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Bozovic, Radomir Todor

ACQUISITION OF FAST FREQUENCY HOPPING AT HF

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ACQUISITION OF FAST FREQUENCY HOPPING AT HF

by

Radomir Bozovic

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ACQUISITION OF FAST FREQUENCY HOPPING AT HF

by

Radomir Bozovic

Adviser: Professor Donald L. Schilling

To my wife Snezana and my daughter Ana

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FIGURE

1. INTRODUCTION

The characteristics of a communication system depend strongly on the kind of the channel, e.g. HF, tropospheric scatter, microwave, etc. Many of the channels used for spread-spectrum communications have time-varying characteristics. The channel considered in this work is the HF ionospheric channel, and the spread spectrum technique considered is frequency hopping spread-spectrum. The characteristics of such a spread spectrum system are determined and limited by the multipath propagation and fading in the HF channel as well as by additive noise.

A signal transmitted at HF propagates to the receiver via several paths determined by single and multiple reflection conditions of the ionospheric layers. Since the propagation time associated with each of the paths is different due to their different optical path lengths, the received signal suffers time spread distortion that degrades the performance of the communication system. The heights of the reflecting ionospheric layers are time variable, and hence the signal is Doppler shifted on reflection, thus causing different frequency shifts on the various multipath components. The presence of ionization irregularities causes variable diffractive fadings on each of the multipath components and resulting fading of the composite received signal. The frequency shift and the fadings combine to cause frequency-spread distortion that degrades the performance of the system.

To obtain immunity to intentional interferers one can design a system which can hop over a very large band of frequencies. As a result of the large frequency separation between hopping frequencies we assume that each hopping frequency propagates via statistically independent HF ionospheric channels. For a frequency-hopping communication system operating via the HF ionospheric channel described above we propose an optimum receiver and acquisition scheme whose characteristics are analyzed.

2. STATEMENT OF THE PROBLEM

In this work we deal with the analysis of a frequency-hopped spread-spectum communication system over a non specular Rayligh fading multipath channel. We assume that the frequency separation between each hopping frequency is large enough to ensure that they propagate via statistically independent channels. This means that each transmitted signal, with different hopping (carrier) frequency, will suffer statistically independent time and frequency spreading.

Suppose now that we transmit a signal $s_i(t)$ whose hopping frequency is f_i. This signal will propagate to the receiver via several paths. The receiver receives the signal $s_i(t)$ after the time delay of τ_i seconds. This delay is the propagation delay and depends on the optical path lengths associated with each path.

The signal $s_j(t)$ with hopping frequency f_j, propagates to the receiver via several paths whose lengths are different than the lengths in the previous case. Thus the propagation delays τ_i and τ_j associated with the signals $s_i(t)$ and $s_j(t)$ with carrier frequencies f_i and f_j respectively, are different.

Let us now transmit $s_i(t)$ with carrier frequency f_i at t_1 and t_2. Because the heights of the reflecting layers are time variable, the path lengths associated with

$s_i(t)$ at t_1 and t_2 are different.

In a frequency hopping spread spectrum system the transmitter carrier frequency is determined by the output sequence of a pseudo random generator. The number of different hopping frequencies is determined by the length of the pseudo random sequence. If the pseudo random generator is a maximum length linear feedback shift register (LFSR) generator the length is given as

$$L=2^n-1 \tag{2-1}$$

where n is the number of stages in the LFSR generator. So, each hopping frequency will be repeated after

$$T=t_2-t_1=(2^n-1)T_h \tag{2-2}$$

where T_h is the hopping period. Therefore we can assume that the time delay between the same hopping frequencies is sufficiently large to ensure that there is no correlation between path lengths at t_1 and t_2.

According to these assumptions each hopping frequency f_i $(1\leqslant i\leqslant L)$ will propagate to the receiver with random delay time τ_i. Each τ_i $(1\leqslant i\leqslant L)$ is a random variable with probability density function

$$p_{\tau_i}(t) = \begin{cases} \dfrac{1}{2\Delta t} & T_{nd}-\Delta t\leqslant t\leqslant T_{nd}+\Delta t \\ 0 & \text{elsewhere} \end{cases} \tag{2-3}$$

where T_{nd} is the nominal or mean delay time, and Δt is the maximum deviation from T_{nd}. Since any two frequencies propagate via statistically independent channels, the random variables τ_i and τ_j are uncorrelated i.e.

$$E\{(\tau_i - \overline{\tau_i})(\tau_j - \overline{\tau_j})\}=0 \qquad i \neq j \qquad (2-4)$$

Also the random variables τ_i at t_1 and t_2 are uncorrelated i.e.

$$E\{[\tau_i(t_1) - \overline{\tau_i(t_1)}][\tau_i(t_2) - \overline{\tau_i(t_2)}]\}=0 \qquad (2-5)$$

The nominal delay time T_{nd}, in the general case, is a function of frequency. Fig. 2-1 gives typical curves for T_{nd} versus frequency for three different paths.

In Fig. 2-2a,b,c,d and e are sketches of measurement results of the time delay of a signal propagating via ionospheric multipath channel. As we can see from Fig. 2-2c,d and e it is possible to identify typical curves for nominal delay time T_{nd}. But it is impossible to recognize the typical curves of the nominal delay time from Fig. 2-2a and b. The reason for this is that the path lengths are random variables and therefore the delay time associated with each path is a random variable whose mean value is T_{nd}, which of course is a function of frequency.

Suppose now that the frequency hopping rate is of the same order as Δt (maximum deviation of the delay time from T_{nd}). In such a case there will be significant overlapping between two, three or more hopping frequencies at the receiver side. With such overlapping of multiple adjacent hopping frequencies, fading, and white additive Gaussian noise, standard FH receivers will neither acquire or track the received signal, nor will such a receiver detect the data with a reasonable error rate. The problem of optimum detection and acquisition is solved in this work.

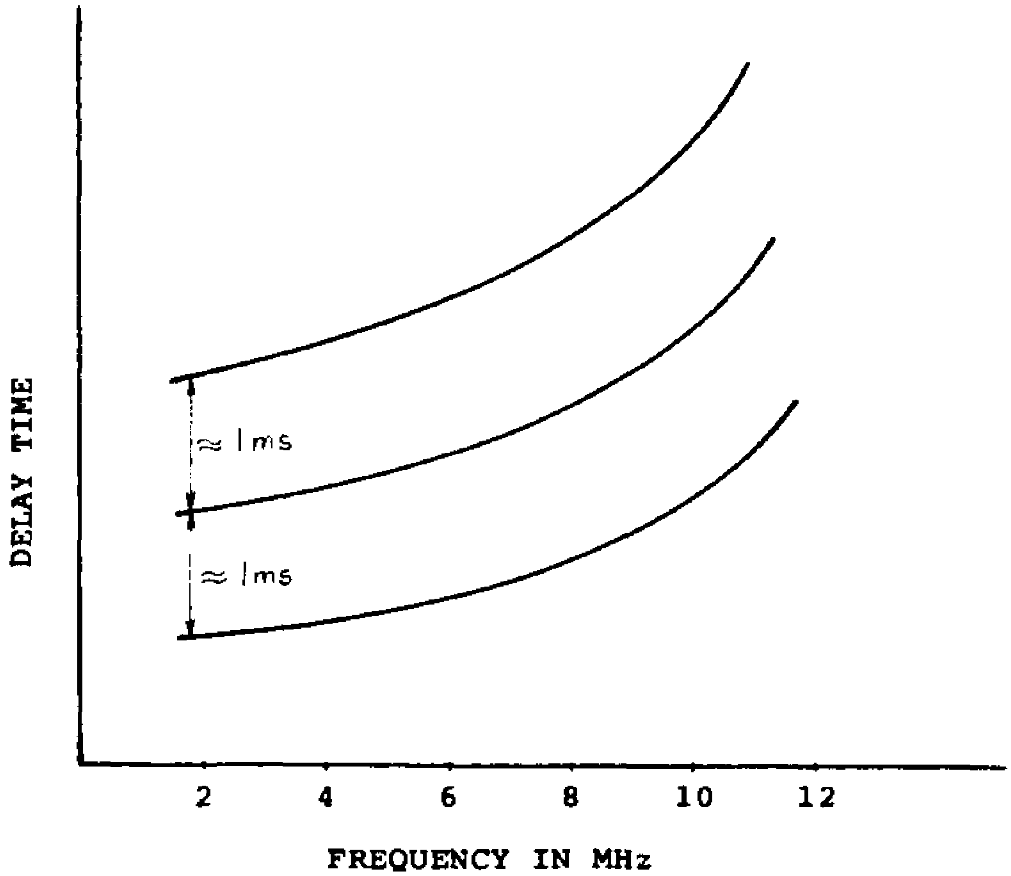
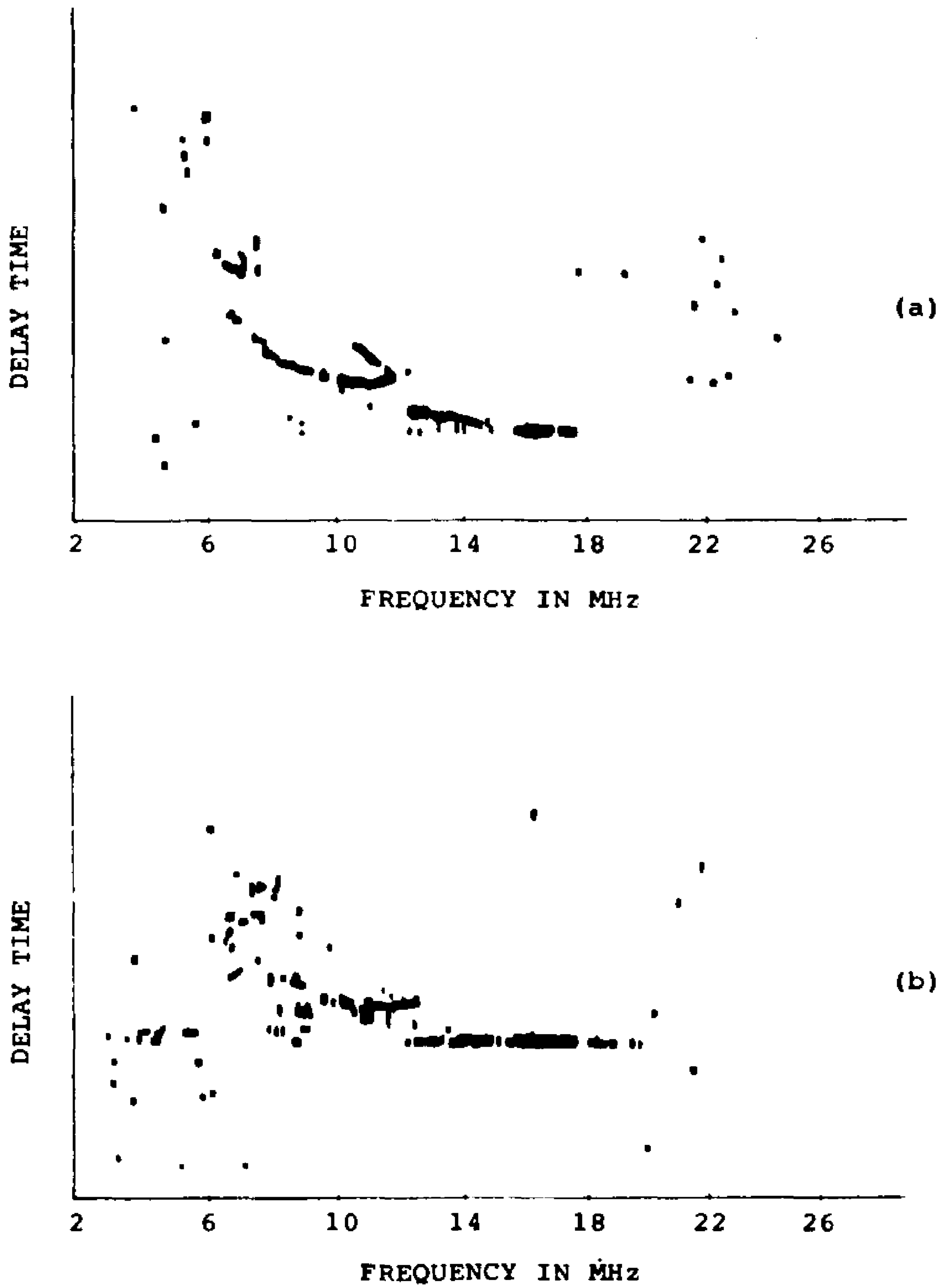
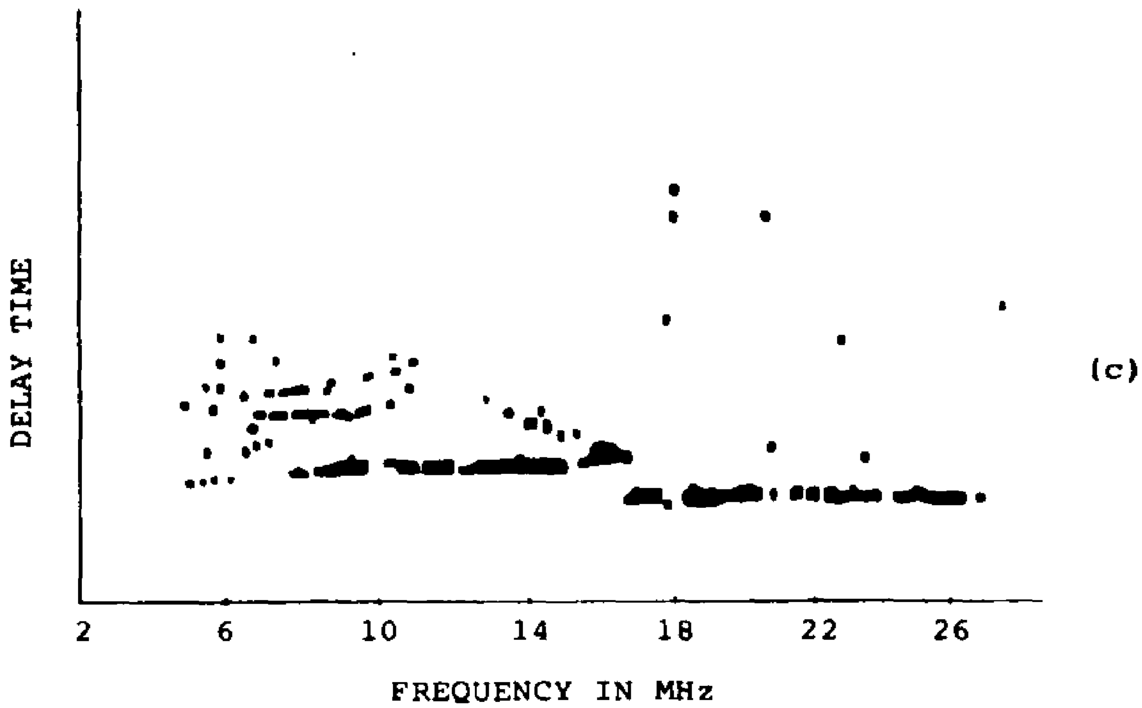


Fig. 2-1



DELAY TIME
(a)
2
6
10
14
18
22
26
FREQUENCY IN MHz
DELAY TIME
(b)
2
6
10
14
18
22
26
FREQUENCY IN MHz

Fig. 2-2


DELAY TIME
2
6
10
14
18
22
26
FREQUENCY IN MHz
(c)

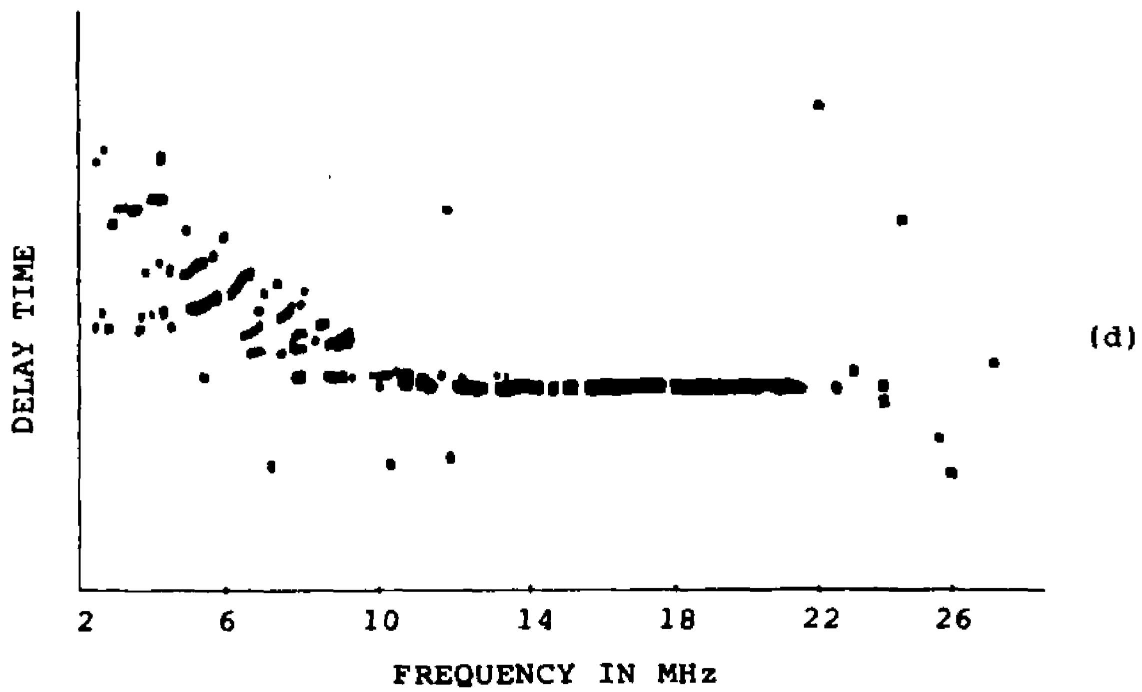

DELAY TIME
2
6
10
14
18
22
26
FREQUENCY IN MHz
(d)

Fig. 2-2

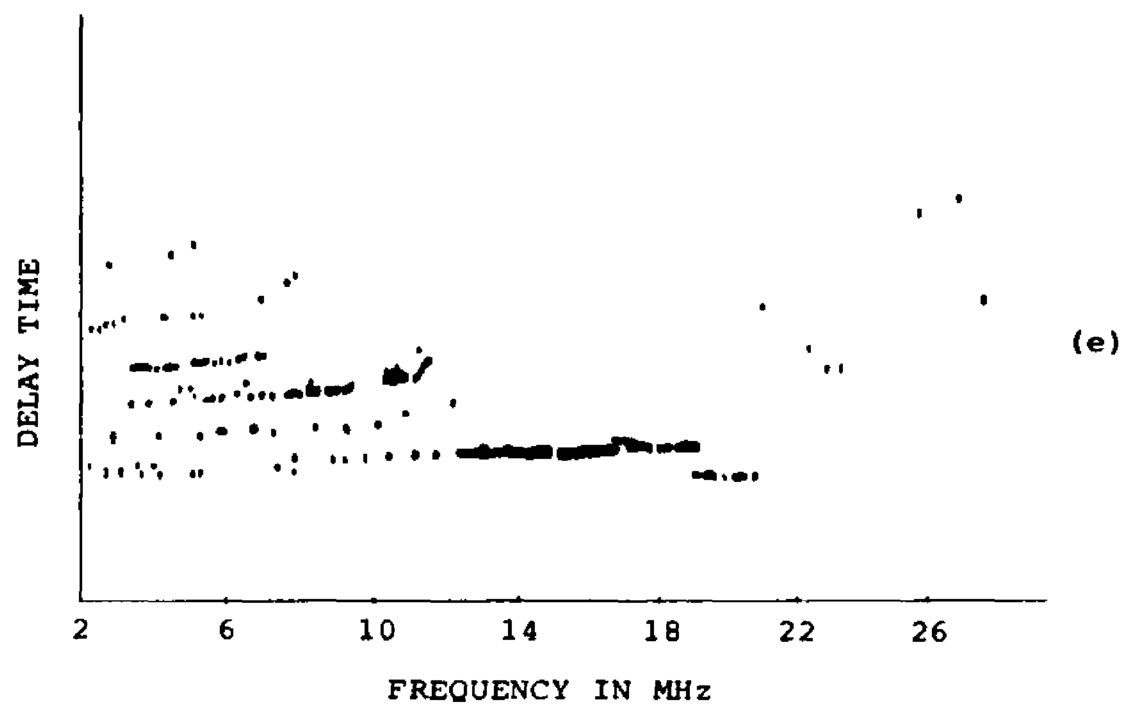


Fig. 2-2

3. REVIEW OF PRIOR WORK

The synchronization problem is extremely important in any spread-spectrum communication system. The initial synchronization or acquisition is the most difficult of all. Specific synchronization requirements depend mostly on the intended application, and often, the synchronizing scheme has to be designed for the worst case condition. Suitable measures of system performance are the mean time of acquisition and the probability of successfully acquiring an anticipated spread-spectrum signal. The scheme to be employed generally depends on the application, the amount of time allowed, and the amount of uncertainty involved. The probability of acquisition is a suitable performance measure in situations where the transmitter continuously emits the spread spectrum signal that the receiver must acquire. The mean time to acquire is a good criterion for burst spread-spectrum systems.

Many techniques for achieving synchronization have evolved. Below, we review some of the frequency-hopping acquisition schemes.

3.1 Matched filter scheme

A near-optimum Matched Filter for detection of the frequency-hopped signal sequence is given in Fig. 3-1a. Each arm of the filter contains a mixer, a bandpass filter, a square law detector and delay line. Assume that the spread-spectrum signal hops over M distinct frequencies, and that the frequency hopping sequence is $f_1, f_2, \ldots, f_M$ which repeats itself. The acquisition scheme then consists of M mixers each followed by a square law detector and delay line. The delay lines are inserted so that when the correct sequence appears, the maximum of the voltages $V_1, V_2, \ldots, V_M$ will occur at the same time. Therefore, the output voltage of the adder will exceed the threshold level with high probability, indicating synchronization of the receiver to the signal.

3.2 Stepped serial search scheme

The stepped serial search acquisition scheme is given in Fig. 3-1b. This scheme is used for acquisition of long pseudo random codes. In a stepped serial acquisition scheme the locally generated pseudo random frequency hopped signal is correlated with the incoming frequency-hopped signal. When the local hopping is aligned with that of the incoming signal, the input to the envelope detector is

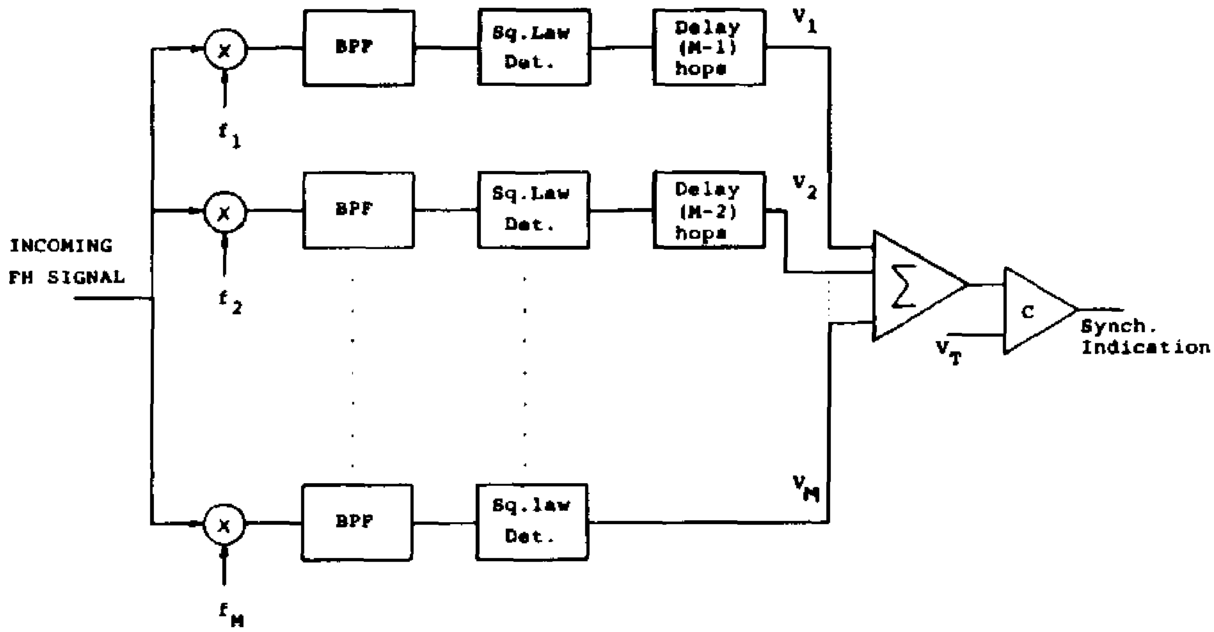


Fig. 3-1a Matched filter scheme

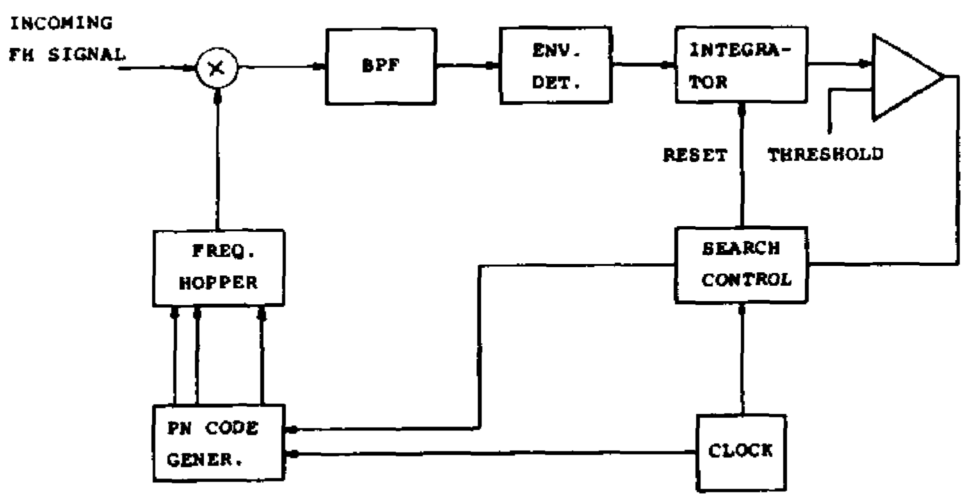


Fig. 3-1b Stepped serial search scheme

ideally a sinusoid at the intermediate frequency, and the output of the integrator will exceed the threshold with high probability.

Both schemes are analyzed in [1], and their performances are given. The performance measure is the probability that, at the end of the examination interval, the desired signal code sequence is not detected. The miss probabilities versus signal to noise ratio for a Ricean fading channel are given.

These two acquisition schemes are basic acquisition schemes for frequency-hopped spread spectrum signal. A two level threshold scheme is also analyzed in [1] and [2]. In [3] and [4] the sequential estimation technique is described.

The commonality among these analysis of acquisition schemes is that they use the same model for the HF channel. It is assumed that the propagation delay time is constant for all hopping frequencies and that the spreading time is not a function of time and frequency. This assumption is acceptable for slow frequency-hopped signal where the overlapping (due to difference in delay time) between two adjacent frequencies can be neglected.

Next we discuss the model of the HF channel with constant delay time.

3.3 The channel model

Let us denote the channel input and output signals by s(t) and r(t) respectively. Suppose that

$$s(t)=Re\{\tilde{u}(t)\exp(jw_o t)\} \qquad (3.3-1)$$

where $\tilde{u}(t)$ is the complex envelope of s(t), Re denotes "the real part" of the indicated quantity, and w_o denotes the carrier frequency in radians per seconds.

To simulate Rayleigh fading multipath channel the tapped delay line model shown in Fig. 3-2 is used. As ilustrated in Fig. 3-2, $\tilde{u}(t)\exp(jw_o t)$ is fed into delay line, and the signal emerging from each tap is multiplied by a complex tap-gain function $C_i(t)$. The resulting signals are then added together and their real part is extracted to obtain the channel-output signal r(t). According to this model the received process can be expressed in the form

$$r(t)=Re\{\sum_{i=1}^{N} C_i(t)\tilde{u}(t-\Delta t_i)\exp[jw_o(t-\Delta t_i)]\} \qquad (3.3-2)$$

where $C_i(t)$ are complex functions, and N is the number of taps.

For fading dispersive channel we can assume that r(t) and $C_i(t)$ are zero mean Gaussian random processes. This implies that the statistical properties of r(t) are

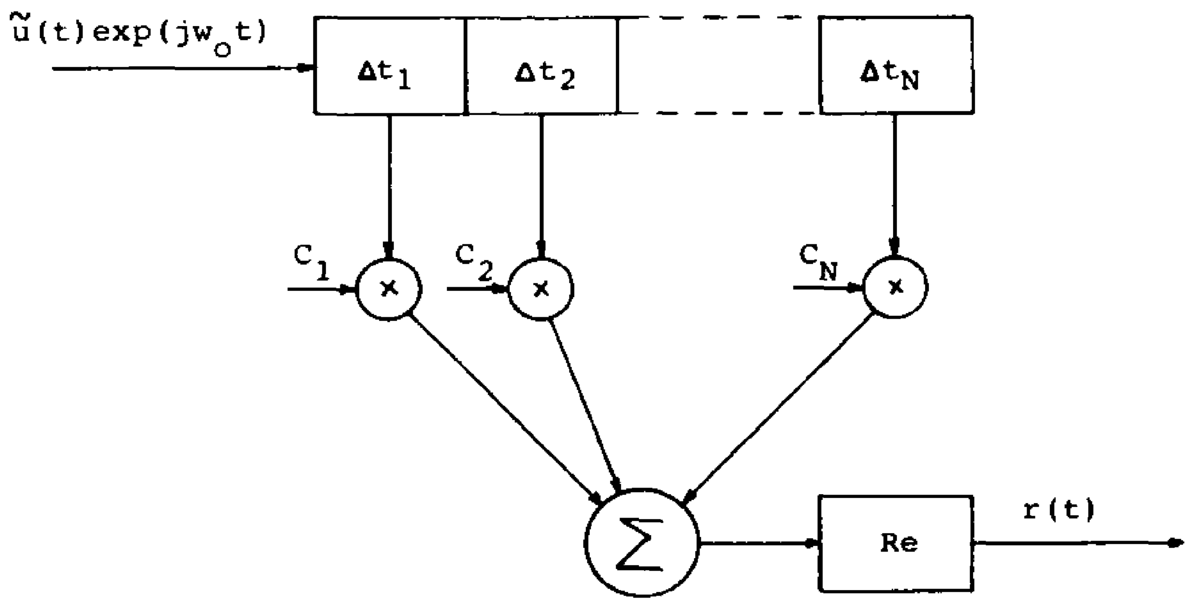


Fig. 3-2

completely specified by the correlation function $R_r(t_1,t_2)$. Furthermore it can be shown thet the tap gain functions associated with different taps are uncorrelated stationary random processes. Also the real and imaginary parts of each $C_i(t)$ are uncorrelated Gaussian processes with the same correlation function (for general discussion see [13], [14] and [15]). This implies that

$$E\{C_i(t_1)C_i(t_2)\}=0 \qquad (3.3\text{-}3)$$

for all i, t_1 and t_2.

Using the preceding assumptions and (3.3-3) we can find the correlation function of the output signal r(t) as

$$R_r(t_1,t_2)=E\{r(t_1)r(t_2)\} \qquad (3.3\text{-}4)$$

or

$$R_r(t_1,t_2)=\mathrm{Re}\{0.5\sum_{i=1}^{N} R_{c_i}(t_1-t_2)\tilde{u}(t_1-\Delta t_i)\tilde{u}^*(t_2-\Delta t_i)\exp[jw_o(t_1-t_2)]\} \qquad (3.3\text{-}5)$$

where $R_{c_i}(t_1-t_2)$ is the i-th tap-gain correlation function, that is

$$R_{c_i}(t_1-t_2) = E\{C_i(t_1)C_i^*(t_2)\} \qquad (3.3\text{-}6)$$

The average power $R_r(t,t)$ of the output signal r(t) is given as

$$P_{av} = R_r(t,t) = 1/2 \sum_{i=1}^{N} R_c(0)\left|\tilde{u}(t- t_i)\right|^2 \qquad (3.3\text{-}7)$$

and assuming that

$$\left|\tilde{u}(t-\Delta t_i)\right|^2 = 1 \tag{3.3-8}$$

we have

$$P_{av} = 1/2 \sum_{i=1}^{N} R_{c_i}(0) \tag{3.3-9}$$

Since $C_i(t)$ are Gaussian random processes with the same variance, and assuming that $P_{av} = 1$ we can find the variance $\sigma_{c_i}^2$ of each $C_i(t)$, that is,

$$1 = 1/2 \sum_{i=1}^{N} \sigma_{c_i}^2 \tag{3.3-10}$$

$$\sigma_{c_i}^2 = 2/NTAPS \tag{3.3-11}$$

where NTAPS is the number of taps.

The Doppler shifts are modeled by the bandwidth of $C_i(t)$ where the rapidity of changes relates to the Doppler spread. It can be shown that a close relation exists between the rapidity of fading and the Doppler spread.

Since the functions $C_i(t)$ are random Gaussian processes, they are generated by passing the Gaussian white noise through low pass filter whose cutoff frequency is equal to the fading rate $1/B_d$ (B_d is the Doppler spread). The variance of the Gaussian process at the output of the low pass filter is adjusted to the value

$$\sigma^2 = 2/NTAPS \tag{3.3-12}$$

Defining the multipath spread for a channel as T_m

and the Doppler spread as B_d, another important characteristic of a fading channel is the spread factor

$$L = T_m B_d \qquad (3.3\text{-}13)$$

If one, for example, attempts to determine the state of the medium by sending a sounding pulse periodically, the period between pulses must be at least T_m to avoid ambiguities. On the other hand, the medium changes state in a time of the order $1/B_d$. Hence the "instantaneous" state of the medium is measurable using periodic pulses only if

$$T_m << \frac{1}{B_d} \qquad (3.3\text{-}14)$$

i.e., the spread factor must be well below unity. Another important interpretation of the spread factor is as follows. If signal pulses are of length T, one must have $T >> T_m$ to avoid signifficant intersymbol interference, since only then will the energy arriving at any instant all primarily relate to one signal pulse. On the other hend to avoid severe time-fading distortion of the pulse (or equivalently, destruction of coherence over the pulse owing to channel phase fluctuation within one pulse duration) one must also have $T << 1/B_d$. Both condition can be satisfied only if

$$T_m << T << 1/B_d \qquad (3.3\text{-}15)$$

which again requires a spread factor well below unity.

4. SUMMARY OF RESEARCH COMPLETED

In this work we propose an acquisition scheme for a frequency-hopped SS signal received over a Rayligh fading multipath channel, whose delay time versus frequency is given in Fig. 2-2. We consider relatively fast FH with a hopping rate of 1 KHz, and 127 different hopping frequencies. We assume that the probability density function of the channel delay time is given by (2-3), with the maximum deviation from T_{nd} of 1 msec. The received signal is embedded in white Gaussian noise. While the above problem appears to be specific the approach given below is general and can be generalized further.

In this case, ignoring the spreading effect, there will be a possible overlaping between only three adjacent frequencies. For such a case we propose an optimum receiver and acquisition scheme whose characteristics are analyzed mathematicaly and by computer simulation.

We characterize the performance of the acquisition scheme by the probability of detection for given probability of false acquisition when the input signal is white Gaussian noise only. The probability of detection is the probability that, at the end of the observation period, the desired signal code sequence is detected.

5. RESEARCH

The system we consider is shown in Fig. 5.1-1. The transmitter consists of a frequency synthesizer whose output frequency is determined by n+m bits. The m-bits are data bits and the n-bits are bits generated by the LFSR generator. Actually this is a case of an m-ary FSK frequency-hopped SS system.

When there is no input data the output frequency is determined by the LFSR generator only. This case is assumed when acquisition is to be performed.

5.1 Receiver

Let us for a moment assume that the channel shown in Fig. 5.1-1 consists of a variable delay line only, and that the nominal delay time T_{nd} is not a function of frequency. Actually we assume that the curve T_{nd} versus frequency is known (Fig. 2-1).

The transmitted signal is given by

$$s_i(t)=A\cos\{(w_i+w_j)t\} \qquad 1\leq i\leq L \ ; \ 1\leq j\leq 2^m \qquad (5.1\text{-}1)$$

For this case the received signal for $\Delta t=0$ is shown in Fig. 5.1-2(a) and the received signal for $-T_h\leq\Delta t\leq T_h$ is shown in Fig. 5.1-2(b). In this figure A means that a frequency, say f_i can arrive advanced by a time as much as T_h and D means

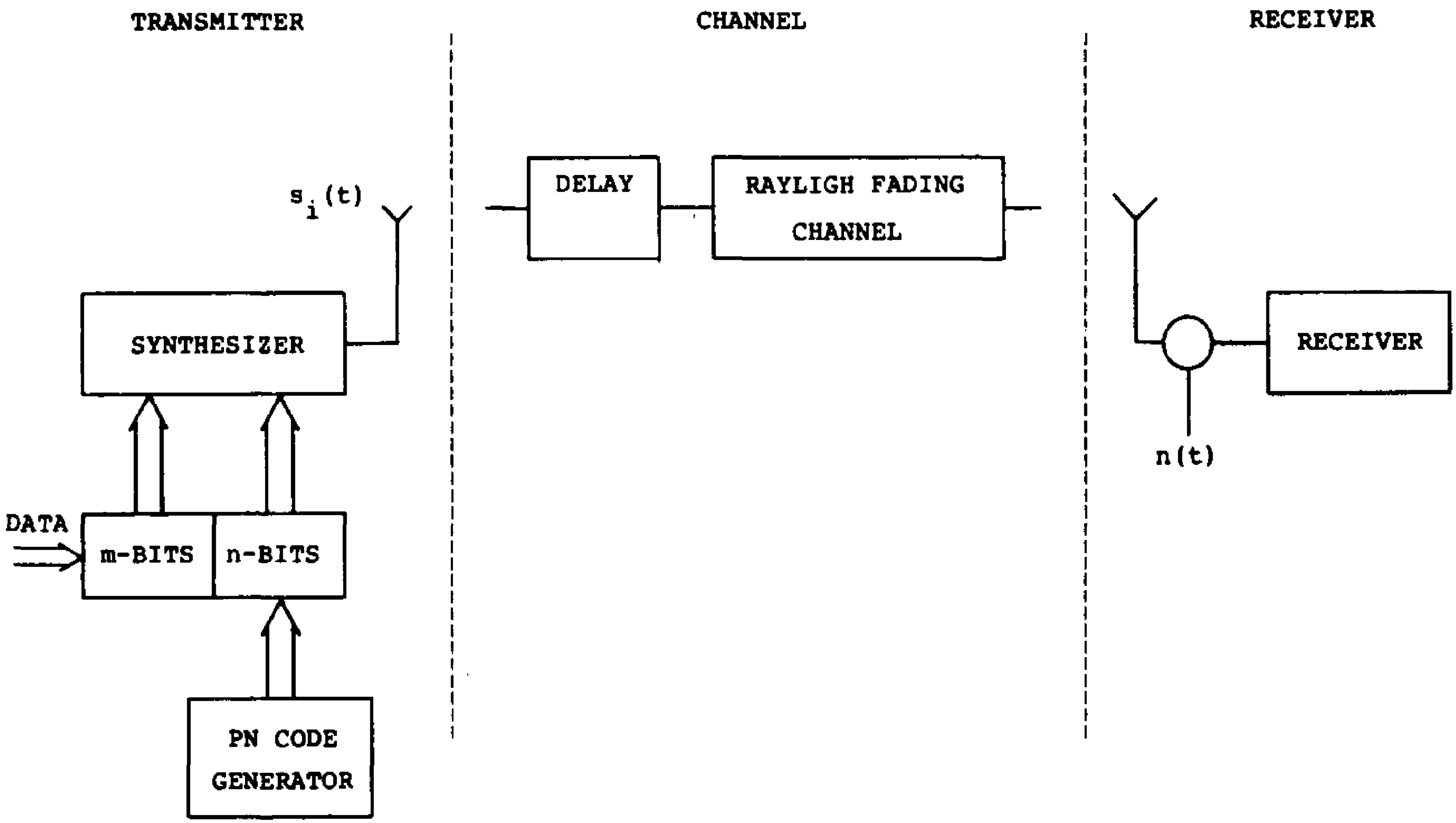
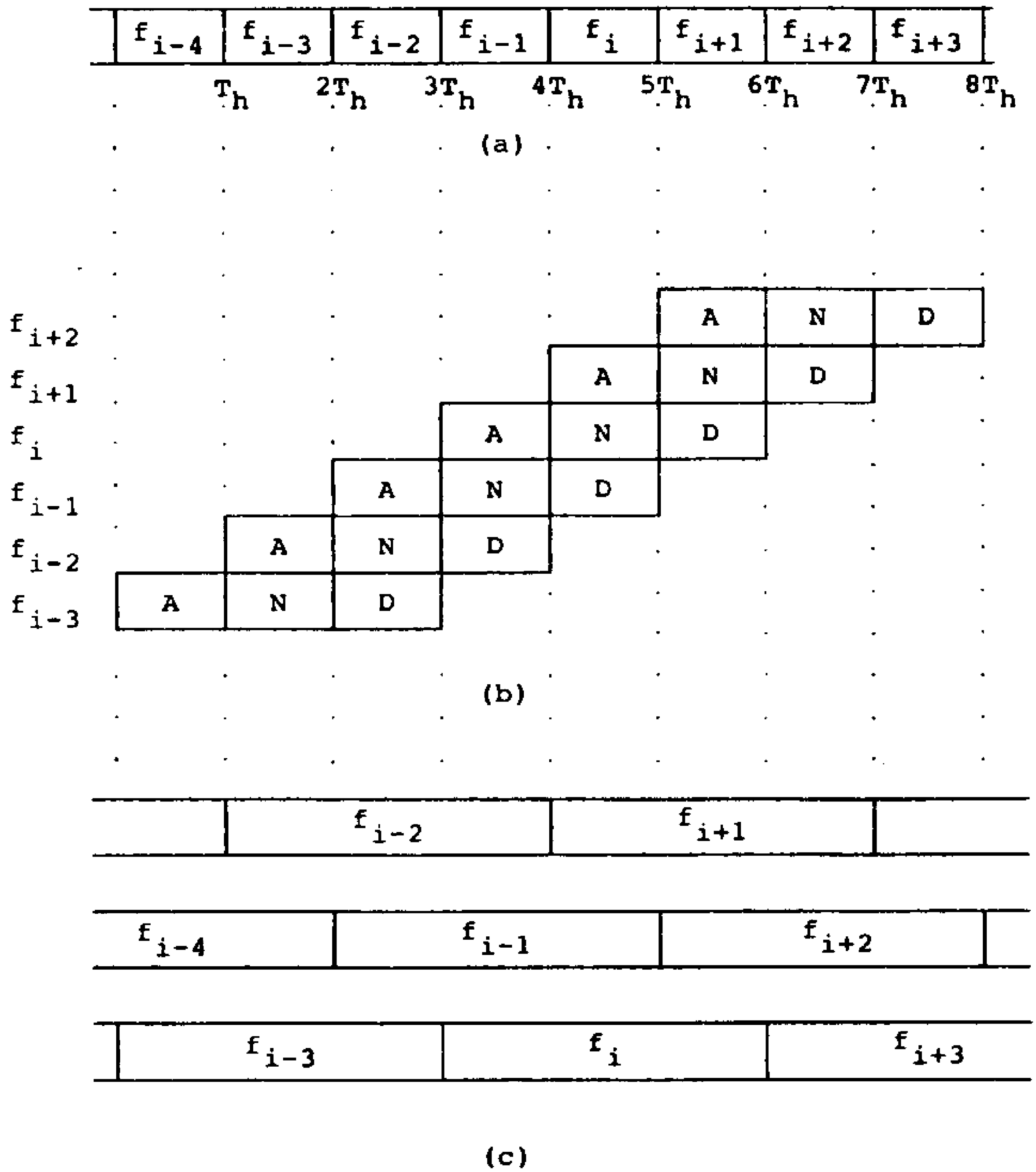

TRANSMITTER
CHANNEL
RECEIVER
$s_i(t)$
DELAY
RAYLIGH FADING
CHANNEL
SYNTHESIZER
RECEIVER
n(t)
DATA
m-BITS
n-BITS
PN CODE
GENERATOR

Fig. 5.5-1



f i-4
f i-3
f i-2
f i-1
f i
f i+1
f i+2
f i+3
Th
2Th
3Th
4Th
5Th
6Th
7Th
8Th
(a)
f i+2
f i+1
f i
f i-1
f i-2
f i-3
A
N
D
(b)
f i-2
f i+1
f i-4
f i-1
f i+2
f i-3
f i
f i+3
(c)

Fig. 5.1-2

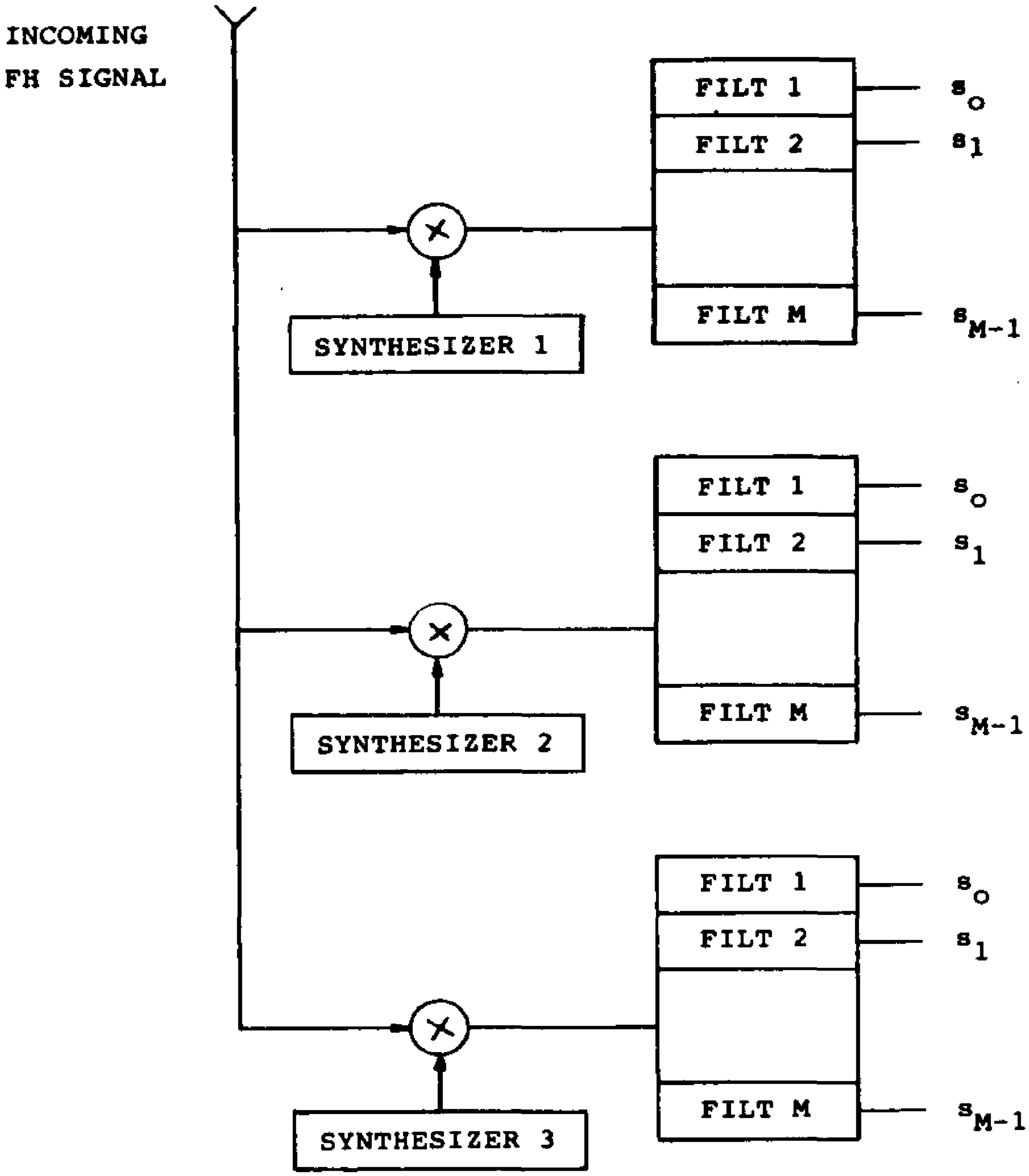

INCOMING
FH SIGNAL
FILT 1
FILT 2
FILT M
SYNTHESIZER 1
s_o
s_1
s_M-1
FILT 1
FILT 2
FILT M
SYNTHESIZER 2
s_o
s_1
s_M-1
FILT 1
FILT 2
FILT M
SYNTHESIZER 3
s_o
s_1
s_M-1

Fig. 5.1-3

that it can be delayed by as much as T_h.

We can see from Fig. 5.1-2(b) that there is possible overlapping between three adjacent frequencies only. In the time interval $3T_h$ to $4T_h$, for example, there is possible overlapping between f_{i-2}, f_{i-1} and f_i. So at the same time we can receive three signals with three different carrier frequencies. In order to receive complete signal without this overlapping we need three receivers R_1, R_2 and R_3 whose input circuits are adjusted to f_{i-1}, f_i and f_{i+1} recpectively. Assuming perfect synchronization i.e. assuming that we know exact time of arrival, for each frequency, for $\Delta t=0$ (no variable delay), the receiver which will detect all signals without overlapping is shown in Fig. 5.1-3.

Synthesizers 1,2 and 3 will hop to the new frequency each $3T_h$ seconds following the sequence shown in Fig. 5.1-2(c).

Next we calculate the probability of error for a receiver given in Fig. 5.1-3, assuming perfect sinchronization i.e. assuming that we know exact time of arrival, for each frequency, for $\Delta t=0$.

5.2 Probability of error

We calculate the brobability of error for binary FSK frequency-hopped spread spectrum system.

First we consider binary FSK signal with known time of arrival, frequency and amplitude but with random phase. Both signals

$$
\begin{aligned}
s_o(t) &= A\sin(w_o t + \Phi) \qquad 0<t<T_h \\
s_1(t) &= A\sin(w_1 t + \theta) \qquad 0<t<T_h
\end{aligned}
\qquad (5.2\text{-}1)
$$

are transmitted with equal probability. Φ and θ are random phases with uniform distribution in the interval (0,2 The optimum receiver for this case according to the theory of optimum detection [9] is given in Fig. 5.2-1. The output signal is sampled each T_h seconds and compared with zero. If the output voltage at T_h is greater than zero we assume that $s_1(t)$ has been sent, and if the output is less than zero we assume that $s_o(t)$ was sent.

In order to calculate the probability of error we can assume that $s_1(t)$ was sent. Conditional probability of error for this case is given as

$$P\ (q_1) = \int_{q_1}^{\infty} p_o(q_o/q_1)\,dq_o \qquad (5.2\text{-}2)$$

where $p(q_o/q_1)$ is the probability density function of q_o

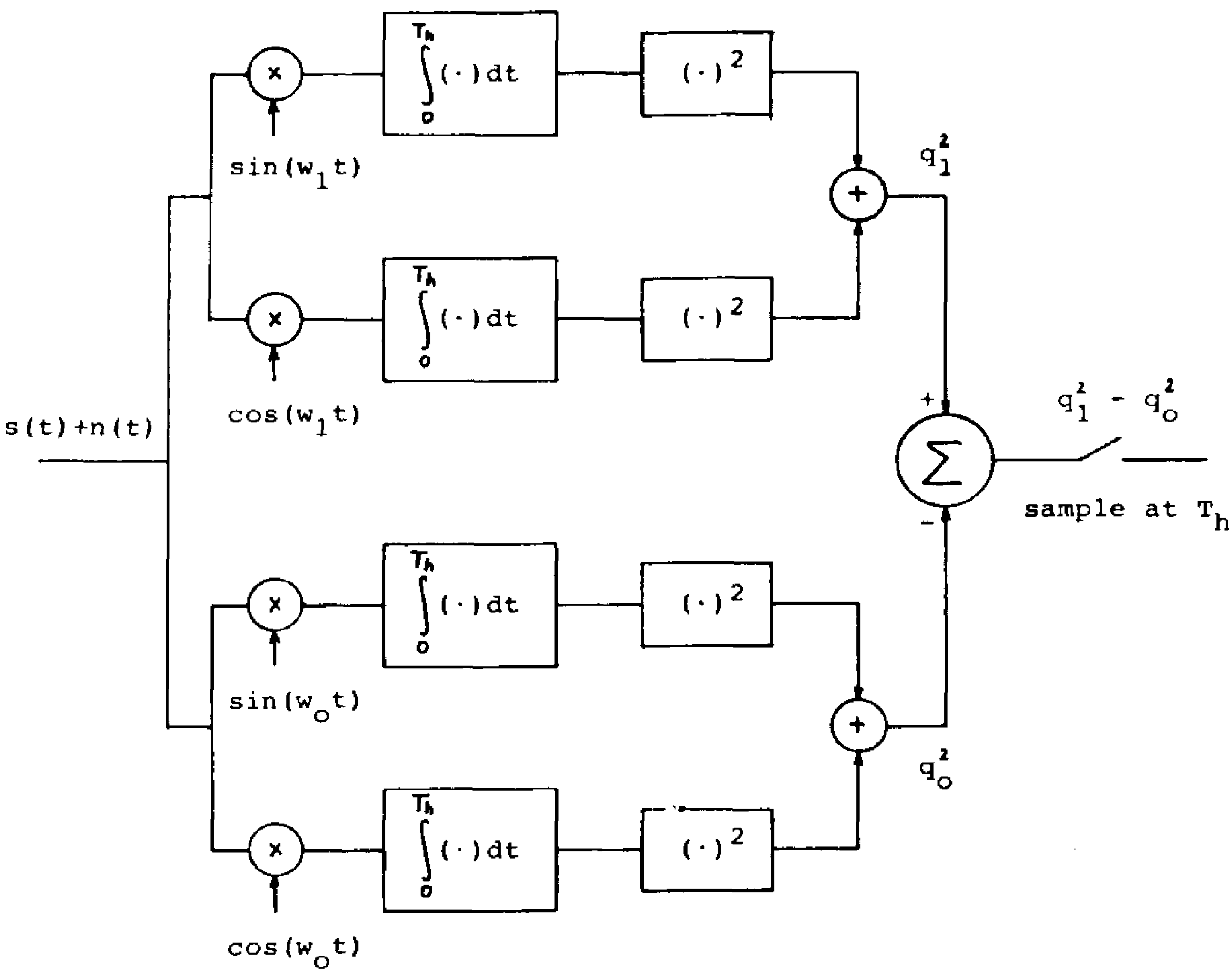


Fig. 5.2-1

when $s_1(t)$ is transmitted. Averaging over all values for q_1 we get the probability of error as

$$P_\varepsilon = \int_0^\infty P\ (q_1) p_1(q_1) dq_1 \qquad (5.2\text{-}3)$$

or

$$P_\varepsilon = \int_0^\infty p_1(q_1) dq_1 \int_{q_1}^\infty p_o(q_o/q_1) dq_o \qquad (5.2\text{-}4)$$

Since in our case the frequency separation between two adjacent frequencies is sufficiently large we can neglect the spillover and we can write that

$$p_o(q_o/q_1) = p_o(q_o) \qquad (5.2\text{-}5)$$

or substituting (5.2-5) into (5.2-4) we have that

$$P_\varepsilon = \int_0^\infty p_1(q_1) dq_1 \int_{q_1}^\infty p_o(q_o) dq_o \qquad (5.2\text{-}6)$$

It can be shown [9] that

$$p_1(q_1) = \frac{q_1}{\sigma_T^2} \exp\left[\frac{q_1^2 + (AT_h/2)^2}{-2\,\sigma_T^2}\right] I_o\left(\frac{q_1 AT_h}{-2\,\sigma_T^2}\right) \qquad (5.2\text{-}7)$$

and

$$p_o(q_o) = \frac{q_o}{\sigma_T^2} \exp\left(-\frac{q^2}{2\,\sigma_T^2}\right) \qquad (5.2\text{-}8)$$

where

$$\sigma_T^2 = \frac{N_o T_h}{4} \qquad (5.2\text{-}9)$$

is the variance of the noise at the output of the integrators in Fig. 5.2-1.

Substituting (5.2-7) and (5.2-8) in (5.2-6) we find for the probability of error

$$P_\varepsilon = \int_0^\infty dq_1 \int_{q_1}^\infty \frac{q_o q_1}{\sigma_T^4} \exp\left[\frac{q^2 + q^2 + (AT_h/2)^2}{-2\sigma_T^2}\right] I_o\left(\frac{q_1 AT_h}{2\sigma_T^2}\right) dq_o \qquad (5.2\text{-}10)$$

The solution of this integral is given in [9] and is

$$P_\varepsilon = \frac{1}{2} \exp(-E/2N_o) \qquad (5.2\text{-}11)$$

where

$$E = \frac{A^2 T_h}{2} \qquad (5.2\text{-}12)$$

is the energy of the signal.

The probability of error P_ε (5.2-11) versus signal to noise ratio is plotted in Fig. 5.2-2.

Now we calculate the probability of error for a signal with known frequency and time of arrival but unknown amplitude and phase. The received signals are

$$s_0(t) = A\sin(w_0 t + \Phi)$$
$$s_1(t) = B\sin(w_1 t + \theta) \qquad (5.2\text{-}13)$$

where Φ and θ are again random phases uniformly distributed in the interval $(0, 2\pi)$. A and B are random amplitudes with Rayleigh density functions

$$p(A) = \frac{A}{A_o^2}\exp\left(-\frac{A^2}{2A_o^2}\right)$$
$$p(B) = \frac{B}{A_o^2}\exp\left(-\frac{B^2}{2A_o^2}\right) \qquad (5.2\text{-}14)$$

The optimum receiver is the same as in the previous case (Fig. 5.2-1), and the decision is made on the same way.

Using (5.2-11) we can find the conditional probability of error as

$$P_\varepsilon(A) = \frac{1}{2}\exp\left(-\frac{E}{2N_o}\right) = \frac{1}{2}\exp\left(-\frac{A^2 T_h}{4N_o}\right) \qquad (5.2\text{-}15)$$

or averaging over all values of A we have

$$P_\varepsilon = \int_0^\infty P_\varepsilon(A)\,p(A)\,dA \qquad (2.5\text{-}16)$$

The solution of this integral is

$$P = \frac{1}{2 + \varepsilon} \qquad (5.2\text{-}17)$$

where

$$\mathcal{E} = \frac{A^2 T}{N_o} = \frac{E_{av}}{N_o} \qquad (5.2\text{-}18)$$

This probability of error P_ε (5.2-17) is also plotted in Fig. 5.2-2.

Now we calculate the probability of error for a signal with known frequency and amplitude but unknown phase and time of arrival. We do not know the exact time of arrival but we know that signal has to come in the interval $(0,3T_h)$. This would be the probability of error for a frequency-hopped spread spectrum system given in Fig. 5.1-1, assuming that synchronization is perfect and that the channel consists only of the variable delay line. The near optimum receiver is given in Fig. 5.2-1 with the integration period $(0,3T_h)$.

Integrating from 0 to $3T_h$ we will receive complete signal as in the case with known time of arrival. Also, the average value of the sample at the end of the integration period is same as in the case with known time of arrival. The only difference is that the variance of this sample will be three times greater, because the integration period is $3T_h$. Thus, taking into account all these changes and using (5.2-11) we find the probability of error for this case is

$$P = \frac{1}{2} \exp(- \frac{E}{6N_o}) \qquad (5.2\text{-}19)$$

This curve (5.2-19) is also plotted in Fig. 5.2-2.

Using same reasoning we find that the probability of error for unknown time of arrival and Rayleigh fading is given as

$$P = \frac{1}{2 + \mathcal{E}/3} \qquad (5.2\text{-}20)$$

This curve (5.2-20) is also plotted in Fig. 5.2-2.

From Fig. 5.5-2 we can see the effect of unknown time of arrival for a channel without and with Rayleigh fading. If the expected time of arrival is increased from $3T_h$ to $4T_h$, $5T_h$ ... that will result in further decrease of the system performance.

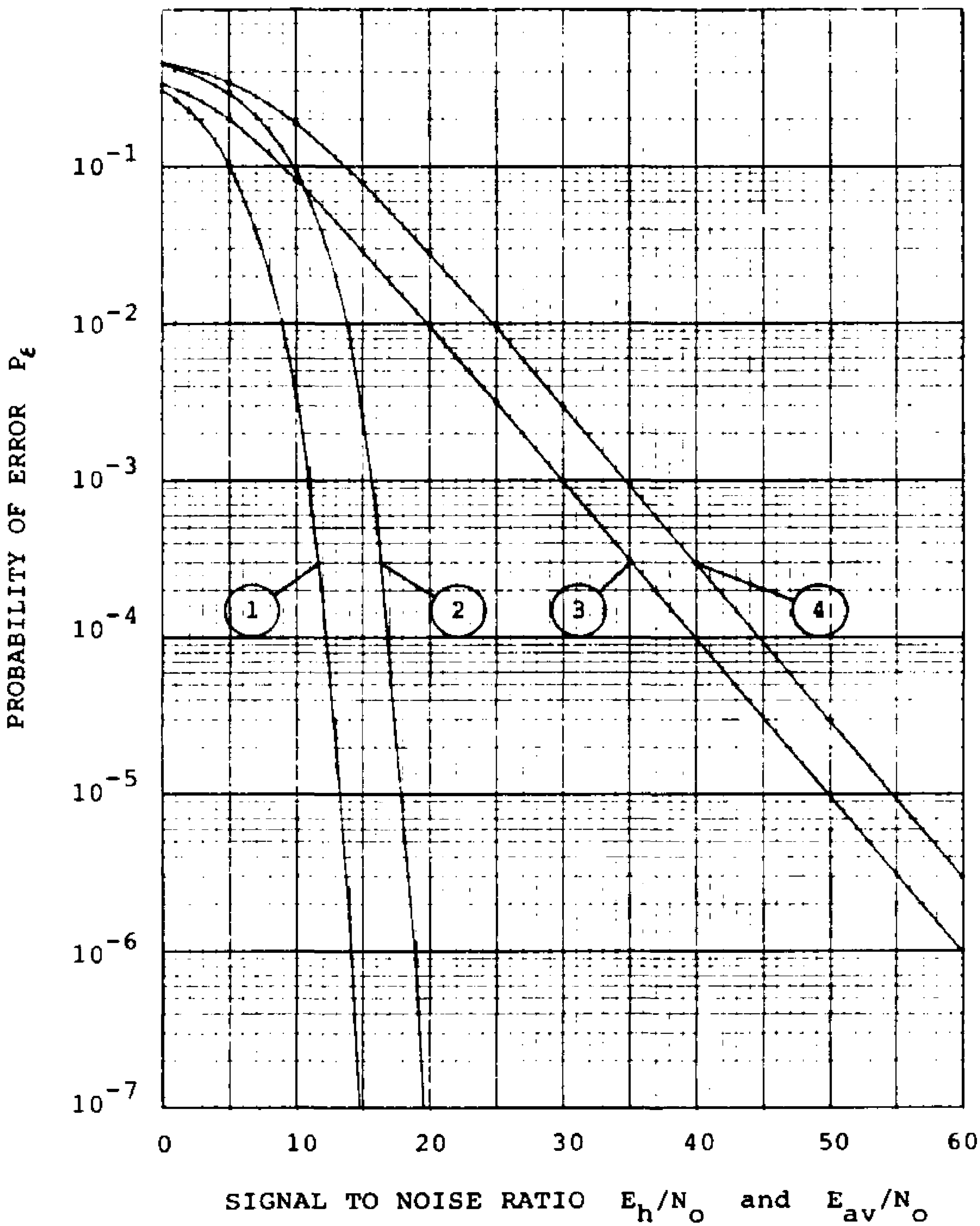


Fig. 5.2-2 1. KNOWN TIME OF ARRIVAL - NO FADING
2. UNKNOWN TIME OF ARRIVAL - NO FADING
3. KNOWN TIME OF ARRIVAL - WITH FADING
4. UNKNOWN TIME OF ARRIVAL - WITH FADING

5.3 Acquisition scheme

The basic function of an acquisition circuit is to bring the incoming PN sequence and the locally generated sequence into sufficiently close alignment so that the difference is within the pull-in range of the fine-acquisition (tracking) loop.

Let us again assume that the channel shown in Fig. 5.1-1 can be modeled by a variable delay line. We also assume that during the initial acquisition no data is transmitted, i.e. the transmitted signal is

$$s_i(t)=A\cos(w_i t) \qquad 0\leqslant t\leqslant T_h, \quad 1\leqslant i\leqslant L \qquad (5.3-1)$$

In this case, the receiver does not know the exact time of arrival of the signal, but the receiver knows that the signal has to arrive in the time interval from 0 to $(2^n-1)T_h+2T_h$ seconds. Also the receiver knows the sequence of the transmitted frequencies. Thus if we are able to determine the time of arrival for any frequency for $\Delta t=0$, the synchronization will be perfect.

So, we have a case of detection of a signal with unknown time of arrival, amplitide and phase. The optimum receiver for this case is shown in Fig. 5.3-1(a). There are three quadrature receivers because there is possible overlapping between three adjacent frequencies only. For the transmitted sequence given in Fig. 5.3-1(b), the outputs

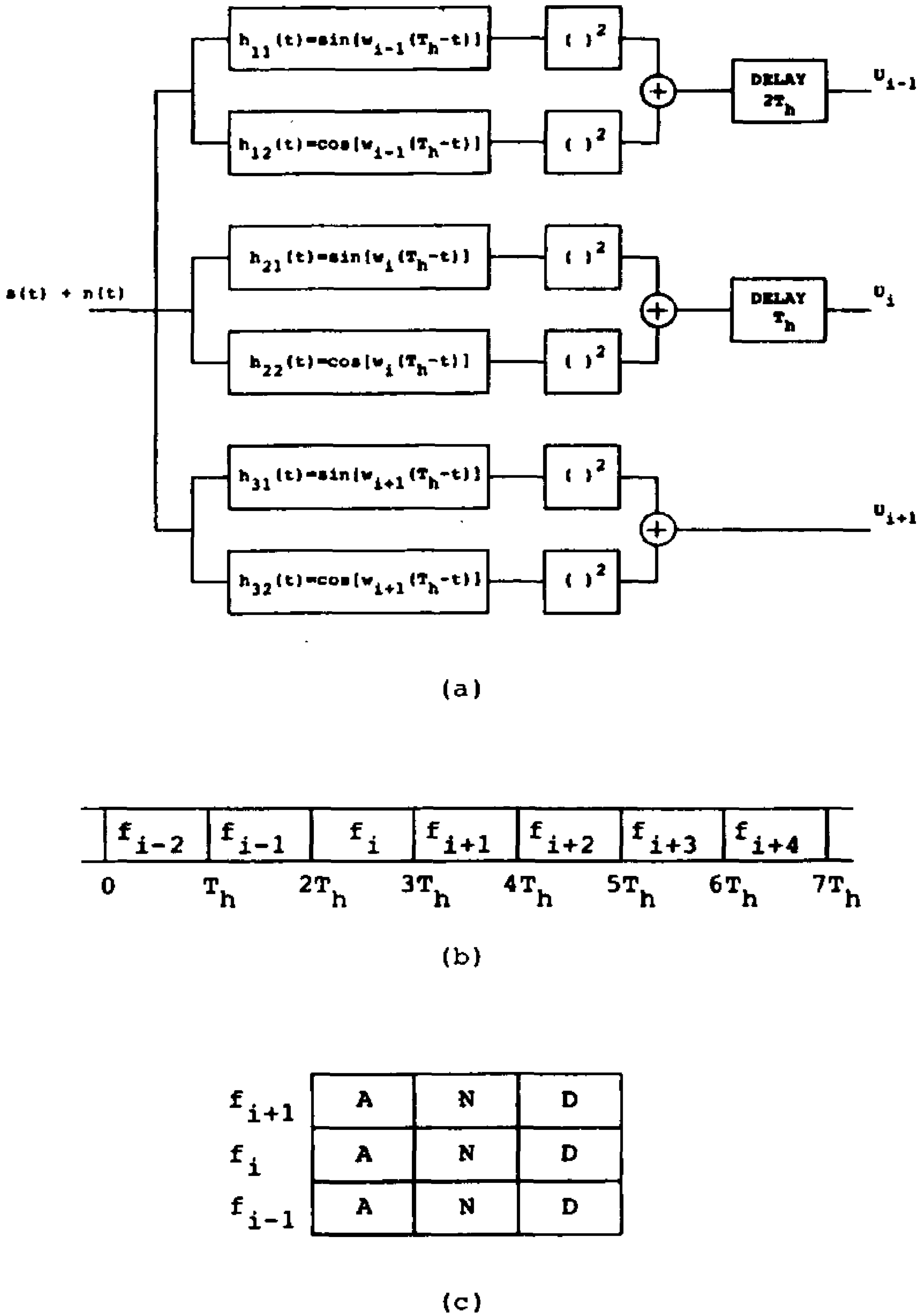

s(t) + n(t)
h11(t)=sin(wi-1(Th-t))
h12(t)=cos(wi-1(Th-t))
h21(t)=sin(wi(Th-t))
h22(t)=cos(wi(Th-t))
h31(t)=sin(wi+1(Th-t))
h32(t)=cos(wi+1(Th-t))
( )2
DELAY 2Th
DELAY Th
Ui-1
Ui
Ui+1
(a)
fi-2 fi-1 fi fi+1 fi+2 fi+3 fi+4
0 Th 2Th 3Th 4Th 5Th 6Th 7Th
(b)
fi+1 A N D
fi A N D
fi-1 A N D
(c)

Fig. 5.3-1

U_{i-1}, U_i and U_{i+1} are shown in Fig. 5.3-1(c). The delay lines of T_h and $2T_h$ will ensure that, for a noiseless channel, the maxima of U_{i-1}, U_i and U_{i+1} will occur in the interval $3T_h$ to $5T_h$.

5.4 Acquisition strategy

During the time interval $T=(2^n-1)T_h+2T_h$ the output signals U_{i-1}, U_i and U_{i+1} are observed continuously and compared with the threshold TR. If any output signal axceeds the threshold TR the maximum is detected and the time of its occurance T_i (i=1,2,3) is determined.

If only one maximum is detected we assume that this maximum is due to the noise only, and we assume that acquisition has not occurred.

In the case when two maxima are detected and when the time distance between them is greater then $3T_h$ we again declare that acquisition has not occurred. If this distance is less then $3T_h$ we calculate the acquisition time as

$$\text{ACQUISITION TIME} = \frac{T_i + T_j}{2} \qquad (5.4\text{-}1)$$

When all three maxima are detected and when the time distance between $T_{i,max}$ and $T_{j,max}$ ($i \neq j$) is less then $3T_h$ we calculate the acquisition time as

$$\text{ACQUISITION TIME} = \frac{T_1 + T_2 + T_3}{3} \qquad (5.4\text{-}2)$$

If $T_{i,max} - T_{j,min}$ is greater then $3T_h$ and if there is no time distance between any two maxima less then $3T_h$, we declare that acquisition has not occurred. If there is such a distance the acquisition time is calculated as in the case with two maxima.

5.5 False-alarm probability

The probability of false acquisition P_{facn} is the probability that acquisition will occur if the input signal to the receiver of Fig. 5.3-1(a) is the noise only. Before we calculate P_{facn} we have to find the false alarm probability for a system that observed the output q(t) of a quadrature receiver over an interval of time $0<t<T$ which is much longer than the duration of the signal. A false alarm occurs in this system if the output q(t) exceeds the threshold level TR at any time during the observation interval, when the input to the system consists of noise alone. Let us denote the probability of this event by P_{fa}. Stated otherwise this probability is given by

$$P_{fa} = 1 - P(T) \qquad (5.5\text{-}1)$$

where

$$P(T) = \Pr\{q(t)<TR,\ 0\leq t\leq T\} \qquad (5.5\text{-}2)$$

is the probability that the receiver output q(t) does not appear above the threshold TR at all during the interval $0<t<T$. We can assume that at $t=0$, $q(0)=0$, i.e., that the receiver output will start from zero. In this case for $TR>0$, $P(0)=1$ and $P(\infty)=0$.

The negative derivative

$$p(t) = -\frac{\partial P}{\partial t} \qquad 0<t<\infty \qquad (5.5\text{-}3)$$

is the so called "first passage-time probability density function"; $p(t)dt$ is the probability that the output $q(t)$ crosses the threshold TR from below for the first time in the interval t to $t+dt$. The problem is to calculate the density function $p(t)$. This problem can be solved for only a few types of stochastic processes.

The first passage time probability density function can be found for the case of a one dimensional Markov process.

It can be shown that the output of the quadrature receiver given in Fig. 5.5-1 is a Markov process whose transitional probability density function is

$$p(q_\tau/q) = \frac{q_\tau}{\sigma^2(1-\mu^2)} \exp\left(-\frac{q_\tau^2+\mu^2 q^2}{2\sigma^2(1-\mu^2)}\right) I_o\left(\frac{\mu \cdot q \cdot q_\tau}{(1-\mu^2)\sigma^2}\right) \quad (5.5\text{-}4)$$

where σ^2 is the variance of the noise at the output of each RC-filter, and $\mu=\exp(-t)$. It can be shown [7],[8] that for normalized variance $\sigma^2=1$, (5.5-4) satisfies a Fokker-Planck differential equation

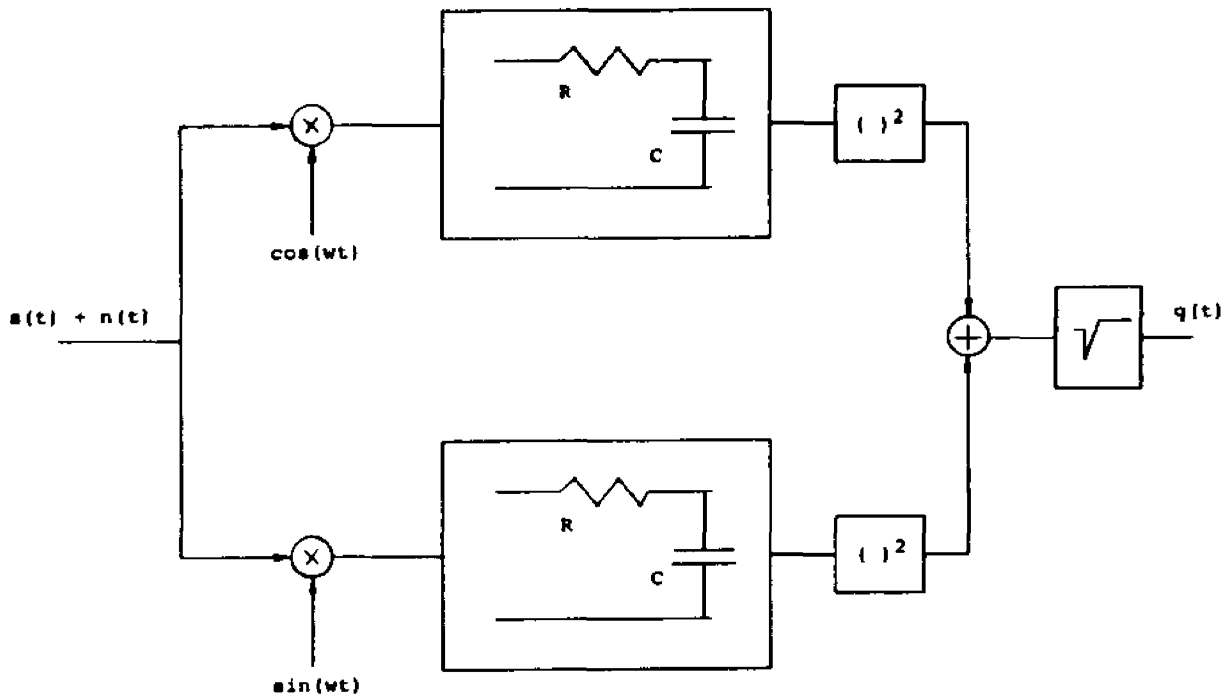


Fig. 5.5-1

$$\frac{\partial p}{\partial t} = -\frac{\partial}{\partial q}\{K_1(q)p(q)\} + \frac{1}{2}\frac{\partial^2}{\partial q^2}\{K_2(q)p(q)\} \qquad (5.5\text{-}5)$$

with

$$K_1(q) = \frac{1}{q} - q \qquad K_2(q) = 2 \qquad (5.5\text{-}6)$$

The Fokker-Planck equation can be used to find the probability density function $p(TR,t/q_o)$ of the first time t that the envelope q(t) crosses the threshold level q=TR given the value q_o at t=0. The first passage problem can be solved by the method of Siegert [5]. According to Siegert, the Laplace transform of the first-passage-time p.d.f. p(TR,t/q) is

$$\mathcal{L}\{p(TR,t/q_o)\} = \frac{M(\frac{S}{2};1;\frac{q_o^2}{2})}{M(\frac{S}{2};1;\frac{TR^2}{2})} \qquad (5.5\text{-}7)$$

where M(a;b;z) is a confluent hypergeometric function and according to [6] is given by

$$M(a;b;z) = \frac{az}{b} + \frac{a(a+1)}{b(b+1)}\frac{z^2}{2!} + \cdots + \frac{a(a+1)\cdots(a+n-1)}{b(b+1)\cdots(b+n-1)}\frac{z^n}{n!} + \cdots \qquad (5.5\text{-}8)$$

In order to find the probability density function itself we must find the inverse Laplace transform of (5.5-7). By the usual technique one can obtain a series of the form

$$p(TR,t/q_o) = \sum_n A_n \exp(S_n t) \qquad (5.5\text{-}9)$$

where the A_n are the residues of (5.5-7) at the roots S_n of

$$M\left(\frac{S_n}{2}; 1; \frac{TR^2}{2}\right) = 0 \qquad (5.5\text{-}10)$$

For values of $(TR^2/2) > 1$ the largest zero S_1 of (5.5-10) is greater than -1 and such that

$$S_1 \gg S_2 \qquad (5.5\text{-}11)$$

where S_2 is the second largest zero. In this case the equation (5.5-9) can be approximated by

$$p(TR,t/q_o) = A_1 \exp(S_1 t) \qquad (5.5\text{-}12)$$

From (5.5-9) and (5.5-3) we can find the probability that the output of a quadrature receiver from Fig. 5.5-1 will not cross the threshold level TR during the interval $0<t<T$, will be

$$P(T) = -\sum_n \frac{A_n}{S_n} \exp(S_n T) \qquad (5.5\text{-}13)$$

or using the approximation (5.5-12)

$$P(T) = -\frac{A_1}{S_1} \exp(S_1 T) \qquad (5.5\text{-}14)$$

and finally using (5.5-1), the probability of false alarm is given as

$$P_{fa} = 1 + \frac{A_1}{S_1} \exp(S_1 T) \qquad (5.5\text{-}15)$$

Usually we need to know the value of the threshold for a given probability of false alarm.

5.6 Calculation of the threshold TR for a given P_{fa}

Using (5.5-13) we can express the probability of false alarm as

$$P_{fa} = 1 + \sum_n \frac{A_n}{S_n} \exp(S_n T) \qquad (5.6\text{-}1)$$

where A_n are the residues of (5.5-7) at the roots S_n of (5.5-10). In this case the probability of false alarm is given and the period of observation T is known. We also assume that $q_o=0$. It follows that in order to find TR we have to determine $M(S/2;1;TR^2/2)$ so that the roots of this confluent hypergeometric function and the residues of (5.5-7) at these roots satisfy (5.6-1) for a given P_{fa} and T.

This problem has been solved numerically since it is impossible to find a solution in closed form.

For different values of P_{fa} and for T = 129 msec the normalized threshold

$$TR_n = \frac{TR}{\sigma} \qquad (5.6\text{-}2)$$

has been calculated and is given in Table 1. and is plotted in Fig. 5.6-1.

Table 1

P_{fa}	TR_n	TR_{eq}
0.9	3.65	3.46
0.5	4.02	3.84
0.25	4.26	4.10
0.1	4.52	4.37
0.075	4.60	4.44
0.05	4.70	4.54
0.025	4.86	4.71
0.01	5.07	4.92
0.0075	5.13	4.99
0.005	5.21	5.08
0.0025	5.36	5.22
0.001	5.54	5.41

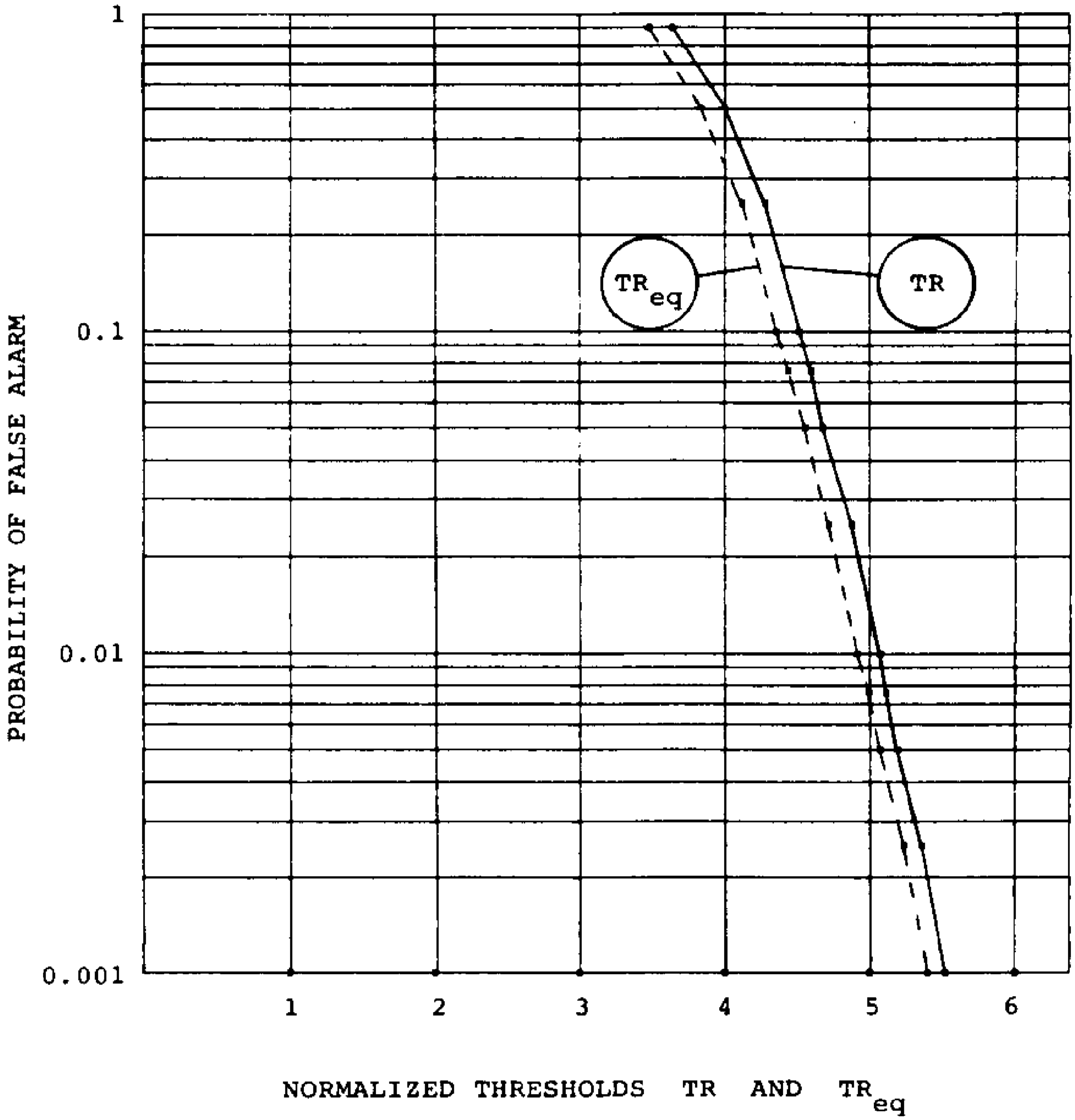

1
0.1
0.01
0.001
PROBABILITY OF FALSE ALARM
TR
TReq
1
2
3
4
5
6
NORMALIZED THRESHOLDS TR AND TReq

Fig. 5.6-1

5.7 Equivalent threshold

In order to simulate the first-passage problem using digital computer and to get reasonable results we have to introduce the equivalent threshold TR_{eq}.

In digital simulations we deal with the samples of analog signals. The sampling rate has a finite value and in our case the sampling period is

$$T_s = \frac{T_h}{10} \qquad (5.7\text{-}1)$$

where T_h is hopping period. So for T_h = 1 msec the sampling rate is 10 KHz. Suppose now that for this sampling rate we have just the noise at the input and that we want to find P_{fa} for given TR and T (the observation period). We will observe the output of our receiver and if the output signal exceeds the threshold TR during the interval (0,T) we say that first passage has occurred. We make N such observations and we calculate the probability of the first passage i.e. the probability of false alarm as

$$P_{fa} = \frac{N_{fp}}{N} \qquad (5.7\text{-}2)$$

where N_{fp} is the number of the first passages occurred in N observations.

The accuracy of the simulation depends on N and T_s. If T_s approaches to zero the digital approximation of an analog signal is beter but the simulation time will go to infinity. For reasonable values of T_s we will get reasonable simulation time, but the simulated P_{fa} will be far away from the calculated value. The reason for this is that we do not know anything about the signal between two samples.

Let us consider the example given in Fig. 5.7-1. We can see that the first passage has occurred but, because of the finite sampling rate, the digital simulation will miss this first passage. In order to keep the same relatively low sampling rate and to be able to simulate the first passage problem we introduce the equivalent threshold.

Let us denote the sampled values of a continuous random proces q(t) by $q(nT_s)$. If sampled value of q(t) at kT_s is $q(kT_s)$ and if the probability that the random proces will cross the threshold TR during the time interval $[kT_s,(k+1)T_s]$ is greater than 0.5 we will assume that the first passage has occurred. Since the sampling perion is constant we can find the value of TR_{eq} such that if the value of any sample is greater than TR_{eq} the probability that the random process will cross the threshold TR, during the next sampling period, is greater than or equal to 0.5.

From Fig. 5.7-1(a) we observe that we have missed the first passage because of the finite sampling rate. But, introducing the equivalent threshold TR_{eq} as we can see from Fig. 5.7-1(b) the first passage will be detected.

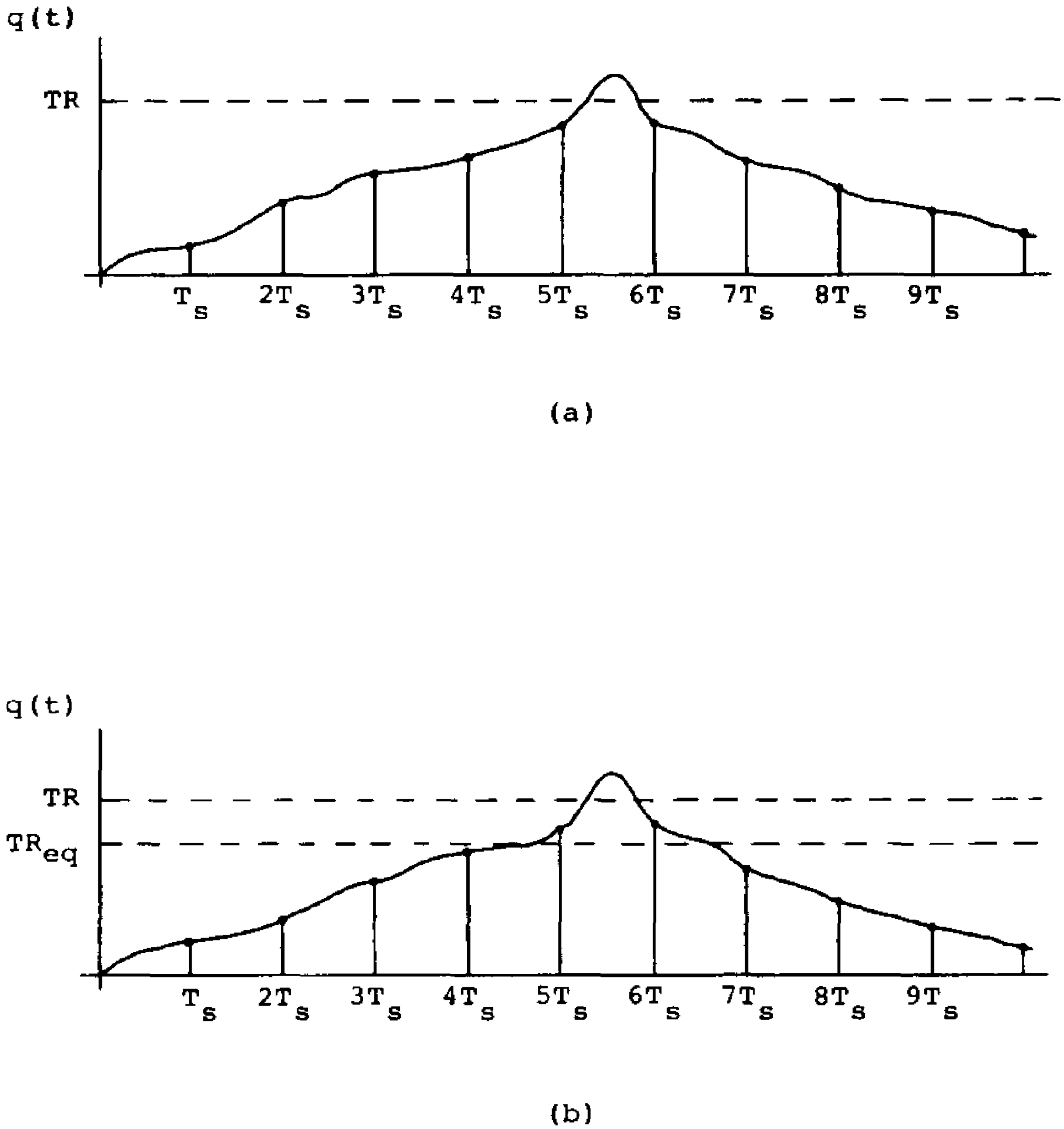

q(t)
TR
T_s
$2T_s$
$3T_s$
$4T_s$
$5T_s$
$6T_s$
$7T_s$
$8T_s$
$9T_s$
(a)
q(t)
TR
TR_{eq}
T_s
$2T_s$
$3T_s$
$4T_s$
$5T_s$
$6T_s$
$7T_s$
$8T_s$
$9T_s$
(b)

Fig. 5.7-1

Next we calculate the value of the equivalent threshold for given probability of false alarm. This value is used in the simulation of the first passage and the simulation of the entire system.

5.8 Calculation of the equivalent threshold TR_{eq}

To find the equivalent threshold TR_{eq} we use (5.6-1) on the following way

$$0.5 = 1 + \sum_n \frac{A_n}{S_n} \exp(S_n T_s) \qquad (5.8-1)$$

or we can write this as

$$\sum_n \frac{A_n}{S_n} \exp(S_n T_s) = -0.5 \qquad (5.8-2)$$

Now we have to solve this equation so that A_n are residues of

$$\frac{M(\frac{S_n}{2};1;\frac{TR_{eq}}{2})}{M(\frac{S_n}{2};1;\frac{TR^2}{2})} \qquad (5.8-3)$$

and S_n are the roots of

$$M\left(\frac{S_n}{2};1;\frac{TR^2}{2}\right) \qquad (5.8-4)$$

This has been done numerically and normalized value of TR_{eq} for different values of P_{fa} is given in Table 1. and is plotted in fig. 5.6-1.

5.9 Probability of false acquisition P_{facn}

False acquisition will occur if the time delay between any two maxima exceeding the threshold is less then or equal to $3T_h$, when the input signal is noise alone. Let us denote by $M_i(t_1)$ (i=1,2,3) the maximum due to noise only occurring at t_1, and with $M_j(t_2)$ (j=1,2,3;i≠j) the maximum due to the noise only occurring at t_2. The probability of false acquisition can then be calculated as

$$P_{facn} = 3P\{|t_1-t_2| \leqslant 3T_h\}\ P\{M_1(t_1)>TR\}\ P\{M_2(t_2)>TR\} \quad (5.9\text{-}1)$$

where

$$P\{M_i(t_1)>TR\} = P\{M_j(t_2)>TR\} = P_{fa} \quad (5.9\text{-}2)$$

is actually the probability of false alarm. For TR=0 the probability of false alarm is $P_{fa}=1$ and (5.9-1) becomes

$$P_{facn} = 3P\{|t_1-t_2| \leqslant 3T_h\} \quad (5.9\text{-}3)$$

Since the probability of false acquisition P_{facn} is usually given, we have to set up the threshold TR so that this given condition is satisfied. In order to calculate the threshold TR for given P_{facn} we first calculate P_{fa} from (5.9-1). The result is

$$P_{fa} = \sqrt{\frac{P_{facn}}{3P\{|t_1 - t_2| \leqslant 3T_h\}}} \qquad (5.9\text{-}4)$$

When we have P_{fa} for a given P_{facn}, the normalized threshold can be found from Fig. 5.6-1, and by using (5.6-2) the true value of the threshold can be calculated.

5.10 Probability of detection

The input signal r(t) to the quadrature receiver of Fig. 5.5-1, is given by

$$r(t) = s(t - \tau) + n(t) \qquad (5.10\text{-}1)$$

where n(t) is Gaussian white noise and

$$s(t - \tau) = A \cos\{w(t - \tau) + \theta\} \qquad 0 \leqslant t \leqslant T_h \qquad (5.10\text{-}2)$$

where θ is a random phase with uniform distribution between 0 and 2π and τ is a random time of arrival with probability density function

$$p(\tau) = \begin{cases} \dfrac{1}{T} & 0 < \tau < T \\ 0 & \text{elsewhere} \end{cases} \qquad (5.10\text{-}3)$$

According to the theory of optimum detection of a signal with random time of arrival [9] we have to observe the output of a quadrature receiver (Fig. 5.5-1) from 0 to

T, to detect the maximum and the time of its occurrence. The maximum is to be compared with the threshold TR, and if that value exceeds the threshold we say that signal is present and the time at which the maximum has occurred is the time of arrival. The value of the threshold is to be chosen according to the given value of false alarm probability P_{fa}.

In order to make the calculation possible we assume that the maximum due to the noise and signal occurs at $\tau + T_h$ i.e. T_h seconds after the signal appears at the input of the quadrature receiver. The value of this maximum is a random variable with a Ricean probability density function

$$p(q) = \frac{q}{\sigma^2} \exp\left(-\frac{q^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{qA}{\sigma^2}\right) \qquad (5.10\text{-}4)$$

where A_0 is the maximum value of the signal at the output when there is no noise at the input; σ^2 is the variance of the noise at the output of the RC filter.

The probability that the signal will be detected is given as

$$P_d = P\{\text{sig+noise>TR and noise<sig+noise}\} \qquad (5.10\text{-}5)$$

The probability that the niose will not reach the value of the envelope at $t = \tau + T_h$ during the time interval $0 \leq t \leq T$ is

$$P(q) = -\sum_n \frac{A_n}{S_n} \exp(S_n T) \qquad (5.10\text{-}6)$$

q - is the value of the signal plus noise at $t=\tau+T_h$ and is a random variable with Rice density function p(q). Averaging over all values for q greater than the threshold TR we get the probability of detection as

$$P_d = \int_{TR}^{\infty} P(q)\,p(q)\,dq \qquad (5.10\text{-}7)$$

or

$$P_d = -\int_{TR}^{\infty} \sum_n \frac{A_n}{S_n} \exp(S_n T)\,p(q)\,dq \qquad (5.10\text{-}8)$$

In this case A_n and S_n are functions of q. Equation (5.10-8) has been solved numerically. For different values of P_{fa} the probability of detection P_d has been calculated and the calculated results have been compared with the results obtained by computer simulation. In the simulation we have assumed that the signal and its time of arrival are detected if the maximum due to both the signal and the noise is in the interval

$$\tau + T_h/2 \leqslant t \leqslant \tau + 3T_h/2 \qquad (5.10\text{-}9)$$

is greater than any maximum due to the noise only. Simulated and calculated results are given in Fig. 5.10-1,2,3 and 4.

5.11 Probability of acquisition

The probability of acquisition P_{acq} can be shown to be

$$P_{acq} = 3(1-P_d)P_d^2 + P_d^3 \qquad (5.11\text{-}1)$$

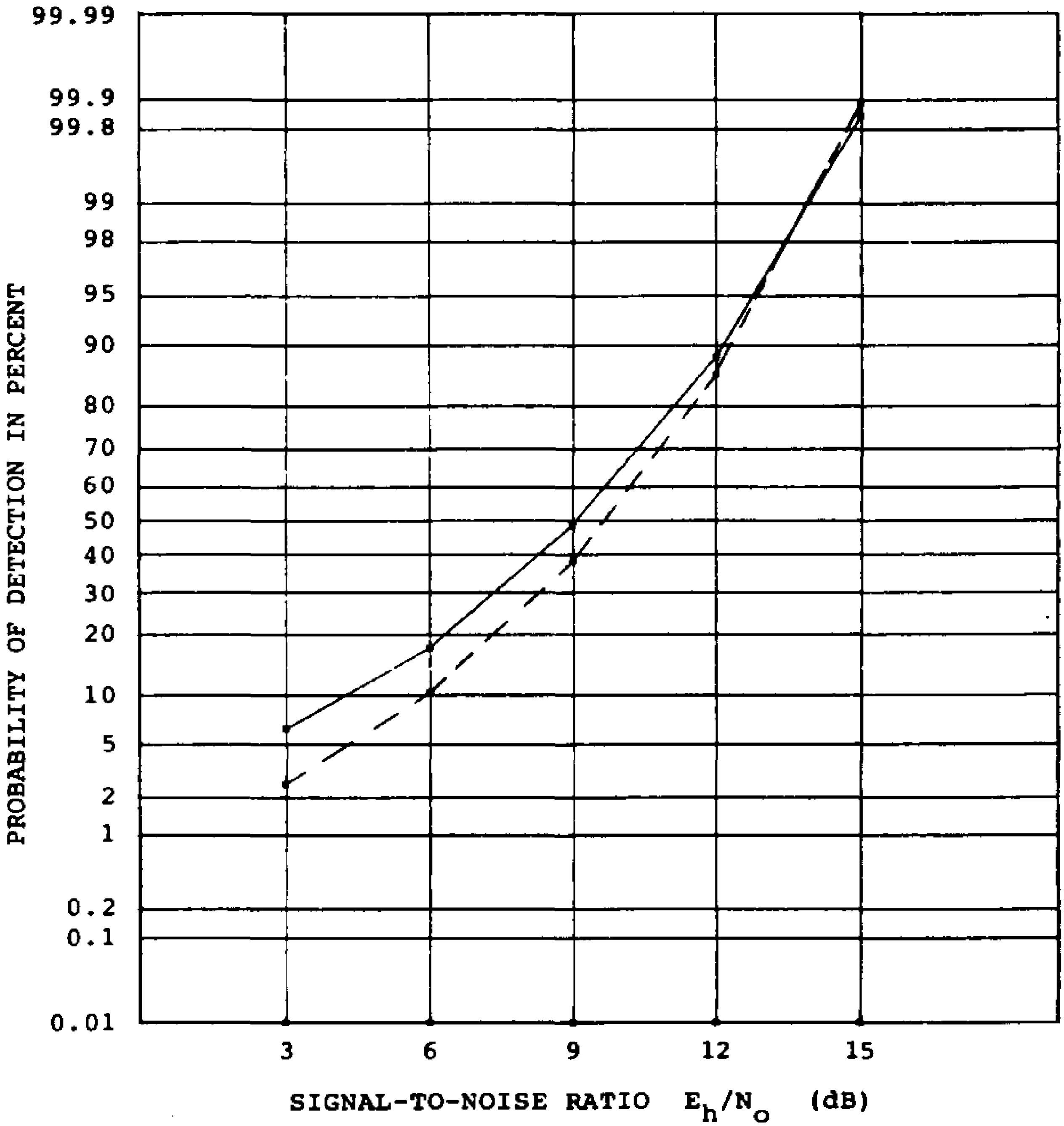


Fig.5.10-1 Probability of detection versus signal to noise ratio for P_{fa}= 1

– – – – calculated results

——— simulated results

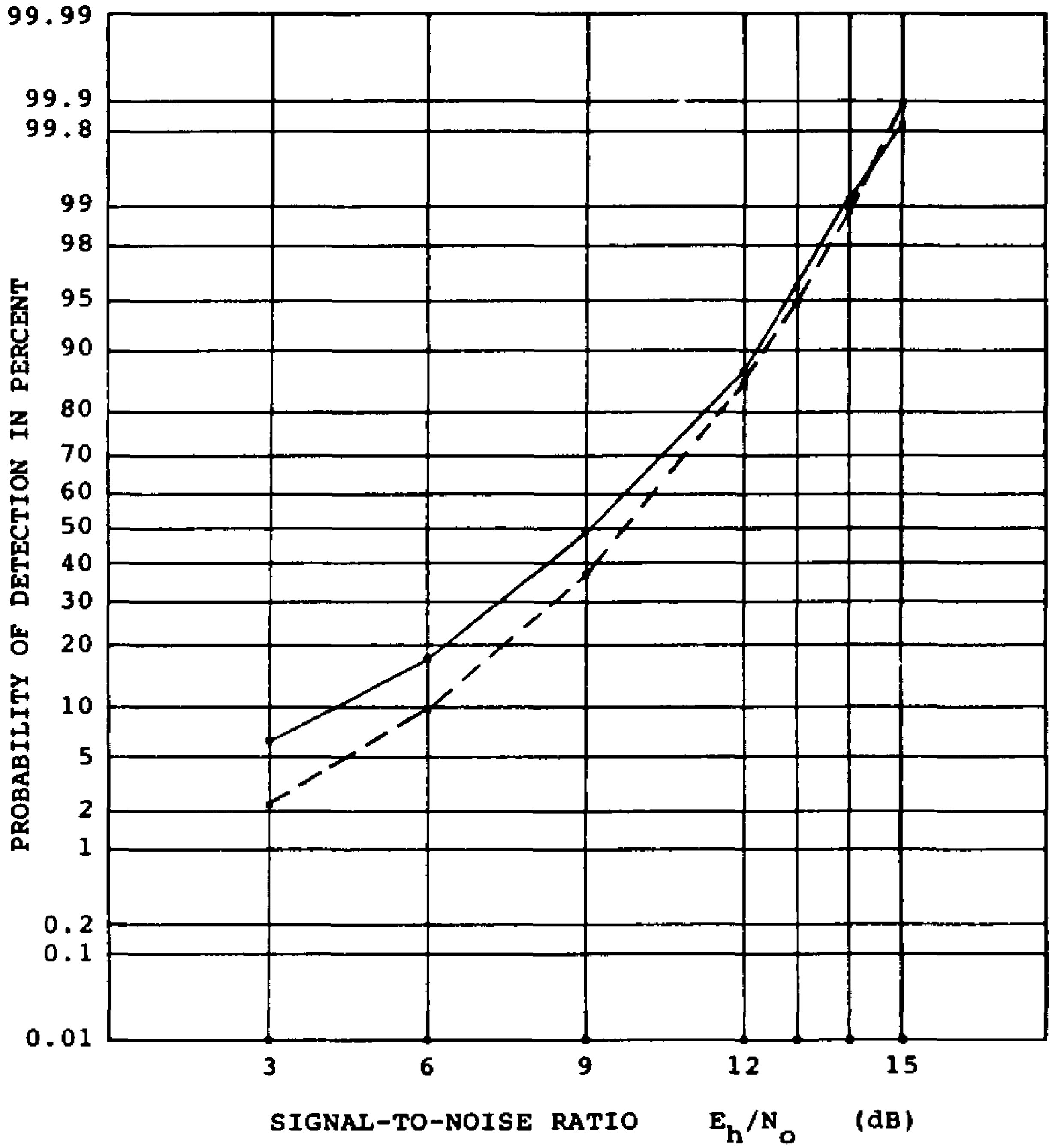


Fig. 5.10-2 Probability of detection versus signal to noise ratio for P_{fa}=0.8483

- - - - calculated results

——— simulated results

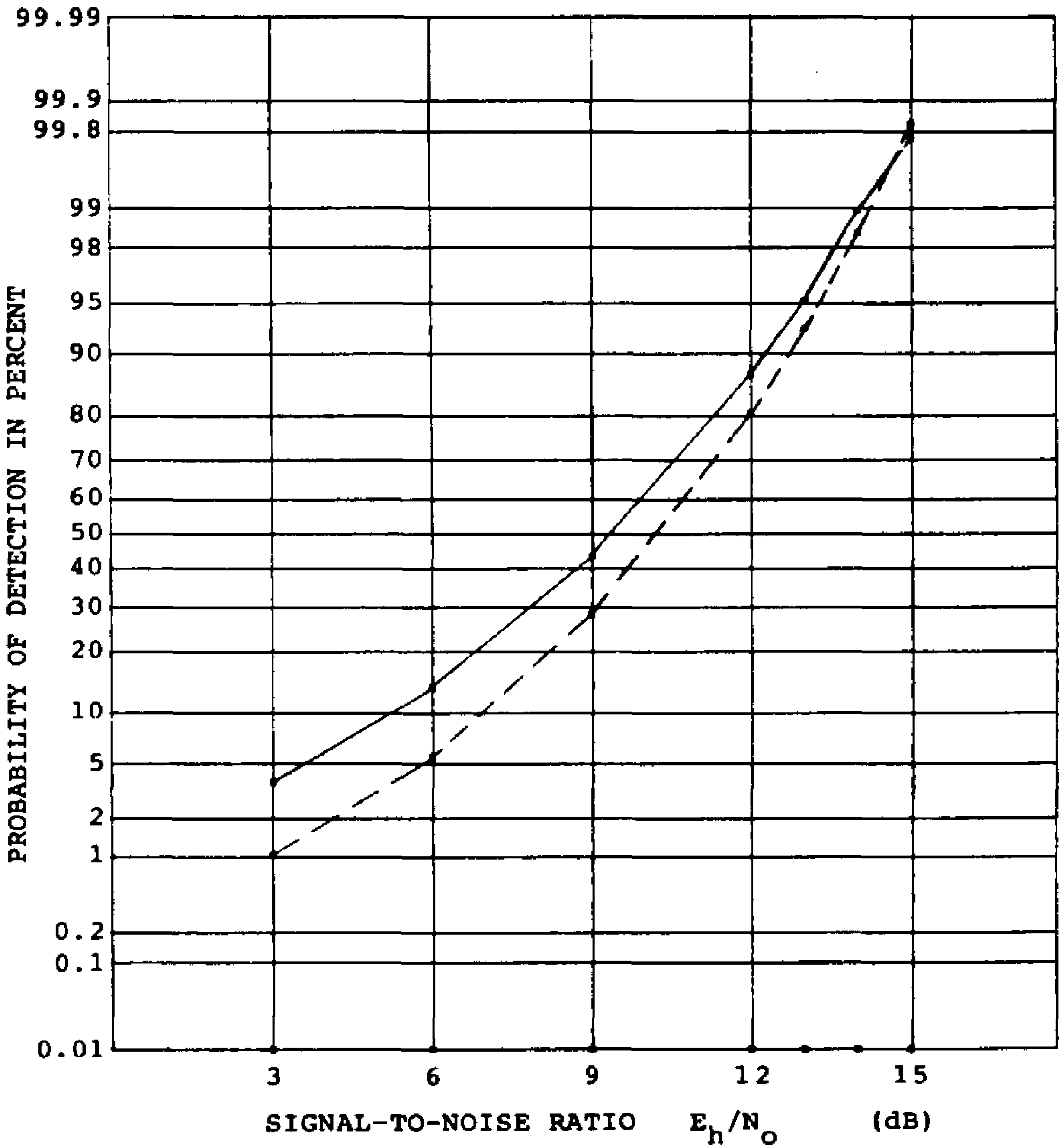


Fig. 5.10-3 Probability of detection versus signal to noise ratio for P_{fa}=0.268

---- calculated results

——— simulated results

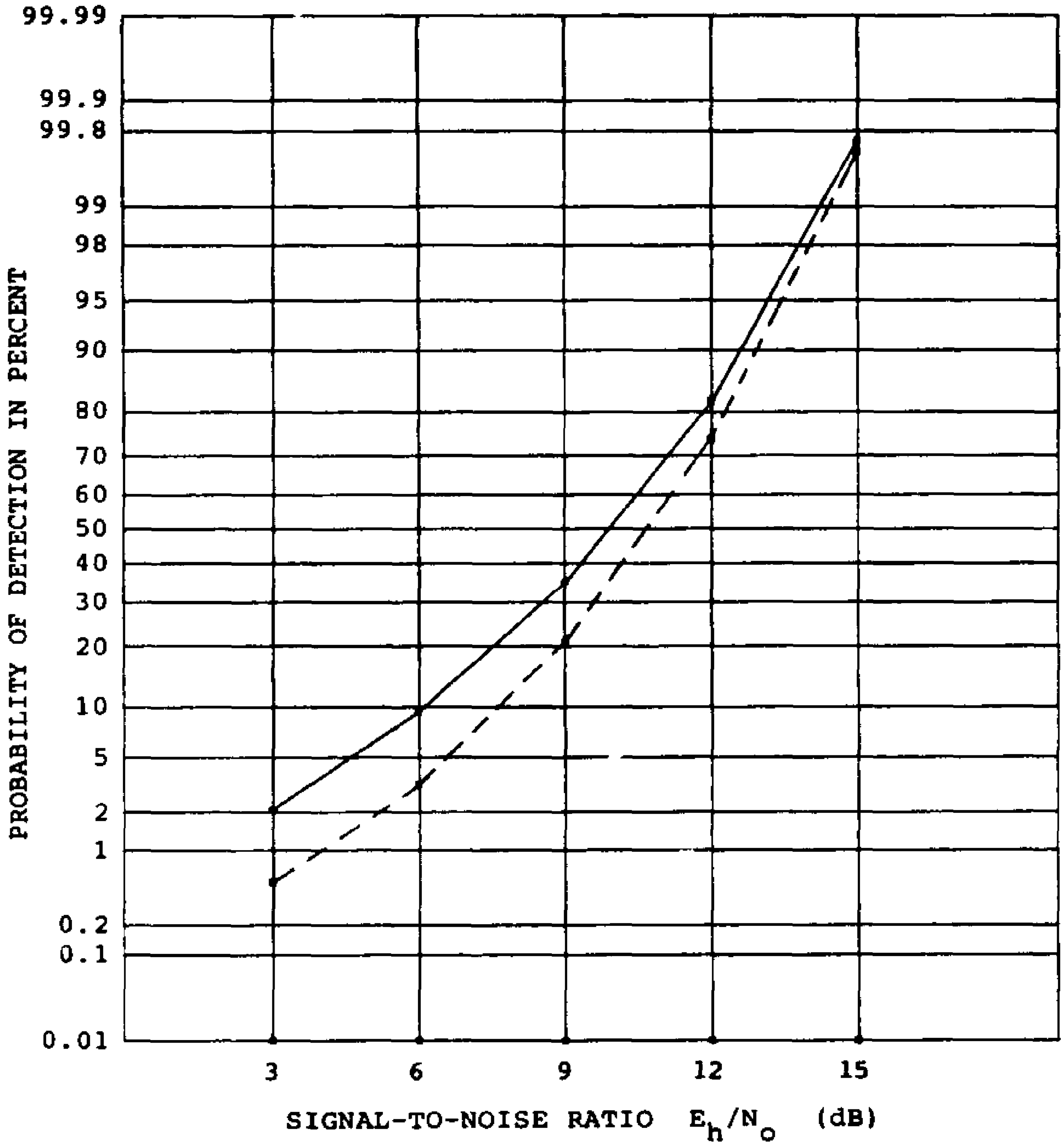


Fig. 5.10-4 Probability of detection versus signal to noise ratio for $P_{fa} = 0.1$

- - - - calculated results

——— simulated results

Calculated and simulated results are given in Fig. 5.11-1,2 and 3.

5.12 Probability of false acquisition, P_{fac}

False acquisition will occur if at least two detected maxima are due to the noise only when the signal is present, and their time distance is less than $3T_h$. In order to calculate P_{fac} we have to find the probability that the noise is greater than threshold and greater than signal plus noise at $t=\tau+T_h$. We can write this as follows

$$P_{fas}=P\{sig+noise>TR \text{ and } noise>sig+noise\} + P\{sig+noise<TR \text{ and } noise>TR\} \quad (5.12\text{-}1)$$

This probability can be calculated as

$$P_{fas}=\int_{TR}^{\infty}[1-P(q)]p(q)dq + P_{fa}\int_{0}^{TR}p(q)dq \quad (5.12\text{-}2)$$

and finally after few manipulations we have

$$P_{fas}=(P_{fa}-1)\int_{0}^{TR}p(q)dq + 1 - \int_{TR}^{\infty}P(q)p(q)dq \quad (5.12\text{-}3)$$

Now we calculate the probability of false acquisition P_{fac} as

$$P_{fac} = (P_{fas})^2 0.139 \quad (5.12\text{-}4)$$

where 0.139 is P_{facn} for TR=0.

Calculated and simulated results for P_{fas} and P_{fac} are given in Fig. 5.12-1,2 and Fig. 5.11-1,2,3 respectively.

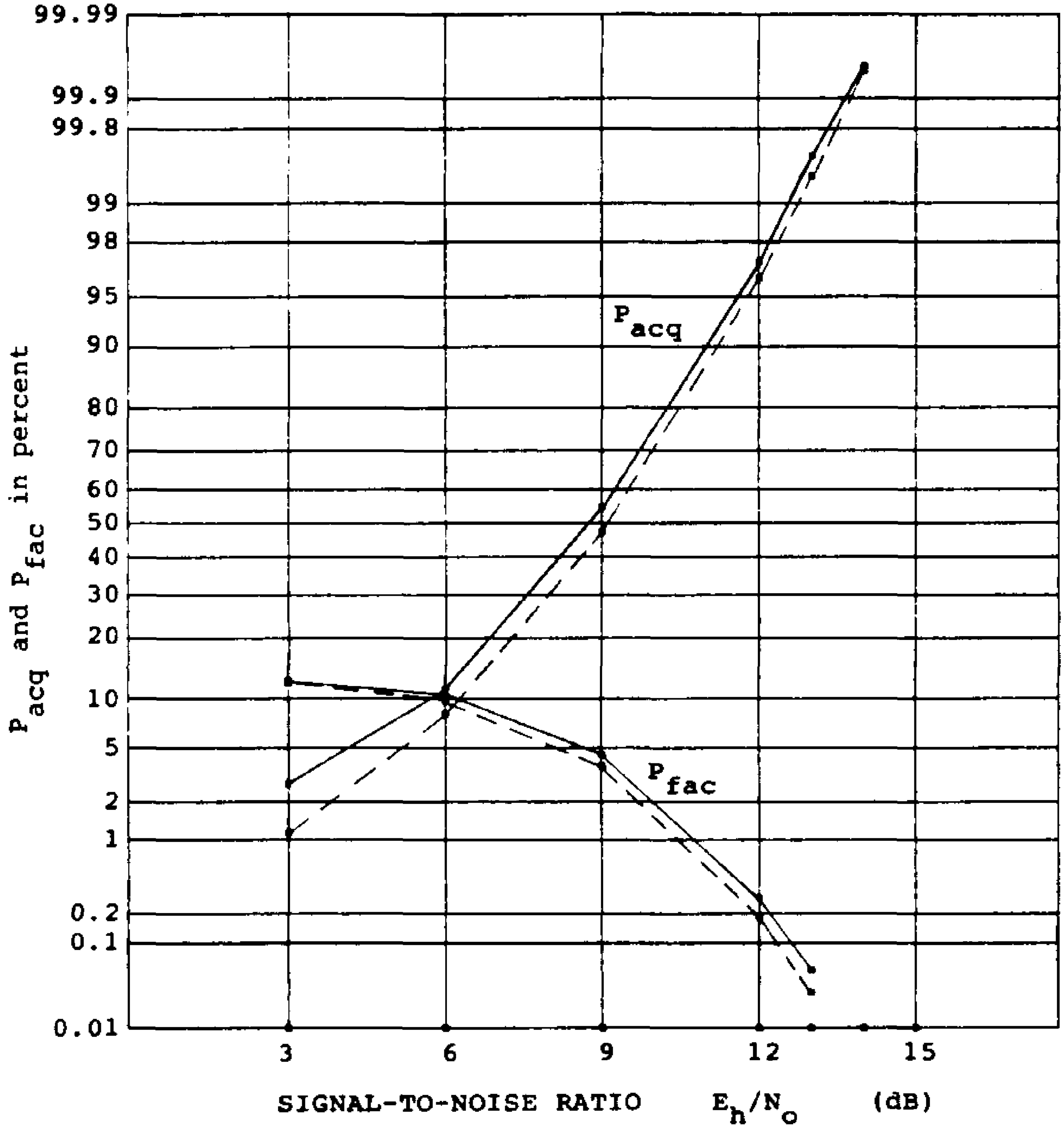


Fig. 5.11-1 Probability of acquisition P_{acq} and probability of false acquisition P_{fac} for given probability of false alarm $P_{fa}=1$.

- - - - calculated results

——— simulated results

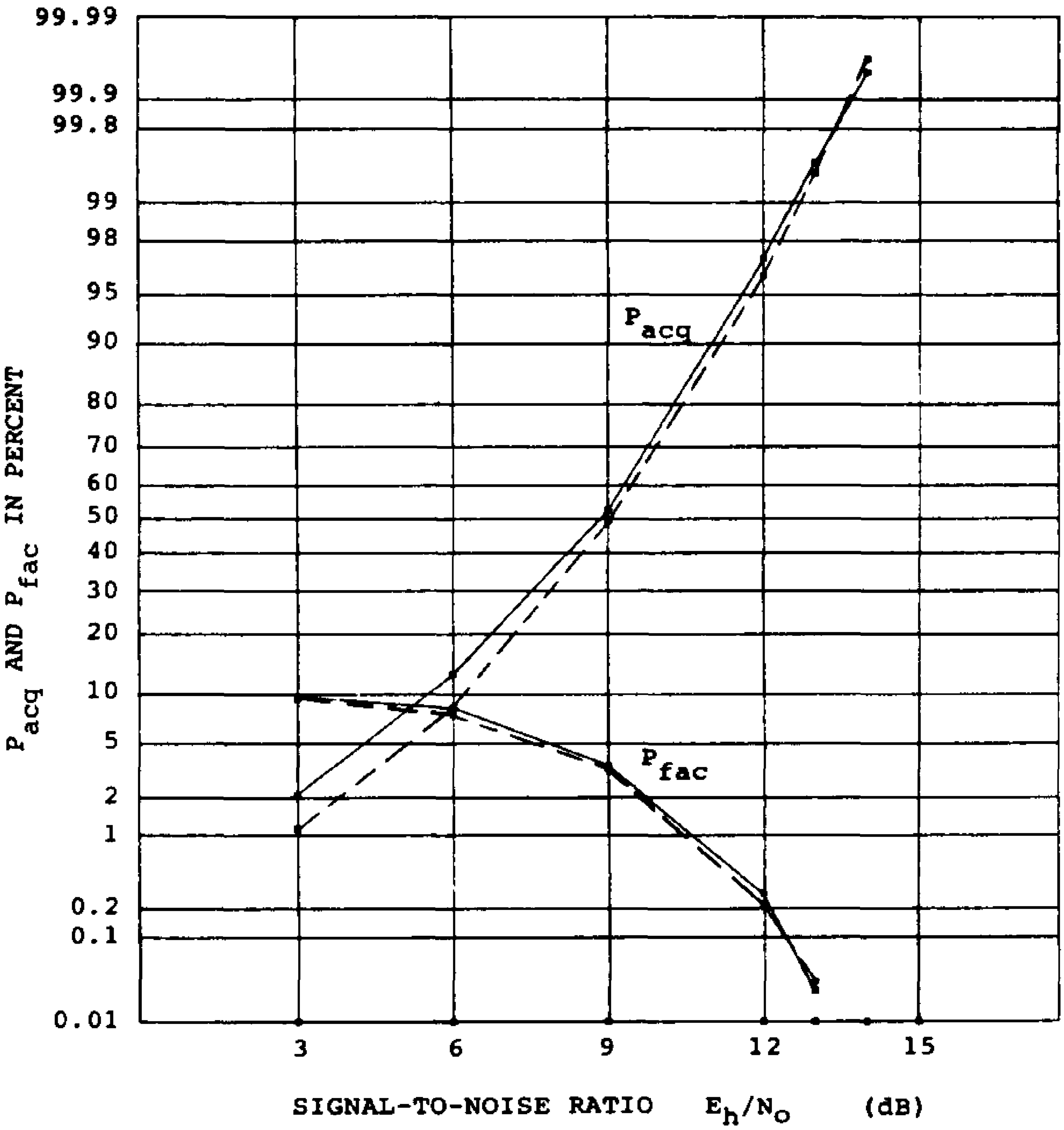


Fig. 5.11-2 Probability of acquisition P_{acq} and probability of false acquisition P_{fac} for $P_{facn} = 0.1$

– – – – calculated results

———— simulated results

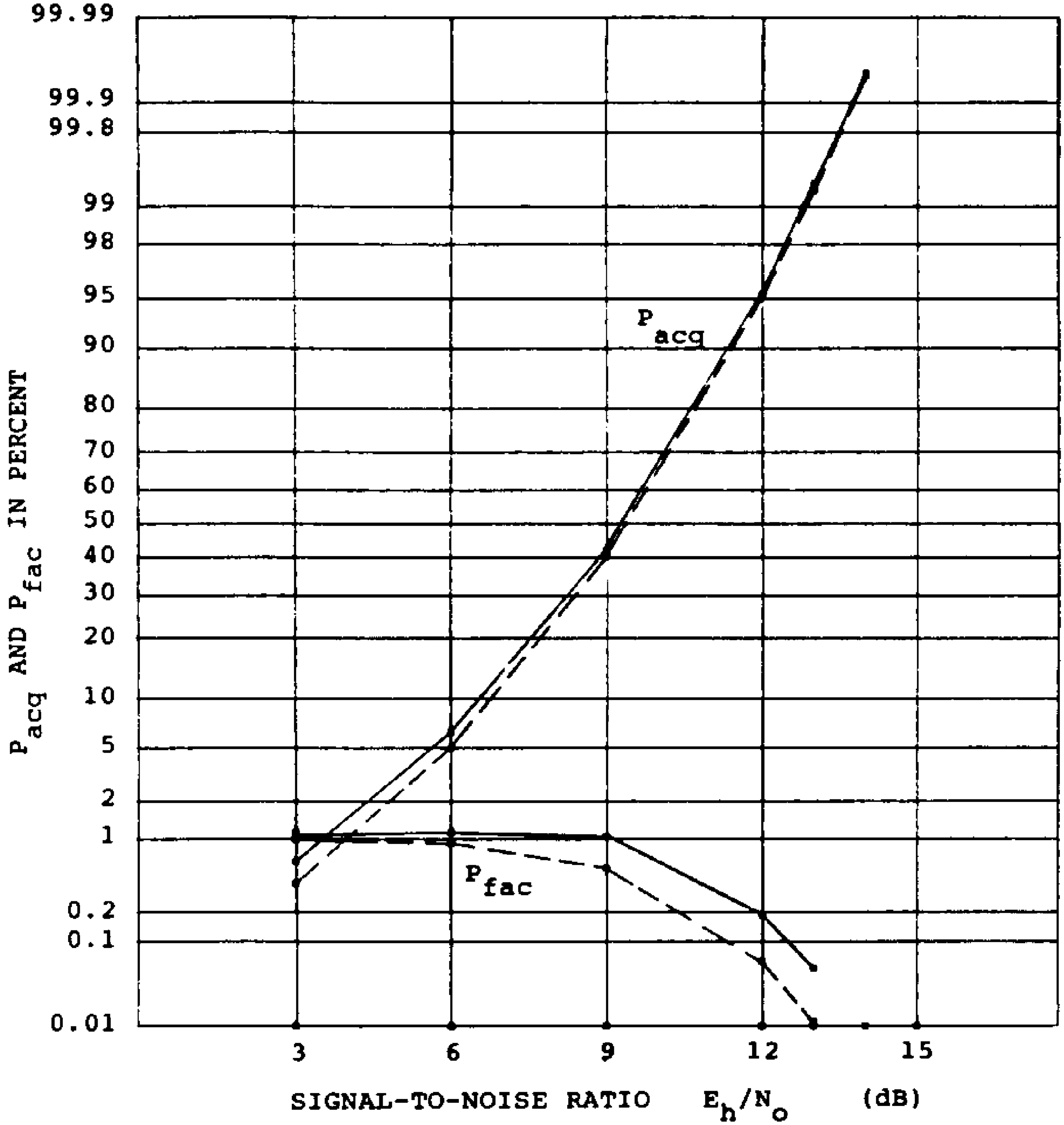


Fig. 5.11-3 Probability of acquisition P_{acq} and probability of false acquisition P_{fac} for $P_{facn} = 0.01$

– – – – calculated results

——— simulated results

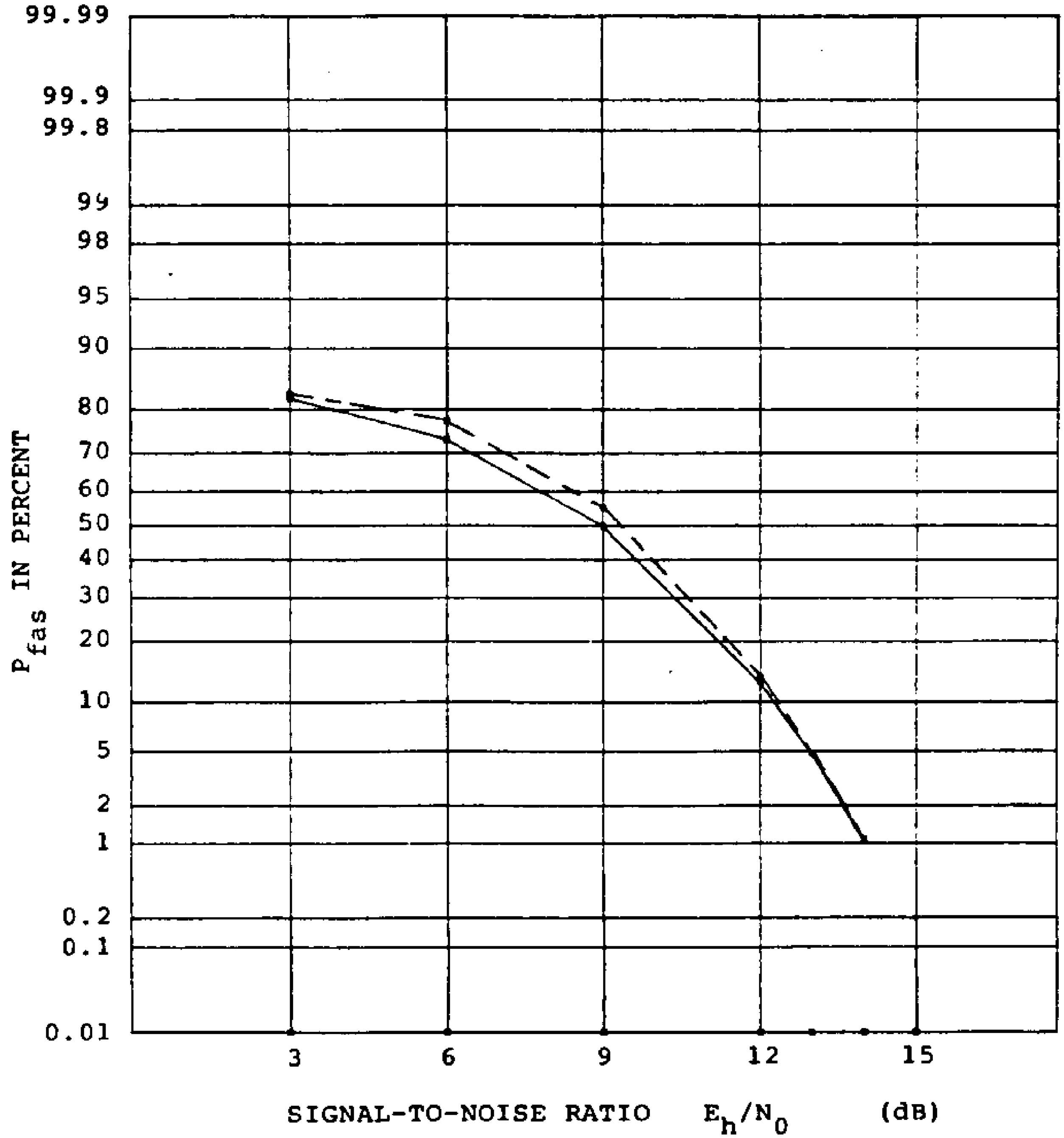


Fig. 5.12-1 P_{fas} versus signal-to-noise ratio for P_{fa}=0.848
----- calculated results
——— simulated results

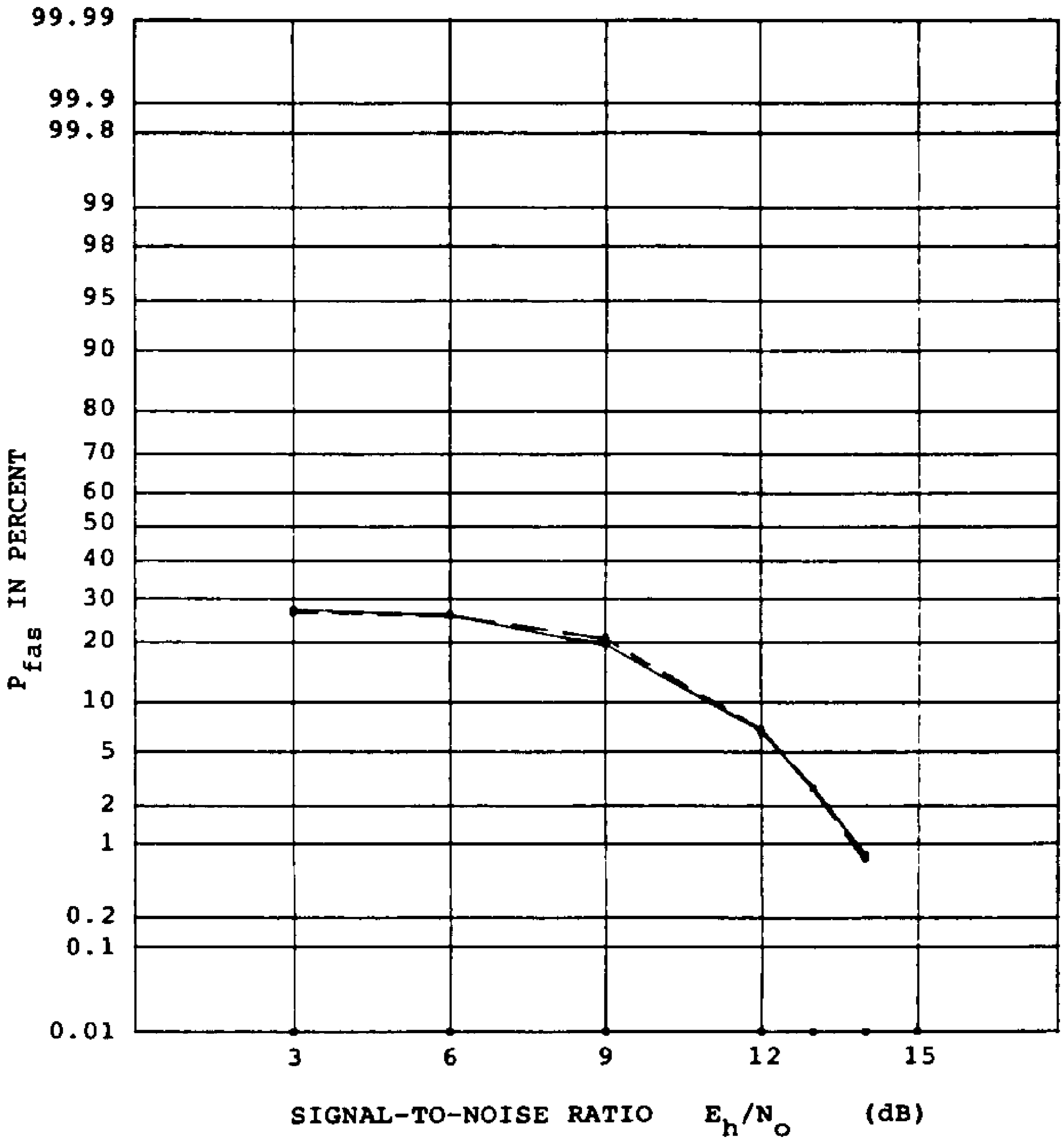


Fig. 5.12-2 P_{fas} versus signal-to-noise ratio for P_{fa}=0.268

---- calculated results

——— simulated results

5.13 Simulation results for a Rayleigh fading multipath channel

For a Rayleigh fading multipath channel we do a simulation to estimate the probability of acquisition and the probability of false acquisition when signal is present.

The channel model of a Rayleigh fading multipath channel used in this simulation is given in Fig. 5.13-1. After each delay line we insert a variable delay line of τ seconds. Each τ is a random veriable with probability density function given by (2-3).

Simulated results are given in Fig. 5.13-2,3 and 4, for different values of P_{facn}. If we compare this results with the results given in Fig. 5.11-1,2 and 3 we will see that there is small difference for small signal to noise ratio, and very big difference for large signal to noise ratio. We could expect this difference, because it is known that fading affects the system much more at large signal to noise ratio than at small signal to noise ratio. The same effect can be seen in Fig. 5.2-2.

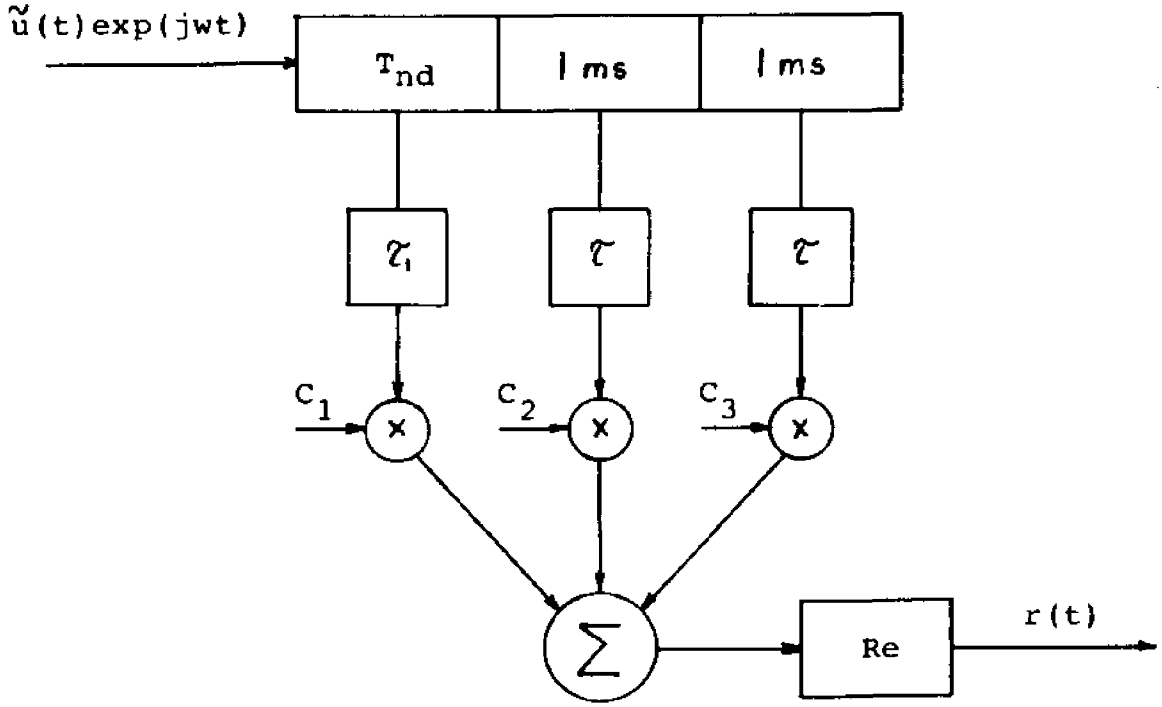


Fig. 5.13-1

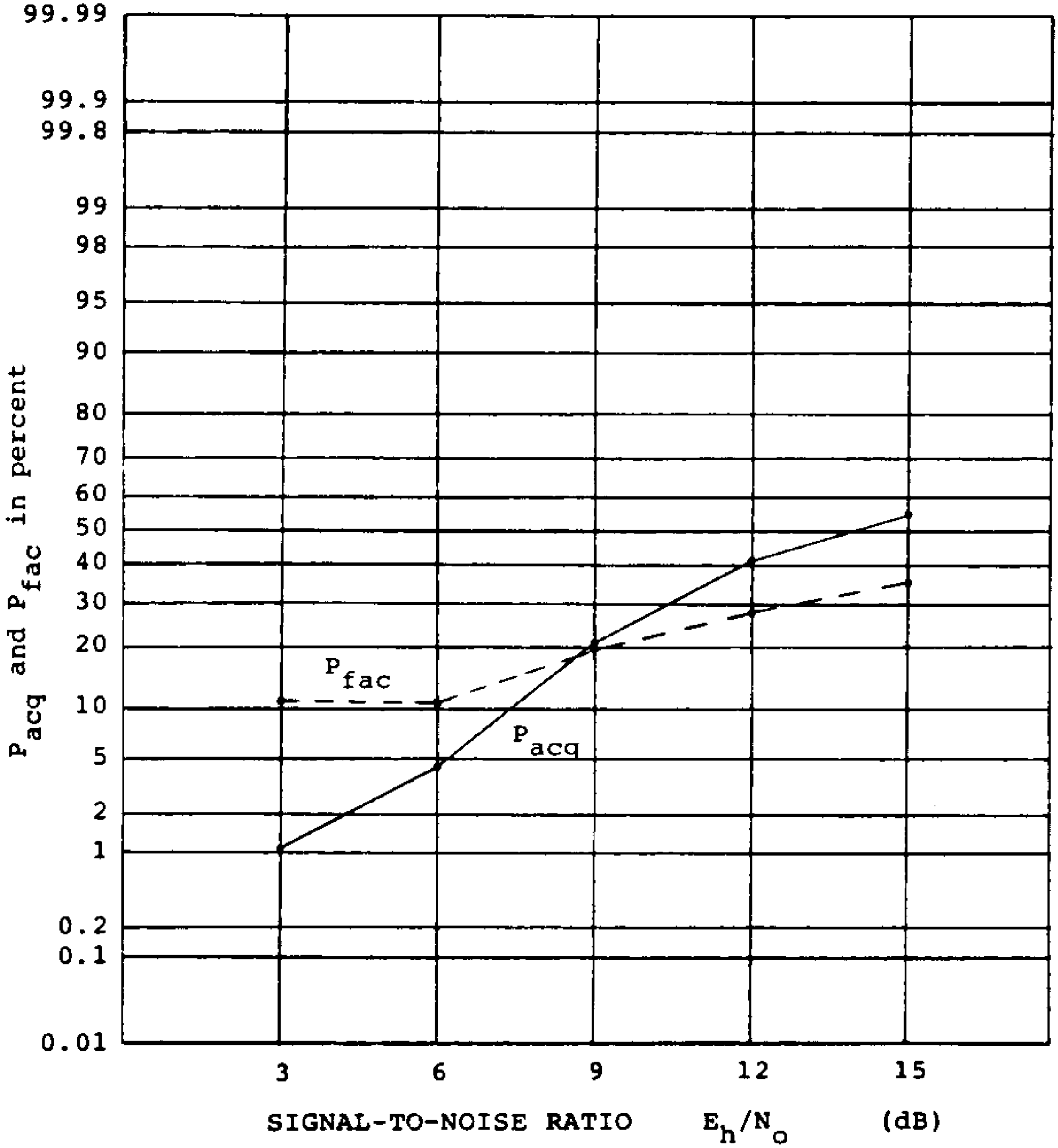


Fig. 5.13-2 Probability of acquisition P_{acq} and probability of false acquisition P_{fac} for $P_{fa} = 1$.

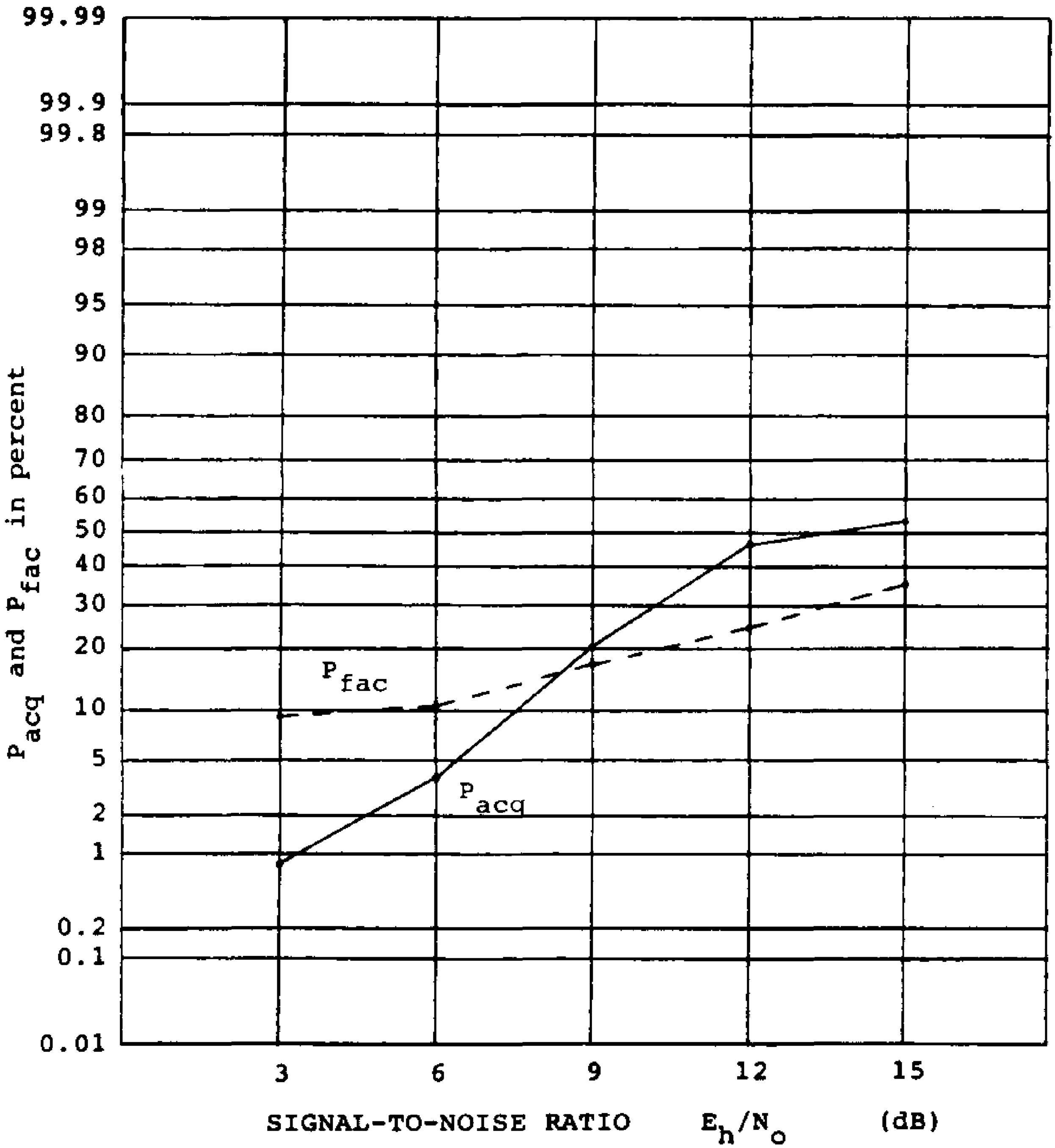


Fig. 5.13-3 Probability of acquisition P_{acq} and probability of false acquisition P_{fac} for $P_{facn} = 0.1$

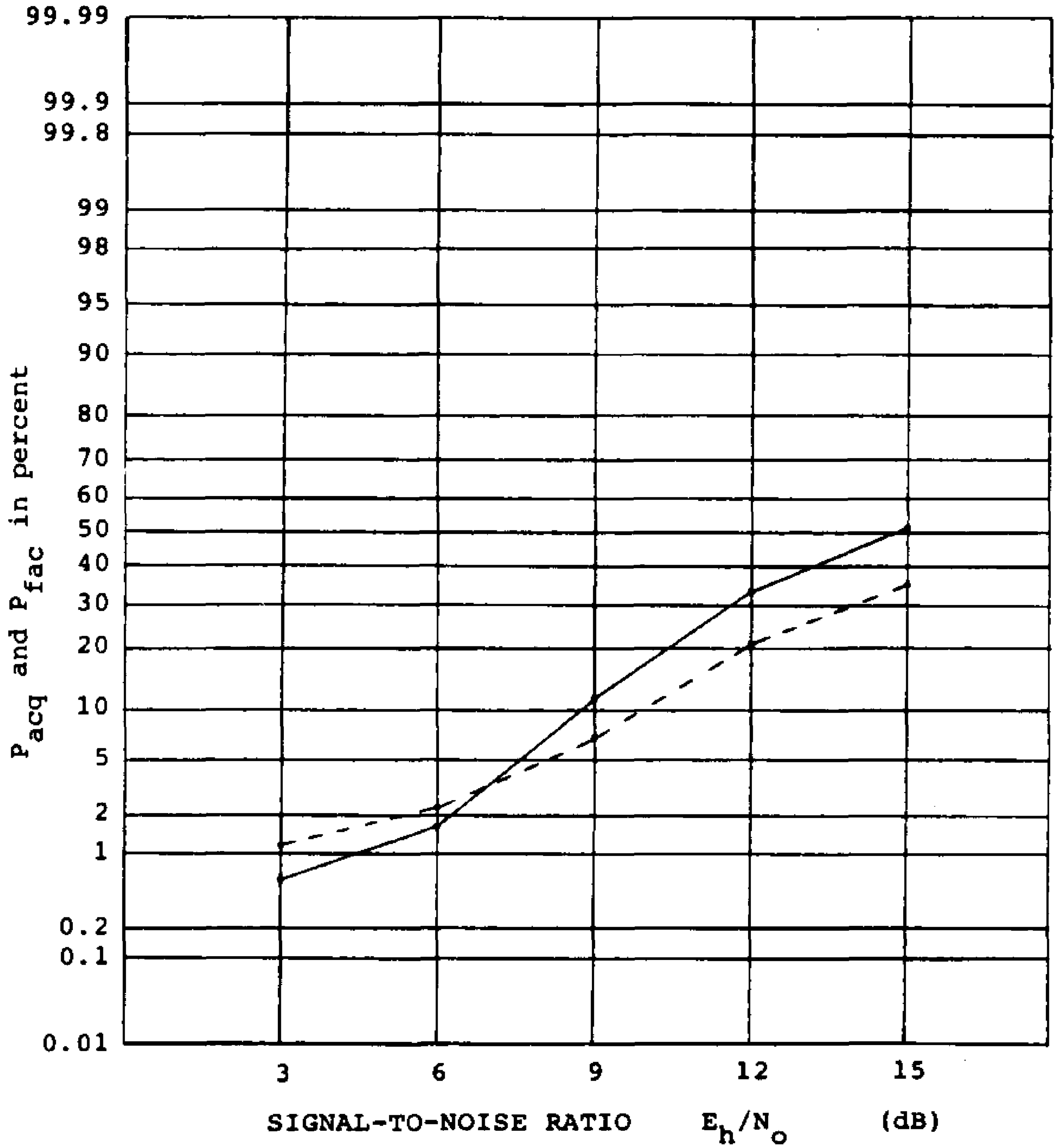


Fig. 5.13-4 Probability of acquisition P_{acq} and probability of false acquisition P_{fac} for $P_{facn} = 0.01$

CONCLUSION

The maximum probability of false acquisition P_{facn} is for TR=0, and this value is 0.139, and is same for all values of signal to noise ratio.

As we can see from the diagrams showing P_d for given probability of false alarm P_{fa}, the difference between simulated and calculated results is acceptable in the entire range of signal to noise ratio. The difference is larger for lower signal to noise ratio. The reason for this is that for small signal to noise ratio there is more probability that the maximum due to the signal+noise will be shifted to the left or to the right from $t=\tau+T_h$. For high signal to noise ratio the difference is almost negligible.

For the same reason, the difference between calculated and simulated results for P_{acq} is larger for lower signal to noise ratio.

Since for a Rayleigh fading multipath channel with variable delay time it is impossible to calculate the probability of acquisition P_{acq} and the probability of false acquisition P_{fac}, computer simulation has been done and simulated results are presented.

FUTURE WORK

Since in this work we have analyzed only the acquisition scheme for a frequency-hopped SS signal received over an HF channel, the future work should include the development and analysis of a tracking scheme. Because of the overlaping between two, three or more adjacent frequencies, the tracking problem is much more difficult than in the case of constant delay time.

With the scheme we have proposed the minimum acquisition time is approximately equal to the period of pseudorandom sequence. For very long sequences the acquisition time can be unacceptable. Therefore, future work should also include the development of rapid acquisition schemes.

In this work we have analized the acquisition scheme for possible overlapping of at most three adjacent frequencies. The case with overlapping of more than three frequencies (faster frequency hopping) has to be analyzed.

APPENDIX A

The first passage time probability density function can be faund for the case of one dimensional Markov process. For this case we derive the Fokker-Planck differntial equation.

Let $x(t)$ be a random process, and let $x(t_1)$, $x(t_2),\ldots,x(t_n)$ be a set of its values at the consecutive time instants $t_1>t_2>\ldots>t_n$. Consider the probability density function of the value of $x(t)$ at the most recent time t_1

$$p_x[x(t_1)/x(t_2),\ldots,x(t_n)]=\frac{p_x[x(t_1),\ldots,x(t_n)]}{p_x[x(t_2),\ldots,x(t_n)]} \qquad \text{(A-1)}$$

The process $x(t)$ is said to be Markov process if the conditional densiti function (A-1) depends only on the last value $x(t_2)$ and not on the preceding values $x(t_3),\ldots,x(t_n)$ where $t_2>t_3>\ldots>t_n$, and hence for a Markov process we can write

$$p_x[x(t_1)/x(t_2),\ldots,x(t_n)]=\frac{p_x[x(t_1),x(t_2)]}{p_x[x(t_2)]} \qquad \text{(A-2)}$$

Let us consider now the relation

$$p_x(x_1,x_2) = p_x(x_1/x_2)\cdot p_x(x_2) \qquad \text{(A-3)}$$

where $p_x(x_i)$ stands for $p_x[x(t_i)]$. Integrating (A-3) over x_2

we have

$$p_x(X_1) = \int p_x(x_1/x_2)p_x(x_2)dx_2 \qquad (A-4)$$

To converte (A-4) into a differential equation we chose

$$t_2=t \qquad t_1=t+\tau \qquad x_2=x \qquad x_1=x_\tau \qquad (A-5)$$

and now we can write (A-4) in the form

$$p_x(x_\tau) = \int p_x(x_\tau/x)p_x(x)dx \qquad (A-6)$$

Let us introduce the characteristic function

$$W(w;x)=E\{\exp[jw(x_\tau-x)]\}= \int \exp[jw(x_\tau-x)]p_x(x_\tau/x)dx_\tau \qquad (A-7)$$

of the random increment $x_\tau-x$ which occurs during the time interval $[t,t+\tau]$, given that $x(t)=x$. Substituting the inverse transform

$$p_x(x_\tau/x) = \frac{1}{2\pi}\int \exp[-jw(x_\tau-x)W(w;x)dw \qquad (A-8)$$

into (A-4) we have

$$p_x(x_\tau) = \frac{1}{2\pi}\iint \exp[-jw(x_\tau-x)]W(w;x)p_x(x)dwdx \qquad (A-9)$$

and using the formula for the characteristic function

$$W(w;x) = 1 + \sum_{n=1}^{\infty}\frac{(jw)^n}{n!}m_n(x) \qquad (A-10)$$

where $m_n(x)$ are moments

$$m_n(x) = E[(x_\tau - x)^n] \tag{A-11}$$

of the increment $x_\tau - x$. It follows that

$$p_x(x_\tau) = \sum_{n=0}^{\infty} \frac{1}{2\pi n!} \iint \exp[-jw(x_\tau - x)](jw)^n m_n(x) p_x(x) dw dx \tag{A-12}$$

However since

$$\frac{1}{2\pi} \int \exp[jw(x_\tau - x)](jw)^n dw = (-\frac{\partial}{\partial x_\tau})^n [\frac{1}{2\pi} \int \exp[-jw(x_\tau - x)] dw]$$

(A-13)

$$= (-\frac{\partial}{\partial x_\tau})^n \delta(x_\tau - x)$$

we find that

$$p_x(x_\tau) = p_x(x) + \sum_{n=1}^{\infty} \frac{1}{n!} (-\frac{\partial}{\partial x_\tau})^n [m_n(x_\tau) p(x_\tau)] \tag{A-14}$$

or dividing by τ and taking the limit $\tau \to 0$, we obtain

$$\frac{dp(x)}{dt} = \sum_{n=1}^{\infty} \frac{1}{n!} (-\frac{\partial}{\partial x_\tau})^n [K_n(x) p(x)] \tag{A-15}$$

where

$$K_n(x) = \lim_{\tau \to 0} \frac{m_n(x)}{\tau} \tag{A-16}$$

are so called intensity cofficients. Equation (A-15) is

For continuous Markov processes higher order intensity cofficients K_3, K_4,... are equal to zero. In this case equation (A-15) takes form

$$\frac{dp(x)}{dt} = - \frac{\partial}{\partial x} [K_1(x)p(x)] + \frac{1}{2} \frac{\partial^2}{\partial x^2} [K_2(x)p(x)] \quad (A-17)$$

and is called the Fokker-Planck equation.

APPENDIX B

In this appendix we derive two-dimensional probability density function of the envelope of zero mean narrowband Gaussian noise.

Narrowband noise can be represented by

$$n(t) = x(t)\cos(w_o t) - y(t)\sin(w_o t) \tag{B-1}$$

where x(t) and y(t) are quadrature components. It can be shown that if the spectrum density function of n(t) is symmetric around w_o, then x(t) and y(t) are uncorrelated random processes, i.e.

$$R_{x,y}(t_1,t_2) = 0 \tag{B-2}$$

for all values of t_1 and t_2. Also it can be shown that

$$R_x(t_2-t_1) = R_y(t_2-t_1) \tag{B-3}$$

and

$$\sigma^2 = \sigma_x^2 = \sigma_y^2 \tag{B-4}$$

Let us denote by x_1, x_2, y_1 and y_2 the samples of the quadrature components at the time instances t_1 and t_2. Also we denote the samples of the envelope at t_1 and t_2 by q_1 and q_2.

First we form the joint density function of x_1,

x_2, y_1 and y_2

$$p(x_1,x_2,y_1,y_2)=\frac{1}{(2\ )^{n/2}|\Lambda|^{1/2}}\exp[-\frac{1}{2}X^T\Lambda^{-1}X] \qquad \text{(B-5)}$$

where

$$X=\begin{vmatrix}x_1\\x_2\\y_1\\y_2\end{vmatrix} \qquad \text{(B-6)}$$

and the covariance matrix Λ is given as follows

$$\Lambda=\begin{vmatrix}\sigma^2 & R_{12} & 0 & 0\\ R_{21} & \sigma^2 & 0 & 0\\ 0 & 0 & \sigma^2 & R_{12}\\ 0 & 0 & R_{21} & \sigma^2\end{vmatrix} \qquad \text{(B-7)}$$

Determinant of Λ is found and is given as

$$\det\Lambda=(\sigma^4-R_{12}^2)^2 \qquad \text{(B-8)}$$

and inverse matrix of Λ is

$$\Lambda^{-1}=\frac{1}{\sigma^4-R_{12}^2}\begin{vmatrix}\sigma^2 & -R_{12} & 0 & 0\\ -R_{12} & \sigma^2 & 0 & 0\\ 0 & 0 & \sigma^2 & -R_{12}\\ 0 & 0 & -R_{12} & \sigma^2\end{vmatrix} \qquad \text{(B-9)}$$

Substituting (B-6) and (B-9) in (B-1) we find

$$p(x_1,x_2,y_1,y_2)=\frac{1}{(2\pi)^2(\sigma^4-R_{12}^2)}\exp[-\frac{1}{2(\sigma^4-R_{12}^2)}[\sigma^2x_1^2+\sigma^2x_2^2+\sigma^2y_1^2+\sigma^2y_2^2 -2x_1x_2R_{12}-2y_1y_2R_{12}]] \qquad \text{(B-10)}$$

Substituting

$$x_1 = q_1\cos\theta_1 \qquad y_1 = q_1\sin\theta_1$$
$$x_2 = q_2\cos\theta_2 \qquad y_2 = q_2\cos\theta_2 \tag{B-11}$$

in (B-10) we find

$$p(q_1,q_2,\theta_1,\theta_2)=\frac{1}{(2\pi)^2(\sigma^4+R_{12}^2)}\exp[-\frac{1}{2(\sigma^4-R_{12}^2)}[\sigma^2q_1^2+\sigma^2q_2^2 -2R_{12}q_1q_2\cos(\theta_1-\theta_2)]]|J| \tag{B-12}$$

where J is the Jacobian of this transformation of variables and is found to be

$$|J| = q_1q_2 \tag{B-13}$$

In order to find $p(q_1,q_2)$ we perform the next integration

$$p(q_1,q_2) = \int_0^{2\pi}\int_0^{2\pi} p(q_1,q_2,\theta_1,\theta_2)\,d\theta_1 d\theta_2 \tag{B-14}$$

and after simple calculation we find

$$p(q_1,q_2)=\frac{q_1q_2}{\sigma^4-R_{12}^2}\exp[-\frac{\sigma^2(q_1^2+q_2^2)}{2(\sigma^4+R_{12}^2)}]I_o(\frac{R_{12}q_1q_2}{\sigma^4-R_{12}^2}) \tag{B-15}$$

which is the result that we were looking for.

Now we find the transitional probability density function $p(q_2/q_1)$ of the envelope

$$p(q_2/q_1)=\frac{p(q_1,q_2)}{p(q_1)} \tag{B-16}$$

Using (B-15) and introducing μ as

$$\mu = \frac{R_{12}}{\sigma^2} \tag{B-17}$$

we get for transitional probability density function

$$p(q_2/q_1) = \frac{q_1}{\sigma^2(1-\mu^2)} \exp\left[-\frac{q_1^2+\mu^2 q_2^2}{2\sigma^2(1-\mu^2)}\right] I_o\left[\frac{\mu q_1 q_2}{\sigma^2(1-\mu^2)}\right]$$

This result has been used for calculation of the first passage time probability density function.

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