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Bell's theorem without inequalities

Choi, Hyung Sup, Ph.D.

City University of New York, 1991

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BELL'S THEOREM WITHOUT INEQUALITIES

by

Hyung Sup Choi

A dissertation submitted to the Graduate
Faculty in Physics in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy, The City University
of New York

1991

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This manuscript has been read and accepted for the Graduate Faculty in Physics in satisfaction of the dissertation requirement for the degree of Doctor of philosophy.

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ABSTRACT

BELL'S THEOREM WITHOUT INEQUALITIES

by

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Advisor: Professor Daniel Greenberger

A stronger refutation of EPR type local realism than that of Bell's theorem is presented based on the Greenberger-Horne-Zeilinger (GHZ) gedanken experiment. The essence of Bell's theorem and that of the GHZ theorem are given along with their difference. The GHZ theorem is extended to three particle spin-1/2 correlations and to photon polarization correlations. Furthermore, the theorem is extended back to two particle system.

A few practically realizable experiments are proposed using nonlinear optical parametric down conversion of light. A theory of multi-photon spontaneous parametric down conversion and its use in a new field of many particle interferometry are introduced and developed for the purpose.

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PREFACE

While quantum mechanics enjoys unprecedented success in its practice, its conceptual or foundational problems have bothered many physicists. Even some founders of the theory, such as Einstein, Schroedinger, and de Broglie, were not satisfied with the implications of the theory at its fundamental level. Hence quests have continued to find a more complete theory, hoping to bring about other interpretations free of conceptual problems.

One of the highlights of the arguments concerning the foundation problems of quantum mechanics was given by Einstein, Podolsky, and Rosen. In their famous EPR paper, they questioned quantum theory as a complete theory by invoking the reality argument. But it turned out that the paper affected virtually the whole area of the conceptual foundations of quantum mechanics.

Because of the practical success of the theory, the majority of physicists followed the standard, so called "Copenhagen", interpretation of the theory, due to Bohr, Heisenberg, von Neumann, and Born, while contending with the idea that the commitment to either position was just a matter of taste.

Thirty years after the EPR paper, Bell came up with his celebrated analysis of EPR experiment that showed that essentially all local theories that are based on realism

might be tested against quantum mechanics by using a single experimental arrangement.

Various types of Bell's inequalities that are suited for different experimental arrangements have been tested and generally recognized to favor the predictions of quantum mechanics.

However, EPR type local realism has not been totally silenced since the experiments have provided various loopholes. Moreover, Bell's theorem says nothing about some special cases where Einstein's argument is most compelling, since, in those special extreme classical cases, local realistic results agree with quantum mechanics.

Most recently, a more powerful theorem was presented by Greenberger, Horne, and Zeilinger, which even refutes even those extreme classical reality cases. Unlike Bell's theorem, this theorem does not rely on the statistical nature of the outcome of the system, therefore it is free of inequalities and provides an "all or nothing" refutation of EPR-type local realism.

This thesis is primarily concerned with this new theorem, its extensions, and corresponding physical realizations.

Considering that one can now realize many fundamental experiments which previously were possible only in the minds of physicists, such as EPR paradox, Schroedinger's Cat, delayed choice experiment, and de Broglie Pilot wave theory etc., the author believes that this is a good time to

revisit and explore once again the magical world of quantum mechanics.

This is the motivation for this thesis which concerns itself with EPR type experiments, illustrating some of the most bizarre aspects of the physical world revealed to us by quantum mechanics.

June, 1991

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CHAPTER I

INTRODUCTION

1.1 Historical Background of The Theorem

In 1935, Einstein, Podolsky and Rosen¹ (EPR) used a reasoning based on a thought experiment to conclude that quantum mechanics is not a complete theory since it does not describe some of the elements of reality that a physical system seems to have. This famous EPR paper represented dissatisfaction among physicists toward quantum mechanics that seemed settling into a set of rules to calculate the probabilities of observable events in the microscopic worlds.

In the same year, Bohr replied in his successive papers^{2,3} that EPR's arguments and their criteria were still classically oriented and could not be applied to conclude that quantum mechanics is not a complete theory. He emphasised the non-intuitive nature of quantum mechanics to defend the theory. Bohr's argument was that Einstein's concepts of physical reality could not be compatible with the quantum mechanics, but it was not entirely clear whether Bohr recognized what really bothered Einstein, even for a long time after the EPR paper^{4,5}. Einstein worried about the implications of quantum mechanics whereby a measurement carried out on one particle would change the state of the other particle, with which it interacted in the past, no

matter how far away the other particle was. This appeared to "violate" relativity, and Einstein called this "spooky action at a distance"⁶, it is now widely referred to as "quantum nonlocality" or "quantum nonseparability".

There had been so called "impossibility proof" by von Neumann⁷ and others^{8,9} which claimed to have proven that it is impossible to introduce "hidden variables" (or, "elements of reality" in Einstein's term, or supplementary parameters) to quantum mechanics. In 1952, Bohm circumvented this proof and explicitly constructed the hidden variable theory which produced all the quantum mechanical results¹⁰. Bell reexamined the von Neumann's proof and found that some of its essential axioms were unreasonable, he also found that Bohm's hidden variable theory had a grossly non-local structure.

In 1964, Bell went further to prove, in his celebrated paper¹¹ that essentially all local deterministic theories of natural phenomena that are based on EPR type realism may be tested in a single experimental arrangement against quantum mechanics and that these two alternatives necessarily lead to observably different predictions. The theorem made use of Bohm's version¹² of EPR experiment, and showed that deterministic local hidden variable theories must satisfy certain inequality, known as "Bell's inequality".

When Bell's theorem was published, quantum mechanics was a very well established theory, supported by very large

number of experiments. One could have regarded Bell's theorem as a proof of the impossibility of supplementary parameters. But it was realized that no data were available for the conflict situations and real experiments were devised along with other Bell-type inequalities which fit various experimental situations. There were a number of real experiments followed with increasing accuracy and technical improvements¹³⁻¹⁵, among them were famous Aspect's experiments^{16,17} in 1982.

Since the Bell's theorem and subsequent experimental results not only have defied classically oriented intuitions but also seemed to support the non-local nature of quantum world, some started to investigate into its possibility of conflict with special relativity¹⁸ and this has been subjected to publicity¹⁹.

But controversy over hidden variable theory was not quite over as Bell-type experiments have provided various loopholes for the advocates of those theories. Bell's theorem itself also has essential weakness in that it should resolve the matter through inequality therefore gives much room for hidden variable theory, and also in the fact that it tells nothing for the extreme classical case on which Einstein's "reality" argument applies and is most compelling, because its predictions agree with those of hidden variable theories.

Recently, more powerful refutation of EPR type local realism than that of Bell's theorem that was based on a

gedanken experiment was devised by Greenberger, Horne and Zeilinger²⁰. This new theorem is even capable of refuting those extreme classical cases and, furthermore, it provides "all or nothing" refutation of EPR type local realism unlike Bell's theorem which relies on inequality.

We expanded this theory, which originally given in four particle spin-1/2 correlations, into more realizable three particle case²¹ and further into photon polarization correlations. We also tried to transform the original gedanken experiment into real experiment and made it feasible by using multi-photon down conversion. For this purpose, we have developed a quantum theory of multi-photon down conversion²² and suggest a few examples of how to carry out the experiments.

1.2 The Scope of The Thesis

We view the present status of EPR experiments as a process of improvements in our understanding of the non-intuitive nature of quantum worlds, not as an effort to settle the controversy which was started by a paper that was written some fifty five years ago.

The scope of this thesis is, therefore, not with the rounding up of its historical or philosophical accounts that include ours, but is on the clarifications and the future directives of *physical* aspects of the problems caused by EPR related papers and subsequent experiments.

We tried to be self-contained as possible, as far as we think it is essential to deal with the mainline physical problems of the issues. Since this paper is primarily concerned with the extension and the realization of Greenberger-Horne-Zeilinger theorem which is more powerful than that of Bell's on EPR issue, we try to unfold our arguments along the line.

In chapter II, we review the essence of original EPR argument and that of Bell's as they will serve as necessary backgrounds. In chapter III, we review most of the important experiments carried out to date on Bell's inequality. In the following chapter we discussed the implications of those experimental results. In chapter V, Greenberger-Horne-Zeilinger's (GHZ) theorem is reviewed in its original form along with its implications. From chapter VI and on we present the extension and physical realization of GHZ theorem which are mostly the author's work unless we show proper credits for each. In chapter VI, there presented various extension of the theorem in their gedanken level, and in the following chapter theoretical study for the non-ideal situations was made, and in Chapter VIII possible experimental models are suggested. Though we have mentioned various types of possible experiments, we see, at this moment, spontaneous multi-photon down conversion as the most ideal and promising way of realization of the theory, and we developed a quantum theory of spontaneous parametric multi-photon down conversion for that purpose. The introductory

theory of multiphoton down conversion is given in Appendix B.

The final two Chapters, IX and X are concerned with more recent development of the theorem that deals with multi-pathed two particle system. In Chapter IX, Bernstein-Greenberger-Horne-Zeilinger's idea on the two particle GHZ theorem is presented and in Chapter X, a more general proof is given along with the ideas of how to implement the theorem.

We also present, in Appendix A, a more coherent way of measuring, counting and calculating EPR type photon polarization correlations which are comparable to the spin- $1/2$ measurements and would serve to eliminate some unnecessary confusions and the loopholes provided by the previous photon polarization experiments. Finally, in Appendix B, as we mentioned earlier, we present quantum theory of generalized nonlinear spontaneous parametric down conversion and its use in many particle interferometry which are necessary to implement the works presented in this thesis.

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CHAPTER II

EPR PAPER AND BELL'S THEOREM

2.1 Einstein, Podolsky and Rosen Argument on Quantum Mechanics

The original model of EPR¹ involved a system of two particles 1 and 2 which was prepared in a state in which the relative position $\vec{r}_1 - \vec{r}_2$ and total momentum $\vec{P}_1 + \vec{P}_2$ have definite values. This is possible since

$$[\vec{P}_1 + \vec{P}_2, \vec{r}_1 - \vec{r}_2] = 0. \quad (2.1)$$

Therefore, we can have simultaneous eigenfunctions of those two physical observables. After this state is prepared, the two particles no longer interact with each other.

Since we know the state at $t=0$, we can calculate the state of combined system 1+2 at any subsequent time with the help of Schroedinger equation. As a result, by measuring the position of particle 1, we can predict with certainty the position of particle 2. But because of the fact that the two particles are spatially separated (therefore, no interaction), the measurement made on particle 1 does not disturb particle 2 in any way, position of particle 2 is an element of reality according to EPR criteria².

Alternatively, we could do the same thing with momentum. By making a measurement of momentum on particle 1,

we could predict with certainty the momentum of particle 2 without anyway disturbing it. Hence, both the position and momentum of particle 2 are the elements of reality though we are prevented by the uncertainty principle to have simultaneously sharp measurement values of both.

So, EPR said, quantum description of a state is not a complete description of physical reality.

Here, Einstein's conclusion is the direct result of his strong belief in locality principle described in his own term: "But on one supposition we should, in my opinion, absolutely hold fast: the real factual situation of the system 2 is independent of what is done with the system 1, which is spatially separated from the former."

Bohr's reply³ to the EPR paper argued that in quantum mechanics one cannot think of two particles separately, since one needed to consider the non-local entities such as experimental arrangements and boundary conditions to completely specify a state.

2.2 Bohm's Version of EPR Experiment⁴

It becomes clear that non-locality that is inherent in quantum description is *not* based on nonrelativistic analysis of the situation when it uses the properties of spin and polarization only, because the properties of these are essentially identical in both the relativistic and nonrelativistic theories.

Also it is much easier to prepare the state which uses strong correlation of spin or polarization to exhibit the same feature of EPR argument than to prepare a position momentum eigenstate which original EPR paper used to invoke the argument.

A simpler version of EPR gedanken experiment was given by Bohm⁴ using measurements of spin components of spin-1/2 particles that dealt with discrete states rather than continuous ones used by EPR.

He considered a pair of spatially separated spin-1/2 particles produced in a singlet state by dissociation of a spin-0 system.

The spin part of the state vector is given by

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|+\rangle_1 |-\rangle_2 - |-\rangle_1 |+\rangle_2). \quad (2.2)$$

Here $|+\rangle$ and $|-\rangle$ each describes a state in which particle has spin "up" or "down", along the direction n . and their subscripts denote particles 1 and 2, respectively. Since we may take any direction as n , if we take n along x direction, even though Eq.(2.2) does not give any predetermined value of outcome of the measurement on particle 1, as soon as we find that particle 1 has spin up then Eq.(2.2) tells us that particle 2 has spin down. Thus, the experimenter can arrange his apparatus so that he can predict the value of the x component of spin of particle 2 without interacting with it and the same argument applies to the spin component of the system. So one may argue that all components of spin of each

particle are definite, which is not so in quantum mechanical description. In quantum mechanics only one component of spin can be measured at any one time so, if we postulate the existence of others, we are giving them the status of hidden variables. Hence, if we agree with Einstein's requirement of a complete theory, hidden variable description seems required.

2.3 EPR "Puzzle" with Photon Polarization Correlation

EPR represents the issues more than those of the elements of reality or completeness of quantum mechanics. EPR concerns more than that⁵, though it is not expressed explicitly in the paper. We try to show the "puzzle" expressed in Einstein's term, so called, "spooky action at a distance".

Suppose a spin-0 particle decays into two photon such as π^0 -meson decay.

$$\pi^0 \rightarrow \gamma + \gamma$$

Both photons must have same helicity by the requirement of angular momentum conservation. Therefore the two photon polarization state can be expressed by

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|R\rangle_1 |R\rangle_2 \pm |L\rangle_1 |L\rangle_2). \quad (2.3)$$

In π^0 -meson decay, it takes up minus sign since the π^0 is pseudoscalar under inversion of the coordinates, therefore

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|R\rangle_1 |R\rangle_2 - |L\rangle_1 |L\rangle_2) \quad (2.4)$$

or alternatively, in terms of linear polarization along x and y axis

$$|\Psi\rangle = \frac{i}{\sqrt{2}} (|X\rangle_1 |Y\rangle_2 + |Y\rangle_1 |X\rangle_2). \quad (2.5)$$

So, the same arguments hold here, instead of using position and momentum, by using circular and plain polarizations as following. If we measure photon 1 to be right handed, we can predict with certainty that photon 2 will be also right handed: the same is true for left handed case.

Alternatively, if we measure photon 1 to be x polarized, we can predict with certainty that photon 2 will be y polarized and vice versa.

Here's what bothered Einstein: Suppose we decide what kind of measurement we want to make (i.e., whether we would make circular polarization measurement or linear polarization measurement) long after the photons went apart so that no interaction is possible between them due to space-like separation, The decision we make on the measurement on photon 1 should not affect the results of measurement of photon 2. But if we use a circular analyzer for photon 1 then we know for certain that photon 2 is circularly polarized, meaning that it is a *random* half and half composer of x and y polarization, on the other hand, if

we use a linear analyzer for photon 1 then we know that photon 2 can only take up the linear polarization *which is orthogonal* to that of particle 1.

Now the question is: "How does photon 2 know which type of analyzer photon 1 will encounter and adjust its own polarization accordingly?"

If we just accept quantum mechanics and go by its rules, then there is no problem in obtaining the values of what we want. Quantum mechanics simply works⁶. But as soon as we start to ask about underlying meaning of those rules we find that quantum mechanics has many strange features⁷.

2.4 Bell's Theorem

Nonclassical nature of quantum mechanics becomes clearer and our hope to have a more complete theory in a classical way gets dimmer as we look into the implications of Bell's theorem⁸ and its experimental results to date⁹.

Those results tell us that we cannot hope to construct a more complete theory of quantum mechanics as we had completed thermodynamics with statistical mechanics by introducing supplementary variables such as position and momentum of individual particles. We may or may not someday construct a better theory but that will not be done by classical solutions since the quantum world is essentially different from the classical ones.

It was Bell's theorem that showed us that any model which is based on classical concepts of locality and reality is doomed to fail to produce *all* the quantum mechanical results. In other words, if we try to introduce supplementary parameters to the quantum mechanics we will have conflicting physically measurable results with quantum mechanics as it is accepted now.

Bell took Bohm's model of two spin-1/2 particles system that are produced in a singlet state by dissociation of a spin-0 system and placed two counters to detect the spins along two adjustable directions, $\hat{a}$ and $\hat{b}$, and looked for the correlations $E(\hat{a}, \hat{b})$ between them as one was set along $\hat{a}$, and the other along $\hat{b}$ as in Fig. 2.1. Since spin would take up values only "up" or "down" (i.e., +1 or -1) for each individual set i of particles, $E_i(\hat{a}, \hat{b})$ would be +1 or -1. We would get average value $E(\hat{a}, \hat{b})$ after many such individual pairs.

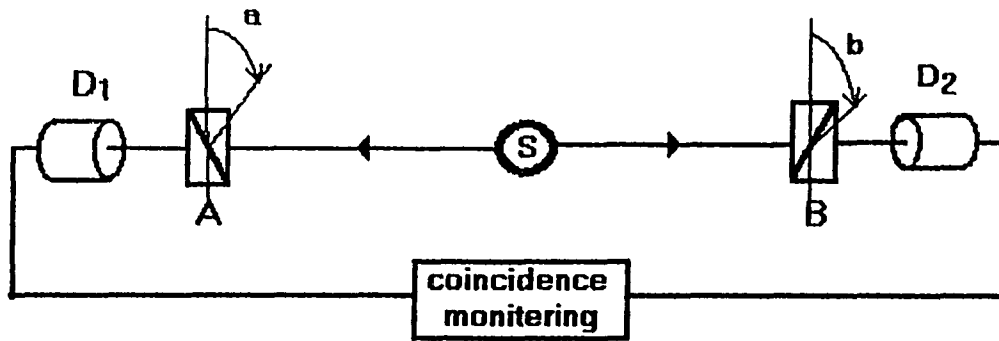


FIG. 2.1

Typical EPR experimental scheme. A source S emitting coherent particle pairs is viewed by two analyzers. The analyzers have externally adjustable parameters a and b. In this example, they are angles between the analyzer axis and a fixed reference axis.

D1, D2 are detectors. A and B are the analyzers such as Stern-Gerlach magnets or linear photon polarization analyzers.

He assumed a local deterministic "hidden" (or "supplementary" or whatever we may call) variable theory in which the state of each particle at the detector is determined at their dissociation by those variables and each detector value will be determined only locally through those variables which are represented by λ in their totality. (The logical result will be same regardless of whether λ is a single variable or a set of variables or is of functional form.)

The measurement by the detectors A,B would yield

$$A_{\lambda}(\hat{a}) = \pm 1, \quad B_{\lambda}(\hat{b}) = \pm 1 \quad (2.6)$$

and

$$A_{\lambda}(\hat{a}) B_{\lambda}(\hat{a}) = -1 \quad \text{for any } \lambda \quad (2.7)$$

$$A_{\lambda}(\hat{a}) B_{\lambda}(-\hat{a}) = +1 \quad \text{for any } \lambda. \quad (2.8)$$

So one gets

$$B_{\lambda}(-\hat{a}) = -B_{\lambda}(\hat{a}), \text{ etc.} \quad (2.9)$$

Since we are dealing with statistical average, we give the various values of λ an arbitrary weighting $\rho(\lambda)$, such that

$$\rho(\lambda) > 0, \quad \int_{\Gamma} \rho(\lambda) d\lambda = 1, \quad (2.10)$$

where Γ is the total λ space.

Thus, the result with one counter set at $\hat{a}$ and the other $\hat{b}$, would be

$$E(\hat{a}, \hat{b}) = \int_{\Gamma} \rho(\lambda) A_{\lambda}(\hat{a}) B_{\lambda}(\hat{b}) d\lambda. \quad (2.11)$$

From (2.6) through (2.11), it follows:

$$[A_{\lambda}(\hat{a})]^2 = 1, \quad B_{\lambda}(\hat{a}) = -A_{\lambda}(\hat{a}) \quad (2.12)$$

then

$$\begin{aligned} E(\hat{a}, \hat{b}) - E(\hat{a}, \hat{c}) &= \int \rho(\lambda) [A_{\lambda}(\hat{a}) B_{\lambda}(\hat{b}) - A_{\lambda}(\hat{a}) B_{\lambda}(\hat{c})] d\lambda \\ &= \int \rho(\lambda) A_{\lambda}(\hat{a}) B_{\lambda}(\hat{b}) [1 + A_{\lambda}(\hat{b}) B_{\lambda}(\hat{c})] d\lambda \end{aligned} \quad (2.13)$$

Since $\rho(\lambda) > 0$, $A_{\lambda}(\hat{a}) B_{\lambda}(\hat{b}) = \pm 1$, $|A_{\lambda}(\hat{a}) B_{\lambda}(\hat{b})| = +1$,
and $1 + A_{\lambda}(\hat{b}) B_{\lambda}(\hat{c}) > 0$,

$$\begin{aligned} |E(\hat{a}, \hat{b}) - E(\hat{a}, \hat{c})| &= \left| \int \rho(\lambda) A_{\lambda}(\hat{a}) B_{\lambda}(\hat{b}) [1 + A_{\lambda}(\hat{b}) B_{\lambda}(\hat{c})] d\lambda \right| \\ &\leq \int |\rho(\lambda) A_{\lambda}(\hat{a}) B_{\lambda}(\hat{b}) [1 + A_{\lambda}(\hat{b}) B_{\lambda}(\hat{c})]| d\lambda \end{aligned}$$

$$\begin{aligned}
&= \int \rho(\lambda) |A_\lambda(\hat{a})B_\lambda(\hat{b})| [1 + A_\lambda(\hat{b})B_\lambda(\hat{c})] d\lambda \\
&= \int \rho(\lambda) [1 + A_\lambda(\hat{b})B_\lambda(\hat{c})] d\lambda \\
&= 1 + E(\hat{b}, \hat{c}). \tag{2.14}
\end{aligned}$$

We therefore have the original Bell's inequality

$$|E(\hat{a}, \hat{b}) - E(\hat{a}, \hat{c})| \leq 1 + E(\hat{b}, \hat{c}). \tag{2.15}$$

Any local deterministic hidden variable theories should satisfy this inequality while quantum mechanical results are not restricted by it.

One can show, how this type of inequality may discriminate those theories by explicitly constructing a hidden variable model for the case.

One of the popular ways to construct the model¹⁰ is to assume that the hidden variable λ is an angle variable, so that

$$A_\lambda(\hat{a}) = A(\lambda, \hat{a}), \quad B_\lambda(\hat{b}) = B(\lambda, \hat{b}) \tag{2.16}$$

If we treat all angles equally for λ , as in Bohm's spherically symmetric $S=0$ case, then we have

$$A(\lambda, \hat{a}) = A(\lambda - \hat{a}) \text{ and } B(\lambda, \hat{b}) = B(\lambda - \hat{b}) \tag{2.17}$$

with

$$\rho(\lambda) = \text{constant} = \frac{1}{2\pi} \tag{2.18}$$

then

$$\begin{aligned}
E(\hat{a}, \hat{b})_{H.V.} &= \frac{1}{2\pi} \int_0^{2\pi} d\lambda A(\lambda - \hat{a}) B(\lambda - \hat{b}) \\
&= E(\hat{a} - \hat{b}) \\
&= E(\theta)
\end{aligned} \tag{2.19}$$

where θ is the angle between $\hat{a}$ and $\hat{b}$.

Now, let's assume that we take a simple specific model which may be called "real spin" hidden variable model, in which hidden variable λ represent the "real" spin so that it produces the values

$$\begin{aligned}
A(\lambda - \hat{a}) &= +1 \quad \text{if} \quad |\lambda - \hat{a}| < \frac{\pi}{2} \\
&= -1 \quad \text{if} \quad |\lambda - \hat{a}| > \frac{\pi}{2}
\end{aligned} \tag{2.20}$$

similarly for $B(\lambda - \hat{b})$.

From these, we have

$$E(\theta) = \frac{1}{2\pi} \int_0^{2\pi} d\lambda A \cdot B = -1 + \frac{2|\theta|}{\pi} \tag{2.21}$$

where $-\pi < \theta < +\pi$ with the period of 2π .

Meanwhile, quantum mechanically, we can calculate the expectation value of joint detection probability of these spin observables:

$$E(\hat{a}, \hat{b})_{Q.M.} = \langle \Psi | (\sigma \cdot \hat{a}) \otimes (\sigma \cdot \hat{b}) | \Psi \rangle \tag{2.22}$$

where $|\Psi\rangle$ satisfies Eq.(2.2).

If we assume the directions a and b be in the x - y plane as we have shown in Fig 2.1, then quantum expectation value will take up a simple form:

$$E(\hat{a}, \hat{b})_{Q.M.} = -\cos(\hat{a} - \hat{b}) = -\cos\theta. \quad (2.23)$$

The closest value this hidden variable theory can produce to this quantum value is the one provided by the equality sign in Eq.(2.15) and those two expectation values are shown in Fig. 2.2a as a function of time. Particular attention may be given to the special directions where two analyzers are parallel or anti-parallel to each other, i.e., $\theta = 0, \pi$. These are the angles with which we can predict with certainty the value of the other side of the measurement therefore Einstein's definition of reality applies and also, at these angles, quantum mechanics and hidden variable take up the same expectation value

$$E(\hat{a}, \hat{a})_{Q.M.} = -1 = E(\hat{a}, \hat{a})_{H.V.} \quad (2.24)$$

$$E(\hat{a}, -\hat{a})_{Q.M.} = +1 = E(\hat{a}, -\hat{a})_{H.V.} \quad (2.25)$$

for all $\hat{a}$.

Another angle at which local hidden variable theory agrees with quantum mechanics is $\theta = \pi/2$; where it takes up the average of above two values.

So, in the frame of Bell's theorem, quantum mechanical results agree with those of hidden variable theory at these special angles. But at another angles, hidden variable

expectation value and quantum expectation value disagree and we can discriminate them experimentally.

We may also see how we can discriminate hidden variable theory and quantum mechanics by using Bell's inequality in this simple model.

Original Bell's inequality for this case may be written as

$$| E(\hat{a}-\hat{b}) - E(\hat{a}-\hat{c}) | - E(\hat{b}-\hat{c}) \leq 1. \quad (2.26)$$

Since $\hat{a}$, $\hat{b}$, $\hat{c}$ are all in the x-y plane, we may choose $\hat{a}=0$ without losing generality. Then

$$| E(\phi_i) - E(\phi_j) | - E(\phi_i-\phi_j) \leq 1 \quad (2.27)$$

for the hidden variable model.

To compare this with quantum mechanical prediction, we put quantum mechanical expectation values into Eq.(2.27) and see if there is any violation of the inequality.

We may choose specifically $\phi_i = 2\phi_j$ so that

$$| E(2\phi) - E(\phi) | - E(\phi) \leq 1. \quad (2.28)$$

Therefore, the predictions of quantum mechanics could be consistent with those of hidden variable only if

$$| \cos 2\phi - \cos \phi | + \cos \phi \leq 1. \quad (2.29)$$

This inequality is clearly violated in the region $0 < \phi < \pi/2$. Figure 2.2b shows the violation of the inequality by quantum mechanics. We can use the angle of maximum violation to test against a whole class of local hidden variable theories.

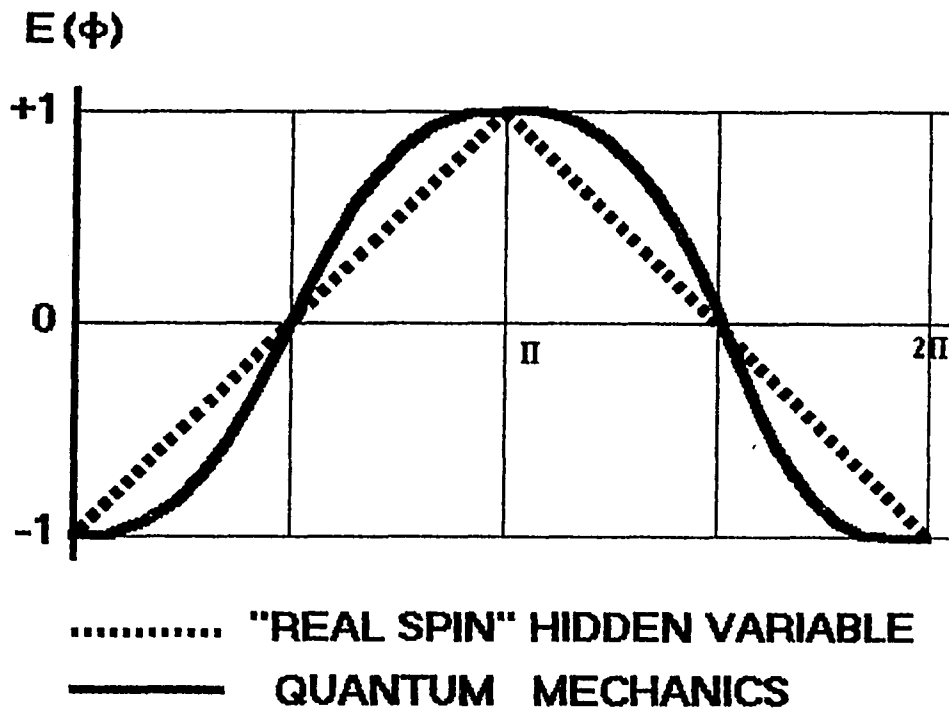


FIGURE 2.2a

$$|E(2\phi) - E(\phi)| - E(\phi)$$

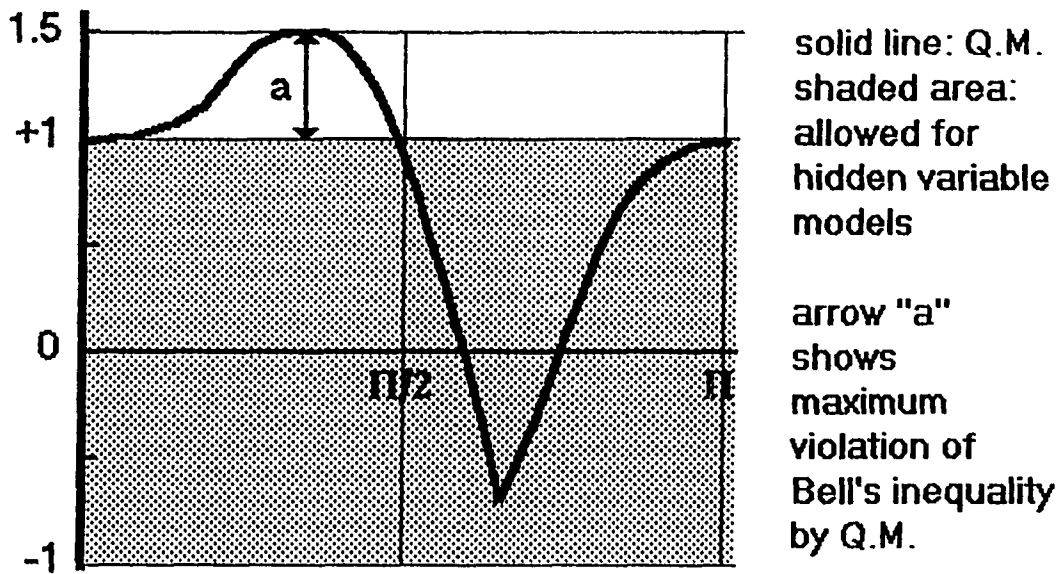


FIGURE 2.2b

E. Other Bell Type Inequalities¹¹

Unfortunately, in any real experimental situation we cannot expect the results given by Eq.(2.24) or Eq.(2.25). For the practical application of the theorem, Clauser, Horne, Shimony and Holt¹² generalized the original Bell's inequality to the situation where perfect correlation measurement cannot be achieved.

The inequality results

$$|E(\hat{a}, \hat{b}) - E(\hat{a}, \hat{c})| \leq 2 - E(\hat{b}', \hat{b}) - E(\hat{b}', \hat{c}). \quad (2.30)$$

In this model, for example, for some b' and b , we may have

$$E(\hat{b}', \hat{b}) = 1 - \delta, \quad (2.31)$$

where $0 < \delta < 1$. we may choose δ close to 0 but not equal to 0 for real experimental situations.

Eq. (2.30) reduces to Eq. (2.15) as we take $\delta = 0$.

In 1974, Clauser and Horne¹³ proved Bell's theorem for general local realistic theories, including stochastic ones. The result was given in a new type of inequality:

$$-1 \leq E_{12}(\hat{a}, \hat{b}) - E_{12}(\hat{a}, \hat{b}') + E_{12}(\hat{a}', \hat{b}) + E_{12}(\hat{a}', \hat{b}') - E_1(\hat{a}') - E_2(\hat{b}) \leq 0 \quad (2.27)$$

Here expectation values are defined in a different way in which one can take into account of both single detector counts and joint coincidents counts.

Several other Bell type inequalities were also devised for various physical situations.

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2. Einstein's Criteria of Reality cf. Quantum Mechanical Reality:
 Usual theory of quantum mechanics on reality: *If operators corresponding to two physical quantities do not commute, then those two quantities cannot have simultaneous reality.*
 Einstein on reality: *If, without in anyway disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.*
3. See chapter I references 2,3
4. See chapter I reference 12
5. See chapter I reference 6
6. This one of the popular attitude toward quantum mechanics is that of diehard pragmatists: Quantum mechanics is simply an algorithm for predicting the expectation values of all possible observables. It is meaningless to look for underlying causes, since there's no underlying principle other than those already stated by quantum mechanics. Quantum mechanics works, so does "no peeking principle".
 For example, see
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7. For example, see D.M.Greenberger, The Neutron Interferometer
 as a device for illustrating the strange behaviour of quantum systems, *Rev.Mod.Phys.* 55, 875 (1983)
8. See chapter I reference 11
9. This will be discussed in chapter III including most recent EPR experiments
10. Greenberger, Horne, Zeilinger, unpublished, (1988)
11. Since Clauser and Shimony's review article provides excellent accounts on this, we will just list few of them here.
 see chapter I reference 13

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CHAPTER III

EXPERIMENTAL TESTS OF BELL'S INEQUALITIES

Various Bell type inequalities have been tested in more than a dozen experiments and generally shown to be violated in support of quantum mechanics.

Most of the experiments performed and published to date on Bell type inequalities have been carried out on photon pair correlations from various sources except for the one by Laméhi-Rachti and Mittig¹ who tested the spin correlation in the proton-proton scattering.

Only two of these experimental results^{2,3} have disagreed with quantum mechanics and shown to satisfy the Bell's inequality where quantum mechanics predicts a violation. Both of the discordant cases were repeated with care, but the discrepancies with quantum mechanics could not be reproduced.

3.1 Pre-Bell Experiments

A limited test of EPR concerning completeness of quantum mechanics was proposed and discussed by Inglis⁴ using charge conjugation correlations of theta particles. There also was a discussion by Aharonov and Bohm⁵ to show the existence of EPR phenomena by making use of the experiment with positronium annihilation photon by Wu and

Shaknov⁶ upon which, later, several other Bell type experiments have rooted.

In 1967, Papaliolios⁷ performed a preliminary test of a hidden variable theory by making use of Bohm-Bub theory⁸. The answer was positive with quantum mechanics.

3.2 Atomic Cascade Experiments

Following the suggestion by Clauser et al.⁹, the polarization correlation between two photons emitted in the same atomic cascade have been performed first by Freedman and Clauser¹⁰, in 1972, who used $(J=0) \rightarrow (J=1) \rightarrow (J=0)$ two photon cascade from calcium, followed by Holt and Ripkin², Clauser¹¹, and Fry and Thompson¹².

In 1981 and 1982, Aspect et al.¹³⁻¹⁵ reported three experiments using atomic cascade photons. The last two of these are considered as the most decisive tests of this type of experiment by many. The second one¹⁵ made use of the two-channel polarizer suggested by Garuccio and Rapisarda¹⁶ and the last one met the Einstein locality condition by using a long distance separation and the special polarizers (Fig.3.1) the orientations of which would change rapidly and independently by acousto-optical switches in a time (10 ns) shorter than the time for a light signal to pass from one instrument to the other (40 ns). The correlations were found to be the same as in the experiment with static settings of the analyzers in agreement with quantum theory.

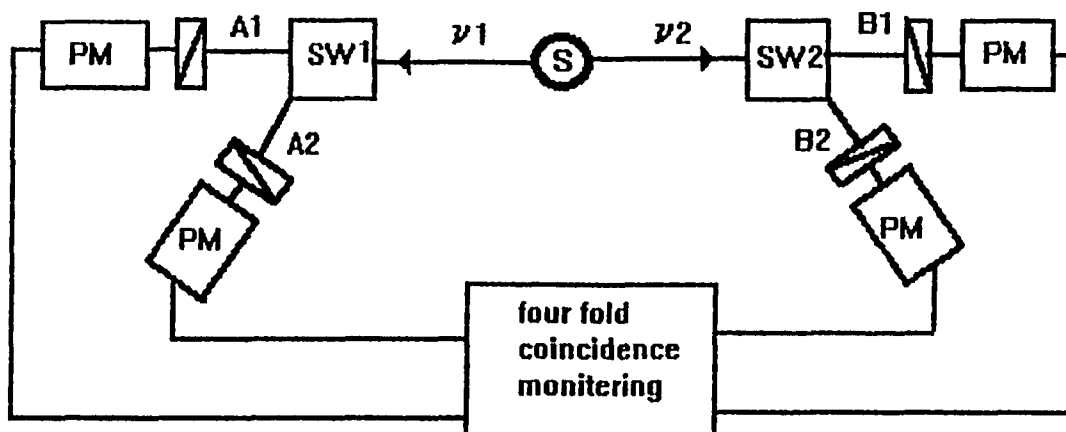


FIG. 3.1

ASPECT'S EXPERIMENTS:

Each of two photons from atomic cascade directed by acousto-optical switches SW1 or SW2 toward two different polarizer settings. Both switches are driven separately rapidly in a time shorter than the time for a light signal to pass from one side to the other. Then through the photomultiplier, their four joint detection rates are monitored.

3.3 Spin Correlation Experiment by Proton Proton Scattering

Test of Bell's inequality in spin correlations of two protons has been carried out by Laméhi-Rachti and Mittig¹ following the suggestion made by Fox¹⁷. Agreement with quantum mechanics is reported within the limited statistics of this experiment.

3.4 Positronium Annihilation Photon Experiments

Test of local realism with positronium annihilation, looked like a favorite case since the relative directions of the two photons are well constrained by momentum conservation and their polarizations are strongly correlated by angular momentum and parity conservations. But unlike the cascade photon case whose efficient low energy photon filter is readily available, efficient polarization filters do not exist for these high energy photons therefore the polarizations can only be inferred indirectly through Compton scattering of electrons. These experiments are less decisive¹⁸ as a refutation of the family of local realistic theories than are the cascade photon experiments.

Experiments of this kind have been performed by Kasday, Ullman and Wu¹⁹ followed by Faraci et al.³, by Wilson et al.²⁰, and by Bruno et al.²¹. With the exception of Faraci et al., good agreements have been found with the predictions of quantum mechanics.

3.5 Deuterium Photon Experiments

More recently, metastable atomic deuterium is used to produce strongly correlated photons by Kleinpoppen et al.^{22,23} In the most recent one, reported in 1989, they used three linear polarizers, instead of two, to check the

validity of the additional assumption made by Clauser and Horne²⁴, so called "no-enhancement assumption", and reported positively for the assumption. Furthermore, the result of the experiment also have been used against the local realistic models²⁵ which were capable of explaining all *then* existing two polarizer type experiments, and known to provide conclusive evidence against such theories.

3.6 Parametric Frequency Down Converted Photon Experiments

Practically ideal entangled state of two photons can be obtained by nonlinear spontaneous parametric frequency down conversion technique.^{27,28} These photons can be produced simultaneously in a nonlinear media into well defined directions and frequencies. With the help of these well defined emissions, this technique enabled experimenters to do some new types of EPR tests such as position correlation, and position and time correlation, as well as polarization correlation experiments.

Shih and Alley^{29,30} used this technique to test Bell's inequality for the first time, followed by similar test by Ou and Mandel³¹. Both of them used KDP crystal to produce down converted photons and adapted a beam splitter to produce an entangled state which has been subjected to small controversies³². In those experiments Bell's inequalities are reported to be violated by 3 and 6 standard of deviations, respectively. Fourth order interference of light

experiment that made use of position variables instead of usual polarization angles to show the violation of local realism has been carried out by Ou and Mandel³³ along the line with Horne and Zeilinger³⁴ who extended the idea of Bell's theorem to linear momentum correlation. And in 1990, Mandel et al.³⁵ carried out new type of Bell's inequality in position and time which was proposed by Franson³⁶ and reported in support of quantum mechanics.

3.7 Other Experiments (Suggested)

Other than those actually performed experiments majority of which have been done with photons, few authors suggested some interesting cases in particle physics which allow a formulation of EPR test of local realism vs. quantum mechanics: among them are Törnquist³⁷ (Λ -decay), Six³⁸, Selleri³⁹ (both of them considered $K^0-\bar{K}^0$ system), Datta and Home⁴⁰ ($B^0-\bar{B}^0$), Datta, Home, Raychaudhuri⁴¹ ($K^0-\bar{K}^0$ system inclusion of CP violation).

Lo and Shimony⁴², also, proposed a molecular test with Bell type inequality to block the loophole which was caused by inefficiency of detectors.

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CHAPTER IV

IMPLICATIONS AND LOOPHOLES OF BELL TYPE EXPERIMENTS

Having looked at the results, we may safely conclude that local realism of EPR type lost its ground to withstand even though it has not been totally silenced because of various loopholes provided by the tests¹.

On the other hand, Bell type inequalities and their subsequent tests brought considerable impacts on our understanding of the quantum world. Some important conceptual standpoints of "old" days, which remained as a matter of preference before the arrival of Bell's theorem, have been put to physical tests and proved to be false, while strange and non-intuitive natures of quantum theory, such as non-locality, have been revealed and established by the tests^{2,3}.

Though nonlocal feature of EPR does not violate special relativity, in its information communication level⁴⁻⁶, possible conceptual conflicts in a deeper level have been studied.

Meanwhile, many school of thoughts have attempted and suggested several ways of explaining this phenomenon.

4.1 Implication on Conceptual Problems in Quantum Mechanics

4.1.1 On Reality:

A naive realism that treats spin, polarizations, position or momentum of particles, and even particle itself etc. as real objective entities should be discarded, along with Einstein type realism that does not require measurements on those quantum entities, since those are the macroscopic manifestations of quantum entities through the interactions with measuring apparatus. A radical revision of our attitude towards physical reality is required.

4.1.2 On The Completeness Of Quantum Mechanics:

One can not introduce supplementary parameters (either deterministic or stochastic) to make the theory more complete in such a way that satisfies Einstein's locality condition.

4.1.3 On The Quantum Measurement Theory:

As we still have controversies over the necessity of the concept of collapse of wavefunction in proper interpretation of quantum mechanics and controversy over when it happens, EPR provides the information as to how the measurement made on one particle can "affect" or "influence" (or, "collapse" if there is any collapse) the state of the other at long distance away. EPR system exhibits the correlation between two distant particles in a symmetric way (for example, of the form of $\cos(\hat{a}-\hat{b})$ where $\hat{a}$ and $\hat{b}$ are the

measuring angles of each polarizer⁷) and, thus, insensitive to their time ordering when their separation is spacelike. It seems harder now to introduce the concept of collapsing of wavefunction about which one may ask a question: "which one collapses the other subsystem⁸?"

4.1.4 On Locality And Compatibility With Special Relativity:

This was in the heart of the EPR about which Einstein called "spooky action at a distance" Actually EPR "paradox", i.e., derivation of a contradiction of quantum mechanics, can be achieved without introducing collapse of wavefunction⁹, or determinism, even without reality argument³. H. Stapp² has shown that the essential feature in the Einstein's argument was locality principle which was strongly motivated by special relativity.

But the way non-locality in quantum mechanics works is not contradictory to the assertion of relativity which guarantees that no communication is possible between the events which are spacelike-separated. Each counter reads completely random results so that one must take time to get the other side of information to find the correlation of individual pairs provided by quantum mechanics. Some have shown⁴⁻⁶ that quantum correlations cannot be used to transmit messages at superluminal speeds. However it is not clear whether there is a deeper incompatibility between quantum

mechanics and relativity¹⁰. It has been worried that our concepts on space-time events should be dramatically revised¹¹ as quantum mechanics asserts that seemingly two different space-time events do not have an independent existence.

4.2 Weakpoints and Loopholes in Bell Type Theories and Experiments

A lot of challenges have been made against many of the Bell type inequalities and the tests performed on them. Although Bell's theorem proved the points of great physical importance, it sets its own reality and locality conditions which are different from EPR's original conditions. Hence there are some cases which are not covered by Bell's theorem but still implicated in EPR.

There also are some assumptions made in addition to the original Bell's inequality to include various practical experiments which provided with loopholes to those who did not agree with those additional assumptions.

4.2.1 Einstein's Original Locality Condition:

"The real factual situation of the system S_2 is independent of what is done with the system S_1 , which is spatially separated from the former."¹² The only experiment that met this condition of space-like separation to test the

Einstein's locality argument was the last experiment of Aspect's¹³ in which he employed switches that changes the channels in a time shorter than the flight of two particles. But it was not truly random, as it was admitted¹⁴, though it could be regarded as random enough to sway away most of the doubts.

4.2.2 Einstein's Original Reality Condition¹⁵:

"If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." As we have shown in section II.D, Bell's theorem uses other angles than the ones with which we can predict the value of the spin of the other particle with probability one. For those cases with which Einstein made his case Bell's theorem cannot say anything about possible violations.

4.2.3 Inefficient Counters:

Bell's theorem has also been extended realistically to the case of inefficient counters¹⁶ because being inefficient provides local realism with much greater flexibility. For example, one can assign certain values of the hidden variables to instruct the counters not to count for certain

circumstances if one doesn't like the results, therefore one can handle many more situations than one could handle with 100% efficient detectors. In fact, if the counter is sufficiently ineffective, one can violate Bell's inequality and get the quantum-mechanics-like results.

It had been found that one needs an efficiency of more than 82.8%^{17,18} in order to rule out local deterministic theories *without* making use of the additional assumptions by CHSH and CH. (see next section)

To date, the best experiments have been done with counters that were less than 15% efficient, though if one makes the reasonable assumption that we only have to count those cases in which at least one counter fires, then one can sufficiently lower this requirement to include the latest experiments *with* the help of additional no-enhancement assumption.

4.2.4 Additional Assumptions:

Bell's theorem, originally based on a gedanken experiment has been implemented with the help of additional assumptions. These assumptions, when they are employed, can change the critical values (with which we discriminate local realism and quantum mechanics) significantly so that some data can be used against some types of local realism, otherwise they cannot be discriminated¹⁹ from quantum mechanics. Among them are Clauser-Horne-Shimony-Holt (CHSH)

assumption²⁰: "Given a pair of photons emerging from two regions of space where two polarizers can be located, the probability of their joint detection from two photomultipliers does not depend upon the presence and the orientation of the polarizers." and Clauser-Horne (CH) additional assumption²¹ or, commonly known as no-enhancement assumption: "The detection probability of a photon in the state λ with a polarizer in place on its trajectory is less than or equal to the detection probability with the polarizer removed, for each state λ ." T. Marshall et al.²² was able to construct a local realistic model that produces all the then existing experimental results within their experimental error except for the second experiment by Aspect et al. (see Appendix A) performed by abandoning the no-enhancement assumption though it might seem somewhat contrived. Later, it was suggested²³ that this type of model which allows enhanced detection could be put to a test using three polarizer instead of usual two and test result was reported²⁴ to provide conclusive evidence against those local realistic theories.

4.3 Other Attempts to Explain EPR Experimental Results

Other attempts have been made by various schools of thoughts to explain EPR phenomena and its implications.

Among them is nonlocal hidden variable theory that admits nonlocal nature of quantum mechanics but insists on

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realism in pursuit of more complete theory. D.Bohm's nonlocal causal interpretation²⁵ and nonlocal stochastic interpretation²⁶ of quantum mechanics are of this type. But these theories often involves as many conceptual problems as quantum mechanics if not more.

A new interpretation,²⁷ so called transactional interpretation, is presented by Cramer in which he explains EPR as an "interaction" through the medium of *transaction*, a Wheeler-Feynman exchange of advanced and retarded wave, which does not necessarily carry the net energy.

Others have considered delayed determinism,²⁸ superluminal connections through Dirac's Aether²⁹, or quantum zero point fluctuations³⁰, or Dirac-Feynman negative probability³¹, or the determinism by common causes³² (i.e., through the intersectional area between the two backward lightcones so that everything remain causal, hence, in this view, there are really no such things as "random" or "independent" settings. Everything in the Aspect experiment could lie in the forward light-cone of some common causes, so the condition for Bell's theorem would not apply), etc.^{33,34} as possible solutions.

4.4 Remarks

This EPR phenomenon turned out to be a macroscopic quantum phenomenon that cannot be reduced to a classical result nor can be resolved by classical intuition.

We do not regard the negation of local realism as the completion of EPR program. Instead, we take the same view as Mehlhop's³⁵ on this point: "EPR still represents a puzzle, maybe more than ever."

We believe that the EPR problem should be investigated more aggressively both theoretically and experimentally with the expectation that it might lead us to some important new knowledge of fundamental nature of the physical world.

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CHAPTER V

BELL'S THEOREM WITHOUT INEQUALITIES

5.1 Going Beyond Bell's Theorem

As it was pointed out in Sec. 4.2, Bell's theorem has some weakpoints and limitations.

Among them are the fact that Bell's theorem cannot tell anything about the cases where Einstein's original reality argument applies, and that, with Bell's theorem, one must go by inequality. (For example, in a simple version of EPR gedanken experiment given by Mermin¹, local hidden variable requires a class of outcomes to occur at least 55.5% of the time, while quantum mechanics allows them to occur only 50% of the time. The inequality therefore permits a loophole through which local realism could fill in the discrepancies between the two values by devising special models that take advantage of the inefficiency of current photon detectors.)

These two defects in Bell's theorem can be corrected and original EPR program might be completed by the new theorem introduced by Greenberger, Horne, and Zeilinger².

5.2 Why Do We Have Inequality?

Bell's theorem restricts local realism by inequality, i.e., local realism is bounded by upper and/or lower limits while quantum mechanics is not.

We have shown in Sec. 2.4 an example of typical hidden variable model that followed closely quantum mechanical predictions (Fig. 2.2a) but could not produce sinusoidal curve by being limited by its own linear character.

This quantum mechanical cosine term such as Eq.(2.19) comes from the interference between particle 1 and 2.

$$E(\hat{a}, \hat{b}) = \langle \Psi | (\sigma_1 \cdot \hat{a}) \otimes (\sigma_2 \cdot \hat{b}) | \Psi \rangle \quad (5.1)$$

where

$$(\sigma \cdot \hat{n}_i) = \begin{pmatrix} \cos \theta_i & \sin \theta_i \cdot e^{-i\phi_i} \\ \sin \theta_i \cdot e^{i\phi_i} & -\cos \theta_i \end{pmatrix} \quad (5.2)$$

and

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \{ |+\rangle_1 |-\rangle_2 - |-\rangle_1 |+\rangle_2 \}. \quad (5.3)$$

Therefore,

$$E_{q.m.}(\hat{a}, \hat{b}) = -\cos \theta_a \cos \theta_b - \sin \theta_1 \sin \theta_2 \cos(\phi_a - \phi_b). \quad (5.4)$$

For $\theta = \pi/2$,

$$E_{q.m.}(\hat{a}, \hat{b}) = -\cos(\phi_a - \phi_b) = -\cos(\Delta\phi) \quad (5.5)$$

On the other hand, the joint probability of any truly independent space-time events (therefore, local) A and B

must be expressed by the product of each probability: for example, $P(A \cap B) = P(A) \cdot P(B)$.

A local theory cannot have the cross-terms like Eq.(5.5) because it does not have the property like the superposition of probability amplitudes, and each measurement A, B takes up values only locally. Therefore, through the absence of this quantum-mechanics-like freedom, the local realistic result is strongly restricted and limited.

The reason Bell was able to come up with the inequality is that he chose the right model that could produce the maximum interference effect, i.e., maximally strong correlation. (We call such a state as an "entangled state", the state that cannot be further factorized: such a state as

$$\frac{1}{\sqrt{2}}(|+\rangle_1 |-\rangle_2 \pm |-\rangle_1 |+\rangle_2).$$

In the heart of the conflict between local realism and quantum mechanics, there exists a chief difference. Local realism assumes elements of reality which necessarily be expressed by ordinary number, therefore follows mathematical probability formulation in which the expectation value (or, the average) of the product $A \cdot B$ can be described by

$$\int \rho(\lambda) A(\lambda, \hat{a}) B(\lambda, \hat{b}) d\lambda, \text{ where } \int \rho(\lambda) d\lambda = 1.$$

Quatum mechanics treats observables not as ordinary numbers but as operators, therefore follows the peculiar rule of quantum mechanics which says: it is amplitude that

combines like mathematical probabilities rather than probability themselves.

So, what do all these say?

If local realism is so restricted by its own assumptions of locality and reality that it cannot even produce a simple sinusoidal curve in a situation where there is only one degree of freedom is involved - namely, the angle between two detectors - , then one might ask the following question.

Can local realism survive the case that involves more than one degree of freedom? In other words, how would it work in the case where quantum mechanics predicts an interference among three or more particles?

The answer is "No, it cannot survive, nor can it work." This time, local realism cannot even have a luxury of being restricted by an inequality, it simply breaks down. If the local realism tries to imitate the quantum mechanical results, it would find itself contradictory.

5.3. New Theorem By Greenberger, Horne and Zeilinger (GHZ)

GHZ^{2,3} noticed that the chief limitation on the Bell's theorem is that the system examined is too simple and the limitation imposed on local realism would become much more severe if we use the system with more degrees of freedom than one.

If we use strong correlation among three or more particles, then we can derive many powerful Bell-type inequalities. But actually we may do better.

GHZ considered a model⁴ which can produce strong quantum correlation among four spin-1/2 particles originated from single spin-1 particle in its $m=0$ state. At first particle 0 decays into two spin-1 particles 1 and 2.

$$|1,0\rangle \rightarrow \frac{1}{\sqrt{2}} (|1,-1\rangle - |1,1\rangle) \quad (5.6)$$

where $|1,0\rangle$ refers to the state $|J,m\rangle$ of particle 0, while on the right hand side we refer to the $|m_1,m_2\rangle$ states of particles 1 and 2.

Subsequently particle 1 decays into two particles A and B, while particle 2 decays into two particles C and D, each of which is spin-1/2, so that the final state will be

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|+\rangle_a |+\rangle_b |-\rangle_c |-\rangle_d - |-\rangle_a |-\rangle_b |+\rangle_c |+\rangle_d) \quad (5.7)$$

where $|+\rangle$, $|-\rangle$ refers to the $m=+1/2$, $m=-1/2$ states and subscripts denote each particle A, B, C, and D.

We assume the particle 0 was at rest and decaying of all particles along z direction to avoid the complication by spin-orbit coupling. (Fig. 5.1)

We calculate average spin products along the direction of 4 detectors, $\hat{n}_a$ for detector D_A , etc.,

$$X = \langle \Psi | (\sigma \cdot \hat{n}_a) \otimes (\sigma \cdot \hat{n}_b) \otimes (\sigma \cdot \hat{n}_c) \otimes (\sigma \cdot \hat{n}_d) | \Psi \rangle \quad (5.8)$$

where $(\sigma \cdot \hat{n}_a)$ is given by

$$(\sigma \cdot \hat{n}_a) = \begin{pmatrix} \cos \theta_a & \sin \theta_a \cdot e^{-i\phi_a} \\ \sin \theta_a \cdot e^{i\phi_a} & -\cos \theta_a \end{pmatrix}, \quad (5.9)$$

and similarly for the other terms. Then this gives

$$\begin{aligned} X = & \cos \theta_a \cos \theta_b \cos \theta_c \cos \theta_d \\ & - \sin \theta_a \sin \theta_b \sin \theta_c \sin \theta_d \cos(\phi_a + \phi_b - \phi_c - \phi_d). \end{aligned} \quad (5.10)$$

If we set all detectors in the x-y plane, then $\theta_i = 90^\circ$ and we have

$$X = -\cos(\phi_a + \phi_b - \phi_c - \phi_d): \quad (5.11)$$

which is remarkably same form as Eq.(2.19) in two particles case. But, this time, we have two additional continuous degrees of freedom to yield the values $X = +1$ or -1 , to which Einstein's original reality criterion applies. (As was defined by EPR, if we know the spins of any three particles we can predict with certainty the value of the spin of the other one. We'll call this case as "super-classical", adopting the original term by GHZ.)

In this four particle case, because of these extra degrees of freedom to form a super-classical case, one doesn't need to go to the angles other than those that give $X=+1$ or -1 to find the discrepancies between local realism and quantum mechanics. (That was what we did in Bell's theorem.)

Quantum mechanics predicts that we can have these objectively definite super-classical case as follows:

$$X = -1 \quad \text{for} \quad \phi_a + \phi_b - \phi_c - \phi_d = 0 \quad (\pm 2\pi) \quad (5.12)$$

and

$$X = +1 \quad \text{for} \quad \phi_a + \phi_b - \phi_c - \phi_d = \pi \quad (\pm 2\pi). \quad (5.13)$$

Unlike Bell's case where if we know ϕ_a , then ϕ_b is determined for the specific expectation values of $X = +1$ or -1 , here, though we know the angle ϕ_a , we have a double continuum for ϕ_b and ϕ_c , and we can still choose ϕ_d to satisfy $X = +1$ or -1 . So quantum mechanics predicts much richer freedom of measurement here.

Now, let's try to make a deterministic local model for these super-classical cases expressed by Eqs.(5.12) and (5.13) to see what happens when local realism tries to imitate the quantum mechanical results.

As we have done in Bell's case, imagine that as particle 0 decays, there is a set of hidden variables λ_0 that determine the behaviour of particles 1 and 2. As particle 1 decays, there is a further set λ_1 determining the

behaviour of particles A and B, and also set λ_2 from the decay of particle 2 that determines the behaviour of C and D.

Then, the behaviour of the four detectors D_i will be given by

$$A_{\lambda_0\lambda_1}(\phi_a) \cdot B_{\lambda_0\lambda_1}(\phi_b) \cdot C_{\lambda_0\lambda_2}(\phi_c) \cdot D_{\lambda_0\lambda_2}(\phi_d). \quad (5.14)$$

For the general case of $X \neq \pm 1$, one needs to integrate over the possibilities as we have done in Bell's case (see Eqs. (2.9) and (2.10) in Chapter II), but for the superclassical case, we do not have to. Here we have

$$A \cdot B \cdot C \cdot D = -1 \quad \text{If } \phi_a + \phi_b = \phi_c + \phi_d \quad (\pm 2\pi) \quad (5.15)$$

and

$$A \cdot B \cdot C \cdot D = +1 \quad \text{If } \phi_a + \phi_b = \phi_c + \phi_d + \pi \quad (\pm 2\pi), \quad (5.16)$$

where

$$A = \pm 1, B = \pm 1, C = \pm 1, D = \pm 1. \quad (5.17)$$

Note also that

$$A^2 = B^2 = C^2 = D^2 = +1. \quad (5.18).$$

If we specifically take the case of (5.12), then

$$A_{\lambda_0\lambda_1} B_{\lambda_0\lambda_1} = - C_{\lambda_0\lambda_2} D_{\lambda_0\lambda_2}. \quad (5.19)$$

Since LHS of the equation doesn't depend on λ_1 and RHS on λ_2 , we may drop subscripts to have

$$A_\lambda(\phi_a) \cdot B_\lambda(\phi_b) \cdot C_\lambda(\phi_c) \cdot D_\lambda(\phi_d) = -1. \quad (5.20)$$

For a given specific λ , this implies

$$A_\lambda(0) \cdot B_\lambda(0) \cdot C_\lambda(0) \cdot D_\lambda(0) = -1, \quad (5.21)$$

and

$$A_\lambda(\phi) \cdot B_\lambda(0) \cdot C_\lambda(\phi) \cdot D_\lambda(0) = -1, \quad (5.22)$$

also

$$A_\lambda(\phi) \cdot B_\lambda(0) \cdot C_\lambda(0) \cdot D_\lambda(\phi) = -1. \quad (5.23)$$

From Eqs.(5.21) and (5.22) we get

$$A_\lambda(\phi) \cdot C_\lambda(\phi) = A_\lambda(0) \cdot C_\lambda(0). \quad (5.24)$$

From Eqs.(5.21) and (5.23) we get

$$A_\lambda(\phi) \cdot D_\lambda(\phi) = A_\lambda(0) \cdot D_\lambda(0) \quad (5.25)$$

Now from Eqs.(5.24) and (5.25) we have

$$\frac{C_\lambda(\phi)}{D_\lambda(\phi)} = \frac{C_\lambda(0)}{D_\lambda(0)} \quad (5.26)$$

By Eq.(5.18), we see that

$$\{D(\phi)\}^2 = \{D(0)\}^2 = 1. \quad (5.27)$$

Thereby we have

$$C_{\lambda}(\phi) \cdot D_{\lambda}(\phi) = C_{\lambda}(0) \cdot D_{\lambda}(0). \quad (5.28)$$

Now look at the special case of (5.20) in the constraint of Eq.(5.12);

$$A_{\lambda}(2\phi) B_{\lambda}(0) C_{\lambda}(\phi) D_{\lambda}(\phi) = -1, \quad (5.29)$$

then, by Eq.(5.28) we have

$$A_{\lambda}(2\phi) \cdot B_{\lambda}(0) \cdot C_{\lambda}(0) \cdot D_{\lambda}(0) = -1. \quad (5.30)$$

Finally comparing this with (5.21), we obtain

$$A_{\lambda}(2\phi) = A_{\lambda}(0), \quad (5.31)$$

which means

$$A = \text{constant} \quad (5.32)$$

for any angle. The same argument applies to B,C,D also, so that

$$A, B, C, D = \text{constant}. \quad (5.33)$$

But this result contradicts with Eq.(5.17) that states each of A,B,C,D can take sometimes +1 and sometimes -1.

For example, while we meet the condition $\phi_a + \phi_b = \phi_c + \phi_d$ of

Eq.(5.12), we can have

$$A_{\lambda}(\phi_a) \cdot B_{\lambda}(\phi_b) \cdot C_{\lambda}(\phi_c) \cdot D_{\lambda}(\phi_d) = -1. \quad (5.34)$$

We certainly can meet the same condition with the other angle, namely $\phi_a + \pi$,

$$A_{\lambda}(\phi_a + \pi) \cdot B_{\lambda}(\phi_b) \cdot C_{\lambda}(\phi_c + \pi) \cdot D_{\lambda}(\phi_d) = -1. \quad (5.35)$$

But we see, from Eq.(5.13), that we must have

$$A_{\lambda}(\phi_a + \pi) \cdot B_{\lambda}(\phi_b) \cdot C_{\lambda}(\phi_c) \cdot D_{\lambda}(\phi_d) = +1. \quad (5.36)$$

By comparing Eq.(5.34) with Eq.(5.36), we must have

$$A_{\lambda}(\phi_a + \pi) = -A_{\lambda}(\phi_a). \quad (5.37)$$

Hence $A_{\lambda}(\phi_a + \pi)$ in Eq.(5.35) must have been a negative value of $A_{\lambda}(\phi_a)$: which is a contradiction to equation Eq.(5.32). Therefore we see that there is no local deterministic model that can yield the results of Eqs.(5.12) and (5.13). So, even the super-classical case, which survived the Bell's theorem, cannot be modelled by a local realism. Moreover, we do not have to resort to an inequality to refute it.

5.4. Implications Of GHZ Theorem

GHZ theorem represents significant breakthrough in EPR problem by going right back to the original EPR criteria,

yet using no more complicated model than that of Bell's except for the degrees of freedom involved, and succeeds in refuting local realism not by an inequality but through an algebraic contradiction.

The logic used by GHZ assumes that quantum mechanics works then shows that local realism faces contradiction if they try to imitate quantum-mechanics-like freedom to yield super-classical correlations. Therefore, if we can prove quantum mechanics by an experiment, then local realism immediately and automatically become disqualified.

In comparison with Bell's version of EPR which expects a certain class of outcomes to occur $x\%$ with local realism and $y\%$ with quantum mechanics, GHZ's version expects a certain class of outcomes to occur all the time with one theory but never with the other.⁵

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CHAPTER VI
EXTENSION OF GHZ THEOREM

6.1 Theorem With Three Particle Spin Correlation

In the previous chapter we have used a special model which produced a quantum mechanical expectation value $X = -\cos(\phi_a + \phi_b - \phi_c - \phi_d)$ in order to reach the contradictory result for local hidden variable theory. In the process of the proof, we held fixed ϕ_b at 0 throughout. Therefore the proof would go through even if there were only three angles (that means three particles) involved, provided that X is symmetrical enough, such as in a model which could yield

$$X = -\cos(\phi_a \pm \phi_b \pm \phi_c). \quad (6.1)$$

This requires the model to be in a perfectly entangled state.

6.1.1 Three spin-1/2 particles system

For three spin-1/2 particles system, we can form eight of the perfectly entangled states. Namely,

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|m_1, m_2, m_3\rangle \pm |-m_1, -m_2, -m_3\rangle), \quad (6.2)$$

where m_i can take up one of the two values from $+1/2$ or $-1/2$. Any one of such combinations could yield strong correlation given by Eq.(6.1).

Let's take one of them for example

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|+ + +\rangle - |- - -\rangle) \quad (6.3)$$

here again + and - denote our choice of specific spin "up" or

spin "down". Then we have a correlation:

$$\begin{aligned} X &= \langle \Psi | (\sigma \cdot \hat{n}_1) \otimes (\sigma \cdot \hat{n}_2) \otimes (\sigma \cdot \hat{n}_3) | \Psi \rangle \\ &= \cos\theta_1 \cos\theta_2 \cos\theta_3 - \sin\theta_1 \sin\theta_2 \sin\theta_3 \cos(\phi_1 + \phi_2 + \phi_3) \end{aligned} \quad (6.4)$$

by setting $\theta_i = \pi/2$,

$$X = -\cos(\phi_1 + \phi_2 + \phi_3). \quad (6.5)$$

Then by using the following relations:

$$A_\lambda(0)B_\lambda(0)C_\lambda(0) = -1, \quad (6.6a)$$

$$A_\lambda(\phi)B_\lambda(-\phi)C_\lambda(0) = -1, \quad (6.6b)$$

$$A_\lambda(\phi)B_\lambda(0)C_\lambda(-\phi) = -1, \quad (6.6c)$$

and

$$A_\lambda(2\phi)B_\lambda(-\phi)C_\lambda(-\phi) = -1, \quad (6.6d)$$

we can proceed with the same logic we have used in four particle system to deduce the contradictory behaviour of local deterministic hidden variable models.

The above model doesn't contradict anything with quantum premises, hence places no problem in gedanken consideration. But unlike four particle case, each of the two eigenstates $|m_1, m_2, m_3\rangle$ and $|-m_1, -m_2, -m_3\rangle$ represents different total spin angular momentum states therefore cannot be a practical model for simple decaying of one particle into three spin-1/2 particles unless one uses a special technique of flipping the spin of the half of the beam in a workable way.

6.1.2 System Mixed With Spin-1 And Spin-1/2¹

Our more practical model consists of one spin-1 particle and two spin-1/2 particles.

Let's consider the case where a spin-1 particle in its $m=0$ state decays into two spin-1 particles and one of them further decays into spin-1/2 particles.

The spin state of the intermediate system and final system can be described by

$$|1, 0\rangle = \frac{1}{\sqrt{2}} (|1, -1\rangle - |-1, 1\rangle)_{\text{intermediate}}, \quad (6.7a)$$

and

$$|\Psi\rangle_{\text{final}} = \frac{1}{\sqrt{2}} (|1, -, -\rangle - |-1, +, +\rangle) \quad (6.7b)$$

We further assume that the original particle is at rest and decaying of all particles along z direction to avoid the complication, as before.

Then for spin-1 particle, measurement of spin $(\hat{J} \cdot \hat{n})$ will yield

$$\left\{ \begin{array}{ll} J_z = +1 & \text{as } +1 \\ J_z = 0 & \text{as } 0 \\ J_z = -1 & \text{as } -1 \end{array} \right.$$

along an arbitrary axis.

Quantum correlation, in this way, would yield the correlation of less than 1, from which superclassical cases cannot be obtained: i.e.,

$$X = \langle \Psi | (\hat{J} \cdot \hat{n}_1) \otimes (\hat{\sigma} \cdot \hat{n}_2) \otimes (\hat{\sigma} \cdot \hat{n}_3) | \Psi \rangle < 1. \quad (6.8)$$

This is because some of the cross-terms (or, off-diagonal matrix elements) will have no contributions here:

$$\langle + | (\hat{J} \cdot \hat{n}) | - \rangle = \langle - | (\hat{J} \cdot \hat{n}) | + \rangle = 0. \quad (6.9)$$

We can fix the situation by introducing a new counting method

$$\left\{ \begin{array}{ll} J_z = +1 \text{ or } -1 & \text{as } +1 \\ J_z = 0 & \text{as } -1 \end{array} \right.$$

Here, no physics has changed; we simply project one set of value onto another. The proper measurement in this case can be described² by $\{2(\hat{J} \cdot \hat{n})^2 - 1\}$, instead of $(\hat{J} \cdot \hat{n})$.

Hence we can have the correlation of joint spin measurement of the particles 1,2,3:

$$X = \langle \Psi | W | \Psi \rangle, \quad (6.10)$$

where

$$W = \{2(\hat{J} \cdot \hat{n}_1)^2 - 1\} \otimes (\hat{\sigma} \cdot \hat{n}_2) \otimes (\hat{\sigma} \cdot \hat{n}_3), \quad (6.11)$$

and

$$(\hat{J} \cdot \hat{n}) = \begin{pmatrix} \cos \theta & \frac{1}{\sqrt{2}} \sin \theta e^{-i\phi} & 0 \\ \frac{1}{\sqrt{2}} \sin \theta e^{i\phi} & 0 & \frac{1}{\sqrt{2}} \sin \theta e^{-i\phi} \\ 0 & \frac{1}{\sqrt{2}} \sin \theta e^{i\phi} & -\cos \theta \end{pmatrix}. \quad (6.12)$$

Since

$$\langle 1 | (\hat{J} \cdot \hat{n})^2 | 1 \rangle = \langle -1 | (\hat{J} \cdot \hat{n})^2 | -1 \rangle = \cos^2 \theta + \frac{1}{2} \sin^2 \theta, \quad (6.13a)$$

$$\langle 1 | (\hat{J} \cdot \hat{n})^2 | -1 \rangle = \frac{1}{2} \sin^2 \theta e^{-2i\phi}, \quad (6.13b)$$

and

$$\langle -1 | (\hat{J} \cdot \hat{n})^2 | 1 \rangle = \frac{1}{2} \sin^2 \theta e^{+2i\phi}. \quad (6.13c)$$

We have

$$X = \frac{1}{2} (\langle 1, -, - | \langle -1, +, + | [\{2(\hat{J} \cdot \hat{n}_1)^2 - 1\} \otimes (\hat{\sigma} \cdot \hat{n}_2) \otimes (\hat{\sigma} \cdot \hat{n}_3)] \\ (|1, -, - \rangle | -1, +, + \rangle)$$

$$= \cos^2 \theta_1 \cos \theta_2 \cos \theta_3 - \sin^2 \theta_1 \sin \theta_2 \sin \theta_3 \cos(2\phi_1 - \phi_2 - \phi_3) \quad (6.14)$$

For $\theta_1=\theta_2=\theta_3=\pi/2$,

$$X = \langle \Psi | W | \Psi \rangle = -\cos(2\phi_1 - \phi_2 - \phi_3) \quad . \quad (6.15)$$

Once we have this superclassical expectation, the proof goes through the same way as in the previous case of three spin-1/2 particles: except for the following trivial replacements

$$\phi_1 \rightarrow 2\phi_1 \quad \text{and} \quad \phi_2, \phi_3 \rightarrow -\phi_2, -\phi_3.$$

6.2 Theorem With Photon Polarization Correlation

Original four particle spin-1/2 correlation can be extended to the case of photon polarization correlation.

But, unlike the extension of Bell's inequality into photon polarization case, we need two things to be adjusted. The first thing is that, to achieve superclassical case, we need to use new counting system, analogous to spin case, that counts orthogonal polarizations as +1 and 1, instead of conventional counting of +1 and 0 which is represented by Q.

$$Q = \begin{pmatrix} \cos^2\theta & \cos\theta \sin\theta \\ \cos\theta \sin\theta & \sin^2\theta \end{pmatrix} \quad (6.16)$$

This is because of the fact that if we involve 0 countings (or, non-countings) in our correlation, then we will not be able to achieve $X = +1$ or -1 .

Second thing to notice is that we will need one more spatial angle degree of freedom in adjustable measurement angles, as it has been shown in Appendix A, to achieve super-classical correlation with the same abundant freedom as we had in spin-1/2 case. This can be solved by inserting retarders in front of analyzers.

Now, going back to the first task, let's find out the operator T that has the property

$$T|0\rangle = +1 |0\rangle \quad \text{and} \quad T|0_{\perp}\rangle = -1 |0_{\perp}\rangle, \quad (6.17)$$

unlike usual operator Q, which yields

$$Q|0\rangle = +1 |0\rangle \quad \text{and} \quad Q|0_{\perp}\rangle = 0 |0_{\perp}\rangle. \quad (6.18)$$

Where $|0\rangle = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$ is a normalized polarization state along

arbitrary direction θ in the representation in which we take x-polarization as

$|+\rangle$ or $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and y-polarization as $|-\rangle$ or $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, and $|0_{\perp}\rangle = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$

is a polarization state which is orthogonal to $|0\rangle$.

By requiring T be hermitian, we have

$$T = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \quad (6.19)$$

which is unitary, as it must be, just as $(\hat{\sigma} \cdot \hat{n})$ is, since T does not involve non-countings unlike Q.

What, then, is T in terms of practical measurement apparatus? We can see the practical meaning of T immediately from linear analyzer Q as

$$T = 2Q - I \quad (6.20)$$

where I is an identity matrix.

So, we may regard this operation as either

(i) measurement with linear analyzer with their measurement

values mapped into new values, i.e.,

$+1 \rightarrow +1$ and $0 \rightarrow -1$.

or,

(ii) measurement with a birefringence analyzer which separates light beam into two orthogonal linear polarization and counts them as $+1$ and -1 .

Now, let's consider one of the simplest kind of four photon polarization entangled state

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|+\rangle + - \rightarrow - |-\rangle - + \rightarrow) \quad (6.21)$$

where $|+\rangle$ represents x-polarization and $|-\rangle$ for y-polarization.

If we try to achieve super-classical case on this entangled state, we find that neither

$$X_{old} = \langle \Psi | Q_1 \otimes Q_2 \otimes Q_3 \otimes Q_4 | \Psi \rangle \quad (6.22)$$

nor

$$X_{new} = \langle \Psi | T \otimes T \otimes T \otimes T | \Psi \rangle \quad (6.23)$$

can do the job.

This is because polarization of photon provides only one adjustable measurement angle, namely θ , due to its transversality, while in spin measurement we had two adjustable measurement angles, namely θ and ϕ , one of which has been used for adjusting to the proper angle for maximum cross-term effect while the other one has been used for providing the freedom in super-classical correlations.

We can solve this by placing retarders in front of each analyzer. If we insert retarders with their fast axis $\rho=0$ and retardance δ , then retarder can be represented by a matrix³

$$R = \begin{pmatrix} e^{-i\delta/2} & 0 \\ 0 & e^{+i\delta/2} \end{pmatrix} \quad (6.24)$$

then we have

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \{ e^{-i(\delta_1+\delta_2-\delta_3-\delta_4)/2} |++--\rangle - e^{i(\delta_1+\delta_2-\delta_3-\delta_4)/2} |--++\rangle \}.$$

$$(6.25)$$

The expectation value of the product of four polarization measurements is given by

$$\begin{aligned}
 X &= \langle \Psi | T \otimes T \otimes T \otimes T | \Psi \rangle \\
 &= \cos 2\theta_1 \cos 2\theta_2 \cos 2\theta_3 \cos 2\theta_4 \\
 &\quad - \frac{1}{2} (e^{i\lambda} + e^{-i\lambda}) \sin 2\theta_1 \sin 2\theta_2 \sin 2\theta_3 \sin 2\theta_4
 \end{aligned}
 \tag{6.26}$$

where $\lambda = \delta_1 + \delta_2 - \delta_3 - \delta_4$.

For $\theta = \pi/4$,

$$X = -\cos(\delta_1 + \delta_2 - \delta_3 - \delta_4) \tag{6.27}$$

This completes the super-classical correlations.

Identical proof will go through this case as it did for the original theorem. It should be noted that the same argument should apply to the system that involves any number of photons (for $n > 2$).

We also like to mention that the method described in this section is not the only way to achieve super-classical correlations. We will present, in Sec. 8.2.3.3 three photon polarization correlation GHZ experiment in a different way.

6.3 Theorem With Position-Momentum Correlation By Greenberger, Horne, Shimony, and Zeilinger⁴

Bell's theorem or GHZ theorem does not necessarily run on spin or photon polarization. These theorems don't hinge on intrinsic properties of those.

The gedanken model by Greenberger, Horne, Shimony, and Zeilinger uses position and momentum as key variables. The model is based on an extension to three particles of a two particle interferometer considered by Horne and Zeilinger⁵ which gave a two photon entangled state.

They consider a particle that decays into three particles of equal mass or into three photons of the same energy. Then, by momentum conservation, they will be emitted 120° apart. The particles pass through two sets of apertures which surrounds the central source in such a way that either they emerge through a set of aperture a, b, c which are separated by 120°, or through a', b', c' which are also 120° apart, but rotated slightly from the set of directions for a, b, c.

Thus the state of the three particle system beyond the apertures can be described by

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|a\rangle_1 |b\rangle_2 |c\rangle_3 + |a'\rangle_1 |b'\rangle_2 |c'\rangle_3) \quad (6.28)$$

where $|a\rangle_1$ denotes the particle 1 through aperture a, etc. The equal energy requirement can be enforced by placing energy filters at the detectors.

Beyond the apertures, beams through a and a' are totally reflected so as to overlap at a 50-50 beam splitter⁶, and after beam a' is allowed to pass through a

phase shifter ϕ_1 , the two outgoing beams are monitored by detectors,.etc.

Then we have the expectation value for the product of the outcome as:

$$E(\phi_1, \phi_2, \phi_3) = \sin(\phi_1 + \phi_2 + \phi_3). \quad (6.29)$$

We again have super-classical case here. Thus the arguments apply unchanged. We will discuss this model further in Sec.8.2.3.1.

6.4 Works By Mermin et al.

Mermin⁷ gave a simple version of GHZ theorem using Clifton et al.'s⁸ argument. The theorem is identical with our proof for three spin-1/2 particles system given in Sec. 6.1.1 except for the fact that Mermin used special set of angles (the angles that are mutually orthogonal to each other) to show the incompatibility of local realism with quantum mechanics, while our model works for general angles.

However, the way it was presented is simple and somehow illuminating, so we will describe here. Then, later, we will show that it is identical with the special case of our version.

We consider a set of measurement of spin of three spin-1/2 particles system represented by

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|+++ \rangle - |-- \rangle) \quad (6.30)$$

where $|+\rangle$, $|-\rangle$ denote spin "up" or "down" states in z-direction.

The measurements are represented by a set of commuting hermitian operators A, B, C.

$$A = \sigma_x^1 \sigma_y^2 \sigma_y^3 \quad (6.31a)$$

$$B = \sigma_y^1 \sigma_x^2 \sigma_y^3 \quad (6.31b)$$

$$C = \sigma_y^1 \sigma_y^2 \sigma_x^3 \quad (6.31c)$$

where A denotes joint spin measurements of particle 1 in x-direction and particles 2,3 in y-direction, etc. A, B, C all have eigenvalue of +1 when they are acted on $|\Psi\rangle$.

Therefore we have *quantum mechanical* expectation values

$$\langle A \rangle = \langle B \rangle = \langle C \rangle = +1. \quad (6.32)$$

Now, if the local realism were to have the same results, each of x, and y components of the spin of the particles 1, 2, 3 must possess a definite value (or, elements of reality) of either +1 or -1 (a fixed value). That is because, for example, in case of measurement A, we can predict with certainty and without disturbing it anyway

the spin value of one particle by looking at the other two spin values, etc.

So let's denote those six elements of reality by

$$m^1_x, m^1_y, m^2_x, m^2_y, m^3_x, m^3_y$$

to each of which the value +1 or -1 should be assigned.

Then measurement A is a result of our measurement on reality $m^1_x m^2_y m^3_y$, and B on $m^1_y m^2_x m^3_y$, and C on $m^1_y m^2_y m^3_x$.

Therefore,

$$m^1_x m^2_y m^3_y = +1, \quad (6.33a)$$

$$m^1_y m^2_x m^3_y = +1, \quad (6.33b)$$

and

$$m^1_y m^2_y m^3_x = +1. \quad (6.33c)$$

Now, what if we decide to measure the product of elements of reality $m^1_x m^2_x m^3_x$? The product must be +1 from Eqs. (6.33a-c) since for any m^i_α , where $i=1,2,3$ and $\alpha=x,y$,

$$(m^i_\alpha)^2 = +1. \quad (6.34)$$

On the other hand, quantum mechanically, this measurement corresponds to

$$D = \sigma^1_x \sigma^2_x \sigma^3_x = -A B C \quad (6.35)$$

with eigenvalue -1, or

$$D|\Psi\rangle = -1 |\Psi\rangle \quad (6.36)$$

Therefore, the hypothesis that elements of reality exist can be refuted by a single measurement of the three x-components of the system. The local realism requires the product of the three outcomes invariably to be +1, but quantum mechanics says the product of the three outcomes is invariably -1.

Now, we may see that Mermin's version is identical to our version of three spin-1/2 particles system.

Our result for correlation of spin on the system described by Eq.(6.30) was given by Eq.(6.5)

$$X = -\cos(\phi_1 + \phi_2 + \phi_3). \quad (6.37)$$

Since the only restriction for choosing a set of ϕ_i s is to meet the super-classical condition:

$$\phi_1 + \phi_2 + \phi_3 = 0, \pi \quad (\pm 2\pi), \quad (6.38)$$

instead of choosing four sets of angles as in (6.6a-d), we can choose the other four sets of angles $\{\phi_1, \phi_2, \phi_3\}$, namely,

$$\{0, \pi/2, \pi/2\} \quad \text{for } A \quad (6.39a)$$

$$\{\pi/2, 0, \pi/2\} \quad \text{for } B \quad (6.39b)$$

$$\{\pi/2, \pi/2, 0\} \quad \text{for } C \quad (6.39c)$$

to yield $\phi_1 + \phi_2 + \phi_3 = \pi$ or $X=+1$, and

$$\{0, 0, 0\} \quad \text{for } D \quad (6.39d)$$

to yield $\phi_1 + \phi_2 + \phi_3 = 0$ or $X=-1$.

Since ϕ_i is an angle from the x-axis in the x-y plane, $\phi=0$ measurement represents σ_x , and $\phi=\pi/2$ for σ_y .

Therefore we see this special choice of A, B, C and D is identical with Mermin's choice of

$$A = \sigma_x^1 \sigma_y^2 \sigma_y^3, \quad B = \sigma_y^1 \sigma_x^2 \sigma_y^3, \quad C = \sigma_y^1 \sigma_y^2 \sigma_x^3$$

and

$$D = \sigma_x^1 \sigma_x^2 \sigma_x^3 \quad (6.40)$$

And as is the case in our model, measurement D in Mermin's case - which yields different eigenvalues from A, B, C - also provides the contradictory behaviour in local realistic models.

REFERENCES FOR CHAPTER VI

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2. For spin 1, If we want to count $J_z=+1$ as +1, $J_z=0$ as 0, $J_z=-1$ as -1, then we need $(\hat{J} \cdot \hat{n})$ along an arbitrary axis. If we want to count $J_z=+1$ and $J_z=-1$ as +1, and $J_z=0$ as 0, then we need J^2 along z-axis therefore $(\hat{J} \cdot \hat{n})^2$ along an arbitrary axis. If we want to count $J_z=+1$ and $J_z=-1$ as +1 and $J_z=0$ as -1, then we need $\{2J_z^2-1\}$ along z-axis therefore $\{2(\hat{J} \cdot \hat{n})^2-1\}$ along an arbitrary axis.
3. We follow Jones matrix representation of polarization. See, A.Yariv and P.Yeh, Optical Waves in Crystals, chapter 5, John Wiley and Sons (1984)
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8. R.K.Clifton, M.L.G.Redhead, and J.M.Butterfield, Generalization of The Greenberger-Horne-Zeilinger Algebraic Proof of Nonlocality, submitted to Found.Phys. (1990)

CHAPTER VII

GHZ THEOREM FOR IMPERFECT CORRELATIONS AND INEFFICIENT COUNTERS

7.1 GHZ Theorem For Imperfect Correlation

One of the essential reasons for the failure of local deterministic hidden variable theory in reproducing quantum results in GHZ theorem is that it cannot meet the degree of freedom that is provided by the quantum theory. Thus, one may expect that the hidden variable models might fail again even in some non-superclassical cases where expectation values $E_{\max} = +1$ or -1 , provided that the quantum model still have the same degree of freedom given by the original GHZ theorem. We found that some group of the hidden variable models (in fact, most of the hidden variable models suggested so far) fail again if $E_{\max} > 1/2$.

Original GHZ theorem employed a system of four spin- $1/2$ particles that are disintegrated from a particle with $J=1$, $m=0$ and provided a final state wavefunction

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|+ + - \rightarrow - | - - + + \rangle) \quad (7.1)$$

with the expectation value for $\theta_1 = \pi/2$

$$E = -\cos(\phi_1 + \phi_2 - \phi_3 - \phi_4). \quad (7.2)$$

With this, GHZ were able to reach a contradiction for the hidden variable theory when the theory tried to immitate the quantum mechanical results.

Now we may think of more general case where the correlations between particles are not perfect, for example,

$$E = -E_{\max} \cos(\phi_1 + \phi_2 + \phi_3 + \phi_4) \quad (7.3)$$

where $E_{\max}$ is constant and $0 < E_{\max} < 1$. This means we have the expectation values of the spin product as

$$E(\phi_1, \phi_2, \phi_3, \phi_4) = -E_{\max} \quad \text{for } \phi_1 + \phi_2 = \phi_3 + \phi_4 \pmod{+2\pi}, \quad (7.4)$$

$$E(\phi_1, \phi_2, \phi_3, \phi_4) = +E_{\max} \quad \text{for } \phi_1 + \phi_2 = \phi_3 + \phi_4 + \pi \pmod{+2\pi}. \quad (7.5)$$

We needed, in the ideal case where $E_{\max}$ takes up the values +1 or -1, four different sets of angles $\{\phi_1, \phi_2, \phi_3, \phi_4\}$ to reach the unacceptable conclusion for local hidden theory that each spin measurement must yield the constant values regardless of their measurement angles. A little inspection into the logic in GHZ theorem¹ shows that only four different sets of angles would suffice in refutation of local realism regardless of any specific model. (For example², we only need the following four sets of angles $\{0, 0, 0\}$, $\{\phi, -\phi, 0\}$, $\{\phi, 0, -\phi\}$, and $\{2\phi, -\phi, -\phi\}$ for the model which yield quantum mechanical expectation value $E = -\cos(\phi_1 + \phi_2 + \phi_3)$ to refute local realism.)

Essential difference between superclassical case and non-superclassical case is that we have to average over many trials for the latter since we are to have two different possible outcomes.

Let's take a specific case of three particle correlation which yields quantum expectation value of the outcome of the joint measurements of spin value of the system as

$$E_{Q.M.} = +E_{\max} \cos(\phi_1 + \phi_2 + \phi_3). \quad (7.6)$$

For this type of non-superclassical case, the hidden variable counterpart must be summed (or integrated) over the different values of hidden variable to yield the average

$$E_{H.v.}(\phi_1, \phi_2, \phi_3) = \int \rho(\lambda) A_{\lambda}(\phi_1) B_{\lambda}(\phi_2) C_{\lambda}(\phi_3) d\lambda. \quad (7.7)$$

Suppose we take the measurements n times. Since the product of the measurements of spin value ABC takes up only $+1$ or -1 , we may equally write the average value as

$$E_{H.v.} = \frac{1}{n} \{ a \times (ABC)^+ + b \times (ABC)^- \} \quad (7.8)$$

where a denotes number of times we had $+1$ results and b for -1 results, $(ABC)^+$ represents the products ABC that resulted in $+1$ and $(ABC)^-$ for the products that resulted in -1 .

We may further symplify Eq.(7.8) by using proportionality constants $\alpha = a/n$ and $\beta = b/n$ to have

$$E_{H.v.} = \alpha(ABC)^+ + \beta(ABC)^- \quad (7.9)$$

and

$$\alpha + \beta = 1. \quad (7.10)$$

In order for the hidden variable theory to yield quantum mechanical results given by Eq.(7.6), Eq.(7.7) must satisfy

$$\begin{aligned} E(0,0,0) &= E(\phi, -\phi, 0) = E(\phi, 0, -\phi) = E(2\phi, -\phi, -\phi) \\ &= E_{\max}. \end{aligned} \quad (7.11)$$

Therefore for any set of angles above, from Eq.(7.9), we have

$$E_{H.v.} = \alpha(+1) + \beta(-1) = \alpha - \beta = E_{\max}. \quad (7.12)$$

From Eqs.(7.10) and (7.12), we get

$$\alpha = \frac{1}{2} (1+E_{\max}) \quad \text{and} \quad \beta = \frac{1}{2} (1-E_{\max}). \quad (7.13)$$

Therefore for a given the value $E_{\max}$, α and β are uniquely determined. So we may write down the hidden variable expectation values for the four different sets of angles measurements as

$$E(0,0,0) = \alpha(ABC)^{+\lambda_1} + \beta(ABC)^{-\lambda_1} \quad (7.14a)$$

$$E(\phi, -\phi, 0) = \alpha(ABC)^{+\lambda_2} + \beta(ABC)^{-\lambda_2} \quad (7.14b)$$

$$E(\phi, 0, -\phi) = \alpha(ABC)^{+\lambda_3} + \beta(ABC)^{-\lambda_3} \quad (7.14c)$$

$$E(2\phi, -\phi, -\phi) = \alpha(ABC)^{+\lambda_4} + \beta(ABC)^{-\lambda_4} \quad (7.14d)$$

where λ_i , λ_i' each denotes the range of λ that produces +1 and -1 respectively for the given set of angles. It should be noted that each set of $\{\lambda, \lambda'\}$ may takes up different values for the different sets of angles, but should span the total λ space since total distribution of λ does not depend on specific settings of measurement angles.

Now if we restrict ourselves to the hidden variable model in which the hidden variable density is constant i.e.,

$$\rho(\lambda) = \text{constant}, \quad (7.15)$$

as is true for the most of the hidden variable models suggested to date. Then, depending on the given value $E_{\max}$, we may have a certain common range of λ_c which yields +1 for all four sets of angles: i.e.,

$$\begin{aligned} [A(0)B(0)C(0)]_{\lambda_c} &= [A(\phi)B(-\phi)C(0)]_{\lambda_c} = [A(\phi)B(0)C(-\phi)]_{\lambda_c} \\ &= [A(2\phi)B(-\phi)C(-\phi)]_{\lambda_c} = +1. \end{aligned} \quad (7.16)$$

Fig. 7.1a to 7.1d shows that there exists common range of hidden variable λ_c that yields the same value +1

for all four different sets of measurement angles if α is greater than $3/4$ or equivalently, by Eq.(7.13), $E_{\max} > 1/2$.

Particular attention must be given to Fig. 7.1d where existence of λ_c is unavoidable. The range of λ_c as a portion of the whole range of λ can be given as

$$(\int d\lambda_c)/(\int d\lambda) = 1 - 4\beta = -3 + 4\alpha \quad (7.17)$$

for $3/4 < \alpha < +1$,
or equivalently as

$$(\int d\lambda_c)/(\int d\lambda) = -1 + 2E_{\max} \quad (7.18)$$

for $1/2 < E_{\max} < +1$.

Here it should be noted that range of λ occupied by same value, either $+1$ or -1 , doesn't necessarily be continuous. We see here, in the case of $0 < E_{\max} < 1/2$, hidden variable has a room to avoid the range λ_c for all four set of angles but there's no room to avoid the same thing in the case of $E_{\max} > 1/2$. Therefore, in these situations the theory faces the same logical failure as it did in superclassical cases.

From Fig. 7.1a we see that hidden variables have room to arrange the regions for $+1$ for each set of measurement angles to avoid the overlapping of all four $+1$ regions. This is possible until α increases up to $3/4$, for which Fig. 7.1b is given. And Fig. 7.1c shows critical value up to

which the construction of local hidden variable model may be possible.

The last one shows the fact that there exists a region in which the overlapping of all four +1 region occurs and thereby the failure of local hidden variable model is guaranteed.

Fig. 7.1 Hidden Variable Model for an Imperfectly Correlated System

It develops contradictory behaviour for $E_{\max} > 1/2$.

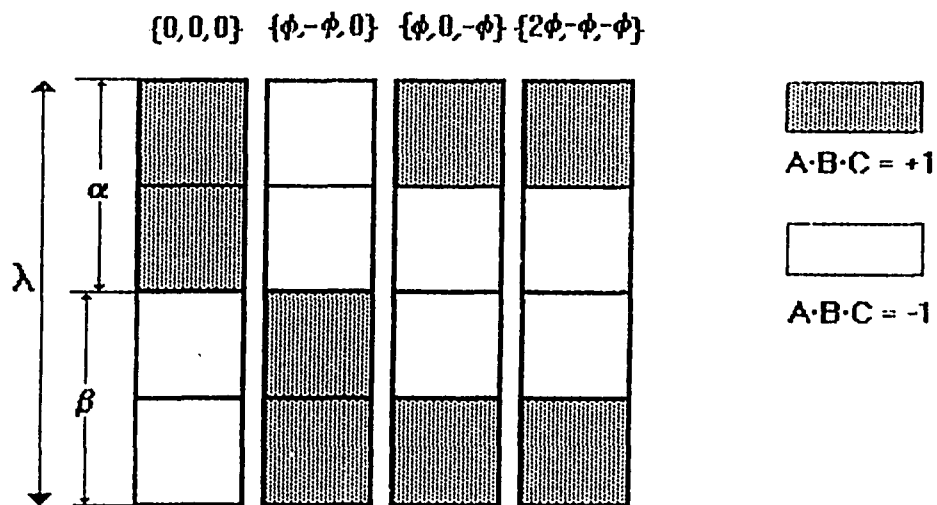


FIG. 7.1a $E_{\max} = 0 : \alpha = \beta = \frac{1}{2}$

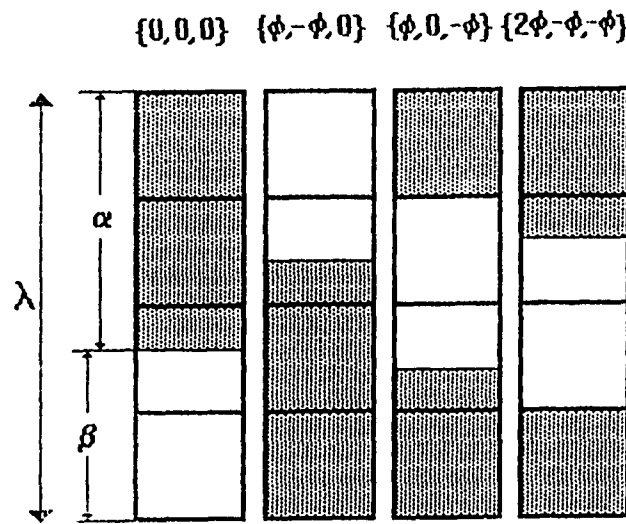


FIG. 7.1b $0 < E_{\max} < \frac{1}{2}$

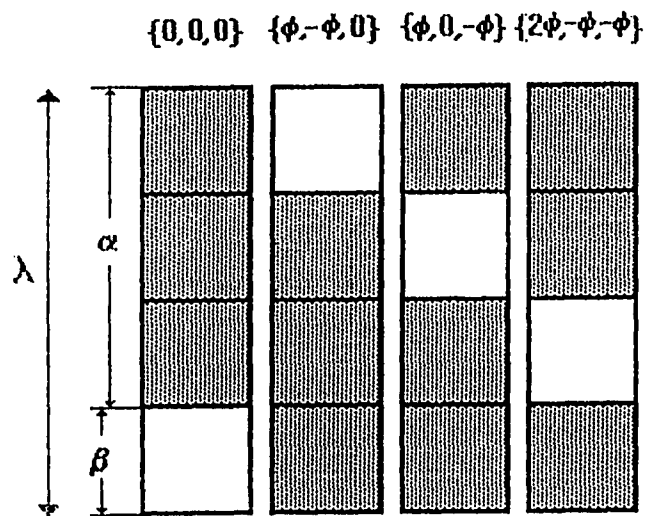


FIG. 7.1c $E_{\max} = +\frac{1}{2}$

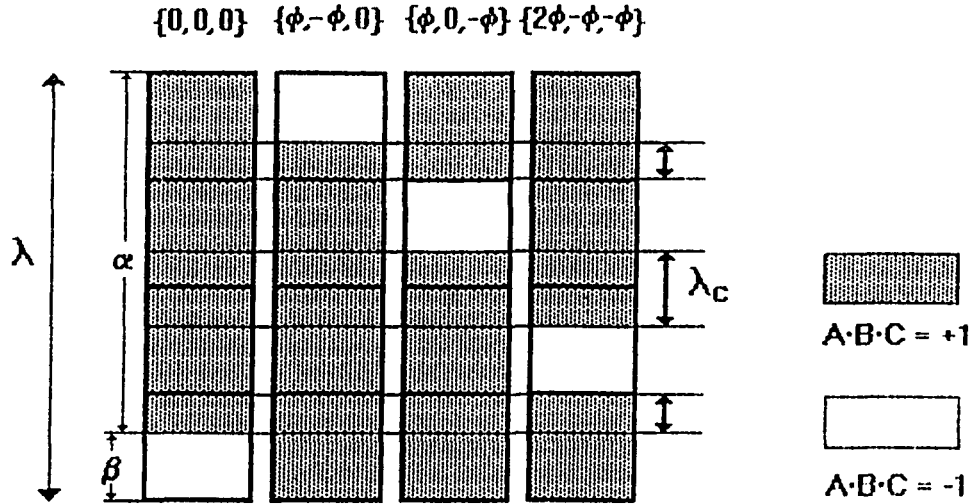


FIG.7.1d $E_{\max} > \frac{1}{2}$

Here, $E_{\max}$ is given in increasing order. From the last one we notice that as $E_{\max}$ approaches to +1, overlapping area λ_c starts to cover the whole range of λ .

It should be noted that the only change in the case of $E = -E_{\max}$, from the case of $E = +E_{\max}$ is that the area for +1 switches to -1, and vice versa.

We may summarize our result as following: GHZ contradictions exist also in some non-superclassical cases where quantum expectation value takes up the form $E = +E_{\max} \cos(\phi_1 + \phi_2 + \phi_3)$, where $E_{\max} > 1/2$, if the hidden variable model has constant $\rho(\lambda)$. The contradiction occurs because of the fact that there exists some range of hidden variable λ_c that provide four equations that we need to reach the

contradiction. We only need a small range that exhibits contradiction to show the theory is inconsistent.

7.2 Inefficient Counters

We also may see the relation between this argument for the non-superclassical cases and inefficiency problem in the superclassical cases. The GHZ theorem implicitly assumed the detectors are perfect as was the case of the original Bell's theorem. Therefore if we let the efficiency of the whole system as η , then we have quantum mechanical expectation value as

$$E = +\eta \cos(\phi_1 + \phi_2 + \phi_3). \quad (7.19)$$

The expectation values given by inefficient counter takes the same form as non-superclassical case, except for the fact that this time hidden variable could result in 0 as well as +1 and -1, given the set of measurement angles. Therefore the same argument goes through against local hidden variable theories, though the situation is little more complicated here.

Let's make a reasonable assumption as follows. This is usually called "fair sampling" assumption³. *"The triples actually detected by any three detectors with any specified angles can be regarded as a fair sample of the set of triples that would have been detected if the detectors were perfect."* Then we may use the following argument⁴ to

have a useful inequality that can be used in any real experimental situation.

As we have seen in Eqs. (6.6a-d) we needed four sets of angles for hidden variable models to be led to a logical contradiction. The fact implies that at least one of the four equations is false. We can express this implication in the language of set theory:

$$(\bar{\Lambda}_1 \cup \bar{\Lambda}_2 \cup \bar{\Lambda}_3) \supseteq \Lambda_4 \quad (7.20)$$

where Λ_i denotes the set of all λ 's that i th equation ($i = 1, 2, 3$, and 4) hold, and $\bar{\Lambda}_i$ represents the set of all λ 's that does not belong to Λ_i .

From Eq.(7.20), the probability measure ρ on the space of complete states satisfies

$$\rho(\Lambda_4) \leq \rho(\bar{\Lambda}_1 \cup \bar{\Lambda}_2 \cup \bar{\Lambda}_3) \quad (7.21)$$

And by standard probability theory

$$\rho(\bar{\Lambda}_1 \cup \bar{\Lambda}_2 \cup \bar{\Lambda}_3) \leq \rho(\bar{\Lambda}_1) + \rho(\bar{\Lambda}_2) + \rho(\bar{\Lambda}_3) \quad , \quad (7.22)$$

so that

$$\rho(\Lambda_4) \leq \rho(\bar{\Lambda}_1) + \rho(\bar{\Lambda}_2) + \rho(\bar{\Lambda}_3) \quad . \quad (7.23)$$

Since

$$\rho(\bar{\Lambda}_1) = 1 - \rho(\Lambda_1), \quad (7.24)$$

we have

$$\rho(\Lambda_1) + \rho(\Lambda_2) + \rho(\Lambda_3) + \rho(\Lambda_4) \leq 3. \quad (7.25)$$

Given the fair sampling assumption we discussed earlier, in a good experimental condition, each term of the RHS must yield very close to unity, thereby the inequality can be violated.

It also can be shown that the fair sampling assumption would be dispensible if the detector efficiency is high enough. In Bell's inequality, it has been shown⁵ that individual detector efficiency must be higher than 82.8%.

The similar conclusion can be drawn here also. Let's say overall detector efficiency is α and we allow the hidden variable theory exploit this detector inefficiency. Then we would have

$$\alpha \leq \rho(\Lambda_1) \quad (7.26)$$

and

$$\rho(\bar{\Lambda}_1) \leq 1 - \alpha. \quad (7.27)$$

Therefore,

$$\alpha \leq \rho(\Lambda_4) \leq \rho(\bar{\Lambda}_1) + \rho(\bar{\Lambda}_2) + \rho(\bar{\Lambda}_3) \leq 3(1-\alpha). \quad (7.28)$$

This inequality is violated if $\alpha = \eta^3 > 0.75$ or individual efficiency $\eta > 0.908$. The demand on detector efficiency is

more stringent than in Bell's theorem if the fair sampling assumption is to be avoided. This is a very general argument at this stage of the theorem but we expect to have stronger restrictions for the hidden variable theories are possible at the experimental level.

Indeed, Mermin⁶ derived a simple Bell-type inequality for n-particle GHZ state (though it may not seem to be an realizable state) and showed that the correlation inequalities would be more strongly violated by the quantum theoretical predictions in GHZ experiments than in Bell's. The data predicted by the quantum theory exceed the classical bound by a factor that grows exponentially with n.

A clever hidden variable model was devised by GHZ⁷ in order to duplicate the quatum results for four particle GHZ model with inefficient counters. They assumed hidden variables are angles and constructed a model that works for each counter A,B,C,D = 0 for 50% of the time. It has been shown that those kind of classical models can be automatically ruled out by employing quantum detectors whose individual efficiency is more than 43.7%.

In general, GHZ-type experiments provide hidden variables a much smaller room than that of Bell-type experiments because there are more degrees of freedom to be met by local realism. So, more artificial models should be employed, rather than intuitive ones *for which local realism originally intended*, if there would be any hope to duplicate quantum results.

I don't regard it as reasonable that local realism is to be saved solely by a failure in detecting the whole ensemble of photon pairs by our current limitation of detectors, while experiments yield the exact results which quantum mechanics predicts.

REFERENCES FOR CHAPTER VII

1. Regardless of the degrees of freedom that are involved in the model, we only need four sets of angles to deduce the contradictory behavior of local realism. We needed equations (5.21-23) and (5.29) for the original four particle cases, eqs.(6.6a-6.6d) for three particle case in Sec.6.1.1, and eqs.(6.43a-6.43d) for Mermin and Redhead model in Sec.6.4. This is partly because the minimum number of freedom needed for proof is three and any additional degree of freedom that is provided by a model that involves more than three particles are redundant.
2. See Sec.6.1.1
3. D.M.Greenberger, M.Horne, A.Shimony, and A.Zeilinger, Bell's theorem without inequalities, *Am.J.Phys.* 58, 1131 (1990)
4. We follow the argument given in Ref.3
5. N.D.Mermin, The EPR experiment - thoughts about the 'Loophole', in D.M.Greenberger (ed.), *New techniques and Ideas in Quantum Measurement Theory*, New York Academy of Sciences, N.Y. (1986) p.422.
6. N.D.Mermin, Extreme Quantum Entanglement in a Superposition of Macroscopically Distinct States, *Phys.Rev.Lett.* 65, 1838 (1990)
7. D.M.Greenberger, M.Horne, and A.Zeilinger, unpublished (1988)

CHAPTER VIII

IMPLEMENTATION OF GHZ THEOREM

8.1 Introduction

If the four particle system given by GHZ in Ch.V or the three particle system given by Greenberger and Choi in Ch.VI should be found in the real world, the GHZ experiment could be done in the original context of a spin correlation system. But it would not be so easy to find an useful spin model for the experiments as we have learned from the experience in a simpler case of two particle system for the Bell's inequality. Hence we would like to use once again the similarity¹ between spin-1/2 system and the photon polarization system for the implementation of the GHZ theorem.

Though it was shown in Appdendix A that the photon polarization version of GHZ experiment is possible through the measurement of general elliptical polarizations, the possibility is not limited to polarization correlation experiments due to recently introduced other interferometry techniques.

8.2 Photon Version of GHZ Experiment

8.2.1 Cascade Photon Experiment

GHSZ suggested a good example of possible (though it is gedanken) photon cascade experiment to test the theorem by following the example of Bell-type photon experiments.

Suppose we have an atom undergoing electric dipole transitions that carry the atom from a state of zero total angular momentum through two intermediate states of angular momentum state one back to a state of zero total angular momentum in a short time. That is a $J=0 \rightarrow J=1 \rightarrow J=1 \rightarrow J=0$ three photon cascade. Suppose that the three detectors select three photons that propagate in a plane along directions at angles 120° from each other. Then by conservation of angular momentum the three photon state is

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|R\rangle_1 |R\rangle_2 |R\rangle_3 + |L\rangle_1 |L\rangle_2 |L\rangle_3), \quad (8.1)$$

which has the required entanglement to exhibit GHZ polarization correlations.

8.2.2 Positronium Annihilation Experiment

A singlet state ($^1S_0$ of $J=0$) of positronium's decay into two photons has been used to implement the Bell's theorem (Ref.19, 20, and 21 in Chapter III). Now we may use the triplet State ($^3S_1$ of $J=1$; ortho-Ps) of positronium that decays into three photons. Since a positronium, which is a bound state of an electron and a positron (e^+e^-) decays only

from the ground state $l=0$ (S-wave), we don't have any problem of selecting the right sets of triplets from the detectors that are 120° apart from each other and we may further impose a restriction to the possible hidden variables such as isotropicity.

Besides the fact that the event selection techniques are now improving², this triplet state has an advantage of much longer decaying time³ ($\tau[3\gamma] = 1.4 \times 10^{-7}$ sec) than the singlet state. This means that it also may be possible to do a generalized Horne-Shimony-Zeilinger⁴ (HSZ) momentum-position interference experiment which we will discuss in the next section. The ideas and the schemes would be same as the three photon parametric down conversion case, although we would have higher energy photon triplets in this positronium decay case.

8.2.3 n-Photon Parametric Down Conversion

We studied a generalized theory of spontaneous parametric down conversion of light in Appendix B that gives us highly correlated n photons from a coherent light beam through the nonlinear optical process. The study shows that at least a low rate of conversion may be obtained.

One can show that a low rate of conversion may be enough to realize some of the interferometers that can be used for the fundamental study of the quantum mechanically entangled states.

We here show that this source of correlated photon triplet can be used to implement three particle GHZ theorem as photon pair from parametric down conversion has been used for the two particle Bell's theorem.

Here we suggest schemes for obtaining three different types of correlated photon triplets using well defined time and direction of emission of the correlated photons from optical down conversion.

8.2.3.1 Generalized Horne-Shimony-Zeilinger⁴ (HSZ) Interferometer

HSZ suggested to form a quantum mechanical entangled state by using two different possible paths for a set of correlated particles, and their idea was implemented by Rarity and Tapster⁵ by employing a two photon down conversion technique. Recently, Greenberger, Horne, Shimony, and Zeilinger⁶ proposed a Gedanken experiment that involves three correlated photons to test against a family of EPR type local realism. Their experimental setup may be regarded as a generalized version of HSZ interferometer, and is depicted in Fig.8.1. It is readily realizable through a three photon down conversion technique which we described in the earlier sections.

A set of degenerated three photons from SPDC source S will emerge either through the apertures 1,2, and 3 or through 1', 2', and 3'. The unprimed beams will goes through

the phase shifters Φ_i before they are combined with the primed beams on the lossless beam splitters BS_i .

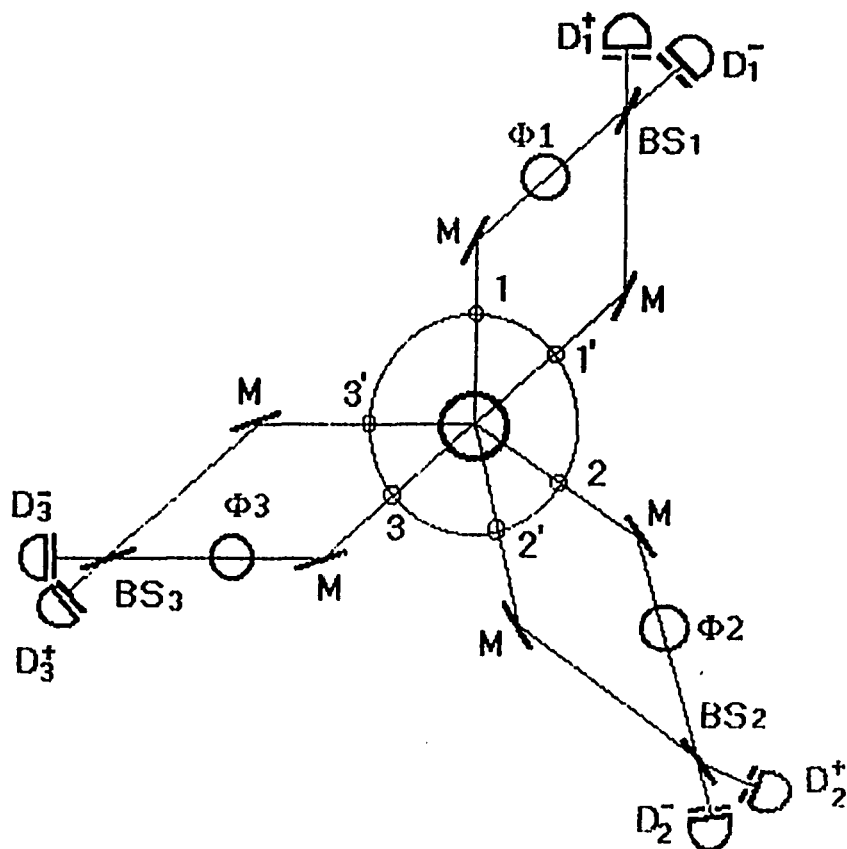


Fig.8.1:

A generalized version of the two particle Horne-Shimony-Zeilinger interferometry as was given by Greenberger et al. All it takes to be realized in a real experiment is to use a three-photon down converter as a source. Then this diagram for the setup may be regarded as a two dimensional view toward the center of the downconverter from the output side. The source emits a triple of particles 1,2, and 3. The beams

through the apertures 1 and 1' are recombined on a beamsplitter 1 before illuminating detectors D_1^+ and D_1^- . Likewise for the particles 2 and 3.

We suppose a perfect entangled state after the apertures, i.e.;

$$|\Psi\rangle_{\text{initial}} = 2^{-1/2} \{ |1,2,3\rangle + |1',2',3'\rangle \}. \quad (8.2)$$

Since the evolution of the wavefunctions for the each downconverted photon may be described by

$$|i\rangle \rightarrow 2^{-1/2} e^{i\Phi_i} (|-\rangle_i + |+\rangle_i) \quad (8.3)$$

and

$$|i'\rangle \rightarrow 2^{-1/2} (|+\rangle_i + |-\rangle_i), \quad (8.4)$$

$$(i = 1, 2, 3)$$

where $|+\rangle_i$ and $|-\rangle_i$ denote the states just before the detectors D_i^+ and D_i^- .

Then the final state of the whole system can be written as

$$\begin{aligned} |\Psi\rangle_{\text{final}} = \frac{1}{4} \{ & (1 - ie^{i\Delta\Phi}) |+++ \rangle + (i - e^{i\Delta\Phi}) |++- \rangle \\ & + (i - e^{i\Delta\Phi}) |+-+ \rangle + (-1 + ie^{i\Delta\Phi}) |+-- \rangle \\ & + (i - e^{i\Delta\Phi}) |-++ \rangle + (-1 + ie^{i\Delta\Phi}) |-+- \rangle \\ & + (-1 + ie^{i\Delta\Phi}) |--+ \rangle + (-i + e^{i\Delta\Phi}) |--- \rangle \} \end{aligned} \quad (8.5)$$

where $\Delta\Phi = \Phi_1 + \Phi_2 + \Phi_3$.

If we count a detection of a photon in D_1^+ as +1 and the detection in D_1^- as -1, then we would have a quantum three photon joint detection expectation value as

$$\begin{aligned} E(\Phi_1, \Phi_2, \Phi_3) &= \eta^3 \langle \Psi |_{\text{final}} V_1 \otimes V_2 \otimes V_3 | \Psi \rangle_{\text{final}} \\ &= \eta^3 \sin(\Phi_1 + \Phi_2 + \Phi_3). \end{aligned} \quad (8.6)$$

where η is an individual quantum detector efficiency, and

$V_i = \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix}$ represents our counting method in the

representation of $|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

Hence the jointing detection expectation value maximally oscillates depending upon $\Phi_1 + \Phi_2 + \Phi_3$.

8.2.3.2 Generalized Franson Interferometer

Franson⁷ ingeniously devised a gedanken interferometer that uses an interference between two possible states, each of which belongs to a different emission time. In the interferometer, two correlated particles fly apart from a common source which has a relatively long coherent time. Each of the two paths in which a particle travels consists of two branches, one of which is long and the other is short. By making the path difference between long and short paths longer than the coherence time of the particle (or, wavepacket of the particle), one can form an entangled state

between longer paths and shorter paths. The experiment was done by Mandel et al.⁸ and by Kwiat et al.⁹, both of who have made use of Michelson interferometer in place of Mach-Zender interferometer which was originally proposed by Franson.

Our generalized three-photon Franson interferometry depicted in Fig.8.2 follows Mandel et al.'s work with one more arm attached.

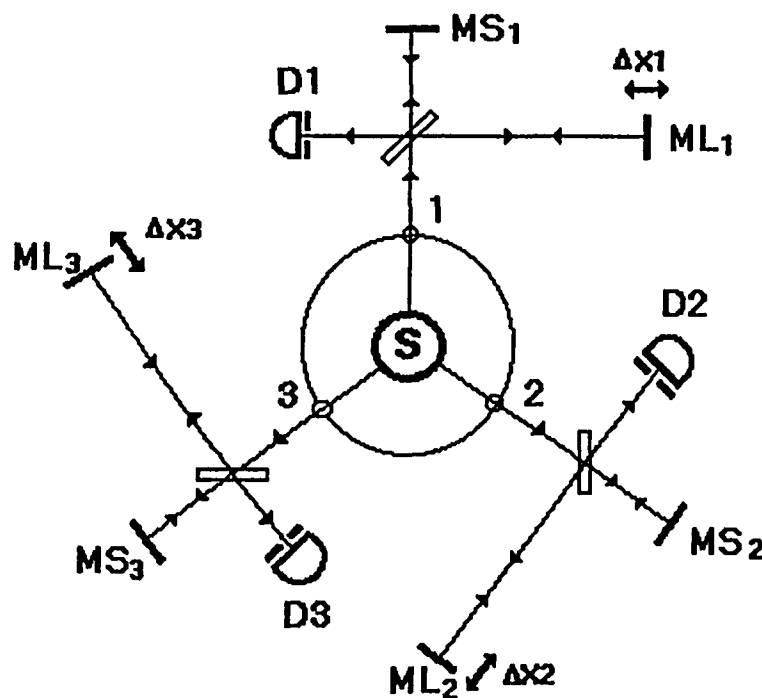


Fig.8.2:

A generalized Franson-type position-time interferometer. Here, only one set of aperture is used instead of two, therefore the beams never mix once they are parted from the

source S. Each particle will either take a shorter path (via the mirror MS_i) or take a longer path (via the mirror ML_i) to reach the coincidence detector D_i through the beam splitters.

Ignoring the polarization of the photons, the field operator for the down converted photon can be written as

$$E^{(+)}(r) = i \sum_k N(k) \hat{a}_k e^{ik \cdot r}, \quad (8.7)$$

and

$$E^{(-)}(r) = [E^{(+)}(r)]^\dagger \quad (8.8)$$

where $N(k)$ denotes the normalization coefficient.

In degenerated three photon down conversion in the parametric approximation, time development of the field operator within the nonlinear material may be given by

$$E^{(+)}(r, t) = e^{iHt} E^{(+)}(r) e^{-iHt} \quad (8.9)$$

with

$$H = \kappa (\hat{a}^{+3} + \hat{a}^3) \quad (8.10)$$

Once the down converted photons leave the nonlinear medium, they will simply propagate at the speed of light, so that

$$E^{(+)}(r + c\Delta t, t) = E^{(+)}(r, t - \Delta t) \quad (8.11)$$

for t greater than the interaction time.

We also notice from the fact that the down converted photons are created simultaneously that

$$E^{(+)}(r_i, t)E^{(+)}(r_j, t)E^{(+)}(r_k, t \pm \Delta t)|0\rangle = 0 \quad (8.12)$$

$$E^{(+)}(r_i, t)E^{(+)}(r_j, t \pm \Delta t)E^{(+)}(r_k, t \pm \Delta t)|0\rangle = 0 \quad (8.13)$$

where r_i , r_j , and r_k are the detector positions for the photon triplet 1, 2, and 3 in any order. The detector distances are the same from the source and Δt is greater than coherence time of the down converted photons. In our experimental setup, Δt represents path difference between a longer path and a shorter path.

The field at the detectors D_i at r_i can be written as

$$E^{(+)}(r_i, t) = \frac{1}{2} E_o^{(+)}(r_i, t) + \frac{1}{2} e^{i\Phi_1} E_o^{(+)}(r_i + c\Delta t, t) \quad (8.14)$$

where E_o represents the field at the same optical distance from the source as the detector r_i without beamsplitters or mirrors, and Φ_1 is the phase shift associated with the time difference $2\Delta x_1/c$ at the middle frequency ω_1 . The variable distances $2\Delta x_1$ in the longer arms are much shorter than the coherence time $1/(\Delta \omega_1)$ of the down converted photons.

The coincident rate R_c among three photon detectors will then be given by

$$\begin{aligned}
R_c &= \eta^3 \times \\
\langle 0 | E^{(-)}(r_1, t) E^{(-)}(r_2, t) E^{(-)}(r_3, t) E^{(+)}(r_3, t) E^{(+)}(r_2, t) E^{(+)}(r_1, t) | 0 \rangle \\
&= \frac{1}{32} R_0 \{ 1 + \cos(\Phi_1 + \Phi_2 + \Phi_3 + \omega_0 \Delta t) \} \\
&= \frac{1}{16} R_0 \cos^2\{(\Phi_1 + \Phi_2 + \Phi_3 + \omega_0 \Delta t)/2\} \quad (8.15)
\end{aligned}$$

where ω_0 denote the pump beam frequency and R_0 represents the coincidence rate among three detectors at the same optical distances from the source as the detectors r_1 without beamsplitters or mirrors. It exhibits a sinusoidal oscillation that depends upon the optical path differences: $\Delta x_1 + \Delta x_2 + \Delta x_3$.

One may notice that the maximum correlation in this case is much less than unity, but this is simply because of the fact that we dump the part of the beam from the each arm. We don't need to dump the beam if we don't want to, by using original Franson type Mach-Zender interferometer configuration and the proper counting system.

8.2.3.3 Polarization Experiment

Finally, we construct a third type of entangled state formed by two orthogonal polarizations of photons for a three particle polarization interferometer. Fig.8.3 shows our scheme for the experiment. Two channel linear polarization analyzers are used to yield an optimum value

for an interference effect. We suppose that the intensity of the beam is so weak that there would be only one correlated photon triple in the entire setup at any time.

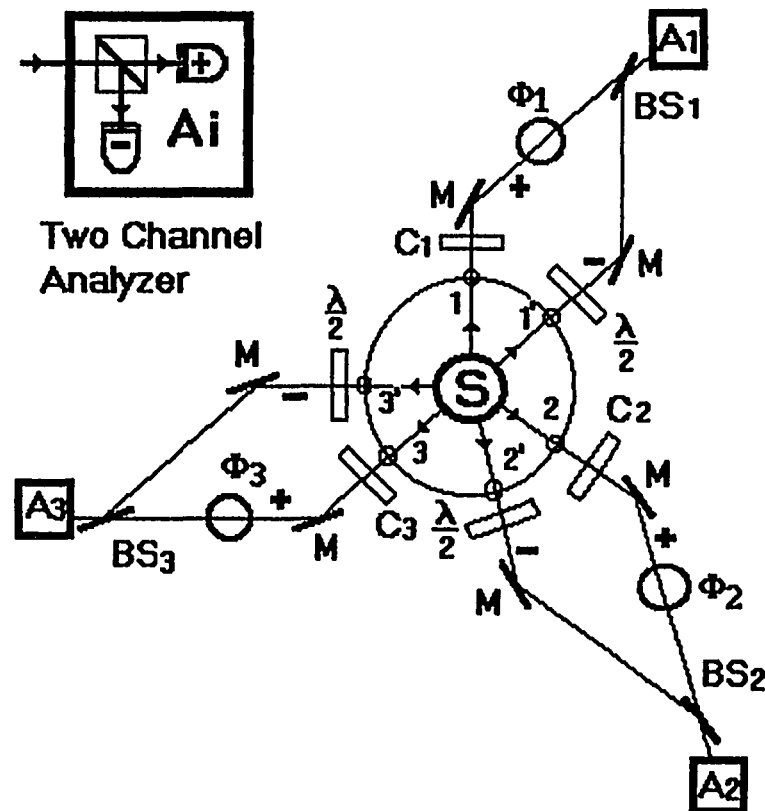


Fig.8.3: A setup for a three photon polarization correlation experiment. We assume (or enforce it by filters) that all the emerging beams are x-polarized. Later, one set of beam (primed ones) will be turned into y-polarized ones through the half wave retarders $\lambda/2$ while the other set (unprimed ones) goes through the compensators C_i ($i = 1, 2, 3$). The x-polarized set of beam will again goes through variable phase shifters Φ_i before they are combined on the

lossless beam splitters BS_i . The combined beam will be detected by two channel analyzers A_i s.

Suppose all three down converted photons are x-polarized. (We could in principle enforce this by placing x-filters after the apertures) In one set of paths (primed ones) we place half wave plates and in the other set of paths (unprimed ones) we place compensators C_i and the variable phase shifters Φ_i , so that the entire down converted photon system after the lossless beam splitters (here they work as beam combiners), may be described by a state vector:

$$|\Psi\rangle = 2^{-1/2} (|-_1-_-2-_-3\rangle + e^{i(\Phi_1+\Phi_2+\Phi_3)}|+_1+_2+_3\rangle), \quad (8.16)$$

where $|+_i\rangle$ and $|-_i\rangle$ denote the x and y-polarizations respectively.

The inverse use of the lossless beamsplitters (Glans-Thomson type) BS_i , in principle, do not impose any problem in completely combining beams, since we have mutually orthogonally polarized beams in each incoming arm, and furthermore, we have assumed that only one photon was present in a beamsplitter at any time.

If we count a detection of a x-polarized photon as +1 and a y-polarized photon as -1 using two channel analyzers, then we would have a three-photon joint detection probability as¹

$$\begin{aligned}
E(\Phi_1, \Phi_2, \Phi_3; \theta_1, \theta_2, \theta_3) &= \eta^3 \langle \Psi | T(\theta_1) \otimes T(\theta_2) \otimes T(\theta_3) | \Psi \rangle \\
&= \eta^3 \sin 2\theta_1 \sin 2\theta_2 \sin 2\theta_3 \cos(\Phi_1 + \Phi_2 + \Phi_3)
\end{aligned}
\tag{8.17}$$

where θ_1 denotes the angle of the axis of the analyzers, and T denotes a two channel polarization measurement operator which may be written as

$$T = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}, \tag{8.18}$$

in the representation in which x-polarization can be expressed as

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and y-polarization as } \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

If we set the axes of the analyzers at $\theta_1 = \pi/4$, then we finally have a sinusoidal polarization correlation among the three down converted photons:

$$E(\Phi_1, \Phi_2, \Phi_3) = \eta^3 \cos(\Phi_1 + \Phi_2 + \Phi_3). \tag{8.19}$$

We have shown that GHZ experiment may be done in several different ways using several different sources and/or methods though we have focused on parametric down conversion technique.

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CHAPTER IX

TWO PARTICLE GHZ THEOREM

9.1 Going Back To The Original EPR

Most recently, Bernstein, Greenberger, Horne, and Zeilinger¹ (BGHZ) extended the results of many particle Greenberger-Horne-Zeilinger (GHZ) theorem to the original two particle system proposed by Einstein, podolsky, and Rosen (EPR). This system uses spatial degree of freedom in the form of two particle interferometer and identifies them as the elements of reality, thereby imposes stronger restrictions on local realism than the Bohm-Bell-type² two particle spin system. As we shall show later, that the restriction given by the spatial degree of freedom is so powerful that even the local realistic model that takes advantage of the counter inefficiency and/or imperfect correlation should also be subjected to a contradiction. The theorem seems to provide a conclusive proof against the EPR type local realism after some fifty five years since the publication of EPR paper. In a sense, if this two particle GHZ theorem could be proved in a future experiment, as we expect that should be the case, then, we may consider this as the completion of "the project, initiated by Bell, of proving that the original program as envisioned by EPR is not viable."³

The essential new feature of the two particle GHZ theorem is that it makes use of multiple paths for the particle pair as elements of reality according to the Einstein's criteria.

The logic that BGHZ employed on a pair of spinless particles can be extended to a spin-1/2 experiment or a photon-polarization measurement experiment, provided that the systems involve multiple paths, or even to the system with only one set of opening at the source instead of two as we will discuss them later. The photon versions of BGHZ gedanken experiment may readily be implemented by a parametric down conversion of light technique.

9.2 Quantum Consideration Of Two Particle Interferometer

Let's consider the two particle gedanken interferometer depicted in Fig. 9.1 which is similar to the ones recently introduced by Horne and Zeilinger⁸, and Horne, Shimony, and Zeilinger⁹, and implemented by Rarity and Tapster⁴.

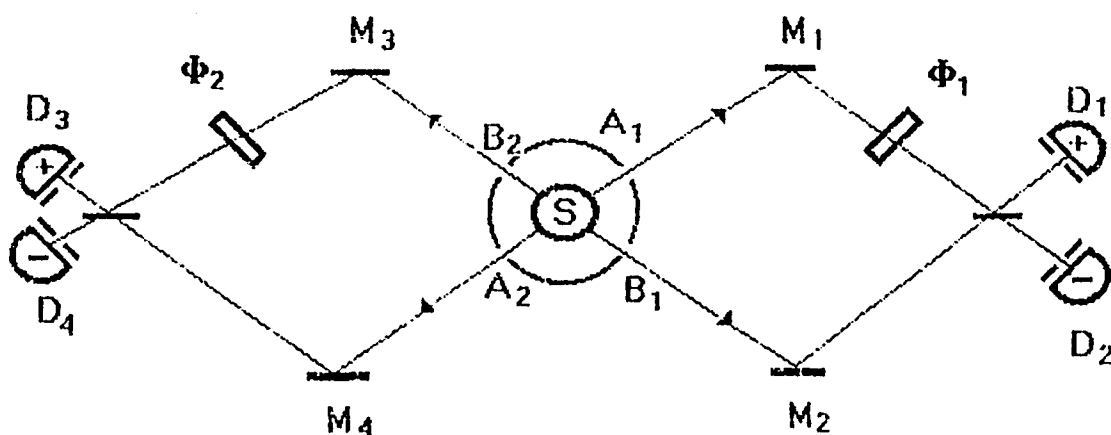


FIG.9.1 EXPERIMENTAL SET UP FOR TWO PARTICLE GHZ EXPERIMENT

The initial particle at S decays into two particles 1 and 2 which fly apart in opposite directions. A filter limits them to either of two sets of directions. Either particle 1 goes through the opening A_1 and particle 2 goes through the opening A_2 , or particle 1 goes through B_1 and particle 2 goes through B_2 . The two paths of the particle 1 are then made to interfere with each other by a set of mirrors M_1 and M_2 and by a 50-50 beam splitter at C_1 , and similarly for particle 2. There are two counters D_1 and D_2 , which can detect particle 1. Obviously, only one of the two counters will fire at a time. Counter D_1 will pick up the coherent combination of the reflected part of the component A_1 and the transmitted part of the component B_1 , while the opposite is true for counter D_2 . The same applies to the

particle 2 and the detectors D_3 and D_4 . Finally we add phase shifters F_1 and F_2 in the paths A_1 and B_2 , respectively.

Since the initial particle decays into two particles 1 and 2 and the filters limit them to either of two sets of directions, we may write down the wavefunction of the system as

$$|\Psi\rangle_0 = \frac{1}{\sqrt{2}} (|A_1\rangle|A_2\rangle + |B_1\rangle|B_2\rangle) \quad (9.1)$$

The two paths of each particle are made to interfere with each other by a set of mirrors and by 50-50 beam splitters.

There are two counters, which can detect each particle. Counter D_1 will pick up the coherent combination of the reflected part of the component A_1 and the transmitted part of the component B_1 , while the opposite is true for counter D_2 . The same applies to the particle 2 and the detectors D_3 and D_4 .

And we add phase shifters Φ_1 and Φ_2 in the paths A_1 and B_2 , respectively as in Fig.9.1.

After the phase shifters, the wavefunction given in Eq.(8.1) becomes

$$|\Psi\rangle_{p.s.} = \frac{1}{\sqrt{2}} (e^{i\Phi_1} |A_1\rangle|A_2\rangle + e^{i\Phi_2} |B_1\rangle|B_2\rangle) \quad (9.2)$$

After the beam splitters, we finally have the final state of wavefunction as

$$\begin{aligned} |\Psi\rangle &= \frac{1}{2\sqrt{2}} e^{i\Phi_1} (|D_1\rangle + i|D_2\rangle) (|D_3\rangle + i|D_4\rangle) \\ &+ \frac{1}{2\sqrt{2}} e^{i\Phi_2} (|D_2\rangle + i|D_1\rangle) (|D_4\rangle + i|D_3\rangle) \end{aligned} \quad (9.3)$$

or equivalently,

$$\begin{aligned} |\Psi\rangle &= \frac{1}{2\sqrt{2}} e^{i\Phi_2} \{ |D_1D_3\rangle (e^{i\Delta\Phi}-1) + i|D_1D_4\rangle (e^{i\Delta\Phi}+1) \\ &+ i|D_2D_3\rangle (e^{i\Delta\Phi}+1) - |D_2D_4\rangle (e^{i\Delta\Phi}-1) \} \end{aligned} \quad (9.4)$$

where $\Delta\Phi = \Phi_1 - \Phi_2$.

Since we want to have a dichotomy of values +1 and -1, just as we did with spin-1/2 case, we count the measurement value as +1 if a particle lands in counters D_1 or D_3 , and count -1 if it lands in counters D_2 or D_4 . This counting measurement can be expressed by operation $V_{12} \otimes V_{34}$, where

$$V_{12} = \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix} \quad (9.5)$$

in the representation of $|D_1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, and $|D_2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

and

$$V_{34} = \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix} \quad (9.6)$$

in the representation of $|D_3\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, and $|D_4\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

Then we have quantum mechanical expectation value

$$\begin{aligned}
 E_{Q.M.} &= \langle \Psi | V_{12} \otimes V_{34} | \Psi \rangle \\
 &= + \frac{1}{8} | e^{i\Delta\Phi} - 1 |^2 - \frac{1}{8} | e^{i\Delta\Phi} + 1 |^2 \\
 &\quad - \frac{1}{8} | e^{i\Delta\Phi} + 1 |^2 + \frac{1}{8} | e^{i\Delta\Phi} - 1 |^2 \\
 &= -\cos(\Delta\Phi) \\
 &= -\cos(\Phi_1 - \Phi_2). \tag{9.7}
 \end{aligned}$$

We note that we have perfect correlations

$$\begin{aligned}
 E &= -1 \quad \text{if } \Phi_1 = \Phi_2 \quad \text{and} \\
 E &= +1 \quad \text{if } \Phi_1 = \Phi_2 + \pi. \tag{9.8}
 \end{aligned}$$

This means that we have perfectly correlated system, and we can assign certain "realities" to these properties in the Einsteinian sense. For example, if $\Phi_1 = \Phi_2$ then, the product of the values representing which counter the particle enters will be -1. If particle 1 lands in detector D_1 , to which we assign the value +1, then particle 2 will land in detector D_4 , to which we assigned -1. If particle 1 lands in detector D_2 , which gives -1, then one can again predict with certainty that particle 2 will land in detector D_3 , that gives +1, and so forth. So this satisfies precisely

the Einstein reality criterion. So far quantum mechanically the result is not different from the spin model considered by Bohm and Bell, but there is a different type of freedom in this model: The freedom of the paths a particle pair may take to reach the last part of the interferometer, or the beam splitters. Quantum mechanically, we don't need to (in fact, we cannot) choose specific path to reach the beam splitters, but in a EPR type local realism we must choose, by its own definition of the theory, which path it takes to reach the beam splitters.

9.3 Local Realism For The Two Particle Interferometer

Classically or quantum mechanically, if we remove the beam splitters C_1 and C_2 from their places, then individual particle pair will fire one and only one set of the detectors $\{D_2 \text{ and } D_3\}$ or $\{D_1 \text{ and } D_4\}$. This is an experimental fact.

Quantum mechanically, this is an entirely different experiment, since quantum mechanical results depend on the entire setup. But, for the local realism that advocates "reality" which is supposed to exist locally, regardless of what kind of measurement we are going to make in the future, the physical contents must be same locally since our decision of whether we are going to use the beam splitters or not does not need to be made until long after the pair of particle leave the source. This is kind of double delayed

choice experiment that could exhibit many interesting features of quantum world. In this set up, we may use either one of the beam splitter, or both, or none of them.

For any one of those setups, local realism is forced to say that there are the same elements of reality in the particle-pair system before they reach the places for the beam splitters. Among the possible elements of realities, we can see, the path(s) in which a pair of particles travel constitute(s) one of them.

Suppose we place no beam splitters. If the detector D_3 is fired by particle 2, we know the particle 1 has taken the path A_1 with 100% certainty without disturbing it, and if the detector D_4 fires we know that particle 1 has taken the path B_1 with 100% certainty without in any way disturbing the particle. Therefore, in Einsteinian sense, this establishes the path for particle 1 as an element of reality.

Obviously, the particles in local realistic theory must take either one of the paths, thus we assign a hidden variable (or a set of hidden variables) which determines the path at the point of decay. We may denote this hidden variable collectively as λ and the fact that this λ establishes two mutually exclusive elements of reality as

$$\lambda = \lambda_a + \lambda_b. \quad (9.9)$$

Then local hidden variable theory yields the expectation value

$$\begin{aligned} E_{H.v.}(\Phi_1, \Phi_2) &= \int d\lambda \rho(\lambda) M_\lambda(\Phi_1) N_\lambda(\Phi_2) \\ &= \int d\lambda_a \rho(\lambda_a) M_{\lambda_a}(\Phi_1) N_{\lambda_a} + \int d\lambda_b \rho(\lambda_b) M_{\lambda_b} N_{\lambda_b}(\Phi_2). \end{aligned}$$

(9.10)

Where M and N denote the counter measurement values triggered by particle 1 and 2 respectively. It is important to notice that the first integral is over all hidden variables that lead to the path A therefore depends only one phase shifter Φ_1 , while the second integral is over the hidden variables that lead to the path B and therefore depend on only Φ_2 .

This kind of separation of integral was not possible in the spin model since each particle takes only one path in that case. Since this separation of hidden variables expressed in Eq.(9.9) imposes such a tremendous restriction on local realism, we will see that the construction of any self-consistent local realistic model is not possible.

We now will follow the same type of argument that was used for many particle GHZ theorem to prove that local realism should yield contradictory results as it tries to immitate the results of quantum mechanics.

9.4 Proof For Two Particle GHZ System

To meet the superclassical results given by Eq. (9.8), the hidden variable counterpart Eq.(9.10) must yield the same expectation value. Therefore,

$$\begin{aligned} E_{H.v.}(\Phi, \Phi) &= \int d\lambda_a \rho(\lambda_a) M_{\lambda_a}(\Phi) N_{\lambda_a} + \int d\lambda_b \rho(\lambda_b) M_{\lambda_b} N_{\lambda_b}(\Phi) \\ &= -1 \end{aligned} \quad (9.11)$$

and

$$\begin{aligned} E_{H.v.}(\Phi, \Phi+\pi) &= \int d\lambda_a \rho(\lambda_a) M_{\lambda_a}(\Phi) N_{\lambda_a} + \int d\lambda_b \rho(\lambda_b) M_{\lambda_b} N_{\lambda_b}(\Phi+\pi) \\ &= +1 \end{aligned} \quad (9.12)$$

for any angle Φ .

Since

$$\int \rho(\lambda_a) d\lambda_a + \int \rho(\lambda_b) d\lambda_b = 1, \quad (9.13)$$

each term in Eqs.(9.11) and (9.12) must take its own extreme values.

I.e.,

$$M_{\lambda_a}(\Phi) N_{\lambda_a} = -1 \quad (9.14a)$$

$$M_{\lambda_b} N_{\lambda_b}(\Phi) = -1 \quad (9.14b)$$

and

$$M_{\lambda_a}(\Phi) N_{\lambda_a} = +1 \quad (9.15a)$$

$$M_{\lambda_b} N_{\lambda_b}(\Phi+\pi) = +1 \quad (9.15b)$$

for any angle Φ .

Eqs.(9.14a) and (9.15a) immediately yield a contradiction since $-1 \neq +1$. The same is true for Eqs.(9.14b) and (9.15b) as we replace Φ by $\Phi+\pi$ in Eqs.(9.14a,b).

Here again we see, even in two particle system, that local realism develops contradictory behavior. Therefore, if we assume (or if any future experiment confirms) that quantum mechanical predictions are correct, then the system cannot be modeled by EPR realism.

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CHAPTER X

IMPLICATION AND IMPLEMENTATION OF TWO PARTICLE GHZ THEOREM

10.1 Introduction

If we analyze the two particle GHZ system given in chapter 9, we will see that the system given by BGHZ is powerful enough to block almost all the loopholes that were exploited by the advocates of the local realism. The model is free from controversy of "no enhancement" assumption and, more importantly, it can refute the EPR type local realism even in the case of inefficient counters.

We also see that the system used in two particle GHZ theorem is capable of carrying out double delayed choice experiments that could exhibit many interesting features of quantum world.

We will show that there can be a variety of simple system which is not any more complicated than the one given in Chapter IX but generate the same result. We give in Sec.10.4 a specific model of that kind with a particle pair system using only one set of openings, instead of two.

The logic that was employed by BGHZ may be easily extended back to a polarization measurement experiment, provided that the system involves multiple paths. Some of these experiments that involve a pair of correlated photons

may be readily implemented by a parametric down conversion of light technique.

The review of the analysis of the Rarity-Tapster¹ (RT) EPR experiment, to which the BGHZ gedanken experiment is most similar, shows that the experiment is little wanting in their data clarification for the two particle GHZ proof as we shall point it out in the next section.

Many different versions of two particle GHZ experiment are possible with only one set of opening at a source, which may be regarded as even simpler ones than that of BGHZ original model, such as a model proposed for an EPR experiment by Franson². Again, the experimental works done for the Franson's model by Ou, Zou, Wang, and Mandel³ (OZWM) and Kwiat, Vreken, Hong, Nathel, and Chiao⁴ (KVHNC) which were originally performed for the case of Bell's theorem, can be used to prove the two particle GHZ theorem, with slight variations.

As an yet another and the final version of the two particle GHZ model, we give a model for the polarization measurement as it was the most common type of experiment for the Bell's theorem.

10.2 Implications Of The Two Particle GHZ Theorem

By using the perfect correlation and anticorrelation case of $\Phi_1 - \Phi_2 = 0$ or π , or $E = \pm 1$, with the help of the

reality of multi-paths, BGHZ were able to prove the contradiction for local realism.

We now show the fact that the system given by BGHZ is powerful enough to block the loopholes that were exploited most by the advocates of the local realism.

For the generality of the argument, let us assume that in a real non-ideal experiment, the quantum mechanical joint detection expectation value may given by

$$E_{Q.M.} = -\eta \cos(\Phi_1 - \Phi_2) \quad (10.1)$$

in the place of Eq.(9.10), with $0 < \eta < 1$.

Here, η may represent the efficiencies of the counters and/or the filters, or the visibilities, or imperfect correlations or the combination of those or any other reason that may contribute to the term.

Then, by the same reason given by BGHZ, any local realistic theory must satisfy

$$\int \rho(\lambda_a) d\lambda_a M_{\lambda_a}(\Phi_1) \cdot N_{\lambda_a} + \int \rho(\lambda_b) d\lambda_b M_{\lambda_b} \cdot N_{\lambda_b}(\Phi_2) = -\eta \cos(\Phi_1 - \Phi_2) \quad (10.2)$$

and

$$\int \rho(\lambda_a) d\lambda_a M_{\lambda_a}(\Phi_1) \cdot N_{\lambda_a} + \int \rho(\lambda_b) d\lambda_b M_{\lambda_b} \cdot N_{\lambda_b}(\Phi_2 + \delta) = -\eta \cos(\Phi_1 - \Phi_2 - \delta) \quad (10.3)$$

for any arbitrary independent angles Φ_1 , Φ_2 , and δ .

Since Φ s are arbitrary, by replacing Φ by $\Phi + \delta$, we have

$$\begin{aligned} & \int \rho(\lambda_a) d\lambda_a M_{\lambda_a}(\Phi_1 + \delta) \cdot N_{\lambda_a} + \int \rho(\lambda_b) d\lambda_b M_{\lambda_b} \cdot N_{\lambda_b}(\Phi_2 + \delta) \\ & = -\eta \cos(\Phi_1 - \Phi_2) \end{aligned} \quad (10.4)$$

and

$$\begin{aligned} & \int \rho(\lambda_a) d\lambda_a M_{\lambda_a}(\Phi_1 + \delta) \cdot N_{\lambda_a} + \int \rho(\lambda_b) d\lambda_b M_{\lambda_b} \cdot N_{\lambda_b}(\Phi_2) \\ & = -\eta \cos(\Phi_1 - \Phi_2 + \delta). \end{aligned} \quad (10.5)$$

Now, by subtracting (10.2) from (10.3), we have

$$\begin{aligned} & \int \rho(\lambda_b) d\lambda_b M_{\lambda_b} \cdot N_{\lambda_b}(\Phi_2 + \delta) - \int \rho(\lambda_b) d\lambda_b M_{\lambda_b} \cdot N_{\lambda_b}(\Phi_2) \\ & = -\eta \cos(\Phi_1 - \Phi_2 - \delta) + \eta \cos(\Phi_1 - \Phi_2) \end{aligned} \quad (10.6)$$

and, by subtracting (10.5) from (10.4), we have

$$\begin{aligned} & \int \rho(\lambda_b) d\lambda_b M_{\lambda_b} \cdot N_{\lambda_b}(\Phi_2 + \delta) - \int \rho(\lambda_b) d\lambda_b M_{\lambda_b} \cdot N_{\lambda_b}(\Phi_2) \\ & = -\eta \cos(\Phi_1 - \Phi_2) + \eta \cos(\Phi_1 - \Phi_2 + \delta) \end{aligned} \quad (10.7)$$

Finally from Eqs.(10.6) and (10.7), we have

$$\begin{aligned} & -\eta \cos(\Phi_1 - \Phi_2 - \delta) + \eta \cos(\Phi_1 - \Phi_2) \\ & = -\eta \cos(\Phi_1 - \Phi_2) + \eta \cos(\Phi_1 - \Phi_2 + \delta) \end{aligned}$$

or,

$$\begin{aligned} 2\cos(\Phi_1 - \Phi_2) & = \cos(\Phi_1 - \Phi_2 - \delta) + \cos(\Phi_1 - \Phi_2 + \delta) \\ & = 2\cos(\Phi_1 - \Phi_2) \cdot \cos \delta, \end{aligned} \quad (10.8)$$

regardless of the value η .

If we choose $\Delta\Phi = \Phi_1 - \Phi_2$ not equal to $\pm \pi/2$, we must have

$$\cos \delta = +1 \quad (10.9)$$

for arbitrary value δ . But this cannot be true. Therefore we proved that, for any angle $0 < \delta < 2\pi$, local realism develops contradictory behaviour.

Hence, If we have an experimental result of cosine type, the only case local realism can immitate the result of quantum mechanics is when the overall quantum efficiency is identically zero: that's when we don't do the experiment.

This two particle GHZ theorem, therefore, can be regarded as an altogether more powerful theorem than other theorems that preceeded it.

Now, we like to specify what to measure experimentaly for the above proof. From Eqs.(10.2)-(10.5) it is clear that we need the following four sets of measurements to verify the theorem experimentally.

$$\{\Phi_1, \Phi_2\}, \{\Phi_1, \Phi_2 + \delta\}, \{\Phi_1 + \delta, \Phi_2 + \delta\}, \text{ and } \{\Phi_1 + \delta, \Phi_2\} \quad (10.10)$$

Though we tried not to use any specific values of Φ_i s and δ for the generality of the argument, we can take any of the allowed four different sets of measurements that are convenient to the specific experiment. For example, by choosing $\Phi_1 = \Phi_2 = \Phi$ and $\delta = \pm\pi$, we can use optimal correlation and anticorrelation of the system while in some experimental situations where it is hard to define the absolute values of the phases, it may be more convenient to

use a nonspecific value of δ as a measure of the unknown phase angles .

The above proof goes through for an even more general case of *imperfectly* correlated system which would yield a quantum mechanical expectation value of the form

$$E_{q.m.} = a + f_1(\Phi_1) + f_2(\Phi_2) + b \cos(\Phi_1 \pm \Phi_2). \quad (10.11)$$

where a , b are the constants and f_i s are the functions of the respective arguments.

It should be noticed that the type of proof given above can be applied to any type of two particle multi-path correlation experiments that yield an expectation value as a function of $|\Phi_1 - \Phi_2|$. The function does not necessarily be an sinusoidal and may take up any form that allows any possible experimental adjustments, as far as the result can be expressed as a function of $|\Phi_1 - \Phi_2|$.

We will give a proof that the only function of this type that any local hidden variable theory can produce is a flat straight line: i.e.,

$$E_{h.v.} = \text{constant}. \quad (10.13)$$

[Proof]

As we discussed in the preceeding proof, any local realistic theory that follows EPR realism must yield an expectation value

$$\int \rho(\lambda_a) d\lambda_a M_{\lambda_a}(\Phi_1) N_{\lambda_a} + \int \rho(\lambda_b) d\lambda_b M_{\lambda_b} N_{\lambda_b}(\Phi_2) = f(|\Phi_1 - \Phi_2|) \quad (10.13)$$

where we replaced the cosine function with a more general function f that depends only on the difference between two arguments: $|\Phi_1 - \Phi_2|$.

Hence the relation may be rewritten in an even more general form as

$$g(x) + h(y) = f(x-y); \quad (10.14)$$

with

$$f(x-y) = f(y-x) \quad (10.15)$$

where g , h , f are the functions of the respective arguments and x , y are the independent variables that replace Φ_1 , Φ_2 .

Eq.(10.15) shows a requirement that the function f must be symmetric in x and y and doesn't depend on their order. Here we restrict ourselves on a minus sign for the present proof, as the same argument can be proceeded for the case of a plus sign, too.

By substituting 0 for y in Eq.(10.14), and the same for x , we have

$$\begin{aligned} g(x) &= f(x) - h(0) \\ h(y) &= f(-y) - g(0) \end{aligned} \quad (10.16)$$

And, since

$$g(0) + h(0) = f(0), \quad (10.17)$$

the function f must satisfy

$$f(x) + f(-y) = f(x-y) + f(0). \quad (10.18)$$

And from Eq.(10.18), we have

$$f(-x) = -f(x) + 2f(0). \quad (10.19)$$

From Eqs.(10.18) and (10.19),

$$f(x-y) = f(x) - f(y) + f(0) \quad (10.20)$$

But from (10.15),

$$f(x) - f(y) + f(0) = -f(x) + f(y) + f(0)$$

or,

$$f(x) = f(y). \quad (10.21)$$

Since the LHS of Eq.(10.21) is a function of x only and the RHS is a function of y only, both side must be equal to a same constant: $f(0)$.

Figure 10.1 gives an illustration of a typical quantum (or, experimental) result for the correlation of two particle in BGHZ experiment against the same of the local realism for a given experimental result: in this case, a cosine function.

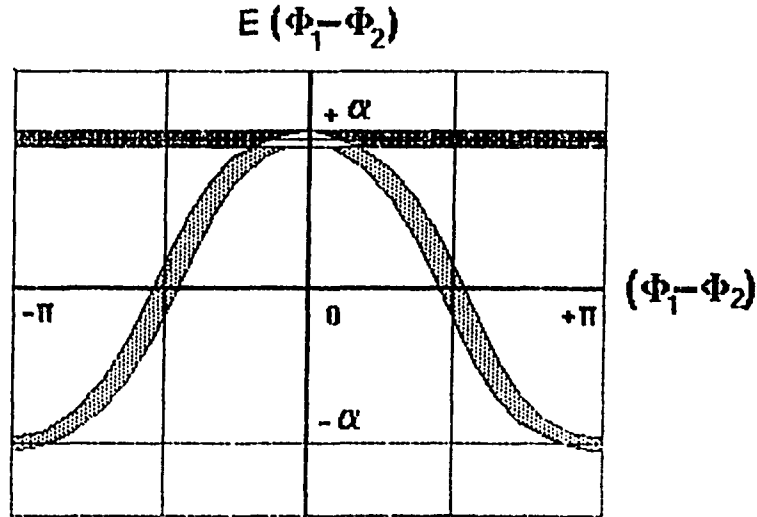


FIG.10.1 Typical quantum mechanical result against a hidden variable result in a real experimental situation.

Once the dependence of $\Phi_1 \pm \Phi_2$ is established within the error, the finiteness of the experimental error should allow us to describe the data in a form:

$$E(\Phi_1, \Phi_2)_{\text{Experiment}} = f(\Phi_1 \pm \Phi_2) \pm \varepsilon \cdot m(\Phi_1, \Phi_2) \quad (10.22)$$

where ε denotes a small parameter connected to the experimental accuracy and m represents any possible systematic error.

On the other hand, a hidden variable theory is expected to yield an experimental result as

$$E(\Phi_1, \Phi_2)_{\text{H.V.}} = f(0) \pm \varepsilon' \cdot m'(\Phi_1, \Phi_2). \quad (10.23)$$

The light shadow areas in Fig.10.1 represent quantum mechanical result within the experimental error, while dark shadow areas represent the local realistic one. What we show here is that it is simply impossible for any local realistic model to produce the result that are represented by the light shadow area even in a real experiment.

10.3 Allowing "Wrong" Counts

Finally, we shall allow local realistic theories to exploit the cases that are prohibited in principle by the law of momentum conservation, yet practically occur due to some unpredictable bouncings of the particles or the misfiring of the counters. These are the cases in which a particle pair takes either the path A_1 and B_1 or the path A_2 and B_2 (Fig.9.1).

Since we allow local realism to dictate the above possible events, the Eq.(10.13) takes the form:

$$\begin{aligned} & \int \rho(\lambda_a) d\lambda_a M_{\lambda_a}(\Phi_1) \cdot N_{\lambda_a} + \int \rho(\lambda_b) d\lambda_b M_{\lambda_b} \cdot N_{\lambda_b}(\Phi_2) \\ & + \int \rho(\lambda_c) d\lambda_c M_{\lambda_c}(\Phi_1) \cdot N_{\lambda_c}(\Phi_2) + \int \rho(\lambda_d) d\lambda_d M_{\lambda_d} \cdot N_{\lambda_d} = \eta f(\Phi_1 - \Phi_2). \end{aligned} \quad (10.24)$$

This equation can be rewritten in an even more general form:

$$p(x) + q(y) + \epsilon r(x,y) + s = \eta f(x-y) \quad (10.25)$$

Since the RHS of Eq.(10.25) is a function of only one variable: $x-y$, the function r in the LHS must also be the function of $x-y$.

Hence:

$$p(x) + q(y) + \varepsilon r(x-y) + s = \eta f(x-y) \quad (10.26)$$

where, from Eq.(10.24),

$$-1 \leq p, q, r, s \leq +1. \quad (10.27)$$

By moving the term $\varepsilon r(x-y)$ to the RHS, we have

$$p(x) + q(y) = g(x-y) \quad (10.28)$$

where

$$g(x-y) = \eta f(x-y) - \varepsilon r(x-y). \quad (10.29)$$

From Eqs.(10.27) and (10.28), one can deduce that function g must be a straight line:

$$g(z) = az + b \quad (10.30)$$

where $z = x-y$ and a, b are the arbitrary constants.

From Eqs.(10.26) and (10.30), we have

$$az + c + \varepsilon r(x-y) = \eta f(x-y) \quad (10.31)$$

where $c = b + s$ is a constant.

From Eq.(10.27), we know

$$-1 \leq az + c \leq +1 \quad (10.32)$$

and

$$-\epsilon \leq \epsilon r(x-y) \leq +\epsilon \quad (10.33)$$

$$-\eta \leq \eta f(x-y) \leq +\eta. \quad (10.34)$$

Inequalities (10.32)-(10.34) impose tremendous limits on the values ϵ and η for Eq.(10.31) to be satisfied.

In order to see this we may plot the RHS (typical experimental result: here we used cosine curve) of the Eq.(10.31) against the LHS (the area in which any function r of a hidden variable prediction can be contained: in shadow) in Fig.10.2.

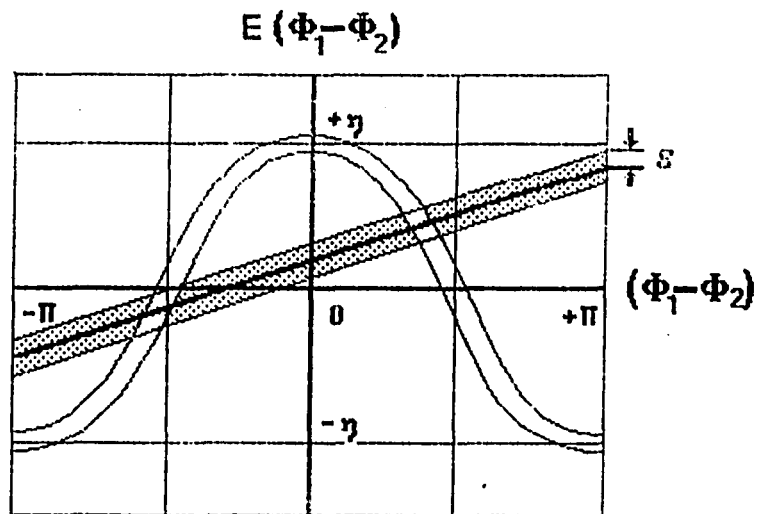


Fig.10.2 A local hidden variable prediction vs. a typical experimental data

From Eq.(10.31), it is clear that any deviation from the straight line for a local hidden variable model must come from the term $\epsilon \cdot r(x-y)$ which is subjected to an inequality (10.33). In other words, in order for any local hidden variable theory to be compatible with the experimental results, the any part of the experimental (or, quantum mechanical) result, $\eta f(x-y)$, that cannot be deduced from a straight line must come from the term $\epsilon r(x-y)$. This type of exploitation of a loophole by hidden variable theory is extremely unreasonable since, while we know that the value η is determined by many reasons (for example, by inefficiency of the counters and/or the filters, visibilities, or imperfect correlations, etc.), the value ϵ of hidden variable theory must be determined by one reason (wrong count).

In our specific model, we have an ideal quantum prediction as

$$f(x-y) = \cos \theta \quad (10.35)$$

then we have

$$-\eta \leq E(\theta)_{\text{Exp.}} \leq +\eta \quad (10.36)$$

where $E(\theta)$ is a continuous function between E_{max} and E_{min} .

Suppose ϵ is smaller than η . Then the range of $r(\theta)$ is smaller than the range covered by $f(\theta)$, therefore, there is no way Eq.(10.31) can be satisfied for the rest of the area as we can see from Fig.10.2 and it should be concluded that

the experimental result cannot be modeled by any local hidden variable theory.

The maximum possible value ϵ may easily be determined by supplementary experiments by inserting coincidence detectors in front of each opening.

It should be mentioned that, when we experimentally determine the optimum value of ϵ , coincidence firing due to double pair or triple pair within the resolution time must be excluded since hidden variable term $\epsilon r(x-y)$ cannot be attributed to them. Those multiple pair coincidence can be statistically calculated and removed from the contribution to the value of ϵ .

The conclusion is therefore that local realism cannot produce any other function than the one that gives a small variation of a maximum width ϵ around a straight line. This conclusion is so general that there can be no loopholes for any reason once the experiment verifies the result as a function of $x \pm y$, or $\Phi_1 \pm \Phi_2$ for our model.

Therefore a very small room is left for the local realism as to how we establish the experimental result as a function of $\Phi_1 \pm \Phi_2$ and how we discriminate the best fit straight line with a maximum variation of ϵ of local realism from the experimental (or, quantum mechanical) function f , which is generally expected to be a curved one, allowing the experimental errors.

Considering the accuracy of the current experimental techniques, we should be able to discriminate those two

results and this should serve as a conclusive experimental proof against local realistic theories that has persisted for a long time since the inceptive work by EPR.

10.4. Implementation Of Two Particle GHZ Experiment

Two particle GHZ theorem may be implemented in many ways. Practically any system with which we can perform Bell-type experiment may also be used for GHZ experiment, provided that we can have a pair of correlated particles in two well defined different directions. One can perform two particle GHZ experiment using particle spins, or polarizations of photons, or positions, which is combined with momentum (or, paths). One may also use different sources of correlated particles, such as photon cascades and parametric down conversion of light. Some experiments that are already performed for the Bell type experiments may be used again for a two particle GHZ experiment, among which the Rarity-Tapster (RT) EPR experiment and the model recently proposed by Franson and implemented by Ou, Zou, Wang, and Mandel (OZWM) and Kwiat, Vareka, Hong, Nathel, and Chiao (KVHNC) are the most promising ones, though some of their setups need slight variations for the new measurements.

10.4.1 RARITY-TAPSTER EPR EXPERIMENT

The analysis of RT experiment to which the BGHZ gedanken experiment is most similar, shows that the experiment appears to have the data we need to prove the two particle GHZ theorem. They reported the four sets of measurement that we showed in List (10.10) that we need to disprove local realism except the "wrong counting loophole" we've just discussed in the previous section.

Since RT's data already shows the cosine curve for the experiment, all we need in addition to their results to disapprove local realism is the additional measurement to determine the optimum value (ϵ) of the probability that a particle pair might accidentally take up the path toward the phase shifters Φ_1 and Φ_2 .

As was mentioned in RT's paper, this experiment may be improved by a rearrangement of the setup as shown in Fig.10.3 that is topologically equivalent to the performed RT's but is spatially separated for a delayed choice experiment.

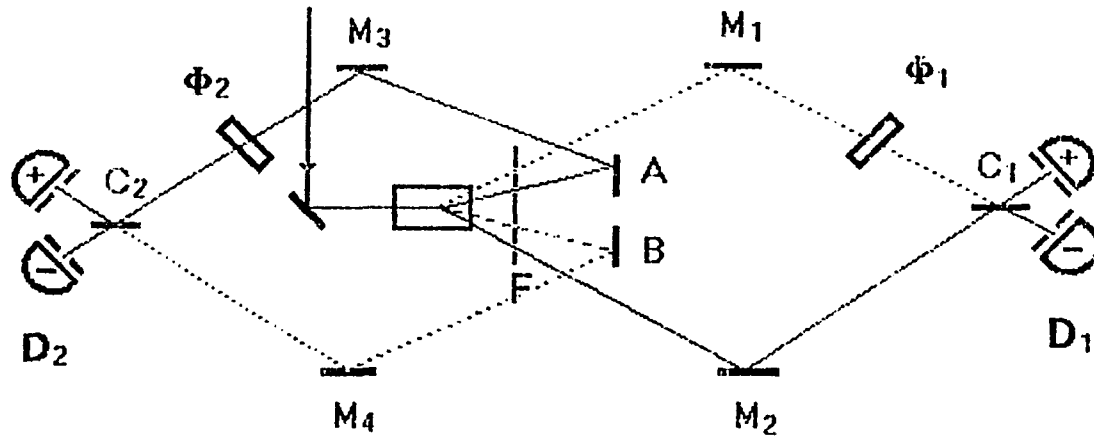


FIG.10.3 Topologically equivalent experimental setup to Rarity-Tapster EPR-Bell experiment. A particle pair takes either the path shown in dotted lines or the path given in solid lines through the filter F to reach the beam splitters C_1 and C_2 .

10.4.2 FRANSON'S MODEL FOR EPR

Many different version of two particle GHZ experiment are possible with only one set of opening at a source, which may be regarded as even simpler than that of BGHZ original model. Among them, we'd like to discuss an experimental work by Mandel et.al (OZWM) for the Franson's model and we shall point out that we can construct the experimental setup for this model that does not allow a "wrong" counting term we discussed in Sec.10.3.

The experiment performed by OZWM, however, is not sufficient for the two particle GHZ proof and we will show the way to an improvement for the new experiment.

Franson made use of "the time taken for traveling by a photon" as an element of reality in showing the nonlocality of quantum mechanics. He made use of a gedanken model given in Fig.10.4 in which a photon pair flies apart from a source through a pair of holes and each photon takes one of two alternate paths.

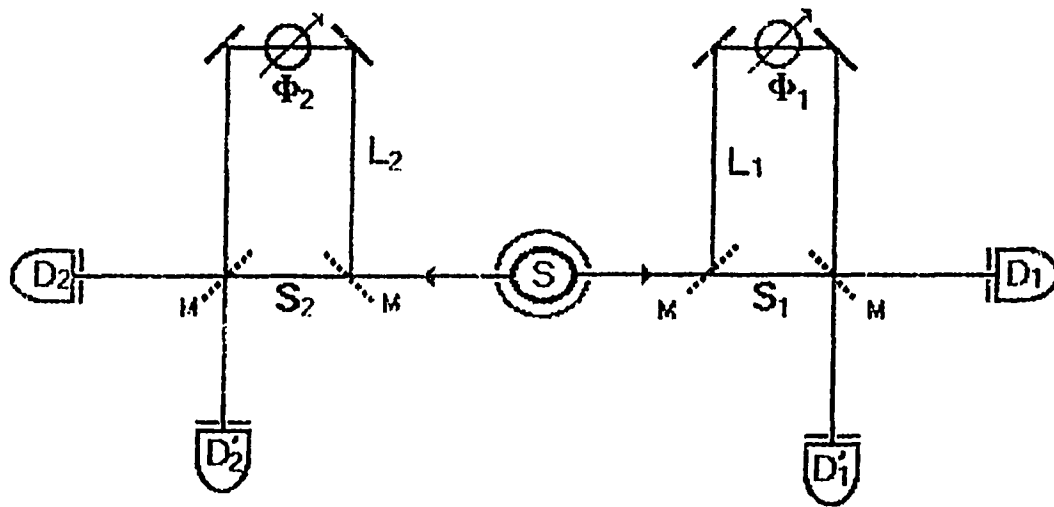


FIG.10.4 Photon coincident measurements devised by Franson which includes the interference between the amplitudes along the shorter paths, S_1 and S_2 , and the longer paths L_1 and L_2 .

In this experiment, we assume the path difference between the shorter path and the longer path is much greater than the length of the wavepacket of the photon involved and greater than the distance needed for the resolution time of the detectors. In a classical reasoning, there can be no interference between the two paths.

However, a *coincidence* measurement of *the two* photons exhibits an interference fringes since it is fundamentally impossible to distinguish the two possible processes:

(a) both photons take the shorter paths (S-S)

(b) both photons take the longer paths (L-L).

The other possibility that one photon takes a shorter path and the other takes a longer path is a distinguishable process and one can block this possibility by making the path difference long enough so that the coincidence of this type would violate the special relativity on which the local realism based on. Therefore, the two possible processes described above (a) and (b) constitutes the elements of reality in Einsteinian sense and thereby two particle GHZ theorem works for this case also.

One may establish these elements of reality by placing very fast shutters at the openings and by measuring the time taken for a pair of photons to travel to the coincidence counters. If the photon 1 is found to have taken the shorter path, we know that the photon 2 has also taken the shorter path, with certainty and without in any way disturbing the path of the particle. And the same thing holds for the longer paths.

The "wrong" counting loophole discussed in Sec.10.3 in principle does not apply to this particular case. However, if *either* the path difference is not long enough or the resolution time of the detectors is not fast enough, we

must take into account the "wrong" counting possibility also.

The experiment performed by OZWM used a variable path instead of variable phase shifters with the fixed paths to produce a cosine interference fringe between the longer and the shorter paths. The OZWM set-up may be regarded as sufficient for the Bell-type experiment but is not sufficient for implementing the two particle GHZ theorem since, as we have shown in Sec.II, one of the essential feature of the theorem is that it must use two independently variable phase shifters (or, equivalently, two independently variable paths) for the two elements of reality. In other words, the proof relies on the interference between the two independently variable phase shifters each of which belongs to one of the two possible elements of reality. In this way, local realism cannot produce an interference fringe between them because those two events are mutually exclusive and belong to different hidden variables.

A similar experiment performed by KVNHC may also be modified for the new experiment for the same reason.

Once the elements of reality are established and an independently variable phase shifter is assigned to each of those elements of reality, then any one of the three different versions of two particle GHZ theorem we have discussed in Sec.II can be applied.

We show the modified version of Franson's experiment for the purpose of the new experiment in Fig.10.5 and Fig.10.6.

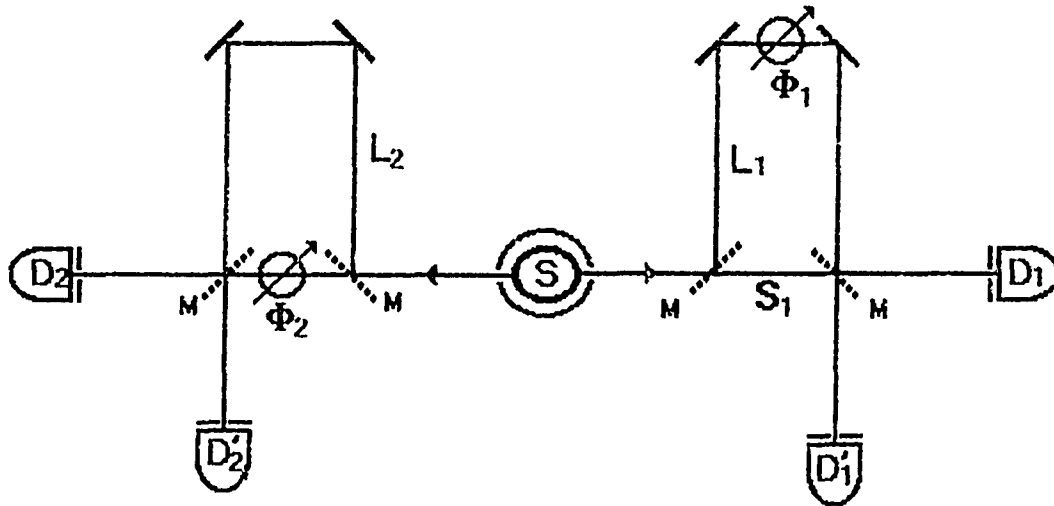


Fig.10.5 Modified version of Franson's experiment for the two particle GHZ theorem. One of the two phase shifters is moved to a shorter path so that each path (S-S or L-L) has its own variable phase shifter. Since we are hoping to block all the possible loopholes, we suggest that this experiment be done in a spatially separated situation.

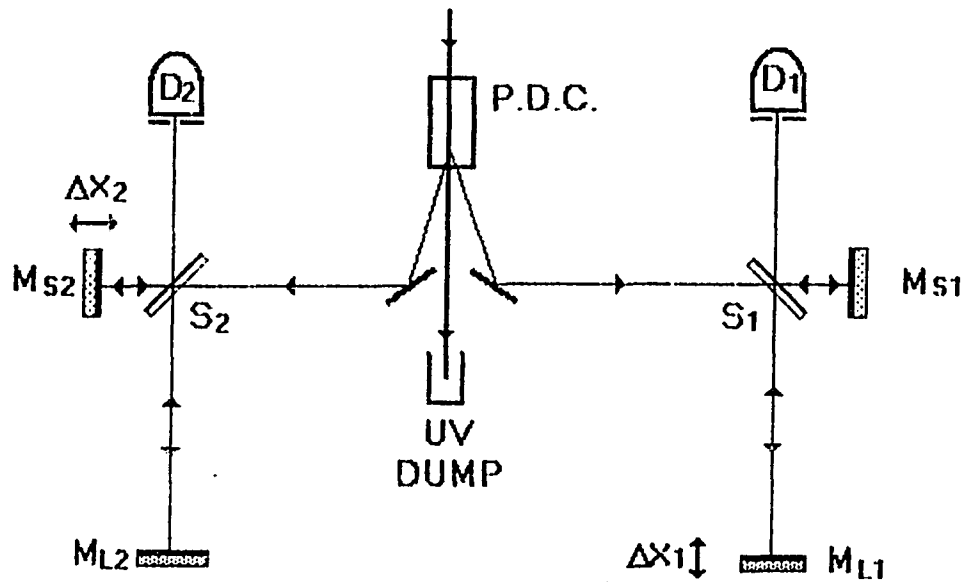


Fig.10.6 Modified Double Michelson Interferometer of OZWM and KVHNC for two particle GHZ experiment. It should be noted that we have two movable mirrors: one in a longer path, the other in a shorter path.

10.4.3 MULTI-PATH PHOTON POLARIZATION GHZ EXPERIMENT

As an yet another and the final version of the two particle GHZ model, we give a model for the polarization measurement as it has been the most common type of experiment for the Bell's theorem.

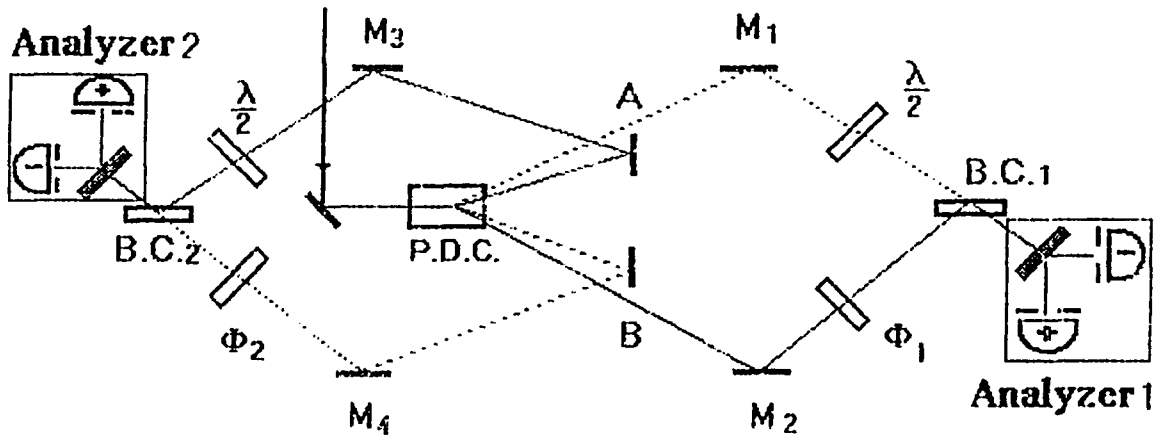


Fig.10.7 Two Particle GHZ Experiment Using Polarization and Multiple Paths Implemented by Parametric Down Conversion of Light Technique: Two channel polarization analyzers are employed after the same colored beams are recombined. Here, dotted lined beam represents the path $k-k'$ and solid lined beam represents the path $l-l'$.

Suppose we have all x-polarized photons that are down converted from an input photon through a nonlinear optical spontaneous parametric process as we show in Fig.10.7. Since we have two sets of openings which lead to the paths $k-k'$ or $l-l'$, we may write down the wavefunction of these two particle system as

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \{ |k+, k'+\rangle + |l+, l'+\rangle \}. \quad (10.37)$$

After half wave plates and phase shifters, it becomes

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{ e^{i\Phi_1} |k-, k'+\rangle + e^{i\Phi_2} |l+, l'+\rangle \}. \quad (10.38)$$

Finally after the beams are combined,

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{ e^{i\Phi_1} |-\rangle_1 |+\rangle_2 + e^{i\Phi_2} |+\rangle_1 |-\rangle_2 \}. \quad (10.39)$$

Quantum mechanical expectation value for the joint polarization measurement becomes

$$\begin{aligned} E(\theta_1, \theta_2; \Phi_1, \Phi_2) &= \langle \Psi | T_1 \otimes T_2 | \Psi \rangle \\ &= \sin 2\theta_1 \sin 2\theta_2 \cos(\Phi_1 - \Phi_2) \end{aligned} \quad (10.40)$$

where θ_i are the angles for the analyzer axes and T_i are the operators for the two channel polarization analyzer measurement.

If we set $\theta_i = \pi/4$, then we finally have

$$E(\Phi_1, \Phi_2) = \cos(\Phi_1 - \Phi_2) \quad (10.41)$$

And the rest of the proof for this case is subject to the same argument as the other cases.

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CHAPTER XI

CONCLUSION

Starting from the original Einstein-Podolsky-Rosen paper, we studied the implications of Bell's Inequalities and the subsequent experiments. It is shown that quantum mechanical predictions are indeed different from that of EPR-type local realism, but it is also stated that the conclusion that "the local realistic program is not viable" is not the final one since a few loopholes are still exist in Bell-type experiments.

We have shown that a more direct and more powerful investigation against the original EPR program is possible using a quantum mechanically entangled state which consists of three or more correlated particles as was proposed recently by Greenberger, Horne, and Zeilinger. We showed the important implications of GHZ theorem and generalized the original four particle GHZ theorem to three particle system, and also to more practically realizable photon systems. The photon polarization analogue of GHZ spin-1/2 gedanken experiment is established using general elliptical polarization measurements.

A study is made also on the experimental verification of the theorem, including the cases of non-perfect correlated system and/or non-ideal detectors to accommodate real experimental situations.

Though we have mentioned various possible experiments for the theorem, we made a special effort to look into the possibility of using n -photon down conversion phenomena for our purpose. We concluded that quantum theory in principle should allow the multi-photon spontaneous down conversion phenomena to occur. We proposed to use the three photon down conversion phenomena for the implementation of GHZ theorem in the form of a few different types of interferometers: a three photon momentum-position interferometer, a time-position interferometer and a polarization interferometer.

Finally, we considered a multi-pathed two particle system newly proposed by Bernstein-Greenberger-Horne-Zeilinger as an extension of GHZ system back to two particle system which has been used by EPR, Bohm, and Bell. The system uses paths as "elements of reality" in Einsteinian sense and leads to a total failure of local realism by not providing any existing loopholes. We gave the general proof for the non-ideal situations and proposed three different types of experiments including Rarity-Tapster-type, Franson-type non-polarization interference experiment, and a multi-path polarization experiment.

We may conclude from the study that the EPR program originally designed to question that "quantum mechanics is a complete theory", and its subsequent local realistic (or hidden variable) theories are not compatible with the quantum mechanical prediction provided that the experiments

on GHZ theorem should turn out to confirm quantum mechanical predictions as we believe it is indeed the case.

After the study made so far on the subject, the author doesn't feel that the negation of local realism is the completion of EPR program. On the contrary, he feels that it may be the beginning of the real problem.

The author thinks that the subject of nonlocality or non-separability¹⁻⁸ in quantum mechanics which has been in the heart of EPR problem must be investigated further both theoretically and experimentally as it still bothers many concerned physicists as it had bothered Einstein. Though this nonlocal quantum correlation which is strange to our classical intuitions cannot be used to transmit messages at superluminal speeds, it is presently not clear whether there is a deeper incompatibility between quantum mechanics and relativity.⁹⁻¹⁴

So the author feels that we should aggressively investigate this EPR problem further along this direction with the expectation that it might lead us to some new important knowledge of fundamental nature of the physical world.

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APPENDIX A

COINCIDENT COUNTING PROPERTIES
OF PHOTON POLARIZATION ANALYZERS AND STERN-GERLACH APPARATUS
IN E-P-R EXPERIMENTS

EPR-type correlation counting properties of a single channel and a two-channel photon polarization analyzers, and a Stern-Gerlach analyzer, are discussed. A general scheme that makes a photon polarization measurement mathematically comparable to a three dimensional spin-1/2 measurement is presented. An example is given using a GHZ gedanken experiment.

Due to its similarity in quantum mechanical properties to that of the spin of spin-1/2 particle, the polarization of photons has been used in implementing Bell's theorem¹, which was originally written for a pair of correlated spin-1/2 particles. Even though the polarization of both the photon and spin-1/2 systems have two degrees of freedom, they have been treated differently because the analyzers we use most for those measurements have different counting properties. For example, with single channel linear polarization analyzers which have been employed in most of the Einstein-Podolsky-Rosen(EPR) type experiments², we count the polarization parallel to the optical axis of the analyzer as +1 and its orthogonal polarization as 0, because the photons of the latter would be absorbed by the polarizer and therefore cannot trigger the counter. On the other hand, in Stern-Gerlach measurements for a spin-1/2 particle we count the mutually orthogonal spin-polarizations as +1 and -1 (in unit of $\hbar/2$).

The employment of a single channel analyzer in photon polarization measurement leads to two major defects when it comes to Bell-type experiments where we need to measure coincident counts of the two correlated particle pairs. The first defect is that it limits the maximum value of the correlation considerably because of zero countings so that, in general, the correlation should be subjected to different inequalities that may require some additional assumption(s).

The second defect is that it provides more loopholes for local realism even in the ideal experiments, since one cannot distinguish the zero counting (or non-counting) due to a polarization orthogonal to the optical axis of the analyzer, from the same result due to counter inefficiency, since the counter will not fire in either case.

These defects can be removed by employing a two channel analyzer, such as an ideal birefringent crystal polarizer that separates a light beam into two mutually orthogonal linear polarizations, and thereby we may assign them the values +1 for the one and -1 for the other as we do in a spin-1/2 measurement.

In fact, in 1981, Garuccio and Rapisarda³ suggested a new definition of the correlation function that made use of a two channel analyzer

$$E(\hat{a}, \hat{b}) = \frac{D(\hat{a}_+, \hat{b}_+) - D(\hat{a}_+, \hat{b}_-) - D(\hat{a}_-, \hat{b}_+) + D(\hat{a}_-, \hat{b}_-)}{D(\hat{a}_+, \hat{b}_+) + D(\hat{a}_+, \hat{b}_-) + D(\hat{a}_-, \hat{b}_+) + D(\hat{a}_-, \hat{b}_-)} \quad (\text{A.1})$$

where $D(\hat{a}, \hat{b})$ denotes the number of detections of a joint counting measurement with a pair of two-channel analyzers whose axes are along the angle directions $\hat{a}$ and $\hat{b}$, and the subscripts denote two orthogonal polarizations. With this, they obtained a different inequality from the one that was derived for a single channel analyzer. The experiment was carried out in the Aspect's second experiment in 1982⁴ with

a stronger violation of inequality than the previous result⁵ with single channel analyzers. In retrospect, it was no wonder that the hidden variable model that was suggested by Marshall et al.⁶, with which they claimed that their model could fit all the then existing data including that of Aspect's third experiment in 1982⁷, could not reproduce the result of Aspect's second experiment which employed the new counting method, as they admitted. It should be noted that Aspect's third experiment used two paths, each of which led to a single channel polarizer for each particle, but did not employ two channel polarizers.

A.1. ONE CHANNEL VS. TWO CHANNEL ANALYZER IN EPR EXPERIMENTS

The operator for the measurement with a one channel linear polarization analyzer may be written as

$$Q(\theta) = \begin{pmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{pmatrix} \quad (\text{A.2})$$

in the representation in which x-polarization can be written as $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and y-polarization as $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

It has the property, for any angle θ ,

$$Q(\theta) |\theta\rangle = +1 |\theta\rangle \quad \text{and} \quad Q(\theta) |\theta_\perp\rangle = 0 |\theta_\perp\rangle \quad (\text{A.3})$$

where the two orthogonal eigenstates may be written as

$$|0\rangle = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \quad \text{and} \quad |0_{\perp}\rangle = \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}. \quad (\text{A.4})$$

The quantum mechanical expectation value of coincidence counts at a measurement angle θ_1 for photon 1 and θ_2 for photon 2 for a state $|\Psi\rangle$ may be given by

$$E_{\text{Q.M.}}(\theta_1, \theta_2) = \langle \Psi | Q(\theta_1) \otimes Q(\theta_2) | \Psi \rangle. \quad (\text{A.5})$$

Now we may compare this with that of a two channel analyzer T which has the property

$$T(\theta) |0\rangle = +1 |0\rangle \quad \text{and} \quad T(\theta) |0_{\perp}\rangle = -1 |0_{\perp}\rangle \quad (\text{A.6})$$

for any angle θ . By demanding that T be hermitian, we can obtain

$$T(\theta) = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} \quad (\text{A.7})$$

which is unitary as we expect, since a measurement T, unlike Q, does not in principle involve zero countings, thus preserving its unitarity.

Hence, in this case we have a different expectation value for the coincident measurement of polarizations for a correlated pair of photons in the same state:

$$E'_{\text{Q.M.}}(\theta_1, \theta_2) = \langle \Psi | T(\theta_1) \otimes T(\theta_2) | \Psi \rangle. \quad (\text{A.8})$$

Let's compare the consequences of Eqs. (A.5) and (A.8) in the case of perfectly entangled states, which is equivalent to the one used in Bell's original inequality. This state, in principle, can be readily obtained, for example, from para-positronium decay:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\{ |+\rangle_1 |-\rangle_2 - |-\rangle_1 |+\rangle_2 \} \quad (\text{A.9})$$

where $|+\rangle_1$ denotes the x-polarization state of photon 1 and $|-\rangle_2$ denotes the y-polarization state of photon 2, etc.

By putting this state into Eqs.(A.5) and (A.8), we have

$$E_{\text{Q.M.}} = \frac{1}{2} \sin^2(\theta_1 - \theta_2) \quad (\text{A.10})$$

for one channel polarizers, and

$$E_{\text{Q.M.}} = -\cos(2\theta_1 - 2\theta_2) \quad (\text{A.11})$$

for two channel polarizers.

One should notice that the maximum and the minimum in the result (A.10) are severely limited by the inescapable zero countings of single channel polarizers.

Bell's original inequality for the ideal experiment of this type was given by

$$|E(a,b) - E(a,c)| \leq 1 + E(b,c). \quad (\text{A.12})$$

And in order for us to discriminate local realism from quantum mechanics, the quantum expectation value must violate this inequality for at least some angles. It is well known from the spin-1/2 case that the expectation value of the type of Eq.(A.11) violates the inequality for an extended region of angle, but it is easy to see that Eq.(A.10) cannot violate the inequality (A.12) at any angle, while we know that the expectation value of Eq.(A.10) violates the Clauser-Horne inequality^b which is subjected to an additional assumption, commonly known as the "no-enhancement assumption".

A.2. TWO CHANNEL PHOTON POLARIZATION ANALYZER VS. STERN-GERLACH APPARATUS IN EPR

In the spin-1/2 case, the operator equivalent to the two channel linear polarization analyzer T may be written as

$$(\sigma \cdot \hat{n}) = \begin{pmatrix} \cos\theta & \sin\theta e^{-i\phi} \\ \sin\theta e^{i\phi} & -\cos\theta \end{pmatrix}. \quad (A.13)$$

When this is applied to the correlated spin system that is equivalent to Eq.(A.9), where, this time $|+\rangle$ and $|-\rangle$ correspond to spin "up" and spin "down" respectively, we have

$$\begin{aligned} E(\hat{n}_1, \hat{n}_2) &= \langle \Psi | (\sigma \cdot \hat{n}_1) \otimes (\sigma \cdot \hat{n}_2) | \Psi \rangle \\ &= -\cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 \cos(\phi_1 - \phi_2) \end{aligned}$$

$$= - \hat{n}_1 \cdot \hat{n}_2, \quad (A.14)$$

where $\hat{n}_i$ denotes an arbitrary measurement direction (θ_i, ϕ_i) in three dimensional space. If we measure them in the x-y plane, or $\theta = \frac{\pi}{2}$, then this becomes

$$E(\phi_1, \phi_2) = -\cos(\phi_1 - \phi_2) \quad (A.15)$$

or, if we measure along any angle with $\phi_1 = \phi_2$, then this becomes

$$E(\theta_1, \theta_2) = -\cos(\theta_1 - \theta_2). \quad (A.16)$$

Needless to say, due to their transversality photons cannot have this additional correlation of (A.16) that spin-1/2 particles have.

Recently, a theorem that makes use of this full three dimensional degree of freedom of spin-1/2 particles was devised by Greenberger, Horne, and Zeilinger⁹ (GHZ) in a more fundamental refutation of local realism than that of Bell¹⁰. The theorem given by GHZ uses a system of four correlated spin- $\frac{1}{2}$ particles that can be represented by

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \{ |+\rangle_1 |+\rangle_2 |-\rangle_3 |-\rangle_4 - |-\rangle_1 |-\rangle_2 |+\rangle_3 |+\rangle_4 \}. \quad (A.17)$$

The expectation value for the coincident spin measurement of four particles along arbitrary directions n_i can then be given by

$$\begin{aligned} E &= \langle \Psi | (\sigma \cdot \hat{n}_1) \otimes (\sigma \cdot \hat{n}_2) \otimes (\sigma \cdot \hat{n}_3) \otimes (\sigma \cdot \hat{n}_4) | \Psi \rangle \\ &= \cos\theta_1 \cos\theta_2 \cos\theta_3 \cos\theta_4 \end{aligned}$$

$$- \sin\theta_1 \sin\theta_2 \sin\theta_3 \sin\theta_4 \cos(\phi_1+\phi_2-\phi_3-\phi_4) \quad (\text{A.18})$$

If we set all Stern-Gerlach detectors along the x-y plane to maximize the effect of cross terms, then we have

$$E = - \cos(\phi_1+\phi_2-\phi_3-\phi_4). \quad (\text{A.19})$$

This shows that in quantum mechanics we have a great degree of freedom to achieve the optimum values +1 or -1. For example, even if we set one detector in a specific direction ϕ_1 , we still have the freedom to vary two other angles ϕ_2 and ϕ_3 independently and continuously, since we always have a fourth angle ϕ_4 to adjust to achieve +1 or -1. What GHZ proved essentially was that it is impossible for local realism to produce this kind of rich correlation.

Now the question is: Can we have this kind of freedom in the photon polarization case? By using analyzers alone it is impossible, as we can see below.

With the one channel analyzer Q we have

$$\begin{aligned} E_Q &= \langle \Psi | Q_1 \otimes Q_2 \otimes Q_3 \otimes Q_4 | \Psi \rangle \\ &= \frac{1}{2} (\cos\theta_1 \cos\theta_2 \sin\theta_3 \sin\theta_4 - \sin\theta_1 \sin\theta_2 \cos\theta_3 \cos\theta_4)^2. \end{aligned} \quad (\text{A.20})$$

If we use two channel polarizers instead, then we would have

$$E_T = \langle \Psi | T_1 \otimes T_2 \otimes T_3 \otimes T_4 | \Psi \rangle$$

$$= \cos 2\theta_1 \cos 2\theta_2 \cos 2\theta_3 \cos 2\theta_4 - \sin 2\theta_1 \sin 2\theta_2 \sin 2\theta_3 \sin 2\theta_4. \quad (\text{A.21})$$

Use of one channel polarizers cannot yield +1 or -1, as was also the case with two photon correlation. In the two channel analyzer case, we may have these extreme values at some discrete sets of angles but not at continuously changable angles.

But one can do better than this by introducing retarders before the two channel analyzers.

The retarder with its azimuthal angle $\rho = 0$ and retardance δ may be represented by a matrix¹¹

$$W = \begin{pmatrix} e^{-i\delta/2} & 0 \\ 0 & e^{+i\delta/2} \end{pmatrix} \quad (\text{A.22})$$

where we suppressed an overall retarder phase factor as it will not affect our result. Then with this retarder-analyzer combination, the polarization measurement can be expressed by an operator

$$U = (W^\dagger T W) = \begin{pmatrix} \cos 2\theta & \sin 2\theta e^{+i\delta} \\ \sin 2\theta e^{-i\delta} & -\cos 2\theta \end{pmatrix} \quad (\text{A.23})$$

which is hermitian and unitary, and has a form remarkably similar to that of the spin measurement operator $(\sigma \cdot \hat{n})$ of Eq.(A.13). With this we can immediately have

$$\begin{aligned} E_U &= \langle \Psi | U_1 \otimes U_2 \otimes U_3 \otimes U_4 | \Psi \rangle \\ &= \cos 2\theta_1 \cos 2\theta_2 \cos 2\theta_3 \cos 2\theta_4 \end{aligned}$$

$$- \sin 2\theta_1 \sin 2\theta_2 \sin 2\theta_3 \sin 2\theta_4 \cos(\delta_1 + \delta_2 - \delta_3 - \delta_4). \quad (\text{A.24})$$

By setting $\theta_i = \frac{\pi}{4}$, we finally have

$$E = - \cos(\delta_1 + \delta_2 - \delta_3 - \delta_4). \quad (\text{A.25})$$

This provides exactly same form of expectation value as the spin-1/2 case and the GHZ proof would go through for the photon system as well.

By expanding the operator U in terms of Pauli spin matrices, we may see more clearly the fact that the retarder provides another angle degree of freedom to a photon polarization measurement through the retarder angle δ and thereby imitates the three dimensional measurement freedom of spin-1/2 particles:

$$(\sigma \cdot \hat{n}) = \sigma_x \sin\theta \cos\phi + \sigma_y \sin\theta \sin\phi + \sigma_z \cos\theta \quad (\text{A.26})$$

whereas

$$U = \sigma_x \sin 2\theta \cos(-\delta) + \sigma_y \sin 2\theta \sin(-\delta) + \sigma_z \cos 2\theta. \quad (\text{A.27})$$

Here we see that a retarder-two-channel-analyzer combination in photon polarization measurements works exactly same way as a Stern-Gerlach apparatus does in spin-1/2 measurements, with the variables of the latter, θ , and ϕ , being replaced by 2θ , $-\delta$. The factor of 2 here simply

shows the fact that photon polarization changes sign twice while the spin-1/2 changes its sign once, as we rotate the axes of the analyzers 180° .

Finally, it should be mentioned that this scheme, which makes a photon polarization measurement mathematically comparable to a three dimensional spin-1/2 measurement, is generally applicable not only to EPR but to any polarization measurement experiments that do not prohibit the use of retarders.

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APPENDIX B

MULTI-PHOTON SPONTANEOUS PARAMETRIC DOWN CONVERSION
AND ITS APPLICATION TO
MULTI-PARTICLE INTERFEROMETRY

A nonlinear optical spontaneous parametric down conversion of a photon into more than two photons is studied using a realistic Hamiltonian in a multi-mode. The general equations for the down converted photon number operator per mode are obtained both for the nondegenerate case and for the degenerated case, while particular attention is given to a three particle down conversion case. The study shows that at least a low rate of conversion may be obtained. Various practical problems associated with the production and the detection of the down converted photons, including phase matching problems are discussed.

Applications of the phenomenon to several multi-particle interference experiments, including a generalized Shimony-Horne-Zeilinger momentum-position interferometry, a generalized version of Franson's time-position interferometry, and a generalized version of Shih-Alley or Ou-Mandel type polarization interferometry, are proposed.

B.1. INTRODUCTION

A nonlinear optical *spontaneous* parametric down conversion¹ (SPDC) of a photon into two photons has been known to be an effective source of highly correlated photon pairs that exhibit many interesting nonclassical properties, such as the photon anti-bunching, the squeezed quantum states, and the violation of Bell and other classical inequalities. And some of its remarkable features, such as a fairly large joint detection probability, opened a possibility of some important applications such as near noise-free optical measurements and improved optical information transmissions^{2,3}.

A few⁴⁻⁷ have inquired into a possibility of generalizing two photon SPDC into many photon SPDC, mostly in the context of seeking generalized squeezed states. They found there exists a photon number divergence problem when the usual method of parametric approximation was employed and that was the subject of a number of studies⁴⁻⁷. Hillery⁶ recently cleared a way for a multi-photon down conversion by formally showing that the divergence problem does not occur when the pump is quantized.

Our independent study⁸ which starts with more realistic Hamiltonian which reduces to Hillery's when the system is degenerated and meets an ideal phase matching conditions also confirms Hillery's work on the issue.

Although some practical problems of phase matching, competing processes, and overheating of the media, etc. still exist for a decent conversion, as we shall discuss them in this paper, it may be argued that it should be possible to have the phenomenon at least in the low conversion rate limit. Considering that many important nonclassical applications in the two particle case does not require a high conversion rate, we should be able to use the multi-photon spontaneous parametric down conversion in the similar situations. In this low conversion limit, which is far below the divergent time, it can be shown that parametric approximation still holds as a very good approximation.

One of its immediate areas of application we consider is a multi-particle interferometry in which we measure the correlation properties of more than two particles.

A two-particle interferometry was recently opened⁹⁻¹⁵ and a number of experiments have been performed using a two particle parametric down conversion technique. We extend, in Sec.V, the two particle interferometry into a n-particle one and show that it can be implemented by using the multi-particle down conversion described in Sec.II-IV. These experiments can be done either in the generalized form of a two particle momentum-position interferometry proposed by Horne, Shimony, and Zeilinger⁹ and realized by Rarity and Tabster¹⁰, or in the generalized form of a position-time interferometry proposed by Franson¹¹ and implemented by Ou et al.¹² and by Kwiat et al.¹³, or in the generalized form of

Shih and Alley¹⁴ or Ou and Mandel¹⁵ type of polarization correlation experiments.

B.2 MULTI-PHOTON PARAMETRIC DOWN CONVERSION

Though we are interested in all the higher order processes, we will start with a Hamiltonian involving third order nonlinearity, as it is the next higher order nonlinear process beyond the usual two particle down conversion. In the parametric approximation, the interaction Hamiltonian¹⁶ which allows multiple mode down conversion can be written as

$$\begin{aligned}
 H_I = & \int_V d\mathbf{v} \sum_{\vec{k}_1, \alpha_1} \sum_{\vec{k}_2, \alpha_2} \sum_{\vec{k}_3, \alpha_3} \tilde{K} \chi_{oijl}(\omega_o; \omega_{\vec{k}_1}, \omega_{\vec{k}_2}, \omega_{\vec{k}_3}) \\
 & \times (\hat{e}_{\vec{k}_1 \alpha_1})_i (\hat{e}_{\vec{k}_2 \alpha_2})_j (\hat{e}_{\vec{k}_3 \alpha_3})_l \\
 & \times \{ \hat{a}_{\vec{k}_1 \alpha_1} \hat{a}_{\vec{k}_2 \alpha_2} \hat{a}_{\vec{k}_3 \alpha_3} e^{-i\Lambda \vec{k} \cdot \mathbf{r} + i\omega_o t} + \hat{a}_{\vec{k}_1 \alpha_1}^+ \hat{a}_{\vec{k}_2 \alpha_2}^+ \hat{a}_{\vec{k}_3 \alpha_3}^+ e^{i\Lambda \vec{k} \cdot \mathbf{r} - i\omega_o t} \},
 \end{aligned}
 \tag{B.1}$$

where $\tilde{K}$ is a coupling that is proportional to the pump field amplitude E_o , and contains the quantization factors $\left[\frac{\hbar \omega_{\vec{k}_1}}{2\epsilon_1} \right]^{1/2}$,

$\left[\frac{\hbar \omega_{\vec{k}_2}}{2\epsilon_2} \right]^{1/2}$, and $\left[\frac{\hbar \omega_{\vec{k}_3}}{2\epsilon_3} \right]^{1/2}$, and $\hat{e}_{\vec{k}\alpha}$ is a unit polarization

vector which depends on the wave vector $\vec{k}$ and the polarization α ($\alpha = 1, 2$), and $\hat{a}_{\vec{k}\alpha}$, $\hat{a}_{\vec{k}\alpha}^+$ are the photon annihilation and creation operators for the mode $\vec{k}, \alpha$ of frequency $\omega_{\vec{k}}$.

And also,

$$\Delta \vec{k} = \vec{k}_0 - \vec{k}_1 - \vec{k}_2 - \vec{k}_3, \quad (\text{B.2})$$

and v is the volume of the interaction region of the nonlinear medium.

If a nonlinear material is spatially uniform and rectangular parallelepiped centered at $\vec{r}_0$ with sides L_1 , L_2 , L_3 , we would have a total Hamiltonian of the system after the space integration as

$$\begin{aligned} H = H_0 + H_I = & \sum_{\vec{k}, \alpha} \hbar \omega_{\vec{k}} \hat{a}_{\vec{k}\alpha}^+ \hat{a}_{\vec{k}\alpha} \\ & + \sum_{\vec{k}_1, \alpha_1} \sum_{\vec{k}_2, \alpha_2} \sum_{\vec{k}_3, \alpha_3} \hbar \chi_{oijl}(\omega_0; \omega_{\vec{k}_1}, \omega_{\vec{k}_2}, \omega_{\vec{k}_3}) \\ & \times (\hat{e}_{\vec{k}_1\alpha_1})_i (\hat{e}_{\vec{k}_2\alpha_2})_j (\hat{e}_{\vec{k}_3\alpha_3})_l \\ & \times \{ \hat{a}_{\vec{k}_1\alpha_1} \hat{a}_{\vec{k}_2\alpha_2} \hat{a}_{\vec{k}_3\alpha_3} e^{i\omega_0 t} + \hat{a}_{\vec{k}_1\alpha_1}^+ \hat{a}_{\vec{k}_2\alpha_2}^+ \hat{a}_{\vec{k}_3\alpha_3}^+ e^{-i\omega_0 t} \} \\ & \times \prod_{m=1}^3 \frac{\sin[(\Delta \vec{k}_m \cdot \vec{L}_m)/2]}{\Delta \vec{k}_m/2} \end{aligned} \quad (\text{B.3})$$

For the limit $L_m \gg \Delta \vec{k}$, the last multiplication factor in Eq.(B.3) may be approximated by a δ function, $\delta(\vec{k}_0 - \vec{k}_1 - \vec{k}_2 - \vec{k}_3)$.

We may obtain an expression for a time development of the operators $\hat{a}$, $\hat{a}^+$ using Heisenberg equation of motion:

$$\dot{\hat{a}}_{\vec{k}\alpha} = -i\omega_{\vec{k}} \hat{a}_{\vec{k}\alpha} - i \sum_{\vec{k}_1, \alpha_1} \sum_{\vec{k}_2, \alpha_2} \hbar \chi_{oijl}(\omega_0; \omega_{\vec{k}_1}, \omega_{\vec{k}_2}, \omega_{\vec{k}_3}) \hat{a}_{\vec{k}_1\alpha_1}^+ \hat{a}_{\vec{k}_2\alpha_2}^+ e^{-i\omega_0 t} \delta(\vec{k}_0 - \vec{k} - \vec{k}_1 - \vec{k}_2) \quad (\text{B.4a})$$

and

$$\dot{\hat{a}}_{\vec{k}\alpha}^+ = i\omega_{\vec{k}} \hat{a}_{\vec{k}\alpha}^+ + i \sum_{\vec{k}_1, \alpha_1} \sum_{\vec{k}_2, \alpha_2} \kappa \hat{a}_{\vec{k}_1\alpha_1} \hat{a}_{\vec{k}_2\alpha_2} e^{i\omega_0 t} \delta(\vec{k}_0 - \vec{k} - \vec{k}_1 - \vec{k}_2) \quad (\text{B.4b})$$

where

$$\begin{aligned} \kappa = \frac{\tilde{\kappa}}{\hbar} \chi_{oijl}(\omega_0; \omega_{\vec{k}}, \omega_{\vec{k}_1}, \omega_{\vec{k}_2}) \{ & (\hat{e}_{\vec{k}\alpha})_i (\hat{e}_{\vec{k}_1\alpha_1})_j (\hat{e}_{\vec{k}_2\alpha_2})_l \\ & + (\hat{e}_{\vec{k}_1\alpha_1})_i (\hat{e}_{\vec{k}\alpha})_j (\hat{e}_{\vec{k}_2\alpha_2})_l + (\hat{e}_{\vec{k}_1\alpha_1})_i (\hat{e}_{\vec{k}_2\alpha_2})_j (\hat{e}_{\vec{k}\alpha})_l \} \end{aligned} \quad (\text{B.5})$$

and we used the fact that $\sum_{\vec{k}_1, \alpha_1}$, $\sum_{\vec{k}_2, \alpha_2}$, and $\sum_{\vec{k}_3, \alpha_3}$ are the summations over dummy indices, and the symmetric property of $\chi^{(3)}$ with respect to the three downconverted frequencies:

$$\begin{aligned} \chi_{oijl}(\omega_0; \omega_{\vec{k}}, \omega_{\vec{k}_1}, \omega_{\vec{k}_2}) &= \chi_{oijl}(\omega_0; \omega_{\vec{k}}, \omega_{\vec{k}_2}, \omega_{\vec{k}_1}) \\ &= \chi_{oijl}(\omega_0; \omega_{\vec{k}_1}, \omega_{\vec{k}}, \omega_{\vec{k}_2}) = \text{etc.} \end{aligned} \quad (\text{B.6})$$

A major difference of Eqs.(B.4) in a higher order down conversion from the equivalent of two photon case is that, this time, δ function at the end of Eqs.(B.4) cannot eliminate the summations (or, integrals, for continuum) over $\vec{k}_1$, $\vec{k}_2$. In other words, for a given $\vec{k}$, $\vec{k}_1$ and $\vec{k}_2$ cannot be uniquely determined unless we have a special selection mechanism such as photon resonances for the specific down converted frequencies.

We temporarily assume that this selection mechanism exists and therefore $\vec{k}_2$ and $\vec{k}_3$ are uniquely determined for a given $\vec{k}_1$, as we shall discuss this ideal phase matching little further in the next section. Only then, the

summations drop out to yield a simpler form of Hamiltonian with which Hillery⁶ and others^{4,5,7} have worked earlier.

Then we have simpler expressions of the time development for the operators $\hat{a}$, $\hat{a}^\dagger$:

$$\dot{\hat{a}}_{\vec{k}_i\alpha_i} = -i\omega_{\vec{k}_i} \hat{a}_{\vec{k}_i\alpha_i} - iK' \hat{a}_{\vec{k}_j\alpha_j}^\dagger \hat{a}_{\vec{k}_l\alpha_l}^\dagger e^{-i\omega_0 t}, \quad (\text{B.7a})$$

and

$$\dot{\hat{a}}_{\vec{k}_i\alpha_i}^\dagger = i\omega_{\vec{k}_i} \hat{a}_{\vec{k}_i\alpha_i}^\dagger + iK' \hat{a}_{\vec{k}_j\alpha_j} \hat{a}_{\vec{k}_l\alpha_l} e^{i\omega_0 t}, \quad (\text{B.7b})$$

where

$$K' = \frac{K}{\hbar} \chi_{oijl}(\omega_0; \omega_{\vec{k}_1}, \omega_{\vec{k}_2}, \omega_{\vec{k}_3}) \{ (\hat{e}_{\vec{k}_1\alpha_1})_i (\hat{e}_{\vec{k}_2\alpha_2})_j (\hat{e}_{\vec{k}_3\alpha_3})_l \\ + (\hat{e}_{\vec{k}_1\alpha_1})_i (\hat{e}_{\vec{k}_3\alpha_3})_j (\hat{e}_{\vec{k}_2\alpha_2})_l + (\hat{e}_{\vec{k}_2\alpha_2})_i (\hat{e}_{\vec{k}_3\alpha_3})_j (\hat{e}_{\vec{k}_1\alpha_1})_l \\ + (\hat{e}_{\vec{k}_2\alpha_2})_i (\hat{e}_{\vec{k}_1\alpha_1})_j (\hat{e}_{\vec{k}_3\alpha_3})_l + (\hat{e}_{\vec{k}_3\alpha_3})_i (\hat{e}_{\vec{k}_1\alpha_1})_j (\hat{e}_{\vec{k}_2\alpha_2})_l \\ + (\hat{e}_{\vec{k}_3\alpha_3})_i (\hat{e}_{\vec{k}_2\alpha_2})_j (\hat{e}_{\vec{k}_1\alpha_1})_l \}, \quad (\text{B.8})$$

and i, j , and l are cyclic in 1, 2, and 3.

From these equations, we have an usual Manley-Rowe relation

$$\frac{d}{dt} (\hat{a}_{\vec{k}_1\alpha_1}^\dagger \hat{a}_{\vec{k}_1\alpha_1}) = \frac{d}{dt} (\hat{a}_{\vec{k}_2\alpha_2}^\dagger \hat{a}_{\vec{k}_2\alpha_2}) = \frac{d}{dt} (\hat{a}_{\vec{k}_3\alpha_3}^\dagger \hat{a}_{\vec{k}_3\alpha_3}). \quad (\text{B.9})$$

Since we are looking for a spontaneous down conversion in which

$$\langle N_1(0) \rangle = \langle N_2(0) \rangle = \langle N_3(0) \rangle = 0, \quad (\text{B.10})$$

we will have the same number of down converted photons for each mode

$$\langle N_1(t) \rangle = \langle N_2(t) \rangle = \langle N_3(t) \rangle. \quad (B.11)$$

By taking second derivatives of N , we have the equations for the down converted photon number operators.

$$\dot{N} = 2\kappa_{(2)}^2 (2N+1) \quad (2\text{-photon}) \quad (B.12a)$$

$$\dot{N} = 2\kappa_{(3)}^2 (3N^2+3N+1) \quad (3 \text{ photon}) \quad (B.12b)$$

$$\dot{N} = 2\kappa_{(4)}^2 (4N^3+6N^2+4N+1) \quad (4\text{-photon}) \quad (B.12c)$$

and

$$\dot{N} = 2\kappa_{(m)}^2 \{ (N+1)^m - N^m \} \quad (m \text{ photon}) \quad (B.12d)$$

for the non-degenerated downconverted photons, and

$$\dot{N} = 8\kappa'_{(2)}^2 (2N+1) \quad (2 \text{ photon}) \quad (B.13a)$$

$$\dot{N} = 18\kappa'_{(3)}^2 (3N^2+3N+2) \quad (3 \text{ photon}) \quad (B.13b)$$

$$\dot{N} = 32\kappa'_{(4)}^2 (4N^3+6N^2+14N+6) \quad (4 \text{ photon}) \quad (B.13c)$$

and

$$\dot{N} = 2m^2\kappa'_{(m)}^2 \sum_{i=N-m+2}^{N+1} i(i+1)(i+2)\cdots(i+m-2) \quad (m \text{ photon}) \quad (B.13d)$$

for the degenerated down-converted photons. Here we used photon number operator N for simpler expressions rather than average photon number $\langle N \rangle$. The relation between $\langle N^m \rangle$ and $\langle N \rangle^m$ may be found in reference 6.

Except for the two photon case of Eq.(B.12a) which has a well known analytic solution $N = \sinh^2 \kappa t$ that diverges at

infinity and the corresponding non-degenerated case of Eq.(B.13a), we may see from the Eqs. (B.12) and (B.13) that in all higher order cases photon number diverges faster than it does in the two photon case, since N is positive and the double derivative which represent the change of the slope takes up a greater value as N increases. Fig.B.1 gives a numerical plot of the divergence of the degenerated photon number as a function of interaction time for the first few equations of Eqs.(12). The photon number diverges at finite time in all higher order processes.

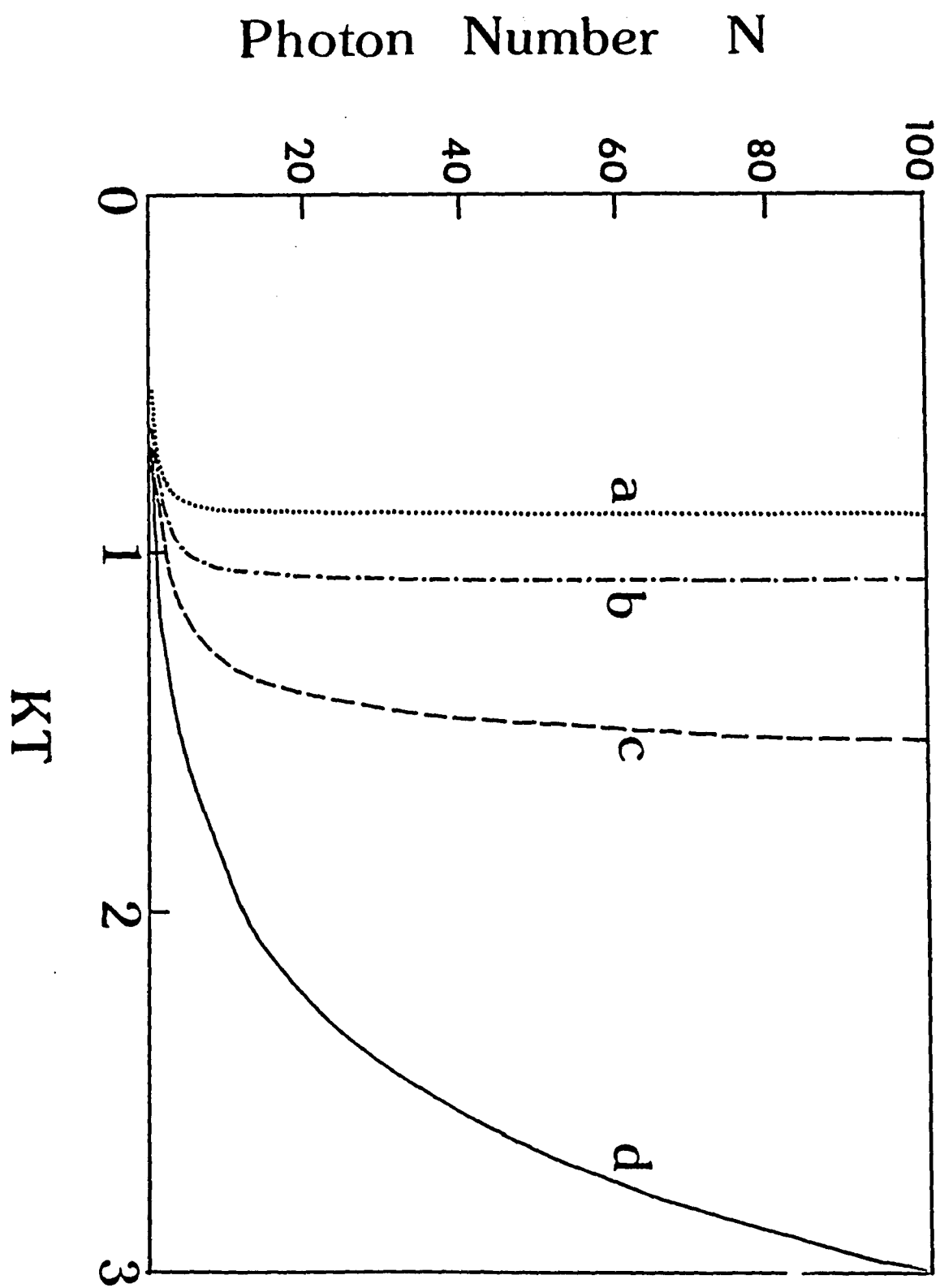


Fig.B.1: The first few equations of Eqs.(B.12), the photon number as a function of Kt in a multi-photon down conversion, is solved numerically. All $K(m)$ are treated as equal, to show the relative slope of the curves. a,b,c, and d represent the curves for the five, four, three, and two photon cases respectively. The photon number scale is dramatically adjusted ($N_0 = 100$).

B.3 QUANTIZED PUMP

If we quantize the pump field, the interaction Hamiltonian H_I in Eq.(B.1) becomes

$$\begin{aligned}
 H_{IQ} = & \int d\mathbf{v} \sum_{\vec{k}_0, \alpha_0} \sum_{\vec{k}_1, \alpha_1} \sum_{\vec{k}_2, \alpha_2} \sum_{\vec{k}_3, \alpha_3} \tilde{K}_Q \chi_{oijl}(\omega_0; \omega_{\vec{k}_1}, \omega_{\vec{k}_2}, \omega_{\vec{k}_3}) \\
 & \times (\hat{e}_{\vec{k}_1 \alpha_1})_i (\hat{e}_{\vec{k}_2 \alpha_2})_j (\hat{e}_{\vec{k}_3 \alpha_3})_l (\hat{e}_{\vec{k}_0 \alpha_0})_o \\
 & \times \{ \hat{a}_{\vec{k}_1 \alpha_1} \hat{a}_{\vec{k}_2 \alpha_2} \hat{a}_{\vec{k}_3 \alpha_3} \hat{a}_{\vec{k}_0 \alpha_0}^+ e^{-i\Lambda \vec{k} \cdot \vec{r}} + \hat{a}_{\vec{k}_1 \alpha_1}^+ \hat{a}_{\vec{k}_2 \alpha_2}^+ \hat{a}_{\vec{k}_3 \alpha_3}^+ \hat{a}_{\vec{k}_0 \alpha_0} e^{i\Lambda \vec{k} \cdot \vec{r}} \},
 \end{aligned}
 \tag{B.14}$$

where $\tilde{K}_Q$ is a quantum pump equivalent to K in parametric approximation and contain a factor $\left(\frac{\hbar \omega_0}{2\epsilon_0} \right)^{1/2}$, instead of the pump field amplitude E_0 .

Similar calculation to the previous section yields the time derivatives of the operators $\hat{a}^+$, $\hat{a}$ as

$$\dot{\hat{a}}_{\vec{k}_i \alpha_i} = -i\omega_{\vec{k}_i} \hat{a}_{\vec{k}_i \alpha_i} - iK'_0 \hat{a}_{\vec{k}_j \alpha_j}^+ \hat{a}_{\vec{k}_l \alpha_l}^+ \hat{a}_{\vec{k}_0 \alpha_0}, \tag{B.15a}$$

and

$$\dot{\hat{a}}_{k_i \alpha_i}^+ = i\omega_{k_i} \hat{a}_{k_i \alpha_i}^+ + iK'_Q \hat{a}_{k_j \alpha_j} \hat{a}_{k_l \alpha_l} \hat{a}_{k_o \alpha_o}^+, \quad (\text{B.15b})$$

in which i, j, and l are cyclic in 1, 2, and 3,

and

$$\dot{\hat{a}}_{k_o \alpha_o} = -i\omega_{k_o} \hat{a}_{k_o \alpha_o} - iK'_Q \hat{a}_{k_i \alpha_i} \hat{a}_{k_j \alpha_j} \hat{a}_{k_l \alpha_l}^+, \quad (\text{B.16a})$$

and

$$\dot{\hat{a}}_{k_o \alpha_o}^+ = i\omega_{k_o} \hat{a}_{k_o \alpha_o}^+ + iK'_Q \hat{a}_{k_i \alpha_i}^+ \hat{a}_{k_j \alpha_j}^+ \hat{a}_{k_l \alpha_l}^+, \quad (\text{B.16b})$$

where K'_Q is a quantum pump equivalent to a K' in Eq.(B.8) of parametric approximation. From these, a time development of the down converted photon number can be calculated. For example, for three photon degenerated case we have

$$\dot{N} = 18 K'_Q{}^2 \{ (3N^2 + 3N + 2)N_0(t) - (N^3 + 5N + 2) \}, \quad (\text{B.17})$$

where $N_0(t)$ gives the expression for the depleted pump beam and is related to a down converted beam as

$$\langle N_0(t) \rangle = \langle N_0(0) \rangle - \langle N(t) \rangle. \quad (\text{B.18})$$

The extra term with a negative sign in Eq.(B.17) will slow down the change of the slope of the curve when the pump depletion becomes significant as we can see from Fig.2. Notice that the expression in the quantum pump reduces to a parametric approximation for $N_0(t) \gg N$. The photon number will eventually oscillate greatly in large $K'_Q t$.

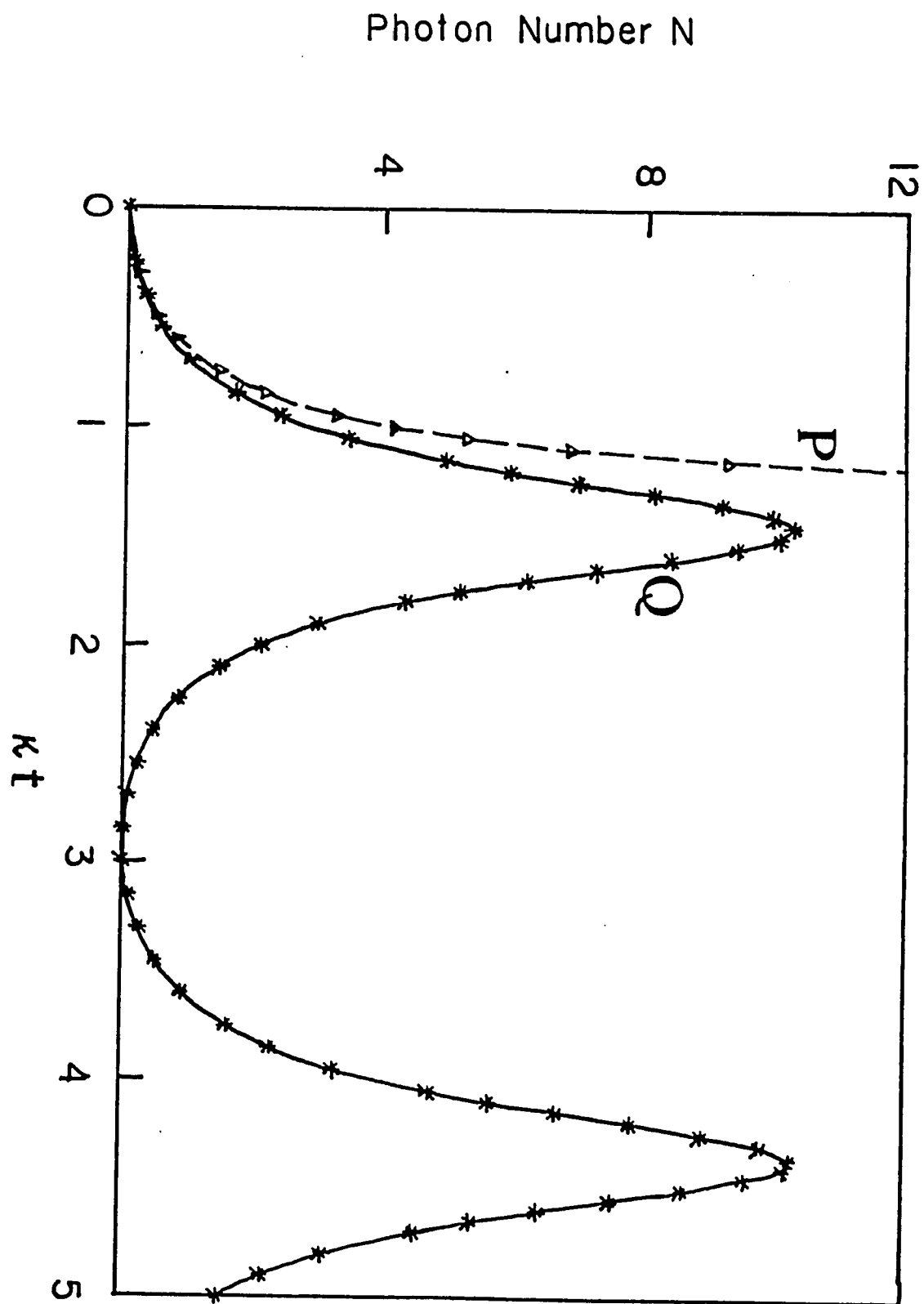


Fig.2: A down converted photon number as a function of interaction time for the case of degenerated three photon down conversion is solved numerically both in a quantized pump (Q; solid line) and in parametric approximation (P; broken line). The parametric approximation would serve as a good approximation in a low conversion limit. The scale for the photon number has been dramatically adjusted ($N_0(0) = 10$) to show the differences effectively.

B.4 GENERATION OF HIGHER ORDER DOWNCONVERTED PHOTONS

Although, for any given material, we expect nonlinear susceptibility $\chi^{(3)}$ for three photon down conversion would take a value similar to that for third harmonic generation (THG) assuming no resonant conditions, there are some practical problems associated with the generation of the multi-photon down conversion that make the phenomenon harder to achieve than THG.

The first and seemingly the biggest problem is that of phase matching. To obtain a strong conversion would require phase-matching to be satisfied, but it is not difficult to see that we would have some combinations of the down converted frequencies. There may be too many of such combinations of the down-converted frequencies to get a decent signal for any one of them unlike the cases of THG or

stimulated parametric down conversion (or, four wave different frequency mixing).

Another major problem arises from the high order nature of the phenomenon that requires a high intensity input but, this time, for the higher frequency one, which usually is more damaging to the material. The ratio of the magnitude of nonlinear polarization in successive order may be approximately given by $|\vec{P}_{(n+1)}/\vec{P}_{(n)}| \simeq |\vec{E}|/|\vec{E}_{\text{atom}}|$, where $\vec{E}$ denotes the input field amplitude. This would yield a value of 3×10^{-3} for a pump intensity of 10^{10} Watt/cm², while the typical dielectric breakdown intensities of gases, which are generally more endurable than solids and liquids, are of the order of 10^8 - 10^{13} Watt/cm². A more energetic pump would also invite more of the competing processes such as Raman and Brillouin scatterings, multi-photon absorptions, multi-photon ionizations, Stark shifts or Kerr effect, that limit the conversion efficiency considerably, or may break a phase matching of the system¹⁷.

The situation, on the other hand, is not so bad for the case of three photon down conversion, since practically any system could exhibit the phenomenon, whereas the two photon case requires absence of inversion symmetry. And we already have many materials that exhibit effective third harmonic generation which, in fact, is an inverse process of the phenomenon in a quantum point of view. For the problem of phase matching, it is easy to see that the ideal phase matching assumption we made in deriving Eqs.(B.12) and

(B.13) is not essential and a similar calculation of the photon number can be carried out for the cases in which the summations in Eq.(B.4) are not eliminated. Hence the solutions for the photon number equations of the type of Eqs.(B.12), (B.13) will exist for the non-ideal phase matching case.

Also, one may increase the intensity of down converted photons by decreasing possible number of combinations of down converted frequencies that meet the index matching conditions. One way of achieving this may be to use a material in which index curve changes rapidly at the vicinity of the desired down converted frequencies. Manipulation of refractive index may be possible either through electro-optical effects for the crystals, or through the use of mixture or anomalous dispersion for the liquids and gases¹⁷. A proper use of two photon resonance would also significantly increase the intensity of down converted photons as was the case for THG. Some gases and/or metal vapors that have been used for the generation of non-tunable (therefore, a smaller number of combinations for the down converted frequencies) infrared radiation through a $\chi^{(3)}$ process should be among the first to be considered.

Some^{15,19} reported an unusually high third order susceptibility $\chi^{(3)}$ as high as 10^{-7} - 10^{-8} esu with a very fast reponse time in some semiconductors. Though the theories for these materials are still not very well established, it is an encouraging development since a higher

$\chi^{(3)}$ means that considerably lower pump intensity is required to produce the same amount of converted photons.

A use of a birefringent material for the phase matching in THG has been reported by Terhune et al.^{20,21} In calcite, they had a phase matching $2\vec{k}_o(\omega) + \vec{k}_e(\omega) = \vec{k}_e(3\omega)$ at 57° from the trigonal axis, where the subscripts o and e refer to the ordinary and extraordinary modes of polarization. Hence we have a candidate with a good phase matching. They have used the Q-switched laser with an intensity just below the critical value for damages to obtain 10^4 photons per laser pulse with $\chi^{(3)} = 10^{-15}$ esu.

B.5 MULTI-PARTICLE INTERFEROMETRY

Two fundamental relations for a multi-photon spontaneous parametric down conversion, i.e.

$$\vec{k}_0 = \vec{k}_1 + \vec{k}_2 + \cdots + \vec{k}_n, \quad (\text{B.19})$$

and

$$\omega_0 = \omega_1 + \omega_2 + \cdots + \omega_n, \quad (\text{B.20})$$

along with the fact that the pump beam with $\vec{k}_0$ and ω_0 is a coherent one and that the n-photon down converted state is represented by the *product* of the individual photon states, tells us that each individual down converted photon doesn't have a definite phase and that only the total system carries a definite phase information. Hence we will observe an interference between n photons, but not from a single photon

nor between 2, 3, $\dots$, $n-1$ photons. This fact was indeed verified in two-photon SPDC by the absence of second order interference and the strong presence of fourth order interference²². This n -photon correlation property opens a new field of multi-photon interferometry in which one measures the joint detection probability of n photons.

Our general scheme for a multi-photon interferometry starts with forming a quantum mechanically entangled state²³. Though it doesn't necessarily have to be a perfectly entangled one, we will give a typical perfectly entangled state for a maximum effect.

$$|\Psi\rangle = 2^{-1/2} \{ |_{+1+2\cdots+n}\rangle + e^{i(\Phi_1+\Phi_2+\cdots+\Phi_n)} |_{-1-2\cdots-n}\rangle \}$$

(B.21)

where $|_{+i}\rangle$ and $|_{-i}\rangle$ refer to the two different possible states of photon i and Φ_i represents the phase difference between those two states. It is a matter of indifference that if $|_{+i}\rangle$ and $|_{-i}\rangle$ states are switched for any particle(s) i .

These two possible states may represent different momentum states, as was suggested by Horne-Shimony-Zeilinger⁹ (HSZ) in their two particle interferometry, or the states that belong to two different emission times as in Franson¹¹ experiment, or two orthogonal polarization states of photon as in Shih-Alley¹⁴ or Ou-Mandel¹⁵ type EPR experiments.

Now if our measurement M on the system involves off-diagonal matrix elements in the representation of $|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

and $|-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$,

i.e., if it involves a measurement of the mixture of two possible states, then the quantum mechanical expectation value $\langle \Psi | M | \Psi \rangle$ for the measurement will generally contain a term that oscillate sinusoidally with $\Phi_1 + \Phi_2 + \dots + \Phi_n$.

These off diagonal elements or the mixing of the states may be achieved by making use of beam splitters (as was the case of HSZ and Franson experiments) or through the polarization analyzer whose axis lies in between the two orthogonal polarization axes. The use of beam splitters are practically very convenient in producing the off-diagonal terms, but it is not an essential feature of a multi-particle interferometry as it has been demonstrated by Greenberger et al.²⁴⁻²⁶ in their three or four spin-1/2 particle proof of Bell's theorem which proceeded without recombining beams.

The next task for the realization of a multi-particle interference experiment using n photon down conversion technique is how to pin down the directions of emissions of a set of correlated n photons.

In the two photon case, a set of correlated photon is always emitted in the opposite directions (180° apart) when we look toward the center of the nonlinear material from the

output side, both in degenerated case and in non-degenerated case as is required by momentum conservation. In the higher order processes, in general, more degrees of freedom in the possible directions of emission for the down converted photons would be involved, therefore, we would not have well defined directions of emitted down converted photons except for the three photon case.

The requirement given by Eq.(19) reduces to

$$(\vec{k}_1 + \vec{k}_2 + \cdots + \vec{k}_n)_{x-y \text{ plane}} = 0, \quad (\text{B.22})$$

when the pump beam is directed along z-direction, with the additional conditions

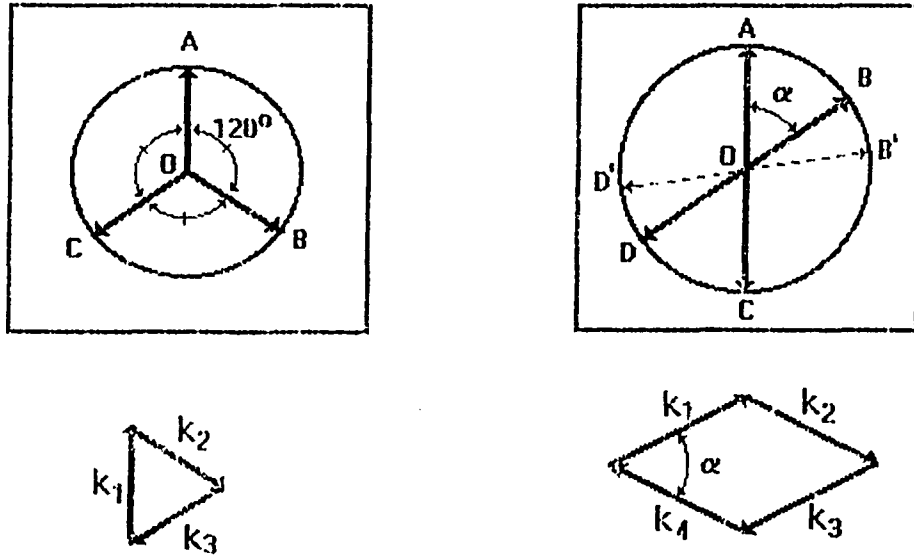
$$|\vec{k}_1| = |\vec{k}_2| = \cdots = |\vec{k}_n| \quad (\text{B.23})$$

for a degenerated case, and

$$|\vec{k}_1| \neq |\vec{k}_2| \neq \cdots \neq |\vec{k}_n| \quad (\text{B.24})$$

for a non-degenerated case.

Fig.B.3a and B.3b show the vector diagrams for the momenta of the degenerated down converted photons in the three and four photon cases. The circle represents a slice parallel to the x-y plane from the cone along which down converted photons are emitted.



[a] Three Photon Case

[b] Four Photon Case

Fig.B.3: A two dimensional vector diagram that satisfies the momentum conservation in the x-y plane in case of three and four photon degenerated down conversions according to Eq.(22). The vector diagram closes and the all sides have equal length in the degenerated case of Eq.(23), so that it forms a equitriangle for the three-photon case and a Rhombus of arbitrary internal angles for the four-photon case. Therefore, we may get the correlated triples from the apertures A,B,C in Fig.1,2,3 that are 120° apart from each other, but we wouldn't get any definite set of four correlated particles from any particular set of four apertures A,B,C,D in Fig.3b.

In a degenerated three photon case, since the vector diagram forms an equilateral triangle, we have three correlated photons in a circle with 120° apart from each other as we can see in Fig.B.4.

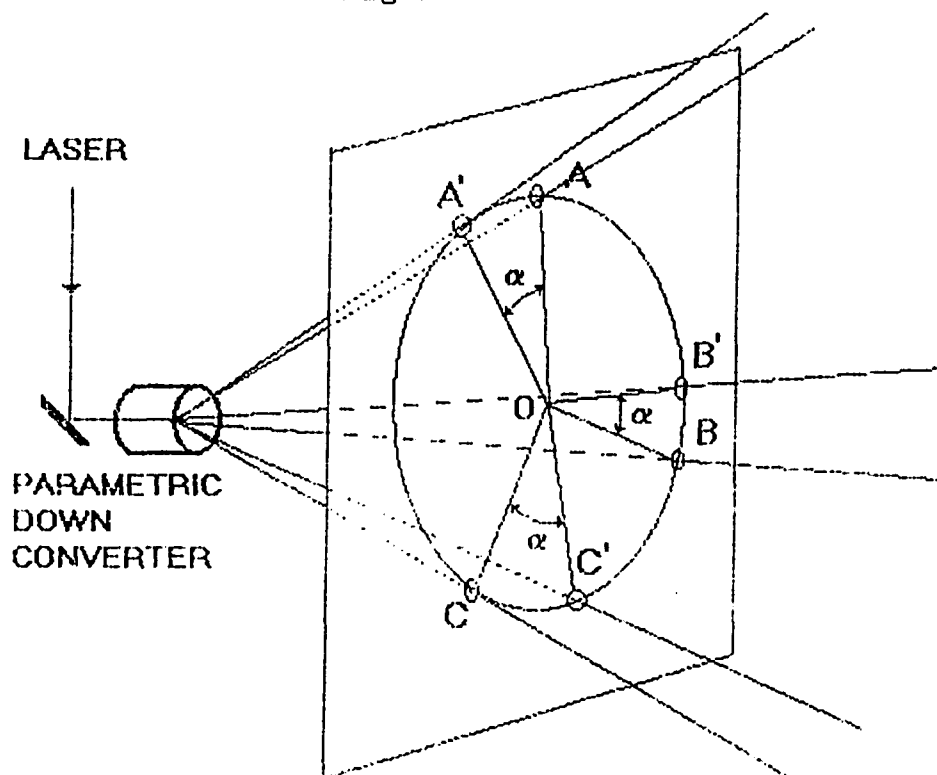


Fig.B.4: A three dimensional view of two sets of directions of emissions for the downconverted triples. A two set of pinholes A,B,C and A',B',C' select six beams from the output of a down converter. The apertures A,B,C are located at the equidistance from the center O toward which the pump beam emerges, and are 120° apart from each other, likewise for the apertures A',B',C'. The angle α between the two sets of apertures may be taken arbitrary.

In a non-degenerated case, we would have three different concentric circles each of which represents the directions for the down converted photons with different momentum and frequency, therefore the triangle formed by three vectors of the given magnitudes is decided within the two variations. Care must be taken when we determine the exact positions for the apertures in the x-y plane for this case, because the three photons will reach the plane at different times because of optical path differences among them.

A trouble arises when we consider the higher order processes than the three photon case, since it is generally impossible to pin down the directions of emission for the correlated photons.

Even in the simplest higher order case of degenerated four photon conversion in which all photons come out through the same circle, the momentum vector diagram will form a Rhombus with an arbitrary angle. Hence, even though we select one point for the photon 1, we will have a continuous degree of freedom in choosing the direction of the photon 2 in order to pin down the directions of the photons 3 and 4. This implies that we have an extremely small chance of getting a right set of correlated particles through a set of four openings.

(1) Generalized Horne-Shimony-Zeilinger⁹ interferometer:

The two photon momentum-position interferometer proposed

by HSZ has been implemented by Rarity and Tabster¹⁰. Recently, a three particle version of HSZ experiment was proposed by Greenberger, Horne, Shimony, and Zeilinger²⁶ to test against a family of local realism. Their gedanken three particle setup may be realizable through the three photon SPDC which we described in the previous sections. They have shown that one would have an expectation value for the three photon joint detection that oscillates sinusoidally with $\Phi_1 + \Phi_2 + \Phi_3$.

(2) Generalized Franson Interferometer:

Franson¹¹ devised a two particle gedanken interferometer that uses the interference between two possible states each of which belongs to a different emission time. The experiment was implemented by Ou et al.¹² and by Kwiat et al.¹³ A generalized three-photon Franson interferometry may use the Ou et al.-type of setup with three Michelson interferometers. The same analysis should go through as in the two photon case and the expectation value for the coincidence counting rate of three photons will exhibit a sinusoidal oscillation that depends upon the accumulated optical path differences between two possible paths.

(3) Polarization Interferometer:

Finally, we construct a third type of entangled state formed by two orthogonal polarizations of photons for a three photon polarization interferometer. We suppose all

three down converted photons are x-polarized. (We could in principle enforce this by placing x-filters after the apertures) In one set of paths we place half wave plates and in the other set of paths we place compensators and the variable phase shifters Φ_i . Then we combine the beams on the beam splitters so that the polarization states may be mixed before they are registered by two channel linear polarization analyzers. If we count a detection of an x-polarized photon as +1 and a y-polarized photon as -1, using two channel analyzers, then we would have a three-photon joint detection probability:²⁷

$$E(\Phi_1, \Phi_2, \Phi_3) \propto \cos(\Phi_1 + \Phi_2 + \Phi_3). \quad (\text{B.25})$$

B.6 CONCLUSION

Starting with a realistic Hamiltonian, we have shown that a multi-photon parametric down conversion is in principle possible and derived the equations for the time development of the photon number operators for both the degenerated case and the nondegenerated case. We have also discussed various problems associated with a practical generation of a multi-photon spontaneous down conversion. Three different types of multi-particle interferometry were proposed. This multi-particle interferometry is immediately applicable to the recent Greenberger-Horne-Zeilinger

gedanken experiment²⁴⁻²⁶ concerning Einstein-Podolsky-Rosen²⁸ problem.

Mermin^{29,30} and Greenberger et al.²⁴⁻²⁶ already have shown that the system that involves higher number of correlated particles exhibits more of the non-classical properties. We expect to have stronger violations of many of the classical inequalities through this multi-photon interferometry and to have some valuable practical applications through this remarkable feature of a multi-particle correlation that is provided by the simultaneously produced correlated photons.

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