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**Multistage capacitated lot-sizing for assembly structure
manufacturing systems**

Eftekharzadeh, Seyyed Mohammad Reza, Ph.D.

City University of New York, 1990

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A

**MULTI-STAGE CAPACITATED LOT-SIZING
FOR
ASSEMBLY STRUCTURE MANUFACTURING SYSTEMS**

BY

SEYYED M. REZA EFTEKHARZADEH

A dissertation submitted to the Graduate Faculty in
Business in partial fulfillment of the requirements
for the degree of Doctor of Philosophy, The City
University of New York.

1990

1990

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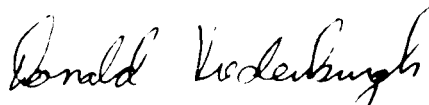
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ABSTRACT

MULTI-STAGE CAPACITATED LOT-SIZING FOR ASSEMBLY STRUCTURE MANUFACTURING SYSTEMS

by

Seyyed M. Reza Eftekharzadeh

Advisor: Professor David G. Dannenbring

The capacitated lot-sizing problem is concerned with determining the timing and size of production lots for all items over a finite planning horizon on all levels (stages) of manufacturing process. An assembly structure is one in which parts, subassemblies, and raw materials (items) combine with other parts, subassemblies, and raw materials to eventually comprise a finished product (end item). These structures can have many predecessors but at most one successor for each item.

The capacitated multi-stage lot-sizing problem can be formulated as a mixed-integer, binary linear program as per period and per stage for an assembly structure. Demand is assumed to be deterministic over a finite horizon and costs are limited to inventory holding cost and setup costs. This formulation is known to be NP-hard which is unlikely to be solvable in polynomial time with respect to the dimensionality of the problem. The computational

complexity of these problems has stimulated major efforts toward developing heuristic solution procedures.

Lot-sizing with infinite capacity has been studied extensively, with the result that many optimal and efficient heuristic procedures are available for that less challenging situation. The consideration of capacity for a multi-stage problem adds a new dimension of complexity that has received little research attention.

The multi-stage capacitated problem in an assembly structure is important for many manufacturing operations. This Study is a step in analyzing the complex, capacity-constrained lot-sizing problem.

Three heuristic solution procedures are developed to find a feasible solution (acceptable production schedules) while attempting to minimize the total cost.

This study evaluates the impact of the product structure (depth and size), variability of demand (represented by coefficient of variation), number of periods, and capacity loading (represented by a ratio of total demand/total capacity) on the performance of these heuristics.

Computational results for a variety of small and large problems are presented. Conclusions and directions for future work are also reported.

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vii

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TABLE OF CONTENTS

ABSTRACT	iv
ACKNOWLEDGEMENTS	vi
TABLE OF CONTENTS	vii
LIST OF TABLES	x
LIST OF FIGURES	xiv
CHAPTER	Page
I. INTRODUCTION	1
1.1 Introduction	1
1.2 Nature of the System	1
1.3 Scope of the Research	1
1.4 Multi-Stage Structures	3
1.5 Importance of the Topic	8
1.6 Objectives of the Research	10
1.7 Contents of the Study	11
II. LITERATURE REVIEW	13
2.1 Single-Stage Lot-Sizing	19
2.1.1 Single-Item, Uncapacitated Lot-Sizing	19
2.1.2 Single-Item, Capacitated Lot-Sizing	20
2.1.3 Multiple Items, Uncapacitated Lot-Sizing	21
2.1.4 Multiple Items, Capacitated Lot-Sizing	21
2.2 Multi-Stage Lot-Sizing	28
2.2.1 Multi-Stage With No Capacity Constraints	31
2.2.1.1 Uncapacitated Multi-Stage, Serial	31
2.2.1.2 Uncapacitated Multi-Stage, Assembly	32
2.2.2 Multi-Stage With Capacity Constraints	38
2.2.2.1 Capacitated Multi-Stage Serial Structure	39

2.2.2.2	Capacitated Multi-Stage Assembly Structure	42 45
2.3	Summary	45
III.	PROBLEM DESCRIPTION AND FORMULATION	47
3.1	Problem Description	47
3.2	Model Formulation	48
3.2.1	Assumptions	48
3.2.2	Notation	49
3.2.3	Development of the Formulation	51
3.2.3.1	Formulation	53
3.2.3.2	Modified Formulation (P-E)	57
3.2.3.3	Problems (P-I) and (P-E) are Equivalent .	58
3.2.3.4	Size of the Problem ..	61
3.3	Solution Techniques	61
3.4	Difficulty of the Problem	65
IV.	Heuristic Procedures	67
4.1	Classification of the Heuristics	68
4.2	Selective Enumeration Procedure (SE) .	68
4.2.1	Summary of the SE Algorithm	73
4.3	Modified Per Stage Procedure (MS)	85
4.3.1	Summary of the MS Algorithm	87
4.4	Modified Per Period Procedure (MP) ...	95
4.4.1	Method 1	95
4.4.2	Method 2	102
4.5	The Blackburn and Millen Procedure ...	107
4.6	Summary	111
V.	COMPUTATIONAL TESTING	112
5.1	Statistical Testing	112
5.1.1	Experimental Conditions	112
5.1.2	Solution Criteria	113
5.2	ANOVA Statistical Tests	114
5.3	Hypotheses of the Research	115
5.4	Research Questions	115
5.5	Product Structures	116
5.6	Computation of the Lower Bound	120
5.7	Computational Testing	121
5.7.1	Mean Costs of Structures	122
5.7.2	Computational CPU Time	123
5.7.3	Computational Difficulty	128
5.7.4	Lower Bound Results	128

5.7.5	Comparison of the Heuristics for Factor CU	132
5.7.6	Comparison of the Heuristics for Factor CV	132
5.7.7	Comparison of the Heuristics for Factor T	133
5.7.8	Comparison of the Heuristics for Factor H	133
5.7.9	Comparison of the Heuristics for Interaction CV * CU	133
5.7.10	Comparison of the Heuristics for Interaction D * T	140
5.7.11	Comparison of the Heuristics for Interaction D * S	140
5.7.12	Comparison of the Heuristics for Interaction D * CAP	140
5.7.13	Comparison of the Heuristics for Interaction S * H	141
5.7.14	Comparison of the Heuristics for Interaction N * T	141
5.7.15	Comparison of the Heuristics for Best Ratios	141
5.8	Summary	142
VI.	CONCLUSION AND FUTURE RESEARCH DIRECTION ...	149
6.1	Summary	149
6.1.1	Summary of the Results	151
6.2	Contribution of the Study	153
6.3	Future Research Direction	154
APPENDIX		
A-1.	Expansion of the Formulation	157
A-2.	Schedule Selection Procedure	159
B.	Test Problems	163
C.	Tables of the Statistical Results	168
D.	Key Terms and Abbreviations	194
BIBLIOGRAPHY		196

LIST OF TABLES

Table	Page
1.1 Literature Review	18
4.1 Data Set for a 5-Period Problem	69
4.2 Demand and Capacity for a 5-Period Problem .	69
4.3 Key Components of Each Heuristic	70
4.4 Schedules and Cost for the 1, 2, 3 and 4-Period Problem	82
4.5 Complete Set of schedules for a 3-Period Problem	82
4.6 12 Possible Combination for a 3-stage Problem	83
4.7 Modified Schedules and Cost for the 1, 2, 3, and 4-Period Problem	83
4.8 Schedules and Cost for the 5-Period Problem	84
4.9 Feasible Solution for the 5-Period Problem	84
4.10 Schedules and Cost for the 1, 2, 3, and 4-Period Problem	93
4.11 Production Schedules and Cost for the 5-Period Problem	93
4.12 Final Solution for the 5-Period Problem ...	93
4.13 Alternative Solution for the 5-Period Problem	94
4.14 The 5-Period Infeasible Problem	99
4.15 Modified Production Schedules for the 5-Period Problem	99
4.16 Cost Coefficients for 3 stage Problem	99

4.17	The 5-Period Modified Solution	100
4.18	The Updated Cost Coefficients	100
4.19	The 5-Period Modified Feasible Solution ...	101
4.20	The Final Updated Cost Coefficients	101
4.21	The 5-Period Feasible Problem	109
4.22	The 5-Period Shifting Production Problem ..	109
4.23	The 5-Period Final Feasible Problem	109
5.1	List of Factors to be Tested	117
5.2	Mean Costs and Ratios for All Heuristics ..	124
5.3	Comparison of CPU time	125
5.4	CPU Time for All Heuristic	126
5.5	CPU Time for Each Heuristic	126
5.6	Statistical Results for Each Heuristic	127
5.7	Statistical Results for All Heuristics	127
5.8	Average Cost (Lower Bound)	130
5.9	The Ratio of (Heuristic / LB)	131
5.10	The Ratio of (Heuristic / LB) for Different Values of CV	131
5.11	The Cost Ratio (MP/SE) for CU Factor	135
5.12	The Cost Ratio (MS/BM) for CU Factor	135
5.13	The Cost Ratio (MP/SE) for CU Factor for Structures 1-6	136
5.14	The Cost Ratio (MP/SE) for CV Factor for Structures 1-6	136
5.15	Mean Costs and Ratios for T Factor	137
5.16	The Cost Ratio (MP/SE) for CU * CV Interaction for Structures 1-6	138

5.17	The Cost Ratio (MS/BM) for CU * CV Interaction for Structures 1-6	139
5.18	The Cost Ratio (MP/SE, MS/BM) for D * T Interaction for Structures 1-6	143
5.19	The Cost Ratio (MP/SE, MS/BM) for D * S Interaction for Structures 1-6	143
5.20	The Cost Ratio (MP/SE, MS/BM) for CU * CV Interaction for Structures 1-6	144
5.21	The Cost Ratio (MP/SE, MS/BM) for S * H Interaction for Structures 1-6	144
5.22	The Cost Ratio (MP/SE, MS/BM) for T * N Interaction for Structures 1-6	145
5.23	Statistical Results for All Effects	146
5.24	% of Trials That MP Performs Best	147
5.25	% of Trials That MS Performs Best	148

LIST OF TABLES IN THE APPENDIX C

A.1	Average Cost of Each Heuristic for CV Effect	169
A.2	CU * CV Effect for Structures 1-3	170
A.3	CU * CV Effect for Structures 4-6	171
A.4	Comparison of All Heuristics for CU Effect .	172
A.5	Comparison of All Heuristics for CV Effect .	173
A.6	Comparison of the Heuristics for Interaction (S/H) * CV for Structure 1-3 ..	174
A.7	Comparison of the Heuristics for Interaction (S/H) * CV for Structure 4-6 ..	175
A.8	Statistical Results for the Structure 1 ...	176
A.9	Statistical Results for the Structure 2 ...	176
A.10	Statistical Results for the Structure 3 ...	176
A.11	Statistical Results for the Structure 4 ...	177

A.12	Statistical Results for the Structure 5 ...	177
A.13	Statistical Results for the Structure 6 ...	177
A.14	Statistical Results for the Structure 1 ...	178
A.15	Statistical Results for the Structure 2 ...	178
A.16	Statistical Results for the Structure 3 ...	178
A.17	Statistical Results for the Structure 4 ...	179
A.18	Statistical Results for the Structure 5 ...	179
A.19	Statistical Results for the Structure 6 ...	179
A.20	Statistical Results for the Structure 1 ...	180
A.21	Statistical Results for the Structure 2 ...	180
A.22	Statistical Results for the Structure 3 ...	180
A.23	Statistical Results for the Structure 4 ...	181
A.24	Statistical Results for the Structure 5 ...	181
A.25	Statistical Results for the Structure 6 ...	181
A.26	(S / H) * CU Effect for Structures 1-6	182
A.27	(S / H) * CU Effect for Structures 1-6	183
A.28	Cost Ratios for S/H Effect	184
A.29	Cost Ratios for CU * D * S Interaction	185
A.30	Cost Ratios for CU * H Interaction for Structure 1-3	186
A.31	Cost Ratios for CU * H Interaction for Structure 4-6	187
A.32	Cost Ratios for H Effect	188
A.33	Cost % for H Effect	189
A.34	CV * H Interaction for Structures 1-3	190
A.35	CV * H Interaction for Structures 4-6	191
A.36	CU * H Interaction for Structures 1-3	192

A.37	CU * H Interaction for Structures 4-6		193
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LIST OF FIGURES

FIGUURE		Page
1.1	Assembly Structure	2
1.2	Types of Product Structures	17
1.3	Types of Manufacturing Structures Serial, Parallel	29
1.4	Types of Manufacturing Structures Assembly, General	30
4.1	The Three Stage Assembly	69
4.2	Shortest Path Problem for the (1, 2, 3, 4, and 5)-Period Problem	71
4.3	Network for a 2-Period Problem	79
4.4	Network for a 3-Period Problem	80
4.5	Network for a 4-Period Problem	81
4.6	The 2-Period Path Problem	92
4.7	The 4-Period Shortest Path Problem	92
4.8	The 5-Period Path Problem	94
4.9	The 2-Period Path Problem	108
4.10	The 5-Period Shortest Path Problem	108
5.1a	Structure No. 1	118
5.1b	Structure No. 2	118
5.1c	Structure No. 3	118
5.1d	Structure No. 4	118
5.1e	Structure No. 5	119
5.1f	Structure No. 6	119

CHAPTER I

INTRODUCTION

This thesis presents and tests three heuristic procedures for the capacitated, multi-stage lot-sizing problem.

1.1 Introduction

The capacitated lot-sizing problem is concerned with determining the timing and size of production lots for all items over a finite planning horizon on all levels (stages) of manufacturing process.

1.2 Nature of the System

A multi-stage production structure consists of a series of manufacturing stages (machining, assembly, etc.). The partial product that emerges at the completion of a manufacturing stage is called an item. Items must be combined in a specified order until completion.

Figure 1.1 represents an assembly structure where item 1 is the end item. Items 2-6 represent intermediate parts or components.

1.3 Scope of the Research

In this study, the capacitated multi-stage lot-sizing

problem is formulated as a mixed-integer, binary linear program. Demand is assumed to be deterministic over a finite

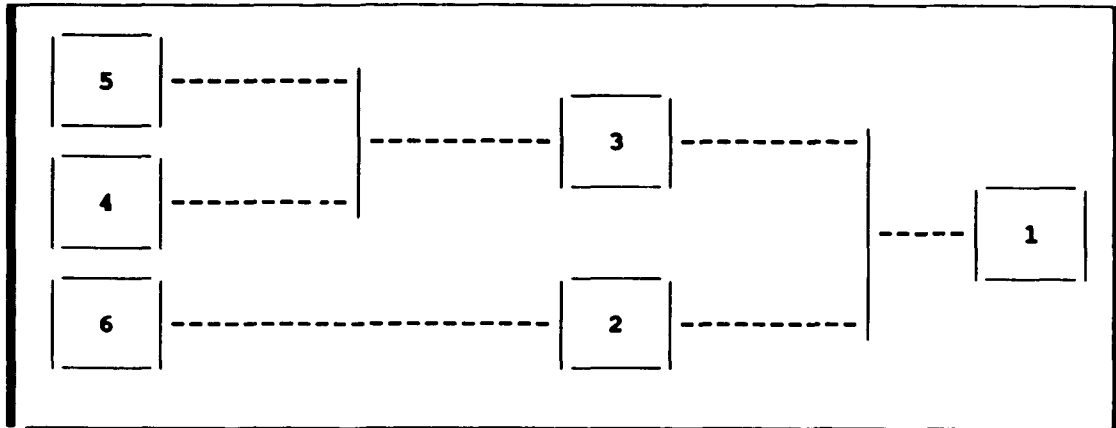


FIGURE 1.1: ASSEMBLY STRUCTURE

horizon and costs are limited to inventory holding cost and setup costs.

The formulation of the capacitated multi-stage lot-sizing problem is known to be NP-hard. The term NP-hard denotes that the problem is unlikely to be solvable in polynomial time with respect to the dimensionality of the problem. The computational complexity of such problems has stimulated major efforts by researchers toward developing heuristic solution procedures.

The following three new heuristic procedures are developed in this study:

- 1 - Selective Enumeration heuristic is a conversion of the multi-stage uncapacitated lot-sizing procedure of Afentakis [6] to multi-stage

capacitated lot-sizing problem.

2 - Modified Per Stage heuristic is an adoption of the single-stage capacitated lot-sizing of Florian and Klien [75] to multi-stage capacitated lot-sizing problem.

3 - Modified Per Period heuristic is a modified LFL version of a multi-stage uncapacitated problem.

This study evaluates the impact of the product structure (depth and width), variability of demand (represented by coefficient of variation), length of the planning horizon, capacity loading (represented by a ratio of total demand/total capacity), and the ratio of setup cost/holding cost on the performance of these heuristics.

The heuristics are tested over a large number of problems of different sizes, their performances are compared with the Blackburn and Millen procedure [43] and the mixed-integer programs solved using LINDO [157]. The results are analyzed using different statistical tests.

1.4 Multi-Stage Structures

Multi-stage structures can be organized in several ways. An assembly structure is one in which parts, subassemblies, and raw material (items) combine with other parts, subassemblies, and raw material to eventually comprise a finished product (end item). These structures can have many predecessors but at most one successor for each item. A

serial structure is one where each item has at most one successor and one predecessor. A serial structure is, thus, a special case of an assembly structure.

MRP Techniques and their Limitations

Material Requirements Planning (MRP) systems process information sequentially across stages, starting at the final stage and working down through the product structure from the end item to the raw material stage.

Information on the cost and capacity related consequences of decisions at higher stages on lower stages is needed. Otherwise, infeasibility may occur at the lower stages and the result may be severe diseconomies. Infeasibility can be avoided by making anticipatory adjustments to the master shedule, or by shifting batches from one period to another. All these alternatives can be used to generate a feasible solution.

In its basic form, an MRP system, schedules production for each work center as if there were infinite capacity; thus, the term "infinite loading" is used to describe this process. The capacity constraints are not explicitly considered in the planned lead times nor in the development of the master schedule. The result is infeasible schedules for work centers with capacity less than the actual loads determined by MRP.

Within MRP, Capacity Requirements Planning (CRP) is

used to provide load summaries for each work center, and to show what capacity will be required in each period. When the capacity available is insufficient, frustration arises because the production planner has only a limited number of alternatives available, namely job rescheduling, overtime, and additional shifts. Rescheduling one job in a complex assembly process often merely shifts the capacity problem to another work center, and the decision process provides no guidelines as to how a feasible schedule can be obtained. Overtime and extra shifts, when feasible, may not be cost effective.

Finite loading procedures have attracted attention as a way of resolving this problem. If capacity constraints are violated, the schedule must then be rearranged in the search for a feasible plan. Once the problem at one stage is resolved, other stages must be examined. If insufficient capacity exists at any stage, a rescheduling is necessary.

Multi-stage, multi-item production scheduling problems are common both in practice and in management science literature [88, 42, 34]. As they progress from raw material to end use, most products must pass through several levels, or stages of production and shipment. At each level, and in every period, there are interrelated decisions to be made about lot-sizing, timing, and stockkeeping. A product's structure need not be very

complex before the potential number of scheduling alternatives is large, and the problem of finding the lowest cost plan appears a formidable, yet very important, task.

The difficulty of solving the general case suggests that there is a need for a catalog of specific algorithms designed to solve specific situations. The solutions to such cases could be useful for two reasons. First, because the algorithms are often designed for specific problem assumptions. Second because an efficient solution technique may be used as a subproblem in solving a more general problem.

Actual applications may involve enough additional deep structure that the optimization algorithms are needlessly costly. Moreover, the immense size of these real life applications and the impressive nature of many of their costs and demands further suggests that good heuristic procedures may be as desirable as (presumably) more costly optimization algorithms. The need for sensitivity analysis may make the ability to quickly get good approximate solutions to such problems more valuable than an ability to eventually find a single optimal solution.

The lack of optimal solution procedures has limited the ability of researchers to evaluate the effectiveness of these heuristics [131, 132, 171]. In fact, in the uncapacitated case, which is relatively easier to solve compared to the capacitated problem, a study by Blackburn

and Millen [42] on a set of test problems revealed that one of the popular heuristics that has been introduced in MRP systems (Part-Period Balancing), produced solutions that deviated from optimality by an average of 16%.

The comparisons used relatively small and simple product structures. Blackburn and Millen [44] found that, since the performance of P-PB depends heavily on the number of levels in the product structure, an inferior performance can be expected on systems with many items, levels, and stages in the production plan.

Forecasting difficulties caused by fluctuating demand, shifts in the product mix, changes in cycle, and seasonality make it very difficult to perform detailed capacity planning. Certain machine centers become overloaded, and are unable to meet the requirements generated by the master production schedule. As a result, some of the scheduled parts cannot be manufactured at the designated capacity constrained machine centers.

One option to solve the problem is to purchase additional machines. However, because it is difficult to predict the longevity of the change in the machine load, it may not be appropriate to purchase additional machinery. Even if these heavy loads appear to be a long-term change, it may be difficult to convince managers to make a large capital investment or it may be financially infeasible to do so. Another option is subcontracting which may increase

lead times and become very costly for the firm. A third option is to revise the existing master production schedule so that the new MPS can be executed using existing capacity.

1.5 Importance of the Topic

This dissertation is motivated by the limited amount of research that has been conducted on this problem, and by the emergence of capacity-constrained lot sizing systems as an important management problem [88]. Lot-sizing with infinite capacity has been studied extensively, with the result that many optimal and efficient heuristics procedures are available for that less challenging situation [3, 4, 5, 15, 42].

The consideration of capacity for a multi-stage problem adds a new dimension of complexity that has received little research attention.

MRP systems are used to schedule a large number of different products made in a general structure; however, the issue of capacity has not been adequately researched [31, 143]. MRP's Capacity Requirement Planning is a tool which assumes infinite capacity. Capacity requirements can be anticipated, adjustments made to the master schedule, or a simple batch-shift heuristic can be used to remove the loads that exceed the available capacity. None of these methods are as useful as one that could finite-load all production planning on a feasible solution with minimum cost.

The lack of an adequate capacity-constrained lot-sizing procedure is a major weakness in MRP systems. In their study of the disaggregation problem, Krajewski and Ritzman [116] conclude that in regard to multi-stage, capacitated lot-sizing for assembly structures, consideration for capacity is absent. Goldratt [88] provides a brief description of a proprietary system for finite scheduling and claims that significant cost benefits are accrued to users. Little information is available concerning the algorithmic design. Steinberg and Napier [161] place a capacity constraint on production of every item, but capacity for every period is assumed unlimited. Blackburn and Millen [43] define the problem in the same manner, but their procedure is limited to situations where capacity utilization is less than 50%.

Billington et al. [31] show how a product structure can be compressed to reduce the size of the problem, but no heuristic solution procedures are discussed in detail. Goldratt [88] claims that capacity constrained machine scheduling is a relatively common problem in practice. When capacity is limited at each stage of production or at each period of the planning horizon, this situation can lead to low throughput and low customer service.

Thus, research on the capacitated multi-stage assembly problem is limited. When this problem is examined, restrictions are usually placed on some aspects of the

problem, so that solution procedures can be developed.

The multi-stage capacitated problem is important for many manufacturing operations. This dissertation is a step in analyzing the complex, capacity-constrained lot-sizing problem. It formulates the multi-stage capacitated lot sizing problem using an assembly structure. Three heuristic solution procedures will be introduced and tested.

1.6 Objectives Of The Study

This study examines the capacitated, multi-stage lot-sizing problem and addresses the following objectives:

- 1) develop three heuristic procedures to find a feasible solution (acceptable production schedules) while attempting to minimize the total cost,
- 2) test the solution procedures on a variety of test problems,
- 3) compare these heuristics to other available procedures, and
- 4) extend the result to different product structures.

Each of these four objectives will be discussed in detail in the following chapters.

The problem to be studied is a multi-stage assembly structure with one end item which can easily be extended to the multi-item case. There are costs of carrying inventory over to the next period and of setup in each period in which there is production of an item. There

are no restrictions on the form of the costs (other than echelon costs must be non-negative) and on consumption of capacity in every period or stage of production.

1.7 Contents of This Research

The dissertation includes six chapters that cover the main concept of the problem, other related works in this area, the formulation, the evaluation procedures, the computational results, and the conclusion and future research directions.

Chapter II includes a literature review for lot-sizing procedures. The lot-sizing procedures are grouped into four categories: one product, multiple products, serial, and assembly, where each category consists of two subgroups: uncapacitated and capacitated approaches. The heuristic solution procedures in each subgroup are discussed in detail.

Assumptions, notation, and mathematical formulation of the model as a mixed-integer linear program are presented in chapter III. Chapter III also discusses existing solution procedures for solving any mixed-integer program (MIP) and their shortcomings.

In chapter IV, the three new heuristics and their formulations are discussed and explained in detail.

Chapter V presents the experimental design, the performance measures, the hypotheses for testing, and finally, the results of this research.

In chapter VI, the summary, the conclusions, and directions for future research are proposed.

The most important finding of this disseratation is that the modified per period and the modified per stage heuristics perform better than the selective enumeration and the Blackburn and Millen approaches. The computation time for the selective enumeration approach was much higher than other heuristics.

CHAPTER II

LITRATURE REVIEW

In this chapter, we examine and summarize the main body of research in the area of discrete lot sizing, emphasizing those aspects that are relevant to the present study.

Introduction

From the earliest work in basic inventory problem, the research has expanded in eight directions. Each of the first four categories are known as single-stage production problems, where these items have no predecessors or successors. Figure 1.2 illustrates product structures for the following eight cases.

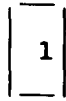
- 1) Single Item, Uncapacitated Lot-Sizing; where a product is manufactured at single stage (facility), with unspecified or unlimited capacity.
- 2) Single Item, Capacitated Lot-Sizing; where a product is produced by a single production facility with a pre-specified capacity.
- 3) Multiple Items, Uncapacitated Lot-Sizing; multiple products are all manufactured by a single facility with unlimited capacity. When setup time and cost is independent of processing order, the problem easily decomposes into a set of independent single-item problems.

- 4) Multiple Items, Capacitated Lot-Sizing; all items are produced by a single constrained stage.

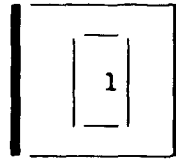
The next four categories involve more complex manufacturing and are called multi-stage problems. A distinction is made between serial and assembly structures.

- 5) Uncapacitated Multi-Stage, Serial Process; each stage has at most one successor and one predecessor and capacities at all stages are unlimited.
- 6) Multi-Stage, Serial Process With a Single Capacity constraint for all stages (part 6a of Figure 1.2) or one stage (part 6b of Figure 1.2). In the first case, there is one work center, the bottleneck, and its capacity is constrained. In the second case, capacity is constrained in only one stage of production and other stages have unlimited capacity.
- 7) Uncapacitated Assembly Structure: Production of an item in different stages of production where capacity is infinite and each item has one successor and many predecessors
- 8) Assembly Structure with Capacity Constraints at one level (part 8a of Figure 1.2) or all levels (part 8b of Figure 1.2). In the first case (part 8a), capacity for the work center is constrained in one level and operation at each stage is performed independently. In the second case (part 8b), there are two capacity-constrained work centers, each at separate level of manufacturing structure.

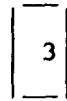
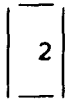
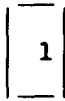
The key research conducted in the area of lot-sizing is summarized in Table 1.1. We will next discuss each of these areas in detail.



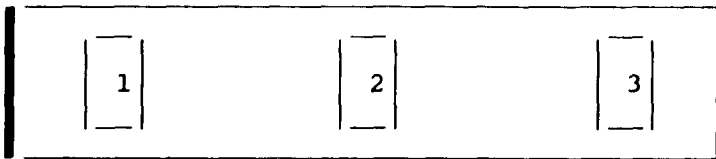
1: SINGLE-ITEM



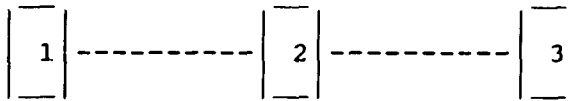
2: CAPACITATED SINGLE-ITEM



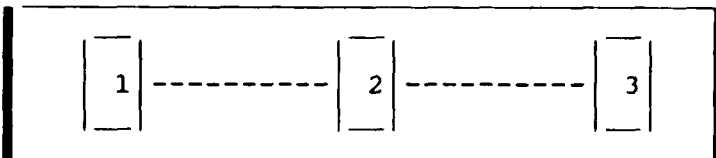
3: MULTI-ITEMS



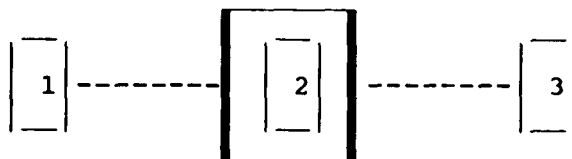
4: CAPACITATED MULTI-ITEMS



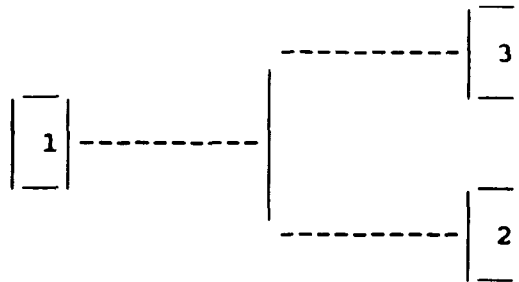
5: SERIAL MULTI-STAGE



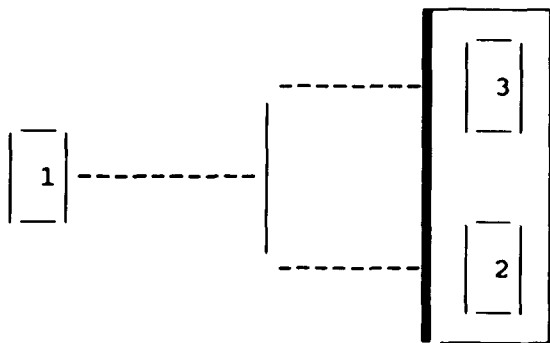
6.a: CAPACITATED SERIAL MULTI-STAGE (CAPACITY AT ALL STAGES)



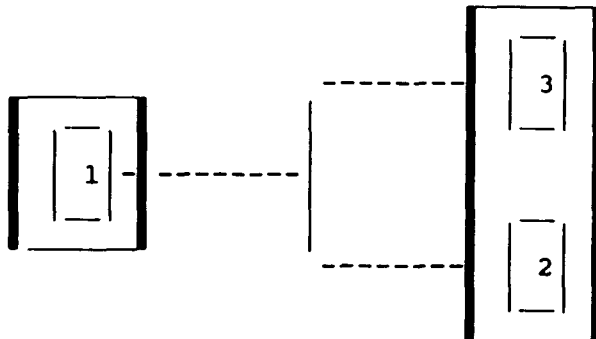
6.b: CAPACITATED SERIAL MULTI-STAGE (CAPACITY AT ONE STAGE)



7: ASSEMBLY STRUCTURE



8.a: CAPACITATED ASSEMBLY STRUCTURE (CAPACITY AT ONE LEVEL)



8.b: CAPACITATED ASSEMBLY STRUCTURE (CAPACITY AT ALL STAGES)

FIGURE 1.2: TYPES OF PRODUCT STRUCTURE

UNCAPACITATED		CAPACITATED		
ONE ITEM	Wagner & Within [177]	1	Florian & Klien [75]	2
	Silver & Meal [163]		Baker et al. [15]	
	Gaither [80]		Bitran & Yanasee [38]	
	Gorham [89]		Florian et al. [77]	
ITEM	Groff [92]			
	Freeland & Golley [78]			
	Blackburn & Millen [39]			
	Decomposes to one Item Problem	3	karmarkar et al. [106]	4
SIMULTANEOUS			Manne [130]	
			Dzielinski & Gomory [67]	
			Lasdon & Terjung [124]	
			Newson [152]	
			Eisenhut [68]	
			VanNunen & Wessels [173]	
			Lambrecht et al. [123]	
			Dixon & Silver [63]	
			Bitran & matsuo [37]	
			VanWasse. & Debodt [174]	
			Newson [153]	
			Dogramaci et al. [64]	
			Bahl [11],[12]	
			Bahl & Ritzman [13]	
			Karni & Roll [110],[111]	
			Barany et al. [19]	
SERIAL			Thizy & VanWassenhove [171]	
			Maes & VanWass.[131],[132]	
			Eppen & Martin [71]	
	Lambrecht et al. [120]	5	Lambrecht et. [120,118]	6
	Zangwill [189]		Ramsay [156]	
	Love [128]		Zahorik et al. [185]	
			Billington et al. [34],[32]	
			Billington [30]	
			Lambrecht et al. [121]	
			Gabby [79]	
	Afentakis [3],[5],[6]	7	Hamilton [94]	8
	Berry [24]		McClain et al. [41]	
	Oricky [154]		Blackburn & Millen [43]	
	Veinott [175]		Steinberg & Napier [167]	
	Crowston et al. [55-6],[57]		Goldratt [88]	
	Afentakis et al. [4]		Billington et al. [31]	
Graves [90]				
Williams [181],[182]				
Blackburn & Millen [42]				
Mclaren [142]				
Mclaren & Whybark [143]				

TABLE 1.1: A CLASSIFICATION OF PREVIOUS RESEARCH

2.1 SINGLE-STAGE LOT-SIZING

Heuristics for Dynamic Lot Sizing in an MRP environment have received a lot of attention and discussion in the literature. They can be categorized as follows: 1) demand related (lot-for-lot) [93,154]: Batching pattern matches the demand pattern in order to eliminate inventory, 2) size related (fixed batch quantities) [166,170]: Batches are of uniform or fixed size, 3) time related (fixed period lot-sizing) [93,166,170]: Batches are determined at uniform or regular intervals, and 4) cost related (classical order quantity) [129,154]: Batch size and release intervals are dictated by setup and holding cost considerations only.

2.1.1 Single Item, Uncapacitated Lot-Sizing

The Wagner and Whitin [177] algorithm is one of the earliest works in this category. Given a single item with deterministic (but not necessarily constant) demand and no capacity constraint, they develop an efficient dynamic programming algorithm that finds the optimal lot sizes. A fixed setup cost is charged for each period in which there is production, and holding cost is charged for each unit of inventory remaining at the end of each period. The major limitations of the Wagner-Whitin method are: it is limited to the single level uncapacitated problem, computation time required is high for problems of realistic size, and the procedure is perceived as complex by many managers who

consequently avoid its use [31].

Some of the efficient, easily understood heuristics include Silver and Meal [163] (Minimum-Cost-Per-Period method), Gaither [80], Gorham [89], Groff [92], and Freeland and Colley [78].

Blackburn and Millen [39,40] test several heuristics over different problem parameters and show that the least costly heuristic solution is highly dependent on the problem parameters, such as setup, holding cost, and level of capacity available at each stage. They also show that some of the more widely used heuristics can yield solutions with costs far from optimal.

2.1.2 Single Item, Capacitated Lot-Sizing

A more difficult problem results when a capacity constraint is included on production in each period. Florian and Klein [75] developed important optimal properties for this type of problem. They showed that when demand is assumed to be deterministic over a finite horizon and the costs of producing and of holding inventory are concave functions, regeneration points can be found which mark the limits of a capacity constrained sequence in which production in at most one period is positive but less than capacity, and in all other periods is at capacity or zero.

Baker, Dixon, Magazine, and Silver [15] use Florian and Klein's results to develop a tree search algorithm to find the

optimal solution for the same problem. The $O(2T)$ algorithm (T is the number of periods) was found to be fairly efficient for this problem which Florian, Lenstra and Rinnooy Kan [77] have shown to be NP-hard; that is, it is unlikely that an efficient and optimal polynomial-time algorithm exists for this type of problem.

Bitran and Yanasse [38] discuss the complexity of single- and multi-item capacity constrained problems. They show that even relatively small one and two item problems are NP-hard.

In most practical production scheduling problems, several items are produced on the same facility. Research on such problems will be discussed next.

2.1.3 Multiple Items, Uncapacitated Lot-Sizing

Multi-item problems with no capacity constraint and separable costs decompose directly to a series of single item problems that can be solved independently using Wagner-Whitin or any single item heuristic.

2.1.4 Multiple Items, Capacitated Lot-Sizing

The capacitated problem is more difficult to solve. All the items are produced in one capacity-constrained work center and the demand for each item is deterministic. Generally, the costs considered include those of production, setup, and holding inventory.

Dzielinski and Gomory [67] point out that Manne's formulation results in a very large linear programming problem because of the large number of columns. They apply a Dantzig and Wolfe decomposition to find optimal solutions.

Lasdon and Terjung [124] use a column generation technique on Manne's problem and show that it is more efficient than the Dantzig and Wolfe decomposition method. The more recent research on the CLSP centers on heuristics and has no restriction that the number of items must be much larger than the number of periods.

Newson [152] develops a heuristic procedure which uses a series of shortest route problems, one for each item. Infeasible arcs are deleted and the next best path found. The procedure continues until the first feasible solution is found. It is not stated whether the cumulative demand in every period for each item must be less than the available capacity, but it appears that the method would not find a feasible solution if that were not the case.

Van Nunen and Wessels [173] approach the problem by first calculating the optimal lot size for each item assuming infinite capacity. Infeasibilities to the original capacitated problem are then removed with a "capacity adapting procedure." Their heuristic is developed primarily for a problem with a large number of items, each of which has irregular demand in relatively small amounts. No computational testing is reported.

Eisenhut [68] develops a heuristic procedure which relies on an incremental cost reduction to determine which items should have future demands batched with those for the period being scheduled. This method suffers from the problem encountered by Newson: it is possible that feasible solutions may not be found.

Lambrecht and Vanderveken [123] extend Eisenhut's method to remove capacity infeasibilities. If an infeasibility results, production is shifted back to an earlier time period. As long as cumulative capacity is greater than cumulative demand for every period, the procedure should produce a feasible result.

Dixon and Silver [63] develop a procedure similar to the Lambrecht and Vanderveken, and much like the Silver-Meal [163] heuristic for the uncapacitated single item problem. They propose additional "fix-up" procedures that improve the solutions, which are shown to be 2.5% above optimal for small problems and within 6.3% of a lower bound for larger problems. Testing on larger problems is limited, and comparison testing with other procedures is not included.

Most of the research on the CLSP ignores the concept of setup time. Manne [130] does include setup time but the problem must have a large number of items relative to the number of periods. Newson [152] does include setup time in the formulation, but computational results are not presented. Van Wassenhove and DeBodt [174] introduce setup

time in their formulation but then remove the variable by reducing the capacity available each period by the average amount of setup time actually observed from historic records. If demand has high variability, the capacity reduction will be too small in capacity constrained periods and too large in periods with slack.

Newson [153] extended the CLSP heuristic to include the variable capacity. His research coordinates aggregate and detailed production schedules (ADSP). The objective of the ADSP heuristic is to eliminate the principal disadvantages of the Dzielinski and Gomory [67] and Lasdon and Terjung [124] formulation (i.e. the existence of split lots), while retaining the advantages of selective column generation and decomposition.

Dogramaci, Panayiotopolos and Adam [64] present a forward pass algorithm. A forward pass algorithm is one which determines the lot sizes for earlier periods before moving to study the later periods.

Bahl [11] and Bahl & Ritzman [13] extended the approach by Manne [130], Lasdon & Terjung [124], and Dzielinski and Gomory [67]. They relax the 0-1 conditions for a set partitioning problem to create a linear programming problem and then round off the resulting solution. Since the maximum number of fractional schedules is limited by the number of capacity constraints, their approach will yield good solutions when the number of items is much greater

than the number of periods.

Bahl [12] describes two heuristic algorithms that are based upon Lasdon's column generation algorithm. Specifically, the Wagner-Whitin conditions in the column generation algorithm are replaced by simpler heuristic routines known as part-period balancing (PPB) and part-period balancing with look ahead and look back (PPB-LA-LB) Berry [24] and Orlicky [154].

Maes and Van Wassenhove [131] presented extensive computational results for the heuristics of Lambrecht and Vanderveken [123], Dixon and Silver [63], and Dogramaci, Panayiotopolos and Adam [64]. They concluded that the difference in performance between the heuristics over a large set of problems with different structures was smaller than anticipated. The cost structure, as reflected in average time between orders, is an important parameter, as well as tightness of the capacity. They proved that when Time Between Orders (TBO for an item is the expected time between orders found by dividing the economic order quantity value by the average demand for the item) increases and capacity becomes tighter the differences between the heuristics become larger and Dogramaci, Panayiotopolos, and Adam (DPA)[64], outperform Dixon and Silver (DS)[63], Lambrecht and Vanderveken (LV)[123].

Maes and Van Wassenhove [132] presented extensive computational results based on comparison of the same three

heuristics (DPA, DS, and LV) in rolling horizon conditions. A heuristic is applied to a T-period problem where T is called "time fence". The first period's decisions are implemented and the second period's production lots are recorded. The horizon is then rolled forward one period and a new T period problem is solved. In this problem demand for each item is adjusted for production in the previous period and demand for a new period (T+1) is added. Again only the current period's decisions are implemented and the next period's decisions are recorded.

The Karni & Roll [110] heuristic is based upon transferring or shifting production to the earlier or later periods in order to reduce costs and eliminate infeasibility. The shifts are either (1) the minimum amount required to reduce or eliminate infeasibility, or (2) shifting the max amount possible to reduce or eliminate carrying costs, or (3) the total available amount to eliminate a setup cost or reduce carrying cost without incurring a further setup cost. They tested 100 examples of 2 to 10 items for 6 to 12 time periods and found that their approach resulted in total costs that were less than 1% higher than optimality in 17 cases.

Barany, Van Roy and Wolsey [19] solve lot sizing using a commercial mixed-integer code. Their test problems were up to 20 items and 13 periods.

Thizy & VanWassenhove [171] use a subgradient

algorithm to minimize the total costs based on a Lagrangian relaxation of the capacity constraints imposed on the resources. The method incorporates a primal partitioning scheme with a network flow subproblem to obtain a good feasible solution.

Eppen and Martin [71] propose the linear programming version of the capacitated lot-sizing. They prove that bounds generated using their methods are either equal or better than those generated by Lagrangian relaxation or column generation.

Karmarkar, Kere, and Kere [106] formulate a capacitated dynamic lot sizing problem with startup and reservation costs. A fixed startup cost is the usual set up cost and represents the explicit or implicit time costs of setting up the machine for the product.

In the multi-item context, startup cost can be interpreted as a changeover expense. The second reservation cost represents the implicit cost of tying up the machine so that other products can not be made on it. They used Lagrangian relaxation with a branch and bound algorithm. They reported good results.

Balinski [16], Everett [73], Geoffrion [83] and Held, Wolfe and Crowder [99] have shown that Lagrangian relaxation and subgradient optimization are theoretically sound and are useful for solving large scale problems. Fisher [74] provides a basic description of the Lagrangian

relaxation methodology and a more complete discussion of applications of the method.

Eppen and Gould [70] limit their work to the single item one machine problem. The optimal solution procedure relies on the assumption that the production cost function is continuously differentiable.

Kleindorfer and Newson [113] consider the one facility, multi-item capacitated production problem. The relaxation results in a series of Wagner-Whitin lot-size problems. They use this property to develop lower bounds on the optimal solution of the main problem, but do not show how to solve the main problem by removing the capacity infeasibilities.

2.2 MULTI-STAGE LOT-SIZING

A multi-stage production structure consists of a series of manufacturing stages (machining, assembly, etc.). The completed product that emerges at the completion of all manufacturing stages is known as the final good (end product). The lot-sizing problem is concerned with determining the timing and size of production lots for all items on all stages.

A. SERIES : SINGLE PRODUCT, PRODUCED IN A SERIES OF STEPS.



B. PARALLEL: MULTI ITEM SERIES STRUCTURE

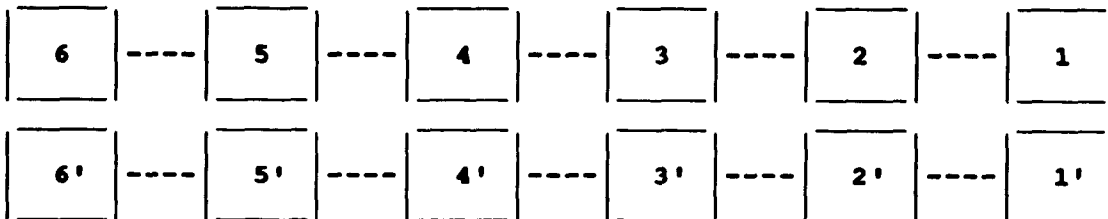
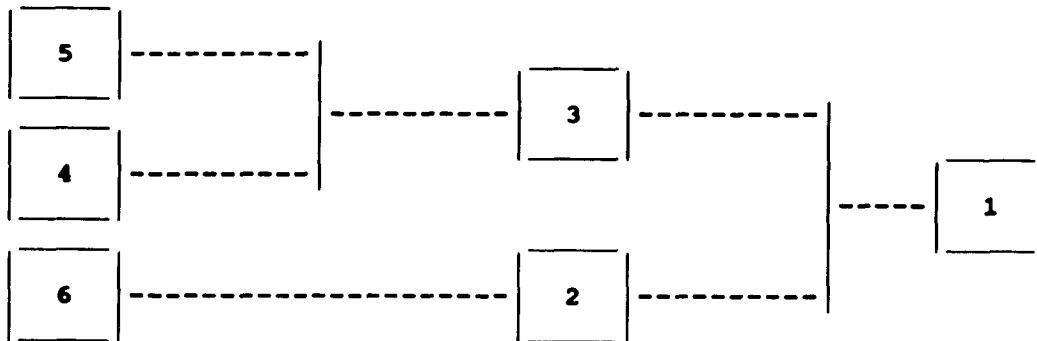


**FIGURE 1.3: TYPES OF MANUFACTURING STRUCTURES
(SERIAL AND PARALLEL)**

A. ASSEMBLY: A PRODUCT MADE BY A COMPLEX ASSEMBLY PROCESS



B. GENERAL: COMMONALITY OF COMPONENTS, OR MATERIALS ARE USED IN MORE THAN ONE SUCCESSORS ITEM

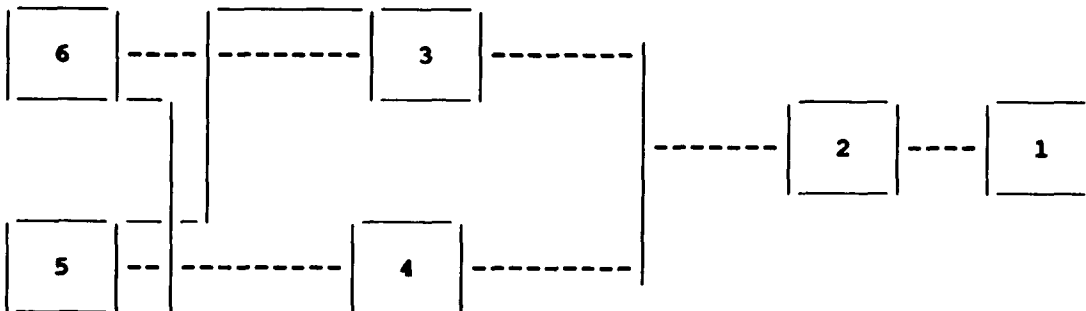


**FIGURE 1.4: TYPES OF MANUFACTURING STRUCTURES
(ASSEMBLY AND GENERAL)**

2.2.1 MULTI-STAGE WITH NO CAPACITY CONSTRAINT

The multi-stage problem is a natural extension of the single item lot-sizing problem, particularly with the popularity of MRP systems. Some of the multi-stage research resulted from the failure of single-item heuristics to provide low cost solutions when applied in a multi-level setting.

2.2.1.1 Uncapacitated Multi-Stage, Serial Structure

Figure 1.3(a) illustrates a serial structure for a single product. When no capacity constraints exist, Zangwill [189] and Love [128] use a concave cost structure (which allows for set-up cost and linear holding cost) and present relatively efficient solution techniques. Under these assumptions it is shown that this type of structure can be represented as a single source network. This discovery leads to the "extreme flow" property for optimal solutions: for any item, either production is positive or incoming inventory is positive, or neither, but never both at the same time. Zangwill then develops a dynamic optimal solution procedure that exploits the extreme flow property. A heuristic approach to lot-sizing in this situation is given by Lambrecht, Vander Eecken and Vanderveken [120].

Love [128] considers the same serial problem of Zangwill, but with different cost assumptions. If the production costs are non-increasing over time and the

echelon holding costs are all positive, then two nesting properties result: 1) if production of an item occurs in a period, then all successor items must also be produced in that period; 2) if there is no production of an item in a period, there will be no production in that period of any predecessor. A dynamic programming algorithm is developed that exploits these nesting properties. Computation time is reduced compared to the Zangwill algorithm because many possible alternatives need not be considered.

2.2.1.2 Uncapacitated Multi-Stage, Assembly Structure

Lot-sizing decisions in such systems have been studied by many authors, including Veinott [175], Crowston and Wagner [55] and Crowston, Wagner and Williams [56]. The complexity added by considering multiple products and capacity constraints that cut across products has not been adequately analyzed.

Veinott [175] extends the work of Zangwill to include more complex structures such as assembly (multiple predecessors only) and arborescence (multiple successors only). Recursive dynamic programming algorithms are presented to find optimal solutions to a wide range of production structures.

Crowston and Wagner [55] extend the work of Love to the assembly structure. They prove that the nesting properties defined by Love also hold for the assembly

structure given the same assumptions on costs.

Crowston, Wagner and Williams [57] study the assembly structure problem, but demand is assumed to be constant and deterministic over an infinite number of periods. They show that the optimal lot size of an item is an integral multiple of its successor's optimal lot size.

Figure 1.4(a) represents a product made by a complex assembly process. However, each stage (item) is assumed to have exactly one successor.

Figure 1.4(b) depicts a more general structure with commonality of components, where components or materials are used in more than one successor item. (Figure 1.4(b) does not illustrate components used in multiple finished products, but that is also possible. This is the general form of an MRP problem. To take full advantage of commonality, a list of successors (also called parents or user-items) of each component must be maintained. The logical place for this information is in the bill-of-material describing the multiple-use item. Many researchers have studied the problems of lot-sizing in such settings. (See, e.g., Berry [24], Biggs [27], Orlicky [154] and Newson [151]. A special case involves lot-sizing rules for a one-stage production process. (See, e.g., Blackburn and Millen [39, 40]. These results, while interesting, do not necessarily generalize to the multi-stage case.

Krajewski and Ritzman [116], Bahl [11] and Bahl and

Ritzman [10] also present mathematical programming formulations, without considering lead times.

Heuristic procedures for problems with finite horizons and deterministic (but not necessarily constant) demands were developed by Crowston, Wagner and Henshaw [56] that exploit this integrality property. Computational study shows that the heuristic provides results close to optimal for a limited set of problems. However, it is easy to develop examples where the integrality property results in very poor solutions [56].

Graves [90] develops an iterative procedure that solves the multi-level problem level-by-level, item-by-item using a single item lot sizing heuristic. The procedure then revises the schedule level-by-level using the previously determined schedule. The iterative procedure continues until convergence at a local optimum.

McLaren and Whybark [143] and McLaren [142] develop a single pass heuristic that uses setup costs adjusted to account for the optimal time between orders (TBO) based on a standard EOQ calculation. Testing shows that this adjustment provides lower cost solutions compared to unadjusted one-item heuristics.

Blackburn and Millen [42] develop several cost modifications to existing efficient single pass heuristics. Extensive testing indicates that the ability of heuristic procedures to find near optimal solutions diminishes in

direct relation to the number of levels: the more levels, the further from optimal.

Lambrecht, Vander Eecken and Vanderveken [120] develop a forward single pass heuristic for the serial problem. Rather than solving the problem item-by-item across all periods, this heuristic solves the problem period-by-period across all items by calculating coefficients that measure the cost of alternate paths through a network. This heuristic is applicable to the serial structure only. The heuristic averaged 1.84% above optimal on tests of problems with four to ten periods and four to ten items.

It should be mentioned that any of the assembly structure heuristics can be applied to the serial problem since it is a special case of the assembly structure.

Williams [181] and [182] develops heuristic procedures for determining production batch sizes in an assembly network and distribution batch sizes in a conjoined distribution network. The demand rate is assumed to be constant and extensions to non-constant demand are not presented.

Biggs [26] and [27] and Biggs, Hahn and Pinto [28] run large scale simulations of MRP systems to test various hypotheses concerning lot-sizing, sequencing, and capacity. It is shown that certain combinations of lot-sizing and sequencing rules perform better than other combinations. The capacity of the system in these studies is considered

the capacity of the labor force for the entire plant; bottlenecks are not considered. They show that as capacity of a system is decreased, the number of stockouts increases dramatically due to the dependent nature of the system.

Banerjee [17,18] uses simulation to study the effect of placement of safety stock at various levels in an MRP scheduled system. The results, although not conclusive, suggest that the selection of a safety stock policy is less critical than the determination of effective methods for lot sizing and sequencing. The simulations are similar to Biggs [27] and consider capacity only through the overall labor availability for the plant.

Afentakis, Gavish, and Karmarkar [2] reformulate the general assembly problem in echelon stocks. A Lagrangian relaxation results in a decomposition to a series of single item problems easily solved to optimality. The Lagrangian relaxation and subgradient optimization are imbedded in a branch and bound procedure that is shown to find optimal solutions in a large number of problems very rapidly.

A considerable amount of research on the uncapacitated lot-sizing problem has resulted in many optimal algorithms and efficient heuristics. The major limitation is the lack of capacity restrictions. Without capacity constraints, the solutions develop "nice" properties, such as the nesting property discovered by Love, the extreme flow property

developed by Zangwill, or the integrality property developed by Crowston et al. These properties will not always hold when capacity constraints are introduced.

Afentakis [5] showed that the lot-sizing and the sequence problem in multi-stage systems with capacity constraints can be solved in one step. The model transforms the general production planning problem into an equivalent job-shop scheduling problem. The solution to that problem will satisfy the capacity constraints and will also determine the sequence in which the machines must be loaded.

Maxwell & Muckstadt [137] studied the problem with a constant demand rate and proposed an efficient algorithm for the special case in which the schedules are nested and the reorder intervals are power-of-two multiples of a basic period.

Afentakis, Gavish & Karmarker [4] developed a new method for optimal lot sizing in multistage assembly systems. Their main contribution was the transformation of the constraint matrix of the traditional formulation into an equivalent matrix with a block-diagonal structure linked by a set of coupling constraints. After relaxing the coupling constraints the "dependent" demand at each stage becomes independent of the production schedules of the succeeding stages. The natural decomposition of the problem leads to a set of a single-stage production planning problems linked by a set of dual prices.

Afentakis & Gavish [5] have developed branch-and-bound based algorithms for solving lot-sizing problems for products that have a complex product structure. Their results suggest that the formulation of the lot-sizing problem in terms of its echelon stocks, and the use of a Lagrangean relaxation, yield efficient algorithms.

Afentakis [6] uses the parallel approach to compute the lot sizes for the individual stages. This method does not require the schedules for an assembly to be determined before any decision on the production schedules for the sub-assemblies and components can be taken. Instead, the schedules are computed simultaneously for all items in a forward fashion (one period is solved first, then a two period problem, then three period problem, so on).

2.2.2 MULTI-STAGE WITH CAPACITY CONSTRAINTS

The capacity constrained multi-level problem is a logical extension of two directions of lot-sizing research: 1) uncapacitated multi-level to capacity constrained multi-level; and 2) single level capacity constrained to multi-level capacity constrained. Both approaches have influenced research on capacity constrained multi-level lot-sizing.

2.2.2.1 Capacitated Multi-Stage, Serial Structure

Figure 1.3(b) illustrates a multi-item series structure, where several items proceed through the same

production steps. Without a limit on resources, each item can be treated independently. Lambrecht and Vander Eecken [121] also present a model which includes one capacity constraint, an upper limit on production at the finished product level.

Gabby [79] includes a parallel serial problem with capacity constraints on every level. Such a system would describe a series of machines, where each product must go through each machine. Setup costs and setup time are assumed to be zero. He presents an efficient optimization algorithm that depends on a very restrictive assumption about the production time required by each item on each machine. (His assumption requires that a job take the same fraction of available time on each machine; i.e., a "one pound" product requires half as much time on each machine as a "two pound" product). Each item of a product structure must consume the same relative amount of time to be produced on each level. If this assumption does not hold, the algorithm may not, and in many cases does not, produce an optimal schedule. The method is also limited in the ability to avoid infeasibilities in the first period. An infeasibility may be found after a large number of calculations, but a "fix-up" routine to remove it is not provided.

Zahorik, Thomas and Trigerio [185] study the parallel serial problem with constraints on total throughput (bundle

constraints). Production costs are linear, i.e. setup costs are zero, and they show that the problem is a network when there are only three periods and when there are restrictions on the number or location of the bundle constraints. The three period result is then used as the basis for a heuristic.

Lambrecht and Vander Eecken [118, 120] study a single serial structure with a capacity constraint on the production of the final finished good. They use the Florian and Klein results to show that there are only a finite number of solutions for the capacitated end item problem alone. Given a solution for the end item, the schedule for the predecessor items can be solved optimality using any convenient method. This is repeated for each of the finite number of solutions for the capacitated end item problem. Total costs can then be calculated, and the optimal solution to the entire problem is shown to be the least costly solution to the two part problem. More complex structures are not studied, nor are capacity constraints placed on any items other than the end items.

Ramsay [156] includes capacity constraints on every item in a single serial structure. He develops a branch and bound procedure based on the Lagrangian relaxation of the capacity constraint into the objective function. Computational testing shows that the capacity relaxed formulation generally gives better lower bounds than the integer-relaxed linear

programming solution, but the Lagrangian relaxed branch and bound could not find feasible solutions for some problems.

Billington [33] has formulated a heuristic for the multi-level lot-sizing problem with a single bottleneck. The heuristic uses Lagrangean relaxation. Billington et al. [34] deals with the case of multi-item, multi-stage capacitated lot-sizing problem, where demands are known and satisfied without backorders. Their heuristic is based on the Lambrecht and Vander Eecken [120], Dixon and Silver [63], and the algorithms of Dogramaci, Panayiotopolos and Adam DPA [64]. The period-by-period heuristics [119,63,64] are unidirectional in that they proceed by constructing a schedule period by period, starting with period 1. The priority indices (cost coefficients) for this problem were based on [163]. The four-step algorithm by Dogramaci, Panayiotopolos and Adam [64] uses a completely different approach. It starts with the lot-for-lot schedule for each item. Karni and Roll [110] start with the optimal Wagner-Whitin solution for each item. DPA [64] looks at all possible lot combinations and merges two lots which yield the largest absolute saving. This is repeated until further improvements by merging two lots are no longer possible.

Research has been extended to the capacity constrained assembly structure, considered next.

2.2.2.2 Capacitated Multi-Stage, Assembly Structure

The capacity planning literature for processes such as Figure 1.4(b) does not deal with how to plan capacity, but rather how to get the information necessary to plan capacity. (See, e.g., Wemmerlov [178] and Belt [22, 23]). Belt's description of the dilemma of the planner makes it clear that MRP and capacity planning have not been well integrated.

Collier [51] studies the interaction of lot-sizing and capacity planning. He finds that certain lot-sizing techniques can lead to more erratic capacity usage. However, Billington, McClain and Thomas [29] point out that batching may either help or hurt the capacity usage pattern. Thus, techniques in which batching decisions are influenced by capacity might be of value. Very few results are available on the three-way interaction of lead-times, lot sizing, and actual capacity usage with the complex product structure of, figure 1.4(b).

Figure 1.4(b) does not represent all of the complex structures that can occur. In many situations although the products pass through the same machine(s), different tools may be required. Changing tools may involve a major set-up, while changing products for a given tool may involve only a minor set-up cost. For this sequence dependent, multi-stage production problem, heuristic methods based on optimization techniques are given by Caie and Maxwell [46] and Maxwell and Muckstadt [137]. In a one-facility example,

Brown, Geoffrion and Bradley [45] use Lagrangian relaxation to get a very efficient solution technique. The approach does not easily generalize to multi-stage manufacturing systems. Some authors take a broader view of the system, recommending a hierarchical solution that accounts for sequence-dependent set-ups and that matches the managerial authority in the organization (Hax and Meal [96] and Bitran, Haas and Hax [35], [36], Krajewski and Ritzman [116]).

Hamilton [94] allows work center capacity constraints in virtually any arrangement of single item or multiple items. The problem is formulated with production, holding, and setup costs, but is solved as both a planning and scheduling problem. The solution procedure first determines the amount of resource that would be required for a feasible solution, then schedules production assuming that the capacity can be adjusted on a period-by-period basis by moving labor and/or equipment. The ability to find feasible solutions may be dependent on the ability to plan different resource levels each period. There is no cost for shifting the resources between periods.

Steinberg and Napier [167] propose a near-network formulation of the multi-level capacitated lot-size problem. There are capacity constraints placed on the production of every item, but no work center bundling constraints. It is not a true network: Side constraints are used to insure that several items can be combined in an assembly node to

produce one new item. Solution attempts are limited to a time-consuming mixed integer binary procedure which would not be feasible for problems of any realistic size.

Bahl [9] considers the multi-level assembly problem with capacity constraints and setup time but without setup costs. The model allows unlimited overtime and hire/fire decisions. Several procedures are developed that are shown to provide solutions less costly than capacity insensitive methods which adjust the master schedule until a feasible solution results.

Blackburn and Millen [43] extend their uncapacitated lot sizing methods to the problem where each item can have a capacity constraint. Capacity must be an integer multiple of the average demand. Thus, capacity is two times demand, three times demand, etc., and capacity utilization will always be less than 50%, on average.

Billington, McClain and Thomas [31] formulate a multi-level lot-sizing problem with multiple work centers. Product structure compression is introduced to reduce the size of realistic problems while retaining salient features. McClain, Thomas and Weiss [141] show how the decomposition method of Dantzig and Wolfe can be used to solve the problem, but computational testing is not reported.

Goldratt [88] briefly describes an MRP lot-sizing system that does consider capacity when scheduling production. The computer code is proprietary and no

information is provided about how the algorithm works.

Most of the research in this area relies on one or more simplifying assumptions to make the solution relatively easy. These include assumptions that capacity is large, setup costs are zero, there are restrictions on location of the capacity constrained work center, and so forth. Except for Bahl [9], Billington et al. [3] and McClain et al. [141] setup time has not been considered. Ramsay [156] has few restrictions, but the problem is only a single item serial problem, and feasible solutions are not found for some problems.

Blackburn & Millen [42] modified a procedure, which originally was developed by McLaren [135]. Their approach is based upon capacity information at each stage.

Bahl and Ritzman [95] develop a simple doubling/halving lot-sizing adjustment used after the first MRP schedule is produced and infeasibilities are identified. A simulation study indicates that this simple adjustment can reduce past due customer demand and inventory in certain types of productive systems.

2.3 Summary

Various lines of research approach the problem of multi-stage lot sizing with capacity constraints from different directions.

Multi-level lot sizing has been studied but little

research has concentrated on the problem with capacity for multiple periods and multiple stages. What work has been done relies on very restrictive assumptions.

In the following chapter, the multi-stage capacitated lot sizing problem is formulated as an mixed-integer program (MIP) and in chapter IV some solution procedures are developed.

CHAPTER III

PROBLEM DESCRIPTION & FORMULATION

In this chapter, we describe the multi-stage capacitated problem, assumptions, notation, and formulation of the problem as a mixed-integer, binary linear program, using echelon inventory cost and echelon inventory. Some solution procedures are discussed for solving a mixed integer programs (MIP), including their limitations.

3.1 Problem Description

A production system consists of multiple stages, or levels, which could follow an assembly, serial or general structure with commonality of items. Parts or sub-assemblies combine with other parts to eventually comprise a finished product (end item). The items are numbered hierarchically, starting with end items. Thus, a successor of an item i , has number less than i . End items start with 1; purchased items and raw materials have the largest numbers. Capacity for each stage or period is known and external demand for end item for every period is also deterministic and known.

The objective is to determine the amount of production of every item, in every period, in assembly manufacturing structure, while attempting to minimize the total cost of

setup and inventory.

In the following sections, assumptions, notation, and formulation of the multi-stage capacitated lot-sizing problem are listed and discussed in detail.

3.2 MODEL FORMULATION

In this section, assumptions, notation, and development of the formulation are discussed.

3.2.1 Assumptions

The following assumptions characterize the problem.

- There is a finite horizon consisting of discrete periods, with a certain fixed length t , such as a week, a day, or a month.
- Demand for the end item is external to the system, deterministic, but not necessarily constant at each period over the horizon.
- External demand for all items except the end item is assumed to be zero; demand for these items is known as dependent demand, and is derived from demand for predecessor items.
- Backordering to meet demand is not allowed. This implies that on-hand (available) inventory must be non-negative at all times or each period's demand must be met during or before the period in which the demand occurs.
- The set-up time is assumed to be zero.

- Item 1 is the end product which corresponds to the only stage without a successor.
- The inventory level can be positive in every period except the last where it is zero.
- Capacity is equal and constant in every period, and additional resources are not available. Hiring/firing and overtime decisions will not be considered.
- Lead times for supplying a stage from its immediate predecessors such as waiting time from the moment a product is ordered until it is available are assumed to be zero.
- There is a fixed set-up charge incurred when a resource is changed over to produce a different item. The set-up charges are independent of subsequent levels of production and of the prior production configuration of the facility. The set-up charges may be carried over from period to period.
- Variable production and holding costs are linear for each item and they vary among items.

3.2.2 Notation

The notation used to formulate the multi-stage capacitated problem is defined below.

S_i	= The production set up cost for stage i
d_t	= External demand for end item in period t
h_i	= Inventory holding cost for stage i
I_{it}	= Inventory of item i carried from period t to period t + 1
R_{it}	= Requirements of i in period t
P_{it}	= Quantity produced at stage i in period t
CAP_t	= Capacity (in time units) available in period t
b_i	= Time required to produce one unit of i or capacity absorption per unit in stage i
M	= Large number
N	= Total # of items (stages) in the manufacturing process
q_{ij}	= Quantity of item i used to produce an item j
x_{ij}	= Total requirements of item i needed to produce item j
$A(i)$	= Sets of immediate successors of item i
$B(i)$	= Sets of immediate predecessors of item i
$S(i)$	= Sets of all successors of item i
$P(i)$	= Sets of all predecessors of item i
E_{it}	= Echelon inventory for item i in period t
e_i	= Cost charged to each unit of inventory or cost added at stage i.
$\delta(P_{it})$	= 0-1 integer variable
$\hat{S}_i$	= Adjusted setup cost for stage i
$\hat{e}_i$	= Adjusted echelon holding cost for stage i
T	= Number of periods in the production problem.
$\bar{d}$	= Average demand
$N(i)$	= Number of generated schedules for stage i.

3.2.3 Development of the Formulation

The objective is to minimize the total cost of setup, inventory, and production.

Since the variable per-unit production cost is constant, and the total external demand is known, the total variable production cost is constant over the horizon and is not included in the objective function. It is easy to see that the objective function is

$$\text{MIN} \quad \sum_{i=1}^N \sum_{t=1}^T S_i * \delta(P_{it}) + h_i * I_{it} \quad (3.1)$$

where:

$$\text{Total setup cost} = \sum_{i=1}^N \sum_{t=1}^T S_i * \delta(P_{it}) \quad (3.2)$$

$$\text{Total inventory cost} = \sum_{i=1}^N \sum_{t=1}^T h_i * I_{it} \quad (3.3)$$

$\delta(P_{it})$ = Production indicator;

$$= \begin{cases} 1 & \text{if } P_{it} \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (3.4)$$

The formulation is constrained by inventory balance requirements and capacity constraints. The requirements of item i in period t will be the sum of external demands or the internal (dependent) demands from successor items.

$$R_{it} = q_{i,A(i)} * P_{A(i),t} \quad \begin{matrix} i=2, \dots, N \\ t=1, \dots, T \end{matrix} \quad (3.5)$$

$$R_{it} = d_t \quad \begin{matrix} i=1 \\ t=1, \dots, T \end{matrix} \quad (3.6)$$

The on-hand inventory at the end of any period is equal to the beginning inventory plus any production that becomes available in that period less the requirements for the item.

$$I_{it} = I_{i,t-1} + P_{i,t} - R_{it} \quad \begin{matrix} i=2, \dots, N \\ t=1, \dots, T \end{matrix} \quad (3.7)$$

Substituting (3.6) into (3.7):

$$I_{i,t-1} - I_{it} + P_{i,t} - d_t = 0 \quad \begin{matrix} t=1, \dots, T \\ i=1 \end{matrix} \quad (3.8)$$

Substituting (3.5) into (3.7):

$$I_{i,t-1} - I_{it} + P_{i,t} - q_{i,A(i)} * P_{A(i),t} = 0 \quad \begin{matrix} i=2, \dots, N \\ t=1, \dots, T \end{matrix} \quad (3.9)$$

At each time period , there will be a maximum capacity that can be used for production (the consumption of the productive resources in period t cannot exceed the capacity available).

$$\sum_{i=1}^N b_i * P_{it} \leq CAP_t \quad t=1, \dots, T \quad (3.10)$$

Items can be produced only in the periods selected for production (incurring a setup cost).

$$P_{it} \leq \delta(P_{it}) * M \quad \begin{matrix} i=1, \dots, N \\ t=1, \dots, T \end{matrix} \quad (3.11)$$

3.2.3.1 Formulation

The objective is to minimize the sum of total set-up and holding cost of all items over T periods of time considering a capacity limitation in every period. The complete formulation is shown below.

PROBLEM P-I

$$\text{MIN} \quad \sum_{i=1}^N \sum_{t=1}^T S_i * \delta(P_{it}) + h_i * I_{it} \quad (3.1)$$

ST.

$$I_{1,t-1} + P_{1t} - I_{1t} = d_t \quad t=1, \dots, T \quad (3.8)$$

$$I_{i,t-1} + P_{it} - I_{it} - q_{i,A(i)} * P_{A(i),t} = 0 \quad \begin{matrix} t=1, \dots, T \\ i=2, \dots, N \end{matrix} \quad (3.9)$$

$$\sum_{i=1}^N b_i * P_{it} \leq CAP_t \quad t=1, \dots, T \quad (3.10)$$

$$P_{it} \leq \delta(P_{it}) * M \quad \begin{matrix} i=1, \dots, N \\ t=1, \dots, T \end{matrix} \quad (3.11)$$

$$\delta(P_{it}) = \begin{cases} 1 & \text{if } P_{it} \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad \begin{matrix} t=1, \dots, T \\ i=1, \dots, N \end{matrix} \quad (3.4)$$

$$P_{it} \geq 0 \quad I_{it} \geq 0 \quad \text{For all } i \text{ \& } t \quad (3.12)$$

Problem (P-I) can be easily modified for capacity per stage formulation by substituting (3.13) for (3.10).

$$b_i * P_{it} \leq CAP_t \quad \begin{matrix} t=1, \dots, T \\ i=1, \dots, N \end{matrix} \quad (3.13)$$

The complete formulation is shown below.

PROBLEM P-II

$$\text{MIN} \quad \sum_{i=1}^N \sum_{t=1}^T S_i * \delta(P_{it}) + h_i * I_{it} \quad (3.1)$$

ST.

$$I_{1,t-1} + P_{1t} - I_{1t} = d_t \quad t=1, \dots, T \quad (3.8)$$

$$I_{i,t-1} + P_{it} - I_{it} - q_{i,A(i)} * P_{A(i),t} = 0 \quad \begin{matrix} t=1, \dots, T \\ i=2, \dots, N \end{matrix} \quad (3.9)$$

$$b_i * P_{it} \leq CAP_t \quad \begin{matrix} t=1, \dots, T \\ i=1, \dots, N \end{matrix} \quad (3.13)$$

$$P_{it} \leq \delta(P_{it}) * M$$

$$\begin{matrix} i=1, \dots, N \\ t=1, \dots, T \end{matrix}$$

(3.11)

$$\delta(P_{it}) = \begin{cases} 1 & \text{if } P_{it} \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{matrix} t=1, \dots, T \\ i=1, \dots, N \end{matrix}$$

(3.4)

$$P_{it} \geq 0 \quad I_{it} \geq 0 \quad \text{For all } i \text{ \& } t$$

(3.12)

A more useful form of inventory can be obtained by redefining the inventory (I_{it}) and inventory holding cost (h_i) in terms of the echelon inventory (E_{it}) concept introduced by Clarck and Scarf [48] for multi-stage inventory systems and subsequently employed by Blackburn and Millen [42], Crowston et al [56], and Schwarz and Schrage [159]. Given I_{it} , the inventory at stage i , the echelon stock at i , E_i , is defined as

$$E_{it} = I_{it} + E_{A(i),t} \quad \text{where, } E_{A(1),t} = 0$$

$$\begin{matrix} i=1, \dots, N \\ t=1, \dots, T \end{matrix}$$

(3.14)

Therefore, the following equations can be obtained

$$E_{it} = I_{it} + \sum_{j(A(i))} x_{ij} * I_{jt}$$

$$\begin{matrix} i=1, \dots, N \\ t=1, \dots, T \end{matrix}$$

(3.15)

$$h_i = e_i + \sum_{j \in B(i)} h_j * q_{ji} \quad i=1, \dots, N \quad (3.16)$$

The total holding cost of inventory of any item is the value added at that stage of the manufacturing process plus the holding cost of all its predecessors. Therefore, rearranging in terms of echelon holding cost, equation (3.15) becomes (3.17).

$$e_i = h_i - \sum_{j \in B(i)} h_j * q_{ji} \quad \begin{matrix} i=1, \dots, N \\ t=1, \dots, T \end{matrix} \quad (3.17)$$

E_{it} can be also expressed by those units which have been produced at stage i , but which have not been sold.

$$E_{it} = \sum_{r=1}^t (P_{ir} - R_{ir}) \quad \begin{matrix} i=1, \dots, N \\ t=1, \dots, T \end{matrix} \quad (3.18)$$

One consideration is the ability to model the problem of scheduling the production operations at stage i as a Wagner-Whitin problem.

Appendix A-1 shows the detail of how problem (P-I) can be transferred to (P-E) using expressions (3.15), (3.17), and (3.18). This leads to the following formulation:

3.2.3.2 Modified Formulation: P-E

$$\begin{array}{ll} \text{ST:} & \text{MIN } \sum_{i=1}^N \sum_{t=1}^T S_i * \delta(P_{it}) + e_i * E_{it} \end{array} \quad (3.19)$$

$$\begin{array}{l} E_{i,t-1} + P_{i,t} - E_{it} = x_{i1} * d_t \\ t=1, \dots, T \\ i=1, \dots, N \end{array} \quad (3.20)$$

$$\begin{array}{l} E_{i,t-1} + q_{i,A(i)} * E_{A(i),t} \leq 0 \\ i=2, \dots, N \\ i=1, \dots, N \end{array} \quad (3.21)$$

$$\begin{array}{l} \sum_{i=1}^N b_i * P_{it} \leq CAP_t \\ t=1, \dots, T \end{array} \quad (3.10)$$

$$\begin{array}{l} P_{it} \leq \delta(P_{it}) * M \\ i=1, \dots, N \\ t=1, \dots, T \end{array} \quad (3.11)$$

$$\begin{array}{l} E_{it} \geq 0 \\ t=1, \dots, T \end{array} \quad (3.22)$$

$$\delta(P_{it}) = \begin{cases} 1 & \text{if } P_{it} \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad \begin{array}{l} t=1, \dots, T \\ i=1, \dots, N \end{array} \quad (3.4)$$

$$P_{it} \geq 0 \quad \text{For all } i \text{ \& } t \quad (3.12)$$

Constraint (3.20) indicates that in every period and at every node, the sum of the entering echelon inventory plus the production must equal the sum of the demand for that period plus the ending echelon inventory.

Constraint (3.21) states that the echelon stock at every node and in every period must be greater than or equal to the sum of the echelon stocks of the successors.

Constraint (3.22) indicates that for node 1 (the end product), for which the set of successors is empty, it is required that the echelon inventory stock be nonnegative.

Solving the above problem for N items must satisfy the capacity limitations and also satisfy some additional restrictions between nodes to insure that predecessor items for item i are available when item i is produced.

The objective is to choose lot sizes for all items in the product structure so as to satisfy the external demand for the end item without exceeding capacity at each time period and to minimize the total cost of setup, and holding.

3.2.3.3 Problems P-I and P-E Are Equivalent

Problems P-I and P-E are equivalent. This can be shown by making a linear transformation in the set of constraints and in the objective function of problem P. The concept behind this transformation is better known in the MRP literature as "explosion". The explosion proceeds

from end item to raw materials in the network for every period t . We start by processing node 2. Multiplying (3.8) by q_{21} and adding the result to (3.9) for node 2 ($i=2$) we have:

$$[I_{i,t-1} + q_{21}I_{1,t-1}] + P_{2t} - [I_{2t} + q_{21}I_{1t}] = q_{21}d_t = x_{21}d_t \quad (3.23)$$

In general, if we continue to multiply (3.8) for fixed indices j , where $j=B(i)$, by q_{ij} , $i=2, \dots, N$, and add the result to (3.9) which is related to node i , we can obtain expression (3.24):

$$[I_{i,t-1} + \sum_{j \in S(i)} x_{ij}I_{j,t-1}] + P_{it} - [I_{it} + \sum_{j \in S(i)} x_{ij}I_{jt}] = x_{i1}d_t \quad (3.24)$$

By making use of (3.14) we obtain (3.21), expression (3.14) can be easily solved for I_{it} :

$$I_{it} = E_{it} - q_{i,A(i)} * E_{A(i),t} \quad \begin{matrix} t=1, \dots, T \\ i=1, \dots, N \end{matrix} \quad (3.25)$$

The nonnegativity constraints of I_{it} in problem (P-I) lead directly to the inequalities in (3.22), by substituting (3.25) in the objective function of problem (P-I), rearranging by E_{it} , and using (3.14), we obtain (3.19) (i.e., the objective function of problem (P-E)).

Property 1: $P_{it} > 0$ implies $P_{A(i),t} > 0$

Property 2: $P_{it} > 0$ implies $P_{k,t} > 0 \quad k \in \text{Path}(i)$

Referring to Veinott [175] there is an optimal solution in which $P_{it} \cdot I_{i,t-1} = 0$. It can be seen by combining property 2 and expression 3.25 that in optimal solution $P_{it} > 0$ implies

$$E_{i,t-1} = q_{i,A(i)} \cdot E_{A(i),t-1}.$$

Proposition I: $E_{i,t-1} \cdot P_{it} = 0$, Wagner-Whitin [177].

The schedules generated using the proposition are called Wagner-Whitin schedules. Indeed, when the capacity constraints are relaxed, problem (P-E) decomposes into N single item uncapacitated dynamic lot-sizing problems for which the Wagner-Whitin algorithm provides an effective solution method, but there are problem instances [35] for which there are no feasible solutions satisfying this condition.

This Proposition states that Cost can never be reduced at any stage by both producing and having positive entering echelon inventory in a period. The search for an optimal solution can be limited to a subset of all feasible solutions, when this subset is limited to those production schedules in which production of item i in period t can take place if and only if, the echelon stock of item i at the end of period $t-1$ was zero. Based on Veinott [175], Crowston and Wagner [57], Manne [130], and Wagner and Whitin [177] production can take place at a facility only if there is no entering inventory (or never carry inventory into a period

in which production is scheduled). In terms of multi-item uncapacitated lot sizing, this proposition provides an effective solution method.

Therefore, the lot scheduling problem can be modeled as a set of N shortest path subproblems (acyclic networks). Between each pair of nodes $0 \leq p < q < T$ there is an edge which represents a production setup in period $p+1$ which satisfies the demand accumulated in period $p+1$ to q .

Solving the above problem for N items must satisfy the capacity limitations and also satisfy some additional restrictions between nodes to insure that predecessor items for item i are available when item i is produced.

3.2.3.4 Size of the Problem

Allowing for the fact that P_{it} is fixed for $t \leq 0$ and P_{it} will be zero for $t \geq T+1$, the number of variables is:

$$\sum_{i=1}^N (T) = N \cdot T$$

There will be $N \cdot T$ production variables, $N \cdot T$ 0-1 integer setup variables, $N \cdot T$ inventory constraints and T capacity constraints.

3.3 Solution techniques

The formulation (P-E) is a mixed-integer binary linear program; mixed integer programming (MIP) models are

typically not used to solve real world production scheduling problems. One reason is that the standard MIP formulation for these problems often requires exorbitant solution times. At this point, it is necessary to explain some general concepts of MIP models. The general MIP model is:

Minimize cx

St: $Ax \geq a$

$Bx \geq b$

$x \geq 0$ x_j integer for all $j \in J$

where c, x, a, b , are column or row vectors, depending on context, A and B are matrices of dimension and J is an index set of integer variables. We assume that upper bounds on all of the variables are contained in the $Bx \geq b$ constraints. Problem LPMIP, the linear programming relaxation of MIP, is obtained by deleting the integrality requirements on all x_j . $Ax \geq a$ constraints can be referred to as coupling or complicating constraints and $Bx \geq b$ as special structure constraints. The complicating constraints typically include capacity constraints, load leveling constraints and so forth. The special structure constraints include demand and setup forcing constraints.

Solving MIP with an LP-based branch and bound code typically requires a great deal of computing time if the optimal solution value of LPMIP is not close to optimal

solution value of MIP. Much of the research devoted to solving MIP involves enumerative methods.

Perhaps the most successful approach for solving MIP to date is the Lagrangian relaxation. Magnanti, Shapiro and Wagner [134] have shown the equivalence of the optimal values of the Lagrangian dual and column generation methods. Examples of papers using Lagrangian relaxation include Afentakis, Gavish and Karmarkar [4], Graves [91], Kleindorfer and Newson [113], Karmarkar, Kekre and Kerkre [106], Karmarkar and Schrage [107] and Thizy and VanWassenhove [171]. Examples of column generation include Dzielinski and Gomory [67], Lasdon and Terjung [124], Manne [130], McClain, Thomas and Weiss [141], and Schrage [157].

Another approach used for solving MIP is dynamic programming. Examples include Bitran and Matsuo [37], Florian and Klien [75], Lambrecht and Vanderveken [119], Veinott [175], Zangwill [187], Wagner and Whitin [177]. Research on better formulations includes exploiting the underlying network structure, as, for example, Evans [72], Steinberg and Napier [167], and Zahorik, Thomas and Trigerio [185], or exploiting hierarchical structure, such as Hax and Meal [96]. Almost all of the above MIP approaches have been either for the single level or the uncapacitated version of the problem (except for Lagrangian relaxation).

Next, some solution methods for multi-level capacitated lot size scheduling will be discussed.

The first method, a widely available branch and bound procedure, utilizes the integer-related linear programming solution to find lower bounds. With $N \times T$ 0-1 integer variables, the size of the problem is large even for small structures. This method is used in an attempt to find optimal solutions to small problems for comparison purposes. It has been proved that a limited number of optimal solutions and feasible solutions could be generated with this method, Geoffrion [84].

The second method utilizes multi-level lot sizing procedures. There are not many multi-level capacitated lot sizing papers. Therefore, one approach could be: use a multi-level lot sizing heuristic item-by-item, moving down the product structure from finished products to raw materials; when the work center is to be scheduled, use a capacitated lot-sizing heuristic; return to the multi-level method to continue scheduling all items down to the raw material level. The difficulty with this method is insuring that the lots for successor items are not batched in such a way that the production requirements are greater than available capacity in the early periods.

The third method is a branch and bound using Lagrangian relaxation to find a lower bound. The basic concept behind this approach is to place some constraints into the objective function using lagrangian multipliers. If the relaxed constraints are selected cleverly, the

resulting problem should be much easier to solve than the original problem. The lower bound from the relaxed problem, in many cases, is much better than the lower bound from the integer-relaxed linear programming solution, and also it is possible to find optimal or near optimal solutions to a problem.

3.4 Difficulty of the Problem

The multi-stage capacitated lot-sizing problem has been formulated as a mixed-integer linear program. Florian, Lenstra, and Kan [77] in their extensive research showed that this problem is known to be NP-hard. This indicates that the problem is unlikely to be solvable in polynomial time with respect to the dimensionality of the problem. The problem is hard in a practical sense as well since methods that provide optimal solutions are unable to do so for all but very small problems within reasonable computation times. The computational complexity of those problems has stimulated major efforts by most researchers toward developing heuristic solution procedures. The lack of optimal solution procedures has limited the ability of researchers to evaluate the effectiveness of these heuristics. In fact, in the uncapacitated case which is relatively easier than the capacitated one, a study by Blackburn and Millen [42] on a set of test problems revealed that one of the popular heuristics that has been

used in MRP systems (Part-Period Balancing), produced solutions that deviated from optimality by an average of 16%. The comparisons used relatively small and simple product structures. Blackburn and Millen [44] found that, since the performance of this heuristic depends heavily on the number of levels in the product structure, an inferior performance can be expected on systems with many items, levels and stages in the production plan. Moreover, the immense size of these problems and the impressive nature of many of their costs and demands further suggests that good heuristic procedures may be more desirable than more costly (presumably) optimization algorithms. The need for sensitivity analysis may make the ability to quickly get good approximate solutions to such problems more valuable than an ability to find the optimal solution.

In the following chapter three new heuristics are introduced to solve these complex problems.

CHAPTER IV

HEURISTICS

In this chapter three new heuristics for the multi-stage capacitated lot-sizing problem are introduced and discussed in detail.

The first of these is called selective enumeration and is based upon the conversion of the uncapacitated lot-sizing procedure of Afentakis [6] to a capacitated multi-stage procedure. Feasible schedules for different stages of production are selectively generated using a unique algorithm. The generated schedules are based upon satisfying feasibility conditions and the final solution is the feasible solution with the minimum cost.

The second heuristic is called the modified per stage heuristic. It is based on the single-stage capacitated lot-sizing procedure of Florian and Klien [75]. This heuristic is useful when capacity constraints apply to each separate stage. Schedules are selectively generated with this heuristic; they are all feasible. The schedule with the minimum cost is chosen as final solution.

The third method is called the modified per period heuristic. It is an extension of a multi-stage uncapacitated procedure. A single capacity constraint applies to all items for each period rather than separate constraints for each stage. Feasibility can be achieved

using a shifting procedure to move production lots from periods with insufficient capacity to those periods to the right or to the left.

Numerical Example

Figure 4.1 is a simple three stage assembly structure, where stage 1 is the end product and stages 2 and 3 are the predecessors of stage 1. Data for this structure and the 5-period lot-sizing problem are shown in Tables 4.1 and 4.2. When capacity is limited for each stage, 13 units are available per stage. When capacity applies to all items, a total of 39 units is available each period.

We will carry this example throughout the chapter to illustrate the basic steps of each of the methods.

4.1 Classification of The Heuristic Procedures

Table 4.3 illustrates the key components of each heuristic with regards to the capacity, planning horizon, lot-splitting, and originality of the model.

4.2 Selective Enumeration (SE)

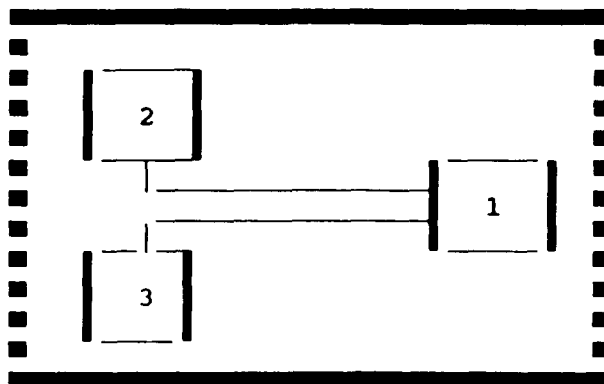
The selective enumeration (SE) heuristic produces schedules for the t -period problem by augmenting the solution to the $(t-1)$ -period problem. A set of schedules is first obtained for each stage. From each set of generated schedules one is selected so that the chosen

Stage	Successor	Predecessor	Setup Cost	Echelon Holding Cost	Stage Capacity
i	$A(i)$	$B(i)$	S_i	e_i	CAP_i
1	-	2,3	10	1	13
2	1	-	20	1	13
3	1	-	8	1	13

TABLE 4.1: DATA FOR A 5-PERIOD PROBLEM

Period	Demand	Period Capacity
t	d_t	CAP_t
1	8	39
2	15	39
3	4	39
4	8	39
5	12	39

TABLE 4.2: DEMAND AND CAPACITY FOR A 5-PERIOD PROBLEM



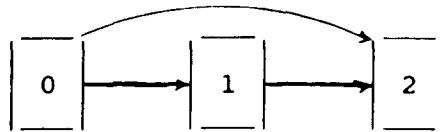
$$q_{21} = q_{31} = 1$$

FIGURE 4.1: THE THREE STAGE ASSEMBLY

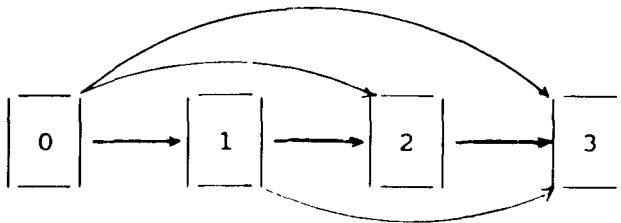
HEURISTICS

COMPONENT	SE: SELECTIVE ENUMERATION	MS: MODIFIED PER STAGE	MP: MODIFIED PER PERIOD
CAPACITY	PER PERIOD	PER STAGE	PER PERIOD
LOT-SPLITTING	NOT ALLOWED	ALLOWED	ALLOWED
FEASIBILITY	NOT ALWAYS	ALWAYS	ALWAYS
MODIFICATION	MULTI-STAGE UNCAPACITATED	SINGLE-STAGE CAPACITATED	MULTI-STAGE UNCAPACITATED
APPLICABLE MODEL	P-I	P-II	P-I

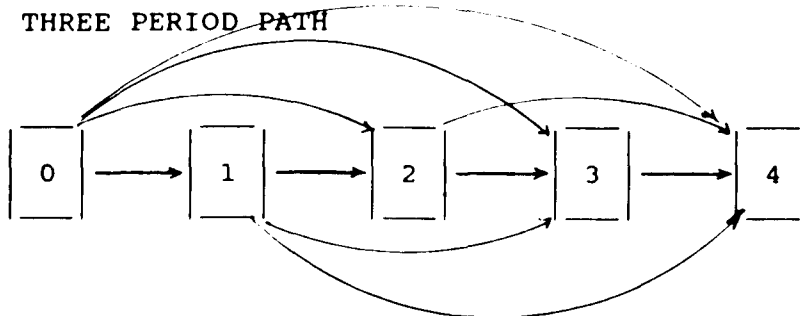
TABLE 4.3: KEY COMPONENTS OF EACH HEURISTIC



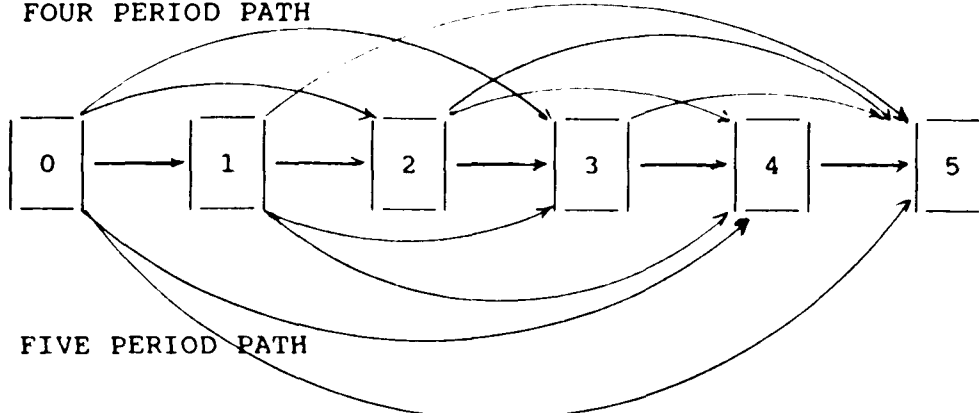
a). TWO PERIOD PATH



b). THREE PERIOD PATH



c). FOUR PERIOD PATH



d). FIVE PERIOD PATH

FIG. 4.2: SHORTEST PATH PROBLEM FOR THE (1-5)-PERIOD PROBLEM

schedules are feasible for the entire system and the total cost is minimized.

The t period problem can be represented as a directed network. At each stage, there are 2^{t-1} production paths (schedules) and $t(t+1)/2$ arcs (fig. 4.2) in the network, where t is the number of periods (i.e., for $t=5$, there are 16 paths and 15 arcs). Each arc (p,q) is assigned a cost $c(p,q)$ which is equal to the cost of producing at the very end of period p (instantaneously) to cover demands for periods $p+1$ to q inclusive. It includes the cost of set-up, production, and holding inventory from $p+1$ to q . Some of the $t(t+1)/2$ arcs, may be infeasible in that the production quantity exceeds the period capacity. Such arcs should be deleted.

There are additional requirements that the schedules of the different stages should not violate. First of all, the independent demand for all stages must be satisfied without backorders. This extends the assumption already made with respect to the final stage. A second requirement is that the capacity needed for the generated schedules in period t must not exceed the capacity available in that period. A third requirement is that the production schedules of any two adjacent stages must satisfy the following system condition:

SYSTEM CONDITION: "Cumulative production for stage i (excluding the final) at the end of each period t must at

least equal cumulative production for its successor stage $A(i)$." This is less restrictive than the condition used by Afentakis [6]. This system condition guarantees the feasibility of the production schedules, (i.e., enough components exist when needed for the production of an assembly).

In the following section, the procedure for developing feasible schedules and calculation of their costs is discussed in detail.

4.2.1 Summary of the SE Algorithm

The H1 heuristic can be summarized as follows:

- Step 1- Create a network (as described above) for each stage i , ($i=1, \dots, N$) for each possible q -period problem, ($q=1, \dots, t$).
- Find the lowest cost solution for each of the t q -period problems at each of the N stages using the shortest path procedure.
- Step 2- Starting with the first period, check feasibility with regards to the system condition and capacity requirement by combining the separately generated schedules for each of the stages in step 1.
- Repeat for each period. If the generated schedules for any period are not feasible, enumerate the possible schedules for each stage, determine those combinations that are feasible

with regards to the system condition and capacity,
and select the combined solution with minimum
cost.

Step 3- Generate $N(i) = t + 2 - t_i$ schedules for stage i
($i=1, \dots, N$) for period $t+1$. Denote by t_i the last
production period in the schedules $V(i, t)$. The
 k^{th} schedule is constructed by adopting the
schedule computed by the algorithm for periods 1
through $t_i + k - 2$ and adding a final setup in
period $t_i + k - 1$ to cover the demand for periods
 $t_i + k - 1$ through $t+1$.

$$W(i, k) = (V(i, t_i + k - 2), \sum_{r=t_i+k-1}^{t+1} R_{ir}, 0, 0..)$$

$$k=1, \dots, N(i)$$

$$i=1, \dots, N$$

$$(4.1)$$

For stage i ($i=1, \dots, N$), the cost of k^{th} schedule
is calculated as follows:

$$W^*(i, k) = (V^*(i, t_i+k-2) + S_i + e_i * \sum_{r=t_i+k}^{t+1} (r-t_i-k+1) R_{ir})$$

$$k=1, \dots, N(i)$$

$$i=1, \dots, N$$

$$(4.2)$$

where:

$V(i, t)$ is the production schedule for the t -period
problem in stage i and $W(i, k)$ is the k^{th} production
schedule for stage i in the $(t+1)$ -period problem.

$v^*(i,t)$ is the cost for the t -period problem for stage i and $w^*(i,k)$ is the cost of the k^{th} generated schedule for stage i in the $(t+1)$ -period problem.

0 in (4.1) indicates no production.

Step 4- Determine those combinations of stage-schedules that are feasible for the entire system, and select the one with minimum cost. (See Appendix A-2 for a complete description of the schedule selection procedure).

Numerical Example

For step 1, we start by using the shortest path procedure to solve each of the 4 q -period problems, ($q=1,\dots,4$) for stages 1, 2, and 3. Figures 4.3, 4.4, and 4.5 illustrate all possible schedules for the 2, 3, and 4 period problem, respectively. The minimum cost production schedules for each problem along with the costs are summarized in Table 4.4. The schedules are computed separately for each stage in a forward fashion. That is, the one period problem is solved first, then the two period problem, then the three period problem, and finally the four period problem.

Following step 2, feasibility is checked. As we can see from Table 4.4, the combined schedules for periods 3 and 4 are not feasible.

$$\begin{aligned} V(1,3) + V(2,3) + V(3,3) &= 43 \text{ units} \\ V(1,4) + V(2,4) + V(3,4) &= 65 \text{ units} \end{aligned}$$

Since these schedules are not feasible, they will be deleted from the network and we select the next lowest cost solution. Consider first the 3-period problem. We generate all schedules for each stage. There will be $3 \times 4 = 12$ schedules in the entire system, four for each stage. We calculate the cost for each schedule and list them in an ascending order. Table 4.5 shows all possible schedules with their cost for the 3-period problem. Enumerate the possible combinations for all stages that are feasible with regards to the system condition and capacity, select the one with minimum cost. If there is a tie, select the schedule with more setups to avoid infeasibility in later periods. Table 4.6 illustrates a portion of a complete enumerated possible combination of all schedules. Table 4.7 illustrates a feasible set of schedules along with the cost for each of the t-period problems.

The next step is to generate schedules for a 5-period problem, $W(i,k)$ by following step 3. The number of candidate schedules for each stage is computed as follows:

$$\begin{aligned} N(i) &= t + 2 - t_i & i=1, \dots, 3 \\ N(1) &= 4 + 2 - 4 = 2 \\ N(2) &= 4 + 2 - 3 = 3 \\ N(3) &= 4 + 2 - 4 = 2 \end{aligned}$$

Next, Let's construct the schedules using (4.1).

$$\begin{aligned} W(1,1) &= \{V(1,4 + 1 - 2), 20, 0\} = \{8, 15, 4, 20, 0\} \\ W(1,2) &= \{V(1,4 + 2 - 2), 12\} = \{8, 15, 4, 8, 12\} \\ W(2,1) &= \{V(2,4 + 1 - 3), 24, 0, 0\} = \{23, 0, 24, 0, 0\} \end{aligned}$$

$$\begin{array}{ll}
W(2,2) = \{V(2,3), 20,0\} & = \{23,0,4,20,0\} \\
W(2,3) = \{V(2,4), 12\} & = \{23,0,12,0,12\} \\
W(3,1) = \{V(3,3), 20,0\} & = \{8,19,0,20,0\} \\
W(3,2) = \{V(3,4), 12\} & = \{8,19,0,8,12\}
\end{array}$$

The complete set of schedules and their cost is shown in Table 4.8.

Step 4 calls for finding the lowest cost combined schedule that meets the system condition. From the set of candidate schedules for each stage in Table 4.8, select the one that is feasible for the entire system and for which the total cost is minimized.

$G_2W(1,1)$ is the set of schedules in stage 2 which are feasible with schedule $W(1,1)$,

$$G_2W(1,1) = W(2,1)$$

$G_2W(1,2)$ is the set of schedules in stage 2 which are feasible with schedule $W(1,2)$, and

$$G_2W(1,2) = \{W(2,1), W(2,2), W(2,3)\}$$

$G_3W(1,2)$ is the set of schedules in stage 3 which are feasible with schedule $W(1,2)$.

$$G_3W(1,2) = \{W(3,1), W(3,2)\}$$

Therefore, schedules $W(1, 2)$, $W(2, 3)$, and $W(3, 2)$ are selected because they are feasible for the system and cost is minimized. Table 4.9 illustrates the final feasible solution for the 5-period problem with total cost of 169. (See Appendix A-2 for the description of schedule selection procedure)

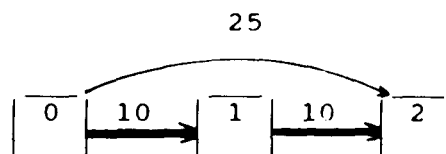
Heuristic 1 does not generate all possible schedules for each stage of production; it uses selective rather

than complete enumeration. For every t -period problem ($t=1, \dots, T$), the number of generated schedules is based upon the schedules for the $(t-1)$ -period problem.

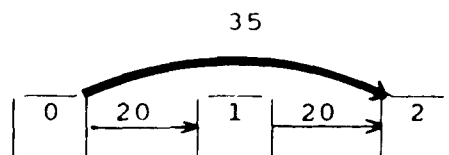
Generating all possible schedules (optimal solution or complete enumeration) is not practical for large size problems. This would require substantial computer time.

In the next section, heuristic 2 will be discussed in detail.

$i = 1$



$i = 2$



$i = 3$

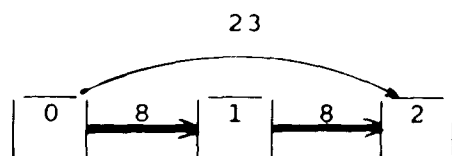
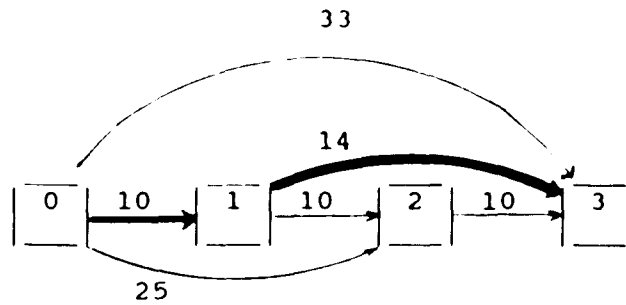
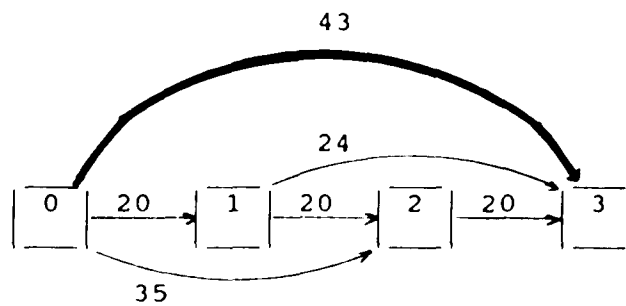


FIGURE 4.3: NETWORK FOR A 2-PERIOD PROBLEM

$i = 1$



$i = 2$



$i = 3$

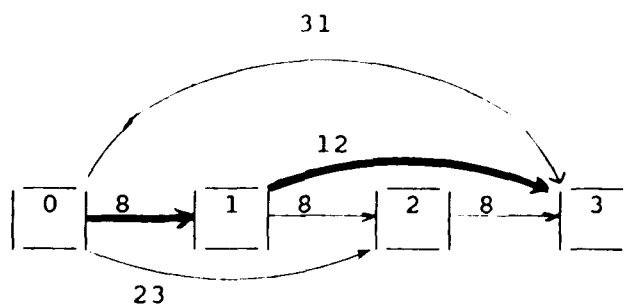
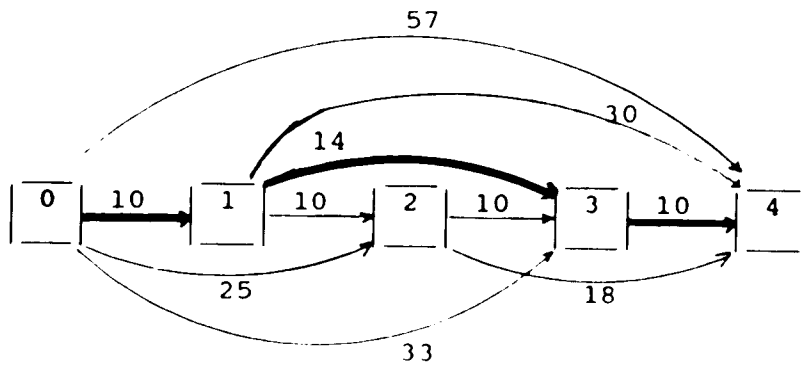
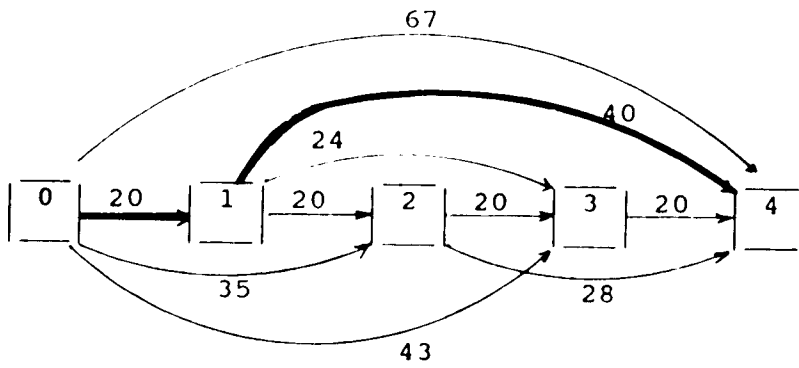


FIGURE 4.4: NETWORK FOR A 3-PERIOD PROBLEM

$i = 1$



$i = 2$



$i = 3$

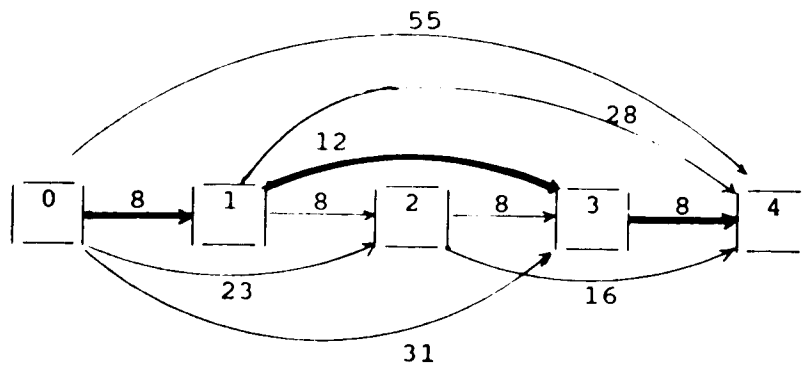


FIGURE 4.5: NETWORK FOR A 4-PERIOD PROBLEM

stage i	period, t							
	1	2	3	4				
1	(8)[10]	(8,15)	[20]	(8, 19,0)	[24]	(8,19,0,8)		[34]
2	(8)[20]	(23,0)	[35]	(27,0, 0)	[43]	(8,27,0,0)		[60]
3	(8)[8]	(8,15)	[16]	(8, 19,0)	[20]	(8,19,0,8)		[28]
Total	(24)[38]	(39,30)	[71]	(43,38,0)	[87]	(16,65,0,16)	[122]	
				*		*		

* Infeasible: Capacity exceeded

TABLE 4.4: SCHEDULES AND COST FOR THE 1, 2, 3, AND 4-PERIOD PROBLEM

Stage, i		
1	2	3
1, (8 ,19, 0) [24]	1, (27, 0, 0) [43]	1, (8 ,19, 0) [20]
2, (8 ,15, 4) [30]	2, (8 ,19, 0) [44]	2, (8 ,15, 4) [24]
3, (27, 0, 0) [33]	3, (23, 0, 4) [55]	3, (23, 0, 4) [31]
4, (23, 0, 4) [35]	4, (8 ,15, 4) [60]	4, (27, 0, 0) [31]

TABLE 4.5: COMPLETE SET OF SCHEDULES AND COST FOR A 3-PERIOD PROBLEM

where:

() = Production Schedule
[] = Production cost

Combination	Stage 1	Stage 2	Stage 3	Feasibility	Cost
1	1	1	1	"No-System"	87
2	1	1	2	"No-System"	91
3	1	1	3	"No-System"	98
4	1	1	4	"No-System"	98
5	2	1	1	"No-Capacity"	93
6	2	2	1	"No-Capacity"	94
7	2	2	2	"No-Capacity"	98
8	2	2	3	Yes	105
9	2	2	4	"No-Capacity"	105
10	2	3	1	Yes	105
11	2	3	2	Yes	109
12	2	3	3	"No-Capacity"	116

TABLE 4.6: 12 POSSIBLE COMBINED SCHEDULES FROM A COMPLETE SET OF 64 (4 x 4 x 4) SCHEDULES.

stage	period, t						
i	1	2	3	4			
1	(8) [10]	(8, 15)	[20]	(8, 15, 4)	[30]	(8, 15, 4, 8)	[40]
2	(8) [20]	(23, 0)	[35]	(23, 0, 4)	[55]	(23, 0, 12, 0)	[63]
3	(8) [8]	(8, 15)	[16]	(8, 19, 0)	[20]	(8, 19, 0, 8)	[28]
Total	(24) [38]	(39, 30)	[71]	(39, 34, 8)	[105]	(39, 34, 16, 16)	[131]

TABLE 4.7: MODIFIED SCHEDULES AND COST FOR THE 1, 2, 3, AND 4-PERIOD PROBLEM

stage	schedules, k		
i	1	2	3
1	(8, 15, 4, 20, 0) [52]	(8, 15, 4, 8, 12) [50]	
2	(23, 0, 24, 0, 0) [87]	(23, 0, 4, 20, 0) [87]	(23, 0, 12, 0, 12) [83]
3	(8, 19, 0, 20, 0) [40]	(8, 19, 0, 8, 12) [36]	

TABLE 4.8: SCHEDULES AND COST FOR THE 5-PERIOD PROBLEM

(8, 15, 4, 8, 12)	=50
(23, 0, 12, 0, 12)	=83
(8, 19, 0, 8, 12)	=36

[39, 34, 16, 16, 36]	=169

TABLE 4.9: FINAL FEASIBLE SOLUTION FOR THE 5-PERIOD PROBLEM

4.3 Modified Per Stage Procedure (MS)

The second heuristic is an extension of SE and is based on Florian and Klien's single stage capacitated procedure extended to generate schedules for the multi-stage capacitated lot sizing problem.

This heuristic follows formulation P-II in chapter 3 (capacity constraints in every stage).

A feasible solution can always be found if cumulative capacity exceeds cumulative demand in each period.

$$\sum_{g=1}^t CAP_{ig} \geq \sum_{g=1}^t R_{ig} \quad \begin{matrix} t=1, \dots, T \\ i=1, \dots, N \end{matrix} \quad (4.8)$$

For a t -period problem, beginning and ending inventory is 0. $I_0 = I_t = 0$. If we assume that inventory is 0 for some intermediate periods.

$I_j = 0$ for some $j \in \{1, \dots, T - 1\}$ and,

$$\sum_{g=j+1}^t CAP_{ig} \geq \sum_{g=j+1}^t R_{ig} \quad t=j+1, \dots, T \quad (4.9)$$

then a solution to the original problem can be found by independent decomposition with solutions to the problems for the first j periods and the last $T - j$ periods. This can be used to solve the problem using dynamic programming

recursion assuming periods $0, \dots, T$ as states. Let f_p be the cost associated with an optimal production plan over periods $p+1, \dots, T$, given that $I_p=0$:

$$f_0 = 0 \quad (4.10)$$

$$f_p = \text{Min}[C(p,q)+f_q] \quad p=0, \dots, T-1 \quad p \leq q \leq T \quad (4.11)$$

$C(p,q)$ = cost of the plan over periods of $p+1, \dots, q$

where $I_p=I_q=0$ and $I_t > 0$ for $t=p+1, \dots, q-1$,

$$0 \leq p < q \leq T \quad)$$

If we assume $CAP_{it} = CAP$, $t=p+1, \dots, q$, then,

$$\sum_{g=p+1}^q R_{ig} = K * CAP + R \quad i=1, \dots, N \quad (4.12)$$

$$0 \leq K \leq q-p$$

where K is a nonnegative integer and CAP_{it} is the capacity available for stage i in period t . $0 \leq R < CAP$.

A feasible production sequence can then be constructed with K periods in which the production levels are equal to CAP , at most one period with production level R , and the remaining periods with zero production levels.

For such schedules, the cumulative amount produced ($X_{ig} = \sum_{t=p+1}^g P_{it}$) over periods $p+1$ to g , can only take on the values:

$$\begin{array}{ll} \text{if } K = 0, R = 0, & X_{ig} = 0 \\ \text{if } K = 0, & X_{ig} = R \end{array} \quad \begin{array}{ll} \text{if } K = 1 & X_{i,g+1} = CAP + R \\ \text{if } R = 0 & X_{i,g+1} = K * CAP \end{array}$$

$$\text{if } K = 1, R = 0, \quad X_{ig} = \text{CAP} \quad \text{otherwise} \quad X_{i,g+1} = K \cdot \text{CAP} + R \quad (4.13)$$

Thus, considering only schedules of this type, a network may be constructed with vertices corresponding to the possible values of X_{ig} for each $g=p+1, \dots, q$. and with directed edges $(X_{ig}, X_{i,g+1})$ defined as :

$$(1) \text{ if } \quad X_{ig} = L * \text{CAP}$$

$$\left| \begin{array}{ll} X_{i,g+1} = X_{ig} + 0 & \text{and } P_{i,g+1} = 0 \\ X_{i,g+1} = X_{ig} + R & \text{and } P_{i,g+1} = R \\ X_{i,g+1} = X_{ig} + \text{CAP} & \text{and } P_{i,g+1} = \text{CAP} \end{array} \right.$$

where $P_{i,g+1}$ is the quantity produced for stage i in period $g+1$.

$$\text{or } (2) \text{ if } \quad X_{ig} = L * \text{CAP} + R$$

$$\left| \begin{array}{ll} X_{i,g+1} = X_{ig} + 0 & \text{and } P_{i,g+1} = 0 \\ X_{i,g+1} = X_{ig} + \text{CAP} & \text{and } P_{i,g+1} = \text{CAP} \end{array} \right. \quad (4.14)$$

The shortest path procedure can be applied to this network to find a low cost solution.

Next, we apply the procedure used in step 3 of heuristic 1 to generate schedules for the $(t+1)$ -period problem based on the solution for the t -period problem.

4.3.1 Summary of the MS Algorithm

The heuristic can be summarized as follows:

Step 1- Set $i = 1, p_i = 1$.

Find the first period $(q_i, q_i < t)$ for which

demand (d_i) exceeds capacity (CAP). If no infeasible period can be found, go to step 2. Create a network for periods p_i through q_i as explained by expressions 4.12, 4.13 and 4.14. All costs are based on the assumption that each stage will follow the same production schedule. Find the lowest cost solution for the $(q_i - p_i)$ -period problem using the shortest path procedure. Set $i = i+1$, $p_i = q_{i-1}$ and repeat step 1 if there are other periods for which demand exceeds capacity.

Step 2- Using the solution to each of the i subproblems in step 1 as subnetworks for the corresponding nodes (p_i, q_i) , create a network for periods 1 through t . Use this network to find the shortest path solutions for each of the t p -period problems ($p = 1, \dots, t$).

Step 3- Generate $N(i) = t + 2 - t_i$ schedules for stage i ($i=1, \dots, N$) for period $t+1$. Denote by t_i the last production period in the schedules $V(i, t)$. The k^{th} schedule is constructed by adopting the schedule computed by the algorithm for periods 1 through $t_i + k - 2$, adding a final setup in period $t_i + k - 1$ to cover the demand for periods $t_i + k - 1$ through $t+1$ (expression 4.1). If infeasibility occurs in period p ($p < t+1$), construct production schedules to cover the demand

for period p through $t+1$ using (4.12).

For stage i ($i=1, \dots, N$), the cost of k^{th} schedule is calculated by expression 4.2.

Step 4- Using the set of candidate schedules found in step 3, determine those combinations of stage-schedules that are feasible for the entire system and select the one that is feasible for the system and for which the total cost is minimized. (Same as step 4 in heuristic 1).

Numerical Example

Following step 1, we look for the first period in which demand exceeds capacity. Period 2 is infeasible since demand for this period is 15 units which exceeds the capacity of 13 units. We create a network for the two-period problem by using (4.12) and find the minimum cost.

Figure 4.6 shows the network for the 2-period problem with a minimum cost equal 82. The solution is to produce 10 and 13 units for each stage in periods 1 and 2, respectively. There are no other infeasible periods in this problem. We next construct a network as described in step 2.

Starting from period 1, we generate schedules by using the shortest path procedure for the one-period problem, the two-period problem, the three period problem, and the four-period problem. The purpose of this forward iterative

approach is to avoid infeasibility in the later stages and also to determine if the total cost can be reduced by finding better schedules. Figure 4.7 illustrates the shortest path for the 4-period problem. Table 4.10 shows the production schedules for each of the 4-period problems, ($q = 1, \dots, 4$).

Table 4.11 shows the candidate schedules for stage i ($i=1, \dots, 3$) for the 5-period problem. These schedules were obtained by following step 3 of the algorithm.

$W(1,1) = \{V(1,2), 24, 0, 0\}$. Since schedule $(24, 0, 0)$ is infeasible because 24 units in period 3 exceeds the available capacity of 13, it is scheduled by using expression (4.12). The total demand of 24 requires production at capacity of 13 for one period with an additional period of 11 units ($24 = 1 \cdot 13 + 11$). Therefore, $W(1,1)$ can be stated as follows:

$W(1,1) = \{V(1,2), 13, 0, 11\}$ or $\{10, 13, 13, 0, 11\}$

$W(1,2) = \{V(1,3), 20, 0\}$. Schedule $(20, 0)$ is infeasible and can also be rescheduled as $(13, 7)$ or $(7, 13)$.

$W(1,2) = \{V(1,3), 13, 7\}$, or $\{10, 13, 4, 13, 7\}$. The remaining schedules are determined as described above.

Step 4 calls for finding the lowest cost combined schedule that meets the system condition. Among those schedules generated in Table 4.11, we enumerate the ones that satisfy the system condition and select the one with the minimum total cost (enumerate possible schedules

similar to step 4 of procedure H1). Table 4.12 shows the final feasible solution with total cost equal to 182.

As the result indicates, the procedure always finds a feasible solution with lowest possible cost from those schedules considered.

In chapter 5, this heuristic is compared with the Blackburn and Millen approach [43]. A description of that procedure is provided later in this chapter.

The following heuristic is not a part of MS procedure and was introduced by Florian and Klien [75]. Figure 4.8 shows the path for the 5-period problem. This network is constructed by forming the vertices as explained in expressions 4.13 and 4.15. If the network is not solved in a forward fashion, the total cost is higher than 182. The feasible solution is to produce at capacity in periods 1, 2, and 3 with a small batch in period 5. Table 4.13 shows schedules for entire system with the total cost of 224.

In the next section of this chapter, heuristic 3 will be discussed.

$$\sum_{t=1}^2 d_t = 23 = 1 \cdot 13 + 10$$

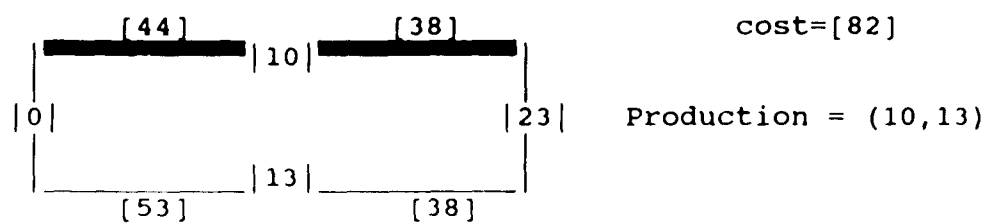


FIGURE 4.6: THE TWO-PERIOD PATH

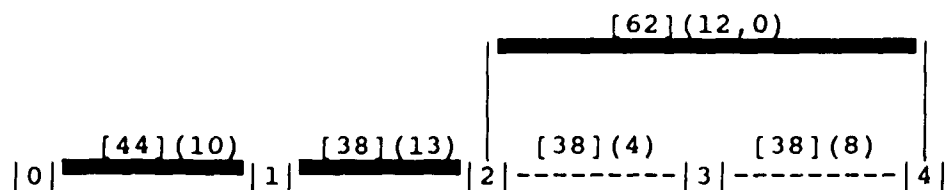


FIGURE 4.7: THE 4-PERIOD SHORTEST PATH PROBLEM

stage	period, t			
i	1	2	3	4
1	(8)[10]	(10,13)[22]	(10,13,4)[32]	(10,13,12,0)[40]
2	(8)[20]	(10,13)[42]	(10,13,4)[62]	(10,13,12,0)[70]
3	(8)[8]	(10,13)[18]	(10,13,4)[26]	(10,13,12,0)[34]
Total	(24)[38]	(30,39)[82]	(30,39,12)[120]	(39,34,16,16)[144]

TABLE 4.10: PRODUCTION SCHEDULES AND COST FOR THE 1, 2, 3, AND 4-PERIOD PROBLEM

stage	schedule, k		
i	1	2	3
1	(10,13,13,0,11)[52]	(10,13,4,13,7)[57]	(10,13,12,0,12)[50]
2	(10,13,13,0,11)[92]	(10,13,4,13,7)[107]	(10,13,12,0,12)[90]
3	(10,13,13,0,11)[44]	(10,13,4,13,7)[49]	(10,13,12,0,12)[42]

TABLE 4.11: SCHEDULES FOR THE 5-PERIOD PROBLEM

(10,13,12,0,12)	=50
(10,13,12,0,12)	=90
(10,13,12,0,12)	=42

[30,39,36,0,36]	=182

TABLE 4.12: SOLUTION FOR THE 5-PERIOD PROBLEM

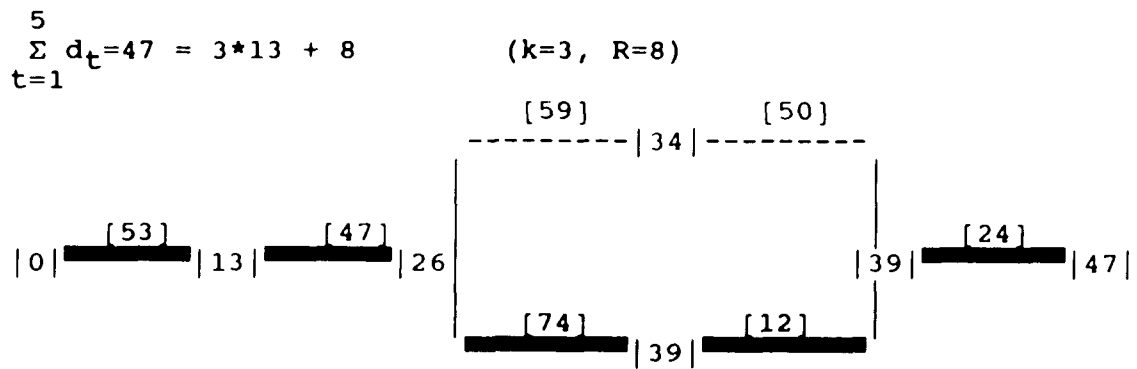


FIGURE 4.8: THE 5-PERIOD PATH PROBLEM

Stage	Production	Cost
1	(13,13,13,0,8)	[64]
2	(13,13,13,0,8)	[104]
3	(13,13,13,0,8)	[56]
Total	(39,39,39,0,24)	[224]

TABLE 4.13: SCHEDULES AND COST THE FOR 5-PERIOD PROBLEM

4.4 Modified Per Period Procedure (MP)

This heuristic follows formulation P-I in chapter 3 (capacity constraints in every period) and is based on a shifting procedure for the uncapacitated multi-stage problem.

Two methods are discussed here. Method 1 begins with a lot-for-lot schedules and feasibility is achieved by moving production lots from periods with insufficient capacity to those periods to the left. Method 2 begins with a feasible production schedules similar to MS and attempts to reduce the total cost by shifting production lots to the right or to the left. Method 2 is not included in the computational testing of chapter 5.

4.4.1 Method 1

Step 1- Start with a production schedule that produces an amount equal to demand in each period (lot-for-lot) and compute the amount of infeasibility (overproduction) for each period.

$$INF_t = \sum_{i=1}^N R_{it} - CAP \quad t=1, \dots, T \quad (4.15)$$

Go to step 3 if the condition for a feasible solution ($INF_t \leq 0$, ($t=1, \dots, T$)) is satisfied. Otherwise, go to step 2.

Step 2- Select the first infeasible period ($INF_q > 0$)

and call that period q . Among the lots that can be left-shifted one period from q to p ($p=q-1$) without violating feasibility (system condition and capacity requirement for p), choose the one with maximum benefit (B_i):

$$\text{MAX}_i B_i = (S_i - h_i * R_{iq})$$

Shift the requirement for the lot with maximum B_i from period q to period p .

If other infeasible periods remain, repeat step 2.

Step 3- Start from the last period; set $q = T$

Step 4- Compute INF_q using 4.15:

Among the products that can be left-shifted without violating the system condition, compute Z_{ipq} as follows:

$$\text{If } -INF_p \geq P_{iq}; \quad Z_{ipq} = S_i - (q - p) * h_i * P_{iq}$$

$$\text{Otherwise,} \quad Z_{ipq} = - (q - p) * h_i * INF_p$$

If at least one Z_{ipq} is positive, find the largest positive Z_{ipq} and move the $\text{Min}\{-INF_p, P_{iq}\}$ from q to p and repeat step 4.

Otherwise go to step 5.

Step 5- Set $q = q-1$ and if $q > 1$, go back to step 4;

Otherwise stop.

If Infeasibility occurs in the first period, there are no earlier periods into which production can be shifted; therefore, the solution is infeasible, as backorders are not allowed. In this case, increasing the capacity in the

first period is the only way to generate a feasible solution.

Numerical Example

First (step 1), we establish a requirement (lot-for-lot) table for the three stages and five periods. Table 4.14 illustrates the total requirement and capacity available in every period. The total cost is 190 and there is a setup in every period. The solution shown in table 4.14 is not feasible due to the overproduction in period 2 ($INF_2 = 6$ units).

Following step 2, we identify the lots that can be shifted from period 2 to period 1 and compute the benefit:

$$B_1 = 10 - 1 * 15 = -5$$

$$B_2 = 20 - 1 * 15 = 5$$

$$B_3 = 8 - 1 * 15 = -7$$

The only positive benefit is for stage 2. Shifting is done by moving 15 units in period 2 to period 1 for stage 2. The result of this shift is shown in Table 4.15.

The solution shown in Table 4.15 is feasible with a total cost of 185. Now we can investigate if the cost can be reduced by shifting other production lots to earlier periods without violating the system condition or the capacity constraints.

Set $q = 5$, and calculate Z_{ipq} for all stages. For $p = 4$ there are:

$$Z_{154} = 10 - (5 - 4) * 12 = -2$$

$$Z_{254} = 20 - (5 - 4) * 12 = 8$$

$$Z_{354} = 8 - (5 - 4) * 12 = -4$$

The remaining benefits are calculated as described above. Table 4.16 summarizes all the Z_{ipq} .

We shift the lot with maximum benefit. As table 4.16 shows, the maximum benefit of 12 occurs for stage 2 when units are shifted from period 4 to period 3. Thus, we can move 8 units of item 2 from period 4 to 3. Table 4.17 shows the schedules after the 8 units are shifted with a total cost of 173. We repeat step 4 to determine if more shifting is possible or cost can further be reduced. The updated INF_t values are shown in Table 4.17. Table 4.18 lists the newly updated Z_{ipq} . Only one positive benefit exists, for stage 3 from period 3 to period 2. Thus, we shift 4 units for stage 3 from period 3 to 2.

Table 4.19 illustrates the final solution with a total cost of 169. Table 4.20 indicates that cost can not be reduced any further, as there are no positive benefits.

In the following section, we introduce another procedure as an alternative to method 1. The following procedure is slightly different from method 1. Both these methods yield a feasible solution. Method 2 begins with a feasible problem and then shifts production in either direction to improve the total cost.

stage i	period, t					cost
	1	2	3	4	5	
1	8	15	4	8	12	50
2	8	15*	4	8	12	100
3	8	15	4	8	12	40
Schedules	24	45	12	24	36	190
Capacity	39	39	39	39	39	
INF_t	-15	6	-22	-15	-3	

TABLE 4.14: THE 5-PERIOD PRODUCTION PROBLEM
(SETUP FOR EVERY PERIOD)

stage i	period, t					cost
	1	2	3	4	5	
1	8	15	4	8	12	50
2	23	0	4	8	12	95
3	8	15	4	8	12	40
Schedules	39	30	12	24	36	185
Capacity	39	39	39	39	39	
INF_t	0	-9	-22	-15	-3	

TABLE 4.15: MODIFIED PRODUCTION SCHEDULES
FOR THE 5-PERIOD PROBLEM

z_{ipq}						$p = q - 1$
Stage, i	Period, q					
	1	2	3	4	5	
1	x	x	x	x	x	
2	x	x	-4	12 [*]	8	
3	X	x	4	0	-4	

TABLE 4.16: COST - BENEFIT FOR EACH ITEM

stage	period, t					
i	1	2	3	4	5	cost
1	8	15	4	8	12	50
2	23	0	12	0	12	83
3	8	15	4	8	12	40
Schedules	39	30	36	0	36	173
Capacity	39	39	39	39	39	
INF_t	0	-9	-3	-39	-3	

TABLE 4.17: THE 5-PERIOD MODIFIED FEASIBLE SOLUTION

Z_{ipq}

Stage, i	Period, q				
	1	2	3	4	5
1	x	x	x	x	x
2	x	x	-4	x	x
3	x	x	4*	0	-4

TABLE 4.18: COST - BENEFIT FOR EACH ITEM

stage	period, t					
i	1	2	3	4	5	cost
1	8	15	4	8	12	50
2	23	0	12	0	12	83
3	8	19	0	8	12	36
Schedules	39	34	16	16	36	169
Capacity	39	39	39	39	39	
INF _t	0	-5	-23	-23	-3	

TABLE 4.19: THE 5-PERIOD FINAL FEASIBLE SOLUTION

z_{ipq}

Stage, i	Period, q				
	1	2	3	4	5
1	x	x	x	x	x
2	x	x	x	x	x
3	x	x	x	0	-4

TABLE 4.20: COST - BENEFIT FOR EACH ITEM

4.4.2 Method 2

This method is different from method 1. It begins with a feasible solution and shifts production to the right or left to reduce costs. The possible solutions to the problem can be represented as a network which includes selected but not all possible production schedules. It is limited to only a number of feasible schedules as described in MS.

A feasible solution can always be found if cumulative capacity exceeds cumulative demand in each period.

$$\sum_{i=1}^N \sum_{g=1}^t \text{CAP}_{ig} \geq \sum_{g=1}^t \sum_{i=1}^N R_{ig} \quad t=1, \dots, T \quad (4.16)$$

If we assume that the capacity is constant in each period, then for $g = p+1, \dots, q$, we can write

$$\sum_{i=1}^N \sum_{g=p+1}^q R_{ig} = K * \text{CAP} + R \quad 0 \leq K \leq q-p \quad (4.17)$$

where K is a nonnegative integer and CAP is the capacity available for each period. $0 \leq R < \text{CAP}$.

One feasible production sequence has K periods in which the production levels are equal to CAP , at most one period with production level R , and the remaining periods have zero production levels.

Therefore, the cumulative amount produced

$(X_{ig} = \sum_{t=p+1}^g \sum_{i=1}^N P_{it})$ over periods $p+1$ to g , can only take on the values:

$$\begin{aligned} \text{if } K = 0, R = 0, \quad X_{ig} &= 0 & \text{if } K = 1 \quad X_{i,g+1} &= \text{CAP} + R \\ \text{if } K = 0, \quad X_{ig} &= R & \text{if } R = 0 \quad X_{i,g+1} &= K \cdot \text{CAP} \\ \text{if } K = 1, R = 0, \quad X_{ig} &= \text{CAP} & \text{otherwise} \quad X_{i,g+1} &= K \cdot \text{CAP} + R \end{aligned}$$

Thus, a network may be constructed with vertices corresponding to the possible values of X_{ig} for each $g = p+1, \dots, q$. and with directed edges $(X_{ig}, X_{i,g+1})$ defined as :

$$(1) \quad \text{if} \quad X_{ig} = L * \text{CAP}$$

$$\left| \begin{array}{ll} X_{i,g+1} = X_{ig} + 0 & \text{and} \quad \sum_{i=1}^N P_{i,g+1} = 0 \\ X_{i,g+1} = X_{ig} + R & \text{and} \quad \sum_{i=1}^N P_{i,g+1} = R \\ X_{i,g+1} = X_{ig} + \text{CAP} & \text{and} \quad \sum_{i=1}^N P_{i,g+1} = \text{CAP} \end{array} \right. \quad (4.18)$$

where $\sum_{i=1}^N P_{i,g+1}$ is the cumulative quantity produced in period $g+1$.

$$\text{or } (2) \quad \text{if} \quad X_{ig} = L * \text{CAP} + R$$

$$\left| \begin{array}{ll} X_{i,g+1} = X_{ig} + 0 & \text{and} \quad \sum_{i=1}^N P_{i,g+1} = 0 \\ X_{i,g+1} = X_{ig} + \text{CAP} & \text{and} \quad \sum_{i=1}^N P_{i,g+1} = \text{CAP} \end{array} \right. \quad (4.19)$$

Summary of Method 2

Step 1- Set $\alpha = 1$, $p_\alpha = 1$.

Step 2- Find the first period (q_α , $q_\alpha < T$) in which the total requirements for all stages exceed capacity for period q_α . If no infeasible periods can be found, go to Step 3. Create a network for periods p_α through q_α as explained by expressions 4.18, 4.19. Find the shortest path and cost for the $(q_\alpha - p_\alpha)$ -period problem.

If there are other periods for which demand exceeds capacity, set $\alpha = \alpha + 1$, $p_\alpha = q_{\alpha-1}$ and repeat step 2. Otherwise go to step 3.

Step 3- Including the solution (if any) to each of the α subproblems in step 2 as subnetworks for the corresponding nodes (p_α, q_α) , create a network for periods 1 through T . Solve this problem for the p -period ($p=1, \dots, T$) problem using the shortest path procedure.

Step 4- Identify those lots that can be shifted to the left (earlier in the schedule) without violating feasibility conditions (system condition). For every stage, i ($i=1, \dots, N$), calculate C_{ipq} .
 C_{ipq} = Inventory cost for a lot produced at stage i in period p and held until period q which is produced.

$$C_{ipq} = (q-p) * h_i * R_{iq} \quad (4.20)$$

Among the alternatives found, choose the one with maximum benefit (Z_{ipq}):

$$Z_{ipq} = \underset{ipq}{\text{MAX}} (S_i - C_{ipq}) \quad (4.21)$$

Move the production from period q to period p . If no benefit can be achieved, then stop.

Step 5- Calculate the infeasibility for every period:

$$(\text{INF}_t = \sum_{i=1}^N P_{it} - \text{CAP}) \quad t=1, \dots, T$$

Let t represent the first infeasible period ($\text{INF}_t > 0$). If all $\text{INF}_t < 0$, go to step 4.

Step 6- Set $\beta = 1$.

Step 7- If capacity is not exceeded in period $t+\beta$, ($\text{INF}_{t+\beta} < 0$) for each i ($i=1, \dots, N$), compute: ($O_{it} = P_{it} - R_{it}$) and choose the one with maximum benefit :

$$F_{it,t+\beta} = \underset{i}{\text{MAX}} (\beta * h_i * O_{it})$$

Move O_{it} from period t to period $t + \beta$;

Otherwise, $\beta=\beta+1$ and repeat step 5.

If Infeasibility occurs in any given period, when, after smoothing, more production is required in that period than there is available capacity, increasing the capacity in the that period is the only way to generate a feasible solution.

Numerical Example

For step 1 we set $\alpha = 1$ and $p_\alpha = p_1 = 1$. Then we start step 2 by finding the first infeasible period. The cumulative requirement for three stages in period 1 is 24 and in period 2 is 45 units, respectively. Period 2 is infeasible since capacity is only 39 units. We create a network for a two-period problem and find the minimum total cost using the shortest path procedure. Figure 4.9 shows the path for the 2-period problem; the total cost is 82. The feasible solution is to produce 30 and 39 in periods 1 and 2, respectively. There are no other periods for which total requirements exceed capacity, so we can go to step 3.

Next, using the solution found in step 2, we create a network for periods 1 to 5 and use the shortest path procedure to determine if the cost can be reduced. As is shown in figure 4.10, the total cost is 182 with the production schedules of (30,39,36,0,36). Table 4.21 illustrates the production quantity for every period and each stage. Next we move to step 4.

C_{ipq} and Z_{ipq} are computed similar to method 1. C_{212} is the cost of shifting production from period 2 to period 1 for stage 2. (C_{212} is selected for shifting because none of the other C_{ipq} are economical).

C_{112} is infeasible, it violates the predecessor condition.

$$C_{212} = (2 - 1) * 1 * 13 = \$13 \quad Z_{212} = 20 - 13 = 7$$

$$C_{312} = (2 - 1) * 1 * 13 = 13 \quad Z_{312} = 8 - 13 = -5$$

Table 4.22 indicates a total cost of 177 but infeasibility occurs in period 1 ($INF_1 = 43 - 39 = 4$ units). We calculate O_{i1} and F_{i12} for $i=1, \dots, 3$:

$$O_{11} = 10 - 8 = 2, \quad O_{21} = 15 - 15 = 0, \quad O_{31} = 10 - 8 = 2,$$

$$F_{112} = 1 * 1 * 2 = 2, \quad F_{212} = 1 * 1 * 2 = 2, \quad \text{and}$$

$$F_{312} = 1 * 1 * 2 = 2. \quad \text{Thus, we shift 2 units from period 1 to 2 in stage 1 and also shift 2 units from period 1 to 2 in stage 3. Stage 2 is not feasible.}$$

Table 4.23 shows the final feasible schedules with the total cost of 171.

In the next section, we provide a brief explanation of the procedure by Blackburn and Millen. It is used for performance comparisons with heuristic 2 (MS) in chapter 5.

4.5 Description of Blackburn and Millen Procedure

The lot-sizing approach contributed by Blackburn and Millen [43] determines a set of order intervals for each stage which minimizes the average cost per period.

Let n_i = order interval for item i

$K_i = n_i / n_{A(i)}$ K_i is integer for $i=1, \dots, n$

K_i represents the number of orders from stage $A(i)$ which are combined into a single order at stage i .

The lot-sizing problem reduces to finding a set of order intervals for each stage (n_i) which minimizes the average cost per period.

$$\sum_{t=1}^2 \sum_{i=1}^3 R_{it} = 69 = 1 \cdot 39 + 30$$

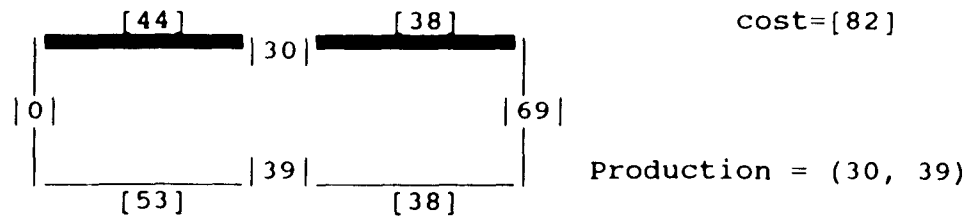


FIGURE 4.9: THE TWO-PERIOD PATH

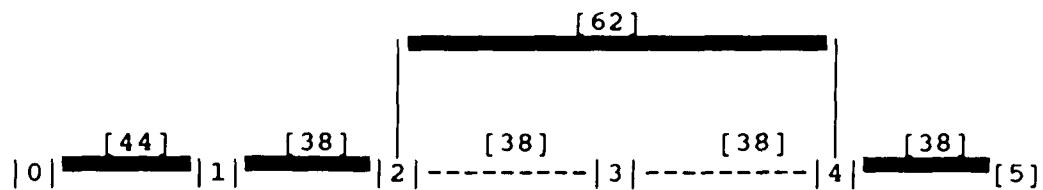


FIGURE 4.10: THE 5-PERIOD SHORTEST PATH PROBLEM

$$C_{02} + C_{23} + C_{45} = 182^*$$

$$\text{Production} = (30, 39, 36, 0, 36)$$

stage	period, t					
i	1	2	3	4	5	cost
1	10	13	12	0	12	50
2	10	13	12	0	12	100
3	10	13	12	0	12	42
Schedules	30	39	36	0	36	182

TABLE 4.21: THE 5-PERIOD FEASIBLE PROBLEM

stage	period, t					
i	1	2	3	4	5	cost
1	10	13	12	0	12	50
2	23	0	12	0	12	85
3	10	13	12	0	12	42
Schedules	43	30	36	0	36	177
Capacity	39	39	39	39	39	

(Infeasible)

TABLE 4.22: THE 5-PERIOD SHIFTING PROBLEM

stage	period, t					
i	1	2	3	4	5	cost
1	8	15	12	0	12	48
2	23	0	12	0	12	83
3	8	15	12	0	12	40
Schedules	39	30	36	0	36	171
Capacity	39	39	39	39	39	

TABLE 4.23: THE 5-PERIOD FEASIBLE PROBLEM

$$\text{Min. } [S_i/n_i + e_i D_i n_i / 2]$$

ST.

$$n_i = K_i * n_{A(i)} \quad \text{for } i=1, \dots, n \quad (4.22)$$

$$(4.23)$$

$$n_i \leq CAP_1 / D_i \quad (4.24)$$

$$n_i \text{ \& } K_i \geq 1 \text{ and integer} \quad (4.25)$$

The implication is that with estimates of K_i 's,

expression (4.22) can be used to make lot-sizing decisions at stage 1 with an adjusted set-up cost:

$S_1 + S_2/K_2 + S_3/K_3$ and adjusted echelon holding cost of $e_1 + e_2 K_2 + e_3 K_3$ for the three item problem under study.

Since the K_j values are estimates of the number of orders from stage $B(j)$ which will be combined into a single order at stage j , these adjusted costs introduce into the lot-sizing decision at stage 1 the economic effects of that decision on the other components in the product structure. Taking partial derivatives of (A) with respect of K_1, K_2 , and K_3 :

$$n_1 = K_1 = (2S_1 / De_1)^{1/2} \quad (4.26)$$

$$K_i = ((S_i / S_{A(i)}) * h_{A(i)} / h_i)^{1/2}$$

or

$$K_i = ((S_i / S_{A(i)}) * e_{A(i)} / e_i)^{1/2} \quad (4.27)$$

The values of the K_j determined are then used to determine

the set of revised costs to be employed at each stage, which are calculated as follows:

$$\hat{S}_i = S_i + \sum_{j \in B(i)} S_j / K_j \quad (4.28)$$

$$\hat{e}_i = e_i + \sum_{j \in B(i)} e_j K_j \quad (4.29)$$

where $\hat{S}_j$ and $\hat{e}_j$ denote the revised setup and holding costs, respectively, to be employed in the lot-sizing algorithms. The K_j values could be calculated in some several ways as explained in the Blackburn and Millen paper [43].

4.6 Summary

In this chapter, three new heuristics were introduced and analyzed in detail. These procedures will find a feasible schedule (while attempting to minimize the total cost) if one exists, or they will identify where infeasibilities are found. A numerical example has been used to illustrate each technique step by step.

In the following chapter, all these procedures are tested on a variety of test problems and their performances are evaluated.

CHAPTER V

COMPUTATIONAL TESTING

This chapter reports testing of the three heuristics on a number of the literature problems with different experimental conditions. Heuristic 1 and heuristic 2 are compared to heuristic 3 and BM, respectively to discover if and under what parameters they can find lower cost solutions or find comparable solutions faster than the available procedures.

5.1 Statistical Testing on Experimental Results

Several performance measures will be calculated to determine the solution ability of the heuristics being tested and these measures are subjected to ANOVA (Analysis of Variance).

5.1.1 Experimental Design (Conditions)

All the heuristics and degree of difficulty of the problem can be evaluated based upon the following factors. These were used by other researchers as Afentakis [4, 6], Billington [33, 34], Thizy and Van Wassenhove [171], and Blackburn and Millen [39, 42].

- 1- The number of periods $|T|$.
- 2- The structure of an assembly (Medium, Shallow, and

Deep). These structures also range in size from small to large.

3- The average capacity load or utilization:

$$CU = \left[\sum_{i=1}^N \sum_{t=1}^T d_{it} \right] / \sum_{t=1}^T CAP_t \quad (5.1)$$

4- The Ratio (S/H) of the total setup cost to inventory costs.

$$S/H = [2 * S_i / (h_i * d_t)]^{1/2}$$

or

$$S/H = [2 * S_i / (h_i * \bar{d})]^{1/2} \text{ where, } \bar{d} \text{ is the average demand.} \quad (5.2)$$

$$i=1, \dots, N$$

$$t=1, \dots, T$$

The S/H ratios (TBO's) are important in determining the economically optimal frequency of setups. The lower the ratio, the more frequently that production is set up.

5. Coefficient of variation: Fluctuation in demand pattern.

The ratio of standard deviation to the mean = $(\sigma/\bar{d})\%$.

5.1.2 Solution Criteria (Performance Measures)

Heuristic performance was judged using four standard criteria. These are:

1- Cost Ratio is defined as the ratio of the total cost for the best heuristic technique which is used in this paper to the total cost found from the compared heuristic or optimal solution. Cost Ratio values less than 1.00

indicate that the solution in the numerator was better than the compared heuristic in the denominator. This ratio will also provide the percentage that the solution cost is better or worse than the compared procedure.

If a ratio is 1.030, then the solution cost is 3% greater than the solution cost from the compared procedure.

- 2- Number of times that a particular method achieves the lowest cost.
- 3- Number of times that capacity has to be expanded in order to obtain a feasible solution.
- 4- CPU Time: Although computation times were quite short for the heuristics tested, they are included here. Timing begins with reading of the raw data file and ends with the termination of the heuristic procedure.

5.2 ANOVA Statistical Tests

ANOVA will be used to test the null hypothesis that the performance measure means for different methods are equal for a given factor (or the factor has no impact on the algorithm's performance). If the null hypothesis is rejected, the factor has some impact on the performance of the heuristic.

The main benefit of the ANOVA results will be to identify those factors that may effect the performance of the method. SAS V5.18 was used for this testing.

In general, the null and alternative hypotheses are defined as:

H_0 : All μ_{ik} are equal

H_1 : At least 1 μ_{ik} is different (or not all equal)

where $i=i^{\text{th}}$ algorithm, $k=k^{\text{th}}$ experimental design.

5.3 Hypotheses of the Research

The performance of four lot-sizing heuristics (SE, MS, MP, and BM) is examined under different circumstances, including different product structures, capacity utilization, coefficient of variation, and number of periods. The impact of the factors (conditions) on the performance measures used to evaluate the solutions are investigated.

5.4 Research Questions

This research will attempt to answer the following questions:

- 1) Are the solutions for SE and MS be as good as (or better than) solutions found by MP and BM, respectively.
- 2) Will these procedures based on the following circumstances result in better (or worse) solutions?
 - a) Impact of using different product structures (shallow, medium, and deep) on performance of the heuristics.
 - b) Impact of capacity utilization (low, medium, and

- high) on performance of the heuristics.
- c) Impact of demand pattern (CV = low, medium, and high).
 - d) Impact of the problem size (number of periods).
 - e) Impact of the interaction effect between capacity utilization and coefficient of variation.
- 3) How close are the solutions to optimal (if there is one)?
- 4) Are there cases where these heuristics can not find a feasible solution?

Table 5.1 details the list of factors for the six product structures. Each structure has different sets of holding cost, setup cost, coefficient of variation, capacity utilization, and the number of periods . The three heuristics are compared on the basis of cost, cpu time, and the number of times that capacity is increased.

5.5 Product Structures

The product structures considered in this chapter are all assembly structure where each part (or stage) is the input to at most one successor stage (or parent) but can employ the output from multiple predecessor stages (or children). Six different structures ranging from small to large size are exhibited in fig. 1 as examples.

| EXPERIMENTAL DESIGN |

Problem Type		Capacity Utilization	Set-up/ Holding	Coefficient of Variation
Structure	Periods	(CU)	(S/H)	(CV)
SMALL	5	50%, 75%, 95%	1, 12	CV<.20, .20≤CV≤.40 CV>.40
MEDIUM	5	50%, 75%, 95%	1, 12	CV<.20, .20≤CV≤.40 CV>.40
MEDIUM SHALLOW	5	50%, 75%, 95%	1, 12	CV<.20, .20≤CV≤.40 CV>.40
MEDIUM	8	50%, 75%, 95%	1, 12	CV<.20, .20≤CV≤.40 CV>.40
LARGE, 7	12	50%, 75%, 95%	1, 12	CV<.20, .20≤CV≤.40 CV>.40
LARGE, 9	7	50%, 75%, 95%	1, 12	CV<.20, .20≤CV≤.40 CV>.40

TABLE 5.1: LIST OF FACTORS TO BE TESTED.

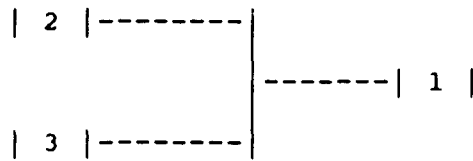


FIGURE 5.1a: STRUCTURE NO.1, SMALL SIZE ASSEMBLY (THREE STAGE)

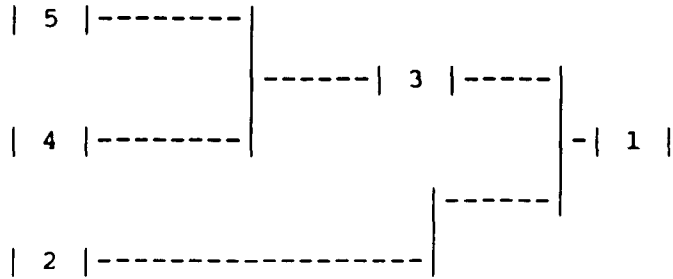


FIGURE 5.1b: STRUCTURE NO. 2, MEDIUM ASSEMBLY (FIVE STAGE)

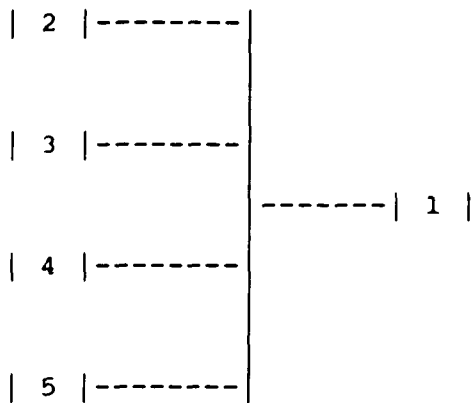


FIGURE 5.1c: STRUCTURE NO. 3, MEDIUM SHALLOW ASSEMBLY (FIVE STAGE)

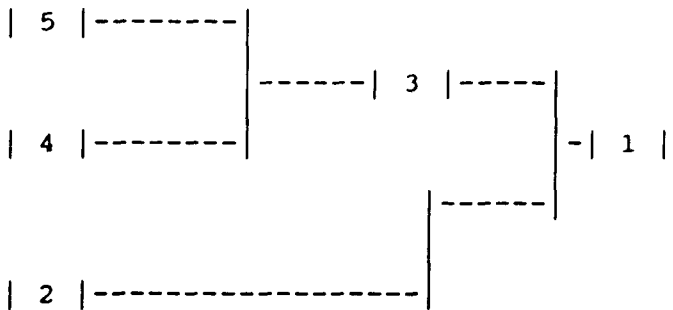


FIGURE 5.1d: STRUCTURE NO. 4, MEDIUM ASSEMBLY (FIVE STAGE, 8 PERIODS)

5.6 Computation of the Lower Bounds

For problems with unknown optimal solutions, lower bounds were used to calculate an upper bound to how far the heuristic solutions are from optimal. Optimal solutions to the problems would be preferred, but the CPU time to find the optimal solution is so large that this would be an extremely time consuming, and impractical, task.

One method for calculating a lower bound is to relax the 0-1 integer constraints on the setup variables of the MIP problem; the optimal LP solution is a lower bound on the original problem. An alternative method is to relax (remove) the capacity constraint. This converts the problem into a set of uncapacitated multi-stage problems, one for each of the end items. These problems can then be solved by an optimal multi-level lot sizing algorithms. Since only the 0-1 integer constraint is relaxed, the LP solution will be feasible with regard to the inventory balance constraints and the capacity constraints. The LP solution is feasible, but the cost will be low since the 0-1 variables forcing the setup costs in the objective function may not be 1 and will not charge the full setup cost.

The quality of the Lower Bound should improve as capacity utilization decreases. As capacity utilization approaches zero, the optimal solution would approach the

lower bound. The ratio of the optimal solution to the lower bound should increase as the capacity utilization increases.

5.7 Computational Testing

A total of 324 problems were used with 6 different product structures. The largest problems contained 9 items and 12 time periods. The capacity utilization (CU) is grouped into low (50%), medium (75%), and high (95%). The coefficient of variation (CV) is also categorized as low ($CV < 0.20$), medium ($0.20 \leq CV \leq 0.40$), and high ($CV > 0.40$). The ratio of setup/holding cost (S/H) is grouped into low (1) and high (12).

There are no other solution procedures available to compare heuristic 1 (SE) and heuristic 3 (MP), since they are per period procedures (capacity is constant in every period), therefore, they will be compared to each other. Heuristic 2 (MS) and Blackburn and Millen (BM) are per stage procedures (capacity is constant in every stage), and they can be compared to each other. All these heuristics are compared to the results obtained from solving the models by mixed-integer programming (MIP, LINDO).

The test problems used in this research were obtained from the randomly generated data sets in literature, Afentakis [6], Florian and Klien [75], and Billington [30].

A Fortran program was used to compare the heuristic performance with the Blackburn and Millen heuristic procedure. The tests were run on a mainframe VAX 8800 system with VMS V5.1 operating system. SAS (V5.18) of ANOVA was used to test the hypotheses.

5.7.1 Mean Costs for Product Structures

The average cost for each heuristic, as applied to the six different product structures, is shown in Table 5.2.

The ratio of average cost for heuristic 2 (MS) to the Blackburn and Millen [43] procedure and also the ratio of two other heuristics are also shown in Table 5.2. As this result indicates, heuristic 1 and heuristic 3 yield almost the same solution. The Blackburn and Millen approach yields a significantly higher cost than heuristic 2. This is at least partly due to the fact that they assume capacity for each stage should be a constant multiplier of an average demand ($D, 2*D, 3*D, 4*D, \dots, k*D$). Furthermore, they use five different cost modification procedures to calculate the order interval; we coded only one of these in our model. The null hypotheses on difference in means for all structures were rejected. For all size problems heuristic 2 performs better than Blackburn and Millen (BM).

Table 5.3 shows the statistical results of the average cost of all structures. As this table indicates, the null hypothesis was rejected in every structure.

5.7.2 Computational CPU Time

Table 5.4 is the average cpu time on all test problems tested for the three heuristics as a group and Blackburn and Millen individually. Table 5.5 indicates all the heuristic's cpu time individually. It should be mentioned that the cpu time in Table 5.5 is the the average cpu time of running only 30 test problems in each structure.

Impact of the problem size (number of stages) on cpu time was tested for all heuristics. As shown in Table 5.5, the cpu time for heuristic 1 increases as the number of stages are increased. Table 5.6 shows the statistical results for cpu time in every structure, the null hypotheses for structures 5 and 6 (large) were rejected, which shows a higher number of stages has an impact on cpu time (only on heuristic 1).

Table 5.7 illustrates the overall impact of all structures and also the impact of setup, demand, (number of stages)*(setup), and (demand)*(setup).

Tables A.20 through A.25 in Appendix C illustrates the results of statistical testing of cpu time for each individual structure.

PRODUCT STRUCTURE	# OF TEST Problems	SE	MS	MP	BM	MS/BM	MP/SE
1	48	158	157	152	169	.9289	.9620
2	72	4495	5590	4412	6347	.8807	.9815
3	48	257	249	250	282	.8829	.9727
4	54	7488	9025	7363	10266	.8791	.9833
5	54	11676	11720	11659	13690	.8560	.9985
6	48	4485	4561	4526	4909	.9291	1.0084

TABLE 5.2: MEAN COSTS AND COST RATIOS FOR ALL HEURISTICS.

STRUCTURE	SE - MP		MS - BM		RESULTS
	F VALUE	PR > F	F VALUE	PR > F	
(1)	3.62	0.0021	33.16	0.0001	REJECT
(2)	9.94	0.0001	33.45	0.0001	REJECT
(3)	35.19	0.0001	16.30	0.0001	REJECT
(4)	37.10	0.0001	29.54	0.0001	REJECT
(5)	61.55	0.0001	67.73	0.0001	REJECT
(6)	5.61	0.0007	3.68	0.0012	REJECT

TABLE 5.3: STATISTICAL RESULTS OF AVERAGE COST
FOR ALL STRUCTURES.

PRODUCT STRUCTURE	SE-MP	BM
1	2.879	2.320
2	3.162	2.331
3	2.996	2.352
4	3.929	2.340
5	68.205	2.370
6	81.324	2.480

TABLE 5.4: COMPARISON OF CPU TIME BETWEEN HEURISTICS.

PRODUCT STRUCTURE	SE	MS	MP	BM
1	2.31	2.44	2.51	2.33
2	2.40	2.41	2.44	2.32
3	2.41	2.36	2.35	2.38
4	2.38	2.49	2.43	2.36
5	58.11	2.51	2.55	2.41
6	65.19	2.46	2.53	2.40

TABLE 5.5: CPU TIME FOR EACH STRUCTURE

STRUCTURE	SE - MP		MS - BM		RESULTS
	F VALUE	PR > F	F VALUE	PR > F	
(1)	0.56	0.7826	0.89	0.5254	ACCEPT
(2)	2.61	0.0197	1.12	0.3638	ACCEPT
(3)	0.60	0.7538	0.46	0.8596	ACCEPT
(4)	1.81	0.1292	2.61	0.0365	ACCEPT
(5)	11.23	0.0001			REJECT
(5)			0.09	0.4928	ACCEPT
(6)	9.06	0.0001			REJECT
(6)			0.95	0.4564	ACCEPT

TABLE 5.6: STATISTICAL RESULTS OF CPU TIME FOR EACH STRUCTURE.

STRUCTURE	SE - MP		MS - BM		RESULTS
	F VALUE	PR > F	F VALUE	PR > F	
(1) - (6)	7.80	0.0001			REJECT
(1) - (6)			1.30	0.1125	ACCEPT
N*S	6.13	0.0005			REJECT
S	3.58	0.0595	4.00	0.0464	ACCEPT
D	0.23	0.8736	0.37	0.7729	ACCEPT
D*S	0.32	0.8098	0.57	0.6326	ACCEPT

TABLE 5.7: STATISTICAL RESULTS OF CPU TIME FOR ALL STRUCTURES.

5.7.3 Computational Difficulty

The results from testing 324 problems indicated that when:

- 1) the coefficient of variation is very high and capacity is tight in every period, a feasible solution is not found in 36 cases (11%), simply because there were high demands in early periods of the test problems. To avoid this problem, capacity should be increased.
- 2) as the number of stages are increased, the number of schedules for each stage also increased dramatically for heuristic 1 and it takes a substantial computer time to select the best schedules for the problem.

5.7.4 Lower Bound Results

216 problems were solved using LINDO (Mixed Integer Program) for the purpose of determining a lower bound for the modified per period and the modified per stage approaches. Overall, the lower bounds were better than cost obtained by our heuristic by an average of 20%, but computer CPU times for the integer programs were very high. CPU times were steadily increasing as the number of periods and stages increased. The major factor contributing to the increased cpu time were the number of constraints and the number of integer variables (0-1 setup variables). For instance, for structure 5 with 7 stages

and 12 time periods, there were 86 0-1 integer variables, 146 non-integer variables, and 260 constraints. For this problem the average cpu time was 43 minutes and the solution was found after 7922 iteration.

When capacity utilization is very high, there must be production in every period, since available slack is insufficient to permit even one period without production. In such cases, the 0-1 relaxed LP solution is nearly a lot-for-lot solution.

Two problem structures were used with LINDO. One assumes a fixed capacity for every period (heuristic 1 and heuristic 3) and the other assumes a fixed capacity per stage (heuristic 2 and BM). The Mixed Integer Program (MIP) results for LINDO are tabulated in Table 5.8 for every product structure.

Table 5.9 illustrates the ratio of average cost for every heuristic to the lower bound values. For SE, this ratio increases as the number of stages increases (product structures 4, 5, and 6), and for MS and BM, the ratio increases for product structure 5 and 6. Table 5.10 shows the ratio of average cost for each heuristic to lower bound in each CV category.

PRODUCT STRUCTURE	# OF TEST PROBLEMS	PER PERIOD FORMULATION	PER STAGE FORMULATION
1	30	141	146
2	54	3223	4435
3	36	221	240
4	36	5369	6321
5	36	8269	8729
6	24	3610	3617

TABLE 5.8: AVERAGE COST (LOWER BOUND) FOR EVERY
STRUCTURE

PRODUCT STRUCTURE	TEST PROBLEMS	SE/LB	MS, LB	MP/LB	BM/LB
1	30	1.14510	1.11640	1.07090	1.15750
2	54	1.17289	1.17650	1.14920	1.29020
3	36	1.17660	1.15250	1.11764	1.28520
4	36	1.25030	1.22010	1.19519	1.31580
5	36	1.33000	1.26070	1.28030	1.47221
6	24	1.27940	1.31010	1.23170	1.58103

TABLE 5.9: THE RATIO OF (AVERAGE COST FOR EACH HEURISTIC/LB)

HEURISTIC / LB	SMALL CV	MEDIUM CV	HIGH CV
SE/LB	1.1416	1.1731	1.2957
MS/LB	1.0912	1.1324	1.2457
MP/LB	1.1122	1.1551	1.2820
BM/LB	1.1622	1.2651	1.4129

TABLE 5.10: THE RATIO OF (AVERAGE COST FOR EACH HEURISTIC / LB) FOR DIFFERENT VALUES OF CV.

5.7.5 Comparison of the Heuristics: (FACTOR = CU)

Tables 5.11 and 5.12 illustrate the cost ratio performance (MP/SE, MS/BM) for all the heuristics categorized by structure and capacity utilization. The capacity utilization (CU) factor has a strong effect on the performance of the heuristic. As CU decreases, the average cost decreases. Almost in all cases MP performs better than SE particularly in tight CU. MS is also performing better than BM in all cases particularly when CU is low. Table A.4 in Appendix C shows the ratio of average cost to the lowest cost obtained by the heuristic at every level of this factor (CU). The lowest cost is found in a loose capacity for every structure.

5.7.6 Comparison of the Heuristics: (FACTOR = CV)

At a 1% level of significance, the null hypotheses that the demand structure (high, medium, and high CV) has no impact on the performance of these heuristics or for different CV values, the ratio of (SE/MP)=1 and (MS/BM)=1 were rejected. This indicates that MP and MS are superior to SE and BM, respectively. Tables 5.13 and 5.14 illustrate the performance ratio of MP/SE and MS/BM, respectively. As this table indicates, when the coefficient of variation (CV) is high, both MP and MS perform very well relative to SE and BM, respectively. For large size problems, SE is better than MP in low and

medium CV. Table A.1 and A.5 show the average cost and the ratio of average cost to the minimum cost obtained by each heuristic in every category of CV, respectively.

5.7.7 Comparison of the Heuristics: (FACTOR = T)

Table 5.15 illustrates the average cost for each heuristic and cost ratios of MP/SE and MS/BM. As this table indicates, in most cases, MP and MS are superior to SE and BM, respectively. Furthermore, the null hypothesis that this factor has some impact over the heuristics was accepted, but the null hypothesis for the D*T interaction was rejected as is shown in Table 5.23.

5.7.8 Comparison of the Heuristics: (FACTOR = H)

The null hypothesis that the echelon holding cost has some impact on the solution performance of each heuristic was accepted. Table 5.23 indicates the result of the statistical testing for this factor. Tables A.32 and A.33 in Appendix C illustrate the comparison ratios and the lowest cost ratios, respectively.

5.7.9 Comparison of the Heuristics: (FACTOR = CV*CU) or Interaction of Demand and Capacity Utilization

Tables 5.16 and 5.17 show the results of the interaction between capacity utilization and coefficient of variation (CU*CV). As these tables indicate in almost all

cases MP performs better than SE except in product structure 5 when the number of periods are increased, MS is also superior to BM. In structure 5, MS is much better than BM in high CU and low CV. MP is superior to SE in high CU and all values of CV. As is shown in Table 5.23, the null hypothesis for this interaction was rejected

Tables A.2 and A.3 in Appendix C illustrate the ratio of average cost at each level of this interaction to the minimum cost achieved by the heuristic. As this table indicates, the minimum cost is obtained when capacity is very loose.

PRODUCT STRUCTURE	CU		
	H	M	L
1	***	0.9622	0.9615
2	0.9703	0.9899	0.9909
3	***	0.9543	0.9720
4	0.9684	0.9918	0.9961
5	0.9490	0.9887	0.9911
6	0.9989	0.9853	0.9876

TABLE 5.11: THE COST RATIOS (MP/SE) FOR ALL STRUCTURES
AT DIFFERENT CAPACITY UTILIZATION (CU).

PRODUCT STRUCTURE	CU		
	H	M	L
1	***	0.9705	0.9352
2	0.9757	0.8992	0.8718
3	***	0.9734	0.9184
4	0.9531	0.8841	0.8966
5	0.8449	0.7951	0.7751
6	0.8832	0.8995	0.9598

TABLE 5.12: THE COST RATIOS (MS/BM) FOR ALL STRUCTURES
AT DIFFERENT CAPACITY UTILIZATION (CU).

*** : solution could not be found due to the tight capacity

PRODUCT STRUCTURE	CV		
	H	M	L
1	0.9617	0.9493	0.9806
2	0.9819	0.9825	0.9877
3	0.9248	0.9960	0.9880
4	0.9816	0.9853	0.9828
5	0.9804	0.9780	1.1092
6	0.9721	1.0620	0.9781

TABLE 5.13: THE COST RATIOS (MP/SE) FOR ALL STRUCTURES
FOR DIFFERENT LEVEL OF (CV).

PRODUCT STRUCTURE	CV		
	H	M	L
1	0.8882	0.9529	0.9467
2	0.8544	0.8784	0.8616
3	0.8333	0.9397	0.8829
4	0.8505	0.9469	0.8385
5	0.8661	0.8677	0.8391
6	0.8513	0.8775	0.9003

TABLE 5.14: THE COST RATIOS (MS/BM) FOR ALL STRUCTURES
FOR DIFFERENT LEVEL OF (CV).

# OF PERIODS	SE	MS	MP	BM	MP/SE	MS/BM
5	2477	2512	2006	2849	0.8090	0.8817
8	7488	9025	7363	10266	0.9833	0.8791
9	4488	4561	4526	4909	1.0081	0.9291
12	11676	11720	11659	13690	0.9980	0.8560

TABLE 5.15: MEAN COSTS AND COST RATIOS FOR ALL HEURISTICS FOR T FACTOR.

FACTOR (STRUCTURE)		CU		
		H	M	L
(1) CV	H	***	0.9570	0.9802
	M	***	0.9605	0.9386
	L	***	0.9620	0.9867
(2) CV	H	0.9756	0.9897	0.9891
	M	0.9746	0.9874	0.9885
	L	0.9787	0.9930	0.9838
(3) CV	H	***	0.9330	0.9657
	M	***	0.9651	0.9754
	L	***	0.9606	0.9635
(4) CV	H	0.9674	0.9903	0.9912
	M	0.9664	0.9927	0.9911
	L	0.9716	0.9918	0.9916
(5) CV	H	0.7403	1.0987	1.1320
	M	0.6539	1.2222	1.1609
	L	0.9721	1.3236	0.9959
(6) CV	H	0.9369	0.9861	0.9790
	M	0.9834	0.9932	0.9913
	L	0.9558	0.9780	0.9937

TABLE 5.16: CU*CV EFFECT FOR (MP/SE) FOR STRUCTURES 1-6.

FACTOR (STRUCTURE)		CU		
		H	M	L
(1) CV	H	0.8529	0.9235	0.9001
	M	0.9000	1.0000	0.9473
	L	0.8764	1.0000	0.9640
(2) CV	H	0.9825	0.8418	0.8013
	M	0.9179	0.8917	0.8017
	L	1.0000	0.9294	0.7146
(3) CV	H	0.7090	0.9184	0.8794
	M	0.8014	1.0000	0.8546
	L	0.8220	1.0000	1.0000
(4) CV	H	0.9174	0.8612	0.7784
	M	0.9721	0.9612	0.9063
	L	0.9698	0.8300	0.6851
(5) CV	H	0.9677	0.8785	0.7347
	M	0.9597	0.8020	0.8191
	L	1.0040	0.6811	0.7738
(6) CV	H	0.8858	0.8994	0.8625
	M	0.8622	0.8793	0.8946
	L	1.0350	0.8431	0.9184

TABLE 5.17: CU*CV EFFECT FOR (MS/BM) FOR STRUCTURES 1-6.

5.7.10 Comparison of the Heuristics: (FACTOR = D*T) or

Interaction of Demand and Number of Periods

The null hypotheses for the ratio of $(MP/SE)=1$ and $(MS/BM)=1$ for D*T effect were rejected which indicates that this interaction has some impact on the solution ability of the heuristics. As table 5.18 indicates when the number of periods increases, SE performs better than MP (structures 5 , where Ratio > 1).

5.7.11 Comparison of the Heuristics: (FACTOR = D*S) or

Interaction of Demand and Setup

The null hypotheses for the ratio of $(MP/SE)=1$ and $(MS/BM)=1$ for D*S effect were accepted which indicates that this interaction does not effect the relative solution ability of the heuristics (table 5.19). The results shown in Table 5.19 are similiar to those in Table 5.18. Tables A.6 and A.7 in Appendic C illustrate the ratio of average cost to the minimum cost computed by each heuristic for structures 1 through 3 and structures 4 through 6, respectively. As tables A.6 and A.7 indicate, the minimum cost is achieved for a low CV.

5.7.12 Comparison of the Heuristics: (FACTOR = D*CAP,

D*CAPU) or Interaction of Demand and Capacity

The null hypotheses for the ratio of $(MP/SE)=1$ and $(MS/BM)=1$ for D*CAP and D*CAPU interactions, respectively,

were rejected. As these effects suggest in Table 5.20, for all interaction of capacity per stage (CAPU) and demand pattern (CV) in all structures, MS is better than BM. Table 5.20 also suggests that for the fixed capacity per period (CAP) and demand pattern (CV) interaction, MP is superior to SE.

5.7.13 Comparison of the Heuristics: (FACTOR = S*H) or Interaction of Setup and Holding Cost

Table 5.21 shows the result for the interaction between setup and echelon holding cost. This shows that as the structure gets deeper (structure 4, 5, and 6) MS shows improvement relative to BM. The null hypothesis for this interaction was rejected as is shown in Table 5.23.

5.7.14 Comparison of the Heuristics: (FACTOR = N*T)

Table 5.22 illustrates the average cost for all heuristics and their ratios for the interaction between the number of stages and number of periods (size). As we can see in this table, SE outperforms H3 for large size product structures.

5.7.15 BEST Ratios

Tables 5.24 and 5.25 show the percentage of trials that the MP and MS achieved the minimum cost (BEST), respectively.

5.8 Summary

In this chapter, we discussed the procedures for evaluating the heuristics, the evaluation measures, the experimental design, and the statistical tests. The chapter also discussed the hypotheses of the research and the statistical tests used in this study. The results of the evaluation are also presented in this chapter. These results, in summary, indicate that the two heuristics MP and MS perform better than SE and BM, respectively.

Table 5.23 illustrates the statistical summary of all factors and their interactions.

In Appendix C, the detailed analysis of all the different interactions and their impact on performance of each heuristic are illustrated in tables A.1 - A.37. In summary, almost the majority of these tables support the finding for this chapter.

Chapter VI concludes the summary for this research and recommends future research directions.

PRODUCT STRUCTURE	MP/SE	MS/BM
1	.9671	.9291
2	.9813	.8813
3	.9742	.8836
4	.9842	.8788
5	1.0164	.8556
6	.9985	.9236

TABLE 5.18: RATIO OF MP/SE AND MS/BM FOR D*T EFFECT FOR DIFFERENT STRUCTURES.

PRODUCT STRUCTURE	MP/SE	MS/BM
1	.9724	.9183
2	.9816	.8814
3	.9701	.8831
4	.9764	.8476
5	.9854	.8652
6	.9978	.9150

TABLE 5.19: RATIO OF MP/SE AND MS/BM FOR D*S EFFECT FOR DIFFERENT STRUCTURES.

PRODUCT STRUCTURE	MP/SE	MS/BM
1	.9921	.9277
2	.9820	.8817
3	.9764	.8836
4	.9623	.8703
5	1.0120	.8540
6	.9910	.9295

TABLE 5.20: COST RATIO OF MP/SE AND MS/BM FOR CU*CV
INTERACTION IN DIFFERENT PRODUCT STRUCTURES.

PRODUCT STRUCTURE	MP/SE	MS/BM
1	.9919	.9292
2	.9820	.8792
3	.9937	.8823
4	.9833	.8791
5	1.0121	.8501
6	.9978	.8670

TABLE 5.21: COST RATIO OF MP/SE AND MS/BM FOR S*H
INTERACTION FOR DIFFERENT PRODUCT STRUCTURES.

(STRUCTUR * T)	SE	MS	MP	BM	MP/SE	MS/BM
1, 5	158	157	152	169	0.9620	0.9289
2, 5	4495	5590	4412	6347	0.9815	0.8807
3, 5	257	249	250	282	0.9727	0.8829
4, 8	7488	9025	7363	10266	0.9833	0.8791
5, 9	11676	11720	11659	13690	0.9980	0.8560
6, 12	4488	4561	4526	4909	1.0084	0.9291

TABLE 5.22: MEAN COSTS AND COST RATIOS FOR ALL HEURISTICS
FOR T*N INTERACTION EFFECT.

STRUCTURE	SE - MP		MS - BM		RESULTS
	F VALUE	PR > F	F VALUE	PR > F	
T	2.00	0.1149	0.93	0.4271	ACCEPT
T*D	4.09	0.0006	5.24	0.0013	REJECT
CAP	5.22	0.0001	4.31	0.0001	REJECT
H	0.62	0.5388	0.41	0.6667	ACCEPT
D*CAP	6.25	0.0001	14.23	0.0001	REJECT
D	5.63	0.0001	4.52	0.0001	REJECT
N*D	0.16	0.9859	0.22	0.9690	ACCEPT
S	13.57	0.0001	24.52	0.0001	REJECT
N*S	37.27	0.0001	16.33	0.0001	REJECT
D*S	0.35	0.7882	0.73	0.5346	ACCEPT
S*CAP	8.74	0.0001	13.82	0.0001	REJECT
S*H	1.94	0.1321			ACCEPT
S*H			5.82	0.0033	REJECT
T*D*S	0.92	0.4779	1.00	0.4266	ACCEPT
N*D*S	0.01	1.0000	0.05	0.9995	ACCEPT

TABLE 5.23: STATISTICAL RESULTS OF AVERAGE COST FOR DIFFERENT EFFECTS.

EFFECT	% MP BEST
STRUCTURE	0.8651
CAP	0.8821
D	0.9145
S	0.8752
CAP*STRUCTURE	0.9662
S*STRUCTURE	0.8642
D*S*STRUCTURE	0.9567
CAP*S*STRUCTURE	0.9475
S*H	0.9136
S*H*D	0.8272
H*STRUCTURE	0.8150

TABLE 5.24: % OF TRIALS THAT MP PERFORMS BETTER THAN SE FOR EACH EFFECT.

EFFECT	% MS BEST
CAP	0.9691
CAP*S	0.9567
D*STRUCTURE	0.9444
D	0.8823
CAP*STRUCTURE	0.9567
D*CAP*STRUCTURE	0.9598
D*S*STRUCTURE	0.9322
STRUCTURE	0.9629
S*H	0.9087
S*H*D	0.8754
H*STRUCTURE	0.8612

TABLE 5.25: % OF TRIALS THAT MS PERFORMS BETTER THAN
BM FOR EACH EFFECT.

CHAPTER VI

CONCLUSIONS AND FUTURE RESEARCH DIRECTION

6.1 SUMMARY

This research, has studied the multi-level capacitated lot-sizing problem for assembly structure manufacturing. In a manufacturing setting an assembly product structure is one in which parts, subassemblies, and raw materials (items) combine with other parts, subassemblies, and raw materials to eventually comprise a finished product (end item). An assembly structure for a single end item is one in which each item can have multiple predecessors but at most one successor. This study does not consider situations in which a part can be a predecessor for more than one item.

A considerable amount of research on the uncapacitated lot-sizing problem has resulted in many optimal algorithms and efficient heuristics. The problem becomes very complicated when capacity is included, little research has concentrated on the problem with capacity in every period or in every stage of production.

The multi-stage capacitated lot-sizing problem was formulated as a mixed-integer linear programming model. This problem is known to be NP-hard so that the problem is unlikely to be solvable in polynomial time with respect to the dimensionality of the problem. The problem is hard in

a practical sense, as well, since methods that provide optimal solutions are unable to do so for all but very small problems within reasonable computation times. The computational complexity of those problems has directed the research toward developing heuristic solution procedures.

Three new heuristics have been introduced. The first of these is called selective enumeration and is based upon modification of the multi-stage uncapacitated procedure of Afentakis [6]. The second heuristic is a modification of the single-stage capacitated procedure of Florian and Klien [75], and the third one is a modification of the multi-stage uncapacitated procedure using shifting techniques to obtain a feasible solution. The performance of the first and second heuristics was compared with the performance of the third and the Blackburn, Millen procedures, respectively.

The effect of the following factors on the relative performance of the selected heuristic were studied in this research:

- 1- The effect of product structure.
- 2- The effect of the number of periods.
- 3- The effect of the demand pattern or coefficient of variation.
- 4- The effect of the capacity utilization.
- 5- The effect of the ratio (setup cost/holding cost).

All these factors were examined and analyzed using different evaluation measures. A program written in Fortran was used to solve all the test problem on VAX 8800, VMS (V5.1) operating system.

6.1.1 Summary of the Results

In this research 324 problems were selected from the literature and used for testing the three heuristics. On average, MP yields a lower cost compared to SE, and also MS is superior to BM.

All these heuristics were also compared to LB results which were obtained using LINDO (MIP section) on a relaxed problem. On average, the ratio of (Heuristic/LB) is 1.1123 (heuristics are 11.23% worse than lower bound). The disadvantage of running LP problems is computer time and it increases dramatically as the number of 0-1 integer variables increases. The average time of our heuristic was 3.41 seconds compared to LP CPU time of 972 seconds on a VAX-VMS 8800 system. The result is consistent with the other studies such as Veinott and Crowston [175], Karni [110], Mclauren's integer code [142], and Thizy & Wassenhove [171]. They also experiment long CPU time as the number of stages is increased.

The null hypotheses in most cases were rejected meaning factors setup/holding cost (S/H), coefficient of variation (C''), and capacity utilization (CU) and some of

the interactions have effects on the solution ability of these heuristics. Both MS and BM performed poorly compared to the lower bound as the number of stages increased or when the structure gets deeper like product structures 5 and 6. The ratio of the (average cost/LB) is increased as N increases.

The null hypothesis that SE and MP perform the same for each level of the coefficient of variation (CV) was rejected which indicates that the CV is significant. For $CV > 0.40$, both MP and MS outperform SE and BM, respectively, but for $CV < 0.20$, SE is better than MP.

With respect to capacity utilization (CU), the null hypothesis that two heuristics (SE and MP) are the same for different CU was also rejected, which suggests that with high CU, the average cost for MP is smaller than SE due to tight capacity (lots cannot be split among different periods), while the cost for H3 was equal to SE for loose capacity situation ($CU=50\%$). The null hypothesis that two heuristics (MS and BM) are the same for different CU and CV was rejected. This suggests that for high CU and high CV, MS outperforms BM.

The null hypothesis for the CPU time for MS & BM was accepted which suggest in all cases that the average CPU time was the same except for SE and was rejected for SE and MP in structures 5 and 6.

The null hypotheses that the mean ratios of

(SE/LB, MS/LB, MP/LB, and BM/LB) are equal for factors CV, CU, S/H, T*CV, and CV*CU were rejected. All these factors individually and some of their interactions affect the cost ratio. Factors T, H have no effect on solution performance of the heuristics, but CV, CU, and structure of problem have some impact. S/H has no impact on SE-MP, but has some impacts on MS-BM performances.

When capacity utilization is high (95%), lot-for-lot is the only solution that will allow a feasible schedule at the work center. As capacity utilization decreases, more batching is allowed in each period, costs are reduced and the ratios improved. When utilization is low, there is more opportunity for batching, and the problem is easier to solve (it becomes close to the uncapacitated problem). When utilization is high, the possible schedules are limited since setups may have to occur in almost every period.

A complete factorial design on all factors that could influence the heuristic's solution ability would be impractical due to the number of trials that would be required.

The results indicated by ANOVA suggest that each factor has a significant effect. When the coefficient of variation (CV) is high, both heuristics MS and BM are considerably greater than the lower bound. It is not clear whether the lower bounds are far from optimal or the

heuristics are not well designed for cases with a high coefficient of variation.

Heuristic 1 can easily be modified to find the best solution in the multi-level lot-sizing (MLLS) category with no capacity limitation. When CV was high and CU was tight, infeasibility occurred in early periods which could only be eliminated by splitting a lot among two periods or increasing capacity.

6.2 Contributions of the Research

This research extends the lot-sizing problem in two directions: (1) it includes capacity per period and per stage of the production; (2) relatively simple heuristics were introduced that can be easily modified to other types of manufacturing structures (serial, general).

6.3 Future Research Directions

More extensive testing of this problem could be undertaken. The following items are suggested as avenues of potential expansion:

1. The program could be modified to work with serial structure and complex general structure. Research in capacitated complex structures is absent. Since the results of this research are promising, the next step is to test SE, MS, and MP on other types of manufacturing structures.

2. Consider heuristics based on Set Partitioning algorithms. A different approach in which the candidate production plans are generated by some well-known multi item capacitated lot-sizing heuristics. The LP relaxation of the set partitioning model is solved and rounded to obtain an integer solution. A number of heuristics are subsequently applied to improve upon this initial solution.
3. Setup time could be included in the model. Setup time in a manufacturing process is an important and realistic assumption, there has been only one research paper that has dealt with setup time and that is in the serial structure not in the assembly.
4. Lagrangian Relaxation (LR) and Duality Theory could be used for calculating lower bounds. Afentakis et al [4] and Billington [33] have found good lower bounds using the LR ability for capacitated serial and uncapacitated assembly structures, respectively.
5. More than one external item could be considered, since most companies fit in this category of production system. This research assumes only one end item and the heuristic performance is superior to others (BM).
6. The transportation approach could be applied as a capacity per period heuristic where setup cost is allocated approximately to each cost cell. It is a relatively simple

approach and was used by Thizy and Wassenhove [171] for the single-level multi-item structure without setup cost.

7. The Afentakis multi-level assembly structure [4] can be modified to treat capacitated work centers. Since their approach is superior for the uncapacitated version of the problem this would seem to be a natural extension.

8. Large scale problems are closer to realistic manufacturing settings and need to be studied.

APPENDIX A-1
EXPANSION OF THE FORMULATIONS

APPENDIX A-1:

$$E_{i0} + P_{i1} - E_{i1} = D_{i1} \quad \text{or} \quad E_{i1} = E_{i0} + P_{i1} - D_{i1}$$

$$E_{i1} + P_{i2} - E_{i2} = D_{i2} \quad \text{or} \quad E_{i2} = E_{i1} + P_{i2} - D_{i2}$$

$$E_{i2} + P_{i3} - E_{i3} = D_{i3} \quad \text{or} \quad E_{i3} = E_{i2} + P_{i3} - D_{i3}$$

$$E_{i3} + P_{i4} - E_{i4} = D_{i4} \quad \text{or} \quad E_{i4} = E_{i3} + P_{i4} - D_{i4}$$

$$E_{i1} = P_{i1} - D_{i1}$$

$$E_{i2} = P_{i1} - D_{i1} + P_{i2} - D_{i2} = \sum_{r=1}^{t=2} (P_{ir} - D_{ir})$$

$$E_{i3} = P_{i1} - D_{i1} + P_{i2} - D_{i2} + P_{i3} - D_{i3} = \sum_{r=1}^{t=3} (P_{ir} - D_{ir})$$

$$E_{i4} = P_{i1} - D_{i1} + P_{i2} - D_{i2} + P_{i3} - D_{i3} + P_{i4} - D_{i4} = \sum_{r=1}^{t=4} (P_{ir} - D_{ir})$$

$$(4) \text{-----} (2) \text{-----} (1)$$

$$E_{4t} = I_{4t} + (X_{42} \cdot I_{2t} + X_{41} \cdot I_{1t})$$

$$E_{2t} = I_{2t} + X_{21} \cdot I_{1t}$$

$$E_{1t} = I_{1t}$$

$$\begin{aligned} I_{4t} &= E_{4t} - X_{42}(E_{2t} - X_{21} \cdot E_{1t}) - X_{41} \cdot E_{1t} \\ &= E_{4t} - X_{42} \cdot E_{2t} + X_{42} \cdot X_{21} \cdot E_{1t} - X_{41} \cdot E_{1t} \\ &= E_{4t} - X_{42} \cdot E_{2t} \\ &= E_{4t} - q_{42} \cdot E_{2t} \end{aligned}$$

APPENDIX A-2
SCHEDULE SELECTION PROCEDURE

Appendix A-2:

The problem of selecting the best schedules can be described as follows:

For each stage i , its successor s and candidate schedule $W(i,k)$, define the following set:

$G_S(i,k) = \{ (s,j) / s \in B(i) \text{ and the pair } W(i,k), W(s,j) \text{ satisfying the system condition, } j = 1, \dots, N(B(i)) \}$.

$G_S(i,k)$ contains the labels from among the candidate schedules of the predecessor stage s which are compatible with schedule $W(i,k)$.

The problem of selecting the best schedules can be stated as:

PROBLEM A-D

$$\begin{aligned} \text{MIN} \quad & \sum_{i=1}^N \sum_{k=1}^{N(i)} W^*(i,k) * \delta(P_{ik}) \\ \text{ST:} \quad & \end{aligned} \tag{1}$$

$$\begin{aligned} & \sum_{K=1}^{N(i)} \delta(P_{ik}) = 1 \\ & i=1, \dots, N \end{aligned} \tag{2}$$

$$\begin{aligned} \delta(P_{ik}) \leq & \sum_{(s,j) \in G_S(i,k)} \delta(P_{sj}) \\ & k=1, \dots, N(i) \\ & i=1, \dots, N \\ & j=1, \dots, N(B(i)) \end{aligned} \tag{3}$$

$$\sum_{i=1}^N f_{ikt} * \delta(P_{ik}) \leq CAP$$

$$\begin{matrix} t=1, \dots, t+1 \\ k=1, \dots, N(i) \end{matrix}$$

(4)

$$\delta(P_{ik}) = \begin{cases} 1 & \text{when schedule } k \text{ for item } i \text{ is chosen.} \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{matrix} k=1, \dots, N(i) \\ i=1, \dots, N \end{matrix}$$

(5)

where:

f_{ikt} = capacity needed for the selected schedule $W(i,k)$ in period t .

(1) represent total costs for all stages.

(2) requires exactly one schedule be selected for each stage.

(3) the selected schedules are feasible for the entire system.

(4) the capacity needed for generated schedules at each period.

Problem A-D can be solved by following step 4a through 4e.

Step 4a- Set $\alpha = N + 1$

Step 4b- Set $\alpha = \alpha - 1$, $k = 1$, $i = A(\alpha)$. If $\alpha = 0$: Stop
where $A(\alpha)$ is the immediate successor of α .

Step 4c- If $\sum_{i=1}^N f_{ikt} \geq CAP$, $k = k + 1$. Otherwise go to 4d.

$$\begin{matrix} t=1, \dots, t+1 \\ k=1, \dots, N(i) \end{matrix}$$

Step 4d- Set $W(i,k) = W(i,k) + \min_j \{W(\alpha, j) / (\alpha, j) \in G_\alpha(i,k)\}$ $j \in B(i)$ $j=1, \dots, N(j)$

Step 4e- If $k < N(i)$ go to Step 4c, otherwise go to Step 4d.

APPENDIX B
TEST PROBLEMS

TEST DATA:

D1=8,15,4,8,12
D2=8,4,8,15,12
D3=6,9,8,12,5
D4=6,10,12,11,4

S1=10,20,8
S2=10,15,5

CAP_i=39/13,48/16,30/10

H1=1,1,1
H2=2,1,1

D1=86,100,87,105,98
D2=80,100,125,100,150
D3=50,80,180,80,93
D4=92,92,92,92,92

S1=27,17,134,294,139
S2=199,55,178,497,1069

CAP=800/150,900/200,560/112

H1=0.1,0.5,1.0,0.5,1.0
H2=0.15,0.46,0.26,0.14,0.09
H3=0.10,0.50,0.50,2.0,1.0

D1=8,15,4,8,12
D2=8,4,15,12
D3=6,9,8,12,5
D4=6,10,12,11,4

S1=10,20,8,10,15
S2=10,15,5,8,12

CAP=65/13,75/15,50/10

H1=1,1,1,1,1

H2=2,1,1,1,1

D1=50,80,180,80,93,93,100,150
D2=86,100,87,105,98,86,83,79
D3=80,100,125,100,150,150,100,125

S1=27,17,134,294,139
S2=199,55,178,497,1069

CAP=800/150,900/200,600/120

H1=0.1,0.5,1.0,0.5,1.0
H2=0.15,0.46,0.26,0.14,0.09
H3=0.10,0.50,0.50,2.0,1.0

D1=75,82,73,124,93,83,111,81,69,121,101,97
D2=89,68,116,62,132,114,131,135,59,68,55,100
D3=96,144,89,151,98,31,67,139,61,115,83,72

H1=0.15,0.46,0.26,0.21,0.18,0.33,0.25
H2=0.10,0.50,0.50,2.0,1.0,0.3,0.6
H3=0.1,0.4,0.3,0.5,0.5,0.2,0.8

S1=55,199,115,497,178,260,185
S2=27,17,134,294,139,247,168

CAP=1100/250,900/200,700/100

D1=75,75,82,73,124,93,81,69
D2=10,10,15,20,70,180,250
D3=50,80,180,80,93,83,100

CAP=1200/250,1300/270,1400/300,900/150

S1=54,134,39,145,139,88,168,189,150
S2=58,71,62,111,180,101,28,46,189

H1=0.1,0.5,0.5,0.15,0.8,1.0,0.18,0.20,0.36
H2=0.15,0.46,0.26,0.14,0.09,0.17,0.09,0.11,0.10

Capacity Levels: 30, 39, 48, 50, 65, 75, 560, 600, 700,
800, 900,1100, 1200, 1300, 1400
[10, 13, 15, 16, 100, 112, 120, 150, 200, 250, 270, 300]

McKareen and Whybark [143] used the following sets.

S _i diff. sets						
i	e _i	1	2	3	4	5
1	.15	199	55	178	497	1069
2	.46	111	180	207	575	598
3	.26	52	195	101	418	272
4	.14	58	62	71	358	310
5	.09	27	17	134	294	139

Carlson et al. [47] used four demand streams. These four sets have an average demand of 92 each period. TBO's ranging from 2.03 to 12.4.

Demand Set												Period				$\bar{d}$	σ	CV
1	10	10	15	20	70	180	250	270	230	40	0	10	92.1	102	1.113			
2	50	80	180	80	0	0	100	150	10	180	180	95	92	66	0.718			
3	80	100	125	100	150	150	100	125	125	100	50	100	92	27.2	0.252			
4	92	92	92	92	92	92	92	92	92	92	92	92	92	0.0	0			

Blackburn and Millen [42] and Graves [90] used the following sets.

1	0.1	150	1.0	600	0.1	300	0.1	600	0.1	150
2	0.5	600	0.5	1500	0.5	1500	0.1	150	0.5	600
3	0.5	1500	0.5	1500	1.0	600	1.0	1500	2.0	1500
4	2.0	600	1.0	300	0.5	150	1.0	300	2.0	300
5	1.0	300	1.0	1500	1.0	1500	1.0	1500	1.0	600

Four demand streams are used by Lambrecht, Vander Eecken [121].

Demand Set	Period												$\bar{d}$	σ	CV
1	86	100	87	105	98	86	83	79	97	102	94	87	92	8.0	0.087
2	75	82	73	124	93	83	111	81	69	121	97	97	92.2	17.7	0.192
3	89	68	116	62	132	114	131	135	59	68	51	93	93.2	30.1	0.323
4	96	144	89	151	98	31	67	139	61	115	78	93	96.8	34.4	0.355

Vanderveken [123] used the following data set, they have short horizons (4 and 6 periods).

		Period							
i	h_i	S_i	$d_t = 1$	2	3	4	5	6	
1	1.0	30	28	30	40	35	25	30	
2	1.0	50	40	50	45	55	70	45	
3	1.0	40	15	10	20	15	15	20	
4	1.0	60	0	35	25	20	30	35	
CV=0.188		CAP=270		250	260	250	260	250	
1	2.0	150	0	54	49	47			
2	3.0	100	34	29	27	32			
3	2.0	100	14	12	15	15			
4	1.0	50	27	29	33	28			
CV=0.100		CAP=240		250	230	180			
1	2.0	150	-	54	-	96			
2	3.0	100	-	63	-	59			
3	2.0	100	-	26	-	30			
4	1.0	50	-	56	-	61			
CV=0.715		CAP=240		250	230	180			

APPENDIX C
TABLES OF STATISTICAL RESULTS

CV STRUCTURE		SE	MS	MP	BM
(1)	H	157.7	162.9	151.3	170.0
	M	158.3	160.3	152.8	170.3
	L	155.0	151.2	150.3	169.1
(2)	H	4802.1	5076.4	4718.7	5941.4
	M	4403.9	5874.7	4349.3	6687.6
	L	4108.6	5460.6	4034.6	6337.2
(3)	H	266.3	265.6	253.0	282.5
	M	256.8	249.7	249.5	282.5
	L	250.6	235.9	246.8	282.5
(4)	H	7236.2	9798.3	7103.0	10347.0
	M	8271.8	8651.3	8150.3	10317.7
	L	6941.6	8628.0	6822.7	10144.4
(5)	H	11914.4	11976.5	11681.2	13827.0
	M	12364.6	11984.8	12092.8	13810.2
	L	10054.4	11254.4	11152.0	13411.4
(6)	H	4819.3	4243.4	5118.3	4984.1
	M	4812.2	4947.7	4707.2	5637.5
	L	4016.9	4581.1	3904.8	5088.8

TABLE A.1: THE AVERAGE COST FOR EACH HEURISTIC AT DIFFERENT DEMAND PATTERN (CV) IN EVERY STRUCTURES.

CV = H: (CV > .40)
CV = M: (.20 ≤ CV ≤ .40)
CV = L: (CV < .20)

COEFFICIENT OF VAR. CAPACITY UTILIZATION		SE	MS	MP	BM
(1) CV = H	CU = H	*	1.0000	1.0060	*
	CU = M	1.0710	1.0821	1.0541	1.0000
	CU = L	1.0000	1.0551	1.0000	1.0000
	CU = H	*	1.0000	1.0412	1.0000
	CU = M	1.0720	1.1109	1.0471	1.0000
	CU = L	1.0000	1.0058	1.0000	1.0050
	CU = H	*	1.1400	1.0961	*
(2) CV = H	CU = M	1.0461	1.1410	1.0481	1.0170
	CU = L	1.0000	1.0000	1.0000	1.0000
	CU = H	1.8640	1.4580	1.8582	1.1880
	CU = M	1.0910	1.0909	1.0910	1.0380
	CU = L	1.0000	1.0000	1.0000	1.0000
	CU = H	2.0422	1.3910	2.0100	1.2150
	CU = M	1.3370	1.5911	1.3361	1.4300
(3) CV = H	CU = L	1.0000	1.0000	1.0000	1.0000
	CU = H	1.4590	1.4940	1.4341	1.0680
	CU = M	1.1111	1.3890	1.1151	1.0680
	CU = L	1.0000	1.0000	1.0000	1.0000
	CU = H	*	1.0000	1.6861	*
	CU = M	1.0220	1.2951	1.3856	1.0000
	CU = L	1.0000	1.2401	1.0000	1.0000
(4) CV = H					
	CU = H	*	1.0000	1.1511	*
	CU = M	1.0571	1.2470	1.0461	1.0000
	CU = L	1.0000	1.0661	1.0000	1.0000
	CU = H	*	1.0000	1.1132	*
	CU = M	1.0280	1.2159	1.0250	1.0000
	CU = L	1.0000	1.2158	1.0000	1.0000

TABLE A.2: CU AND CV INTERACTION FOR STRUCTURES 1-3.

CU = H: (95%)
CU = M: (75%)
CU = L: (50%)

COEFFICIENT OF VAR. CAPACITY UTILIZATION		SE	MS	MP	BM
(4) CV = H	CU = H	1.5510	1.2750	1.5171	1.0820
	CU = M	1.2040	1.1970	1.2031	1.0820
	CU = L	1.0000	1.0000	1.0000	1.0000
CV = M	CU = H	1.4260	1.1000	1.3720	1.0260
	CU = M	1.0760	1.0881	1.0641	1.0260
	CU = L	1.0000	1.0000	1.0000	1.0000
CV = L	CU = H	1.4810	1.4720	1.4511	1.0400
	CU = M	1.0200	1.2600	1.0230	1.0400
	CU = L	1.0000	1.0000	1.0000	1.0000
(5) CV = H	CU = H	1.3090	1.5360	1.2711	1.1660
	CU = M	1.1441	1.2420	1.1141	1.0390
	CU = L	1.0000	1.0000	1.0000	1.0000
CV = M	CU = H	1.4990	1.4370	1.4410	1.2260
	CU = M	1.1740	1.1151	1.1601	1.1380
	CU = L	1.0000	1.0000	1.0000	1.0000
CV = L	CU = H	1.0812	1.7130	1.3451	1.2480
	CU = M	1.0074	1.0561	1.0051	1.0740
	CU = L	1.0000	1.0000	1.0000	1.0000
(6) CV = H	CU = H	1.2300	1.0770	1.1401	1.2620
	CU = M	1.0910	1.0530	1.0911	1.2280
	CU = L	1.0000	1.0000	1.0000	1.0000
CV = M	CU = H	1.0913	1.1750	1.3361	1.2770
	CU = M	1.0880	1.1210	1.0881	1.2770
	CU = L	1.0000	1.0000	1.0000	1.0340
CV = L	CU = H	1.3860	1.0530	1.3351	1.1710
	CU = M	1.0180	1.0430	1.0640	1.0750
	CU = L	1.0000	1.0000	1.0000	1.0000

TABLE A.3: CU AND CV INTERACTION FOR STRUCTURES 4-6

HIGH (95%)
CU = MEDIUM (75%)
LOW (50%)

HIGH (CV < .20)
CV = MEDIUM (.20 ≤ CV ≤ .40)
HIGH (CV > .40)

CAPACITY UTILIZATION		SE	MS	MP	BM
(1)	CU =	H	*	1.0200	*
		M	1.0192	1.0131	1.0059
		L	1.0000	1.0000	1.0000
(2)	CU =	H	1.6875	1.4481	1.1131
		M	1.2118	1.2105	1.2113
		L	1.0000	1.0000	1.0000
(3)	CU =	H	*	1.053	1.0000
		M	1.0521	1.0411	1.0000
		L	1.0000	1.0000	1.0000
(4)	CU =	H	1.4832	1.4342	1.0510
		M	1.0869	1.0821	1.0521
		L	1.0000	1.0000	1.0000
(5)	CU =	H	1.4137	1.3536	1.2129
		M	1.1193	1.1166	1.0888
		L	1.0000	1.0000	1.0000
(6)	CU =	H	1.2334	1.2687	1.2199
		M	1.0664	1.0638	1.1637
		L	1.0000	1.0000	1.0130

TABLE A.4: COMPARISON OF THE HEURISTICS FOR CU FACTOR.

H = HIGH (95%)

M = MEDIUM (75%)

L = LOW (50%)

COEFFICIENT OF VARIATION (CV)		SE	MS	MP	BM
(1) CV =	H	1.0381	1.0720	1.0198	1.0059
	M	1.0190	1.0590	1.0133	1.0001
	L	1.0000	1.0000	1.0000	1.0000
(2) CV =	H	1.1689	1.0000	1.1695	1.0000
	M	1.0731	1.1133	1.0780	1.1255
	L	1.0000	1.0348	1.0000	1.0666
(3) CV =	H	1.0361	1.1276	1.0284	1.0000
	M	1.0041	1.0590	1.0243	1.0000
	L	1.0000	1.0000	1.0000	1.0000
(4) CV =	H	1.0421	1.1356	1.0411	1.0170
	M	1.1916	1.0026	1.1946	1.0029
	L	1.0000	1.0000	1.0000	1.0000
(5) CV =	H	1.1876	1.0639	1.0470	1.0310
	M	1.2297	1.0570	1.0842	1.0297
	L	1.0000	1.0000	1.0000	1.0000
(6) CV =	H	1.1999	1.0000	1.3120	1.0000
	M	1.1982	1.1659	1.2056	1.3470
	L	1.0000	1.0796	1.0000	1.2150

TABLE A.5: COMPARISON OF THE HEURISTICS FOR CV EFFECT

CV = H: (CV > .40)
CV = M: (.20 ≤ CV ≤ .40)
CV = L: (CV < .20)

DEMAND * (SETUP/HOLDING COST)		SE	MS	MP	BM
S/H = H (1)	CV= H	1.0496	1.0350	1.0361	1.0000
	CV= M	1.0421	1.0390	1.0361	1.0065
	CV= L	1.0000	1.0000	1.0000	1.0000
S/H = L	CV= H	1.0480	1.0310	1.0371	1.0165
	CV= M	1.0006	1.0932	1.0000	1.0115
	CV= L	1.0000	1.0000	1.0561	1.0000
S/H = H (2)	CV= H	1.1350	1.0002	1.1446	1.0000
	CV= M	1.0576	1.0700	1.0611	1.0244
	CV= L	1.0000	1.0000	1.0000	1.0240
S/H = L	CV= H	1.0640	1.0420	1.0540	1.0135
	CV= M	1.0126	1.0580	1.0006	1.0380
	CV= L	1.0000	1.0000	1.0000	1.0000
S/H = H (3)	CV= H	1.0210	1.0530	1.0870	1.0000
	CV= M	1.0112	1.1250	1.0551	1.0000
	CV= L	1.0000	1.0000	1.0000	1.0000
S/H = L	CV= H	1.0550	1.0630	1.0271	1.0000
	CV= M	1.0111	1.0000	1.0000	1.0000
	CV= L	1.0000	1.0000	1.0561	1.0000

TABLE A.6: (S/H) * DEMAND INTERACTION FOR STRUCTURES 1-3.

DEMAND ★ (SETUP/HOLDING COST)		SE	MS	MP	BM
S/H = H (4)	CV= H	1.0003	1.0000	1.0000	1.0600
	CV= M	1.1166	1.1216	1.1366	1.0604
	CV= L	1.0000	1.0806	1.0010	1.0000
S/H = L	CV= H	1.2640	1.1010	1.2641	1.0380
	CV= M	1.1351	1.2095	1.1341	1.0270
	CV= L	1.0000	1.0000	1.0000	1.0000
S/H = H (5)	CV= H	1.1890	1.0502	1.0490	1.0035
	CV= M	1.1540	1.0379	1.0330	1.0260
	CV= L	1.0000	1.0000	1.0000	1.0100
S/H = L	CV= H	1.2430	1.0502	1.0916	1.0035
	CV= M	1.1890	1.0350	1.0490	1.0000
	CV= L	1.0000	1.0000	1.0000	1.0100
S/H = H (6)	CV= H	1.1980	1.0000	1.0000	1.0000
	CV= M	1.1681	1.1410	1.3151	1.2995
	CV= L	1.0000	1.0472	1.1760	1.2125
S/H = L	CV= H	1.1820	1.0000	1.2471	1.0000
	CV= M	1.1491	1.1340	1.1880	1.3240
	CV= L	1.0000	1.0609	1.0000	1.1408

TABLE A.7: (S/H) ★ DEMAND INTERACTION FOR STRUCTURES 4-6.

FACTOR	SE - MP		RESULTS
	F VALUE	PR > F	
CAP	0.64	0.4370	ACCEPT
D	0.74	0.5451	ACCEPT
D*CAP	2.75	0.0767	ACCEPT
S	17.63	0.0001	REJECT
S*CAP	2.41	0.1401	ACCEPT
D*S	0.93	0.4470	ACCEPT
D*S*CAP	0.06	0.9787	ACCEPT

TABLE A.8: STATISTICAL RESULTS FOR STRUCTURE 1.

FACTOR	SE - MP		RESULTS
	F VALUE	PR > F	
CAP	20.53	0.0001	REJECT
D	6.35	0.0010	REJECT
D*CAP	4.63	0.0009	REJECT
S	15.01	0.0001	REJECT
S*CAP	23.78	0.0001	REJECT
D*S	0.71	0.5509	ACCEPT
D*S*CAP	1.81	0.1165	ACCEPT

TABLE A.9: STATISTICAL RESULTS FOR STRUCTURE 2.

FACTOR	SE - MP		RESULTS
	F VALUE	PR > F	
CAP	20.76	0.0001	REJECT
D	13.55	0.0001	REJECT
D*CAP	3.83	0.0306	ACCEPT
S	22.17	0.0001	REJECT
S*CAP	1.35	0.2616	ACCEPT
D*S	6.74	0.0038	REJECT
D*S*CAP	3.29	0.0480	ACCEPT

TABLE A.10: STATISTICAL RESULTS FOR STRUCTURE 3.

FACTOR	SE - MP		RESULTS
	F VALUE	PR > F	
CAP	26.35	0.0001	REJECT
D	6.32	0.0044	REJECT
D*CAP	0.46	0.7679	ACCEPT
S	14.04	0.0001	REJECT
S*CAP	12.83	0.0001	REJECT
D*S	3.04	0.0603	ACCEPT
D*S*CAP	0.40	0.8083	ACCEPT

TABLE A.11: STATISTICAL RESULTS FOR STRUCTURE 4.

FACTOR	SE - MP		RESULTS
	F VALUE	PR > F	
CAP	31.65	0.0001	REJECT
D	2.18	0.1295	ACCEPT
D*CAP	1.72	0.1833	ACCEPT
S	12.06	0.0001	REJECT
S*CAP	0.67	0.5213	ACCEPT
D*S	0.60	0.5562	ACCEPT
D*S*CAP	0.59	0.6279	ACCEPT

TABLE A.12: STATISTICAL RESULTS FOR STRUCTURE 5.

FACTOR	SE - MP		RESULTS
	F VALUE	PR > F	
CAP	5.81	0.0044	REJECT
D	13.06	0.0002	REJECT
D*CAP	1.36	0.2574	ACCEPT
S	21.66	0.0001	REJECT
S*CAP	0.63	0.6033	ACCEPT
D*S	0.70	0.5063	ACCEPT
D*S*CAP	0.13	0.9841	ACCEPT

TABLE A.13: STATISTICAL RESULTS FOR STRUCTURE 6.

FACTOR	MS - BM		RESULTS
	F VALUE	PR > F	
CAP	15.73	0.0001	REJECT
D	12.54	0.0001	REJECT
D*CAP	1.09	0.3980	ACCEPT
S	21.08	0.0001	REJECT
S*CAP	13.10	0.0001	REJECT
D*S	4.25	0.0152	ACCEPT
D*S*CAP	2.84	0.0312	ACCEPT

TABLE A.14: STATISTICAL RESULTS FOR STRUCTURE 1.

FACTOR	MS - BM		RESULTS
	F VALUE	PR > F	
CAP	21.71	0.0001	REJECT
D	15.37	0.0001	REJECT
D*CAP	22.69	0.0001	REJECT
S	19.31	0.0001	REJECT
S*CAP	4.96	0.0110	ACCEPT
D*S	0.98	0.4109	ACCEPT
D*S*CAP	1.36	0.2506	ACCEPT

TABLE A.15: STATISTICAL RESULTS FOR STRUCTURE 2.

FACTOR	MS - BM		RESULTS
	F VALUE	PR > F	
CAP	19.97	0.0001	REJECT
D	29.46	0.0001	REJECT
D*CAP	19.12	0.0001	REJECT
S	15.49	0.0001	REJECT
S*CAP	1.48	0.2471	ACCEPT
D*S	2.00	0.1403	ACCEPT
D*S*CAP	1.22	0.3283	ACCEPT

TABLE A.16: STATISTICAL RESULTS FOR STRUCTURE 3.

FACTOR	MS - BM		RESULTS
	F VALUE	PR > F	
CAP	20.18	0.0001	REJECT
D	25.17	0.0001	REJECT
D*CAP	6.16	0.0007	REJECT
S	17.72	0.0001	REJECT
S*CAP	30.48	0.0001	REJECT
D*S	17.32	0.0001	REJECT
D*S*CAP	3.37	0.0192	ACCEPT

TABLE A.17: STATISTICAL RESULTS FOR STRUCTURE 4.

FACTOR	MS - BM		RESULTS
	F VALUE	PR > F	
CAP	17.83	0.0001	REJECT
D	6.56	0.0038	REJECT
D*CAP	12.70	0.0001	REJECT
S	19.83	0.0001	REJECT
S*CAP	18.76	0.0001	REJECT
D*S	0.82	0.4491	ACCEPT
D*S*CAP	0.69	0.6071	ACCEPT

TABLE A.18: STATISTICAL RESULTS FOR STRUCTURE 5.

FACTOR	MS - BM		RESULTS
	F VALUE	PR > F	
CAP	6.35	0.0025	REJECT
D	18.30	0.0001	REJECT
D*CAP	0.78	0.6119	ACCEPT
S	17.28	0.0001	REJECT
S*CAP	0.42	0.7438	ACCEPT
D*S	0.84	0.4448	ACCEPT
D*S*CAP	0.33	0.9125	ACCEPT

TABLE A.19: STATISTICAL RESULTS FOR STRUCTURE 6.

FACTOR	SE - MP		MS - BM		RESULTS
	F VALUE	PR > F	F VALUE	PR > F	
D	0.82	0.4923	0.94	0.4248	ACCEPT
S	1.05	0.3108	0.58	0.4504	ACCEPT
D*S	0.14	0.3108	0.93	0.4336	ACCEPT
STRUCTURE (1)	0.56	0.9345	0.89	0.5254	ACCEPT

TABLE A.20: STATISTICAL RESULTS OF CPU TIME FOR STRUCTURE 1.

FACTOR	SE - MP		MS - BM		RESULTS
	F VALUE	PR > F	F VALUE	PR > F	
D	2.61	0.0197	1.12	0.3638	ACCEPT
S	1.66	0.1844	0.66	0.5804	ACCEPT
D*S	10.63	0.0018			REJECT
D*S			4.90	0.0305	ACCEPT
STRUCTURE (2)	0.88	0.4555	0.31	0.8151	ACCEPT

TABLE A.21: STATISTICAL RESULTS OF CPU TIME FOR STRUCTURE 2.

FACTOR	SE - MP		MS - BM		RESULTS
	F VALUE	PR > F	F VALUE	PR > F	
D	0.60	0.7538	0.46	0.8596	ACCEPT
S	0.54	0.6580	0.18	0.9105	ACCEPT
D*S	1.52	0.2245	1.66	0.2051	ACCEPT
STRUCTURE (3)	0.35	0.7906	0.33	0.8006	ACCEPT

TABLE A.22: STATISTICAL RESULTS OF CPU TIME FOR STRUCTURE 3.

FACTOR	SE - MP		MS - BM		RESULTS
	F VALUE	PR > F	F VALUE	PR > F	
D	1.81	0.1292	1.81	0.1292	ACCEPT
S	0.02	0.9811	0.02	0.9811	ACCEPT
D*S	5.54	0.0228	5.54	0.0228	ACCEPT
STRUCTURE (4)	1.73	0.1878	1.73	0.1878	ACCEPT

TABLE A.23: STATISTICAL RESULTS OF CPU TIME FOR STRUCTURE 4.

FACTOR	SE - MP		MS - BM		RESULTS
	F VALUE	PR > F	F VALUE	PR > F	
D	1.05	0.4005	0.09	0.4928	ACCEPT
S	0.31	0.7346	0.05	0.9505	ACCEPT
D*S	3.74	0.0590	0.23	0.6303	ACCEPT
STRUCTURE (5)	0.44	0.6466	0.07	0.9364	ACCEPT

TABLE A.24: STATISTICAL RESULTS OF CPU TIME FOR STRUCTURE 5.

FACTOR	SE - MP		MS - BM		RESULTS
	F VALUE	PR > F	F VALUE	PR > F	
D	1.04	0.4071	0.95	0.4564	ACCEPT
S	0.82	0.4470	0.03	0.9699	ACCEPT
D*S	3.48	0.0693	2.96	0.0929	ACCEPT
STRUCTURE (6)	0.04	0.9600	0.88	0.4243	ACCEPT

TABLE A.25: STATISTICAL RESULTS OF CPU TIME FOR STRUCTURE 6.

(S/H) * CU		SE	MS	MP	BM
S/H = H (1)	CU = H	*	1.0630	1.0570	*
	CU = M	1.0000	1.0421	1.0000	1.0000
	CU = L	1.0130	1.0000	1.0071	1.0060
S/H = L	CU = H	*	1.0000	1.0000	*
	CU = M	1.0420	1.2610	1.0441	1.0160
	CU = L	1.0000	1.2121	1.0060	1.0000
S/H = H (2)	CU = H	1.3680	1.1750	1.3220	1.0220
	CU = M	1.0000	1.1660	1.0030	1.0000
	CU = L	1.0010	1.0000	1.0000	1.0220
S/H = L	CU = H	1.6922	1.4350	1.6600	1.0530
	CU = M	1.0860	1.2911	1.0791	1.0530
	CU = L	1.0000	1.0000	1.0000	1.0000
S/H = H (3)	CU = H	*	1.0000	1.0040	1.0000
	CU = M	1.0430	1.3381	1.0351	1.0210
	CU = L	1.0000	1.2601	1.0000	1.0053
S/H = L	CU = H	*	1.0000	1.0981	1.0000
	CU = M	1.0550	1.2410	1.0301	1.0632
	CU = L	1.0000	1.1711	1.0000	1.0234
S/H = H (4)	CU = H	1.2541	1.1100	1.1171	1.0531
	CU = M	1.0370	1.1160	1.2031	1.0531
	CU = L	1.0000	1.0000	1.0000	1.0000
S/H = L	CU = H	1.5600	1.3110	1.2300	1.0506
	CU = M	1.1058	1.1851	1.0641	1.0506
	CU = L	1.0000	1.0000	1.0000	1.0000
S/H = H (5)	CU = H	1.4190	1.4760	1.3681	1.2990
	CU = M	1.1181	1.1110	1.1111	1.0640
	CU = L	1.0000	1.0000	1.0000	1.0000
S/H = L	CU = H	1.4090	1.4567	1.4342	1.2159
	CU = M	1.1202	1.1251	1.1204	1.1030
	CU = L	1.0000	1.0000	1.0000	1.0000
S/H = H (6)	CU = H	1.2650	1.1320	1.3041	1.1740
	CU = M	1.0250	1.0691	1.0190	1.0000
	CU = L	1.0000	1.0000	1.0000	1.0139
S/H = L	CU = H	1.1770	1.0610	1.2061	1.2300
	CU = M	1.0205	1.0281	1.0260	1.0300
	CU = L	1.0000	1.0000	1.0000	1.0000

TABLE A.26: (S/H) * CU INTERACTION FOR EACH HEURISTIC

(S/H) * CU		MP/SE	MS/BM
(1)	S/H = H CU = H	*	1.0000
	CU = M	0.9661	0.9811
	CU = L	0.9596	0.9311
(2)	S/H = L CU = H	*	*
	CU = M	0.9596	0.9610
	CU = L	0.9708	0.9421
(3)	S/H = H CU = H	0.9496	0.9590
	CU = M	0.9871	0.9710
	CU = L	0.9813	0.8160
(4)	S/H = L CU = H	0.9782	0.9800
	CU = M	0.9997	0.8821
	CU = L	0.9970	0.7190
(5)	S/H = H CU = H	*	0.7320
	CU = M	0.9587	0.9811
	CU = L	0.9661	0.9241
(6)	S/H = L CU = H	*	0.7770
	CU = M	0.9514	0.9650
	CU = L	0.9746	0.9011
(7)	S/H = H CU = H	0.9745	0.9320
	CU = M	0.9861	0.9320
	CU = L	0.9478	0.8850
(8)	S/H = L CU = H	0.9671	0.9590
	CU = M	0.9932	0.8671
	CU = L	0.9940	0.7680
(9)	S/H = H CU = H	0.9606	0.9720
	CU = M	0.9880	0.8810
	CU = L	0.9940	0.8560
(10)	S/H = L CU = H	0.9425	0.9809
	CU = M	0.9899	0.7401
	CU = L	0.9900	0.7200
(11)	S/H = H CU = H	1.0030	0.8690
	CU = M	0.9881	0.9641
	CU = L	0.9930	0.8864
(12)	S/H = L CU = H	1.0070	0.9020
	CU = M	0.9896	1.0431
	CU = L	0.9832	1.0450

TABLE A.27: COST RATIOS FOR (S/H) * CAP INTERACTION

SETUP/HOLDING COST (STRUCTURE)		MP/SE	MS/BM
(1)	S/H = H	0.9436	0.9970
	S/H = L	0.9792	0.8940
(2)	S/H = H	0.9688	0.9086
	S/H = L	0.9862	0.8710
(3)	S/H = H	0.9577	0.8810
	S/H = L	0.9900	0.8860
(4)	S/H = H	0.9870	0.9190
	S/H = L	0.9823	0.8869
(5)	S/H = H	1.0101	0.9110
	S/H = L	0.9959	0.8219
(6)	S/H = H	0.9842	0.9653
	S/H = L	1.0000	0.8960

TABLE A.28: COST RATIOS FOR ALL HEURISTICS FOR S/H EFFECT

PRODUCT STRUCTURE	MP/SE	MS/BM
1	.8960	.9609
2	.8872	.9601
3	.9135	.9511
4	.8942	.9454
5	.8325	.9322
6	1.0071	.9276

TABLE A.29: RATIOS OF MP/SE AND MS/BM FOR CU*D*S
INTERACTION FOR ALL PRODUCT STRUCTURES.

HOLDING COST * CU		SE	MS	MP	BM
(1) H = 1.0	CU = H	*	1.0789	1.0541	1.0171
	CU = M	1.0390	1.0461	1.0401	1.0170
	CU = L	1.0000	1.0000	1.0000	1.0000
H = 1.33	CU = H	*	1.1591	1.0132	1.0000
	CU = M	1.0000	1.1041	1.0066	1.0000
	CU = L	1.0120	1.0000	1.0000	1.0110
(2) H = 0.62	CU = H	1.7650	1.5560	1.7140	1.0260
	CU = M	1.1180	1.3630	1.1175	1.1710
	CU = L	1.0000	1.0000	1.0000	1.0000
H = 0.22	CU = H	1.7482	1.4034	1.7036	1.0000
	CU = M	1.0130	1.2731	1.0049	1.0620
	CU = L	1.0000	1.0000	1.0000	1.0620
H = 0.82	CU = H	1.5590	1.6598	1.5471	1.4870
	CU = M	1.3671	1.3790	1.3701	1.1749
	CU = L	1.0000	1.0000	1.0000	1.0000
(3) H = 1.0	CU = H	*	1.2460	1.0571	1.0000
	CU = M	1.0560	1.1731	1.0331	1.0000
	CU = L	1.0000	1.0000	1.0000	1.0000
H = 1.2	CU = H	*	1.3201	1.0611	1.0000
	CU = M	1.0480	1.2290	1.0416	1.0000
	CU = L	1.0000	1.0000	1.0000	1.0000

TABLE A.30: CAPACITY * HOLDING COST EFFECT FOR STRUCTURES
1 - 3.

H = Average echelon holding cost

HOLDING COST * CU UTILIZATION		SE	MS	MP	BM
(4) H = .62	CU = H	1.7480	1.3260	1.6421	1.0311
	CU = M	1.2540	1.1580	1.2231	1.0311
	CU = L	1.0000	1.0000	1.0000	1.0000
H = .22	CU = H	1.3800	1.2010	1.3680	1.0710
	CU = M	1.0430	1.1711	1.0501	1.0710
	CU = L	1.0000	1.0000	1.0000	1.0000
H = .82	CU = H	1.3810	1.2550	1.3501	1.0510
	CU = M	1.0130	1.0480	1.0100	1.0510
	CU = L	1.0000	1.0000	1.0000	1.0000
(5) H = .263	CU = H	1.2550	1.3700	1.2091	1.4080
	CU = M	1.0544	1.0670	1.0491	1.0630
	CU = L	1.0000	1.0000	1.0000	1.0000
H = .714	CU = H	1.6780	1.6670	1.5890	1.0400
	CU = M	1.1810	1.1341	1.1751	1.0170
	CU = L	1.0000	1.0000	1.0000	1.0000
H = .4	CU = H	1.3760	1.5780	1.3231	1.2460
	CU = M	1.1531	1.1610	1.1551	1.1580
	CU = M	1.0000	1.0000	1.0000	1.0000
(6) H = .421	CU = H	1.3220	1.1910	1.3631	1.3910
	CU = M	1.0000	1.0751	1.0000	1.0210
	CU = L	1.0210	1.0000	1.0120	1.0000
H = .174	CU = H	1.1650	1.0370	1.1891	1.0710
	CU = M	1.0230	1.0000	1.0160	1.0000
	CU = L	1.0000	1.0120	1.0000	1.0150

TABLE A.31: CAPACITY * HOLDING COST EFFECT FOR STRUCTURES
4 - 6.

HOLDING COST * STRUCTURE		SE	MS	MP	BM
(1)	H = 1.0	1.0000	1.0128	1.0000	1.0059
	H = 1.33	1.0450	1.0000	1.0061	1.0000
(2)	H = .62	1.0998	1.0129	1.1025	1.0800
	H = .22	1.0000	1.0000	1.0000	1.0000
	H = .82	1.3230	1.1880	1.3390	1.1578
(3)	H = 1.0	1.0039	1.0000	1.0000	1.0000
	H = 1.2	1.0000	1.0740	1.0081	1.0800
(4)	H = .62	1.1660	1.0610	1.1572	1.0123
	H = .22	1.0000	1.0000	1.0000	1.0000
	H = .82	1.3048	1.0998	1.2986	1.0060
(5)	H = .263	1.0000	1.0000	1.0000	1.0000
	H = .714	1.2204	1.0839	1.2021	1.1860
	H = .40	1.1149	1.0369	1.1060	1.1036
(6)	H = .421	1.2117	1.0133	1.2719	1.0171
	H = .174	1.0000	1.0000	1.0000	1.0259

TABLE A.32: HOLDING COST EFFECT FOR EACH STRUCTURE

HOLDING COST (STRUCTURE)		MP/SE	MS/BM
(1)	H = 1.0	0.9813	0.9346
	H = 1.33	0.9442	0.9170
(2)	H = .62	0.9777	0.8906
	H = .22	0.9976	0.8350
	H = .82	0.9891	0.9140
(3)	H = 1.0	0.9689	0.8860
	H = 1.2	0.9813	0.8810
(4)	H = .62	0.9876	0.8370
	H = .22	0.9893	0.9079
	H = .82	0.9803	0.8928
(5)	H = .263	0.9914	0.9809
	H = .714	1.0060	0.7620
	H = .40	0.9989	0.8519
(6)	H = .421	1.0080	0.8860
	H = .174	1.0075	0.9783

TABLE A.33: COST RATIOS FOR ALL HEURISTICS FOR H EFFECT

HOLDING COST*DEMAND		SE	MS	MP	BM
(1) CV = H	H = 1.0	1.0000	1.0000	1.0000	1.0000
	H = 1.33	1.0250	1.0006	1.0060	1.0000
CV = M	H = 1.0	1.0000	1.0000	1.0000	1.0000
	H = 1.33	1.0930	1.0118	1.0271	1.0209
CV = L	H = 1.0	1.0000	1.0000	1.0000	1.0000
	H = 1.33	1.0150	1.0820	1.0100	1.0179
(2) CV = H	H = .62	1.3488	1.1310	1.3575	1.0610
	H = .22	1.0000	1.0000	1.0000	1.0000
	H = .82	1.6210	1.1350	1.6400	1.2818
CV = M	H = .62	1.1128	1.0240	1.1260	1.0820
	H = .22	1.0000	1.0000	1.0000	1.0000
	H = .82	1.1170	1.1900	1.1320	1.2250
CV = L	H = .62	1.0310	1.0589	1.0330	1.0150
	H = .22	1.0000	1.0000	1.0000	1.0000
	H = .82	1.1470	1.1310	1.1510	1.2480
(3) CV = H	H = 1.0	1.0000	1.0000	1.0000	1.0000
	H = 1.2	1.0070	1.0190	1.0000	1.0000
CV = M	H = 1.0	1.0000	1.0110	1.0000	1.0000
	H = 1.2	1.0209	1.0000	1.0081	1.0000
CV = L	H = 1.0	1.0000	1.0000	1.0000	1.0000
	H = 1.2	1.0169	1.0200	1.0110	1.0000

TABLE A.34: CV * H INTERACTION FOR STRUCTURES 1-3.

HOLDING COST*DEMAND			SE	MS	MP	BM
(4) CV = H	H = .62		1.3200	1.1323	1.3140	1.0820
	M = .22		1.0000	1.0000	1.0000	1.0000
	H = .82		1.4498	1.1408	1.4496	1.0820
CV = M	H = .62		1.1560	1.0350	1.1302	1.0263
	H = .22		1.0000	1.0001	1.0000	1.0000
	H = .82		1.2998	1.0431	1.2750	1.0260
CV = L	H = .62		1.0180	1.0020	1.0170	1.0070
	H = .22		1.0000	1.0000	1.0000	1.0000
	H = .82		1.1548	1.0261	1.1650	1.1240
(5) CV = H	H = .263		1.0000	1.0000	1.0000	1.0000
	H = .714		1.1614	1.0799	1.1621	1.1550
	H = .40		1.0889	1.0333	1.0820	1.0931
CV = M	H = .263		1.0000	1.0000	1.0000	1.0000
	H = .714		1.3054	1.1199	1.2451	1.2270
	H = .40		1.1831	1.0369	1.1290	1.0926
CV = L	H = .263		1.0000	1.0000	1.0000	1.1837
	H = .714		1.3150	1.0529	1.1981	1.0000
	H = .40		1.0831	1.0369	1.1020	1.1276
(6) CV = H	H = .421		1.1857	1.0593	1.1909	1.2029
	H = .174		1.0000	1.0000	1.0000	1.0000
	H = .421		1.1827	1.1053	1.1249	1.0000
CV = M	H = .174		1.0000	1.0000	1.0000	1.2151
	H = .421		1.2087	1.1803	1.2219	1.0029
	H = .174		1.0000	1.0000	1.0000	1.0000

TABLE A.35: CV * H INTERACTION FOR STRUCTURES 4 - 6.

HOLDING COST * CAPACITY STRUCTURE		MP/SE	MS/BM
(1) H = 1.0	CU = H	*	*
	CU = M	0.9746	0.9640
	CU = L	0.9737	0.9521
H = 1.2	CU = H	*	*
	CU = M	0.9505	0.9821
	CU = L	0.9451	0.9241
(2) H = .62	CU = H	0.9624	0.9660
	CU = M	0.9900	0.9100
	CU = L	0.9910	0.7860
H = .22	CU = H	0.9661	0.9897
	CU = M	0.9832	0.8440
	CU = L	0.9910	0.6636
H = .82	CU = H	0.9832	0.9600
	CU = M	0.9645	0.9219
	CU = L	0.9823	0.8260
(3) H = 1.0	CU = H	*	0.7760
	CU = M	0.9469	0.9680
	CU = L	0.9680	0.9111
H = 1.2	CU = H	*	0.7420
	CU = M	0.9624	0.9711
	CU = L	0.9765	0.9241

TABLE A.36: COST RATIOS FOR H*CU INTERACTION FOR
HEURISTICS 1 - 3.

HOLDING COST * CAPACITY UTILIZATION		MP/SE	MS/BM
(4) H = .62	CU = H	0.9614	0.9465
	CU = M	0.9980	0.8260
	CU = L	0.9970	0.7350
H = .22	CU = H	0.9764	0.9480
	CU = M	0.9745	0.9240
	CU = L	0.9842	0.8450
H = .82	CU = H	0.9680	0.9640
	CU = M	0.9881	0.9011
	CU = L	0.9910	0.8000
(5) H = .263	CU = H	0.9541	0.9649
	CU = M	0.9850	0.9941
	CU = L	0.9900	0.9910
H = .714	CU = H	0.9398	0.9830
	CU = M	0.9869	0.6841
	CU = L	0.9920	0.6130
H = .40	CU = H	0.9541	0.9840
	CU = M	0.9939	0.7790
	CU = L	0.9920	0.7770
(6) H = .421	CU = H	1.0183	0.8170
	CU = M	0.9900	1.0251
	CU = L	0.9810	0.9334
H = .174	CU = H	1.0138	0.9570
	CU = M	0.9881	0.9891
	CU = L	0.9941	0.9859

TABLE A.37: COST RATIOS FOR H*CU INTERACTION
FOR HEURISTICS 4 - 6.

APPENDIX D
KEY TERMS AND ABBREVIATIONS

CU: Capacity Utilization
CV: Coefficient of Variation
LV: Lambretch and Vandervecken
MLLS: Multi-Level Lot Sizing
CLSP: Capacitated Lot Sizing Problem
BM: Blackburn and Millen
SE: Heuristic 1
MS: Heuristic 2
MP: Heuristic 3
LB: Lower Bound
Block: Product Structure
CAP: Capacity per period
CAPU: Capacity per stage
MIP: Mixed Integer Program
AD: Average Demand

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