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**Exchange rates and formation of expectations: Are nonlinear models appropriate?**

**Causse, Dominique, Ph.D.**

**City University of New York, 1989**

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**Exchange Rates and Formation of Expectations:  
Are Nonlinear Models Appropriate ?**

A

by

Mr. Dominique Causse

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## Abstract

**Exchange Rates and Formation of Expectations:  
Are Nonlinear Models Appropriate ?**

by

Mr. Dominique Causse

Advisor Professor: Salih Neftci

This dissertation studies *daily* changes of the French-deutschmark exchange rate over 9 years (1979-1987). The question of expectation formation and the effect of important shocks (elections, devaluations...) on this market, are addressed using *nonlinear time series models*. One classes of model is given special attention: the *Thresholds models*. The AIC criterion is used to identify the best model in this classe and the results are compared to an ARMA model. The dissertation gives some insights on the use of nonlinear models to characterize certain aspect of the exchange rate market. For example we are able to address the effect of interventions on exchange rate markets, by the Central Banks, on the french-deutschmark rate. Furthermore, the methodology used (nonlinear techniques ) sheds some light on the hypotheses of linearity and normality in econometric models.

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## Table of Contents

List of tables.....	vii
List of Graphs.....	vii
Introduction.....	1
Chapter 1. A Review of the Literature.....	7
Section 1.1. Structural Models.....	7
Section 1.1.1. Monetary Models.....	8
Section 1.1.2. Asset Market Models.....	10
Section 1.2. Structural Models in Question.....	12
Section 1.2.1 Structural Models versus Random Walk Models.....	13
Section 1.2.2. Filter Rules.....	15
Section 1.3. Nonlinear Models and Exchange Rates Market.....	16
Section 1.3.1. Nonlinear Time series Models of Exchange Rates.....	17
Section 1.3.2. Nonlinear Structural Models of Exchange Rates.....	21
Chapter 2. The European Monetary System.....	23
Section 2.1. The European Monetary System Institutions.....	23
Section 2.1.1. The EMS Institutional Factors.....	24
Section 2.1.2. The EMS Operating System.....	25
Section 2.1.3. The ECU.....	29
Section 2.2. Exchange Rate Developments.....	30
Section 2.2.1. Overview: 1979 - 1988.....	30
Section 2.2.2. Variability of the EMS exchange rates.....	31
Section 2.2.3. Econometric forecasting of the EMS exchange rates.....	32
Chapter 3. Methodology.....	34
Section 3.1. Data Analysis and Nonlinear Models.....	34
Section 3.2. Identification.....	37
Section 3.4. Diagnostic Checking.....	39

Chapter 4. Data Analysis.....	40
Section 4.1. The Rate of Change.....	42
Section 4.1.1 Statistical analysis.....	45
Section 4.1.2 Looking for nonlinearity.....	50
Section 4.2. The Indexed Spot Rate .....	53
Section 4.2.1 Statistical analysis.....	57
Section 4.1.2 Looking for nonlinearity.....	61
Chapter 5. Threshold Models.....	63
Section 5.1. Presentation of the Threshold Model.....	64
Section 5.2. Identification and Estimation.....	68
Section 5.2.1. The Rate of Change.....	70
Section 5.2.2. The Indexed Rate.....	75
Section 5.3. Simulation Results.....	82
Section 5.3.1. The Rate of Change.....	82
Section 5.3.2. The Indexed Rate.....	86
Conclusion.....	90
Bibliography.....	92

### List of tables

Table #2-1	EMS Realignments.....	31
Table #4-1	Summary statistics for the rate of change .....	44
Table #4-2	Summary Statistics for the indexed rate .....	56
Table #5-1	Threshold Model and the Rate of Change.....	72
Table #5-2	The ARMA Model and the Rate of Change.....	73
Table #5-3	Threshold Model and the Indexed Rate.....	77
Table #5-4	The ARMA Model and the Indexed Rate.....	78
Table #5-5	Statistics on the Setar Simulated rate of change.....	83
Table #5-6	Statistics on the Setar Simulated Sindt.....	87

### List of Graphs

Graph#4-1	The Spot FDM Exchange Rate .....	41
Graph#4-2	Plot of the FDM rate of change .....	43
Graph#4-3	Correlation and Partial Correlation function of rate of change.....	48
Graph# 4-4	Histogram of rate of change.....	49
Graph# 4-5	Plot of the Indexed FDM rate of change .....	55
Graph# 4-6	Correlation and Partial Correlation function of Indexed Rate.....	59
Graph# 4-7	Histogram of Indexed Rate.....	60
Graph#5-1	Correlation function of the residuals from the threshold model applied to rate of change.....	74
Graph#5-4-	Correlation function of the residuals from the threshold model applied to Indexed Rate .....	79
Graph#5-5	Correlation function of the squared residuals from the threshold model applied to Indexed Rate .....	80

Graph#5-6	Correlation function of the residuals from the ARMA model applied to Indexed Rate .....	81
Graph #5-7	Simulation of rate of change using a Threshold model.....	84
Graph#5-8	Simulation of rate of change using an ARMA model.....	85
Graph#5-9	Simulation of Indexed Rate using a Threshold model .....	88
Graph#5-10	Simulation of Indexed Rate and the ARMA model -.....	89

## Introduction

Data on daily exchange rates are of practical interest to any economic agent involved in international transactions, either to cover himself against a risk of change or to make a speculative profit. Recently the extreme variations of the dollar have sensibilized an increasing number of people, from economists to politicians, to exchange rates. It is accepted by most that extreme variations in the exchange rates have negative effects on growth and trade. In particular, short-run variability is said to be costly in the sense that the associated uncertainty reduces the volume of trade and investment. Furthermore it has also been recognized that real exchange rates of major currencies have been subject to large and persistent misalignments, interpreted as deviation from the assumed equilibrium level<sup>1</sup>.

But little consensus appears on the evaluation of the associated cost to misalignment and volatility and on the way to regulate those markets. A

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<sup>1</sup> Misalignment has been measured as the departure of nominal exchange rates from the implied Purchasing Power Parity equilibrium level, deviation of real exchange rates from its trend (based on the implicit assumption that only gradual changes should take place) and deviation of exchange rates from the level that would yield an equilibrium in the balance of payments. See chapter 1: Review of the Literature, for more details on these points.

particular controversial point is the necessity of intervention by Central Banks. This explains the considerable interest in modeling and projecting exchange rates, especially since the floating exchange rates system was adopted in 1973.

Yet it seems that most econometric models of exchange rates have not been able to give an accurate description of the underlying dynamics of these markets. Deviations from equilibrium for extremely long periods and sudden bursts remain unexplained. These models all have in common the assumption that the dynamics of the exchange markets can well be approximated by a linear model. But in the past two decades and more recently in economics there has been an increasing interest for the use of nonlinear models. For example, such models have been applied to stock market price series with some success.

There are no compelling reasons to believe that economic agents on the exchange rate market, construct their expectations as a linear forecast based on market fundamentals or on recent variations in the exchange rate. In fact, there are many reasons to consider that the process generating the exchange rate is nonlinear. Nonlinear process may imply non-symmetry, non-reversibility, the existence of thresholds and sudden bursts. If expectations are rational and therefore are based on the "true model", fitting a linear model might leave too much information in the error term and test of rational expectation could be bias. In the following paragraph we provide examples, from the FDM exchange rate market, which shows that the underlying process generating the exchange rates may very well be nonlinear.

## Symmetry

Participants on the market might react differently to an increase or decrease of the spot rate. This seems to be the case for the FDM rate. We will see that the largest positive fluctuations outweigh, on the average, the largest negative fluctuations.

## Thresholds

The possible existence of thresholds, which is a very important concept in Nonlinear Theory, is not to be ignored. The application of the "% rule" or filter rules, could imply such thresholds. Under such a rule, one does not sell (buy) a currency *gradually* as the rate goes down (up) but only as the decrease (increase) reaches a certain level or *threshold*.

Another possible reason for thresholds to exist is institutional. In the European monetary System, Central Banks must intervene on the market when their currency deviates more than 2.25% from an official level. The intervention limit could act as a threshold. Knowing that the Central Bank will try to "lean against the wind", expectations from the participant of the exchange rate market might be generated differently. This could explain the existence of positive filter rules profits. We will further develop this point in the following chapters.

## Sudden jumps

The market is certainly influenced by political events (elections) or announcements and changes in the monetary policy, exchange rates controls and sudden realignments. These shocks imply important jumps in the day-to-day rate of change which may not be correctly modeled as a linear process. For example, Flood and Garber (1983) show, that if the monetary authority is expected to change its exchange rate policy, and the timing of the switch is unknown, then if economic agents are rational, one must include in the model expectation of this switch in policy. The resulting exchange rate equation will be nonlinear.

Finally, even if individual forecasts were linear or slightly nonlinear, it is possible that, at the aggregate level, the overall forecast would not be linear. May(1976) has shown that the output of a system of very simple deterministic equations, could be highly nonlinear and even chaotic<sup>2</sup>.

---

<sup>2</sup> Study of chaotic model is a fairly recent area of research in economics. Reference articles on "chaos" include Grandmont JM (1985) in economics and Grassberger P and Procaccia I (1983) in physics. Readers interested may also refer to a technical overview of these models by Gumowski I and Mira C (1980) and, for an example of an application in economics, to the paper of Brock W A and Sayers C I (1986).

In the next chapters we would like to determine whether nonlinear models, as compared to linear models, are more accurate when modeling exchange rates. This work is original in the following ways:

- First, we will use a class of Threshold models which are able to capture a wide variety of nonlinearities. More specifically they are able to model sudden jumps and switches in the driving process.
- Second, since these models only use past values of the dependent variable and error terms as regressors, we are able to use daily data. The most important filter rule profits were found for such data.
- Finally the exchange rate studied in this paper is the french-deutschmark exchange rate (FDM) which allows us to study the effect of important shocks, such as devaluations and systematic interventions from the central banks on the process of expectations formation.

In Chapter 1, we will go over the literature on both structural models and time series models. We will also look at some of the more recent literature, which attempts to apply nonlinear models to exchange rate models. Chapter 2 goes over the history of the European Monetary System, its institutions and organization. Chapter 3 presents the Methodology used in this paper for data analysis and model building. Chapter 4 analyzes the data. Chapter 5 studies the Threshold model and shows how it can be used to look both at the effect of volatility and intervention of the Central banks. It also gives some simulation results and compares the results to

the ones obtained by an ARMA model. Finally some conclusions and directions for further research are given.

## **Chapter 1. A Review of the Literature**

In the next section we will go over the literature on structural exchange rate models and see some of the most important critics that have been addressed to these models. One of these critics refers to the assumption that the "best" forecast is a linear forecast. We will then look at some of the more recent work, which uses nonlinear models, showing that they can characterize a wide variety of behavior which could not have been represented under a linear model.

### **Section 1.1. Structural Models**

Structural models try to represent an economic behavior based on some theoretical aspect. Therefore, they generally include behavior equations to model the behavior of economic agents, structural equations based on what we know of the economy, and equilibrium equations which are there to ensure the overwhole homogeneity of the model.

### Section 1.1.1. Monetary Models

The Purchasing Power Parity (PPP) concept has served as a building block for many exchange rates models of the past two decades. The basic notion is that exchange rates between any two currencies should converge in equilibrium to a ratio of aggregate price indices of the two countries, multiplied by a constant scaling factor. It can also be expressed in the following way:

the percent change in the exchange rate ( $e_t$ ) of a domestic currency (d) against a foreign currency (f) should be equal to the difference between the percent rate of inflation of the domestic country ( $\pi_t^d$ ) and of the foreign country ( $\pi_t^f$ ):

$$(1-1) \quad e_t = \pi_t^d - \pi_t^f$$

### Short Run Purchasing Power Parity (SRPPP)

The first models, which used a short run or continuous version of the PPP hypothesis, appeared in the 1970's. In these models the PPP hypotheses were used as a building block for flexible price monetary models by Frenkel (1976), Bilson (1978), Bilson (1979) and Hodrick (1978). This version assumes that good prices are perfectly flexible and thus that PPP holds instantaneously. The spot exchange rate ( $s_t$ ) is then defined as the ratio of the price of domestic currency ( $p^d$ ) to foreign currency ( $p^f$ ):

$$(1-2) \quad s_t = \frac{p^d}{p^f}$$

But at the end of the 1970's, it was clear that these types of models were rejected by the data. See for example Frenkel (1981) or Isard (1977).

### Long Run Purchasing Power Parity (LRPPP)

A weak form of the PPP hypothesis was therefore proposed. It was based on the assumption of time in-variant expectation of the long run PPP level ( $s_t^*$ ) defined as the nominal exchange rate multiplied by the ratio of the national price levels:

$$(1-3) \quad s_t^* = s_t \cdot \frac{p^f}{p^d}$$

This assumption was used in sticky price models developed by Dornbush (1976) and tested by Frankel (1979), Frankel (1981). In these models PPP is assumed to hold only in the long run. Test of the LRPPP hypothesis have focused on the coefficient of serial correlation between deviations of the real exchange rate from the sample mean or the trend line of the spot rate. This coefficient is interpreted as one minus the speed of adjustment towards equilibrium. But, as pointed out by Isard (1987), finding that the coefficient is less than unity, does not imply that LRPPP holds. This is due to the fact that these tests could not reject the overshooting hypothesis<sup>3</sup> (see section 1.2.1. for a discussion of other possible tests applied to the PPP hypothesis ).

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<sup>3</sup> The overshooting hypothesis state that shocks may cause even greater jumps to the level of exchange rate observed in the short run, then to the real exchange rate expected to prevail in the long run.

### Section 1.1.2. Asset Market Models

Another hypothesis, which has served as in the building block of many exchange rates models, is the Interest Parity Rule.

#### Covered Interest Parity Rule (CIP)

The concept of interest rate parity was first developed by Keynes (1923). It assumes that investors have a choice between holding assets denominated in domestic currencies, yielding an interest rate ( $r^d$ ) as opposed to foreign currencies, yielding an interest rate ( $r^f$ ). An arbitrage, therefore, takes place between [1] investing on the domestic market to accumulate  $(1+r^d)$  units of domestic currencies and [2] converting the domestic assets at the spot rate ( $s_t$ ) to invest in foreign assets, converting it back to the principal plus interests at the foreign exchange rate  $f$  to obtain  $s(1+r^f)/f$  units of domestic currencies. The arbitrageur would then follow the

Covered Interest Parity (CIP) rule:

$$(1-4) \quad (1 + r^d) = s (1 + r^f) / f$$

$$(1-5) \quad (f - s) / s = (1 + r^f) / (1 + r^d) - 1$$

$$(1-6) \quad (f - s) / s \approx r^f - r^d$$

### Uncovered Interest Parity Rule (UIP)

If investors are assumed to be able to deliver (or obtain)  $f$  units of foreign currencies at some future date in exchange for 1 unit of domestic currency, they will be tempted to take an uncovered position on the forward market if the expected future spot rate  $s^e$  offers them a chance to convert it back to a foreign (domestic) currency with an expected positive profit of:

$$(1-7) \quad s^e - f \quad (\text{or: } 1/s^e - 1/f)$$

Under the hypothesis of risk neutrality, we obtain the Uncovered Interest Parity rule (UIP):

$$(1-8) \quad s^e = f$$

$$(1-9) \quad (s^e - s) / s = (1 + r^f) / (1 + r^d)$$

$$(1-10) \quad (s^e - s) / s \approx r^f - r^d$$

It is usually accepted that a deviation from CIP reflects transaction costs, capital control or the fact that the data on interest rates are not sufficiently compatible (when Euro-currency are used, the deviation is usually negligible).

UIP remains more controversial. Tests, assuming rational expectations, which implies that the forward rate is an unbiased predictor of the future spot rate, and assuming that the unexplained variance can be counted for a risk premium effect, have been applied to the data. But Cumby and Obstfeld (1984) using weekly data and techniques to take into account the non-stationarity of the foreign exchange rate data, are able to reject that

hypothesis. A survey data set<sup>4</sup> analyzed by Frankel and Froot (1985) shows that the magnitude and the variance of the exchange rate risk premiums has been large, although not as large as the expected change of the spot exchange rate as found by Fama (1984). Frankel and Froot findings are not subject to peso problems or other problems related to indirect testing of the UIP hypothesis ( for example Fama's tests are based on the hypothesis of market rationality). Of course the quality of the survey can be put in question but on the whole, it seems that the magnitude of the risk premium makes it difficult to support the UIP hypothesis.

### **Section 1.2. Structural Models in Question**

Two particular points have raised questions on the ability for structural models to explain the observed variations of exchange rates:

- Their performances as compared to random walk models.
- Persistent positive filter rule profits.

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<sup>4</sup> The surveys are :

the Annex Bank Review survey which contains data collected from 11 surveys conducted between January 1976 and Jun 1984 on financial participants and economists, asked to record their expectation for the dollar-mark,pound,yen,suisse franc and french francs for a 6 month horizon.

the Economist surveys on 13 international major banks between Jun 1981 and March 1985 for the same currencies at a 3 and 6 month horizon.

### **Section 1.2.1 Structural Models versus Random Walk Models**

Statistical test of time in-variant expectation of the long run level real exchange rates are difficult to establish. The LRPPP hypothesis could still be rejected if there were clear evidence that real exchange rate follows a random walk, or exhibits no serial correlation.

Meese and Rogoff (1983) have compared the fit of various structural models of systematic exchange rate behavior to the random walk model. The period studied is March 1973 to Jun 1981 for the dollar/mark, the dollar/pound, the dollar/yen and a trade weighed dollar exchange rate. The models tested are the flexible price monetary model of Frenkel (1976) and Bilson (1978), Bilson (1979), a sticky price monetary model as developed by Dornbush (1976) , Frankel (1979) and Frankel (1981) and a hybrid specification of a trade balance and a sticky price monetary model close to the Hooper and Morton (1982) model. Rolling regression is used to construct post sample forecast based on combining the estimated parameters values with the realized values of the explanatory variables. A search over a wide range of possible parameter values is conducted, rejecting the possibility that the low performance of these models could be due to poor estimated parameter values. They show that these models could not outperform the random walk model in post sample forecasting tests for a one to twelve month horizon. This conclusion holds even when the forecasts were based on the ex-post realized values of the explanatory variables.

Meese and Rogoff (1985) extended their work to the other dollar currencies and non dollar rates (deutschmark-yen and deutschmark-pound) and to nominal and real exchange rates using November 1980 to Jun 1984 as a post-sample forecasting period. Compared to Meese and Rogoff (1983), the results are only "slightly more favorable" to the structural models. Woo (1985) uses an alternative specification of the monetary model which combines a classical long run money demand specification with a short run partial adjustment hypothesis. For the dollar-deutschmark exchange rate, over a forecasting period of March 1980 to October 1981, he finds that the model outperforms the random walk model. Finally Schinasi and Swamy (1986) reexamined the post-sample forecasting period tested in the first paper of Meese and Rogoff (1983); and found that under certain specification changes (non fixed coefficients over time for example) structural models could outperform forecast of random walk models.

This work has been extended to the asset type exchange rates models. Backus (1984) tested the portfolio balance model including stocks of outside assets and net foreign assets positions for the US dollar and the Canadian dollar. Backus finds that his model is outperformed by a random walk model as well.

Although these last results show some improvements, the fact that structural models explain little of the variance of exchange rates remains.

### **Section 1.2.2. Filter Rules**

It is interesting that filter rules<sup>5</sup> seem able to exhibit systematic profits on the foreign exchange rates markets, especially in regard to the low explanatory power of structural models of systematic exchange rates behavior noted in the precedent section.

Dooley and Shafer (1976) find that 15% to 20% filter rules applied to daily exchange rate data give positive annualized profits even when adjusting for transaction costs. Using a CAPM model, Sweeney (1986) has developed a test of the statistical significance of risk adjusted profits from filter rules. These have been shown to be positive even when taking into account transaction costs. Furthermore, he shows that, when dividing his sample into two sub-samples, the best filter rules of the first sub-sample still give significant profits in the second sub-sample. Since the test assumes constant risk premium, Sweeney's results imply that either the positives profits can be explained by time varying risk premium, but we have seen that test found little evidence for risk premium, or by evidence of significant speculative profits and inefficiency.

The apparent contradiction of positive filter rules and the low explanatory power of the above structural models could be due to the fact that most models developed to this date do not take into account an important feature of the data at hand. In fact, they assume that linear projections are

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<sup>5</sup> Under such a rule one sells (buys) a foreign currency each time it falls (rises) of more than X% under (above) it's previous value.

the "best" projections. But exchange rates time series have been shown to exhibit skewness and kurtosis<sup>6</sup>. See for example Friedman and Vandersteal (1982). Furthermore, tests on foreign exchange rates models of both weekly and monthly data, have usually rejected homoscedasticity error structure, see Cumby and Ofsfeld (1984), Gregory and McCurdy (1986) and Hasen and Hodrick (1983). The implication is that usual statistical and econometric procedures based on normality hypothesis will be inappropriate and that nonlinear techniques should therefore be used<sup>7</sup>. In fact linear models are often seen as a first approximation of reality easy to implement. But these models may leave aside important features of the process generating the exchange rates, especially in regard to expectation formation.

### **Section 1.3. Nonlinear Models and Exchange Rates Market**

Growing interest for nonlinear models has appeared in many scientific fields and more recently in economics. One reason for this gain in interest is the combined development of Non Linear mathematics and numerical technics allowed by more and more powerful computers. Another reason

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<sup>6</sup> Coefficient of skewness and kurtosis refer to the symmetry and the spread of the probability mass of a distribution. See chapter 4: Data Analysis, for more details.

<sup>7</sup> Filter rules are consistent with non-linear models. They may be seen as crude indicators of non-linearity.

is the general acceptance that most physical, biological and even social or economic events can not be correctly modeled as linear process.

The use of nonlinear models raises two major questions for the modeler:

- Face to the great variety of possible nonlinear models, which one is the most appropriate in a particular situation.
- Is the additional cost generated by using nonlinear techniques worth the gain in information on a particular question.

Of course the answer to these questions depends on the context in which they are stated. For now, we will look at some of the recent literature on nonlinear time series and structural models applied to exchange rates theory.

### **Section 1.3.1. Nonlinear Time series Models of Exchange Rates**

Recently alternative approaches, using nonlinear time series analysis, have been proposed. We will review in this section the Auto Regressive Conditional Heteroscedasticity (ARCH) model and Bubbles models.

#### **ARCH models**

Domowitz and Hakkio (1985) have tried to apply an Auto Regressive Conditional Heteroscedasticity (ARCH) model, developed by Engle to exchange rates. It models daily price changes as conditional heteroscedasticity which is a function of recent "news" or forecast errors.

The aim is to take into account a possible time-varying risk premium. The model can be specified as follows:

$$(1-11) \quad (Y_t/I_{t-1}) \sim N(X_{t-1}\beta, h_t^2) \quad , \quad h_t^2 \text{ is the variance of } Y$$

$$(1-12) \quad e_t = Y_t - X_{t-1}\beta$$

$$(1-13) \quad h_t^2 = f(e_t, Z_t) \\ = a_0 + a_1 e_{t-t}^2 + \dots + a_q e_{t-q}^2 \\ + g(X_{t-i}^2)$$

The  $Z_t$  are predetermined variables which enter the information set at  $t-1$ . The squared regressor  $X_{t-i}^2$  may be considered as a particular subset of the  $Z_{t-1}$ . Domowitz and Hakkio test the unbiasedness of the thirty day forward premium by predicting the change in the future spot rate. But they found little support from monthly exchange rate data.

McCurdy and Morgan(1985) have also applied the ARCH model to exchange rates time series. But their study differs in several points from Domowitz and Hakkio's work. First they use daily data of the deutschmark dollar spot and future rates. The period studied is March 15, 1983 to Jun 17, 1985 covering 9 contracts and giving them 562 observations. Finally they use a more general version of the ARCH model, named GARCH (General Auto Regressive Conditional Heteroscedasticity) based on the work of Bollerslev. The model, which extends equation (1-13) to include past variances in the regressors, can be represented as follow:

$$(1-11)' \quad (Y_t/I_{t-1}) \sim N(X_{t-1}\beta, h_t^2) \quad , \quad h_t^2 \text{ is the variance of } Y$$

$$(1-12)' \quad e_t = Y_t - X_{t-1}\beta$$

$$(1-13)' \quad h_t^2 = f(e_t, h_{t-j}, Z_t) \\ = a_0 + a_1 e_{t-t}^2 + \dots + a_q e_{t-q}^2 \\ + \beta_1 h_{t-t}^2 + \dots + \beta_p h_{t-p}^2$$

$$+ g(X_{t,i}^2)$$

They estimate the model with maximum likelihood techniques and find that it is not rejected by the data. It must be noticed however, that although the aim of the above specification of time-varying conditional variance is to model time varying risk premium associated with turbulent periods, it is well known that it may also be proxying omitted variables or structural changes.

### Bubble models

Another line of research has been developed by Meese, Frankel, Froot and other. It deals with the existence of rational and non rational bubbles first studied on the stocks markets. A bubble can be defined as a speculative run acting on the price of a particular asset, not based on the fundamental value of the asset, to which is associated a probability that the bubble will "burst", returning the value of the asset or currency to its original value. We should notice that arbitrage does not prevent bubbles and, as shown by Blanchard and Watson (1982), that rational asset market bubbles may exist in a world of risk averse economic agents with heterogeneous information.

Dornbush (1982) was one of the first to apply the concept of stochastic rational bubbles to exchange rates to explain the strong dollar. Meese (1986) has presented a model which seems to give some evidence for the existence of stochastic rational bubbles. He tests the existence of bubbles for the dollar-pound, deutschmark and yen exchange rates for the period of October 1973 to November 1982 using monthly data which gives him 110

observations. The test, based on the methodology developed by West (1984) who applies it to asset markets, is based on the hypothesis that the driving process for the monetary money supply and real incomes are both stable over the floating rate period and that a hybrid exchange rate monetary model represents well the data.

Recently there has been some interest for non-rational bubbles, mainly to explain the dollar long appreciation until mid 1985. Frankel and Froot (1986) define an irrational bubble as one for which " the true probability of collapsing may be different from the expected probability that investors build into the forward discount". Marris (1985) and Krugman (1985) have proposed the idea that the increasing US debt was clearly going to force the dollar down but that the small discount or interest differential in 1985 , did not correctly reflect this expected depreciation. The model developed by Frankel and Froot is not based on rational expectations. Three classes of economic agents can influence the market: the fundamentalists, the chartists and the portfolio managers, each of them using a different model to build their forecasts, a Dornbush overshooting model, an autoregressive model and a mixture of both models respectively. Agents are not rational in the current sense since they do not use the "true" model but are rational under some constraint of limited information. One interesting feature of the model is its stability even when far away from equilibrium and unstablility near equilibrium. They found that the model is able to explain the long deviation from equilibrium of the dollar: it was due to a lack of confidence in the fundamentalist forecast which would have started to regain influence only in 1985.

### **Section 1.3.2. Nonlinear Structural Models of Exchange Rates**

Few nonlinear structural models have been studied and even fewer tested. Let us consider a world where Central banks or governments adopt policy rules so as to set certain key variables under control. These policies can be seen as defining certain bounds within which the target variable is to remain. If the variable does cross one of the bounds, the authorities will switch to a new policy. Given that prices such as exchange rates are usually influenced by expectations of future events and assuming that expectations are rational, a model of exchange rates should include the probability of such switches.

Flood and Garber (1983) have developed a model in order to take into account such switches based on the historical experience of the French and British exchange rates in the 1920's. During this period, it was expected that Britain would return to the gold standard at the pre-war parity of \$4.86/£ which is what they did on April 28, 1925.

They use a monetary model developed by Frenkel (1978) and Bilson (1978) which assumes the PPP and UIP hypothesis. They further assume that, if economic agents know that at a future time the exchange rate will be fixed at a known level, the driving process of the exchange rate will reflect this knowledge. To incorporate the expectation of such a switch of policy in the exchange rate equation, they use the probability density function (pdf) of a

first passage of the exchange rate through a certain bound defined by the policy authorities.

The resulting exchange rate equation is nonlinear<sup>8</sup>, implying that using linear models when modelling exchange rates will not give accurate results if expectations are subject to policy switches. Clearly this methodology may be applied to a wide variety of problems.

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<sup>8</sup> Flood and Garber have not derive a complete solution for their equation which contains a double integral. They point out however that it may in principal be estimated by a combination of nonlinear techniques and numerical integrated subroutines.

## **Chapter 2. The European Monetary System**

### **Section 2.1. The European Monetary System Institutions**

The European Monetary System, to which we will refer from there on as the EMS, came into operation in March 1979. It was the outcome of the adoption of the Resolution of December 5, 1978 by the European Council, composed of the nine member countries of the European Communities (EC). It succeeded to the European Common Margin Arrangements also called the Snake. The aim was to create "a zone of monetary stability in Europe" so as to favor trade between participant countries. This goal was to be reached by pushing the participating countries to adopt convergent monetary policies and by allowing central bank interventions when a currency deviates by more than  $\pm 2.25\%$  (6% for the Italian Lira) from an official "Central Rate".

In the following section we will analyze how the system operates and its institutional framework. We will then summarize the exchange rates developments during the EMS period.

### **Section 2.1.1. The EMS Institutional Factors**

The EMS operates on the basis of provisions laid down in the Resolution of the European Council of December 5, 1978, in the Agreement between the EC central banks of the EMS on March 13, 1979 (EMS Agreement) and in its amendments of June 1985. But these rules only serve as a framework to the operations of interventions, financing, and as to the role of the European Currency Unit (ECU) and the significance of the divergence indicator. In fact the system has proven to ensure considerable flexibility. In this section we will briefly review problems of membership to the EMS, Intervention mechanism, Credit facilities and the role of the ECU.

#### **Membership in the EMS**

In March 1979 all member countries of the European Community<sup>9</sup> (EC) but United Kingdom, became members of the EMS and signed the EMS Agreement. Although United Kingdom never joined the EMS, the pound sterling was used from the beginning to constitute a basket of currencies for the ECU and, on July 6, 1979, the United Kingdom followed the other EMS countries, on a voluntary basis, by depositing 20% of its gold and dollar assets with the European Monetary Cooperation Fund (EMCF) in exchange of an equivalent amount of ECUs. Greece became member of the EC on January 1, 1981. The Greek drachma was included in the ECU basket

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<sup>9</sup> The participating countries were: Belgium-Luxembourg, Denmark, France, The Federal Republic of Germany, Ireland, Italy and Netherlands

on September 17, 1984 and Greece signed the EMS Agreement on June 10, 1985 and deposited 20% of its gold and dollar reserve in exchange of ECUs on January 1, 1986 but did not become a participant in the mechanism of the EMS. Portugal and Spain became themselves members of the EC on January 1, 1986 but are still not members of the EMS. The main reason for Greece, Portugal and Spain for not becoming full members of the EMS is due to the important structural differences between their economies and the other member countries of the EC, implying the need for gradual adjustments. For the United Kingdom arguments against full membership are as follow: first, the large export of oil has created a "petro-currency effect"; second, as for the deutschmark, the pound behaves differently from other EC currencies at time of large swing of the dollar. All these factors, combined with the important role of the pound as a trading currency and the abolishment of exchange rate controls, imply that necessary interventions to limit fluctuations of the pound could be much larger than for other currencies.

### **Section 2.1.2. The EMS Operating System**

#### **The intervention mechanism**

The EMS is centered around a system of fixed but adjustable exchange rate, each participating currency being tied to the other currencies by a bilateral central rate. A fluctuation margin of 2.25% (6% for the Italian lira) around the central rate determines the intervention limits. When these limits are reached an intervention is obligatory and potentially unlimited in amount. The funds needed for such interventions are provided by mutual

credit lines between central banks on a very short run basis. The debtor is entitled to use ECU for up to 50% of the amount due.

### The divergence indicator

A "divergence indicator" was created to induce change in policies before the intervention limits were reached. It shows the movements of the exchange rate of each participating currency against the ECU, a weighed average of the other participating currency. The indicator scales the exchange rate to values between zero and a maximum allowed deviation from the ECU central rate . When the indicator reaches 75% of this maximum, it is said to have passed the "threshold of divergence" and the concerned country has to take some policy actions. The maximum divergence allowed varies from one country to another, depending on the weight of their currency within the ECU. For example<sup>10</sup> a currency like the french franc, which has a weight of about 20% within the ECU and a maximum deviation of 2.25%, would be allowed a maximum deviation from the ECU central rate in french francs of :

$$2.25 * (100-20) / 100 = 1.8\%$$

and a threshold of divergence from the central ECU rate of:

$$1.8 * 0.75 = 1.35\%$$

An important difference with the official bilateral limits of intervention is that a particular country may reach the threshold of divergence without any other country reaching it. This is due to the fact that the weight of each

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<sup>10</sup> The following example is taken from Banque De France (1979)

EMS currency in the ECU may be different. But the indicator only reached partially its role. Still, most central banks have chosen to keep their currencies well inside the limit margin and minimize the fluctuation of their currencies with respect to key currencies of the EMS. This has implied an increasing "intramarginal intervention" as compared to "intervention at the margin". Some amendments to the EMS Agreement were adopted in June 1985 to favor intramarginal intervention. There was a temporary reversal of this trend after the April 7, 1986 realignment which moved the deutschmark at the bottom of the band. Nevertheless systematic intramarginal intervention has been applied by an increasing number of Central banks since 1983 last realignment. The Threshold model developed in chapter 5 will take into account the interventions by Central banks.

## Credit facilities

The "very short term facility" for financing obligatory intervention is limited to the participants of the EMS. Other facilities are open to all members of the EC. These include the Short Term Monetary Support (STMS) which is quasi-automatic, the Medium Term Financial Assistance (MTFA) and the Community loan mechanism who are subject to conditionality. For example, France requested a loan of ECU 4 billion under the Community loan mechanism after the March 1983 realignment. The loan was granted by the Council of Ministers on the basis of an adjustment program which included a reduction of the public sector deficit and the target growth of the money supply, encouragement of private saving and reduction of indexation of costs and prices.

### Section 2.1.3. The ECU

#### The ECU

The ECU was introduced in 1979 as a mean of settlements, a reserve asset and a unit of account for financial transaction in the EMS and for EMS related institutions as well as EC activities. It succeeded to the European Unit of Account (EUA) introduced in 1975. As a mean of settlement its role remained limited especially in recent years since intramarginal intervention does not give rise to the very short term financing facilities reserved to obligatory intervention.

The ECU was originally defined as a fixed amount of the nine participating currencies. Revision of its composition takes place periodically<sup>11</sup>.

Now in discussion is the second phase of the EMS. Increasing the role of the ECU has already been proposed. Also in 1992 all capital transactions will be liberalized within the EC member countries. Liberalization of capital movements should put pressure on the EMS countries to adopt even more convergent economic policies. There is also talks about the creation of a European Monetary Fund which would replace the EMCF and could act as a European Central bank "creating ECU, affecting exchange rates and monetary policies of the member states, and regulating the money supply", John H Works (1986).

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<sup>11</sup> There also exists a private ECU not related to the official ECU creation which has gained a large success on the exchange rate markets.

## **Section 2.2. Exchange Rate Developments**

### **Section 2.2.1. Overview: 1979 - 1988**

The EMS goal was not to return to a fixed exchange rate system and adjustments did take place through devaluations. But fears that the system of fixed or even partly fixed rate will break down or degenerate in a system of frequent small adjustments have not materialized. There were 11 realignments, of which 6 affected the FDM rate, over the ten years of existence of the EMS (table 2-1).

**- Table # 2-1 - EMS Realignments**

**Percentage Changes in the Central Rate\***

	Sep 24 1979	Nov 30 1979	Mar 23 1981	Oct 5 1981	Feb 22 1982	Jun 14 1982	Mar 21 1983	July 22 1985	Apr 7 1986	Aug 1986	Jan 1987
Belgium and Luxembourg						-8.5		+1.5	+2.0	+1.0	
Francs											
Danish Krone	-2.9	-4.8			-3.0		+2.5	+2.0	+1.0		
Deutschmark	+2.0			+5.5		+4.25	+5.5	+2.0	+3.0		+3.0
French Franc				-3.0		-5.75	-2.5	+2.0	-3.0		
Italian Lira			-6.0	-3.0		-2.75	-2.5	-6.0			
Irish Pound							-3.5	+2.0		-8.0	
Netherlands Guilder			+5.5		+4.25	+3.5	+2.0	+3.0			

\* The figures are calculated as the percentage change against the group of currencies whose bilateral rate remained unchanged in the realignments.

Sources: Commission of the European Communities, the IMF (Occasional Paper 48, Dec 1986) and Michel Develle (1988)

**Section 2.2.2. Variability of the EMS exchange rates**

Between 1979 and the realignment of 1983 the adjustments increased both in size and frequency. But, during the second half of the EMS existence, the convergence of economic policies and a rising dollar have allowed greater exchange rate stability. In fact, the EMS seems to have accomplished its task by decreasing volatility as shown by two consecutive studies: Ungerer (1983), Ungerer (1986) of the IMF. In his paper, Ungerer uses different measures of variability of the exchange rates. The first is the Coefficient of Variation (CV = standard deviation / mean). It can be seen as a measure of stability around the average (or the trend) which is the best

measure if one is to test the ability of the EMS to stabilize bilateral nominal rates. A second approach looks at the variability in the natural logarithm of the spot rate measured by the standard deviation in the log of the rates (an extension of this measure takes into account the covariance of bilateral rates) used as a proxy of unexpected changes in the trend. These studies show that, when compared to both past EMS exchange rates and to non-EMS exchange rates, volatility of current EMS currencies has decreased. The dollar began declining in 1985 after a peak of DM 3.47 per US\$ 1 on February 26, 1985. But only in the second part of 1985 did the decline of the dollar begin to affect the stability of the EMS currencies, due to increasing report on the deutschmark taking place after that date.

Over the whole EMS period, the deutschmark continually appreciated against the french franc. There was 7 realignments between the two countries, 4 of which took place during the first part of the EMS period before mid 1983, ranging from 2% to 10%. The central rate raised from 230.95 to 325.617 DM per 1 FF which represents an increase of 41%. Therefore, although intervention did take place to support the deutschmark, it was mostly aimed at supporting the french franc.

### **Section 2.2.3. Econometric forecasting of the EMS exchange rates**

Few models have used bilateral EMS exchange rates for econometric testing. Instead, most studies use bilateral US versus foreign currencies. This is due, in part, to the fact that there is a certain difficulty to incorporate in the usual model the effect of institutional constraints which have ruled exchange rates management under the EMS period. A

set of theoretical and econometric problems arise from the switches in institutional regimes and systematic interventions on these markets. These problems can be addressed by using nonlinear models such as the one developed by Flood and Garber (1983) But we have seen in chapter 1 that these models are usually not as easy to estimate then linear models.

Basevi and Calzolari (1982) have attempted to specify reaction functions that take into account these constraints. They also provide estimates for their model. Giavazzi and Giovannini (1986) give an interesting analysis of the role of the french and italian capital controls within the EMS by comparing interest differential between the free Eurocurrency markets and national markets. The study clearly shows important differential during exchange rate crisis.

Important statistical studies on the EMS currencies include the IMF papers by Ungerer (1983), and Ungerer (1986), noted above, and an important work by Rogoff (1985).

## **Chapter 3. Methodology**

In this chapter, we will first go over some of the motivations for searching nonlinear data structures in the exchange rate series and the mean to do so by an accurate data analysis. If this analysis implies a nonlinearity of the data then linear model such as ARIMA models should not be used.

Second we will go over Aikes criterion which we will use for identification of the "best" model among a specified class of nonlinear models. Finally we will present some of the diagnostic checking that one may apply to these models.

### **Section 3.1. Data Analysis and Nonlinear Models**

ARIMA modeling, as defined by Box and Jenkins (1970) , allows to choose among different models within the general class of ARIMA models. It uses an iterative process of model building defined by the three steps of identification, estimation and diagnostic checking. These models are well suited for estimating stationary Gaussian process but by Wold's

decomposition they also give the "best" linear forecast even if the process is nonlinear. See Malinvaud (1983) for a discussion on this point.

But ARIMA models are no longer "best" in the Mean Square Error (MSE) sense when compared to nonlinear forecast for data generated by a nonlinear process. Some limitations of the ARIMA models are given by Tong (1983):

- (1) If errors terms  $\{e_t\}$  have zero variance, the only possible stationary solution for an ARIMA model is a constant steady state.
- (2) ARIMA models have symmetric joint distribution and are therefore inappropriate for non symmetric distributed data which seems to be the case of the FDM rate.
- (3) Sudden and important jumps in the data can not be understood with an ARIMA model and we will see that such jumps are frequent in the exchange rate series.
- (4) Since the auto covariance function only measures linear relationship between  $r_{t-i}$  and  $r_{t-j}$  some information may be left out, for example the possible cyclicity of the data.
- (5) Finally ARIMA model, having a symmetric Gaussian distribution, are time reversible and are therefore not ideal for data presenting time irreversibility.

As a result, if the process is nonlinear, fitting an ARIMA model to the data might leave too much "information" in the error term. In fact, even if a series  $\{X_t\}$  appears to be white noise it may contain valuable information that can be used for forecasting  $\{X_t\}$ . The following example, presented by Priestly (1981) is an illustration of this point.

Suppose  $\{X_t\}$  is a third order stationary series, generated by a nonlinear process of the following form:

$$(3-1) \quad X_t = \beta e_{t-1} e_{t-2} + e_t, \quad E[e_t] = 0, \quad E[e_t^2] = s^2$$

where  $e_t$  is an independently and Normally distributed random variable.

Although the first and second order moments of  $X_t$  (M1 and M2) will be zeros, the third order moment (M3) will not vanish.

$$M1 = E[X_t]$$

$$= \beta E[e_{t-1} e_{t-2}] + E[e_t]$$

$$= 0$$

$$M2 = E[X_t X_{t-1}]$$

$$= \beta^2 E[e_{t-1} e_{t-2}^2 e_{t-3}] + \beta E[e_{t-1}^2 e_{t-2}] + \beta E[e_t e_{t-2} e_{t-3}] + E[e_t e_{t-1}]$$

$$= 0$$

All terms vanish since the  $e_t$  are i.i.d. For example:

$$\beta^2 E[e_{t-1} e_{t-2}^2 e_{t-3}] = \beta^2 s^2 E[e_{t-1} e_{t-3}] = 0$$

But this is not true for at least one term of the third order moments of  $X_t$  :

$$M3 = E[X_t X_{t-1} X_{t-2}]$$

$$= \beta E[e_{t-1}^2 e_{t-2}^2]$$

$$= \beta s^4$$

Therefore the  $X_t$ 's may not be considered as independent although they are uncorrelated. We call a "white noise series" one for which the first and second order moments are all zero and a "purely random process" a series

which is white noise and has third order moments equal to zeros or equivalently when the series is Gaussian. This demonstrates that even if  $X_{t-i}$  and  $X_{t-j}$  are uncorrelated some valuable information might still be available for forecasting if the series is not generated by a Gaussian process.

As a result, if one is to model exchange rates, one should test for nonlinearity. Our data analysis will include the following steps:

- (1) Computation of standard statistics (mean, variance, extremums...) and plot of the series.
- (2) Analysis of the volatility using the coefficient of variation and the standard deviation of the log exchange rate.
- (3) The correlation and partial correlation function.
- (4) Histogram of the data which is compared to the histogram of a normal distribution with same mean and variance.
- (5) Check for nonlinearity by calculating the ratios of skewness and kurtosis.

### **Section 3.2. Identification**

We have seen that Box and Jenkins (1970) have developed an iterative process of model building defined by the three stages of identification, estimation and diagnostic checking. This approach allows flexibility as to the choice of the candidate model which remains largely subjective.

Akaike (1973) has developed a different approach. He replaces the problem of identification by an optimization problem with respect to some prespecified criterion and over a preselected class of models. The approach is an extension of the principle of entropy maximization and of the likelihood estimation of parameters to the likelihood estimation of models. For a discussion of this approach one can refer to the original series of papers by Akaike (1973), Akaike (1974), Akaike (1977) and to Tong (1983) for an extension of these results to Threshold models.

Akaike's Information Criterion (AIC) is defined as follows:

$$(3-2) \quad \text{AIC}(L) = -2\ln(\mathcal{L}) + 2(k)$$

where  $\mathcal{L}$  is the maximized likelihood,  $k$  is the number of independently adjusted parameters of the model and  $L$  is an index for the different competing models. The model  $L^*$  which minimizes  $\text{AIC}(L)$  is to be chosen.

The first and second term on the right hand side of (3-2) are measures of the badness of the fit and of the complexity of the models respectively. (3-2) is therefore a loss function giving a trade-off between those two measures. This implies that even if the true model does not belong to the class of hypothetical models under investigation, AIC can still be a useful criterion  $\mathcal{L}$  being a measure of goodness of the fit.

There is some limitation to the AIC criteria that one must bear in mind. First, AIC is not a good measure when comparing different classes of models. This is due to the fact that we usually do not know the true joint distribution of the  $X$ 's and therefore can only use a likelihood conditional

on some available observations  $\{X_1, X_2, \dots, X_p\}$  giving us only an approximated AIC. The approximation might have different effects from one class of models to another. Furthermore using a "too high" order of  $k$  could imply a loss of optimality. Finally AIC is only a global measure of the goodness of the fit and diagnostic checks should not be disregarded. Keeping all of this in mind, we will use AIC in the identification stage of the Threshold model applied to the FDM exchange rate.

### **Section 3.4. Diagnostic Checking**

Diagnostic checking of the model is the last stage of model building. It should include three steps:

- (1) Check of the statistical significance of the coefficients by standard tests.
- (2) Testing for the Gaussianity of the residuals. This stage includes most of the tests mentioned in the data analysis: the correlation function, standard plots such as histograms, and calculation of the coefficient of skewness and kurtosis.
- (3) Simulation of the model to see if the generated data conserves the structure of the original data. Specifically, it is interesting to see if the non linear structure of the data has been conserved by the model.

## Chapter 4. Data Analysis

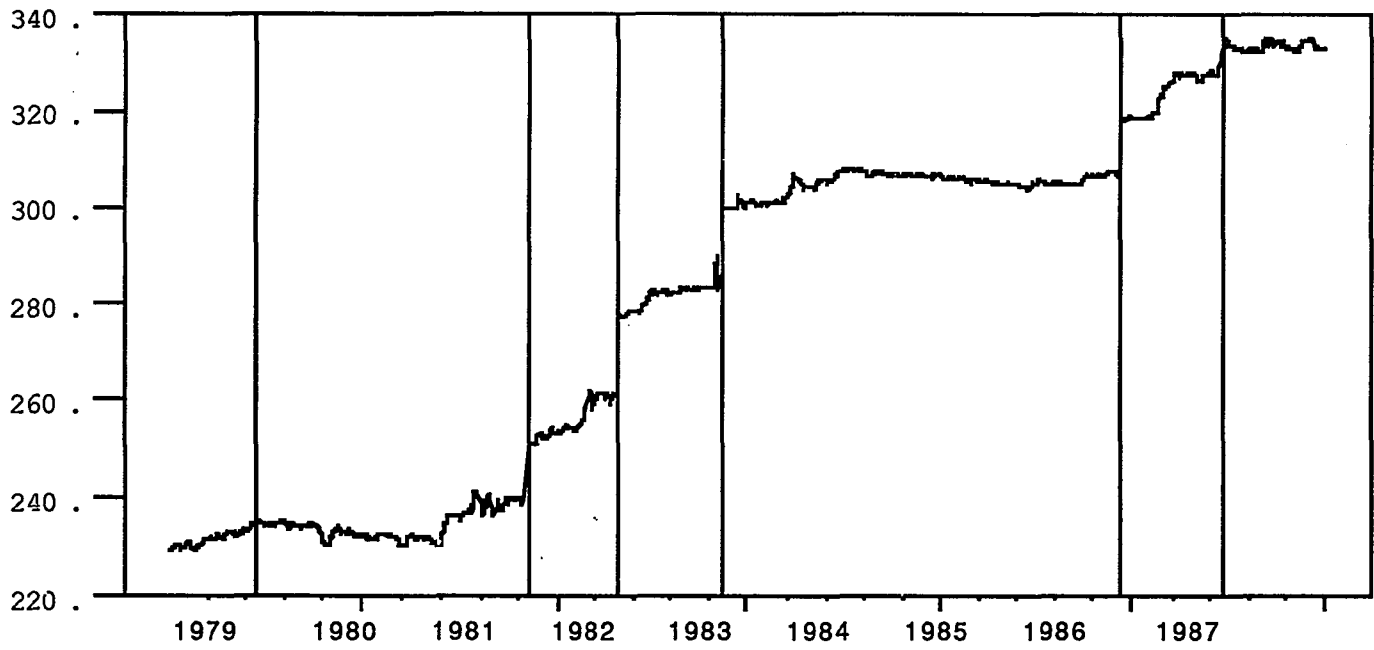
In this chapter, we will analyze the spot exchange rate between the french franc and the deutschmark in details. The data used in this paper are the observed spot exchange rates between the french franc and the deutschmark (FDM). They were obtained from the French Central Bank (Banque de France) and the Paris Stock Exchange (France), where official rating by "fixing", of the main foreign currencies, takes place every open day between 1.30pm and 2.00pm.

### The data

The period covered by the data is January 1, 1979 to October 15, 1987. The starting date of January 1979 was chosen to cover the whole European Monetary System (EMS) period in order to ensure homogeneity of the data set.

The data has a daily periodicity. This gives us 2143 observations which have been checked in detail for possible errors.

A plot of  $S_t$  is given in graphs #4-1. The vertical lines represent devaluation dates.



- Graph #4-1 The Spot FDM Exchange Rate-

We use two different transformations of the spot rate data:

- the percentage daily change or "rate of change"
- the percentage daily deviation from the central bilateral rate of the french deutschmark rate or "indexed rate".

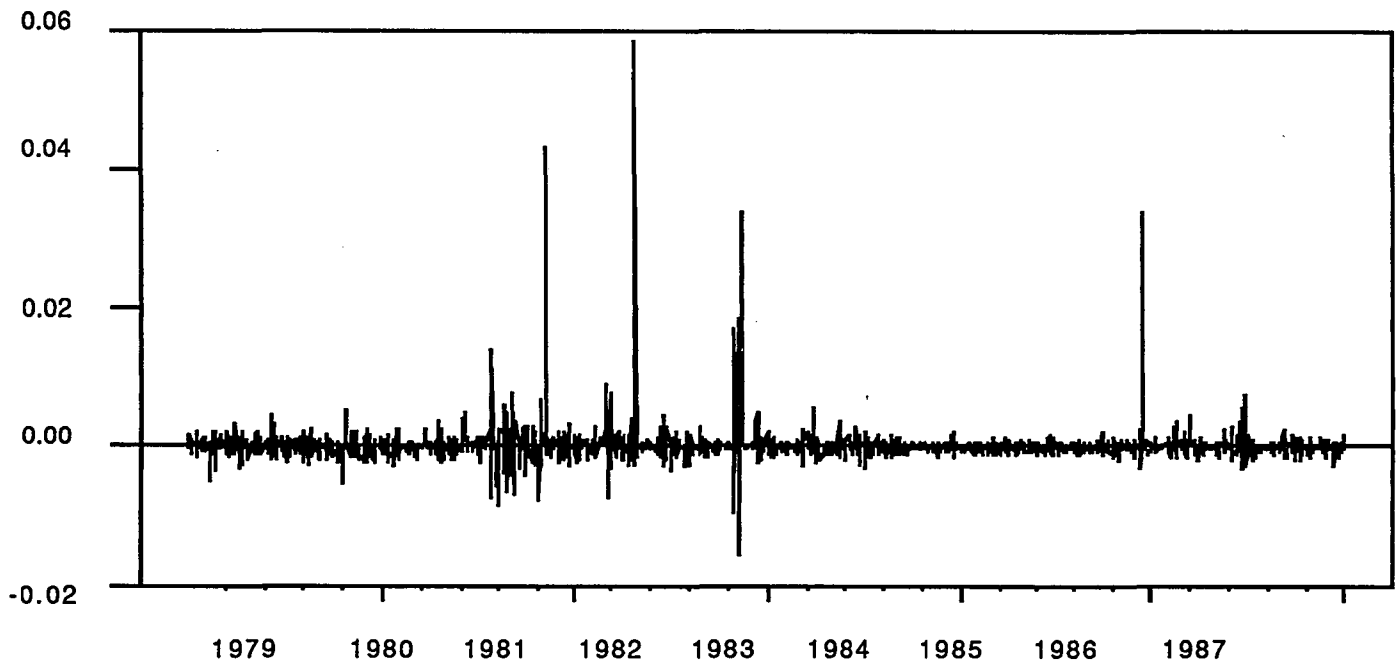
Both of these are studied in the next two sections.

### Section 4.1. The Rate of Change

The rate of change is obtained by taking the first difference of the log of the spot rate  $S_t$  multiplied by 100 :

$$r_t = \log \left( \frac{S_t}{S_{t-1}} \right) \cdot 100$$

This allows us to study the deviation from the trend. If the market is efficient  $r_t$  is usually supposed to follow a random walk. A plots of  $r_t$  is given in graph #4-2. Statistics on  $r_t$  are summarized in table #4-1.



-Graph#4-2 Plot of the FDM rate of change-

**- Table #4-1 Summary statistics\* for the rate of change -**  
(1979-1987)

	average%	max%	min%	sd%	coef-v	skew	kurt
1979	0.0088	0.4396	-0.49503	0.1141	0.0085	-0.0925	2.6395
	<i>0.0068</i>	<i>0.3536</i>		<i>0.1103</i>		<i>-0.3212</i>	<i>2.3450</i>
1980	-0.0031	0.5238	-0.51845	0.1108	0.0052	0.2399	3.5760
1981	0.0357	4.2945	-0.80993	0.3500	0.0298	7.5004	90.4050
	<i>0.0182</i>	<i>1.3876</i>		<i>0.2195</i>		<i>0.5951</i>	<i>8.4794</i>
1982	0.0453	5.8294	-0.7016	0.4005	0.0440	12.4180	179.1231
	<i>0.0216</i>	<i>0.9020</i>		<i>0.1527</i>		<i>0.9112</i>	<i>8.3696</i>
1983	0.0313	3.3919	-1.5599	0.3305	0.0263	5.2963	52.5470
	<i>0.175</i>	<i>1.8634</i>		<i>0.2510</i>		<i>2.3882</i>	<i>30.2900</i>
1984	0.0006	0.2969	-0.2727	0.0691	0.0023	0.3052	3.0119
1985	0.0003	0.1995	-0.1112	0.0522	0.0021	0.7716	1.4584
1986	0.0353	3.4089	-0.2929	0.2413	0.0255	11.4459	157.9766
	<i>0.0184</i>	<i>0.5510</i>	<i>-0.29294</i>	<i>0.0961</i>		<i>1.3163</i>	<i>6.3788</i>
1987	-0.0017	0.2391	-0.2543	0.0734	0.0019	-0.0262	1.8747
1979-82	0.0220	5.8294	-0.8099	0.2809	0.0708	13.0172	246.5335
	<i>0.0110</i>	<i>1.3876</i>		<i>0.1559</i>		<i>0.7118</i>	<i>11.8584</i>
1983-85	0.0106	3.3919	-1.5599	0.1971	0.0198	8.5104	143.5648
	<i>0.0061</i>	<i>1.8634</i>		<i>0.1530</i>		<i>3.6801</i>	<i>76.7675</i>
1986-87	0.0193	3.4089	-0.2929	0.1887	0.0275	13.7046	242.5920
	<i>0.0097</i>	<i>0.5510</i>		<i>0.0875</i>		<i>1.0634</i>	<i>6.1685</i>
1979-87	0.0175	5.8294	-1.5599	0.2370	0.0131	12.8326	261.9391
	<i>0.0090</i>	<i>1.8634</i>		<i>0.1435</i>		<i>2.0007</i>	<i>41.9570</i>
						<i>( 37.75 )</i>	<i>(395.64)</i>

\* - Values of the coefficients, when devaluation dates are set to zero, are given in *Italic* except when these are not affected  
- Z statistic in parenthesis

### Section 4.1.1 Statistical analysis

#### Daily changes

The mean rate of change is small: 0.017% and equal to zero with a t-statistic of 3.418 and a significance level of 0.0006, but daily changes can be quite important 5.8% (0.009% and 1.8% respectively when devaluation dates are set to zeros).

#### Volatility

Following Ungerer who has analyzed volatility of the EMS currencies, Ungerer (1984) and Ungerer (1986), we have measured volatility by:

- (1) the standard deviation of the rate of change
- (2) the coefficient of variation of the spot rate  
(CV= average spot rate /standard deviation).

The first is a measure of dispersion around the trend. It assumes that the economic agents on the exchange rate markets expect that an underlying trend will persist in the near future due, for example, to a persisting inflation differential. In that case the standard deviation of the rate of change is the best measure of the risk that these agents support.

The second measure assumes that the exchange rate varies around a constant level; in this sense, CV is a measure of dispersion around the

mean which is appropriate if the aim of the EMS is to stabilize bilateral nominal rates.

We now look at the results for the french deutschmark rate of change which are reported in table # 4-1. In accordance with the IMF studies, the FDM exchange rate volatility tends to decrease after March 1983's last realignment. The standard deviation is 0.281 until 1983, 0.197 and 0.188 for the two periods following 1983. The coefficient of variation is 0.0708, 0.0198 and 0.0275 for the three periods, showing again a decrease in volatility after 1983. This is partly explained by the long appreciation of the dollar until the mid 1985's years. It favors stability between the EMS currencies, no report on the DM tacking place. But it may also be due to the increasing , and more systematic, intramarginal intervention after 1983's last realignment as we have noted above. In the following chapters, we will try to see if this apparent decrease in volatility is related to a shift in the dynamic process of the FDM rate.

### Correlation coefficients

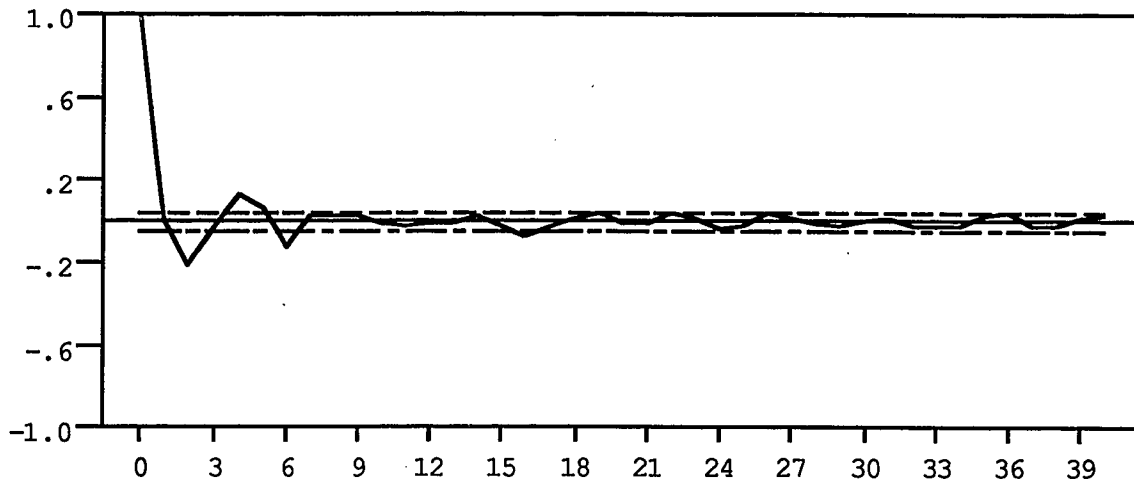
Looking at the correlation function, once devaluation dates are set to zeros, we find that lags 2, 4 and 6 are significant in the sense that they lie outside the limits of two standard deviations measured by  $2/\sqrt{n}$  (graph 4-3). Notice that this measure assumes normality, see Anderson (1942),. They then die out rapidly, implying that no further differentiating is needed to obtain stationarity. For the partial correlation function, only lag 2 and 4 seem significant (graph 4-3'). This could indicate that an ARMA

model of order (4,6) might be a good first approximation of the generating process.

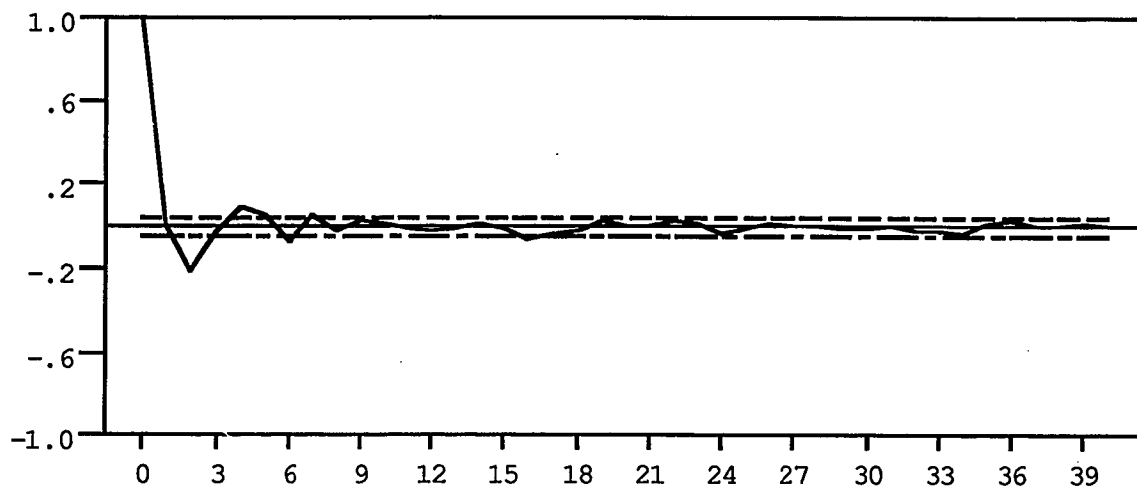
But we have seen in chapter 3, that if the process is nonlinear, fitting an ARMA model to the data will leave to much "information" in the error term. In fact, even if a series  $\{X_t\}$  appears to be white noise it might contain valuable information that could be used for forecasting  $\{X_t\}$ . We must therefore investigate more deeply the nature of the data and test for non linearity.

**- Graph # 4-3 Correlation and Partial Correlation function of  $r_t$ -**

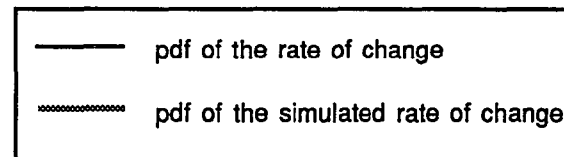
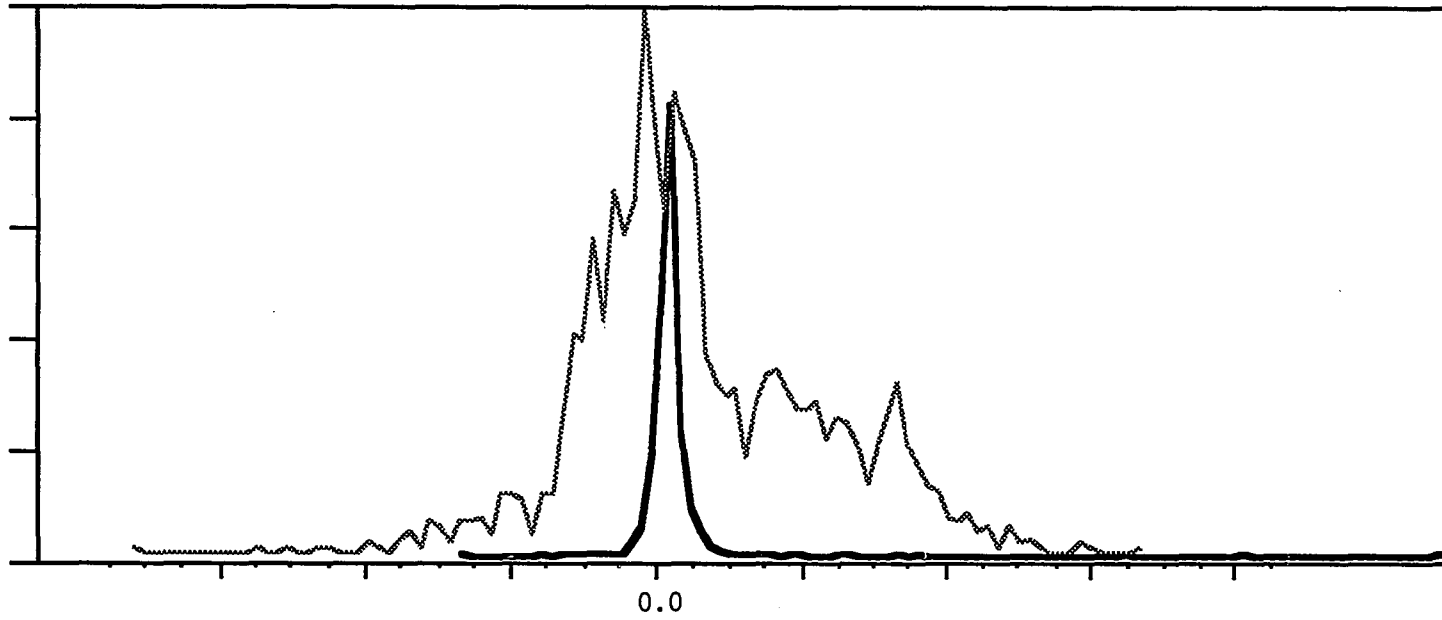
Correlation function of the rate of change  $r_t$



Partial correlation function of the rate of change  $r_t$



Histogramme of the EDM rate of change



- Graph # 4-4 Histogram of  $\pi$  -

### Section 4.1.2 Looking for nonlinearity

#### The Histogram

A first step is to look at the histogram of the series. The histogram of  $r_t$  is plotted for comparison with the histogram of a random variable drawn from a Normal distribution with same mean and variance then  $r_t$  and for the same number of observations then the original data series (graph #4-4). They indicates possible skewness (non symmetry) and leptokurtosis ("fat tails" and sharper central picks as compared to the Normal distribution). This is only a crude information about normality and we must now look at the coefficient of skewness and kurtosis<sup>12</sup>.

#### Skewness

The coefficient of skewness is defined as the first moment around the mean divided by the cube of the standard deviation. The following formulas, used in estimating the coefficient of skewness, are taken from Kendall and Stuart (1958) :

Moments :

$$(4-2) \quad m_k = \frac{1}{n} \sum_{i=1}^n (r_t - \mu)^k$$

---

<sup>12</sup> We could also have computed the kolmogorov-Smirnov statistic to test for normality of the histogram.

Skewness :

$$(4-3) \quad Sk = \frac{n^2}{(n-1)(n-2)} \frac{m_3}{s^3}$$

The significance level is computed

for a two tailed normal test using: (4-4) 
$$Z = Sk \sqrt{\frac{(n-1)(n-2)}{6n}}$$

Sk should be zeros for symmetric distributions, but is equal to 12.3 and 2.0 when devaluation dates are set to zero , with a significance of zero for the whole period. The Z statistic as define above is 37.75 when devaluation dates are set to zero. Such positive skewness suggest that the upper tails of the distribution are larger and longer then the lower . This implies that the largest upward fluctuations outweigh the largest downward shifts or, in other words, that the probability of large positive shifts is greater than that of large negative shifts. This is confirmed by comparing the maximum and minimum percentage of daily variations.

But except for the 3 years where the most important devaluations took place, this coefficient is not significantly different from zero. We must know look at Kurtosis.

### Kurtosis

The coefficient of kurtosis is defined as the fourth moment around the mean divided by the square of the variance. We estimate it by the following formulas, taken from Kendall and Stuart (1958):

Kurtosis:

$$(4-5) \text{ Ku} = \frac{n^2}{(n-1)(n-2)(n-3)} \frac{(n+1)m_4 - 3(n-1)m_2^2}{s^4}$$

The significance level is computed

for a two tailed normal test using:

$$(4-6) \text{ Z} = \text{Ku} \sqrt{\frac{(n-1)(n-2)(n-3)}{24n(n+1)}}$$

It measures the spread of the probability "mass" at the tails. Ku should be 0 for a normal distribution<sup>13</sup> and decrease the more the mass of the probability is concentrated near  $(\mu+s^2)$  and  $(\mu-s^2)$ . For the FDM rate it takes the value of 261.4 and 41.9 when devaluation dates are set to zero, with a zero % level of significance. The Z statistic as define above is 395.64 when devaluation dates are set to zero.

This reveals strong leptokurtosis with larger tails and sharper central peak than normal distribution with same mean and variance. It indicates that we can not apply the usual econometric techniques based on the stationarity and the normality hypothesis to exchange rates, i.e. significance test such as test involving the t-test. The result agrees with most statistical studies on exchange rates. See Friedman and Vandersteal (1982), .

---

<sup>13</sup> The skewness and kurtosis tests will be invalid if used on series which are serially correlated. We assume here, that we have achieved stationarity by differentiating the exchange rate series.

Therefore, although the values of these coefficients are not stable from one period to another and are very sensible to shocks such as devaluations, it seems that the hypothesis that the FDM daily rate of change is distributed as a Gaussian process, should be rejected.

#### **Section 4.2. The Indexed Spot Rate**

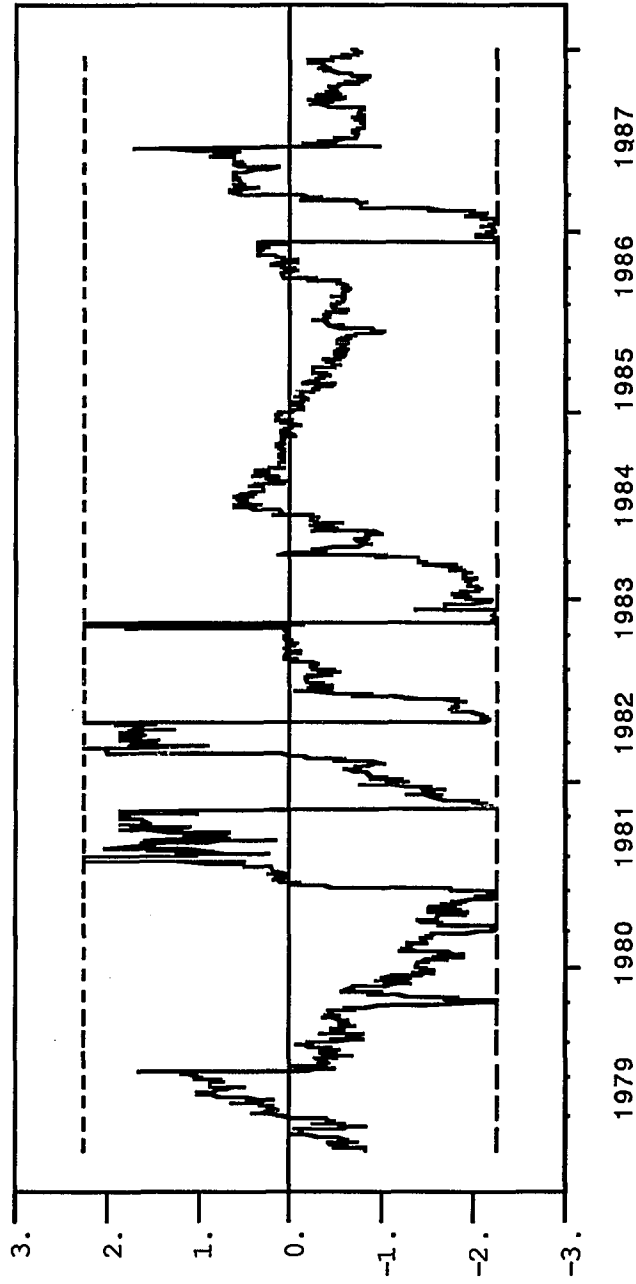
We will also use an indexed spot rate :  $Sind_t$  defined as the difference of the log of the spot rate  $S_t$  and the log of the current bilateral central rate  $C_i$ :

$$(4-7) \quad Sind_t = \log \left( \frac{S_t}{C_i} \right) \cdot 100$$

The motivation for computing this transformation is that it allows a very interesting application of the Threshold model and estimation of the effect of the intervention limits. We could have used the deviation from the ECU based on the fact that it is used to define the "threshold of divergence". But as seen earlier, it has not been successful as a systematic instrument since it was not completely adopted by the Central Banks. Furthermore, participants on the market might not be as sensible to variations of the deviation of the french franc/ECU or deutschmark/ECU as compared to the french franc/deutschmark rate from the Central bilateral rate. Since expectations of future price play a central role on the

exchange rate markets, we will use the deviation of the FDM rate from the bilateral central rate.

- Graph # 4-5 Plot of the Indexed FDM rate of change -



**- Table #4-2 Summary Statistics for the indexed  
rate\*  
(1979-1985)**

	average	max	min	sd	skew	kurt
1979	0.0209	1.6448	-0.8392	0.6014	0.59130	-0.93480
1980	-1.2800	-0.3351	-2.2487	0.5146	0.13319	-0.83344
1981	-0.0593	2.2507	-2.2546	1.4793	-0.10652	-1.45610
1982	-0.2224	2.2530	-2.1817	1.2343	0.38014	-0.73570
1983	-1.0649	2.2518	-2.2418	0.9985	0.70081	0.19714
1984	0.1380	0.5989	-0.3031	0.2143	0.43028	-0.38217
1985	-0.4582	0.8215	-1.0319	0.2081	0.43827	1.05060
1986	-0.3885	1.3371	-2.2508	1.1382	-0.72661	-1.19220
1987	-0.5183	1.7001	-0.9786	0.3968	4.02490	19.68000
1979-82	-0.398	2.2530	-2.2546	1.1744	0.41481	-0.68409
1983-85	-0.479	2.2518	-2.2418	0.7890	-0.67828	0.57827
1985-87	-0.436	1.7001	-2.2508	0.8733	-0.55230	-0.04719
1979-87	-0.433	2.2530	-2.2546	0.9964	0.15058 (2.842)	-0.07866 (-0.742)

- Z statistic in parenthesis

A plot of  $Sind_t$  is given in graph 4-5 with intervention limits of 2.25% indicated in dashed lines. In this graph the bilateral central rate is equal to zero since:

$$\text{Log}\left(\frac{C_i}{C_i}\right) * 100 = \text{Log}(1) * 100 = 0 .$$

This plot is extremely different from the rate of change plot studied earlier. At time of tension, deviation from central rate seems to accumulate until a realignment takes place and that the rate jumps to the lower bound of the intervention limit. The statistics on  $Sind_t$  are summarized in table #4-2 and studied in the following pages.

### Section 4.2.1 Statistical analysis

#### Volatility

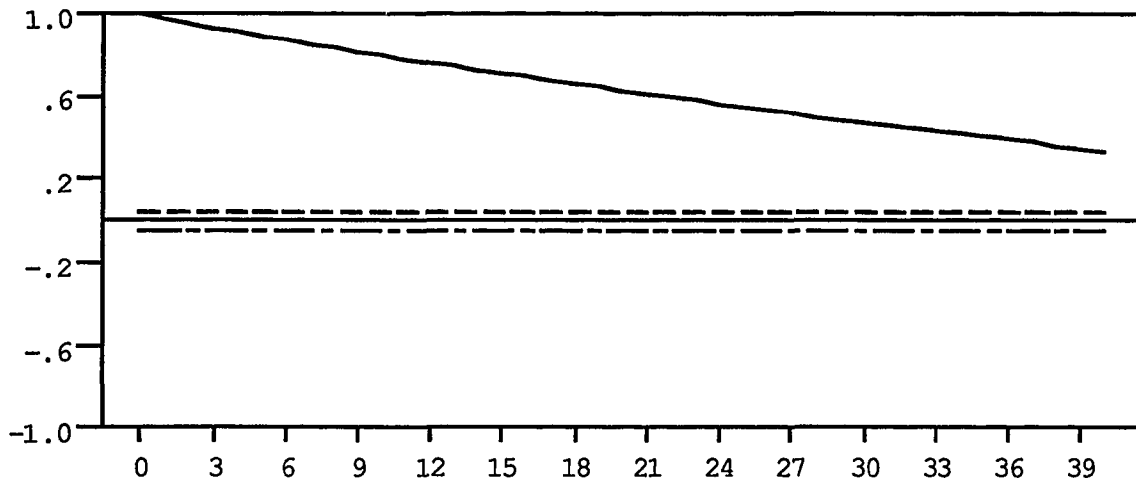
As for the rate of change, volatility decreased after the 1983's realignment. The standard deviation of  $Sind_t$  is equal to 1.1744 in the first period, 0.7890 and 0.8733 after 1983. But this decrease is not as important then for the rate of change. The mean deviation from the central bilateral rate is -0.431 which is equal to zero with a t-statistic of -16.296 and a significance level of 0.106E-58 . The maximum and minimum daily deviation from the Central rate are 2.253 and -2.254 respectively. They are, therefore, well contained by the intervention margins which are +/- 2.25%.

## Correlation functions

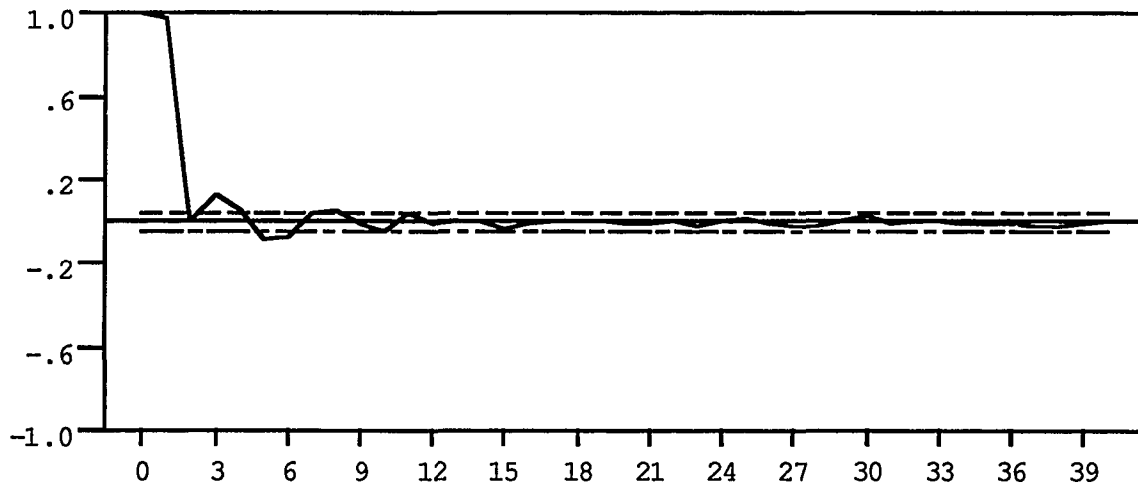
Again we look at the correlation functions. The correlation function of the indexed rate smoothly decreases towards zero, but all coefficients are significant (graph 4-6). As for the partial correlation function, coefficient 1,3 and 5 are highly significant (graph 4-6'), i.e. they stand out of the band of  $\pm 2/\sqrt{n}$ . This implies that a AR model of order (5) or (3) could be appropriate for modelling the indexed rate of change if one assumes that the generating process is linear.

**- Graph # 4-6 Correlation and Partial Correlation function of sindt-**

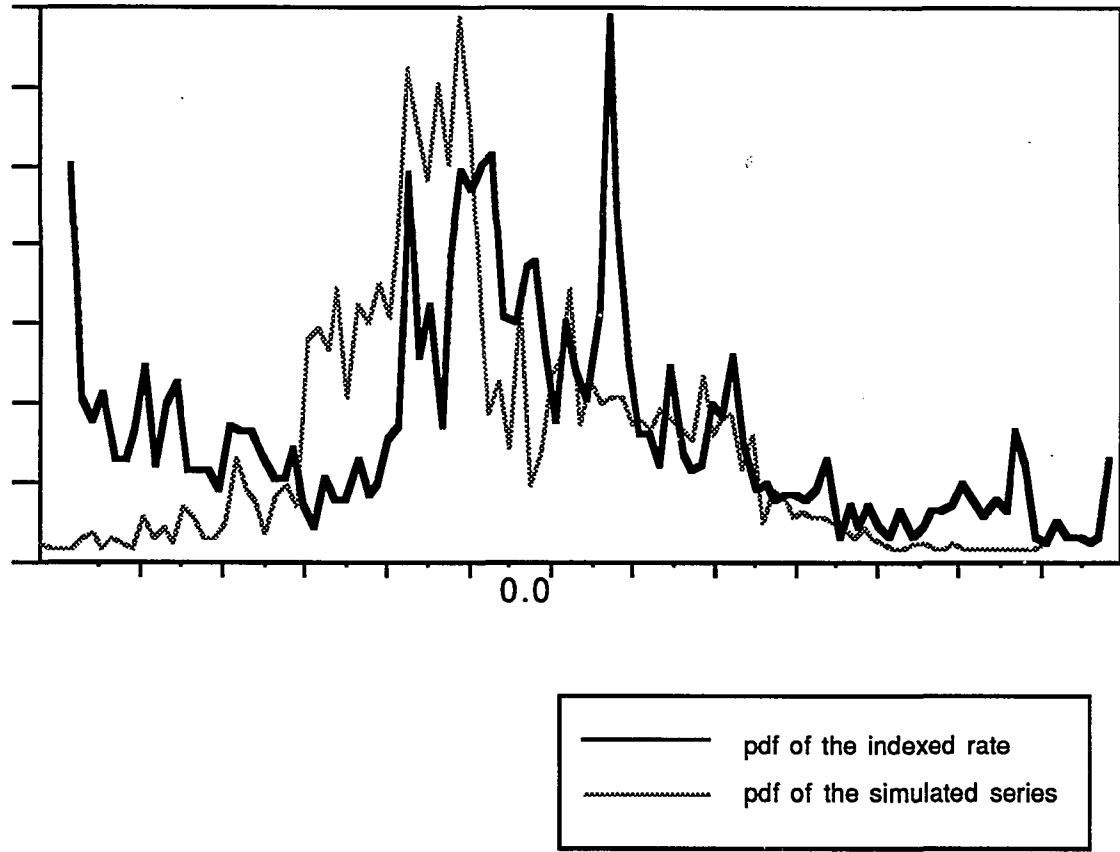
Correlation function of the indexed rate of change



Partial correlation function of the indexed rate of change



Histogramme of the indexed FDM rate



- Graph # 4-7 Histogram of sind-

### Section 4.1.2 Looking for nonlinearity

#### The Histogram

The histogram has not the usual bell shape of the normal distribution. It is rather unsmooth. The probability mass is not concentrated around zero and the tails are "anormally" fat. This is specially the case for the lower tail (graph 4-7). Therefore skewness and kurtosis are likely to be far away from their normal value.

#### Skewness

Again we check for kurtosis and skewness. The coefficient of skewness, defined in the above section, and which should be zero for symmetric distributions, is equal to 0.15058 for the whole period. The Z statistic as is 2.842 when devaluation dates are set to zero. This coefficient is not significantly different from zero so that the distribution seems to be approximately symmetric. It is interesting to notice that the coefficient is positive before 1983 when most devaluations took place, and negative afterwards.

#### Kurtosis

The coefficient of kurtosis, as measured above, should be zero for a normal distribution. It measures the spread of the probability "mass". For

the FDM indexed rate it takes the value of  $-0.0787$  with a significance level of  $0.4593$ . The Z statistic is  $-0.742$  when devaluation dates are set to zero. At the opposite of the rate of change for which kurtosis was above the normal value it is now under its normal value of zeros. This suggests again that we can not use the usual Gaussian hypothesis. Although departure from normality is not as drastic as with the rate of change, we will apply the threshold model and compare the results with a fit of the same data set to an ARMA model.

## Chapter 5. Threshold Models

Threshold models have received an increasing attention and have been applied to a variety of problems. In physics, for example, Andronov, Vitt and Khaikin (1966) have studied the triode oscillators showing that the saturation of the triode valve acts as a threshold. These models have been widely applied to the study of biologic populations. The population growth is usually a function of the available food supply which comes in limited amounts implying a saturation type function. May and Oster (1976) have developed such models. The famous predator-prey models can also exhibit cyclical phenomena related to thresholds.

In economics Chien and Chan (1979) have proposed an extension of the well known Leontief model:

$$X - AX = Y$$

where  $X$  is the output vector,  $Y$  the demand vector and  $A$  a matrix of coefficients. They develop a piecewise version of this model replacing  $AX$  by  $g(X)$  which is piecewise linear in  $X$ .

Further references and examples of applications can be found in Tong (1983) which has developed a general framework for the identification and estimation of such models. In the following sections we will see that these models have very interesting applications with regard

to exchange rates markets and more specifically when modeling systematic intervention on such markets.

### Section 5.1. Presentation of the Threshold Model

We first define some terms which will be used to describe the threshold model:

- $\{X_t\}$  is a k-dimensional time series
- $j_t$  an observable random variable indicator taking the integer values  $\{1,2,\dots,m\}$ .  $j_t$  indicates "the mode" of the dynamic system and is defined as a function of  $\{X_t\}$ . We will see that it is closely related to the concept of threshold and delay.
- The ordered subset of real numbers  $\{r_1, r_2, \dots, r_{m-1}\}$  are the thresholds parameters
- $\partial$  some fixed integer is the the delay parameter.

A general canonical form of the threshold model is then

$$(5-2) X_t = C^{(j_t)} + B^{(j_t)} X_t + A^{(j_t)} X_{t-1} + H^{(j_t)} e_t$$

Where  $A^{(j_t)}$ ,  $B^{(j_t)}$  and  $H^{(j_t)}$  are  $k \times K$  matrix of non-random coefficients,  $C^{(j_t)}$  is a  $k \times 1$  vector of constants, and  $\{e_t\}$  is a sequence of i.i.d.

k-dimensional random vectors with zero mean and a covariance matrix  $\Sigma$ .

In the following sections we will use a special case of the threshold model which we know describe.

We first define  $j_t$  as a function of  $\{X_t\}$  for all  $t$ , such that  $j_t$  takes the values of  $j=1,2,\dots,m$  conditional on  $X_{t-\partial}$  included in  $(r_{j-1}, r_j] = R_j$ ,

we then have:

$$(5-3) \quad X_t = a_0^{(j)} + \sum_{i=1}^k a_i^{(j)} x_{t-i}^{(j)} + e_t^{(j)}$$

conditional on  $X_{t-\partial}$  in  $R_j$   
and  $a_i^{(j)} = 0$  for  $i = k_j+1, k_j+2, \dots, k$

Tong has named this model Self-Exciting Threshold model of order  $(m; k_1, k_2, \dots, k_m)$  with threshold parameters  $\{r_1, r_2, \dots, r_{m-1}\}$  and delay  $\partial$ . Or in short SETAR( $m; k_1, k_2, \dots, k_m$ ). The  $\{e_t^j\}$  series is an heterogeneous white noise sequence.

#### Generality of the threshold models:

Note that the SETAR(1,K) is the usual AR(k) model. As for the AR(k) model extension of the SETAR model, to include a moving average part or other explanatory variables, is straight forward. Other possibilities include considering  $j_t$  as a Markove chain of an appropriate order and state space  $(1,2,\dots,m)$ . Tong (1983) also shows that if one consider general non-linear first order autoregressive model NLAR(1), of the form

$$(5-4) \quad X_t = f(X_{t-1}) + e_t$$

and uses approximation theorems of the Weierstrass-type, then a SETAR(m,k,k,...k) could approximate arbitrarily well a NLAR(1) model.

In a more general approach it can be shown that threshold models emerge "naturally" from catastrophe theory developed by Thom (1982),

Catastrophe theory is a framework for studying discontinuous decision processes. It implies that sudden qualitative changes in a model are brought about by an accumulation of small quantitative changes. The notion of threshold is embedded in these models.

#### Determining the threshold parameter ( $r_i$ ):

In view of the discussion in the precedent chapters, several possible interpretations of thresholds in exchange rate theory can be proposed. These can be defined either with respect to volatility or more interestingly with respect to intervention on the exchange rate markets.

In the following section we calculate two threshold parameters, defining three regions, by: (5-6)  $r_i = \pm \rho * sd$

were:

- $\rho$  is a constant measuring the sensibility of expectations with respect to exchange rate volatility or intervention on the exchange rate market.
- $sd$  is the standard deviation of the time series under scrutiny

A grid search, using AIK's criterion is done on the different values of  $\rho$ .

Possible extensions of this definition include making  $sd_t$  time dependant such that:

$$(5-7) \quad \left| \begin{array}{l} r_i = \pm \rho * sd_t \\ \text{and } sd_t \text{ define on the interval } (t-N, t-1], N > 0 \end{array} \right.$$

which could express the fact that the variance of exchange rate increases at time of intervention.  $\rho$  itself could be made to vary depending on the sign of  $r_i$  in view of the none symmetric structure of exchange rates time series pointed out in the data analysis. Other possibilities include defining  $\rho$  as a function of exogenous variable such as the growth rate, import/export balance, differential of inflation and interest rate for some period. Clearly there is wide variety of formulation of the threshold parameter that one may choose, but further work will be needed to see if some, if any, of these are relevant with respect to exchange rates modeling.

#### The delay parameter ( $\theta$ ):

The delay parameter is an important notion for threshold models and more specifically to the understanding of cyclical phenomena. In fact, few feedback mechanisms would produce instantaneous responses. Of course in the case of discreet time data series, if the time delay is negligible relatively to the sampling period, then it will be reasonable to assume that the delay parameter is zero for all practical purposes. This might be the case for exchange rates markets where the delay of reaction, from the different economic agents on the market, could very well be less then a day. Since the time periodicity of the FDM exchange rate time series used

for this work is a day, the delay parameter estimated by the Setar model might come out to be zero.

Maximum number of AR lags (K) :

For each region, defined by the threshold parameters, we fit an AR regression. The maximum number of lags allowed for the grid search is  $k_{\max}$ . This does not imply that lags for different regions are the same. In fact the maximum lag retained for two regions will be different as we will see in the following section.

**Section 5.2. Identification and Estimation**

We estimate two models:

- a SETAR model applied to the rate of change  $r_t$
- a SETAR model applied to the deviation from the bilateral central rate  $\text{Sind}_t$ .

We have used AIC's criterion, reviewed in chapter 3, in the identification process of the SETAR models applied to the FDM exchange rate. The best values of the NAIC for different values of the delay and threshold parameters is given along with the best corresponding lags for each region ( the normalized AIC criterion: NAIC is defined below). The range for the grid search of the different parameters is of course subject to

some arbitrary, but it seems that no gain of information could be obtained by increasing them. The following ranges were used:

- [0.5 , 1.5] for  $\rho$ ....., the "sensitivity parameter"
- [0 , 6 ] for  $\partial$ ....., the delay parameter
- [0 , 8 ] for  $k_j$ ....., the number of lags for a given region

We now proceed as follow:

(1) given the initial values  $\rho_0$  and  $\partial_0$  sort the data in  $m$  subset and find the best AR model, for the  $m$  regions, by computing the AIC criterion varying  $k_j$  over it's range. The AIC criterion can be computed by :

$$AIK(k_j) = N_i * \ln \left( \frac{||e_{k_j}||^2}{N_i} \right) + 2(k_j+1)$$

which we minimize with respect to  $k_j$  for each regions obtaining :

$$AIK(r_0, \partial_0) = AIK(\bar{k}_1) + AIK(\bar{k}_2) + AIK(\bar{k}_3)$$

(2) same as in step (1) keeping  $\partial_0$  fixed and allowing  $\rho$  to vary over it's range and minimize the AIC criterion such that:

$$AIK(\bar{r}, \partial_0) = \min_r AIK(r, \partial_0)$$

(3) search over the range of  $\partial$  and find the best model, i.e. which minimize the normalized AIC criterion (NAIC) defined by:

$$NAIC(\partial) = \frac{1}{(N-N_{\partial})} AIK(\bar{r}, \partial)$$

Where  $(N-N_{\partial})$  is number of observation used for the regression. We know turn to the results<sup>14</sup>.

### Section 5.2.1. The Rate of Change

We first apply the threshold model to the rate of change. The results are given in Table #5-1. The best model is the one which minimizes the NAIC. For the rate of change, this model is given for a delay parameter  $\partial$  equal to zero. As noted above this could be due to the fact that the market reacts extremely rapidly to variation in the exchange rate. The sensibility coefficient  $\rho$  is 0.25, implying a threshold of  $\pm 0.5927$  E-03. This small threshold values are understandable when one looks back to the histogram of the rate of change. It shows that most of the probability mass is concentrated around the mean. The best lags, for each regions defined by the two thresholds, are 6 for the lower region, 3 for the middle region and 1 for the upper region. The Spooled Residual Sum of Square (SRSS) is  $0.29544$ E-02 and  $R^2 = 0.06795764$ . The correlation function of residuals for the "best" threshold model identified above, has been computed and plotted (Graph #5-1).

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<sup>14</sup> The model was estimated using the RATS econometric package available on mainframes, IBM PC and Macintosh.

For comparison an ARMA model of order ARMA(2,4) is fitted to the data set with devaluation dates set to zero (table #5-2). The best results were found for these lags. The RSS is 0.42052601E-02 which is higher than the value found for the threshold model, and a  $R^2 = 0.05153415$  slightly less than for the threshold model. The Naic is -13.150 implying that the threshold model is the "best model".

**- Table #5-1 Threshold Model and the Rate of Change**

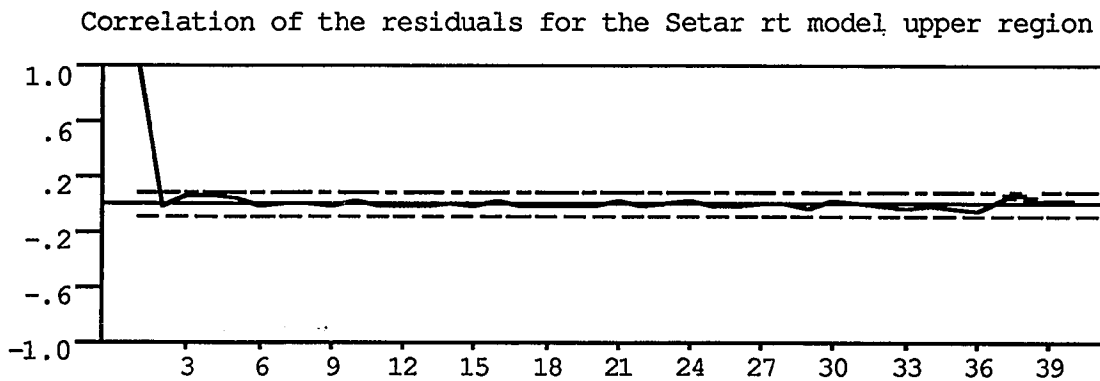
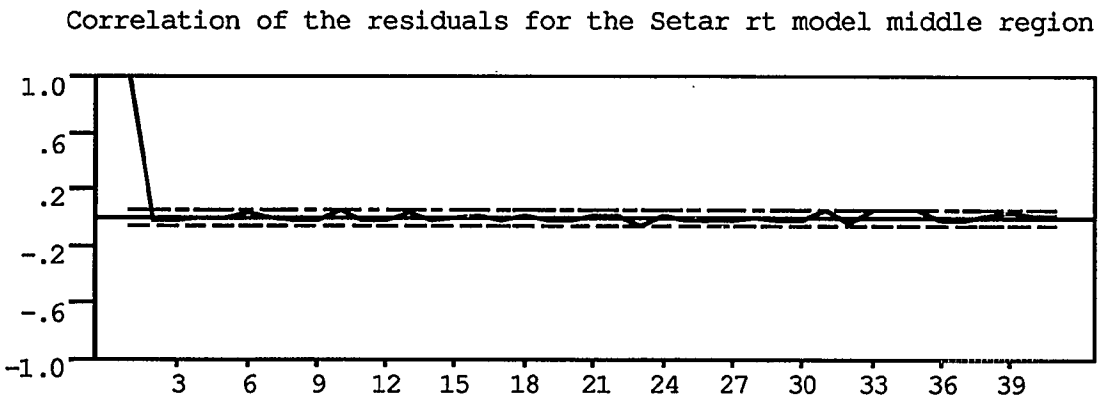
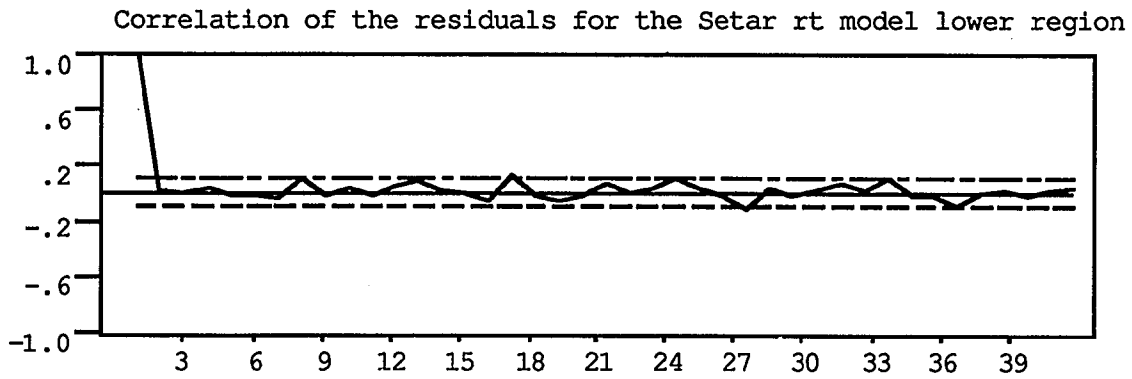
$\partial$	$\rho$	$r_i$	$K_1$	$K_2$	$K_3$	NAIC
0	0.25	$\pm 0.59271E-03$	6	3	1	-14.684
	0.50	$\pm 0.11854E-02$	4	1	1	-14.519
	1.00	$\pm 0.23708E-02$	6	6	1	-14.134
	1.50	$\pm 0.35563E-02$	6	6	6	-13.955
1	0.50	$\pm 0.11854E-02$	1	5	1	-12.554
	1.00	$\pm 0.23708E-02$	1	2	1	-12.575
	1.50	$\pm 0.35563E-02$	6	2	1	-12.565
2	0.25	$\pm 0.59271E-03$	2	2	2	-12.169
	0.50	$\pm 0.11854E-02$	3	1	2	-12.138
	1.00	$\pm 0.23708E-02$	1	4	2	-12.209
	1.50	$\pm 0.35563E-02$	1	2	1	-12.239
4	0.25	$\pm 0.59271E-03$	2	5	1	-12.341
	0.50	$\pm 0.11854E-02$	2	6	1	-12.462
	1.00	$\pm 0.23708E-02$	2	1	1	-12.218
	1.50	$\pm 0.35563E-02$	4	2	2	-12.247
6	0.25	$\pm 0.59271E-03$	1	5	1	-12.128
	0.50	$\pm 0.11854E-02$	1	5	1	-12.226
	1.00	$\pm 0.23708E-02$	2	3	1	-12.248
	1.50	$\pm 0.35563E-02$	6	3	1	-12.291

SDXX 0.2370839684E-02

- Table #5-2- The ARMA Model and the Rate of Change

DEPENDENT VARIABLE	rt			
TOTAL OBSERVATIONS	2121	SKIPPED/MISSING	0	
USABLE OBSERVATIONS	2121	DEGREES OF FREEDOM	2115	
R**2	0.05153415	RBAR**2	0.04929191	
SSR	0.42052601E-02	SEE	0.14100719E-02	
DURBIN-WATSON	1.99909456	SIGNIFICANCE LEVEL	0.441473E-02	
Q(138)	181.503	NAIC	-13.150	
LABEL	LAG	COEFFICIENT	STAND ERROR	T-STATISTIC
AR	1	0.01367163	0.1026987	1.331236
AR	2	-0.5918685	0.1022836	-5.786544
MA	1	-0.1352683	0.1045783	-1.293464
MA	2	0.4111923	0.1042232	3.945307
MA	3	0.7039114E-02	0.3362839E-01	0.2093206
MA	4	0.2299846E-01	0.3343701E-01	0.6878147

**- Graph #5-1 Correlation function of the residuals  
from the threshold model applied to rt**



### Section 5.2.2. The Indexed Rate

We now apply the same model on the indexed rate of change ( $Sind_t$ ). The results are given in Table #5-3. We found that the best model is given for a delay parameter  $\partial$  again equal to zero. The sensibility coefficient is 0.5 implying a threshold parameter of  $\pm 0.4981$ . It is interesting to note that these threshold values are much smaller than the limits of intervention respectively  $\pm 2.25\%$ . The best lags, for each region defined by the two threshold, are 1 for the lower region, 3 for the middle and for the upper region. The pooled Residual Sum of Square (SSR) = 15.7642 and the pooled  $R^2=0.86053$ .

We have also fitted an ARMA model of order AR(3) to the data set. The results are reported in table #5-4. The value  $R^2$  is 0.94722006 which is sensibly higher than for the threshold model. The Naic for this ARMA model is -2.948, way above the Naic found for the best threshold model<sup>15</sup>.

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<sup>15</sup> The best model is the one that minimize the Naic coefficient.

The correlation function of the residuals and squared residuals are computed and plotted (graph #5-4 and #5-5). The correlation function of residuals show no significant coefficients. This is not the case however for the squared residuals In particular coefficient 1 through 8 of the upper region are significant. This implies that our model has not captured all the structure of the data set. One might consider adding a Moving average part to the model. For now we will keep this model and compare its simulation results to the ARMA model. For the ARMA model (graph #5-6) coefficient 4 and 5 of residuals are slightly significant. None of the coefficients of the correlation function of the squared residuals are significant.

- Table #5-3 Threshold Model and the Indexed Rate

$\partial$	$\rho$	$r_i$	$K_1$	$K_2$	$K_3$	NAIC
0	0.5	$\pm 0.4981$	1	3	3	-4.042
	1.0	$\pm 0.9962$	1	5	1	-3.862
	1.5	$\pm 1.4942$	2	3	2	-3.682
2	0.5	$\pm 0.4981$	1	5	3	-3.095
	1.0	$\pm 0.9962$	1	5	5	-3.076
	1.5	$\pm 1.4942$	1	4	5	-3.077
4	0.5	$\pm 0.4981$	1	6	5	-2.952
	1.0	$\pm 0.9962$	4	5	5	-2.964
	1.5	$\pm 1.4942$	1	6	5	-2.851
6	0.5	$\pm 0.4981$	2	5	1	-2.856
	1.0	$\pm 0.9962$	1	6	1	-2.814
	1.5	$\pm 1.4942$	5	6	2	-2.795

$sd = 0.9962$

- Table #5-4 The ARMA Model and the Indexed Rate

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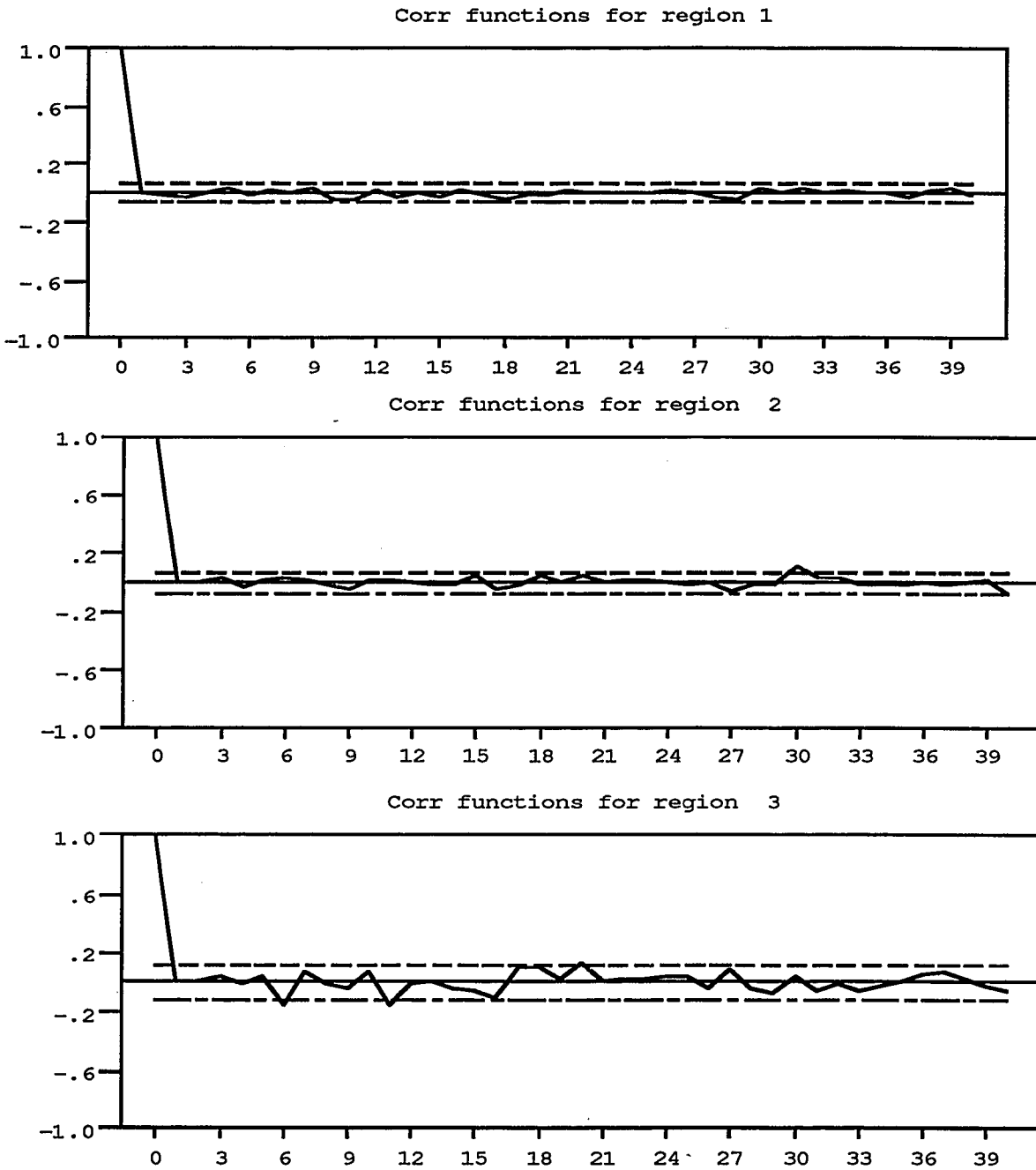
DEPENDENT VARIABLE	Sindt		
TOTAL OBSERVATIONS	2128	SKIPPED/MISSING	0
USABLE OBSERVATIONS	2128	DEGREES OF FREEDOM	2125
R**2	0.94722006	RBAR**2	0.94717038
SSR	111.59269	SEE	0.22915978
DURBIN-WATSON	2.01429091	SIGNIFICANCE LEVEL	0.273898
Q(138)	147.986	NAIC	-2.948

---

VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC
AR	1	0.9684053	0.2152094E-01	44.99829
AR	2	-0.1139522	0.2997209E-01	-3.801944
AR	3	0.1257200	0.2151971E-01	5.842088

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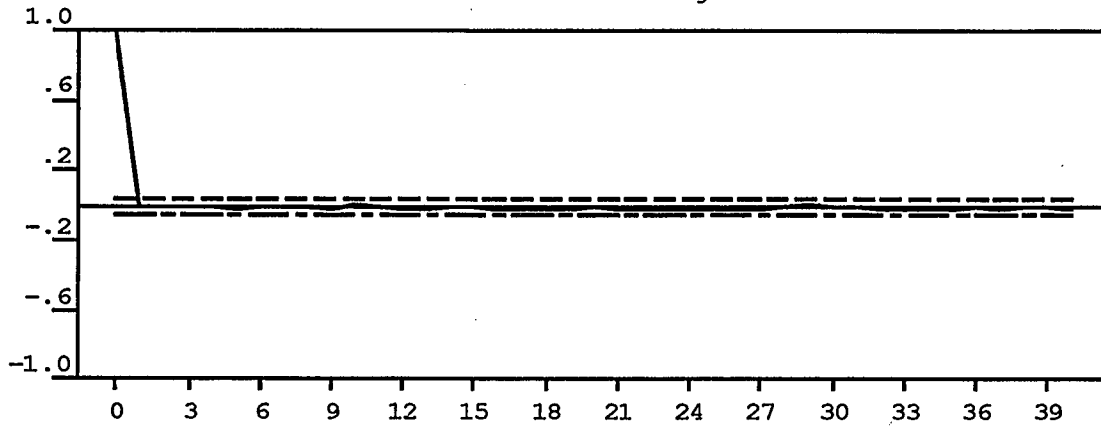
- Graph #5-4- Correlation function of the residuals  
from the threshold model applied to Sindt



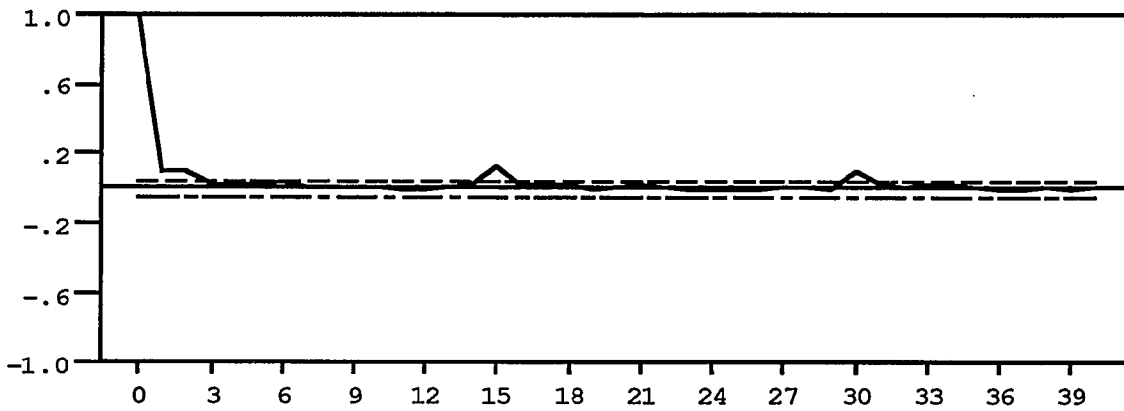
**Graph #5-5 Correlation function of the squared  
residuals from the threshold model applied to**

**Sindt**

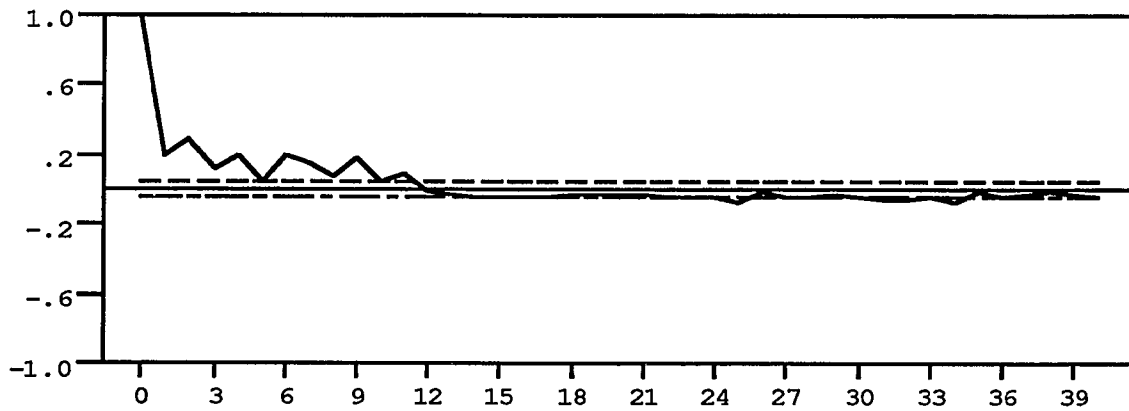
Corr functions for region 1



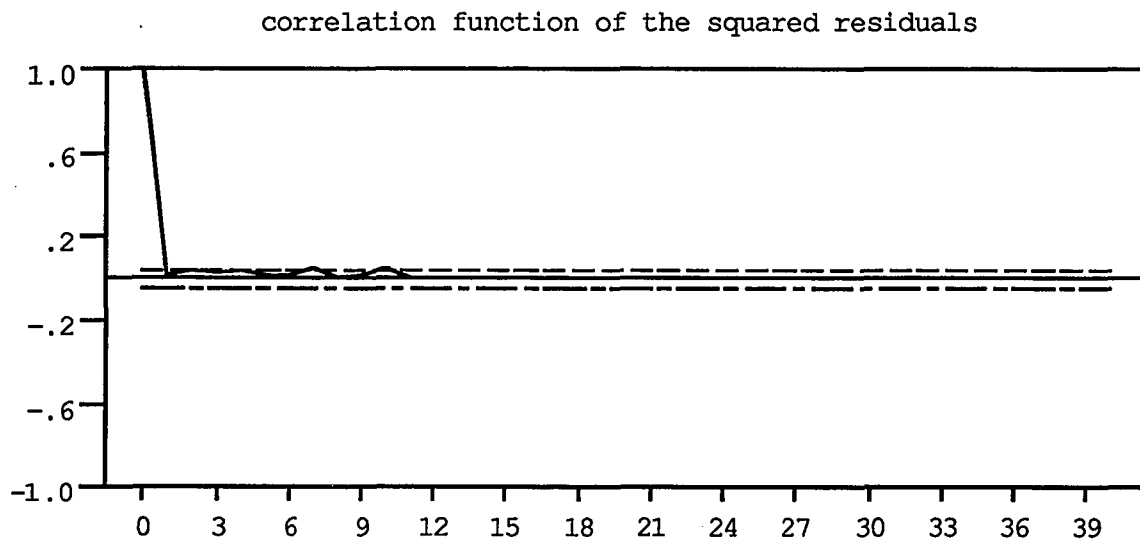
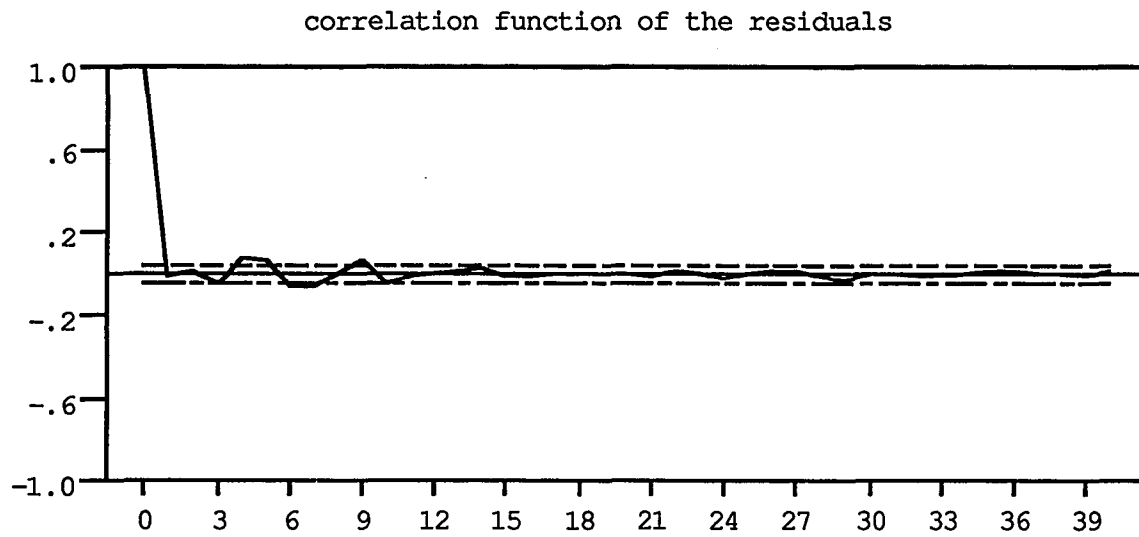
Corr functions for region 2



Corr functions for region 3



**Graph #5-6 Correlation function of the residuals  
from the ARMA model applied to Sindt**



### **Section 5.3. Simulation Results**

We now turn to some simulation results. If the fitted model can generate a data set exhibiting the same structure then the original series, then this model is said to represent well the data. In the following sections we compare the results of the threshold model and the ARMA model for both the rate of change  $r_t$  and the indexed rate of change  $S_{int}$ .

#### **Section 5.3.1. The Rate of Change**

Statistics on the simulated rate of change are reported in table #5-5. The sample mean of the simulated data is much smaller as compared to the original data set. This is also the case of the maximum and minimum values, as well as the standard deviation even when compared with the results obtained with the devaluation dates set to zero. Skewness, although positive is not as high as for the original data set. More important Kurtosis is way under the original data set value. It seems therefore that the model has not been able to capture the generating process of the rate of change.

A plot of the simulated data is given in Graph # 5-7, along with its correlation and partial correlation function.

**Table #5-5- Statistics on the Setar Simulated rt**

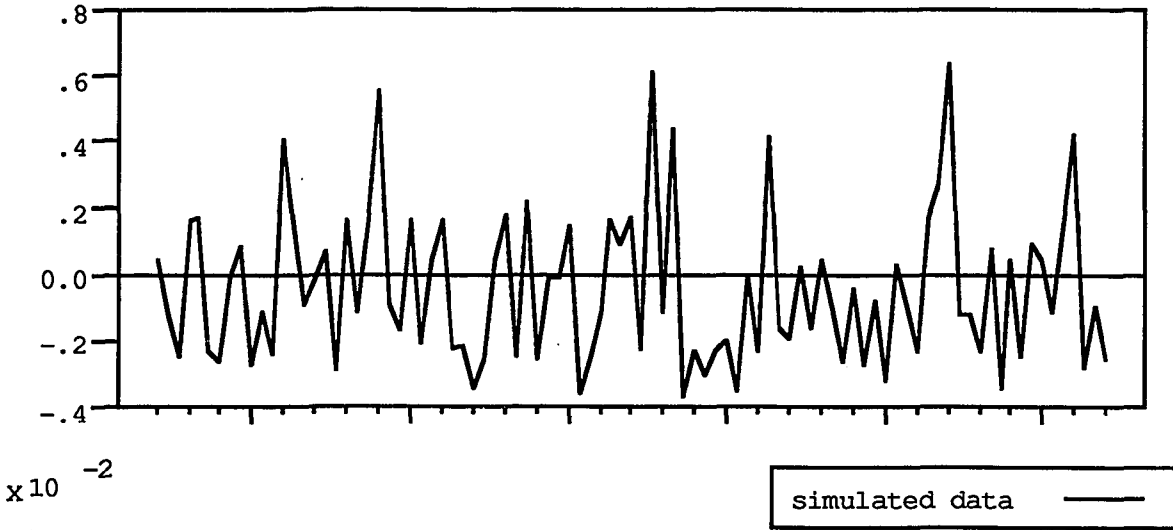
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SAMPLE MEAN	0.1265267E-03	VARIANCE	0.23636E-05
STANDARD DEVIATION	0.1537421E-02		
STAN. DEV. OF MEAN	0.1343251E-03		
T-STAT FOR MEAN=0	0.9419433	SIGNIFICANCE LEVEL	0.3462217
SKEWNESS	0.3052940	SIGNIFICANCE LEVEL	0.1584875
KURTOSIS	-0.5502718E-01	SIGNIFICANCE LEVEL	0.9004072
MINIMUM VALUE IS	-0.3622259E-02		
MAXIMUM VALUE IS	0.4694872E-02		

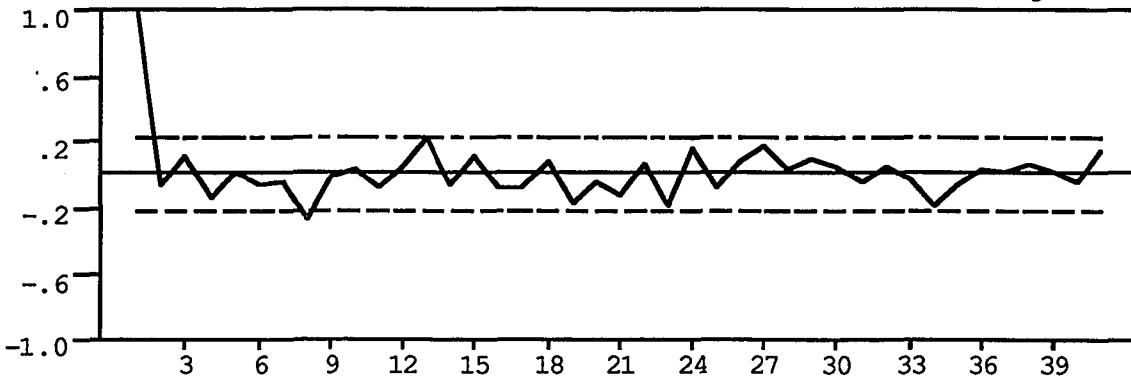
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**Graph #5-7- Simulation of  $\pi$  using a Threshold model**

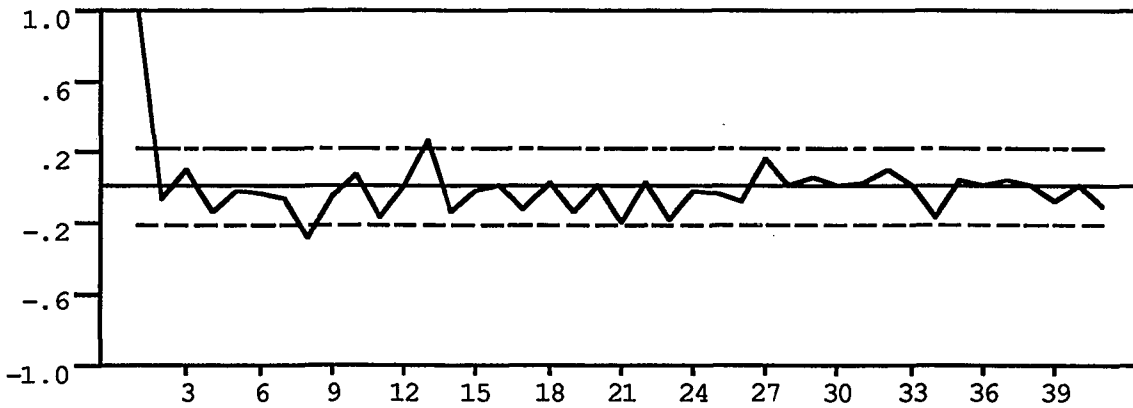
Setar simulation of the rate of change



Correlation of the Setar simulated rate of change

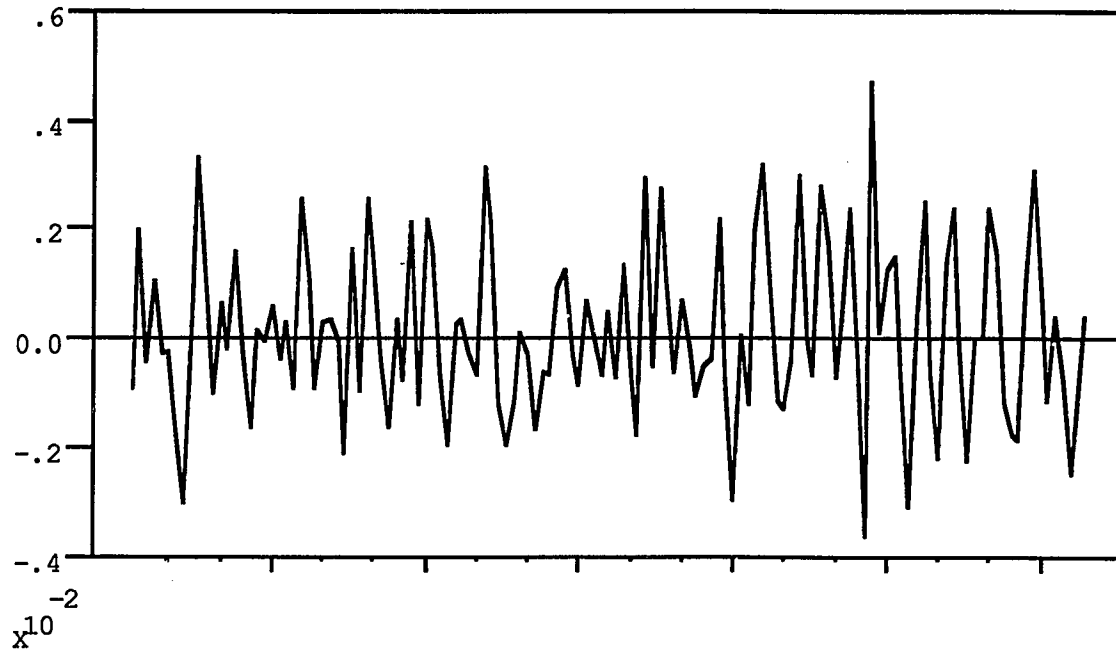


partial correlation of the Setar simulated rate of change



**Graph #5-8- Simulation of rt using an ARMA  
model**

Plot of of a simulated rt series  
using an ARMA(2,4) model (300 points)



### **Section 5.3.2. The Indexed Rate**

We repeat the simulation exercise of the above section with the indexed rate. Statistics are computed on the data set simulated by the threshold model. The results are reported in table #5-6 below. We may compare these statistics to the one obtained for the original data set. The sample mean is in both cases negative, but in the case of the simulated series, which is equivalent to the official bilateral central rate, it is much farther away from zero than the original data set. The minimum values are approximately equal while the maximum value is lower in the case of the simulated series. It seems that the model has generated a bias increasing the importance of the lower region of the original data series. Standard deviation is quite close to its original value. Skewness is slightly higher and kurtosis is positive instead of negative.

This model seems to capture more accurately the underlying dynamical process of the indexed rate as compared to the model identified for the rate of change. Still some aspect of the data structure has not been captured and further research will be needed to see if possible extension of this model can give a more accurate description of the data.

**Table #5-6 Statistics on the Setar Simulated Sindt**

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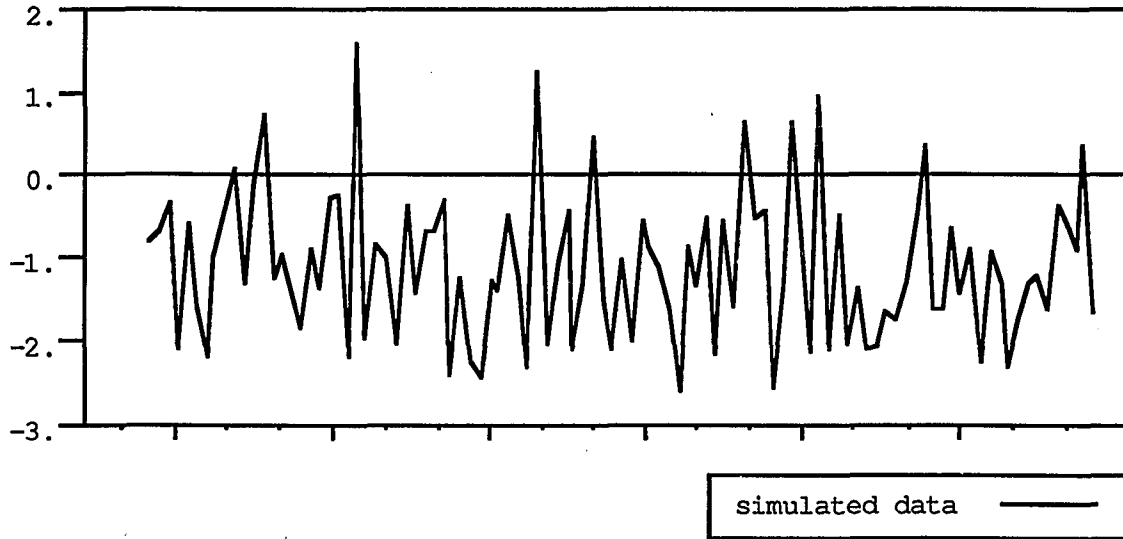
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SAMPLE MEAN	1.109257	VARIANCE	0.7908321
STANDARD DEVIATION	0.8892874	STAN. DEV. OF MEAN	0.8937675E-01
T-STAT FOR MEAN=0	-12.41102	SIGNIFICANCE LEVEL	0.0000000
SKEWNESS	0.6505139	SIGNIFICANCE LEVEL	0.9259297E-02
KURTOSIS	0.2434325	SIGNIFICANCE LEVEL	0.6333062
MINIMUM VALUE IS	-2.583533		
MAXIMUM VALUE IS	1.585408		

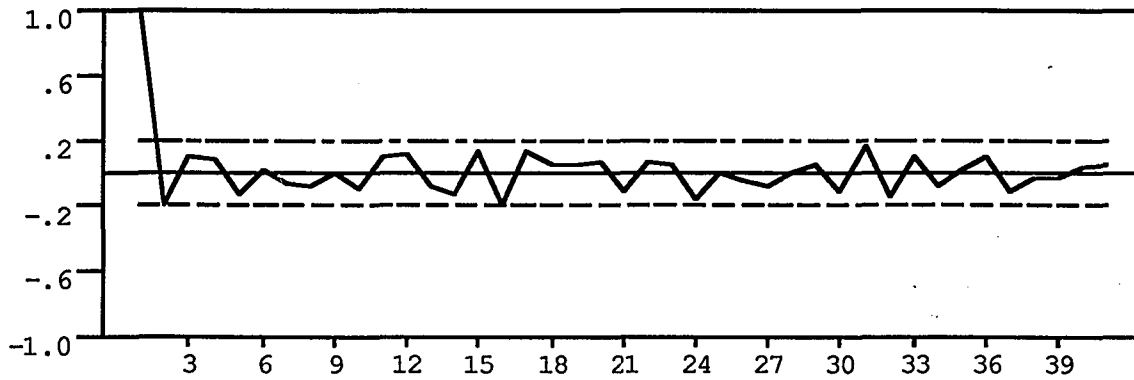
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**Graph #5-9- Simulation of  $Sind_t$  using a Threshold model**

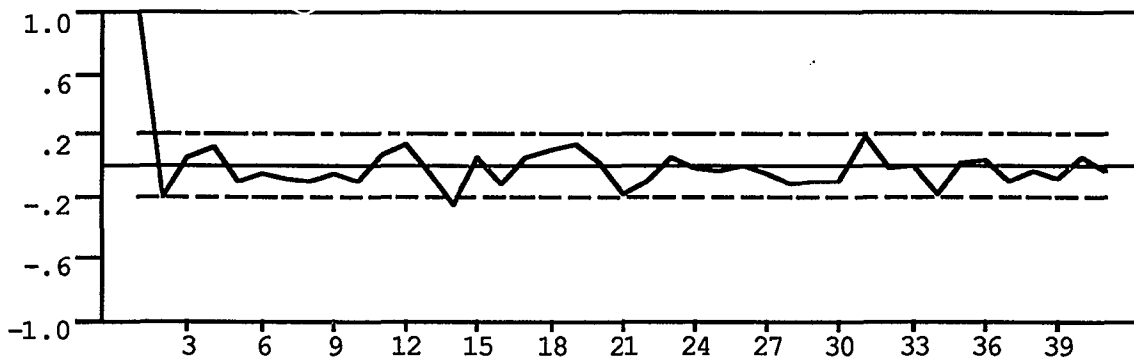
Setar simulation of the indexed rate



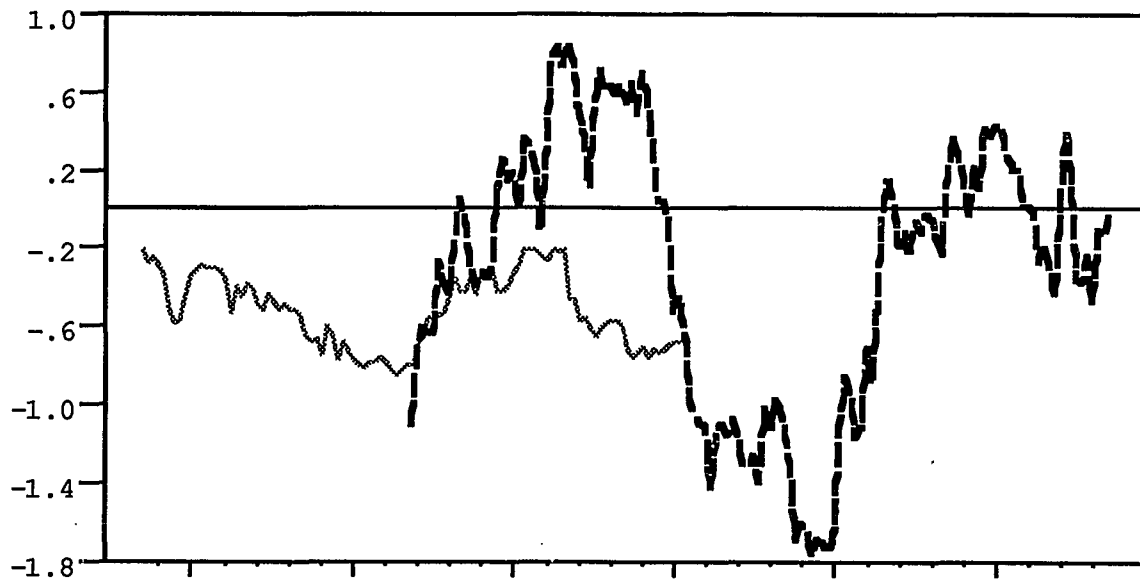
correlation of the setar simulated  $Sind_t$  series



partial correlation of setar simulated  $Sind_t$  series



- Graph #5-10 Simulation of Sindt and the ARMA  
model -



## Conclusion

The main goal we assigned ourselves, was to investigate the use of nonlinear models, as compared to linear model, when modeling econometric time series. A secondary goal was to see if the Threshold model developed by Tong could help characterize the french-deutschmark exchange rate generating process. In particular, we were interested in the effect of intervention.

The data series has been shown to exhibit skewness and leptokurtosis. This casts doubts on the usefulness of the Gaussian hypotheses when modeling exchange rates and suggests the use of nonlinear models. We show that there are many reasons to believe that the driving process is indeed nonlinear. In particular we show that the specificity of the EMS exchange rate markets may imply "thresholds" and sudden jumps due to devaluations.

We then present a threshold model. The threshold parameter is computed as the standard deviation of the data set multiplied by a sensibility parameter, and identified by AIC's criterion. We apply the model, both to the rate of change and the indexed rate. The results are reported and compared to a ARMA model. The best results are obtained for the indexed rate. But the model doesn't seem to be able to characterize completely the data. One immediate extension of the model, is the inclusion of an MA part to the threshold model used in chapter 5.

Further research topics include testing different threshold parameters. As noted in chapter 5, an interesting extension would be to make this coefficient time dependent through the standard deviation. One may also fit different nonlinear model such as a Bilinear model to exchange rate series. Finally it will be interesting to apply nonlinearity test such as the Hinich and Patterson (1985) bispectrum test or the whiteness test developed in Brock and Sayers (1986) to our data series.

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