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BRST anomalies and mass renormalisation with anomalous $U(1)$ gauge symmetries in string theory

Lee, Christopher Ji-Hua, Ph.D.

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U·M·I
300 N. Zeeb Rd.
Ann Arbor, MI 48106

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BRST ANOMALIES AND MASS RENORMALISATION WITH ANOMALOUS $U(1)$ GAUGE SYMMETRIES IN STRING THEORY

by

CHRISTOPHER LEE

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ABSTRACT

BRST ANOMALIES AND MASS RENORMALISATION WITH ANOMALOUS $U(1)$ GAUGE SYMMETRIES IN STRING THEORY

by

CHRISTOPHER LEE

Adviser: Professor Michael Dine

The heterotic string, when compactified from ten to four dimensional spacetime, has a gauge group that often includes apparently anomalous $U(1)$ gauge symmetries which lead to the generation of masses at one-loop. It is shown how the masses, and fermion masses in particular, can be calculated consistently from the effective action, field theory and in the string theory. The resulting mass shifts violate symmetries, the BRST symmetry in particular. Mass renormalisation is carried out in the BRST framework for both bosons and fermions using local properties and is shown to be a consistent procedure by explicit calculations up to one-loop.

Acknowledgements

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Chapter 1 : Introduction

1.1 Introduction

String theory is a theory of fundamental interactions based on one-dimensional objects as opposed to traditional field theories where the basic objects are points. Each fundamental object in string theory can be a string with two loose ends(open string) or when the ends are joined, forming a closed loop(closed string). The spacetime coordinates of a string are X^μ with $\mu = 0$(time coordinate); or $1, \ldots D-1$(space coordinates), and D is the number of spacetime dimensions. Such strings have length and evolve in time. X^μ is thus a function of two parameters, σ and τ, one of which can be considered as position along the length of the string and the other is a 'time' which parametrises the evolution of the string. The evolution of a string in this 'time' generates a two-dimensional surface as in Fig.1.

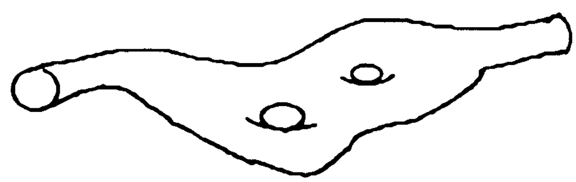

Fig.1 Propagation of a closed string in spacetime generating a genus 2 surface

Each surface is equivalent to the surface of a sphere with handles attached, and the initial and final string states correspond to two external states as shown in Fig.2.

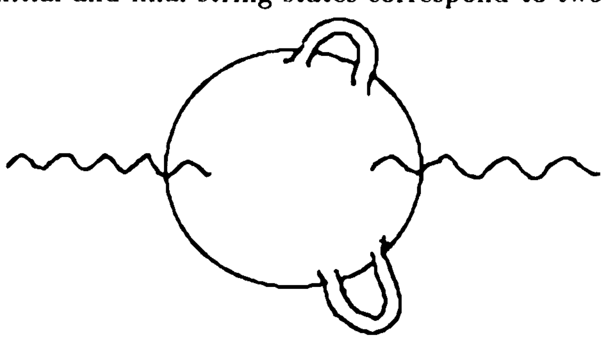

Fig.2 Equivalent picture with 2 handles and 2 external states attached to a sphere

If the area of this surface is invariant with respect to the choice of coordinates on the surface[1], and because for a sufficiently small neighbourhood on the surface one can always choose σ and τ to be Cartesian coordinates, it can be shown[2] that the functions $X^\mu(\tau,\sigma)$ satisfy,

$$[\partial_\tau^2 - \partial_\sigma^2]X^\mu(\tau,\sigma) = 0 \quad . \tag{1}$$

For a fixed length, say L, of the string, the boundary conditions are that $X^\mu(\sigma + L) = \lambda X^\mu(\sigma)$ where $\lambda = e^{i\phi}$. This leads to classical normal modes of vibration with expansion,

$$X^\mu = \sum_{n=1}^{\infty} \left[c_n^\mu e^{(2n\pi+\phi)i\sigma/L} + c_{-n}^\mu e^{-(2n\pi-\phi)i\sigma/L} \right] \quad . \tag{2}$$

The string modes evolve in 'time' τ with factor $e^{\pm 2\pi i\tau/L}$. At the classical level, the two sets of modes for the choice of $\pm$ signs are independent with independent functions $X^\mu(\tau \pm \sigma)$. In addition, if the functions X^μ commute, $[X^\mu, X^\nu] = 0$, one has a bosonic string, whereas if they anti-commute (they are denoted ψ^μ), $\{\psi^\mu, \psi^\nu\} = 0$, the string is fermionic[3],[4].

For periodic boundary conditions, $\phi = 2m\pi$ with m an integer, and there is an additional mode, the zero mode, which reflects translational invariance along the string. Periodic boundary conditions are consistent with the properties of two-dimensional bosons, whereas two-dimensional fermions are expected to be anti-periodic. An arbitrary choice of ϕ corresponds to two-dimensional parabosons or parafermions without the zero mode.

In the literature, at the classical level a simple choice defines the closed bosonic string with periodic boundary conditions, and the closed fermionic string with two possible sectors[3],[4] with periodic and antiperiodic conditions. In a two-dimensional system with both X^μ and ψ^μ, there is a symmetry between the bosonic and fermionic string modes which is called a supersymmetry; a two-dimensional supersymmetry[5].

The string normal modes defined by coefficients c_n^μ and c_{-n}^μ carry spacetime indices, μ. They define many body states of a two-dimensional theory but can be interpreted as single particle states in the D-dimensional spacetime. The latter are spacetime states and are constrained by the symmetries of the two-dimensional theory which manifest themselves as spacetime symmetries on the spectrum of spacetime states. For example, the existence of two-dimensional supersymmetry (with two generators, $N = 2$) between fermionic and bosonic string modes is sufficient for and is a necessary consequence of the existence of spacetime supersymmetry (with one generator, $N = 1$) between bosonic and fermionic spacetime states[8]. The relationship between two-dimensional and D-dimensional spacetime symmetries is not always obvious.

Historically, the original serious objections to the identification of spacetime states derived from the bosonic string with higher angular momentum hadronic resonance states are that, (i) there exist massless states with spacetime spin 0,1 and 2 in the physical spacetime spectrum, which disagrees with the experimentally known hadronic states, and (ii) the ground state has negative mass square, i.e. is a tachyon. The former problem rules out string theory as simply a strong interaction theory, but it was suggested[9],[10] that the theory could describe a more fundamental theory that included electrodynamics, weak and strong interactions as well as gravity, by identifying massless spin 1 modes with gauge bosons mediating interactions as in the standard model and spin 2 bosons with gravitons†. The fundamental length scale of the unified theory would then be the Planck length $L_{\rm P} \approx M_{\rm P}^{-1}$ ($L_{\rm P} \approx 10^{-31}$ m, or $M_{\rm P} \approx 10^{19}$ GeV). The tachyon problem was resolved with the discovery of two-dimensional supersymmetry[5] in a string theory which contains two-dimensional bosons and fermions. It was subsequently discovered[12] that there were spacetime bosonic and fermionic states. The classical symmetry between the fermionic and bosonic spacetime states, a spacetime

† One could try to identify some of the massless spin 0 bosons with Higgs bosons but supersymmetry breaking mechanisms(for example introducing soft breaking mass terms) do not generally prevent massless scalar fields from gaining large masses at the large scales of, say, 10^{14} GeV necessary to solve the hierarchy problem[11].

supersymmetry, eliminates the tachyon from the physical spacetime spectrum.

1.2 First Quantisation

1.2.1 Quantum Theory

String theory can be quantised at the simplest level by quantising a two-dimensional field theory, where fields become local operators, $X^\mu(\sigma^m) \rightarrow \hat{X}^\mu(\sigma^m)$ and $\psi^\mu(\sigma^m) \rightarrow \hat{\psi}^\mu(\sigma^m)$. The c-number coefficients in eqn. (2) representing classical string normal modes are promoted to operators, i.e. $c^\mu_{-n} \rightarrow \hat{a}^{\mu\dagger}_n$ and $c^\mu_n \rightarrow \hat{a}^\mu_n$, creating and annihilating normal modes. It was the remarkable discovery of Green and Schwarz[13] that a first quantised string theory with gauge symmetries and spacetime supersymmetry has no quantum anomalies and can thus be a consistent theory. This is, however, true only if the spacetime dimension is 10 and the gauge group is $SO(32)$. It led to renewed interest in string theory. Quantisation of a classical theory can in general lead to violations, so-called anomalies, of the classical symmetries. The elimination of local gauge anomalies is a non-trivial requirement on the consistency of the quantum theory. Soon after the Green-Schwarz discovery, Gross, Harvey, Martinec and Rohm[14] discovered a hybrid theory, with supersymmetry in only one of the two independent sectors of the normal modes. This theory is also anomaly free, can accomodate the gauge group, $E_8 \otimes E_8$ or $SO(32)$, and is consistent in ten spacetime dimensions. Since the above gauge groups contain the gauge symmetries of the standard model, $SU(3) \otimes SU(2) \otimes U(1)$, and the spacetime Lorentz group in ten dimensions $SO(1,9)$ contains the four-dimensional Lorentz symmetry $SO(1,3)$, superstring theory can conceivably be a unified theory of the four known fundamental interactions. The theory in ten spacetime dimensions must be reduced to obtain the standard model and gravity in four spacetime dimensions by 'compactifying' six dimensions. A serious unsolved problem is the non-uniqueness of 'compactified' theories none of which lead naturally to a theory consistent with known physics.

It is possible that the non-perturbative aspects of string theory need to be understood first. The only exact non-perturbative solutions known are toy models which exist in less than or equal to one spacetime dimension[15]. One hopes that a better understanding of perturbative expansions in string theory may be useful. Here, we will only consider a string perturbative expansion scheme, and only up to the quantum field theory equivalent of one-loop.

We will show that for the simplest example of strings with spacetime supersymmetry, there is a two-dimensional $U(1)$ gauge symmetry which allows spacetime supersymmetry to be restored when broken by small corrections [16]. This is achieved by altering classical boundary conditions. In particular, periodic boundary conditions at tree-level are changed and the translational zero mode is eliminated. The two-dimensional symmetry known as superconformal symmetry (to be discussed below) is essential for consistency, and when broken can be restored in the quantum theory. In the example considered, the spectrum of physical spacetime states, determined by the (super)conformal symmetry, is thus stable against small quantum mass corrections.

It will first be shown how generic interactions lead to quantum corrections that shift the masses of certain spacetime states that are massless at the classical(tree) level. These mass shifts violate classical(tree) two-dimensional and related spacetime symmetries of the string. One can, in general, find a suitable gauge transformation to restore all classical worldsheet and spacetime symmetries needed for the consistency of the theory.

1.2.2 String Loop Perturbation

One way of doing a perturbative expansion in string theory is about a vacuum for which the coupling constant is small. The coupling constant is related to the vacuum expectation value of a scalar field, the dilaton ϕ, that belongs to the gravitational multiplet of states which is a generic feature in string theory. The multiplet as its name suggests includes the graviton. The coupling constant appears in first quantised string correlation functions in two ways. The normalisation is chosen so that each interaction vertex contributes a factor of a coupling constant. For example, in quantum electrodynamics, the cubic interaction of the photon field $\hat{A}_\mu$ with a bilinear in fermion fields $\hat{\bar{\Psi}}\gamma^\mu\hat{\Psi}$ carries a factor proportional to the electron charge e. In an effective first quantised string theory, the cubic coupling above is analogous to an insertion, into the string correlation function, of a 'vertex' operator, $\hat{V}$, with coefficient proportional to the coupling constant, $g \sim \langle\phi\rangle$. The vertex operator possesses the symmetries and spacetime momentum of the corresponding external spacetime state and is constructed from normal ordered products of two-dimensional fields. For a theory with only closed strings, i.e. closed one-dimensional loops, a correlation function for n-external states is characterised by the topology of the surface generated by the evolution of n closed strings in spacetime(e.g. Fig.1 is a two-point function).

Besides contributions from vertex operators, correlation functions at each genus are normalised by a genus dependent factor, $g^{-\chi}$, with the coupling constant raised to the power of a topological quantity, the Euler characteristic χ of the surface, which depends linearly on the genus, $\chi = 2 - 2h$. The number of handles or the genus h reduces in the field theory limit to the number of loops in Feynman diagrams. The effective n-point correlation function is a perturbative sum over the quantum fluctuations, for small coupling constant, of genus dependent n-point functions. We will assume the existence of a stable ground state, at genus zero or with topology of a sphere, about which such an expansion can be made. It will be shown why such a supersymmetric ground state would be stable against small quantum fluctuations, i.e. perturbative effects. Supersymmetry, if it exists in nature, is broken at presently achieved energies. This is non-trivial for the supersymmetric string since supersymmetry breaking is in general an unsolved problem. We assume a vacuum for which the coupling constant is small so that perturbative calculations can make sense. We will consider string correlation functions up to one-loop, i.e. with the topology of a torus.

Correlation functions that correspond to scattering amplitudes of spacetime states in spacetime can be evaluated in an operator formalism as correlation functions, in a two-dimensional space, of vertex operators representing external states.

Thus an n-point correlation function is

$$\langle \prod_{i=1}^{n} \hat{V}_i \rangle_h \quad , \tag{3}$$

where $\langle \ldots \rangle_h$ includes integrals $\prod_{i=1}^{n} \int dz_i d\bar{z}_i$ over the positions, with local coordinates $z_i = \sigma_i^0 + i\sigma_i^1$ and $\bar{z}_i = z_i^*$, of the i'th vertex operator $\hat{V}_i(z_i, \bar{z}_i)$ on the surface. It also includes the correct normalisation and symmetry factors to account for the symmetries of the surface of genus h, which is equivalent to the surface of a sphere with h handles. The vertex operators are in general composite operators of a two-dimensional field theory, satisfying conformal or superconformal invariance. Figure 2, for example, corresponds to a two-point function $\langle \hat{V}_1 \hat{V}_2 \rangle_2$. The effective two-point function is thus $\langle \hat{V}_1 \hat{V}_2 \rangle_{\text{effec}} = \sum_{h=0}^{\infty} \langle \hat{V}_1 \hat{V}_2 \rangle_h$.

The symmetries of the theory are symmetries of the surfaces. These include two-dimensional coordinate reparametrisation invariance, two-dimensional supersymmetry and symmetries of the two-dimensional metric tensor. In particular, in the formulation of the string theory due to Polyakov[20], there is explicit local scale invariance which is central to the theory. Up to finite order in the genus or the string scale $l = \sqrt{\alpha'}$, cancellation of quantum anomalies which would otherwise break scale invariance leads to consistent spacetime equations of motion[17].

Although strings are one-dimensional objects, short distance worldsheet divergences occur in string theory when the size of strings shrink to zero. In contrast, there are logarithmic singularities which appear due to massless bosonic string modes propagating on the worldsheet. Such modes appear when there are massless spacetime states in the physical spectrum. The former singularities are pole singularities, $1/z^\alpha$, and do not have the direct physical interpretation of the log singularities. The nonlogarithmic short distance divergences are generic and break conformal symmetry. They can occur with the coincidence of vertex operator positions on the sheet and when handles become small[22],[23]. These nonlogarithmic divergences become sources of the breakdown of two-dimensional reparametrisation invariance. For the purpose of calculations, one must control divergences, i.e.

regularise the theory. We will regularise naively by having a short distance cut-off ϵ on the surface. Since we will be dealing with massless spacetime states, the cut-off also leads to logarithmic contributions $\ln \epsilon$. This will be shown to be sufficient for tree-level and one-loop calculations involving massless spacetime states with the result that physical quantities will be finite and independent of the arbitrary cut-off ϵ in the limit as $\epsilon \to 0$. A general regularisation scheme that would be defined unambiguously for all surfaces with different numbers of handles is not known.

1.2.3 (Super)Virasoro Algebra

To simplify the analysis, one can make a "gauge choice" to reduce two dimensional general coordinate invariance. A suitable choice is the "conformal gauge", the name derived from the symmetry after gauge-fixing, the conformal symmetry. Conformal symmetry refers to invariance under angle preserving coordinate transformations. At a more fundamental level,it comes from global(position independent) and local(position dependent) scale invariance[24]. The symmetry group of the quantum theory in two-dimensions is generated by an infinite number of quantum operators $\hat{L}_n$, $n = 0, \pm 1, \pm 2, \ldots$ which obey the Virasoro algebra defined by commutators[25],

$$[\hat{L}_m, \hat{L}_n] \;=\; (m-n)\hat{L}_{m+n} \;+\; \frac{c}{12}m(m^2-1)\delta_{m+n,0} \quad . \tag{4}$$

The $\hat{L}_m$ are Fourier modes of the energy-momentum-stress tensor $\hat{T}(z)$ in two-dimensions. The commutators would form a closed algebra except for the second term, with central charge c. It represents a quantum anomaly since the classical two-dimensional modes L_m form a closed algebra with $c = 0$. In a system of $n_{\rm b}$ two-dimensional bosons, $c = n_{\rm b}$. The gauge choice can be implemented formally in the quantum theory by introducing "ghost" fields. Unlike physical fields, ghosts fields have negative norm. The combined system of physical and ghost fields has total energy-momentum-stress tensor $\hat{T}(z)^{\rm tot}$ with modes $\hat{L}_m^{\rm tot}$ satisfying eqn. (4) without a central term, i.e. the resultant central charge of the system of physical and ghost fields vanishes, $c^{\rm tot} = 0$. Conformal symmetry is maintained on quantising the two-dimensional theory.

In the case of the supersymmetric string, one can make a "superconformal" gauge choice and the corresponding $N = 1$ super-Virasoro algebra includes $\hat{L}_m$ which are quantum modes of the super-stress tensor $\hat{T}(z)_{\text{sup}}$ and additional generators, $\hat{G}_r$ which are quantum modes of a single two-dimensional supersymmetry generator $\hat{G}(z)$; hence $N = 1$. The $\hat{L}_m$'s also have commutators as in eqn. (4) but with central charge, $c = n_b + n_f/2$, accounting for the presence of n_f two-dimensional fermions. Two-dimensional supersymmetry requires $n_f = n_b$. As the supersymmetric counterparts of $\hat{L}_m$, the $\hat{G}_r$'s have anti-commutators

$$\{\hat{G}_r, \hat{G}_s\} = 2\hat{L}_{r+s} + \frac{\hat{c}}{2}\left(r^2 - \frac{1}{4}\right)\delta_{r+s,0} \quad , \tag{5}$$

where the re-defined central charge $\hat{c}$ ($\hat{c} = 2c/3$) is equal to the spacetime dimension, $\hat{c} = D$. "Mixing" between the two types of generators is expressed through commutators,

$$[\hat{L}_m, \hat{G}_r] = \left(\frac{m}{2} - r\right)\hat{G}_{m+r} \quad . \tag{6}$$

The two possible choices of periodic and antiperiodic boundary conditions[3],[4] for two-dimensional fermions $\hat{\psi}^\mu$ lead to integral and half-integral values for indices r, s. As with the Virasoro algebra, the superconformal gauge choice can be implemented as constraint conditions by introducing ghosts as well as now including "super-ghosts".[26] The combined physical and ghost system has super-stress tensor $\hat{T}(z)^{\text{tot}}_{\text{sup}}$ with modes $\hat{L}^{\text{tot}}_m$ and supersymmetry generator $\hat{G}(z)^{\text{tot}}$ with modes $\hat{G}^{\text{tot}}_r$ so that the resulting super-Virasoro algebra, defined by eqns. (4), (5) and (6), has zero quantum anomaly, i.e. total central charge $\hat{c}^{\text{tot}} = 0$. Thus, superconformal symmetry is maintained on quantising the two-dimensional theory.

In the model to be considered, we assume that the heterotic theory has been compactified with a surviving spacetime supersymmetry with one generator, $N = 1$, in four-dimensions. A necessary and sufficient condition for the spacetime supersymmetry is the existence of a two-dimensional current $J(z)$ of a $U(1)$ symmetry[8]. This current, which is conserved, implies the division of the spacetime spectrum into states with integral(bosons) and half-integral(fermions) $U(1)$ charges and leads to two supersymmetry generators $\hat{G}$ and $\hat{\bar{G}}$ in two-dimensions with $U(1)$ charges of opposite signs. The $N = 1$ super-Virasoro algebra above thus turns into an $N = 2$ super-Virasoro algebra. The latter has in addition global and local $U(1)$ invariance[27]. We will show, in chapter 3, that small mass quantum corrections on the spacetime spectrum are equivalent to global $U(1)$ transformations on the super-Virasoro algebra. The local $U(1)$ then allows superconformal symmetry to be restored, i.e. the super-Virasoro algebra is left unchanged. This is possible at least up to the one-loop level but breaks classical boundary conditions in two-dimensions for string modes.

1.2.4 BRST Quantisation

A gauge-choice can, in general, be made with the simultaneous construction of a symmetry called the (super)BRST symmetry in which ghosts fields play an integral role. As stated earlier, any gauge choice is equivalent to imposing constraints. Constraints can be implemented formally by introducing ghost fields. In general it can be shown that a symmetry exists between the ghost fields and the physical fields. This is the BRST symmetry. The original BRST symmetry named after Becchi, Rouet and Stora[18], and Tyupin[19] was first discovered and proved useful for simplifying the analysis of the renormalisation of non-Abelian gauge theories after fixing gauge symmetries. The main disadvantage of gauge-fixing is that the gauge symmetry is not explicit and leads to non-local expressions in the Lagrangian, but this can be ameliorated by constructing the BRST symmetry which converts non-local terms into local ones. The latter symmetry has a single fermionic (anticommuting) generator with a conserved charge, the BRST charge, Q. Classically, one finds that

$$Q^2 = 0 \quad . \tag{7}$$

Although the (super)BRST symmetry is essentially a consequence of constraints, and its physical significance remains unclear, it is convenient to exploit it because the physical spectrum of states can be clearly defined, as those states annihilated by Q. Quantisation of a theory with complicated symmetries can sometimes be implemented more simply by using its (super)BRST invariance. This quantisation prescription is of central importance here and we will assume as a basic tenet that the (super)BRST symmetry constructed in the theory must be maintained upon quantisation and when broken, it will be shown that the symmetry can be restored in a self-consistent and unambiguous way. The breaking of the (super)BRST symmetry that is constructed with the choice of the (super)conformal gauge, can be traced back to the breaking of (super)conformal symmetry. It is due to divergences that occur on the two-dimensional surface. Such violations of the (super)BRST symmetry will be called BRST anomalies.

We will consider an example where it is known that world-sheet simple pole divergences lead to the generation of masses for both spacetime bosonic and fermionic states that are originally, i.e. classically, massless states. An analysis in the BRST formulation has been done formally for the bosonic string[21] and thus only for spacetime bosonic states. We will consider explicit calculations for an example in which the heterotic string is compactified in 4-dimensions with minimal spacetime supersymmetry and an apparently anomalous $U(1)$ gauge symmetry. We will verify the validity of the BRST procedure for the renormalisation of states that have masses generated by string one-loop corrections and show that it includes fermionic states. We hope that this will lead to a better understanding of the (super)BRST symmetry not only for string theory, but perhaps in a more general context.

1.3 Summary of Thesis

We will study a certain class of theories with an anomalous $U(1)$ gauge symmetry. Besides the breaking of superconformal symmetry, the $U(1)$ gauge symmetry is itself violated by one-loop quantum corrections, because we will consider theories with chiral spacetime fermions, i.e. with asymmetry between the left and right handed four-dimensional chirality fermions.

As a comparison, the vector $U(1)$ gauge symmetry of quantum electrodynamics is non-anomalous to be consistent with electric charge conservation. In the electroweak $SU(2) \otimes U(1)$ theory, the vector $U(1)$ must for the same reason be non-anomalous. Chiral fermions in this theory means that there is a non-trivial axial $U(1)$ symmetry. The latter is unavoidably broken by one-loop corrections. The heterotic string theory in ten dimensions reduces in four dimensions, at energies small compared to the Planck energy, to an effective field theory with a gauge group that may include $U(1)$'s in addition to the gauge group $SU(3) \otimes SU(2) \otimes U(1)$ of the standard model. It is possible for one of the additional $U(1)$'s to be anomalous when there are chiral fermions in four dimensions. Since the original theory in ten dimensions has no anomalies, the four dimensional theory must also be anomaly free. Indeed, one finds that there is a natural mechanism within the theory to remove the axial $U(1)$ anomaly when it arises so that the reduced theory is consistent with the original theory in ten dimensions.

In chapter 2, we will show the details of the computations needed for the cancellation of axial $U(1)$ anomalies and that this leads to couplings quadratic in fields.

In chapter 3, vertex operators corresponding to spacetime states with mass shifts will be renormalised for masses generated when an anomalous axial $U(1)$ is present. We will show that the procedure first used by A.Sen[21] for the bosonic string is applicable to the heterotic string compactified with an anomalous axial $U(1)$ gauge symmetry. Just as the gauge invariance of electrodynamics must be preserved in the quantum theory, one also expects the same for the two-dimensional reparametrisation invariance of string theory.

Chapter 2 Masses from an anomalous $U(1)$ gauge symmetry

2.1 Overview

In supersymmetric field theories, in which supersymmetry is unbroken at tree level, non-renormalization theorems usually guarantee that particles which are massless at tree level remain massless to all orders of perturbation theory[36]. The only exception to this rule occurs in theories in which Fayet-Iliopoulos D terms are generated at one loop. In a supersymmetric field theory, states can be grouped together into supermultiplets, where states within each supermultiplet can be transformed into linear combinations of each other by supersymmetry transformations generated by the supersymmetry charge†. A supermultiplet of gauge fields, also called a vector multiplet because it includes the gauge boson, contains an auxiliary field, D. Because the lagrangian contains no terms involving derivatives of D, this field can be eliminated by its equation of motion. A Fayet-Iliopoulos term is a term in the lagrangian linear in D. This can lead to masses for some particles, and supersymmetry breaking. Such terms arise at one-loop in the case of gauge theories with $U(1)$ factors, in which the $U(1)$ charge has a non-vanishing trace. The resulting expression for the D-term is quadratically divergent. In fact, however, in conventional field theory model building, these terms never arise; if the trace of the $U(1)$ generator is non-vanishing, there is a gravitational anomaly.

In string theory, many classical solutions are known in which supersymmetry is unbroken at tree level. For these,a non-renormalization theorem very similar to that in field theory holds[37]. Again, this theorem insures that supersymmetry remains unbroken and massless particles do not gain mass provided Fayet-Iliopoulos terms are not generated. However, in many string compactifications, the gauge group possesses a $U(1)$ factor with apparent gauge and gravitational anomalies.

† This is called the spacetime supersymmetry charge as opposed to the supersymmetry charge on the world-sheet that is proportional to the fermionic stress tensor T_F.

In particular, the $U(1)$ charge has a non-vanishing trace. As explained in Ref.[28] these anomalies can be cancelled by a Green-Schwarz mechanism. Due to supersymmetry, the counterterms required to cancel the anomalies are accompanied by a Fayet-Iliopoulos D term. In the original vacuum, this term leads to masses for the $U(1)$ gauge boson and the charged scalars at one loop and a cosmological constant at two loops[35]. Explicit string calculations have verified the existence of the scalar[30,29] , and gauge boson[31] masses, and there has been some progress in understanding the two loop dilaton tadpole[35].

It is not true, in general, that this D term leads to a breakdown of supersymmetry. Typically, it is possible to find a new, supersymmetric vacuum by giving small expectation values to some massless fields for which the F and D terms vanish[30,29]. From a stringy point of view, this provides an example of the Fischler-Susskind mechanism[22,23,33]. In world sheet terms, the shift of the scalar field introduces an operator whose tree level (super)-conformal anomaly cancels the one loop anomaly. In chapter 3, this anomaly will be reinterpreted as an anomaly of the BRST symmetry. The arguments of Ref.[28] guarantee that this can be carried out to all orders of the string loop expansion.

In this chapter, we verify the presence of the one loop fermion mass terms. We use both the light-cone Green-Schwarz formalism and the covariant RNS formalism. Apart from completing the picture presented in Ref.[28], this calculation nicely illustrates the role of supersymmetry. For example, earlier arguments that bosonic masses could not be generated in perturbation theory involved manipulations of fermion vertex operators. In this calculation, we can see to what extent these arguments are valid, and where they break down. In the next section, we review the analysis of Ref.[28].

2.2 Effective Theory

In a general supersymmetric compactification there are a variety of massless fields. These include various gauge fields, grouped into vector multiplets, V, matter fields charged under the gauge group, Φ_i, and assorted neutral fields. Among these neutral fields there is always a supermultiplet, Y, consisting of the dilaton, an axion, and a chiral fermion ("dilatino").

In superspace,

$$Y \; = \; \phi^{-2} \; + \; ia \; + \; \sqrt{2}\theta\lambda \; + \; \theta^2 F \quad . \tag{8}$$

Superspace is a generalisation of 4-dimensional spacetime and consists of the superspace coordinates $(x^\mu, \theta_\alpha, \bar{\theta}^{\dot{\alpha}})$. x^μ are the usual spacetime coordinates with θ_α and $\bar{\theta}^{\dot{\alpha}}$ as their supersymmetry counterparts. The latter coordinates are anti-commutating Grassmann variables with spinor indices corresponding to the 2 chiralities, $\alpha, \dot{\alpha} = 1, 2$. The vertex operators for these fields involve only free two dimensional fields; thus at tree level, the Lagrangian for these fields has a universal structure, independent of the details of the internal conformal field theory. In particular, the kinetic term for the field Y is

$$-\int \mathrm{d}^4\theta \ln\,(Y \; + \; Y^*) \quad , \tag{9}$$

and the Y multiplet couples to the gauge fields through

$$\frac{1}{4}(\int \mathrm{d}^2\theta Y W^\alpha W_\alpha \; + \; \int \mathrm{d}^2\bar{\theta} Y^* \bar{W}_{\dot{\alpha}} \bar{W}^{\dot{\alpha}}) \quad , \tag{10}$$

where W_α is the usual supersymmetric gauge field strength. By suitable rescalings of the fields, Φ_i, the kinetic terms for these fields can be brought to the standard form,

$$\int d^4\theta \Phi_i^* e^{2V} \Phi_i \quad . \tag{11}$$

The charged matter fields, Φ_i, are expanded as

$$\Phi_i \ = \ C_i \ + \ \sqrt{2}\theta\Psi_i \ + \ \theta^2 F_i \quad . \tag{12}$$

For modular invariant compactifications, there should be no anomaly. Indeed, as noted in Refs.[28] and [31], it is always possible to cancel the gauge and gravitational anomalies by assigning a non-linear transformation law to the the axion field, a, under the $U(1)$ gauge symmetry. This is essentially the four dimensional version of the Green-Schwarz anomaly cancellation mechanism. Assigning to the axion field a non-linear transformation law under the gauge symmetry corresponds to letting

$$Y \ \rightarrow \ Y \ + \ c\Lambda \ , \tag{13}$$

where Λ is the chiral gauge transformation parameter and c is a constant to be determined. In order that the full Lagrangian be gauge invariant, we must modify the Y kinetic term,

$$-\int d^4\theta \ln \left(Y \ + \ Y^* \ + \ cV\right) \quad . \tag{14}$$

In Wess-Zumino gauge,

$$V \ = \ -\theta\sigma^\mu\bar{\theta}A_\mu \ + \ i(\theta^2\bar{\theta}\bar{\chi} \ - \ \bar{\theta}^2\theta\chi) \ + \ \theta^2\bar{\theta}^2\frac{D}{2} \tag{15}$$

V transforms as $V \ \rightarrow \ V - \Lambda - \Lambda^*$. In terms of the usual gauge parameter $\omega \ = \ i(\lambda \ - \ \lambda^*)$, where λ is the scalar component of Λ , $a \rightarrow a - ic(\lambda - \lambda^*)/2$. The $U(1)^3$ and gravitational anomalies, for example, are given by $-\omega F\tilde{F}\,\mathrm{Tr}\,Q^3/6\times 16\pi^2$ and $\omega R\tilde{R}\,\mathrm{Tr}\,Q/384\pi^2$ (in the first expression there is a factor of six due to Bose symmetry, while the latter is the conventional gravitational anomaly with one external gauge boson line[38]). Since a has couplings[14] $-aF\tilde{F}/4$ and $aR\tilde{R}/4$, for the anomalies to cancel it is necessary that $c = \mathrm{Tr}\, Q^3/12\pi^2 = \mathrm{Tr}\, Q/48\pi^2$. Performing the θ integrals yields the component Lagrangian

$$L = L_{\mathrm{tree}} + L_{\mathrm{loop}}$$

where

$$L_{\mathrm{tree}} = -\frac{1}{4}\phi^{-2}F_{\mu\nu}F^{\mu\nu} - i\phi^2\chi\sigma^\mu\partial_\mu\bar{\chi} - \frac{1}{4}aF_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{1}{2}\phi^{-2}D^2$$

$$- (\frac{\partial_\mu\phi}{\phi})^2 - \frac{1}{4}\phi^4(\partial_\mu a)^2 - \frac{i}{4}\phi^4\lambda\sigma^\mu\partial_\mu\bar{\lambda}$$

$$+ |\mathrm{D}_\mu C_i|^2 + q_i|C_i|^2 D - i\Psi_i\sigma^\mu \mathrm{D}_\mu\bar{\Psi}_i \tag{16}$$

where the covariant derivative $\mathrm{D}_\mu = \partial_\mu + iq_iA_\mu$, and

$$L_{\mathrm{loop}} = -\frac{c}{4}\phi^4 A_\mu\partial^\mu a - \frac{c^2}{16}\phi^4 A_\mu A^\mu - \frac{c}{4}\phi^2 D - \frac{ic}{4\sqrt{2}}\phi^4(\chi\lambda - \bar{\chi}\bar{\lambda}) \ . \tag{17}$$

The auxiliary field, D, can be eliminated by its equation of motion

$$D = \phi^2[-q_i|C_i|^2 + \frac{c}{4}\phi^2] \tag{18}$$

We have divided the Lagrangian into tree and loop effects. One can easily determine at which order in the string perturbation expansion each term should arise as follows. Expanding the dilaton field about its expectation value, $\phi = \phi_0 + \delta\phi$, one can rescale the fields so that there are no factors of ϕ in the kinetic terms.

Then, keeping only terms which are quadratic in fluctuating fields yields

$$L_{\text{tree}} = -\frac{1}{4}F_{\mu\nu}^2 - i\chi\sigma^\mu\partial_\mu\bar{\chi} - \frac{\sqrt{2}}{4}aF_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{D^2}{2}$$

$$-\frac{1}{2}(\partial_\mu a)^2 - i\lambda\sigma^\mu\partial_\mu\bar{\lambda} - |D_\mu C_i|^2 - i\Psi_i\sigma^\mu D_\mu\bar{\Psi}_i$$

and

$$L_{\text{loop}} = -\frac{c\sqrt{2}}{4}\phi_0^3 A_\mu\partial^\mu a - \frac{c^2}{16}\phi_0^6 A_\mu^2 - \frac{c}{4}\phi_0^3 D$$

$$+ q_i\phi_0|C_i|^2 D - \frac{ic}{4\sqrt{2}}\phi_0^3(\chi\lambda - \bar{\chi}\bar{\lambda}) \tag{19}$$

Now

$$D = \frac{c}{4}\phi_0^3 - q_i\phi_0|C_i|^2 \ . \tag{20}$$

In these expressions, the relevant energy scale is at Planck mass which is related to the string scale M_S by, $M_P = \sqrt{2}M_S/\phi_0 = \sqrt{2}/\sqrt{\alpha'}\phi_0$. This relation can be deduced by examining the scaling behaviour of the classical Lagrangian of the low energy spectrum of the string modes. So, since ϕ_0 is the string loop expansion parameter, we see that the scalar and fermion masses, and the axion-gauge boson mixing arise at one loop, while the dilaton tadpole and the gauge boson mass-squared terms arise at two loops.

In the remainder of this chapter, we verify the existence of the fermion mass terms. In section 2.3, we calculate these masses using the low energy field theory, employing a "stringy" cutoff due to Polchinski[39]. For the scalar masses, such a calculation has already been performed in Ref.[29]. In section 2.4, we calculate these masses directly in string theory, using the light-cone Green-Schwarz formalism. Finally in section 2.5, the masses are calculated using the covariant RNS formalism. There are close parallels between the field theoretic and string calculations. In both calculations, there are "supersymmetric" pieces, i.e. pieces which give the same contribution for both bosons and fermions. There are also pieces, proportional to D term tadpoles[40] , which break supersymmetry in the original classical vacuum, and give different contributions to the masses of bosons and fermions in this vacuum. (Of course, in the true supersymmetric vacuum, the masses are equal. As mentioned above, this can be understood as an example of the Fischler-Susskind mechanism). The supersymmetric pieces are related by the contour integral manipulations of the second reference of [37]. The supersymmetry breaking terms are associated with contact interactions, as discussed in Ref.[40]. Thus those pieces of the calculation which are holomorphic respect the arguments of Ref. [26] for unbroken supersymmetry, while those which involve δ-function singularities do not.

2.3 Field Theory Calculation

In this section, we calculate the fermion masses from field theoretic considerations, using Polchinski's stringy cutoff[39]. A similar calculation for scalar masses has been given in Ref.[29]. We review this calculation and give also the calculation for the mixing of the axion and the gauge boson. These calculations have close parallels in the fermion computation. The diagrams for the scalar mass, gauge boson-axion mixing, and gaugino-dilatino mixing are shown in Figs.3 and 4 below, and in Figs.5a and 5b at the end of this section.

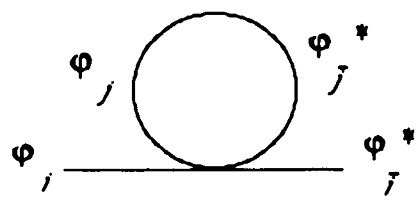

Fig. 3 Charged scalar mass correction

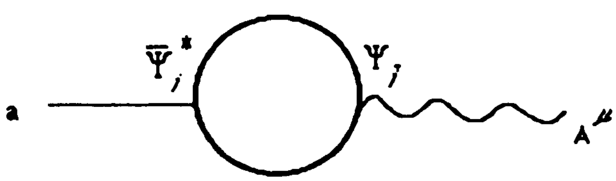

Fig.4 axion - gauge boson coupling

The couplings which appear in these diagrams are universal. For the gauge couplings this is well known. For the couplings of the matter fields to the space-time axion and the dilatino, this is perhaps not quite so obvious. It can be seen directly from the string theory, however. The vertex operators for the space-time axion, dilaton, and dilatino, as well as for the massless gauge bosons, gauginos, and graviton are constructed entirely out of free two dimensional fields (for the case of the dilatino, one uses the right-moving $U(1)$ current in the $N = 2$ supersymmetry multiplet). The vertex operators for the various matter fields ("quarks", "leptons", etc.) involve products of operators in the "internal" conformal field theory, $\hat{O}_i$, times operators built from free fields. Thus the couplings of these fields to the gauge bosons, axions, etc. is determined by the coefficient of the unit operator in the operator product expansion of $\hat{O}_i\hat{O}_j$. This, however, can be absorbed in the normalization of the vertex operators. Thus all of these couplings are universal.

Since the couplings are universal, we can determine them in whatever manner is convenient. We can, for example, evaluate the required operator product expansions directly. Alternatively, we will use the simple dimensional reduction procedure of Witten[41] to determine these couplings. In this approach one simply uses the standard ten-dimensional supergravity action (which is reviewed in Ref.[45]) to determine these couplings. One obtains a theory with $N = 1$ supersymmetry in four dimensional spacetime by truncating a theory with $N = 1$ supersymmetry in ten dimensions. Before the truncation, there is an $SO(6)$ symmetry which acts on the "internal" spacetime indices, and an $E_8 \otimes E_8$ gauge symmetry. $SO(6)$ has an obvious $SU(3)$ subgroup. E_8 contains an $SU(3) \otimes E_6$ symmetry. Witten's procedure is to take all fields to be independent of the internal coordinates, and to drop all fields from the Lagrangian which are not invariant under the direct sum of the two $SU(3)$'s. This yields a four dimensional $N = 1$ supergravity theory, with gauge group E_6 and a single generation, and with two neutral supermultiplets, one associated with the superfield Y of equation (1.1) with precisely the Kähler potential of equation (1.2), and one associated with the radial dilaton. Expanding about a particular expectation value for the two dilatons (ϕ_0, R_0), we can rescale all the fields so as to bring their kinetic terms to the standard form. The relevant terms in the ten dimensional action are[45]

1. For the kinetic terms of the various fields, as well as auxiliary D^2 terms and the Yukawa couplings of the gauge and matter fields:

$$\begin{aligned} L_1 \;=\; & \frac{-1}{4\tilde{\phi}^2} F^a_{MN} F^{MNa} \;-\; \frac{1}{2}\bar{\chi}^a \Gamma^M \mathrm{D}_M \chi^a \;-\; 4\left(\frac{\partial_M \tilde{\phi}}{\tilde{\phi}}\right)^2 \\ & -\frac{3}{8\tilde{\phi}^4} H^2_{MNP} \;-\; \frac{1}{2}\bar{\lambda}\Gamma^M \mathrm{D}_M \lambda \quad , \end{aligned} \qquad (21)$$

where we have set the ten dimensional gravitational coupling to one and have absorbed the ten dimensional gauge coupling into the definition of $\tilde{\phi}$. $\tilde{\phi}$ is the ten dimensional dilaton; the four dimensional field, ϕ, appearing in eqn.(1.1), is given by $\phi = \tilde{\phi} R^{-3/2}$. In making the reduction to four dimensions, one divides the ten-dimensional Minkowski index into the ordinary Minkowski indices, μ, $\nu = 0$, 1, 2, 3 and six internal indices (which it is convenient to take complex), $i, \bar{i} = 1,2,3$. The coordinates of M^4 will be denoted by x^μ. Then the metric with internal indices becomes $g_{i\bar{j}} = \delta_{i\bar{j}} R(x)$, and the $SU(3)$ singlet piece of the gauge bosons is (a is an E_6 index in the **27** representation) $A^{a\bar{i}}_{i} = C^a(x)$. The internal components of the antisymmetric tensor become $b_{i\bar{j}} = b(x)\epsilon_{i\bar{j}}$. The other $SU(3)$ bosonic fields which are $SU(3)$ singlets have no internal indices: the graviton, the field ϕ, the space-time axion (related by a duality transformation to $B_{\mu\nu}$) and the E_6 gauge bosons. All other fields are simply erased from the Lagrangian. For our purposes, the field R may simply be frozen at some value, (e.g. 1), and the field b may be ignored.

2. For the coupling of the axion to the matter fields:

$$L_2 = \frac{1}{16\tilde{\phi}^2}\bar{\chi}^a \Gamma^{MNP} \chi^a H_{MNP} \quad . \tag{22}$$

3. For the coupling of the dilatino to the matter fields:

$$L_3 = -\frac{\sqrt{2}}{8\tilde{\phi}}\bar{\chi}^a \Gamma^{MN} \lambda F^a_{MN} \quad . \tag{23}$$

In reducing the fermions, we can think of the $SO(1,9)$ spinors as tensor products of $SO(1,3)$ and $SO(6)$ spinors. $SO(6)$ is isomorphic to $SU(4)$; the spinor of $SO(6)$ decomposes as a **4** and a $\bar{\mathbf{4}}$. Under $SU(3)$, $\mathbf{4} = \mathbf{3} + \mathbf{1}$; in Witten's reduction, we keep only the singlets. The Γ matrices in 10-dimensions are 32 by 32 dimensional matrices and can be taken to be direct products of gamma matrices in 4-dimensional spacetime and those corresponding to the compactified internal manifold :

$$\Gamma^\mu = \gamma^\mu \otimes 1 \quad , \qquad \Gamma^k = \gamma_5 \otimes \gamma^k \ , \tag{24}$$

where γ^μ are the usual four dimensional gamma matrices, and γ^k are $SO(6)$ gamma matrices. These latter matrices are conveniently constructed, for example, using creation and annihilation operators. In a complex basis, in particular, the six matrices γ^i, $\gamma^{\bar{j}}$ obey the anticommutation relation, $\{\gamma^i, \gamma^{\bar{j}}\} = 2g^{i\bar{j}}$.

Performing the reduction and the necessary rescalings, the relevant couplings turn out to be:

1. axion - two charged fermions

$$-\frac{\sqrt{2}}{4}\partial_\mu a \Psi_i \sigma^\mu \bar{\Psi}_{\bar{i}} \quad , \tag{25}$$

where a is the axion field, ϕ_i is the i'th charged scalar field, and Ψ_i is its fermionic partner, and we have written the result in two component notation.

2. dilatino - charged scalar - charged fermion

$$\frac{\sqrt{2}}{4}(\Psi_i \sigma^\mu \bar{\lambda} \partial_\mu \phi_{\bar{i}} \ + \ \lambda \sigma^\mu \bar{\Psi}_{\bar{i}} \partial_\mu \phi_i) \ , \tag{26}$$

where λ is the dilatino field.

3. gaugino - charged scalar - charged fermion

$$i\phi \Psi_i \chi^a \phi_j T^a_{ij} \ , \tag{27}$$

where T^a are the matrix generators of the gauge group.

4. dilatino - gaugino - two charged scalars

$$\frac{i\sqrt{2}}{8}(\chi^a \lambda \ - \ \bar{\chi}^a \bar{\lambda}) D^a \quad , \tag{28}$$

where D^a is the auxiliary field of the gauge supermultiplet. The other couplings needed in these diagrams, which involve the gauge fields are conventional; note that the gauge coupling constant is $g = \phi_0$.

It is now straightforward to write down the Feynman integrals for each of these cases. However, each of the integrals is divergent. To cure these divergences, we follow Polchinski, and write

$$\int \frac{d^4p}{p^2} = \frac{\pi}{\alpha'} \int \frac{d^2\tau}{\tau_2^2} = \frac{\phi^2\pi}{2} \int \frac{d^2\tau}{\tau_2^2} M_P^2 \tag{29}$$

Now we interpret τ as the modular parameter of the torus in string theory. Restricting the integration to the fundamental region gets rid of the divergence as $\tau_2 \to 0$, and yields a finite result. In this way one finds, for the mass of the i'th scalar[29],

$$m_i^2 = \frac{\phi^2}{48\pi^2} q_i \,\mathrm{Tr}\, Q\ M_S^2 \quad , \tag{30}$$

where $M_S = 1/\sqrt{\alpha'}$, and the trace sums over the $U(1)$ charges of the chiral multiplets. For the axion-photon mixing, the coefficient of $\partial_\mu a A^\mu$, we find

$$m_{a-A} = \frac{\phi^2}{2 \times 48\pi^2} \mathrm{Tr}\, Q\ M_S \quad , \tag{31}$$

while the dilatino-gaugino mass term is given by

$$m_{\chi-\lambda} = -\frac{\sqrt{2}\phi^2}{48\pi^2}\left(\frac{\sqrt{2}}{8} - \frac{\sqrt{2}}{4}\right) \mathrm{Tr}\, Q\ M_S \quad . \tag{32}$$

Here, the mass scale of the original 10-dimensional Lagrangian, M_P, has been replaced by the string mass scale, M_S. We have indicated here the contributions of the diagrams in Figs.5a and 5b, respectively.

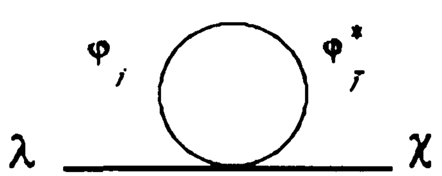

Fig.5a dilatino - gaugino - D term coupling

The first can be interpreted as a contribution to the mass from the expectation value of the D term. This corresponds to Fig.5a.

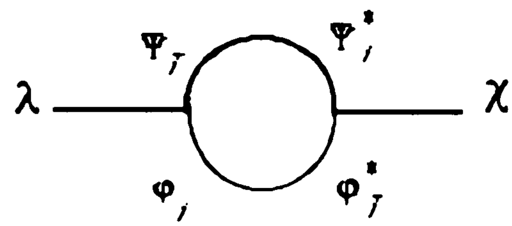

Fig.5b dilatino - gaugino coupling

The second is identical to the axion-photon mixing term, and can be thought of as the supersymmetric counterpart to this term, i.e. Figs.4 and 5b are supersymmetric counterparts. These masses agree with those obtained from the effective lagrangian. We will see that the same structure arises in the full string computation.

2.4 Light Cone Gauge Calculation

In this section, we describe the calculation of the fermion masses directly in string theory, using the light cone gauge. It is helpful to first review the calculation of the scalar masses (note that the calculation of the gauge boson-axion mixing is similar to the calculation of anomalies in that it involves an epsilon tensor. It is thus difficult to perform in light cone gauge). It is simplest to discuss the light cone gauge calculation either in weakly coupled σ-models or in free theories such as orbifolds. For the discussion of this section, we limit ourself to these cases. For definiteness, we consider in this section the spin(32)/Z_2 heterotic string compactified on a Calabi-Yau space at large radius (so that the sigma model approach makes sense) or on a left-right symmetric orbifold, such as the "Z orbifold". More general cases can be analyzed using the methods of Ref. 42. For the sigma-model or orbifold, it is natural to decompose the spinor fields, S, of Green and Schwarz as a **3**, S^{i}, a $\bar{\mathbf{3}}$, $S^{\bar{i}}$, and two singlets, S^{o} and $S^{\hat{o}}$ under the internal $SU(3)$. Similarly, the left moving fermions of the fermionic formulation of the heterotic string decompose as λ^i, $\lambda^{\bar{i}}$, and λ^a, where a is an $SO(26)$ index of the manifest $SO(26)$ gauge symmetry.

The calculation of the scalar masses in light cone gauge is relatively simple. The vertex operators for the charged scalar fields have the form

$$V_\phi \;=\; \frac{g}{2\pi} b_{i\bar{j}} \lambda^a \lambda^i (\partial X^{\bar{j}} \;-\; i S^o S^{\bar{j}} k^-) e^{ik\cdot X} \tag{33}$$

For the CPT conjugate fields, one has

$$V_{\phi^*} \;=\; \frac{g}{2\pi}\; b_{\bar{i}j} \lambda^a \lambda^{\bar{i}} (\partial X^j \;+\; i S^{\bar{o}} S^j k^+) e^{ik\cdot X} \tag{34}$$

Here, $k^\pm \;=\; k^1 \pm i k^2$. Only terms in the vertex operators involving the fields S^o and $S^{\bar{o}}$ have been retained here. Note we have been careful here about the normalizations. In particular, the wave functions $b_{i\bar{j}}(y)$ are normalized to unity, $\int \mathrm{d}^6 y \sqrt{g(y)} b_{i\bar{i}} b_{j\bar{j}} g^{i\bar{j}} g^{\bar{i}j} \;=\; 1$. These normalizations can be checked, for example, by studying the factorization of four point functions on massless states. The vertex operators are separately normalised to have unit coefficient for the most divergent term $1/|z \;-\; w|^4$ in their operator product expansions. The scalar mass is obtained by examining the two point function,

$$\int \mathrm{d}^2\tau \mathrm{d}^2 z \mathrm{d}^2 w \;\; \mathrm{Tr}\; [V_{\phi_i}(z,\bar{z}) V_{\phi_{\bar{i}}}(w,\bar{w}) \; q^{L_o} \bar{q}^{L_o - 1}]$$

$$= \int \mathrm{d}^2\tau \mathrm{d}^2 z \;\; \mathrm{Tr}[V_{\phi_i}(z,\bar{z}) V_{\phi_{\bar{i}}}(0) \; q^{L_o} \bar{q}^{L_o - 1}] \tag{35}$$

where $q \;=\; \exp(2\pi i \tau)$. The V's are the vertex operators on the torus. In light cone gauge, we will use vertex operators involving the transverse coordinates only. The argument of the second vertex operator has been set to 0 using the translational symmetry of the torus. On the torus, the fields S^o and $S^{\bar{o}}$ each have a zero mode. Thus to obtain a non-vanishing result for the scalar mass, one must take the pieces bilinear in these S's for each vertex operator. Since these involve factors of momenta, one might imagine that they could not contribute to the mass. However, care is required; the integrations over the locations of the vertex operators can yield factors of $1/k^2$. These can arise only when the two vertex operators are very close together. In this limit, we can use the operator product expansion to isolate the pole :

$$S^i(z)S^j(w) \ = \ \frac{-g^{ij}}{z \ - \ w} \ + \ O(1)$$

$$\lambda^a(\bar{z})\lambda^b(\bar{w}) \ = \ \frac{-\delta^{ab}}{\bar{z} \ - \ \bar{w}} \ + \ O(1)$$

$$g^{ji}b_{j\bar{j}}\lambda^{\bar{j}}(\bar{z})b_{ik}\lambda^k(\bar{w}) \ = \ q_i j_{\rm L}(\bar{w}) \ + \ O(\bar{z} \ - \ w) \tag{36}$$

where $j_{\rm L}(\bar{w})$ is the $U(1)$ gauge current, $j_{\rm L}(\bar{w}) \ = \ g_{i\bar{j}}\lambda^i\lambda^{\bar{j}}/\sqrt{3}$. The factor of $\sqrt{3}$ has been included to give unit coefficient for the central term in the Kac-Moody algebra, for the gauge boson vertex operator. Thus we obtain

$$\frac{g^2}{4pi^2\Omega_1} q_i \int d^2 z \frac{\delta^{ab} k^{i2}}{|z|^2} \chi(z, \ \bar{z})^{2k\cdot k'} \ {\rm Tr} \ [j_L S^o S^{\bar{o}} q^{L_o} \bar{q}^{L_o - 1}] \tag{37}$$

where χ arises from the correlation function of the $exp(ik \cdot X)$ factors on the torus. The normalisation constant for 1 loop amplitudes is $\Omega_1 \ = \ i/\pi^2$. This can be found by inserting a correctly normalised propagator into a tree amplitude with the normalisation for the sphere and summing over the same asymptotic in and out external states including a sum over zero modes, the loop momenta, resulting in the trace. One now takes the limit $z \rightarrow 0$ so that $\chi \rightarrow |z|$. It is only after the factors of k^2 cancel that the second limit $k \cdot k' \ = \ -k^2 \rightarrow 0$ is allowed.

As explained in Ref. 4, the trace receives contributions only from massless states, from which one obtains ${\rm Tr}\, Q$, as in field theory. In the light cone gauge, there are two spacetime supersymmetry charges proportional to the zero modes of S^o and $S^{\bar{o}}$. Explicitly, $\tilde{Q}^o \ = \ \sqrt{2p^+}S^o$ and $\tilde{Q}^{\bar{o}} \ = \ \sqrt{2p^+}S^{\bar{o}}$. There are two which are non-trivial, Q^o and $Q^{\bar{o}}$. These operators are each a sum of a term from the internal conformal theory and a piece involving the two transverse coordinates in four dimensions. It is convenient to subtract from each of these operators the term involving the zero mode of S^o or $S^{\dot{o}}$. Call the resulting operators $\hat{Q}^o$ and $\hat{Q}^{\dot{o}}$. These operators satisfy the commutation relations $\{\hat{Q}^o,\ \hat{Q}^{\dot{o}}\} = 2M^2/p^+$, where M^2 is the mass squared operator, and commute with L_o and $\bar{L}_o$. Massless states are annihilated by the $\hat{Q}$'s. Massive states are not. For a given mass level, we can consider the state, $|\psi\rangle$, annihilated by S^o and $\hat{Q}^o$. Then there are three other states at this mass level, obtained by acting with $S^{\dot{o}}$ and $Q^{\dot{o}}$. It is easy to see that the trace of $S^oS^{\dot{o}}$ vanishes on these four states. To obtain the trace on massless states, consider for simplicity the case of the $SO(32)$ theory. The spectrum for massless modes consists of fields in the **26** of $SO(26)$,

$$\frac{1}{\sqrt{3}} g_{ij} \lambda^a_{-1/2} \lambda^i_{-1/2} |0\rangle_{\rm L} \otimes |\bar{j}\rangle_{\rm R}$$

and $SO(26)$ singlets,

$$\frac{1}{\sqrt{3!}} \epsilon_{ijk} \lambda^i_{-1/2} \lambda^j_{-1/2} |0\rangle_{\rm L} \otimes |\bar{k}\rangle_{\rm R} \qquad (38)$$

The right moving vacuum is defined so that it is annihilated by S^o. The fermionic counterparts of the above bosonic fields have right movers replaced by $S^{\dot{o}}|\bar{i}\rangle_{\rm R}$. The trace in eqn. (3.7) picks up only the bosonic zero modes. The **26** can be seen to have charge $1/\sqrt{3}$ and the singlets $-2/\sqrt{3}$ under the action of the $U(1)$ gauge current, $j_{\rm L}$. Then $\mathrm{Tr}\, Q = 24/\sqrt{3}$ for each generation. Integration over loop momentum, implicit in the trace in (3.7), gives a factor of $4/\tau_2^2$. Thus, the final result is as in the field theory calculation

$$\frac{q_i g^2}{16 p i^3 \alpha'} \int \frac{d^2\tau}{\tau_2^2} \mathrm{Tr}\, [j_{\rm L} S^o S^{\dot{o}}] = \frac{q_i g^2}{48\pi^2} \mathrm{Tr}\, Q\ M_S^2 \qquad (39)$$

This agrees with the expectations described in the introduction, as well as with the field theory calculation, and the results of Ref.[29].

The calculation of the fermion masses introduces certain new features. The scalar masses were computed by studying a two point function on shell (or nearly on shell). We would like to do the same thing for the fermions. However, even if a mass is generated, the two point function vanishes, using the Dirac equation for the external massless fermions. This is a consequence of momentum and angular momentum conservation. This problem can be avoided by inserting a soft dilaton in the diagram. A similar problem arises in the covariant computation of the axion-gauge boson mixing. Since the algebra is a bit simpler in that case, we postpone a detailed discussion of this problem to the next section. Suffice it to say that the upshot of this analysis is that one can compute the mass from the two point function, ignoring the fact that the final product of the external spinors vanishes. Thus we need to study

$$\int \mathrm{d}^2\tau \mathrm{d}^2 z \mathrm{d}^2 w \;\; \mathrm{Tr}[V_\lambda(z,\bar{z}) \; V_\chi(w,\bar{w}) \; q^{L_o}\bar{q}^{L_o - 1}] \tag{40}$$

where λ denotes the dilatino and χ the gaugino. Before writing down the vertex operators, we need to discuss the external spinors. These again have a natural decomposition as triplets and singlets of the internal $SU(3)$. In particular, the dilatinos and gauginos are singlets of this $SU(3)$. We denote the 2 components of opposite 4-dimensional chiralities as λ^o , $\lambda^{\bar{o}}$ for the dilatino, and χ^o, $\chi^{\bar{o}}$ for the gaugino. Then the dilatino vertex operator is given by

$$\begin{aligned} V_\lambda(z,\bar{z}) \;=\; & \frac{g\sqrt{2}}{8\pi}\frac{i}{2} \, [\sqrt{p^+/2}\{\bar{\partial}X^-(\bar{z})S^o\lambda^{\bar{o}} \;-\; \bar{\partial}X^+(\bar{z})S^{\bar{o}}\lambda^o\} \\ & + \frac{1}{\sqrt{2p^+}}\{S^{\bar{o}}\lambda^o\bar{\partial}X^-(\bar{z})\partial X^+(z) \;-\; S^o\lambda^{\bar{o}}\bar{\partial}X^+(\bar{z})\partial X^-(z) \\ & + \frac{i}{2}(S^{\bar{o}}\lambda^o S^o S^{\bar{o}} k^+\bar{\partial}X^-(\bar{z}) \;+\; S^o\lambda^{\bar{o}}S^{\bar{o}}S^o k^-\bar{\partial}X^+(\bar{z}))\}]\mathrm{e}^{ik\cdot X(z,\bar{z})} \quad . \end{aligned} \tag{41}$$

The gravitational coupling is the same as the gauge coupling constant g except for a factor of $\sqrt{2}/4$. Note that the dilatino vertex is similar to the gravitino vertex operator, except that the external spinor, $U^i = \Gamma^i\lambda$, does not satisfy $\Gamma^i U^i = 0$. The gaugino vertex operator is given by

$$V_\chi(w,\bar{w}) = \frac{g}{2\pi} j_L(\bar{w})\,[\sqrt{p^+/2}\{-S^{\dot{o}}\chi^o + S^o\chi^{\dot{o}}\} + \frac{1}{\sqrt{2p^+}}\{S^{\dot{o}}\chi^o\partial X^+(w)$$

$$- S^o\chi^{\dot{o}}\partial X^-(w) + \frac{i}{2}(S^{\dot{o}}\chi^o S^o S^{\dot{o}} k'^+ + S^o\chi^{\dot{o}} S^{\dot{o}} S^o)k'^-\}\,]\,e^{ik'\cdot X(w,\bar{w})} \quad , \qquad (42)$$

where $\bar{\partial}X^\pm = \bar{\partial}X^1 \pm \bar{\partial}X^2$, not to be confused with light cone coordinates. To derive these expressions, one can proceed directly using the four dimensional supersymmetries. Alternatively, one can start with the ten dimensional expressions for the vertex operators, and use the explicit form of the external wave functions corresponding to the various states.In this case, it is helpful to construct the basis for the Dirac matrices by writing these as creation and annihilation operators. There are three pairs of such operators, b^i and $\bar{b}^i$ associated with the internal coordinates, and one associated with the transverse coordinates, a and a^*. S, χ and λ can then be expanded in this basis, and it is a simple matter to obtain the expressions in eqns. (3.10) and (3.11). Here we have dropped terms which do not contain S^0 or which contain too many powers of momentum to contribute to the final result.

In order to obtain a non-vanishing result from the p^+ integration, it is necessary to have no overall factor of p^+, i.e. to take one factor of $\sqrt{p^+}$ and one of $1/\sqrt{p^+}$. Thus we need to evaluate

$$\int d^2\tau d^2z \;\text{Tr}\;[V_\lambda(z,\bar{z})\; V_\chi(0) q^{L_o}\bar{q}^{L_o-1}]$$

$$= \frac{ig^2\sqrt{2}}{64pi^2\Omega_1}\int d^2\tau d^2z\;(\chi^o\lambda^{\dot{o}} - \chi^{\dot{o}}\lambda^o)\lim_{w\to 0}\text{Tr}\;[j_L S^{\dot{o}} S^o$$

$$\times\{\bar{\partial}X^i(\bar{z})\partial X^i(z) + \bar{\partial}X^i(\bar{z})\partial X^i(w)\}\; e^{ik\cdot X(z,\bar{z})}\; e^{ik'\cdot X(w,\bar{w})} q^{L_o}\bar{q}^{L_o-1}] \qquad (43)$$

From the S's in this expression, we have taken only the zero modes. The ∂X terms can either be contracted with the exponential factors or with each other.

The contraction with the exponentials yields a result essentially identical to the scalar mass calculation above. One obtains explicit factors of external momenta giving k^2, which are cancelled by a factor of $1/k^2$ coming from the integration over z. The contraction of these terms with each other yields, essentially a factor of the transverse integration momentum, p^{i2}. We may obtain the result by differentiating the expression for the scalar propagator on the torus :

$$\frac{ig^2\sqrt{2}}{64\pi^2\Omega_1}(\bar{\chi}\bar{\lambda} - \chi\lambda)\int \mathrm{d}^2\tau \mathrm{d}^2 z \ \mathrm{Tr}\ [j_L S^{\hat{o}} S^o] \lim_{w\to 0}(\partial_{\bar{z}}\partial_z\langle X^i(z)X^i(z)\rangle_{\text{torus}}$$

$$+ \partial_{\bar{z}}\partial_w\langle X^i(\bar{z})X^i(w)\rangle_{\text{torus}} - \frac{2k^{i\,2}}{|z - w|^2})\chi(z,\bar{z})^{2k\cdot k'} \tag{44}$$

where $\langle X^i(z,\bar{z})X^j(w,\bar{w})\rangle_{\text{torus}} = -2\delta^{ij}\ln\chi(z - w, \bar{z} - \bar{w})$, is the scalar propagator on the torus, and in the second term one allows $z \to 0$. One notes that

$$\partial_{\bar{z}}\partial_w\langle X^i(\bar{z})X^i(w)\rangle_{\text{torus}} = -\frac{2\pi}{\tau_2} \tag{45}$$

Thus eqn. (3.13) is

$$\frac{g^2\sqrt{2}}{16}(\bar{\chi}\bar{\lambda} - \chi\lambda)\int\frac{\mathrm{d}^2\tau}{(2\pi)^4\tau_2^2}\mathrm{Tr}\ [j_L^o S^{\hat{o}}S^o]\int \mathrm{d}^2 z(-\frac{4\pi}{\tau_2} - \frac{2k^{i2}}{|z|^{2-2k\cdot k'}}) \tag{46}$$

Again we use the fact that only massless modes contribute and in the second term take the limit as $z \to 0$ before taking $k\cdot k' \to 0$. These two contributions are very similar to the two contributions in the field theory case. Putting all of these factors together we obtain

$$\frac{ig^2\sqrt{2}}{16\pi^4}\mathrm{Tr}\ [j_L^o S^{\hat{o}}S^o](\bar{\chi}\bar{\lambda} - \chi\lambda)\int\frac{\mathrm{d}^2\tau}{\tau_2^2}\frac{1}{16}[8\pi - 4\pi] \tag{47}$$

The trace is precisely the one encountered in the scalar mass computation. The integration over τ yields finally

$$m_{\mathrm{F}} = \frac{g^2}{4\times 48\pi^2}\mathrm{Tr}\, QM_{\mathrm{S}} \tag{48}$$

2.5 Covariant Computation

In this section, we compute the fermion mass using the covariant RNS formulation. It will be helpful to review briefly the covariant calculation of the scalar masses[29]. For simplicity again we restrict ourselves to the spin(32)/Z_2 examples. In this case, we divide the right-moving fermions, ψ^M, into internal fermions, $\psi^i, \psi^{\bar{i}}$, and free fermions associated with M^4, ψ^μ. The vertex operators for the scalars (again in the **26** of the unbroken $SO(26)$) have the form

$$V_{\phi_i} \;=\; \frac{g}{2\pi} b_{i\bar{j}} \lambda^a \lambda^i (\partial X^{\bar{j}} \;+\; ik \cdot \psi\psi^{\bar{j}})\; e^{ik \cdot X} \tag{49}$$

while those for their CPT conjugates have the form

$$V_{\phi^{\bar{i}}} \;=\; \frac{g}{2\pi} b_{\bar{i}j} \lambda^a \lambda^{\bar{i}} (\partial X^j \;-\; ik \cdot \psi\psi^j)\; e^{-ik \cdot X} \tag{50}$$

The scalar masses are then obtained by studying the two point function,

$$\int d^2\tau d^2z d^2w \;\; \mathrm{Tr}\; [V_{\phi_i}(z,\; \bar{z})\; V_{\phi^{\bar{i}}}(w,\; \bar{w})\; q^{L_0} \bar{q}^{L_0}] \tag{51}$$

on the torus with a sum over spin structures. Insertions of ghosts and antighosts, c and b, needed to soak up the ghost zero modes but with ghost charge conserved on the torus are implicit. Now there are only fermion zero modes for the PP spin structure, so it is not quite as simple to evaluate the trace as in the computation with Green-Schwarz fermions. However, there are many important simplifications here as well. The term quadratic in ∂X 's vanishes after the sum over spin structures[30],[29]. Cross terms involving only one ∂X vanish because they are proportional to $\langle \psi^\mu \rangle \;=\; 0$. Thus one has to look at the terms in the vertex operators containing two right moving fermions. Again, due to the explicit factors of momenta in these operators, one must look for factors of $1/k^2$ arising from short distance singularities. In this way, the expression for the mass simplifies

$$\frac{g^2}{4\pi^2\Omega_1}q_i \int \mathrm{d}^2\tau \mathrm{d}^2 z \frac{k^2}{|z|^2 + 2k^2} \mathrm{Tr}\ [j_{\mathrm{L}} j_{\mathrm{R}}\ q^{L_\circ}\bar{q}^{L_\circ}] \quad , \tag{52}$$

where j_{R} is the right moving current in the $N\ =\ 2$ superconformal algebra, and j_{L} is the left moving $U(1)$ gauge current,

$$j_{\mathrm{R}}\ =\ g_{i\bar{i}}\psi^i\psi^{\bar{i}} \qquad\qquad j_{\mathrm{L}}\ =\ \frac{g_{i\bar{i}}}{\sqrt{3}}\lambda^i\lambda^{\bar{i}} \quad . \tag{53}$$

In expressions (4.1) and (4.2), vertex operators are normalised to create properly normalised states. As in Ref. [29], g has been defined so that it can be identified with the physical gauge coupling.

One can show[30],[29] that only massless states contribute to the trace. Thus the momentum integral is readily performed, giving

$$m_i^2\ =\ \frac{g^2}{16\pi^3}q_i \int \frac{\mathrm{d}^2\tau}{\tau_2^2} \mathrm{Tr}\ [j_{\mathrm{L}} j_{\mathrm{R}}]\ M_{\mathrm{S}}^2 \quad , \tag{54}$$

where now the trace does not include the integral over momenta. It can be shown that this expression gives the same result as we found in the field theory and light cone calculations[30],[29].

It will be instructive also to briefly review the calculation of the axion-gauge boson[31] mixing. This calculation is most conveniently performed with the vertex operators in the 0 and -1 ghost number pictures. In the 0 ghost picture the vertex operator for the axion ($=\ B_{\mu\nu}$) is

$$V_a\ =\ \frac{g\sqrt{2}}{8\pi}\epsilon_{\mu\nu}\bar{\partial}X^\mu\ (\ \partial X^\nu\ +\ ik\cdot\ \psi\psi^\nu\ )\ \mathrm{e}^{ik\cdot X} \quad , \tag{55}$$

where the polarisation tensor, $\epsilon_{\mu\nu}$, is antisymmetric. The vertex operator for the gauge boson is

$$V_A\ =\ \frac{g}{2\pi}\ \zeta_\mu j_{\mathrm{L}}\ \mathrm{e}^{-\phi}\psi^\mu \mathrm{e}^{ik'\cdot X} \quad , \tag{56}$$

where ϕ is the bosonised superconformal ghost.

We seek the mixing term, $A_\mu \partial^\mu a$. Note that this vanishes on shell; as in the fermion calculation above, it is in principle necessary to insert a low momentum dilaton and consider 3 point amplitudes to obtain a non-vanishing result. The point is that the effective Lagrangian of eqns.(16)–(20) implies a coupling of λ, χ, and the dilaton, does not vanish provided the dilaton carries some momentum. So we insert the dilaton vertex operator

$$V_{\text{dilaton}} = \frac{1}{2} g_{\mu\nu}\, \bar{\partial} X^\mu \left(\partial X^\nu + i\epsilon \cdot \psi\psi^\nu \right) e^{i\epsilon \cdot X} \quad , \tag{57}$$

with external momentum ϵ into the three point function,

$$\int d^2\tau d^2 z_1 d^2 z_2 d^2 z_3 \,\mathrm{Tr}\, [V_1(z_1, k)\, V_2(z_2, k')\, V_{\text{dilaton}}(z_3, \epsilon)\, q^{L_o} \bar{q}^{\bar{L}_o}]$$

$$= \int d^2\tau d^2 z_1 d^2 z_2 \,\mathrm{Tr}\, [V_1(z_1, k)\, V_2(z_2\, ,\, k')\, \frac{1}{2} \bar{\partial} X_\mu (\partial X^\mu + i\epsilon \cdot \psi\psi^\mu)\, e^{i\epsilon \cdot X} q^{L_o} \bar{q}^{\bar{L}_o}] \ . \tag{58}$$

Here translational invariance on the torus has been used to put the dilaton vertex operator at the origin. There are a variety of contributions in the limit of very small ϵ. All are proportional to the naive two point function of the axion and the gauge boson, except that the external wave functions of these particles now carry momenta which differ by ϵ . For example, there is a contribution where the $\bar{\partial} X^\mu$ and ∂X^ν terms in the vertex operator are contracted with the exponential factor in V_1 or V_2 . This gives a factor of $k \cdot k'$, which is cancelled by a similar factor after the integration over z_1 or z_2 . Similar terms arise if the $\epsilon \cdot \psi\psi^\mu$ term is contracted with terms in V_1 or V_2. There are also terms where the holomorphic and antiholomorphic derivatives of X^μ are contracted with each other. These again yield results proportional to the two point function. One can check that the overall coefficient is that expected from the effective Lagrangian.

The only piece of the two point function of the axion and the gauge boson containing the parity violating epsilon tensor in 4 dimensions comes from the PP sector of the right movers

$$\int d^2\tau d^2z d^2w \ \mathrm{Tr}\ [V_a(z,\ \bar{z})\ V_A(w,\bar{w})\ T_{\mathrm{F}}(z_0)\ q^{L_o}\bar{q}^{L_o}\ ]_{\mathrm{PP}} \quad , \tag{59}$$

where T_{F} is the two dimensional supersymmetry current. In this sector, there are four fermion zero modes, so we must again use the pieces of the right moving vertex operators to soak them up. In particular, we note that the relevant part of T_{F} is $\exp(\phi)\psi^\mu\partial X_\mu$ and use the identity

$$\langle\ \hat{O}\psi^\mu\psi^\nu\psi^\rho\psi^\sigma\ \rangle_{\mathrm{PP}} \ = \ \frac{i}{4}\epsilon^{\mu\nu\rho\sigma}\langle\ \hat{O}\gamma_5\ \rangle_{\mathrm{PP}} \quad , \tag{60}$$

for an operator $\hat{O}$ not containing further ψ 's (in our conventions, $\psi_o^\mu \ = \ \gamma^\mu/\sqrt{2}$). To obtain a non-vanishing result, it is necessary to contract ∂X^μ of the gauge boson vertex with $\bar{\partial}X^\nu$ from the axion vertex operator. This corresponds to integration of loop momentum in the field theory calculation, and in the light cone calculation is equivalent to the supersymmetric contribution to the fermion mass.

Massive modes do not contribute to the trace. To see this, we can first ignore the bosonic and fermionic zero modes associated with the non-compact directions. Then examine the supersymmetry operator, G_o in the Ramond sector, constructed from T_{F}. We can split this operator into three pieces, one referring to the ghosts, one to (say) the 0 and 1 directions in the uncompactified dimensions, and the third referring to the rest (the 3 and 4 directions and the compactified dimensions). This last piece is essentially the light cone supersymmetry generator. We will call these G_o^{gh}, G_o^{ext} and G_o^{lc}, respectively. The trace over the non-zero modes factorizes into a product of three terms, again corresponding to the ghosts, the "external" coordinates and the light cone piece. G_o^{lc} satisfies

$$(G_o^{\mathrm{lc}})^2 \ = \ \hat{L}_o \ - \ \frac{1}{2} \ . \tag{61}$$

Here $\hat{L}_o$ denotes the light cone Virasoro generator with the zero modes (momenta) removed. Now in the trace over the light cone variables, since G_o^{lc} commutes with

$\hat{L}_o$ but anticommutes with $(-1)^F$, states which are not annihilated by G_o^{lc} cancel pairwise. States annihilated by G_o^{lc} are massless, since they have $\hat{L}_o = 1/2$. To evaluate the trace over massless states, we can use the following trick. We can add to this trace the term arising from the AP sector, since this term vanishes. This gives the usual GSO projector. The remaining trace over physical massless states is easily evaluated, $\mathrm{Tr}\,[j_L\,\gamma_5]_{PP} = -4\,\mathrm{Tr}\,Q$.

After performing the momentum integral,

$$-\frac{g^2\sqrt{2}}{4\pi^2\Omega_1} i k_\mu \zeta_\sigma \epsilon_{\rho\nu}\epsilon^{\mu\nu\rho\sigma}\ \mathrm{Tr}\,[j_L]_{PP} \int \frac{d^2\tau}{(2\pi)^4\tau_2^2} \int d^2 z(-\frac{\pi}{\tau_2}) \ . \tag{62}$$

With the duality relation $\epsilon^{\mu\nu\rho\sigma}k_\nu\epsilon_{\rho\sigma} = ik^\mu$ (from the relation $*dB = da$), and remaining off-shell, one finds that the axion-gauge boson coupling is

$$-\frac{g^2\sqrt{2}}{2\times 16\pi^2} k_\mu\zeta^\mu\ \mathrm{Tr}\,Q \int \frac{d^2\tau}{\tau_2^2} = -\frac{g^2}{2\times 48\pi^2} k_\mu \zeta^\mu\ \mathrm{Tr}\ Q\ M_S \ , \tag{63}$$

the same result as that obtained from the field theory argument[31].

Now let us consider the fermion mass. In theories with $N = 1$ supersymmetry in spacetime, the internal conformal field theory always possesses a right moving $N = 2$ superconformal invariance[8]. This symmetry implies the existence of a conserved right-moving $U(1)$ current, j_R, which can be written as the derivative of a free boson, $j_R = i\partial H$. This boson is normalized so that, on the plane, $\langle H(z)H(w)\rangle = -3\ln(z - w)$. The four free fermions, ψ^μ, can also be bosonized in terms of two free bosons, ϕ_1 and ϕ_2 . The space-time supersymmetry generators are then integrals of four currents, J and $\bar{J}$, given by

$$J_\alpha = e^{-\frac{\phi}{2}} S_\alpha e^{\frac{1}{2}H} = e^{-\frac{\phi}{2}} e^{\pm\frac{1}{2}(\phi_1 + \phi_2) + \frac{1}{2}H}$$

$$\bar{J}^{\dot{\alpha}} = e^{-\frac{\phi}{2}} \bar{S}^{\dot{\alpha}} e^{-\frac{1}{2}H} = e^{-\frac{\phi}{2}} e^{\pm\frac{1}{2}(-\phi_1 + \phi_2) - \frac{1}{2}H} \ , \tag{64}$$

with an even number of $-$ signs in the exponential of non-ghost fields corresponding to a choice of positive 10-dimensional chirality. The vertex operators for the gaugino, gravitino, and dilatino, are built from these operators and from other free fields. For the fermion mass calculation, it is convenient to take one vertex operator in the picture with ghost number 1/2, the other with ghost number −1/2. For the gaugino vertex operator, we have

$$V_\chi \; = \; \frac{g}{2\pi}\, j_{\rm L}\, e^{-\frac{\phi}{2}} \bar{S}\chi\, e^{ik'\cdot X} \; = \; \frac{g}{\pi}\, j_{\rm L}\, e^{-\frac{\phi}{2}}\, (S^\alpha \chi_\alpha\, e^{\frac{1}{2}H} \; + \; \bar{S}_{\dot\alpha} \bar\chi^{\dot\alpha}\, e^{-\frac{1}{2}H})\, e^{ik'\cdot X} \quad . \qquad (65)$$

The dilatino vertex operator in the +1/2 superconformal ghost picture is

$$V_\lambda \; = \; \frac{ig\sqrt{2}}{8\pi} e^{\frac{\phi}{2}} \frac{1}{2} \bar\partial X_\mu (\partial X_\nu - ik\cdot\psi\psi_\nu) \bar{S}\Gamma^\nu \Gamma^\mu \lambda\; e^{ik\cdot X}$$

$$= \; \frac{ig^2\sqrt{2}}{16\pi}\, e^{\frac{\phi}{2}}\; \bar\partial X_\mu (\partial X_\nu \; - \; ik\cdot\psi\psi_\nu)$$

$$\times (S^\alpha [\sigma^\nu \bar\sigma^\mu]_\alpha^\beta\; \lambda_\beta\, e^{-\frac{1}{2}H} \; + \; \bar{S}_{\dot\alpha} [\bar\sigma^\nu \sigma^\mu]^{\dot\alpha}_{\dot\beta}\; \bar\lambda^{\dot\beta}\, e^{\frac{1}{2}H})\, e^{ik\cdot X} \quad . \qquad (66)$$

The overall factor is again determined by the requirement that the vertex operator create properly normalized states. The coefficient of the term bilinear in 2-dimensional fermions differs from that of the ten dimensional vertex operator (Ref. [7]). Indeed, this coefficient depends on the uncompactified spacetime dimension d (here $\mu,\ \nu\ =\ 0,\ .\ .\ ,3$). The fermion mass is given by

$$\int d^2\tau d^2 z d^2 w\, {\rm Tr}\, [V_\lambda(z,\ \bar z)\ V_\chi(w,\bar w)\ q^{L_o} \bar q^{\bar L_o}] \quad . \qquad (67)$$

There are two types of non-vanishing contributions here. First, the $\bar\partial X_\mu$ and ∂X_ν factors of the dilatino vertex can be contracted with the $e^{ik'\cdot X}$ factor from the gaugino vertex, each yielding an external momentum k' . In order to cancel the resultant factor of k^2 which will be eventually taken to zero we must, as usual, look for short distance singularities, and it is enough to examine the leading terms in operator product expansions (OPE). In particular,

$$S_\alpha(z)S_\beta(w) \;=\; \frac{1}{(z\;-\;w)^{1/2}}\,\epsilon_{\alpha\beta} + \frac{1}{8}(z\;-\;w)^{1/2}\,[\sigma^\mu,\,\bar\sigma^\nu]_\alpha^\gamma\epsilon_{\beta\gamma}\psi_\mu\psi_\nu \;+\;\ldots$$

$$\bar S^{\dot\alpha}\,(z)\bar S^{\dot\beta}\,(w) \;=\; \frac{1}{(z\;-\;w)^{1/2}}\,\epsilon^{\dot\alpha\dot\beta} \;+\; \frac{1}{8}(z\;-\;w)^{1/2}\,[\bar\sigma^\mu,\,\sigma^\nu]_{\dot\gamma}^{\dot\alpha}\epsilon^{\dot\beta\dot\gamma}\psi_\mu\psi_\nu \;+\;\ldots \qquad (68)$$

$$e^{+-\frac{1}{2}H(z)}e^{-(+-)\frac{1}{2}H(w)} \;=\; \frac{1}{(z\;-\;w)^{3/4}} \;\pm\; \frac{1}{2}(z\;-\;w)^{1/4}j_{\mathrm{R}}(w)\;\;.\;+\;\ldots \qquad (69)$$

The coupling is thus

$$-\frac{ig^2\sqrt{2}}{32\pi^2\Omega_1}(\chi\lambda\;-\;\bar\chi\bar\lambda)\;\mathrm{Tr}\;[\;\frac{1}{2}j_{\mathrm{L}}j_{\mathrm{R}}\;\int \mathrm{d}^2\tau\;\mathrm{d}^2z\frac{k'^2}{|z|^{2\;-\;2k\cdot\;k'}}\;q^{L_o}\bar q^{L_o}\;] \qquad (70)$$

A further short distance singularity comes from contraction of ∂X_μ with the exponential factor, with the term bilinear in 2 dimensional fermions. By examining the OPE's of the latter together with the spin field at the same point z and the spin field from the gaugino vertex at w , one finds 2 ways of obtaining singular terms with $1/(z\;-\;w)^{3/2}$ divergence. When combined with the first term of the OPE of the ghost fields and the finite term from the OPE of the internal fields in (4.17), the result is a $1/|z\;-\;w|^2$ pole that can yield $1/k^2$ after integration. Here we need the OPE

$$\psi^\mu(z)\psi^\nu(z)S_\alpha(w) \;=\; -\frac{1}{4(z\;-\;w)}\;[\sigma^\mu\bar\sigma^\nu\;-\;\sigma^\nu\bar\sigma^\mu]_\alpha^\beta S_\beta(w)\;+\;\ldots \qquad (71)$$

The above leads to

$$\psi^\mu(z)\psi^\nu(z)S_\alpha(z)S_\beta(w) \;=\; -\;\frac{1}{2(z\;-\;w)^{3/2}}[\sigma^\mu\bar\sigma^\nu\;-\;\sigma^\nu\bar\sigma^\mu]_\alpha^\gamma\epsilon_{\beta\gamma}\;+\;\ldots \qquad (72)$$

By using identities for 4 dimensional gamma matrices, $\sigma_\rho\bar\sigma^\mu\sigma^\rho \;=\; -2\sigma^\mu$, and $[\sigma_\mu\bar\sigma^\mu]_\alpha^\beta \;=\; 4\delta_\alpha^\beta$, we can find a second term proportional to k^2

$$-\frac{ig^2\sqrt{2}}{32\pi^2\Omega_1}(\chi\lambda\;-\;\bar\chi\bar\lambda)\;\mathrm{Tr}\;[\frac{1}{2}j_{\mathrm{L}}j_{\mathrm{R}}\;\int \mathrm{d}^2\tau\;\mathrm{d}^2z\frac{-3k^2}{|z|^{2\;-\;2k\cdot\;k'}}q^{L_o}\bar q^{L_o}]\;\;. \qquad (73)$$

The final contribution is found by contracting $\bar\partial X_\mu(\bar z)$ and $\partial X_\nu(z)$. The remaining fields have at most a $1/(z\;-\;w)$ singularity which when summed over all spin structures must be cancelled[43] . By examining the OPE's, we find that the only coordinate independent term has coefficient proportional to the right moving current j_R, as for the singular contributions. Assembling all these terms we get

$$-\frac{ig^2\sqrt{2}}{32\pi^2\Omega_1}(\chi\lambda \ - \ \bar{\chi}\bar{\lambda}) \ \mathrm{Tr}\ [j_L j_R \frac{1}{2} \int d^2\tau$$

$$\times \ (-\frac{4\pi}{\tau_2} \int d^2 z \ - \int \frac{d^2 z 2k^2}{|z|^2}) \ \chi(z,\ \bar{z})^{2k\cdot\ k'} q^{L_o} \bar{q}^{L_o} \ ] \ . \qquad (74)$$

The first term in (4.21) is identical to the contribution from the axion-gauge boson coupling and is related to it by supersymmetry. To see this, one can first rewrite the gaugino vertex in (4.16) as a commutator of the supersymmetry charge Q and the gauge boson vertex operator. A factor of momentum appears in this relation. This reflects the fact that the vertex operators correspond to the usual in and out fields of field theory times inverse propagators (see, for example, Ref.[29]). The commutator can be expressed as a contour integral of the supersymmetry current in (4.13) and the gauge boson operator, the contour enclosing the position of the latter vertex. Ignoring possible singularities, the contour may be deformed to enclose the position of the dilatino vertex; the resulting commutator gives the axion vertex operator. The gamma matrix manipulations, along with the factor of momentum mentioned above, yield precisely the $A_\mu \partial^\mu a$ coupling. These contour manipulations are not valid when the vertex operator positions coincide. At such points, there are additional δ- function singularities corresponding precisely to the k^2/k^2 terms found earlier. Of course, one can also transform the boson vertex operators in (4.9) to obtain (4.16).

The reader may be confused by one point. In our earlier computation, the fermion mass received contributions from various spin structures; the axion-gauge boson mixing, on the other hand, came only from the PP sector (on the right). In Ref.[29], it was shown using general arguments that Tr $[j_L j_R]$ is proportional to Tr $[j_L]_{PP}$, where one sums over spin structures of both left and right sectors in the former but in the latter on the right, only the PP spin structure is included. Thus the results described here are consistent with one another.

The short distance singularity contribution to the fermion mass, from the second term in eqn. (4.21), is

$$m_{\mathrm{F}}^{(1)} = -\frac{g^2}{4 \times 48\pi^2} \mathrm{Tr}\, Q\, M_{\mathrm{S}} \ . \tag{75}$$

Again, this agrees with (2.12) and (3.16). The contribution equivalent to that in the axion-gauge boson computation corresponds to the supersymmetric terms in eqns. (2.12) and (3.16),

$$m_{\mathrm{F}}^{(2)} = \frac{g^2}{2 \times 48\pi^2} \mathrm{Tr}\, Q\, M_{\mathrm{S}} \tag{76}$$

Thus all four approaches yield the same result for this mass.

In the next chapter, we will only work with covariant string vertex operators. We will show how these mass corrections allow ultraviolet divergences on the world-sheet to manifest themselves as BRST anomalies. We will then show how the anomalies can be cancelled with the renormalisation of the vertex operators involved in the two-point functions. The vertex operators for physical states are BRST invariant operators and their BRST invariance is only true on shell. In fact, the BRST commutator or anti-commutator of vertex operators for massless states results in contributions which vanish only if polarisation tensors are transverse to the external momenta, or in the case of fermion vertex operators, if the external spinors satisfy the massless Dirac equation.

Chapter 3 BRST Anomalies and Mass Renormalisation

3.1 Introduction

In this chapter, we will use the BRST symmetry of string theory as a guiding principle in the mass renormalsation of string vertex operators representing states that have gained mass by quantum corrections. We will consider the simpler case of the bosonic string before moving on to the compactified heterotic string in 4-dimensions. Since the physical constraints of superconformal and therefore BRST invariance of physical states[47],[48] at tree level depend on tree level masses, one would expect that BRST invariance will be affected by loop generated masses.

A.Sen[21] has shown that, in the bosonic string, non-vanishing two-point functions at one-loop(mass corrections) lead to BRST anomalies at tree level, and at one and two-loops. Then, correlation functions that include null or BRST exact states do not always vanish. BRST exact states correspond to trivial states and are set to zero, since their inclusion in any correlation function involving physical states should lead to the vanishing of the correlation function. Non-vanishing correlation functions that contain these null states are thus BRST anomalous and break BRST invariance explicitly. It was shown that the BRST anomalies can be cancelled exactly by modifying the vertex operators, using renormalised vertex operators for states with mass corrections. The renormalisation of a vertex operator includes changing the external momentum k to incorporate the mass shift. This leads to the modification of other physical constraints on the vertex operator, to maintain the BRST invariance of the vertex operator.

In general, quantisation of a system subject to constraints in the BRST framework requires constraint conditions to be replaced by the BRST constraint condition that the BRST charge Q annihilate physical states[49],[50]. BRST invariant interactions involving physical states should lead to non-vanishing correlation functions which are consistent with BRST symmetry and thus with the constraints of a system. This is true at tree-level. Loop corrections, which are otherwise BRST invariant, can generate modifications inconsistent with tree-level constraints and break BRST invariance implicitly. In string theory, tadpoles break BRST invariance and can be understood, as shown in Refs.[22] and [23], to be due to divergences that arise when handles become small or equivalently when all vertex operators approach. BRST symmetry can also be broken by singularities that occur when all except one vertex operator approach each other causing non-vanishing two-point functions to factorise. In the scheme to be used here, the implicit breaking of BRST invariance by mass corrections is reformulated as explicit breaking by the non-vanishing of correlation functions that include null states.

We will show how this prescription can be used to carry out mass renormalisation in a systematic way, for both boson and fermion vertex operators, when the heterotic string is compactified to 4 dimensions with an anomalous $U(1)$ gauge symmetry. A partial analysis of this model was carried out in Ref. 46 by using global properties of the world-sheet. It is well-known[34],[35] that there are ambiguities related to the insertion of picture changing operators on the world-sheet. Our analysis here is local and we find that potential ambiguities can be removed.

3.2 BRST anomalies in the Bosonic String

We review Ref.[21] for the bosonic string and compare this with a conformally equivalent procedure. An n-point function on the torus, $\langle \prod_{i=1}^{n-1} V_i c\bar{c}V\rangle_{\text{torus}}$, that includes a BRST exact state, $c\bar{c}V = [Q, \bar{c}\hat{V}]$, where Q is the BRST charge, can be non-zero[21]. $\hat{V}$ must have conformal dimensions $(1,0)$ and without loss of generality, $\hat{V}$ can be chosen with zero ghost charge†, and V to be Virasoro primary. With $k^2 = 0$,

$$\hat{V}(z,\bar{z}) = \hat{O}(\bar{z})e^{ik\cdot X(z,\bar{z})} \quad . \tag{77}$$

$\hat{V}$ satisfies the physical Virasoro constraints, except that $[L_0, \hat{V}] = 0$. The commutator with the BRST charge rewritten as a contour integral can be deformed and leads to a total derivative in the modular parameter that is related to the existence of tadpoles.

It can also give contributions not related to tadpoles. The n-point correlation function becomes a sum and is then factorised,

$$- \sum_{j=1\ (j\neq i)}^{n-1} \langle \prod_{i=1}^{n-2} V_i \int d^2 z_j \partial_{z_j} [cV_j]\ \bar{c}\hat{V}\ \rangle_{\text{torus}}$$

$$= -\frac{2\pi i \Omega_0}{a^2} \sum_l \langle\ V\ c\bar{c}V_l\ \rangle_{\text{torus}} \langle\ c\partial c\bar{c}\bar{\partial}\bar{c}V_l\ cV_j \prod_{i=1}^{n-3} V_i\ \bar{c}\hat{V}\ \rangle_{\text{sphere}} \quad . \tag{78}$$

$(n-2)$ vertex operators on the left hand side are restricted to a disc in the neighbourhood of $\bar{c}\hat{V}$. This corresponds to a choice of kinematics that leads to the factorisation of a one-loop two-point function from one-loop n-point($n \geq 1$) correlation functions. Sen showed[21], that the total derivative contributions are non-vanishing if the integrand contains[21] a $1/\bar{t}$, which gives †, $2\pi i\delta^{(2)}(t)$ and

† This argument is valid only because we will choose to use vertex operators V_i that have zero ghost charge.

thus a finite contribution interpreted as due to the boundary of the moduli space of parameter t at $t = 0$. Each total derivative of cV_j can thus be factorised. $\Omega_0 = a^2/4\pi i$ is the normalisation constant on the sphere and a is the coupling constant for each vertex operator.

These are BRST anomalous contributions if the factorised two-point function on the torus and the n-point function on the sphere are both non-vanishing. One sums j over all $(n-1)$ factorised contributions. Three-point functions on the torus that include a BRST exact state generally vanish when summed over different contributions because $\hat{O}$ is a conserved current. For example, $\hat{O}$ could be the left moving momentum current $\bar{\partial}X$ (or, as will be seen for the heterotic string, a left moving $U(1)$ charge current j_L). With 4 or more vertex operators, there are additional poles which give a non-vanishing sum.

Conformally equivalent manipulations on the torus lead to poles like $1/\bar{z}$ which creates a Dirac delta function at the location of the vertex operator V_i. One examines OPE's to find

$$V_i(z,\bar{z})V_j(w,\bar{w}) = \frac{C_{ijk}}{\bar{z}-\bar{w}}V_k(w,\bar{w}) + \ldots \quad , \tag{79}$$

where C_{ijk} is a coefficient, proportional to the corresponding three-point function. A candidate for V_j is the $(1,0)$ operator $\hat{V}$. The singularity at the boundary of the moduli space of a long tube is equivalent to this singularity when the positions of these vertex operators coincide. There can be up to $(n-1)$ contributions of the type

$$\langle \prod_{l=1}^{n-2} V_l \; C_{ijk}c\bar{c}V_k \;\rangle_{\text{torus}} \quad . \tag{80}$$

All vertex operators V_l, i.e. except for one which has non-vanishing two-point functions with some operator $\tilde{V}$, are allowed to approach $c\bar{c}V_k$ simultaneously.

† There is a phase ambiguity which we find convenient to fix by the phase choice in $\delta^{(2)}(z) = \delta(z)\delta(\bar{z}) = -i\delta(x)\delta(y)/2$, where $z = x+iy$, $\bar{z} = x - iy$. This determines the phase in $d^2z = dzd\bar{z} = 2idxdy$ and $\partial_z(1/\bar{z}) = 2\pi i\delta^{(2)}(z)$. However, one will see that the amplitudes to be calculated are independent of this choice.

The use of OPE's to bring $(n-2)$ vertex operators close together can be justified by a suitable choice of kinematics. This will lead to an amplitude that corresponds to Fig.6.

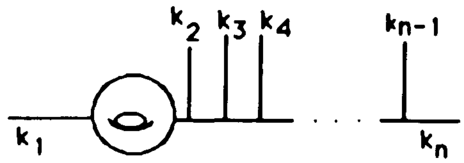

Fig. 6 Factorisation of 1-loop 2-point function from 1-loop n-point function

A non-vanishing two-point function can be obtained if $\tilde{V}$ results from the OPE's of these operators. The poles of the propagators of intermediate states that arise in the OPE's, when massless, lead to generic log divergences when the world-sheet coordinates are integrated. Factorised one and two-point functions do not vanish in general. Factorised three and higher point functions vanish. All vertex operators coming close lead to a one-point function on the torus, i.e. a tadpole, and is removed by shifting tree-level fields[22],[23].

A vertex operator renormalised for the mass correction[33] $V_i^{ren}(k_{ren})$ is given by $\Lambda^{-\delta k^2} V_i(k_{ren})$ where $k_{ren}^2 = k_{tree}^2 + \delta k^2 = -m_{tree}^2 - \delta m^2$. Λ is the world sheet ultraviolet momentum cut off. We use the convention where $\alpha' = 2$. As shown in Ref.[33], the prefactors are normal ordering factors which appear due to anomalous dimensions in vertex operators with external momenta incorporating the mass shift. If a tree-level vertex operator is normalised to unity, the OPE of the dimension $(1,1)$ vertex operator with its CPT conjugate, $V(z,\bar{z},k)V^{\dagger}(0,-k) \sim 1/|z|^4$, is also normalised to unity for the renormalised vertex operator. The renormalised vertex operator does not scale as a dimension $(1,1)$ tree-level operator, and the Virasoro operator L_0 will be shifted as the mass is shifted, $L_0^{ren} = L_0^{tree} - \delta m^2$. Scale invariance can however be recovered as will be shown in the last section.

The tree level BRST anomalous contribution is

$$\langle\, c\bar{c}V_l^{\mathrm{ren}} \prod_{i=1}^{n-3} V_i \; c\bar{c}V_j \; [Q, \bar{c}\hat{V}] \,\rangle_{\mathrm{sphere}} = \frac{\delta k^2}{2} \langle\, c\partial c\bar{c}V_l^{\mathrm{ren}} \prod_{i=1}^{n-3} V_i \; c\bar{c}V_j \bar{c}\hat{V} \,\rangle_{\mathrm{sphere}} \; + \; \ldots, \tag{81}$$

with the same external states as in the one-loop calculation. One has deformed the BRST commutator as before and ... represents contributions which always vanish because they have unrestricted momentum flow through connecting tubes. The non-vanishing term has exactly the same form as in eqn. (78). It will be shown later that the renormalisation prefactors for vertex operators result in tree-level contributions that are finite up to logarithmic divergent factors common to both the tree-level BRST anomalous correlations and the one-loop BRST anomalous correlations they are to cancel. The log divergences are due to propagators of massless intermediate states[51]. The identification, $\delta k^2 = \langle\, V \; c\bar{c}V_i \,\rangle_{\mathrm{torus}} = -\delta m^2$., has also been used. Thus, eqn. (81) is the tree-level anomalous term that cancels the anomalous one-loop contribution.

Just as for the one-loop calculation, one finds a suitable choice of kinematics, using OPE's, such that the first term in eqn. (81) is evaluated by allowing $(n-2)$ vertex operators to come close together. The required anomalous tree amplitude needed to cancel that of Fig.6 is shown in Fig.7 below.

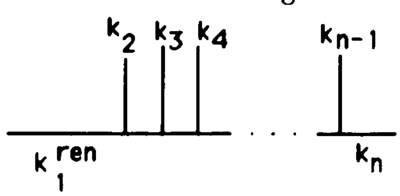

Fig.7 Kinematics for anomalous tree-level n-point function

3.3 BRST anomalies in the Heterotic String from an anomalous $U(1)$

We now implement the procedure of the previous section for the heterotic string where, instead of pinching, one allows all but one vertex operator on the world sheet to come close to each other. As is well-known, this is conformally equivalent to the pinching process. Mass corrections at one-loop occur in the presence of an anomalous $U(1)$ gauge symmetry.

3.3.1 Charged Scalar Mass-Renormalisation

Consider the non-vanishing two-point function at one-loop for a charged scalar ϕ_i and its CPT conjugate $\phi_{\bar{i}}$ with $U(1)$ charges q_i^+ and q_i^- respectively,

$$m_i^2 \; = \; \langle \; V_{\phi_i} c\bar{c} V_{\phi_{\bar{i}}} \; \rangle_{\mathbf{torus}} \; = \; \frac{q_i g^2}{48\pi^2} \, \mathrm{Tr}\, Q \; \mathrm{M}_{\mathrm{S}}^2 \quad , \tag{82}$$

where $q_i \; = \; |q_i^{\pm}|$ is the $U(1)$ charge and the string scale $\mathrm{M_S} \; = \; 1/\sqrt{\alpha'}$. The sign of m_i^2 depends only on that of $\mathrm{Tr}\, Q$. The vertex operators for gauge fields have been normalised to have coupling constant $a \; = \; g/2\pi$, where g is identified with the 4-dimensional gauge coupling.

The simplest non-vanishing BRST anomalous correlation function at one-loop is a 4-point function,

$$\langle \; V_{\phi_i}(z_1, \bar{z}_1) \; V_{\phi_{\bar{i}}}(z_2, \bar{z}_2) \; V_A(z_3, \bar{z}_3) \; c\bar{c}V(z_4, \bar{z}_4) \; \rangle_{\mathbf{torus}} \quad , \tag{83}$$

where the null state is, $c\bar{c}V = [Q, \bar{c}\hat{V}]$. This corresponds to Fig.8a.

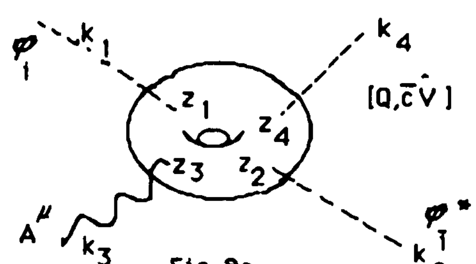

Fig. 8a

All vertex operators† are in the zero superconformal ghost picture. V_A is the vertex operator for the $U(1)$ gauge boson,

$$V_A(z_3,\bar{z}_3) \;=\; \zeta_\mu j_{\rm L}(\partial X^\mu \;+\; ik_3\cdot\psi\psi^\mu)e^{ik_3\cdot X}. \tag{84}$$

BRST invariance at tree-level demands the transversality of the polarisation tensor, $k_3\cdot\zeta = 0$.

A suitable choice for $\hat{V}$ is,

$$\hat{V} \;=\; j_{\rm L}(\bar{z}_4)e^{ik_4\cdot X(z_4,\bar{z}_4)} \quad , \tag{85}$$

This operator was also used in Ref.[46].

The correlation function in eqn. (83) includes a sum over the 4 spin structures on the torus. In the PP sector, integration over the supermodulus[26] and the 4 zero modes of the world sheet fermions, ψ_o^μ, is also included. In fact, the PP sector in eqn. (83) vanishes since there can only be at most 3 world sheet fermions from the vertex operators and picture changing operator. The supermodulus is assumed to have been integrated out to give an insertion of the picture changing operator[26], $Y(\bar{z})$. The vertex operators are initially in the zero superconformal ghost picture except for, say, V_A in the -1 picture. Here, the only contribution of the picture changing operator is its matter piece, $e^\phi T_{\rm F}$, which would change V_A back to the zero ghost picture. Thus, for our purposes we can conveniently choose vertex operators in certain ghost pictures on the torus so as to have total zero superconformal ghost charge for non-vanishing correlation functions. This choice can be made because the picture changing operator gives vanishing contributions in the following 2 cases. Firstly, physical vertex operators which are BRST closed up to a total derivative are neither BRST exact nor trivial and may contribute at the boundary of moduli

† The convention will be that the i'th vertex operator $V(z_i,\bar{z}_i)$ carries external momentum, k_i, labelled by index i, unless otherwise stated.

space through their total derivatives. The picture changing operator is BRST exact and trivially closed and cannot contribute this way. Secondly, there are null state vertex operators which are BRST exact but non-trivial because they can soak up total derivatives of other vertex operators. These null states lead to non-vanishing contributions because they carry left moving currents that can give rise to a $1/\bar{z}$ pole which soaks up total derivative terms as was shown for the bosonic string (this continues to be true for the heterotic string). The picture changing operator, being constructed from purely holomorphic fields on the world sheet, cannot create a pole of this type.

There remains the question of the dependence of the correlation function on the position of the supermodulus insertion on the world-sheet. A dependence of this type leads to ambiguities.

As before, we deform the contour integral with BRST current. As there are no tadpoles at one-loop in these theories, the contributions from antighost b insertions which lead to total derivatives in the modular parameter vanish. The BRST commutators with vertex operators lead to 3 contributions which are total derivatives with respect to the vertex positions. We note that

$$V_{\phi_i}(z_1,\bar{z}_1)\hat{V}(z_4,\bar{z}_4) \;=\; \frac{q_i^+}{\bar{z}_1 \;-\; \bar{z}_4} V_{\phi_i}(z_4,\bar{z}_4,k_1+k_4)\,|z_1 \;-\; z_4|^{2k_1\cdot k_4} + \ldots \quad , \tag{86}$$

gives rise to the required pole. The OPE of $V_{\phi_{\bar{i}}}$ with $\hat{V}$ is the same except that the charge is q_i^-. The OPE of V_A with $\hat{V}$ does not contain such a pole and its contribution vanishes. Thus, total derivative contributions from charge neutral vertex operators always vanish for this choice of $\hat{V}$. The result is that eqn. (83) becomes

$$-2\pi i\{\; q_i^+\langle\; V_{\phi_{\bar{i}}}(z_2,\bar{z}_2)\; V_A(z_3,\bar{z}_3)\; c\bar{c}V_{\phi_i}(z_4,\bar{z}_4,k_1+k_4)\;|z_1 \;-\; z_4|^{2k_1\cdot k_4}\;\rangle_{\text{torus}}$$

$$+\; q_i^-\langle\; V_{\phi_i}(z_1,\bar{z}_1)\; V_A(z_3,\bar{z}_3)\; c\bar{c}V_{\phi_{\bar{i}}}(z_4,\bar{z}_4,k_2+k_4)\;|z_2 \;-\; z_4|^{2k_2\cdot k_4}\;\rangle_{\text{torus}}\;\}. \tag{87}$$

If in eqn. (87), one exchanges the dummy labels 1 and 2 in the second term, it is easy to see that the 2 correlation functions are the same except that $V_{\phi_{\bar{i}}}$ and

V_{ϕ_i} have their locations interchanged. Since the interchange does not introduce any phases, then except for poles between V_A and the other vertex operators, the 2 correlation functions are identical and would cancel. The point here is that the contributions of the 2 correlation functions with the position of V_A integrated over the analytic part of the torus, i.e. away from the other vertex operators, cancel exactly because of charge conservation. Without the gauge boson vertex, one finds 2 contributions which cancel exactly. The only non-vanishing momentum independent contribution comes when V_A is allowed to approach z_4. We can safely restrict our analysis, in the absence of tadpoles at one-loop, to the case when all but one vertex operator approach z_4.

Since V_A carries the left moving $U(1)$ gauge current, its OPE's with the charged scalar vertex operators result in

$$4\pi i \ln \epsilon \langle \; \{V_{\phi_i}(z_2) c\bar{c} V_{\phi_i}(z_4, -k_2) i(k_1 \; + \; k_4)_\mu$$

$$+V_{\phi_i}(z_1) \; c\bar{c} V_{\phi_i}(z_4, -k_1) i(k_2 + k_4)_\mu\} \zeta^\mu \; \rangle \qquad (88)$$

To regularise the limits as all but one vertex operator tend to coincide in their positions, one can define small but finite quantities, $\epsilon \; = \; |z_3 \; - \; z_4|$ and $\epsilon' \; = \; |z_1 \; - \; z_4|$, for the first term. Then, before allowing the limits $\epsilon \rightarrow \; 0$ and $\epsilon' \; \rightarrow \; 0$, consider $\epsilon^{2k_3 \cdot (k_1 + k_4)} \; \epsilon'^{2k_1 \cdot k_4}$. If the 2 small quantities can be identified, i.e. $\epsilon \; = \; \epsilon'$, which can be taken to mean that the limits are taken simultaneously and at the same rate, the sum of the exponents of ϵ vanish identically by momentum conservation and as these tree level states are massless. The same argument is also applicable to the second contribution. The first contribution is obtained by taking the successive limits $z_1 \rightarrow z_4$ and $z_3 \rightarrow z_4$, and for the second $z_2 \rightarrow z_4$ before $z_3 \rightarrow z_4$. For both contributions, the first limiting process leads to an intermediate state. The vertex operator corresponding to the intermediate state appears in the OPE and gives rise, in the second limit, to the pole of the propagating intermediate state. This state is massless and its pole leads to a logarithmic divergence. The two contributions correspond to the amplitudes in the 's' and 't' channels shown in Figs.8b and 8c below.

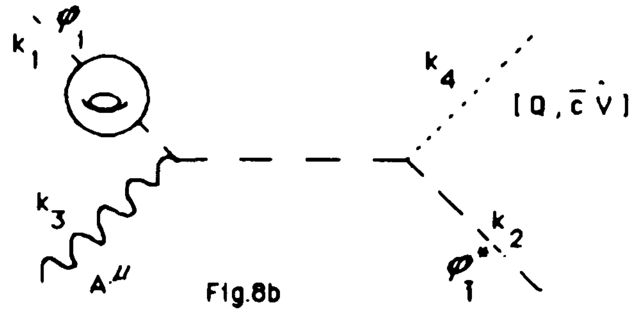

Fig.8b

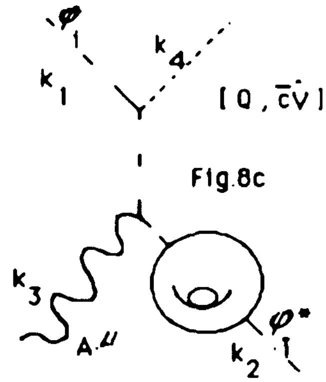

Fig.8c

Using eqn.(82) and choosing $z_4 = 0$, for convenience, one adds the 2 contributions and thus obtains

$$-4\pi i a^2 m_i^2 |q_i|^2 i k_4 \cdot \zeta(-4\pi i \ln \epsilon) \quad . \tag{89}$$

We now examine the BRST anomalous correlation functions at tree-level required to cancel the anomalous contribution in eqn. (89). The required correlations are the four-point functions,

$$\langle\, c\bar{c}V^{\rm ren}_{\phi_i}(z_1, \bar{z}_2)\; c\bar{c}V_{\phi_i^*}(z_2, \bar{z}_2)\; V_A(z_3, \bar{z}_3)\; c\bar{c}V(z_4, \bar{z}_4)\,\rangle_{\rm sphere} \quad ,$$

$$\langle\, c\bar{c}V_{\phi_i}(z_1, \bar{z}_2)\; c\bar{c}V^{\rm ren}_{\phi_i^*}(z_2, \bar{z}_2)\; V_A(z_3, \bar{z}_3)\; c\bar{c}V(z_4, \bar{z}_4)\,\rangle_{\rm sphere} \quad , \tag{90}$$

with the renormalised vertex operators $V^{\rm ren}_{\phi_i}$ and $V^{\rm ren}_{\phi_i^*}$, as well as V_A in the -1 picture. The renormalised vertex operators for the charged scalar fields are the same as tree level operators but with new momenta incorporating the mass, $k_i^{\rm ren\;2} = -m_i^2$, and prefactors $\Lambda^{-k_i^{\rm ren\;2}/2}\,\bar{\Lambda}^{-k_i^{\rm ren\;2}/2}$. As will be seen shortly, these changes are necessary to cancel BRST anomalies. Deforming the BRST contour leads to 4 terms of which only 2 survive,

$$\frac{k_i^{\text{ren}^2}}{2}\langle\, c\partial c\bar{c}V^{\text{ren}}_{\phi_i}\; c\bar{c}V_{\phi_i}\; V_A\; \bar{c}\hat{V}\,\rangle_{\text{sphere}} + \frac{k_i^{\text{ren}^2}}{2}\langle\, c\bar{c}V_{\phi_i}\; c\partial c\bar{c}V^{\text{ren}}_{\phi_i}\; V_A\; \bar{c}\hat{V}\,\rangle_{\text{sphere}} \quad , \tag{91}$$

with $k_i^{\text{ren}} = k_1^{\text{ren}}$ and $k_i^{\text{ren}} = k_2^{\text{ren}}$. To obtain non-zero contributions, it is necessary to let $z_2, z_3, z_1 \rightarrow z_4$, in that order, for the first correlation function, and $z_1, z_3, z_2 \rightarrow z_4$ for the second correlation function. They lead to the 's' and 't' channel contributions in Figs.8d and 8e below.

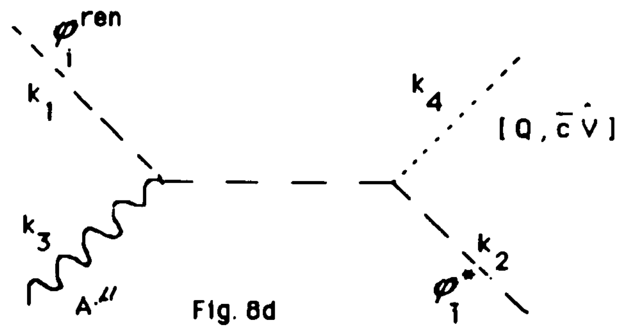

Fig. 8d

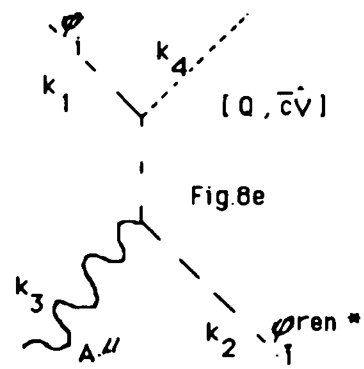

Fig.8e

The OPE's produce factors,

$$\epsilon^{-2k_1^2} \quad , \tag{92}$$

where as in the one-loop calculation, we allow the vertex operator positions to approach z_4 at the same rate. The resultant factor $\epsilon^{-k_i^{\text{ren}\;2}}$ cancels the pre-factor $\Lambda^{-k_i^{\text{ren}\;2}}$. Thus,

$$\langle\, c\partial c\bar{c}V^{\text{ren}}_{\phi_i}\; c\bar{c}V_{\phi_i}\; V_A\; \bar{c}\hat{V}\,\rangle_{\text{sphere}} \;=\; \frac{a^4}{\Omega_0}\frac{k_i^{\text{ren}\;2}}{2}|q_i|^2 ik_2\cdot\zeta\lim_{\epsilon\rightarrow 0}(-4\pi i\ln\epsilon)\;|\Lambda\epsilon|^{-2k_i^{\text{ren}\;2}} \quad , \tag{93}$$

where for a contribution finite up to the log divergence, the limits are taken so that

$$\lim_{\Lambda\rightarrow\infty,\epsilon\rightarrow 0}\Lambda\epsilon \;=\; 1 \quad . \tag{94}$$

The prefactors $\Lambda^{-k_i^{\text{ren}\;2}}$ for the renormalised vertex operators are thus necessary for the tree and one-loop anomalies to cancel. The 2 contributions in eqn. (91)

add to give

$$\frac{a^4}{\Omega_0} k_i^{\mathrm{ren}\ 2} |q_i|^2 i(k_1 + k_2) \cdot \zeta(-4\pi i \ln \epsilon) \quad , \tag{95}$$

with $z_4 = 0$ for convenience. The coupling constants have been reinserted with the tree normalisation constant $\Omega_0 = a^2/4\pi i$. Using $(k_1 + k_2) \cdot \zeta = -k_4 \cdot \zeta$, cancellation of eqns. (89) and (95) is consistent with the identification

$$k_i^{\mathrm{ren}\ 2} = -m_i^2 = -\frac{q_i g^2}{48\pi^2} \mathrm{Tr}\, Q\ \mathrm{M}_{\mathrm{S}}^2 \quad . \tag{96}$$

We have thus cancelled off the anomalous contributions of Figs.8b with 8d, and 8c with 8e.

3.3.2 Axion - Gauge Boson Mass Renormalisation

In compactified models with an anomalous $U(1)$, the apparent anomaly can be cancelled[28] in the effective Lagrangian by shifting the free 4-dimensional axion field a by a well-defined constant. The non-linear transformation couples the axion to the $U(1)$ gauge boson through a derivative coupling. In the string computation one finds the non-vanishing two-point function

$$-\epsilon_{\mu\nu\rho\lambda} \zeta^\mu \epsilon^{\nu\rho} k^\lambda m_{a-A} = \langle\, Y(\tilde{z})\ V_a\ c\bar{c} V_A \,\rangle^{\mathrm{PP}}_{\mathrm{torus}} \quad , \tag{97}$$

where $m_{a-A} = (g^2/96\pi^2)\, \mathrm{Tr}\, Q\ \mathrm{M_S}$, and $\epsilon_{\mu\nu\rho\lambda}$ is the antisymmetric tensor in 4-dimensions. The axion vertex operator is in the -1 picture,

$$V_a = \epsilon_{\mu\nu} \bar{\partial} X^\mu \mathrm{e}^{-\phi} \psi^\nu \mathrm{e}^{ik\cdot X} \quad . \tag{98}$$

Only the PP sector of the spin structures contributes. With the gauge boson vertex operator in the zero ghost picture, only the matter piece of the picture changing operator[26],[7] Y inserted at $\tilde{z}$ is relevant. This is also the only part of Y that can give rise to the antisymmetric tensor.

Consider the anomalous four-point function

$$\langle\, e^{\phi}T_{\rm F}(\tilde z)V_a(z_1,\bar z_1)\; V_{\Psi_i}(z_2,\bar z_2)\; V_{\bar\Psi_{\bar i}}(z_3,\bar z_3)\; c\bar c V(z_4,\bar z_4)\,\rangle^{\rm PP}_{\rm torus} \quad , \qquad (99)$$

corresponding to Fig.9a.

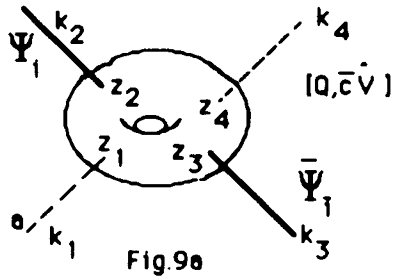

Fig.9a

The gauge fermion vertex operators V_{Ψ_i} and $V_{\bar\Psi_{\bar i}}$ are in the $-1/2$ ghost picture. The vertex operator for the charged fermion, which belongs to the same supermultiplet as the charged scalar, V_{ϕ_i}, is

$$V_{\Psi_i}(z_2,\bar z_2) \;=\; b_{i\bar i}\, j^{a i}\, e^{-\phi/2}\Psi_i^{\alpha} S_\alpha\; e^{iH/6}\theta^{i} e^{ik_2\cdot X} \quad . \qquad (100)$$

It carries odd half-integral internal $U(1)$ charge $+1/2$ as required for a physical fermion vertex and transforms correctly under spacetime supersymmetry. 2-component spinors are used with spinor indices α and $\dot\alpha$ for the two 4-dimensional chiralities. $V_{\bar\Psi_{\bar i}}$ is the vertex operator for the CP conjugate of V_{Ψ_i}, with opposite quantum numbers and opposite chirality,

$$V_{\bar\Psi_{\bar i}}(z_3,\bar z_3) \;=\; b_{\bar i i}\, j^{a\bar i}\, e^{-\phi/2}\bar\Psi_{\bar i\dot\alpha} S^{\dot\alpha}\; e^{-iH/6}\theta^{i} e^{ik_3\cdot X} \quad . \qquad (101)$$

The null state is as before using $\hat V$ from eqn. (85). The correlation function leads to two non-vanishing terms, using eqn. (86),

$$-a2\pi i q_i^{+}\langle\, e^{\phi}T_{\rm F}\; V_a\; V_{\bar\Psi_{\bar i}}\; c\bar c V_{\Psi_i}|z_2 \;-\; z_4|^{2k_2\cdot k_4}\,\rangle^{\rm PP}_{\rm torus}$$

$$-\; a2\pi i q_i^{-}\langle\, e^{\phi}T_{\rm F}V_a\; V_{\Psi_i}\; c\bar c V_{\bar\Psi_{\bar i}}|z_3 \;-\; z_4|^{2k_3\cdot k_4}\,\rangle^{\rm PP}_{\rm torus} \quad . \qquad (102)$$

These two contributions correspond to Figs.9b and 9c.

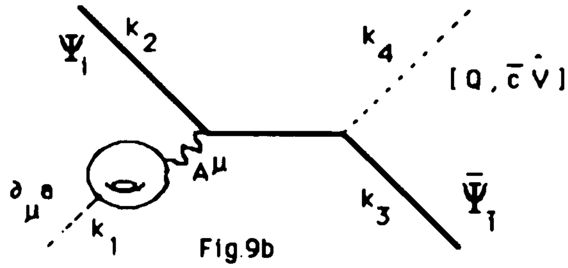

Fig.9b

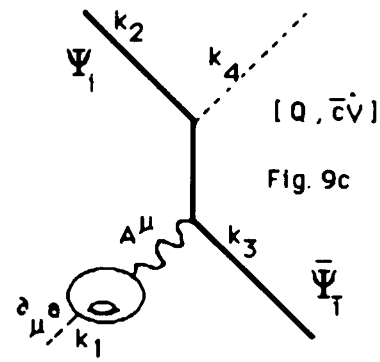

Fig. 9c

We use the OPE's

$$V_{\Psi_i}^{(-1/2)}(z_2,\bar z_2)V_{\bar\Psi_i}^{(-1/2)}(z_4,\bar z_4) \;=\; \frac{\zeta'_\mu}{|z_2 \;-\; z_4|^{2-2k_2\cdot(k_3+k_4)}}V_A^\mu(z_4,\bar z_4,-k_1)+\ldots \quad ,$$

$$V_{\bar\Psi_i}^{(-1/2)}(z_2,\bar z_2)V_{\Psi_i}^{(-1/2)}(z_4,\bar z_4) \;=\; -\frac{\zeta'_\mu}{|z_2 \;-\; z_4|^{2-2k_2\cdot(k_3+k_4)}}V_A^\mu(z_4,\bar z_4,-k_1)+\ldots \quad , \tag{103}$$

with V_A in the -1 picture and polarisation tensor $\zeta'^\mu \;=\; \Psi_i\sigma^\mu\bar\Psi_{\bar i}$. The analogous expression for the second term has the opposite sign. This can be interpreted as due to the exchange of the 2 fermion vertex positions on the world-sheet. We have used the OPE's

$$S_\alpha(z)\bar S_{\dot\alpha}(w) \;=\; \sigma^\mu_{\alpha\dot\alpha}\psi_\mu(w) \;+\; \ldots \quad ,$$

$$\bar S^{\dot\alpha}(z)S^\alpha(w) \;=\; -\bar\sigma^{\dot\alpha\alpha\mu}\psi_\mu(w) \;+\; \ldots \quad . \tag{104}$$

and relation[54], $\bar\Psi_{\bar i}\bar\sigma^\mu\Psi_i \;=\; -\Psi_i\sigma^\mu\bar\Psi_{\bar i}$. Note that the minus sign in the second OPE in eqn. (104) is consistent with the anticommutation of the supersymmetry charges Q_α and $\bar Q^{\dot\alpha}$.

Eqn. (102) becomes,

$$-a^2 2\pi i(-4\pi i\ln\epsilon)\big[q_i^+\langle\, V_a\, c\bar c\zeta'\!\cdot\! V_A e^\phi T_{\rm F}\,\rangle^{\rm PP}_{\rm torus} + q_i^-\langle\, V_a\, c\bar c\zeta'\!\cdot\! V_A e^\phi T_{\rm F}\,\rangle^{\rm PP}_{\rm torus}\big] \quad , \tag{105}$$

We have used momentum conservation and the fact that these states are massless at tree level. Using (97), one finds

$$-4\pi i a^2 q_i(-\epsilon_{\mu\nu\rho\lambda}\zeta^{\mu\prime}\epsilon^{\nu\rho}k_1^\lambda m_{a-A})(-4\pi i \ln\epsilon) \;=\; 4\pi i a^2 q_i \zeta' \cdot k_1 m_{a-A}(-4\pi i \ln\epsilon) \quad , \tag{106}$$

where we have used the duality condition, $\epsilon^{\mu\nu\rho\lambda}k_{1\ \nu}\epsilon_{\rho\lambda} \;=\; k_1^\mu$, from the relation between the antisymmetric tensor $B_{\mu\nu}$ and the axion a, $*\mathrm{d}B \;=\; \mathrm{d}a$.

The expected tree level anomalous correlations are four-point functions,

$$\langle\; c\bar{c}V_A^{\mathrm{ren}}(z_1,\bar{z}_1)\; c\bar{c}V_{\Psi_i}(z_2,\bar{z}_2)\; V_{\bar{\Psi}_i}(z_3,\bar{z}_3)\; c\bar{c}V(z_4,\bar{z}_4)\;\rangle_{\mathrm{sphere}} \quad ,$$

$$\langle\; c\bar{c}V_a^{\mathrm{ren}}(z_1,\bar{z}_1)\; c\bar{c}V_{\Psi_i}(z_2,\bar{z}_2)\; V_{\bar{\Psi}_i}(z_3,\bar{z}_3)\; c\bar{c}V(z_4,\bar{z}_4)\;\rangle_{\mathrm{sphere}} \quad , \tag{107}$$

where we use renormalised vertex operators for the gauge boson and axion vertex operators [33]. The renormalised vertex operators are in the -1 ghost charge picture with the fermion operators in the $-1/2$ ghost picture. The null state vertex operator is the same as before. One needs to add the corresponding contributions to (107) where, instead of Ψ_i, the $\bar{\Psi}_i$ vertex is accompanied by the reparametrisation ghosts $c\bar{c}$. In a sense, one averages over the possible positions of $c\bar{c}$ and thus includes a factor of $1/2$. This causes irrelevant total derivative contributions from the charged vertex operators to cancel. The tree level transversality of the polarisation tensor for the axion vertex, $\epsilon_{\mu\nu}k_1^\mu = 0 = \epsilon_{\mu\nu}k_1^\nu$, is also valid for the BRST invariant renormalised vertex operator. In addition, the tree-level constraint on the gauge boson polarisation tensor, $\zeta_\mu k_1^\mu \;=\; 0$, continues to hold. This is in fact necessary for BRST invariance of the renormalised gauge boson vertex operator.

The anomalous contributions with the renormalised axion vertex in eqn. (107) cancel. Thus, we consider only the first term in eqn. (107) and its counterpart with $c\bar{c}$ at z_3 instead of z_2. Rewriting the BRST contour in the 2 correlation functions as before, one finds

$$\frac{k_1^{\mathrm{ren}\,2}}{2}\langle\; c\partial c\bar{c}V_A^{\mathrm{ren}}\; c\bar{c}V_{\Psi_i}\; V_{\bar{\Psi}_i}\; \bar{c}\hat{V}\;\rangle_{\mathrm{sphere}} + \frac{k_1^{\mathrm{ren}\,2}}{2}\langle c\partial c\bar{c}V_A^{\mathrm{ren}}\; V_{\Psi_i}\; c\bar{c}V_{\bar{\Psi}_i}\; \bar{c}\hat{V}\rangle_{\mathrm{sphere}} \tag{108}$$

The four-point correlation functions are then evaluated, as shown in section 2.1, by taking the limits $z_2, z_3, z_1 \to z_4$, in that order, in the first term, and $z_3, z_2, z_1 \to z_4$ for the second term. This results in the 4-point tree amplitudes in Figs.9d and 9e.

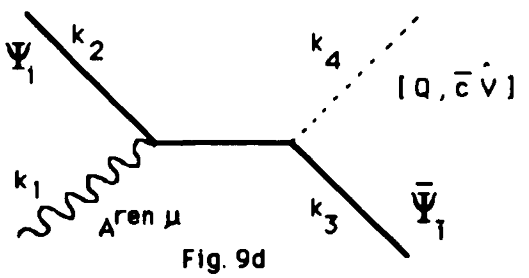

Fig. 9d

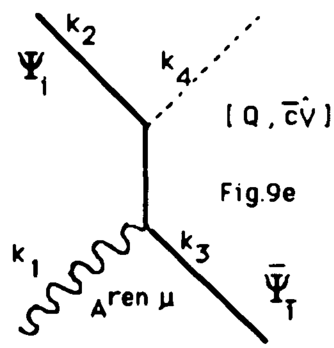

Fig.9e

The total contribution is,

$$\frac{a^4}{\Omega_0} q_i \, \frac{\sqrt{2}}{4} \zeta \cdot \zeta' \; k^2_{\rm ren}(-4\pi i \ln \epsilon) \quad , \tag{109}$$

where a factor of $1/2$ has been inserted in the averaging over the positions of ghost insertion $c\bar{c}$. This contribution cancels that of eqn. (106) only if we make the following identifications :

$$\zeta_\mu m_{a-A} \;=\; i2\sqrt{2} k_{1\,\mu} \quad ,$$

$$k^2_{\rm ren} \;=\; -m^2_{a-A} \tag{110}$$

This is equivalent to the identification $\partial_\mu a \;=\; \sqrt{2} m_{a-A} A_\mu / 4$. Thus, the anomalies cancel only if the axion field becomes the longitudinal component of the gauge boson. The anomalous amplitudes of Figs.9b and 9d, as well as Figs.9c and 9e cancel each other.

In the mass renormalisation of boson vertex operators considered, the anomalous tree-level correlations are proportional to mass square. The one-loop mass corrections for charged scalars are also proportional to mass square and are quadratic in the gauge coupling, $O(g^2)$. It is thus possible for the 2 contributions to cancel each other. In the case just discussed, the axion - gauge boson coupling is linear in mass and $O(g^2)$, whereas the tree-level anomalous contributions are quadratic in mass. However, the axion coupling is a derivative coupling, and by identifying the axion derivative with an appropriately normalised product of the mass and the gauge boson field, we find that the anomalies at tree and one-loop levels cancel. A similar situation appears for fermions in the next section as the one-loop mass correction for fermions is linear in mass and also $O(g^2)$. Since the tree-level correlations found so far are mass square terms, one expects that the BRST anomalous contributions for fermions at one-loop and tree-level cannot cancel. It is expected that there will be two-loop BRST anomalous corrections to cancel these tree-level contributions at order $O(g^4)$ in the coupling constant, but this will not be considered here. In the next section, one finds that only the picture changing operator, T_F, in the piece of BRST charge $Q^{(1)}$ with +1 superconformal charge, can give a contribution linear in mass that is non-vanishing and which has the correct structure to cancel the one-loop contribution.

3.3.3 Dilatino - Gaugino Mass Renormalisation

The coupling of the gaugino χ to the dilatino λ is linear in mass and of order g^2,

$$\langle\, V_\chi \; c\bar{c}V_\lambda \,\rangle_{\rm torus} \;=\; i(\bar{\chi}\bar{\lambda} \;-\; \chi\lambda)\; m_{\chi-\lambda} \quad , \tag{111}$$

with spinors in 2-component notation and $m_{\chi-\lambda} \;=\; (g^2/192\pi^2)\,{\rm Tr}\,Q\;{\rm M_S}$.

Consider the anomalous one-loop four-point correlation function,

$$\langle\, V_\lambda(z_1,\bar{z}_1)\; V_{\Psi_i}(z_2,\bar{z}_2)\; V_{\phi_i}(z_3,\bar{z}_3)\; c\bar{c}V(z_4,\bar{z}_4)\,\rangle_{\rm torus} \quad , \tag{112}$$

which corresponds to Fig.10a.

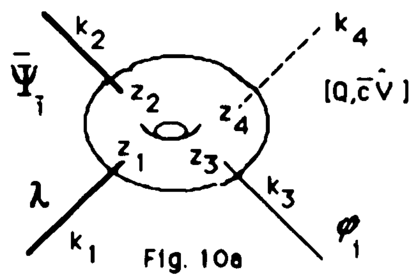

Fig. 10a

The fermion vertex operators are in the $\pm 1/2$, and the boson vertex operators 0 superconformal ghost pictures respectively. A sum over all spin structures is implicit. Here, the appropriate null state is as before with $\hat{V}$ from eq. (85). The tree-level dilatino vertex in the $-1/2$ picture is

$$V_\lambda \;=\; \zeta_{\mu\nu}\bar{\partial}X^\mu e^{-\frac{\phi}{2}}[\lambda\bar{\sigma}^\nu\bar{S}e^{i\frac{H}{2}} \;+\; \bar{\lambda}\sigma^\nu S e^{-i\frac{H}{2}}]e^{ik_1\cdot X} \quad . \tag{113}$$

The corresponding vertex in the $+1/2$ picture is

$$V_\lambda \;=\; \zeta_{\mu\nu}\bar{\partial}X^\mu e^{+\frac{\phi}{2}}(\partial X_\rho \;-\; i\frac{k_1}{2}\cdot\psi\psi_\rho)[\lambda\bar{\sigma}^\nu\sigma^\rho S e^{i\frac{H}{2}} \;+\; \bar{\lambda}\sigma^\nu\bar{\sigma}^\rho\bar{S}e^{-i\frac{H}{2}}]e^{ik_1\cdot X} \quad . \tag{114}$$

The gaugino vertex in the $-1/2$ picture is

$$V_\chi \;=\; j_L e^{-\frac{\phi}{2}}[\bar{\chi}\bar{S}e^{i\frac{H}{2}} \;+\; \chi S e^{-i\frac{H}{2}}]e^{ik\cdot X} \quad . \tag{115}$$

BRST invariance of the tree-level vertex operators, in this case due to the picture-changing part of the BRST charge carrying the world-sheet supersymmetry current, requires that the spinors satisfy the massless Dirac equation, $k_{\mu\ 1}\sigma^\mu\bar{\lambda} \;=\; 0$ and $k_{\mu\ 1}\bar{\sigma}^\mu\lambda = 0$. Similarly for the gaugino spinors χ and $\bar{\chi}$.

Deforming the BRST contour one finds three terms one of which vanishes as it is charge neutral. The remaining 2 terms give

$$-2\pi i a\langle\, q_i^+ V_\lambda\, V_{\bar{\Psi}_i}\, c\bar{c}V_{\phi_i}|z_3 - z_4|^{2k_3\cdot k_4} + q_i^- V_\lambda\, V_{\phi_i}\, c\bar{c}V_{\bar{\Psi}_i}|z_2 - z_4|^{2k_2\cdot k_4}\,\rangle_{\rm torus} \quad , \tag{116}$$

where the coupling constant for gauge fields is $a \;=\; g/2\pi$ and $\sqrt{2}g/8\pi$ for the dilatino. We allow the charged fermion and scalar vertex positions to come close and use the OPE's,

$$V_{\bar{\Psi}_{\bar{i}}}^{(-1/2)}(z_2,\bar{z}_2)V_{\phi_i}^{(0)}(z_4,\bar{z}_4) \;=\; \frac{\sqrt{2}ik_{3\ \mu}\bar{\Psi}_{\bar{i}\ \dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\beta}}{2|z_2\ -\ z_4|^{2-2k_2\cdot(k_3+k_4)}}V_{\chi\ \beta}^{(-1/2)}(z_4,\bar{z}_4,-k_1)+\dots \quad ,$$

$$V_{\phi_i}^{(0)}(z_3,\bar{z}_3)V_{\bar{\Psi}_{\bar{i}}}^{(-1/2)}(z_4,\bar{z}_4) \;=\; -\frac{\sqrt{2}ik_{3\ \mu}\bar{\Psi}_{\bar{i}\ \dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\beta}}{2|z_3\ -\ z_4|^{2-2k_2\cdot(k_2+k_4)}}V_{\chi\ \beta}^{(-1/2)}(z_4,\bar{z}_4,-k_1)+\dots . \tag{117}$$

We note that there is a minus sign difference between the two OPE's which comes from a sign difference between the simple poles in : $g^{i\bar{j}}b_{i\bar{i}}b_{\bar{j}j}j^{ai}(\bar{z})j^{a\bar{j}}(0) = \bar{z}^{-2} + q_i\bar{z}^{-1}j_L(0) + \dots$ and $g^{i\bar{j}}b_{i\bar{i}}b_{\bar{j}j}j^{a\bar{i}}(\bar{z})j^{ai}(0) = \bar{z}^{-2} - q_i\bar{z}^{-1}j_L(0) + \dots$. The absence of this sign difference would allow exact cancellation of the 2 contributions in eqn. (116). We have used OPE's,

$$\psi^{\mu}(z)S_{\alpha}(0) = \frac{1}{\sqrt{2}z}\sigma^{\mu}_{\alpha\dot{\alpha}}S^{\dot{\alpha}} + \dots \tag{118}$$

and, $\theta^{i}(z)\theta^{\bar{j}}(w) \;=\; g^{i\bar{j}}/(z\ -\ w)^{2/3}\dots$ Eqn. (116) is now

$$\frac{1}{4}2\pi i a^2 ik_{3\ \mu}\bar{\Psi}_{\bar{i}\ \dot{\alpha}}\bar{\sigma}^{\mu\dot{\alpha}\beta}\langle\ q_i^- V_\lambda\ c\bar{c}V_{\chi\ \beta} - q_i^+ V_\lambda\ c\bar{c}V_{\chi\ \beta}\ \rangle_{\text{torus}} \tag{119}$$

By taking the limits as the positions z_2 and z_3 approach z_4, we find non-vanishing contributions which correspond to Figs.10b and 10c.

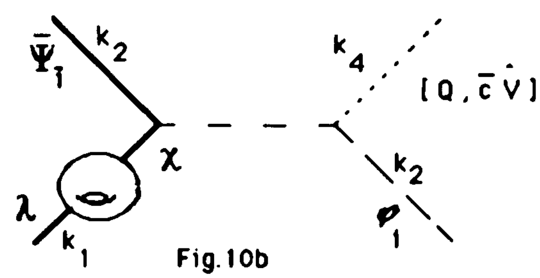

Fig.10b

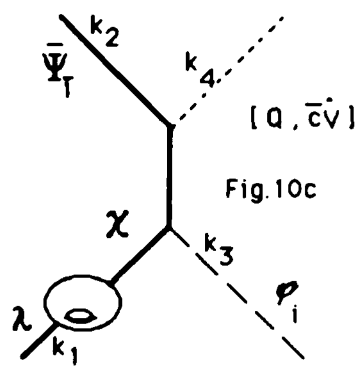

Fig.10c

Thus, using eqn. (111), the result is

$$\pi i^2 a^2 q_i k_{3\ \mu} \bar{\Psi}_{i\ \dot\alpha} \bar{\sigma}^{\mu\dot\alpha\beta} \lambda_\beta (-4\pi i \ln \epsilon)\ m_{\chi-\lambda} \quad . \tag{120}$$

The expected tree level anomalous correlation functions are,

$$\langle\ c\bar{c}V_\lambda^{\mathrm{ren}(-3/2)}(z_1,\bar{z}_1)\ c\bar{c}V_{\Psi_i}^{(-1/2)}(z_2,\bar{z}_2)\ V_{\phi_i}^{(0)}(z_3,\bar{z}_3)\ c\bar{c}V^{(0)}(z_4,\bar{z}_4)\ \rangle_{\mathrm{sphere}} \quad ,$$

$$\langle\ c\bar{c}V_\chi^{\mathrm{ren}(-3/2)}(z_1,\bar{z}_1)\ c\bar{c}V_{\Psi_i}^{(-1/2)}(z_2,\bar{z}_2)\ V_{\phi_i}^{(0)}(z_3,\bar{z}_3)\ c\bar{c}V^{(0)}(z_4,\bar{z}_4)\ \rangle_{\mathrm{sphere}} \quad , \tag{121}$$

which would give mass square contributions in the same way as for boson vertex operators. These should be cancelled by two-loop contributions. Note that the renormalised vertex operators for fermions are the same as the tree-level vertex operators except that the spinors now satisfy the massive Dirac equation. They also carry prefactors as for the renormalised boson vertex operators.

We expect, then, to find linear mass contributions from the following correlation functions where the BRST exact state carries +1 superconformal ghost charge. This null state thus depends on the +1 ghost piece of the BRST charge and is proportional to the world sheet supersymmetry current responsible for picture changing vertex operators.

$$\langle\ c\partial c\bar{c}V_\lambda^{\mathrm{ren}(-3/2)}(z_1,\bar{z}_1)\ c\bar{c}V_{\Psi_i}^{(-3/2)}(z_2,\bar{z}_2)\ V_{\phi_i}^{(0)}(z_3,\bar{z}_3)\ \bar{c}V'^{(1)}(z_4,\bar{z}_4)\ \rangle_{\mathrm{sphere}} \quad ,$$

$$\langle\ c\partial c\bar{c}V_\chi^{\mathrm{ren}(-3/2)}(z_1,\bar{z}_1)\ c\bar{c}V_{\Psi_i}^{(-3/2)}(z_2,\bar{z}_2)\ V_{\phi_i}^{(0)}(z_3,\bar{z}_3)\ \bar{c}V'^{(1)}(z_4,\bar{z}_4)\ \rangle_{\mathrm{sphere}} \quad . \tag{122}$$

The dilatino, charged fermion, charged scalar and null state vertex operators are respectively in the $-3/2$, $-3/2$, 0 and +1 superconformal ghost pictures. The charged fermion state remains massless up to one loop. As for the axion-gauge boson calculation, one must add the contribution where V_{ϕ_i} carries $c\bar{c}$ instead of V_{Ψ_i}. It can be shown that contributions involving the dilatino vertex eliminate each other. The remaining contributions come from the gaugino correlation functions in eqn. (122).

The choice of the tree level contributions used in this section can be seen to be consistent with a pinching procedure. in which a long tube connecting the torus to the sphere pinches off in the limit of an infinitely long thin tube. The tube, in the limit as its length becomes infinite, is equivalent to a two-point function on a sphere in the limit (in the complex plane) as one state approaches the origin and its conjugate is taken to infinity. The 2 conjugate vertex operators must carry reparametrisation ghosts in the combination c and its conjugate $c\partial c$ to soak up the tree level backgound charge†. The same applies to the antiholomorphic ghosts. The fermionic string allows at least 2 possibilities where in addition to the reparametrisation ghost charges, the total superconformal ghost charge must add up to -2. For a tube with a pair of conjugate bosons, the -1 ghost picture would be consistent with non-vanishing correlations. For fermions, it is consistent to use conjugate states in the $-1/2$ and $-3/2$ pictures. In general, in the degeneration of a genus g Riemann surface into 2 pieces of genera g_1 and g_2, with $g_1 + g_2 = g$, it is unclear which pair of conjugate states, i.e. which picture, is preferred especially since picture changing operators can be located on the connecting tube. We have avoided this problem by not using the pinching procedure. It is found to be sufficient but not necessary, to choose contributions with vertex operators in certain superconformal ghost pictures, for cancellation of BRST anomalies.

The null state operator in the $+1$ ghost picture is a $(1,0)$ operator,

$$V'^{(1)} = [Q^{(1)}, \hat{V}] = \frac{\gamma i k_4 \cdot \psi}{2} j_L e^{ik_4 \cdot X} \quad , \tag{123}$$

where $Q^{(1)} = (-1/2) \oint \gamma\psi \cdot \partial X$ is the part of the BRST charge with $+1$ superconformal ghost charge, and $\gamma = e^{\phi}\eta$. The null states used previously and in (123) are in fact related. This is clear in a superspace formulation[26],[7], with

$$\{Q, \hat{V}(\tilde{z}, \bar{z})\} = \oint dz \partial \left[c(z)\hat{V}^{(0)}(z, \bar{z}) - V'^{(1)}(z, \bar{z}) \right] \quad . \tag{124}$$

$\tilde{z} = (z, \theta)$, where θ is the supersymmetric counterpart of coordinate z and the super-

† Again we choose to use only vertex operators that have zero reparametrisation ghost charge and which are primary.

conformal ghost pictures are indicated. The null state, $\hat{V}(\tilde{z},\bar{z}) = j_L(\bar{z})e^{ik_4\cdot X(\tilde{z},\bar{z})}$. It is necessary, because of the presence of the η fermion[26] in eqn. (123), that for non-vanishing correlations there be another ξ field in the correlator in addition to one already present implicitly to absorb the ξ zero mode. With this fact, we will use the following relations for a massive fermion vertex operator,

$$\{Q^{(1)}, V_{\rm F}^{(-3/2)}\} = 0$$

$$[Q^{(1)}, \xi V_{\rm F}^{(-3/2)}] = im\frac{\sqrt{2}}{4}V_{\rm F}^{\dagger(-1/2)} \quad , \tag{125}$$

with superconformal ghost charges indicated. The coefficient of $\sqrt{2}/4$ is due to the normalisation of $Q^{(1)}$ and eqn. (118). The fermion vertex operator $V_{\rm F}^{\dagger(-1/2)}$ is the same vertex operator as $V_{\rm F}^{(-3/2)}$ but has the opposite 4-dimensionality chirality. This result uses Dirac's equation for a massive 2-component spinor $U_\alpha(k,\ k^2 = -m^2)$,

$$k_\mu \bar{\sigma}^{\mu\dot{\alpha}\alpha}U_\alpha(k) = m\bar{U}^{\dot{\alpha}}(k) \quad , \tag{126}$$

which relates the two 4-dimensional chiralities.

The first correlation function in eqn. (122) involving the dilatino has contributions which cancel exactly. For the remaining correlation function, the contour integral is deformed with 2 surviving non-vanishing terms.

$$-i\frac{\sqrt{2}}{4}\langle\ c\partial c\bar{c}V_\chi^{\prime{\rm ren}(-1/2)}\ c\bar{c}V_{\Psi_i}^{(-3/2)}V_{\phi_i}^{(0)}\ \bar{c}\hat{V}^{(0)}\ \rangle_{\rm sphere}$$

$$-i\frac{\sqrt{2}}{4}\langle\ c\partial c\bar{c}V_\chi^{\prime{\rm ren}(-1/2)}\ V_{\Psi_i}^{(-3/2)}\ c\bar{c}V_{\phi_i}^{(0)}\ \bar{c}\hat{V}^{(0)}\ \rangle_{\rm sphere} \quad , \tag{127}$$

where the spinor in $V_\chi^{\prime{\rm ren}(-1/2)}$ is now $k_{1\nu}\chi^\alpha\sigma^\nu_{\alpha\dot{\alpha}}S^{\dot{\alpha}}$, and similarly for the other chirality. We have used eqns. (125) and (126). One can now evaluate the correlation functions by making a choice of kinematics as before, by taking the limits

$z_2, z_3, z_1 \rightarrow z_4$, in the given order, for the first term, and for the second term, $z_3, z_2, z_1 \rightarrow z_4$. One obtains the four-point amplitudes for the 's' and 't' channels in Figs.10d and 10e.

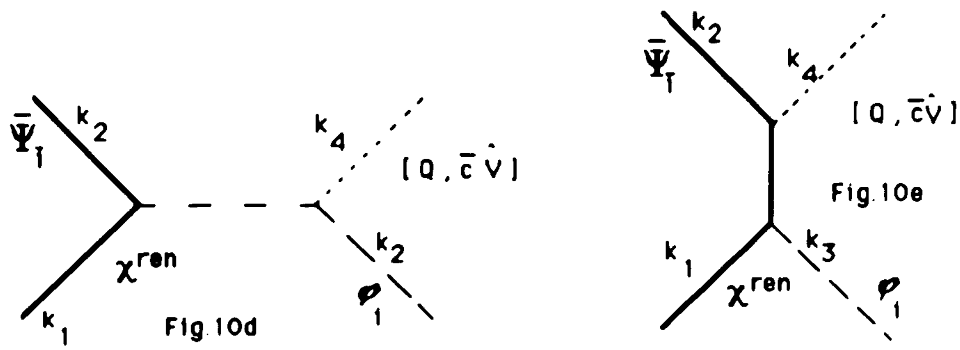

Fig 10d Fig 10e

Finally, adding the two contributions with a factor 1/2 to average over the positions of $c\bar{c}$ as before, one finds

$$\frac{1}{2}\frac{a^2}{2\Omega_0} q_i k_{3\mu} k_{1\nu} \bar{\Psi}_i \bar{\sigma}^{\mu} \sigma^{\nu} \bar{\chi}(-4\pi i \ln \epsilon) \quad . \tag{128}$$

One must identify the spinors,

$$k_{1\nu}\sigma^{\nu}\bar{\chi} = m_{\chi-\lambda}\lambda \quad , \tag{129}$$

in order for the one-loop anomaly of eqn. (120) to cancel the tree-level anomaly of eqn. (128) above. The massless 2-component spinors for the gaugino and the dilatino thus pair up to form a massive 4-component spinor in 4-dimensions. One can say that these anomalous contributions have been eliminated by cancelling contributions between amplitudes in Figs.10b with 10d, and Figs.10c with 10e.

3.4 BRST invariance of renormalised vertex operators

It is well-known[27] that the $N = 2$ superconformal algebra(SCA) possesses global and local $U(1)$ symmetries that result in an isomorphism between SCA's differing by phase shifts η (we review here the arguments of Ref.[27] and adopt their notation). More precisely, the world-sheet supersymmetry generators satisfy the periodicity conditions,

$$G^+(z) \rightarrow e^{2\pi i\eta}G^+(e^{2\pi i}z) \ , \ G^-(z) \rightarrow e^{-2\pi i\eta}G^-(e^{2\pi i}z) \ . \tag{130}$$

The local $U(1)$ symmetry allows the modes of the $U(1)$ charge generator and stress tensor to be simultaneously shifted as

$$J_n \rightarrow J_n - \frac{\hat{c}}{2}\eta\delta_{n,0} \ \ , L_n \rightarrow L_n - \eta J_n + \frac{\hat{c}}{4}\eta^2\delta_{n,0} \ , \tag{131}$$

in order to remove the phase shift in the modes of the supersymmetry generators, i.e.

$$\begin{aligned} &G_n^+ \underset{\text{global}}{\longrightarrow} e^{2\pi i\eta}G_n^+ \rightarrow G_{n+\eta}^+ \underset{\text{local}}{\longrightarrow} G_n^+ \ , \\ &G_n^- \underset{\text{global}}{\longrightarrow} e^{-2\pi i\eta}G_n^- \rightarrow G_{n+\eta}^- \underset{\text{local}}{\longrightarrow} G_n^- \ , \end{aligned} \tag{132}$$

and thus leave the SCA unchanged.

For parity conserving transformations, the SCA's of different η's are isomorphic with shifts in conformal dimension h and $U(1)$ charge q by

$$h \rightarrow h' = h - \eta q + \eta^2\frac{\hat{c}}{4} \ , \ q \rightarrow q' = q - \frac{\hat{c}}{2}\eta \ , \tag{133}$$

where central charge $\hat{c} = 2c/3$. In the mass renormalisation of vertex operators, the conformal dimensions of the operator is shifted, $h \rightarrow h' = h + \delta k^2 = h - \delta m^2$. If one interpretes the small mass shift as equivalent to a local $U(1)$ transformation, the

(2,2) SCA's are invariant under sufficiently small mass renormalisation. Solving for η, we find

$$\eta = \frac{2q}{D}\left(1 \pm \sqrt{1 - \frac{D\delta m^2}{q^2}}\right) \quad , \tag{134}$$

and the dimension of the manifold, $D = \hat{c}$. We note that the unitarity constraint on tree-level physical states is $h \geq q/2$, with equality for chiral or anti-chiral primary fields[55]. It can be seen, from eqn. (133), that $h' \geq q'$, with equality only if $\eta = 0$ and for primary tree-level states. As η must remain real, if $\delta m^2 > 0$, we find a bound on the bare 4-dimensional gauge coupling g_0, using for example δm^2 for the charged scalars in eqn. (82), $g_0^2 \leq 48\pi^2 q^2/N_{\rm gen} D q_i \,{\rm Tr}\, Q$. $N_{\rm gen}$ is the number of light generations after compactification to 4-dimensions. In the compactification of the $O(32)$ theory to $SO(26) \otimes U(1)$, for one generation ${\rm Tr}\, Q = 3 \times 24/\sqrt{3}$. This provides a relatively large upper bound, $g_0^2 \leq 2\pi^2 q^2/N_{\rm gen} D$. Thus, for three light generations, one finds an order of magnitude estimate, $g_0^2 \lesssim 1$. We have of course assumed, for the calculations here, a perturbation expansion about small g_0. The above condition is thus consistent with perturbation theory. On the other hand, one finds shifted $U(1)$ charges, $q' = q - {\rm O}(\delta m^2)$ or ${\rm O}(\delta m^2)$. The renormalised states now do not satisfy the tree-level condition for chiral primary states, i.e. $h = q/2$ but $h' > q'/2$.

It has been shown[56] that when supersymmetry is broken by one-loop mass corrections in a compactified theory with an anomalous $U(1)$ gauge symmetry, the boson-fermion mass splitting for both massless and massive states is proportional to g_0^2. One can thus regain BRST invariant vertex operators in the same way for massive states as well.

The mass shift for the charged scalars lead to 'twists' in the left and right moving $N = 2$ internal SCA's. The internal world-sheet supercharges pick up non-trivial phases when circulated around the charged scalar vertex positions. For the axion and gauge boson vertex operators, the corresponding mass shift leads to phases in the $N = 2$ SCA for free transverse coordinates that is manifest in the light cone gauge. The renormalised vertex operators for the charged scalars, axion and gauge boson are thus BRST invariant only if their respective SCA's are phase shifted to isomorphic SCA's. A similar interpretation for the gaugino and dilatino vertex operators would also be necessary with the additional constraint that the renormalised fermion vertex operator is BRST invariant only in the $-3/2$ superconformal ghost picture. This is because the fermion vertex operator in the $-1/2$ ghost picture can only be BRST invariant if its spinors satisfy the massless Dirac equation.

Conclusions

We have shown that the BRST formalism can be used to renormalise bosonic and fermionic vertex operators for masses generated at one loop. One can consistently impose the constraint that BRST anomalies in correlation functions must be eliminated between contributions up to one-loop. The BRST symmetry of the theory, or equivalently the (super)conformal symmetry, is thus maintained under small mass corrections.

It is found that, at least up to one loop, operator product expansions can be used to evaluate correlation functions in a procedure equivalent to pulling off a sphere(with vertex operators) from the world-sheet until the connecting tube pinches off. The use of OPE's is local in contrast to the latter procedure involving global deformations of the world-sheet. The use of OPE's is feasible in one-loop computations because tree amplitudes can be factorised locally. Higher loop calculations involve the factorisation of loops and it not clear how to induce this locally.

Another problem, specific to the supersymmetric string, is the question of supermoduli and the ambiguity of correlation functions because of their dependence on the positions of supermoduli insertions on the world-sheet. This is known as a 'fermionic' ambiguity since it appears directly as a consequence of integration over anti-commuting or Grassmann variables. There is no clear physical basis for such a dependence. However, the commuting supersymmetric partners of the supermoduli, the moduli of the world-sheet, are well-understood. The moduli parametrise inequivalent two-dimensional surfaces with handles, and physical observables must be invariant with respect to the symmetries of the parameters or moduli. Modular symmetry is related to unitarity and is needed for the finiteness of string correlation functions. For a supersymmetric string, modular symmetry is needed for unbroken spacetime supersymmetry. Modular invariance is thus crucial for the consistency of the theory.

In the present analysis, we find that the cancellation of BRST anomalous contributions is independent of the 'fermionic ambiguity'. This is because the position dependence of the supermodulus at one-loop can be reduced to a logarithmic dependence, $\ln \epsilon$, present in both anomalous contributions which cancel exactly. Since such a dependence would violate modular invariance the procedure used here is thus modular invariant.

A second problem is the extension of the BRST analysis to higher loops, independent of fermionic ambiguities and OPE's. The procedure outlined by Sen is not easily extended to arbitrary loops for the bosonic string because renormalised vertex operators do not have conformal dimensions $(1,1)$. This problem, first pointed out by Sen, prevents the cancellation of BRST anomalous contributions between different loop levels because loss of conformal invariance of vertex operator insertions implies that a uniform choice of the two-dimensional metric for different genus surfaces is not possible. This is necessary to compare local coordinates on surfaces with different genus. In the case of the heterotic string, $N = 2$ superconformal invariance enables one to regain BRST invariance for renormalised vertex operators and the above obstruction does not hold for the cancellation of higher loop BRST anomalous contributions.

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