Pricing of Credit-Risk Derivatives: A numerical implementation

by

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Abstract

Pricing Credit-Risk Derivatives: A numerical implementation.

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Based on expanding the Heath-Jarrow-Morton term-structure model to allow for defaultable debt, this study examines the implementation of the Das and Sundaram (1999) framework for modeling risky debt and valuing credit-risk derivatives on a simple portfolio of risky debt. The flexible framework of the Das-Sundaram model allows implementation, based on observables, to the maximum extent possible, while working directly with the evolution of spreads; rather than following the procedure of implying out the behavior of spreads from assumptions concerning the default process. This implementation uses Garbade (1988) value of a basis point-weighted average of the spreads with the industry practice of using the value-weighted averages of a position in the portfolio. The study takes advantage of the recursive representation of the risk-neutral drifts in the Das-Sundaram framework that facilitates implementation and makes it possible to handle path-dependence and early exercise features without difficulty. Further more, a simple statistical test of whether default rates follow a Markov process, finds that they do not. This result has implications for work that typically assumes that default rates follow a Markov process.

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My first-reader, discussant... buddy and wife, Assistant Professor (tenured) Dr. Chevy Alford...no words!

The dissertation is dedicated to Norman, from whom I inherited a love of life and a (Bradshaw) perspective on the world; and Ucilla, 'mummy', from whom I inherited perseverance and diligence.

> Your enviroment does not matter. Everything starts with you. You must forge yourself through your own efforts. Iurge each of you to create something, start something and make a success of something. That is the essence of human existence, the challenge of youth. Herein lies a wonderful way of life always aiming for the future (Ikeda, 1999, p. 43).
>
> *For Today & Tomorrow: Daily Encouragement* by Daisaku Ikeda (1999). World Tribune Press: Santa Monica, CA.

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CHAPTER I.

INTRODUCTION

This research examines the issues involved in the numerical implementation of the Das-Sundaram (1999) arbitrage-free, discrete-time model, based on the Heath-Jarrow-Morton (HJM) (1992) framework, for value credit-risk derivative instruments on a small portfolio of bonds. The Das-Sundaram framework is flexible and simple to implement, and is to the maximum extent possible, based on observables. The approach used in this research works directly with forward rates, forward spreads and their volatilities observed in the market. By modeling observed default probabilities of the various bonds, the portfolio forward spreads, and volatilities, are derived. A non-recombining, arbitrage-free tree is then generated.

A crucial input to any model to price credit-risky debt is default rates data. Many financial models are assumed to have the Markov property (Wilmott, 1998, p. 57). Hence, a simple test is performed to determine if default rates follow a Markov process.

The study is organized as follows. The rest of this chapter discusses credit-risk derivatives. Chapter II provides a short background on finance theory. The literature review on credit derivative models and fixed income securities markets follow in chapter III. The Das-Sundaram (1999) discrete-time model is described in chapter IV. Implementation of the model for a simple portfolio of bonds is described in chapter V. Chapter VI describes Moody's default rate data and describes a simple test to determine whether default rates follow a Markov process. The statistical results of the simple test, follows in chapter VII and, the study's conclusions and recommendations for further study are presented in chapter VIII.

A. Credit-Risk Derivatives.

Credit derivatives are revolutionizing the financial industry (Tavakoli, 1998, p.1). Neal (1996) states that estimates from industry sources suggest that the credit derivatives market has grown from virtually nothing in 1994 to about $20 billion of transactions in 1995. Tavakoli (1998) puts the global estimates at $100 to $200 billion at the end of 1996. This growth has been driven by the ability of credit derivatives to provide valuable new methods for managing credit risk—the risk that a borrower may default. *Credit-risk* derivatives are financial contracts that provide insurance against credit-risk-related losses. Neal (1996), Das (1997), Reoch (1997) and Tavakoli (1998) provide information on the rationale for, and use of, credit derivatives and show how credit derivatives can help manage credit risk.

Credit risk is the probability that a borrower will default. Default occurs when the borrower can not fulfill key financial obligations i.e., make interest payments to bond investors or repay bank loans. If a firm defaults, neither banks nor investors will receive their promised payments. Das (1997) and Neal (1996) suggest that the various methods for managing credit risk have typically been insufficient to reduce credit risk to levels lenders are willing to hold.

Credit risk affects any party making or receiving a loan or debt payment. These include bond issuers, bond investors and commercial banks. Traditional methods of managing credit risk include loan underwriting standards and diversification. After careful review of the prospective borrower's financial statements, and the condition of the borrower's industry, the bank underwrites the loan, upon a favorable review. The bank would manage its credit risk exposure by (i) controlling the terms of the loan, (ii) setting limits on the size of the loan, (iii) establishing a repayment schedule, and (iv) requiring additional collateral for higher risk loans. The next step in the traditional approach is diversification.

Diversification is based on the premise of offsetting risks. The earnings from some loans will offset the losses from defaulted loans, thereby reducing the likelihood that, on net, that the bank will lose money. While diversification and underwriting standards are necessary first steps for managing credit risk, their ability to reduce credit risk is often limited by scarcity of diversification opportunities (Neal, 1996). An alternative approach, developed over the last ten years, has focused on selling assets with credit risk.

Securitization and loan sales are additional methods for managing credit-risky portfolios. In the asset securitization approach, bonds or loans with credit risk are pooled and sold to outside investors. From an investor's perspective, purchasing a part of a package is attractive because the diversification across many loans reduces the overall credit risk. In addition, to the extent that returns from the package are not closely correlated with the investor's other holdings, diversification allows the investor to reduce the credit risk of the overall portfolio. From the seller's perspective, selling the loan eliminates the bank's credit risk exposure to the loans.

Unfortunately, the securitization approach is only suited for loans that have standardized payment schedules and similar credit risk characteristics, such as home mortgages, automobile loans and now, credit cards. Loans for commercial and industrial purposes, in contrast, have diverse credit risks. Consequently, it is difficult for banks to securitize these loans or sell them to institutional investors.

In cases such as these, a more promising way to manage the credit risk is through the third method of managing credit risk, credit derivatives. Credit derivatives are financial contracts that provide insurance against credit-related losses. Three basic structures of credit derivatives, used to hedge risk, are credit default swaps, total rate of return swaps, and credit-spread options.

B. The structures and applications of credit-risk derivatives

B.1.a Credit (default) swaps

Credit (default) swaps are used for capital management, investment management and risk management. They are financial contracts in which the protection buyer pays, either up-front or periodically, a fee expressed in basis point per annum on the notional amount of the credit swap, in return for a contingent payment by the protection seller following a credit event with respect to the credit entity. The relevant obligations, the definitions of a credit event, and the settlement mechanism used to determine the contingent payment are flexible and determined by negotiation between the counterparties at the inception of the transaction.

The relevant obligation can be a loan, bond, sovereign risk arising from cross-border commercial transactions, or even credit exposure due to a derivative contract, such as counterparty credit exposure in a cross-currency swap transaction. Credit protection can be linked to an individual credit, or to a basket of credits.

A credit event can be defined in various ways, but typically it signifies a major deterioration in credit quality of the reference entity. This might include some or all of the following:

(i) failure to meet payment obligations when due

(ii) bankruptcy (for non-sovereign entities) or moratorium (for sovereign entities only)

(iii) repudiation

(iv) restructuring of debt obligations that is materially detrimental to the debt holder

(v) credit rating downgrades

A cash payment, or physical settlement, can effect the contingent payment. Cash settlement is designed to mirror the loss incurred by creditors of the reference entity following a credit event. This payment is calculated as the fall in price of the reference obligation below par at some pre-designated point in time after the credit event. As an alternative, counterparties can fix the contingent payment as a pre-determined sum.

Finally, the protection buyer makes physical delivery of a portfolio of specified delivery obligations in return for payment of their face amount. Deliverable obligations may be the reference obligation, or one or more of a broad class of obligations meeting certain specifications, such as any senior unsecured claim against the reference entity. The physical settlement option is not always available since credit swaps are often used to hedge exposures to assets that are not readily transferable, or create short positions for users who do not own a deliverable obligation.

B.1.b. Credit swaps to address illiquidity

Credit swaps can be used to address *illiquidity*. Any number of factors, both internal and external to the organization can cause illiquidity of credit positions in question. Internally, in the case of bank loans and derivative transactions, relationship concerns often lock portfolio managers into credit exposure arising from key client transactions. Corporate borrowers prefer to deal with smaller lending groups and typically place restrictions on transferability and on which entities can have access to that group. Credit derivatives allow users to reduce credit exposure without physically removing assets from their balance sheet. Loan sales or, the assignment or unwinding of derivative contracts, typically require the notification and/or the consent of the customer. In contrast, a credit derivative is a confidential transaction that the customer need neither be party to, nor aware of, thereby separating relationship management from risk management decisions.

Similarly, the tax or accounting position of an institution can create significant disincentives to the sale of an otherwise liquid position. Purchasing default protection via a credit swap can hedge the credit exposure of such positions without triggering a sale for either tax or accounting purposes. Lately, credit swaps have been employed in such situations to avoid unintended adverse tax or accounting consequences of otherwise sound risk management decisions (Financial Engineering Ltd., 1999, p. 17).

More often, illiquidity results from factors external to the institution in question. The secondary market for many loans and private placements is not deep, and in the case of certain forms of trade receivables or insurance contracts, may not exist at all. Some forms of credit exposure, such as the business concentration risk to key customers faced by many corporates—not only the default risk on receivables, but also the risk of customer replacement cost, or the exposure employees face to their employers in respect to non-qualified deferred compensation, are simply not transferable at all. In all these cases, credit swaps can provide a hedge of exposure that would otherwise be achievable through the sale of an underlying asset.

B.1.c. Credit swaps to exploit a funding advantage or avoid a disadvantage

When an investor owns a credit-risky asset, the return for assuming that credit risk is only the net spread earned after deducting that investor's cost of funding the asset on its balance sheet. Therefore, it makes little sense for an A-rated bank funding at Libor[1] flat to lend to an AAA-rated entity that borrows at Libid[2]. After funding cost, the A-rated bank takes a loss but still takes on risk. Consequently, entities with high funding levels often buy risky assets to generate income. Moreover, since there is little or no up-front principal outlay required for most protection sellers when assuming a credit swap position, these provide an opportunity to take on credit exposure in off-balance sheet positions that do not need to be funded. Credit swaps are therefore fast becoming an important source of investment opportunity and portfolio diversification for banks, insurance companies, and other institutional investors who would otherwise continue to accumulate concentrations of lower quality assets due to their own high funding costs.

[1] Libor: London Interbank Offer Rate.

[2] Libid: London Interbank Bid Rate.

On the other hand, institutions with low funding costs may capitalize on this advantage by funding assets on the balance sheet and purchasing default protection on those assets. The premium for buying default protection on such assets may be less than the net spread such an institution would earn over it funding costs. Therefore a low-cost investor may offset the risk of the underlying credit but still retain a net positive income stream. Of course, the counterparty risk to the protection seller must be covered by this residual income. However, the combined credit quality of the underlying asset and the credit protection purchased, even from a lower-quality counterparty, may often be very high--since two defaults must occur before losses are incurred. Even then, losses will be mitigated by recovery rate on claims against both entities (Financial Engineering Ltd., 1999; Tavakoli, 1998).

B.2.a Total (rate of) return swaps (TROR)

Total (rate of) return swaps are also designed to transfer credit risk between parties. However, a total return swap is different from a credit swap in that it allows the exchange of the total economic performance of a specified asset for the cash flow from another asset. So, payments between the counterparties involved in a TROR swap are based on the changes in the market value of a specific credit instrument, irrespective of whether a credit event has occurred.

In particular, the TROR payer pays the TROR receiver the total return on the reference obligation. Total return includes the sum of interest, fees, and any change-in-value payment equal to any appreciation or depreciation in market value of the reference obligation. Change in value is usually determined on the basis of a poll of reference dealers. A net depreciation in value results in a payment to the TROR payer.

Change-in-value payments may be made at maturity or on a periodic interim basis. An alternative to cash settlement of the change-in-value payment is that TROR swaps can allow for physical delivery of the reference obligation on maturity. The TROR payer physically delivers the reference obligation in return for a payment of the reference obligation" initial value by the TROR receiver. Maturity of the TROR swap is not required to match that of the reference obligation, and in practice rarely does. In return, the TRORS receiver typically makes regular floating payment of Libor plus a spread.

B.2.b. Synthetic financing: motivation for using TROR swaps

By entering into a TROR swap on an asset residing in its portfolio, the TROR payer has effectively removed all economic exposure to the underlying asset. This risk transfer is effected with confidentiality, and without the need for a cash sale. Typically, the TROR payer retains the servicing and voting rights to the underlying asset, although occasionally certain rights may be passed through to the TROR receiver under the terms of the swap (Financial Engineering Ltd., 1999, p. 20). The TROR receiver has exposure to the underlying asset without the initial cash outlay required to purchase it.

The key determinant of pricing the spread on a TROR swap is the cost to the TROR payer of financing and servicing the reference obligation on its own balance sheet; which has, in effect, been lent to the TROR receiver for the term of the swap. Counterparties with high funding cost can make use of other low-cost balance sheets through TROR swaps, thereby facilitating investment in assets that diversify the portfolio of the receiver away from more affordable but riskier assets.

Since the maturity of a TROR swap does not necessarily match the maturity of the underlying reference obligation, the TROR receiver in a swap may benefit from the positive carry associated with being able to roll forward short-term synthetic financing of a longer-term investment. The TROR payer may benefit from being able to purchase protection for a limited period without having to liquidate the asset permanently. At the maturity of a TROR swap whose term is less than that of the reference obligation, the TROR payer essentially has the option to reinvest in that asset, by continuing to own it, or to sell it at market price. At this time, the TROR payer has no exposure to the market price since a lower price will lead to a higher payment by the TROR receiver under the terms of the TROR swap.

TROR swaps make new asset classes accessible to investors for whom administrative complexity or lending group restrictions imposed by borrowers has traditionally presented barriers to entry. For example, insurance companies and levered fund managers have made use of TROR swaps to access bank loan markets (Financial Engineering Ltd., 1999, p. 21; Tavakoli, 1998, p.25).

B.3.a. Credit-spread options

Credit-spread options are a third type of credit derivative used to insure against the risk of adverse changes in credit quality. *Credit-spread options* are call or put options on the price of an asset swap, which consist of a package of credit-risky instruments with any payment characteristics and a corresponding derivative contract that exchanges the cash flow of that instrument for a floating rate cash flow stream.

With a credit-spread option on an asset swap package, the put buyer pays a premium for the right, but not the obligation, to sell to the put seller a specified reference asset. The put buyer simultaneously enters into a swap in which the put seller pays the coupons on the reference asset and receives three- or six-month Libor, plus a predetermined spread (the strike spread). The put seller makes an up-front cash payment of par for this combined package upon exercise.

Credit options may be American or European. They may be structured to survive a credit event of the issuer or guarantor of the reference asset (in which case both default risk and credit spread risk are transferred between the parties), or to knock-out upon a credit event (in which case only credit spread risk changes hands).

As with other options, the credit-spread option premium is sensitive to the volatility of the underlying asset market price (in this case driven primarily by credit spreads rather than the outright level of yields, since the underlying instrument is a floating rate asset or asset swap package). The credit-spread option premium is also sensitive to the extent to which the strike spread is in- or out-of-the money relative to the applicable current forward credit-spread curve. Therefore, the premium is greater for more volatile credits, tighter strike spreads in the case of puts, and wider strike spreads in the case of calls.

Note that the extent to which a strike spread on a one-year credit option on a five-year asset is in- or out-of-the money will depend upon the implied five-year credit spread in one year's time (or the 'one by five' year credit spread). The 'one by five' year credit spread, in turn, would have to be backed out from current one- and six-year spot credit spreads (Financial Engineering Ltd., 1999, p. 22).

B.3.b. Yield enhancement and credit spread protection

Credit options can be used as a source of yield enhancements. In buoyant market environments, with credit spread products in tight supply, credit market investors frequently find themselves under-invested. Consequently, the ability to write credit options, whereby investors collect current income in return for the risk of owning (in the case of a put) or losing (in the case of a call) an asset at a specified price in the future, is an attractive enhancement to inadequate current income.

Buyers of credit options, on the other hand, are often institutions such as banks and dealers who are interested in hedging their mark-to-market exposure to fluctuations in credit spreads: hedging long positions with puts, and short positions with calls. For such institutions, which often run leveraged balance sheets, the off-balance-sheet nature of the positions created by credit options is an attractive feature. Credit options can also be used to hedge exposure to downgrade risk. Credit swaps and credit options can both be tailored so that payments are triggered upon a specified downgrade event. Such options have been attractive for portfolios that are forced to sell deteriorating assets, where pre-emptive measures can be taken by structuring credit derivatives to provide down grade protection. This reduces the risk of forced sales at distressed prices and consequently enables portfolio managers to own assets of marginal credit quality at lower risk. Where the cost of such protection is less than the pickup in yield of owning weaker credits, portfolio risk-adjusted returns are improved.

B.3.c. Hedging future borrowing cost

Credit options also have applications for borrowers wishing to lock in future borrowing costs without inflating their balance sheets. A borrower with a known future funding requirement could hedge exposure to outright interest rates using interest rate derivatives. Prior to the advent of credit derivatives however, exposure to changes in the level of the issuer's borrowing spreads could not be hedged without issuing debt immediately and investing the funds in other assets. This had the adverse effect of inflating the current balance sheet unnecessarily and exposing the issuer to reinvestment risk and, often, negative carry. Today, issuers can enter into credit options on their own name and lock in future borrowing costs with certainty. Essentially, the issuer is able to buy the right to put its paper to a dealer at a pre-arranged spread. In a further recent innovation, issuers have sold puts or downgrade puts on their own paper, thereby providing investors with credit enhancements in the form of protection against a credit deterioration that falls short of outright default (whereupon such a put would of course be worthless). The objective of the issuer is to reduce borrowing costs and boost investor confidence.

C. Credit derivative risk

Das (1997c) and Tavakoli (1998) suggest that while providing a valuable tool for managing credit risk, credit derivatives can also expose the user to new financial risks. Like other over-the-counter derivative securities, credit derivatives are privately negotiated (as compared to exchange traded) financial contracts. These contracts expose the user to operational risk, counterparty risk, liquidity risk, and legal risk. "For the most part these risks are either controllable or relatively small and therefore unlikely to restrict the development of the credit derivatives market" (Neal, 1996, p.24).

CHAPTER II.

BACKGROUND

A. Motivation

This research examines the issues involved in developing and implementing a model for pricing credit risk derivatives on a portfolio of bonds. Credit-risk derivatives, financial contracts, provide insurance against counterparties that may default on their contractual obligations—the failure of the 'no-counterparty risk' assumption of finance theory.

Financial Economics studies the allocation of scarce resources over time. Finance theory helps to guide/inform the decision-maker's thinking about the allocation of resources over time and consists of a set of quantitative models to help evaluate the various choices that emanate from this thinking. Bodie and Merton (1998) state two features that distinguish financial decisions from other resource allocation decisions. Costs and benefits are (1) spread out over time (intertemporal), and (2) are known in advance with certainty, neither by the decision makers or anyone else (stochastic). To implement their decisions people use the financial system—the set of markets and other institutions used for the exchange of assets and risks.

To define the concepts and build the models, Jarrow and Turnbull (1997) list five assumptions typically imposed in the pricing of financial assets and/or derivatives:

1. No market frictions: There are no transaction costs, no bid/ask spread, no margin requirements, no restriction on short sales, and no taxes.
2. No counterparty risk: Counterparties will not default on any contracts they undertake.
3. Competitive markets: Market participants act as price takers.

4. Market participants prefer more wealth to less.
5. No-arbitrage: Prices adjust so that there are no arbitrage opportunities.

Many practitioners, in particular Wilmott (1998), argue that the second assumption is the one that most obviously goes against the everyday evidence of the daily business media. Bankruptcy and non-execution of contracts are a major concern in all business transactions, including in the trading of financial instruments, especially in over-the-counter [non-exchange backed] contracts (Jarrow & Turnbull, 1996, p.555). The no-counterparty risk assumption implies that there is only one rate—the default-free rate—for borrowing and lending. In fact, there are many rates and they differ based on the perceived ability, or probability, that a borrower defaults on a loan (Jarrow & Turnbull, 1996; Musiela & Ruthowski, 1997;Wilmott, 1998).

CHAPTER III.

LITERATURE REVIEW

A. Credit-risky Debt Valuation Models

Models for the valuation of credit risky debt can be placed into three categories. The Merton (1974) category of models views the firm's liabilities as a contingent claim on the firm's underlying assets. Payoff to all the firm's liabilities is completely specified in these models. Bankruptcy is determined via the evolution of the firm's assets in conjunction with its debt covenants [e.g., Black and Cox (1976), Merton (1974)]. Implementing this approach will be difficult given that all a firm' assets are not tradable or observable (e.g., goodwill). Jones, Mason and Rosenfeld (1984) work suggests that it is very difficult to specify the complex priority structure of the payoff to all of a firm's liabilities. However, in the Merton (1974) type models this specification needs to be done when implementing the model.

The second category of models views risky debt as paying off an exogenously specified fraction of each promised dollar, in the event of bankruptcy. Bankruptcy is determined when a firm's underlying assets hit some exogenously specified boundary [cf., Hull and White (1991); Longstaff and Schwartz (1996)]. This approach simplifies the first category of models by exogenously specifying the cash flow to risky debt in the event of bankruptcy. These models, however, still require the valuation of the firm's entire underlying assets, which is unobservable. Lando (1995) points out that in the case of municipalities, it is not clear what 'firm value' to use (p.376). Also, they cannot handle various credit derivatives whose payoff depends on the credit rating of the debt issue.

The third category, which has developed over the last few years, directly models the default process of risky debt. The value of risky debt can be determined by combining the bankruptcy process with a term structure model, and a recovery rate in the event of default. Das and Turfano (1995), and Jarrow, Lando and Turnbull (1997) employ a credit-rating based approach in which a Markov transition matrix drives the gradual change in ratings. Duffie and Singleton (1996) do not refer to the credit ratings when they model the default process.

Lando (1995) provides a more comprehensive and readable review of this literature.

B. Fixed Income Securities Market.

Credit derivatives are one of the many new interest rate dependent securities that have proliferated over the last twenty years. The value of these new interest rate dependent products—bond futures, bond options, swaps—depends in some way on the level of interest rates. Interest rates are used for discounting and for defining the payoff of derivatives. When constructing models to evaluate these products, it is crucial to incorporate the stochastic movement of interest rates (Campbell et. al., 1997; Jarrow, 1996; Luenberger, 1998; Kwok, 1998; Pliska, 1997; Rebonato, 1998).

Pliska (1997) states that the securities market model for the valuation of risky fixed income securities, such as bonds, and interest rate derivative products is called a *term structure model*. Three things are required for a securities market model to be a term structure model. First, the model must incorporate multiple periods. Second, the interest rate must be a strictly positive, predictable process—the interest rate for borrowing/lending over the next period is known in the current time. Usually, B(0)=1 and $B(t) = B(0)\exp\left\{\sum_{k=1}^{\frac{t}{h}-1} r(kh)h\right\}$. This interest rate, *r(t)*, one of several, is called the *spot interest rate*.

Finally, and "most importantly", zero coupon or discount bonds must be included among the risky securities (Pliska, 1997, p.200). The zero-coupon bonds are defined for each T, such that 1≤T≤τ. The zero-coupon bond with maturity T is the security whose price at time T is certain to be one. The term structure model includes a zero-coupon bond, P(t,T), for every maturity T, satisfying T=1,2...τ. Hence at each time-t there is a collection {P(t, t+1), P(t, t+2)..., P(t, T)} of zero coupon bond prices. This collection is called the *term structure of zero-coupon bond prices*.

The term structure model must be free of arbitrage opportunities, so there must exist a risk neutral probability measure Q under which discounted prices of the zero c coupon bonds are martingales. That is, there must exist some probability measure Q, with Q(ω)>0 for all ω ε Ω such that, for every T,

$$P(s,T)/B(s) = E_s^Q[P(t,T)/B(t)] \quad 0 \le s \le t \le T \qquad 3.1$$

But P(T,T)=1 and $B(t)/B(s) = \exp\left\{\sum_{k=s+1}^{T} r(k)\right\}$, so taking t = T we get that zero-coupon bonds must satisfy the important relationship:

$$P(s,T) = E_s^Q[B(s)/B(T)] = E_s^Q\left[\exp\left\{-\sum_{k=s+1}^{T} r(k)\right\}\right] \; 0 \le s \le T \qquad 3.2$$

given any risk neutral probability measure Q. Since r(t)>0, this implies, for fixed time-s, that as a function of maturity T, P(t, T) is a strictly decreasing function with P(s, s+1)<1. Note, taking T=s+1 gives:

$$\exp\{r(s)\} = 1/(P(s,s+1) \qquad s = 0,1,2,\ldots,T-1. \qquad 3.3$$

The main essence is to model the prices of the interest rate securities [products] as functions of one or, a few state variables; for example, spot interest rates or spot forward rates (Luenberger, 1998; Kwok, 1998). "In the so called no-arbitrage interest rate models, the consistencies with the observed initial term structures of interest rates and/or volatilities of interest rates are enforced" (Kwok, 1998, p.313).

In no-arbitrage interest rate models, the initial term structure of interest rates are taken as inputs to the model. Hence, values of contingent claims obtained from these models are automatically consistent with these inputs. These no-arbitrage models include parameters that are functions of time, and these parameters have to be determined from the current market data. The Heath, Jarrow, Morton (1992) (HJM) framework, used in this study, incorporates most of the models popularly used by practitioners as a special case (Jarrow, 1995).

CHAPTER IV.

THE MODEL

The discrete-time model for valuing credit-risk derivatives of Das and Sundaram (1999) is the basis for this research. The model "possesses the advantage of simple implementation mechanics and requires as input on easily available information" (Das and Sundaram, 1999, p.1). To value credit-risky debt, the approach extends the term structure framework of Heath, Jarrow, Morton (1987) (HJM), in discrete-time [c.f., Heath, Jarrow & Morton, 1990; Jarrow, 1997], by adding a "forward spread" process to the forward rate process for default-free bonds (Das and Sundaram, 1999).

This chapter will describe a general asset-pricing framework and then derive the HJM-framework from this general framework. After, we will derive the Das-Sundaram model and explain how it fits into the HJM-framework. A key element of the Das-Sundaram model is a recursive structure for prices. The recursive structure is imbedded naturally in the stochastic differential equation (SDE) of the model. The paper will illustrate, with an example, how the parameters of the SDE can be calculated recursively.

A. General asset-pricing framework

The money market account can be defined as

$$B(0,t) \equiv \exp\left\{\int_0^t r(s)ds\right\} \qquad 4.1.$$

Equation (4.1) says that the rolled-up money market account at time-t is the value of $1 invested at time-0 at the prevailing instantaneous short rate, and reinvested (with the *accrued* interest) over each infinitesimal time step dt out to the final time-t, always at the prevailing instantaneous short rate. One immediately notices from the definition that the money market account always has a strictly positive value and therefore the positivity condition (required for an asset to be a possible numeraire) is satisfied. Relative price with respect to this numeraire are then given by:

$$\tilde{Z}(0,t) = \frac{S_n(t)}{B(0,t)} \qquad 4.2$$

where the tilde is a reminder that the expectations and relative prices refer to the numeraire B.

If no arbitrage is to be allowed, a unique equivalent measure $\tilde{Q}$, implicitly defined by this particular numeraire, must exist such that the martingale condition holds:

$$\tilde{E}\left[\tilde{Z}_n(t,u) \middle| \Im(t)\right] = \left[\frac{S_n(u)}{B(0,t)} \middle| \Im(t)\right] = \tilde{Z}_n(0,t) \qquad \forall u \geq t \qquad 4.3$$

where $\tilde{E}[\;]$ indicates expectation taken in the measure $\tilde{Q}$. With (4.3) one is standing at time-t, up to which time the information $\Im(t)$ has accumulated. At the very least, this information will consist of the value of all the assets at time-t, but it could in also include the past prices up to time-t. In addition, the money market account has been rolled up from some previous arbitrary time-0 to time-t. So $B(0,t)$ is a known quantity at time-t. By the no-arbitrage theorem the expectation at time-t of the ratio of the future cash asset price at time-u, $S_n(u)$, to the future cash value of the money market account also at time-u, $B(0,u)$, is equal to the ratio of the known value of $S_n(t)$ to $B(0,t)$.

Since $B(0,u)$ can be looked at as the money market account rolled up from time-0 to time-t, and then from time-t to time-u, i.e.

$$B(0,u) = B(0,t) \cdot B(t,u), \qquad 4.4$$

hence, (4.3) can be written:

$$\tilde{E}\left[\frac{S_n(u)}{B(0,u)}\middle|\Im(t)\right] = \tilde{E}\left[\frac{S_n(u)}{B(0,t)\cdot B(t,u)}\middle|\Im(t)\right] = \frac{S_n(t)}{B(0,t)} \qquad \forall u \geq t \qquad 4.3'$$

Given that at time-t, $B(0,t)$ is a known (non-stochastic) quantity and, as such, can be taken out of the expectation operator, one can cancel it from both sides of the (4.3') and write:

$$\tilde{E}\left[\frac{S_n(t)}{B(t,u)}\middle|\Im(t)\right] = S_n(t). \qquad 4.3''.$$

For the special case where the original asset price is the time-t price of a bond of maturity $u = T$, $P(t,T)$, (4.3") readily gives:

$$P(t,T) = \tilde{E}\left[\frac{P(T,T)}{B(t,T)}\middle|\Im(t)\right] = \tilde{E}\left[\frac{1}{B(t,T)}\middle|\Im(t)\right]$$
$$= \tilde{E}\left[\exp\left\{-\int_{s=t}^{T} r(s)ds\right\}\middle|\Im(t)\right] \qquad 4.5$$

where $P(T,T) = 1$. That is, the time-t price of a T-maturity bond is equal to the expectation under $\tilde{Q}$ of the reciprocal of the money market account at time-t.

Equation (4.5) is one of the most useful and important tools in option pricing, both conceptually and for practical implementations. This will be appreciated in the context of all the pricing methodologies, which make use (explicitly or implicitly) of the money market account as numeraire. It is by enforcing condition (4.5) that numerical procedures ensure that expectations of future option payoffs are taken with respect to the correct, but *a priori* unknown, measure $\tilde{Q}$ (Rebonato, 1998, p.167)[3].

In our future application, because our model is a linear, stochastic differential equation, it will be very important to derive the drift on any cash asset price under the measure $\tilde{Q}$. To do this we start with (4.2) and write

$$S_n(t) = Z_n(0,t)\cdot B(0,t) \qquad 4.6.$$

After taking the differential we get:

$$dS_n(t) = Z_n(0,t)\cdot dB(0,t) + dZ_n(0,t)\cdot B(0,t) + dB(0,t)\cdot dZ_n(0,t) \qquad 4.7$$

which yields:

$$\begin{aligned} dB(0,t) &= d\left(\exp\left\{\int_{s=0}^{t} r(s)ds\right\}\right) \\ &= \exp\left\{\int_{s=0}^{t} r(s)ds\right\}\cdot\frac{d}{dt}\left(\int_{s=0}^{t} r(s)ds\right)\cdot dt \\ &= \exp\left\{\int_{s=0}^{t} r(s)ds\right\}\cdot r(t)\cdot dt \\ &= B(0,t)\cdot r(t)\cdot dt \qquad 4.8 \end{aligned}$$

since the value of the money market account is known at time-t (i.e. $dB(0,t)$ is locally, non-stochastic at time-t). By the Ito multiplication table, mixed stochastic and deterministic deferentials are zero, so (4.7) becomes

$$\begin{aligned} dS_n(t) &= \tilde{Z}_n(0,t)\cdot B(0,t)\cdot r(t)\cdot dt + d\tilde{Z}_n(0,t)\cdot B(0,t) \\ &= r(t)\cdot B(0,t)\cdot\frac{S_n(t)}{B(0,t)}\cdot dt + B(0,t)\cdot d\tilde{Z}_n(0,t) \\ &= r(t)\cdot S_n(t)\cdot dt + B(0,t)\cdot d\tilde{Z}_n(0,t) \qquad 4.7' \end{aligned}$$

[3] See also, Neftci (2000) Chapters 18 & 19 for a slow, very clear account.

Dividing through by $S_n(t)$ and using the reciprocal of definition (4.2), we now have:

$$\frac{dS_n(t)}{S_n(t)} = r(t)\cdot dt + \frac{d\tilde{Z}_n(0,t)}{\tilde{Z}_n(0,t)} \qquad 4.7''.$$

Equation (4.7") shows the very important result: *Under the measure $\tilde{Q}$, all assets earn the same return, given by the instantaneous short rate.* This is the same rate that would be earned in a deterministic economy, hence justifying the name *'risk-neutral'* for this particular measure–obtained by using the rolled up money market account as the numeraire (Rebonato, 1998, p. 138).

To be more specific about the process (4.7") the Brownian motion assumption is invoked, i.e. one requires that the shocks to the prices can be modeled by a diffusive (no jumps) process. If this is the case one can write in integral form

$$\tilde{Z}_n(0,t) = \tilde{Z}_n(0,0)\cdot \exp\left\{\int_{s=0}^{t} \tilde{\sigma}(s)d\tilde{W}(s) - \tfrac{1}{2}\int_{s=0}^{t} |\tilde{\sigma}(s)|^2 ds\right\} \qquad 4.9$$

or, using

$$\tilde{Z}_n(0,t) = S_n(t)/B(0,t),$$

$$S_n(t) = S_n(0)\cdot B(0,t)\cdot \exp\left\{\int_{s=0}^{t} \tilde{\sigma}(s)d\tilde{W}(s) - \tfrac{1}{2}\int_{s=0}^{t} |\tilde{\sigma}(s)|^2 ds\right\}$$

$$= S_n(0)\cdot \exp\left\{\int_{s=0}^{t} r(s)ds\right\}\cdot \exp\left\{\int_{s=0}^{t} \tilde{\sigma}(s)d\tilde{W}(s) - \tfrac{1}{2}\int_{s=0}^{t} |\tilde{\sigma}(s)|^2 ds\right\} \qquad 4.10$$

where $\tilde{\sigma}$ indicates the volatility of relative rates under the measure $\tilde{Q}$.

Equation (4.9) can be written in differential form as

$$\frac{d\tilde{Z}_n(0,t)}{\tilde{Z}_n(0,t)} = \tilde{\sigma}(t)\cdot d\tilde{W}(t) \qquad 4.9'$$

which allows one to write Equation 4.7'' as

$$\frac{dS_n(t)}{S_n(t)} = r(t)\cdot dt + \tilde{\sigma}(t)\cdot d\tilde{W}(t) \qquad 4.7'''$$

By (4.9'), the variable $\tilde{Z}_n(0,t)$ follows a lognormal distribution. This distributional assumption is compatible with both the cash asset and the numeraire being also lognormally distributed. Hence, you have:

$$\tilde{\sigma}(t) = \sqrt{\sigma(t)^2 + \sigma_B(t)^2 - 2\rho\sigma(t)\sigma_B(t)} \qquad 4.11$$

where $\sigma_B(t)$ indicates the percentage volatility of the numeraire asset. However, since the rolled-up money market is locally deterministic, the percentage volatility of the numeraire is equal to zero; i.e. $\sigma_B(t) = 0$. Therefore, with the rolled-up money market account as numeraire, we have $\tilde{\sigma}(t) = \sigma(t)$, which says that *the instantaneous percentage volatilities of the relative prices equal the percentage volatilities of the cash prices* (Rebonato, 1998, p. 169).

Another important result that can be obtained under this measure, $\tilde{Q}$, concerns the link between the discount factor and a discount bond. More precisely, if one assumes an asset to have evolved from any arbitrary time-0 to time-T, and one imposes that (4.10) should hold for any asset, and hence, in particular for a discount bond of maturity T, one gets:

$$\frac{P(T,T)}{B(0,T)} = \frac{1}{B(0,T)} = \exp\left\{-\int_{s=0}^{T} r(s)ds\right\}$$

$$= P(0,T)\cdot\exp\left\{\int_{s=0}^{T} v(s,T)d\tilde{W}(s) - \tfrac{1}{2}\int_{s=0}^{T} |v(s,T)|^2 ds\right\} \qquad 4.12$$

where, $v(s,T)$ is used to denote volatilities when assets are discount bonds, and indicates the volatilities at time-s of a discount bond of maturity T. *Expression (4.12) therefore provides the link between the discount factor in the money market numeraire account* $1/B(0,T)$ *and the price of a discount bond.* At a very simple level, (4.12) shows that it is not correct, under the risk-neutral measure $\tilde{Q}$, to discount a payoff occurring at time-T from an asset which has grown at the short rate $r(t)$ by the discount bond $P(0,T)$ (Rebonato, 1998, p.170).

At this point, we can examine the difference between future and forward prices under the risk-neutral measure, $\tilde{Q}$. Solving Equation (4.10) for $B(0,t)$ one can write:

$$B(0,t) = \frac{S_n(t)}{S_n(0)} \cdot \exp\left\{ -\int_{s=0}^{t} \tilde{\sigma}(s) d\tilde{W}(s) + \tfrac{1}{2} \int_{s=0}^{t} \left| \tilde{\sigma}(s) \right|^2 ds \right\} \qquad 4.13.$$

Substituting (4.13) in Equation (4.12) and solving for $S_n(t)$ gives:

$$S_n(t) = \frac{S_n(t)}{S_n(0)} \cdot \exp\left\{ \int_{s=0}^{t} (\tilde{\sigma}(s) - v(s,t)) d\tilde{W}(s) - \tfrac{1}{2} \int_{s=0}^{t} \left(\left| \tilde{\sigma}(s) \right|^2 - \left| v(s,t) \right|^2 \right) ds \right\} \qquad 4.14$$

or, for the special case of $S_n(t)$ being a discount bond maturing at time-T $(T > t)$,

$$P(t,T) = \frac{P(0,T)}{P(0,t)} \cdot \exp\left\{ \int_{s=0}^{t} (v(s,T) - v(s,t)) d\tilde{W}(s) - \tfrac{1}{2} \int_{s=0}^{t} \left(\left| v(s,T) \right|^2 - \left| v(s,t) \right|^2 \right) ds \right\} \qquad 4.1.14'$$

show that *under the risk-neutral measure* , $\tilde{Q}$, *future prices* $(S_n(t)$ *or* $P(t,T))$ *differ from the forward prices* $(S_n(t)/P(0,t))$ *or* $(P(0,T)/P(0,t))$, *by the exponential terms in (4.14) and (4.14') respectively* (Rebonato, 1998, p.170).

B. The Heath-Jarrow-Morton (1992) (HJM) framework

The above section showed that, if the chosen numeraire is the money market account, *all assets grow at the riskless rate* (4.7''), and *forward rates are not martingales* but display a non-zero drift term. This drift term can generally be derived within the framework of the Heath-Jarrow-Morton (1992) approach.

Derivation, typically, follows two different formulations. The price-based formulation takes the dynamics of the discount bonds as the fundamental building block. The second formulation, forward based, obtains the no-arbitrage, stochastic differential equations obeyed by forward rates. Rebonato (1998) shows that the two approaches are equivalent. Starting with the no-arbitrage condition shown in the previous section to hold for the dynamics of any asset in order to arrive at the HJM approach, and then obtain the equivalence of this to the forward-rate-based approach. Clewlow and Strickland (1998), Neftci (2000) and Rebonato (1998) point out that this approach is probably more intuitively clear. However, it must be noted that, historically, the HJM results were first obtained in the forward-rates context in Heath-Jarrow-Morton (1992).

The starting point for any implementation of the HJM approach is the observed yield curve, as described either by the collection of discount bonds given at time-0, $P(0,T)$, or by the instantaneous forward rates, $f(0,T)$, linked by

$$P(0,T) = \exp\left\{-\int_{s=0}^{T} f(0,s)ds\right\} \qquad 4.15$$

$$f(0,T) = -\frac{\partial P(0,T)}{\partial T} \qquad 4.15'.$$

Either the discount bonds or the forward rates can be taken as equivalent building blocks. The approach, in either case, hence recovers by construction any given market yield curve. If one uses the rolled-up money market account as numeraire, all assets instantaneously grow at the riskless (short) rate. Hence, for discount bonds, $P(t,T)$, one can write

$$dP(t,T) = r(t) \cdot P(t,T)dt + v(t,T,P(t,T)) \cdot P(t,T)d\tilde{W}^T(t) \qquad 4.16$$

where $\tilde{W}^T(t)$ is a Wiener process with respect to the risk-neutral probability measure $\tilde{Q}$.

Three points about this SDE, (4.16), are emphasized by Neftci (2000, p.438) (cf. Rebonato, 1998, p.374). First, the diffusion parameter is written in terms of percentage bond volatility, but is not generally of geometric form (i.e. lognormally distributed)– $v(t,T,P(t,T))$ depend on $P(t,T)$ as well, hence percentage bond volatility is not constant here. Second, a Wiener process indexed by T drives the SDE. This means that every bond with different maturity is allowed to be influenced by a different shock. Finally, it should be noted that the diffusion parameter is explicitly made a function of the maturity T. Hence, the maximum generality has been allowed for the price volatility of the discount bond. Neftci (2000) and Rebonato (1998) note that in (4.16) the drift component is totally specified by the no-arbitrage condition, and that using the money market account as numeraire leads to all assets growing by the instantaneous riskless short rate under the risk-neutral measure, $\tilde{Q}$.

Let us apply Ito's lemma to $\ln P(t,T)$:

$$d(\ln P(t,T)) = \left[r(t) - \tfrac{1}{2} v(t,T,P(t,T))^2\right]dt + v(t,T,P(t,T))d\tilde{W}(t) \qquad 4.17$$

Hence,

$$\begin{aligned} d\left[\ln P(t,T+h) - \ln P(t,T)\right] = {} & \tfrac{1}{2}\left[v(t,T,P)^2 - v(t,T+h,P)^2\right]dt \\ & + \left[v(t,T+h,P) - v(t,T,P)\right]d\tilde{W}(t) \qquad 4.18 \end{aligned}$$

Define the continuously compounded time-t forward rate that span the discrete period $[T,T+h]$, $f(t,T,T+h)$, as:

$$f(t,T,T+h) \equiv -\frac{\ln P(t,T+h) - \ln P(t,T)}{T+h-T} \qquad 4.19.$$

Using (4.18) with this definition leads to:

$$d[f(t,T,T+h)] = \tfrac{1}{2}\frac{\left[v(t,T,P)^2 - v(t,T+h,P)^2\right]}{(T+h)-T}dt + \frac{\left[v(t,T+h,P) - v(t,T,P)\right]}{(T+h)-T}d\tilde{W}(t) \qquad 4.20$$

Taking the limit as $h \to 0$, the discrete forward rate approaches the instantaneous forward rate $f(t,T)$:

$$f(t,T) = -\frac{\partial \ln P(t,T)}{\partial T} = -\frac{1}{P}\cdot\frac{\partial P(t,T)}{\partial T} \qquad 4.21$$

and recalling that $\partial\left(f(x)^2\right)/\partial x = 2f(x)\partial f(x)/\partial x$, we finally obtain:

$$\begin{aligned} df(t,T) &= v\cdot\left[\left(\frac{\partial v}{\partial T}\right)_P + \left(\frac{\partial v}{\partial P}\right)_T\cdot\frac{\partial P}{\partial T}\right]dt + \left[\left(\frac{\partial v}{\partial T}\right)_P + \left(\frac{\partial v}{\partial P}\right)_T\cdot\frac{\partial P}{\partial T}\right]d\tilde{W}(t) \\ &= v\cdot\left[\left(\frac{\partial v}{\partial T}\right)_P - \left(\frac{\partial v}{\partial P}\right)_T\cdot f(t,T)\cdot P\right]dt + \left[\left(\frac{\partial v}{\partial T}\right)_P - \left(\frac{\partial v}{\partial P}\right)_T\cdot f(t,T)\cdot P\right]d\tilde{W}(t) \end{aligned} \qquad 4.22$$

where use is made of (4.21) in the second line, and $(\partial f(x)/\partial x)_z$ indicates the partial derivative of the function f with respect to x, *holding* z *constant.*

Equation (4.22) therefore shows that:

(i) when the money market account is used as numeraire, forward rates are not martingales, and

(ii) establishes the link that must exist in a risk-neutral world, if no-arbitrage condition is to hold, between the percentage volatility of discount bonds and the drift of forward rates.

Rebonato (1998) notes that the relationships obtained so far are completely general, and embody nothing more than the conditions of no-arbitrage.

Condition (4.22) can be re-expressed in terms of the volatilities of the forward rates. If one denotes the stochastic terms in square brackets in (4.22) by σ_f:

$$\sigma_f = \left[\left(\frac{\partial v}{\partial T}\right)_P - \left(\frac{\partial v}{\partial P}\right)_T \cdot f(t,T)\cdot P\right] \qquad 4.23$$

then the no-arbitrage condition (4.22) can be rewritten as:

$$df = \sigma_f \cdot v dt + \sigma_f d\tilde{W}(t) \qquad 4.24\ .$$

It is clear, from the expressions (4.22) and (4.24) above, that σ_f --the volatility of the forward rates and, v --the percentage volatility of the price, are not independent of each other. To establish the link between the two, one can start from the definition:

$$P(t,T) \equiv \exp\left\{-\int_{s=t}^{T} f(t,s)ds\right\} \equiv \exp\left\{-\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1} f(t,kh)\cdot h\right\} \qquad 4.25$$

where the last sum is carried out over the *discrete forward rates* of different expiry dates kh as seen from the time-t yield curve. Then applying Ito's lemma, one obtains:

$$dP = r(t)\cdot P(t,T)dt + v(t,T,P(t,T))\cdot P(t,T)d\tilde{W}$$

$$= r(t)\cdot P(t,T)dt + \sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}\left[\frac{\partial P(t,kh)}{\partial f(t,kh)}\cdot \sigma_f(t,kh)\right]d\tilde{W}$$

$$= r(t)\cdot P(t,T)dt + \sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}\left[\frac{\partial \exp\left\{-\sum_{l=\frac{t}{h}+1}^{\frac{T}{h}-1} f(t,lh)\cdot h\right\}}{\partial f(t,kh)}\cdot \sigma_f(t,kh)\right]d\tilde{W}$$

$$= r(t)\cdot P(t,T)dt + P(t,T)\cdot \sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}\left[h\cdot \sigma_f(t,kh)\right]d\tilde{W} \qquad 4.26$$

where the last line makes use of the fact that $\frac{\partial}{\partial f_k(t,kh)}\left[\sum_l f(t,lh)\cdot h\right] = h$.[4]

Taking the limit as $h \to 0$, the term

$$\sum_l \sigma_f(t,lh)\cdot h \quad \to \quad \int_t^T \sigma_f(t,s)ds$$

and therefore

$$v(t,T,P(t,T))\cdot P(t,T) = P(t,T)\cdot \int_t^T \sigma_f(t,s)ds \qquad 4.27$$

Using result (4.27), after dividing both sides by the price, in condition (4.24), we obtain:

$$df = \sigma_f \cdot v dt + \sigma_f d\tilde{W}(t)$$

$$= \left[\sigma_f(t,T,f(t,T))\cdot \int_t^T \sigma_f(t,s,f(t,s))ds\right]dt + \sigma_f(t,T,f(t,T))d\tilde{W} \qquad 4.28$$

[4] Note that two different indices are used for the summations, k and l.

Equation (4.28) provides us with the so-called HJM drift condition, under the risk-neutral (martingale) measure $\tilde{Q}$. If we assume that the forward rates are specified directly under a martingale measure $\tilde{Q}$ as:

$$df(t,T) = m(t,T,f(t,T))dt + \sigma_f(t,T,f(t,T))d\tilde{W} \qquad 4.29$$

where $\tilde{W}$ is a $\tilde{Q}$-Wiener process. Then to impose some sort of consistency relationship between $m(t,T,f(t,T))$ and $\sigma_f(t,T,P(t,T))$ in the forward rate dynamics, the following condition must hold:

$$m(t,T,f(t,T)) = \sigma_f(t,T,f(t,T)) \cdot \int_t^T \sigma_f(t,s,f(t,s))ds \qquad 4.30.$$

Expression (4.30) links the drift of the time-t instantaneous forward rate of expiry time-T with the volatility of the forward rate expiring at time-T, and with the volatilities of all the forward rates expiring between time-t and time-T (Rebonato, 1998, p.378). In the HJM framework, when the forward rate dynamics are specified the volatility structure may be freely specified. The drift parameters are then uniquely determined by the HJM drift condition (Bjork, 1998; Heath, Jarrow, Morton, 1990 & 1992; Neftci, 2000).

C. The Model [Das and Sundaram (1998)]

Das and Sundaram (1998) is closely followed unless otherwise indicated. Consider a finite time interval $[0,\tau]$. Discrete periods are taken to be of length $h>0$. Hence, a typical point t has the form kh for integer k.

Assume the following:

Assumption #1: At all times t, a full range of default-free zero-coupon bonds trades, as well as a full range of risky zero-coupon bonds.

Assumption #2: Markets are arbitrage-free, so there exists an equivalent martingale measure, Q, for this economy [cf., Pliska (1997); Musiela and Ruthowski (1997)].

For any given pair of time points (t,T), where $0 \le t \le T \le \tau - h$, let $f(t,T)$ denote the forward rate on the default-free bonds applicable to the period $[t,T+h]$, i.e.,

$$f(t,T,T+h) \equiv f(t,T).$$

In other words, $f(t,T)$ is the rate, as viewed from time-t, for a default-free transaction over the interval $[T,T+h]$. Note, $f(t,t) \equiv r(t)$ is called the short rate.

Assumption #3: Evolution of the forward rate curve (default-free bonds).

The forward rate evolves according to the process

$$f(t+h,T) = f(t,T) + \alpha(t,T)h + \sigma(t,T)X_1\sqrt{h} \qquad 4.31$$

$\alpha(.,.) \equiv$ drift coefficient of the process,

$\sigma(.,.) \equiv$ volatility coefficient of the process,

and $X_1 \equiv$ a random variable.

Both $\alpha(.,.)$ and $\sigma(.,.)$ may depend on other information available at time-t.

Let $\varphi(t,T)$ denote the "forward rate" on the risky bonds implied from the spot yield curve. The forward spread, $s(t,T)$ on the risky bonds is then defined as:

$$s(t,T) \equiv \varphi(t,T) - f(t,T).$$

Assumption #4: Evolution of the forward spreads (and thus of the forward rates of risky bonds).

The forward spread evolves according to the process

$$s(t+h,T) = s(t,T) + \beta(t,T)h + \eta(t,T)X_2\sqrt{h} \qquad 4.32$$

$\beta(.,.) \equiv$ drift coefficient of the process,

$\eta(...) \equiv$ volatility coefficient of the process,

and $X_2 \equiv$ a random variable.

At this point, no restrictions are placed on the joint distribution of X_1 and X_2.

Denote by $P(t,T)$ the time-t price of a default-free, zero-coupon bond with maturity $T \geq t$, and by $\Pi(t,T)$ a risky, zero-coupon bond with maturity $T \geq t$. By definition

$$P(t,T) \equiv \exp\left\{ -\sum_{k=\frac{t}{h}}^{\frac{T}{h}-1} f(t,kh).h \right\} \qquad 4.33$$

and

$$\Pi(t,T) \equiv \exp\left\{ -\sum_{k=\frac{t}{h}}^{\frac{T}{h}-1} \varphi(t,kh).h \right\} \qquad 4.34$$

Spreads on risky bonds represent the cost of default, and as such depend on both the probability of default as well as the amount that bondholders expect to recover in the event of default. Denote by $\lambda(t)$ the probability (under measure $\tilde{Q}$) of default by time-$t+h$, given that default has not occurred at time-t. Concerning the recovery rate, Das and Sundaram (1998) adopts the "Recovery of Market Value" (RMV) condition of Duffie and Singleton (1996), while Jarrow and Turnbull (1995) use a recovery of treasury assumption. Let $\Phi(t)$ denote the recovery amount in the event of default at time-t. The RMV condition then states that conditional upon the default occurring at time-$t+h$, the time-t expectation $\tilde{E}^t\left[\Phi^{t+h}\right]$ of the amount bondholders will receive is given by:

$$\tilde{E}^t\left[\Phi^{t+h}\right] = \phi(t)\tilde{E}\left[\Pi(t+h,T)\right] \qquad 4.35$$

where $\phi(t)$ denotes the "recovery rate."[5] As with $\lambda(t)$, $\phi(t)$ may depend on all information in the model up to and including period t.

Objective: The objective is to develop a risk-neutral lattice for pricing risky debt. This is done in three steps:

Step #1: The default-free interest rates lattice is generated by solving for the risk-neutral drifts so that relative prices of all default-free securities (i.e.. discounted prices) are martingales.

Step #2: A lattice for credit spreads is superimposed on the default-free lattice, and risk-neutral drifts are computed for the forward spread process so as to make the discounted prices of risky debt martingales.

Step #3: The recursive structure of the model is used, together with a specific assumption about the default process, to illustrate implementation of the model.

D. Identifying the Risk-Neutral Drifts.

Recursive expressions for the drifts $\alpha(.,.)$ and $\beta(.,.)$ of the forward-rate and spread processes, respectively, are obtained in terms of volatilities $\sigma(.,.)$ and $\eta(...)$.

Define $B(t)$, the time-t value of a money-market account that uses an initial investment of \$1, and rolls over the proceeds at the default free short rate:

$$B(t) \equiv \exp\left\{\sum_{k=0}^{\frac{T}{h}-1} r(kh).h\right\} \qquad 4.36$$

[5]Jarrow & Turnbull (1995) assumes $\phi(t)\tilde{E}[P(t+h,T)]$, i.e. that the recovery amount is a percentage of treasury (risk-free valuation). They view risky bonds as equivalent to treasury/ risk-free bonds after default has occurred.

Assumption #5: With out lost of generality, the equivalent martingale measure $\tilde{Q}$ is defined with respect to $B(t)$ as numeraire.

Hence, under Q all asset prices in the economy which are discounted using B(t) will be martingales.

The risk-neutral drifts α(.,.) of the default-free forward rates can be identified in terms of the volatilities σ(.,.) of these rates. Let $Z(t,T)$ denote the price of the default-free bond discounted by the using $B(t)$:

$$Z(t,T) \equiv \frac{P(t,T)}{B(t)} \qquad 4.37$$

Since $Z(.,.)$ is a martingale under Q, for any $t \leq T$, we must have:

$$\tilde{E}^t[Z(t+h,T)] = Z(t,T)$$
$$\Rightarrow \quad E^t\left[\frac{Z(t+h,T)}{Z(t,T)}\right] = 1 \qquad 4.38$$

But:

$$\frac{Z(t+h,T)}{Z(t,T)} = \frac{P(t+h,T)\big/B(t+h)}{P(t,T)\big/B(t)}$$
$$= \frac{P(t+h,T)}{P(t,T)} \cdot \frac{B(t)}{B(t+h)} \qquad 4.39$$

So, using definition (4.33):

$$\frac{P(t+h,T)}{P(t,T)} = \exp\left\{ - \sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1} [f(t+h,kh) - f(t,kh)]h + f(t,t)h \right\} \qquad 4.40$$

Using definition (4.36), we get:

$$\frac{B(t)}{B(t+h)} = \exp(-r(t)h) = \exp(-f(t,t)h) \qquad 4.41$$

since $r(t) \equiv f(t,t)$, i.e. the time-t short rate, for the period $[t, t+h]$ is equivalent to the time-t forward rate for the same period.

Substituting (4.40) into (4.38) and using (4.41):

$$\tilde{E}^t\left[\frac{Z(t+h,T)}{Z(t,T)}\right] = \tilde{E}^t\left[\exp\left\{-\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}[f(t+h,kh) - f(t,kh)]h\right\}\right] \qquad 4.42$$

$$= 1 \qquad 4.43$$

Using $[f(t+h, kh)-f(t, kh)]$ from (4.31) we now have:

$$\tilde{E}^t\left[\exp\left\{-\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}\left[\alpha(t,kh)h^2 + \sigma(t,kh)X_1 h^{3/2}\right]\right\}\right] = 1 \qquad 4.44.$$

Equation (4.44) is one equation in two unknowns, for each period $[t, t+h]$. Once we specify one, the other variable is known. That is, given that $\sigma(t,.)$ is specified, then $\alpha(t,.)$ known[6].

Since $\alpha(t,.)$ is known at time-t, so:

$$\exp\left\{\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}\alpha(t,kh)h^2\right\} = \tilde{E}^t\left[\exp\left\{-\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}\left[\sigma(t,kh)X_1 h^{3/2}\right]\right\}\right]$$

Taking logarithms:

$$\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}\alpha(t,kh) = \frac{-1}{h^2}\ln\left(\tilde{E}^t\left[\exp\left\{-\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}\left[\sigma(t,kh)X_1 h^{3/2}\right]\right\}\right]\right) \qquad 4.45$$

We now want to identify the drifts β(t,T), but first we state a preliminary result that relates short spreads to the default probabilities and recover rates under martingale measure Q.

Result #1: Under the martingale measure $\tilde{Q}$, risky short spreads are a logarithm function of default rates (probabilities) and recovery rates; i.e.,

$$s(t,t) = \frac{-1}{h}\ln\left((1-\lambda(t)) + \lambda(t)\phi(t)\right) \qquad 4.46$$

Now pick any $t \leq T$ and consider a 1-period investment in $\Pi(t,T)$ at time-t. Viewed from time-t, there are two possibilities regarding expected cash flow at time-($t+h$). If the bond has not defaulted by time-($t+h$), there is an expected cash flow of $E^t[\Pi(t+h, T)]$; if the bond has defaulted, on the other hand, the expected cash flow is $\phi(t)E^t[\Pi(t+h, T)]$. Since the probability of default by time-($t+h$) is $\lambda(t)$, the expected cash flow at time-($t+h$) is:

$$\begin{aligned}&(1-\lambda(t))\tilde{E}^t\left[\Pi(t+h,T)\right] + \lambda(t)\phi(t)\tilde{E}^t\left[\Pi(t+h,T)\right]\\ &\left[(1-\lambda(t)) + \lambda(t)\phi(t)\right]\tilde{E}^t\left[\Pi(t+h,T)\right]\end{aligned} \qquad 4.47$$

By definition of the risk-neutral martingale measure Q, when discounted at the short rate r(t), this expected time-(t+h) cash flow must equal the current (time-t) price Π(t,T), so we have:

$$\tilde{E}^t\left[\frac{\left[(1-\lambda(t)) + \lambda(t)\phi(t)\right]\Pi(t+h,T}{\exp\{r(t)\}\Pi(t,T)}\right] = 1 \qquad 4.48$$

[6] Hence the reason that Das-Sundaram (1998) is identified with the HJM-framework.

Result #2:

$$\frac{\Pi(t+h,T)}{\exp\{r(t)\}\Pi(t,T)}=\exp\left\{-\left(\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}[\varphi(t+h,kh)-\varphi(t,kh)]h\right)+s(t,t)\right\} \qquad 4.49$$

Equation (4.49) implies:

$$\frac{\Pi(t+h,T)}{\exp\{[-s(t,t)+f(t,t)]h\}\Pi(t,T)}=\exp\left\{-\left(\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}[\varphi(t+h,kh)-\varphi(t,kh)]h\right)\right\} \qquad 4.49a$$

However, by Result #1, i.e., (4.46), we know:.

$[(1-\lambda(t)+\lambda(t)\phi(t)]=\exp\{-s(t,t)h\}$

So using (4.46), (4.48), and (4.49) we get

$$\tilde{E}'\left[\exp\left\{-\left(\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}[\varphi(t+h,kh)-\varphi(t,kh)]h\right)\right\}\right]=1$$

Result #3:

$$\exp\left\{\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}[\alpha(t,kh)+\beta(t,kh)]h^2\right\}=$$

$$\tilde{E}'\left[\exp\left\{-h^{3/2}\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}[\sigma(t,kh)X_1+\eta(t,kh)X_2]\right\}\right] \qquad 4.50$$

Since we have solved for $\alpha(t,.)$ in terms of $\sigma(.,.)$ using (4.45), we now use (4.50) to solve for $\beta(.,.)$ in terms of $\eta(.,.)$ and $\sigma(.,.)$:

$$\exp\left\{\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}\beta(t,kh)h^2\right\} = \tilde{E}^t\left[\exp\left\{\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}\sigma(t,kh)X_1h^{3/2}\right\}\right]\tilde{E}^t\left[\exp\left\{-\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}(\sigma(t,kh)X_1+\eta(t,kh)X_2)h^{3/2}\right\}\right]$$

$$\Rightarrow \sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}\beta(t,kh) = \frac{-1}{h^2}\ln\left(\tilde{E}^t\left[\exp\left\{\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}\sigma(t,kh)X_1h^{3/2}\right\}\right]\tilde{E}^t\left[\exp\left\{-\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}(\sigma(t,kh)X_1+\eta(t,kh)X_2)h^{3/2}\right\}\right]\right)$$

$$= \frac{-1}{h^2}\ln\left(\tilde{E}^t\left[\exp\left\{\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}\eta(t,kh)X_2h^{3/2}\right\}\right]\right) \qquad 4.51$$

where in (4.51) we again recognize that once the volatility parameters--for both the risk-free and risky spread--are specified, the drift parameters are known.

The recursive relation (4.45) and (4.50) play a key role in facilitating implementation of the model.

E. A Recursive Representation of Risky Bond Prices.

Analogous to the risk-neutral drifts, the prices of risky bonds in our model also have a recursive representation, which leads, in turn, to a representation in terms of bond prices of short maturities (i.e., of the form $\Pi(\tau,\tau+h)$). This representation is described in the following.

Under our assumptions:

$$\tilde{E}^t\left[\exp\left\{-\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}\left[\varphi(t+h,kh)-\varphi(t,kh)\right]h\right\}\right]=1 \qquad 4.52$$

$$\tilde{E}^t\left[\frac{\exp\left\{-\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}\varphi(t+h,kh)h\right\}}{\exp\left\{-\sum_{k=\frac{t}{h}+1}^{\frac{T}{h}-1}\varphi(t,kh)h\right\}}\right]=1$$

$$\tilde{E}^t\left[\frac{\Pi\Pi(t+h,T)}{\exp\{\varphi(t,t)h\}\Pi(t,T)}\right]=1 \qquad 4.53$$

(See equation (4.49)).

Rearranging terms and using the fact that:

$$\exp\{-\varphi(t,t)h\}=\Pi(t,t+h)$$

we get

$$\Pi(t,T)=\Pi(t,t+h)\tilde{E}^t\left[\Pi(t+h,T)\right] \qquad 4.54$$

Iterating (4.54)

$$\begin{aligned}\Pi(t,T)&=\Pi(t,t+h)\tilde{E}^t\left[\Pi(t+h,t+2h)\tilde{E}^{t+h}\left[\Pi(t+2h,T)\right]\right]\\&=\Pi(t,t+h)\tilde{E}^t\left[\Pi(t+h,t+2h)\tilde{E}^{t+h}\left[\Pi(t+2h,3h)\tilde{E}^{t+2h}[\ldots]\ldots\right]\right]\end{aligned} \qquad 4.55$$

Das and Sundaram state that the recursive structure of prices of risky bonds embodied in (4.55) facilitates computation of these prices. They also note that since all terms on the right-hand side of (4.55) have the form $F(\tau,\tau+h)$, use can be made of:

$$\begin{aligned}\Pi(t,t+h)&=\exp\{-(f(t,t)+s(t,t))\cdot h\}\\&=\exp(-f(t,t)\cdot h)\cdot\left[1-\lambda(t)+\lambda(t)\cdot\phi(t)\right]\end{aligned} \qquad 4.56$$

where we use (4.45).

to employ the forward spread components (i.e., the default and recovery rates) in this process.

CHAPTER V.

TOWARD IMPLEMENTATION OF THE MODEL FOR A SIMPLE PORTFOLIO OF BONDS

We propose to price a credit derivative on a basket of high-yield bonds. To keep the analysis simple, we consider a portfolio of two bonds.

In the Das and Sundaram (1999) model the discount rate per (semi-annual) period on a risky bond is decomposed into one risk-free forward rate and a risky forward spread. In a binomial model, this results in each node having $(2^2 =)4$ branches. For a model with 2 spreads, this means that each branch will have $(2^3 =)$ 8 branches. Even with the recursive algorithm suggested by Das and Sundaram (1999), the numerical implementation is expensive with regard to computer memory. One solution is to approximate the forward spreads of the individual risky bonds that are included in the model, by one rate or risky forward spread, which will be the aggregate risky forward spread of the portfolio. That is, the portfolio will be modeled as if it is a single bond. Yet, when the portfolio's behavior depends crucially on the behavior of the individual, constituent bond, focus shifts to the way each bond affects the portfolio. Hence, the behavior of the portfolio, when individual bonds default, will be a point of focus.

A. The risky forward spread of the portfolio

We need to approximate the forward spread on the portfolio. The portfolio forward spread will be some average of the forward spread of the bonds that are included in the portfolio. The portfolio forward spread we derive is an extension of an analysis suggested by Garbade (1988) for approximating the yield on a portfolio of several different bonds from the yields on the individual bonds in the portfolio. We first review Garbade (1988) analysis for yields.

B. Review of Garbade (1988)

The yield on a bond is defined as the discount rate that makes the present value of the bond's future cash flows equal to the market price of the bond. By extension, the yield on a portfolio of bonds is the discount rate that makes the present value of the portfolio's future cash flows equal to the total market value of the portfolio. Assume that a bond will make a total of K semi-annual payments in the future, where the Kth payment is for the amount C_K=\$100 and will occur in x_K semi-annual periods. Let B(0,K) denote the market price of the bond.

The semi-annually compounded yield on the bond is the value of R that satisfies the equation:

$$B(0,K)=\sum_{k=1}^{K} C_k (1+R)^{-x_k} \qquad 5.1$$

Thus the yield R is the discount interest rate that makes the present value of the future cash flows of the bond equal to the market price of the bond.

Consider now, a portfolio of N different bonds. Let B_i denote the market price per hundred dollars of principal value, of the *i*th bond, and let A_i denote the quantity of the *i*th bond denominated in hundreds of dollars of principal value. Finally, let $g_i(.)$ denote the present value function for \$100 principal value of the *i*th bond so that $g_i(R)$ is the present value of the bond's future cash flow discounted at the interest rate R.

It follows from (5.1) the yield on bond *i* is defined as the value of R_i that satisfies the equation:

$$B_i = g_i(R_i), \qquad 1 = 1,2,\ldots,N \qquad 5.2\ .$$

By extension the yield on the portfolio of bonds is the value of R that satisfies the equation:

$$\sum_{i=1}^{N} A_i \cdot (B_i) = \sum_{i=1}^{N} A_i \cdot g_i(R_i) \qquad 5.3\,.$$

The left-hand side of (5.3) is the total market value of the portfolio. The right-hand side of the equation is the total present value of the portfolio's future cash flows. where every payment is discounted at the common interest rate R. Thus the yield on the portfolio is the discount interest rate which makes the present value of the portfolio's future cash flows equal to the total market value of the portfolio (Garbade, 1996, p. 254).

B.1.*a. Market value-weighted average* (MVA)

Garbade (1988) views equation (5.3) as the *exact definition* of the yield on a portfolio of many different bonds. However, Garbade observes that (5.3) is not very useful in most practical situations because it require the individual yield-dependent discount (valuation) functions $g_1(.), g_2(.),\ldots, g_N(.)$. What is needed is a method to "average" the yields on the individual bonds $R_1, R_2,\ldots, R_N$, as define by (5.2), to approximate the portfolio yield (Garbade, 1996, p. 254).

The common way to approximate the yield on a portfolio is to weigh the yield on a bond by the total market value of that bond:

$$R_{MVA} = \frac{\sum_{i=1}^{N} A_i \cdot B_i \cdot R_i}{\sum_{i=1}^{N} A_i \cdot B_i} \qquad 5.4$$

where R_{MVA} is the market value-weighted average yield and A_i is the quantity of the *i*th bond denominated in hundreds of dollars of principal value.

The approximation of portfolio yield as a value-weighted average has the intuitively appealing attribute that the larger the position in a bond the more the yield on that bond 'counts'. Indeed, this attribute is often taken, as so obvious that equation (5.4) has become the conventional definition of portfolio yield. Many analyst have lost sight of the fact that (5.4) is really only an approximation to the true definition in (5.3) (Garbade, 1996, p.255).

B.1.*b* *Aggregate value of a basis point* (AVBP)

It is not disputed that individual bond yields should be weighted; however, it is *aggregate value of a basis point*, rather than its aggregate market value that should be used (Garbade, 1996, p.256). Thus, looking at equations (5.2) and (5.3) again, we may think of the problem as that of computing a set of yield increments $\Delta R_1, \Delta R_2, ..., \Delta R_N$ such that:

$$R_1 + \Delta R_1 = R_2 + \Delta R_2 = \ldots = R_N + \Delta R_N \qquad 5.5a$$

and such that :

$$\sum_{i=1}^{N} A_i \cdot (B_i) = \sum_{i=1}^{N} A_i \cdot g_i(R_i) \qquad 5.5b.$$

The portfolio yield R is any of the yields $R_i + \Delta R_i$ for i=1 or 2...or N (which are all the same from equation (5.5a). It follows from equation (5.5b) that this yield satisfies the implicit definition, equation (5.3).

To compute ΔR_i, let us consider a first-order approximation to the right-hand side of equation (5.5b):

$$\sum_{i=1}^{N} A_i \cdot (B_i) = \sum_{i=1}^{N} A_i \cdot g_i(R_i) + \sum_{i=1}^{N} A_i \cdot g_i'(R_i) \cdot \Delta R_i \qquad 5.6$$

where $g_i'(R_i)$ is the derivative of the yield-dependent value function $g_i(\cdot)$ evaluated at the yield R_i. (This is proportional to the value of a basis point for \$100 principal amount of the bond.) However, condition (5.5b) must hold, hence equation (5.6) becomes:

$$0 = \sum_{i=1}^{N} A_i \cdot g_i'(R_i) \cdot \Delta R_i \qquad 5.7$$

Since $R = R_i + \Delta R_i$ for i=1,2,..., N, this gives:

$$0 = \sum_{i=1}^{N} A_i \cdot g_i'(R_i) \cdot (R - R_i) \qquad 5.8$$

or

$$R \cdot \sum_{i=1}^{N} A_i \cdot g_i'(R_i) = \sum_{i=1}^{N} A_i \cdot g_i'(R_i) \cdot R_i \qquad 5.9$$

Therefore, we can write the approximate yield on the portfolio as:

$$R_{AVBP} = \frac{\sum_{i=1}^{N} A_i \cdot g_i'(R_i) \cdot R_i}{\sum_{i=1}^{N} A_i \cdot g_i'(R_i)} \qquad 5.10$$

where AVBP≡aggregate value of a basis point.

The approximation of portfolio yield in equation (5.10) is similar to that in equation (5.4). The difference is that the aggregate value of a basis point in that bond $(A_i \cdot g_i'(R_i))$, rather than the aggregate market value of the position $(A_i \cdot B_i)$, weights the yield on a bond. This shows that value sensitivity, rather than simple value, is a better way to weight bond yields when approximating portfolio yield. Since interest rates are the crucial factor in bond valuation, bond prices are sensitive to changes in interest rates.

C.1. Estimating the portfolio one-period forward spread

Using Garbade (1988) insight, that value sensitivity, rather than simple value, is a better way to weight bond yields when approximating portfolio yield, consider the $[T,T+h]$ period weighted portfolio forward spread, $s_{pfo}(T,T+h)$. Let us denote by A_i, the number of units of the ith zero-coupon risky bond per \$100 of payoff at time-$T+h$, expected at time-t for the period $[T,T+h]$. Denote by $s_i(T,T+h)$ the forward spread and, $g_i(s_i(T,T))$ the discount function of the forward spread, for the period $[T,T+h]$. The aggregate portfolio spread for this period is:

$$s_{pfo}(T,T)=\frac{\sum_{i=1}^{N} A_i \cdot g_i'(s_i(T,T))\cdot s_i(T,T)}{\sum_{i=1}^{N} A_i \cdot g_i'(s_i(T,T))} \qquad 5.11$$

The risky forward spread on a zero-coupon bond for the period $[T,T+h]$ is weighted by the aggregate value of a basis point in that individual bond $(A_i \cdot g_i'(s_i(T,T)))$. Value sensitivity is used when approximating portfolio forward spreads.

Note that since we are considering here the rate of change of the discount function with respect to a change in rate, it is not stochastic. Hence, if

$$g_i(s_i(T,T))=\exp\{-s_i(T,T)\cdot h\} \qquad 5.12$$

then

$$g_i'(s_i(T,T))=-h\cdot\exp\{-s_i(T,T)\cdot h\}=-h\cdot g_i(s_i(T,T)) \qquad 5.13$$

Therefore, for N=2, we have:

$$s_{pfo}(T,T)=\frac{A_1\cdot g_1'(s_1(T,T))\cdot s_1(T,T)+A_2\cdot g_2'(s_2(T,T))\cdot s_2(T,T)}{A_1\cdot g_1'(s_1(T,T))+A_2\cdot g_2'(s_2(T,T))} \quad 5.14$$

$$=\frac{A_1\cdot -h\cdot \exp\{-s_1(T,T)\cdot h\}\cdot s_1(T,T)+A_2\cdot -h\cdot \exp\{-s_2(T,T)\cdot h\}\cdot s_2(T,T)}{A_1\cdot -h\cdot \exp\{-s_1(T,T)\cdot h\}+A_2\cdot -h\cdot \exp\{-s_2(T,T)\cdot h\}}$$

$$=\frac{A_1\cdot -h\cdot g_1(s_1(T,T))\cdot s_1(T,T)+A_2\cdot -h\cdot g_2(s_2(T,T))\cdot s_2(T,T)}{A_1\cdot -h\cdot g_1(s_1(T,T))+A_2\cdot -h\cdot g_2(s_2(T,T))} \quad 5.15$$

Hence, the weight for risky forward spread $s_i(T,T)$ is

$$w_i(T,T)=\frac{A_i\cdot -h\cdot g_i(s_i(T,T))\cdot s_i(T,T)}{\sum_{i=1}^{2} A_i\cdot -h\cdot g_i(s_i(T,T))} \quad 5.16$$

C.2. Estimating the portfolio volatility of the one-period forward spread

The volatility of the one-period forward-spread of the portfolio is the weighted average of the individual one-period forward-spreads. Using the same weights as for the portfolio forward-spread, we have:

$$\eta_{pfo}^2(T,T)=w_1^2(T,T)\cdot\eta_1^2(T,T)+w_2^2(T,T)\cdot\eta_2^2(T,T)$$
$$+\eta_1(T,T)\cdot\eta_2(T,T)\cdot\rho_{12}(T,T) \quad 5.17$$

where

$\eta_{pfo}^2(T,T)\equiv$ *volatility squared of the portfolio one – period forward spread*
$\eta_i^2(T,T)\equiv$ *volatility squared of one – period forward* $s_i(T,T)$
$\rho_{12}(T,T)\equiv$ *one – period correlation between forward spreads* s_1 *and* s_2

Notice that the volatility of the one-period forward-spread of the portfolio will be greatest when the correlation between the constituting bonds is one and positive. Hence, the portfolio one-period forward-spread will take its largest value. The opposite will hold for a correlation that is one and negative. If the correlation is zero, the individual forward-spreads do not move together.

C.3. Observation about the portfolio weights

One advantage of using the value of a basis point as the weight to obtain the portfolio one-period forward spreads is that it adjusts for small, or large, values of the forward spread in each period. The adjustment occurs without the investor having to change the proportion of his/her wealth invested in each bond that make up the portfolio. If the value-weight is used the investor would have to change the proportion of wealth invested in each bond at the beginning of each period: There is no automatic adjustment for large values of the forward spreads compared to small values of the forward spreads.

C.4. Obtaining the term structure of credit spreads.

Ideally, to price a particular, risky coupon bond we obtain the term structure of credit spread for the particular bond. However, many bonds do not trade. Some bonds are offered to investors wanting to buy them, *a public issue*, and once issued they can be freely traded. Yet many of these public issues rarely trade. In a *private placement*, the issue is sold directly to a small number of qualified buyers. The debt cannot be resold to individuals, only to qualified institutional investors (Brealey &. Myers, 1996, p.359). In effect, there are no public price quotes on these bonds. So an investor may hold, in her/his portfolio, bonds for which there are no up-to-date price quotes.

A path around this problem is to compare the issue with that of an issue with similar characteristics: from the same industry, risk class, and maturity. In the end this is just an approximation since some of the characteristics of the issue are specific to the issuer.

C.5. *Example*

Bond A: Class 1 with coupon 8% per annum, paid semi-annually on a face value of $1,000 and maturity of 2 years.

Bond B: Class 2 with coupon 9% per annum, paid semi-annually on a face value of $1,000 and maturity of 2 years.

An investor holds a portfolio comprised of 400 unit of bond A and 600 unit of bond B. Given the term structure of credit spreads, the term structure of credit spreads relevant to the portfolio can be determined.

Period	Class 1 spread	Class 2 spread	Pfo spread
1	0.015890	0.015990	0.015953
2	0.017370	0.017470	0.017433
3	0.018620	0.018820	0.018746
4	0.020790	0.021010	0.020928

$$s_{pfo}(0,1,1) = \frac{400 \cdot 40 \cdot \exp(0.01589 \cdot 1) \cdot 0.01589 + 600 \cdot 45 \cdot \exp(0.01599 \cdot 1) \cdot 0.01599}{400 \cdot 40 \cdot \exp(0.01589 \cdot 1) + 600 \cdot 45 \cdot \exp(0.01599 \cdot 1)}$$
$$= 0.015953$$

$$s_{pfo}(0,1,2) = \frac{400 \cdot 40 \cdot \exp(0.01737 \cdot 1) \cdot 0.01737 + 600 \cdot 45 \cdot \exp(0.01747 \cdot 1) \cdot 0.01747}{400 \cdot 40 \cdot \exp(0.01737 \cdot 1) + 600 \cdot 45 \cdot \exp(0.01747 \cdot 1)}$$
$$= 0.017433$$

$$s_{pfo}(0,1,3) = \frac{400 \cdot 40 \cdot \exp(0.01862 \cdot 1) \cdot 0.01862 + 600 \cdot 45 \cdot \exp(0.01882 \cdot 1) \cdot 0.01882}{400 \cdot 40 \cdot \exp(0.01862 \cdot 1) + 600 \cdot 45 \cdot \exp(0.01882 \cdot 1)}$$
$$= 0.018746$$

$$s_{pfo}(0,1,4) = \frac{400 \cdot 40 \cdot \exp(0.02079 \cdot 1) \cdot 0.02079 + 600 \cdot 45 \cdot \exp(0.02101 \cdot 1) \cdot 0.02101}{400 \cdot 40 \cdot \exp(0.02079 \cdot 1) + 600 \cdot 45 \cdot \exp(0.02101 \cdot 1)}$$
$$= 0.020928$$

C.6. Binomial model of forward spread

So in our binomial model of a risk-free forward rate and a risky-portfolio forward spread we have:

$$X_1, X_{pfo} = \begin{cases} (+1,+1) & w.p. \quad (1+\rho_{pfo})/4 \\ (+1,-1) & w.p. \quad (1-\rho_{pfo})/4 \\ (-1,+1) & w.p. \quad (1-\rho_{pfo})/4 \\ (-1,-1) & w.p. \quad (1+\rho_{pfo})/4 \end{cases} \qquad 5.18$$

where ρ_{pfo} is the correlation between the risky-portfolio forward spread, X_{pfo}, and a risk-free forward rate. Duffee (1998) reports that treasury-yield spreads and risky yield spreads are negatively correlated over the period 1985-1995.

C.7. Estimating the correlation between portfolio term-structure of risky forward-spreads and the risk-free term-structure.

It is very important to note, at this time, that the portfolio does not exist in the market place, though the bonds that are included in the portfolio does exist. The correlation between the portfolio term-structure of risky forward-spreads and the risk-free term-structure may be estimated using historical data. For consistency, one can use a weighted-average of the correlation of the individual bond term-structure of forward-spreads with the risk-free term-structure.

Hence, to obtain the correlation between the risky-portfolio forward-spread, ρ_{pfo}, as used in the previous section, the following relationship can be used:

$$\rho_{pfo} = w_1 \cdot \rho_1 + w_2 \cdot \rho_2 \qquad 5.19$$

where

$$\rho_i \equiv correlation\, between\, bond\, i\, and\, the\, risk-free\, term-structure, i = 1,2$$

One observes, from (5.19) and (5.17), that as the weights change the correlation changes, and so too do the volatilities. In section (C.3), it was observed when the value of a basis point is used to obtain portfolio weights. These weights adjust for small, or large, changes in the risky forward spread each period. Therefore the portfolio volatilities and spreads calculated using these weights also adjust.

D. Estimation of the default probability for X_{pfo}

D.*a. Decomposition of Spread*

Das and Sundaram (1999), following Wilson (1997), use a Logit model to estimate the default probabilities (or default rates)—the number of corporates that default divided by the number of corporates that could have defaulted. Like them, we assume that the default probability $\lambda(t)$ at time-t is a function of interest rates in the model at time-t. So letting $F\ and\ S$ denote the entire risk-free forward and risky spread curves, a Logit specification is:

$$\lambda(F,S) = \frac{1}{e^x + 1}, \quad x = a + b \cdot F + c \cdot S \qquad 5.20.$$

We could allow for the default probabilities to also depend on the slope of the yield curve. Flat or inverted yield curves are features associated with recessions in the economy and hence, to the extent that such a relationship holds, we would expect default rates to increase as the slope of the term structure decreases (Das & Sundaram, 1999, p.9).

Logit specification for portfolio default probabilities: Our portfolio forward spread is a weighted-average of two forward spreads S_A *and* S_B such that $X_{pfo} = w_A \cdot S_A + w_B \cdot S_B$. Moreover, the identity of which security defaults is very important in a model of default risk. Hence, we specify a Logit model of default probabilities for each of the risky securities that are included in our portfolio. Thus, if $\lambda_j(t) \quad (j = A, B)$, is the probability that bond j defaults then $(1-\lambda_j(t))$ is the probability that bond j does not default. Therefore, assuming that bond A default is independent of the probability that bond B defaults, as is implicit in a binomial model, we have:

$$(S_A, S_B) = \begin{cases} (+D,+D) & w.p. \quad (1-\lambda_A)\cdot(1-\lambda_B) \quad -- \quad \textit{neither bond A nor bond B defaults} \\ (+D,-D) & w.p. \quad (1-\lambda_A)\cdot\lambda_B \\ (-D,+D) & w.p. \quad \lambda_A \cdot (1-\lambda_B) \qquad\qquad \vdots \\ (-D,-D) & w.p. \quad \lambda_A \cdot \lambda_B \qquad\qquad -- \quad \textit{both bond A and bond B default.} \end{cases} \qquad 5.21$$

Default probability $\lambda_A(t)$ is estimated by the Logit model:

$$\lambda_A(t) = \frac{1}{e^{x_A}+1} \qquad x_A = a_A + b_A \cdot F + c_A \cdot S_A \qquad 5.22a$$

and default probability $\lambda_B(t)$ is estimated by:

$$\lambda_B(t) = \frac{1}{e^{x_B}+1} \qquad x_B = a_B + b_B \cdot F + c_B \cdot S_B \qquad 5.22b.$$

Alternative Logit specifications for portfolio default probabilities: However, it is not reasonable to assume that the risky spreads in our portfolio are not correlated in some way. If a risky bond defaults, it is very likely that its neighbors, other risky bonds of the same class or a class close to it, may also default. With this insight in mind, we consider two alternative Logit specifications for portfolio default probabilities:

Case 1. Probability of default influenced by risky neighbor's spread

The default probability $\lambda_i(t)$ is estimated by the Logit model:

$$\lambda_i(t) = \frac{1}{e^{x_i} + 1} \quad x_i(t) = a_i + b_i \cdot F(t) + c_i \cdot S_i(t) + d_i \cdot S_j(t) \qquad i \neq j \quad 5.22c$$

That is, the probability that bond *i* defaults is explained by the risk-free forward rate, bond *i*'s risky forward spread, S_i, and the risky forward spread of its neighbor bond *j*, S_j.

Case 2. Probability of default influenced by risky neighbor lagged probability of default

The default probability $\lambda_i(t)$ is estimated by the Logit model:

$$\lambda_i(t) = \frac{1}{e^{x_i} + 1} \quad x_i(t) = a_i + b_i \cdot F(t) + c_i \cdot S_i(t) + d_i \cdot \lambda_j(t-1) \qquad i \neq j \quad 5.22d$$

That is, the probability that bond *i* defaults is explained by the risk-free forward spread, bond *i*'s risky forward spread, S_i, and the probability of default of its neighbor bond *j* in the past period, $\lambda_j(t-1)$.

D.*b. Risk-neutral probability of default*

Estimates of the parameters of the process is in equations (5.20), (5.22a) and (5.22b) are based on real world data. That is, the probabilities of default λ_A *and* λ_B are real world probabilities, which we, from now on, denote by λ_A^P *and* λ_B^P. Yet, our model is set in a risk-neutral world, so we must translate the actual probabilities to a risk-neutral measure. Following Das and Sundaram (1999), we assume the recovery rates are the same in both worlds. Letting $\xi_A(t)$ *and* $\xi_B(t)$ denote the time-t premium for bearing default risk, for bond A and bond B respectively, then equation (3.9) under actual probabilities become:

$$\exp\{-s_j(t,t)\cdot h\} = \exp\{-\xi_j(t)\cdot h\}\cdot\left[\left(1-\lambda_j^P(t)\right)+\phi_j(t)\cdot\lambda_j^P(t)\right] \qquad 5.23$$

where $\phi_j(t)$, $j=$ A,B is the recovery rate if bond j default at time-t. The difference between (3.9) and (5.23) is that (3.9) is a relationship developed in the risk-neutral world, where, by definition, there is no premium for taking on risk. Hence, expression (5.23) would follow the same derivation as (3.9), except that it is set in the real world.

In the real world, we would expect the risk-premium term to be positive for risky securities j, ($j=$ A,B). So from (5.23) we get:

$$\lambda_j(t) = \lambda_j^P(t)\cdot\left[\frac{1-\exp\{-s_j(t,t)\cdot h\}}{1-\exp\{-\left(s_j(t,t)-\xi_j(t)\right)\cdot h\}}\right] \qquad 5.24$$

after comparison with (3.9). Das and Sundaram (1999) notes that expression (5.24) implies the intuitive condition that $\lambda_j(t) > \lambda_j^P(t)$ whenever the risk-premium ξ_j is positive.

Expressions (5.23) and (5.24) can be used to estimate the parameters of (5.22a) and (5.22b). From (5.23) we have:

$$\phi_j(t) = \frac{1}{\lambda_j^P(t)}\cdot\left[\exp\{-\left(s_j(t,t)-\xi_j(t)\right)\cdot h\}-\left(1-\lambda_j^P(t)\right)\right] \qquad 5.25.$$

Expression (5.20) can be rewritten as:

$$\ln\left(\frac{1}{\lambda_j^P(t)}-1\right)=a_j+b_j\cdot F+c_j\cdot S_j \qquad 5.26.$$

Letting $\overline{\phi_{j,Av}(t)}$ be the average recovery rate in the data for class j bonds, we have the following minimization problem:

$$Minimize_{a_j,b_j,c_j} \sum \varepsilon_j^2(t)$$

$$subject\ to \quad \ln\left(\frac{1}{\lambda_j^P(t)}-1\right)=a_j+b_j\cdot F+c_j\cdot S_j+\varepsilon_j \qquad 5.27$$

$$\overline{\phi_{j,Av}(t)}=\left[\exp\{-(s_j(t,t)-\xi_j(t))\cdot h\}-(1-\lambda_j^P(t))\right]\Big/\lambda_j^P(t).$$

Thus, given the value of parameters (a_j,b_j,c_j) and the risk premium ξ_j, the actual default probability at time-t may be obtained from (5.20). We obtain the recovery rate $\phi_j(t)$ by using (5.25) and the risk-neutral probability $\lambda_j(t)$ by using (5.24).

E. Conclusion

The model, developed in this research, is implemented using an adaptation of Das-Sundaram algorithm. However, since it is used to make valuations based on a simple portfolio of two bonds, the implementation is more complicated. The following summarizes a comparison of this model with the Das-Sundaram algorithm at the key points where portfolio implementation requires changes:

1. Initialization

Das-Sundaram imports information at the initial step that includes the vector of risk-free rates and vector of volatilities for the risk-free rates; vector of the risky spread and vector of volatilities of the risky-spread; the correlation of the risk-free rates with the risky spread; the size of the time intervals; the exercise price; the parameters a1, b1, and c1 to generate default rates; and finally, the risk premium. The portfolio implementation requires all of the same and more.

The parameters to generate default rates for each bond in the portfolio is required; as well as the vector of the weighted average risky-spreads--the risky portfolio spreads; and the recovery rates for each bond.

2. Cumulative default rates

For Das-Sundaram, calculation of the cumulative default rates is easy. Their algorithm follows the branch where default does not occur. The probability of not defaulting is one minus the probability of default. For the portfolio, there are three instances of possible default (cf. Expression 5.22). So again, the probability of no default is one minus the probability of default, where, the probability of default is the sum:

$$\lambda_A + \lambda_B - \lambda_A \cdot \lambda_B \qquad 5.28.$$

The program below, by Das-Sundaram, prices a credit spread option written on an underlying credit spread. The contract pays off at some defined maturity if the spread is trading above a strike level K, the exercise price.

```
(** Program to generate the HJM Tree with default risk recursively.**).
CRD[f0_, fsig0_, s0_, ssig0_, rho_, h_, exprice_, a_, b_, c_, xi_] := Module [
        {n, puu, pud, pdu, pdd},
 (* Number of levels is equal to the length of the risk-free vector*)
n=length[f0];
puu=(1+rho)/4; pud=(1-rho)/4; pdu=(1-rho)/4; pdd=(1+rho)/4;
CRVAL[level_, f_, fsig_, s_, ssig_, cumdef] :=
CRVAL[level, f, fsig, s, ssig, cumdef]=
Module[{i, m, j, alpha, beta, fuu ,fud, fdu, fdd, suu, sud, sdu,
          sdd, fsigma, ssigma, pd, recov, cumd},
```

```
(*Careful about n or n-1; this is where the check for the last level occurs*)
        If [level==n-1,
           result=Max[0, s[[1]]-exprice]*100;
           ];
        If [level < n-1,
           m=length[f]-1;
           fuu=Take[f, -m]; fud=fuu; fdu=fud; fdd=fdu; (*Initialize next level f*)
           suu=Take[s, -m]; sud=suu; sdu=sud; sdd=sdu; (*Initialize next level s*)
           fsigma=Take[fsig, -m];
           ssigma=Take[ssig, -m];
           alpha=Table[0, {k,m}];
           beta=Table[0,{k, m}];
            For [j=1, j<=m, j++,
               If [j==1,
                       alpha[[j]]=Log[0.5*Exp[-fsigma[[j]]*h*Sqrt[h]]+
                                   Exp[fsigma[[j]]*h*Sqrt[h]])]/h^2;
                      beta[[j]]=Log[puu*Exp[(-fsigma[[j]]-ssigma[[j]])*h*Sqrt[h]]+
                                    pud* Exp[(-fsigma[[j]]=ssigma[[j]])*h*Sqrt[h]]+
                                    pdu* Exp[(fsigma[[j]]-ssigma[[j]])*h*Sqrt[h]]+
                                    pdd* Exp[(fsigma[[j]]+ssigma[[j]])*h*Sqrt[h]]]/h^2-
                                        alpha[[j]];
                 ];
                 If [j>1,
                     alpha[[j]]=Log[0.5*
                          (Exp[-Sum[fsigma[[k]], {k,j}]*h*Sqrt[h]+
                           Exp[Sum[fsigma[[k]], {k, j}]*h*Sqrt[h]])]/h^2-
                           Sum[alpha[[k]], {k, j-1}];
                    beta[[j]]=Log[
                          puu*Exp[Sum[(-fsigma[[j]]-ssigma[[j]])*h*Sqrt[h], {k,j}]]+
                                  pud* Exp[Sum[(-fsigma[[j]]+ssigma[[j]])*h*Sqrt[h], {k, j}]]+
                                  pdu* Exp[Sum[(fsigma[[j]]-ssigma[[j]])*h*Sqrt[h], {k, j}]]+
                                  pdd* Exp[Sum(fsigma[[j]]+ssigma[[j]])*h*Sqrt[h], {k, j}]]]/h^2-
                                     Sum[alpha[[k]], {k, j}]-Sum[beta[[k]], {k, j-1}];
                   ];
               ];
           fuu=fuu+alpha*h+fsigma*Sqrt[h];
           fud=fud+alpha*h+fsigma*Sqrt[h];
           fdu=fdu+alpha*h-fsigma*Sqrt[h];
           fdd=fdd+alpha*h-fsigma*Sqrt[h];
           suu=suu+beta*h+ssigma*Sqrt[h];
           sud=sud+ beta*h-ssigma*Sqrt[h];
           sdu=sdu+ beta*h+ssigma*Sqrt[h];
           sdd=sdd+ beta*h-ssigma*Sqrt[h];


(** Calculate the cumulative default rates.**)
           cumd=cumdef+(1-cumdef)/(1+Exp[a+b*f[[1]]+c*s[[1]]])*
                  (1-Exp[-s[[1]]*h])/(1-Exp[(xi*s[[1]]-s[[1]]*h);
           result=Exp[-(f[[1]])*h]*
                                  (puu*CRVAL[level+1, fuu, fsigma, suu, ssigma, cumd]+
                                   pud* CRVAL[level+1, fud, fsigma, sud, ssigma, cumd]+
                                   pdu* CRVAL[level+1, fdu, fsigma, sdu, ssigma, cumd]+
                                    pdd* CRVAL[level+1, fdd, fsigma, sdd, ssigma, cumd]];
        (** End IF Level < n-1**)
         ];
         Return[result];
```

```
        ]:
Return[CRVAL[0. f0. fsig0. s0. ssig0. 0]]:
];
```

The required modifications are the following:

a. Initialization

```
CRD[f0_,fsig0_, s0_, ssig0_, rho_. h_. exprice_. a1_,b1_. c1_. a2_. b2_. c2_. xi1_. xi2]
        := Module [{n. puu. pud. pdu. pdd}.
```

and

b. Calculation of the cumulative default rates

```
cumd=cumdef+(1-cumdef)*{[1/(1+Exp[a1+b1*f[[1]]+c1*s[[1]]])*
 (1-Exp[-s[[1]]*h])/(1-Exp[(x1i*s[[1]]-s[[1]]*h)]+[ 1/(1+Exp[a2+b2*f[[1]]+c2*s[[1]]])*
(1-Exp[-s[[1]]*h])/(1-Exp[(x2i*s[[1]]-s[[1]]*h)]-
 [[1/(1+Exp[a1+b1*f[[1]]+c1*s[[1]]])* (1-Exp[-s[[1]]*h])/(1-Exp[(x1i*s[[1]]-
s[[1]]*h)]*[ 1/(1+Exp[a2+b2*f[[1]]+c2*s[[1]]])*(1-Exp[-s[[1]]*h])/(1-Exp[(x2i*s[[1]]-
s[[1]]*h)]};.
```

Notice that expression (5.26) is implemented to calculate the default probabilities. However, for simplification, use is made of the portfolio spread. While strictly speaking, the spread for each bond is required. In addition, as a further simplification, use could be made of the weighted-average risk premium of the portfolio. Therefore, the model is implementable for pricing a credit spread call option for a portfolio of two bonds, with only minor changes in the algorithm. However, the modification for credit default swaps will prove to be much more complicated.

Chapter VI.

Default Rates

In this, the empirical chapter, we examine historical default rates. We are concerned with whether default rates are Markov. In any model of credit risk, default rates/probabilities must play an important role. The Das-Sundaram (1999) model utilizes a Logit specification for default rates. As an alternative, to make the model more consistent with market prices, default rates themselves could be modeled as a random process (c.f. Wilmott, 1998, p. 569). The Markov property of random processes is "of fundamental importance in modeling in finance" (Wilmott, 1998, p. 57). It essentially states that the expected value of a random variable, conditional upon all of the past events, only depends on the previous value of the random variable.

A. Data

Our data for default rates is extracted from Moody's Investors Services January 1999 report, *Historical Default rates of Corporate Bond Issuers, 1920-1998*. Moody's bases the results of its study on a proprietary database of ratings and defaults for industrial and transportation companies, utilities, financial institutions, and sovereigns that have issued long-term debt to the public. Municipals, structured finance transactions, private placements, and issuers with only short-term debt rating are excluded.

Moody's defines a *bond default* as any missed or delayed disbursement of interest and/or principal, bankruptcy, receivership, or distressed exchange where (i) the issuer offered bondholders a new security, or package of securities, that amount to a diminished financial or, (ii) the exchange had the apparent purpose of helping the borrower avoid default (Moody's Investor Services, 1999, p. 10).* Moody's rating incorporates assessments of both the likelihood and the severity of default. Hence, in order to calculate the default rates, which are the assessments of the likelihood of default, severity considerations are held constant. This is done by considering the rating on each company's senior unsecured debt or, if there is none, by statistically implying such a rating from rated subordinated or secured debt. Since the likelihood of default is essentially the same for all of a firm's public debt issues, irrespective of size, Moody's believes that weighing their statistics by the number of bond issues or their par amounts will simply bias their results towards the characteristics of large issuers. Therefore, the issuer is their unit of study rather than individual debt instruments or outstanding dollar amounts of debt.

The default rates calculated are fractions in which the numerator represents the number of issuers that defaulted in a particular time period, and the denominator is the number of issuers that could have defaulted in that time period. Thus, one-year default rates for any rating classification--say B rating--is the number of Moody's B-rated issuers that defaulted over the following one-year period divided by the number of Moody' B-rated issuers that could have defaulted over that period.

Moody's employs a cohort approach to calculating multi-year default rates. A cohort consists of all issuers holding a given senior implied rating at the start of a given year. These issuers are then followed through time, keeping track of when they default or leave the rated universe for non-credit-related reasons (e.g. maturity of debt). Thus the cohorts are dynamic and allow the estimation of cumulative default probabilities over multi-year horizons. Hence, by forming and tracking cohorts of all Moody's rated issuers with debt outstanding as of January 1 of each year, Moody's replicate the experience of a portfolio of both seasoned and new-issue bonds purchased in a given year. Moody's believes cohort-based default rates can answer questions such as, "What was the probability that a Baa-rated issuer with bonds outstanding as of January 1, 1985 would default by 1998?" When the default probabilities are extracted from the cumulative rates for the 1985 cohort, single period default probabilities are obtained. Hence, cohort-based default rates can answer questions such as, " What was the likelihood that a Baa-rated issuer with bonds outstanding as of January 1, 1985 would have defaulted in the period, say 1995-1996?" (Moody's Investor Services, 1999, p. 10).

B. Methodology

The two simple Markov tests performed in this research utilize linear models on the cohort-based one-period default rates. The 1971 to 1979 cohorts are used because they include the most observations, twenty. So the 1970 cohort would have twenty observations from 1970 to 1989 and, the 1979 cohort has observations from 1979 to 1998. The first model, of a Markov process, is a linear auto-regressive model of order one and, the second model uses the previous cohort as the independent variable.

Auto-regressive (AR) models: The auto-regressive models are of the form:

$$y_t = \rho_1 \cdot y_{t-1} + \rho_2 \cdot y_{t-2} + \cdots + \rho_n \cdot y_{t-p} + u_t \qquad 6.2$$

That is, it is AR(p), where p is the order of the process. The variable y_t is the current (time-t) value of the random variable y, and is related to its values in the past

$1, 2, \cdots, p$ periods. The variable u_t is the white-noise error term assumed to have a mean equal to zero, variance equal to a constant, and uncorrelated with past values of itself. More specifically, a Markov process is an AR(1) process represented by:

$$y_t = \rho_1 \cdot y_{t-1} + u_t \qquad 6.3$$

Hence the empirical model actually tested can be written in the form:

$$\hat{y}_t = \hat{\rho}_0 + \hat{\rho}_1 \cdot \hat{y}_{t-1} + \hat{u}_t \qquad 6.4$$

which says that the current value of the dependent variable is a linear function of a constant and one lag of the dependent variable, and the value of the current disturbance. For the model to be Markov, the coefficient of the lagged dependent variable must be significantly different from zero.

Previous-cohort models: Given the manner in which Moody's builds their default rates data base, the previous-cohort model that we describe is a more logical representation of Markov processes than the auto-regressive model.

Recall, from our previous description of the data, that a cohort formed at *time-t* *includes* any issuers at time-t, *excluding* the ones that go out of existence between the periods t and t-1, and *including* the new issuers during this period.

Previous-cohort models are of the form:

$$y_t^{cohort} = \rho_1 \cdot y_{t-1}^{cohort} + \rho_2 \cdot y_{t-2}^{cohort} + \ldots + \rho_p \cdot y_{t-p}^{cohort} + u_t \qquad 6.5$$

which say that the time-t cohort is a linear function of several previously formed cohorts, and u_t the white-noise error term assumed to have mean equal to zero, variance equal to a constant, and uncorrelated with past values of itself. More specifically, in this research, the previous-cohort model is of the form:

$$y_t^{cohort} = \rho_1 \cdot y_{t-1}^{cohort} + u_t \qquad 6.6$$

Since each cohort contains any surviving old issuers and new issuers, this is a reasonable Markov model.

Hence the empirical model actually tested can be written in the form:

$$\hat{y}_t^{cohort} = \hat{\rho}_0 + \hat{\rho}_1 \cdot \hat{y}_{t-1}^{cohort} + \hat{u}_t \qquad 6.7$$

which says that the current value of the dependent variable (the current cohort) is a linear function of a constant, the cohort formed in the previous period, and the value of the current disturbance. For the model to be Markov, the coefficient of the previous cohort must be significantly different from zero.

Therefore, in the empirical tests the two models will be estimated. If the coefficient of the lagged variable is found to be statistically, significantly different from zero, it will be concluded that the model follows a Markov process.

CHAPTER VII.

RESULTS

A. Linear-Autoregressive AR(1) processes.

First is an examination of the statistical output (c.f. Table VII.A[7] or Appendix A) from testing the linear-autoregressive processes of order one, AR(1) processes. One of the first things examined in the output is the autocorrelation check of residuals. If the *Q*-statistic has significant p-values, i.e. $p \leq 0.05$, then we may conclude that the estimated model provides a poor fit to the data. There are nineteen included observations, so the degree of freedom for the *t*-statistic is $m=17$. Hence the critical, two-tailed *t*-value, at an $\alpha = 0.05$ level of significance, is $t(m=17)=2.11$. If the *t*-statistic is less than $t=2.11$, we cannot reject the null hypothesis, at an $\alpha = 0.05$ level of significance, that the coefficient of the first lagged variable is not statistically different from zero.

Tested is an AR(1) model for cohorts formed in 1972 through to cohorts formed in 1979, for high-yield bonds in classes Baa, Ba, and B. For all the models tested, the *Q*-statistic indicates that the models were a good fit of the data, since all the *p*-values for the *Q*-statistic were much larger than 0.05, indicating insignificant *Q*-values. In each case, the hypothesis that the coefficient of the lagged variable is not statistically different from zero could not be rejected. The two exceptions are the models of Baa class bonds for the cohorts formed in 1972 and 1978.

---

[7] Table VII.A. summarizes within the chapter the information provided in Appendix A.

Summary table of the diagnostic statistics for the AR(1) models of Cohort 1979 Class Baa through Cohort 1971 Class B presented in Appendix A.; 19 observations are included.

Table VII. A.

	Statistics							
No.	Models	Constant C(1)	t-stat of C(1)	Coefficient C(2)	t-stat of C(2)	F-stat*	Prob(Q-stat)	
1	Baa79	0.651520	2.793715	-0.161246	-0.673651	0.453806	Insign.	
2	Ba79	1.583715	0.696820	0.242631	0.522473	0.272978	Insign.	
3	B79	2.849196	2.199290	0.109626	0.44742	0.206790	Insign.	
4	Baa78	0.284288	1.653095	0.466785	2.176240	4.736022	Insign.	
5	Ba78	1.274250	2.038689	0.224686	0.933977	0.872312	Insign.	
6	B78	2.793920	2.144385	0.090642	0.372281	0.138593	Insign.	
7	Baa77	0.292146	1.693272	0.439232	1.969705	3.879736	Insign.	
8	Ba77	1.354794	2.273475	0.117868	0.484513	0.234752	Insign.	
9	B77	2.005815	1.334441	0.332253	1.438943	2.070556	Insign.	
10	Baa76	0.343909	1.868877	0.412386	1.866403	3.483459	Insign.	
11	Ba76	1.495745	2.386007	0.088598	0.360563	0.130006	Insign..	
12	B76	1.772341	1.338585	0.325161	1.417712	2.009908	Insign.	
13	Baa75	0.369306	1.961004	0.370689	1.645630	2.708098	Insign.	
14	Ba75	1.535071	2.404910	0.110334	0.449242	0.201819	Insign.	
15	B75	1.390277	1.396762	0.236632	1.030010	1.060920	Insign.	
16	Baa74	0.459843	2.419239	0.265180	1.133962	1.285869	Insign.	
17	Ba74	1.608063	2.345331	0.171698	0.732109	0.535984	Insign.	
18	B74	0.553905	0.963638	0.128480	0.745055	0.555107	Insign.	
19	Baa73	0.427741	1.999809	0.383830	1.639805	2.688960	Insign.	
20	Ba73	1.551286	2.534385	0.087237	0.367632	0.135153	Insign.	
21	B73	0.650146	1.018727	-0.069170	-0.297401	0.088447	Insign.	
22	Baa72	0.234514	1.346822	0.800201	3.306480	10.93281	Insign.	
23	Ba72	1.139042	2.000490	0.351724	1.057393	1.118081	Insign.	
24	B72	0.637969	1.028535	-0.067290	-0.327826	0.107470	Insign.	
25	Baa71	0.423310	2.776810	0.269820	1.133875	1.285674	Insign.	
26	Ba71	0.809035	2.103163	0.238589	0.998538	0.997078	Insign.	
27	B71	0.865315	1.181971	-0.008436	-0.035716	0.001276	Insign.	

*Critical F-stat= $F(1,18) = 4.41$

For these cases, the 1972 and 1978 cohorts of Baa class bonds, again the Q-statistics indicate the models are a good fit of the data. However, the conclusion is that the coefficients of the lagged variables are statistically different from zero. These are the two cases of the default rates following a Markov process.

B. Previous-Cohort AR(1) processes.

Examination of the statistical output (c.f. Table VII.B[8] or Appendix B) of testing the previous-cohort AR(1) processes proceeds in the same manner as above, for the Q-statistic. In these models, there are twenty included observations, so the degree of freedom for the t-statistic is m=18. Hence the critical, two-tailed t-value, at an $\alpha = 0.05$ level of significance, is $t(m=17)=2.101$. If the t-statistic is less than t=2.101, we cannot reject the null hypothesis, at an $\alpha = 0.05$ level of significance, that the coefficient of the first lagged variable is not statistically different from zero.

Tested are AR(1) models for cohorts formed in 1971 through to cohorts formed in 1979, for high-yield bonds in classes Baa, Ba, and B. For all the models tested, the Q-statistic indicates that the models were a good fit of the data, since all the p-values for the Q-statistic were much larger than 0.05; indicating insignificant Q-values. In each case, the hypothesis that the coefficient of the lagged variable is not statistically different from zero could not be rejected. The three exceptions are the models of Baa class bonds for the cohorts formed in 1972, 1977, 1978 and 1979.

---

[8] Table VII.B. summarizes within the chapter the information provided in Appendix B.

Summary table of the diagnostic statistics for the previous-cohort models of default-rates for the cohorts formed 1971 through 1979, high yield bonds in class Baa, Ba, and B presented in Appendix B; 20 observations are included.

Table VII. B.

	Statistics							
No.	**Models Dep.**	**Prev**	**Constant C(1)**	**t-stat of C(1)**	**Coefficient C(2)**	**t-stat of C(2)**	**F-stat***	**Prob(Q-stat)**
1	Baa79	Baa78	0.188985	0.905865	0.679200	2.544166	6.472779	Sign.
2	Ba79	Ba78	0.577762	1.115083	0.719337	3.505738	12.36042	Insign.
3	B79	B78	3.785839	3.035786	-0.231304	-0.967398	0.935859	Insign.
4	Baa78	Baa77	0.233967	1.381834	0.525112	2.338811	5.470038	Insign.
5	Ba78	Ba77	1.416668	2.449569	0.143992	0.594449	0.353370	Insign.
6	B78	B77	2.716636	2.513066	0.163432	0.959262	0.920183	Insign.
7	Baa77	Baa76	0.277002	1.773652	0.435248	2.262294	5.117975	Insign.
8	Ba77	Ba76	1.293101	2.252130	0.121028	0.524142	0.274725	Insign.
9	B77	B76	2.173886	1.589740	0.374194	1.539711	2.370711	Insign.
10	Baa76	Baa75	0.317560	1.839989	0.427695	2.019372	4.077862	Insign.
11	Ba76	Ba75	1.410326	2.366596	0.120350	0.511580	0.261714	Insign.
12	B76	B75	1.727937	1.299198	0.359617	1.141814	1.303740	Insign.
13	Baa75	Baa74	0.346968	1.723156	0.354132	1.393319	1.941337	Insign.
14	Ba75	Ba74	1.102488	2.488268	0.160970	0.821174	0.674327	Insign.
15	B75	B74	1.818920	2.016705	0.262390	0.945177	0.893360	Insign.
16	Baa74	Baa73	0.331029	1.827508	0.378550	1.861366	3.464572	Insign.
17	Ba74	Ba73	1.595266	2.376390	0.143847	0.543387	0.295270	Insign.
18	B74	B73	1.067839	1.453127	0.169725	0.617708	0.381563	Insign.
19	Baa73	Baa72	0.527610	2.635959	0.252648	1.087773	1.183249	Insign.
20	Ba73	Ba72	1.238720	2.081729	0.248840	0.950312	0.903093	Insign.
21	B73	B72	0.672738	1.064048	0.096462	0.449371	0.201934	Insign.
22	Baa72	Baa71	0.354204	1.914338	0.589794	2.152532	4.633394	Insign.
23	Ba72	Ba71	1.493704	2.760943	-0.011287	-0.032737	0.001072	Insign.
24	B72	B71	0.845605	1.210144	0.068471	0.296044	0.087642	Insign.
25	Baa71	Baa70	0.382108	2.547132	0.284540	1.266421	1.603822	Insign.
26	Ba71	Ba70	0.838219	2.195955	0.167254	0.746203	0.556819	Insign.
27	B71	B70	0.792106	1.146647	0.123065	0.987195	0.974553	Insign.

*Critical F-stat= $F(1, 19) = 4.38$

In these exceptional cases, the 1972, 1977, 1978 and 1979 cohorts of Baa class bonds, again the *Q*-statistic indicates the models are a good fit of the data. However, the conclusion is that the coefficients of the lagged variables are statistically different from zero. These are the four cases of the default rates following a Markov process.

Several other tests were also conducted on the default rates data to determine if default rates follow an autoregressive process. Test were also conducted on AR(2) and AR(3) models. The statistical output, specifically the *Q*-statistic suggests that both types (linear auto-regressive and previous cohort) of autoregressive models are a good fit of the data. However, with the exception of the class Ba bonds in the cohort formed in 1979, the estimated parameters were found to be individually, and simultaneously statistically insignificant from zero. It should be noted that in most cases the intercept parameter, individually, was significantly different from zero. Hence, default rates may maintain an average level over time.

C. Default rates/probabilities conclusion

Given the results of the statistical tests, one must therefore conclude that default rates do not follow a Markov process. This is a very important conclusion, since the Markov assumption is often made about default rates, either implicitly or explicitly, in the literature. If the Markov assumption is indeed a bad assumption, then practitioners will get the wrong results. This will eventually lead to very large losses in the market place.

Yet, the Markov assumption is a very convenient assumption, since there are many results and techniques known about Markov processes that may simplify the practitioners work. In this regard, the exceptions provide some evidence to warrant a closer examination of the default rates data using more sophisticated tests to determine if they follow a Markov process.

CHAPTER VIII

CONCLUSION

This work concludes that the Das-Sundaram model is implementable for a small portfolio of bonds. However, the binomial tree rapidly becomes extremely large and complicated, with the addition of each risky forward-spread. Security defaults are very important in a model of default risk; hence, a model of default risk has to be specified for each individual bond in the portfolio. Thus, although the model takes advantage of the recursive pricing structure, the binomial tree grows exponentially with the addition of each risky forward-spread.

Modifying the Das-Sundaram algorithm, to find the price of a credit spread call option, is relatively simple. Yet, the modification for credit default swaps will prove to be much more complicated. Modification for credit default swaps requires that the recovery rate of each bond is considered, and what happens to the bond that does not default.

There are several points that call for further investigation:

1. Correlation is used as a measure of the relationship among spreads. In addition, weighted average correlation is used to measure the relationship between the portfolio risky forward-spread and the risk-free forward rate. Correlation assumes that the relevant distributions are normal, and hence, the relationships are linear. However, it is not clear that there is much support for the normality assumption. Some recent researchers suggest that copulas may be a better measure (cf. Li, David. X. (2000); Artzner, Delbaen, and Heath (1997).

2. In addition, there needs to be an empirical investigation of the choice of investors' strategies. For instance, a determination of whether to buy one credit derivative to protect the portfolio, or buy a credit derivative to protect each bond in the portfolio. One of the criteria in the investigation should be cost-effectiveness.
3. Furthermore, an investigation should be undertaken of how well the prices produced by this model compare with other prices in the market.
4. From the results of the statistical test, one must conclude that default rates do not follow a Markov process. Yet, the Markov assumption is a very convenient assumption, since there are many results and techniques known about Markov processes that may simplify the practitioners work. In this regard, the exceptions provide some evidence to warrant a closer examination of the default rates data using more sophisticated test to determine if they follow a Markov process.
5. Finally, Glasserman and Zhao (2000) find that the simple Euler scheme (cf. Equations (4.31) and (4.32)), used to discretize the model of forward rates and risky forward-spreads, leads to a discrete-time bias. Given their work, the reconsideration of the Das-Sundaram model is necessary.

APPENDIX A.

Test for a Markov process in Moody's default rate data for the cohorts formed in the years 1971 through 1979, high-yield bonds in class Baa, Ba, B. The process is modeled as a first order, linear auto-regressive process.

Auto-Regressive model of Cohort79 Class Baa

LS // Dependent Variable is BAAR79
Date: 04/18/00 Time: 16:34
Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BAAR79 =C(1)+C(2)*BAAR79(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.651520	0.233209	2.793715	0.0125
C(2)	-0.161246	0.239362	-0.673651	0.5096

R-squared	0.026000	Mean dependent var	0.561053
Adjusted R-squared	-0.031294	S.D. dependent var	0.818365
S.E. of regression	0.831071	Akaike info criterion	-0.270779
Sum squared resid	11.74154	Schwarz criterion	-0.171365
Log likelihood	-22.38743	F-statistic	0.453806
Durbin-Watson stat	1.877265	Prob(F-statistic)	0.509589
Critical-t		Critical-F	

Date: 04/18/00 Time: 16:41
Sample: 2 20
Included observations: 19

```
      ACF                   PACF            AC     PAC     Q-Stat     Prob

  .    |  .    |      .    |  .   |1    0.038  0.038    0.0321    0.858
  .    |**.    |      .    |**.   |2    0.324  0.323    2.4942    0.287
  .    |  .    |      .    |  .   |3    0.063  0.048    2.5942    0.459
  . ** |  .    |      .*** |  .   |4   -0.291 -0.446    4.8520    0.303
  .  * |  .    |      .  * |  .   |5   -0.084 -0.146    5.0543    0.409
  .*** |  .    |      .  * |  .   |6   -0.361 -0.121    9.0453    0.171
  .    |  .    |      .    |* .   |7   -0.023  0.161    9.0629    0.248
  .  * |  .    |      .    |  .   |8   -0.066  0.054    9.2197    0.324
  .  * |  .    |      .*** |  .   |9   -0.163 -0.337   10.285     0.328
  .    |* .    |      . ** |  .   |10   0.073 -0.219   10.520     0.396
  .  * |  .    |      .    |* .   |11  -0.095  0.071   10.971     0.446
  .  * |  .    |      .    |  .   |12  -0.063 -0.009   11.200     0.512
  .    |  .    |      .  * |  .   |13  -0.002 -0.098   11.200     0.594
  .    |  .    |      .    |  .   |14   0.038 0.014    11.312     0.661
  .    |  .    |      .  * |  .   |15   0.023 -0.128   11.366     0.726
  .    |  .    |      .  * |  .   |16   0.018 -0.137   11.407     0.784
  .    |  .    |      .    |  .   |17   0.053 -0.003   11.967     0.802
```

Auto-Regressive models Cohort 1979 Class Ba

LS // Dependent Variable is BAR79
Date: 04/18/00 Time: 17:18
Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BAR79 =C(1)+C(2)*BAR79(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.583715	0.696820	2.272774	0.0363
C(2)	0.126768	0.242631	0.522473	0.6081

R-squared	0.015804	Mean dependent var	1.817368
Adjusted R-squared	-0.042090	S.D. dependent var	2.281792
S.E. of regression	2.329318	Akaike info criterion	1.790452
Sum squared resid	92.23727	Schwarz criterion	1.889866
Log likelihood	-41.96912	F-statistic	0.272978
Durbin-Watson stat	2.007458	Prob(F-statistic)	0.608083

Date: 04/18/00 Time: 17:22
Sample: 2 20
Included observations: 19

```
      ACF               PACF           AC      PAC     Q-Stat    Prob

  .   |   .   |    .   |   .   |1  -0.032 -0.032   0.0227   0.880
  .   |*  .   |    .   |*  .   |2   0.069  0.068   0.1341   0.935
  .  *|   .   |    .  *|   .   |3  -0.111 -0.108   0.4426   0.931
  .  *|   .   |    .  *|   .   |4  -0.081 -0.093   0.6157   0.961
  . **|   .   |    . **|   .   |5  -0.310 -0.308   3.3480   0.646
  .   |*  .   |    .   |*  .   |6   0.177  0.168   4.3050   0.635
  .   |*  .   |    .   |*  .   |7   0.096  0.142   4.6093   0.708
  .   |   .   |    .   |   .   |8   0.050 -0.037   4.6984   0.789
  .   |   .   |    .  *|   .   |9  -0.024 -0.082   4.7214   0.858
  .   |   .   |    .  *|   .   |10 -0.038 -0.101   4.7861   0.905
  .  *|   .   |    .   |   .   |11 -0.150 -0.020   5.9156   0.879
  . **|   .   |    . **|   .   |12 -0.252 -0.247   9.5316   0.657
  .   |   .   |    .  *|   .   |13 -0.031 -0.112   9.5972   0.726
  .   |   .   |    .   |   .   |14  0.006 -0.036   9.6004   0.791
  .   |   .   |    .   |   .   |15  0.024 -0.056   9.6593   0.841
  .   |   .   |    .  *|   .   |16  0.034 -0.064   9.8137   0.876
  .   |   .   |    .  *|   .   |17  0.046 -0.108   10.233   0.894
```

Auto-Regressive models of Cohort 1979 Class B

```
LS // Dependent Variable is BR79
Date: 04/18/00   Time: 19:19
Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BR79=C(1)+C(2)*BR79(-1)
```

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	2.849196	1.295507	2.199290	0.0420
C(2)	0.109626	0.241074	0.454742	0.6550

R-squared	0.012018	Mean dependent var	3.200000
Adjusted R-squared	-0.046099	S.D. dependent var	4.435572
S.E. of regression	4.536657	Akaike info criterion	3.123682
Sum squared resid	349.8814	Schwarz criterion	3.223096
Log likelihood	-54.63481	F-statistic	0.206790
Durbin-Watson stat	1.863866	Prob(F-statistic)	0.655048

```
Date: 04/18/00   Time: 19:23
Sample: 2 20
Included observations: 19
```

```
     ACF              PACF              AC       PAC     Q-Stat   Prob

  .   |   .   |    .   |   .   |1   0.036    0.036    0.0281   0.867
  .   |   .   |    .   |   .   |2  -0.009   -0.010    0.0299   0.985
  .   |   .   |    .   |   .   |3   0.016    0.017    0.0363   0.998
  .   |***    |    .   |***    |4   0.370    0.369    3.6721   0.452
  .   |   .   |    .   |   .   |5   0.047    0.027    3.7360   0.588
  .   |   .   |    .   |   .   |6   0.042    0.054    3.7912   0.705
  . **|   .   |    . **|   .   |7  -0.241   -0.290    5.7251   0.572
  .   |   .   |    . **|   .   |8  -0.027   -0.189    5.7507   0.675
  .  *|   .   |    . **|   .   |9  -0.134   -0.229    6.4709   0.692
  .   |   .   |    .   |   .   |10  0.006   -0.026    6.4723   0.774
  .  *|   .   |    .   |   .   |11 -0.183    0.021    8.1382   0.701
  .  *|   .   |    .   |   .   |12 -0.141   -0.006    9.2682   0.680
  .  *|   .   |    .   |*  .   |13 -0.066    0.144    9.5593   0.730
  .   |   .   |    .  *|   .   |14 -0.052   -0.086    9.7713   0.779
  .   |   .   |    .   |   .   |15 -0.036    0.028    9.9040   0.826
  .  *|   .   |    .  *|   .   |16 -0.066   -0.130   10.492    0.840
  .   |   .   |    .  *|   .   |17 -0.031   -0.065   10.679    0.873
```

Auto-Regressive models of Cohort 1978 Class Baa

LS // Dependent Variable is BAAR78
Date: 04/18/00 Time: 19:43
Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BAAR78=C(1)+C(2)*BAAR78(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.284288	0.171973	1.653095	0.1167
C(2)	0.466785	0.214491	2.176240	0.0439

R-squared	0.217888	Mean dependent var	0.533158
Adjusted R-squared	0.171882	S.D. dependent var	0.615224
S.E. of regression	0.559860	Akaike info criterion	-1.060836
Sum squared resid	5.328536	Schwarz criterion	-0.961421
Log likelihood	-14.88189	F-statistic	4.736022
Durbin-Watson stat	1.987637	Prob(F-statistic)	0.043927

Date: 04/18/00 Time: 19:48
Sample: 2 20
Included observations: 19

```
      ACF              PACF               AC      PAC     Q-Stat   Prob
  .   |   .   |    .   |   .   |  1  -0.009  -0.009   0.0018   0.966
  .   |   .   |    .   |   .   |  2  -0.004  -0.004   0.0021   0.999
  .   |   .   |    .   |   .   |  3  -0.053  -0.053   0.0714   0.995
  .  *|   .   |    .  *|   .   |  4  -0.133  -0.134   0.5400   0.969
  .   |   .   |    .   |   .   |  5  -0.030  -0.035   0.5657   0.990
  .  *|   .   |    .  *|   .   |  6  -0.071  -0.078   0.7193   0.994
  .   |   .   |    .   |   .   |  7  -0.016  -0.035   0.7278   0.998
  .  *|   .   |    .  *|   .   |  8  -0.090  -0.118   1.0210   0.998
  .   |*  .   |    .   |*  .   |  9   0.193   0.177   2.5109   0.981
  . **|   .   |    . **|   .   | 10  -0.238  -0.279   5.0141   0.890
  .  *|   .   |    .  *|   .   | 11  -0.153  -0.188   6.1843   0.861
  .   |   .   |    .  *|   .   | 12  -0.056  -0.102   6.3648   0.897
  .   |   .   |    .   |   .   | 13   0.043   0.051   6.4856   0.927
  .   |   .   |    .  *|   .   | 14   0.034  -0.094   6.5785   0.950
  .   |   .   |    .  *|   .   | 15  -0.009  -0.079   6.5870   0.968
  .   |   .   |    .   |   .   | 16   0.046  -0.042   6.8624   0.976
  .   |   .   |    .   |   .   | 17   0.030   0.027   7.0459   0.983
```

Auto-Regressive models of Cohort 1978 Class Ba

LS // Dependent Variable is BAR78
Date: 04/18/00 Time: 20:04
Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BAR78=C(1)+C(2)*BAR78(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.274250	0.625034	2.038689	0.0573
C(2)	0.224686	0.240569	0.933977	0.3634

R-squared	0.048808	Mean dependent var	1.660000
Adjusted R-squared	-0.007144	S.D. dependent var	2.037624
S.E. of regression	2.044890	Akaike info criterion	1.529988
Sum squared resid	71.08676	Schwarz criterion	1.629403
Log likelihood	-39.49472	F-statistic	0.872312
Durbin-Watson stat	1.928478	Prob(F-statistic)	0.363393

Date: 04/18/00 Time: 20:09
Sample: 2 20
Included observations: 19

```
    ACF              PACF                AC      PAC     Q-Stat     Prob

 .   |  .   |     .   |  .    |1   0.008   0.008   0.0015    0.969
 .  *|  .   |     .  *|  .    |2  -0.066  -0.066   0.1028    0.950
 .  *|  .   |     .  *|  .    |3  -0.160  -0.159   0.7399    0.864
 .   |  .   |     .   |  .    |4  -0.016  -0.020   0.7467    0.945
 .   |**.   |     .   |**.    |5   0.213   0.198   2.0406    0.843
 .  *|  .   |     . **|  .    |6  -0.172  -0.213   2.9504    0.815
 .  *|  .   |     .  *|  .    |7  -0.145  -0.137   3.6487    0.819
 .   |  .   |     .   |* .    |8   0.011   0.077   3.6533    0.887
 .   |  .   |     .  *|  .    |9   0.002  -0.070   3.6534    0.933
 .  *|  .   |     . **|  .    |10 -0.137  -0.273   4.4805    0.923
 .  *|  .   |     .  *|  .    |11 -0.138  -0.058   5.4267    0.909
 .  *|  .   |     .  *|  .    |12 -0.132  -0.143   6.4157    0.894
 .   |  .   |     .  *|  .    |13  0.040  -0.159   6.5209    0.925
 .   |  .   |     .  *|  .    |14  0.001  -0.092   6.5211    0.952
 .   |  .   |     .   |* .    |15  0.051   0.069   6.7788    0.964
 .   |  .   |     .   |  .    |16  0.061  -0.041   7.2744    0.968
 .   |  .   |     .   |  .    |17  0.050  -0.031   7.7738    0.971
```

Auto-Regressive models of Cohort 1978 Class B

LS // Dependent Variable is BR78
Date: 04/18/00 Time: 20:20
Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BR78=C(1)+C(2)*BR78(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	2.793920	1.302900	2.144385	0.0467
C(2)	0.090642	0.243478	0.372281	0.7143

R-squared	0.008087	Mean dependent var	3.101579
Adjusted R-squared	-0.050261	S.D. dependent var	4.284232
S.E. of regression	4.390578	Akaike info criterion	3.058222
Sum squared resid	327.7120	Schwarz criterion	3.157637
Log likelihood	-54.01294	F-statistic	0.138593
Durbin-Watson stat	1.858112	Prob(F-statistic)	0.714284

Date: 04/18/00 Time: 20:22
Sample: 2 20
Included observations: 19

```
      ACF                  PACF                 AC      PAC      Q-Stat     Prob

  .   |   .     |     .   |   .     |1   0.042   0.042   0.0399     0.842
  . **|   .     |     . **|   .     |2  -0.204  -0.206   1.0126     0.603
  . **|   .     |     . **|   .     |3  -0.207  -0.196   2.0787     0.556
  .   |**.      |     .   |**.      |4   0.222   0.210   3.3945     0.494
  .  *|   .     |     .  *|   .     |5  -0.061  -0.175   3.4992     0.624
  .  *|   .     |     .  *|   .     |6  -0.175  -0.143   4.4364     0.618
  .  *|   .     |     .   |   .     |7  -0.073  -0.002   4.6144     0.707
  .  *|   .     |     . **|   .     |8  -0.086  -0.276   4.8841     0.770
  .   |*  .     |     .   |**.      |9   0.173   0.200   6.0805     0.732
  .   |   .     |     .   |   .     |10  0.042  -0.012   6.1585     0.802
  .   |   .     |     .   |   .     |11  0.022  -0.049   6.1835     0.861
  .  *|   .     |     .   |   .     |12 -0.149  -0.004   7.4483     0.827
  .   |   .     |     .  *|   .     |13 -0.015  -0.172   7.4632     0.877
  .   |   .     |     .   |   .     |14 -0.012  -0.052   7.4748     0.915
  .   |   .     |     .  *|   .     |15 -0.042  -0.068   7.6480     0.937
  .   |   .     |     .   |   .     |16 -0.004  -0.029   7.6503     0.959
  .   |   .     |     .   |   .     |17 -0.004   0.032   7.6528     0.973
```

Auto-Regressive models of Cohort 1977 Class Baa

LS // Dependent Variable is BAAR77
Date: 04/19/00 Time: 11:15
Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BAAR77=C(1)+C(2)*BAAR77(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.292146	0.172533	1.693272	0.1086
C(2)	0.439232	0.222994	1.969705	0.0654

R-squared	0.185813	Mean dependent var	0.532105
Adjusted R-squared	0.137920	S.D. dependent var	0.573562
S.E. of regression	0.532542	Akaike info criterion	-1.160886
Sum squared resid	4.821218	Schwarz criterion	-1.061471
Log likelihood	-13.93141	F-statistic	3.879736
Durbin-Watson stat	1.923978	Prob(F-statistic)	0.065389

Date: 04/19/00 Time: 11:29
Sample: 2 20
Included observations: 19

ACF	PACF		AC	PAC	Q-Stat	Prob
.	.		.	.	1	0.028	0.028	0.0169	0.896
.	.		.	.	2	0.033	0.032	0.0421	0.979
. **	.		. **	.	3	-0.231	-0.233	1.3692	0.713
. *	.		. *	.	4	-0.188	-0.185	2.3052	0.680
.	.		.	* .	5	0.054	0.082	2.3875	0.793
. *	.		. **	.	6	-0.184	-0.245	3.4282	0.754
.	* .		.	* .	7	0.146	0.072	4.1353	0.764
.	* .		.	* .	8	0.104	0.132	4.5276	0.807
.	* .		.	.	9	0.070	-0.032	4.7240	0.858
. **	.		. **	.	10	-0.223	-0.305	6.9351	0.732
. **	.		. *	.	11	-0.246	-0.139	9.9495	0.535
. *	.		. *	.	12	-0.077	-0.073	10.291	0.590
.	.		. *	.	13	0.020	-0.076	10.318	0.668
.	.		. *	.	14	0.030	-0.150	10.389	0.733
.	* .		.	.	15	0.074	0.009	10.940	0.757
.	.		. *	.	16	0.050	-0.105	11.278	0.792
.	.		. *	.	17	0.033	-0.093	11.492	0.830

Auto-Regressive Models of Cohort 1977 Class Ba

LS // Dependent Variable is BAR77
Date: 04/19/00 Time: 11:35
Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BAR77=C(1)+C(2)*BAR77(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.354794	0.595913	2.273475	0.0363
C(2)	0.117868	0.243271	0.484513	0.6342

R-squared	0.013621	Mean dependent var	1.539474
Adjusted R-squared	-0.044401	S.D. dependent var	1.953758
S.E. of regression	1.996662	Akaike info criterion	1.482254
Sum squared resid	67.77322	Schwarz criterion	1.581669
Log likelihood	-39.04125	F-statistic	0.234752
Durbin-Watson stat	1.950057	Prob(F-statistic)	0.634206

Date: 04/19/00 Time: 11:36
Sample: 2 20
Included observations: 19

ACF	PACF		AC	PAC	Q-Stat	Prob
. | .	. | .	1	0.011	0.011	0.0025	0.960
. *| .	. *| .	2	-0.130	-0.130	0.3963	0.820
. **| .	. **| .	3	-0.215	-0.216	1.5532	0.670
. | .	. | .	4	-0.007	-0.027	1.5547	0.817
. |**.	. |**.	5	0.247	0.204	3.2988	0.654
. | .	. | .	6	-0.003	-0.051	3.2991	0.770
. **| .	. **| .	7	-0.211	-0.189	4.7745	0.687
. *| .	. *| .	8	-0.156	-0.086	5.6604	0.685
. | .	. | .	9	0.042	0.009	5.7308	0.767
. *| .	.***| .	10	-0.141	-0.329	6.6138	0.761
. | .	. *| .	11	-0.015	-0.097	6.6252	0.829
. *| .	. *| .	12	-0.096	-0.065	7.1452	0.848
. | .	. *| .	13	0.004	-0.092	7.1462	0.894
. | .	. *| .	14	0.012	-0.161	7.1570	0.928
. | .	. |* .	15	0.064	0.073	7.5671	0.940
. | .	. | .	16	0.052	0.028	7.9223	0.951
. | .	. *| .	17	0.036	-0.077	8.1807	0.963

Auto-Regressive models of Cohort 1977   Class B

LS // Dependent Variable is BR77
Date: 04/19/00   Time: 11:58
Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BR77=C(1)+C(2)*BR77(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	2.005815	1.503112	1.334441	0.1997
C(2)	0.332253	0.230901	1.438943	0.1683

R-squared	0.108573	Mean dependent var	3.092632
Adjusted R-squared	0.056137	S.D. dependent var	5.830722
S.E. of regression	5.664700	Akaike info criterion	3.567809
Sum squared resid	545.5101	Schwarz criterion	3.667223
Log likelihood	-58.85401	F-statistic	2.070556
Durbin-Watson stat	1.761602	Prob(F-statistic)	0.168325

Date: 04/19/00   Time: 11:59
Sample: 2 20
Included observations: 19

```
      ACF              PACF                AC       PAC     Q-Stat    Prob

  .    |* .     |     .    |* .     |1   0.115    0.115   0.2948   0.587
  . ** |  .     |     . ** |  .     |2  -0.272   -0.289   2.0254   0.363
  .   *|  .     |     .   *|  .     |3  -0.176   -0.113   2.7949   0.424
  .   *|  .     |     .   *|  .     |4  -0.087   -0.143   2.9971   0.558
  .   *|  .     |     . ** |  .     |5  -0.149   -0.234   3.6320   0.604
  .   *|  .     |     . ** |  .     |6  -0.104   -0.191   3.9633   0.682
  .   *|  .     |     .*** |  .     |7  -0.158   -0.373   4.7978   0.685
  .    |* .     |     .   *|  .     |8   0.126   -0.086   5.3731   0.717
  .    |**.     |     .    |  .     |9   0.303    0.005   9.0247   0.435
  .    |  .     |     .   *|  .     |10  0.034   -0.185   9.0760   0.525
  .    |  .     |     .   *|  .     |11 -0.045   -0.061   9.1761   0.606
  .   *|  .     |     . ** |  .     |12 -0.096   -0.253   9.7030   0.642
  .    |  .     |     .    |  .     |13  0.038   -0.015   9.8010   0.710
  .    |  .     |     .   *|  .     |14 -0.035   -0.186   9.9006   0.769
  .    |  .     |     .    |  .     |15 -0.004   -0.021   9.9019   0.826
  .    |  .     |     .    |  .     |16  0.001   -0.003   9.9021   0.872
  .    |  .     |     .   *|  .     |17  0.010   -0.131   9.9224   0.907
```

Auto-Regressive models of Cohort 1976 Class Baa

LS // Dependent Variable is BAAR76
Date: 04/19/00 Time: 12:07
Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BAAR76=C(1)+C(2)*BAAR76(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.343909	0.184019	1.868877	0.0790
C(2)	0.412386	0.220952	1.866403	0.0793

R-squared	0.170062	Mean dependent var	0.585263
Adjusted R-squared	0.121242	S.D. dependent var	0.608773
S.E. of regression	0.570676	Akaike info criterion	-1.022566
Sum squared resid	5.536411	Schwarz criterion	-0.923152
Log likelihood	-15.24545	F-statistic	3.483459
Durbin-Watson stat	1.878934	Prob(F-statistic)	0.079332

Date: 04/19/00 Time: 12:08
Sample: 2 20
Included observations: 19

ACF	PACF		AC	PAC	Q-Stat	Prob
. | .	. | .	1	0.049	0.049	0.0540	0.816
. | .	. | .	2	-0.041	-0.043	0.0927	0.955
. **| .	. **| .	3	-0.233	-0.230	1.4442	0.695
. **| .	. **| .	4	-0.281	-0.277	3.5470	0.471
. |* .	. |* .	5	0.115	0.120	3.9231	0.561
. *| .	. **| .	6	-0.152	-0.260	4.6292	0.592
. |* .	. |* .	7	0.171	0.081	5.6004	0.587
. |* .	. | .	8	0.102	0.061	5.9785	0.650
. | .	. | .	9	0.013	-0.012	5.9854	0.741
. *| .	. **| .	10	-0.170	-0.283	7.2732	0.699
. *| .	. | .	11	-0.180	-0.003	8.8880	0.632
. *| .	. *| .	12	-0.082	-0.175	9.2667	0.680
. | .	. *| .	13	-0.022	-0.126	9.2990	0.750
. |* .	. *| .	14	0.088	-0.096	9.9138	0.768
. | .	. | .	15	0.055	-0.017	10.214	0.806
. | .	. **| .	16	0.047	-0.189	10.509	0.839
. | .	. | .	17	0.015	0.009	10.556	0.879

Auto-Regressive models of Cohort 1976   Class Ba

LS // Dependent Variable is BAR76
Date: 04/19/00   Time: 12:16
Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BAR76=C(1)+C(2)*BAR76(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.495745	0.626882	2.386007	0.0289
C(2)	0.088598	0.245722	0.360563	0.7229

R-squared	0.007589	Mean dependent var	1.646316
Adjusted R-squared	-0.050788	S.D. dependent var	1.988099
S.E. of regression	2.037959	Akaike info criterion	1.523198
Sum squared resid	70.60569	Schwarz criterion	1.622613
Log likelihood	-39.43021	F-statistic	0.130006
Durbin-Watson stat	1.943191	Prob(F-statistic)	0.722867

Sample: 2 20
Included observations: 19

```
      ACF               PACF               AC        PAC      Q-Stat   Prob

  .   |   .    |     .   |   .    |1    0.011     0.011    0.0025   0.960
  .  *|   .    |     .  *|   .    |2   -0.159    -0.159    0.5926   0.744
  . **|   .    |     . **|   .    |3   -0.223    -0.225    1.8333   0.608
  .   |   .    |     .   |   .    |4   -0.022    -0.054    1.8464   0.764
  .   |***     |     .   |**.     |5    0.362     0.313    5.5713   0.350
  .   |   .    |     .   |   .    |6   -0.002    -0.055    5.5714   0.473
  . **|   .    |     .  *|   .    |7   -0.224    -0.185    7.2357   0.405
  . **|   .    |     .  *|   .    |8   -0.196    -0.090    8.6331   0.374
  .   |   .    |     .   |   .    |9   -0.001    -0.025    8.6331   0.472
  .   |   .    |     .***|   .    |10  -0.052    -0.322    8.7536   0.556
  .   |   .    |     .  *|   .    |11   0.007    -0.085    8.7562   0.644
  .  *|   .    |     .  *|   .    |12  -0.123    -0.063    9.6245   0.649
  .   |   .    |     .   |   .    |13  -0.023    -0.045    9.6586   0.722
  .   |   .    |     .  *|   .    |14   0.004    -0.139    9.6598   0.787
  .   |   .    |     .   |*  .    |15   0.062     0.097   10.047    0.817
  .   |   .    |     .   |   .    |16   0.046     0.002   10.325    0.849
  .   |   .    |     .   |   .    |17   0.022    -0.034   10.422    0.885
```

Auto-Regressive models of Cohort 1976   Class B

LS // Dependent Variable is BR76
Date: 04/19/00   Time: 12:25
Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BR76=C(1)+C(2)*BR76(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.772341	1.324041	1.338585	0.1983
C(2)	0.325161	0.229356	1.417712	0.1743

R-squared	0.105729	Mean dependent var	2.626316
Adjusted R-squared	0.053125	S.D. dependent var	5.281732
S.E. of regression	5.139520	Akaike info criterion	3.373220
Sum squared resid	449.0494	Schwarz criterion	3.472635
Log likelihood	-57.00542	F-statistic	2.009908
Durbin-Watson stat	1.701668	Prob(F-statistic)	0.174344

Sample: 2 20
Included observations: 19

```
      ACF                PACF                AC        PAC      Q-Stat    Prob

  .    |* .      |     .    |* .    |1    0.141     0.141    0.4421    0.506
  .***|   .      |     .***|   .    |2   -0.350    -0.377    3.3170    0.190
  .   *|  .      |     .    |   .   |3   -0.157    -0.042    3.9292    0.269
  .   *|  .      |     .  **|   .   |4   -0.156    -0.299    4.5735    0.334
  .    |  .      |     .    |   .   |5   -0.035    -0.050    4.6083    0.466
  .    |  .      |     .  **|   .   |6   -0.056    -0.292    4.7055    0.582
  .   *|  .      |     .  **|   .   |7   -0.155    -0.284    5.5075    0.598
  .    |* .      |     .    |   .   |8    0.136    -0.054    6.1747    0.628
  .    |***      |     .    |*  .   |9    0.352     0.110   11.124     0.267
  .    |  .      |     .  **|   .   |10  -0.047    -0.233   11.222     0.341
  .   *|  .      |     .    |   .   |11  -0.134    -0.002   12.114     0.355
  .   *|  .      |     .  **|   .   |12  -0.124    -0.266   12.997     0.369
  .    |  .      |     .    |   .   |13  -0.011     0.048   13.006     0.447
  .    |* .      |     .   *|   .   |14   0.095    -0.176   13.732     0.470
  .    |  .      |     .    |   .   |15  -0.006     0.021   13.735     0.546
  .    |  .      |     .    |   .   |16   0.011    -0.032   13.751     0.617
  .    |  .      |     .   *|   .   |17   0.004    -0.129   13.755     0.684
```

Auto-Regressive models of Cohort 1975 Class Baa

LS // Dependent Variable is BAAR75
Date: 04/19/00 Time: 13:16
Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BAAR75=C(1)+C(2)*BAAR75(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.369306	0.188325	1.961004	0.0665
C(2)	0.370689	0.225257	1.645630	0.1182

R-squared	0.137410	Mean dependent var	0.586842
Adjusted R-squared	0.086670	S.D. dependent var	0.611792
S.E. of regression	0.584679	Akaike info criterion	-0.974083
Sum squared resid	5.811448	Schwarz criterion	-0.874668
Log likelihood	-15.70604	F-statistic	2.708098
Durbin-Watson stat	1.916497	Prob(F-statistic)	0.118201

Sample: 2 20
Included observations: 19

```
      ACF               PACF              AC       PAC     Q-Stat    Prob

 .   |   .    |     .   |   .    |1   0.018    0.018   0.0074    0.931
 .  *|   .    |     .  *|   .    |2  -0.069   -0.069   0.1181    0.943
 . **|   .    |     . **|   .    |3  -0.223   -0.222   1.3589    0.715
 .  *|   .    |     .  *|   .    |4  -0.142   -0.150   1.8974    0.755
 .   |*  .    |     .   |   .    |5   0.072    0.042   2.0468    0.843
 .   |   .    |     .  *|   .    |6   0.005   -0.067   2.0476    0.915
 .   |   .    |     .   |   .    |7   0.034   -0.026   2.0860    0.955
 .   |   .    |     .   |   .    |8   0.013    0.015   2.0923    0.978
 .   |   .    |     .   |   .    |9  -0.028   -0.025   2.1231    0.989
 .  *|   .    |     . **|   .    |10 -0.184   -0.210   3.6259    0.963
 .   |   .    |     .   |   .    |11 -0.017   -0.020   3.6411    0.979
 .  *|   .    |     .  *|   .    |12 -0.110   -0.165   4.3298    0.977
 .   |*  .    |     .   |   .    |13  0.094   -0.016   4.9236    0.977
 .  *|   .    |     . **|   .    |14 -0.088   -0.201   5.5445    0.977
 .   |   .    |     .  *|   .    |15 -0.005   -0.070   5.5475    0.986
 .   |*  .    |     .   |   .    |16  0.075   -0.003   6.2913    0.985
 .   |   .    |     .   |   .    |17  0.031   -0.025   6.4837    0.989
```

Auto-Regressive models of Cohort 1975 Class B

LS // Dependent Variable is BAR75

Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BAR75=C(1)+C(2)*BAR75(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.535071	0.638307	2.404910	0.0278
C(2)	0.110334	0.245601	0.449242	0.6589

R-squared	0.011732	Mean dependent var	1.732105
Adjusted R-squared	-0.046401	S.D. dependent var	1.976157
S.E. of regression	2.021485	Akaike info criterion	1.506965
Sum squared resid	69.46880	Schwarz criterion	1.606380
Log likelihood	-39.27600	F-statistic	0.201819
Durbin-Watson stat	1.916790	Prob(F-statistic)	0.658931

Sample: 2 20
Included observations: 19

```
      ACF                PACF             AC       PAC     Q-Stat   Prob

  .    |    .   |    .    |    .   |1    0.018    0.018   0.0074   0.932
  . ** |    .   |    . ** |    .   |2   -0.204   -0.204   0.9814   0.612
  . ** |    .   |    . ** |    .   |3   -0.209   -0.209   2.0684   0.558
  .    |*   .   |    .    |*   .   |4    0.177    0.148   2.8983   0.575
  .    |***     |    .    |**  .   |5    0.331    0.278   6.0155   0.305
  .   *|    .   |    .   *|    .   |6   -0.079   -0.070   6.2078   0.400
  . ** |    .   |    .   *|    .   |7   -0.235   -0.105   8.0502   0.328
  . ** |    .   |    . ** |    .   |8   -0.241   -0.219  10.154    0.254
  .    |*   .   |    .    |    .   |9    0.133   -0.040  10.862    0.285
  .    |    .   |    . ** |    .   |10  -0.055   -0.275  10.998    0.358
  .    |    .   |    .    |    .   |11  -0.055   -0.045  11.152    0.431
  .   *|    .   |    .    |    .   |12  -0.137   -0.018  12.218    0.428
  .    |    .   |    .    |    .   |13  -0.029    0.010  12.273    0.505
  .    |    .   |    .   *|    .   |14   0.018   -0.068  12.299    0.582
  .    |    .   |    .    |    .   |15   0.006    0.015  12.303    0.656
  .    |    .   |    .    |    .   |16   0.034   -0.005  12.454    0.712
  .    |    .   |    .    |    .   |17   0.014    0.005  12.492    0.769
```

Auto-Regressive Models of Cohort 1975 **Class B**

LS // Dependent Variable is BR75

Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BR75=C(1)+C(2)*BR75(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.390277	0.995358	1.396762	0.1805
C(2)	0.236632	0.229738	1.030010	0.3174

R-squared	0.058741	Mean dependent var	1.921579
Adjusted R-squared	0.003373	S.D. dependent var	3.716882
S.E. of regression	3.710608	Akaike info criterion	2.721692
Sum squared resid	234.0665	Schwarz criterion	2.821107
Log likelihood	-50.81591	F-statistic	1.060920
Durbin-Watson stat	1.652745	Prob(F-statistic)	0.317438

Sample: 2 20
Included observations: 19

```
      ACF                  PACF              AC       PAC     Q-Stat   Prob

 .    |* .     |     .    |* .     | 1    0.152    0.152   0.5135   0.474
 .***|   .     |    .***|    .     | 2   -0.361   -0.393   3.5671   0.168
 .   *|   .    |     .    |    .   | 3   -0.152   -0.019   4.1401   0.247
 .    |   .    |     .   *|    .   | 4   -0.038   -0.175   4.1787   0.382
 .   *|   .    |     .  **|    .   | 5   -0.146   -0.219   4.7879   0.442
 .   *|   .    |     .  **|    .   | 6   -0.125   -0.190   5.2693   0.510
 .   *|   .    |     .***|    .    | 7   -0.115   -0.333   5.7067   0.574
 .    |* .     |     .    |    .   | 8    0.160    0.012   6.6314   0.577
 .    |***     |     .    |* .     | 9    0.389    0.158  12.672    0.178
 .    |   .    |     .   *|    .   |10   -0.015   -0.175  12.682    0.242
 .   *|   .    |     .    |    .   |11   -0.168    0.041  14.094    0.228
 .    |   .    |     .    |    .   |12   -0.003   -0.051  14.094    0.295
 .    |   .    |     .    |* .     |13    0.062    0.080  14.348    0.350
 .   *|   .    |     .   *|    .   |14   -0.161   -0.125  16.414    0.289
 .   *|   .    |     .    |* .     |15   -0.068    0.081  16.880    0.326
 .    |   .    |     .    |    .   |16    0.047    0.018  17.171    0.375
 .    |   .    |     .   *|    .   |17    0.025   -0.124  17.298    0.434
```

Auto-Regressive models of Cohort 1974 Class Baa

LS // Dependent Variable is BAAR74

Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BAAR74=C(1)+C(2)*BAAR74(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.459843	0.190077	2.419239	0.0271
C(2)	0.265180	0.233853	1.133962	0.2725

R-squared	0.070320	Mean dependent var	0.625789
Adjusted R-squared	0.015633	S.D. dependent var	0.532909
S.E. of regression	0.528727	Akaike info criterion	-1.175264
Sum squared resid	4.752395	Schwarz criterion	-1.075849
Log likelihood	-13.79482	F-statistic	1.285869
Durbin-Watson stat	1.926689	Prob(F-statistic)	0.272549

Sample: 2 20
Included observations: 19

ACF	PACF		AC	PAC	Q-Stat	Prob
. | .	. | .	1	0.014	0.014	0.0046	0.946
. *| .	. *| .	2	-0.091	-0.091	0.1998	0.905
. | .	. | .	3	0.006	0.009	0.2008	0.977
. *| .	. *| .	4	-0.120	-0.130	0.5859	0.965
. | .	. | .	5	0.009	0.015	0.5883	0.989
. |* .	. | .	6	0.071	0.047	0.7433	0.994
. | .	. | .	7	-0.011	-0.009	0.7477	0.998
. *| .	. *| .	8	-0.066	-0.072	0.9071	0.999
. *| .	. *| .	9	-0.120	-0.122	1.4849	0.997
. *| .	. **| .	10	-0.186	-0.190	3.0158	0.981
. | .	. | .	11	0.020	-0.006	3.0352	0.990
. *| .	. **| .	12	-0.146	-0.219	4.2541	0.978
. |* .	. | .	13	0.081	0.061	4.6864	0.981
. | .	. *| .	14	0.008	-0.086	4.6919	0.990
. *| .	. | .	15	-0.077	-0.052	5.2769	0.990
. |* .	. | .	16	0.078	0.033	6.0946	0.987
. | .	. | .	17	0.031	0.005	6.2884	0.991

Auto-Regressive models of Cohort 1974 Class Ba

LS // Dependent Variable is BAR74

Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BAR74=C(1)+C(2)*BAR74(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.608063	0.685644	2.345331	0.0314
C(2)	0.171698	0.234525	0.732109	0.4741

R-squared	0.030565	Mean dependent var	1.922632
Adjusted R-squared	-0.026461	S.D. dependent var	2.298792
S.E. of regression	2.329007	Akaike info criterion	1.790185
Sum squared resid	92.21264	Schwarz criterion	1.889599
Log likelihood	-41.96659	F-statistic	0.535984
Durbin-Watson stat	1.923817	Prob(F-statistic)	0.474075

Sample: 2 20
Included observations: 19

```
        ACF                 PACF                AC       PAC     Q-Stat   Prob

  .    |    .    |    .    |    .   | 1    0.036    0.036   0.0293   0.864
  .  **|    .    |    .  **|    .   | 2   -0.290   -0.292   2.0069   0.367
  .   *|    .    |    .   *|    .   | 3   -0.168   -0.157   2.7108   0.438
  .    |**  .    |    .    |**  .   | 4    0.280    0.228   4.7900   0.310
  .    |***      |    .    |**  .   | 5    0.349    0.293   8.2632   0.142
  .   *|    .    |    .   *|    .   | 6   -0.161   -0.084   9.0541   0.171
  .  **|    .    |    .    |    .   | 7   -0.213   -0.013  10.566    0.159
  .    |    .    |    .    |    .   | 8   -0.008   -0.040  10.568    0.227
  .    |*   .    |    .   *|    .   | 9    0.157   -0.078  11.556    0.240
  .   *|    .    |    .  **|    .   |10   -0.158   -0.304  12.657    0.243
  .   *|    .    |    .    |    .   |11   -0.113    0.015  13.296    0.274
  .   *|    .    |    .   *|    .   |12   -0.068   -0.086  13.561    0.330
  .    |    .    |    .   *|    .   |13    0.006   -0.104  13.564    0.405
  .   *|    .    |    .   *|    .   |14   -0.076   -0.079  14.020    0.448
  .   *|    .    |    .    |*   .   |15   -0.071    0.085  14.521    0.486
  .    |    .    |    .   *|    .   |16   -0.015   -0.064  14.552    0.558
  .    |    .    |    .    |    .   |17    0.011   -0.002  14.577    0.626
```

Auto-Regressive models of Cohort 1974 Class B

LS // Dependent Variable is BR74

Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BR74=C(1)+C(2)*BR74(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.553905	0.574806	0.963638	0.3487
C(2)	0.128480	0.172443	0.745055	0.4664

R-squared	0.031621	Mean dependent var	0.715789
Adjusted R-squared	-0.025343	S.D. dependent var	2.290779
S.E. of regression	2.319625	Akaike info criterion	1.782112
Sum squared resid	91.47122	Schwarz criterion	1.881526
Log likelihood	-41.88989	F-statistic	0.555107
Durbin-Watson stat	1.503306	Prob(F-statistic)	0.466415

Sample: 2 20
Included observations: 19

```
       ACF                 PACF               AC       PAC    Q-Stat   Prob

   .   |**.    |     .    |**.    |1    0.227     0.227  1.1453   0.285
   .  *|  .    |     . **|   .    |2   -0.135    -0.197  1.5743   0.455
   .  *|  .    |     .    |   .   |3   -0.082     0.000  1.7413   0.628
   .  *|  .    |     .  *|   .    |4   -0.085    -0.098  1.9344   0.748
   .  *|  .    |     .  *|   .    |5   -0.089    -0.063  2.1578   0.827
   .  *|  .    |     .  *|   .    |6   -0.156    -0.162  2.9001   0.821
   . **|  .    |     .  *|   .    |7   -0.194    -0.170  4.1575   0.761
   .   |  .    |     .    |   .   |8   -0.002     0.016  4.1577   0.843
   .   |  .    |     .  *|   .    |9   -0.030    -0.149  4.1940   0.898
   .   |  .    |     .    |   .   |10  -0.034    -0.052  4.2439   0.936
   .   |  .    |     .  *|   .    |11  -0.044    -0.137  4.3408   0.959
   .   |  .    |     .    |   .   |12   0.005    -0.047  4.3422   0.976
   .   |  .    |     .  *|   .    |13   0.028    -0.093  4.3950   0.986
   .   |  .    |     .  *|   .    |14   0.025    -0.062  4.4441   0.992
   .   |  .    |     .    |   .   |15   0.021    -0.048  4.4901   0.996
   .   |  .    |     .  *|   .    |16   0.018    -0.081  4.5337   0.998
   .   |  .    |     .    |   .   |17   0.015    -0.048  4.5771   0.999
```

Auto-Regressive models of Cohort 1973 Class Baa

LS // Dependent Variable is BAAR73

Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BAAR73=C(1)+C(2)*BAAR73(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.427741	0.213891	1.999809	0.0618
C(2)	0.383830	0.234071	1.639805	0.1194

R-squared	0.136572	Mean dependent var	0.708947
Adjusted R-squared	0.085782	S.D. dependent var	0.582770
S.E. of regression	0.557214	Akaike info criterion	-1.070310
Sum squared resid	5.278290	Schwarz criterion	-0.970896
Log likelihood	-14.79188	F-statistic	2.688960
Durbin-Watson stat	1.664470	Prob(F-statistic)	0.119417

Sample: 2 20
Included observations: 19

ACF	PACF		AC	PAC	Q-Stat	Prob
.	.		.	.		1	0.011	0.011	0.0025	0.960
. *	.		. *	.		2	-0.104	-0.105	0.2585	0.879
.	.		.	.		3	0.035	0.037	0.2886	0.962
.	.		.	.		4	-0.001	-0.013	0.2886	0.991
.	**.		.	**.		5	0.210	0.221	1.5480	0.907
.	.		. *	.		6	-0.049	-0.063	1.6208	0.951
. *	.		.	.		7	-0.098	-0.050	1.9380	0.963
.	.		.	.		8	0.017	-0.013	1.9482	0.983
. *	.		. *	.		9	-0.148	-0.165	2.8271	0.971
. *	.		. **	.		10	-0.171	-0.221	4.1253	0.942
.	.		.	.		11	0.004	-0.006	4.1261	0.966
. **	.		. **	.		12	-0.193	-0.220	6.2598	0.902
.	* .		.	* .		13	0.123	0.156	7.2602	0.888
. *	.		. *	.		14	-0.080	-0.085	7.7662	0.901
. *	.		.	.		15	-0.082	0.060	8.4330	0.905
.	.		. *	.		16	-0.005	-0.097	8.4362	0.935
.	.		.	.		17	-0.055	0.020	9.0354	0.939

Auto-Regressive models of Cohort 1973 Class Ba

LS // Dependent Variable is BAR73

Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BAR73=C(1)+C(2)*BAR73(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.551286	0.612096	2.534385	0.0214
C(2)	0.087237	0.237294	0.367632	0.7177

R-squared	0.007887	Mean dependent var	1.692105
Adjusted R-squared	-0.050472	S.D. dependent var	2.030459
S.E. of regression	2.081069	Akaike info criterion	1.565064
Sum squared resid	73.62439	Schwarz criterion	1.664478
Log likelihood	-39.82794	F-statistic	0.135153
Durbin-Watson stat	1.980327	Prob(F-statistic)	0.717685

Sample: 2 20
Included observations: 19

```
     ACF                 PACF                AC       PAC      Q-Stat   Prob

  .   |   .    |     .   |   .    |1    0.003    0.003    0.0002   0.988
  . **|   .    |     . **|   .    |2   -0.196   -0.196    0.8975   0.638
  .  *|   .    |     .  *|   .    |3   -0.170   -0.175    1.6163   0.656
  .   |** .    |     .   |*  .    |4    0.225    0.196    2.9648   0.564
  .   |** .    |     .   |** .    |5    0.309    0.276    5.6799   0.339
  .   |   .    |     .   |   .    |6   -0.054    0.002    5.7704   0.449
  . **|   .    |     .  *|   .    |7   -0.212   -0.079    7.2612   0.402
  .  *|   .    |     .  *|   .    |8   -0.071   -0.059    7.4447   0.489
  .   |** .    |     .   |   .    |9    0.209    0.054    9.1828   0.421
  .   |   .    |     . **|   .    |10  -0.054   -0.196    9.3114   0.503
  .  *|   .    |     .  *|   .    |11  -0.140   -0.086   10.288    0.505
  .  *|   .    |     .   |   .    |12  -0.124   -0.029   11.160    0.515
  .   |   .    |     .  *|   .    |13   0.002   -0.083   11.161    0.597
  .  *|   .    |     . **|   .    |14  -0.066   -0.195   11.507    0.646
  .  *|   .    |     .   |   .    |15  -0.083   -0.047   12.193    0.664
  .   |   .    |     .   |   .    |16  -0.040    0.034   12.410    0.715
  .   |   .    |     .  *|   .    |17  -0.045   -0.071   12.817    0.748
```

Auto-Regressive models of Cohort 1973   Class B

LS // Dependent Variable is BR73

Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BR73=C(1)+C(2)*BR73(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.650146	0.638194	1.018727	0.3226
C(2)	-0.069170	0.232582	-0.297401	0.7698

R-squared	0.005176	Mean dependent var	0.594737
Adjusted R-squared	-0.053343	S.D. dependent var	2.592398
S.E. of regression	2.660643	Akaike info criterion	2.056436
Sum squared resid	120.3434	Schwarz criterion	2.155851
Log likelihood	-44.49598	F-statistic	0.088447
Durbin-Watson stat	1.986073	Prob(F-statistic)	0.769764

Sample: 2 20
Included observations: 19

```
      ACF                 PACF               AC        PAC      Q-Stat   Prob

  .   |   .   |      .   |   .   |1    0.005     0.005    0.0005   0.983
  .  *|   .   |      .  *|   .   |2   -0.072    -0.072    0.1231   0.940
  .  *|   .   |      .  *|   .   |3   -0.076    -0.076    0.2664   0.966
  .  *|   .   |      .  *|   .   |4   -0.079    -0.085    0.4338   0.980
  .  *|   .   |      .  *|   .   |5   -0.083    -0.096    0.6293   0.987
  .  *|   .   |      .  *|   .   |6   -0.086    -0.109    0.8582   0.990
  .  *|   .   |      .  *|   .   |7   -0.090    -0.126    1.1266   0.993
  .  *|   .   |      .  *|   .   |8   -0.068    -0.120    1.2939   0.996
  .   |   .   |      .  *|   .   |9   -0.034    -0.104    1.3402   0.998
  .   |   .   |      .  *|   .   |10  -0.031    -0.116    1.3826   0.999
  .   |   .   |      .  *|   .   |11   0.027    -0.067    1.4179   1.000
  .   |   .   |      .  *|   .   |12   0.023    -0.075    1.4484   1.000
  .   |   .   |      .  *|   .   |13   0.020    -0.074    1.4740   1.000
  .   |   .   |      .  *|   .   |14   0.016    -0.074    1.4947   1.000
  .   |   .   |      .  *|   .   |15   0.013    -0.072    1.5105   1.000
  .   |   .   |      .  *|   .   |16   0.009    -0.070    1.5214   1.000
  .   |   .   |      .  *|   .   |17   0.006    -0.067    1.5276   1.000
```

Auto-Regressive Models of Cohort 1972 Class Baa

LS // Dependent Variable is BAAR72

Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BAAR72=C(1)+C(2)*BAAR72(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.234514	0.174124	1.346822	0.1957
C(2)	0.800201	0.242010	3.306480	0.0042

R-squared	0.391397	Mean dependent var	0.701579
Adjusted R-squared	0.355597	S.D. dependent var	0.552834
S.E. of regression	0.443786	Akaike info criterion	-1.525526
Sum squared resid	3.348080	Schwarz criterion	-1.426111
Log likelihood	-10.46734	F-statistic	10.93281
Durbin-Watson stat	2.236289	Prob(F-statistic)	0.004171

Date: 04/19/00 Time: 17:40
Sample: 2 20
Included observations: 19

```
      ACF                PACF              AC       PAC      Q-Stat   Prob

 . **|  .  |      . **|  .  |1    -0.230   -0.230   1.1717   0.279
 .   |* .  |      .   |* .  |2     0.183    0.138   1.9602   0.375
 .  *|  .  |      .  *|  .  |3    -0.130   -0.067   2.3810   0.497
 .   |* .  |      .   |  .  |4     0.121    0.063   2.7729   0.597
 .   |* .  |      .   |**.  |5     0.195    0.280   3.8618   0.569
 .  *|  .  |      .   |  .  |6    -0.076   -0.024   4.0367   0.672
 .   |  .  |      .  *|  .  |7    -0.051   -0.143   4.1235   0.765
 .  *|  .  |      .  *|  .  |8    -0.106   -0.105   4.5306   0.806
 .   |* .  |      .   |  .  |9     0.126    0.065   5.1683   0.819
 .  *|  .  |      . **|  .  |10   -0.169   -0.195   6.4283   0.778
 .  *|  .  |      . **|  .  |11   -0.105   -0.213   6.9748   0.801
 .  *|  .  |      .  *|  .  |12   -0.174   -0.120   8.6963   0.729
 .   |* .  |      .   |* .  |13    0.102    0.101   9.3891   0.743
 .   |  .  |      .   |  .  |14   -0.057   -0.033   9.6452   0.788
 .  *|  .  |      .   |  .  |15   -0.068   -0.055  10.101    0.813
 .  *|  .  |      .   |  .  |16   -0.135   -0.029  12.521    0.707
 .   |  .  |      .   |  .  |17   -0.035   -0.042  12.765    0.752
```

Auto-Regressive models of Cohort 1972 Class Ba

LS // Dependent Variable is BAR72

Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BAR72=C(1)+C(2)*BAR72(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.139042	0.569382	2.000490	0.0617
C(2)	0.351724	0.332633	1.057393	0.3051

R-squared	0.061711	Mean dependent var	1.560000
Adjusted R-squared	0.006517	S.D. dependent var	1.780181
S.E. of regression	1.774371	Akaike info criterion	1.246192
Sum squared resid	53.52264	Schwarz criterion	1.345607
Log likelihood	-36.79866	F-statistic	1.118081
Durbin-Watson stat	1.353239	Prob(F-statistic)	0.305127

Sample: 2 20
Included observations: 19

```
      ACF                PACF                AC       PAC     Q-Stat   Prob

  .   |   .   |     .   |   .   |1    0.057    0.057    0.0717   0.789
  . **|   .   |     . **|   .   |2   -0.213   -0.217    1.1380   0.566
  .  *|   .   |     .  *|   .   |3   -0.127   -0.105    1.5392   0.673
  .   |   .   |     .   |   .   |4    0.030   -0.004    1.5623   0.816
  .   |*  .   |     .   |*  .   |5    0.194    0.153    2.6356   0.756
  .   |*  .   |     .   |   .   |6    0.074    0.055    2.8028   0.833
  .   |   .   |     .   |   .   |7   -0.042    0.026    2.8622   0.897
  .  *|   .   |     .   |   .   |8   -0.100   -0.045    3.2282   0.919
  .   |*  .   |     .   |*  .   |9    0.091    0.110    3.5600   0.938
  .   |   .   |     .   |   .   |10   0.029   -0.043    3.5968   0.964
  .   |   .   |     .   |   .   |11  -0.055   -0.055    3.7483   0.977
  .  *|   .   |     .  *|   .   |12  -0.163   -0.164    5.2581   0.949
  .   |   .   |     .   |   .   |13  -0.036   -0.027    5.3437   0.967
  .   |   .   |     .  *|   .   |14  -0.039   -0.148    5.4679   0.978
  .  *|   .   |     .  *|   .   |15  -0.093   -0.151    6.3362   0.974
  .   |   .   |     .   |   .   |16  -0.003   -0.032    6.3373   0.984
  .   |   .   |     .   |   .   |17  -0.034   -0.026    6.5675   0.988
```

Auto-Regressive models of Cohort 1972 Class B

LS // Dependent Variable is BR72

Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BR72=C(1)+C(2)*BR72(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.637969	0.620269	1.028535	0.3181
C(2)	-0.067290	0.205261	-0.327826	0.7470

R-squared	0.006282	Mean dependent var	0.573158
Adjusted R-squared	-0.052172	S.D. dependent var	2.498337
S.E. of regression	2.562680	Akaike info criterion	1.981408
Sum squared resid	111.6446	Schwarz criterion	2.080823
Log likelihood	-43.78321	F-statistic	0.107470
Durbin-Watson stat	1.993349	Prob(F-statistic)	0.747046

Sample: 2 20
Included observations: 19

ACF	PACF		AC	PAC	Q-Stat	Prob
. | .	. | .	1	0.001	0.001	4.E-05	0.995
. *| .	. *| .	2	-0.074	-0.074	0.1276	0.938
. *| .	. *| .	3	-0.077	-0.078	0.2768	0.964
. *| .	. *| .	4	-0.081	-0.088	0.4513	0.978
. *| .	. *| .	5	-0.085	-0.099	0.6555	0.985
. *| .	. *| .	6	-0.088	-0.113	0.8947	0.989
. *| .	. *| .	7	-0.092	-0.131	1.1757	0.991
. *| .	. *| .	8	-0.096	-0.154	1.5071	0.993
. | .	. *| .	9	-0.046	-0.128	1.5930	0.996
. | .	. *| .	10	0.029	-0.070	1.6304	0.998
. | .	. *| .	11	0.026	-0.079	1.6650	0.999
. | .	. *| .	12	0.023	-0.080	1.6943	1.000
. | .	. *| .	13	0.019	-0.081	1.7183	1.000
. | .	. *| .	14	0.015	-0.081	1.7372	1.000
. | .	. *| .	15	0.012	-0.080	1.7509	1.000
. | .	. *| .	16	0.008	-0.078	1.7596	1.000
. | .	. *| .	17	0.004	-0.073	1.7636	1.000

Auto-regressive Model of Cohort 1971 Class Baa

LS // Dependent Variable is BAAR71
Date: 05/16/00 Time: 12:48
Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BAAR71=C(1)+C(2)*BAAR71(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.423310	0.152445	2.776810	0.0129
C(2)	0.269820	0.237963	1.133875	0.2726

R-squared	0.070310	Mean dependent var	0.557368
Adjusted R-squared	0.015623	S.D. dependent var	0.422793
S.E. of regression	0.419477	Akaike info criterion	-1.638191
Sum squared resid	2.991340	Schwarz criterion	-1.538777
Log likelihood	-9.397014	F-statistic	1.285674
Durbin-Watson stat	2.067515	Prob(F-statistic)	0.272584

Sample: 2 20
Included observations: 19

```
      ACF                  PACF                AC       PAC    Q-Stat   Prob

  .   *|   .    |    .   *|   .    |1   -0.120   -0.120   0.3199  0.572
  .    |*  .    |    .    |*  .    |2    0.170    0.157   0.9950  0.608
  .    |   .    |    .    |   .    |3   -0.039   -0.003   1.0320  0.793
  .    |***     |    .    |***     |4    0.375    0.358   4.7668  0.312
  .    |*  .    |    .    |*  .    |5    0.082    0.191   4.9578  0.421
  .   *|   .    |    .   *|   .    |6   -0.058   -0.149   5.0614  0.536
  .  **|   .    |    .***|    .    |7   -0.261   -0.403   7.3216  0.396
  .    |   .    |    .  **|   .    |8    0.061   -0.222   7.4556  0.488
  .    |*  .    |    .    |*  .    |9    0.070    0.079   7.6518  0.570
  .   *|   .    |    .    |   .    |10  -0.178   -0.017   9.0546  0.527
  .  **|   .    |    .    |   .    |11  -0.226   -0.009  11.599   0.394
  .   *|   .    |    .    |   .    |12  -0.069    0.045  11.869   0.456
  .    |   .    |    .   *|   .    |13  -0.008   -0.096  11.873   0.538
  .   *|   .    |    .  **|   .    |14  -0.116   -0.209  12.939   0.531
  .    |   .    |    .    |*  .    |15  -0.057    0.083  13.259   0.582
  .   *|   .    |    .    |   .    |16  -0.140    0.054  15.874   0.462
  .    |*  .    |    .    |   .    |17   0.096    0.050  17.697   0.408
```

Auto-Regressive Models of Cohort 1971 Class Ba

LS // Dependent Variable is BAR71

Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BAR71=C(1)+C(2)*BAR71(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.809035	0.384675	2.103163	0.0506
C(2)	0.238589	0.238939	0.998538	0.3320

R-squared	0.055402	Mean dependent var	1.069474
Adjusted R-squared	-0.000162	S.D. dependent var	1.232380
S.E. of regression	1.232480	Akaike info criterion	0.517358
Sum squared resid	25.82313	Schwarz criterion	0.616772
Log likelihood	-29.87473	F-statistic	0.997078
Durbin-Watson stat	1.748694	Prob(F-statistic)	0.332021

Sample: 2 20
Included observations: 19

```
      ACF                PACF               AC       PAC     Q-Stat  Prob

  .    |* .    |    .    |* .    |1   0.096    0.096    0.2056 0.650
  ****|   .    |   ****|   .    |2  -0.493   -0.507    5.9031 0.052
  .    |* .    |    .    |**.    |3   0.076    0.267    6.0484 0.109
  .    |**.    |    .   *|   .    |4   0.229   -0.127    7.4459 0.114
  .   *|   .    |    .    |   .    |5  -0.109    0.040    7.7841 0.169
  .  **|   .    |    .  **|   .    |6  -0.234   -0.227    9.4587 0.149
  .    |   .    |    .    |   .    |7   0.003    0.023    9.4589 0.221
  .    |   .    |    .   *|   .    |8   0.053   -0.186    9.5620 0.297
  .   *|   .    |    .   *|   .    |9  -0.159   -0.116   10.568  0.307
  .    |   .    |    .    |   .    |10 -0.051   -0.022   10.684  0.383
  .    |* .    |    .    |   .    |11  0.178    0.043   12.262  0.344
  .    |   .    |    .   *|   .    |12 -0.021   -0.131   12.287  0.423
  .   *|   .    |    .    |   .    |13 -0.114    0.049   13.159  0.436
  .    |   .    |    .  **|   .    |14 -0.011   -0.225   13.168  0.513
  .    |   .    |    .    |   .    |15  0.016   -0.010   13.195  0.587
  .    |   .    |    .   *|   .    |16  0.009   -0.122   13.205  0.658
  .    |   .    |    .    |   .    |17  0.010    0.063   13.226  0.721
```

Auto-Regressive Model of Cohort 1971 **Class B**

LS // Dependent Variable is BR71

Sample(adjusted): 2 20
Included observations: 19 after adjusting endpoints
BR71=C(1)+C(2)*BR71(-1)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.865315	0.732095	1.181971	0.2535
C(2)	-0.008436	0.236184	-0.035716	0.9719

R-squared	0.000075	Mean dependent var	0.856316
Adjusted R-squared	-0.058744	S.D. dependent var	2.911859
S.E. of regression	2.996166	Akaike info criterion	2.293968
Sum squared resid	152.6092	Schwarz criterion	2.393382
Log likelihood	-46.75252	F-statistic	0.001276
Durbin-Watson stat	2.061399	Prob(F-statistic)	0.971925

Sample: 2 20
Included observations: 19

```
    ACF              PACF                AC        PAC      Q-Stat   Prob

 .  *|  .    |    .  *|  .    |1   -0.066    -0.066    0.0967   0.756
 .  *|  .    |    .  *|  .    |2   -0.080    -0.085    0.2468   0.884
 .  *|  .    |    .  *|  .    |3   -0.085    -0.097    0.4265   0.935
 .  *|  .    |    .  *|  .    |4   -0.090    -0.113    0.6409   0.958
 .  *|  .    |    .  *|  .    |5   -0.095    -0.132    0.8964   0.970
 .  *|  .    |    .  *|  .    |6   -0.100    -0.157    1.2009   0.977
 .  *|  .    |    . **|  .    |7   -0.105    -0.191    1.5640   0.980
 .  *|  .    |    . **|  .    |8   -0.109    -0.240    1.9936   0.981
 .   |  .    |    . **|  .    |9   -0.041    -0.235    2.0621   0.990
 .   |**.    |    .   |* .    |10   0.275     0.091    5.4216   0.861
 .   |  .    |    .  *|  .    |11   0.018    -0.101    5.4384   0.908
 .   |  .    |    .  *|  .    |12   0.011    -0.104    5.4456   0.941
 .   |  .    |    .  *|  .    |13   0.006    -0.103    5.4483   0.964
 .   |  .    |    .  *|  .    |14   0.001    -0.100    5.4485   0.979
 .   |  .    |    .  *|  .    |15  -0.003    -0.092    5.4497   0.988
 .   |  .    |    .  *|  .    |16  -0.008    -0.077    5.4589   0.993
 .   |  .    |    .   |  .    |17  -0.013    -0.052    5.4940   0.996
```

APPENDIX B.

Test for a Markov process in Moody's default rate data for the cohorts formed in the years 1971 through 1979, high-yield bonds in class Baa, Ba, B. The process is modeled as a first order, previous-cohort process.

Previous-Cohort Model

LS // Dependent Variable is BAAR79
Sample: 1 20
Included observations: 20
BAAR79=C(1)+C(2)*BAAR78

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.188985	0.208624	0.905865	0.3770
C(2)	0.679200	0.266964	2.544166	0.0203

R-squared	0.264489	Mean dependent var	0.533000
Adjusted R-squared	0.223627	S.D. dependent var	0.806357
S.E. of regression	0.710497	Akaike info criterion	-0.588940
Sum squared resid	9.086518	Schwarz criterion	-0.489367
Log likelihood	-20.48937	F-statistic	6.472779
Durbin-Watson stat	3.052207	Prob(F-statistic)	0.020346

```
     ACF               PACF              AC       PAC     Q-Stat    Prob

  ****|   .    |   ****|   .    |1   -0.530   -0.530    6.5059   0.011
    .  |**.    |     . *|   .    |2    0.213   -0.094    7.6171   0.022
    .  |* .    |     .  |**.     |3    0.108    0.255    7.9204   0.048
   ***|   .    |    ***|   .    |4   -0.407   -0.335   12.473    0.014
    .  |**.    |     . *|   .    |5    0.318   -0.124   15.440    0.009
   ***|   .    |    .**|   .    |6   -0.332   -0.218   18.904    0.004
    .  |   .   |    .**|   .    |7    0.044   -0.214   18.971    0.008
    .  |**.    |     .  |* .     |8    0.218    0.114   20.720    0.008
    .**|   .   |     .  |   .    |9   -0.247    0.038   23.170    0.006
    .  |**.    |     . *|   .    |10   0.218   -0.148   25.267    0.005
    . *|   .   |     . *|   .    |11  -0.119   -0.18    25.955    0.007
    .  |   .   |     .  |* .     |12   0.054    0.079   26.116    0.010
    .  |   .   |     . *|   .    |13  -0.039   -0.132   26.212    0.016
    .  |   .   |     .  |   .    |14   0.007    0.048   26.216    0.024
    .  |   .   |     .  |   .    |15  -0.002   -0.022   26.216    0.036
    .  |   .   |     . *|   .    |16  -0.017   -0.116   26.249    0.051
    .  |   .   |     . *|   .    |17   0.006   -0.131   26.254    0.070
    .  |   .   |     .  |   .    |18   0.002    0.062   26.254    0.094
```

Previous-Cohort Model

LS // Dependent Variable is BAR79
Sample: 1 20
Included observations: 20
BAR79=C(1)+C(2)*BAR78

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.577762	0.518133	1.115083	0.2795
C(2)	0.719337	0.204605	3.515738	0.0025

R-squared	0.407123	Mean dependent var	1.751000
Adjusted R-squared	0.374185	S.D. dependent var	2.240679
S.E. of regression	1.772566	Akaike info criterion	1.239496
Sum squared resid	56.55586	Schwarz criterion	1.339070
Log likelihood	-38.77373	F-statistic	12.36042
Durbin-Watson stat	2.108254	Prob(F-statistic)	0.002469

```
      ACF                 PACF                  AC       PAC      Q-Stat   Prob

  .  *|   .   |     .  *|   .   |1   -0.064   -0.064   0.0939   0.759
  .  *|   .   |     .  *|   .   |2   -0.107   -0.112   0.3742   0.829
  .  *|   .   |     .  *|   .   |3   -0.072   -0.088   0.5068   0.917
  .   |*  .   |     .   |*  .   |4    0.141    0.119   1.0503   0.902
  ***|    .   |     ***|    .   |5   -0.378   -0.391   5.2326   0.388
  .   |   .   |     .   |   .   |6    0.011   -0.003   5.2365   0.514
  .   |** .   |     .   |*  .   |7    0.221    0.187   6.8966   0.440
  .   |*  .   |     .   |   .   |8    0.069   -0.009   7.0696   0.529
  .  *|   .   |     .   |   .   |9   -0.099    0.041   7.4616   0.589
  .   |   .   |     .  *|   .   |10  -0.044   -0.176   7.5470   0.673
  .   |   .   |     .  *|   .   |11  -0.018   -0.083   7.5631   0.752
  .  *|   .   |     .   |   .   |12  -0.125   -0.006   8.4216   0.751
  .  *|   .   |     .  *|   .   |13  -0.096   -0.131   9.0012   0.773
  .   |   .   |     .  *|   .   |14  -0.014   -0.101   9.0166   0.830
  .   |   .   |     .  *|   .   |15   0.034   -0.103   9.1212   0.871
  .   |   .   |     .   |   .   |16   0.002   -0.054   9.1216   0.908
  .   |   .   |     .   |   .   |17   0.015    0.013   9.1542   0.935
  .   |   .   |     .   |   .   |18   0.015   -0.045   9.2020   0.955
```

Previous-Cohort Model

LS // Dependent Variable is BR79
Sample: 1 20
Included observations: 20
BR79=C(1)+C(2)*BR78

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	3.785839	1.247070	3.035786	0.0071
C(2)	-0.231304	0.239099	-0.967398	0.3462

```
R-squared                 0.049423   Mean dependent var       3.040000
Adjusted R-squared       -0.003387   S.D. dependent var
4.376164S.E. of regression     4.383569   Akaike info criterion
3.050366
Sum squared resid         345.8822   Schwarz criterion        3.149939
Log likelihood           -56.88243   F-statistic              0.935859
Durbin-Watson stat        1.468784   Prob(F-statistic)        0.346165
```

```
      ACF                PACF              AC          PAC    Q-Stat    Prob

  .   |**.     |     .   |**.     |1    0.236      0.236   1.2882    0.256
  .   |* .     |     .   |  .     |2    0.097      0.043   1.5161    0.469
  .   |  .     |     .   |  .     |3    0.030     -0.002   1.5399    0.673
  .   |***     |     .   |***     |4    0.349      0.359   4.8831    0.300
  .   |* .     |     .   |  .     |5    0.165      0.011   5.6838    0.338
  .   |  .     |     . *|  .      |6   -0.002     -0.101   5.6840    0.460
  .**|   .     |     ***|  .      |7   -0.314     -0.347   9.0158    0.252
  . *|   .     |     . *|  .      |8   -0.151     -0.180   9.8490    0.276
  . *|   .     |     . *|  .      |9   -0.122     -0.133  10.440     0.316
  .   |  .     |     .   |  .     |10  -0.049      0.031  10.545     0.394
  .**|   .     |     .   |  .     |11  -0.298     -0.055  14.897     0.187
  . *|   .     |     .   |* .     |12  -0.169      0.115  16.460     0.171
  . *|   .     |     .   |* .     |13  -0.107      0.074  17.178     0.191
  .   |  .     |     . *|  .      |14  -0.056     -0.126  17.410     0.235
  .   |  .     |     .   |  .     |15  -0.046      0.013  17.599     0.284
  . *|   .     |     . *|  .      |16  -0.069     -0.131  18.118     0.317
  .   |  .     |     .   |  .     |17  -0.018     -0.027  18.164     0.379
  .   |  .     |     . *|  .      |18  -0.004     -0.097  18.168     0.445
```

Previous-Cohort model of Cohort78 Class Baa

LS // Dependent Variable is BAAR78
Sample: 1 20
Included observations: 20
BAAR78=C(1)+C(2)*BAAR77

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.233967	0.169316	1.381834	0.1839
C(2)	0.525112	0.224521	2.338811	0.0311

R-squared	0.233065	Mean dependent var	0.506500
Adjusted R-squared	0.190457	S.D. dependent var	0.610567
S.E. of regression	0.549355	Akaike info criterion	-1.103380
Sum squared resid	5.432245	Schwarz criterion	-1.003807
Log likelihood	-15.34497	F-statistic	5.470038
Durbin-Watson stat	1.940018	Prob(F-statistic)	0.031081

```
      ACF                 PACF                 AC        PAC      Q-Stat   Prob

   .   |   .    |     .   |   .    | 1     0.012     0.012    0.0033   0.954
   .   |*  .    |     .   |*  .    | 2     0.068     0.068    0.1167   0.943
   . *|    .    |     . *|    .    | 3    -0.092    -0.094    0.3358   0.953
   .**|    .    |     .**|    .    | 4    -0.194    -0.198    1.3663   0.850
   .   |   .    |     .   |   .    | 5     0.030     0.048    1.3922   0.925
   . *|    .    |     . *|    .    | 6    -0.149    -0.136    2.0896   0.911
   .   |   .    |     .   |   .    | 7    -0.001    -0.044    2.0896   0.955
   .   |   .    |     . *|    .    | 8    -0.056    -0.071    2.2043   0.974
   .   |*  .    |     .   |*  .    | 9     0.173     0.176    3.3954   0.947
   . *|    .    |     .**|    .    |10    -0.180    -0.266    4.8242   0.903
   .**|    .    |     .**|    .    |11    -0.208    -0.263    6.9368   0.804
   . *|    .    |     . *|    .    |12    -0.124    -0.127    7.7774   0.802
   .   |   .    |     .   |   .    |13    -0.025     0.040    7.8157   0.855
   .   |*  .    |     . *|    .    |14     0.074    -0.115    8.2181   0.878
   .   |   .    |     . *|    .    |15     0.027    -0.058    8.2821   0.912
   .   |   .    |     .   |   .    |16     0.053    -0.047    8.5908   0.929
   .   |   .    |     .   |   .    |17     0.043    -0.019    8.8607   0.945
   .   |   .    |     . *|    .    |18     0.033    -0.147    9.0976   0.957
```

Previous-Cohort model of Cohort78 Class Ba

LS // Dependent Variable is BAR78
Sample: 1 20
Included observations: 20
BAR78=C(1)+C(2)*BAR77

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.416668	0.578334	2.449569	0.0248
C(2)	0.143992	0.242228	0.594449	0.5596

R-squared	0.019254	Mean dependent var	1.631000
Adjusted R-squared	-0.035232	S.D. dependent var	1.987513
S.E. of regression	2.022223	Akaike info criterion	1.503034
Sum squared resid	73.60892	Schwarz criterion	1.602607
Log likelihood	-41.40911	F-statistic	0.353370
Durbin-Watson stat	1.793942	Prob(F-statistic)	0.559610

ACF	PACF		AC	PAC	Q-Stat	Prob
. |* . |	. |* .	1	0.088	0.088	0.1803	0.671
. | . |	. *| .	2	-0.053	-0.061	0.2485	0.883
. *| . |	. *| .	3	-0.150	-0.141	0.8315	0.842
. | . |	. | .	4	-0.031	-0.009	0.8586	0.930
. |* . |	. |* .	5	0.194	0.189	1.9636	0.854
. *| . |	.**| .	6	-0.163	-0.233	2.7988	0.834
. *| . |	. *| .	7	-0.150	-0.115	3.5613	0.829
. | . |	. |* .	8	-0.017	0.066	3.5722	0.894
. | . |	. *| .	9	-0.004	-0.069	3.5730	0.937
. *| . |	.**| .	10	-0.161	-0.291	4.7158	0.909
. *| . |	. | .	11	-0.161	-0.055	5.9874	0.874
. *| . |	. *| .	12	-0.142	-0.131	7.0924	0.851
. | . |	. *| .	13	0.000	-0.172	7.0924	0.897
. | . |	. *| .	14	0.010	-0.089	7.0992	0.931
. | . |	. |* .	15	0.047	0.082	7.2947	0.949
. |* . |	. | .	16	0.083	-0.038	8.0509	0.947
. | . |	. | .	17	0.065	-0.043	8.6801	0.950
. | . |	. | .	18	0.038	0.004	9.0017	0.960

Previous-Cohort model of Cohort78 Class B

LS // Dependent Variable is BR78
Sample: 1 20
Included observations: 20
BR78=C(1)+C(2)*BR77

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	2.716636	1.081004	2.513066	0.0217
C(2)	0.163432	0.170373	0.959262	0.3501

R-squared	0.048635	Mean dependent var	3.224500
Adjusted R-squared	-0.004219	S.D. dependent var	4.206044
S.E. of regression	4.214906	Akaike info criterion	2.971894
Sum squared resid	319.7778	Schwarz criterion	3.071467
Log likelihood	-56.09771	F-statistic	0.920183
Durbin-Watson stat	2.042507	Prob(F-statistic)	0.350136

```
      ACF            PACF              AC       PAC     Q-Stat   Prob

   .  |  .   |    .  |  .   |1   -0.041   -0.041   0.0389   0.844
   . *|  .   |    . *|  .   |2   -0.162   -0.164   0.6775   0.713
   . *|  .   |    . *|  .   |3   -0.134   -0.153   1.1453   0.766
   .  |** .  |    .  |** .  |4    0.283    0.252   3.3422   0.502
   . *|  .   |    . *|  .   |5   -0.095   -0.129   3.6096   0.607
   . *|  .   |    . *|  .   |6   -0.129   -0.085   4.1361   0.658
   . *|  .   |    . *|  .   |7   -0.116   -0.088   4.5933   0.709
   . *|  .   |    .**|  .   |8   -0.110   -0.281   5.0345   0.754
   .  |* .   |    .  |** .  |9    0.185    0.217   6.3968   0.700
   .  |  .   |    .  |  .   |10   0.005   -0.042   6.3978   0.781
   .  |  .   |    .  |  .   |11  -0.007    0.021   6.4002   0.845
   . *|  .   |    .  |  .   |12  -0.098    0.030   6.9274   0.862
   .  |* .   |    . *|  .   |13   0.078   -0.152   7.3140   0.885
   .  |  .   |    .  |  .   |14  -0.042   -0.032   7.4424   0.916
   . *|  .   |    . *|  .   |15  -0.058   -0.105   7.7361   0.934
   .  |  .   |    .  |  .   |16  -0.024   -0.016   7.7979   0.955
   .  |  .   |    .  |  .   |17  -0.024    0.022   7.8798   0.969
   .  |  .   |    . *|  .   |18   0.009   -0.064   7.8961   0.980
```

Previous-Cohort model of Cohort77 Class Baa

LS // Dependent Variable is BAAR77
Sample: 1 20
Included observations: 20
BAAR77=C(1)+C(2)*BAAR76

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.277002	0.156176	1.773652	0.0930
C(2)	0.435248	0.192393	2.262294	0.0363

R-squared	0.221385	Mean dependent var	0.519000
Adjusted R-squared	0.178129	S.D. dependent var	0.561332
S.E. of regression	0.508887	Akaike info criterion	-1.256417
Sum squared resid	4.661396	Schwarz criterion	-1.156844
Log likelihood	-13.81460	F-statistic	5.117975
Durbin-Watson stat	1.990686	Prob(F-statistic)	0.036287

ACF	PACF		AC	PAC	Q-Stat	Prob
. | . |	. | .	1	-0.004	-0.004	0.0003	0.986
. | . |	. | .	2	0.047	0.047	0.0551	0.973
. *| . |	. *| .	3	-0.184	-0.184	0.9332	0.817
.**| . |	.**| .	4	-0.204	-0.214	2.0767	0.722
. |* . |	. |* .	5	0.103	0.124	2.3882	0.793
.**| . |	.**| .	6	-0.220	-0.251	3.9040	0.690
. |* . |	. |* .	7	0.163	0.089	4.8069	0.684
. |* . |	. |* .	8	0.093	0.133	5.1216	0.744
. | . |	. *| .	9	-0.005	-0.089	5.1227	0.823
.**| . |	***| .	10	-0.232	-0.348	7.4987	0.678
.**| . |	. *| .	11	-0.227	-0.083	10.010	0.529
. | . |	. *| .	12	-0.049	-0.096	10.145	0.603
. | . |	. *| .	13	0.026	-0.087	10.188	0.679
. | . |	. *| .	14	0.031	-0.114	10.258	0.743
. |* . |	. | .	15	0.077	0.019	10.779	0.768
. | . |	. *| .	16	0.047	-0.136	11.024	0.808
. | . |	. | .	17	0.030	-0.026	11.159	0.848
. | . |	. |* .	18	0.007	0.074	11.169	0.887

Previous-Cohort model of Cohort77 Class Ba

LS // Dependent Variable is BAR77
Sample: 1 20
Included observations: 20
BAR77=C(1)+C(2)*BAR76

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.293101	0.574168	2.252130	0.0370
C(2)	0.121028	0.230906	0.524142	0.6066

R-squared	0.015033	Mean dependent var	1.488500
Adjusted R-squared	-0.039687	S.D. dependent var	1.915264
S.E. of regression	1.952900	Akaike info criterion	1.433270
Sum squared resid	68.64870	Schwarz criterion	1.532843
Log likelihood	-40.71147	F-statistic	0.274725
Durbin-Watson stat	1.940542	Prob(F-statistic)	0.606574

```
     ACF                  PACF                   AC        PAC    Q-Stat    Prob

  .  |  .     |       .  |  .     |1    0.012     0.012    0.0032   0.955
  . *|  .     |       . *|  .     |2   -0.104    -0.104    0.2686   0.874
  .**|  .     |       .**|  .     |3   -0.197    -0.196    1.2719   0.736
  .  |  .     |       .  |  .     |4    0.007    -0.003    1.2734   0.866
  .  |**.     |       .  |**.     |5    0.234     0.203    2.8736   0.719
  .  |  .     |       .  |  .     |6    0.011    -0.026    2.8778   0.824
  . *|  .     |       . *|  .     |7   -0.186    -0.164    4.0491   0.774
  . *|  .     |       . *|  .     |8   -0.167    -0.100    5.0759   0.749
  .  |  .     |       .  |  .     |9   -0.012    -0.039    5.0814   0.827
  . *|  .     |       .**|  .     |10  -0.136    -0.292    5.8999   0.824
  .  |  .     |       . *|  .     |11  -0.010    -0.090    5.9044   0.880
  . *|  .     |       . *|  .     |12  -0.092    -0.079    6.3744   0.896
  .  |  .     |       .  |  .     |13   0.002    -0.053    6.3747   0.931
  . *|  .     |       . *|  .     |14  -0.069    -0.175    6.7229   0.945
  .  |* .     |       .  |* .     |15   0.069     0.067    7.1396   0.954
  .  |  .     |       .  |  .     |16   0.047     0.007    7.3812   0.965
  .  |  .     |       .  |  .     |17   0.052    -0.055    7.7801   0.971
  .  |  .     |       .  |  .     |18   0.023    -0.050    7.8956   0.980
```

Previous-Cohort model of Cohort77 Class B

LS // Dependent Variable is BR77
Sample: 1 20
Included observations: 20
BR77=C(1)+C(2)*BR76

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	2.173886	1.367447	1.589740	0.1293
C(2)	0.374194	0.243029	1.539711	0.1410

R-squared	0.116378	Mean dependent var	3.107500
Adjusted R-squared	0.067288	S.D. dependent var	5.675598
S.E. of regression	5.481322	Akaike info criterion	3.497332
Sum squared resid	540.8080	Schwarz criterion	3.596905
Log likelihood	-61.35209	F-statistic	2.370711
Durbin-Watson stat	1.692842	Prob(F-statistic)	0.141026

```
      ACF                  PACF                  AC       PAC     Q-Stat   Prob

  .  |*  .      |      .  |*  .      |1    0.148    0.148   0.5062  0.477
  .**|   .      |      ***|   .      |2   -0.298   -0.327   2.6780  0.262
  . *|   .      |      . *|   .      |3   -0.162   -0.063   3.3570  0.340
  . *|   .      |      . *|   .      |4   -0.103   -0.183   3.6501  0.455
  . *|   .      |      .**|   .      |5   -0.142   -0.200   4.2437  0.515
  . *|   .      |      .**|   .      |6   -0.126   -0.222   4.7423  0.577
  . *|   .      |      ***|   .      |7   -0.172   -0.377   5.7455  0.570
  .  |*  .      |      . *|   .      |8    0.110   -0.097   6.1896  0.626
  .  |***       |      .  |   .      |9    0.332    0.004   10.587  0.305
  .  |   .      |      . *|   .      |10   0.059   -0.184   10.743  0.378
  .  |   .      |      . *|   .      |11  -0.042   -0.059   10.828  0.458
  . *|   .      |      .**|   .      |12  -0.091   -0.258   11.283  0.505
  .  |   .      |      .  |   .      |13   0.042    0.012   11.393  0.578
  .  |   .      |      . *|   .      |14  -0.013   -0.166   11.406  0.654
  .  |   .      |      .  |   .      |15  -0.030    0.011   11.484  0.718
  .  |   .      |      .  |   .      |16  -0.005    0.044   11.487  0.779
  .  |   .      |      . *|   .      |17   0.004   -0.071   11.489  0.830
  .  |   .      |      .  |   .      |18  -0.005   -0.008   11.494  0.872
```

Previous-Cohort model of Cohort76 Class Baa

LS // Dependent Variable is BAAR76
Sample: 1 20
Included observations: 20
BAAR76=C(1)+C(2)*BAAR75

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.317560	0.172588	1.839989	0.0823
C(2)	0.427695	0.211796	2.019372	0.0586

R-squared	0.184704	Mean dependent var	0.556000
Adjusted R-squared	0.139409	S.D. dependent var	0.606816
S.E. of regression	0.562931	Akaike info criterion	-1.054557
Sum squared resid	5.704041	Schwarz criterion	-0.954984
Log likelihood	-15.83320	F-statistic	4.077862
Durbin-Watson stat	1.942282	Prob(F-statistic)	0.058594

```
     ACF                 PACF               AC       PAC     Q-Stat   Prob

 .   |   .   |       .   |   .   |1    0.011    0.011   0.0029   0.957
 .  *|   .   |       .  *|   .   |2   -0.059   -0.059   0.0890   0.956
 .  *|   .   |       .  *|   .   |3   -0.138   -0.137   0.5829   0.900
 . **|   .   |       . **|   .   |4   -0.223   -0.230   1.9516   0.745
 .   |*  .   |       .   |*  .   |5    0.111    0.097   2.3142   0.804
 .  *|   .   |       . **|   .   |6   -0.162   -0.225   3.1397   0.791
 .   |*  .   |       .   |*  .   |7    0.154    0.124   3.9412   0.787
 .   |*  .   |       .   |*  .   |8    0.119    0.068   4.4604   0.813
 .   |   .   |       .   |   .   |9   -0.034   -0.022   4.5066   0.875
 . **|   .   |       . **|   .   |10  -0.201   -0.282   6.2878   0.791
 .  *|   .   |       .   |   .   |11  -0.155   -0.023   7.4660   0.760
 .  *|   .   |       .  *|   .   |12  -0.068   -0.175   7.7201   0.807
 .   |   .   |       .  *|   .   |13  -0.014   -0.082   7.7317   0.861
 .   |   .   |       .  *|   .   |14  -0.002   -0.168   7.7319   0.903
 .   |   .   |       .   |   .   |15   0.018   -0.045   7.7620   0.933
 .   |*  .   |       .  *|   .   |16   0.076   -0.125   8.3985   0.936
 .   |   .   |       .   |   .   |17   0.028    0.034   8.5166   0.954
 .   |   .   |       .   |   .   |18   0.020   -0.021   8.6075   0.968
```

Previous-Cohort model of Cohort76 Class Ba

LS // Dependent Variable is BAR76
Sample: 1 20
Included observations: 20
BAR76=C(1)+C(2)*BAR75

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.410326	0.595930	2.366596	0.0294
C(2)	0.120350	0.235252	0.511580	0.6152

R-squared	0.014331	Mean dependent var	1.614500
Adjusted R-squared	-0.040428	S.D. dependent var	1.940297
S.E. of regression	1.979130	Akaike info criterion	1.459954
Sum squared resid	70.50517	Schwarz criterion	1.559527
Log likelihood	-40.97831	F-statistic	0.261714
Durbin-Watson stat	1.977633	Prob(F-statistic)	0.615164

```
      ACF              PACF                AC        PAC      Q-Stat   Prob

 .   |   .     |    .   |   .    |1   -0.005    -0.005    0.0005   0.981
 . * |   .     |    . * |   .    |2   -0.158    -0.158    0.6088   0.738
 .** |   .     |    .** |   .    |3   -0.215    -0.222    1.8055   0.614
 .   |   .     |    . * |   .    |4   -0.018    -0.057    1.8148   0.770
 .   |***      |    .   |**.     |5    0.370     0.322    5.8405   0.322
 .   |   .     |    .   |   .    |6   -0.005    -0.042    5.8412   0.441
 .** |   .     |    . * |   .    |7   -0.202    -0.155    7.2170   0.407
 . * |   .     |    . * |   .    |8   -0.186    -0.083    8.4798   0.388
 .   |   .     |    .   |   .    |9    0.003    -0.021    8.4803   0.487
 . * |   .     |    *** |   .    |10  -0.079    -0.346    8.7570   0.555
 .   |   .     |    . * |   .    |11   0.012    -0.074    8.7642   0.644
 . * |   .     |    . * |   .    |12  -0.112    -0.065    9.4581   0.663
 .   |   .     |    .   |   .    |13  -0.021    -0.047    9.4846   0.735
 .   |   .     |    . * |   .    |14   0.003    -0.120    9.4853   0.799
 .   |   .     |    .   |*  .    |15   0.012     0.086    9.4984   0.850
 .   |   .     |    .   |   .    |16   0.047     0.004    9.7376   0.880
 .   |   .     |    .   |   .    |17   0.020    -0.034    9.7979   0.912
 .   |   .     |    .   |   .    |18   0.021    -0.055    9.8995   0.935
```

Previous-Cohort model of Cohort76 ClassB

LS // Dependent Variable is BR76
Sample: 1 20
Included observations: 20
BR76=C(1)+C(2)*BR75

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.727937	1.330003	1.299198	0.2103
C(2)	0.359617	0.314952	1.141814	0.2685

R-squared	0.067538	Mean dependent var	2.495000
Adjusted R-squared	0.015735	S.D. dependent var	5.174294
S.E. of regression	5.133425	Akaike info criterion	3.366186
Sum squared resid	474.3369	Schwarz criterion	3.465759
Log likelihood	-60.04063	F-statistic	1.303740
Durbin-Watson stat	1.617097	Prob(F-statistic)	0.268494

```
      ACF              PACF              AC       PAC     Q-Stat   Prob

 .  |* .    |      .  |* .    |1    0.172    0.172   0.6847   0.408
 .**|  .    |      ***|  .    |2   -0.312   -0.352   3.0624   0.216
 . *|  .    |      .  |  .    |3   -0.155   -0.026   3.6830   0.298
 . *|  .    |      .**|  .    |4   -0.120   -0.222   4.0799   0.395
 . *|  .    |      . *|  .    |5   -0.077   -0.088   4.2557   0.513
 . *|  .    |      .**|  .    |6   -0.121   -0.258   4.7151   0.581
 . *|  .    |      .**|  .    |7   -0.111   -0.192   5.1307   0.644
 .  |* .    |      .  |  .    |8    0.135   -0.012   5.8031   0.669
 .  |***    |      .  |* .    |9    0.352    0.193  10.750    0.293
 .  |  .    |      . *|  .    |10  -0.007   -0.173  10.752    0.377
 . *|  .    |      .  |  .    |11  -0.147    0.029  11.812    0.378
 . *|  .    |      . *|  .    |12  -0.108   -0.166  12.459    0.410
 .  |  .    |      .  |* .    |13   0.036    0.137  12.540    0.484
 .  |  .    |      .**|  .    |14  -0.032   -0.201  12.617    0.557
 . *|  .    |      .  |  .    |15  -0.097    0.061  13.444    0.568
 .  |  .    |      .  |  .    |16   0.058   -0.024  13.815    0.612
 .  |  .    |      . *|  .    |17   0.013   -0.121  13.841    0.678
 .  |  .    |      . *|  .    |18   0.007   -0.133  13.851    0.739
```

Previous-Cohort model of Cohort75 Class Baa

LS // Dependent Variable is BAAR75
Sample: 1 20
Included observations: 20
BAAR75=C(1)+C(2)*BAAR74

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.346968	0.201356	1.723156	0.1020
C(2)	0.354132	0.254165	1.393319	0.1805

R-squared	0.097352	Mean dependent var	0.557500
Adjusted R-squared	0.047205	S.D. dependent var	0.609762
S.E. of regression	0.595196	Akaike info criterion	-0.943091
Sum squared resid	6.376641	Schwarz criterion	-0.843518
Log likelihood	-16.94786	F-statistic	1.941337
Durbin-Watson stat	1.823986	Prob(F-statistic)	0.180492

ACF	PACF		AC	PAC	Q-Stat	Prob
. |* . |	. |* . |	1	0.069	0.069	0.1107	0.739
. *| . |	. *| . |	2	-0.073	-0.078	0.2419	0.886
. *| . |	. *| . |	3	-0.156	-0.146	0.8697	0.833
. *| . |	. *| . |	4	-0.095	-0.082	1.1162	0.892
. | . |	. | . |	5	0.057	0.048	1.2113	0.944
. | . |	. | . |	6	0.039	-0.002	1.2589	0.974
. | . |	. | . |	7	-0.005	-0.027	1.2597	0.989
. | . |	. | . |	8	0.039	0.053	1.3167	0.995
. | . |	. | . |	9	-0.049	-0.043	1.4128	0.998
.**| . |	.**| . |	10	-0.194	-0.197	3.0752	0.980
. | . |	. | . |	11	-0.010	0.015	3.0801	0.990
. *| . |	. *| . |	12	-0.146	-0.187	4.2445	0.979
. | . |	. | . |	13	0.063	0.013	4.4978	0.985
. *| . |	.**| . |	14	-0.134	-0.217	5.8143	0.971
. *| . |	. *| . |	15	-0.103	-0.121	6.7490	0.964
. | . |	. | . |	16	0.034	-0.013	6.8753	0.976
. |* . |	. | . |	17	0.097	0.039	8.2549	0.961
. | . |	. | . |	18	0.047	-0.017	8.7371	0.966

Previous-Cohort model of Cohort75   Class Ba

LS // Dependent Variable is BAR75
Sample: 1 20
Included observations: 20
BAR75=C(1)+C(2)*BAR74

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.402488	0.563640	2.488268	0.0229
C(2)	0.160970	0.196024	0.821174	0.4223

R-squared	0.036110	Mean dependent var	1.696500
Adjusted R-squared	-0.017440	S.D. dependent var	1.930029
S.E. of regression	1.946786	Akaike info criterion	1.426999
Sum squared resid	68.21957	Schwarz criterion	1.526572
Log likelihood	-40.64876	F-statistic	0.674327
Durbin-Watson stat	2.028475	Prob(F-statistic)	0.422290

```
   ACF               PACF                 AC       PAC      Q-Stat  Prob

  .  |  .   |      .  |  .   | 1   -0.036   -0.036   0.0300  0.863
  . *|  .   |      . *|  .   | 2   -0.184   -0.185   0.8546  0.652
  .**|  .   |      .**|  .   | 3   -0.194   -0.216   1.8287  0.609
  .  |* .   |      .  |* .   | 4    0.187    0.139   2.7951  0.593
  .  |**.   |      .  |**.   | 5    0.325    0.299   5.8903  0.317
  . *|  .   |      . *|  .   | 6   -0.107   -0.058   6.2499  0.396
  .**|  .   |      . *|  .   | 7   -0.236   -0.124   8.1374  0.321
  .**|  .   |      .**|  .   | 8   -0.215   -0.224   9.8313  0.277
  .  |* .   |      .  |  .   | 9    0.158   -0.053  10.832   0.287
  . *|  .   |      .**|  .   |10   -0.076   -0.284  11.088   0.351
  . *|  .   |      . *|  .   |11   -0.069   -0.065  11.322   0.417
  . *|  .   |      .  |  .   |12   -0.132    0.000  12.277   0.424
  .  |  .   |      .  |  .   |13   -0.009    0.016  12.282   0.505
  .  |  .   |      .  |  .   |14    0.030   -0.052  12.350   0.578
  .  |  .   |      .  |  .   |15    0.014    0.044  12.367   0.651
  .  |  .   |      .  |  .   |16    0.005   -0.019  12.369   0.718
  .  |  .   |      .  |  .   |17    0.017   -0.006  12.410   0.775
  .  |  .   |      . *|  .   |18    0.012   -0.109  12.444   0.823
```

Previous-Cohort model of Cohort75 Class B

LS // Dependent Variable is BR75
Sample: 1 20
Included observations: 20
BR75=C(1)+C(2)*BR74

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.818920	0.901927	2.016705	0.0589
C(2)	0.262390	0.277609	0.945177	0.3571

R-squared	0.047284	Mean dependent var	2.133000
Adjusted R-squared	-0.005644	S.D. dependent var	3.739261
S.E. of regression	3.749799	Akaike info criterion	2.738044
Sum squared resid	253.0979	Schwarz criterion	2.837617
Log likelihood	-53.75921	F-statistic	0.893360
Durbin-Watson stat	1.507905	Prob(F-statistic)	0.357084

```
      ACF                PACF                AC        PAC      Q-Stat    Prob

   .  |**.    |     .  |**.    |1    0.234     0.234    1.2717   0.259
   .**|  .    |     ***|  .    |2   -0.300    -0.375    3.4671   0.177
   . *|  .    |     .  |  .    |3   -0.181     0.004    4.3145   0.229
   . *|  .    |     . *|  .    |4   -0.093    -0.182    4.5505   0.337
   . *|  .    |     . *|  .    |5   -0.157    -0.186    5.2728   0.384
   . *|  .    |     . *|  .    |6   -0.086    -0.102    5.5062   0.481
   . *|  .    |     .**|  .    |7   -0.124    -0.300    6.0244   0.537
   .  |  .    |     .  |  .    |8    0.039     0.016    6.0797   0.638
   .  |***    |     .  |* .    |9    0.345     0.174   10.851    0.286
   .  |  .    |     .**|  .    |10   0.044    -0.258   10.935    0.363
   . *|  .    |     .  |  .    |11  -0.144     0.063   11.946    0.368
   .  |  .    |     . *|  .    |12  -0.046    -0.127   12.063    0.441
   .  |  .    |     .  |  .    |13   0.012    -0.017   12.072    0.522
   . *|  .    |     . *|  .    |14  -0.123    -0.144   13.182    0.512
   .  |  .    |     .  |  .    |15  -0.003     0.018   13.182    0.588
   .  |* .    |     .  |  .    |16   0.077     0.040   13.838    0.611
   .  |  .    |     . *|  .    |17   0.015    -0.122   13.869    0.676
   .  |  .    |     . *|  .    |18   0.001    -0.092   13.869    0.738
```

Previous-Cohort model of Cohort74 Class Baa

LS // Dependent Variable is BAAR74
Sample: 1 20
Included observations: 20
BAAR74=C(1)+C(2)*BAAR73

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.331029	0.181137	1.827508	0.0843
C(2)	0.378550	0.203376	1.861336	0.0791

R-squared	0.161409	Mean dependent var	0.594500
Adjusted R-squared	0.114820	S.D. dependent var	0.537239
S.E. of regression	0.505456	Akaike info criterion	-1.269949
Sum squared resid	4.598746	Schwarz criterion	-1.170375
Log likelihood	-13.67928	F-statistic	3.464572
Durbin-Watson stat	2.027584	Prob(F-statistic)	0.079109

```
      ACF                 PACF                  AC       PAC     Q-Stat   Prob

 .  |  .    |       .  |  .    |1   -0.053   -0.053   0.0651  0.799
 . *|  .    |       . *|  .    |2   -0.084   -0.087   0.2367  0.888
 .  |  .    |       . *|  .    |3   -0.049   -0.059   0.2982  0.960
 .**|  .    |       .**|  .    |4   -0.215   -0.232   1.5753  0.813
 .  |* .    |       .  |* .    |5    0.143    0.110   2.1741  0.825
 .  |* .    |       .  |* .    |6    0.096    0.071   2.4640  0.872
 .  |  .    |       .  |  .    |7    0.040    0.053   2.5194  0.926
 .  |  .    |       .  |  .    |8   -0.037   -0.055   2.5700  0.958
 . *|  .    |       .  |  .    |9   -0.117   -0.055   3.1163  0.960
 . *|  .    |       . *|  .    |10  -0.170   -0.180   4.3817  0.928
 .  |  .    |       .  |  .    |11   0.018   -0.028   4.3974  0.957
 . *|  .    |       .**|  .    |12  -0.104   -0.204   4.9936  0.958
 .  |* .    |       .  |  .    |13   0.111    0.054   5.7695  0.954
 .  |  .    |       . *|  .    |14  -0.018   -0.099   5.7933  0.971
 . *|  .    |       .  |  .    |15  -0.099   -0.041   6.6479  0.967
 .  |  .    |       . *|  .    |16  -0.016   -0.082   6.6769  0.979
 . *|  .    |       .  |  .    |17  -0.102   -0.057   8.1996  0.962
 .  |* .    |       .  |  .    |18   0.119    0.040  11.316   0.880
```

Previous-Cohort model of Cohort74   Class Ba

LS // Dependent Variable is BAR74
Sample: 1 20
Included observations: 20
BAR74=C(1)+C(2)*BAR73

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.595266	0.671298	2.376390	0.0288
C(2)	0.143847	0.264723	0.543387	0.5935

R-squared	0.016139	Mean dependent var	1.826500
Adjusted R-squared	-0.038520	S.D. dependent var	2.278407
S.E. of regression	2.321875	Akaike info criterion	1.779389
Sum squared resid	97.03983	Schwarz criterion	1.878962
Log likelihood	-44.17266	F-statistic	0.295270
Durbin-Watson stat	1.812428	Prob(F-statistic)	0.593530

ACF	PACF		AC	PAC	Q-Stat	Prob
. |* . |	. |* .	1	0.081	0.081	0.1506	0.698
.**| . |	.**| .	2	-0.243	-0.251	1.5973	0.450
. *| . |	. *| .	3	-0.144	-0.106	2.1329	0.545
. |**. |	. |**.	4	0.289	0.271	4.4300	0.351
. |*** |	. |**.	5	0.380	0.320	8.6679	0.123
. *| . |	. *| .	6	-0.114	-0.077	9.0745	0.169
.**| . |	. | .	7	-0.198	-0.014	10.406	0.167
. | . |	. *| .	8	-0.052	-0.087	10.504	0.231
. |* . |	. *| .	9	0.163	-0.073	11.571	0.239
. *| . |	.**| .	10	-0.126	-0.312	12.266	0.268
. *| . |	. | .	11	-0.147	-0.035	13.325	0.273
. *| . |	. *| .	12	-0.157	-0.152	14.681	0.259
. | . |	. | .	13	0.003	-0.049	14.681	0.328
. | . |	. *| .	14	-0.044	-0.070	14.826	0.390
. *| . |	. |* .	15	-0.068	0.135	15.239	0.434
. | . |	. | .	16	-0.045	0.043	15.466	0.491
. *| . |	. | .	17	-0.099	-0.033	16.912	0.460
. | . |	. | .	18	0.021	0.033	17.014	0.522

Previous-Cohort model of Cohort74   Class B

LS // Dependent Variable is BR74
Sample: 1 20
Included observations: 20
BR74=C(1)+C(2)*BR73

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.067839	0.734856	1.453127	0.1634
C(2)	0.169725	0.274766	0.617708	0.5445

R-squared	0.020758	Mean dependent var	1.197000
Adjusted R-squared	-0.033644	S.D. dependent var	3.098830
S.E. of regression	3.150528	Akaike info criterion	2.389780
Sum squared resid	178.6649	Schwarz criterion	2.489353
Log likelihood	-50.27657	F-statistic	0.381563
Durbin-Watson stat	1.477293	Prob(F-statistic)	0.544504

```
        ACF               PACF                AC       PAC     Q-Stat   Prob

   .   |   .    |     .   |   .    |1    0.051    0.051    0.0599   0.807
   .  *|   .    |     .  *|   .    |2   -0.130   -0.133    0.4705   0.790
   .  *|   .    |     .  *|   .    |3   -0.089   -0.076    0.6753   0.879
   .  *|   .    |     .  *|   .    |4   -0.095   -0.107    0.9254   0.921
   .  *|   .    |     .  *|   .    |5   -0.102   -0.119    1.2290   0.942
   .  *|   .    |     .  *|   .    |6   -0.108   -0.142    1.5963   0.953
   .   |*  .    |     .   |*  .    |7    0.123    0.085    2.1057   0.954
   .   |***     |     .   |***     |8    0.405    0.361    8.1069   0.423
   .  *|   .    |     . **|   .    |9   -0.150   -0.211    9.0049   0.437
   .  *|   .    |     .   |   .    |10  -0.064    0.033    9.1841   0.515
   .  *|   .    |     .  *|   .    |11  -0.082   -0.080    9.5102   0.575
   .   |   .    |     .   |   .    |12  -0.033    0.026    9.5699   0.654
   .   |   .    |     .   |   .    |13  -0.013    0.034    9.5807   0.728
   .   |   .    |     .   |   .    |14  -0.020    0.004    9.6087   0.790
   .   |   .    |     .  *|   .    |15  -0.026   -0.188    9.6678   0.840
   .   |   .    |     . **|   .    |16  -0.032   -0.202    9.7825   0.878
   .   |   .    |     .   |*  .    |17  -0.039    0.109   10.002    0.904
   .   |   .    |     .  *|   .    |18  -0.045   -0.120   10.449    0.916
```

Previous-Cohort model of Cohort73 Class Baa

LS // Dependent Variable is BAAR73
Sample: 1 20
Included observations: 20
BAAR73=C(1)+C(2)*BAAR72

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.527610	0.200159	2.635959	0.0168
C(2)	0.252648	0.232262	1.087773	0.2910

R-squared	0.061681	Mean dependent var	0.696000
Adjusted R-squared	0.009553	S.D. dependent var	0.570174
S.E. of regression	0.567445	Akaike info criterion	-1.038585
Sum squared resid	5.795882	Schwarz criterion	-0.939011
Log likelihood	-15.99292	F-statistic	1.183249
Durbin-Watson stat	1.559230	Prob(F-statistic)	0.291047

```
      ACF                PACF                AC       PAC     Q-Stat   Prob

  .   |*  .    |     .   |*  .    |1    0.117    0.117   0.3154   0.574
  .   |   .    |     .   |   .    |2   -0.027   -0.041   0.3333   0.846
  .   |   .    |     .   |*  .    |3    0.063    0.072   0.4348   0.933
  .   |   .    |     .   |   .    |4    0.059    0.042   0.5310   0.970
  .   |**.     |     .   |**.     |5    0.226    0.224   2.0340   0.844
  .   |   .    |     .  *|   .    |6   -0.007   -0.063   2.0355   0.916
  .  *|   .    |     .  *|   .    |7   -0.092   -0.075   2.3227   0.940
  .   |   .    |     .   |   .    |8   -0.030   -0.050   2.3560   0.968
  .  *|   .    |     .  *|   .    |9   -0.159   -0.185   3.3672   0.948
  .**|    .    |     .**|    .    |10  -0.207   -0.235   5.2531   0.874
  .   |   .    |     .   |   .    |11  -0.052   -0.007   5.3849   0.911
  .**|    .    |     .**|    .    |12  -0.221   -0.207   8.0707   0.780
  .   |*  .    |     .   |*  .    |13   0.068    0.188   8.3606   0.819
  .  *|   .    |     .   |   .    |14  -0.081   -0.040   8.8454   0.841
  .  *|   .    |     .   |*  .    |15  -0.094    0.087   9.6186   0.843
  .  *|   .    |     .  *|   .    |16  -0.060   -0.099  10.020    0.866
  .  *|   .    |     .   |   .    |17  -0.092   -0.020  11.269    0.842
  .   |*  .    |     .   |   .    |18   0.075   -0.048  12.519    0.819
```

Previous-Cohort model of Cohort73 Class Ba

LS // Dependent Variable is BAR73
Sample: 1 20
Included observations: 20
BAR73=C(1)+C(2)*BAR72

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.238720	0.595044	2.081729	0.0519
C(2)	0.248840	0.261850	0.950312	0.3545

R-squared	0.047775	Mean dependent var	1.607500
Adjusted R-squared	-0.005127	S.D. dependent var	2.012197
S.E. of regression	2.017348	Akaike info criterion	1.498207
Sum squared resid	73.25447	Schwarz criterion	1.597780
Log likelihood	-41.36084	F-statistic	0.903093
Durbin-Watson stat	2.130093	Prob(F-statistic)	0.354540

```
      ACF                  PACF                AC        PAC      Q-Stat   Prob

 . *|   .     |      . *|   .     |1    -0.090    -0.090    0.1892   0.664
 . *|   .     |      . *|   .     |2    -0.166    -0.175    0.8608   0.650
 . *|   .     |      . *|   .     |3    -0.132    -0.172    1.3134   0.726
 .  |**.      |      .  |*  .     |4     0.198     0.141    2.3876   0.665
 .  |**.      |      .  |**.      |5     0.269     0.281    4.5150   0.478
 .  |   .     |      .  |*  .     |6    -0.040     0.077    4.5655   0.601
 . *|   .     |      .  |   .     |7    -0.169    -0.048    5.5306   0.595
 . *|   .     |      . *|   .     |8    -0.060    -0.072    5.6634   0.685
 .  |*  .     |      .  |   .     |9     0.186     0.046    7.0537   0.632
 .  |   .     |      . *|   .     |10   -0.036    -0.139    7.1096   0.715
 . *|   .     |      . *|   .     |11   0.090    -0.079     7.5065   0.757
 . *|   .     |      . *|   .     |12   -0.123    -0.073    8.3448   0.758
 .  |   .     |      . *|   .     |13   -0.019    -0.094    8.3673   0.819
 .  |   .     |      . *|   .     |14   -0.053    -0.177    8.5750   0.857
 .  |   .     |      . *|   .     |15   -0.054    -0.093    8.8334   0.886
 .  |   .     |      .  |   .     |16   -0.018     0.017    8.8709   0.919
 .  |   .     |      .  |   .     |17   -0.022     0.023    8.9409   0.942
 . *|   .     |      . *|   .     |18   -0.105    -0.094   11.366    0.878
```

Previous-Cohort model of Cohort73 Class B

LS // Dependent Variable is BR73
Sample: 1 20
Included observations: 20
BR73=C(1)+C(2)*BR72

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.672738	0.632244	1.064048	0.3014
C(2)	0.096462	0.214659	0.449371	0.6585

R-squared	0.011094	Mean dependent var	0.761000
Adjusted R-squared	-0.043845	S.D. dependent var	2.630529
S.E. of regression	2.687578	Akaike info criterion	2.071921
Sum squared resid	130.0154	Schwarz criterion	2.171494
Log likelihood	-47.09798	F-statistic	0.201934
Durbin-Watson stat	2.242819	Prob(F-statistic)	0.658528

ACF	PACF		AC	PAC	Q-Stat	Prob
. *| . |	. *| .	1	-0.148	-0.148	0.5060	0.477
. *| . |	. *| .	2	-0.060	-0.084	0.5940	0.743
. *| . |	. *| .	3	-0.063	-0.088	0.6983	0.874
. *| . |	. *| .	4	-0.067	-0.100	0.8215	0.936
. *| . |	. *| .	5	-0.070	-0.115	0.9671	0.965
. *| . |	. *| .	6	-0.074	-0.135	1.1388	0.980
. *| . |	. *| .	7	-0.077	-0.160	1.3416	0.987
. *| . |	.**| .	8	-0.081	-0.195	1.5815	0.991
. |* . |	. |* .	9	0.194	0.075	3.0902	0.961
. *| . |	. *| .	10	-0.061	-0.105	3.2523	0.975
. | . |	. *| .	11	0.015	-0.070	3.2630	0.987
. | . |	. *| .	12	0.011	-0.060	3.2699	0.993
. | . |	. | .	13	0.008	-0.054	3.2737	0.997
. | . |	. | .	14	0.004	-0.046	3.2751	0.998
. | . |	. | .	15	0.001	-0.035	3.2752	0.999
. | . |	. | .	16	-0.003	-0.021	3.2759	1.000
. | . |	. | .	17	-0.006	-0.009	3.2815	1.000
. | . |	. *| .	18	-0.010	-0.071	3.3018	1.000

Previous-Cohort model of Cohort72   Class Baa

LS // Dependent Variable is BAAR72
Sample: 1 20
Included observations: 20
BAAR72=C(1)+C(2)*BAAR71

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.354204	0.185027	1.914338	0.0716
C(2)	0.589794	0.274000	2.152532	0.0452

R-squared	0.204715	Mean dependent var	0.666500
Adjusted R-squared	0.160532	S.D. dependent var	0.560491
S.E. of regression	0.513536	Akaike info criterion	—1.238232
Sum squared resid	4.746941	Schwarz criterion	—1.138659
Log likelihood	-13.99645	F-statistic	4.633394
Durbin-Watson stat	1.486259	Prob(F-statistic)	0.045170

```
   ACF            PACF               AC       PAC     Q-Stat   Prob

 .  |* .   |   .  |* .   |1    0.090    0.090    0.1879   0.665
 .  |* .   |   .  |* .   |2    0.161    0.154    0.8197   0.664
 . *|  .   |   . *|  .   |3   -0.076   -0.106    0.9710   0.808
 .  |* .   |   .  |* .   |4    0.183    0.181    1.8912   0.756
 .  |* .   |   .  |* .   |5    0.189    0.197    2.9394   0.709
 .  |  .   |   . *|  .   |6   -0.032   -0.146    2.9717   0.812
 . *|  .   |   . *|  .   |7   -0.123   -0.148    3.4834   0.837
 . *|  .   |   . *|  .   |8   -0.138   -0.083    4.1848   0.840
 .  |* .   |   .  |* .   |9    0.135    0.135    4.9170   0.841
 . *|  .   |   .**|  .   |10  -0.166   -0.223    6.1249   0.805
 . *|  .   |   . *|  .   |11  -0.082   -0.061    6.4513   0.842
 .**|  .   |   .  |  .   |12  -0.210   -0.010    8.8858   0.713
 .  |  .   |   .  |  .   |13   0.015    0.011    8.9001   0.780
 . *|  .   |   . *|  .   |14  -0.065   -0.098    9.2109   0.817
 . *|  .   |   .  |  .   |15  -0.059   -0.002    9.5159   0.849
 . *|  .   |   . *|  .   |16  -0.181   -0.083   13.110    0.665
 . *|  .   |   .  |  .   |17  -0.076   -0.052   13.948    0.671
 .  |  .   |   .  |  .   |18   0.025   -0.008   14.085    0.724
```

Previous-Cohort model of Cohort72 Class Ba

LS // Dependent Variable is BAR72
Sample: 1 20
Included observations: 20
BAR72=C(1)+C(2)*BAR71

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	1.493704	0.541012	2.760943	0.0129
C(2)	-0.011287	0.344776	-0.032737	0.9742

R-squared	0.000060	Mean dependent var	1.482000
Adjusted R-squared	-0.055493	S.D. dependent var	1.767465
S.E. of regression	1.815844	Akaike info criterion	1.287740
Sum squared resid	59.35119	Schwarz criterion	1.387313
Log likelihood	-39.25617	F-statistic	0.001072
Durbin-Watson stat	1.140835	Prob(F-statistic)	0.974245

```
      ACF              PACF              AC       PAC     Q-Stat  Prob

  .  |* .    |     .  |* .    |1    0.165    0.165   0.6280  0.428
  . *|  .    |     . *|  .    |2   -0.154   -0.186   1.2041  0.548
  . *|  .    |     . *|  .    |3   -0.122   -0.065   1.5883  0.662
  .  |* .    |     .  |* .    |4    0.142    0.158   2.1407  0.710
  .  |**.    |     .  |**.    |5    0.289    0.222   4.5825  0.469
  .  |  .    |     .  |  .    |6    0.061    0.008   4.7002  0.583
  . *|  .    |     .  |  .    |7   -0.060    0.026   4.8206  0.682
  .  |  .    |     .  |  .    |8   -0.035    0.009   4.8657  0.772
  .  |* .    |     .  |  .    |9    0.078    0.019   5.1084  0.825
  . *|  .    |     . *|  .    |10  -0.058   -0.173   5.2576  0.873
  . *|  .    |     . *|  .    |11  -0.094   -0.074   5.6894  0.893
  . *|  .    |     . *|  .    |12  -0.156   -0.167   7.0213  0.856
  . *|  .    |     . *|  .    |13  -0.130   -0.163   8.0756  0.839
  . *|  .    |     .**|  .    |14  -0.145   -0.208   9.6171  0.790
  .  |  .    |     .  |  .    |15  -0.050   -0.013   9.8378  0.830
  .  |  .    |     .  |* .    |16   0.038    0.075   9.9959  0.867
  .  |  .    |     .  |  .    |17  -0.043    0.040  10.269   0.892
  . *|  .    |     .  |* .    |18  -0.091    0.071  12.077   0.843
```

Previous-Cohort model of Cohort72   Class B

LS // Dependent Variable is BR72
Sample: 1 20
Included observations: 20
BR72=C(1)+C(2)*BR71

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.845605	0.698763	1.210144	0.2419
C(2)	0.068471	0.231287	0.296044	0.7706

R-squared	0.004845	Mean dependent var	0.915000
Adjusted R-squared	-0.050441	S.D. dependent var	2.872334
S.E. of regression	2.943884	Akaike info criterion	2.254099
Sum squared resid	155.9962	Schwarz criterion	2.353672
Log likelihood	-48.91976	F-statistic	0.087642
Durbin-Watson stat	1.999262	Prob(F-statistic)	0.770585

```
    ACF            PACF                 AC        PAC      Q-Stat  Prob

 . *|  .    |       . *|   .   |1   -0.129    -0.129    0.3839  0.536
 . *|  .    |       . *|   .   |2   -0.064    -0.082    0.4829  0.785
 . *|  .    |       . *|   .   |3   -0.068    -0.090    0.6034  0.896
 . *|  .    |       . *|   .   |4   -0.073    -0.104    0.7492  0.945
 . *|  .    |       . *|   .   |5   -0.077    -0.121    0.9249  0.968
 . *|  .    |       . *|   .   |6   -0.082    -0.143    1.1361  0.980
 . *|  .    |       . *|   .   |7   -0.087    -0.172    1.3897  0.986
 . *|  .    |       .**|   .   |8   -0.091    -0.213    1.6944  0.989
 . *|  .    |       .**|   .   |9   -0.119    -0.303    2.2647  0.987
 .  |***    |       .  |** .   |10   0.454     0.300   11.326   0.333
 .  |  .    |       .  |   .   |11  -0.030    -0.021   11.369   0.413
 .  |  .    |       .  |   .   |12  -0.001    -0.035   11.369   0.498
 .  |  .    |       .  |   .   |13  -0.005    -0.029   11.371   0.580
 .  |  .    |       .  |   .   |14  -0.010    -0.018   11.378   0.656
 .  |  .    |       .  |   .   |15  -0.014    -0.003   11.396   0.724
 .  |  .    |       .  |   .   |16  -0.019     0.016   11.435   0.782
 .  |  .    |       .  |   .   |17  -0.023     0.040   11.516   0.829
 .  |  .    |       .  |*  .   |18  -0.028     0.071   11.689   0.863
```

Previous-Cohort model of Cohort71   Class Baa

LS // Dependent Variable is BAAR71
Sample: 1 20
Included observations: 20
BAAR71=C(1)+C(2)*BAAR70

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.382108	0.150015	2.547132	0.0202
C(2)	0.284540	0.224680	1.266421	0.2215

R-squared	0.081812	Mean dependent var	0.529500
Adjusted R-squared	0.030801	S.D. dependent var	0.429975
S.E. of regression	0.423302	Akaike info criterion	-1.624702
Sum squared resid	3.225315	Schwarz criterion	-1.525128
Log likelihood	-10.13175	F-statistic	1.603822
Durbin-Watson stat	1.986959	Prob(F-statistic)	0.221506

```
        ACF              PACF                AC        PAC     Q-Stat   Prob

   . *|  .    |     . *|  .     |1   -0.082    -0.082    0.1555   0.693
   .  |* .    |     .  |* .     |2    0.084     0.078    0.3297   0.848
   .  |  .    |     .  |  .     |3   -0.003     0.010    0.3299   0.954
   .  |***    |     .  |***     |4    0.416     0.415    5.0845   0.279
   .  |* .    |     .  |* .     |5    0.094     0.193    5.3424   0.376
   . *|  .    |     . *|  .     |6   -0.081    -0.130    5.5473   0.476
   .**|  .    |     ***|  .     |7   -0.256    -0.408    7.7725   0.353
   .  |* .    |     . *|  .     |8    0.144    -0.180    8.5307   0.383
   .  |* .    |     .  |* .     |9    0.081     0.067    8.7962   0.456
   . *|  .    |     .  |* .     |10  -0.086     0.145    9.1189   0.521
   .**|  .    |     .  |  .     |11  -0.309    -0.018   13.788    0.245
   .  |  .    |     . *|  .     |12  -0.008    -0.068   13.791    0.314
   .  |  .    |     . *|  .     |13   0.027    -0.158   13.839    0.385
   . *|  .    |     .**|  .     |14  -0.141    -0.317   15.305    0.358
   . *|  .    |     .  |  .     |15  -0.140     0.035   17.041    0.316
   . *|  .    |     .  |* .     |16  -0.152     0.133   19.571    0.240
   .  |  .    |     .  |* .     |17   0.046     0.175   19.880    0.280
   .  |  .    |     .  |  .     |18  -0.049    -0.003   20.417    0.310
```

Previous-Cohort model of Cohort71 Class B

LS // Dependent Variable is BAR71
Sample: 1 20
Included observations: 20
BAR71=C(1)+C(2)*BAR70

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.838219	0.381710	2.195955	0.0414
C(2)	0.167254	0.224140	0.746203	0.4652

R-squared	0.030006	Mean dependent var	1.037000
Adjusted R-squared	-0.023882	S.D. dependent var	1.208270
S.E. of regression	1.222613	Akaike info criterion	0.496621
Sum squared resid	26.90610	Schwarz criterion	0.596194
Log likelihood	-31.34498	F-statistic	0.556819
Durbin-Watson stat	1.580667	Prob(F-statistic)	0.465180

```
     ACF              PACF                AC       PAC     Q-Stat   Prob

 .   |*  .    |    .   |*  .    |1    0.166    0.166    0.6344   0.426
 ***|    .    |    ***|    .    |2   -0.405   -0.444    4.6389   0.098
 .   |   .    |    .   |**.     |3    0.043    0.274    4.6875   0.196
 .   |*  .    |    .  *|   .    |4    0.187   -0.111    5.6533   0.227
 .  *|   .    |    .   |   .    |5   -0.065    0.044    5.7783   0.328
 .**|    .    |    .**|    .    |6   -0.199   -0.194    7.0187   0.319
 .   |   .    |    .   |   .    |7   -0.027    0.042    7.0439   0.424
 .   |*  .    |    .  *|   .    |8    0.072   -0.090    7.2328   0.512
 .  *|   .    |    .  *|   .    |9   -0.106   -0.092    7.6831   0.566
 .  *|   .    |    .   |   .    |10  -0.063    0.044    7.8570   0.643
 .   |*  .    |    .   |   .    |11   0.106   -0.008    8.4050   0.677
 .   |   .    |    .   |   .    |12  -0.002   -0.048    8.4052   0.753
 .  *|   .    |    .   |   .    |13  -0.069    0.011    8.7069   0.795
 .  *|   .    |    .**|    .    |14  -0.109   -0.219    9.5832   0.792
 .  *|   .    |    .  *|   .    |15  -0.144   -0.132   11.420    0.722
 .   |   .    |    .   |   .    |16   0.024   -0.011   11.483    0.779
 .   |   .    |    .  *|   .    |17   0.051   -0.065   11.869    0.808
 .   |   .    |    .   |   .    |18  -0.004    0.064   11.872    0.854
```

Previous-Cohort model of Cohort71 Class B

LS // Dependent Variable is BR71
Sample: 1 20
Included observations: 20
BR71=C(1)+C(2)*BR70

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.792106	0.690802	1.146647	0.2665
C(2)	0.123065	0.124662	0.987195	0.3366

R-squared	0.051361	Mean dependent var	1.013500
Adjusted R-squared	-0.001341	S.D. dependent var	2.920069
S.E. of regression	2.922026	Akaike info criterion	2.239194
Sum squared resid	153.6883	Schwarz criterion	2.338767
Log likelihood	-48.77071	F-statistic	0.974553
Durbin-Watson stat	2.260781	Prob(F-statistic)	0.336632

```
      ACF               PACF                AC       PAC     Q-Stat   Prob

   . *|  .     |     . *|  .     |1   -0.133   -0.133   0.4083   0.523
   . *|  .     |     . *|  .     |2   -0.067   -0.087   0.5196   0.771
   . *|  .     |     . *|  .     |3   -0.071   -0.094   0.6495   0.885
   . *|  .     |     . *|  .     |4   -0.075   -0.109   0.8039   0.938
   . *|  .     |     . *|  .     |5   -0.079   -0.127   0.9870   0.964
   . *|  .     |     . *|  .     |6   -0.083   -0.150   1.2040   0.977
   . *|  .     |     . *|  .     |7   -0.087   -0.181   1.4612   0.984
   . *|  .     |     .**|  .     |8   -0.096   -0.231   1.7984   0.987
   . *|  .     |     .**|  .     |9   -0.063   -0.262   1.9555   0.992
   .  |**.     |     .  |  .     |10   0.254    0.055   4.8052   0.904
   .  |  .     |     . *|  .     |11   0.023   -0.068   4.8315   0.939
   .  |  .     |     . *|  .     |12   0.008   -0.099   4.8347   0.963
   .  |  .     |     . *|  .     |13   0.006   -0.102   4.8366   0.979
   .  |  .     |     . *|  .     |14   0.001   -0.100   4.8368   0.988
   .  |  .     |     . *|  .     |15  -0.003   -0.093   4.8374   0.993
   .  |  .     |     . *|  .     |16  -0.007   -0.080   4.8423   0.996
   .  |  .     |     . *|  .     |17  -0.011   -0.058   4.8595   0.998
   .  |  .     |     .  |  .     |18  -0.017   -0.027   4.9198   0.999
```

BIBLIOGRAPHY

Anderson, N., F. Breedon, A. M. Derry, and G. Murphy., 1996. *Estimating and Interpreting the Yield Curve*. John Wiley and Sons: Chichester, U.K.

Artzner, P., F. Delbaen, J. M. Eber, and D. Heath, 1997. "Thinking Coherently", *RISK* vol. 10, 68-71.

Bjork, T., 1998. *Arbitrage Theory in Continuous Time*. Oxford University Press Inc.: New York.

Black, F. and J. Cox, 1976. "Valuing corporate Securities: Some Effects of Bond Indenture Provisions," *Journal of Finance*, 31(2), 361-367.

Bodie, Z., and R. C. Merton, 1998. *Finance*. Prentice Hall, Inc.: New Jersey.

Brealey, R. A. and S. C. Myers, 1996, *Principles of Corporate Finance*. McGraw-Hill: New York.

Clewlow, L. and C. Strickland, 1998. *Implementing Derivative Models*. John Wiley and Sons: Chichester, U.K.

Cooper, I. and A. Mello., 1991. "The Default Risk of Swaps." *Journal of Finance* 46(2), 597-620.

Das, S. R. and R. K. Sundaram., 1999. "A Direct Approach to Arbitrage-Free Pricing of Credit Derivatives." **Working Paper**. Harvard Business School.

Das, S. R. and P. Turfano, 1995. "Pricing Credit-Sensitive Debt when Interest Rates, Credit Ratings and Credit Spreads are Stochastic." **Working paper**. Harvard Business School.

Das, S., 1997. "Credit Derivatives: Applications", in *Credit Derivatives*. (Ed) S. Das. John Wiley and Sons (Asia) Pte Ltd.

Duffee, G., 1996. "On Measuring Credit Risks of Derivative Instruments." *Journal of Banking Finance* 20, 805-833.

Duffee, G., 1998. " The Relation between Treasury Yields and Corporate Bond Yield Spreads." *Journal of Finance* 53(6), 2225-2241.

Duffie, D., and K. Singleton, 1997. "An Econometric Model of the Term Structure of Interest-Rate Swap Yields." *Journal of Finance*. 52 (4) 1287-1321.

Duffie, D., 1996b. "Special Repo Rates." *Journal of Finance* 51(2) 493-526.

Duffie, D. and M. Huang, 1996. "Swap Rates and Credit Quality." *Journal of Finance* 51 (3), 921-949.

Financial Engineering Ltd., 1999. *The J. P. Morgan Guide to Credit Derivatives. RISK* Publications, 1999, Grange Press, Southwick, UK.

Garbade, K. D., 1988. "Approximating the Yield on a Portfolio of Bonds: Value-Weighted Averages vs. Value of a Basis point-Weighted Averages", in K. D. Garbade, ed., 1996: *Fixed Income Analytics*, The MIT Press, Cambridge, Massachusetts.

Glasserman, P. and X. Koa, 2000. "Arbitrage-free discretization of lognormal forward Libor and swap rate models." *Finance and Stochastics* 4(1) 35-68.

Heath, D., R. A. Jarrow, and A. Morton, 1992. "Bond Pricing and the Term Structure of Interest Rates: A new methodology for contingent claim valuation." *Econometrica* 60(4) 77-106.

Heath, D., R. A. Jarrow, and A. Morton, 1990. "Bond Pricing and the Term Structure of Interest Rates: A Discrete Time Approximation." *Journal of Financial and Quantitative Analysis* 25(4) 419-440.

Hull, J. and A. White, 1995. "The Impact of Default Risk on the Prices of Options and other Derivatives." *Journal of Banking Finance* 19, 299-322.

Jarrow, R. A., 1997. *Modeling Fixed Income Securities and their Derivatives*. MacGraw-Hill: New York.

Jarrow, R. A., 1995. "Pricing Interest Rate Options", in: R. Jarrow, V. Maksimovic and W.T. Ziemba ,eds.: *Finance*, Handbooks in Operation Research and Management Science, vol. 9, Elsivier, Amsterdam.

Jarrow, R. A., and S. M. Turnbull., 1995. "Pricing Options on Derivative Securities Subject to Credit Risk." *Journal of Finance* 50(1), 53-85.

Jarrow, R.A., D. Lando, and S.M. Turnbull, 1997. "A Markov Model for the Term Structure of Credit Risk Spreads." *Review of Financial Studies* 10(1), 481-523.

Jones, E., S. Mason and E. Rosenfeld, 1984. "Contingent claims analysis of corporate capital structures: An empirical investigation." *Journal of Finance* 39(), 611-625.

Kwok, Y. K., 1998. *Mathematical Models of Financial Derivatives*. Springer-Verlag: Singapore.

Lando, D., 1997. Modeling Bonds and Derivatives with Default Risk, in: M. A. Dempster and S.R. Pliska, eds.: *Mathematics of Derivative Securities* , Publications of the Newton Institute: Cambridge University Press, Cambridge, U.K.

Li, D. X. (2000). "On Default Correlation: A Copula Function Approach". **Working Paper** 99-07, RiskMetrics Group.

Longstaff, F. and E. Schwartz, 1995. "A Simple Approach to Valuing Risky Fixed and Floating Rate Debt. *Journal of Finance* 50(3) 789-819.

Luenberger, D., 1998. *Investment Science*. Oxford University Press: New York.

Merton, R. C, 1974. "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates." *Journal of Finance* 2(2), 449-470 and in R.C. Merton, ed. 1991:*Continuous Time Finance*.

Musiela, M. and M. Ruthwoski, 1997. *Martingale Methods in Finance*. Springer-Verlag: Berlin.

Neal, R. S., 1996. "Credit Derivatives: New Financial Instruments for Controlling Credit Risk." *Economic Review*. No.2, 1996 Federal Reserve Bank of Kansas.

Neftci, S., 2000. *An Introduction to the Mathematics of Financial Derivatives, 2nd Edition*. Academic Press: New York.

Pliska, S. R., 1997. *An Introduction to Mathematical Finance: Discrete Time Models*: Blackwell, Oxford, UK.

Rebonato, R., 1998. *Interest-Rate Option Models, 2nd Edition*. John Wiley and Sons: Chichester, U.K.

Reoch, R., 1997. "The Market for Credit Derivatives," in S. Das, ed. 1998: *Credit Derivatives*. John Wiley and Sons (Asia) Pte Ltd.

Tavakoli, J., 1998. *Credit Derivatives*. John Wiley and Sons: New York.