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ESTIMATION OF THE COBB-DOUGLAS AND CES PRODUCTION FUNCTIONS IN KOREA (1957-1975)

by

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A dissertation submitted to the Graduate Faculty in Economics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

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ABSTRACT

ESTIMATION OF THE COBB-DOUGLAS AND CES PRODUCTION FUNCTIONS IN KOREA (1957-1975).

by

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Adviser: Professor Michael Grossman

This study attempts to provide an analytical review of the Korean productions such as manufacturing, agriculture, construction, transcommunication, and mining sectors separately from 1957 to 1975 sample periods.

The kind of questions which was answered by production functions are degree of returns to scale, technology effect, and the degree of substitutability between inputs. The methodologies of specific func tional form which were used to estimate those parameters are the various Cobb-Douglas and CES formulations.

The present study applies the ordinary least squares, indirect least squares principles with unconstrained returns to scale and the straight regression under the assumptions of the constant returns to scale to the Cobb-Douglas type ,and applies the Two-Stage least squares and Taylor expansion regressions principles to the CES type.

The assumptions throughout this paper are as follows; I was worked with Hicks-neutral disembodied technological change for the estimation of specific production functions, Cobb-Douglas and CES, and the homogenity condition of output and inputs during sample periods.

The essential to the derivation of the Cobb-Douglas and CES is that prices of per unit of capital and labor service, r and w respectively, are exogeneously determined.

The results of the findings are as follows; The results of nonlinear method(CES) may, in some cases, be quite different from those obtained by linear(Cobb-Douglas), and the elasticities of substitution are extremely sensitive to difference in measurement and the data construction. This study shows that, there are evidence of increasing returns to scale during economic development plan periods for most sectors and associated with this phenomenon somewhat lower rates of neutral technical progress than Solow's residual which was constrained returns to scale. In this respect, the results of the study support the Walters(1963) findings.

In the discussion of qualification of the results, some estimates turned out to be unacceptable due to the multicollinearity, and also due to the business cycle effect some estimates were overestimated.

ACKNOWLEDGEMENTS

I wish to acknowledge the immeasurable assistance from my advisors, Professor Michael Grossman, Elliot Zupnick, and Damador Gujarati.

Throughout all the various stages of my study, professor Michael Grossman has contineously guided me and was always willing to confer on any difficulty that developed, no matter how trivial. For this, and for what I was able to learn while working with him, I shall always be grateful. I am also grateful to professor Damador Gujarati. His guidance and advice rescued me from commiting errors as well as averting impasses, and providing me the empirical foundation of this study through his invaluable and thorough lectures which he had given to us in his econometric seminar. Needless to say, I would like to pay an extra thanks to professor Elliot Zupnick for his kindness and helpful advice.

Thanks also to my parents and brothers for their encouragement and willingness to sacrifice all too many thinks so that I might con tinue my study. To all these individuals, and to many others whom I am unable to mention, I express my sincere gratitude and heart feld thanks.

Without the assistance of those people this dissertation could not have been completed. But the author alone bears final responsibility for its numerous errors which might be still contain.

HACHEONG YEON

TABLE OF CONTENTS

ACKNOWLEDGEMENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER		Page
I.	INTRODUCTION	1
II.	THEORETICAL ASPECTS	
	1. Measurement of Productivity	9
	1-1. Indexes of Factor Productivity	9
	1-2. Technical Change and The Cobb-Douglas and CES.	12
	1-3. Hicks and Harrod Neutral	14
	2. Two Production Function Models and Estimation	21
	2-1. Estimation of The Cobb-Douglas Function	21
	2-2. Indirect Least Square Estimate	25
	2-3. The Cobb-Douglas with Constant Returns to Scale	26
	2-4. The Alternative Estimation for Cobb-Douglas	28
	2-5. The Estimation of CES Production Function	32
	2-6. The Two-Step Procedure	35
	2-7. Taylor Expansion Method	35
	2-8. One Alternative Estimation Method for CES (Modified Indirect Least Square)	38
III.	EMPIRICAL RESULTS	
	1. Introduction to Empirical Section	41
	2. Technical Progress and Productivity Change	43
	3. Results of Estimation in Manufacturing	52
	3-1. The Cobb-Douglas Type	52
	3-2. Estimated Results of CES	57
	4. Results of Estimation in Agriculture	60
	4-1. The Cobb-Douglas Type	60
	4-2. The Estimated Results of CES	62
	5. Results of Estimation in Construction	63
	5-1. The Cobb-Douglas Type	63
	5-2. The Estimated Results of CES	65
	6. Results of Estimation in Transcommunication	66
	6-1. The Cobb-Douglas Type	66
	6-2. The Estimated Results of CES	68

7. Results of Estimation in Mining 69
7-1. The Cobb-Douglas Type 69
7-2. The Estimated Results of CES 70

IV. THE QUALIFICATION OF THE RESULTS AND CONCLUSION 72

V. APPENDICES

APPENDIX A Summary of Tables for Regression Results 78

APPENDIX B MATHEMATICAL APPENDIX
(Estimation of Parameters and Standard Errors in ILS.) 96
(The Method of Calculation of Standard Errors in Taylor Expansion CES) 100

APPENDIX C Properties of The CES Production Function 103

BIBLIOGRAPHY 106

LIST OF TABLES

TABLE		Page
Ma-I.	The Partial Productivity Indexes and The Solow's Residual (Manufacturing)	78
Ma-II.	Estimated Results of The Cobb-Douglas for Manufacturing	79
Ma-III.	Estimated Results of CES for Manufacturing	80
Ag-I.	The Partial Productivity Indexes and The Solow's Residual (Agriculture)	81
Ag-II.	Estimated Results of The Cobb-Douglas for Agriculture	82
Ag-III.	Estimated Results of CES for Agriculture	83
C-I.	The Partial Productivity Indexes and The Solow's Residual (Construction)	84
C-II.	Estimated Results of The Cobb-Douglas for Construction	85
C-III.	Estimated Results of CES for Construction	86
TC-I.	The Partial Productivity Indexes and The Solow's Residual (Transcommunication)	87
TC-II.	Estimated Results of The Cobb-Douglas for Transcommunication	88
TC-III.	Estimated Results of CES for Transcommunication	89
MI-I.	The Partial Productivity Indexes and The Solow's Residual (Mining)	90
MI-II.	Estimated Results of The Cobb-Douglas for Mining	91
MI-III.	Estimated Results of CES for Mining	92
IV.	Testing for Equality between Sets of Coefficient in Two Different Regressions (CHOW Test)	93
V.	Rate of Growth(%) by Industrial Origin	94

LIST OF FIGURES

FIGURE		Page
1.	Hicks Neutral Technical Change	15
2.	Harrod Neutral Technical Change	17
3.	A Scatter of ΔA/A against Capital Labor Ratio	44
4.	The Cumulated Effect of A(t) over Time	45
5.	Rate of Growth(%) by Industrial Origin	95

CHAPTER I.

INTRODUCTION

I. INTRODUCTION.

For nearly a half century, the Cobb-Douglas form for the production function has been popular, in both theoretical and empirical analysis. A good summary of much of the earlier work with aggregate Cobb-Douglas production functions and an effort to continue and extent earlier studies of the production function appear in Paul Douglas (1947 and 1976).[1] There have been many applications of the Cobb-Douglas production function to industries.[2]

In recent years, more general form of production function appeared. In,[3] Arrow, Chenery, Minhas, and Solow (herein abbreviated, ACMS) derived a family of production function in which the elasticity of substitution is an unspecified constant. This function has both the Cobb-Douglas and a fixed proportions (Leontief) production function as special cases.[4]

---

1. Douglas,Paul H. "Are there laws of production?" American Economic Review, XXXVIII, No.1 (March,1948) pp.1-41.
   "The Cobb-Douglas production function once again: its history, its testing, and some new empirical values." J.P.E. vol.4, No.5, Oct.,1976

2. Brown,Murry, and Joel Popkin. "A measure of technological change and returns to scale." The Review of Economics and Statistics, XLIV, No.4 (Nov.,1962) pp.402-411
   Griliches,Zvi. "Research expenditures, education, and aggregate agricultural production function." American Economic Review, LIV,No.6 Dec.,1964.
   Kurz, Mordecai, and Alan S. Manne. "Engineering estimates of capital labor substitution in metal machining." AER LIII, No.4 (Sept.,1963)

3. Arrow,Kenneth, Hollis B. Chenery, Bagicha Minhas, and Robert M.Solow. "Capital Labor substitution and economic efficiency." The Review of Economics and Statistics, XLIII, No.3, (Aug.,1961)

The former is obtained when the elasticity of substitution is equal to unity and the latter emerges in the limit as the elasticity of substitution approaches to zero. Thus the Cobb-Douglas function was threatened, not merely, technological obsolences, but with the indignity of being swallowed as a special case of a more general formulation. ACMS presented both cross-sectional and time series evidence that the elasticity of substitution was significantly different from both zero and unity.

The kind of questions which may be answered by production functions are the following. 1. Returns to scale. 2. Allocation efficiency. 3. Returns to factors. 4. The degree of substitutability between inputs. The pioneering econometric estimation of a production function by Cobb-Douglas was undertaken precisely for these purposes. However, in recent years, there has been some doubt about the validity of an aggregate production function and the meaningfulness of empirical estimates using such a production function.[5]

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4. See the appendix for discussion of this special cases.
Both popular parametric forms were discovered by others than those whose names are associated with the familiar formulas. It has been known that Knut Wicksell(1934) predated Cobb and Douglas with the loglinear form, while J.K. Witaker has established(1964) H.D. Dickinson's priority of discovery for the CES (Constant Elasticity of Substitution) function.

5. Marschak J., and W.H. Andrews. "Random Simultaneous equations and the theory of production." Econometrica, 12. (July-Oct.,1944) pp.143-205.
They indicated that "Single equation estimates are biased when the equation is a member of a system of equations in the following way: the system is such that some of the "independent" variables, as well as the dependent variable, are functions of the disturbance in the given equation. This contradicts the assumptions underlying single equation regression since the presumed independent variables are in fact correlated with the disturbance. The resulting bias may be called the simultaneous equation bias."

In this study, for empirical study, we will follow both approaches, Cobb-Douglas and CES production functions applying the ordinary least square principle, indirect least square method with unconstrained labor and capital elasticity, and the straight regression with the labor and capital exponents constrained to add to unity for the Cobb-Douglas type and the Taylor expansion regression and the Two-Step regression method for the CES production function. These estimation procedures will be applied to the Korean economy, manufacturing, agriculture, construction, transportation and communication, and mining sectors separetaly.

Throughout the 1960's to 1975, the Korean economy sustained high growth and achived resounding successes in various sectors, including the highly improved industrial structure, expanded exports, increased domestic savings, agricultural development, expansion of social overhead capital and the laying of a ground work for economic stability. From 1960 through 1975, the growth rate of the gross national product at an average annual rate of 9.1 per cent, thus increasing by 2.6 times from 1960. Per capita GNP increased by 2.3 times during the 15 years. The growth rate was much higher than the average annual economic growth rate of 4.8 per cent 1950's (1954-1959). An accelerated high rate of economic growth during second and third five year economic development plan period provide the propelling force needed to accomplish the take-off.

First 5-year plan (1962-1966)

The Korean economy completed successfully its first five year economic development plan in 1966. During the plan period, the Korean economy recorded an annual average growth rate of 7.8 per cent. The final year of the First five year plan the primary industry grew by 5.3 per cent annually on the average during the same period the secondary industry by 15 per cent and tertiary industry by 8.9 per cent respectively. The share of the primary industry in the GNP formation dropped from 36.9 per cent in 1960 to 36.1 per cent in 1966, while that of the secondary industry went up from 15.7 per cent in 1960 to 19.6 per cent in 1966. The ratio of the tertiary industry to the GNT formation also declined from 47.4 per cent in 1960 to 45.2 per cent in 1966.

The annual average investment scale during the 1962-1966 period was 16.9 per cent of the GNP; and 54.8 per cent of the total investment requirements was met with foreign funds and the remaining 45.2 per cent with domestic funds. Of the total investment during 1962-66, 57.9 per cent was for the tertiary industry, 24.1 per cent for the secondary industry and 9.6 per cent for the primary industry.

The nation's commodity exports which reached only $41 million in 1961, soared by 44 per cent annually during the 1962-66 period- to $250,334,000 in 1966. Of the total export ammount in 1966, agricultural products accounted for 9.5 per cent, fishery products for 14.7 per cent, mineral products for 13.4 per cent and manufactured goods for 62.4 per cent, as compared with 29.1 per cent, 13.9 per cent, 38 per cent and 19 per cent in 1960, respectively. The population also jumped by 2.7 per cent annually during the plan period from 24,600,000 in 1960 to 29,100,000 in 1966. Of the labor force of 9,700,000 persons in 1966, some 9,100,000 were employed.

Second 5-year plan (1967-1971).

The Korean economy grew by 8.9 per cent in 1967, 13.3 per cent in 1968, 15.9 per cent in 1969, 9.7 per cent in 1970 and 9.2 per cent in 1971. The primary industry, which dipped by 5.5 per cent in 1967, rose again by 1.2 per cent in 1968, 11.9 per cent in 1969 and only 2 per cent in 1970, while secondary industry grew by 22.5 per cent in 1967, 25.9 per cent in 1968, 21 per cent in 1969 and 17.4 per cent in 1970. The tertiary industry recorded a growth rate of 15.4 per cent in 1967, 15.9 per cent in 1968, 15.6 per cent in 1969 and 10.2 per cent in 1970. The share of the primary industry in the GNP formation dropped from 32.8 per cent in 1967 to 26.4 per cent in 1970, while that of the secon dary industry expanded from 22.3 per cent in 1967 to 27.7 per cent in 1970. The ratio of the tertiary industry to the GNP formation slightly increased from 44.9 per cent in 1967 to 45.9 per cent in 1970.

The annual average investment scale during the plan period was set at 29.5 per cent of the GNP and only 38.2 per cent of the total investment requirements was met with foreign funds and the remaining 61.8 per cent with domestic capital. Of the total investment during 1967-1969 period, 25.3 per cent was used for the secondary industry, 6.9 per cent for the primary industry and 67.8 per cent for tertiary industry.

The nation's commodity exports went up by 40 per cent on the average during the 1967-1970 period to record $358,593,000 in 1967, and $1,000 million in 1971. Of the total export amount in 1969, agriculture products accounted for 4.2 per cent, fishery products for 9.4 per cent, mineral products for 7.4 per cent and manufactured goods for 79.0 per cent, as compared 9.5 per cent, 14.7 per cent, 13.4 per cent and 62.4 per cent in

1967 respectively. The number of foreign countries buying Korean products increased 70 in 1966 to 98 in 1969. The annual population growth rate was arrested at 2 per cent during 1967-1970 period from 29,100,000 in 1966 to 31,793,000 as in 1970.

Third 5-year plan (1972-1976).

Third five year economic development plan (1972-76) under which an annual average economic growth of 10.6 per cent was recorded. With the aim of achieving a harmonized balance between "growth" and "stability", the plan places emphasis on improving the rural economy, boosting exports and developing heavy and chemical industries.

During the period, the primary industry was grown by 4.5 per cent annually, the secondary industry by 20.8 per cent and tertiary industry by 8.3 per cent. The share of primary industry in the GNP formulation, which stood at 36.1 per cent in 1966,was 27.7 per cent in 1971 and again to 21.1 per cent in 1975, while that of the secondary industry was risen from 19.6 per cent in 1966 to 23.6 per cent in 1971 and to 30.7 per cent in 1975. But the ratio of the tertiary industry was left almost unchanged- 45.2 per cent in 1966, 49.2 per cent in 1971 and 46.1 per cent in 1975.

The annual average investment scale during the 1972-75 period was 26.2 per cent of the GNP. Of the total investments scale, domestic funds was 77.2 per cent and 22.8 per cent with foreign capital. This means increased tax burden during the plan period. The annual population increase rate was kept at 1.5 per cent during the periods, as compared with 1.9 per cent in 1969. The nation's exports was reached $5,081 million in 1975 compare to $1,000 million in 1971.

The plan of this paper is as follows. In section II, I present theoretical aspects of this study, discussion of measurement problem of technological change and introducing various estimation procedures applicable to the Cobb-Douglas and CES production functions. In section III, after brief description of the data employed, I present and interpret the empirical results. In section IV, a summary of principal conclusion will be devoted. In section V, general statistics of the Korean economy and the parameters estimated. Some mathematical solutions for standard error calculations of restricted regressions and discussion of special cases of CES production function will be contained in section VI as an appendix.

For the estimation of the Cobb-Douglas and CES production functions, we will work with Hicks-neutral disembodied technological change. Thus the output obtainable from a given combination of labor and capital is assumed to grow exponentially over time, in such a manner that the marginal rate of substitution between the unchanged amounts of these two factors remains constant. We thus make no allowance for technological change which can be utilized only if it is embodied in capital equipment of a current vintage and which has been in recent paper by Solow and Pheleps.[6]

For our empirical work some adjustment for underutilization of

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6. Solow, Robert M. "Investment and technical progress." in Kenneth J. Arrow, Samuel Karlin, and Patrick Suppes eds., Mathematical methods in the social sciences 1959. (Stanford: Stanford Univ. Press 1960_

   Pheleps, Edmund S. "The new view of investment: A neoclassical analysis." Quartely Journal of Economics, LXXVI, No.4 (Nov.,1962) pp. 548-67.

the capital stock would appear to be necessary, since we strongly believe that in the "real world" plant and equipment like labor force, can be less than fully employed and to construct an index of labor services, it was thought that corrections should be made with respect to age, sex, schooling ,and OJT (on the job training). However, the data are not available for this study. I used number of person and capital stock as an unit of inputs under the assumption of homogenity during sample period rather than the person hours and machine hours. Thus I entail some amount of misspecification in this respect. In terms of the Cobb-Douglas and CES production functions which we examine, with this assumption, the parameters of these functions will be stable over the period of estimation.

CHAPTER II.

THEORETICAL ASPECTS

II. THEORETICAL ASPECTS

II-I. Measurement of Productivity: Technical Change.

More than a decade ago M. Abramovitz(1956), Kendrick(1961), and Solow(1957) established on the basis of measures that conventionally measured inputs, capital and labor, leave a large portion of the growth of output unexplained. Since then considerable research on the measurement, determinants and consequences of factor productivity has been undertaken. Just to see the empirical behavior in the Korean economy for 1957-1975, I will discuss the partial factor productivity and the Kendrick's arithmetic measure and Robert Solow's geometric index as the "residual" or the index of "technical progress."

II-I-1. Indexes of Factor Productivity.

Productivity is often measured as a ratio of output to inputs, there are as many indices of productivity as there are factors of production. While each index has its own use, the most important and most often used are the partial productivity indices of labor and capital and the total productivity index. The partial indices are simply the average products of labor, or capital. While total factor productivity, often referred to as the "residual" or the index of "technical progress" is defined as output per unit of labor and capital combined. These indices are (a). Partial indices

$$AP_L = Y/L$$

$$AP_k = Y/K$$

(b) Total Productivity Index.

$A = Y/(aL + bK)$

Where Y, L, and K are, respectively, the aggregate level of output, labor, and capital inputs and a and b are some appropriate weights.

There are many ways of measuring total productivity, but two indices most often use in empirical research are Kendrick's arithmetic measure and R. Solow's geometric index. Kendrick approaches measurement of dA/A, he assumes a homogeneous production function and the Euler condition to obtain following measure

$$\frac{dA}{A} = \frac{Y_1/Y_o}{(wL_1+rK_1)/(wL_o+rK_o)} - 1 \qquad (1)$$

Where w and r are the wage rate and the rate of return on capital respectively, and variables with the subscript 1 refer to the current period and those with the subscript o refer to the base period. In empirical estimates the weights for calculating equation (1) are often permitted to change smoothly over time.

Without using a specific production function, Solow's tech nical change refers to any kind of shift in the production function of the firm. The aggregate production function can be written as

$$Y = F(K, L; t) \qquad (2)$$

The variable t for time appears in F to allow for technical change. Solow was using the phrase "technical change" as a short hand expression for any kind of shift in the production function. Thus, slowdowns, speedups, improvements in the education of the labor force, and all sorts of things will appear as "technical change." Shift in the produc tion function are defined as neutral if they leave marginal rates of substitution untouched but simply increase or decrease the output attainable from given inputs, the production function can be written as

$$Y = A(t)F(K, L) \qquad (3)$$

A(t) measures the cumulated effect of shift overtime. Differentiate equation (3) totally with respect to time and devide by Y, we have

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + A\frac{\partial f}{\partial K}\frac{\dot{K}}{Y} + A\frac{\partial f}{\partial L}\frac{\dot{L}}{Y} \qquad (4)$$

Where dots indicate time derivatives. Now define $w_k = (\partial Y/\partial K)(K/Y)$ and $w_L = (\partial Y/\partial L)(L/Y)$ the relative shares of capital and labor, and substitute in the equation (4) (note: $(\partial Y/\partial K) = A(\partial f/\partial K)$, etc.,), and these results

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + w_k\frac{\dot{K}}{K} + w_L\frac{\dot{L}}{L} \qquad (5)$$

From time series of $\dot{Y}/Y$, w_k, $\dot{K}/K$, w_L, $\dot{L}/L$ or their discrete year to year analogues, we could estimate $\dot{A}/A$ and thence A(t) itself. If all factor inputs are classified either as K or L, then the available figures always show w_k and w_L adding up to one ($w_k + w_L = 1$). Since we have assumed that factors are paid their marginal products, this amounts to assuming the hypotheses of Euler's theorem. That is the production function is homogeneous of degree one.

Let $Y/L = q$: $K/L = k$, and $w_L = 1 - w_k$: note that $\dot{q}/q = \dot{Y}/Y - \dot{L}/L$, then we have

$$\frac{\dot{q}}{q} = \frac{\dot{A}}{A} + w_k \frac{\dot{K}}{K} \tag{6}$$

In equation (6), given the time series of k, q, and w_k we can get A(t) series which is a profit of technical change.[1] And q/A(t) gives the output per labor unit if there had been no shift in production function. Solow calls it "corrected" output per unit labor, or output per labor unit "net of technical change."

II-I-2. Technical Change and the Cobb-Douglas and CES.

Differentiate equation (3) totally with respect to time and devide by Y, it can be expressed

$$\frac{dA}{A} = \frac{dY}{Y} - \left[\frac{L}{Y} F_L \frac{dL}{L} + \frac{K}{Y} F_k \frac{dK}{K} \right] \tag{7}$$

Where F_L and F_k are partial derivatives of output with respect to L and K and the variables proceeded with d refer to time derivatives of the variables.[2] It is clear from equation (7), the magnitude of the residual, dA/A and its stability over time depend on: 1. the form of the production function that governs the behavior of F_L and F_k.

---

1. In empirical study, assuming the hypotheses of Euler's theorem, we will use the following formula.

$dA/A = dq/q - w_k\, dk/k$, here $q=Y/L$ and $k=K/L$

$A(t+1) = A(t)(1 + dA(t)/A(t))$

2. Proper measurement of L and K and adjustment for their quality changes, and 3. the importance of variables other than K and L such as the entrepreneurial ability that are left out of the production function.

Suppose the production function is the Cobb-Douglas type, then equation (7) reduces to

$$\frac{dA}{A} = \frac{dY}{Y} - \left[\alpha \frac{dL}{L} + \beta \frac{dK}{K}\right] \qquad (8)$$

If the share of labor, α, is assumed to be invatiant with respect to dL/L and dK/K and constant returns to scale prevail ($\alpha + \beta = 1$). Thus any errors will spill over into the measure of dA/A.

Suppose the underlying production function is CES and not the Cobb-Douglas and the input shares are assumed constant, we may write the equivalent of equation (7) as

$$\frac{dA}{A} = \frac{dY}{Y} - \left[\alpha \frac{dL}{L} + (1-\alpha)\frac{dK}{K}\right] - \frac{1}{2}\alpha(1-\alpha)\left[\frac{\sigma-1}{\sigma}\right]\left[\frac{dK}{K} - \frac{dL}{L}\right]^2 \qquad (9)$$

Where σ is the elasticity of substitution between K and L. Thus the measure of technical change equation (8) differs from equation (9) by:

$$-\frac{1}{2}\alpha(1-\alpha)[(\sigma-1)/\sigma](dK/K - dL/L)^2$$

If $\sigma < 1$ and $dK/K > dL/L$ then Y grows at a lower rate than the index of K and L combined.

2. For simple derivative, see Nadri, M. Ishag. "Some approaches to the theory and measurement of total productivity." Journal of Economic Literature, Dec.,1970. p.1140.

II-I-3. Hicks and Harrod Neutral Technical Change.

The purpose of a definition of neutral technical progress is to indicate characteristics of technical progress which will in some sense leave unchanged the balance between labor and capital, and so will permit steady growth. Technical progress shifts the entire produc tion function, and therefore an index-number type of problem arises in deciding which point on the old production function to compare with which point on the new one. This problem has led to the formulation of a number of alternative definitions of neutral technical progress. (see Harrod,1948: Hicks,1932: Kennedy,1961,1962: Robinson,1938: Uzawa, 1961).

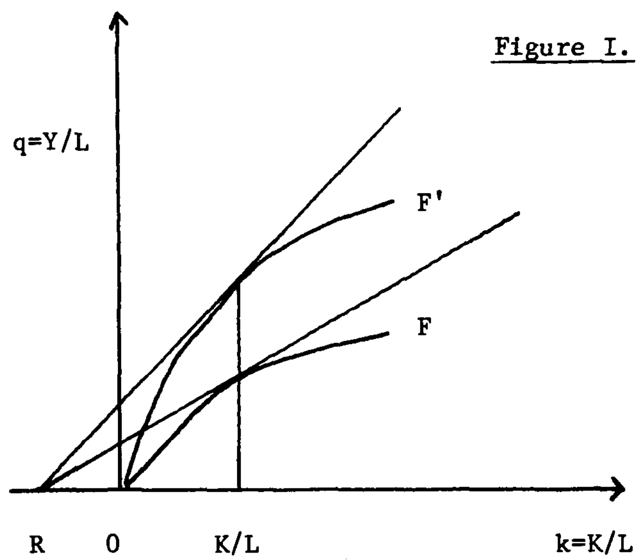
Two definitions of neutral technical change will be discussed in this section, since we will use Hicks neutral in our empirical study. They are known after authors as Hicks neutral and Harrod neutral. There is no agreement on the precise definition of bias in technical change. There are several ways of defining technical bias. Change in relative shares of the inputs is often used as a measure of technical bias. The Hicksian definition measures the bias along a constant capital-labor ratios; the Harrodian definition measures the bias along a constant capital-output ratio; and Solow's definition measures the bias along a constant labor-output ratio.

Assuming constant returns to scale, the definition of neutral progress given by Hicks(1932) is based on comparing points on the two functions, where the capital-labor ratio, K/L is constant. Neutral progress is defined as taking place if the ratio of the marginal product of labor to the marginal product of capital is unchanged when K/L is unchanged. (Technical progress is capital saving if the marginal product of labor is raised by more than that of capital, given K/L, and it is labor saving in the opposite.)

Symbolically;

$$\left[\frac{\partial\,(F_k K)/(F_L L)}{\partial\, t}\right] \gtreqless 0 \quad \text{K/L constant} \quad \text{Hicks}\begin{cases}\text{labor-saving}\\ \text{neutral}\\ \text{Capital-saving}\end{cases}$$

Figure I.



In figure I, we have shown that the production function before and after Hicks neutral technical change, with k=K/L, and q=Y/L.

We know that OR measures the wage-rental ratio; i.e., the ratio of the marginal products.(for a proof, see p.819 of the source: Economic Journal, vol.74). The distinctive feature of Hicks-neutral technical progress is that for any given value of the capital-labor ratio, k this remains unchanged by the technical change.

It can be shown that in this case the curve F' can be derived from F by raising output per man for all values of k in the new proportion. In order for technical progress to be Hicks-neutral, the production function must be[3]

$$Y = A(t)F(K, L)$$

The Harrod definition of neutrality is based on the comparision of points on the production function at different times where the marginal product of capital assumed equal to the rate of profit, ρ is constant. With K/L unchanged, the technical progress will normally raise the marginal product of capital. For the marginal product of capital to remain constant in face of technical progress, K/L must normally rise. Technical progress is neutral in the Harrod sense if the level of K/L which causes ρ to remain constant after a technical improvement is such to cause the capital-output ratio(K/Y) to remain constant.

---

3. Let F(K,L;t) be the production function with constant returns to scale. Let F_k and F_L be the partial derivative of F with respect to capital and labor.
   (1). Since F_k and F_L are the marginal product of K and L, Hicks-neutrality implies $\partial \ln F_k = \partial \ln F_L = C$. C; constant.

   (2). Consider $a = F_t/F$. By constant returns to scale a is homogeneous of degree zero in K and L.

$$\frac{\partial a}{\partial K} = \frac{\partial F_t/\partial K}{F} - \frac{F_t F_K}{F^2} = \frac{F_K}{F}(C - F_t/F)$$

   So,

$$K\frac{\partial a}{\partial K} + L\frac{\partial a}{\partial L} = \frac{(F_K K + F_L L)}{F}(C - F_t/F) = C - F_t/F$$

   Since, a is homogeneous of degree zero in K and L , $C - F_t/F = 0$, and so $\partial a/\partial K = 0$. Hence the production function be written as A(t)F(K,L) where $dA(t)/A = a$.

Technical progress is labor saving(capital using) or capital saving (labor using) if a constant ϱ is associated with a higher or lower capital-output ratio respectively.

Symbolically:

$$\left(\frac{\partial (F_k K)/(F_L L)}{\partial t}\right)_{K/Y \text{ constant}} \gtrless 0 \qquad \text{Harrod} \begin{cases} \text{labor-saving} \\ \text{neutral} \\ \text{capital-saving} \end{cases}$$

$$\text{Harrod} \begin{cases} \text{neutral} \\ \text{capital-saving} \end{cases}$$

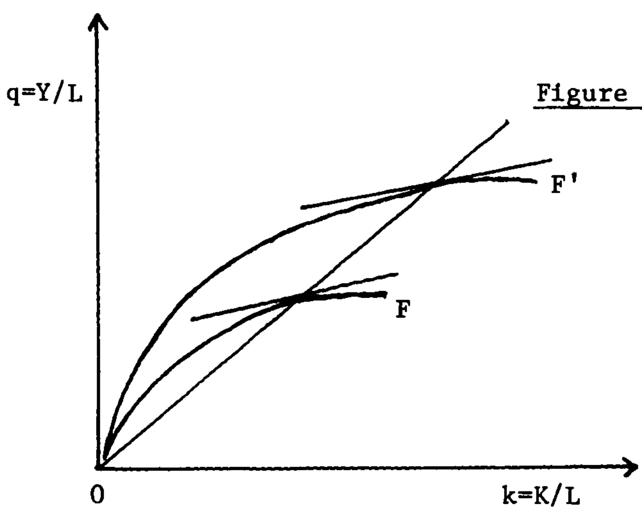


Figure II.

Using the digram, we depict Harrod-neutral technical progress in figure II. Here F' is derived from F, on the ray through the origin the slope of OF' is the same as the slope of OF. This is so because the slope of the line through the origin is the ratio of output to capital, and technical progress is such that if this ratio is constant, then so is the rate of profit. (the slope of the tangent). The general form of Harrod-neutral technical change with two factors of production can be written as;

$$Y = G(a(t)L,\ K)$$

The t term is thus prefixed to L instead of to F(K,L), as in the Hicks-neutral case. With such a production function, assuming constant returns to scale, it is apparent that an equal rise in K and a(t)L must lead to an equal proportional rise in Y. (for a proof that it is a necessary and sufficient condition of Harrod-neutral that production function should have this form, see Uzawa,1961).

Consider two linear homogeneous production functions

$$Y = F(a(t)L, \quad a(t)K) = a(t)F(K, L) \tag{10}$$

$$Y = G(a(t)L, K) \tag{11}$$

Equation (10) defines a Hicks neutral innovation, while equation(11) defines a Harrod neutral innovation.[4]

The cost minimization condition associated equation (10) is

$$\frac{MP_L}{MP_k} = \frac{w}{r}$$

Where w is the wage per person and r is the rental price of capital.

---

4. Both the Hicks and Harrod definitions of neutrality amount to the same if the production function is the Cobb-Douglas type.

$$Y = AL^{\alpha} K^{1-\alpha} \quad , A=e^{\lambda t} \quad , Y = e^{\lambda t} L^{\alpha} K^{1-\alpha}$$

Harrod neutrality at rate m:

$$Y = (\bar{L}^{\alpha}) K^{1-\alpha}, \text{ here, } \bar{L} = L e^{mt}$$

$$Y = (L e^{mt})^{\alpha} K^{1-\alpha} \quad , Y = e^{m\alpha t} L^{\alpha} K^{1-\alpha} : \lambda = m\alpha$$ causes Harrod neutrality.

Hicks neutrality at rate m:

$$Y = F(aL, aK), \quad Y = \bar{L}^{\alpha} \bar{K}^{1-\alpha} \quad , Y = (Le^{mt})^{\alpha} (Ke^{mt})^{1-\alpha}$$

$Y = e^{mt} L^{\alpha} K^{1-\alpha}$, $\lambda = m$ causes Hicks-neutrality. In both cases, as A(t) is growing, we have no effect on the ratio of marginal products of K and L.

When we are assuming w/r is fixed, as increased in a(t) has no effect on the optimal ratio of K to L, $(k=K/L)$.

The cost minimization condition associated with equation (11) is

$$\frac{MP_{\bar{L}}}{MP_k} = \frac{\bar{w}}{r}$$

Where $\bar{L} = a(t)L$, and $\bar{w}$ is the wage of one efficiency unit of labor, since

$MP_L = a(t)MP_{\bar{L}}$, $(\because \partial Y/\partial L = (\partial Y/\partial \bar{L})(\partial \bar{L}/\partial L))$

$$\frac{MP_L/a(t)}{MP_k} = \frac{\bar{w}}{r} \quad \text{,or} \quad \frac{MP_L}{MP_k} = \frac{a(t)\bar{w}}{r} = \frac{w}{r}$$

When we are assuming w/r is fixed, if a(t) is rising, this means that $\bar{w}$ is falls, under the assumption of a fixed w/r, $k = K/L$ would vary with a(t) if the elasticity of substitution in production function is not equal to unity.[5]

---

5. Let us see the CES production function.

$$Y = (\alpha K^{-\rho} + (1-\alpha)\bar{L}^{-\rho})^{-1/\rho}, \text{ here } \bar{L} = a(t)L.$$

First order condition show us $MP_k/MP_{\bar{L}} = r/\bar{w}$.

Using the notation (...) as a shorthand for $(\alpha K^{-\rho} + (1-\alpha)L^{-\rho})$, we have

$$F_L = \partial Y/\partial L = A(-1/\rho)(\ldots)^{-(1/\rho)-1}(1-\alpha)(-\rho)L^{-\rho-1}$$

$$= (1-\alpha)A(\ldots)^{-(1+\rho)/\rho}L^{-(1+\rho)}$$

$$= (1-\alpha)A^{1+\rho}/A^{\rho}(\ldots)^{-(1+\rho)/\rho}L^{-(1+\rho)}$$

continue on next page

If, technical change is "embodied" in capital and labor the bias in technical change will depend on the elasticity of substitution, σ. "Embodiment" means that because of technological advance the new inputs are more efficient than old inputs. In particular,

$$\frac{\partial k}{\partial a} \lesseqgtr 0 \qquad , \text{ as } \sigma \gtrless 1.$$

$$= (1-\alpha)/A^{\rho} \; (Y/L)^{1+\rho}$$

And similarly, $F_k = \partial Y/\partial K = \alpha/A^{\rho}(Y/K)^{1+\rho}$

Thus the slope of an isoquant and the ratio between marginal products (with K plotted vertically and L horizontally) is

$$\frac{dK}{dL} = - \frac{F_L}{F_K} = \frac{-(1-\alpha)}{\alpha} \left[\frac{K}{L}\right]^{1+\rho}$$

Therefore, $MP_k/MP_{\bar{L}} = (\alpha/1-\alpha)(K/L)^{-1-\rho} = r/\bar{w}$, and the relative demand curve for labor is:

$\ln \bar{L}/K = \sigma \ln r/\bar{w} + \sigma \ln(1-\alpha/\alpha)$, we know $\bar{L} = aL$ and $\bar{w} = w/a$
therefore,
$\ln L/K + \ln a = \sigma \ln r/w + \sigma \ln a + \sigma \ln(1-\alpha/\alpha)$
$\ln L/K = \sigma \ln r/w + (\sigma - 1) \ln a + \ln (1-\alpha/\alpha)$, here $\sigma = 1/(1+\rho)$

$$\frac{\partial \ln L/K}{\partial \ln a} = \sigma - 1$$

$$\therefore \frac{\partial \ln L/K}{\partial \ln a} \gtrless 0 \qquad \text{as } \sigma \gtrless 1.$$

II-II. Two Production Function Models and Estimation.

The explicit production functions such as the Cobb-Douglas and CES offer us in a convenient form of one or two parameters of main interest, returns to scale or elasticity of substitution.

II-II-1. Estimation of the Cobb-Douglas Production Function.

The Cobb-Douglas production function with provision for dis embodied neutral technical change can be written as

$$Y = Ae^{\lambda t} L^{\alpha} K^{\beta} u \qquad (1)$$

Where Y is real output, L is the labor, K is the capital input, u is a random disturbance, and A, λ, α, and β are parameters. Equation (1) is nearly always treated as a linear relationship by making a logarithmic transformation, which yields

$$\ln Y = \ln A + \lambda t + \alpha \ln L + \beta \ln K + \ln u \qquad (2)$$

Where ln u is treated as an additive random error with a zero mean and constant variance.

One of the objective of this study is to see whether simple O.L.S. and I.L.S.[6] estimation of the Cobb-Douglas type which are not constrained labor and capital elasticity yield significantly different results from, the straight regression under the assumption of constant returns to scale.

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6. This method was suggested by Henri Theil. See Hoch (p.572. footnote #12).

We turn, however, to the problem of identifiability of the production function. In discussing identifiability we shall confine ourselves to a production function producing a homogeneous output(Y) using two homogeneous inputs (L and K) which are being bought and sold in competitive conditions and therefore their respective prices are exogeneously given.

In a special case which is indicated in the literature as the Marschak-Andrews problem, the production function becomes under-identified and cannot be meaningfully estimated by a straightforward OLS regression.[7] The production function is a technical relationship between output and inputs. But the actural quantities of inputs used and output produced are a result of economic or behavioral decisions. In order to see the problem of under-identification, let us set up the profit maximizing behavior given input and output prices and a production function. If the production function is of the Cobb-Douglas type with two inputs,[8] the production function would be represented as follows

$$Y = AL^{\alpha}K^{\beta} \qquad (4)$$

$$\pi = PY - wL - P_k K \qquad \text{: profit definition} \qquad (5)$$

$$\left.\begin{aligned} \frac{\partial \pi}{\partial L} &= 0 \\ \frac{\partial \pi}{\partial K} &= 0 \end{aligned}\right\} \qquad \text{:Maximizing condtions} \qquad (6)$$

Here, π represents profits, Y, L, and K represent quantities of output

7. Marschak, J., and W.J. Andrew. "Random simultaneous equations and the theory of production." Econometrica, 12 (oct.,1944)

8. A two input case is used here in order to economized on exposition. The argument and estimation methods can be extended to any number of inputs.

and labor and capital inputs respectively, and P, w, and P_k their respective prices. Following Marschak and Andrews, the traditional representation of the production model has been

$$\alpha = MP_L/AP_L \quad : \quad \beta = MP_k/AP_k \qquad (7)$$

$$\alpha(Y/L) = w/P \quad : \quad \beta(Y/K) = P_k/P \qquad (8)$$

By manipulating the equation (8), we have

$$L = [\alpha(P/w)]\ Y \qquad (9)$$

$$K = [\beta(P/P_k)]\ Y \qquad (10)$$

$$\ln L = \ln[\alpha(P/w)] + \ln Y + v_1 \qquad (11)$$

$$\ln K = \ln[\beta(P/P_k)] + \ln Y + v_2 \qquad (12)$$

$$\ln Y - \alpha \ln L - \beta \ln K = \lambda_0 + u_o \qquad (13)$$

$$\ln Y - \ln L = \lambda_1 + v_1 \qquad (14)$$

$$\ln Y - \ln K = \lambda_2 + v_2 \qquad (15)$$

Where $\lambda_0 = \ln A$, and u_o, v_1, and v_2 are stochastic disturbances satisfying the usual conditions about zero means and homoscedastic variances. We have observation on Y, L, and K for the sample, we wish to estimate α, β, and A. The parameters λ_1 and λ_2 are given by

$$\lambda_1 = \ln\ (wR_1/P\alpha) \qquad (16)$$

$$\lambda_2 = \ln\ (P_kR_2/P\beta) \qquad (17)$$

It should be noted that λ_1 and λ_2 are the same for all firms, since it is assumed that prices of output and inputs are the same for all firms. It assumes that the entrepreneur has full knowledge of his production function, that means he knows given the factor inputs, exactly what out put he will obtain. The parameters R_1 and R_2 suggested by Hoch[9], are introduced to allow for the possibility that firms in the sample may exhibit systematic errors, perhaps as a result of satisfying the first order conditions. Of course, if $R_1 = R_2 = 1$, such systematic errors are absent.

$$\lambda_1 = \ln (w/P\alpha) \tag{18}$$

$$\lambda_2 = \ln (p_k/P\beta) \tag{19}$$

The random disturbances v_1 and v_2 in the traditional model are introduced to allow for random, nonsystematic errors on the part of entrepreneures in their attemps to adjust inputs to satisfy the necessary conditions for profit maximization. On the other hand, the interpretation of u_o has not as clear cut in the literature. Marschak and Andrews descrive u_o as reflecting "technical efficiency and depending on the technical knowledge, the will, effort and luck of a given entrepreneur."[10]

It is clear from equation (13) to (15) that L and K are not independent of u_o, since each input is a function of all disturbances of the system. Consequently, classical least square (OLS) estimates of the production function parameters will be, in general, biased and inconsistent.

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9. Hoch, Irving. "Simultaneous equation bias in the context of the Cobb-Douglas production function." Econometrica, 26, (oct.,1958)

10. see footnote #7 p.145 and p.156.

This conclusion led to the development of a number of alternative estimation methods based on various assumptions concerning the profit maximization conditions. I will discuss indirect least square method, and the straight regression under the assumption of constant returns to scale which are used in my empirical study in Korean economy to estimate the key parameters of the Cobb-Douglas production type, and will introduce the Klein's[11] and Hoch's[12] methods as the alternative estimation procedures.

II-II-2. Indirect Least Square Estimate.

The logarithmic form of the Cobb-Douglas production function, extended by the inclusion of the "technical"disturbances is

$$\ln Y = \ln A + \alpha_1 \ln K + \alpha_2 \ln L + \ln u \qquad (20)$$

By deducing $\ln Y(\alpha_1+\alpha_2)$ from both sides of equation (20) and deviding by $(1 - \alpha_1 - \alpha_2)$, we obtain

$$\ln Y = \beta_0 + \beta_1 Z_1 + \beta_2 Z_2 + e \qquad (21)$$

Where, $Z_1 = \ln K - \ln Y$,and $Z_2 = \ln L - \ln Y$

$\beta_r = \alpha_r/(1-\alpha_1-\alpha_2)$,here r=(1,2)

and $e = u/(1-\alpha_1-\alpha_2)$

Since the profit maximizing decision equations lead to

11. Klein, L.R. "A textbook of econometrics" New York, Row,peterson and co., 1953 pp.193-96.
12. Hoch, Irving: "Simultaneous equation bias in the context of the Cobb-Douglas production function." Econometrica, 26. (oct.,1958)

$$\ln L - \ln Y = \text{constant} + v_1 \qquad (22)$$
$$\ln K - \ln Y = \text{constant} + v_2$$

Where v_1, v_2 are the "economic" disturbances, least square estimates of based on equation (21) will be consistent as long as $E(u_0 v_r) = 0$. From these we can obtain consistent estimates of α_r using simultaneous equations solution (see appendix for this). It would appear that efficient estimation of the production function parameters will be obtained through the use of this two step procedure only when the stochastic terms in the two, technical and behavioral, equations are independent or at least uncorrelated. If one of the inputs is fixed, we have only one decision equation, and the appropriate substitution leads to

$$\ln Y = \beta_0 + \beta_1 Z + \beta_2 \ln L + e \qquad (23)$$

Where, $\beta_r = \alpha_r/(1 - \alpha_1)$:r=(1.2)

$Z_1 = \ln K - \ln Y$

and $e = u/(1 - \alpha_1)$

Here again, least squares estimates of β_r based on equation (23) will lead to consistent estimates of α_r providing $E(uv_1) = 0$. r=1,2.

II-II-3. The Cobb-Douglas with Constant Returns to Scale.

With constant returns to scale, the sum of α and β is equal to unity and the Cobb-Douglas production function can be written as

$$\frac{Y}{L} = A\, e^{\lambda t} \left(\frac{K}{L}\right)^{1-\alpha} u \qquad (24)$$

With constant returns to scale, we may make the stronger assumptions that each of the two factors of production will receive a factor price which is just equal to the value (in a competitive market) of the respective factor's marginal product. As is well known, this assumption will, by virtue of Euler's theorem, aside from stochastic disturbances, ensure that the total output is exhausted by both of these factor payments. Assuming that output Y is the relevant component in factor pricing, we have,

$$\frac{\partial Y}{\partial L} = \alpha \frac{Y}{L}$$

Next we may insert this expression for the level of labor's marginal productivity into a behavioral relationship which equates the real wage to this expression with allowance for a multiplicative disturbances. This gives,

$$\frac{w}{P} = \frac{\partial Y}{\partial L}$$

Where P is the price of final product and w/P presents "real wage". [13]

By simple algebraic reduction, we obtain,

$$\frac{Y}{L} = \frac{1}{\alpha} \frac{w}{P} \qquad (25)$$

Equations (24) and (25) are our complete systems, as return to capital is functionally dependent on the return to labor by Euler's theorem.

---

13. Factor share,

$$\alpha = \frac{L}{P}\frac{w}{Y} = \frac{L\ VMP_L}{PY} = \frac{L\ MP_L P}{PY} = \frac{MP_L}{AP_L}$$

$$MP_L = \partial Y/\partial L = \alpha(Y/L)$$

Since, $w = MP_L P$, $MP_L = w/P$ Therefore, $Y/L = 1/\alpha\ (w/P)$.

They are nearly treated as a linear, linear relationship in ratio variables with unknown parameters lnA, λ , and $(1-\alpha)$, by making a logarithmic transformation, which yields,

$$\ln (Y/L) = \ln A + \lambda t + (1-\alpha)\ln (K/L) + \ln u \qquad (26)$$

$$\ln (Y/L) = \ln (1/\alpha) + \ln (w/P) \qquad (27)$$

II-II-4. The Alternative Estimation Methods in the Literature.

A method of estimating α and β without a direct resort to OLS was suggested by Klein(1953). This assumes absence of profit maximizing restraints,[14] and the assumption that v_1 and v_2 are distributed independently of u_o. The method involves using the input demand equations in equation (8) to obtain estimate of α and β as follows

$$\alpha = wL/PY \qquad ; \qquad \beta = P_k K/PY \qquad (28)$$

α and β are the shares of labor and capital in total output. Given our sample observations, we can derive estimates of α and β from the geometric mean of the share of ith input over all firms.

$$\alpha_i = \prod_{j=1}^{n} \frac{w_j L_j}{P_j Y_j} \qquad (29)$$

$$\text{or,} \quad \ln \alpha_i = 1/n \sum_{j=1}^{n} \ln \left[(w_j L_j) - (P_j Y_j)\right] \qquad (30)$$

---

14. See equations (16) and (17), in Kleins method does not assuming $R_1=1$, $R_2=1$ in a priori.

15. However, Hoch indicated that the following,in his work(1958). "If, u_o represents "technical knowledge, the will, effort, and the luck of a given entrepreneur" as Marschak and Andrews state, the assumption that U_o and v_1 and v_2 are independently distributed may be questionable."

If the production function is assumed to obey constant returns to scale then of the two only need be estimated $\sum_{i=1}^{2} \alpha_i = 1$.

The Klein estimator has been shown by Dhrymes (1962)[16] to be asymtotically unbiased and of such estimators it has the minimum variance. Klein has implicitely used two non-linear restrictions which can be seen if we rearrenge the equation (28), we have

$$\ln (wL_j) = \ln \alpha_1 + \ln(PY_j) + v_1 \qquad (31)$$

$$\ln (P_k K_j) = \ln \alpha_2 + \ln (PY_j) + v_2 \qquad (32)$$

$$\ln Y_j = \ln A + \alpha_1 \ln L_j + \alpha_2 \ln K_j + u_o \qquad (33)$$

The two non linear restrictions are that the constant terms in the first two equations, are logarithms of the two parameters α_1 and α_2 in the third equation. Thus while the Klein procedures does not estimate the production function directly, it use the information that the input demand equations are jointly derived along with the production function in the courses of a profit maximizing exercise. Klein also assumes that the input and output prices are not only exogeneous constants but that they are also observable. Thus we derive our estimates of physical productivity from data on value shares. In fact, for the Klein estimates we need no data on physical outputs and inputs only total revenue and input payments.

The stimulus to the Monte-Carlo study[17] came from Hochs article(1958) on the estimation of parameters of the Cobb-Douglas produc

16. Dhrymes, P. "On devising unbiased estimators for the parameters of the Cobb-Douglas production function." Econometrica, vol.30 No,2 (April,1962)

tion function. Hoch specified the extent of the single equation least squares bias in infinitely large samples and proposed an estimation procedure which removes the bias from OLS estimates. Hoch specifies two different cases.

Case I. In the case where the disturbances are not correlated, $E(u_o v_1) = E(u_o v_2) = E(v_1 v_2) = 0$, and where both inputs are variable. Hochs estimates of α_1 and α_2 are given by

$$\alpha_r = \hat{\alpha}_r \left[1 + \frac{\hat{\hat{S}}_{oo}}{\hat{S}_{11}} + \frac{\hat{\hat{S}}_{oo}}{\hat{S}_{22}}\right] - \frac{\hat{\hat{S}}_{oo}}{\hat{S}_{rr}} \quad , (r=1,2) \qquad (34)$$

Where $\hat{\alpha}_r$ is the OLS estimate of α_r ,and $\hat{\hat{S}}_{oo}$, $\hat{S}_{11}$, and $\hat{S}_{22}$ are the estimates of the error variances $E(u_o^2)$, $E(v_1^2)$, and $E(v_2^2)$.

The estimates of the error variances can be obtained from the sample moments as follows

$$\hat{S}_{rr} = C_{oo} + C_{rr} - 2C_{or} \quad , (r=1,2) \qquad (35)$$

$$\hat{\hat{S}}_{oo} = \frac{\hat{S}_{oo}}{1 - (\hat{S}_{oo}/\hat{S}_{11}) - (\hat{S}_{oo}/\hat{S}_{22})} \qquad (36)$$

Where C_{oo} is the sample variance of Y_o, C_{rr} is the sample variance of inputs X_r, C_{or} is the sample covariance of Y_o and X_r, and

$$\hat{S}_{oo} = C_{oo} - \hat{\alpha}_1 C_{o1} - \hat{\alpha}_2 C_{o2}$$

17. Kmenta,J.,and M.E. Joseph. "A monte-carlo study of alternative estimates of the Cobb-Douglas production function." Econometrica, 31 (July,1963) pp. 363-389.

When input 2 is taken as exogeneously determined and the remaining disturbances are not correlated, $E(u_o v_1) = 0$, Hoch's estimates are;

$$\alpha_1 = \hat{\alpha}_1 \left[1 + \frac{\hat{\hat{S}}_{oo}}{\hat{S}_{11}}\right] - \frac{\hat{S}_{oo}}{\hat{S}_{11}} \qquad (37)$$

$$\alpha_2 = \hat{\alpha}_2 \left[1 + \frac{\hat{\hat{S}}_{oo}}{\hat{S}_{11}}\right] \qquad (38)$$

$\hat{\hat{S}}_{oo}$ is now given by $\hat{\hat{S}}_{oo} = \hat{S}_{oo} / [1 - (\hat{S}_{oo}/\hat{S}_{11})]$

and other symbols remain unchanged.

Case II. Hoch points out, but does not elaborate, an estimation procedure for the situation where v_1 and v_2 are correlated with each other but not with u_o, and both inputs are variable. This is the case where the economic disturbances are correlated with each other but not with the technical disturbances. The estimators in this case are

$$\alpha_r = \hat{\alpha}_r \left[1 + \frac{\hat{\hat{S}}_{oo} (\hat{S}_{11} + \hat{S}_{22} - 2\hat{S}_{12})}{\hat{S}_{11}\hat{S}_{22} - \hat{S}_{12}^2}\right] - \frac{\hat{S}_{oo}(\hat{S}_{pp} - \hat{S}_{12})}{\hat{S}_{11}\hat{S}_{22} - \hat{S}_{12}^2}$$

$$(r=1,2;\ p=1,2;\ p \neq r) \qquad (39)$$

Here $\hat{S}_{12} = C_{oo} + C_{12} - C_{o1} - C_{o2}$, where C_{12} is the sample covariance of inputs X_1 and X_2 , and

$$\hat{\hat{S}}_{oo} = \frac{\hat{S}_{oo} (\hat{S}_{12}^2 - \hat{S}_{11}\hat{S}_{22})}{\hat{S}_{oo} (\hat{S}_{11} + \hat{S}_{22} - 2\hat{S}_{12}) + (\hat{S}_{12}^2 - \hat{S}_{11}\hat{S}_{22})}$$

Other symbols are defined as in equation (34).

Klein and Hoch, they differ in making use of different bits of infor mation contained in equation (11) and (12). Hoch neglects the information on prices as well as the non-linear restrictions on the constant terms in the input demand equations. Klein on the other hand uses no information about the error terms.

II-II-5. The Estimation of CES Production Function.

The CES production function with provision for disembodied neutral technical change can be written as

$$Y = Ae^{\lambda t}\left[\delta K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-v/\rho}$$

Where Y denotes output in constant prices, K and L are measured for capital and labor inputs, and A, λ, δ, ρ, and v are parameters, the meaning of which are supposed to be sufficiently known to the reader. The non-linearity posed a problem in the estimation of CES production function. The initial attempt at fitting the CES function was made by ACMS(1961). That method was restricted to the case of constant returns to scale. With this restriction it is possible to estimate the elasticity of substitution from the marginal productivity condition by regressing the value of production per worker on wage rate (both variables measured in logarithms). If, however, the CES production function in generialized to allow for the possibility of non-constant returns to scale, this method is no longer feasible. We shall follow their approach here. ACMS fit CES production function on the assumption of constant returns to scale , Thus we have;

$$Y = A[\delta K^{-\rho} + (1-\delta) L^{-\rho}]^{-1/\rho} \qquad (1)$$

setting

$$\frac{\partial Y}{\partial X_1} = P_1/P \qquad (2)$$

The marginal product of X_1 input is

$$\frac{\partial Y}{\partial X_1} = A^{-\rho}\delta_1[\frac{Y}{X_1}]^{1+\rho}$$

Therefore, we have, $P_1/P = A^{-\rho}\delta_1 (Y/X_1)^{1+\rho}$

At this stage (P_1/P) is the exogeneously given variable. By taking logarithms and rearrange, we get

$$\ln (Y/X_1) = \ln (A^{\rho}/\delta_1) + \sigma \ln (P_1/P) \qquad (3)$$

A crucial hypothesis to be tested here is whether $\sigma = 1$, i.e., if the industry obeys a Cobb-Douglas production function. The method for this is computing t-statistics $\hat{\sigma} - 1 / S_{\hat{\sigma}}$, the term $S_{\hat{\sigma}}$ being the standard error of $\hat{\sigma}$. ACMS study did not dwell on the estimation of A and δ_1, these can be easily obtained. One way is to assume, as they did, that A = 1 which is valid for a cross section, and derive $\hat{\sigma}$ from equation (3), or with obtained from equation (3) go back to equation (1) and derive A and $\hat{\delta}$ from the resulting linear equation. ACMS method has the advantage that it does not need data on the second input capital. Capital data are usually either unobtainable or non comparable over time. Dhrymes (1965) discovered that for his study, in most cases the ACMS method yielded the results that $\hat{\sigma}$ was significantly different from one, while with the same data using ln(Y/K) as dependent variable resulted in $\hat{\sigma}$ not being different from value of one.[18]

The ACMS estimator therefore appeared to be biased downward. One reason for the downward bias in the ACMS estimate to be biased their assump tion of perfection in factor market, if the labor market is imperfectly competitive, equation (2) is no longer valid.

An alternative exploration of the ACMS method has been made by Maddala and Kadane(1966).[19] Their approach is to reverse the relationship of dependent and independent variables and regress $\ln(P_1/P)$ on $\ln(Y/X_1)$ to obtain estimate of $(1+\rho)$. They first show the ACMS estimate of σ_{S_1} will always be lower than their estimate of σ_{S_2}.[20]

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18. Phoebus J. Dhrymes. "Some extensions and tests for the CES classes of production function." The Review of Economics and Statistics, vol. 37, 1965 pp.357-66.

19. G. S. Maddala and Joseph B. Kadane. "Some notes on the estimation of the constant elasticity of substitution production function." The Review of Economics and Statistics, vol. 48, 1966 pp.340-44.

20. See the comparision of statistics, S_1 and S_2. The former is the ACMS estimate and is the regression coefficient obtained from the regression of ln(Y/L) on $\ln(P_1/P)$. The latter is the reciprocal, the regression of $\ln(P_1/P)$ on ln(Y/L). If we denote ln(Y/L) by Y and $\ln(P_1/P)$ by X, then we have

$$S_1 = \Sigma(Y-\bar{Y})(X-\bar{X}) \ / \ \Sigma(X-\bar{X})^2$$

$$S_2 = \Sigma(Y-\bar{Y})^2/\Sigma(X-\bar{X})(Y-\bar{Y})$$

It is easy to see from the Cauchy Schwartz inequality,

$$[\Sigma(X-\bar{X})(Y-\bar{Y})]^2 \leqslant [\Sigma(X-\bar{X})^2][\Sigma(X-\bar{Y})^2]$$

that S_2 will always be greater than S_1.

II-II-6. The Two-Step Procedure.

$$Y = Ae^{\lambda t}\left[\delta K^{-\rho} + (1-\delta) L^{-\rho}\right]^{-v/\rho} \qquad (4)$$

With the CES formulation, the marginal productivity relationship based on cost minimization, may be written as,

$$P_k/w = \frac{(\partial Y/\partial K)}{(\partial Y/\partial L)} = \frac{\delta}{1-\delta}\left[\frac{K}{L}\right]^{-(\rho+1)} \qquad (5)$$

The two-step procedure for estimation of CES function would be to trans formed equation (5) logarithmically into,

$$\ln P_k/w = \ln \frac{\delta}{1-\delta} - (\rho + 1) \ln K/L + \ln u \qquad (6)$$

And to compute estimates of $\delta/1-\delta$ and $(\rho + 1)$ as regression coefficients. With estimates of δ and ρ ($\hat{\delta}$ and $\hat{\rho}$,respectively) from this regression of logarithmic ratio variables, we can form,

$$\ln Y = \ln A + \lambda t - v/\rho \ln\left[\hat{\delta} K^{-\hat{\rho}} + (1-\hat{\delta}) L^{-\hat{\rho}}\right] + \ln u \qquad (7)$$

It is then possible to estimate A, λ, and v from a second regression either of ln Y and t and $\ln\left[\hat{\delta} K^{-\hat{\rho}} + (1-\hat{\delta}) L^{-\hat{\rho}}\right]$ of the computed variable on the first two.

II-II-7. Taylor Expansion Method.

An obvious starting point is to consider estimates obtained by fitting the production function to observation on output and inputs alone. These estimates are consistent if the input variables are non-stochastic or, if stochastic, independent of the disturbance in the production function. The CES function can be written in the form by taking logarithms of both sides of the equation (4).

$$\ln Y = \ln A + \lambda t - v/\rho \ln\left[\delta K^{-\rho} + (1-\delta) L^{-\rho}\right] + \ln u \qquad (8)$$

The stochastic error term assumed to be independently and normally distributed with zero mean and constant variance. The parameters of equation (8) could be estimated by non-linear least squares methods for which computer programs are now available. An alternative method,based on simple least square estimation is possible if we replaces equation(8) by its approximation which is linear in ρ. This can be derived by using Taylor's formula for expansion around $\rho = 0$.[21] After disregarding the terms of third and higher orders, expansion lead to,

$$\ln Y = \ln A + \lambda t + v\delta \ln K + v(1-\delta) \ln L$$
$$-\tfrac{1}{2}\rho v \delta (1-\delta)(\ln K - \ln L)^2 \qquad (9)$$

---

21. Let us say, production function is f(X) that is contineous and has a contineous pth derivative can be written as,

$$f(X) = f(a) + (X-a)f'(a) + (X-a)/2!\ f''(a) + (X-a)/3!\ f'''(a) +$$
$$+ \ldots + (X-a)/p!\ f^{p}(a) + R_{p+1} \qquad (A)$$

Where a is any fixed number in the domain of X and

$$f(a) = f(X)\big/_{X=a}\ ,\quad f'(a) = \partial f(X)/\partial X\big/_{X=a}\ ,\quad f''(a) = \partial^2 f(X)/\partial X^2\big/_{X=a}$$

R_{p+1} = remainder. The series given in (A)is called the Taylor's series expansion of f(X) about point X=a.

The approximation to the CES function given by equation (9) can then be regarded into two parts, one corresponding to the Cobb-Douglas form and one representing a "correction" due to the departure of ρ from zero. The latter term, given by the term $\frac{1}{2}\rho v \delta(1-\delta)(\ln K - \ln L)^2$ will disappear if $\rho = 0$.

The estimation of the parameter of equation (9) is the same as in the case of estimation with non-linear restrictions under exact identification.[22]

The "unrestricted" version of equation (9) is,

$$\ln Y = \beta_1 + \lambda t + \beta_2 \ln K + \beta_3 \ln L + \beta_4 (\ln K - \ln L)^2 \quad (10)$$

Which represents an linear regression model. If the estimate of β_4 is not significantly different from zero, we would reject the CES model in favor of the Cobb-Douglas model. The parameters of equation (9) are related to the coefficients of (10) as follows (see appendix for this).

$$A = \text{antilog } \beta_1$$

$$\delta = \frac{\beta_2}{\beta_2+\beta_3}$$

$$v = \beta_2 + \beta_3 \quad , \quad \rho = \frac{-2\beta_4(\beta_2+\beta_3)}{\beta_2\beta_3}$$

Thus we can use ordinary least squares estimates of the β_A to obtain estimates of the parameters of equation (9).

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22. See page 443. Jan Kmenta. "Elements of econometrics" <u>The MacMillian Company 1971</u>.

The estimated standard errors can be calculated by using the approximation formula[23] (see appendix for this).

For illustrative purposes I will apply this method to the empirical study in the following section and will test the Cobb-Douglas hypotheses by examining the significance of the coefficient attached to $(\ln K - \ln L)^2$.

II-II-8. One Alternative Estimation Method in the Literature, Modified Indirect Least Square.[24]

We are assuming firms which operate under perfectly competitive conditions and obtained their inputs at fixed prices in the same market. Given that the production model may be specified by the following relationship,

$$\ln Y = \ln A - v/\rho \ln\left[\delta L^{-\rho} + (1-\delta) K^{-\rho}\right] + \ln u_0 \qquad (11)$$

$$(\rho/v + 1)\ln Y - (\rho + 1)\ln L = \ln\left[P_1 A^{\rho/v}(Pv\delta)^{-1}\right] + \ln v_1 \qquad (12)$$

$$(\rho/v + 1)\ln Y - (\rho + 1)\ln K = \ln\left[P_2 A^{\rho/v}(Pv)^{-1}(1-\delta)^{-1}\right] + \ln V_2 \qquad (13)$$

Where P is price of product, P_1 is wage rate and P_2 is price of capital input. The model is formally equivalent to the traditional model of

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23. For the formula of calculating standard errors, see Klein p.258. Klein L.R. "A textbook of econometrics" New York, Row and Peterson and Co., Inc. 1953.
When the estimator, say $\hat{\alpha}$ is a function of k other estimators such as,

$$\hat{\beta}_1, \hat{\beta}_2, \ldots\ldots \hat{\beta}_k \text{etc.} \quad \hat{\alpha} = f(\hat{\beta}_1, \hat{\beta}_2, \ldots \hat{\beta}_k)$$

Then the sample variance of $\hat{\alpha}$ can be approximated as,

$$\mathrm{Var}(\hat{\alpha}) = \sum_k \left[\partial f / \partial \hat{\beta}_k\right]^2 \mathrm{Var}(\hat{\beta}_k) + 2\sum_{j<k}\left[\partial f/\partial \hat{\beta}_j\right]\left[\partial f/\partial \hat{\beta}_k\right] \mathrm{Cov.}(\hat{\beta}_j, \hat{\beta}_k)$$

production analysis, except that ususal Cobb-Douglas production function has been replaced by the CES function. Equations (12) and (13) are the profit maximizing marginal productivity conditions. In the system above we clearly see that the production reduces to the Cobb-Douglas for $\rho = 0$.

The important thing is that even if $\rho \neq 0$, and $v \neq 1$, we have three equations which are log linear in the parameters. Compare to the Cobb-Douglas case, we have a larger number of parameters to estimate and therefore we should carefully examine the possibility of identification by prior restrictions on the errors. J. Kmenta (1967) was assuming that the variance-covariance matrix of the disturbance is diagonal. Then, the model of (11),(12) and (13) could be handled by non-linear information method,

$$\ln Y - v\delta \ln L - v(1-\delta)\ln K + \tfrac{1}{2}\rho v\delta(1-\delta)(\ln K - \ln L)^2 = \ln \lambda_0 + u_0 \qquad (14)$$

$$(\rho/v + 1)\ln Y - (\rho + 1)\ln L = \ln \lambda_1 + v_1 \qquad (15)$$

$$(\rho/v + 1)\ln Y - (\rho + 1)\ln K = \ln \lambda_2 + v_2 \qquad (16)$$

The disturbances u_0 , v_1 , and v_2 are assumed to be normally distributed with zero means and constant variances. The constant terms $\lambda_r (r=1,2)$ are defined with reference to equation (11) through equations (12) and (13). Given the assumption of independence of v_1 and v_2 , the two input

24. This method was developed by J. Kmenta. "On estimation of the CES production function." International Economic Review, vol.8,No.2, June,1967.

demand equations can be used to estimate $(\rho/v+1)/(\rho+1)$ consistently. In fact, each of them is just identified and we could use either of them. But the estimates derived from either have to be agreement with each other since they involve the same parameters. Therefore multiplies them together to get in terms of sample moments, a quadratic equation,

$$\hat{F}^2 \widehat{var} \ln Y + \hat{F}(\widehat{cov} \ln Y \ln L + \widehat{cov} \ln Y \ln K) + \\ + \widehat{cov} \ln L \ln K = 0 \qquad (17)$$

Here, $\hat{F} = (\rho/v+1)/(1+\rho)$ derived by solving the quadratic equations. Let us first introduce new variables, defined as follows.

$$\ln Z_1 = F \ln Y - \ln L$$
$$\ln Z_2 = F \ln Y - \ln K$$
$$\ln Z_3 = (\ln K - \ln L)^2$$

Next, we form a new regression equation,

$$\ln Y = a_o + a_1 \ln Z_1 + a_2 \ln Z_2 + a_3 \ln Z_3 + e \qquad (18)$$

Because of the definition of Z's, the coefficient of equation (18) can be identified with those of the production functions (14), (15), and (16).

$$a_o = \ln A/(1-Fa_1-Fa_2)$$
$$v\delta = a_1/(1-Fa_1-Fa_2)$$
$$v(1-\delta) = a_2/(1-Fa_1-Fa_2)$$
$$-\tfrac{1}{2}\rho v\delta(1-\delta) = a_3/(1-Fa_1-Fa_2)$$

And e is proportional to u_o. Once we have estimated F, then we form Z_1, Z_2, and Z_3 and proceed to estimate the coefficients a_1, a_2 and a_3 which will give us the parameter estimates. These estimates are consistent if error variance matrix is diagonal as assumed.

CHAPTER III.

EMPIRICAL RESULTS

III. EMPIRICAL RESULTS.

III-I. Introduction to Empirical Section.

The present study applies the ordinary least square principle, indirect least square method with unconstrained labor and capital elas ticity, and the straight regression with the labor and capital exponents constrained to add to unity to the Cobb-Douglas type as a log linear and the Taylor expansion regression, and the two step regression method to the constant elasticity of substitution production function as a non-linear function.

These estimation procedures are applied to the Korean economy, particularly to manufacturing, agriculture, construction, communication and transportation, and mining sectors.

Section 2 explaines the applications of the Solow's residual and growth accounting concepts, in section 3 the discussion of the estimated results of both production functions and gives some qualification to the results obtained in this study. For estimation of the Cobb-Douglas and CES production functions, annual data were collected from 1957-1975 and the sources of the data are as followings; 1. capital stock series, K; Fixed capital stock series were calculated from the K/dY series in various sectors through 1957 to 1975. The source of this is the national income in Korea, published by The Bank of Korea, 1976. pp.255-257.
2. output series, Y; The output figures were obtained from value added by various sectors given in the economic statistics yearbook, published

by Bureau of statistics, economic planning board, 1976. The production value index (1970 at the constant price) was used to extend the real value added figures from 1957 to 1975.

3. Labor series, L; From 1957 to 1975, the sources are the same as those of output series, economic statistics yearbook, 1976 by Bureau of statistics, economic planning board. Labor: Economically active population employed during survey period, includes only those 14 years of age or over who are able to contribute to producing economic goods and services by supplying labor for wages. 4. The price indexes to estimate the parameters in the Cobb-Douglas (in factor share method case) ,and CES production functions; the price index of various sectors was taken from "the general wholesale price indexes" and the sources are same as those of output series. The series are based on 1970=100. Figures from 1957 to 1969 are the 1965 base indexes linked to the 1970 base indexes. This index is designed to measure the average price level of all types of commodities transacted between domestic establishment. The type of price series included is the prices actually applied at the first stage of distribution, i.e., producer's price. When it is not possible or inconvenient to collect such price data, however, the prices actually applied at the later stages of distribution are used.

Price data are collected only at a city or cities (15 in all, including Seoul, capital city) where transactors of a certain commodity amount to considerable volume. This index is calculated by the Laspeyres formula, weighted arithmetic mean of price relatives. 5. Wages, w; To estimate the parameters in the Cobb-Douglas (in factor share method case), and

CES production functions; the sources of data are the same as in output series and wages are deflated by 1970 at the constant price.

III-2. Technical Progress and Productivity Change.

In order to isolate shifts of the aggregate production function from movements along it, three times series are needed; output per unit of labor, (Y/L), capital per unit of labor, (K/L) ,and the share of capital. The conceptually cleanest measure of aggregate output would be real net national product, but NNP series are hard to come by, so I have used value added series instead. A(t) series in table I's (Ma-I, Agr-I, C-I, TC-I ,and Mi-I) are meant to be a rough profile of technical change at least looks reasonable. Because we can see the trend is strongly upward, in manufacturing, A(t) goes from 1.00 in 1957 to 1.9476 in 1975, 1.00 to 1.325 in agriculture, 1.00 to 2.2083 in mining, 1.00 to 1.4791 in construction ,and 1.00 to 2.1699 in transcommunication sector.
Of course, by arbitrarily setting A(1957) = 1.00 and the using the fact that,

$$A(t+1) = A(t)\left[1+\Delta A(t)/A(t)\right]$$

There are sustained rises for A(t) in manufacturing, construction ,and transcommunication during economic development plan periods compare to the previous years (see Figure II and column 9 in table Is).

In figure I, a scatter of $\Delta A(t)/A(t)$ against K/L indicates no trace of a relationship, so we may state it, as a formal conclusion

Figure 1. A SCATTER OF $\Delta A/A$ AGAINST K/L

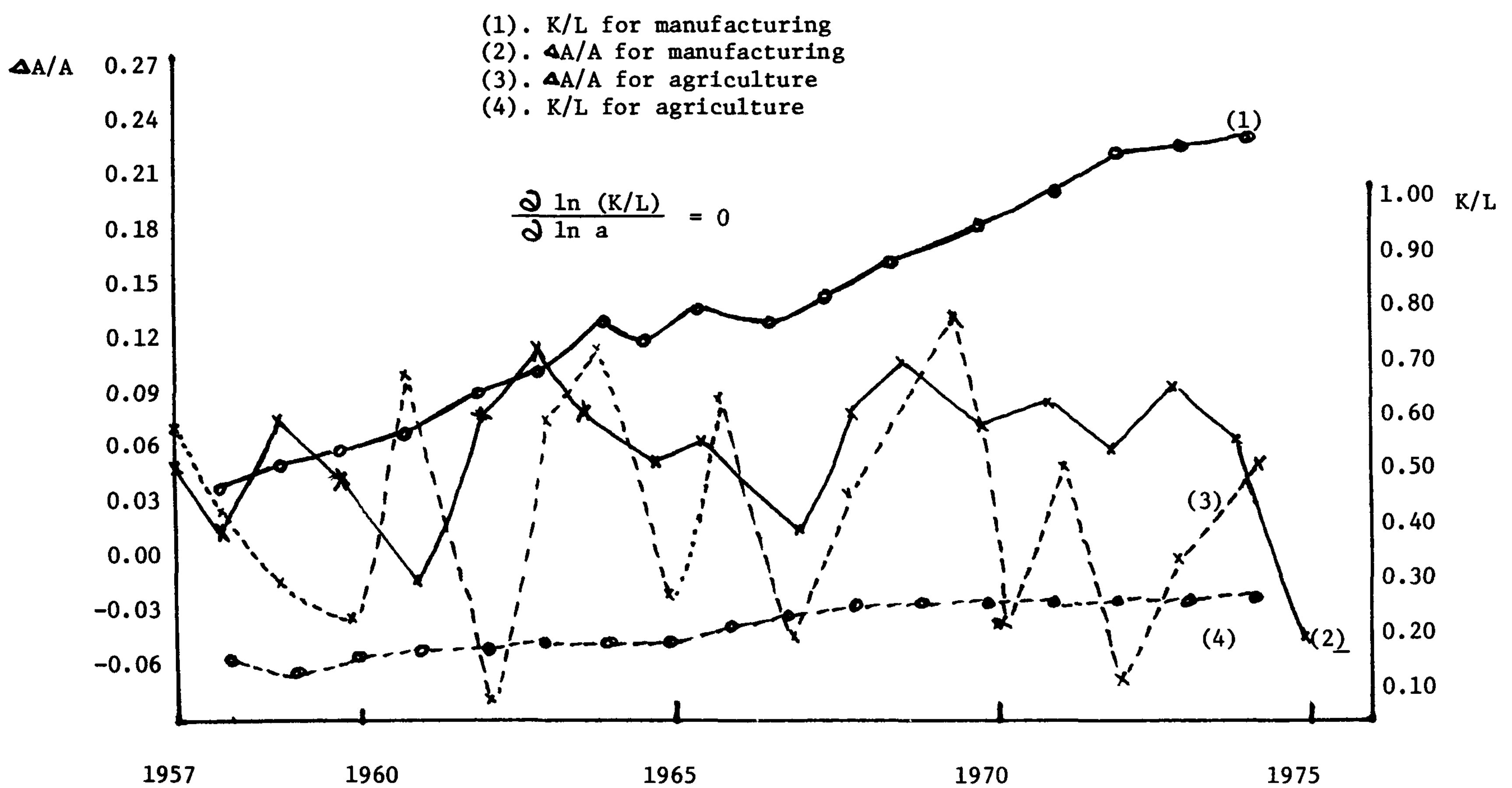
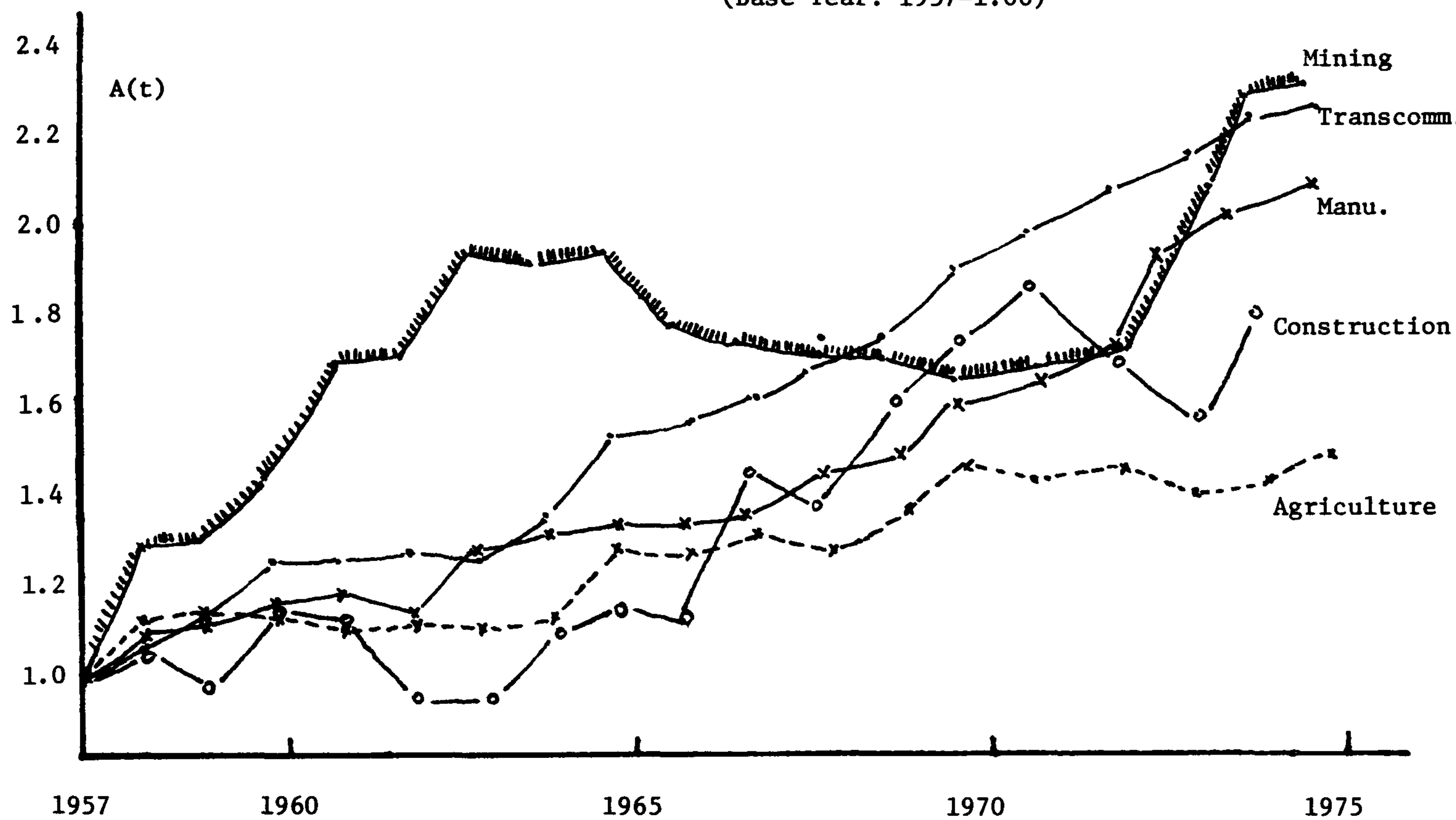


FIGURE II. The Cumulated Effect of A(t) Over Time.
(Base Year: 1957=1.00)



The Growth rate of $\Delta A(t)/A(t)$

Note: These statistics come from table Is(column 8).

	1957-61	1962-66	1967-71	1972-75	Ave.
Manufacturing	0.0275	0.0522	0.0780	0.0270	0.0461
Agriculture	0.0263	0.0317	0.0109	0.0092	0.0190
Construction	-0.0110	0.0910	0.0402	-0.0402	0.0266
Transcommunication	-0.0591	0.0730	0.0488	0.0651	0.0610
Mining	0.1587	0.0022	-0.0010	0.1001	0.0671

that over the sample period from 1957 to 1975, a shift in the aggregate production function netted out to be approximately neutral (see figure I for this). In figure I, I draw the curves for only manufacturing and agriculture sectors to avoid the visual complexity, however the same arguments can be applied to the other sectors such as construction, tran communication, and mining. We should recall that we have defined neutrality to mean that the shifts were pure scale changes, leaving marginal rate of substitution at given capital-labor ratio. Not only is $\Delta A(t)/A(t)$ uncorrelated with K/L, but also we might conclude from figure I and II that $\Delta A(t)/A(t)$ is essentially constant in time, exhibiting more or less random fluctuations about a fixed mean.

There is some evidence that the average rate of progress in the year 1957-61 was smaller than that from 1962-1975 except agriculture and mining sectors. The first five year relative shifts average about 0.0275 (2.7%) while the last fifteen years average 0.0524(5.2%) in manufaturing, the first five year relative shifts average -0.011(-1.1%), while, the last fifteen year relative shifts average 0.0280(2.8%) in construction, and the first five year relative shifts average 0.0591 (5.9%), while the last fifteen year average 0.0623(6.2%) in transcommunication. These figures indicate us that during the first (1962-66), second (1967-71), and third (1972-76) five year economic development plan periods the technology are sharp rise compare to the previous periods in manufacturing, construction, and transcommunication sectors.

The over all results for the whole sample periods are the average upward shift of the production function about 4.6 per cent in manufacturing, 1.9 per cent in agriculture, 2.6 per cent in construc tion, 6.1 per cent in transcommunication, and 6.7 per cent in mining and over 19 year period output per man approximately increased 41 per cent in agriculture, 2.5 times in construction, trippled in manufacturing, 4.7 times in transcommunication, and 3.4 times in mining.

At the same time, according to table Is, the cumulative upward shift in the production function was about 94 per cent in manufacturing. It is possible to argue that about 30 per cent of the total increase in output is traceable to increased capital per man, and remaining 70 per cent to technological change. The reasoning is this; The real output per man increased from 0.1756 in 1957 to 0.5451 in 1975 (see table Ma-I). Devide the 0.5451 by 1.9476 which is the 1975 value of A(t). This result is a "corrected" output per man, net technical change, of 0.2798 billion won. Thus about 0.1042 (0.2798-0.1756=0.1042), billion won of the 0.3695 (0.5451-0.1756=0.3695), increase can be imputed to increase capital intensity and the remainder to increased productivity.

In the agriculture, the cumulative upward shift in the production was about 32 per cent, see table Agr-I. It is possible to argue that about 1/3 of the total increase in output is traceable to increase capital per man, and the remaining 2/3 to technical change. Because, the real output per man increased from 0.1040 in 1957 to 0.1645 in 1975. Divide the 0.1645 by 1.3250 which is the 1975 value for A(t). Therefore, the "corrected" output per man is 0.1242. Thus about 0.0202 (0.1242-0.1040=0.0202) billion won increase can be imputed to increase in capital intensity and the remainder to increased productivity.

Similiarly, it is possible to argue that about 48 per cent of the total increase in output is traceable to increase capital per man, and the remaining 52 per cent to technical change in construction, 31 per cent of the total increase in output is traceable to increase capital per man and the remainder, 69 per cent to technical change in transcommunication, and 24 per cent of the total increase in output is traceable to increase capital per man and the remaining 76 per cent to technical change in mining.[1]

Let us see the identification of the sources of economic growth in different sectors once again using the following formula as a comparision with Solow's residual concept.

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1. R.M. Solow's finding (1957) for U.S. aggregate production (1909-49) is following .... "about one eighth of the total increase is traceable to increased capital per man hour, and the remaining seven eighth to technical change."

$$\tilde{Y} = \alpha \tilde{L} + (1-\alpha)\tilde{K} + R$$ [2]

Over time (1957-75), in manufacturing aggregate output has been rising 15.8 per cent, 4.3 per cent in agriculture, 12.9 per cent in construction, 14.3 per cent in transcommunication, and 8.2 per cent in the mining sectors. There may be three courses, i.e., 1. $\alpha\tilde{L}$ gives the amount due to increase in labor input. 2. $(1-\alpha)\tilde{K}$ gives the amount due to increasing in capital input, and 3. R gives the amount due to the technology.

Rate of Growth in Output and Inputs.

Average rate of GROWTH	$\tilde{Y}=(dY/dt)(1/Y)$	$\tilde{L}=(dL/dt)(1/L)$	$\tilde{K}=(dK/dt)(1/K)$
Over time(1957-75)			
Manufacturing	0.158	0.069	0.119
Agriculture	0.043	0.014	0.032
Construction	0.129	0.074	0.091
Transcommunication	0.143	0.042	0.110
Mining	0.082	0.017	0.051

Source: from table Is

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2. The aggregate production function is, Y = F(K, L; T)

$$\frac{dY}{dt} = MP_L \frac{dL}{dt} + MP_k \frac{dK}{dt} + MP_T \frac{dT}{dt}$$

$$\frac{dY}{dt}\frac{1}{Y} = \frac{L\ MP_L}{Y}\frac{dL}{dt}\frac{1}{L} + \frac{K\ MP_K}{Y}\frac{dK}{dt}\frac{1}{K} + \frac{T\ MP_T}{Y}\frac{dT}{dt}\frac{1}{T}$$

and define $\tilde{Y} = \alpha_L\tilde{L} + \alpha_K\tilde{K} + R$

Here, $\alpha_L = L\ MP_L/Y$, $\alpha_K = K\ MP_k/Y$, and $R = (T\ MP_T/Y)(dT/dt)(1/T)$

If the production function is linear homogeneous, then $\alpha_K = (1-\alpha_L)$ and if there is perfect competition in the factor markets, the $MP_L = w$ so, $L\ MP_L/Y = wL/Y$ = labor share.

Therefore, we have, $\tilde{Y} = \alpha_L\tilde{L} + (1-\alpha_L)\tilde{K} + R$

In manufacturing,

$$\tilde{L} = 0.07 \qquad \tilde{K} = 0.119$$

$$\alpha = 0.53^{*} \qquad (1-\alpha) = 0.46^{*}$$ [3]

Therefore, $\alpha\tilde{L} + (1-\alpha)\tilde{K} = (0.53)(0.07) + (0.46)(0.119) = 0.0918$

Therefore, of the 15.8 per cent of rate of increase in output, $\tilde{Y}$ about 9.1 per cent was due to increase in labor and capital inputs. That leaves 7.7 per cent due to technology, or labor and capital account for 56 per cent of the growth in output while technology account for 44 per cent of the growth in output.

In agriculture,

$$\tilde{L} = 0.014 \qquad \tilde{K} = 0.032$$

$$\alpha = 0.78^{*} \qquad (1-\alpha) = 0.21^{*}$$

So, $\alpha\tilde{L} + (1-\alpha)\tilde{K} = (0.78)(0.014) + (0.21)(0.032) = 0.0176$

Therefore, of 4.3 per cent of the rate of increase in output, about 1.7 per cent was due to increase in labor and capital inputs, that leaves 2.6 per cent due to technology, or labor and capital account for 40 per cent of the growth in output while technology accounts for 60 per cent of the growth in output.

In similar way, labor and capital account for 53 per cent of the growth in output while technology accounts for 47 per cent of the growth in output in construction. In transcommunication, labor and capital account for 51.1 per cent while technology accounts for 48.9 per cent

3. The figures with * are come from the regression results of the S.CRS estimation in table II's.

of the growth in output, and in mining labor and capital account for 50.3 per cent while technology accounts for 49.7 per cent of the growth in output.[4]

4. Nelson analysed U.S. data from 1929-60, over the periods labor and capital account for 32 per cent while technology accounts for 68 per cent of the growth in output.

III-3. Results of Estimation in Manufacturing.

III-3-1. The Cobb-Douglas Type.

Let us turn to the substantive results of our investigation which are presented in table Ma-II. I employ the same symbols in this section as in the theoretical part. Table Ma-II presents estimates of the parameters of the Cobb-Douglas production function with unconstrained labor and capital elasticities (see rows 1 and 2 of the table), while row 3 gives estimates of the Cobb-Douglas parameter with the labor and capital exponents constrained to add to unity ($\alpha + \beta = 1$). In table Ma-II, the upper figure is the parameter estimates itself, while the figure below it, in parentheses, is the associated standard error. The R^2 is the coefficient of multiple determination which adjusted for degree of freedom, and D.W. is the Dubin Watson statistics.

In all three rows in table Ma-II, the first row is a straight O.L.S. (linear in the logarithms) regression of the production function with a multiplicative error, while the second row is an I.L.S. regression of this relationship. For instances, $\beta = \hat{\beta} / (1 - \hat{\alpha} - \hat{\beta})$ from equation 21. Here $\hat{\alpha}$ and $\hat{\beta}$ are the parameters of O.L.S. In table Ma-II, the dependent variable in the first and second rows are total output, while in third row, the dependent variable is output per man, the average productivity of labor.

The statistical pictures which emerges are different in two different estimation methods, O.L.S. and I.L.S. which are selected in first and second rows. It seems that I.L.S. estimates are not well suited for small sample estimation and can be highly unstable in small samples.

We shall be more interested in the statistical significance of individual parameter estimates. In general, the residuals of the production equation are not significantly autocorrelated, as indicated by the generally high enough values of the Durbin-Watson statistics as compare to the lower bound theoretical values at the 95 per cent level[5] Thus our statistical tests of significance performed below, are confident by the presence of this phenomenon, and also in many cases the t ratios are high enough that one might still be willing to place a fair amount in the statistical test under examination. We may now proceed to a discussion of the picture of the economic structure which emerges in table Ma-II.

The paremeter estimates of the unconstrained regression are quite different from those generated by the straight regression under assumption of the constant returns to scale. Before rejecting the existence of the stronger assumption that $(\alpha+\beta = 1)$, and each of the two factors of production will receive a factor price which is just equal to the value (in competitive product market) of the respective factor's marginal product, we should note that the picture of economic structure

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5. J. Durbin and G.S. Watson. "Testing for serial correlation in least squares regression." Biometrika, vol. 38, 1951
If, D.W. $< d_L$: We reject the null-hypotheses. Ho: Auto correlation is not present. Here, D.W. is

$$D.W. = \sum_{t=1}^{n} (u_t - u_{t-1})^2 / \sum_{t=1}^{n} u_t^2$$

I accept null-hypo. in most cases at the 95 per cent significant level.

which emerges from these O.L.S. and I.L.S. regression is hardly believable. I will discuss this point in the results of the CES production function, discussing on the significance of the coefficient of correction part, $(\ln K - \ln L)^2$ in Taylor expansion regression.

Both unconstrained returns Cobb-Douglas regressions show increasing returns to scale in three different epoch periods, 1957-75, 1957-66 ,and 1967-75. The contribution of the capital inputs is significant compare to the labor inputs in three different sample periods. In other words, here it is the capital input which appears to do all the work, as indicated by the parameters estimate of β =0.9579, β=0.9877 ,and β =0.7814. However, during 1957-75, the constant returns to scale regression shows that both labor and capital inputs are significant. Here, both capital and labor inputs which appear to do the work, as indicated by the parameters estimates of α =0.53 ,and β =0.46.

From the regression results, it seems that there is a clear division in the production characteristic between the first ten years (1957-66) and the last nine years (1967-75), see table Ma-IV for Chow test. The first ten years were associated with the downward neutral tech nical shift[6] in the production function, with an increasing returns to scale and with both inputs positive contribution to the production process.

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6. Throughout this study, some estimates turned out to be unacceptable (e.g., a negative rate of disembodied technical progress). This is probably due to multicollinearity. Since the degree of correlation bet ween explanatory variables are highly correlated.

The later nine years (1967-75) shows that upward neutral technical shift in the production function with increasing returns to scale and with both inputs positive contribution to the production process. One might explain these poor results which diverge rather widely from the empirical results of α =0.75 and β =0.25 for U.S. and the other advanced western economy.[7]

Next, we may turn to the question of returns to scale, we found the evidence of increasing returns to scale during 1957-75 ($\alpha+\beta$ =1.522), during 1957-66 ($\alpha+\beta$ =1.252) ,and during 1967-75 ($\alpha+\beta$ =1.123).[8] Thus, for the Cobb-Douglas function, the sum of the exponents in table Ma-II is significantly greater than unity. We might conclude, during the first, second ,and third five year economic develop ment plan periods, the Korean economy have enjoyed the increasing returns to scale in manufacturing sector. However, the implications of increasing returns in the aggregate production function are not al all clear. It is, however, quite easy to say what does not necessarily follow: It is not, for example, necessarily implied that the production function of a number of firms in the economy exhibits increasing returns.

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7. Paul H. Douglas. "The Cobb-Douglas production function once again: it's history, it's testing, and some new empirical values. J.P.E. Oct.,1976.

8. Null-hypotheses, Ho: $\alpha+\beta$ = 1. Using t ratio test, I reject null-hypo. at the 5 per cent significance level. Therefore, we can say there are increasing returns to scale in manufacturing sector during sample periods.

The effect on the aggregate relation may be the results of external economies to firms that are not internal economies to any group of firms- such as improvements in the skill's of the labor due to education, etc. One suspects, however, that the aggregate function simply reflects the growth of these industries that do enjoy economies of scale.

We may now examine the implied rates of disembodied, neutral technological progress (see column 4 in table Ma-II and column 8 in table Ma-I). Our preferred point estimates of this growth rate come from the unconstrained regression estimation results of table Ma-II. We take this rate to be 0.027 for 1957-75, -0.019 for 1957-66 ,and 0.040 for 1972-75. As compare these data to the Solow's residual which is in table Ma-I column 8, the average of technological progress is approximately 2.7 per cent per annum, however, the Solow's technical index in 1975 is 1.94 (0.94/18 = 0.046), i.e.,4.6 per cent per annum. As Walters (1963) have pointed out, allowing for increasing returns to scale reduces one's estimates of the pace of neutral technological progress of the disembodied type in a period such as this one during which factor inputs increased rapidly.[9]

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9. Walters A.A. "A note on economics of scale." The Review of Economics and Statistics. XLV, No.4 (nov.,1963) pp. 425-427.
Walters pointed out that " Clearly, these data discredit the hypotheses of constant returns to scale. In all tests, the sums of the coefficients are significantly greater than unity, and compared with Solow's results, the effect of neutral technical progress is reduced from the annual rate of 1.5 to 1.8 per cent to about 1.0 to 1.25 per cent."

III-3-2. Estimated Results of CES.

Table Ma-III presents the estimated results for manufacturing sector for two different methods of estimation.

(1). Nonlinear ordinary least squares estimates using Taylor's Expansion. In this case, the CES production function can be rewritten again;

$$\ln Y = \ln A + \lambda t + v\delta \ln K + v(1-\delta)\ln L - \frac{1}{2}\rho v \delta(1-\delta)(\ln K-\ln L)^2$$

(2). Nonlinear two-stage least squares estimates with $\hat{\delta}$ and $\hat{\rho}$ computed at the first stage;

Stage one: $\ln (P_k/w) = \ln(\delta/1-\delta) - (\rho + 1) \ln K/L$

Stage two: $\ln Y = \ln A + \lambda t - v/\rho \ln\left[\hat{\delta} K^{-\hat{\rho}} + (1-\hat{\delta}) L^{-\hat{\rho}}\right]$

The view of economic structure which the Taylor expansion regression presents, while not consistent with the two-step CES regress -ion presents, is hardly satisfactory. Here, it is the capital input which appears to do all the work, as indicated by a parameter estimate of δ significantly larger than unity. In our computer program δ is not constrained to be ($0 \leq \delta \leq 1$).

The point estimates of the most important parameter, σ vary considerably for different sets of data, industries, and levels of aggregation. Furthermore, they are sensitive to cyclical fluctuations of demand. The only tentative conclusion possible is that most of the time-series estimates of σ are below unity.[10]

10. Nerlove(1967) has summarized the evidence on the time series estimates of the CES function for the post war period of U.S. "Generally the time series estimates of σ are less than unity." Recent papers reporting various estimates of σ which are distinctly less than unity include V.K.Chetty(1969), J. Morney(1967) ,and Nadiri-Rosen(1969).

In our results, both in Taylor and two-step CES regressions, the elas ticity of substitution is smaller than unity (see table Ma-III, column 2). However, it is relatively bigger than zero. Thus, our estimates of the elasticity of substitution are significantly different from both the Leontief special case of zero value and the Cobb-Douglas case of unitary value.[11]

If we test the significance of coefficient of the computed variable $(\ln K-\ln L)^2$ which is the correction part of the Taylor expan sion regression, the coefficient of correction part is significant in t = 0.05 level.[12] However, in the two-step CES regression (see column 2 again in table Ma-III), σ , the elasticity of substitution is relatively close to the unitary value which would correspond to unitary elasticity of substitution (the Cobb-Douglas case). The results thus provide no evidence against the Cobb-Douglas model in this case.

For either the Cobb-Douglas (as indicated previously) or CES formulation, we find evidence of increasing returns to scale. Thus, for the CES function, we need examine the estimates of returns to scale parameter, v and the significance of the different of these estimates from unity.

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11. We may estimate the standard error(denoted S.E.) of σ by applying the following approximation formula.

$$S.E.(\sigma) = S.E.(1/1+\rho) = 1/(1+\rho)^2\ S.E.(\rho)$$

12. For 1957-66; $0.41069(\ln K - \ln L)^2$ (0.2154) $t=1.995$ $t_o=1.86$
    For 1957-75; $-1.87109(\ln K - \ln L)^2$ (0.2916) $t=-6.448$ $t_o=1.74$
    For 1967-75; $0.63389(\ln K- \ln L)^2$ (0.1464) $t=4.341$ $t_o=1.89$

For my results in table Ma-III, column 4, we have point estimates of returns to scale of 1.646, 1.094, and 1.135 (with two-step CES) and 0.87, 1.0027, and 1.710 (with Taylor Expansion CES). In both cases, in generally speaking, these point estimates are significantly above the constant returns to scale value of unity.

The examination of the implied rates of disembodied, neutral technical progress in CES formulation of the production function is also valid compare to the Solow's residual concept. Using the results (see in table Ma-III, column 5), both in Taylor Expansion and the Two-Step CES regressions, the rates of neutral technical progress are equal to 5.9 per cent and 4.6 per cent respectively per year, which are approxmately equal to the Solow's residual (see table Ma-I, column 8), over the sample periods it was increased 94.7 per cent.

In previous discussion, I raised the question of comparing the results of the two different estimation methods. It is time to return to this question. The procedures would appear to yield different results (compare ρ, σ, and δ in table Ma-III). Therefore, it seems quite possible that some of these parameter estimates could differ in different estimation procedure in CES production function.[13] In general our results indicate that no single estimation procedure is satisfactory in all circumstances.

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13. For this argument, see Ronald G. Bodkin and Lawrence R. Klein. "Nonlinear estimation of aggregate production function." The Review of Economics and Statistics. XLIX, (Feb.,1976) p.40.

III-4. Results of Estimation in Agriculture.

During the sample period (1957-75), agriculture grew at an annual average of 4.3 per cent. However, the contribution of this primary industry to the gross national product fell from 36.9 per cent in 1957 to 21.1 per cent in 1975, largely beacuse of the modernization of the industrial structure of the country during that period.

III-4-I. The Cobb-Douglas Type.

Table Ag-II (rows 1 and 2) presents estimates of the parameters of the Cobb-Douglas production function with unconstrained labor and capital elasticity, while row 3 gives estimates of the Cobb-Douglas parameters with the labor and capital exponents constrained to add to unity. The explanation of the table in this sector is same as that of the manufacturing sector.

In general, the residuals of the production function equation are not significantly autocorrelated, as indicated by the generally high enough values of Durbin Watson statistics compare to the theoretical low bound d_L value at the 95 per cent level. The t ratios are not high enough compare to the theoretical value in some cases, however in many cases the t ratios are high enough. Thus, we are willing to place a fair amount of confidence in the statistical test under examination. As compare to the OLS and ILS which are not constrained to the constant returns to scale, the parameter estimates of the straight regression under the assumption of the constant returns to scale are quite different from those of unconstrained regressions. In OLS and ILS system, during the entire sample period (1957-75), the results show the contribution of capital input is statistically significant compare to the labor input, and the sector is behaving under a situation characterized by diminishing returns to scale.

From the regression results, it seems that there is a clear division in the production characteristic between the first ten years (1957-66) and the last nine years (1967-75)(see table IV for CHOW test). The first ten years were associated with the downward neutral technical shift in the production function, with an increasing returns to scale and with both inputs positive contribution to the production process. The later nine years shows that upward neutral technical shift in the production.function with diminishing returns to scale and with negative contribution of labor to the production. This phenomenon can be interpreted to reflect the existence of abundant labor force (disguised unemployment) and of a capital scarcity in the agricultural sector during that period. Thus we might conclude, only during 1957-66, Korean economy have enjoyed returns to scale in agricultural sector.

We may now examine the implied rates of disembodied, neutral technical progress (see column 4 in table Ag-II and column 8 in table Ag-I). Our preferred point estimates of this growth rate come from the unconstrained regression estimation results of table Ag-II. We take this rate to be 0.037 for 1957-75, -0.015 for 1957-66, and 0.036 for 1967-75. As compared those data to the Solow's residual which is in table Ag-I, the average of technical progress is approximately 3.7 per cent per annum which is somewhat larger than the Solow's residual, 0.325/18 = 0.019, 1.9 per cent per annum. This is not against Walters (1963), since the agricultural sector did not enjoy the increasing returns to scale during the sample period (1957-75).

III-4-2. The Estimated Results of CES.

Table Ag-III presents the estimated results for agriculture in CES framework. The view of economic structure which the Taylor Expan sion regression presents, while not consistent with the two-step CES regression. Here, it is not the capital input which do all the work in the production process, as indicated by a parameter estimate of (0.56, 0.46, 0.46 respectively)δ, during three different periods. However, the elasticity of substitution, σ is more or less similar between estimation procedures (see column 2 in table Ag-III). Our estimate of the elasticity of substitution are significantly different from zero which is the special case of Leontief. But during 1967-75, the elasticity of substitution are not significantly different from unity which is the Cobb-Douglas case.

If we test the significance of the coefficient $(\ln K - \ln L)^2$, which is the correction part of the Taylor expansion regression, the Coefficient of the correction part is not significant in t_o=0.05 level, even though regression coefficients show us large numbers.[14]

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14. For 1957-75: $-0.22157(\ln K - \ln L)^2$ (0.9346) t=-0.2370 t_o=1.86
For 1957-66: $-10.0681(\ln K - \ln L)^2$ (26.08) t=-0.2860 t_o=1.89
For 1967-75: $-0.4109(\ln K - \ln L)^2$ (3.10) t=0.1325 t_o=1.74

The results thus provide no evidence against the Cobb-Douglas model in this sense. For either the Cobb-Douglas or CES formulation, we find evidence of diminishing returns to scale during 1957-75 and 1967-75. For our results in table Ag-III, we have point estimates of returns to scale of -0.096, 4.39, -0.387 with Taylor expansion CES, or 0.057, 2.28, and -0.498 with two-step CES regression. In both cases, in general these estimates are significantly different with the constant returns to scale value of returns to scale parameter, v equal to unity.

The examination of the implied rates of disembodied, neutral technical progress in the CES formulation of the production function is also valid to Walters finding(1963) in this sector. Using the results (see column 5 in table Ag-III), both in Taylor and two-step CES regression, a rate of neutral technical progress equal to 3.6 per cent per year in average which is larger than the Solow's residual over the sample time period it was increased 32.5 per cent. It seems that the difference is due to the returns to scale parameter, because agriculture did not enjoyed the increasing returns to scale. In previous sector(manufacturing), we knew that the two procedures would appear to yield different results (compare ρ, σ, and δ in table Ag-III), in this sector we also find these parameter estimates could differ in different estimation procedure in CES function.

III-5. The Results of Estimation in Construction.

III-5-1. The Cobb-Douglas Type.

The explanation of the tables in this sector is same as that of the manufacturing sector. In general, the residuals of the production function equations are not significantly autocorrelated, as indicated by the high enough values of Durbin-Watson statistics compare to the Theoretical low bound d_L value at the 95 per cent level except 1957-75 regression. The t ratios are not high enough compare to the theoretical value in labor input, however for capital input and neutral technology the t ratios are high enough. Thus we are willing to place a fair amount of confidence in the statistical test under examination. Once again, the parameter estimates of the straight regression under the assumption of the constant returns to scale are quite different those of the uncons trained regression, OLS and ILS.

In the unconstrained regression systems, during the sample periods,(1957-75) and (1957-66), the results show the contribution of the capital input is not statisticallly significant compare to the labor input, and the sector is behaving under a situation characterized by diminishing returns to scale. However, in the sample period (1967-75), the results show the contribution of capital input is statistically significant compare to the labor input, and the sector is behaving under a situation by increasing returns to scale.

From the regression results, it seems that there is a clear division in the production characteristics between the first ten years (1957-66) and the last nine years (1967-75)(see table IV for CHOW test). The first ten years was associated with the upward neutral technical shift in the production function, with a diminishing returns to scale.

The later nine years shows that downward neutral technical shift in the production function with increasing returns to scale.

We may now examine the implied rates of disembodied, neutral technical progress (see column 4 in table C-II and column 8 in table C-I). Our preferred point estimates of this growth rates come from the unconstrained regression results of table C-II. We take this rate to be 0.1234 for entire sample period of 1957-75. As compare this to the Solow's residual which is in table C-I, the average of technical progress is approximately 12.3 per cent which is larger than the Solow's residual (0.4791/18 =0.026), 2.6 per cent per annum. This is not against Walters finding(1963), since the construction sector did not enjoyed the increasing returns to scale during the sample period.

III-5-2. The Estimated Results of CES.

Our estimates of the elasticity of substitution are significantly different from zero which is the special case of Leontief. But during 1957-66 in Taylor CES and during 1967-75 in two-step CES, the elasticities of substitution are not significantly different from unity which is the Cobb-Douglas case. If we test the significance of the coefficient $(\ln K-\ln L)^2$, which is the correction part of the Taylor expansion regression, the coefficient of the correction part is not significant at the t_o=0.05 level for 1967-75, even though regression coefficient shows us large number. Thus the results provide no evidence against the Cobb-Douglas model in this case.[15]

15. For 1957-75; $-0.8349(\ln K-\ln L)^2$
(0.5694) t=-1.46 t_o=1.86
For 1957-66; $-0.7636(\ln K-\ln L)^2$
(0.6768) t=-1.12 t_o=1.89
For 1967-75; $0.1944(\ln K-\ln L)^2$
(1.4392) t=0.135 t_o=1.74

For either the Cobb-Douglas or CES formulation, we found the evidence of diminishing returns to scale during 1957-75 and 1957-66. For our results in table C-III, we have point estimates of returns to scale of 0.836, 1.20 ,and 2.29 with Taylor expansion CES, or 0.397, 0.097 ,and 2.140 with two-step CES regressions. In both cases, in generally these estimates are significantly different with the constant returns to scale value of returns to scale parameter, v equal to unity.

For the implied rates of disembodied, neutral technical change, we take the rate to be 6.1 per cent in Taylor expansion CES, or 11.1 per cent in Two-Step CES which are larger than the Solow's residual (2.7%). We might explain this as because of diminishing returns to scale during the sample period which was indicated previously in the discussion of the Cobb-Douglas production function.

III-6. The Results of Estimation in Transcommunication.

III-6-1. The Cobb-Douglas Type.

As previous sectors, the residuals of the production function of this sector are not significantly autocorrelated as indicated by the high values of Durbin-Watson statistics compare to the theoretical lower limit of d_L value at the 95 per cent level. The t ratios are not high enough compare to the theoretical value in some cases (particularly in labor input), however, in many cases the t ratios are high enough. Thus we are willing to place a fair amount of confidence in the statistical test under examination.

Generally again, as compare to the results of the unconstrained regression, the parameter estimates of the straight regression under the assumption of the constant returns to scale are quite different from those of unconstrained regression. In the OLS, during the entire sample period (1957-75), the results show the contribution of capital input is statistically significant compare to the labor input, and the sector is behaving under a situation characterized by diminishing returns to scale.

From the regression results, it seems that there is a clear division in the production characteristic between the first ten years (1957-66) and the last nine years (1967-75). The first ten years were associated with the downward neutral technical shift in the production function with an increasing returns to scale. The later nine years show that downwards neutral technical shift in the production function with diminishing returns to scale (see Chow test in appendix). Thus, we might conclude only the first and second five year economic development plan periods Korean economy had enjoyed increasing returns to scale in trans communication sector.

We may now examine the implied rates of diembodied, neutral technical progress (see column 4 in table TC-II and column 8 in table TC-I). Our preferred point estimates of this growth rate come from the unconstrained regression estimation results of table TC-II. We take this rate to be 0.0487 for 1957-75. As compare this to the Solow's residual which is in table TC-I, the average of technical progress is approximately 4.8 per cent per annum which is somewhat smaller than the Solow's residual (1.11/18 -0.0610, 6.1 per cent per annum).

This results is against Walters finding(1963), since the transcommunication sector did not enjoy the increasing returns to scale during the sample period, however one might explain this as because of multicollinearity since the explanatory variables are highly correlated.

III-6-2. The Estimated Results of CES.

Table TC-III presents the estimated results for trancommunication in CES framework. The view of economic structure which the Taylor expansion regression presents, while not consistent with the two-step CES regression. Here, it is not the capital input which do all the work in the production process, as indicated by the parameter estimates of in Taylor expansion, 0.54, 0.59 ,and 0.48 for three different sample per iods. The parameter estimates of δ in two-step regression are slightly larger than that of Taylor expansion CES. However, the elasticity of substitution, σ is smaller in two-step CES case. Our estimates of the elasticity of substitution are significantly different from zero which is the special case of Leontief production, and they are not significantly different from unity which is the Cobb-Douglas case. If we test the significance of the coefficient $(\ln K-\ln L)^2$ which is the correction part of the Taylor expansion regression, the coefficient of the correc tion part is significant in t_o=0.05 level.[16]

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16. For 1957-75; $-0.2094(\ln K - \ln L)^2$

For 1957-75;	$-0.2094(\ln K - \ln L)^2$		
	(0.0707)	t=-2.96	t_o=1.86
For 1957-66	$-0.2080(\ln K - \ln L)^2$		
	(0.8332)	t=-0.249	t_o=1.89
For 1967-75	$-0.7669(\ln K - \ln L)^2$		
	(0.2649)	t=-2.89	t_o=1.74

Thus the results provide evidence against the Cobb-Douglas model in that sense. For our results in table TC-III, we have point estimates of returns to scale of 0.711, 1.46 ,and 0.178 with Taylor expansion CES and 0.763, 1.07 ,and 0.375 with two step CES regression. In both cases, these estimates are significantly different with the constant returns to scale value of returns to scale parameter, v equal to unity. The examination of the implied rate of disembodied, neutral technical progress in CES formulation of the production function in the two-step procedure(5.7%) is also valid to the Solow's residual (6.1%).

III-7. The Results of Estimation in Mining.

III-7-1. The Cobb-Douglas Type.

The residuals of the production functions are not significantly autocorrelated except for 1957-75 sample period as indicated by the values of Durbin-Watson statistics at the 95 per cent level, and the t ratios are not high enough in some cases(specially in labor input), however, in many cases that ratios are high enough. Thus again we are willing to place a fair amount of confidence in the statistical test under examination.

In the OLS and ILS system, during the entire sample period (1957-75), the results show that the contribution of capital input is significantly strong compare to the labor input, and the sector is behav -ing under a situation characterized by increasing returns to scale. From the regression results, it seems that there is a clear division in the production characteristic between the first ten years(1957-66) and the last nine years (1967-75)(see appendix for CHOW test). The first ten years were associated with the upward neutral technical shift in the production function with diminishing returns to scale and with both inputs negative contribution to the production process. The later nine years shows that downward neutral technical shift in the production function with diminishing returns to scale and with positive contribution of capital and negative contribution of labor to the production process.

Our preferred point estimates of the implied rates of disembodied, neutral technical progress come from the unconstrained regression estimation results of table MI-II. We take this rate to be 0.0115 for 1957-75 sample period. As compared this to the Solow's residual which is in table MI-I, the average of technical progress is approximately 1.2 per cent per annum which is smaller than the Solow's residual(1.20/18 =0.0671, 6.7%). This result supports the Walters findings(1963).

III-7-2. The Estimated Results of CES.

Our estimates of the elasticity of substitution are significantly different from zero which is the special case of Leontief production. However, during 1957-66, the elasticity of substitution in Taylor expansion CES is not significantly different from unity which is the Cobb-Douglas case. If we test the significance of the coefficient $(\ln K - \ln L)^2$, which is the correction part of the Taylor expansion regression, the coefficient of the correction part is not significant at the level of t_0=0.05, even though the regression coefficients show us large numbers. The results thus provide no evidence against the Cobb-Douglas model in this sense.[17]

For either the Cobb-Douglas or CES formulation, we find the evidence of diminishing returns to scale during 1957-66 sample period and 1967-75 sample period. In both cases, these estimates are significantly different with the constant returns to scale value of the returns to scale parameter, v equal to unity.

The examination of the implied rate of disembodied, neutral technical progress in the CES production function (1.4%) is smaller than the Solow's residual (6.7%) as indicated in the discussion of the Cobb-Douglas type.

17. For 1957-75; $0.1554(\ln K - \ln L)^2$ (0.5848) $t=0.2657$ $t_o=1.86$

For 1957-66; $5.2081(\ln K - \ln L)^2$ (3.0420) $t=1.73$ $t_o=1.89$

For 1967-75; $0.1536(\ln K - \ln L)^2$ (0.2757) $t=0.5571$ $t_o=1.74$

CHAPTER IV.

THE QUALIFICATION OF THE RESULTS AND CONCLUSION

IV. The Qualification of The Results and Conclusion.

I have had a twofold objective in this study. On the one hand, I wanted to have another look at substantive issues in the measurement of the aggregate Korean production functions for their bearing on the measurement of technical progress, the degree of returns to scale, and the magnitude of the elasticity of substitution. As to the second objective, I have found that the results of nonlinear methods (Taylor expansion CES and Two-Step CES) may, in some cases, be quite different from those obtained by linear methods such as O.L.S., I.L.S., and S.CRS which were discussed in section II. As we can see that the statistical pictures which emerges are different in two different estimation methods, O.L.S. and I.L.S. which are selected in the tables II. J.Kmenta and M.E. Joseph pointed out in their empirical study[1] that I.L.S. estimates are not well suited for small sample estimation, and some extent, indirect least squares estimates can be highly unstable in small samples.

Throughout this study, some estimates turned out to be unacceptable (e.g. a negative rate of disembodied technical progress and negative sign of coefficient of parameters). The multicollinearity between labor, capital, and time might be responsible for some of the negative regression coefficients, in other words, interaction among labor and capital be quite substantial and their adjustments may not be independent of each other. This relationship between inputs is against one of the basic assumptions of the general linear model that is no linear dependence

1. J. Kmenta and M.E. Joseph. "A monte Carlo study of alternative estimates of the Cobb-Douglas production function" Econometrica, Vol. 31. No.3, (July, 1963) pp. 384-85.

exists between the explanatory variables. Thus many of the specific results may have the fall in precision or errors, and estimates of coefficients become very sensitive to particular sets of sample data, and the addition of a few more observations can sometimes produce dramatic shifts in some of the coefficients due to this multicollinearity.

There were constraints and restraints, large and small, in the process of economic development plan periods (sample periods) in Korean economy. These shocks may reflect inexactness in the model, since random shocks are the prime causes of business cycle[2] and also there are "unseen cycles"[3] that lies hidden within the movements of economic aggregate. The use of number of people and capital stock which I used rather than person hours and machine hours might cause overestimation of parameters. Hours per person and hours per machine (utilization of capital) tend to vary a lot in response to temporary (cyclical) fluc tuations in output.

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2. See Figure 5 in appendix (Rate of Growth by industrial Origin).

3. During each expansion in total activity, there is first a rise and then a fall in the scope of the expansion, and during each contraction in total activity there is first a fall and then a rise in the scope of contraction.
   Expansionary movements, which begin during a contraction in aggregate activity, spread from firm to firm, industry to industry, region to region, and from one aspect of economic activity to another, and their acumulative process takes time.
   Constractions in their turn spread in a similar way, beginning while aggregate activity is still rising.

Number of persons and number of machines vary much less. The rate of change of output exceeds the rate of change of employment and capital stock over the business cycle. This is due to fixed cost of hiring and training men. Therefore, as the inputs, number of men and capital stock overestimate the specific results.[4]

Thus I might say the results of estimation admit some misspecification in these respect which I have discussed above and we are not absolutely certain that the different estimates are firm in the sense of representing the solution to the problem of finding true production behavior.

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4. Let the true production function be

$$\ln Y = a \ln PH + (1-a) \ln K\phi$$

P = persons employed, H = hours/persons
K = capital stock ϕ = utilization of capital

$$\ln Y = a \ln P + a \ln H + (1-a) \ln K + (1-a) \ln\phi$$

If lnH and $\ln\phi$ are omitted and if they are positively related to lnP and lnK,

$$\frac{\partial \ln Y}{\partial \ln P} = a + a \frac{\partial \ln H}{\partial \ln P}$$

$$= a + a\delta_1$$

$$\frac{\partial \ln Y}{\partial \ln K} = (1-a) + (1-a) \frac{\partial \ln\phi}{\partial \ln K}$$

$$= (1-a) + (1-a)\,\delta_2$$

If H and ϕ omitted and if $\delta_1 > 0$, and $\delta_2 > 0$, one would overestimate the output elasticities of P and K.

Reviewing the growth rate by industry, the agriculture sector which had been lagging behind other sectors marked a growth rate of a little less than 5.0 percent (4.3%). Although the growth rate was still higher than average growth rates of agriculture in other developing countries, it was relatively low compared with the growth rate of 15.8 percent growth of manufacturing, 12.9 percent growth of construction, 14.2 percent growth of transcommunication, and 8.3 percent growth of mining sectors. This reflects the fact that the high growth in the past two decades was led by rapid industrialization. By industrial origin of GNP the relative share of agriculture sector declined, while the shares of manufacturing, construction, transcommunication, and mining showed rapid increases, thus resulting in great improvement in industrial structure. This change in industrial structure is also reflected in the structure of employment by industry. Employment in agriculture was increased 1.4 percent per annum, on the contrary, the employment increased by 6.9 percent, 7.4 percent, 4.2 percent, and 1.7 percent in manufacturing, construction, transcommunication, and mining respectively. The rapid growth and improvement of industrial structure discussed above was made possible by the following basic factors. (1) non-economic factors such as the high level of education and (2) economic factors such as export expansion, and the expansion of social overhead capital to support industrial development that could be financed by mobilization of domestic savings and the smooth inflow of foreign capital (savings from the foreign sector through active international economic cooper-ation). The availability of abundant, cheap and skilled labor force, including scientific and technical manpower accelerated investment activities in industrial sectors.

Our most sobering results are that, first of all, estimates of elasticities of substitution are extremely sensitive to difference in measurement and data construction. In this respect we are with Nerlove who pointed out that "even slight variations in the period or concepts tend to produce drastically different estimates of the elasticity (1967)." Secondly, the most sectors of this study except transcommunication, support the Walters finding (1963).

I have found the evidence of increasing returns to scale during economic development plan periods for most sectors and associated with this phenomenon somewhat lower rates of neutral technical progress than the Solow's residual which was constrained returns to scale. It is our belief that the Korean economy does function under increasing returns to scale, as discussed in section III.

The estimates of the elasticity of substitution from our CES model vary from approximately from zero to 1.3. In the most cases, the estimates are quite different from the zero (Leontief case) and are about 0.4 to 0.9 and significantly less than unity.

Frequently, economists are faced with the take of attempting to reconcile inconsistent empirical evidence. A common approach is to investigate possible errors in functional and stochastic specification. The implication of this study is that although errors in stochastic specification may be of considerable importance, "all econometric results are no better than the data that went into them" (Griliches,1967. p.308).

The data used in this study have been constructed with considerable care, but no doubt some errors remain. Two potential sources of error are particularly worthy of further consideration. First, the statistical data for new series on NNP originating, capital utilization, and employees by type (different efficiency by schooling and on the job traing) are not now available for this study, if it is, one could do a better job of production estimation by using them. Secondly, if we could drop the assumption of neutral technical progress, we could obtain additional improvements, besides those associated with generalization of the estimation procedures. Considerable theoretical work still remains to be done on appropriate indexing procedures for inputs and output when technical change is not of the simple neutral form. For example, input prices in terms of efficiency units, rates of return to durable inputs, and measure of value added will be affected when nonneutral technical change impinges on inputs.

CHAPTER V.

APPENDICES

Appendix A Summary of Tables for Regression Results
Appendix B Mathematical Appendix
Appendix C Properties of The CES Function
Bibliography

TABLE Ma-I. THE PARTIAL PRODUCTIVITY INDEXES AND THE SOLOW's RESIDUAL(MANUFACTURING).

All the monetary terms are billion Korean won at the 1970 constant price.

Year	Real Output (Bil. won)	Labor employed 1,000 prs.	Real capital (bil.won)	Real output per man	Employed capital per man	Share of property in income	ΔA(t)/A(t)	A(t)	q/A
1957	94.65	539	201.67	0.1756	0.3757	0.212	0.0401	1.0000	0.1756
1958	103.25	552	230.92	0.1837	0.4183	0.241	0.0187	1.0401	0.1756
1959	112.77	561	255.29	0.2010	0.4550	0.260	0.0652	1.0588	0.1898
1960	121.99	572	281.39	0.2132	0.4919	0.264	0.0374	1.1240	0.1896
1961	125.78	598	307.26	0.2103	0.5138	0.235	-0.0237	1.1624	0.1810
	0.078	0.02	0.09	0.0459	0.0816		0.0275	(Ave. Rate of Growth)	
1962	142.34	602	341.87	0.2364	0.5678	0.244	0.0872	1.1377	0.2077
1963	166.95	610	386.91	0.2736	0.6342	0.214	0.1135	1.2249	0.2233
1964	177.86	637	431.51	0.2792	0.6744	0.220	0.0060	1.3384	0.2086
1965	213.34	772	492.90	0.2736	0.6384	0.240	0.0043	1.3444	0.2055
1966	249.87	833	597.74	0.2996	0.7175	0.259	0.0499	1.3487	0.2142
	0.157	0.07	0.12	0.0771	0.0891		0.0521	(Ave. rate of Growth)	
1967	306.77	1,021	701.87	0.3004	0.6874	0.284	0.0151	1.3986	0.2147
1968	389.66	1,176	841.13	0.3313	0.7152	0.272	0.0828	1.4137	0.2343
1969	473.03	1,232	999.53	0.3839	0.8113	0.287	0.1109	1.4965	0.2565
1970	560.01	1,284	1,161.30	0.4361	0.9044	0.297	0.0890	1.6074	0.2713
1971	659.21	1,336	1,326.96	0.4934	0.9932	0.267	0.0922	1.6964	0.2908
	0.215	0.07	0.07	0.1065	0.0705		0.0780	(Ave. rate of Growth)	
1972	762.78	1,445	1,489.58	0.5278	1.0308	0.251	0.0561	1.7886	0.2951
1973	998.75	1,774	1,789.25	0.5629	1.0084	0.249	0.0678	1.8447	0.3051
1974	1,173.12	2,012	2,033.37	0.5831	1.0106	0.241	0.0351	1.9125	0.3048
1975	1,202.10	2,205	2,083.23	0.5451	0.9447	0.281	-0.0510	1.9476	0.2798
	0.179	0.13	0.10	0.0267	-0.0469		0.0270	(Ave. rate of Growth)	
Total Ave. rate of Growth.									
	0.158	0.069	0.119	0.0640	0.0485		0.0461		

Notes: Column(4): Column(1)/Column(2), q=Y/L and Column(5); Column(3)/column(2). k=K/L.
Column(7),(8), and (9) are the calculated figures. See the calculation procedures in footnote #1 in section II.

TABLE Ma-II. Fitting the Production Function in the Cobb-Douglas Type. (MANUFACTURING)

Year	Regression Coefficient Labor(α)	Capital(β)	($\alpha+\beta$)	λ	R^2	D.W.	F
1957-75							
O.L.S.	0.5940	0.9579	1.522	0.0270	0.99	0.604	1827.36
	(0.1673)	(0.2724)		(0.0283)			
I.L.S.	0.4092	2.0263	2.435	0.0159	0.98	0.642	1447.01
	(0.3424)	(0.5002)		(0.0132)			
S.CRS.	0.5374	0.4626	1.00	0.0451	0.97	0.509	387.83
	(0.1978)			(0.0112)			
1957-66							
O.L.S.	0.2645	0.9877	1.252	-0.0197	0.99	1.800	315.11
	(0.2699)	(0.5244)		(0.0511)			
I.L.S.	-0.6869	4.8668	4.7757	0.02067	0.98	1.734	254.39
	(0.7124)	(1.2124)		(0.03124)			
S.CRS.	0.3233	0.6766	1.00	0.0139	0.97	1.88	178.69
	(0.2400)			(0.0172)			
1967-75							
O.L.S.	0.3419	0.7814	1.123	0.0408	0.99	3.135	7337.94
	(0.0603)	(0.0818)		(0.0145)			
I.L.S.	0.5500	1.11365	1.6636	0.0375	0.98	2.13	599.34
	(0.2321)	(0.7942)		(0.0079)			
S.CRS.	0.0813	0.9189	1.00	0.0399	0.99	1.92	558.90
	(0.0877)			(0.0049)			

Note: The parentheses present the associated standard errors.

TABLE Ma-III. ESTIMATES OF THE PARAMETERS OF THE CES PRODUCTION FUNCTION(MANUFACTURING).

	ρ	$\sigma=\frac{1}{1+\rho}$	δ	v	λ	R^2	D.W.	F
1957-75								
Taylor Ex.	26.03	0.037	1.053	0.878	0.059	0.98	1.38	2995.6
CES	(8.02)	(0.11)	(0.49)	(0.37)	(0.027)			
Two-Step	0.225	0.818	0.687	1.646	0.046	0.89	0.58	2646.0
CES	(0.090)	(0.06)		(0.492)	(0.020)			
1957-66								
Taylor Ex.	2.002	0.333	1.312	1.003	0.051	0.98	2.70	273.5
CES.	(0.98)	(0.21)	(0.519)	(0.782)	(0.005)			
Two-Step	-0.191	1.246	0.649	1.094	0.009	0.60	1.78	536.67
CES.	(0.229)	(0.35)		(0.472)	(0.020)			
1967-75								
Taylor Ex.	2.37	0.296	1.584	1.710	0.026	0.98	2.52	19.7
CES.	(1.00)	(0.09)	(0.780)	(0.902)	(0.003)			
Two-Step	0.047	0.954	0.671	1.135	0.041	0.92	2.73	1234.7
CES.	(0.11)	(0.10)		(0.520)	(0.013)			

Note: The associated standard error is in parenthesis.

Taylor Expansion CES.

$$\ln Y = \ln A + \lambda t + v\delta \ln K + v(1-\delta) \ln L - -\tfrac{1}{2}\rho v\delta(1-\delta)(\ln K - \ln L)^2$$

Two-Step CES.

$$(1).\ \ln (P_k/w) = \ln (\delta/1-\delta) - (\rho + 1) \ln (K/L)$$

$$(2).\ \ln Y = \ln A + \lambda t - v/\hat{\rho}\ \ln\left[\hat{\delta} K^{-\hat{\rho}} + (1-\hat{\delta}) L^{-\hat{\rho}}\right]$$

TABLE A-I. THE PARTIAL PRODUCTIVITY INDEXES AND THE SOLOW's RESIDUAL(AGRICULTURE).

All the monetary terms are billion Korean won at the 1970 constant price.

Year	Real output (bil.won)	Labor employed 1,000 prs.	Real capital (bil.won)	Real output per man	Employed capital per man	Share of property in income	$\Delta A(t)/A(t)$	A(t)	q/A
1957	450.15	4,328	851.73	0.1040	0.1967	0.212	0.0624	1.0000	0.1040
1958	478.19	4,476	866.03	0.1060	0.1934	0.241	0.0221	1.0624	0.1005
1959	472.51	4,543	850.55	0.1040	0.1872	0.260	-0.0183	1.0845	0.0958
1960	466.57	4,628	861.61	0.1008	0.1861	0.264	-0.0272	1.0662	0.0945
1961	522.20	4,698	873.58	0.1111	0.1859	0.235	0.0929	1.0390	0.1069
Ave. rate	0.04	0.02	0.006	0.0176			0.0263		
1962	492.16	4,742	887.12	0.1037	0.1870	0.244	-0.0856	1.1319	0.0916
1963	532.04	4,744	904.55	0.1121	0.1906	0.214	0.0709	1.0463	0.1071
1964	614.59	4,825	923.86	0.1273	0.1914	0.220	0.1185	1.1117	0.1145
1965	602.65	4,810	947.60	0.1252	0.1970	0.240	-0.0235	1.2375	0.1013
1966	667.91	4,876	982.40	0.1369	0.2014	0.259	0.0785	1.2122	0.1129
Ave. rate.	0.05	0.007	0.024	0.0453	0.01216		0.03176		
1967	634.78	4,811	1,013.55	0.1329	0.2106	0.284	-0.0489	1.2907	0.1029
1968	650.08	4,801	1,049.27	0.1354	0.2185	0.272	0.0160	1.2418	0.1090
1969	731.48	4,825	1,089.16	0.1516	0.2257	0.287	0.0976	1.2578	0.1205
1970	724.59	4,916	1,141.16	0.1473	0.2340	0.297	-0.0371	1.3554	0.1086
1971	748.40	4,876	1,197.47	0.1534	0.2455	0.267	0.0272	1.3183	0.1164
Ave. rate.	0.03	0.002	0.04	0.0244	0.0404		0.01090		
1972	760.92	5,346	1,270.12	0.1423	0.2375	0.251	-0.0696	1.3455	0.1057
1973	802.95	5,569	1,345.80	0.1441	0.2416	0.249	0.0082	1.2759	0.1129
1974	847.56	5,584	1,444.21	0.1527	0.2586	0.241	0.0409	1.2841	0.1181
1975	892.43	5,425	1,513.27	0.1645	0.2789	0.281	0.0574	1.3250	0.1241
Ave. rate.	0.05	0.028	0.058	0.0193	0.0333		0.00925		
Total ave. rate of Growth.	0.043	0.014	0.032	0.0266	0.0175		0.0190		

Notes: Column (4); column(1)/column(2) and column(5); column(3)/column(2).
Column(7),(8), and(9) are the calculated figures. see the calculation procedures in footnote#1 in section II.

TABLE Ag-II. Fitting the Production Function in the Cobb-Douglas Type.(AGRICULTURE)

Year	Regression Coefficients Labor(α)	Capital(β)	($\alpha+\beta$)	λ	R^2	D.W.	F
1957-75							
O.L.S.	-0.2692	0.1216	-0.1476	0.0376	0.96	2.169	153.12
	(0.2737)	(0.1076)		(0.0065)			
I.L.S.	0.0052	0.2840	0.2892	0.0275	0.98	1.450	94.24
	(0.0049)	(0.2178)		(0.0042)			
S.CRS.	0.7866	0.2123	1.00	0.0227	0.88	1.310	70.75
	(0.2598)			(0.0058)			
1957-66							
O.L.S.	1.2907	2.742	4.032	-0.015	0.90	2.935	28.03
	(2.04)	(1.33)		(0.042)			
I.L.S.	1.1512	1.4631	2.6143	0.0119	0.99	2.69	26.72
	(0.0129)	(0.2428)		(0.0012)			
S.CRS.	-0.563	1.563	1.00	0.0246	0.84	2.99	24.88
	(0.53)			(0.0049)			
1967-75							
O.L.S.	-0.463	0.257	-0.179	0.036	0.94	2.46	43.83
	(0.366)	(0.17)		(0.037)			
I.L.S.	-0.421	-1.6912	-2.112	0.0437	0.98	1.25	39.48
	(0.541)	(0.1242)		(0.0072)			
S.CRS.	-0.3567	1.3567	1.00	-0.022	0.79	2.21	16.14
	(0.396)			(0.012)			

Note: The Parentheses present the associated standard errors.

TABLE AGRI-III. ESTIMATES OF THE PARAMETERS OF THE CES PRODUCTION FUNCTION (AGRICULTURE)

	ρ	$\sigma = \frac{1}{1+\rho}$	δ	v	λ	R^2	D.W.	F
1957-75								
Taylor Ex.	1.852	0.351	0.562	-0.096	0.0365	0.95	2.179	107.63
CES.	(0.245)		(0.132)	(0.007)	(0.0082)			
Two-Step	0.571	0.639	0.702	0.057	0.0368	0.96	2.06	235.98
CES.	(0.06)		(0.063)	(0.012)	(0.0064)			
1957-66								
Taylor Ex.	0.0862	0.079	0.466	4.39	-0.0193	0.88	3.043	18.08
CES.	(0.062)		(0.401)	(2.28)	(0.0468)			
Two-Step	1.239	0.4476	0.691	2.28	0.0071	0.90	2.98	46.38
CES.	(0.522)		(0.126)	(1.99)	(0.0138)			
1967-75								
Taylor Ex.	-0.158	1.187	0.463	-0.387	0.0476	0.92	2.45	26.42
CES.	(0.124)		(0.064)	(0.089)	(0.0093)			
Two-Step	-0.168	1.203	0.6517	-0.498	0.0613	0.94	2.12	67.11
CES.	(0.1004)		(0.043)		(0.026)			

Note: The standard error is in parenthesis.

Taylor Ex.
CES.

$$\ln Y = \ln A + \lambda t + v\delta \ln K + v(1-\delta) \ln L -$$

$$-\tfrac{1}{2}\rho v \delta (1-\delta)(\ln K - \ln L)^2$$

Two-Step
CES. (1). $\ln (P_k/w) = \ln (\delta/1-\delta) - (\rho+1) \ln (K/L)$

(2). $\ln Y = \ln A + \lambda t - v/\hat{\rho} \ln\left[\hat{\delta} K^{-\hat{\rho}} + (1-\hat{\delta}) L^{-\hat{\rho}}\right]$

TABLE C-I. THE PARTIAL PRODUCTIVITY INDEXES AND THE Solow's RESIDUAL (CONSTRUCTION).

All the monetary terms are million Korean won at the 1970 constant price.

Year	Real Output (mil.won)	Labor employed 1,000 prs.	Real capital (mil.won)	Real output per man	Employed capital per man	Share of property in income	$\Delta A(t)/A(t)$	A(t)	q/A
1957	21,075	149	27,339	0.1414	0.1834	0.212	0.0014	1.0000	0.1414
1958	21,348	152	28,911	0.1404	0.1902	0.241	-0.0015	1.0014	0.1402
1959	25,559	154	29,902	0.1662	0.1914	0.260	0.1497	0.9999	0.1662
1960	25,519	169	30,942	0.1510	0.1830	0.264	-0.0849	1.1496	0.1313
1961	28,618	164	32,157	0.1745	0.1960	0.235	-0.1191	1.0647	0.1638
Ave. rate	0.0761	0.0253	0.0414	0.0602	0.0178		-0.0110		
1962	33,246	179	36,368	0.1857	0.2031	0.244	0.0519	0.9456	0.1963
1963	38,377	181	39,805	0.2120	0.2199	0.214	0.1078	0.9975	0.2125
1964	41,547	183	42,189	0.2270	0.2305	0.220	0.0559	1.1053	0.2053
1965	50,731	238	45,195	0.2131	0.1898	0.240	-0.0137	1.1612	0.1835
1966	63,706	209	48,712	0.3048	0.2330	0.259	0.2528	1.1475	0.2656
Ave. rate	0.1751	0.06	0.0866	0.1543	0.0436		0.0910		
1967	72,297	259	52,056	0.2791	0.2009	0.284	-0.0465	1.4003	0.1993
1968	102,629	316	65,750	0.3247	0.2080	0.272	0.1313	1.3538	0.2398
1969	143,612	337	79,467	0.4267	0.2358	0.287	0.2042	1.4851	0.1587
1970	150,201	284	87,505	0.5288	0.3081	0.297	0.1031	1.6895	0.3130
1971	150,592	348	95,760	0.4327	0.2751	0.267	-0.1902	1.7924	0.2416
Ave. rate	0.2004	0.1067	0.1471	0.0901	0.0461		0.0402		
1972	148,473	392	104,167	0.3787	0.2657	0.251	-0.1336	1.6022	0.2363
1973	180,719	371	118,229	0.4871	0.3163	0.249	0.1810	1.4686	0.3316
1974	182,272	450	126,533	0.4050	0.2811	0.241	-0.1705	1.6496	0.2455
1975	191,123	511	135,214	0.3740	0.2646	0.281	-0.0653	1.4791	0.2528
Ave. rate	0.0651	0.1053,	0.0903	-0.0208	-0.0031		-0.0471		
Total Ave. rate of the Growth	0.1292	0.0743	0.09135	0.0709	0.0261		0.0266		

Notes: Column (4); column(1)/column(2), q=Y/L and column(5); column(3)/column(2), k=K/L. Column(7),(8), and (9) are the calculated figures. See the calculation procedures in footnote #1 in section II.

TABLE C-II. Fitting the Production Function in the Cobb-Douglas Type. (CONSTRUCTION)

Year	Regression Coefficients Labor(α)	Capital(β)	($\alpha+\beta$)	λ	R^2	D.W.	F
1957-75							
O.L.S.	0.1572	0.1249	0.2821	0.1234	0.96	0.573	164.71
	(0.4337)	(0.1069)		(0.0285)			
I.L.S.	-0.5921	0.4752	-0.1209	0.0874	0.98	1.42	491.26
	(0.4728)	(0.5211)		(0.0139)			
S.CRS.	0.0538	0.9461	1.00	0.0480	0.90	0.419	83.01
		(0.0010)		(0.0113)			
1957-66							
O.L.S.	0.0952	0.0946	0.1898	0.1175	0.96	1.388	82.33
	(0.391)	(0.0555)		(0.0192)			
I.L.S.	0.5182	0.0567	0.5749	0.0879	0.97	1.689	127.45
	(0.7282)	(0.0372)		(0.0212)			
S.CRS.	0.0185	0.9815	1.00	0.0542	0.93	1.569	61.84
		(0.3415)		(0.0102)			
1967-75							
O.L.S.	0.0256	2.2270	2.2526	-0.1513	0.97	1.506	111.04
	(0.1990)	(0.2740)		(0.036)			
I.L.S.	0.2212	2.521	2.7422	-0.1138	0.98	1.79	250.73
	(0.7141)	(1.002)		(0.0075)			
S.CRS.	-0.43	1.43	1.00	-0.0209	0.83	1.47	20.98
		(0.246)		(0.014)			

Note: The parentheses present the associated standard errors.

TABLE C-III. ESTIMATES OF THE PARAMETERS OF THE CES PRODUCTION FUNCTION(CONSTRUCTION)

	ρ	$\sigma=\frac{1}{1+\rho}$	δ	v	λ	R^2	D.W.	F
1957-75								
Taylor Ex.	0.1649	0.8569	0.526	0.836	0.061	0.96	0.62	133.54
CES.	(0.0579)	(0.0425)	(0.018)	(0.210)	(0.050)			
Two-Step	0.3357	0.7489	0.690	0.3976	0.114	0.96	0.46	262.14
CES.	(0.047)	(0.026)		(0.082)	(0.019)			
1957-66								
Taylor Ex.	0.0252	0.9754	0.502	1.20	0.053	0.96	1.43	64.87
CES.	(0.012)	(0.019)	(0.182)	(0.79)	(0.060)			
Two-Step	-0.6484	2.844	0.586	0.097	0.121	0.96	1.30	142.67
CES.	(0.5389)	(0.160)		(0.012)	(0.007)			
1967-75								
Taylor Ex.	-4.160	-0.31	0.042	2.29	-0.155	0.97	1.50	66.93
CES.	(6.721)	(1.60)			(0.050)			
Two-Step	0.0482	0.9539	0.6708	2.140	-0.104	0.84	1.67	23.49
CES.	(0.0002)	(0.0001)		(0.387)	(0.09)			

Note: The standard error is in parenthesis.

Taylor Ex.
CES. $\ln Y = \ln A + \lambda t + v\delta \ln K + v(1-\delta)\ln L - \frac{1}{2}\rho v\delta(1-\delta)(\ln K - \ln L)^2$

Two-Step
CES. (1). $\ln (P_k/w) = \ln (\delta/1-\delta) - (\rho+1)\ln (K/L)$

(2). $\ln Y = \ln A + \lambda t + v/\hat{\rho}\ \ln\left[\hat{\delta} K^{-\hat{\rho}} + (1-\hat{\delta}) L^{-\hat{\rho}}\right]$

TABLE TC-I. THE PARTIAL PRODUCTIVITY INDEXES AND THE Solow's RESIDUAL (TRANSCOMMUNICATION).

All the monetary terms are million Korean won at the 1970 constant price.

Year	Real output (mil.won)	Labor employed 1,000 prs.	Real capital (mil.won)	Real output per man	Employed capital per man	Share of property in income	ΔA(t)/A(t)	A(t)	q/A
1957	24,282	820	214,553	0.02961	0.26125	0.212	0.0558	1.0000	0.0296
1958	27,114	847	237,004	0.03201	0.27986	0.241	0.0593	1.0558	0.0303
1959	31,658	862	263,481	0.03672	0.30566	0.260	0.1065	1.1623	0.0329
1960	35,030	870	283,353	0.04026	0.32569	0.264	0.0708	1.2331	0.0330
1961	35,342	889	310,926	0.03975	0.34974	0.235	0.0034	1.2365	0.0308
Ave. rate	0.0467	0.0204	0.0972	0.0529	0.0756		0.0591		
1962	39,668	892	346,784	0.04447	0.38877	0.244	0.0815	1.2399	0.0343
1963	46,562	954	391,774	0.04880	0.41066	0.214	0.0775	1.3214	0.0354
1964	56,002	966	436,764	0.05797	0.45213	0.220	0.1379	1.3989	0.0398
1965	63,927	1,036	510,516	0.06170	0.49278	0.240	0.0407	1.5368	0.0387
1966	74,742	1,166	597,133	0.06410	0.51212	0.259	0.0276	1.5775	0.0392
Ave. rate	0.1569	0.0567	0.1096	0.1014	0.0795		0.0730		
1967	90,619	1,211	718,846	0.07482	0.59359	0.284	0.1044	1.6051	0.0451
1968	112,574	1,289	871,170	0.08733	0.67584	0.272	0.1101	1.7095	0.0495
1969	132,751	1,466	1,093,076	0.09055	0.74561	0.287	0.0086	1.8196	0.0483
1970	149,657	1,599	1,287,418	0.09359	0.80514	0.297	0.0106	1.8282	0.0497
1971	165,206	1,696	1,495,300	0.09741	0.88166	0.267	0.0160	1.8388	0.0514
Ave. rate	0.1730	0.0803	0.1719	0.0891	0.2401		0.0488		
1972	183,282	1,763	1,722,768	0.10396	0.97717	0.251	0.0385	1.8548	0.0542
1973	232,019	1,635	1,968,403	0.14190	1.20391	0.249	0.2205	1.8933	0.0750
1974	247,265	1,703	2,260,211	0.14519	1.32719	0.241	0.0002	2.1138	0.0721
1975	251,312	1,768	2,296,379	0.14214	1.29885	0.281	0.0015	2.1140	0.0710
Ave. rate	0.1144	0.0117	0.1147	0.1085	0.1053		0.0651		
Total ave. rate of growth.	0.1428	0.0423	0.1104	0.0879	0.1251		0.0610		

Notes: Column(4); column(1)/column(2), q=Y/L and column(5); column(3)/column(2), k=K/L.
Column(7), (8), and (9) are the calculated figures. See the calculation procedures in footnote#1 in section II.

TABLE TC-II. Fitting the Production Function in the Cobb-Douglas Type. (TRANSCOMMUNICATION)

Year	Regression Coefficients Labor(α)	Capital(β)	($\alpha+\beta$)	λ	R^2	D.W.	F
1957-75							
O.L.S.	0.0969	0.6077	0.7046	0.0487	0.99	1.17	1451.21
	(0.3216)	(0.2372)		(0.0225)			
I.L.S.	-1.7241	1.0021	-0.7220	0.0498	0.98	0.98	1242.11
	(0.8213)	(0.7092)		(0.0318)			
S.CRS.	0.4996	0.5004	1.00	0.0437	0.98	1.45	774.66
		(0.2517)		(0.0237)			
1957-66							
O.L.S.	-0.4893	1.7162	1.2269	-0.0522	0.98	1.77	251.69
	(1.0578)	(1.1875)		(0.0987)			
I.L.S.	-0.9425	2.9432	2.0005	-0.0582	0.98	0.80	262.43
	(1.4324)	(2.145)		(0.0424)			
S.CRS.	-0.3877	1.3877	1.00	-0.0190	0.97	1.61	218.78
		(0.9920)		(0.0768)			
1967-75							
O.L.S.	-0.6651	1.2319	0.5688	-0.0253	0.99	2.59	427.93
	(0.2358)	(0.2440)		(0.0295)			
I.L.S.	-0.8912	1.0512	0.1600	-0.0505	0.98	1.32	398.24
	(0.4217)	(0.7241)		(0.0552)			
S.CRS.	0.4453	0.5547	1.00	-0.0675	0.96	1.91	105.37
		(0.3859)		(0.0411)			

Note: The parentheses present the standard errors.

TABLE TC-III. ESTIMATES OF THE PARAMETERS OF THE CES PRODUCTION FUNCTION(TRANSCOMMUNICATION).

	ρ	$\sigma=\frac{1}{1+\rho}$	δ	v	λ	R^2	D.W.	F
1957-75								
Taylor Ex.	-0.0221	1.0226	0.5495	0.7117	-0.0248	0.99	1.30	1654.1
CES.	(0.0071)	(0.0070)	(0.1272)	(0.0088)	(0.0308)			
Two-Step	0.0199	0.9805	0.6709	0.7632	0.0567	0.99	1.36	2286.19
CES.	(0.0012)	(0.0010)		(0.1452)	(0.0163)			
1957-66								
Taylor Ex.	-0.0437	1.0457	0.5970	1.468	-0.0891	0.98	1.86	159.28
CES.	(0.0320)	(0.8420)	(0.2902)	(0.9111)	(0.1828)			
Two-Step	0.2051	0.8298	0.6497	1.0783	0.0556	0.98	1.77	359.31
CES.	(0.2090)	(0.3365)		(0.3752)	(0.0237)			
1967-75								
Taylor Ex.	0.0002	0.9998	0.4898	0.1784	0.0222	0.99	1.99	796.63
CES.	(0.0001)	(0.0001)	(0.1124)	(0.0528)	(0.0249)			
Two-Step	0.3754	0.7271	0.6150	0.3754	0.1124	0.96	1.87	129.54
CES.	(0.1066)	(0.0410)		(0.1268)	(0.0207)			

Note: The standard error is in parenthesis.

Taylor Ex.
CES.

$$\ln Y = \ln A + \lambda t + v\delta \ln K + v(1-\delta) \ln L - \frac{1}{2}\rho v \delta(1-\delta)(\ln K - \ln L)^2$$

Two-Step
CES.

$$(1).\ \ln (P_k/w) = \ln (\delta/1-\delta) - (\rho+1) \ln (K/L)$$

$$(2).\ \ln Y = \ln A + \lambda t - v/\hat{\rho}\ \ln\left[\hat{\delta} K^{-\hat{\rho}} + (1-\hat{\delta}) L^{-\hat{\rho}}\right]$$

TABLE MI-I. THE PARTIAL PRODUCTIVITY INDEXES AND THE Solow's RESIDUAL(MINING).

All the monetary terms are million Korean won at the 1970 constant price.

Year	Real Output (mil.won)	Labor employed 1,000 prs.	Real capital (mil.won)	Real output per man	Employed capital per man	Share of property in income	$\Delta A(t)/A(t)$	A(t)	q/A
1957	9,785	49	34,586.67	0.19969	0.70585	0.212	0.3404	1.0000	0.1996
1958	9,796	48	36,720.67	0.20408	0.76501	0.241	0.0026	1.3404	0.1522
1959	11,175	51	37,898.33	0.21961	0.74310	0.260	0.0762	1.3431	0.1631
1960	14,801	49	39,574.26	0.30206	0.80763	0.264	0.2534	1.4193	0.2128
1961	15,193	49	39,763.40	0.31006	0.81149	0.235	0.0088	1.6727	0.1853
Ave. rate.	0.1230	0.0007	0.0365	0.1249	0.0366		0.1587		
1962	19,351	47	41,062.77	0.41172	0.87365	0.244	0.2295	1.6815	0.2448
1963	20,107	50	42,947.70	0.40214	0.85895	0.214	-0.0201	1.9111	0.2104
1964	22,207	53	44,215.89	0.41900	0.83426	0.220	0.0696	1.8909	0.2215
1965	24,109	77	46,532.71	0.31313	0.60431	0.240	-0.2480	1.9603	0.1595
1966	24,747	80	49,763.22	0.30936	0.62204	0.259	-0.0199	1.7123	0.1806
Ave. rate	0.0981	0.1167	0.04598	0.0254	-0.0538		0.0022		
1967	27,254	94	53,315.64	0.28993	0.56718	0.284	-0.0393	1.6924	0.1713
1968	27,027	106	58,391.36	0.25497	0.55086	0.272	-0.0129	1.6531	0.1542
1969	26,562	114	62,677.00	0.23300	0.54944	0.287	-0.0942	1.6402	0.1420
1970	30,728	111	66,164.35	0.27682	0.59607	0.297	0.0142	1.5460	0.1785
1971	31,208	92	70,949.95	0.33921	0.77119	0.267	0.1252	1.5606	0.2174
Ave. rate	0.0454	0.0161	0.07358	0.0287	0.0472		-0.0010		
1972	31,210	54	70,911.95	0.57796	1.31318	0.251	0.3094	1.6854	0.3429
1973	36,849	47	76,550.95	0.78402	1.62874	0.249	0.2150	1.9948	0.3130
1974	38,965	50	84,358.99	0.77930	1.68711	0.241	-0.0014	2.2098	0.3526
1975	39,785	60	85,884.19	0.66308	1.43140	0.281	-0.1237	2.2083	0.3002
Ave. rate.	0.0468	-0.0462	0.04768	0.2263	0.2068		0.1001		
Ave. rate of the Growth(Total period)	0.0828	0.0173	0.0509	0.1013	0.0592		0.0671		

Notes: Column(4); column(1)/column(2) and column(5); column(3)/column(2).
Column(7),(8), and (9) are all the calculated figures. See the calculation procedures in footnote#1 in section II.

TABLE MI-II. Fitting the Production Function in the Cobb-Douglas Type. (MINING)

Year	Regression Coefficients Labor(α)	Capital(β)	(α+β)	λ	R^2	D.W.	F
1957-75							
O.L.S.	0.0822	1.1835	1.2657	0.0115	0.84	0.489	34.12
	(0.1425)	(0.1938)		(0.0113)			
I.L.S.	0.4721	0.4972	0.9693	0.0295	0.84	1.358	32.88
	(0.0324)	(0.0421)		(0.0091)			
S.CRS.	0.1862	0.8138	1.00	0.0286	0.90	0.460	88.71
	(0.0884)			(0.0074)			
1957-66							
O.L.S.	-0.241	-1.526	-1.77	0.1836	0.96	2.97	89.68
	(0.209)	(1.69)		(0.0560)			
I.L.S.	0.1321	-0.7024	-0.5703	0.0459	0.99	2.53	2057.33
	(0.4212)	(0.5428)		(0.0072)			
S.CRS.	-0.4418	1.4418	1.00	0.0856	0.93	2.41	60.85
	(0.2004)			(0.0085)			
1967-75							
O.L.S.	-0.1094	0.7334	0.6240	-0.0007	0.86	2.108	17.76
	(0.0855)	(0.1956)		(0.0039)			
I.L.S.	-0.0332	0.0574	0.0242	0.0002	0.72	1.46	7.96
	(0.0041)	(0.5472)		(0.0056)			
S.CRS.	-0.0886	1.0886	1.00	-0.0124	0.99	1.79	552.76
	(0.0792)			(0.0129)			

Note: The parentheses present the associated standard errors.

TABLE MI-III. ESTIMATES OF THE PARAMETERS OF THE CES PRODUCTION FUNCTION(MINING)

	ρ	$\sigma=\frac{1}{1+\rho}$	δ	v	λ	R^2	D.W.	F
1957-75								
Taylor Ex.	0.1694	0.8551	0.8634	1.1914	0.0129	0.83	0.492	24.0
CES.	(0.0923)	(0.0428)	(0.7240)	(0.0129)	(0.0094)			
Two-Step	-0.8449	6.451	0.3745	1.1894	0.0137	0.85	0.515	53.46
CES.	(0.0048)	(0.2)		(0.0095)	(0.0102)			
1957-66								
Taylor Ex.	0.0045	0.995	0.5026	-0.73	0.1268	0.97	2.49	89.63
CES.	(0.0037)	(0.942)			(0.0589)			
Two-Step	0.2450	0.8031	0.401	0.5093	0.1392	0.96	2.76	141.41
CES.	(0.1930)	(0.0185)		(0.0834)	(0.01204)			
1967-75								
Taylor Ex.	0.075	0.9302	0.401	0.6636	0.0007	0.84	2.07	11.45
CES.	(0.062)	(0.0482)	(0.092)	(0.078)	(0.0051)			
Two-Step	-0.850	6.471	0.3138	-10.99	0.0004	0.29	0.973	2.71
CES.					(0.0089)			

Note: The standard error is in parenthesis.

Taylor Ex. CES.

$$\ln Y = \ln A + \lambda t + v\delta \ln K + v(1-\delta) \ln L - \frac{1}{2}\rho v\delta(1-\delta)(\ln K - \ln L)^2$$

Two-Step CES.

(1). $\ln (P_k/w) = \ln (\delta/1-\delta) - (\rho+1) \ln (K/L)$

(2). $\ln Y = \ln A + \lambda t - v/\hat{\rho} \ln\left[\hat{\delta} K^{-\hat{\rho}} + (1-\hat{\delta}) L^{-\hat{\rho}}\right]$

TABLE IV. TESTING FOR EQUALITY BETWEEN SETS OF COEFFICIENT IN TWO DIFFERENT REGRESSION (1957-66 and 1967-75) CHOW TEST

	1957-75 S_1	1957-66 S_2	1967-75 S_3	$S_4=S_2+S_3$	$S_5=S_1-S_4$	F	F(0.05) F(0.01)
O.L.S.							
Manufacturing	0.0381	0.00578	0.00045	0.00623	0.03183	22.15	3.34
Agriculture	0.0275	0.01089	0.00378	0.01467	0.01291	3.36	5.56
Construction	0.3571	0.02920	0.01150	0.04070	0.31630	36.23	
Transcommuni.	0.0394	0.00970	0.00400	0.01370	0.02570	7.81	
Mining	0.4512	0.02580	0.01690	0.04280	0.40840	112.81	
S.CRS.							
Manufacturing	1.3187	0.00628	0.00252	0.00880	1.30990	1129.13	3.68
Agriculture	0.04759	0.01234	0.00545	0.01778	0.02981	7.26	6.36
Construction	0.3126	0.02850	0.03840	0.06690	0.24560	27.53	
Transcommuni.	0.0485	0.01030	0.01320	0.02350	0.02500	7.81	
Mining	0.3811	0.03880	0.02070	0.05860	0.32140	42.82	
Taylor Ex. CES.							
Manufacturing	0.0162	0.00417	0.01980	0.02390	-0.00760	-0.88	3.66
Agriculture	0.0275	0.01058	0.00376	0.01434	0.01313	3.96	5.67
Construction	0.3095	0.02960	0.08810	0.11770	0.19180	4.48	
Transcommuni.	0.0242	0.00960	0.00130	0.01080	0.01330	3.30*	
Mining	0.4489	0.01623	0.01570	0.03180	0.41710	36.05	
Two-Step CES.							
Manufacturing	0.0420	0.00594	0.00049	0.00643	0.04060	21.05	3.68
Agriculture	0.0286	0.01148	0.00441	0.01589	0.01271	3.63*	6.36
Construction	0.3589	0.02340	0.01140	0.03490	0.32410	69.84	
Transcommuni.	0.0400	0.01190	0.02330	0.03510	0.00480	1.04*	
Mining	0.4596	0.02830	0.10380	0.13210	0.32750	18.60	

Notes: S_i is sums of squares residuals.
The F value with star(*) means that there is not significant difference between two different epoch periods regressions at the 95 per cent level.

Here, $$F = \frac{S_5/k}{S_4/(N_1 + N_2 - 2k)}$$ with degree of freedom of k and $(N_1 + N_2 - 2k)$.

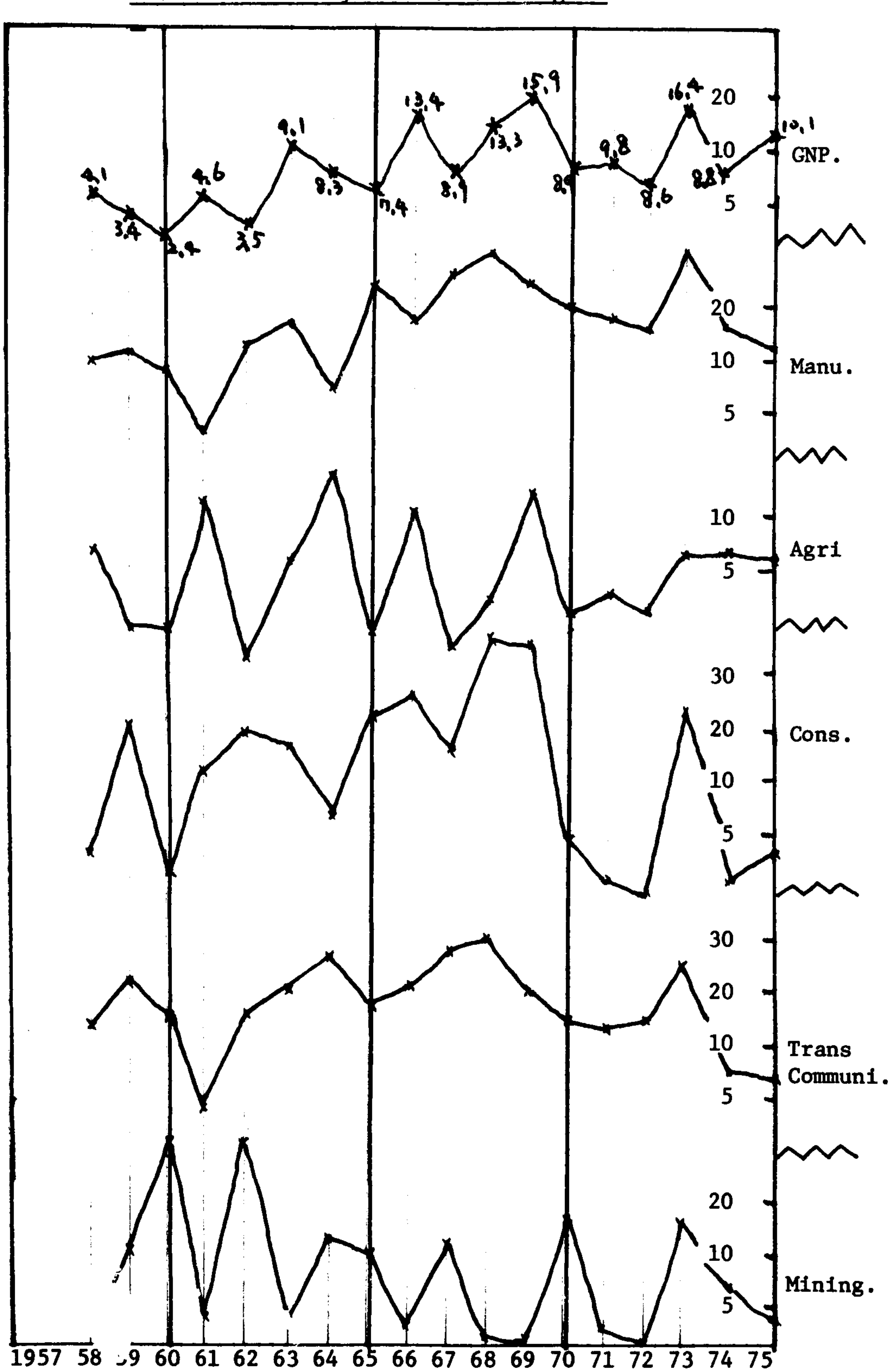
If, $F > F_a$ for significance level a, we reject the hypothesis that the parameters are the same in two sets of observations. Most of cases in this study reject null-hypothesis.

TABLE V. RATE OF GROWTH(%) BY INDUSTRIAL ORIGIN.

Source: Figures are calculated from table I

	1958	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	AVE.
Manu.	8.8	9.2	8.1	3.1	13.1	17.5	6.6	20.3	16.9	22.8	27.1	21.5	18.4	17.7	15.6	30.9	17.5	12.5	15.8
Agri.	6.2	-1.2	-1.2	12.0	-5.7	8.1	15.4	-1.9	10.8	-4.9	2.5	12.4	0.9	3.3	1.6	5.5	5.6	5.3	4.3
Cons.	1.3	19.7	-0.1	12.1	16.1	15.4	8.2	22.1	25.5	13.4	41.9	39.9	4.5	0.2	-1.4	21.7	0.8	4.8	12.9
Trans Comm.	11.6	16.7	10.6	0.8	12.2	17.3	20.2	14.1	16.9	21.2	24.2	17.9	12.7	10.3	10.9	26.5	6.5	1.6	14.2
Mini.	0.1	14.0	32.4	2.6	27.3	3.9	10.4	8.5	2.6	10.1	-0.8	-1.7	15.6	1.5	0.1	18.0	5.7	2.1	8.2

FIGURE III.
Rate of Growth by Industrial Origin.



APPENDIX B.

Estimation of Parameters and Standard Errors in ILS.

A. For Parameters.

From the OLS regression, we have,

$$\ln Y = \ln A + \alpha \ln K + \beta \ln L \tag{A}$$

By deducing $\ln Y(\alpha + \beta)$ from both sides of equation (A), and deviding through by $(1 - \alpha - \beta)$, we obtain,

$$\frac{\ln Y - (\alpha+\beta)\ln Y}{(1-\alpha-\beta)} = \frac{\ln A}{(1-\alpha-\beta)} + \frac{\alpha}{(1-\alpha-\beta)} \ln K + \frac{\beta}{(1-\alpha-\beta)} \ln L - \frac{(\alpha+\beta)}{(1-\alpha-\beta)} \ln Y$$

$$\ln Y = \frac{\ln A}{(1-\alpha-\beta)} + \frac{\alpha}{(1-\alpha-\beta)} \ln K + \frac{\beta}{(1-\alpha-\beta)} \ln L - \frac{\alpha}{(1-\alpha-\beta)} \ln Y - \frac{\beta}{(1-\alpha-\beta)} \ln Y$$

Therefore we have,

$$\ln Y = \frac{\ln A}{(1-\alpha-\beta)} + \frac{\alpha}{(1-\alpha-\beta)} (\ln K - \ln Y) + \frac{\beta}{(1-\alpha-\beta)} (\ln L - \ln Y) \tag{B}$$

$$\therefore \ln Y = a + b_1(\ln K - \ln Y) + b_2(\ln L - \ln Y) \tag{C}$$

From equations (B) and (C), we have,

$$b_1 = \alpha/(1-\alpha-\beta) \tag{1}$$

$$b_2 = \beta/(1-\alpha-\beta) \tag{2}$$

Using simultaneous equation solution from equations (1) and (2) we have;

$$b_1 - (b_1 + 1)\alpha - b_1\beta = 0 \qquad (1)'$$

$$b_2 - b_2\alpha - (b_2 + 1)\beta = 0 \qquad (2)'$$

Multiply equation (1)' by b_2 and equation (2)' by $(b_1 + 1)$, we have,

$$b_1b_2 - b_2(b_1 + 1)\alpha - b_1b_2\beta = 0 \qquad (1)''$$

$$b_2(b_1 + 1) - b_2(b_1 + 1)\alpha - (b_1 + 1)(b_2 + 1)\beta = 0 \qquad (2)''$$

By deducing (2)" from (1)" we have,

$$b_1b_2 - b_2(b_1 + 1) - b_1b_2\beta + (b_1 + 1)(b_2 + 1)\beta = 0$$

$$b_1b_2 - b_2b_1 - b_2 - \left\{b_1b_2 - (b_1 + 1)(b_2 + 1)\right\}\beta = 0$$

$$- b_2 - (- b_1 - b_2 - 1)\beta = 0$$

$$\therefore \beta = \frac{b_2}{1 + b_1 + b_2}$$

Similarly, multiply equation (1)' by $(b_2 + 1)$ and equation (2)' by b_1 we have,

$$b_1(b_2 + 1) - (b_1 + 1)(b_2 + 1)\alpha - b_1(b_2 + 1)\beta = 0 \qquad (1)''$$

$$b_1b_2 - b_1b_2\alpha - (b_2 + 1)b_1\beta = 0 \qquad (2)''$$

By deducing (2)" from equation (1)" we have,

$$b_1(b_2 + 1) - b_1b_2 - (b_1 + 1)(b_2 + 1)\alpha + b_1b_2\alpha = 0$$

$$b_1b_2 + b_1 - b_1b_2 + \left\{b_1b_2 - (b_1 + 1)(b_2 + 1)\right\}\alpha = 0$$

$$b_1 + \left\{b_1b_2 - b_1b_2 - b_1 - b_2 - 1\right\}\alpha = 0$$

$$b_1 - \left\{b_1 + b_2 + 1\right\}\alpha = 0$$

$$\therefore \alpha = \frac{b_1}{1 + b_1 + b_2}$$

B. For Standard Errors.

From the results of simultaneous equation we have,

$$\beta = \frac{b_2}{1 + b_1 + b_2}$$

$$\text{Var}(\beta) = (\partial f/\partial b_1)^2 \text{var}(b_1) + (\partial f/\partial b_2)^2 \text{var}(b_2) + 2 (\partial f/\partial b_1)(\partial f/\partial b_2) \text{cov}(b_1 b_2) \quad \ldots\ldots\text{(A)}$$

Here,

$$\frac{\partial f}{\partial b_1} = \frac{(1 + b_1 + b_2)\partial b_2/\partial b_1 - b_2 \partial/\partial b_1 \ (1 + b_1 + b_2)}{(1 + b_1 + b_2)^2}$$

$$= -b_2/(1 + b_1 + b_2)^2$$

$$\frac{\partial f}{\partial b_2} = \frac{(1 + b_1 + b_2)\partial b_2/\partial b_2 - b_2 \partial/\partial b_2 \ (1 + b_1 + b_2)}{(1 + b_1 + b_2)^2}$$

$$= (1 + b_1)/(1 + b_1 + b_2)^2$$

Therefore substitute $(\partial f/\partial b_1)$ and $(\partial f/\partial b_2)$ into the equation (A), we have;

$$\text{var}(\beta) = \left[\frac{-b_2}{(1 + b_1 + b_2)^2}\right]^2 \text{var}(b_1) + \left[\frac{(1+b_1)}{(1 + b_1 + b_2)^2}\right]^2 \text{var}(b_2)$$

$$+ 2 \left[\frac{-b_2}{(1 + b_1 + b_2)^2}\right] \left[\frac{(1 + b_1)}{(1 + b_1 + b_2)^2}\right] \text{cov}(b_1 b_2)$$

Similarly, from the results of the simultaneous equation, we have;

$$\alpha = b_1/(1 + b_1 + b_2)$$

$$\text{var}(\alpha) = (\partial f/\partial b_1)^2 \text{var}(b_1) + (\partial f/\partial b_2)^2 \text{var}(b_2)$$
$$+ 2\,(\partial f/\partial b_1)(\partial f/\partial b_2)\ \text{cov}(b_1 b_2) \qquad \text{(B)}$$

Here,

$$\frac{\partial f}{\partial b_1} = \frac{(1 + b_1 + b_2)\,\partial b_1/\partial b_1 - b_1\,\partial/\partial b_1\ (1 + b_1 + b_2)}{(1 + b_1 + b_2)^2}$$

$$= \frac{(1 + b_1 + b_2 - b_1)}{(1 + b_1 + b_2)^2} = \frac{(1 + b_2)}{(1 + b_1 + b_2)^2}$$

$$\frac{\partial f}{\partial b_2} = \frac{(1 + b_1 + b_2)\,\partial b_1/\partial b_2 - b_1\,\partial/\partial b_2\ (1 + b_1 + b_2)}{(1 + b_1 + b_2)^2}$$

$$= \frac{-b_1}{(1 + b_1 + b_2)^2}$$

Therefore substitute $(\partial f/\partial b_1)$ and $(\partial f/\partial b_2)$ into equation (B), we have;

$$\text{var}\ (\alpha) = \left[\frac{1 + b_2}{(1 + b_1 + b_2)^2}\right]^2 \text{var}(b_1) + \left[\frac{-b_1}{(1 + b_1 + b_2)^2}\right]^2 \text{var}(b_2)$$

$$+ 2\left[\frac{(1 + b_2)}{(1 + b_1 + b_2)^2}\right]\left[\frac{-b_1}{(1 + b_1 + b_2)^2}\right] \text{cov}(b_1 b_2)$$

Note:

Kmenta, "Elements of Ecnometrics." The MacMillan Company, New York, 1971, p.444 for the approximation formula (A) and (B) of standard errors of the estimator, say α, is a function of k other estimators such as b_1 and b_2 in our case.

C. The Method of Calculation of Standard Errors in Taylor Ex.CES.

The following formula refers to the general case where an estimator, say a, is a function of k other estimators such as b_1, b_2.. ..... b_k.

$$a = f(b_1, b_2, \ldots . b_k)$$

Then, the sample variance of a can be approximated as;

$$Var(a) \approx \sum_{K} \left[\partial f/\partial b_k\right] var(b_k) + 2\sum_{j<K} \left[\partial f/\partial b_j\right]\left[\partial f/\partial b_k\right] \; cov(b_j b_k)$$

$$(j,k = 1,2,\ldots k)$$

From the results of Taylor Ex. CES, show us;

$$\delta = \frac{b_2}{b_2 + b_3}$$

$$Var(\delta) = (\partial f/\partial b_2)^2 \, var(b_2) + (\partial f/\partial b_3)^2 \, var(b_3) + 2(\partial f/\partial b_2)(\partial f/\partial b_3) \; cov(b_2 b_3)$$

Here,

$$\frac{\partial f}{\partial b_2} = \frac{(b_2 + b_3)\partial b_2/\partial b_2 - b_2 \; \partial/\partial b_2 \; (b_2 + b_3)}{(b_2 + b_3)^2}$$

$$= \frac{(b_2 + b_3 - b_2)}{(b_2 + b_3)^2} = \frac{b_3}{(b_2 + b_3)^2}$$

$$\frac{\partial f}{\partial b_3} = \frac{(b_2 + b_3)\partial b_2/\partial b_3 - b_2 \; \partial/\partial b_3 \; (b_2 + b_3)}{(b_2 + b_3)^2}$$

$$= \frac{-b_2}{(b_2 + b_3)^2}$$

$$\therefore Var(\delta) = \left[b_3/(b_2 + b_3)^2\right]^2 var(b_2) + \left[-b_2/(b_2 + b_3)^2\right]^2 var(b_3) + 2\left[b_3/(b_2+b_3)^2\right]\left[-b_2/(b_2+b_3)^2\right] cov(b_2 b_3)$$

From the regression results, we have;

$$v = b_2 + b_3$$

$$Var(v) = (\partial v/\partial b_2)^2 \ var(b_2) + (\partial v/\partial b_3)^2 \ var(b_3) + 2\ (\partial v/\partial b_2)(\partial v/\partial b_3)\ cov(b_2 b_3)$$

$$\therefore Var(v) = var(b_2) + var(b_3) + 2\ cov(b_2 b_3)$$

From the regression results, we also have;

$$\rho = \frac{-2\ b_4(b_2 + b_3)}{b_2 b_3}$$

$$Var(\rho) = (\partial f/\partial b_2)^2 \ var(b_2) + (\partial f/\partial b_3)^2 \ var(b_3) + (\partial f/\partial b_4)^2 \ var(b_4) + 2\ (\partial f/\partial b_2)(\partial f/\partial b_3)\ cov(b_2 b_3) + 2\ (\partial f/\partial b_2)(\partial f/\partial b_4)\ cov(b_2 b_4) + 2\ (\partial f/\partial b_3)(\partial f/\partial b_4)\ cov(b_3 b_4) \ldots\ldots (A)$$

Here,

$$\frac{\partial f}{\partial b_2} = \frac{b_2 b_3(-2b_4) - b_3(-2b_4)(b_2 + b_3)}{(b_2 b_3)^2}$$

$$= \frac{-2b_2 b_3 b_4 + 2b_2 b_3 b_4 + 2\ b_3^2 b_4}{(b_2 b_3)^2} = \frac{2\ b_4}{b_2^2}$$

$$\frac{\partial f}{\partial b_3} = \frac{b_2 b_3(-2\ b_4) - b_2 \left[-2\ b_4(b_2 + b_3)\right]}{(b_2 b_3)^2}$$

$$= \frac{-2\ b_2 b_3 b_4 + 2\ b_2\ b_4 + 2\ b_2 b_3 b_4}{(b_2 b_3)^2} = \frac{2b_4}{b_3^2}$$

$$\frac{\partial f}{\partial b_4} = \frac{-2(b_2 + b_3)}{b_2 b_3}$$

Substitute $(\partial f/\partial b_2)$, $(\partial f/\partial b_3)$,and $(\partial f/\partial b_4)$ into the equation (A), we have;

$$\mathrm{Var}(\rho) = \left[\frac{2\,b_4}{b_2^2}\right]^2 \mathrm{var}(b_2) + \left[\frac{2\,b_4}{b_3^2}\right]^2 \mathrm{var}(b_3)$$

$$+ \left[\frac{-2(b_2 + b_3)}{b_2 b_3}\right]^2 \mathrm{var}(b_4) + 2\left[\frac{2\,b_4}{b_2^2}\right]\left[\frac{2\,b_4}{b_3^2}\right] \mathrm{cov}(b_2 b_3)$$

$$+ 2\left[\frac{2\,b_4}{b_2^2}\right]\left[\frac{-2(b_2 + b_3)}{b_2 b_3}\right] \mathrm{cov}(b_2 b_4) + 2\left[\frac{2\,b_4}{b_3^2}\right]$$

$$\left[\frac{-2\,(b_2 + b_3)}{b_2 b_3}\right] \mathrm{cov}(b_3 b_4)$$

Note:

Notation b in this appendix is equivalent to the notation β in theoretical part and notation a is equivalent to α.

Appendix C.

Properties of the CES Production Function.(in the case of σ = 1 and σ = 0)

$$Y = A\left[\delta K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-1/\rho} \qquad (A)$$

A change in the parameter A changes the output for any given set of inputs in the same proportion. It will therefore be referred to as the (neutral) efficiency parameter. The parameter ρ is a transform of the elasticity of substitution ($1/1+\rho = \sigma$) and will be termed the substitution parameter. It will be seen below that for any given value of σ (equivalently, for any given value of ρ), the functional distribution of income is determined by δ, the distribution parameter.

The lowest admissible value for ρ is -1; this implies an infinite elasticity of substitution and therefore straight line isoquants. One varifies this by putting ρ = -1 in equation (A). For values of ρ between -1 and zero we have elasticities of substitution greater than unity. The case of ρ = 0 yields an elas ticity of substitution of unity and should, therefore, lead back to the Cobb-Douglas function. This can be seen by direct application of L'HOPITAL's rule to equation (A).

L'HOPITAL's rule which states that, if,

$$\lim_{Z \to b} h(Z) = 0 \quad \text{and} \quad \lim_{Z \to b} g(Z) = 0$$

and if

$$\lim_{z \to b} \frac{h'(Z)}{g'(Z)} = \delta$$

then

$$\lim_{z \to b} \frac{h(Z)}{g(Z)} = \delta$$

write the natural logarithm of equation (A) as the quotient of two functions of ρ :

$$\ln Y - \ln A = \frac{-\ln\left[\delta K^{-\rho} + (1-\delta)L^{-\rho}\right]}{\rho}$$

$$= \frac{h(\rho)}{g(\rho)}$$

where $h(\rho) \to 0$ and $g(\rho) \to 0$ as $\rho \to 0$

taking the derivative of the numerator

$$h'(\rho) = \frac{\delta K^{-\rho} \ln K + (1-\delta)L^{-\rho} \ln L}{\delta K^{-\rho} + (1-\delta) L^{-\rho}}$$

which converges to $\delta \ln K + (1-\delta)\ln L$ as $\rho \to 0$

Finally, L'HOPITAL's rule the limiting case is

$$\ln Y - \ln A = \delta \ln K + (1-\delta) \ln L$$

and $Y = A K^{\delta} L^{1-\delta}$ which is the Cobb-Douglas function at $\rho = 0$.

This special case reinforces singling-out of δ as a distribution parameter.

For $0 < \rho < \infty$, which is empirically interesting case, we have

$\sigma < 1$. Whenever $\rho > -1$, the isoquants have the right curvature ($\rho = -1$ is the case of straight line isoquants, and $\rho < -1$ is ruled out precisely because the isoquants have the wrong curvature). And as $\rho \to \infty$, the elasticity of substitution tends to zero and we approaches the case of fixed proportions. We may prove this by making the appropriate limiting process on equation (A).[1]

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1. G.H. Hardy, J.E. Littlewood, and G. Polya. "Inequality" Cambridge, England. 1934

   They proved this by their theorem 4.
   The general theory of mean values assures us that as a mean value of order $-\infty$ we have

$$\lim_{\rho \to \infty} A\left[\delta K^{-\rho} + (1-\delta)L^{-\rho}\right]^{-1/\rho}$$

$$= A \min.(K,L)$$

$$= \min.\left[\frac{K}{A^{-1}}, \frac{L}{A^{-1}}\right]$$

   This represents a system of right angled isoquants with coners lying on a 45° line from the origin.

Note. See for the L'HOPITAL's rule, W. Rudin. "Principle of mathematical analysis" New York McGraw-Hill, 1953 pp.82--83.

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