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DOUBLE THRESHOLD ACQUISITION SCHEME IN FREQUENCY HOPPING SPREAD SPECTRUM

BY

QIANYI JIANG

A dissertation submitted to the Graduate Faculty in Electrical Engineering in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

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Abstract

DOUBLE THRESHOLD ACQUISITION SCHEME

IN FREQUENCY HOPPING SPREAD SPECTRUM

by

Qianyi Jiang

Adviser: Professor Donald L. Schilling

In this dissertation a double threshold acquisition scheme in frequency hopping spread spectrum is proposed and has been studied to minimize the mean acquisition time. The dissertation consists of two parts, in part one the acquisition scheme is applied to the nonfading channel, in part two the acquisition scheme is applied to the fading channel.

For the nonfading channel the equations to calculate the two thresholds are derived. The probability of false acquisition and dismissal are analyzed mathematically. The theoretical mean value of acquisition time is calculated. A computer simulation program of the communication system is used to determine the mean and the variance of acquisition time. The experiments are repeated more than 50 times to get the statistical value. The results of the

experiments are coincident with the theory. The comparision among the single threshold scheme and the different double threshold schemes is made analytically and experimentally.

In the second part the double threshold acquisition scheme working in the fading channel has been studied. The signal in the fading channel has been analyzed by theoretical calculation and also been investigated by the experiments of simulation. The variance of the signal and the probability density function of the signal envelope in the fading channel are presented. The equations of the two thresholds are derived. The computer simulation is also done to determine the mean and the variance of acquisition time for the optimum P_{FA} and P_D and also for the different hopping rates. The results are compared with the nonfading channel.

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Chapter 1

INTRODUCTION

1.1 Definition of Acquisition in Frequency Hopping Spread Spectrum (FH SS)

Frequency hopping spread spectrum is becoming more and more popular in communication system due to its strong ability of guarding against the interference (the jammer or the other friendly user who is stronger in power or closer to the receiver) and the fading channel. In FH communication system the transmitting carrier frequency is controlled by a pseudonoise sequence generated by linear feedback shift registers. The state of the shift registers controls a frequency synthesizer which provides the transmitting carrier frequencies. The state of the shift registers changes in accordance with the timing clock so that the transmitting carrier frequency hops in pace with the timing clock as well. The jammer doesn't know the PN sequence that we used, hence it can only jam a certain frequency but not the total signal. In the receiver there must be such a PN sequence replica which controls the local carrier. The two PN sequence should be synchronized such that the local carrier frequency synthesizer provides exactly the same hopping frequency and hops at the same rate as the transmitter in order to dehop the signal and recover the data from demodulation such as FSK.

The synchronization usually contains two parts, coarse acquisition (initial acquisition) and fine acquisition. The coarse acquisition brings the system into the pull-in range of the tracking loop and then the tracking loop brings the local hopper completely in phase with the incoming signal.

The tracking loop used is shown in Fig. 1-1.

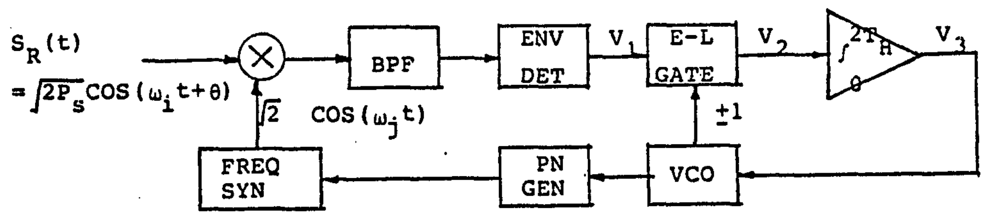

Fig. 1-1 The tracking loop

The incoming signal having frequency ω_i is correlated with the local carrier frequency ω_j. If $\omega_i \neq \omega_j$, the output of the LPF is noise only. If $\omega_i = \omega_j$, the output of the LPF is signal plus noise. Such signal or noise presents at the input of the envelope detector. The output of ENV DET is always postive but the magnitude is large if $\omega_i = \omega_j$, or the magnitude is very small if $\omega_i \neq \omega_j$ as shown in Fig. 1.2. This signal passes through an Early-Late Gate which is a multiplier multiplied by the VCO voltage of ±1 for each half period. The output of the E-L Gate is also shown in Fig. 1.2. After integration the output of the integrator is related to the

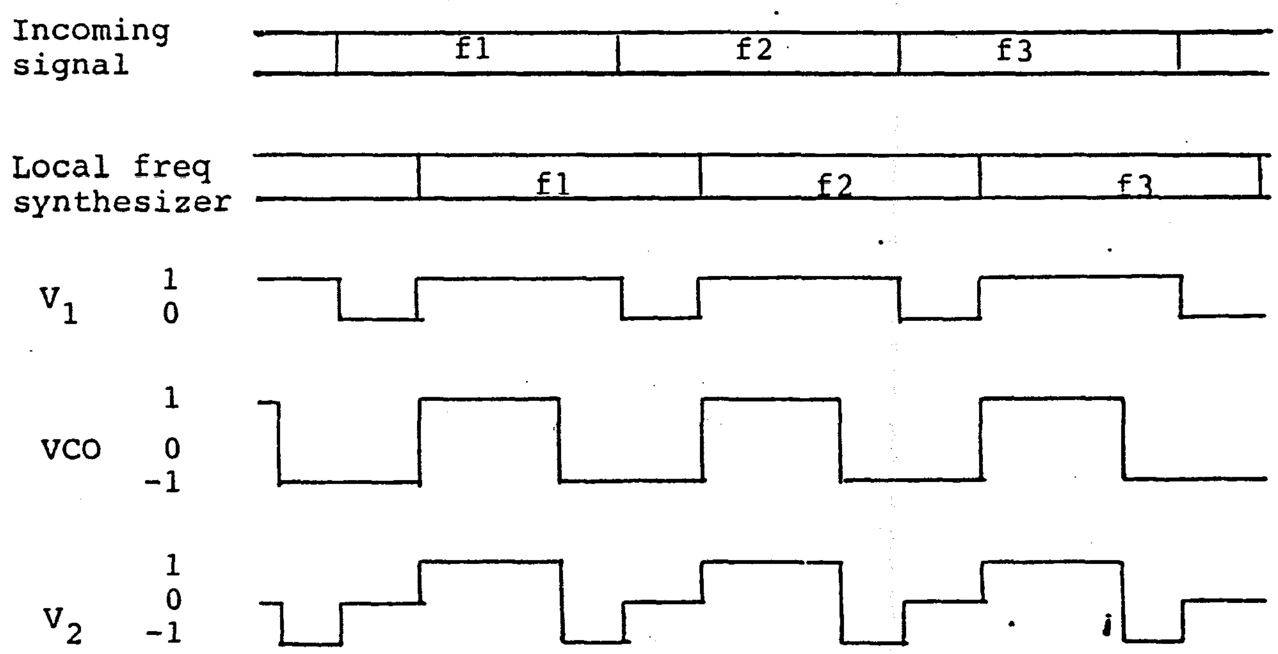

Fig. 1-2 Operation of the E-L Gate

delay time between the local carrier and the incoming signal. The voltage controls VCO and brings the transition of the local frequency synthesizer exactly coincident with the incoming signal. The fine acquisition is fulfilled.

From Fig. 1-3 we saw if the allignment between the incoming signal and the local frequency synthesizer is greater than half of the hop duration, fine acquisition can succeed. Hence, the task of the coarse acquisition is to bring the system into the pull-in range of the tracking loop; that is, half of the hop duration.

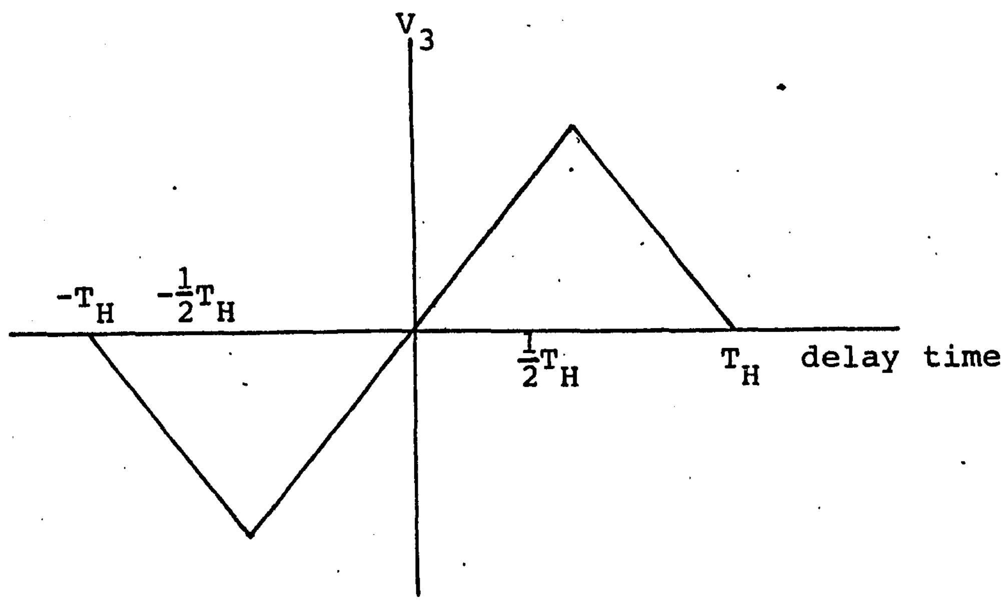

Fig. 1-3 VCO control voltage versus the delay time between the incoming signal and VCO

1.2 Importance of the Problem

Before the system starts to work, it must be synchronized. Therefore, people can not communicate until the synchronization has occurred. The time needed for synchronization is greatly concerned. Because the tracking loop works in the same way as written in many other literatures,here we emphysize the strategy of coarse acquisition. The performance of coarse acquisition can be represented by the mean acquisition time. How to minimize the mean acquisition time is the goal that we strive for. In this dissertation we presented a double threshold acquisition scheme. The theoretical calculation and also the experiments show that the mean acquisition time of this scheme is 1/2 to 1/4 of that of the single threshold scheme. The implementation of this scheme is as simple as the single threshold scheme. Therefore, its advantage is evident.

1.3 Summary of Prior Work

Coarse acquisition can be broadly divided into correlation techniques and sequential estimation techniques. The various typies of methods which can be used for DS SS can also be used for FH SS similarly. In this section I only illustrated them briefly and the advantage and the disadvantage of these techniques will also be mentioned.

1.3.1 Correlation Technique

Correlation technique can also be considered as serial search mode, parellel search mode and the combination of both.

(1) Serial Search Mode, Single Threshold Scheme

For DS SS refer to Fig. 1-4

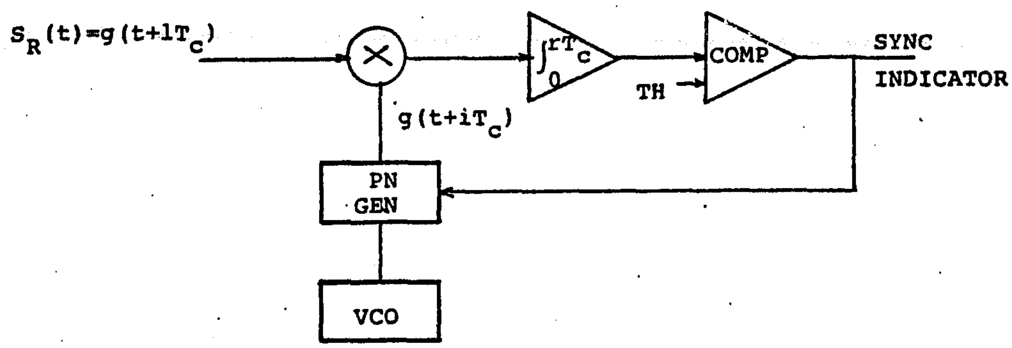

Fig. 1-4 Receiver in DS SS system
(serial search mode, single threshold)

The incoming signal is correlated with a local PN generator which generates a PN sequence replica as in the transmitter. If these two PN sequences are synchronized the output of the integrator will be greater than some threshold and coarse acquisition is declared. If the output of the integrator is below the threshold, we say that the system is not synchronized. The comparator outputs a signal which changes the state of the PN generator and further processing is continued until the true acquisition occurs.

In order to reduce the probability of false alarm to a certain value the integration time for each comparision is relatively long. The number of comparision is of the order of the code length. Hence, the disadvantage of this scheme is time consuming but the advantage of this scheme is that the implementation of such a system is simple.

For FH SS refer to Fig. 1-5.

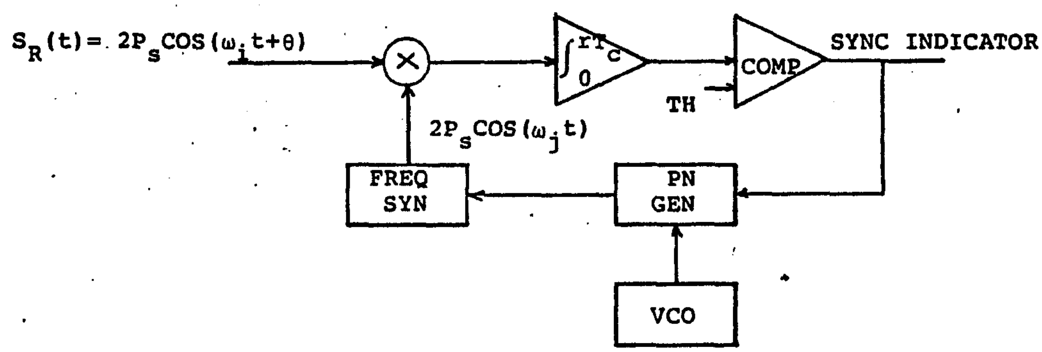

Fig. 1-5 Receiver in FH SS system
(serial search mode, single threshold)

This scheme can also be used to FH SS. Instead of comparing the incoming signal with the local PN sequence, the incoming signal is correlated with a frequency synthesizer which is controlled by the PN sequence. The principle of the acquisition strategy is the same as DS SS.

(2) Serial Search Mode, Double Threshold Scheme

In reference [1], a double threshold acquisition scheme is suggested for DS SS. This acquisition scheme has the advantage of the simplicity as the serial search mode and the rapid acquisition performance. The price to pay is that the number of false acquisition will increase a little bit, If the assigned probability of false acquisition is low enough for each comparision, the result is not bad.

An improved double threshold acquisition scheme for FH SS is proposed in this dissertation. We use the conditional probability to calculate the two thresholds. As a result, We keep the rapid acquisition performance and the number of false acquisition is also reduced to the minimum. This scheme will be described in more detail in chapter 2.

(3) Parellel Search Mode

In reference [5] a two level coarse code acquisition scheme is suggested and in reference [11] the matched filters, passive correlators,are used to search the starting epoch in real time. These schemes cost a lot of hardware to save time.

1.3.2 Sequential Estimation Technique

In reference [7] recurrion aided rapid acquisition by sequential estimation (RARASE) is suggested.

Sequential estimation technique is much faster than correlation technique but the drawback of any sequential estimation technique is that it is more vulnerable to jamming. Thus it is suitable as a rapid synchronization method only for system designed to operate at medium signal to noise ratios.

In reference [9] and [10] the sequential estimation technique used in FH SS is described. In FH system one sequential estimation technique is to sample the received signal at more than twice the maximum hopping frequency, Nyquist Rate, and use sequential power spectral estimation techniques to obtain the received frequencies during n consecutive signaling intervals. N is the number of LFSR. The initial timing uncertainty necessitates the sampled data to be devided into a number of segment within the hop interval, so that the transient change in estimated frequency during the segment containing a transition from one frequency to another can be discarded. The mean value of the estimated frequency in the remaining segments in each hop interval is used to obtain an estimate of the frequency during the hop interval. Once we know the frequency we know the vector of the LFSR, at least we know the MSB of the vector. IF the estimate is accurate enough, we may determine more than one MSB in one hop interval. Therefore, at most n successive frequency estimates are used to obtain the initial state of the LFSR, with which tracking is attempted.

1.4 Summary of Research Completed

The research completed is that we applied the double threshold acquisition scheme to both nonfading and fading channel. In chapter 2 we present the performace of this scheme working in the nonfading channel. In section 2.1 the acquisition strategy is described in detail. In section 2.2 we illustrate how to determine the two thresholds by assigning the same P_{FA} and P_D as we did to the single threshold scheme. In section 2.3 we layout the performance of this scheme; that is, the probability of making an error and the probability of missing the signal. We also calculate the expected value and the variance of number of chips to be integrated for each decision which will be compared with the single threshold scheme. In section 2.4 we show the theoretical calculation of mean acquisition time. In section 2.5 we show the results of computer simulation. The experiments are repeated 50 times for specific signal to noise ratios. The results are compared with the theoretical calculation. In section 2.6 we make a comment about the different acquisition schemes.

In chapter 3 we present the performace of this scheme working in the fading channel. Section 3.1 is the general introduction to this chapter. In section 3.2 we describe the computer simulation model of the fading channel. In section 3.3 we demonstrate the signal in the fading channel by theoretical calculation and computer simulation.

In section 3.4 we calculate the variance of the received signal (energy of the signal). In section 3.5 we mention the operation of the early-late gate in the fading channel. In section 3.6 we determine the probability density function of the signal envelope in the fading channel and from the probability density function we assign a certain value to P_{FA} and P_D to determine the two thresholds. Section 3.7 gives us the computer simulation result which is compared to the nonfading channel.

Chapter 2

DOUBLE THRESHOLD ACQUISITION SCHEME IN THE NONFADING CHANNEL

2.1 Acquisition Strategy

In this dissertation we suggested a rapid acquisition scheme for FH SS system; that is, the double threshold acquisition scheme.

The acquisition strategy of FH signal is the following: Refer to Fig. 2-1.

The incoming signal having frequency ω_i and phase θ is embedded in noise. The local frequency synthesizer is multiplied by the incoming signal and the product passes through a BFP which eliminates the sum frequency components and presents the difference component $\omega_i-\omega_j$ and the noise term to the integrator. ω_i and ω_j are selected such that the output of the integrator

$$\sqrt{P_s} \int_0^{rT_c} \cos(\omega_i-\omega_j)t \, dt = 0, \quad \text{for } \omega_i \neq \omega_j \tag{2.1-1}$$

$$\text{ie,} \quad (f_i-f_j)T_c = \text{integer} \tag{2.1-2}$$

$$\text{and} \quad \sqrt{P_s} \int_0^{rT_c} \cos(\omega_i-\omega_j)t \, dt = rT_c\sqrt{P_s} \quad \text{for } \omega_i = \omega_j \tag{2.1-3}$$

as shown in Fig. 2-2.

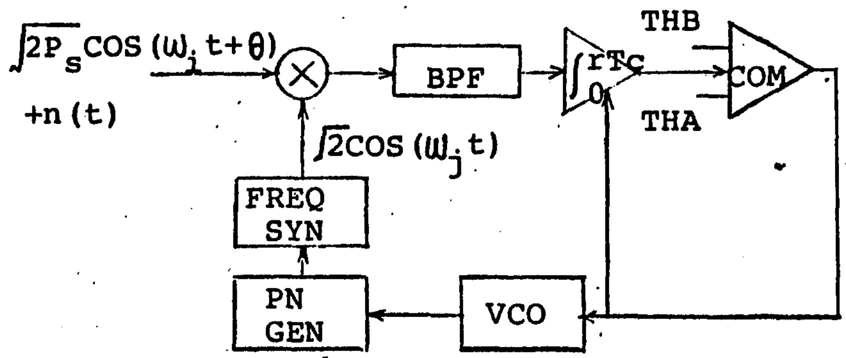

Fig.2-1 The receiver in FH SS

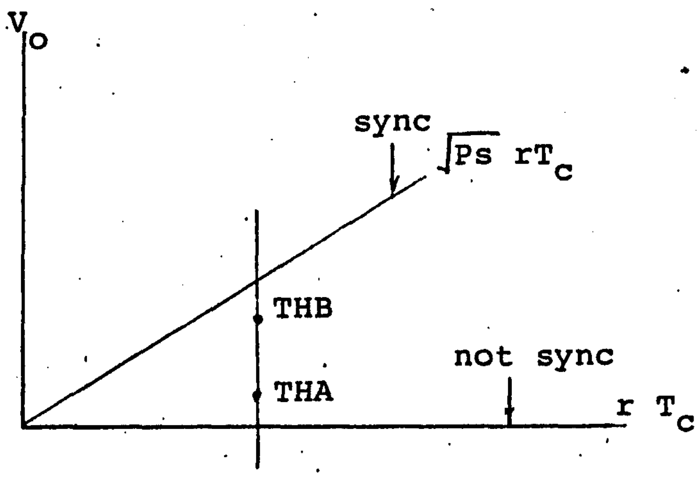

Fig. 2-2 The output voltage of the integrator and the two thresholds

The noise term present at the output of the integrator has a mean value of 0 and the variance σ^2 is proportional to $\eta r T_c/2$.

The output of the integrator is compared with two thresholds THA and THB. If V_0 < THA, we say that the incoming signal is not synchronized with the local frequency synthesizer. The comparator outputs a signal which controls the local PN generator to shift to the next state, thus the frequency synthesizer also hops to the next frequency. If V_0 is greater than THB we say that pre-synchronization has occurred. The frequency synthesizer and the PN generator remain in the old state until the incoming signal hops to the next frequency. At this time V_0 again drops below THA, the comparator declares that true acquisition has occurred, the PN generator shifts to next state and the frequency synthesizer hops to next frequency which gaurantees that the alignment between the incoming signal and the local frequency synthesizer is longer than half of the hop duration as shown in Fig. 2-3. At this moment the tracking loop starts to work which brings the local hopper completely in phase with the incoming signal and thus, fine acquisition is fulfilled. In Fig. 2-3

At time t_1, V_0 < THA, receiver hops to f_1

t_2, V_0 > THB, pre-acquisition is declared.

t_3, V_0 < THA, receiver hops to f_2, true acquisition is declared.

If V_0 is between THA and THB, no decision is made and more samples must be taken. At the end of the second rT_c another comparision is made.....etc. This procedure is repeated until a decision is made.

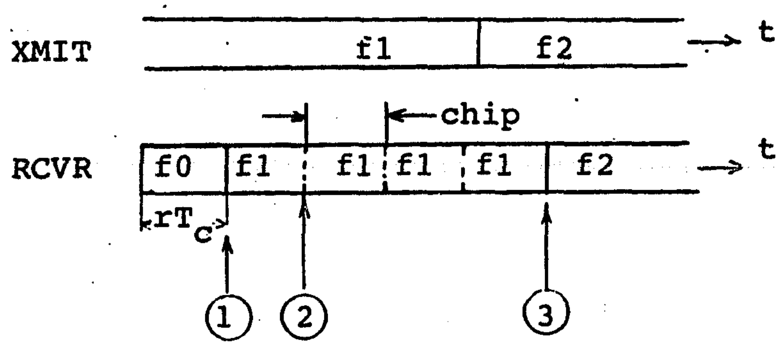

Fig. 2-3 Hopping frequency slot

2.2 Mathematical Analysis of Determination of the Thresholds

2.2.1 Single Threshold

For single threshold scheme we set the threshold at the middle of synchronization and nonsynchronization levels.

$$T_H = \frac{\sqrt{2}r_n S_c}{2} \tag{2.2-1}$$

We assign the probability of error so as to determine the number of chips r_n that we have to integrate to meet the required probability of error. The probability of error is the probability that the noise is greater than the threshold when we are not synchronized, which we call the probability of false acquisition, or the signal plus noise is below the threshold when we are synchronized, which we call the probability of dismissal. Refer to Fig. 2-4. Suppose the noise is white and Gaussian.

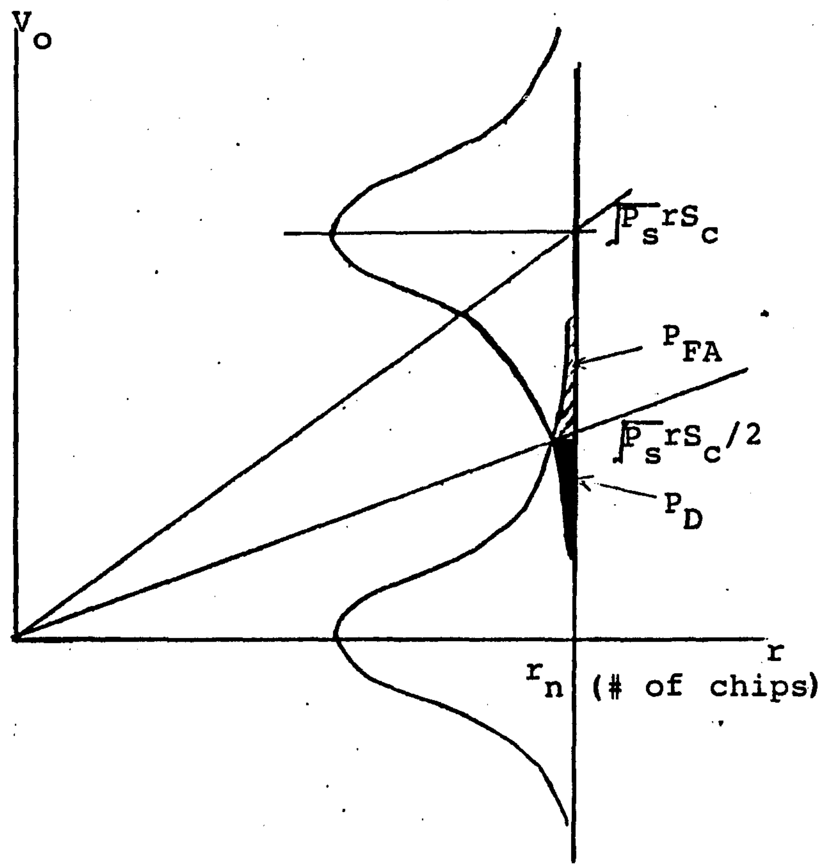

Fig. 2-4 Prob of false acq and dismissal at r_n chips

$P_{FA} = P(\text{noise} > T_H)$, when the system is not synchronized

$$= \int_{TH}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_{rn}} e^{-n^2/2\sigma_{rn}^2} dn = \frac{1}{2} \text{ erfc } \frac{T_H}{\sqrt{2}\sigma_{rn}}$$

$$= \frac{1}{2} \text{ erfc } \frac{r_n S_c}{2\sigma_{rn}} \tag{2.2-1}$$

$P_D = P(\text{signal} + \text{noise} < T_H)$, when the system is synchronized

$$= \int_{-\infty}^{T_H} \frac{1}{\sqrt{2\pi}\sigma_{rn}} e^{-(V_0-\sqrt{2}r_n S_c)^2/2\sigma_{rn}^2} dV_0$$

Let $\delta = V_0-\sqrt{2}r_n S_c$, $d\delta = dV_0$

$$P_D = \int_{-\infty}^{T_H-\sqrt{2}r_n S_c} \frac{1}{\sqrt{2\pi}\sigma_{rn}} e^{-\delta^2/2\sigma_{rn}^2} d\delta$$

$$= \int_{\sqrt{2}r_n S_c-T_H}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_{rn}} e^{-\delta^2/2\sigma_{rn}^2} d\delta$$

$$= \frac{1}{2} \text{ erfc } \frac{\sqrt{2}r_n S_c-T_H}{\sqrt{2}\sigma_{rn}}$$

$$= \frac{1}{2} \text{ erfc } \frac{r_n S_c}{2\sigma_{rn}} \tag{2.2-2}$$

We assign $P_{FA} = P_D$ (2.2-3)

σ_{rn}^2 = variance of noise for r_n chips

σ_0^2 = variance of noise for 1 chip

$$\sigma_0^2 = \frac{\eta}{2} S_c \qquad (2.2\text{-}4)$$

$$\sigma_{rn}^2 = r_n \sigma_0^2 \qquad (2.2\text{-}5)$$

S_c = number of samples/chip

From eqs (2.2-1) and (2.2-2) we have

$$r_n = \frac{2\sigma_{rn}}{S_c} \operatorname{erfc}^{-1} [2P_{FA}] \qquad (2.2\text{-}6)$$

or

$$r_n = \frac{2\sigma_{rn}}{S_c} \operatorname{erfc}^{-1} [2P_D] \qquad (2.2\text{-}7)$$

2.2.2 Unconditional Double Thresholds

In order to gaurantee P_{FA} and P_D lower than a certain value we have to seperate the two levels of synchronization and nonsynchronization in the single threshold scheme. That means r_n must be very large. Can we make a decision prior to r_n chips? The answer is definite. Suppose we compare at the end of r_1 chips, if we assign the same probability of error as the single threshold scheme to such comparision,what will be the thresholds for P_{FA} and P_D? Refer to Fig. 2-5.

P_{FA} = P(noise in r_1 chips > THB1)

$$P_{FA} = \int_{THB1}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_{r1}} e^{-n^2/2\sigma_{r1}^2} dn$$

$$= \frac{1}{2} \text{ erfc } \frac{THB1}{\sqrt{2}\sigma_{r1}}$$

$$\sigma_{r1}^2 = r_1 \sigma_0^2$$

$$P_{FA} = \frac{1}{2} \text{ erfc } \frac{THB1}{\sqrt{2r_1}\sigma_0} \qquad (2.2\text{-}8)$$

P_D = P(signal+noise in r_1 chips < THA1)

$$= \int_{-\infty}^{THA1} \frac{1}{\sqrt{2\pi}\sigma_{r1}} e^{-(V_0 - \sqrt{2}r_1 S_c)^2/2\sigma_{r1}^2} dV_0$$

$$= \frac{1}{2} \text{ erfc } \frac{\sqrt{2}r_1 S_c - THA1}{\sqrt{2r_1}\sigma_0} \qquad (2.2\text{-}9)$$

If we assign the same probability of error as we did to single threshold scheme, then the arguments must be equal.

$$\frac{THB1}{\sqrt{2r_1}\sigma_0} = \frac{r_n S_c}{2\sigma_{rn}}$$

$$THB1 = \sqrt{\frac{r_1 r_n}{2}} S_c \qquad (2.2\text{-}10)$$

$$\frac{\sqrt{2}r_1 S_c - THA1}{\sqrt{2r_1}\sigma_0} = \frac{r_n S_c}{2\sigma_{rn}}$$

$$THA1 = \sqrt{2}r_1 S_c - \sqrt{\frac{r_1 r_n}{2}}\, S_c \qquad (2.2\text{-}11)$$

If V_0 is between THA1 and THB1, we cannot make a decision at r_1, we will compare at r_2 using the thresholds THA2 and THB2.

$$THA2 = \sqrt{2}r_2 S_c - \sqrt{\frac{r_2 r_n}{2}}\, S_c \qquad (2.2\text{-}12)$$

$$THB2 = \sqrt{\frac{r_2 r_n}{2}}\, S_c \qquad (2.2\text{-}13)$$

The thresholds THA and THB are sketched in Fig.2-5. THA and THB will intersect at r_n chips, which means we can make a decision prior to or equal to r_n chips, no further processing is needed.

2.2.3 Conditional Double Thresholds

In the above scheme when we cannot make a decision at r_1 we compare once more at r_2 using the thresholds THA2 and THB2 with the same probability of error P_{FA} and P_D. Why we ignore the information that we already have, ie, we cannot make a decision at r_1 (V_0 is between THA1 and THB1)? We should take the advantage of the existing information to the greatest extent; therefore, we should use the conditional probability to calculate the double thresholds. The calculation is shown as follow:

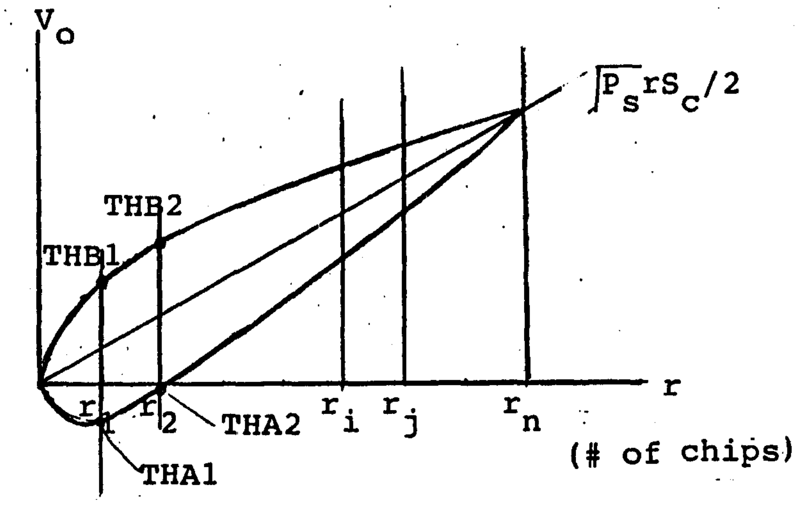

Fig. 2-5 Double thresholds in unconditional scheme

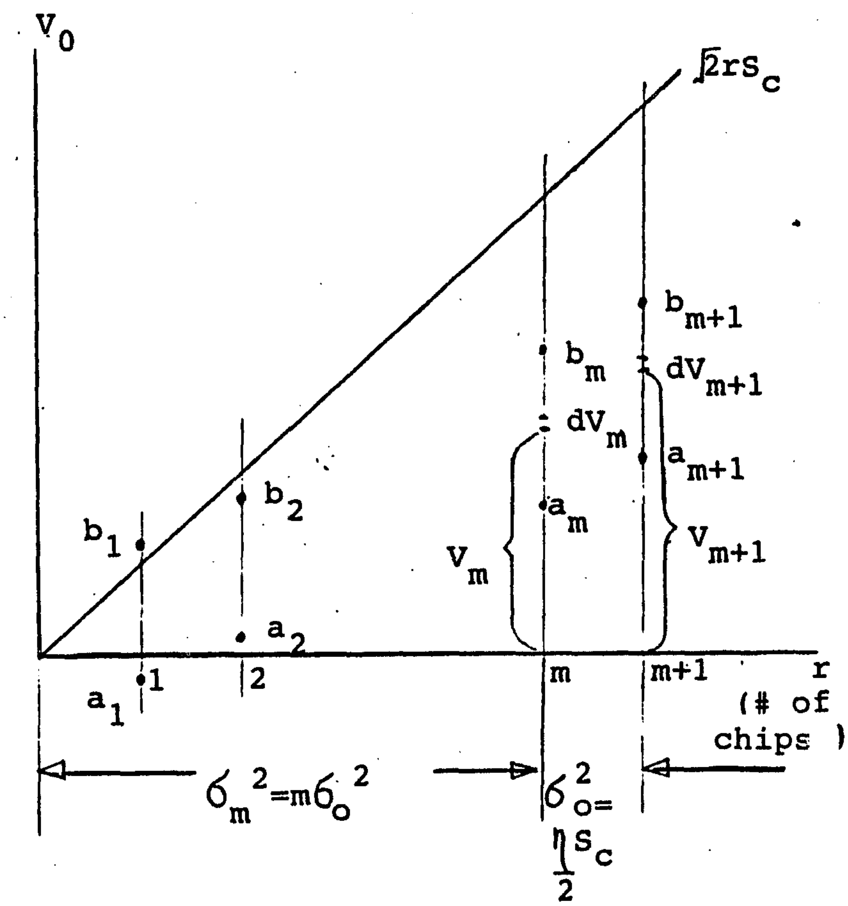

Fig. 2-6 Double thresholds in conditional scheme

Refer to Fig. 2-6.

For the first chip:

$$P_{FA1} = P(\text{noise} > b_1)$$

$$= \frac{1}{2} \operatorname{erfc} \frac{b_1}{\sqrt{2}\sigma_0} \qquad (2.2\text{-}14)$$

$$P_{D1} = P(\text{signal} + \text{noise} < a_1)$$

$$= \frac{1}{2} \operatorname{erfc} \frac{\sqrt{2}S_c - a_1}{\sqrt{2}\sigma_0} \qquad (2.2\text{-}15)$$

a_1 and b_1 are the two thresholds at r_1.

S_c is the number of samples/chip

σ_0^2 is the variance of noise for one chip.

We assign P_{FA1} and P_{D1} to calculate a_1 and b_1.

Suppose we cannot make a decision at the end of the first chip, we will compare at the end of the second chip. In general, if we cannot make a decision at the end of the m^{th} chip, the probability of making a wrong decision at the end of the $m+1^{th}$ chip is the following:

$$P_{FAm+1} = P(V_{m+1} > b_{m+1} \,/\, a_m < V_m < b_m)$$

$$= \frac{P(V_{m+1} > b_{m+1},\, a_m < V_m < b_m)}{P(a_m < V_m < b_m)}$$

$$= \frac{\int_{a_m}^{b_m} \frac{1}{\sqrt{2\pi}\sigma_m} e^{-V_m^2/2\sigma_m^2} dV_m \int_{b_{m+1}}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_0} e^{-(V_{m+1}-V_m)^2/2\sigma_0^2} dV_{m+1}}{\int_{a_m}^{b_m} \frac{1}{\sqrt{2\pi}\sigma_m} e^{-V_m^2/2\sigma_m^2} dV_m}$$

Let $\delta = V_{m+1} - V_m$

$$P_{FAm+1} = \frac{\int_{a_m}^{b_m} \frac{1}{\sqrt{2\pi}\sigma_m} e^{-V_m^2/2\sigma_m^2} dV_m \int_{b_{m+1}-V_m}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_0} e^{-\delta^2/2\sigma_0^2} d\delta}{\frac{1}{2} \operatorname{erfc} \frac{a_m}{\sqrt{2}\sigma_m} - \frac{1}{2} \operatorname{erfc} \frac{b_m}{\sqrt{2}\sigma_m}}$$

$$= \frac{\int_{a_m}^{b_m} \frac{1}{\sqrt{2\pi}\sigma_m} e^{-V_m^2/2\sigma_m^2} * \frac{1}{2} \operatorname{erfc} \frac{b_{m+1}-V_m}{\sqrt{2}\sigma_0} dV_m}{\frac{1}{2} \operatorname{erfc} \frac{a_m}{\sqrt{2}\sigma_m} - \frac{1}{2} \operatorname{erfc} \frac{b_m}{\sqrt{2}\sigma_m}} \quad (2.2\text{-}16)$$

$$P_{Dm+1} = P(V_{m+1} < a_{m+1} / a_m < V_m < b_m)$$

$$= \frac{P(V_{m+1} < a_{m+1}, a_m < V_m < b_m)}{P(a_m < V_m < b_m)}$$

$$P_{Dm+1} = \frac{\int_{a_m}^{b_m} \frac{1}{\sqrt{2\pi}\sigma_m} e^{-(V_m-\sqrt{2}mS_c)^2/2\sigma_m^2} dV_m \int_{-\infty}^{a_{m+1}} \frac{1}{\sqrt{2\pi}\sigma_0} e^{-(V_{m+1}-V_m-\sqrt{2}S_c)^2/2\sigma_0^2} dV_{m+1}}{\int_{a_m}^{b_m} \frac{1}{\sqrt{2\pi}\sigma_m} e^{-(V_m-\sqrt{2}mS_c)^2/2\sigma_m^2} dV_m}$$

$$= \frac{\int_{a_m}^{b_m} \frac{1}{\sqrt{2\pi}\sigma_m} e^{-(V_m-\sqrt{2}mS_c)^2/2\sigma_m^2} * \frac{1}{2} \operatorname{erfc} \frac{V_m+\sqrt{2}S_c-a_{m+1}}{\sqrt{2}\sigma_0} dV_m}{\frac{1}{2} \operatorname{erfc} \frac{a_m-\sqrt{2}mS_c}{\sqrt{2}\sigma_m} - \frac{1}{2} \operatorname{erfc} \frac{b_m-\sqrt{2}mS_c}{\sqrt{2}\sigma_m}} \qquad (2.2\text{-}17)$$

Where V_m and V_{m+1} are the outputs of the integrator at the end of the m^{th} and $m+1^{th}$ chips respectively.

a_m and b_m, a_{m+1} and b_{m+1} are the two thresholds for the m^{th} and $m+1^{th}$ comparision.

σ_m is the variance of noise for m chips which equals

$$\sigma_m^2 = m\sigma_0^2 \qquad (2.2\text{-}18)$$

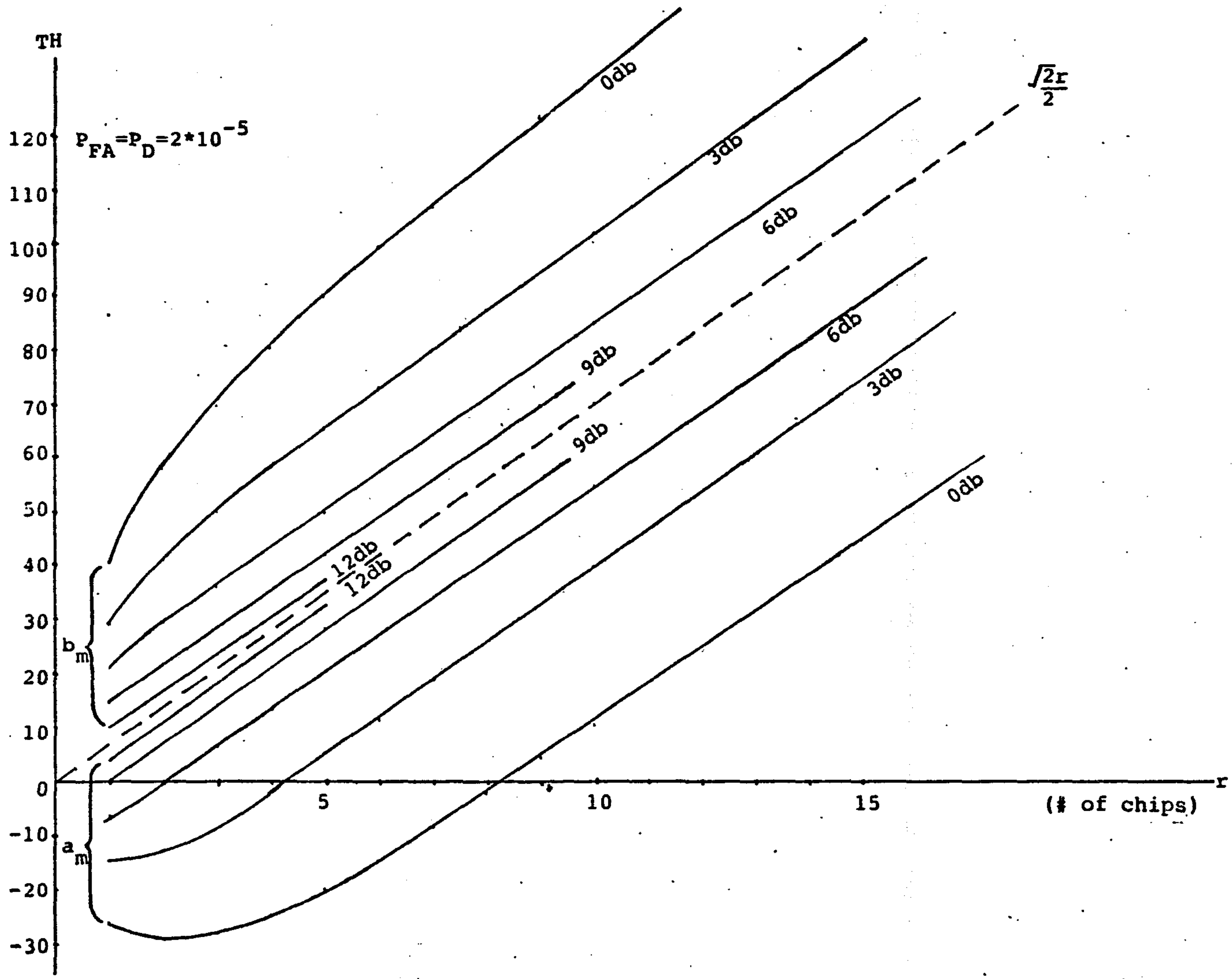

Fig. 2-7 Conditional double thresholds for different E_b/η

Given P_{FAm+1} and P_{Dm+1} we can calculate a_{m+1} and b_{m+1}. Eqs (2.2-16) and (2.2-17) can only be calculated using a computer. The curves of the two thresholds for the different signal to noise ratios are shown in Fig. 2-7. The assigned P_{FA} and P_D are $2*10^{-5}$.

From Fig. 2-7 we saw that the two thresholds never intersect. It seems as if there is a possibility that we may be unable to make a decision. The results of theoretical calculations (see section 2.3 Table 2.3-1 to 2.3-5) shows us that the probability of making a correct decision is so large that we should be satisfied with such a practical system. The computer simulation also shows that in the majority of cases we can make a decision prior to the r_n chips which are needed by a single threshold scheme. In only a very few instances are decisions made later than r_n chips. The reason is that although the noise variance increases with m, if we accumulate sufficient information, the mean value of the noise is 0. However, we can find a time that the output of the integrator goes outside the two thresholds and a decision is made. Besides, it is also possible to design a system that after a certain time of comparision we make a hard decision.

2.3 Mathematical Analysis of Probability of Making an Error Combined with Acquisition Strategy and the Expected Value and the Variance of Number of Chips to be Integrated for Each Decision

2.3.1 Conditional Double Threshold Scheme

In the previous section we assigned the probability of error P_{FA} and P_D for each comparision to determine the thresholds. Each decision is made after many comparisions. The total probability of making a wrong decision is calculated as follows. Refer to Fig. 2-7.

If the system is not synchronized, for the first chip

$$P_{FA1} = P(\text{noise} > b_1)$$

$$= \frac{1}{2}\,\text{erfc}\,\frac{b_1}{\sqrt{2}\sigma_0} \qquad (2.3\text{-}1)$$

$$P_{C1} = P(\text{noise} < a_1)$$

$$= \frac{1}{2}\,\text{erfc}\,\frac{-a_1}{\sqrt{2}\sigma_0} \qquad (2.3\text{-}2)$$

P_{C1} = Probability of correct decision at r_1

P_{ND1} = Probability of no decision is made at r_1

$$P_{ND1} = 1-P_{FA1}-P_{C1} \qquad (2.3\text{-}3)$$

In general case, if the system is not synchronized, the probability of correct decision at the end of the $m+1^{th}$ chip is:

$$P_{Cm+1} = P(V_{m+1} < a_{m+1}/a_m < V_m < b_m)$$

$$= \frac{P(V_{m+1} < a_{m+1}, a_m < V_m < b_m)}{P(a_m < V_m < b_m)}$$

$$= \frac{\int_{a_m}^{b_m} \frac{1}{\sqrt{2\pi}\sigma_m} e^{-V_m^2/2\sigma_m^2} dV_m \int_{-\infty}^{a_{m+1}} \frac{1}{\sqrt{2\pi}\sigma_0} e^{-(V_{m+1}-V_m)^2/2\sigma_0^2} dV_{m+1}}{\int_{a_m}^{b_m} \frac{1}{\sqrt{2\pi}\sigma_m} e^{-V_m^2/2\sigma_m^2} dV_m}$$

$$= \frac{\int_{a_m}^{b_m} \frac{1}{\sqrt{2\pi}\sigma_m} e^{-V_m^2/2\sigma_m^2} \cdot \frac{1}{2} \operatorname{erfc} \frac{V_m - a_{m+1}}{\sqrt{2}\sigma_0} dV_m}{\frac{1}{2} \operatorname{erfc} \frac{a_m}{\sqrt{2}\sigma_m} - \frac{1}{2} \operatorname{erfc} \frac{b_m}{\sqrt{2}\sigma_m}} \quad (2.3\text{-}4)$$

Probability of false acquisition at the end of the $m+1^{th}$ chip P_{FAm+1} is given by equ (2.2-16).

Probability of no decision at the end of the $m+1^{th}$ chip is:

$$P_{NDm+1} = 1 - P_{FAm+1} - P_{Cm+1} \quad (2.3\text{-}5)$$

Probability of making a wrong decision is:

$$PE = P_{FA1} + P_{ND1}P_{FA2} + P_{ND1}P_{ND2}P_{FA3} + \cdots \quad (2.3\text{-}6)$$

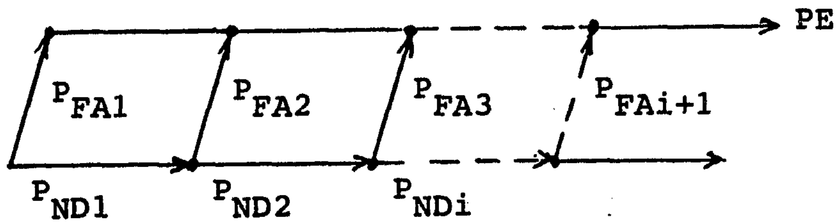

Fig. 2-8 Flow graph of calculating PE using Markov chain

Probability of making a correct decision is:

$$P_C = P_{C1} + P_{ND1}P_{C2} + P_{ND1}P_{ND2}P_{c3} + \cdots\cdots \qquad (2.3\text{-}7)$$

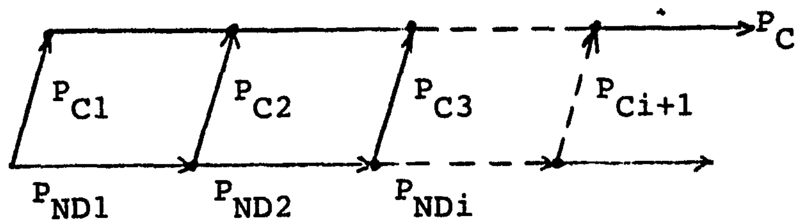

Fig. 2-9 Flow graph of calculating P_C using Markov chain

Expective value of number of chips for each decision is:

$$\overline{N} = (P_{C1}+P_{FA1}) + 2P_{ND1}(P_{C2}+P_{FA2}) + 3P_{ND1}P_{ND2}(P_{C3}+P_{FA3}) + \cdots\cdot \qquad (2.3\text{-}8)$$

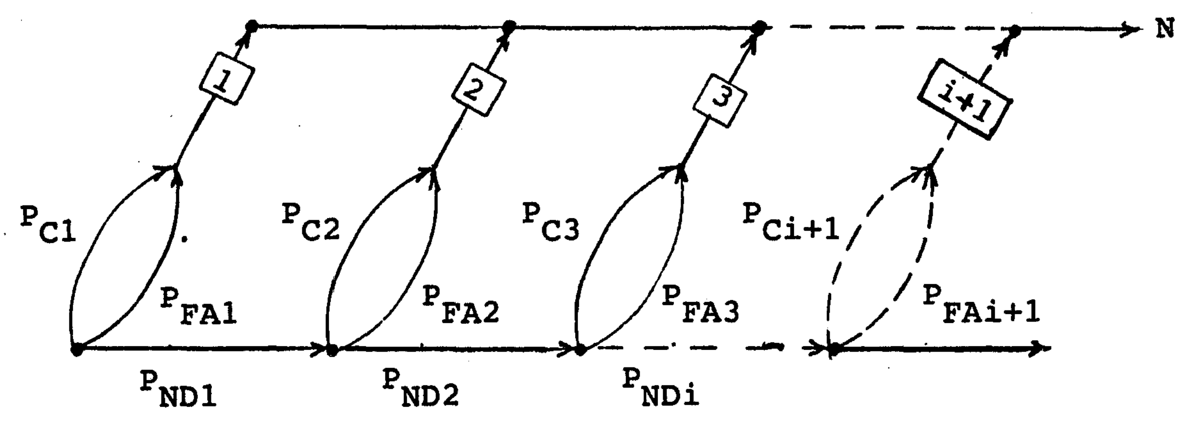

Fig. 2-10 Flow graph of calculating the expected value of number of chips for each decision

The expected value of N^2 is:

$$\overline{N^2} = (P_{C1}+P_{FA1}) + 2^2 P_{ND1}(P_{C2}+P_{FA2}) + 3^2 P_{ND1}P_{ND2}(P_{C3}+P_{FA3}) + \ldots. \quad (2.3\text{-}9)$$

The variance of N is:

$$VARN = \overline{N^2} - \overline{N}^2$$

$$= (P_{C1}+P_{FA1}) + 2^2 P_{ND1}(P_{C2}+P_{FA2}) + 3^2 P_{ND1}P_{ND2}(P_{C3}+P_{FA3})$$

$$+\ldots\ldots -[(P_{C1}+P_{FA1}) + 2P_{ND1}(P_{C2}+P_{FA2}) + P_{ND1}P_{ND2}(P_{C3}$$

$$+P_{FA3}) + \ldots\ldots]^2 \quad (2.3\text{-}10)$$

Table 2.3-1

Results of theoretical calculation for conditional double threshold scheme, E_b/η=12db

```
WNL=12.DB    INTPER=  1    PFA=0.00002

        PC( 1)= 0.9359074
        PC( 2)= 0.9925640
        PC( 3)= 0.9861559
        PC( 4)= 0.9895250
        PC( 5)= 0.9943901
AVERAGE NUMBER OF BITS FOR EACH DECISION = 1.0645313
VARIANCE OF NUMBER OF BITS FOR EACH DECISION = 0.0613890
NORMALIZED STANDARD DEVIATION OF NUMBER OF BITS FOR EACH DECISION = 0.2327484
MEAN ACQUISITION TIME = 1.0100508
VARIANCE OF ACQUISITION TIME =  0.0506611
NORMALIZED STANDARD DEVIATION OF ACQUISITION TIME = 0.2228404
TOTAL PE =  0.0000210
TOTAL PC =  0.9999790
```

Table 2.3-2

Results of theoretical calculation for conditional double threshold scheme, E_b/η=9db

```
WNL= 9.DB    INTPER=  1    PFA=0.00002

        PC( 1)= 0.4508333
        PC( 2)= 0.8579457
        PC( 3)= 0.8851499
        PC( 4)= 0.8913127
        PC( 5)= 0.8959100
        PC( 6)= 0.8985181
        PC( 7)= 0.9042680
        PC( 8)= 0.8976351
        PC( 9)= 0.9138591
        PC(10)= 0.9720911
AVERAGE NUMBER OF BITS FOR EACH DECISION = 1.6371374
VARIANCE OF NUMBER OF BITS FOR EACH DECISION = 0.4298496
NORMALIZED STANDARD DEVIATION OF NUMBER OF BITS FOR EACH DECISION = 0.4004729
MEAN ACQUISITION TIME = 1.5343027
VARIANCE OF ACQUISITION TIME =  0.3678131
NORMALIZED STANDARD DEVIATION OF ACQUISITION TIME = 0.3952779
TOTAL PE =  0.0000320
TOTAL PC =  0.9999678
```

Table 2.3-3

Results of theoretical calculation for conditional double threshold scheme, E_b/η=6db

```
WNL= 6.DB    INTPER=  1    PFA=0.00002

        PC( 1)= 0.0992023
        PC( 2)= 0.3931423
        PC( 3)= 0.5759943
        PC( 4)= 0.6314732
        PC( 5)= 0.6634035
        PC( 6)= 0.6793132
        PC( 7)= 0.6895344
        PC( 8)= 0.6966513
        PC( 9)= 0.7018917
        PC(10)= 0.7058983
        PC(11)= 0.7090526
        PC(12)= 0.7116306
        PC(13)= 0.7137462
        PC(14)= 0.7207978
        PC(15)= 0.7225325
        PC(16)= 0.7394765
        PC(17)= 0.7657728
        PC(18)= 0.7729753
        PC(19)= 0.8375034
        PC(20)= 0.8653485
 AVERAGE NUMBER OF BITS FOR EACH DECISION = 2.8065071
 VARIANCE OF NUMBER OF BITS FOR EACH DECISION = 1.4505978
 NORMALIZED STANDARD DEVIATION OF NUMBER OF BITS FOR EACH DECISION = 0.4291483
 MEAN ACQUISITION TIME = 2.6542158
 VARIANCE OF ACQUISITION TIME =  1.4258871
 NORMALIZED STANDARD DEVIATION OF ACQUISITION TIME = 0.4498900
 TOTAL PE =  0.0000548
 TOTAL PC =  0.9999447
```

Table 2.3-4

Results of theoretical calculation for conditional double threshold scheme, E_b/η=3db

WNL= 3.DB   INTPER= 1   PFA=0.00002

```
PC( 1)= 0.0174074
PC( 2)= 0.0900336
PC( 3)= 0.1924137
PC( 4)= 0.2788709
PC( 5)= 0.3433337
PC( 6)= 0.3803560
PC( 7)= 0.4074118
PC( 8)= 0.4285497
PC( 9)= 0.4419194
PC(10)= 0.4562495
PC(11)= 0.4607902
PC(12)= 0.4714485
PC(13)= 0.4731342
PC(14)= 0.4817951
PC(15)= 0.4859533
PC(16)= 0.4895468
PC(17)= 0.4926606
PC(18)= 0.4912331
PC(19)= 0.4976033
PC(20)= 0.4997771
PC(21)= 0.5017142
PC(22)= 0.5034755
PC(23)= 0.5050641
PC(24)= 0.5065137
PC(25)= 0.5078235
PC(26)= 0.5090455
PC(27)= 0.5143685
PC(28)= 0.5155321
PC(29)= 0.5208204
PC(30)= 0.5135472
```

Table 2.3-4 (cont'd.)

AVERAGE NUMBER OF BITS FOR EACH DECISION = 4.9564867
VARIANCE OF NUMBER OF BITS FOR EACH DECISION = 5.0456390
NORMALIZED STANDARD DEVIATION OF NUMBER OF BITS FOR EACH DECISION = 0.4531940
MEAN ACQUISITION TIME = 4.9270153
VARIANCE OF ACQUISITION TIME = 6.5190887
NORMALIZED STANDARD DEVIATION OF ACQUISITION TIME = 0.5182144
TOTAL PE = 0.0000972
TOTAL PC = 0.9999017

Table 2.3-5

Results of theoretical calculation for conditional double threshold scheme, E_b/η=0db

WNL= 0.DB   INTPER= 1   PFA=0.00002

PC(1)= 0.0035356
PC(2)= 0.0162474
PC(3)= 0.0401154
PC(4)= 0.0699345
PC(5)= 0.1032389
PC(6)= 0.1339711
PC(7)= 0.1602859
PC(8)= 0.1848244
PC(9)= 0.2026836
PC(10)= 0.2218084
PC(11)= 0.2339811
PC(12)= 0.2461710
PC(13)= 0.2550213
PC(14)= 0.2647423
PC(15)= 0.2737483
PC(16)= 0.2776349

Table 2.3-5 (cont'd)

```
         PC(17)= 0.2850778
         PC(18)= 0.2898476
         PC(19)= 0.2940871
         PC(20)= 0.2978850
         PC(21)= 0.3012940
         PC(22)= 0.3067907
         PC(23)= 0.3073348
         PC(24)= 0.3098843
         PC(25)= 0.3146671
         PC(26)= 0.3144953
         PC(27)= 0.3189433
         PC(28)= 0.3209033
         PC(29)= 0.3202212
         PC(30)= 0.3242976
         PC(31)= 0.3258769
         PC(32)= 0.3273474
         PC(33)= 0.3261961
         PC(34)= 0.3299227
         PC(35)= 0.3311468
         PC(36)= 0.3322958
AVERAGE NUMBER OF BITS FOR EACH DECISION = 8.7822418
VARIANCE OF NUMBER OF BITS FOR EACH DECISION =16.6429998
NORMALIZED STANDARD DEVIATION OF NUMBER OF BITS FOR EACH DECISION = 0.4645250
MEAN ACQUISITION TIME = 9.8828068
VARIANCE OF ACQUISITION TIME = 39.1710205
NORMALIZED STANDARD DEVIATION OF ACQUISITION TIME = 0.6332893
TOTAL PE =  0.0001728
TOTAL PC =  0.9997994
```

Eqs (2.3-6), (2.3-7), (2.3-8), (2.3-9) and (2.3-10) are calculated by using computer. The computer program is given in Appendix A. The results are shown in Table 2.3-1 to 2.3-5 for the different signal to noise ratios.

The probability of missing the signal is related to the acquisition scheme. For example, suppose there are 48 chips /hop, comparision is made at the end of each chip. refer to Fig. 2-11.

At time t_1 when the output of the integrator goes high, pre-acquisition is declared. The receiver remains at the same state until the transmitter hops to the next frequency. At time t_2 the output of the integrator drops below the threshold, the receiver hops to the next frequency and true acquisition is declared. Suppose between time t_1 and t_2, due to large noise, the output of the integrator drops below the threshold. Then the receiver would assume that the transmitter has already hopped to the next frequency; therefore, the receiver will also hop to the next frequency. However, the transmitter is still transmitting f_2 but the receiver already will have hopped to f_3. In that case we really miss the signal and we have to wait until the next peroid which equals the code length. In order to avoid such situation we have to maintain synchronization from t_1 to t_2. The probability of missing the signal is considered as the following:

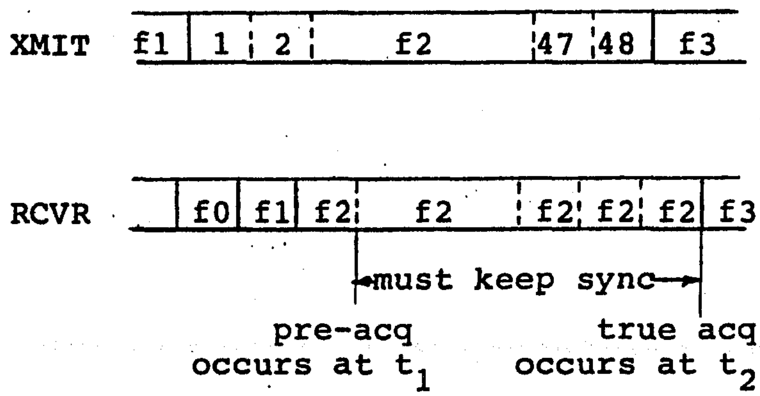

Fig. 2-11 Hopping freq slot when the system is going to synchronize

Pre-acquisition can occur at the end of any chip from 1 to NCH. The probability is uniformly distributed.

NCH = number of chips/hop.

If pre-acquisition occurs at the end of the first chip, the synchronization must keep NCH chips. The probability of missing the signal is:

$$P_{M1} = 1-(1-P_D)^{NCH} \qquad (2.3\text{-}11)$$

If pre-acquisition occurs at the end of the second chip, the synchronization must keep NCH-1 chips.

$$P_{M2} = 1-(1-P_D)^{NCH-1} \qquad (2.3\text{-}12)$$

etc. Hence, the total probability of missing the signal is:

$$P_M = \frac{1}{NCH} [P_{M1}+P_{M2}+P_{M3}+\ldots\ldots+P_{MNCH}]$$

$$= \frac{1}{NCH} [1-(1-P_D)+1-(1-P_D)^2+1-(1-P_D)^3+\ldots\ldots+1-(1-P_D)^{NCH}]$$

$$= \frac{1}{NCH} [NCH-[(1-P_D)+(1-P_D)^2+(1-P_D)^3+\ldots+(1-P_D)^{NCH}]]$$

$$= 1-\frac{1}{NCH} [\sum_{n=0}^{NCH} (1-P_D)^n - 1]$$

$$= 1-\frac{1}{NCH} [\frac{1-(1-P_D)^{NCH+1}}{P_D} - 1] \qquad (2.3\text{-}13)$$

2.3.2 Unconditional Double Threshold Scheme

For this scheme the two thresholds are already determined by equation (2.3-12) and (2.3-13). If we cannot make a decision at i^{th} comparision, what is the probability that a wrong decision is made at the j^{th} comparision $P_{ij}(j=i+1)$? Different from conditional double threshold scheme, $P_{ij} \neq$ constant.

$$THA_i = \sqrt{\frac{r_n r_i}{2}}\, S_c \qquad (2.3\text{-}12)$$

$$THB_i = \left[\sqrt{2}\, r_i - \sqrt{\frac{r_n r_i}{2}}\,\right] S_c \qquad (2.3\text{-}13)$$

If the system is not synchronized,

$$PE_{ij} = P(V_{0j} > THB_j \,/\, THA_i < V_{0i} < THB_i)$$

$$= \frac{\displaystyle\int_{THA_i}^{THB_i} \frac{1}{\sqrt{2\pi}\sigma_i}\, e^{-V_{0i}^2/2\sigma_i^2}\, dV_{0i} \cdot \int_{THB_j}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_0}\, e^{-(V_{0j}-V_{0i})^2/2\sigma_0^2}\, dV_{0j}}{\displaystyle\int_{THA_i}^{THB_i} \frac{1}{\sqrt{2\pi}\sigma_i}\, e^{-V_{0i}^2/2\sigma_i^2}\, dV_{0i}}$$

$$= \frac{\int_{THA_i}^{THB_i} \frac{1}{\sqrt{2\pi}\sigma_i}\, e^{-V_{0i}^2/2\sigma_i^2} \cdot \frac{1}{2}\,\mathrm{erfc}\,\frac{THB_j - V_{0i}}{\sqrt{2}\,\sigma_0}\, dV_{0i}}{\frac{1}{2}\,\mathrm{erfc}\,\frac{THA_i}{\sqrt{2}\sigma_i} - \frac{1}{2}\,\mathrm{erfc}\,\frac{THB_i}{\sqrt{2}\sigma_i}} \qquad (2.3\text{-}14)$$

$$P_{Cij} = P(V_{0j} < THA_j \,/\, THA_i < V_{0i} < THB_i)$$

$$= \frac{\int_{THA_i}^{THB_i} \frac{1}{\sqrt{2\pi}\sigma_i}\, e^{-V_{0i}^2/2\sigma_i^2}\, dV_{0i} \int_{-\infty}^{THA_j} \frac{1}{\sqrt{2\pi}\sigma_0}\, e^{-(V_{0j}-V_{0i})^2/2\sigma_0^2}\, dV_{0j}}{\int_{THA_i}^{THB_i} \frac{1}{\sqrt{2\pi}\sigma_i}\, e^{-V_{0i}^2/2\sigma_i^2}\, dV_{0i}}$$

$$= \frac{\int_{THA_i}^{THB_i} \frac{1}{\sqrt{2\pi}\sigma_i} e^{-V_{0i}^2/2\sigma_i^2} \cdot \frac{1}{2} \operatorname{erfc} \frac{V_{0i}-THA_j}{\sqrt{2}\sigma_0} dV_{0i}}{\frac{1}{2} \operatorname{erfc} \frac{THA_i}{\sqrt{2}\sigma_i} - \frac{1}{2} \operatorname{erfc} \frac{THB_i}{\sqrt{2}\sigma_i}} \qquad (2.3\text{-}15)$$

Similarly, PE, PC, P_M and N can be calculated as in the previous section. The computer program is given in appendix B. The results are shown in Table 2.3-6 to 2.3-10.

If the system is synchronized, because of symetry PE_{ij} is the same.

For this scheme when i and j become larger and larger, r_i and r_j close to r_n, the conditional probability of false acquisition is very large. Only $r_j < \frac{2}{3}r_n$ is acceptable.

Table 2.3-6

Results of theoretical calculation for unconditional double threshold scheme, E_h/η=12db

```
WNL=12.DB    IRN= 3


PE( 0, 1) = 0.0000005          PC( 0, 1) = 0.7746634
PE( 1, 2) = 0.0000023          PC( 1, 2) = 0.9954886
PE( 2, 3) = 0.0004568          PC( 2, 3) = 0.9994019
AVERAGE NUMBER OF BITS FOR EACH DECISION = 1.2263479
VARIANCE OF NUMBER OF BITS FOR EACH DECISION = 0.1771507
NORMALIZED STANDARD DEVIATION OF NUMBER OF BITS FOR EACH DECISION = 0.3432083
MEAN ACQUISITION TIME = 1.1553164
VARIANCE OF ACQUISITION TIME =  0.1460733
NORMALIZED STANDARD DEVIATION OF ACQUISITION TIME = 0.3308145
TOTAL PE =  0.0000015
TOTAL PC =  0.9899983
```


Table 2.3-7

Results of theoretical calculation for unconditional double threshold scheme, E_h/η=9db

```
WNL= 9.DB    IRN= 4


PE( 0, 1) = 0.0000335          PC( 0, 1) = 0.5000000
PE( 1, 2) = 0.0000617          PC( 1, 2) = 0.9037451
PE( 2, 3) = 0.0005568          PC( 2, 3) = 0.9667510
PE( 3, 4) = 0.0127673          PC( 3, 4) = 0.9872702
AVERAGE NUMBER OF BITS FOR EACH DECISION = 1.5493746
VARIANCE OF NUMBER OF BITS FOR EACH DECISION = 0.3500810
NORMALIZED STANDARD DEVIATION OF NUMBER OF BITS FOR EACH DECISION = 0.3818862
MEAN ACQUISITION TIME = 1.4548035
VARIANCE OF ACQUISITION TIME =  0.2960396
NORMALIZED STANDARD DEVIATION OF ACQUISITION TIME = 0.3739991
TOTAL PE =  0.0001113
TOTAL PC =  0.9998888
```

Table 2.3-8

Results of theoretical calculation for unconditional double threshold scheme, E_b/η=6db

WNL= 6.D8   IRN= 9

PE(0, 1) = 0.0000115	PC(0, 1) = 0.0791405
PE(1, 2) = 0.0000117	PC(1, 2) = 0.3597876
PE(2, 3) = 0.0000163	PC(2, 3) = 0.5822854
PE(3, 4) = 0.0000345	PC(3, 4) = 0.7062992
PE(4, 5) = 0.0001030	PC(4, 5) = 0.7771493
PE(5, 6) = 0.0004017	PC(5, 6) = 0.8213552
PE(6, 7) = 0.0019365	PC(6, 7) = 0.8525896
PE(7, 8) = 0.0110566	PC(7, 8) = 0.8843728
PE(8, 9) = 0.0649183	PC(8, 9) = 0.9351714

AVERAGE NUMBER OF BITS FOR EACH DECISION = 2.8480968
VARIANCE OF NUMBER OF BITS FOR EACH DECISION = 1.1930809
NORMALIZED STANDARD DEVIATION OF NUMBER OF BITS FOR EACH DECISION = 0.3835130
MEAN ACQUISITION TIME = 2.6871786
VARIANCE OF ACQUISITION TIME = 1.1456785
NORMALIZED STANDARD DEVIATION OF ACQUISITION TIME = 0.3983226
TOTAL PE = 0.0000674
TOTAL PC = 0.9999325

Table 2.3-9

Results of theoretical calculation for unconditional double threshold scheme, E_b/η=3db

WNL= 3.DB   IRN=17

PE(0, 1) = 0.0000190	PC(0, 1) = 0.0169773
PE(1, 2) = 0.0000180	PC(1, 2) = 0.0870517
PE(2, 3) = 0.0000176	PC(2, 3) = 0.1869603
PE(3, 4) = 0.0000193	PC(3, 4) = 0.2834220
PE(4, 5) = 0.0000241	PC(4, 5) = 0.3641925
PE(5, 6) = 0.0000339	PC(5, 6) = 0.4288877
PE(6, 7) = 0.0000527	PC(6, 7) = 0.4804147
PE(7, 8) = 0.0000897	PC(7, 8) = 0.5218502
PE(8, 9) = 0.0001652	PC(8, 9) = 0.5556456
PE(9,10) = 0.0003255	PC(9,10) = 0.5837365
PE(10,11) = 0.0006808	PC(10,11) = 0.6075843
PE(11,12) = 0.0015027	PC(11,12) = 0.6285819
PE(12,13) = 0.0034848	PC(12,13) = 0.6485587
PE(13,14) = 0.0084842	PC(13,14) = 0.6710171
PE(14,15) = 0.0217687	PC(14,15) = 0.7041175
PE(15,16) = 0.0583102	PC(15,16) = 0.7625386
PE(16,17) = 0.1495774	PC(16,17) = 0.8504854

AVERAGE NUMBER OF BITS FOR EACH DECISION = 4.8348742
VARIANCE OF NUMBER OF BITS FOR EACH DECISION = 3.9776459
NORMALIZED STANDARD DEVIATION OF NUMBER OF BITS FOR EACH DECISION = 0.4125037
MEAN ACQUISITION TIME = 4.7669525
VARIANCE OF ACQUISITION TIME = 4.7930145
NORMALIZED STANDARD DEVIATION OF ACQUISITION TIME = 0.4592652
TOTAL PE = 0.0001495
TOTAL PC = 0.9998500

Table 2.3-10

Results of theoretical calculation for unconditional double threshold scheme, E_b/η=0db

WNL= 0.DB   IRN=34

PE(0, 1) = 0.0000186	PC(0, 1) = 0.0033752
PE(1, 2) = 0.0000174	PC(1, 2) = 0.0152299
PE(2, 3) = 0.0000158	PC(2, 3) = 0.0357639
PE(3, 4) = 0.0000148	PC(3, 4) = 0.0626292
PE(4, 5) = 0.0000144	PC(4, 5) = 0.0928779
PE(5, 6) = 0.0000145	PC(5, 6) = 0.1240594
PE(6, 7) = 0.0000152	PC(6, 7) = 0.1545415
PE(7, 8) = 0.0000165	PC(7, 8) = 0.1834043
PE(8, 9) = 0.0000185	PC(8, 9) = 0.2102265
PE(9,10) = 0.0000214	PC(9,10) = 0.2348897
PE(10,11) = 0.0000256	PC(10,11) = 0.2574452
PE(11,12) = 0.0000313	PC(11,12) = 0.2780288
PE(12,13) = 0.0000393	PC(12,13) = 0.2968140
PE(13,14) = 0.0000506	PC(13,14) = 0.3139763
PE(14,15) = 0.0000665	PC(14,15) = 0.3296891
PE(15,16) = 0.0000891	PC(15,16) = 0.3441179
PE(16,17) = 0.0001216	PC(16,17) = 0.3573895
PE(17,18) = 0.0001690	PC(17,18) = 0.3696594
PE(18,19) = 0.0002387	PC(18,19) = 0.3810282
PE(19,20) = 0.0003422	PC(19,20) = 0.3916342
PE(20,21) = 0.0004977	PC(20,21) = 0.4015474
PE(21,22) = 0.0007339	PC(21,22) = 0.4109135
PE(22,23) = 0.0010962	PC(22,23) = 0.4198524
PE(23,24) = 0.0016580	PC(23,24) = 0.4285536
PE(24,25) = 0.0025382	PC(24,25) = 0.4372008
PE(25,26) = 0.0039342	PC(25,26) = 0.4461734
PE(26,27) = 0.0061762	PC(26,27) = 0.4560165
PE(27,28) = 0.0098327	PC(27,28) = 0.4676786

Table 2.3-10 (cont'd)

PE(28,29) = 0.0159177	PC(28,29) = 0.4828316
PE(29,30) = 0.0263221	PC(29,30) = 0.5045582
PE(30,31) = 0.0447531	PC(30,31) = 0.5381487
PE(31,32) = 0.0783468	PC(31,32) = 0.5912681
PE(32,33) = 0.1384906	PC(32,33) = 0.6691488
PE(33,34) = 0.2352905	PC(33,34) = 0.7647218

AVERAGE NUMBER OF BITS FOR EACH DECISION = 8.7823973
VARIANCE OF NUMBER OF BITS FOR EACH DECISION =13.9990692
NORMALIZED STANDARD DEVIATION OF NUMBER OF BITS FOR EACH DECISION = 0.4260264
MEAN ACQUISITION TIME = 9.7495441
VARIANCE OF ACQUISITION TIME = 28.5277710
NORMALIZED STANDARD DEVIATION OF ACQUISITION TIME = 0.5478348
TOTAL PE = 0.0001933
TOTAL PC = 0.9998056

2.4 Calculation of Theoretical Mean Acquisition Time and Variance of Acquisition Time

On the average the difference between the transmitter and the receiver is half of the code length L/2. A decision is made by comparing N chips. (for one chip/bit it also equals N bits)

The time that the receiver catches up L/2 hops equals $L/2*N*T_b$. But during this time interval, the transmitter also finishes (L/2)(N/NBH) hops. NBH = number of bits/hop. The receiver also has to catch up, the time needed is $(L/2)(N/NBH)N\ T_b$. During this time interval, the transmitter finishes (L/2)(N/NBH)(N/NBH) hops and so on. The total time needed is:

$$\frac{L}{2} N\ T_b + \frac{L}{2} \frac{N}{NBH} N\ T_b + \frac{L}{2} \frac{N}{NBH} \frac{N}{NBH} N\ T_b + \ldots.$$

Let α = N/NBH

$$\text{The total time} = \frac{L}{2} N\ T_b (1+\alpha^2+\alpha^3+\ldots..)$$

$$= \frac{L}{2} N \frac{1}{1-\alpha} T_b \qquad (2.4\text{-}1)$$

Besides, when the system is going to synchronize , it must keep synchronization from t_1 to t_2 as shown in Fig.2-11.

The mean value of $t_2 - t_1 = 1/2 * NBH * T_b$ (2.4-2)

In addition we also have false acquisition and dismissal, each false acquisition takes additive $N * T_b$ and each dismissal takes $(L)(N)T_b/1-\alpha$; that is, if we miss the signal, we have to catch up another code length.

Number of false acquisition = $(L/2)PE/(1-\alpha)$ (2.4-3)

Number of dismissal = P_M (2.4-4)

Hence, for single threshold scheme the acquisition time T_{acqN} is:

$$T_{acqN} = [\frac{L}{2} N \frac{1}{1-\alpha}(1+PE+2P_M) + \frac{1}{2}NBH]\ T_b \qquad (2.4\text{-}5)$$

For double threshold scheme

$$\text{Mean acquisition time} = P_{C1}\ T_{acq1} + P_{ND1}\ P_{C2}\ T_{acq2} + P_{ND1}\ P_{ND2}\ P_{C3}\ T_{acq3} + \ldots\ldots \qquad (2.4\text{-}6)$$

The expected value of (acquisition time)^2 is:

$$P_{C1}T_{acq1}^{\ 2} + P_{ND1}P_{C2}T_{acq2}^{\ 2} + P_{ND1}P_{ND2}P_{C3}T_{acq3}^{\ 2} + \ldots$$

The variance of acquisition time VARTacq

= the expected value of $(\text{acquisition time})^2 - (\text{mean acq time})^2$

$$= P_{C1}T_{acq1}^{2} + P_{ND1}P_{C2}T_{acq2}^{2} + P_{ND1}P_{ND2}P_{C3}T_{acq3}^{2} + \cdots$$

$$-[P_{C1}T_{acq1} + P_{ND1}P_{C2}T_{acq2} + P_{ND1}P_{ND2}P_{C3}T_{acq3} + \ldots]^{2} \quad (2.4\text{-}7)$$

Eqs (2.4-5), (2.4-6) and (2.4-7) are also calculated by using the computer for the different E_b/η. The results are shown in Table 2.3-1 to Table 2.3-10 and also in Fig. 2-12. The computer program is atteched in appendices A and B.

2.5 Mean and Variance of Acquisition Time, The Results of Computer Simulation

The performance of an acquisition scheme can be represented by the mean acquisition time which can be achieved by statistical experiments.* A computer simulation of a communication system is used to determine the mean and the variance of the acquisition time. The acquisition strategy is already described in section 2.1. After initial acquisition is declared a subroutine is used to test if the acquisition is false. If so, the system returns to search until true acquisition is verified. In the computer simulation we test for false or true acquisition by comparing the transmitting vector and the local vector of the PN generator. The test doesn't take time. In a real system there must be some penalty for false acquisition. But the time needed to test the false acquisition is usually much less than the acquisition time and therefore, can be neglected.

The parameters used for the simulation are:

Bit rate = 300 bits/S

10 samples/chip

1 chip/bit

48 bits/hop

Code length $2^9-1 = 511$

$\sigma_N = \sigma$/mean acquisition time, normalized standard deviation

*: Appendix F

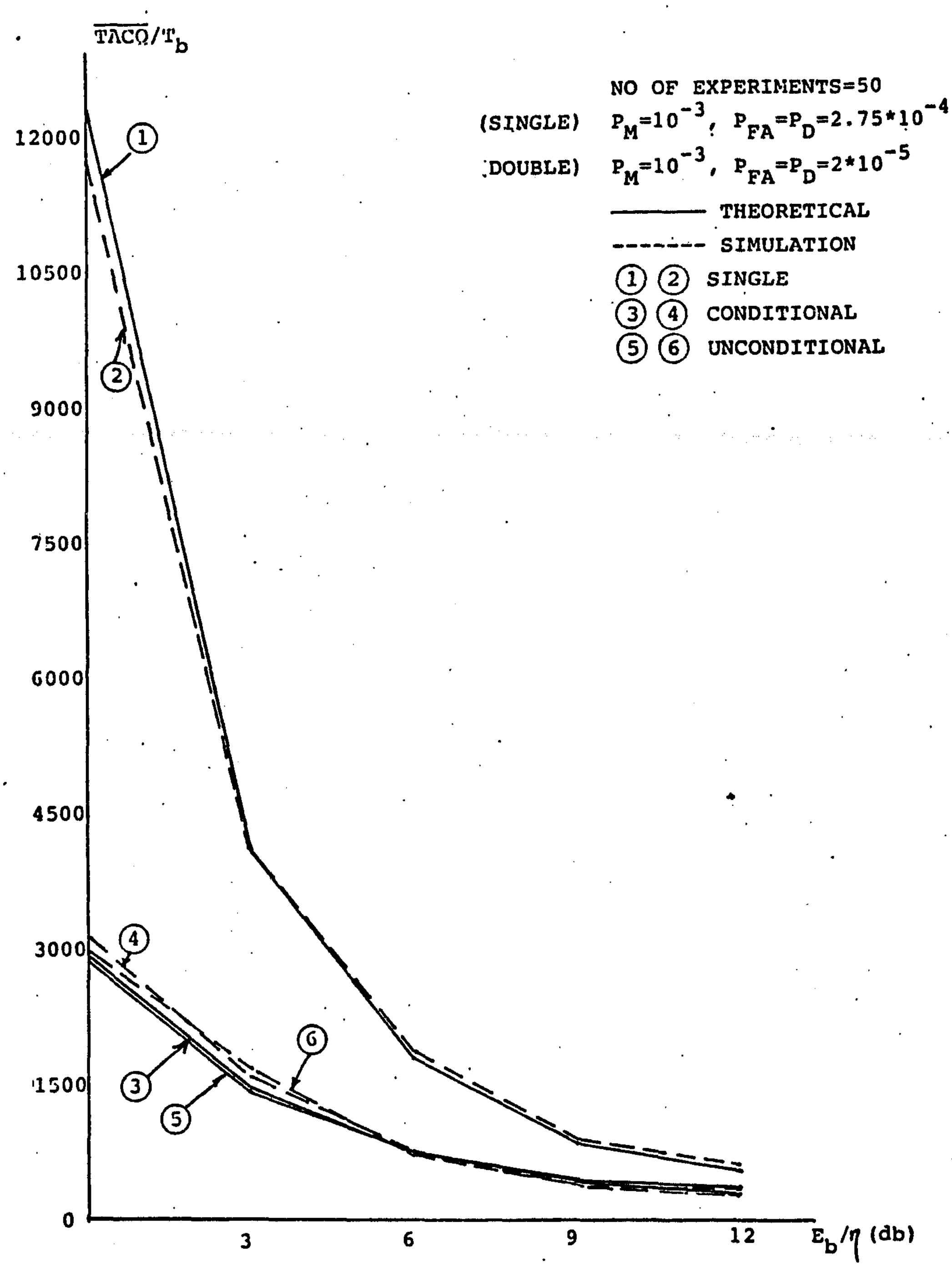

Fig. 2-12 Mean acquisition time versus E_b/η

σ^2 = variance of acquisition time

The results of mean acquisition time are shown in Fig. 2-12. the variances of acquisition time are shown in Table 2.5-1, 2.5-2, 2.5-3 for single threshold scheme, conditional double threshold scheme and unconditional double threshold scheme.

comments:

From Fig. 2-12 and from comparing Table 2.3-1 to Table 2.3-10 with Table 2.5-1 to Table 2.5-3 we found that the results of the experiments for mean acquisition time completely agree with the theoretical calculation. As for the variance when the signal to noise ratio is low, the results of experiments are very close to the theoretical calculation. But when the signal to noise ratio goes high, say, E_b/η = 12db, the variance of experiments is greater than the theory. Why? The reason is: when we did the simulation, we found that 93% of the decisions are made in one bit time which means that we didn't chop the bit time to many chips to make them short enough. The problem is that in this computer simulation program the local hopper can only hop at the end of the bit time. For example, if the mean acquisition time is 1.5, sometimes the decisions are made in one bit, sometimes they are made in 2 bits. This situation doesn't affect the mean acquisition time. The mean acquisition time is still 1.5. However, it does affect the variance because $1^2+2^2 > 1.5^2+$

1.5^2. Therefore, the variance of the experimental results is greater than the theoretical results when the signal to noise ratio increases. Besides, equ (2.4-7) is an approximation. When we calculated T_{acqN} we assumed that the difference between the transmitter and the receiver has the mean value of L/2 hops. Hence, T_{acqN} is an average value. The result of equ(2.4-7) is smaller than the practical system.

Table 2.5-1

Variance of Acquisition Time for Single Threshold Scheme (Experimental Result)

E_b/η (db)	Variance	Normalized Standard Deviation
12	1.469871	0.58880
9	2.452550	0.53130
6	11.80217	0.55176
3	58.62646	0.54509
0	432.6261	0.53291

Table 2.5-2

Variance of Acquisition Time for Conditional Double Threshold Scheme (Experimental Result)

E_b/η (db)	Variance	Normalized Standard Deviation
12	0.3649368	0.56010
9	0.8245974	0.67194
6	1.789268	0.52269
3	6.822891	0.49109
0	27.40433	0.50093

Table 2.5-3

Variance of Acquisition Time for Unconditional Double Threshold Scheme (Experimental Result)

E_b/η (db)	Variance	Normalized Standard Deviation
12	0.5498991	0.65351
9	0.5801373	0.53640
6	2.250667	0.60878
3	7.175140	0.48489
0	30.55341	0.54902

2.6 Comparision Among the Different Acquisition Schemes of FH Signal

From Fig. 2-12 the double threshold schemes are obviously much better than the single threshold scheme, especially when the signal to noise ratio is low. the mean acquisition time is reduced to 1/2 to 1/4 depending on the different S/N ratio. The conditional and the unconditional double threshold schemes are very close to each other. The reason is: when the assigned P_{FA} and P_D are very low for unconditional scheme the intersection point of the two thresholds goes far away. In the region where the acquisition occurs the two thresholds are very close to those for conditional scheme. If the assigned P_{FA} and P_D are large, say, 10^{-3}, the unconditional scheme has a lot of false acquisitions. The mean acquisition time will increase if there is penalty for false acquisition. Besides, the unconditional double thresholds are easy to calculate but hard to implement. The conditional double thresholds are easy to implement because the two thresholds are almost two parellel straight lines. Even if the channel changes, the signal to noise ratio changes, we only have to adjust the DC voltage of the thresholds. On the other hand, the unconditional double threshold scheme makes noncoherent FSK technique and using the fading channel to be feasible. But we have to keep in mind that the assigned probability of false acquisition must be lower than, say, at least 10^{-4}.

Chapter 3

DOUBLE THRESHOLD ACQUISITION SCHEME IN THE FADING CHANNEL

3.1 Introduction

Double threshold acquisition scheme will also be used to the fading channel. Due to the fading the signal seperates into two parts, in phase and quadrature. The method used to calculate the conditional double thresholds becomes very much complicated. Based on the previous experience if we assign very low probability to the unconditional scheme, the unconditional scheme becomes the conditional scheme approximately. The unconditional double thresholds are possible to be calculated even in the fading channel. Therefore, in this chapter we depict the situation which is the unconditional double threshold acquisition scheme being used to the fading channel. The signal in the fading channel has been studied. The probability density function of the envelope square is given. The equations for the two thresholds are derived. The results of the computer simulation are analyzed and compared to the nonfading channel.

3.2 The Model of the Fading Channel

The fading channel is represented by two tapped delay lines, one representing the in phase component and another representing the quadrature component. To represent a HF channel the taps are spaced about 1ms apart. There are either

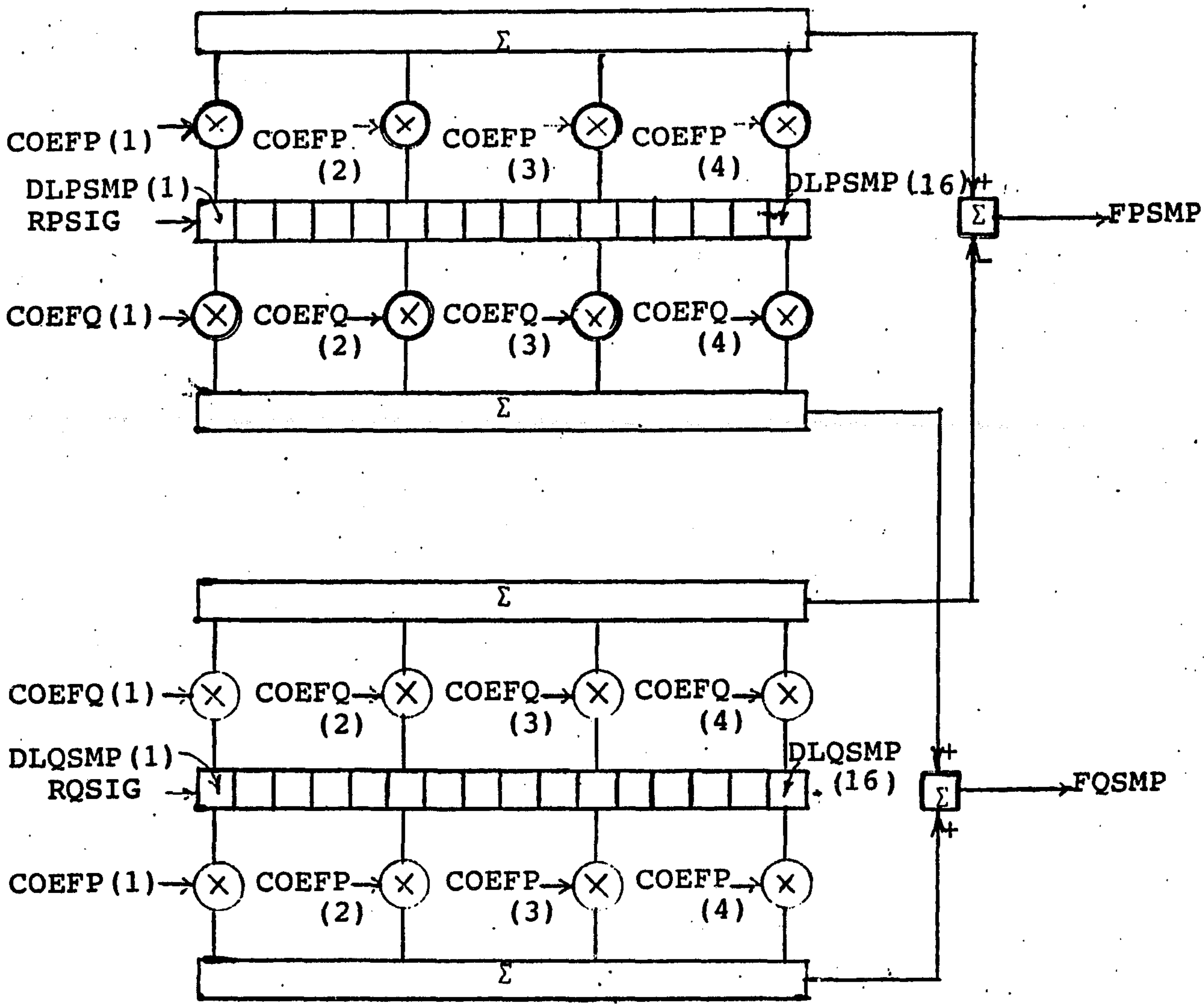

FIG. 3-1 MODEL OF FADING CHANNEL

3 or 4 taps employed. The signal samples from those taps are multiplied by independent Gaussian noise processed as shown in Fig. 3-1. The output signal from the fading channel is then represented by two equations.

$$\text{FPSMP} = \sum_{j=1,\text{TDLSMP},\frac{\text{NSC}}{2}} \text{DLPSMP}(j)*\text{COEFP}(j)-\text{DLQSMP}(j)*\text{COEFQ}(j) \tag{3.2-1}$$

$$\text{FQSMP} = \sum_{j=1,\text{TDLSMP},\frac{\text{NSC}}{2}} \text{DLQSMP}(j)*\text{COEFP}(j)+\text{DLPSMP}(j)*\text{COEFQ}(j) \tag{3.2-2}$$

In equations (3.2-1) and (3.2-2)

FPSMP: Fading in phase sample

FQSMP: Fading quadrature sample

TDLSMP: Total number of delayed sample, in phase or quadrature

$$\text{TDLSMP} = [(\text{NTAPS}-1) * \frac{\text{NSC}}{2}] + 1 \tag{3.2-3}$$

NTAPS: Total number of taps, in phase or quadrature

$$\text{NTAPS} = \text{chip rate} * \text{channel spacing} * \text{spread time} * 2 \tag{3.2-4}$$

In this chapter we give two examples.

(1). chip rate = bit rate = 300 chips/s

channel spacing for FSK = 2

spread time = 1ms

$NTAPS = 300 * 2 * 10^{-3} * 2 = 1.2$

We takes NTAPS as 2.

$TDLSMP = [(2-1) * \frac{10}{2}] + 1 = 6$

(2). chip rate = bit rate = 1000 chips/s

channel spacing for FSK = 2

spread time = 1ms

$NTAPS = 1000 * 2 * 10^{-3} * 2 = 4$

$TDLSMP = [(4-1) * \frac{10}{2}] + 1 = 16$

NSC: number of samples/chip. In these examples NSC = 10 samples/chip

DLPSMP(j): Individual in phase delayed sample

DLQSMP(j): Individual quadrature delayed sample

COEFP(j): Individual in phase coefficient

COEFQ(j): Individual quadrature coefficient

COEFP and COEFQ are random variables which represent the characteristic of the fading channel. COEFP or COEFQ equals a Gaussian random number multiplied by its standard deviation and then passes through a low pass filter whose cut off frequency equals the fading rate. The data passing through the low pass filter will lose energy and therefore its variance is adjusted accordingly. The adjustment is proportional to the sampling frequency and inversely proportional to the fading rate. The constant of proportionality is about 1/3.* The variance of COEFP and COEFQ equals the total normalized power divided by the number of taps** and multiplied by the factor of the adjustment.

$$\text{variance before filtering} = \frac{1}{2*\text{NTAPS}} * \frac{\text{sampling rate}}{\text{fading rate}} * \frac{1}{3} \qquad (3.2\text{-}5)$$

$$\text{variance after filtering} = \frac{1}{2*\text{NTAPS}} \qquad (3.2\text{-}6)$$

In the case of FSK, RQSIG=0. Eqs (3.2-1) and (3.2-2) become Eqs (3.2-7) and (3.2-8).

$$\text{FPSMP} = \sum_{j=1,\text{TDLSMP},\frac{\text{NSC}}{2}} \text{DLPSMP}(j) * \text{COEFP}(j) \qquad (3.2\text{-}7)$$

$$\text{FQSMP} = \sum_{j=1,\text{TDLSMP},\frac{\text{NSC}}{2}} \text{DLPSMP}(j) * \text{COEFQ}(j) \qquad (3.2\text{-}8)$$

RPSIG: Transmitting in phase signal sample

RQSIG: Transmitting quadrature signal sample

---

*: Appendix C

**: In the case of FSK, RQSIG = 0, the total signal power in RPSIG should be distributed between the in phase and the quadrature components; that is, 1/2NTAPS.

3.3 Signal in the Fading Channel

The fading channel is represented by two delay lines. Therefore, when the transmitter hops to a new frequency, the signal is passed through a new fading channel. Consider the in phase component only. (the quadrature component is the same). Refering to Fig. 3-2,

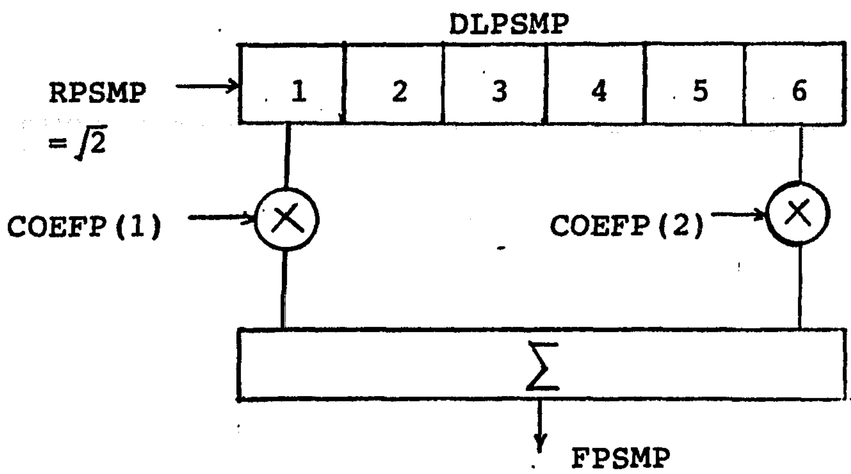

Fig. 3-2 Fading channel using FSK

For a two-tap model we see that the first to the fifth fading samples are:

$$FPSMP(1) = \sqrt{2} * COEFP(1) \tag{3.3-1}$$

$$\vdots$$

$$FPSMP(5) = \sqrt{2} * COEFP(1)$$

$$= \sqrt{2} * \sqrt{\frac{1}{2NTAPS}} = \sqrt{\frac{1}{2}} = 0.707$$

The sixth to tenth fading samples are:

$$FPSMP(6) = \sum_{j=1}^{2} \sqrt{2}\ COEFP(j) \tag{3.3-2}$$

Because COEFP's are Gaussian random variables with a variance of 1/2NTAPS and a frequency bandwidth of 1 H_z, the summation among the different taps will be calculated according to power. Hence,

$$FPSMP(6) = \sqrt{2} \sum_{j=1}^{2} COEFP(j)$$

$$= \sqrt{2} * \sqrt{2} * \sqrt{\frac{1}{2NTAPS}} = 1 \qquad (3.3\text{-}3)$$

For FPSMP(11) to FPSMP(100), since all the delay lines are filled with signal, the magnitude of FPSMP will be 1.

At the 101th sample, the transmitter hops to the next frequency and the delay lines are gradually filled by new values. Nevertheless, some delay lines are still filled with the old frequency signal, however, the magnitude of the old frequency signal decreases gradually according to the same rule.

For the 101^{th} to 105^{th} samples

$$FPSMP(101) = \sqrt{2}\ COEFP(2) = \sqrt{2} * \sqrt{\frac{1}{2NTAPS}} = 0.707 \qquad (3.3\text{-}4)$$

For a four-tap model, NTAPS = 4

$$FPSMP(1) \text{ to } FPSMP(5) = \sqrt{2} * COEFP(1)$$

$$= \sqrt{2} * \sqrt{\frac{1}{2NTAPS}} = 0.5$$

$$\text{FPSMP(6) to FPSMP(10)} = \sum_{j=1}^{2} \sqrt{2}\ \text{COEFP(j)}$$

$$= \sqrt{2} * \sqrt{2} * \sqrt{\frac{1}{2\text{NTAPS}}} = 0.707$$

$$\text{FPSMP(11) to FPSMP(15)} = \sum_{j=1}^{3} \sqrt{2}\ \text{COEFP(j)}$$

$$= \sqrt{2} * \sqrt{3} * \sqrt{\frac{1}{2\text{NTAPS}}} = 0.866$$

$$\text{FPSMP(16) to FPSMP(100)} = \sum_{j=1}^{4} \sqrt{2}\ \text{COEFP(j)}$$

$$= \sqrt{2} * \sqrt{4} * \sqrt{\frac{1}{2\text{NTAPS}}} = 1$$

FPSMP(101) to FPSMP(105) = 0.866

FPSMP(106) to FPSMP(110) = 0.707

FPSMP(111) to FPSMP(115) = 0.5

The signal variation with time in the fading channel is shown in Fig. 3-3 to Fig. 3-7 for the different hopping rates.

3.4 Variance of the Received Signal at the Output of the Integrator

When the system is not synchronized, the received detected output consists only of noise. When the receiver hops to the transmitting frequency, there are ten possible

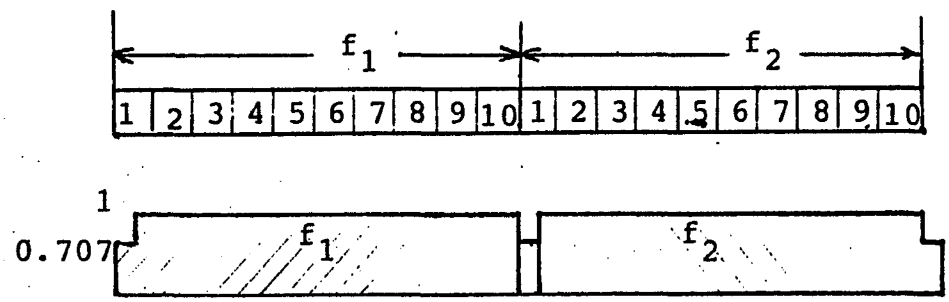

Fig.3-3 Signal in fading channel (theoretical)
(30 HOPS/S, 300 BITS/S)

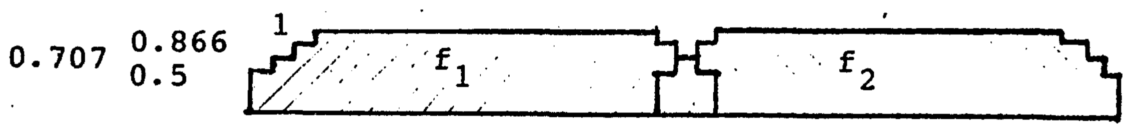

Fig. 3-4 Signal in fading channel (theoretical)
(100 HOPS/S, 1000 BITS/S)

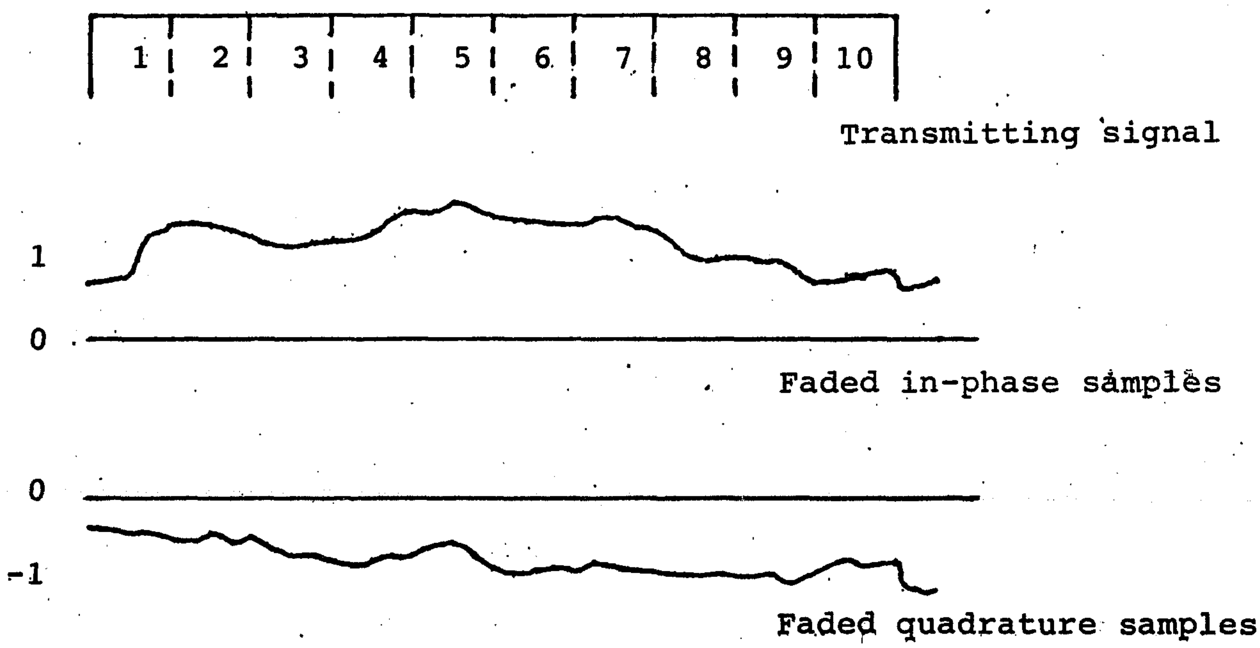

Fig.3-5 Signal in the fading channel (simulation example 1)

Data of simulation: 30 hops/s
10 bits/hop
10 samples/bit
spread time 1ms
fading rate 1Hz
2-tap model

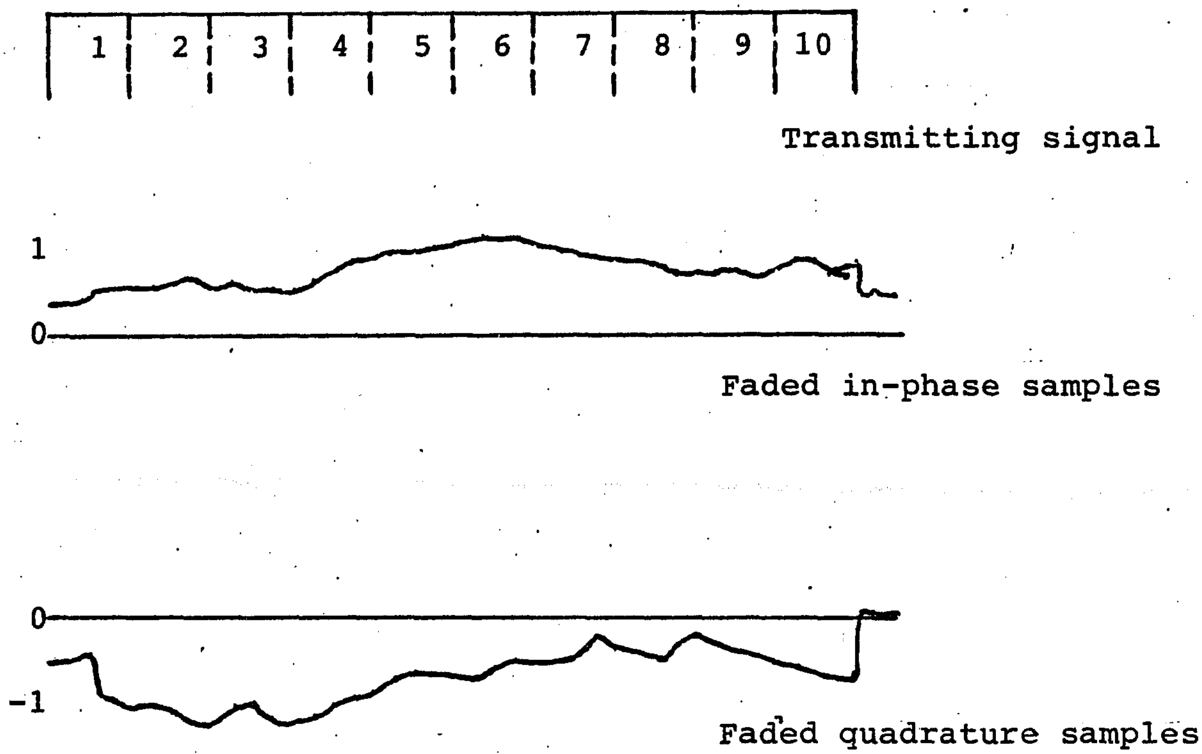

Fig. 3-6 Signal in the fading channel (simulation example 2)

Data of simulation: 30 hops/s
10 bits/hop
10 samples/bit
spread time 1ms
fading rate $1H_z$
2-tap model

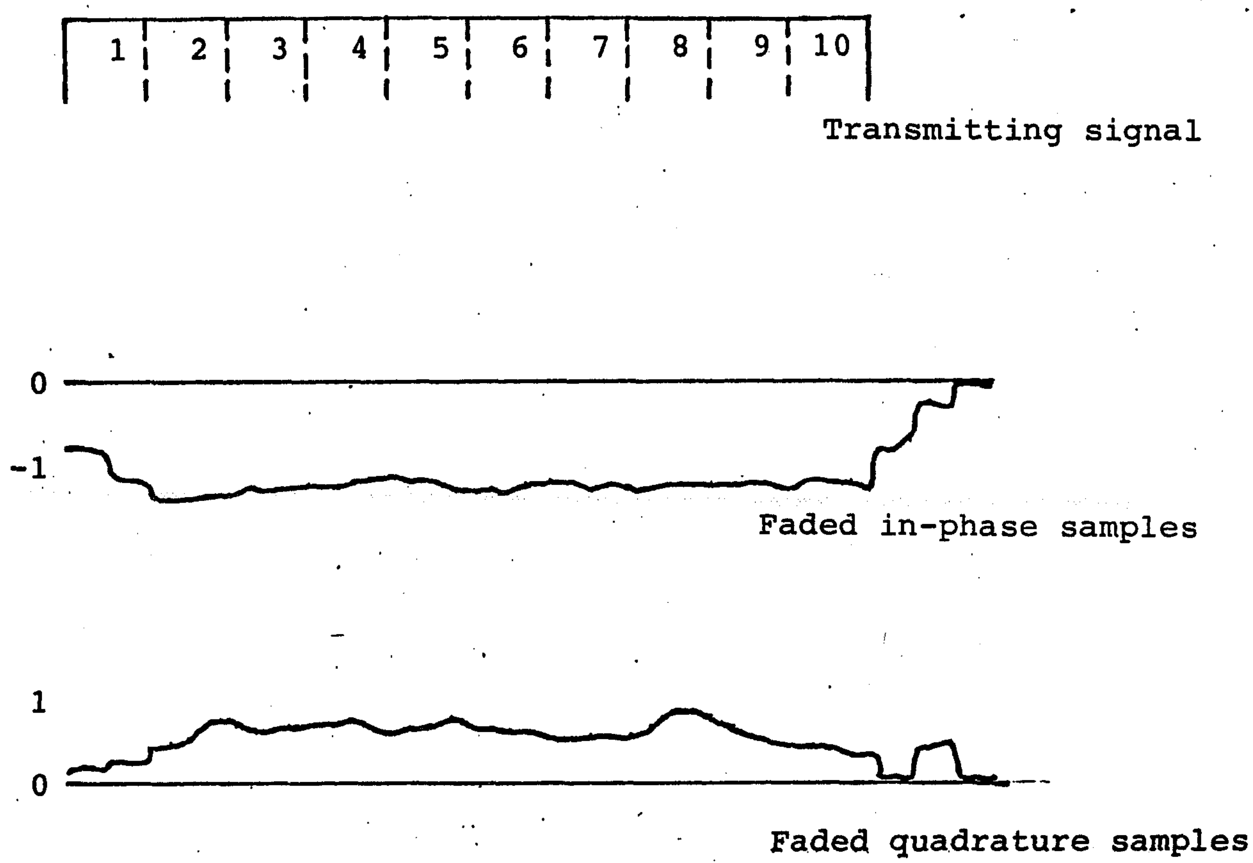

Fig. 3-7 Signal in the fading channel
(simulation example 3)

Data of simulation: 100 hops/s
10 bits/hop
10 samples/bit
spread time 1ms
fading rate $1H_z$
4-tap model

situations if we have ten bits in one hop. The receiver can only hop at the end of the bit time. If the transmitter and the receiver hop simultaneously, there is no signal in the channel; otherwise, some samples are already in the fading channel.* Now we consider the situation that both the transmitter and the receiver hop simultaneously. At the end of the first bit the receiver receives 10 samples. The first 5 samples are correlated because they have the same COEFP(1). Due to the low fading rate these five samples add almost linearly.** Sample 6 and sample 10 also add linearly. But sample 1 to sample 5 and sample 6 to sample 10 only add partly linearly because they have the common coefficient COEFP(1) but sample 1 to 5 don't have COEFP(2). For simplicity we suppose all the samples add linearly. Of course, this is an approximation. But the influence of such approximation will be partly cancelled if the signal is already in the fading channel when the receiver hops to the transmitting frequency. Therefore, for a two-tap model at the end of the first bit the standard deviation of the received signal at the output of the integrator is :

$$\sigma_{s1} = [\ \sqrt{P_s}\ \sqrt{\frac{1}{2NTAPS}} * \frac{NSC}{2} + \sqrt{P_s}\ \sqrt{\frac{2}{2NTAPS}} * \frac{NSC}{2}\ ]$$

$$= \frac{NSC}{2} \sqrt{\frac{1}{NTAPS}}\ [\ 1 + \sqrt{2}\ ] \qquad (3.4\text{-}1)$$

*: Appendix D

**: Appendix C

From bit 2 to bit 10, because all the delay lines are filled with signal; hence,

$$\sigma_{s2} = \sigma_{s1} + NSC$$

$$\sigma_{s3} = \sigma_{s2} + NSC$$

$$\vdots$$

$$\sigma_{s10} = \sigma_{s9} + NSC \qquad (3.4\text{-}2)$$

For bit 11, the signals in the delay lines gradually move out; hence,

$$\sigma_{s11} = \sigma_{s10} + \frac{NSC}{2}\sqrt{\frac{1}{NTAPS}}\,(\sqrt{1} + 0) \qquad (3.4\text{-}3)$$

For a 4-tap model:

$$\sigma_{s1} = \frac{NSC}{2}\sqrt{\frac{1}{NTAPS}}\,[1+\sqrt{2}] \qquad (3.4\text{-}4)$$

$$\sigma_{s2} = \frac{NSC}{2}\sqrt{\frac{1}{NTAPS}}\,[\sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4}] \qquad (3.4\text{-}5)$$

$$\sigma_{s3} = \sigma_{s2} + NSC$$

$$\vdots$$

$$\sigma_{s10} = \sigma_{s9} + NSC \qquad (3.4\text{-}6)$$

$$\sigma_{s11} = \sigma_{s10} + \frac{NSC}{2}\sqrt{\frac{1}{NTAPS}}\,[\sqrt{3} + \sqrt{2} + \sqrt{1} + 0] \qquad (3.4\text{-}7)$$

$$\sigma_{s12} = \sigma_{s11} + \frac{NSC}{2} \sqrt{\frac{1}{NTAPS}} [\sqrt{1} + 0] \quad (3.4\text{-}8)$$

As for the noise:

At the end of the M^{th} bit, the standard deviation of the noise is:

$$\sigma_{NM} = \sqrt{M} * NSC * NCB * 10^{-\frac{WNL}{20}} \quad (3.4\text{-}9)$$

WNL: E_b/η in db

NCB: number of chips/bit, which equals 1.

At the end of the M^{th} bit, the standard deviation of the signal plus noise is:

$$\sigma_{SNM} = \sqrt{(\sigma_{SM})^2 + (\sigma_{NM})^2} \quad (3.4\text{-}10)$$

3.5 The Operation of the Early-Late Gate in the Fading Channel

The circuit diagram of the tracking loop is shown in fig. 3-8. The operation of the early late gate can be illustrated graphically as shown in Fig. 3-9.

Fig. 3-9a represents the signal from the transmitter.

Fig. 3-9b represents the local carrier in the receiver. Suppose the transmitter and the receiver hop simultaneously.

Fig. 3-9c represents the incoming signal (refer to Fig. 3-3).

Fig. 3-9d shows us the output of the integrator which means that the tracking loop will almost lock at the vicinity of the 0^{th} bit (1/4 bit delay).

Fig. 3-9e demonstrates that the receiver hops to a new frequency at the end of the first bit.

Fig. 3-9f shows the input to the integrator. The output of the integrator will pull the local carrier toward the 1/4 bit and lock at the 1/4 bit.

Fig. 3-10 gives us the operation of the early late gate working in a 4-tap model fading channel. The control voltage of the early late gate is shown by Fig. 3-11. Fig. 3-11a shows us the curve for 30 hops/s, 300 bits/s (2-tap model). Fig. 3-11b gives us the curve for 100 hops/s, 1000 bits/s (4-tap model).

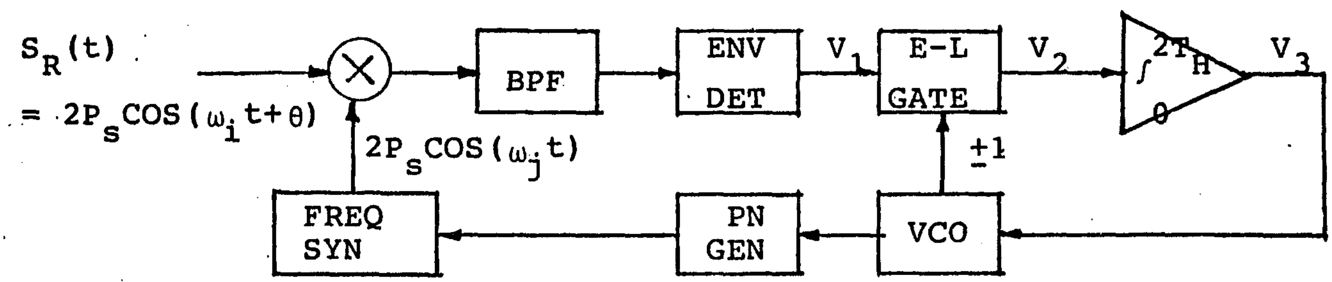

Fig.3-8 Tracking Loop

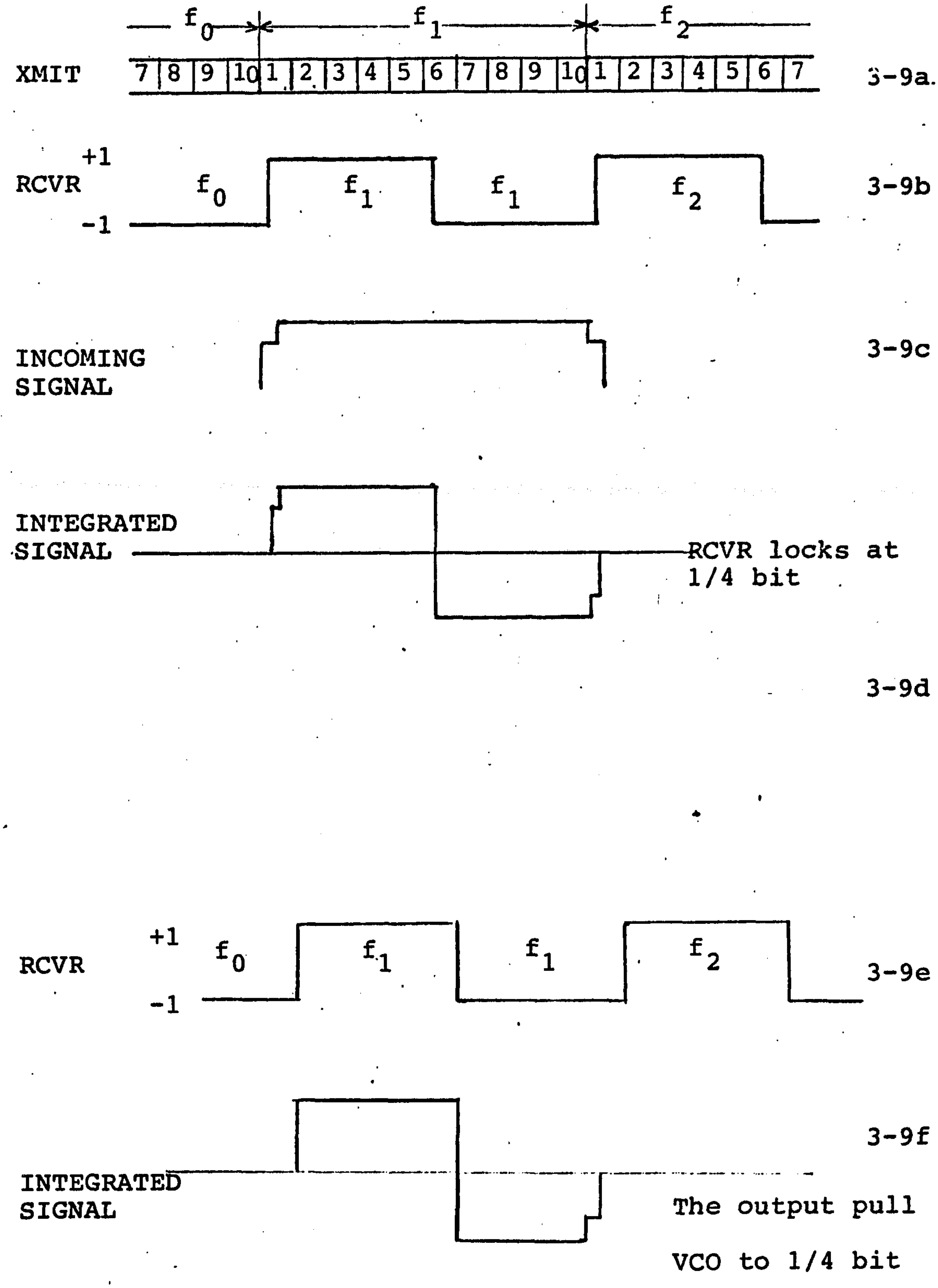

Fig. 3-9 Early-late gate working in the fading channel (2-tap model)

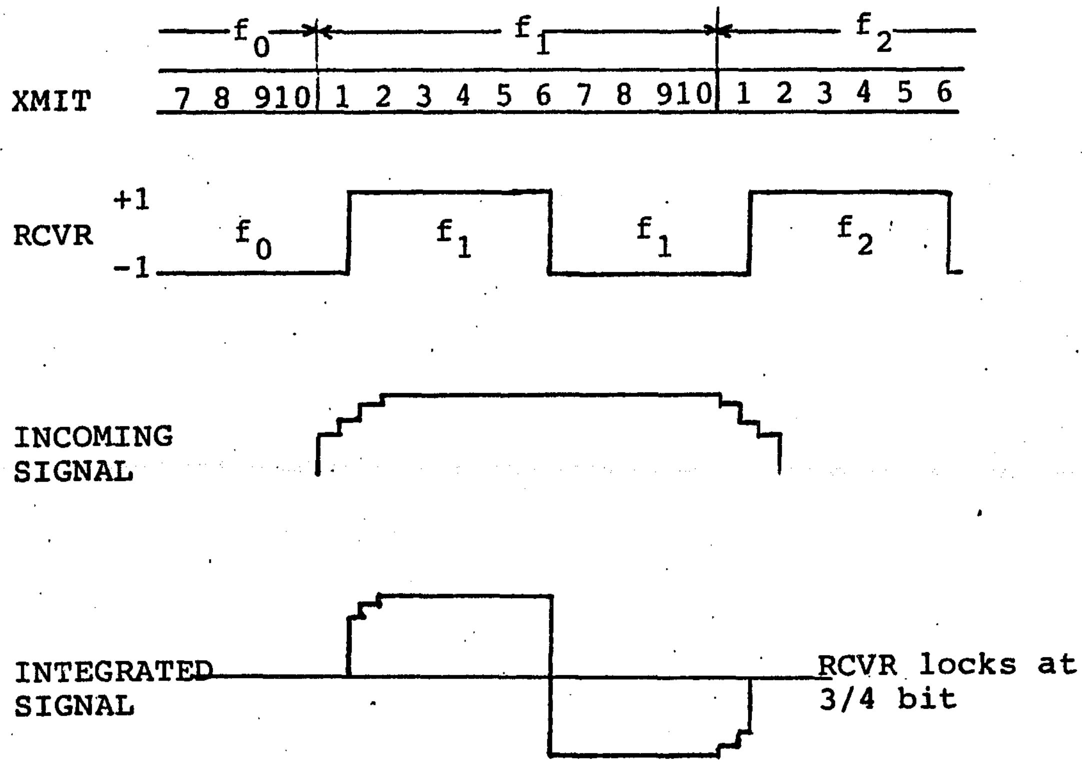

Fig. 3-10 Early-late gate working in fading channel (4-tap model)

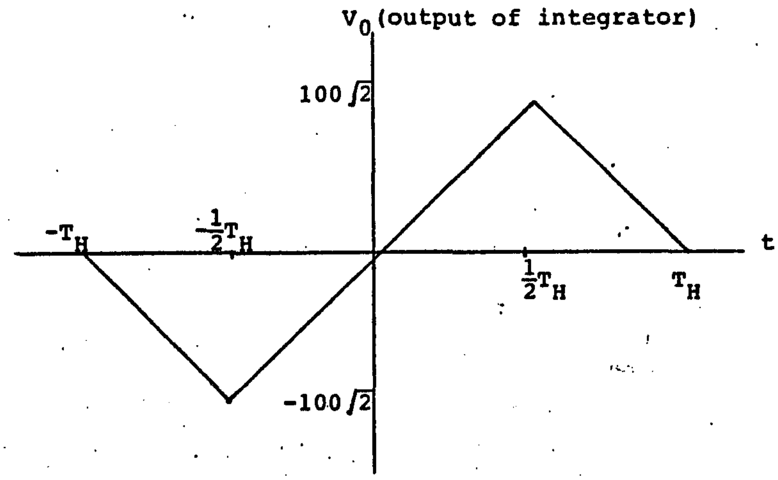

Fig. 3-11a   Control voltage of E-L gate

(integrate 2 hops, 2-tap model)

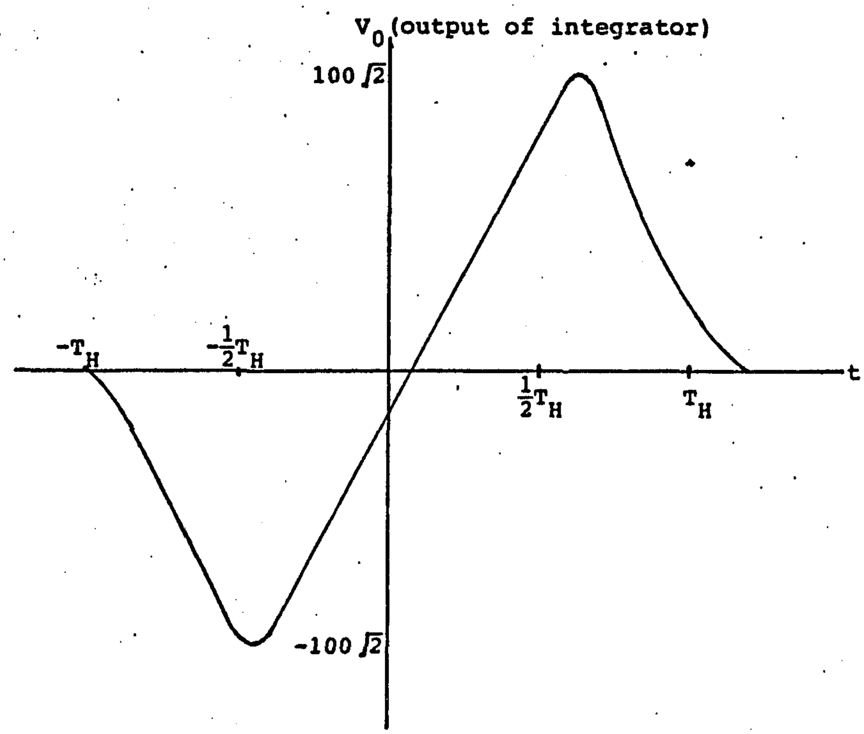

Fig. 3-11b   Control voltage of E-L gate

(integrate 2 hops, 4-tap model)

For fading channel if the number of taps is less than 4, the operation of the early-late gate is not very different from that of the nonfading channel. The only difference is that the operating point is not exactly at the origin. The pull in range of the tracking loop is almost half of the hop duration. The acquisition strategy used by the nonfading channel also works effectively for the fading channel. The flow chart of the computer simulation is shown in Fig. 3-12. However, if the number of taps is greater than 4, the operating point of the early-late gate will move to the right hand side and the pull in range of the tracking loop is greater than half of the hop duration. The acquisition strategy must be modified accordingly. But usually the number of taps is less than 4, we don't have to worry about this situation.

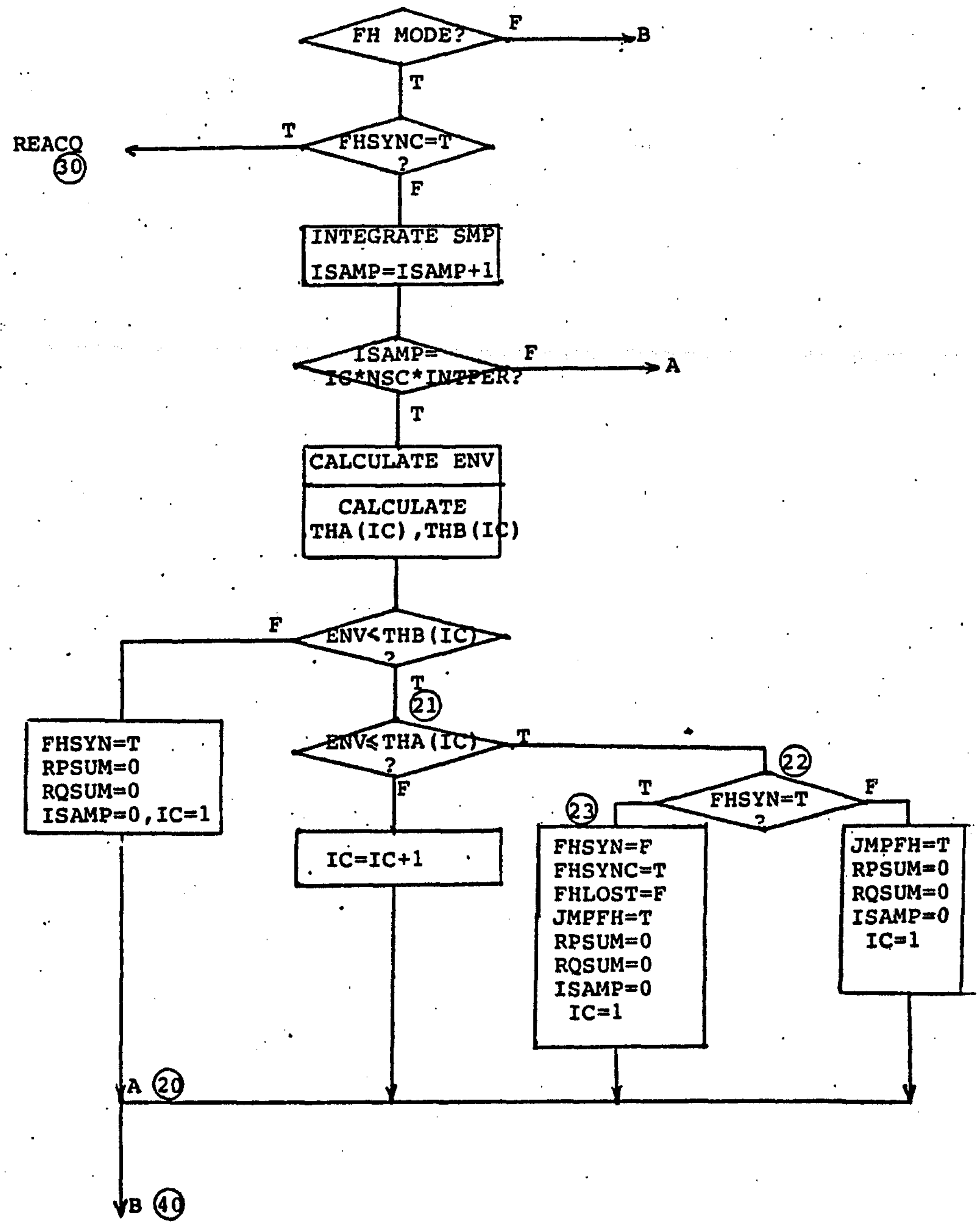

Fig. 3-12 ACQ strategy for fading channel

(300 bits/S, 30 hops/S, spread time=1ms)

3.6 Probability Density Function of the Signal Envelope in the Fading Channel

The mean value of the coefficients of the fading channel is zero so we say that the fading channel without specular component is Rayleigh channel. As a matter of fact, when the system is going to synchronize , we only investigate a very short time. Due to the low fading rate, the signal samples add almost linearly. In this sense it looks like a Ricean channel. However, we consider it as Rayleigh channel with large variance when the system is going to synchronize.

X and Y are Gaussian random variables with variance σ^2.

$$Z = \sqrt{X^2 + Y^2} \quad .$$

$$f(z) = \frac{z}{\sigma^2} e^{-z^2/2\sigma^2} U(z) \qquad \text{———Rayleigh Density} \qquad (3.6\text{-}1)$$

U(z) = unit step function

For convenience we consider the envelope square. we define

$W = Z^2$, W = envelope square

Probability density function of W

$$f(W) = \frac{1}{2\sigma^2} e^{-W/2\sigma^2} U(W) \qquad (3.6\text{-}2)$$

The probability that W is between α and β is:

$$P(\alpha \leq W < \beta) = \int_{\alpha}^{\beta} f(W)dW = e^{-\alpha/2\sigma^2} - e^{-\beta/2\sigma^2} \qquad (3.6\text{-}3)$$

Before synchronization we have noise only. The variance equals $\sigma_N{}^2$. When the system is going to synchronize, we have signal and noise. The variance equals $\sigma_{SN}{}^2$.

Refer to Fig. 3-13, when the system is not synchronized, for each comparision we make an error if the noise is greater than THB.

$$P_{FA} = P(\text{noise} \geq \text{THB}) = \int_{\text{THB}}^{\infty} f(W)\ dW$$

$$= e^{-\text{THB}/2\sigma_N{}^2} \qquad (3.6\text{-}4)$$

When system is going to synchronize, we miss the signal if signal + noise is less than THA.

$$P_D = P(\text{signal + noise} \leq \text{THA}) = \int_{0}^{\text{THA}} f(W)\ dW$$

$$= 1 - e^{-\text{THA}/2\sigma_{SN}{}^2} \qquad (3.6\text{-}5)$$

From eqs (3.6-4) and (3.6-5) we have

$$\text{THA} = -2\sigma_{SN}{}^2 \ln(1 - P_D) \qquad (3.6\text{-}6)$$

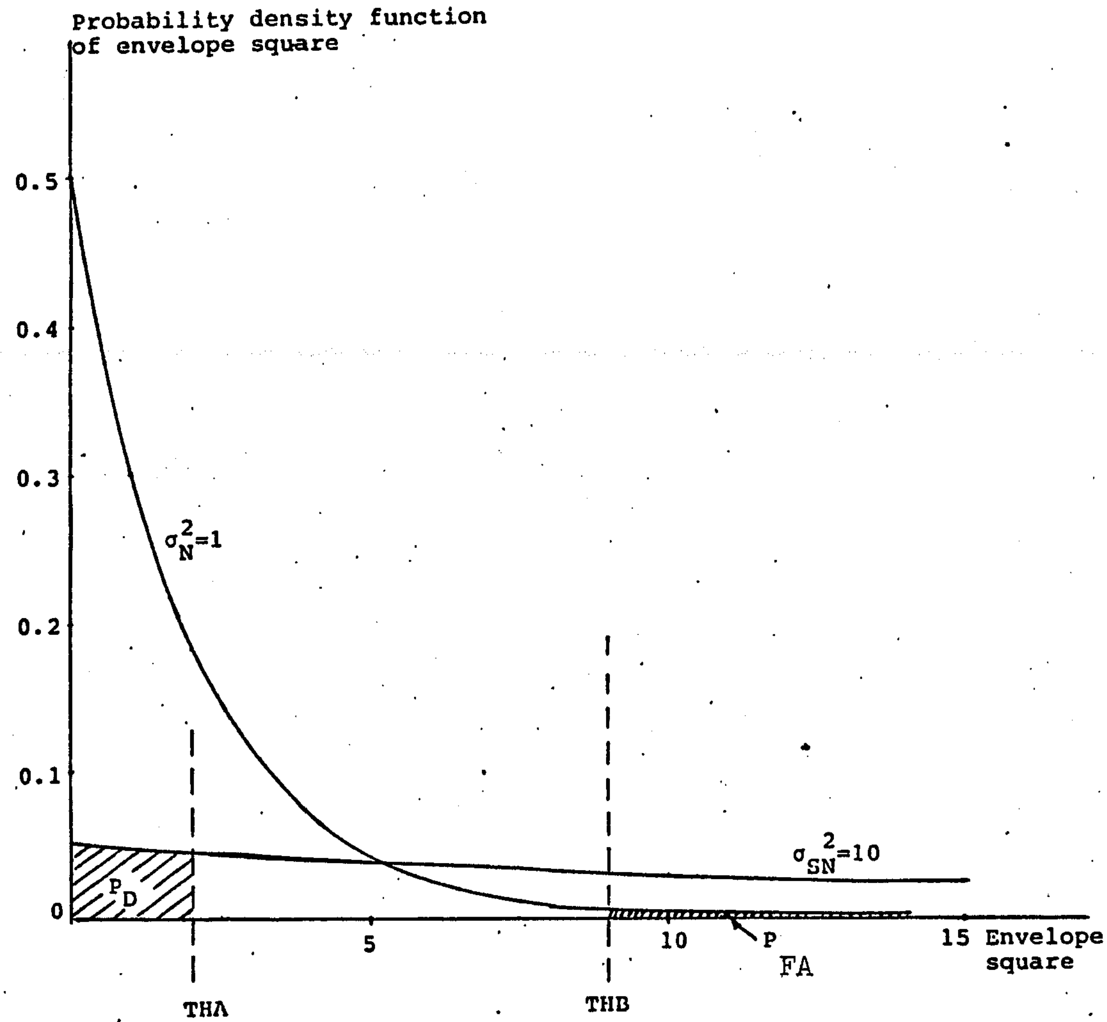

Fig. 3-13 Probability density function of envelope square, P_{FA} and P_D

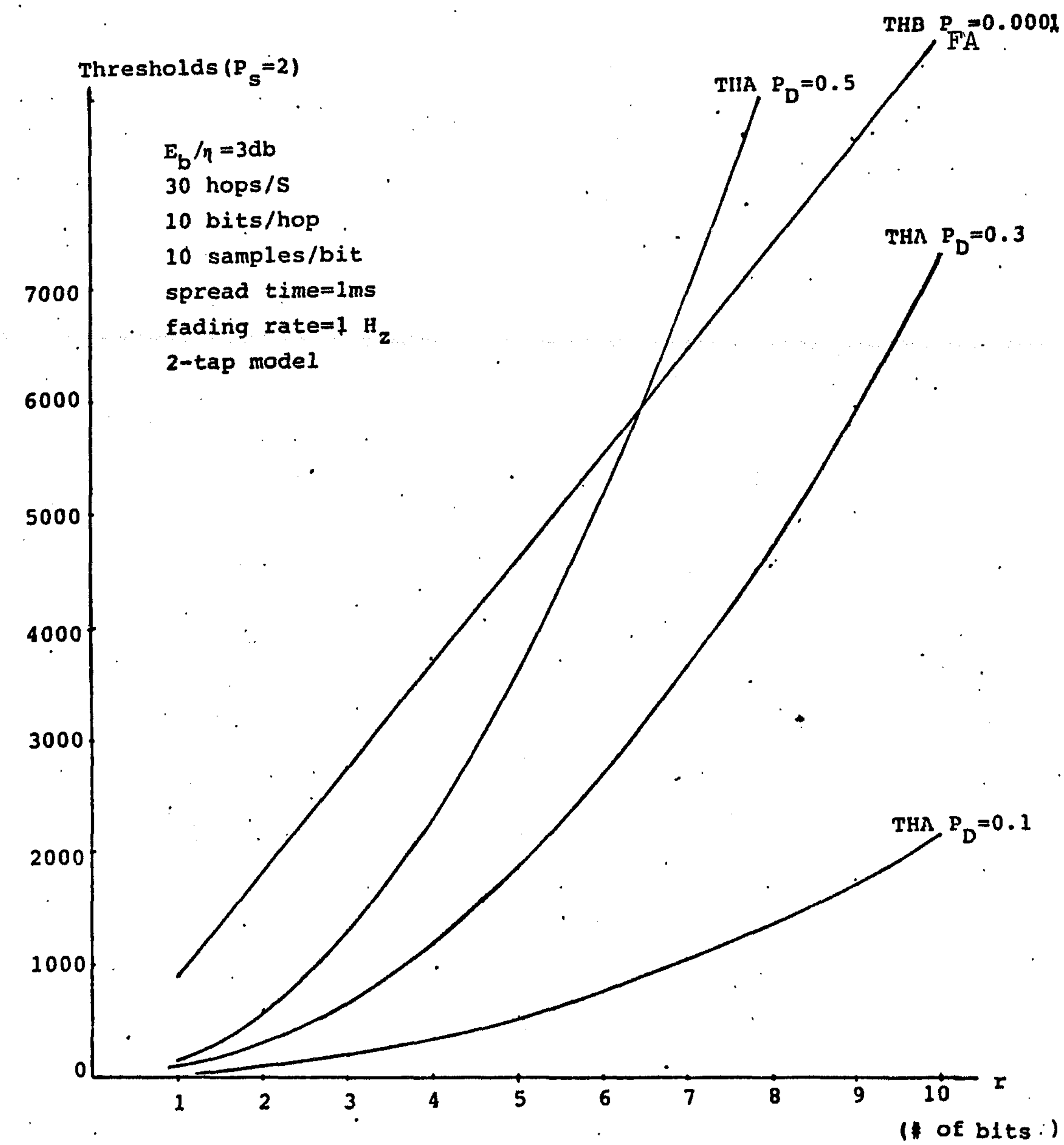

Fig. 3-14 Thresholds versus NO. of bits to be integrated

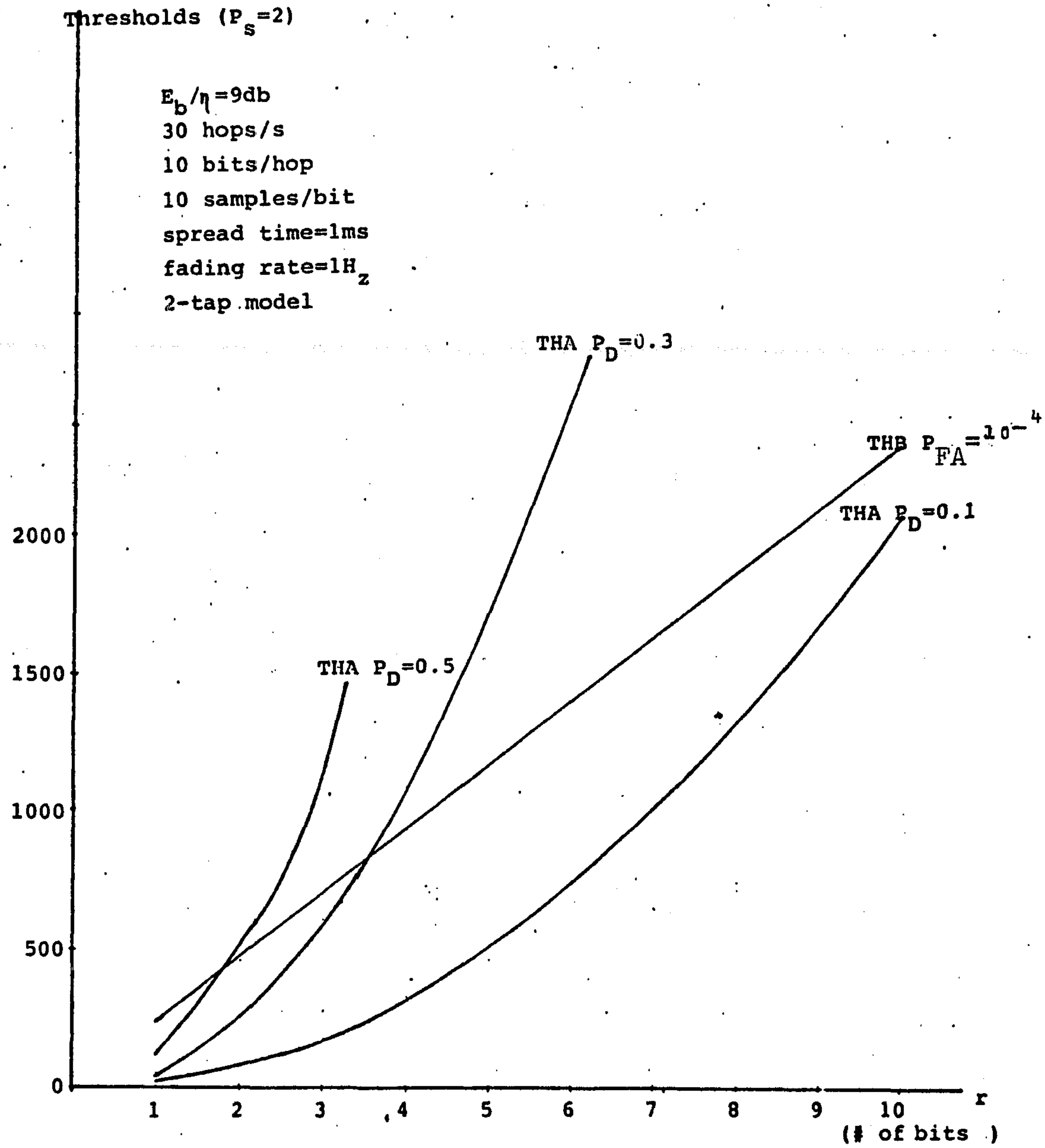

Fig. 3-15 Thresholds versus NO. of bits to be integrated

$$THB = -2\sigma_N^2 \ln P_{FA} \tag{3.6-7}$$

Fig. 3-13 shows the probability density function of envelope square, P_{FA} and P_D. Fig. 3-14 and Fig. 3-15 give THA and THB versus number of bits being integrated for the different signal to noise ratio E_b/η.

In order to minimize the probability of false acquisition P_{FA} equals 0.0001 is an appropriate value. P_D affacts the mean acquisition time quite a lot. With a small P_D most of the noise will be above THA and we always can not make a decision which increases the mean acquisition time. If P_D is too large signal will also lie below THA and we miss it. Owing to the slow fading rate the mean value of the signal is not zero and also varies very slowly in a short time interval. Such situation allows us to select a relative large P_D and signal would not drop below THA. Besides, the value of P_D is true only if the transmitter and the receiver hop to a new frequency simultaneously. Most of the cases when receiver hops to a new frequency, the transmitter already transmits a few bits which are already in the delay lines of the fading channel (from 0 bit to 9 bits are all possible). The received signal power is greater than the variance which we use to calculate the threshold THA. The results of simulation show us that there is an optimum P_D for the different signal to noise ratio. As a matter of fact, the effective P_D is smaller than that

Value. For example: E_b/η = 6db, P_D=0.3, the effective P_D is 0.2485 (for 2-tap model).*

3.7 Computer Simulation Results——Mean Acquisition Time and Variance Versus E_b/η for Optimum P_{FA} and P_D

The normalized mean acquisition time versus E_b/η for the different P_D are shown in Fig.3-16(for 30 hops/S) and Fig.3-17 (for 100 hops/S). From these figures we observe that $\overline{T_{acq}}$ doesn't depend on the hop rate and mostly depends on energy.

Comparing with the nonfading channel, refer to Fig.3-16 and Fig.2-12, we observe that when E_b/η=3db, fading increases $\overline{T_{acq}}$ 22% only because comparing to the noise fading is not important. When E_b/η=30db, fading doesn't affect $\overline{T_{acq}}$ at all. The mean acq time is close to the minimum possible value which is $\frac{L}{2}$*1.1=281 bits. The factor 1.1 is due to the hopping rate of the transmitter during acquisition. But when E_b/η=12db, fading does increase $\overline{T_{acq}}$ about 60 % which is serious. When E_b/η=15db, P_D=0.3 and 0.5 (points A and B), THA and THB intersect at the vicinity of one bit, double threshold scheme becomes single threshold scheme, $\overline{T_{acq}}$ goes high. Point C (P_D= 0.1) is still the case of double threshold. Hence, this is the improvement of double threshold scheme. Again at points D and E double threshold becomes single threshold. Point F is the improvement.

The variances of acquisition time are listed in Table 3.7-1 and 3.7-2 for the different model of fading channel and P_D.

*: Appendix E

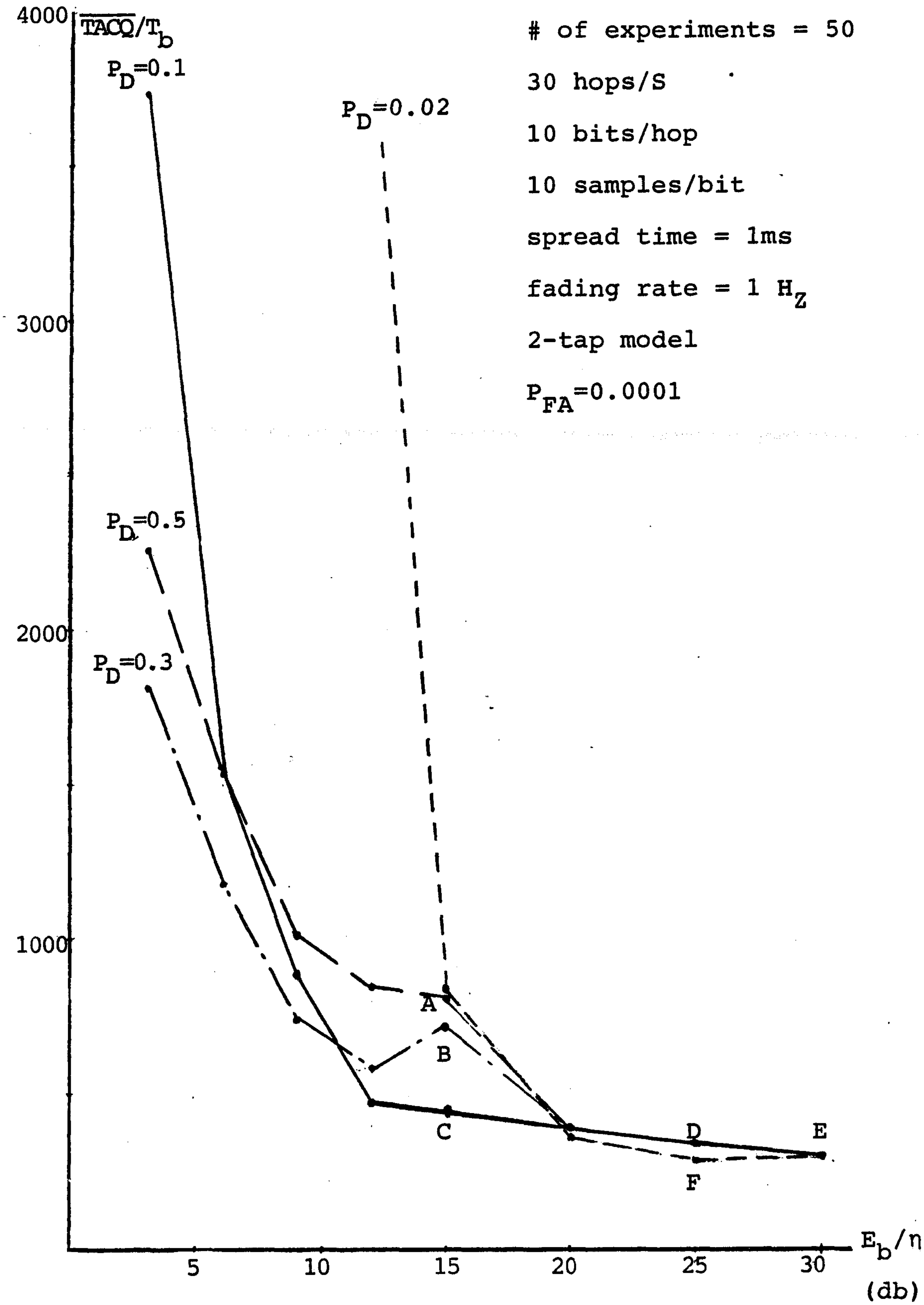

Fig. 3-16 Mean acquisition time versus E_b/η

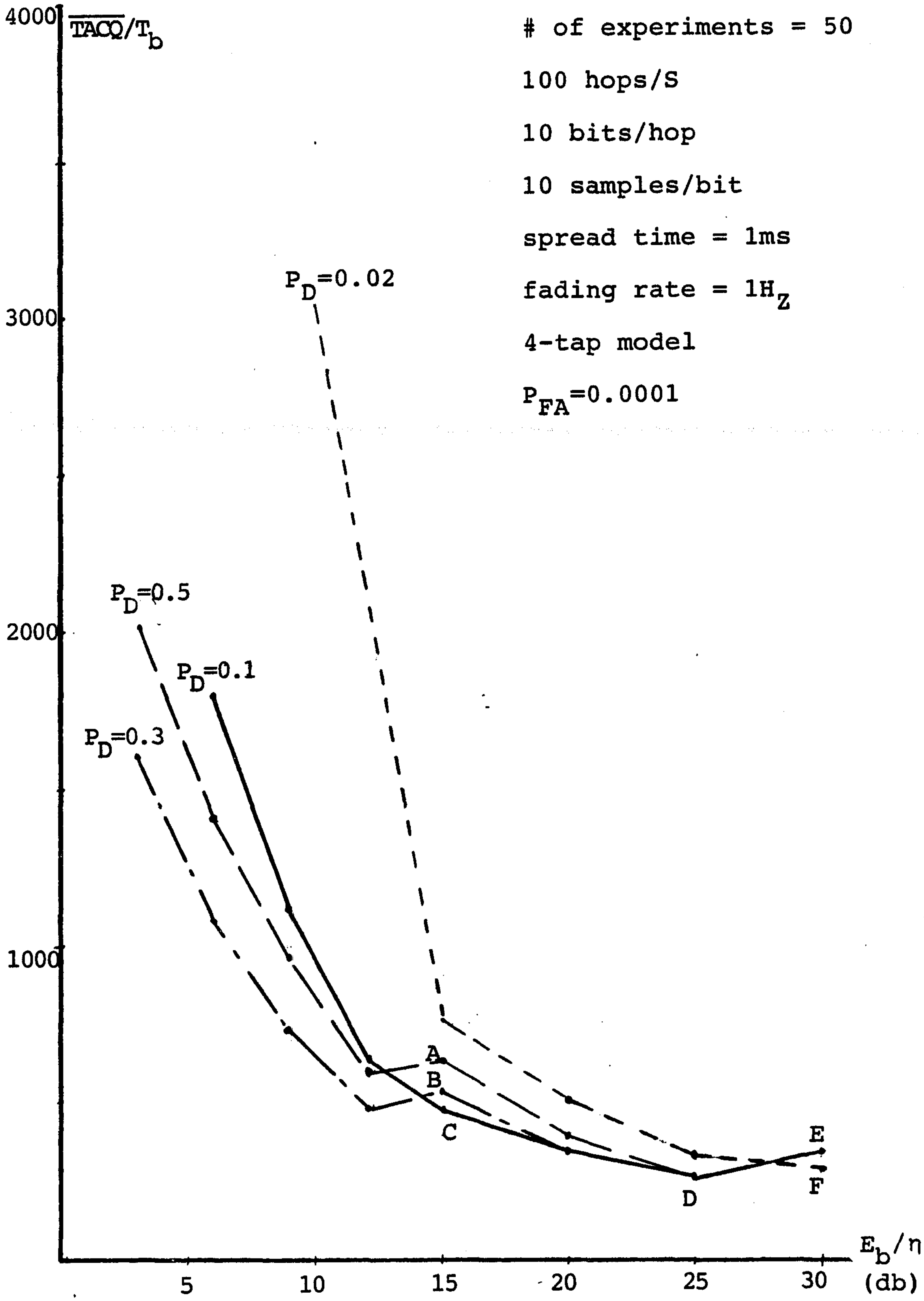

Fig. 3-17 Mean acquisition time versus E_b/η

Table 3.7-1

Variance of Acquisition Time (Experiment Result)

Hop rate = 30 hops/S, 2-tap Model

E_b/η (db)	P_D	Variance	Normalized Standard Deviation
30	0.1	0.418418	0.62966
	0.02	0.418418	0.62966
	0.01	0.303081	0.55344
	0.005	0.275799	0.55099
	0.002	0.435575	0.56009
25	0.1	0.455856	0.61307
	0.02	0.337374	0.50216
	0.01	0.412955	0.55545
	0.005	0.921933	0.62320
	0.002	2.070953	0.51520
20	0.1	0.524299	0.65931
	0.02	0.778812	0.69792
	0.01	1.200690	0.63962
15	0.5	6.588859	0.93680
	0.3	5.536296	0.97243
	0.1	1.629105	0.82622
	0.02	6.980358	0.95249

Table 3.7-1 (Cont'd.)

E_b/η (db)	P_D	Variance	Normalized Standard Deviation
12	0.5	6.665808	0.89386
	0.3	2.210138	0.76763
	0.1	0.840163	0.56669
9	0.5	9.171375	0.89559
	0.3	4.471619	0.83897
	0.1	7.753166	0.93249
6	0.5	16.20247	0.77684
	0.3	13.74216	0.94278
	0.1	18.79460	0.84126
3	0.5	36.24931	0.79713
	0.3	48.56720	1.15495
	0.1	146.13722	0.97152

Table 3.7-2

Variance of Acquisition Time (Experiment Result)

Hop rate = 100 hops/S, 4-tap Model

E_b/η (db)	P_D	Variance	Normalized Standard Deviation
30	0.1	0.056443	0.68907

Table 3.7-2 (Cont'd.)

E_b/η (db)	P_D	Variance	Normalized Standard Deviation
30	0.02	0.028083	0.55752
	0.01	0.039172	0.60551
	0.005	0.039606	0.51557
25	0.1	0.022601	0.56278
	0.02	0.033616	0.52921
	0.01	0.069021	0.59486
20	0.1	0.045693	0.62705
	0.02	0.106365	0.64079
15	0.5	0.308951	0.86610
	0.3	0.289192	1.00024
	0.1	0.146960	0.80117
12	0.5	0.358209	0.99068
	0.3	0.144358	0.77637
	0.1	0.201937	0.69824
9	0.5	0.811281	0.92320
	0.3	0.485079	0.94808
	0.1	0.388513	0.55792
6	0.5	1.514624	0.87441
	0.3	1.176961	1.00146
	0.1	2.291038	0.84144
3	0.5	3.302562	0.90081
	0.3	2.015820	0.88359

Chapter 4

SUGGESTION FOR FUTURE WORK

4.1 Consideration of the Fading Channel As A Ricean Channel

When we studied the fading channel without specular component we considered the channel as a Rayleigh channel because the addition among the different taps is random and each coefficient of the tap has a mean value of zero (refer to Fig. 3.1). If we only investigate a very short time, due to the slow fading rate those coefficients only change slightly. Therefore, in time domain the integration is a linear summation. The energy of the signal is the variance of the signal. Can we consider the channel as a Ricean channel with the mean value of σ (standard deviation of signal component) and the variance of σ_N^2 (variance of noise)? If we assign small value for both P_{FA} and P_D, what will be the result of simulation?

4.2 Consideration of the Different Spread Time for the Different Hopping Frequencies in the Fading Channel

In chapter 3 we consider the spread time of the fading channel for all the different hopping frequencies is constant. What is the situation if the spread time varies with respect to the different frequencies such as in

Fig.4-1 ?

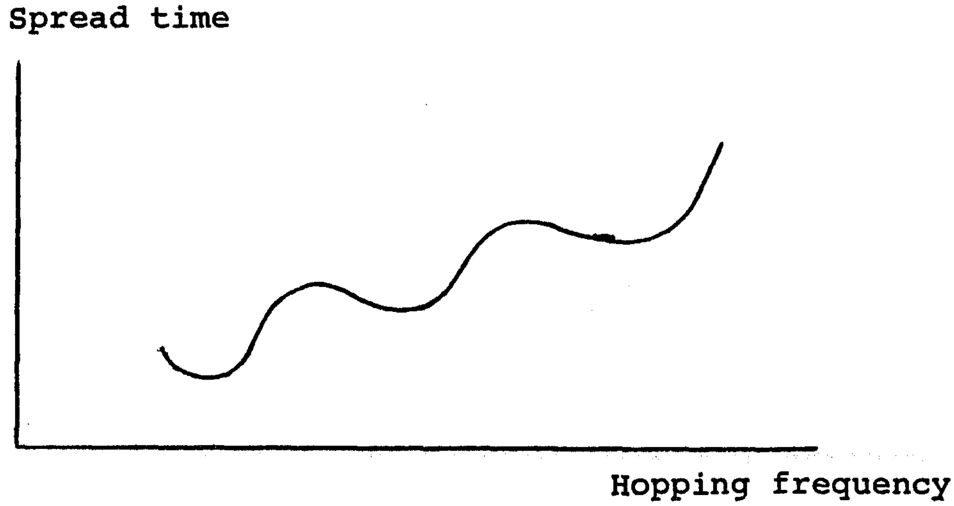

Fig. 4-1 Spread time with respect to hopping frequencies

In this case the number of taps of the delay line is different for the different hopping frequencies. Two problems will arise.

1. Is the acquisition strategy compatible to the different hopping frequencies?

2. How does the phase lock loop work? There will be a big jitter in the timing clock. How to solve this problem?

For the first problem we say that this acquisition strategy is still compatible if the number of taps of the fading channel for the different hopping frequencies is no greater than 20. The signal of a 20-tap model is shown in

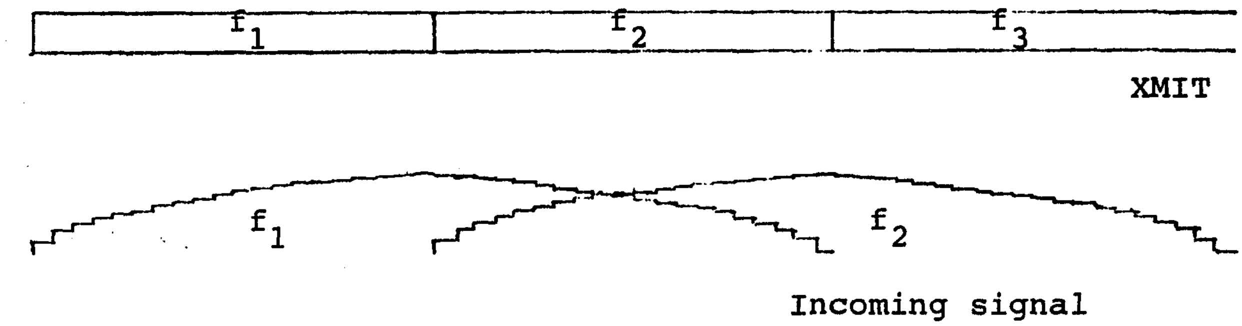

Fig. 4- 2 Signal of a 20-tap model

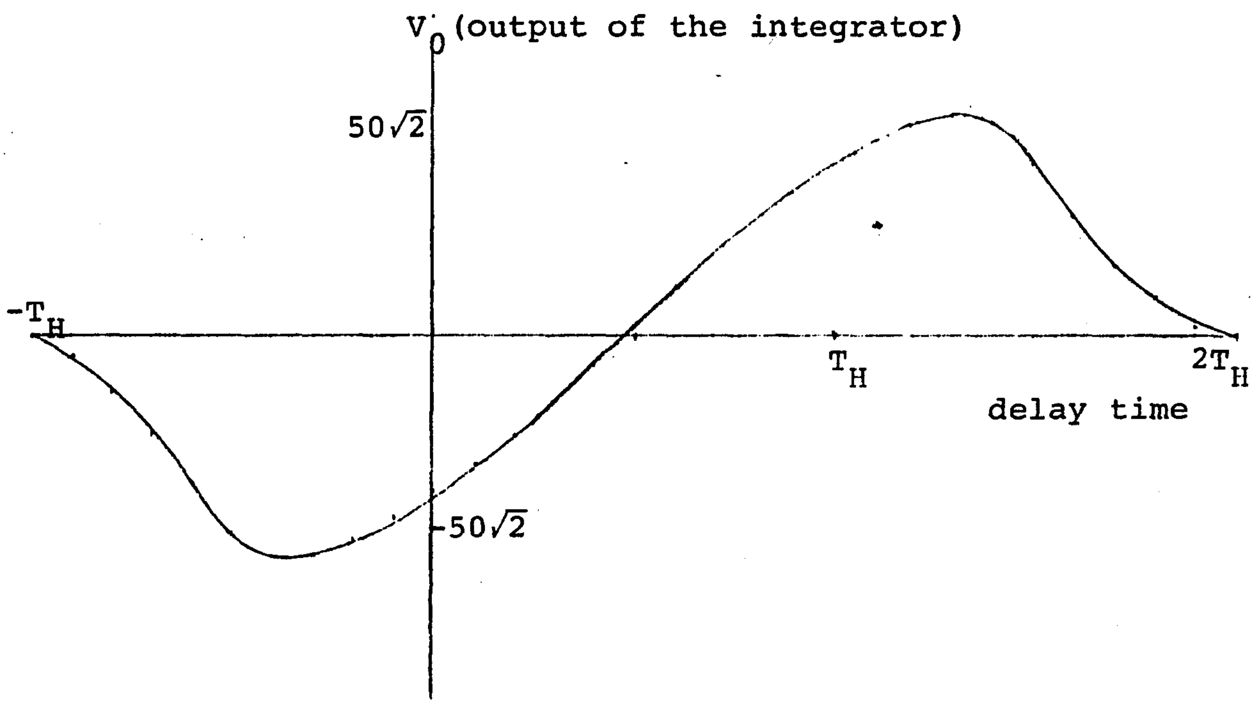

Fig. 4- 3 Operation of E-L Gate of a 20-tap model

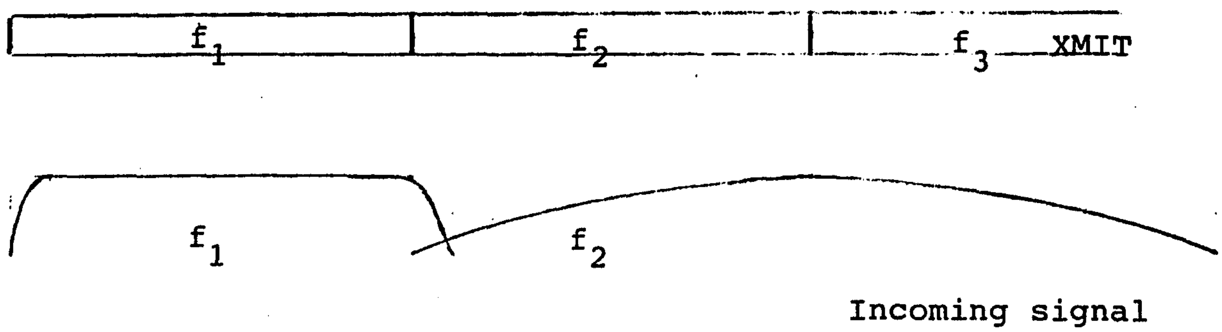

Fig. 4-4 Signal having different spread time

Fig. 4-2. From Fig. 4-2 we saw if the number of taps is greater than 20,after pre-acquisition has declared the local frequency of the receiver remains at f_1 until the output of the integrator again drops below the threshold. At that moment the transmitter has already hopped to f_3; therefore, true acquisition will never occur. Hence, only if that the number of taps is less than 20 is acceptable.

The second problem is more critical. The operation of the E-L Gate for a 20-tap model is shown in Fig. 4-3. The phase lock loop will lock at 4.75 bit. But for two-tap model the phase lock loop will lock at 1/4 bit (refer to Fig. 3-11). Hence, there will be a big jitter in the timing clock when the local hopper hops from one frequency to another. Especially if one of the hopping frequency having a short spread time is followed by another hopping frequency which has a long spread time as shown in Fig. 4-4. When f_1 passed away, the most of the energy of f_2 hasn't arrived yet. The phase lock loop will lose the signal. Even if we select a very large time constant for the low pass filter, the error rate of the data will be very large. In this sense only the number of the taps less than 4 is acceptable. How to solve this problem? Discard those frequencies which have a long spread time or .....?

4.3 The Ability of Guarding against the Jammer

If there is a jammer embedded in the signal, when the local hopper hops to that frequency pre-acquisition will declare. Since then the local hopper will remain at this frequency and acquisition can not keep on going. Therefore, we have to design such a system that after a certain times of comparision we make a hard decision. In this case the mean acquisition time would not be affected too much by the jammer.

APPENDIX A

COMPUTER PROGRAM OF THEORETICAL CALCULATION OF THE TWO THRESHOLDS, PROBABILITY OF MAKING AN ERROR, PROBABILITY OF MAKING A CORRECT DECISION, THE EXPECTED VALUE AND VARIANCE OF NUMBER OF CHIPS TO BE INTEGRATED FOR EACH DECISION AND THE MEAN AND VARIANCE OF ACQUISITION TIME FOR CONDITIONAL DOUBLE THRESHOLD SCHEME

```
// JOB
//*MAIN LINES=19,TIME=4
// EXEC FORTG,REGION=150K
//FORT.SYSIN DD *
       DIMENSION THA(40),THB(40)
       WNL=9.
       INTPER=1
       WRITE(6,104)WNL,INTPER
  104  FORMAT(1X,'WNL=',F3.0,'DB',4X,'INTPER=',I3,4X,'PFA=0.00002',//)
       SIGW9=SQRT(10.)*10.**(-WNL/20.)
       NSC9=10
       SIGMAO =SIGW9 * SQRT(FLOAT(NSC9*INTPER))
       B=SQRT(2.0)*FLOAT(NSC9*INTPER)/2.0
  11   PE1=ERFC(B/1.414/SIGMAO)/2.
       IF(PE1.GT.0.00002) GO TO 13
       GO TO 12
  13   B=B+0.01
       GO TO 11
  12   A=SQRT(2.0)*FLOAT(NSC9*INTPER)-B
         PC1=ERFC(-A/1.4142/SIGMAO)/2.0
         BMEAN=PC1
       SQBM=PC1
       PEAVE=PE1
```

```
      PND=1.0-PE1-PC1
        PM=1.0-(1.0-PEAVE)**48
        TACQ=((1.0+PEAVE+2.*PM)/(1.0-1./48.)*511./2.+24.)/300.*PC1
       SQTACQ=TACQ**2/PC1
        PCSUM=PC1
      WRITE(6,102)A,B,PC1
  102 FORMAT(9X,'THA( 1)=',F7.2,/,9X,'THB( 1)=',F7.2,/,9X,'PC( 1)=',
     1     F10.7)
      MC=2
      DO 60 K=1,9
       SIGMAM=SIGW9*SQRT((FLOAT(MC)-1.)*FLOAT(NSC9*INTPER))
C     WRITE(6,100)NSC9,SIGMAO,SIGMAM
  100 FORMAT(1X,'NS3=',I3,4X,'SIGMAO=',F7.2,4X,'SIGMAM=',F7.2)
       FACTR2=1/(ERFC(A/SQRT(2.0)/SIGMAM)-
     1              ERFC(B/SQRT(2.0)/SIGMAM))
       TEMP=SQRT(2.)*(FLOAT(MC)-1)*FLOAT(NSC9*INTPER)
       FACTR1=1/(ERFC((A-TEMP)/SQRT(2.)/SIGMAM)
     1             -ERFC((B-TEMP)/SQRT(2.)/SIGMAM))
       C=SQRT(2.0)*FLOAT(MC)*FLOAT(NSC9*INTPER)/2.0
       D=C
    5  SUMA=0.0
       SUMB=0.0
        SUMC=0.0
       STEP=(B-A)/100.
       VM=A+STEP/2.0
       DO 10 I=1,100
       SUMA=SUMA+ERFC((VM+SQRT(2.0)*FLOAT(NSC9*INTPER)-C)/SQRT(2.0)/
     1SIGMAO)*EXP(-(VM-TEMP)**2/2./SIGMAM**2)*STEP/SQRT(6.2832)/SIGMAM
         ANY=EXP(-VM*VM/2./SIGMAM/SIGMAM)*STEP/SQRT(6.2832)/SIGMAM
       SUMB=SUMB+ERFC((D-VM)/SQRT(2.)/SIGMAO)*ANY
       SUMC=SUMC+ERFC((VM-C)/SQRT(2.)/SIGMAO)*ANY
       VM=VM+STEP
   10  CONTINUE
       PEA=FACTR1*SUMA
       PEB=FACTR2*SUMB
        PC=FACTR2*SUMC
```

```
C
      IF((PEA.GT.0.00002).AND.(PEB.GT.0.00002)) GO TO 20
      IF((PEA.GT.0.00002).AND.(PEB.LE.0.00002)) GO TO 30
      IF((PEA.LE.0.00002).AND.(PEB.GT.0.00002)) GO TO 40
      GO TO 50
C
 20   C=C-0.1
      D=D+0.1
      GO TO 5
C
 30   C=C-0.1
      GO TO 5
 40   D=D+0.1
      GO TO 5
C
  50  WRITE (6,103)MC,C,MC,D,MC,PC
  103 FORMAT(9X,'THA(',I2,')=',F7.2,/,9X,'THB(',I2,')=',F7.2,
     1       /,9X,'PC(',I2,')=',F10.7)
      BMEAN=BMEAN+PND*PC*MC
      SQBM=SQBM+PND*PC*MC**2
      PEAVE=PEAVE+PND*PEB
      PM=1.0-(1.0-PEAVE)**(48./MC)
      ACQ=((1.0+PEAVE+2.*PM)/(1.0-FLOAT(MC)/48.)*MC*511./2.+24.)
     1     /300.
      TACQ=TACQ+ACQ*PND*PC
      SQTACQ=SQTACQ+ACQ**2*PND*PC
      PCSUM=PCSUM+PND*PC
      PND=PND*(1.0-PEB-PC)
      A=C
      B=D
      MC=MC+1
  60  CONTINUE
      VARN=SQBM-BMEAN**2
      VARTACQ=SQTACQ-TACQ**2
      STDBM=SQRT(VARN)/BMEAN
      STDTACQ=SQRT(VARTACQ)/TACQ
```

```
      WRITE(6,105) BMEAN,VARN,STDBM,TACQ,VARTACQ,STDTACQ,PEAVE,PCSUM
105   FORMAT(1X, 'AVERAGE NUMBER OF BITS FOR EACH DECISION =',F10.7,/,
     1   1X,'VARIANCE OF NUMBER OF BITS FOR EACH DECISION =',F10.7,/,
     2   1X 'NORMALIZED STANDARD DEVIATION OF NUMBER OF BITS'
     3      ' FOR EACH DECISION =',F10.7,/,1X,
     4       'MEAN ACQUISITION TIME =',F10.7,/,1X,
     5       'VARIANCE OF ACQUISITION TIME = ',F10.7,/,1X,
     6       'NORMALIZED STANDARD DEVIATION OF ACQUISITION TIME =',
     7       F10.7,/,1X,'TOTAL PE = ',F10.7,/,1X,'TOTAL PC = ',F10.7)
      STOP
      END
```

APPENDIX B

COMPUTER PROGRAM OF THEORETICAL CALCULATION OF PROBABILITY OF MAKING AN ERROR, PROBABILITY OF MAKING A CORRECT DECISION, THE EXPECTED VALUE AND VARIANCE OF NUMBER OF CHIPS TO BE INTEGRATED FOR EACH DECISION AND THE MEAN AND VARIANCE OF ACQUISITION TIME FOR UNCONDITIONAL DOUBLE THRESHOLD SCHEME

```
// JOB
//*MAIN LINES=19,TIME=4
// EXEC FORTG,REGION=150K
//FORT.SYSIN DD *
      WNL=0.
      IRN=34
      WRITE(6,104)WNL,IRN
 104  FORMAT(1X,'WNL=',F3.0,'DB',4X,'IRN=',I2,//)
      SIGW9=SQRT(10.)*10.**(-WNL/20.)
      NSC9=10
      SIGMAO =SIGW9 * SQRT(FLOAT(NSC9))
        IRNP=IRN-1
      B=SQRT(IRN/2.0)*FLOAT(NSC9)
 11   PE1=ERFC(B/1.414/SIGMAO)/2.
 12   A=SQRT(2.0)*FLOAT(NSC9)-B
        PC1=ERFC(-A/1.4142/SIGMAO)/2.
        BMEAN=PC1
        SQBM=PC1
        PEAVE=PE1
        PND=1.0-PE1-PC1
        PM=1.0-(1.0-PEAVE)**48
        TACQ=((1.0+PEAVE+2.*PM)/(1.0-1./48.)*511./2.+24.)/300.*PC1
        SQTACQ=TACQ**2/PC1
        PCSUM=PC1
```

```
                WRITE(6,102)PE1,PC1
      102       FORMAT(1X,'PE( 0, 1) =',F10.7,10X,'PC( 0, 1) =',F10.7)
                MC=2
                DO 60 K=1,IRNP
                 SIGMAM=SIGW9*SQRT((FLOAT(MC)-1.)*FLOAT(NSC9))
C               WRITE(6,100)NSC9,SIGMAO,SIGMAM
      100       FORMAT(1X,'NS3=',I3,4X,'SIGMAO=',F7.2,4X,'SIGMAM=',F7.2)
                 FACTR2=1/(ERFC(A/SQRT(2.0)/SIGMAM)-
     1                     ERFC(B/SQRT(2.0)/SIGMAM))
C                TEMP=SQRT(2.)*(FLOAT(MC)-1)*FLOAT(NSC9)
C                FACTR1=1/(ERFC((A-TEMP)/SQRT(2.)/SIGMAM)
C    1                    -ERFC((B-TEMP)/SQRT(2.)/SIGMAM))
                 D=SQRT(IRN*MC/2.0)*NSC9
                 C=SQRT(2.0)*MC*NSC9-D
    5            SUMA=0.0
                 SUMB=0.0
                SUMC=0.0
                 STEP=(B-A)/100.
                 VM=A+STEP/2.0
                 DO 10 I=1,100
C                SUMA=SUMA+ERFC((VM+SQRT(2.0)*FLOAT(NSC9)-C)/SQRT(2.0)/
C    1SIGMAO)*EXP(-(VM-TEMP)**2/2./SIGMAM**2)*STEP/SQRT(6.2832)/SIGMAM
                  ANY=EXP(-VM*VM/2./SIGMAM/SIGMAM)*STEP/SQRT(6.2832)/SIGMAM
                 SUMB=SUMB+ERFC((D-VM)/SQRT(2.)/SIGMAO)*ANY
                 SUMC=SUMC+ERFC((VM-C)/SQRT(2.)/SIGMAO)*ANY
                 VM=VM+STEP
   10            CONTINUE
C                PEA=FACTR1*SUMA
                 PEB=FACTR2*SUMB
                  PC=FACTR2*SUMC
                 BMEAN=BMEAN+PND*PC*MC
                  SQBM=SQBM+PND*PC*MC**2
                 PEAVE=PEAVE+PND*PEB
                PM=1.0-(1.0-PEAVE)**(48./MC)
                ACQ=((1.0+PEAVE+2.*PM)/(1.0-FLOAT(MC)/48.)*MC*511./2.+24.)
     1              /300.
```

```
          TACQ=TACQ+ACQ*PND*PC
          SQTACQ=SQTACQ+ACQ**2*PND*PC
       PCSUM=PCSUM+PND*PC
       PND=PND*(1.0-PEB-PC)
C
        M=MC-1
   50 WRITE (6,103)M,MC,PEB,M,MC,PC
  103 FORMAT(1X,'PE(',I2,',',I2,') =',F10.7,10X,'PC(',I2,',',I2,') =',
     1 F10.7)
      A=C
      B=D
      MC=MC+1
   60 CONTINUE
       VARN=SQBM-BMEAN**2
       VARTACQ=SQTACQ-TACQ**2
       STDBM=SQRT(VARN)/BMEAN
       STDTACQ=SQRT(VARTACQ)/TACQ
       WRITE(6,105) BMEAN,VARN,STDBM,TACQ,VARTACQ,STDTACQ,PEAVE,PCSUM
  105  FORMAT(1X, 'AVERAGE NUMBER OF BITS FOR EACH DECISION =',F10.7,/,
     1     1X,'VARIANCE OF NUMBER OF BITS FOR EACH DECISION =',F10.7,/,
     2     1X,'NORMALIZED STANDARD DEVIATION OF NUMBER OF BITS'
     3        ' FOR EACH DECISION =',F10.7,/,1X,
     4         'MEAN ACQUISITION TIME =',F10.7,/,1X,
     5         'VARIANCE OF ACQUISITION TIME = ',F10.7,/,1X,
     6         'NORMALIZED STANDARD DEVIATION OF ACQUISITION TIME =',
     7         F10.7,/,1X,'TOTAL PE = ',F10.7,/,1X,'TOTAL PC = ',F10.7)
      STOP
      END
```

APPENDIX C

NOISE COEFFICIENT OF THE FADING CHANNEL

In reference [2] coefficients are generated by a subroutine which is used to generate Gaussian random variable. The FFT of the Gaussian random variables seems to be white. The coefficients pass through the Doppler filter where the frequency bandwidth is gradually cut off at one H_z refer to Fig. 3-18. By running DO LOOP 900,000 times (300 times of F_s), I found that the power attenuation is about sampling rate/fading rate/3 which must be compensated for. I accumulated every 10 coefficients and found the variance which is almost 100 times of the variance of the coefficient. The precision is up to the third digit. That means the integration of the signal samples can be considered as a linear summation.

There is a mistake in reference [2]. That is when the receiver hops to a new frequency, all the coefficients are reset. Because the Doppler filter has a narrow bandwidth, at least 300 samples are needed to set up the steady state. If the parameters we used are 10 bits/hop, 10 samples/bit, we have only 100 samples in one hop so that the fading channel never works at the steady state situation and the signal output is much lower than it should be. Therefore, when we reset the fading channel, we should not only reset the coefficients but also XPDEL.

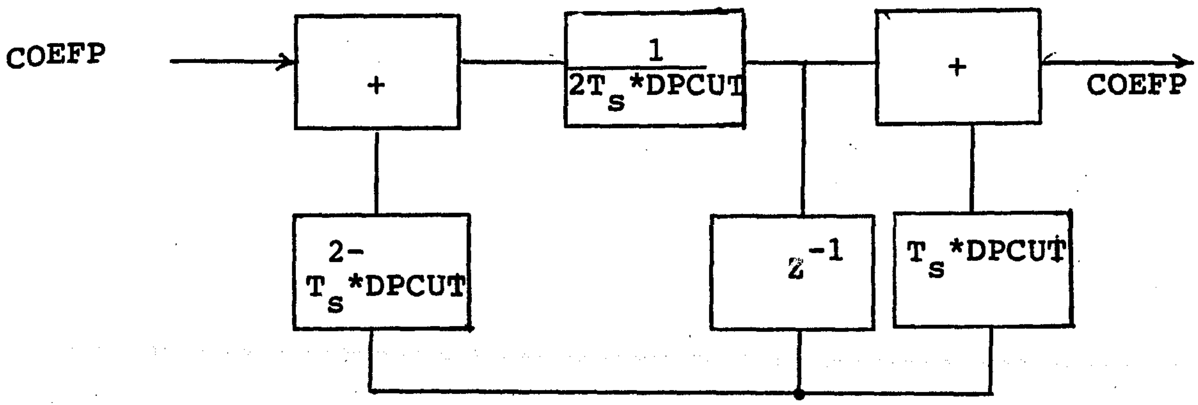

Fig. C-1 Doppler filter

DPCUT = 2π*fading rate

XPDEL = a random variable * standard deviation

The standard deviation of XPDEL is found by statistics.

APPENDIX D

RESET THE DELAY LINE

There is another mistake in the reference [2]. That is: when the receiver hops to a new frequency all the delay lines are simply reset to 0. That is not true. When the receiver hops to the transmiting frequency, if the transmitter already transmits a certain bits, some signal samples are already in the delay lines. Hence, the delay lines should be reset according to the specific situation. The flow chart of resetting the delay lines is given in Fig. D-1

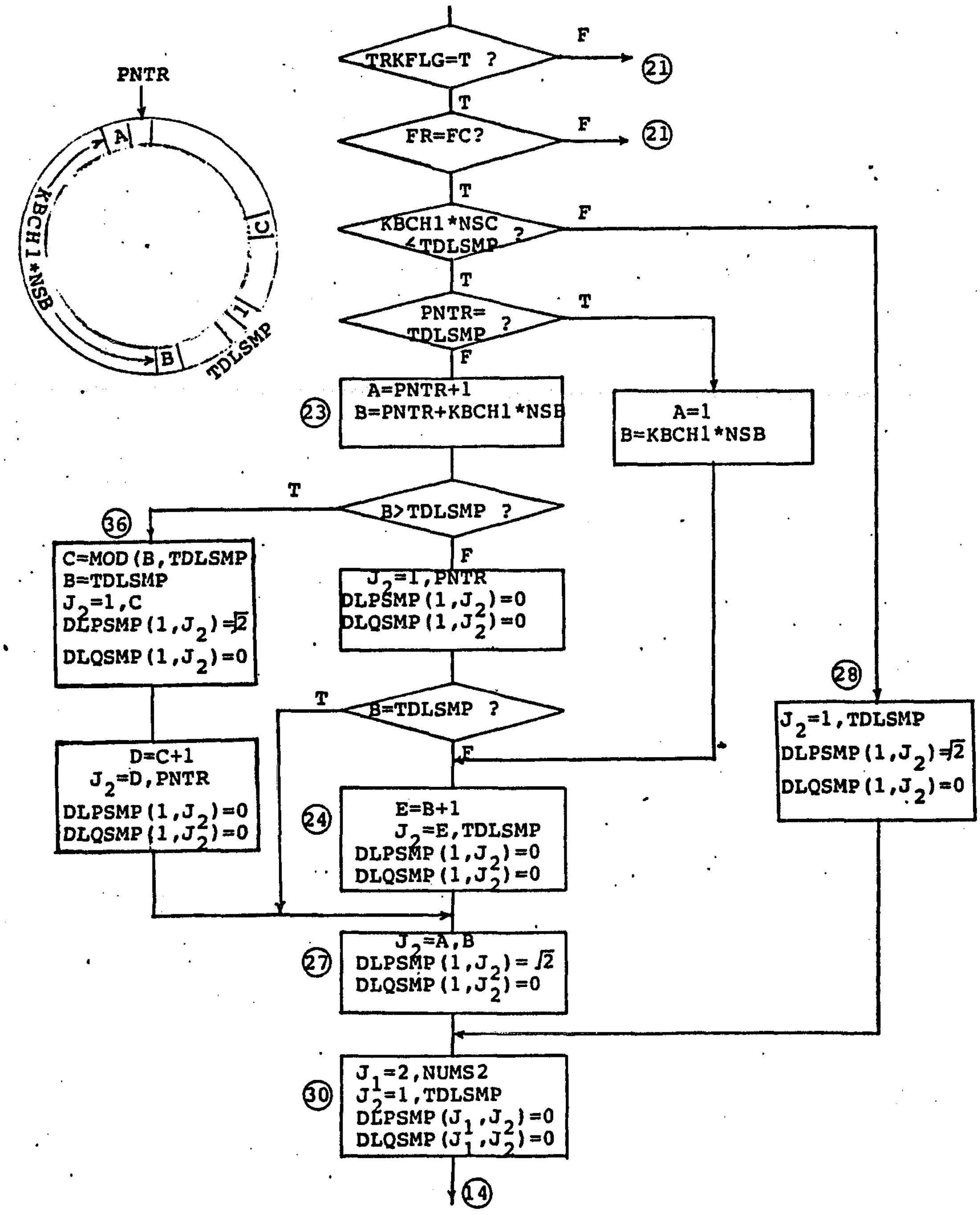

Fig. D-1 Flowchart: Reset delay line (when local carrier hops to a new frequency during acquisition and tracking)

APPENDIX E

THE EFFECTIVE P_D

Refer to fig. 3-3 when the receiver and the transmitter hop to a new frequency simultaneously, the standard deviation of the signal which we got at the output of the integrator at the end of one bit is 5 * (1+0.707) = 8.53 (2-tap model). The standard deviation of the noise in one bit time is 5.01 (E_b/η = 6db).

$$\sigma_{SN1} = \sqrt{(8.53)^2 + (5.01)^2} = 9.89$$

We use this value to calculate THA

$$THA = -2\,\sigma_{SN}^{2}\ \ln(1-P_D)$$

P_D = 0.3, THA = 69.77

P_D = 0.5, THA = 135.6

TABLE E-1

The Effective P_D

The receiver hops at the beginning of	σ_{S1}	σ_{SN1}	P(signal+noise<THA) for P_D=0.3 $=1-e^{-THA/2\sigma_{SN}^2}$ $=1-e^{-69.77/2\sigma_{SN}^2}$	P(signal+noise<THA) for P_D=0.5 $=1-e^{-THA/2\sigma_{SN}^2}$ $=1-e^{-135.6/2\sigma_{SN}^2}$
1	8.53	9.89	0.3	0.5
2	10	11.18	0.2428	0.4186
3	10	11.18	0.2428	0.4186
4	10	11.18	0.2428	0.4186
5	10	11.18	0.2428	0.4186
6	10	11.18	0.2428	0.4186
7	10	11.18	0.2428	0.4186
8	10	11.18	0.2428	0.4186
9	10	11.18	0.2428	0.4186
10	10	11.18	0.2428	0.4186
Effective P_D			$\frac{1}{10}$ P = 0.2485	$\frac{1}{10}$ P = 0.4286

APPENDIX F

VALIDITY OF THE SIMULATION WITH RESPECT TO THE NUMBER OF EXPERIMENTS

If $t_1, t_2, \ldots\ldots t_n$ are independent random variables with mean value of $\overline{T}$, normalized standard deviation σ_n, the variance $\sigma^2 = \sigma_N^{\,2}\overline{T}^2$

We define $\quad T_n = \dfrac{t_1+t_2+\ldots\ldots+t_n}{n}$

$$\overline{T_n} = E\left[\frac{t_1+t_2+\ldots\ldots+t_n}{n}\right] = \overline{T}$$

$$\sigma_{T_n}^{\;2} = \frac{1}{n}\sigma^2 = \frac{1}{n}\sigma_N^{\;2}\,\overline{T}^2, \qquad \sigma_{T_n} \doteq \frac{1}{\sqrt{n}}\sigma_N\,\overline{T}$$

$$P\{|T_n-\overline{T_n}| \geq \varepsilon\overline{T_n}\} = 1\text{-}2\text{erf}\,\frac{\varepsilon\overline{T_n}}{\sigma_{T_n}}$$

$$= 1\text{-}2\text{erf}\,\frac{\varepsilon\overline{T}}{\frac{1}{\sqrt{n}}\sigma_N\,\overline{T}}$$

$$= 1\text{-}2\text{erf}\,\frac{\varepsilon\sqrt{n}}{\sigma_N}$$

If we assign 10^{-3} to this probability,

$$\frac{\varepsilon\sqrt{n}}{\sigma_N} = 3$$

In our simulation the normalized standard deviation of the mean acquisition time σ_N is about 0.6.

If we did the experiments 50 times, n=50

$$\varepsilon = \frac{3\ \sigma_N}{\sqrt{n}} = \frac{3*0.6}{\sqrt{50}} = 0.254$$

Therefore, the accuracy is 25.4 %

If n=100,

$$\varepsilon = \frac{3*0.6}{\sqrt{100}} = 0.18$$

The accuracy is 18 %

In order to save CPU time we must have some tradeoff. Therefore, we took the number of experiments as 50.

BIBLIOGRAPHY

[1]. Sorin Davidovici, Laurance B. Milstein and Donald L. Schilling, "A Fast PN Acquisition Scheme,"

[2]. M. M. Drossman, C. Ziegler, B. Coetzer, P. Handley, D. L. Schilling, "Digitat Computer Simulation of Spread Spectrum Communication System," ICC 1981

[3]. Phillip M. Hopkin, "A Unified Analysis of Pseudonoise Synchronization by Envelope Correlation," IEEE Trans. on Communication VOL.COM-25 Aug,1977

[4]. A. Papoulis, "Probability, Random Variables and Stochastic Processes," McGraw-Hill, 1965

[5]. Stephens Rappaport and Donald L. Schilling, "A Two-level Coarse Code Acquisition Scheme for Spread Spectrum Radio," IEEE Trans. on Communication, VOL.COM-28, NO.9, Sep, 1980

[6]. R. B. Ward, "Acquisition of Pseudonoise Signal by Sequential Estimation," IEEE Trans. on Communication, VOL.COM-13, NO.4, Dec, 1965

[7]. R. B. Ward and Kap Yiu, "Acquisition of Pseudonoise Signals by Recussion-Aided Sequential Estimation," IEEE Trans. on Communication, VOL.COM-25, NO.8, Aug, 1977

[8]. J. A. Ponnusamy and M. D. Srinath, "On Initial Acquisition of FH/TH Spread Spectrum Signal," IEEE Trans. 1981

[9]. A. K. Elhakoem, G. S. Takhar and S. C. Gupta, "New Code Acquisition Techniques in Spread Spectrum Communication," IEEE Trans. on Communication, VOL.COM-28, Feb,1980

[10]. J. A. Ponnusamy and M. D. Srinath, "Acquisition of Pseudonoise Codes in FH Systems," IEEE Trans. on Aerospace and Elec. Systems, VOL.AES-17, May, 1981

[11]. C. A. Putman, S. S. Rappaport and D. L. Schilling, "Comparision of Schemes for Coarse Acquisition of Frequency Hopped Spread Spectrum Signals," IEEE Trans. on Communication. VOL.COM-31, NO.2, Feb, 1983

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