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**Essays in the economics of market making**

**Liu, Hengzhong, Ph.D.**

**City University of New York, 1993**

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# ESSAYS IN THE ECONOMICS OF MARKET MAKING

by

**Hengzhong Liu**

A dissertation submitted to the Graduate Faculty in  
Economics in partial fulfillment of requirements for the  
degree of Doctor of Philosophy, The City University of  
New York.

1993

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This manuscript has been read and accepted for the Graduate Faculty in Economics in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

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Abstract  
**Essays in the Economics of Market Making**  
by  
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This dissertation is composed of three self-contained essays. It is mainly concerned with the economics of market making in the specialist-organized stock exchange.

Under different configurations of the specialist's market structure and his risk attitude, the first essay studies the competitive and informational properties of the specialist's pricing. It demonstrates the following. The greater his monopolistic power and/or the more averse his risk attitude, the higher the sale price and the lower the purchase price the specialist would quote, and hence the less likely his special information would be revealed through his pricing. The specialist could possibly make abnormal profits in providing transaction service by using price as a wand to manipulate the stock exchange. Furthermore, the Grossman-Stiglitz(1980) impossibility rises only when traders' expectations become homogenized and traders share the same belief with the specialist.

By integrating asymmetric-information and inventory-control theoretic approaches, the second essay derives a general transaction price functional that synthesizes diverse effects of trading volumes on transaction prices. This price functional for a monopoly specialist is found to be the average of his estimate about the trader's reservation price of demanding for transaction service and his own reservation price of providing transaction service. Under assumptions that the specialist is risk neutral and his inventory policy is linear, this price functional is estimated for 10 most heavily traded stocks in TORQ database. In spite of the fact that some estimates appear inconsistent across stocks, the overall estimation gives the general transaction price functional a very encouraging empirical support.

Most existing studies on price–volume relation are based on the specialist’s supply function of transaction service, addressing the positive correlation between changes in transaction prices and signed volumes. In contrast, chapter three introduces and estimates a systematic market model for both bid and ask submarkets of stock exchange from both demand and supply sides of the specialist’s transaction service. The model is derived by integrating rational expectations approach and asymmetric information approach. It is estimated by the so–called disequilibrium methods. The empirical results reliably support the asymmetric information hypothesis, the market disequilibrium hypothesis and infinite elasticity hypothesis. They are also consistent, to some extent, with the mean–variance investment theory.

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## Table of Contents

Title.....	i.
Copyright.....	ii.
Approval.....	iii.
Abstract.....	iv.
Acknowledgements.....	vi.
Table of Contents.....	vii.
List of Tables.....	viii.
List of Figures.....	ix.
 <b>Introduction</b> .....	 1.
 <b>Chapter One Competitive and Informational Properties of the Specialist's Pricing</b>	
I    The Model.....	7.
II   The Preliminaries.....	13.
III  The Results.....	17.
IV   An Example.....	24.
 <b>Chapter Two A Systemic Model of Transaction Price Formation</b>	
I    The Systematic Model.....	36.
II   Estimation Techniques.....	46.
III  Empirical Results.....	55.
 <b>Chapter Three Estimating the Bid and Ask Submarkets of Stock Exchange</b>	
I    A Model for the Bid and Ask Submarkets of Stock Exchange.....	66.
II   Estimation Techniques.....	71.
III  Major Empirical Results.....	75.
 <b>Appendix</b> .....	 84.
 <b>References</b> .....	 91.

## List of Tables

Table II–A	Descriptive Statistics.....	59.
Table II–B	Estimates of the Linear Specialist pricing model.....	60.
Table II–C	Decomposition of the Effect of Trades on Prices and Measured Bid–Ask Spreads.....	61.
Table III–A	Descriptive Statistics.....	81.
Table III–B	2SLS Estimates of Bid and Ask Submarkets.....	82.
Table III–C	OLS Estimates for Price and Trade Equations.....	83.

## List of Figures

Figure 1	The Specialist's Speculation in Providing Transaction Services.....	30.
Figure 2	The bid and Ask Submarkets of the Specialist-organized Stock Exchange.....	80.

## Introduction

Studies in security pricing are traditionally conducted from the point of view of security valuation. Among the well-known security valuation approaches are contingent claim theory due to Arrow(1964) and Debrue(1959), capital asset pricing model due to Sharp(1964), Lintner(1965) and Mossin(1965), the arbitrary pricing theory due to Ross(1976, 1977). All those valuation theories are established under the the assumption that securities are exchanged in standard Walrasian markets. Consequently, pricing securities is attributed to the "invisible hands". Transaction price formation and stock exchange mechanisms are essentially ignored in those theories, and so is the price-quantity relation. As Ross(1989) comments: *"All of the tests" on security pricing "are very much in the spirit of the pricing models themselves, which is to say that they use price data. There is no serious theories as yet on volume data. This is in large part because we have no serious theories that support to explain the volume of trade, . . ."* *"Surely there can be nothing more embarrassing to an economist than the ability to explain the price in a market while being completely silent on the quantity," ". . ., in my mind the failure to explain the volume of trade looms as the major dark continent for explorers to this terrain."* (Ross, 1989, pp94) When we further ask the question of how security prices are formed after having answered question of how securities are valued in the market, we are not able to ignore market micro-structure and trading quantity any longer.

The market micro-structure thus becomes a fast growing field in contemporary economics. Studies in this field bring out insights into how exchanges proceed in different market frameworks. The specialist system is one of major market making

frameworks of stock exchange.

**The New York Stock Exchange (NYSE for short) is a typical example of the specialist system. NYSE is the oldest and largest continuous, two way auction market for trading of securities in the United States of America. The fundamental nature of this market is that it requires almost all orders in Exchange's listed stocks to be channeled to a central location — the trading floor. Here, buy and sell orders meet and are matched by the specialist on the basis pricing priority. It is the role of the specialist system to provide price continuity and liquidity to this market. The specialist is "an exchange member who is registered as his accountable to the Exchange and the investing public for the quality of the Exchange markets in the securities in which he is registered and responsible for fostering and acting to maintain liquid and continuous two-sided auction markets on the Exchange floor in those securities. This is accomplished by his acting as agent and principal in such securities, in accordance with the provisions of Federal and Exchange rules and policies, to help insure that such markets are fair, orderly and operationally efficient in the public interest, and competitive with non-Exchange markets in those securities."**

**This dissertation is devoted to studying one respect of the economics of market making — how transaction prices are formed or set by the specialist in a stock market like NYSE from the point of view of price-volume relation.**

**In literature there exist three major theoretical approaches to the specialist's pricing. The immediacy service approach relates transaction price to order quantity by assuming liquidity demanders are willing to pay a proper service premium to the liquidity suppliers. This approach is due to Demsetz(1964) and Tinic(1973) among others. The inventory control approach associates transaction price with order quantity by that assuming the specialist should claim a risk premium and/or add an adjustment fee to the price because he keeps inventory for maintaining liquidity in the presence of stock value uncertainty and order flow uncertainty. This approach is due to**

Stoll(1978), Ho and Stoll(1981, 1983), Amihud and Mendelson(1980), Oldfield and O'Hara(1986) among others. The asymmetric information approach connects transaction price with order quantity by assuming that larger orders should be committed at worse prices than smaller orders because the former are often initiated by superior information. This approach is due to Bagehot(1971), Copeland and Galai(1983), Kyle(1985), Glosten and Milgrom(1985), Easley and O'Hara(1987,1992), Glosten(1989), Admati and Pfleiderer(1989), Deppi(1991) among others. On the other hand, there have emerged a tremendous number of empirical studies on price-volume relation. The primitive studies are surveyed by Karpoff(1987). Their general finding is the positive correlation between changes in transaction price and order quantity. The contemporary studies has been focusing on testing the three analytical approaches and disentangling diverse effects of volume on price. Good examples are Glosten and Harris(1988), Hasbrouck(1988,1991), Stoll(1989), Madhavan and Smidt(1991) among others.

**This dissertation is not to pursue a whole picture** of transaction price formation. Instead, it is intended to deal with three respects of the specialist's pricing from the view of point of price-volume relation. Each respect is examined in a self-contained essay.

**The data** used in the empirical studies is from the TORQ database. This database is prepared by the New York Stock Exchange and distributed by the Institute of Study in Stock Market (ISSM in short). It contains transactions, quotes, order processing data and audit trail data. It covers 144 NYSE stocks for the three months November, 1990 through January, 1991.

**The basis methodology this dissertation employs** is integrating the rational expectations approaches conventionally applied to Walrasian stock market and the asymmetric information approach conventionally applied to the specialist's pricing. Obviously, merging these two approaches together is desirable for investigating how

transaction price and order quantity relate to each other, especially in respect of information revelation. To be specific, first of all, it is desirable for describing a complete market information communication process. This process may be dogmatically thought to consist of two stages. At the first stage, the information privately available for individuals is disclosed through individuals' trading demands to the market or the market organizer who is supposed to be responsible to establish the market prices. At the second stage, prices publicly disseminate the information aggregated by the aggregate demand back to individuals. Secondly, it is desirable for explaining the implementation of market equilibrium. For traders, an ex ante optimal trading strategy selected with ignorance of its informational role may turn out not ex post optimal. For the specialist, an ex ante optimal price quote posted without respect to its information effects on demand may turn out not ex post optimal. Generally speaking, a pair of strategies of buyer and seller could possibly be realized as a non-regret equilibrium only when the seller has considered the information contained in the buyer's demand and the buyer has considered the information contained in the seller's price. Finally, it is desirable for elaborating the informational linkage between transaction prices and trading volumes. For example, the relationship between price revealing and quantity revealing, and the relationship between price process and order process can only be analyzed properly when both price and volume are assumed simultaneously informative and mutually feedback.

**Chapter one** presents the first essay. What are the competitive and informative properties of the specialist's pricing under different configurations of the specialist's market structure and the specialist's risk attitude? Through examining a simple stock market where the specialist presumably possesses superior information and where both trading volume and transaction price are hypothetically information, Chapter one demonstrates the following. (i) The greater his monopolistic power and/or the more averse his risk attitude, the higher sale price and the lower purchase price the specialist

would quote, and hence the less likely that his superior information would be revealed through his pricing. (ii) The Stiglitz–Grossman (1980) impossibility arises only when traders' expectations become homogenized and traders share the same belief with the specialist. (iii) The monopolistic specialist can make profits by using price as a wand to manipulating stock exchange. (iv) Unlike the risk averse specialist's pricing, the competitive and informational properties of the risk neutral specialist's pricing depend upon the number of traders in the market.

**Chapter two** presents the second essay. This essay builds and tests a synthetic model of transaction price formation. By integrating asymmetric-information and inventory-control theoretic approaches, it derives a general transaction price functional that synthesizes diverse effects of trades on prices for the specialist-organized stock market like NYSE. The optimal price for a monopoly specialist is found to be the average of his estimate about the trader's reservation price of demanding transaction service and his own reservation price of providing transaction service. The optimal bid–ask spread is decomposed into five parts: order processing costs, adverse selection effects, inventory control impacts, monopoly profits and risk adjustments for demand order uncertainty. Under assumptions of risk neutrality and linear inventory control rule, a linear transaction price functional is estimated for 10 most heavily traded stocks in TORQ database. It is found that in setting prices, the specialist does consider order processing costs, asymmetric information effects, inventory control impacts and the price elasticity of demand simultaneously.

**Chapter three** presents the third essay. According to Demsetz (1968) and others, the specialist-organized stock exchange is composed of two submarkets: the ask submarket where public purchases versus the specialist's sales and the bid submarket where public sales versus the specialist's purchases. However, most existing studies on price–volume relation are based on the specialist's supply function of transaction service, addressing the positive correlation between changes in transaction prices and

signed volumes. These studies are subject to debates over whether stock markets persistently operate at equilibriums and whether stock price elasticities are infinite. This chapter introduces and estimates a systematic market model for both bid and ask submarkets of stock exchange from both demand and supply sides of the specialist's transaction service. The model is derived by integrating the rational expectations approach conventionally applied to the Walrasian Market and the asymmetric information approach conventionally applied to the specialist's pricing. Using transaction data, the model is tested by the so-called disequilibrium method. The empirical results support the asymmetric information hypothesis, the market disequilibrium hypothesis and infinite elasticity hypothesis. They are also consistent, to some extent, with the mean-variance investment theory.

## Chapter One

# Competitive and Informational Properties of the Specialist Pricing

What are the competition and information properties of the specialist's pricing under different configurations of the specialist's market structure and his risk attitude? This research is intended to examine this problem by studying a stock market where traders presumably acquire costly differentiated information and use price to revise their respective prior beliefs, and where the specialist presumably possesses superior information and is able to learn exactly what private information his customers have obtained. The answer is not straightforward partly because of informativeness of both price and quantity, and partly on account of the interactive effects of the market structure and risk attitude of the specialist.

This research relates to two popular approaches in the literature of financial economics. One is the rational expectations approach that is conventionally used in studying the information property of the price in Walrasian financial markets. The other is the asymmetric information approach (or the adverse selection approach) that is currently used in investigating the information property of the trading volume in the specialist-organized markets. To present some linkage between this research and the existing literature, we brief three works below. Through elaborating a stock market that is organized by a Walrasian auctioneer Grossman and Stiglitz(1980) reach the following conclusions. (i)The competitive equilibrium price equals the expected asset liquidation value conditional on all the information traders could preposterously have. (ii)It is impossible for stock market to be informationally efficient in the sense that "price reflects all the information available in the market" because a fully revealing

price functional makes no incentives for traders to collect costly information. Feeling that "the Walrasian auctioneer assumption is in a fundamental sense inconsistent with the spirit of rational expectation models", Gould and Verrecchia(1985) consider a market that is operated by a price-setting agent. They demonstrate the existence of rational expectations equilibrium that maximizes respective utilities of both traders and the specialist in their market, and show how the information contents of the price quotation vary with different configurations of information acquisition and risk attitudes. The informativeness of trading demands, however, is not considered in their study. Through investigating an exchange market that is manipulated by an originally-uninformed monopolistic specialist, Madrigal and Scheinkman(1990) allow the specialist infer traders' information from trading activity, finding that a monopolistic specialist never sets price that is equal to his expectation, and that fully reveals his own information. Nevertheless, these works have been focusing on how the price quotation is determined rather than how the specialist's pricing differs with different characteristics of the specialist. The latter, as we mentioned in the very beginning, is the subject of this research.

Considering a stock market which has a large number of competitive traders, excluding noise trading, allowing trading volume fully revealing, and assuming the specialist possesses superior information distinguish our modeling from Madrigal and Scheinkman(1990). we differ from Grossman and Stiglitz(1980) in that it is a specialist instead of a Walrasian auctioneer who operates the market. Also, unlike Gould and Verrecchia(1985), we let the specialist infer traders' information from trading volume.

Our methodology employed in this research is characterized by the following four respects. First, both price and quantity are assumed informative. This allows the specialist to infer his customers' information from transaction volume, and allow traders to learn the information of the specialist from price. Accordingly and secondly, as Madrigal and Scheinkman(1990) realize, an ex ante optimal price strategy for the

specialist may turn out not ex post optimal for him while an ex ante optimal demand policy for a trader may turn out not ex post optimal for that trader. To ensure the existence of the non-regret equilibriums, we assume there is a bargaining and learning process prior to the settlement of trades. Thirdly and especially, by viewing the specialist's expectation pertaining to the asset liquidation value to be his marginal cost (or the reservation price) of providing transaction service, we are able to treat the specialist as an ordinary seller/buyer. This makes it much easier to analyze the competitive and informational properties of the specialist's pricing. Fourthly, for simplicity, we assume that all traders use the mean-variance terminology to make their trading policies.

The major findings of this research are the following. (i) The greater his monopolistic power and/or the more averse his risk attitude, the higher price at his sale and lower price at his purchase the specialist would quote, and hence the less amount of his superior information would be revealed through his pricing. For example, similar to what Madrigal and Scheinkman (1990) obtained, both because of his using price as a wand to manipulate the market in favor of his speculation and because of his adding a risk premium markup to the price, a monopolistic specialist never commits a trading on his own account at the competitive level, and never quotes a price that spreads all his superior information to the market even though from trading volume, he is able to learn all the information traders possess. (ii) Quantity fully revealing (requires perfect competitiveness of traders' market as a necessary condition) alone is not sufficient to guarantee price fully revealing, the other necessary conditions are perfect competitiveness of the specialist's market and the neutrality of the specialist's risk attitude. Virtually, A Walrasian auctioneer is implicitly "risk neutral" and has no "market power". That is why we see from Grossman and Stiglitz (1980) and others that, price is fully revealing as long as transaction demand is perfect informative. The Grossman-Stiglitz (1980) impossibility arises in the specialist-organized stock market

not only because of the homogenization of traders' expectations, as noticed by Grossman and Stiglitz(1980), but also on account of traders sharing the same belief with the specialist. What is so striking is that, the risk averse specialist is less confident to use his monopolistic advantages than the risk neutral. While the competitive and informational properties of the risk neutral specialist's pricing do not depend on the number of traders participating in the market, the competitive and informational properties of the risk averse specialist's pricing do. By adding a premium markup to the price, a risk averse specialist earns less profits than a risk neutral specialist .

The rest of our research is arranged as follows. We build up a very simple specialist-organized stock exchange model in the first section and derive some preliminaries in the second section, then obtain main results in the third section and establish rational expectations equilibriums in a mean-variance framework in the fourth section. Finally, we conclude the note with a brief discussion on the modeling and the results.

## I. The Model

Considering a specialist-organized stock market where there are a large number of traders who speculate on a single risky financial asset. The market opens only once to carry out one shot trading in a given period. Once the specialist opens the market and posts tentative quotes, traders with differential information arrive the specialist's post all together at the same time<sup>1</sup> and submit either limit orders and/or market orders based on their own expectations and the price posted by the specialist. Then, the

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<sup>1</sup> A specialist organized market essentially differs from a Walrasian market in two respects. One is that the specialist maintains the market liquidity by trading on his own account while a Walrasian does not. The other is that orders arrive and are proceeded sequentially in the specialist-organized market rather than simultaneously as in the Walrasain market.

specialist clears up the residual market demand by trading on his own account after matching out public buy orders against public sale orders. Let  $\tilde{V}$  denote the liquidation value of the risky asset in the next period. Let  $p$  the current trading price that the specialist sets. Let  $x_i$  the  $i$ th speculator's trading policy. Let  $x_s$  the amount of the asset the specialist would like to trade on his own account. Suppose that each trader is endowed with a prior information set  $\Omega_i$  and the specialist is endowed with  $\Omega_s$ . To begin with, we make the following assumptions.

**Assumption I.** There are no operational costs and no taxes related to transactions. There are no regulations and no short sale restrictions for both traders and specialists.

**Assumption II.** Every information-based trader are relatively small enough to be regarded as a price-taker.

**Assumption III.** The specialist and traders have monotonically increasing and strictly concave utility functions.

**Assumption IV.**  $\tilde{V} \sim N(m, \sigma_v^2)$ . There is no net return on risk free asset.

**Assumption V.** Before trades take place, trader  $i$  observes a signal for  $\tilde{V}$ , denoted by  $y_i = \tilde{V} + \epsilon_i$  with  $\epsilon_i \sim N(0, \sigma_\epsilon^2)$ . The specialist acquired a revelation  $v_s$  pertaining to  $\tilde{V}$  with no error. That is,  $E[\tilde{V} | \Omega_i] = E[\tilde{V} | Y_i] = m\sigma_\epsilon^2 / (\sigma_v^2 + \sigma_\epsilon^2) + y_i\sigma_v^2 / (\sigma_v^2 + \sigma_\epsilon^2) = v_i$ ,  $\text{Var}[\tilde{V} | \Omega_i] = 1/\sigma_v^2 + 1/\sigma_\epsilon^2$ ,  $E[\tilde{V} | \Omega_s] = E[\tilde{V} | V_s] = v_s$  and  $\text{Var}[\tilde{V} | \Omega_s] = \sigma^2$ . Especially,  $U_{i=1}^n \Omega_i \subset \Omega_s$ .

A single trader has no significant influence on price as compared with the whole market where there are an infinite number of traders. This argument may defend Assumption II. By assumption II, traders neglect their own effects of demand policies on prices. This fact is needed to guarantee quantity fully revealing.

$U_{i=1}^n \Omega_i \subset \Omega_s$  does not mean at all that the specialist has a perfect prior knowledge

about  $(v_1, v_2, \dots, v_n)$  before trades take place. What it implies is that the combination of traders' information sets is not finer than the specialist's information set. Thus, trading volume is useless for the specialist to update his prior belief about  $\tilde{V}$ , but is necessary for him to infer what information traders actually have. Assumption  $U_{i=1}^n \Omega_i \subset \Omega_s$  is made simply for focusing on the theme of this research. Actually, more realistic and complex specification of information endowments will not reverse the results.

A specialist differs from a Walrasian auctioneer to the extent that the former not only organizes the market operation but also manipulates the market to earn profits through trading on his own account. Therefore, the market clearing condition becomes the following.

$$\sum_{i=1}^n x_i + x_s = 0. \quad (1)$$

When  $x_s$  is negative, the specialist takes a short position and receives a positive price for a unit of share. Otherwise he takes a long position and pays out a positive price for a unit of share. Below, we define a rational expectations equilibrium that we will refer to throughout.

**Definition** A rational expectations equilibrium in a specialist organized market under Assumptions I–V is a  $n+2$  tuple  $(x_1, x_2, \dots, x_n, x_s, p)$  such that (i) each trader's expected utility is maximized:  $E_i[U_i(\cdot, x_i) | \Omega_i, p] \geq E_i[U_i(\cdot, x'_i) | \Omega_i, p]$  for all  $i$ ; (ii) the specialist's expected utility is maximized:  $E_s[U_s(\cdot, p) | \Omega_s, \sum_{i=1}^n x_i] \geq E_s[U_s(\cdot, p') | \Omega_s, \sum_{i=1}^n x_i]$ ; (iii) the market clears:  $\sum_{i=1}^n x_i + x_s = 0$ .

The equilibrium is assumed to be realized through a bargaining and learning process. At the very beginning, the specialist posts a tentative price which is subject to

change afterwards, traders submit their respective tentative orders according to their respective priori. After the specialist gets to know  $\sum_{i=1}^n v_i/n$  and traders revise their respective priori conditional on the price, if no one regrets his trading decision announced previously, then the transaction is carried out and a rational expectations equilibrium is said to have been implemented. Otherwise, the bargaining and revision continue until an agreement that no one regrets is reached.

By the way, since our major task is to make a descriptive comparison between various types of specialists in terms of competitive behavior and informational revelation, whether a specialist market is monopolistic or competitive<sup>2</sup> and whether a specialist is risk averse or risk neutral are simply beyond our scope.

## II. The Preliminaries

In this section, we derive five lemmas that will be used to establish main results in the next section. The first is regarding individuals' preposterous expectations about  $\tilde{V}$  conditional on  $y_i$  and  $p$ . The second states the specialist's preposterous expectation about  $\tilde{V}$  conditional on his superior information and transaction volume. The third describes the market demand functional of traders for the specialist's transaction service. The fourth is concerned with the fully revealing property of trading volume. And the fifth is about the determination of the mean–variance speculation policy.

**Lemma 1.** In the market under Assumptions I–V, (i)  $E_i[\tilde{V}|\Omega_i, p]$

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<sup>2</sup> According to Glosten(1989), Madrigal and Scheinkman(1990), a specialist is acting as a monopolist and/or a monoposony in the financial market because of his institutional position and his solely access to some information sources, i.g. the book and the trading history, etc.. Anderson(1987) attributes the specialist's monopoly position to the scales of economy. The competitive feature of the specialist is originally due to Demsetz(1968). Glosten and Milgrom(1985) and Kyle(1985) postulate a Bertrand type price competition in the specialist's market.

$= (1-\xi)E_i[\tilde{V}|\Omega_i] + \xi p$ , where  $\xi = (\sigma_v^2 \sigma_\delta^2 + \sigma_\epsilon^2 \sigma_\delta^2) / (\sigma_v^2 \sigma_\delta^2 + \sigma_\epsilon^2 \sigma_\delta^2 + \sigma_v^2 \sigma_\epsilon^2) \in [0,1]$ . Hence, (ii)  $p \geq \sum_{i=1}^n E_i[\tilde{V}|\Omega_i, p] / n \geq \bar{v}$ , where  $\bar{v} \equiv \sum_{i=1}^n E_i[\tilde{V}|\Omega_i] / n$ .

The proof of Lemma 1 is provided in Appendix I. It is straightly based on Bayesain expectation. Note that in establishing Lemma I, we have assumed that the price  $p$  is a signal about  $\tilde{V}$  such that  $P = \tilde{V} + \delta$  with  $\delta \sim \text{IID}(0, \sigma_\delta^2)$ .  $E_i[\tilde{V}|\Omega_i, p]$ , individual  $i$ 's posterior expectation conditional on his private information set  $\Omega_i$  and  $p$ , is a weighted average of  $E_i[\tilde{V}|\Omega_i]$  and  $p$ . Both  $E_i[\tilde{V}|\Omega_i]$  and  $p$  are revelations of  $\tilde{V}$ . Further, the price greater (less) than  $\bar{v}$  is also greater (less) than  $\sum_{i=1}^n E_i[\tilde{V}|\Omega_i, p] / n$ , i.e.,  $p \geq \sum_{i=1}^n E_i[\tilde{V}|\Omega_i, p] / n \geq \bar{v}$  as  $p \geq \bar{v}$ , but  $|v_s - \sum_{i=1}^n E_i[\tilde{V}|\Omega_i, p] / n| \leq |v_s - \sum_{i=1}^n E_i[\tilde{V}|\Omega_i] / n|$ . For the specialist, this is a bad news in that the informativeness of his price reduces his opportunity of making profits. But it is a good news as well on the other hand because his price is more likely to be partially revealing. To this extent, he could still make some profits. Expressing  $E_i[\tilde{V}|\Omega_i, p]$  as the linear combination of  $E_i[\tilde{V}|\Omega_i]$  and  $p$  instead of as the direct linear combination of  $m$ ,  $y_i$  and  $v_s$  enables us to study the specialist's pricing in a textbook market framework.

**Lemma 2.** In the market under Assumptions I–V,  $E_s[\tilde{V}|\Omega_s, \sum_{i=1}^n x_i(\cdot, p)] = v_s$ .

Lemma 2 is verified in Appendix I. By Assumption VI, the specialist presumably observes a realization of  $\tilde{V}$  with no error, i.e.,  $V_s = \tilde{V}$ . So no any other additional information could possibly increase the precision of the specialist's prior belief about  $\tilde{V}$ . While we do not have any intention to find out why the specialist's information is so perfectly precise, the whole point we have raised here is that the specialist has some special information sources which is not available to traders so that  $v_s$  is not necessarily equal to  $\bar{v}$ , or  $v_s$  is superior to  $\bar{v}$  in some positive probability.

**Lemma 3.** In the market under Assumptions I–V, trader  $i$ 's mean–variance trading policy is expressed by

$$x_i(\cdot, p) = (1-\xi) \frac{E_i[\tilde{V}|\Omega_i] - p}{\sigma^2 U''/U'} \quad (2)$$

which is a continuous, monotonic decreasing function of  $p$ , where  $\sigma^2 = \text{Var}[\tilde{V}|\Omega_i, p]$ .

We verify Lemma 3 in Appendix 1. Note that the price in  $x(\cdot, p)$  is both as a budget parameter and as a piece of information. In this connection, trader  $i$ 's demand function for transaction service is "Bayesian". Note also that the variances do not involve any specific realizations of relating random variables.  $U''/U'$  indicates the risk attitude of trader  $i$ .  $\xi$  stands for the weight trader  $i$  puts on the specialist's price in forming his posterior belief. Other things being equal,  $x(\cdot, p)$ , the ex post optimal demand policy of trader  $i$  is negatively related to  $\xi$ .

**Lemma 4.** In the market under Assumptions I–V, the aggregate demand of all traders in the market is fully revealing.

Lemma 4 is proven in Appendix I. Aware of his insignificance, an individual prefers to use one piece of information to commit only one–shot trade, this piece of information hence could be possibly fully revealed via one–shot trade at a point in time. Moreover, since all trades are made in the mean–variance framework, there is no noise trades that would possibly give rise to specialist's Bayesian inference error. Therefore, the aggregate individual demand for transaction service fully discloses traders' information to the specialist. This is what Lemma 4 establishes. Postulating quantity fully disclosing distinguishes our setup from Madrigal and Scheinkman(1990) and others. It also enables us to concentrate on the competitive and informational properties of specialist's strategic pricing.

Another important advantage of Eq(2) is that it expresses the Bayesian demand for transaction service  $x(\cdot, p)$  of individual  $i$  as a function of  $E_i[\tilde{V} | \Omega_i] - p$  instead of  $E_i[\tilde{V} | \Omega_i, p] - p$ . This implies that providing is known, a trader's pre-posterior optimal trading policy can be directly explained by  $E_i[\tilde{V} | \Omega_i] - p$ . This is summarized in Lemma 5.

**Lemma 5.** In the market under Assumptions I–V, given §, the mean–variance trading policy  $x_i$  depends only upon  $E_i[\tilde{V} | \Omega_i] - p$ , irrelevant of  $p - m$  and  $E_i[\tilde{V} | \Omega_i] - m$ . Accordingly, the market residual demand  $\sum_{i=1}^n x_i$  depends only upon  $\sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n - p$  and the specialist's speculation policy  $x_s$  depends only upon  $v_s - p$ .

*Proof.* From Eq(2),  $x(\cdot, p)$  is a function of  $E_i[\tilde{V} | \Omega_i] - p$ . Let us take trader  $i$ 's speculation policy as an example. First, suppose  $E_i[\tilde{V} | \Omega_i] = m$ . If  $p \geq \Sigma E_i[\tilde{V} | \Omega_i] = m$ , he goes short; if  $p \leq E_i[\tilde{V} | \Omega_i] = m$ , he takes a long position. Suppose  $E_i[\tilde{V} | \Omega_i] > m$ . if  $p \geq E_i[\tilde{V} | \Omega_i] > m$ , he sells high; if  $p \leq E_i[\tilde{V} | \Omega_i]$  and either  $p \geq m$  or  $p \leq m$ , he buys low. Suppose  $E_i[\tilde{V} | \Omega_i] < m$ . If  $p \geq E_i[\tilde{V} | \Omega_i]$  and either  $p \geq m$  or  $p \leq m$ , his rational policy is to commit a sale. If  $p \leq E_i[\tilde{V} | \Omega_i] < m$ , his rational policy is to commit a purchase. So, no matter what configuration of  $E_i[\tilde{V} | \Omega_i]$ ,  $m$  and  $p$  is, as long as  $p > E_i[\tilde{V} | \Omega_i]$ , trader  $i$  absolutely prefers selling short because the price is higher than his valuation. Conversely, as long as  $p < E_i[\tilde{V} | \Omega_i]$ , going long is evidently profitable for trader  $i$  since the price is lower than his valuation. By the same token, the residual market demand is solely determined by  $\bar{v} - p$ , independent of both  $p - m$  and  $\bar{v} - m$ . Also, the specialist speculates on the asset only according to  $v_s - p$ , not to  $p - m$  and  $\bar{v} - m$  at all. Q.E.D.

With  $\bar{v}$  fixed, the residual demand, i.e.  $\sum_{i=1}^n x_i$  or  $x_s$ , is a single-valued real function of  $p$ . Whether  $\sum_{i=1}^n x_i$  is positive or negative depends upon the difference  $\bar{v} - p$ . By marking up or down the price appropriately, a monopolistic specialist can possibly

make the residual demand positive or negative. This is the crux that a monopolistic specialist could possibly make profits by choosing optimal price strategy. This is the essential characteristic that a monopolistic market differs from a competitive one in competitive behavior and information revelation. So this is the starting point for our analysis in the this research.

### III. The Results

Three propositions are derived in this section. The first proposition specifies the upper limit and the lower limit of the rational expectations price when both price and quantity are assumed informative. The second proposition states the competitive properties of the specialist's pricing under different configurations of the specialist's market structures and his risk attitude. The third proposition summaries the information properties of the specialist's pricing under different configurations of the specialist's market structure and risk attitude.

**Proposition 1.** Under assumptions I–V, (i). A rational expectations equilibrium price is such that  $p \in [\bar{v}, v_s]$  when  $\bar{v} \leq v_s$  and  $p \in [\bar{v}_s, \bar{v}]$  when  $\bar{v} \geq v_s$ . (ii) At the equilibrium,  $x_i \geq 0$  as  $v_i \geq v_s$  and  $\sum_{i=1}^n x_i \geq 0$  as  $\bar{v} \geq v_s$ .

**Proof.** Considering the case  $\bar{v} \leq v_s$  first. If  $p \geq v_s \geq \bar{v}$ , i.e.  $p \geq E_s[\tilde{V} | \Omega_s, \Sigma_{i=1}^n x_i(\cdot, p)] = v_s$  and  $p \geq \sum_{i=1}^n E_i[\tilde{V} | \Omega_i, p] / n \geq \bar{v}$ , the public would conduct sales in general and so would the specialist by lemma 5. This leads to no actual transaction taking place on the specialist own account. Thus,  $\sup\{p: x_s \neq 0, \bar{v} \leq v_s\} \leq v_s$ . If  $p \leq \bar{v} \leq v_s$ , i.e.  $p \leq E_s[\tilde{V} | \Omega_s, \Sigma_{i=1}^n x_i(\cdot, p)] = v_s$  and  $p \leq \sum_{i=1}^n E_i[\tilde{V} | \Omega_i, p] / n \leq \bar{v}$ , from Lemma 1, the public prefers going long to taking short, so does the specialist. It is accordingly impossible for

the trades on the specialist own account to occur. Hence,  $\inf\{p:x_s \neq 0, \bar{v} \leq v_s\} \geq \bar{v}$ . Similarly,  $\sup\{p:x_s \neq 0, \bar{v} \geq v_s\} \leq \bar{v}$  and  $\inf\{p:x_s \neq 0, \bar{v} \geq v_s\} \geq v_s$ . Additionally, in the case of  $x_s=0$ , it implies  $p=v_s=\bar{v}$  in the market where traders are based on mean-variance criteria. Furthermore, if  $\bar{v} > p > v_s$ , i.e.,  $\bar{v} > \sum_{i=1}^n E_i[\tilde{V} | \Omega_i, p] / n > p > v_s$ , according to lemma 5, the public conducts purchase and the specialist prefers sale. Else, if  $\bar{v} < p < v_s$ , i.e.,  $\bar{v} < \sum_{i=1}^n E_i[\tilde{V} | \Omega_i, p] / n < p < v_s$ , the public commits sale and the specialist prefers purchase. Therefore,  $p \in [\bar{v}, v_s]$  as  $\bar{v} \leq v_s$  and  $p \in [v_s, \bar{v}]$  as  $\bar{v} \geq v_s$  if  $p$  is one of rational expectations equilibrium prices in the specialist-organized market. (ii) obviously follows from Lemma 5 and (i) of this proposition. Q.E.D.

Three comments on Prop.1 are in order. As long as  $\bar{v} \neq v_s$ , a monopolistic specialist can always make his profits by selecting price between  $\bar{v}$  and  $v_s$  properly.  $|\bar{v}-v_s|$ , not  $|\bar{v}-p|$ ,  $|\bar{v}-m|$ ,  $|v_s-p|$ ,  $|v_s-m|$  and  $|p-m|$ , measures the opportunity of specialist's making profits<sup>3</sup>. Secondly, the price is a wand for the specialist to commit his own speculation profitably in his providing intermediary services. When  $v_s > \bar{v}$ , by naming price obeying  $v_s > p > \bar{v}$ , he makes the aggregate net demand of traders negative and hence grabs profits via a purchase. Conversely, when  $v_s < \bar{v}$ , posting price obeying  $v_s < p < \bar{v}$  brings out a positive market residual demand, the specialist then takes a short position for profits. Thirdly and more interestingly, whether a specialist will speculate by going long or taking short and whether an individual will demand sale transaction service or purchase transaction service could be determined by the difference of prior beliefs between the specialist and the trader, besides by the difference of their preposterous beliefs. That is to say, regardless of posterior beliefs, a monopolistic specialist can always make profits as long as he learns that his prior belief is different from and truly superior to the prior belief of traders as a whole.

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<sup>3</sup> If price is assumed uninformative,  $|\sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n - v_s|$  measures the opportunity of making profits for the specialist.

Let  $p^c$  be the price named by a risk neutral competitive specialist,  $p^m$  the price by a risk-neutral monopolist,  $p^r$  the price by a risk-averse competitive specialist and  $p^o$  the price by a risk-averse monopolistic specialist. Also let  $\pi_s$  stand for the profit the specialist earns from his own trading, we have Proposition 2.

**Proposition 2.** Under assumptions I–V, suppose  $v_s \neq \bar{v}$  and  $x(\cdot, p) \neq 0$  in some positive probability, if the specialist commits a purchase, then  $0 < p^o \leq p^m \leq p^c = v_s$  and  $0 < p^r \leq p^c = v_s$  in some positive probability; If the specialist commits a sale, then  $p^o \geq p^m \geq p^c = v_s > 0$  and  $p^r \geq p^c = v_s > 0$  in some positive probability.

*Proof.* From Lemma 3,  $\pi_s = x_s(\cdot, p)(p - \bar{V})$  and  $E_s[\pi_s | \Omega_s] = x_s(\cdot, p)(p - v_s)$  are monotonic, continuous and concave function of  $p$ . If the specialist is risk neutral,  $E_s[U_s(\cdot, \pi_s) | \Omega_s] = U_s(\cdot, E_s[\pi_s | \Omega_s]) = U_s(\cdot) + U'_s E_s[\pi_s | \Omega_s]$ . That is,  $\text{Max} E_s[U_s(\cdot, \pi_s) | \Omega_s]$  is equivalent to  $\text{Max} E_s[\pi_s | \Omega_s]$ . Write  $E[\pi_s | \Omega_s]$  in its general form, i.e.  $E[\pi_s | \Omega_s] = x_s(\cdot, p)(p - v_s)$ . Various competitions among specialists and others necessarily drive  $E[\pi_s | \Omega_s]$  to zero. With  $x_s(\cdot, p) \neq 0$ , it has  $p^c = v_s$ . Providing the specialist is a monopolist, the first-order condition for his expected profit maximization is  $[\partial x_s(\cdot, p) / \partial p](p - v_s) + x_s(\cdot, p) = 0$ . This results in the monopolistic price to be  $p^m = v_s - [\partial p / \partial x_s(\cdot, p)] x_s(\cdot, p)$ , where  $\partial p / \partial x_s(\cdot, p) < 0$  and the sign of  $x_s(\cdot, p)$  is contingent upon the type of transaction the specialist commits on his own account. Be the transaction a specialist's sale. it has  $x_s \leq 0$  and accordingly  $p^m \geq v_s = p^c$ . Be the transaction a specialist's purchase, it has  $x_s \geq 0$ , and accordingly  $p^m \leq v_s = p^c$ .

Suppose the specialist is risk averse. And suppose he purchases  $x_s$  shares at price  $p^r$ . For making the specialist indifferent between holding the cash  $x_s p^r$  and  $x_s$  shares, it must hold that  $E_s[U_s(\cdot, x_s p^r) | \Omega_s] = E_s[U_s(\cdot, x_s \bar{V}) | \Omega_s]$ , But  $E_s[U_s(\cdot, x_s \bar{V}) | \Omega_s] \leq U_s(\cdot, x_s E_s[\bar{V} | \Omega_s])$ . This implies that  $x_s p^r \leq x_s E_s[\bar{V} | \Omega_s]$ . Namely,  $p^r \leq v_s = p^c$  for  $U_s$  is monotonic increasing and  $E_s[U_s(\cdot, x_s p^r) | \Omega_s] = U_s(\cdot, x_s p^r)$ . In contrast, if the specialist

sells  $-x_s$  shares at price  $p^r$ , it must hold that  $E_s[U_s(\cdot, -x_s p^r) | \Omega_s] = E_s[U_s(\cdot, -x_s \tilde{V}) | \Omega_s]$ . This leads to that  $p^r \geq v_s = p^c$ .

Now assume the specialist is a risk-averse monopolist. Let  $|U''/U'| \text{Var}[\pi_s | \Omega_s]/2$  be the Pratt-Arrow risk premium with  $\text{Var}[\pi_s | \Omega_s] = x_s^2(\cdot, p)\sigma^2$ , then,

$$E_s[U_s(\cdot, \pi_s) | \Omega_s] = U_s(\cdot, +E_s[\pi_s | \Omega_s] - |U''/U'| \text{Var}[\pi_s | \Omega_s]/2).$$

We thus have the first-order condition for maximization of  $E_s[U_s(\cdot, \pi_s) | \Omega_s]$ .

$$\frac{\partial E_s[U_s(\cdot, \pi_s) | \Omega_s]}{\partial p} = U' \left[ \frac{\partial E_s[\pi_s | \Omega_s]}{\partial p} - x_s(\cdot, p) |U''/U'| \sigma^2 \frac{\partial x_s(\cdot, p)}{\partial p} \right] = 0.$$

That is

$$\frac{\partial E_s[\pi_s | \Omega_s]}{\partial p} = x_s(\cdot, p) |U''/U'| \sigma^2 \frac{\partial x_s(\cdot, p)}{\partial p}.$$

If  $U''=0$ , i.e. the specialist is risk neutral, we have  $E_s[\pi_s | \Omega_s]/\partial p=0$ . Hence,  $p^0=p^m$ . If  $U'' \neq 0$ ,  $|U''/U'| > 0$ ,  $\sigma^2 > 0$  and  $\partial x_s(\cdot, p)/\partial p < 0$ , that is  $|U''/U'| \sigma^2 \partial x_s(\cdot, p)/\partial p < 0$ . When the specialist commits a purchase, i.e.,  $x_s(\cdot, p) > 0$ , it has  $E_s[\pi_s | \Omega_s]/\partial p < 0$  and  $p^0 < p^m$ . Conversely, when the specialist commits a sale, i.e.,  $x_s(\cdot, p) < 0$ , it has  $E_s[\pi_s | \Omega_s]/\partial p > 0$  and  $p^0 > p^m$ . Q.E.D.

Proposition 2 says that, a specialist posts higher price at his sale and lower price at his purchase in a monopolized market than in a perfect competitive market, a risk averse specialist quotes higher price at his sale and lower price at his purchase than a risk neutral specialist does, and a monopolistic risk-averse specialist sets higher price at his sale and lower price at his purchase than a risk neutral monopolist does and a perfect risk-averse competitor does. The intuition behind Prop.2 is rather obvious. A risk averse specialist evaluates the current cash higher than the expected value of a risky asset, and thus expects a proper premium to compensate him for taking risk in his speculation. This is why there is a risk premium component included in the specialist's quotes. As regards the monopolistic profit component, it is simply because a monopolist can always set up the price at the level most favorable to himself.

By Prop.2, the deviation of transaction price from the public expectation consists of three parts, one is the risk premium  $|p^0 - p^m|$ . One is the monopolistic profit  $|p^m - p^c|$  and the other is the mean reversion  $|E_s[\tilde{V}|\Omega_s] - m|$ . In summary,  $|p^0 - m| \leq |p^0 - p^m| + |p^m - E_s[\tilde{V}|\Omega_s]| + |E_s[\tilde{V}|\Omega_s] - m|$ . The explanation for the presence of  $|E_s[\tilde{V}|\Omega_s] - m|$  can be found in the literature that uses adverse selection approach<sup>4</sup> to study the the informational effects of transaction volumes on transaction prices. According to this approach, in order to recoup the loss from trading with insiders by obtaining some profits from liquidity motivated trading, the specialist has to post his price equal to his reservation price. The inventory effects and order processing costs are not reflected in the price simply because our model ignores them from the very beginning.

Define the relative information revelation index  $RIRI^5 = |p - \sum_{i=1}^n E_i[\tilde{V}|\Omega_i]/n| / |v_s - \sum_{i=1}^n E_i[\tilde{V}|\Omega_i]/n|$ , where  $|p - \sum_{i=1}^n E_i[\tilde{V}|\Omega_i]/n|$  is the indicator of information amount that price  $p$  actually reflects with reference to  $\sum_{i=1}^n E_i[\tilde{V}|\Omega_i]/n$  and  $|v_s - \sum_{i=1}^n E_i[\tilde{V}|\Omega_i]/n|$  is the indicator of information amount that the specialist privately obtains superior to  $\sum_{i=1}^n E_i[\tilde{V}|\Omega_i]/n$ . So  $RIRI$  is the ratio of information price conveys to that the specialist possesses with respect to the information of all traders as a whole. By definition,  $0 \leq RIRI \leq 1$ . The higher the  $RIRI$ , the more information the price conveys. Proposition 3 naturally follows from Prop. 2.

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<sup>4</sup> For detail information, we refer interested readers to Bagehot(1971), Copland and Galai(1983), Glosten and Milgrom(1985), Kyle(1985) and Easley and O'Hara(1987,1992)

<sup>5</sup> One may also define the relative information revelation index as  $RIRI' = |p - m| / |v_s - m|$ , namely, the information revelation index is the ratio of information that price conveys to that the specialist possesses with reference to the mean of  $\tilde{V}$  the representative of publicly known information. Yet, what we are interested in here is how much of the specialist's superior information relative to the combination of traders' information, rather than how much of the absolute amount of the specialist's information, would be revealed through the the specialist's pricing. Thus, we prefer  $RIRI$  to  $RIRI'$  in this note. By the way, as can be easily illustrated,  $RIRI'$  and  $RIRI$  have the same sign although their magnitude may differ.

**Proposition 3.** Under assumptions I–V, it has  $1 \geq RIRI^0 \geq RIRI^m \geq RIRI \geq 0$  and  $1 \geq RIRI^c \geq RIRI^r$ .

*Proof.* First of all, we verify the fact  $p^0 \geq \bar{v} \Leftrightarrow p^m \geq \bar{v}$  to facilitate establishing the statement. Taking  $p \geq \bar{v}$  as an example. By Prop.1,  $p^0 \geq \bar{v}$  implies  $v_s \geq p \geq \bar{v}$  and hence  $\inf\{p: x_s \neq 0\} \geq \bar{v}$ . So it must have  $p^m \geq \bar{v}$ . Reversely,  $p^m \geq \bar{v}$  means  $v_s \geq \bar{v}$  and consequently gives rise to  $p^0 \geq \bar{v}$ . The same reasoning is applicable to the case  $p \leq \bar{v}$ . Now considering the statement. By definition and by Lemma 1.

$$RIRI^0 - RIRI^m = \left\{ \left| p^0 - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n \right| - \left| p^m - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n \right| \right\} \\ / \left| v_s - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n \right|$$

$$RIRI^m - RIRI^c = \left\{ \left| p^m - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n \right| - \left| p^c - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n \right| \right\} \\ / \left| v_s - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n \right|$$

$$RIRI^r - RIRI^c = \left\{ \left| p^r - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n \right| - \left| p^c - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n \right| \right\} \\ / \left| v_s - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n \right|$$

Since  $|v_s - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n|$  is positive, whether  $RIRI^c \geq RIRI^m$ ,  $RIRI^m \geq RIRI^0$  and  $RIRI^r \geq RIRI^0$  depend on whether the differences  $|p^0 - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n| - |p^m - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n|$ ,  $|p^m - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n| - |p^c - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n|$  and  $|p^r - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n| - |p^c - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n|$  are positive or negative, respectively. In the case that the specialist commits purchases,  $p^0 \leq p^m \leq p^c = v_s$  from Prop.2, this implies  $\bar{v} \leq p^0 \leq p^m \leq p^c = v_s$  from Prop.1. Accordingly,  $|p^0 - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n| \leq |p^m - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n| \leq |p^c - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n|$ . In the case that the specialist commits sales,  $p^0 \geq p^m \geq p^c \geq v_s$  from Prop.2. This implies  $\bar{v} \geq p^0 \geq p^m \geq p^c = v_s$  from Prop.1. Accordingly,  $|p^0 - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n| \leq |p^m - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n| \leq |p^c - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n|$ . The same reasoning verifies  $|p^r - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n| \leq |p^c - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n|$ . Q.E.D.

According to Proposition 3, the transaction prices do transmit some superior information of the specialist to traders. But, the price set by a monopolistic specialist is

less informative than that by a competitive specialist, and the price set by a risk averse specialist is less informative than that by a risk neutral specialist. Generally speaking, the greater his monopolistic power and/or the more averse his risk attitude, the less information is revealed through the specialist's pricing. In order to compensate himself for taking risk and to grab monopolistic profits, a specialist never posts price  $p=v_s$  so that his superior information expressed by  $|v_s - \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n|$  is fully reflected in his price.

Granted that trading volume discloses all the information from traders to the specialist and specialist is risk neutral in the perfectly competitive market, a specialist would quote price equal to his conditional expected value, spreading all his information both from traders and from his own private channel to the market. Meanwhile, traders who take price as additional information find  $p$  containing superior information and realize their own information redundant, they hence share the identical beliefs about the realization of  $\tilde{V}$  with the specialist. As a consequence, neither traders nor the specialist would collect costly private information, no speculative trades both on and not on the specialist's own account would take place, thus there would not exist any rational expectations equilibrium and the market might shut down. If we believe the existence of rational expectations equilibrium, we have to accept that quantity fully revealing, perfect competitiveness of the market and risk neutrality of the specialist do not and/or can not coexist since their coexistence leads to price fully revealing. Notice that the impossibility of informational efficient specialist market is not only because of expectation homogenization of traders, as noticed by Grossman and Stiglitz(1980), but also on account of traders sharing the same belief with the specialist. In effect, a competitive risk-neutral specialist would do the same thing as a Walrasian auctioneer does if he has no any private information.

To the contrary, in the circumstance that quantity is fully revealing while the specialist is risk averse and/or monopolistic, the transaction price is some sort of

combination of  $\bar{v}$  and  $v_s$  that are random to the public, traders have to recourse to their private information since they have no way to induce either  $\bar{v}$  or  $v_s$  perfectly. In the result, traders' information and the specialist's information remain differentiated after traders use price to update their prior beliefs respectively, there hence exists rational expectations equilibrium with quantity fully revealing and price partially revealing.

#### IV. An Example

In this section, we establish and analyze a rational expectations equilibrium in a simple mean–variance framework. First, let us restate Assumption III as follows.

**Assumption III'.** Both the specialist and all traders are thought to have negative exponential preference function  $W = -\exp(-\alpha C)$  with  $\alpha$  standing for the constant relative risk averse coefficient.

In conformity with Assumptions I, II, III' and IV, a competitive trader  $i$ 's mean–variance trading strategy is written as follows.

$$x_i = (1-\xi)(E_i[\tilde{V} | \Omega_i] - p) / \psi \sigma^2. \quad (5)$$

A positive  $x_i$  means a purchase while the negative stands for sale. The aggregate demand function for transaction service is

$$\sum_{i=1}^n x_i = (1-\xi)(\sum_{i=1}^n E_i[\tilde{V} | \Omega_i] - np) / \psi \sigma^2. \quad (6)$$

**Proposition 4.** Under Assumptions I, II, III' and IV, (i) the optimal strategy

pair a risk-averse monopolistic specialist adopts or the rational expectations equilibrium is

$$p^0 = \lambda \bar{v} + (1-\lambda)v_a; \quad x_s^0 = (1-\lambda)n(v_s - \bar{v})/\psi\sigma^2.$$

Where,  $\lambda = [\psi + \phi(1-\xi)n]/[2\psi + \phi(1-\xi)n]$ . (ii) the optimal strategy pair a risk-neutral monopolistic specialist adopts or the rational expectations equilibrium is

$$p^m = 1/2 (\bar{v} + v_s). \quad x_s^m = n(v_s - \bar{v})/2\psi\sigma^2.$$

(iii) granted that  $m$  specialists are identical in both market operations and rational expectations, the optimal strategy pair a risk-neutral competitive specialist adopts or the rational expectations equilibrium is

$$p^c = v_s. \quad x_s^c = \sum_{i=1}^m (v_i - v_s)/m\psi\sigma^2.$$

where,  $\psi$  is the common constant relative risk averse coefficient of traders and  $\phi$  is the constant relative risk averse coefficient of the specialist.

Proof. The problem a monopolistic specialist faces is to maximize his expected profits conditional on his pre-posterior information set with respect to price (or amount of service  $x_s^m$ ) subject to the market clearing condition. Namely,

$$\text{Max } \pi_s = x_s(v_s - p) - \phi x_s \sigma^2 / 2 \quad \text{subject to} \quad \sum_{i=1}^m x_i + x_s = 0.$$

Differentiating  $\pi_s$  with respect to  $p$  yields the first order condition below.

$$[2n + \phi(1-\xi)n^2/\psi]P - [1 + \phi(1-\xi)n/\psi]\sum_{i=1}^m v_i - n v_s = 0.$$

That is  $p^0 = [\psi + \phi(1-\xi)n]\bar{v}/[2\psi + \phi(1-\xi)n] + v_s\psi/[2\psi + \phi(1-\xi)n]$ . Substituting the expression for  $p^0$  and Eq(2) into Eq(3) gives  $x_s^0$ . The proof of (ii) is identical to the proof of (i) except for absence of the second moment term. In fact,  $p^m = p^0(\phi=0)$  and  $x_s^m = x_s^0(\phi=0)$ . Now considering (iii). The competition among specialists necessarily drives  $\pi_s$  in general to zero. It hence turns out  $p^c = v_s$  on account of risk neutrality. In the sequel,  $m$  specialists share the total residual demand  $\sum_{i=1}^m x_i$ .

It is easy to verify that the optimal strategy pairs the specialist employs are rational expectations equilibriums, too. First,  $x_i(\cdot, p)$  is preposterous optimal because

it, by construction, maximizes individual  $i$ 's expected utility based on  $\Omega_i$  and  $P$ .  $X_s^0$ ,  $p^0$ ,  $X_s^m$ ,  $p^m$ ,  $X_s^c$ ,  $p^c$  are optimal from the way they are derived. Finally, it can be shown that  $\sum_{i=1}^n x_i(\cdot, p^m) = x_s^m$ ,  $\sum_{i=1}^n x_i(\cdot, p^0) = x_s^0$ ,  $\sum_{i=1}^n x_i(\cdot, p^c) = x_s^c$ . This completes our proof. Q.E.D.

The rest of this section is devoted to understating Proposition 4. To do this, remember that  $\lambda \leq \psi/2$ ,  $\psi = 1/2 < 1$  and  $\lim_{\eta \rightarrow 0} \lambda = 1$ . First, consistent with Proposition 2, the example shows that when  $\bar{v} \geq v_s$ , i.e., the market residual demand is negative, it has  $p^0 = \lambda \bar{v} + (1-\lambda)v_s \geq p^m = 1/2(\bar{v} + v_s) \geq p^c = v_s$ . Inversely, when  $\bar{v} \leq v_s$ , i.e., the market residual demand is positive, it has  $p^0 = \lambda \bar{v} + (1-\lambda)v_s \leq p^m = 1/2(\bar{v} + v_s) \leq p^c = v_s$ . The liquidation value of an asset next period is conventionally hypothesized to be independent of type and amount of its transactions this period. At this regard,  $v_s$  is both the marginal cost and average cost per unit asset for the specialist. As the reservation price of the specialist,  $v_s$  is the upper limit price that the specialist would bid and the lower limit that he would ask. Similarly, as the constant term in the linear market demand function Eq(6),  $\bar{v}$  is the maximum price that traders as a whole are willing to pay for the asset and the minimum price at which traders as a whole are willing to sell the asset. Therefore, transaction among the specialist and traders can only take place at the price which is between  $\bar{v}$  and  $v_s$ . Using the interpretation of  $\bar{v}$  and  $v_s$  given above, Prop.4 are illustrated in Figure 1. In order to include a risk premium into the price, the risk averse monopolist has to give up some expected profits. This can be seen from  $\pi_s^0 = (p^0 - v_s)x_s^0 \leq (p^m - v_s)x_s^m = \pi_s^m$ . Note that  $\pi^m$  is the highest possible expected profit a monopolist can possibly seize given the market residual demand function Eq(6) and the the specialist's marginal cost curve  $MC = v_s$ . We thus claim that a risk averse specialist has less confident of utilizing his private information to make profits than a risk neutral specialist does. Trivially,  $\pi_s^0$  is composed of two parts. One is  $(p^m - v_s)x_s^0$ , the pure monopolistic profit; The other is

$(p^0 - p^m)x_s^0$ , the premium to compensate the specialist for taking the risk in speculation.

Secondly, consistent with Prop.1, Prop.4 clearly shows  $x_s^0$ ,  $x_s^m$  and  $x_s^c$  are dependent on  $v_s - \bar{v}$ . Moreover,  $\partial x_s^0 / \partial (v_s - \bar{v}) = (1 - \lambda)n / \psi \sigma^2 \geq 0$ ;  $\partial x_s^m / \partial (v_s - \bar{v}) = n / 2\psi \sigma^2 \geq 0$ ;  $\partial x_s^c / \partial (v_s - \bar{v}) = n / m\psi \sigma^2 \geq 0$ .  $\partial x_s^0 / \partial (v_s - \bar{v}) \leq \partial x_s^m / \partial (v_s - \bar{v})$  for  $(1 - \lambda) \leq 1/2$  while  $\partial x_s^m / \partial (v_s - \bar{v}) \geq \partial x_s^c / \partial (v_s - \bar{v})$  for  $m \geq 2$ . That means, the difference  $v_s - \bar{v}$  does have positive effect on the specialist's speculation. However, the effect is larger to a risk neutral specialist than to a risk averse specialist, and is greater to a monopolist than to a perfect competitor.

Finally, consistent with Prop.3, in our example,  $0 \leq RIRI^0 \leq RIRI^m \leq RIRI^c = 1$ . As compared with  $p^m$ ,  $p^0$  is closer to  $\bar{v}$  and farther from  $v_s$ . As compared with  $p^c$ ,  $p^m$  is closer to  $\bar{v}$  and farther from  $v_s$ . We see that  $p^0$  reveals nothing to the market if the number of speculative traders goes to infinity, i.e.  $\lim_{n \rightarrow \infty} RIRI^0 = 0$  since  $\lim_{n \rightarrow \infty} \lambda = 1$  and  $\lim_{n \rightarrow \infty} \sum_{i=1}^n E_i[\tilde{V} | Y_i] / n = \lim_{n \rightarrow \infty} \bar{v} = m$ . However  $RIRI^m$  is independent of  $n$ . This is because a risk averse specialist has to put more weights on the public opinion as more and more information-based traders enter the market, on the contrast, a risk neutral specialist do not worry about the risk born from his prior belief.  $p^c$  spreads all the superior information of the specialist to traders under circumstances that market is competitive and the specialist is risk neutral.

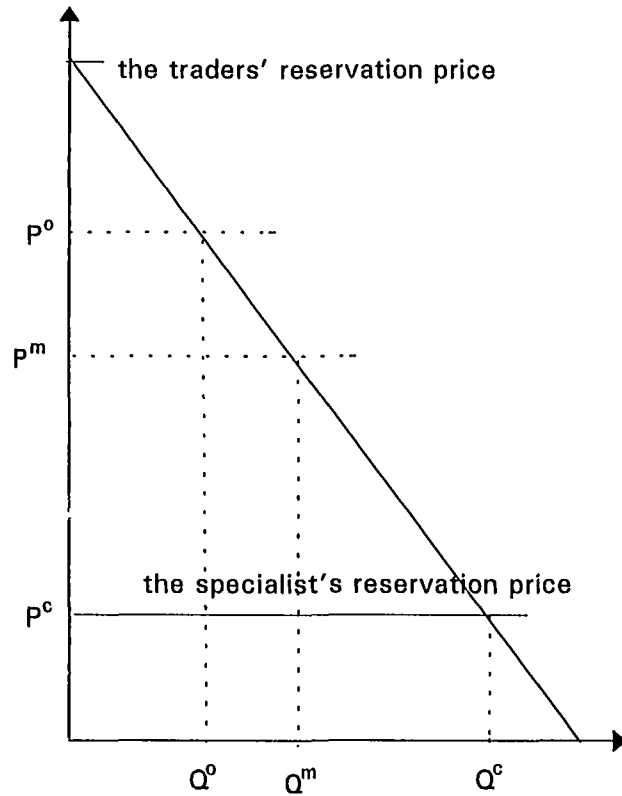
### Conclusion

This research has been examining the informational and competitive properties of the specialist's pricing under various configurations of the market structure and the risk attitude of the specialist. As economists usually expect, the absolute difference between the equilibrium price posted by the specialist and the specialist's prior belief  $E_s[\tilde{V} | \Omega_s]$  positively relate to the monopolistic market power and the risk averse degree of the specialist. We also see that the coexistence of quantity fully revealing, perfect

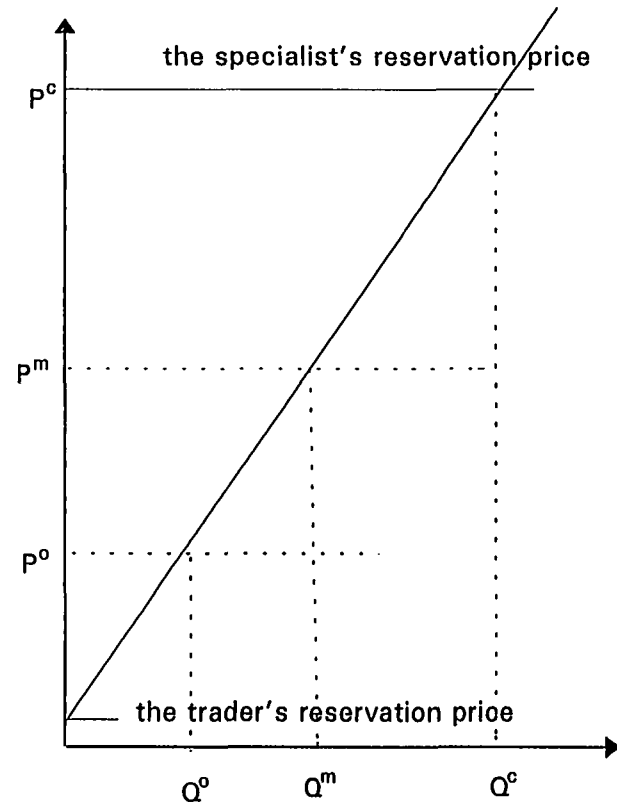
competitiveness of the specialist's market and neutrality of the specialist risk attitude are necessary conditions for price to be perfect informative. Namely, With risk averse attitude of the specialist and non-competitiveness of the specialist's market, rational expectations could possibly exist in the speculative market under quantity fully revealing (note, perfect competitiveness of traders' market is implied here). The speculative market may shut down when operated by a risk-neutral competitive specialist who has to post price equal to his preposterous expectation. Besides, As the number of market participants increases to infinity at a point in time, the price set by a risk-averse specialist approaches to the public expectation and transmits no superior information to the market.

The model we analyze in this research is a static mean-variance framework with trading, where quantity is hypothetically fully revealing, all market participants are price-takers and the specialist obtains error free superior information. It thus can be significantly extended in several different ways. As Kyle(1985) has modeled, an insider, who is large enough to influence the specialist's pricing, may employ one piece of information to commit sequential trading strategically over time, his demand for transaction service at a point in time hence may only transmits partial information to the specialist. It is interesting to see the dynamic properties of the specialist pricing in terms of competitive behavior and information revelation when inside traders speculate strategically and transaction volume is partially revealing. In the market there also exists what is called noise trading. The presence of noise trading would lead to partial revelation of transaction volume because the randomness of the noise trading makes it impossible that the specialist infers perfect aggregate information of traders by just inverting trader's aggregate demand functional. Thus, incorporating noise trading into the model will bring out more sophisticated implications that are closer to the reality. At last, it is too hard to imagine that the specialist's information set is error free. If assuming the specialist observes some sort of disturbed random signal about  $\tilde{V}$ , the

preposterous expectation of  $\tilde{V}$  conditional on that disturbed random signal and traders' aggregate demand volume will become much more complicated, and so will be the whole analysis presented in this paper.



(a) When the traders' reservation price is greater than the specialist's, the specialist is a seller.



(a) When the traders' reservation price is lower than the specialist's, the specialist is a buyer.

**Figure 1 The Specialist's Speculation in Providing Transaction Services**

In panel (a), a risk-neutral competitive specialist sets price equal to his expectation and earns zero profits. A risk-neutral monopolist sets price at the ordinary monopolistic level and earns the ordinary monopoly profits. A risk-averse monopolistic specialist sets price higher than the ordinary monopoly price and earns less profits than the ordinary monopolist.

In panel (b), a risk-neutral competitive specialist sets price equal to his expectation and earns zero profits. A risk-neutral monopolist sets price at the ordinary monopolistic level and earns the ordinary monopoly profits. A risk-averse monopolistic specialist sets price lower than the ordinary monopoly price and earns less profits than the ordinary monopolist.

## Chapter Two

# A Synthetic Model of Transaction Price Formation

In the specialist-organized stock market (for example, the New York Stock Exchange), trades are generally thought to have diverse effects on specialist pricing. For example, there are order processing effect, asymmetric information (adverse selection) effect, inventory control (inventory holding) effect and demand elasticity effect. This paper is intended to synthesize these effects in an analytical model of transaction price formation and disentangle them empirically by estimating a linear transaction price functional.

The first objective of this paper is to integrate inventory-control and asymmetric-information approaches into a single theoretical framework and hence to derive a price functional that incorporates diverse price effects of trades. This objective is motivated by the following observations. Existing literature has already well explained diverse impacts of trades on prices in theory through different analytical approaches. To be specific, the order processing effect is attributed to the cost of providing immediacy service (e.g., Demsetz(1968), Tinic(1973) among others). The information effect is explained by adverse selection approach (e.g., Copeland and Galai(1983), Glosten and Milgrom(1985), Kyle(1985), Easley and O'Hara(1987,1992) among others). The inventory effect is analyzed through dynamic programming approach (e.g., Stoll(1978), Amihud and Mendelson(1980), Ho and Stoll(1981,1983), Oldfield and O'Hara(1986) among others). Meanwhile, the price elasticity of investors' demands for transaction services is considered in the studies that investigate how demands respond to prices (e.g., Glosten and Milgrom(1985) and Glosten(1989) among

others) and how the specialist manipulates the market (Madrigal and Scheinkman(1991)). Since each of those approaches explains one effect, it is accordingly desirable to integrate them into a single analytical model. But no such model has appeared yet. On the other hand, there have emerged many works that disentangle various price impacts of trades in a single **empirical** model. For example, Ho and Macris(1984) estimate the bid–ask spread and the inventory effect according to Ho and Stoll(1981)'s dynamic inventory model. Glosten and Harris(1988) estimate the bid–ask spread and the information effect using an adverse selection model. Hasbrouck(1988) examines the nature of inventory control and asymmetric information effects by regressing the revision in the average of bid and ask prices on the trade innovation history. Stoll(1989) infers adverse information costs, inventory holding costs and order processing costs as components of the bid–ask spread. Madhavan and Smidt(1991) integrate inventory and information theoretic approaches in a single empirical model to examine diverse effects of trades on intraday pricing. Nevertheless, in all these works, the price functional and the additivity of processing, inventory and information effects are simply assumed rather than derived analytically<sup>1</sup>.

The logic of our model for specialist pricing can be summarized as follows. Assume the specialist maximizes his expected utility of trading profits over an infinite time horizon conditional upon his information set at each point in time. Assume some anonymous traders possess superior information to the specialist. Then in choosing optimal price strategy, the specialist must consider all the processing, inventory, information and elasticity effects of trades on prices at the same time. Putting this

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<sup>1</sup> Hasbrouck(1988,pp236) note that "Since no formal solution has been posed for the dealer control problem when both inventory–control and asymmetric–information features are present, the possibility of interactions that belie the simple additive solution must be considered." Madhavan and Smidt(1991) estimate a Bayesian pricing model, this model is derived directly from a price functional in which diverse effects of trades on prices are assumed additive. So they state that they integrate both inventory and information theoretic approaches in a single **empirical** model (not an analytical model).

problem in a stationary Markov decision framework, the optimal price functional for the specialist at each point in time is found to be the weighted average of his estimate about the trader's reservation price and his own reservation price. The trader's reservation price is his expectation about the underlying asset's value. The specialist's reservation price is composed of three elements: the expected marginal utility generated by the coverage of order processing cost, the expected marginal utility generated by the liquidating value of the underlying asset and the expected marginal future utility generated by the current price strategy. According to this price functional, the optimal bid-ask spread is decomposed into five parts: order processing costs, adverse selection effects, inventory control impacts, monopolistic profits and a risk adjustment for demand order uncertainty. In particular, when the specialist is risk neutral and his optimal inventory control policy is assumed to be linear in inventory, this price functional degenerates to a special form in which the processing, inventory, information and elasticity effects become additive. This special form serves the foundation of most existing empirical models for the specialist's pricing, and it will also be the base of our empirical studies in this paper.

In using dynamic programming to model specialist pricing, our approach differs from Ho and Stoll(1981) and O'Hara and Oldfield(1986) in the following respects. First, they wish to determine how both order flow and stock value uncertainty influence a risk averse specialist's pricing from the point of view of inventory control. Thus in their model the specialist is not supposed to update his prior conditional upon the incoming order and as a result the asymmetric information effect is not analyzed<sup>2</sup>. In contrast, we assume that trades contain private information and the specialist must

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<sup>2</sup> In deriving the optimal price that maximizes the dealer's expected utility of terminal wealth, Ho and Stoll(1981) assume some fixed, intrinsic value for the stock. In deriving the optimal price that maximize the dealer's expected utility of trading profits, the progress that O'Hara and Oldfield(1986) make towards impounding asymmetric information effects in inventory control framework is to allow the dealer's pricing strategy to depend on his expectation of the future stock price.

infer this information in making optimal pricing policy. Consequently, we introduce asymmetric information into the conventional inventory control framework. Second, since they do not intend to deal with asymmetric information, they postulate the transaction demand of a trader as a linear function of transaction price in which no specific term represents the trader's expectation. Similar to Madhavan and Smidt(1991), we formulate the transaction demand of a trader as a linear function of the difference between the the trader's prior belief about the underlying asset's value and the price quoted by the specialist. This type of demand function is consistent with the mean–variance framework<sup>3</sup> and allows the specialist to infer the trader's reservation price from the trader's demand order. Third, they are primarily concerned with the bid and ask quote setting and thus allow multiple transactions at a point in time. Our major concern is with transaction price formation, so only one transaction is allowed at each point in time. This price represents the ask price if the trade is initiated by a public buyer and the bid price otherwise.

The second objective of this paper is to test our synthetic model of transaction price formation. Based on the linear form of the transaction price functional that is constructed from our synthetic model, the test is conducted by regressing the transaction price change on the following explanatory variables: (i)the change in signs of trades, (ii)the deviation of the observed trade from the anticipated, (iii)the trade which affects the specialist's inventory, and (iv)the difference between current and last trades. The first explanatory variable measures the source of order processing cost. The second stands for the innovation of asymmetric information. The third is the source of inventory control effect. The last represents the response of trades to price changes from the point of view of demand elasticity.

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<sup>3</sup> As Easley and O'Hara (1992) point out, demand functions can be very badly behaved in an asymmetric information economy. At this point, the investment demand function of mean–variance type should be viewed as an assumption rather than a general formulation.

There have appeared several ways to disentangle various effects of trades on prices. Stoll(1989)'s technique is to model the serial covariance of price changes as a function of the probability of price reversal and the magnitude of price change. Hasbrouck(1988)'s method is to regress quote revisions on trade innovations. In Glosten and Harris(1985) and Madhavan and Smidt(1991), the approach is to model and estimate a price functional. As has been described above, our empirical study resembles the last approach with certain important differences. (i)In Glosten and Harris(1985) and Madhavan and Smidt(1991), each trade as a whole is viewed as of information revealing. In building our price regression equation we follow Hasbrouck(1988, 1991), assuming only the unanticipated component of the trade reveals new information. Our argument is as follows. If the market is informationally efficient and if sequentially arrived trades are serially correlated, the specialist should not be shocked at the predictable component of a trade and the information that the predictable component of a trade carries should have already been reflected in previous prices. (ii)If the market is non-competitive, a utility-maximizing specialist needs to take into account the demand elasticity of his price. This fact has not received direct attention in the empirical literature although it is demonstrated theoretically in Glosten and Milgrom(1985), Madrigal and Scheinkman(1989) and Glosten(1989). In this paper, by introducing the change in trade as an explanatory variable into the price regression equation, we are able to estimate the elasticity effect of trades on prices under the assumption that the specialist possesses some degree of power of manipulating the market. (iii)Roll(1984) postulates the effective bid-ask spread to be pure order processing costs. From a Bayesian model, Madhavan and Smidt(1991) compute the effective bid-ask spread by adding up the asymmetric information impacts and order processing costs. In contrast, we calculate the bid-ask spread as composed of four components: order processing costs, asymmetric information effect,

inventory control effect and demand elasticity effect.<sup>4</sup>

We have done the regression mentioned above for 10 most heavily traded NYSE stocks in TORQ database. Order processing, asymmetric information, inventory control and demand elasticity all display statistically significant effects on transaction price movements in spite of the fact that inventory control and demand elasticity do not show consistent effects across stocks.

The rest of this paper is arranged as follows. Section I develops a synthetic model of transaction price formation. Section II derives a regression equation for transaction price changes. Section III describes the data. Section IV presents the empirical results. The paper is concluded in Section V.

### I. The Synthetic Model

We consider the following stock market which opens at discrete points in time. In this market, there are two securities to be traded among agents: a risk-free treasury bond and a common stock. The risk-free bond is the numeraire and has a constant return of certain units in every period, while the common stock has a random value  $\tilde{V} \sim N(m, \sigma_V^2)$ . There are two types of agents: a specialist who makes the market and some number of traders who initiate transactions. As an agent and a principal as well, the market maker or the specialist is responsible for maintaining liquid and continuous two-side auction market on the exchange floor in the security in which he is registered. He is supposed to maximize his profits in providing traders and fostering the market with intermediary services. Traders include those who are information-motivated

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<sup>4</sup> Glosten and Harris(1988) do realize the spread is composed of four elements: information effects, inventory costs, specialist monopoly power and clearing costs. But they only decompose the spread into two components: the permanent component due to adverse selection and the transitory component due to inventory costs, specialist monopoly power and clearing costs.

(such as speculators and insiders) as well as those who are liquidity-motivated (such as noise investors and investment institutions). They are assumed to maximize their respective expected utility through exchanging their asset holdings directly via the specialist. For simplicity, we assume the market to be a continuous auction in an infinite time horizon, ignoring intermittent trading, overnight markets and non-trading periods. Besides, we assume there is one and only one public order that arrives and is processed in each trading period. In other words., we follow the transaction time instead of clock time.

#### A. The Trader's Demand for Transaction Service

Suppose a trader's information set just prior to the trade in time  $t$  consists of a series of signals he obtained at some cost —  $\{Z_{t-k}, k=0,1,\dots\}$  and a historical sequence of transaction prices —  $\{P_{t-k}, k=1,2,\dots\}$ . His expectation about  $\tilde{V}$  conditional on all the up-to-date information available to him prior to the trade in time  $t$  is given by  $y_t \equiv E_t[\tilde{V} | Z_t, Z_{t-1}, P_{t-1}, \dots]$ .  $y_t$  is apparently very complicated. But for simplicity, we suppose it possesses all nice properties that are necessary to our analysis and we use it as a unitary variable. Economically,  $y_t$  serves as the reservation price of the trader. When the ask price is less than or equal to  $y_t$  his expected order is a purchase, otherwise it is a sale.

Let  $p_t$  be the transaction price made by the specialist at time  $t$ .  $p_t$  is the ask price if the transaction is public purchase and the bid price otherwise. Let  $q_t$  denote the order flow at time  $t$ . A positive  $q_t$  represents a public purchase while a negative stands for a public sale. The demand function of a trader for transaction service<sup>5</sup> or the order flow at time  $t$  is stipulated as<sup>6</sup>

<sup>5</sup> As in Ho and Stoll(1981), we view both the public demand to sell securities and the public demand to buy securities as demands for transaction services. Symmetrically, that the specialist sells securities to and purchases securities from the public is regarded as supply of transaction services.

<sup>6</sup> To see a derivation of this demand function, the interested reader is referred to Madhavan

$$q_t = \varphi(y_t - p_t) + \tilde{x}_t, \quad (1)$$

where  $\varphi$  is a parameter. The non-discretionary trade component<sup>7</sup>  $\tilde{x}_t$  is a white noise to the specialist and is characterized by  $\tilde{x}_t \sim \text{IIN}(0, \sigma_{\tilde{x}}^2)$ . Assumption of  $E[\tilde{x}_t] = 0$  is made for tractability for it is very difficult, if it is not impossible, to single out the noise trades from observed transaction process. (1) differs from the transaction demand functions in Zable(1981) and O'Hara and Oldfield(1986), but resembles the order flow function in Madhavan and Smidt(1991).  $\varphi$  deserves four related interpretations. (i) If (1) is the policy which is made by a risk averse trader in the mean-variance framework,  $\varphi$  includes the "risk discounted rate". As in Madhavan and Smidt(1991), this rate is  $1/\phi \text{Var}[\tilde{V} | \cdot]$  — the reciprocal of the risk averse coefficient times the conditional variance of the underlying security value. (ii) In Kyle(1985),  $\varphi$  measures the degree to which information-based traders use their superior information to commit trades in time  $t$ . In this connection,  $\varphi$  represents the response of trades to prices. (iii) Write the expectation of a trader about  $\tilde{V}$  conditional on all the information available to him when he commits trade in time  $t$  as  $E_t[\tilde{V} | (Z_t, Z_{t-1}, P_{t-1}, \dots), P_t] = \zeta y_t + (1-\zeta)p_t$ , where  $1-\zeta$  is the weight the trader in forming his posterior belief gives to the current price. It follows that  $\{E_t[\tilde{V} | (Z_t, Z_{t-1}, P_{t-1}, \dots), P_t] - p_t\} / \phi \text{Var}[\tilde{V} | \cdot] + \tilde{x}_t = (\zeta / \phi \text{Var}[\tilde{V} | \cdot])(y_t - p_t) + \tilde{x}_t$ . This implies  $\zeta$  is a component of  $\varphi$ . (iv) As the absolute value of the slope of the transaction demand curve,  $\varphi$  is the deterministic parameter in the function of demand elasticity.

Taking the specialist's price into account, traders submit their orders randomly upon stochastic arrivals of news and their liquidity needs for optimizing portfolios. The

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and Smidt(1991).

<sup>7</sup> Admati and Pfleiderer(1988) define non-discretionary traders as those liquidity traders who have no discretion with regard to the timing of their trades, and discretionary traders as those liquidity traders who choose the timing of their transactions strategically subject to the constraint of trading a particular number of shares within a given period of time.

first term on the LHS of (1) not only represents the order that is generated by private information but also reflects so-called discretionary trade that is submitted by investment institutions. This follows because investment institutions are usually said to reconstruct their portfolios optimally by minimizing the aggregate variances of stock returns. On the other hand, the non-discretionary liquidity trade  $\bar{x}_t$  is independent of price, which implies that noise traders believe perfect informational efficiency of asset pricing.

### B. The Specialist's Supply of Transaction Service

The specialist's problem in each trading period  $t$  is assumed to choose optimal transaction price  $p_t$  so as to maximize his expected utility over an infinite horizon conditional on all the information available to him in time  $t$ . In solving this problem, he has to consider the following five elements that are interesting to us.

First, he has to incorporate a reasonable component into the price to cover all the expenses incurred in providing order processing services. In reality, the order processing cost may be charged according to volume. Yet following the convention and for convenience, we stipulate it is independent of order size. Let  $\psi$  denote the order processing cost. The price net of processing costs is  $p+\psi$  for one unit of public sale and  $p-\psi$  for one unit of public purchase. Let  $D_t$  be an indicator function, where  $D_t=+1$  if the transaction is a public buy and  $D_t=-1$  otherwise. we write the price net of processing costs compactly as  $p-\psi D_t$ .

Second, according to the theoretic approach of asymmetric information, the specialist faces adverse selection problem in making the market. In order to recoup his losses suffered from inside trading by gains from liquidity trading, the specialist needs to add a special component that reflects the effect of asymmetric information into the bid-ask spread. For example, a public block purchase may signal a good news and a positive component needs to add into the ask price, while a public block sale may

indicate a bad news and a negative component needs to add into the bid price. This necessitates the specialist to infer what is the current true value of the underlying security. Let  $\mathcal{F}_t$  be the up-to-date information set available to the specialist in time  $t$ .  $\{\mathcal{F}_t, t \in T\}$  is an increasing sequence of  $\sigma$ -fields contained in  $\mathcal{F}$ . With  $\tilde{V}_t$  being economically significant and  $\mathcal{F}_t$ -measurable in  $\mathbb{R}^+$ , the specialist's expectation about  $\tilde{V}$  conditional on his information at time  $t$  is well defined to be  $E_t[\tilde{V} | \mathcal{F}_t]$ . Other thing being equal, setting price such that  $p_t = E_t[\tilde{V} | \mathcal{F}_t]$  solves the specialist's adverse selection problem. This follows from the following arguments. When a trade is initiated by a public buyer,  $E_t[\tilde{V} | \mathcal{F}_t]$  positively relates to order size. Reversely, when a trade is initiated by a public seller,  $E_t[\tilde{V} | \mathcal{F}_t]$  negatively relates to order size.

Third, assuming all trades are committed against the specialist's own account, the specialist's inventory position subsequent to the trade in time  $t-1$  and prior to the trade in time  $t$  is an accumulative algebraic sum of all past trades up to time  $t-1$ , i.e.,  $I_{t-1} = \sum_{k=1}^{t-1} (-q_{t-k})$ , and his inventory position before the trade in time  $t+1$  and after the trade in time  $t$  is  $I_t = I_{t-1} - q_t(p_t)$ . According to pure inventory control theory, the specialist lowers transaction prices when his inventory exceeds the desired level so as to induce public buys and inhibit additional public sales. Reversely, he raises transaction prices when his inventory is below the desired level so as to induce public sales and inhibit additional public buys. Note also that carrying previous inventory to future trading periods involves stock value uncertainty and order flow uncertainty. Therefore, in order to acquire a compensation for his taking the risk of holding inventory and to adjust his existing inventory to the desired level, the specialist needs to add a component as inventory control effects and inventory holding costs to transaction price. Moreover, since current control policy (i.e., current transaction price) is a function of previous inventory position, since current trade is a function of current transaction price, and since the inventory position of a specialist after current trade is equal to his previous inventory position plus his current trade, inventory position

innately serves as the state variable in specialist's dynamic problem.

Fourth, if his market is non-competitive and the market demand for his intermediary service is elastic, the specialist is able to make abnormal profits by setting transaction price at somewhere between his own reservation price and his inference about the trader's reservation price. In order to do so optimally, he needs to infer the trader's reservation price as precisely as he can and consider the demand elasticity of his price.

Finally, transaction price has to be risk-adjusted so as to compensate a risk-averse specialist with a proper risk premium for his taking risk in making the market. Ho and Stoll(1981) show that this risk premium ultimately depends upon the specialist's fundamental attitude towards risk, the stock return uncertainty and market transaction uncertainty. O'Hara and Oldfield(1986) formulize this risk premium to be a risk adjustment component in the bid-ask spread for market order and inventory value uncertainty. Both of these two works relate the specialist's risk compensation to his inventory control problem.

Let  $U_t$  be a Von-Neumann Morganstern utility function for the specialist. It is increasing, concave, bounded and twice differentiable with respect to the profit the specialist obtains in time  $t$ . It is also additively separable in time. Meanwhile, let  $\Lambda_t$  be the value function in Bellman's equation. It is increasing, concave, bounded and twice differentiable with respect to the specialist's profits. To make things easily workable, we also hypothesize that functions  $U_t$  and  $\Lambda_t$  are well defined so that specialist's improvable strategy  $p_t$  is linear with respect to trade  $q_t$  and that the specialist's problem is of stationary Markov. Granted the specialist has no other income than profits from serving stock exchange, his aggregate utility in the current time is composed of two major parts: (i)the utility of the profits that will be made in current trade; (ii)the utility of the profits that will be generated by future trades. Given that all the above assumptions hold, the specialist's problem in time  $t$  is expressed by the

following dynamic programming equation<sup>8</sup>.

$$\Lambda_t(I_{t-1}, \mathcal{F}_t) = \text{Max}_{\{p_t\}} E_t \left[ U_t((p_t - D_t \psi - \tilde{V})[\varphi(y_t - p_t) + \tilde{x}_t]) + \theta \Lambda_{t+1}(I_t, \mathcal{F}_{t+1}) \mid \mathcal{F}_t \right]^9 \quad (2)$$

$\theta$  is the discount factor and lies in  $(0,1)$ .  $(p_t - D_t \psi - \tilde{V})[\varphi(y_t - p_t) + \tilde{x}_t]$  is the random profits that will be made in time  $t$ . Notice that whenever the specialist receives (pays out) cash from selling (buying) share, his inventory in dollar value occurs a corresponding decrease (increase). The difference between the change in cash flow and the associating change in inventory value yields the specialist's profits.  $I_t = I_{t-1} - [\varphi(y_t - p_t) + \tilde{x}_t]$  is the inventory position and  $-[\varphi(y_t - p_t) + \tilde{x}_t]$  is the change in inventory position after the trade in time  $t$  has taken place. In summary,  $U_t((p_t - D_t \psi - \tilde{V})[\varphi(y_t - p_t) + \tilde{x}_t])$  represents the utility of the profits that will be made in current trade,  $\theta \Lambda_{t+1}((I_{t-1} - [\varphi(y_t - p_t) + \tilde{x}_t])$  represents the discounted utility of the profits that future trades will generate. Notice that the value function  $\Lambda_{t+1}$  hinges on the specialist's current price strategy. This is because future profits depends upon future

<sup>8</sup> To make (2) be a problem of intraday specialist pricing as in O'Hara and Oldfield(1986), one only needs to remember that all transactions settle at the end of each trading day. The specialist's last decision in a trading day of  $n$  transactions is

$$\text{Max}_{\{p_n\}} E_n \left[ \sum_{t=1}^{n-1} U_t(\Pi_t) + U_n((p_n - D_n \psi)[\varphi(y_n - p_n) + \tilde{x}_n] - r \tilde{V}((I_{n-1} - [\varphi(y_n - p_n) + \tilde{x}_n]) + \theta \Lambda((I_{n-1} - [\varphi(y_n - p_n) + \tilde{x}_n]) \mid \mathcal{F}_n) \right], \quad (2')$$

where,  $r$  is the intraday interest rate. Comparing with the Equation (6) in O'Hara and Oldfield(1986), (2') has the following distinctions. (i)The specialist maximizes his intraday utility conditional on his up-to-date information set. (ii)The trader's expectation about the stock value is explicitly modeled in his demand function for transaction service. (iii)Only one transaction at one price is modeled at each point in time.

<sup>9</sup> One of the assumptions implied in (2) is that the specialist participates all transactions and trades on his own account. While this assumption is standard in most relating literature, it is worthwhile noting that it should be removed wherever possible. In their sample of 16 stocks, Madhavan and Smidt(1991) find the specialist's participation rate is not more than 70%. We keep this assumption here because the following reasons. first, we have no intent to model the specialist's participation rate either exogenously or endogenously. Second, we do not have data available to compute the specialist's participation rate.

price strategy, and because future price strategy is influenced by current post-trade inventory position which is in turn affected by current transaction price.

### C. A General Transaction Price Functional

Define  $U' \equiv \partial U / \partial \Pi_t$  and  $\Lambda'_{t+1} \equiv (\partial \Lambda_{t+1} / \partial \Pi_t)(\partial \Pi_t / \partial q_t) \equiv \partial \Lambda_{t+1} / \partial q_t$  for  $t \in \mathcal{T}$ . The solution for optimal price for the trade in time  $t$  is derived from the first-order condition of (2) and is given by

$$p_t^* = \frac{1}{2} \left[ \frac{D_t \psi E_t[U' | \mathcal{F}_t] + E_t[\tilde{V}U' | \mathcal{F}_t] - \theta E_t[\Lambda'_{t+1} | \mathcal{F}_t]}{E_t[U' | \mathcal{F}_t]} \right] + \frac{1}{2} \left[ \frac{E_t[y_t U' | \mathcal{F}_t] + E_t[\tilde{x}_t U' | \mathcal{F}_t] / \varphi}{E_t[U' | \mathcal{F}_t]} \right]. \quad (3)$$

$p_t^*$  is the specialist's price strategy for the trade in time  $t$ . It is optimal for all trades in the entire time horizon granted that control policies will be optimally selected thereafter. The interpretation of (3) is quite intuitive although it is not a close-form solution for  $p_t$ . First of all,  $\hat{y}_t \equiv E_t[y_t U' | \mathcal{F}_t] / E_t[U' | \mathcal{F}_t] + E_t[\tilde{x}_t U' | \mathcal{F}_t] / E_t[U' | \mathcal{F}_t]$  is the specialist's inference about the trader's prior belief about the stock value or the trader's reservation price of demand for transaction service. Note that  $E_t[\tilde{x}_t U' | \mathcal{F}_t] / E_t[U' | \mathcal{F}_t]$  functions as the specialist's inference error term. On the other hand,  $E_t[\tilde{V}U' | \mathcal{F}_t] / E_t[U' | \mathcal{F}_t]$  represents the specialist's rational expectation about  $\tilde{V}$  conditional on  $\mathcal{F}_t$ .  $E_t[\Lambda'_{t+1} | \mathcal{F}_t] / E_t[U' | \mathcal{F}_t]$  represents the marginal expected future utility that is generated by the trade in time  $t$ . The cost of providing dealership service of one additional unit of stock to the market includes the "true value" of the stock, order processing costs and inventory holding costs. So the marginal cost of transaction service or the the specialist's reservation price for providing dealership service is given by  $MC_t \equiv E_t[\tilde{V}U' | \mathcal{F}_t] / E_t[U' | \mathcal{F}_t] + \psi D_t - \theta E_t[\Lambda'_{t+1} | \mathcal{F}_t] / E_t[U' | \mathcal{F}_t]$ . In what follows, we delivery several remarks on (3).

The optimal transaction price for a monopoly specialist is the average of his estimate about the trader's reservation price of demand for transaction service and his own reservation price of providing transaction service. This holds because in our modeling the specialist is implicitly assumed monopolistic. A public sale could be possibly carried out only when  $MC_t \geq p_t \geq \hat{y}_t$ . Reversely,  $MC_t \leq p_t \leq \hat{y}_t$  is necessary for the occurrence of a public purchase. Hence, the transaction price for a monopoly specialist is a linear combination of  $MC_t$  and  $\hat{y}_t$ , and actually is  $(MC_t + \hat{y}_t)/2$  in our case. Furthermore, we see that price could be a magic wand for the specialist to obtain profits through trading on his own account. Knowing  $MC_t > \hat{y}_t$ , he sets price such that  $MC_t \geq p_t \geq \hat{y}_t$  and earns expected profit from a public sale. Otherwise, he names price such that  $MC_t \leq p_t \leq \hat{y}_t$  and earns expected profit from a public purchase. In contrast, if the specialist's market is perfect competitive, competition forces from other dealers and/or limit orders will drive the price to be such that  $E_t[\Pi_t | \mathcal{F}_t] = 0$ . With non-zero order flow, this means  $p_t = MC_t$ . That is to say, in the competitive market, transaction price equals the specialist's reservation price and  $\hat{y}_t$  plays no role in transaction price formation. Therefore, when  $MC_t > \hat{y}_t$ , it must have  $MC_t \geq p_t = (MC_t + \hat{y}_t)/2 \geq \hat{y}_t$ , traders sell stocks to the specialist at prices that are below the competitive level. Inversely, when  $MC_t < \hat{y}_t$ , it must have  $MC_t \leq p_t = (MC_t + \hat{y}_t)/2 \leq \hat{y}_t$ , traders buy stocks from the specialist at prices that are above the competitive level. It turns out that monopoly feature widens bid/ask spread and makes price more favorable to the specialist.

Both quantity and price are not fully information-revealing in (3). The presence of noise component  $\tilde{x}$  in demand function (1) makes demand order  $q_t$  not fully disclosing the trader's private information to the specialist. Why does price not fully reveal the specialist's information to the market? At the first place, as has been discussed above, the price for a monopoly specialist is a weighted average of his reservation price  $MC_t$  and his inference about the trader's reservation price  $\hat{y}_t$ . But both  $MC_t$  and  $\hat{y}_t$  are random to the trader. At the second place, even the specialist's

market is perfect competitive and  $p_t = MC_t$ , a risk averse specialist might add a markup to his reservation price  $MC_t$  for risk compensation. The partially revealing property of quantity and price protects stock market from Grossman and Stiglitz(1980)'s impossibility.

Risk aversion makes stock market less liquid. This can be easily seen from the following reasoning. The more risk averse a trader, the smaller the parameter  $\varphi$ , hence the less amount he is willing to trade for a given price. Likewise, the more risk averse the specialist, the larger risk premium markup will be added to his reservation price. In the consequence, higher price would be set and less order flow will arrive for execution.

Under assumptions of additivity of utility over time, normality of random variables, linearity of transaction demand and adaptivity of information sequence, the effects of order processing, inventory control and asymmetric information are additive in the specialist's optimal price functional.

(3) makes many existing specialist pricing model to be its special cases. For example, in Glosten and Milgrom(1985), the specialist's optimal price is given by  $p_t^* = E_t[\tilde{V} | \mathcal{F}_t] + \psi D_t$ . This is the special case of (3) when the specialist is risk neutral, his market is competitive and he does not worry about inventory. The intraday specialist pricing formula that O'Hara and Oldfield(1986) derive is in our terms that  $p_t^* = \text{constant}/2\varphi + E_t[\tilde{x}_t U'] / 2\varphi E_t[U'] + E_t[\tilde{V} U'] / 2E_t[U'] + E_t[\Lambda'_{t+1}] / 2E_t[U']$ . This is a special case of (3) when the specialist does not face an adverse selection problem, no cost is incurred in processing order and the specialist does not infer information from trades.

Let  $\mathcal{F}_t^b$  be the  $\sigma$ -field in which  $q_t$  is a public purchase and  $\mathcal{F}_t^s$  be the  $\sigma$ -field in which  $q_t$  is a public sale. The bid-ask spread is computed by subtracting the bid price  $p_t^*(\mathcal{F}_t^b)$  from the ask price  $p_t^*(\mathcal{F}_t^s)$ . The optimal spread is

$$p_t^*(\mathcal{F}_t^s) - p_t^*(\mathcal{F}_t^b)$$

$$\begin{aligned}
&= -\frac{1}{2} \left\{ 2\psi + \left( \frac{E_t[\tilde{V}U' | \mathcal{F}_t^+]}{E_t[U' | \mathcal{F}_t^+]} - \frac{E_t[\tilde{V}U' | \mathcal{F}_t^-]}{E_t[U' | \mathcal{F}_t^-]} \right) \right. \\
&\quad - \theta \left( \frac{E_t[\Lambda_{t+1}' | \mathcal{F}_t^+]}{E_t[U' | \mathcal{F}_t^+]} - \frac{E_t[\Lambda_{t+1}' | \mathcal{F}_t^-]}{E_t[U' | \mathcal{F}_t^-]} \right) + \left( \frac{E_t[y_t U' | \mathcal{F}_t^+]}{E_t[U' | \mathcal{F}_t^+]} - \frac{E_t[y_t U' | \mathcal{F}_t^-]}{E_t[U' | \mathcal{F}_t^-]} \right) \\
&\quad \left. + \frac{1}{\varphi} \left( \frac{E_t[\tilde{x}_t U' | \mathcal{F}_t^+]}{E_t[U' | \mathcal{F}_t^+]} - \frac{E_t[\tilde{x}_t U' | \mathcal{F}_t^-]}{E_t[U' | \mathcal{F}_t^-]} \right) \right\}^{10}. \tag{4}
\end{aligned}$$

(4) decomposes the bid–ask spread into five parts. The first term on the left hand side covers order processing costs. The second term grouped by brackets is due to adverse selection. The third term grouped by brackets represents the inventory holding costs. The fourth term grouped by brackets measures monopolistic profits. The fifth term grouped by brackets captures demand order uncertainty.

To sum up, (3) is a quite general expression for specialist pricing. it integrates various existing theoretical approaches and synthesizes diverse effects of trades on prices. However, it is not a closed–form solution. More detail interpretations about transaction price formation rely upon specifications of utility function, information sets and value function.

## II. The Econometric Model

Based on the synthetic model developed in the previous section, this section derives the econometric model that will be used for empirical studies. From now on, the specialist is assumed to be risk neutral throughout. With  $E_t[\bar{\Lambda}'_{t+1} | \mathcal{F}_t]$  standing for

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<sup>10</sup> In our terms, the bid–ask spread given by the Equation (8) in O'Hara and Oldfield(1986) is written as:

$$\text{the spread} = \text{constant} + (E[\tilde{x}_t U' | q_t > 0] - E[\tilde{x}_t U' | q_t < 0]) / 2\varphi E[U'].$$

It is clear that this formula is only a special case of (4). What it mainly catches is the effect of market order uncertainty on bid–ask spreads. The adverse selection term and the trader's belief do not show up in this formula because O'Hara and Oldfield assume the specialist's expectation about stock value does not vary with trades.

the expected marginal future profit generated by the trade in time  $t$ , we rewrite (3) as follows,

$$p_t = (1/2)E_t[y_t | \mathcal{F}_t] + (1/2)\{E_t[\tilde{V} | \mathcal{F}_t] + D_t\psi - \theta E_t[\bar{\Lambda}'_{t+1} | \mathcal{F}_t]\}. \quad (5)$$

(5) states that the transaction price functional for a risk-neutral monopoly specialist is an average of his estimate of the trader's reservation price and his own reservation price with no risk premium adjustment .

To make (4) econometrically worktable, we need to specify the information set  $\mathcal{F}_t$ . First, the specialist's information set contains the sequence of past transaction prices  $\{p_{t-k}, k \in 1, 2, \dots\}$ . Second, suppose prior to the trade in time, the specialist observes a signal about  $\tilde{V}$ . Denote this signal by  $s_t = \tilde{V} + \epsilon_t$  with assumption of  $\epsilon_t \sim \text{IIN}(0, \sigma_\epsilon^2)$ . Accordingly, the specialist's knowledge about  $\tilde{V}$  includes a up-to-date series of privately-observed signals  $\{s_{t-k}, k=0, 1, \dots\}$ <sup>11</sup>. Third, The past sequence of trades  $\{q_{t-k}, k=1, 2, \dots\}$  of course is a element in  $\mathcal{F}_t$ . The problem is whether the order that is currently under processing should be included in  $\mathcal{F}_t$ . The answer hinges on which trading protocol is assumed. In NYSE, block trades usually take place upstairs through negotiation while small trades are commonly committed downstairs through "retail". These two different trading protocols<sup>12</sup> give rise to two different definitions of the specialist's information set and two different concepts of equilibriums. In block trading, because both price and order are finally determined at the time of trade execution, the specialist is able to utilize the current order to update his prior while the trader has

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<sup>11</sup> Glosten(1989) and madrigal and Scheinkman(1990) attribute the specialist's "monopolistic" knowledge about the underlying asset value to his sole access to the "book" and his special knowledge about trading activity.

<sup>12</sup> This classification is directly motivated by O'Hara and Oldfield(1986,p363). They distinguish two types of call markets. One is what they named short lived call market where "the market maker sets price at the beginning of the trading period and executes all trades at the period's end." The other is what they named standard call market where "both price setting and execution occur at the end of the period."

chance to employ the current prices to revise his prior. So the specialist's information set in time  $t$  contains the trade in time  $t$ , and it is given by  $\{(q_t, s_t), (q_{t-1}, s_{t-1}, p_{t-1}), \dots\}$ . Bayesian rational expectations approach is usually employed to build the equilibrium. Good examples are Kyle(1985), Easley and O'Hara(1987) and Madhavan and Smidt(1991). In "retail" trading, prices are set up by the specialist before each transaction actually takes place, and at those prices the specialist is obligated to absorb all incoming market orders. The specialist is thus not able to utilize the information contained in the current order while traders have chance to "hit the quotes". So the specialist's information set in time  $t$  in "retail" trading protocol does not include the order in time, and it is given by  $\{(s_t), (q_{t-1}, s_{t-1}, p_{t-1}), \dots\}$ . The not-regret rational expectations approach is usually used to establish the equilibrium. Good examples are Glosten and Milgrom(1985), O'Hara and Oldfield(1986) and Stoll(1989). As O'Hara and Oldfield(1986) notice, there is no single depiction of the specialist's market clearing process since trading arrangements differ among exchanges. Following Madhavan and Smidt(1991), we choose Bayesian approach to formulate specialist pricing, and hence assume the specialist's information set to be  $\mathcal{F}_t = \{(q_t, s_t), (q_{t-1}, s_{t-1}, p_{t-1}), \dots\}$ .

### A. Bayesian Price Formation

By Bayesian price formation, we mean transaction price is formed through bilateral negotiation between trader and the specialist. The formation protocol of Bayesian rational expectations equilibrium price is supposed to be a bargaining process. At the very beginning of each transaction, the specialist announces his tentative prices that are subject to change afterwards, the prospective trader submits his tentative order according to his prior and the specialist's tentative prices. After updating his beliefs based on the trader's tentative order, if the specialist does not regret his tentative prices, then the trade is committed and the Bayesian rational expectations

equilibrium implemented. Otherwise, bargaining and revision continue until an agreement that no one regrets is reached. In conformity with this price formation scenario, this subsection obtains a testable price functional by specifying  $E_t[\bar{\Lambda}'_{t+1} | \mathcal{F}_t]$ ,  $E_t[Y_t | \mathcal{F}_t]$ , and  $E_t[\tilde{V} | \mathcal{F}_t]$ .

Following Hasbrouck(1988) and Madhavan and Smidt(1991), we postulate that the optimal control policy causes the inventory level to be a stationary process such that

$$E_t[\bar{\Lambda}'_{t+1} | \mathcal{F}_t] = -\iota[q_t - (I_{t-1} - I^*)], \quad (6)$$

where  $\iota$  is a parameter,  $I^*$  is the long-run mean or the desired level of inventory position at each trading period and  $I_{t-1} - I^*$  is the gap in time  $t$  between the pre-trade inventory and the desired level. When  $q_t > I_{t-1} - I^*$  ( $q_t < I_{t-1} - I^*$ ), the marginal future profits that is expected to generated by the trade in time  $t$  is negative (positive). It means the trader demands to buy more (less) shares from the specialist than the specialist is willing to sell or the trader wants to sell less (more) shares to the specialist than what the specialist needs to keep his inventory in the desire level. (6) implies that the best inventory control policy is such that current trades offset previous trades in expectation<sup>13</sup>

Knowing the trader's policy rule characterized by (1), the specialist can infer the trader's reservation price from the trader's order based on (7) given below.

$$E_t[y_t | \mathcal{F}_t] = p_t + q_t / \varphi. \quad (7)$$

Note that the noise component  $\bar{x}_t$  is assumed to be uncorrelated with  $\{q_{t-k}, s_{t-k}, s_{t-k}\}$  for

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<sup>13</sup> See Hasbrouck(1988) for more detail reasoning.

all  $k \in \mathcal{T}$  and  $q_t$  is in  $\mathcal{F}_t$ . The logic behind (7) is very intuitive. When a trader demands to buy (sell) shares at price  $p_t$  his reservation price is more likely to be greater (less) than  $p_t$ . The more his reservation price is higher (lower) than  $p_t$ , the more he wants to buy. Therefore, associating with a public purchase (sale), the specialist's inference about the trader's reservation price is larger (smaller) than the price and positively (negatively) relates to order size.

Let  $\lambda_{t-k}$  be the Bayesian weight for  $q_{t-k} - E_t[q_{t-k}]$  and  $\gamma_{t-k}$  be the Bayesian weight for  $s_{t-k} - E_t[s_{t-k}]$ . Using multivariate normal distribution theory to compute  $E_t[\tilde{V} | \mathcal{F}_t]$  results in Equation (8).

$$E_t[\tilde{V} | \mathcal{F}_t] = m_t + \sum_{k=0}^{\infty} \lambda_{t-k} (q_{t-k} - E_t[q_{t-k}]) + \sum_{k=0}^{\infty} \gamma_{t-k} (s_{t-k} - E_t[s_{t-k}]). \quad (8)$$

Note that  $\lambda_{t-k}$ 's and  $\gamma_{t-k}$ 's are composed of variances and covariances of  $\tilde{V}$ ,  $q_t$ ,  $s_t$ ,  $q_{t-k}$ ,  $s_{t-k}$ ,  $\dots$  but independent of their realizations.  $m_t$  is the unconditional stock value in time  $t$ . Term  $\sum_{k=0}^{\infty} \lambda_{t-k} (q_{t-k} - E_t[q_{t-k}])$  represents the information that is disclosed by trade innovations and reflected in prices. Term  $\sum_{k=0}^{\infty} \gamma_{t-k} (s_{t-k} - E_t[s_{t-k}])$  represents the information that is carried by non-trade innovations and incorporated in prices. In summary, (9) expresses the specialist's posterior expectation about the underlying stock value.

Substituting (6), (7) and (8) into (5) and rearranging the result yield the following linear transaction price functional.

$$p_t = m_t + \sum_{k=0}^{\infty} \lambda_{t-k} (q_{t-k} - E_t[q_{t-k}]) + \sum_{k=0}^{\infty} \gamma_{t-k} (s_{t-k} - E_t[s_{t-k}]) + D_t \psi + \iota \theta [q_t - (I_{t-1} - I^*)] + q_t / \varphi. \quad (9)$$

(9) is a linear transaction price functional. it is clear in (9) that the asymmetric information effect of quantity on price is persistent but the transaction cost, inventory

control and demand elasticity effects are transitory. It is also clear that under some proper assumptions order processing, asymmetric information, inventory control and demand elasticity are additive.

As has been mentioned in the introduction, the existing empirical studies in price–trade informational relations generally connect price with the trade as a whole. In contrast, (9) associates prices only with the innovation parts of trades – the deviations of trades from the viewpoint of the specialist. This notion is implicitly modeled theoretically by Easley and O’Hara(1987,1992) and examined empirically by Hasbrouck(1988,1991) and Jang and Venkatesh(1991). There are two possible explanations for the presence of inventory impacts  $\iota\theta[q_t-(I_{t-1}-I^*)]$  in (9). From the point of view of inventory control, a public purchase (sale) is more likely to cause the specialist’s inventory below (above) the desired level, so the specialist needs to add a positive (negative) component as inventory control instrument to the price for a public purchase (sale). From the point of view of inventory holding, the larger the size of an order, the larger the risk will be involved in absorbing this order and holding it as inventory. So as a risk premium, inventory holding costs associate with sizes of purchase orders positively and with the sizes of sale orders negatively. however, since the specialist has been assumed risk neutral, we interpret the term  $\iota\theta[q_t-(I_{t-1}-I^*)]$  as inventory control effect.

It might be for the first time in the literature that the term  $q_t/\varphi$  appears as a component of the trader’s reservation price in transaction price function. For referring convenience and in the sense that  $\varphi$  is the absolute value of slope of the trader’s demand schedule, let us call  $1/\varphi$  the demand elasticity effect of trades on prices. Since by construction  $\varphi = \zeta/\phi\text{Var}[\tilde{V} | \mathcal{F}_t]$ , this effect varies with the volatility of stock value, the risk attitude of the trader and the weight of the transaction price in forming trader’s posterior belief. Being more risk averse makes the trader less confident to make use of his private information and hence more sensitive to the change in

transaction price. Thus, from the point of view of demand elasticity, risk averse traders are charged higher price than risk neutral traders. On the other hand, a larger  $\zeta$  implies the trader draws he information contained in price more heavily and hence his posterior is closer to the effective transaction price and to the specialist's reservation price. Namely, a big  $\zeta$  narrows the gap in beliefs between trader and the specialist and consequently reduces the specialist's opportunity of earning profits. To maintain his profitable opportunity as great as possible when  $\zeta$  is close to one, the better strategy for the specialist is to lower ask price and raise bid price appropriately. To see this, recall that necessary condition for a public purchase (sale) to be committed on the specialist's account is  $MC_t > p_t > \zeta p_t + (1-\zeta)p_t > y_t$  ( $MC_t < p_t < \zeta p_t + (1-\zeta)p_t < y_t$ ).

### B. The Regression Equation

By the assumption that the specialist's problem is of stationary Markov, the price strategy expressed in (5) only depends upon the state where the specialist finds himself, and it has the same analytical form in all trading periods. That is to say, it also has

$$p_{t-1} = m_{t-1} + \sum_{k=1}^{\infty} \lambda_{t-k} (q_{t-k} - E_t[q_{t-k}]) + \sum_{k=1}^{\infty} \gamma_{t-k} (s_{t-k} - E_t[s_{t-k}]) + D_{t-1} \psi + \iota \theta [q_{t-1} - (I_{t-2} - I^*)] + q_{t-1} / \varphi.$$

In the sequel, let  $\delta_{t-k} \equiv s_{t-k} - E_t[s_{t-k}] \sim \text{IIN}(0, \sigma_\delta^2)$ , the change in transaction price  $\Delta p_t \equiv p_t - p_{t-1}$  can be written as,

$$\Delta p_t = \sum_{k=0}^{\infty} \lambda_{t-k} (q_{t-k} - E_t[q_{t-k}]) + \psi(D_t - D_{t-1}) + \theta \iota q_t + \varphi^{-1}(q_t - q_{t-1}) + \tilde{\mu}_t.$$

where  $\tilde{\mu} = \sum_{k=0}^{\infty} \gamma_{t-k} \delta_{t-k} + \tilde{\nu}.$  (10)

(10) is our basic econometric model for "Bayesian" price setting. It states that if the specialist is risk neutral, the successive change in transaction price is made of five components: adjustment for asymmetric information  $\sum_{k=0}^{\infty} \lambda_{t-k} (q_{t-k} - E_t[q])$ , adjustment for order processing cost  $\psi(D_t - D_{t-1})$ , adjustment for inventory control  $\theta \iota q_t$ , adjustment for demand elasticity  $\varphi^{-1}(q_t - q_{t-1})$  and inclusion of non-trade innovations  $\sum_{k=0}^{\infty} \gamma_{t-k} \delta_{t-k}$ .

The reason we treat  $\sum_{k=1}^{\infty} \gamma_{t-k}(s_{t-k} - E_t[s_{t-k}])$  as moving average disturbance process is because we have no direct knowledge about the specialist's private information other than trading history.

$\lambda$ 's are the marginal responses of price to the adverse information disclosed by order sequence. Positive  $\lambda$ 's imply that the more surprising the orders are to the specialist, the greater transaction price revision will be. Moreover, Easley and O'Hara(1987) demonstrate that both trade size and trade sequence matters (or both order flow uncertainty and information event uncertainty) influence price-trade relationship. (10) is apparently consistent with this notion. And what is more, it shows that trade sequence associating with information events matters when and only when orders are serially correlated over time. To see this, suppose the trade sequence is serially independent and we only consider the information effect of trades, (10) degenerates to  $p_t - p_{t-1} = \lambda_t(q_t - E_t[q_t])$ . This implies that only the current trade affects the current price movement.<sup>14</sup> As the pure order processing cost,  $\psi$  is expected to be positive.  $\theta u_{q_t}$  means that if the specialist's inventory control policy is assumed to be linear the transaction price revision due to inventory control only relates to the current trade  $q_t$ . In other words, from the point of view of inventory control, changes in prices are caused by changes in inventory position but not inventory position itself.  $\varphi^{-1}(q_t - q_{t-1})$  represents the demand elasticity effect of trades on prices. With respect to traders' demands for transaction services, it has  $(q_t - q_{t-1}) / (p_t - p_{t-1}) = \Delta q_t / \Delta p_t = -\varphi < 0$ . This is to say, holding other things constant, the corresponding

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<sup>14</sup> For a simple illustration, consider the following example in which only adverse information problem matters in specialist's pricing for the very first two periods. From (9), we write  $p_1 = m + \lambda_{11}(q_1 - E[q_1])$ ,  $p_2 = m + \lambda_{21}(q_1 - E[q_1]) + \lambda_{22}(q_2 - E[q_2])$ . Hence  $p_2 - p_1 = \{m + \lambda_{21}(q_1 - E[q_1]) - p_1\} + \lambda_{22}(q_2 - E[q_2])$ . If  $q_1, q_2$  are uncorrelated, i.e.,  $\lambda_{21} = \lambda_{11} = \text{Cov}[V, q_1] / \text{Var}[q_1]$ , it has  $\Delta p_2 = \lambda_{22}(q_2 - E[q_2])$ . However, this will not hold if  $q_1, q_2$  are correlated. To obtain more intuition on this issue, the interested reader is referred to O'Hara and Oldfield(1987).

change in transaction price will be  $-\varphi^{-1}(q_t - q_{t-1})$  if public order goes from  $q_{t-1}$  to  $q_t$ .

To gain more understanding about (10) and to see its distinctive features, a good way is to compare (10) with some existing well-known models. To do so, we first put these models in our terms. Equation (11) reproduces Madhavan and Smidt(1991)'s model, in which  $\pi$  denotes the weight the specialist places on his own prior belief when he uses order to build his posterior. Equation (12) is Roll(1984)'s model. Equation (13) reproduces Glosten and Harris(1988) model. And Equation (14) represents Ho and Macris(18) model with  $\xi(\tau_t - \tau_{t-1})$  standing for the time trend effect.

$$\Delta p_t = \kappa + \lambda q_t - (\nu/\pi)I_t + \mu I_{t-1} + (\psi/\pi)D_t - \psi D_{t-1}. \quad (11)$$

$$\Delta p_t = \psi(D_t - D_{t-1}). \quad (12)$$

$$\Delta p_t = \lambda q_t + \psi(D_t - D_{t-1}). \quad (13)$$

$$\text{Ln}(p_t/p_{t-1}) = \kappa + \mu(I_{t-1} - I_t) + \psi(D_t - D_{t-1}) + \xi(\tau_t - \tau_{t-1}). \quad (14)$$

Comparing with (11)–(14), Model (10) clearly has the following distinctive features. First, (10) includes the demand elasticity effect  $\varphi^{-1}(q_t - q_{t-1})$  but (11)–(14) do not. (11)–(14) model the specialist as a perfect competitor, assuming he earns zero profits on average. (10) models the specialist as a monopolist, assuming he uses quote as a wand to make profits. Second, the asymmetric information effect of trades on prices is generated by the demand order as a whole in (11)–(14). Instead, it is resulted from the relative demand order  $q_t - E_t[q_t]$  in (10). We believe that not all trades but surprising ones contain new information and drive the specialist to revise his prior. Third, (10) associates current price changes with past trade sequence. This is similar to Hasbrouck(1991) and consistent with the notion suggested in Easley and O'Hara(1978,1992). In addition, since the specialist is assumed to participate all transactions, (10) is simply attributes the price changes resulted from inventory control to the current trade.

### C. The Bid–Ask Spread

Roll(1984) considers a special case where there are no inventory effect and private information. Under the additional assumption that the sign of trade is serially independent and has zero mean, he formulates the implied bid–ask spread to be

$$s = 2\psi = 2(\text{Cov}(\Delta p_t, \Delta p_{t-1}))^{1/2}. \quad (15)$$

Madhavan and Smidt(1991) define the bid–ask spread to be the difference between the price if the order were to buy and the price if the order were to sell. Their formula for the implied bid–ask spread is

$$s(q_t) = 2(\psi + \lambda |q_t|), \quad (16)$$

where  $\lambda$  represents the information effect in their model. Following Madhavan and Smidt, from (9), we formulate the implied bid–ask spread to be

$$s(q_t) = 2[\psi + (\lambda + 1/\varphi + \iota) |q_t|], \quad (17)$$

According to (17), not only the information effect, but the inventory effect and the elasticity effect also present in the implied bid–ask spread.

## III. Empirical Results

### A. Data Description

The database used in the empirical studies is TORQ database, which is prepared at the NYSE and distributed by ISSM. It covers 144 NYSE stocks for the three months

November 1990 through January 1991. 10 most heavily traded stocks are selected for our investigation.

The method we employ to classify buy and sale transactions is similar to Ananth Madhavan and Smidt(1991). First, a transaction is classified as a public buy (sell) if the price is greater (less) than or equal to the last BBO prevailing ask (bid), or closer to the ask (bid) than to the bid (ask). Second, a transaction which is not identifiable according to the last prevailing BBO quotes is classified as a buy (sell) if the price is greater (less) than or equal to the next BBO prevailing ask (bid), or closes to the ask (bid) than to the bid (ask). Third, Trades which are identifiable neither by the last quotes nor the next quote are classified by the tick test. That is, at the first place, compared with the most recent transaction price, trades with positive (negative) price changes are assumed initiated by public buyers (sellers). At the second place, if its preceding trade were classified as a public buy (sell) and its price is equal to the price of its preceding trade, then, it is also assumed to be a public buy (sell).

Table II–A gives some descriptive statistics about the data. Several features of the data can be drawn from this table. (i)For all ten stocks except XON, the average number of public purchase is larger than the average number of public sale per day. This implies that our data sample reflects the Wall Street adage that the bull market is heavy while the bear market is light. (ii) $\Delta p$  has expected correlations with all its explanatory variables. That is,  $\text{Cov}(\Delta p, \Delta D) > 0$ ,  $\text{Cov}(\Delta p, E(q)) < 0$  and  $\text{Cov}(\Delta p, q) > 0$ . (iii)All explanatory variable are not seriously correlated. (iv)What looks puzzling is that the  $\text{Cov}(q, E[q])$  is positive for some stocks but negative for the other.

## B. Coefficient Estimates

For estimation, we rewrite (10) as follows.

$$\Delta p_t = \alpha + \beta q_t + \psi \Delta D_t + \lambda \theta_t + (1/\varphi) q_{t-1} + \bar{\mu}_t + \epsilon_t, \quad (18)$$

where  $\tilde{\mu}_t = \eta_t \tilde{\mu}_{t-1} + \tilde{\nu}_t$ .

Several remarks are given below. First note that  $\beta$  is the aggregate marginal effect of the trading volume on the transaction price. It is the sum of  $\lambda_t$ ,  $\theta_t$  and  $1/\varphi$ . Since this is the specialist's supply curve, it is expected to be positive. Second, the AR(1) process is assumed to take place of the non-trade innovations because we do not have this type of information available. Third,  $\theta_t$  replaces  $E_t[q_t]$  to represent the specialist's anticipation about the incoming trades. It is constructed in the following way. Let  $E_t[\hat{q}_t]$  be the anticipated trade of time  $t$  that the specialist obtains from the past trade sequence. Then  $\theta_t = E_t[\hat{q}_t]$  when  $|q_t| > |E_t[\hat{q}_t]|$ . Otherwise,  $\theta_t = q_t$ . This method is actually motivated by the following idea suggested by Easley and O'Hara (1987). Small trades are more likely carry no information, so they do not drive the specialist to revise his expectation. In contrast, large trades are more likely to be information-based, and hence cause higher ask prices and/or lower bid prices

The model is estimated using Box-Jenkins ARIMA method. The coefficient estimates are presented in Table II-B. The following points can be drawn from this table. (i) The change in the order sign has the most significant effects on the change in transaction prices. This effect, conventionally viewed as the order processing cost, is about 5 cents across all ten stocks. (ii) The aggregate marginal effect of the current trading volume do generate significant effect on price changes, but its magnitude varies from one stock to the other. The maximum is about 2.5 cents while the minimum is only about .3 cents. (iii) The pure information effect of trades  $-\lambda$  is reliably positive, implying block trades truly receive the worse price from the point of view of the asymmetric information hypothesis. (iv) What looks puzzling is the estimate of  $1/\varphi$ . Some are significantly positive while some are significantly negative. (v) As regards the inventory effects, it can be obtained by subtracting  $1/\varphi$  and  $\lambda$  from  $\beta$ . The results show that this effect is rather weak and the signs are inconsistent across stocks.

In summary, the empirical results do generally support the following statement: the trade volume generate the order processing effect, asymmetric information effect, inventory control effect and demand elasticity effect on the transaction price.

### **C. Decomposition of the Bid-ASk Spread**

Using the coefficients presented in Table II-b, we are able to decompose the aggregate effects of trades on prices into the order processing effect, asymmetric information effect, inventory control effect and demand elasticity effect. The results are recorded in Table II-C. The order processing effects and asymmetric information effects for all stocks are consistent with what we expect, but the inventory control effect and demand elasticity effects are again not so encouraging. Using (17), we compute the implicit bid-ask spreads and recorded them in Table II-C. The results show that the effective bid-ask spread measured by (17) for the median size of trades is roughly 11 cents, which is about 88 percent of the quoted bid-ask spread 12.5 cents.

### **Conclusion**

By introducing the asymmetric information approach to the conventional dynamic programming method, we have developed a synthetic model of transaction price formation. This model incorporates diverse price effects of trades on prices using a rather general price functional. It is so general that it treats most existing pricing models for the specialist as its special cases. Furthermore, using TORQ database and ARIMA method, we have estimated the above price functional under some proper assumptions. Despite some effects appear rather vague in estimation, the overall estimation does give our synthetic model for transaction price formation a very encouraging support from the point of empirical view.

## Table II-A Descriptive Statistics

This table provides descriptive statistics for the 10 stocks that are studied in this research. These stocks are most heavily traded in NYSE among the 144 stocks contained in the TORQ database. The time period is from November 1, 1990 through January 31, 1991. Using Lee-Ready criteria, trades are classified as public purchases and public buys.  $\Delta p$  is the change in transaction prices.  $\Delta D$  is the change in the sign of trades.  $q_t$  is the trade or the signed volume in time  $t$ .  $\theta_t$  is the specialist's prediction about the trade of time  $t$ .

Ticket Symbols	BA	FLP	FNM	GE	GLX	IBM	MO	SLB	T	XOM
<b>Average Number of Trades Per Day</b>										
Public Purchases	172.7	131.8	167.0	350.5	140.8	298.8	307.8	99.2	322.9	104.3
Public Sales	128.9	52.5	142.0	274.7	88.4	295.5	306.2	80.5	307.0	165.7
Sum	301.6	184.3	309.1	625.4	229.2	594.3	613.9	179.7	629.9	269.9
<b>Average Size Per Order</b>										
<b>Public Purchases:</b>										
the First Quartile	200	200	600	200	200	700	200	300	100	300
Median	1000	300	1800	500	400	1500	700	900	300	1000
Third Quartile	2500	700	5000	6000	1100	3200	2500	2300	1000	4700
<b>Public Sales:</b>										
First Quartile	300	200	500	250	200	400	200	300	100	200
Median	1000	500	1200	450	400	1000	500	1000	300	550
Third Quartile	3000	1200	4000	5000	1400	2300	2000	2800	1000	2000
<b>Both:</b>										
First Quartile	300	200	500	200	200	500	200	300	100	200
Median	1000	400	1500	500	400	1100	600	900	300	800
Third Quartile	2700	800	5000	5500	1200	3000	2000	2500	1000	2800
<b>Pearson Correlation Coefficients</b>										
Cov( $\Delta p, \Delta D$ )	.7167	.9206	.6809	.8168	.8521	.5196	.8861	.6769	.9327	.8737
Cov( $\Delta p, \theta_t$ )	-.2042	-.3291	-.1929	-.2129	-.3314	-.1751	-.3371	-.3156	-.2953	-.2977
Cov( $\Delta p, q_t$ )	.4053	.1565	.3773	.2442	.1939	.3577	.2244	.2758	.1663	.1736
Cov( $\Delta D, \theta_t$ )	-.2631	-.3408	-.2801	-.1998	-.3800	-.2180	-.3528	-.3414	-.2991	-.2957
Cov( $\Delta D, q_t$ )	.1867	.1077	.1853	.1098	.1574	.1541	.1335	.1515	.0843	.1177
Cov( $\theta_t, q_t$ )	-.0359	-.0087	.0184*	-.0567	.0652	-.0109	-.0210	-.0288	-.0078	-.0088*

Data Source: TORQ Database.

\*: significant at probability .01 or above.

Program for computing the correlation: the CORR procedure in SAS.BASICS.

Table II-B Coefficient Estimates

This Table reports the coefficient estimates (with t-values in the parentheses) for the model (10)

$$\Delta p_t = \alpha + \beta q_t + \psi \Delta D_t + \lambda \theta_t + (1/\phi) q_{t-1} + \eta_t / (1-\phi) + \epsilon_t$$

where  $\Delta p_t$  is the observed transaction price change in time  $t$ .  $q_t$  and  $q_{t-1}$  are the observed trades (the signed volumes) in time  $t$  and  $t-1$ , respectively.  $\theta_t$  is the specialist's prediction about the trade of time  $t$ . When  $|q_t| > |E_t[q_t]|$ ,  $\theta_t = E_t[q_t]$ , otherwise  $\theta_t = q_t$ , where  $E_t[q_t]$  is the anticipated trade of time  $t$  obtained by the stepwise autoregressive method of the FORECAST procedure in SAS.  $\Delta D_t$  is the change in transaction type or volume sign.  $\eta_t$  is the AR(1) innovation of  $\Delta p_t$ . The model is estimated using Box-Jenkins ARIMA method. The sample contains ten most heavily traded NYSE stocks in TORQ database which includes 144 stocks and covers period of November 1990 through January 1991. Volume is scaled by  $10^4$ , and the coefficients  $\beta$ ,  $\lambda$  and  $1/\phi$  should be interpreted accordingly.

Symbol	$\alpha$	$\psi$	$\beta$	$\lambda$	$1/\phi$	$\phi$	OBS/R <sub>2</sub>
BA	0.0025 ( 4.2)	0.050 (125)	0.0238 (59.8)	-0.013 ( 5.5)	0.002 ( 5.9)	-0.100 (13.8)	18650 0.510
FLP	0.0015 ( 4.3)	0.059 (228)	0.0050 (16.5)	-0.018 ( 5.8)	0.001 ( 4.4)	-0.092 ( 9.9)	11506 0.848
FNM	0.0013 ( 2.2)	0.041 (112)	0.0125 (50.9)	-0.006 ( 3.6)	-0.000 ( 1.5)	-0.104 (14.3)	18671 0.460
GE	0.0023 ( 9.5)	0.053 (266)	0.0109 (53.5)	-0.017 (13.5)	-0.001 ( 6.7)	-0.096 (19.1)	30383 0.669
GLX	0.0010 ( 2.0)	0.056 (172)	0.0046 (12.3)	-0.007 ( 3.1)	-0.005 (13.0)	-0.052 ( 6.2)	14261 0.726
IBM	0.0104 (15.7)	0.040 ( 99)	0.0241 (65.6)	-0.052 (18.5)	0.001 ( 1.4)	-0.144 (26.7)	33855 0.272
MO	0.0032 (10.9)	0.055 (329)	0.0077 (47.3)	-0.017 (13.2)	-0.001 ( 3.4)	-0.146 (28.8)	37958 0.785
SLB	0.0159 (12.3)	0.055 ( 82)	0.0179 (26.4)	-0.084 (14.1)	0.001 ( 1.5)	-0.015 ( 1.6)	11039 0.465
T	0.0014 ( 8.7)	0.059 (479)	0.0045 (50.0)	-0.010 (11.8)	-0.000 ( 4.1)	-0.124 (24.9)	39451 0.870
XON	0.0051 (11.1)	0.053 (207)	0.0031 (20.4)	-0.020 (12.4)	0.000 ( 1.8)	-0.180 (23.6)	16796 0.765

Data Source: the TORQ database described in Table II-A.  
Computing Program: ARIMA procedure in SAS.ETS.

**Table II-C Decomposition of the Effect of Trades on Prices and Measured Bid-Ask Spreads**

Using the coefficient presented in Table II-B, this table first decomposes the aggregate effect of trades on prices into the order processing effect, asymmetric information effect, inventory control effect and demand elasticity effect. It then computes the bid-ask spread according to the Equation (15) in the text and the measure provided by Madhavan and Smidt(1991). The quoted bid-ask spread is also included in the table for reference. To be specific, the order processing effect is given by  $\psi$  estimate. The asymmetric information is given by  $-\lambda$ . The inventory effect is computed by  $\iota = \beta - \lambda - 1/\phi$ . The demand elasticity effect is given by  $1/\phi$ . All effects are adjusted by the median order size. Equation (15) is:

$$\text{the Spread} = 2[\psi + (\lambda + 1/\phi + \iota)q].$$

Madhavan and Smidt(1991)'s Measure is:

$$\text{the Spread} = 2[\psi + \lambda q].$$

Symbol	Decomposition of The Effects of Trades on Prices				Measured Bid-Ask Spreads By		
	Order Processing	Asymmetric Information	Inventory Control	Demand Elasticity	Equation (15)	Madhavan & Smidt	Bid-Ask Quotes
BA	0.5012	0.0130	-0.000032	-0.0024	0.1050	0.0976	0.125
FLP	1.4758	0.0184	0.000016	-0.0013	0.1185	0.1166	0.125
FNM	0.2707	0.0062	0.000002	0.0004	0.0850	0.0794	0.125
GE							
GLX	1.4010	0.0073	-0.000037	0.0048	0.1125	0.1115	0.125
IBM	0.3649	0.0522	0.000014	-0.0005	0.0856	0.0688	0.125
MO	0.9151	0.0169	-0.000005	0.0006	0.1107	0.1078	0.125
SLB	0.6129	0.0843	0.000067	-0.0010	0.1136	0.0952	0.250
T	1.9647	0.0104	-0.000002	0.0004	0.1182	0.1173	0.125
XON	0.6678	0.0198	0.000009	-0.0003	0.1073	0.1037	0.125

Data Source: The data described in Table II-A and the coefficients reported in Table II-B.

### Chapter Three

## Estimating the Bid and Ask Submarkets of Stock Exchange

According to Demsetz(1968), the specialist–organized stock exchange like NYSE is composed of two submarkets. As shown in Figure 1, one is the ask submarket where traders purchase shares from the the specialist. The other is the bid submarket where traders sell shares to the specialist. Each submarket has its own demand and supply curves and hence has its own equilibrium. Estimating an analytical model of those two submarkets from both demand and supply sides of dealership services is the major task of this research. This type of study has not been attempted in existing literature.

In the past three decades, there have appeared an immense number of empirical works on price–volume relationship in the dealer/specialist market. All of those works are essentially born to find positive correlation between transaction price (changes) and (signed) trading volumes based on the specialist’s supply functional of dealership service. For example, the central concern of numerous empirical studies reviewed by Karpoff(1987) is with whether the transaction price change in absolute value or per se is significantly associated with trading volume. Most recent works are based upon the specialist’s pricing functional that attempt to disentangle the asymmetric information, inventory control and order processing impacts of trades on prices (Glosten and Harris(1988), Hasbrouck(1988,1991), Stoll(1989), Madhavan and Smidt(1991) among others).<sup>1</sup>

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<sup>1</sup> There have emerged three major theoretical approaches to studying price–volume relation. One of such is what we called information revelation approach. This approach includes sequential information arrival hypothesis (Copland(1976,1977), Jennings, Starks and Fellingham(1981)) and Jennings and Barry(1984)), mixture of distribution hypothesis (Epps and Epps(1976), Clark(1973),

The eligibility of measuring the price–volume relation by estimating the specialist’s price functional of some form rests on the assumption that stock exchanges always take place along the specialist’s supply curve of transaction service. For this assumption to hold, it is required that stock markets always operate at their equilibriums or that traders’ demands for transaction services are perfectly elastic with respect to prices. Notwithstanding, the persistence hypothesis of stock market equilibrium does not possess much guaranty from theoretical point of view and has been receiving continuous empirical challenges (e.g., Beja and Goldman(1979), Garbade and Lieber(1977), Cohan, Maier and Schwartz(1977) among others). Likewise, the infiniteness hypothesis of stock price elasticities is also ambiguous on both theoretical and empirical grounds. A series of works has evidenced imperfect elasticities of stock prices (Asquith and Mullins(1986), Barclay and Litzenberger(1988), Loderer, Sheehan and Kadlec(1990), Rozeff(1986), Ritter(1988) and Lakonishok and Vermaelen(1986)). As a matter of empirical research, if there are a remarkable portion of observed transactions that actually lie in the curve of traders’ demands for immediacy services, estimating the price–volume relation solely based on the specialist’s supply curve of immediacy service would be either somewhat misleading or even meaningless.

However, estimating the price–volume relation from both sides of traders’ demands for and the specialist’s supply of immediacy service does not require hypotheses of equilibrium persistence and elasticity infiniteness. And by doing so, not only can we understand how specialist strategically serves stock exchanges, but we also

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Harris(1983)) and asymmetric information or adverse selection hypothesis (Bagehot(1971), Copland and Galai(1983), Glosten and Milgrom(1985), Kyle(1985), Easley and O’Hara(1987), Admati and Pfleiderer(1987), Glosten(1989), Foster and Viswanathan(1989), Seppi(1990), and Madrigal and Scheinkman(1990)). One is inventory control approach (Stoll(1978), Amihud and Mendelson(1980), Zible(1981), Ho and Stoll(1981, 1983), and O’Hara and Oldfield(1986)). The other is order processing approach (Demsetz(1968), Tinic(1973), Roll(1984)). All those three approaches are born to assert how the specialist pricing is affected by trades.

can obtain some knowledge about how traders actually behave in security markets; not only can we know how prices are affected by order flows, but we also can gain some insights into how order flows are influenced by prices. Meanwhile, it could also shed lights on testing hypotheses of equilibrium persistence and elasticity infiniteness. In brief, estimating both demand and supply schedules of transaction service simultaneously can fulfill four tasks at the same time.

The first task of course is the major conventional subject of empirical researches in market microstructure — estimating the specialist's pricing functional, finding out the price–volume relation from the specialist's perspective and testing various theories about the specialist's pricing. What differs from the conventional approach is that we carry out this task in a systematic market framework rather than along the side of the specialist's supply.

The second task is to estimate trader's demand curve for transaction service. As far as we are aware of, this task has not been attempted by existing empirical microstructure literature although there have appeared some empirical works that relate trading volume to traders' expectations in the reduced–form. (e.g., Ajinkya, Atiase and Gift(1987), Comiskey, Walkling and Weeks(1987), Park(1988), Ziebart(1987), Lang and Litzenberger(1989) among others) The fulfillment of this task allow us to test some respect of the mean–variance investment theory using transaction data and to see how trading orders are generated in the stock market. Moreover, how trading volume and transaction price are determined and affect each other are investigated simultaneously from the perspectives of both traders and the specialist.

The third task is to test the persistence hypothesis of market equilibrium. The debate over whether stock exchanges always operate at equilibriums is pretty much a matter of empirical investigation. Some works have been devoted to show the persistent deviation of observed prices from their equilibrium values (e.g., Beja and Goldman (1979), Garbade and Lieber(1977), Cohen, Maier, Schwartz(1977) among

others). Those works are generally carried out from the time-series properties of transaction price process. In contrast, we test the equilibrium hypothesis by examining how sensitive are transaction prices to order flows.

The fourth task is to test the finiteness hypothesis of stock price elasticity. Traditional tests have rested on the belief that a negative relation between price and quantity changes at the time of new stock offering indicates downward sloping of demand schedules (Loderer, Cooney and Drunen(1991)). These tests are usually conducted under the assumption that the demand curve for a common stock is inert to effects of information, transaction cost and what the like. This method involves several difficulties. First, it is evidenced that stock offerings do on average convey adverse information about the firms' value. Second, no relation might be found between price and quantity if the negative effects of an outward shift of supply curve on price is approximately offset by the positive effects of an accompanying outward movement of demand curve on price<sup>2</sup>. However, through estimating both demand and supply curves simultaneously, we are able to test if demand curve is downward sloping in secondary market without involving such difficulties. The other advantage of our approach is its capable of investigating stock price elasticities of both demand and supply at the same time.

The traders' transaction policy and the specialist's pricing rule have been well analyzed in the literature. The theoretical contribution of this research is that it introduces a systematic market model for bid and ask submarkets of the sepcialist-organized stock exchange. In empirical investigation, we employ the disequilibrium methods suggested by Fair and Jaffee(1972) and others to facilitate estimating the market models. The empirical contribution of our paper first lies in its estimating both demand and supply schedules of bid and ask submarkets

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<sup>2</sup> Seeing those serious difficulties, Lodere, Cooney and Drunen try to eliminating information effects when conducting the traditional tests in their 1991's paper.

simultaneously.

The rest of this paper is arranged as follows. Section I introduces a market model for bid and ask submarkets of stock exchange. Section II deals with some estimation techniques. Section III presents major empirical results. The paper is finally concluded with some comments on the limitations of our technology and suggestions for further research.

### I. A Model for Bid and Ask Submarkets of Stock Exchange

Consider a stock market where there are a common stock and a risk-free treasury bond that are traded among individual investors through a market organizer — the specialist. The common stock has a random payoff  $\tilde{V} \sim N(m_v, \sigma_v^2)$  and the risk-free bond yields a constant return  $R$ . Transactions take place according to the following protocol. At a point in time, a trader (a group of traders may also be considered) with a demand schedule in his mind arrives at the specialist's post and requests for transaction service. His demand schedule is made ready to reflect the information that could be possibly revealed by the specialist's prices. On the trader's request, the specialist provides the trader with a price schedule. This schedule allows the specialist to incorporate the asymmetric information and inventory control effects of the incoming trade into the transaction price. By matching the specialist's price schedule with his own demand schedule, the trader then decides how much to buy from or to sell to the specialist. After the transaction, both the trader and the specialist may revise their respective strategies for the subsequent trade conditional upon new information revealed by the transaction and/or newly-arrived signals. According to this framework, in what follows, we first formulate the trader's trading strategy, then derive the specialist's pricing rule, and finally integrate them together to obtain a

market model for bid and ask submarkets.

Let  $A$  be the ask transaction price and  $B$  the bid transaction price. Symbolize demand for stock by  $D$  and supply of stock by  $S$ . Let subscript  $a$  stand for the submarket and  $b$  for the submarket. Also let subscript  $i$  indicate trader and  $m$  the specialist.

### A. The Trader's Trading Strategy

Individual investor  $i$  is assumed to have negative exponential preference. He initially holds  $X_i$  shares of the common stock and  $Z_i$  amount of treasury bond at Pareto optimal level. Prior to trade at time  $t$ , he may observe a private signal  $s_i = \tilde{V} + \epsilon_i$  with  $\epsilon_i \sim N(0, \sigma_\epsilon^2)$ . Upon this signal, he may arrive at the specialist's post for updating his initial optimal portfolio holdings  $(X_i, Z_i)$  in order to maximize the expected value of his terminal wealth  $\tilde{W}_i$  in the next period. On the account of the normal distribution of  $\tilde{V}$ , investor  $i$ 's problem is represented by the following mean–variance expression.

$$\begin{aligned} & \text{Max}_{\{x_i\}} -\text{Exp}\{-E_i[\tilde{W}_i | \Omega_i] - \psi_i \text{Var}[\tilde{W}_i | \Omega_i]\} \\ & \text{subject to } \tilde{W}_i = [X_i + (I_a - I_b)x_i]\tilde{V} + (Z_i - z_i)R, \\ & \text{and } z_i = x_i(I_a A - I_b B). \end{aligned} \quad (1)$$

$I_a$  and  $I_b$  are indicator functions. To be specific,  $I_a = 1$  if  $E_i[\tilde{V} | \Omega_i] > A$ , and  $I_a = 0$  otherwise.  $I_b = 1$  if  $E_i[\tilde{V} | \Omega_i] < B$  and  $I_b = 0$  otherwise. With  $A \geq B$ ,  $\{I_a = 1\} \cap \{I_b = 1\} \in \emptyset$ .  $\Omega_i$  is investor  $i$ 's information endowment. The current price of risk–free bond is normalized as one.  $X_i + (I_a - I_b)x_i$  is the optimal holdings of the risk security, of which  $x_i$  is investor  $i$ 's demand for transaction service in time  $t$ . A positive  $x_i$  means purchase and a negative  $x_i$  means sale. In the Appendix, we derive the public purchase  $D_i$  and the public sale  $S_i$  at a point in time as follows.

$$D_{ai} = \frac{E_{ai}[\tilde{V}|\Omega_{ai}] - RA}{\psi \text{Var}[\tilde{V}|\Omega_{ai}]} - X_{ai} \quad \text{as } I_a=1, \quad (2A)$$

$$S_{bi} = -\frac{E_{bi}[\tilde{V}|\Omega_{bi}] - RB}{\psi \text{Var}[\tilde{V}|\Omega_{bi}]} + X_{bi} \quad \text{as } I_b=1. \quad (2B)$$

Subscript ai stands for investor i who demands for service of stock purchase and bi for investor i who demand for service of stock sale.  $D_{ai}=x_i$  if investor i commits purchase while  $S_{bi}=-x_i$  if he wants to sell shares.

A few notes on (2) are in order. In deriving these two equations, the following two assumptions have been implicitly made for not to involving complication. Regardless of his initial holdings, a trader buys no shares when his expectation is lower than the ask price and sell no shares when his expectation is higher than the bid price. The bid and ask submarkets are supposed to have no interaction and possess separate equilibriums. Second,  $A \geq B$  implies that  $E_{ai}[\tilde{V}|\Omega_{ai}] \geq E_{bi}[\tilde{V}|\Omega_{bi}]$ . Third, in (2), transaction order is simply the difference between the optimal and initial holdings.

## B. The Specialist's Pricing Rule

Assume that the specialist exhibits risk-neutral preference and his market is competitive. He is initially uninformed but knows traders' general trading strategy. Hypothetically, the specialist uses Bayesain rule to set his price quote conditional upon all the up-to-date information available to him in time t. Because of his risk neutrality and the competitiveness of his market, the specialist's prudent pricing rule must obey the principle that price equals the conditional expectation value of the underlying asset. So the crux of deriving the specialist's pricing rule is to formulate his pre-posterior expectation about  $\tilde{V}$ . To remind, the specialist has no knowledge about the trader's pre-trade holdings although he presumably knows the trader's optimal demand function. In light of Kyle(1985), Glosten and Milgrom(1985), Easley and O'Hara(1987) and Glosten(1989), we derive the specialist's pricing rule in the

following.

First, let  $E_m[E_{ai}[\tilde{V}|\Omega_{ai}]|D_{ai}]$  and  $E_m[E_{bi}[\tilde{V}|\Omega_{bi}]|D_{bi}]$  be the specialist's inference about the buyer's and the seller's expectations conditional upon their demand orders, respectively. To specialist, these inferences are signals for the underlying stock value. It thus has,

$$E_m[E_{ai}[\tilde{V}|\Omega_{ai}]|D_{ai}] = \tilde{V} + \nu_a \quad \nu_a \sim \text{IIN}(0, \sigma_{\nu_a}^2), \quad (3A)$$

$$E_m[E_{bi}[\tilde{V}|\Omega_{bi}]|S_{bi}] = \tilde{V} + \nu_b \quad \nu_b \sim \text{IIN}(0, \sigma_{\nu_b}^2). \quad (3B)$$

Second, the specialist will simply not to provide any service if doing so makes him a loss in expectation. On the other hand, any strategy that would bring any positive profits to the specialist cannot be implemented in a Bertrand-type competitive market. Therefore, only the strategies that would bring zero expected profits from providing transaction services are both acceptable to the specialist and realizable in the competitive environment. With positive order flow almost surely, these strategies should be characterized in the equilibrium by

$$A = E_m[\tilde{V}|D_{ai}]/R, \quad (4A)$$

$$B = E_m[\tilde{V}|S_{bi}]/R. \quad (4B)$$

Finally, using (3) and (4), the following rules for the specialist are derived in the Appendix.

$$A = \frac{1}{R} \{m_v + k_a(S_{am} + E_m[\tilde{X}_{ai}|D_{ai}])\}, \quad (5A)$$

$$B = \frac{1}{R} \{m_v - k_b(D_{bm} + E_m[\tilde{X}_{bi}|S_{bi}])\}, \quad (5B)$$

where  $k_a = \sigma_{\nu_a}^2 \psi \text{Var}[\tilde{V}|\Omega_{ai}]/\sigma_v^2$  and  $k_b = \sigma_{\nu_b}^2 \psi \text{Var}[\tilde{V}|\Omega_{bi}]/\sigma_v^2$ .  $m_v$  is the specialist's

expectation prior to the trade in time  $t$ . Note that the trader's demand associates with the specialist's supply and the trader's supply is associates with the sepcialist's demand. With the specialist being uninformed initially, (5) reveals the essence of the asymmetric information approach to the specialist's pricing — transaction prices are the functions of order flows. It holds that  $A \geq M_v \geq B$ . Parameters  $k_a$  and  $k_b$  measure the "depth" of the market. The larger the variance of  $\nu$ , the more risk averse of the trader and the less precise the trader's private signal, the more information a given order flow might carry, so the wider the deviation of transaction price from  $m_v$ .

### C. Equilibriums of Bid and Ask Submarkets

Bringing (2) and (5) together, we obtain two equation systems.

For the ask submarket,

$$D_{ai} = \frac{E_{ai} [\tilde{V} | \Omega_{ai}] - RA}{\psi \text{Var} [\tilde{V} | \Omega_{ai}]} - X_{ai}, \quad (6-1)$$

$$A = \frac{1}{R} \{m_v + k_a(S_{ai} + E_m[\tilde{X}_{ai} | D_{ai}])\}, \quad (6-2)$$

$$Q_{ai} = \text{Min}\{S_{ai}, D_{ai}\}. \quad (6-3)$$

For the bid submarket,

$$S_{bi} = \frac{E_{bi} [\tilde{V} | \Omega_{bi}] - RB}{\psi \text{Var} [\tilde{V} | \Omega_{bi}]} + X_{bi}, \quad (7-1)$$

$$B = \frac{1}{R} \{m_v - k_b(D_{bi} + E_m[\tilde{X}_{bi} | S_{bi}])\}, \quad (7-2)$$

$$Q_{bi} = \text{Min}\{S_{bi}, D_{bi}\}. \quad (7-3)$$

In the sequel, the market equilibrium price functionals, if ever exists any, would be characterized by the following equations.

For the ask submarket,

$$RA^* = \omega_a m_v + (1 - \omega_a) \{E_{ai}[V_{ai} | \Omega_{ai}] - \psi \text{Var}[\tilde{V} | \Omega_{ai}](X_{ai} - E_m[\tilde{X}_{ai} | \Omega_{ai}])\}, \quad (8-1)$$

$$Q_a^* = \frac{E_{a_i}[\tilde{V} | \Omega_{a_i}] - m_v}{\psi \text{Var}[\tilde{V} | \Omega_{a_i}] + k_a} - \{\omega_a \bar{X}_{a_i} + (1 - \omega_a) E_m[\bar{X}_{a_i} | \Omega_{a_i}]\}. \quad (8-2)$$

For the bid submarket,

$$RB^* = \omega_b m_v + (1 - \omega_b) \{E_{b_i}[V_{b_i} | \Omega_{b_i}] - \psi \text{Var}[\tilde{V} | \Omega_{b_i}](X_{b_i} - E_m[\bar{X}_{b_i} | \Omega_{b_i}])\}. \quad (9-1)$$

$$Q_b^* = \frac{E_{b_i}[\tilde{V} | \Omega_{b_i}] - m_v}{\psi \text{Var}[\tilde{V} | \Omega_{b_i}] + k_b} - \{\omega_b \bar{X}_{b_i} + (1 - \omega_b) E_m[\bar{X}_{b_i} | \Omega_{b_i}]\}. \quad (9-2)$$

Equations (8) and (9) describe the equilibriums of the bid and ask submarkets, in which  $\omega_a = \psi \text{Var}[\tilde{V} | \Omega_{a_i}] / \{\psi \text{Var}[\tilde{V} | \Omega_{a_i}] + k_a\}$  and  $\omega_b = \psi \text{Var}[\tilde{V} | \Omega_{b_i}] / \{\psi \text{Var}[\tilde{V} | \Omega_{b_i}] + k_b\}$ . Be the specialist uninformed, the possible equilibrium price of a common stock exchanged through the specialist's service is a discounted weighted average of the specialist's prior expectation  $m_v$ , and the trader's conditional expectation plus (or minus) a specialist's inference error term  $\psi \text{Var}[\tilde{V} | \Omega_{a_i}](X_{a_i} - E_m[\bar{X}_{a_i} | \Omega_{a_i}])$  or  $\psi \text{Var}[\tilde{V} | \Omega_{b_i}](X_{b_i} - E_m[\bar{X}_{b_i} | \Omega_{b_i}])$ . The equilibrium transaction volume is a function of the difference between the trader's expectation and the specialist's expectation. Non-zero  $X_{a_i} - E_m[\bar{X}_{a_i} | \Omega_{a_i}]$  and  $X_{b_i} - E_m[\bar{X}_{b_i} | \Omega_{b_i}]$  play a crucial role in protecting the specialist-organized market from Stiglitz-Grossman impossibility (Stiglitz and Grossman(1980)).

## II. Estimation Techniques

In this section, we cope with some technical problems so that the models derived in the last section can be actually estimated. Among these problems are the formation of expectations, disequilibrium transformation of the bid and ask submarket models and classification of transactions in the data.

### A. Expectation Formation

It is not easy, if not impossible at all, to acquire traders' and the specialist's expectations for the time the trade under consideration is committed. Some proxies must be used. It is generally admitted that public buyers have higher expectation about the underlying asset's value than public sellers (Milgrom and Glosten 1985). Traders do not share the same expectation with the specialist, otherwise, there would be no trades on the specialist's own account taking place. In this connection, we take the PDLREG forecast about the average ask price of in the periods subsequent to the trade in time  $t$  as the proxy of public buyers's expectation, and the PDLREG forecast about the average bid price as the proxy of public seller's expectation. The specialist's expectation prior to the trade in time  $t$  is stipulated to be the PDLREG forecast of the average of ask and bid quotes in the subsequent periods. Thus, we have

$$E_{ai}[\tilde{V} | \Omega_{ai}] \equiv \hat{V}_a = \hat{\eta}_0 + \sum_{j=1}^n \hat{\eta}_{t-j} A_{t-j}, \quad (10)$$

$$E_{bi}[\tilde{V} | \Omega_{bi}] \equiv \hat{V}_b = \hat{\zeta}_0 + \sum_{j=1}^n \hat{\zeta}_{t-j} B_{t-j}, \quad (11)$$

$$E_m[\tilde{V} | \Omega_m] \equiv \hat{V}_m = \hat{\xi}_0 + \sum_{j=1}^n \hat{\xi}_{t-j} q_{t-j}, \quad (12)$$

where,  $V_a = (\sum_{j=1}^n A_{t+j}/n)$ ,  $V_b = (\sum_{j=1}^n B_{t+j}/n)$ ,  $V_m = (\sum_{j=1}^n (q_{t+j}/n))$  with  $q$  being the average of bid and ask quotes. It virtually holds that  $\hat{V}_a \geq \hat{V}_m \geq \hat{V}_b$ .

## B. Disequilibrium Transformation

Following the convention of disequilibrium estimation modeling, we simply convert systems (6) and (7) into (13) and (14). In constructing, we have excluded the trader's pre-trade holdings by assuming it equal to zero.

For the ask submarket:

$$D_{ait} = \phi_{ai} + \theta_{ai} \hat{V}_{at} + \rho_{ai} A_t + \tilde{u}_{ait}, \quad (13-1)$$

$$A_t = \phi_{am} + \theta_{am} \hat{V}_{mt} + \rho_{am} S_{amt} + \tilde{u}_{amt}. \quad (13-2)$$

$$Q_{at} = \text{Min}\{D_{ait}, S_{amt}\}, \quad (13-3)$$

$$A_t - A_{t-1} = (D_{ait} - S_{amt}) / \lambda_{ai}, \quad \text{as } D_{ait} > S_{amt}, \quad (13-4)$$

$$A_t - A_{t-1} = \lambda_{am}(D_{ait} - S_{amt}) \quad \text{as } D_{ait} < S_{amt}. \quad (13-5)$$

For the bid submarket:

$$S_{bit} = \phi_{bi} + \theta_{bi} \hat{V}_{bt} + \rho_{bi} B_t + \tilde{u}_{bit}, \quad (14-1)$$

$$B_t = \phi_{bm} + \theta_{bm} \hat{V}_{mt} + \rho_{bm} D_{bmt} + \tilde{u}_{bmt}. \quad (14-2)$$

$$Q_{bt} = \text{Min}\{D_{bmt}, S_{bit}\}, \quad (14-3)$$

$$B_t - B_{t-1} = (D_{bmt} - S_{bit}) / \lambda_{bi} \quad \text{as } D_{bmt} > S_{bit}, \quad (14-4)$$

$$B_t - B_{t-1} = \lambda_{bm}(D_{bmt} - S_{bit}) \quad \text{as } D_{bmt} < S_{bit}, \quad (14-5)$$

$\phi_s$ ,  $\theta_s$ ,  $\rho_s$  and  $\lambda_s$  are parameters. To be specific,  $\theta_{ai} = \rho_{ai} = 1 / (\psi \text{Var}[\tilde{V} | \Omega_{ai}])$ ,  $\theta_{bi} = \rho_{bi} = 1 / (\psi \text{Var}[\tilde{V} | \Omega_{bi}])$ ,  $\theta_{am} = \theta_{bm} = 1/R$ ,  $\rho_{am} = k_a$  and  $\rho_{bm} = k_b$ .  $\tilde{u}_{ai}$ ,  $\tilde{u}_{am}$ ,  $\tilde{u}_{bi}$  and  $\tilde{u}_{bm}$  are standard white noises. (13-3) and (14-3) are the Min conditions usually imposed in disequilibrium setting. Taking the public purchase as an example. If public purchase demand is greater than the specialist's supply, the observed transaction then lies in the specialist's supply curve. Inversely, if the public purchase demand is less than the specialist's supply, the observed transaction then lies in the trader's demand curve. (13-4)–(13-5) and (14-4)–(14-5) depict price responses to the disequilibrium of the stock exchange. Note that we allow traders' response to market innovations is different from the specialist's, and the price adjustment speed of the demand side different from that of the supply side. Namely, we do not restrict  $\lambda_{ai} = \lambda_{am} = \lambda_{bi} = \lambda_{bm}$ . Also note that no disturbance terms have been added to (13-3)–(13-5) and (14-3)–(14-5). This leads systems (13) and (14) to what Fair and Jaffee(1972) called the quantitative models. In these models, sample separation between demand and supply are supposed to be definitely determined by the changes of associating prices. We choose this type of model simple because its easy tractability. Following Jaffee(1974) again, we obtain two simultaneous equation systems (15) and (16) that can be estimated by non-maximum

likelihood methods.

For the public purchase submarket:

$$Q_{at} = \phi_{ai} + \theta_{ai} \hat{V}_{at} + \rho_{ai} A_t + \lambda_{ai} \Delta A_{ait} + \tilde{u}_{ait}, \quad (15-1)$$

$$A_t = \phi_{am} + \theta_{am} \hat{V}_{mt} + \rho_{am} Q_{at} + \lambda_{am} \Delta A_{amt} + \tilde{u}_{amt}. \quad (15-2)$$

For the public sale submarket:

$$Q_{bt} = \phi_{bi} + \theta_{bi} \hat{V}_{bt} + \rho_{bi} B_t + \lambda_{bi} \Delta B_{bit} + \tilde{u}_{bit}, \quad (16-1)$$

$$B_t = \phi_{bm} + \theta_{bm} \hat{V}_{mt} + \rho_{bm} Q_{bt} + \lambda_{bm} \Delta B_{bmt} + \tilde{u}_{bmt}. \quad (16-2)$$

In the above equations,  $\Delta A_{ait} = -(A_t - A_{t-1})$  if  $A_t > A_{t-1}$  and  $\Delta A_{ait} = 0$  otherwise.  $\Delta A_{amt} = A_t - A_{t-1}$  if  $A_t < A_{t-1}$  and  $\Delta A_{amt} = 0$  otherwise. Similarly,  $\Delta B_{bit} = -(B_t - B_{t-1})$  if  $B_t > B_{t-1}$  and  $\Delta B_{bit} = 0$  otherwise.  $\Delta B_{bmt} = B_t - B_{t-1}$  if  $B_t < B_{t-1}$  and  $\Delta B_{bmt} = 0$  otherwise.

### C. Transaction Classification

The method we employ to classify buy and sale transactions is similar to Ananth Madhavan and Smidt(1991) or the method suggested by Lee and Ready(1991). First, a transaction is classified as a public buy (sell) if the price is greater (less) than or equal to the last BBO prevailing ask (bid), or closer to the ask (bid) than to the bid (ask). Second, a transaction which is not identifiable according to the last prevailing BBO quotes is classified as a buy (sell) if the price is greater (less) than or equal to the next BBO prevailing ask (bid), or closes to the ask (bid) than to the bid (ask). Third, Trades which are identifiable neither by the last quotes nor the next quote are classified by the tick test. That is, at the first place, compared with the most recent transaction price, trades with positive (negative) price changes are assumed initiated by public buyers (sellers). At the second place, if its preceding trade were classified as a public buy (sell) and its price is equal to the price of its preceding trade, then, it is also assumed to be a public buy (sell).

### III. Major Empirical Results

#### A. Data Description

Five large stocks are drawn from ISSM database distributed by the Institute for the Study in Stock market. This data used in our analysis cover period January 1988 through march 1989 (except stock FNM which is only for the the quarter of 1989). Table III–A gives the average values of the variables. From this table, it is seen that the following relation holds. The buyer's expectation is on average greater than the transaction price which is greater the specialist's expectation. Reversely, the seller's expectation is on average lower than the transaction price which is lower than the specialist's expectation.

#### B. Estimation of Market Models

The 2SLS estimates for models (15) and (16) are presented in Table III–B. We summarize some principle implications of the results in this section.

(1) As we expect, the coefficients of the trader's expectation appear reliably positive in his purchase function and reliably negative in his sale function. In contrast, prices are estimated to have negative effects on public purchases and positive effects on public sales. As to the specialist's side, the estimates show that the specialist sets prices higher for larger public purchases and lower for larger public sales.

(2) Consistent with the mean–variance investment theory, the estimates of the price and expectations coefficients are pretty much the same in magnitude in the trader's function of demand for transaction service. Also, consistent with the prediction by the asymmetric information approach to the specialist's pricing, the coefficients of the specialist's expectation prior to the trade in time  $t$  turn out to be close to one.

(3) The slopes of both inverse demand functions for and supply functions of transaction services are estimated to be reliably rather small, ranging roughly from

.001 to .01. This fact indicates that, stock price elasticities may not be so small that it is necessary to drop infiniteness hypothesis in all cases, but may also not be so large that their infiniteness property can be ignored in all occasions.

(4) Recognizing that it is not easy to test whether a market is of disequilibrium type or equilibrium type, Quandt(1978) suggests a way to check the disequilibrium property of a market from the estimates obtained by disequilibrium methods. If parameters of adjustments to disequilibrium are found to depart from zero the equilibrium model would tend to be rejected (Quandt(1978), Maddala(1987)). In our estimation, all but a few adjustment parameter estimates are found significantly from zero, implying existence of market disequilibrium. Moreover, the price adjustment speeds of traders, i.e.,  $1/\lambda_{ai}$  and  $1/\lambda_{bi}$  appear in regression are far less than that of the specialist, i.e.,  $1/\lambda_{am}$  and  $1/\lambda_{bm}$ . Additionally, some  $1/\lambda_{am}$  and  $1/\lambda_{bm}$  estimates are even not reliably depart from zero. This implies that the specialist's responses to market innovations is rather faster than the traders'. Hence stock market disequilibrium, if ever exists, largely biases towards the trader's side.

(5) What is peculiar is that  $R^2$ s for dealer's equations are extremely high while that for trader's equations are extremely low. One technical reason is the changes in prices (i.e.,  $\Delta A_a$ ,  $\Delta A_m$ ,  $\Delta B_b$  and  $\Delta B_m$ ) as explanatory variables are higher correlated with transaction prices than with trading volumes. In addition, it is not surprising that when used as the proxy of the specialist's expectation, the PDLREG forecasts of the average bid/ask quotes have a great power to explain transaction prices. More essentially, if trades are for updating holdings, the difference between trader's expectation and the transaction price alone would not show much power of interpreting trading volume in the regression. High  $R^2$ s would not be obtained without trader's pre-trade holdings and liquidity-motivated trades being present as independent variables.

### C. The Estimation of Reduced-Form Models

We convert equilibrium functionals (8) and (9) into estimation equations (17) and (18).

For the ask submarket:

$$Q_{at} = \phi_{ai} + \varphi_{ai}\hat{V}_{at} - \varphi_{am}\hat{V}_{mt} - \gamma_{ai}\hat{\beta}_{ait} + \tilde{\mu}_{ait}, \quad (17-1)$$

$$A_t = \phi_{am} + \delta_{ai}\hat{V}_{at} + \delta_{am}\hat{V}_{mt} + \gamma_{am}\hat{\beta}_{amt} + \tilde{\mu}_{amt}. \quad (17-2)$$

For the bid submarket:

$$Q_{bt} = \phi_{bi} + \varphi_{bi}\hat{V}_{bt} - \varphi_{bm}\hat{V}_{mt} - \gamma_{bi}\hat{\beta}_{bit} + \tilde{\mu}_{bit}, \quad (18-1)$$

$$B_t = \phi_{bm} + \delta_{bi}\hat{V}_{bt} + \delta_{bm}\hat{V}_{mt} + \gamma_{bm}\hat{\beta}_{bmt} + \tilde{\mu}_{bmt}. \quad (18-2)$$

The OLS results are presented in Table III-C, and summarized below.

(1) Except the volume equation for Stock 3, all major estimates have signs that agree with what equilibrium functionals (7) and (8) predict. That is, the higher both the trader's and the specialist's expectations, the higher the stock price would be (all  $\hat{\varphi}_{ai}$ ,  $\hat{\varphi}_{am}$ ,  $\hat{\varphi}_{bi}$  and  $\hat{\varphi}_{bm}$  are positive). The equilibrium public purchase volumes are positively related to the trader's expectation (all  $\hat{\delta}_{ai}$ 's are positive), but negatively related to the specialist's (all  $\hat{\delta}_{am}$ 's are negative) expectation. Reversely, the equilibrium public sale volume is negatively related to the trader's expectation (all  $\hat{\delta}_{bi}$ 's are negative), but positively related to the specialist's ( $\hat{\delta}_{bm}$ 's are positive) expectation.

(2) The coefficients of the trader's and the specialist's expectations in the equilibrium price equations, as predicted by (7) and (8), approximately amount to one in sum (i.e.,  $\hat{\varphi}_{ai} + \hat{\varphi}_{am}$  and  $\hat{\varphi}_{bi} + \hat{\varphi}_{bm}$  are approximately equal to one). In the equilibrium volume equations, coefficients of the trader's and the specialist's expectations turn out to be almost the same in magnitude.

(3) The liquidity-motivated trades display extremely small effects on the

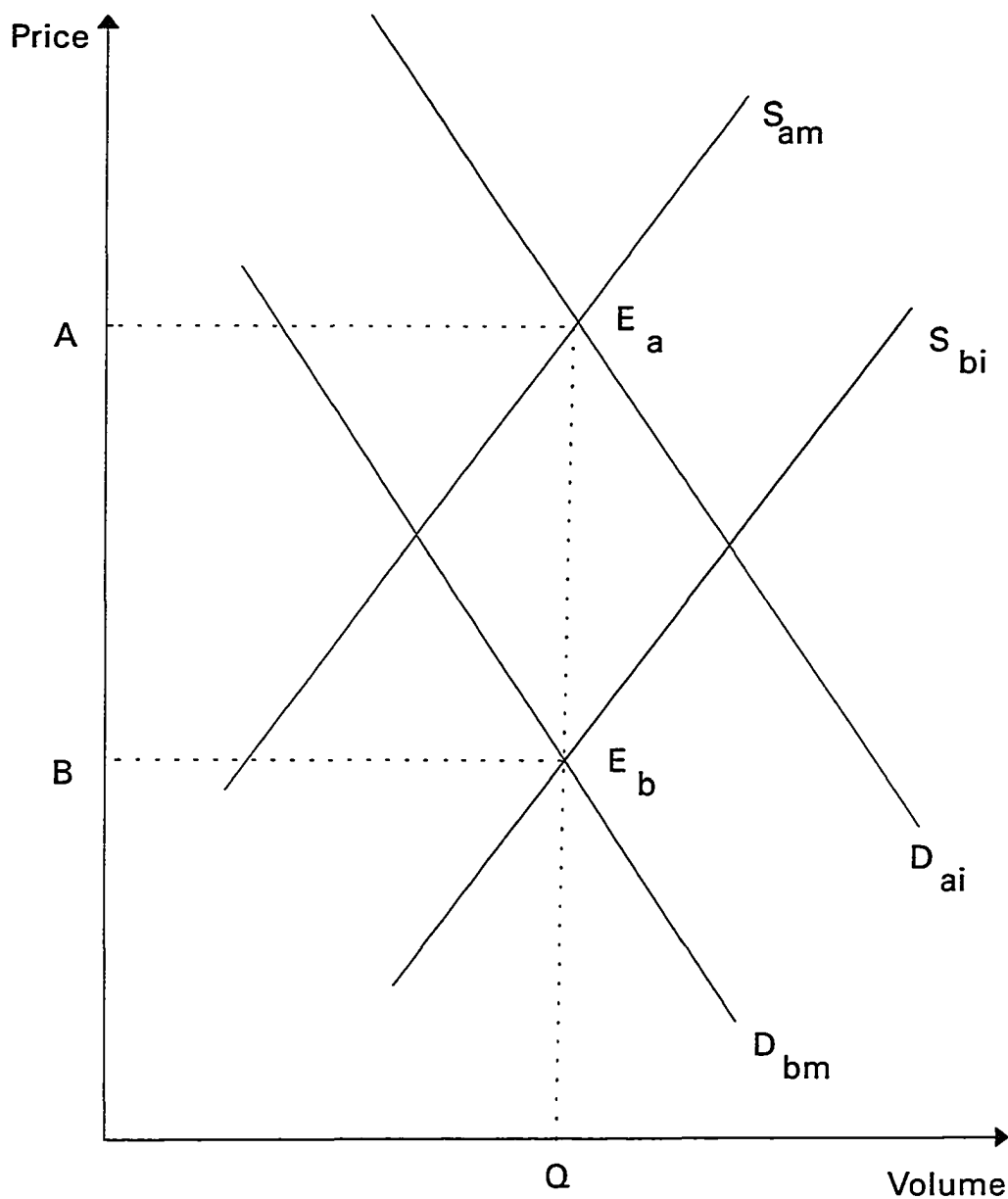
equilibrium prices. Namely, the  $\gamma$ 's in price equations are reliably close to zero, signaling the specialist's inference error about liquidity trades does not matter very much in stock price formation. And as it should be, the  $\gamma$ 's in volume equations are close to one.

### Conclusion

We have introduced an analytical market model for the bid and ask submarkets of the specialist-organized stock exchange. This model has been estimated by the disequilibrium method using ISSM database. Our findings are consistent with the mean-variance investment theory and the asymmetric information approach to the specialist's pricing. They also support the market disequilibrium hypothesis and elasticity infiniteness hypothesis.

Our approach to describing and estimating bid and ask submarkets possesses several methodological, technical and data limitations. What we have provided is only an atemporal framework in which bid and ask submarkets are treated non-interactive. It would be interesting to see a dynamic model in which bid and ask submarkets are assumed interactive. In estimation, we have postulated that sample separation are known. Although using two-stage linear regression method is good enough for our estimation purpose in this paper, it is expected that more reliable results might be generated by maximum likelihood method with assumption that sample separation is not known. Finally, our empirical examination is unfortunately constrained by lack of data for trades' security holdings prior to commitment of their trades and non-speculative order flow. These constraints, as we have already noted, are responsible for low  $R^2$ s of estimations for traders' market behavior, and we believe, if not impossible at all, are extremely difficult to overcome.

In this paper, we have followed Demsetz(1968) to build the market model for stock exchange according to the demand and supply of the underlying stock in each bid and ask submarkets. Another way to look at the specialist-organized stock exchange is to based on the supply and demand of transaction services. This way, the specialist's demand for shares in the bid submarket and his supply of shares in the ask submarket will be described by one supply function of transaction service. The traders' demands for shares in the ask submarket and their supply of shares in the bid submarket will be represented by one demand function of transaction services. Developing and testing this type of model for stock exchange is another direction to extend the present research.



**Figure 2 The Bid and Ask Submarkets of the Specialist-Organized Stock Exchange**

- $D_{ai}$  -- the trader's demand schedule in the ask sunmarket.
- $S_{am}$  -- the specialist's supply schedule in the ask sunmarket.
- $S_{bi}$  -- the trader's supply schedule in the bid sunmarket.
- $D_{bm}$  -- the specialist's demand schedule in the bid sunmarket.

**Table III-A Means of Variables**

This table gives the means of the variables used in the empirical studies. The data are drawn from the ISSM database and cover the period January 1988 through March 1989. Transactions are classified into public purchases and public sales using Lee and Ready method. The expectations of buyers, sellers and the specialist are constructed by PDLREG procedure in SAS.

**Panel A The Public Purchase Submarket**

VARIABLES \ STOCKS	HNZ	FNM	GWF	IPC	ITT
VOLUME ( $Q_p$ ), number of shares	15.469	32.855	48.445	21.448	19.903
ASK PRICE (A), \$	43.785	60.174	14.763	44.756	49.317
BUYER'S EXPECTATION ( $V_p$ ), \$	43.812		14.756	44.762	49.343
SPECIALIST'S EXPECTATION ( $V_M$ ), \$	43.710		14.694	44.680	49.252
NOISE TRADE ( $\beta_p$ ), number of shares	- 6.649		-27.651	-11.132	- 8.320
NUMBER OF OBSERVATIONS	7201	3567	3937	12923	9583

**Panel B The Public Sale Submarket**

VARIABLES \ STOCKS	HNZ	FNM	GWF	IPC	ITT
VOLUME ( $Q_B$ ), number of orders	15.071	33.232	47.463	22.809	17.233
BID PRICE (B), \$	43.740	60.172	14.696	44.569	49.328
SELLER'S EXPECTATION ( $V_B$ ), \$	43.771		14.694	44.575	49.349
SPECIALIST'S EXPECTATION ( $V_M$ ), \$	43.843		14.762	44.653	49.419
NOISE TRADE ( $\beta_B$ ), number of shares	- 9.614		-23.824	-11.636	-10.246
NUMBER OF OBSERVATIONS	6125	3505	3309		

HNZ -- Heinz (H.I.) Co.  
 FNM -- Federal National Mortgage Association  
 GWF -- Great Western Financial  
 IPC -- International Paper Corporation  
 ITT -- ITT Corp.

**Table III-B 2SLS Estimates of Bid and Ask Submarkets**

This table presents 2SLS estimates for the bid and ask submarkets according to the following model.

For the ask submarket,

$$Q_a = \phi_{ai} + \theta_{ai}V_a + \rho_{ai}A + \lambda_{ai}\Delta A_{ai} + \mu_{ai}, \quad (15-1)$$

$$A = \phi_{am} + \theta_{am}V_m + \rho_{am}Q_a + \lambda_{am}\Delta A_{am} + \mu_{am}; \quad (15-2)$$

For the bid submarket,

$$Q_b = \phi_{bi} + \theta_{bi}V_b + \rho_{bi}B + \lambda_{bi}\Delta A_{bi} + \mu_{bi}, \quad (16-1)$$

$$B = \phi_{bm} + \theta_{bm}V_m + \rho_{bm}Q_b + \lambda_{bm}\Delta A_{bm} + \mu_{bm}. \quad (16-2)$$

$V_a$  and  $V_b$  are the predicted expectations of public buyer and public seller about the stock value, respectively.  $V_m$  is the predicted expectation of the specialist about the stock value prior to the trade taking place.  $A$  and  $B$  are ask and bid transaction prices, respectively.  $Q_a$  and  $Q_b$  are the public purchase and sale orders, respectively.  $\Delta A_{ai} = -(A_t - A_{t-1})$  if  $A_t > A_{t-1}$ ,  $\Delta A_{ai} = 0$  otherwise; reversely,  $\Delta A_{am} = (A_t - A_{t-1})$  if  $A_t < A_{t-1}$ ,  $\Delta A_{am} = 0$  otherwise; Similarly,  $\Delta B_{bi} = -(B_t - B_{t-1})$  if  $B_t > B_{t-1}$ , otherwise  $\Delta B_{bi} = 0$ ; reversely,  $\Delta B_{bm} = (B_t - B_{t-1})$  if  $B_t < B_{t-1}$ ,  $\Delta B_{bm} = 0$  otherwise. Trades are grouped in the ask and bid submarket according to Lee and Ready criteria.

**Panel A The Public Purchase Submarket**

Stocks	Trader's Purchase					Specialist's Sale				
	$\phi_{ai}$	$\theta_{ai}$	$\rho_{ai}$	$\lambda_{ai}$	Obs/R <sup>2</sup>	$\phi_{am}$	$\theta_{am}$	$\rho_{am}$	$\lambda_{am}$	Obs/R <sup>2</sup>
HNZ	-38.343 (2.245)	88.631 (3.433)	-87.511 (3.433)	-95.354 (11.27)	7160 .019	-0.4378 (5.286)	1.0084 (544.7)	0.0096 (12.29)	0.0683 (1.116)	7160 .977
FNM	-123.34 (2.872)	106.10 (3.721)	-103.71 (3.717)	-230.04 (10.91)	3567 .043	-0.4728 (19.64)	1.0072 (2541.)	0.0011 (11.11)	-0.0806 (3.212)	3567 .999
GWF	-181.19 (0.797)	135.33 (0.497)	-119.99 (0.466)	-193.26 (2.696)	3894 .004	0.1343 (0.333)	0.9669 (30.61)	0.0086 (3.529)	0.3407 (0.840)	3894 .305
IPC	-165.63 (6.829)	278.51 (7.514)	-274.43 (7.507)	-145.35 (15.50)	12960 .018	-0.3058 (6.061)	1.0056 (904.2)	0.0064 (16.06)	0.0842 (1.987)	12960 .984

**Panel B The Public Sale Submarket**

Stocks	Trader's Sale					Specialist's Purchase				
	$\phi_{bi}$	$\theta_{bi}$	$\rho_{bi}$	$\lambda_{bi}$	Obs/R <sup>2</sup>	$\phi_{bm}$	$\theta_{bm}$	$\rho_{bm}$	$\lambda_{bm}$	Obs/R <sup>2</sup>
HNZ	104.96 (4.293)	-152.67 (4.616)	150.62 (4.623)	-89.014 (8.269)	6084 .011	-0.5150 (4.066)	1.0136 (352.6)	-0.0122 (8.684)	-0.1166 (1.312)	6084 .953
FNM	122.88 (2.864)	-133.78 (4.650)	112.28 (4.700)	-541.02 (21.28)	3505 .137	-0.5358 (22.38)	1.0082 (2516.)	-0.0008 (17.46)	0.0983 (4.258)	3505 .999
GWF	263.82 (1.270)	-231.64 (1.094)	216.75 (1.096)	-173.56 (3.363)	3349 .003	-0.5226 (2.192)	1.0493 (67.92)	-0.0060 (3.354)	-0.7386 (2.936)	3138 .579
IPC	294.20 (7.697)	-344.46 (6.786)	338.27 (6.770)	-178.42 (12.64)	11324 .017	-0.2762 (4.269)	1.0064 (720.5)	-0.0043 (13.22)	-0.1885 (3.941)	11327 .980

\*The number in parentheses are student t-values. \*\*The R<sup>2</sup>s are adjusted ones.

### Table III-C OLS Estimates for Price and Trade Equations

This table reports OLS estimates of the reduced-form equations for prices and trades in the bid and ask submarkets according to the following model.

For the ask submarket:

$$Q_a = \phi_{ai} + \varphi_{ai}V_a + \varphi_{am}V_m + \gamma_{ai}\beta_{ai} + \eta_{ai}, \quad (17-1)$$

$$A = \phi_{am} + \delta_{ai}V_a + \delta_{am}V_m + \gamma_{am}\beta_{am} + \eta_{am}. \quad (17-2)$$

For the bid submarket:

$$Q_b = \phi_{bi} + \varphi_{bi}V_b + \varphi_{bm}V_m + \gamma_{bi}\beta_{bi} + \eta_{bi}, \quad (18-1)$$

$$B = \phi_{bm} + \delta_{bi}V_b + \delta_{bm}V_m + \gamma_{bm}\beta_{bm} + \eta_{bm}. \quad (18-2)$$

A and B are ask and bid transaction prices, respectively.  $Q_a$  and  $Q_b$  are public purchase and sales, respectively.  $V_a$  and  $V_b$  are the predicted expectations of public purchaser and seller about the stock value.  $V_m$  is the predicted expectation of the specialist about the stock value prior to the trade taking place.  $\beta$ 's are the constructed noise trades.

#### Panel A The Public Purchase Submarket

Stocks	Price					Volume				
	$\phi_a$	$\varphi_{ai}$	$\varphi_{am}$	$\gamma_a$	Obs/R <sup>2</sup>	$\phi_a$	$\delta_{ai}$	$\delta_{am}$	$\gamma_a$	Obs/R <sup>2</sup>
HNZ	-0.5212 (68.41)	0.8777 (167.7)	0.1340 (25.75)	-0.0002 (16.86)	7160 .999	66.616 (10.45)	17.665 (4.034)	-19.021 (4.365)	-0.9672 (78.09)	7160 .465
ITT	-0.7632 (108.8)	0.8295 (186.2)	0.1860 (42.20)	-0.0002 (16.21)	9623 .999	100.87 (12.79)	48.593 (9.710)	-50.483 (10.20)	-0.9557 (69.46)	9623 .341
GWF	-0.8638 (143.1)	1.0387 (347.9)	0.0199 (7.300)	-0.0000 (3.390)	3894 .999	275.99 (7.737)	-4.0496 (0.248)	-13.252 (0.823)	-0.9825 (46.31)	3894 .356
IPC	-0.5523 (121.4)	0.8760 (232.9)	0.1365 (36.56)	-0.0001 (19.32)	12960 .999	76.148 (12.25)	48.235 (9.385)	-49.784 (9.757)	-0.9509 (101.9)	12960 .450

#### Panel B The Public Sale Submarket

Stocks	Price					Volume				
	$\phi_b$	$\varphi_{bi}$	$\varphi_{bm}$	$\gamma_b$	Obs/R <sup>2</sup>	$\phi_b$	$\delta_{bi}$	$\delta_{bm}$	$\gamma_b$	Obs/R <sup>2</sup>
HNZ	-0.6854 (74.41)	0.8662 (135.3)	0.1488 (23.32)	0.0001 (7.806)	6084 .999	-53.917 (9.759)	-32.740 (8.526)	34.041 (8.898)	-0.9869 (124.7)	6084 .721
ITT	-0.7986 (147.2)	0.8939 (246.6)	0.1219 (33.92)	0.0001 (9.587)	10237 .999	-88.102 (14.62)	-22.325 (5.549)	24.217 (6.068)	-0.9837 (131.3)	10237 .632
GWF	-1.0227 (190.9)	1.0019 (288.0)	0.0671 (19.64)	0.0000 (4.640)	3349 .999	-128.67 (11.17)	1.5012 (0.092)	19.209 (1.196)	-0.9819 (68.27)	3349 .582
IPC	-0.6867 (138.7)	0.8306 (213.1)	0.1844 (47.57)	0.0000 (9.584)	11325 .999	-76.899 (9.344)	-57.023 (8.799)	58.895 (9.137)	-0.9873 (152.1)	11325 .674

\*The number in parentheses are student t-values.

\*\*The R<sup>2</sup>s are adjusted ones.

## Appendix of Chapter One

### Appendix I

#### Proof of of Lemma 1.

First of all, an individual's preposterous expectation about  $\tilde{V}$  conditional on  $Y_i$  and  $P$  is calculated as follows.

$$\begin{aligned} & E_i[\tilde{V} | \tilde{Y}_i=y_i, P=p] \\ &= m + (\sigma_{vy} \sigma_{vp}) \begin{bmatrix} \sigma_y^2 & \sigma_{yp} \\ \sigma_{yp} & \sigma_p^2 \end{bmatrix}^{-1} \begin{bmatrix} y_i - m \\ p - m \end{bmatrix} \\ &= \frac{\sigma_y^2 \sigma_p^2 - \sigma_{yp}^2 - \sigma_{vp} \sigma_y^2 + \sigma_{vp} \sigma_{yp}}{\sigma_y^2 \sigma_p^2 - \sigma_{yp}^2} \left[ m + \frac{\sigma_{vy} \sigma_p^2 - \sigma_{vp} \sigma_{yp}}{\sigma_y^2 \sigma_p^2 - \sigma_{yp}^2 - \sigma_{vp} \sigma_y^2 + \sigma_{vp} \sigma_{yp}} (y_i - m) \right] \\ &\quad + \frac{\sigma_{vp} \sigma_y^2 - \sigma_{vp} \sigma_{yp}}{\sigma_y^2 \sigma_p^2 - \sigma_{yp}^2} p, \end{aligned}$$

where  $E[P]=m$ ,  $\text{Var}[P]=\sigma_p^2$ ,  $\text{Cov}[Y_i, P]=\sigma_{yp}^2$ ,  $\text{Cov}[\tilde{V}, Y_i]=\sigma_{vy}^2=\sigma_v^2$ ,  $\text{Cov}[\tilde{V}, P]=\sigma_{vp}^2$ . Let  $\xi=(\sigma_{vp}\sigma_y^2-\sigma_{vp}\sigma_{yp})/(\sigma_y^2\sigma_p^2-\sigma_{yp}^2) \in [0,1]$ .

Given (i) proved above, we have  $\sum_{i=1}^n E_i[\tilde{V} | \Omega_{i,p}]/n = (1-\xi)\bar{v} + \xi p$  with  $\xi \in [0,1]$ . That implies,  $\sup_{\xi \in [0,1]} \{\sum_{i=1}^n E_i[\tilde{V} | \Omega_{i,p}]/n\} = \text{Max}\{\bar{v}, p\}$  and  $\inf_{\xi \in [0,1]} \{\sum_{i=1}^n E_i[\tilde{V} | \Omega_{i,p}]/n\} = \text{Min}\{\bar{v}, p\}$ . Granting  $p \geq \bar{v}$ ,  $\sup_{\xi \in [0,1]} \{\sum_{i=1}^n E_i[\tilde{V} | \Omega_{i,p}]/n\} = p$  while  $\inf_{\xi \in [0,1]} \{\sum_{i=1}^n E_i[\tilde{V} | \Omega_{i,p}]/n\} = \bar{v}$ . Inversely, let  $p \leq \bar{v}$ ,  $\sup_{\xi \in [0,1]} \{\sum_{i=1}^n E_i[\tilde{V} | \Omega_{i,p}]/n\} = \bar{v}$  while  $\inf_{\xi \in [0,1]} \{\sum_{i=1}^n E_i[\tilde{V} | \Omega_{i,p}]/n\} = p$ . This completes our proof. Q.E.D.

#### Proof of Lemma 2.

By assumption, information contained in  $\sum_{i=1}^n x_i(\cdot, p)$  is disclosure of  $U_{i=1}^n \Omega_i$ , which is belong to  $\Omega_s$ . So  $E_s[\tilde{V} | \Omega_s, \sum_{i=1}^n x_i(\cdot, p)] = E_s[\tilde{V} | \Omega_s, U_{i=1}^n \Omega_i] = E_s[\tilde{V} | \Omega_s] = v_s$ . Q.E.D.

**Proof of Lemma 3.**

Employing Arrow-Pratt method to proceed the following,

$$\begin{aligned} & E_i[U(\cdot, x_i(\cdot, p)(\tilde{V}-p) | \Omega_i, p)] \\ &= U\{\cdot, x_i(\cdot, p)(E_i[\tilde{V} | \Omega_i, p]-p) - \frac{x_i(\cdot, p)\sigma^2}{2} \frac{U''}{U'}\}. \end{aligned}$$

Where,  $\sigma^2 = \text{Var}[\tilde{V} | \Omega_i, p]$ . Differentiating  $E_i[U | \Omega_i, p]$  with respect to  $x_i$  yields

$$\frac{\partial E_i[U | \Omega_i, p]}{\partial x_i} = U'\{E_i[\tilde{V} | \Omega_i, p]-p - x_i(\cdot, p)\sigma^2 U''/U'\} = 0.$$

$$\text{That is } x_i(\cdot, p) = \frac{E_i[\tilde{V} | \Omega_i, p]-p}{\sigma^2 U''/U'}.$$

From Lemma 1(i),

$$x_i(\cdot, p) = (1-\xi) \frac{E_i'[\tilde{V} | \Omega_i, p]-p}{\sigma^2 U''/U'}.$$

**Proof of Lemma 4.**

Using Eq(1) to sum up  $x_i(\cdot, p)$  given by Eq(2) over  $i$  and rearranging the resulting equation yield

$$\bar{v} = \sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n = \frac{x_i(\cdot, p)\sigma^2 U''/U' + p}{(1-\xi)}.$$

This equation is deterministic expressions of  $\sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n$ , respectively. That means, once orders arrive at his post, the specialist has all the information needed to infer the exact value of  $\bar{v}$  and in turn the exact value of  $\sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n$ . Further, Grossman(1976) has verified that  $\sum_{i=1}^n E_i[\tilde{V} | \Omega_i] / n$  is a sufficient statistic for  $(v_1, v_2, \dots, v_n)$ , so knowing  $\bar{v}$  is equivalent to knowing every trader's information in the market, i.e.  $(v_1, v_2, \dots, v_n)$ . Q.E.D.

**Appendix II**

Proportion VI is actually a special case of the following statement.

**In the financial market operated by  $m$  specialists who are identical in operation and expectations, under Assumptions I-V, the optimal strategy pair a risk-averse specialist adopts is given by**

$$x_j(m, n, \phi) = [1 - \lambda(m, n, \phi)]n(v_s - \bar{v})/\psi\sigma^2;$$

$$p(m, n, \phi) = \lambda(m, n, \phi)\bar{v} + [1 - \lambda(m, n, \phi)]v_s.$$

Where,  $\lambda(m, n, \phi) = [\psi + mn(1 - \xi)\phi] / [(1 + m)\psi + mn(1 - \xi)\phi]$ .

Clearly, assigning  $m=1$  gives us (i) in Prop. VI, letting  $m=1$  and  $\phi=0$  yields (ii), and when  $n \rightarrow \infty$  and  $\phi=0$  we have (iii). To prove the statement, we only need to assume specialists taking  $x_j(\sum_{j=1}^m x_j = x_s)$  as strategy to commit Cournot quantity competition.

Proof. At the first place, when  $m=1$ , from Prop. 4,  $x_s = [1 - \lambda(m=1, n, \phi)]n(v_s - \bar{v})/\psi\sigma^2$ ,  $p = \lambda(m=1, n, \phi)\bar{v} + [1 - \lambda(m=1, n, \phi)]v_s$ , where,  $\lambda(m=1, n, \phi) = [\psi + 1n(1 - \xi)\phi] / [2\psi + 1n(1 - \xi)\phi]$ . At the second place, when  $m=2$ , let  $j=1, 2$  be the index of the first and second specialist respectively. Then, the  $j$ th specialist's profit function can be stated as below.

$$\pi_j = (x_1 + x_2)[v_s - (\bar{v} + \psi\sigma^2(x_1 + x_2)/n)] - (x_1 + x_2)^2\phi\sigma^2/2, \quad j=1, 2.$$

Taking partial derivative of  $\pi_j$  with respect to  $x_j$  generates the following Cournot–Nash reaction function system.

$$x_1 = A - bx_2; \quad x_2 = A - bx_1.$$

Where,  $A = n(v_s - \bar{v}) / [2\psi + n(1 - \xi)\phi]\sigma^2$ ,  $B = [\psi + n(1 - \xi)\phi] / [2\psi + n(1 - \xi)\phi]$ . Solving this system for  $x_j$  and  $p$ , we have,

$$x_j(m=2, n, \phi) = [1 - \lambda(m=2, n, \phi)]n(v_s - \bar{v})/\psi\sigma^2;$$

$$p(m=2, n, \phi) = \lambda(m=2, n, \phi)\bar{v} - [1 - \lambda(m=2, n, \phi)]v_s.$$

Where,  $\lambda(m=2, n, \phi) = [\psi + n(1 - \xi)\phi] / [2\psi + n(1 - \xi)\phi]$ . Finally, making use of mathematical induction obtain the desired result. Q.E.D.

However, a specialist is usually thought using price instead of quantity as competition strategy in that he posts quotes rather than volumes and, unlike in the commodity market, there is no quantity constraints in stock market (The short sale restriction may not be regarded as the quantity constraint in the restrict sense). We are thus thrown into a dilemma—a reasonable result induced from a unreasonable premise. Further researches evidently need to be done into this matter. Liu(1990)

argues that under market share constraint both price strategic competition and quantity strategic competition yield the same equilibrium.

## Appendix of Chapter Two

### Derivation of Equation (3)

The specialist's problem is

$$\text{Max}_{\{p_t\}} E_t \left[ \sum_{k=1}^{\infty} U_{t-k}(\Pi_{t-k}) + U_t((p_t - D_t \psi - \tilde{V})[\varphi(y_t - p_t) + \tilde{x}_t]) \right. \\ \left. + \delta \Lambda_{t+1}((I_{t-1} - [\varphi(y_t - p_t) + \tilde{x}_t]) \mid \mathcal{F}_t) \right].$$

Taking the partial derivative with respect to  $p_t$  returns the first order condition for optimal transaction price.

$$E_t[(\partial U_t / \partial \Pi_t)(\varphi y_t - 2\varphi p_t + \tilde{x}_t + \varphi \psi D_t + \varphi \tilde{V}) + \delta \varphi (\partial \Lambda_{t+1} / \partial I_t)(\partial I_t / \partial q_t) \mid \mathcal{F}_t] = 0.$$

Define  $U' \equiv \partial U_t / \partial \Pi_t$  and  $\Lambda'_{t+1} \equiv (\partial \Lambda_{t+1} / \partial I_t)(\partial I_t / \partial q_t) \equiv \partial \Lambda_{t+1} / \partial q_t$ . Distributing the expectation operator  $E_t$  yields,

$$\varphi E_t[U' y_t \mid \mathcal{F}_t] - 2\varphi p_t E_t[U' \mid \mathcal{F}_t] + E_t[U' \tilde{x}_t \mid \mathcal{F}_t] + \varphi \psi D_t E_t[U' \mid \mathcal{F}_t] \\ + \varphi E_t[U' \tilde{V} \mid \mathcal{F}_t] + \delta \varphi E_t[\Lambda'_{t+1} \mid \mathcal{F}_t] = 0$$

Solving for  $p_t$  gives

$$p_t^* = \frac{1}{2} \left[ \frac{D_t \psi E_t[U' \mid \mathcal{F}_t] + E_t[\tilde{V} U' \mid \mathcal{F}_t] + \delta E_t[\Lambda'_{t+1} \mid \mathcal{F}_t]}{E_t[U' \mid \mathcal{F}_t]} \right] \\ + \frac{1}{2} \left[ \frac{E_t[y_t U' \mid \mathcal{F}_t] + E_t[\tilde{x}_t U' \mid \mathcal{F}_t] / \varphi}{E_t[U' \mid \mathcal{F}_t]} \right].$$

### Appendix of Chapter Three

#### Derivation of Equations (2-A) and (2-B).

The trader's problem is

$$\begin{aligned} & \text{Max}_{\{x_i\}} -\text{Exp}\{-E_i[\tilde{W}_i | \Omega_i] - \psi_i \text{Var}[\tilde{W}_i | \Omega_i]\} \\ & \text{subject to} \quad \tilde{W}_i = [X_i + (I_a - I_b)x_i]\tilde{V} + (Z_i - z_i)R, \end{aligned} \quad (\text{A-1})$$

$$\text{and } z_i = x_i(I_a A - I_b B).$$

Note that

$$E_i[\tilde{W}_i | \Omega_i] = X_i E_i[\tilde{V}_i | \Omega_i] + I_a x_i (E_i[\tilde{V}_i | \Omega_i] - AR) - I_b x_i (E_i[\tilde{V}_i | \Omega_i] - BR) + Z_i R, \quad (\text{A-2})$$

and

$$\text{Var}[\tilde{W}_i | \Omega_i] = [X_i + (I_a - I_b)x_i]^2 \text{Var}[\tilde{V}_i | \Omega_i]. \quad (\text{A-3})$$

Now taking the first-order derivative with respect to  $x_i$  gives the optimal transaction demand functions for trader  $i$ .

$$x_i = I_a \frac{E_{ai}[\tilde{V} | \Omega_{ai}] - RA}{\psi \text{Var}[\tilde{V} | \Omega_{ai}]} - I_b \frac{E_{bi}[\tilde{V} | \Omega_{bi}] - RB}{\psi \text{Var}[\tilde{V} | \Omega_{bi}]} - (I_a - I_b)X_i. \quad (\text{A-4})$$

So when  $I_a=1$ , it has,

$$x_i = \frac{E_{ai}[\tilde{V} | \Omega_{ai}] - RA}{\psi \text{Var}[\tilde{V} | \Omega_{ai}]} - X_{ai}. \quad (\text{A-5})$$

When  $I_b=1$ , it has,

$$x_i = -\frac{E_{bi}[\tilde{V} | \Omega_{bi}] - RB}{\psi \text{Var}[\tilde{V} | \Omega_{bi}]} + X_{bi} \quad (\text{A-6})$$

#### Derivation of Equations (5-A) and (5-B).

Take the ask submarket as an example. First, using (A-5) and noting that  $X_{ai}$

is random to the specialist, the specialist's inference about the buyer's  $[E_{ai}[\tilde{V} | \Omega_{ai}]$  for a given  $D_{ai}$  is written as

$$E_m[E_{ai}[\tilde{V} | \Omega_{ai}] | D_{ai}] = \psi \text{Var}[\tilde{V} | \Omega_{ai}] (D_{ai} + E_m[\tilde{X}_{ai} | D_{ai}]) + RA. \quad (\text{A-7})$$

Second, since by construction  $E_m[E_{ai}[\tilde{V} | \Omega_{ai}] | D_{ai}] = \tilde{V} + \nu_a$  with  $\nu_a \sim \text{IIN}(0, \sigma_{\nu_a}^2)$ , it has  $E[E_m[E_{ai}[\tilde{V} | \Omega_{ai}] | D_{ai}]] = m_v$  and  $\text{Var}[E_m[E_{ai}[\tilde{V} | \Omega_{ai}] | D_{ai}]] = \sigma_v^2 + \sigma_{\nu_a}^2$ .

Third, conditional on (A-7), the specialist's expectation about the underlying stock value is as follows.

$$\begin{aligned} E_m[\tilde{V} | E_m[E_{ai}[\tilde{V} | \Omega_{ai}] | D_{ai}]] &= \psi \text{Var}[\tilde{V} | \Omega_{ai}] (D_{ai} + E_m[\tilde{X}_{ai} | D_{ai}]) + RA \\ &= E_m[\tilde{V} | D_{ai}] \\ &= [\sigma_v^2 / (\sigma_v^2 + \sigma_{\nu_a}^2)] m_v + [\sigma_{\nu_a}^2 / (\sigma_v^2 + \sigma_{\nu_a}^2)] \{ \psi \text{Var}[\tilde{V} | \Omega_{ai}] (D_{ai} + E_m[\tilde{X}_{ai} | D_{ai}]) + RA \} \end{aligned}$$

Finally, remember that in the competitive market, an uninformed risk-neutral specialist sets his ask price such that  $A = E_m[\tilde{V} | D_{ai}] / R$ . In the result,

$$A = \frac{1}{R} \{ m_v + k_a (S_{am} + E_m[\tilde{X}_{ai} | D_{ai}]) \}, \quad (\text{A-9})$$

where  $k_a = \sigma_{\nu_a}^2 \psi \text{Var}[\tilde{V} | \Omega_{ai}] / \sigma_v^2$ .  $S_{am}$  replaces  $D_{ai}$  for making (A-9) to be the specialist's supply function.

Similarly, the specialist's demand function is derived as follows,

$$B = \frac{1}{R} \{ m_v - k_b (D_{bm} + E_m[\tilde{X}_{bi} | S_{bi}]) \}. \quad (\text{A-10})$$

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