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City University of New York PH.D. 1986

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ANALYSIS OF CERTAIN DESIGN PROBLEMS IN THE CONTEXT OF THE FLEXIBLE MANUFACTURING SYSTEM (FMS)

by

HELEN WANG

A dissertation submitted to the Graduate Faculty in Business in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

1986

This manuscript has been read and accepted for the Graduate Faculty in Business in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

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Abstract

ANALYSIS OF CERTAIN DESIGN PROBLEMS IN THE CONTEXT OF THE FLEXIBLE MANUFACTURING SYSTEM

by

Helen Wang

Adviser:Professor Georghios P. Sphicas

A Flexible Manufacturing System (FMS) is a highly automated system equipped with robots or numerically controlled (NC) machines which can process a variety of jobs with very little changeover time and setup cost. Automated material handling devices (transporters) allow jobs to move between any machine and are the factor which limits the number of jobs in a system. We model the FMS system as a single stage birth and death queue with finite states. The transition rate for each state depends on system throughput which is derived through the Closed Queueing Network theory. A balanced workload for all its machines is considered in this study. Four profit maximization models are proposed. The net profit, composed of profit per unit produced, inventory carrying cost, storage size cost, additional machine cost and loss production cost, is used to choose the optimal combination of number of machines, batch size (buffer size), and reorder point.

Model I is a single stage birth and death queue with a variable transition rate for finite Q states. The transition rate at each state depends on the throughput rate and the number of transporters in the system. Lower bound solutions to the optimal batch size are derived, and the sensitivity of the optimal number of transporters to the model's parameters is also studied.

Model II has a fixed transition rate for all its states. The closed form solution to the optimal batch size is derived for this model.

In Model III we assume the number of machines and number of transporters are fixed. The variable transition rate and loss of production are allowed to determine batch size (Q) and reorder point (s). The model is very complicated and it appears impossible to derive a closed form solution. A computer search is used to study the model and an approximation formula for Q and s are derived. It is verified that the approximation solution is very accurate.

In Model IV, lead time for delivery of material from a central warehouse or supplier is considered in the model to determine optimal batch sizing and reorder points. A normal approximation formula is derived for the system.

Acknowledgement

The author wishes to thank all the people who, because of their time, thoughts, encouragement, friendship and understanding enabled me to complete this research.

First, much thanks go to the Chairperson of this dissertation, Professor Georghios P. Sphicas for his guidance, professionalism, and unselfish committment of time to this research. Thanks are also due to the remaining members of the committee for their support and valuable assistance, including their friendship: Professor Lloyd Rosenberg and Professor Chao-Hsien Chu of Baruch College of the City University of New York. Special thanks go to Professor David D. Yao from Columbia University for his valuable suggestions and time spent in the guidance of the direction of this research. The author is deeply appreciative.

In addition, appreciation is expressed to Baruch College, The City University of New York and Pace University and the many faculty members for their support and encouragement. The author also would specifically like to thank Professor Howard Blum for his valuable discussions, and Professor Joseph Seif and Mr. Allen Kong for their computer assistance.

Thanks go especially to Ms. Sheryl Iwanski for various assistance in finishing this thesis.

Finally, the author expresses sincere gratitude to my husband, Chao-fu Wang, and my daughters, Sophia Wang, and Lauren Wang for their understanding, support, encouragement and love.

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LIST OF SYMBOLS

M = Number of Machines
N = Number of Transporters (Pallets)
L = Total Mean Load Imposed to the Network by a Single Job.
Cp = Profit per Unit Produced
C_s = Cost of Additional Buffer.
C_i = Inventory carrying cost per Unit per Unit of Time
C_t = Transportation Cost (Order Cost).
C_m = Cost of Additional Facilities
s = Reorder Point
Q = Batch Size
  Q_s = Low Bound Solution to Optimal Batch Size
  Q_a = Adjusted Low Bound Solution for Large Batch Size
  Q_c = Crude Model for Large Batch Size
U_n = Throughput Rate for n Jobs in the System
P_n = Probability that Total Number of jobs in System is n
M_L = Mean Production Rate During the Lead Time
q_L = Actual Production Rate During the Lead Time
$P_L(q_L)$ = Probability of Actual Production Rate During the Lead Time is q_L.
I(s,Q) = Expected Inventory
T(s,Q) = Expected Transportation Cost
b(s) = Expected loss of product for a given reorder point, s.
L'(.) = Unit Normal Loss Integral
σ = Standard Deviation of the production rate
Q_n = Batch Size at n^{th} iteration (Model IV)
s_n = Reorder Point at n^{th} iteration (Model IV)
F'(s) = Cumulative probability for throughput rate greater than s
TH(N) = Throughput Rate for N Jobs in the System
$\Delta_N G$ = Change in G(.) with respect to N
$\Delta_Q G$ = Change in G(.) with respect to Q
G(N,Q) = Net Profit Function for N Jobs and Q Batch Size

Note: Number of parts and number of jobs, and throughput rate and production rate are used interchangeablly throughout the thesis.

CHAPTER 1: INTRODUCTION

1.1 WHAT IS AN FMS

A Flexible Manufacturing System (FMS) is defined as an integrated, computer-controlled complex of numerically controlled machine tools and automated material handling devices which can simultaneously process medium-sized volumes of a variety of part types (Stecke (1983)). Because eighty percent of the world's manufactured goods are now made by batch rather than mass production methods, this results in a penalty of five to twenty times higher costs (Taylor (1983)). A Flexible Manufacturing System can improve this situation by reducing throughput times and lead times from weeks to days, which reduces the cost of labor and work-in-process. The machines in an FMS can be productive eighty to ninety percent of the time. Without an FMS, they may be productive only twenty to thirty percent of the time (Bradt and Allred(1983)).

FMS systems were first developed in the metal working industry, which encompasses approximately fifty percent of the dollar volume of all manufacturing. Seventy-five percent of the dollar volume is manufactured in batches (Cook(1975)). The growth of the industry spawned technological improvements over time which spurred research into developing efficient means of small batch production-FMS.

An FMS can be set up for both large-batch or small-batch production in which semi-automated and fully automated work stations are served by an automated materials-handling system. The system is under computer control. It controls the activities at the work stations, as well as the flow of work through the stations and storage areas. It provides the flexibility to produce mid-sized batches of different parts or products with the efficiency of automated mass production and the flexibility of a job shop. The FMS accomplishes that through the technological hardware and software developments which virtually eliminate the set-up time for each individual batch.

1.2 THE COMPONENTS OF FMS

An FMS is composed of machine tools, a materials handling system, a storage area for in-process inventory, and supervisor computer control (Browne, et. al. (1984)). The complexity of the FMS is dependent on the types of machine tools and material handling system used. The process of an FMS cell is described as follows (Sims (1983)): The material from which the component will be made is mounted on pallets or some other convenient carrier (transporter). The pallets are coded and are called up from the marshalling area on the conveyor by the central computer in accordance with the production schedule. The conveyor system may take many forms: roller or belt conveyors for light components over short track distances, wire guided trucks for heavy pieces over relatively long distances, or an automated monorail system, and in the case of assembly, a queue system on a guided track.

The conveyor takes the job to each work station in turn, as specified by the job planning program and the manufacturing instructions called up from the central computer. In the case of a machining or sheet-metal cell, the cutter location file is sent to the machine tool and post processed to suit the local machine control system.

The conveyor system is designed to deliver the component precisely oriented so that it may be placed by the machine by a robot on the pallet, correctly aligned to the machining or assembly datum. As soon as the last piece has left, the new component can be entered, correctly located and centered so that operations can start immediately without the delay associated with manual control, thereby increasing machine utilization three or four fold.

A number of ancillary devices are available, or are becoming available to give reliable minimum manning with maximum output. Data on tool life under a wide variety of conditions and materials will be collected and correlated. This information will permit the central computer to log the life of each tool and predict when it should be replaced for optimum cutting conditions. Other devices give an early warning of gross tool wear or incipient failure in order to avoid the production of out of tolerance components. A great deal of attention has also been given to machine fault diagnosis by scanning sensors which are designed to give an early warning of machine malfunction.

1.3 HOW THE FMS DEVELOPED

FMS has developed as the most recent step in the manufacturing evolutionary process. The development of FMS, described in Hitomi (1979), started with Henry Ford's conveyor assembly line in 1913. The next step was the development of the Transfer Machine by Morris Motors in 1924. Then, H.H. Aiken introduced his Automatic Sequence-Controlled Calculator. In 1946 came the Group Technology concept from S.P. Mitrofanov of the U.S.S.R.. Until the mid-fifties, metal cutting machine tools were totally manually operated. Requirements for high precision led to the development of numerically controlled (NC), general purpose machine tools. The first NC machine was developed in 1954 by J. Parsons at M.I.T.. Production had to be halted to set up the machine tool. Each tool was manually changed, and had to be adjusted for direction and angle. At the same time, the set-up procedure might include refixturing of the part and manual part movement.

After this came office automation in 1953 from J.Lyons and Co. of the U.K.; G.C. Devol's 1954 patent for a playback manipulator; and the 1955 introduction of the auto-programming language, APT, in the U.S. In 1958, Kearney & Trecker built the first NC machine centre with automatic tool interchange. Additional developments included the Unimate industrial robot by Unimation, Inc. (1959); Bendix Inc.'s adaptive-controlled milling machine (1960); Western Electric's 1961 computer controlled automation for making carbon resistors; AMF's Versatran industrial robot with a cylindrical configuration (1962); the CAD sketch pad developed by I.E. Sutherland (1963); the I.B.M. 360, Universal electronic computer (1964); the direct digital control process (D.D.C.) for process automation (c.1965); the introduction of EXAPT, an auto-programming language from W. Germany (1966); the I.B.M. Product Information and Control System (PICS) (1968); the Molins Company (U.K.) introduction of System 24 Direct Numerical Control (1968); General Motors Robots-line operation for welding car bodies (1970); and I.B.M.'s

Communications- oriented Production Information and Control System (1972).

In 1968, Sundstrand built the first computer controlled system called a directly numerically controlled system (DNC), in which the computer replaced punched tape for controlling all machining operations (Brosheer (1967), Freinberg (1968)). The first DNC integrated manufacturing system with automatic material handling and storage was built in East Germany and demonstrated at the 1971 Leipzing Spring Fair (Schmoll (1971)). At about the same time, the Ingersoll-Rand Heavy Machining Center, consisting of six machine tools interfaced with a conveyor delivery of parts, was installed by Sundstrand in Roanoke, Virginia. Such computer controlled, integrated, batch manufacturing systems have been called Computerized Manufacturing Systems (CMS) and Flexible Manufacturing Systems (FMS) (Stecke (1981)). The entire production process has been quickened through the utilization of computers to supervise work by continually receiving progress reports from the work stations, analyzing them and transmitting timely operation orders back to each of the work stations, on an on-line real time basis, thereby creating an efficient total factory management system (Groover (1980), Young (1981), Thompson (1982), among others). Software control strategies and decision algorithms are important for implementing and controlling FMS.

By 1976, four FMSs were in operation in the U.S. There were also several operating in Europe. Japan leads in the number of FMSs. By 1981 there were sixty FMSs operating in the world (Wisnosky (1981)). At their plants in Belgium and France, Catepillar has made massive investments in FMS which include linked lines, integrated manufacturing systems, and flexible transfer systems (Lincoln (1983)). Flexible manufacturers in the UK are developing a machine called the Matchmaker FMS 3, which allows companies to invest in the new technology with less costs (Barash (1978), Houston (1982)). Recently General Electric has been developing a generic "factory cell controller", and intends to provide a generic software base for the development of customized FMS control systems (Adamchick (1984)). A new automatic detection method of turning machine tool failure is also available now using acoustic emission and statistical tool control (Gee (1984), Beer (1984)).

It is estimated that about 50,000 manufacturing companies will automate part or all of their facilities by 1990, and 100,000 will make similar attempts to automate portions of their facilities by the year 2000 (Searls (1984)).

The most sophisticated system to date, reported in Thompson and Paris (1982), is at Messerschmidt, West

Germany. It has twenty-four machines, most of which have four or five axis heads. Tools are transferred from the tool crib automatically in overhead pallets. Raw materials and finished goods move to and from the machines automatically. Robotic cranes stack tools, raw materials, and finished parts. The system runs itself. It is reported (Winsnosky (1981)) that after instaling the FMS, Messerschmidt had increased machine utilization by 44 percent, reduced the work force by 30 percent, increased throughput by 25 percent, and increased its annual return by 24 percent.

Today, FMS has developed to the stage of realizing control of a group of machines by computer with the flexibility of program changes. Several examples of automated manufacturing in general are explored in the perspective of the factory of the future in Toshio (1983), White (1982), and Gunn (1982). Operations research on FMSs and related analytical models is called for in order to improve FMS performance. Various research groups have been brought together (e.g. ORSA/TIMS conference on FMS at Michigan 1984, etc.), and models and issues in system design and operations are continually being explored to increase the understanding of FMS's behavior and to enhance the future success of operating the system.

1.4 THE MOTIVATION AND OBJECTIVE OF THE THESIS

The motivation for this research comes from attending the ORSA/TIMS Conference on FMS in 1984 where research framework for FMS issues was discussed and research papers on design and lot sizing problems were called for. Because the new automated FMS system is very different from the conventional job shop manufacturing system, many well known inventory policy and lot sizing models for conventional manufacturing (e.g., Wagner (1975) and Hadley and Whitin (1963)) may not apply to the new FMS system. At the conference, research problems in FMS were identified. For example, Stecke (1984) classified FMS problems into design, planning, scheduling and control problems. In the design aspect, the determination of the appropriate number of machine tools, the capacity of the material handling system and the size of the buffer is specified.

From some experimental studies (Wang (1985), it is shown that the system throughput rate is seriously affected by a good system configuration, i.e., the combination of machines, transporters, and batch size. The modelling of an optimal configuration design becomes one of the important issues in FMS success. Suri (1984) also pointed out that important decisions in the context of FMS are (1) number of machines; (2) number of load and unload stations; (3) alternative routing; (4) number of transporters; (5)

buffer sizes; (6) system layout; (7) tool allocation; (8) types of fixtures; and (9) operating policies. In this research we are pursuing some of these issues, and focusing on the design of system configuration and lot sizing problem.

In the life-time of the FMS, an organization goes through many phases of decision making: initial design, configuration design, and on-going modification of the system. In the initial design and operation of such a system, it is important to determine the system configuration. Several questions need to be answered, such as: for a given number of machines and work load conditions, how many transporters are needed; what will be the optimal storage size (batch size); and when should a new batch be shipped to the system. This research attempts to answer some of the above questions by studying the optimal system configuration in terms of the best combination of number of machines, number of transporters, batch size and reorder point based on costs and profit trade-off analysis.

Four optimization models for different conditions are examined in this paper. Model I examines the initial design stage, where the number of machines and transporters is not fixed. The model determines the best combination of number of machines, number of transporters and batch size for a fixed work load condition and given costs and profit parameters. The throughput rate depends on the state of the system range from N to N+Q. Model II examines the optimal batch size at a design stage where number of machines and transporters is fixed, and work load conditions and costs and profit parameters are known. Reorder point is when the number of parts in storage reachs zero. This model considers that the throughput rate is a constant and the cost of lost production is very high and is not allowed. The major difference between Model I and Model II is that Model I considers a state dependent throughput rate and Model II considers a constant throughput rate for states of the system and rangeing from N to N+Q. In Model III we consider that loss of production is allowed. Batch size and reorder point are determined based on the known parameters -- number of machines, number of transporters, cost of inventory, cost of transporting a batch to the system, cost of additional storage size, and cost of lost production. Model IV considers the lead time factor (the time between placing an order and the time that the order arrives) in the model for determining the optimal batch size and reorder point.

1.5 DESCRIPTIONS OF THE MODELS

This research focuses on the design issue of determining the optimal system configuration of an FMS, namely, in number of machines, number of transporters, batch size and reorder point. Four models at different design stages and under different conditions are investigated.

The controllable and uncontrollable variables in the models are:

Controllable variables:

- Storage size (Q);
- Minimum number of jobs in the system (reorder point) (s);
- Number of machines and/or robots (M) (Model I);
- Number of transporters or pallets (N).

Uncontrollable variables are cost parameters:

- Profit per unit (Cp);
- Inventory cost per unit per unit of time (Ci);
- Buffer size cost (Cs);
- Transportation cost per batch transfered (Ct);
- In Models II, III and IV, the number of machines is fixed.

A flexible manufacturing system composed of M machine groups, N transportation vehicles, and Q storage spaces in the load and unload area as depicted in Figure 3.1, with balanced work load on all its machines is considered. For choosing the best order point and order quantity, the objective function composed of profit per unit, cost of inventory, order cost, and buffer cost is considered in Models I and II. Model I considers a state dependent throughput rate and Model II considers a constant throughput rate for all its states. Model III considers the additional cost of loss of production in choosing the order point and order quantity. Model IV is formulated with the additional consideration of lead time for transporting parts from a central warehouse to the FMS. (See Figure 6.1)

Model I: We examine a case where the number of NC machines or robots (M) is equal to the number of transportation vehicles (N) and also to the reorder point (s). The part is mounted on the transportation vehicle, and must stay there until it is finished for all the processes and is unmounted at the unloading area. Then the vehicle is free to pick up another part to take to the machine area. The throughput rate is state dependent; it depends on the number of jobs in the system. In this model we consider that the reorder point is only equal to the number of vehicles. In other words, as soon as the storage area is empty, a new batch is sent over with no lead time. The two decision variables in this model are storage size (Q) and number of machines in a cell (M). The objective of the model is to help answer questions in the initial design stage in order to choose the best configuration of number of machines and number of transporters for a given constant work load, profit, and cost parameters. How each of these parameters (work load, profit per unit, cost of inventory, cost of buffer, and order cost) effects the decision for selecting the number of machines and number of transporters is also studied.

Model II: We examine a case where s=N: a system with an existing set of NC machines or robots and with transportation vehicles installed. The objective of this model is to find the optimal batch size in a given system. The problem can be viewed as a single stage birth and death queue with constant work load, and the throughput rate of the system as a Poisson distribution with rate of Un. (The throughput rate Un is determined by analyzing the subsystem FMS with the network of queues with M machines, N pallets, and a balanced workload in the system.)

Model III: We consider a situation where the possibility of lost production is taken into account. The system is formulated as a single stage birth and death queue with finite Q states and with a state dependent throughput rate (allowing for loss of production to occur). The objective of this model is to determine the reorder point and the storage size based on a given number of machines and transporters and a given set of cost and profit parameters.

Model IV: In this model we extend Model III to consider lead time for replenishment from warehouse and suppliers. The model is formulated similarly to stochastic (s,S) types of inventory policy. In our model, demand arrives when a unit is finished and the transporter arrives for a new part. When the number of units in the system drops to a certain level (s), the reorder point, an order is placed and with some lead time, a new batch will arrive. Determination of the reorder point and batch size depends on cost parameters and lead time demand assumed.

The system has a total of Q states, where the transition rate at each state depends on the throughput rate Un calculated from the closed queueing network subsystem. The system throughput rate derived from the closed queueing network with balanced work load results is described in Sections 3.1 and 3.2. The derivation of the steady state probability of the queueing system is shown in Section 3.3.

1.6 ORGANIZATION OF THE THESIS

This thesis is organized into seven chapters. Chapter 2 provides a literature review of analytical models on FMS.

How to model the system using a queueing network, the assumptions of the model, and the relationship of system configuration and lot sizing problem are discussed. In Chapter 3, the closed queuing network, the product form solution results and the birth-death queue for modelling the lot sizing problem in the context of FMS are described. Chapter 4 describes the formulation of Model I, giving the lower bound approximation and the results of the study. Chapter 5 describes the formulation of Models II and III, the exact solution to Model II, the methodology to solve Model III, the research design of Model III, and a test of the performance of Model III. Chapter 6 describes the formulation of Model IV and the solution by normal approximation. Examples of the normal approximation are given. Chapter 7 provides a summary of the models and future research.

CHAPTER 2: LITERATURE REVIEW OF ANALYTICAL MODELS

2.1 CURRENT STATUS OF FMS

Currently FMS studies are looking at the integration of (1) numerically controlled machines or robots which could be programmable to perform different tasks, (2) a materials handling system which is automated and flexible enough to permit jobs to move between any pair of machines so that any job routing can be followed, (3) computerized job routing algorithms to direct the routing of jobs through the system and to track the status of all jobs in progress so it is known where each job is to go next, (4) a set of linked computers which can pass instructions for the processing of each operation to each station and ensure that the right tools are available for the job, (5) computers to provide essential monitoring of the correct performance of operations, and (6) analytical models to solve planning, design, implementation, and control problems in order to increase system performance.

Some of the systematic approaches for selecting the appropriate types of robots, NC machines and range of parts to produce can be found in Chen (1985), Stauffer (1985), Hollingum (1983), Skoog (1983), Redford and Killeen (1983), Berger (1982), Ottinger (1981), Klahorst (1981), Fitch and Bryce (1981), and Houtzeel (1981). Group technology and strategic implications for operations to increase flexibility can be found in Groover and Hughes (1981), Desai (1981), Wang (1981), Thompson and Paris (1982), Browne et. al. (1984) among others. Various aspects of analytical modelling in system configuration, system design, system performance evaluation, operational policy, and routing algorithms are explored in the operations research literature. A general reviews in robotics, conveyor theory, transfer line, storage alternatives, palletizing, automated storage and retrieval systems can be found in Matson and White (1982). In the following section the analytical decision models are reviewed.

2.2 ANALYTICAL MODELS IN FMS

There are several problems associated with managing an FMS: planning, design, performance evaluations, scheduling and control. Various research groups (e.g. Harvard Univ., Toronto Univ., Michigan Univ., Purdue Univ., MIT, Draper Labs, among others) have made several contributions in solving these problems. A review on the developments and the analytical models formulated by these various research groups appears in Buzacott and Yao (1985a). Broadly speaking, the models used in FMS can be classified as Queueing Network Models, Perturbation

Analysis, and Simulation.

Queueing Network Models

Queueing Network Models have been used extensively for modelling FMS system operation. A state-of-the-art review on Queueing Network Models of FMS can be found in Buzacott and Yao (1985b).

The closed queueing network with exponential servers was first studied by Gordon and Newell (1967a, 1967b). Buzen (1973) developed a computational algorithm for calculating the normalizing constant. Several different notions of closed queueing networks have been studied by many researchers. Posner and Bernholtz (1968) considered the closed finite queueing network with time lags. Chandy, Muntz, Baskett, and Palacios (1975) extended the study to consider several job-streams, and non-exponential holding times. Chandy, Howard, and Towsley (1977) investigated the relationship of product form and local balance in queueing networks. Schweitzer (1977) compared maximum throughput in finite and infinite capacity open queuing networks with product-form solutions. Solberg (1977) proposed the closed network of queues (CNQ) model to study FMS throughput with n pallets, and infinite jobs arrival in first come first served queues. Buzacott and Shanthikumar (1980) modelled the FMS as a open network of queues. Then the open network was solved by presenting it as a single server queue with a state dependent service rate equal to TH(N), which is the throughput for a closed network of queue with N jobs. They used the model to investigate various dispatching rules. Yao and Buzacott (1985g) developed a technique to transform an FMS network with general servers into an equivalent network with exponential servers. Koenigsberg and Mamer (1982) considered a flexible assembly system which consists of a work transporter to feed work stations and a conveyor to store work in process. The system was modelled as a set of cyclic queues and analyzed through decomposition. Seidmann and Schweitzer (1982) considered the real-time on-line control of an FMS cell feeding a set of parallel downstream stations. Zahorjan et. al.(1982) studied the performance of the balanced job bound. Stecke and Solberg (1982) examined the optimality of unbalancing both workload and machine group size in closed queueing networks of multi-server queues. Vinod and Solberg (1984) studied FMS performce with unreliable machines. Using the closed queueing network model, system throughput under different conditions were compared.

Olsder and Suri (1982) considered the time optimal control and developed feedback policies on part routing. Buzacott (1982) examined hierarchical control and optimal operating rules at each level. Gershwin (1981) considered the CNQ model to study the optimal routing and loading of an FMS. Cavaille and Dubois (1982) used mean value analysis (MVA) to waive the exponential assumption of the CNQ model. Reiser (1979) used mean residual service time to approximate non-exponential stations.

There have been different types of structures proposed to analyze the FMS. Hildebrant (1980) proposed an hierarchical control algorithm for overall production planning and control problems of FMS. Kimemia (1982) considered the optimal control of an FMS work center. The structure of controls is separated into three levels: (1) flow control, (2) routing control, and (3) sequence control. Suri and Whitney (1984) provided a framework and structure of components for decision support system parallels to the organization activities. Three levels of decisions are identified, and detailed issues are discussed. Stecke and Solberg (1981) and Stecke (1983) classified FMS problems into five sub-problems. Some of the sub-problems are analyzed through the queueing network.

Several researchers Hughes and Moe (1973), Buzen (1975), Giammo (1976), Lipsky and Church (1977), and Rose (1978) have validated the use of queueing network models through empirical studies, and have verified that the models reproduce observed quantities with great accuracy.

Suri (1983) has also shown the robustness of the queueing network formulas for predicting the performance of queueing networks.

Perturbation Analysis

Another approach used in analyzing FMS is perturbation analysis. The approach allows one to evaluate the sensitivity of a system, with several parameters contributing to the system performance. The approach views the system as a stochastic dynamic system evolving in time, and observes a sample realization of its trajectory. In general, in order to optimize certain system parameters, the central problem is to derive the corresponding gradients, and to study the sensitivity of the system performance to these parameters. This approach was first developed by Ho, Eyler, and Chien (1979), and entitled SPEEDS (sample path perturbation extrapolation of discrete event systems). The basic idea is to observe a given sample path (the nominal path) obtained from a detailed simulation and to consider if the occurrence of specific events in the nominal path were perturbed, and the consequence of this perturbed path to the system performance measure of interest.

Many research papers have been published on this subject, e.g., Suri (1983), Ho and Cao (1983), and Ho and

Cassandras (1983). The approach is applicable to evaluating performance in a production system with respect to many parameters. Ho, Eyler and Chen (1979) used this approach to evaluate throughput with respect to the various buffer sizes in a production line. Ho, Suri, Cao, Diehl, Dille and Zazanis (1984) studied the sensitivity of network performance with respect to various decision parameters with simulations and perturbation analysis. Tay and Suri (1984) studied the error bounds for performance prediction on queueing networks. Zazanis and Suri (1985) developed a perturbation analysis algorithm to estimate second derivatives of the mean system time with respect to parameters of the interarrival and service distribution from the observation of a single sample path. This approach is very useful when a system is complex and analytical solutions are not available.

Simulation

A pure simulation model mimics the detailed operation of the system through a computer-based discrete event simulation, which effectively steps through each event that would occur in the FMS. Simulation models can be made to be very accurate to the actual system without many assumptions made. At present, simulation is a widely used computer-based performance evaluation tool for FMS practitioners. Many computer packages are available. For example, GPSS, GPSS/H, and SIMSCRIPT are powerful simulation languages in modelling a typical FMS, and MAP/1, SLAM, SIMAN are specially tailored to the FMS to ease the model building and data input efforts. However, the major drawbacks of simulation study are three-fold: (1) it is more expensive to use simulation studies, (2) if the situation changes, a new simulation study is needed, and (3) it is difficult to perform sensitivity analyses of system parameters. Therefore, to overcome some of these difficulties a Hybrid Simulation/Analytic Model (Shanthikumar and Sargent (1983)) is recommended.

2.3 MODELLING FMS CONFIGURATION WITH QUEUEING NETWORK THEORY

The FMS is a network of automated NC machines or robots linked by a common computer controlled material handling device to transport work pieces from one work station to another. The special feature in FMS which enables the system to be evaluated under Closed Queueing Network theory is that the number of jobs in the system is constrained by the available number of transporters (automated material handling devices) circulating within the system. Whenever a job finishes all the processing requirements and leaves the system, another job is immediately picked up by the transporter. One underlying assumption is that jobs to be processed are always available to be released into the system. Insertion of the job is triggered by a job leaving the system. Therefore, at any given point in time the total number of jobs in the system is fixed as if there were no arrivals and departures.

The Closed Queueing Network with product form solution has been used extensively in studying computer system performance. Chandy (1972) showed that networks with certain queueing disciplines (Processor Sharing and Last Come First Served Preemptive Resume) have product form solution and satisfy local balance. Baskett (1972) showed that the steady state behavior of such models depends only on the mean CPU service time and not on higher order moments. Buzen (1973) developed computational procedures for determining the normalizing constant, steady state probabilities, throughputs and queue length distributions for single class locally balanced networks. Subsequently, this has been extended by many researchers, e.g. Reiser and Kobayashi (1975); Chandy, Muntz, Baskett, and Palacios (1975); Chandy, Howard, and Towsley (1977); Schweitzer (1976, 1977); and Zahorjan, Sevcik, and Eager (1982), who studied the effects of computer system configurations and system throughput, and established the balanced bound of a queueing network. A large class of service distributions was found to also have product form solutions, for which throughput rates can be derived with simple formulation. Buzacott, Yao, and Shanthikumar extended the research of analytical queueing network models to study FMS and the dynamic job shop. Queueing Network models for machining stations, routing, optimal design, and operational control of FMS were studied (Yao (1983a,b), Yao(1985), Yao and Buzacott (1985a,b,c,d,e,f). Open Queueing Network models, dispatch policies, and time spent in a Dynamic job shop are analyzed in Shanthikumar (1979, 1982, 1984), and Shanthikumar and Buzacott (1980,1981,1984), Shanthikumar and Stecke (1984). A detailed review of the models can be found in Buzacott and Yao (1985a,b).

Suri and Diehl (1983) studied limited buffer spaces under the closed queueing networks with the possibility of blocking. A variable buffer size model under a closed queueing network condition with product-form solution is considered. The decomposability theory of Courtois (1977) is used to decompose the queueing system into a set of states and to compute the production rate of a finite buffer system for three and five stages with two and three buffer locations. The results of numerical examples show the buffer size and number of transporters have a significant impact on the average production rate and mean response time. The result of this research motivated further research for determining optimal configuration for buffer size and reorder point.

Vinod and Solberg (1985) considered the optimal configuration of an FMS and formulated a cost minimization model. The objective is to minimize a set of costs given a number of machines (servers) and transporters (jobs) in the system, subject to achieving a desired system throughput. A lower and upper bound is specified for the desired throughput rate. A Binary Search procedure and implicit enumeration algorithm are used to find the optimal solution for both reliable and unreliable closed network models of an FMS.

Yao (1985) examined the optimal storage model for an FMS. He examined an FMS equipped with N pallets and a storage area which accommodates parts waiting to be inserted into the system. Parts to be processed are delivered in batches from a central warehouse to the storage area and then inserted into the FMS whenever a pallet is available. A continuous review (s,Q) type of model is developed to decide the optimal order point s, and the order quantity Q. The model assumes the following are given: pallets (N), inventory holding cost (when parts are waiting in the storage area) and loss of production cost (when the number of parts in the shop drops below N). The objective of the model is to minimize the expected total cost of ordering, holding and loss of production. The production rate depends on the number of parts within the shop. The length of time between the departure of parts is assumed to be independent of each other. The paper considers constant lead time and exponentially distributed inter-departure time with production rate Un which is derived from the closed queueing network model. The exact solution for the model is derived. The paper was extended to consider stochastic lead time and general distribution of inter-departure time, and the approximation solution procedure is derived. The convexity of the objective function and uniqueness of the optimal solution are also discussed. The solution procedure requires solving a collection of s and finding the minimum as the optimal solution.

Our research is aimed at deriving a good simple closed form approximation for the optimal solution of s and Q, and at enhancing the understanding of system configuration with respect to the various parameters.

CHAPTER 3: MODELLING THE LOT SIZING PROBLEM

An FMS cell consists of machining stations which are controlled by computer with the flexibility of producing different part types without much change-over time. Transporters (pallets) are automated and are allowed to move jobs between machines as directed by the job routing program. The feature of flexibility in machine capability and routing possibility allows one to consider that the network is separable at each node and service time at each station is exponential. The system throughput rate at steady state can be computed based on Closed Queueing Network with product form solution. Figure 3.1 depicts the FMS cell flow diagram.

3.1 PRODUCT FORM AND THROUGHPUT RATE DETERMINATION

Product form was introduced by Jackson (1963), who considered only exponential service distributions. The closed queueing system with exponential servers was examined by Gordon and Newell (1969), who showed that closed queueing networks with queue length dependent service rates and exponential service times also had product form solution. The number of customers, N, in a closed queueing system is fixed since customers pass repeatedly through the M stages with neither entrances nor exits permitted.

A flexible manufacturing system can be modelled as a closed network of queues as depicted in Figure 3.1 where the average processing time of machine i is ti. Since the system is closed (has a fixed number of transporters) there is always a fixed number of parts in the system, N. When a part is finished, it is taken to a load and unload station where another part immediately enters the system. The work load can be measured by the visit frequency multiplied by the average processing time. For a particular system which satisfies the product form solution, the throughput rate can be defined as a function of G(M,N), which in turn is a funtion of the assigned workload. G(M,N) is the normalizing constant with M machines and N jobs in the system. The expected production rbte, Pr, is equal to (see Stecke (1981), Gelenbe et. al. (1980)):

$$Pr = \frac{G(M,N-1)}{G(M, N)} \qquad (3.1)$$

Buzacott and Yao (1985a) provide a state-of-the art review in subsequent developments in studying FMSs using analytical queueing network models. The paper includes Jackson's networks, reversible networks and approximate models of non-product-form network.

3.2 BALANCED WORKLOAD CONDITION

In this study, many proposed system configurations must be evaluated with respect to their throughput rates. It is infeasible to compute the exact solution of the general class of queueing network models; instead, balanced workload condition is assumed to compute system throughputs. There are several reasons for us to choose the product form solution with balanced workload to the machines: (1) the closed form solution can be derived, (2) the expression of the closed form solution is simple and can be used for modelling optimal design problems, and (3) the result is roubust to many types of queue networks (Suri (1983b)).

Asymptotic Bound Analysis (ABA) for throughput and response time was studied by Denning and Buzen (1978), Sevcik et. al (1980), Reiser and Lavenberg (1980), Buzacott and Shanthikumar (1980), and Zahorjan, et.al (1982), and was proven to be a valuable and inexpensive technique in actual modelling of computer systems and FMSs. Under a balanced workload condition the throughput rate TH(N) can be derived through Eq.(3.1), and is shown below in Eqs. (3.2) and (3.3):

$$TH(N) = \frac{\sum_{n1+n2+\cdots=N-1} \left[\prod_{i=1}^{M} (L/M)^{n_i}\right]}{\sum_{n1+n2+\cdots=N} \left[\prod_{i=1}^{M} (L/M)^{n_i}\right]} \quad (3.2)$$

$$= \frac{\binom{N-1+M-1}{N-1} (L/M)^{N-1}}{\binom{N+M-1}{N} (L/M)^{N}}$$

therefore,

$$TH(N) = \frac{N}{(M+N-1)} * \frac{M}{L} \qquad (3.3)$$

Where: N is the number of jobs (parts) in the system.

M is the number of machine centers with single server.

L is the total mean load imposed on a machine i by a single job. (i=1,2,.....M)

M/L is the mean load imposed on each machine center by a single job.

For example: N=3, M=2, L=.1

$$TH(3) = \frac{(.1/2)^1 (.1/2)^1 + (.1/2)^2 (.1/2)^0 + (.1/2)^0 (.1/2)^2}{(.1/2)^0 (.1/2)^3 + (.1/2)^2 (.1/2)^1 + (.1/2)^1 (.1/2)^2 + (.1/2)^3 (.1/2)^0}$$

$$= \frac{3(.1/2)^2}{4(.1/2)^3} = \frac{N}{M+N-1} * \frac{M}{L}$$

Shanthikumar (1982) proved that balancing the workloads on all machines will maximize the throughput rate and stochastically minimize the number of jobs in the system under the restriction that the mean load imposed on each machine by a single job is a constant. Other proofs have been established in Stecke (1981) and Yao (1985c) concerning maximum expected production by balancing the workload on all machines. Stecke and Solberg (1985) showed that balancing the workload per machine need not maximize the expected production of FMSs having a different number of machines in different machine centers. Shanthikumar and Stecke (1984) also showed that maintaining a balanced workload on each machine over time stochastically minimizes the work-in-process inventory under three different dispatching policies with finite and infinite common input buffer storage, and an ample buffer at each machine.

Equation (3.3) provides an inexpensive means of calculating the throughput of a balanced network. Therefore, we used this result for our modelling of system design and the lot sizing problem.

3.3 MODELLING LOT SIZE WITH BIRTH AND DEATH QUEUE

To extend the model to consider the lot sizing problem we model the system as a single stage birth and death queue with state dependent arrival rate (exponentially distributed), and bulk arrival of size Q (order quantity). The assumptions are that the FMS cell has equal loading to all its machines, and overall loading is a constant, and the network is separable and has an independent exponential inter-departure time. The arrival and departure at each state depends on the state of the system (number of jobs in the FMS cell). The FMS cell here refers to the Closed Queueing Network system with M machines and N transporters in the cell, and includes finite storage space. The system is depicted in Figure 3.2a.

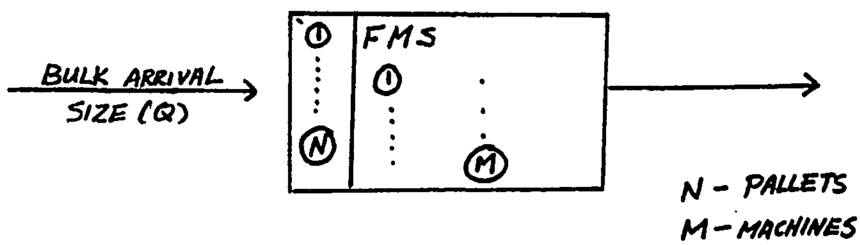


Figure 3.2a FMS System Diagram

The system has a total of Q states and the transition rate diagram at each state is shown in Figure 3.2b.

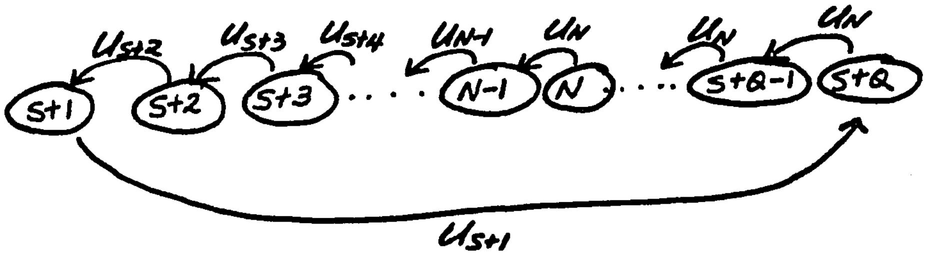


Figure 3.2b: Transition Rate Diagram

Where:Q is the total number of states in the system (shown inside the circles)

Un is the transition rate at a given state. Un (throughput rate with n jobs in the cell) is derived from the Closed Queueing Network with the assumption of a balanced work load and constant loading rate, and has independent exponential inter-departure time.

The steady state probability of the single stage Birth and Death Queue system (Pn) can be calculated through the following balanced equations shown below.

$$
\begin{aligned}
U_{s+1} P_{s+1} &= U_{s+Q} P_{s+Q} && \text{for state s or s+Q} \\
U_{s+2} P_{s+2} &= U_{s+1} P_{s+1} && \text{for state s+1} \\
U_{s+3} P_{s+3} &= U_{s+2} P_{s+2} && \text{for state s+2} \\
&\vdots && \vdots
\end{aligned}
\qquad (3.4)
$$

$$U_{s+Q} P_{s+Q} = U_{s+Q-1} P_{s+Q-1} \quad \text{for state } s+Q-1$$

$$\sum_{n=s+1}^{s+Q} P_n = 1 \tag{3.5}$$

Solving this system of equations recursively yields the following: (For detailed proof, see Appendix A)

$$P_n = \frac{1/U_n}{1 \Big/ \sum_{k=s+1}^{s+Q} U_k} \quad \text{for } n=s+1.....s+Q \tag{3.6}$$

where P_n = the steady state probability of n jobs in the system.

U_n = the transition rate (product rate) when there are n jobs in the system.

The objective of Models II, III and IV is to maximize the net profit function (which includes profit per unit produced, inventory cost, order cost, buffer cost, and loss of production cost) with respect to batch size (Q) and reorder point (s) for different configurations and system design. Details of each model and results are discussed in Chapters 4, 5 and 6.

CHAPTER 4: MODEL I

4.1 INTRODUCTION

Model I studies the FMS cell in which the system requires that the number of transporters equals the number of machines, and that there are always parts to process. In this model we do not allow for loss of production and no loss of production cost is assumed--as soon as the last part is processed, there will be a new batch arriving instantaneously or within a negligible amount of time. The objective of this model is to study optimal lot sizing in an FMS cell where system configuration and cost parameters are known.

4.2 NOTATIONS AND ASSUMPTIONS

The f llowing notations and assumptions are used in the model formulation:

(1) M = number of machines in the system.

(2) N = number of transporters - we assume the system uses automated transporters to move material from the loading area to work stations where machines can process the job.

(3) s = reorder point, where a new batch of size Q will be sent to the loading area.

(4) In this model we assume the system requires M=N=s, the

number of machines equals the number of vehicles and no loss of production is allowed.

(5) L = Total mean load imposed on the network by a single job.

(6) The total number of jobs in the system at maximum is equal to s+Q and at minimum is equal to s.

(7) The system has a total of Q states. (n=s+1....s+Q)

(8) Pn is the steady state probability of the cell when there are n jobs in the system.

(9) The cost coefficients are:

Cp = profit per unit produced;

Cs = fixed cost for the local buffer at the cell per unit of time;

Ct = transportation cost per trip from the central storage in the cell;

Cm = cost of additional facilities per unit of time.

The system operates as a birth and death queue type of model with a total of Q states from state N+1 to state N+Q. The system changes its state when a unit leaves the system. This process is continued until the system reaches state N, at which time a new batch of jobs will be sent over and the system starts from N+Q again, as depicted in Fig. 3.2a,b. The steady state probability of the system at state n can be derived by Eq. (3.6) in Chapter 3, Sec. 3.3. The transition rate at each state is based on the balanced queueing network where product form solution or reversibility theorem applies (Gelenbe and Mitrani (1980)). The throughput rate is as follows (For detailed derivation see Chapter 3, Sec.3.2):

$$U_K = \frac{kM}{L(k+M-1)}$$

where K equals the number of jobs (transporters) in the system

L equals the work load condition

M equals number of machines.

The decision variables of this model are lot size (Q), the number of machines (M) and transportation vehicles (N), and reorder point (s) with the assumption that the system requires M=N=s.

The objective of this model is to determine batch size and number of machines which will maximize the profit.

4.3 FORMULATION

Given: M=N=s

$$\text{Max. } G(N,Q) = \sum_{n=N+1}^{N+Q} U_n P_n (Cp - Ct/Q) - Ca(Q) - Cm(N) \qquad (4.1)$$

Where:

$$U_{N+1} < U_{N+2} < \ldots\ldots < U_{N+Q} \quad \text{for } N>0 \qquad (4.2)$$

$$L<1$$

and

$$U_n = n.M/L(n+M-1) \quad \text{for } n=N+1, N+2, \ldots. N+Q \qquad (4.3)$$

$$P_n = (U_{N+1}/U_n)\, P_{N+1} \text{ for } n=N+2, N=3, \ldots N+Q \qquad (4.4)$$

$$P_{N+1} = [1+(U_{N+1}/U_{N+2})+\ldots.(U_{N+1}/U_{N+Q})]^{-1} \quad (4.5)$$

At Equilibrium, Rate in = Rate out

$$U_{N+1}P_{N+1} = U_{N+2}P_{N+2} = \ldots. = U_{N+Q}P_{N+Q} \qquad (4.6)$$

Substituting Equations (4.3), (4.4) and (4.5) into (4.1) we get:

$$G(N,Q) = Q(U_{N+1}P_{N+1})[CP-(Ct/Q)] - Ca(Q) - Cm(N) \qquad (4.7)$$

$$G(N+1,Q) = Q(U_{N+2}P_{N+2})[CP-(Ct/Q)] - Ca(Q) - Cm(N+1) \qquad (4.8)$$

$$\Delta_N G = G(N+1,Q) - G(N,Q)$$

To find N optimal, we let $\Delta_N G=0$ or when

$$G(N+1,Q)-G(N,Q)<0 \text{ and } G(N-1,Q)<0$$

and

$$\Delta_N G=Q[Cp-(Ct/Q)]\{[(N+2)N/L(2N+1)]*\sum_{j=N+2}^{N+Q+1}(U_{N+1}/U_j)^{-1}$$

$$-[((N+1)N/L(2N))*\sum_{j=N+1}^{N+Q}(U_{N+1}/U_j)]^{-1}\}-Cm \qquad (4.9)$$

4.4 DETERMINATION OF OPTIMAL NUMBER OF MACHINES

To derive the exact formulation for N* is very complicated, if not impossible. Therefore, we use a computer search approach to find the optimal equipment for the model, and to analyze the sensitivity of the optimal equipment to the given parameters. This approach is useful for complex formulations in which a closed form solution is difficult to find. The approach can also be used when the formulation or the FMS environment changes to avoid going through the entire derivation. A computer program is written to analyze the behavior of the NG function, and how each parameter--Q, L, Cs, Cm, Cp, Ct--impacts the optimal M&N decision.

Computer Search Program for Optimal Machine Configuration

The program is written in the interactive mode in TURBO PASCAL on an IBM-PC, and is very efficient and easy to use. The program will ask for the given parameters -- L, N, M, Q, Cp, Ct, Cs, Ci. Once the parameters are entered, the program will automatically compute the G(N,Q), Eq. (4.9), for the given batch size Q. The program will search for a sequence of M and N with an increment of 1 each time, for a total of 18 sets. The 18 results will appear on the screen and the program asks "Do you want to continue?". If the optimal N is not found in those 18 sets, one answers "yes", and the program will continue to search for the next 18 sets. An optimal configuration can be found in one or two steps.

Sensitivity Analysis of ΔNG

The sample printout of the interactive computer search results are shown in Appendix B, and Table 4.1 shows the summary of computer search results of NG on several configuration designs. It is shown that NG is very sensitive to Cp, Cs, Q, and L, and completely insensitive to Cm. The following summarizes how each parameter affects the optimal N, while holding the other parameters constant:

(1) Optimal N increases as Cp increases, and as Cp decreases optimal N also decreases. For example, keeping

other factors constant and varying Cp from 200 to 20, optimal N changes from 37 to 6 (see cases 1 and 3). There are exceptions: when Cs is very large, then varying Cp does not affect N and M (see cases 5 and 6).

(2) Optimal N is also significantly affected by the Cs parameter. As Cs increases, optimal N decreases and as Cs decreases, optimal N increases. For example, as Cs increases from 100 to 1000, N decreases from 37 to 2. However, when Cs increases to some limit, NG becomes a strictly decreasing function (see cases 1 and 6).

(3) As Ct increases, the optimal N increases moderately and vise versa. For example, as Ct increases from 100 to 1000, N increases from 37 to 46 (see cases 9 and 11, and cases 1 and 10).

(4) As Cm increases or decreases, G(N,Q) changes, but the optimal N is not changed (see cases 1 and 2, cases 3 and 4, cases 10 and 12, and cases 8 and 9).

(5) As batch size (Q) increases, N also increases, and as Q decreases, N also decreases (see cases 1,14,15 16 17 and 20).

(6) As L increases, N decreases, and vise versa (see cases 15 and 18).

4.5 DETERMINATION OF OPTIMAL BATCH SIZE

To find Q*, we let:

$$\Delta_Q G = G(Q+1,N) - G(Q,N) \quad \text{where}$$

$$G(Q+1,N) = [(Q+1)*U_{N+1}(\sum_{J=1}^{Q+1} U_{N+1}/U_{N+J})^{-1}](CP-(Ct/Q+1))$$

$$-Cs(Q+1) - Cm*N \qquad (4.10)$$

$$G(Q,N) = [Q\ U_{N+1}(\sum_{J=1}^{Q} U_{N+1}/U_{N+J})^{-1}(Cp-(Ct/Q)) - Cs(Q)$$

$$-Cm*N \qquad (4.11)$$

$$QG = U_{N+1}\ ([(Q+1)\sum_{j=1}^{Q+1}(U_{N+1}/U_{N+J})^{-1}(Cp-(Ct/Q+1)] -$$

$$[Q(\sum_{J=1}^{Q} U_{N+1}/U_{N+J})^{-1}(Cp-(Ct/Q)]) - Cs \qquad (4.12)$$

Conditions for Q* are when:

$$\Delta_Q G = G(Q-1,N) - G(Q,N) < 0 \quad \text{and,}$$

$$\Delta_Q G = G(Q+1,N) - G(Q,N) < 0$$

Deriving the exact solution for Q* is very difficult because it includes a finite sum series; therefore, we use approximation to calculate the lower bound solution.

Lower Bound Approximation

Since

$$U_{N+1}<U_{N+2}<U_{N+3}.......<U_{N+Q+1}$$

Therefore:

$$(U_{N+1}/U_{N+2}) > (U_{N+1}/U_{N+3}) >...>(U_{N+1}/U_{N+Q+1})$$

And

$$U_{N+1}/U_{N+Q+1}=[(N+1)M/L(N+M)]*[L(M+N+Q)/M(N+Q+1)]$$
$$=[2N^2+2N+Q(N+1)]/[2N^2+2N+2NQ] \Rightarrow 1$$

If we let:

$$U_{N+1}/U_{N+j}=1 \text{ for } j=1,2,....Q+1$$

then

$$[\sum_{j=1}^{Q+1}(U_{N+1}/U_{N+j})]^{-1}>=1/Q+1 \qquad (4.13)$$

Substituting Eq.(4.13) to Eq. (4.12), we get:

$$\Delta_Q G=U_{N+1}[(Ct/Q+1)-(Ct/Q)]-Cs=0 \qquad (4.14)$$

This substitution provides the lower bound for optimal batch size Q. Because QG is a decreasing function with respect to Q, and we obtain a QG with a lower value than the actual value of QG, the resulting batch size Qs becomes a lower bound solution of the optimal batch size Q*.

Solving Eq.(4.14), the result of the lower bound approximation is:

$$Q(Q+1) = Ct(U_{n+1})/Cs = (Ct\ N+1)/(2L*\ Cs)$$

The optimal Q is approximated as follows:

$$Q_s = \{[Ct*(N+1)]/(2L*Cs)\}^{1/2} \qquad (4.15)$$

Test for Lower Bound Solution Approximation

In order to test the lower bound approximation for Q optimal, a PASCAL computer search program is written to search for the optimal solution and to compare it to the lower bound solution derived from the formula. Table 4.2 compares a system which produces expensive items at low volumes. Table 4.3 compares a system which produces inexpensive items with a short processing time. Three levels of transportation costs -- 100 vs. 500 vs. 1000--and three levels of equipment configurations -- 5 vs. 10 vs. 20 --are chosen. Storage cost and profit per unit are kept at one level. Profit per unit is set at

\$200 and \$20 per unit; work load is set at 0.1, 0.25, 0.5 and 0.01, 0.02, 0.05 for Table 4.2 and Table 4.3 respectively. A total of 54 combinations are evaluated. To derive computed Q, Eq. (4.15) was used for Tables 4.2 and 4.3. The results are also plotted in graph form as shown in Figs. 4.1, 4.2,4.3 and 4.4. From the test data it is shown that Eq. (4.15) gives a tighter approximation when batch size Q is small than for a large batch size situation. This is because we assumed UN+1/UN+j is approaching 1 for j=1, 2, ...Q, but, in effect, UN+1/UN+j is less than one and when j gets large the ratio gets smaller. Therefore when Q gets large the error term also gets large, and the difference between optimal and approximated becomes large. In order to improve the approximation for a large batch size situation, we develop another formula.

Large Batch Size Lower Bound Approximation

To account for large batch size, we let:

$U_{N+1}/U_{N+j}=1-k$, where k is very small $0<k<1$

and

$$\sum_{j=1}^{Q+1}(U_{N+1}/U_{N+j}) = (Q+1)*(1-k) \qquad (4.16A)$$

Then substituting Eq. (4.16A) into Eq. (4.12) and expanding and collecting terms, we get the following formula: (A detailed proof is shown in Appendix A.)

$$Q(Q+1) \cong \{[Ct*(N-1)]/[Cs*2L*(1-k)]\} \tag{4.16}$$

where k is approximately between $0<k<0.2$. K depends on several parameters: work load, number of machines, and batch size.

Adjusted Lower Bound Solution

In order to understand more about what is the major factor among batch size, work load, and number of machines, which causes the difference between the optimal solution and the lower bound approximation, we conduct a statistical analysis using 54 known optimal batch sizes and computed batch sizes derived by lower bound solution. (The output of the statistical analysis is shown in Appendix B.) We found batch size (Q) to be the major factor -- it explains 94.3 percent of the variation. Machine configuration and work load factors only explain an additional 0.4 percent, which is relatively insignificant. It is further proof that the lower bound solution did account for most of the variation in machine and work load factors. The main deviation between Q and Q* is due to the batch size factor. The solution for large batch size is as follows:

$$Q_a = .9690\ (Q_s)^{1.073353} \qquad (4.17)$$

where Qs is the Lower Bound Approximation and

Qa is the Adjusted Lower Bound Approximation for a large batch size when Q>10.

Therefore, the procedure for finding an approximation of the optimal batch size is to first use Eq.(4.15) to find Qs. If Qs is larger than 10, then apply Eq.(4.17) to find the Adjusted Lower Bound solution. The answer 10 was heuristically found to be satisfactory. This procedure improves the approximation for optimal Q size significantly as shown in Table 4.4A.

The other alternative adjusting formula for a large batch size situation is:

$$Qc = Qs + (Qs)^{.5} \qquad (4.18)$$

where Qc is the crude approximation for the adjusted lower bound approximation.

Results from Eqs. (4.15),(4.17) and (4.18) are compared to the optimal Q found from the search model and are shown in Table 4.4A.

The lower bound approximation formulas, Eqs. (4.15), (4.16), (4.17) and (4.18), are very simple and easy to calculate. They are useful for generating guidelines for determining system configuration and lot size in the design stage, and for providing sensitivity analyses of various configurations in practice. They are also useful for serving as a starting point for simulation modelling and optimal configuration search.

CHAPTER 5: MODEL II AND MODEL III

5.1 INTRODUCTION

In this chapter we will discuss the formulation of Model II and Model III together. This is because for Model II we have derived the exact solution for optimal Q which serves as a crude model to analyze Model III, a more complex situation where an analytical solution cannot be derived. For Model III, we used numerical analysis to derive the Power Approximation formulas for optimal s and Q. Methodology and research design for Power Approximation are described in the chapter. The performance of the Power Approximation formula is evaluated in a wide variety of parameter ranges, and the results are discussed in this chapter. From testing the model under different settings, we have found that the model is very promising and has a high degree of accuracy.

5.2 MODEL II

Model II studies an FMS cell with an existing number of machines and transporters, and where the cost of carrying inventory per unit per unit of time is known. The model assumes no lost production is allowed and that the system has a reorder point s=N. Whenever the last part is taken out of storage, a new batch will arrive instantaneously. Therefore, the number of jobs in processing is maintained at N, which is equal to the number of transporters available in the system at all times. In this system, the throughput rate is maintained at UN for all possible states. The decision variable in this model is the optimal batch size, while holding the number of machines and number of transporters fixed. The model has a constant average transition rate, and the closed form can be derived.

Notations and Assumptions

(1) M = the number of machines is fixed and is an uncontrollable variable;
(2) N = the number of transporters is fixed and is also an uncontrollable variable;
(3) Production capacity is fixed for all n=s,s+1....s+Q;
(4) Ci = inventory carrying cost per unit of time.

Model II Formulation

The profit maximizing function is as follows:

$$\text{Max. } G(s,Q)=U_N(Cp-Ct/Q)-Ci(s+Q/2)-Cs(s+Q-N) \qquad (5.1)$$

where $U_n = U_N$ for all states, N, N+1, N+2,....N+Q
At steady state, rate in= rate out

and Pn=1/Q for all n= N, N+1, ... N+Q.

Eq. (5.1) becomes:

$$Max.G(s,Q)=U_N[Cp-(Ct/Q)]-Ci[s+(Q/2)]-Cs(s+Q-N)$$

$$=U_NCp+NCs-(Ci+Cs)s-[(Ci/2)+Cs]Q-(Ct*U_N)/Q \quad (5.2)$$

The exact solution for the optimal Q derived through differentiation of Eq. (5.2),(A detailed proof is shown in Appendix A.)

$\partial G(s,Q)/\partial Q=0$ We get:

$$(Ci/2)+Cs-(Ct*U_N)/Q^2=0$$

$$Q*=((Ct*U_N)/(.5Ci+Cs))^{.5} \quad (5.3)$$

5.2 MODEL III

Model III studies an FMS system where the number of machines and number of vehicles is known and fixed. The system allows under capacity, i.e., the number of jobs in the system can be less than the number of vehicles. The transition rates for states N to s+Q are equal to UN, and the transition rates for states s to N-1 depend on the number of jobs in the system. A heuristic approximation approach is developed to find the near optimal solution for different configurations. We find the result of Model II, Eq. (5.3), is very useful for solving the more complicated situation that is investigated in Model III -- where average transition rate depends on the state of the system, Un (n=s+1, s+2,.....N).

Formulations

The profit function is formulated as follows:

$$MaxG(s,Q)=\sum_{n=s+1}^{s+Q} (Cp*U_nP_n-Ci*n*P_n-(Ct/Q)*U_nP_n$$

$$-Cs(s+Q-N) \qquad (5.4)$$

where

$$Ps+1=[1+(Us+1/Us+2)+(Us+1/Us+3)+.....+(Us+1/Us+Q)]^{-1}$$

$$=(1/Us+1)*[(s+Q-N)(1/U_N)+\sum_{n=s+1}^{N}(1/U_n]^{-1}$$

$$P_n=(Us+1/Un)(Ps+1) \quad \text{for } n=s+2,s+3,.........,s+Q-1$$

$$Un=\frac{M*n}{L(M+n-1)} \quad \text{for } n= s+1,\ s+2,......,N-1$$

$$Un=\frac{M*N}{L(M+N-1)} \quad \text{for } n= N,N+1,........,N+Q$$

therefore,

$$(U_{s+1})(P_{s+1})=(U_{s+1})\{(1/U_{s+1})*[(s+Q-N)(1/U_N) + \sum_{n=s+1}^{N}(1/U_n)]\}^{-1} \qquad (5.5)$$

Substituting UN into Equation (5.5) above, we get:

$$(U_{s+1})(P_{s+1})$$

$$=(M/L)\{(s+Q-N)[1+((M-1)/N)]+(N-s)+(M-1)\sum_{n=s+1}^{N}(1/n)\}^{-1}$$

At steady state, rate in = rate out, and

$(U_n)(P_n)=(U_N)(P_N)$ for all n= s+1, s+2,......,s+Q.

$$\sum_{n=s+1}^{s+Q} U_n*P_n=(M/L)*(Q)*[(s+Q-N)(1+((M-1)/N)]$$

$$+(N-s)+(M-1)\sum_{n=s+1}^{N}(1/n)\}^{-1} \qquad (5.6)$$

$$\sum_{n=s+1}^{s+Q} n*P_n =$$

$$[(s+1)P_{s+1}]+[(s+2)P_{s+2}]+........+[(s+Q)P_{s+Q}]$$

$$= \sum_{n=s+1}^{N}(n/U_n)[(U_{s+1})(P_{s+1})] + \sum_{n=N+1}^{s+Q}(N/U_N)[(U_{s+1})*(P_{s+1})] \qquad (5.7)$$

It can be verified that Equation (5.7) equals:

$$\sum_{n=s+1}^{s+Q} n*Pn=$$

$$\frac{(N-s)[(M-1)+(N-s-1)/2]+(s+Q-N)(s+Q+N-1)[(.5+(M-1)/(Q*N))]}{(N-s)+(s+Q-N)[1+(m-1/N)]+(M-1)N\sum_{n=s+1}(1/n)} \quad (5.8)$$

Substituting Equations (5.6) and (5.8) into (5.4), we get:

$$G(s,Q)=(((M/L)*Q(Cp-(Ct/Q)-Ci((N-s)(M-1+(N+s-1)/2)+(s+Q-N)*(s+Q+N-1)(.5+(m-1)/(2N)))/((N-s)+(s+Q-N)((1+((M-1)/N))+(M-1)*N\sum_{n=s+1} 1/n)-Cs(s+Q-N) \quad (5.9)$$

Methodology

This model is much more complicated. Finding s*,Q* through analytical methods is very cumbersome. Our approach uses numerical analysis to fit a power series to G(.) by using known optimal policies as data. The optimal policies are generated through a computer search algorithm.

The method of the computer search is based on the direct search using a grid method. The search algorithm is written in TURBO PASCAL on an IBM-PC. In order to ensure that the point that generated from the search is a global maximum, we search all the positive integers within a wide range. From our search results, we found the objective function is concave and has a unique optimal for s and Q in every case. Our conjecture is that the profit function is concave and has a unique global optimal.

The procedure for the search is not complicated. The program provides freedom to select value of the parameters and the intervals that one wishes to search. The program is interactive and each configuration search can be found within a short time. The search procedure is summarized as follows:

(1) Select a set of parameter values; (2) Identify the search interval top to bottom for s, and right to left for Q. In order to come up with an integer coordinate point, the interval must be divisible by 5. For example, if one sets the interval for Q at the right extreme of 90 and the left extreme of 40, and the interval for s at 35 for the upper bound and 10 for the lower bound, these intervals are divisible by 5. The objective functions for the following discrete points will be computed.

(10,40),(10,50),(10,60),(10,70),(10,80),(10,90)

(15,40),(15,50),(15,60),(15,70),(15,80),(15,90)

(20,40),(20,50),(20,60),(20,70),(20,80),(20,90)

(25,40),(25,50),(25,60),(25,70),(25,80),(25,90)

(30,40),(30,50),(30,60),(30,70),(30,80),(30,90)

(35,40),(35,50),(35,60),(35,70),(35,80),(35,90)

At total of 36 points are found each time. The program is efficient especially for this FMS design problem which generally has a reasonable range for Q and s.

Here, we propose a systematic algorithm for reaching the optimal solution which proceeds as follows: (a) Base points are picked in a large interval and the objective functions are evaluated at each point (s,Q). For selecting a starting point for Q, the crude model (Model II) is used. A total of thirty-six grid points are calculated for each run. (b) If a potential optimal is observed, local search will be made by contracting the interval of Y in each direction of s and Q, and the objective function of each pair is evaluated to see if the potential optimal is within that range. If a potential optimal is within that range, we contract the interval for s and Q again until all integer values of s and Q around this potential optimal are evaluated. If the objective function value for each pair in the grid is increasing, we will search higher values of s and Q by selecting a higher value interval for the local search. If the values of the objective function in the grid are decreasing, the potential optimal is not in the range. A new interval for s and Q is then selected with smaller values of s and Q and the search continues until a specific pair which yields the highest value for the objective function is found. For a small configuration of M and N, the optimal s and Q can be found in one or two steps. For a large configuration of M and N, several interactive search steps may be required. Since the program is interactive there is always room for intelligent human input in the search process.

Research Design

Four types of configurations for machines and transportation vehicles are used: 10 machines and 15 vehicles; 10 machines and 30 vehicles; 20 machines and 25 vehicles; and 20 machines and 40 vehicles. Three levels of cost parameters on transportation cost, inventory cost, and buffer cost are used to generate a grid of 108 items for analysis. Table 5.1 lists the parameter settings.

The 108 items of optimal policy, Table 5.2, are derived by a computer direct grid search program written in TURBO PASCAL-PC (Some sample outputs from the direct search program are shown in Appendix B). In our data set the optimal policies have values ranging from 2 to 38 for optimal s, and values ranging from 9 to 104 for optimal Q. The resulting 108 sets of values for s and Q are the data utilized for the regression fit.

Regression Formulation

We first construct a regression model for Q*. We generalize the expression to the multiplicative form

$$Qp = C\,(2Ct/Ci+2Cs)^{i}\,(MN/L(N+M-1))^{k} \qquad (5.10)$$

where C,i,and k are constants to be fitted. The variables (2Ct/Ci+2Cs) and (MN/L(N+M-1)) appear in the crude model which is derived from Model II. We construct a regression model for s*. We generalize the expression to the multiplicative form

$$Sp = (L(N+M-1))^{j}\,((Cp/Ci+Cs)Q)^{z}\,CC \qquad (5.11)$$

where CC, j, s, and z are constants to be fitted. The variables (N-M), and L(N+M-1) are developed by theoretical analysis and intuition, because when we have more transportation vehicles and machines and the production rate is higher, we will need more safety stock and the reorder point will be higher. The result of the regression analysis shows the variable (L(N+M-1)) alone explains 83% of the variation, and the combination of L(N+M-1) and Cp/(Ci+Cs)Q explains 94.7% of the total variation. On the other hand, for the Qp approximation formula, the cost factor is more important. Variable

(2Ct/Ci+2Cs) explains 89.4% of the total variations; the combination of this variable with NM/L(N+M-1) explains 96.3% of the variation.

We form a linear model by taking the logarithm of Equations (5.10) and (5.11), and using least-squares regression to fit the model, we obtain the coefficients of Sp and Qp as follows: (For SPSS output, see Appendix A.)

$$Qp = .9073289 \left(\frac{2Ct}{Ci+2Cs}\right)^{.4507591} \left(\frac{MN}{L(M+N-1)}\right)^{.5255085} \quad (5.12)$$

$$sp = .519086(N+M-1)^{1.2132}(Cp/(Ci+Cs)Q)^{.19768} \quad (5.13)$$

The result of the regression analysis (see Appendix B) shows for variable s, the system configuration of work load, number of machines and number of transporters, (N+M-1), explains 83% of the total variation for the data set. Cost parameters (Cp/(Ci+Cs)Q) explain another 11.7% of the variation, and the combination of (N+M-1) and Cp/[(Ci+Cs)Q] explains 94.7% of the total variation. On the other hand, for the Qp approximation formula, the cost factor is more important. Cost parameters [2Ct/(Ci+2Cs)] explain 89.4% of the total variation, average production rate (N*M)/[L(N+M-1)] explains 96.3% of the total variation.

The regression fit Equations (5.12) and (5.13) have coefficients of determination, R2, equal to 0.95 for Sp and 0.963 for Qp. Multiple R is equal to 0.973 for Sp and 0.981 for Qp.

Test of the Approximation

1. With original 108 items data set:

We proceed with a thorough analysis of the performance of the power approximation with the set of 108 items that are used to derive the power approximation. Let Pp and p* be the expected total profit per period for an item when controlled using the power approximation and the optimal policy, respectively. Our performance measure for a single item is

$$p=100\%(P^*-Pp)/P^*$$

namely, the percentage by which the optimal profit exceeds the power approximation profit. Table 5.3 shows the summary of error frequencies for the original 108 data set. Detailed comparisons of optimal Q to computed Q for each configuration are shown in Figures 5.1, 5.2, 5.3 and

5.4, and comparisons of optimal s vs. computed s for the design data are shown in Figures 5.5, 5.6, 5.7 and 5.8.

Table 5.3 reveals that the power approximation yields an average of 0.5% below optimal total profit per period, and 95% yield less than or equal to 3% compared to optimal search.

2. With random combinations of parameters:

We test the model against some random combinations of parameters for M ranging from 3 to 600 and N from 7 to 1000. The results are quite encouraging. The percentage difference between optimal and approximate is very small: for large M and N the differences are less than .5% and for small M and N the differences are higher, reaching 2%. The results are shown in Table 5.4

The summary statistics for the random data set is shown in Table 5.4A. It reveals that the power approximation yields 48.7% below 3% difference in profit per period, and 85.4% is below 7% difference in profit compared to the optimal search results.

3. With profit parameter

In our design data set, we set one level for profit and work load parameters. Therefore, the regression fit does not take into account the variation of these parameters. It was our curiosity to test the performance of our model against these factors to see the robustness of the model. Table 5.5 reveals that, for the configurations of 5 machines and 9 transporters, and varying Cp in a range from 60 to 200, the results are still within the error bound of 5%.

4. With work load factor

Table 5.6 shows the model's performance under a work load factor varying from 0.01 to 0.05 and for configurations of five and ten machines and nine and sixteen transporters. The result shows the error bound is within 10%. It is more sensitive than the Cp factor.

As shown in Tables 5.3 and 5.4, the model provides a good approximation to a wide range of parameters in machine configuration, inventory cost, transportation cost and buffer cost. The model also appears to be quite robust to varying profit parameters. In some ranges, the varying work load factor seems to produce good results, but for large variations, the error bound increases.

This model is easy to compute and provides a good source of information to decision makers in the configuration design of an FMS. For a more complex design situation the computer search algorithm will still be applicable for searching optimal configurations efficiently without requiring major changes.

CHAPTER 6: MODEL IV

6.1 INTRODUCTION

In Model IV we take into account lead time for replenishing an order. The objective of this model is to investigate an optimal design of two decision variables -- batch size and reorder point -- in the context of an FMS system, with the assumption that the system has a given number of machines, transporters, and cost parameters -- inventory cost, transportation cost, lost production cost, and buffer cost. Determination of optimal batch size and reorder point in this model has some similarity to the traditional (Q,r) type of inventory policy with stochastic demand and constant lead time. This type of inventory policy has been well studied by many researchers, e.g., Hadley and Whitin (1963), Wagner (1975) and Ehrhardt (1979). In our model the demand is generated through the completion of one part in the system, and an order arrival is when a new batch of parts is sent over to the cell. The model is developed under the context of FMS where throughput rate is state-dependent and lead time is constant, and the results of the model are compared to Ehrhardt's model (1984), a traditional inventory model.

6.2 SYSTEM DIAGRAM AND MODEL ASSUMPTIONS

The FMS system under study for this model is depicted in Figure 6.1.

Notations and Model Assumptions

(1) M = number of machines (given);

(2) N = Number of transporters (given);

(3) L = Work Load (total processing time/number of machines)

(4) lt = Lead time for transporting material from warehouse to FMS cell.

(5) U_n = throughput rate- Un=UN for n=N;
Un=Uk for k=s to N-1;

(6) M_L = Mean production rate during lead time;
(The mean production rate can be derived by using Yao's (1985f) result. But in this paper we assume that it is a given parameter.)

(7) q_L = Actual production rate during lead time;

(8) $P_L(q_L)$ = Probability of actual production rate during the lead time is qL;

(9) I(s,Q) = Expected inventory;

(10)R(s,Q) = Expected revenue;

(11)T(s,Q) = Expected transportation cost;

(12)G(s,Q) = Expected profit;

(13)B(s,Q) = Buffer cost;

(14)Cost coefficients are the same as in Model III.

6.3 MODEL FORMULATION

The objective function is:

$$Max.G(s,Q)=R(s,Q)-I(s,Q)-T(s,Q)-B(s,Q) \qquad (6.1)$$

where

$$I(s,Q)=Q/2-M_L+s+(M_L/2Q)\sum_{q_L>s}(q_L-s)P_L(q_L)$$

$$R(s,Q)=U_NC_p-C_p\sum_{q_L>s}(q_L-s)P_L(q_L)$$

$$T(s,Q)=C_t/Q(U_N-\sum_{q_L>s}(q_L-s)P_L(q_L)$$

$$B(s,Q)=Cs(s+Q-N)$$

Substituting I(s,Q),R(s,Q),T(s,Q) and B(s,Q) to Eq.(6.1) we get:

$$G(s,Q)=((U_NC_p)-C_pb(s))-c_i((q/2)\ -M_L+s$$
$$+(M_L/2Q)b(s))-(C_t/Q)(U_N-b(s))-C_s(s+Q-N) \quad (6.2)$$

where $b(s)=\sum_{qL>s}(qL-s)\ PLqL$

To Maximize G(s,Q), we solve:

$$\partial G(s,Q)/\partial Q=0 \qquad \partial G(s,Q)/\partial s=0$$

$$Q^{*}=([(2C_tU_N+M_L-2C_t)b(s)]/(C_i+2C_s)).5 \quad (6.3)$$

$\partial G(s,Q)/\partial s=0$;

$$\partial b(s)/\partial s=-C_i+C_s/Cp-(C_t/Q)+(C_i M_L/2Q) \quad (6.4)$$

$$b(s)=\int_s^{\infty}(x-s)f(x)\ d(x) = -F'(s)$$

Using normal approximation (Veinott and Wagner (1965)), for Un>5, Normal Distribution can be used as an approximation for Poisson Distribution. b(s) can be found in normal loss integral L'(.).

$$b(s)=\sigma L'((s-U/\sigma)) \quad (6.5)$$

The iterative procedure to find the optimal pair (s*,Q*) is:

(1)Assume b(s)=0, compute Q with Eq.(6.3), find Q1;
(2)Use Q1 with Eq.(6.4) to find s1;
(3)Use s1 with Eq.(6.5) find Q2. Continue procedures (2) and (3) until convergence occurs.

6.4 NUMERICAL EXAMPLES OF MODEL IV-NORMAL APPROXIMATION

The following examples show the computation of the model:

Example 1:

M=3,N=7, Ct=800, Ci=100, Cs=54, Cp=200, L=.01, =.05

U_N = MN/L(M+N-1)= 233

Substituting the above paramaters to Eq.(6.3) we get:

Q_1= 42.33

Then, we substitute Q1 to Eq. (6.4),

F'(s)= .79

s=12-.81(12)=2

Using normal loss integral table we found:

b(s)= 11

substitute b(s)=11 to Eq.(6.3) to get:

Q_2=42

Then, we substitute Q2 to Eq. (6.4),

F'(s)=.79

s=2

The solution converges and $(Q^*, s^*) = (42,2)$

We calculate six examples, the results of which are shown in Table 6.1. Comparing Example 1 to Example 2, we find that batch size (Q) is sensitive to order cost (Ct). When the order cost increases the order quantity also increases. On the other hand, the reorder point (s) is sensitive to the inventory carrying cost (Ci) and buffer size cost (Cs). For an FMS system with high inventory cost and buffer size cost increases, the reorder point should be small, and for a system with relatively low cost of inventory, the reorder point policy should be high. Model IV provides the analysis for inventory policy in an FMS environment which incorporate the additional consideration of system configuration of machines, transporters, and loading factor compared to traditional inventory policy. In the following section we list several well known inventory policies to compare how inventory policy changes under the modern manufacturing technology.

6.5 COMPARISON OF MODEL IV AND TRADITIONAL INVENTORY MODELS

Several well known traditional inventory policies are chosen to compare to our Model IV which is derived under the context of FMS. The models chosen are: (1) Wagner's Normal Approximation Model; (2) Roberts's Asymptotic Renewal Theory; (3) Ehrhardt and Mosier's Power Approximation.

Model IV - Under the Context of FMS

$$Q=\{(2Ct*U_N+[Ci*M_L-2Ct]*b(s)\}/(Ci+2Cs)\}^{.5}$$

$$F'(s)= (Ci+Cs)/[Cp-(Ct/Q)+(Ci*M_L/2Q)]$$

$$b(s)= \sigma L'[(s-U/\sigma)]$$

The notations can be found at the beginning of Chapter 6.

Wagner's Normal Approximation

$$Q=\{[2KD/h]+[D_L+2D\pi/h] * b(s)\}^{.5}$$

$$F'(s)= hQ/[(hM_L/2)+D]$$

Robert's Asymptotic Renewal Theory Approximation

$$Q=[(2KD)/h]^{.5}$$

$$s=(L+1)\mu_L +\sigma(L+1).5 * G\{ Q/[1+(Q/h)\sigma[L+1].5\}$$

Ehrhardt and Mosier's Power Approximation

$$Q=1.3\ U^{.494}\ (k/h)^{.506}\ (1+\sigma_L^2/\mu^2)^{.116}$$

$$s=.973\ \mu_L+\sigma_L[(.183/z)+1.063-2.192z]$$

$$z=[Q/(p/h)]^{.5}$$

where p is the penalty cost.

$G(x)= -F(x)$

μ_L is mean demand during the lead time

σ_L is the standard deviation during the lead time

h is the holding cost

k is the order cost

D is demand rate per unit of time

π is the penalty cost

b(s) is the probability that demand during the lead time is greater than reorder point s.

In comparing these formulas, we found they appear different from Model IV. We tried some of the numerical results of Model IV (which is derived under FMS assumptions) in Ehrhardt and Mosier's Model, and found that for optimal batch size (Q) the results are quite close. However, for reorder point the result of the FMS model is significantly different. One of the reasons may be due to the different context of manufacturing technology, therefore, the formulas developed under traditional job shop assumptions are not quite applicable to FMS.

CHAPTER 7: CONCLUSIONS

In this research we analyze certain design problems in the context of FMS under four different conditions. For the first three conditions, we view the system in a closed system perspective, assuming instantaneous replenishment with no lead time for sending a batch from warehouse or supplier. Model IV allows lead time for replenishment. An exact solution and approximations are derived and the results of the approximations are compared to the computer direct search optimal solution. Table 7.1 summarizes the solutions of the four models.

In Model I, we consider an initial design problem when the system is still at its planning stage, where the decision is made regarding central buffer size needed for a given number of machines and transporters. A lower bound solution is derived for finding the minimum storage size for a given system. Tighter bounds are proposed for finding the optimal batch size for large and small batch systems. Properties of system configuration relating to profit and costs parameters are analyzed. The model assumes the number of machines and transporters are equal. It is found that the optimal batch size is affected mainly by transportation cost, number of transporters, storage cost and work load parameters. The optimal number of transporters has a positive relationship to the parameters of unit profit, and transportation cost, and a negative relationship to the storage cost, but no relationship with the cost of equipment.

For Model II, we find the optimal batch size has a positive relationship to the throughput rate of the system and transportation cost and a negative relationship to the inventory and storage costs. The exact closed form solution to the model is derived. The variable Q is treated as a continuous variable in the solution.

For Model III we find that reorder point (s) is strongly affected by number of machines, transporters and work load. These parameters account for eighty three percent of the variation in reorder point of our data. The cost parameters-- inventory cost, buffer size cost and order quantity-- have a negative relationship to the optimal reorder point, and profit per unit has a positive relationship to the reorder point. The cost and profit parameters make very little contribution to the reorder point. From our data analysis these parameters only account for an additional eleven percent of the total variation of the data set. For the optimal order quantity (Q), the contribution of the variation of the order quantity comes from the cost parameters--order cost, inventory carrying cost and buffer size cost. These parameters contribute eighty-nine percent of the total variation in our data base. Throughput rate, which is determined by the system configuration of number of machines and number of transporters and workload condition, only accounts for an additional seven percent of the total variation in our data base. There are other approximation formulas we derived with a very complex form and the predictivity only increases by a very small percentage. One of our objectives is to try to keep solutions as simple as possible with reasonable accuracy. For the trade-off of simplicity and accuracy, we decided to propose the approximation shown in Eqs.(5.12) and (5.13). These formulas account for more than ninety six percent of the total variation. We feel it is reasonable in achieving our objective.

For Model IV the solution is complicated and requires interactive procedures to find the solution. Normal approximation method is used to determine order quantity and reorder point, and lead time for replenishment is considered in the model. The formulas are compared with the traditional well known inventory policies, and we found that there is some difference in the ordering and inventory policy between the conventional job shop and the FMS system.

This research examines four optimal design problems in the context of FMS where product form solution applies, the number of machines is evenly distributed to each station, workload to all machines is equal, and overall loading is a constant. There are several aspects that can be extended in future research:

(1) Extend the models to the non-product form solution types of system, and compare and evaluate the sensitivity of the models' performance to non-product form type of systems.

(2) Extend the models to an unequal loading and machine grouping situation and perform a sensitivity analyses for those systems.

(3) Extend the research to a hierarchical model for a multi-echelon lot sizing problem.

Table 4.1

Model I: Sensitivity Analysis for N & M

	Q	L	Cs	Cm	Cp	Ct	Optimal N & M
1.	11	0.1	100	400	200	100	37
2.	11	0.1	100	4000	200	100	37
3.	11	0.1	100	400	20	100	6
4.	11	0.1	100	4000	20	100	6
5.	11	0.1	1000	400	20	100	2
6.	11	0.1	1000	400	200	100	2
7.	11	0.1	100	400	200	100	37
8.	11	0.1	500	400	200	100	5
9.	11	0.1	500	4000	200	100	5
10.	11	0.1	100	400	200	1000	46
11.	11	0.1	500	4000	200	1000	11
12.	11	0.1	100	4000	200	1000	46
13.	11	0.1	500	400	200	100	5
14.	1	0.1	100	400	200	100	3
15.	6	0.1	100	400	200	100	16
16.	11	0.1	100	400	200	100	37
17.	15	0.1	100	400	200	100	53
18.	6	0.2	100	400	200	100	9
19.	6	0.1	100	400	200	100	16
20.	6	0.1	100	40	200	100	18
21.	6	0.2	100	900	200	100	9

Table 4.2

Model I:Comparison of Lower Bound Approximation

For Expensive and Low Volume Items

For Cs=100 and Cp=200

N & M	L	Ct	(1) Comp Q	(2) Q*	Profit Comp Q	Profit of Q*
5	0.5	1000	8	9	-782	-778
5	0.5	500	6	6	-317	-317
5	0.5	100	3	3	255	255
10	0.5	100	5	4	616	628
10	0.5	500	8	9	-120	-115
10	0.5	1000	11	12	-724	-715
20	0.5	1000	15	17	-354	-328
20	0.5	500	11	12	448	461
20	0.5	100	7	6	1433	1445
5	0.25	1000	11	13	-40	- 7
5	0.25	500	8	10	598	621
5	0.25	100	5	5	1373	1373
10	0.25	500	11	14	1797	1854
10	0.25	1000	15	19	980	1058
10	0.25	100	7	7	2784	2784
20	0.25	100	10	11	5662	5667
20	0.25	1000	20	28	3287	3495
20	0.25	500	15	21	4364	4504
5	0.1	100	6	11	5047	5222
5	0.1	1000	16	18	3028	3286
5	0.1	500	12	18	4014	4211
10	0.1	100	10	20	9860	10178
10	0.1	1000	23	36	7466	7953
10	0.1	500	17	29	8600	9051
20	0.1	100	15	37	19306	20[illegible]97
20	0.1	1000	32	59	16458	17586
20	0.1	500	23	49	17688	18863

1. Comp Q is the batch size derived from model I.

2. Q* is the optimal batch size derived from computer search.

Table 4.3

Model I:Comparison of Lower Bound Solution to Optimal

For Inexpensive and High Volume Items

For Cs=100 and Cp=20

M & N	L	Ct	(1) Comp.Q	(2) Q*	Profit Comp. Q	Profit Q*
5	0.05	100	8	9	-782	-778
5	0.05	500	17	19	-2907	-2880
5	0.05	1000	25	27	-4570	-4548
10	0.05	500	23	27	-3539	-3491
10	0.05	1000	33	38	-5785	-5727
10	0.05	100	11	12	-724	-715
20	0.05	500	32	37	-4076	-4003
20	0.05	1000	45	55	-7096	-6998
20	0.05	100	15	17	-354	-328
5	0.02	500	24	27	-3030	-2959
5	0.02	1000	39	46	-5561	-5464
5	0.02	100	12	15	405	468
10	0.02	100	14	19	929	1058
10	0.02	500	33	40	-2902	-2783
10	0.02	1000	53	64	-5866	-5684
20	0.02	500	51	66	-142	183
20	0.02	100	21	33	5202	5063
20	0.02	1000	72	91	-4827	-4442
5	0.01	500	39	48	-1443	-1271
5	0.01	100	17	24	3102	3286
5	0.01	1000	54	67	-5270	-5101
10	0.01	1000	74	95	-3225	-2725
10	0.01	100	33	36	7279	7953
10	0.01	500	52	69	1735	2189
20	0.01	1000	103	138	3161	4102
20	0.01	500	72	109	9482	10454
20	0.01	100	32	59	16458	17586

(1) Batch size derived from Model I.
(2) Optimal batch size derived from computer search.

Table 4.4

Summary of MODEL I: 54 Item Data Set from

Computer Search and Formula

For Cs=100 and Cp=200

M & N	L	Ct	Computed Q	Optimal Q
5	0.5	1000	8	9
5	0.5	500	6	6
5	0.5	100	3	3
10	0.5	100	5	4
10	0.5	500	8	9
10	0.5	1000	11	12
20	0.5	1000	15	17
20	0.5	500	11	12
20	0.5	100	7	6
5	0.25	1000	11	13
5	0.25	500	8	10
5	0.25	100	5	5
10	0.25	500	11	14
10	0.25	1000	15	19
10	0.25	100	7	7
20	0.25	100	10	11
20	0.25	1000	20	28
20	0.25	500	15	21
5	0.1	100	6	11
5	0.1	1000	16	24
5	0.1	500	12	18
10	0.1	100	10	20
10	0.1	1000	23	36
10	0.1	500	17	29
20	0.1	100	15	37
20	0.1	1000	32	59
20	0.1	500	23	49

Table 4.4
Continue

For Cs=100 and Cp=20

M & N	L	Ct	Computed Q	Optimal Q
5	0.05	100	25	27
5	0.05	500	17	19
5	0.05	1000	8	9
10	0.05	500	11	12
10	0.05	1000	23	27
10	0.05	100	33	38
20	0.05	500	45	55
20	0.05	1000	32	37
20	0.05	100	15	17
5	0.02	500	39	46
5	0.02	1000	24	27
5	0.02	100	12	15
10	0.02	100	33	40
10	0.02	500	53	64
10	0.02	1000	14	19
20	0.02	500	21	33
20	0.02	100	72	91
20	0.02	1000	51	66
5	0.01	500	17	24
5	0.01	100	54	67
5	0.01	1000	39	48
10	0.01	1000	33	36
10	0.01	100	74	95
10	0.01	500	52	69
20	0.01	1000	32	59
20	0.01	500	103	138
20	0.01	100	72	109

Table 4.4A
Model I: Comparison of LBS Adjusting Formula for

Large Qa vs. Q* vs. Qs + $Qs^{.5}$

(Qs) LBS-Q	(Qa) Power Formula	$Qs+Qs^{.5}$	Q*
8	9.02	10.80	9
11	12.70	14.00	12
23	28.00	27.70	27
33	41.00	38.00	38
45	57.00	51.70	55
32	39.98	37.65	37
15	17.72	18.87	17
39	49.44	45.20	46
24	29.36	28.89	27
53	68.72	60.28	64
21	25.44	25.58	33
72	95.00	80.48	91
51	65.00	58.14	66
17	20.27	21.12	24
54	70.00	61.34	67
39	49.00	45.24	48
33	41.00	38.74	36
74	98.00	82.60	95
52	67.32	59.21	69
32	39.90	37.65	59
103	140.00	113.00	138
72	95.00	80.00	109

Computed Qa = $.969 \cdot (Qs)^{1.073353}$
Q* is the optimal batch size from computer search.

Table 5.1

Parameter Settings for Model III

Factor	Level	No. of Levels
Machines	10, 20	2
Transport vehicles	(15,25) (30,40)	2
Transport cost	100,500,1000	3
Inventory cost	10, 50, 100	3
Buffer cost	100,500,1000	3
Loading factor	.01	1
Profit per unit	200	1

Table 5.2

Model III: Original 108 Item Data Set

For M = 10 and N = 15

Ct	Ci	Cs	Cp	L	S*	Q*	Comp S	Comp Q
500	100	1000	200	0.01	10	18	9.86	19.40
100	10	500	200	0.01	13	11	12.31	13.33
500	100	500	200	0.01	11	24	10.51	25.58
100	100	100	200	0.01	13	21	13.35	22.40
500	100	100	200	0.01	12	46	11.68	44.61
1000	10	100	200	0.01	12	78	12.02	69.91
500	50	1000	200	0.01	10	19	9.93	19.60
1000	100	100	200	0.01	11	65	11.03	60.02
500	50	500	200	0.01	11	25	10.65	26.10
100	10	100	200	0.01	14	24	12.17	48.23
500	50	100	200	0.01	12	53	12.17	48.23
1000	100	500	200	0.01	10	35	9.93	34.42
500	10	1000	200	0.01	10	18	9.99	19.77
1000	50	100	200	0.01	11	72	11.48	64.89
500	10	100	200	0.01	13	54	12.73	51.97
100	10	1000	200	0.01	12	8	11.42	9.93
500	10	500	200	0.01	11	26	10.77	26.54
1000	10	1000	200	0.01	10	26	9.43	26.59
100	100	500	200	0.01	12	15	12,02	12.85
1000	10	500	200	0.01	10	36	10.17	35.70
100	50	50	200	0.01	14	29	14.42	30.14
1000	50	500	200	0.01	10	35	10.06	35.11
100	50	1000	200	0.01	12	8	11.35	9.84
1000	100	1000	200	0.01	8	25	9.31	26.10
100	10	1000	200	0.01	12	8	11.42	9.93
1000	50	1000	200	0.01	10	25	9.38	26.37
100	50	100	200	0.01	13	24	13.90	24.22

Table 5.2
Continued

For M = 10 and N = 30

Ct	Ci	Cs	Cp	L	S*	Q*	Comp S	Comp Q
100	50	1000	200	0.01	22	10	20.26	11.07
1000	100	500	200	0.01	16	40	17.71	38.72
1000	10	500	200	0.01	17	41	18.15	40.16
100	100	500	200	0.01	25	13	21.45	14.45
500	10	50	200	0.01	26	84	24.23	77.10
100	50	500	200	0.01	24	13	21.73	14.74
100	10	100	200	0.01	27	30	25.98	29.36
1000	100	1000	200	0.01	10	30	16.62	29.36
1000	50	100	200	0.01	21	81	20.50	72.99
1000	100	100	200	0.01	20	74	19.68	67.51
500	100	1000	200	0.01	16	22	17.60	21.82
1000	10	1000	200	0.01	14	30	16.83	29.91
500	50	1000	200	0.01	16	22	17.73	22.05
1000	10	100	200	0.01	23	87	21.46	78.65
500	50	100	200	0.01	23	57	21.71	54.26
100	100	1000	200	0.01	22	10	20.12	10.96
500	10	100	200	0.01	24	62	22.73	58.46
1000	50	500	200	0.01	17	41	17.95	39.50
100	10	1000	200	0.01	22	11	20.38	11.17
500	50	500	200	0.01	20	28	19.01	29.36
100	10	500	200	0.01	24	15	21.98	14.99
1000	50	1000	200	0.01	13	30	16.74	29.66
500	10	1000	200	0.01	16	22	17.83	22.24
500	100	500	200	0.01	19	29	18.76	28.78
100	50	100	200	0.01	26	26	24.82	27.25

Table 5.2
Continued

For M = 20 and N = 25

Ct	Ci	Cs	Cp	L	S*	Q*	Comp S	Comp Q
1000	10	500	200	0.01	20	48	19.48	50.10
500	100	1000	200	0.01	20	31	18.89	27.23
500	100	500	200	0.01	20	33	20.14	35.91
500	100	100	200	0.01	21	65	22.37	62.61
100	10	500	200	0.01	23	15	23.59	18.70
100	10	1000	200	0.01	22	11	21.87	13.93
100	50	100	200	0.01	23	31	26.63	33.99
100	100	1000	200	0.01	22	11	21.59	13.67
100	50	1000	200	0.01	22	11	21.74	13.81
1000	10	1000	200	0.01	20	30	18.07	37.32
100	50	50	200	0.01	22	40	27.62	42.30
1000	100	1000	200	0.01	20	36	17.84	36.63
100	100	500	200	0.01	22	16	23.02	18.03
1000	50	100	200	0.01	21	100	22.00	91.06
500	10	500	200	0.01	20	35	20.64	37.24
100	10	100	200	0.01	24	33	27.88	36.63
500	10	100	200	0.01	23	74	24.39	72.93
100	100	100	200	0.01	23	25	25.57	31.44
500	10	1000	200	0.01	19	24	19.14	27.74
1000	100	100	200	0.01	20	88	21.12	84.23
500	50	100	200	0.01	22	69	23.30	67.69
1000	50	1000	200	0.01	20	34	17.96	37.01
500	50	500	200	0.01	20	35	20.40	36.63
1000	10	100	200	0.01	22	104	23.03	98.12
1000	100	500	200	0.01	20	46	19.01	48.30
1000	50	500	200	0.01	20	47	19.26	49.28
500	50	1000	200	0.01	20	25	19.02	27.51

Table 5.2
Continued

For M = 20 and N = 40

Ct	Ci	Cs	Cp	L	S*	Q*	Comp S	Comp Q
1000	10	1000	200	0.01	25	40	25.36	41.25
500	10	100	200	0.01	36	81	34.23	80.62
1000	100	100	200	0.01	32	97	29.64	93.10
500	10	1000	200	0.01	25	40	26.86	30.66
100	10	100	200	0.01	36	41	39.13	40.49
500	50	100	200	0.01	35	74	32.70	74.82
100	100	100	200	0.01	36	33	35.89	34.75
500	50	500	200	0.01	32	35	28.63	40.19
1000	100	1000	200	0.01	24	39	25.03	40.49
500	50	1000	200	0.01	28	28	26.70	30.41
100	50	1000	200	0.01	34	13	30.52	15.27
500	100	100	200	0.01	34	69	31.40	69.20
100	50	500	200	0.01	35	17	32.73	20.33
500	100	500	200	0.01	31	37	28.26	39.69
500	10	500	200	0.01	32	38	28.96	41.16
500	100	1000	200	0.01	28	28	26.51	30.09
100	10	1000	200	0.01	34	13	30.70	15.40
1000	50	500	200	0.01	29	53	27.03	54.47
100	10	100	200	0.01	38	36	39.13	40.49
1000	50	100	200	0.01	33	104	30.87	100.66
100	100	500	200	0.01	35	17	32.30	19.93
1000	50	1000	200	0.01	25	39	25.21	40.91
100	100	1000	200	0.01	34	13	30.30	15.11
1000	10	100	200	0.01	36	113	32.32	108.45
100	50	50	200	0.01	38	43	38.76	46.76
1000	100	500	200	0.01	28	52	26.68	53.39
1000	10	500	200	0.01	29	54	27.34	55.38

Table 5.3

Summary of Error Frequencies - for Original 108 Items

Range-for p	No. of Items	Cumulative Percentage
.0%- .1%	18	16.6%
.1%- .5%	35	49.0%
.5%- 1.0%	31	77.7%
1.0%- 2.0%	12	88.8%
2.0%- 3.0%	7	95.3%
3.0%- 4.0%	1	96.2%
4.0%- 5.0%	2	98.0%
5.0%- 6.0%	1	99.0%
6.0%- 7.0%	1	100.0%
	108	

Table 5.4

Model III Test Data Set

For M = 5 N = 9 Ct = 1000
Ci = 80 Cs = 120

Cp	L	Com.Q	Com.S	Cru.Q	s*	Q*	Pft*	Com.Pft
200	.02	31.10	5.91	32.89	4	34	23901	23452
200	.03	25.13	6.16	26.25	4	28	14394	14157
200	.04	21.61	6.35	23.26	3	24	9845	9492
200	.05	19.21	6.50	20.80	3	22	7238	6785
180	.01	44.76	5.39	46.51	5	48	47152	46915
150	.01	44.76	5.20	46.51	5	47	36829	36714
120	.01	44.76	4.97	46.51	4	47	26540	26119
100	.01	44.76	4.80	46.51	4	47	19694	19385
200	.01	44.76	5.5	46.51	5	48	54034	53709
80	.01	44.76	4.59	46.51	3	47	12901	12644
60	.01	44.76	4.33	46.51	2	47	6165	5898
130	.02	31.10	5.43	32.89	3	34	11997	11717
160	.03	25.13	5.90	26.85	3	31	10591	9455
200	.01	44.76	5.50	46.51	5	48	54034	53709

For Cs = 90 (All other parameters the same)

Cp	L	Com.Q	Com.S	Cru.Q	s*	Q*	Pft*	Com.Pft
160	.03	27.60	5.89	29.79	3	31	10590	10188

For Ct = 100 Ci = .80 Cs = .10
(All other parameters the same)

Cp	L	Com.Q	Com.S	Cru.Q	s*	Q*	Pft*	Com.Pft
10	.02	148.32	6.99	186.50	6	187	1539	1530
200	.01	213.49	11.75	263.12	8	265	68960	66605

For M = 10 N = 16 Ct = 1000
Ci = 180 Cs = 200

Cp	L	Com.Q	Com.S	Cru.Q	s*	Q*	Pft*	Com.Pft
200	.01	47.29	10.59	46.98	10	49	99062	98353
200	.02	32.85	11.39	33.22	9	35	43459	42899
200	.03	26.55	11.87	27.12	7	29	25964	24875
200	.04	22.82	12.24	23.49	6	26	17656	16618
200	.05	20.30	12.52	21.01	6	23	12929	11511
100	.01	49.29	9.24	46.98	7	49	35987	35654
100	.02	32.85	9.93	33.22	5	35	12673	11550

For Ct = 200 Ci = .60 Cs = .02
(All other parameters the same)

Cp	L	Com.Q	Com.S	Cru.Q	s*	Q*	Pft*	Com.Pft
2	.03	226.26	10.58	292.12	3	294	132	119

For M = 20 N = 60 Ct = 60
Ci = 1.8 Cs = .2

Cp	L	Com.Q	Com.S	Cru.Q	s*	Q*	Pft*	Com.Pft
10	.06	100.84	57.49	117.51	43	132	2185	2163
20	.06	100.84	65.93	117.51	49	122	4708	4605
40	.06	100.84	75.61	117.51	54	120	9766	9357
10	.01	77.10	60.62	91.02	36	100	1237	1200
20	.01	77.10	69.52	91.02	46	96	2744	2621
40	.01	77.10	79.73	91.02	52	94	5775	5378
4	.06	100.84	47.96	117.51	24	129	690	662
4	.08	86.69	49.42	101.77	19	126	476	443

For M = 40 N = 70 Ct = 750
Ci = 8.0 Cs = 4.0

Cp	L	Com.Q	Com.S	Cru.Q	s*	Q*	Pft*	Com.Pft
10	.04	209.94	51.57	245.37	19	252	2354	2086
10	.01	435.00	44.65	490.74	37	501	17507	17410
60	.01	129.71	80.83	155.19	49	162	12504	11796
80	.01	129.71	85.56	155.19	54	160	17611	16502
100	.01	129.71	89.41	155.19	57	160	22729	21152
200	.01	129.71	102.54	155.19	63	160	48377	73811

Table 5.5

Sensitivity Analysis of Varying Profit Parameter of Model III Power Approximation for s and Q

M	N	Ct	Ci	Cs	Cp	L	(P*-Pp)/P*
5	9	1000	80	120	60	.01	.0434
5	9	1000	80	120	80	.01	.0199
5	9	1000	80	120	100	.01	.0157
5	9	1000	80	120	120	.01	.0159
5	9	1000	80	120	150	.01	.0031
5	9	1000	80	120	180	.01	.0050
5	9	1000	80	120	190	.01	.0060

Table 5.6

Sensitivity Analysis of Varying Work Load Factors

For Model III Power Approximaiton of s and Q

M	N	Ct	Ci	Cs	Cp	L	(P*-Pp)/P*
5	9	1000	80	120	200	.01	.0060
5	9	1000	80	120	200	.02	.0188
5	9	1000	80	120	200	.03	.0165
5	9	1000	80	120	200	.04	.0359
5	9	1000	80	120	200	.05	.0626
10	16	1000	180	120	200	.01	.0072
10	16	1000	180	200	200	.02	.0129
10	16	1000	180	200	200	.03	.0419
10	16	1000	180	200	200	.04	.0588
10	16	1000	180	200	200	.05	.1097

Table 6.1

Model IV
Numerical Examples Using Normal Approximation Method

M	N	Ct	Ci	Cs	Cp	UN	L	lt	Normal S_N	Approx. Q_N
3	7	800	100	54	200	233	.01	.05	2	42
3	7	80	5	50	100	233	.1	1	19	6
4	8	100	10	20	180	7.	.4	2	1	6
6	12	100	2	8	20	14.	1	3	3	8
10	15	200	2	2	30	7.	.8	1	0	22
20	40	300	8	2	60	27	1	1	0	23

L - Total Mean Work Load to the Network by a Single Job.

lt- Lead time for replenishment.

Table 7.1

SUMMARY OF RESULTS OF THE FOUR MODELS

Assumptions Solution Method	Formulation and Results
	MODEL I
M=N=s	$G(N,Q)=\sum_{n=N+1}^{N+Q} UnPn[Cp-(Ct/Q)]-Cs(Q)-Cm(N)$
Lower Bound Solution	(1) Optimal Batch Size for Small Q: Qs $Qs= \frac{Ct*(N-1)}{2L * Cs}$
Adjucted Lower Bound Solution Power Approximation	(2) Optimal Batch Size for Large Q: Qa $Q_a = .9690\ (Q_s)^{1.073353}$ R2 = .943 R=.971
Crude Model	(3) $Q_c = Q_s + Q_s{}^{.5}$ Property of NG Function: (1) Cp increase; N increase, (2) Cs decrease; N increase. (3) Ct increase; N increase. (4) Cm has no effects on N optimal

SUMMARY RESULTS OF THE MODEL II

Assumptions Solution Method	Formulation and Results
s =N and N,M are Fixed	$G(s,Q)= U_N[Cp-(Ct/Q)]-Ci[s+(Q/2)]-Cs(s+Q-N)$
	Optimal Batch Size:
Exact Solution	$Q2= \dfrac{Ct\ UN}{(Ci/2)+Cs}$
	Where UN = (N*M)/ L (N+M-1)

SUMMARY OF RESULTS OF MODEL III

Assumption Solution Method	Formulation and Results

s<N<N+Q and M & N are fixed

$$G(s,Q)=\sum_{n=s+1}^{s+Q}[CpUnPn-Ci*n*Pn-(Ct/Q)UnPn]-Cs(s+Q-N)$$

Optimal Batch Size:

Power Approximation $Q=.907328\left(\frac{2Ct}{Ci+2Cs}\right)^{.4507591}\left(\frac{MN}{L(M+N-1)}\right)^{.5255085}$

R2 = .963

R = .9813

Optimal Reorder Point:

$$s=.519086(N+M-1)^{1.2132}\left(\frac{Cp}{(Ci+Cs)Q}\right)^{.19768}$$

R2 = .947

R = .973

SUMMARY OF RESULTS OF THE MODEL IV

Assumptions Solution Method	Formulation and Results

Assume Lead time (LT)

$$G(s,Q)= [U_N Cp - Cp \sum_{q_L>s} (q_L - s) P_L(q_L)]$$

$$- Ci [Q/2 - M_L + s + (M_L/2Q) \sum_{q_L>s} (q_L - s) P_L(q_L)]$$

$$- (Ct/Q) [U_N - \sum_{q_L>s} (q_L - s) P_L(q_L)] - Cs(s+Q-N)$$

Optimal Batch Size and Reorder Point:

Normal Approximation

$$Q^2 = \frac{2CtU_N + (M_L - 2Ct) b(s)}{Ci + 2Cs} \quad \ldots(1)$$

$$F'(s) = \frac{Ci + Cs}{(Cp - Ct/Q) + CiM_L/2Q} \quad \ldots.(2)$$

$$b(s) = \sigma L'(s - U/\sigma) \quad \ldots\ldots.(3)$$

Solution Procedure:

Use iterative procedure to find the optimal (s*,Q*)

(1) Assume b(s)=0, compute Q with Eq. (1). Find Q_1
(2) Substitute Q1 into Eq. (2). Find s1
(3) Substitute s1 into Eq. (3). Find b(s)
(4) Substitute b(s) to Eq. (1). Find Q_2 Continue procedure (2) until convergence occurs.
$Q_{n-1} \simeq Q_n$ or $s_{n-1} \simeq s_n$

Figure 3.1 FMS System Diagram for Model I, II, and III

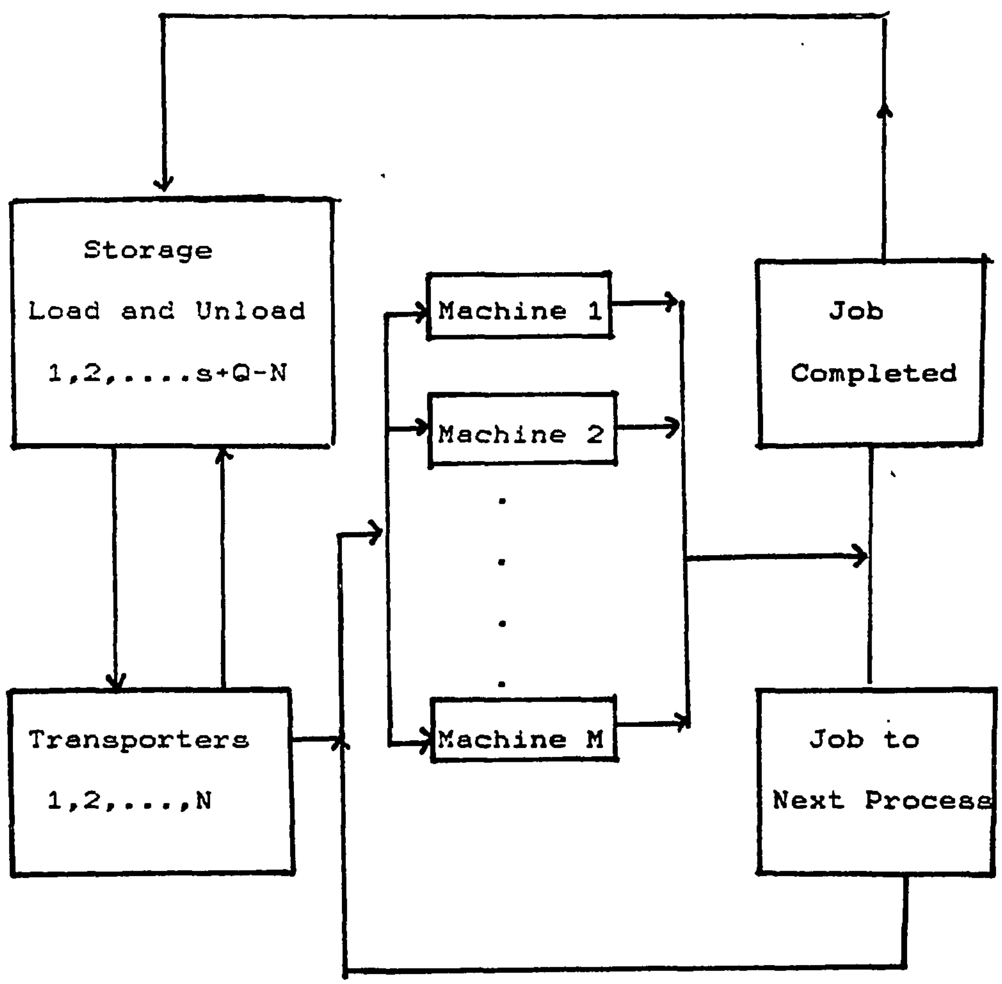


Figure 4.1

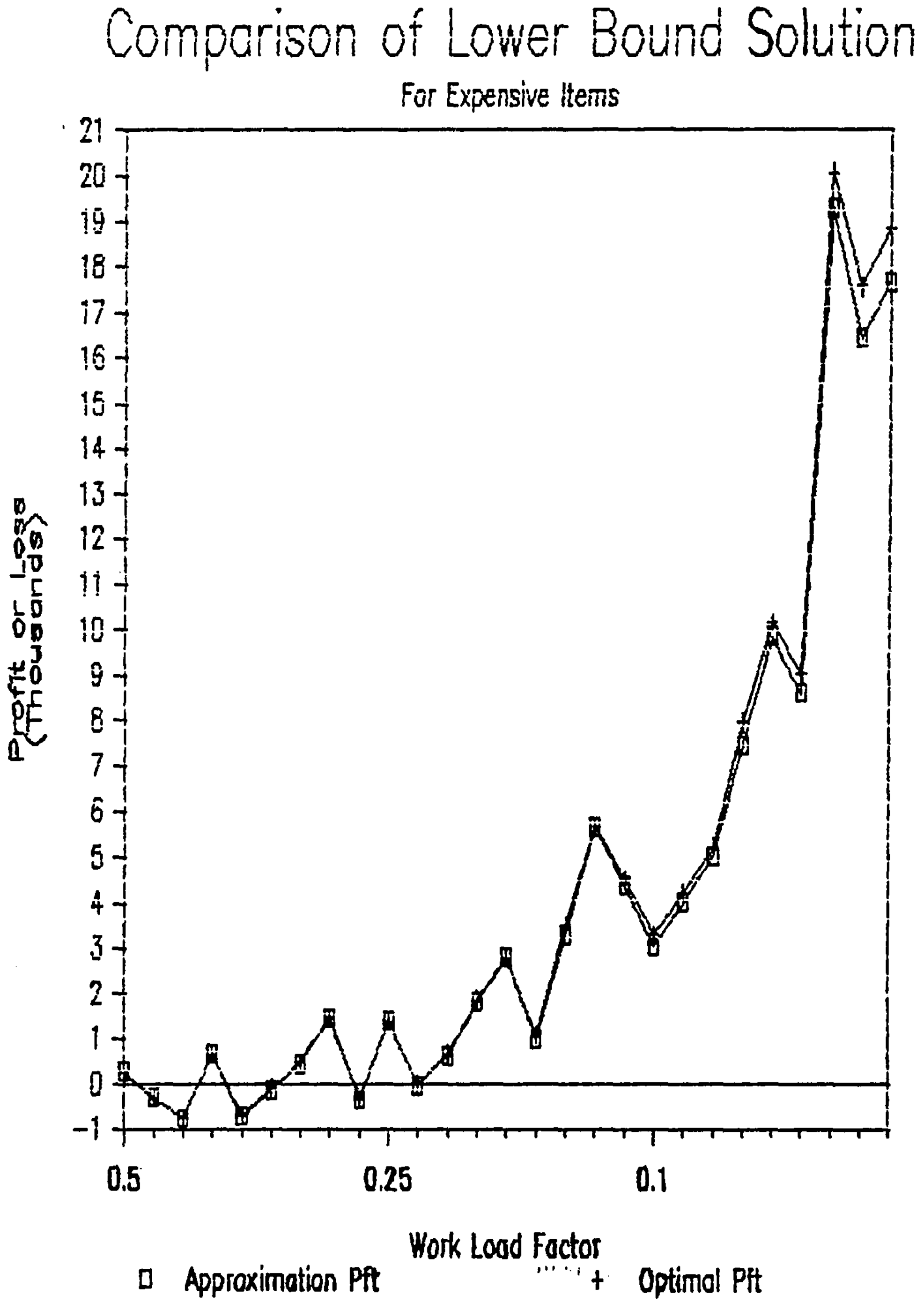

Comparison of Lower Bound Solution
For Expensive Items
Profit or Loss
(Thousands)
21
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
-1
0.5
0.25
0.1
Work Load Factor
Approximation Pft
Optimal Pft

Figure 4.2

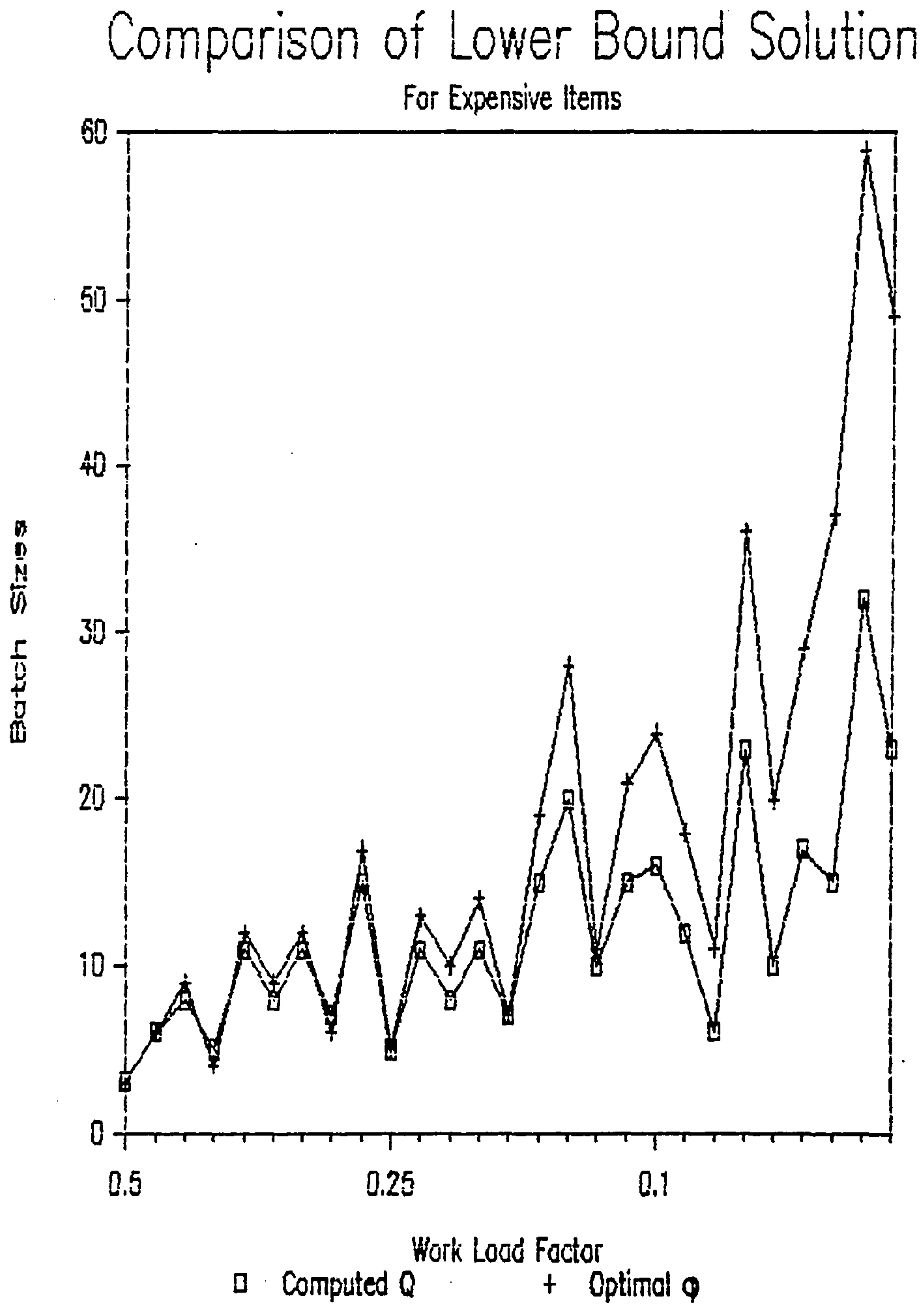

Comparison of Lower Bound Solution
For Expensive Items
60
50
40
30
20
10
0
Batch Sizes
0.5
0.25
0.1
Work Load Factor
Computed Q
Optimal φ

Figure 4.3

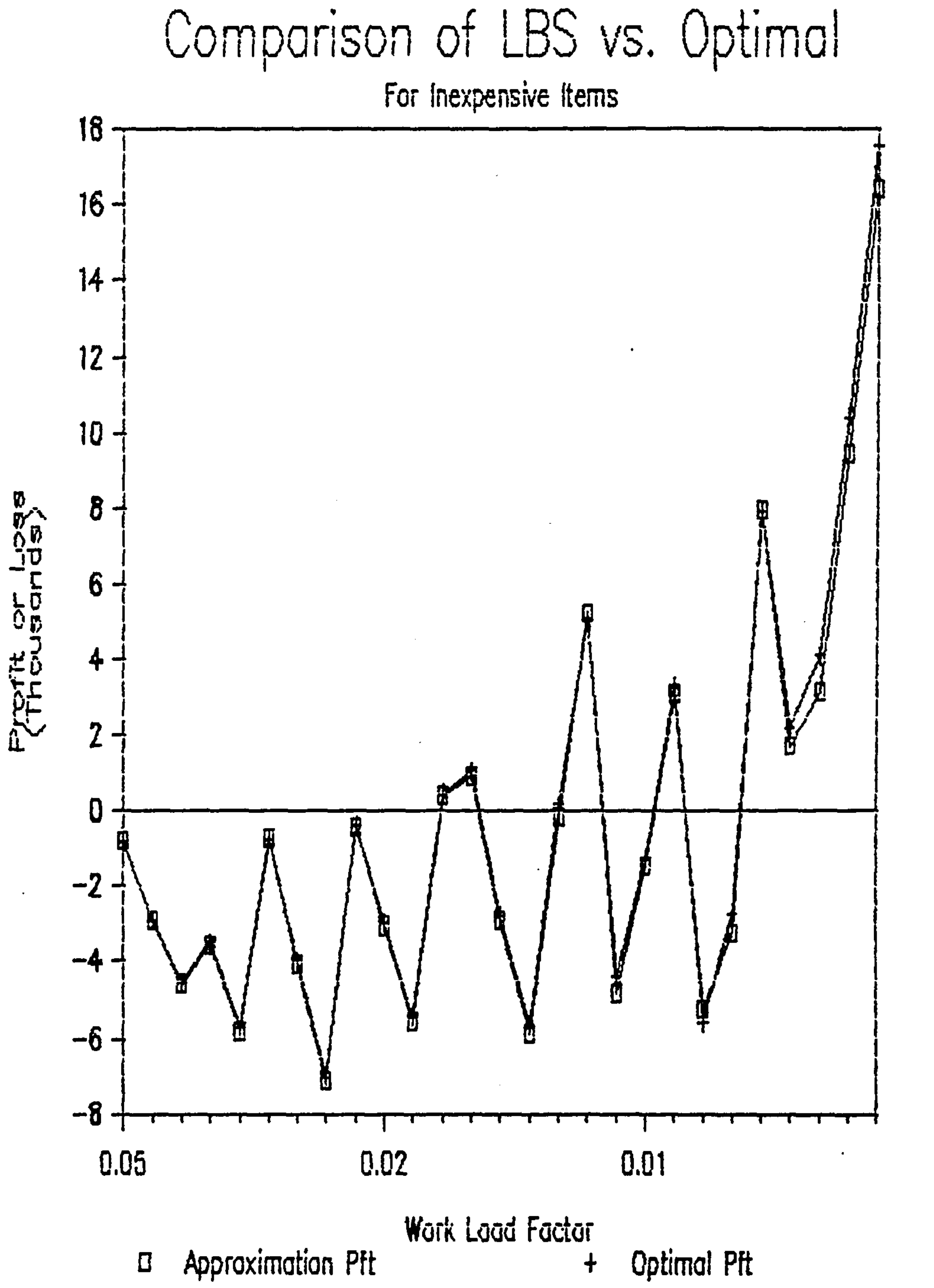

Comparison of LBS vs. Optimal
For Inexpensive Items
18
16
14
12
10
8
6
4
2
0
-2
-4
-6
-8
Profit or Loss (Thousands)
0.05
0.02
0.01
Work Load Factor
Approximation Pft
Optimal Pft

Figure 4.4

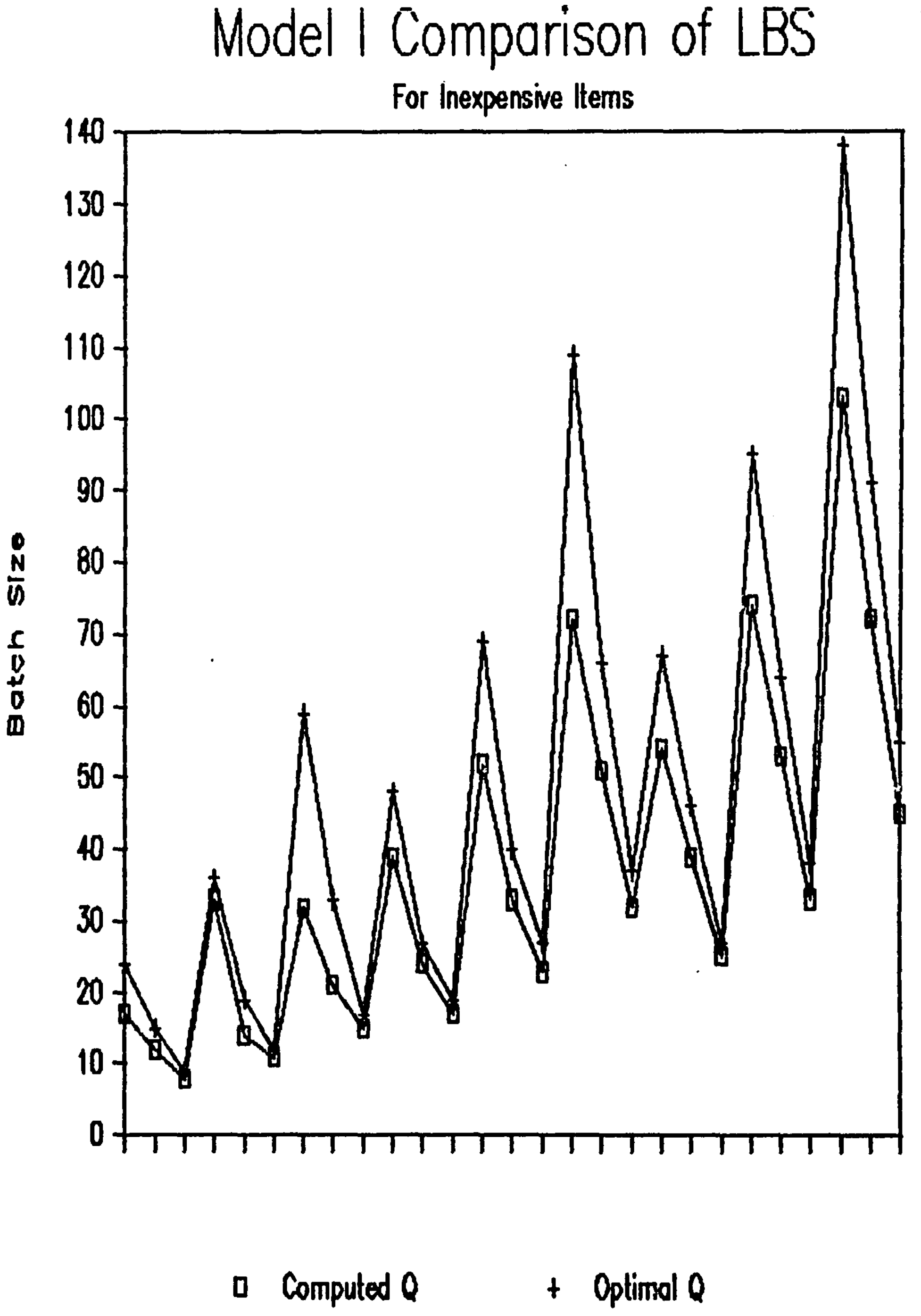

Model I Comparison of LBS
For Inexpensive Items
Batch Size
140
130
120
110
100
90
80
70
60
50
40
30
20
10
0
Computed Q
Optimal Q

Figure 4.5

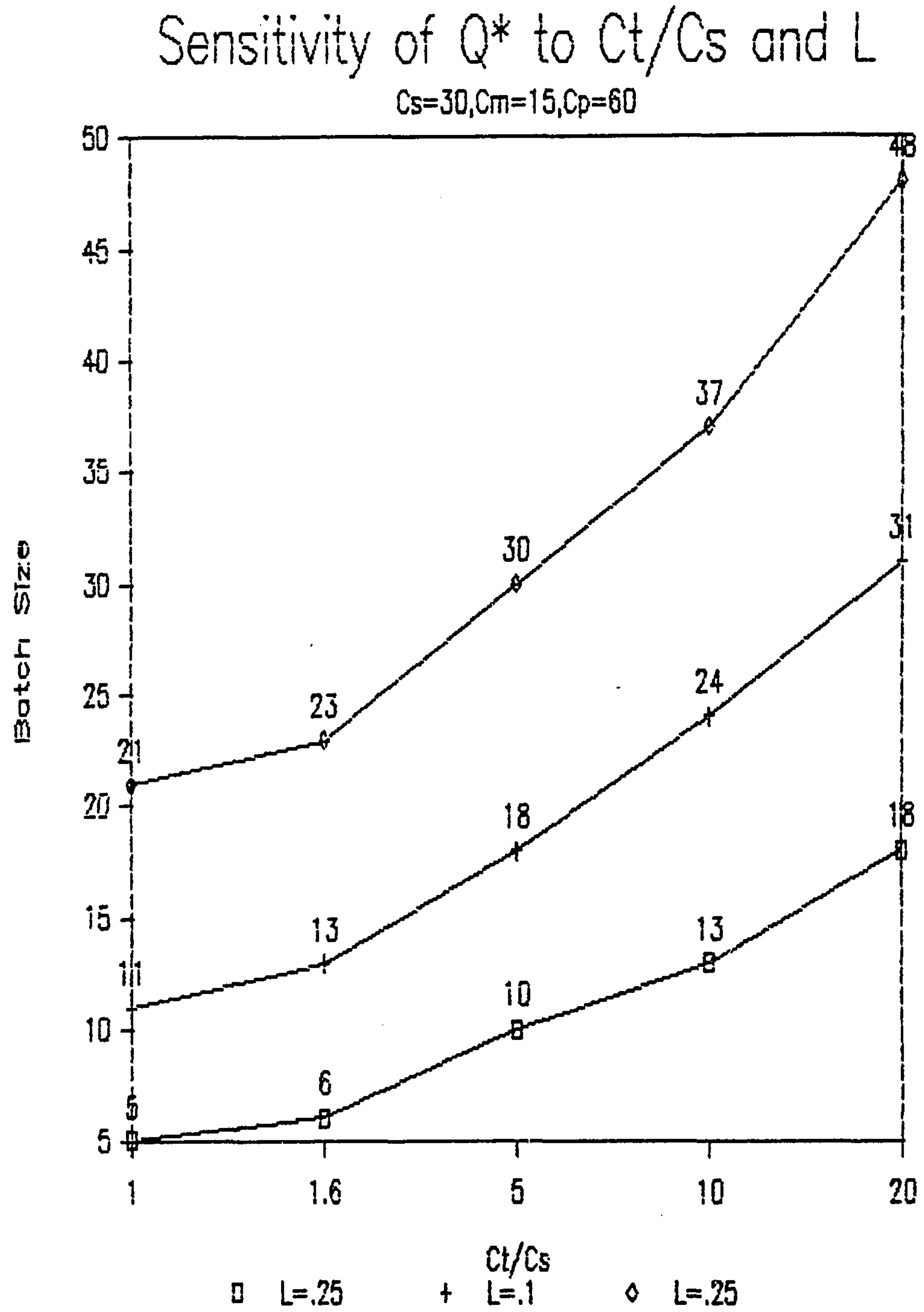

Sensitivity of Q* to Ct/Cs and L
Cs=30,Cm=15,Cp=60
Batch Size
50
45
40
35
30
25
20
15
10
5
1
1.6
5
10
20
Ct/Cs
21
23
30
37
48
11
13
18
24
31
5
6
10
13
18
L=.25
L=.1
L=.25

Figure 4.6

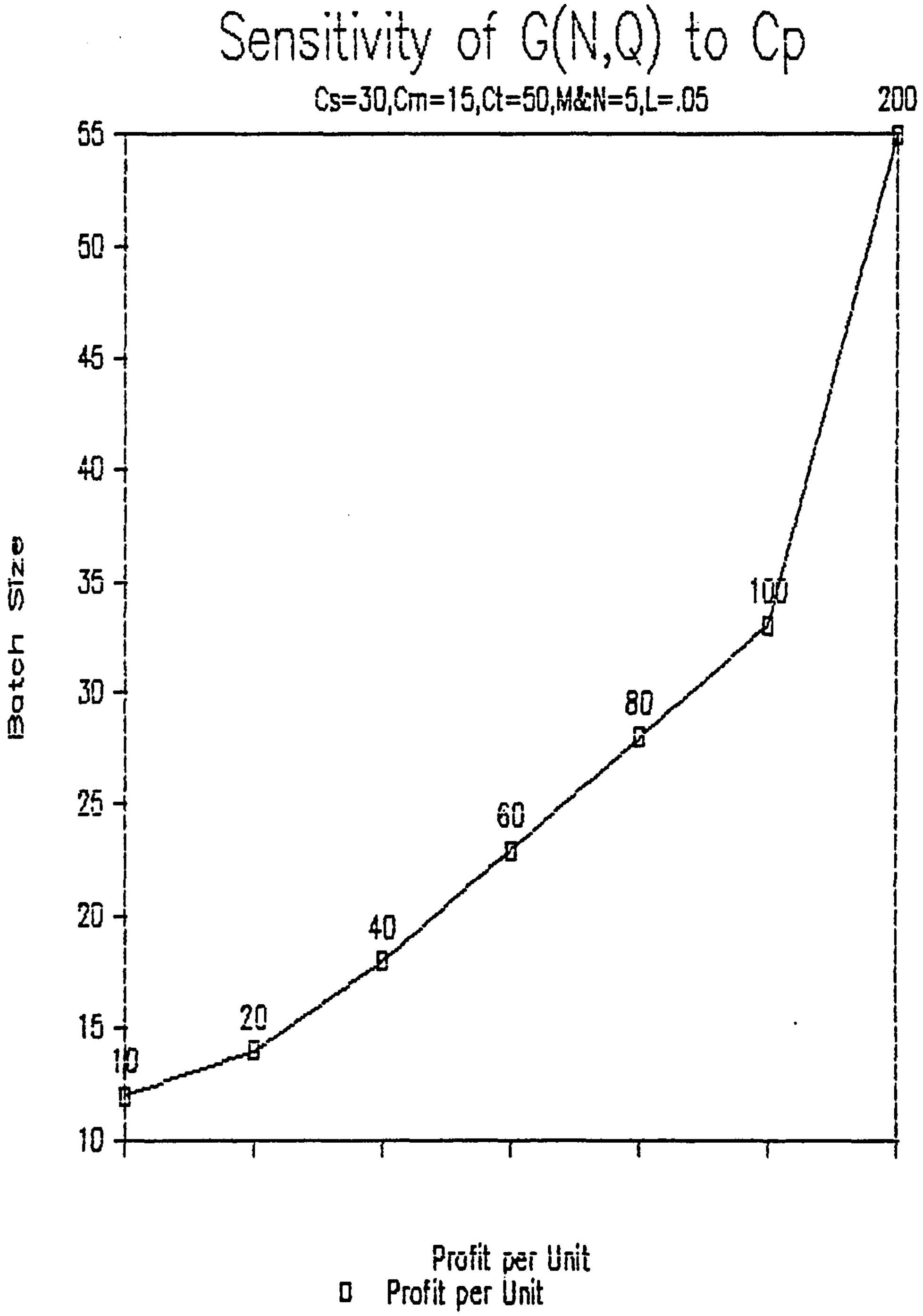

Sensitivity of G(N,Q) to Cp
Cs=30,Cm=15,Ct=50,M&N=5,L=.05
Batch Size
55
50
45
40
35
30
25
20
15
10
10
20
40
60
80
100
200
Profit per Unit
Profit per Unit

Figure 5.1

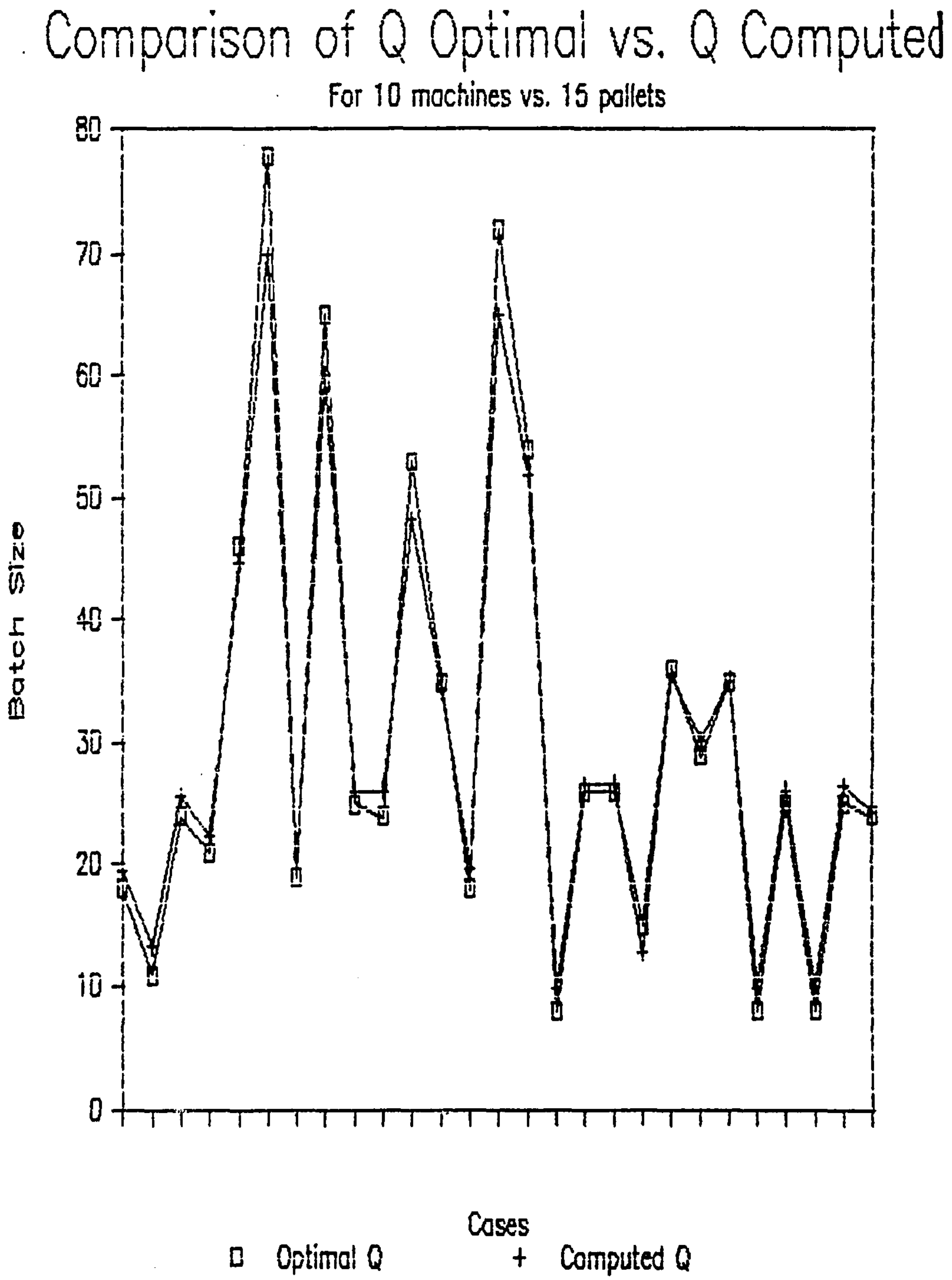

Comparison of Q Optimal vs. Q Computed
For 10 machines vs. 15 pallets
Batch Size
80
70
60
50
40
30
20
10
0
Cases
Optimal Q
Computed Q

Figure 5.2

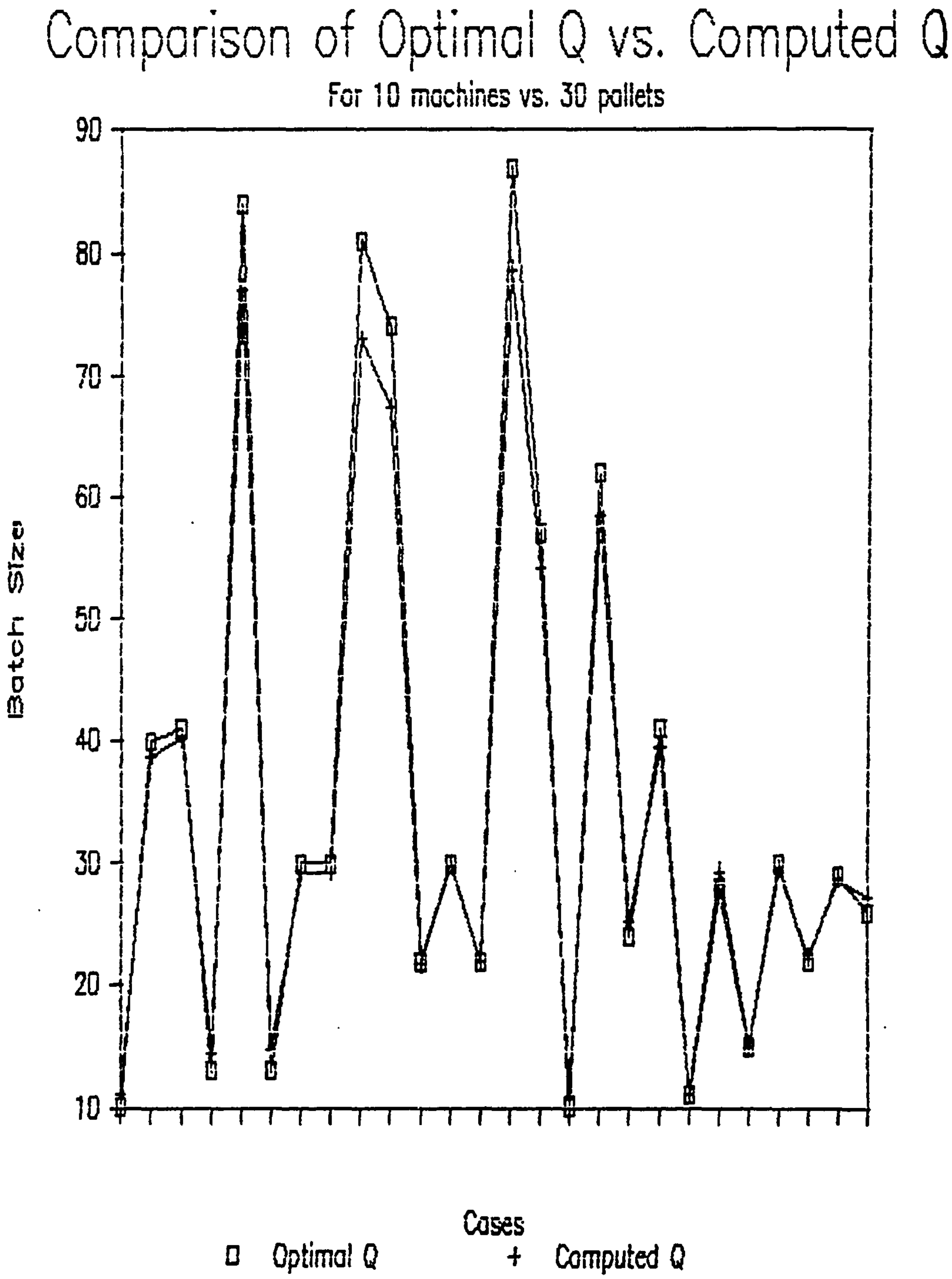

Comparison of Optimal Q vs. Computed Q
For 10 machines vs. 30 pallets
Batch Size
90
80
70
60
50
40
30
20
10
Cases
Optimal Q
Computed Q

Figure 5.3

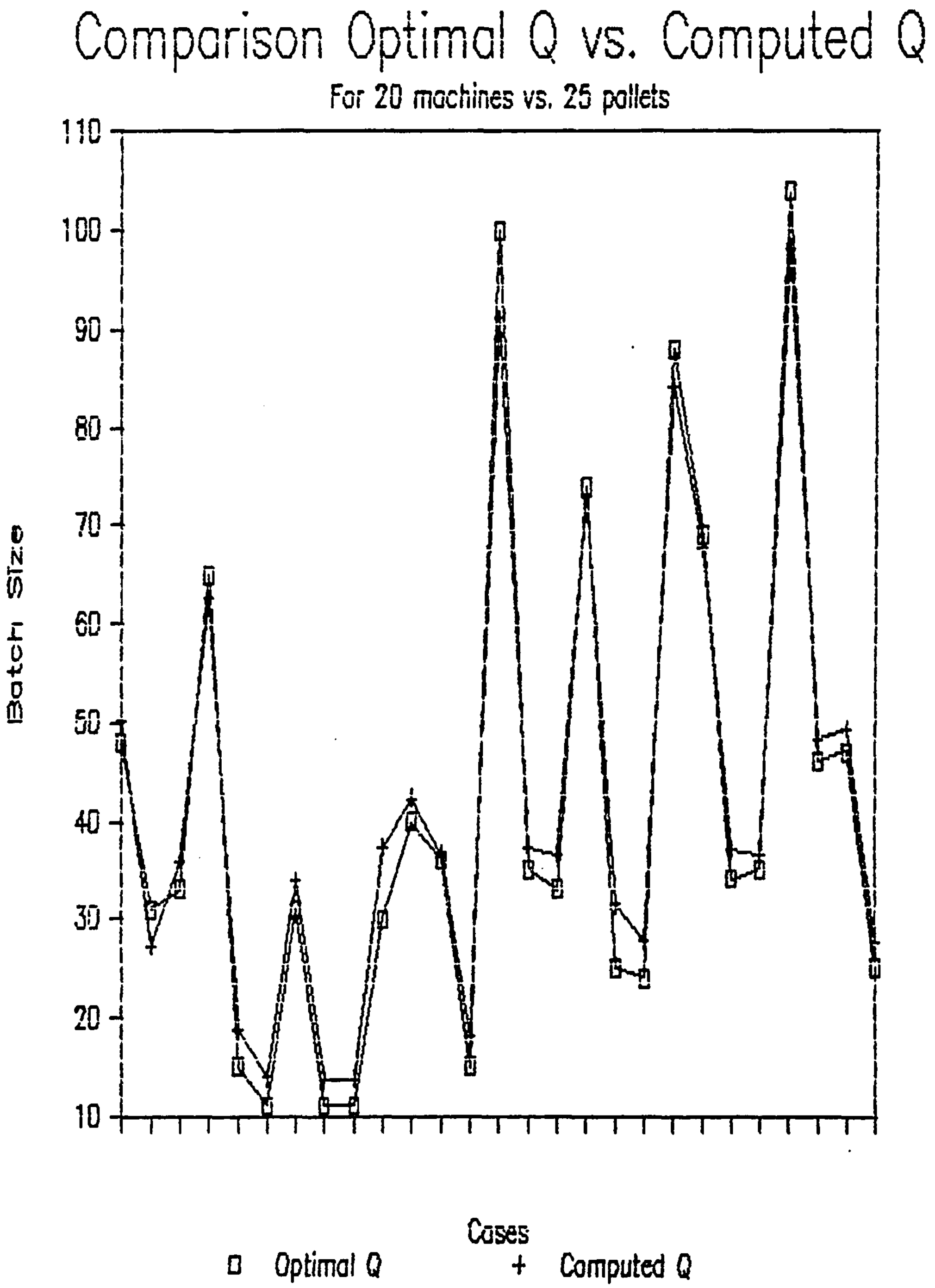

Comparison Optimal Q vs. Computed Q
For 20 machines vs. 25 pallets
Batch Size
110
100
90
80
70
60
50
40
30
20
10
Cases
Optimal Q
Computed Q

Figure 5.4

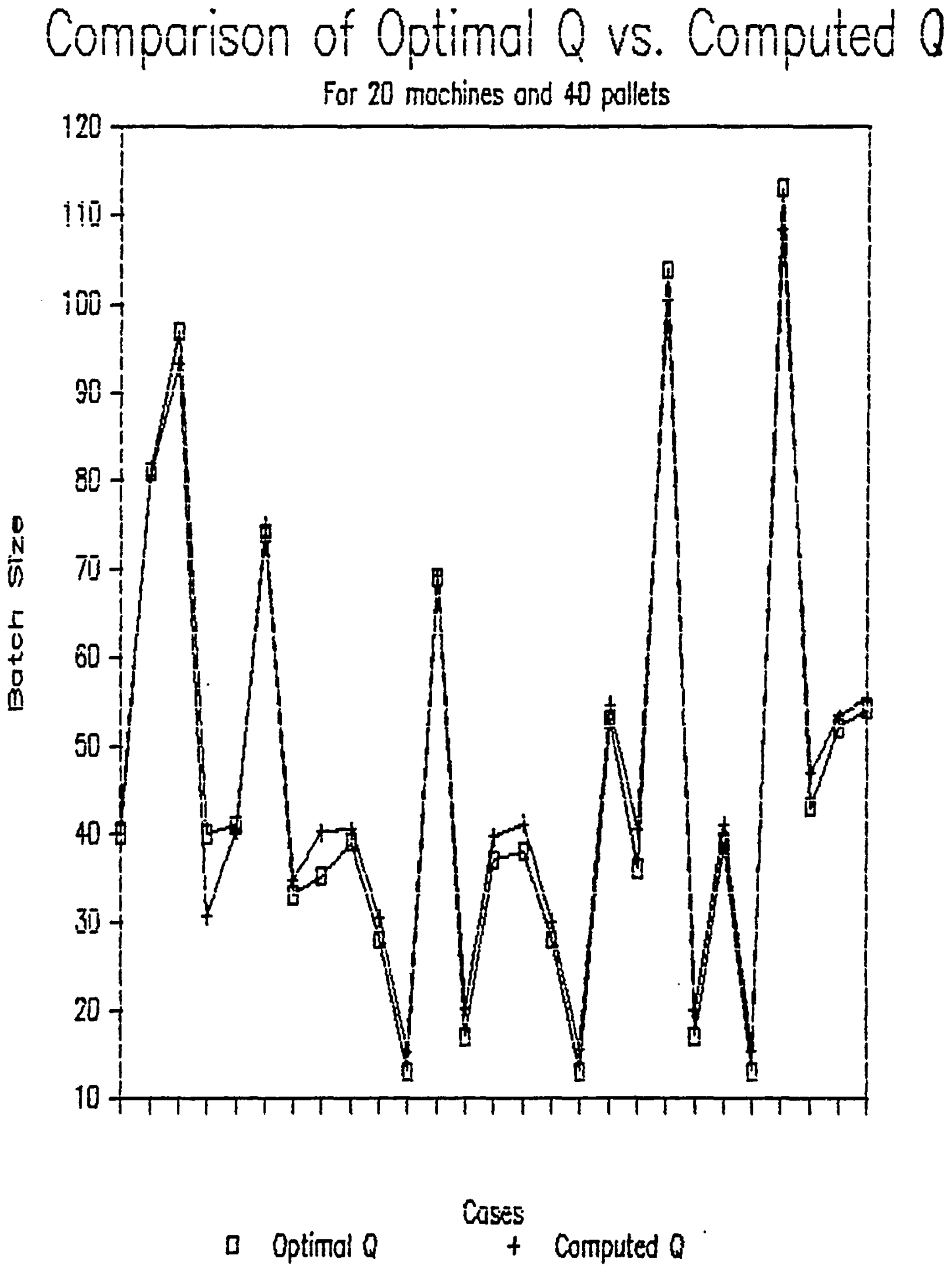

Comparison of Optimal Q vs. Computed Q
For 20 machines and 40 pallets
Batch Size
120
110
100
90
80
70
60
50
40
30
20
10
Cases
Optimal Q
Computed Q

Figure 5.5

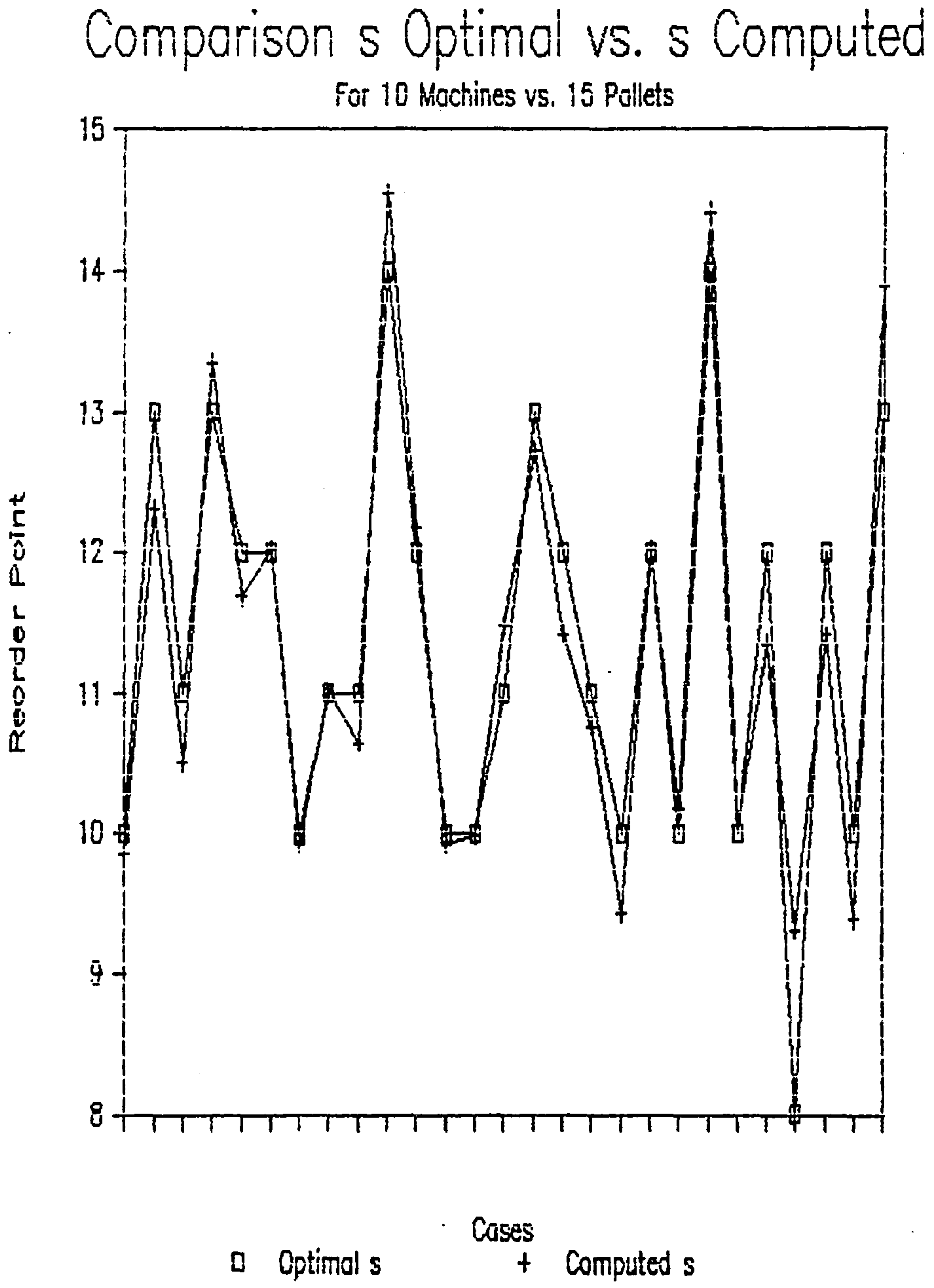

Comparison s Optimal vs. s Computed
For 10 Machines vs. 15 Pallets
15
14
13
12
11
10
9
8
Reorder Point
Cases
Optimal s
Computed s

Figure 5.6

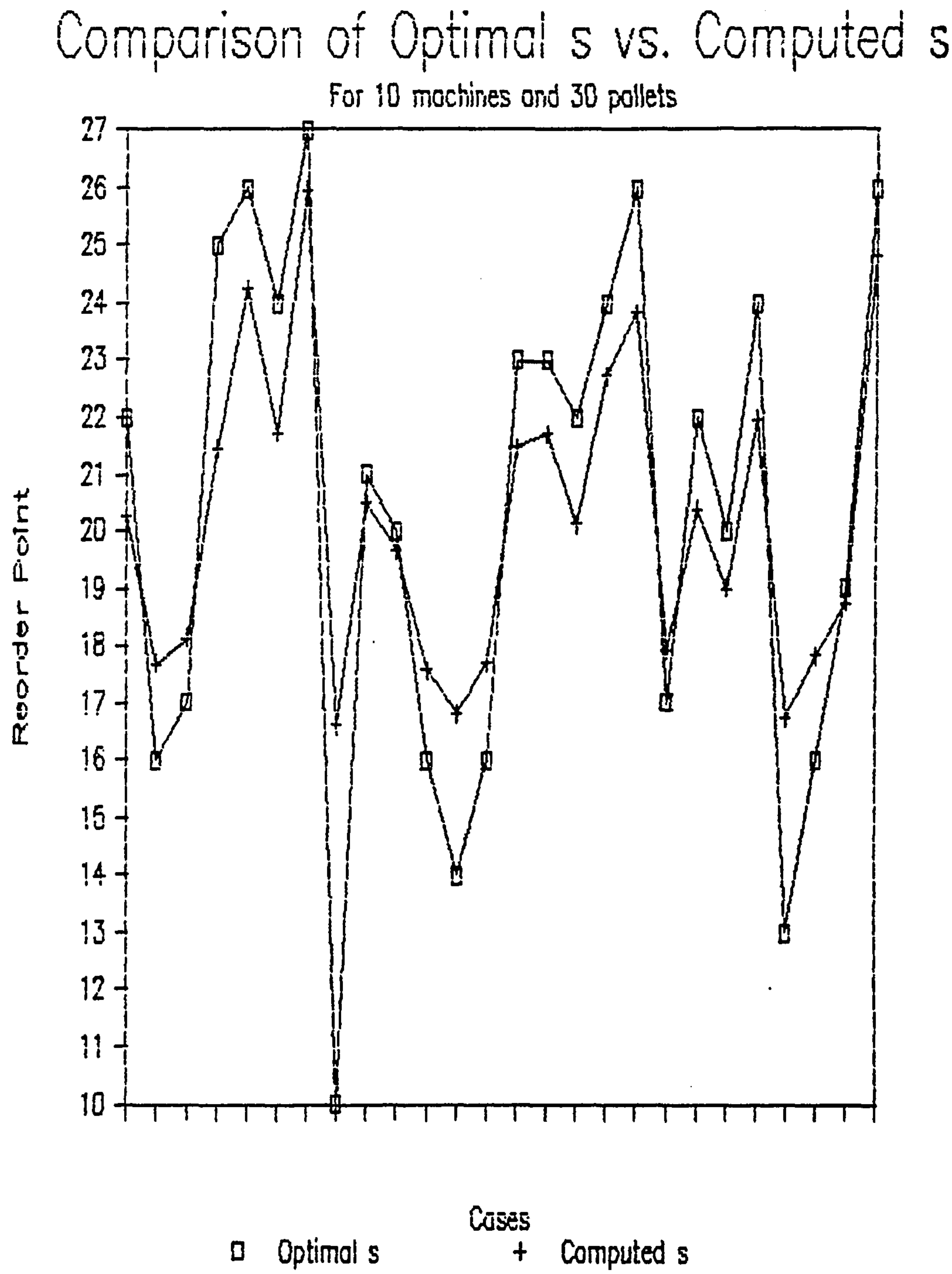

Comparison of Optimal s vs. Computed s
For 10 machines and 30 pallets
Reorder Point
27
26
25
24
23
22
21
20
19
18
17
16
15
14
13
12
11
10
Cases
Optimal s
Computed s

Figure 5.7

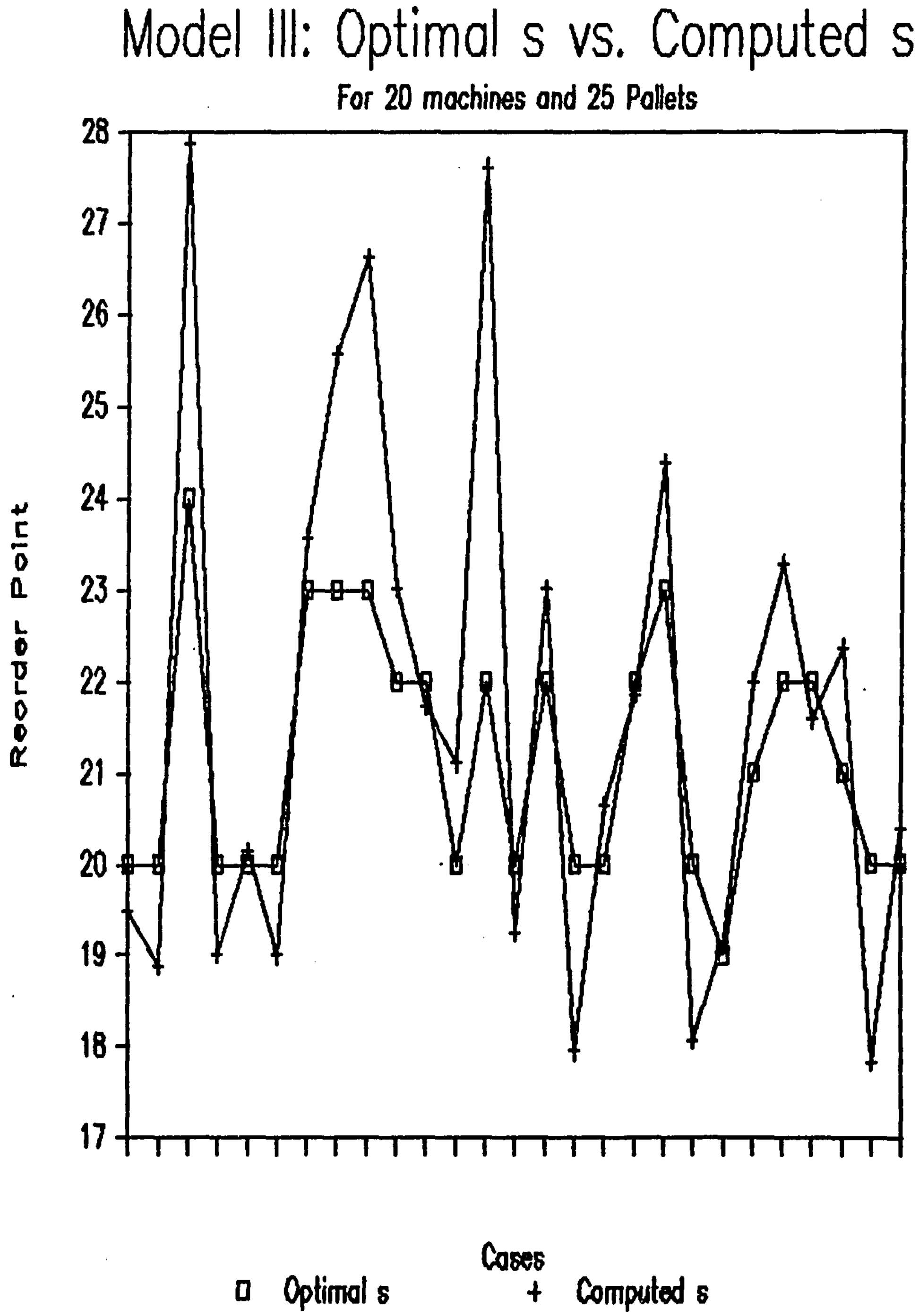

Model III: Optimal s vs. Computed s
For 20 machines and 25 Pallets
Reorder Point
28
27
26
25
24
23
22
21
20
19
18
17
Cases
Optimal s
Computed s

Figure 5.8

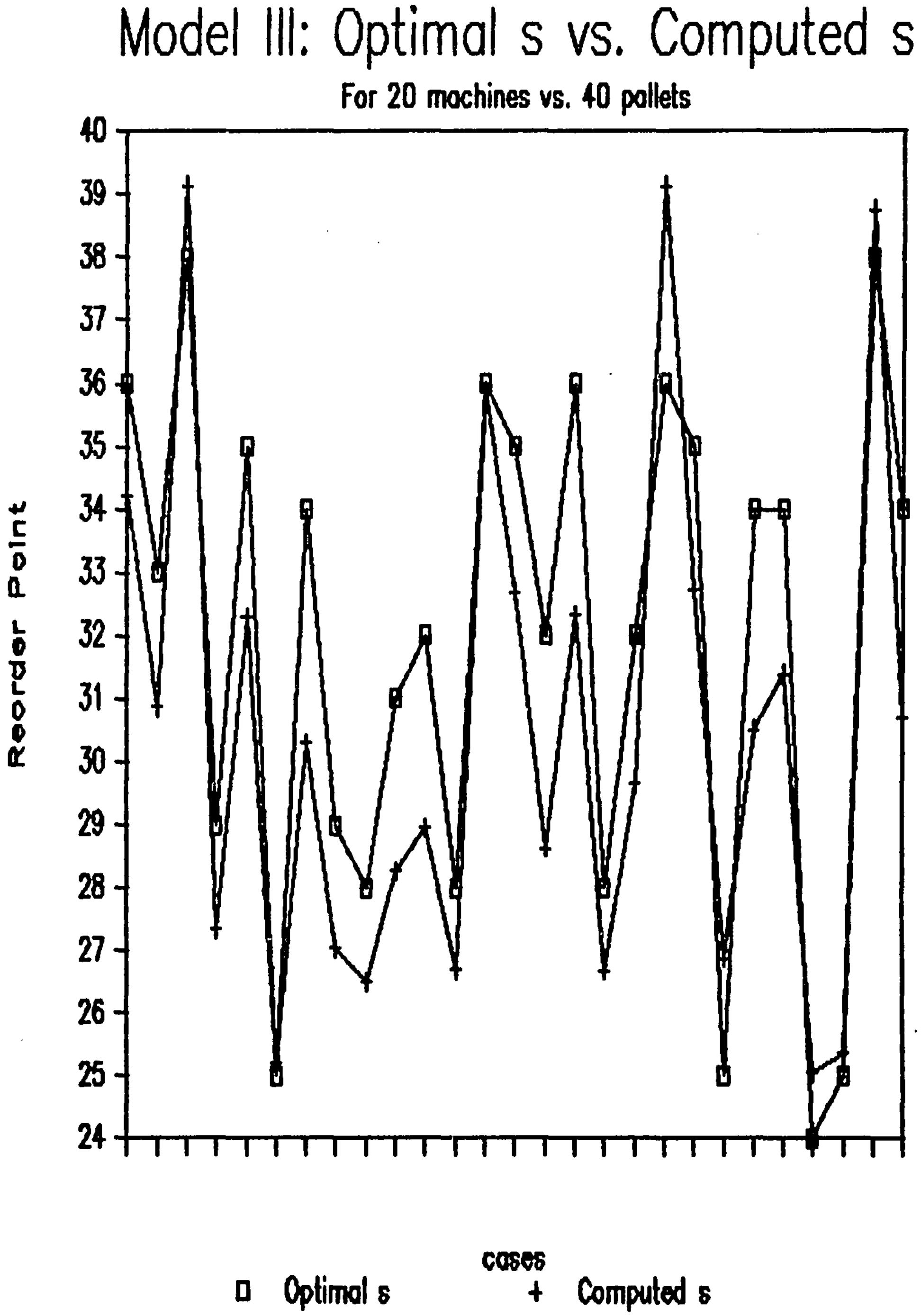

Model III: Optimal s vs. Computed s
For 20 machines vs. 40 pallets
Reorder Point
40
39
38
37
36
35
34
33
32
31
30
29
28
27
26
25
24
cases
Optimal s
Computed s

Figure 5.9

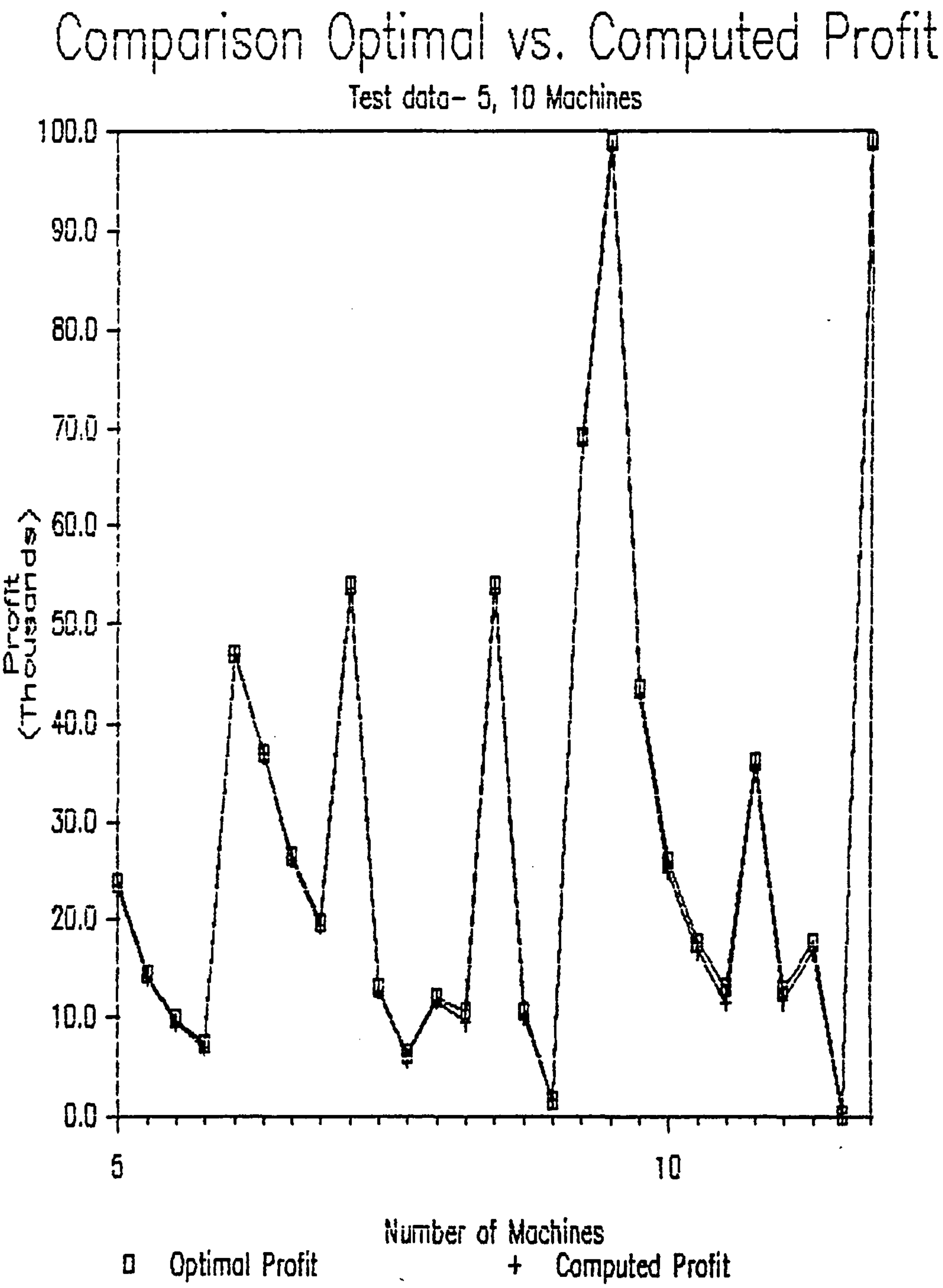

Comparison Optimal vs. Computed Profit
Test data- 5, 10 Machines
100.0
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
Profit
(Thousands)
5
10
Number of Machines
Optimal Profit
Computed Profit

Figure 5.10

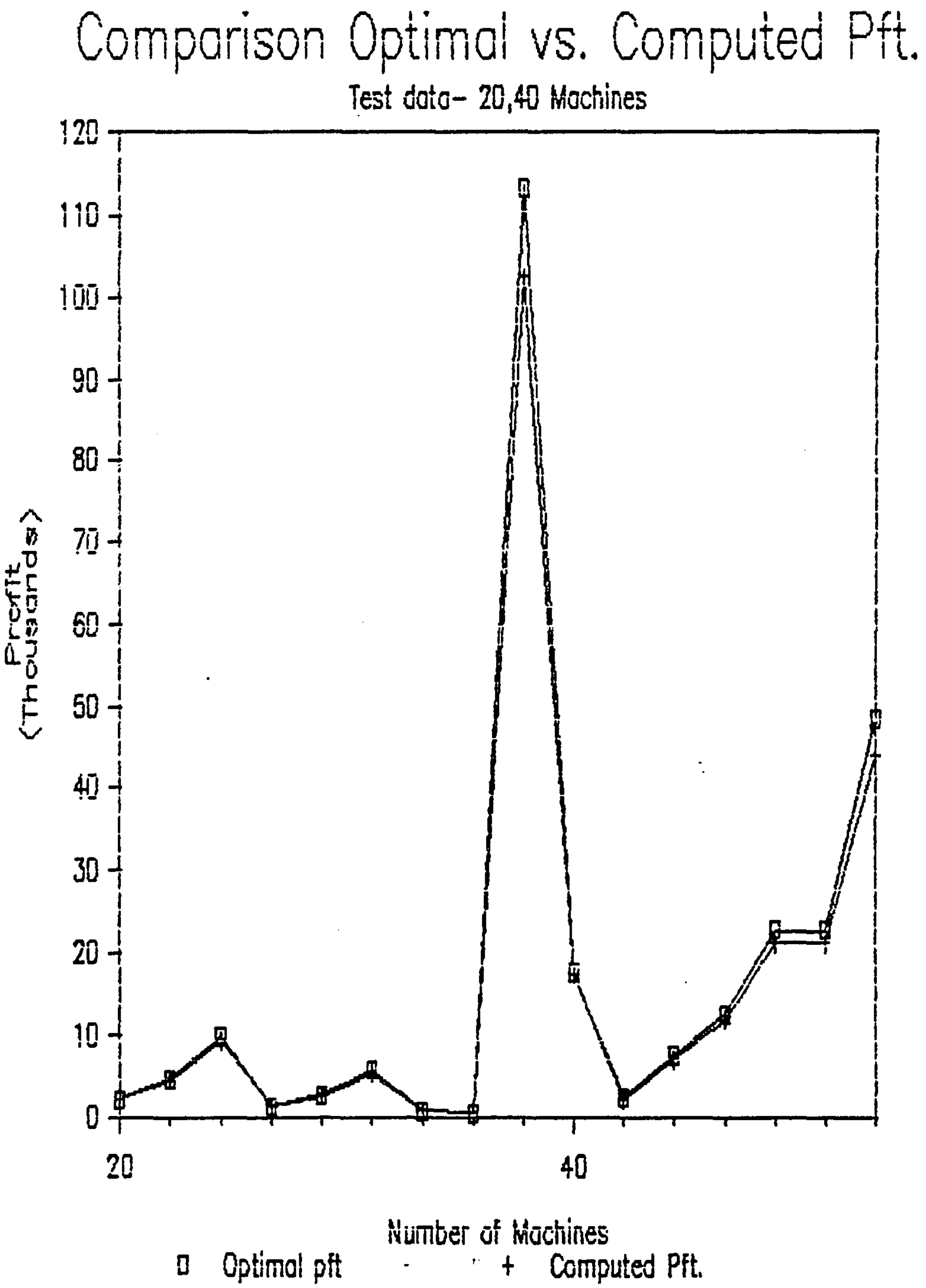
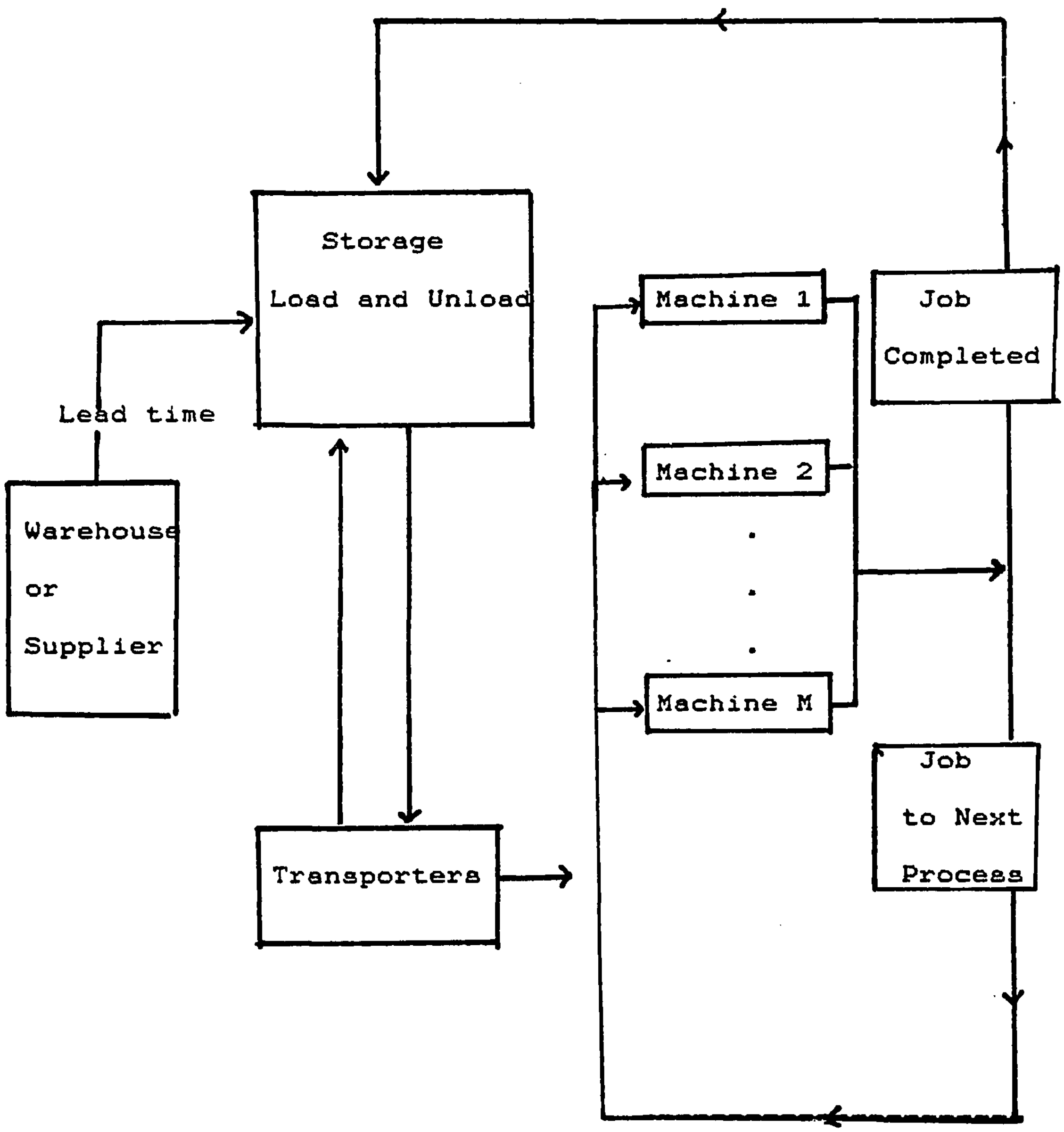

Comparison Optimal vs. Computed Pft.
Test data- 20,40 Machines
Profit (Thousands)
120
110
100
90
80
70
60
50
40
30
20
10
0
20
40
Number of Machines
Optimal pft
Computed Pft.

Figure 6.1 System Diagram for Model IV.



APPENDIX A: MODEL FORMULATIONS AND PROOFS

MODEL I:

Assume M=N=s

Objective: Find Optimal N and Q

M = No. of NC machines   s = Reorder point
N = No. of transporters   Q = Batch size (Buffer size)

$$\text{Max. } G(N,Q) = \sum_{n=N+1}^{N+Q} [U_nP_nC_p - (C_t/Q)U_nP_n] - C_sQ - C_mN$$

To find N*, let

$$\Delta_N G = G(N+1,Q)-G(N,Q)<0 \quad \text{and} \quad \Delta_N G = G(N-1,Q)-G(N,Q)<0$$

$$\text{or} \quad \Delta_N G = G(N+1,Q)-G(N,Q) \rightarrow 0$$

where

$$G(N+1,Q) = [(\sum_{n=N+1}^{N+Q+1} U_nP_n)(C_p-(C_t/Q))]-C_sQ-C_m(N+1) \ldots\ldots(1)$$

$$G(N,Q)=[\sum_{n=N+1}^{N+Q} U_nP_n)(C_p-(C_t/Q))]-C_sQ-C_m(N) \ldots\ldots\ldots\ldots(2)$$

where $U_n = \dfrac{n\ M}{L(n+M-1)}$ for n=N+1, N+2,....N+Q, M=N

$$P_{N+1} = \sum_{j=N+1}^{N+Q} \left(\frac{U_{N\pm1}}{U_j}\right)^{-1}$$

$$P_n = \frac{U_{N\pm1}}{U_n} P_{N+1}$$

Substituting Un, P_{N+1} and P_n into Eqs. (1) and (2) we get:

$$G(N,Q) = \{(Q(N+1)N/L*2N)[\sum_{j=N+1}^{N+Q} (U_{N+1}/U_j)^{-1}]*$$

$$(C_p-(C_t/Q))\}-C_s*Q-C_m*N$$

$$G(N+1,Q)=((Q(N+2)(N+1)/2L(N+1))[\sum_{j=N+2}^{N+Q+1}(U_{N+1}/U_j)^{-1}]*$$

$$(Cp-(Ct/Q))\}-Cs*Q-Cm*N$$

$$G(N,Q)=(Cp(Ct/Q)Q)\{[(N+2)N/L(2N+1)][\sum_{j=N+2}^{N+Q+1}(U_{N+1}/U_j)]-$$

$$[(N(N+1)/L(2N)][\sum_{j=N+1}^{N+Q}(U_{N+1}/U_j)]\}-Cm$$

Let Eq. (3) approach 0 to find optimal N.

To derive the exact formulation for N optimal is formidable; therefore, computer search is used to analyze the behavior of $\Delta_N G$.

To find Q*, we let

$\Delta_Q G$ = G(Q+1,N) - G(Q,N), where

$$G(Q+1,N) = \{(Q+1)*U_{N+1}[\sum_{j=1}^{Q+1}(U_{N+1}/U_{N+j})^{-1}]\}$$

$$(Cp-(Ct/Q+1))-Cs*(Q+1)-Cm*N$$

$$G(Q,N) = (Q*U_{N+1}[\sum_{j=1}^{Q}(U_{N+1}/U_{N+j})^{-1}]*$$

$$(Cp-(Ct/Q))-Cs*Q-Cm*N$$

$$\Delta_Q G=U_{N+1}\{(Q+1)[\sum_{j=1}^{Q+1}(U_{N+1}/U_{N+j})]^{-1}(Cp-(Ct/Q+1))\}-$$

$$\{Q[\sum_{j=1}^{Q}(U_{N+1}/U_{N+j})]^{-1}(Cp-(Ct/Q))\}-Cs...(4)$$

Conditions for Q* are when:

$\Delta_Q G$ = G(Q-1,N)-G(Q,N)<O and $\Delta_Q G$ = G(Q+1,N)-G(Q,N)<O

To derive the exact solution for Q* is very complicated, so we use approximation to derive the lower bound solution.

Since:

$$U_{N+1} < U_{N+2} < U_{N+3} <...... < U_{N+Q+1}$$

so $\frac{U_{N+1}}{U_{N+2}} > \frac{U_{N+1}}{U_{N+3}} > .... \frac{U_{N+1}}{U_{N+Q+1}}$

and

$$\frac{U_{N+1}}{U_{N+Q+1}} = \frac{(N+1)M}{L(N+M)} \cdot \frac{L(M+N+Q)}{M(N+Q+1)} = \frac{MN+M+N^2+N+Q(N+1)}{MN+M+N^2+N+MQ+NQ} \rightarrow 1$$

Let $\frac{U_{N+2}}{U_{N+j}}$ = 1 for j=1,....., Q+1

then $(\sum_{j=1}^{Q+1}\frac{U_{N+1}}{U_{N+j}})^{-1} = \frac{1}{Q+1}$

and

$$\left(\sum_{J=1}^{Q} \frac{U_{N+1}}{U_{N+J}} \right)^{-1} = \frac{1}{Q}$$

$\Delta_Q G = U_{N+1}\{[Cp-(Ct/Q+1)]-[Cp-(Ct/Q)]\}- Cs$

When $\Delta_Q G \rightarrow 0$

$Q(Q+1) \rightarrow Ct\ (U_{N+1})/Cs = (Ct * N+1)/(Cs*2L)$ ....(5)

where $U_{n+1} = M(N+1)/L(M+N+1-1)$ and $M=N$

MODEL II:

Assumptions:

1) M - No. of machines is fixed and is an uncontrolled variable.

2) N - No. of transporters is fixed and is also an uncontrolled variable.

3) Production capacity is fixed $U_n = \frac{N\ M}{L(N+M-1)}$

for all n = s, s+1, ..., s+Q

4) Reorder point s > N

5) Let additional cost coefficient Ci = inventory carrying cost per part in the cell per unit of time.

The objective in this case:

$$\text{Max. } G(s,Q) = \sum_{n=s}^{s+Q} [Cp-U_nP_n-Ci*n*P_n-(Ct/Q)U_nP_n] -Cs(s+Q-N)$$

$$= \sum_{n=s}^{s+Q} U_nP_n[Cp-(Ct/Q)-(Ci*n)/U_n]-Cs(s+Q-N)$$

since loss of production is not allowed.

Therefore, s = N, and

$U_n = U_N$ for all states, s, s+1, ..., s+Q

Eq. (6) becomes Eq. (7)

$$\text{Max } G(s,Q) = U_N[Cp-(Ct/Q)]-Ci(s+(Q/2))-Cs(s+Q-N)$$

$$= U_NCp-NCs-(Ci+Cs)s-[(Ci/2)+Cs)]Q-CtU_N/Q] \quad \ldots\ldots\ldots\ldots(7)$$

$$\frac{\partial G(s,Q)}{\partial Q} = 0$$

Therefore $(C_i/2)+C_s-(C_t*U_N)/Q^2 = 0$

$$Q* = \{(C_t*U_N)/[(C_i/2)+C_s]\}^{.5} \ldots. \quad (8)$$

Model III: proof

Max. $G(s,Q)=$

$$\sum_{n=s+1}^{s+Q} \{(Cp*Un*Pn)-(Ci*n*Pn)-[(Ct/Q)*Un*Pn]\}-Cs(s+Q-N)\ldots(9)$$

where $Ps+1=$

$$[1+(Us+1/Us+2)+(Us+1/Us+3)+\ldots\ldots+(Us+1/Us+Q)]^{-1}$$

$$=(1/Us+1)*[(s+Q-N)(1/U_N)+\sum_{n=s+1}^{N}(1/U_N]^{-1}$$

$P_n=(Us+1/Un)(Ps+1)$ for $n=s+1,s+2,\ldots\ldots\ldots,s+Q-1$

$Un=\frac{M*n}{L(M+n-1)}$ for $n= s+1, s+2,\ldots\ldots,N-1$

$Un=\frac{M*N}{L(M+N-1)}$ for $n= N,N+1,\ldots\ldots\ldots,N+Q$

therefore,

$(Us+1)(Ps+1)=$

$$(Us+1)\{(1/Us+1)*[(s+Q-N)(1/U_N)+\sum_{n=s+1}^{N}(1/Un)]\}^{-1}\ldots\ldots\ldots\ldots(10)$$

Subsititute U_N into above equation (2), we get:

$(Us+1)(Ps+1)=$

$$(M/L)\{(s+Q-N)[1+((M-1)/N)]+(N-s)+(M-1)\sum_{n=s+1}^{N}(1/n\ )\}^{-1}$$

At steady state, rate in= rate out, and

$(Un)(Pn)=(U_N)(P_N)$ for all $n= s+1, s+2,\ldots\ldots,s+Q.$

$$\sum_{n=s+1}^{s+Q} Un*Pn=$$

$$(M/L)*(Q)*[(s+Q-N)(1+((M-1)/N)]+(N-s)+(M-1)\sum_{n=s+1}^{N}(1/n)\}^{-1}\ldots(11)$$

$$\sum_{n=s+1}^{s+Q} n*Pn$$

$$= [(s+1)Ps+1]+[(s+2)Ps+2]+........+[(s+Q)Ps+Q]$$

$$= \sum_{n=s+1}^{N} (n/Un)[(Us+1)(Ps+1)] + \sum_{n=N+1}^{s+Q} (n/U_N)[(Us+1)*(Ps+1)] \quad ......(12)$$

It can be varified that equation (4) equals:

$$\sum_{n=s=1}^{s+Q} n*Pn=$$

$$\frac{(N-s)[(M-1)+(N-s-1)/2]+(s+Q-N)(s+Q+N-1)[(.5+(M-1)/(Q*N\ )]}{(N-s)+(s+Q-N)[1+(m-1/N)]+(M-1)\sum_{n=s+1}^{N}(1/n)} \quad ....(13)$$

Subsititute equation (3) (5) to (1), we get:

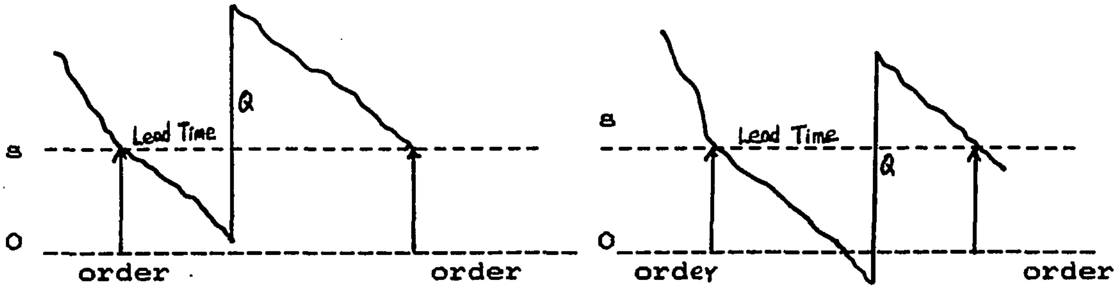
G(s,Q)=

$$\{(M/L)(Q)[Cp-(Ct/Q)]-Ci[(N-s)(M-1+(N+s-1)/2)+(s+Q-N)(s+Q+N+1)$$

$$(.5+(M-1)/2N)\}/\{(N-s)+(s+Q-N)(1+((M-1)/N))+(M-1)\sum_{n=s+1}^{N}(1/n)\}$$

$$-Cs(s+Q-N)$$

MODEL IV: Proof



(1) $s > q_L$ (2) $s < q_L$

Expected Average Inventory During Lead Time

$$= \sum_{q_L=0}^{s} 1/2[s+(s-q_L)]P_L(q_L) + \sum_{q_L>s} 1/2(s)P_L(q_L)$$

$$= 1/2[s + \sum_{q_L=0}^{s} (s- q_L)P_L(q_L)]$$

Expected Average Inventory Level after Replenishment until next order

$$= 1/2[s-M_L+Q)+s] - 1/2[2s-M_L+Q]$$

Expected Average Inventory Level

$$=M_L/2Q\ [-2s+M_L-Q+s+\sum_{q_L=0}^{s} (s-q_L)P_L(q_L)]$$

$$+[1/2(2s-M_L+Q)]\sum_{q_L>s}^{s} (s-q_L)P_L(q_L) \quad \ldots(14)$$

$$= s-M_L-\sum_{q_L>0}^{s} (s-q_L)P_L(q_L) \quad \ldots(15)$$

Substitute Eq. (10) into Eq. (9)

$$I(s,Q)= M_L/2Q[-2s+M_L-Q+2s-M_L$$

$$-\sum_{q_L>s} -(s-q_L)P_L(q_L)] \; -[1/2(2s-M_L+Q)]$$

$$= (Q/2)-M_L+s+M_L/2Q \sum_{q_L>s}(q_L-s)P_L(q_L) \quad \ldots\ldots(16)$$

$$R(s,Q) = [U_N Cp - Cp\sum_{q_L>S}(q_L-s)P_L(q_L\;)] \quad \ldots\ldots(17)$$

$$T(s,Q)=Ct/Q[U_N-\sum_{q_L>s}(q_L\;-s)P_L(q_L)]-Cs(s+Q-N) \quad \ldots(18)$$

$$G(s,Q) =$$

$$[U_N Cp-Cp\sum_{q_L>S}(q_L-s)P_L(q_L)]-Ci[Q/2-M^L+s+M_L/2Q$$

$$\sum_{q_L>s}(q_L-s)P_L(q_L)]-Ct/Q[U_N-\sum_{q_L>s}(q_L\;-s)P_L(q_L)] \quad \ldots(19)$$

To maximize G(s,Q), we solve

$$\frac{\partial G(S,Q)}{\partial Q} = 0 \qquad \frac{\partial G(s,Q)}{\partial s} = 0$$

$$\frac{\partial G(S,Q)}{\partial Q} = (-Ci/2)+[(CiM_L/2Q^2)-(Ct/Q^2)]*$$

$$[\sum_{q_L>s}(q_L-s)P_L q_L]-Cs+(Ct/Q^2)U_N=0$$

and

$$\frac{2CtU_N}{2Q^2} + \frac{(C_iM_L\;C_t\;)}{2Q^2}[\sum_{q_L>s}(q_L-s)P_L q_L]$$

$$= (Ci+2Cs)/2$$

therefore,

$$Q^2 = \frac{(CiM_L-2Ct)[\sum_{q_L>s}(q_L-s)P_L(q_L)]+2CtU_N}{Ci + 2Cs}$$

$$Q = \sqrt{\frac{2Ct*U_N+(CiM_L-2Ct)\sum_{q_L>s}(q_L-s)P_L(q_L)}{Ci + 2Cs}} \quad ....(20)$$

Let $\sum_{q_L>s}(q_L-s)P_L(q_L) = b(s)$

$$Q = \sqrt{\frac{2CtU_N+(CiM_L-2Ct)b(s)}{Ci + 2Cs}} \quad ....(21)$$

$$\frac{\partial^2 G(s,Q)}{\partial Q^2} = \left[\frac{(-CiM_L)}{2Q^3} + \frac{Ct}{Q^3} b(s)\right] - \frac{CtU_N}{Q^3} < 0$$

$$\frac{CtU_N}{Q^3} > \frac{-M_L}{2Q^3} + \frac{Ct}{Q^3}$$

Since $\frac{\partial^2 G(s,Q)}{\partial Q^2} < 0$ Maximize

$\frac{\partial G(s,Q)}{\partial s} = 0$ we get

$$-Cp\ (\partial b(s)/\partial s)-Ci-(CiM_L/2Q)(\partial b(S)/\partial s) +(Ct/Q)(\partial b(s)/\partial s)-Cs=0$$

$$\partial b(s)/\partial s=[2Q(Ci+Cs)]/[2QCp+CiM_L-(Ct/Q)]$$

$$=-(Ci+Cs)/[Cp-(Ct/Q)+(CiM_L)/2Q]$$

$$..........(22)$$

$$\partial b(s)/\partial s = \int_s^{\infty}(x-s) f(x)dx = -\int_s^{\infty} (x)dx = -F'(s)$$

$$F'(s) = \frac{Ci + Cs}{Cp-(Ct/Q)+(CiM_L/2Q)} \quad ....(23)$$

(Assume $Cp-Ci-Cs-(Ct/Q) > 0$)

$$b(s) = \int_s^{\infty}(x-s) f(x)dx$$

or $\sum_{q_L=s}^{\infty} (q_L-s)P_L(q_L)$

Using Normal Approximation (Veinott and Wagner):

For $U_N > 5$, Normal Distribution can be used as an approximation for Poisson Distribution. b(s) can be expressed as a unit normal loss integral L'(.).

$$b(s) = \sigma L'[(s-u)/\sigma] \quad ....(24)$$

Use iterative procedure to find the optimal pair (s^*,Q^*) as follows:

(1) Assume b(s) = 0. Compute Q with Eq. (21). Find Q_1.
(2) Use Q_1 with Eq. (23) to find s_1.
(3) Use s_1 with Eq. (24). Find Q_2.
Continue procedure (2) above until convergence occurs.

Appendix B

MODEL I- TURBO.PASCAL COMPUTER PROGRAM FOR SEARCH

G(N*,Q) AND G(N,Q*)

```
program equation;
const size=100;
type num=record
xn,xm,xq,xcs,xcm,xcp,xct:integer;
xu,xp,xl,xg:real
end;
var
yy,a,count,n,m,q,cs,ct,cp,cm:integer;
gng,pnl,un,l:real;
yyy,want:char;
list:array [1..size] of num;
{-----------------------------------}
function u(n,m:integer;l:real):real;
begin
    u: = n*m /(l * (n+m-1));
end;
{-----------------------------------}
function p(n,m:integer;l:real;q:integer):real;
var
  i:integer;
  k:real;
begin
  k:=0;
  for i:=1 to q do
   begin
     k:= k +(u(n=1,m,l) / u (n+i,m,l))
   end;
   p:= 1/k
end;
{-------------------------------------}
function g(q,n,m:integer;l:real;cs,cm,cp,ct:integer):real;
var
  p1,p2,p3:real;
begin
   p1:= q * u(n+1,m,l) * p(n,m,l,q) * (cp-(ct/q));
   p2:= cs*q;
   p3:= cm*n;
   g:= p1-p2-p3
end;
{---------------------------------------}
procedure makearray;
begin
      with list [count] do
        begin
          xn:=n;
```

```
          xm:=m;
          xl:=l;
          xq:=q;
          xcs:=cs;
          xcm:=cm;
          xcp:=cp;
          xct:=ct;
          xu:=un;
          xp:=pnl;
          xg:=gnq
        end;
end;
{-------------------------------------------}
procedure printlist;
begin
   gotoxy(1,1);
   textcolor(2);
   writeln('q n m l cs cm cp ct U(n+1) P(n+1) G(N Q')
   textcolor(3);
   for a:=1 to count-1 do
    begin
     gotoxy(1,a+1);
     with list [a] do
       begin
         write (xq:3,xn:5,xm:5,xl:8:3,xcs:5,xcm:6,xcp:6);
         write (xct:6,xu:11:4,xp:11:4,xg:13:4);
       end;
    end
end;
{-----------------------------------------------}
{main program}
begin
   want:='y'; yyy:='y';
   yy:=0;
   writeln(' input: q n m l cs cm cp ct');
   read(q,n,m,l,cs,cm,cp,ct);
   while yyy= 'y' do
     begin
      count:=1;
      while (want='y') and ( yy<10) do
       begin
        yy:=yy+1;
        un:= u(n+1,m,l);
        pnl:=p(n,m,l,q);
        gnq:=g(q,m,n,l,cs,cm,cp,ct);
        makearray;
        count:= count+1;
        q:=q+1;
       end;
       textcolor(5);
       clrscr;
       printlist;
       write(' do you want to continue? Y/N ');
```

```
        read(yyy);
        yy:=1;
        clrscr;
    end;
end.
```

SAMPLE OUTPUTS OF MODEL I

INTERACTIVE COMPUTER SEARCH FOR N*

n	q	l	cs	cm	cp	ct	U(N+1)	P(N+1)	G(N,Q)
3	6	.1	100	400	200	100	20.00	.1877	3431.51
4	6	.1	100	400	200	100	21.42	.1820	3786.26
5	6	.1	100	400	200	100	22.50	.1784	4030.89
6	6	.1	100	400	200	100	23.33	.1760	4211.93
7	6	.1	100	400	200	100	24.00	.1742	4350.91
8	6	.1	100	400	200	100	24.55	.1729	4459.61
9	6	.1	100	400	200	100	25.00	.1720	4545.22
10	6	.1	100	400	200	100	25.38	.1712	4613.48
11	6	.1	100	400	200	100	25.71	.1706	4664.73
12	6	.1	100	400	200	100	26.00	.1701	4704.44
13	6	.1	100	400	200	100	26.25	.1697	4733.45
14	6	.1	100	400	200	100	26.47	.1693	4753.24
15	6	.1	100	400	200	100	26.66	.1691	4764.96
16	6	.1	100	400	200	100	26.84	.1688	4769.58
17	6	.1	100	400	200	100	27.00	.1686	4767.87
18	6	.1	100	400	200	100	27.14	.1684	4760.50
19	6	.1	100	400	200	100	27.27	.1683	4748.02
20	6	.1	100	400	200	100	27.39	.1682	4730.92

Multiple Regression Summary for Model I

T1= ln (Q)
T2 = ln (M)

Model I
Multiple Regression Outputs - 54 Item Data Set

-----Variables in the Equation for Q at Step 1---

Variable	B (Slope)	Std Error of B	F
T1	1.073353	.03638	870.461
Constant	.314543D-01		

----Variables in the Equation for Q at Step 2------

Variable	B (Slope)	Std Error of B	F
T1	1.057249	.03853	752.896
T2	.6585112D-01	.05387	1.494
Constant	-.7355344D-01		

Summary of Multiple Correlation and R Square

Dependent Variable: Batch Size(Q)

Variable	Multiple R	R Square
T1	0.97141	0.94363
T2	0.97223	0.94523

APPENDIX B- MODEL III

MODEL III- TURBO.PASCAL PROGRAM FOR GRID SEARCH

```
program search;
const tn= 5;
      nn=3; (*n=m+1 *)
      lw=8;
      ll=0.1; mm=5; nnn=10; ct=150.0; ci=100.0; cs=40.0;
      cp=200.0;
type vector =array[1..nn] of real;
      grd = array [0..tn,0..tn] of real;
      index = 0..255;
var   mh,adj,ss,l,r,t,b,h1,h2: real;
      i,j:index;
      m,n:1..nn;
      din: text;
      dsc:string[80];
      sq: grd;
      st: string[80];
      ch: char;
procedure gl;
  var i,j:integer;
  begin
    adj:=0; mh:999.0;
    b:=95;t:=100;l:=85;r:=90;
    h2:=(r-l)/tn; h1:=(t-b)/tn;
    write('   !'); for j:=1 to 60 do write('-'); writeln;
  end;

procedure sor (var x: vector);
  var ss, kn:integer;
       q,s,sum,g,gl:real;
  begin
    s:=x[1];
    q:=x[2];
    g:=(mm/ll)*q*(cp-ct/q)-ci*((nnn-s)*(mm-1+(nnn+s-1)/2)
       +(s+q-nnn)*(s+q+nnn+1)*(.5+(mm-1)/(2*nnn)));
   ss:=trunc(s+1);if ss=0 then ss:=1; sum:=0;
   for kn:=ss to nnn do sum:=sum+(1/kn);
   gl:=(nnn-s)+(s+q-nnn)*(1+(mm-1)/nnn)+(mm-1)*sum;
   g:=g/gl-cs*(s+q-nnn);
   x[3]:=g
  end;
procedure grid;
   var i,j: integer;
       x:vector;
   begin
    gl;
    for i:=tn down to 0 do
      begin
       x[1]:=b +i*h1;
       write(x[1]:9:2,'!');
```

```
          for j:=0 to tn do
             begin
                x[2]:=1 + j*h2;
                sor(x);
                write(x[3]:10:2);
             end;
           writeln;
          end;
   for j:=1 to 70 do write ('-'); writeln;
       end;
begin
   m:=2;n:=3;
   grid;
   write ('L=',ll:5:2,' n=',nnn:2,' m=',mm:2, ' cp=',cp:6:1)
   writeln('ci=',ci:6:1,  'ct=',ct:6:3, " cs=", cs:6:3)
   end.
```

SAMPLE OUTPUTS OF MODEL III

COMPUTER DIRECT TWO DIMENSIONAL GRID SEARCH

s\Q :	10.0	12.0	14.0	16.0	18.0	20.0
11.0	12.2	768.8	1280.0	1636.1	1888.1	2066.7
10.0	135.7	908.5	1426.3	1784.6	2036.6	2214.2
9.0	262.9	1052.2	1576.2	1936.3	2188.0	2364.3
8.0	380.3	1168.3	1691.5	2050.9	2302.1	2477.9
7.0	492.8	1278.2	1800.1	2159.0	2409.9	2585.7
6.0	599.2	1380.3	1900.6	2258.9	2509.9	2685.9

L=.0.01 n=10 m=5 cp=20.0 ci=100.0 ct=150.0 cs=10.0

MODEL III: SPSS PROGRAM FOR NONLINEAR MODELS

```
RUN NAME        DETATL ANALYSIS OF FMS DESIGN
FILE NAME       MODEL III- LOGLINEAR MODEL
VARIABLE LIST   M,N,CT,CI,CS,S,Q
N OF CASES      108
INPUT FORMAT    FREEFIELD
COMPUTE         T1=LN((N*M)/(.01*(N+M-1)))
COMPUTE         T2=LN(2*CT/(CI+2*CS))
COMPUTE         T3=LN(((CI+CS)/200)*Q)
COMPUTE         T4=LN(.01*(N+M-1))
COMPUTE         Y1=LN(Q)
COMPUTE         Y2=LN(S)
REGRESSION      VARIABLES = Y1,Y2,T1,T2,T3,T4/
                REGRESSION = Y1 WITH T1,T2/
                REGRESSION = Y2 WITH T3,T4,T5/
```

FORTRAN PROGRAM FOR MODEL III

COMPARISON OF OPTIMAL SEARCH VS. APPROXIMATION MODEL

```
/LOAD FORTG1
       DOUBLE PRECISION X1,X2,X3,Q1,Q2
       WRITE(6.18)
       WRITE(6,22)
11     READ(5,10,END=100) XM,XN,CT,CI,CS,CP,WL,MS,MQ,OP
10     FORMAT(5F8.1,F7.1,F6.4,2I4,F9.1)
C COMPUTE CRUDE MODEL FOR Q
       QQ=SQRT(((2*CT)/(CI+2*CS))*((XN*XM)/(WL*
      *(XN+XM-1))))
       NQ=IFIX(Q)
C COMPUTE Q BY POWER APPROXIMATION
       Q1=ALOG(2*CT/(CI+2*CS))
       Q2=ALOG((XN*XM).(WL*(XN+XM-1)))
       Q=DEXP( .4507591*Q1 + .5255085*Q2 - .0972503)
C COMPUTE S BY POWER APPROXIMATION
       X1=ALOG(WL*(XN+XM-1))
       X2=ALOG(((CI+CS).CP)*q)
       X3=ALOG(XN-XM)
       S=DEXP(1.162389*X1 -.1960183*X2 +.039015*X3
      *+4.787315)
       NS=IFIX(S)
       DS=MS-S
       DQ=MQ-QQ
       PS=DS/MS
       PQ=DQ/MQ
       DQ1=MQ-Q
       PQ1=DQ1/MQ
       SUMN=0
       P1=(XM/WL)*Q*(CP-CT/Q))
       P2=CI*(((XN-S)*(XM-1+(XN+S-1)/2))+(S+Q-XN)
      **(S+Q+XN+1)*(.5+(XM-1)/(2*XN)))
       NNN=IFIX(XN)
       NN=NS+1
       DO 2 N =NN,NNN,1
       SUMN=SUMN + (1/FLOAT(N))
2      CONTINUE
```

```
       P3=(NNN-S)+((S+Q-NNN)*(1+((XM-1)/NNN)))+((XM-1)
      **SUMN
       PP=((P1-p2)/p3)-CS*(S+Q-XN)
       PPP=(OP-PP)/OP
       WRITE (6,20) XM,XN,CT,CI,CS,CP,WL,Q,S,QQ,MS
      *MQ,PS,PQ1,PQ,PPP
18     FORMAT (///2X,1HM,8X,1HN,6X,2HCT,6X,2HCI,
      *6X,2HCS,6X,2HCP,6X,1X,6HCOMP.Q,2X,6HCOMP.S,3X,
      *5HCRU.Q,2X,5HOPT.S,2X,5HOPT.Q,2X,6H%DIF.S,2X,
      *6H%DIF.Q,2X,7H%DIF.CQ,2X,8H%DIF.PFT)
22     FORMAT (4H----,2X,6H------,2X,5H-----,2X,6H------,
      *6H------,2X,6H------,2X,6H------,2X,6H------,2X,
      *6H------,2X,5H-----,2X,5H-----,2X,6H------,2X,
      *6H------,2X,6H------,2X,6H------,2X,8H--------)
20     FORMAT (F5.1,F7.1,4F8.0,F8.3,3F8.2,2I7,4F8.4)
       GO TO 11
100    STOP
       END
/DATA
```

Multiple Regression Outputs Summary for Model III

Let: T1= ln {(N*M)/[L*(N+M-1)]}
T2= ln {[(2*Ct)/[Ci+(2*Cs)]}
T3= ln {Cp/[(Ci+Cs)*Q]}
T4= ln (N+M-1)

Multiple Regression Outputs - 108 Items Data Set

----Variables In The Equation for Batch Size (Q)---

Variable	B (Slop)	Standard Error of B	F
T2	.4507591	.00895	2539.101
T1	.5255085	.03763	194.998
Constant	-.9725027D-01		

-Variables In The Equation for Reorder Point (s)--

Variable	B (Slop)	Standard Error of B of B	F
T4	1.213214	0.02831	1936.187
T3	.1976827	0.01310	227.580
(Constant)	-.6556857		

Summary Table for Multiple Correlation and R Square

A. Dependent Variable: Batch Size(Q)

Variable	Multiple R	R Square
T2	0.94546	0.89389
T1	0.98126	0.96286

B. Dependent Variable : Order Point (s)

Variable	Multiple R	R Square
T4	.91186	.83148
T3	.97303	.94680

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