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A Voting Paradox and the Deficit: Arrow's Impossibility Theorem at Work

or

Why a legislature full of Stubborn Conservatives Would Run a Deficit

by

Karl Phillip Widerquist

A dissertation submitted to the graduate faculty in Economics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

1996

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Abstract

A Voting Paradox and the Deficit: Arrow's Impossibility Theorem at Work

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Karl Phillip Widerquist

Adviser: Sali Neftci

This paper uses a game-theoretic approach to demonstrate that a voting paradox could create deficit bias in government budget decisions. Many political economy models show deficit bias. This model has the advantage of having all three of the following characteristics. (1) The deficit is unintentional. (2) All agents in the model are rational. (3) The deficit is the result of the inability of government to resolve disagreement among citizens about the composition of government spending. Alessina and Tabellini (1989, 1990) demonstrate that a deficit results from citizens' disagreement about the composition of government spending. However, this disagreement is combined with the willingness of the current government to leave a deficit in order to limit the spending options of future governments, making the deficit is ex post intentional. This paper, however, demonstrates that if preferences are sufficiently diverse, an Arrow type voting paradox could create contradictory political pressure. The government's inability

to resolve this conflict efficiently creates the deficit, rather than the desire of the current government to create a deficit.

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Chapter 1: Introduction

Disagreement about the composition of government spending can deficit bias. Despite unanimous agreement about the size of government spending and unanimous desire to have a balanced budget, citizens' disagreement about the composition of government spending can cause a voting paradox. Representatives' inability to compromise to resolve such a paradox can cause bias against a balanced budget.

This shows the deficit to be the unintentional result of disagreement among rational voters about the composition of government spending. Other political economy models have shown that disagreement among voters can indirectly lead to deficit bias by giving the current government an incentive to leave a deficit in order to limit the spending options of future governments (Alesina and Tabellini, 1990). There is bias, in the sense that the government has more incentive to create a deficit than it otherwise would, but the deficit in this model is intentional, it is exactly as large as the current government wants. This model, however, explains the deficit as the direct result of government's inability to compromise to resolve disagreement among voters about the composition of government spending. Representatives have no desire to create a deficit, they would be rewarded by citizens for balancing the budget, the political cost of doing so may be prohibitive.

Kenneth Arrow (1951), in what has often been called his "Impossibility theorem," demonstrated the conditions necessary to create a voting paradox, that is a situation in which majority rule cannot produce consistent results on a series of

issues, even though all voters vote rationally and consistently on every issue. This happens because the group of voters that constitutes majority varies from issue to issue. This model shows an example of a voting paradox in which it is impossible for majority voting to make consistent decisions about taxes, the deficit, and the level and composition of government spending. Majority voting can pass more total spending than anyone is willing to pay taxes to support, even if there is unanimous agreement about the optimal level of taxes and unanimous agreement that the budget should be balanced, and every citizen votes for the same amount of taxes spending. This happens, not because anyone wants more spending than taxes, but because each voter demands different spending from the same limited amount of taxes.

Suppose there are three citizens, Moe, Larry, and Curly, who must decide the level of taxes and the level of spending on three public goods, parks, highways, and railroads, each of which costs $1. Suppose that they unanimously agree that the optimal level of spending is $2, and so they unanimously agree that the optimal level of taxes $2. Moe votes for parks and highways, but against railroads. Larry votes for parks and railroads, but against highways. Curly votes for highways and railroads, but against parks. Thus, spending totaling $3 can pass by majority vote, even though everyone votes consistently with the belief that the optimal budget is balanced with $2 in taxes and $2 in total spending.

	Moe:	Larry:	Curly:	Majority:	Vote:
Optimal level taxes	2	2	2	2	3 to 0
x	yes	yes	0	pass	2 to 1
y	yes	0	yes	pass	2 to 1
z	0	yes	yes	pass	2 to 1

Sum of spending with majority support	3
Taxes that the majority is willing to pay	2

This does not mean that there will be a deficit, but it does mean that it is *impossible* for "the majority" to rule on every issue. One, or a combination of the following things must happen: taxes will be larger than the majority wants, or spending will be smaller than the majority wants, or the deficit will be larger than the majority wants.

If citizens unanimously prefer a balanced budget to achieving their goals on spending and taxes, some compromise balanced budget is optimal, involving either higher taxes than the majority think is optimal or lower spending on each issue than the majority think is optimal, but a representative democracy may not be able to reach a compromise that balances the budget. Information problems may make representatives more concerned with the process of voting than with the outcome, this may prohibit them from compromising and deficit bias could result. Citizens cannot observe the next best possible compromise, they only observe the actual compromise and the votes of their representatives. Many political economy models show governments gaining votes maximizing the utility of their

constituents[1]. But, making a compromise to increase the utility of citizens will only increase votes, if citizens can compare the marginal utility of that compromise to the next best politically possible outcome. But, citizens do not observe the next best possible outcome they observe only the actual outcome and the voting record of their representatives. If citizens do not know anyone else's preferences, including their representative's, and do not know what compromises are politically possible, they won't be able to evaluate a compromise negotiated by their representative - they will not know if it is the best achievable compromise or not.

Because of this informational problems, citizens must decide how to vote (i.e. how to reward their representatives) based on how well their representatives voting record approximates their optimum and how well the outcome of the legislature's decision approximate their optimum. This means that representative leaders must play two, often contradictory, roles. One, as a representative, is to vote for their constituents optimum given the economic constraints, that is represent the views of their constituents. The other, as a leader or decision maker, is to determine the political constraints and to construct the best possible compromise if their constituents' optimum is politically impossible.

A person negotiating only for himself will make any compromise that increases his utility, but representatives are often better off fighting for their

[1] See Downs (1957), Mayhew (1974). There is no fundamental difference here from that approach, this is a vote maximization model, "parties formulate policies in order to win elections." The difference here is how the representatives are judged. Representatives are judged as individuals, not as members of ruling parties. Representatives do not themselves control the outcome, and are judged partly by how they vote on each issue whether that vote is a winner or not.

constituents' optimum and being defeated (to prove their dedication to their constituents' ideals), than they are compromising to improve the result (but risking that their constituents will suspect that they are not truly dedicated to achieving their optimum). If Moe, Larry, and Curly represent millions of constituents they can vote for their constituents' optimal budget by voting for $2 in taxes and $2 in spending, but they cannot achieve it, they will create a deficit. Any compromise that makes their constituents better off by balancing the budget would force the representatives to vote *against* their constituents' optimum. The better leader they are, the poorer representative they are. Thus, even though a compromise makes their constituents better off, it involves a political cost. Unless citizens know exactly what political compromises are possible, or have complete trust in their representatives, the political cost of compromising will create a bias against a balanced budget.

Chapter 2 reviews the literature on this topic, showing how this model fits in. Chapter 3 uses constrained optimization demonstrating that the voting paradox is derived from the optimization behavior of rational voters. Politically feasible budgets are compared to the budget of a benevolent social planner,. Chapter 4 uses game theory to examine the voting behavior of a legislature. The necessary conditions for such a voting paradox to exist are examined, and the necessary conditions for political costs to prohibit a balanced budget are examined. Chapter 5 briefly considers the effect of possible decisions rules on this problem.

Chapter 2: Literature Review

This approach to the deficit has a number of desirable characteristics. Each of which will be discussed in turn. (1) This model explains observations about the budget are often explain by using irrational behavior of voters. (2) The model does not rely on selfish rent seeking behavior, but finds similar results as models that do. (3) The bias in this model can only produce a deficit, not a surplus. (4) The deficit is the unintentional result of disagreement about the composition of government spending (5) This paper shows that the median voter theorem may by applicable to the budget because it may not be possible to break down the budget to a one dimensional issue. (6) This paper is consistent with certain observations about government decision making.

2.1

All agents in this model, are rational but the outcome is deficit bias. Much of the literature showing deficit bias relies on fiscal illusion, an irrationality on the part of voters.[2] Citizens attempting to receive as much in government services as possible, but pay as little as possible, reward their representatives for increasing spending, but punish them for raising taxes. True or false, this is assumption inconsistent with one of the most common assumptions in economic literature - rational behavior - voters fail to see the consequences for their behavior. In this model citizens as a whole demand more spending than they are willing to pay taxes while individually each of them demands the same amount of taxes and spending.

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[2] See Buchanan and Wagner (1977).

Thus this model explains the type of observations of the fiscal illusion models without any individual being irrational.

This model also explains perception that voters suffer from fiscal illusion. The majority of voters prefer a balanced budget, but a majority of citizens oppose any specific plan to balance the budget implies that the majority of citizens are irrational. Unless the breakdown of voting is known, and the voting paradox is revealed, each citizens may perceive that the other citizens prefer more spending than taxes. If a voting paradox exists, a majority of voters support *some* plan to balance the budget, but only a minority supports any particular plan. The problem is not irrational voters who do not see the consequences of a deficit, but rational voters who can't agree on strategy to balance the budget.

Some authors, explain the deficit itself as rational.[3] It is certainly true that some level of debt may be optimal at certain times. This paper does not intend to show that budget balance is desirable. However, even if some level of deficit is desirable, the question remains whether is it possible that a democracy would create a larger than optimal deficit.

If the deficit can be described as entirely rational then polls showing voters overwhelming disapproval of the deficit demonstrate that voters have a different type of fiscal illusion, the opposite of what fiscal illusion models suppose, perhaps it should be called "fiscal hysteria." That is, voters over-estimate the cost of the deficit. But, if they do over estimate the cost of the deficit, the question becomes

[3] See Barro (1979) and Lucas and Stokey (1983) for example.

why do we have an optimal deficit, when voters prefer an irrationally large deficit? Thus even if the deficit is optimal, there is a decision making problem at work. If the deficit is itself rational, then all the time politicians spend worrying about the deficit is irrational. Thus, it is impossible to explain *both* the deficit *and* our behavior regarding the deficit as rational.

This model does not address any issues of how much debt is optimal, it merely ask the question is it possible for the deficit to be larger than people want.

2.2

Many political economy models obtain inefficient government decision making as a result of rent seeking, selfish utility maximizers who seek to benefit even at the expense of the welfare of society as a whole. [4] This model assumes that all agents are trying to maximize the benefit to society, but many the same problems arise because of disagreement about what is best. Although this paper makes quite the opposite assumption about human motivations, it does not contradict those models, it is better thought of as an extension of those models. The results of this paper are mathematically identical whether citizens are thought to be selfishly maximizing their own conflicting utility functions or they are thought to be responsibly trying to find the best outcome for society, but have a different idea of what is best for society.

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[4] Buchanan, Tollison, and Tullock (1980) for application of rent-seeking to government decision-making.

2.3

Persson and Svensson (1989) in "Why a Stubborn conservative would run a deficit..." show that disagreement among current and future majorities about the size of government spending can give the current government a incentive to leave a deficit in order to limit the ability of future governments to increase spending. However, the higher spending government would have an equal incentive to leave a surplus and so this model Persson and Svensson's model has been criticized for failing to explain persistent deficits (Alesina and Perotti, 1994) It also should be noted that in the United States the political ideology of lower spending is often endorsed by politicians who also endorse balanced budgets as an ideology, although the behavior may not be as consistent as the rhetoric.

The model presented in this paper is similar to Persson and Svensson's in that deficit bias arises out of conflict about government spending. But, the conflict is over the composition the size rather than of spending, although as will be explained the conflict may appear as a conflict about the size of spending. One advantage of this model is that it can be used to explain a deficit, but not a surplus.

2.4

The deficit in this model is unintentional; a problem the government would like to solve but is politically constrained from doing so. Alessina and Tabellini (1989 & 1990) reach the similar conclusions as this paper as they put it, "citizens' disagreement rather than their myopia... may generate a deficit bias." However, in

their model the disagreement is between current and future majorities, while in this paper it is disagreement among today's citizens that causes the deficit. They find that the conflict gives the current government an incentive to leave a deficit for future governments while this model finds that the conflict creates a political barrier to balancing the budget, despite the fact that the current government would like to do so.

In Alesina and Tabellini the current majority government expects that it will someday be replaced by a another majority government that will have a different preference about the composition of government spending. They demonstrate that such a government would have an incentive to leave a deficit in order to limit the options of future governments. This predicts that deficits would be produced by politically polarized successive governments, but not by divided governments. Because the deficit is sub-optimal, and used by one ruling party to limit the options of the next ruling party, coalition governments should not have an incentive to create deficits. Yet, the United States and other nations experienced large growth in deficits under divided governments. Presumably the democratic congress under Reagan did not intend to limit the spending options of the next democratic president. The model presented in this paper explains why citizens disagreement within today's government can causes a deficit.

2.5

This paper looks at the demand for government spending as separate demands for individual programs that cannot be aggregated together into a one dimensional issue. This prevents the median voter theorem (Bowen, 1943) from producing the optimal quantity of spending. If one attempted to model this as a one dimensional issue, the demand for government services and the political cost of tax and deficit financing[5] may always be in dis-equilibrium, if different spending programs cannot be aggregated together to create a single demand for government spending.

2.6

The voting paradox in this paper explains a number of interesting observations about the deficit. It is consistent with the common observation that U.S. citizens are unhappy with Congress, but happy with their own representative. This model is consistent with the increase in government deficits over the last few decades, this being a result of increased political disagreement about the composition of spending.

Many have concluded that the majority of citizens are irrational about the deficit citing polls that show a large majority of U. S. voters support a balanced budget but a majority opposes any specific plan to eliminate the deficit (New York Times 1987). This model shows how those preference could be explained by

[5] See Lee and Vedder, 1992 for example.

rational behavior. James Buchanan, Milton Freedman, and others seem to have concluded that citizens are irrational about deficits, rewarding their governments for spending all the taxes they receive and as much more as they can get away with.

This perception is reasonable, but if it is false, this perception itself could make the budget more difficult to balance. The poll data quoted above could be explained by irrational citizens or by rational citizens and a voting paradox. If citizens believe the deficit is caused by rational behavior and a voting paradox, it would be irrational for them to continue to make votes year after year that lead to deficits.[6] A tax increase will eliminate the deficit once and for all. However, the same is not true if citizens believe that "the" majority of citizens are irrational. If the irrational majority allows the government to spend all it receives in taxes and as much more as it can get away with, a tax increase will not eliminate the deficit. Instead, it will be followed by a spending increase, leaving the same deficit at a higher level of spending. However, if citizens are rational, but they suspect each other of being irrational, they will not increase taxes to balance the budget. The very belief that others want a deficit, could itself cause the deficit.

[6] see Aumann (1976), and Fudenburg and Tirole (1992, p. 548 - 550)

Chapter 3: The Voting Paradox

3.1: Assumptions

1. There are three citizens [1, 2, 3], call them Moe (1), Larry (2), and Curly(3).[7] They each have different utility functions with the same general form:

$$U_i = u_i(x, y, z, c, d)$$

Where the utility function of person i depends the budget deficit (d), the one consumption good (c), and on the three public commodities [x, y, z], call them highways (x), railroads (y), parks (z).

2. The following assumptions specify the utility functions. The consumption good (c) enters positively and the deficit (d) enters negatively into each person's utility function. Two of the public commodities enter positively into each person's utility function and one enters negatively, that to each citizen is there are two public goods and one public bad.[8] Unfortunately, each person has a different idea of which commodity is the public bad. Moe, an industrialist, favors highways and railroads, but not parks. Larry, a suburbanite, favors highways and parks, but not railroads. Curly, a green, supports railroads and parks, but opposes highways. Thus, the three utility functions are.

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[7] These can be thought of as three equal size groups of citizens with three different utility functions.

[8] Note that preferences are not normally distributed. There is a bimodal distribution of preferences. all citizens who believe a particular commodity is a public good desire the same quantity of it. All those who believe a particular commodity to be a public bad, dislike it by the same amount. This is purely for the purpose of illustration. It does not effect the decision making problem although it does effect the socially optimal quantity of each good,. The deficit bias still exists if preferences are normally distributed. The generality of the results will be discussed in the last chapter.

(1) $U_1 = A_{1x}\ln(x) + A_{1y}\ln(y) - B_{1z}\ln(z) + C_{1c}\ln(c) - D_{1d}\ln(d)$

(2) $U_2 = A_{2x}\ln(x) - B_{2y}\ln(y) + A_{2z}\ln(z) + C_{2c}\ln(c) - D_{2d}\ln(d)$

(3) $U_3 = - B_{3x}\ln(x) + A_{3y}\ln(y) + A_{3z}\ln(z) + C_{3c}\ln(c) - D_{3d}\ln(d)$

Where,

$A_{ij} > 0$

$- B_{ij} < 0$, that is, $B_{ij} > 0$

$- D_{ij} < 0$, that is, $D_{ij} > 0$[9]

A_{ij}, $- B_{ij}$, C_{ij}, and $- D_{ij}$ are parameters showing the value each citizen puts on the three public commodities, the private commodity, and the deficit. The following simplifying assumptions will allow us to leave off some of the subscripts:

$C_{1c} = C_{2c} = C_{3c} = C$

$D_{1d} = D_{2d} = D_{3d} = D$

3. The resource constraint for this economy is as follows:

(4) $I = P_x x + P_y y + P_z z + P_c c$

4. The budget constraint for government spending is taxes (t) plus the budget deficit equal the sum of the cost of the government programs:

[9] The parameters are displayed with negative signs for emphasis.

(5) $t + d = P_x x + P_y y + P_z z$

Substituting from equation 4,

(5a) $t + d = I - P_c c$

5. Each citizen has identical income, each citizen pays equal taxes, and each citizen receives equal amounts of the public commodities.

6. Each citizen knows resource and budget constraints and their own utility function, but has no information about the utility function of others, except what is revealed by the outcome of the votes. In other words citizens know the economic constraints, but not the political constraints. This can be explained by transaction costs to negotiation.

7. Each citizen is civic minded, that is they vote for what is best for society AS THEY SEE IT. Each voter is concerned with the consumption of other citizens, though not necessarily with the preferences of other citizens. That is, on issues of public goods, they vote for what they prefer others to consume, not for what others prefer consume. Therefore, each citizen has a social welfare function in mind when they vote, but it is what would be the social welfare function if all voters had preferences identical to his. The optimal composition of government spending to any one voter, maximizes the net gain to society weighing the benefit

of each public good, as they see it, against the cost of producing public goods given by the resource constraint. In the political process, citizens will try to obtain the best approximation of their optimum that is politically achievable.[10]

It must be understood that give the fact that it is already agreed that the payment and receipt of public goods is shared equally by all citizens, the assumption that citizens vote for what they believe is best for society is not terribly different than the assumption that they vote selfishly.[11] Rather than conflicting with models using that assumption this paper should be thought of as an extension of them. Many of the problems of joint decision making that exist when citizens vote selfishly, still exist when citizens vote for what's best for society, but disagree about what best means. All of the problems of free-riders and voters trying to pay less than their share of taxes do not exist, but the problems of not being able to make a rational joint decision remain.[12]

Assumptions about the political process will be specified in the next chapter. All decisions are made by majority vote of elected representatives. In this chapter, each citizen determines his optimum based on his preferences and the

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[10] Citizens behavior could still be described as selfish because they do not take others dislike into account when they vote for a public good. Whether this behavior is truly civic minded or selfish is unimportant. The reason for this assumption is not to because I believe people actually are civic minded, but to abstract away for problems of revealed preference and free riding to focus on the problem of disagreement about what is best for society. The decision making problems in this paper exist whether or not voters are selfish, but if voters are selfish these problems exist along side others problems that are not the focus of this paper.

[11] Most opportunities for parasitic behavior are ruled out by assumption 6. Given that all citizens have the same income, pay the same taxes, and receive the same quantity of public goods, maximizing one's private benefit from public consumption is mathematically identical to maximize the public benefit from public consumption, if all voters had the same preferences.

[12] Although we will allow citizens to suspect that others are attempting to free-ride.

resource constraint. Ideally, he would take into account the political constraints as well. However, because the preferences of other citizens are assumed unobservable, the political constraints and the political optimum are unobservable. It will be left up to the legislature to determine the political optimum.

3.2 Individual Optimization

Based on his own preferences and the available resources each citizens determines his optimum by maximizing his version of the social welfare function.[13] This determines what he sees as the optimal mix of government spending and taxes, and it is the ideal vision of government that he has in mind when entering the political process. If a majority of voters have preferences identical to his, he will be able to achieve his physical optimum. If not, he will try to achieve the best politically possible budget, his political optimum.

Because each citizen favors two and opposes one of the public commodities, each faces a similar optimization problem.

Voter 1's objective function:

$$(1) \qquad \text{Max}U_1 = A_{1x}\ln(x) + A_{1y}\ln(y) - B_{1z}\ln(z) + C_{1c}\ln(c) - D_{1d}\ln(d)$$

subject to,

[13] Note that, this would the social welfare function if all voters had preferences identical to his.

(4) $I = P_x x + P_y y + P_z z + P_c c$

To, Moe, the optimal quantities of d and z are clearly zero, simplifying his optimization problem to:

(6) $\text{Max } U_1 = A_{1x}\ln(x) + A_{1y}\ln(y) + C\ln(c)$

subject to,

(7) $I = P_x x + P_y y + P_c c$

Larry's and Curly's optimization problems are analogous to Moe's, except that Larry's omits y and d but includes instead of z, while Curly's omits x and d.

Voter 1's optimal quantities are as follows:

(8) $$x_1^* = \frac{IA_{1x}}{P_x(A_{1x} + A_{1y} + C)}$$

(9) $$y_1^* = \frac{IA_{1y}}{P_y(A_{1x} + A_{1y} + C)}$$

(10) $z_1^* = 0$

(11) $$c^* = \frac{IC}{P_c(A_{1x} + A_{1y} + C)}$$

(12) $$t^* = I - \frac{IC}{P_c(A_{1x} + A_{1y} + C)}$$

Larry and Curly will reach analogous conclusions. The following table summarizes each voters optimal level of each variable.

	1	2	3
x_i^*	$\frac{IA_{1x}}{P_x(A_{1x} + A_{1y} + C)}$	$\frac{IA_{2x}}{P_x(A_{2x} + A_{2z} + C)}$	0
y_i^*	$\frac{IA_{1y}}{P_y(A_{1x} + A_{1y} + C)}$	0	$\frac{IA_{3y}}{P_y(A_{3y} + A_{3z} + C)}$
z_i^*	0	$\frac{IA_{2z}}{P_z(A_{2x} + A_{2z} + C)}$	$\frac{IA_{3z}}{P_x(A_{3y} + A_{3z} + C)}$
c_i^*	$\frac{IC}{P_c(A_{1x} + A_{1y} + C)}$	$\frac{IC}{P_c(A_{2x} + A_{2z} + C)}$	$\frac{IC}{P_c(A_{3y} + A_{3z} + C)}$
t_i^*	$I - \frac{IC}{P_c(A_{1x} + A_{1y} + C)}$	$I - \frac{IC}{P_c(A_{2x} + A_{2z} + C)}$	$I - \frac{IC}{P_c(A_{3y} + A_{3z} + C)}$
d_i^*	0	0	0

If we make the following specifications, we can determine a numerical value for each citizens optimal level of c, x, y, z, t, and d.

$A_{ij} = 1$

$P_j = \$1$

$C_j = 1$

$I = \$3$

Making Moe's ideal quantity of each variable:

$x_1^* = 1$

$y_1^* = 1$

$z_1^* = 0$

$d_1^* = 0$

$c_1^* = 1$

$t_1^* = 2$

The following table summarizes each citizen's optimal values of each variable, given the above specifications:

	1	2	3
x_i^*	1	1	0
y_i^*	1	0	1
z_i^*	0	1	1
c_i^*	1	1	1
t_i^*	$2	$2	$2
d_i^*	0	0	0

It is difficult to illustrate this problem graphically because five variables enter each citizen's utility function (x, y, z, c, and d). We will use a three dimensional graph in x-y-z space. The indifference map is drawn to reflect the opportunity cost of producing the public commodities in terms of lost private consumption making his indifference map a set of concentric spheres. The center of the concentric spheres is Moe's optimum in x-y-z space, point M* [(x_i^*, y_i^*, z_1^*), that is (1, 1, 0)]. The indifference spheres show Moe's utility decreasing if the government spending moves any direction away from M*. Using this method the utility of x, y, z, and c can all be all be shown on one three dimensional graph.

Figure 1: **Citizen 1's indifference map in two dimensions**
In x-y space assuming $z = z_1^*$ (i.e. assume $z = 0$).

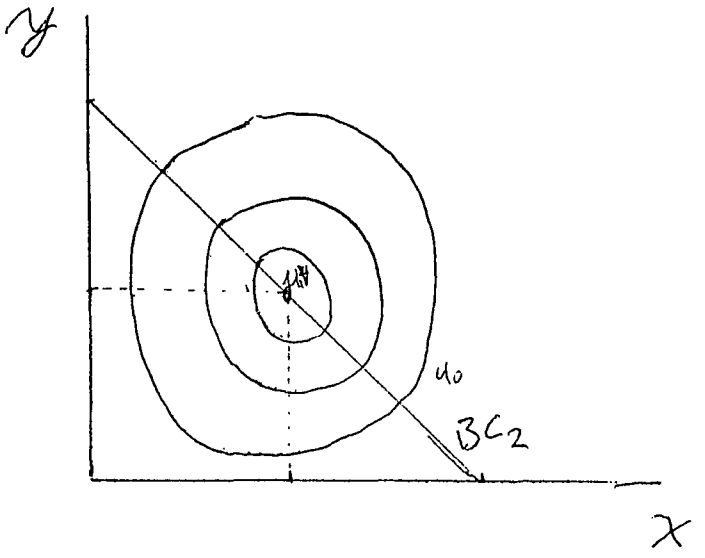
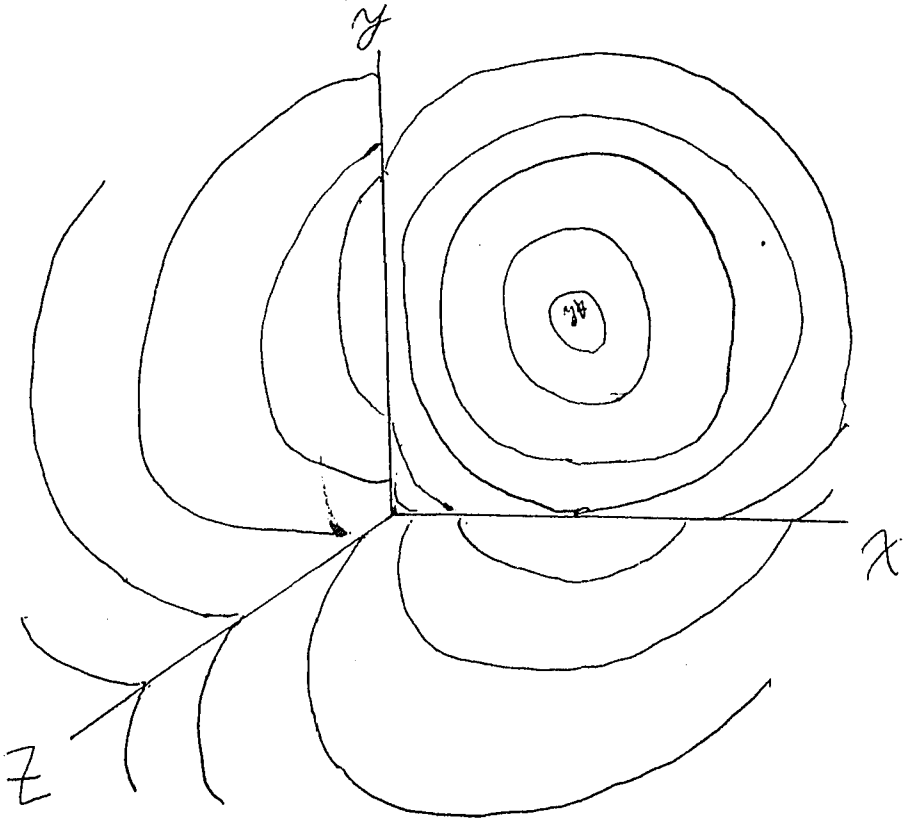


Figure 2: **Citizen 1's indifference map in three dimensions**
In .x-y-z spacc



The budget constraint for government spending, from equation 5, given the above specifications, simplifies to:

(13) $t + d = x + y + z$

This has five variables and so isn't very useful in x-y-z space. If citizens set the deficit at it's optimal level [$(d = d^*)$ at $(d = 0)$] we have the tax constraint:

(14) $t = x + y + z$

If citizens fix taxes at their optimal level ($t = 2$) the tax constraint becomes usable in x-y-z space:

(15) $2 = x + y + z$

This is not however a binding constraint. The only biding constraint is the resource constraint (equation 4). It is a constraint only in the sense that citizens would like to make this constraint binding on their government. It is possible to produce inside the tax constraint by running a surplus, or outside the tax constraint by running a deficit. It is also possible to move the tax constraint by changing the level of taxes. Producing anywhere except on the constraint implies a loss of

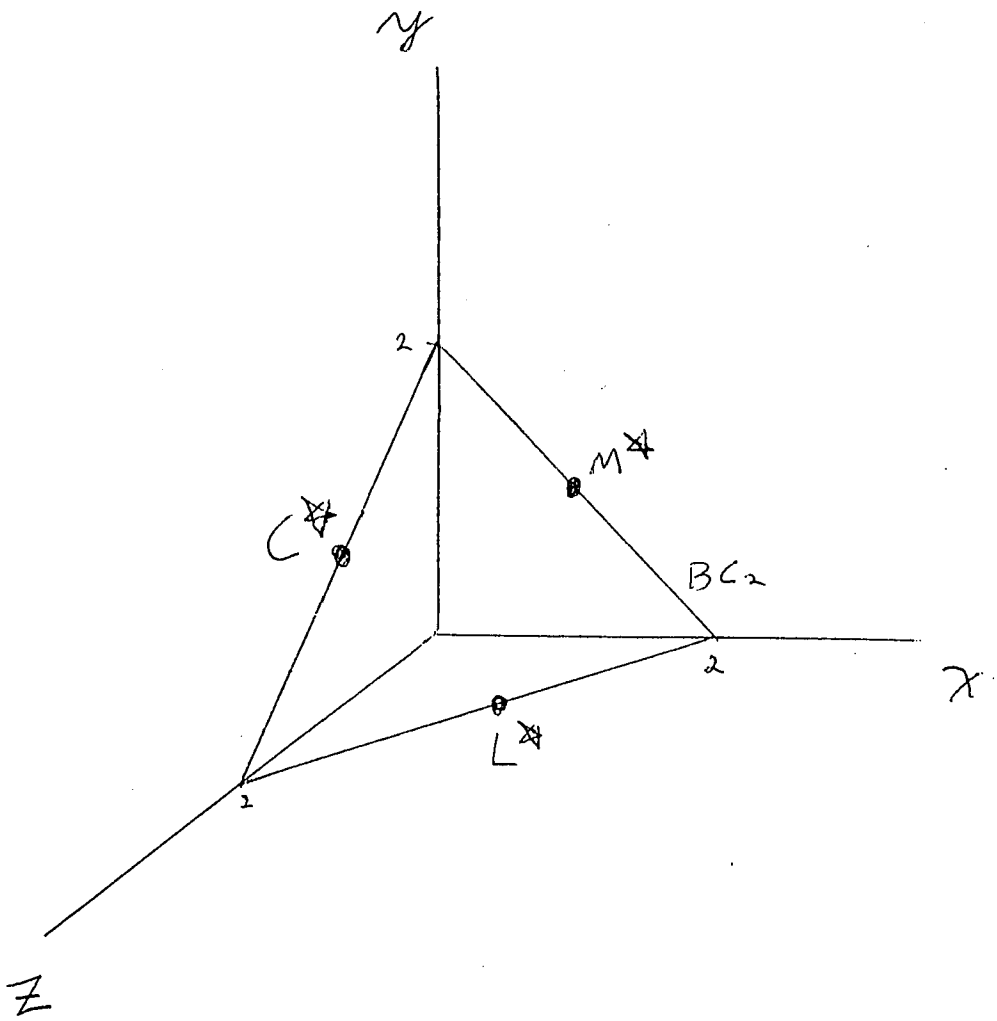
utility,[14] thus it devalues the indifference curves but does not change their position. A surplus, producing inside the tax constraint, reflects a greater loss in the private consumption good than is necessary to produce the given level of public goods, and so also reduces utility. Increasing taxes, shifting the tax constraint to the left, implies a similar loss in utility. A deficit, producing outside the constraint, produces a greater loss in utility because the deficit enters negatively in citizens' utility function.

A graphic interpretation is that an increase in taxes shits out the tax constraint, but revalues the indifference curves showing lower utility from lost consumption. Running a deficit, spending outside the taxes constraint, revalues the indifference curves by a greater amount reflecting both the loss in private consumption and the disutility of the deficit.

Figure 3 illustrates the tax constrain in three dimensions. BC is the tax constraint for government spending implied by $2 in spending and budget balance. Points M*, L*, and C* are Moe, Larry, and Curly's optimal points, respectively points (1, 1, 0), (1, 0, 1), and (0, 1, 1). The tax constraint is a pyramid bounded by the points (2,0,0,), (0,2,0), and (0,0,2), showing all combinations of spending on x, y, and z totaling $2.

[14] This follows from equations 1, 2, 3, and 4.

Figure 3: **The tax constraint with Each Citizen's optimum in x-y-z space.**
BC is the tax constraint given taxes and the deficit fixed at their optimal levels (t=2, d=0).



3.3 The political problem.

The following table surveys public opinion on each issue, showing what each person perceives to be the optimal quantities of the public commodities and the optimal level of taxes needed for optimal consumption of public and private goods. The last two columns summarize the results of a majority vote on each. The fourth row is the sum of the first three rows.

	Moe:	Larry:	Curly:	Majority:	Ratio:
t	2	2	2	2	3 to 0
d	0	0	0	0	3 to 0
x	1	1	0	1	2 to 1
y	1	0	1	1	2 to 1
z	0	1	1	1	2 to 1
$\Sigma x,y,z$	2	2	2	3	

There is no disagreement among citizens about the size of government spending. However, although the disagreement among citizens centers only on the composition of government spending, it manifests itself as a disagreement about the size and financing of government spending. A unanimous majority wants $2 in spending. A unanimous majority wants no deficit. Each citizen favors spending on only 2 public goods for a total of $2 in spending and a balanced budget. Yet, all 3 public commodities are favored by 2 to 1 majority for a total of $3. Despite the fact that every citizens behaves consistently with a balanced budget, majority voting can pass more spending than any individual is willing to pay takes to support.

It is tempting, but incorrect, to suppose that "the solution" is simply that citizens will instantly become willing to pay more taxes to balance the budget. This is no more true than supposing that citizens will instantly become willing to accept spending on each public commodity to balance the budget.

The society must try to find a second best solution, (i.e. the best politically possible compromise given that no one's first best budget is politically possible). The existence of a second best will be discussed in the next section. Whether the political process can actually reach the second best solution will be discussed in the next chapter.

3.4 The Second Best Solution

Each citizen's optimal budget is their version of "first best," the best possible outcome. However, because only a minority supports each optimal budget, none of the "first bests" are politically viable in a majority rule democracy.[15] The political process must look for the second best solution; the best politically possible solution, the best possible compromise. A "second best" can be determined from the perspective of a benevolent social planner who maximizes a social welfare function, a weighted sum of the individuals utility functions, subject to the resource constraint.

The social welfare function will give us a unique solution in this case, but on difficult theoretical grounds because this solution involves making an

[15] The fist best solutions are, however, politically possible under plurality rule democracy, democracy by lot, or simple dictatorship, but these solutions all have their own problems.

interpersonal comparison of utility. The social planner's answer depends on B_i the parameter that was left out in every individuals optimization problem. This maximization problem weighs "how much" one person dislikes a public commodity against "how much" another person likes it. However, because we are not allowing for any differences in the distribution of income, and because of the symmetrical nature of the likes and dislikes for public commodities, most, but not all of the theoretical problems with a social welfare function disappear.[16] The socially optimal outcome is theoretically reachable by voluntary agreement, if citizens force each other to address the question of how much of the public bads are they willing to accept to receive the public goods that they want. In other words if citizens internalize the political constraints. If nothing else, the planner's social optimum is at least useful as a benchmark against which to compare the political outcome.

The planner's optimization problem is as follows.

$$(16)\ \text{Max } W=(A_{1x}+A_{2x}-B_{3x})\ln(x)+(A_{1y}+A_{3y}-B_{2y})\ln(y)+(A_{2x}+A_{3z}-B_{1z})\ln(z)+3C\ln(c)$$

subject to equation (4),

---

[16] If we allow A and B to vary among voters the optimum of the social welfare function could be anywhere in xyz space. For example, if one person had a much higher value for B than the others, the social optimum, would approach his optimum. However, there is no reliable method to judge the relative size of one person's dislike for one commodity to another person's like. This would give citizens an incentive to overstate their likes and dislikes.

It is also possible, for any two citizens to maximize their joint utility, creating a ruling majority, while ignoring the preferences of the third citizen. Giving the third citizen an incentive to offer one of the others a better deal, resulting in cycling. This problem, however, is not the subject of this paper. For more on cycling see Mueller (1989).

$$I = P_x x + P_y y + P_z z + P_c c$$

Like in the individuals' optimization, the optimal level of deficit is zero and can be left out of the optimization equation. The optimal quantity of each good is following:

$$(17) \quad c^{**} = \frac{I3C}{P_c[(A_{1x} + A_{2x} - B_{3x}) + (A_{1y} + A_{3y} - B_{2y}) + (A_{2z} + A_{3z} - B_{1z}) + 3C]}$$

$$(18) \quad x^{**} = \frac{I(A_{1x} + A_{2x} - B_{3x})}{P_x[(A_{1x} + A_{2x} - B_{3x}) + (A_{1y} + A_{3y} - B_{2y}) + (A_{2z} + A_{3z} - B_{1z}) + 3C]}$$

$$(19) \quad y^{**} = \frac{I(A_{1y} + A_{3y} - B_{2y})}{P_y[(A_{1x} + A_{2x} - B_{3x}) + (A_{1y} + A_{3y} - B_{2y}) + (A_{2z} + A_{3z} - B_{1z}) + 3C]}$$

$$(20) \quad z^{**} = \frac{I(A_{2z} + A_{3z} - B_{1z})}{P_y[(A_{1x} + A_{2x} - B_{3x}) + (A_{1y} + A_{3y} - B_{2y}) + (A_{2z} + A_{3z} - B_{1z}) + 3C]}$$

Using the same specifications as above, and specifying $-B_i = -1$ gives the following numerical values to the socially optimal quantities:

$c^{**} = 1.5$

$x^{**} = 0.5$

$y^{**} = 0.5$

$z^{**} = 0.5$

Making the optimal level of taxes

$t^{**} = \$1.5$

The optimal composition of spending follows from the fact that all three public commodities have equally strong support. Whatever the level of total spending in the budget, the level of spending on each public good (x, y, and z) is an equal amount on each.

The optimal method of financing is independent of the level and composition of spending. The parameter - D demands that tax financing is always preferred to deficit financing. Whatever level and composition of spending are determined, taxes should be set equal to the level of spending. The inability of the political decision making process to separate the question of financing from the question of the level and composition of spending will cause problems in the next chapter.

The optimal level of spending, and therefore taxes, is less obvious than it may first appear. It is not a simple question, of should taxes be brought up to the level of spending or should spending be brought down to the level of taxes. Faced with a budget that must contain equal amounts of x, y, and z citizens would choose less than what was previously their optimal level of spending and taxes. The reason for this is given that government spending must contain a bad, citizens would prefer less of it.

The socially optimal level of spending depends on the values of -B and A. Had we specified - B = 0, the optimal quantity of each good would be 1 and the optimal level of taxes be $2. Depending on the relative sizes of -B and A, the optimal level of spending on each good can be anywhere in the range from zero to the quantity of spending approved by the majority:

$$\text{if } B_{3j} > A_{1j} + A_{2j},\ j^{**} = 0$$

The is an externality problem: the optimal quantity for society depends on the relationship between how much the good increase the utility of some to how much it decreases the utility of others. However, the difference between this and most externality problems is that the public commodity does not impose a quantifiable cost on those it harms. They simply do not consider it a public good. Were these commodities private, no externality would exist, people who do not value a commodity would simply not consume it. Because a public commodity must be consumed by everyone or no one, public commodities have a greater ability to create externalities. For example, on private land, a railroad creates an externality if it spews sparks onto neighboring property. On public land, a railroad produces an externality when a minority of citizens simply dislike railroads.[17]

[17] The preferences of a minority may not be considered an eternality by a ruling majority, but to a social planner they are. It is only the fact that the three minorities that oppose each public commodity, together constitute a majority that opposes some public spending that brings this problem to the surface. If Larry opposed all three public commodities, while Curly and Moe supported all three, the social planner's decision would not change. However, the political problem would disappear. The majority would pass spending on all three goods, and the minority would have no recourse.

3.5 What is the Optimal Size of Government Spending?

Depending how this question is asked, there could be three different definitions of the optimal size of government spending (call them G_1, G_2, and G_3). Definition 2: Government spending is defined as the sum of spending that a majority favors on each issue; the optimal size is $G_1^* = \$3$. Definition 2: Government spending is defined as the amount of spending a majority would favor given their own optimal composition of government spending; the optimal size is $G_2^* = \$2$. Definition 3: Government spending is defined as the amount the majority would choose given that they must accept equal portions of spending on each issue; the optimal size is $G_3^* = \$1.5$.[18] The third definition is the social planner's definition. This definition would be chosen by a majority vote if the question were asked in this way. However, G_3^* conflicts with the G_1^* because G_3^* incorporates the externality of the minorities who dislike the some of the spending, while G_1^* does not.

The fundamental problem underlying the decision making problem in the next chapter is the failure of the political process to ask the following question in a consistent manner. What is the optimal size of government spending? The quantity of each public good is determined by an individual vote on each issue, which implies definition 1. There is no vote on the total size of government

[18] It should be noted that if political opinion on each issue were normally distributed, these three definitions would be the same, and it would become possible to consistently determine the optimal size of government. However, this would not effect the deficit bias discussed in the next chapter. For discussion see unpublished work available from the author.

spending. As for taxes, there is a vote on the overall size, but not a vote on what composition of spending the taxes are for. [19] This implies that taxes will be set at level corresponding to definition 2 or 3.[20] If $G_3^* < G_1^*$ the taxes the majority is willing to pay, will be less than the sum of the spending a majority favors on each issue. The next chapter discusses the ability of a legislature to solve this problem.

3.6 A Graphic illustration

A three dimensional graph allows us to compare the three individual optima to G_1^*, G_2^*, and G_3^*. In x-y-z space, Moe's optimum is the point [M*= (1, 1, 0)]. Larry's optimum is [L*= (1, 0, 1)]. Curly's optimum is [C* = (0, 1, 1)]. The majority's optimum according to definition 1 is the point [G_1^* = (1, 1, 1)]. According to definition 2 the optimum is [G_2^* = (2/3, 2/3, 2/3)]. The social optimum, definition 3 is [G_3^* = (0.5, 0.5, 0.5)].

If taxes are set according to definition 1, taxes = 3. The tax constraint, BC_1, is the triangle connecting the points (3, 0, 0,), (0, 3, 0), and (0, 0, 3). G_1^* lies on BC_1. If taxes are set according to definition 2, taxes = $2. The tax constraint, BC_2, is the triangle connecting the points (2, 0, 0,), (0, 2, 0), and (0, 0, 2). This puts M*, L*, C*, and G_2^* on the tax constraint. G_1^* is outside the constraint and G_3^* is inside the constraint. If taxes are set according to definition 3, taxes = $2. The tax constraint, BC_3, is the triangle connecting the points (1.5, 0, 0,), (0, 1.5, 0), and (0, 0, 1.5).

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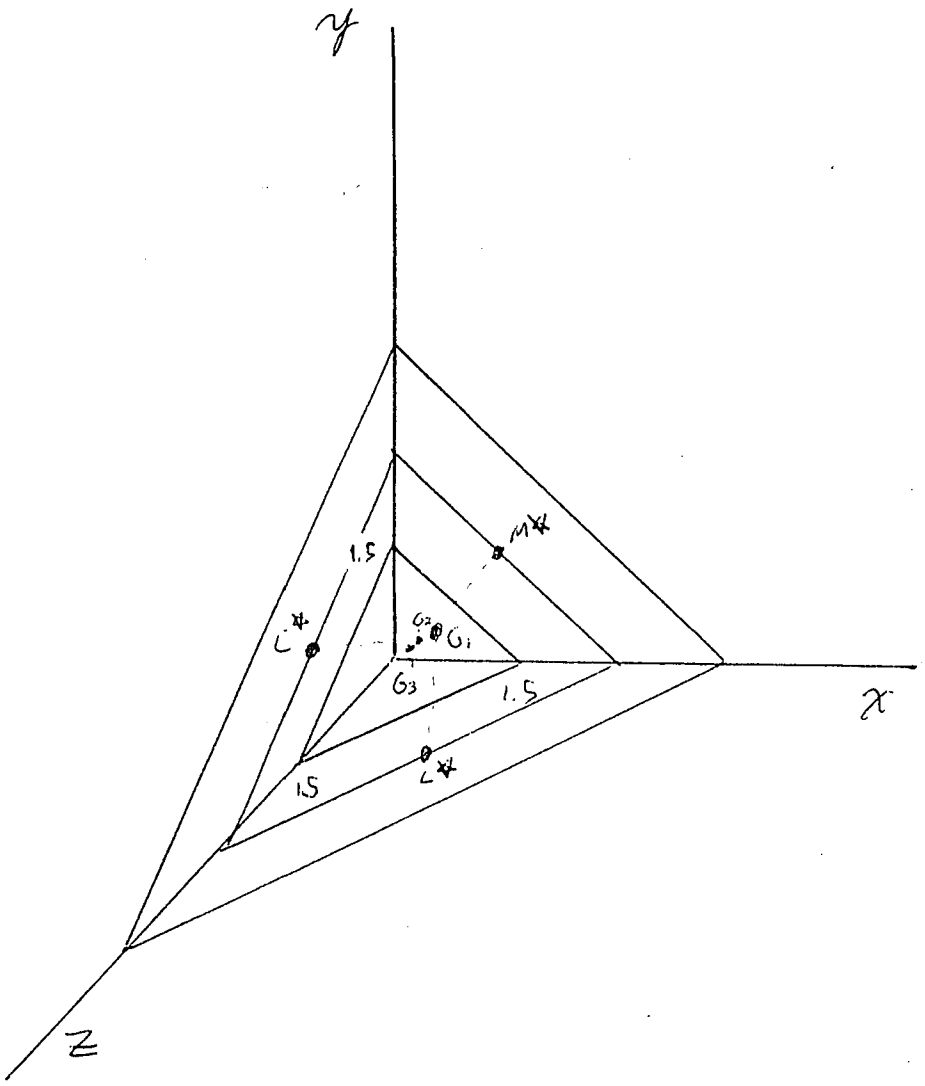
[19] Except for a few car-marked programs such as social security.

[20] Uncertainty about the composition of spending at the time of the vote on taxes makes it questionable just what spending is being voted on.

Graphically, a balanced budget is simple. Simply choose the optimal point, then set taxes at the level that equates the tax constraint and the optimal point. The legislature, however, will have difficulty separating these questions and deficit bias will result.

Figure 4: **The individual optima and three possible compromises**



3.7 The necessary conditions

It is clear from the above example that the sum of the spending that will pass by a majority vote can be greater than the level of taxes the majority is willing to pay. However, as long as each citizen votes for the same amount of taxes and spending there must be some relationship. What is the general case? If the number of citizens and the number of public commodities goes to infinity, then the sum of government spending that passes by a majority vote (G) can be anywhere from equal to, to twice the amount of taxes (t) that the majority is willing to pay:

$$t \leq G < 2t$$

If the same groups of voters passes every spending bill, a majority of voters favor all the spending in the government budget and $t = G$. However, if slightly different majorities pass each bill, G approaches 2t, as more and more voters find something in the budget that they aren't willing to pay for. Notice that $t = G$ is a special case and is only true when the same group constitutes a majority on every issue. In any other case, the sum of the minorities that oppose spending on each issue, constitute a majority that oppose some spending. In the terms of the example above, the problem exists unless there is a majority of Moes, a majority of Larrys or a majority of Curlys. In other words, this problem exists unless a majority of the population has the identical opinion on *every* issue.

Chapter 4: The Games

4.1 Assumptions.

8. There are millions of citizens, equal numbers have preferences like Moe, Larry, and Curly. Transactions costs exist making it impossible for citizens to negotiate directly with each other, and so all negotiations must be done by elected representatives, who have the same names as their constituents, Moe, Larry, and Curly.

10. Citizens have the following information: Citizens know their own preferences, but do not know the preferences of other voters or the preference of their representatives.[21] Citizens observe the voting record of their own representative and the outcome of the decisions made by the legislature, but they do not observe the voting record of individual representatives other than their own. Citizens do not know what other political outcomes might have been possible. As before, citizens know the physical constraints and are able to determine what they perceive to be the best outcome for society given their own preferences. That is they know their ideal budget. Citizens do not observe the negotiation process in the legislature and so, once a decision is made, citizens do not know what other outcomes may have been politically possible. In other words, citizens know the physical constraints but not completely know the political constraints.

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[21] In other words, the Curly citizens don't know for sure that their representative is really a Curly.

11. The voting of representatives in a legislature is modeled with game theory. Citizens do not play the games; they provide the rewards. Competition for office insures that each representative maximizes his chance for reelection. To do this he maximizes his constituents' satisfaction with his performance. That is, the representative's objective is, not simply to maximize his constituents' utility, but to maximize his constituents' satisfaction with him as a representative. If he was a single decision maker there would be no difference between the two, but because he is only a part of a legislature and is not fully responsible for the decisions of the legislature, the reward system is more complicated than simply maximizing the utility of his constituents.

12. Citizens reward representatives for their vote on each issue. This reward is assumed to be proportional to the net benefit of that issue [e.g. $N(x_v)$]. But that reward is split into two parts. Part 1 is the reward for how the representatives themselves vote. Part 1 rewards the legislature for attempting to obtain the constituency's physical optimum (i.e. the reward as the *representative* of the constituency's ideals). Part 2 is the reward for the outcome of the decision made by the majority of legislature. Part 2 is the reward for achieving the best politically possible outcome if the physical optimum is not possible (i.e. the reward as a *decision maker* for the constituency.)

Reward for X = representative's vote + legislature's decision

For example,

$$R_{1x} = vN(X_v) + (1-v)N(x)$$

M_{rx} is Moe's reward for X. x_v is the quantity voted by the representative, and x is the quantity passed by the legislature. N( ) is the NET benefit function of x given the resource constraints and the preference of the constituency. The parameter v shows the weight constituents put on the two parts of the reward.

$$0 < v < 1$$

If v = 1, citizens hold their representative responsible only for his own vote. If v = 0, citizens are concerned only with the outcome of the decision made by the legislature. Representatives cannot receive any part of the reward for the legislature's decision on an issue, if they themselves did not vote for that issue. The size of "v" depends on the trust that voters give their representative to bargain for them, and the probability that one legislator can affect the final decision made by the legislature, and the willingness of voters to compromise. A very small v is probably unlikely, it would punish a representative for decisions that are out of his own control. If the representative is in the minority, he may be a very faithful representative of his constituents, but may be able to get none of their legislation passed.

13. The tax reward is zero if the representative votes for the optimal level of taxes, and negative he votes for too much or too little spending. This is because the cost of the spending in terms of lost private consumption was factored into the net benefit function for each public commodity. Therefore voting for the optimal level of taxes, produces no additional benefit, and so no additional reward, the representatives are punished if they vote for anything but the optimal level. The optimal level of taxes is assumed to be the amount needed to provide the public goods desired by his constituency and passed by the legislature.

$$R_{It} = vT(x + y - t_v) + (1-v)T(x + y - t),$$

where $t > x+y$

R_{It} is Moe's reward for taxes. T(t) represents citizens dislike of being overtaxed. If $t > x+y$, R_{It} is less than zero showing that representatives are punished for deviating from the optimal level of taxes.

14. The deficit reward is more complicated because representatives do not take a direct vote on the deficit, and so citizens must judge if their representative voted for a balanced budget by judging if his votes were consistent with a balanced budget. Representatives are rewarded for voting against a deficit if they voted for the same amount of taxes as spending. Representatives are rewarded for achieving a balanced budget if taxes passed by the legislature equal spending passed by the

legislature. Like taxes the reward for the balanced budget is zero if the budget is balanced and negative otherwise.

$R_{1d} = vd(t_v - x_v - y_v - z_v) + (1-v)d(t - x - y - z)$

if $t < x + y + z$

R_{1d} is Moe's reward for the deficit. d represents the voters dislike for any deficit spending. By assumption, $d > c$ showing that voters always prefer tax financing to deficit financing, representatives will decrease lowering taxes and creating a deficit.

15. Legislators can vote and re-vote on any issue until the majority votes to close the discussion.

16. The following simplification assumptions make all benefit functions alike, and all loss functions alike:

$A_{ij} = A$, for all A_{ij}

$-B_{ij} = B$, for all $-B_{ij}$

N(j) is the net benefit function of any public good, showing the net benefit to society of any public commodity as see by those who believe commodity j is a public good. -L(j) is the net loss function, showing the net loss to society of any public commodity as see by those who believe it is a public bad. -T(t) is the net loss function for over taxation. -D(d) is the net loss function of the deficit, showing that any level of deficit decreases the utility of voters.

Two of these assumptions are designed to cancel each other out. The assumption that there are only three groups, three representatives, and three public commodities, is designed to make it easy for us to see that a voting paradox exists. However, it makes it less believable that citizens are unable to determine the political constraints. Actually there are millions of utility functions, thousands of government spending bills, and hundreds of representatives. Given that, it is much more believable that voters are unable to determine the political constraints. The more public commodities there are, the easier it is for citizens to believe that there is waste in the budget that can be cut to eliminate the deficit. And, the more complex the budget and society is, the more likely it is that a voting paradox can exist.

4.2

The following table shows Moe's constituency's values for the optimal quantity of each public commodity, the cost of producing that quantity in terms of lost consumption, and the net benefit function of each good. It is from the net benefit function that the rewards will be derived.

Issue	Optimal Quantity	Cost	Net benefit
X	$x^* = \frac{IA_x}{P_x(A_x + A_y + C)}$	$P_x x^* = \frac{IA_x}{(A_x + A_y + C)}$	$N_x(x) = A_x\ln(x) - P_x x^*$
Y	$y^* = \frac{IA_y}{P_y(A_x + A_y + C)}$	$P_y y^* = \frac{IA_y}{(A_x + A_y + C)}$	$N_y(y) = A_y\ln(y) - P_y y^*$
Z	0	0	$L_z(z) = - P_z z - B_z\ln(z)$
T	$t_1^* = P_x x + P_y y$	0	$-T(t) = C\ln(t^*) - C\ln(t)$, if $t > t^*$
D	0	0	$-D(d) = -D\ln(x + y + z - t)$ if $x + y + z > t$

The net benefit functions for Larry and Curly's constituents are identical except that the Larrys has a loss function for y and the Curlys have a loss function for z. Everyone has the identical negative net benefit from the rewards, however the net loss function for taxes is different for everyone:

Larry,

$t_2^* = P_x x + P_z z$

Curly,

$t_3^* = P_y y + P_z z$

The reward for each issue depends on the net benefit (or net loss) functions of that issue. Therefore the rewards for each of the legislators are as follows:

R_{1j}	Moe's [representative 1's] rewards	
Issue:	his vote	+ the majority's decision
x	$vB(x_v)$	$+(1-v)B(x)$
y	$vB(y_v)$	$+(1-v)B(y)$
z	$-vL(z_v)$	$-(1-v)L(z)$
t	$-vT(t_v-t_1^*)$	$-(1-v)T(t-t_1^*)$
d	$-vD(x_v+y_v+z_v-t_v)$	$-(1-v)D(x+y+z-t)$

R_{2j}	Larry's [rep. 2's] rewards	
Issue:	his vote:	+the majority's decision:
x	$vB(x_v)$	$+(1-v)B(x)$
y	$vL(y_v)$	$-(1-v)L(y)$
z	$-vB(z_v)$	$+(1-v)B(z)$
t	$-vT(t_v-t_2^*)$	$-(1-v)T(t-t_2^*)$
d	$-vD(x_v+y_v+z_v-t_v)$	$-(1-v)D(x+y+z-t)$

R_{3j}	Curly's [rep. 3's] rewards	
Issue:	his vote:	+the majority's decision:
x	$-vL(x_v)$	$-(1-v)L(x)$
y	$vB(y_v)$	$+(1-v)B(y)$
z	$vB(z_v)$	$+(1-v)B(z)$
t	$-vT(t_v-t_3^*)$	$-(1-v)T(t-t_3^*)$
d	$-vD(x_v+y_v+z_v-t_v)$	$-(1-v)D(x+y+z-t)$

In other words, Moe's optimization problem is:

$$\text{Max} \quad U=vB(x_v)+(1-v)B(x)+vB(y_v)+(1-v)B(y)-vL(z_v)-(1-v)L(z)-$$
$$vT(t_v-t_1^*)-(1-v)T(t_v-t_1^*)-vD(x_v+y_v+z_v-t_v)-(1-v)D(x+y+z-t)$$

These rewards are based on all the information citizens have available to them. They allow a representative to trade votes to improve the outcome, but to monitor his loyalty they reward him for voting for their ideal, for their physical optimum. By trading, Moe can improve the outcome portion of his vote (weighted by 1-v), but will cost him the vote portion of his reward weighted by "v" for whatever he sacrifices. The logic behind this is that any vote trade, even if it improves the outcome, means that Moe will have to vote for something other than his constituents ideal (M*). This makes him suspect. His constituents do can not

observe whether he is doing this because M* is impossible to achieve or because he does not really believe in M*. Becoming a better decision maker means becoming a poorer representative.

Moe's constituents would prefer him to maximize their utility out of public expenditure decisions, the following equation:

$$U = B(x) + B(y) - L(z) - T(t_v - t_1^*) - D(x+y+z-t)$$

But, they simply don't have the information to monitor this. To enforce this, they would need to know the opportunity cost of any political outcome. If the representatives make a compromise, (G_3, t = 1.5 for example) citizens would need to know what the next best possible compromise is, in order to decide whether the representative has maximized their utility function. They do not have this information, they can only evaluate Moe's voting record and the majority decisions. This, unfortunately, does not give Moe the incentive to always maximize his constituents utility.

The inability of constituents to evaluate compromises has created a bias against compromise. As long as any part of the representative's reward depends on how he votes, whether it passes or not, there is a biased against making a political compromise even if it increase the utility of constituents. However, rewarding him only for the outcome is not desirable either. Suppose v = 0. Legislators are only rewarded for the outcome of the majority decision by the

legislature, but once they witness the outcome they have nothing to compare it with expect their ideal. This could mean voting out of office a very faithful representative who is in the minority in the legislature.

Moe's constituents could set v=1 and reward him only for how he votes. This makes his objective function:

$$U = B(x_v) + B(y_v) - L(z_v) - T(t_v - t_1^*) - D(x_v + y_v + z_v - t_v)$$

This insures that he will vote for their physical optimum, but it gives him no ability to compromise to improve the political outcome if their ideal is not reachable.

It is reasonable to suppose that citizens will reward their representative based partly on how he votes and partly on the outcome of the majority decision in the legislature. This allows him to make Pareto improving vote trades and to monitor his loyalty.

Obviously the above game is rather complex. Many outcomes are possible. Cycling is possible, but the bias against compromise actually helps to make cycling less likely.[22] The next section simplifies the game to focus only on the problem of building a coalition to balance the budget.

[22] See unpublished work be the author for discussion of cycling.

4.3 Prisoner's Dilemma

A simple model of the problem facing the legislature can be maintained by assuming that political pressure in favor of the majority preferred level of spending is too strong to be changed and that the legislatures are faced with the choice of balancing the budget by increasing taxes or be leaving the deficit. In other words spending is at G_1 and the legislature is attempting to building a coalition to increase the tax constraint to BC_1.[23]

The Game is thought of as a positive-sum game beginning with the default that everyone votes for their constituents ideal, M*, L*, C*, and BC_2. The combination of these votes makes the spending point G_3, which is out side or the tax constraint BC_2. A "yes" vote is a vote to agree to the compromise the budget to raise taxes to t = 3, achieving a balanced budget at G_3/BC_3. A "no" means no agreement, a default deficit. A majority vote determines the outcome, so any cell in which two or more of the representatives vote yes balances the budget.

The simplifications:

No matter what the vote on taxes the legislators all receive the same rewards for the spending issues. So we can define that as a constant:

$$S = B(x) + B(y) - (1-a)L(z),$$

[23] Note, this game can be used to represent any method of balancing the budget. The pattern of the pay-offs would be the same if we were cutting a bit from each program, to achieve G_2/BC_2 or G_3/BC_3..

This allows us to write the payoffs in a simple game that is similar to a three player version of the prisoners dilemma.

Curly votes yes	Larry votes yes	Larry votes no
Moe votes yes	S-T(1) S-T(1) S-T(1)	S-T(1), S-(1-v)T(1) S-T(1)
Moe votes no	S-(1-v)T(1) S-T(1) S-T(1)	S-(1-v)D(1) S-(1-v)D(1) S-(1-v)D(1)- T(1)

Curly votes no	Larry votes yes	Larry votes no
Moe votes yes	S-T(1) S-T(1) S-(1-v)T(1)	S-(1-v)D(1)-T(1) S-(1-v)D(1) S-(1-v)D(1)
Moe votes no	S-(1-v)D(1) S-(1-v)D(1)-T(1) S-(1-v)D(1)	S-(1-v)D(1) S-(1-v)D(1) S-(1-v)D(1)

This game is still complex. Every cell includes the reward S which is the sum of the spending awards. The rewards for taxes and the deficit are only negative, the reward for delivering the optimal budget is S, the negative rewards represent punishment for deviating from s.

The lower right hand corner, shows all three representatives voting for their constituents' ideal level of taxes, they receive their reward for S and vote for t* so they are reward for voting for the optimal level of taxes. They are punished for the deficit. However, they lose only part of the deficit reward. Because each representative voted for the same amount of spending and taxes they receive the reward for vote portion [(v)D(0)], but they lose the majority decision portion (1-

v)D(1). This loss gives them the opportunity to improve their reward by making an agreement to raise taxes and balance the budget. However, to obtain this reward representatives must vote against their constituents optimal level of taxes. This action gains the remaining portion of the balanced budget reward, but loses the entire tax reward. Thus, an agreement to balance the budget improves the reward of the representatives if:

$$(1\text{-}v)D(1) > T(1)$$

but, an agreement to balance the budget is socially improving if

$$D(1) > T(1)$$

If $D(1) > T(1)$, all else equal, voters would rather pay a given level of taxes than have a budget deficit. Therefore the coefficient (1-v) shows the bias against a balanced budget. The reason for the bias is simple. The responsibility for taxes and spending is clear, there is a direct vote on these issues. Taxes and spending will be set at whatever level a majority of representatives decides. But, the deficit happens because of the *pattern* of voting. Each legislature can tell their constituents that they voted for a balanced budget, but were overruled by the majority. It appears that the majority voted for extra spending, but not extra taxes. Thus, each representative is less directly responsible for the deficit than he is for the level of spending and taxes, making him less likely to eliminate the deficit, despite the popularity of a balanced budget.

The games can by simplified further, if leave out the constant S and positivize the rewards by only looking at the *difference* in the reward caused by the vote on financing:

Curly votes yes	Larry votes yes	Larry votes no
Moe votes yes	(1-v)D(1)	(1-v)D(1)
	(1-v)D(1)	D(1) + (v)T(1)
	(1-v)D(1)	(1-v)D(1)
Moe votes no	D(1) + (v)T(1)	T(1)
	(1-v)D(1)	T(1)
	(1-v)D(1)	0

Curly votes no	Larry votes yes	Larry votes no
Moe votes yes	(1-v)D(1)	0
	(1-v)D(1)	T(1)
	D(1) + (v)T(1)	T(1)
Moe votes no	T(1)	T(1)
	0	T(1)
	T(1)	T(1)

A specific version of the game, with $(1-v)D(1) < T(1)$

$D(1) = 4$

$v = 0.5$

$T(1) = 2$

Curly votes yes

	Larry votes yes	Larry votes no
Moe votes yes	1 1 1	1 4.5 1
Moe votes no	4.5 1 1	2 2 0

Curly votes no

	Larry votes yes	Larry votes no
Moe votes yes	1 1 4.5	0 2 2
Moe votes no	2 0 2	2 2 2

A specific version of the game with (1-v)..D(1) < T(1), set

D(1) = 4

v = 0.25

T(1) = 2

Curly votes yes	Larry votes yes	Larry votes no
Moe votes yes	3	3*
	3	5.5*
	3	3*
Moe votes no	5.5*	2
	3*	2
	3*	0

Curly votes no	Larry votes yes	Larry votes no
Moe votes yes	3*	0
	3*	2
	5.5*	2
Moe votes no	2	2*
	0	2*
	2	2*

* Nash equilibrium

In the first example, an agreement to balance the budget is impossible. The penalty representatives receive for voting against their constituents first best ideal outcome is too great to balance the budget. In this case it pays representatives to vote for their constituents ideal and not to compromise despite the fact that the compromise outcome would benefit his constituents.

In the second example, citizens preference for balancing the budget is exactly the same, only the value of v has changed. The amount that voters hold representatives responsible for the outcome of the legislature's decision is larger. Despite the fact that voter's preference for a balanced budget is no different, the

decrease in v makes it rewarding to balance the budget. But, it does not insure a balanced budget. The game above has four Nash equilibria, non of which involves all three representatives voting for the compromise to balance the budget.

When all three representatives vote for the compromise, the rewards are 3, 3, and 3. But, if one representative defects he receives a 4.5 while the other representatives still receive 3. This is the outcome where a compromise is found to balance the budget, but one representative holds out for a little lower taxes financed by cuts in what his constituents consider to be "bad" spending. This representative is in the position where it appears that the majority prefers more taxes and more spending than he does. Each representative would rather be the one defecting, but is no worse of if only one other representative defects. It is possible that the desire to defect could cause all representatives to vote against the compromise, leaving a deficit even though the reward for balancing the budget is greater.

Chapter 5: Conclusion

The purpose of the above model is to show that if there is a voting paradox and citizens do not know the political constraints, representatives will have a bias against a compromise that will balance the budget. When a voting paradox like the one above exists, balancing the budget requires compromise, some agreement on how to balance the budget. Unless "v" is extremely low, or the disutility of deficits extremely high, only citizens can compromise to balance the budget, representatives don't have the power to impose a compromise on them. This means that voters must have enough information to determine that there is a voting paradox and then determine the second best compromise to balance the budget (i.e. raise taxes or cut spending). It would be unreasonable to assume that votes have all of this information. If voters can determine the second best solution, why do they need a legislature? The role of a legislature is to determine political constraints and compromise solutions, when transaction costs are too high to make direct negotiations impossible. But, the mere fact that representatives must make decisions for someone else who will evaluate their decision based on limited information. impairs their ability to make optimal decisions.

It is obvious that a decision rule could solve the deficit bias, but most decision rules lead to another bias. A balanced budget amendment could solve this problem, but would create a bias against deficits, if they are desired. Ear-marked taxes would connect the votes on spending and taxes, but would lead to bias toward the higher level of spending. Each spending issue would be funded

adequately, but the majority of voters would be unable to enforce their preference for of the optimal size of government spending. A unanimous voting rule would eliminate this problem, but in turn create a bias towards a lower level of spending than desired by the majority. Super-majority rules would lessen the likelihood of this scenario, but not eliminate it, but in the example above each issue passes by a 2/3 majority.

In the case of a voting paradox, the question is not how to eliminate bias, but which bias is least damaging.

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Autobiographical Statement

So I couldn't resist. This is the biography of the idea, rather than my own biography, which may not be that interesting to read, but it's interesting for me to try to figure out where this idea came from. So feel free not to read this. Of course, this is a history that I'm piecing together long after the fact, so don't expect it to be accurate. It is true to the best of my knowledge, but we can't discount the possibility that somebody told me this exact idea and forgot the telling.

Well, you see, it all started, when I was watching television. Back in the 1970's, when I was a kid, I saw this old Shurly Temple movie. An adult showed Shirly this cartoon of Uncle Sam surrounded by a bunch of people all pulling him in different directions. The adult explained that everybody wants something from Uncle Sam, and he can't be all things to everyone.

Then in the '80's, when I was in college. I read, "In the Belly of the Beast" by Jack Henry Abbot. [He's the guy who wrote the gripping prison book while in prison, so they let him out, and he killed somebody] He told a story about how the Americans in his prison thought the Mexican's were disgusting slobs, and the Mexican's thought the American's were disgusting slobs. In Mexico the water pressure is so low that you can't throw paper in the toilet or it will clog. So in Mexico they have waste paper baskets next to the toilet. They get into a U.S. prison and nobody tells them about our powerful water pressure, they find to waste baskets so they throw the toilet paper on the floor. Creating a disgusting mess. They think it's the Americans fault for having to baskets, the Americans think the Mexicans just like to throw used toilet paper on the floor. The problem was not the people. It was the situation. But, everyone blamed the people. So I think I incorporated the value to look for a faulty situation first, before you blame the people. I don't know if it started when I read that or if that just re-enforced a value I already had. Usually, people are not as bad as you think they are if you understand their situation. Maybe it was Ann Frank, she said wrote something about how she still believes in the basic goodness of people, just before she was captured.

Another day at college I was at an anti-apartheid demonstration when some guy came up and tried to tell me that he knew my stands on economic issues just by the fact that I was at a left wing demonstration. At the time I was a Libertarian voter, he was so far off base that he left convinced that I was lying just to contradict him. But, this made me think that the idea of left and right is way to simplistic. Even if there is a left and right on every issue. Many political issues are not logically connected meaning that political views could only be graphed in multi-dimensional space.

In the Spring of 1991 We covered Arrow's Impossibility Theorem in Microeconomics II at the Graduate Center.

The fall of 1992, I'm in grad school and I'm teaching Principles of Macroeconomics to undergraduates at Hunter college. We get to a section on the budget deficit and I came up with the core idea for this as a discussion idea to explain the often quoted poll result that "voters tend to overwhelmingly support a balanced budget, but a majority opposes any specific plan to balance the budget." Therefore, voters don't know what they want, right. Maybe it's not the people it's the situation. Maybe they don't all want more things than they're willing to put into the budget. Maybe they all what different things out of the Uncle Sam's limited budget. The class talked about how the government would have difficulty trying to solve this problem. One student said the government should get the extra money by taxing resident aliens.

I had no idea that it was an original idea at the time, and actually I'm still skeptical, but I couldn't find anything in the journals about it, so my advisor said go ahead.