. Figure 2.6-8 shows a chip, fabricated for voltage profile measurements, containing a device with the extended contacts (from $y=0$ to $y=Y$) and another device with the split contact geometry. Measurements on these devices show that the voltage differences between the measured and theoretical profiles are reduced to approximately one tenth of those measured on the devices with the



—→— CURRENT FLOW LINES
- - - - - EQUIPOTENTIAL LINES

FIGURE 2.67 CURRENT CROWDING



(1 inch $\approx$ 23.81 mils) 20 mils

FIGURE 2.6-8 ALTERNATE BIASING CONTACT GEOMETRY WITH
VOLTAGE PROFILE MEASURING CONTACTS

short biasing contact geometry. These small discrepancies are difficult to observe and are, therefore, not shown.

The input volt-ampere characteristics were measured using a Tektronix type 575 transistor curve-tracer oscilloscope and are shown in Fig. 2.6-9. The volt-ampere characteristic at the biasing contact at $z=0$ was displayed while the voltage at the biasing contact at $z=z_1$ was stepped. Excellent agreement between the theoretical and measured characteristics were obtained and is shown for two values of $Y(z_1)$ in Fig. 2.6-10. The introduction of a small bottom contact resistance, which would occur with a poor ohmic contact, produces a shifting of the characteristics in the positive $Y(0)$ direction for $Y(z_1) \gg 25$. Figure 2.6-11 shows how the introduction of a bottom contact resistance of approximately $.02 \Omega\text{-cm}^2$, determined from test diode measurements, effects the characteristics. This shifting effect has a deleterious effect on the operation of the structure as a protection network since $Y(z_1)$ no longer remains constant with changes in $Y(0)$ for large values of $Y(0)$. The importance of providing a low resistance bottom contact cannot be overemphasized.

The total resistance of the diffused p layer between biasing contacts with the p-v junction reverse biased may be derived from the measured volt-ampere characteristics. Measurement of the slope of the input volt-ampere characteristics for $Y(0) < 0$ and $Y(z_1) < 0$ reveals a p layer resistance of 400Ω which corresponds to a $200 \Omega/\square$ sheet resistance of the p



Horizontal: $V_T Y(0)$, .2 V/DIV
 Vertical: I_0 , 2 mA/DIV
 Steps: $V_T Y(0)$, .2 V/STEP
 Top Curve: $V_T Y(0) = -1.2$ V
 Curve Through Origin: $V_T Y(0) = 0$ V

FIGURE 2.6-9 DISPLAYED INPUT VOLT-AMPERE CHARACTERISTICS

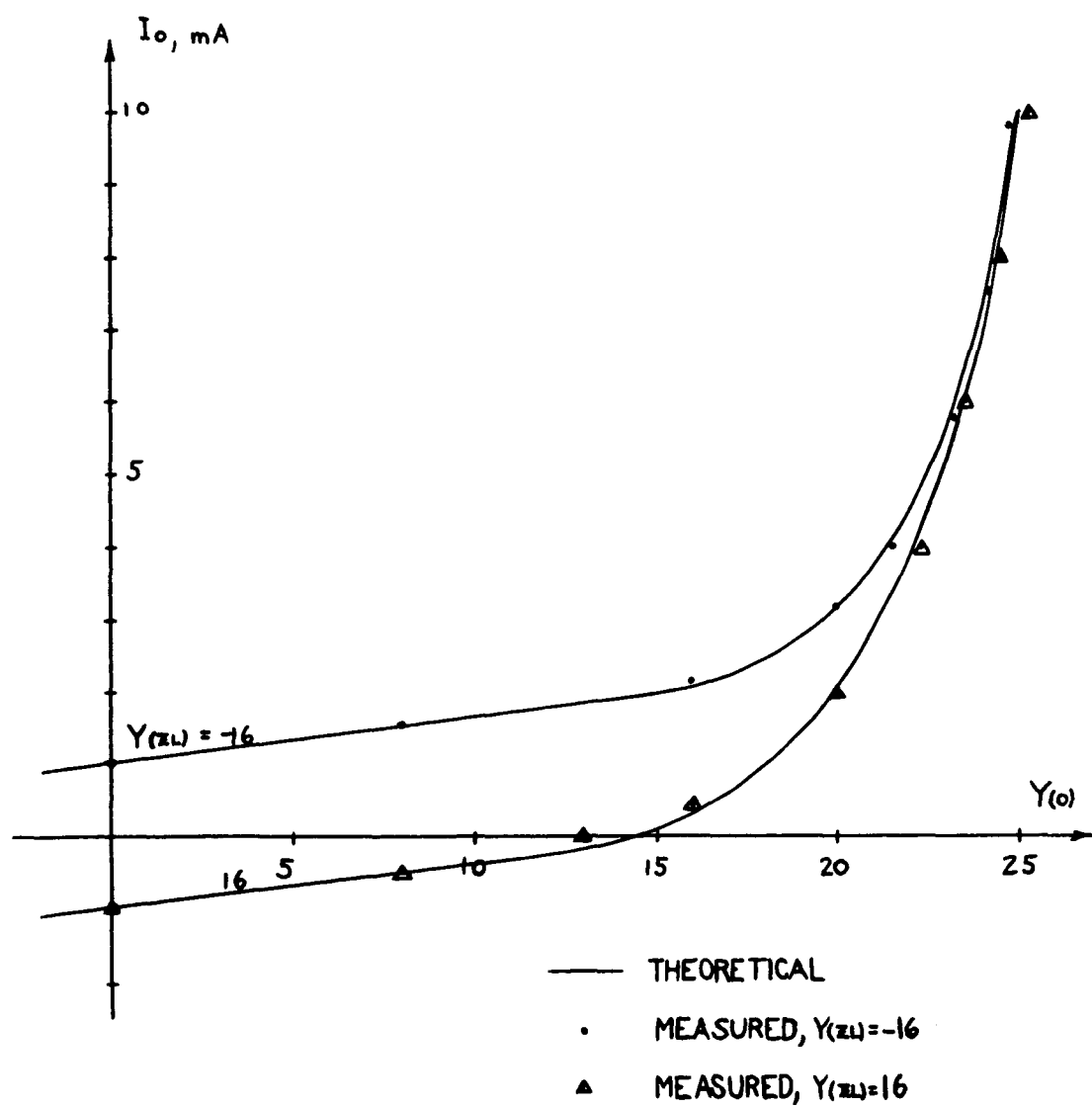
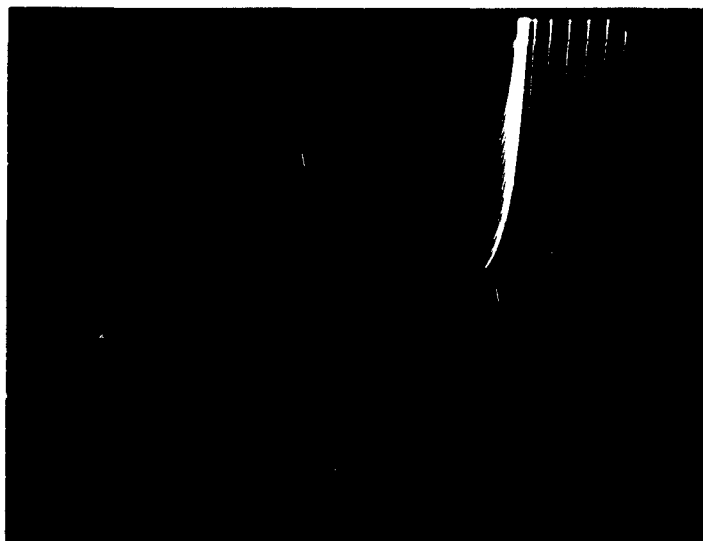


FIGURE 2.6-10 MEASURED INPUT VOLT-AMPERE CHARACTERISTICS



Horizontal: $V_T Y_{(0)}$, .2 V/DIV
 Vertical: I_0 , 2 mA/DIV
 Steps: $V_T Y_{(ZL)}$, .2 V/STEP
 Top Curve: $V_T Y_{(ZL)} = -1.8$ V
 Curve Through Origin: $V_T Y_{(ZL)} = 0$ V

Bottom Contact Resistance = $.02 \Omega\text{-cm}^2$

FIGURE 2.6-11 EFFECT OF BOTTOM CONTACT RESISTANCE

layer, for the geometry used.

Figure 2.6-12 shows the excellent agreement between the measured and calculated transfer characteristics, supporting the usefulness of the structure as a protection network.

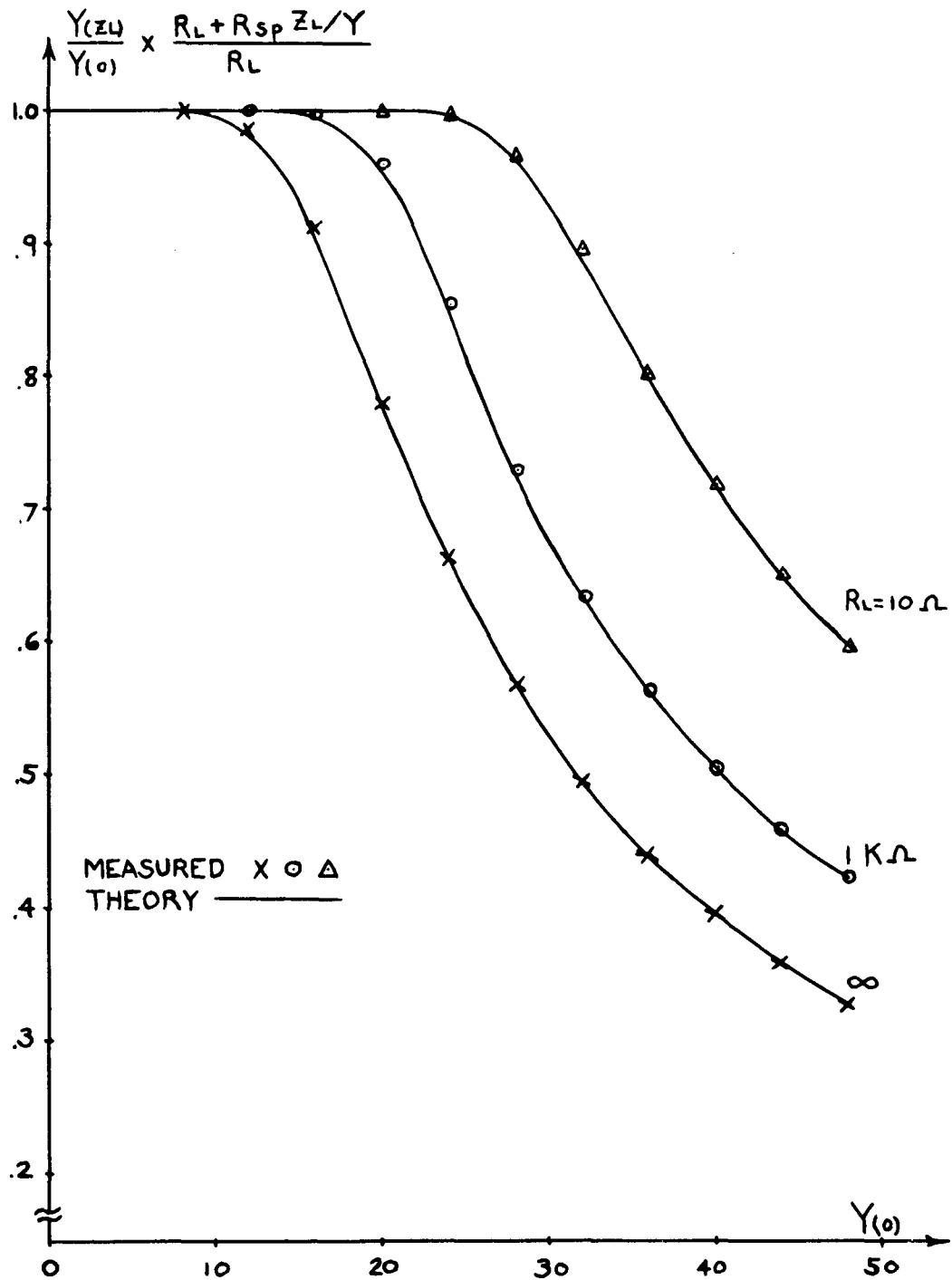


FIGURE 2.6-12 MEASURED TRANSFER CHARACTERISTICS

CHAPTER 3 - RF ANALYSIS OF THE M-O-D P/N DEVICE

The purpose of this chapter is to develop a small signal rf model characterizing electromagnetic wave propagation on the microstrip-oxide-distributed p/n device. The driving point input impedance and input-output voltage transmission coefficient, of the microstrip line, is evaluated as a function of dc biasing voltages, excitation frequency and device parameters. Experimental results are compared to the theory and the feasibility of the use of the device as a dc voltage controllable stub tuner is discussed.

3.1 Electromagnetic Wave Propagation on a Microstrip-Oxide-Distributed P/N Structure

Electromagnetic wave propagation on a microstrip-oxide-distributed p/n structure (referred to as the m-o-d structure), shown in Fig. 3.1-1, is investigated by modeling the variation of the wave propagation characteristics in the z direction due to the variation of the p layer dc surface voltage as a cascade of short m-o-d structures, each having uniform propagation characteristics. The overall operation of the device is characterized by a voltage-current transmission matrix representation which is reduced to a simplified form. Figure 3.1-1 illustrates the rf voltages and currents of interest, allowing one to write

$$V_{in} = A V_L + B I_L \quad 3.1$$

$$I_{in} = C V_L + D I_L \quad 3.2$$

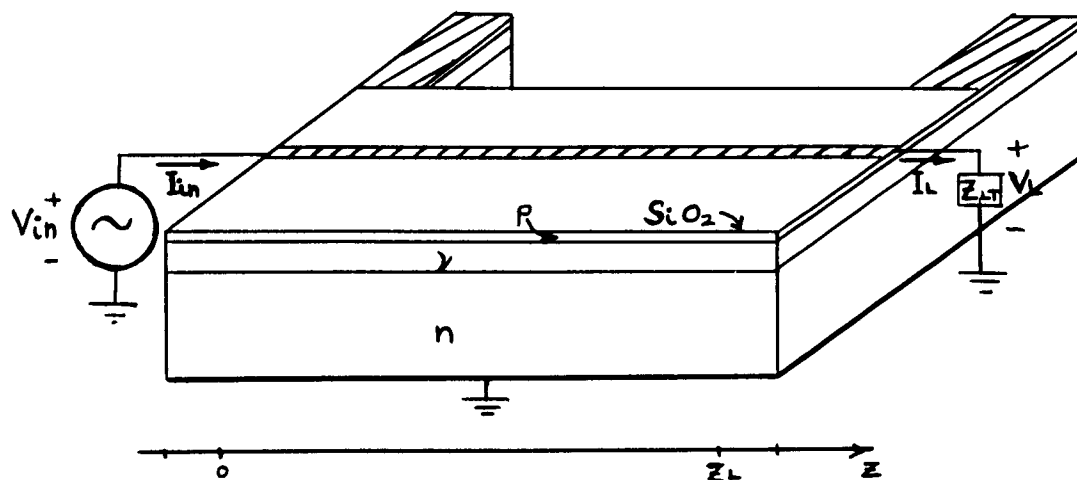


FIGURE 3.1-1 VOLTAGES AND CURRENTS OF INTEREST

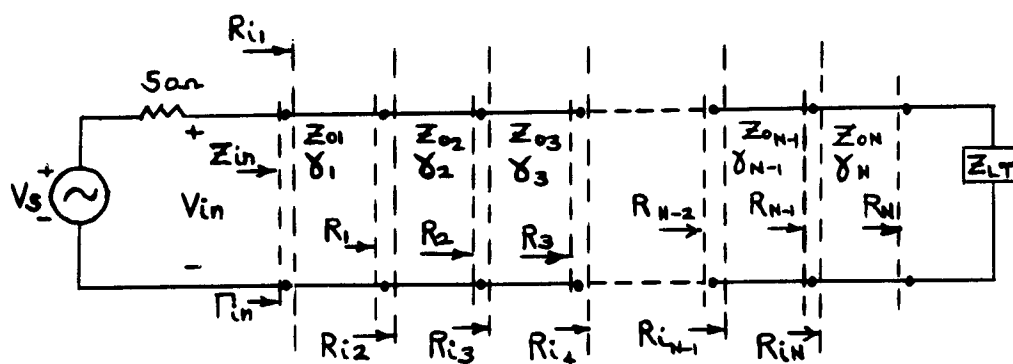


FIGURE 3.1-2 N CASCADED SEGMENT MODEL

Since $V_L = I_L Z_{LT}$, Eq. 3.1 is expressed as

$$\frac{V_L}{V_{in}} = \frac{1}{A + B/Z_{LT}} \quad , \quad 3.3$$

where V_L/V_{in} is the input-output voltage transmission coefficient, T_{io} . Similarly, substitution of Eq. 3.3 into Eq. 3.2 results in

$$\frac{V_{in}}{I_{in}} = \frac{A + B/Z_{LT}}{C + D/Z_{LT}} \quad 3.4$$

the input driving point impedance, Z_{in} .

As with the dc model, the device is divided into N uniform segments of length $DZ = z_L/N$, in the z direction. The i^{th} segment is assumed to have a uniform p layer dc surface voltage equal to the voltage at the i^{th} node (the junction between the i^{th} and $i+1^{th}$ segments), found from the dc solution, and is characterized by a complex propagation constant, γ_i , and a characteristic impedance, Z_{oi} . The driving point impedance and the input-output voltage transmission coefficient are evaluated from the N cascaded segment model, shown in Fig. 3.1-2.

The reflection coefficient seen by the N^{th} segment at $z = z_L$, looking into the terminating impedance Z_{LT} , is

$$R_N = (Z_{LT} - Z_{oN}) / (Z_{LT} + Z_{oN}) \quad . \quad \text{The reflection coefficient at the input to the } N^{th} \text{ segment, found from}$$

$R_N = R_N \exp(-2 \gamma_N DZ)$, allows the driving point impedance

seen looking into the N^{th} segment to be evaluated as

$$Z_N = Z_{oN} [(1 + R_{in}) / (1 - R_{in})] \quad . \quad \text{In turn, the reflec-}$$

tion coefficient seen by the $N-1^{\text{th}}$ segment at $z=z_L-DZ$ due to Z_N , found from $R_{N-1}=(Z_N-Z_{0N-1})/(Z_N+Z_{0N-1})$, allows R_{N-1} and Z_{N-1} to be obtained. Repetition of this process until the input to the first segment is reached results in the evaluation of the driving point impedance seen looking into the input of the microstrip line.

The input-output voltage transmission coefficient is found by multiplying the voltage transmission matrices of each of the cascaded two-port sections, defined within the dotted boundaries of Fig. 3.1-3, together. The distinction between a two-port section and a parallel-plate waveguide segment cannot be overemphasized. As discussed earlier in this section, a parallel-plate waveguide segment has uniform dc surface voltage, characteristic impedance and propagation constant. However, each two-port section contains one parallel - plate waveguide segment and its junction with one adjacent segment. For example, the i^{th} two-port section, contained within the dotted boundaries of Fig. 3.1-4, consists of the i^{th} parallel-plate waveguide segment and the junction of the $i-1^{\text{th}}$ and i^{th} segments. Therefore, the input-output characterization of the i^{th} two-port section considers the effect of the discontinuity in characteristic impedance between the $i-1^{\text{th}}$ and i^{th} segments in addition to the phase shift and attenuation due to propagation through the i^{th} segment. The forward and backward traveling voltage waves at the input of the i^{th} two-port section, V_{i-}^+ and V_{i-}^- ,

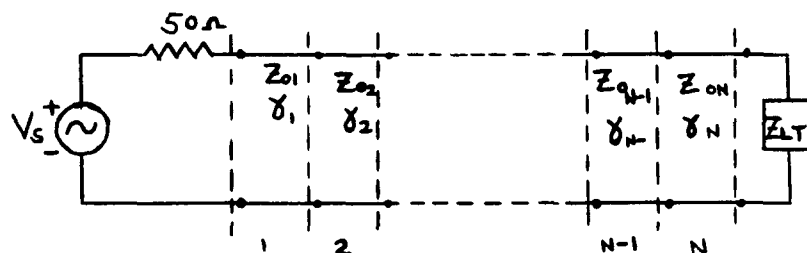


FIGURE 3.1-3 CASCADE OF TWO-PORT SECTIONS

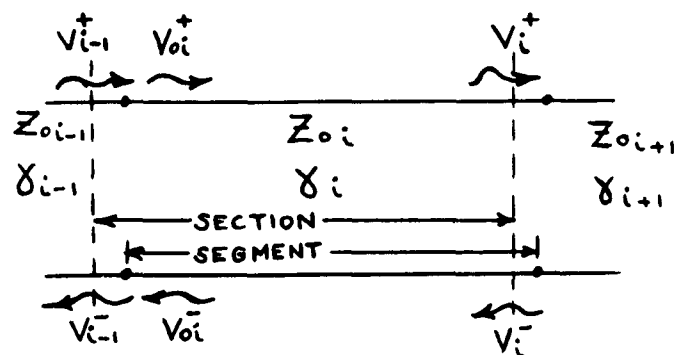


FIGURE 3.1-4 THE i^{th} TWO-PORT SECTION

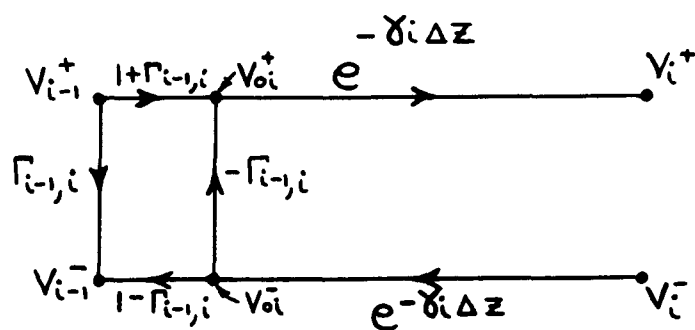


FIGURE 3.1-5 SIGNAL FLOW GRAPH OF i^{th} SECTION

respectively, are also the forward and backward traveling voltage waves at the end of the $i-1^{\text{th}}$ waveguide segment closest to the load. Similarly, V_i^+ and V_i^- , the forward and backward waves at the output of the i^{th} two-port section are also the forward and backward waves at the end of the i^{th} waveguide segment closest to the load. In addition, V_{oi}^+ and V_{oi}^- , the forward and backward waves at the end of the i^{th} waveguide segment closest to the source are internal to the i^{th} two-port section. These voltages are summarized in Table 3.1-1.

VOLTAGE WAVE	LOCATION IN TWO-PORT SECTION	LOCATION IN SEGMENT
V_{i-1}^+ , V_{i-1}^-	Input of i^{th} section	Side of $i-1^{\text{th}}$ segment closest to load
V_{oi}^+ , V_{oi}^-	Internal to i^{th} section	Side of i^{th} segment closest to source
V_i^+ , V_i^-	Output of i^{th} section	Side of i^{th} segment closest to load

TABLE 3.1-1

The voltage transmission matrix formulation for the i^{th} two-port section of the device, shown in Fig. 3.1-4, is

$$\begin{bmatrix} V_{i-1}^+ \\ V_{i-1}^- \end{bmatrix} = \begin{bmatrix} a_{11}^i & a_{12}^i \\ a_{21}^i & a_{22}^i \end{bmatrix} \begin{bmatrix} V_i^+ \\ V_i^- \end{bmatrix} \quad 3.5$$

and is easily evaluated using signal flow techniques. The signal flow graph of the i^{th} two-port section is shown in

Fig. 3.1-5. The top nodes represent forward traveling voltage waves while the bottom nodes represent backward traveling waves. The two left hand nodes are the input voltages, V_{i-1}^+ and V_{i-1}^- , of the i^{th} two-port section. The two right hand nodes are the output voltages, V_i^+ and V_i^- , of the i^{th} two-port section. The two middle nodes are the voltages V_{oi}^+ and V_{oi}^- , internal to the i^{th} two-port section.

The reflection coefficient that V_{i-1}^+ sees due to the discontinuity in characteristic impedance at the $(i-1)^{\text{th}}$ segment- i^{th} segment junction is $\Gamma_{i-1,i} = (Z_{oi} - Z_{oi-1}) / (Z_{oi} + Z_{oi-1})$; similarly, the backward traveling voltage wave at the i^{th} segment side of the junction, V_{oi}^- , sees $-\Gamma_{i-1,i}$. The transmission coefficients of the forward and backward voltage waves across the junction are $1 + \Gamma_{i-1,i}$ and $1 - \Gamma_{i-1,i}$, respectively. The factor $e^{-\gamma_i \Delta z}$ accounts for the propagation of the forward and backward waves over the i^{th} segment. Adding the com-

ponents contributing to V_i^+ yields

$$V_i^+ = (1 + \Gamma_{i-1,i}) e^{-\gamma_i \Delta z} V_{i-1}^+ - \Gamma_{i-1,i} e^{-2\gamma_i \Delta z} V_{i-1}^- \quad 3.6$$

which is written as

$$V_{i-1}^+ = \frac{1}{1 + \Gamma_{i-1,i}} \left[e^{\gamma_i \Delta z} V_i^+ + \Gamma_{i-1,i} e^{-\gamma_i \Delta z} V_i^- \right] \quad 3.7$$

Similarly, summing the components of V_{i-1}^- gives

$$V_{i-1}^- = \Gamma_{i-1,i} V_{i-1}^+ + (1 - \Gamma_{i-1,i}) e^{-\gamma_i \Delta z} V_i^- \quad 3.8$$

which reduces to

$$V_{i-1}^- = \frac{1}{1 + \Gamma_{i-1,i}} \left[\Gamma_{i-1,i} e^{\gamma_i \Delta z} V_i^+ + e^{-\gamma_i \Delta z} V_i^- \right], \quad 3.9$$

upon substitution of Eq. 3.7 into Eq. 3.8. Letting $T_{i-1,i} = 1 + \Gamma_{i-1,i}$ in Eqs. 3.7 and 3.9 yields the result that

$$a_{i1}^i = \frac{e^{\gamma_i \Delta z}}{T_{i-1,i}}, \quad a_{i2}^i = \frac{\Gamma_{i-1,i}}{T_{i-1,i}} e^{-\gamma_i \Delta z},$$

$$a_{21}^i = \frac{\Gamma_{i-1,i}}{T_{i-1,i}} e^{\gamma_i \Delta z} \quad \text{and} \quad a_{22}^i = \frac{e^{-\gamma_i \Delta z}}{T_{i-1,i}} .$$

Multiplication of the cascaded matrices results in the relationship

$$\begin{bmatrix} V_{in}^+ \\ V_{in}^- \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} V_N^+ \\ V_N^- \end{bmatrix} , \quad 3.10$$

where the matrix elements are evaluated on a digital computer. The total input voltage is written as

$$V_{in} = V_{in}^+ + V_{in}^- = (A_{11} + A_{21}) V_N^+ + (A_{12} + A_{22}) V_N^- . \quad 3.11$$

The total output voltage is written as

$$V_L = V_N^+ + V_N^- \quad 3.12$$

Using the relationship that

$$V_N^- = R_N V_N^+ \quad 3.13$$

Eq. 3.12 is written as

$$V_L = (1 + R_N) V_N^+ . \quad 3.14$$

Substitution of Eqs. 3.13 and 3.14 into Eq. 3.11 yields

$$T_{io} = \frac{V_L}{V_{in}} = \frac{1 + R_N}{A_{11} + A_{21} + (A_{12} + A_{22}) R_N} . \quad 3.15$$

3.2 Parallel-Plate Waveguide Model of Segment

To evaluate the expressions for the driving point impedance and input-output voltage transmission coefficient of the m-o-d structure, developed in the last section, it is necessary to obtain the propagation constant and characteristic impedance as a function of dc surface voltage. While an exact electromagnetic analysis of the m-o-d segment is desirable it is not achievable because the boundary conditions that must be satisfied require an infinite number of modes. The structure is approximated by a parallel-plate waveguide filled with the material between the microstrip line and the groundplane in the actual device, as shown in Fig. 3.2-1. A parallel-plate waveguide filled with silicon dioxide on silicon has been analyzed by Guckel et. al.¹⁸ and by Hasegawa et. al.¹⁹ who show that the lowest order mode is an E mode; the structure shown in Fig. 3.2-1 is analyzed by an extension of their method. The solution of the problem at hand is complicated by the existence of five layers and the anisotropic complex permittivity of the py space charge region. The effective complex permittivity, of the py space charge region, transverse to the junction, ϵ_{3t}^* , is found from the small signal admittance of the py junction in Appendix II. The effective complex permittivity parallel to the junction, ϵ_{3l}^* , is much more difficult to analyze since the carrier concentrations in the space charge region are difficult to evaluate. For -

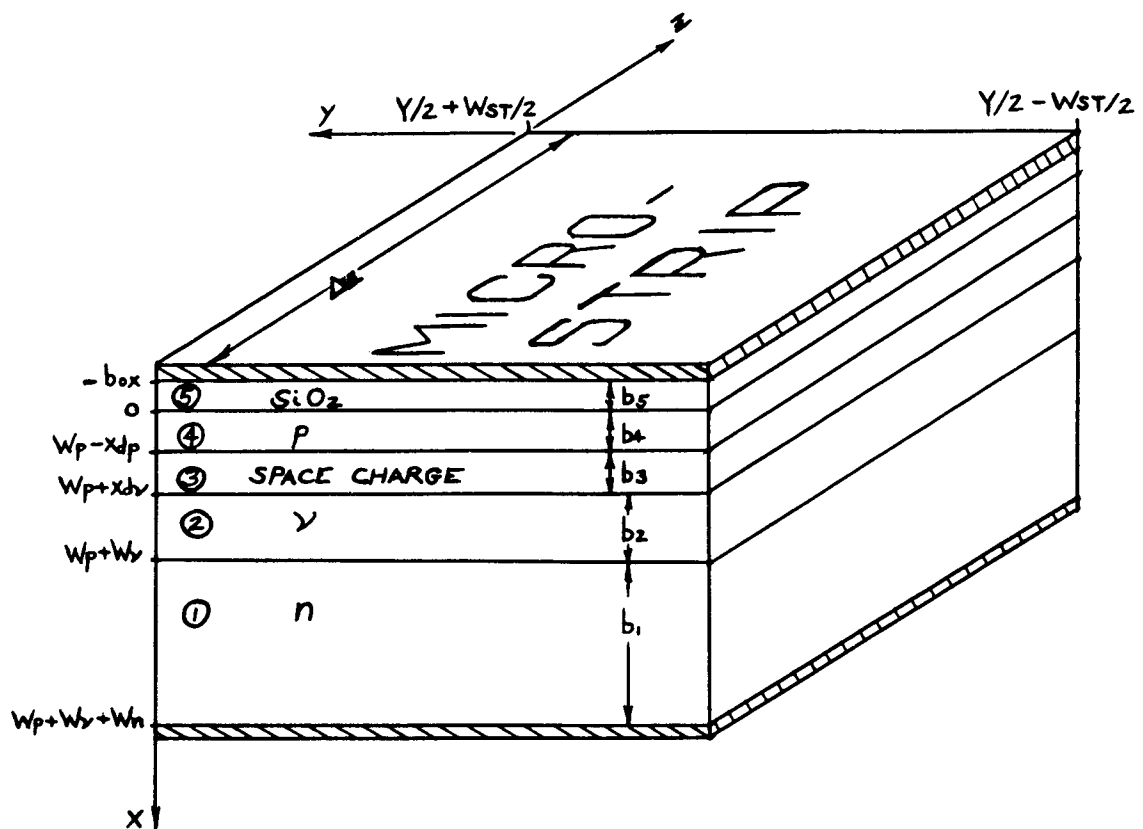


FIGURE 3.2-1 PARALLEL-PLATE WAVEGUIDE SEGMENT

Unfortunately, the numerical results have demonstrated that the propagation characteristics are nearly independent of the value of ϵ_3^* , allowing the approximation that ϵ_3^* is equal to the complex permittivity of the ν layer, ϵ_2^* .

The injected carrier concentrations and width of the ν layer are functions of dc surface voltage. Injected carrier concentration distributions are assumed to be uniform in the x direction in the ν layer so that the layer may be considered to be homogeneous. For dc surface voltages less than zero the injected carrier concentration is zero and the conductivity is

$$\sigma_2 \doteq e \mu_{nv} N_{0\nu} . \quad 3.16$$

For low level injection, the excess hole concentration is determined from Eq. I-7 and the conductivity is written as

$$\sigma_2 \doteq e \left[\mu_{nv} (N_{0\nu} + p'(W_p + x_{dv})) + \mu_{pv} (n_i^2 / N_{0\nu} + p'(W_p + x_{dv})) \right] . \quad 3.17$$

For forward dc surface voltages greater than or equal to the value at the onset of high level injection, the excess carrier concentrations are determined from the diode dc current density, J_0 . Since the total dc current density at $x = W_p + x_{dv}$ is equal to the dc hole current density in Eq. I-22, by assuming total excess carrier recombination in the ν layer, one may write

$$J_0 = Q_{p\nu} / \tau_{\nu n} , \quad 3.18$$

where Q_{py} is the total hole charge per unit area in the y layer; or similarly for electrons, from Eq. I-24,

$$J_0 = Q_{ny} / \tau_{yn} . \quad 3.19$$

Approximating Q_{py} as

$$Q_{py} \doteq e p(w_p + x_{dy}) [w_y - x_{dy}] \quad 3.20$$

and by substituting into Eq. 3.18 gives

$$p(w_p + x_{dy}) \doteq J_0 \tau_{yn} / e (w_y - x_{dy}) , \quad 3.21$$

which, along with the condition that $p \approx n$, is used to determine the y layer conductivity for high level injection. The space charge layer width into the y layer is determined from the depletion model of the abrupt py junction and for reverse bias through low level injection is assumed to follow the well known relationship²⁰

$$x_{dy} = x_{dy0} (1 - V/\phi_{py})^{1/2} , \quad 3.22$$

where

$$x_{dy0} \doteq \left[(2 \epsilon_i \phi_{py}) / (e N_{dy}) \right]^{1/2} , \quad 3.23$$

and ϕ_{py} is the barrier potential of the py junction. Near the onset of high level injection in the y layer, the depletion approximation breaks down because of the high injected carrier density across the py space charge region and x_{dy} is set equal to zero (at $V=V_c$) with negligible dis-

continuity in the calculated propagation characteristics observed.

The excess carrier concentrations of the p and n layers are assumed to be much less than the ionized impurity atom concentrations for all bias levels considered, resulting in

$$\sigma_i \doteq e \mu_{nn} N_{0n} \quad 3.24$$

and

$$\sigma_s \doteq e \mu_{pp} N_A . \quad 3.25$$

The νn space charge region is a localized electron accumulation over the bulk equilibrium value, with resulting increased conductivity, at the ν side of the νn junction. In the rf analysis, the gradual variation of conductivity over the νn space charge region is neglected with the transition from the ν side to the n side being treated as abrupt, although, the νn space charge region must be considered in the dc evaluation because of its "blocking" property (Appendix I).

By neglecting the fringing fields at the edges of the plates, a field configuration identical to that existing in a similar structure of infinite extent in the y direction is derived. Therefore, for a TM mode plane wave propagating in the z direction

$$\frac{\partial}{\partial y} = 0 , \quad 3.26$$

$$E_y = 0 , \quad 3.27$$

$$H_x = 0 \quad 3.28$$

and

$$H_z = 0 \quad 3.29$$

Using Eqs. 3.26-3.29 Maxwell's Equations for the i^{th} layer are written as

$$\left(\frac{\partial}{\partial x} \hat{a}_x + \frac{\partial}{\partial z} \hat{a}_z \right) \times (E_{xi} \hat{a}_x + E_{zi} \hat{a}_z) = -j\omega\mu_0 H_{yi} \hat{a}_y, \quad 3.30$$

$i = 1, 2, 3, 4, 5$

$$\left(\frac{\partial}{\partial x} \hat{a}_x + \frac{\partial}{\partial z} \hat{a}_z \right) \times H_{yi} \hat{a}_y = j\omega\epsilon_i^* (E_{xi} \hat{a}_x + E_{zi} \hat{a}_z), \quad 3.31$$

$i = 1, 2, 4, 5$

$$\frac{\partial H_{y3}}{\partial x} = j\omega\epsilon_{3t}^* E_{z3}, \quad 3.32$$

and

$$\frac{\partial H_{y3}}{\partial z} = j\omega\epsilon_{3t}^* E_{x3}, \quad 3.33$$

where $\epsilon_i^* = \epsilon_{si} - j\sigma_i/\omega$, ϵ_{si} = permittivity of silicon, σ_i is the conductivity of the i^{th} layer (for $i \neq 3, 5$) and $\epsilon_5^* = \epsilon_{sio_2}$. Equations 3.30 and 3.31 are expanded to obtain

$$\frac{\partial E_{xi}}{\partial z} - \frac{\partial E_{zi}}{\partial x} = -j\omega\mu_0 H_{yi} \quad i = 1, 2, 3, 4, 5, \quad 3.34$$

$$\frac{\partial H_{yi}}{\partial x} = j\omega\epsilon_i^* E_{zi} \quad i = 1, 2, 4, 5 \quad 3.35$$

and

$$\frac{\partial H_{yi}}{\partial z} = -j\omega\epsilon_i^* E_{xi} \quad i = 1, 2, 4, 5. \quad 3.36$$

Substitution of Eqs. 3.35 and 3.36 into 3.34 give

$$\frac{\partial^2 H_{yi}}{\partial z^2} + \frac{\partial^2 H_{yi}}{\partial x^2} = -\omega^2 \mu_0 \epsilon_i^* H_{yi} \quad i=1,2,4,5 . \quad 3.37$$

Similarly, substitution of Eqs. 3.32 and 3.33 into 3.34 results in

$$\frac{1}{\epsilon_{3t}^*} \frac{\partial^2 H_{y3}}{\partial z^2} + \frac{1}{\epsilon_{3l}^*} \frac{\partial^2 H_{y3}}{\partial x^2} = -\omega^2 \mu_0 H_{y3} . \quad 3.38$$

Assuming a field dependence of the form

$$H_{yi} \propto \exp(\pm k_i x \pm \gamma z) \quad 3.39$$

reduces Eqs. 3.27 and 3.28 to

$$\gamma^2 + k_i^2 = -\omega^2 \mu_0 \epsilon_i^* \quad i=1,2,4,5 \quad 3.40$$

and

$$\frac{\gamma^2}{\epsilon_{3t}^*} + \frac{k_3^2}{\epsilon_{3l}^*} = -\omega^2 \mu_0 . \quad 3.41$$

Application of the electromagnetic boundary conditions at each interface yields (see Appendix III)

$$\frac{k_1^2 b'_1}{\epsilon_1^*} + \frac{k_2^2 b'_2}{\epsilon_2^*} + \frac{k_3^2 b'_3}{\epsilon_{3l}^*} + \frac{k_4^2 b'_4}{\epsilon_4^*} + \frac{k_5^2 b'_5}{\epsilon_5^*} = 0 , \quad \text{III-29}$$

where $b'_i = (\tanh k_i b_i) / k_i$.

Substitution of Eqs. 3.40 and 3.41 into III-29 results in

$$\gamma^2 = \frac{-\omega^2 \mu_0 [b'_1 + b'_2 + b'_3 + b'_4 + b'_5]}{\left[\frac{b'_1}{\epsilon_1^*} + \frac{b'_2}{\epsilon_2^*} + \frac{b'_3}{\epsilon_{3t}^*} + \frac{b'_4}{\epsilon_4^*} + \frac{b'_5}{\epsilon_5^*} \right]} , \quad 3.42$$

which is solved together with Eqs. 3.40 and 3.41 using a secant method ²² in subroutine RF. Using the result of

Guckel et. al.²³ that initialization of the eigenvalues to the Quasi-TEM case results in convergence to the lowest order TM mode for the lossy system, $b'_i = b_i$ ($i=1,2,3,4,5$) is initially chosen. More rapid convergence to the lowest order mode is found by choosing the initial conditions of $b'_i = b_i$ ($i=2,3,4,5$) and $k_1^2 \gg \gamma^2$, which gives $k_1^2 \approx -\omega^2 \mu_0 \epsilon_1^*$ and $b'_1 \approx 1/k_1$. This improvement is made possible by the very thick and highly doped substrate ($Wn \approx 225 \mu$ and $N_{Dn} \approx 10^{19} \text{ cm}^{-3}$). A flow chart of subroutine RF is in Appendix V.

The characteristic impedance of the m-o-d segment is found by taking the ratio of the forward (or backward) traveling voltage wave to the forward (or backward) traveling current wave. The transmission line voltage is defined as

$$V_t = \int_{x=W_p+W_v+W_n}^{x=-b_0} E_x dx \quad 3.43$$

and the transmission line current as

$$I_t = \oint_C \vec{H} \cdot d\vec{l} = \int_{\substack{y=Y/2+W_s/2 \\ x=0^-}}^{y=Y/2+W_s/2} H_y dy, \quad 3.44$$

with the path of integration, C, shown in Fig. 3.2-2. Strictly speaking, since the mode considered is non-TEM, no unique characteristic impedance exists. This is shown by the definition of current, which is dependent on the choice of path, C, since H_y is a function of x , although V_t is unique for any path from the bottom to top plate for a constant value of z . By selecting the path, C, in Eq. 3.44 to cross between the plates at different values of x , over the range

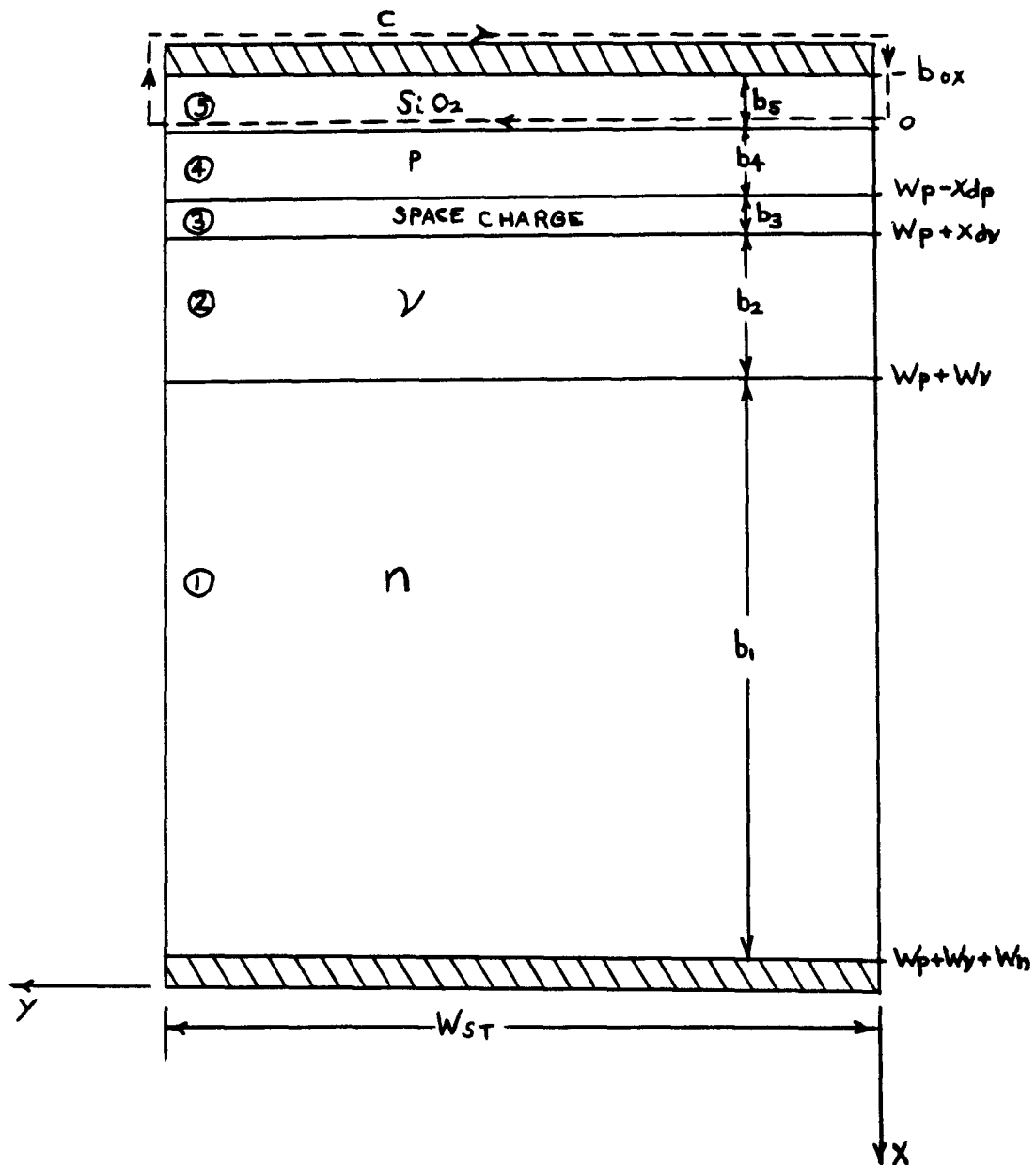


FIGURE 3.2-2 FINDING THE TRANSMISSION LINE CURRENT

$-b_{ox} \leq x \leq W_p + W_v$, different expressions for I_t are obtained. However, due to the nature of the material, $|k_i b_i| \ll 1$ for $i \neq 1$, Eq. IV-4, and the current integral may always be simplified to Eq. IV-2. Paths crossing into the n layer are not considered since the n layer is viewed as a lossy conductor extension of the bottom groundplane. Equation 3.44 is equivalent to the definition of current given by Hasegawa et. al.²⁴ since his lossy substrate is replaced by the multiple layer lossy silicon substrate under the oxide in the m-o-d structure. In the Quasi-TEM limit, as the layers become lossless, $|k_i b_i| \rightarrow 0$, $\left| \frac{E_{xi}}{E_{xi}} \right| \rightarrow 0$ (see Appendix III) and Eq. 3.44 becomes independent of C, as expected. Using Eq. 3.44 as the definition of current for the characteristic impedance, reasonably good agreement with measured results are obtained for the structure. Evaluation of Eqs. 3.43 and 3.44 in Appendix IV gives

$$Z_o^2 = \frac{\mu_o}{W_{ST}} \left[\frac{b'_1}{\epsilon_1^*} + \frac{b'_2}{\epsilon_2^*} + \frac{b'_3}{\epsilon_{3t}^*} + \frac{b'_4}{\epsilon_4^*} + \frac{b'_5}{\epsilon_5^*} \right] \left[b'_1 + b'_2 + b'_3 + b'_4 + b'_5 \right] \quad \text{IV-10}$$

3.3 Electromagnetic Properties of a Microstrip-Oxide-Distributed P_NN Segment

In this section, the effect of device parameters on the propagation constant and the characteristic impedance is investigated from the numerical solutions of Eqs. 3.40, 3.41, 3.42 and IV-10.

It is desirable to keep the oxide layer as thin as possible, while maintaining high reliability against voltage breakdown at oxide pinholes, to ensure tight coupling between the microstrip and its p_NN substrate. Changing the oxide thickness from 500 Å to 1000 Å produces less than a 2% change in the magnitude and phase of γ and Z_0 . Oxide thicknesses greater than 1000 Å reduce the maximum obtainable change in γ and Z_0 with a change in dc surface voltage noticeably. Thus, b_{ox} is selected to be between 500 Å and 1000 Å.

The thickness and doping density of the p layer are constrained by practical processing considerations. The thickness is determined from the surface source impurity concentration, diffusion time and temperature, and background concentration of the ν epi layer. The boron diffusion source maintains a surface concentration equal to the solid solubility of boron in silicon during the predeposition diffusion, to ensure p layer sheet resistance uniformity. The minimum and maximum drive in diffusion times are determined from the thickness of the oxide grown and

the amount of n substrate layer outdiffusion into the ν layer allowed. From these considerations, p layer thicknesses lie in the range $.1\mu$ to 1μ with surface impurity concentrations 10^{19} cm^{-3} to 10^{20} cm^{-3} . Over this range of p layer parameter variation there is a negligible variation in the propagation characteristics. The supply of readily available silicon epitaxial wafers have substrate thicknesses from approximately 225μ to 250μ and resistivities from approximately $.005 \Omega\text{-cm}$ to $.1 \Omega\text{-cm}$. A low substrate resistivity is desirable to minimize the parasitic spreading resistance of the n layer, as discussed in Chapter 2. Variations in n layer parameters over this range produce less than 5% variation in the magnitude and phase of γ and Z_o .

Numerical solutions show that γ and Z_o are by far most sensitive to changes in the ν layer parameters. The choice of ν layer thickness and doping density has a dramatic effect on the attenuation of the m-o-d segment, as shown in Fig. 3.3-1 for both $Y=30$ and $Y=-40$ normalized volts. For the m-o-d structure to be useful as a dc voltage controllable variable impedance device, the attenuation of a reverse biased segment must be kept to a minimum so that the reflected wave from the plane of strong reflection reaches the input of the microstrip line with minimum attenuation. This requires a low ν layer impurity concentration. A practical impurity concentration of 10^{14} cm^{-3} is selected as a modeling parameter. Figure 3.3-2 shows the sensitivity of the normal-

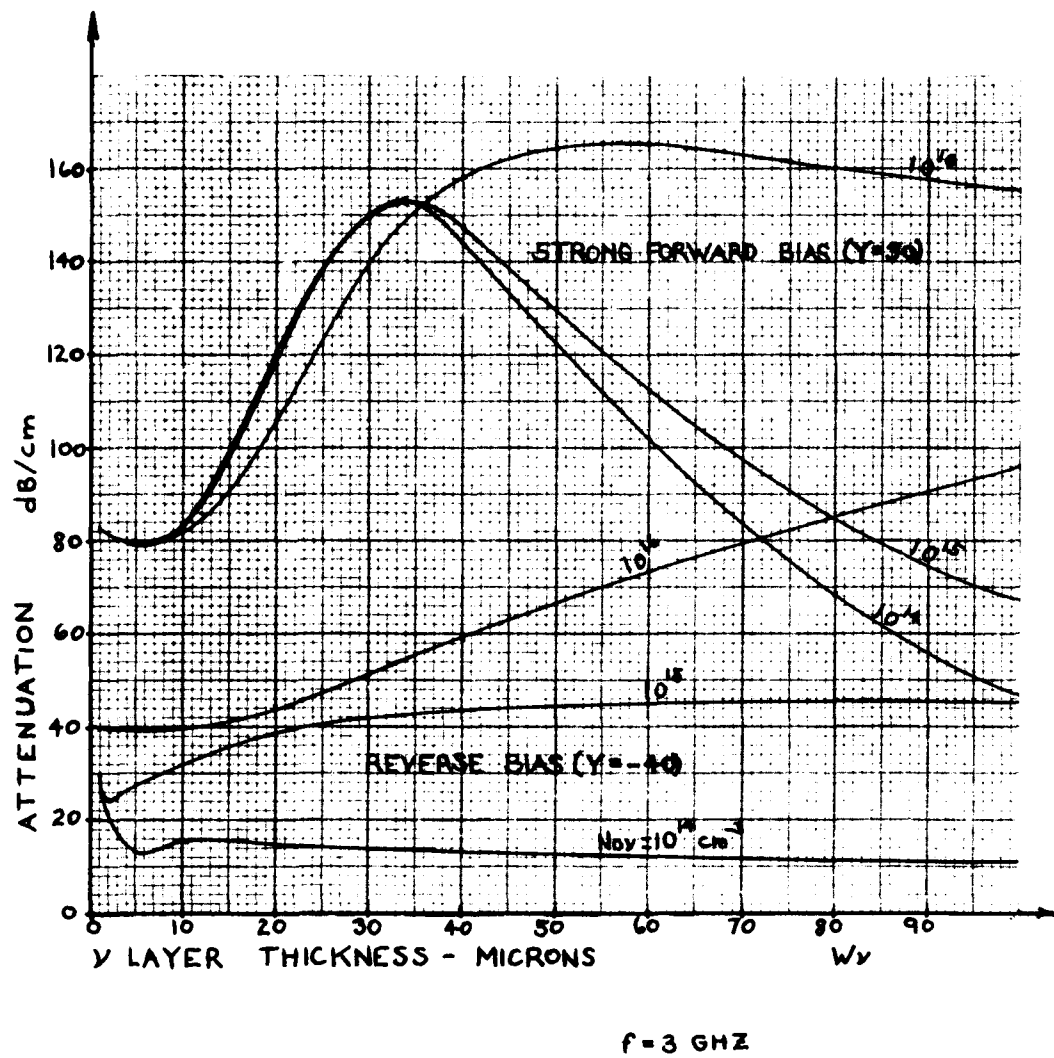
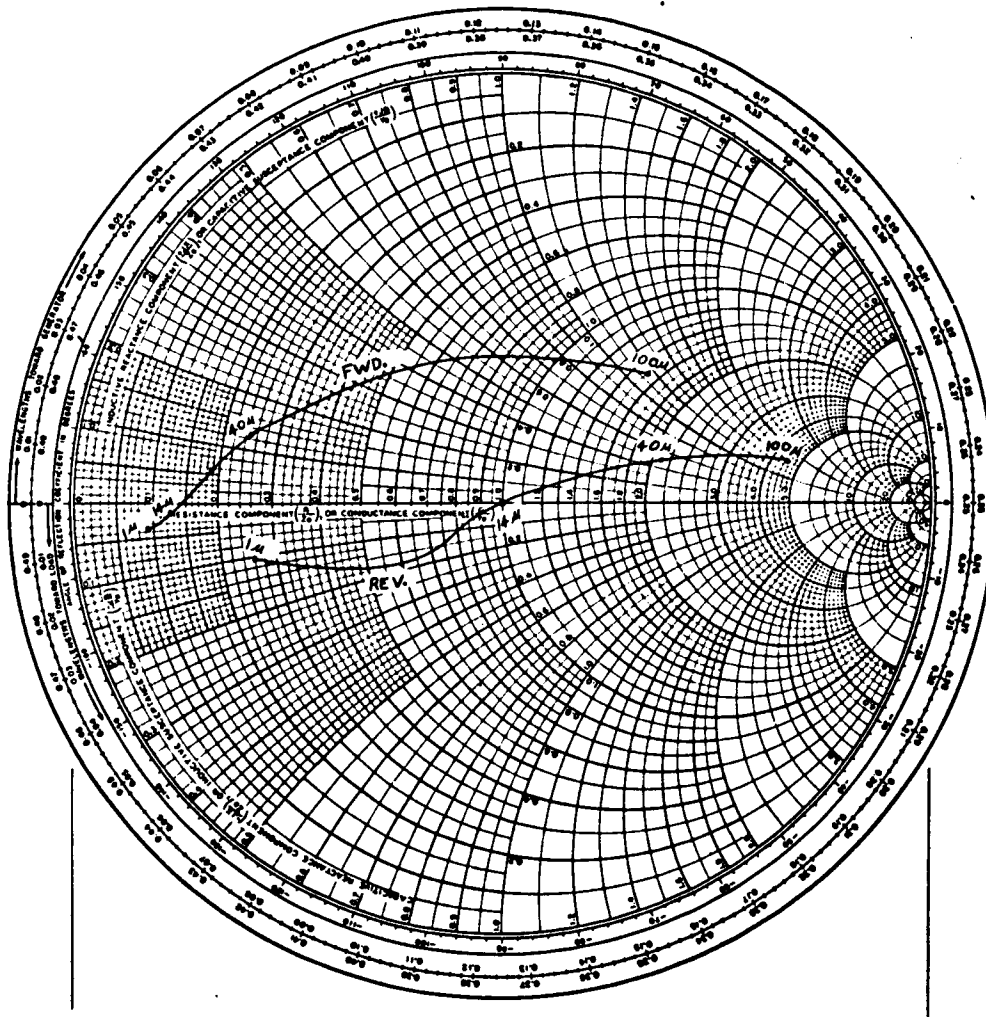


FIGURE 3.3-1 ATTENUATION VS V LAYER PARAMETERS



$$f = 3 \text{ GHz} \quad N_{\text{Dv}} = 10^{14} \text{ cm}^{-3} \quad W_{\text{ST}} = 1.55 \text{ mils}$$

FIGURE 3.3-2 Z_o vs EPI LAYER THICKNESS

ized characteristic impedance to changes in ν layer thickness at 3 GHz. To achieve a good match between the device package and the input of the microstrip line, for Y_0 maintained at -40, the reverse bias normalized characteristic impedance must be as close to $1+j0$ as possible. A ν layer thickness of 14μ produces a nearly zero imaginary part of Z_0 and, with a microstrip width, W_{ST} , of 1.55 mils, a characteristic impedance of 50Ω .

An interesting property of the attenuation as a function of dc surface voltage is observed with large forward biases. From approximately $Y=15$ to 26 normalized volts the attenuation increases rapidly, as expected, because of the increase in stored charge in the ν layer, shown in Fig.3.3-3. But, at approximately $Y=27$ the attenuation reaches a maximum and then decreases with increasing dc surface voltage. This corresponds to the type of behavior predicted by Guckel et. al.²⁵ at 3 GHz, where with increasing substrate conductivity the attenuation reaches a maximum and then decreases. The increase in stored charge in the ν layer with increasing dc surface voltage corresponds to the increased substrate conductivity in Guckel's model. Increased stored charge, and hence, conductivity of the ν layer also produces a rapid decrease in the normalized wavelength with increasing dc surface voltage, as shown in Fig. 3.3-4. At a normalized surface voltage of -40 the m-o-d segment wavelength is 1.79 cm at 3 GHz. Movement of the plane of strong

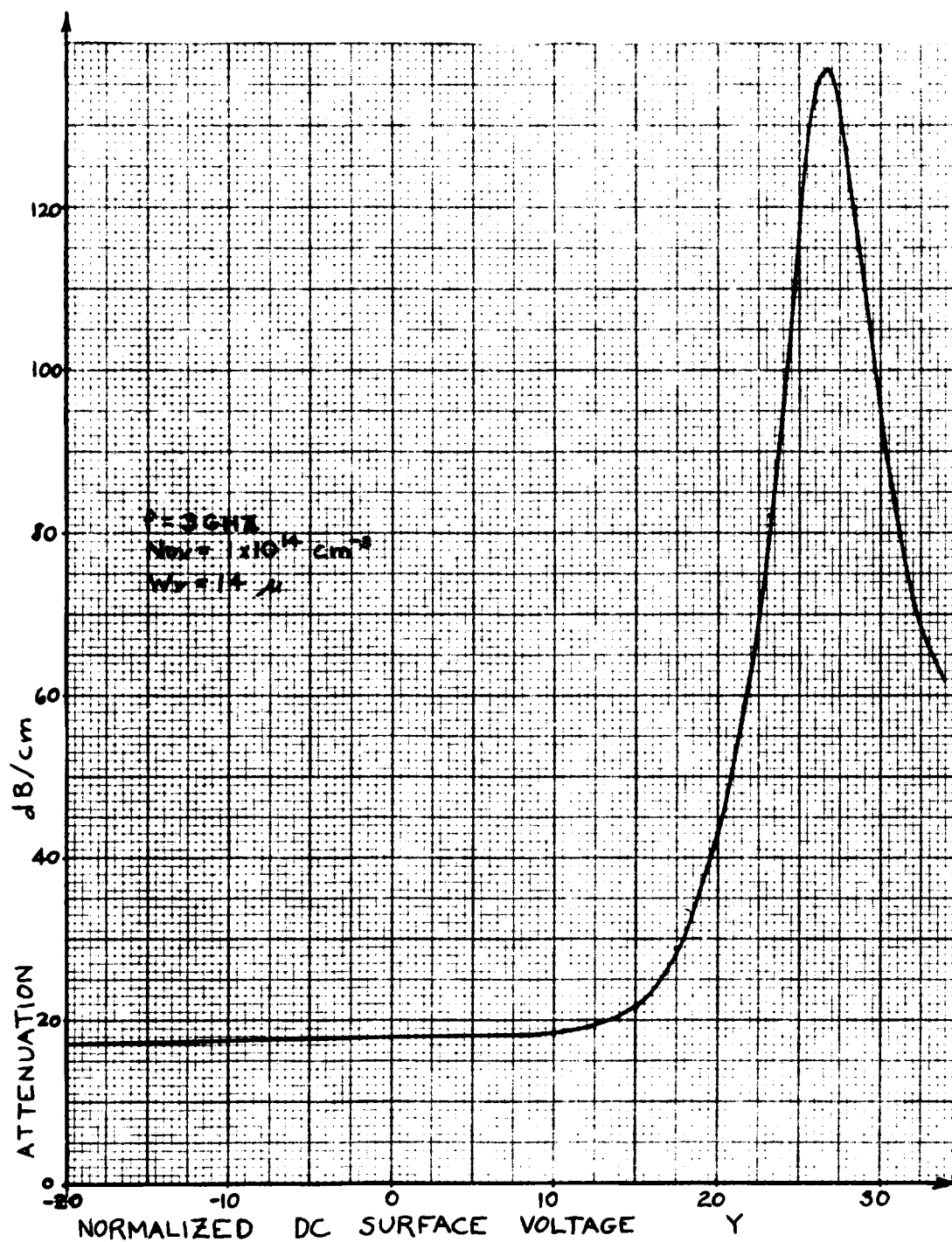


FIGURE 3.3-3 ATTENUATION VS SURFACE VOLTAGE

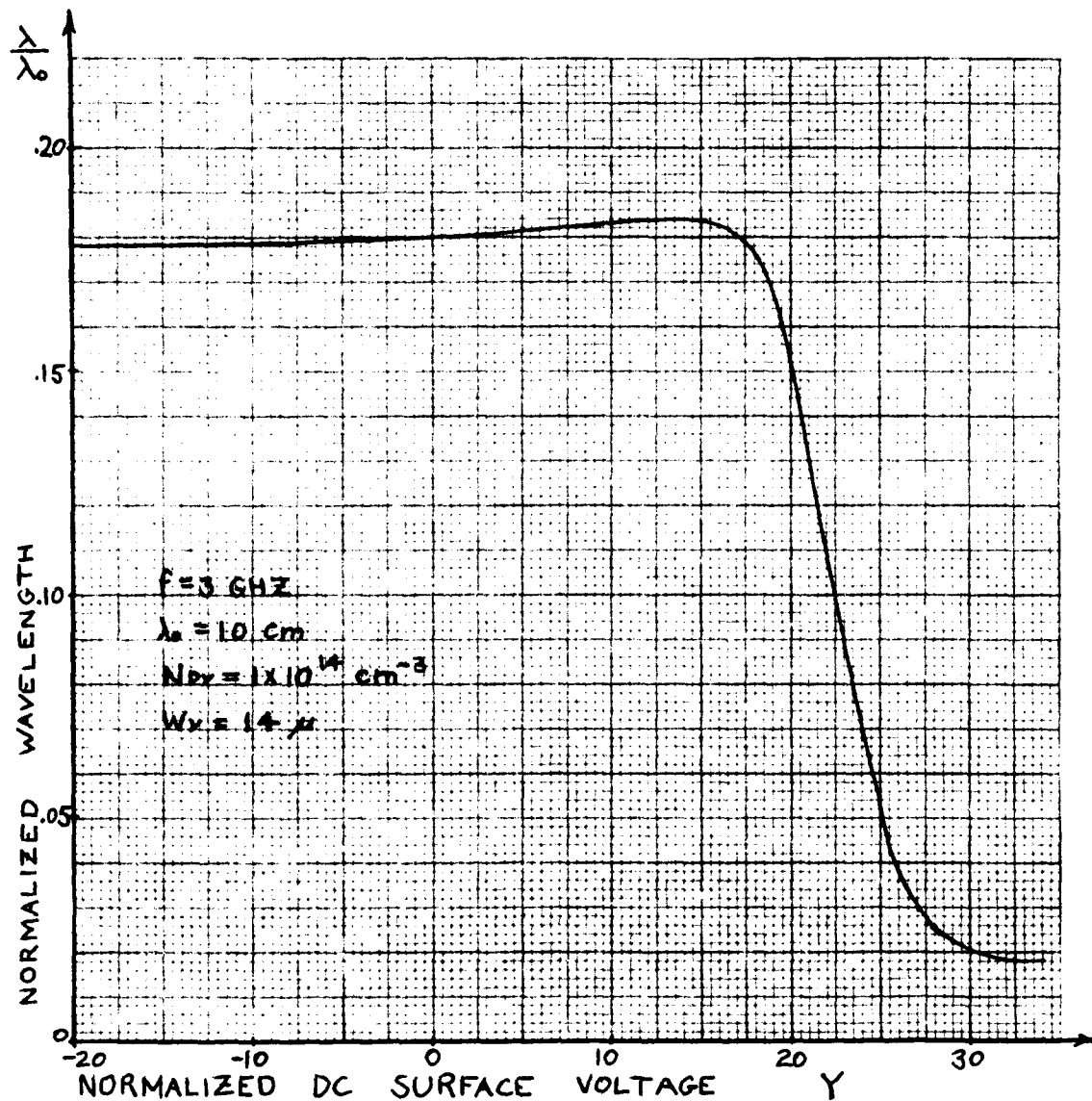


FIGURE 3.3-4 NORMALIZED WAVELENGTH VS
 SURFACE VOLTAGE

reflection corresponding to a tenth of a reverse biased wavelength at 3 GHz is obtainable with the device dimensions used (see Sec. 2.4). Since the reverse biased m-o-d segment wavelength is found to decrease with frequency, a change in microstrip line driving point impedance, due to movement of the plane of strong reflection, is observable at 3 GHz or higher. As shown in Fig. 3.3-5, the attenuation of the m-o-d segment increases with increasing frequency, making the effects of the reflection from the plane of strong reflection more difficult to observe.

Figure 3.3-6, which shows the locus of the normalized characteristic impedance as a function of normalized dc surface voltage at 3 GHz, reveals that the largest rate of change of Z_0 with Y occurs between $Y=20$ and 25. For a linear dc surface voltage profile on the device, the plane of strong reflection is expected to occur between $Y(z) = 20$ and $Y(z) = 25$. The locus of the reverse biased characteristic impedance as a function of frequency is found to be confined within the $|\Gamma| = .05$ circle of the Smith Chart, from $f=2$ GHz to 12 GHz, which is desirable to assure a minimum reflection from the junction of the package stripline and the microstrip line, when $Y(0) = -40$, for these idealized parameters.

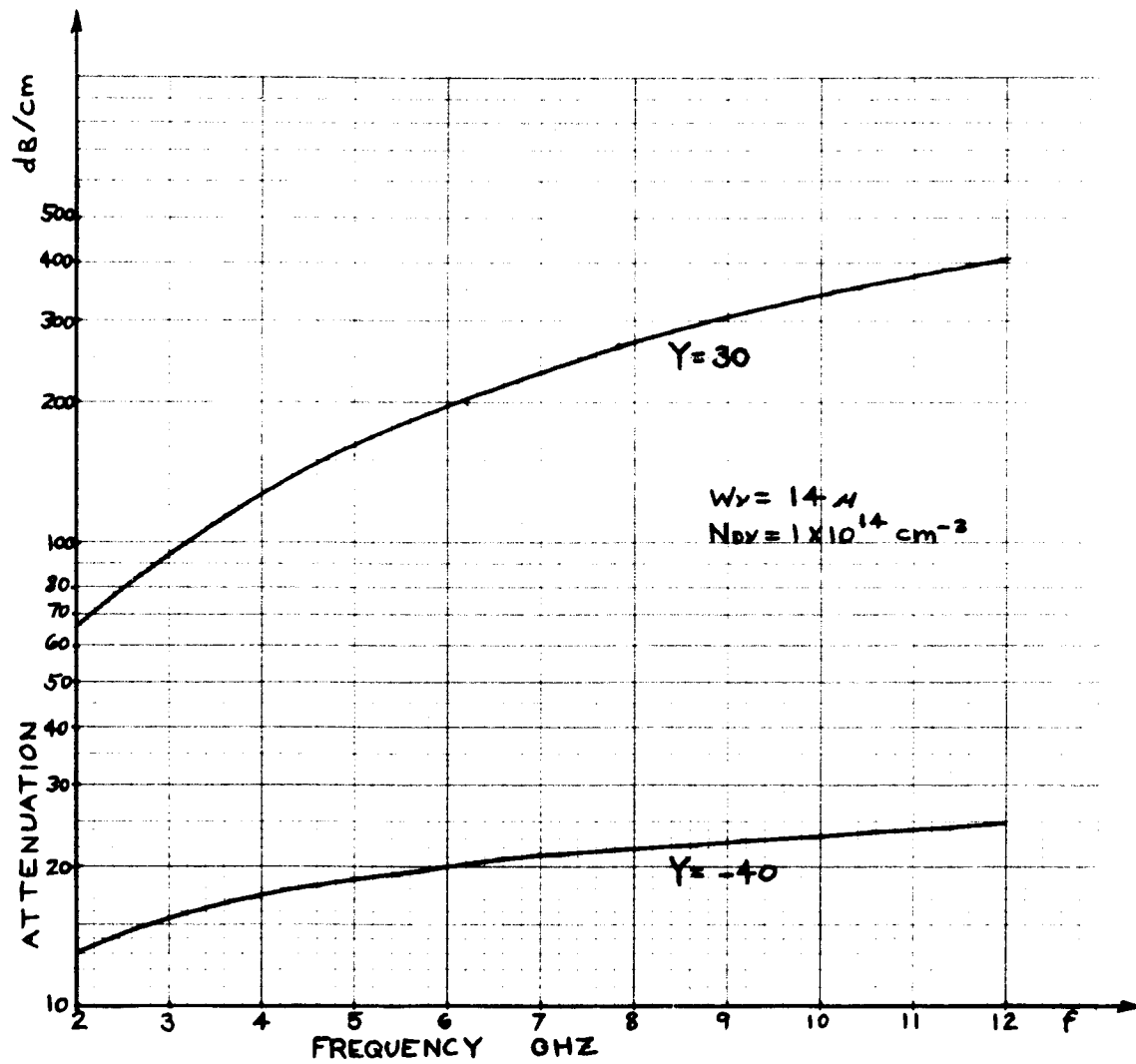
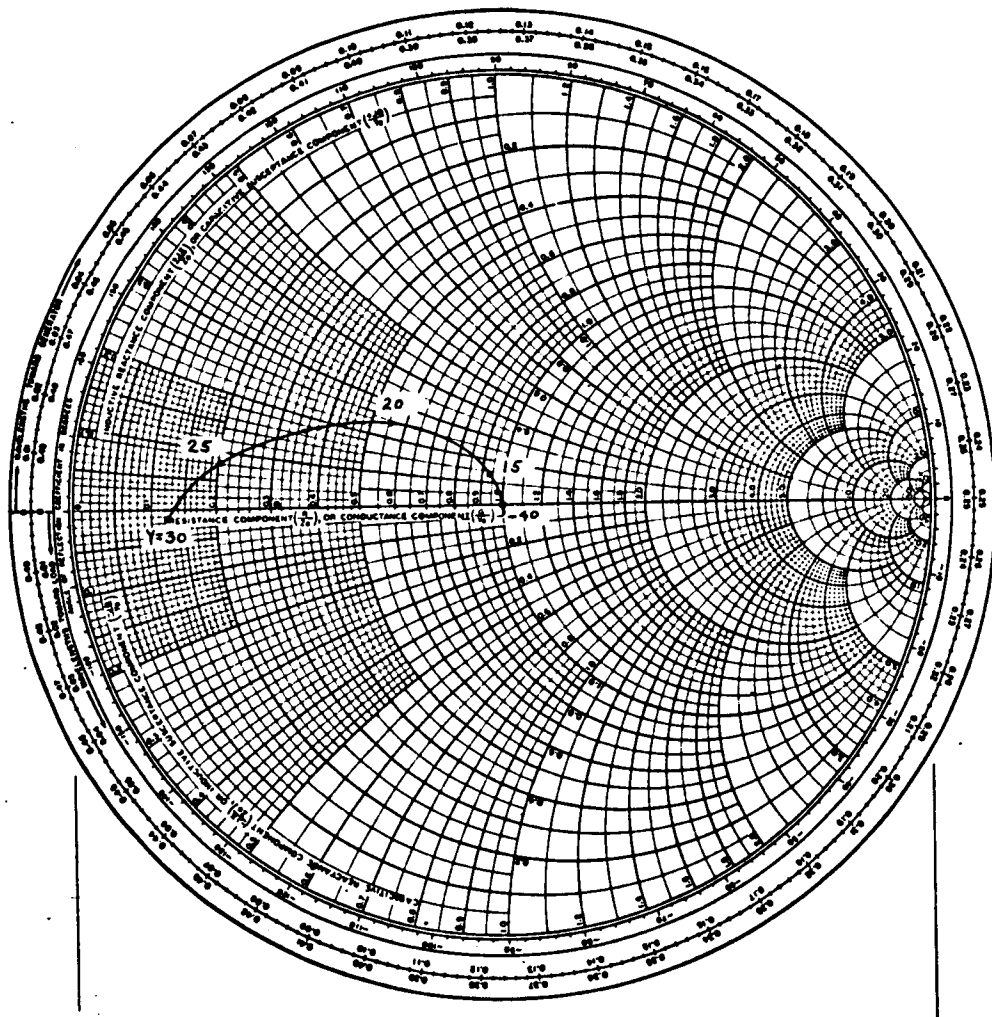


FIGURE 3.3-5 ATTENUATION VS FREQUENCY



$$f = 3 \text{ GHz} \quad N_{\text{Dv}} = 10^{14} \text{ cm}^{-3} \quad W_{\text{ST}} = 1.55 \text{ mils}$$

FIGURE 3.3-6 Z_s vs SURFACE VOLTAGE

3.4 Measurement of the Microwave Operation of the Microstrip-Oxide-Distributed P/N Device

The purpose of this section is to demonstrate the validity of the model of the microstrip-oxide-distributed p/n device, developed in previous sections, with measurements on actual devices at microwave frequencies. As discussed in Sec. 3.1, the behavior of the device is characterized by its input-output voltage transmission coefficient, T_{io} , and driving point impedance, Z_{in} . Visual display of T_{io} and the input reflection coefficient, Γ_{in} , (which is equivalent to the driving point impedance) is obtained from the HP Microwave Network Analyzer in the setup shown in Fig. 3.4-1. The device to be tested is mounted on a 1000 mil x 1000 mil photolithographically etched piece of Rexotherm material (which consists of a 10 mil thick dielectric sandwiched by a 1 oz copper top and a thick aluminum bottom), shown in Fig. 3.4-2, with a relative dielectric constant of 2.3. A transmission line width of 30 mils provides a 50Ω characteristic impedance. The device is mounted in a recess cut in the dielectric so that the back contact and the substrate is conductively epoxied to the Rexotherm groundplane. The transmission and biasing lines are ultrasonically bonded to the chip and external connections are made with OSM connectors. A second piece of Rexotherm material which has a transmission line running straight through, and no device,



FIGURE 3.4-1 MICROWAVE NETWORK ANALYZER TEST SETUP

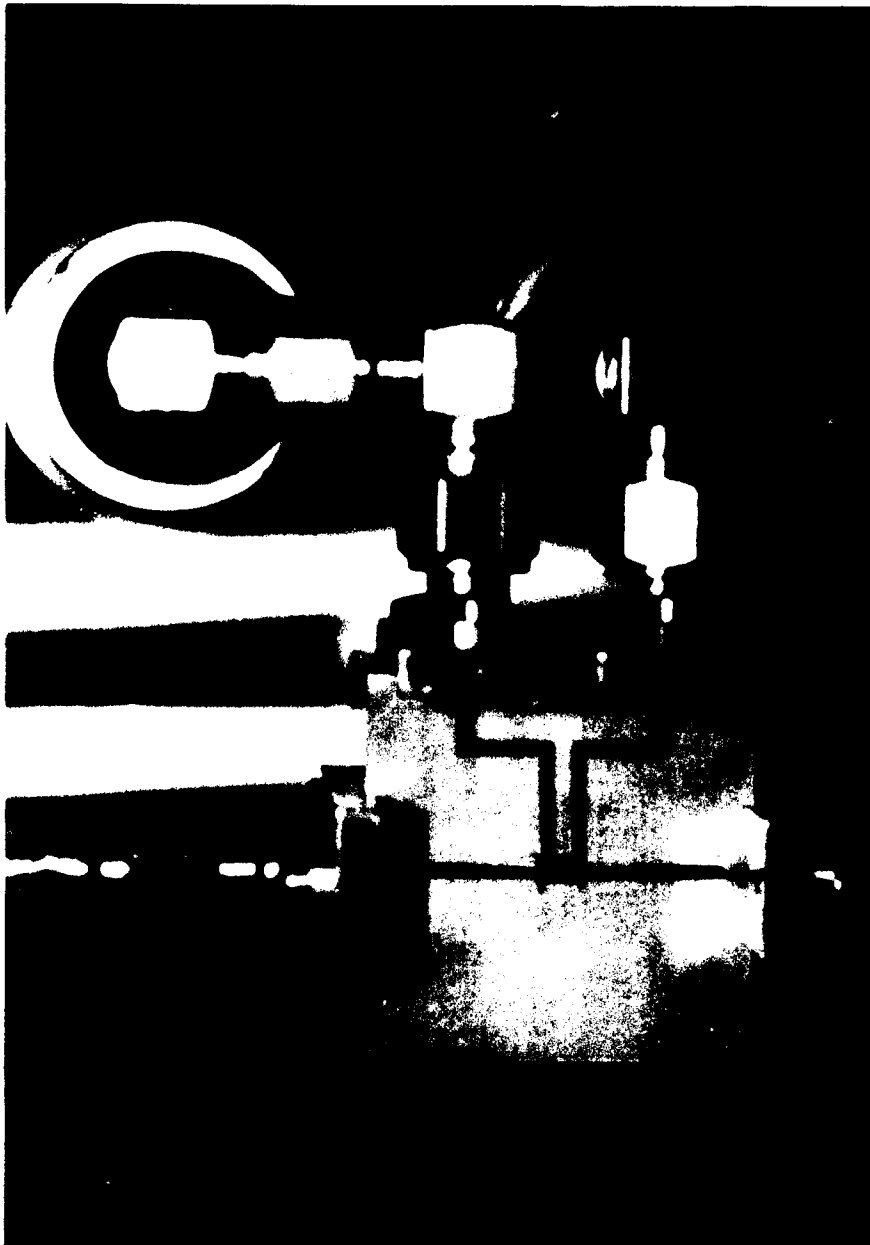
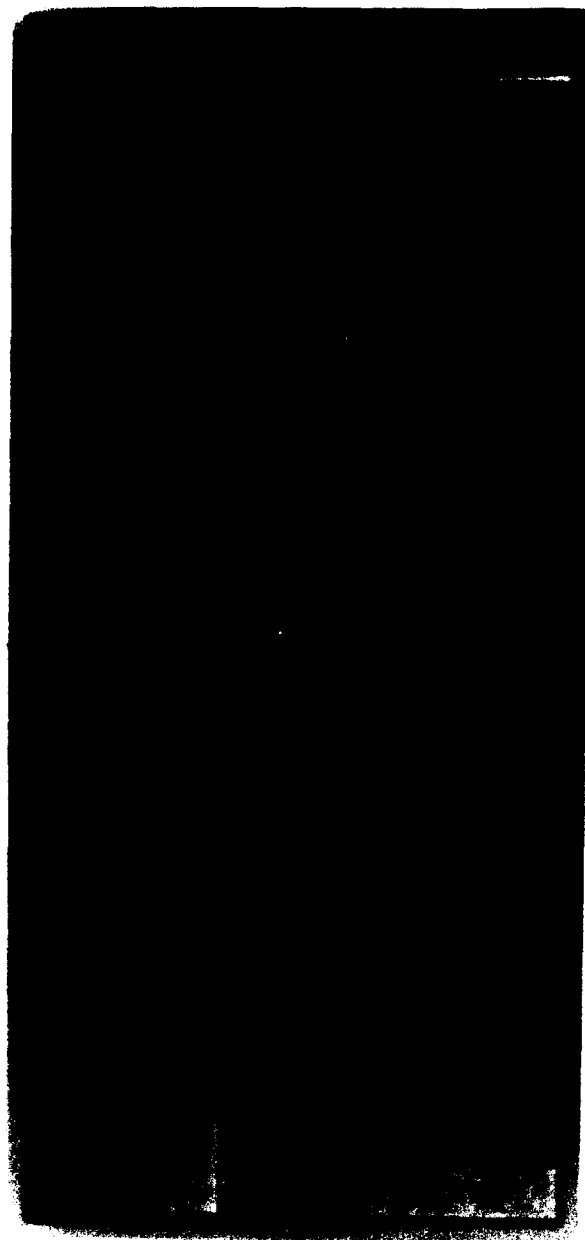


FIGURE 3.4-2 DEVICE WITH RF MEASUREMENT JIG

is used to determine the amount of phase shift and attenuation introduced into the measurements by the device holding jig.

To properly model the device being tested, the parameters of each layer must be determined. The quantities of interest are obtained from dc and pulse measurements made on the device and its corresponding test diodes. A typical chip, containing a device and a test diode, which was fabricated in The City College Semiconductor Device Laboratory is shown in Fig. 3.4-3. The silicon ν on n material used for fabricating these devices (Batch 810E25) has vendor specified ν and n layer thicknesses of 25μ and 225μ , respectively. Four point probe measurements²⁶ on the unprocessed silicon wafer reveal ν and n layer impurity concentrations of approximately $5 \times 10^{14} \text{ cm}^{-3}$ and $1.5 \times 10^{18} \text{ cm}^{-3}$, respectively. To make a low resistance ohmic Al-Si contact to the back of the wafer it is necessary to degenerately dope the surface of the n layer (see Sec. 2.6). A high concentration ($1 \times 10^{21} \text{ cm}^{-3}$) phosphorous diffusion into the back of the wafer had previously resulted in ν layer contamination manifesting itself as a narrow, buried, high concentration n layer near the p ν junction. Microwave experiments on these devices show that the buried n layer greatly reduces the amount of change obtainable in T_{co} and Γ_{in} with changes in dc biasing voltage. Precautions were taken in Batch 810E25 to prevent this contamination by modifying the sequence of processing



(1 inch $\approx$ 15.15 mils) 10 mils

FIGURE 3.4-3 DEVICE AND TEST DIODE

steps. To demonstrate that no buried layer is present, the ν layer impurity profile was obtained from C-V measurements on the test diodes using the relationships ²⁷

$$N_{D\nu}(x_{D\nu}) = \frac{-2}{e\epsilon_{si} A^2} \cdot \frac{dV}{d(1/C^2)} \quad 3.45$$

and

$$x_{D\nu} = \frac{\epsilon_{si} A}{C}, \quad 3.46$$

where A is the $6.25 \times 10^{-4} \text{ cm}^2$ cross sectional area of the test diode. The $1/C^2$ - V curve obtained from the C-V data is shown in Fig. 3.4-4; the corresponding impurity profile obtained from the $1/C^2$ - V curve and Eqs. 3.45 and 3.46 being shown in Fig. 3.4-5. No buried layer is present and the impurity concentration far from the junction approaches the value measured on the unprocessed wafer.

The minority carrier lifetime in the ν layer was investigated using two methods; the method used in Sec. 2.6 involving the solution of Eqs. 2.30 and I-33 and a pulse technique using the measurement of the storage time ²⁸. A high level injection leakage current density, $J_{0\nu}$, of $4 \times 10^{-6} \text{ A/cm}^2$ was measured which along with the solution of Eqs. 2.30 and I-33 results in $\frac{Wb}{L^*} = .529$ ($Wb = 25 \mu$) yielding $\tau_{p\nu} = 1.4 \times 10^{-6}$ sec. Using the pulse technique, $\tau_{p\nu}$ was found to be 7.2×10^{-7} sec. The discrepancy is mainly due to the approximations that $\tau_{p\nu} = \tau_{\nu H}$, in the first method, and that $\tau_{p\nu}$ is independent of injected carrier concentration, in the second method.

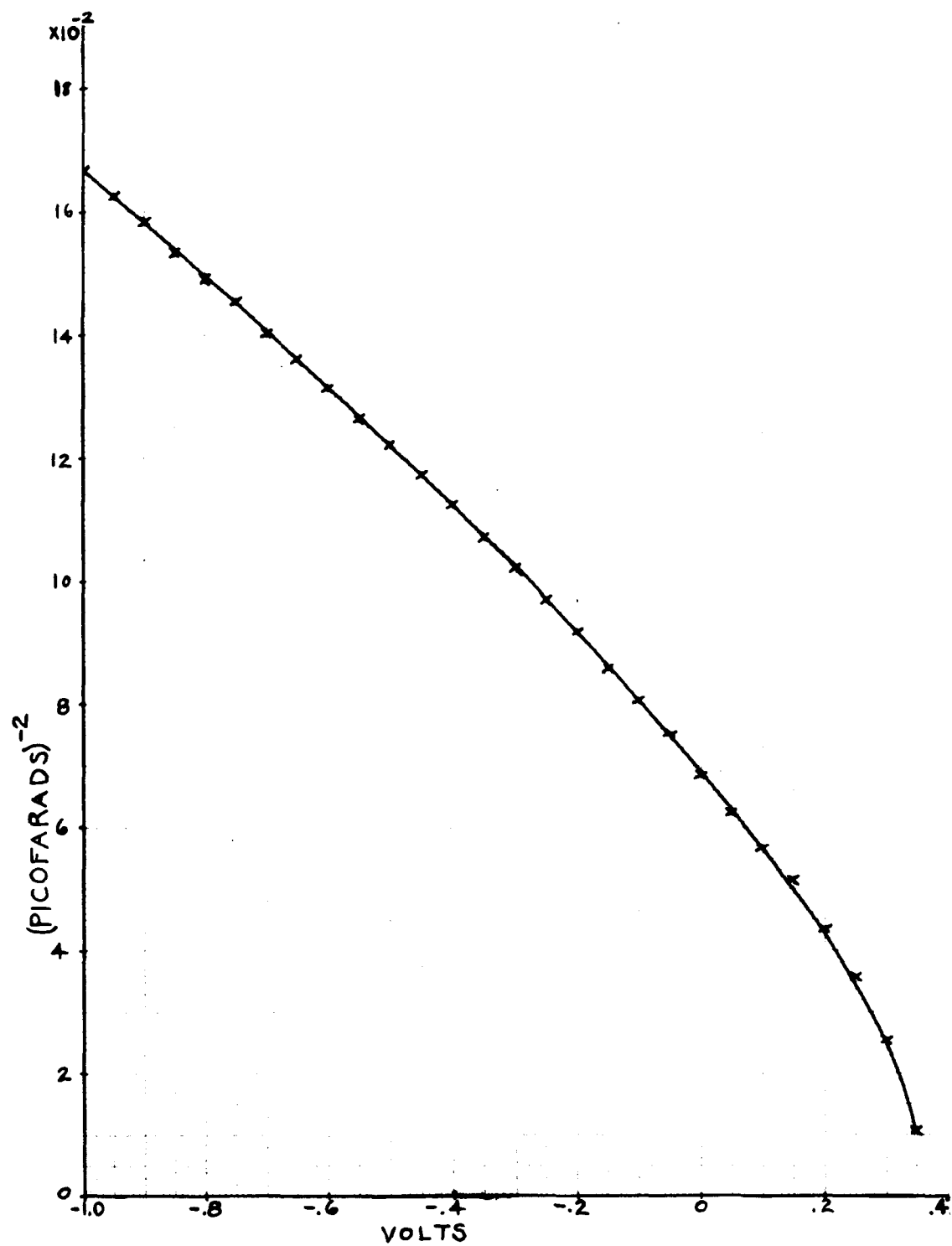


FIGURE 3.4-4 MEASURED $1/C^2$ VS V CURVE

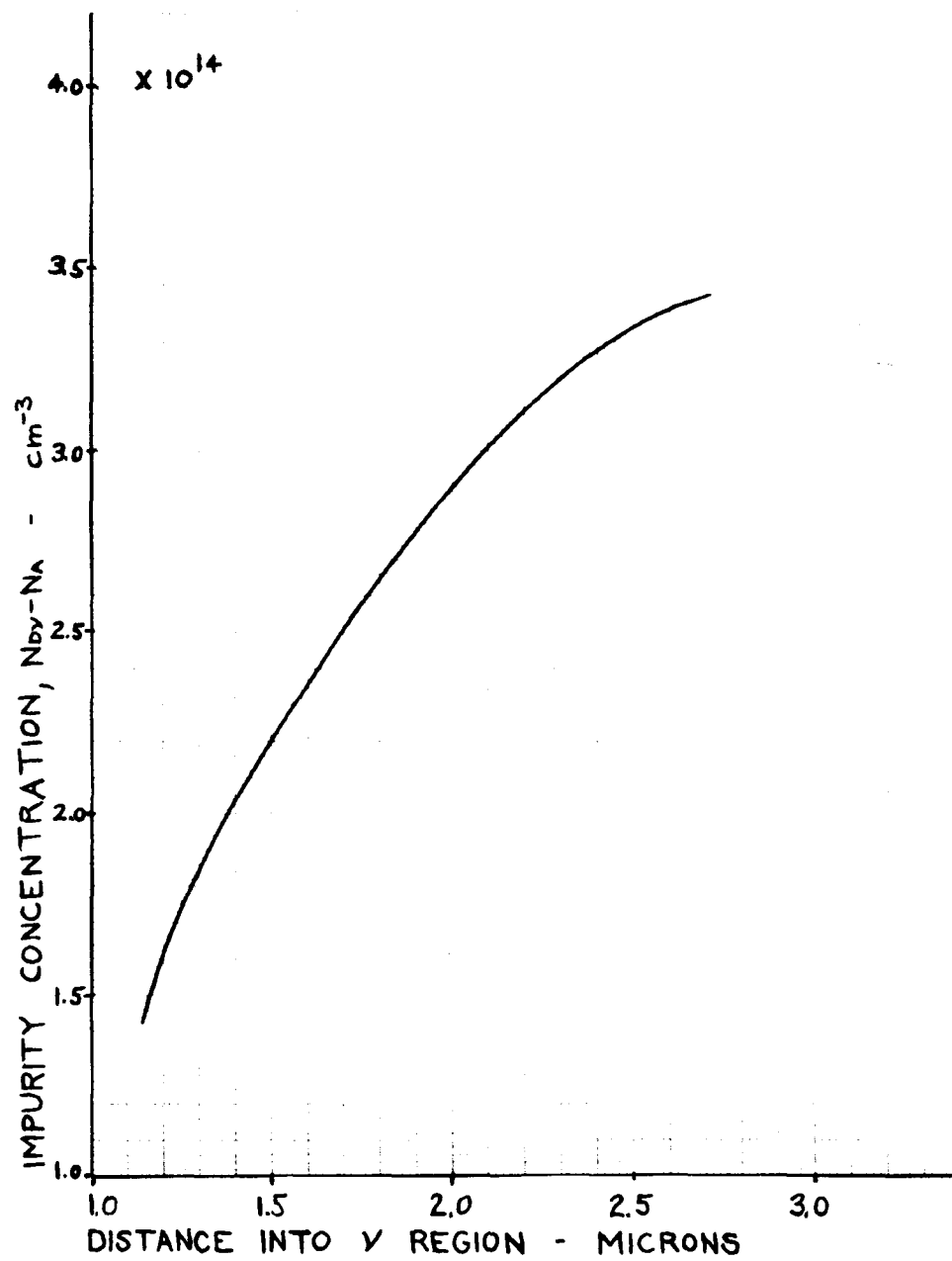


FIGURE 3.4-5 EXPERIMENTALLY DETERMINED
IMPURITY PROFILE

The $p\text{-}n$ junction depth, which is calculated²⁹ from the diffusion source impurity concentration, diffusion time and temperature and background impurity concentration, is found to be approximately $.78\ \mu$. The resistance measured between the biasing contacts with the $p\text{-}n$ junction reverse biased is used to calculate a p layer sheet resistance of $64\ \Omega/\square$. A p layer average resistivity of approximately $5 \times 10^{-3}\ \Omega\text{-cm}$ is found from the product of the sheet resistance and the p layer depth. This value of average resistivity corresponds to a surface impurity concentration of $1.5 \times 10^{20}\ \text{cm}^{-3}$, obtained by assuming an approximately Gaussian distribution³⁰.

These measured parameters are used in the device model and compared with measured rf results. The calculated results are very sensitive to the value of minority carrier lifetime used since the stored n layer charge greatly influences the rf characteristics (Sec. 3.3). The magnitude and phase of T_{10} vs biasing voltage, calculated for $Y(\omega) = Y(\omega)$ at 3 GHz, is shown in Figs. 3.4-6 and 3.4-7 for different values of $\tau_{p\text{-}n}$, illustrating this effect. A minority carrier lifetime of 5×10^{-7} sec produced the best agreement between measured and calculated results. Table 3.4-1 shows the values of the parameters used in the model. Figures 3.4-8 through 3.4-15 show actual x-y plotter recordings of the output of the HP Microwave Network Analyzer for measured T_{10} and Γ_{1n} as a function of $Y(\omega)$ and $Y(\omega)$ ($Y(\omega) = Y(\omega)$) at 2, 3, 4, 8, 10 and 12 GHz. Calculated points are drawn on the same axes for com-

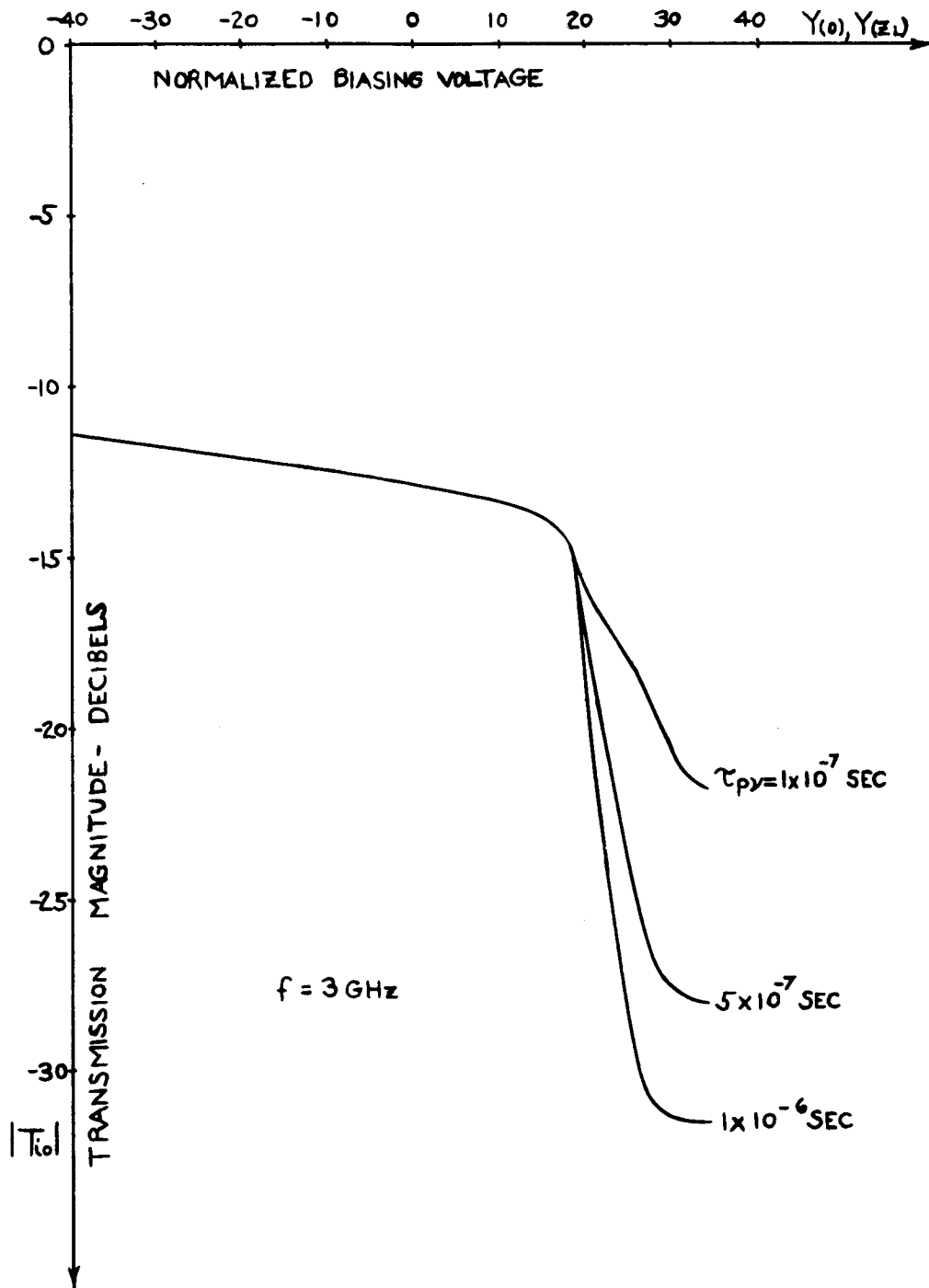


FIGURE 3.4-6 EFFECT OF MINORITY CARRIER LIFETIME ON CALCULATED TRANSMISSION MAGNITUDE

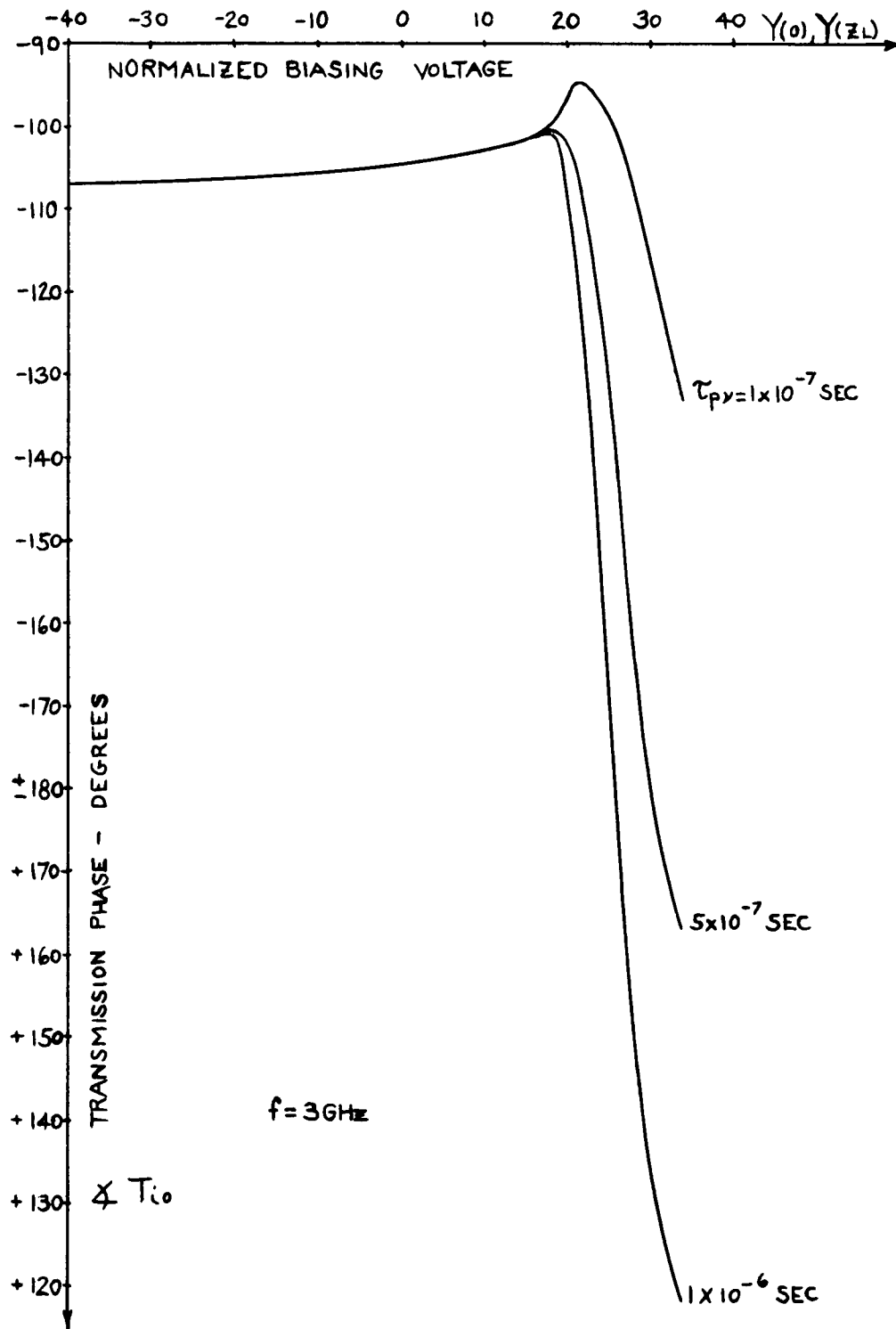


FIGURE 3.4-7 EFFECT OF MINORITY CARRIER LIFETIME ON CALCULATED TRANSMISSION PHASE

$$\begin{aligned}
 \alpha &= 1.62 \\
 b_{ox} &= 1000 \text{ \AA} \\
 J_o &= 3.6 \times 10^{-10} \text{ A/cm}^2 \\
 J_{on} &= 4.0 \times 10^{-6} \text{ A/cm}^2 \\
 N_A &= 1.5 \times 10^{20} \text{ cm}^{-3} \\
 N_{Dn} &= 1.5 \times 10^{18} \text{ cm}^{-3} \\
 N_{Dv} &= 5.0 \times 10^{14} \text{ cm}^{-3} \\
 \bar{\rho}_p &= 5.0 \times 10^{-3} \Omega\text{-cm} \\
 W_n &= 225 \mu \\
 W_v &= 25 \mu \\
 W_p &= .78 \mu \\
 W_{st} &= 5.0 \text{ mils} \\
 Z_L &= 80.0 \text{ mils} \\
 Z_{LT} &= 50 + j0 \Omega
 \end{aligned}$$

Table 3.4-1 Important Parameters Used In Model

parison. Curves and/or points are omitted at one frequency in some figures for clarity. Figures 3.4-16 through 3.4-23 show measured and calculated T_{10} and Γ_{10} for $-40 \leq Y(\omega) \leq 34$ and $Y(z_1) = 34$. For a plane of strong reflection to exist, for possible stub tuner application, $Y(\omega)$ must be less than zero. Figures 3.4-20 through 3.4-23 show that the device is useless as a stub tuner. To be useful as a stub tuner, the reflection magnitude must be close to 1 (0 dB) with a reflection phase proportional to the distance of the plane of strong reflection from the input. Figures 3.4-20 and 3.4-22 show that when the input of the device is well matched to the package stripline ($Y(\omega) < 0$) the reflection magnitude is less than -6 dB, indicating that the wave traveling from the device input to the plane of strong reflection and back to the input is strongly attenuated. The reflection magnitude increases sharply for strong forward bias ($Y(\omega) \geq 20$) due to the large discontinuity in characteristic impedance between the package stripline and the input of the device. Similarly, most of the change in reflection phase occurs when no plane of strong reflection exists and results from the change in Z_0 at the device input. Figures 3.4-21 and 3.4-23 show that the reflection phase changes very little with frequency for $Y(\omega) < 0$, which is unexpected if the phase shift is due to propagation to and from the plane of strong reflection. In addition, the voltage at $z=z_1$ has a small effect on reflection phase and magnitude (Figs. 3.4-12,13,14,15 compared to

3.4-20,21,22,23), indicating again that most of the reflection occurs at the device input.

There is excellent agreement between measured and calculated transmission phase and magnitude, but only fair to good agreement for reflection phase and magnitude. One possible way to improve the agreement between measured and calculated results is to model the region where the package stripline is bonded to the device microstrip. In this region the bonding wires introduce series inductance and the wide package stripline (30 mils) contributes fringing capacitance to the device input, which is a function of V_0 . The dc voltage V_0 determines the amount of charge (due to depletion or injection) at the junction of the p layer and the ν back-ground at the silicon surface ($x=0$) and, therefore, modulates the fringing capacitance. For a more exact rf model, a dc voltage dependent lumped element network should be developed and included between the package stripline and the device input.

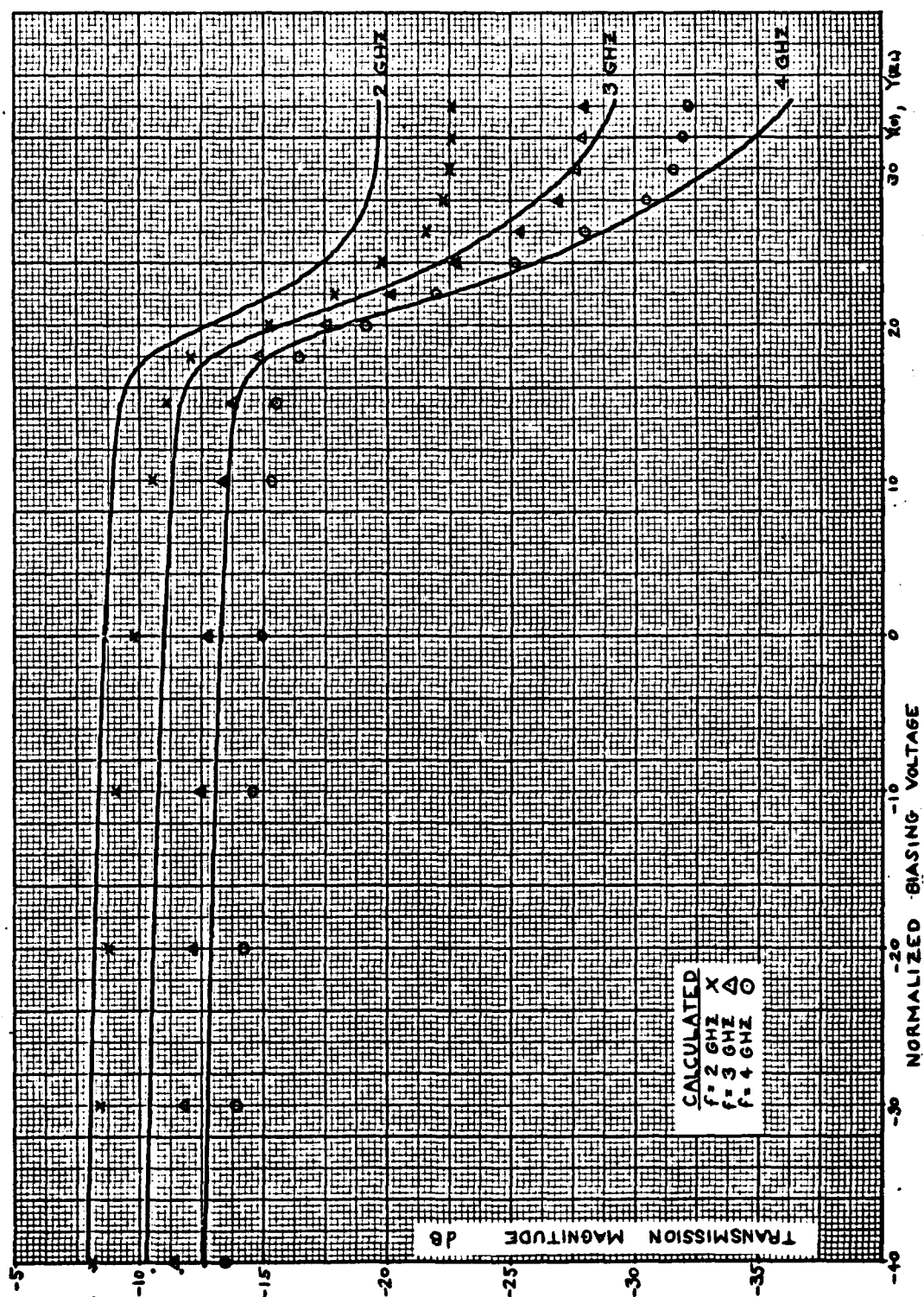


FIGURE 3.4-8 TRANSMISSION MAGNITUDE, $Y(z_L) = Y(o)$, 2, 3 AND 4 GHz

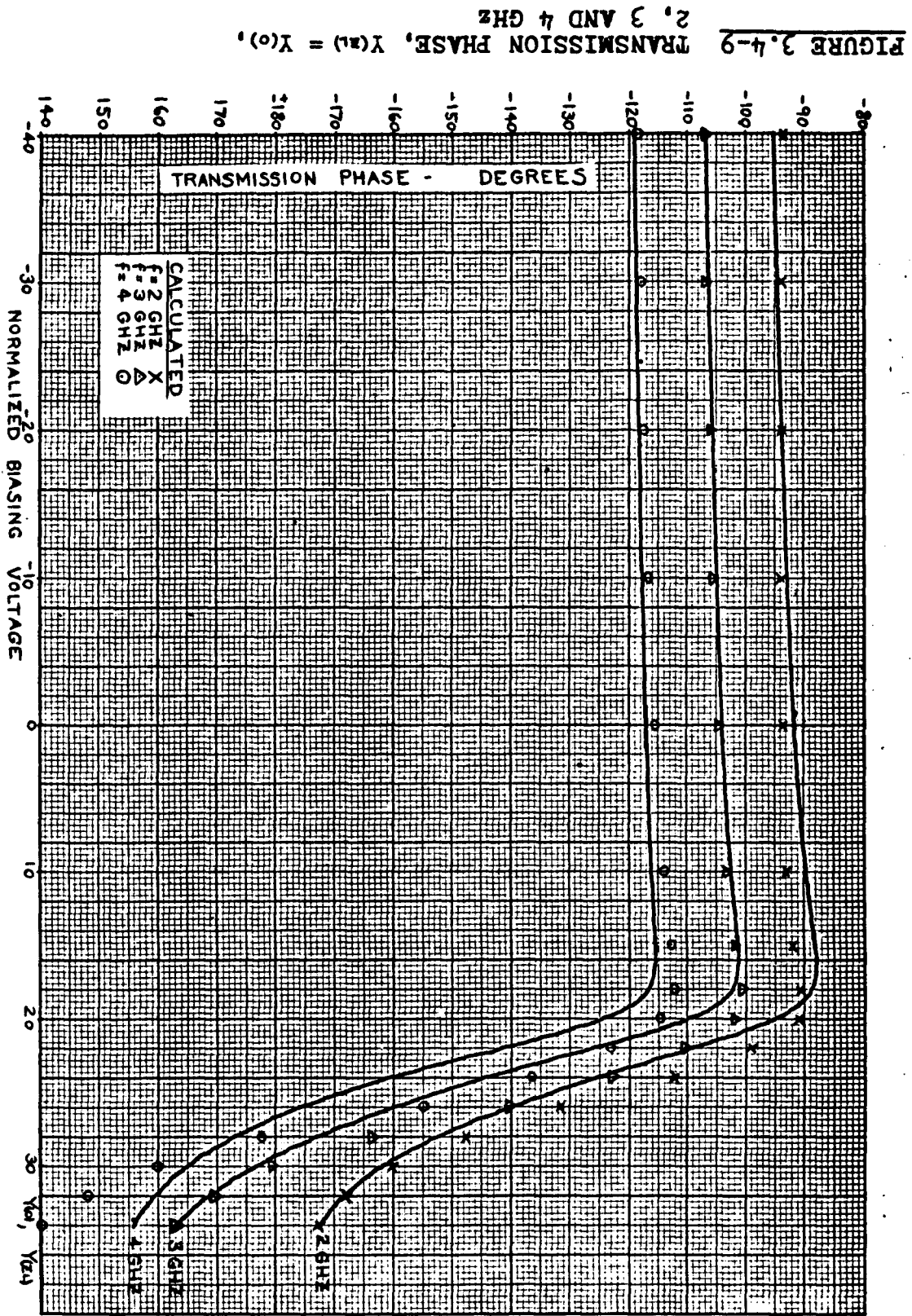
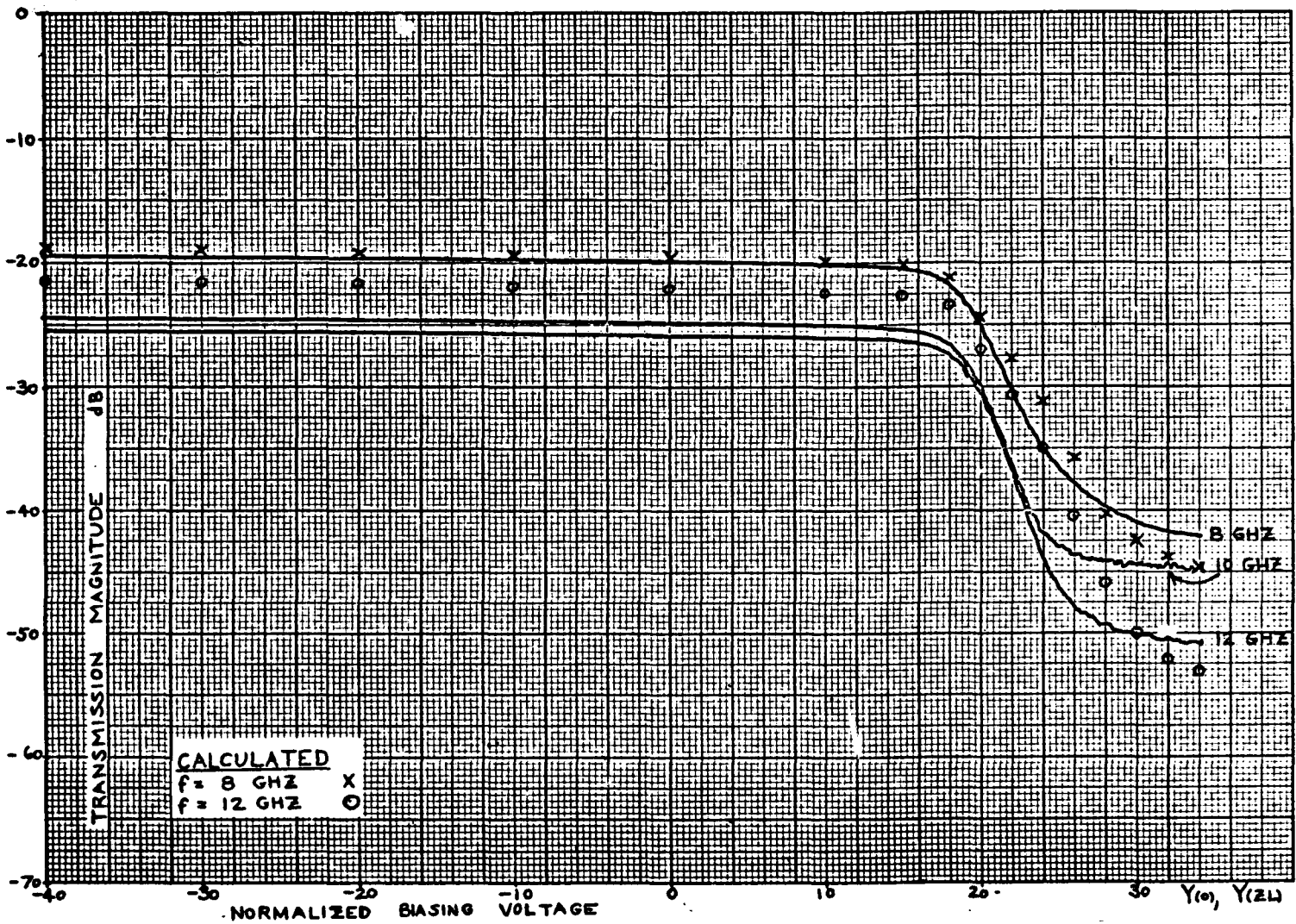


FIGURE 3.4-10 TRANSMISSION MAGNITUDE, $Y(\omega) = Y(0)$,
8, 10 AND 12 GHz



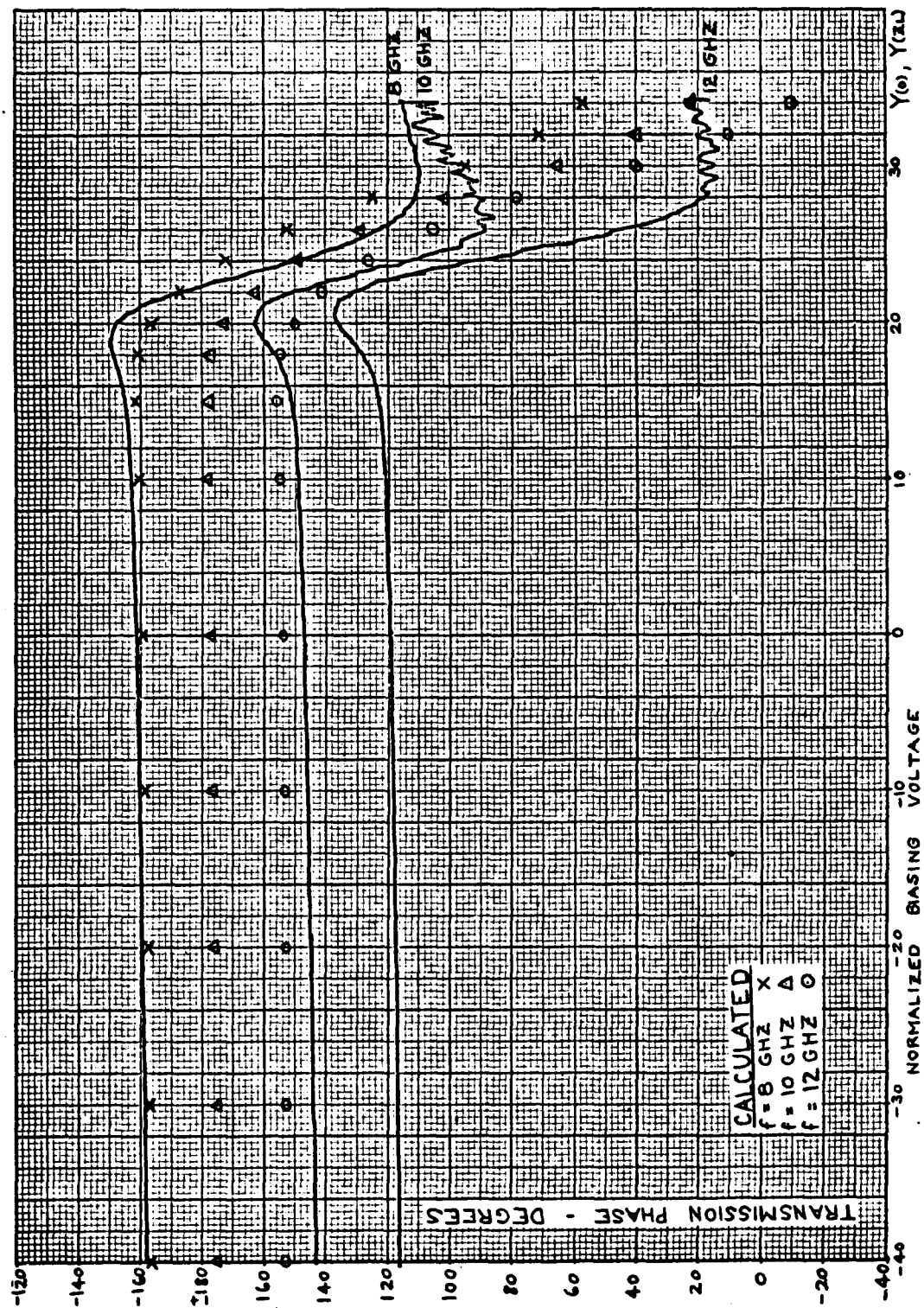


FIGURE 3.4-11 TRANSMISSION PHASE, $Y(2L) = Y(0)$,
8, 10 AND 12 GHz

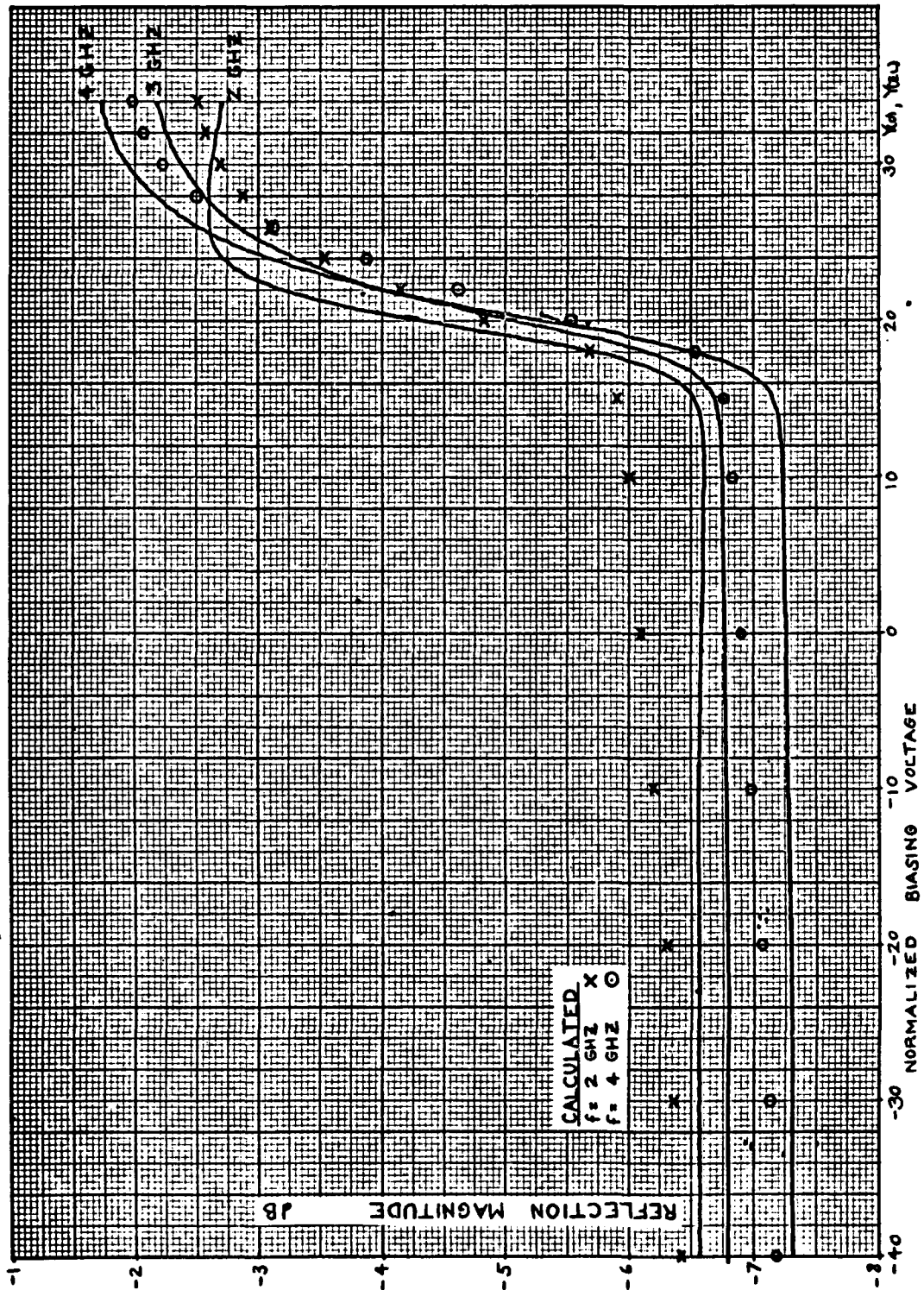
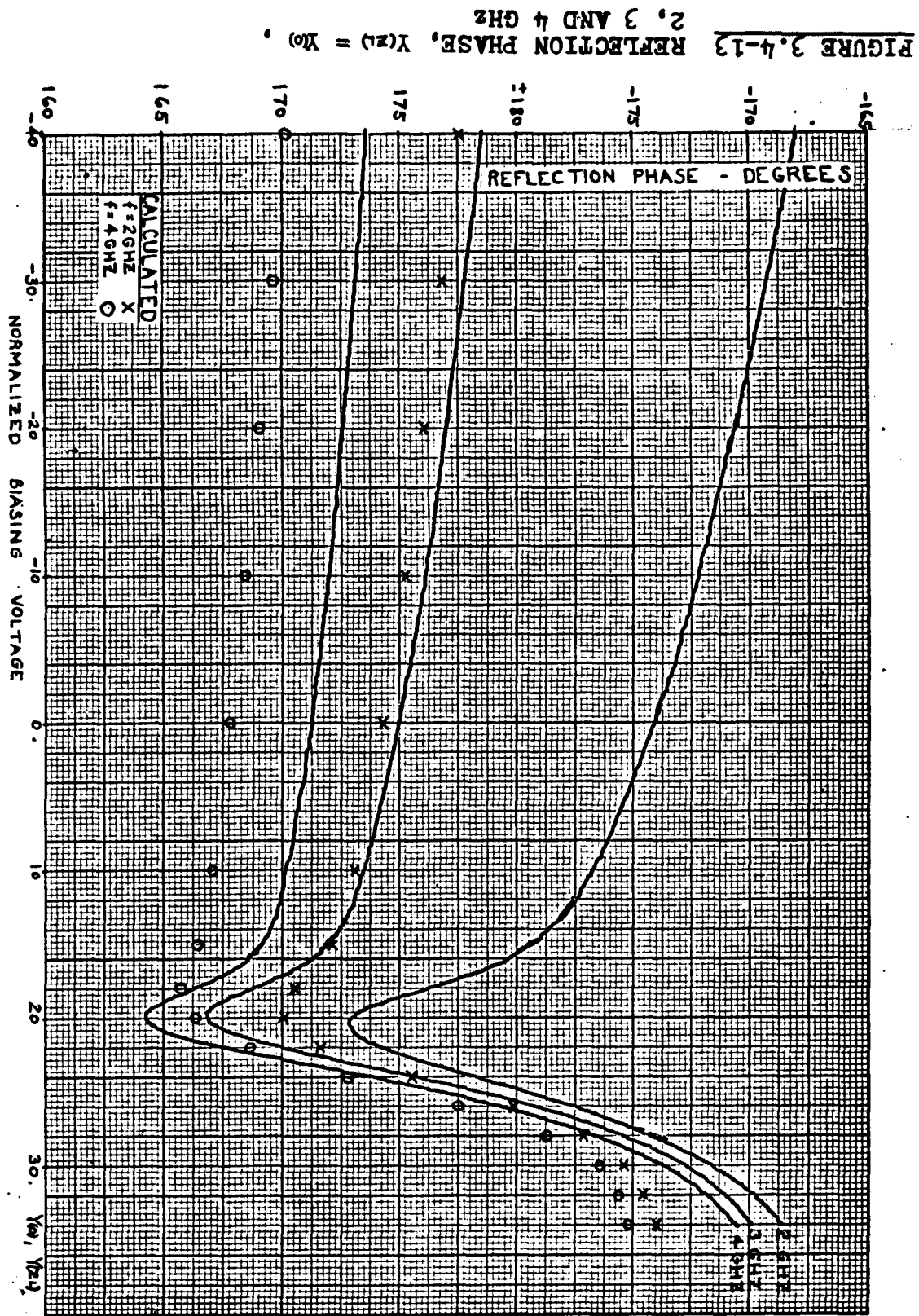


FIGURE 3.4-12 REFLECTION MAGNITUDE, $Y(\infty) = Y(0)$,
2, 3 AND 4 GHz



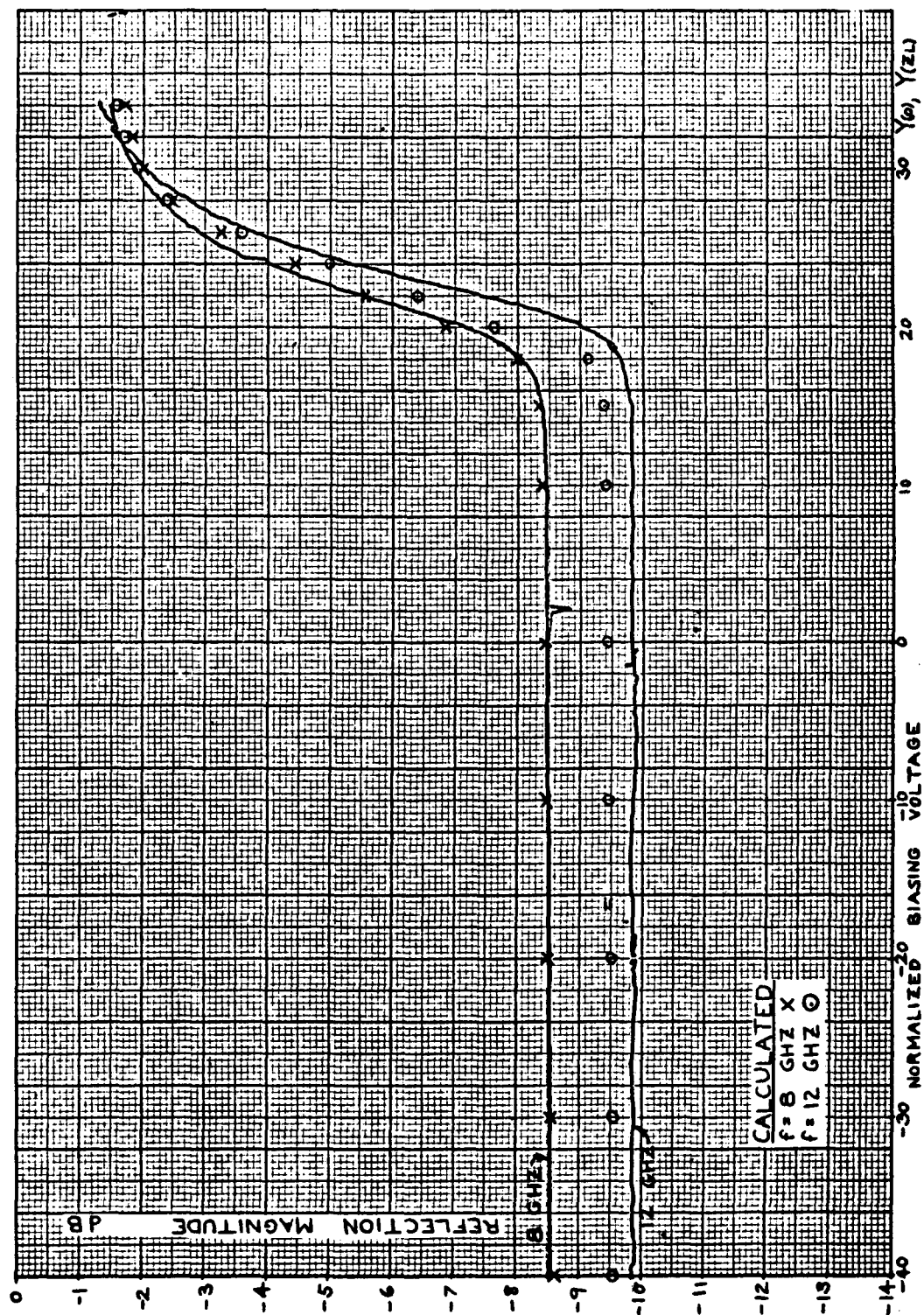


FIGURE 3.4-14 REFLECTION MAGNITUDE, $Y(z) = Y(o)$,
8 AND 12 GHz

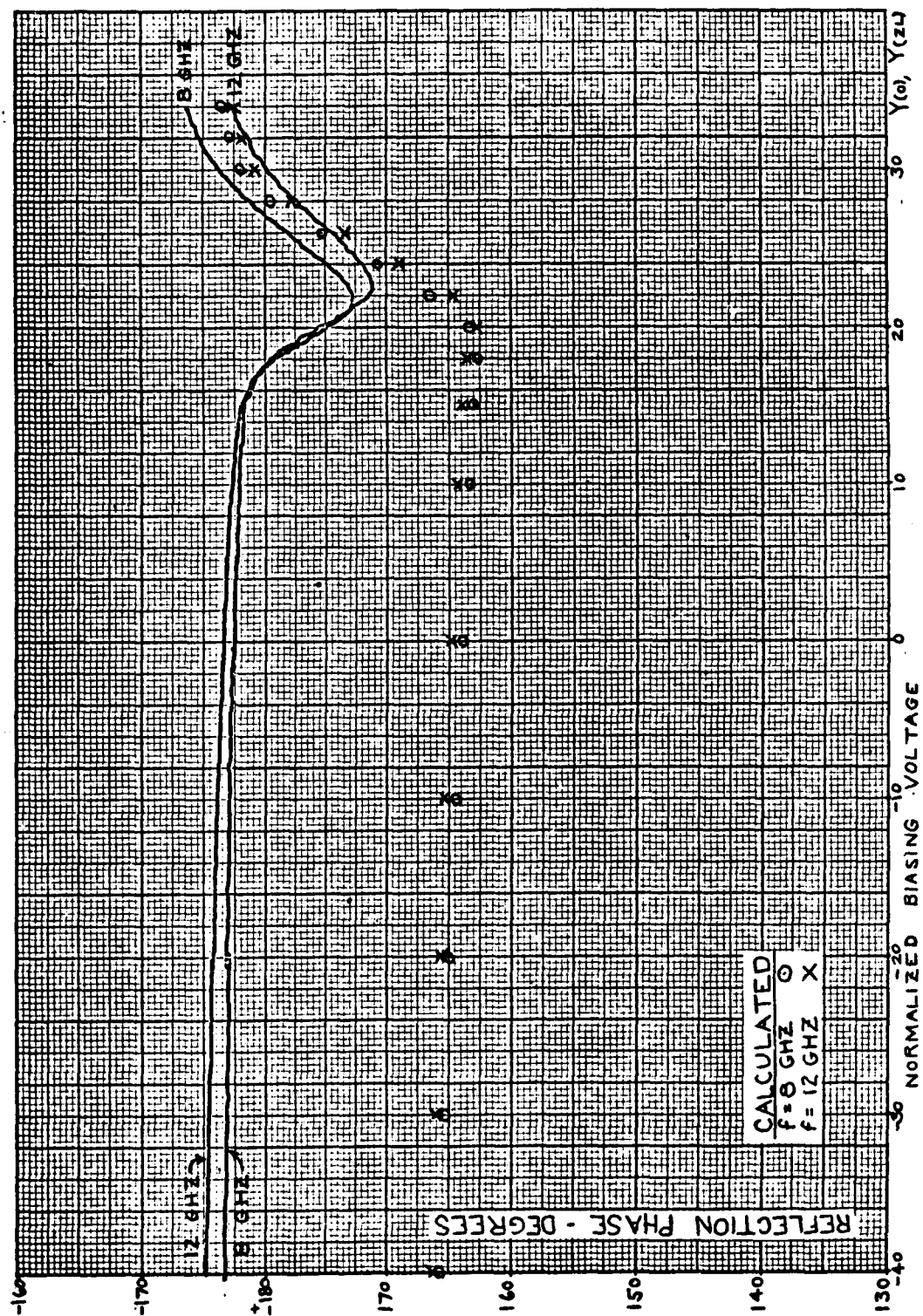


FIGURE 3.4-15 REFLECTION PHASE, $Y(z) = Y(o)$,
8 AND 12 GHz

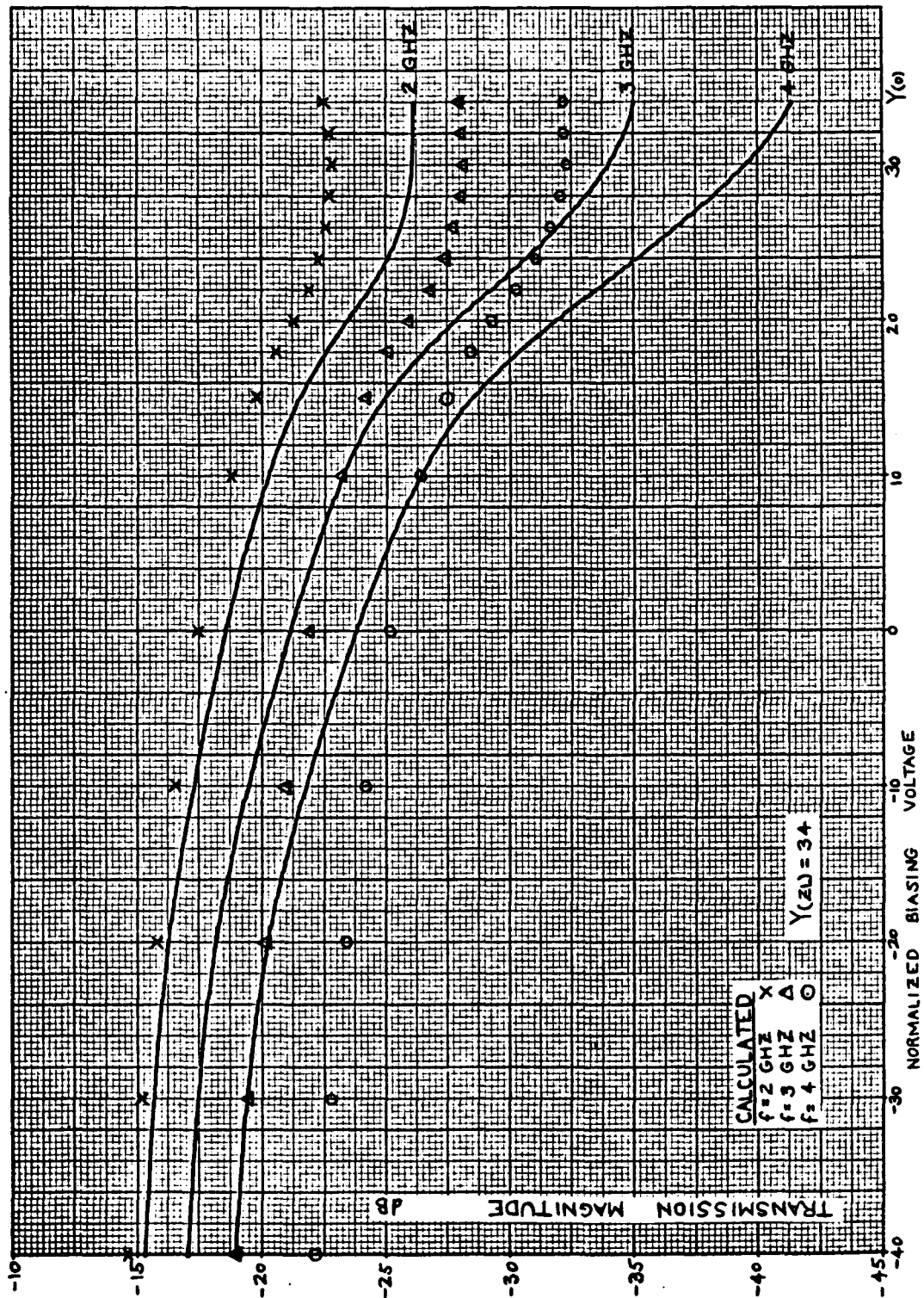


FIGURE 3.4 -16 TRANSMISSION MAGNITUDE, $Y(z_0) = 34$, 2, 3 AND 4 GHz

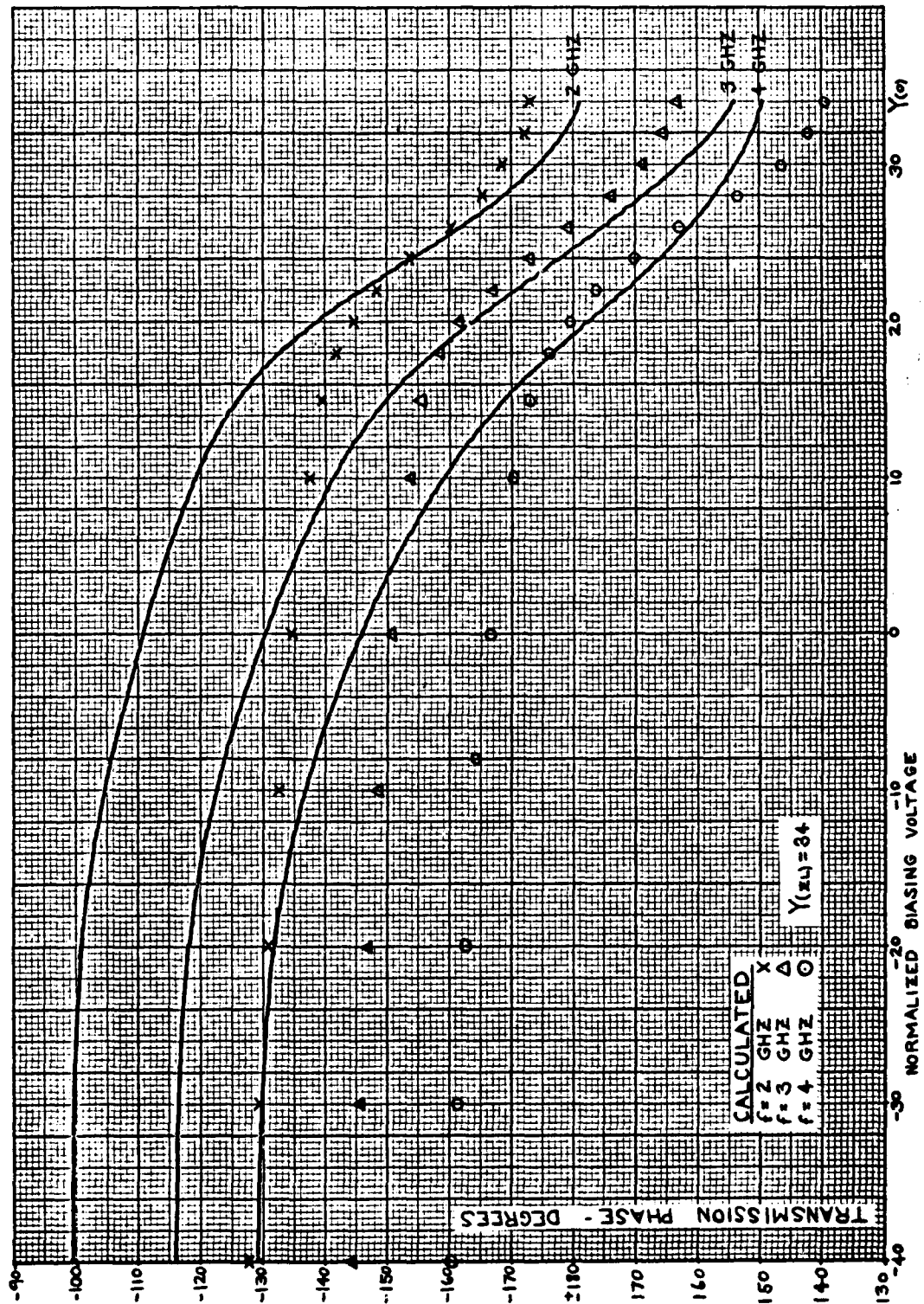


FIGURE 3.4-17 TRANSMISSION PHASE, $Y(z_1) = 34$, 2, 3 AND 4 GHz

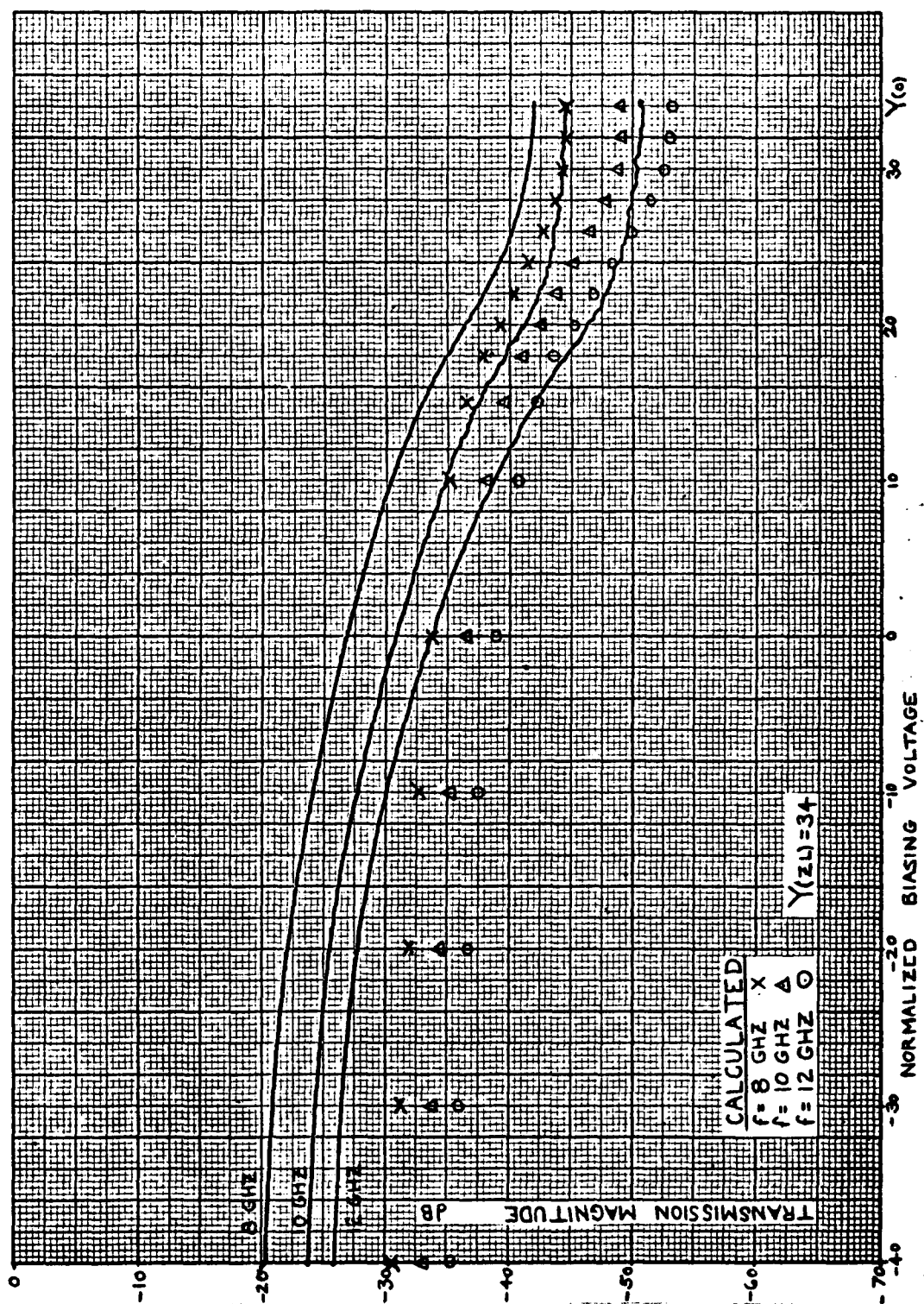
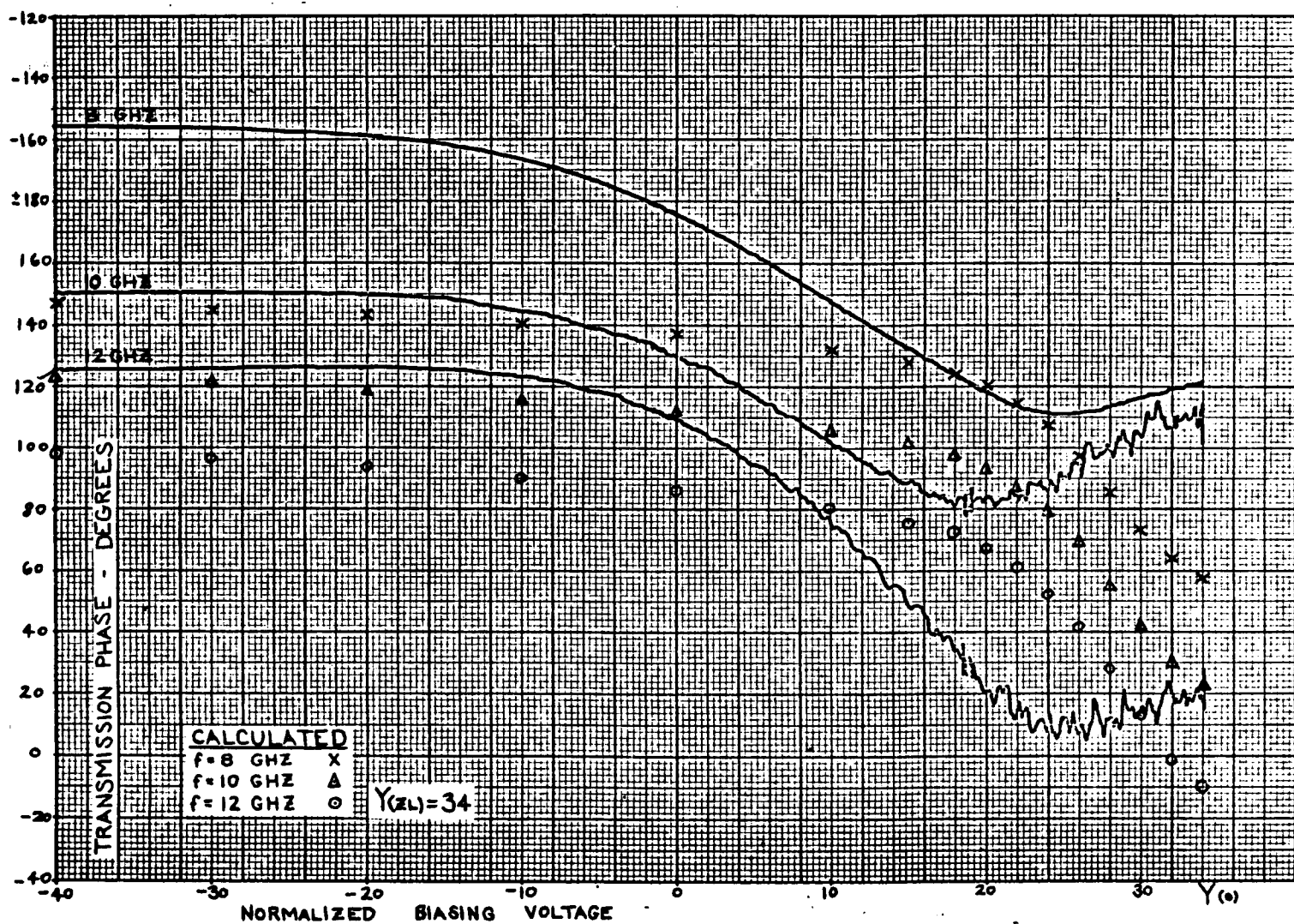


FIGURE 3.4-18 TRANSMISSION MAGNITUDE, $Y(z) = 34$, 8, 10 AND 12 GHz

FIGURE 3.4-19 TRANSMISSION PHASE, $Y(\omega) = 34$, 8, 10 AND 12 GHz



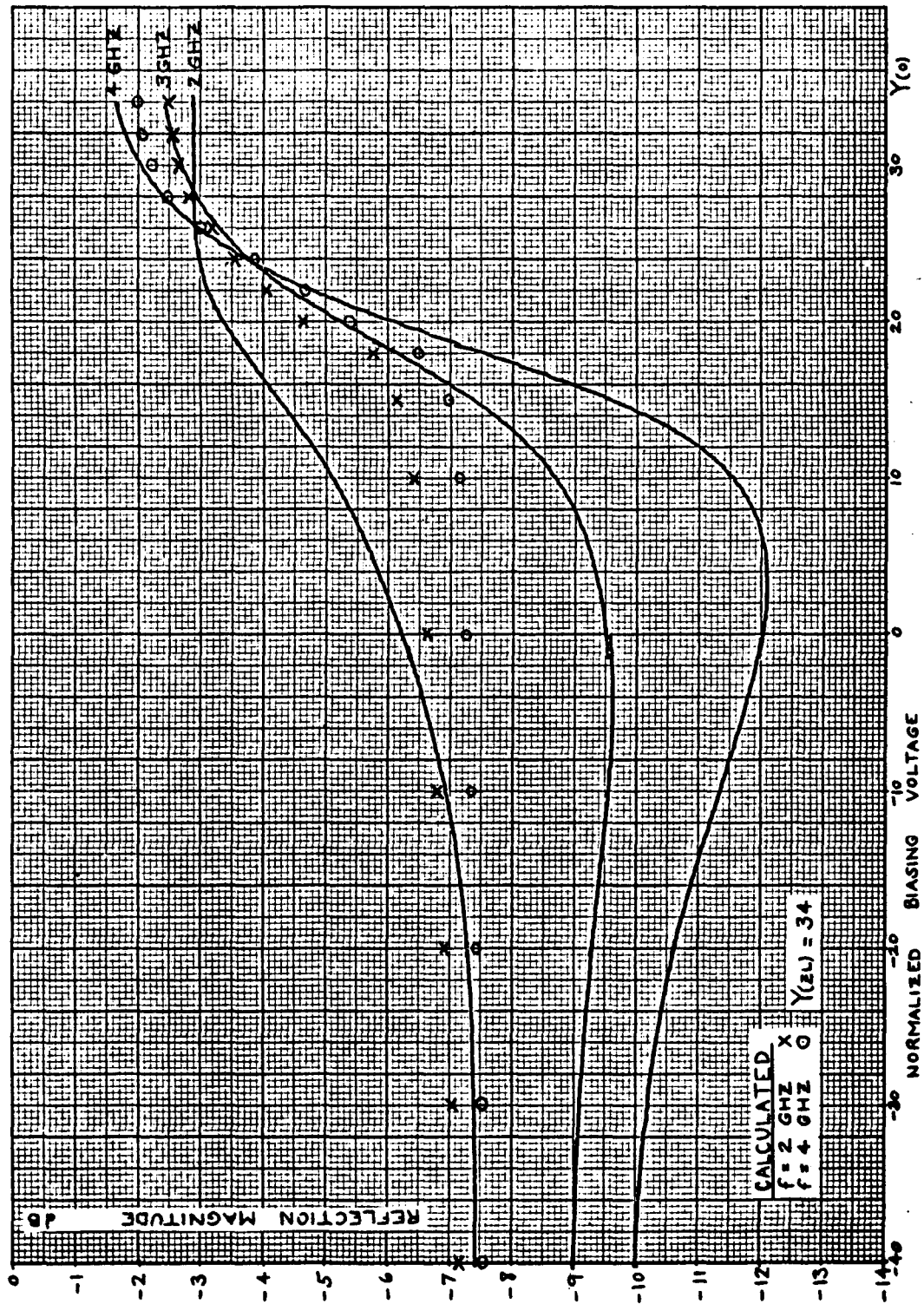


FIGURE 3.4-20 REFLECTION MAGNITUDE, $Y(0) = 34$, 2, 3 AND 4 GHz

FIGURE 3.4-21 REFLECTION PHASE, $\gamma(z_L) = 34^\circ$, 2 AND 4 GHz

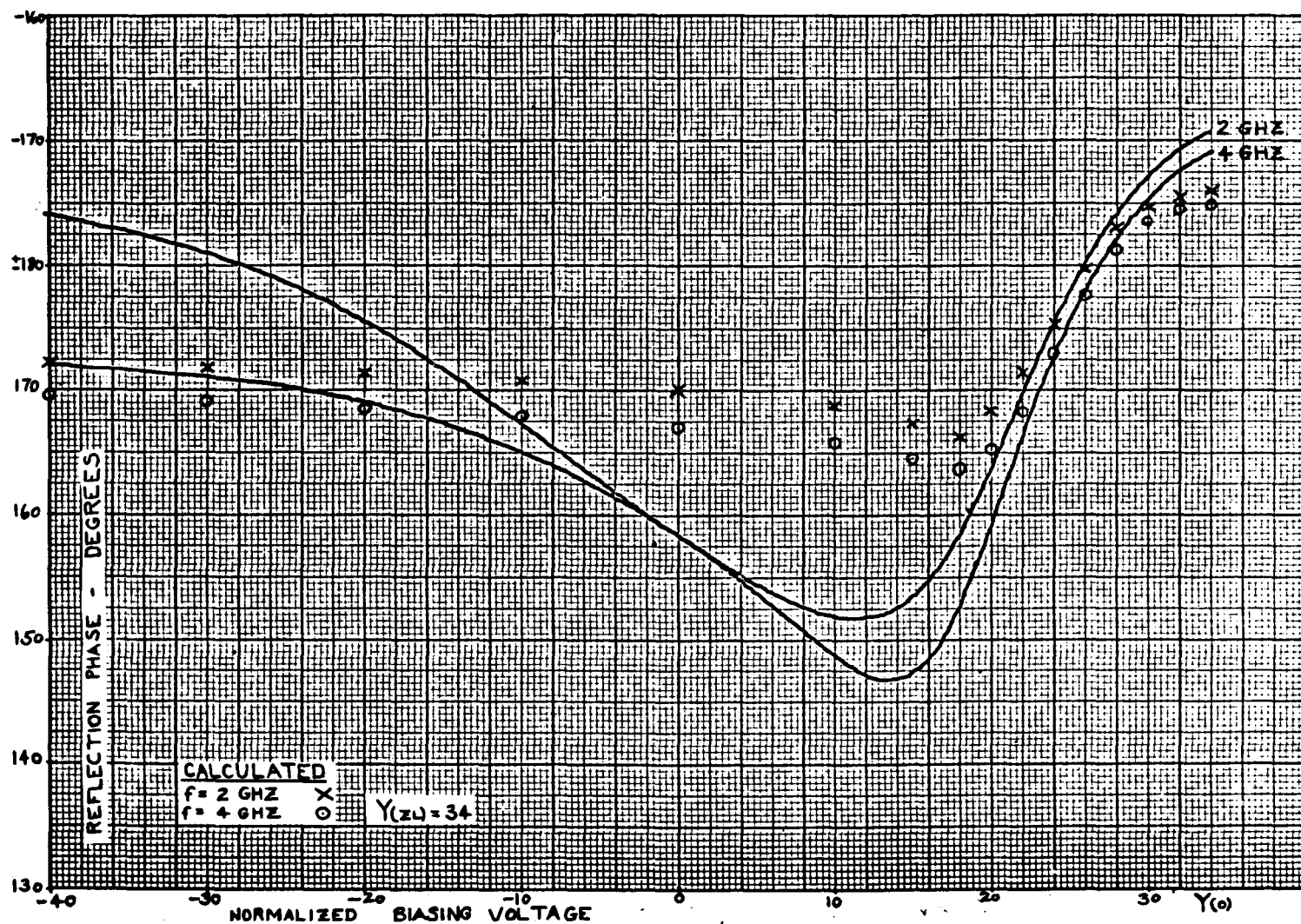
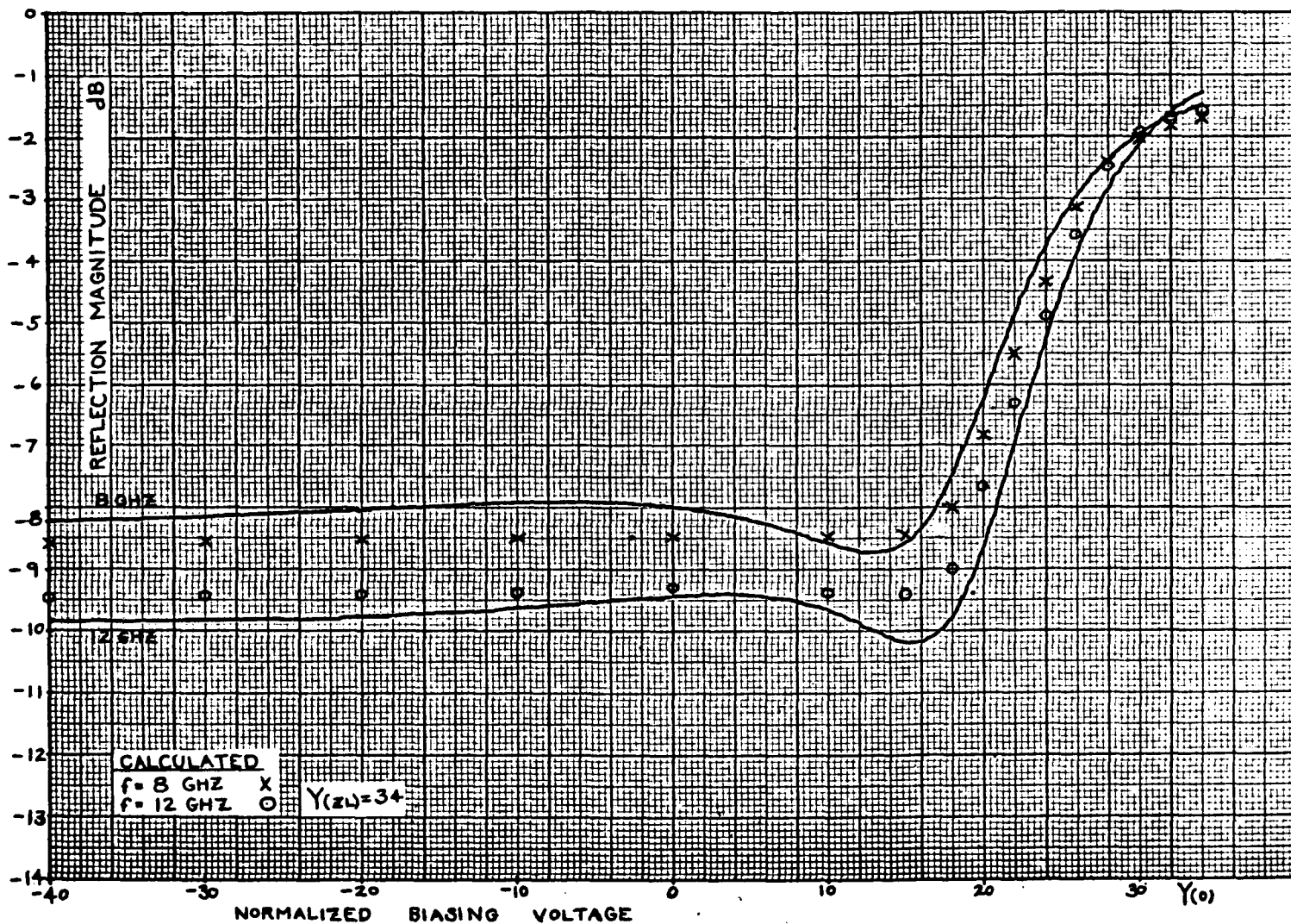


FIGURE 3.4-22 REFLECTION MAGNITUDE, $\gamma(z_L) = 34$, 8 AND 12 GHz



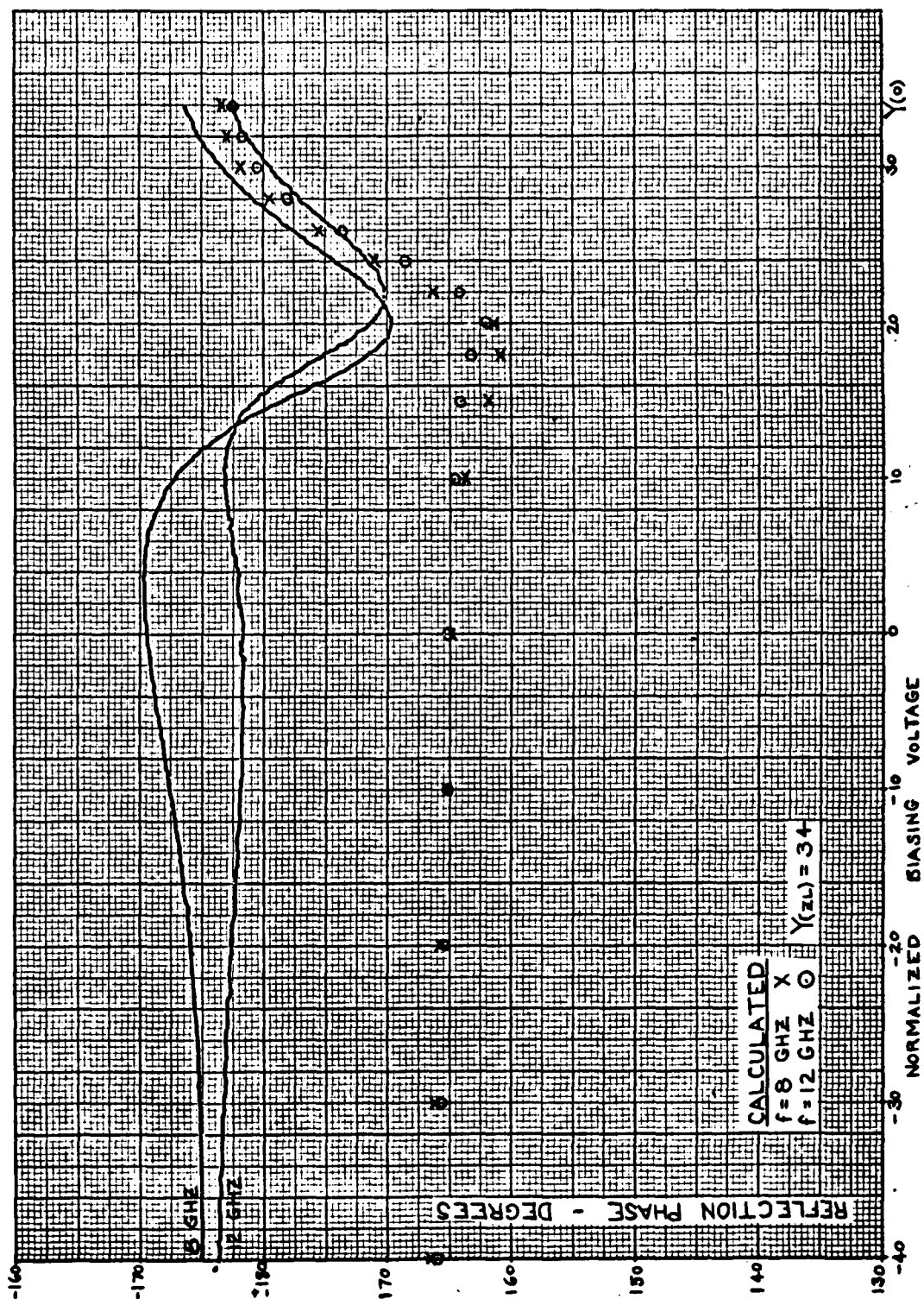


FIGURE 3.4-23 REFLECTION PHASE, $Y(2L) = 34$, 8 AND 12 GHz

CHAPTER 4 - SUMMARY AND CONCLUSIONS

The purpose of this chapter is to summarize the major problems solved in this dissertation, draw conclusions from its contents and outline suggestions for further work.

The major problems solved in this dissertation are the development of one-dimensional dc and rf models of the microstrip-oxide-distributed pyn device and the fabrication of devices used in experiments at dc, S and X bands. The dc model was developed with the use of approximations regarding the current density and potential distributions within the structure, based on the electrical properties of the material. The effect of biasing contact geometry on the approximations of uniform surface potential in the y direction and uniform z directed p layer current density was demonstrated experimentally. The dc voltage transfer function of the distributed pyn diode was numerically evaluated and confirmed by experiments, demonstrating its usefulness as a gate-oxide protection network. Above all, the dc surface voltage profile, evaluated from the dc model, sets the quiescent conditions for the rf analysis.

The rf characteristics of the device have been evaluated by cascading segments, each having uniform dc surface voltage, characteristic impedance and propagation constant. A parallel-plate waveguide model of the segment, supporting a dominant TM mode, has been used to evaluate Z_0 and γ . The transverse charge dynamics of the p_v junction have been char-

acterized by an effective complex permittivity derived from the small signal admittance of the junction. The effect of changes in parameters on propagation characteristics has been studied showing that Z_0 and γ are, by far, most sensitive to changes in ν layer parameters. Good to excellent agreement between experiments and theory was obtained for transmission coefficients at S and X bands. However, only fair to good agreement was obtained for reflection coefficients.

Measurements and calculated results show that use of the device as a dc voltage controllable semiconductor analog of a microwave stub tuner is impractical with currently available materials. For $-40 \leq Y_0 < 0$ and $Y(x) = 34$ (see Sec. 3.4) changes in the location of the plane of strong reflection had a disappointingly small effect on the input reflection coefficient. The main reasons for this are the attenuation in the reverse biased region, between the plane of strong reflection and the input, high attenuation near the plane of strong reflection, due to large amounts of stored charge (see Sec. 3.3), and insufficient values of $|\nabla Z_0|$ near the plane of strong reflection. The attenuation reduces the magnitudes of the reflected waves reaching the input, while the small values of $|\nabla Z_0|$ excite reflected waves which are undesirably small in magnitude. Increasing the magnitudes of the biasing voltages increases the values of $|\nabla Z_0|$ near the plane of strong reflection and results in larger values of Γ_{in} . However, this improvement is limited by the power dissipation of the device

and satisfactory operation is still unattainable. Changes in Γ_n when $Y(o)$ and $Y(z_L)$ are varied from -40 to 34 are, mainly, the result of changes in Z_o near the input of the device, producing a reflected wave from the discontinuity in characteristic impedance of the jig and driving point impedance of the microstrip. Although transmission through the device produces large phase shifts, its use as a phase shifter (with $Y(o)=Y(z_L)$) is unsatisfactory because of excessive propagation attenuation.

The analytical methods used in this dissertation may be applied to structures of different semiconductor substrate doping profiles but with similar contact and microstrip line geometry. An improved cascaded rf segment model may be used which considers the lumped impedance at the package stripline-device microstrip discontinuity (Sec. 3.4). A case which may be interesting to consider for future work is a substrate doping profile which produces active diodes. This device may provide amplification when used in the transmission mode. The junction of a tunnel diode, in such a structure, may be characterized as by Scott³¹ who investigates transmission on a strip Esaki diode (no oxide layer). The heavily doped pn substrate may propagate a slow wave mode at certain frequencies^{32,33} and, hence, amplify over many wavelengths.

Another area for future work is the modeling of the distributed pyn structure to obtain the dynamic response of

a MOSFET gate-oxide protection network. The device may be modeled by a cascade of sections, each being characterized by hole and electron charge control equations, for x directed current, and a continuity equation for the z direction. The system of equations may then be solved, using an appropriate numerical method on the digital computer, for the stored charge and surface voltage as functions of distance and time. A less involved approach may be to develop a resistor-capacitor-diode network directly from the charge dynamic equations, which may then be solved using an ECAP simulation.

APPENDIX I

STATIC CURRENT-VOLTAGE CHARACTERISTIC OF PYN DIODES

The calculation of the current density, $J_D(z)$, into the groundplane for a voltage, $V(z)$, existing at the surface of the p layer is a one dimensional diode problem presented here because of the unusual boundary conditions present. The structure, shown in Fig. I-1, is an epitaxial pyn diode without a metallic contact to the p region. The length in the z direction, $z_2 - z_1$, is made arbitrarily small so that $V(z)$ may be approximated by the constant voltage V from $z=z_1$ to $z=z_2$. The diode current density is supplied from the z directed current in the p layer and is

$$J_D(z) = \frac{1}{Y} \lim_{z_2 \rightarrow z_1} \left[\frac{I(z_1) - I(z_2)}{z_2 - z_1} \right] .$$

The evaluation of the static I-V characteristic is divided into three regions: low level injection in the y region, where the current density is proportional to $\exp(V/V_T)$, high level injection in the y region, where the current density is proportional to $\exp(V/2V_T)$, and the ohmic region, where voltage drops in the p and n regions dominate other voltage drops and the current density is proportional to V . To evaluate $J_D(z)$, referred to as J_D in this section, the carrier distributions in the y region as a function of V are sought.

The ambipolar transport equation ³⁴ in terms of the excess hole concentration in the y region, p' , assuming $p' = n'$, is

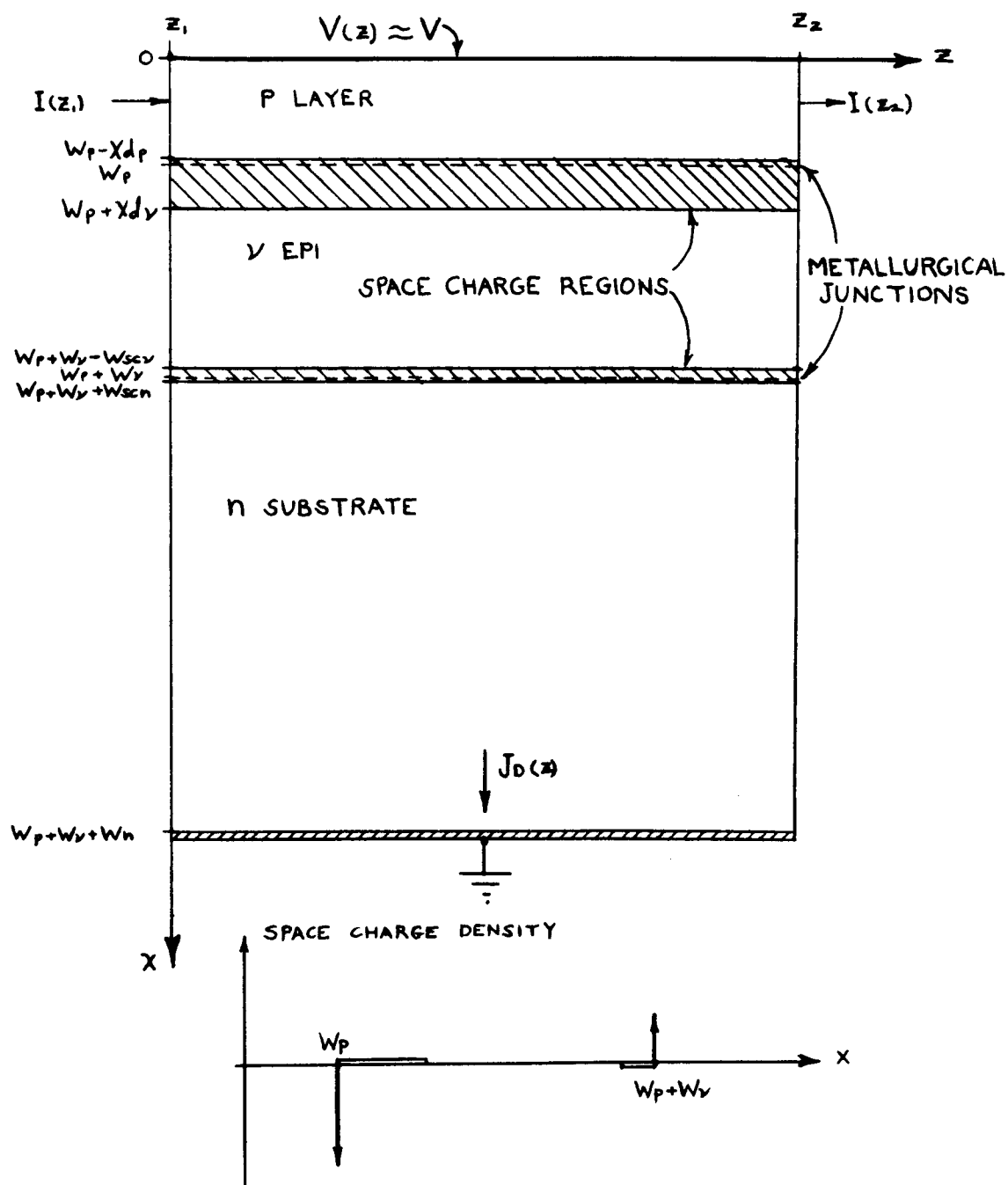


FIGURE I-1 STRUCTURE FOR THE EVALUATION OF THE J_0 -V CHARACTERISTIC

$$D_v^* \nabla^2(p') - \mu_v^* \vec{E} \cdot \nabla(p') + g'_v - \frac{p'}{\tau_v} = \frac{\partial p'}{\partial t} \quad \text{I-1}$$

where the ambipolar diffusion constant,

$$D_v^* = \frac{(n_{v0} + p_{v0} + 2p') D_{nv} D_{pv}}{(n_{v0} + p') D_{nv} + (p_{v0} + p') D_{pv}} \quad , \quad \text{I-1a}$$

and the ambipolar mobility,

$$\mu_v^* = \frac{(n_{v0} - p_{v0}) \mu_{nv} \mu_{pv}}{(n_{v0} + p') \mu_{nv} + (p_{v0} + p') \mu_{pv}} \quad . \quad \text{I-1b}$$

Carrier concentration variations in the y and z directions are not considered for this one dimensional diode and Eq.

I-1 is rewritten as

$$D_v^* \frac{\partial^2 p'}{\partial x^2} - \mu_v^* \frac{\partial p'}{\partial x} E_x + g'_v - \frac{p'}{\tau_v} = \frac{\partial p'}{\partial t} \quad . \quad \text{I-2}$$

Under forward bias conditions the carrier recombination rate, $\frac{p'}{\tau_v}$, is much greater than the thermal generation rate, g'_v , in the v region ³⁵, $\frac{\partial p'}{\partial t} = 0$ in the dc steady state and Eq. I-2 reduces to

$$D_v^* \frac{d^2 p'}{dx^2} - \mu_v^* \frac{dp'}{dx} E_x - \frac{p'}{\tau_v} = 0 \quad . \quad \text{I-3}$$

Low Level Injection in the v Region:

For low level injection conditions in the v region, $p' \ll N_{bv}$, the minority carrier diffusion term, $D_v^* \frac{d^2 p'}{dx^2}$, is much greater than the minority carrier drift term, $\mu_v^* \frac{dp'}{dx} E_x$, and Eq. I-3 is approximated by ³⁶

$$D_v^* \frac{d^2 p'}{dx^2} - \frac{p'}{\tau_{vl}} = 0, \quad \text{I-4}$$

where τ_{vl} is the low level injection minority carrier lifetime. Furthermore, since $n_{vo} \gg p'$ and $n_{vo} \gg p_{vo}$, $D_v^* \approx D_{pv}$ and Eq. I-4 is written as

$$D_{pv} \frac{d^2 p'}{dx^2} - \frac{p'}{\tau_{vl}} = 0. \quad \text{I-5}$$

Since the acceptor concentration in the p region is much greater than the donor concentration in the v region ($N_A \approx 10^{20} \text{ cm}^{-3}$, $N_{dv} \approx 10^{14} \text{ cm}^{-3}$) the current at the pv junction is dominated by hole diffusion, i.e. the total current density is approximately the hole diffusion current density. It is important to note that since the electron diffusion current at the pv junction is negligible compared to the hole diffusion current, knowledge of the excess electron distribution in the p region is unnecessary and the minority carrier surface recombination velocity at $x=0$ does not enter into the expression for J_0 .

The first boundary condition for the solution of Eq. I-5 is obtained by expressing the excess hole concentration at the edge of the pv space charge region, $x=W_p+x_d$, in terms of p_{vo} and the voltage dropped across the pv space charge region, V_1 , i.e. ³⁷

$$p'(W_p+x_d) = p_{vo} [\exp(V_1/V_T) - 1], \quad \text{I-6}$$

where $p_{y0} = n_i^2 / N_{Dy}$ and $V_T = kT/e$. For an epitaxial pyn diode with a similar doping profile Dutton and Whittier³⁸ assume that all of the applied voltage is dropped across the py space charge region, which is verified by Choo's³⁹ numerical analysis. As a result Eq. I-6 is written as

$$p'(W_p + x_{dy}) = p_{y0} [\exp(V/V_T) - 1] \quad \text{I-7}$$

The second boundary condition is obtained by evaluating the hole current density at the edge of the high-low junction space charge region, $x = W_p + W_y - W_{scy}$. The hole current density at $x = W_p + W_y - W_{scy}$ consists of the current density due to hole recombination in the yn space charge region plus the current density due to hole recombination in the n substrate. The current density due to hole recombination in the substrate is found by evaluating the hole diffusion current density at $x = W_p + W_y + W_{scn}$. The transport equation for excess holes in the substrate, $D_{pn} \frac{d^2 p'}{dx^2} - \frac{p'}{\tau_n} = 0$, is solved subject to the boundary conditions at $x = W_p + W_y + W_{scn}$ and $x = W_p + W_y + W_n$. The excess hole concentration at $x = W_p + W_y + W_n$ is assumed to be zero due to a large surface recombination velocity at the bottom contact and since $W_n \gg L_{pn}$ for the doping density used in the substrate ($W_n \approx 250 \mu$, $L_{pn} \approx 9.4 \mu$)⁴⁰. Solving the transport equation for the excess hole distribution in the substrate and evaluating

$J_{pn} = -e D_{pn} \left. \frac{dp'}{dx} \right|_{x=W_p+W_y+W_{scn}}$, the current density due to hole recombination in the substrate, yields

$$J_{pn} = e \frac{D_{pn}}{L_{pn}} p'(W_p + W_v + W_{scn}). \quad I-8$$

Since all of the external voltage is assumed to be dropped across the p_v space charge region, the voltage drop across the v_n space charge region is the barrier potential of the v_n junction and $p'(W_p + W_v + W_{scn})$ is related to $p'(W_p + W_v - W_{scv})$ by ⁴¹

$$p'(W_p + W_v + W_{scn}) = p'(W_p + W_v - W_{scv}) \frac{N_{dv}}{N_{dn}}. \quad I-9$$

Substituting Eq. I-9 into I-8 yields

$$J_{pn} = \frac{e D_{pn} N_{dv}}{L_{pn} N_{dn}} p'(W_p + W_v - W_{scv}). \quad I-10$$

The current density due to hole recombination in the v_n space charge region is evaluated by integrating the recombination rate across the v_n space charge region. With the impurity concentrations considered, nearly all of the v_n space charge region lies in the v material and the hole recombination current integral is approximated by integration over the v material side of the v_n space charge region. The recombination rate in the v material, from the Shockley-Read-Hall theory is ^{42, 43}

$$U = \frac{pn - n_i^2}{\tau_{pv}(n + n_i) + \tau_{nv}(p + n_i)}, \quad I-11$$

where τ_{pv} is the excess hole lifetime for the condition $n \gg p$ and τ_{nv} is the excess electron lifetime for the cond-

ition $p \gg n$, and the recombination centers are assumed to lie at the intrinsic Fermi level. In the νn space charge region $n \gg p$ and $n \gg n_i$ which reduces Eq. I-11 to (assuming τ_{pv} and τ_{nv} are the same order of magnitude)

$$U = U_{sc2} = \frac{p - n_i^2/n}{\tau_{pv}} \quad . \quad \text{I-12}$$

Assuming that $p \gg \frac{n_i^2}{n}$ and $n \approx N_{D\nu}$, $p' = p - \frac{n_i^2}{N_{D\nu}} \approx p$ and Eq. I-12 is written as

$$U_{sc2} = \frac{p'}{\tau_{pv}} \quad . \quad \text{I-13}$$

The excess hole concentration is approximated by

$$p'(x) \approx p(x) = n_i \exp\left[(\phi_i(x) - \phi_{fp}(x))/V_T\right], \quad \text{I-14}$$

where $\phi_i(x)$ is the intrinsic Fermi level and $\phi_{fp}(x)$ is the quasi-Fermi level for holes. As a first order approximation the potential is assumed to vary linearly and the quasi-Fermi level for holes is assumed to be constant across the νn space charge region⁴⁴. Integration of Eq. I-13 across the νn space charge region yields

$$J_{psc2} = e \int_{W_p+W_v-W_{scv}}^{W_p+W_v} U_{sc2} dx = \frac{e W_{scv} V_T p'(W_p+W_v-W_{scv})}{\tau_{pv} \phi_{\nu n}}, \quad \text{I-15}$$

where $\phi_i(W_p+W_v) - \phi_i(W_p+W_v-W_{scv}) \approx \phi_i(W_p+W_v+W_{scn}) - \phi_i(W_p+W_v-W_{scv}) = \phi_{\nu n}$, the barrier potential of the νn junction. The total current density at $x = W_p+W_v-W_{scv}$ is

$$-e D_{pv} \left. \frac{dp'}{dx} \right|_{x=W_p+W_v-W_{scv}} = J_{psc2} + J_{pn} \quad . \quad \text{I-16}$$

Substituting Eqs. I-15 and I-10 into I-16 results in the second boundary condition needed to solve Eq. I-5

$$\left. \frac{dp'}{dx} \right|_{x=W_p+W_y-W_{scv}} = -\frac{\alpha}{L_{pv}} p'(W_p+W_y-W_{scv}), \quad \text{I-17}$$

where the blocking parameter $\alpha = \frac{W_{scv} V_T}{\tau_{pv} \phi_{yn}} + \frac{D_{pn} N_{dv}}{L_{pn} N_{on}}$
and $L_{pv} = (D_{pv} \tau_{pv})^{1/2}$ as obtained by Dutton and Whittier⁴⁵.

Equation I-5 is solved and J_0 is evaluated from

$$J_0 = -e D_{pv} \left. \frac{dp'}{dx} \right|_{x=W_p+x_{dv}} \quad \text{resulting in}$$

$$J_0 = \frac{e D_{pv} n_i^2}{L_{pv} N_{dv}} \left\{ \frac{\sinh\left(\frac{W_b}{L_{pv}}\right) + \alpha \cosh\left(\frac{W_b}{L_{pv}}\right)}{\cosh\left(\frac{W_b}{L_{pv}}\right) + \alpha \sinh\left(\frac{W_b}{L_{pv}}\right)} \right\} [\exp(V/V_T) - 1], \quad \text{I-18}$$

where $W_b = W_y - W_{scv} - x_{dv}$, the width of the v region between the two space charge regions (base width). The current density is very nearly proportional to $(\exp(V/V_T) - 1)$ since W_b is a much weaker function of V than is $\exp(V/V_T)$, resulting in the desired characteristic for low level injection.

The blocking parameter, α , specifies the effect of the vn junction on the injected hole concentration distribution in the base and, hence, the leakage current density. The vn junction is blocking if the excess hole concentration at $x = W_p + W_y - W_{scv}$ is greater than the excess hole concentration, at the same value of x , that would result from an infinite base width. Dutton and Whittier show that $0 < \alpha < 1$ corresponds to a blocking vn junction. An $\alpha = 1$ corresponds to an injected hole concentration distribution in the base that would result from an infinite base width and is called a neutral vn

junction. For $1 < \alpha < \infty$ the excess hole concentration at the edge of the νn space charge region is less than the concentration, at the same value of x , that would result from an infinite base width, and the νn junction is non-blocking. An $\alpha \rightarrow \infty$ corresponds to the case where the n substrate is replaced by a metal resulting in infinite recombination velocity and zero excess hole concentration at $x = W_p + W_{\nu} - W_{sc\nu}$. The blocking parameter, α , may also be thought of as the normalized recombination velocity for holes at the edge of the νn space charge region. The νn junction may, therefore, cause a hole accumulation at the edge of the νn space charge region.

For large differences in impurity concentration between the base and the substrate, $\frac{N_{bv} D_{pn}}{N_{bn} L_{pn}} \ll \frac{W_{sc\nu} V_T}{\tau_{pv} \phi_{\nu n}}$ as verified from Choo's numerical results⁴⁶, and α reduces to

$$\alpha = \frac{W_{sc\nu} V_T L_{pv}}{\tau_{pv} \phi_{\nu n} D_{pv}} \quad . \quad \text{I-19}$$

For the device doping densities under consideration, $\frac{N_{bv}}{N_{bn}}$ lies in the range 10^{-4} to 10^{-6} and Eq. I-19 is a good approximation. For increasingly larger values of $\frac{N_{bv}}{N_{bn}}$ the νn space charge width decreases resulting in less recombination in the νn space charge region relative to the recombination in the substrate.

The parameter α is experimentally determined by comparing the measured I-V characteristic of a test diode, fabricated on the same chip as and in close proximity to the

microstrip-oxide-distributed pvn device, with the I-V characteristic obtained analytically by setting $\alpha \rightarrow \infty$ in Eq. I-18. For $\alpha \rightarrow \infty$ the current density from Eq. I-18 becomes

$$J_0 = \frac{e D_{pv} n_i^2}{L_{pv} N_{bv}} \coth\left(\frac{W_b}{L_{pv}}\right) [\exp(V/V_T) - 1] \quad \text{I-20}$$

The ratio of Eq. I-18 to I-20 yields

$$\frac{J_0}{J_0} = \frac{J_0}{J_0} = \beta = \tanh\left(\frac{W_b}{L_{pv}}\right) \left\{ \frac{\sinh\left(\frac{W_b}{L_{pv}}\right) + \alpha \cosh\left(\frac{W_b}{L_{pv}}\right)}{\cosh\left(\frac{W_b}{L_{pv}}\right) + \alpha \sinh\left(\frac{W_b}{L_{pv}}\right)} \right\} \quad \text{I-21}$$

where J_0 and $J_0_{\alpha \rightarrow \infty}$ are the leakage current densities of Eqs. I-18 and I-20 arbitrarily evaluated as $V \rightarrow 0$.

Equation I-18 specifies J_0 as a function of V for low level injection conditions in the v region. As V is increased the excess hole and electron concentrations in the v region increase until the low level injection condition, $p' \ll N_{bv}$, is no longer satisfied and Eq. I-18 no longer accurately describes the J_0 vs V characteristic.

High Level Injection in the v Region:

For high level injection in the v region the hole and electron concentrations are greater than or comparable to the equilibrium electron concentration and the voltage dropped across the vn junction is no longer negligible compared to the voltage dropped across the pv junction. The voltage dropped in the v region due to ohmic effects and carrier gradients must also be considered. However, at all but the largest applied voltages, low level injection still exists in the heavily doped p and n regions, making the voltage

dropped in these regions negligible when compared with the applied voltage, V , between the p layer and the groundplane. The current density due to recombination in the ν region is much larger than the current density due to recombination in the other regions, since the excess carrier concentrations relative to the equilibrium majority carrier concentration is much larger in the ν region than in the other regions. The current densities due to recombination in the space charge regions may be neglected as shown by Choo's ⁴⁷ numerical results and Herlet's ⁴⁸ analytical treatment. Thus, the total current density, J_0 , is due to recombination in the ν region and is equal to the hole current density at $x=W_p+x_{dv}$ or the electron current density at $x=W_p+W_\nu-W_{sc\nu}$, as treated by McKelvey ⁴⁹, which allows one to write

$$J_0 = J_p(W_p+x_{dv}) = p(W_p+x_{dv})e\mu_{pv}E(W_p+x_{dv}) - eD_{pv}\left.\frac{dp}{dx}\right|_{x=W_p+x_{dv}}, \quad \text{I-22}$$

$$J_n(W_p+x_{dv}) = 0 = n(W_p+x_{dv})e\mu_{nv}E(W_p+x_{dv}) + eD_{nv}\left.\frac{dn}{dx}\right|_{x=W_p+x_{dv}}, \quad \text{I-23}$$

$$J_0 = J_n(W_p+W_\nu-W_{sc\nu}) = n(W_p+W_\nu-W_{sc\nu})e\mu_{nv}E(W_p+W_\nu-W_{sc\nu}) + eD_{nv}\left.\frac{dn}{dx}\right|_{x=W_p+W_\nu-W_{sc\nu}}, \quad \text{I-24}$$

and

$$J_p(W_p+W_\nu-W_{sc\nu}) = 0 = p(W_p+W_\nu-W_{sc\nu})e\mu_{pv}E(W_p+W_\nu-W_{sc\nu}) - eD_{pv}\left.\frac{dp}{dx}\right|_{x=W_p+W_\nu-W_{sc\nu}}. \quad \text{I-25}$$

For strong high level injection in the ν region $p', n' \gg N_{D\nu}$ and

$$p' = n' = n = p \quad . \quad \text{I-26}$$

Substituting Eq. I-23 into I-22 and using I-26 yields

$$\left. \frac{dp'}{dx} \right|_{x=W_p+W_{sv}} = -J_0 / (2eD_{p\nu}) \quad ; \quad \text{I-27}$$

similarly, substituting Eq. I-25 into I-24 with I-26 yields

$$\left. \frac{dp'}{dx} \right|_{x=W_p+W_v-W_{scv}} = J_0 / (2eD_{p\nu}\mu_{nv}/\mu_{pv}) \quad . \quad \text{I-28}$$

Since under strong high level injection in the ν region, $p' \gg n_{v0}$, the ambipolar mobility, μ_{ν}^* , is much smaller than its value for low level injection and the drift term in Eq. I-3 is much smaller than the diffusion term. This does not mean that the carriers do not drift, but that the carrier concentration distribution does not drift significantly ⁵⁰. As a result Eq. I-3 reduces to

$$D_{\nu}^* \frac{d^2 p'}{dx^2} - \frac{p'}{\tau_{vH}} = 0 \quad . \quad \text{I-29}$$

In the limit as $p' \rightarrow p$ the ambipolar diffusion constant, D_{ν}^* , becomes $D_{\nu}^* = \frac{2D_{nv}D_{pv}}{D_{nv}+D_{pv}}$. Equation I-29 is solved subject to the boundary conditions specified by Eqs. I-27 and I-28 resulting in the excess hole distribution in the ν region as a function of J_0 . Boundary condition I-6 is used along with the boundary condition ⁵¹ $p'(W_p+W_v-W_{scv}) = N_{D\nu} [\exp(V_2/V_T) - 1]$ and the expression for $p'(x)$ in terms of J_0 to find V_1 and V_2 in terms of J_0 , where V_2 is the portion of V dropped across

the γ n space charge region. The voltage dropped across the γ region, V_γ , due to its resistivity and carrier concentration gradients, is found by integrating the electric field across the γ region. The electric field is found from the general expression for J_b , in the γ region, in terms of $E(x)$ and $\frac{dp'(x)}{dx}$, and is simplified by substituting $\frac{dp'(x)}{dx}$ as a function of J_b into the expression⁵². By expressing V as $V = V_1(J_D) + V_2(J_D) + V_\gamma$, J_D is found to be⁵³

$$J_D = J_{on} \exp(V/2V_T), \quad \text{I-30}$$

where

$$J_{on} = \frac{e n_i 2 D_{ny} D_{py}}{L_y^* (D_{ny} + D_{py})} \exp[-\Phi_b(h)], \quad \text{I-31}$$

$$h = \frac{W/b}{L_y^*}, \quad \text{I-32}$$

$$\begin{aligned} \Phi_b(h) = & \ln \left[\frac{\sqrt{(1+b \cosh(h))(b+\cosh(h))}}{(b+1) \sinh(h)} \right] \\ & + \frac{2b}{1+b} \frac{\sinh(h) \tan^{-1} \left[\frac{\sinh(h/2) \sqrt{1 - \left(\frac{b-1}{b+1}\right)^2 \tanh^2(h/2)}}{\sqrt{1+b^2+2b \cosh(h)}} \right]}{\sqrt{1+b^2+2b \cosh(h)}} \\ & - \frac{1}{2} \frac{b-1}{b+1} \ln \left[\frac{b+\cosh(h)}{1+b \cosh(h)} \right], \end{aligned} \quad \text{I-33}$$

$$L_y^* = \tau_{yn} D_y^*, \quad \text{I-34}$$

and

$$b = \mu_{ny} / \mu_{py} \quad . \quad \text{I-35}$$

The value of J_{oh} used in the numerical solution is obtained from measurements on test diodes fabricated on the same chip as, and close proximity to, the device and is discussed in Sec. 2.6.

Ohmic Region:

For still higher values of forward voltage a significant portion of V is dropped in the substrate and the J_D - V characteristic begins to appear resistive. Above the voltage $V = V_R$ the J_D - V characteristic is approximated as

$$J_D = (V - V_R) G_D + J_D \Big|_{V=V_R} \quad \text{I-36}$$

where the constant G_D is a dc incremental conductance per unit area.

Summary:

The J_D - V characteristic used in the dc model is therefore the piecewise approximation

$$J_D = \begin{cases} J_0 [\exp(V/V_T) - 1] & V \leq V_c \\ J_{oh} \exp(V/2V_T) & V_c < V \leq V_R \\ (V - V_R) G_D + J_D \Big|_{V=V_R} & V_R < V \end{cases} \quad , \quad \text{I-37}$$

where J_0 is the low level injection leakage current density, J_{oh} is the high level injection "leakage" current density,

and V_c is the critical voltage, defined as the voltage between the surface of the p layer and the groundplane at which the transition from low level to high level injection occurs. V_c may be approximated by the voltage at which the low level injection current density function is equal to the high level injection current density function,

$$V_c \approx 2 V_T \ln \left(\frac{J_{oh}}{J_o} \right) . \quad \text{I-38}$$

G_o and V_R are selected for best fit to the measured I-V characteristics of the test diodes at very large forward bias.

APPENDIX II
EFFECTIVE TRANSVERSE COMPLEX PERMITTIVITY OF THE
PV SPACE CHARGE REGION

The evaluation of the effective complex permittivity of the pv space charge region for an electric field normal to the plane of the junction requires a knowledge of the junction small signal admittance and transition capacitance.

The x directed small signal current density of the structure shown in Fig. I-1 is expressed as

$$J_x = J_{ip}(W_p + x_{dv}) + J_{in}(W_p + x_{dv}) . \quad \text{II-1}$$

Neglecting recombination in the space charge region, as in the static analysis, enables Eq. II-1 to be written as

$$J_x = J_{ip}(W_p + x_{dv}) + J_{in}(W_p - x_{dp}) . \quad \text{II-2}$$

Since $N_A \gg N_D$, the small signal hole current density is much greater than the small signal electron current density and Eq. II-2 simplifies to

$$J_x = J_{ip}(W_p + x_{dv}) . \quad \text{II-3}$$

Assuming that $J_{ip}(W_p + x_{dv})$ consists entirely of a diffusion term, Eq. II-3 is evaluated from the small signal hole concentration distribution, $p(x)$, in the y region.

As expressed in Appendix I the ambipolar transport equation is written

$$\frac{\partial p'}{\partial t} + \mu_v^* E \frac{\partial p'}{\partial x} - D_v^* \frac{\partial^2 p'}{\partial x^2} = -\frac{p'}{\tau_{pv}} \quad , \quad \text{II-4}$$

where

$$\mu_v^* = \frac{(n_{v0} - p_{v0}) \mu_{nv} \mu_{pv}}{p \mu_{pv} + n \mu_{nv}} \quad \text{II-4a}$$

and

$$D_v^* = \frac{(2p' + n_{v0} + p_{v0}) D_{pv} D_{nv}}{p D_{pv} + n D_{nv}} \quad . \quad \text{II-4b}$$

For low level dc hole injection in the v region $p' \ll n_{v0}$ and $\mu_v^* E \frac{\partial p'}{\partial x} \ll D_{pv} \frac{\partial^2 p'}{\partial x^2}$, and since $p_{v0} \ll n_{v0}$ Eq. II-4 becomes

$$\frac{\partial p'}{\partial t} - D_{pv} \frac{\partial^2 p'}{\partial x^2} = -\frac{p'}{\tau_{pv}} \quad . \quad \text{II-5}$$

Applying a p layer surface voltage with respect to the groundplane of

$$V = V_0 + V_a e^{j\omega t} \quad \text{II-6}$$

gives rise to an excess hole concentration of the form

$$p' = p'_0(x) + p_1(x) e^{j\omega t} \quad , \quad \text{II-7}$$

where $p_1(x) \ll p'_0(x)$ when $V_a \ll V_T$. Substitution of Eq. II-7 into II-5 and separating out the ac part gives

$$D_{pv} \frac{\partial^2 p_1(x)}{\partial x^2} - p_1(x) \frac{(1 + j\omega \tau_{pv})}{\tau_{pv}} = 0 \quad \text{II-8}$$

having the solution

$$p_1(x) = A e^{-x/L'_{pv}} + B e^{x/L'_{pv}}, \quad \text{II-9}$$

where

$$L'_{pv} = \frac{(D_{pv} \tau_{pv})^{1/2}}{(1 + j\omega \tau_{pv})^{1/2}} = \frac{L_{pv}}{(1 + j\omega \tau_{pv})^{1/2}}. \quad \text{II-9a}$$

Boundary conditions at $x=W_p+x_{dv}$ and $x=W_p+W_v-W_{scv}$ are used to obtain A and B in Eq. II-9. Substitution of Eq. II-6 into I-7 yields

$$p_1(W_p+x_{dv}) \doteq p_{v0} \frac{V_a}{V_T} \exp\left(\frac{V_0}{V_T}\right). \quad \text{II-10}$$

The small signal hole current density at $x=W_p+W_v-W_{scv}$, which consists of the sum of the small signal hole recombination current densities in the substrate and the n space charge region, is evaluated in the manner used for the dc current density in Appendix I, obtaining the boundary condition at $x=W_p+W_v-W_{scv}$. The ambipolar transport equation for the small signal hole concentration in the substrate is written as

$$\frac{\partial^2 p_1(x)}{\partial x^2} - \frac{p_1(x)}{(L'_{pn})^2} = 0. \quad \text{II-11}$$

The evaluation of

$$J_{1pn} = -e D_{pn} \left. \frac{\partial p_1(x)}{\partial x} \right|_{x=W_p+W_v+W_{scn}}, \quad \text{II-12}$$

the small signal hole recombination current density in the substrate, from the solution of Eq. II-11, subject to the boundary condition $p_1(W_p+W_v+W_n)=0$ (with $W_n \gg L'_{pn}$), yields

$$J_{ipn} = \frac{e D_{pn}}{L'_{pn}} p_i (W_p + W_y + W_{scn}) . \quad \text{II-13}$$

Use of Eq. I-9 enables Eq. II-13 to be written as

$$J_{ipn} = \frac{e D_{pn} N_{Dn}}{L'_{pn} N_{Dn}} p_i (W_p + W_y - W_{scn}) . \quad \text{II-14}$$

Equation I-15 together with $p'(W_p + W_y - W_{scn}) = p'_0(W_p + W_y - W_{scn}) + p_i(W_p + W_y - W_{scn})e^{j\omega t}$ is used to obtain

$$J_{ipscn} = \frac{e W_{scn} V_T}{\tau_{pn} \phi_{yn}} p_i (W_p + W_y - W_{scn}) . \quad \text{II-15}$$

The small signal hole recombination current density at $x = W_p + W_y - W_{scn}$ is written as

$$-e D_{py} \left. \frac{dp_i(x)}{dx} \right|_{x=W_p+W_y-W_{scn}} = J_{ipscn} + J_{ipn} , \quad \text{II-16}$$

to obtain the boundary condition

$$\left. \frac{dp_i(x)}{dx} \right|_{x=W_p+W_y-W_{scn}} = \frac{-\alpha_1}{L'_{py}} p_i (W_p + W_y - W_{scn}) , \quad \text{II-17}$$

where

$$\alpha_1 = \left[\frac{W_{scn} V_T}{\tau_{py} \phi_{yn}} + \frac{D_{pn} N_{Dn}}{L'_{pn} N_{Dn}} \right] \bigg/ \frac{D_{py}}{L'_{py}} . \quad \text{II-18}$$

Since

$$\alpha \approx \frac{W_{scn} V_T L_{py}}{\tau_{py} \phi_{yn} D_{py}} \quad \text{I-19}$$

α_1 is re-expressed as

$$\alpha_1 = \frac{\alpha}{(1+j\omega\tau_{pv})^{1/2}} + \frac{D_{pn}}{D_{pv}} \frac{L'_{pv}}{L'_{pn}} \frac{N_{dv}}{N_{dn}} \quad . \quad \text{II-19}$$

Use of boundary conditions II-10 and II-17 in Eq. II-9 and application of $J_1 = -eD_{pv} \left. \frac{dp_1(x)}{dx} \right|_{x=W_p+x_{dv}}$ gives

$$J_1 = \frac{eD_{pv}n_i V_a}{L'_{pv} N_{dv} V_T} \left\{ \frac{\sinh\left(\frac{W_b}{L'_{pv}}\right) + \alpha_1 \cosh\left(\frac{W_b}{L'_{pv}}\right)}{\cosh\left(\frac{W_b}{L'_{pv}}\right) + \alpha_1 \sinh\left(\frac{W_b}{L'_{pv}}\right)} \right\} [\exp(V_0/V_T) - 1] \quad , \quad \text{II-20}$$

where $W_b = W_p - x_{dv} - W_{ecv}$. The small signal admittance,

$$Y_1 \equiv \frac{J_1}{V_a} = g + j\omega C_0 \quad , \quad \text{II-21}$$

from which $g = \text{Re } Y_1$ and $C_0 = \frac{I_m Y_1}{\omega}$, where C_0 is the diffusion capacitance, is obtained. The total capacitance consists of C_0 plus the transition capacitance, C_T , resulting from modulation of the pv depletion region width by the applied voltage.

The transition capacitance for the abrupt pv junction is well known and is ⁵⁴

$$C_T = \left(\frac{e\epsilon_s N_{dv}}{2 \Phi_{pv}} \right)^{1/2} \left/ \left(1 - \frac{V_0}{\Phi_{pv}} \right)^{1/2} \right. \quad \text{f/cm}^2 \quad . \quad \text{II-22}$$

The pv space charge region is therefore represented as a homogeneous layer of conductivity σ_3 and permittivity ϵ_3 , defined as

$$\sigma_3 = g x_{dv} \quad , \quad \text{II-23}$$

and

$$\epsilon_3 = (C_D + C_T) x_{dv} \quad , \quad \text{II-24}$$

since x_{dv} is very nearly the total space charge layer width. Thus, the complex permittivity of the pn space charge layer for an electric field normal to the plane of the junction is

$$\epsilon_{3t}^* = (C_D + C_T) x_{dv} - j \frac{q x_{dv}}{\omega} \quad . \quad \text{II-25}$$

APPENDIX III

DERIVATION OF MULTIPLE-LAYER EIGENVALUE EQUATION USING ELECTROMAGNETIC BOUNDARY CONDITIONS AND TRANSVERSE RESONANCE METHODS

Boundary Condition Method:

The z directed electric field in the silicon dioxide layer, layer 5, is expressed as

$$E_{z5} = E_0 \sinh k_5(x+b_{ox}), \quad \text{III-1}$$

to satisfy the condition that $E_{z5}(-b_{ox})=0$. Substituting Eq. III-1 into Eq. 3.35 and integrating with respect to x gives

$$H_{y5} = \frac{j\omega\epsilon_5^*}{k_5} E_0 \cosh k_5(x+b_{ox}) . \quad \text{III-2}$$

Substituting Eq. III-2 into Eq. 3.36 yields

$$E_{x5} = -\frac{\gamma}{k_5} E_0 \cosh k_5(x+b_{ox}) . \quad \text{III-3}$$

A similar procedure for layers 4, 2, and 1 gives

$$E_{z4} = A_4 \sinh k_4 x + B_4 \cosh k_4 x , \quad \text{III-4}$$

$$H_{y4} = \frac{j\omega\epsilon_4^*}{k_4} [A_4 \cosh k_4 x + B_4 \sinh k_4 x] , \quad \text{III-5}$$

$$E_{x4} = -\frac{\gamma}{k_4} [A_4 \cosh k_4 x + B_4 \sinh k_4 x] , \quad \text{III-6}$$

$$E_{z2} = A_2 \sinh k_2(x-Wp-x_d) + B_2 \cosh k_2(x-Wp-x_d) , \quad \text{III-7}$$

$$H_{y2} = \frac{j\omega\epsilon_2^*}{k_2} [A_2 \cosh k_2(x-Wp-x_d) + B_2 \sinh k_2(x-Wp-x_d)] , \quad \text{III-8}$$

$$E_{x2} = -\frac{\gamma}{k_2} [A_2 \cosh k_2(x-Wp-x_d) + B_2 \sinh k_2(x-Wp-x_d)] , \quad \text{III-9}$$

$$E_x = A_1 \sinh k_1 (x - W_p - W_y - W_n) , \quad \text{III-10}$$

$$H_y = \frac{j\omega\epsilon_1^*}{k_1} A_1 \cosh k_1 (x - W_p - W_y - W_n) , \quad \text{III-11}$$

and

$$E_x = -\sum \frac{A_i}{k_i} \cosh k_i (x - W_p - W_y - W_n) . \quad \text{III-12}$$

The z directed electric field in the py space charge layer is expressed as

$$E_{x3} = A_3 \sinh k_3 (x - W_p + x_{dp}) + B_3 \cosh k_3 (x - W_p + x_{dp}) . \quad \text{III-13}$$

Substituting Eq. III-13 into Eq. 3.32 and integrating with respect to x gives

$$H_{y3} = \frac{j\omega\epsilon_3^*}{k_3} [A_3 \cosh k_3 (x - W_p + x_{dp}) + B_3 \sinh k_3 (x - W_p + x_{dp})] . \quad \text{III-14}$$

Substitution of Eq. III-14 into Eq. 3.33 yields

$$E_{x3} = -\sum \frac{\epsilon_3^*}{k_3 \epsilon_{st}^*} [A_3 \cosh k_3 (x - W_p + x_{dp}) + B_3 \sinh k_3 (x - W_p + x_{dp})] . \quad \text{III-15}$$

Since there are no current sheets, the boundary conditions of continuous tangential electric field and continuous tangential magnetic field are applied at each interface to obtain

$$B_4 = E_0 S_5 , \quad \text{III-16}$$

$$A_4 = \frac{Z_4}{Z_5} E_0 C_5 , \quad \text{III-17}$$

$$B_3 = \left[\frac{Z_4}{Z_5} S_4 C_5 + S_5 C_4 \right] E_0 , \quad \text{III-18}$$

$$A_3 = \left[\frac{Z_3 C_4 C_5}{Z_5} + \frac{Z_3 S_4 S_5}{Z_4} \right] E_o , \quad \text{III-19}$$

$$B_2 = \left[\frac{Z_3 S_3 C_4 C_5}{Z_5} + \frac{Z_3 S_3 S_4 S_5}{Z_4} + \frac{Z_4 S_4 C_3 C_5}{Z_5} + S_5 C_3 C_4 \right] E_o , \quad \text{III-20}$$

$$A_2 = \left[\frac{Z_2 C_3 C_4 C_5}{Z_5} + \frac{Z_2 S_4 S_5 C_3}{Z_4} + \frac{Z_2 Z_4 S_3 S_4 C_5}{Z_3 Z_5} + \frac{Z_2 S_3 S_5 C_4}{Z_3} \right] E_o , \quad \text{III-21}$$

$$-A_1 = A_2 \frac{S_2}{S_1} + B_2 \frac{C_2}{S_1} \quad \text{III-22}$$

and

$$A_1 = \frac{Z_1}{Z_2} \left[A_2 \frac{C_2}{C_1} + B_2 \frac{S_2}{C_1} \right] , \quad \text{III-23}$$

where the simplified notation of

$$S_i = \sinh k_i b_i ,$$

$$C_i = \cosh k_i b_i ,$$

$$Z_i = \frac{k_i}{j\omega \epsilon_i} \quad i \neq 3, \text{ and}$$

$$Z_3 = \frac{k_3}{j\omega \epsilon_3^* l}$$

has been used. Addition of Eqs. III-22 and III-23 gives

$$A_2 \left[\frac{S_2}{S_1} + \frac{Z_1 C_2}{Z_2 C_1} \right] + B_2 \left[\frac{C_2}{S_1} + \frac{Z_1 S_2}{Z_2 C_1} \right] = 0 , \quad \text{III-24}$$

which upon substitution of Eqs. III-20 and III-21 and some algebra results in the condition

$$\begin{aligned} & Z_1 T_1 \left[1 + \frac{Z_3 T_2 T_3}{Z_2} + \frac{Z_4 T_2 T_4}{Z_2} + \frac{Z_5 T_2 T_5}{Z_2} + \frac{Z_4 T_3 T_4}{Z_3} + \frac{Z_5 T_3 T_5}{Z_3} \right. \\ & \left. + \frac{Z_5 T_4 T_5}{Z_4} + \frac{Z_3 Z_5 T_2 T_3 T_4 T_5}{Z_2 Z_4} \right] + Z_2 T_2 \left[1 + \frac{Z_4 T_3 T_4}{Z_3} + \frac{Z_5 T_3 T_5}{Z_3} \right. \\ & \left. + \frac{Z_5 T_4 T_5}{Z_4} \right] + Z_3 T_3 \left[1 + \frac{Z_5 T_4 T_5}{Z_4} \right] + Z_4 T_4 + Z_5 T_5 = 0 , \end{aligned} \quad \text{III-25}$$

where $T_i = \tanh k_i b_i$. The solution of Eqs. 3.40, 3.41 and

3.42 show that the following inequalities are valid for the physical parameters investigated:

$$\left| \frac{Z_3 T_2 T_3}{Z_2} + \frac{Z_4 T_2 T_4}{Z_2} + \frac{Z_5 T_2 T_5}{Z_2} + \frac{Z_4 T_3 T_4}{Z_3} + \frac{Z_5 T_3 T_5}{Z_3} + \frac{Z_5 T_4 T_5}{Z_4} + \frac{Z_3 Z_5 T_2 T_3 T_4 T_5}{Z_2 Z_4} \right| \ll 1, \quad \text{III-26}$$

$$\left| \frac{Z_4 T_3 T_4}{Z_3} + \frac{Z_5 T_3 T_5}{Z_3} + \frac{Z_5 T_4 T_5}{Z_4} \right| \ll 1, \quad \text{III-27}$$

and

$$\left| \frac{Z_5 T_4 T_5}{Z_4} \right| \ll 1. \quad \text{III-28}$$

Application of Eqs. III-26, III-27, and III-28 to Eq. III-25 with some additional algebra gives the multiple-layer eigenvalue equation

$$\frac{k_1^2 b_1'}{\epsilon_1^*} + \frac{k_2^2 b_2'}{\epsilon_2^*} + \frac{k_3^2 b_3'}{\epsilon_3^*} + \frac{k_4^2 b_4'}{\epsilon_4^*} + \frac{k_5^2 b_5'}{\epsilon_5^*} = 0, \quad \text{III-29}$$

where $b_i = (\tanh k_i b_i)/k_i$.

Transverse Resonance Method:

A transverse resonance ⁵⁵ method is used to derive Eq. III-25 in an alternate manner. A transverse cross section of the parallel-plate waveguide segment is modeled by a cascade of transmission lines, on which a TM mode propagates in the x direction, as shown in Fig. III-1. The driving point impedance at the arbitrarily chosen plane of the interface of the p and p' space charge layers is evaluated look-

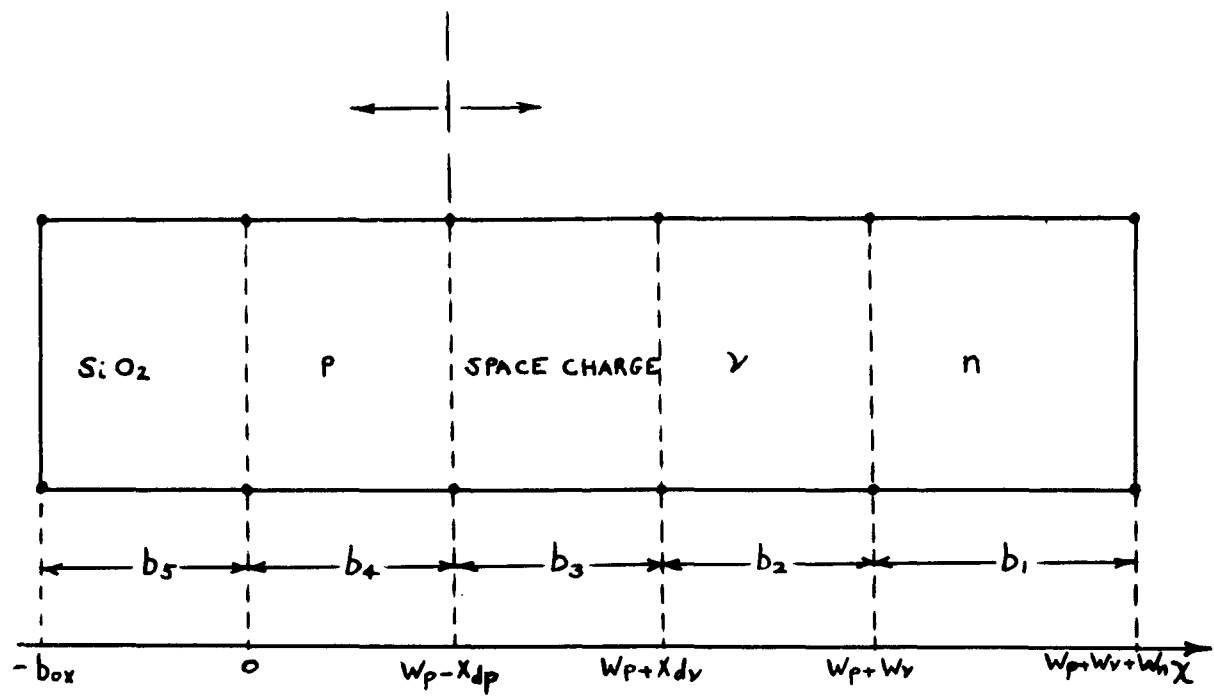


FIGURE III-1 TRANSVERSE TRANSMISSIONLINE

ing in both the +x and -x directions. The transverse resonance condition requires that the sum of the driving point impedances be zero, which is equivalent to the conditions of continuous tangential electric and magnetic fields.

Reflection of the equivalent short at the top plate ($Z_L=0$) to the SiO_2 -p layer interface using the relationship

$$Z_{\text{SiO}_2\text{-p}}^- = Z_S \left[\frac{Z_L + Z_S T_S}{Z_S + Z_L T_S} \right] \quad \text{III-30}$$

gives

$$Z_{\text{SiO}_2\text{-p}}^- = Z_S T_S, \quad \text{III-31}$$

where $Z_S = \frac{k_S}{j\omega\epsilon_S^*}$ as defined earlier in this appendix, is actually the TM wave impedance in the x direction (E_{xS}/H_{yS}).

Similarly, reflection of $Z_{\text{SiO}_2\text{-p}}^-$ to the p-space charge layer interface results in

$$Z_{\text{p-sc}}^- = Z_4 \left[\frac{Z_S T_S + Z_4 T_4}{Z_S T_4 T_S + Z_4} \right]. \quad \text{III-32}$$

Reflection of the equivalent short at the bottom plate to the γ -n layer interface yields

$$Z_{\gamma\text{-n}}^+ = Z_1 T_1. \quad \text{III-33}$$

Reflecting $Z_{\gamma\text{-n}}^+$ to the space charge- γ layer interface gives

$$Z_{\text{sc-}\gamma}^+ = Z_2 \left[\frac{Z_1 T_1 + Z_2 T_2}{Z_1 T_1 T_2 + Z_2} \right]. \quad \text{III-34}$$

Reflecting Z_{sc}^+ to the p-space charge layer interface results in

$$Z_{p-sc}^+ = Z_3 \left[\frac{Z_2 [Z_1 T_1 + Z_2 T_2 + Z_3 T_3]}{Z_2 [Z_1 T_1 T_3 + Z_2 T_2 T_3 + Z_3]} + \frac{Z_1 Z_3 T_1 T_2 T_3}{Z_1 Z_3 T_1 T_2} \right]. \quad \text{III-35}$$

Use of the condition that $Z_{p-sc}^- + Z_{p-sc}^+ = 0$ and some algebra yields Eq. III-25 which is reduced to Eq. III-29 with the use of the inequalities III-26, III-27, and III-28.

APPENDIX IV

EVALUATION OF THE CHARACTERISTIC IMPEDANCE

Evaluation of Eq. 3.43 using the expressions for the transverse electric field in Appendix III gives

$$V_t = \gamma \left[\frac{A_1 S_1}{k_1^2} + \frac{A_2 S_2}{k_2^2} + \frac{B_2 (C_2 - 1)}{k_2^2} + \frac{A_3 \epsilon_{3l}^* S_3}{k_3^2 \epsilon_{3t}^*} + \frac{B_3 \epsilon_{3l}^* (C_3 - 1)}{k_3^2 \epsilon_{3t}^*} \right. \\ \left. + \frac{A_4 S_4}{k_4^2} + \frac{B_4 (C_4 - 1)}{k_4^2} + \frac{E_0 S_5}{k_5^2} \right], \quad \text{IV-1}$$

where $S_i = \sinh k_i b_i$ and $C_i = \cosh k_i b_i$. Evaluation of Eq. 3.44 using the expression for H_y gives

$$I_t = \frac{W_{ST} E_0 C_5}{Z_5}, \quad \text{IV-2}$$

where $Z_5 = \frac{k_5}{j\omega \epsilon_5^*}$. Taking the ratio of V_t to I_t gives

$$Z_0 = \frac{\gamma}{W_{ST}} \left[\frac{A_1 S_1 Z_5}{E_0 k_1^2 C_5} + \frac{A_2 S_2 Z_5}{E_0 k_2^2 C_5} + \frac{B_2 (C_2 - 1) Z_5}{E_0 k_2^2 C_5} + \frac{A_3 \epsilon_{3l}^* S_3 Z_5}{E_0 k_3^2 \epsilon_{3t}^* C_5} \right. \\ \left. + \frac{B_3 \epsilon_{3l}^* (C_3 - 1) Z_5}{E_0 k_3^2 \epsilon_{3t}^* C_5} + \frac{A_4 S_4 Z_5}{E_0 k_4^2 C_5} + \frac{B_4 (C_4 - 1) Z_5}{E_0 k_4^2 C_5} + \frac{S_5 Z_5}{k_5^2 C_5} \right]. \quad \text{IV-3}$$

With the substitution of Eqs. III-16 through III-22 into Eq. IV-3 and the inequalities and approximations

$$|k_i b_i| \ll 1 \quad i \neq 1, \quad \text{IV-4}$$

$$S_i \doteq T_i \quad i \neq 1, \quad \text{IV-5}$$

$$|(C_2 - 1) Z_1 T_1| \ll |Z_2 T_2|, \quad \text{IV-6}$$

$$|(C_3 - 1)(Z_4 T_4 + Z_5 T_5)| \ll |Z_3 T_3|, \quad \text{IV-7}$$

and

$$|Z_5 T_5 (C_4 - 1)| \ll |Z_4 T_4| , \quad \text{IV-8}$$

which have been verified numerically from the solutions, along with Eqs. III-27 and III-28 one obtains the result

$$Z_o = \frac{\gamma}{j\omega W_{ST}} \left[\frac{b'_1}{\epsilon_1^*} + \frac{b'_2}{\epsilon_2^*} + \frac{b'_3}{\epsilon_{st}^*} + \frac{b'_4}{\epsilon_4^*} + \frac{b'_5}{\epsilon_5^*} \right] . \quad \text{IV-9}$$

Squaring Eq. IV-9 and substituting γ^2 from Eq. 3.42 gives the final result

$$Z_o^2 = \frac{\mu_o}{W_{ST}} \left[\frac{b'_1}{\epsilon_1^*} + \frac{b'_2}{\epsilon_2^*} + \frac{b'_3}{\epsilon_{st}^*} + \frac{b'_4}{\epsilon_4^*} + \frac{b'_5}{\epsilon_5^*} \right] \left[b'_1 + b'_2 + b'_3 + b'_4 + b'_5 \right] . \quad \text{IV-10}$$

It is noteworthy to mention that the transmission line current obtained for any path of integration crossing between the plates within layers 2, 3, 4, and 5 is numerically very nearly equal to (worst case is within 1%) the value obtained from Eq. IV-2. This is evident from Eq. IV-4 which physically means that H_y is nearly constant within layers 2, 3, 4, and 5. Layer 1, the n substrate, is considered as a lossy bottom conductor and is excluded from the path of current integration.

APPENDIX VLISTING OF COMPUTER PROGRAMS

1. Main Program
2. Subroutine SIGN
3. Subroutine ITER
4. Subroutine RF
5. Subroutine MULT

```

1 REAL*8 C(205),YI(205),DYDZ2,DYDZ,A,CI
2 REAL LP
3 COMPLEX ZO,GAM,R,REFL,ZI,ZIN,PH,RI,CEXP,PHT,TR,ZO2,CABS
4 COMPLEX RL,T,A11,A12,A21,A22,AL11,AL12,AL21,AL22,TIJ,RIJ
5 COMPLEX ZLT
6 INTEGER O,Q
7 COMMON YI,YC,YL,AH,AJOH
8 REAL*8 YR,YC
9 YR=34.
10 WRITE(6,50)
11 050 FORMAT(1H,'WHAT IS RESISTIVITY OF P-LAYER? OHM-CM')
12 READ(5,*) RHOP
13 WRITE(6,51)
14 051 FORMAT(1H,'WHAT IS DEPTH OF P-LAYER? MICRONS')
15 READ(5,*) WPO
16 WP=WPO*1.E-04
17 WRITE(6,600)
18 600 FORMAT(1H,'WHAT IS THICKNESS OF EPI REGION? MICRONS')
19 READ(5,*) WNM0
20 WNM=WNM0*1.E-04
21 WRITE(6,601)
22 601 FORMAT(1H,'DOPING DENSITY OF P MATERIAL? CM**-3')
23 READ(5,*) DNA
24 WRITE(6,602)
25 602 FORMAT(1H,'DOPING DENSITY OF EPI MATERIAL? CM**-3')
26 READ(5,*) DND
27 WRITE(6,605)
28 605 FORMAT(1H,'HOLE LIFETIME IN EPI? SEC.')
29 READ(5,*) TP
30 WRITE(6,606)
31 606 FORMAT(1H,'DP IN EPI MATERIAL? (CM**2)/SEC')
32 READ(5,*) DP
33 WRITE(6,831)
34 831 FORMAT(1H,'LOW INJECTION JO? A/CM**2')
35 READ(5,*) AJO
36 WRITE(6,666)
37 666 FORMAT(1H,'HIGH INJECTION JO? A/CM**2')
38 READ(5,*) AJOH
39 WRITE(6,680)
40 680 FORMAT(1H,'LENGTH OF P REGION? CM')
41 READ(5,*) ZL
42 WRITE(6,52)
43 052 FORMAT(1H,'WIDTH OF P REGION? CM')
44 READ(5,*) WT
45 WRITE(6,102)
46 102 FORMAT(1H,'WIDTH OF MICROSTRIP? CM')
47 READ(5,*) WST
48 WRITE(6,490)
49 490 FORMAT(1H,'LOAD IMPEDANCE? OHMS COMPLEX')
50 READ(5,*) ZLT
51 WRITE(6,106)
52 106 FORMAT(1H,'FREQUENCY OF EXCITATION? GHZ')
53 READ(5,*) F
54 JI=0
55 5000 WRITE(6,2)
56 JI=JI+1
57 002 FORMAT(1H,'WHAT IS THE VALUE OF Y(0)?')
58 WRITE(6,3)
59 003 FORMAT(1H,'HOW MANY SECTIONS ON THE INTERVAL ZL?')
60 104 WRITE(6,108)
61 108 FORMAT(1H,'SPEC. VOLTAGE Y(ZL).')
62 107 PNO=2.25E20/DND
63 LP=SQRT(DP*TP)
64 PHI0=.025*ALOG(DNA*DND/2.25E20)
65 XDNO=SQRT((24.*8.85E-14*PHI0)/(1.6E-19*DND))
66 WN=WNM-XDNO
67 YC=2.*ALOG(AJOH/AJO)
68 AJOI=1.6E-19*PNO*DP/(LP*TANH(WN/LP))
69 A=(AJO*RHOP)/(.025*WP)
70 AH=A*AJOH/AJO

```

```

71 BT=AJ0/AJ01
72 ALP=(TANH(WN/LP)-BT/TANH(WN/LP))/(BT-1.)
73 READ(5,*) Y0
74 005 FORMAT(F7.2)
75 READ(5,*) N
76 006 FORMAT(I3)
77 109 READ(5,*) YL
78 019 CONTINUE
79 021 FORMAT(E12.3)
80 IF(N.LT.205) GO TO 7
81 WRITE(6,11)
82 011 FORMAT(1H,'NO. SECTIONS SPECIFIED IS TOO LARGE.')
83 WRITE(6,180)
84 180 FORMAT(1H,'HOW MANY SECTIONS ON THE INTERVAL ZL?')
85 READ(5,*) N
86 007 CONTINUE
87 IREV=0
88 DZ=ZL/N
89 N=N+1
90 NN=N
91 IF(Y0.GT.0.) GO TO 260
92 IF(Y0.NE.YL) GO TO 260
93 DO 261 I=1,N
94 261 YI(I)=Y0
95 CI=-2.*A*(EXP(Y0)-Y0)
96 GO TO 1000
97 260 CONTINUE
98 CI=-2.*A
99 CALL SIGN(Y0,YL,S,IREV)
100 YI(1)=Y0
101 CALL ITER(ZL,ZS,CI,N,DZ,S,A)
102 IF(DABS(YI(NN)-YL).LT..00001) GO TO 1000
103 301 IF(YI(1).GE.YL) GO TO 302
104 IF(YI(NN).LT.YL)GO TO 303
105 GO TO 304
106 302 IF(YI(NN).GT.YL)GO TO 303
107 GO TO 304
108 303 C(1)=CI
109 CI=CI+((ABS(Y0)+ABS(YL))/ZL)*((ABS(Y0)+ABS(YL))/ZL)
110 CALL ITER(ZL,ZS,CI,N,DZ,S,A)
111 305 IF(YI(1).GE.YL) GO TO 306
112 IF(YI(NN).GT.YL) GO TO 307
113 GO TO 303
114 306 IF(YI(NN).LT.YL) GO TO 307
115 GO TO 303
116 304 C(1)=CI
117 CI=CI-((ABS(Y0)+ABS(YL))/ZL)*((ABS(Y0)+ABS(YL))/ZL)
118 CALL ITER(ZL,ZS,CI,N,DZ,S,A)
119 406 IF(YI(1).GE.YL) GO TO 405
120 IF(YI(NN).LT.YL) GO TO 307
121 GO TO 304
122 405 IF(YI(NN).GT.YL) GO TO 307
123 IF(DABS(YI(NN)-YL).LT..00001) GO TO 1000
124 GO TO 304
125 307 C(2)=CI
126 M=3
127 310 IF(DABS(YI(NN)-YL).LT..00001) GO TO 1000
128 O=2
129 Q=2
130 311 IF(YI(1).GT.YL) GO TO 312
131 IF(YI(NN)-YL) 313,1000,314
132 312 IF(YI(NN)-YL) 314,1000,313
133 314 O=2
134 C(M)=(C(M-1)+C(M-Q))/2.
135 Q=Q+1
136 GO TO 315
137 313 Q=2
138 C(M)=(C(M-1)+C(M-Q))/2.
139 O=O+1
140 315 CI=C(M)

```

```

141 CALL ITER(ZL,ZS,CI,N,DZ,S,A)
142 316 IF(DABS(C(M)-C(M-1)).LT.1.E-15) GO TO 1000
143 M=M+1
144 IF(DABS(YI(NN)-YL).LT..00001) GO TO 1000
145 GO TO 311
146 1000 WRITE(6,501)
147 IF(IREV.EQ.0) GO TO 800
148 DO 500 IRE=1,N
149 C(IRE)=YI(N-IRE+1)
150 500 CONTINUE
151 DO 800 IRE=1,N
152 YI(IRE)=C(IRE)
153 800 CONTINUE
154 PHT=(0.,0.)
155 T=(1.,0.)
156 NQ=NN-1
157 I=NQ
158 CALL RF(TP,DND,WNMO,RHOP,DNA,DP,WPO,AJO,AJOH,WST,YI(I),YC,YR,F,ZO,GA

```

M,ALP)

```

159 R=(ZLT-ZO)/(ZLT+ZO)
160 RL=R
161 TR=2.*ZLT/(ZLT+ZO)
162 Z02=ZO
163 PHT=-GAM*DZ
164 PH=2.*PHT
165 RI=R*CEXP(PH)
166 ZI=ZO*(1.+RI)/(1.-RI)
167 A11=CEXP(-PHT)
168 A22=CEXP(PHT)
169 I=NQ-1
170 CALL RF(TP,DND,WNMO,RHOP,DNA,DP,WPO,AJO,AJOH,WST,YI(I),YC,YR,F,Z ,GA

```

M,ALP)

```

171 TIJ=2.*Z02/(Z02+ZO)
172 T=T*TIJ
173 RIJ=(Z02-ZO)/(Z02+ZO)
174 A12=RIJ*A22
175 A21=RIJ*A11
176 PHT=-GAM*DZ
177 PH=2.*PHT
178 Z02=ZO
179 R=(ZI-ZO)/(ZI+ZO)
180 RI=R*CEXP(PH)
181 ZI=ZO*(1.+RI)/(1.-RI)
182 DO 823 I=3,NQ
183 NI=NN-I
184 AL11=CEXP(-PHT)
185 AL22=CEXP(PHT)
186 CALL RF(TP,DND,WNMO,RHOP,DNA,DP,WPO,AJO,AJOH,WST,YI(NI),YC,YR,F,ZO,G

```

AM,ALP)

```

187 TIJ=2.*Z02/(Z02+ZO)
188 T=T*TIJ
189 RIJ=(Z02-ZO)/(Z02+ZO)
190 AL12=RIJ*AL22
191 AL21=RIJ*AL11
192 PHT=-GAM*DZ
193 PH=2.*PHT
194 Z02=ZO
195 CALL MULT(A11,A12,A21,A22,AL11,AL12,AL21,AL22)
196 R=(ZI-ZO)/(ZI+ZO)
197 RI=R*CEXP(PH)
198 ZI=ZO*(1.+RI)/(1.-RI)
199 823 CONTINUE
200 AL11=CEXP(-PHT)
201 AL22=CEXP(PHT)
202 AL12=(0.,0.)
203 AL21=(0.,0.)
204 CALL MULT(A11,A12,A21,A22,AL11,AL12,AL21,AL22)

```

```

205 REFL=(ZI-50.)/(ZI+50.)
206 ZIN=ZI
207 TR=T*TR/(A11+A21+(A12+A22)*RL)
208 501 FORMAT(///,1H,'INPUT AND CALCULATED DATA')
209 DO 319 IY=1,NN
210 IF(YI(IY)*YI(IY+1).LE..0) GO TO 320
211 319 CONTINUE
212 320 NZS=IY
213 ZS=NZS*DZ
214 321 ETA=1.
215 AJ=AJ0
216 IF(YI(1).GT.YC) ETA=2.
217 IF(YI(1).GT.YC) AJ=AJ0H
218 SIO=9.4E-04*AJ*(DEXP(YI(1)/ETA)-1.)+(YI(1)-YI(2))*WP*WT/(RHOP*DZ*40.
)
219 IF(YI(1).GT.YR) SIO=9.4E-04*((YI(1)-YR)*40.+AJ0H*DEXP(YR/2.))+(YI(1)
-YI(2))*WP*WT/(RHOP*DZ*40.)
220 ETA=1.
221 AJ=AJ0
222 IF(YI(N).GT.YC) ETA=2.
223 IF(YI(N).GT.YC) AJ=AJ0H
224 SIL=9.4E-04*AJ*(DEXP(YI(N)/ETA)-1.)+(YI(N)-YI(N-1))*WP*WT/(RHOP*DZ*4
0.)
225 IF(YI(N).GT.YR) SIL=9.4E-04*((YI(N)-YR)*40.+AJ0H*DEXP(YR/2.))+(YI(N)
-YI(N-1))*WP*WT/(RHOP*DZ*40.)
226 WRITE(6,502) YI(1)
227 502 FORMAT(1H,/,1H,'Y0=',1PE12.3)
228 YL=YI(N)
229 WRITE(6,318) YL
230 318 FORMAT(1H,'YL=',1PE12.3)
231 IF(JI.NE.1) GO TO 545
232 317 WRITE(6,503) A
233 503 FORMAT(1H,'A=',1X,1PE12.3,'CM**-2')
234 WRITE(6,201) AJ0
235 201 FORMAT(1H,'LEAKAGE CURRENT DENSITY=',1PE12.3,'AMPS/CM**2')
236 WRITE(6,649) ALP
237 649 FORMAT(1H,'BLOCKAGE PARAMETER, ALPHA=',F7.2)
238 WRITE(6,629) AJ0H
239 629 FORMAT(1H,'JOH=',1PE12.3,'AMPS/CM**2')
240 WRITE(6,659) YC
241 659 FORMAT(1H,'NORMALIZED BREAKPOINT VOLTAGE=',F7.2)
242 WRITE(6,504) ZL
243 504 FORMAT(1H,'ZL=',1PE12.3,1X,'CM')
244 545 CONTINUE
245 WRITE(6,505) ZS
246 505 FORMAT(1H,'ZS=',1PE12.3,1X,'CM')
247 WRITE(6,506) NQ
248 506 FORMAT(1H,'N=',1X,13,1X,'SECTIONS')
249 WRITE(6,508) NZS
250 508 FORMAT(1H,'THE NODE NEAREST THE SHORT, NZS=',13)
251 WRITE(6,507) CI
252 507 FORMAT(1H,'C=',1PE12.3,'CM**-2')
253 WRITE(6,53) SIO
254 053 FORMAT(1H,'THE INPUT CURRENT IS, IO=',1PE12.3,'AMPS')
255 WRITE(6,54) SIL
256 054 FORMAT(1H,'THE OUTPUT CURRENT IS, IL=',1PE12.3,'AMPS')
257 ZI=ZI/50.
258 WRITE(6,778) ZI
259 778 FORMAT(1H,'NORMALIZED DRIVING PT IMPEDANCE=',2(1PE12.3))
260 X1=REAL(REFL)
261 X2=AIMAG(REFL)
262 X3=CABS(REFL)
263 X3=20.*ALOG10(X3)
264 WRITE(6,220) X3
265 220 FORMAT(1H,'MAG OF INPUT REFL COEFF=',1PE12.3,'DB')
266 X4=180.*ATAN2(X2,X1)/3.141592728

```

```

267 WRITE(6,221)X4
268 221 FORMAT(1H,'ANGLE OF INPUT REFL COEFF=',1PE12.3,'DEGREES')
269 X1=REAL(TR)
270 X2=AIMAG(TR)
271 X3=CABS(TR)
272 IF(X3.EQ.0.) GO TO 205
273 X3=20.*ALOG10(X3)
274 WRITE(6,222) X3
275 222 FORMAT(1H,'MAG OF I/O V TRANS COEFF=',1PE12.3,'DB')
276 X4=180.*ATAN2(X2,X1)/3.141592728
277 WRITE(6,223) X4
278 223 FORMAT(1H,'ANGLE OF I/O V TRANS COEFF=',1PE12.3,'DEGREES')
279 205 IF(CABS(TR).LE.0.) WRITE(6,206)
280 206 FORMAT(1H,'I/O V TRANS COEFF=-INFINITY DB')
281 IF(CABS(ZLT).EQ.0.) ATT=0.
282 IF(CABS(ZLT).EQ.0.) GO TO 279
283 ATT=CABS(TR)*CABS(TR)*CABS(ZIN)*CABS(ZIN)*REAL(ZLT)/(REAL(ZIN)*CABS(
ZLT)*CABS(ZLT))
284 279 CONTINUE
285 IF(ATT.EQ.0.) WRITE(6,207)
286 207 FORMAT(1H,'I/O POWER RATIO=-INFINITY DB')
287 IF(ATT.EQ.0.) GO TO 208
288 ATT=10.*ALOG10(ATT)
289 WRITE(6,224) ATT
290 224 FORMAT(1H,'I/O POWER RATIO =',1PE12.3,'DB')
291 208 CONTINUE
292 IL=1
293 IU=N
294 INC=(N-1)/10
295 IB=1
296 WRITE(6,610)
297 610 FORMAT(1H,////,1H,'TYPE 1 TO SKIP VOLTAGE PROFILE PRINT')
298 READ(5,*) IPR
299 IF(IPR.EQ.1) GO TO 716
300 GO TO 907
301 900 WRITE(6,901)
302 WRITE(6,920)
303 WRITE(6,921)
304 IB=IB+1
305 901 FORMAT(1H,////,1H,'IF MICROPOSITION INFO IS DESIRED TYPE 1')
306 920 FORMAT(1H,'TO BEGIN AGAIN, TYPE 2')
307 921 FORMAT(1H,'TO EXIT, TYPE 3')
308 READ(5,*) IM
309 902 FORMAT(11)
310 IF(IM.EQ.1) GO TO 903
311 IF(IM.EQ.2) GO TO 5000
312 IF(IM.EQ.3) GO TO 17
313 903 WRITE(6,908)
314 908 FORMAT(1H,'WHAT IS THE LOWER NODE NUMBER?-13')
315 READ(5,*) IL
316 904 FORMAT(13)
317 WRITE(6,905)
318 905 FORMAT(1H,'WHAT IS THE UPPER NODE NUMBER?-13')
319 READ(5,*) IU
320 WRITE(6,909)
321 909 FORMAT(1H,'WHAT IS THE INCREMENT VALUE?-13')
322 READ(5,*) INC
323 WRITE(6,906)
324 906 FORMAT(1H,'MICROPOSITION INFORMATION')
325 IL=IL+1
326 IU=IU+1
327 907 WRITE(6,705)
328 705 FORMAT(1H,////,1H,'THE CALCULATED RESULTS ARE:')
329 WRITE(6,720)
330 WRITE(6,710)
331 WRITE(6,720)
332 710 FORMAT(1H,1X,'N',6X,'Z',11X,'Y',14X,'Z0',20X,'GAM')
333 IF(IR.EQ.1) GO TO 685
334 IF(IM.EQ.1) GO TO 685

```

```

335 716 CONTINUE
336 IF(IPR.EQ.1) GO TO 802
337 685 DO 670 I=IL,IU,INC
338 IK=I-1
339 Z=FLOAT(IK)*DZ
340 CALL RF(TP,DND,WNM0,RHOP,DNA,DP,WPO,AJO,AJOH,WST,YI(1),YC,YR,F,ZO,GA
M,ALP)
341 WRITE(6,715)(IK,Z,YI(1),ZO,GAM)
342 670 CONTINUE
343 802 CONTINUE
344 715 FORMAT(1H,I3,1PE10.3,5(1X,1PE10.3))
345 628 CONTINUE
346 IF(IB.EQ.1) GO TO 900
347 910 WRITE(6,911)
348 911 FORMAT(1H,///,1H,'IF MORE MICRO INFO IS DESIRED TYPE 1')
349 WRITE(6,912)
350 912 FORMAT(1H,'IF NONE TYPE 2')
351 READ(5,*) IR
352 913 FORMAT(1I)
353 IF(IR.EQ.1) GO TO 903
354 IF(IR.EQ.2) GO TO 801
355 WRITE(6,914)
356 914 FORMAT(1H,'WRONG MICRO CODE')
357 GO TO 910
358 801 CONTINUE
359 720 FORMAT(1H,1X,/)
360 030 WRITE(6,14)
361 014 FORMAT(1H,///,1H,'TYPE 1 TO BEGIN AGAIN')
362 WRITE(6,16)
363 16 FORMAT(1H,'TYPE 2 TO EXIT')
364 READ(5,*) II
365 015 FORMAT(1I)
366 IF(II.EQ.1) GO TO 5000
367 IF(II.EQ.2) GO TO 17
368 WRITE(6,20)
369 020 FORMAT(1H,'WRONG COMPLETION CODE')
370 GO TO 30
371 017 STOP
372 END
373 SUBROUTINE SIGN(YO,YL,S,IREV)
374 IF(YO.LT.0.) GO TO 440
375 IF(YL.LT.0.) GO TO 441
376 IF(YO.LT.YL) GO TO 443
377 S=-1.
378 GO TO 450
379 443 YK=YL
380 YL=Y0
381 Y0=YK
382 IREV=1
383 S=-1.
384 GO TO 450
385 440 IF(YL.LT.0.) GO TO 442
386 S=1.
387 GO TO 450
388 441 S=-1.
389 GO TO 450
390 442 IF(Y0.GT.YL) GO TO 444
391 S=1.
392 GO TO 450
393 444 S=-1.
394 450 RETURN
395 END
396 SUBROUTINE ITER(ZL,ZS,C1,N,DZ,S,A)
397 REAL*8 YI(205),DYDZ2,A,C1,ARG,A1,A2,A3,A4,D2,AA
398 REAL*8 CC,CR,YR,YC
399 YR=34.
400 COMMON YI,YC,YL,AH,AJOH
401 CC=C1+2.*AH*(DEXP(YC)-YC)-2.*AH*(2.*DEXP(YC/2.)-YC)
402 CR=CC+2.*AH*(2.*DEXP(YR/2.)-YR)+(AH/AJOH)*(40.*YR*YR)-2.*AH*(DEXP(YR

```

```

/2.)-1.)*YR
403 P=0.
404 B=1.
405 DO 1 I=1,N
406 IF(YI(I).GE.174.) GO TO 860
407 ARG=2.*A*(DEXP(YI(I))-YI(I))+CI
408 IF(YI(I).GT.YC) ARG=2.*AH*(2.*DEXP(YI(I)/2.)-YI(I))+CC
409 860 CONTINUE
410 IF(YI(I).GT.348.) YI(I)=348.
411 IF(YI(I).GT.YR) ARG=(2.*AH/AJOH)*(40.*YI(I)*YI(I)/2.-40.*YR*YI(I))+2
    .*AH*(DEXP(YR/2.)-1.)*YI(I)+CR
412 IF(ARG.LE.0.) ARG=0.
413 DYDZ2=S*B*DSQRT(ARG)
414 250 A1=DYDZ2*DZ
415 AA=YI(I)+A1/2.
416 IF(AA.GE.174.) GO TO 861
417 ARG=2.*A*(DEXP(AA)-AA)+CI
418 IF(AA.GT.YC) ARG=2.*AH*(2.*DEXP(AA/2.)-AA/2.))+CC
419 861 CONTINUE
420 IF(AA.GT.348.) AA=348.
421 IF(AA.GT.YR) ARG=(2.*AH/AJOH)*(40.*AA*AA/2.-40.*YR*AA)+2.*AH*(DEXP(Y
R/2.)-1.)*AA+CR
422 IF(ARG.LE.0.) ARG=0.
423 DYDZ2=S*B*DSQRT(ARG)
424 251 A2=DYDZ2*DZ
425 AA=YI(I)+A2/2.
426 IF(AA.GE.174.) GO TO 862
427 ARG=2.*A*(DEXP(AA)-AA)+CI
428 IF(AA.GT.YC) ARG=2.*AH*(2.*DEXP(AA/2.)-AA/2.))+CC
429 862 CONTINUE
430 IF(AA.GT.348.) AA=348.
431 IF(AA.GT.YR) ARG=(2.*AH/AJOH)*(40.*AA*AA/2.-40.*YR*AA)+2.*AH*(DEXP(Y
R/2.)-1.)*AA+CR
432 IF(ARG.LE.0.) ARG=0.
433 DYDZ2=S*B*DSQRT(ARG)
434 252 A3=DYDZ2*DZ
435 AA=YI(I)+A3
436 IF(AA.GE.174.) GO TO 863
437 ARG=2.*A*(DEXP(AA)-AA)+CI
438 IF(AA.GT.YC) ARG=2.*AH*(2.*DEXP(AA/2.)-AA/2.))+CC
439 863 CONTINUE
440 IF(AA.GT.348.) AA=348.
441 IF(AA.GT.YR) ARG=(2.*AH/AJOH)*(40.*AA*AA/2.-40.*YR*AA)+2.*AH*(DEXP(Y
R/2.)-1.)*AA+CR
442 IF(ARG.LE.0.) ARG=0.
443 DYDZ2=S*B*DSQRT(ARG)
444 253 A4=DYDZ2*DZ
445 IF(YI(I).GE.174.) GO TO 864
446 D2=A*(DEXP(YI(I))-1.)
447 IF(YI(I).GT.YC) D2=AH*(DEXP(YI(I)/2.)-1.)
448 864 CONTINUE
449 IF(YI(I).GT.YR) D2=(AH/AJOH)*(40.*(YI(I)-YR))+AH*(DEXP(YR/2.)-1.)
450 DYDZ2=(A1+2.*A2+2.*A3+A4)/(6.*DZ)
451 IF(DABS(.5*D2*DZ).GE.DABS(DYDZ2)) B=-1.
452 YI(I+1)=YI(I)+DYDZ2*DZ
453 IF(DYDZ2.EQ.0.) YI(I+1)=YI(I)+.5*D2*DZ*DZ
454 001 CONTINUE
455 008 RETURN
456 END
457 SUBROUTINE RF(TP,DND,WN,RHOP,DNA,DP,WP,AJO,AJOH,A,YI,YCI,YRI,F,ZO,GA
M,ALP)
458 REAL*8 YRI,YCI,YI
459 Y=YI
460 YR=YRI
461 YC=YCI

```

```

462 COMPLEX EP1,EP2,EP3,EP4
463 COMPLEX EPP3
464 REAL IMM3
465 REAL IM1,IM2,IM3,IM4
466 REAL LP,LPN
467 COMPLEX K1,K2,K3,K4,K5,GAM,KISQ
468 COMPLEX GAMSQ,T1,CSQRT,CEXP
469 COMPLEX T2,T3,T4,T5,YD,RD,GAMSQ1,GAMSQ2,F1,F11,FPRI
470 COMPLEX J,LPI,LPNI,ALPI,MN,MD,M,ZO,BIPRI,B2PRI,B3PRI,B4PRI
471 COMPLEX B5PRI,BPRI,BEPRI
472 002 FORMAT(F7.2)
473 004 FORMAT(E12.3)
474 PHIO=ALOG(DND*DNA/2.25E20)
475 DNDN=1.5E18
476 TPNN=4.5E-07
477 RHONN=1.5E-02
478 BNN=250.-WN-WP
479 BOX=.1
480 DPN=5.
481 DNU=2.*11.*35./(11.+35.)
482 ALPHA=(2.*12.*8.85E-14*.025*PHIO*DND)/(1.6E-19*DNA*DNA)
483 IF(Y.GT.YC) XDP=0.
484 IF(Y.GT.YC) XDN=0.
485 IF(Y.GT.YC) GO TO 26
486 XDP=SQRT(ALPHA*(1.-Y/PHIO))
487 XDN=DNA*XDP/DND
488 IF(XDN.GE.WN*1.E-4) XDN=1.E-4*WN
489 26 B1=BNN*1.E-4
490 B2=WN*1.E-4-XDN
491 B3=XDN+XDP
492 B4=WP*1.E-4-XDP
493 B5=BOX*1.E-4
494 SIG1=1./RHONN
495 SIG4=1./RHOP
496 LP=SQRT(TP*DP)
497 LPN=SQRT(TPNN*DPN)
498 OMEGA=44.*1.E09*F/7.
499 J=CMPLX(0.,1.)
500 LPI=LP/CSQRT(1.+J*OMEGA*TP)
501 LPNI=LPN/CSQRT(1.+J*OMEGA*TPNN)
502 ALPI=ALP/CSQRT(1.+J*OMEGA*TP)+DPN*LPI*DND/(DP*LPNI*DNDN)
503 352 CONTINUE
504 AJ01=1.6E-19*2.25E20*DP/(LP*DND)
505 IF(Y.GT.YC) GO TO 55
506 RD=.025/(AJ01*EXP(Y))
507 IF(REAL(B2/LPI).GE.174.) GO TO 200
508 MN=(1.+ALPI)*CEXP(B2/LPI)+(ALPI-1.)*CEXP(-B2/LPI)
509 MD=(1.+ALPI)*CEXP(B2/LPI)+(1.-ALPI)*CEXP(-B2/LPI)
510 M=MN/MD
511 GO TO 201
512 200 M=(1.,0.)
513 201 CONTINUE
514 YD=M*CSQRT(1.+J*OMEGA*TP)/RD
515 IF(Y.LT.0.) YD=(0.,0.)
516 GJ=REAL(YD)
517 CJ=AIMAG(YD)/OMEGA
518 CO=SQRT((1.6E-19*12.*8.85E-14*DNA*DND)/(2.*.025*PHIO*(DNA+DND)))
519 CTRAN=CO*SQRT(ALPHA)*DNA/(DND*XDN)
520 CJ=CJ+CTRAN
521 SCP=(2.25E20/DND)*(EXP(Y)-1.)
522 55 CONTINUE
523 CP=2.25E20*EXP(Y)/DND
524 IF(Y.GT.YC) CP=2.*TP*AJ0H*EXP(Y/2.)/(1.6E-19*B2)
525 IF(Y.GT.YR) CP=2.*TP*(40.*(Y-YR)+AJ0H*EXP(YR/2.))/(1.6E-19*B2)
526 CN=DND+CP
527 SIG2=1.6E-19*(1.5E03*CN+4.E02*CP)
528 IF(Y.GT.YC) GO TO 56
529 SIG3=1.6E-19*1900.*SCP
530 IMM3=SIG3/OMEGA
531 56 CONTINUE

```

```

532 EPSI=12.*8.85E-14
533 IM1=SIG1/OMEGA
534 IM2=SIG2/OMEGA
535 IF(Y.GT.YC) GO TO 57
536 IM3=B3*GJ/OMEGA
537 RE3=B3*CJ
538 57 CONTINUE
539 IM4=SIG4/OMEGA
540 EP1=CMPLX(EPSI,-IM1)
541 EP2=CMPLX(EPSI,-IM2)
542 IF(Y.GT.YC) GO TO 58
543 EP3=CMPLX(RE3,-IM3)
544 EPP3=CMPLX(EPSI,-IMM3)
545 58 IF(Y.GT.YC) EP3=1.
546 EP4=CMPLX(EPSI,-IM4)
547 EP5=4.*8.85E-14
548 K1SQ=-OMEGA*OMEGA*88.E-09*EP1/7.
549 K1=CSQRT(K1SQ)
550 IT=0
551 IG=1
552 IF(REAL(K1*B1).GE.174.) T1=(1.,0.)
553 IF(REAL(K1*B1).GE.174.) GO TO 350
554 T1=(CEXP(K1*B1)-CEXP(-K1*B1))/(CEXP(K1*B1)+CEXP(-K1*B1))
555 350 CONTINUE
556 B1PRI=T1/K1
557 B2PRI=B2
558 B3PRI=B3
559 B4PRI=B4
560 B5PRI=B5
561 BPRI=B1PRI+B2PRI+B3PRI+B4PRI+B5PRI
562 BEPRI=B1PRI/EP1+B2PRI/EP2+B3PRI/EP3+B4PRI/EP4+B5PRI/EP5
563 GAMSQ=-OMEGA*OMEGA*88.E-09*BPRI/(7.*BEPRI)
564 FI=(0.,0.)
565 GAMSQ1=(0.,0.)
566 GAMSQ2=(0.,0.)
567 GAM=CSQRT(GAMSQ)
568 028 CONTINUE
569 IF(IT.EQ.2) GO TO 100
570 FI=FI
571 GAMSQ2=GAMSQ1
572 IG=IG+1
573 GAMSQ1=GAMSQ
574 K1=CSQRT(-OMEGA*OMEGA*88.E-09*EP1/7.-GAMSQ1)
575 K2=CSQRT(-OMEGA*OMEGA*88.E-09*EP2/7.-GAMSQ1)
576 IF(Y.GT.YC) GO TO 59
577 K3=CSQRT(-OMEGA*OMEGA*88.E-09*EPP3/7.-GAMSQ1+EPP3/EP3)
578 59 CONTINUE
579 K4=CSQRT(-OMEGA*OMEGA*88.E-09*EP4/7.-GAMSQ1)
580 K5=CSQRT(-OMEGA*OMEGA*88.E-09*EP5/7.-GAMSQ1)
581 IF(REAL(K1*B1).GE.174.) T1=(1.,0.)
582 IF(REAL(K1*B1).GE.174.) GO TO 351
583 T1=(CEXP(K1*B1)-CEXP(-K1*B1))/(CEXP(K1*B1)+CEXP(-K1*B1))
584 351 CONTINUE
585 T2=(CEXP(K2*B2)-CEXP(-K2*B2))/(CEXP(K2*B2)+CEXP(-K2*B2))
586 IF(Y.GT.YC) GO TO 60
587 T3=(CEXP(K3*B3)-CEXP(-K3*B3))/(CEXP(K3*B3)+CEXP(-K3*B3))
588 60 CONTINUE
589 T4=(CEXP(K4*B4)-CEXP(-K4*B4))/(CEXP(K4*B4)+CEXP(-K4*B4))
590 T5=(CEXP(K5*B5)-CEXP(-K5*B5))/(CEXP(K5*B5)+CEXP(-K5*B5))
591 B1PRI=T1/K1
592 B2PRI=T2/K2
593 IF(Y.GT.YC) GO TO 61
594 B3PRI=T3/K3
595 61 IF(Y.GT.YC) B3PRI=0.
596 B4PRI=T4/K4
597 B5PRI=T5/K5
598 BPRI=B1PRI+B2PRI+B3PRI+B4PRI+B5PRI
599 BEPRI=B1PRI/EP1+B2PRI/EP2+B3PRI/EP3+B4PRI/EP4+B5PRI/EP5
600 FI=GAMSQ1+OMEGA*OMEGA*88.E-09*BPRI/(7.*BEPRI)
601 XTEST=TEST

```

```

602 IF(CABS(GAMSQ1).EQ.0.) GO TO 541
603 TEST=CABS(FI/GAMSQ1)
604 541 IF(CABS(GAMSQ1).EQ.CABS(GAMSQ2)) GO TO 115
605 FPRI=(FI-FI1)/(GAMSQ1-GAMSQ2)
606 IF(IG.EQ.2) FPRI=(1.,0.)
607 IF(CABS(FPRI).EQ.0.) GO TO 537
608 GAMSQ=GAMSQ1-FI/FPRI
609 537 IF(CABS(FPRI).EQ.0.) GAMSQ=0.
610 GAM=CSQRT(GAMSQ1)
611 IF(IT.EQ.1) GO TO 112
612 IF(IG.LT.50) GO TO 110
613 IF(TEST/XTEST.LE.1.) IT=1
614 110 IF(TEST.GT.0.) GO TO 28
615 GO TO 100
616 112 IF(TEST/XTEST.LT.1.) GO TO 28
617 115 GAMSQ=GAMSQ2
618 GAM=CSQRT(GAMSQ2)
619 TEST=XTEST
620 IT=2
621 GO TO 28
622 100 CONTINUE
623 Z0=CSQRT(88.E-09*BEPRI*BPRI/(A*A*7.))
624 RETURN
625 END
626 SUBROUTINE MULT(A11,A12,A21,A22,AL11,AL12,AL21,AL22)
627 COMPLEX A11,A12,A21,A22,AL11,AL12,AL21,AL22
628 COMPLEX B11,B12,B21,B22
629 B11=AL11*A11+AL12*A21
630 B12=AL11*A12+AL12*A22
631 B21=AL21*A11+AL22*A21
632 B22=AL21*A12+AL22*A22
633 A11=B11
634 A12=B12
635 A21=B21
636 A22=B22
637 RETURN
638 END

```

APPENDIX VI
COMPUTER PROGRAM FLOW CHARTS

Notation Used:

$Y(0)$, $Y(z_1)$ -Specified normalized voltages at $z=0$ and
 $z=z_1$.

$Y(IDZ)$ - Calculated normalized voltage at $z=IDZ$.

CI - The constant C (see Eq. 2-17).

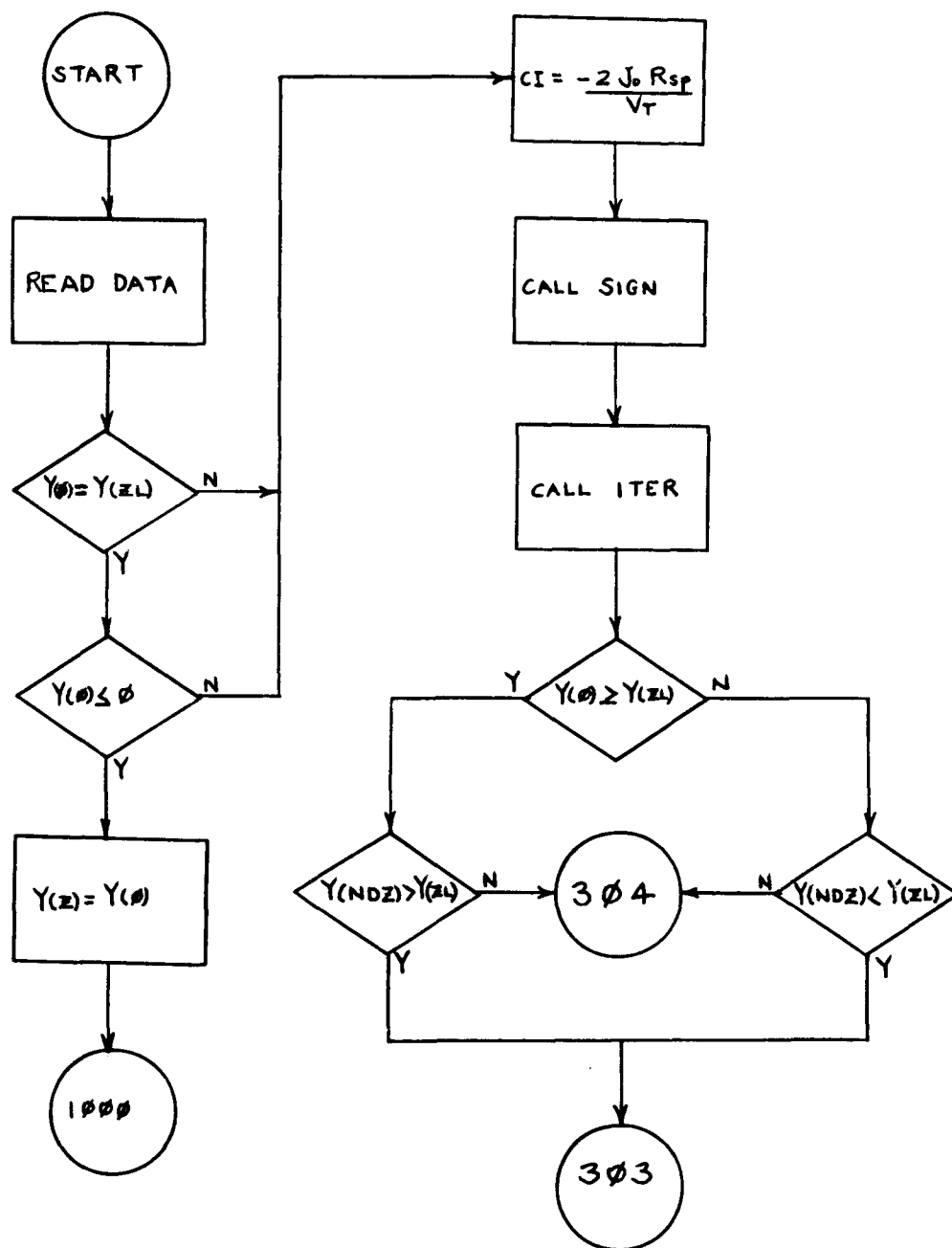


FIGURE VI-1 MAIN PROGRAM

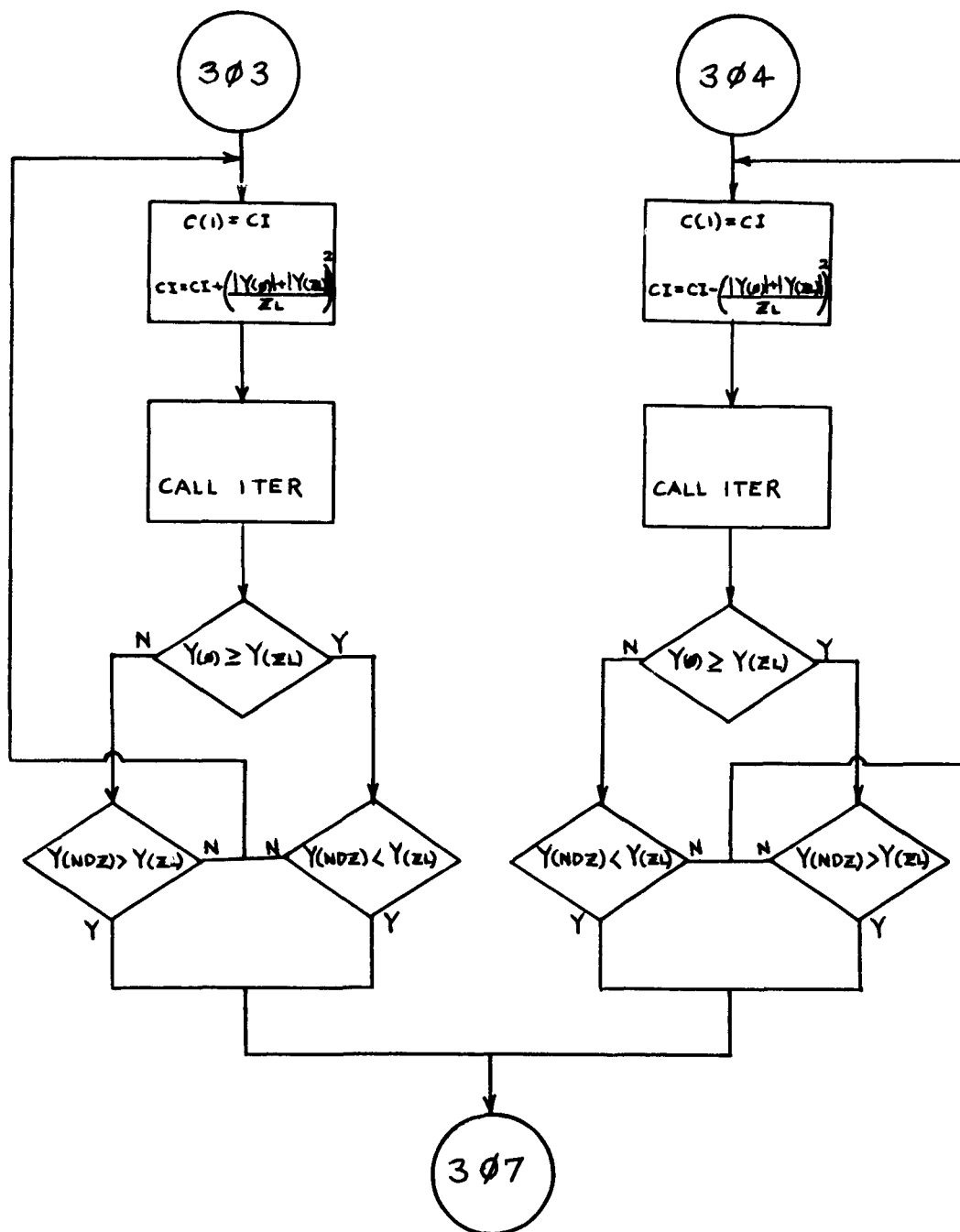


FIGURE VI-2 MAIN PROGRAM (cont. 1)

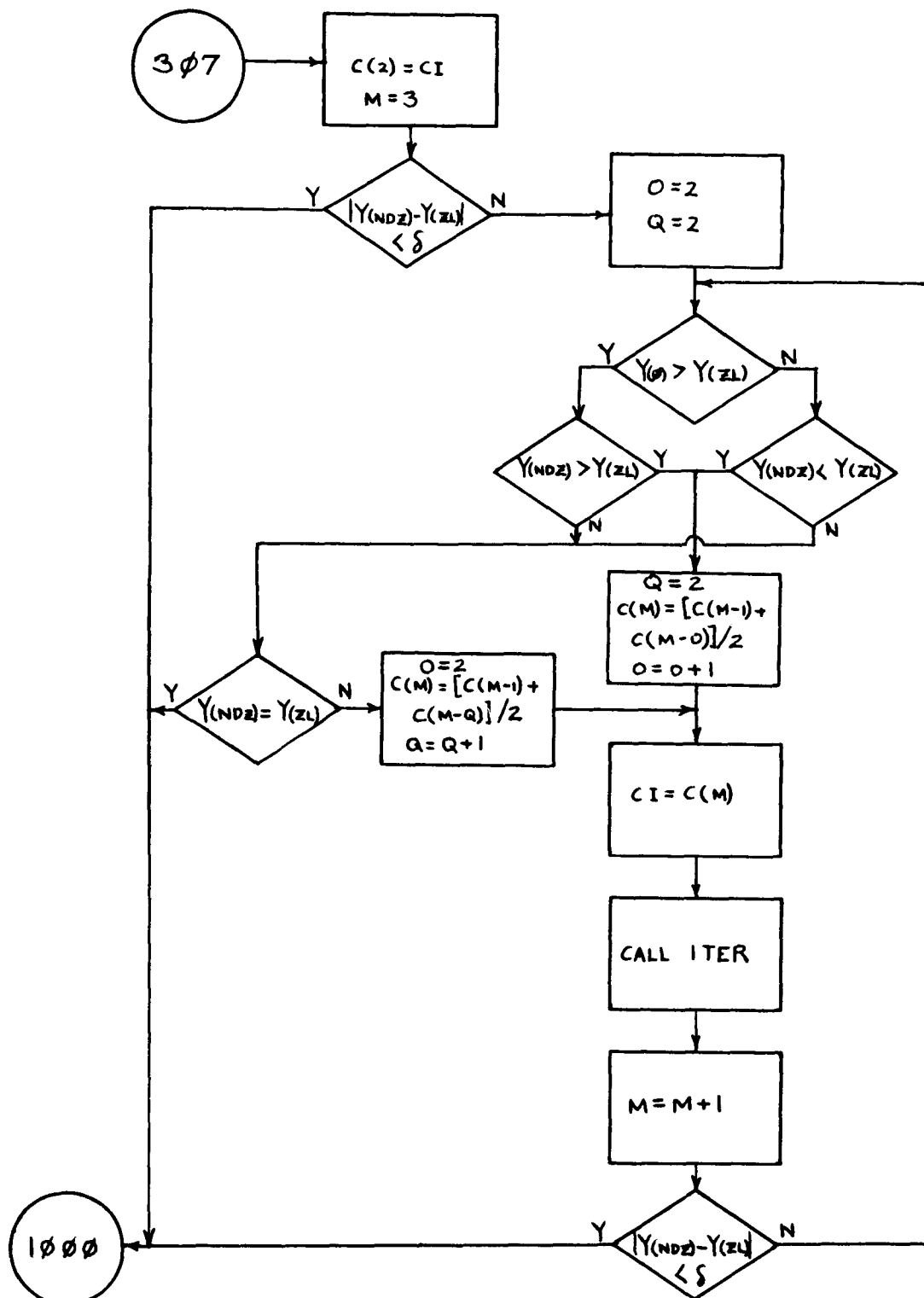


FIGURE VI-3 MAIN PROGRAM (cont. 2)

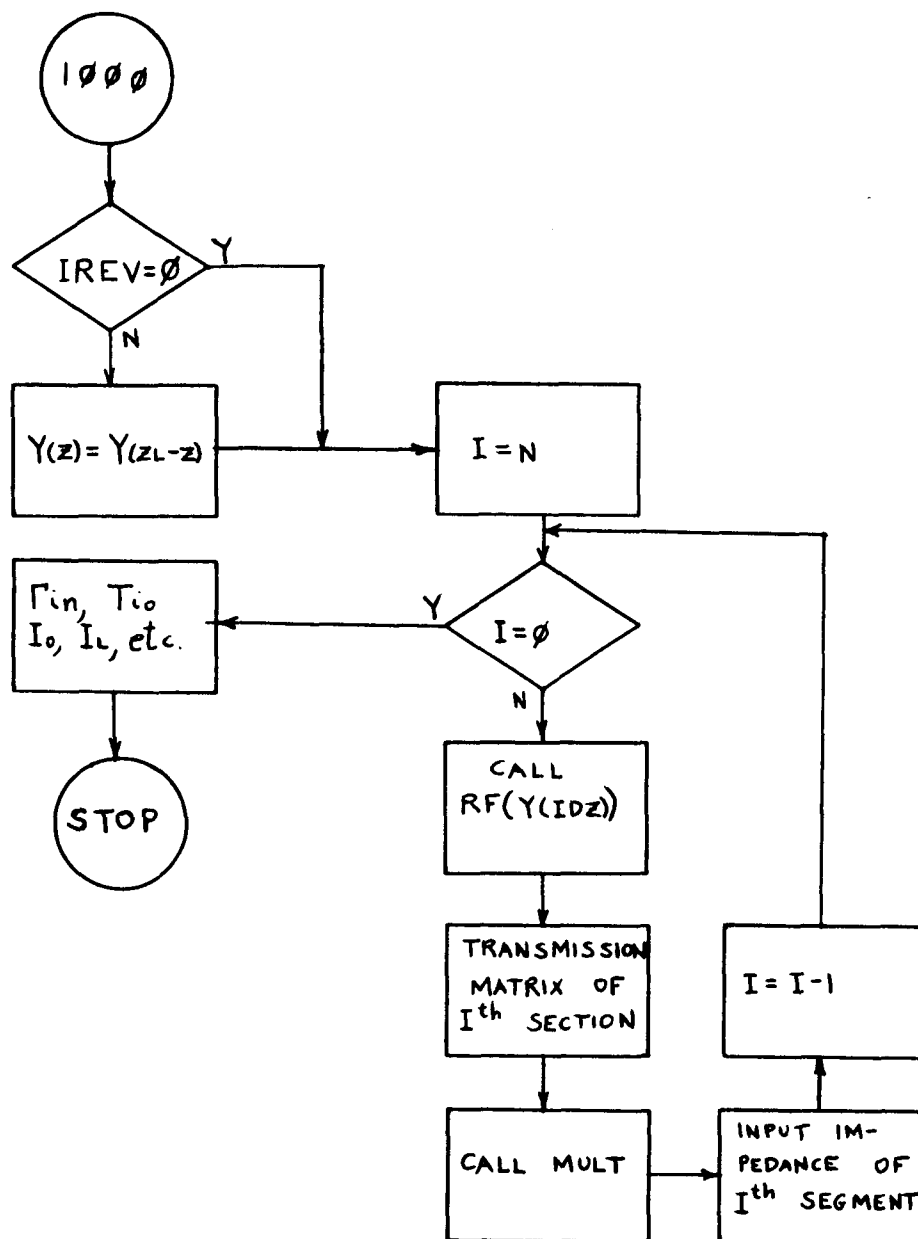


FIGURE VI-4 MAIN PROGRAM (cont. 3)

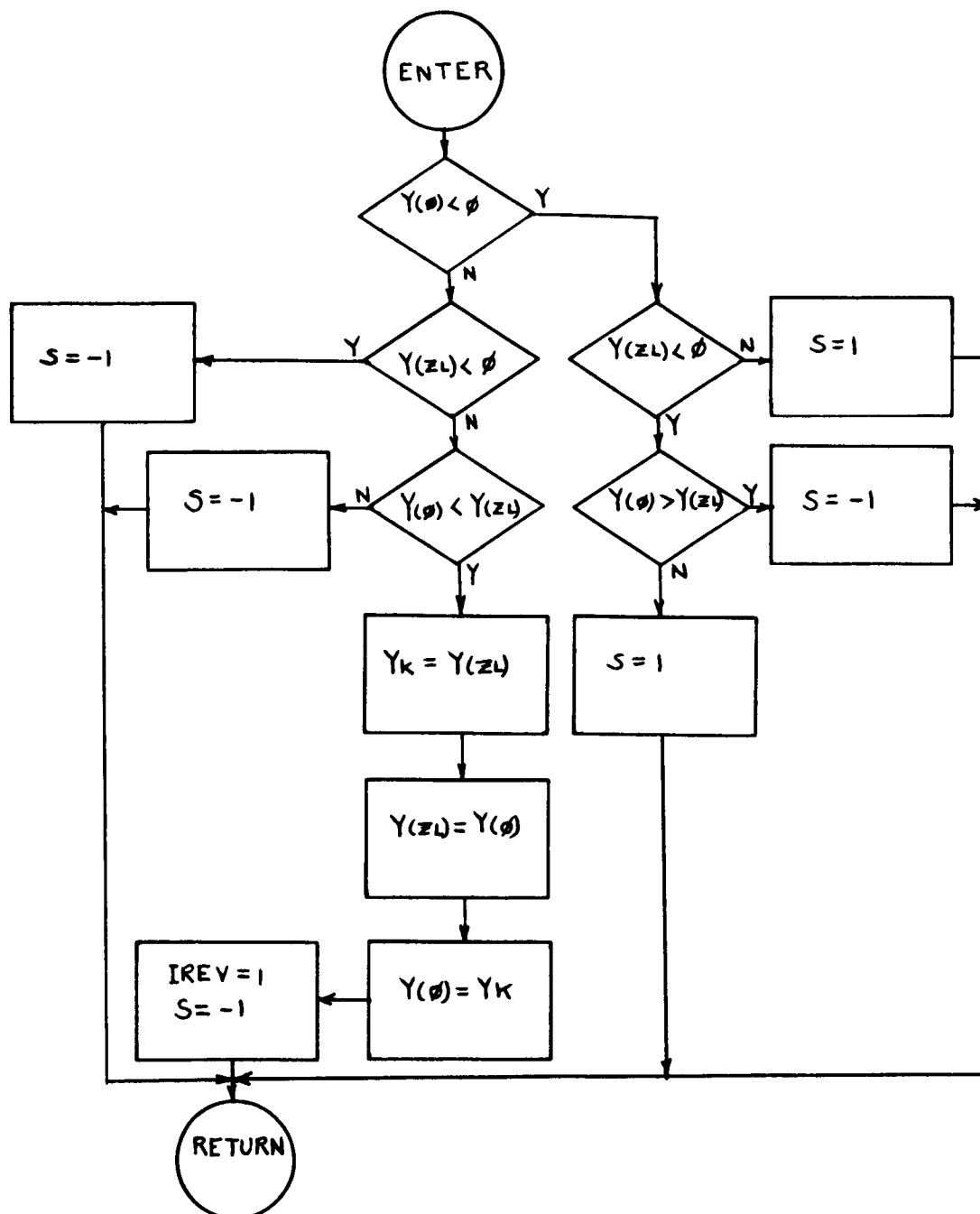


FIGURE VI-5 SUBROUTINE SIGN

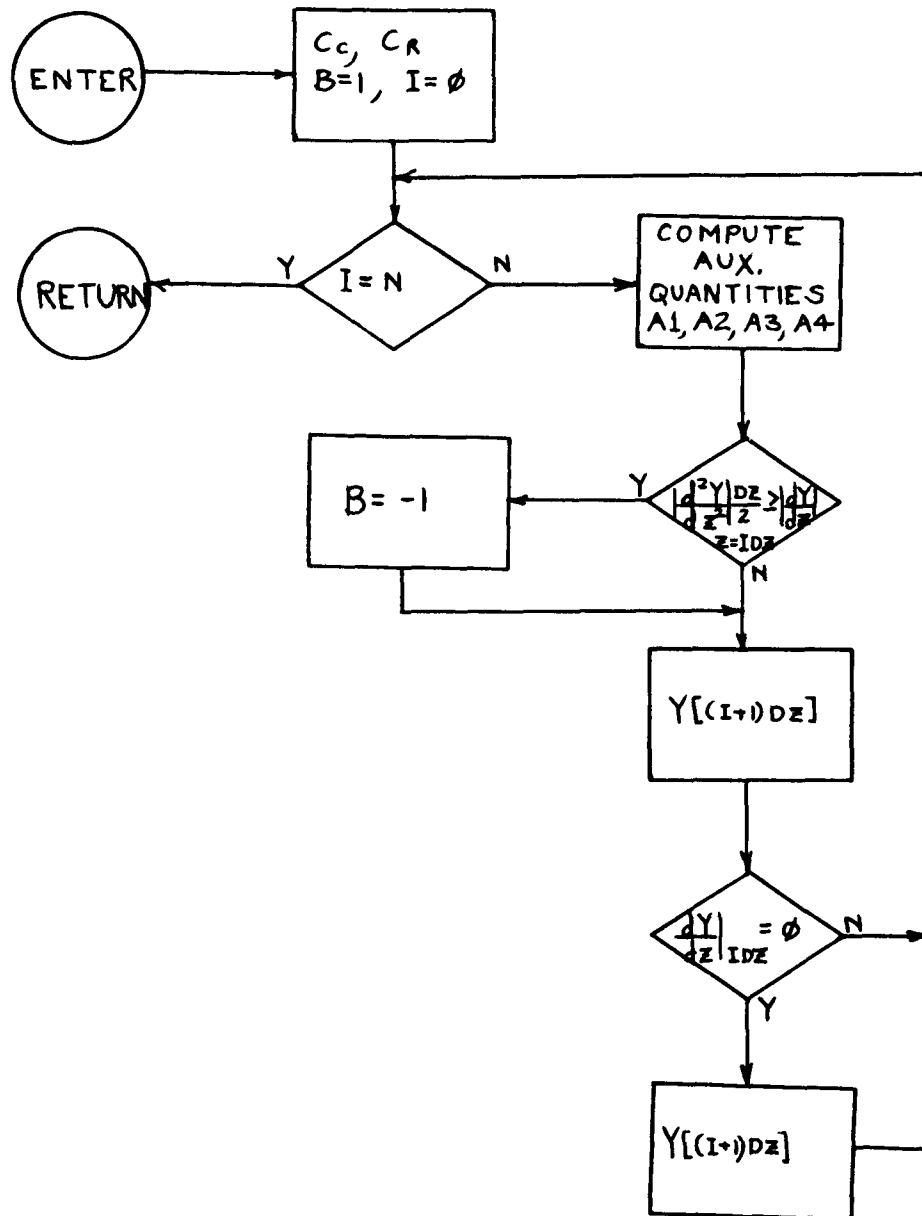


FIGURE VI-6 SUBROUTINE ITER

APPENDIX VII
DEVICE FABRICATION PROCEDURE

Microstrip-oxide-distributed pvn devices were fabricated from 2 inch diameter epitaxially grown ν on n antimony doped, $\langle 111 \rangle$ orientation, silicon wafers. The following processing steps were used at room temperature (25° C) unless otherwise stated: .

1. Wafer cleaning cycle
 - a. 3 min. in trichloroethylene (TCE) in ultrasonic bath at 40°C.
 - b. 3 min. in acetone in ultrasonic bath at 40°C.
 - c. 3 min. in isopropyl alcohol in ultrasonic bath at 40°C.
 - d. Dry with N₂.
2. Remove any SiO₂ in buffered HF etch for 2 min.
3. Rinse in de-ionized (DI) water.
4. Rinse in isopropyl alcohol.
5. Repeat cleaning cycle (step 1).
6. Grow wet oxide: furnace at 1000°C, water bath at 95°C, O₂ carrier gas at 1 liter/min., 35 min.
7. Spin on Shipley AZ-111 photoresist at 3000 rpm, 10 sec.
8. Prebake at 80°C, 15 min.
9. Expose photoresist through diffusion mask, 3 min.
10. Develop photoresist in 1 part Shipley AZ-303 developer diluted with 4 parts DI water at 20°C, 20 sec.
11. Rinse in DI water, 3 min.
12. Dry with N₂.

13. Postbake at 110°C, 30 min.
14. Etch windows in buffered HF, 5 min.
15. Rinse in DI water, 3 min.
16. Dry with N₂.
17. Remove photoresist with acetone, 1 min.
18. Rinse in clean acetone, 2 min.
19. Rinse in isopropyl alcohol, 2 min.
20. Rinse in clean isopropyl alcohol, 2 min.
21. Dry with N₂.
22. Spin on Emulsitone Borosilicafilm 5257 diffusion source ($\approx 4 \times 10^{20} \text{ cm}^{-3}$ surface concentration) 3000 rpm, 10 sec.
23. Bake at mouth of diffusion furnace set at 1000°C, 15 min., gas flows at .5liter/min O₂ and 4.5liter/min Ar.
24. Slowly push boat into center of furnace (3"/min) and diffuse for 15 min. with same conditions as in step 23.
25. Remove boat at 3"/min.
26. Spin on Emulsitone Silicafilm, 3000 rpm, 10 sec.
27. Densify at mouth of furnace set at 1000°C, 1liter/min Ar, 20 min.
28. Repeat step 7.
29. Repeat step 13.
30. Etch oxide from back of wafer in buffered HF, 2 min.
31. Rinse in DI water.
32. Dry with N₂.
33. Repeat steps 17-21.
34. Spin on Emulsitone Phosphorosilicafilm (surface concentration of 10^{21} cm^{-3}) to back of wafer.

35. Prebake 120°C, 15 min.
36. Diffuse at 1000°C, 1 liter/min O₂, 3 liter/min Ar, 15 min.
37. Repeat step 25.
38. Etch in buffered HF, 5 min.
39. Etch in concentrated HF in ultrasonic bath to remove boron skin, 2 min.
40. Rinse in DI water.
41. Rinse in isopropyl alcohol.
42. Dry with N₂.
43. Dry oxidation and drive in: 1000°C, 5 liter/min O₂, 100 min.
44. Repeat steps 7 and 8.
45. Repeat step 9 using contact window mask.
46. Repeat steps 10-13.
47. Etch windows in buffered HF, 3 min.
48. Repeat steps 15-21.
49. Evaporate Al at 10⁻⁶ torr.
50. Repeat steps 7 and 8.
51. Repeat step 9 using metallization mask.
52. Develop in 1 part AZ-303 to 8 parts DI water at 20°C, 30 sec.
53. Rinse in DI water, 3 min.
54. Check to see if resist is completely developed; if not, repeat step 52 for 10 sec and steps 53 and 54.
55. Repeats steps 12 and 13.
56. Etch Al in Al etch (400 ml H₃PO₄ + 60 ml H₂O + 25 ml HNO₃ + 15 ml glacial acetic acid) until pattern is well defined and no aluminum haze exists.
57. Rinse in DI water, 3 min.

58. Dry with N_2 .
59. Repeat steps 17-21.
60. Evaporate Al on back of wafer at 10^{-6} torr.
61. Alloy at 480°C , 1 liter/min Ar, 15 min.
62. Scribe, dice and package.

It is noteworthy to mention that by doing the boron predeposition, on the front of the wafer (step 24), before the phosphorous predeposition, on the back of the wafer (step 36), no buried layer is formed under the p region (see Sec. 3.4). This sequence gives the boron impurity profile the necessary lead over any phosphorous diffusing into the front of the wafer during dry oxidation. The phosphorous doped back surface of the wafer ensures that a good ground-plane ohmic contact is formed.

REFERENCES

1. R.R. Painter, Gate-Oxide Protection Circuit in RCA COS/MOS Digital Integrated Circuits, RCA Solid State Databook Series, pp. 351-352.
2. H. Hasegawa, M. Furukawa and H. Yanai, Properties of Microstrip Line on Si-SiO₂ System, IEEE Trans. Microwave Theory and Techniques, Vol. MTT-19, No. 11, 1971, pp. 869-881.
3. H. Guckel, P. Brennan and I. Palocz, A Parallel-Plate Waveguide Approach To Microminiaturized, Planar Transmission Lines For Integrated Circuits, IEEE Trans. Microwave Theory and Techniques, Vol. MTT-15, No. 8, 1967, pp. 468-476.
4. A. Van der Ziel, Solid State Physical Electronics, Prentice Hall, New York, 1968, p.101.
5. H. Wolf, Semiconductors, John Wiley and Sons, New York, 1971, pp. 418-440.
6. D.J. Hamilton, F.A. Lindholm and A.H. Marshak, Principles and Applications of Semiconductor Device Modeling, Holt, Rhinehart and Winston, New York, 1971, Chapt. 2.
7. Ibid.
8. Ibid.
9. H. Ghosh, A Distributed Model of the Junction Transistor and Its Application in the Prediction of the Emitter-Base Diode Characteristic, Base Impedance, and Pulse Response of the Device, IEEE Trans. Electron Devices, Vol. ED-12, No. 10, 1965, pp. 513-531.
10. H.K. Gummel, A Self-Consistent Iterative Scheme For One-Dimensional Steady State Transistor Calculations, IEEE Trans. Electron Devices, Vol. ED-11, No. 10, 1964, pp. 455-465.
11. Ibid.
12. S.D. Conte, Elementary Numerical Analysis, McGraw Hill, New York, 1965, pp. 220-224.
13. A.S. Grove, Physics and Technology of Semiconductor Devices, John Wiley and Sons, New York, 1967, pp. 186-189.

14. S.C. Choo, Numerical Analysis of a Forward-Biased Step-Junction P-L-N Diode, IEEE Trans. Electron Devices, Vol. ED-18, No. 8, 1971, pp. 574-586.
15. A. Herlet, The Forward Characteristic of Silicon Power Rectifiers at High Current Densities, Solid State Electronics, Vol. 11, 1968, pp. 717-742.
16. A.S. Grove, op. cit., p. 133.
17. H. Wolf, Silicon Semiconductor Data, Pergamon Press, New York, 1969.
18. H. Guckel, P. Brennan, and I. Palocz, op. cit.
19. H. Hasegawa, M. Furukawa, and H. Yanai, op. cit.
20. H.A. Watson, Microwave Semiconductor Devices and Their Circuit Applications, McGraw Hill, New York, 1969, pp. 102-104.
21. R.E. Collin, Field Theory of Guided Waves, McGraw Hill, New York, 1960, pp. 227-228.
22. S.D. Conte, op. cit., pp. 39-42.
23. H. Guckel, P. Brennan, and I. Palocz, op. cit.
24. H. Hasegawa, M. Furukawa, and H. Yanai, op. cit.
25. H. Guckel, P. Brennan, and I. Palocz, op. cit.
26. H. Wolf, Semiconductors, John Wiley and Sons, New York, 1971, pp. 503-510.
27. A. Van der Ziel, op. cit., pp. 269-270.
28. J. Millman, and H. Taub, Pulse, Digital and Switching Waveforms, McGraw Hill, New York, 1965, pp. 755-756.
29. H. Wolf, Silicon Semiconductor Data, Pergamon Press, New York, 1969.
30. Ibid.
31. A.C. Scott, Distributed Effects in Large-Area Esaki Diodes, IRE Trans. Electron Devices, Vol. ED-9, No. 9, 1962, pp. 417-422.
32. H. Hasegawa, M. Furukawa, and H. Yanai, op. cit.

- 33. H. Guckel, P. Brennan, and I. Palocz, op. cit.
- 34. J.P. McKelvey, Solid State and Semiconductor Physics, Harper and Row, New York, 1966, pp. 327-328.
- 35. A.S. Grove, op. cit., p. 182.
- 36. Ibid., p. 124.
- 37. Ibid., p. 185.
- 38. R.W. Dutton and R.J. Whittier, Forward Current-Voltage and Switching Characteristics of p^+-n-n^+ (Epitaxial) Diodes, IEEE Trans. Electron Devices, Vol. ED-16, No. 5, 1969, pp. 458-467.
- 39. S.C. Choo, Theory of a Forward-Biased Diffused-Junction P-L-N Rectifier - Part I: Exact Numerical Solutions, IEEE Trans. Electron Devices, Vol. ED-19, No. 8, 1972 pp. 954-966.
- 40. H. Wolf, Silicon Semiconductor Data, Pergamon Press, New York, 1969.
- 41. R.W. Dutton and R.J. Whittier, op. cit.
- 42. W. Shockley and W.J. Read, Statistics of the Recombination of Holes and Electrons, Phys. Rev., Vol. 87, 1952, pp. 835-842.
- 43. R.N. Hall, Electron-Hole Recombination in Germanium, Phys. Rev., Vol. 87, 1952, p. 387.
- 44. R.W. Dutton and R.J. Whittier, op. cit.
- 45. Ibid.
- 46. S.C. Choo, op. cit.
- 47. Ibid.
- 48. A. Herlet, op. cit.
- 49. J.P. McKelvey, op. cit., pp. 448-456.
- 50. Ibid., p. 330.
- 51. Ibid., p. 451.
- 52. Ibid., p. 453.

53. Ibid., pp. 453-454.
54. H.A. Watson, op. cit., pp. 109-110.
55. R.E. Collin, op. cit., pp. 227-228.

AUTOBIOGRAPHICAL STATEMENT

Jack Allan Mandelman, born in New York, N.Y. on May 25, 1946, attended elementary schools in Manhattan and Queens. He graduated from Brooklyn Technical High School with a Technical Diploma in the electronics course of study in June 1964.

In May 1966, upon completion of his pre-engineering studies at Queens College, Mr. Mandelman received the John T. Dollard Engineering Award for excellence in technical scholarship. At City College, Mr. Mandelman majored in electrical engineering and participated in a special research project involving the study of semiconductor-traveling wave interactions. Upon graduation with the B.E.E. degree (magna cum laude) in June 1969 he entered the doctoral program at the City University of New York.

In September 1969, Mr. Mandelman became a Lecturer in the Department of Electrical Engineering and pursued his interest in microwave semiconductor devices. He received the M.E.E. degree and was awarded a National Science Foundation Summer Traineeship in 1971. In the summer of 1974 he held a City University of New York Research Assistantship.

Mr. Mandelman is presently employed by the General Instrument Corporation, Microelectronics Division, as a Senior Process Engineer in research and development.

He and his wife, Antoinette, reside in Queens.