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THE ASSESSMENT OF FORMAL OPERATIONS: ORGANISMIC AND TASK CONSTRAINTS

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THE ASSESSMENT OF FORMAL OPERATIONS:

ORGANISMIC AND TASK CONSTRAINTS

by

SOL MAGZAMEN

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Abstract

ASSESSMENT OF FORMAL OPERATIONS:

ORGANISMIC AND TASK CONSTRAINTS

by

Sol Magzamen

Adviser: Professor David Bearison

Three tasks having the underlying structure of Piaget's INRC group were given to 157 subjects from 11 to 16 years of age as individuals and dyads. Subjects' strategies were predicted from Piaget's theories of formal operations, causality and equilibration. Pascual-Leone's construct of m-power was utilized in predicting performances of different groups on four dependent measures. The results supported the hypotheses in general. In particular, the predicted control of variables strategy, called the operator strategy, was found to be the predominant means used by high m-power 13- to 14-year-olds. Eleven to 12-year-olds utilized a concrete operational approach called the pattern strategy. An unpredicted combinatorial strategy, consistent with Piaget's theory, was found at 15 to 16 years. Further, results for the three tasks were in accord with their relative levels of difficulty as determined by their so-called m-demand. Subjects consistently utilized their age-characteristic operative strategies to attempt solutions of the different tasks, despite producing different levels of performance on other measures in accord with the complexity of the tasks. Also, sex differences in competence and performance were found at all

ages. Results were discussed in the context of individual differences in development as determined by subjects' levels of m-power and their gender. Educational implications of the results were discussed.

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INTRODUCTION TO THE PROBLEM

Piaget (1957) argued that logic can be thought of from two points of view: as an operational algebra with its calculational procedures and structures; or as a science of truth conditions called pure or axiomatic logic. Since his purpose was not to formalize psychological theory, but, on the contrary, it was to separate out logical structures from mental facts, he considered formal logic useless to him for three reasons: one, that adult thought is unformalizable; two, that axiomatics reverses the genetic order of construction of operations; and, three, that axiomatic logic is atomistic in character and its order of demonstration is linear or deductive. Piaget maintained that psychological facts are such that axioms are the product of conscious intuition or reflection directed by underlying operations. Further, he maintained that operational mechanisms are made up of structured wholes, the parts of which are cyclically interconnected and irreducible to a linear system. Thus, in his theorizing he started from operational structures and applied axiomatic logic only in order to construct a psychologic that would then provide a deductive theory to explain the results of experiments in psychology.

Thus, Piaget (1949, 1952) introduced his psychologic in an effort to determine whether the operational algebra of logic can be used to discover structures which correspond to operational structures as may be found in psychology. In his role as a psychologist, he said, he welcomed the qualitative character of logic since it facilitates analysis of the structures underlying mental processes.

He contrasted this approach with the quantitative treatment of the behavioral outcome of mental operations, which he considered to be a no more mathematically or scientifically rigorous approach than his own.

As a result of his efforts in this direction, Piaget arrived at a view which held that cognitive development was a matter of achieving closure within a sufficiently large system of operations. The ontogenesis of ideas was not explained by a cumulative history of experience, but by a future stable logical equilibrium. This equilibrium comes into psychological being during the acquisition of what Piaget called the stage of formal operations. Inhelder and Piaget (1958) presented their experimental tests of this equilibrium theory of formal operations. Their work has led, in turn, to a number of other studies of their conclusions.

Further, Piaget (1974) reconsidered the ontogenesis of the concept of causality, a subject he had studied earlier. The new book on causality presented a theory of the application of the internal logical structures to the external world. Also, later books on equilibration (Piaget, 1977; 1980) make more explicit the framework of the growth of the stage of formal operations. In his last works, Piaget (1980) was revising his psychologic in terms of his so-called logic of meaning. His new model of thought is an extension of the older psychologic within which the older model is regarded as valid only in its limited domain of application.

The present study was a test of the older model of psychologic (Piaget, 1949, 1957) in the context of the newer ideas on causality

(Piaget, 1974) and equilibration (Piaget, 1977). Moreover, based on the results of others, Genevan and non-Genevan, the present study was intended to provide a test of predictions derived from Piaget's theory.

Review of the Literature

Aspects of Formal Operational Theory

Piaget argued (1957) and demonstrated in his studies with Inhelder (1958) that children in the approximate age range from seven to eleven do not engage in systematic experimentation when handling data. They set up correspondences, order or classify, but do not systematically isolate factors. Piaget stated that, on the other hand, after the age of twelve, adolescents intuitively try to discover as many possible combinations of propositions in solving a problem as are necessary in order to select what is true and discard what is false. In the course of this activity adolescents construct what may be described as a combinatorial system.

Piaget distinguished this combinatorial system from the so-called "elementary groupements" of concrete operations which were based only on simple additive sets or product sets. He based the propositional structures upon what he called a set of all subsets or "n" by "n" combinations taken among the product sets. This set has, in Piaget's logic, the dual structure of a complete lattice and a group. The lattice has a "join" (pvq) and "meet" (p.q) and gives 16 possible arrangements into which any type of relation between two propositions may be translated. The group is a representation of

the so-called group of four transformations (or the Klein group). The elements of the group are the four transformations: Identity, Negation, Reciprocal and Correlative (INRC). The 16 arrangements or combinations are related to each other by the four transformations of the INRC group (Inhelder and Piaget, 1958).

Thus, propositions appear in the adolescent's thought as a unified system structured by the operations. Piaget (1957) contended that these operations may not be regarded as the mere cumulative product of past experience since they are unconscious and used intuitively. Further, because they appear so late in the course of development, he maintained that they cannot be "a priori" forms of the mind. Moreover, he pointed out that they cannot be regarded as the result of the late maturation of neural connections, for they do not appear in their entirety during thinking. Only a part of a structure may be actualized in the course of solving a particular problem, the remainder of the structure remaining as possible but determinative transformations. This "causality of the possible" characterized adolescent thought for Piaget:

> "From the physical point of view, only reality is of a causal nature...possibility is only an instrument of calculation or deduction...(but) in the case of operational equilibrium... reality and possibility are both of a psychological nature... (I)n a state of psychological equilibrium possibility plays as important a causal role as real operations. It could even be maintained that the whole of mental life is dominated by this sort of causality of the possible." (Inhelder & Piaget, 1958, p. 262-63.)

Thus, the INRC group and the lattice characterize an equilibrium attained by the activity of thinking, an equilibrium which may be described by a structure, or a set of related transformations.

These transformations causally determine the thought of the adolescent, enabling him to proceed from the possible to the actual.

The Genesis of Possibility and Necessity

Piaget characterized adolescent thought in its most general sense by formal statements about possibilities. He traced the development of the set of possible operations to its final state of equilibrium in the stage of formal operations. In the earlier preoperational state of the system perceptions and representations of static situations dominate transformations and remain uncoordinated. However, Piaget saw here the beginnings of psychological organization in what he called the regulations of perception and action. With systematization of the regulations, an elementary form of equilibrium is achieved: the stage of concrete operations. In this stage static situations are subordinated to transformations and transformations are assimilated to operations (reversible transformations), a process implied, for Piaget, by a state of equilibrium. In the concrete operational stage reality is extended by thought in the direction of the potential. But, the concept of "possibility" implicit in the notion of concrete operations is based upon the repeated application of operations: it is not sufficiently dissociated from physical action to be generalized into a set of related hypotheses or structures. Such structures come into existence only with construction of the stable equilibrium of formal operations and the onset of the full cognitive subordination of reality to possibility.

Because the psychological system tends toward equilibrium, in

Piaget's theory "possibility" is not arbitrary.

> "...starting from an actually performed propositional operation, or endeavoring to express the characters of a given situation by such an operation, he cannot proceed in any way he likes. He finds himself, as it were, in a field of force governed by the laws of equilibrium, carrying out transformations or operations determined...by the laws of the whole operational field..." (Piaget, 1957, p. 41.)

The field of possibilities is limited by the closure which characterizes equilibrium. The freedom implied by an unlimited number of possibilities seems constrained by the imposition of equilibrium. But, for Piaget, almost paradoxically, the freedom implied by possibilities arises only in the context of the completeness of the set of possibilities attained in equilibrium. And out of the lack of arbitrariness a sense of necessity seems to arise. For Piaget, just as the notion of possibility is indispensable to the consideration of psychological equilibrium, so too is deductive necessity the logical correlate of formal possibility. Formal operational thought is, for him, the final integration in which progressively constructed syntheses of "the possible" and "the necessary" subordinate and structure reality. In other words, "reality is plunged into a universe of possibilities that...are susceptible to being linked by necessity. As a consequence, one sees in this stage a striking expansion in both causal explanation and logico-mathematical reasoning." (Piaget and Voyat, 1979.)

The Genesis of Causality

Piaget (1974; Inhelder, 1980) extended his notion of necessity into a more general operatory scheme, that of causality. Causality is an expression of the interaction between objects. The operations

of thought are transformations of reality which correspond to the subject's action on objects either through direct manipulation or through mental deduction. Piaget supposes that throughout development there is an intimate relation between the operations and causality. If such a relation did not exist, logical operations would be useless in comprehending reality.

Causal transmissions from the initiator to the effect are assumed to be unobservable. "X" caused "y" assumes that "y" necessarily implies "x", but that the necessity is not observable. According to Piaget, subjects create models of necessity which do not merely describe reality as accurately as possible; on the contrary, they explain our understanding of how objects causally act upon each other and, in particular, explain how these interactions possess an inherent necessity.

A concept of causality, then, presupposes the construction of a model of necessity in which objects have the property of being operators. A causal model, then, is defined as a system of operations attributed to objects. This definition stems from the three characteristics of "causality" described by Piaget (1974): it goes beyond the observable; it reaches necessity; and it constitutes systems. He states, "On the whole, operations constitute, so to speak, a causality applicable to extratemporal forms and physical causality" (Piaget, 1974, p. 137). In other words, in the period of formal operations, the concept of causality and the operations are sufficiently differentiated to allow the separate progress and application of each. At this level there is the maximum possible

flexibility and extension between conceptions of reality and the mental instruments. Equilibrium, necessity and possibility are manifested in the operatory scheme of causality.

The Genesis of Equilibrium

Piaget describes five models of equilibration, three of which may be related to the present study and the context of causality the study provides. The three are models of interactions in which inferences by subjects are necessary if they are to complete the cycle of coordinating their observations of themselves with their observation of what happened when they acted. Piaget (1977) utilized succesive applications of any of the models to describe subjects' acquisitions of knowledge regarding how and why things happened the way they did on a given task.

The first model is utilized in describing subjects' acquisition of knowledge regarding the causal effects of their own actions. In this model the subject may observe his or her own actions and what happens to the objects in the study. The subject may observe, too, a relation between what was done and what happened. That is a child learns by observing the effects of actions on objects. After this observation, in this model, the subject may then make an inferential coordination about his or her own actions as well as inferences about what happened to the objects. The subject may then relate the inferences made regarding his or her actions to those made with respect to the objects. This relation is causal and, in Piaget's theory, necessarily proceeds in the direction from action to occurrence because a child can understand causal relationships only

through the mediation of his or her own operatory schemes. This is an explication of Piaget's (1977) model IIA of interaction.

The next model of interest is Piaget's (1977) model IIB. This model has the same form as the previous one, but instead of observing his or her own actions, the subject now observes his or her own plans, ideas or operations. Observations of the objects then serve as verifications of the ideas or plans. The coordinations are the same as above and are supposed to lead via an experimental procedure to a newly equilibrated level.

In model IIC (Piaget, 1977) of interaction the observations become observations of two objects (or a system of objects). Utilizing the same procedure as above causal inferences are made in this model regarding the relations between the two objects.

In the course of the process of equilibration, contradictions may occur. In Piaget's (1977, 1980a) theory, adjustments to the disturbance created by the contradiction may take the form of three kinds of compensation: these are the so-called alpha, beta and gamma behaviors. Here, we are primarily interested in two of these: beta behavior occurs when a subject modifies an assimilating scheme in order to accommodate a disturbing object; and gamma behavior occurs when possibilities are anticipated. Thus, in gamma behavior, a new object requires no alteration of existing schemes because the compensation had been made when the new possibility was anticipated. That is, no contradiction is experienced because of the anticipation.

Further, Piaget distinguishes among three forms of equilibration of which two are of interest here. First, in the equilibration among subsystems, knowledge which may be true in one subsystem and may not be true in another is reconciled after contradiction between the two subsystems has occurred. The equilibration may occur through either alpha, beta or gamma behavior. Second, in the equilibration between parts and the whole a larger entity assimilates the parts. Again, the whole may be constructed by means of alpha, beta or gamma behavior.

The stable equilibrium of formal operations must arise then in accord with model IIB. The system of full formal operations must be characterized by gamma behavior and a relation of the parts of any system to a subsuming totality in model IIB. As seen by Piaget, model IIC is a manifestation of model IIB in the representation of the external world of real objects.

Studies of Formal Operations

The studies of Inhelder and Piaget (1958) on formal operations has been the focus of attention by others studying adolescent cognition. There has been little or no attention paid to Piaget's later work as it pertains to formal operational theory. The present study will be concerned with formal operational theory as it was first presented by Inhelder and Piaget. However, some account will be taken of later developments too. Also, this study will take account of important factors that have emerged from the many studies done by others.

Piaget's logic. Brainerd (1976) and Ennis (1975, 1976) among

others criticize Piaget for reversing the proper order of development of the logic of propositions and that of classes and relations. In Piaget's theory the logic of propositions derives from that of classes and relations. From the standpoint of axiomatic logic the logic of classes should be deduced from that of propositions. Further, Parsons (1960), Strauss and Kroy (1970) and Braine (1978) have suggested alternative logical models to that of Piaget. It must first be pointed out to the critics of Piaget's logic that in his earliest writings on his logic (Piaget, 1949, 1957) he attempted to justify his approach and to contrast it to other views on logic. Also, the ability to solve problems in propositional logic is not necessarily evidence for the attainment of formal operations. Nor is the inability to solve such problems evidence against the attainment of formal operations. Piaget is concerned with the unconscious algebra of thought in the minds of subjects, not with their discovery of the logic in situations or tasks presented to them.

On the other hand, there do appear to be defects in Piaget's logical model. Agreeing with Piaget, Apostel (1980) believes that operations must be a fundamental concept in formal operational theory, yet he points out that propositional calculus must be considered an inadequate basis of formulation because propositional calculus does not offer operations beyond those offered by class calculus. He argues, however, that because operations must be fundamental concepts in the explanation of thought, "possibility" must be a fundamental notion in the concept of a formal stage. He has sought new formulations of the stages in terms of modal logic

and recently in the logic of action. As noted above, Piaget (1980b), too, was revising his logical system in his work on the so-called logic of meaning. In Piaget's new logical model the older model was still regarded as valid in its domain.

Attainment of formal operations. An important body of criticism of the theory of formal operations is the work which constitutes its apparent empirical disconfirmation. These studies utilize Inhelder's or related tasks in assessing formal operational performance. This type of research often found that only a relatively small percentage of subjects apparently perform on a formal operational level. Also, a low cross-task consistency was frequently found in the results of these studies. This body of research has been reviewed thoroughly by Blasi and Hoeffel (1974), Neimark (1975) and Modgil and Modgil (1976), Monnier and Wells (1980) and Kinsler (1984).

Various explanations for these findings have been proposed. Danner and Day (1977) and Kuhn and Angelev (1976) consider the low percentage of success to be a result of the frequent use of non-directive instructions. Danner and Day, and Kuhn and Angelev increase the percentage of success by giving more specific instructions to subjects. Neimark (1975, 1979, 1981) also argues that less ambiguous instructions increase success. She believes that in order to assess competence, any conditions which evoke individual variations should be ameliorated.

Another factor believed to evoke individual variation in performance is cognitive style. Neimark (1979) concludes that

field-independent subjects attain a significantly higher level of performance on the Inhelder-Piaget tasks. However, because individual response to ambiguity may be related to cognitive style, the two variables are confounded. However, Neimark contends that her position is supported by results of a number of studies in which ambiguous instructions were not a factor (Lawson, 1976; Linn, 1978; Neimark, 1975; Pascual-Leone, 1970; Pascual-Leone & Goodman, 1979; Saarni, 1973; Stone & Day, 1980).

Further, the phenomenon of horizontal decalage is believed to be a factor affecting formal operational performance across tasks (Voyat, 1982). Horizontal decalages in the period of concrete operations may be viewed as a lack of interface between the transformations of the groupings and the transformational nature of the task contents. The lattice and group structures and the operational schemas of formal operations are more general than the concrete operational structures. Therefore, any engagement between the formal operations and the constraints imposed by the task content should be dependent upon an appropriately constructed match or interface between the two (de Ribaupierre & Pascual-Leone, 1979). The interface between formal operational structures and task content must be more complex than that of concrete operational structures and content. Thus, in this view, horizontal decalages should be more prevalent in the formal stage than in the concrete stage. The low generalizability and level of performance on the Inhelder tasks are, in this view, consistent with the constructive character of Piaget's theory.

From the above argument we may conclude also that the issue of task content as it relates to performance on tasks goes beyond mere ambiguity of directions. The issue appears also to involve both familiarity and complexity of the subject matter, both of which may not be confounded with cognitive style.

Attainment of formal operations: Pascual-Leone's theory. Previous research, as noted above, has isolated task factors such as ambiguity of directions, familiarity and complexity, and an organismic factor, field-dependence, as influences on formal operational performance. Pascual-Leone (1980) takes account of all these factors in his theory as well as in studies with his colleagues (Scardamalia, 1977; de Ribaupierre, 1975). His work poses another significant issue, as well, for Piaget's theory of operations. That is, can subjects' apparent operational level be accounted for by careful quantitative analyses of the complexity of Piaget's tasks and measures of the processing levels of subjects.

Before presenting the results of studies by Pascual-Leone and his colleagues, some of the essential elements of his theory will be presented here since his theory is not widely known . Pascual-Leone (Pascual-Leone and Goodman, 1979) calls his theory the "theory of constructive operators" (TCO). The TCO is composed of two interacting systems: the so-called subjective system and the silent system. The subjective system is composed of sub-systems of active schemes all of which are, to some degree, situation specific. These include Piaget's operational schemes. The silent system is composed of metaconstructs or silent operators which can increase or decrease

the activation weights of schemes. The silent operators are content-free and can only be described by their organismic function. The interaction among these operators may be used to represent the step-by-step temporal mental process that is in this theory the construction of performance. Metasubjective task analysis is the term used for the representation of this mental processing: it describes the strategies used by subjects in coping with particular tasks.

The particular silent operator of interest here is called "M". M denoted what Pascual-Leone describes as mental attentional energy or mental space. This construct is an explication of the limited capacity of centration at different ages. M can take on values from one to seven. The value of this measure (called M-power) is defined as the maximum number of different schemes that can be increased in activation in a single operation (or centration). The allocation of M to schemes is presumed to be directed by a set of executive schemes which at any moment directs the application of M energy onto those schemes that are relevant to its implementation (de Ribaupierre & Pascual-Leone, 1979).

"M demand" (Md) of a task denotes the minimum M-power necessary to solve a problem with a given strategy. Different strategies for the same task may require different values of Md. "M reserve" (Mr) denotes the maximum value of M-power available to a subject at a given age. Mr grows developmentally, increasing regularly with age in relation to the Piagetian stages. At age 11 Mr is equal to five.

At ages 13 and 15 years, the years during which the formal operational stage may be attained, Mr is six and seven, respectively. Mr does not grow beyond this value. Individuals, however, do not always use their full M reserve. Field-dependent persons characteristically function at values of functional M (Mf) less than their theoretical Mr.

A minimum Mf is required to solve any given Inhelder-Piaget task. Md is five for the pendulum task and six or greater for any of the other tasks, as determined by metasubjective task analysis. Thus, a Mf of six or seven is necessary, but not sufficient, for solving any task except the pendulum, which requires a Mf of at least five.

Let us examine the TCO assessment of performance on the Inhelder-Piaget tasks. De Ribaupierre (1975) conducted an extensive study in which the TCO was utilized to make predictions of the performance of 12- and 15-year-olds on seven tasks. The possible correct and incorrect strategies on the tasks were analyzed to determine their Md requirements. The M capacity of subjects was assessed. Their M-power estimates were used to predict subjects' performances on the tasks in accord with the predetermined Md of the strategies and tasks. Their performances were compared with the predictions.

Subjects' operational levels were assessed, as well, utilizing a controlled clinical-interview. First, let us note that, in accord with prior studies, the percentage of subjects at the formal level

on any task was quite small and very few subjects performed consistently at the formal level on all the tasks. However, Piaget's theory was supported in that 77 percent of the 15-year-olds, as contrasted with 33 percent of the 12-year-olds, performed at a formal operational level on more than one of the tasks. Further, 39 percent of the 15-year-olds, as contrasted with 3 percent of the 12-year-olds, performed at the formal level on more than four tasks.

The predictions of the TCO were supported by the data. Only one of 30 12-year-olds and three of 39 15-year-olds performed beyond the level predicted by their Mf. These four subjects all scored higher on M-power reassessments given after all the tasks, a result, then, in accord with their performance. Also, field-independence was compared with M-power as a predictor of performance. Whereas Mf scores significantly predicted performances unattainable by subjects, field-independence did not. In a further comparison, a multiple regression analysis showed that, for 12-year-olds, M-power was the best single predictor of performance. This was reasonable because for 12-year-olds, this factor is a scarce resource relative to the Md of the tasks: they lacked the Mr necessary for efficient solution of the formal tasks. On the other hand, 15-year-olds had the Mr necessary to solve the problems. Task-related and organismic factors other than Mr predicted their performances: IQ and cognitive style were the best predictors of their performances. IQ is a task-related learning factor in Pascual-Leone's theory. It is related to his so-called LM structures (Pascual-Leone and Goodman, 1979) and to a lesser degree to the LC structures. These are

necessary for the construction of the interface between the logical operators which organize performance and the content of the tasks. Also, short-term memory tasks were very poor predictors in all cases. This is consistent with Pascual-Leone's analysis of the differences between the constructs of short-term memory and M-power. Short-term memory measures involve complex factors other than M-power (Burtis, 1976, 1982).

STATEMENT OF THE PROBLEM

In the above discussion we have seen that, although much previous research has shown that formal operational behavior has not been found or has been difficult to assess on many tasks (Kinsler, 1984; Neimark, 1975, 1979), evidence is available for the existence of a formal stage of thought (de Ribaupierre and Pascual-Leone, 1979; Kinsler, 1984; Markovits, 1983, 1984; Monnier and Wells, 1980; Neimark, 1981). In recent research, investigators have utilized methodological ploys with some degree of success in attempting to enhance subjects' formal operational performances. In these studies, some investigators have manipulated instructions to subjects or the task itself (Danner and Day, 1977; Kinsler, 1984), some have attended to organismic factors such as m-power and field independence (de Ribaupierre, 1975; Lawson, 1976) and others have studied so-called conflicts or transactions in the social context of problem solving (Berkowitz and Gibbs, 1983; Dimant, 1984). In the present study task and organismic factors were utilized in order to predict formal operational performance from Piaget's theory.

Piaget's Theory and the Problem

Based upon his conception of operations, Piaget (1949, 1952, 1957) concluded that a stage of formal thought necessarily exists. For, in his theory, it is only in that stage that operations achieve a sufficient degree of flexibility to remain in stable equilibrium in the face of conflict or perturbation. Further, for Piaget, a stable equilibrium among operations implies that the logic underlying mental activity contains all possible operations within a very

general but logically complete structure. The completeness or closure of the algebraic structures called groups and, in particular, the properties of the Klein group, served for Piaget as the embodiment of a structure possessing the requisite elements necessary to describe the organization of thought.

Beyond his theory, Piaget's method of verifying the existence of his proposed abstract structures of thought have not been widely accepted. Both his selection of tasks and his analyses of subjects' solutions to them have been the subject of controversy. As noted above, others' uses of the Piagetian tasks have sometimes not led them to conclusions in agreement with Piaget's. But, it must be noted, the goals of others' studies have often been subtly or very different from those of the Genevans (Pascual-Leone, 1976) and one's methods are designed to suit one's purposes.

When Piaget and his collaborators study the unobservable structures that determine thought, they attempt to abstract from their subjects' efforts the most general principles that can explain subjects' solutions to problems given them. In doing this, they attempt to demonstrate that these abstract structures are, in every case, their proposed theoretical structures. In accord with their intent, the Genevans have chosen tasks for their studies that differentiate subjects from each other in ways that are amenable to their stage analyses.

In the present study, although the operations of thought as postulated by Piaget were the object of study, the method used and the choice of task were different in character from those of the

Genevans. Here, the strategies used by subjects at different operational levels were predicted from the theory prior to giving subjects the tasks. In order for it to be possible to make such predictions a task had to be designed that was presumed to enable "direct contact" with the proposed underlying operations as well as to have a concrete causal structure which would facilitate such "direct contact". In Piagetian studies a structure is presumed to operate internally on the elements of a task in generating a solution.

The above thoughts may be expressed in the context of Montangero's model of Piaget's theory. Montangero (1980) presented a four level model of Piaget's theory in which the successive inclusions of forms and contents lead from operatory structures at level one to "representations of reality" at level four. Each level served as content for the next higher level. In other words, in Piaget's theory, structures do not operate on aspects of reality unless they are appropriately organized. Table 1 presents the four levels of the model. In the terms of the model, a task is normally operated upon by schemes at level 3 and level 4 and possibly at level 2, depending upon a subject's familiarity with the content and level of development of interiorised logical schemes.

In Inhelder and Piaget's studies of formal operations each subject was given many tasks and solved some of them at the levels necessary to manifest formal operations, that is at a level manifesting a stable organization of thought at least at level 2.

Table 1

Forms and Contents in Cognition
at Four Levels According to Montangero's
Model of Piaget's Theory

Level 1	Operatory Structures	
Level 2	Logical or operations	Infralogical
Level 3	class, relation number	intervals, space-time, measurement
Level 4	characteristics of objects; objects	points in space or time; figures or events

Subjects often give solutions at level 3 or level 4 (the phenomenon of horizontal decalage). In the present study, it was intended to make more direct contact with logical schemes at level 2 or level 1 of Montangero's model and even to stimulate coordinations leading to new schemes at either of these two levels. Thus, the task described below was designed to bridge the gap from level 4 to level 2 in accord with Piaget's theory by reflecting the underlying operations directly and possessing a direct and obvious causal relation to the subject or the subject's actions.

M-power and the Problem

Although the problems of unfamiliarity and vagueness associated with the Piagetian tasks may have been avoided in the present study by the particular task design, organismic limitations would not have been sufficiently accounted for by the task if accurate predictions of competence or performance were to be made. That is, organismic constraints may impede the sought bridging of the gap between level one or two and level four. In their study of formal operational performance, de Ribaupierre and Pascual-Leone (1979) utilized the organismic variable m-power, the mental-attentional factor from Pascual-Leone's theory. As described above, they concluded that a formal operational stage does exist. They predicted subjects' performances on the basis of their m-power and the m-demand of the tasks. Further, in Pascual-Leone's theory, equilibrated structures need not be the generators of thought. Novel thinking in his theory is a constructive process that arises from the normal activity of schemes and the coordinations influenced by the situation-free

organismic operators such as m-power. The level achieved by novel mental activity is constrained by the level of m-power.

Thus, in testing Piaget's theory in the present study, the organismic level of functioning was taken account of by utilizing the construct of m-power. Introducing the measure of m-power into this study meant that, in turn, a closer look at the proposed task was required. For Pascual-Leone has predicted subjects' performances on tasks not only on the basis of their m-power, but also upon the basis of a task analysis of the m-demand of the tasks. In this study, Piaget's and Pascual-Leone's theories were coordinated in order to investigate Piagetian formal operational structures. In his analyses, Pascual-Leone has not made direct use of the operations proposed by Piaget. So, just as Piaget's methodology was altered for the purpose of this study, so too was it necessary to alter Pascual-Leone's.

Rather than utilizing an absolute m-demand value for predicting subjects' performances, a notion of relative m-demand was utilized as follows. In addition to the main task given to subjects, two ancillary tasks were designed: one had a lower m-demand than the central task and the other had a higher m-demand. The m-demand was manipulated by controlling the perceptual salience of factors necessary for the solution of the task. In Pascual-Leone's method of task-analysis, a perceptually salient factor does not require the utilization of a unit of m-power at every step in the solution of a problem. Factors which must be maintained mentally, rather than being perceptually available, require the utilization of m-power at

every step, even when they are not directly used. In this way, then, three tasks with identical structures, but with three levels of m-demand, were utilized. Thus, the manifestations of the Piagetian operations in competence and performance were available for study from tasks at three levels of complexity.

Social Interaction and the Problem

The particular form of coordination of Piaget's with Pascual-Leone's theories utilized in this study made possible and perhaps made necessary one further test of both theories. Conflict between schemes is an important generator of constructuve activities in both theories of development. Here, our main focus is on Piaget's theory, but it should be said that in Pascual-Leone's theory the resolution of conflicts appears in at least two fundamental ways in a manner that is consistent with Piaget's theory: one, it is inherent in his method of task analysis in the specification of the steps in the solution of problems and, two, it is present in internal actions determining the construction of new schemes by the situation-free organismic operators such as m-power.

In Piaget's theory, conflicts are the content of resolutions constructed by the "general coordination of actions". Coordinations of actions may be interindividual as well as intraindividual (Piaget, 1971). In fact, in Piaget's theory, the mastery of interindividual coordinations on the social plane leads the way for mastery of intraindividual coordinations (Bearison, et al, 1984; Piaget, 1950). This may be understood in the context of the present study by observing that interindividual coordinations would have to

occur in any interaction on level four of Montangero's model. The decentrations and coordinations necessary for internalization of social conflict at level 4 are dependent upon the mental organization at deeper levels, but so too is the level of organization at levels 1, 2 or 3 dependent upon prior experiences internalized at level 4. But, before any logical deepening or interiorization of an experience occurs, it must first be internalized at level 4. From Piaget's theory, then, one might expect an enhancement of performance in some instances on level 4 when a task is solved socially, say by dyads. An apparent increase in competence might be manifested on a task solved by dyads if there is an existing or about to be formed structure which can assimilate or "find" correspondences with the interindividual coordinations internalized at level 4. From the limitations imposed at every level by the construct of m-power, one would predict a limitation on the enhancement of competence, even when performance is enhanced by social interaction. That is to say, m-power is, in this view, a necessary condition for the development of the operational structures of the factor called competence. In addition, task constraints are relevant: while some advancement may occur when a task with an m-demand less than or equal to subject's m-power is solved socially, if the m-demand of a task is greater than any subject's m-power, one would not expect advances to occur even in the face of attempted social problem solving.

Studies of the enhancement of subjects' operational levels have been conducted at the level of concrete operations (Mugny and Doise,

1978; Bearison, 1982; Bearison, et al, 1984). Very little research has been done on the influence of social interaction on cognitive development during adolescence (Dimant, 1984). In the present study a dyadic condition was included. In accord with the views expressed above, it was expected that in the dyadic condition performance might be enhanced but that competence and performance would be constrained by the functional m-power of members of the dyads relative to the m-demand of the task.

The Tasks

There were three tasks, one central one called the "INRC line" task and two variant tasks called the "INRC large square" task and the "INRC small square" task. All three had an identical structure underlying their operation. They were all embodiments of the Klein group, or the INRC group, as it was called by Piaget. Only the configuration of the lights varied on the three tasks.

The main task: the INRC line. The device used in the main task is portrayed in figure 2. The task was a game played with two such identical looking electronic devices. Subjects were required to figure out how the switches operated the lights on each device. Since the rules which each device embodied were the rules of composition of the INRC group, a subject had to articulate these rules in order to have solved the problem at the highest level.

The device consisted of a box with two sets of four switches each and one set of four lights, all on the top surface. The four switches were evenly spaced along two parallel lines, each close to opposite edges of the top surface; the four lights were evenly

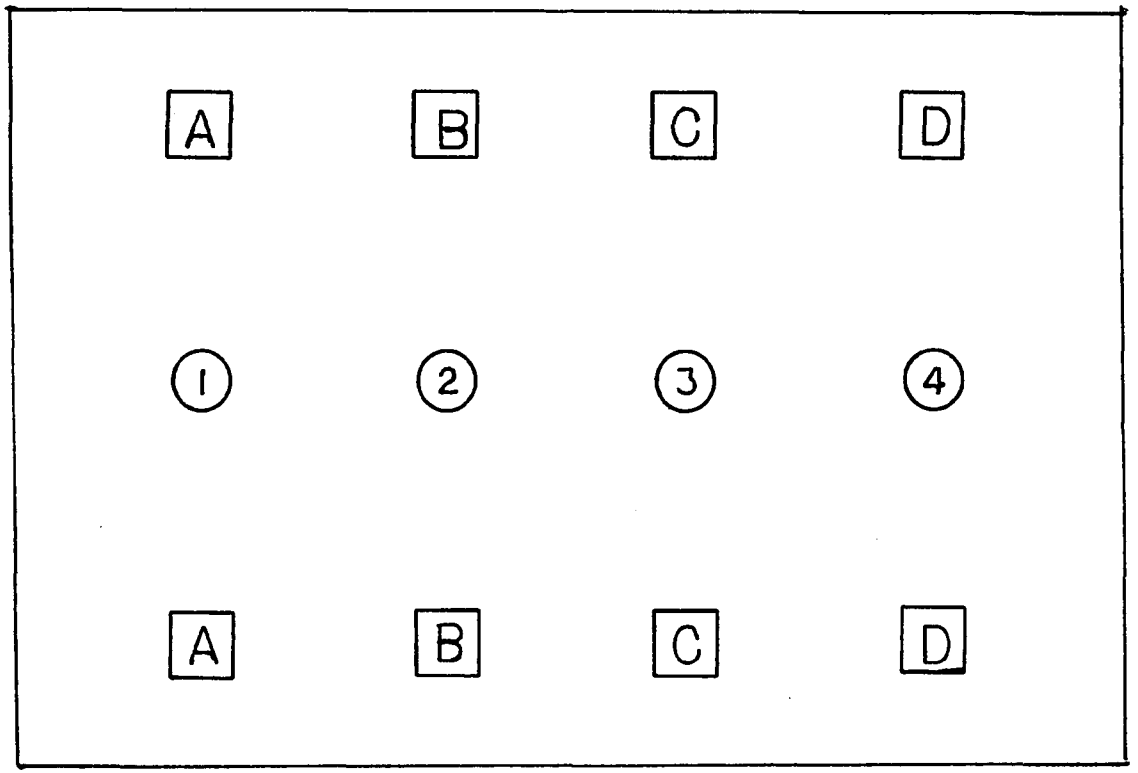


Figure 1. The top of the INRC line task device

spaced along a line between and parallel to the two edges.

The switches operated the lights in the following way: one light of the four was always on; the light that was turned on by any switch depended upon both the switch depressed and the light that was on immediately prior to depressing the switch;

(1) Switch "A" did not shift the light at all when depressed;

(2) Switch "B" alternated the light on between "1" and "2" or between "3" and "4";

(3) Switch "C" alternated the light on between "1" and "4" or between "2" and "3";

(4) Switch "D" alternated the light on between "1" and "3" or between "2" and "4".

Each set of four switches performed identical functions, the corresponding switches opposite each other being interconnected.

Two identical looking boxes were used to play the game as follows: on each of the boxes the subject operated one set of four switches and the experimenter operated the second set. Each round consisted of four plays as follows: the experimenter played first; he pressed any one of his set of four switches on the first box. Prior to the experimenter taking his turn, the same light was on on both boxes. The lighted bulb on the first box may have changed when he pressed the switch of his choice. The subject played next: he or she pressed any one of the second set of four switches on the same box. The light on may have shifted again. The subject then was required to predict the switch on the second box which he believed would light the same bulb on the second box as was lighted

on the first. The subject made this prediction by pressing one of his set of switches on the second box. The single press made next by the experimenter on the second box depended upon the two presses made on the first and was, in fact, completely determined by them according to the rules of composition of the INRC group. The rules of composition are presented in table 2. The subject's task was to be able to predict every combination correctly. The game ended when he or she had made 16 consecutive correct predictions or had played 120 rounds.

The secondary tasks: the INRC squares. There were two additional tasks: the "INRC large square" and the "INRC small square". The "INRC large square" was designed to have a smaller m-demand than the "INRC line" task. This was achieved by arranging the lights in a salient square configuration. Thus subjects did not have to mentally construct a model to describe the movement of the lights. The square configuration was perceptually available and suitable, as well, as a model because one switch shifted the lighted bulb horizontally, another shifted the light on vertically and the third moved the light diagonally. The device is portrayed in figure 2.

The "small square" had a greater m-demand than the "line" because first, the square was not perceptually salient; second, the fact that the four lights were different colors was perceptually misleading with regard to constructing a square configuration; and third, the fact that the switches were not identified by color meant that a model for distinguishing them had to be kept in mind. These

Table 2

The Rules of Composition of the INRC Group:

The 16 Possible Combinations of
Plays by Subject and Experimenter

Subject's Plays	Experimenter's Plays			
	A	B	C	D
A	A	B	C	D
B	B	A	D	C
C	C	D	A	B
D	D	C	B	A

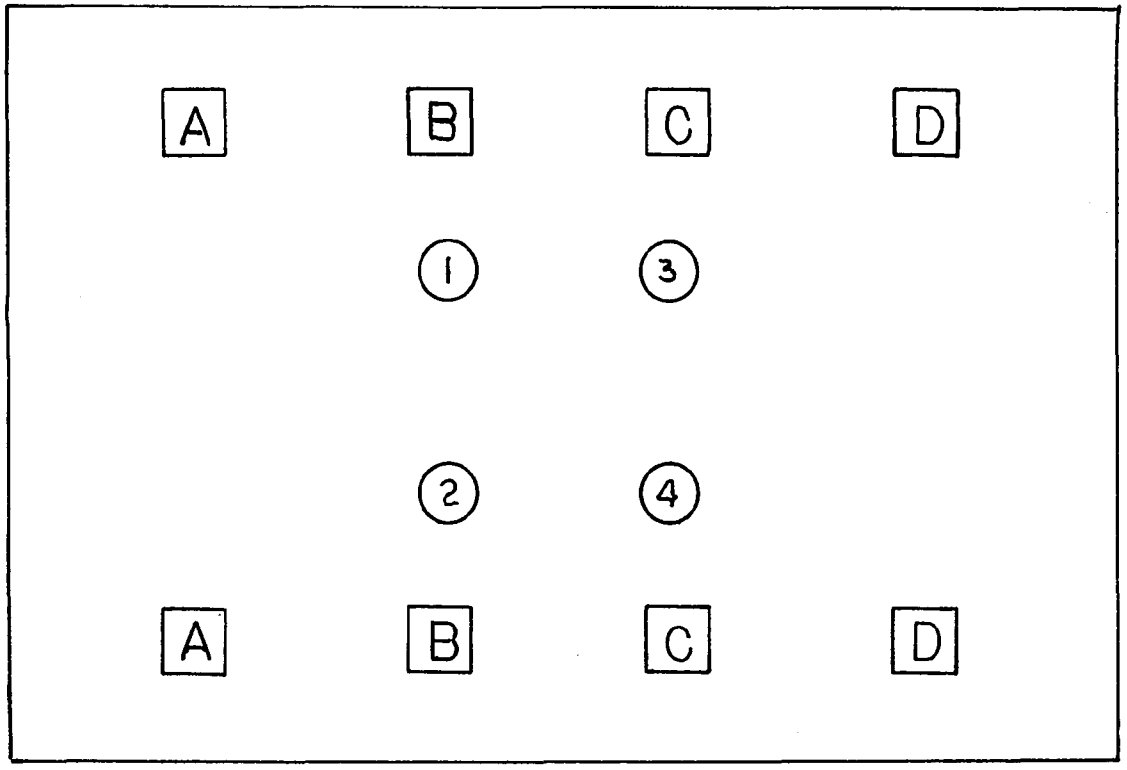


Figure 2. The top of the INRC large square task device

three factors made the "small square" a difficult task. The switches operated the lights on the "small square" in the same way as on the large square. The "small square" device is portrayed in figure 3.

The Strategies

Dienes and Jeeves (1965, 1970) noted two strategies, among others, utilized by children and adults in attempting to solve problems based upon algebraic group structures. They called these strategies the "operator" and the "pattern", these names deriving from the mathematical notions of states and operators.

In the present study, the operator and pattern strategies were used to denote levels of competence with respect to operations as defined by Piaget's theory. Subjects' strategies were inferred from the sequence of their presses on the first box in the course of playing the game. Their strategies, derived in this way from their performances, were regarded as evidence of their conception of the task, that is as a direct manifestation of their underlying competence. Piaget, in his studies, elicited from his subjects their conceptions of the tasks they performed and from these conceptions he inferred subjects' levels of competence. The use herein of subjects' conceptions stems from Piaget's approach, although the method utilized in this study was different from his.

Subjects who regarded their plays as experiments (Piaget, 1977) to determine the relations between the switches and lights saw the functioning of the box as separate from their own actions or the experimenter's. In Piaget's (1974) terms, such subjects were

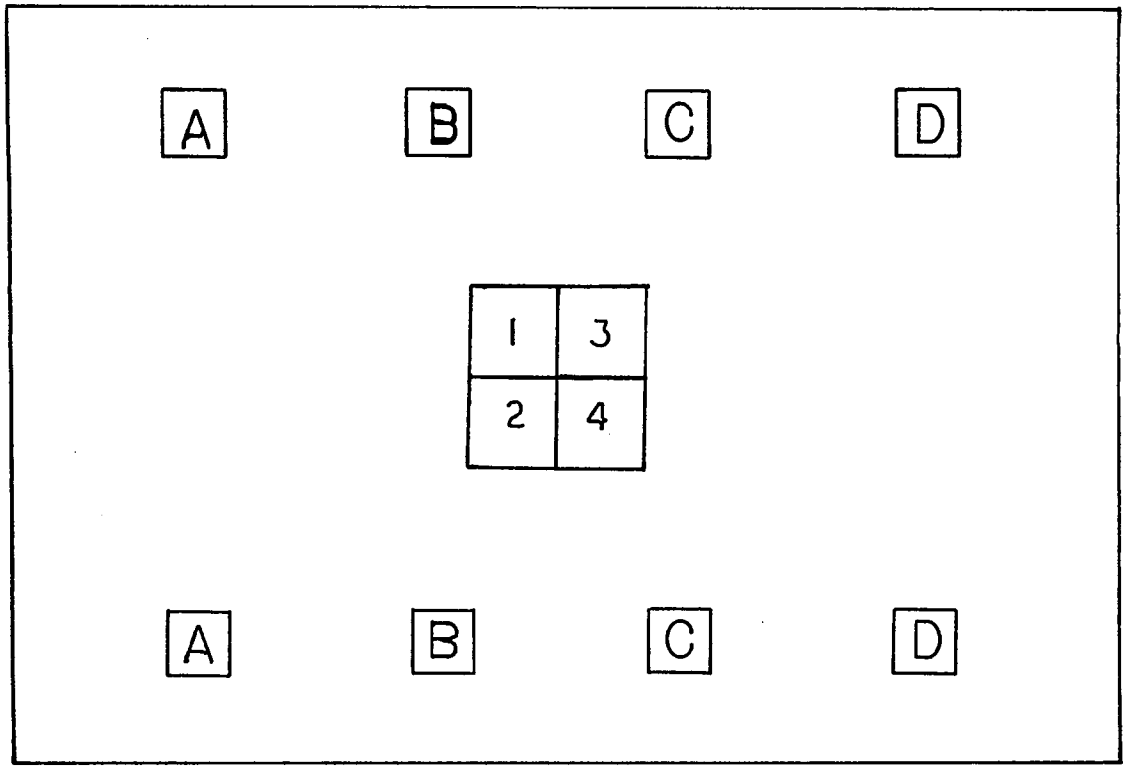


Figure 3. The top of the INRC small square task device

searching for the explicity causal relations between the switches and lights, and regarded the switches as operators. A subject who performed in this way, in terms of Piaget's theory, was performing at the formal operational level. According to Piaget, at the highest level, subjects would necessarily regard the operators, or switches, as part of an inter-related system. They would have sought for the relations between the two presses on one box and the one press on the second. Such a search would have been conducted in an efficient combinatorial manner if it were characteristic of full formal operations. Because it reflects an efficient approach as well as a causal experimental approach the operator strategy of Dienes and Jeeves (1965, 1970) was adopted as a measure for this study. The repeated playing of a single button characterized this strategy. It will be defined further below.

But, although the operator strategy was an efficient causal approach to the task, knowledge of the individual switches acquired by means of that approach did not imply an understanding of the relations among the four operations. The operator strategy is a control of variables scheme, but not necessarily a systematic combinatorial approach. An integrated combinatorial strategy would reflect both the operator approach and a causal model of coordination of the operators or switches.

Another strategic behavior defined by Piaget's theory (1977, 1980a) is one in which subjects divide the elements of a task into sub-systems and may reach equilibrium between the sub-systems. In the case of such a strategy, subjects may be seen, for instance, as

projecting their conceptions of their behavior onto the devices. This may be interpreted as alpha or beta behavior in the context of an interaction described by Piaget's model IIA (Piaget, 1977). As a result, in the present study, they may not see the two electronic devices used as functionally independent of each other, that is as individual objects whose functions are separately and internally determined. Thus, such a strategy would not reflect formal operational thinking. The best manifestation of an equilibrium between parts which could lead to solution in this study was the division by subjects of the 16 possible combinations of play into sub-systems or regions and the making of repeated plays within a given sub-system or region. This strategic behavior was called the pattern strategy (Dienes and Jeeves, 1965, 1970). This may be interpreted as gamma behavior within model IIA of interaction. This strategy will be defined further below.

Plays by subjects which were not counted as operator or pattern strategy plays were counted in a third category called the random strategy.

Hypotheses

The hypotheses of this study were:

(1) The operator strategy would be used significantly more by the older groups, the 13- and 15-year-olds, than by the youngest group, the 11-year-olds. Further, the use of the operator strategy was expected to increase significantly with increasing m-power within the 13-year and older age groups.

(2) The pattern strategy would be used significantly more by the eleven-year-olds than by older age groups. The use of the pattern strategy would decrease with increasing m-power within age groups.

(3) The use of the random strategy would decrease significantly with increasing m-power.

(4) The number of plays required to complete the game would decrease significantly with increasing m-power.

(5) The number of correct items on the posttest would increase significantly with m-power.

(6) In the dyadic condition, high m-power dyads will have significantly larger posttest scores than low m-power dyads.

(7) The 11-year-old dyads will have significantly higher game scores, lower posttest scores and lower strategy difference and use scores than 13- and 15-year-old dyads.

METHOD

Subjects

One hundred fifty seven children and adolescents ranging in age from eleven years, four months to sixteen years, ten months participated in this study. The Figural Intersection Task was administered to all subjects and their scores on this test served as a criterion according to which they were selected to participate.

The Figural Intersection Task provided estimates of subjects' m-power. The Figural Intersection Task was designed by Pascual-Leone in the mid-1960's and has been extensively researched since (Johnson, 1982). Each item consists of a number of overlapping figures. The number of overlapping figures varies from three to nine and the number of figures defines the equivalence class of each item. The task is to place a dot in the one location that is common to or inside of all the figures. Since the correct location is embedded among often misleading cues, the Figural Intersection Task is a measure of M-capacity and (secondarily) of field-independence. The Figural Intersection Task was used in group testing sessions without a time limit.

Four scoring systems (Johnson, 1982) were used and the m-power estimate for each subject was taken as the mean of the four scores. One basic system was to find the highest class in which a subject had 80 percent of the items correct and to assign the value of that class as the m-power estimate. A second basic system was to count the number of items correct and if a subject had correct a number of items at least equal to 100 percent of the number of items in a

given class plus 100 percent of all the items in lower classes, the value of the highest such class was assigned as the estimate of m-power. There were irrelevant figures in some items. There were two ways to treat such figures: one was to include the irrelevant figure in the count of figures in a given item; another was to ignore the irrelevant shape in the count. Both of these methods were used in both basic systems, giving four scores.

Four hundred sixty eight subjects were tested with the Figural Intersection Task. Eighty seven percent of the mean scores were within the desired ranges of m-power as specified for each age level below. Subjects were excluded from participation based first upon their m-power scores and their ages. Subjects were then selected randomly from the eligible groups.

The eleven-year-old group. Fifty six of the subjects attended two public schools in New York City and were from eleven years, four months to twelve years, five months of age. Twenty two of these subjects had an m-power estimate of four. These twenty two subjects were eleven males with a mean age of eleven years, nine months and eleven females with a mean age of eleven years, ten months. These subjects will be referred to as the low m-power 11-year-olds. Thirty four subjects had m-power estimates from five to five and one-half. Seventeen of these subjects were males with a mean age of eleven years, eight months and seventeen were females with a mean age of 11 years, eight months. These subjects will be referred to as the high m-power 11-year-olds.

The thirteen-year-old group. There were fifty two subjects who

attended a public school in New York City and who were from thirteen years, three months to fourteen years, four months of age. Twenty two of them had m-power estimates of five to five and one-half: there were eleven males, mean age thirteen years, eight months and eleven females, mean age thirteen years, eight months. These subjects will be referred to as the low m-power 13-year-olds. Thirty subjects had m-power estimates of six to six and one-quarter. Fifteen were males with mean age thirteen years, eleven months, and fifteen were females with mean age thirteen years, nine months. These subjects will be referred to as the high m-power 13-year-olds.

The fifteen-year-old group. There were forty nine subjects from fifteen years, three months to sixteen years, ten months. They attended a public school in New York City and two private schools in Stamford, Connecticut. Twenty two of them had m-power estimates of six to six and one-quarter: eleven were males with mean age fifteen years, eleven months and eleven were females with mean age sixteen years, one month. These subjects will be referred to as the low m-power 15-year-olds.

Twenty two subjects had m-power estimates from six and one-half to six and three-quarters. Eleven were males with mean age sixteen years, one month and eleven were females with mean age sixteen years one month. They were referred to as the high m-power fifteen-year-old groups.

Five males had m-power estimates from seven to seven and one-half. Their mean age was fifteen years, nine months. They will be referred to as the very high m-power fifteen-year-old males.

Task and condition assignment. Within each of the three age levels 44 subjects worked on the main task used in this study, the "INRC line" task. Of these 44 subjects at each age level, 22 had low m-power estimates and 22 had high m-power estimates. Among all 22 subjects in low or high groups, 10 subjects, 5 male and 5 female, were assigned randomly to the individual condition and 12 subjects, 6 male and 6 female, were assigned to the dyadic condition. Members of each dyad were of the same sex. There were no mixed sex dyads. Thus at the low and high m-power levels at three age levels there were 5 male and 5 female individuals and 3 male-male and 3 female-female dyads who were given the "INRC line" task. In addition, the 5 very high m-power 15-year-old males were given the "INRC line".

There was one additional group of subjects at the 11 year and at the 13 year age levels. Twelve high 11-year-olds, 6 male and 6 female, were given the "INRC large square" to solve. Eight high 13-year-olds, 4 male and 4 female, were given the "INRC small square" to play.

All subjects in the study used computers in school at least one period each day, often more. All 13-year-olds and 15-year-olds were students in special science programs and the 11-year-olds were students in a "talented and gifted" program or a science program.

Materials

The apparatus utilized by subjects consisted of two identical boxes (described below), each of which had eight switches and four lights on it. The two boxes were connected by wires to a panel which provided an interface for the boxes with a Timex/Sinclair 1000

computer, a Timex/Sinclair 2040 printer and Panasonic 9 inch black and white screen monitor.

The boxes on which the game was played are portrayed in figures 1, 2 and 3. The two boxes utilized by each subject were identical in appearance. Each was 2 inches high and had a top surface that was 5 inches by 7 inches. Each had two rows of four square head red press-switches along the long edges of the box. The switches were each 1.3 cm wide, were each 1.9 cm from the edge and 3.2 cm apart. The boxes for the "INRC line" task had a row of four green lights parallel to the rows of switches. Each light was a 0.6 cm high cylinder that was 0.6 cm in diameter across the top. The lights were 4.4 cm apart, from center to center.

The "INRC large square" boxes had four red lights arranged in a square array that was centered on the top of the box. The lights were separated by 3.2 cm along the sides of an imagined square. Each light was a 1.0 cm high cylinder with a 0.8 cm diameter circular top.

The "INRC small square" boxes each had a 2.5 cm by 1.9 cm rectangular plastic window set into the center of the box. The window was divided into four equal parts by two perpendicular white, 0.03 cm thick, lines. Each of the four equal parts was a different color: red, green, yellow and white. Behind each of the four parts was a light which illuminated only one sector at a time.

When playing the game the subject sat facing the experimenter, the two identical boxes between them. The remainder of the equipment was approximately 61 cm to the side of the subject and

experimenter. At the start of a session the subject gave his name and birthdate and saw this information entered into the computer and on the monitor screen. After all identifying information was entered, the screen remained blank during actual play. The computer recorded all button presses and this information was printed out, one line at a time, during play. The printout could not be read by the subject.

Prior to the beginning of each session, colored dots were placed on the switches. Only four colors were needed as each set of four switches performed the same four operations on the lights. The same function was identified in each of the four sets of switches by placing the same color dot on all four functionally corresponding switches. The function of each switch was identifiable by its position as well; the switches which caused the same transformation were all in corresponding positions within each set. Thus there were four dots of the same color in corresponding positions in each set. In the instructions, subjects were informed that switches in corresponding positions had the same color dot and this implied that they caused the same change in the lights. Colors were changed after each session.

Procedure

All subjects were first given the Figural Intersection Task. They were selected for the study upon the basis of their ages and their scores as desribed above. Each subject or dyad selected was given one version of the INRC task. After the task was completed a posttest was given to each individual or dyad. Then an interview

was conducted.

The INRC game. Prior to playing the game subjects were given the following instructions:

> "We are going to play a game with these two boxes (point). Let me show you how they work. First, each box has two sets of switches, one set is yours and one is mine on each box (point to each). Each set does exactly the same thing to the lights. Your (yellow) one does the same thing as mine (demonstrate). So all four (yellows) on both boxes mean that the switches do the same thing. The same is true for all the (greens) (point). Your task will be to figure out what the four different switches do.
>
> Here's how we play the game. First, notice that the light in the same place is on, on both boxes. I always go first on this box. I will press any of the four switches I want to press. Whenever I press one of these the choice is mine to press what I feel like pressing - nothing that has happened before in the game forces me to make a choice (press). Now it's your choice to press one of these, any one at all, go ahead, press one.
>
> The light on the second box was in the same place as the light on the first box. We moved the light on the first box by pressing two buttons. Now, by pressing only one button on the second box, we must get the lights into the same position again. Your task is to tell me which button I should press. Tell me by pressing one of your four buttons on the second box (point). Tell me which one should I press. Now you will see if you are right (press).
>
> Then we start over again on the first box. (Play 3 practice rounds). The two presses we make on the first box completely determine the press on the second box. Any questions?
>
> The lights are operated by a computer program (point to the equipment). There are no tricks. Each switch will always do the same thing whenever you press it. This is just an electronic machine.
>
> Remember, your task is to figure out how all the switches work. When you know that, you will be able to predict which button I will press on the second box.
>
> Questions? Let's play!"

In the course of the game, the experimenter always played the same color button on the first box as he had just played in his immediately preceding press on the second box. This was done to allow the subject to control the game as well as to control the

experimenter's plays with different subjects. The game was stopped by the computer if the subject used 120 prediction presses or made 16 consecutive correct predictions.

Subjects in the dyadic condition were instructed to work together on the task and were told that they were expected to agree on how to play the game and on their presses. They were given the same instructions, but, during practice rounds, they were asked if they agreed with each press made by their partners. They were encouraged to discuss their plays prior to making them. On the posttest and interview they were told they had to agree on their responses.

The posttest. Immediately following the conclusion of the game, a posttest was administered. On the posttest all 16 possible combinations of two presses on the first box were systematically tested. The lights did not operate during the posttest but the boxes were referred to during the questioning:

> "If I press orange and you press blue, which button should I press on the second box?"

The order of questions went from left to right, from the first row to the last, in sequence, as portrayed in Table 1.

The interview. After the posttest was completed, each subject was interviewed. The following questions were asked and the responses were audiotaped:

(1) "How does the game (program) work?"

(2) "While you were playing the game, what strategies did you use to figure it out?"

(3) If the switches are labelled A, B, C and D, the subjects were asked verbally, referring to the boxes, (not in equation form) what "X" is in the following equations: For example, in question (a) subjects were asked - "if I pressed blue (press) on the first box and you wanted me to press green (press) on the second box, what would you press on the first box?" (Questioning was terminated if two consecutive wrong answers were given):

(a) AX = B
(b) CX = A
(c) BX = D
(d) DX = C
(e) DX = D

(4) The following were asked in verbal form:

(a) XY = A
(b) XY = C

(5) If a correct answer was given to one of the items in (4), subjects were asked:

"Suppose you had to press three buttons on the first box and I was going to press
(a) "A"
(b) "B"
on the second box. What could you press on the first box to end up with both lights on in the same place?"

(6) Then, they were asked:

"Suppose you had to press four, five or more buttons on the first box, is there always one button on the second box which will bring the light to the same position on the second box as on the first?"

Measures

Two measures of performance were used, the game score (or the

number of predictions), and the posttest score. Two presumed measures of competence were inferred from subjects' sequences of plays, the pattern strategy score and the operator strategy score. Also a global measure of the field of possible plays was the distribution of a subject's plays over the 16 possible combinations of presses. In accord with the definitions of the strategies, the number of plays were grouped and scored over three mutually exclusive regions of possible play. The measures are defined below:

The game score. In each round of play there were four presses made, two by the subject and two by the experimenter, each of them making one press on each of the boxes. The number of presses on the second box by the subject, the prediction presses, were tallied. The total number of such presses was called the game score or the number of predictions. The game score could have a value from 16 to 120.

The posttest score. The posttest score was the number of correct items on the posttest. The score could have a value from 0 to 16.

The operator strategy score. This score was computed by counting the number of consecutive presses of the same switch made by the subject in runs of three presses or moreon the first box; the number of presses in all such runs was added to obtain the strategy score. The count of consecutive presses in a single run may be continued if a subject has broken the run in order to escape from a closed loop of identical presses and has made a correct prediction in the round of plays used to break the loop. For instance, if the

loop was the following sequence: BB = A; AB = B; BB = A;..., the subject may have played C to break the loop, obtaining IC = C, then, on his next play returned to pressing B to continue the run with CB = D; DB = C;... Also, runs of three or more identical prediction presses were counted if they were all correct and if they were uninterrupted.

The pattern strategy score. This score was computed in runs too, as for the operator score. Counts in a single run were continued across a break here as in the case of the operator score. The runs were scored within regions among the 16 possible combinations of plays. These regions were: the "diagonal" region, in which the correct play on the second box was always A, or in which the subject and the experimenter played the same operation on the first box; the "identity" region, in which the subject or experimenter played A on the first box and the other played B, C, or D; the remaining region, the "triangle", in which all three plays in a round on both boxes were different. The number of presses in all runs across all regions was added to obtain the score P.

The distribution of plays. The distribution of plays was a set of three scores: "T", "I" and "D". "T" was the number of plays by a subject made in the triangle region. "I" was the number of plays made in the identity region. "D" was the number of plays in the diagonal region. The sum of T, I and D was equal to the game score or the number of predictions. The distribution of plays was viewed as another indicator of subjects' performances. It provided an

indication of which regions were played regardless of the strategies used.

It should be noted that, the higher the game score, the poorer the performance. That is, the higher the game score, the greater the number of predictions and the feedback that was required to learn the workings of the devices.

RESULTS

Data analyses were divided into two parts. In the first part, data were examined by age in three groups. In the second, each age group was divided into low and high categories according to the m-power of the subjects as determined by their scores on the Figural Intersection Task. The first set of analyses, that in which the data were examined by age, were a supplement to the second set of analyses. The two purposes of the first set were, first, to obtain results that paralleled the between age results obtained in the second set and, second, to enable the comparison of the results of this study with the results of other studies in which age or operational level and not m-power was the independent variable of interest.

The second set of analyses, the main set of analyses of the hypotheses in this study, had the construct of m-power as the central independent variable. In both sets of analyses gender was introduced as an independent variable, along with age or m-power. In both sets, analyses were done on four main dependent variables: the total number of predictions or the game score; the number of correct posttest items or the posttest score; the strategy difference score; and the strategy use score.

In addition, within each group, as reported below, strategy employment was assessed using the difference and use scores. Also, ancillary analyses were done in which the results for different tasks were analyzed for selected groups.

Age

Age Effects: The Number of Predictions or The Game Score

Two 3X2 analyses of variance using age and gender were done on the game score subjects made, one analysis for subjects working alone and another for dyads solving the problem together. Table 3 presents the game score means and standard deviations for individuals and dyads by age. For the subjects working alone, there was a significant main effect for age, $F(2,54) = 6.739$, $p = .001$. Newman-Keuls post-hoc analyses indicated that the 11-year-old group used significantly more predictions than either the 13-year-old group or the 15-year-old group (means = 119.05, 103.05 and 88.80, respectively, $p < .05$).

For dyads, there were significant main effects for age, $F(2,30) = 8.955$, $p = .001$, and gender, $F(1,30) = 9.779$, $p < .005$. Table 4 presents the game score means and standard deviations by age and gender. Here too, as for individuals, 11-year-old dyads needed significantly more predictions than 13-year-old or 15-year-old dyads (means = 117.92, 88.83 and 92.92, respectively). There were no significant differences between 13- and 15-year-olds. Also, males made fewer predictions than females (90.39 and 109.39, respectively, $p < .05$).

Age Effects: Number of Correct Posttest Items or Posttest Score

Two 3X2 analyses of variance using age and sex were done on the number of correct items on the posttest. Table 3 presents the posttest score means and standard deviations for individuals and

Table 3

Mean Number of Predictions and Mean Number of Correct Items on Posttest and Standard Deviations for 11-, 13- and 15-year-old Individuals and Dyads

Age condition (n)	Predictions (SD)	Posttest (SD)
11 individ (n = 20)	119.05 (4.24)	3.75 (1.61)
dyad (n = 12)	117.92 (4.90)	6.17 (4.32)
13 individ (n = 20)	103.05 (29.11)	6.60 (2.90)
dyad (n = 12)	88.83 (27.71)	12.58 (2.99)
15 individ (n = 20)	88.80 (30.90)	11.20 (3.94)
dyad (n = 12)	92.92 (22.98)	12.83 (2.79)

dyads by age and table 4 presents these by age and gender. There was a significant main effect for age, $F(2,54) = 24.940$, $p < .001$; Newman-Keuls post-hoc analysis showed that 11-year-old subjects had significantly fewer items correct on the posttest than 13-year-old or 15-year-old subjects (means = 3.75, 6.60 and 10.15, respectively, $p < .05$). In addition, 13-year-olds had significantly fewer correct items than 15-year-olds (p .05).

For dyads, too, there was a significant main effect for age, $F(2,39) = 8.955$, $p < .001$, on the number of correct items on the posttest as well as a significant main effect for gender, $F(1,30) = 7.278$, $p = .01$. Newman-Keuls post-hoc tests showed that 11-year-old children had significantly fewer items correct than 13-year-old or 15-year-old children (means = 6.17, 12.58 and 12.83, respectively, $p < .05$). Males had more items correct than females (means = 11.94 and 9.11, respectively).

Age Effects: Strategy Difference Score

Two 3X2 analyses of variance with age and gender were done on the strategy difference scores, one on individual players' scores and one on dyads' scores. Tables 5 and 6 present the means and standard deviations of the strategy difference score by age, and by age and gender, respectively. The strategy difference score was defined as the difference between the natural logarithm of the operator strategy score and the natural logarithm of the pattern strategy score: strategy difference = $\ln(O + .5) - \ln(P + .5)$. There was a significant main effect for age on individuals' scores, $F(2,54) = 9.643$, $p < .001$. Post-hoc analysis determined

Table 4

Mean Number of Predictions and Mean Number of Correct Items on Posttest and Standard Deviations for 11-, 13- and 15-year-old Male and Female Individuals and Male and Female Dyads

Age condition (n)	Predictions (SD)		Posttest (SD)	
	Male	Female	Male	Female
11 individ (n = 10)	118.10 (6.01)	120.00 (0.00)	4.40 (2.01)	3.10 (0.74)
dyad (n = 6)	115.83 (6.52)	120.00 (0.00)	8.50 (5.09)	3.83 (1.47)
13 individ (n = 10)	87.10 (34.93)	119.00 (2.11)	7.50 (3.03)	5.70 (2.63)
dyad (n = 6)	72.00 (26.35)	105.67 (17.74)	14.00 (1.55)	11.17 (3.54)
15 individ (n = 10)	88.90 (20.63)	88.70 (39.89)	10.40 (3.53)	9.90 (4.04)
dyad (n = 6)	83.33 (26.55)	102.50 (15.38)	13.33 (2.73)	12.33 (3.01)

Table 5

Difference Between the Natural Log of the Operator Strategy and the Natural Log of the Pattern Strategy (ln(O + .5) - ln(P + .5)), and the Difference Between the Natural Log of the Sum of the Operator and Pattern Strategies and the Natural Log of the Random Strategy (ln(O + P + .5) - ln(R + .5)) and Their Standard Deviations for 11-, 13- and 15-year-old Individuals and Dyads

Age condition (n)	ln(O+.5) - ln(P+.5) Strategy Difference (SD)	ln(O+P+.5) - ln(R+.5) Strategy Use (SD)
11 individ (n = 20)	-1.008 (0.548)	-0.538 (0.426)
dyad (n = 12)	-0.631 (0.598)	-0.408 (0.219)
13 individ (n = 20)	0.119 (1.129)	-0.316 (0.582)
dyad (n = 12)	0.170 (1.621)	-0.250 (0.527)
15 individ (n = 20)	-0.078 (0.829)	-0.187 (0.465)
dyad (n = 12)	0.490 (0.555)	-0.330 (0.547)

Table 6

Difference Between the Natural Log of the Operator Strategy and the Natural Log of the Pattern Strategy (ln(O + .5) - ln(P + .5)), and the Difference Between the Natural Log of the Sum of the Operator and Pattern Strategies and the Natural Log of the Random Strategy (ln(O + P + .5) - ln(R + .5)) and Their Standard Deviations for 11-, 13- and 15-year-old Male and Female Individuals and Dyads

Age condition (n)	ln(O+.5) - ln(P+.5) Strategy Difference (SD)		ln(O+P+.5) - ln(R+.5) Strategy Use (SD)	
	Male	Female	Male	Female
11 individ (n = 10)	-0.935 (0.527)	-1.080 (0.586)	-0.486 (0.418)	-0.590 (0.450)
dyad (n = 6)	-0.335 (0.696)	-0.927 (0.304)	-0.422 (0.245)	-0.395 (0.213)
13 individ (n = 10)	0.407 (1.262)	-0.169 (0.955)	-0.370 (0.586)	-0.261 (0.603)
dyad (n = 6)	0.794 (1.252)	-0.455 (1.810)	-0.095 (0.334)	-0.406 (0.665)
15 individ (n = 10)	0.253 (0.403)	-0.255 (1.104)	-0.201 (0.320)	-0.173 (0.594)
dyad (n = 6)	0.533 (0.578)	0.448 (0.582)	-0.086 (0.552)	-0.575 (0.459)

that 11-year-old children had significantly lower strategy difference scores than 13-year-old or 15-year-old children (means = -1.01, 0.12 and -0.08, respectively, $p < .05$). There were no significant differences between 13- and 15-year-olds.

For dyads also there was a significant main effect for age, $F(2,30) = 3.942$, $p < .05$. Post-hoc analysis indicated that 11-year-old subjects had significantly lower strategy difference scores than 15-year-old subjects (means = -0.63 and 0.49, respectively, $p < .05$), but not 13-year-olds.

Age Effects: Strategy Use Score

The strategy use score reflected the extent to which subjects used the operator and pattern strategies. It was defined as the difference between the natural logarithm of the sum of the operator and pattern strategy scores and the natural logarithm of the random strategy score: strategy use = $\ln(O + P + .5) - \ln(R + .5)$. Tables 5 and 6 present the means and standard deviations for the strategy use score.

Two 3X2 analyses of variance with age and gender were done on strategy use scores of individuals and dyads. No significant differences were detected at the .05 level.

Strategy Employment at Each Age

Subject's use of the operator and pattern strategies at each age level were compared in two ways: first, the Wilcoxon test for two related groups was used to compare the natural logarithm of the operator score with the natural logarithm of the pattern score; second, the natural logarithm of the sum of the operator and pattern

scores was compared with the natural logarithm of the random strategy score. The first comparison was to determine the predominant use of strategy; the purpose of the second was to determine if the operator and pattern strategies were indeed significant parts of subjects' approaches to the task. Figures 4 and 5 portray the strategy use and difference scores for males and females at three ages.

In the individual condition, 11-year-old children used the random strategy significantly more than the sum of the operator and pattern strategy, $z = -3.584$, $p < .001$. When 11-year-olds used the operator or pattern strategy they used the pattern strategy significantly more ($z = -3.920$, $p < .001$). The 13-year-old subjects used the random strategy significantly more often ($z = -2.017$, $p < .05$), but used the operator strategy as often as the pattern strategy. The 15-year-old subjects showed no significant differences in their use of strategies.

In the dyadic condition, significant differences were found on both comparisons for 11-year-olds. As in the case of individuals, the pattern strategy was used significantly more ($z = -2.589$, $p < .01$) when one of the two strategies was used, and the random strategy was used significantly more than the combined use of the other two ($z = -3.059$, $p < .005$). The 13-year-olds evidenced no significant differences on both comparisons. On the other hand, 15-year-old dyads used the operator strategy significantly more than the pattern strategy ($z = -2.353$, $p < .05$), while utilizing both strategies combined as often as they used the random strategy.

Figure 4. The strategy use score for females and males at 11, 13 and 15 years.

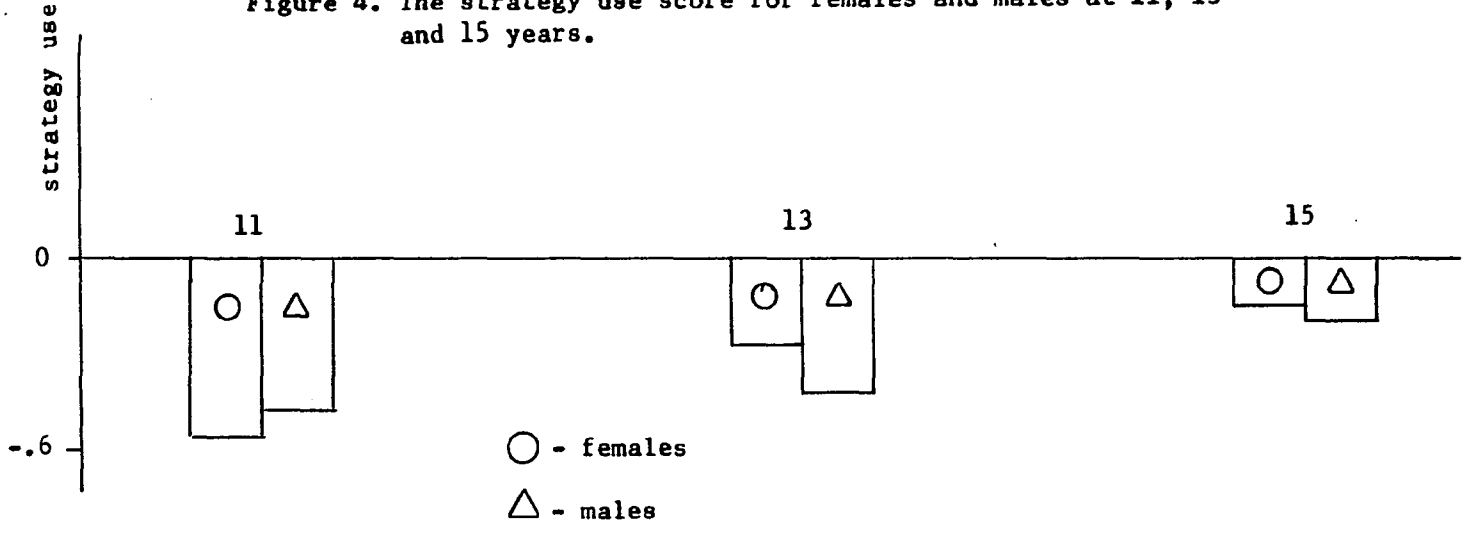
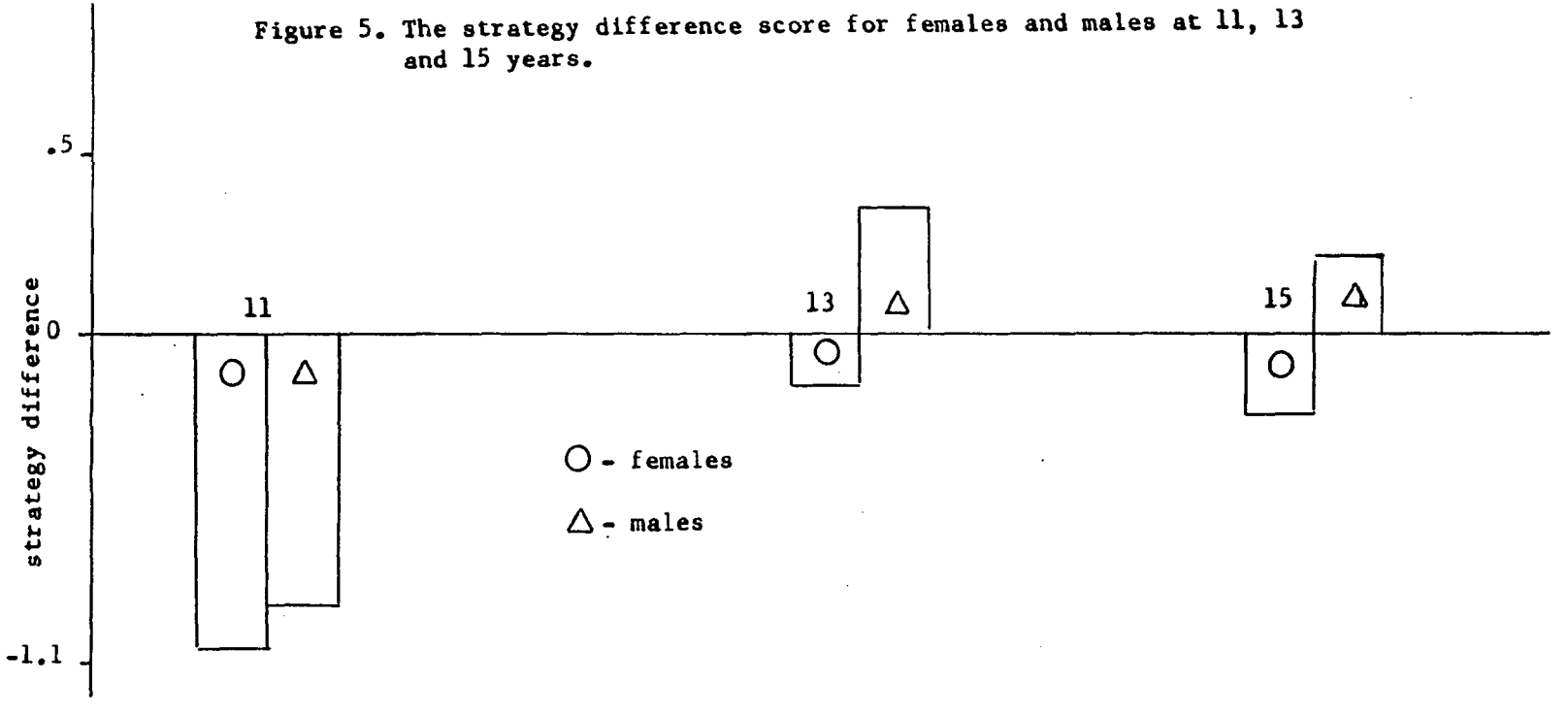


Figure 5. The strategy difference score for females and males at 11, 13 and 15 years.



When the analyses on strategy employment were done for each age group subdivided by gender, it was found that females working individually in the 11-year-old group used the pattern strategy more often than the operator strategy ($z = -2.083$, $p < .005$) and the random strategy more often than the other two combined ($z = -2.803$, $p < .005$). There were no other significant differences for females working alone. Figures 4 and 5 portray the strategy use and strategy difference scores, respectively, for males and females in the three age groups.

Female 11-year-old dyads, too, used the pattern strategy significantly more than the operator strategy ($z = -2.2014$, $p < .05$) and the random strategy more than the other two combined ($z = -2.201$, $p < .05$). Female 15-year-old dyads, too, used the random strategy significantly more ($z = -1.991$, $p < .05$). There were no other differences for females.

For males, when working individually, significant differences appeared only for the 11-year-old group: male 11-year-olds used the pattern strategy significantly more often than the operator strategy ($z = -2.803$, $p < .01$) and the random strategy more than the combined other two ($z = -2.395$, $p < .01$).

Male 11-year-old dyads used the random strategy more than the other two ($z = -2.201$, $p < .05$), but there was no significant difference between their use of the operator and pattern strategies. Male 15-year-old dyads used the operator strategy significantly more often than the pattern strategy ($z = -1.991$, $p < .05$).

Age Effects: The Distribution of Plays by Subjects

In the game, the first two button presses, one by the subject and one by the experimenter, determined the experimenter's second button press. The subject's second button press was his or her prediction of which of the four buttons the experimenter would press. How did subjects distribute their first plays among the possible presses irrespective of their strategies? The following supplementary analyses addressed this question.

There were 16 possible combinations of the first presses made by the subject and experimenter. These 16 possible combinations were classified into three mutually exclusive regions of play: the so-called triangle region (which contained six of the combinations), the identity region (which contained six combinations) and the diagonal region (which contained four combinations). Thus, each first press made by a subject, considered in coordination with the experimenter's first button press, was classified into one of the three regions of play. For each subject, the total number of plays in the three regions was, of course, equal to his or her number of predictions or game score. Profile analyses (Morrison, 1976) were done on three differences defined among the four measures, game score, triangle score, identity score and diagonal score:

$$D(TI) = \text{Difference}(TI) = \ln(\text{triangle} + .5) - \ln(\text{identity} = .5),$$

$$D(ID) = \text{Difference}(ID) = \ln(\text{identity} + .5) - \ln(\text{diagonal} + .5)$$

and $$D(NT) = \text{Difference}(NT) = \ln(N + .5) - \ln(\text{triangle} + .5).$$

A profile analysis separates the sources of difference between the response vectors or profiles of different groups into two parts.

The response vector or profile for each group has as its components the mean values of the dependent variables. A profile analysis separates the sources of differences between groups into differences in shape and differences in level. In addition, profile analyses test for the flatness of the curve formed by the components of the response vector. Thus in a profile analysis three tests are performed: first, a test ot the parallelism of the profiles or of the equality of the slopes or the differences between each pair of adjacent dependent measures is done; second, a test of the differences between groups of the levels of the dependent measures is done; third, a test of the flatness of the profiles or the equality of the grand means is done.

In the present study profile analyses were used to study the auxiliary null hypotheses that differences in age, m-power and gender did not lead to differences in subjects' distributions of plays among the three regions. These analyses were intended to explore the approach of individual subjects to the game, thus, the results for dyads are not presented here.

First, a profile analysis was done with age. The hypothesis of parallel profiles for the three variables at three levels of age was not rejected. The hypothesis of flatness of the profiles was rejected, $F(2,56) = 46.375$, $p < .001$. D(NT) was found to be significantly greater than D(TI), $F(1,57) = 73.814$, $p < .001$. Thus the parallel profiles sloped upward between D(TI) and D(NT) and were flat between D(NT) and D(ID). No significant main effect for age was indicated for the distribution of plays by subjects. Table 7

gives means for the three differences by age.

Profile analyses for males and for females with age were done on the three differences D(NT), D(TI) and D(ID). Table 8 gives means and standard deviations of the three variables for males and females. For females the profiles were found parallel with D(NT) significantly larger than D(TI), $F(1,27) = 18.456$, $p < .001$, and D(NT) significantly smaller than D(ID), $F(1,27) = 8.828$, $p < .01$. For males the profiles were found parallel with D(NT) significantly greater than both D(TI), $F(1,27) = 71.925$, $p < .001$, and D(ID), $F(1,27) = 5.013$, $p < .05$. No significant main effects for age were found for males or females.

M-power and Age

Scores on the Figural Intersection Task were used as the measure of m-power in this study. These scores were used to divide each age group into high and low categories: 11-year-old children with scores from 5 to 5.5 were called "high" and those with scores of 4 were called "low"; 13-year-old children with scores on the FIT from 6 to 6.25 were called "high" and those with scores of 5 to 5.5 were considered "low"; 15-year-olds were divided into 3 groups: those with scores of 6 to 6.25 on the FIT were considered "low" scorers; those with scores from 6.5 to 6.75 were considered "high" and those from 7 to 7.5 were called "very high". There were only 5 subjects, all male, in the "very high" group.

There were three levels of m-power at the 15-year-old age level, rather than two levels as at the 11- and 13-year-old age

Table 7

The Distribution of the Mean Number of Plays in the Three Regions, X, Y, and Z* [Standard Deviation] for 11-, 13- and 15-year-olds in the Individual and Dyadic Conditions

Age condition (n)	(X, Y, Z) [SDX, SDY, SDZ]
11 individ (n = 20)	(0.81, 0.14, 1.05) [0.23, 0.23, 0.70]
dyad (n = 12)	(0.97, -0.01, 0.54) [0.26, 0.37, 0.33]
13 individ (n = 20)	(0.86, 0.12, 0.83) [0.29, 0.45, 0.61]
dyad (n = 12)	(0.87, 0.23, 0.44) [0.32, 0.54, 0.36]
15 individ (n = 20)	(0.84, 0.24, 0.62) [0.28, 0.37, 0.70]
dyad (n = 12)	(0.91, 0.12, 0.40) [0.22, 0.36, 0.20]

* X = D(NT) = ln(N + .5) - ln(T + .5)
  Y = D(TI) = ln(T + .5) - ln(I + .5)
  Z = D(ID) = ln(I + .5) - ln(D + .5)

Table 8

The Distribution of the Mean Number of Plays in the Three Regions, X, Y, and Z* [Standard Deviation] for Male and Female 11-, 13- and 15-year-olds in the Individual and Dyadic Conditions

Age condition (n)	(X, Y, Z) [SDX, SDY, SDZ]	
	Male	Female
11 indivíd (n = 10)	(0.91, 0.02, 0.92) [0.27, 0.13, 0.91]	(0.70, 0.26, 1.17) [0.11, 0.26, 0.43]
dyad (n = 6)	(1.06,-0.15, 0.57) [0.31, 0.38, 0.43]	(0.88, 0.13, 0.52) [0.18, 0.33, 0.22]
13 indivíd (n = 10)	(0.95, 0.08, 0.42) [0.30, 0.41, 0.30]	(0.77, 0.16, 1.23) [0.28, 0.51, 0.58]
dyad (n = 6)	(0.94, 0.13, 0.31) [0.25, 0.31, 0.42]	(0.81, 0.32, 0.58) [0.40, 0.72, 0.25]
15 indivíd (n = 10)	(0.92, 0.18, 0.35) [0.29, 0.25, 0.60]	(0.76, 0.30, 0.88) [0.27, 0.48, 0.72]
dyad (n = 6)	(0.98,-0.01, 0.45) [0.25,-0.40, 0.18]	(0.84, 0.26, 0.36) [0.19, 0.28, 0.23]

* X = D(NT) = ln(N + .5) - ln(T + .5)
  Y = D(TI) = ln(T + .5) - ln(I + .5)
  Z = D(ID) = ln(I + .5) - ln(D + .5)

levels. The decision to include three levels was made during data collection for the following reasons: only one subject scored "7" on the Figural Intersection Task when it was administered to 15-year-olds. This led to further testing with the Figural Intersection Task than was originally planned in order to obtain a high m-power group at the oldest age level. In the course of this further testing there appeared to be three levels of performance on the task: the "lows", who had few items correct at the level of seven or higher; the "very highs", who had all or almost all items correct, even above the level of seven; and an intermediate group, who had many items correct at the higher levels and who had m-power scores of seven on at least two of the four scoring systems used (in this study, subjects were assigned m-power scores that were the means in each case of scores obtained from four scoring systems). Hence, it was decided to administer the INRC task to all three groups.

The resulting seven groups of individuals and the six groups of dyads were compared in accord with the hypotheses of this study within age groups at different levels of m-power and between age groups when m-power differed between groups by zero or one. The analyses were done on the four variables, game score, posttest score, strategy difference score and strategy use score. The employment of strategies within each group and the distribution of plays between groups were compared too. Tables 9, 10 and 11 present the means and standard deviations of the game, posttest, strategy

Table 9

Mean Number of Predictions and Mean Number of Correct Items on Posttest and Standard Deviations for 11-, 13- and 15-year-old Individuals and Dyads With Low and High m-power

Age condition (n)	Predictions (SD)		Posttest (SD)	
	High	Low	High	Low
11 indivíd (n = 10)	118.10 (6.01)	120.00 (0.00)	4.70 (1.77)	2.80 (0.63)
dyad (n = 6)	115.83 (6.52)	120.00 (0.00)	8.67 (4.97)	3.67 (1.21)
13 indivíd (n = 10)	92.60 (34.51)	113.50 (18.86)	7.30 (2.67)	5.90 (3.11)
dyad (n = 6)	84.50 (23.35)	93.16 (33.16)	13.50 (2.81)	11.67 (3.14)
15 indivíd (n = 10)	90.90 (25.52)	86.70 (36.82)	10.90 (3.18)	9.40 (4.20)
dyad (n = 6)	88.67 (12.58)	97.16 (30.99)	14.00 (1.41)	11.67 (3.44)

Table 10

Difference Between the Natural Log of the Operator Strategy and the Natural Log of the Pattern Strategy (ln(O + .5) - ln(P + .5)), and the Difference Between the Natural Log of the Sum of the Operator and Pattern Strategies and the Natural Log of the Random Strategy (ln(O + P + .5) - ln(R + .5)) and Their Standard Deviations for 11-, 13- and 15-year-old Individuals and Dyads With High and Low m-power

Age condition (n)	ln(O+.5) - ln(P+.5) Strategy Difference (SD)		ln(O+P+.5) - ln(R+.5) Strategy Use (SD)	
	High	Low	High	Low
11 individ (n = 10)	-1.024 (0.699)	-0.994 (0.380)	-0.447 (0.494)	-0.629 (0.347)
dyad (n = 6)	-0.465 (0.732)	-0.797 (0.430)	-0.358 (0.232)	-0.459 (0.214)
13 individ (n = 10)	0.607 (1.217)	-0.369 (0.823)	-0.268 (0.551)	-0.364 (0.637)
dyad (n = 6)	0.109 (2.274)	0.231 (0.775)	-0.132 (0.691)	-0.368 (0.317)
15 individ (n = 10)	-0.130 (0.679)	-.025 (0.991)	-0.329 (0.542)	-0.046 (0.343)
dyad (n = 6)	0.372 (0.308)	0.608 (0.741)	0.013 (0.352)	-0.674 (0.501)

Table 11

The Distribution of the Mean Number of Plays
in the Three Regions, X, Y, and Z* [Standard Deviation]
for Male and Female 11-, 13- and 15-year-olds
With High and Low m-power
in the Individual and Dyadic Conditions

Age condition (n)	(X, Y, Z) [SDX, SDY, SDZ]	
	High	Low
11 indivíd (n = 10)	(0.85, 0.15, 0.89) [0.30, 0.25, 0.93]	(0.77, 0.13, 1.20) [0.12, 0.24, 0.36]
dyad (n = 6)	(0.82, 0.20, 0.64) [0.19, 0.32, 0.33]	(1.12, -0.22, 0.45) [0.24, 0.30, 0.32]
13 indivíd (n = 10)	(0.81, 0.22, 0.73) [0.32, 0.46, 0.56]	(0.91, 0.01, 0.92) [0.27, 0.44, 0.68]
dyad (n = 6)	(0.82, 0.40, 0.35) [0.39, 0.66, 0.38]	(0.93, 0.05, 0.54) [0.27, 0.37, 0.35]
15 indivíd (n = 10)	(0.77, 0.28, 0.77) [0.18, 0.31, 0.60]	(0.91, 0.19, 0.46) [0.36, 0.44, 0.79]
dyad (n = 6)	(0.80, 0.30, 0.43) [0.14, 0.21, 0.17]	(1.03, -0.05, 0.37) [0.24, 0.40, 0.15]

* X = D(NT) = ln(N + .5) - ln(T + .5)
  Y = D(TI) = ln(T + .5) - ln(I + .5)
  Z = D(ID) = ln(I + .5) - ln(D + .5)

and distribution scores for low and high groups. Table 12 presents means and standard deviations for the very high 15-year-olds.

M-power and Age: The Number of Predictions or Game Score

Seven groups were ordered by age and by level of m-power: thus, seven groups were arranged from "low" 11-year-olds to "very high" 15-year-olds. A set of planned comparisons with the seven groups was done on the mean game score which compared all adjacent groups of individuals. There were significant differences between (1) low and high 13-year-olds, $t(18) = 1.974$, $p = .05$, (2) low and very high 15-year-olds, $t(13) = 3.016$, $p < .01$, and (3) high and very high 15-year-olds, $t(13) = 3.339$, $p = .001$. The high 13-year-olds needed fewer predictions than the low (means = 92.60 and 113.50). Very high 15-year-old children required fewer predictions than low or high 15-year-olds (means = 47.60, 86.70 and 90.90, respectively).

For dyads, there were no significant differences found between adjacent groups when the same planned comparison was done.

M-power and Age: The Number of Correct Posttest Items or Posttest Score

Again, the seven groups of individuals were ordered by age and m-power: low 11-year-olds first, then high 11-, low 13-, and so on, up to very high 15-year-olds. A set of planned comparisons was done on the posttest scores comparing adjacent groups in the sequence. For individuals, the only significant differences within an age group were that very high 15-year-olds had more correct posttest

Table 12

Mean Number of Predictions, Number of Correct Posttest Items, Strategy Difference Score and Strategy Use Score (Standard Deviations) for Special Groups A, B and C*

	Group A (n = 12)	Group B (n = 8)	Group C (n = 5)
Number of Predictions	92.75 (32.85)	120.00 (0.00)	47.60 (6.99)
Number of Correct Posttest Items	8.67 (4.83)	2.63 (0.74)	15.40 (0.89)
Strategy Use Score	-0.22 (0.58)	-0.28 (0.32)	0.50 (0.15)
Strategy Difference Score	-1.40 (1.24)	-0.56 (1.64)	0.56 (0.17)

* Group A: high 11-year-olds on large square task.
Group B: high 13-year-olds on small square task.
Group C: very high 15-year-olds.

items than high or low 15-year-olds, $t(13) = -2.999$, $p < .005$ (means = 15.40 and 10.90), and $t(13) = -3.999$, $p < .001$ (means = 15.40 and 9.4, respectively).

In the case of dyads, the analyses showed that low 11-year-olds had significantly fewer items correct than high 11-year-olds, $t(10) = -2.791$, $p < .01$ (means = 3.67 and 8.67, respectively).

M-power and Age: Strategy Difference Score

The groups of individuals were ordered by age and m-power in the same way. The set of planned comparisons for adjacent groups was done. Significant differences on the strategy difference score appeared in one place between groups of the same age, that is, between low and high 13-year-olds, $t(18) = -2.796$, $p < .01$, the high group having a higher score (means = 0.607 and -0.369).

M-power and Age: Strategy Use Score

The above sequence for age and m-power was again defined and the set of comparisons for adjacent groups was done on the strategy use score. The contrasts indicated the existence of significant differences within the 15-year-old age group. The very high 15-year-olds had mean scores higher than the low, $t(18) = 2.058$, $p < .05$, and the high 15-year-olds $t(18) = -3.131$, $p < .01$ (means = 0.497, -0.046 and -0.329, respectively).

In the dyadic condition, utilizing the same set of comparisons, it was found that the high 13-year-old group had a significantly higher mean score than the low 15-year-olds, $t(10) = 2.241$, $p < .05$ (means = -0.132 and -0.674, respectively). Further, the high 15-year-old dyads had a greater mean strategy use score than

the low 15-year-old group, $t(10) = -2.843$, $p < .01$ (means = 0.0134 and -0.674, respectively).

Age and M-power: Strategy Employment at Each Level

Subjects' use of the pattern and operator strategies were examined at all levels of m-power at each age. The Wilcoxon test of related groups was done to determine the extent of subjects' use of the pattern and operator strategies as compared with their use of the random strategy and to compare their use of the operator and pattern strategies. Figures 6 and 7 portray the strategy use and difference scores at each age and level of m-power.

Low 11-year-olds used the random strategy significantly more than the sum of the pattern and operator strategies, $z = -2.803$, $p = .005$, and the pattern strategy significantly more than the operator strategy, $z = -2.803$, $p = .005$. High 11-year-old subjects, too, used the random strategy more, $z = -2.293$, $p < .05$, and the pattern strategy more, $z = -2.803$, $p = .005$. There were no other significant differences for individuals. For dyads, the low 11-year-olds, again, used the random strategy significantly more than the combined other two, $z = -2.201$, $p < .05$, and the pattern strategy more than the operator, $z = -2.204$, $p < .05$. The high 11-year-old dyads used the random strategy significantly more too, $z = -2.204$, $p < .05$, as did the low 15-year-olds, $z = -1.991$, $p < .05$. The high 15-year-old dyads used the operator strategy significantly more than the pattern strategy, $z = -1.991$, $p < .05$.

Age and M-power: The Distribution of Plays by Subjects

The distribution of plays by subjects over the triangle,

Figure 6. The strategy use score for low and high m-power scorers at 11, 13, and 15 years

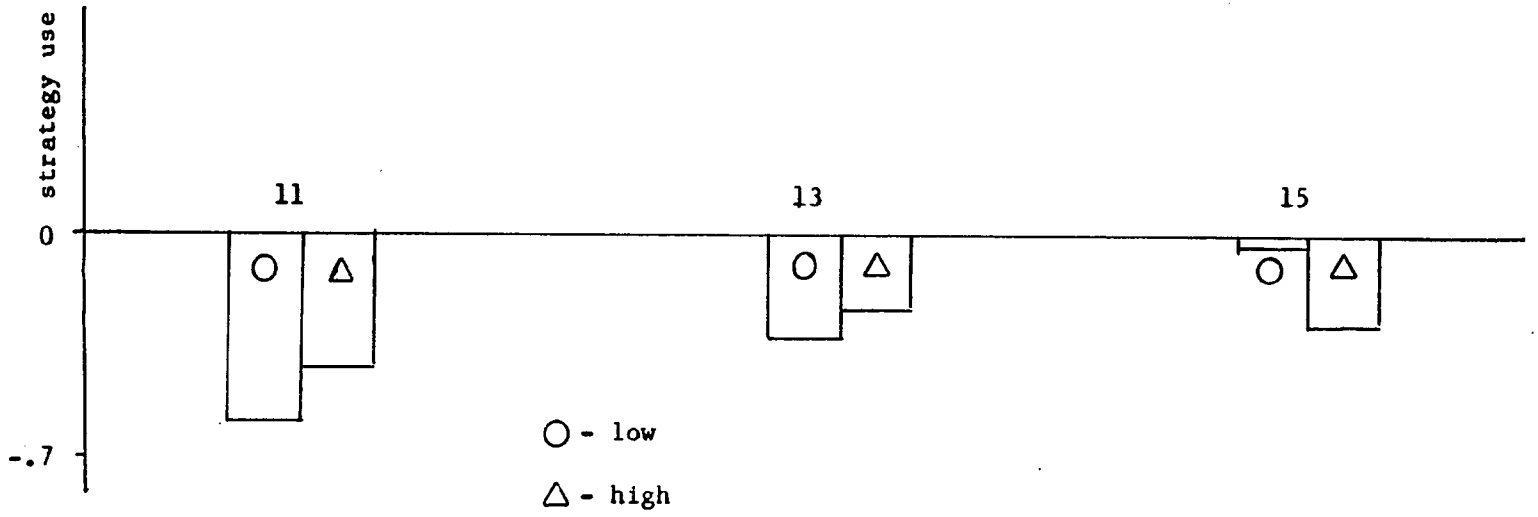
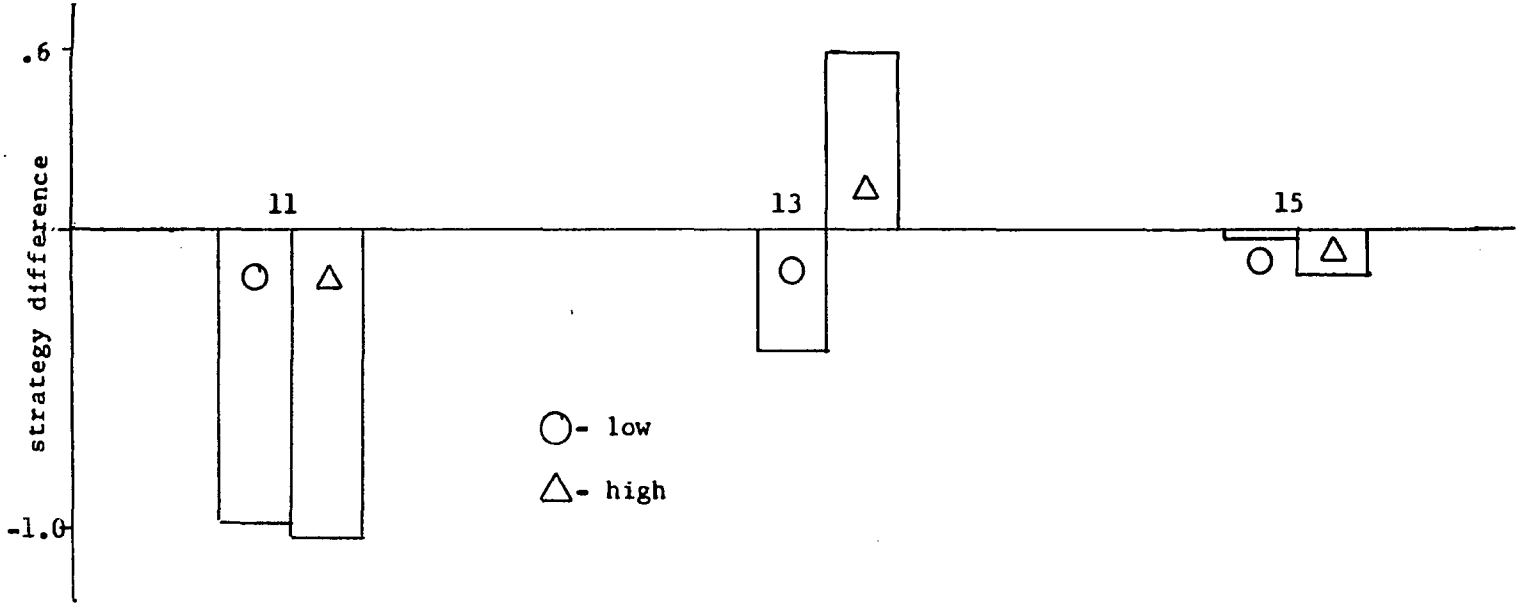


Figure 7. The strategy difference score for low and high m-power scorers at 11, 13 and 15 years



identity and diagonal regions were analyzed by profile analyses done on the three variables D(NT), D(TI) and D(ID). One analysis found that the profiles for high m-power groups at three age levels were parallel. The profile at each level was not flat, D(NT) was found significantly larger than D(TI), $F(1,27) = 30.386$, $p < .001$. A second analysis found analogous results for the profiles of the low m-power groups at three age levels. D(NT) was significantly larger than D(TI), $F(1,27) = 42.251$, $p < .001$, and the hypothesis of parallel profiles was not rejected.

Profile analyses were done comparing groups with different levels of m-power and the same age, and the same m-power at different ages. The results in all cases were consistent with the above two profile analyses. The profiles were judged parallel with D(NT) being found significantly greater than D(TI), but with no significant difference found between D(NT) and D(ID). Thus the profiles sloped up from D(TI) to D(NT) and were flat between D(NT) and D(ID).

Age, M-power and Gender

As originally designed, this study was to investigate the influence of functional m-power and operational level upon task performance. At every level of m-power, half of the subjects were male and half were female. Casual perusal of the data revealed differences between males and females in scores attained on some measures. The results below present the statistical differences related to gender and m-power. Means and standard deviations for the game score, posttest score, strategy difference and use scores,

and the distribution of plays are presented in tables 13, 14 and 15.

Age, M-power and Gender: Number of Predictions or Game Score

The same set of contrasts as above for adjacent groups with either the same m-power or the same age was done on the mean game scores for individual females and for individual males. Recall that a higher game score indicated a poorer performance.

High 13-year-old female individuals had a significantly larger game score than low 15-year-old females, $t(8) = 2.675$, $p = .01$ (means = 119.0 and 79.0, respectively). Low 13-year-old males had a significantly higher game score than high 13-year-old males, $t(8) = 3.393$, $p < .01$ (means = 108.0 and 66.2, respectively). But, low 15-year-old males had a significantly higher game score than high 13-year-old males, $t(8) = 2.289$, $p < .05$ (means = 94.4 and 66.2, respectively). Also, high 15-year-old males required significantly more predictions than very high 15-year-old males, $t(8) = 2.906$, $p < .01$ (means = 83.4 and 47.6, respectively).

Male and female groups were compared to each other using a set of contrasts of adjacent groups. The groups were ordered according to their age and m-power and, within each level of m-power, by gender with females first and males next. The analyses gave one significant difference: that between high 13-year-old males and females. The males required fewer predictions than the females, $t(8) = 3.746$, $p < .001$ (means = 66.2 and 119.0, respectively).

Age, M-power and Gender: Posttest Score

The same set of planned comparisons as done on the game

Table 13

Mean Number of Predictions and Mean Number of Correct Items on Posttest and Standard Deviations for 11-, 13- and 15-year-old Male and Female Individuals and Male and Female Dyads With Low and High M-power

Age condition (n)	Predictions (SD)				Posttest (SD)			
	High Male	High Female	Low Male	Low Female	High Male	High Female	Low Male	Low Female
11								
individ (n = 5)	116.20 (0.00)	120.00 (0.00)	120.00 (0.00)	120.00 (0.00)	5.80 (1.92)	3.60 (0.55)	3.00 (0.71)	2.60 (0.55)
dyad (n = 3)	111.67 (7.37)	120.00 (0.00)	120.00 (0.00)	120.00 (0.00)	13.00 (1.73)	4.33 (1.53)	4.00 (1.00)	3.33 (1.53)
13								
individ (n = 5)	66.20 (30.54)	119.00 (2.24)	108.00 (26.83)	119.00 (2.24)	7.80 (1.79)	6.80 (3.49)	7.20 (4.15)	4.60 (0.55)
dyad (n = 3)	70.00 (11.35)	99.00 (24.56)	74.00 (39.95)	112.33 (7.09)	13.67 (2.31)	13.33 (3.79)	14.33 (0.58)	9.00 (1.73)
15								
individ (n = 5)	83.40 (14.65)	98.40 (33.32)	94.40 (25.83)	79.00 (47.26)	10.60 (3.21)	11.20 (3.49)	10.20 (4.21)	8.60 (4.51)
dyad (n = 3)	79.00 (10.44)	98.33 (2.52)	87.67 (39.95)	106.67 (23.09)	14.67 (1.15)	13.33 (1.53)	12.00 (3.46)	11.33 (4.16)

Table 14

Difference Between the Natural Log of the Operator Strategy and the Natural Log of the Pattern Strategy ($\ln(O + .5) - \ln(P + .5)$), and the Difference Between the Natural Log of the Sum of the Operator and Pattern Strategies and the Natural Log of the Random Strategy ($\ln(O + P + .5) - \ln(R + .5)$) and Their Standard Deviations for 11-, 13- and 15-year-old Male and Female Individuals and Dyads With High and Low m-power

Age condition (n)	$\ln(O+.5) - \ln(P+.5)$ Strategy Difference (SD)				$\ln(O+P+.5) - \ln(R+.5)$ Strategy Use (SD)			
	High Male	High Female	Low Male	Low Female	High Male	High Female	Low Male	Low Female
11								
individ	-0.879	-1.164	-0.992	-0.996	-0.308	-0.587	-0.664	-0.594
(n = 5)	(0.740)	(0.707)	(0.264)	(0.506)	(0.450)	(0.546)	(0.333)	(0.397)
dyad	0.005	-0.935	-0.675	-0.920	-0.295	-0.421	-0.550	-0.368
(n = 3)	(0.817)	(0.084)	(0.441)	(0.473)	(0.111)	(0.332)	(0.298)	(0.034)
13								
individ	1.033	0.182	-0.219	-0.519	-0.050	-0.486	-0.691	-0.036
(n = 5)	(1.185)	(1.216)	(1.095)	(0.518)	(0.512)	(0.551)	(0.505)	(0.624)
dyad	1.683	-1.466	-0.094	0.556	0.116	-0.381	-0.306	-0.431
(n = 3)	(0.706)	(2.234)	(1.025)	(0.365)	(0.174)	(0.989)	(0.338)	(0.354)
15								
individ	0.102	-0.362	0.097	-0.148	-0.340	-0.317	-0.062	0.029
(n = 5)	(0.311)	(0.898)	(0.515)	(1.381)	(0.353)	(0.732)	(0.239)	(0.454)
dyad	0.464	0.280	0.601	0.614	0.269	-0.242	-0.440	-0.908
(n = 3)	(0.277)	(0.367)	(0.862)	(0.792)	(0.208)	(0.267)	(0.584)	(0.350)

Table 15

The Distribution of the Mean Number of Plays in the Three Regions, X, Y, and Z* [Standard Deviation] for 11-, 13- and 15-year-old Male and Female Individuals and Dyads With High and Low m-power

Age condition (n)	(X, Y, Z) [SDX, SDY, SDZ]			
	High Male	High Female	Low Male	Low Female
11 individ (n = 5)	(0.99, 0.03, 0.62) [0.37, 0.12, 1.18]	(0.70, 0.26, 1.19) [0.10, 0.30, 0.54]	(0.82, 0.01, 1.26) [0.08, 0.16, 0.37]	(0.71, 0.25, 1.14) [0.13, 0.25, 0.34]
dyad (n = 3)	(0.79, 0.17, 0.84) [0.11, 0.15, 0.35]	(0.86, 0.22, 0.43) [0.27, 0.49, 0.17]	(1.33,-0.47, 0.30) [0.04, 0.18, 0.34]	(0.91, 0.03, 0.60) [0.10, 0.06, 0.28]
13 individ (n = 5)	(0.90, 0.16, 0.64) [0.41, 0.56, 0.82]	(0.73, 0.29, 0.98) [0.22, 0.39, 0.65]	(1.00,-0.01, 0.35) [0.16, 0.24, 0.34]	(0.81, 0.03, 1.49) [0.35, 0.62, 0.43]
dyad (n = 3)	(1.13,-0.08, 0.04) [0.15, 0.29, 0.23]	(0.51, 0.88, 0.66) [0.24, 0.56, 0.12]	(0.74, 0.35, 0.58) [0.14, 0.15, 0.43]	(1.12,-0.24, 0.50) [0.23, 0.23, 0.35]
15 individ (n = 5)	(0.84, 0.22, 0.66) [0.13, 0.19, 0.36]	(0.71, 0.34, 1.10) [0.21, 0.41, 0.67]	(1.00, 0.13, 0.39) [0.39, 0.31, 1.00]	(0.82, 0.25, 0.67) [0.34, 0.58, 0.77]
dyad (n = 3)	(0.88, 0.18, 0.32) [0.13, 0.14, 0.18]	(0.71, 0.41, 0.53) [0.11, 0.23, 0.06]	(1.08,-0.20, 0.57) [0.34, 0.52, 0.02]	(0.97, 0.10, 0.18) [0.16, 0.26, 0.18]

* X = D(NT) = ln(N + .5) - ln(T + .5)
  Y = D(TI) = ln(T + .5) - ln(I + .5)
  Z = D(ID) = ln(I + .5) - ln(D + .5)

score was done on the mean posttest scores of males and of females. Although both females and males showed an increase in the number of items correct as m-power increased, the only significant difference between adjacent pairs was that between the means for the very high and high 15-year-old males, $T = -2.748$, $p < .01$ (means = 15.40 and 10.6, respectively).

Age, M-power and Gender: Strategy Difference Score

Using the same set of planned comparisons again, the following significant differences were found on the strategy difference scores for males and females: for males, low 13-year-olds had a lower mean strategy difference score than high 13-year-old males, $t(8) = -2.757$, $p = .01$ (means = -0.22 and 1.03, respectively).

Among themselves, females had no significant differences. They exhibited an increase in scores with increasing m-power as did males, but their scores were generally lower. The only significant difference between males and females was that low 15-year-old females had lower mean strategy difference scores than the high 13-year-old males, $t(8) = 2.180$, $p < .05$, (means = -0.15 and 1.03, respectively).

Age, M-power and Gender: Strategy Use

Also, the planned comparisons with age, m-power and gender as factors defining the groups were done on the strategy use score. There were no significant differences among the six groups for females. For males, there were significant differences between the low and high 13-year-olds, $t(8) = -2.634$, $p < .01$, and high and very

high 15-year-olds, $T = -3.444$, $p < .01$. In the case of the 13-year-olds, the high group had a greater mean strategy use score than the low (means = -0.050 and -0.691, respectively). For the 15-year-old boys, the mean score of the very high group was greater than that of the high group (means = 0.497 and -0.340, respectively).

In comparing the mean scores of males and females, the low 13-year-old females were significantly different from the low 13-year-old males, $T = 2.103$, $p < .05$, the females having the higher score (means = -0.036 versus -0.691). There were no other significant differences between males and females.

Age, M-power and Gender: The Dyadic Condition

In the dyadic condition at each age and level of m-power only three pairs of males and three pairs of females were subjects in this study. Therefore, statistical analyses of the data will not be presented here.

Age, M-power and Gender: Employment of Strategies at Each Level

Here again, because of the small number of male and female dyads by age and m-power, statistical analyses of the data for dyads will not be reported. For individuals, low 11-year-old males as well as females used the random significantly more than the combined operator and pattern strategies, $z = -2.022$, $p < .05$, and the pattern more than the operator strategy, $z = -2.022$, $p < .05$. The high 11-year-old males and females, too, used the pattern strategy significantly more often than the operator, $z = -2.022$, $p < .05$. But, only females used the random strategy significantly more than the other two, $z = -2.022$, $p < .05$. The low 13-year-old males did,

however, use the random significantly more than the sum of the operator and pattern strategies, $z = -2.022$, $p < .05$. The very high 15-year-old males had the only other significant differences. They used the operator more than the pattern strategy and their sum of the operator and pattern strategies was greater than their use of the random strategy, both at $z = -2.022$, $p < .05$. Figures 8 and 9 present the strategy use and difference scores by age, m-power and gender.

Age, M-power and Gender: The Distribution of Plays

Profile analyses were done for high females at the three age levels, low females at the three age levels and the high and low males, as well. A fifth profile analysis was done in which the high male group was replaced by the very high group. The dependent variables on which the analyses were done were the three differences defined among the number of button presses in each of three regions and the total number of presses: D(TI), D(ID) and D(NT).

For high females the hypothesis of parallel profiles was not rejected. The test for flatness of the profiles was significant, $F(2,11) = 6.361$, $p = .01$, and there were significant main effects for (D(TI) - D(NT)), $F(1,12) = 8.780$, $p = .01$, and for (D(NT) - D(ID)), $F(1,12) = 5.202$, $p < .05$. Thus the means for the three variables were not equal and D(NT) was significantly different from D(TI) and D(ID). There was no indication of a significant effect for age on the three variables.

For low females the profiles were found to be parallel. The flatness test was significant, $F(2,11) = 8.737$, $p < .01$, and there

Figure 8. The strategy use score for female and male low and high m-power scorers at 11, 13 and 15 years

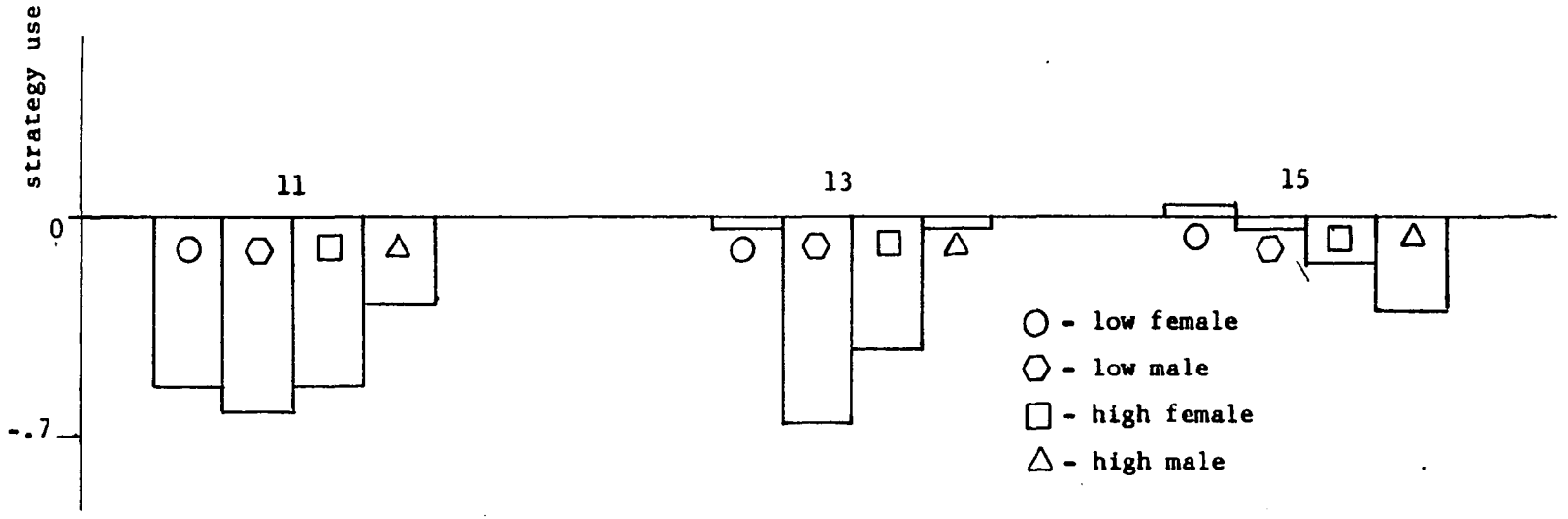
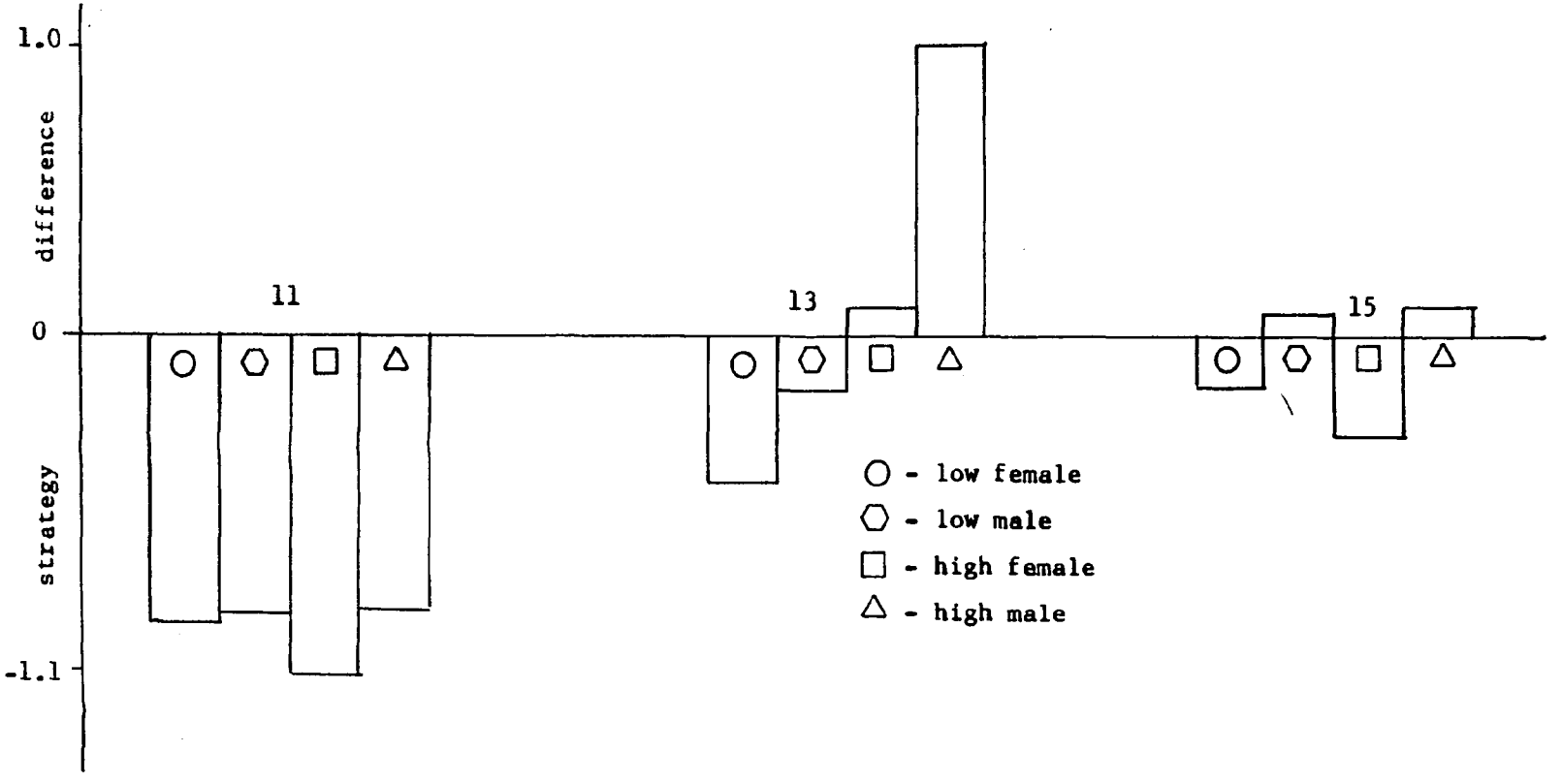


Figure 9. The strategy difference score for female and male low and high m-power scorers at 11, 13 and 15 years



was significant main effect for (D(TI) - D(NT)), $F(1,12) = 8.780$, $p = .01$. There were no significant main effects for age.

For high males, the profiles with age were parallel. Also the profiles were not flat, with a significant difference between D(NT) and D(TI), $F(1,12) = 22.399$, $p < .001$. Again, no significant main effect for age was found.

For low males, the multivariate test for parallelism was significant, $F(4,20) = 2.290$, $p < .10$. The univariate test for (D(NT) - D(DI)) was significant at $p = .05$, $F(2,12) = 3.671$. The lack of parallelism for these variables indicated the existence of an age effect. The flatness test indicated significant differences between D(TI) and D(NT), $F(1,12) = 54.672$, $p < .001$. The group mean difference test indicated the significant age effect to be between 11 years and 13 years, $T = 3.027$, $p = .01$.

When the very high males were substituted for the high males the profiles were found parallel. D(TI) was significantly different from D(NT), $F(1,12) = 17.070$, $p = .001$. And again, no age effects were found.

Additional profile analyses on the same variables were done with high and low m-power at each age level and with age at each level of m-power. The overall analysis for females from m-power of four to seven indicated parallel profiles with D(NT) significantly larger than D(TI), $F(1,24) = 16.986$, $p < .001$, and D(ID) significantly larger than D(NT), $F(1,24) = 8.793$, $p < .01$. The analysis for males indicated parallel profiles with D(NT) significantly greater than D(TI), $F(1,20) = 73.251$, $p < .001$. But, for males

D(NT) was found significantly greater than D(ID), $F(1,28) = 6.355$, $p < .05$.

Further profile analyses were done in order to compare low and high males with low and high females playing of the game. It was found that low 13-year-old females differed significantly from both low and high male 13-year-olds on the transformed dependent variable (D(NT) - D(ID)). Also, it was found that low 13-year-old males differed significantly from low and high 15-year-old females on the same variable.

Ancillary Analysis: The Task as a Variable

The basic design of this study was to consider groups with high and low m-power at three age levels. This was done for individuals and dyads. Data were collected for three additional groups in this study. The results for one of these, the very high 15-year-olds, have already been presented along with the results for the main groups. Results for the additional two groups follows:

First, there was a high 11-year-old group composed of 12 subjects that worked as individuals on a different task, but different only in the sense that the configuration of the lights was different. The lights were at the corners of a large square (the square was not drawn on the apparatus). T-tests were done to compare the performance of this group with that of the other high 11-year-olds on the standard task, the task that was used with the other groups of subjects included in the basic design. There were two significant differences between the groups of high 11-year-olds.

One, the group working with the square configuration had significantly fewer mean number of predictions, $t(20) = 2.40$, $p < .05$, (means = 92.75 and 118.1, respectively). Two, the same group had a significantly greater mean posttest score, $t(20) = -2.46$, $p < .05$, (means = 8.67 and 4.70, respectively).

The second ancillary group had a task in which the lights were arranged in a very small square configuration. This group was composed of eight high m-power 13-year-olds. Again, a series of t-tests was used to compare the results of this group with those of its counterpart. Here too, two significant differences were obtained. This group required significantly more predictions, $T = -2.23$, $p < .05$ (means = 120.0 and 92.6, respectively), and had a significantly lower posttest score, $T = 4.78$, $p < .001$ (means = 2.63 and 7.30, respectively). The strategy scores did not differ significantly between the two groups.

DISCUSSION

The hypotheses of this study were, in general, supported by the data. Eleven-year-old subjects did make greater use of the pattern strategy than the operator strategy and they did utilize the random strategy more than the older groups of subjects. Also, they had poorer game scores and posttest scores than other subjects. Within the 13-year-old and the 15-year-old age groups the game and posttest scores did increase with increasing m-power, as did the use of the operator strategy. However, there was an apparent regression between 13 years and 15 years along with an increased use of the pattern strategy. This will be discussed in a theoretical context below.

Introduction

Piaget proposed a single path for the development of operations up to the level of formal operations. He proposed this path for the development of the epistemic subject, a concept he defined in order to abstract this path from the influence of individual differences in performance. Individual differences have created some problems for the interpretation of the theory. Individual differences in distance traveled along the path, that is differences in the operational level attained by subjects of studies cannot be explained by the theory.

In contrast to the notion of the epistemic subject, Pascual-Leone has introduced the concept of the metasubject. The metasubject is a term used to refer to the whole psychological organism, in particular to refer to the active processing structures responsible

for performance. Interactions among the silent operators of the system underly this processing. One question which will be addressed here is whether these operators, in particular the operator "M", indicate different paths of development or individual differences in so-called competencies. Other questions such as that of the occurrence of decalages will be examined in the following discussion.

The INRC Line Task

It was noted above that the use of the Inhelder tasks results in a low level of incidence of formal operational performance. One purpose in devising the INRC tasks was to attempt to obviate this result by bridging the gap between the interiorised levels of organization as prescribed by Piaget's theory. As stated above, the results of this study indicated that, on the measures of performance, the 11-year-olds required more predictions and had lower posttest scores than both older groups. One strategy measure, the strategy difference showed a significant difference in competence between the 11-year-olds and the two older groups. Also, only the 11-year-olds used the pattern strategy significantly more than the operator strategy. Thus, we may conclude that there was a significant difference in performance and competence measures between the 11-year-old group and the older groups. We may interpret this to mean that formal operational behavior was exhibited by the 13- and 15-year-old groups. There were no significant sex differences indicated here. However, further examination of the results indicates that, between ages, males may undergo greater changes than

females on the game score, posttest score and the strategy difference score.

The Influence of M-power

When subjects at each age level were categorized by their levels of M-power a different pattern from that found for age alone emerged. The 13-year-old age group appeared to be at the center of a cluster of changes. The low 13-year-olds had a larger game score, a poorer performance, than the high 13-year-olds.

The strategy difference score, a competence measure, was much larger for the high 13-year-olds than the score for any other group. A sudden increase in use of the operator strategy occurred for this group with the concomitant sudden fall in game score noted above. The sudden fall in game score did not occur for a low M-power group until the 15-year age level, the low 15-year-olds having the same m-power as high 13-year-olds. The strategy difference score fell at 15 years for the high M-power group, indicating an apparent regression; however, the continued increase in posttest score as age increased indicated an apparently paradoxical increasing level of underlying competence. As noted above for gender, greater change from age level to age level seems to occur for the high than for the low M-power groups on the game score and the strategy difference score. When the results are examined by m-power and gender, the pattern of differences appears more obvious.

The Influence of M-power and Gender

When groups were classified not only by level of M-power, but by gender too, the most differentiated pattern emerged from the

data. The most obvious result was the dramatic changes in game score and strategy difference score undergone by the high males between the three age levels. An even larger fall in game score and rise in strategy difference score than noted above occurred between 11 and 13 years; and an even larger apparent regression occurred between 13 and 15 years, there being a rise in game score and a fall in strategy difference score. If we compare the scores of the very high 15-year-old males with the scores of the other 15-year-old groups, it seems clear that the apparent regression was not a regression at all, but a reorganization at a new level. The very high 15-year-olds had a substantially lower strategy difference score than the high 13-year-olds, yet they were the only group that utilized the combined operator and pattern strategies significantly more than the random strategy. Thus, they played a more strategic game and, moreover, attained an almost perfect posttest score while requiring the fewest number of predictions of any group. Thus we may argue that the operator strategy seems to be consolidated by 13 years of age for high males, indicating an efficient separation of variables strategy, and in turn, we must then argue that another level of competence must be emerging on this task at 15 years.

Further, we may argue for low females that, if the two factors of gender and M-power interact as agents of change, then their scores exhibit the smallest resultant changes and the smallest growth in competence. The high females and low males had scores indicating an intermediate influence of the two factors.

Four Paths of Development

If the interpretation of two organismic factors interacting to produce the scores attained by subjects in this study is correct, then it seems that there may be different paths of development followed by individuals according to their gender and M-power level. Consider a path followed by a hypothetical high male from 11 years to 13 years to 15 years. His decrease in game score and increase in strategy difference score with increasing age (and m-power) would be greater than those of anyone having any of the remaining three combinations of the two factors. It would seem that high females, low males and low females would not attain these extreme values on these two scores. It would then be difficult to argue, for instance, that low males follow the same trace or path of development as high males, but they move more slowly along it. For, they would then somehow have to skip the extreme values on the path while still following it. It may be simpler to argue that there are four different paths of development, one for each combination of the two factors.

Let us briefly note some features of each of the four proposed paths (the paths are portrayed in figures 10 to 13).

The primary indicator of cognitive development, we note, was the strategy difference score. A negative score indicated predominant use of the pattern strategy and a positive score was indicative of greater use of the operator strategy. The posttest score was regarded as a performance factor. The strategy use score and the game score were regarded as indicators of both competence and

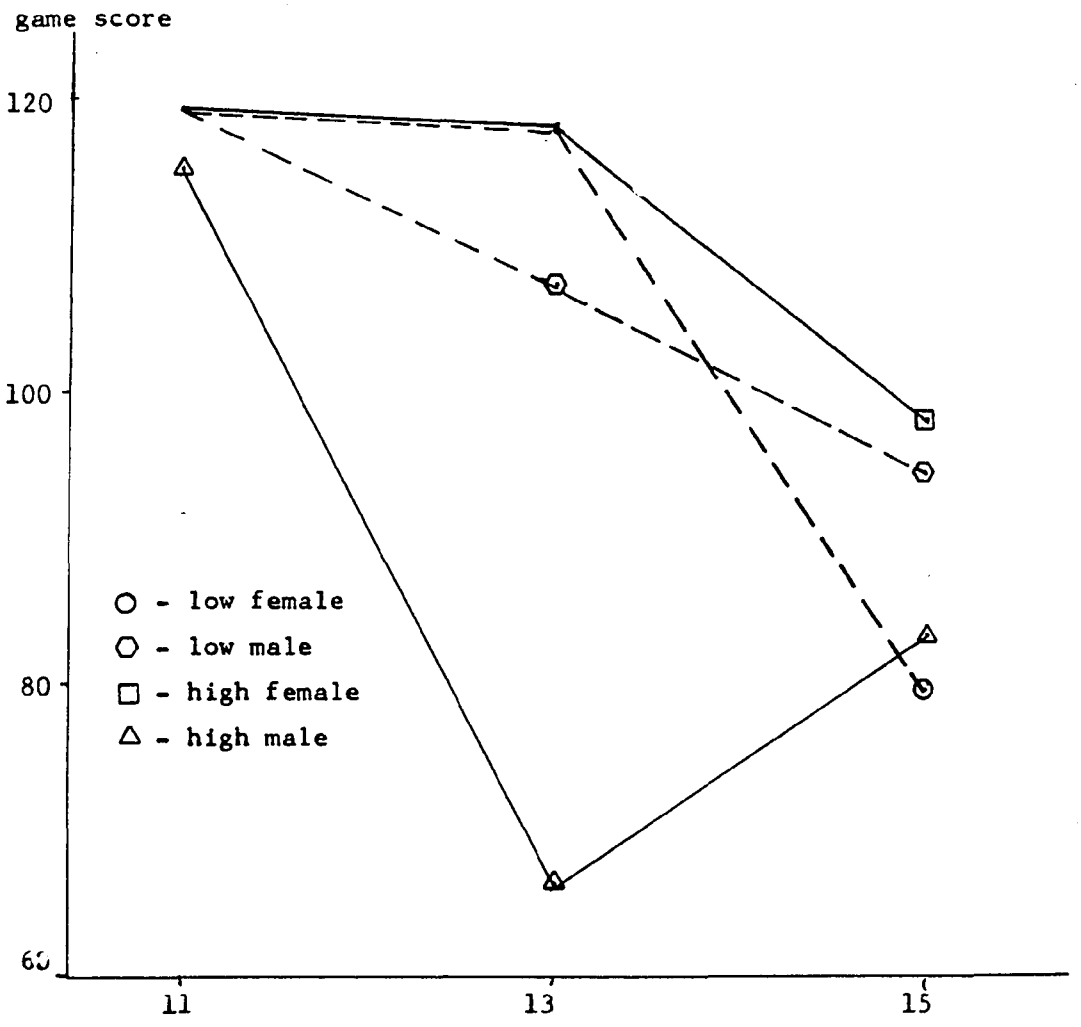


Figure 10. Four paths: the number of predictions or the game score at 11, 13 and 15 years

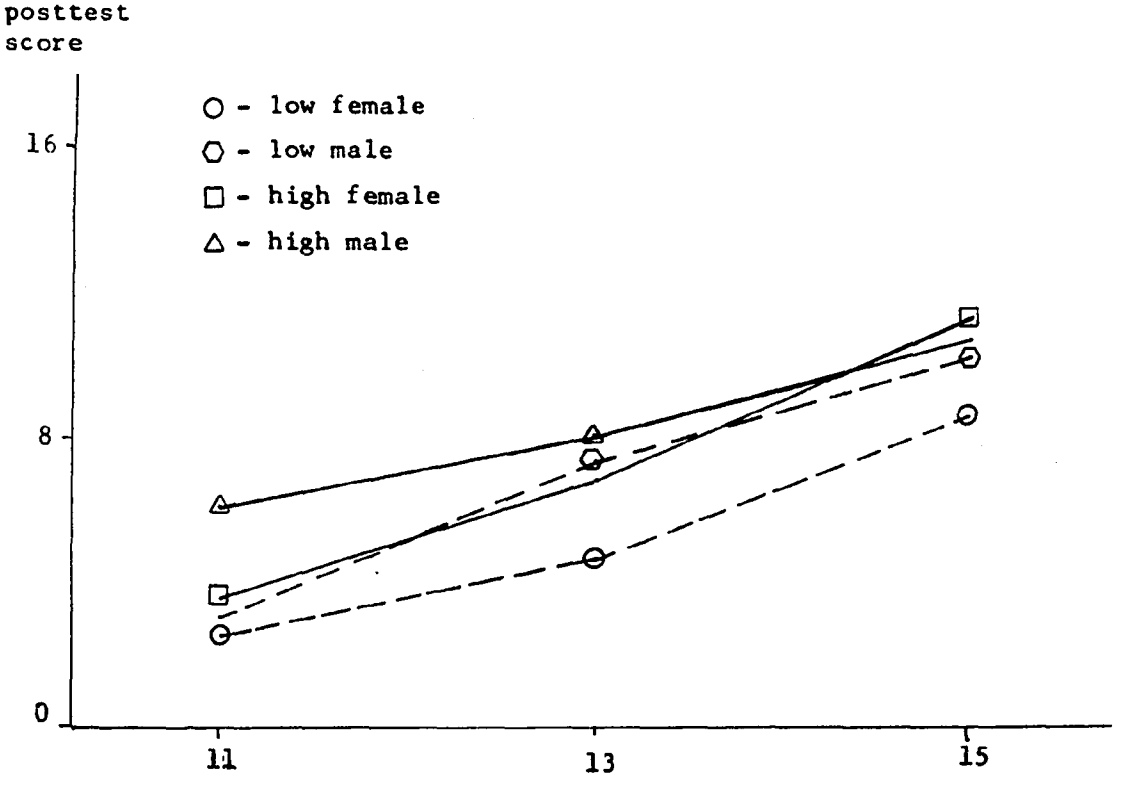


Figure 12. Four paths: the posttest score at 11, 13 and 15 years

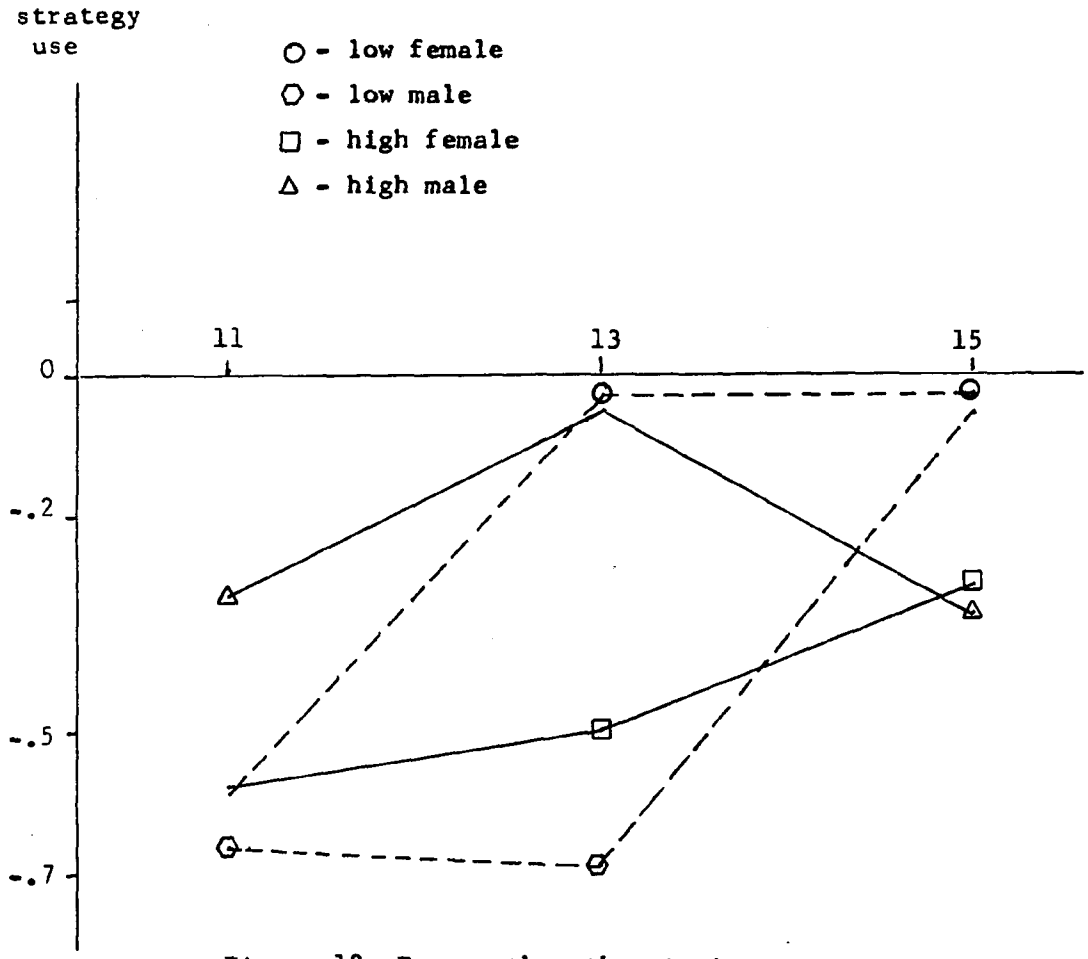


Figure 12. Four paths: the strategy use score at 11, 13 and 15 years

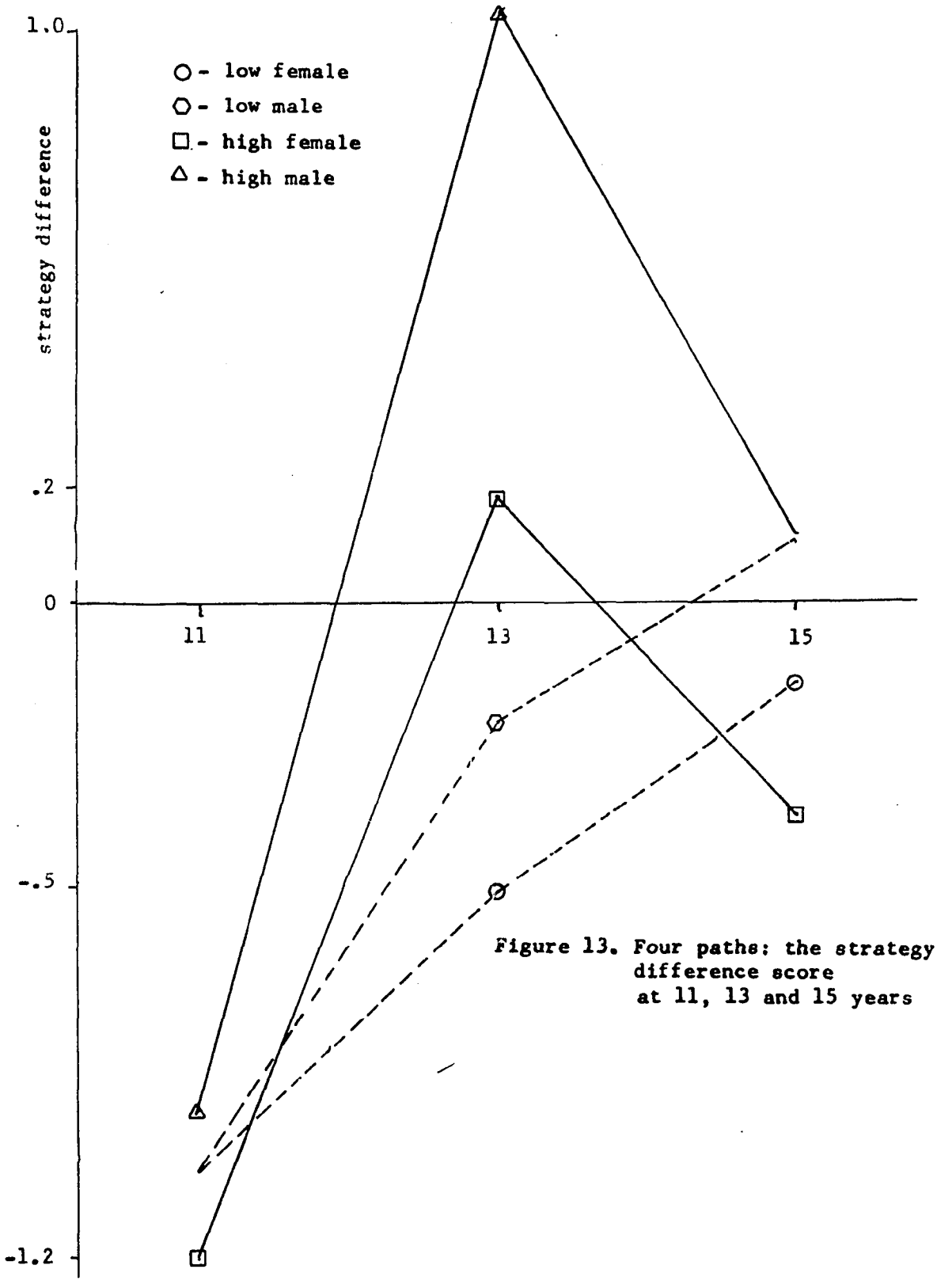


Figure 13. Four paths: the strategy difference score at 11, 13 and 15 years

performance.

Low females appeared to make continued substantial use of the pattern strategy at 13 years, the only group to do so. This delay offered an explanation of their sudden decline in use of the random strategy at 13 years: they were making increased use of a strategy the other groups used only at 11 years. By 15 years they made greater use of the operator strategy, a shift made by the others at 13. Their increase in posttest score thus lagged behind the others at 13; it showed a sudden rise at 15 years to a level more characteristic of the other groups at 13 years. Their prediction score showed a sudden fall characteristic of none of the other groups at 15 years.

The low males showed a rise in strategy difference score at 13 years which was the result of increased use of the operator strategy and decreased use of the pattern strategy. At 15 years they made greater use of the operator strategy and showed an increase in pattern strategy use, the latter result characteristic in its direction of change of the 15-year-old high groups. The low males had smaller changes than the high males. Their prediction score did not show the sudden large fall evidenced by the low females at 15 years and the high males at 13 which was accompanied by the sharp increase in operator strategy use of these groups. This may be explained by the increase in pattern strategy use which seemed to lead to an increase in prediction score at 15 years in the high groups.

The high females at 13 years showed a large increase in operator strategy use and a decrease in pattern strategy use, neither of these changes being as large as those of the high males. As in the case of low males, they did not show a decrease in prediction score. Their posttest score did increase substantially. The size of their change in strategy difference score at 13 years was notably larger than that of low males, but differences between changes in strategy difference scores of the two groups were most notable at 15 years. There, the high females showed a decrease in strategy difference scores, but it was primarily a result of increased use of the pattern strategy rather than a differential increase in use of both strategies. The low males and females showed an increased use of the operator strategy at 15 years, the males demonstrating an accompanying increase in pattern strategy. The high males and females showed a sharply increased use of the pattern strategy.

The high males at 13 years had a decline in strategy use score and a great increase in strategy difference score, both being the result of a substantial increase in operator strategy use. Along with this extreme change in use of strategy, the high males had an extraordinary drop in prediction score, both indicating a growth of competence; but, their posttest score was not different from the other 13-year-old groups, except for the low females. At 15 years, along with a large increase in the use of the pattern strategy, high males substantially increased their random strategy usage as well as their prediction score.

To conclude the description, it seemed that low M-power resulted in a lag in differentiation and reintegration of the use of both strategies. Being female resulted in delayed changes relative to a high or low male counterpart. That is, at each age performance was influenced the two factors, M-power and gender, or their degree of coordination. The path followed by the high males, however, cannot be regarded as optimal. It seemed clear that the very high males performed at another level or on another path.

Piaget's theory is a theory of competence which, by definition of the epistemic subject, is concerned only with a single path which specifies different levels of mental organization necessary for development. In this theory performance is a dependent variable which is a function of competence. Pascual-Leone (1980) regards M-power, an organismic variable, as a necessary determinant of performance. The results of this study appear to indicate that both operational level and M-power are necessary conditions of performance and together determine different paths of developmental growth. In other words, performance is a function of the organization of mental schemes and of M-power.

Although the dependence of performance upon operational or strategic competence was as hypothesized in the present study, this result contradicts the frequent findings of others using Piagetian tasks. This unexpected finding, then, is strong evidence for the operational nature of cognition, stronger than it would have been if the result were expected upon the basis of other studies. The dependence of performance (and competence to some degree) upon level

of m-power conforms with the consistent results of many prior studies (Pascual-Leone and Smith, 1969; de Ribaupierre, 1975).

Gender appeared in this study to be another organismic determinant of competence. Previous research (Neimark 1975, 1979; Pesch, 1984) has found differences between males and females on formal operational performance. But reports of such differences have not been widespread and findings in this regard have been somewhat ambiguous. Inhelder and Piaget's work (1958) does not differentiate between the sexes. Pascual-Leone, in his work on psychological differentiation (1974), provides a possible explanation of the sex differences found here. Pascual-Leone differentiates between cognitive-developmental level and cognitive style mode of psychological differentiation. He argues that cognitive styles reflect optional modes of adaptation, and that no single organismic factor accounts for a particular style dimension such as field independence. In his view, both field dependent and field independent subjects experience the conflicts inherent in certain tasks, but the outcome of the conflict is different. The outcome is determined by a learned complex organismic formula of process-structural factors. In this formula, affective, attentional, executive and field operators interact to produce performance. Pascual-Leone utilizes the constellation of organismic factors to explain both developmental level and cognitive style. Thus different rearing patterns and brain lateralization, for instance, may result in different levels of equilibrium for development and style.

In the present study, the different levels of functional

m-power may result from the low, but significant, correlation of the Figural Intersection Task score with the factor of field independence. Beyond this style factor, the different possible levels determined by gender may be, in Pascual-Leone's terms, a further result of the interaction of learning with the organismic factors he postulates. That is, in the present context, perhaps we should speak of style of differentiation, level of differentiation and path of development of differentiation. Then, for each of these, particular tasks or kinds of interactions may result in performances that are different for the two sexes.

The Distribution of Plays by Individuals

Although the distribution of plays by subjects did not appear to be related to operational level or m-power level, there were differences in the ways in which males and females distributed their presses over the three regions of play. Each play by a subject could be uniquely assigned to one of the three regions: the so-called diagonal, identity or triangle region. The regions were defined by the combination of the first two button presses. Of the 16 possible combinations, four were in the diagonal regions, six in the identity and six in the triangle. The total number of plays by a subject in the game was equal to the sum of the number of plays in each of the three regions.

Profile analyses gave the following results: non-parallel profiles occurred between males and females at 11 years and at 13 years; the profiles of 13-year-old males and of 15-year-old females were not parallel either. All the non-parallel portions were

between D(NT) and D(ID). There was thus a sex effect for the two age groups. Profiles were parallel across the three age levels for males as well as for females.

These results may be understood by examining the analyses of the profiles of low and high males and females. First, across the three age levels profiles were not parallel for low males between D(NT) and D(ID). Profiles were parallel across three age levels for low females, high males and high females. Further analyses showed that, at each age level high males and high females had parallel profiles, but low 13-year-old males had profiles that were not parallel with those of low 13-year-old females, low 15-year-old females and high 15-year-old females. Low 13-year-old females had non-parallel profiles with high 13-year-old males. Thus, it appears that both the low 13-year-old males and females play the game with a somewhat different distribution of moves than some of the other groups of different sex.

Another view of the sex differences was indicated by the profile analyses. The shape of the profiles was different for males and females: profiles for males at low and high M-power in general were flat between D(NT) and D(ID) and those for females were not. They sloped down to D(ID). When high and low male groups were combined, the hypothesis of flatness was rejected for this section of the profile and it sloped in a direction opposite to that of the combined female groups. The part of the profile between D(TI) and D(NT) sloped in the same direction for males and females. Thus the differences in play may have been in the use of the diagonal region.

Task characteristics: levels of difficulty

Comparing data for the three tasks, the INRC line, large square and small square, the idea that M-power and structural level were necessary conditions for development was further supported. Horizontal decalages may be seen in comparing these data. The tasks were structurally identical: the hardware was the same, only the position of the lights was different in each of them; the computer program that operated the lights was identical for all three; the solution to all three problems was formally identical. High 11-year-olds needed only 92.75 predictions to complete the "large square" task and were able to score 8.67 of a possible 16 on the posttest. The high 13-year-olds reached a comparable level of solution on the "line" task, a task not solved at that level by 11-year-olds. Also, the 11-year-olds used very similar strategic approaches to the two tasks, utilizing predominantly the pattern and random approaches. The strategy usage was in accord with the hypotheses of this study. The 11-year-olds functioned at the predicted level of competence and their performances varied with the complexity of the task. The performances by the 11-year-olds on the two tasks may be seen as an instance of a horizontal decalage.

Another horizontal decalage occurred when high 13-year-olds could not solve the "small square" task at a level comparable to that of their efforts on the "line" task. Although their strategy difference score regressed on the more difficult task, they still relied significantly more upon the operator strategy than 11-year-olds did on any other task. Thus, the 13-year-olds, too, performed

in accord with the hypotheses. Some high 15-year-olds (data not reported here) did solve the "small square" task and relied heavily upon the operator strategy in doing so. Within the contexts of Pascual-Leone's theory and Piaget's theory, the decalages may be explained somewhat differently.

Pascual-Leone's theory: horizontal decalages. In Pascual-Leone's theory, schemes are the basic elements (1970, Case, 1974). A scheme consists of at least two components: a releasing component, or an initial set of conditions to which it can apply; and an effecting component, or a set of conditions which it can generate. A scheme which is not activated directly by the presence of perceptual input requires for its activation the application of M-power or mental effort (see also Kahneman, 1973; Globerson, 1983). The three tasks differ in the level of M-power required to activate schemes necessary for solution.

In order to reduce the amount of information required for solution, it seemed necessary for subjects to define a model embodied by the lights and, within the model, to further define relations among the lights which the switches made manifest. The line task offered a model by its very configuration: for instance, a serial ordering, or a closed configuration with inside and outside lights. In the serial ordering one subject saw the switches as moving the lights, or relating, from odd to odd position, even to even and odd to even. On the line task, relations required mental effort to maintain their presence in the course of solution.

The large square, on the other hand, provided not only a model,

but also, a perceptually salient set of relations: for instance, horizontal, vertical and diagonal movement of the lights. Thus, the large square required less mental effort for solution; at least one less unit of information did not require mental boosting during each step of solution.

The "small square" provided the least perceptual information and, thus, required the most M-power for solution. Subjects did not see the configuration as square, nor the colored lights moving in any particular direction. Therefore, they needed to mentally define and maintain a model or configuration as well as the relations among its parts, making the "small square" a more demanding task than the other two in required mental effort.

Thus, because subjects utilized the same competencies or strategies on tasks having identical underlying structures but different m-demands, the differences in performance may be seen as a manifestation of a horizontal decalage.

It may be argued that only m-demand, and not structure, is necessary to explain the decalage of performances. For instance, Scardamalia (1977) studied horizontal decalages in a task requiring "combinatorial reasoning" with 9-, 11- and 15-year-old subjects. She was able to vary M-demand while holding the apparent structure of the task constant. In her study, even the 9-year-old subjects were able to develop an efficient combinatorial strategy when the M-demand of the task was two less than their M-reserve. After developing the strategy and practicing it on six trials of increasing complexity, they were able to apply it to a task which they had

previously been unable to solve, one in which Md was equal to Mr. Scardamalia thus, apparently, demonstrated the possible confounding of the notions of logical structure and M-power. While it is essential that any task be presented with the lowest Md consistent with its logical structure before a judgment about competence is made, at the same time a careful assessment of the structures actually required to solve the task must be made. It may be argued that Scardamalia's so-called combinatorial task required a "counting" algorithm and not a formal operational structure for its solution.

In the present study a strong cue to use an operational strategy was present on the large square task. The same button always moved the lighted bulb in the same very salient direction. Moreover, subjects at 11 years noted this fact irrespective of their level of performance. Yet they persisted in using a pattern strategy and to look for relations among the sets of four switches in a way that was not directly related to the movement of the lights. Their strategy reflected a different underlying organization from that of the 13- and 15-year-olds.

Piaget's theory: horizontal and vertical decalage. The notion of horizontal decalage is usually considered a weak point in Piaget's theory (de Ribaupierre and Pascual-Leone, 1979). Piaget's comments on this idea have not been systematic or extensive, yet there are subtle and insightful comments to be found on this subject in his work. For instance, Piaget (Piaget and Inhelder, 1974) did see in the figurative aspects of thought a cause of decalage. More

recently, the Genevans have been closely studying the subject of decalages (Bovet, et al, 1982; Montangero, 1980). In one of these studies, extending Piaget's thought, Montangero (1980) notes, "the influence of figurative factors results in changes in the degree of complexity or in the number of mental activities" (p. 265). Thus, it would seem that the decalages found above would be of current interest to the Genevans as well as to neo-Piagetians such as Pascual-Leone.

In Montangero's model, presented above (see table 1), structures do not operate on aspects of reality unless the interacting levels are appropriately organized. In this model, the pattern strategy and operator strategy may be seen as representative of different levels of organization of thought derived from subjects' experiences. The pattern strategy may be considered to be a means of establishing correspondences or a spatial scheme at the fourth level or, at best, one level above subjects' representations of interaction with the task. The operator strategy may be considered to be a means or procedure organized at level three or two of the model and to be constructed from infralogical and logical interaction with the spatio-temporal classes and relations constructed at a lower level.

In this model the horizontal decalage for the 11-year-olds may occur because of the figural complexity of the line task as compared to the less complex large square task. Eleven-year-old subjects attempting to solve the line task may have found the correspondences between the switches and lights too difficult to learn. On the

other hand, after establishing some level of correspondences on the large square, they may have gone on to more effective use of their pattern strategy in solving the problem.

In the case of the high 13-year-olds attempting to solve the small square task, again they may have been unable to organize the available information in order to have established correspondences between switches and lights. If they used only the colors, the quantity of information necessary to solve the problem was near impossible to remember. Thus subjects whose dominant strategy, in this case the operator strategy, enabled them to solve the line task efficiently could not solve the figuratively more complex small square.

Further, an unpredicted vertical decalage appeared in the data at the 15-year-old level. Although unpredicted in this study, the occurrence of this reorganization was explainable by Piaget's theory. The pattern strategy was predicted from Piaget's theory as follows: an efficient solution by a concrete operational subject (M-power of five or less) would be to treat the 16 possible combinations as portrayed in table 2 as a multiplication of classes problem with 16 resulting elements. The number of classes could be increased to three, but with all the elements in each class chunked, by organizing the combinations into the triangle, identity and diagonal regions. The equilibration process to solution could be described by Piaget's model IIA in which subjects' inferences would be based upon their actions. The equilibrium reached would be an equilibrium among the three parts and subjects' behaviors in the

face of contradictions could be gamma behavior, in the sense that their solution was complete, if their level of solution was high enough. Only the alpha level was utilized by 11-year-olds on the line task, but the beta and gamma levels were in evidence on the large square task. In the case of alpha behavior an assimilatory scheme or hypothesis was maintained in the face of contradiction. Using this behavior subjects were able to learn few of the buttons and scored low on the posttest. In the case of beta behavior, the assimilatory scheme was altered in the face of contradiction. Eleven-year-olds on the "large square" showed this degree of flexibility by not repeatedly making the same errors. Some 11-year-olds, who used the pattern strategy often, utilized gamma behavior in setting up a classification scheme of the buttons, which, in effect, did not have to be altered in the face of contradiction.

The operator strategy was characterizable in an analogous fashion for 13-year-olds by Piaget's model IIC for causal relations between objects, in this case the switches and the lights. Subjects at this age level too exhibited an equilibrium characterizable as an equilibrium among parts. However, the parts in this instance were the characterization of each of the four switches by its transformation of the state of the lights. Subjects at this level exhibited beta behavior in the face of contradictions of their hypotheses.

Subjects at the 15-year-old age level exhibited the unpredicted vertical decalage. Piaget (1974) concluded that an interaction exists between the notions of causality and the operatory structures

such that the development of one exerts an influence on the development of the other. The conception of the operator strategy in this study was based upon the joint development of operational structures and causal conceptions, and their dialectical interaction. To be consistent with this view, a strongly causal development such as occurred at 13 years should be followed by an increase in operational behavior. The path from one phase to the other at 15 years should contain a transition revealing an integration into an equilibrated operatory system of the prior attempt at 13 to know causally the elements or individual operators. The coordination of the individual elements or switches would be achieved by playing the switches in combination to study their rules of composition. That is to say a final integration might be achieved by playing the game using what would appear to be a pattern strategy. The data, in fact, especially when categorized by gender and level of M-power, revealed that at 15 years the strategy difference score was lower than it was at 13 years. The very high 15-year-olds, in this view, with their increase in overall performance, represent the culmination of this integration of the causal approach into the closure of the operatory structure. This view would explain their increased strategy difference and use scores.

At 15 years, consistent with the INRC structure, Piaget's model IIB of interaction, with the form of equilibrium being that between the parts and the whole, and gamma behavior as the response to contradiction would characterize the very high 15-year-old male performance. Other 15-year-old groups may be characterized by beta

behavior.

The four paths of development discussed above may then be viewed as possible different levels of equilibrium of the causal and operatory structures reached along four different paths. The further implication of this idea is that there may be six paths, the two additional paths being for very high males and very high females. This notion will receive some support when the concept of field-independence is discussed below.

It should be noted that all four groups attained a level of formal operational behavior in this study by the 15-year-old age level. This result is in conflict with the results of prior studies in which only a small percentage of subjects have been found to reach the formal stage. But, different tasks have yielded different results (Lunzer, 1965; Neimark, 1975) in the many studies done. In the present study, the task appears to have played a substantial role in determining the results.

The Interview: What Was Learned by Subjects

After each subject or dyad completed a task, a posttest was administered and each child was interviewed. Few probes were given and the information reported below was that spontaneously offered by subjects. The information was spotty and thus was not analyzed in detail. However, a qualitative picture emerged from the accumulated responses. Only information from interviews given after the line task will be reported.

In general, 11-year-olds offered little information regarding their strategies and did not answer more than one or two specific

items correctly. They made no statements regarding reversibility or coordination among the different presses. They gave no indications of reasoning by possibility; that is, by combinatorial patterns or control of variables. Some of the boys gave elaborate spatial schemes as the basis for their predictions while playing the game.

Reversibilities were often noted by male 13-year-olds: they spontaneously commented in response to a specific question that all buttons, when pressed twice, returned the light to its original position. A few subjects reported systems within which the button presses were coordinated in a way that expressed a rule of composition: one example was a form of clock arithmetic with numbers from zero to three arranged evenly around a clock face. The systems given were never completely correct. Specific questions regarding button presses were often answered correctly only by high males. Such questions were very frequently answered by "state matching", a procedure in which the subject traced the result of each button press in a sequence individually before giving a response. This approach reflects their strategy difference score in which each button, according to the operator strategy, was treated as an operator which caused a change in the state of the lights.

The 15-year-olds as a group gave many noteworthy responses. Reversibility of the state of the lights with regard to repeated button presses was often noted. A few subjects noted that the three switches which changed the state of the lights "all worked together": any two always gave the third as a result. Some reasoned by "possibility" in responding to questions: they eliminated the

incorrect choices, one by one, even after finding a correct response. A few explicitly stated the operator strategy: "you press the same button until you know what it does". In response to questions on using more switches in sequence than were actually used in playing the game, a number of them gave generalized responses, such as, "from any place you can always get to any other place in one press, so it doesn't make any difference how many buttons you press before...". As a group, then, 15-year-olds gave explanations that were reflections of reversibility, operational causality, combinatorial reasoning, control of variables and the closure of subsystems. These responses came predominantly from males.

But, as was stated above, many 15-year-old subjects gave no spontaneous responses which could be categorized as formal operational. One purpose of the interview was to elicit examples of the so-called reflected abstraction. Subjects did reflect upon their approaches to the task, and their expressions of their thoughts were, to a degree, reasonable examples of reflected abstractions. But these examples did not arise very often.

Another point to be made is that the reasoning contained in subjects' responses is consistent with the overall picture given by the description given above of the paths of development at the three age levels. But, it must be emphasized, that the responses given during the interviews did not distinguish the paths from each other.

One final point, the majority of very high 15-year-old males gave more thorough and consistent explanations of the task and their strategies. They stated causal and operational reasons: "the red

button goes odd to odd and even to even, the blue one goes even to odd and odd to even and the yellow one too; so the red one goes by two, and the others go by one from odd to even, but the yellow one uses one and three...no, only three if it wraps around. So we have one, two, three, for each of them...but there's still something missing...". He knew his system was incomplete and wanted to complete it. He never did, but for our purposes it is important to note that he wanted to. An attempt at systematization or closure was made by others in this group. But, these interviews were unlike the previously described interviews in that subjects' responses were probed (in the style of the Piagetian clinical interview). But, even without the probes, subjects' responses in this group were somewhat different from those given by members of other groups.

The dyadic condition

Little research has been done on the influence of social interaction on cognitive development during adolescence (Dimant, 1984). The results obtained in this area with younger children (Bearison, 1982) certainly justify extending this domain of study to older groups. Dimant (1984) found that significant pretest to posttest change on a modified Piagetian test of combinatorial reasoning occurred only when college students in dyads had a sufficiently high level of conflicts. In the present study, the process of interaction was not studied. Here, M-power was the factor proposed as having a possible influence on the results. Would subjects with a low functional M-power for their age be stimulated by the social context of the dyadic condition to perform as subjects

at a higher level of M-power? The results were interesting.

First, when grouped only by age, the pattern of results for dyads on the four dependent variables was almost the same as that for the individuals: on all scores but the strategy use score, where no significant differences occurred, 11-year-olds differed significantly from the two older groups. There was one exception: on the strategy difference score, in the case of dyads, the 11-year-olds scored significantly less than 15-year-olds, but not 13-year-olds. In addition, although it may be a result of sampling effects as much as of social interaction, on the posttest score, dyads at all three age levels scored higher than individuals. Also, the pattern of results between ages was interesting in that dyads at the 13 year and 15 year age levels had similar posttest scores, but the results for individuals at the same age levels were significantly different.

When each age group was subdivided by level of M-power, differences between individuals and dyads on the posttest score occurred at many levels. But, again, it should be noted that the source of the differences is undetermined in this study.

Thus, it appears that the posttest score, a performance score, may be significantly improved by social interaction in dyads. But, subjects' dominant strategic approaches to the tasks were not altered by working in dyads. Also, the interactions between subjects were qualitatively different at the different age levels. There was little interaction among the 11-year-olds. Among the

13-year-olds there were many comments made between subjects regarding which button to press and how a button might cause a change in the lights. Among the 15-year-olds there were more explanations and explicit considerations of alternative presses spontaneously given by subjects to each other. Given the results of this study, dyadic interaction seems a fertile ground in which to study the influence of social interaction on cognitive performance in adolescence.

Conclusion

In conclusion, some issues raised here require further comment. Two of these are the third level of M-power at each age referred to above and the relation of the paths of development to the theories utilized in this study.

First, let us consider the issue of three levels of M-power. The M-power for each child was determined by taking the mean of four scoring systems for the Figural Intersection Task. In order to have scored a given level of M-power on two of these systems, a subject must have had correct a number of items equal to 100 per cent of the total number of items on the test up to and including that level. This characteristic of these two scoring systems made it difficult to find subjects with M-power equal to seven. The search for such subjects led to the observation that there were three different levels of performance. With this knowledge in hand, the scoring system could be altered so that three levels could be discerned at the 11 year and 13 year age levels too.

Support for this suggestion may be found in studies of the relationship between M-power and field dependence-independence.

Globerson (1983) found that two measures of M-power not only gave an inverted U-shaped curve for the relation between M-power and field-independence, but that three distinct levels of M-power were obtained. Pupillary dilation was used as the criterion for assessing m-power. Pascual-Leone (1974) has suggested that Witkins's (1965) "flexible field-independent" subjects, who achieve moderate scores on perceptually misleading tasks, utilize a high degree of M-power. Extreme field independent subjects are those who utilize moderate M-power and are insensitive to misleading cues. Field dependent subjects are especially sensitive to perceptual cues and utilize low M-power. Globerson (1983) found that field-dependent subjects utilize different strategic allocations of M-power than field-independent subjects in solving tasks that are also measures of M-power. In the context of this study, these results are very suggestive regarding the levels of M-power as well as the strategies used by subjects. Although some studies have been done (Parkinson, 1977, van Esch, 1978, Pascual-Leone and deRibaupierre, 1979, Johnson, 1982), further study of the relationships between measures of M-power, field-dependence and task performance should be undertaken.

The second issue that will be commented on is that of the relationships between Pascual-Leone's and Piaget's theories as they pertain to the results of this study. In Piaget's theory, a single developmental path for the epistemic subject is implied. Along this path recursive operators that generate performance develop up to a final point of equilibrium at the level of full formal operations.

Progress along this path constitutes a growing awareness of "contradiction" (Piaget, 1980a) until a full operatory understanding of it is achieved. This full understanding may occur when the INRC structure is achieved in the context of a given task.

In the present study, a set of different paths of development have been proposed: the data do not seem to suggest different rates of development along a single path, for one would have to explain how certain points on the path are skipped as one followed the development of a group from 11 years through 15 years. It may be suggested here that not only might there be different paths of development, but that if we make that assumption, then there may be different final points of equilibrium on each path rather than one. That is, very high males may function well in the subculture of the scientist, while low males may function well within a different subculture.

Pascual-Leone (1980) argues that there are two problems with Piaget's theory: one, "the attempt to make one descriptive-structural model, equilibration, the cause...of the other..., the stages, when in fact both coexist as structural aspects of the data base". Two "is the attempt to make the stage models into causal determinants of performance". That is to say, Pascual-Leone seems to object to the INRC groups as a determinant of performance. But, just as Genevan theory sees "contradiction" as a motor of development, so too does Pascual-Leone see a "principle of internal consistency" as a necessary situation-free organismic factor in his theory. And what is a principle of internal consistency but a

generator of equilibrium from contradictions?

It must be recalled that Pascual-Leone's theory (Pascual-Leone and Goodman, 1979) is more than a theory of M-power. There are a number of situation-free organismic operators in his framework which may serve to explain the Piagetian dialectic of development between causal and operatory structures as well as different paths of development. But, in order to arrive at such an explanation, one would have to simulate within Pascual-Leone's theory the generation of the Piagetian psycho-genetic causal and operative structures and their approach to equilibrium. They could not, in the course of construction, be determinants of performance. But, after being constructed, could they not serve as generators of performance such as that found in this study?

Implications for Education

Much has been written on the subject of Piaget and education (Gallagher and Easley, 1978). Yet, some researchers, even among those studying the acquisition of mathematical structures (Jeeves and Greer, 1983), feel that Piagetian theory is not useful for educational applications. Here, we wish to address the question of the relevance of the present study for education.

Although it has been argued above that this study has demonstrated that one may predict strategies used by subjects on tasks from Piaget's theory of equilibration, that aspect of the study is not considered herein, as an issue relevant to education. This is because the tasks utilized in this study were constructed to "fit"

or validate the theory and not to be suited to educational applications. However, it may now be argued that the study has demonstrated the importance of operations to human thought and to education.

The usefulness of this demonstration is implicit in the fact that it shows learning to be dependent upon both the educational task and the learner or the interaction between the two. In particular, it is believed that the importance of the present study to education has been to demonstrate that what is consistent to a student at a formal level of thought may not appear consistent to a concrete level thinker. In other words, the principle of consistency which resolves contradictions in experience is different at different levels of thought (Collis, 1974; Halford, 1975).

Such abstract Piagetian principles have proven difficult to apply in the past. Another point of importance, but still not one easy to apply, is the application of the notions of m-power and m-demand to education. Their usefulness in educational contexts has been demonstrated before (Case, 1974, 1975). The present study, too, has shown the usefulness of Pascual-Leone's processing constructs. If only one recommendation for education had to be made upon the basis of this study, it would be that the method of task-analysis which leads to a value for the m-demand of a task be more widely taught in order that students' interactions with their task be more clearly understood by educators in operational terms.

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