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ESSAYS IN FINANCIAL ECONOMICS

by

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ABSTRACT

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Essay1: In this essay, we investigate the theoretical and empirical differences between the convexity adjustment method and the LIBOR market model in pricing two-period constant maturity swap (CMS). Using daily data, we obtain the differences (spreads) between the two-period CMS rates calculated from the convexity adjustment and those from the LIBOR market model. The convexity adjustment method yields higher CMS rates than the LIBOR market model does. The spreads are correlated with the cap volatilities and the yield curves.

Essay 2: Developing countries need flexibility in borrowing from the international capital markets, and are susceptible to liquidity risk when foreign capital flow reverses. Contingent credit lines (CCL) contract could be used to inject liquidity and back the exchange reserves held in the central banks. This essay presents a pricing method for the CCL contract signed by sovereign borrowers and banks. CCL can be modelled as a reverse knock-out option whose underlying instrument is credit spread. We apply the LIBOR market model under survival measure to price CCL for three countries: Argentina, Brazil and Mexico.

Essay 3: This essay introduces a conditional volatility estimator based on the skewed fat-tailed generalized error distribution (SGED) within a discrete-time GARCH model. The information content of the SGED-GARCH volatility estimators is compared with those of the implied volatility index (VIX) and the fitted realized volatility models for 1-day-ahead and 20-day-ahead forecasts of the S&P 100 index volatility. The in-sample and out-of-sample performance results based on the R^2 and the mean absolute percentage errors imply superior performance of the SGED-TGARCH and the VIX in capturing time-series variation in realized volatility. The results also suggest that nearly all information is provided by the SGED-TGARCH, the VIX, and the sum of the squared five-minute returns. There is little incremental information in the traditional volatility estimator based on the absolute demeaned daily index returns.

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1.1 Introduction

The purpose of this paper is to use the LIBOR[1] market model to price instruments that require convexity adjustment. The instrument we choose is constant maturity swap (CMS), a popular interest rate derivative. We compare its price calculated from two methods. One is the LIBOR market model, and the other is the standard convexity adjustment method.

We use daily data from April, 1991 to January, 1998 to conduct an empirical investigation on the performances of these two methods in pricing CMS. Our results indicate that for a two-period CMS, there is a significant price difference (spread) between these two methods. We define spread as the CMS rate calculated using the convexity adjustment minus that calculated using the LIBOR market model. The spread reaches 8.49 basis points in some dates. Since the LIBOR market model is more exact in pricing, we conclude that the standard convexity adjustment, although fairly close to the exact price, is still not a substitute for the LIBOR market model.

In addition to this general conclusion we show that the spread is always positive, implying that the convexity adjustment yields higher CMS rate than the LIBOR market model does. Finally, we show that the spread of the CMS rates is highly related with the underlying cap volatilities.

The paper is organized as follows. The following section provides a framework to price CMS. Section 3 has two parts. The first part discusses CMS and the convexity adjustment. Second part introduces the LIBOR market model. Section

[1]Abbreviation for London InterBank Offered Rate.

4 shows the pricing results using daily data. The last section concludes the paper.

1.2 Framework

Three components are required to construct a CMS. The structure of the CMS is based on a default-free environment with zero credit risk.

First we need an n-period forward fixed-payer interest rate swap with swap rate $s(t_0, t_1, t_{n+1})$, price fixed at t_0, start date t_1, settlement beginning at time t_2, and ending at t_{n+1}. δ is the year fraction between t_i and t_{i+1}, $i = 1, ..., n$. This interest rate swap is shown in the figure 1.1. We assume the period $[t_1 - t_0]$ is also equal to δ. The $\{t_1, ..., t_i, ..., t_n\}$ are the reset dates when the relevant LIBOR rates $\{L_{t_1}, ..., L_{t_i}, ..., L_{t_n}\}$ will be determined.

The second component is the $n+1$ default-free zero-coupon bonds whose prices at t_0 are $B(t_0, t_i)$'s, $i = 1, ..., n, n+1$. These prices are the amounts to pay at t_0 in exchange of receiving 1 dollar at the maturity dates t_i, $i = 1, ..., n, n+1$. Hence, $B(t_0, t_i)$ is the discount factor for the time period $[t_0, t_i]$, and

$$B(t_0, t_i) > B(t_0, t_j) \quad \forall\ t_i < t_j. \tag{1.1}$$

In general, $B(t_j, t_i), t_j \leq t_i$, is the discount factor determined at t_j for the time period $[t_j, t_i]$.

The last component is a sequence of forward rate agreements (FRA), which is shown in the figure 1.2. The figure shows the cash flow diagrams of $n-1$ paid-in-arrears FRAs. The FRAs determine the forward rates $F(t_0, t_i, t_{i+1})$'s for the future periods $[t_i, t_{i+1}]$, $i = 1, ..., n$. These forward rates are known at t_0. As t_0 approaches t_i, $F(t_0, t_i, t_{i+1})$ approaches $F(t_i, t_i, t_{i+1})$. $F(t_i, t_i, t_{i+1}) \triangleq L_{t_i}$ is the LIBOR rate known at t_i for the time period $[t_i, t_{i+1}]$. For each FRA, a floating payment is made against a fixed payment with the result of a net payment of $[L_{t_i} - F(t_0, t_i, t_{i+1})]\,\delta$ at t_{i+1}.

Arbitrage-free relationship between the forward rates and the corresponding LIBOR rates implies the following equality:

$$F(t_0, t_i, t_{i+1}) = \mathrm{E}_{t_0}^{p^{t_{i+1}}}\left[L_{t_i}\right] \quad \forall\, i = 1, ..., n, \tag{1.2}$$

where the probability $p^{t_{i+1}}$ represents the t_{i+1} forward measure using $B(t_0, t_{i+1})$ as the numeraire. (1.2) shows that the forward rate is an unbiased forecast of the relevant LIBOR rate under proper forward measure.

Using these relationships we can obtain the following arbitrage-free value of the forward swap rate

$$\begin{aligned} s(t_0, t_1, t_{n+1}) &= \frac{\sum_{i=1}^{n} B(t_0, t_{i+1}) F(t_0, t_i, t_{i+1})}{\sum_{i=1}^{n} B(t_0, t_{i+1})} \\ &= \sum_{i=1}^{n} \omega_i F(t_0, t_i, t_{i+1}), \end{aligned} \tag{1.3}$$

where

$$\omega_i = \frac{B(t_0, t_{i+1})}{\sum_{i=1}^{n} B(t_0, t_{i+1})}.$$

(1.3) shows that the swap rate is a weighted average of paid-in-arrears FRA rates. The weights, w_i's, are obtained from the zero coupon bond prices, which are functions of the forward rates:

$$B(t_0, t_i) = \frac{1}{\prod_{j=0}^{i-1} (1 + \delta F(t_0, t_j, t_{j+1}))} \tag{1.4}$$

and

$$1 + \delta F(t_0, t_i, t_{i+1}) = \frac{B(t_0, t_i)}{B(t_0, t_{i+1})}. \tag{1.5}$$

(1.5) leads to

$$F(t_0, t_i, t_{i+1}) = \frac{1}{\delta}\left[\frac{B(t_0, t_i)}{B(t_0, t_{i+1})} - 1\right]. \tag{1.6}$$

Substituting (1.6) into (1.3), we can get

$$s(t_0, t_1, t_{n+1}) = \frac{B(t_0, t_1) - B(t_0, t_{n+1})}{\delta \sum_{i=2}^{n+1} B(t_0, t_i)}. \tag{1.7}$$

For a forward swap that makes n payments with the start date t_j and the settlement date beginning at t_{j+1}, we have

$$s(t_0, t_j, t_{n+j}) = \frac{B(t_0, t_j) - B(t_0, t_{n+j})}{\delta \sum_{i=j+1}^{n+j} B(t_0, t_i)}. \tag{1.8}$$

Its relevant swap rate with price determined in the future ("future" swap rate) is

$$s(t_j, t_j, t_{n+j}) = \frac{B(t_j, t_j) - B(t_j, t_{n+j})}{\delta \sum_{i=j+1}^{n+j} B(t_j, t_i)}. \tag{1.9}$$

Forward swap rate is an unbiased forecast of the relevant (future) swap rate under the proper forward swap measure $\widetilde{p}$:

$$s(t_0, t_j, t_{j+n}) = \mathrm{E}_{t_0}^{\widetilde{p}}\left[s(t_j, t_j, t_{j+n})\right]. \tag{1.10}$$

CMS is a generalized vanilla interest rate swap. For a vanilla swap, in each settlement period, a fixed swap rate is exchanged against a floating LIBOR rate, the rate for that settlement period. For a CMS, in each settlement period, a fixed leg is also exchanged against a floating leg. Nevertheless, this floating leg has a longer maturity than the LIBOR rate has. Each floating leg is a spot swap rate $s(t_j, t_j, t_{j+n})$ with reset date t_j. t_{j+n} is the maturity date. For example, for the two-period CMS shown in the figure 1.3, its two floating legs are $s(t_1, t_1, t_3)$

and $s(t_2, t_2, t_4)$ which have reset dates t_1, and t_2; and maturity dates t_3, and t_4. These "future" swap rates are the functions of the underlying forward rates as (1.9) and (1.4) indicate. The conditional expectation of one "future" swap rate is the relevant forward swap rate under one proper forward swap measure as (1.10) shows. Nevertheless, this statement does not apply to more complex swaps, such as CMS. Since the conditional expectation of each floating leg of a CMS is based on different forward swap measure, the CMS's fixed leg is not a simple weighted average of its floating legs.

1.3 Pricing Convex Interest Rate Instruments

In practice, there are at least two methods to price convex interest rate instruments. The first method calculates the forward swap rate for each floating CMS leg and then adjusts for the convexity to get the expected value of the "future" swap rate

$$\hat{s}(t_j, t_j, t_{j+n}) = s(t_0, t_j, t_{j+n}) + \mu(t_0 : \sigma_{t_j}, t_j),$$

where $\hat{s}(t_j, t_j, t_{j+n})$ is the expected value of the "future" swap rate $s(t_j, t_j, t_{j+n})$ known at t_j, and $\mu(t_0 : \sigma_{t_j}, t_j)$ is the adjustment factor dependent on the volatility σ_{t_j}, and the reset date t_j.

The second approach employs the LIBOR market model to obtain the arbitrage-free trajectories of the forward LIBOR rates under one measure, and then calculate the CMS rate. Convexity adjustment is not needed since the "future" spot swap rates, i.e., floating legs, are expressed as the functions of these forward LIBOR rates under the same measure.

1.3.1 Market convention: the convexity adjustment

This section has two parts. The first part obtains a convexity adjustment using PV01 as the numeraire, and then obtains the expected "future" swap rate. The second part computes the CMS rate using these rates.

Convexity adjustment to forward swap rate

This section follows the methodology outlined in Section 2. The derivation below follows Hagan (2002). We repeat the following relationships derived before

$$s(t_0, t_j, t_{n+j}) = \frac{B(t_0, t_j) - B(t_0, t_{n+j})}{\delta \sum_{i=j+1}^{n+j} B(t_0, t_i)},$$

$$\mathrm{CMS}_{\text{floating}}(t_j) = s(t_j, t_j, t_{n+j}) = \frac{B(t_j, t_j) - B(t_j, t_{n+j})}{\delta \sum_{i=j+1}^{n+j} B(t_j, t_i)}.$$

$\mathrm{PV01}(t_0)$ and $\mathrm{PV01}(t_j)$ are the numerical durations known at t_0 and t_j:

$$\mathrm{PV01}(t_0) \triangleq \sum_{i=j+1}^{n+j} B(t_0, t_i),$$

$$\mathrm{PV01}(t_j) \triangleq \sum_{i=j+1}^{n+j} B(t_j, t_i). \tag{1.11}$$

We define $s(t_0, t_j, t_{n+j})$ as s_{t_0}, and $s(t_j, t_j, t_{n+j})$ as s_{t_j}. If s_{t_j} were known, the t_0 value of the CMS floating leg reset at t_j would be $\mathrm{CMS}^{t_0}_{\text{floating}}(t_j) = s_{t_j} B(t_0, t_{j+1})$.[2] Nevertheless, s_{t_j} is unknown at t_0. Using $\mathrm{PV01}(t_j)$ as the numeraire, we can rewrite the expected value of $\mathrm{CMS}^{t_0}_{\text{floating}}(t_j)$ under the forward swap measure as

$$\widehat{\mathrm{CMS}}^{t_0}_{\text{floating}}(t_j) = \hat{s}_{t_j} B(t_0, t_{j+1}) = \mathrm{PV01}(t_0)\mathrm{E}^{\tilde{p}}_{t_0}\left[\frac{s_{t_j} B(t_j, t_{j+1})}{\mathrm{PV01}(t_j)}\right]. \tag{1.12}$$

[2]The payment is made at t_{j+1}.

This equation presents the t_0 value of the CMS floating leg. By doing this, we can adjust all the forward swap rates based on the information set available at t_0.

Since

$$\mathrm{E}_{t_0}^{\tilde{p}}\left[s_{t_j}\right] = s_{t_0}$$

and

$$\mathrm{E}_{t_0}^{\tilde{p}}\left[\frac{B(t_j, t_{j+1})}{\mathrm{PV01}(t_j)}\right] = \frac{B(t_0, t_{j+1})}{\mathrm{PV01}(t_0)},$$

by using the simple statistical identity $\mathrm{E}[AB] = \mathrm{E}[A]\,\mathrm{E}[B] + \mathrm{Cov}[AB]$, we can obtain

$$\begin{aligned}
\widehat{\mathrm{CMS}}_{\text{floating}}^{t_0}(t_j) = {} & \mathrm{PV01}(t_0)\mathrm{E}_{t_0}^{\tilde{p}}\left[\frac{s_{t_j}B(t_j, t_{j+1})}{\mathrm{PV01}(t_j)}\right] \\
= {} & s_{t_0}B(t_0, t_{j+1}) \\
& + s_{t_0}B(t_0, t_{j+1})\mathrm{E}_{t_0}^{\tilde{p}}\left[\frac{(s_{t_j} - s_{t_0})}{s_{t_0}}\left(\frac{B(t_j, t_{j+1})\mathrm{PV01}(t_0)}{B(t_0, t_{j+1})\mathrm{PV01}(t_j)} - 1\right)\right] \\
= {} & s_{t_0}B(t_0, t_{j+1})\left\{1 + \mathrm{E}_{t_0}^{\tilde{p}}\left[\frac{s_{t_j} - s_{t_0}}{s_{t_0}}\left(\frac{\mathrm{PV01}(t_0)}{\mathrm{PV01}(t_j)B(t_0, t_j)} - 1\right)\right]\right\}.
\end{aligned} \tag{1.13}$$

Assuming flat yield curve, we can rewrite $\dfrac{\mathrm{PV01}(t_0)}{B(t_0, t_j)}$ and $\mathrm{PV01}(t_j)$ as

$$\begin{aligned}
\frac{\mathrm{PV01}(t_0)}{B(t_0, t_j)} &= \sum_{i=j+1}^{n+j}\frac{\delta}{(1 + \delta s_{t_0})^{(i-j)}} \\
&= \frac{(1 + \delta s_{t_0})^n - 1}{s_{t_0}(1 + \delta s_{t_0})^n},
\end{aligned} \tag{1.14}$$

and

$$\begin{aligned}
\mathrm{PV01}(t_j) &= \sum_{i=j+1}^{n+j}\frac{\delta}{(1 + \delta s_{t_j})^{(i-j)}} \\
&= \frac{(1 + \delta s_{t_j})^n - 1}{s_{t_j}(1 + \delta s_{t_j})^n}.
\end{aligned} \tag{1.15}$$

The Taylor approximation of s_{t_j} around s_{t_0} is

$$\frac{\mathrm{PV01}(t_0)}{\mathrm{PV01}(t_j)B(t_0,t_j)} - 1 = \frac{\dfrac{s_{t_j}(1+\delta s_{t_j})^n}{(1+\delta s_{t_j})^n - 1}}{\dfrac{s_{t_0}(1+\delta s_{t_0})^n}{(1+\delta s_{t_0})^n - 1}} - 1 \tag{1.16}$$

$$\cong \left(1 - \frac{n\delta s_{t_0}}{(1+\delta s_{t_0})\left[(1+\delta s_{t_0})^n - 1\right]}\right)\frac{(s_{t_j} - s_{t_0})}{s_{t_0}}.$$

Hence, before discounting to t_0, the convexity adjusted CMS floating rate reset at t_j is

$$\widehat{\mathrm{CMS}}_{\text{floating}}(t_j) = \hat{s}_{t_j} = s_{t_0}\left\{1 + \left(1 - \frac{n\delta s_{t_0}}{(1+\delta s_{t_0})\left[(1+\delta s_{t_0})^n - 1\right]}\right)\mathrm{E}^{\tilde{p}}_{t_0}\left[\frac{(s_{t_j} - s_{t_0})^2}{s^2_{t_0}}\right]\right\}. \tag{1.17}$$

Since $(1+\delta s_{t_0})^n - 1 \cong n\delta s_{t_0}$ for small δs_{t_0}, and

$$\mathrm{E}^{\tilde{p}}_{t_0}\left[\frac{(s_{t_j} - s_{t_0})^2}{s^2_{t_0}}\right] = \int_0^{t_j} \sigma(\tau)^2_j d\tau,$$

we can finally write $\hat{s}_{t_j}$ as

$$\hat{s}_{t_j} = s_{t_0} + \frac{\delta s^2_{t_0}}{1+\delta s_{t_0}} T_j \overline{\sigma^2_j}, \tag{1.18}$$

where T_j is the time interval between t_0 and t_j, and $\overline{\sigma^2_j}$ is the average variance of the forward swap rate during T_j.

According to this derivation, the accuracy of the convexity adjustment depends on:

1. The level of s_{t_0}. The smaller this forward swap rate, the better the approximation of $(1+\delta s_{t_0})^n - 1 \cong n\delta s_{t_0}$.

2. The size of $|s_{t_j} - s_{t_0}|$. The smaller the size, the better the Taylor approximation.

3. The smoothness of the instantaneous volatility σ_j. If the instantaneous volatility shows spikes over time, then the convexity adjustment will deteriorate.

4. The flatter the yield curve, the more accurate the convexity adjustment due to the assumption of flat yield curve. The approximation will deteriorate if the yield curve becomes steep.

The last point is especially important for CMS since market participants use CMS to take a view on the slope of the yield curve. Nevertheless, when the slope is expected to steepen, the convexity adjustment will deteriorate.

CMS rate using convexity adjustment

We consider a two-period CMS shown in the figure 1.3 where a fixed CMS rate, $\text{CMS}_{\text{fixed}}$, is paid at t_2 and t_3 against the floating two-period swap rates reset at t_1 and t_2.

The arbitrage-free relationship shows

$$0 = \text{CMS}_{\text{fixed}}(B(t_0,t_2)+B(t_0,t_3)) - (\hat{s}(t_1,t_1,t_3)B(t_0,t_2)+\hat{s}(t_2,t_2,t_4)B(t_0,t_3)), \tag{1.19}$$

hence

$$\text{CMS}_{\text{fixed}} = \frac{\hat{s}(t_1,t_1,t_3)B(t_0,t_2)+\hat{s}(t_2,t_2,t_4)B(t_0,t_3)}{B(t_0,t_2)+B(t_0,t_3)}, \tag{1.20}$$

where

$$\begin{aligned} \hat{s}(t_1,t_1,t_3) &= s(t_0,t_1,t_3) + \frac{\delta s^2(t_0,t_1,t_3)}{1+\delta s(t_0,t_1,t_3)} T_1 \overline{\sigma_1^2}, \\ \text{and } \hat{s}(t_2,t_2,t_4) &= s(t_0,t_2,t_4) + \frac{\delta s^2(t_0,t_2,t_4)}{1+\delta s(t_0,t_2,t_4)} T_2 \overline{\sigma_2^2}. \end{aligned} \tag{1.21}$$

$T_1 = [t_0, t_1]$, and $T_2 = [t_0, t_2]$; $\overline{\sigma_1^2}$ and $\overline{\sigma_2^2}$ are the average variances of the forward swap rates in T_1 and T_2. Since a forward swap rate can be expressed as a function of discount bonds indicated in (1.8), the convexity adjustment makes the CMS rate a function of the discount bonds, the average covariance of the forward swap rates and the time intervals.

1.3.2 Market procedure: the LIBOR market model

Each forward LIBOR rate is an unbiased forecast of the relevant LIBOR under a proper forward measure. Pricing CMS rate needs more than one forward rate. Hence, we will first select a "working" forward measure, and then convert each forward LIBOR process to an arbitrage-free process under this working measure. This is the LIBOR market model, first introduced by Brace, Gatarek, and Musiela (1996).

Once the stochastic differential equation (SDE) of each forward rate is written under a single forward measure, Monte Carlo paths can be generated to evaluate any desired expectation expressed by these forward rates.

There are three parts in this section. In the first part, an example of CMS shows that CMS rate can be derived from a group of forward rates. The second part introduces the arbitrage-free SDEs of the forward rates. The last part discusses the Monte Carlo simulation method used to derive the price of the CMS.

Pricing CMS using forward rates

We consider the same two-period CMS introduced before.[3] Under the t_4 forward measure, the arbitrage-free relationship can be expressed as

[3]Since we are generating the forward rates separately using the forward measure, we cannot use a bond pricing equation and obtain the values of zero coupon bonds from an arbitrage pricing formula.

$$0 = \mathrm{E}_t^{p^{t_4}} \left[\frac{x_{t_0} - s(t_1, t_1, t_3)}{(1 + \delta L_{t_0})(1 + \delta F(t_1, t_1, t_2))} \right. \tag{1.22}$$

$$\left. + \frac{x_{t_0} - s(t_2, t_2, t_4)}{(1 + \delta L_{t_0})(1 + \delta F(t_1, t_1, t_2))(1 + \delta F(t_2, t_2, t_3))} \right],$$

where x_{t_0} is the fixed CMS rate. After rearranging (1.22), we have

$$x_{t_0} = \frac{\mathrm{E}_t^{p^{t_4}} \left[\frac{s(t_1, t_1, t_3)}{(1 + \delta L_{t_0})(1 + \delta F(t_1, t_1, t_2))} + \frac{s(t_2, t_2, t_4)}{(1 + \delta L_{t_0})(1 + \delta F(t_1, t_1, t_2))(1 + \delta F(t_2, t_2, t_3))} \right]}{\mathrm{E}_t^{p^{t_4}} \left[\frac{1}{(1 + \delta L_{t_0})(1 + \delta F(t_1, t_1, t_2))} + \frac{1}{(1 + \delta L_{t_0})(1 + \delta F(t_1, t_1, t_2))(1 + \delta F(t_2, t_2, t_3))} \right]}, \tag{1.23}$$

where

$$s(t_1, t_1, t_3) = \omega_1^{t_1} F(t_1, t_1, t_2) + \omega_2^{t_1} F(t_2, t_2, t_3),$$

$$\text{and } s(t_2, t_2, t_4) = \omega_1^{t_2} F(t_2, t_2, t_3) + \omega_2^{t_2} F(t_3, t_3, t_4).$$

The weights ω_1, ω_2 are not fixed, depending on the forward rates as we have seen in the case of convexity adjustment.

(1.23) shows that this CMS rate is determined by the LIBOR rates $F(t_1, t_1, t_2)$, $F(t_2, t_2, t_3)$ and $F(t_3, t_3, t_4)$ known at t_1, t_2, and t_3. We need to simulate the dynamics of the LIBOR processes $F(t, t_1, t_2)$, $F(t, t_2, t_3)$ and $F(t, t_3, t_4)$ in order to get the CMS rate. We choose the same t_4 forward measure, p^{t_4} to simulate these dynamics.

Arbitrage-free SDE for FRA rate

We use a one-factor model, and assume the forward rate $F(t, t_i, t_{i+1})$, $\forall\, t \in [0, t_i]$, follows the SDE

$$dF(t, t_i, t_{i+1}) = \mu_i F(t, t_i, t_{i+1}) dt + \sigma_i F(t, t_i, t_{i+1}) dW_t, \tag{1.24}$$

where $\mu_i F(t, t_i, t_{i+1}) dt$ is the drift, W_t is a Wiener process, and σ_i is the volatility. We use the volatility of i-year cap to approximate the volatility of the forward rate in the simulation. [4]

The two forward rates' SDEs are

$$\begin{aligned} dF(t,t_{i-1},t_i) =& \ \mu_{i-1}F(t,t_{i-1},t_i)dt + \sigma_{i-1}F(t,t_{i-1},t_i)dW_t \forall\ t \in [t_0, t_{i-1}], \\ dF(t,t_i,t_{i+1}) =& \ \mu_i F(t,t_i,t_{i+1})dt + \sigma_i F(t,t_i,t_{i+1})dW_t \qquad \forall\ t \in [t_0, t_i]. \end{aligned} \tag{1.25}$$

The forward rate $F(t, t_i, t_{i+1})$ for the period $[t_i, t_{i+1}]$ is given by:

$$1 + \delta F(t,t_i,t_{i+1}) = \frac{B(t,t_i)}{B(t,t_{i+1})}. \tag{1.26}$$

Under the t_{i+1} forward measure, $p^{t_{i+1}}$, and $B(t_0, t_{i+1})$ as the numeraire, the ratio on the right hand side is a martingale, which means that $F(t, t_i, t_{i+1})$ is a martingale. For some small, but non-infinitesimal, time interval Δt we can write

$$F(t+\Delta t, t_i, t_{i+1}) = F(t,t_i,t_{i+1}) + \sigma_i F(t,t_i,t_{i+1})\Delta W_t^i. \tag{1.27}$$

Under different measure: the t_i forward measure, p^{t_i}, and $B(t_0, t_i)$ as the numeraire, the SDE of $F(t, t_{i-1}, t_i)$ is also driftless

$$F(t+\Delta t, t_{i-1}, t_i) = F(t,t_{i-1},t_i) + \sigma_{i-1} F(t,t_{i-1},t_i)\Delta W_t^{i-1}. \tag{1.28}$$

We have the following expectations under different measures:

$$\begin{aligned} \mathrm{E}_t^{p^{t_i}}\left[\Delta W_t^{i-1}\right] =& \ 0, \\ \mathrm{E}_t^{p^{t_{i+1}}}\left[\Delta W_t^{i}\right] =& \ 0, \\ \text{and } \mathrm{E}_t^{p^{t_{i+1}}}\left[\Delta W_t^{i-1}\right] =& \ \lambda_t^{t_{i+1}}\Delta t, \end{aligned} \tag{1.29}$$

where $\Delta W_t^{i-1} = W_{t+\Delta t}^{i-1} - W_t^{i-1}$, and $\Delta W_t^i = W_{t+\Delta t}^i - W_t^i$. $\lambda_t^{t_{i+1}}$ is a correction that has to be made for the Wiener increment ΔW_t^{i-1}, which is evaluated under

[4] Because of the inherent time structure of the volatility of forward, it is not appropriate to use cap volatility to approximate a forward rate volatility. Nevertheless, this short-term simulation justifies this approximation.

a different measure than its own forward t_i measure, p^{t_i}. See appendix for the description of measure change.

After applying the mechanics of measure change, we get the following two dynamics

$$\begin{aligned} dF(t, t_i, t_{i+1}) = &\ \sigma_i F(t, t_i, t_{i+1}) dW_t^i, \\ dF(t, t_{i-1}, t_i) = &\ -\sigma_{i-1} F(t, t_{i-1}, t_i) \frac{\delta \sigma_i F(t, t_i, t_{i+1})}{1 + \delta F(t, t_i, t_{i+1})}\, dt \\ &+ \sigma_{i-1} F(t, t_{i-1}, t_i) dW_t^i. \end{aligned} \tag{1.30}$$

They are arbitrage-free and can be easily exploited using Monte Carlo simulation. Since both dynamics are expressed under the same measure, the set of SDEs of the forward rates can be applied in pricing all sorts of interest rate instruments, including CMS.

Monte Carlo simulation of CMS swap

The two-period CMS rate can be calculated using Monte Carlo simulation. The right hand side of (1.23) can be approximated as:

$$x_{t_0} = \frac{\frac{1}{M}\sum_{j=1}^{M}\left[\frac{s(t_1,t_1,t_3)^j}{(1+\delta L_{t_0})(1+\delta F(t_1,t_1,t_2)^j)} + \frac{s(t_2,t_2,t_4)^j}{(1+\delta L_{t_0})(1+\delta F(t_1,t_1,t_2)^j)(1+\delta F(t_2,t_2,t_3)^j)}\right]}{\frac{1}{M}\sum_{j=1}^{M}\left[\frac{1}{(1+\delta L_{t_0})(1+\delta F(t_1,t_1,t_2)^j)} + \frac{1}{(1+\delta L_{t_0})(1+\delta F(t_1,t_1,t_2)^j)(1+\delta F(t_2,t_2,t_3)^j)}\right]}, \tag{1.31}$$

where

$$s(t_1,t_1,t_3)^j = \frac{\dfrac{F(t_1,t_1,t_2)^j}{(1+\delta L_{t_0})(1+\delta F(t_1,t_1,t_2)^j)} + \dfrac{F(t_2,t_2,t_3)^j}{(1+\delta L_{t_0})(1+\delta F(t_1,t_1,t_2)^j)(1+\delta F(t_2,t_2,t_3)^j)}}{\dfrac{1}{(1+\delta L_{t_0})(1+\delta F(t_1,t_1,t_2)^j)} + \dfrac{1}{(1+\delta L_{t_0})(1+\delta F(t_1,t_1,t_2)^j)(1+\delta F(t_2,t_2,t_3)^j)}}$$

$$= \frac{F(t_1,t_1,t_2)^j(1+\delta F(t_2,t_2,t_3)^j) + F(t_2,t_2,t_3)^j}{2+\delta F(t_2,t_2,t_3)^j},$$

$$s(t_2,t_2,t_4)^j = \frac{F(t_2,t_2,t_3)^j(1+\delta F(t_3,t_3,t_4)^j) + F(t_3,t_3,t_4)^j}{2+\delta F(t_3,t_3,t_4)^j}.$$

More paths, i.e., larger M, more accurate, but at the expense of speed. This is a dilemma encountered in simulation.

The dynamics of $F(t,t_1,t_2)$, $F(t,t_2,t_3)$, and $F(t,t_3,t_4)$ are

$$\begin{aligned} dF(t,t_1,t_2)^j = & -\sigma_1 F(t,t_1,t_2)^j \frac{\delta\sigma_2 F(t,t_2,t_3)^j}{1+\delta F(t,t_2,t_3)^j} \\ & -\sigma_1 F(t,t_1,t_2)^j \frac{\delta\sigma_3 F(t,t_3,t_4)^j}{1+\delta F(t,t_3,t_4)^j} + \sigma_1 F(t,t_1,t_2)^j dW_t^3, \end{aligned} \tag{1.32}$$

$$dF(t,t_2,t_3)^j = -\sigma_2 F(t,t_2,t_3)^j \frac{\delta\sigma_3 F(t,t_3,t_4)^j}{1+\delta F(t,t_3,t_4)^j} + \sigma_2 F(t,t_2,t_3)^j dW_t^3, \tag{1.33}$$

and

$$dF(t,t_3,t_4)^j = \sigma_3 F(t,t_3,t_4)^j dW_t^3. \tag{1.34}$$

Since $F(t_1,t_1,t_2)^j$ is known at t_1, $F(t_2,t_2,t_3)^j$ at t_2, and both are known earlier than $F(t_3,t_3,t_4)^j$, we have to set stopping time for $F(t_1,t_1,t_2)^j$ and $F(t_2,t_2,t_3)^j$ in the simulation. This means that at $t_1$we stop the simulation of $F(t_1,t_1,t_2)^j$, while keep simulating $F(t_2,t_2,t_3)^j$ and $F(t_3,t_3,t_4)^j$. At t_2, the simulation of $F(t_2,t_2,t_3)^j$ is over. $F(t_3,t_3,t_4)^j$'s ends at t_3.

1.4 Results

We examine a sample consisting of 1,688 daily data of forward swap rates, forward LIBOR, and cap volatilities in the period from April, 1991 to January, 1998. We obtain two groups of two-period CMS rates from two methods: the convexity adjustment method and the LIBOR market model. Figure 1.5 shows the spreads of the CMS rates. The spread is $\text{CMS}_{\text{fixed}} - x_{t_0}$: CMS rates calculated using the convexity adjustment minus those calculated using the LIBOR market model.

Figure 1.5 shows that the spread is positive in most cases, and reaches 8 basis points in some cases, implying that the CMS rate calculated using the convexity adjustment is higher than that using the LIBOR market model. Since the LIBOR market model is more exact, we can say that the standard convexity adjustment overestimates the CMS rate.

1.4.1 Convexity adjustment and yield curve slope

In the following, we use statistical tests to show the deterioration of the convexity adjustment as the yield curve slope increases. First, for all the trading days in the data sample, we calculate the spread between these two methods. Second, we calculate the slope. We use $s(t_0, t_2, t_4) - s(t_0, t_1, t_3)$ as a substitute for the slope of the yield curve. Finally, with the help of the Nadaraya-Watson kernel estimator, we can investigate the correlation between the difference and the yield curve slope.

We define the kernal as

$$K(h, \mu, \alpha) = \frac{1}{\sqrt{2\pi h^2}} \exp\left[-\frac{1}{2}\left(\frac{\alpha - \mu}{h}\right)^2\right],$$

and the data series

$$X = s(t_0, t_2, t_4) - s(t_0, t_1, t_3),$$
$$\text{and } Y = \text{CMS}_{\text{fixed}} - x_{t_0},$$

where X and Y are two $T \times 1$ vectors, $T = 1,688$. The Nadaraya-Watson kernel estimator can be expressed as

$$\widehat{f}_{NW}(i) = \frac{\sum_{j=1}^{T} K(h, X(j), X(i)). \times Y(j)}{\sum_{j=1}^{T} K(h, X(j), X(i))}, \quad i = 1, ..., T. \tag{1.35}$$

The results are displayed in the figure 1.4. We see a very strong positive correlation between the spread and the slope of the yield curve. The spread increases as the slope becomes steeper. This verifies that convexity adjustment deteriorates as yield curve steepens. It is reasonable to conclude that for longer maturity CMS the spread will be larger, since longer maturity means larger convexity in yield curve in most cases.

1.4.2 Other results

We are also interested in the relationship among spread, volatility, and swap rate. Granger causality tests are carried for the spread, the two-year cap volatility and the two-year forward swap rate. 3-day, 5-day and 100-day lags are used in these causality tests. The results are listed in the table 1.1. All the 3-day, 5-day and 100-day lags demonstrate that the spread is highly dependent on the volatility and the swap rate. The calculated linear correlation coefficient between the spread and the one-year cap volatility is 0.8750, and that between the spread and the two-year cap volatility is 0.7939.

1.5 Conclusion

In this paper, we investigate the theoretical and empirical differences between the standard convexity adjustment method and the LIBOR market model. Numeraire and martingale play important roles in both methods. The estimated CMS rates using the former method are higher than those using the latter one. We conclude that the standard convexity adjustment, although fairly close to the rate estimated from the LIBOR market model, overestimates the CMS rate and is still not a substitute for the LIBOR market model. A two-period CMS is applied in this paper. It is reasonable to believe that the spread will be larger for longer maturity CMS, since longer maturity and more settlements mean larger convexity in most cases. Furthermore, the spread is highly dependent on the volatilities.

Table 1.1: Granger Causality Probabilities among the Spread, Two-year Cap Volatility and Two-year Forward Swap Rate

3-day lag Granger causality probabilities			
Dependent variables	Spread	Volatility	Swap
Spread	0.00	0.00	0.00
Volatility	0.54	0.00	0.66
Swap	0.99	0.21	0.00
5-day lag Granger causality probabilities			
Dependent variables	Spread	Volatility	Swap
Spread	0.00	0.00	0.00
Volatility	0.34	0 .00	0.11
Swap	0.95	0.04	0.00
100-day lag Granger causality probabilities			
Dependent variables	Spread	Volatility	Swap
Spread	0.00	0.00	0.00
Volatility	012	0.00	0.56
Swap	0.82	0.06	0.00

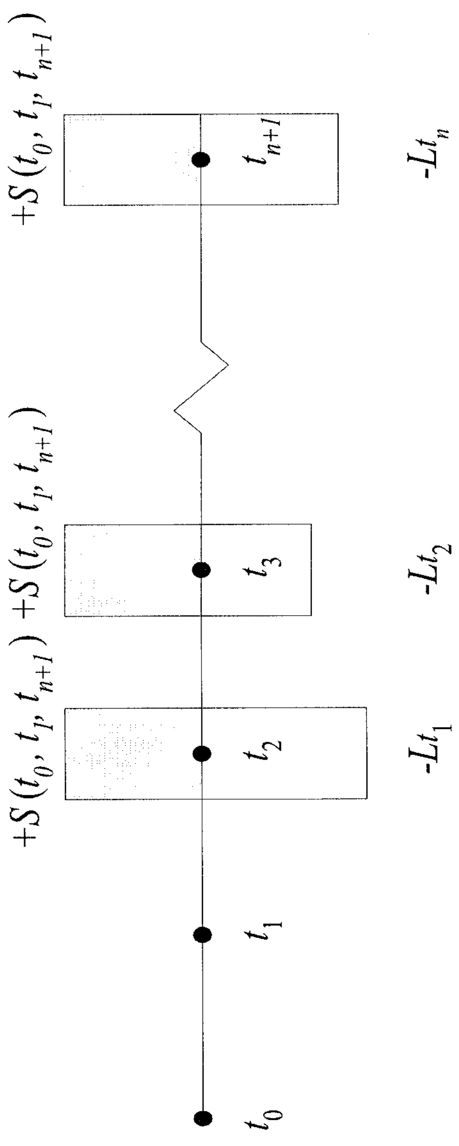

Figure 1.1: An n-period Fixed-receiver Swap

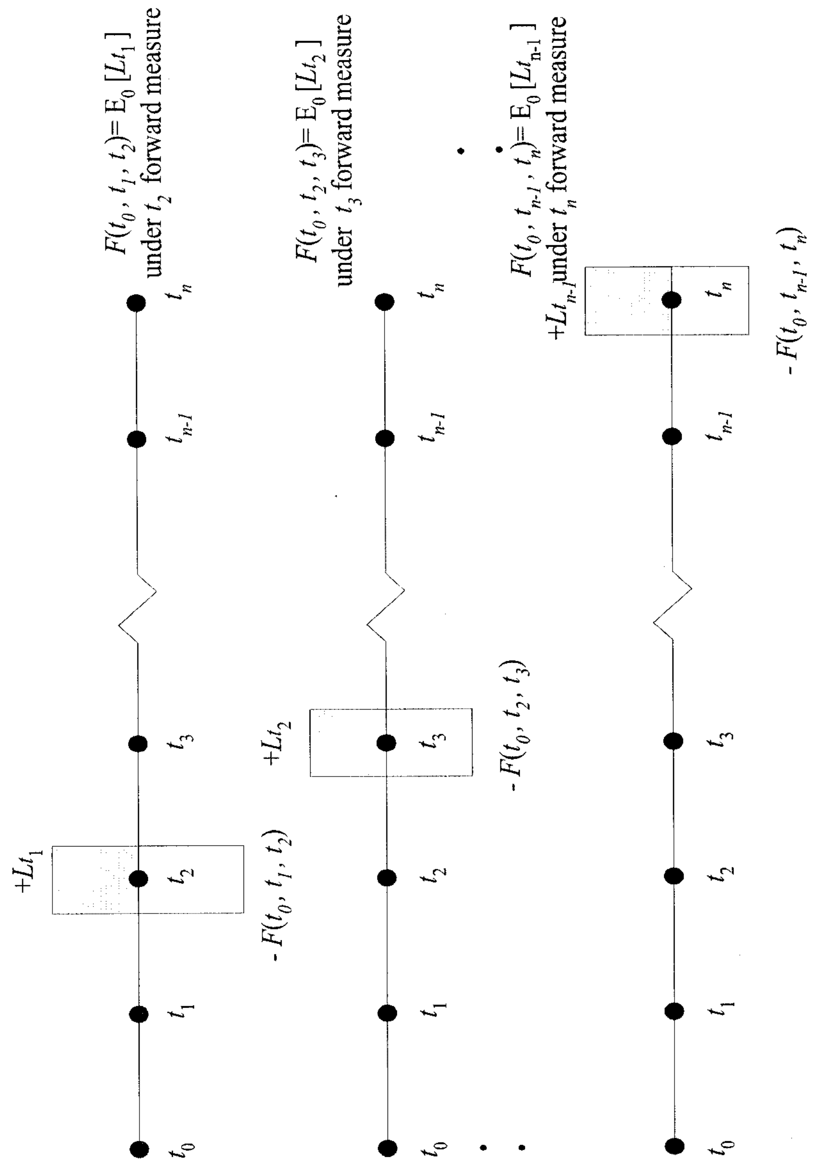

Figure 1.2: An $n-1$ -period Forward LIBOR Structure

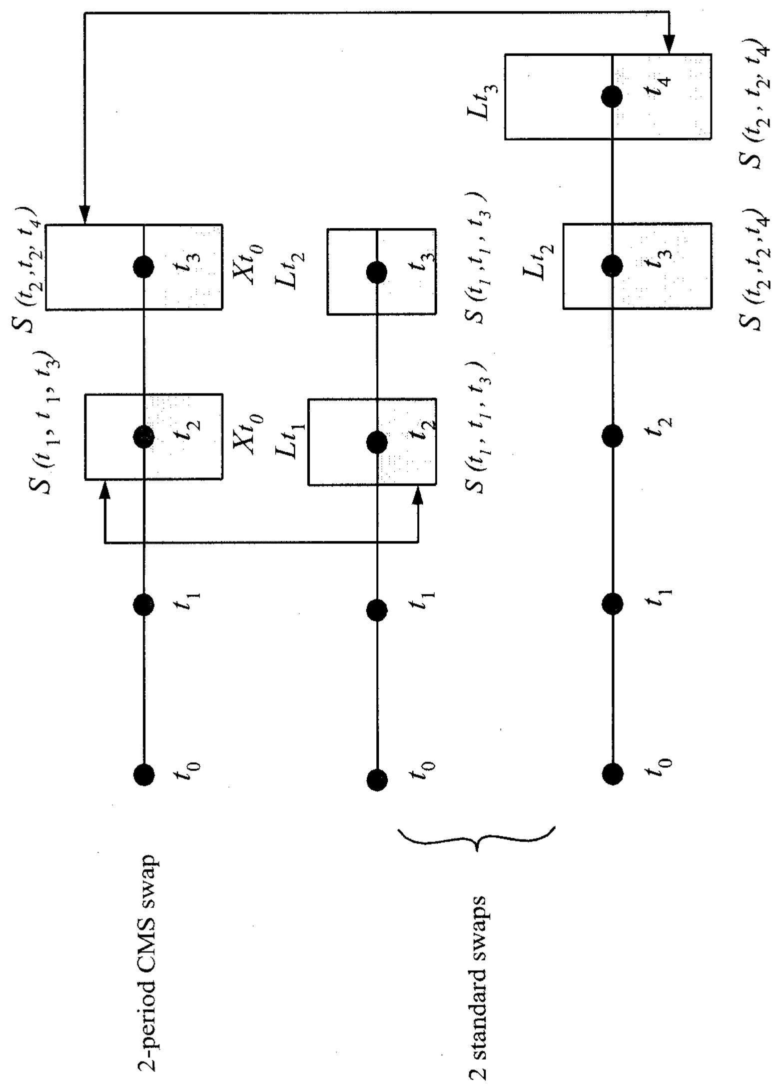

Figure 1.3: A Two-period CMS

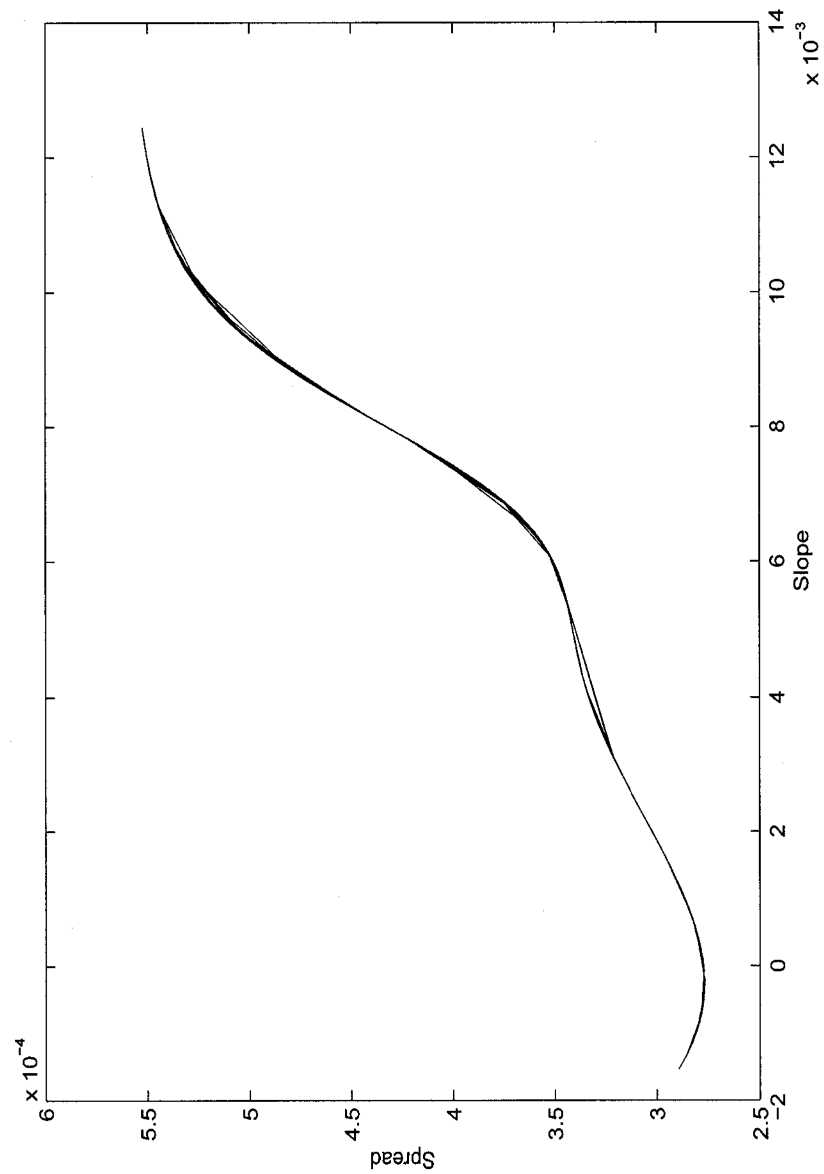

Figure 1.4: Kernel Smooth of the Slope and the Spread

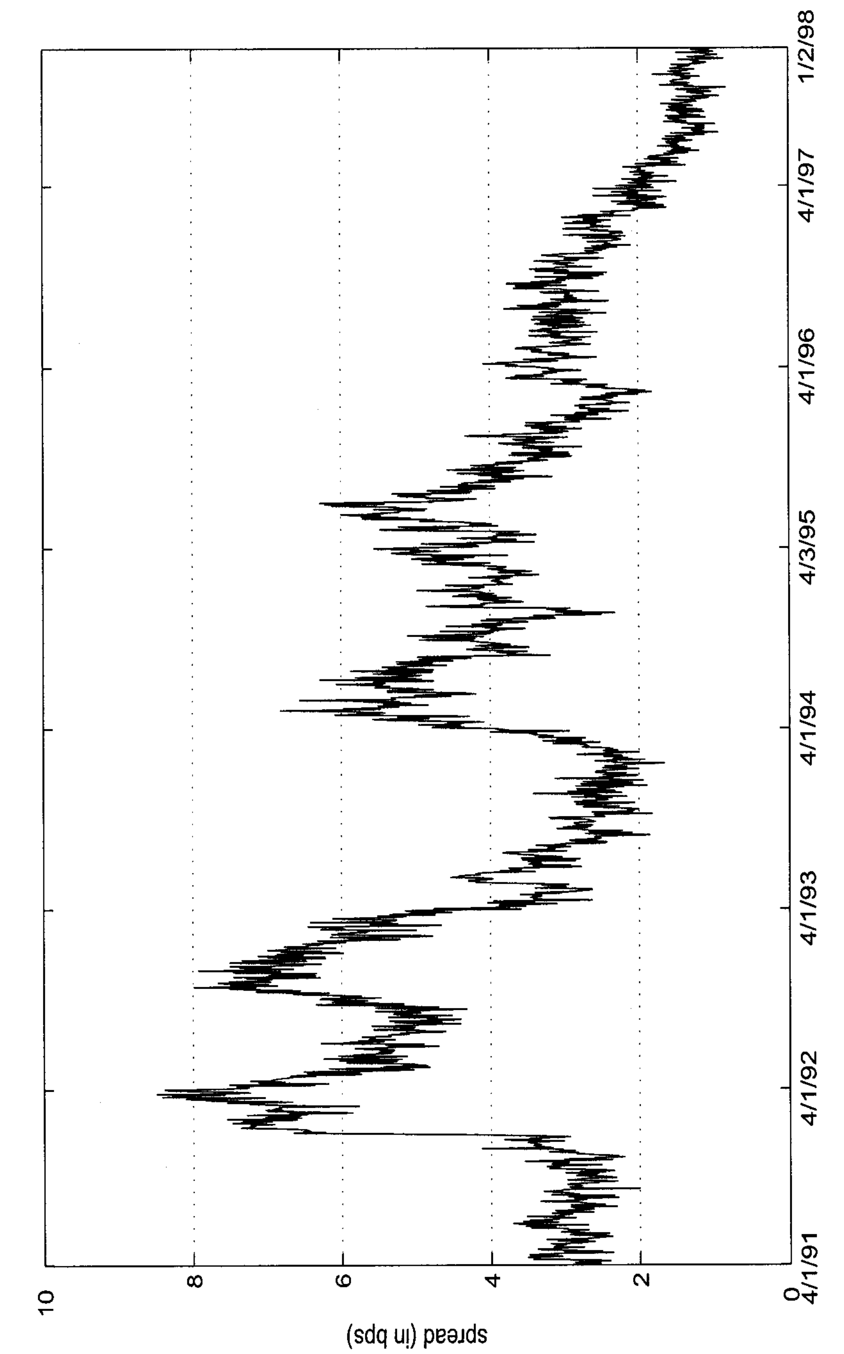

Figure 1.5: Spreads of the CMS Rates

Appendix: The Mechanics of Measure Changes

We use the following discrete expression of the expectation of ΔW_t^{i-1} with K individual states:

$$\begin{aligned}
\mathrm{E}_t^{p^{t_i}}\left[\Delta W_t^{i-1}\right] &= \sum_{j=1}^{K} \Delta W_t^{i-1}[j]\, p^{t_i}[j] \\
&= \sum_{j=1}^{K} \Delta W_t^{i-1}[j] \left[\frac{B(t,t_{i+1})}{B(t,t_{i+1})} \frac{B(t_i,t_{i+1})^j}{B(t_i,t_{i+1})^j}\right] p^{t_i}[j] \\
&= \sum_{j=1}^{K} \Delta W_t^{i-1}[j] \left[\frac{B(t,t_{i+1})}{B(t,t_i)} \frac{1}{B(t_i,t_{i+1})^j}\right] p^{t_{i+1}}[j] \\
&= 0
\end{aligned} \tag{A.1}$$

since $p_i^t[j] = \dfrac{B(t_i,t_i)}{B(t_i,t_{i+1})^j} p^{t_{i+1}}[j]$. By deleting $\dfrac{B(t,t_{i+1})}{B(t,t_i)}$, we can get[5]

$$\sum_{j=1}^{K} \Delta W_t^{i-1}[j] \left[1 + \delta F(t,t_i,t_{i+1})^j\right] p^{t_{i+1}}[j] = 0, \tag{A.2}$$

where $p^{t_i}[j]$, and $p^{t_{i+1}}[j]$ are the probabilities associated with the individual states $j = 1, ..., K$. Rearranging and writing (A.2) in expectation notation, we can get

$$\mathrm{E}_t^{p^{t_{i+1}}}\left[\Delta W_t^{i-1}\right] = -\delta \mathrm{E}_t^{p^{t_{i+1}}}\left[F(t,t_i,t_{i+1})\Delta W_t^{i-1}\right]. \tag{A.3}$$

The left hand side is the desired expectation of the ΔW_t^{i-1} under the new probability $p^{t_{i+1}}$.

Multiplying both sides of

$$F(t+\Delta t, t_i, t_{i+1}) = F(t,t_i,t_{i+1}) + \sigma_i F(t,t_i,t_{i+1}) dW_t^i \tag{A.4}$$

[5]We can eliminate the constant term $\frac{B(t,T+\delta)}{B(t,T)}$ since there is no i term in it.

by ΔW_t^{i-1} and taking expectations, we have

$$\begin{aligned}
&\mathrm{E}_t^{p^{t_{i+1}}}\left[F(t,t_i,t_{i+1})\Delta W_t^{i-1}\right] \\
&= F(t,t_i,t_{i+1})\mathrm{E}_t^{p^{t_{i+1}}}\left[\Delta W_t^{i-1}\right] + \sigma_i F(t,t_i,t_{i+1})\mathrm{E}_t^{p^{t_{i+1}}}\left[\Delta W_t^i \Delta W_t^{i-1}\right] \\
&= -\delta F(t,t_i,t_{i+1})\mathrm{E}_t^{p^{t_{i+1}}}\left[F(t,t_i,t_{i+1})\Delta W_t^{i-1}\right] + \sigma_i F(t,t_i,t_{i+1})\Delta t.
\end{aligned} \tag{A.5}$$

(A.3) and (A.5) lead to

$$\mathrm{E}_t^{p^{t_{i+1}}}\left[F(t,t_i,t_{i+1})\Delta W_t^{i-1}\right] = \frac{\sigma_i F(t,t_i,t_{i+1})\Delta t}{1+\delta F(t,t_i,t_{i+1})}, \tag{A.6}$$

and

$$\mathrm{E}_t^{p^{t_{i+1}}}\left[\Delta W_t^{i-1}\right] = -\frac{\delta\sigma_i F(t,t_i,t_{i+1})\Delta t}{1+\delta F(t,t_i,t_{i+1})}. \tag{A.7}$$

Now we can rewrite the other SDE as

$$\begin{aligned}
&F(t+\Delta t,t_{i-1},t_i) \\
&= F(t,t_{i-1},t_i) + \sigma_{i-1}F(t,t_{i-1},t_i)\Delta W_t^{t-1} \\
&= F(t,t_{i-1},t_i) - \sigma_{i-1}F(t,t_{i-1},t_i)\frac{\delta\sigma_i F(t,t_i,t_{i+1})\Delta t}{1+\delta F(t,t_i,t_{i+1})} + \sigma_{i-1}F(t,t_{i-1},t_i)\Delta W_t^i,
\end{aligned} \tag{A.8}$$

where $\Delta W_t^i = \dfrac{\delta\sigma_i F(t,t_i,t_{i+1})\Delta t}{1+\delta F(t,t_i,t_{i+1})} + \Delta W_t^{i-1}$, and its expected value is zero under $p^{t_{i+1}}$.

Therefore, we obtain the two dynamics

$$\begin{aligned}
dF(t,t_i,t_{i+1}) =&\ \sigma_i F(t,t_i,t_{i+1})dW_t^i, \\
\text{and}\ \ dF(t,t_{i-1},t_i) =&\ -\sigma_{i-1}F(t,t_{i-1},t_i)\frac{\delta\sigma_i F(t,t_i,t_{i+1})}{1+\delta F(t,t_i,t_{i+1})}\,dt \\
&+\sigma_{i-1}F(t,t_{i-1},t_i)dW_t^i.
\end{aligned} \tag{A.9}$$

Part II

Essay 2: Pricing Contingent Credit Lines with Application to Argentina, Brazil and Mexico

2.1 Introduction

Contingent credit lines (CCL) contract is also called loan commitment, which is widely used by companies to prevent liquidity crisis in the commercial paper (CP) market. The CCL contract commits to lending up to a certain amount of money to a borrower under prespecified terms. A 2003 survey by the US Federal Reserve shows that almost three-quarters of bank lending is done using commitment contracts. As of March 2003, the outstanding loan commitments of the US firms were close to \$1.6 trillion, more than doubling those in 1990.

Companies use lines of credit for three general reasons. First, many lines of credit are issued as the backstop facilities that give flexibility to issuers in the capital markets. For example, companies with unused loan commitments will avoid borrowing in the CP market when CP rates temporarily spike in the market due to unforeseen events. Second, having a credit line in place signals that the company has the ability to pay for specific transactions and hence reduces credit risk on short-term borrowing. Third, opening a credit line with a highly reputable bank usually sends a positive signal to other financial market participants. It reduces information asymmetries between the company management and the market about the company's financial condition (see Fama (1985)).

Developing countries also need flexibility when borrowing in the international capital markets, and they face liquidity risk when the markets shut the door to them. Developing countries could use CCL contracts to gain the flexibility and back their foreign exchange reserves. One pioneer in this area is Argentina. On December 20, 1996, the Central Bank of Argentina agreed with 14 international banks on a firm commitment of \$6.1bn liquidity option. Other countries, for example, Mexico, Indonesia, and South Africa had reached similar agreements with international banks. In September 1998, Mexico drew $2.66bn from its loan commitment signed with international banks. Nevertheless, the use of CCL contracts by developing countries is rather limited.

The reasons can be analyzed from both sides of the demand-supply relationship. From the demand side, the major concern of the developing countries is that applying CCL would send a negative signal to the market showing their vulnerability to financial crisis. Moreover, a country's eviction from a CCL contract would trigger even larger adverse consequences. As the major suppliers of the CCL contracts, the international banks feel uneasy about writing blank checks to countries, especially to some developing countries vulnerable to crisis. There is no bankruptcy law for sovereign debtors. In early 2005, Argentina's success in forcing its creditors to accept the lowest recovery rate in the history of sovereign defaults shows the thin restraints on sovereign borrowers. Moreover, compared with a company, a country has much more intricate and volatile economic and political environment, which makes analyzing a sovereign country cumbersome.

The lack of a simple and transparent pricing method for CCL could also be another reason. Instead of using macroeconomic data to analyze and price CCL for sovereign countries, we use the market data, i.e., Eurobond yields which have embedded credit risk. In this paper, we apply the popular term-structure model, the LIBOR market model to the sovereign credit spread and price CCL for 3 Latin American countries: Argentina, Brazil, and Mexico.

This paper is organized as follows. The next section discusses a simple CCL structure. Section 3 presents the pricing model. Section 4 discusses the data. Section 5 shows the pricing results. Section 6 concludes the paper.

2.2 CCL Structure

2.2.1 Fee structure

A typical structure of a CCL contract specifies that the facility provider would lend a certain amount of money to a sovereign country at a fixed markup as long as that country's Eurobond credit spread doesn't reach a certain barrier level. If a country's credit spread is above this fixed markup, it can draw the CCL at a lower cost than it could otherwise get from the market. The barrier level, knock-out, is written in the Material Adverse Change (MAC) clause. During the contract period, if the credit spread jumps above that barrier level, the CCL contract terminates automatically. The CCL contract gives the prospective borrower the right, but not the obligation, to borrow money at a fixed mark-up. This contract is an option; and as an option, it must have a price.

The fees of the CCL contract include a commitment fee, an annual fee, and a usage fee. The future borrower has to pay an up-front commitment fee when signing the contract. During the maturity of the CCL contract, it has to pay an annual fee on the used amount, and a usage fee on the available unused amount. It also has to pay interest on the used amount, which is composed of two parts. One is the London InterBank Offered Rate (LIBOR) and the other is a mark-up, the rate depending on the creditworthiness of the borrower. This mark-up could be either a floating or a fixed rate. In this paper, we assume a fixed mark-up rate.

2.2.2 A simplified CCL contract

Before we present the pricing model, we put froward some assumptions and explanations on the CCL contract:

- The CCL contract in our paper matures at T_1. The borrower can make a decision on whether to use the credit line at T_1 and pay back at T_2, which makes the CCL contract a European-style option. The difference between European-style option and American-style option lies in that the latter can be drawn at any business day before the maturity date. The assumption of a European-style structure instead of an American-style structure is to prevent the borrower from drawing the commitment with the knowledge that his credit spread could jump above the barrier level in the coming days. If allowed to happen, the banks wouldn't have any protection. In order to circumvent the restrictions on fixed drawing date and meet future liquidity requirement, the borrower can write a series of CCL contracts with different maturity dates.

- If the borrower decides to draw the line, it has to draw the total amount to simplify the calculation.

- We are interested in deriving the CCL price, which is defined as the sum of commitment fee, annual fee, and usage fee. We won't derive a specific price for each of the above three since this is beyond our method. CCL fee structures vary from company to company. We refer Ergungor (2001) for the interested readers. The commitment fee is an up-front fee, the annual fee is applied to the used amount, and the usage fee is levied on the unused amount. With the assumption of full percentage draw, the borrower has to pay the up-front commitment fee, and either the annual fee when drawing CCL or the usage fee when not. We combine the up-front commitment fee and the present value of either the annual fee or the usage fee as the CCL price. These fees are expressed as a fraction of the CCL loan amount.

2.2.3 Payment in a CCL contract

CCL contract is a contract between a bank and a sovereign country, signed at time t_0 and matures at T_1. The maximum loan amount, N,[6] which is the maximum amount that can be drawn at T_1. If the sovereign country chooses to draw at T_1, it has to pay back principal plus interest at T_2. The interest rate is the sum of the LIBOR for the borrowing period $[T_1, T_2]$, $L(T_1, T_2)$, and a fixed mark up, $s(t_0)$, which is specified at t_0. Sovereign country's forward credit spread at t, $t \in [t_0, T_1]$, is $c(t, T_1, T_2)$. This is the difference between the risky forward rate $f(t, T_1, T_2)$, and the risk-free forward rate $F(t, T_1, T_2)$. When t reaches T_1, $F(T_1, T_1, T_2)$ becomes the LIBOR rate for the period $[T_1, T_2]$: $L(T_1, T_2)$.[7] We write them as $c(t, T_1)$, $f(t, T_1)$, $F(t, T_1)$, and $L(T_1)$. The contract also contains an MAC that specifies the termination condition of the contract. For example, before T_1, triggered by the downgrade of the borrower's credit rating, the contract could terminate. In stead of credit rating, the contract could specify a credit spread threshold, i.e., $H(t_0)$. If $c(t, T_1)$ exceeds H_0 before T_1, the contract terminates and the contract's payoff is zero.

Suppose $c(t, T_1)$ doesn't exceed H_0 during $[t_0, T_1]$. We will have two possible outcomes. One is that $c(T_1, T_1) > s(t_0)$ at T_1 (in the money), and the other is $c(T_1, T_1) \leq s(t_0)$ (out of the money). If $c(T_1, T_1) > s(t_0)$, the country could draw the line at a lower price than it could get from the market, and the contract yields a payoff of $(T_2 - T_1)(c(T_1, T_1) - s(t_0))$ at T_2. If $c(T_1, T_1) \leq s(t_0)$ at T_1, it will not

[6] $N = 1$.

[7] LIBOR is the funding cost for an AA-rated firm. Nevertheless, the LIBOR market model assumes LIBOR contains zero credit risk.

choose to draw, ending with zero payoff.

Since $H(t_0) > s(t_0)$, we can see that if $c(t, T_1) > H(t_0)$, $c(t, T_1)$ must exceed $s(t_0)$. So its payoff structure shows that CCL has the reverse knock-out characteristic, getting knocked out when it is in the money. The strike is $s(t_0)$, and $H(t_0)$ is the barrier level. The payoff of a CCL contract is shown in the figure 2.1. The CCL contract can also be classified as credit derivatives, since it is written on the credit risk specified by the credit spread.

2.3 Modelling CCL as a Credit Derivative with Reverse Knock-out Characteristic

Credit derivatives are financial instruments with credit risk as the underlying variable. There are two types of credit derivatives. The first type is the default class whose payment is contingent on the default event. Credit default swaps, and total return swaps fall into this class. In the second type, the payment is determined by the changes in the credit quality of the borrower rather than the default event. Spread derivatives and rating-based derivatives are of this type. CCL belongs to the second type, since its payment depends on the changes in the borrower's creditworthiness.

There are two well-established approaches to modelling credit risk. One is structural model, or asset value model, following the seminal paper by Merton (1974). The structural model views the debt as a contingent claim written on the borrower's assets. The lower the debt to asset ratio, the longer the distance to default. Obtaining an accurate value of the borrower's asset is a difficult step, which is magnified in the case of sovereign borrower. Valuing asset involves how to define a country's asset. No sovereign bankruptcy law ever exists. Even if a country's asset can be clearly identified, to model the change of its asset is beyond management bacause it could involve variables such as trade deficit, budget deficit, exchange rate movement, growth rate, and inflation rate, etc.. The obstacles force us to resort to the other method, the reduced-form approach. With the combination of term-structure model, it directly models the credit process of a risky debt. A default event is modelled by a jump intensity process. Since the underlying pricing methodology has already been used in the risk-free term-structure models, the reduced-form approach is more tractable than the structural model as demonstrated by Das and Tufano (1996), Jarrow and Turnbull (1997), Lando (1998), Madan and Unal (1998), and Duffie and Singleton (1999). Maksymiuk and Gatarek (1999) shows that by substituting hazard rates for interest rates, default can be priced in an Heath-Jarrow-Morton (HJM) framework. Pugachevsky (1999) extends the work of Maksymiuk and Gatarek by adding correlation between interest rate and default risk to the HJM framework. Schönbucher (2000) presents a credit risk model based on the LIBOR market model, a popular term-structure model. Schönbucher (2004) shows that credit default swap can be priced using defaultable assets as the numeraires. Here, we use the Schönbucher (2000) framework to price the CCL contract.

2.3.1 The model

Risk-free forward rate

We first consider a risk-free environment. Under the LIBRO market model, we can define the risk-free froward rate for the period $[T_1, T_2]$ as

$$F(t, T_1) = \frac{1}{\delta}\left(\frac{P(t, T_1)}{P(t, T_2)} - 1\right), \tag{2.1}$$

where $P(t, T_1)$ and $P(t, T_2)$ are the prices of the risk-free zero coupon bonds with maturity T_1 and T_2. δ is the year fraction between T_1 and T_2.

According to the LIBOR market model, using $P(t, T_2)$ as the numeraire, $F(t, T_1)$ is a martingale

$$dF(t, T_1) = F(t, T_1)\sigma_t^F dW_2(t) \tag{2.2}$$

under T_2 forward measure. σ_t^F is the instantaneous volatility of the risk-free forward rate. $W_2(t)$ is a Wiener process under T_2 forward measure, p^{T_2}. Changing measure using Girsanov's theorem, $dW_2(t)$ can be expressed as

$$dW_2(t) = dW_Q(t) + \alpha_{T_2}(t)dt, \tag{2.3}$$

where $\alpha_{T_2}(t)$ is defined as the minus volatility of the risk-free bond $P(t, T_2)$. $dW_Q(t)$ is a Brownian motion under the spot measure, which is a forward measure associated with initial date t_0. $\alpha_{T_2}(t)$ can be expressed recursively as

$$\alpha_{T_2}(t) = \alpha_{T_1}(t) + \frac{\delta F(t, T_1)}{1 + \delta F(t, T_1)}\sigma_t^F, \tag{2.4}$$

where $\alpha_{T_1}(t)$ is the minus volatility of the risk-free bond $P(t, T_1)$.

Hence, the dynamic of $F(t, T_1)$ can also be expressed under the spot measure as

$$dF(t, T_1) = F(t, T_1)\sigma_t^F\left[\alpha_{T_2}(t)dt + dW_Q(t)\right]. \tag{2.5}$$

Risky forward rate

Now let us look at the case of the risky market where exists default probability. The time of default is denoted as τ. Let

$$I(t) = \mathbf{1}_{\{\tau > t\}} \tag{2.6}$$

be the survival indicator function, which is one before default, and zero if default happens. The prices of the risky zero coupon bonds with maturity T_1 and T_2 are $\tilde{P}(t, T_1)$, and $\tilde{P}(t, T_2)$

$$\tilde{P}(t, T_1) = I(t)B(t, T_1), \tag{2.7}$$

$$\tilde{P}(t, T_2) = I(t)B(t, T_2), \tag{2.8}$$

where $B(t, T_1)$ and $B(t, T_2)$ are the pre-default bond prices, which do not need to be zero at default, since $I(t)$ would jump to zero, making the risky zero coupon bond worth zero.[8]

The risky forward rate can be expressed as

$$f(t, T_1) = \frac{1}{\delta}\left(\frac{B(t, T_1)}{B(t, T_2)} - 1\right). \tag{2.9}$$

As Schönbucher (2000) points out, under the T_2 survival measure, and risky bond $B(t, T_2)$ as the numeraire, $f(t, T_1)$ is a martingale

$$df(t, T_1) = f(t, T_1)\sigma_t^f d\tilde{W}_2(t), \tag{2.10}$$

where σ_t^f is the instantaneous volatility of the risky forward rate.

After changing measure, $d\tilde{W}_2(t)$ can be expressed as

$$d\tilde{W}_2(t) = dW_Q(t) + \tilde{\alpha}_{T_2}(t)dt, \tag{2.11}$$

[8]We assume zero recovery rate.

where $\tilde{\alpha}_{T_2}(t)$ is the minus volatility of the risky bond $B(t,T_2)$, and can be expressed recursively as

$$\tilde{\alpha}_{T_2}(t) = \tilde{\alpha}_{T_1}(t) + \frac{\delta f(t,T_1)}{1+\delta f(t,T_1)}\sigma_t^f, \tag{2.12}$$

where $\tilde{\alpha}_{T_1}(t)$ is the minus volatility of the risky bond $B(t,T_1)$.

The dynamic of $f(t,T_1)$ can also be expressed under the spot measure as

$$df(t,T_1) = f(t,T_1)\sigma_t^f\left[\tilde{\alpha}_{T_2}(t)dt + dW_Q(t)\right]. \tag{2.13}$$

Dynamics of credit spread

From (2.3) and (2.11) we can conclude that

$$d\tilde{W}_2(t) = dW_2(t) + \left(\tilde{\alpha}_{T_2}(t) - \alpha_{T_2}(t)\right)dt. \tag{2.14}$$

This equation links two measures: the T_2 forward measure and the T_2 survival measure. We can make either $F(t,T_1)$ or $f(t,T_1)$ martingale. We choose to use the T_2 survival measure, under which $f(t,T_1)$ is a martingale

$$df(t,T_1) = f(t,T_1)\sigma_t^f d\tilde{W}_2(t), \tag{2.15}$$

and

$$\begin{aligned} dF(t,T_1) &= F(t,T_1)\sigma_t^F\left[d\tilde{W}_2(t) - \left(\tilde{\alpha}_{T_2}(t) - \alpha_{T_2}(t)\right)dt\right] \\ &= -\left(\tilde{\alpha}_{T_2}(t) - \alpha_{T_2}(t)\right)F(t,T_1)\sigma_t^F dt + F(t,T_1)\sigma_t^F d\tilde{W}_2(t). \end{aligned} \tag{2.16}$$

Since $dc(t,T_1) = df(t,T_1) - dF(t,T_1)$, we can express the forward credit spread, $c(t,T_1)$, as

$$dc(t,T_1) = \left(\tilde{\alpha}_{T_2}(t) - \alpha_{T_2}(t)\right)F(t,T_1)\sigma_t^F dt + \left(f(t,T_1)\sigma_t^f - F(t,T_1)\sigma_t^F\right)d\tilde{W}_2(t). \tag{2.17}$$

2.3.2 Application to CCL contract

(2.17) shows that as long as we know the volatilities of both the risk-free and risky forward rates, and the initial values of both the risk-free and risky forward rates, we can get the dynamics of the forward credit spread under the T_2 survival measure. The price of CCL contract at t_0 can be expressed as

$$\text{CCL} = B(t_0, T_2) * \ \mathbb{E}\left[N\delta \ \text{Max}\left\{(c(T_1, T_1) - s(t_0)), 0\right\} \ \mathbf{1}_{\{c(t,T_1)<H(t_0), \ t_0 \le t \le t_1\}}\right] \tag{2.18}$$

under the T_2 survival measure and $B(t_0, T_2)$ as the numeraire.

When we apply this model to sovereign countries, we make several assumptions:

- Since $\tilde{\alpha}_{T_k}(t)$ and $\alpha_{T_k}(t)$ $(k = 1, 2)$ are derived recursively, we have to set $\tilde{\alpha}_{T_1}(t)$ and $\alpha_{T_1}(t)$ equal to zero. The shorter the time interval between t_0 and T_1, the more justified this assumption is.

- We assume that $[t_0, T_1]$ is one year, and $[t_0, T_2]$ is two years. This assumption follows banking practices. During 2002-2003, 60 percent of US CCLs were marketed as one year (364-day) facilities. As long as the facility is not drawn, a commitment with maturity shorter than one year carries zero risk weighted capital in bank's balance sheets. For commitment with maturity longer than one year, a bank is required to put aside 4% capital on the commitment amount. This explains the bank's preference for maturity shorter than one year.

2.4 Data

The data sets we use are the risk-free forward rates and risky forward rates. The risk-free forward rates are the USD Forward Rate Agreements (FRA), which can be obtained from Reuters. The sovereign countries we are interested in are Argentina, Brazil, and Mexico. We can obtain these countries' benchmark yields of global borrowing from Reuters. Applying the piecewise cubic Hermite interpolation, we can obtain the 1-year forward rates for these three countries. The daily data extend from September 3, 2002 through January 31, 2005. Each data set has 620 observations. We list some descriptive statistics of the USD FRA and the risky forward rates of three countries in the table 2.1.

The "Mean" column of the table 2.1 shows that Argentina has the highest borrowing cost among the three countries. During that period, Brazil's average cost of borrowing is similar to Mexico's. We plot the extrapolated one year forward rates of Argentina, Brazil and Mexico in the figure 2.2.

2.5 Parameter Calculation and Simulation Results

2.5.1 Parameter calculation

The dynamics of the credit spreads and the pricing formula of the CCL show that the instantaneous volatilities of the risk-free and the risky forward rates, the initial values of these rates, and the risky discount factors (bond prices) are required to price the CCL contract. The initial values and the discount factors can be easily extracted from the yields. Volatilities are difficult to get. For the risk-free forward rates, we can obtain the instantaneous volatilities from the caplet data. But caplet market based on risky forward rates doesn't exist, which makes the historical data the only choice. We calculate the annual volatility of the risky forward rate by annualizing the standard deviation of the logarithmic changes of the risky forward rate. To be consistent, we use the same method to get the annual volatility of the risk-free forward rate.

On January 31, 2005, the risky forward rates of Argentina, Brazil and Mexico are 67.80%, 5.27%, and 8.84%. The annualized historical volatilities on the logarithmic daily changes of these rates are 28.27%, 112.43%, and 24.89%. The discount factors are 33.56% for Argentina, 93.56% for Brazil, and 84.03% for Mexico. The historical volatility of the logarithmic change of the risk-free forward rates is 51.44%,[9] and the one year risk-free forward rate is 3.98% on January 31, 2005.

2.5.2 Simulation results

Using the calculated parameters, we can simulate one year dynamics of the credit spreads with start date January 31, 2005, the last date in the sample period for three countries. On January 31, 2005, the forward credit spread $c(t_0)$ of Argentina, Brazil and Mexico is 63.82%, 1.29% and 4.86%. CCL price also depends on the strike price $s(t_0)$ and the threshold $H(t_0)$. Nevertheless, the credit spread differences between Argentina and the other two countries are too large to use the same set of $s(t_0)$ and $H(t_0)$. Hence we use two sets of combinations of $s(t_0)$ and $H(t_0)$. One is for Argentina, and the other is for Brazil and Mexico. The $s(t_0)$ and $H(t_0)$ set for Argentina is $s(t_0) = [0.5 : 0.001 : 1]$,[10] and $H(t_0) = [0.5 : 0.001 : 1]$. The set for Brazil and Mexico is $s(t_0) = [0.01 : 0.0001 : 0.1]$, and $H(t_0) = [0.01 : 0.0001 :$

[9] The volatility from caplet price is 22.45% on January 31, 2005 from Reuters.
[10] [0.5:0.001:1]=[0.5,0.5001,0.5002,...,1].

0.1]. We can compare the CCL prices of Brazil and Mexico under the same $s(t_0)$ and $H(t_0)$ range. For $H(t_0) \leq s(t_0)$, the CCL contract is meaningless with price zero.

The dynamics of $c(t)$ are path-dependent on $H(t_0)$. We resort to the Monte Carlo simulation to obtain the CCL prices based on (2.17) and (2.18). There are 3 possible cases. One is that $c(t)$ doesn't hit $H(t_0)$ before T_1, and it ends in exceeding $s(t_0)$ at T_1. In this case, the borrower will draw the CCL at T_1, and pay back the total amount plus interest at T_2. The second is that $c(t)$ hits the barrier $H(t_0)$ before T_1, leading to the termination of this CCL contract. The last case is that $c(t)$ is always below $s(t_0)$ during $[t_0, T_1]$, and the CCL terminates without drawing. For each country, we draw 100,000 paths of $c(t)$ with the finite time interval of $\frac{1}{10,000}$ year. We do this for each combination of $s(t_0)$ and $H(t_0)$. After discounting the expected payoff, we can get the CCL price.

Figure 2.3 shows the graph of the CCL price for Argentina. Figure 2.4 and figure 2.5 show those for Brazil and Mexico. These figures show that as the strike, $s(t_0)$, increases, the CCL price decreases. Opposite relationship holds for $H(t_0)$ and the CCL price. The reasoning is that the lower the $s(t_0)$, the higher the probability that the CCL contract will end in the money. On the contrary, the lower the $H(t_0)$, the higher the possibility that the CCL contract will be knocked out.

In order to see the effects of country's creditworthiness on the price of CCL, we put the CCL prices of Brazil and Mexico under the same set of $H(t_0)$ and $s(t_0)$. On January 31, 2005, the spreads for Brazil and Mexico are 1.29%, and 4.86%, respectively. Mexico's credit spread is higher than Brazil's. From the figures we can see that under the same $s(t_0)$ and $H(t_0)$ the CCL price of Mexico is higher than that of Brazil. One interesting thing of Mexico's CCL price is that after $H(t_0)$ reaches a level between 0.05 and 0.06, the CCL price doesn't depend on $H(t_0)$. The reason is that the volatility of Mexico's forward rate is 25.18%, which is lower than that of the risk-free forward rate, 51.15%. The probability of Mexico's credit spread jumping to $H(t_0)$ or higher is almost zero.

2.6 Conclusion

In this paper, we apply the LIBOR market model in pricing CCL contracts for three Latin American countries. The CCL contract is modelled as a reverse knock-out option. The pricing results demonstrate that the CCL price is determined mainly by the credit spread, the interest rate mark-up and the MAC.

This paper shows that, instead of using macroeconomic variables to price CCL, we can use market data and interest rate derivative models to price CCL for sovereign countries. The shift of the yields on the sovereign country's bonds reflects the market expectation on the default risk and provide policymaker with updated information on his country's creditworthiness. The banks can also use this information to price the creditworthiness into CCL contract, and design an appropriate fixed markup and a barrier level.

Table 2.1: Descriptive Statistics

This table shows the descriptive statistics for the FRA, and the forward rates of Argentina, Brazil and Mexico. The time period extends from September 3, 2002 through January 31, 2005, yielding a total of 620 daily observations. ρ_j represents the autocorrelation coefficient of order j.

	FRA	Argentina	Brazil	Mexico
Maximum	0.0424	0.7017	0.3103	0.1091
Minimum	0.0158	0.4342	0.0279	0.0599
Mean	0.0296	0.5900	0.0882	0.0816
Std. Dev.[a]	0.0324	0.0178	0.0708	0.0157
ρ_1	0.987	0.982	0.992	0.993
ρ_2	0.975	0.965	0.985	0.987
ρ_3	0.963	0.947	0.979	0.980
ρ_4	0.951	0.930	0.972	0.974
ρ_5	0.940	0.914	0.966	0.969
ρ_{10}	0.888	0.832	0.937	0.940

[a]They are based on the logarithmic daily changes of the rates.

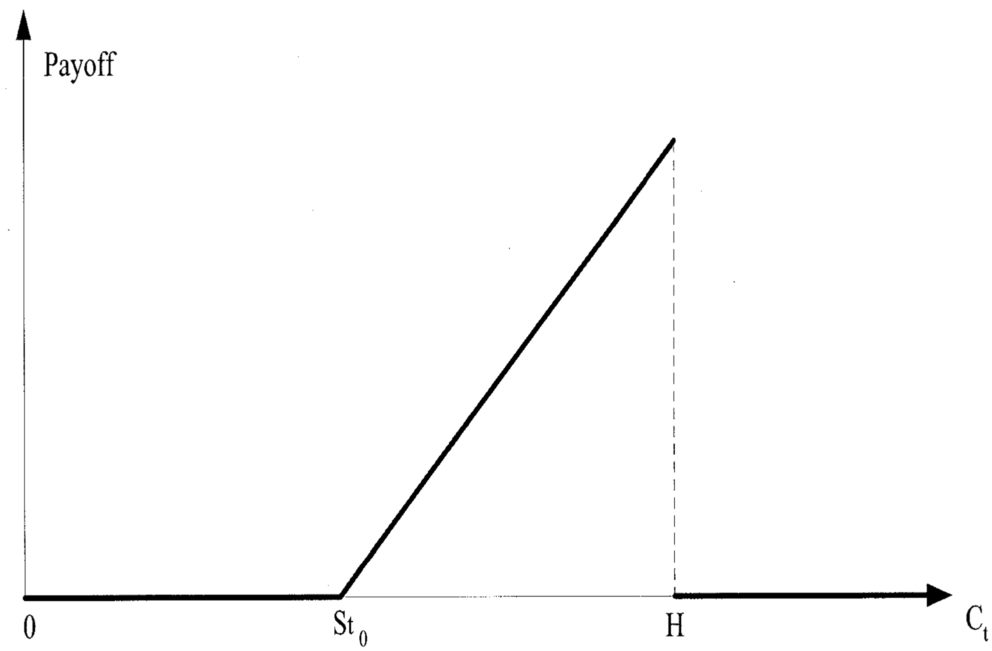

Figure 2.1: Reverse Barrier Option

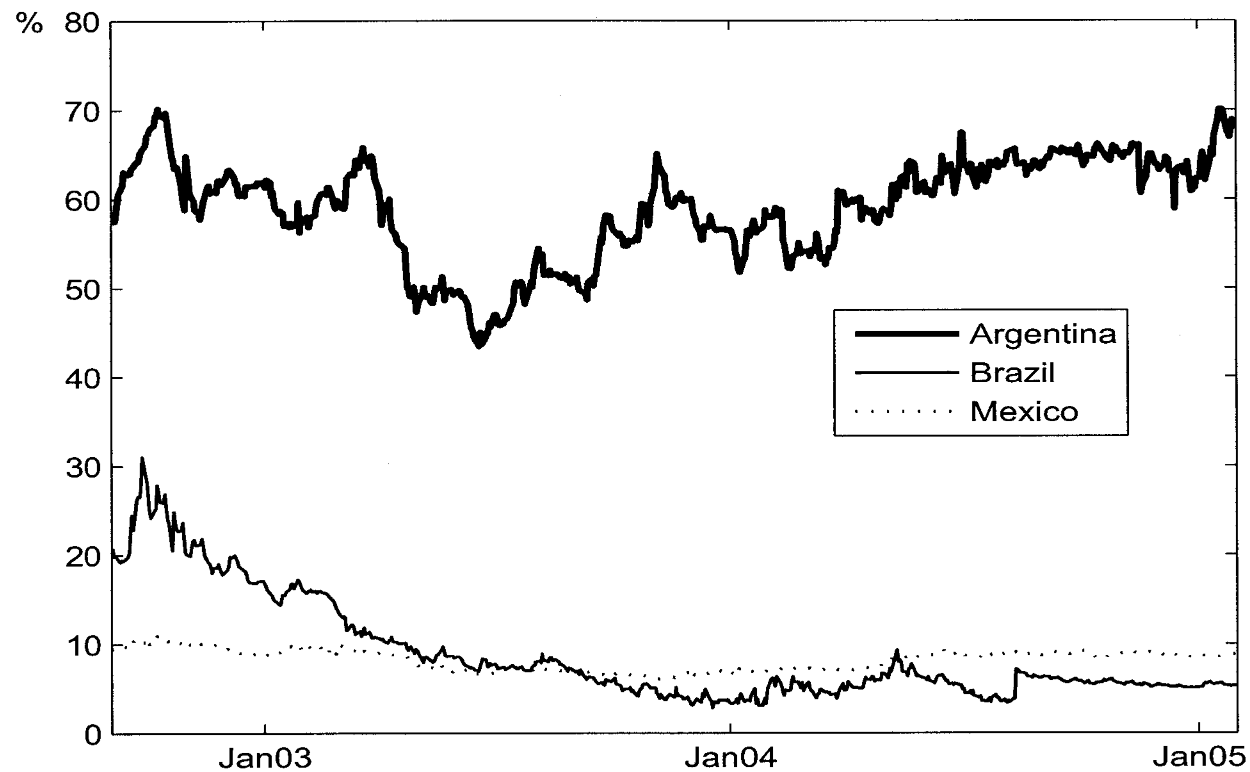

Figure 2.2: Interpolated Forward Rates of Argentina, Brazil and Mexico

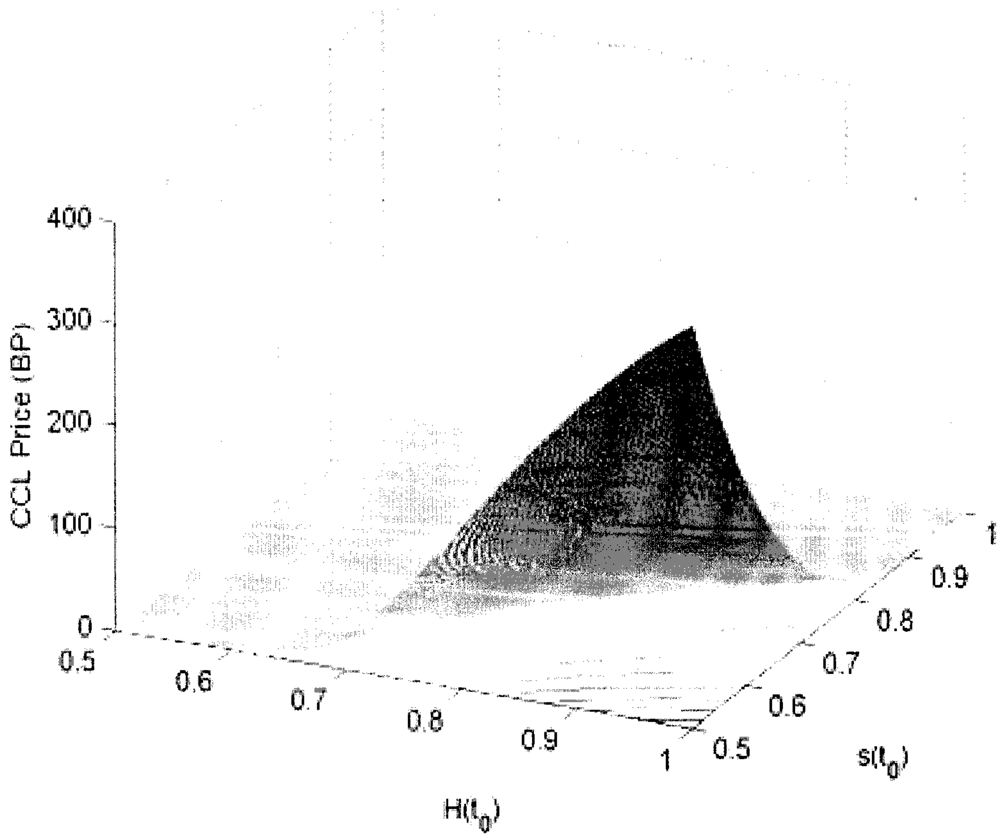

Figure 2.3: CCL Price for Argentina

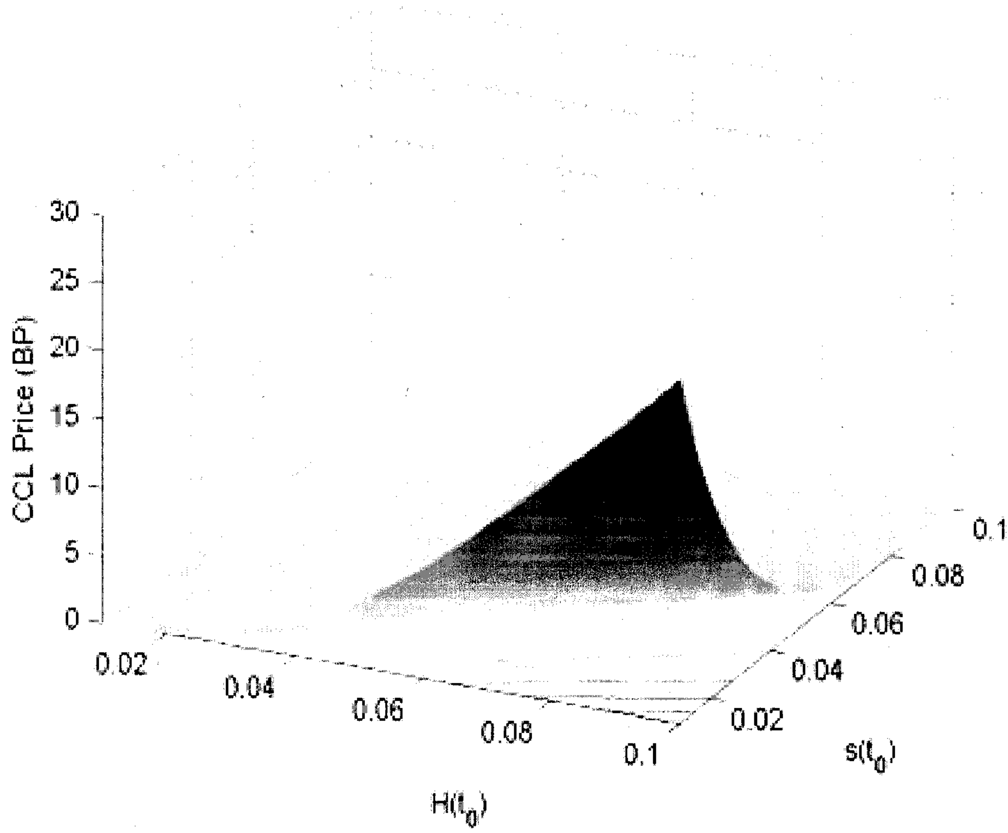

Figure 2.4: CCL Price for Brazil

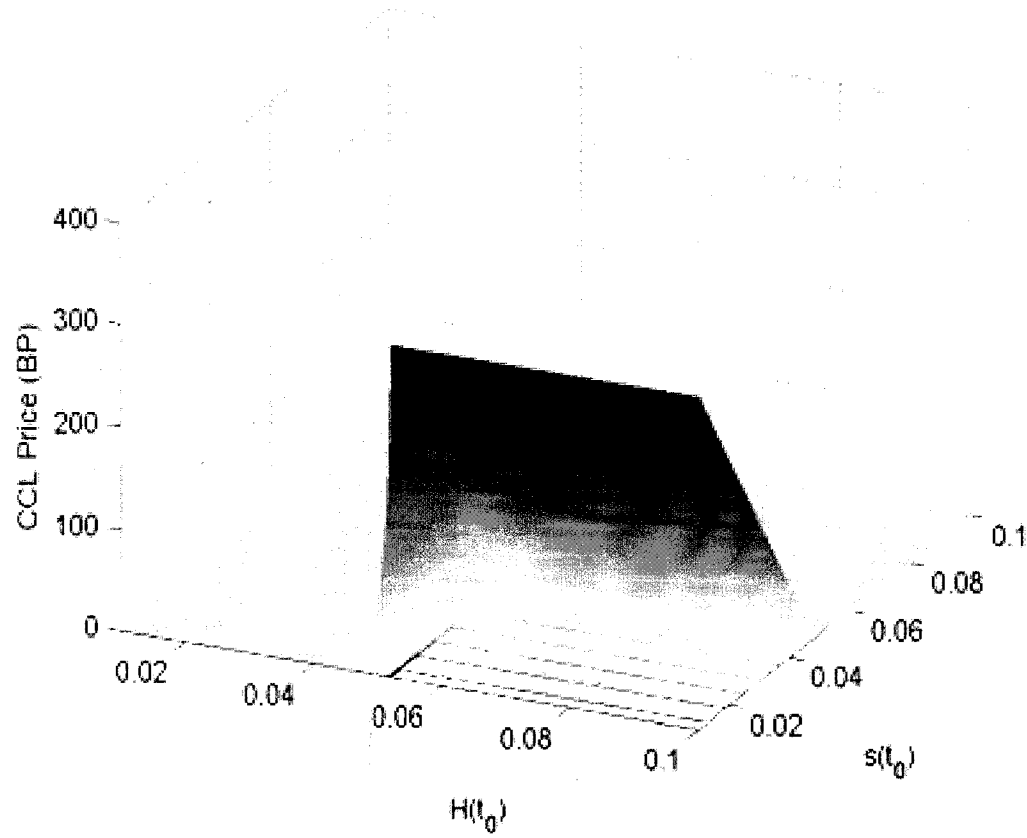

Figure 2.5: CCL Price for Mexico

Part III

Essay 3: Forecasting Stock Market Volatility: The Empirical Performance of the SGED-GARCH, Implied, and Realized Volatility Models

3.1 Introduction

The volatility of equity returns is one of the cornerstones of the theory of modern finance. Volatility is central to capital asset pricing and portfolio analysis, plays a pivotal role in the pricing of contingent claims, and has an important role in the hypothesis testing inherent in event and market microstructure studies. In addition, stock return volatility is an underlying asset for financial derivatives known as volatility swaps. These are the forward contracts on future realized stock volatility, and provide an easy way for investors to gain exposure to the future level of volatility. Investors can use these instruments to speculate on future volatility levels, to trade the difference between realized and implied volatility, or to hedge the volatility exposure of other positions or businesses.

Given these facts, many researchers have been led to model and estimate the conditional distribution of stock returns and stock return volatility for the last two decades. Following the introduction of ARCH processes by Engle (1982) and their generalization by Bollerslev (1986), there have been numerous refinements of this approach to modelling conditional volatility. Most of these refinements have been driven by three empirical regularities of stock prices. First, equity returns are fat-tailed and this leptokurtosis cannot be eliminated by the time-varying variances of GARCH processes because even allowing for changing variances, there remain too many very large events.

A second empirical finding especially for weekly, daily, or higher-frequency returns is volatility clustering: large (small) changes in short term stock returns are followed by large (small) changes of either sign. This type of behavior has been successfully captured by the standard GARCH model of Bollerslev (1986) that models conditional variance as a moving average of lagged squared residuals and the absolute value GARCH model of Taylor (1986) that models conditional standard deviation as a moving average of lagged absolute residuals. A shortcoming of these symmetric GARCH processes is that positive and negative information shocks of the same magnitude produce the same amount of volatility. In other words, these models cannot cope with the skewness of equity returns.

The third stylized fact, asymmetry, seems to be responsible for the plethora of extant GARCH models. Following Black's (1976) exploration of this phenomenon, it is now commonly referred to as the leverage effect: changes in stock prices tend to be negatively related to changes in volatility. In the absence of a good theoretical model for this asymmetry, the GARCH literature has searched for econometric ways to describe the asymmetry. Models such as the Asymmetric GARCH (AGARCH) process of Engle (1990), the Exponential GARCH (EGARCH) introduced by Nelson (1991), the Square-root GARCH (SQR-GARCH) process of Heston (1993), the Nonlinear GARCH (NGARCH) and VGARCH processes of Engle and Ng (1993), the Threshold GARCH models of Glosten, Jagannathan, and Runkle (GJR-GARCH, 1993) and Zakoian (TGARCH, 1994), and the Quadratic GARCH (QGARCH) process of Sentana (1995) are among the most popular asymmetric GARCH models.[11]

Although many studies show that the parameters of different ARCH models are highly significant in-sample, there is mixed evidence that they provide good out-

[11]For a comprehensive survey on ARCH models, see Bollerslev, Chou and Kroner (1992), Bollerslev, Engle and Nelson (1994), Hentschel (1995), Ghysels, Harvey and Renault (1996), Palm (1996), Pagan (1996), Duan (1997), and Bali (2000).

of-sample forecasts of the equity return volatility. Some studies examine the out-of-sample predictive ability of ARCH models, and find that a regression of realized volatility on estimated volatility yields a low R^2 value. Andersen and Bollerslev (1998a,b) show that regression methods will produce low R^2 values when realized volatility is measured by the daily squared returns, even for optimal GARCH forecasts, because squared daily returns are noisy estimates of volatility. They show that intraday returns can be used to construct a realized volatility series that eliminates the noise in measurements of realized volatility. They find remarkable improvements in the forecasting performance of ARCH models when they are used to forecast the new realized series, compatible with theoretical analysis.

An alternative approach to GARCH volatility forecasts is to use implied volatilities from options. Day and Lewis (1992) and Lamoureux and Lastrapes (1993) examine implied volatility (IV) as a source of information. Both studies find that IV contributes a statistically significant amount of information about volatility over the short-term forecasting horizon covered by the models, but they also find that IV does not fully impound the information that the model is able to extract from historical prices. Day and Lewis conclude that IV performs as well but no better than forecasts from ARCH models, and mixtures of two forecasts outperform both univariate forecasts. Lamoureux and Lastrapes examine forecasting volatility through option expiration and find that IV alone is less accurate in 8 out of 10 cases than the models that incorporate historical prices. Canina and Figlewski (1993) provide contrary evidence, and indicate that implied volatilities are poor forecasts of volatility, and simple historical volatilities outperform implied volatilities. Christensen and Prabhala (1998) show for a much longer period that while implied volatilities are biased forecasts of volatility they perform better than historical volatility models. Fleming (1998) also provides evidence that implied volatilities are more informative than daily returns in forecasting equity volatility.

The importance of intraday returns for measuring realized volatility is demonstrated by Andersen and Bollerslev (1998a,b), Andersen et al. (2001a,b; 2003), Barndoff-Nielsen and Shephard (2001; 2002a,b), and Andreou and Ghysels (2002).[12] They take the sum of squared high-frequency intraday returns as an approximation to the daily volatility. The estimate of this quadratic variation is referred to as integrated or realized volatility. Blair et al. (2001) explore the incremental volatility information of high-frequency (5-min) stock index returns. They answer some important empirical questions for the S&P 100 index. Based on their in-sample analysis of low-frequency (daily or weekly) data using GARCH models, Blair et al. find no evidence for incremental information in daily index returns beyond that provided by the implied volatility index (VIX). Their extension of the historical information set to include high-frequency intraday returns suggests that there is only minor incremental information in high-frequency returns, and this information is almost subsumed by implied volatilities. Their out-of-sample comparisons of volatility forecasts show that VIX provides more accurate forecasts than either low- or high-frequency index returns, regardless of the definition of realized volatility and the horizon of the forecasts.

There is substantial empirical evidence that the distribution of stock returns is typically skewed to the left, is peaked around the mode (leptokurtic), and has fat tails. The leptokurtosis is reduced, but not eliminated, when returns are stan-

[12]The integrated volatility has also been used in foreign exchange and equity markets by Andersen and Bollerslev (1997a,b), Taylor and Xu (1997), Blair et al. (2001), Maheu and McCurdy (2001), and Areal and Taylor (2002).

dardized using time-varying estimates for the conditional means and variances.[13] To accommodate skewness, kurtosis, and higher-order moment dependencies, this paper uses a skewed fat-tailed distribution and tests its ability to model the conditional volatility of stock market returns within a discrete time GARCH framework.

It is well known that the residuals from any return generating process within the class of ARCH models with normally distributed errors are non-normal. This result led to the use of fat-tailed distributions in modelling the conditional volatility of financial time series. Bollerslev (1987) and Bollerslev and Wooldridge (1992) use the standardized Student t distribution in estimating conditional variances and covariances. Hsieh (1989) and Nelson (1991) use the fat-tailed generalized error distribution (GED) to model the empirical distribution of exchange rates and stock market returns. It is important to note that both the standardized t and GED are symmetric fat-tailed distributions. Hansen (1994) introduces a generalization of Student t distribution where asymmetries may occur, while maintaining the assumption of a zero mean and unit variance. Theodossiou (2002) introduces an asymmetric (or skewed) version of the generalized error distribution (SGED), which adds an additional moment - skewness - to the GED formulation. Bali (2003) uses SGED and extends the standard option pricing models with lognormal forward rates to accommodate significant kurtosis observed in the interest rate data. Bali and Theodossiou (2003) use SGED to estimate value at risk and expected shortfalls in an unconditional risk management context.

SGED has so far not been used to model the time-varying conditional mean or conditional variance of financial return series. This paper compares the in-sample and out-of-sample performance of the SGED-GARCH models with implied

[13]See Bollerslev, Engle, and Nelson (1994) and the references therein.

and realized volatility models for forecasting the S&P 100 index volatility. The information content of the alternative volatility estimators is examined, and several important questions for forecasting stock market volatility are answered.

The paper is organized as follows. Section 2 provides the alternative volatility models. Section 3 describes the data on daily and 5-minute index returns, and daily realized, implied and SGED-GARCH volatilities. Section 4 presents the estimation results and discusses the in-sample and out-of-sample performance of the alternative volatility models. Section 5 concludes the paper.

3.2 Alternative Volatility Models

3.2.1 Realized volatility

The empirical performance of the alternative volatility models can be evaluated using a measure of the latent volatility process. Since the actual volatility process is not observed, researchers have to use a variety of methods to calculate ex post estimates of volatility, often called realized volatility. The most common method of computing a realized volatility is to square or take the absolute value of the inter-period returns. For example, if one intends to forecast daily volatility, the realized measure is the squared or absolute demeaned daily returns. Nevertheless, Andersen and Bollerslev (1998a,b) show that this method yields inaccurate forecasts for correctly specified volatility models. By sampling more frequently and producing a measure based on intraday data, they find that the noisy component of the realized measure diminishes and that in theory the realized volatility based on the high-frequency data is then much closer to the volatility during a day.

Following a recent set of articles by Andersen and Bollerslev (1998a,b), Andersen et al. (2001a, 2001b, 2003), Barndoff-Nielson and Shephard (2001, 2002a, 2002b), and Andreou and Ghysels (2002), we assume the sum of squared high frequency intraday returns provides a reliable approximation to the daily volatility. We use the 5-minute returns from the S&P 100 index to calculate a measure of realized (or integrated) volatility. To construct the daily realized volatility, we square and then sum the 5-minute returns for the period from 9:30 EST to 16:00 EST.

3.2.2 Implied volatility

Expected future volatility plays a central role in finance theory, and an accurate estimation of this parameter is crucial to financial decision-making. Finance researchers generally use the past behavior of asset prices to develop expectations about volatility, modelling movements in volatility as they relate to the prior volatility and/or variables in the investors' information set. An alternative approach is to use reported option prices to infer volatility expectations. Since option value depends critically on the expected future volatility, the volatility expectation of market participants can be recovered by inverting the option valuation formula.

Implied volatilities are considered to be the market's forecast of the volatility of the underlying asset of an option. To calculate an implied volatility an option valuation model (such as the Black-Scholes (1973) model) is needed as well as inputs for that model (price of the underlying asset, strike price, risk-free rate, time-to-expiration, dividends) and an observed option price.

Following Fleming et al. (1995), we use an implied volatility index (VIX) that mitigates the problems caused by the use of inappropriate option valuation model and relatively infrequent trading of the stocks in the index. VIX is constructed so that it represents a hypothetical option that is at-the-money and has a constant 22 trading days (30 calendar days) to expiration. VIX is a weighted index of American implied volatilities calculated from eight near-the-money, near-to-expiry, S&P 100 call and put options. VIX is constructed in such a way as to eliminate mismeasurement and smile effects. This makes it a more accurate measurement of implied market volatility.[14] Even though VIX is robust to mismeasurement, it is still a biased predictor of subsequent volatility. As indicated by Blair et al. (2001), bias occurs because of a trading time adjustment that typically multiplies[15] conventional implied volatilities by approximately 1.2.[16]

3.2.3 SGED-GARCH models

Subbotin (1923) introduces the generalized error distribution (GED) as a generalized case of Laplace, normal distributions. The symmetric GED density is given as

$$f(\varepsilon_t; v) = \frac{v \exp\left[-\frac{1}{2}\left|\frac{\varepsilon_t}{\Pi}\right|^{v}\right]}{\Pi\, 2^{\frac{v+1}{v}}\, \Gamma\left(\frac{1}{v}\right)},$$

where $\varepsilon_t = \dfrac{R_t - \mu}{\sigma}$, R_t is the return, μ and σ are the mean and standard deviation parameters of the GED. $\Gamma(a) = \int_0^{+\infty} x^{a-1} e^{x} dx$ is the Gamma function, $\Pi =$

[14] VIX uses pairs of near-the-money exercise prices, that are just above the current index price and just below it. It also uses pairs of times to expiry, one that is nearby (at least eight calendar days to expiration) and one that is second nearby (the following contract month). For a detailed explanation of the construction of the VIX index, see Fleming et al. (1995).

[15] VIX multiplies the conventional implied volatility by $(N_c/N_t)^{0.5}$, with N_c and N_t, calendar and trading days until expiry.

[16] Our empirical analysis uses the original version of VIX. The new VIX, introduced by the CBOE on September 22, 2003, is obtained from the S&P 500 index option prices and incorporates information from the volatility skew by using a wider range of strike prices rather than just at-the-money series.

$\left[\frac{2^{-\frac{2}{v}}\Gamma(\frac{1}{v})}{\Gamma(\frac{3}{v})}\right]^{\frac{1}{2}}$, and $v > 0$ is the tail-thickness parameter. For $v = 2$, the GED reduces to normal distribution has. If $v > 2$, the density has thinner tails than normal distribution, whereas for $v < 2$, it has thicker tails. For $v = 1$, it reduces to Laplace distribution.

The GED is used by Box and Tiao (1962) to model prior densities in Bayesian estimation, Nelson (1991) to model the distribution of stock market returns, and Hsieh (1989) to model the distribution of exchange rates. Theodossiou (2002) introduces an asymmetric (or skewed) version of GED. The skewed generalized error distribution (SGED) adds an additional moment, skewness, to the GED formulation. The probability density function for the SGED is

$$f(R_t; \mu, \sigma, v, \lambda) = \frac{C}{\sigma} \exp\left[-\frac{|R_t - \mu + \delta\sigma|^v}{[1 + \text{sign}(R_t - \mu + \delta\sigma)\lambda]^v \theta^v \sigma^v}\right], \qquad (3.1)$$

$$C = \frac{v}{2\theta}\left[\Gamma\left(\frac{1}{v}\right)\right]^{-1},$$

$$\theta = \left[\Gamma\left(\frac{1}{v}\right)\right]^{-\frac{1}{2}}\left[\Gamma\left(\frac{3}{v}\right)\right]^{-\frac{1}{2}} S^{-1},$$

$$\delta = 2\lambda\Omega S^{-1},$$

$$S = \sqrt{1 + 3\lambda^2 - 4\Omega^2\lambda^2},$$

$$\Omega = \Gamma\left(\frac{2}{v}\right)\left[\Gamma\left(\frac{1}{v}\right)\right]^{-\frac{1}{2}}\left[\Gamma\left(\frac{3}{v}\right)\right]^{-\frac{1}{2}},$$

where R_t is the return, $\mu = \mathrm{E}[R_t]$ is the mean of R_t, $\varepsilon_t = R_t - \mu + \delta\sigma$, and σ is the standard deviation of R_t. Sign($\cdot$) is the sign function, which is 1 for positive value and -1 for negative value. $\upsilon > 0$ is the tail-thickness parameter. $-1 < \lambda < 1$ is the skewness parameter to control the rate of descent of the density around R_t's mode, which is $\mu - \delta\sigma$. In the case of $\lambda > 0$, the density function is skewed to the right side (long right tail). This is because R_t is weighted by a greater value than unity for $R_t < \mu - \delta\sigma$; and by a value less than unity for $R_t > \mu - \delta\sigma$. This means that the probability of $R_t < \mu - \delta\sigma$ is lower than that of $R_t > \mu - \delta\sigma$. The opposite is true for negative λ. Note that λ and δ have the same sign, thus, in case of positive λ and δ, Mode(R_t) is less than μ. The parameter δ is called Pearson's skewness. The SGED distribution reduces to GED distribution for $\lambda = 0$, Laplace distribution for $\lambda = 0$ and $\upsilon = 1$, and normal distribution for $\lambda = 0$ and $\upsilon = 2$.

Most asset pricing models postulate a positive relationship between stock market expected return and risk modelled by the variance (or standard deviation) of returns. The following GARCH-in-mean process is used to model the conditional mean and the conditional volatility of the stock market returns:

$$R_t = \mu - 2\lambda\Omega S^{-1}\sigma_t + \varepsilon_t, \ \varepsilon_t = \sigma_t z_t,$$

$$f(\sigma_t) = h(\sigma_{t-1}, \varepsilon_{t-1}; \beta_0, \beta_1, \beta_3) + \beta_2 f(\sigma_{t-1}),$$

$$f(\sigma_t) = \sigma_t, \ \sigma_t^2, \text{ or } \ln\sigma_t^2,$$

where λ, Ω, S, and $\delta = 2\lambda\Omega S^{-1}$ are defined in (3.1). R_t is the S&P 100 index return for the period t. $\mu - 2\lambda\Omega S^{-1}\sigma_t$ is the conditional mean and σ_t is the conditional standard deviation of the returns based on the information available at time $t-1$.

We use 10 GARCH models to estimate the volatility of S&P 100 returns:

1. AGARCH: Asymmetric GARCH model of Engle (1990)

$$\sigma_t^2 = \beta_0 + \beta_1(\varepsilon_{t-1} + \beta_3)^2 + \beta_2\sigma_{t-1}^2;$$

2. EGARCH: Exponential GARCH model of Nelson (1991)

$$\ln \sigma_t^2 = \beta_0 + \beta_1 \left(\left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| - \sqrt{\frac{2}{\pi}} \right) + \beta_2 \ln \sigma_{t-1}^2 + \beta_3 \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right);$$

3. GARCH: Linear symmetric GARCH model of Bollerslev (1986)

$$\sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2;$$

4. GJR-GARCH: Threshold GARCH model of Glosten, Jagannathan, and Runkle (1993)

$$\sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2 - \beta_3 \mathbf{1}_{t-1}^- \varepsilon_{t-1}^2$$

$$\mathbf{1}_{t-1}^- = 1 \text{ if } \varepsilon_{t-1} < 0, \text{ and } \mathbf{1}_{t-1}^- = 0 \text{ otherwise;}$$

5. NGARCH: Nonlinear asymmetric GARCH model of Engle and Ng (1993)

$$\sigma_t^2 = \beta_0 + \beta_1 (\varepsilon_{t-1} + \beta_3 \sigma_{t-1})^2 + \beta_2 \sigma_{t-1}^2;$$

6. QGARCH: Quadratic GARCH model of Sentana (1995)

$$\sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \beta_2 \sigma_{t-1}^2 + \beta_3 \varepsilon_{t-1};$$

7. SQR-GARCH: Square-Root GARCH model of Heston (1993)

$$\sigma_t^2 = \beta_0 + \beta_1 \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_3 \sigma_{t-1} \right)^2 + \beta_2 \sigma_{t-1}^2;$$

8. TS-GARCH: The specification proposed by Taylor (1986)

$$\sigma_t = \beta_0 + \beta_1 |\varepsilon_{t-1}| + \beta_2 \sigma_{t-1};$$

9. VGARCH: A version proposed in Engle and Ng (1993)

$$\sigma_t^2 = \beta_0 + \beta_1 \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \beta_3 \right)^2 + \beta_2 \sigma_{t-1}^2;$$

10. TGARCH: Threshold GARCH model of Zakoian (1994)

$$\sigma_t = \beta_0 + \beta_1 |\varepsilon_{t-1}| + \beta_2 \sigma_{t-1} + \beta_3 \mathbf{1}_{t-1}^{-} \varepsilon_{t-1}$$

$$\mathbf{1}_{t-1}^{-} = 1 \text{ if } \varepsilon_{t-1} < 0, \text{ and } \mathbf{1}_{t-1}^{-} = 0 \text{ otherwise,}$$

where $\beta_0 \geq 0$, $0 \leq \beta_1 < 1$, $0 \leq \beta_2 < 1$. $\beta_3 \leq 0$ because of skewness. We use the SGED distribution to capture the fat tail and the skewness in the error terms. Based on the SGED density in (3.1), we can estimate the parameters in the SGED-GARCH models by maximizing the log-likelihood function

$$\mathrm{LogL}_{\mathrm{SGED}} = n \ln C - \sum_{t=1}^{n} \ln \sigma_t - \sum_{t=1}^{n} \left[\frac{|R_t - \mu + \delta\sigma_t|^{\upsilon}}{[1 + \mathrm{sign}(R_t - \mu + \delta\sigma_t)\lambda]^{\upsilon} \theta^{\upsilon} \sigma_t^{\upsilon}} \right], \qquad (3.2)$$

where C, θ, δ, λ, υ are given by (3.1). μ, β_0, β_1, β_2 and β_3 are the parameters in the conditional mean and volatility of the alternative SGED-GARCH models.

3.2.4 Fitted volatility

The traditional measure of volatility, based on the squared or absolute daily returns, has been criticized in the recent literature (see, e.g., Andersen and Bollerslev (1998a,b) among others), which advocates the use of the sum of squared intraday returns as an essentially model-free and unbiased estimator of volatility. While there certainly are sound reasons to focus on volatility derived from intraday returns, in certain applications, it may be of interest to forecast volatility based upon squared (or absolute) daily returns, and not the integrated volatility. We also think that it would be interesting to see how important the selection of the measure of realized volatility is in evaluating the predictive accuracy of volatility forecasts with skewed-fat tailed distributions. Therefore, we generate the traditional realized volatility measure, which is calculated as the absolute demeaned returns, $|R_t - \overline{R}|$, where R_t is the daily index return and $\overline{R}$ is the sample mean.

We forecast the realized volatility based on the 5-minute returns and the traditional volatility based on the daily returns directly. We use an ARMA(p,q) process to generate the fitted value of the realized and traditional volatility measures and compare the empirical performance of the ARMA-fitted volatility model with the implied and SGED-GARCH volatility estimators. Specifically, the following ARMA(5,5) specifications are used to generate the fitted volatilities based on the daily realized and the traditional volatility models:

$$\mathrm{INTRA}_t = \alpha_0 + \sum_{i=1}^{5} \alpha_i \mathrm{INTRA}_{t-i} + \sum_{i=1}^{5} \omega_i \varepsilon_{t-i} + \varepsilon_t, \tag{3.3}$$

$$\mathrm{TRAD}_t = \alpha_0 + \sum_{i=1}^{5} \alpha_i \mathrm{TRAD}_{t-i} + \sum_{i=1}^{5} \omega_i \varepsilon_{t-i} + \varepsilon_t, \tag{3.4}$$

where INTRA_t is the sum of squared five-minute returns in day t and TRAD_t is the absolute demeaned daily return for day t. Following Andersen et al. (2003), we use the lag polynomials up to 5 days, but the qualitative results remain the same when we use the lag polynomials up to three and six days.

3.3 Data

We use three data sets in this paper: daily S&P 100 index returns, daily implied volatility, and 5-minute index returns. The first data set contains daily closing levels of the S&P 100 index. To compute the stock market returns (R_t), we use a formula: $R_t = \ln P_t - \ln P_{t-1}$, where P_t is the value of the stock market index for time t. The time period of investigation for daily index returns extends from January 2, 1987 through December 31, 2002, yielding a total of 4,029 daily observations. This data set is used to estimate the conditional volatilities based on the alternative GARCH models with SGED density. Because of the autoregressive of order one process in the GARCH models, the 4,029 daily index return generate a time-series of 4,028 daily volatilities from January 5,[17] 1987 through December 31, 2002.

The second data set includes the Chicago Board Options Exchange (CBOE)'s implied volatility index (VIX) from January 2, 1987 to December 31,2002, yielding a total of 4,029 daily observations. VIX provides investors with up-to-the-minute market forecasts of expected volatility by using real-time OEX index option bid/ask quotes. OEX options are American-style options written on the S&P 100 index, and can be exercised on any business day before the expiration date. The CBOE's volatility index is calculated by taking an average time to maturity of 22 trading days (or 30 calendar days). In this study, daily implied volatilities are computed from an annualized implied volatility index as $\mathrm{VIX}/\sqrt{252}$.

The third data set contains high-frequency intraday data from January 2, 1987 through December 31, 2002, giving a total of 315,263 5-minute returns. These 5-minute returns are constructed from index levels recorded every 15 seconds. We use these 5-minute returns from the S&P 100 index to calculate a measure of realized (or integrated) volatility following Blair et al. (2001). To construct the daily realized volatility, we square and then sum 5-minute returns from 9:30 EST to 16:00 EST for each trading day.

Table 3.1 shows the descriptive statistics for the 5-minute returns on the S&P 100. The unconditional mean of the 5-minute returns is about -0.0035% with a standard deviation of 0.1027%. The maximum and minimum values are about 2.7989% and -1.7339%. The skewness and kurtosis are reported to test the assumption of normal distribution. The skewness (0.2348) for the 5-minute returns

[17] January 3 and 4, 1987 are weekends.

is positive. The kurtosis (20.4349) is large and significant at the 1% level, implying that the distribution of the 5-minute returns has much thicker tails than normal distribution has. The Augmented Dickey-Fuller (ADF) statistic indicates strong rejection of the null hypothesis of a unit root for the 5-minute returns. The autocorrelations of the 5-minute returns are generally small and not systematically positive or negative.

Table 3.1 also presents the descriptive statistics for the daily returns on the S&P 100 index. The unconditional mean is about 0.0339% with a standard deviation of 1.2043%. The maximum and minimum values are about 8.5307% and -23.7805%. The skewness (-2.0636) for the daily returns is negative and statistically significant at the 1% level. The kurtosis (45.0103) is considerably large and significant at the 1% level, implying that the distribution of daily returns has much thicker tails than normal distribution has. The ADF statistic shows strong rejection of the null hypothesis of a unit root for the daily returns. Like those of the 5-minute returns, the autocorrelations of the daily returns are small and not systematically positive or negative.

Table 3.1 also provides the descriptive statistics for the daily realized, implied (VIX) and SGED-TGARCH volatility measures. The unconditional means of the realized, VIX, and SGED-TGARCH volatility measures are 0.8524%, 1.3629% and 1.0851%. Their unconditional standard deviations are 0.5217%, 0.5347% and 0.4485%. The (maximum, minimum) values of the realized, VIX and SGED-TGARCH volatility measures are (10.2525%, 0.0747%), (9.4611%, 0.5695%), and (8.4677%, 0.0996%). The skewness and kurtosis of these volatility measures imply that the volatility distribution is skewed toward right and has much thicker tails than normal distribution has. The ADF statistic indicates the rejection of the null hypothesis of a unit root for these volatility estimators. The autocorrelations of these volatility estimators are significant and systematically positive, implying the existence of a strong serial correlation in each of the volatility estimators.

3.4 Empirical Results

3.4.1 Estimation results and model comparisons

Table 3.2 shows the maximum likelihood parameter estimates of the alternative GARCH models with SGED density. The intercept (μ) of the conditional mean equation, $\mu - 2\lambda\Omega S^{-1}\sigma_t$, is not statistically significant at the 5% level in 6 out of the 10 GARCH models according to the t-statistics given in parentheses.

In the conditional standard deviation, variance, or log-variance of the alternative GARCH models, $f(\sigma_t) = h(\sigma_{t-1}, \varepsilon_{t-1}; \beta_0, \beta_1, \beta_3) + \beta_2 f(\sigma_{t-1})$, where $f(\sigma_t) = \sigma_t, \sigma_t^2$, or $\ln \sigma_t^2$, the parameters $(\beta_0, \beta_1, \beta_2)$ are all significant at the 5% level. This result confirms the presence of conditionally heteroskedastic volatility effects in the stock return process. Specifically, the conditional volatility parameters are positive $(\beta_1 > 0, \beta_2 > 0)$, and the sum of β_1 and β_2 is around one for all the SGED-GARCH models. This implies the existence of substantial volatility clustering and persistence in stock market volatility. β_3's are all significantly negative, which implies that an unexpected negative return shock $(\varepsilon_{t-1} < 0)$ will have a greater impact on $f(\sigma_t)$ than the positive return shock $(\varepsilon_{t-1} > 0)$ of the same magnitude will. These results are consistent with the former research on asymmetric GARCH models.

The maximum likelihood estimates of the skewness parameter λ and the tail-thickness parameter υ in the SGED distribution are not only statistically significant, but also confirm our observation of the negative skewness and tail-thickness of the empirical return distribution. The estimates of λ turn out to be negative and significant at the 5% level in 8 out of the 10 GARCH models. This implies negative skewness in the S&P 100 index returns, which can be viewed as the phenomenon that, after the returns have been standardized by subtracting the mean, positive returns of a given magnitude have higher probabilities than negative returns of the same magnitude. Another notable point in the table 3.2 is that the degrees of freedom (or tail-thickness) parameter υ is estimated to be in the range of 1.15 to 1.27, and they are all statistically significant at the 1% level. Normality requires that the estimated υ be equal to 2. From the t-statistics that tests the null hypothesis of $\upsilon = 2$, the estimates of υ turn out to be statistically less than 2 for all the GARCH models, which indicates tail-thickness in the empirical return distribution.

Overall, the maximum likelihood parameter estimates imply that the conditional distribution of the stock index returns is skewed to left, and has fatter tails than normal distribution has. The t-statistics of the parameters λ and υ reject generalized error distribution, Laplace, and normal distributions in favor of skewed generalized error distribution.

We use the determination coefficient (R^2) and the mean absolute percentage error (MA%E) to compare the in-sample forecasting performance of the alternative volatility models. The alternative models include the 10 SGED-GARCH models, the implied volatility index (VIX), and the ARMA-fitted volatility. We label the realized volatility as INTRA_t, which is calculated by taking square root of the sum of squared 5-minute returns on the S&P 100 index. We also use the traditional measure of realized volatility, the absolute demeaned daily index return. Since this is a traditional way of calculating volatility, we label it as TRAD_t or $|R_t - \bar{R}|$.

We report the R^2 values from the following two sets of regressions:

$$\begin{aligned} \mathrm{TRAD}_t = & \ a_0 + a_1\sigma_t + \varepsilon_t, \\ \mathrm{TRAD}_t = & \ a_0 + a_1\sigma_{t-1} + \varepsilon_t \end{aligned} \tag{3.5}$$

$$\begin{aligned} \mathrm{INTRA}_t = & \ a_0 + a_1\sigma_t + \varepsilon_t, \\ \mathrm{INTRA}_t = & \ a_0 + a_1\sigma_{t-1} + \varepsilon_t, \end{aligned} \tag{3.6}$$

where σ_t and σ_{t-1} are the current and lagged estimated SGED-GARCH, ARMA-fitted, and VIX implied volatilities. The MA%E is calculated by

$$\mathrm{MA\%E} = \frac{\sum_{t=6}^{N} \left| \frac{\mathrm{INTRA}_t \text{ or } \mathrm{TRAD}_t - \sigma_t}{\sigma_t} \right|}{N} \times 100\%, \tag{3.7}$$

where $N = 4,029$, and t begins from 6 because the fitted volatility is the forecasted value of the ARMA(5,5) process.

The first four columns of the table 3.3 present the R^2 obtained from the regression (3.5) and the MA%E. The R^2's from the contemporaneous and predictive regressions show that the VIX is the best volatility model. Nevertheless, when we look at the mean absolute percentage errors, the VIX is eclipsed by the SGED-GARCH and the fitted volatility models. The MA%E shows that the fitted volatility performs best. The last four columns of this table report the R^2 from the regression (3.6) and the MA%E. Based on the contemporaneous and predictive relations between the realized and the alternative volatility estimators, the VIX again outperforms all the other volatility models. In the GARCH models, the R^2 value shows that the TGARCH model performs best, and it provides a better forecast than the fitted volatility does. The R^2 values show that the SGED-TGARCH model provides best forecasts among the GARCH models. Since the TGARCH model performs best, we choose it to represent the SGED-GARCH family to compare its forecasting performance with the VIX and the fitted volatility in the following sections.

3.4.2 Comparing in-sample forecasting performance

This section compares the information content of the SGED-GARCH, VIX, traditional, and integrated volatility models for forecasting stock market volatility. The empirical analysis is based on seven different specifications of the conditional volatility process.

Traditional volatility

We first consider traditional volatility. Assuming that the SGED density approximates the empirical return distribution, the conditional mean and the alternative conditional volatility models can be written as

$$
\begin{aligned}
&\text{Mean:} && R_t = && \mu - 2\lambda\Omega S^{-1}\sigma_t + \varepsilon_t, \\
&\text{TGARCH:} && \sigma_t = && \beta_0 + \beta_1|\varepsilon_{t-1}| + \beta_2\sigma_{t-1} + \beta_3\max[\varepsilon_{t-1}, 0], \\
&\text{VIX:} && \sigma_t = && \beta_0 + \beta_4\text{VIX}_{t-1}, \\
&\text{TRAD:} && \sigma_t = && \beta_0 + \beta_5|R_{t-1} - \bar{R}|, \\
&\text{VIX-TRAD:} && \sigma_t = && \beta_0 + \beta_4\text{VIX}_{t-1} + \beta_5|R_{t-1} - \bar{R}|, \\
&\text{VIX-TARCH:} && \sigma_t = && \beta_0 + \beta_1|\varepsilon_{t-1}| + \beta_2\sigma_{t-1} + \beta_3\max[\varepsilon_{t-1}, 0] + \beta_4\text{VIX}_{t-1}, \\
&\text{TGARCH-TRAD:} && \sigma_t = && \beta_0 + \beta_1|\varepsilon_{t-1}| + \beta_2\sigma_{t-1} + \beta_3\max[\varepsilon_{t-1}, 0] \\
& && && +\beta_5|R_{t-1} - \bar{R}|, \\
&\text{GPM}_{\text{trad}}: && \sigma_t = && \beta_0 + \beta_1|\varepsilon_{t-1}| + \beta_2\sigma_{t-1} + \beta_3\max[\varepsilon_{t-1}, 0] \\
& && && +\beta_4\text{VIX}_{t-1} + \beta_5|R_{t-1} - \bar{R}|,
\end{aligned}
\tag{3.8}
$$

where the general parametric model (GPM) nests SGED-TGARCH, VIX, and the forecasted traditional volatility models, as well as their possible combinations.

Table 3.4 presents the maximum likelihood parameter estimates, the asymptotic t-statistics, and the maximized log-likelihood values. For all the specifications of the conditional volatility processes, the skewness parameter λ is found to be negative and statistically different from zero, indicating negative skewness in the S&P 100 index returns. [18] The tail-thickness parameter υ is in the range of 1.08 to 1.39. υ's are significantly less than 2. These results confirm our earlier finding that the empirical distribution of stock index returns is skewed to the left, and has fatter tails than normal distribution has.

In all these models, the coefficient on VIX (β_4) is positive and highly significant. In the VIX and VIX-TRAD models, β_4 is close to 0.88, whereas it reduces to 0.43 in the VIX-TGARCH model and further reduces to 0.27 in the GPM_{trad}. A notable point is that β_4 is significant at the 1% level in all the cases. The coefficients of the TGARCH model indicate significant information content in the past absolute shocks and asymmetric volatilities. Moreover, as shown in the VIX-TGARCH model, when SGED-TGARCH is added to the VIX model, the maximized log-likelihood (Log-L) value increases by 14 points. In the TRAD-model, the coefficient on the absolute demeaned returns (β_5) is significant, suggesting useful information content of the traditional volatility estimator in forecasting the S&P 100 index volatility. Nevertheless, the estimates from the VIX-TRAD model imply that there

[18]The only exception is the TGARCH-TRAD model. We should note that it is difficult to interpret results from the TGARCH-TRAD and GPM_{trad} models when the traditional volatility estimator and the SGED-TGARCH model are used as explanatory variables in the same volatility specification because there is strong multicollinearity between the absolute demeaned returns, $|R_{t-1} - \bar{R}|$, and the absolute information shocks, $|\varepsilon_{t-1}|$.

is no incremental information in the absolute demeaned daily returns and nearly all information is provided by the VIX index. This can be seen from the maximized log-likelihood values as well. When the traditional volatility information is added to the VIX model, the log-likelihood value increases only by 0.03 points.

Overall, the results in the table 3.4 indicate significant information content of the implied volatility index and the SGED-TGARCH model, and almost none or very minor incremental information in the absolute demeaned daily returns.

We use the R^2 and the MA%E values to compare the relative performance of the alternative volatility models used to forecast the conditional volatility of the S&P 100 index returns. We report the R^2 values from the regression

$$\mathrm{TRAD}_t = a_0 + a_1\sigma_t + \varepsilon_t, \tag{3.9}$$

and the MA%E is calculated from

$$\mathrm{MA\%E} = \frac{\sum_{t=2}^{N}\left|\frac{\mathrm{TRAD}_t - \sigma_t}{\sigma_t}\right|}{N} \times 100\%,$$

where $N = 4,029$, and t starts from 2 because of the autoregressive of order one process in the alternative volatility models. The first three columns of the table 3.6 report the estimated R^2 and MA%E values. The results show that among the VIX, TRAD_t, and TGARCH models, the traditional volatility estimator has the lowest R^2 and highest MA%E values, whereas the implied volatility index has the highest R^2 and lowest MA%E values. The R^2 and MA%E values show that the SGED-TGARCH model performs almost as well as the VIX model. As expected, the $\mathrm{GPM}_{\mathrm{trad}}$ model yields the highest R^2 and lowest MA%E values since it incorporates all the explanatory variables.

Realized volatility

We now consider the integrated (or realized) volatility. Assuming that the SGED density approximates the empirical return distribution, the conditional mean and the alternative conditional volatility models can be defined as

$$
\begin{aligned}
&\text{Mean:} & R_t =\ & \mu - 2\lambda\Omega S^{-1}\sigma_t + \varepsilon_t, \\
&\text{TGARCH:} & \sigma_t =\ & \beta_0 + \beta_1|\varepsilon_{t-1}| + \beta_2\sigma_{t-1} + \beta_3 \max[\varepsilon_{t-1}, 0], \\
&\text{VIX:} & \sigma_t =\ & \beta_0 + \beta_4 \text{VIX}_{t-1}, \\
&\text{INTRA:} & \sigma_t =\ & \beta_0 + \beta_5 \text{INTRA}_{t-1}, \\
&\text{VIX-INTRA:} & \sigma_t =\ & \beta_0 + \beta_4 \text{VIX}_{t-1} + \beta_5 \text{INTRA}_{t-1}, \\
&\text{VIX-TGARCH:} & \sigma_t =\ & \beta_0 + \beta_1|\varepsilon_{t-1}| + \beta_2\sigma_{t-1} + \beta_3 \max[\varepsilon_{t-1}, 0] + \beta_4 \text{VIX}_{t-1}, \\
&\text{TGARCH-INTRA:} & \sigma_t =\ & \beta_0 + \beta_1|\varepsilon_{t-1}| + \beta_2\sigma_{t-1} + \beta_3 \max[\varepsilon_{t-1}, 0] \\
& & & + \beta_5 \text{INTRA}_{t-1}, \\
&\text{GPM}_{\text{intra}}\text{:} & \sigma_t =\ & \beta_0 + \beta_1|\varepsilon_{t-1}| + \beta_2\sigma_{t-1} + \beta_3 \max[\varepsilon_{t-1}, 0] \\
& & & + \beta_4 \text{VIX}_{t-1} + \beta_5 \text{INTRA}_{t-1}.
\end{aligned} \tag{3.10}
$$

Table 3.5 presents the maximum likelihood parameter estimates, the asymptotic t-statistics, and the maximized log-likelihood values. For all the specifications of the conditional volatility processes, the skewness parameter λ is found to be negative and statistically different from zero, indicating negative skewness in the S&P 100 index returns. The tail-thickness parameter υ is in the range of 1.18 to 1.40. υ's are significantly less than 2. These results are very similar to those in the table 3.4.

Table 3.5 also shows that the coefficient on VIX (β_4) is positive and highly significant. When the implied volatility index is added to the TGARCH and the TRAD models, the maximized log-likelihood value increases substantially, indicating the significant incremental information content in the SGED-TGARCH model. When SGED-TGARCH is added to the VIX, and the INTRA models respectively, the maximized log-likelihood value increases by 14 and 103 points. In the INTRA-model, the coefficient on the integrated volatility estimator (β_5) is highly significant, suggesting strong information content of the high-frequency intraday returns in forecasting the S&P 100 index volatility. Similarly, the estimates from the combined VIX-INTRA and TGARCH-INTRA models imply the significant incremental information in the five-minute returns. This can be seen from the maximized log-likelihood values as well. When the integrated volatility information is added to the VIX and the SGED-TGARCH models, the log-likelihood value increases by 9 and 101 points, respectively. Overall, the results in the table 3.5 indicate significant information content of the implied volatility index, the integrated volatility estimator based on the five-minute index returns, and the SGED-TGARCH model.

We use the R^2 and the MA%E values to compare the empirical performance of the alternative volatility models for forecasting the conditional volatility of the S&P 100 index returns. We report the R^2 values from the regression

$$\mathrm{INTRA}_t = a_0 + a_1\sigma_t + \varepsilon_t, \tag{3.11}$$

and the MA%E is calculated from

$$\mathrm{MA\%E} = \frac{\sum_{t=2}^{N} \left| \frac{\mathrm{INTRA}_t - \sigma_t}{\sigma_t} \right|}{N} \times 100\%, \tag{3.12}$$

where $N = 4,029$. The last three columns of the table 3.6 report the estimated R^2 and MA%E values. A notable point is the superior performance of the SGED-

TGARCH model. Based on R^2, the TGARCH model outperforms the VIX and INTRA models. Adding VIX and integrated volatility information to TGARCH model does not significantly increase R^2. The R^2 and the MA%E values show that the integrated volatility estimator performs worst, whereas the SGED-TGARCH and the $\text{GPM}_{\text{intra}}$ models are among best.

3.4.3 Out-of-sample performance of alternative volatility models

The out-of-sample performance of alternative volatility models is evaluated based on their 1-day-ahead and 20-day-ahead forecasts of the S&P 100 index volatility. The rolling window is set to be about half of the entire sample period. Specifically, the in-sample period is from 1/2/1987 to 12/30/1994 providing 2,019 daily observations, followed by the out-of-sample period from 1/3/1995 to 12/31/2002, yielding 2,010 1-day-ahead and 1,991 20-day-ahead forecasts.

Time-series forecasts are obtained from rolling maximum likelihood estimation of volatility models given in the equations (3.8) and (3.10). Each conditional standard deviation model is estimated initially over the 2,019 days of the in-sample period from 1/2/1987 to 12/30/1994, and the forecasts of the S&P 100 index volatility are made for the next day, say $t + 1$, using the in-sample parameter estimates. The model and data are then rolled forward one day, deleting the observation(s) at time $t - 2018$ and adding the observation(s) at time $t + 1$, reestimated and a forecast is made for time $t + 2$. This rolling method is repeated until the end of the out-of-sample forecast period. The 1-day-ahead forecasts provide predictions for 1/3/1995 to 12/31/2002 providing time-series of length 2,010. On each day, forecasts are also made for 20-day volatility. [19]

Two measures of realized volatility are predicted. The first measure is the absolute demeaned daily index returns, and the forecasts are made at time t of

$$|R_{t+1} - \bar{R}| \text{ for 1-day-ahead and } \sqrt{\sum_{j=1}^{20}(R_{t+j} - \bar{R})^2} \text{ for 20-day-ahead.}$$

These quantities are predicted because a correctly specified GARCH model provides optimal forecasts of $(R_{t+1} - \bar{R})^2$ for variance and $|R_{t+1} - \bar{R}|$ for standard deviation. The second measure of realized volatility is INTRA which uses 5-minute returns. Forecasts are produced at time t for

$$\text{INTRA}_{t+1} \text{ for 1-day-ahead and } \sqrt{\sum_{j=1}^{20}\text{INTRA}_{t+j}^2} \text{ for 20-day-ahead.}$$

Table 3.7 presents the R^2 values from the regressions of traditional and realized volatilities on the alternative volatility estimators. These R^2 values measure the models' predictive accuracy of the 1-day-ahead and 20-day-ahead forecasts of the S&P 100 index volatility. As shown in the first three columns of the table 3.7, when the realized volatility is measured by the demeaned daily index returns for both the 1-day-ahead and 20-day-ahead forecasts, the VIX implied volatility index performs better than the SGED-TGARCH and traditional volatility estimators. Nevertheless, the difference between the predictive powers of VIX and TGARCH is not substantial based on the R^2 values. The R^2 of the traditional volatility estimator is much inferior to those of the VIX and TGARCH models.

A notable point in the table 3.7 is that the R^2 values of all the models are much higher when the realized volatility is measured by the square-root of the sum of

[19]We follow the forecasting methodology of Blair et al. (2001) to generate 1-day-ahead and 20-day-ahead forecasts of the TGARCH, implied, and realized volatility models.

squared 5-minute returns instead of the absolute demeaned daily returns.[20] The results in the last three columns of this table indicate superior performance of the VIX and the TGARCH volatility estimators against the INTRA in capturing time-series variation in realized volatility. The R^2 of the VIX and TGARCH models are very similar. Specifically, the R^2 values from 1-day and 20-day-ahead forecasts are 38.40% and 15.15% for the VIX, and 38.10% and 13.76% for the SGED-TGARCH model. The R^2 values are 31.97% and 9.40% for the INTRA. Overall, the results indicate that the performance of the SGED-GARCH model is not influenced by different measures of realized volatility and the evidence for incremental forecasting information in traditional and integrated volatility estimators is not significant.

3.5 Conclusion

To accommodate skewness, kurtosis, and higher-order moment dependencies, this paper uses a conditional skewed fat-tailed generalized error distribution and tests its ability to model the conditional volatility of stock market returns within a discrete-time GARCH framework. The paper compares the in-sample and out-of-sample performance of the SGED-GARCH models with the implied volatility index and the ARMA-fitted volatility models for forecasting the S&P 100 index volatility. The information content of the alternative volatility estimators is examined, and two important questions for forecasting stock market volatility are answered: (1) How does the predictive power of volatility forecasts from the SGED-GARCH

[20]This confirms the findings of Andersen and Bollerslev (1998a,b) who indicate that the traditional measures of realized volatility are poor estimators of day-by-day movements in volatility, as the idiosyncratic component of daily returns is large. They show that as the observation frequency increases from a daily to an infinitesimal interval, this measure converges to a genuine measurement to the latent volatility factor.

models compare with forecasts from models that use information contained in implied volatility and intraday returns? (2) How important is the selection of the measure of realized volatility in evaluating the predictive accuracy of volatility forecasts with skewed-fat tailed distributions?

The in-sample and out-of-sample performance results based on the R^2 and the mean absolute percentage errors imply superior performance of the VIX index and the asymmetric GARCH models with SGED density in capturing 1-day and 20-day-ahead forecasts of the S&P 100 index volatility. The results also suggest that nearly all information is provided by the SGED-TGARCH model, the VIX index, and the sum of squared five-minute returns. Hence there is little incremental information in the traditional volatility estimator based on the absolute demeaned daily index returns.

Table 3.1: Descriptive Statistics

This table shows the descriptive statistics for the 5-minute returns on the S&P 100 index, daily logarithmic returns on the S&P 100, and for the daily realized, implied (VIX), and SGED-TGARCH volatility estimators. The time period of investigation extends from January 5, 1987 through December 31, 2002, giving a total of 4,028 daily observations. The number of 5-minute returns in the sample period is 315,263. The skewness and kurtosis statistics are reported for testing the distributional assumption of normality. ADF denotes the Augmented Dickey-Fuller (ADF) unit root statistic with a 1% critical value of -3.44. ρ_j represents the autocorrelation coefficient of order j.

	5-min Return	Daily Return	Realized	VIX	SGED-TGARCH
Maximum	2.7989%	8.5307%	10.2525%	9.4611%	8.4677%
Minimum	-1.7339%	-23.7805%	0.0747%	0.5695%	0.0996%
Mean	-0.0035%	0.0339%	0.8524%	1.3629%	1.0851%
Std. Dev.	0.1027%	1.2043%	0.5217%	0.5347%	0.4485%
Skewness	0.2348	-2.0636	4.3052	3.3015	4.5068
Kurtosis	20.4349	45.0103	47.6869	35.1147	48.8335
ADF statistic	-61.4686	-47.7236	-9.9937	-5.5044	-10.8616
ρ_1	0.123	-0.017	0.686	0.951	0.915
ρ_2	0.011	-0.053	0.601	0.910	0.843
ρ_3	0.001	-0.031	0.539	0.894	0.778
ρ_4	-0.003	-0.025	0.511	0.875	0.717
ρ_5	-0.004	0.018	0.492	0.855	0.676
ρ_{10}	-0.001	0.005	0.407	0.764	0.508
ρ_{21}	0.008	-0.010	0.321	0.674	0.367
ρ_{63}	0.001	0.016	0.201	0.479	0.239
ρ_{126}	-0.001	-0.012	0.173	0.382	0.189
ρ_{200}	-0.001	-0.003	0.143	0.357	0.121

Table 3.2: Maximum Likelihood Estimates of SGED-GARCH Models

This table displays the maximum likelihood estimates of the alternative GARCH models with SGED density. The results are based on the daily S&P 100 index returns that extend from January 2, 1987 through December 31, 2002 (number of observations = 4,029). Because of the autoregressive of order one process in the alternative GARCH models, the 4,029 daily index return generate a time-series of 4,028 daily volatilities from January 5, 1987 through December 31, 2002. The parameter estimates with asymptotic t-statistics are shown in parentheses for each model. The maximized log-likelihood values are reported in the last column.

$$R_t = \mu - 2\lambda\Omega S^{-1}\sigma_t + \varepsilon_t,\ \varepsilon_t = \sigma_t z_t,$$

$$\text{where } \Omega = \Gamma\left(\frac{2}{\upsilon}\right)\left[\Gamma\left(\frac{1}{\upsilon}\right)\right]^{-\frac{1}{2}}\left[\Gamma\left(\frac{3}{\upsilon}\right)\right]^{-\frac{1}{2}} \quad \text{and } S = \sqrt{1+3\lambda^2-4\Omega^2\lambda^2}$$

$$f(\sigma_t) = h(\sigma_{t-1}, \varepsilon_{t-1}; \beta_0, \beta_1, \beta_3) + \beta_2 f(\sigma_{t-1}),$$

$$\text{where } f(\sigma_t) = \sigma_t,\ \sigma_t^2,\ \text{or } \ln\sigma_t^2.$$

Models	μ	β_0	β_1	β_2	β_3	λ	υ	Log-L
AGARCH	0.0002	1.246e-6	0.1018	0.8675	-0.0049	-0.0496	1.2667	12945.52
	(1.4956)	(1.7060)	(10.2517)	(83.6638)	(-6.3794)	(-3.0891)	(-25.3220)[a]	
EGARCH	0.0004	-0.2001	0.1845	0.9774	-0.0655	-0.0279	1.1957	12928.87
	(3.0377)	(-4.9440)	(14.9267)	(225.2147)	(-6.4961)	(-1.9523)	(-60.1002)	
GARCH	0.0003	2.991e-6	0.1046	0.8751		-0.0803	1.2519	12926.54
	(2.2074)	(9.6171)	(10.8518)	(93.0204)		(-4.7961)	(-25.6265)	
GJR-GARCH	0.0007	3.445e-6	0.0331	0.8729	-0.1316	-0.0313	1.2712	12945.84
	(3.9290)	(11.0766)	(2.6668)	(88.2812)	(-7.5172)	(-1.9516)	(-25.3746)	
NGARCH	0.0002	3.357e-6	0.0909	0.8327	-0.7232	-0.0486	1.2736	12956.91
	(1.5590)	(10.2024)	(9.1254)	(69.1557)	(-6.2747)	(-2.8075)	(-25.1399)	
QGARCH	0.0002	3.668e-6	0.1014	0.8681	-0.0010	-0.0497	1.2665	12945.52
	(1.5672)	(10.4026)	(10.2719)	(83.9557)	(-7.2092)	(-3.0910)	(-25.3358)	
SQR-GARCH	6.5266e-5	2.7838e-6	6.7636e-6	0.8105	-127.0316	-0.0233	1.2122	12883.67
	(0.4104)	(9.5601)	(6.5606)	(43.5854)	(-6.7183)	(-1.3887)	(-25.1399)	
TS-GARCH	0.0004	0.0010	0.1720	0.7850		-0.0596	1.1529	12860.07
	(1.9329)	(16.1559)	(12.2735)	59.7494		(-3.6130)	(-29.4016)	
VGARCH	0.0001	-3.447e-6	7.636e-6	0.9081	-0.9346	-0.0178	1.2066	12875.09
	(0.7176)	(-2.5699)	(6.9060)	(90.9887)	(-6.9941)	(-1.0956)	(-31.2161)	
TGARCH	0.0002	0.0010	0.2396	0.7942	-0.1795	-0.0314	1.1796	12895.05
	(1.3800)	(17.1448)	(13.2199)	(58.0286)	(-9.1221)	(-1.8123)	(-29.3574)	

[a]report t-statistics from testing the null hypothesis H_0: $\upsilon = 2$.

Table 3.3: R^2 and MA%E from Regressions of Traditional Volatility and Realized Volatility on Alternative Volatility Models

TRAD_t is the traditional volatility and defined as the absolute demeaned daily returns, $|R_t - \bar{R}|$. INTRA_t is the square root of the sum of squared 5-minute returns on the S&P 100 index. Fitted volatility is the forecasted value of the ARMA(5,5) process of the traditional and realized volatility, respectively. We run two sets of regressions:

$$\begin{cases} \text{TRAD}_t = & a_0 + a_1\sigma_t + \varepsilon_t, \\ \text{TRAD}_t = & a_0 + a_1\sigma_{t-1} + \varepsilon_t, \end{cases}$$

$$\begin{cases} \text{INTRA}_t = & a_0 + a_1\sigma_t + \varepsilon_t, \\ \text{INTRA}_t = & a_0 + a_1\sigma_{t-1} + \varepsilon_t, \end{cases}$$

where σ_t and σ_{t-1} are the current and lagged implied, fitted, and SGED-GARCH volatility models. R^2 and MA%E are reported.

Regression of TRAD_t on estimated volatility				Regression of INTRA_t on estimated volatility			
Explanatory Variables	R^2 (σ_{t-1})	R^2 (σ_t)	MA%E	Explanatory Variables	R^2 (σ_{t-1})	R^2 (σ_t)	MA%E
VIX	19.54%	28.71%	56.23%	VIX	49.42%	57.05%	40.39%
FITTED	14.15%	16.72%	66.68%	FITTED	42.68%	53.03%	25.60%
AGARCH	14.77%	17.12%	58.00%	AGARCH	37.60%	48.70%	30.35%
EGARCH	16.17%	18.27%	57.75%	EGARCH	39.87%	50.28%	30.65%
GARCH	13.89%	16.05%	58.17%	GARCH	36.38%	46.60%	30.63%
GJR-GARCH	14.17%	16.83%	58.03%	GJR-GARCH	36.25%	47.80%	30.11%
NGARCH	15.78%	18.27%	57.83%	NGARCH	38.86%	49.98%	30.09%
QGARCH	14.76%	17.11%	58.00%	QGARCH	37.58%	48.66%	30.34%
SQR-GARCH	13.95%	15.91%	58.54%	SQR-GARCH	34.59%	44.20%	31.72%
TS-GARCH	14.63%	16.05%	58.49%	TS-GARCH	37.98%	49.38%	31.98%
VGARCH	13.46%	15.37%	58.72%	VGARCH	34.62%	44.60%	31.94%
TGARCH	16.63%	18.88%	57.96%	TGARCH	40.18%	54.25%	31.03%

Table 3.4: Maximum Likelihood Estimates of Alternative Models with SGED and Traditional Volatility

Mean:	$R_t = \mu - 2\lambda\Omega S^{-1}\sigma_t + \varepsilon_t,$
TGARCH:	$\sigma_t = \beta_0 + \beta_1\lvert\varepsilon_{t-1}\rvert + \beta_2\sigma_{t-1} + \beta_3\max[\varepsilon_{t-1}, 0]$
VIX:	$\sigma_t = \beta_0 + \beta_4\mathrm{VIX}_{t-1}$
TRAD:	$\sigma_t = \beta_0 + \beta_5\lvert R_{t-1} - \bar{R}\rvert$
VIX-TRAD:	$\sigma_t = \beta_0 + \beta_4\mathrm{VIX}_{t-1} + \beta_5\lvert R_{t-1} - \bar{R}\rvert$
VIX-TARCH:	$\sigma_t = \beta_0 + \beta_1\lvert\varepsilon_{t-1}\rvert + \beta_2\sigma_{t-1} + \beta_3\max[\varepsilon_{t-1}, 0] + \beta_4\mathrm{VIX}_{t-1}$
TGARCH-TRAD:	$\sigma_t = \beta_0 + \beta_1\lvert\varepsilon_{t-1}\rvert + \beta_2\sigma_{t-1} + \beta_3\max[\varepsilon_{t-1}, 0] + \beta_5\lvert R_{t-1} - \bar{R}\rvert$
$\mathrm{GPM}_{\mathrm{trad}}$:	$\sigma_t = \beta_0 + \beta_1\lvert\varepsilon_{t-1}\rvert + \beta_2\sigma_{t-1} + \beta_3\max[\varepsilon_{t-1}, 0] + \beta_4\mathrm{VIX}_{t-1} + \beta_5\lvert R_{t-1} - \bar{R}\rvert$
	where $\Omega = \Gamma\left(\frac{2}{\upsilon}\right)\left[\Gamma\left(\frac{1}{\upsilon}\right)\right]^{-\frac{1}{2}}\left[\Gamma\left(\frac{3}{\upsilon}\right)\right]^{-\frac{1}{2}}$ and $S = \sqrt{1 + 3\lambda^2 - 4\Omega^2\lambda^2}$

Parameters	TGARCH	VIX	TRAD	VIX-TRAD	VIX-TGARCH	TGARCH-TRAD	$\mathrm{GPM}_{\mathrm{trad}}$
μ	0.0002	0.0003	0.0002	0.0003	0.0003	0.0006	0.0003
	(1.3800)	(2.0423)	(1.3749)	(2.0402)	(2.0606)	(5.3700)	(2.3898)
β_0	0.0010	-0.0015	0.0090	-0.0015	-0.0005	0.0001	-0.0002
	(17.1448)	(-5.0202)	(44.6061)	(-4.3252)	(-2.5727)	(4.8258)	(-1.5977)
β_1	0.2396				0.1106	-0.9681	-0.2769
	(13.2199)				(7.1688)	(-3.0323)	(-1.6991)
β_2	0.7942				0.4663	0.9370	0.6385
	(58.0286)				(10.0564)	(126.6523)	(10.1749)
β_3	-0.1795				-0.1809	-0.1202	-0.2185
	(-9.1221)				(-8.5773)	(-8.0355)	(-9.6240)
β_4		0.8829		0.8785	0.4297		0.2688
		(32.3516)		(23.7516)	(10.6591)		(5.1016)
β_5			0.2673	0.0051		1.0929	0.4204
			(12.8860)	(0.3172)		(3.4315)	(2.5360)
λ	-0.0314	-0.0609	-0.0394	-0.0609	-0.0536	0.0062	-0.0409
	(-1.8123)	(-3.2269)	(-2.4703)	(-3.2265)	(-2.8611)	(0.6709)	(-2.3885)
υ	1.1796	1.3506	1.0763	1.3518	1.3765	1.3193	1.3894
	(-29.3574)[a]	(-25.7818)	(-36.3797)	(-22.1002)	(-18.4694)	(-26.7135)	(-18.6066)
Log-L	12895.05	13022.96	12688.84	13022.99	13036.97	12993.85	13043.46

[a]report t-statistics from testing the null hypothesis H_0: $\upsilon = 2$

Table 3.5: Maximum Likelihood Estimates of Alternative Models with SGED and Realized Volatility

Mean:	$R_t = \mu - 2\lambda\Omega S^{-1}\sigma_t + \varepsilon_t,$
TGARCH:	$\sigma_t = \beta_0 + \beta_1\lvert\varepsilon_{t-1}\rvert + \beta_2\sigma_{t-1} + \beta_3\max[\varepsilon_{t-1}, 0]$
VIX:	$\sigma_t = \beta_0 + \beta_4\mathrm{VIX}_{t-1}$
INTRA:	$\sigma_t = \beta_0 + \beta_5\mathrm{INTRA}_{t-1}$
VIX-INTRA:	$\sigma_t = \beta_0 + \beta_4\mathrm{VIX}_{t-1} + \beta_5\mathrm{INTRA}_{t-1}$
VIX-TGARCH:	$\sigma_t = \beta_0 + \beta_1\lvert\varepsilon_{t-1}\rvert + \beta_2\sigma_{t-1} + \beta_3\max[\varepsilon_{t-1}, 0] + \beta_4\mathrm{VIX}_{t-1}$
TGARCH-INTRA:	$\sigma_t = \beta_0 + \beta_1\lvert\varepsilon_{t-1}\rvert + \beta_2\sigma_{t-1} + \beta_3\max[\varepsilon_{t-1}, 0] + \beta_5\mathrm{INTRA}_{t-1}$
$\mathrm{GPM}_{\mathrm{intra}}$:	$\sigma_t = \beta_0 + \beta_1\lvert\varepsilon_{t-1}\rvert + \beta_2\sigma_{t-1} + \beta_3\max[\varepsilon_{t-1}, 0] + \beta_4\mathrm{VIX}_{t-1} + \beta_5\mathrm{INTRA}_{t-1}$
	where $\Omega = \Gamma\left(\frac{2}{\upsilon}\right)\left[\Gamma\left(\frac{1}{\upsilon}\right)\right]^{-\frac{1}{2}}\left[\Gamma\left(\frac{3}{\upsilon}\right)\right]^{-\frac{1}{2}}$ and $S = \sqrt{1 + 3\lambda^2 - 4\Omega^2\lambda^2}$

Parameters	TGARCH	VIX	INTRA	VIX-INTRA	VIX-TGARCH	TGARCH-INTRA	$\mathrm{GPM}_{\mathrm{intra}}$
μ	0.0002	0.0003	0.0002	0.0003	0.0003	0.0003	0.0003
	(1.3800)	(2.0423)	(1.7049)	(1.7213)	(2.0606)	(2.1803)	(2.0790)
β_0	0.0010	-0.0015	0.0036	-0.0012	-0.0005	0.0003	-0.0004
	(17.1448)	(-5.0202)	(14.6085)	(-3.6905)	(-2.5727)	(3.2906)	(-2.3408)
β_1	0.2396				0.1106	0.0614	0.0503
	(13.2199)				(7.1688)	(3.8796)	(2.4242)
β_2	0.7942				0.4663	0.7543	0.4976
	(58.0286)				(10.0564)	(45.5357)	(10.0114)
β_3	-0.1795				-0.1809	-0.1331	-0.1586
	(-9.1221)				(-8.5773)	(-7.8614)	(-7.5030)
β_4		0.8829		0.7334	0.4297		0.3124
		(32.3516)		(16.9293)	(10.6591)		(7.1124)
β_5			0.8380	0.2018		0.2718	0.1868
			(23.4434)	(5.4890)		(16.3227)	(5.1938)
λ	-0.0314	-0.0609	-0.0499	-0.0609	-0.0536	-0.0493	-0.0565
	(-1.8123)	(-3.2269)	(-2.8578)	(-3.2418)	(-2.8611)	(-2.8138)	(-2.9679)
υ	1.1796	1.3506	1.2791	1.3801	1.3765	1.3358	1.3956
	(-29.3574)[a]	(-25.7818)	(-23.4685)	(-20.5711)	(-18.4694)	(-22.4684)	(-17.912)
Log-L	12895.05	13022.96	12893.10	13032.00	13036.97	12996.60	13048.59

[a]report t-statistics from testing the null hypothesis H_0: $\upsilon = 2$

Table 3.6: In-sample Forecasting Performance of Alternative Volatility Models

This table presents the R^2 values from the regressions of the traditional and realized volatilities on the alternative volatility estimators, and the mean absolute percentage errors. TRAD_t is the traditional volatility and defined as the absolute demeaned daily return, $|R_t - \bar{R}|$. INTRA_t is the square root of the sum of squared 5-minute returns on the S&P 100 index. The explanatory variables are the volatility forecasts obtained from alternative volatility models with different information.

Regression of the traditional volatility on estimated volatility			Regression of realized volatility on estimated volatility		
Explanatory Variables	R^2	MA%E	Explanatory Variables	R^2	MA%E
VIX	19.52%	57.25%	VIX	49.43%	29.30%
TRAD	6.57%	61.81%	INTRA	47.03%	31.01%
TGARCH	18.77%	58.49%	TGARCH	54.16%	31.20%
VIX-TRAD	19.54%	57.25%	VIX-INTRA	53.26%	28.42%
VIX-TGARCH	20.57%	57.07%	VIX-TGARCH	52.23%	28.92%
TGARCH-TRAD	18.66%	57.66%	TARCH-INTRA	54.33%	28.91%
GPM_{trad}	20.72%	57.00%	$\text{GPM}_{\text{intra}}$	54.18%	28.31%

Table 3.7: Out-of-sample Forecasting Performance of Alternative Volatility Models

This table presents the R^2 values from the regressions of the traditional and realized volatilities on the alternative volatility estimators. The R^2 values measure the models' predictive accuracy of the 1-day ahead and 20-day ahead forecasts of the S&P 100 index volatility. TRAD_t is the traditional volatility and defined as the absolute demeaned daily returns, $|R_t - \bar{R}|$. INTRA_t is the square root of the sum of squared 5-minute returns on the S&P 100 index. The explanatory variables are the volatility forecasts obtained from the alternative volatility models with different information.

Regression of the traditional volatility on estimated volatility			Regression of realized volatility on estimated volatility		
Explanatory Variable	R^2 (1-day ahead)	R^2 (20-day ahead)	Explanatory Variables	R^2 (1-day ahead)	R^2 (20-day ahead)
VIX	18.16%	7.89%	VIX	38.40%	15.15%
TRAD	5.58%	2.24%	INTRA	31.97%	9.40%
TGARCH	17.31%	6.49%	TGARCH	38.10%	13.76%
VIX-TRAD	18.19%	7.90%	VIX-INTRA	41.31%	15.31%
VIX-TGARCH	19.45%	8.17%	VIX-TGARCH	40.70%	14.56%
TGARCH-TRAD	18.49%	8.12%	TARCH-INTRA	43.43%	14.52%
GPM_{trad}	19.50%	8.16%	$\text{GPM}_{\text{intra}}$	43.40%	14.75%

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