Transformational Voice Leading in Atonal Music

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A dissertation submitted to the Graduate Faculty in Music in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

1997

UMI Number: 9732955

UMI
300 North Zeeb Road
Ann Arbor, MI 48103

This manuscript has been read and accepted for the Graduate Faculty in Music in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

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Abstract

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Since at least the time of Tinctoris, music theorists have been centrally concerned with both "vertical" pitch structures (chords or simultaneities) and the "horizontal" connections (voice leading) among them. In the study of twentieth-century music the former has received extensive coverage, while the latter remains substantially less explored. Over the last two decades, a small, but growing, number of theoretical and analytical works attempt to redress this imbalance, and "Transformational Voice Leading in Atonal Music" is my contribution to that effort.

As a point of departure I explore the analytical ramifications of interpreting operational mappings as contrapuntal voices. Finding that transposition and inversion cannot sufficiently account for the point-to-point motions of most musical surfaces, I probe the recent theoretical literature for alternative transformations. In particular, Klumpenhouwer networks and three singleton transformations—Forte's "unary transformations," Lewin's "if-only," and Straus's "near-transformations"—inspire me to develop and generalize a number of original "dual transformations." These theoretical tools coalesce in a voice-leading model that combines the mappings generated by dual transformations with those implied by recursive Klumpenhouwer networks. This transformational model offers multiple interpretations of musical passages that I organize into non-hierarchical levels of voices called "adjacencies" and "recursive structure."

The remainder of the dissertation tests the analytical viability of my voice-leading model in the context of a wide variety of twentieth-century atonal musical literature. The nine analytical essays examine substantial excerpts or complete compositions by Babbitt, Bartók, Ives, Skryabin, Stravinsky, and Webern.

To my father,

John O'Donnell

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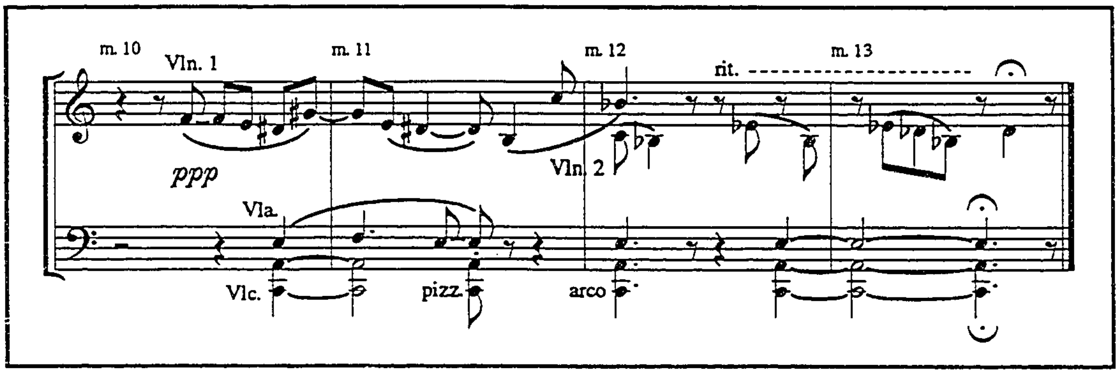
Prelude

Throughout much of modern history—certainly since Tinctoris in the fifteenth century—two central concerns of music theorists have been "vertical" pitch structures (chords or simultanieties) and the "horizontal" connections (voice leading) among them. In the study of twentieth-century music the former has received extensive coverage, while the latter remains substantially less explored. Over the last two decades, a small, but growing, number of theoretical and analytical works attempt to redress this imbalance, and "Transformational Voice Leading in Atonal Music" is my contribution to that effort.

As a point of departure I would like to direct the reader's attention to the Webern excerpt shown below in Example 1. How does one hear and interpret this passage? Certainly there are as many answers to that question as there are listeners, but perhaps I can make a few assumptions. The active violin lines heard against the primarily static and homorhythmic lower strings evokes the traditional texture of melody and accompaniment. Throughout most of the passage four pitches sound simultaneously, implying, or at least resembling, some kind of four-part harmony. Given these seemingly conventional musical properties, one might imagine that answering the following question would be easy: what is the voice leading of this passage?

Example 1: Webern, Five Pieces for String Quartet, Op. 5, No. 2, mm. 10–13



Contrary to the expectations generated by this relatively elementary musical fabric, the issue is not so simple. There are numerous possible interpretations of the voice leading here, and music theorists could spend hours debating the various merits or shortcomings of each. For example, registral or instrumental lines are frequently equated with the concept of voice leading, and such an interpretation meshes well with the four-part melody-plus-accompaniment texture in this excerpt. On the other hand, if the voice leading is no more than the lines played by each of the instruments, why bother discussing it at all? We can already hear and see it in the score itself. Furthermore, equating the registral or instrumental lines with the voice leading offers no explanation of the dynamic musical processes involved. Without additional clarification, this unsatisfactory interpretation yields nothing that increases our understanding of the music in question. Theorists therefore customarily bring more substantial analytical tools to bear on the problem, which in turn yield more sophisticated voice leading interpretations. Despite the multiplicity of possible interpretations, the analytical literature on twentieth-century voice leading tends to cluster around a few central themes, and Joseph Straus specifically identifies "three analytical models for atonal voice leading": prolongational, associational, and transformational.[1]

The prolongational model, growing out of the work of Heinrich Schenker, asserts a differentiation between structural and embellishing tones. Works by proponents of atonal prolongation further subdivide into two categories: those that are obviously extending Schenker's approach, and those that develop alternative prolongational techniques. The former includes analyses by Schenker, Felix Salzer, Roy Travis, Edward Laufer, and James Baker, while the latter includes studies by Joel Lester, Fred Lerdahl, and Paul

[1] Joseph N. Straus, "Voice Leading in Atonal Music," paper presented at the Florida State University Music Theory Forum (February, 1993): 1. A published version of this paper is forthcoming in *Music Theory in Concept and Practice*, ed. James Baker, David Beach, and Jonathan Bernard (Rochester, NY: University of Rochester Press, 1997).

Wilson, among others.[2] In the Webern excerpt, the application of a prolongational methodology could manifest itself in numerous ways. A tonally influenced interpretation might suggest that the entire passage prolongs an A-minor triad. The lower strings sound the complete triad throughout the excerpt, with the viola sounding an F neighboring tone in measure 11. The violin melody freely embellishes the triad with half-step upper and lower neighbors to each member of the A-minor triad. The entire collection of pitch-classes in the excerpt is: {G#→A←Bb, B→C←Db, D#/Eb→E←F}. Example 2 shows one possible graphic representation of this model. In this example black noteheads represent neighboring tones and half notes represent literal or implied (parenthetical) structural tones. This interpretation posits a traditional Schenkerian prolongational object (a triad), but it is nonetheless a radical extension of Schenker's theories in allowing unresolved embellishments or dissonance. Note that almost all of the structural tones are implied, rather than literally present, in the second half of the phrase. An approach less influenced by tonal traditions might allow for a non-triadic prolongational object, such as the frequently sounding {C, A, E, D#/Eb}, the final {C, A, E, Db}, or perhaps even the looser notion of a triad plus any extra chromatic neighboring tone. Melodic tones outside the prolonged object would then be interpreted as some kind of dissonance. The

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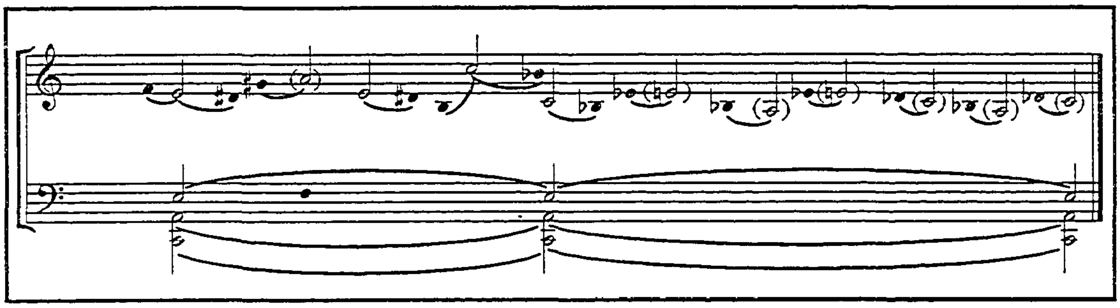
[2] Examples of the more directly Schenkerian-based approach include: Heinrich Schenker "Fortsetzung der Urlinie-Betrachtungen," in *Das Meisterwerk in der Musik: Jahrbuch II* (Munich: Drei Masken Verlag, 1926): 9–42; Felix Salzer, *Structural Hearing: Tonal Coherence in Music* (New York: Dover, 1962); Roy Travis, "Toward a New Concept of Tonality?" *Journal of Music Theory* 3.2 (1959): 257–284, "Directed Motion in Schoenberg and Webern," *Perspectives of New Music* 4.2 (1966): 85–89, and "Tonal Coherence in the First Movement of Bartók's Fourth String Quartet," *Music Forum* 2 (1970): 298–371; Edward Laufer, "Review of Schenker's *Free Composition*," *Music Theory Spectrum* 3 (1981): 158–184, and "Schoenberg's Klavierstück Op. 33a: A Linear Approach," paper presented at the Arnold Schoenberg Institute/Music Theory Society of New York State Joint Conference (Barnard College, 1991); and James M. Baker, "Schenkerian Analysis and Post-Tonal Music," *Aspects of Schenkerian Theory*, ed. David Beach (New Haven: Yale University Press, 1983): 153–188, "Voice-Leading in Post-Tonal Music: Suggestions for Extending Schenker's Theory," *Music Analysis* 9.2 (1990): 177–200, and "Post-Tonal Voice-Leading," *Early Twentieth-Century Music*, ed. Jonathan Dunsby (Oxford: Blackwell, 1993): 20–41. Examples of modified prolongational approaches include: Joel Lester, "A Theory of Atonal Prolongations as Used in an Analysis of the Serenade, Op. 24, by Arnold Schoenberg" (Ph.D. dissertation, Princeton University, 1970); Fred Lerdahl, "Atonal Prolongational Structure," *Contemporary Music Review* 4 (1989): 65–87; and Paul Wilson, "Concepts of Prolongation and Bartók's Opus 20," *Music Theory Spectrum* 6 (1984): 79–89, and *The Music of Béla Bartók* (New Haven: Yale University Press, 1992).

prolongational model captures the persistence of the A-minor triad, as well as the half-step relationship of all the other tones in the passage to that triad. Unfortunately, when so few of the melodic tones are "harmonic," virtually none in the second half of the phrase, it becomes quite difficult to imagine exactly how the embellishing tones are functioning in this context. Without the support of the tonal system's efficient hierarchical structure, it is extremely difficult to produce a satisfactory prolongational analysis, as Straus suggests:

> If you cannot distinguish the structural tones from the non-structural ones, and, even if you could, you still cannot decide how the non-structural tones embellish the structural ones, then you will have trouble producing a convincing prolongational analysis.[3]

Example 2: Prolongational interpretation of the Webern excerpt



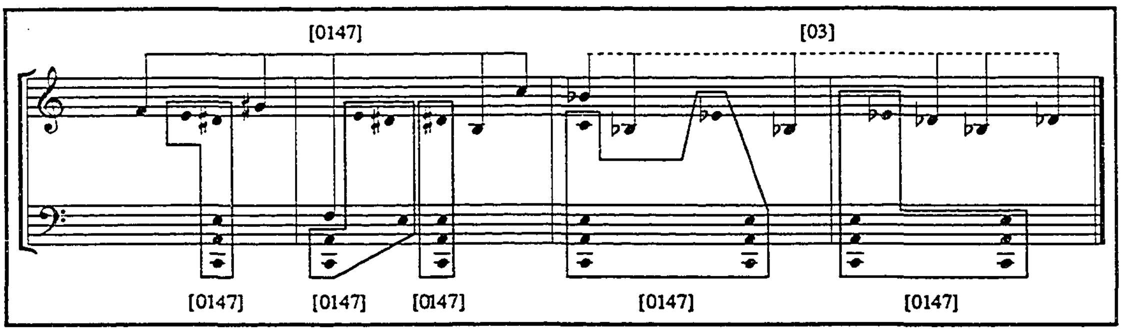
The associational model, growing out of the work of Allen Forte, generally avoids the problem of distinguishing between structural and embellishing tones, and focuses instead on tracing linear projections of important sets of pitches or pitch classes. The analytical literature based on this model includes prominent contributions by Milton Babbitt, Forte, Christopher Hasty, and Straus.[4] Example 3 shows one possible application

[3] Straus, "Voice Leading in Atonal Music," 2. Straus tackles this issue more thoroughly in "The Problem of Prolongation in Post-Tonal Music," *Journal of Music Theory* 31.1 (1987): 1–22.

[4] Examples of the associational approach are found in: Milton Babbitt, *Words About Music*, ed. Stephen Dembski and Joseph N. Straus (Madison: University of Wisconsin Press, 1987); Allen Forte, "Tonality, Symbol, and Structural Levels in Berg's *Wozzeck*," *Musical Quarterly* 71.4 (1985): 474–499, and "New Modes of Linear Analysis," paper presented at the Oxford University Conference on Music Analysis (1988); Christopher Hasty, "Segmentation and Process in Post-Tonal Music," *Music Theory Spectrum* 3 (1981): 54–73, and "On the Problem of Succession and Continuity in Twentieth-Century Music," *Music Theory Spectrum* 8 (1986): 58–74; and Joseph N. Straus, "A Principle of Voice Leading in the Music of Stravinsky," *Music Theory Spectrum* 4 (1982): 106–124, and "The Problem of Prolongation in Post-Tonal Music."

of associational methodology to the Webern excerpt. Note that all of the verticalities encased in polygons are the chord {C, A, E, D#/Eb}, or, using Forte's terminology, set-class 4–18 [0147]. Long stems and beams connect the remaining notes in each half of the phrase. The first group of beamed notes form another instance of [0147], thereby associatively linking the vertical and horizontal musical dimensions. The second set of beamed notes is not another complete linear projection of [0147] (hence the dashed beam), but rather just the important minor 3rd subset, or set-class 2–3 [03]. Combining this [03] with the upper two notes of the lower strings {A, E}, creates another instance of set-class [0147], further linking the two dimensions. This model of the phrase seems to deal with the literal musical surface better than the prolongational model (with all of its parenthetical notes), yet it is still not entirely satisfactory. Straus offers this comment about the associational model: "it gives no sense at all of one thing going to another. It has the static, lifeless quality that many people have complained about in set-theory analyses."[5] In a sense, this Webern analysis is a catalog, and while suggesting a similarity between the chords and melody, it offers no greater insight into the active musical processes than did assuming the registral lines were voices.

Example 3: Associational interpretation of the Webern excerpt



The transformational model originates in the work of David Lewin, with additional significant contributions by Henry Klumpenhouwer, John Roeder, and Straus, among

[5] Straus, "Voice Leading in Atonal Music," 3.

others.[6] I will not offer a sample transformational analysis of the Webern excerpt here for two reasons: in general because the transformational model is the central focus of the body of this dissertation, and more specifically because an extensive transformational exploration of this phrase begins Chapter 3. For now let it suffice to say that the transformational model, like traditional tonal voice leading, focuses on musical processes, rather than musical content. The prolongational model attempts to do this, but when embellishing tones are devoid of function, they become mere content. By this I mean a note a half-step away is simply that; it is no longer a neighbor tone involved in the process of elaboration. The associational model avoids the arbitrary distinction between structural and embellishing tones, but with its emphasis on static collections, it too lacks a sense of musical motion. The transformational model seeks out the musical actions that drive the voices forward. If in the Webern excerpt the first violin line does comprise a voice, how does the opening F move through its series of pitches to arrive at the concluding Bb? If the second violin line continues that voice, how does the opening C eventually arrive at the final Db? And what are the processes relating the two violin lines? These are the types of questions that I seek to answer through the application of transformational techniques.

Throughout this dissertation the term voice leading specifically refers to pitch-class counterpoint. That is, it refers first to the linear connections between two musical "events," such as two adjacent chords or simultaneities, and then to a series of such connections

[6] Examples of the transformational approach include: David Lewin, "Transformational Techniques in Atonal and Other Music Theories," *Perspectives of New Music* 21.1–2 (1982): 312–371, *Generalized Musical Intervals and Transformations* (New Haven: Yale University Press, 1987), "Klumpenhouwer Networks and Some Isographies that Involve Them," *Music Theory Spectrum* 12.1 (1990): 83–120, *Musical Form and Transformation: 4 Analytic Essays* (New Haven: Yale University Press, 1993), and "A Tutorial on Klumpenhouwer Networks, Using the Chorale in Schoenberg's Opus 11, No. 2," *Journal of Music Theory* 38.1 (1994), 79–101; Henry Klumpenhouwer, "A Generalized Model of Voice Leading for Atonal Music" (Ph.D. dissertation, Harvard University, 1991); John Roeder, "A Theory of Voice Leading for Atonal Music" (Ph.D. dissertation, Yale University, 1984), "Harmonic Implications of Schoenberg's Observations of Atonal Voice Leading," *Journal of Music Theory* 33.1 (1989): 27–62, and "Voice Leading as Transformation," *Musical Transformations and Musical Intuition: Eleven Essays in Honor of David Lewin*, ed. Raphael Atlas and Michael Cherlin (Roxbury, MA: Ovenbird Press, 1994): 41–58; and Straus, "Voice Leading in Atonal Music."

creating longer lines or voices. This shares with traditional counterpoint a concern for the path traveled by each note over the course of a musical passage in the context of the paths traveled by the other notes in the same passage. This is distinct from traditional counterpoint in that pitch-class transformations—rather than intervallic consonance and dissonance—generate the sense of linear musical motion. Additional similarities and differences between my definition of voice leading and traditional uses of the term will become apparent as I explain and demonstrate my methods in subsequent chapters.

The purpose of this study is to introduce, explain, expand, and demonstrate by analytical example a transformational model of atonal voice leading. Chapter 1, "Transformational Techniques," explains and expands the model, while the remaining two chapters—Chapter 2 "Analyses I: Homophonic Applications" and Chapter 3 "Analyses II: Linear and Polyphonic Applications"—offer nine analyses illustrating the versatility of the model in the context of a markedly diverse repertoire of non-tonal twentieth-century music. My investigation of transformational voice leading begins with the premise that pitch-class lines created by the operations of traditional set theory (that is, transposition and inversion) are contrapuntal voices, and culminates in the introduction of a few original voice-leading transformations which I currently group under the generic label *dual transformations*. Along the way I also probe recent theoretical literature—most notably the work of Forte, Straus, Lewin, and Klumpenhouwer—for alternative transformations that allow mappings between sets belonging to different set classes. Faced with the myriad possibilities of an expanded transformational universe, it is necessary to impose theoretical and analytical limitations for the model to remain effective. Various musical transformations range from being extremely restrictive to being all-inclusive, as well as spanning the gamut from describing very audible surface progressions to seemingly inaudible deeply embedded structures. The analytical work I find most satisfactory avoids the extremes of these ranges. It falls instead between the poles of exclusivity and promiscuity, that is, between reliance on those transformations that generate a very limited number of sets versus those that allow any set to generate any other set. At the same time, it ranges comfortably between being overly concrete or overly abstract, that is, between models that explain nothing more than the score itself versus those that seem to have absolutely no relation to any potentially perceivable musical experience. Therefore it is perhaps most convenient to imagine my work as an ongoing search for an ideal analytical location on these two related, but non-identical, continua: exclusivity⇔promiscuity and concrete⇔abstract. Current deficiencies in the transformational model result primarily from most of the analytical tools being located at or near the extremes of these continua; my remedy is to seek out a balance.

Chapter 1

Transformational Techniques

> It should be observed that by melodic interval is meant the silent passage made from one sound or step to the next; it is intelligible though inaudible.[1]

Despite its mid-sixteenth-century origins, the above comment by Gioseffo Zarlino captures the essence of transformational voice leading: the *intelligible passage* from one sound to the next. This chapter presents the general principles and guidelines of the transformational model in the following four sections:

- 1.1 Transformations and Mappings
- 1.2 Similarity, Networks, and Singleton Transformations
- 1.3 Klumpenhouwer Networks
- 1.4 Dual Transformations

Section 1.1 introduces the basic premises of the model, with an emphasis on the traditional set-theoretical operations of transposition and inversion. Section 1.2 focuses on attempts to move beyond the exclusivity of the set classes, primarily by allowing the use of alternative transformations. Section 1.3 examines the most significant recent development in transformational voice leading: the powerfully elegant Klumpenhouwer networks. The chapter culminates in Section 1.4 with the introduction and explanation of several original voice-leading transformations grouped under the label dual transformations. These theoretical tools—particularly Klumpenhouwer networks and dual transformations—coalesce in the nine transformational analyses presented in chapters 2 and 3.

[1] Gioseffo Zarlino, *The Art of Counterpoint;* Part Three of *Le Istitutioni harmoniche*, 1558 edition, translated by Guy A. Marco and Claude V. Palisca (New Haven: Yale University Press, 1968): 2.

1.1 Transformations and Mappings

Transposition and inversion, the canonical operations of traditional set theory, offer powerful tools for grouping the many pitch-class sets into a much smaller number of set classes. More specifically, transpositional equivalence reduces the 4096 possible sets (2^{12}, including the null and universal sets) to 352 set classes, and inversional equivalence further decreases the number to 224. The table in Example 1.1–1 outlines the specific number of these equivalence classes within each cardinality type. Although numerous equivalence classes based on other transformational relationships are possible, the term set class henceforth refers specifically to those pitch-class sets related by transposition or inversion, as shown in the final column of Example 1.1–1.[2] All other equivalence classes are hereafter given specific names, for example, Klumpenhouwer classes.

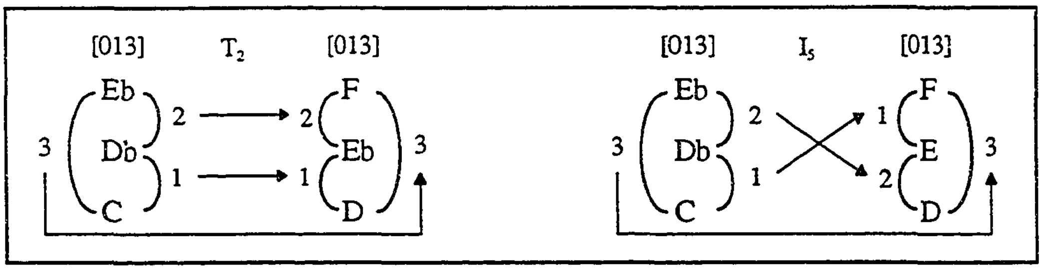
Example 1.1–1: Set classes by cardinality type

card.	pcsets	T_n	T_n or I_n
0	1	1	1
1	12	1	1
2	66	6	6
3	220	19	12
4	495	43	29
5	792	66	38
6	924	80	50
7	792	66	38
8	495	43	29
9	220	19	12
10	66	6	6
11	12	1	1
12	1	1	1
totals	**4096**	**352**	**224**

[2] Allen Forte systematically introduces these set classes in *The Structure of Atonal Music* (New Haven: Yale University Press, 1973). John Rahn coins the term T_n/T_nI-types for these classes, and discusses other equivalence classes, in Chapter Four, "Set Types," of *Basic Atonal Theory* (New York: Longman, 1980): 74–96.

Preservation of the total interval-class content of any pitch-class set under both transposition and inversion—as represented by the set's interval-class vector—is the primary justification for these set classes. Example 1.1–2 demonstrates this intervallic relationship using two trichords from set-class 3–2 [013].[3] The interval-class vector of these sets is [111000], representing one occurrence each of interval classes 1, 2, and 3. The trichords in this example all appear in normal form from bottom to top, thus maintaining the intervals in the same location under the T_n operation (the left side of Example 1.1–2), and very visibly exchanging the inner set of intervals under the I_n operation (the right side of Example 1.1–2). The arrows in the example merely illustrate the location of the interval class in the set after the relevant operation occurs. In notating transposition I use the conventional T_n in which n represents the ordered pitch-class interval between corresponding pitch classes, and for inversion I use the similar notation I_n in which n represents the sum of the corresponding pitch classes.[4]

Example 1.1–2: Interval-class preservation in transposition and inversion



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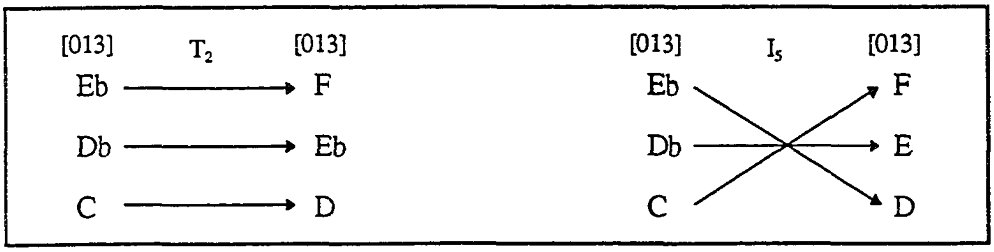
[3] Within each section I introduce set classes by their Forte names, and thereafter use only their prime form to identify them.

[4] I prefer the notation I_n to both the common T_nI or David Lewin's more recent I^p_q. Not only is I_n more succinct than T_nI, but T_nI prioritizes inversion around C (or pitch-class 0), making it a compound or two-stage operation, while I_n directly maps one pitch-class to another along the wedge of the n index. Lewin develops the I^p_q notation in Chapter 3 "Generalized Interval Systems: Formal Features" of *Generalized Musical Intervals and Transformations*, particularly pages 50–56. Like I_n, Lewin's I^p_q notation is a single operation, but it is perhaps too specific for my purposes in its prioritization of the two pitch classes replacing the variables p and q. Admittedly, such notational distinctions are subtle, as the n variables in I_n and T_nI are identical and $p + q = n$ in I^p_q, but nonetheless I feel they suggest slightly different theoretical concepts.

The adoption of these equivalence classes allows the systematic tracing of pitch-class "lines" or mappings that result from the transformation of one member to another of the same set class. For two members X and Y of the same set class, at least one T_n or I_n operation maps each element of set X onto exactly one element of set Y. Example 1.1–3 illustrates these mappings using the same trichords shown in Example 1.1–2. The trichords again appear in normal form in these examples, making the difference between the two processes—transposition and inversion—readily apparent. The horizontal arrows of the transpositional mappings in Example 1.1–3 result from the direct correspondence of the intervals in Example 1.1–2 (left side), and the diagonal arrows of the inversional mappings in Example 1.1–3 result from the interval exchange in Example 1.1–2 (right side).

Example 1.1–3: Pitch-class mappings



At this juncture it is critical to emphasize that the respective T_n and I_n operations are not meant to be understood as mere static measures of distance or passive relationships, but rather as active transformations that dynamically project each element of the first set into its new "location" as a member of the second set. In other words, the process of transposition or inversion—in this case T_2 or I_5—generates the second set by actively transforming each pitch class of the first set. David Lewin has been, and continues to be, the driving force in this pivotal reinterpretation of pitch-class set operations.[5]

Joseph Straus suggests that operational mappings interpreted in this manner "form an underlying pitch-class counterpoint" connecting any two members of the same set class,

[5] See bibliography for references, particularly *Generalized Musical Intervals and Transformations.*

and further suggests that these contrapuntal mappings are the atonal equivalent of voice-leading lines (henceforth voices).[6] Straus's analogy arises from a simple distinction between surface lines (for example, registral or instrumental lines) and transformational voices. Specifically, on one hand there are significant musical lines such as the successive notes played by a first violin, or the consecutive notes in a single register, while on the other hand there are voices such as a leading-tone resolving to a tonic that can cut across registral or instrumental boundaries. In tonal music, voices are often confused with registral or instrumental lines, yet in a V → I progression the leading-tone is understood to map onto (that is, resolve into) the tonic even if the resolution occurs in a different register or instrument. The tonal transformation V → I, or any other progression, consists of certain abstract contrapuntal motions that are understood as voices regardless of any particular musical realization.[7] If, in atonal music, transposition or inversion transforms one pitch-class set into another, then the projection of each element in the first set onto an element in the second set—via either of these operations—is comparable in its abstract contrapuntal nature to tonal voice leading.

The correspondence between pitch-class mappings and any given musical surface varies drastically. To reiterate Straus's distinction, a line is articulated by some surface parameter (register, timbre, etc.), while a voice traces some transformational path that may or may not coincide with the musical surface. The Webern excerpt in Example 1.1–4 traces one-to-one operational mappings through a series of pitch-class sets belonging to set-class 3-3 [014].[8] The transformational voices in this example are identical with both the

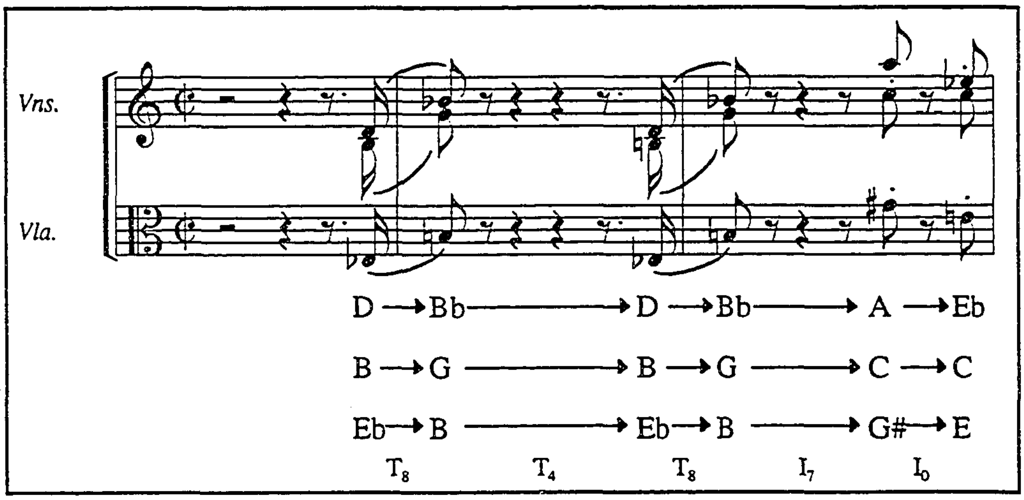
[6] Straus, "Voice Leading in Atonal Music," 4.

[7] For an interesting discussion of abstract counterpoint in tonal music see William E. Benjamin's "Pitch-Class Counterpoint in Tonal Music," in *Music Theory: Special Topics*, ed. Richmond Browne (New York: Academic Press, 1981): 1–32.

[8] Straus uses the opening of this movement as Example 2 in "Atonal Voice Leading." I omit the cello's C# pedal, as only the upper three voices participate in these operations. I offer a more extensive study of this movement in Chapter 3.

instrumental and registral lines. The Stravinsky excerpt in Example 1.1–5 traces operational mappings through a series of alternating tetrachords belonging to set-class 4–8 [0156]. In this progression both transpositional and inversional mappings are possible owing to the symmetrical properties of set-class [0156]; the example illustrates both operations. In sharp contrast to the previous Webern example, the transformational voices in the Stravinsky excerpt do not coincide with either the registral or instrumental lines, or with each other.

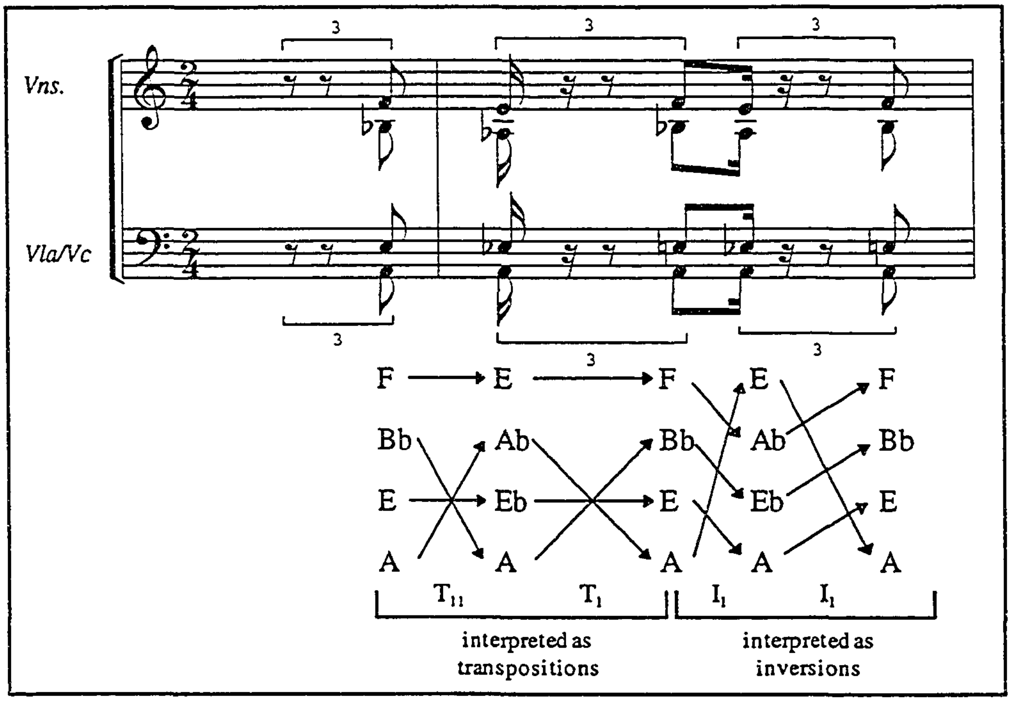
Example 1.1–4: Webern, Five Pieces for String Quartet, Op. 5, No. 3, mm. 1–3



While the surface reinforcement of the transformational voices in the Webern excerpt agrees with my musical intuition, the Stravinsky example raises questions about multiple mappings and criteria for satisfactory readings. The definition of voice leading based on the distinction between surface lines and operational mappings simultaneously elevates two conflicting sets of lines—one generated by T_{11} followed by T_1 and another by I_1—to the status of voices. Preference for either transformation in this progression becomes a matter of contextual analytical discretion, rather than a simple definitional issue, and calls the concrete⇔abstract continuum into play. The inversional mappings seem to me a clear case of overabstraction. Hearing the cello A2 transform into the first violin E4 is particularly counterintuitive as the cello retains the A2 throughout the passage. The I_1

transformation is so far removed from my experience of the musical surface that the voices it generates are analytically unacceptable without further structural justification. The transpositional mappings are not much better, but at least the soprano voice coincides with the first violin line, the tenor voice coincides with the viola line, and the cello A2 moves to the ever so slightly more plausible Ab3 in an alto–bass exchange. These transpositional voices are a little less abstract, but they are still unsatisfactory. In this homorhythmic excerpt I want the voices to interact in the most rich way with the mutually reinforcing registral and instrumental lines. To model my experience of this progression, at the very least I want a bass voice that retains the A2. In other words, I want to move further along the continuum in the direction of concreteness. To achieve this end, it is necessary to move beyond the restrictions of traditional operations by developing new transformations to describe these and similar musical events in a more satisfactory way. I will offer a few alternative interpretations when I return to this passage in the final section of this chapter.

Example 1.1–5: Stravinsky, Three Pieces for String Quartet, II, mm. 0–1



The most significant limitation of a transformational voice-leading model based solely on transposition and inversion is the self-contained nature of the set-class equivalence classes. At most twenty-four sets belong to any single set class—twelve T-related sets and twelve I-related sets—though many classes have even fewer members owing to varying degrees of symmetry (for example, the set class 6–35 [02468a], commonly known as the whole-tone hexachord, has only two members). Therefore, at best, a series of transformational voice mappings can only occur among twenty-four out of the 4096 possible pitch-class sets, placing this prohibitive transformational model at the extremely exclusive end of the exclusivity⇔promiscuity continuum. The stringent nature of these equivalence-classes inevitably leads towards an analytical approach that traces independent voice-leading paths for each prominent set class within a given musical work. Remember, by definition pitch-class sets from different set classes are not related by transposition or inversion. In addition to the analytical fragmentation caused by the unfolding of several simultaneous—yet independent—transformational pathways (one for each significantly recurring set class), large temporal gaps often separate two "adjacent" members of the same set class. The intervening musical passages must be ignored in order to map the voices directly from one member of these equivalence classes to the next. The transformational interpretation of the traditional operations, that is, transposition and inversion as dynamic processes rather than static relationships, is severely weakened in such circumstances. Straus does not find the analytical fragmentation and temporal discontinuity of this model a problem, stating:

> It is evident that I have carefully selected harmonies that are related by transposition or inversion and ignored the rest...My analysis teases out of the musical fabric a single strand and describes the voice leading connections that give that strand its integrity and coherence. The music is comprised of many interwoven strands, of varying content and length...no single strand endures from beginning to end...The remaining notes belong to other strands, other transformational networks, each with its own internal voice leading...My analyses do not purport to describe the voice leading of an entire passage. Indeed, I do not think this music lends itself to single, all-encompassing readings of that kind.[9]

On the contrary, it is the model, rather than the music, which does not lend itself to all-encompassing readings. It is absolutely essential that a successful voice-leading model be capable of describing complete passages, because there must be some aural continuity in these musical gestures or we would not reify them with the term "passages." This concept is so significant that chapters 2 and 3 of this dissertation are devoted purely to analyzing complete musical passages, as well as a few complete works. Transformational voice leading—as a form of pitch-class counterpoint—models musical motion, which I equate foremost with the aural continuity of contiguous events. Perhaps Zarlino's intelligible passage from one sound to the next remains the best description. That is not to say there are never interpolations that interrupt the flow of musical processes; but the overexclusivity of the traditional transformational model makes disruption the norm rather than the exception. Therefore expansion of the transformational machinery is necessary to avoid the potential pitfalls of overabstraction and overexclusivity.

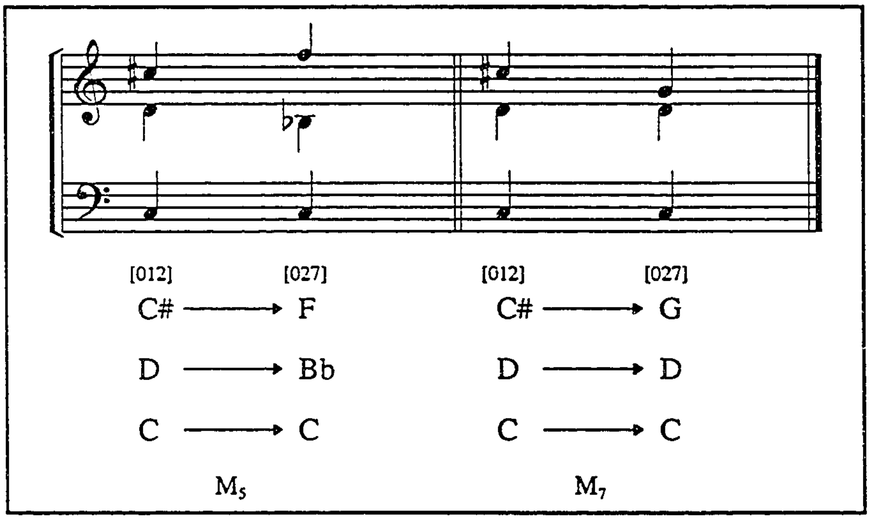
Inclusion of the less frequently discussed multiplication operations, notably M_5 and M_7 (or M and MI), can further reduce the number of set classes to 158 [refer back to Example 1.1–1].[10] But as these operators transform the interval content of a set, exchanging instances of interval-classes 1 and 5, they alter the sound quality of the set

[9] Straus, "Voice Leading in Atonal Music," 7.

[10] For an early discussion of these operators, see Hubert S. Howe's "Some Combinational Properties of Pitch Structures," *Perspectives of New Music* 4.1 (1965): 45–61. John Rahn also includes a brief section titled "Multiplicative Operations" in Chapter 3, "Basic Operations," of *Basic Atonal Theory*, pp. 53–56.

more substantially than T_n or I_n. Therefore it is perhaps best not to consider such related sets as equivalent. As Robert Morris puts it: they "are more controversial with regard to their aural realizations as compared with T_n and T_nI operators."[11] Nonetheless, M_5 and M_7 operations do allow mappings between some pairs of different set classes, for example, 3–1 [012] and 3–9 [027] as shown in Example 1.1–6. The interval vectors of these two trichords clearly illustrate the shift of the interval classes from 1 to 5: [012] [210000] → [027] [010020].

Example 1.1–6: M_5 and M_7 mappings



While these multiplicative operations are powerful in allowing transformations among pairs of set classes, they are much more abstract than transposition and inversion. What exactly does it mean musically to multiply a pitch or pitch class? The abstract quality of these operations make them difficult to conceptualize horizontally. We can trace the transformational mappings, but the "M-ness" is exceedingly difficult to hear as voices, despite the very audible intervallic change in the collections. As my focus is voice leading,

[11] Robert D. Morris, *Composition with Pitch-Classes: A Theory of Compositional Design* (New Haven: Yale University Press, 1987): 148.

specifically referring to the process of getting from one musical event to another, I will abandon multiplicative transformations here.

Once the transformational floodgates are open beyond the traditional limits of transposition and inversion (and multiplication too), Straus's definitional distinction between line and voice is superfluous. Anything becomes possible as one can create a unique transformation to elevate any line to the status of a voice. This leads to the opposite extreme of the exclusivity⇔promiscuity continuum, and John Roeder follows this turn of events to its logical conclusion. He proposes a transformational voice-leading model in which registral lines are once again the voices, and independent transpositions occur within each voice. He states of homophonic textures:

> we can conceive of voice leading as the intervals between registrally corresponding members of simultaneities...when one simultaneity succeeds another of the same size, we hear a *voice* as the succession of pitches in the same registral order position, and we consider the *voice leading* to be the intervals spanning those pitch successions.[12]

Roeder would notate the transformations back in Example 1.1–5 as <0,11,10,11> and <0,1,2,1> in which the integers represent the individual T_n subscripts ordered by register from bass to soprano. Not only does Roeder's model suffer by being too promiscuous, it is also too concrete. His use of angle brackets to turn multiple transpositions into a single transformation works on paper, but it is still quite difficult to hear all of the independent horizontal intervals as a single transformation. Roeder's expansion of the transformational model differs significantly from the one I will develop over the course of this dissertation as it is built upon the assumption that voices are by default registral lines, while my premise is that any set of possible mappings are potential voices. The number of possible one-to-one mappings between two sets is calculated by n!,

[12] John Roeder, "Voice Leading as Transformation," 41. His dissertation, "A Theory of Voice Leading for Atonal Music," explores this approach in great detail.

where n equals the cardinality of the sets. For example, two trichords have 3 x 2 x 1 or 6 possible mappings, and two tetrachords have 4 x 3 x 2 x 1 or 24 possible mappings.[13] To reduce the number of acceptable mappings, I will propose—throughout the rest of this chapter—various compromises between the transformational unity of the traditional operations and the total transformational independence of Roeder's voices. Furthermore, as I will demonstrate in chapters 2 and 3, specific musical contexts and the process of interpretive analysis then narrow the remaining possibilities to the much fewer musically plausible ones.

[13] A very interesting article by Ramon Fuller, "A Study of Interval and Trichord Progressions," *Journal of Music Theory* 16.1–2 (1972): 102–140, considers the voice leading to be the sum of all the possibilities rather than any single mapping. Fuller limits his study to trichords with their six mappings as larger sets become unwieldy.

1.2 Similarity, Networks, and Singleton Transformations

Traditional set theory has developed a substantial number of similarity relations for comparing members of different set classes, even those of different cardinalities. Most of these relations revolve around comparisons of pitch-class and/or total interval content. Allen Forte introduces three R-relations—R_p, R_1, and R_2—to compare members of different set-classes, though still of the same cardinality. R_p relates sets that differ by a single element, either literally ("strongly represented," for example, {C, D, E} and {C, D, F}) or abstractly by equivalence type ("weakly represented," for example, {C, D, E} and {F, Ab, Bb}).[14] R_1 and R_2 represent different degrees of intervallic similarity as illustrated by a pitch-class set's interval-class vector (R_1 = four identical entries and an exchange between the remaining two, while R_2 = four identical entries only). The complement relation is often considered the most significant possible relationship between pitch-class sets of different cardinalities. The interval vectors of two complementary sets, for example, the pentachord 5–10 [01346] [223111] and the septachord 7–10 [0123469] [445332], reflect the proportionally similar interval content of the two sets. The interval vector entries of the larger set are always greater by the difference between the two sets' cardinalities, with the exception of interval-class 6 which is greater by half the difference. Sets of different sizes may also be subsets or supersets of one another, such as the two 3–2 [013] or 3–3 [014] trichords within every 4–3 [0134] tetrachord. These relationships—complementation and inclusion—lead to Forte's set-complexes, K and Kh.[15] The K-relation implies that a pitch-class set can contain or be contained in another set *or* its complement. The Kh-relation refines the K-relation by requiring the pitch-class set to contain or be contained in both the other set *and* its complement. While all of these

[14] Forte, *The Structure of Atonal Music*, particularly Section 1.13 "Similarity Relations," pp. 46–60.

[15] Ibid., pp. 24–29, presents a more fully developed discussion of subsets, and pp. 93ff. introduce K and Kh complexes.

comparative relationships provide a system of measuring some of the similarities and differences between two or more sets, they do not suggest the one-to-one mapping of voices as described above. They are generally interpreted as static measures, rather than dynamic transformations.

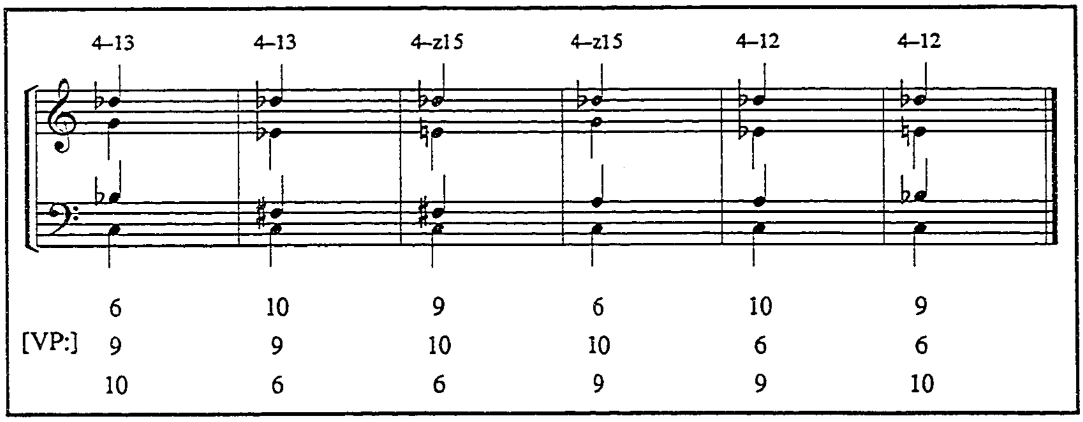
A number of more rigorous similarity functions are currently in circulation. Many of these functions take two sets as arguments and return a single numerical value determined through comparisons of the sets' interval vectors or more extensive subset counts. Perhaps the most well-known of these functions are Robert Morris's SIM, John Rahn's MEMB, and David Lewin's REL.[16] More recently, theorists have attempted to refine these functions and present alternatives; most notable are Eric Isaacson's IcVSIM, Marcus Castrén's RECREL, and Michael Buchler's SATSIM.[17] Although some of these functions model subset structures that relate to the transformations I develop later in this chapter, they still do not represent a dynamic transformational process or suggest specific mappings. In general, these similarity measures provide information that allows an analyst to group sets into more or less similar families, but offer no clues toward deciphering possible contrapuntal voices.

[16] Robert D. Morris, "A Similarity Index for Pitch-Class Sets," *Perspectives of New Music* 18.1–2 (1979-80): 445-460; John Rahn, "Relating Sets," *Perspectives of New Music* 18.1–2 (1979–80): 483–498; and, David Lewin, "A Response to a Response: On Pcset Relatedness," *Perspectives of New Music* 18.1–2 (1979–80): 498–502. Charles Lord's sf, presented in "Intervallic Similarity Relations in Atonal Set Analysis," *Journal of Music Theory* 25.1 (Spring 1981): 91-111, and Richard Teitelbaum's s.i., presented in "Intervallic Relations in Atonal Music," *Journal of Music Theory* 9.1 (1965): 72-127, are also important early similarity functions.

[17] Eric J. Isaacson, "Similarity of Interval-Class Content between Pitch-Class Sets: the IcVSIM Relation," *Journal of Music Theory* 34.1 (Spring 1990): 1-28, and "Issues in the Study of Similarity in Atonal Music," *Music Theory Online* 2.7 (1996); Marcus Castrén, *RECREL: A Similarity Measure for Set-Classes*, Studia Musica 4 (Helsinki: Sibelius Academy, 1994); and Michael Buchler, "An Alternative to the Interval-Class Vector and its use in Relating Abstract PC Sets," paper presented at the annual Music Theory Society of New York State Conference (Buffalo, 1995), and "Relative Saturation of Subsets and Interval Cycles as a Means for Determining Set-Class Similarity," (Ph.D. dissertation, Eastman School of Music, University of Rochester, 1997). Thomas R. Demske's "Relating Sets: On Considering a Computational Model of Similarity Analysis," *Music Theory Online* 1.2 (1995) is another recent work of interest.

Alternatively, a number of recent theoretical models explore the internal structure of pitch-class sets to create networks allowing relations among greater numbers of sets. Alan Chapman introduces AB ("above-bass") and VP ("voice-pair") interval sets to examine the structure of registrally distributed pitch-class sets.[18] AB interval sets are the atonal equivalent of thoroughbass identifications, labeling all the intervals from the lowest voice. The more innovative VP interval sets label the intervals between adjacent voices. As Chapman points out, a "VP set which consists of three different intervals may appear in six distinct permutations," and "a given VP set may be expressed by specific voicings of *more than one* pitch-class set."[19] Example 1.2–1 reproduces Chapman's accompanying example. Note that the resulting pitch-class sets are inversionally-related pairs from three different set classes. As transposition without permutation of these six pitch-class sets does not alter their VP interval set, there are seventy-two members of this VP class. Interactions between set classes and VP classes offer the potential of using VP relations to link different set classes, or the converse, using traditional set classes to link different VP classes.

Example 1.2–1: Chapman's VP interval set 6/9/10

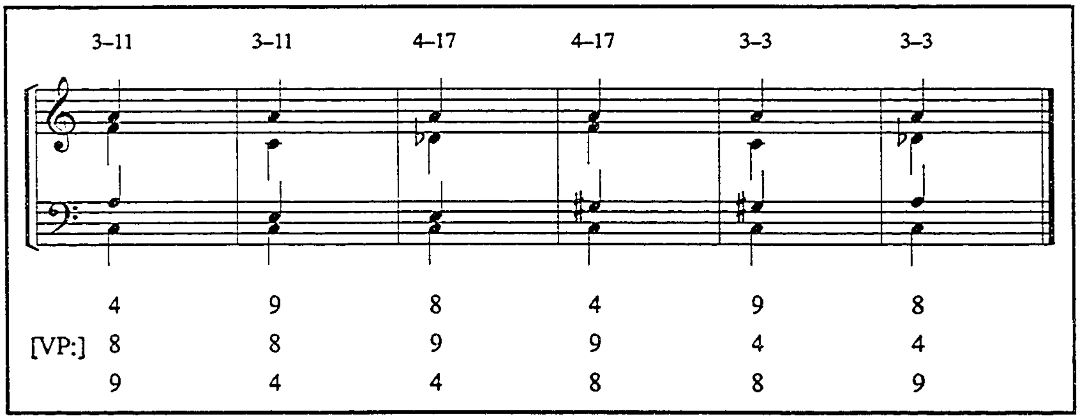


[18] Alan Chapman, "Some Intervallic Aspects of Pitch-Class Set Relations," *Journal of Music Theory* 25.2 (1981): 275–290. For further information see his dissertation "A Theory of Harmonic Structures for Nontonal Music" (Ph.D. dissertation, Yale University, 1978).

[19] Ibid., 280. The three examples I include here are Chapman's Examples 8, 9, and 11, pp. 281–282.

Not only do VP classes cross set-class boundaries, they can also span cardinality types. Example 1.2–2 reproduces Chapman's example illustrating this point. Again there are inversionally-related pairs from three different set classes, but this example includes both trichords and tetrachords. VP-class 4/8/9 still includes seventy-two members, but two-thirds of them are trichords with doublings. Any VP class with a pair of intervals that sum to 0 mod-12 (the 4 and 8 in this case) will have this cardinality-crossing property. Despite the power of the VP interval sets to cross traditional boundaries, Chapman limits his linear exploration of the model to VP interval exchanges. Example 1.2–3 reproduces one of his examples illustrating an interval exchange.[20] Unfortunately, this limitation essentially exchanges VP classes for set classes, ignoring the potential interaction of the two classes, and leaves the analyst with many of the same problems of continuity.

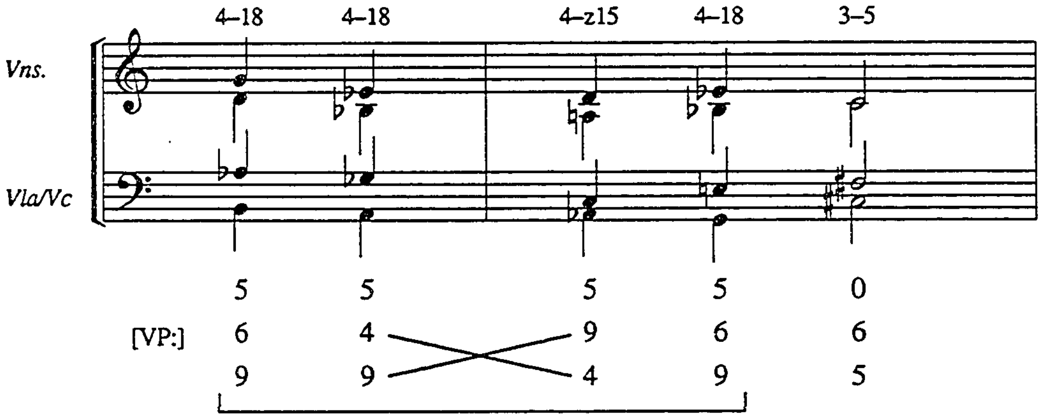
Example 1.2–2: Chapman's VP interval 4/8/9



[20] I offer a more extensive study of this movement in Chapter 2.

Example 1.2–3: Chapman's VP interval exchange

Stravinsky, *Three Pieces for String Quartet*, III, mm. 17–18



Developing similar ideas, David Lewin encourages relations between many musical events of various sizes or configurations through the application of numerous context-specific transformations.[21] Most significantly, he illuminates the dynamic nature of any set's internal structure, and, therefore, its inherent ability to be a transformational model for subsequent musical gestures.[22] Lewin interprets the intervallic content within a set as transformations of pitch classes analogous to the between-set transformations discussed above. The model of the first trichord in Example 1.1–2 above is but one interpretation of that set's interval content; using Lewin's methods there are a total of eight possible models for any trichord, shown in Example 1.2–4. Unlike Chapman's VP interval sets, these transformational models do not relate pitch-class sets from different set classes, but they do emphasize particular transformations within the set class. One might consider the resulting set networks to be specific interpretations of the set's interval-class vector.[23] Transpositions of the whole set by any one of the six internal T_n values begins to suggest a coordination

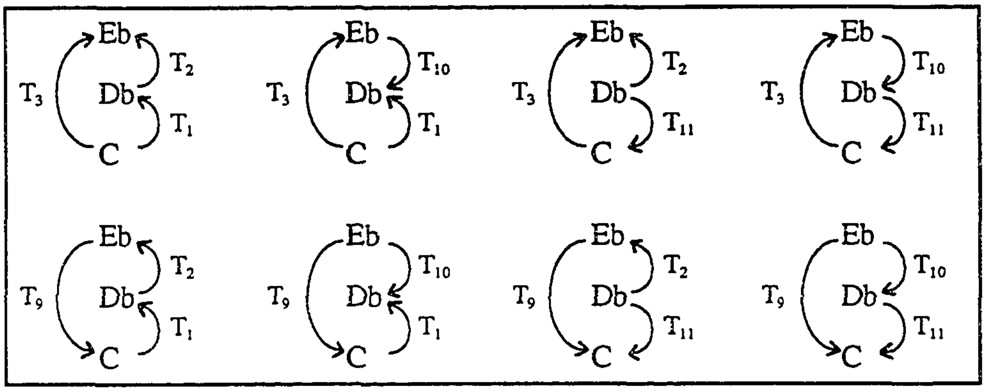
[21] The development and application of these context-sensitive transformations are most thoroughly presented in his book *Generalized Musical Intervals and Transformations.*

[22] Particularly relevant is Lewin's "Transformational Techniques in Atonal and Other Music Theories."

[23] Incidentally, Lewin networks—as graphic models of the interval-class vector—are very useful in illustrating the subtle structural differences between z-related sets not evident from the vector itself.

among multiple levels of structure. A listener's particular reading of a set and its subsequent transformations are mutually influential.

Example 1.2–4: All eight models of [013] using internal transpositions



None of the above theoretical models yield specific pitch-class mappings, but they do relate to the development of three crucial precursors of my work that I call singleton transformations: Forte's "unary voice-leading transformation," Lewin's "if-only," and Straus's "near-transformations."[24] The most restricted of these is Forte's unary transformation. A unary transformation is the "mutation" of a pitch-class set by the transposition of only one of its members, the remaining members being held constant. As Forte points out: "the unary transformation effects the strongest possible pitch-class connexion other than complete identity, since it preserves n-1 pitch classes over two collections of n elements."[25] He further refines the transformation with V_n labels in which n equals the T_n subscript of the singleton. For example, V_2 means the singleton moves by T_2 while the remaining pitch classes stay unchanged. As shown in Example 1.2–5, which reproduces his first example, Forte's analytical goals in this article are not much different than the associational model, the primary distinctions being the animation provided by the

[24] Forte presents "unary transformations" in "New Modes of Linear Analysis," Lewin discusses "if-only" in great detail in "Transformational Techniques," and in less detail in *Generalized Musical Intervals and Transformations*, and Straus introduces "near-transposition and near-inversion" in "Voice Leading in Atonal Music."

[25] Forte, "New Modes of Linear Analysis," 6.

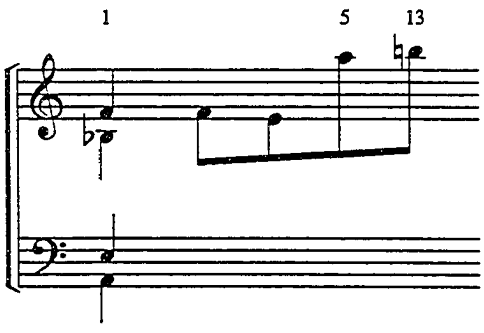
V_1 arrow, and the related ability to incorporate sets from different classes. Not only is this a dynamic form of the associational model, but it is also a transformational interpretation of one of his earlier similarity relations, specifically the strong R_p relation. Forte points out that:

> unary voice-leading transformations are directly related to the various categories of similarity relations, beginning with Rp. However, in their reincarnation here they have a much more dynamic, far less abstract impact on the musical continuum.[26]

Despite the positive impact of this dynamic interpretation on a previously static measure, Forte severely limits the power of this new transformation by modeling only the strong R_p relation. Rather than making a substantial move away from the exclusive end of the exclusivity⇔promiscuity continuum, the pitch-class restrictions on the V_n transformations firmly suppress the number of possible analytical applications. The remaining two singleton transformations—if-only and near-transformations—take the additional step along the continuum and offer a transformational model of the weak R_p relation.

Example 1.2–5: Forte's unary transformation

Stravinsky, *Three Pieces for String Quartet*, No. 2



V_1: 4–8 ⟶ 4–16
[4,5,9,10] [4,5,9,11]

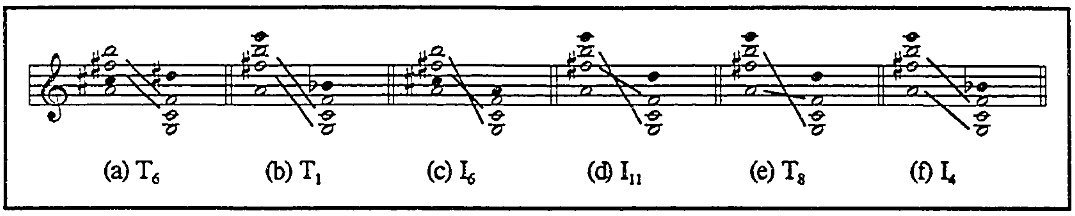
[26] Ibid., 12.

If-only and near-transformations model processes in which all the members of a pitch-class set except a singleton participate in traditional transposition or inversion. Unlike unary transformations, the specific behavior of the non-participatory member is not particularly relevant. The difference between if-only and near-transformations is one of analytical goals. Lewin uses the rogue singleton to set up expectations for later completion of the traditional transformation, while Straus allows the singleton its own transformation by mapping it by default to its counterpart in the next set. Example 1.2–6 reproduces one of Lewin's figures to illustrate if-only.[27] In this example Lewin points out how the progression between the first two chords of Schoenberg's *Piano Piece*, Op. 19, No. 6, closely approximates a number of operations: T_6, T_1, I_6, I_{11}, T_8, and I_4. The open noteheads represent Schoenberg's music, the lines highlight the realized transformational mappings, and the solid noteheads show what the singletons in each chord need to be for a literal T_n or I_n operation. He describes this phenomenon with phrases like "fairly T_6-ish," and discusses the "urges" and "lusts" of the music to complete the suggested operations. Lewin "find[s] it suggestive to think of these generative lusts as musical tensions and/or potentialities which later events of the piece will resolve and/or realize to greater or lesser extents."[28] Example 1.2–7 reproduces a portion of Lewin's next figure to illustrate how Schoenberg's music eventually realizes some of the transformational possibilities suggested by the if-only scenarios in Example 1.2–6. Though I will expand my transformational universe beyond singleton transformations, the lascivious nature of if-only remains valuable and plays an important role in a few analyses in chapters 2 and 3.

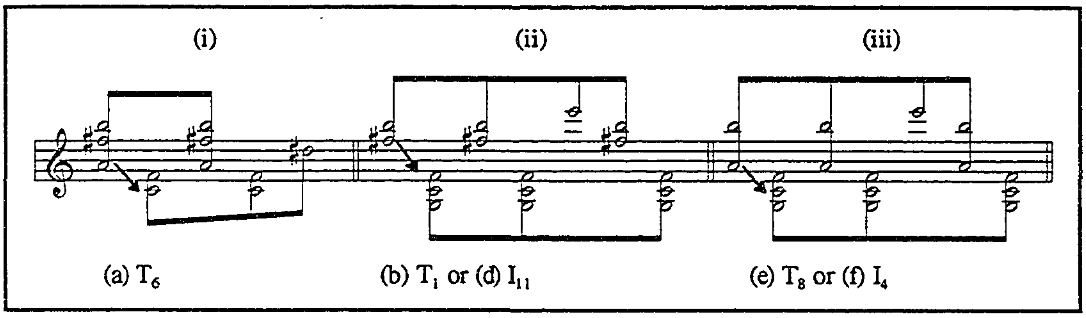
[27] Lewin, "Transformational Techniques," Figure 19, 339. In this reproduction and the next, Figure 20 [340], I replace Lewin's inversion notation with its equivalent I_n.

[28] Ibid., 341.

Example 1.2–6: Lewin's if-only



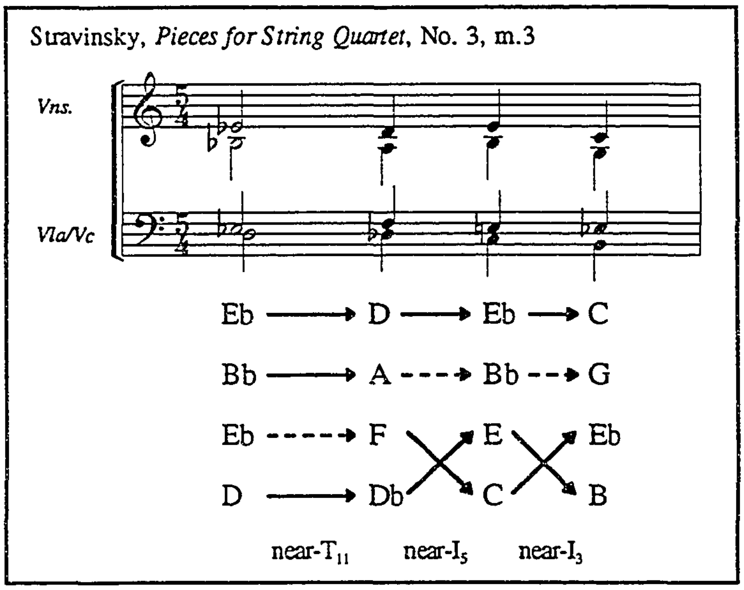
Example 1.2–7: A few completions of the above if-only scenarios



Straus's near-transformations trade Lewin's musical expectations and tensions for the ability to map the music as composed. If-only singletons remain ignored until they reach the goal of the selected traditional operation, sometimes creating problematic gaps or omissions in the mappings. As shown in Example 1.2–8, a reproduction of one of Straus's examples, the default mappings of near-transformations often better account for musical surfaces.[29] In this example solid arrows represent the mappings of the traditional operations, while dashed arrows represent the default mapping of the singletons. If we waited for the viola to reach D because of the "T_{11}-ishness" of the near-T_{11}, or for the second violin to reach Ab or F to fulfill the desires of the near-I_5 or near-I_3, we would be disappointed as the passage cadences without ever reaching those goals. Near-transformations allow continuous voice mappngs despite the lack of resolution of the if-only tensions.

[29] Straus, "Voice Leading in Atonal Music," Example 7a.

Example 1.2–8: Straus's near-transformations



As demonstrated by the previous example, near-transformations are a significant departure from the narrow end of the exclusivity⇔promiscuity continuum. They allow mappings among chords belonging to different set classes and even different cardinality types. While the power of these transformations is theoretically liberating, it is a double-edged sword that once again raises the issue of multiple mappings and criteria for satisfactory readings. Recognizing the increasingly fragile condition of his definitional distinction between voices and lines, Straus points out:

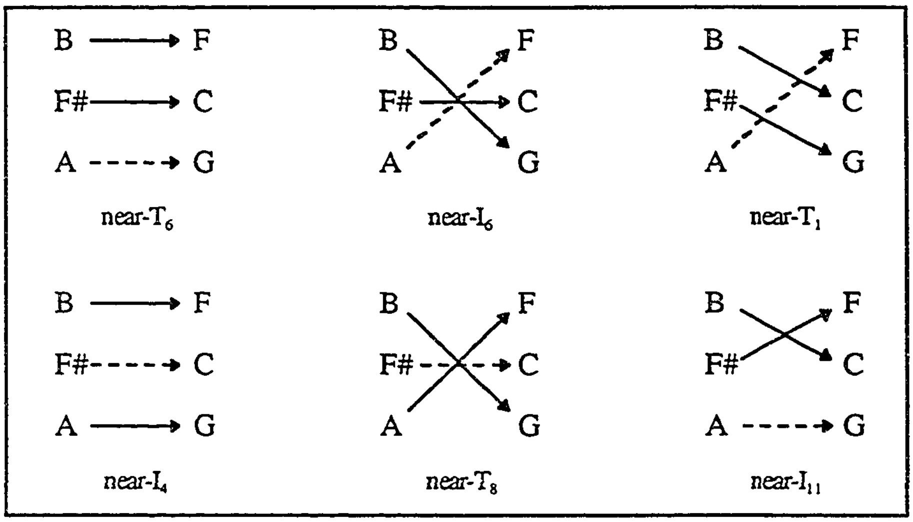
> By increasing the number of transformations we also, of course, increase the number of viable voice leading interpretations... As a general matter, there will be as many different voice-leading interpretations as there are different transformations that could plausibly take us from harmony to harmony.[30]

Example 1.2–9 illustrates some of the resulting problems by reinterpreting the if-only scenarios from Example 1.2–6 as near-transformations. First, keep in mind that there are only six (3!) possible mappings between any two trichords. Then note that these near-

[30] Ibid., 13.

transformations eliminate only two of the six possibilities. Four possible voice leadings are certainly better then none, as we would have had using only traditional operations, but which one, or more, of the four model significant features of this specific progression as Schoenberg realizes it in this particular context? As I hear it, two lines stand out more than the others: B5 → F4 (owing to respective registral placement) and A4 → F4 (owing to registral proximity). Unfortunately, that only rules out near-I_{11}, leaving five near-transformations that include one of those two voices. Further complications arise from having different transformations result in the same set of voices; note the vertically aligned pairs near-T_6 and near-I_4, and near-I_6 and near-T_8 in the example. Now I am left with five different transformations, but only three different mappings. Favoring transposition over inversion, I might decide to eliminate the near-inversions, but that still leaves me with three possibilities: near-T_6, near-T_8, and near-T_1. Isolating this progession I might prefer near-T_6 because of its simplicity, but it is at this point in each application of this model that it becomes necessary to apply context-specific analytical filters. Are there any deeper structural reasons to select a particular interpretation over another? Perhaps the two most important considerations are recurrence and recursion. Does a particular transformation, or even a particular mapping pattern, recur significantly throughout a passage? Is a particular transformation replicated on a different structural level, for example, within the sets themselves or spanning the entire passage? The answers to all of these questions must then be weighed against our musical intuitions in order to arrive at a plausible location on the concrete⇔abstract continuum.

Example 1.2–9: Near-transformations modeling Example 1.2–6



Unary, if-only, and near-transformations pave the way for the final two models I discuss in this chapter: Klumpenhouwer networks and dual transformations. In fact, as instances of the R_p relation, these singleton transformations are subsets of the more generalized Klumpenhouwer and dual transformations.

1.3 Klumpenhouwer Networks

In the most abstract sense, Klumpenhouwer expands Lewin's network model by including I_n operations within the internal structure of a pitch-class set. While transposition alone allows eight ways to model any given trichord (previously illustrated in Example 1.2–4), replacing one or more of the T_n operations with I_n operations allows nineteen additional—for a total of twenty-seven—possible interpretations. Example 1.3–2 presents the nineteen new transformational networks, and Example 1.3–1 reproduces Example 1.2–4 for convenient comparison. Klumpenhouwer's expansion is possible because one element of a dyad can map equally well onto the other via transposition or inversion. This simple abstraction results in the dramatic increase in graphic possibilities shown in these two examples. The practical applications of these new networks, however, are slightly more restricted than their numbers suggest. The table shown in Example 1.3–3 compares the number of possible voice-leading mappings between sets of the same cardinality with the number of possible network models for sets of the given size. The table includes Lewin networks (that is, pre-Klumpenhouwer networks based on T_n transformations only) as well as Klumpenhouwer networks.[31] Obviously, in terms of voice leading, it is excessive to have 32,768 different transpositional interpretations—not to mention having more than fourteen million Klumpenhouwer networks—when there are only 720 different mappings between any two hexachords. The discrepancies among these numbers make it evident that beyond the tetrachord the possibilities become cumbersome, and that beyond the hexachord they become entirely overwhelming. Along with the historical weight of three- and four-

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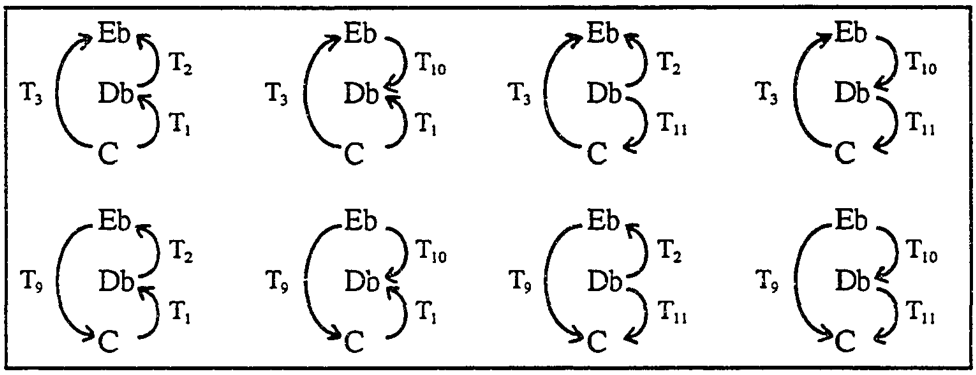
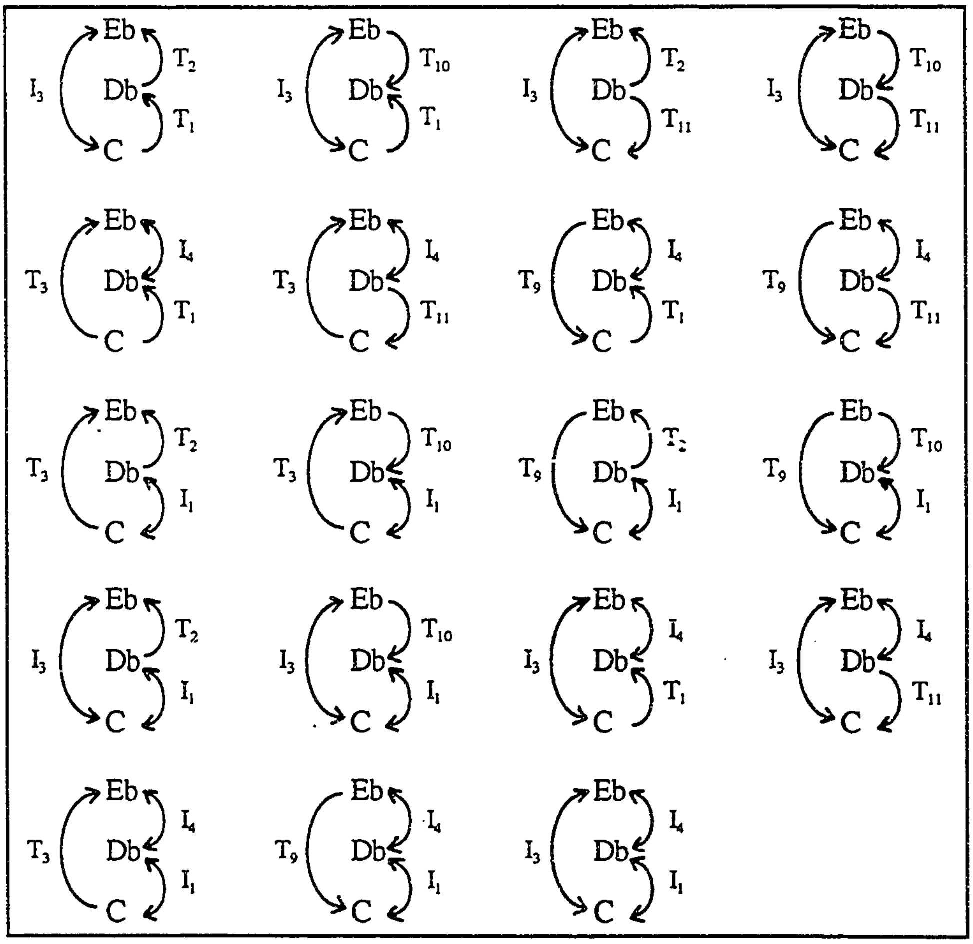
[31] As previously mentioned, the number of possible voice-leading mappings is calculated by n! where n equals the cardinality of the set. The number of possible Lewin networks is calculated by 2^p where p equals the number of interval classes contained in the set. The cardinality n can be used to determine the value of p with the equation $p = n (n - 1) \div 2$. For example: tetrachords contain $p = 4 \times 3 \div 2 = 6$ interval classes, therefore if $p = 6$, then $2^p = 2^6 = 64$ possible Lewin models. The Klumpenhouwer possibilities are calculated by a similar formula 3^p. For example: if $p = 6$ (using tetrachords again) then $3^p = 3^6 = 729$ possible Klumpenhouwer models.

note chords, the exponential unwieldiness of larger collections further explains the emphasis on trichords and tetrachords in the analytical literature.

Example 1.3–1: All eight models of [013] using internal transpositions



Example 1.3–2: Nineteen additional models of [013] using internal inversions



Example 1.3–3: Comparison of possible mappings and networks

Cardinality	Voice-Leading Mappings	Lewin Networks	Klumpenhouwer Networks
dyad	2	2	3
trichord	6	8	27
tetrachord	24	64	729
pentachord	120	1,024	59,049
hexachord	720	32,768	14,348,907
septachord	5,040	2,097,152	10,460,353,203
octachord	40,320	268,435,456	3^{28}
nonachord	362,880	68,719,476,736	3^{36}
decachord	3,628,800	2^{45}	3^{45}
monodecachord	39,916,800	2^{55}	3^{55}
dodecachord	479,001,600	2^{66}	3^{66}

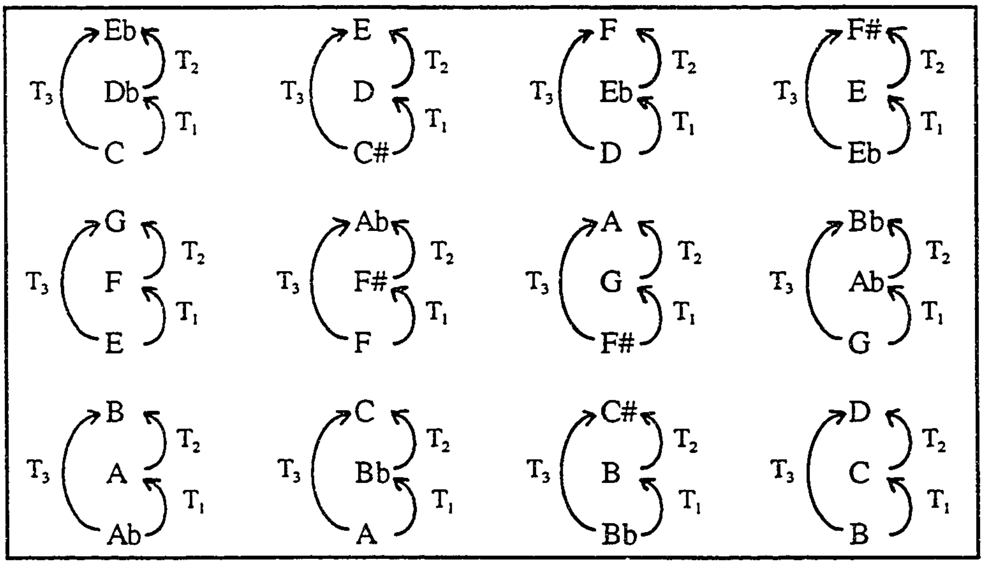
In practice, most of the additional interpretations elicited by this theoretical abstraction do not afford any new transformational relations among pitch-class sets, and the power of Klumpenhouwer networks actually resides in a small subset of these numerous possibilities. Though Lewin and Klumpenhouwer work toward different analytical goals in their applications of these networks, a subject to which I will later return, they both focus on network isography. Isographic networks are interpretations of sets yielding isomorphic graphs, or more simply, networks with the same configuration of nodes and transformational arrows, and a mathematical correspondence among their analogous arrows.[32] Furthermore, if the analogous transformational arrows are identical, networks are said to be strongly isographic. While other types of isography are possible, I will temporarily use strong isography as the measure of a network's potential value. With this criterion it is possible to reduce the multitude of graphic possibilities to a much smaller number of practical Klumpenhouwer networks.

First I will examine the capacity for strong isography among Lewin networks, that is, those networks comprised solely of T_n arrows. Continuing to use [013] as an example,

[32] Lewin explores the concept of network isomorphism much more formally in "Klumpenhouwer Networks and Some Isographies that Involve Them," particularly pages 84–90.

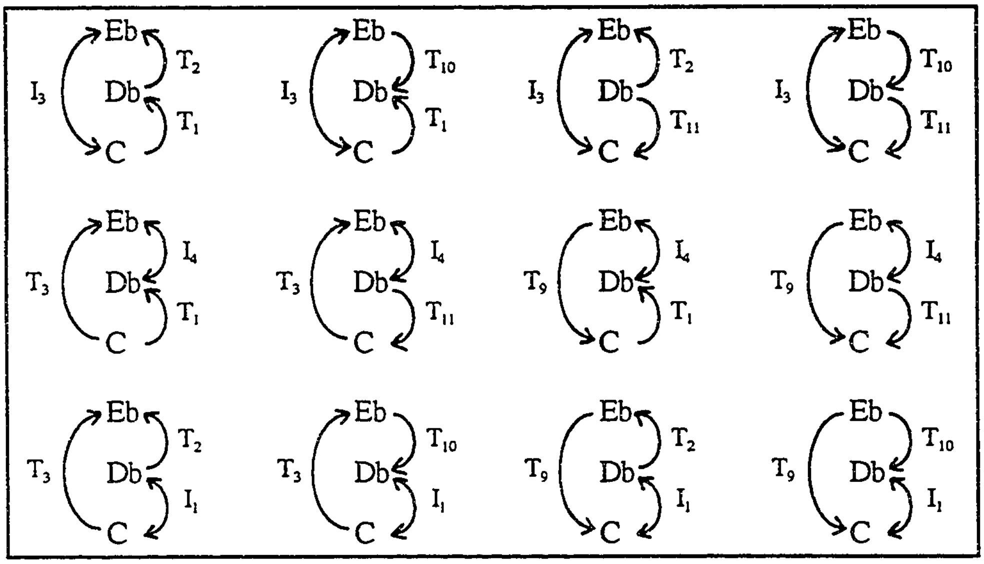
the purely transpositional graphs in Example 1.3–1 do not relate sets beyond traditional set-class boundaries. To exhibit strong isography two Lewin networks must model T_n related pitch-class sets. Example 1.3–4 illustrates this phenomenon building the first network of Example 1.3–1 (chosen arbitrarily) on each of the twelve pitch classes, yielding the twelve members of T_n-type [013]. Any of the other seven networks similarly yield this complete T_n-type. In duplicating a subset of traditional set classes, the T_n-type, these networks are redundant in terms of voice leading transformations, and, as previously mentioned, their primary value is their potential for replication on different structural levels. Given that Lewin networks merely replicate the set classes, I will put them aside and turn to the remaining nineteen possibilities.

Example 1.3–4: Twelve strongly isographic Lewin networks modeling T_n-type [013]

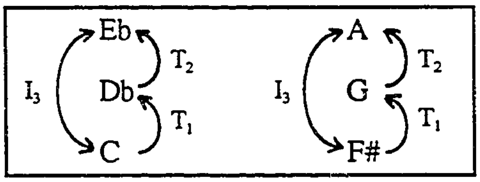


The next group of [013] networks I will examine are those incorporating two T_n arrows and one I_n arrow. There are twelve such networks, as shown in Example 1.3–5 (which reproduces the relevant portion of Example 1.3–2), and they generate even smaller and more exclusive subsets of the set classes. These configurations only generate classes of two strongly isographic members. Example 1.3–6 illustrates the two strongly isographic networks derived from the arbitrarily chosen first graph of Example 1.3–5. Note that these two pitch-class sets are equivalent via T_6. As a result, I will also eliminate these twelve networks from consideration.

Example 1.3–5: [013] networks with two T_n arrows and one I_n arrow

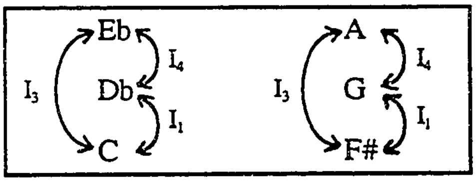


Example 1.3–6: Sample class of strongly isographic networks for the above networks



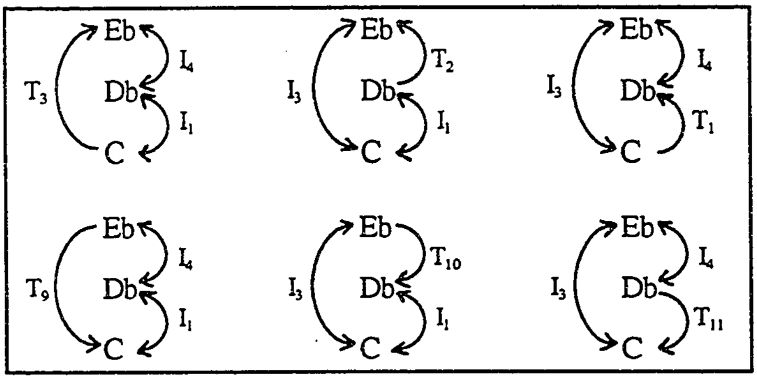
The last [013] network I will expunge is the final graph of Example 1.3–2, which incorporates only I_n arrows. It is as exclusive as the previous group, similarly yielding a class of only two members, as shown in Example 1.3–7. These two graphs are again strongly isographic to each other, and the pitch-class sets are again T_6 transformations of each other. Thus, there are only six more graphic interpretations of [013] to consider.

Example 1.3–7: Sample class of strongly isographic networks using all I_n arrows



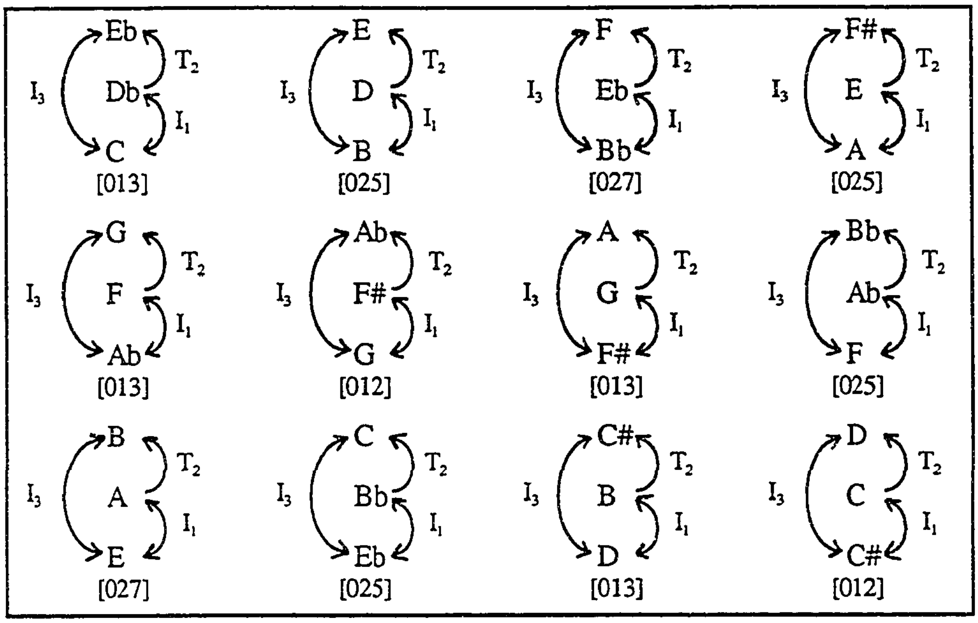
After excluding the eight Lewin networks (those with three T_n arrows), the all-inversion network (the one with three I_n arrows), and the twelve networks of Example 1.3–5 (those with two T_n arrows and one I_n arrow), I am left with the six interpretations shown in Example 1.3–8. Arbitrarily taking the second of these graphs and building identical networks on each pitch class yields the twelve strongly isographic networks shown in Example 1.3–9. Significantly, this group of strongly isographic networks cuts across traditional boundaries, forming a family of similarly structured pitch-class sets not primarily related by transposition or inversion. The prime form of each set is given under its graph in the example. To differentiate these families from traditional set classes, I will label such collections of strongly isographic networks Klumpenhouwer classes.[33] Another Klumpenhouwer class is shown in Example 1.3–10. This class is arbitrarily derived from the third network in Example 1.3–8. Note that this class even crosses traditional cardinality boundaries by incorporating two pitch-class dyads with doublings: {C#, D} and {G, Ab}.

Example 1.3–8: The six practical Klumpenhouwer networks modeling [013]

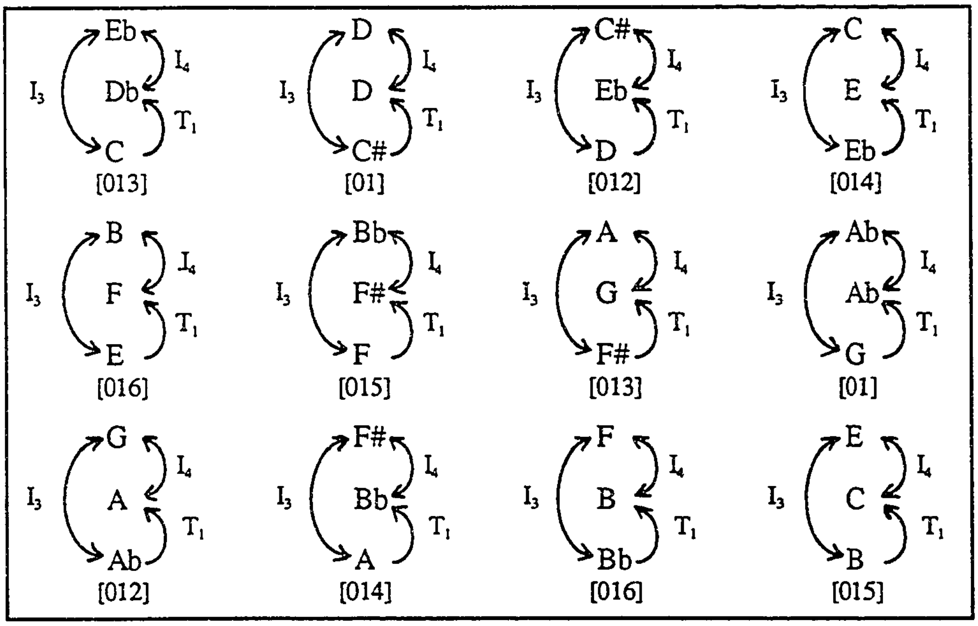


[33] This is not Klumpenhouwer's terminology; he explores network isography in individual analytical cases, rather than as a generalizing force.

Example 1.3–9: Sample Klumpenhouwer class



Example 1.3–10: Another sample Klumpenhouwer class



The power of Klumpenhouwer's abstraction resides in the small subset of networks with the ability to generate classes such as those shown in the previous two examples. Referring back to Example 1.3–8, note that the six networks modeling [013] with this potential all incorporate two I_n arrows and one T_n arrow. In other words, each network contains a transpositional dyad and one other pitch class that is related by inversion to both members of the dyad. Another way to think of this structure is as a partition of the set into two subsets linked by I_n arrows. Note further that the vertically aligned networks are identical except for the reversal of the T_n arrow directions. The left networks both isolate the inversional singleton Db, the central networks isolate C, and the right isolate Eb, thereby exhausting all the partitions of the set other than the set itself. This demonstrates a property that is true for sets of any cardinality, and the number of practical Klumpenhouwer networks arises from the total number of partitions less one (the excluded unity + null partition). The table in Example 1.3–11 presents these more manageable numbers for sets of cardinalities three through six. The formula $2^{n-1} - 1$, where n equals the cardinality of the set, generates the number of possible general Klumpenhouwer networks, while flipping each of the T_n arrows generates the number of specific Klumpenhouwer networks. The direction of the T_n arrows is not significant in terms of developing Klumpenhouwer classes and voice leading, though it can suggest subtle interpretive differences on the level of individual sets and their higher-level replications.

Example 1.3–11: Comparison of possible mappings and practical Klumpenhouwer networks

Cardinality	Mappings	General Klumpenhouwer Networks	Specific Klumpenhouwer Networks
trichord	6	3	6
tetrachord	24	7	44
pentachord	120	15	480
hexachord	720	31	8,704

In terms of voice leading, Klumpenhouwer's work focuses on registral permutations, and he employs network isography to create mappings among sets from different set classes. Example 1.3–12 illustrates his methods with three sets from the opening of the third movement of Webern's *Piano Variations*, Op. 27.[34] The networks immediately below the score demonstrate the strong isography among graphs of the three trichords, while the bottom of the example presents the notes in registral order and superimposes mappings based on the isography. The power of this voice-leading model is readily apparent here in its ability to relate the structures of three trichords of different set classes: 3–4 [015], 3–1 [012], and 3–3 [014]. Significantly, when comparing graphs of pitch-class sets from the same Klumpenhouwer class, the dyads modeled with identical I_n arrows are not related to each other by inversion, but rather merely lie along the same inversional wedge. In Example 1.3–12, the dyads {B, Eb} and {D, C} are both representatives of Klumpenhouwer's internal I_2 transform, but they are clearly members of different interval classes (interval-classes 4 and 2 respectively). The tremendous benefit of this modeling technique is that it allows voice-leading mappings between six of the twelve intervals ("even" or "odd"), as long as they fall along the same inversional wedge, as illustrated in Example 1.3–13. In this example W_0 represents the even wedges, having either C or F# as its axis, though the wedge could also begin on C#, D, Eb, E, or F, generating index numbers 2, 4, 6, 8, or 10, respectively. W_1 similarly represents the odd wedges (index numbers 1, 3, 5, 7, 9, 11). In these examples it is important to remember that the individual elements within the dyads are related to each other by inversion, while the dyads themselves are related to each other by wedging.[35]

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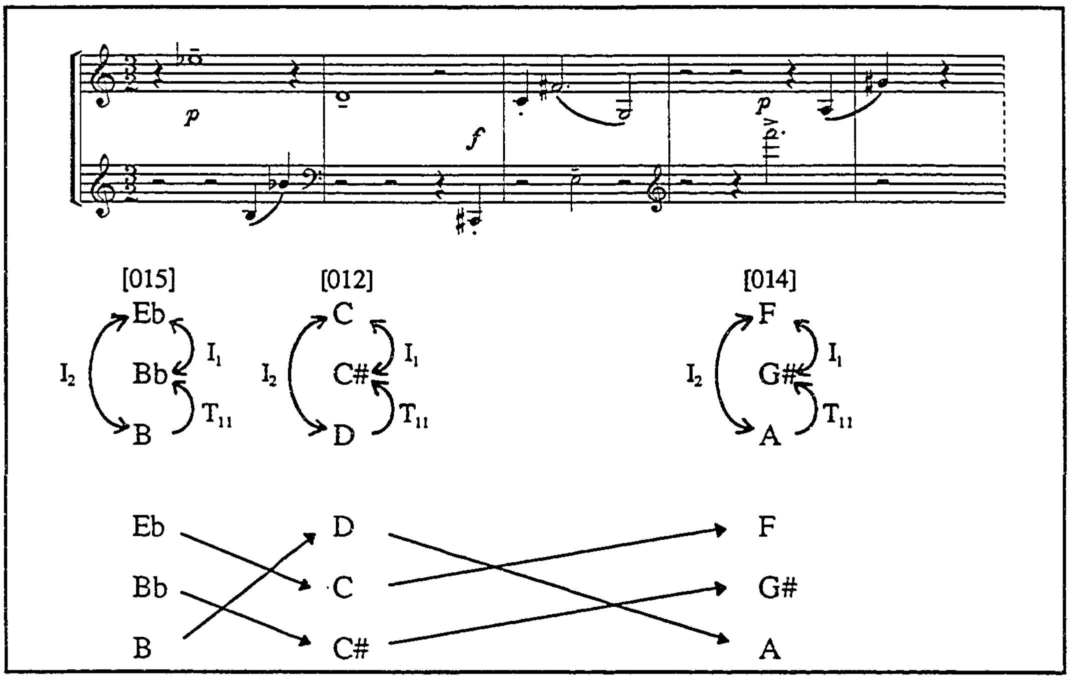
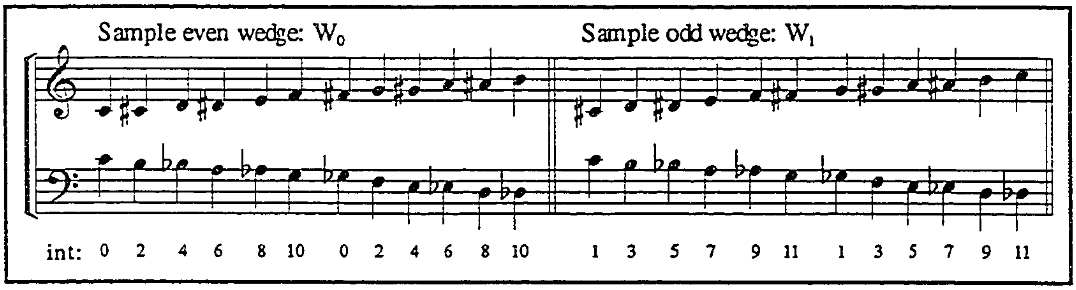
[34] Klumpenhouwer, "A Generalized Model of Voice Leading for Atonal Music," particularly "Chapter 8: Klumpenhouwer Networks and Registral Permutation." My examples are based on his examples 8.11–8.13.

[35] George Perle pioneered the research into these symmetrical relationships decades earlier in much of his theoretical, analytical, and compositional work. Of particular relevance is his work on P/I dyads and sum and difference scales in *Twelve-Tone Tonality*, 2nd ed. (Berkeley: University of California Press, 1996). Perle specifically points out some of the connections between Klumpenhouwer's work and his own in a recent letter to *Music Theory Spectrum* 15.2 (1993): "A Klumpenhouwer network is a chord analyzed in terms of its dyadic sums and differences. This kind of analysis of triadic combinations has been implicit in the concept of the cyclic set...since 1939. Any one of Klumpenhouwer's triadic networks may...be understood as a segment of a cyclic set, and...may be...efficiently and economically represented this way. The analysis of tetrachords into their component dyadic sums and differences as a means of defining the relations between different tetrachords is demonstrated in the third chapter of *Twelve-Tone Tonality*." [300, 302] While a detailed comparison of Perle's and Klumpenhouwer's models is beyond the scope of this study, there are two fundamental differences worth noting here. First, the two models have different

Example 1.3–12: Webern, Piano Variations, Op. 27, III, mm. 1–5



Example 1.3–13: Intervals along even and odd inversional wedges



Example 1.3–14 again models the excerpt from Webern's *Piano Variations*, but now Klumpenhouwer incorporates the measure 3 trichord not included among the networks shown in Example 1.3–12 above. The new network has the same configuration of nodes and arrows, an identical T_n arrow, and analogous I_n arrows that differ by a fixed

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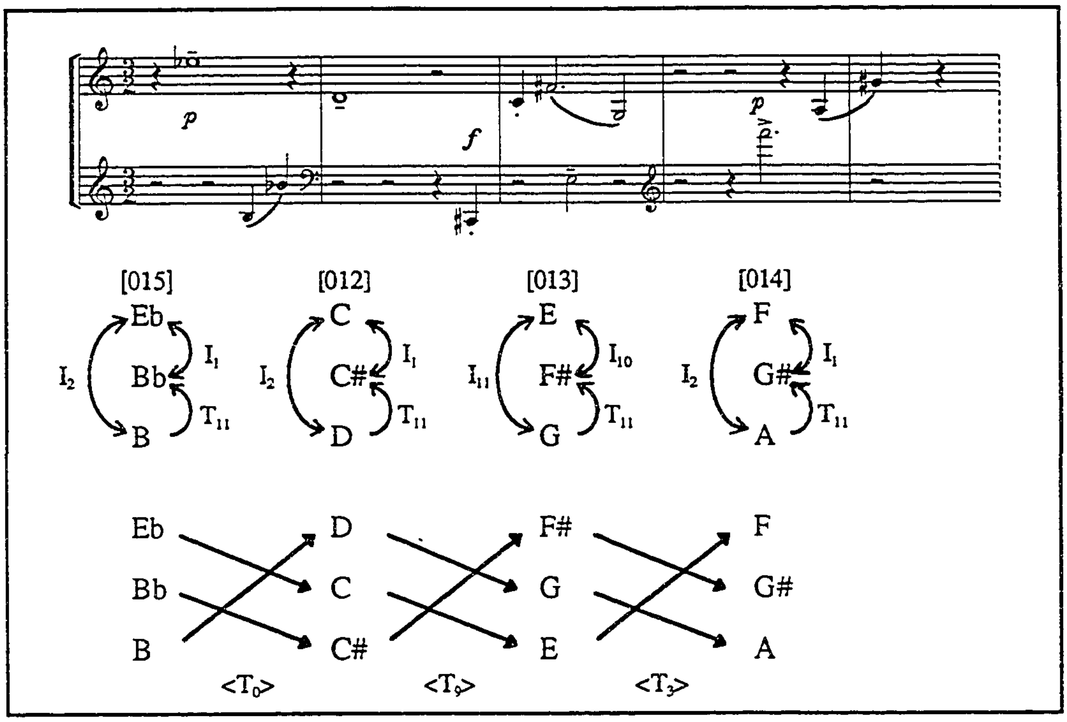
musical orientations. Perle's model is primarily a compositional model or perhaps even a precompositional system, while Klumpenhouwer's is a graphic analytical model. Second, and more importantly in the context of this dissertation, specific mappings are an essential part of Klumpenhouwer's model, while Perle's cyclical sets imply no particular voice leading. The new edition of *Twelve-Tone Tonality* does include a brief chapter on voice leading, "Voice Leading Implications of Sum Tetrachords" (pp. 177–182), but Perle seems to equate voices with registral lines and offers very few musical realizations of the progressions he discusses.

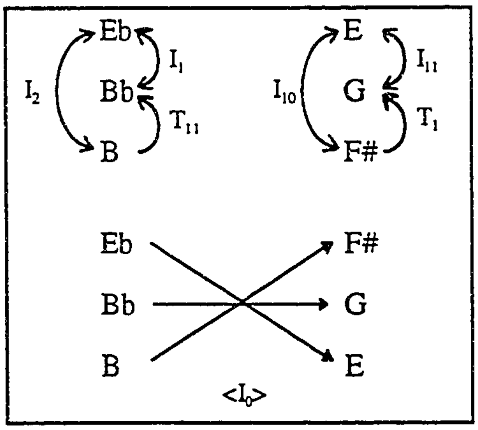
amount (specifically 9 in this case), in comparison to the three previously discussed networks. Lewin defines this relationship as positive isography (strong isography is the subset which has I_n arrows differing by 0), and further defines the transformation among positively isographic networks $<T_n>$, where n equals the difference between analogous I_n arrows.[36] $<T_n>$ is essentially a transposition of the inversional wedge, and the brackets help distinguish it from traditional transposition, which normally operates on pitch classes. Example 1.3–14 notates the series of transformations $<<T_0>,<T_9>,<T_3>>$ under the mappings. Note that $<T_0>$ transforms members of the same Klumpenhouwer class, including the motion away by $<T_9>$ and return by $<T_3>$ as in traditional transposition ($T_9 + T_3 = T_0$). The use of $<T_n>$ transformations groups Klumpenhouwer classes into families of twelve classes—one for each possible value of n (0–11)—for a total of 144 networks. The concept of negative isography further expands the family to 288 networks. Negatively isographic networks have complementary T_n arrows and their analogous I_n arrows sum to the same value. Example 1.3–15 illustrates negative isography using two of the trichords from the Webern excerpt. Negatively isographic networks transform by $<I_n>$, where n equals the sum of the analogous I_n arrows. Klumpenhouwer networks derive their analytical power from these extended families of positively and negatively isographic networks.

[36] Lewin formally introduces positive and negative isography in "Klumpenhouwer Networks and Some Isographies that Involve Them," and further develops the model in "A Tutorial on Klumpenhouwer Networks."

Example 1.3–14: Webern, Piano Variations, Op. 27, III, mm. 1–5



Example 1.3–15: Negative isography



One potential flaw with isography as a transformational model is that while the sets themselves are dynamic with internal transformations, the basis of the mappings between successive sets is equivalency of nodes, rather than an active transformational process like traditional transposition or inversion. The fundamental difference between the two processes in question (that is, traditional operations and isographic Klumpenhouwer networks) is one of internal versus external orientation, or perhaps the more familiar "vertical" versus "horizontal" if thinking in a harmonic context. Traditional transposition transforms a pitch class through a specified external or horizontal motion, and similarly, traditional inversion transforms a pitch class by flipping it externally or horizontally around a given axis. Klumpenhouwer's internal transposition and inversion transform pitch-classes by a vertical distance, or through a vertical axis. This process makes the set dynamic, yet renders the transformation static; the horizontal voice leading results from equivalent locations in the internal or vertical structure rather than from any particular linear transformation. The wedging process does imply a specific kind of contrapuntal linear motion (complementary contrary motion), but the fact that the distance traveled along the wedge is irrelevant to the network process underscores its vertical emphasis. I address these issues further in the next section "1.4 Dual Transformations."

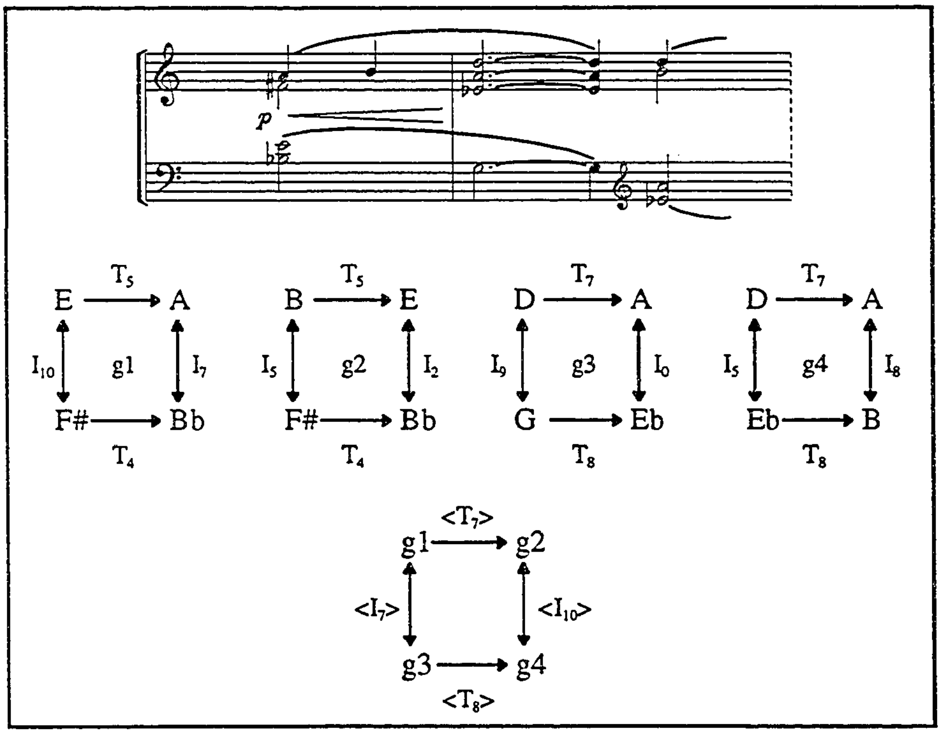
The final topic of this section is network recursion. While Klumpenhouwer's uses of isographic networks emphasize registral permutations, Lewin's applications focus on recursion, that is, isographic supernetworks of the graphs themselves. Example 1.3–16 illustrates network recursion based on Lewin's analysis of Schoenberg's *Three Piano Pieces*, Op. 11, No. 2.[37] Note that there are two pairs of positively isographic networks, g1 → g2 and g3 → g4, and that these pairs are negatively isographic to each other. This pairing suggests that the graphs themselves may follow a transformational path similar to the pitch classes within the individual networks. The supernetwork at the bottom of the example illustrates the isographic relationship of the network of networks to g1 through g4. The supernetwork "thus interprets the *progression* of chords 1–2–3–4 by a K-structure that exactly reproduces, on a higher level, the structure that interpreted each chord."[38] In

[37] Lewin's examples 1, 9, 10, 12, and 13, in "A Tutorial on Klumpenhouwer Networks" are the basis of this example. I have taken the liberty of omitting all of the networks that do not directly generate the sample supernetwork, and I have modified his notation slightly to match my own.

[38] Lewin, "A Tutorial on Klumpenhouwer Networks," 91 (emphasis original). "K-structure" is shorthand for Klumpenhouwer structure.

Lewin's analysis of this work he takes things one step further, generating an even higher level network using <<T_n>> and <<I_n>> to model the overall progression of progressions. Theoretically these nested recursions could go on indefinitely.

Example 1.3–16: Schoenberg, Three Piano Pieces, Op. 11, No. 2, mm. 9–10



Lewin finds great significance in recursive structures, and suggests a potential hierarchical voice leading model through his choice of terminology:

> When a lower-level Klumpenhouwer Network is interpreting a chord, and a higher-level network-of-Networks is interpreting a progression of chords (more precisely, of chord-interpretations), I noted that one could conceive of the higher-level network as "prolonging" the given progression. This potentiality of the system, observed again and again in the article, can afford an especially compelling rationale (albeit a non-phenomenological one) for asserting one particular Klumpenhouwer Network rather than another, to interpret a given chord. I found it suggestively comparable, methodologically, to the ways in which a choice among foreground readings in a Schenkerian analysis can be influenced by middleground considerations.[39]

[39] Lewin, "Klumpenhouwer Networks and Some Isographies that Involve Them," 115.

The importance of recursive supernetworks in Lewin's work raises the inevitable question: how meaningful an isomorphism is network recursion? I cannot offer an unequivocal answer to that question, but any answer is contingent on how one perceives the relation between traditional transformations and Klumpenhouwer's bracketed transformations. In the most abstract sense, both T_n and $<T_n>$ are about differences, while I_n and $<I_n>$ are about sums. Despite this mathematical similarity, one must keep in mind that traditional and bracketed transformations operate on different objects. $<T_n>$ and $<I_n>$ transform Klumpenhouwer arrows among different isographic networks, while T_n and I_n transform pitch classes within a single network. This suggests a substantial functional difference (transformations of transformations versus transformations of pitch classes), but within the context of the graphic model including supernetworks they both transform nodes. Currently I find the isomorphism musically meaningful and useful on a middleground level, and somewhat less convincing on a background level incorporating supernetworks invoking $<<T_n>>$ or $<<I_n>>$. I will return to these and related issues in the next section, but most importantly, it is my dissatisfaction with the graphic model on the foreground level that prompts me to introduce dual transformations. Chapters 2 and 3 further clarify (by example) my conception of the analytical value of Klumpenhouwer networks and supernetworks.

1.4 Dual Transformations

As demonstrated in the previous two sections of this chapter, recent transformational models tend to parse pitch-class sets into subsets to explain transformations among sets belonging to different set classes. The emphasis on the individual subsets ranges from the singleton models, which explicitly isolate the rogue pitch class from the remainder of the set, to Klumpenhouwer's model, which disguises the parsing with its unified graphic representation. Roeder's work similarly parses pitch-class sets into subsets, though into their smallest components (that is, individual pitch classes), and also attempts transformational reunification through representational means. The sizable body of musical works discussed by Roeder, Forte, Lewin, Straus, and Klumpenhouwer in introducing their respective models suggests that this is a fruitful analytical avenue. Furthermore, the number of theorists independently arriving at and exploring similar approaches, as well as the increasing number of current dissertations and theoretical papers on related topics, suggests that something about these subset-oriented models resonates well with us as musicians.[40] In the hopes of further refining the current transformational model, I will conclude this chapter by introducing a number of original voice-leading transformations grouped under the generic label split transformations.

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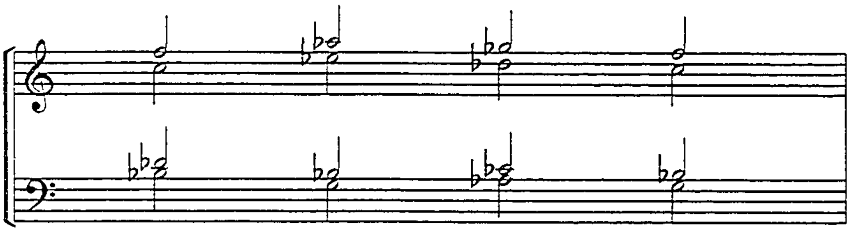
[40] Recent items—within the past two years—include: Brian Alegant, "When Even Becomes Odd: A Partitional Approach to Inversion," paper presented at the Society for Music Theory Annual Conference (Baton Rouge, 1996); the transformational voice leading component of Michael Buchler's "Setmaker" software for Macintosh computers (version 4.9, ©1996); Edward Jurkowski, "A Model for Non-Equivalent Set-Class Associations: The Interval-Difference Network," paper presented at the Music Theory Society of New York State Annual Conference (Stony Brook, 1996), as well as his dissertation-in-progress on the same topic; Stephen Nuss, "Western Instruments, 'Japanese' Music: Issues of Texture and Harmony in Minoru Miki's *Jo-no-kyoku*," paper presented at the Music Theory Society of New York State Annual Conference (Buffalo, 1995), and his dissertation "Tradition and Innovation: The Art Music of Post-War Japan" (Ph.D. dissertation, City University of New York, 1996); Ian Quinn, "Fuzzy Transposition of Pitch Sets," paper presented at the Society for Music Theory Annual Conference (Baton Rouge, 1996); and, Stephen Soderberg, "Z-Related Sets as Dual Inversions," *Journal of Music Theory* 39.1 (1995): 77–100. See the bibliography for a more complete listing of related works, including older sources by Richard Cohn, Michael Friedmann, Andrew Mead, and Robert Morris.

In the most general sense, split transformations simply allow multiple traditional transformations to occur simultaneously. In this broad context, Roeder's model generates extreme split transformations that incorporate a transposition for each member of a given pitch-class set, and the split therefore equals the cardinality of the given set. As I previously mentioned, I find this approach too promiscuous and not particularly sensitive to listener perceptions. While I can envision any logical split being potentially convincing in the right context (for example, parsing hexachords into three independently moving dyads, which in turn yields three simultaneous transformations), for the purposes of the present study I am restricting my model to two simultaneous transformations. In addition to this limitation on the number of simultaneous transformations, I am imposing a restriction on the types of allowable transformations. While there are potentially many useful possibilities, I am limiting my model to various combinations of the traditional operators, transposition and inversion. Such dual operations follow the lead of, and encompass, all of the singleton and Klumpenhouwer models. These two restrictions—no more than two subsets, and only traditional operators—help generate transformations that I find satisfactorily located in the center of the exclusivity⇔promiscuity continuum. The one exception to the operational limitation is my occasional incorporation of wedges. In musical situations involving dyads, I find wedging (represented by W_n) to have some perceptual significance, particularly in pitch space, and I will discuss this in greater detail in subsequent chapters as it becomes analytically important. For now let me offer more formal definitions of the two most important kinds of split transformations: dual transpositions and dual inversions.

Dual transposition is a transformation utilizing two simultaneous transpositions, that is, some voices move at T_n while the remainder move at T_{n+x}, resulting in the composite transformation T_n/T_{n+x}. More formally, imagine two pitch-class sets J and K. Partition each of these sets into two discrete subsets encompassing the entire set, so that they become the partially ordered sets J = <{j1}, {j2}> and K = <{k1}, {k2}>. The cardinalities of these sets and subsets are irrelevant, except that the cardinality of each subset must be at least one. If T_n transforms {j1} into {k1}, and T_{n+x} transforms {j2} into {k2}, then T_n/T_{n+x} transforms J into K. Dual inversions work identically except that I_n transforms {j1} into {k1} and I_{n+x} transforms {j2} into {k2}, and therefore I_n/I_{n+x} transforms J into K. In other words, some voices move at I_n while the remainder move at I_{n+x}. These transformations are best illustrated by musical examples.

Example 1.4–1: Dual transposition



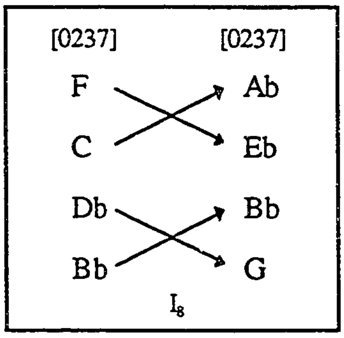
c1	c2	c3	c4
[0237]	[0237]	[0257]	[0257]
F - - →	Ab - - →	Gb - - →	F
C - - →	Eb - - →	Db - - →	C
Db →	Bb →	Cb →	Bb
Bb →	G →	Ab →	G

T_9/T_3 T_1/T_{10} T_{11}/T_{11}

Example 1.4–1 shows a progression of four chords labeled c1–c4. The middle of the progression, c2 → c3, consists of a member of set-class 4–14 [0237] transforming into a member of set-class 4–23 [0257]. By definition, traditional transposition or inversion cannot generate this transformation, but as notated at the bottom of the example below the mappings, the dual transposition T_1/T_{10} can. That is, T_1 transforms the bass-clef dyad, while T_{10} transforms the treble-clef dyad. In this and subsequent examples, solid and dashed arrows distinguish between the two T_n levels in the mappings. Intellectually, this process involves first partitioning the c2 tetrachord {G, Bb, Eb, Ab} into two discrete dyads {G, Bb} and {Eb, Ab}, and then tracing the dyads' independent transpositional paths into the dyadic subsets {Ab, Db} and {Db, Gb} of the c3 tetrachord {Ab, Db, Db, Gb}. While this may sound overly complex, a quick listen reveals that I am simply asking the reader to hear the [03] dyad move up a half step and the [05] dyad move down a whole step.

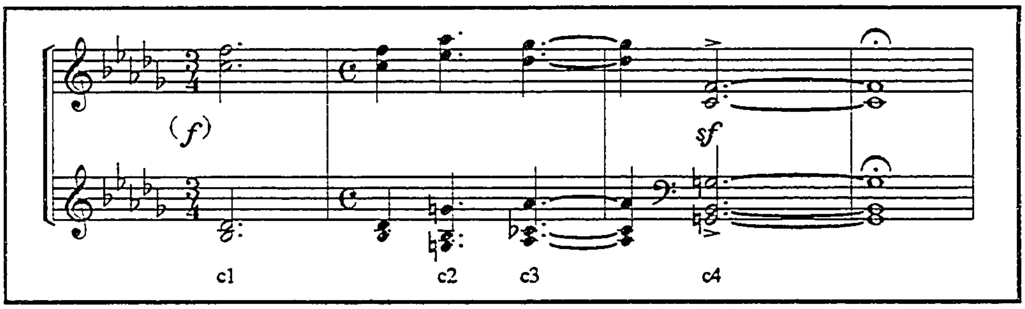
The progression c2 → c3 illustrates the power of dual transpositions to generate mappings among different set classes, but they can also generate mappings among members of the same class, sometimes in ways that are more satisfactory than traditional operations. I could interpret the initial progression c1 → c2 as a traditional I_8, but that would involve hearing the dyads flip as shown by the mappings in Example 1.4–2. Such an interpretation is plausible for the bass-clef dyad as Bb acts as a common tone, but it is less satisfactory for the treble-clef dyad. On the other hand, the dual transposition T_9/T_3 shown in Example 1.4–1 suggests hearing the symmetry of the progression as the two dyads move away from each other by complementary T_n values. It is much easier to hear the motion of the parallel intervals than the pitch-class inversion in this particular case. The final progression c3 → c4 illustrates a situation in which traditional and dual transpositions agree. All the pitches of c3 move down a half step to generate c4, but owing to the emphasis on the independent dyads in the three previous chords, the dyadic partition remains very audible as reflected by the notation T_{11}/T_{11}.

Example 1.4–2: Inversional mapping for c1 → c2



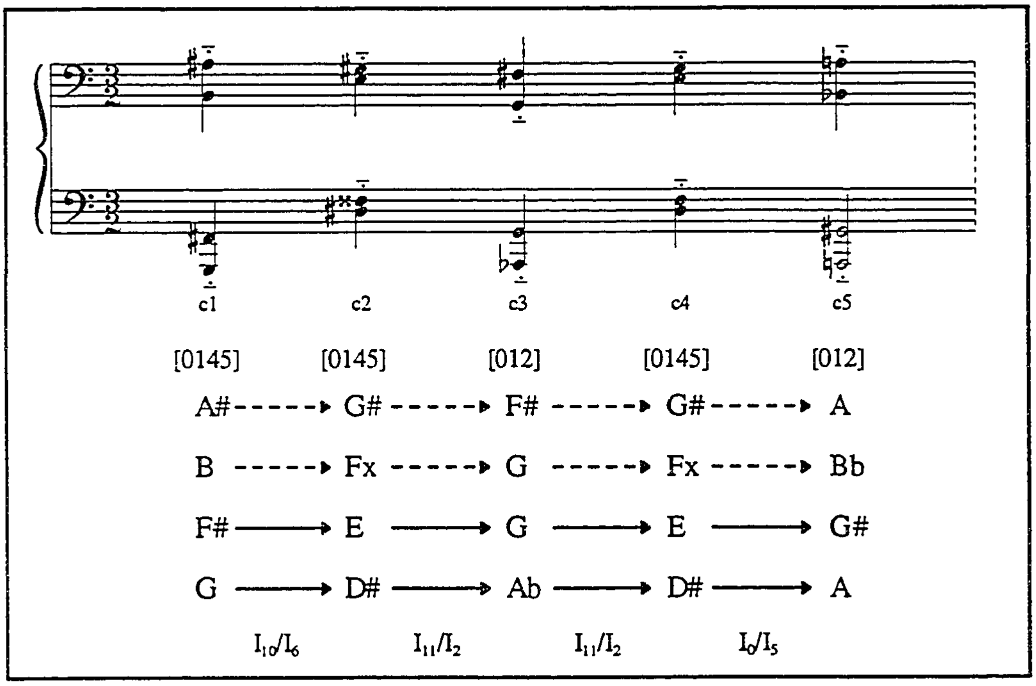
The source of the abstract chord progression in Example 1.4–1 is the opening of Aaron Copland's *Piano Sonata*, shown in Example 1.4–3 below. I normalized the passage in the earlier example for reasons of clarity by removing the octave displacement in the transformation c3 → c4, by omitting the orchestrational doubling in the left hand in chords c2–c4, and by eliminating any metric reference. Henceforth musical examples will be given "as is" with the assumption that the accompanying mappings supply my interpretation of the underlying structure.

Example 1.4–3: Aaron Copland, Piano Sonata, I, mm. 2–5

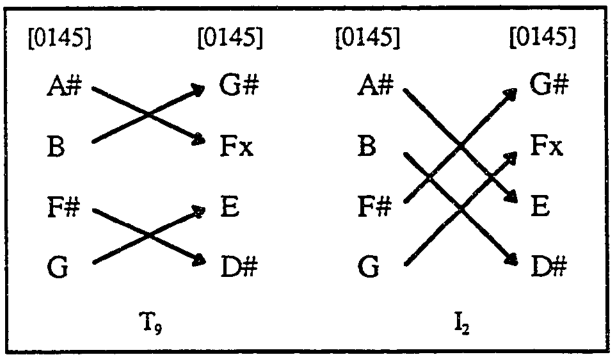


The Bartók excerpt shown in Example 1.4–4 illustrates dual inversions. The chord progression c1–c5 contains members of two different set classes 4–7 [0145] and 3–1 [012]. The transformations among the last four chords reconfirm the power of dual transformations to generate mappings among sets from different classes, and further demonstrate that dual transformations can also span traditional cardinality boundaries. It is important to note that these sets have an equal number of pitches, despite the difference in pitch-class cardinality. In this model it is therefore possible to incorporate pitch-class doublings as elements of independent voices, allowing analysts to distinguish between orchestrational doublings as in the above Copland example and occasional pitch-class duplications between different voices as in the Bartók example. The progression c1 → c2 shows that dual inversion, like dual transposition, can sometimes generate more satisfactory mappings than traditional operations among members of the same set class. Example 1.4–5 presents the two possible operational mappings between c1 → c2 for comparison. As the title of the piece—"Minor Seconds, Major Sevenths"—suggests, it is the inversional transformation of the dyads that is most significant in this excerpt; the tetrachords are fallout from this process, rather than the reverse.

Example 1.4–4: Béla Bartók, No. 144, Mikrokosmos, VI, m. 39

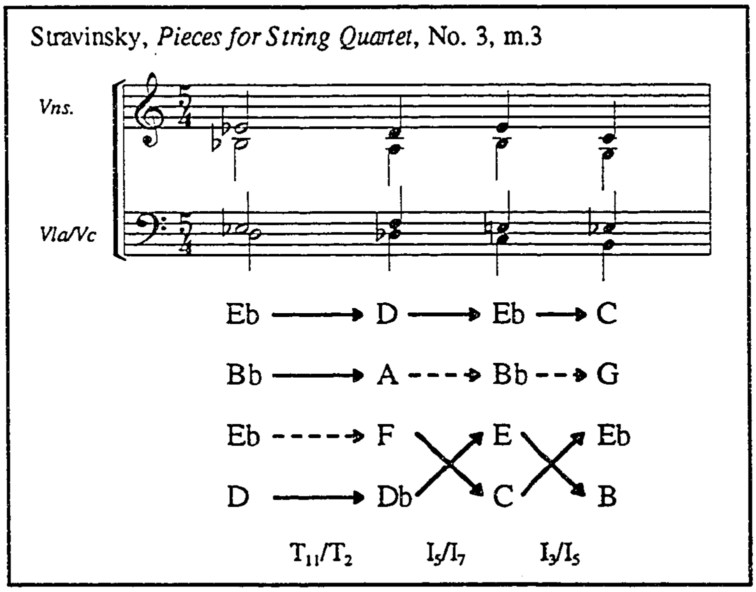


Example 1.4–5: Traditional mappings for c1 → c2



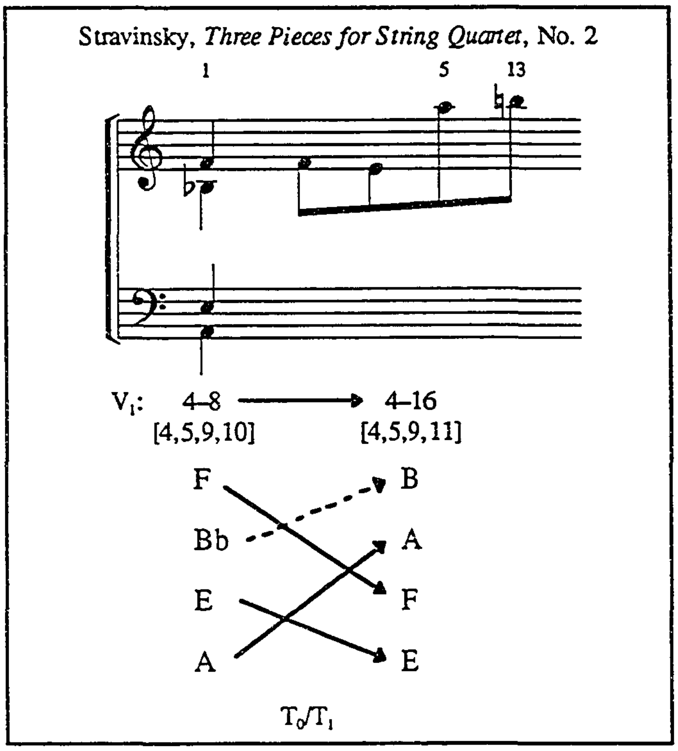
As many readers may already realize, dual transformations are quite closely related to most of the transformations discussed in previous sections of this chapter. First, if x = 0, then T_n/T_{n+x} and I_n/I_{n+x} yield the traditional operations transposition and inversion. Next, if the n+x value affects a singleton, then T_n/T_{n+x} and I_n/I_{n+x} yield the singleton transformations near-transposition and near-inversion, or if-only. Example 1.4–6 illustrates this relation by reproducing Straus's example from my earlier discussion of near-transformations and renotating the progression as three dual transformations.[41] Furthermore, if n+x affects a singleton and n = 0 in a dual transposition (that is, T_0/T_{0+x}), this yields one of Forte's unary transformations. In fact, the value of n in Forte's V_n notation is my x-value. Example 1.4–7 illustrates this correspondence by reproducing Forte's V_1 example from my earlier discussion and reinterpreting it as T_0/T_1.

Example 1.4–6: Straus's near-transformations reinterpreted as dual transformations



[41] The mappings in this example do not represent my interpretation of the voice leading, but merely renotate Straus's interpretation. My interpretation of this Stravinsky passage is given in Chapter 2 as part of a more extensive analysis.

Example 1.4–7: Forte's unary transformation reinterpreted as a dual transposition



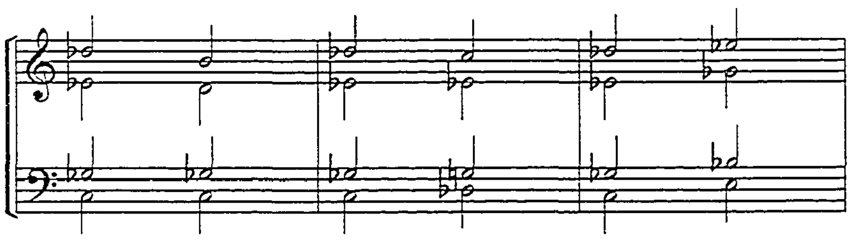
Dual transformations borrow and combine the most powerful aspects of the near-transformational and unary models. As mentioned in Section 1.2, unary transformations model only the strong R_p relation, while near-transformations additionally model the weak R_p relation. In other words, the advantage of near-transformations is that the T_n value can be any interval, rather than the T_0 limitation of unary transformations. On the other hand, near-transformations do not adequately consider the independent motion of the rogue singleton, or its relation to the remainder of the transformation, while unary transformations use the V_n notation to specify the transformation of the singleton. The V_n value is significant because it represents the specific change (if any) in a set's interval-class vector, and therefore determines any change in the set's sound quality and its particular set-class affiliation. The dual transformations T_n/T_{n+x} and I_n/I_{n+x} encompass both of these singleton models by allowing n to equal any interval and by retaining the relation between the two independent transformations with the x-value.

The next three examples illustrate how the x-value corresponds to changes in set-class membership. Example 1.4–8 presents three progressions from c1, a member of set-class 4–13 [0136], to chords c2–c4, all members of set-class 4–8 [0156]. Note that x = 2 in all three of these dual transpositions: T_0/T_2, T_1/T_3, and T_4/T_6. Any T_n/T_{n+2} transformation on this particular partition of a member of set-class [0136] will yield a member of set-class [0156]. Example 1.4–9 shows the same phenomenon in the context of dual inversions. Note that for the same partition of set-class [0136], any I_n/I_{n+5} transformation yields a member of set-class 4–z29 [0137]. Example 1.4–10 illustrates, in much simplified form, exactly how the x-value affects the interval-class vector and therefore sound quality. In this example three dual transpositions transform c1, a member of set-class 3–2 [013], into chords c2–c4. The first progression, T_0/T_1, transforms c1 into a member of set-class 3–3 [014], the second, T_0/T_2, transforms c1 into a member of set-class 3–4 [015], and the third, T_0/T_{10}, transforms c1 into a member of set-class 2–1 [01] with a doubling. Below the pitch-class mappings are interval-class vector mappings illustrating the interval-class content changes caused by each of the dual transformations. Note that the x-value corresponds to the number of places the relevant intervals shift in the vectors. In c1 → c2 x = 1 and the two affected intervals shift to the right one place, while in c1 → c3 x = 2 and the two affected intervals shift to the right two places. Under the last progression the interval-class vectors are slightly modified by including a bracketed location for interval-class zero to accommodate pitch-class doublings. In c1 → c4 x = 10 and the two affected intervals shift to the left two places, which pushes one into the zeroes column (yielding the Db doubling) and the other into the ones column. Similar processes occur in larger sets and partitions though the intervallic shifts become increasingly complex. The first two analyses in Chapter 2 further illustrate the significance of the x-value.

Example 1.4–8: The x-value in dual transpositions (x = 2)

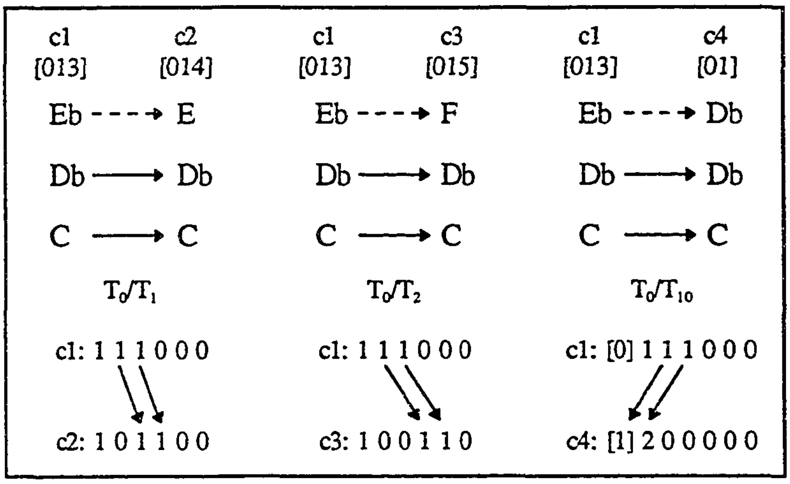
c1	c2	c1	c3	c1	c4
[0136]	[0156]	[0136]	[0156]	[0136]	[0156]
Db ——→	Db	Db ——→	D	Db ——→	F
Eb - - -→	F	Eb - - -→	F#	Eb - - -→	A
Gb ——→	Gb	Gb ——→	G	Gb ——→	Bb
C ——→	C	C ——→	C#	C ——→	E
T_0/T_2		T_1/T_3		T_4/T_6	

Example 1.4–9: The x-value in dual inversions (x = 5)



c1	c2	c1	c3	c1	c4
[0136]	[0137]	[0136]	[0137]	[0136]	[0137]
Db ——→	B	Db ——→	C	Db ——→	Eb
Eb - - -→	D	Eb - - -→	Eb	Eb - - -→	Gb
Gb ——→	Gb	Gb ——→	G	Gb ——→	Bb
C ——→	C	C ——→	Db	C ——→	E
I_0/I_5		I_1/I_6		I_4/I_9	

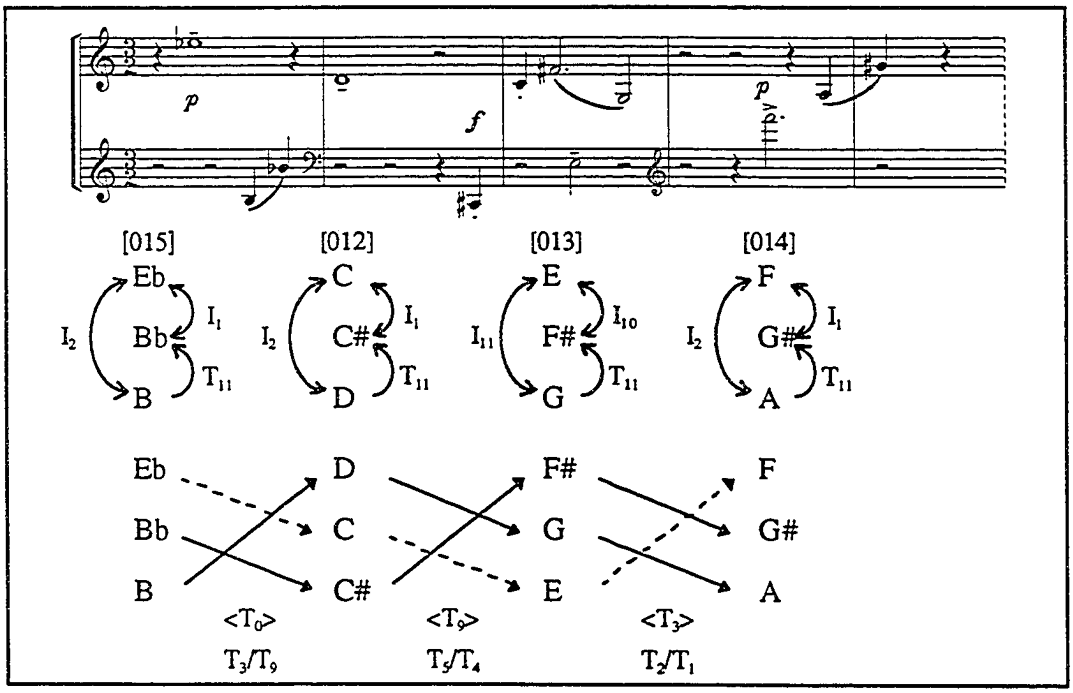
Example 1.4–10: The x-value and the interval-class vector



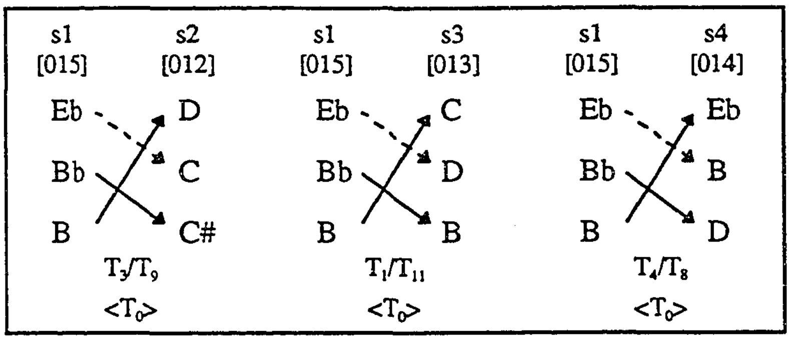
Among the models presented in this chapter, Klumpenhouwer networks have the closest ties to dual transformations. Both Klumpenhouwer networks and dual transformations encompass all three singleton models, and they both expand on those models by allowing partitions in which both subsets are larger than a singleton. In fact, one could notate any of Klumpenhouwer's $<T_n>$ transformations as T_n/T_{n+x}, and likewise one could notate any of his $<I_n>$ transformations as I_n/I_{n+x}. That is, positively isographic networks can model any dual transposition and negatively isographic networks can model any dual inversion. A valid question at this point might be: if dual transformations are merely another way of describing Klumpenhouwer networks, why introduce a new model? The answer is that while the two models have substantial similarities, they are complementary rather than identical. The next two examples introduce one of the most important differences between the two models. Example 1.4–11 reproduces Klumpenhouwer's analysis of Webern's *Piano Variations*, Op. 27, III (measures 1–5), and modifies it in two ways. First, the use of dashed arrows makes the partitioning explicit, and second, each of the transformations include their appropriate T_n/T_{n+x} labels. Note the relation between corresponding $<T_n>$ and T_n/T_{n+x} transformations: the $<T_n>$ subscript equals the sum of the dual subscripts, or $<n> = 2n + x$. The same holds true for $<I_n>$ and

I_n/I_{n+x}. Example 1.4–12 illustrates the significance of this notational difference. This example shows three different $<T_0>$ transformations from the first pitch-class set of the Webern excerpt. The progression s1 → s2 is identical to that in the Webern excerpt. The next two, however, result in pitch-class sets s3 and s4 which are quite different progressions from Webern's actual music. In fact, there are nine other possible $<T_0>$ transformations yielding different second pitch-class sets, as $<T_0>$ represents strong isography, or members of the same Klumpenhouwer class. The dual transpositions, on the other hand, distinguish among these twelve different progressions, and therefore provide a better foreground model of the voice leading. It is the x-value and its effect on set-class metamorphosis that helps characterize the distinct transformations. Note that each of the T_n/T_{n+x} transformations has a different x-value (6, 10, and 4) which results in s2, s3, and s4 being members of different set classes. One further correspondence between these two models illustrated by this example is that if n and n+x are complementary, that is, if 2n + x = 0 (mod–12), then the pitch-class sets are members of the same Klumpenhouwer class.

Example 1.4–11: Klumpenhouwer's $<T_n>$ reinterpreted as dual transposition



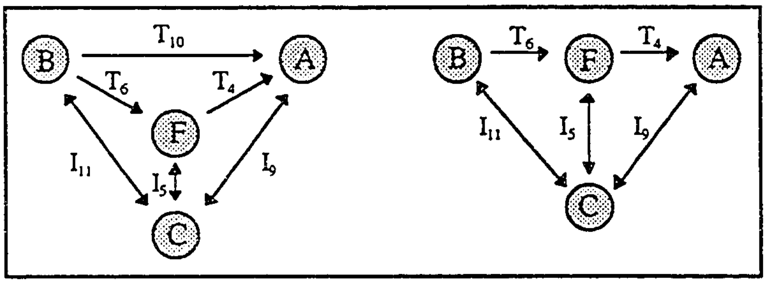
Example 1.4–12: T_n/T_{n+x} *versus* $\langle T_n \rangle$



If dual transformations generally offer a better foreground voice-leading model, as I suggested in the previous paragraph, one might ask what is the advantage of continuing to use Klumpenhouwer networks? There are at least two extremely significant reasons not to displace Klumpenhouwer networks with dual transformations. The first is the extraordinary potential for network recursion. The dual transformational model does not particularly lend itself to replication on different structural levels, while Klumpenhouwer networks provide an elegant recursive model. The second reason is more subtle. In Section 1.3 I point out that one flaw in deriving voices from Klumpenhouwer networks is the model's internal orientation, that is, the mappings result from equivalent nodes rather than any specific linear transformation. Dual transformations remedy this problem by describing the same voice mappings using two simultaneous linear operators; in other words, they render dynamic the between-set transformation. Although I feel that a successful voice-leading model must have an external or horizontal orientation, I also feel that the internal dynamism of the network model captures an invaluable dimension of musical structure that I think of as set integrity, or what we might call chord quality in tonal music. With its horizontal emphasis and explicit partitioning of sets, dual transformations do not satisfactorily model the cohesive quality of individual chords or sets, particularly in homorhythmic contexts. Although Klumpenhouwer networks implicitly embody the same partitions, the graphic model more successfully models the unity of individual sets via what I consider a molecular metaphor.

In fact, I visualize Klumpenhouwer networks as three-dimensional ball-and-stick models with nodes standing in for atoms, and transformations functioning as bonds. Though it is impossible to include a ball-and-stick model in this two-dimensional medium, imagine the left network in Example 1.4–13 as a pyramid with the pitch-class nodes providing the corners, and the transformations forming the sides. This molecular model best captures the way I understand chord or set quality as a dynamic, yet cohesive, network of individual elements (pitch classes).[42] Owing to the incompatibility of a three-dimensional model and a two-dimensional medium, for the remainder of the dissertation I will present Klumpenhouwer networks in the abbreviated form shown on the right-hand side of Example 1.4–13. This abbreviated network attempts to retain the essence of the ball-and-stick model, but aims for visual clarity in two dimensions. The most drastic change in the graph is the omission of the T_{10} arrow from B → A, but it is non-essential as the T_6 and T_4 arrows spanning the top of the network already imply the T_{10} arrow ($T_6 + T_4 = T_{10}$). My two-dimensional Klumpenhouwer networks omit such redundant arrows, but always retain a minimum of two transformations anchoring each node in place.

Example 1.4–13: The ball-and-stick model in two dimensions



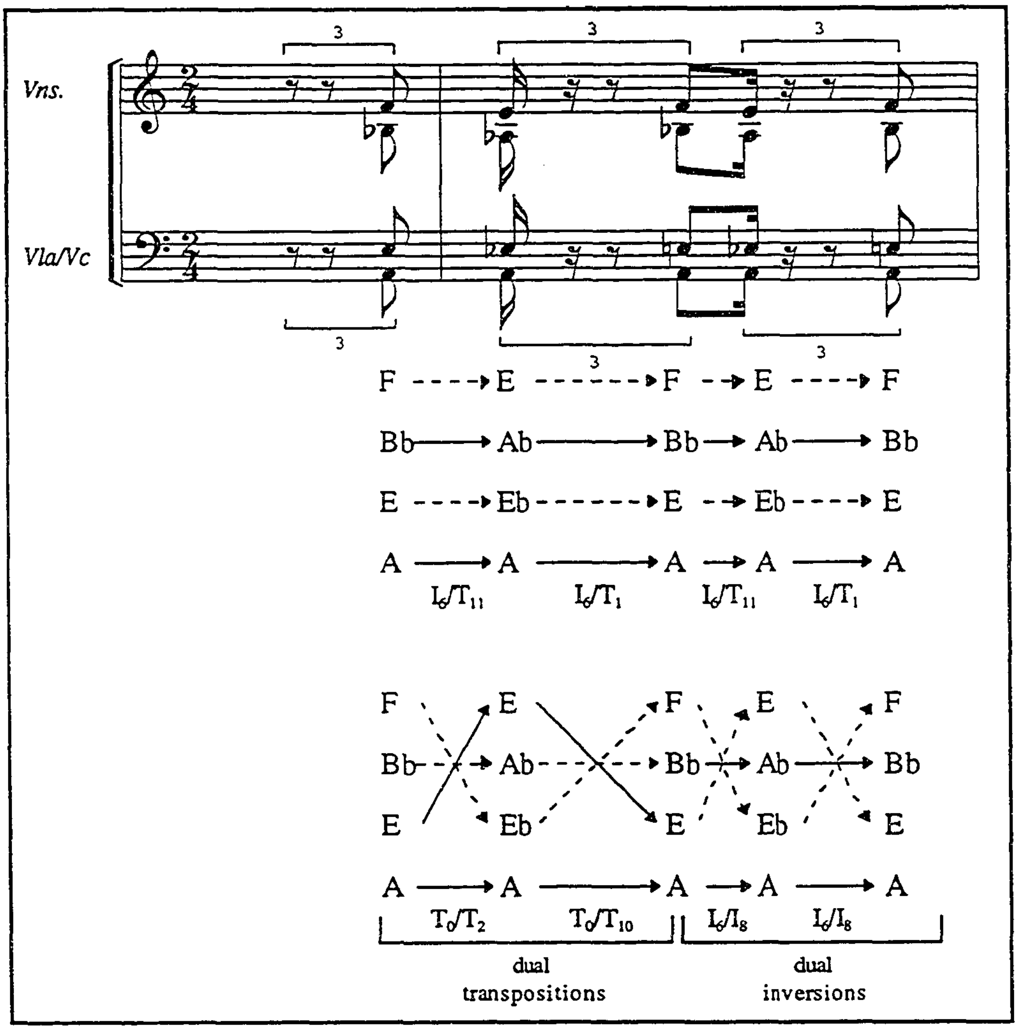
Now that I have defined dual transposition and dual inversion, as well as made a case for the complementary nature of the dual transformational and Klumpenhouwer models, I would like to mention two more potentially significant dual transformations. If

[42] Though I occasionally like to think of the T_n arrows as ionic bonds and the I_n arrows as covalent bonds, the molecular metaphor swiftly breaks down under closer scrutiny.

the premise behind my model is that we can track two simultaneous operations, then the next logical additions are the hybrid transformations T_n/I_{n+x} and I_n/T_{n+x}. These two transformations push the dual transformational model beyond the limits of Klumpenhouwer networks as they have no Klumpenhouwer equivalents. Positive isography corresponds to T_n/T_{n+x} and negative isography corresponds to I_n/I_{n+x}, but the hybrid transformations generate conflicting positive and negative characteristics that render networks graphically dissimilar. The lack of Klumpenhouwer equivalents is both the strength and weakness of the hybrid transformations. They benefit my model by substantially increasing the number of possible mappings, which is particularly useful in modeling foreground voice leading. On the other hand, their main drawback is that they do not have the recursive potential of the pure dual transformations, and therefore they do not offer the same kind of insight into deeper musical structures.

Example 1.4–14 reproduces the Stravinsky excerpt from Section 1.1 to illustrate hybrid transformations. I originally modeled these two alternating tetrachords belonging to set-class 4–8 [0156] as a series of traditional operations, either T_{11} or I_1, but then complained that both mappings were overly abstract and did not satisfactorily match my experience of the passage. I was particularly concerned with the retention of the cello's A2 as a bass voice; I find this to be the most important characteristic of the phrase. The bottom mappings in Example 1.4–14 present two more interpretations of the voice leading generated this time by dual transposition and dual inversion. Note that these two mappings result in the same voices despite different partitions (represented by the solid and dashed arrows), and that they both retain the common tones A and E. I find either of these mappings far superior to the traditional operations presented in Section 1.1, but I still want the voices to correspond even more closely to the concrete musical surface. The hybrid transformations directly below the short score best capture my interpretation of the passage. This interpretation merges the strong half-step motion in the soprano and tenor voices with the importance of A2 as a pedal point. Hearing the alto as a symmetrical transformation around A3 further emphasizes, though indirectly, the A2 pedal.

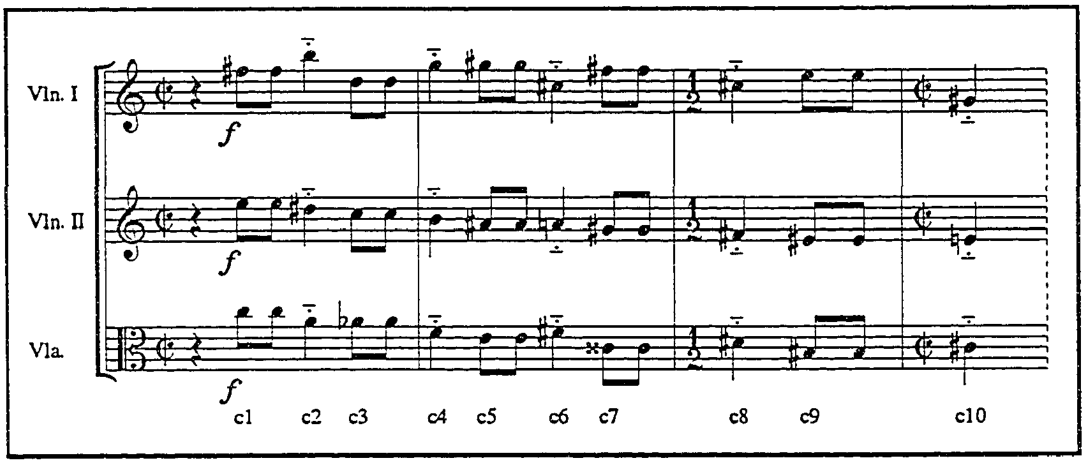
Example 1.4–14: Stravinsky, Three Pieces for String Quartet, II, mm. 0–1



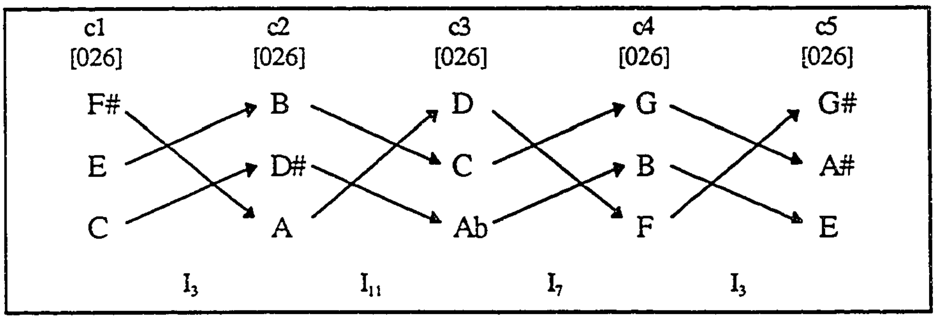
Just as this Stravinsky excerpt raised the issue of multiple mappings and criteria for satisfactory readings in Section 1.1, the addition of three more possible interpretations only increases the need for me to clarify my methods. As the number of possible interpretations steadily increases, a byproduct of moving away from the exclusive end of the exclusivity⇔promiscuity continuum, contextual analytical filters become increasingly important. While these analytical filters may change slightly for each unique musical context, I do have three interpretive biases that can be more-or-less generalized. First, in the absence of any other considerations, I want voices that most richly interact with my experience of the musical surface. In other words, I prefer voices that are reinforced by one or more surface lines, such as registral or instrumental lines. Though many of the examples in this and subsequent chapters present voices that correspond to the registral lines, it is important for the reader to keep in mind the distinction between surface reinforcement and equating registral lines with voices by default. My preference for the $<I_6/T_{11}, I_6/T_1>$ interpretation of the above Stravinsky excerpt arises from the A2 pedal and the 2+2 partition suggested by the half steps, which I find to be the most salient features of the progression. So while my foreground voices may at times coincide exactly with the concrete musical surface, my intention is that they capture musical features beyond the literal surface. In other words, I generally want my foreground voices to be rather concrete, but not located at the extreme end of the concrete⇔abstract continuum that includes mere replication of the score.

My second preference is for motivic partitions or mappings as illustrated by the next three examples. Example 1.4–15 shows an excerpt from the introduction to the third movement of Bartók's *String Quartet No. 1*. The first five chords, c1–c5, of this progression are members of set-class 3–8 [026]. As shown in Example 1.4–16, a series of inversions generates this progression of [026] trichords, and the resulting voice mappings form a very consistent pattern. Repeated registral permutations like those shown in this example are what I call motivic mappings. The second half of the phrase, c6–c10, contains trichords from a mix of set classes, but as shown in Example 1.4–17, it is still possible to interpret the progression with the same motivic mappings. The partitions suggested by the dual inversions are similarly consistent, that is, the singletons form a unified voice, and this illustrates what I call a motivic partition. The mappings in these two examples do not coincide with the registral or instrumental lines, but the consistency of the permutations and partitions nonetheless make a compelling case for this interpretation. Voices not reinforced by surface lines are somewhat more abstract than those voices incorporating one or more surface features, but as long as they do not directly contradict my intuitive experience of a musical passage, I find motivic mappings and partitions analytically significant.

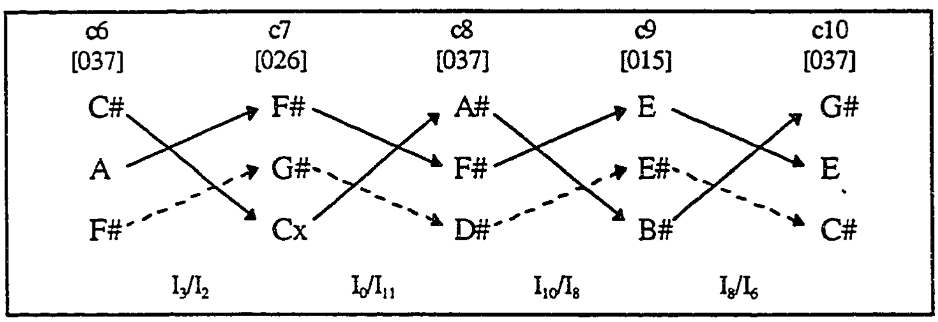
Example 1.4–15: Béla Bartók, String Quartet No. 1, III, mm. 1–4 (Introduzione)



Example 1.4–16: Motivic I_n mappings in c1–c5

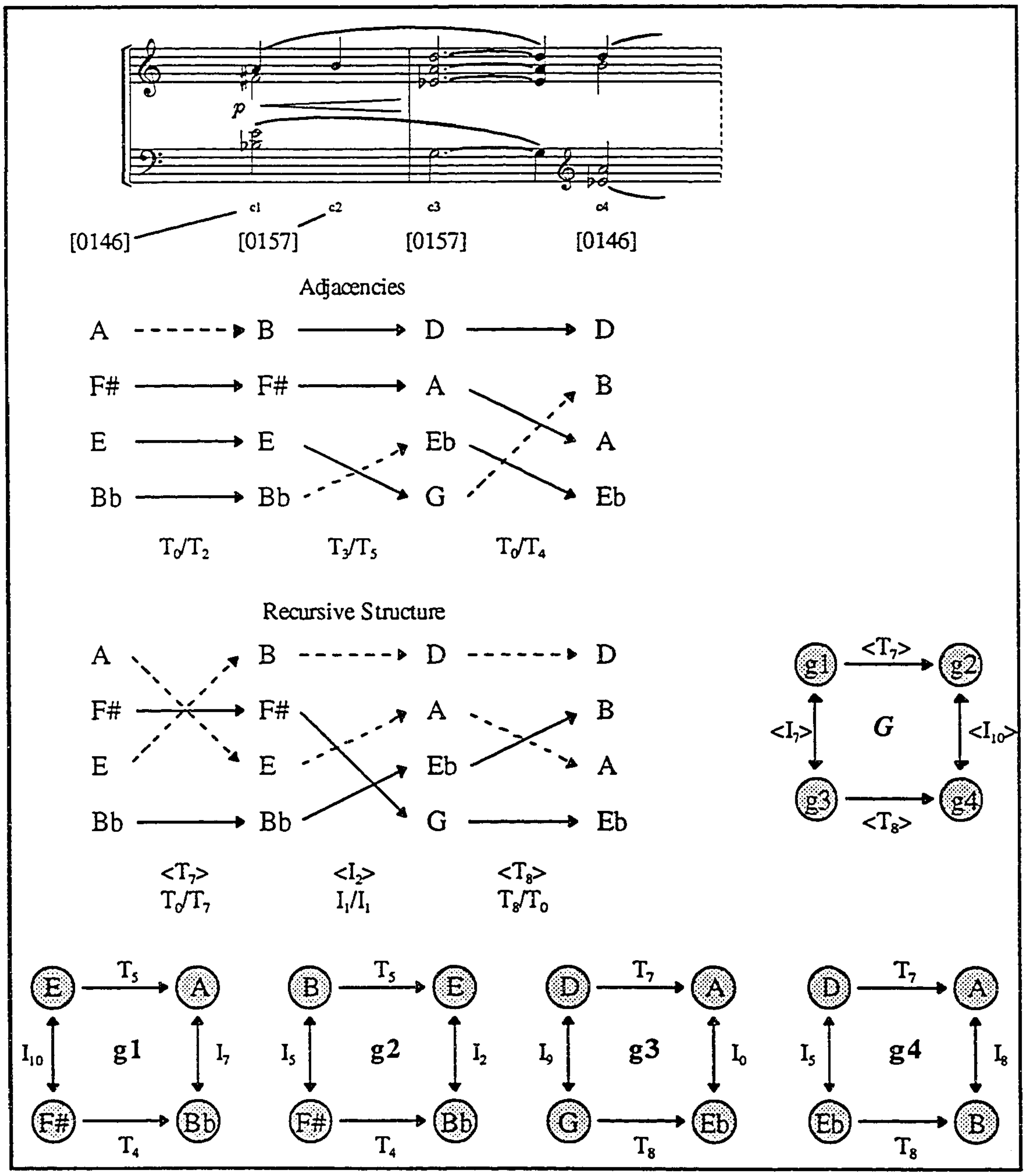


Example 1.4–17: Motivic I_n/I_{n+x} mappings and partitions in c6–c10



If my first preference leans toward the concrete, and my second moves in the direction of abstraction, my third preference—for recursive structures—brings me to the abstract end of the concrete⇔abstract continuum. The analytical elegance of network recursion loses some of its charm if one examines exactly what it implies on the musical surface. The first progression, c1 → c2, in Example 1.4–18 (a reproduction and modification of Lewin's example previously presented in Section 1.3) illustrates this point. Lewin's graphs g1–g4 and his recursive supernetwork, which I am labeling *G*, imply the lower set of mappings in the example. Keep in mind that networks g1–g4 are particular interpretations of chords c1–c4, and that the supernetwork is a particular interpretation of the progression growing out of the isography among graphs g1–g4. The upper set of mappings is a series of dual transpositions that model my interpretation of the musical surface. A comparison of the two sets of mappings for the progression c1 → c2 highlights the abstract nature of the network mappings. The progression consists of a single melodic motion in the highest register heard against a sustained three-voice chord. The T_0/T_2 transformation of my foreground interpretation successfully captures this singleton partition, as well as the non-motion of the lower three voices. The $<T_7>$ mappings, on the other hand, retain the alto and bass voices, but suggest a soprano-tenor exchange and a 2+2 partition. Hearing the soprano A4 map into an already sounding E4, and the E4 map into B4 in the middle of its duration, strongly contradicts my experience of the progression. In another musical situation I might consider the $<T_7>$ mappings a serious case of overabstraction, but in this instance I find them compelling owing to the recursive relationship between this interpretation of the chords and progression. In this musical passage, like many others, I find that both voice leading interpretations describe significant musical attributes despite their contradictory characteristics. Since I find value in both the concrete mappings of surface-reinforced voices and the often abstract mappings of network recursions, I frequently incorporate both into my analytical model in non-hierarchical levels I call "Adjacencies" and "Recursive Structure" as shown in the example. I call these levels non-hierarchical because they model different aspects of the music, and one interpretation does not subsume the next as in a Schenkerian graph. They do, however, suggest varying degrees of distance from the musical surface, or varying locations on the concrete⇔abstract continuum.

Example 1.4–18: Arnold Schoenberg, Three Piano Pieces, Op. 11, No. 2, mm. 9–10



The transformational voice-leading model consists of all the techniques presented in this chapter, but the above example best illustrates my personal methodology. Subsequent chapters test the analytical viability of my transformational model—with its emphasis on dual transformations and Klumpenhouwer networks—in the context of longer musical passages, as well as a few complete works, drawn from a wide variety of twentieth-century atonal musical literature. Throughout the nine analyses that comprise the remainder of this dissertation I will further embellish and clarify many aspects of my model that I hope will be more easily understood within these specific musical contexts.

Chapter 2

Analyses I: Homophonic Applications

Thus far I have presented very brief musical examples to illustrate a variety of transformational tools, and to support my interpretation of transformational mappings as voice-leading lines. In chapters 2 and 3 of this dissertation I apply the analytical models introduced in Chapter 1 to longer musical passages. Chapter 3 explores linear and polyphonic applications of these models, while Chapter 2 exclusively examines homophonic progressions, specifically drawn from the following five works:

- 2.1 Béla Bartók: *Concerto for Piano and Orchestra No. 2*, II
- 2.2 Igor Stravinsky: *Three Pieces for String Quartet*, III
- 2.3 Alexander Skryabin: *Prelude*, Op. 74, No. 4
- 2.4 Charles Ives: "Serenity"
- 2.5 Charles Ives: "The Cage"

Each of these analyses portrays multiple voice-leading interpretations engendered by Klumpenhouwer networks and supernetworks, dual transformations, and/or traditional operations. As demonstrated at the conclusion of the previous chapter, I arrange these mappings in various levels called "Adjacencies," "Recursive Structure," etc., for easy comparison. I urge the reader to keep in mind that despite a sense of varying distance from the musical surface, these are non-hierarchical levels, and they usually model substantially different aspects of the given music.

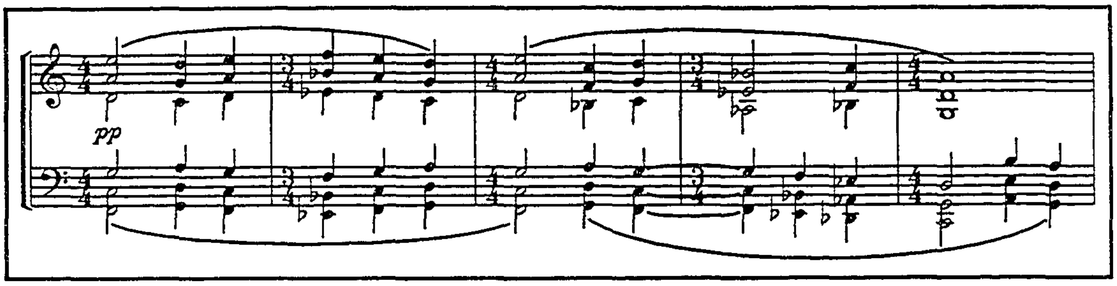
2.1 Béla Bartók: Concerto for Piano and Orchestra No. 2, II

To illustrate the mechanics of applying my transformational model to more substantial musical passages (that is, self-contained phrases or formal sections), I will begin with an analysis of an excerpt from the second movement of Bartók's *Concerto for Piano and Orchestra No. 2*. The first section, spanning 63 bars, of this ternary movement consists of contrasting passages alternating between homorhythmic string progressions and solo piano lines with timpani accompaniment. As outlined in the table in Example 2.1–1, the string passages occur in measures 1–22, 30–38, and 54–62, and make up the bulk of this A section. In each of these three passages, harmonic progressions arise from six—via *divisi* first violins and cellos—registral/instrumental lines. As the resulting progressions yield pitch-class sets of four different cardinalities (trichords, tetrachords, pentachords, and hexachords), the traditional operators transposition and inversion cannot map the chord-to-chord motions of the contrapuntal voices. Despite their awkwardness in pitch-class space, the pitch-class doublings that create these shifts in cardinality fit comfortably into the hybrid musical space inherent in the transformational model developed in Chapter 1. It is the consistent registral division of the six lines into three upper and three lower voices that immediately suggests dual transpositions. Example 2.1–2 presents a short score of the opening five-bar phrase of the movement.

Example 2.1–1: Internal divisions within the first A section

A1: *Adagio* (mm. 1–63)					
a1	b1	a2	b2	a3	b3
strings mm. 1–22	piano/timp. mm. 23–29	strings mm. 30–38	piano/timp. mm. 39–53	strings mm. 54–62	piano/timp. mm. 62–63

Example 2.1–2: Bartók, Concerto for Piano and Orchestra No. 2, II, mm. 1–5



In this example one hears a succession of fifteen simultaneities which includes representatives of four different set classes: 3–9 [027], 4–23 [0257], 5–35 [02479], and 6–32 [024579]. Despite the homorhythmic chorale style of this excerpt, register, contour, phrase articulation, and subset structure emphasize two independent streams in the progression. The listener immediately notices the almost exact mirroring between the upper and lower parts at the outset of the progression, mentally tracing the separate, yet related, paths of these two subsets. The upper strings (Violins I and II) very audibly move in stacked fifths or [027] trichords: first down a whole-step, back up a whole step, then up another half-step, etc. The lower strings also move in stacked fifths, though in contrary motion to the upper strings: up a whole step, back down a whole step, down another whole step, etc. Jonathan Bernard comments on this passage:

> the string parts are disposed in two groups moving for the most part in contrary motion. The symmetry involved in this motion seems at first only approximate, but if all the notes in mm. 1–5 are arranged in a scale, preserving the registral positions in which they actually appear, then a mirror-symmetrical pattern emerges...[1]

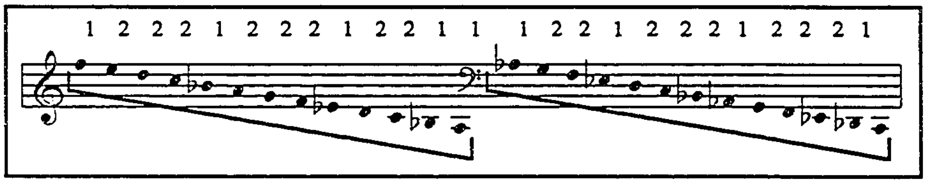
Example 2.1–3 reproduces his supporting example.[2] There are two substantial flaws in this interpretation: the Ab3/A3 axis of the "mirror-symmetrical pattern" is extremely abstract in the context of the actual music, and, more significantly, some of the notes in the passage are omitted from the example. The axis is less than satisfactory as it contradicts the audible

[1] Jonathan W. Bernard, "Space and Symmetry in Bartók," *Journal of Music Theory* 30.2 (1986): 188.

[2] Ibid., Example 4b, 189.

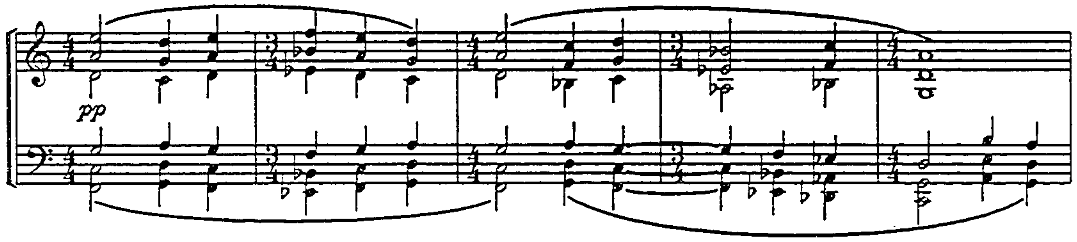
division between upper and lower strings by including some of the upper stream notes in the lower half of the symmetric pattern. In fact, the example dissolves the distinct chords into an amorphous totality that seemingly has no concrete presence in the music. Furthermore, Bernard ignores the penultimate chord in the excerpt, omitting the violas' B3 from the upper segment of his pattern, and omitting the cellos' E3 and A3 (the latter doubled by the basses) from the lower half of his pattern. Incorporating these pitches into the example destroys the mirror-symmetrical pattern. It is possible to interpret the last two chords in measure 5 as an anacrusis to the next phrase, but Bartók's phrase marks contradict such an interpretation.

Example 2.1–3: Bernard's "mirror-symmetrical pattern"



Rather than striving to perceive a unified symmetrical pattern, I prefer hearing the independent motions of the two subsets. In other words, I like to hear one series of transpositions operating on the lower strings' trichord (specifically $<T_2, T_{10}, T_{10}, T_2, T_2, T_{10}, T_2, T_{10}, T_0, T_{10}, T_{10}, T_{11}, T_9, T_{10}>$), and another series of transpositions operating on the upper strings' trichord (specifically $<T_{10}, T_2, T_1, T_{11}, T_{10}, T_2, T_8, T_2, T_8, T_0, T_2, T_9, T_0, T_0>$). Using dual transposition to describe this passage accounts for both the composite progression and the motions of the subsets. The transformation projecting chord 1 onto chord 2 may be described as T_2/T_{10}; T_2 transforms the lower strings, while T_{10} transforms the upper strings. The entire progression can be described as a series of dual transpositions: $<T_2/T_{10}, T_{10}/T_2, T_{10}/T_1, T_2/T_{11}, T_2/T_{10}, T_{10}/T_2, T_2/T_8, T_{10}/T_2, T_0/T_8, T_{10}/T_0, T_{10}/T_2, T_{11}/T_9, T_9/T_0, T_{10}/T_0>$. These transformations and the mappings they yield are shown in Example 2.1–4. The solid lines represent the first T_n value, or the lower strings, while the broken lines represent the second T_n value (T_{n+x}), or the violins. The voices resulting from these transformations coincide exactly with the instrumental/registral lines, the difference being that an octave displacement or reorchestration would destroy the lines, whereas the transformational voices would remain intact.

Example 2.1–4: Dual transpositions and their mappings in the first phrase



	c1	c2	c3	c4	c5	c6	c7	c8	c9	c10	c11	c12	c13	c14	c15
	[024579]	[0257]	[024579]	[027]	[024579]	[0257]	[024579]	[024579]	[0257]	[024579]	[0257]	[024579]	[0257]	[02479]	[027]
	E	D	E	F	E	D	E	C	D	Bb	Bb	C	A	A	A
	A	G	A	Bb	A	G	A	F	G	Eb	Eb	F	D	D	D
	D	C	D	Eb	D	C	D	Bb	C	Ab	Ab	Bb	G	G	G
[027]	G	A	G	F	G	A	G	A	G	G	F	Eb	D	B	A
	C	D	C	Bb	C	D	C	D	C	C	Bb	Ab	G	E	D
	F	G	F	Eb	F	G	F	G	F	F	Eb	Db	C	A	G

T_2/T_{10} T_{10}/T_2 T_{10}/T_1 T_2/T_{11} T_2/T_{10} T_{10}/T_2 T_2/T_8 T_{10}/T_2 T_0/T_8 T_{10}/T_0 T_{10}/T_2 T_{11}/T_9 T_9/T_0 T_{10}/T_0

One of the most interesting aspects of the dual transpositions in this example is how the x-values—a rather abstract construct defined in the previous chapter—become remarkably audible. As shown in Chapter 1, the x-value in any dual transposition (T_n/T_{n+x}) corresponds to the specific changes in the interval vector, and therefore determines the resulting set-class mutation. Owing to the consistent set-class content within each of the two subset streams, Bartók's simultaneities form a closed group of set classes, and the transformations among them are readily apparent. As shown in the table in Example 2.1–5, the union of any two members of set-class [027] will be a member of one of seven possible set classes.[3] The first column of the table shows the T_n relation between the two forms of [027], while the middle column generates their union, and the last column provides set-class information.[4]

Example 2.1–5: [027] Unions

T_0	{027}	U	{027}	=	{027}	3–9 [027]
T_1	{027}	U	{138}	=	{012378}	6–z38 [012378]
T_2	{027}	U	{249}	=	{02479}	5–35 [02479]
T_3	{027}	U	{35a}	=	{02357a}	6–32 [024579]
T_4	{027}	U	{46b}	=	{02467b}	6–z26 [013578]
T_5	{027}	U	{570}	=	{0257}	4–23 [0257]
T_6	{027}	U	{681}	=	{012678}	6–7 [012678]
T_7	{027}	U	{792}	=	{0279}	4–23 [0257]
T_8	{027}	U	{8a3}	=	{02378a}	6–z26 [013578]
T_9	{027}	U	{9b4}	=	{02479b}	6–32 [024579]
T_{10}	{027}	U	{a05}	=	{0257a}	5–35 [02479]
T_{11}	{027}	U	{b16}	=	{01267b}	6–z38 [012378]

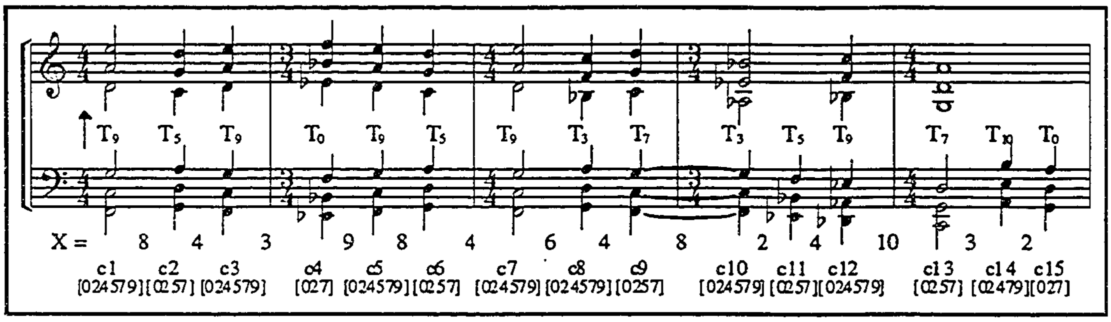
As shown in Example 2.1–4, the first chord is a member of set-class [024579]. This diatonic hexachord results from the T_9 relationship between the lower and upper [027] subsets. The second chord is a member of set-class [0257], resulting from the T_5 relationship between the two [027] subsets. The T_2/T_{10} transformation that creates the progression c1 → c2 has an x-value of 8. It is this x-value that transforms one set class into another; in this case, it transforms the vertical T_9 relationship into the T_5 relationship. The x-value operates as a transposition on the vertical T_n values between the subsets, $T_9 + T_8 = T_5$, and any x = 8 dual transposition on these two subsets (for example, T_3/T_{11} or T_4/T_0) would also result in the progression [024579] → [0257]. Thus the subsequent motion from chord 2 to chord 3, T_{10}/T_2, is a literal return to the first chord because of the complementary

[3] Robert D. Morris offers a rigorous study of such set-class unions in "Pitch-Class Complementation and Its Generalizations," *Journal of Music Theory* 34.2 (1990): 175–245.

[4] Owing to the symmetrical properties of [027], the two subsets could also be interpreted as inversions of each other.

values between the two progressions (T_2/T_{10} + T_{10}/T_2 = T_0), but the abstract progression [0257] → [024579] results from the complementary x-value 4. Example 2.1–6 reproduces Example 2.1–2, but now includes set-class labels, the between subset T_n values (shown between the staves), and the x-values (shown below the staves) that operate on them. The example reveals that there are six between-subset T_n values in this Bartók passage—T_9, T_5, T_0, T_3, T_7, and T_{10}—resulting in four different set classes. Complementary T_n values, specifically T_9 and T_3 or T_5 and T_7 in this case, yield the same set-class union when the two transposed subsets are members of the same set class. The specific succession of set-classes is directly related to the dual transpositional x-values shown in the example.

Example 2.1–6: First phrase, mm. 1–5, with annotations



Dual transpositions model the musical surface in this excerpt very well. The two streams of lower and upper [027] trichords are clearly audible, and the effect of the x-value on their composite set structure is easily identified. Moving beyond the adjacencies level is more problematic as the phrase suggests a number of interesting interpretations. The opening seven chords, which I will call subphrase 1, might be interpreted as "prolonging" a referential sonority via neighboring motions. After all, one hears the first chord {F, C, G, D, A, E} return as c3, c5, and c7, with chord 7 particularly prominent as the conclusion subphrase 1, as well as being the nexus point in Bartók's overlapping slurs. One might go on to suggest that this is a pseudo-diatonic space with all the voices moving by whole or half steps for these seven chords, but perhaps a better way of discussing this prolongation is in terms of the complementary dual transpositions and x-values shown in Example 2.1–6. Note the symmetry of the vertical T_n values <T_9, T_5, T_9, T_0, T_9, T_5, T_9> for c1–c7. The complementary x-values (8–4, 3–9, 8–4) are a kind of abstract inverse with the first value mutating the set class, and the complement returning to the original set class. In this subphrase the dual transpositions are themselves complementary (for example, T_2/T_{10} and T_{10}/T_2), and therefore they act as a literal inverse returning not only to the same set class, but to the same pitch-class set.

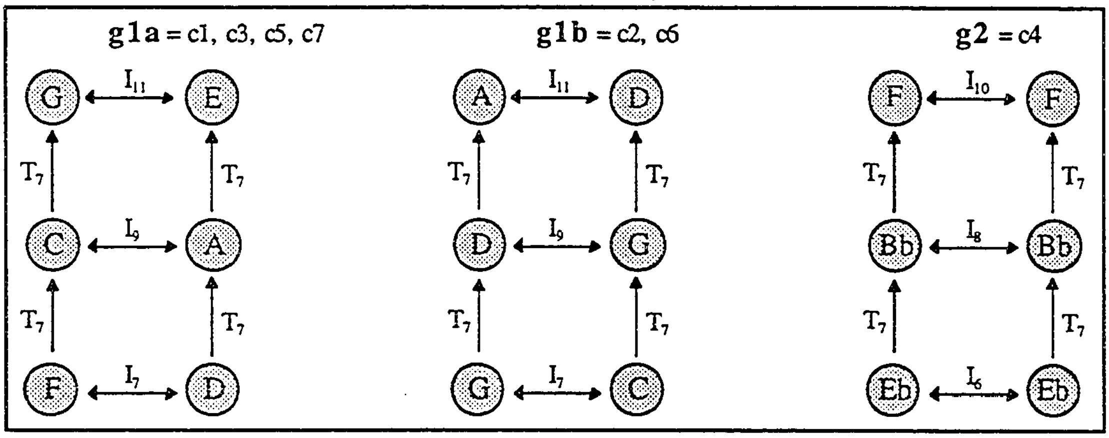
If we follow the concept of a referential [024579] sonority into subphrase 2 (c8–c15), there begins to be some competition for referential status from the prominent [0257] sonorities. There are three occurrences of each sonority within subphrase 2, and [0257] is most notable as the important cadential chord heard on the downbeat of measure 5. Then there is the problem of the two final chords that sound like an afterthought, but not quite like an anacrusis to the next phrase. Chord 14 (the problematic chord omitted from Bernard's example) is unique, being the only member of set-class [02479] within the phrase, and chord 15 is one of only two members (along with c4) of set-class [027]. Is the downbeat [0257] or the final [027] the "real" goal of motion? Either way, a harmony other than [024579] brings the complete phrase to a close as the goal of subphrase 2.

Before moving on to a network approach, I want to highlight a few relations between the two subphrases. First, the initial progressions of both subphrases, c1 → c2 → c3 and c8 → c9 → c10 , incorporate different dual transpositions and the corresponding chords differ in pitch-class content, but they are, however, identical set-class progressions: [024579] → [0257] → [024579]. It is the complementary vertical T_n values—<T_9, T_5, T_9> in subphrase 1 and <T_3, T_7, T_3> in subphrase 2 (refer back to Example 2.1–6)—that generate these corresponding set classes. Furthermore, the progression c5 → c6 → c7 is identical to c1 → c2 → c3, and therefore has the same relationship to c8 → c9 → c10. By the third occurrence of this progression it begins to achieve motivic status. Finally, the progression c11 → c12 → c13 has the vertical T_n values $<T_5, T_9, T_7>$, yielding the converse set-class progression [0257] → [024579] → [0257]. These motivic progressions reinforce the pseudo-sequential, or echoing, nature of the second subphrase.

Modeling this phrase as a series of Klumpenhouwer networks reveals some additional structural information. Example 2.1–7 presents three networks representing all seven chords of the first subphrase. Note that graphs g1a and g1b are strongly isographic; in other words c1, c2, c3, c5, c6, and c7 belong to the same Klumpenhouwer class. These are the chords connected by contrary complementary motion, or exact inversionally symmetric voice leading. The one motion away from this Klumpenhouwer class is the $<T_{11}>$ transformation into c4, represented by network g2, and the immediate return via $<T_1>$. This interpretation of the first subphrase makes its symmetry more obvious, and increases the emphasis on the prolongation of the [024579] hexachord.

It is worthwhile reiterating the perceptual similarities and differences between the two models—dual transformations and Klumpenhouwer networks—at this point. Both models require the listener to divide the passage into two somewhat independent streams, upper and lower strings in this case. Dual transpositions require the listener to trace the two streams independently, though still using the traditional transposition operator, and the trick is to hear the two pathways simultaneously in order to capture the progression as a gestalt. Klumpenhouwer networks require the listener to associate corresponding graphic nodes via a wedging process between the two subsets. Within a single Klumpenhouwer class the wedging is not difficult to perceive, particularly in musical situations like the Bartók phrase that occur in pitch space. On the other hand, the difficulty lies in hearing the bracketed transpositions $<T_n>$ that operate on the wedges. As I mentioned previously, $<T_n>$ is the sum of the two T_n operators of a dual transposition. It would seem that this would represent the gestalt very well, but as a composite the $<T_n>$ values lose their horizontal dimension. The $<T_{11}>$ progression c3 → c4 is specifically T_{10}/T_1, but $<T_{11}>$ could also represent T_9/T_2, T_8/T_3, etc. And as pointed out in Example 2.1–5, each of those very different partitions, with their different x-values, would result in different set-class progressions. The $<T_{11}>$ only indicates that the wedging is one semitone "off," but not how far to wedge, that is, how far the voices move horizontally along the wedge index. This makes it difficult for a listener to hear the skewed wedges as such. Nonetheless, the network model does frequently yield additional insights into underlying transformational structures.

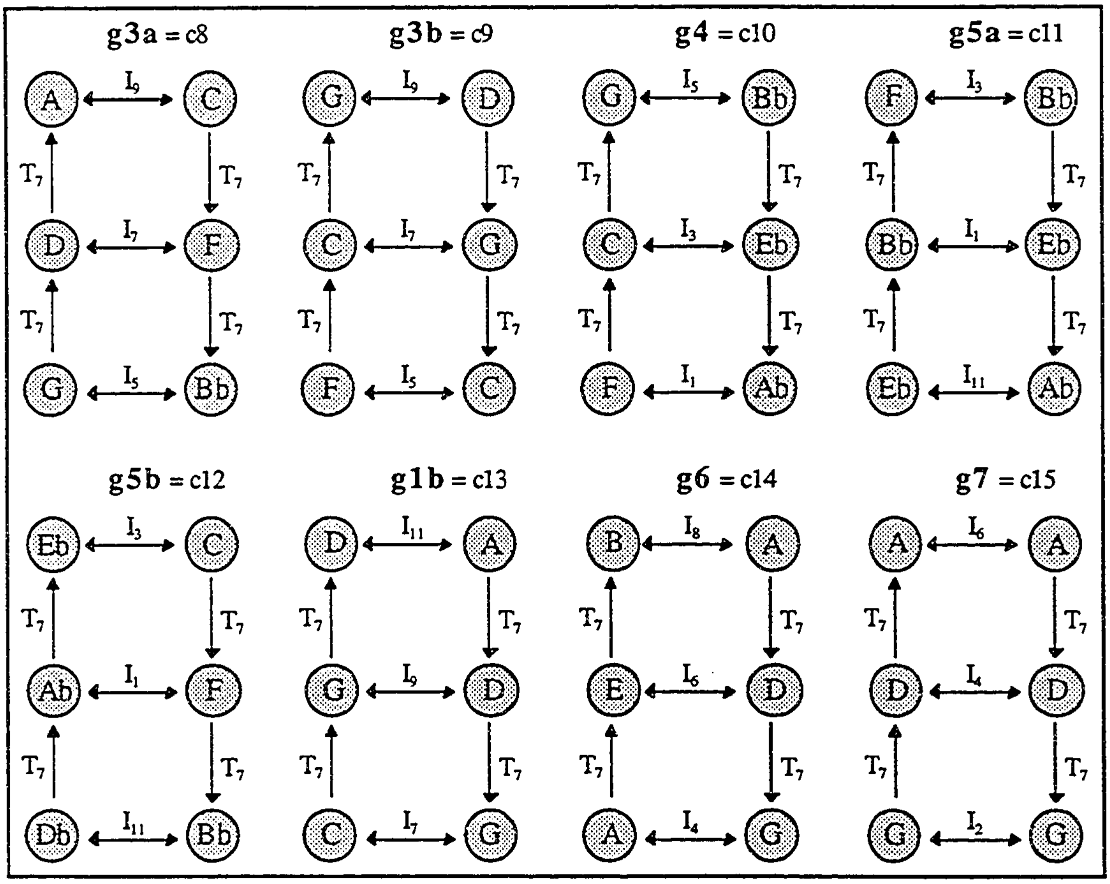
Example 2.1–7: Klumpenhouwer networks modeling the chords in subphrase 1



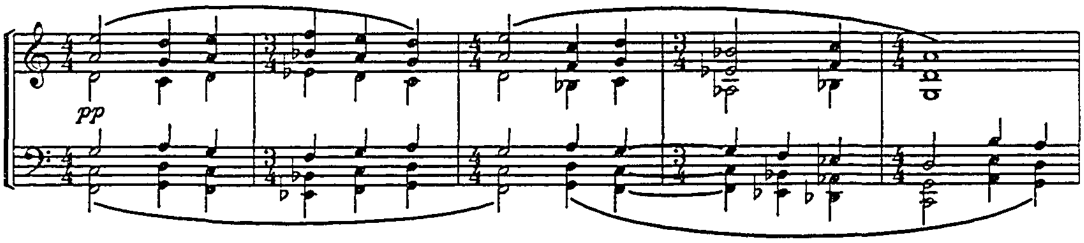
Example 2.1–8 models the subphrase 2 chords as Klumpenhouwer networks. Again, the relationship between the two three-chord units c8 → c9 → c10 and c11 → c12 → c13 stands out. As shown in Example 2.1–10, the latter trio (represented by networks g5a, g5b, and g1b) is the $<T_6>$ transform of the former (represented by networks g3a, g3b, and g4). More significantly, the $<T_6>$ relationship supports the overlapping of the two subphrases implied by Bartók's phrasing. The bottom of the example illustrates the overlapping progressions c7 → c8 → c9 → c10 and c10 → c11 → c12 → c13. The complete series of transformations from chord 7 (the point of overlap between the two subphrases) to chord 13 is $<<T_{10}>, <T_0>, <T_8>, <T_{10}>, <T_0>, <T_8>> = <T_0>$. This series of transformations, like the dual transpositions discussed above, highlights chords 10 and

13 as goals, also confirmed by Bartók's metric notation. The overlap emphasized by the Klumpenhouwer networks further links c7, c10, and c13. A middleground progression emerges prolonging {F, C, G, D, A, E} in chords 1–7, which then progresses <T_6> to chord 10 and returns to the original Klumpenhouwer class via the inverse <T_6> to chord 13 (though c13 is a member of a different set class). It is worth noting that the <T_6> has no recursive power as interval-class 6 is conspicuously absent from all of the set classes involved in this passage. Example 2.1–9 summarizes this interpretation of the phrase using three levels of mappings: adjacencies, subphrase articulations, and phrase span. Transformations are given in both dual transformational and Klumpenhouwer formats, with the more relevant form given first in each level.

Example 2.1–8: Klumpenhouwer networks modeling the chords in subphrase 2



Example 2.1–9: Voice-leading summary



c1 [024579] c2 [0257] c3 [024579] c4 [027] c5 [024579] c6 [0257] c7 [024579] c8 [024579] c9 [0257] c10 [024579] c11 [0257] c12 [024579] c13 [0257] c14 [02479] c15 [027]

Adjacencies

E - -→ D - -→ E - -→ F - -→ E - -→ D - -→ E - -→ C - -→ D - -→ Bb - -→ Bb - -→ C - -→ A - -→ A - -→ A

A - -→ G - -→ A - -→ Bb - -→ A - -→ G - -→ A - -→ F - -→ G - -→ Eb - -→ Eb - -→ F - -→ D - -→ D - -→ D

D - -→ C - -→ D - -→ Eb - -→ D - -→ C - -→ D - -→ Bb - -→ C - -→ Ab - -→ Ab - -→ Bb - -→ G - -→ G - -→ G

G ──→ A ──→ G ──→ F ──→ G ──→ A ──→ G ──→ A ──→ G ──→ G ──→ F ──→ Eb ──→ D ──→ B ──→ A

C ──→ D ──→ C ──→ Bb ──→ C ──→ D ──→ C ──→ D ──→ C ──→ C ──→ Bb ──→ Ab ──→ G ──→ E ──→ D

F ──→ G ──→ F ──→ Eb ──→ F ──→ G ──→ F ──→ G ──→ F ──→ F ──→ Eb ──→ Db ──→ C ──→ A ──→ G

T_2/T_{10} T_{10}/T_2 T_{10}/T_1 T_2/T_{11} T_2/T_{10} T_{10}/T_2 T_2/T_8 T_{10}/T_2 T_0/T_8 T_{10}/T_0 T_{10}/T_2 T_{11}/T_9 T_9/T_0 T_{10}/T_0

$\langle T_0\rangle$ $\langle T_0\rangle$ $\langle T_{11}\rangle$ $\langle T_7\rangle$ $\langle T_0\rangle$ $\langle T_0\rangle$ $\langle T_{10}\rangle$ $\langle T_0\rangle$ $\langle T_8\rangle$ $\langle T_{10}\rangle$ $\langle T_0\rangle$ $\langle T_8\rangle$ $\langle T_9\rangle$ $\langle T_{10}\rangle$

Subphrase articulations

E - - - - - - - - - -→ F - - - - - - - - - -→ E - - - - - - - - - -→ Bb - - - - - - - - - -→ A

A - - - - - - - - - -→ Bb - - - - - - - - - -→ A - - - - - - - - - -→ Eb - - - - - - - - - -→ D

D - - - - - - - - - -→ Eb - - - - - - - - - -→ D - - - - - - - - - -→ Ab - - - - - - - - - -→ G

G ──────→ F ──────→ G ──────→ G ──────→ D

C ──────→ Bb ──────→ C ──────→ C ──────→ G

F ──────→ Eb ──────→ F ──────→ F ──────→ C

$\langle T_{11}\rangle$ $\langle T_1\rangle$ $\langle T_6\rangle$ $\langle T_0\rangle$

T_{10}/T_1 T_2/T_{11} T_0/T_6 T_7/T_{11}

Phrase span

E -→ A

A -→ D

D -→ G

G ──────────────────────────────→ D

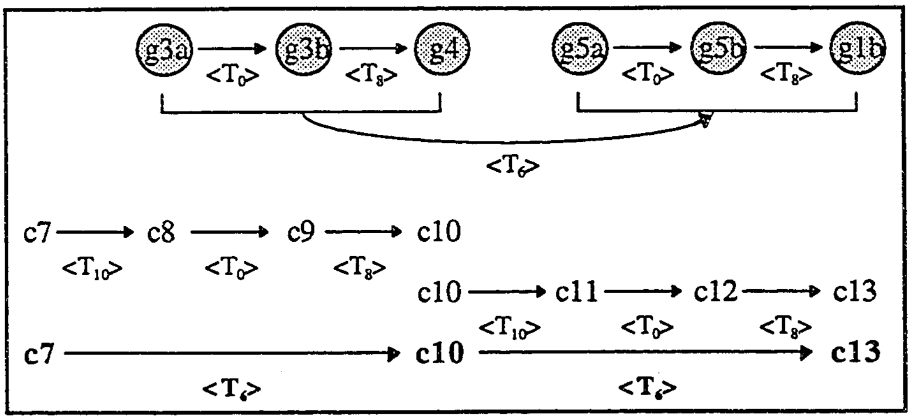
C ──────────────────────────────→ G

F ──────────────────────────────→ C

$\langle T_0\rangle$

T_7/T_5

Example 2.1–10: Klumpenhouwer networks modeling the phrase overlap



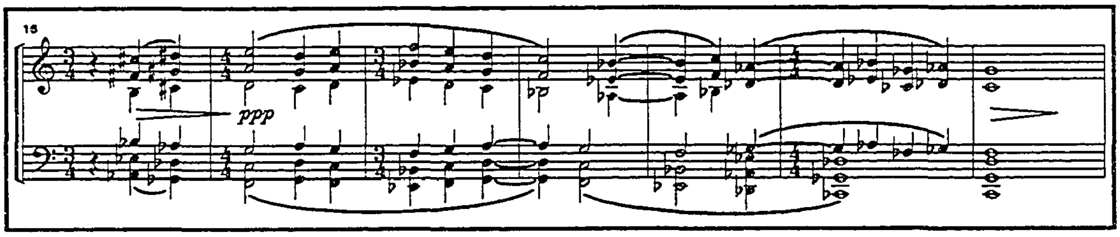
The final two chords of the phrase remain ambiguous. My intuitive impression is that the strong cadence on the downbeat of measure 5 imparts a post-cadential quality to them, but Bartók's slur marking prevents them from being truly anacrustic. Though the <T_9> motion from c13 → c14 is strikingly unusual in this context, it is possible to hear the <T_{10}> progression of c14 → c15 as an echo of the two previous <T_{10}> motions: c7 → c8, and particularly c10 → c11. It is tempting to suggest that reaching [027] as the final sonority is significant as it is the source trichord, but as none of the other string passages cadence with [027], almost always cadencing with [0257], this interpretation is not contextually convincing.[5]

The remainder of this first string passage has only one other unambiguous internal division, and that is the rest on the downbeat of measure 16. The third and final phrase of this passage is shown in Example 2.1–11. Measure 17 begins a recapitulation of the opening material, with some subsequent variations, forming a smaller ternary form within this section. The two chords in measure 16 are reminiscent of the post-cadential echo of measure 5, but in this case the rest that precedes them, and the fact that they belong to the

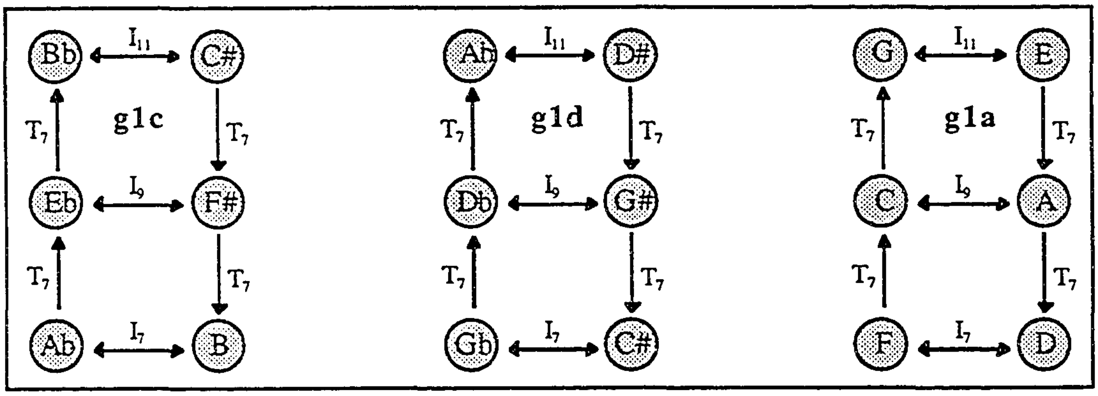
[5] The single exception to the [0257] cadences is the 6–z26 [013578] cadence at the end of the fifth string passage in measure 9 after the B section *Presto*.

same Klumpenhouwer class as the recapitulation material, as illustrated in Example 2.1–12, combine to make their anacrustic function less ambiguous.

Example 2.1–11: Bartók, Concerto for Piano and Orchestra No. 2, II, mm. 16–22

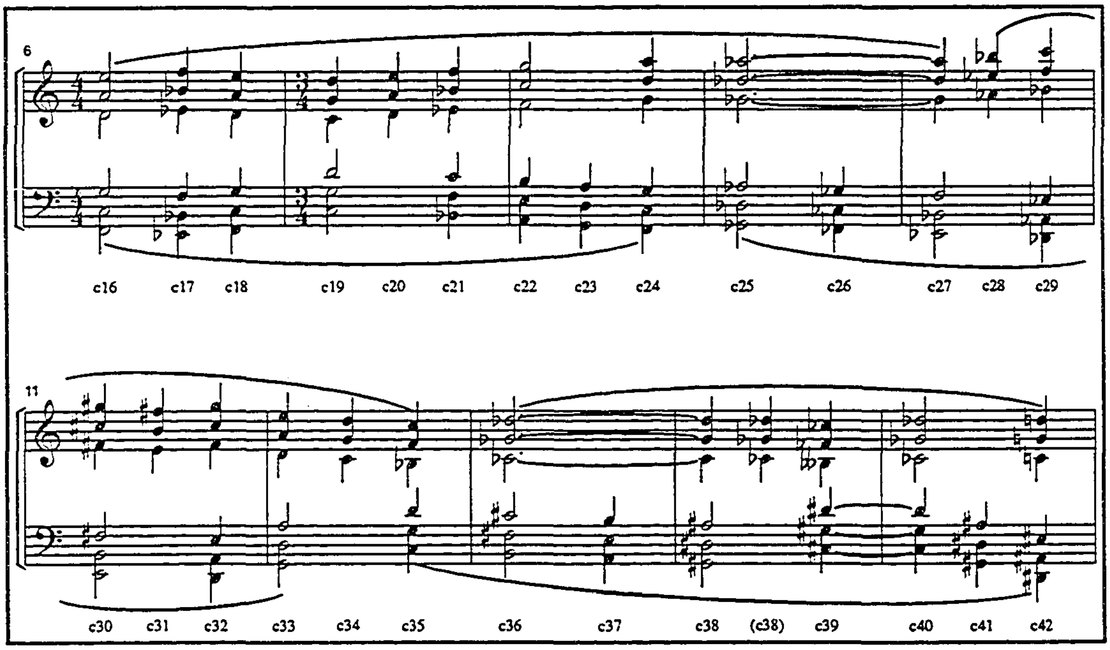


Example 2.1–12: Klumpenhouwer networks modeling the anacrusis to the third phrase

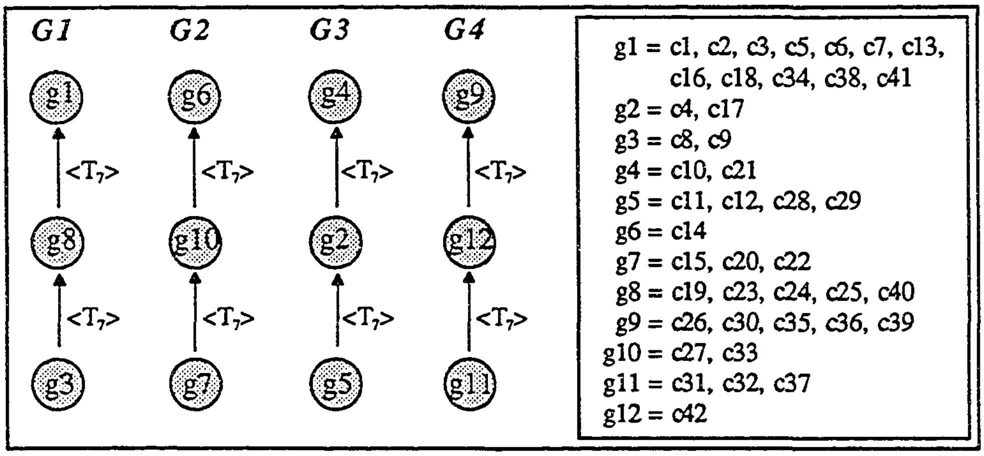


The reason for pointing out the recapitulatory nature of the third phrase is to discuss an interesting aspect of the structure of phrases 1 and 2 as a unit. In the long second phrase spanning measures 6–15, shown in Example 2.1–13, Bartók avoids a definitive goal until the final beat of measure 15. The first phrase included representatives of seven Klumpenhouwer classes, and the second phrase reiterates five of those, while also introducing representatives of the remaining five. The final Klumpenhouwer class representative is the cadential chord simultaneously completing the phrase and the Klumpenhouwer aggregate. As a 7-cycle can generate an aggregate, the Klumpenhouwer aggregate easily partitions into four supernetworks of stacked $<T_7>$ relationships, as shown in Example 2.1–14. The four supernetworks can be further paired as shown in Example 2.1–15 to create two recursive supernetworks that are isographic to the networks of the individual chords of the passage. The example includes network g1a as a reference. As these particular partitions are somewhat arbitrary (the three-chord $<T_7>$-cycle subsets could be rotated), it may be more satisfactory to notate this deep structure as a Klumpenhouwer chain, or, more accurately, a circle in this case. This supernetwork chain is shown in Example 2.1–16.

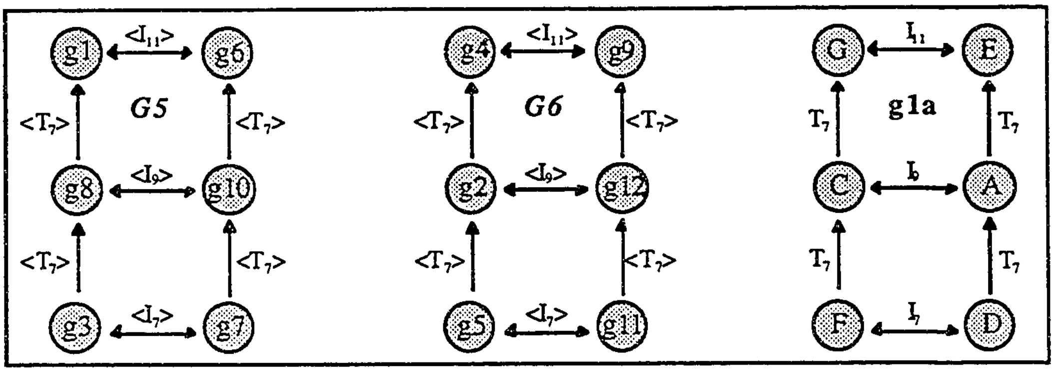
Example 2.1–13: Bartók, Concerto for Piano and Orchestra No. 2, II, mm. 6–15



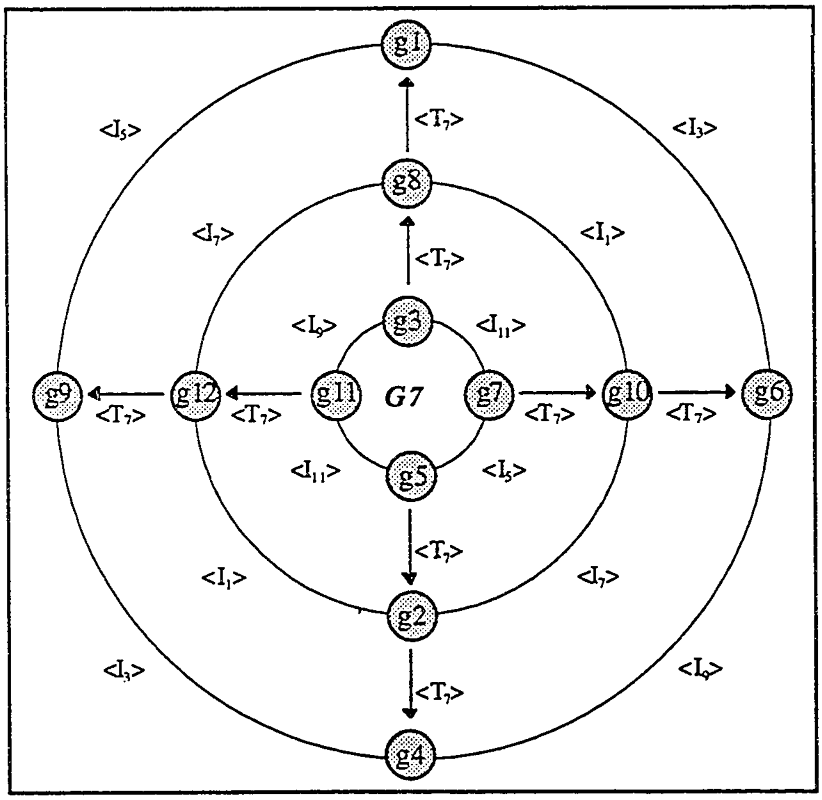
Example 2.1–14: $<T_7>$-cycle supernetworks



Example 2.1–15: Recursive supernetworks



Example 2.1–16: Supernetwork chain

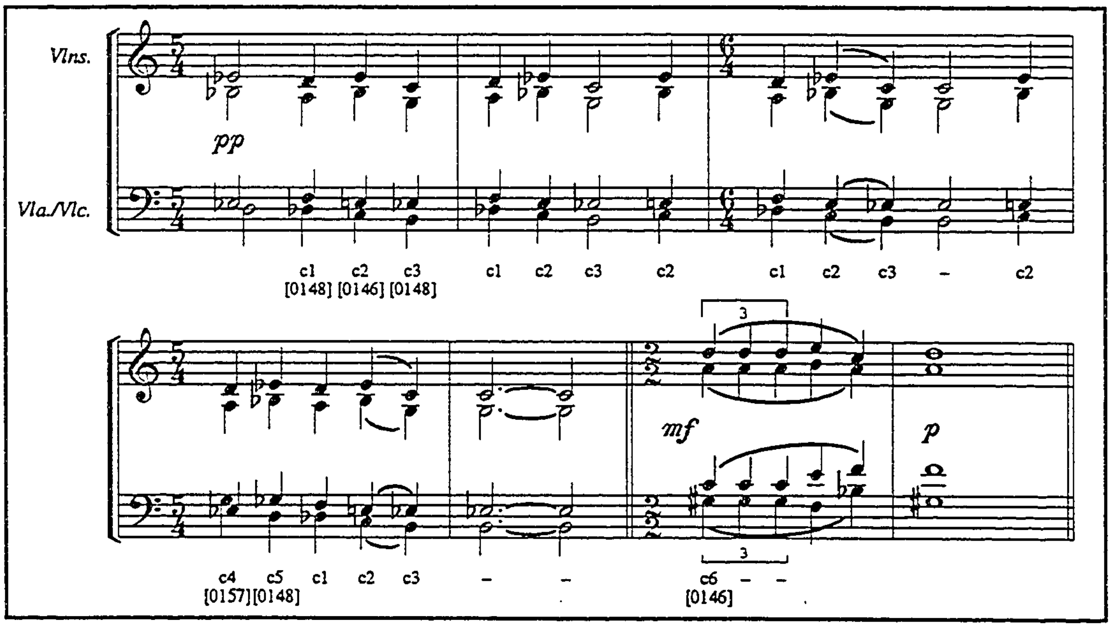


When these string passages return later in the movement, the pitches and often pitch classes are different. The specific progressions are even different, yet I hear each of these sections as a return to this material in a way that goes beyond instrumentation. It is the consistency of the two [027] streams and the resultant dual transpositions that are primarily responsible for this effect.

2.2 Igor Stravinsky: Three Pieces for String Quartet, III

As the third of Stravinsky's *Three Pieces for String Quartet* is almost entirely homorhythmic, it provides another opportunity to examine some of the benefits and ambiguities of my transformational approach across a longer musical span. Example 2.2–1 presents a short score of measures 3–9. The most striking aspect of this passage is the parallel pairs of voices: SA/TB. Though slightly less consistent than the previous Bartók excerpt, the upper and lower strings again generally follow their own streams. The viola and cello perform a series of intervals belonging to set-class 2–4 [04] (except for the unmarked chords in the example), and the violins perform a series of intervals belonging to set-class 2–5 [05] (except for the penultimate chord in the example). Traditional operations cannot generate the mappings between these chords, except for the few occasions when the pairs move at the same T_n value (specifically c3 → c1 and c5 → c1). Klumpenhouwer networks and dual transformations, on the other hand, are particularly useful for mapping such paired voices.

Example 2.2–1: Stravinsky, Three Pieces for String Quartet, III, mm. 3–9



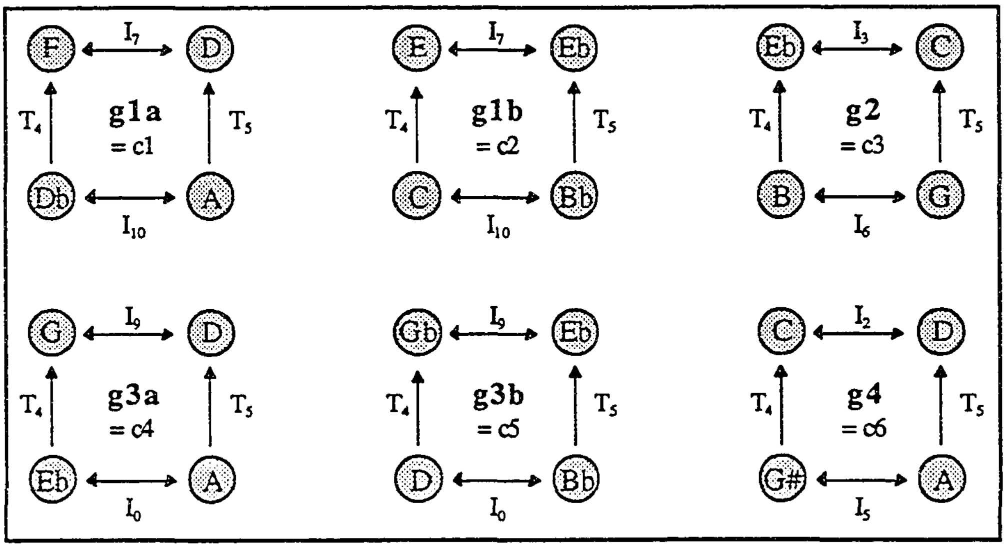
Ignoring the unmarked chords for now, the six chords generated by unions of [04] and [05] dyads (labeled c1–c6 in Example 2.2–1) make up the bulk of this passage.[6] Although the unions of various [04] and [05] dyads potentially generate the six different set classes shown in the table in Example 2.2–2, this excerpt only incorporates three of these possibilities: 4–19 [0148], 4–z15 [0146], and 4–16 [0157]. Unlike the similar table illustrating [027] unions in section 2.1, the first column in this table refers to the transpositional relationship between the bottom notes of the two intervals, rather than the entire sets. Stravinsky's limited harmonic vocabulary has significant structural ramifications. Example 2.2–3 models chords c1–c6 as Klumpenhouwer networks. All six networks are positively isographic, and two pairs—specifically g1 and g3—are strongly isographic, that is, they belong to the same Klumpenhouwer class. Tracing the voices among the corresponding network nodes yields the mappings shown in Example 2.2–4. The dual transpositions listed immediately below the mappings are equivalent to this particular Klumpenhouwer interpretation.

Example 2.2–2: Unions of [04] and [05] dyads

T_0	{04}	U	{05}	=	{045}	3–4 [015]
T_1	{04}	U	{16}	=	{0146}	4–z15 [0146]
T_2	{04}	U	{27}	=	{0247}	4–22 [0247]
T_3	{04}	U	{38}	=	{0348}	4–19 [0148]
T_4	{04}	U	{49}	=	{049}	3–11 [037]
T_5	{04}	U	{5a}	=	{045a}	4–16 [0157]
T_6	{04}	U	{6b}	=	{046b}	4–16 [0157]
T_7	{04}	U	{70}	=	{047}	3–11 [037]
T_8	{04}	U	{81}	=	{0148}	4–19 [0148]
T_9	{04}	U	{92}	=	{0249}	4–22 [0247]
T_{10}	{04}	U	{a3}	=	{034a}	4–z15 [0146]
T_{11}	{04}	U	{b4}	=	{04b}	3–4 [015]

[6] Arnold Whittall systematically catalogs the structure of all the chords in this movement in his essay "Tonality and the Emancipation of the Dissonance: Schoenberg and Stravinsky," *Early Twentieth-Century Music*, ed. Jonathan Dunsby (Oxford: Blackwell, 1993): 1–19. Whittall points out similarities among "chord-types," but he does not offer any interpretation of their linear connections.

Example 2.2–3: Klumpenhouwer networks modeling chords c1–c6



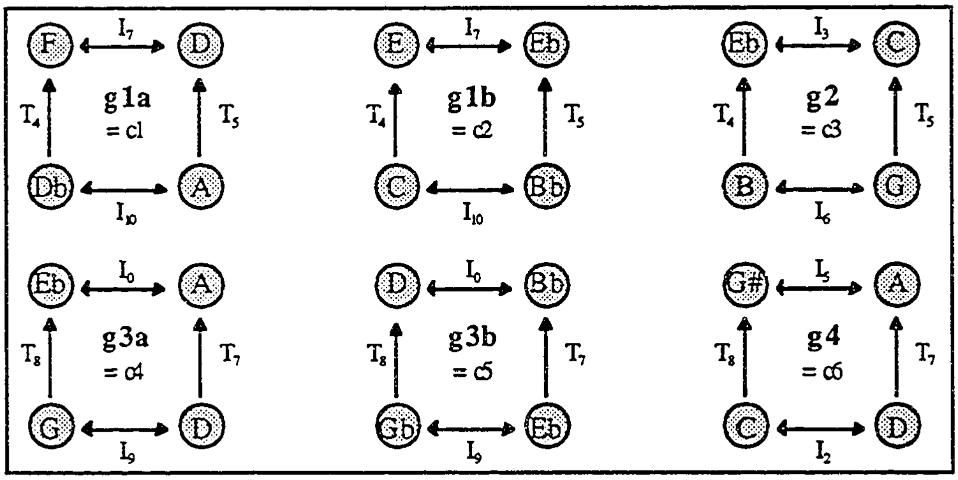
Example 2.2–4: Klumpenhouwer and dual transpositional mappings

m.3			m.4				m.5				m.6				m.7	m.8
c1	c2	c3	c1	c2	c3	c2	c1	c2	c3	c2	c4	c5	c1	c2	c3	c6
D	Eb	C	D	Eb	C	Eb	D	Eb	C	Eb	D	Eb	D	Eb	C	D
A	Bb	G	A	Bb	G	Bb	A	Bb	G	Bb	A	Bb	A	Bb	G	A
F	E	Eb	F	E	Eb	E	F	E	Eb	E	G	Gb	F	E	Eb	C
Db	C	B	Db	C	B	C	Db	C	B	C	Eb	D	Db	C	B	G#
T_{11}/T_1	T_{11}/T_9	T_2/T_2	T_{11}/T_1	T_{11}/T_9	T_1/T_3	T_1/T_{11}	T_{11}/T_1	T_{11}/T_9	T_1/T_3	T_3/T_{11}	T_{11}/T_1	T_{11}/T_{11}	T_{11}/T_1	T_{11}/T_9	T_9/T_2	
$\langle T_0 \rangle$	$\langle T_8 \rangle$	$\langle T_4 \rangle$	$\langle T_0 \rangle$	$\langle T_8 \rangle$	$\langle T_4 \rangle$	$\langle T_0 \rangle$	$\langle T_0 \rangle$	$\langle T_8 \rangle$	$\langle T_4 \rangle$	$\langle T_2 \rangle$	$\langle T_0 \rangle$	$\langle T_{10} \rangle$	$\langle T_0 \rangle$	$\langle T_8 \rangle$	$\langle T_{11} \rangle$	

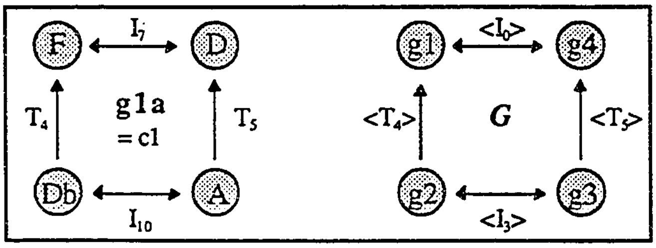
As any two dyads related by transposition are also related by inversion, any member of a series of positively isographic 2+2 Klumpenhouwer networks can be "flipped over," thereby creating a negatively isographic relationship to the remaining networks, as well as to its former self. Example 2.2–5 takes advantage of this relationship to invert graphs g3 and g4. The example includes networks g1 and g2 for reference. This revised interpretation of the harmonic structure of chords c4–c6 may seem counterintuitive at first, but it reveals a significant recursive structure among all of the [04] + [05] chords Stravinsky uses in this phrase. Example 2.2–6 presents this recursive relationship as the supernetwork *G*, and includes network g1a for comparison. What I find most interesting about this supernetwork is the way it unfolds throughout the passage. In measures 3–5

Stravinsky continually reiterates c1, c2, and c3, which I interpret as the supernetwork nodes g1 → g2. In measures 6–7 he includes these three chords and introduces c4 and c5, which I interpret as supernetwork node g3. At the end of the phrase in measure 7, just one more node—in a very specific relationship to the other three—is necessary to complete the supernetwork. The desire to attain the final node of the thus-far incomplete supernetwork recalls Lewin's if-only construct, and the teleological nature of if-only is particularly interesting on these deeper recursive levels. Chord 6, which I interpret as the final supernetwork node g4, begins the "response" phrase on the downbeat of measure 8. I find it extremely significant that fulfilling the if-only desire coincides with the boundary between the two phrases in this excerpt. Changes in surface features such as meter, dynamic, and register, dramatize both the arrival of the new phrase and the completion of the supernetwork.

Example 2.2–5: Inverting networks g3 and g4 for negative isography

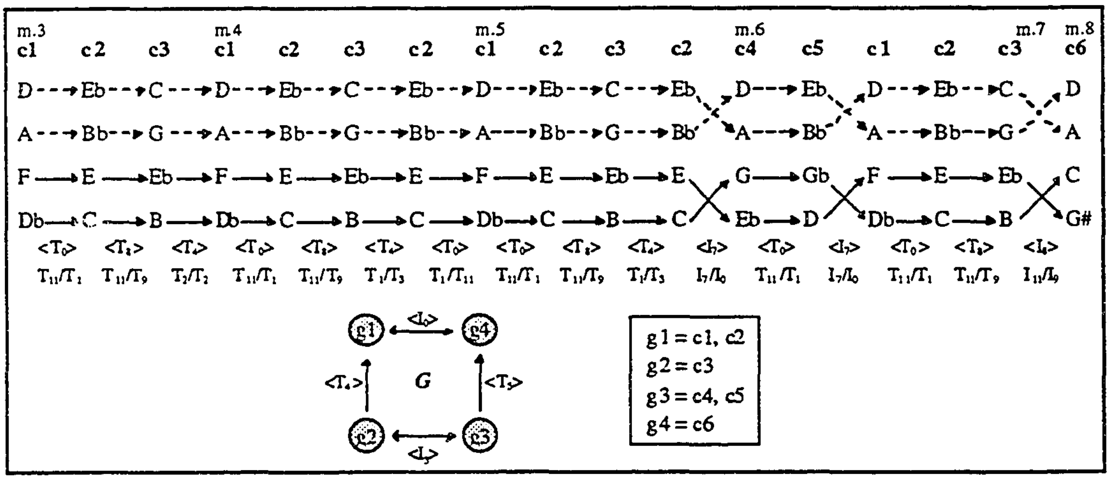


Example 2.2–6: Recursive supernetwork G



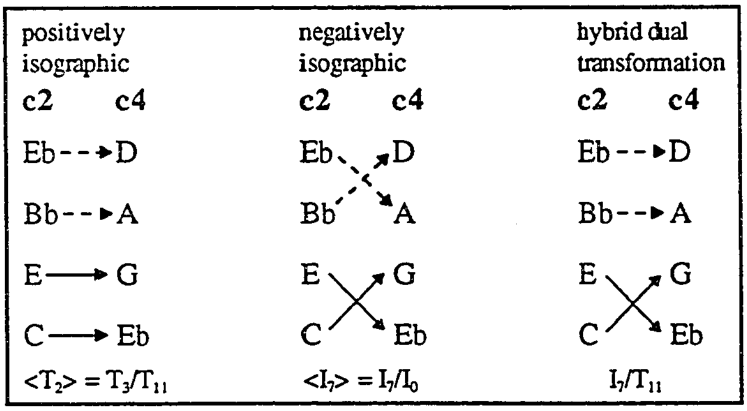
The negatively isographic relationships shown in the supernetwork *G* and in Example 2.2–5 imply a different set of mappings, that is, voice leading, than those shown in Example 2.2–4. Inverting the dyads, via the flipping of the networks, yields the three crossed mappings shown in Example 2.2–7. These <I_n> transformations are equivalent to dual inversion, as shown at the bottom of the example. The reason for listing the Klumpenhouwer transformations first in this example (immediately below the mappings) is that there is little cause for privileging this particular interpretation without taking the recursive supernetwork into account. The voices shown in Example 2.2–4 are identical to the registral—and most of the instrumental—lines, thereby modeling the musical surface very well, while the voices shown in Example 2.2–7 are counterintuitive to the musical surface, resulting more as a byproduct of a deeper musical isomorphism.

Example 2.2–7: Mappings based on the supernetwork G



A couple of surface events support the notion that the transformations into measures 6 and 8 are unusual in the context of this phrase. Most obvious is the start of the new section, the "response," at measure 8 with its dramatic change of register and dynamic. The motion into measure 6 is slightly more subtle: after hearing three measures of repeated chords, we hear a new simultaneity along with a voice crossing in the lower strings. The cello and viola cross voices here for the only time in the movement (excluding the cello harmonics heard later). Registrally the tenor and bass still continue the parallel [04] dyads established by the previous chords as shown in Example 2.2–4, but the crossing instrumental lines support the first negatively isographic mappings shown in Example 2.2–7, at least in the bottom two voices. This illustrates an interesting limitation of Klumpenhouwer networks mentioned in Chapter 1. It is not possible to map the bottom dyad as an inversion and the top dyad as a transposition. Both dyads must be inverted to create the negatively isographic relationship. Example 2.2–8 presents the two possible mappings based on [04] + [05] Klumpenhouwer networks, as well as a third mapping illustrating the additional possibilities of hybrid dual transformations. The hybrid I_7/T_{11}, shown in the example, models the instrumental lines exactly, while maintaining the integrity of the subsets; it is impossible to do this with Klumpenhouwer networks. Though there are significant musical reasons in this particular case to privilege the two mappings previously discussed, it is easy to imagine giving the I_7/T_{11} instrumental lines greater weight here—justified by the unique voice-crossing event—or in another context.

Example 2.2–8: Three interpretations of c2 → c4

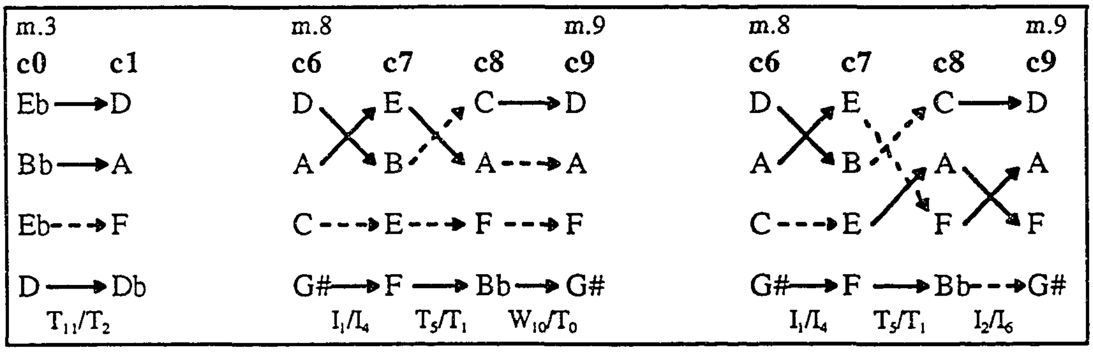


Four chords—the first one and final three (hereafter c0, c7, c8, c9)—in Example 2.2–1, and their four voice-leading connections, remain unaccounted for thus far. Chords c7, c8, and c9 are representatives of set-classes 3–5 [016], 4–14 [0237], and 4–18 [0147] respectively, none of which can partition into the discrete subsets [04] + [05]. The first chord, c0, is a member of set-class 3–4 [015], which can divide into [04] + [05] (refer back to Example 2.2–2), but the doubling here suggests otherwise. A doubled Bb, rather than Eb, would allow this interpretation. Though none of these chords consists of the [04] + [05] structure heard throughout the remainder of the passage, they all do share common discrete subset pairs with adjacent chords, and therefore can map into their neighbors via dual transformations. For example, c0 can map into c1 at T_{11}/T_2 as they share the trichordal subset [015]; see the far left side of Example 2.2–9.[7] There are a number of dual-transformational relationships among the final three chords, but I prefer the two shown in Example 2.2–9 as the voice permutations ultimately arrive back at their original registral location. Furthermore, I prefer the left example as it does not contradict the motivic registral SA/TB voice-leading pairs established by the parallel [04] and [05] dyads. In this case the 3+1 I_1/I_4 transformation (measure 8) exchanges the upper pair of voices, while the subsequent 2+2 progression T_1/T_5 pairs the voices ST/AB, rather than the SA/TB already established, but significantly re-exchanges the upper pair of voices. The final progression, c8 → c9 (measures 8–9), illustrates a case in which I find dyadic wedging perceptually significant. In this progression I hear a repetition of the inner two pitches, while the outer two pitches wedge further apart by a whole tone each. This process literally involves three transpositions: T_0 for the static pitches, and both T_2 and T_{10} for the wedging pitches. However, instead of hearing a split transposition in three parts, I believe that the hybrid dual transformation W_{10}/T_0 best captures my interpretation, with W_{10} representing the vertical wedge index of the outer voices. The SB/AT voice pairing in this transformation digresses from the previously prominent SA/TB pairs, but the voices do still coincide with the surface lines. The mappings shown on the right side of the example cut across the consistent upper-string/lower-string division with the middle (STA) permutation resulting

[7] The dual transposition T_{11}/T_2 is virtually identical to Straus's near-T_{11} interpretation of this progression in "Voice Leading in Atonal Music," which I discussed previously in Chapter 1. Our interpretations of the voice leading diverge after c1 as he maintains the singleton partition, while I prefer the dyadic partition suggested by the salient parallel intervals.

from the reinterpretation of the T_5/T_1 transformation. On the other hand, this second interpretation may be preferable in its inclusion of only traditional operators, specifically excluding the W_n of the previous interpretation. One could model each of these isolated chord pairs as isographic Klumpenhouwer networks, but as the specific shared discrete subsets change with each pair, it is necessary to reconfigure each chord network as one moves through the progression. I discuss this process of *double emploi* more fully in my analysis of Charles Ives's song "Serenity" later in this chapter.

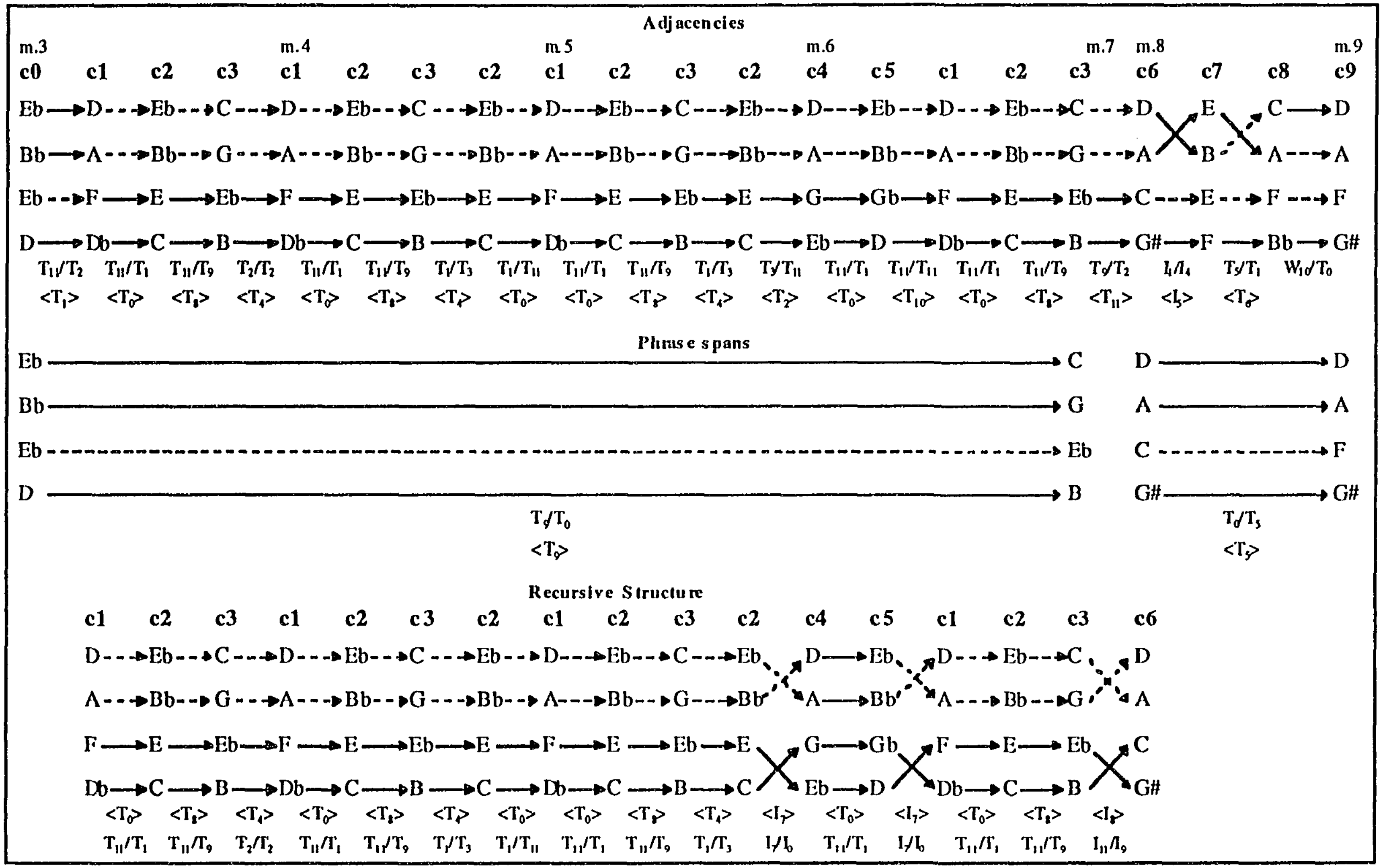
Example 2.2–9: The chords excluded from the supernetwork



Many ambiguities arise owing to the numerous voice-leading possibilities generated by dual transformations, particularly with the inclusion of W_n transformations. In addition to the kind of multiple mappings presented in Example 2.2–8 and Example 2.2–9, there are instances when multiple transformations yield the same set of mappings. For example, two additional dual wedges can relate all of the chord pairs belonging to the same Klumpenhouwer class, for example, one could interpret the motion c1 → c2 as W_3/W_2 (SB/AT) or W_{10}/W_7 (AB/ST). As these transformations do not affect the resultant mappings—only the pairings change—it is more relevant in this case to maintain the dual transpositions, although the SB/AT wedges may prepare the final progression, W_{10}/T_0. It is important to keep in mind that in most musical contexts only a few of the numerous transformational possibilities will have the qualities of coherence, simplicity, continuity, and suggestiveness that make analyses persuasive or interesting.

Example 2.2–10 presents a voice-leading summary of the entire passage. The example includes three levels of voice-leading transformations, as did the Bartók summary in the previous section. Once again it is worth reminding the reader that these are non-hierarchical levels; each "deeper" level does not subsume the previous one. There is a sense that the adjacencies level is closest to the concrete musical surface, and that the recursive level is the most abstract, but beyond that they merely model different aspects of the same musical phenomena.

Example 2.2–10: Voice-leading summary

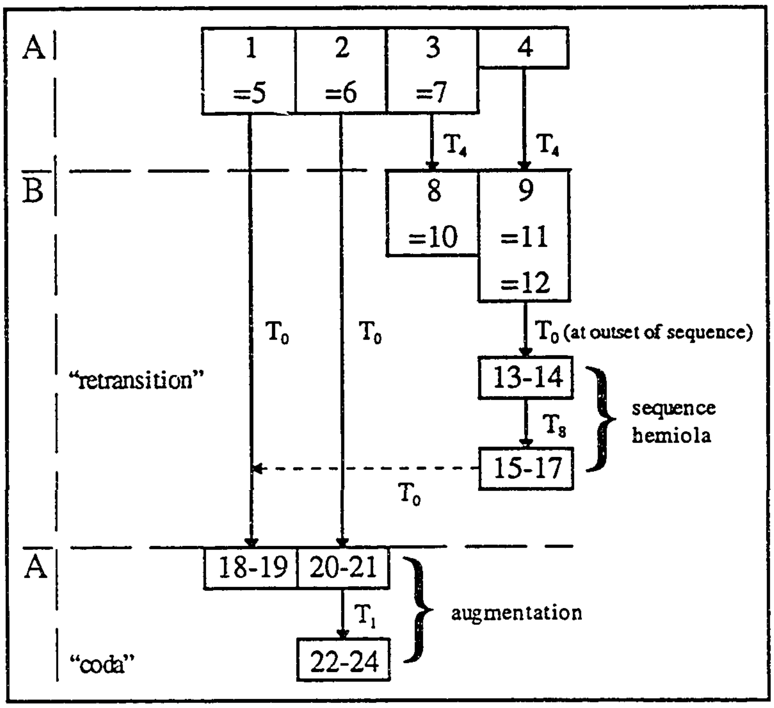


2.3 Alexander Skryabin: Prelude, Op. 74, No. 4

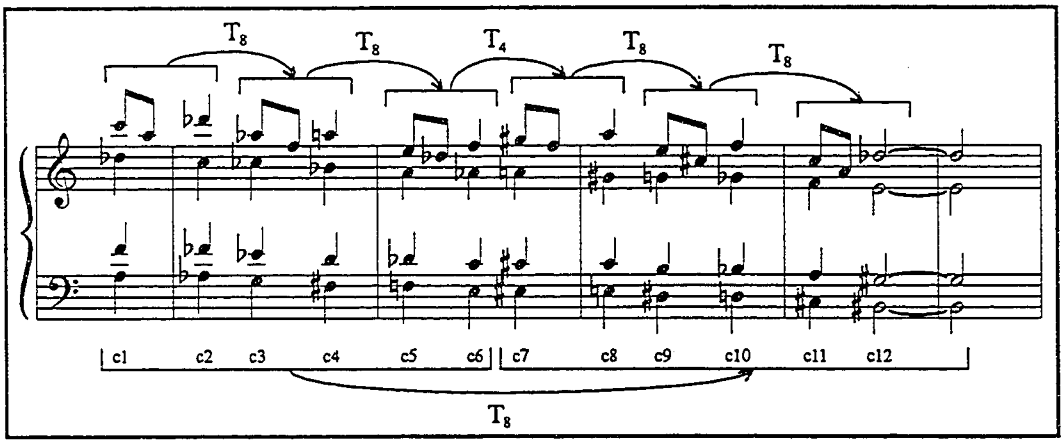
As demonstrated in the previous two analyses, dual transformations and Klumpenhouwer networks provide a useful method of creating mappings and exploring phrase structure in musical passages containing chords from a number of different set classes. They are also useful in situations where all the harmonies are members of the same set class, as I will show in this analysis of an excerpt from Skryabin's *Prelude*, Op. 74, No. 4. Skryabin develops this roughly ternary prelude of twenty-four measures from four bars of material via repetition, slight variation, and transposition. Example 2.3–1 presents a form network showing how transformations of the first four measures generate the entire prelude. The transformations are of the measures, not just their set-class content, with little or no variation.[8] Long-range projection of the motivic set-class 3–3 [014] articulates the form, beginning with the large-scale T_4 transformation in measure 8, continuing with the T_8 return in measure 15, and concluding with the T_1 "coda" of measures 22–24. The passage I wish to explore in greater detail is the sequential "retransition" of measures 13–17, reproduced in Example 2.3–2, that facilitates the return to the original level of transposition. Note the T_8 series of linear [014] trichords in the melody that characterize the sequence. The excerpt includes twelve vertical tetrachords—all members of set-class 4–19 [0148]—divided into two subphrases of six chords each. The set-class homogeneity of this phrase offers an opportunity to compare some of the analytical possibilities of traditional operations, dual transformations, and Klumpenhouwer networks in a single musical context.

[8] For example measures 11 and 12 are identical, but they are slightly different than measure 9.

Example 2.3–1: Form diagram

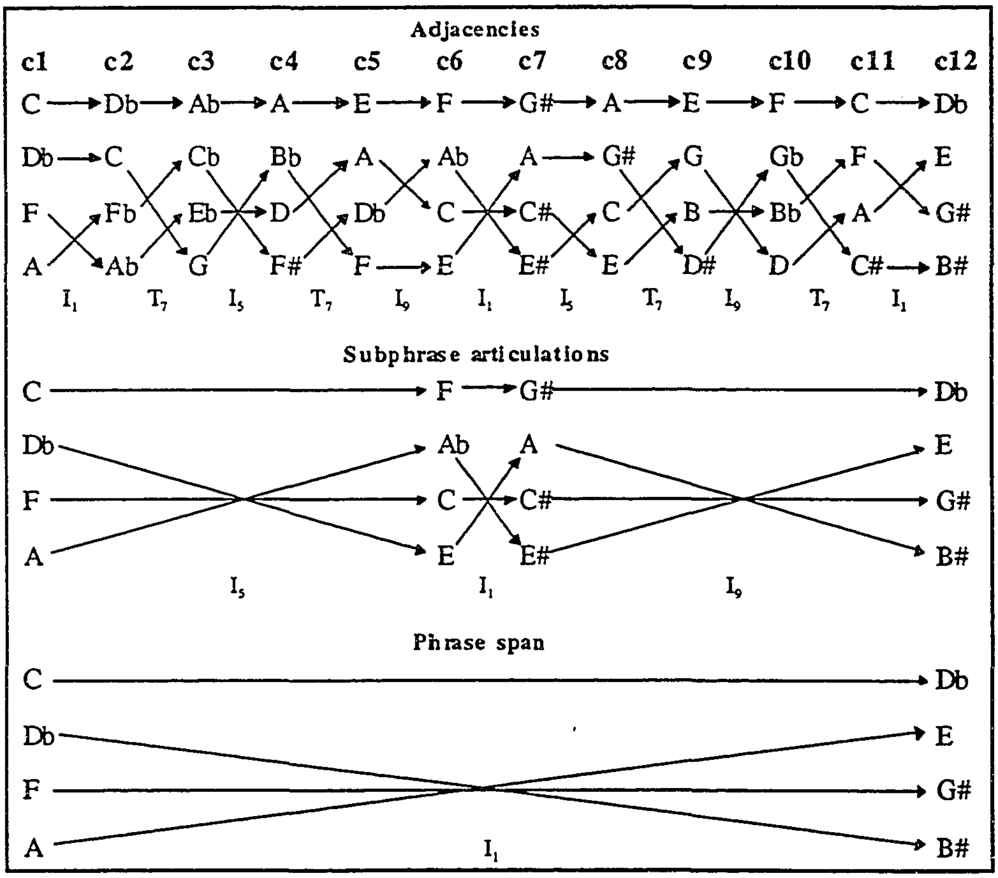


Example 2.3–2: Skryabin, Prelude, Op. 74, No. 4, mm. 13–17



As all the chords are from the same set class, by definition it is possible to use traditional operators to map the voices. Example 2.3–3 illustrates the voices generated by transposition and inversion. Much about this interpretation is satisfactory, particularly the consistency of the melodic line on the adjacencies level, the motivic alto-bass exchange of the subphrase level, and the replication of the initial progression in the background span of the entire sequence. On the other hand, the voices in Example 2.3–3 do not seem to capture the musical surface very well. Despite the set-class uniformity, I hear this phrase partitioned into its two component threads: the harmonic support of descending parallel members of set-class 3–12 [048] (or augmented triads), and the ascending melodic half steps. Example 2.3–4 uses dual transpositions to model this interpretation of the musical surface.[9] These voices do not negate the mappings in Example 2.3–3, but rather provide a more concrete account of the passage, perhaps relegating the traditional mappings to various deeper and more abstract structural levels.

Example 2.3–3: Traditional transformational mappings



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[9] Example 2.3–3 and Example 2.3–4 both omit the weak eighth-note pitches, though the dual transformational model could incorporate them easily enough.

Example 2.3–4: Dual transformational mappings

Adjacencies

c1	c2	c3	c4	c5	c6	c7	c8	c9	c10	c11	c12
C	Db	Ab	A	E	F	G#	A	E	F	C	Db
Db	C	Cb	Bb	A	Ab	A	G#	G	Gb	F	E
F	Fb	Eb	D	Db	C	C#	C	B	Bb	A	G#
A	Ab	G	F#	F	E	E#	E	D#	D	C#	B#

T_{11}/T_1 T_{11}/T_7 T_{11}/T_1 T_{11}/T_7 T_{11}/T_1 T_1/T_3 T_{11}/T_1 T_{11}/T_7 T_{11}/T_1 T_{11}/T_7 T_{11}/T_1

Subphrase articulations

C - - - → F - - → G# - - - → Db
Db ——→ Ab ——→ A ——→ E
F ——→ C ——→ C# ——→ G#
A ——→ E ——→ E# ——→ B#

T_7/T_5 T_1/T_3 T_7/T_5

Phrase span

C - - - → Db
Db ——→ E
F ——→ G#
A ——→ B#

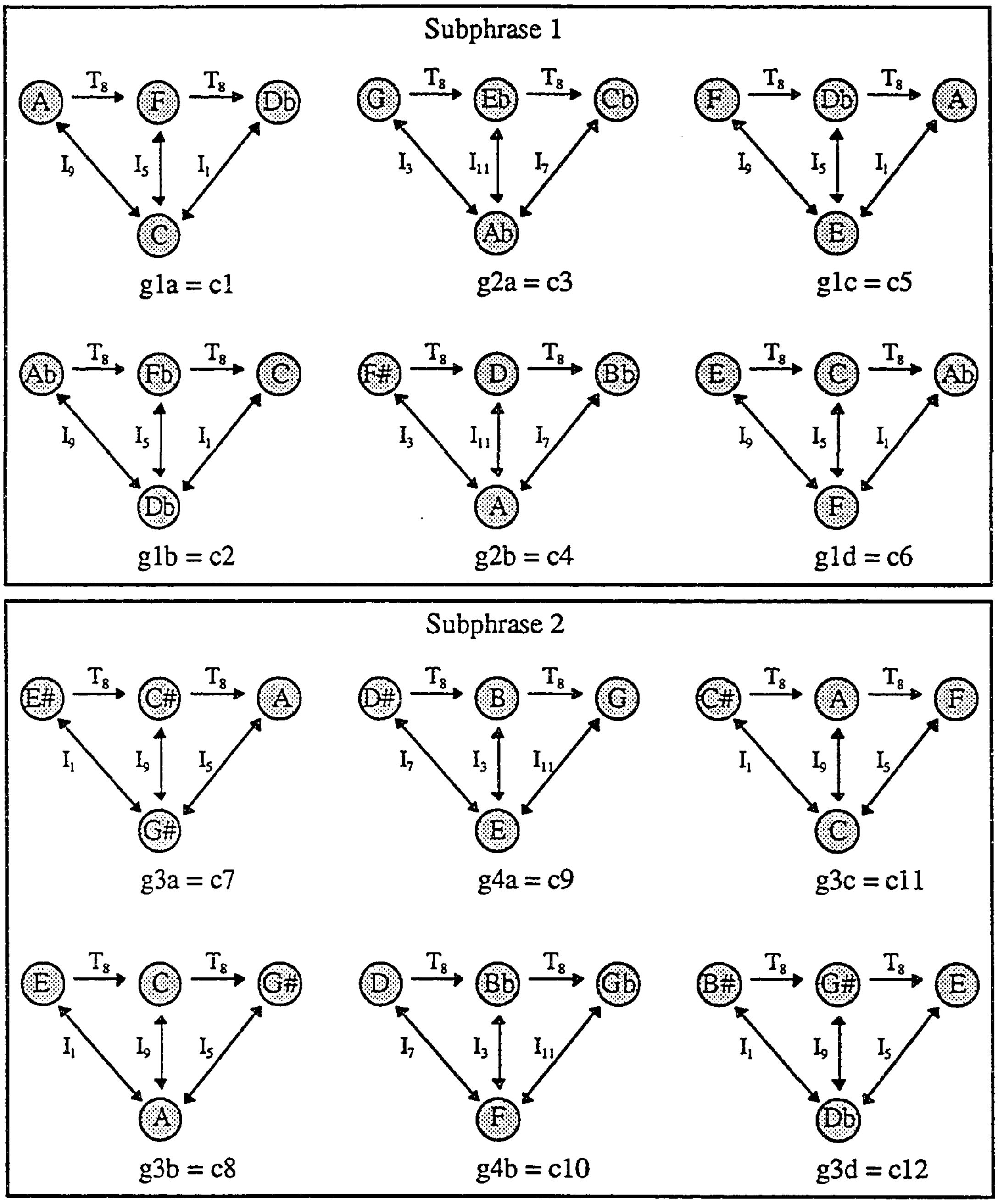
T_3/T_1

As Example 2.3–4 shows, the dual transpositions further emphasize the sequential chord pairs by the complementary dual T_n values (T_{11}/T_1), meaning each of these pairs belong to the same Klumpenhouwer class. The same holds true for the outer chords of the two subphrases, as shown by the complementary values T_7/T_5 in the middleground. The consistent ±2 x-values—in each reiteration of the sequential chord pairs in the foreground, as well as in all the transformations on both deeper levels—illustrate the cohesion between the levels shown in this example.

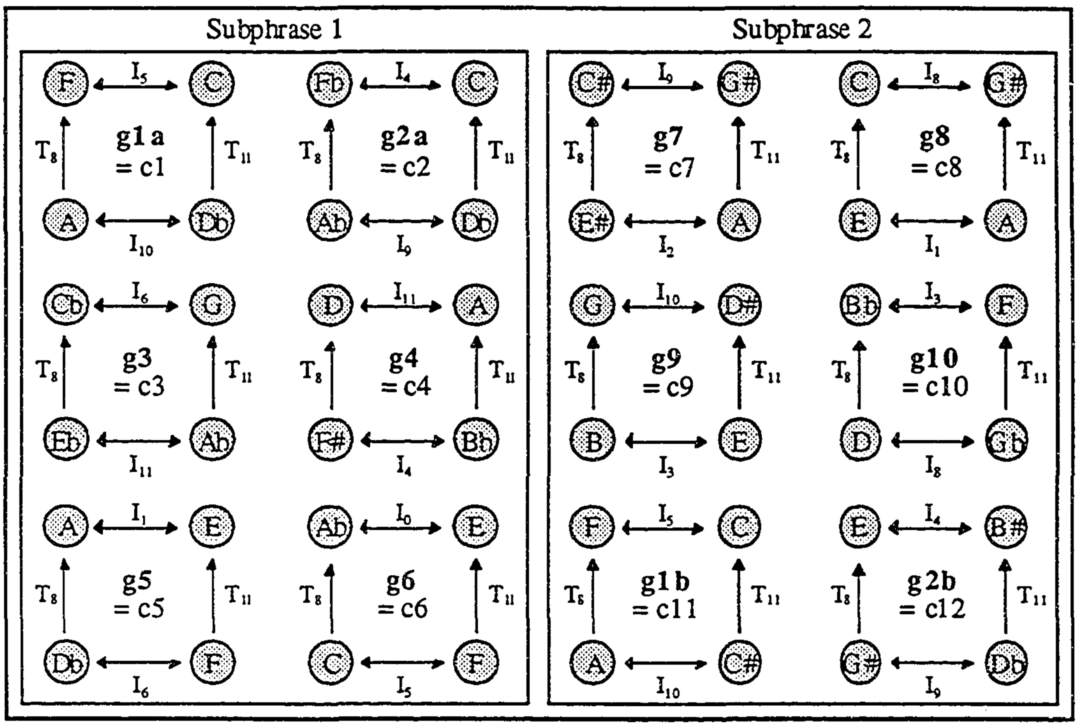
Expressing this sequence of [0148] tetrachords as Klumpenhouwer networks yields some additional information about more abstract levels of structure, and highlights a few otherwise unnoticeable features of the progression. Example 2.3–5 presents the Klumpenhouwer equivalent of the dual transpositions illustrated in Example 2.3–4. The networks again express the two component threads of the passage (that is, the parallel [048] trichords and the melodic singletons), and again emphasize the chord pairs that characterize the sequence. The strong isography between network pairs (g1a → g1b, g2a → g2b, etc., arranged vertically in the example) arises from the half-step wedging between the trichords and singletons, and is the graphic representation of the complementary dual transpositional values (T_{11}/T_1) mentioned above. The strong isography between the outer network pairs of each subphrase of the sequence, specifically all the g1 graphs and all the g3 graphs, is the graphic representation of the complementary dual T_7/T_5 of the subphrase articulations level. That these outer networks belong to the same Klumpenhouwer class stresses the similarity between the subphrase structure and the individual chord pairs in a more obvious manner than the previous two interpretations. Despite the prominent $\langle T_4 \rangle$ relationship between the two subphrases, and the four different graphs (suggesting four network nodes g1–g4), it is not possible to create a complete recursive supernetwork via this particular Klumpenhouwer partition.

Owing to the set-class homogeneity of this progression, each chord has the identical set of discrete subsets, thereby allowing the maximum number of matching Klumpenhouwer partitions, which in the case of tetrachords is seven. Example 2.3–6 illustrates one of the several possible alternative interpretations: a 2+2 partition structured around set-classes 2–4 [04] and 2–1 [01]. All the graphs in this example are positively isographic, but because both subsets are dyads, any number of them can be flipped over rendering them negatively isographic to the rest. Taking advantage of the invertible property of these networks, Example 2.3–7 presents two possible supernetwork chains—*G1* and *G2*—based on the [04] + [01] structure.

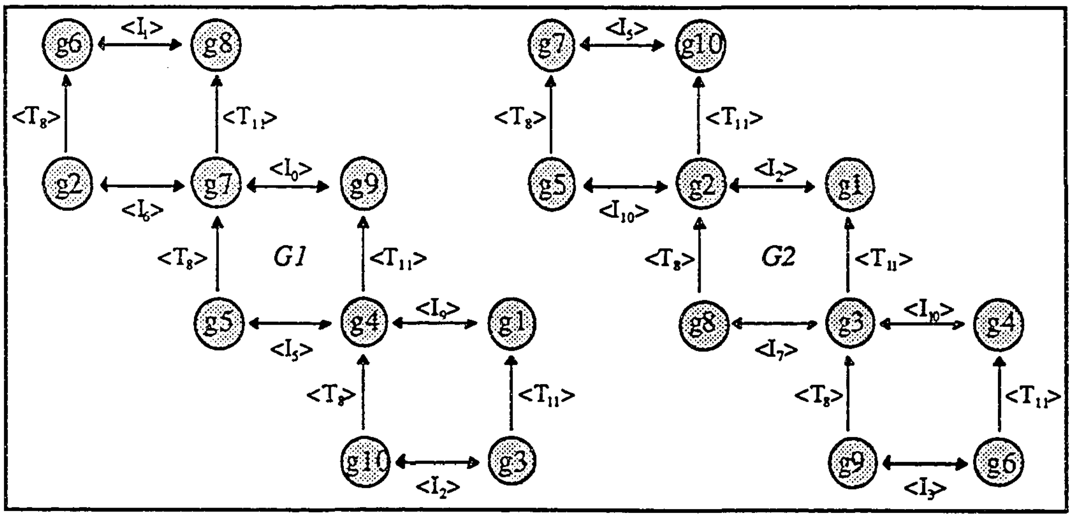
Example 2.3–5: Klumpenhouwer networks around the [048] subsets



Example 2.3–6: Klumpenhouwer networks around the [04] and [01] subsets



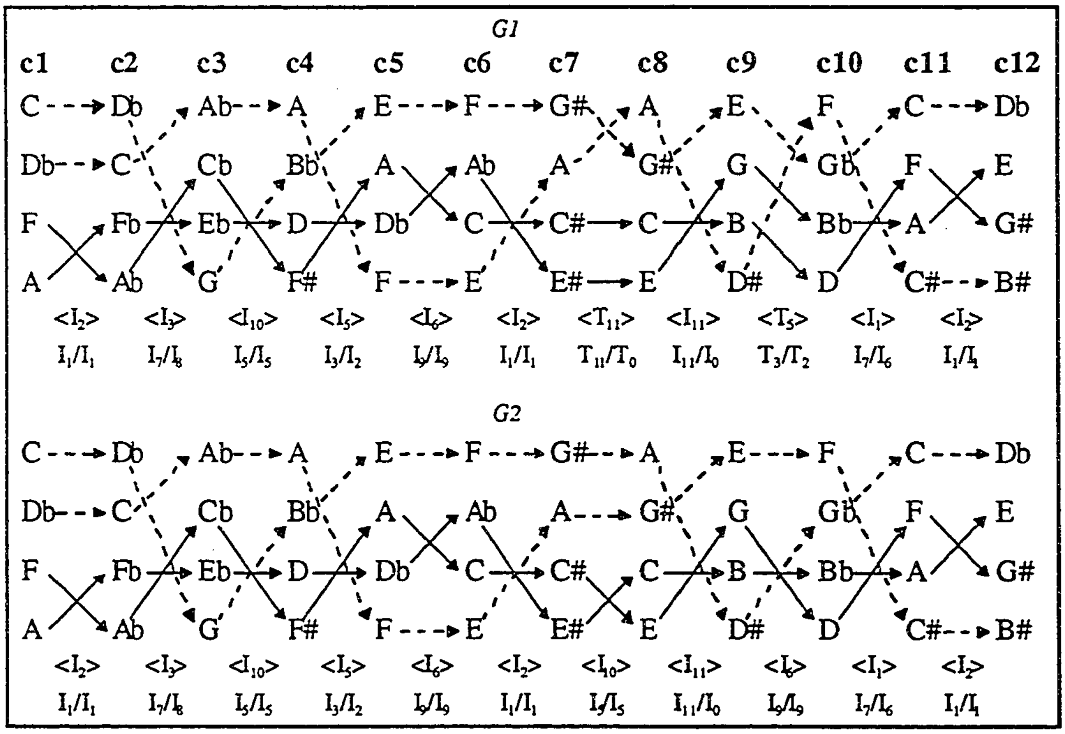
Example 2.3–7: Two supernetwork chains on the [04] + [01] structure



Example 2.3–8 realizes the implied voice leading of the two supernetwork chains *G1* and *G2*. The equivalent dual transformation for each Klumpenhouwer transformation is given below the mappings, and in this case many of them are equivalent to traditional

operations. Unlike Example 2.3–3 and Example 2.3–4, both sets of mappings in Example 2.3–8 are rather unsatisfactory on the foreground level in their destruction of the soprano line's integrity. *G1* is further unsatisfactory in its lack of consistency, including both <I_n> and <T_n> operations, which yield the uncharacteristic mappings in the progressions c7 → c8 and c9 → c10. These two <T_n> transformations are the only differences between the two sets of recursive voices. The most interesting result of these recursive structures is that both sets of mappings suggest the same middleground and background as the traditional operations shown back in Example 2.3–3 with their characteristic alto-bass exchanges.[10] I find the *G2* interpretation more successful than *G1* owing to its greater degree of transformational consistency, with both subphrases implying the same set of mappings.

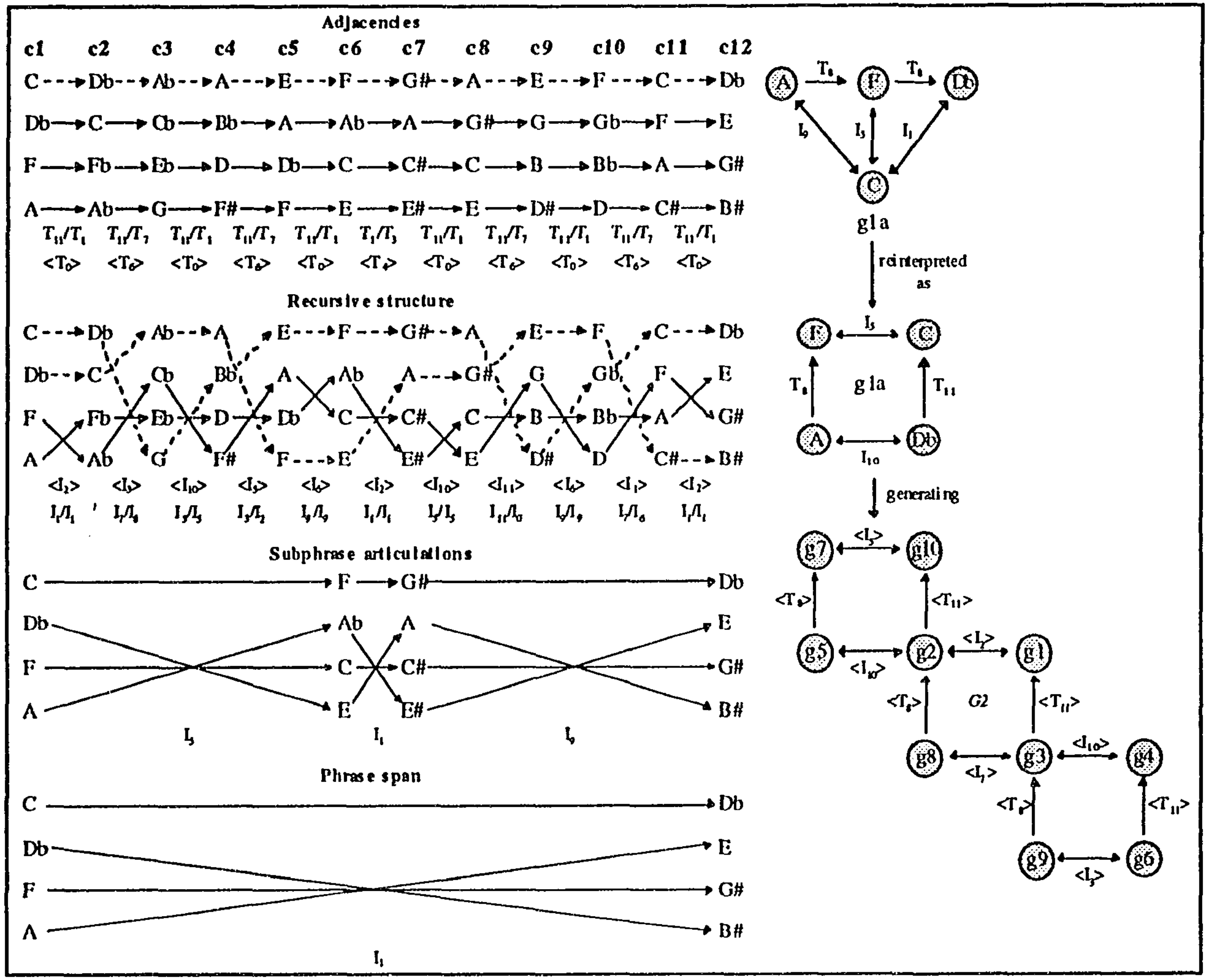
Example 2.3–8: Voice leading implied by supernetworks G1 and G2



[10] In the first subphrase of both *G1* and *G2* mappings: $I_1/I_1 + I_7/I_8 + I_5/I_5 + I_3/I_2 + I_9/I_9 = I_5/I_5$, which is merely a partitioned version of the traditional I_5 operation spanning the first subphrase. Despite the <T_n> operations incorporated into the second subphrase of the *G1* mappings, the same relationship holds true: $T_{11}/T_0 + I_{11}/I_0 + T_3/T_2 + I_7/I_6 + I_1/I_1 = I_5/I_5 + I_{11}/I_0 + I_9/I_9 + I_7/I_6 + I_1/I_1 = I_9/I_9$. Remember that this is only true because the partitions remain consistent throughout the passage.

In summary, each of these analytical passes through Skryabin's sequence of [0148] tetrachords highlights and suppresses different features of the progression. The dual transpositions (Example 2.3–4) and their equivalent [048] + [0] Klumpenhouwer networks (Example 2.3–5) most effectively model the concrete musical surface. The traditional operations (Example 2.3–3) naturally emphasize the set-class uniformity of the passage, and suggest that alto-bass exchanges are motivically significant on the subphrase and phrase levels. The [04] + [01] Klumpenhouwer networks (Example 2.3–6) and their supernetwork chains (Example 2.3–7, particularly *G2*) emphasize the structural integrity of the phrase on an abstract level. Furthermore, the mappings generated by the supernetwork chains (Example 2.3–8) merge with those of the traditional operations at the subphrase level. I find a reading that incorporates elements from all three passes through the phrase most satisfying, as shown in the voice-leading summary in Example 2.3–9. This interpretation attempts to combine the most appealing aspects of all the above models.

Example 2.3–9: Voice-leading summary



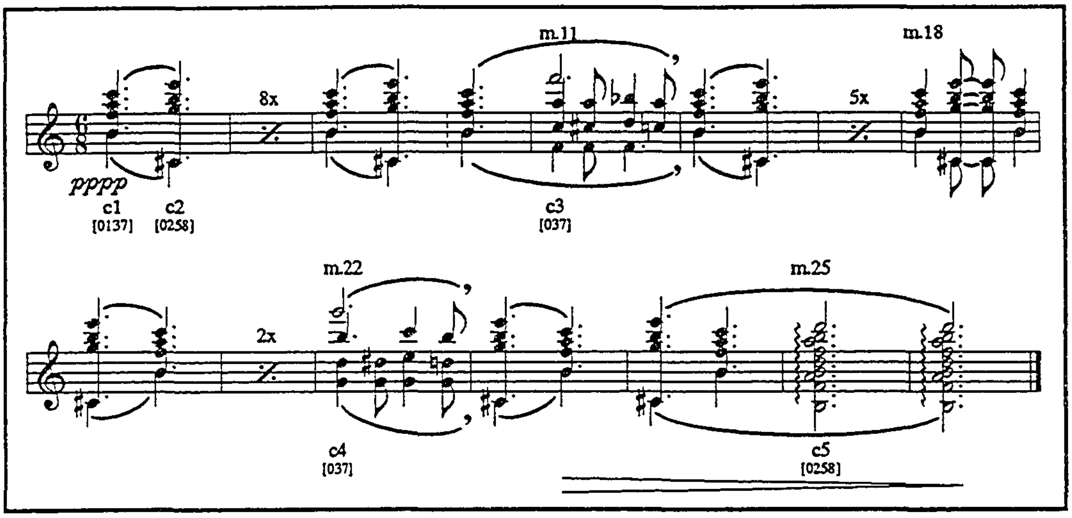
2.4 Charles Ives: "Serenity"

Expanding beyond the phrase-length excerpts contemplated in the three previous analytical essays, the final two analyses in this chapter examine complete works. Both of the Charles Ives songs I discuss—"Serenity" and "The Cage"—are relatively brief, but as complete musical compositions they offer a more comprehensive context in which to apply and further refine my voice-leading model.

A proto-minimalist piano accompaniment supports the chant-like vocal melody in Charles Ives's song "Serenity," shown in Example 2.4–1. The extremely quiet piano chords (*pppp*) maintain a four-voice texture throughout the chant's brief twenty-six bars, the only exception being the two final chords which roll through more than two octaves. Nearly the entire accompaniment consists of two alternating tetrachords, {BFAC} from set-class 4–z29 [0137] (labeled c1 in the example) and {C#GBE} from set-class 4–27 [0258] (labeled c2 in the example), with two interjected "cadential" passages in measures 11 and 22 that borrow triadic material from a hymn tune by William Wallace.[11] The entire harmonic vocabulary of this accompaniment contains only nine different chords, representing four different set classes: the two tetrachords [0137] and [0258], and the two trichords 3–11 [037] and 3–12 [048]. The tetrachords are by far the most significant, as the trichords surface for only two measures, and, furthermore, the [048] trichords sound for a single eighth note within each of those measures. In all but four bars of this song, the piano alternates only c1 and c2, making the progression [0137] → [0258] (and its retrograde [0258] → [0137]) one of the most prominent features of "Serenity." It was a fascination with this simple progression that originally led me to a closer examination of this song's harmonic structure.

[11] J. Peter Burkholder discusses this borrowing in substantial detail on pages 11–13 of his article "'Quotation' and Emulation: Charles Ives's Uses of His Models," *Musical Quarterly* 71.1 (1985): 1–26.

Example 2.4–1: Ives, "Serenity," piano accompaniment

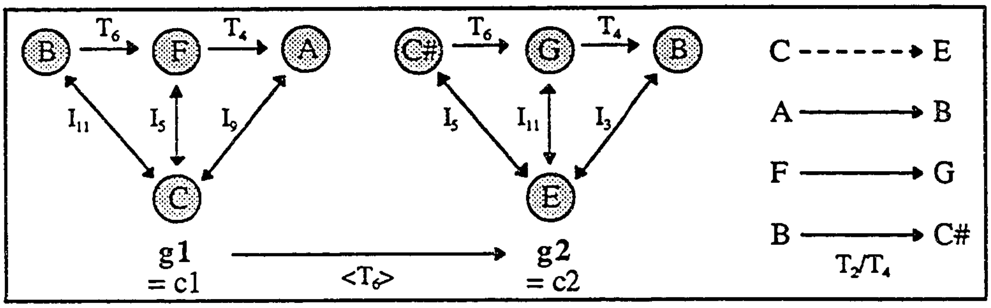


Although c1 and c2 are members of different set classes, they are strikingly similar in this context owing to the correspondence between their registral deployment and their shared trichordal subsets. There are two ways I like to hear this progression, and both involve dual transformations. The first way, my preferred interpretation and the one most salient to my ear, is the lower three voices moving up a whole step, or put more formally, the shared 3–8 [026] subsets being transformed by T_2.[12] Once again borrowing Lewin's if-only concept: if the soprano also moved up a whole step, then the progression would be a traditional T_2 operation. Instead, the soprano moves up by T_4, and I interpret this composite progression as the dual transposition T_2/T_4 illustrated by the mappings on the right side of Example 2.4–2. I'll return to the if-only expectations later on. Perhaps the most elegant way of modeling this harmonic similarity is with Klumpenhouwer networks. The left side of the example portrays chords c1 and c2 as graphs g1 and g2. The positive isography between the graphs results from the T_2 transformation between the two [026] subsets, with the lower-node singletons representing the piano's soprano voice C → E.

[12] Larry Starr points out the conspicuous near-transposition between these two chords in his book *A Union of Diversities: Style in the Music of Charles Ives* (New York: Schirmer, 1992): 145. Douglas Greene similarly focuses on the shared [026] subsets of c1 and c2 when he interprets them as half-diminished chords in his article "A Chord Motive in Ives's 'Serenity'," *In Theory Only* 4.5 (1978): 20–21.

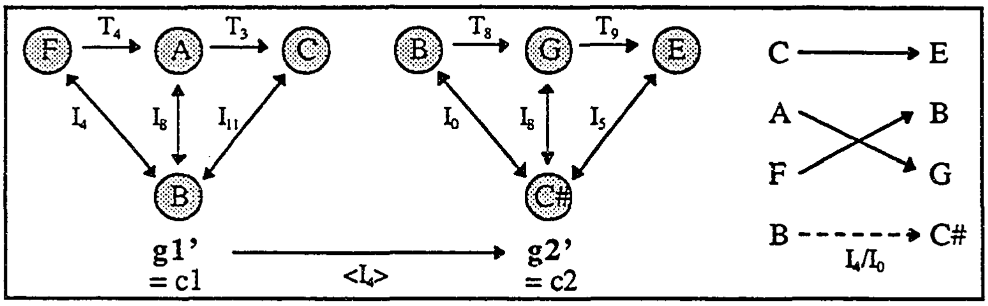
The transformational relationship between g1 and g2 is <T_6>, which mirrors the prominent T_6 transformation within the graphs.

Example 2.4–2: Klumpenhouwer networks around the shared [026] subset



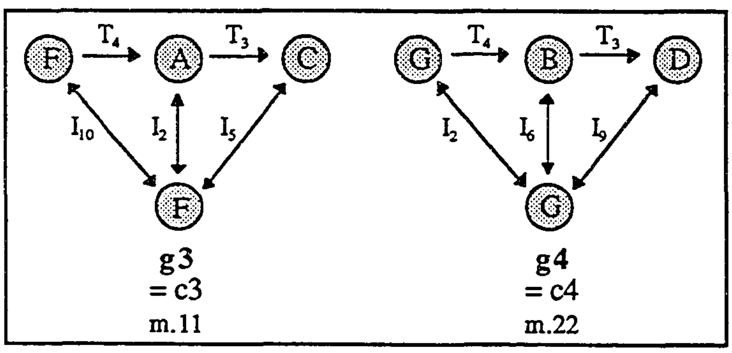
The second way I like to hear this progression is as two chords that are triads plus an extra note. The top three notes of c1 are an F-major triad, while the top three notes of c2 are an E-minor triad. In this interpretation, shown on the right side of Example 2.4–3, the upper voices transform by I_4, while the bass voice transforms by I_0, yielding the dual inversion I_4/I_0. The left side of the example rearranges the Klumpenhouwer networks around the common [037] trichords. The negative isography between these two graphs, now labeled g1' and g2', results from the inversional relationship I_4 between the triadic subsets. In these networks the lower node singletons now represent the bass voice B → C#, and the transformational mappings involve the alto-tenor exchange shown on the right side of the example. This interpretation is slightly more distant from the concrete musical surface than the previous one; I find it generally more difficult to hear dual inversions than dual transpositions, and the alto/tenor exchange of this particular case is no exception. Nonetheless, the emphasis on the triadic subsets in these transformations highlights interesting structural relationships within the work as a whole.

Example 2.4–3: Klumpenhouwer networks around the shared [037] subset



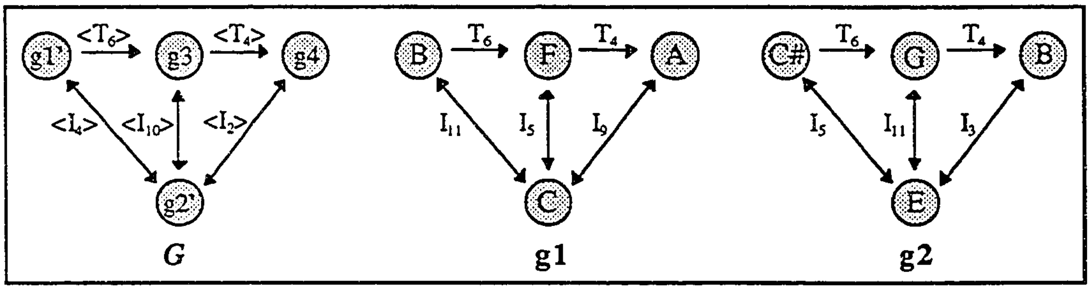
The piano articulates the two stanzas of Whittier's poem with cadential motions in measures 11 and 22; the only two measures that use set classes other than [0258] and [0137]. Although the harmonic vocabulary becomes triadic, employing major and augmented triads, or trichords [037] and [048], these measures maintain the four-voice texture. In fact, the progression is a common tonal gesture embellishing a root position major triad with a neighboring $\substack{6\\4}$ chord, and in which a chromatic passing tone creates a fleeting augmented sonority. Temporarily entering a diatonic space, I interpret each of these two measures as an expansion of a single harmony, specifically the major triads that initiate and conclude each of these cadential gestures. These chords are labeled c3 and c4 back in Example 2.4–1. Klumpenhouwer networks again provide a useful illustration of the striking similarities between these triads with doublings and the pervasive tetrachords. In Example 2.4–4 g3 and g4 represent measures 11 and 22 respectively. Note that these networks are isographic to those in the previous example.

Example 2.4–4: Klumpenhouwer networks representing cadential sonorities

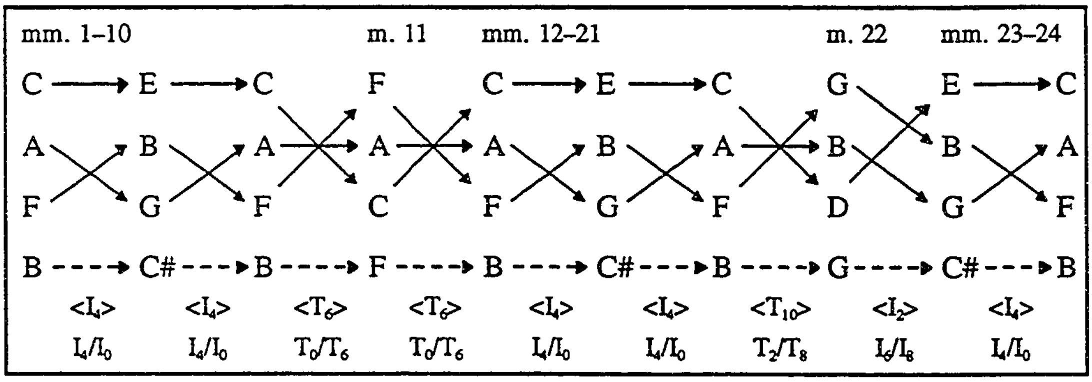


Constructing a supernetwork, *G*, encompassing the four graphs built around the [037] trichord reveals an interesting structural parallel. The positive isography among graphs g1', g3, and g4, and their negatively isographic relationship to g2', generates a 3+1 network similar to those in the previous examples. Significantly, the supernetwork *G*, shown in Example 2.4–5, is itself positively isographic with the original graphs of the tetrachords g1 and g2. Example 2.4–5 includes g1 and g2 for easy comparison. This recursive structure reveals that the transformations among the four most significant harmonies in Ives's piano accompaniment are equivalent to the internal pitch-class transformations within the two principal tetrachords themselves. Example 2.4–6 summarizes the voices implied by supernetwork *G*.

Example 2.4–5: Recursive supernetwork G



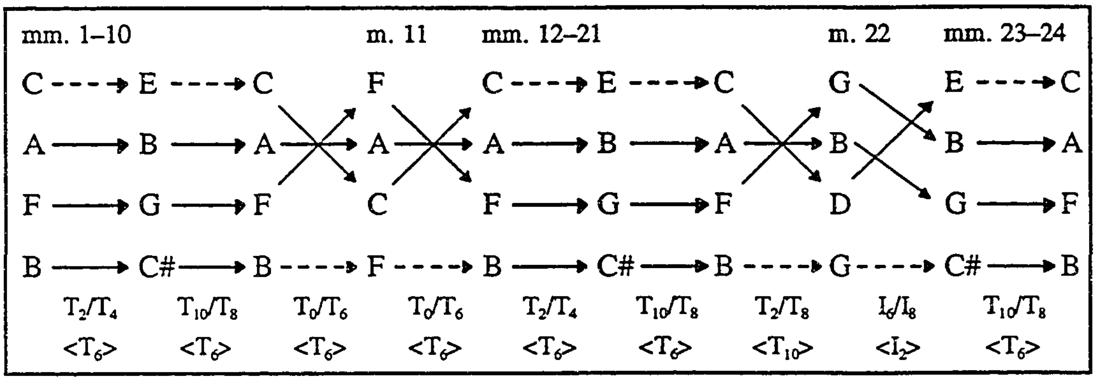
Example 2.4–6: Voice leading implied by supernetwork G (Recursive structure)



The Klumpenhouwer networks in the above examples present a wealth of information about the internal structure of the pitch-class sets, the similarities among the different sets, and the song's deeper harmonic relationships. Significantly, the deepest level of structure—the supernetwork *G*'s recursive relationship to g1 and g2—involves a graphic reinterpretation of the pitch-class sets. The networks emphasizing the [037] trichords yield a supernetwork comparable to the networks that emphasize the [026] trichords. This reinterpretation does not weaken the structural parallel between the relationship among the song's principal harmonies and the relationship among the pitch classes within those harmonies, but it does call the multiplicity of graphic possibilities to our attention. In any practical application of Klumpenhouwer networks such reinterpretations are almost inevitable, and when dealing with foreground chord progressions of any substantial length they are commonplace.

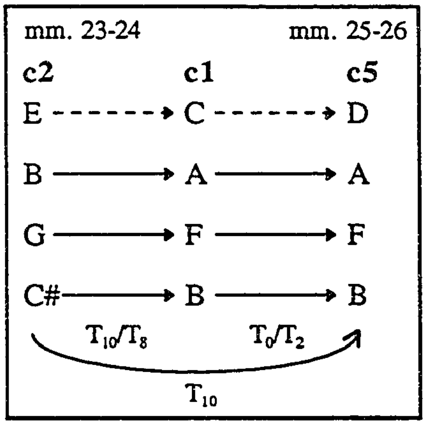
Intuitively I prefer the mappings implied by the [026] subsets shared by g1 and g2 as they coincide with the registral lines, but I do not want to lose the shared [037] subsets that relate g1' and g3. The adjacency voices shown in Example 2.4–7 offer an interpretation that incorporates both relationships. In the first stanza, the T_2/T_4 and T_{10}/T_8 mappings in measures 1–10 are the result of the shared [026] trichords, as in the Klumpenhouwer networks g1 and g2. The T_0/T_6 mapping into measure 11 is the result of the shared [037] trichord, as in the Klumpenhouwer networks g1' and g3. Note that the mapping *into* the third chord of the example arises from one interpretation of the set (specifically the [026] partition), while the mapping *out of* the third chord arises from another interpretation of the set (specifically the [037] partition). Borrowing Rameau's terminology, I call this reinterpretation *double-emploi. Double-emploi* is inherent in a dual transformational approach, but it involves continual graphic "updating" in a network approach. As equivalent nodes are the basis of Klumpenhouwer mappings, each time a chord relates to its predecessor and successor by different pairs of discrete subsets, its graph must be redrawn in the appropriate new configuration. Example 2.4–10, located at the end of this section, illustrates Klumpenhouwer *double-emploi* in the context of the first stanza. The horizontal graphs represent harmonic progressions, while the vertical graphs represent a harmonic reinterpretation. The equivalent dual transpositions are given in the lower left corner of the example for easy comparison.

Example 2.4–7: Adjacency voices



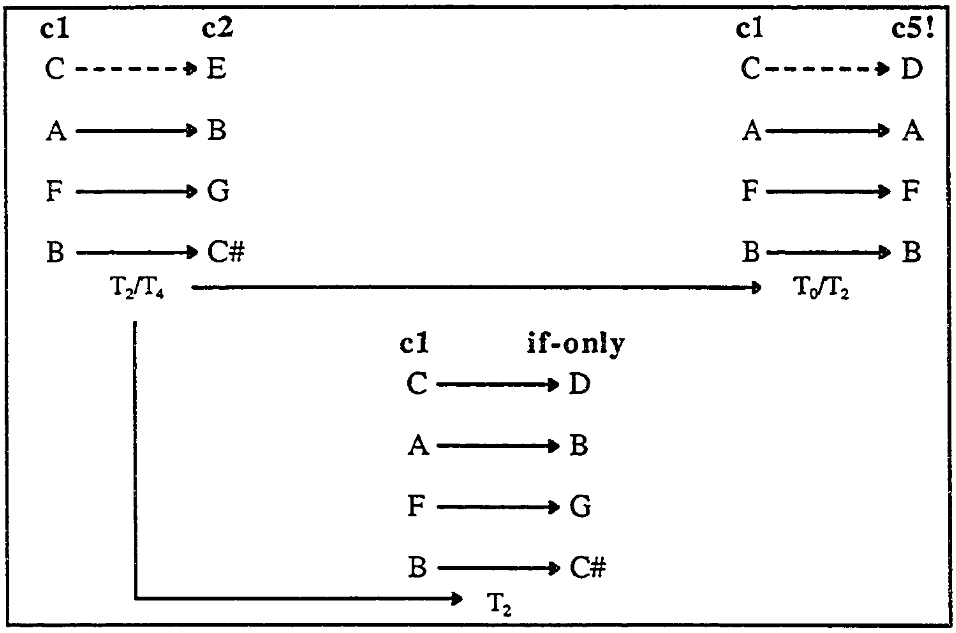
The second stanza is similar to the first until the T_2/T_8 transformation into measure 22, which renders the second cadence a whole step higher than its previous incarnation in measure 11. The large-scale T_2 transformation between the two cadential measures is reminiscent of the near-T_2 of the primary tetrachordal progression c1 → c2. The "coda" (measures 23–26) returns to the pervasive c1 and c2 tetrachords, as shown in Example 2.4–8. The familiar dual transposition T_{10}/T_8 is not, however, followed by its complement T_2/T_4 this time. Instead, the last transformation T_0/T_2 generates c5 {BFAD}, which is the T_{10} transformation of all the previous c2 tetrachords. This T_{10} motion is reminiscent of the near-T_{10} of the primary progression c2 → c1. Within the limited harmonic framework of this song, there is some sense that the T_{10} transformation in the coda balances the earlier T_2 transformation between the two cadential measures, and that this large-scale process is not altogether unlike the continual <near-T_2, near-T_{10}> of the alternating tetrachords.

Example 2.4–8: Dual transpositions modeling the "coda"

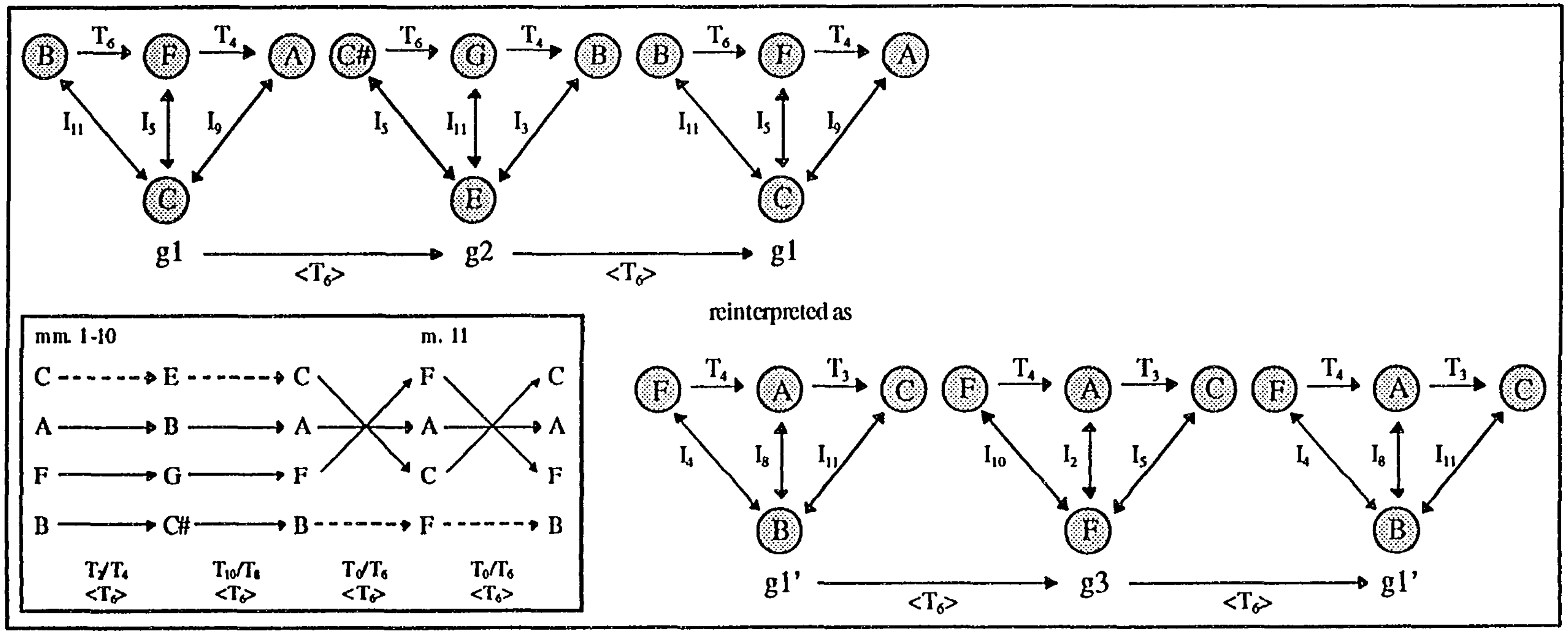


The final progression, c1 → c5, brings me back to the if-only scenario (summarized in Example 2.4–9) as I promised at the outset of this analysis. Recall my desire for the soprano to move by T_2 like the lower three voices in the T_2/T_4 progression c1 → c2. By the time the final sonority—c5 {BFAD}—arrives, this desire has been reinforced more than twenty times. The T_0/T_2 transformation into c5 finally supplies the longed for soprano T_2, but the ironic punch line is that this time the lower three voices do not transform at all!

Example 2.4–9: If-only!



Example 2.4–10: Double-emploi

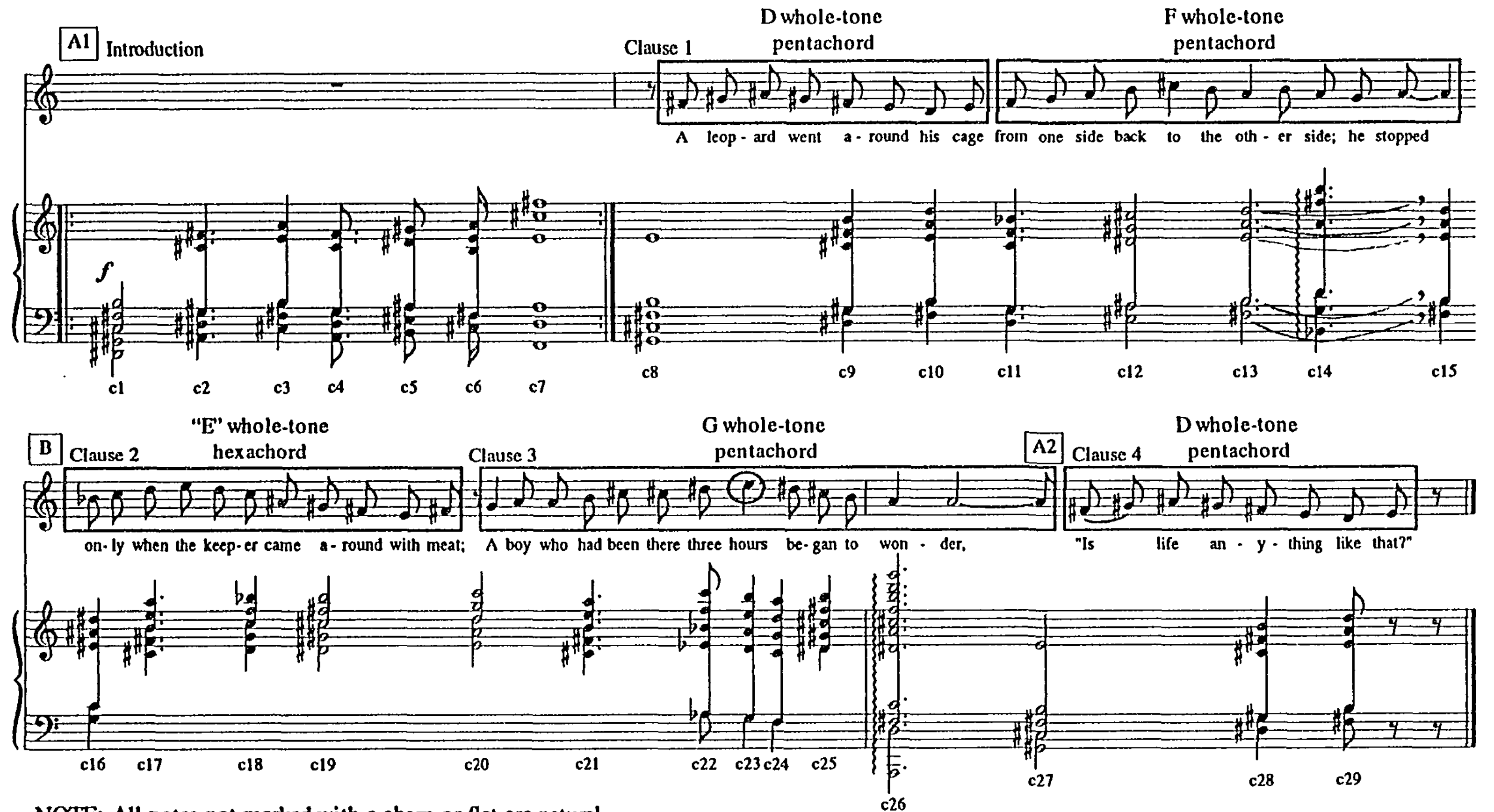


2.5 Charles Ives: "The Cage"

An annotated score of Charles Ives's song "The Cage" is shown in Example 2.5–1. Note that the piano accompaniment consists almost entirely of members of set-class 5–35 [02479]. Of the twenty-nine simultaneities that comprise the accompaniment (labeled c1–c29), twenty-six are from this set class, and each of its twelve members is heard at least once. The three other chords punctuate formal boundaries, as marked in the score. Members of set-class 6–14 [013458] occur at the end of the introduction (c7) and the end of the A1 section (c14), and {b01235679}—a member of set-class 9–8 [01234678a]—occurs at the end of the B section (c26). This extreme emphasis of a single set class across the span of an entire composition offers a somewhat unusual opportunity to further explore the multiple graphic interpretations possible among members of the same set class.

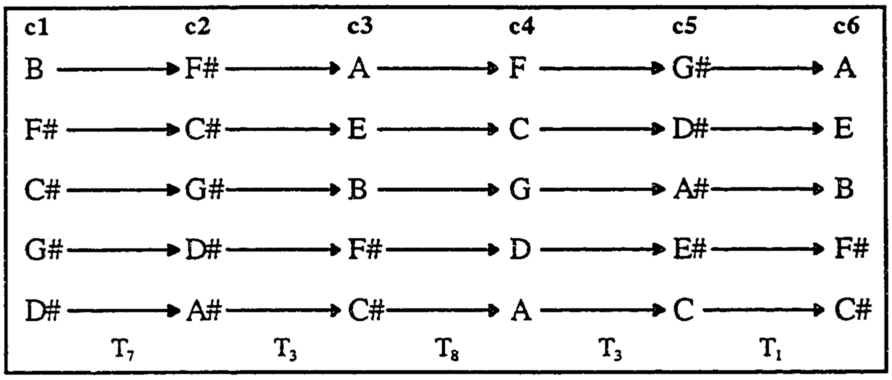
All of the [02479] pentachords are registrally deployed as stacked perfect 4ths, with the exception of c22–c24 which are stacked perfect 5ths. This makes it very easy to hear all five voices moving in parallel motion via the relevant transpositional value. Example 2.5–2 illustrates this interpretation of the voice leading using the six pentachords of the introduction (c1–c6). The extreme parallelism of the transformational voices highlights the intervallic structure of the passage, that is, we can hear the harmonic progression as a series of horizontal intervals. The similarity between the linear structure of the progression and the vertical structure of the chords suggests an alternative interpretation of the mappings. The symmetrical properties of set-class [02479] allow its members to map onto themselves under inversion. This means any member of [02479] can map onto any other member equally well via transposition or inversion. Reinterpreting the progression c3 → c4 as an inversion, I_6 rather than T_8, facilitates the comparison between the chords and the progression shown in Example 2.5–3. This recursive relationship is different then those shown in my previous analyses because the introduction network is not a supernetwork. Its

Example 2.5–1: Ives, "The Cage"

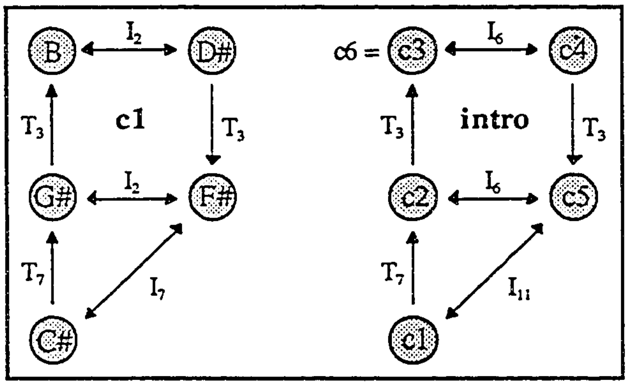


NOTE: All notes not marked with a sharp or flat are natural

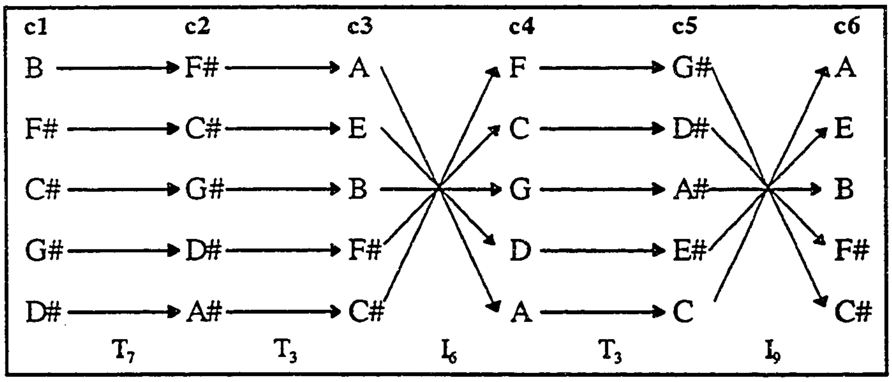
Example 2.5–2: Transpositional voices in the introduction



Example 2.5–3: Recursive networks modeling the chords and the introduction



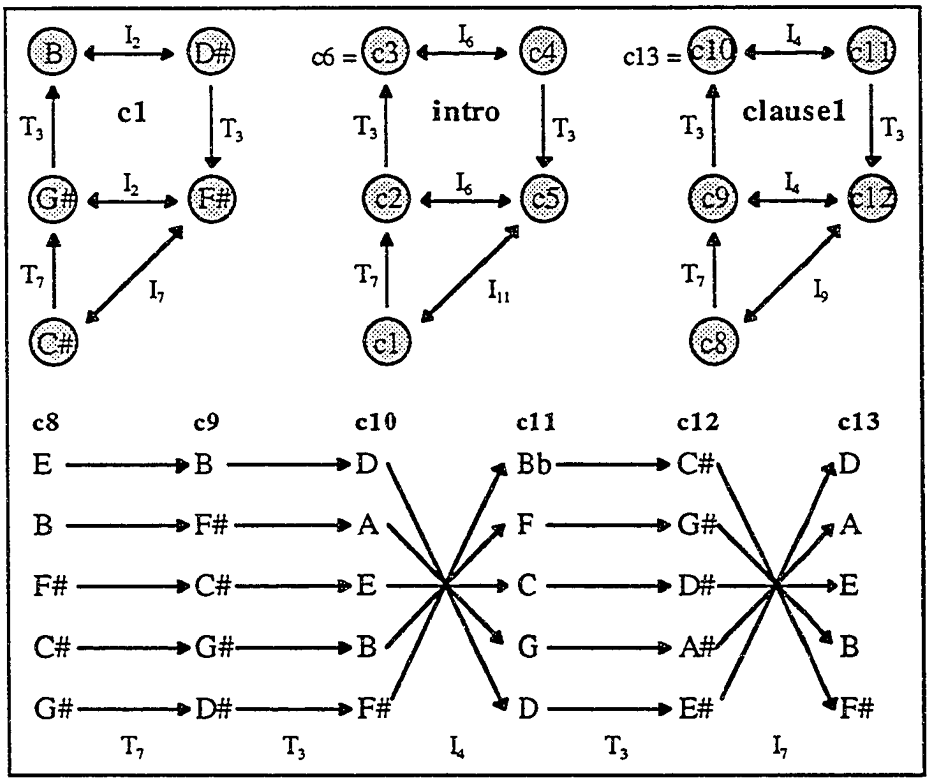
Example 2.5–4: Recursive voices in the introduction



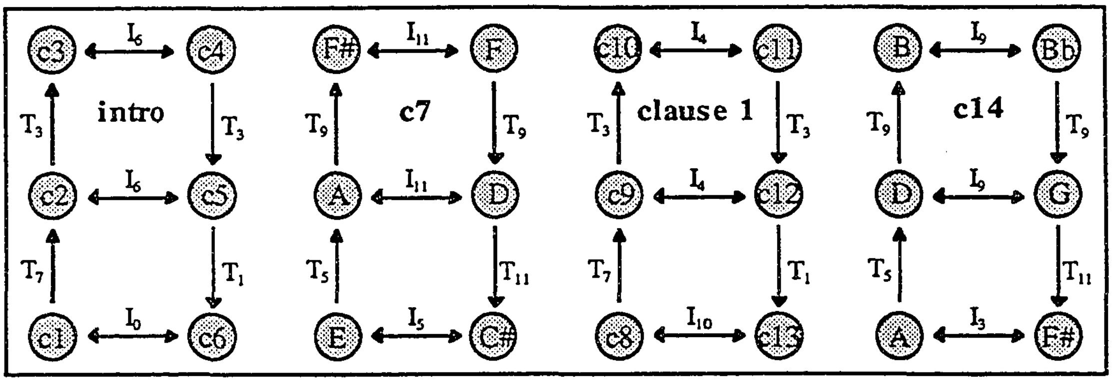
nodes are chords rather than graphs of chords, and its arrows represent operations rather than Klumpenhouwer $\langle T_n \rangle$ and $\langle I_n \rangle$ transformations. Despite these differences, the isography illustrates significant similarities between the horizontal and vertical components of this phrase. Example 2.5–4 shows the voices implied by the introduction network. In this interpretation the more abstract isographic recursion displaces the concrete aural clarity of the transpositional voices.

A rhythmically varied transposition, T_5, of the piano introduction accompanies the next formal section, clause 1 (spanning c8–c15). Example 2.5–5 illustrates the positive isography among graphs of the chords, the introduction, and clause 1. The recursive voices for this section, incorporating the appropriate inversions between c10 → c11 and c12 → c13, are given below the networks in the example. Thus far I am suggesting that we can hear the progressions in the introduction and clause 1 either as the obvious series of parallel voices, or interpret them as having a structure equivalent to the individual chords through the use of Klumpenhouwer networks.

Example 2.5–5: Recursive voices in clause 1



Example 2.5–6: The cadential hexachords and their progressions



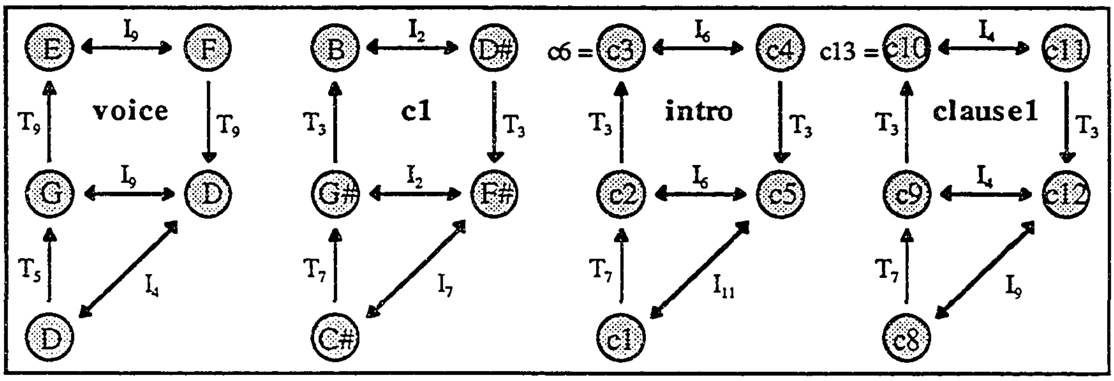
The "cadential" hexachords c7 and c14 are somewhat distinct from the progressions by not being members of set-class [02479], and further by Ives's notation of the accompaniment: chord 14 is rolled through a sustained chord 13, sounding almost like an interpolation. Nonetheless, these two chords are structurally very similar to the preceding progressions. As shown in Example 2.5–6, extending the chordal networks to include the repeated chords c6 and c13 as independent nodes illustrates the negative isography between graphs of the progressions and their respective cadential chords. It is interesting that the same negatively isographic relationship is found between graphs of the vocal line and the pentachords, and therefore the A-section progressions as well. The vocal line consists entirely of whole-tone sets. As annotated on the score, the whole-tone collections progress as follows: 1. a whole-tone pentachord on D {2468a}; 2. a whole-tone pentachord on F {579b1}; 3. the complete even whole-tone collection, spanning an octave E4–E5; 4. a whole-tone pentachord on G {79b13};[13] and a return to the whole-tone pentachord on D.[14]

[13] I have excluded the E5 on the word "hours" from the collection as it is an event in its own right. It is the vocal climax, the first vocal pitch after completion of the vocal aggregate, and it coincides with and is part of the completion of the pentachordal aggregate, that is, the twelfth and final member of set-class [02479].

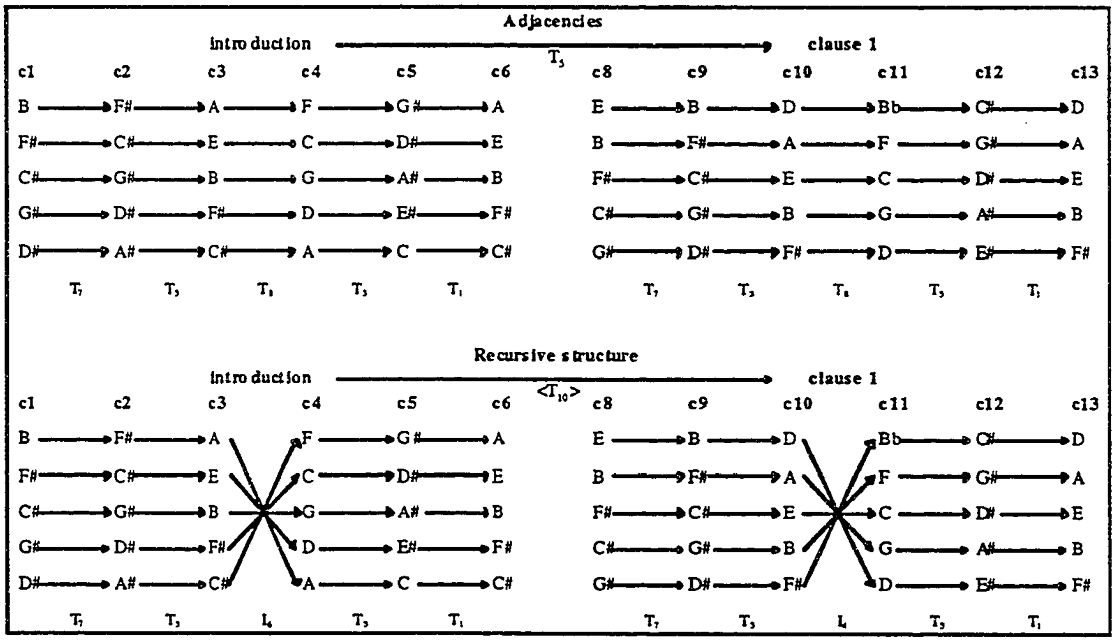
[14] Philip Lambert similarly partitions the vocal line in his essay "Ives and Berg: 'Normative' Procedures and Post-Tonal Alternatives," in *Charles Ives and the Classic Tradition*, ed. Geoffrey Block and J. Peter Burkholder (New Haven: Yale University Press, 1996): 105–130. An extensive analysis of this song is also soon to be published in Lambert's *The Music of Charles Ives* (New Haven: Yale University Press, forthcoming).

Using the lowest tone of each of these vocal collections, Example 2.5–7 illustrates the negatively isographic relationship between graphs of the vocal line and its accompaniment.

Example 2.5–7: Negative isography between graphs of the vocal line and its accompaniment



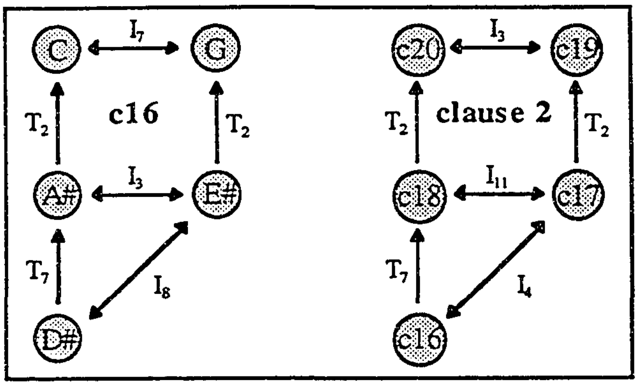
Example 2.5–8: Voice-leading summary of the A-section pentachords



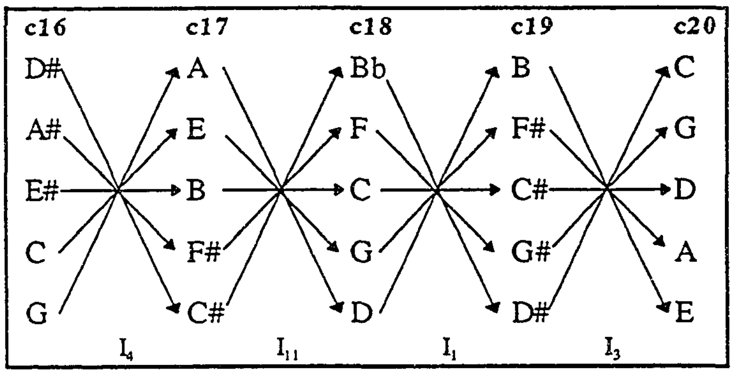
Summarizing the A section, the adjacency voices are the parallel lines generated by the explicit transpositions between pentachords, while the recursive voices are the transformational mappings growing out of the isographic relationships among the pentachords and hexachords. These two levels are shown in Example 2.5–8. Note that I now interpret the final pentachordal progression in each section, c5 → c6 and c12 → c13, as transpositions based on the hexachordal graphs in Example 2.5–6. This new interpretation leaves a single inversion in the middle of each phrase, c3 → c4 and c10 → c11, and concrete musical features such as changes in contour and vocal whole-tone collections reinforce the peculiarity of these two progressions.

The B section accompaniment continues the series of [02479] pentachords, but a new harmonic progression, as well as a change of vocal whole-tone collections, distinguish it from the previous material. The progression supporting clause 2 (c16–c20) again relates to the structure of the pentachords, as in the introduction and clause 1, but in a different way. Example 2.5–9 illustrates the positive isography between graphs of the pentachords and clause 2. Though this is a new progression, it seems to spring from the same source material, namely the ubiquitous [02479] pentachords. The recursive voices implied by this graphic interpretation, shown in Example 2.5–10, at first seem quite removed from the obvious parallelism of the musical surface. Upon closer examination two interlocking progressions begin to stand out: c15 → c17 → c19 and c16 → c18 → c20, both moving $\langle T_7, T_2 \rangle$. Two factors combine to make this alternating progression conspicuous: first, the $\langle T_7, T_2 \rangle$ progression is an inversion of the $\langle T_7, T_3 \rangle$ progression that opens both previous sections, and therefore has a degree of motivic weight; and second, the continuation of the stacked 4ths across adjacent chords c15 → c16, c17 → c18, and c19 → c20 emphasizes the relationships between corresponding members of the three chord pairs. These transformations, summarized in Example 2.5–11, suggest that the surface parallelism is not entirely removed from the recursive level, but that it has shifted to alternating—rather than adjacent—chords.

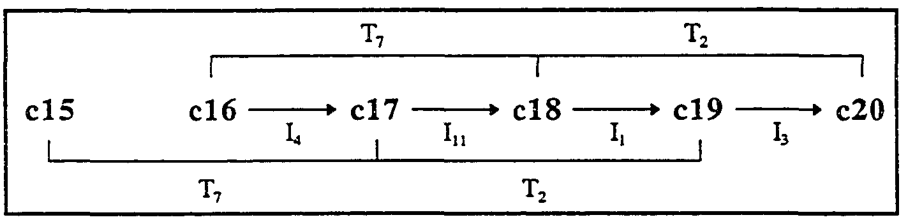
Example 2.5–9: Recursive networks modeling the chords and clause 2



Example 2.5–10: Recursive voices in clause 2



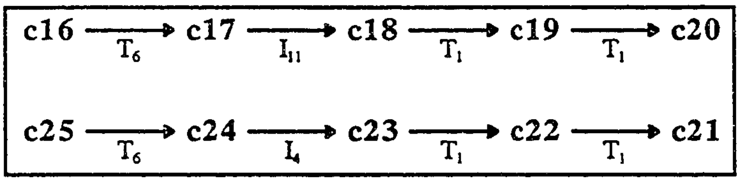
Example 2.5–11: Interlocking motivic progressions linking clauses 1 and 2



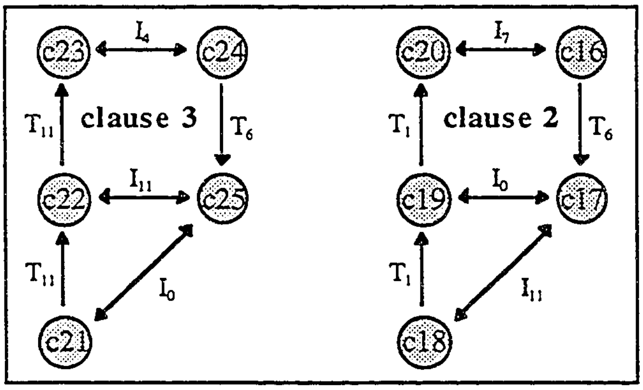
The clause 3 accompaniment does not relate to the pentachordal structure in the same way as the three previous sections, but instead completes the B section with a retrograde of clause 2. Example 2.5–12 summarizes this Klumpenhouweresque retrograde which progresses in 2+3 and 3+2 segments: c16 → c17, c18 → c19 → c20; c21 → c22 →

c23, c24 → c25. The complementary transpositional values of the retrograde yield the negatively isographic relationship between the clause graphs shown in Example 2.5–13.[15]

Example 2.5–12: Retrograde relationship between clauses 2 and 3



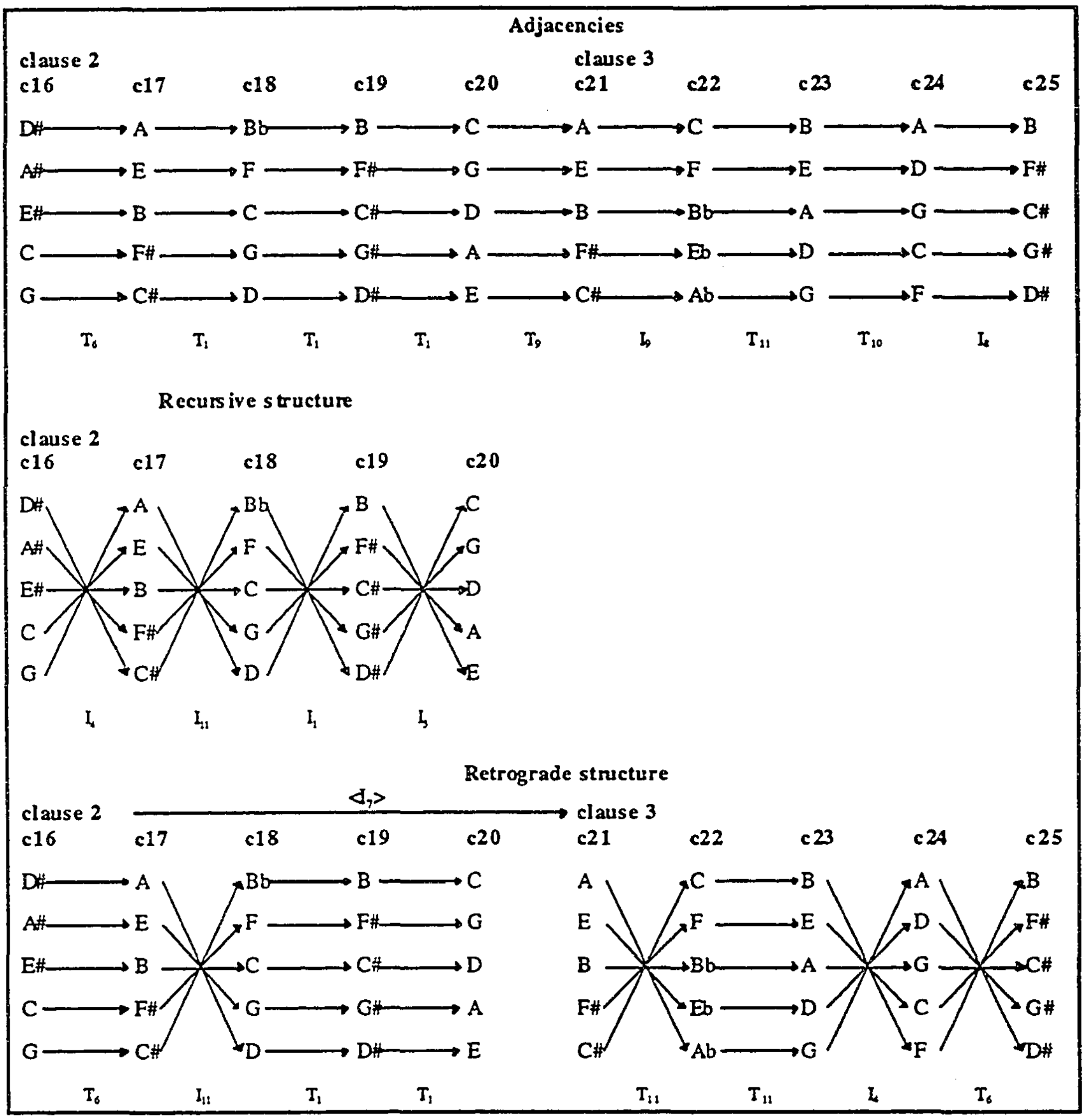
Example 2.5–13: Negative isography between graphs of clauses 2 and 3



Summarizing the B section, the adjacency voices are again the parallel lines generated by the explicit transformations between pentachords, which now include inversions, as chords 22–24 are stacked perfect 5ths. The next level is again the transformational mappings in a recursive relationship to the pentachords, though this only applies to clause 2. A third, slightly deeper, level of transformations arises from the retrograde process across the entire B section. These three levels are shown in Example 2.5–14.

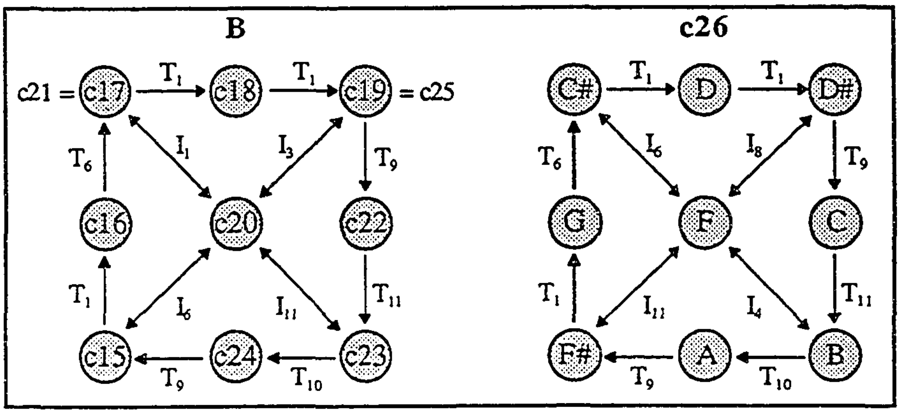
[15] Owing to the symmetrical properties of the two subsets involved, [012] and [06], it is also possible to graph clause 3 so that it is positively isographic to clause 2, but the negative relationship better reflects the retrograde motion of the progression.

Example 2.5–14: Voice-leading summary of the B-section pentachords



The very large cadential chord c26 {b01235679} is such an anomaly that it is difficult to relate it to the rest of the accompaniment. Like the cadential hexachords of the introduction and clause 1, it too sounds like a mysterious interpolation, befitting the text "wonder." Ives's theater orchestration of "The Cage" supports this interpretation, as the strings perform the piano accompaniment with the exception of this chord, which is played by the piano after being silent since the downbeat of clause 1. Also like the previous cadential chords, it is possible to interpret this chord as a summary of the progression that drives toward it. Example 2.5–15 incorporates chords 15–25 (note that c15 is the link from clause 2) into a network that is comparable to the structure of chord 26.[16]

Example 2.5–15: The B section and its cadence

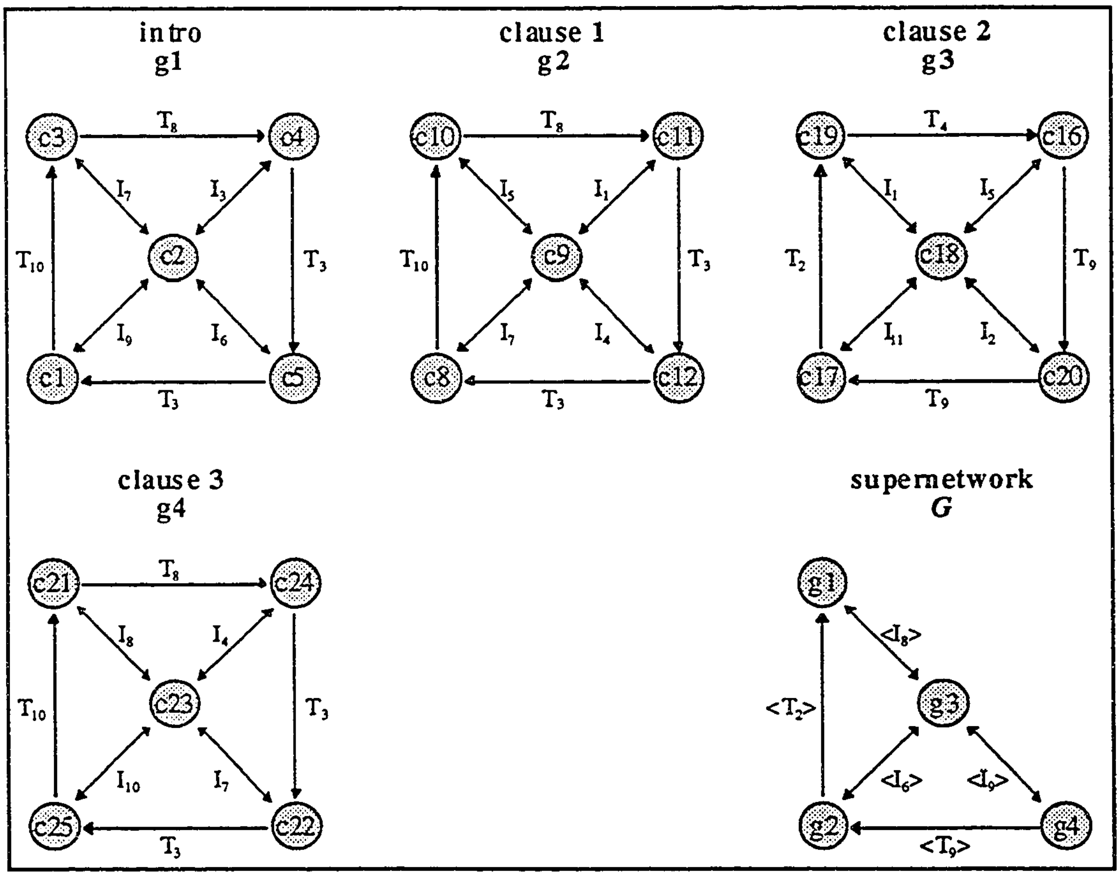


Clause 4 consists only of the first three chords of clause 1 supporting the text: "Is life anything like that?" This return of the A section is so truncated, and the ending is so abrupt, that—apropos the text—one is left with the palpable feeling that something (or even much) is missing. The progression from chord 27 through chord 29 is the familiar motivic transformation <T_7, T_3> from the A section and clause 2, but it is possible to interpret this final gesture in another way that has ramifications on the deepest level of structure. Example 2.5–16 reinterprets all four of the progressions discussed in detail above—the introduction and clauses 1 through 3—and offers networks g1–g4 that emphasize their background similarity. This is the deepest level of structure relating all of the formal segments to one another. Note that g1, g2, and g4 are positively isographic, while g3 is negatively isographic in relation to them. These networks can be grouped into the partial supernetwork *G* shown in the bottom right corner of the example. *G* needs an additional node g5—in a very specific relationship to the other four nodes—to be recursively

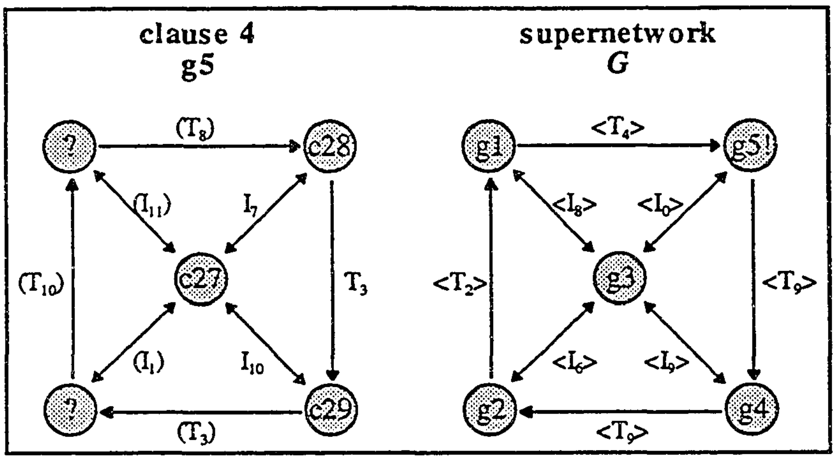
[16] There are at least four variants of this chord in different versions of "The Cage." The chord I use, c26, is derived from the version of the song bundled with Ives's *Theater Orchestra Set*. Another common variant—Lambert's preference in his analyses, and editor Wiley Hitchcock's preference in a forthcoming edition of all of Ives's songs—includes D#6 instead of D6, but as both variations already contain D and D# (specifically D3 and D#4), my network could represent either variant.

isographic to the networks representing all of the formal sections. Rather than interpret clause 4 as a truncated repetition of clause 1 (that is, an incomplete network g2) bringing a return of previously heard material, it is possible to interpret it as a new entity effecting completion of the large-scale formal structure (that is, an incomplete network g5). Example 2.5–17 illustrates the g5 interpretation of clause 4, and also presents the now complete supernetwork *G*. The partial network g5 retains the sense of incompleteness characteristic of the song's ending, yet suggests a stronger sense of closure than a mere repetition of clause 1.

Example 2.5–16: Section networks and a partial background supernetwork



Example 2.5–17: Clause 4 as g5 and the complete recursive supernetwork G



In summary, at least three significant levels of transformational voice leading are unfolding in the accompaniment of "The Cage." The adjacency voices are the parallel lines generated by traditional operations (mostly transpositions), the "middleground" recursive voices are those mappings generated by isographic Klumpenhouwer networks modeling each individual formal section, and the "background" recursive voices are those encapsulated in the networks relating all five formal sections to each other (g1–g5). The recursive supernetwork *G* illustrates the structural similarities among the formal sections despite their middleground differences.

Chapter 3

Analyses II: Linear and Polyphonic Applications

In this chapter I apply the transformational voice-leading model developed in the previous chapters to polyphonic musical examples, specifically drawn from the following four works:

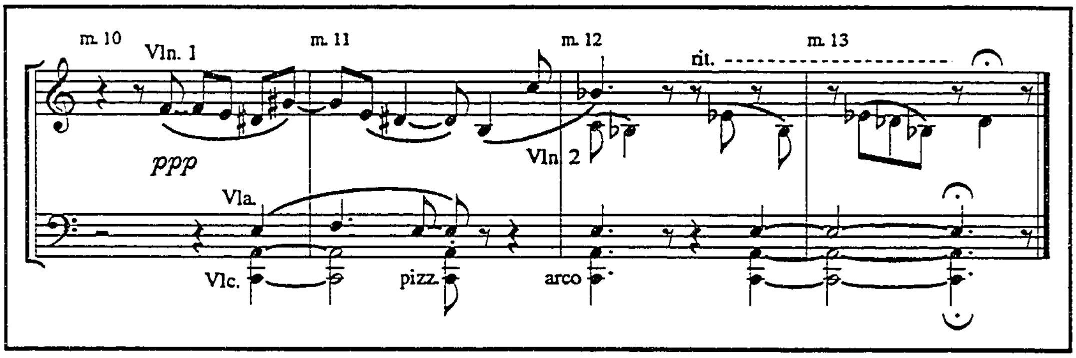
- 3.1 Anton Webern: *Five Pieces for String Quartet*, Op. 5, No. 2
- 3.2 Anton Webern: *Five Pieces for String Quartet*, Op. 5, No. 3
- 3.3 Milton Babbitt: *Semi-Simple Variations*
- 3.4 Igor Stravinsky: *Movements for Piano and Orchestra*, I

These analyses again portray multiple voice-leading interpretations generated by Klumpenhouwer networks and supernetworks, dual transformations, and/or traditional operations. I no longer arrange the voices into levels, as the polyphonic nature of the music raises additional problems regarding partitioning and the transformational interaction among various lines. Each of the four analyses focuses on a different approach to polyphonic texture. My essay on Webern's Op. 5, No. 2, serves as an introduction to the linear application of dual transformations, as well as an exploration of the interplay between melody and harmony. In my study of Webern's Op. 5, No. 3, I use equivalent musical events to parse the musical surface, while my study of Babbitt's *Semi-Simple Variations* begins by interpreting the background array as a reduction of the musical surface. My final analysis, examining Stravinsky's *Movements for Piano and Orchestra*, attempts to apply my voice-leading model more directly to the concrete musical surface.

3.1 Webern: Five Pieces for String Quartet, Op. 5, No. 2

David Lewin presents a transformational analysis of the opening four bars of Anton Webern's Op. 5, No. 2, in which he concentrates on transformations of pitch-class set "X = (G, B, C#)," and its role in both the linear and harmonic dimensions of these measures.[1] Along similar lines, but employing the concepts presented in the previous two chapters, I will explore the final four bars of the piece. Example 3.1–1 is a short score of this concluding passage.[2] I will approach this phrase from a number of different angles; first from a purely linear perspective, and then in a more comprehensive manner.

Example 3.1–1: Webern, Five Pieces for String Quartet, Op. 5, No. 2, mm. 10–13



The violin melody comprises two subphrases; the first played by violin I and the second by violin II. The nine pitch events of the first subphrase can be heard as a progression of three trichords, [012] → [015] → [012], as bracketed in Example 3.1–2. This trichordal parsing contradicts Webern's phrasing, but it is emphasized by repetition (<E, D#>), rhythm, and contour. There are six possible mappings between any pair of trichords, but since two of the trichords are members of the same set class it is worthwhile working with traditional operators as a starting point. With that in mind, the progression

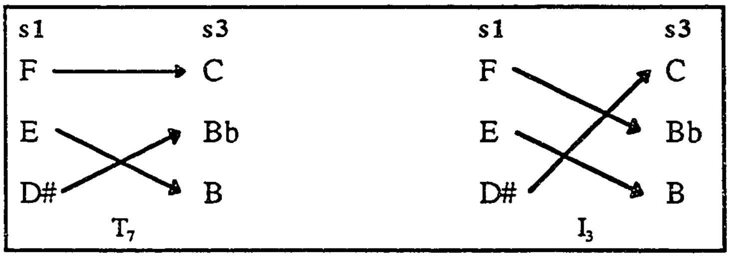
[1] Lewin, "Transformational Techniques."

[2] This score omits the E (violin II) and Eb (cello, *pizzicato*) on the downbeat of measure 10, as they are part of the previous phrase.

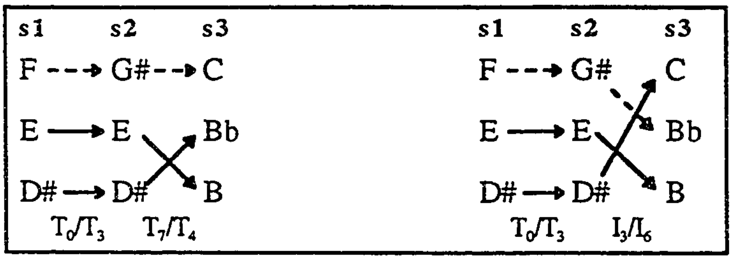
between the outer two [012] trichords is either T_7 or I_3, yielding the mappings shown in Example 3.1–3. s1 and s3 arrange the sets of pitches in registral order, but temporal order would work equally well for my purpose. Dual transformations can incorporate the intervening [015] trichord, thereby creating a pathway between the two [012] trichords. In the first progression, [012] → [015], the shared <E, D#> dyad isolates the motion F → G#, which suggests the dual transposition T_0/T_3 shown in both sides of Example 3.1–4. The serial and registral correspondence of this transformation help make the T_0/T_3 relationship preferable to numerous other possibilities. The return progression, [015] → [012], can be tailored to complete either of the traditional mappings cited above. T_7/T_4 completes the T_7 mapping, while I_3/I_6 completes the I_3 mapping. Significantly, $x = \pm 3$ in all of these progressions.

Example 3.1–2: Webern, Op. 5, No. 2, mm. 10–12, violin I

Example 3.1–3: [012] mappings

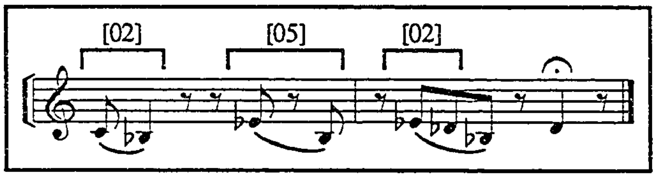


Example 3.1–4: Dual transformations in subphrase 1



Violin II performs the second subphrase, shown in Example 3.1–5, beginning with an overlapping echo of Violin I's final two pitch-classes, <C, Bb>. Another dyad follows, <Eb, Bb>, retaining the Bb (T_0), while transforming the C by T_3. Thus Violin II's [02] → [05] opening gesture echoes Violin I's [012] → [015] which began the first subphrase. The next two pitch-classes continue this pattern by retaining the Eb and transforming the Bb to Db, reversing the progression, [05] → [02], via the transformation T_3/T_0. The motion [02] → [05] → [02] is strikingly similar to the earlier [012] → [015] → [012]. The final two pitches reiterate the previous T_3 transformation, that is, Bb → Db. Example 3.1–6 illustrates these mappings, and once again, x = ±3 in all of the transformations.

Example 3.1–5: Webern, Op. 5, No. 2, mm. 12–13, violin II



Example 3.1–6: Transformations in subphrase 2

s4 s5 s6

C --→ Eb --→ Eb

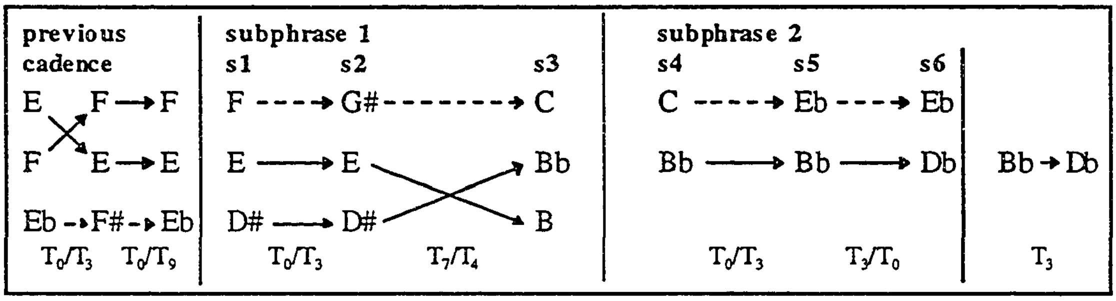
Bb → Bb → Db → Bb → Db

T_0/T_3 T_3/T_0 T_9 T_3

Example 3.1–7 presents the complete melodic phrase, while Example 3.1–8 aligns the mappings below the music. This example also includes the previous cadence, a progression of three [012] trichords (mm. 9–10, not shown in Example 3.1–7) that emphasizes interval-class 3 in a similar fashion. I incorporate T_7/T_4 rather than I_3/I_6 in the progression s2 → s3 as it is more consistent with rest of the transformations by being a dual transposition, and by maintaining the x = <+3, -3> motion.

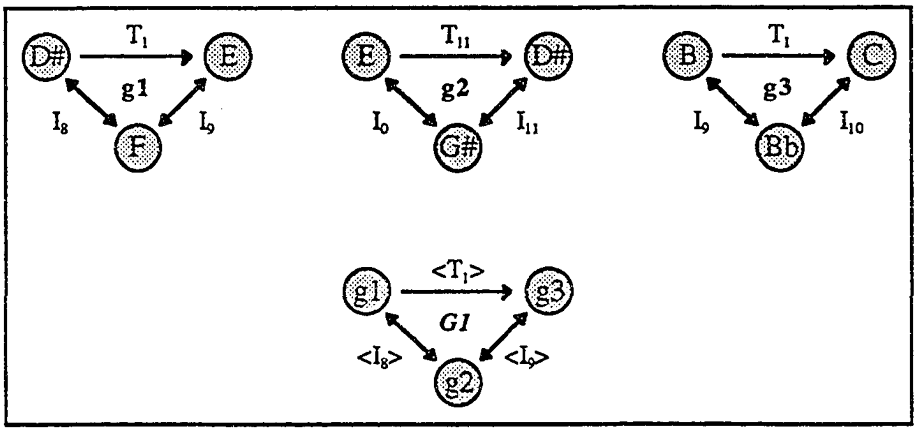
Example 3.1–7: Webern, Op. 5, No. 2, mm. 10–13, violins I & II

Example 3.1–8: Dual transposition summary



Klumpenhouwer networks offer a number of fascinating ways to model the rich web of relations in this phrase. As I am using trichordal partitions, the possibilities are almost overwhelming, but I will limit my discussion to several of the most significant network structures. The first of these is shown in Example 3.1–9. The graphs g1–g3 model each of the trichords of the subphrase 1 melody, previously labeled s1–s3. Among the numerous possibilities, the reasons for this particular interpretation are twofold. As mentioned above, the shared <E, D#> dyad emphasizes the melodic motion F → G#. In my earlier interpretation the G# then mapped onto the C, which—despite a nice registral correspondence—is not quite satisfactory as I prefer to hear Bb as the goal of the G# owing to its metric placement and location within the subphrase. That is, I find Bb particularly salient as it is both a downbeat and the final note of the Violin I melody. Therefore my first criterion in creating a network model of this subphrase is to generate the mapping F → G# → Bb. As a member of set-class 3–7 [025] this grouping has significant resonance in the remainder of the passage, as I will show below. The second reason for this particular configuration is its recursive power. The supernetwork *G1*, included in Example 3.1–9, incorporates g1–g3 into another isographic network, which also happens to be strongly isographic to g1.

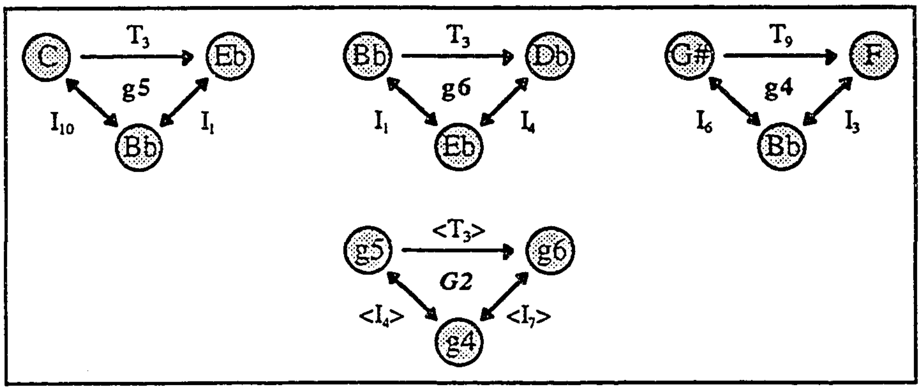
Example 3.1–9: Klumpenhouwer networks modeling the subphrase 1 melody



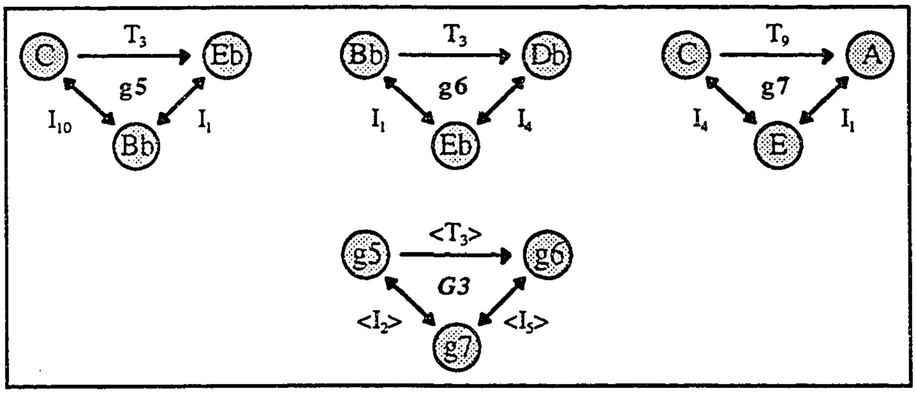
Example 3.1–6 (above) partitions the subphrase 2 melody into dyads, but a trichordal partition (by bar) is also possible, and in some ways preferable. Significantly, the two resulting trichords are members of set-class [025]; the same set class projected over the span of subphrase 1. Example 3.1–10 and Example 3.1–11 model the two trichords of the second subphrase as the isographic Klumpenhouwer networks g5 and g6. The $<T_3>$ transformation creating the progression g5 → g6 suggests a potential supernetwork, but a third negatively isographic network is missing. Two sets have particularly strong ties to this melodic material: the middleground [025] of the previous subphrase, and the accompanying triad in the lower strings. Example 3.1–10 realizes the recursive implications of g5 and g6 by including the negatively isographic network g4 modeling the earlier [025], and generating the recursive supernetwork *G2*. Example 3.1–11 realizes the recursive implications by including the negatively isographic network g7 modeling the sustained accompaniment chord, a member of set-class 3–11 [037], and generating the recursive supernetwork *G3*. It is worth noting that g4, g5, and g6 model members of the same set class, but in this context the expected relations are contradicted. {Bb, C, Eb} and {Bb, Db,

Eb} would typically be interpreted as I_n relations, but here they are $\langle T_n \rangle$ relations, while {Bb, Db, Eb} and {F, G#, Bb} would normally be interpreted as T_n relations, but here they are $\langle I_n \rangle$ relations.

Example 3.1–10: Klumpenhouwer networks modeling the subphrase 2 melody



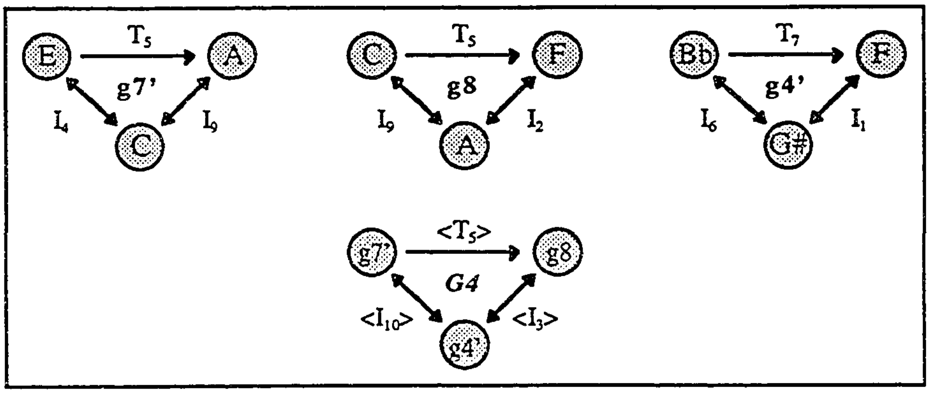
Example 3.1–11: Klumpenhouwer networks modeling subphrase 2 with harmony



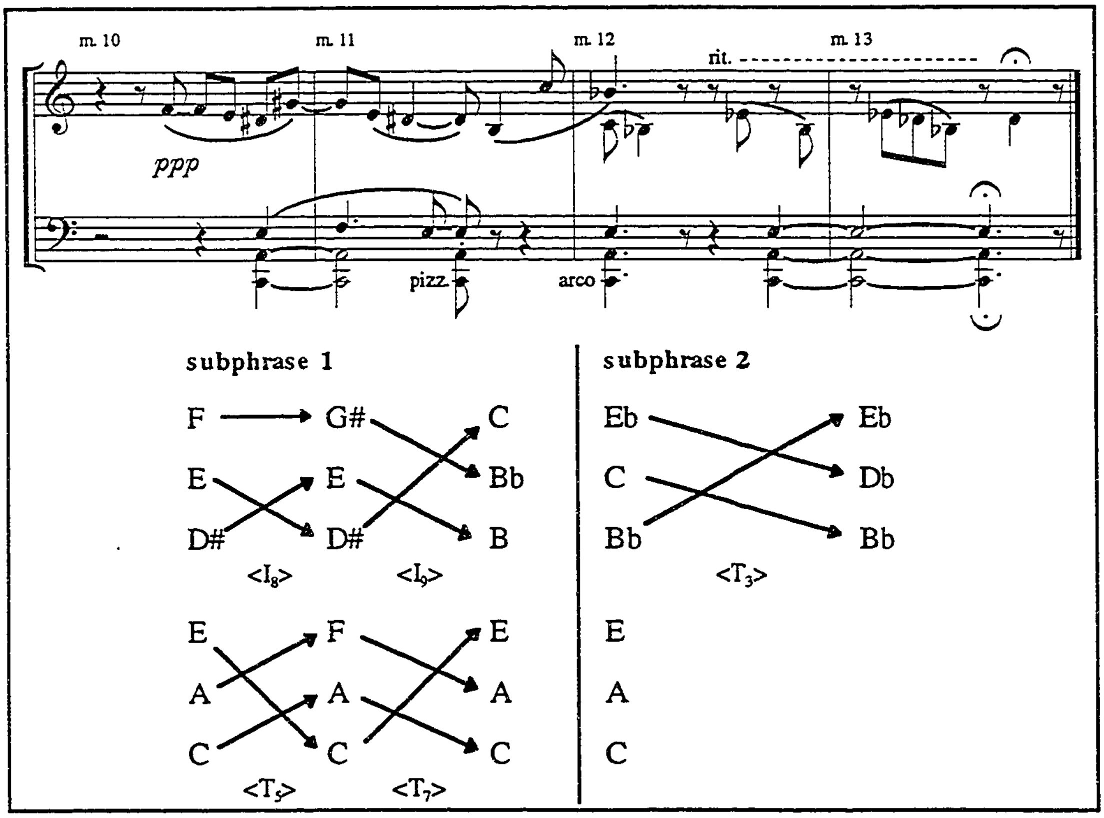
Example 3.1–11 illustrates the possibility of modeling set-classes [025] and [037] as isographic networks. As two members of set-class [037] also provide harmonic support for subphrase 1, it is possible to incorporate these trichords into a similar network. Example 3.1–12 models the two harmonic [037] trichords as g7' and g8. The former is a reinterpretation of g7 (shown previously in Example 3.1–11) as the chord supporting subphrase 2 is identical to the first chord of subphrase 1. The $\langle T_7 \rangle$ transformation generating the progression g7' → g8 suggests another recursive supernetwork, and g4'

provides the necessary negatively isographic network. Therefore the relationships among the pitch classes of the two harmonic sets, the middleground melody, and the interaction of these three musical entities are structurally similar.

Example 3.1–12: Klumpenhouwer networks modeling the subphrase 1 harmony



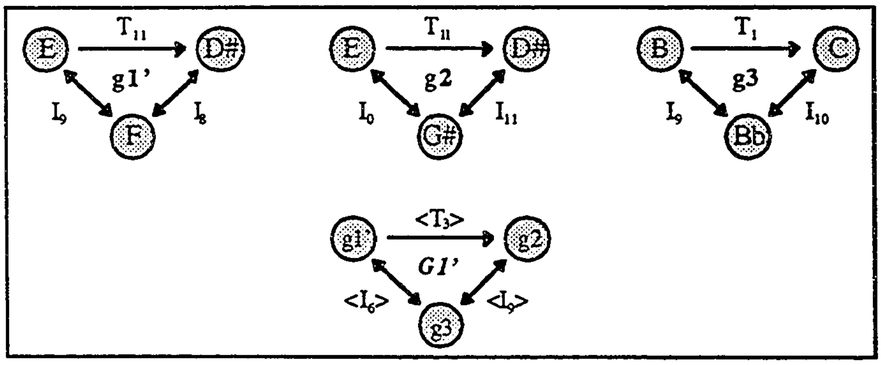
Example 3.1–13: Subphrase recursive structures I



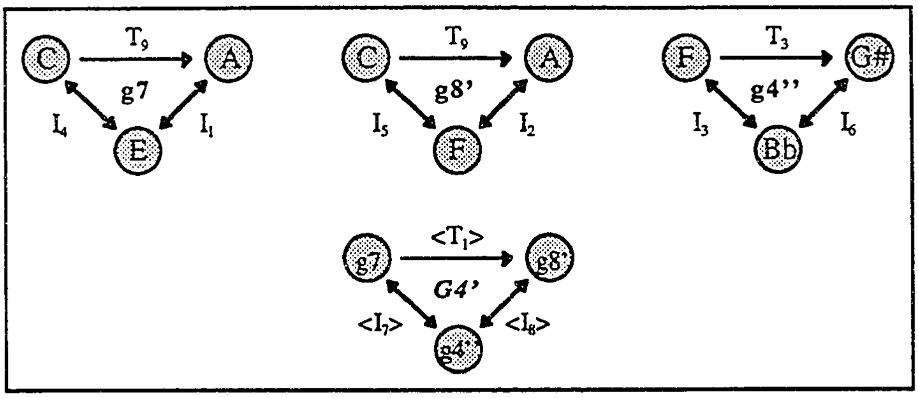
The preceding four network examples construct a recursive structure within each of the two subphrases that incorporates both linear and harmonic progression. Example 3.1–13 converts the relationships implied by supernetworks *G1–G4* into pitch-class mappings among adjacent events. The rhythm in the example is normalized; the displacement between melody and accompaniment is removed. The example also includes the short score for convenient reference.

Although the recursive supernetworks *G1–G4* model some aspects of this complex phrase very well, the implied mappings shown in Example 3.1–13 are somewhat less than satisfactory. Most notably, common tones are not held in common among the sets. It is possible to reinterpret the melody and harmony networks of subphrase 1 so that they sustain the common tones and retain some of their recursive power. Example 3.1–14, corresponding to Example 3.1–9 above, reinterprets network g1 as g1'. The advantage of g1' is the more intuitive mapping of the <E, D#> dyad. Example 3.1–15, corresponding to Example 3.1–12 above, reinterprets networks g8 as g8' and g4 as g4''. g8' retains the common tones C and A, and g4'' is more appealing as it brings out the T_3 motion F → G#. The two examples also illustrate an interesting cross-recursion. The supernetwork *G1'* (in Example 3.1–14) modeling the melodic transformations is isographic to the harmonic networks in Example 3.1–15, while the supernetwork *G4'* (in Example 3.1–15) modeling the harmonies is isographic to the melodic networks in Example 3.1–14.

Example 3.1–14: Another interpretation of the subphrase 1 melody

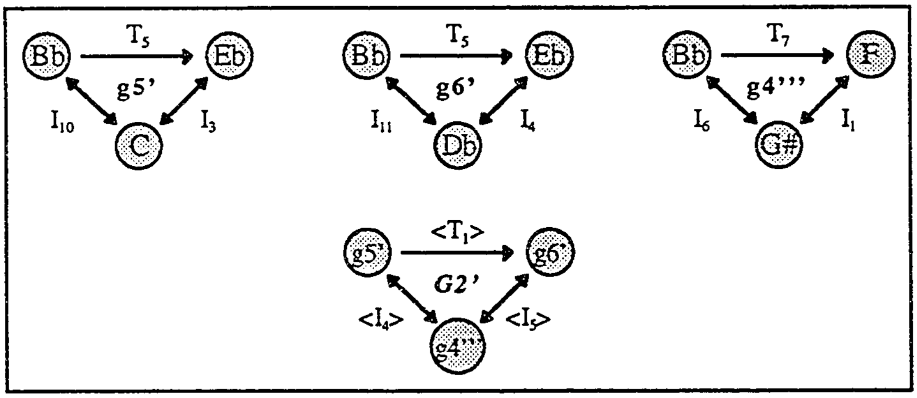


Example 3.1–15: Another interpretation of the subphrase 1 harmony

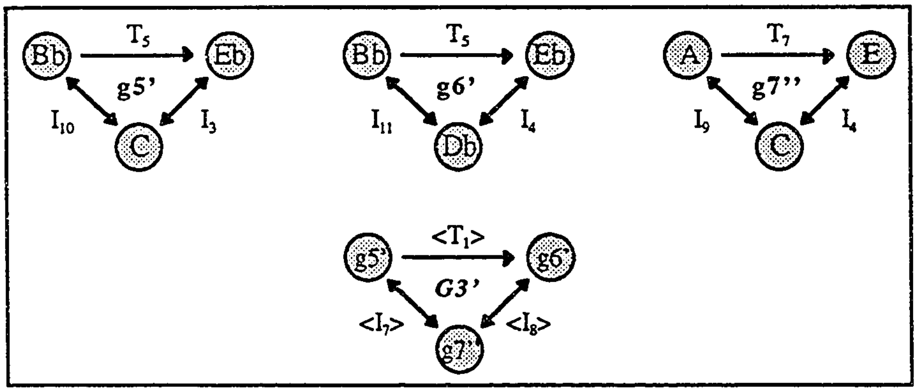


The networks modeling subphrase 2 can also be organized around the common tones; see Example 3.1–16 and Example 3.1–17. Significantly, the resulting supernetworks *G2'* and *G3'* are also isographic to the melodic networks g1–g3.

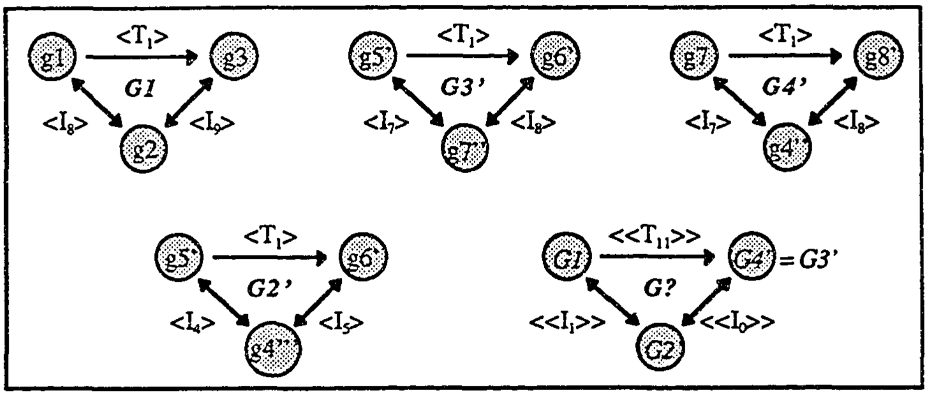
Example 3.1–16: Another interpretation of the subphrase 2 melody



Example 3.1–17: Another interpretation of subphrase 2 with harmony



Example 3.1–18: Possible third-level Klumpenhouwer supernetwork

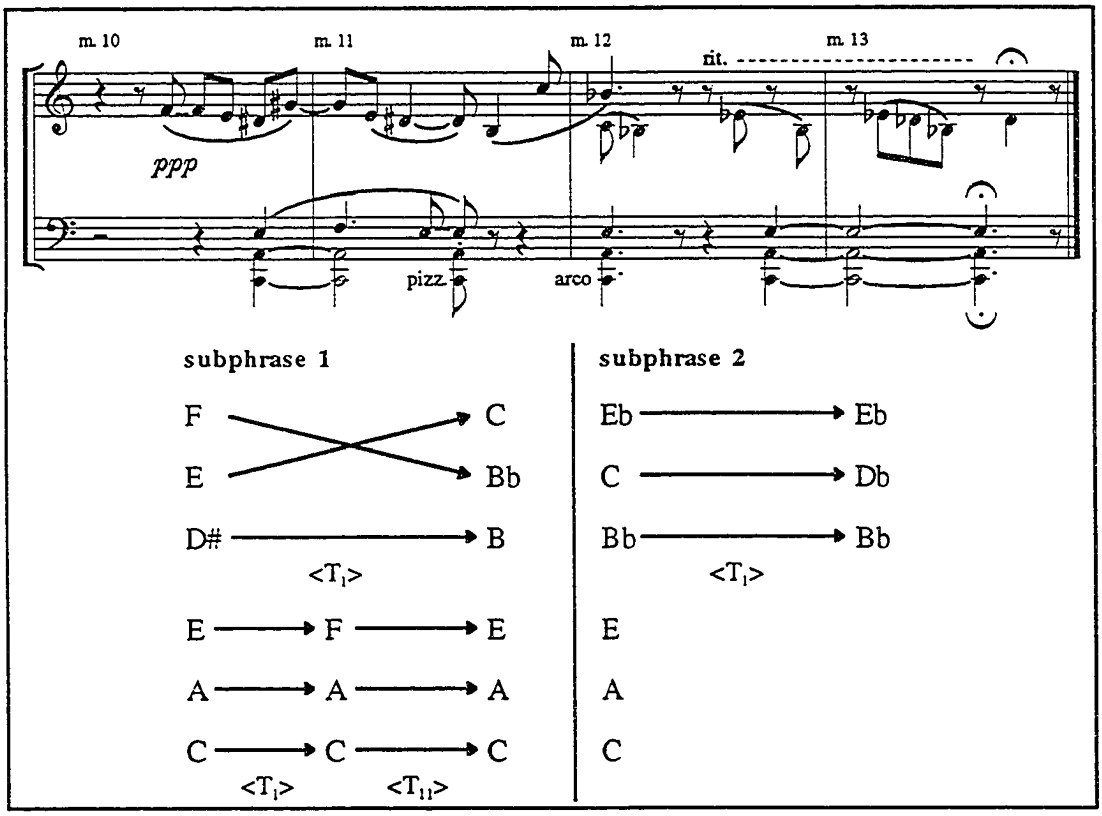


The isography of supernetworks *G1*, *G2'*, *G3'*, and *G4'* (reproduced in Example 3.1–18) makes it tempting to create another level of recursion. The strong isography between graphs *G3'* and *G4'* and their <<T_1>> relationship to *G1* combine to make it possible to generate the recursive supernetwork *G?* shown in Example 3.1–18.[3] While it is striking to find a replication of the initial melodic sets on this level, there are a few problems with this next level of recursion. First, the graph simultaneously incorporates the same objects more than once. Most problematic in this regard is including both the subphrase 1 melody as the *G1* node and its middleground as parts of the *G2'* and *G4'* nodes. More significant, there is no longer any correlation between the musical surface and the transformations included in *G?*. All of the previous supernetworks implied specific horizontal mappings derived from the bracketed transformations, but the doubly-bracketed transformations in this case do not accurately represent specific voice-leading transformations. The problem stems from the different subsets used to generate each supernetwork, thereby creating an inconsistency among the different nodes of supernetwork *G?*. These problems do not invalidate the isography among the supernetworks, they just cannot be joined satisfactorily as nodes in a single network. The musical relevance of the isography is the consistent use of the <T_1> transformation

[3] The supernetwork *G2'* is incorporated in its negatively isographic form (that is, with the upper two nodes reversed) in supernetwork *G?*.

spanning both subphrases melodically and harmonically. Example 3.1–19 aligns these <T_1> mappings with the musical excerpt.

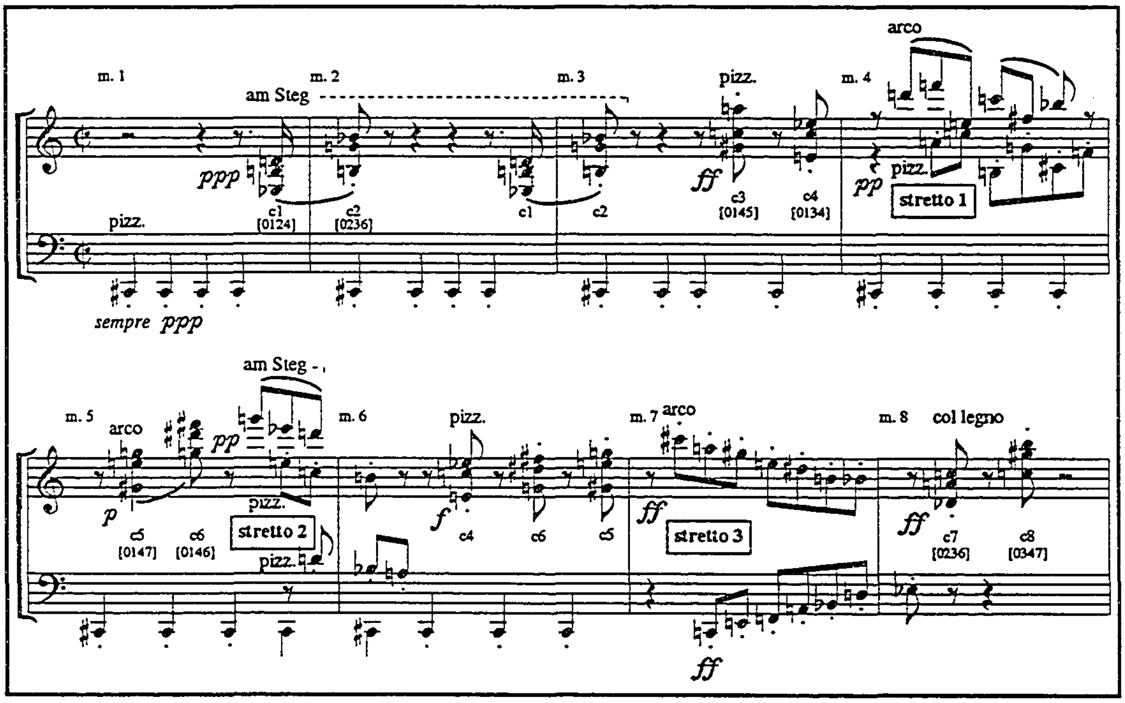
Example 3.1–19: Subphrase recursive structures II



3.2 Webern: Five Pieces for String Quartet, Op. 5, No. 3

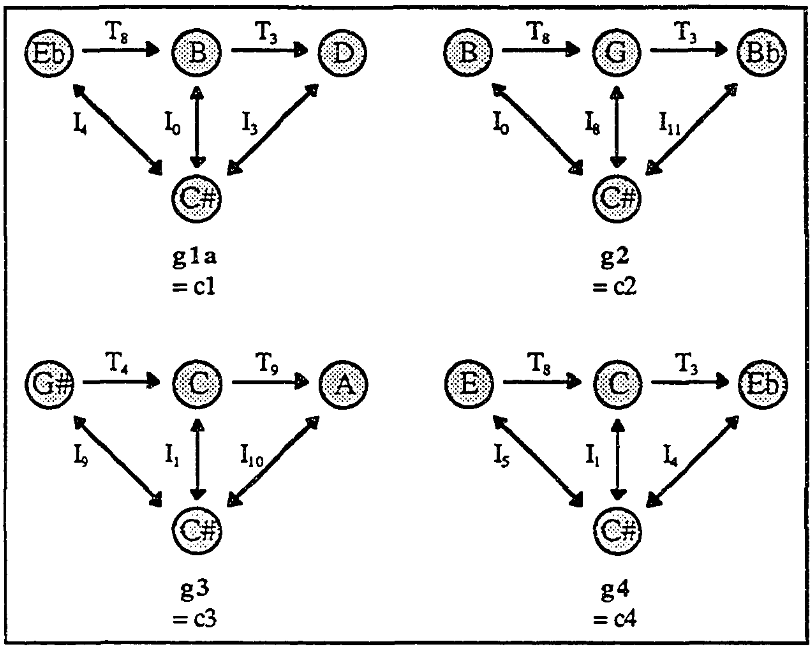
Webern's Op. 5, No. 3 is also no stranger to the transformational approach. Eric Lai offers a rather detailed "account of the unfolding of transformations of two pitch-class sets, 3–3 (014) and 3–4 (015)," in this work.[4] These two set classes do play an important role in the following analysis, though in a substantially different context. In addition, while Lai works his way through the entire piece, I will focus entirely on the first formal section, measures 1–8. A short score of the opening eight bars is shown in Example 3.2–1.

Example 3.2–1: Webern, Five Pieces for String Quartet, Op. 5, No. 3, mm. 1–8



[4] Eric Lai, "Transformational Structures in Webern's Opus 5, No. 3," *Indiana Theory Review*, 10.1–2 (1989): 21–50. Joseph Straus also mentions the "intensive use of set-class 3–3 (014)" in his *Introduction to Post-Tonal Theory* (Englewood Cliffs, NJ: Prentice Hall, 1990): 64.

Example 3.2–2: Klumpenhouwer networks modeling the first four tetrachords

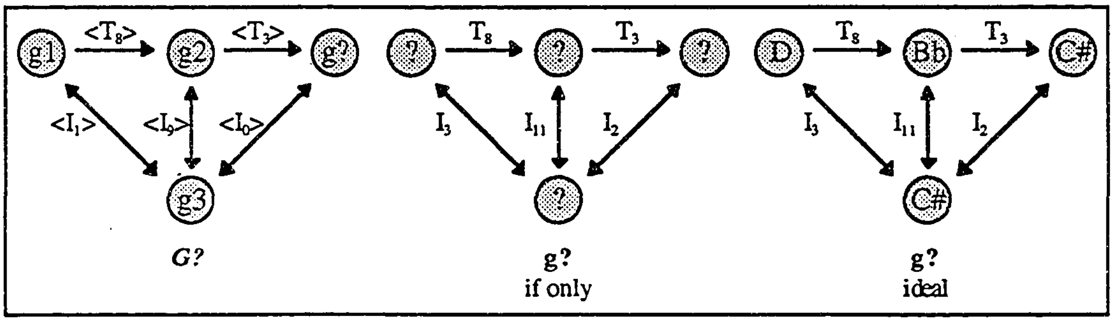


The organization of musical materials in these bars stratifies into two contrasting textures: homorhythmic chords in the upper strings alternating with *stretti* passages. More simply, there is an alternation between a vertical and a horizontal emphasis in terms of pitch organization. All eight different upper-string chords—labeled c1–c8 in Example 3.2–1—are members of set-class 3–3 [014]. Yet when the first six are heard in the context of the persistent *pizzicato* C# pedal in the cello, they form the six different tetrachords listed under the labels c1–c6. According to Straus, through "brute repetition, the C# is established as an important pitch center in the passage. We inevitably hear the other events in the passage in relation to it."[5] Klumpenhouwer networks are again an elegant way of modeling this 3+1 texture; Example 3.2–2 interprets c1–c4 as networks g1–g4. The voice leading implied by these isographies matches the registral/instrumental lines exactly, and the graphs capture the central role of the C# pedal. The <T_8> progression g1 → g2 mirrors the prominent T_8 within the graphs, and combines with the negatively isographic relationship of g3 to

[5] Straus, *Post-Tonal Theory*, 91.

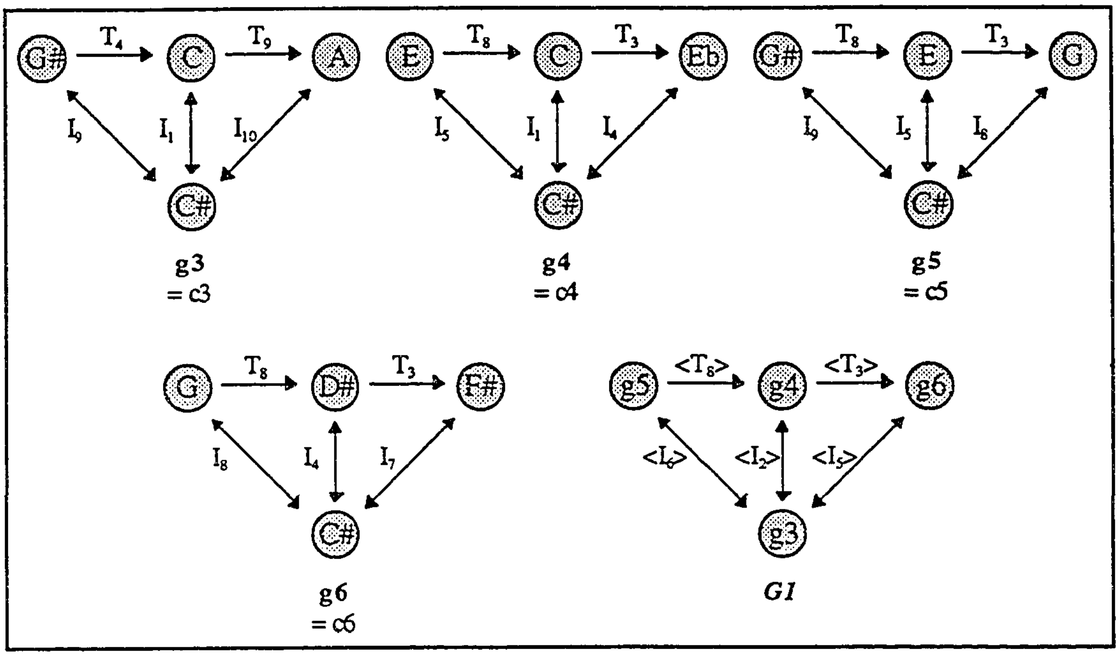
suggest a potential supernetwork that is not realized by g4. The non-realization of this supernetwork sets up the if-only scenario shown in Example 3.2–3. The leftmost graph is the incomplete supernetwork *G?*, the central graph g? is the "missing" if-only network, and the rightmost graph is the ideal realization of the g? network in the context of the C# pedal.

Example 3.2–3: If-only supernetwork



Ignoring the interpolated *stretti* passages for the moment, the next two chordal sections progress c5 → c6 and c4 → c6 → c5. In the latter progression, the inclusion of the previously heard c4, as well as the loud *pizzicato* articulation, supports the association of these chords with the earlier progression c3 → c4. Example 3.2–4 models chords 5 and 6 as Klumpenhouwer networks and reproduces the earlier networks g3 and g4. The relationship among these four networks can be drawn into the supernetwork *G1* also shown in the example. Thus the relationships among all the *pizzicato* chords in this passage are equivalent to the relationships within the tetrachords themselves. In addition, the instrumental lines in the progression c4 → c6 → c5 (measure 6) also create the same sets: violin I performs c6, violin II performs c4, and the viola performs c5. The network also encompasses the *arco* forms of c5 and c6 (measure 5), as networks g5 and g6 also represent them. The realization of this network still does not fulfill the if-only desire generated by the first three tetrachords.

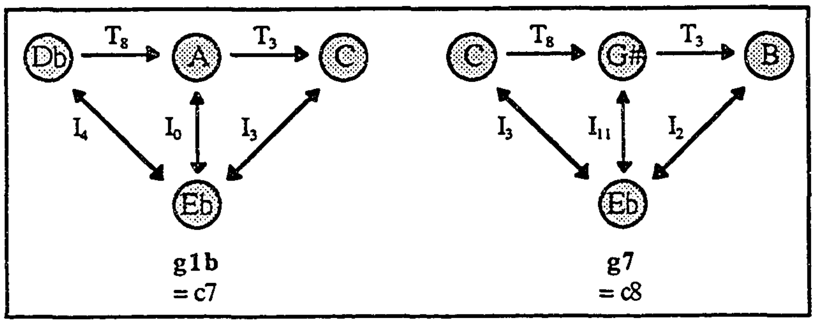
Example 3.2–4: Klumpenhouwer networks modeling tetrachords c3–c6



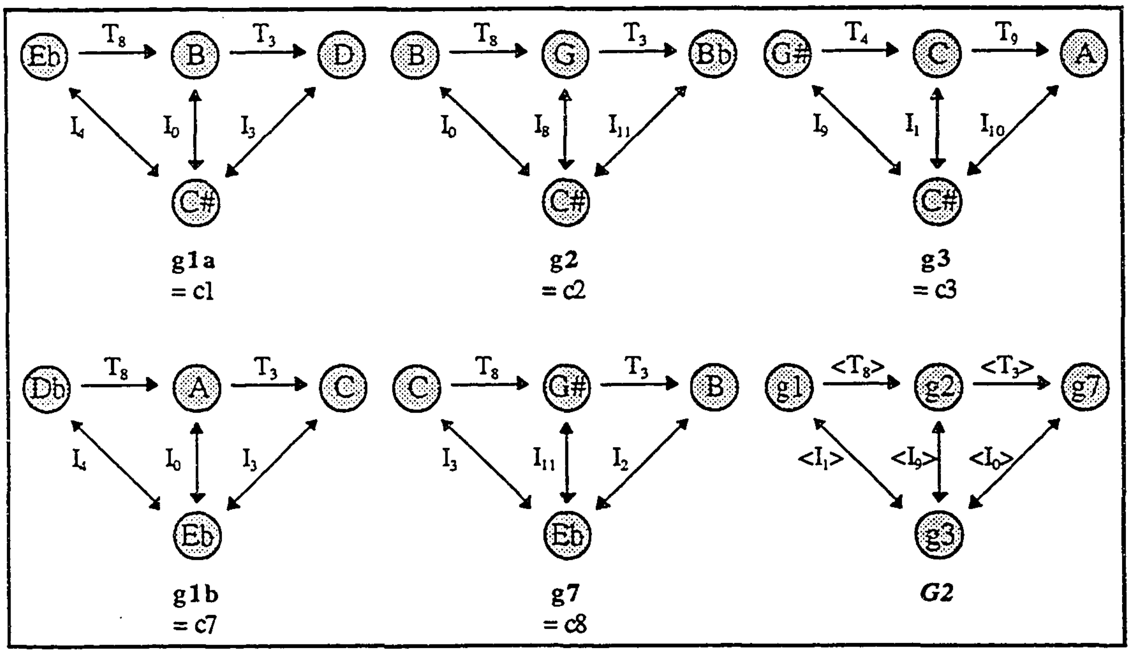
The C# cello pedal does not continue beyond measure 6, and again ignoring *stretto* #3 in measure 7, the first formal section cadences with the progression c7 → c8. These [014] trichords sound over a *pizzicato* Eb in the cello. c7 does so in a manner equivalent to the previous chords—the cello Eb sounds on the strong eighth, while the chord sounds on the subsequent weak eighth—but c8 does so only by inference. Example 3.2–5 models these two chords as Klumpenhouwer networks over their Eb foundation. Note that the graph of c7 belongs to the same Klumpenhouwer class as c1 and is therefore labeled as g1b, and that g7 (the graph of c8) is the if-only network of Example 3.2–3. Significantly, g1b recalls the earlier progression and the incomplete supernetwork, while g7 supplies the previously missing node and brings this section to a close. Example 3.2–6 reproduces graphs g1–g3 and g7 and generates the supernetwork *G2* that models this closure. It is worth noting that though g7 is the if-only network, it is not the ideal realization portrayed in

Example 3.2–3. It is not until the final gesture of the piece—specifically measure 23, shown in Example 3.2–7—that the set {Bb, D, C#} sounds against a *pizzicato* C#.[6]

Example 3.2–5: Klumpenhouwer networks modeling tetrachords c7 and c8.

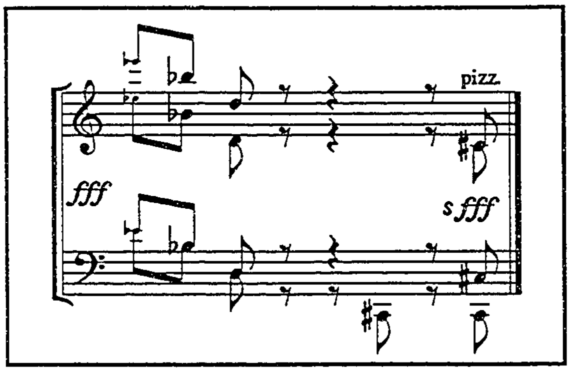


Example 3.2–6: Supernetwork G2 and the realization of the if-only scenario



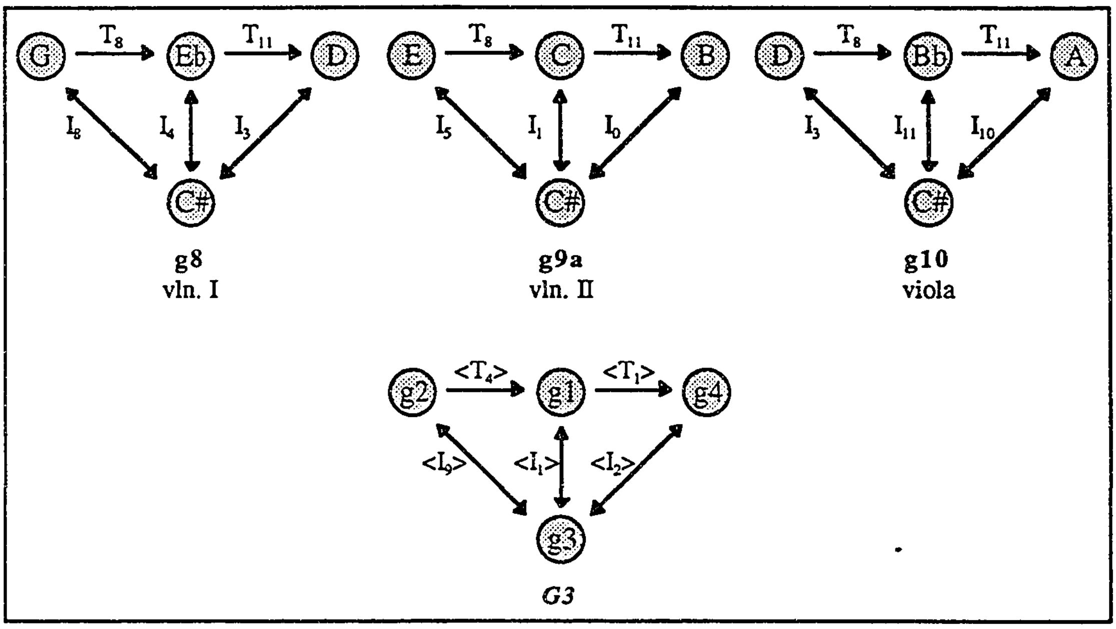
[6] The set {D, Bb, C#} sounds several times before the final gesture, most notably in the first violin in measure 9, in the viola and cello in measures 5–6, and as elements within *stretto* #1. In measure 9 it is not heard against the *pizzicato* C#, and only a single pitch C# is heard. In measures 5–6 the C# pedal still sounds, but no other C# is heard against it. In *stretto* #1 the C# pedals sounds, there is another C# heard against it, but the set is non-contiguous and there is no justification for hearing it as a separate entity.

Example 3.2–7: Webern, Op. 5, No. 3, m. 23



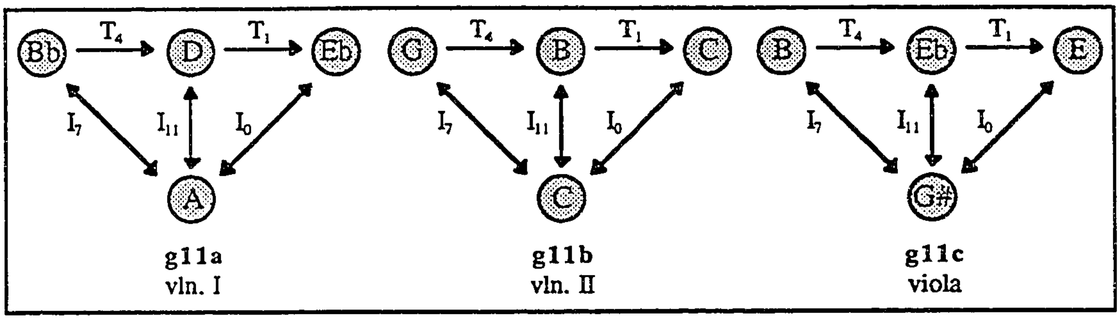
The central canonic passage, *stretto* #2 in Example 3.2–1, highlights set-class 3–4 [015]. Each of the upper three strings play a form of this set-class an eighth-note apart, while the cello continues the C# pedal. A number of relationships exist between this passage and the chordal segments of the excerpt, particularly the opening progression c1 → c2 → c3 → c4. The third statement of the motive, <D, Bb, A>, is a pitch-class literal recurrence of violin I's opening line, that is, the soprano voice of the first three chords. The T_8 motion of the motive is reminiscent of both the vertical T_8s within the chords and the initial progressive gesture <T_8>. Example 3.2–8 facilitates further comparison by modeling each of these canonic voices as Klumpenhouwer networks g8–g10. Note that the prominent T_8 transformation linking the two upper-left nodes on each graph recalls chordal graphs g1–g7. Example 3.2–2 and Example 3.2–3 above illustrated how g4 did not complete the expected recursive supernetwork, thereby setting up the if-only condition. The relationships among g1–g4 are instead similar to *stretto* #2 as shown by the negatively isographic supernetwork *G3* also included in Example 3.2–8.

Example 3.2–8: Klumpenhouwer networks modeling stretto #2

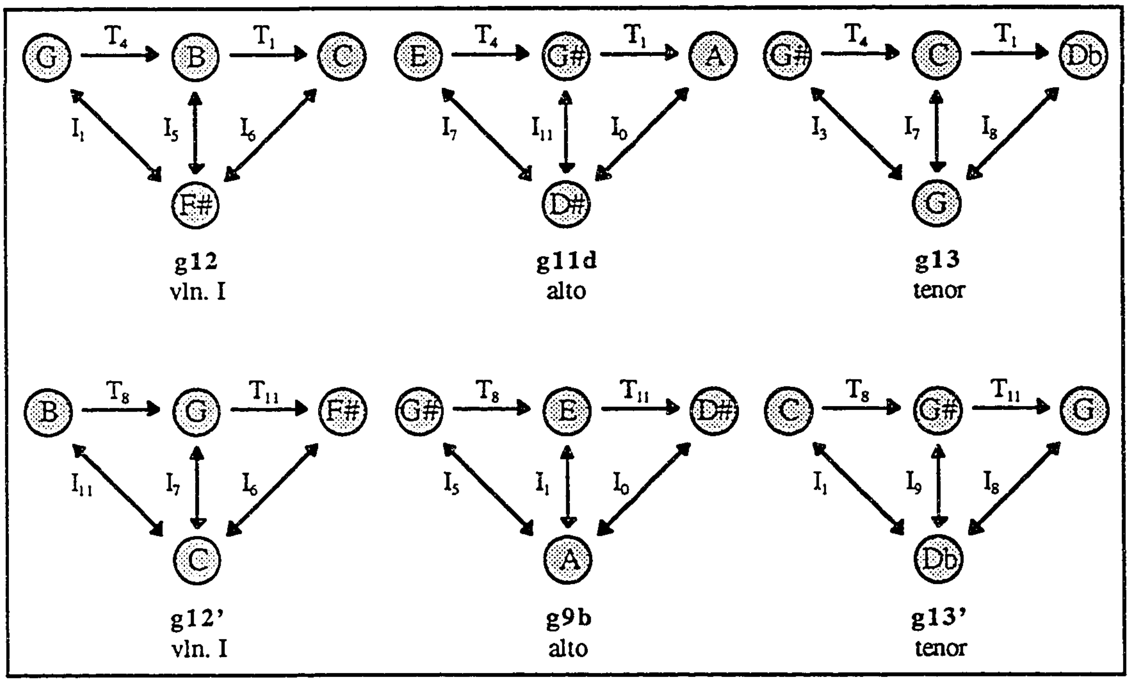


Example 3.2–9 takes this correspondence one step further and explicitly models the upper three transformational voices of the first four chords as networks g11a–c. Significantly, all three of these lines belong to the same Klumpenhouwer class, and are also negatively isographic to networks g8–g10. Graphs g11a and g11c could be rearranged to portray a positive isography to graphs g8–g10, but the interpretation in Example 3.2–9 is preferable as the corresponding nodes generate the vertical chords in this case. Although chords c5–c8 are not contiguous, the upper three transformational voices also yield isographic networks. Example 3.2–10 includes two possible pathways through the progression; one positively and the other negatively isographic to networks g8–g10 modeling *stretto* #2. In summary, networks modeling the transformational voices spanning all eight chords (segmented c1–c4 and c5–c8), the central canonic passage *stretto* #2, and the supernetwork *G3* modeling the chord progression c1–c4 are all structurally similar as illustrated by their isography.

Example 3.2–9: Klumpenhouwer networks modeling the transformational voices of c1–c4

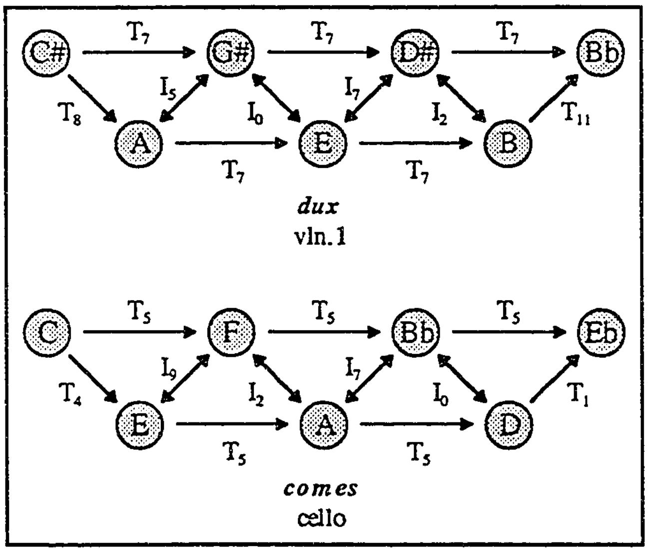


Example 3.2–10: Klumpenhouwer networks modeling the transformational voices of c5–c8



The other two thus far unaccounted for canonic interpolations, *stretto* #1 (measure 4) and *stretto* #3 (measure 7), incorporate only two instrumental lines, and are therefore less easily related to the transformations described above. In the former, the viola imitates the first violin at T_7 (still over the C# cello pedal), and in the latter, the cello imitates the first violin at I_1 with no other instruments sounding. Turning first to *stretto* #3 in a continuation of my consideration of set-class [015], RI-chains in each instrumental voice render any partitioning somewhat arbitrary.[7] So although ten linear forms of set-class [015] sound in *stretto* #3, there is no convincing argument to select specific singleton nodes to allow the generation of 3+1 Klumpenhouwer networks relating to the above graphs. On the other hand, Klumpenhouwer networks do model the complete RI-chains very nicely as shown in Example 3.2–11. Rather than modeling the individual imbricated sets, these networks treat each chain as a *gestalt*, thereby yielding two negatively isographic networks. I_n transformations link the shared dyads, and $<I_2>$ transforms *dux* into *comes*. *Stretto* #1, like *stretto* #3, is neatly modeled as a pair of isographic Klumpenhouwer networks. In this case $<T_7>$ transforms *dux* into *comes*. Note that most of the T_n transformations are familiar from previous networks, the one exception being T_6.[8]

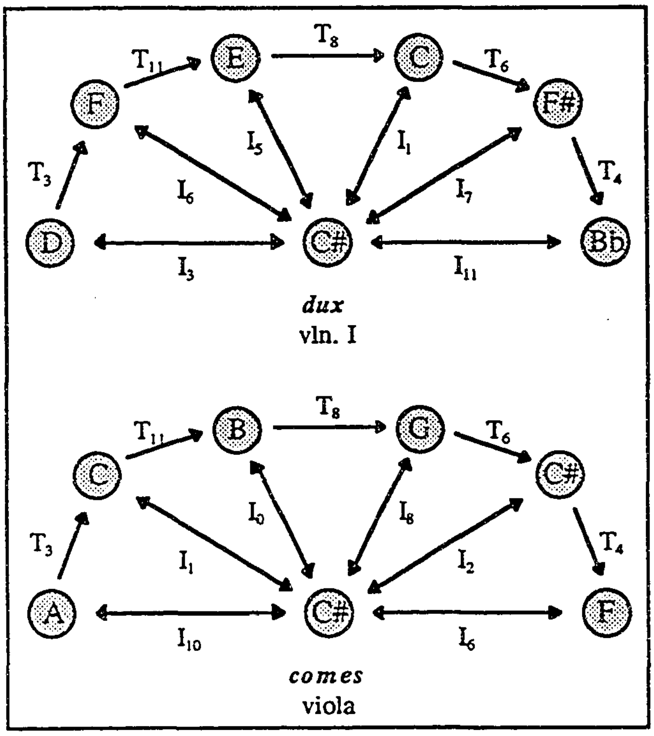
Example 3.2–11: Klumpenhouwer networks modeling stretto #3



[7] Lai, "Transformational Structures," relates this RICH structure to numerous other transformations of set-class [015] in great detail on pp. 29–31ff.

[8] Previously heard tritones were all modeled as I_n transformations.

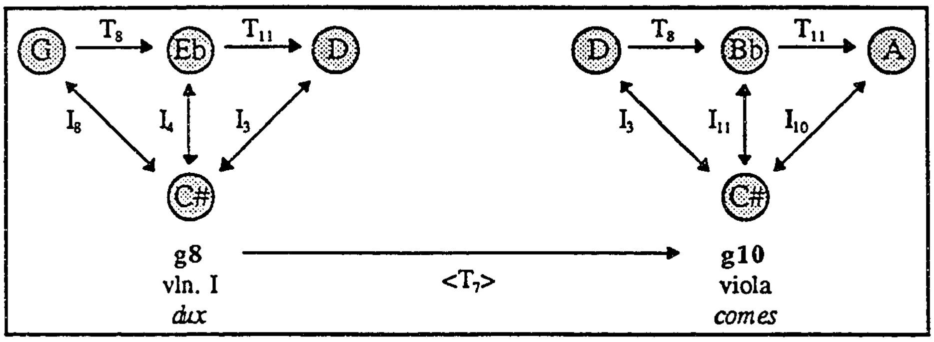
Example 3.2–12: Klumpenhouwer networks modeling stretto #1



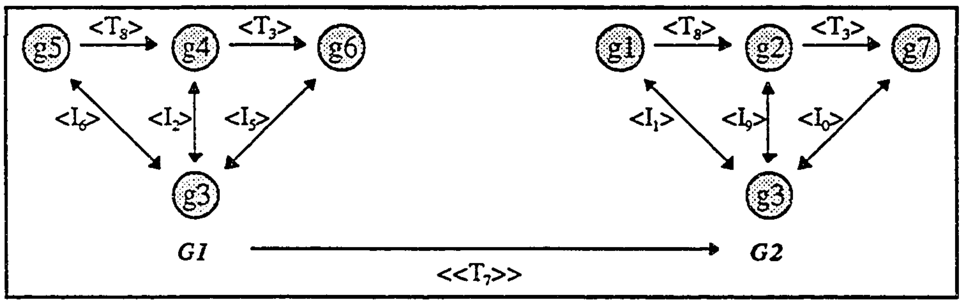
Example 3.2–11 and Example 3.2–12 highlight a transformation that was previously inconspicuous: T_7. In the model of *stretto* #3 it is the important transformation linking alternate statements within the RI-chain, or the TCH-interval described by Lai.[9] In the model of *stretto* #1 <T_7> links the violin I *dux* and the viola *comes*. Although three voices take part in *stretto* #2, <T_7> links the outer statements, once again between violin I and the viola as shown in Example 3.2–13. Example 3.2–14 takes this relationship to another level of abstraction, illustrating the <<T_7>> transformation linking the two chordal supernetworks *G1* and *G2*. The <<T_7>> represents the <T_7> transformations linking each of the graphs represented by corresponding nodes—other than the common-node g3—in the two supernetworks. Therefore all of the chords in the passage relate to at least one other chord by a transformation equivalent to those that play significant roles in each of the *stretti* fragments.

[9] Lai, "Transformational Structures," 30. Lewin describes the calculation of TCH-intervals in *Generalized Musical Intervals and Transformations*, 181.

Example 3.2–13: $<T_7>$ transformation in stretto #2



Example 3.2–14: $<<T_7>>$ transformation linking supernetworks G1 and G2



3.3 Milton Babbitt: Semi-Simple Variations

Andrew Mead's recent book describes Milton Babbitt's music in terms of three compositional periods.[10] *Semi-Simple Variations* is a relatively early work (1956) falling into Babbitt's first period characterized by the exploration of trichordal arrays. Mead provides a concise summary of Babbitt's compositional practices during this period in his first chapter "Milton Babbitt's Compositional World."[11] Scholars frequently discuss works from this period in terms of their precompositional arrays, and I too will begin my examination of *Semi-Simple Variations* from this perspective. My first analytical pass through this work further develops and refines the concept of dual wedges, W_n/W_n, briefly discussed in my analysis of the third movement of Stravinsky's *Three Pieces for String Quartet* in Chapter 2. Then, turning to a graphic approach, I develop a new transformation among Klumpenhouwer networks, $<W_n>$, that corresponds to my dual wedges. My final excursion through the work moves away from the somewhat artificial homophony of the precompositional array, and describes a series of linear supernetworks that better captures the polyphonic musical surface.

A few authors specifically associate the lynes of the trichordal array of *Semi-Simple Variations* with the concept of voices. Charles Burkhart includes this work in his well-known *Anthology for Musical Analysis* and comments:

> The lines, for the most part, do not form "voices" in the conventional sense; instead, members of different lines are associated to form a pointillistic fabric of short sixteenth-note motives. Figure 2 [see the first "measure" of my Example 3.3–1], then, represents a "background" that receives rhythmic elaboration in the "foreground."[12]

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[10] Andrew Mead, *An Introduction to the Music of Milton Babbitt* (Princeton: Princeton University Press, 1994).

[11] Ibid., particularly pages 25–30.

[12] Charles Burkhart, *Anthology for Musical Analysis*, Fourth Edition (New York: Holt, Rinehart and Winston, 1986): 553.

Burkhart's wording suggests that though these may not be conventional voices, they are voices of some kind, perhaps background voices. Elaine Barkin similarly comments:

> Ex. 2 [measures 1–2 of my Example 3.3–1] is a reduction of the four registrally partitioned "voices" of mm. 1–6, preserving the registration but not the repetition of pitches... The top two voices consist of an aggregate as do the bottom two voices.[13]

The scare quotes around the initial voices suggest Barkin has some hesitation in calling the lynes voices, but once adopted she uses the term freely. Christopher Wintle is most explicit, referring to the complete trichordal array as a "voice chart" and continually referring to the lynes as voices.[14] The distinct registral deployment of each of the lynes, as well as the pitch repetition within those registral lines, help make the array structure (particularly the outer voices) easy to hear. In other words, one way a listener might make sense of this music is by tracing registral lines, in effect reducing the complex polyphonic surface to its background homorhythmic lynes.[15] Such a reduction of the first six measures—the theme and first variation—of *Semi-Simple Variations* is shown in Example 3.3–1.[16] Babbitt's surface realization of the Theme and Variation I array is shown in Example 3.3–2 for comparison.

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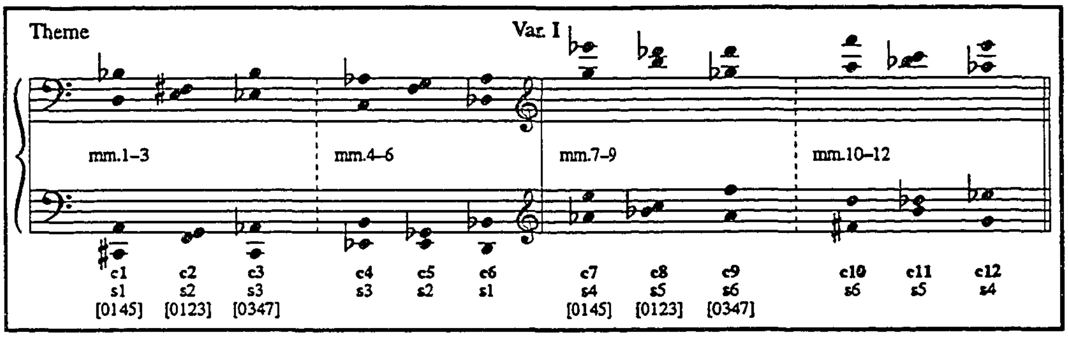
[13] Elaine Barkin, "A Simple Approach to Milton Babbitt's *Semi-Simple Variations*," *The Music Review* 28.4 (1967): 318.

[14] Christopher Wintle, "Milton Babbitt's *Semi-Simple Variations*," *Perspectives of New Music* 14.2–15.1 (1976): 111–154.

[15] An interesting study might involve testing listeners' perception of the background array by first playing a section of *Semi-Simple Variations* and then playing a few series of tetrachords, asking the listeners to select the progression that sounds most representative of the musical excerpt. Nicola Dibben's "The Cognitive Reality of Hierarchic Structure in Tonal and Atonal Music," *Music Perception* 12.1 (1994): 1–25, might provide a suitable model.

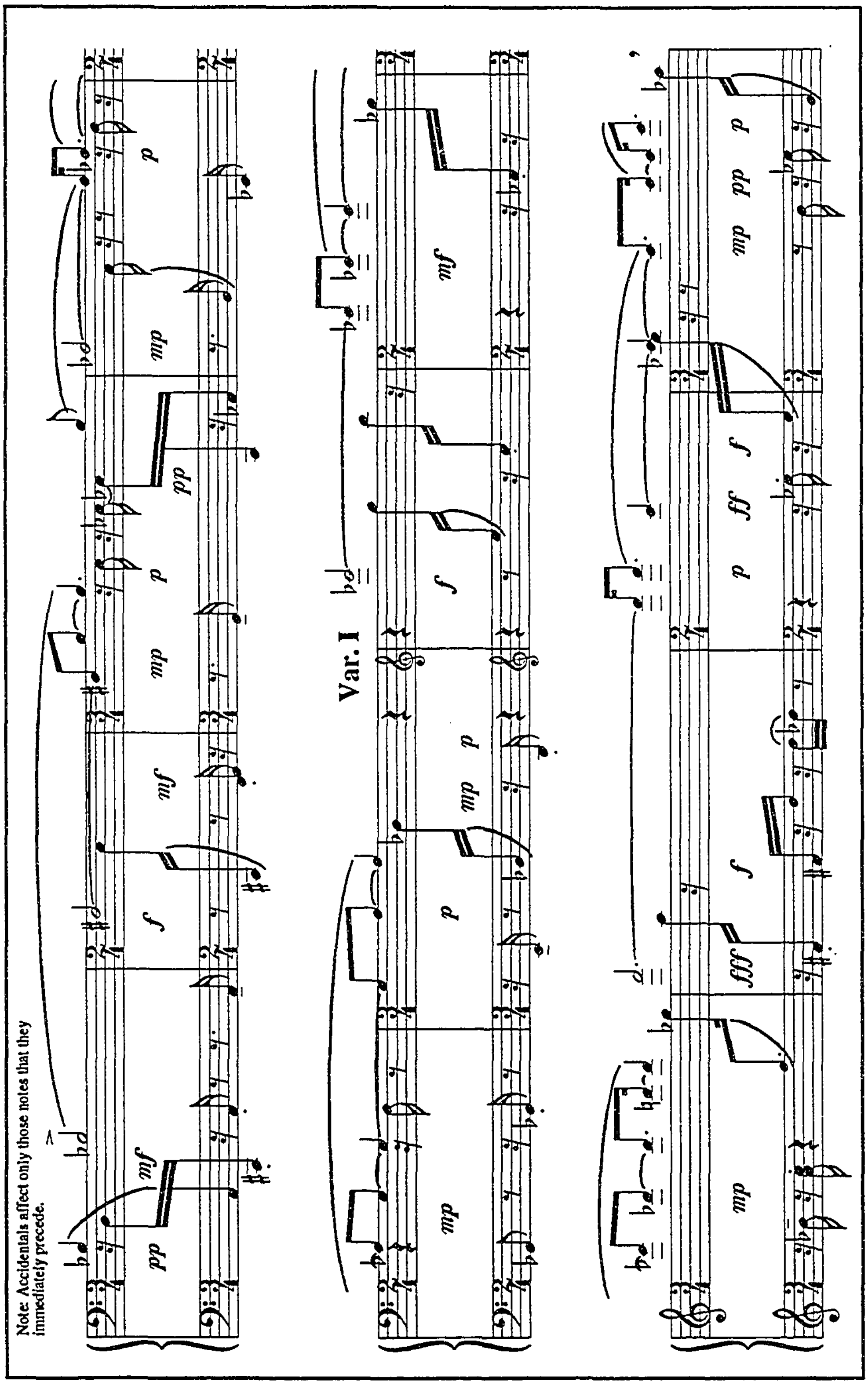
[16] Wintle includes a similar reduction in "*Semi-Simple Variations*," 128. In this and subsequent examples, accidentals affect only those notes that they immediately precede.

Example 3.3–1: Babbitt, Semi-Simple Variations, array (mm. 1–12)

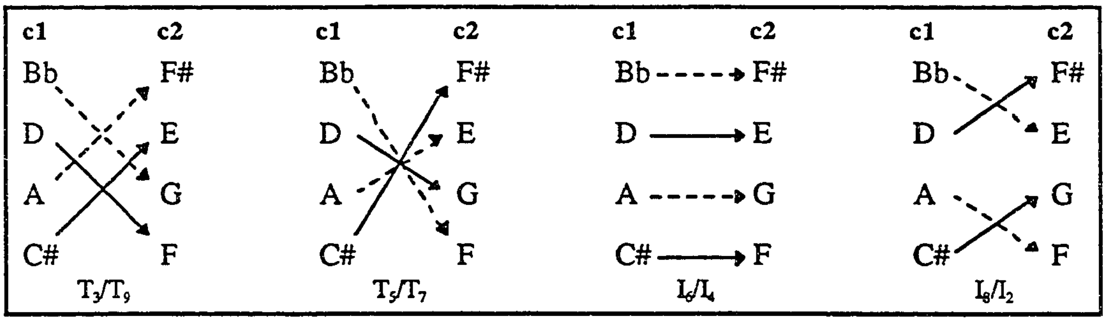


Wintle discusses this work in great detail, particularly the combinatorial processes of aggregate formation, and therefore I will focus on the issues of harmony and voice leading. Each section of *Semi-Simple Variations* is a realization of a four-by-twelve array, or a series of twelve tetrachords. The theme and first variation comprise the first twelve tetrachords, while subsequent variations each span the complete series of chords. Example 3.3–1 consecutively labels these chords c1–c12, and a second set of labels s1–s6 refers to pitch-class content. This progression includes representatives of three different set classes: 4–7 [0145], 4–1 [0123], and 4–17 [0347]. There are a number of dual transformations capable of generating the first progression c1 → c2, or [0145] → [0123]. Example 3.3–3 presents the four possible mappings suggested by the shared discrete [01] dyads keeping the dual transformations pure, that is, not combining transposition and inversion. While it is plausible to imagine any of these mappings as being significant, if we are starting with the assumption that the lynes are voices, then the I_6/I_4 mapping is clearly the most relevant possibility. Example 3.3–4 does the same with chords c1 and c3, though in this case there are even more possibilities. The shared discrete [04] dyads generate the first row of mappings in the example, while the shared [014] subsets generate the second row of mappings. In general, the 2+2 mappings seem more plausible than the 3+1 mappings, but, again, there is still a clear winner among the 2+2 partitions in T_{11}/T_1 as it matches the lynes.

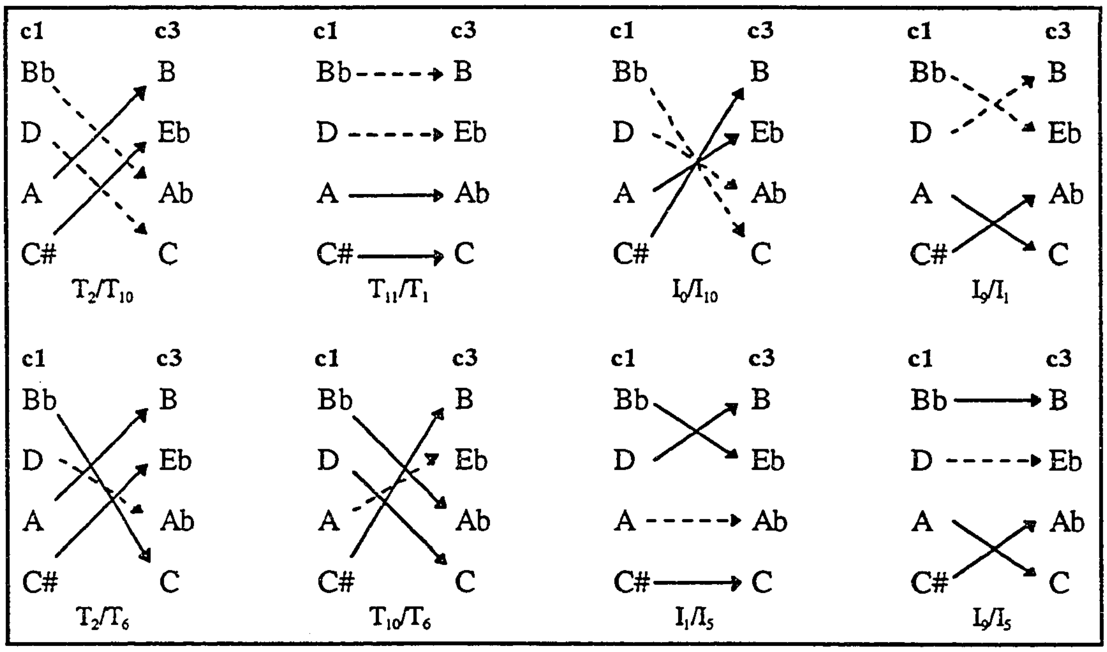
Example 3.3–2: Babbitt, Semi-Simple Variations, mm. 1–12



Example 3.3–3: Some possible dual-transformational mappings for c1 → c2

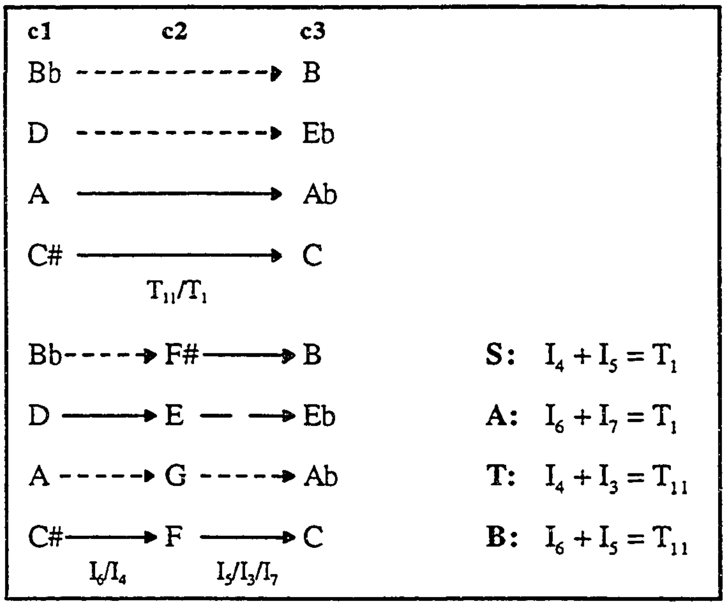


Example 3.3–4: Some possible dual-transformational mappings for c1 → c3



These examples make the structural similarity between c1 (and therefore its counterpart c7) and all the other chords in the progression explicit, but problems arise in progressions between set-classes [0123] and [0347] (specifically c2 → c3, c4 → c5, c8 → c9, and c10 → c11). These set classes are not related by dual transposition or dual inversion; in other words, they cannot be partitioned into two shared discrete subsets. One possible solution to this problem is to use default mappings. For example, if I_6/I_4 generates c1 → c2 and T_{11}/T_1 generates c1 → c3, then theoretically T_{11}/T_1 minus I_6/I_4 should yield the appropriate transformation. Example 3.3–5 illustrates this concept. As shown on the bottom of the example, this yields the transformation $I_5/I_3/I_7$. The left side of the example illustrates the mappings, and the right side shows the derivation of the individual operations. While this coordinates the transformational voices and the lynes, the progression c2 → c3 is no longer a dual transformation as my definition allows no more than two simultaneous operations.

Example 3.3–5: Default mapping between c2 and c3



A more effective way of using dual transformations to model the lynes of this array as voices is via dual wedges. The classic transformations of the thematic hexachord—P, $RI_{11}P$, RP, $I_{11}P$—generate soprano-bass (P–I_{11}P) and alto-tenor (RI_{11}P–RP) dyadic invariances at I_{11}. This property of inversional symmetry is a general characteristic of Babbitt's work as Mead points out: "Generally, Babbitt employs the same row class for both members of a hexachordally combinatorial pair of lynes, related by inversion."[17] Thus W_{11}/W_{11}, representing W_{S+B}/W_{A+T}, describes all of the registral lines shown in Example 3.3–1. As all of this wedging takes place in pitch space, other than the dramatic registral

[17] Mead, *Milton Babbitt*, 27.

shift marking the formal boundary between the Theme and Variation I, the mirror symmetry of the lynes is a comparatively prominent feature of the musical surface.

Up to this point I have used W_n, where n equals the vertical index, to represent wedging, but this notation is rather ambiguous as it does not specify any specific horizontal motion. Now I would like to enhance the notation to include some representation of the horizontal transformation along the wedge index, thereby bringing dual wedges more in line with the other dual transformations. Example 3.3–6 introduces this new notation in the context of the theme. As has been the case in previous examples, the solid arrows represent the mappings generated by the left-hand side of the dual transformations, while the dashed arrows represent the right-hand side. I now include the wedge index to the left of all the transformations, followed by a colon, W_n:, in a fashion similar to key information given in a tonal Roman numeral analysis. I label the individual transformations ±p/±q representing the linear or horizontal motion along the universal wedge index. For example, Wn: ±4/±2 describes the first progression, c1 → c2, in which ±4 represents the soprano-bass dyad wedging apart by interval-class 4, and ±2 represents the alto-tenor dyad wedging apart by interval-class 2. Example 3.3–7 similarly models Variation I.

Example 3.3–6: Dual wedges modeling the Theme

c1 (s1)		c2 (s2)		c3 (s3)		c4 (s3)		c5 (s2)		c6 (s1)
Bb	⟶	F#	⟶	B	⟶	Ab	⟶	G	⟶	A
D	- - - →	E	- - - →	Eb	- - - →	C	- - - →	F	- - - →	Db
A	- - - →	G	- - - →	Ab	- - - →	B	- - - →	Gb	- - - →	Bb
C#	⟶	F	⟶	C	⟶	Eb	⟶	E	⟶	D
W_{11}:	±4/±2		±5/±1		±3/±3		±1/±5		±2/±4	

Example 3.3–7: Dual wedges modeling Variation I

c7	c8	c9	c10	c11	c12
(s4)	(s5)	(s6)	(s6)	(s5)	(s4)
Eb →	Db →	D →	F →	C →	E
G ---→	B ---→	Gb ---→	A ---→	Bb ---→	Ab
E ---→	C ---→	F ---→	D ---→	Db ---→	Eb
Ab →	Bb →	A →	F# →	B →	G
W_{11}: ±2/±4	±1/±5	±3/±3	±5/±1	±4/±2	

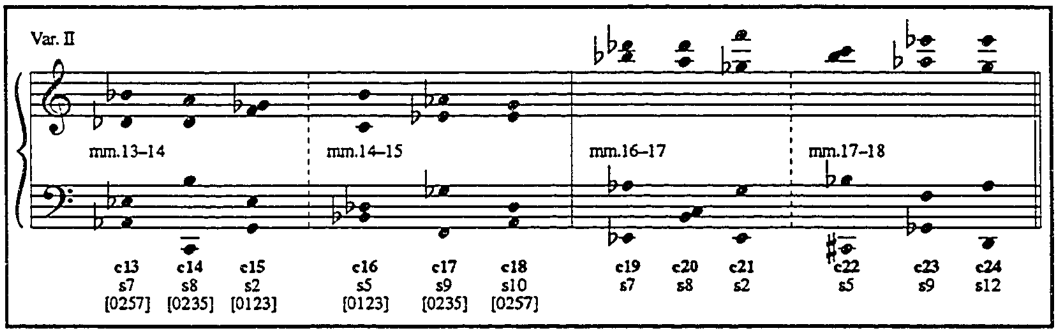
Though this cumbersome wedging notation may suggest four different T_n levels, reminiscent of Roeder's approach, it is specifically intended to be interpreted as two transformations along particular wedging paths.[18] I urge the reader to try to strike a balance between the vertical and horizontal information incorporated into this notation; this will be helpful as I shift modes to a network approach later on in this analysis. Ideally, one should listen for the two horizontal interval classes, without losing sense of the two vertical dyads. In other words, the listener is attempting to trace the exact amount of symmetrical expansion or contraction of the two vertical dyads. For the remainder of my discussion I will use W_n for generic purposes and W_n: ±p/±q when more specific linear information is necessary.

Variation II, shown in Example 3.3–8, maintains the I_{11} dyads, and therefore continues the W_{11}/W_{11} mappings. The inversional pairing is now permuted to soprano-alto and tenor-bass, but the wedgings all still take place in pitch space. Example 3.3–9 shows the dual wedge mappings. Variation III, shown in Example 3.3–10, also continues the dual wedges for the most part. The pairing returns to soprano-bass and alto-tenor, but now some of the wedgings occur in pitch-class space thereby rendering them less perceptible on the musical surface. Example 3.3–11 shows the dual wedge mappings. Note that c34 and

[18] Roeder would model the first transformation in Example 3.3–6 as <4, 10, 2, 8> in "Theory of Voice Leading," representing the T_n levels in ascending order from bass to soprano.

c35 are isolated from the process; this is the point at which Babbitt reverses the pitch-classes Ab and A in the top lyne, thereby beginning the disintegration of the W_{11} wedges. The arrow below the chords circumvents these problematic chords, illustrating the transformation W_{11}: ±5/±1 that links the final statement of c36 to the rest of the passage. There are a few ways to relate c34 and c35 to each other and the preceding chord, two of which are shown in Example 3.3–12. The final c35 → c36 progression is more problematic as no dual transformation can account for any of the twenty-four possible mappings. For this reason I prefer to imagine the dyad switch as an isolated event from which ripples emanate affecting the subsequent variations.

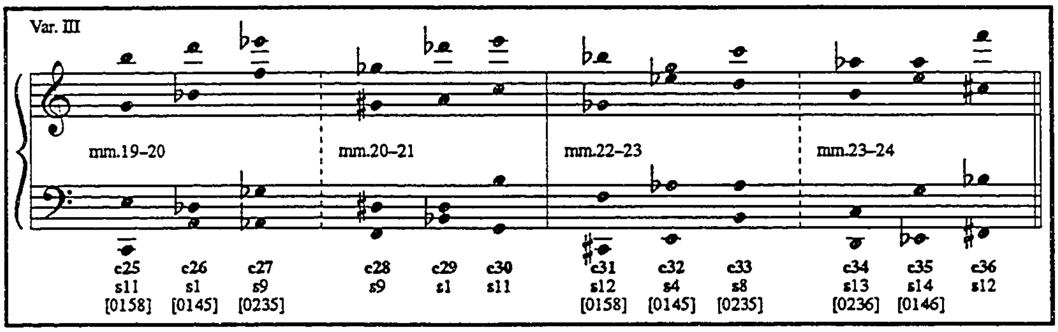
Example 3.3–8: Babbitt, Semi-Simple Variations, Variation II, array (mm. 13–18)



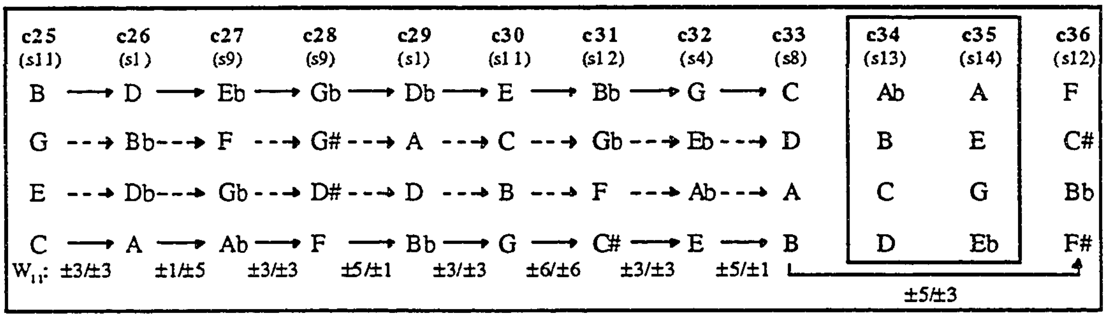
Example 3.3–9: Dual wedges modeling Variation II

	c13	c14	c15	c16	c17	c18	c19	c20	c21	c22	c23	c24
	(s7)	(s8)	(s2)	(s5)	(s9)	(s10)	(s7)	(s8)	(s2)	(s5)	(s9)	(s10)
	Bb ---→	A ---→	Gb ---→	B ---→	Ab ---→	G ---→	Db ---→	D ---→	F ---→	C ---→	Eb ---→	E
	Db ---→	D ---→	F ---→	C ---→	Eb ---→	E ---→	Bb ---→	A ---→	Gb ---→	B ---→	Ab ---→	G
	Eb ⟶	B ⟶	E ⟶	Db ⟶	Gb ⟶	D ⟶	Ab ⟶	C ⟶	G ⟶	Bb ⟶	F ⟶	A
	Ab ⟶	C ⟶	G ⟶	Bb ⟶	F ⟶	A ⟶	Eb ⟶	B ⟶	E ⟶	C# ⟶	Gb ⟶	D
W_{11}:	±4/±1	±5/±3	±3/±5	±5/±3	±4/±1	±6/±6	±4/±1	±5/±3	±3/±5	±5/±3	±4/±1	

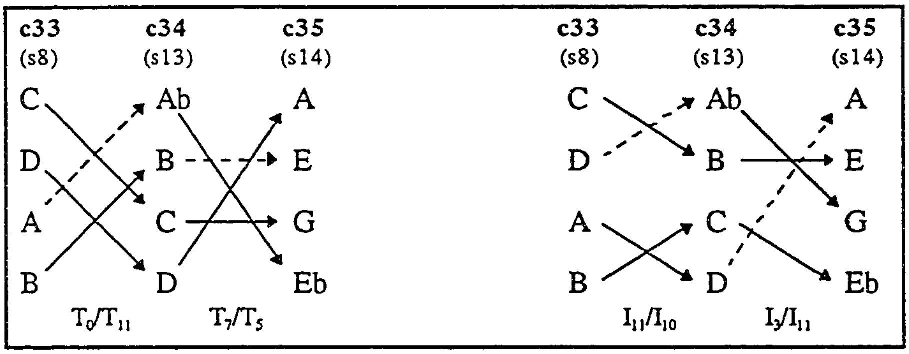
Example 3.3–10: Babbitt, Semi-Simple Variations, Variation III, array (mm. 19–24)



Example 3.3–11: Dual wedges modeling Variation III

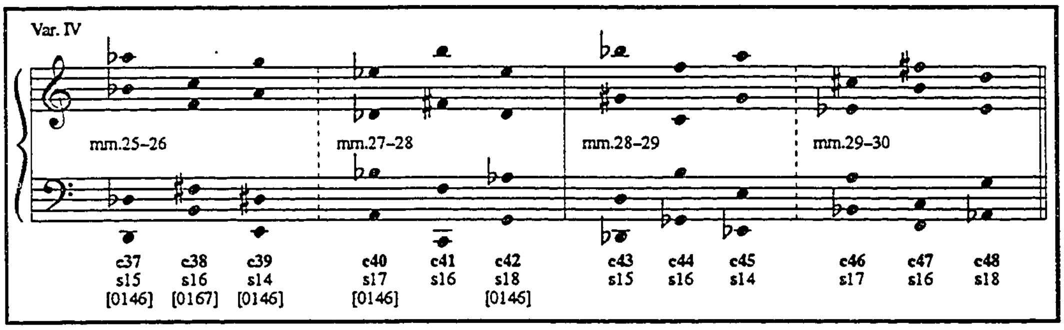


Example 3.3–12: A couple of possible dual transformations for c33 → c34 → c35

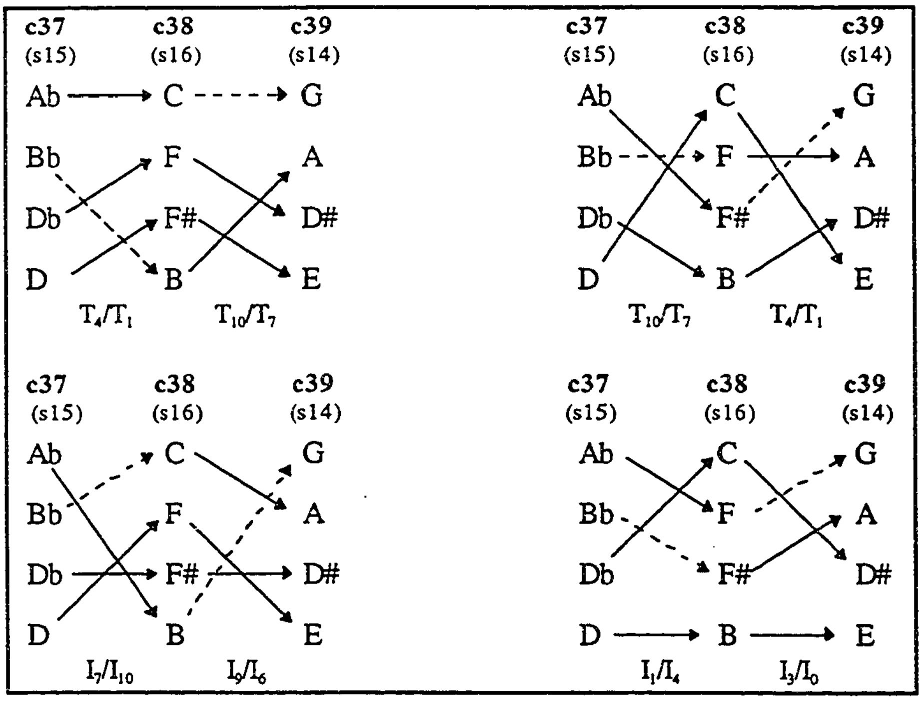


As Variation IV, shown in Example 3.3–13, retains the new dyad pairs Ab-D and A-Eb introduced at the end of Variation III, the voices no longer travel along a common wedge index. The two tetrachords heard in this passage [0146] and [0167] do share the trichordal subset [016] allowing a number of dual transformational mappings. Example 3.3–14 illustrates the four possible symmetrical mappings around the [016] subsets in the context of the first aggregate. These mappings permute the voices in various ways moving from c37 into c38 and "undo" these permutations moving out of c38 and into c39. This has the effect of preserving the lynes as voices between alternate chords in the trichordal aggregates and across the span of the whole variation. Example 3.3–15 shows this property using the first and last mappings of Example 3.3–14. The reason for selecting those two mappings is the continuity of one of the outer voices in each, which avoids the soprano-bass exchange incorporated into the other two mappings. Note that within all of the transformations among these chords $x = \pm 3$.

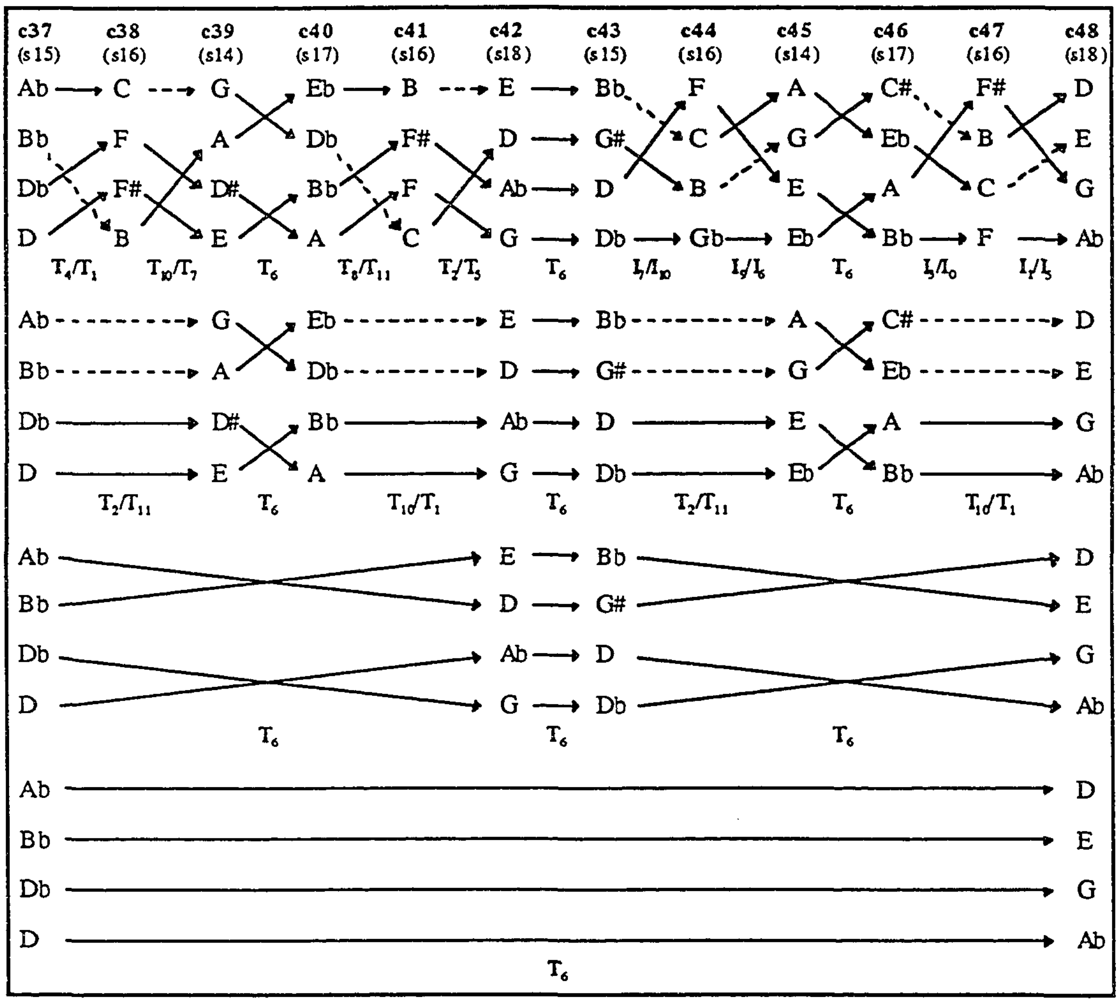
Example 3.3–13: Babbitt, Semi-Simple Variations, Variation IV, array (mm. 25–30)



Example 3.3–14: Four possible mappings around the [016] subsets

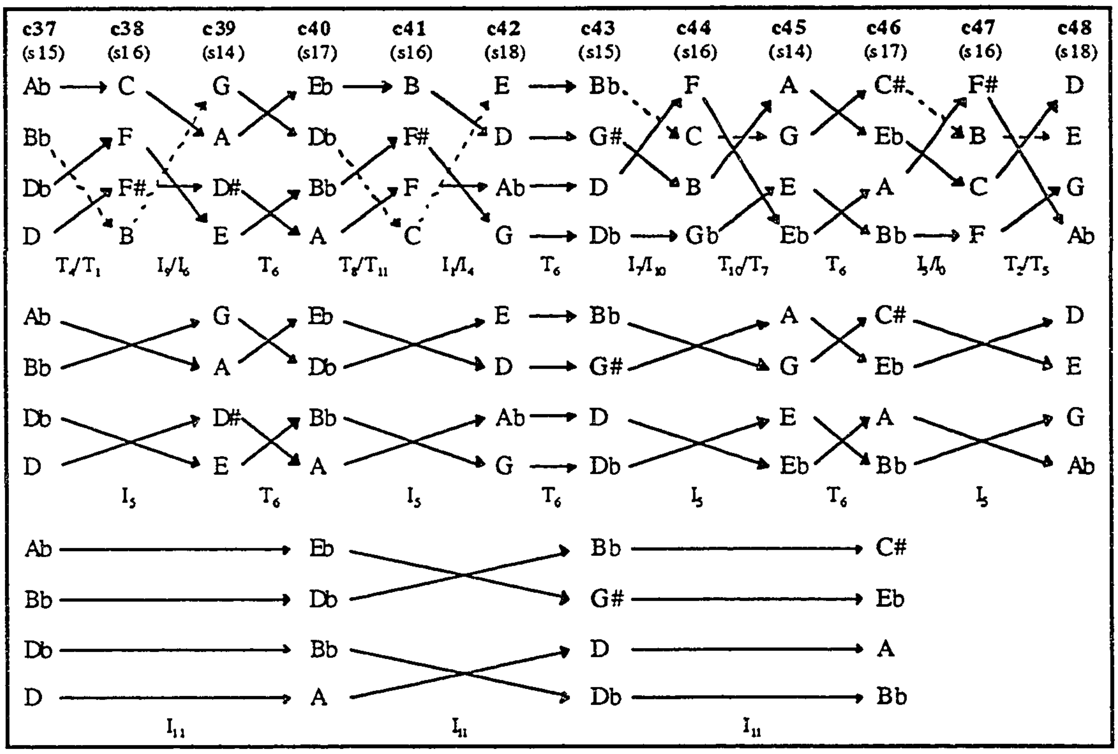


Example 3.3–15: One set of dual transformations modeling Variation IV



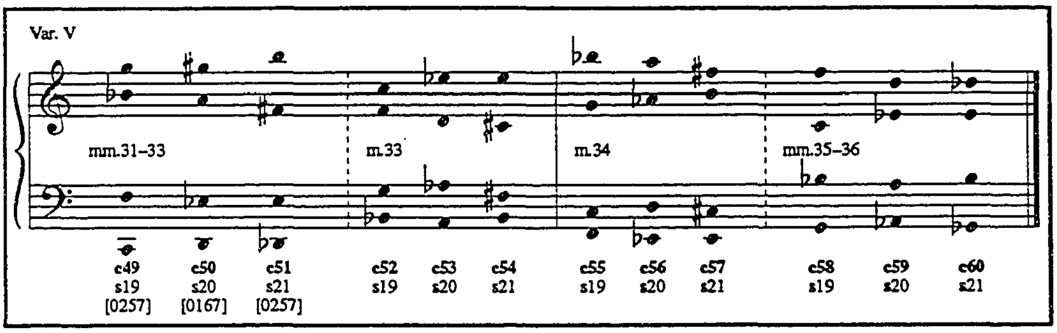
Another option is arranging the mappings to keep the singleton line consistent. One such possibility is shown in Example 3.3–16. This has the advantage of relating all of the [0146] tetrachords by the traditional operations shown in the second level of Example 3.3–16. This second level could yield the bottom two levels shown previously in Example 3.3–15, but I find the third level shown in Example 3.3–16 more interesting. The additional (SA)(TB) permutation within each trichordal aggregate unifies each hexachord with the unpermuted I_{11} mappings. Significantly, this third level, representing the starting points of each trichordal aggregate, projects the previously vertical dyads horizontally. In other words, the W_{11} relationships created by the vertical I_{11} dyads is now projected as the linear voices among the trichordal aggregates. This is a direct byproduct of exchanging pitch-classes Ab and A; one of the more significant effects or ripples originating with the problematic chords of Variation III.

Example 3.3–16: Another set of dual transformations modeling Variation IV



Variation V, shown in Example 3.3–17, returns to a pitch-space wedging process, though not around vertical I_{11} dyads. This variation's dyads wedge along a vertical I_5 axis; the axis containing the important {Ab, A} dyad. In some sense this is the converse of the Variation IV, as a previously linear event is now verticalized. The pairings are soprano-alto and tenor-bass (as in Variation II), and the relevant dual wedges are shown in Example 3.3–18. The transformational wedges in pitch space suggest a return to the "style" of the opening, possibly a "resolution" after the increasing complexity of Variations III and IV, while the new index W_5 suggests a dramatic shift away from the opening—with its W_{11} transformations—resulting from events in Variations III and IV.

Example 3.3–17: Babbitt, Semi-Simple Variations, Variation V, array (mm. 31–36)



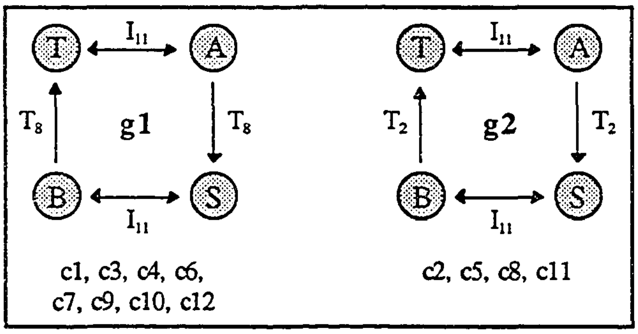
Example 3.3–18: Dual wedges modeling Variation V

c49 (s19)	c50 (s20)	c51 (s21)	c52 (s19)	c53 (s20)	c54 (s21)	c55 (s19)	c56 (s20)	c57 (s21)	c58 (s19)	c59 (s20)	c60 (s21)
G --->	G# --->	B --->	C --->	Eb --->	E --->	Bb --->	A --->	F# --->	F --->	D --->	Db
Bb --->	A --->	F# --->	F --->	D --->	C# --->	G --->	Ab --->	B --->	C --->	Eb --->	E
F ⟶	Eb ⟶	E ⟶	G ⟶	Ab ⟶	F# ⟶	C ⟶	D ⟶	C# ⟶	Bb ⟶	A ⟶	B
C ⟶	D ⟶	Db ⟶	Bb ⟶	A ⟶	B ⟶	F ⟶	Eb ⟶	E ⟶	G ⟶	Ab ⟶	Gb
W_{11}: ±2/±1	±1/±3	±3/±1	±1/±3	±2/±1	±6/±6	±2/±1	±1/±3	±3/±1	±1/±3	±2/±1	

As promised earlier, I will now shift analytical modes to a network approach. Any of the dual transformations described above can be modeled as Klumpenhouwer networks, but a few networks are particularly interesting. All of the tetrachords in the Theme and Variation I can be modeled as one of the two networks shown in Example 3.3–19. The nodes contain the generic labels SATB rather than specific pitch classes in this example, and the corresponding chords represented by each network are listed below the graphs. Note that the two networks share the same I_{11} arrows, but have different T_n arrows. Therefore neither $<T_n>$ nor $<I_n>$ transformations link these two networks. I propose a new Klumpenhouwer transformation $<W_n>$ to describe this process; in this case $<W_6>$ transforms g1 into g2 and vice versa. $<W_n>$ transformations retain the I_n values and add a fixed amount to the T_n values, somewhat like the converse of $<T_n>$ transformations which retain the T_n values and add a fixed amount to the I_n values. Like the other bracketed transformations, $<T_n>$ and $<I_n>$, $<W_n>$ can be thought of as a composite transformation expressing the sum of the two wedges in a dual wedge. For example, the networks in

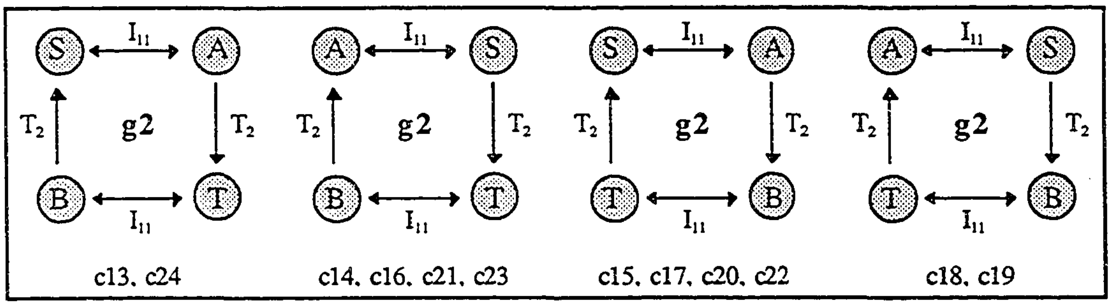
Example 3.3–19 correspond to the dual wedges shown in Example 3.3–6 and Example 3.3–7. The I_{11} network arrows correspond to the dual label W_{11}, and the differences between the network T_n values match the sum of p and q in ±p/±q. In other words, the subscript 6 in <W_6> arises from one dyad wedging by ±4 while the other wedges by ±2. In cases where ±p = ±q, their sum is zero, resulting in the transformation <W_n> or strong isography. This composite is similar to other bracketed transformations in its combination of dual values. It is crucial to note that <W_n> transformations only apply to networks in this specific configuration. Only dyads can wedge, and only two discrete subsets are possible in Klumpenhouwer networks. The relationships among the two omitted I_n arrows that would cross the center of the network are more complex: one increases by the <W_n> subscript and one decreases by the <W_n> subscript. In this situation I omit them not only for visual clarity, as in previous analytical applications, but also for theoretical ease.

Example 3.3–19: Klumpenhouwer networks modeling the Theme and Variation I

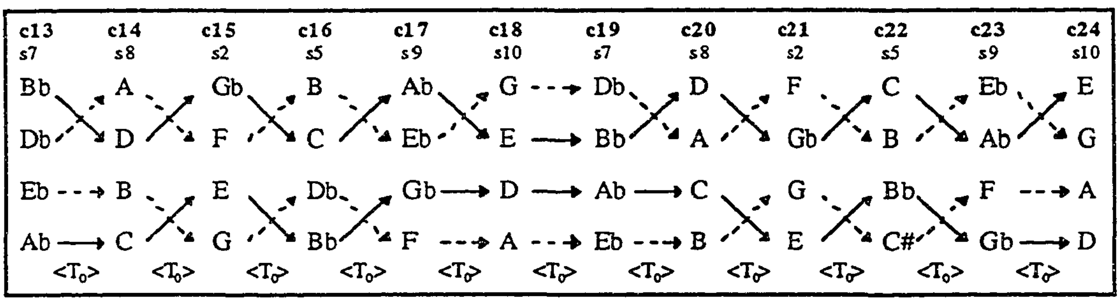


As mentioned above, Example 3.3–19 is a graphic model of Example 3.3–6 and Example 3.3–7. g1 models chords c1, c3, c4, c6, c7, c9, c10, and c12, the outer chords of each trichordal aggregate, and g2 models chords c2, c5, c8 and c11, the inner chords of the trichordal aggregates. The <<W_6>, <W_6>> transformations generate each trichordal aggregate, and each aggregate is linked by <W_0> (or <T_0>). The implied voice leading matches the registral lines exactly without any permutations. A graphic model of the dual wedges in Variation II, shown in Example 3.3–9, is similarly possible, but more interesting is the fact that all the tetrachords in Variation II can be modeled as network g2 of Example 3.3–19 with various registral permutations. Example 3.3–20 shows the four permutations of g2 occurring in Variation II. In this variation all of the chords belong to the same Klumpenhouwer class, related by <T_0>. The extremely active permutations belie the seemingly static network structure. Example 3.3–21 shows the voice leading implied by the g2 networks and their permutations. Note the soprano-alto exchanges in virtually every single progression.

Example 3.3–20: Klumpenhouwer networks modeling Variation II

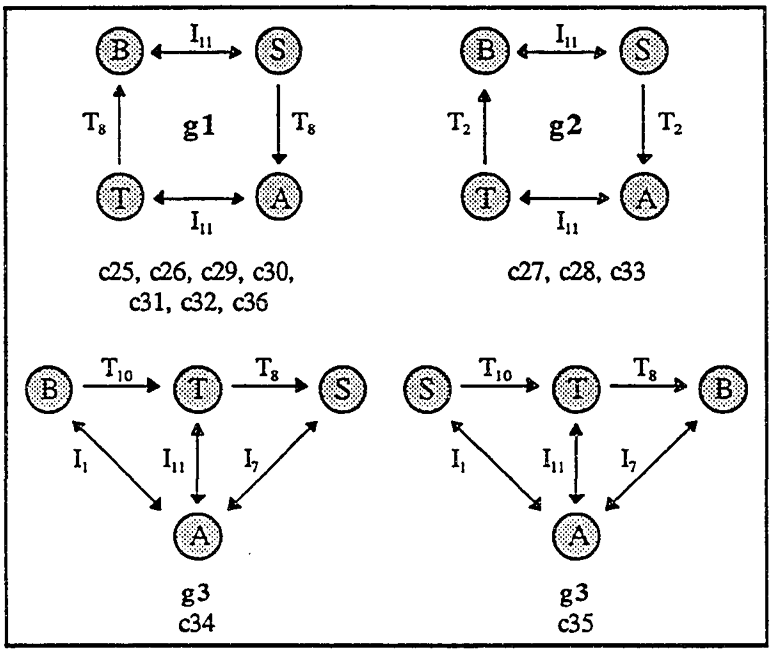


Example 3.3–21: Variation II mappings implied by g2 networks

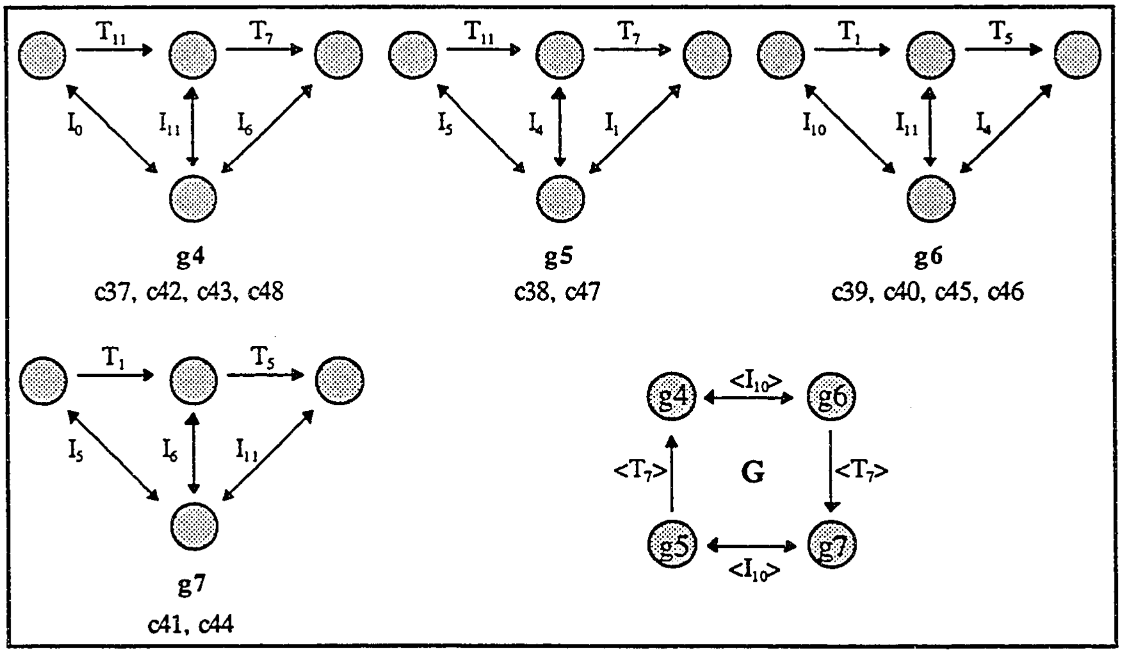


Variation III progresses more like the Theme and Variation I. As shown in Example 3.3–22, it includes both g1 and g2 networks and no registral permutations. The mappings implied by these networks are identical to those shown in Example 3.3–11. Example 3.3–22 also includes network g3 representing the two chords, c34 and c35, altered by the dyad switch. Significantly, the progression primarily alternates <T_0> and <W_6>, and c34 → c35 progresses <T_0> despite the drastic graphic shift to a 3+1 network.

Example 3.3–22: Klumpenhouwer networks modeling Variation III



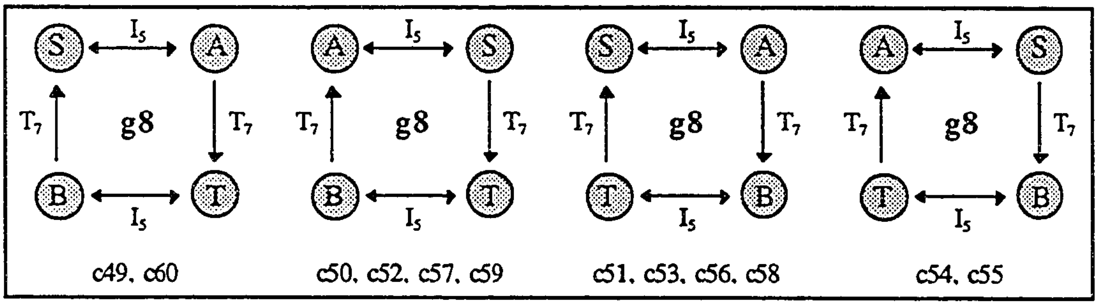
Example 3.3–23: Klumpenhouwer networks modeling Variation IV



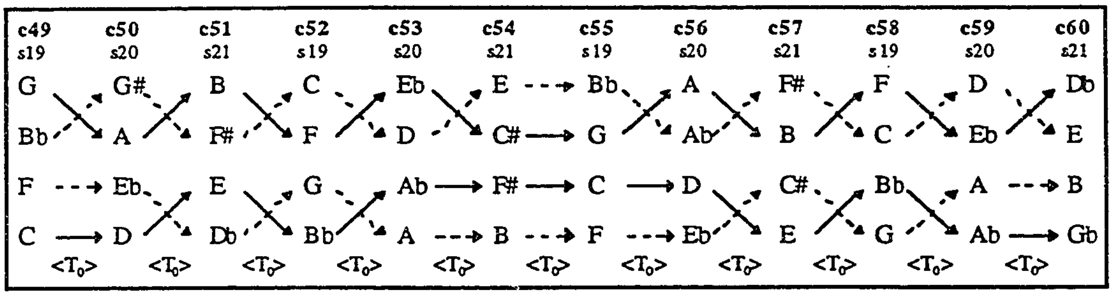
As mentioned above, Variation IV continues the 3+1 transformations introduced by the dyad switch. Example 3.3–23 shows the Variation IV tetrachords as Klumpenhouwer networks corresponding to the series of mappings shown in Example 3.3–16; the interpretation that keeps the singleton line unified. This example includes four different networks: g4–g7. The nodes in these networks are empty rather than duplicating the graphs with varying node content. These four networks are similar to g3, sharing the outer-node T_6 transformation, but are built around a shared [016] subset rather than a [026] subset. Example 3.3–23 also includes a supernetwork incorporating g4 through g7. This supernetwork is positively isographic to the networks modeling Variation V discussed below.

Variation V returns to a dual wedge, but like Variation II, all the chords can be modeled as a single Klumpenhouwer network. This network is shown with its voice permutations—identical to those in Variation II—in Example 3.3–24. As mentioned above, this network is positively isographic to the supernetwork generated by Variation IV. The mappings implied by g8 are shown in Example 3.3–25. Again, like Variation II, the active permutations clash against the static network structure. The information presented in all of the above networks reinforces the dual transformational interpretation which suggests that the variations are paired Theme/I and III, II and V, with IV organized quite differently. Within that framework, Variation III introduces an event that causes, or at least foreshadows, Variation IV, which in turn generates Variation V.

Example 3.3–24: Klumpenhouwer networks modeling Variation V

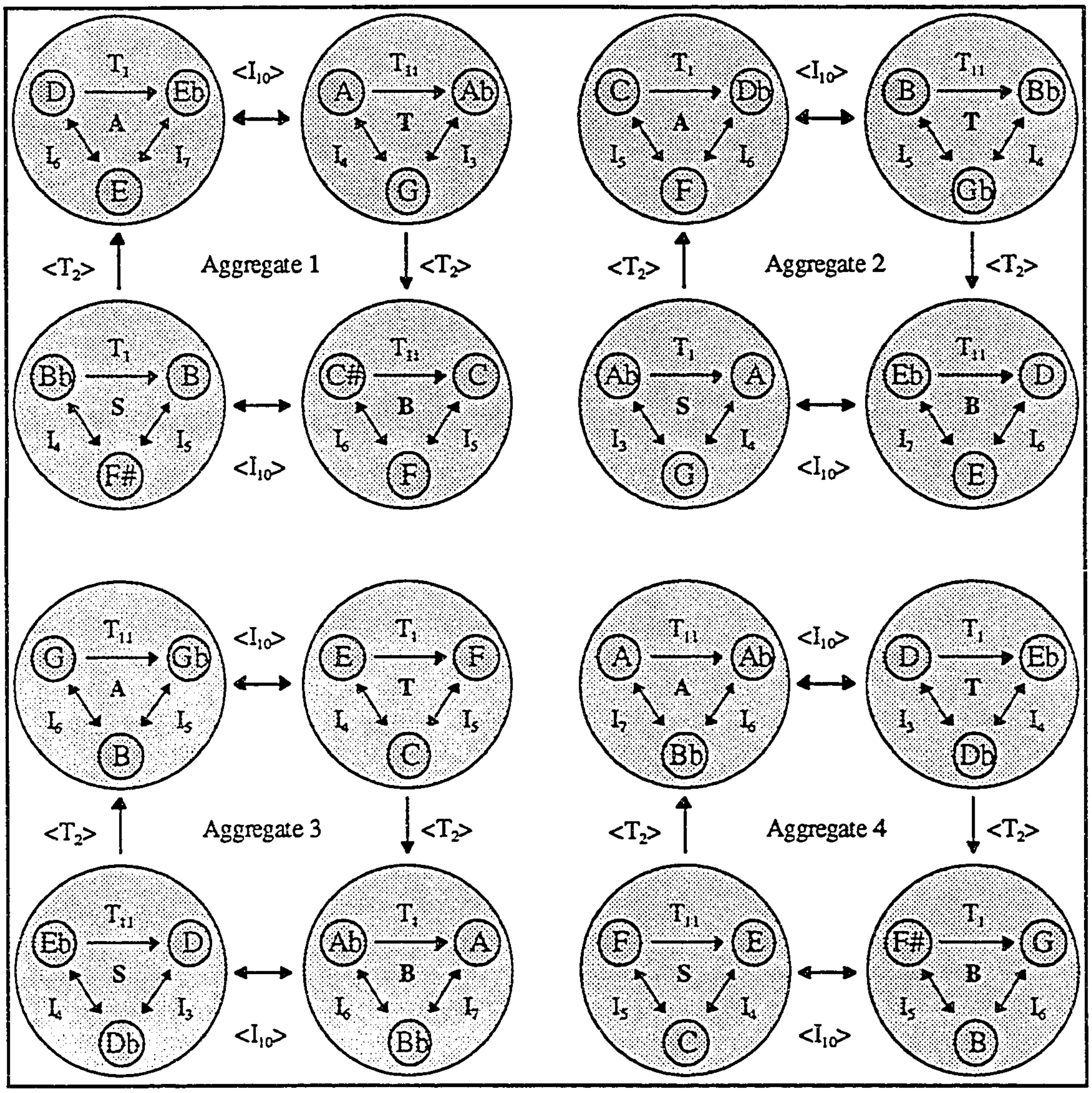


Example 3.3–25: Variation V mappings implied by g8 networks



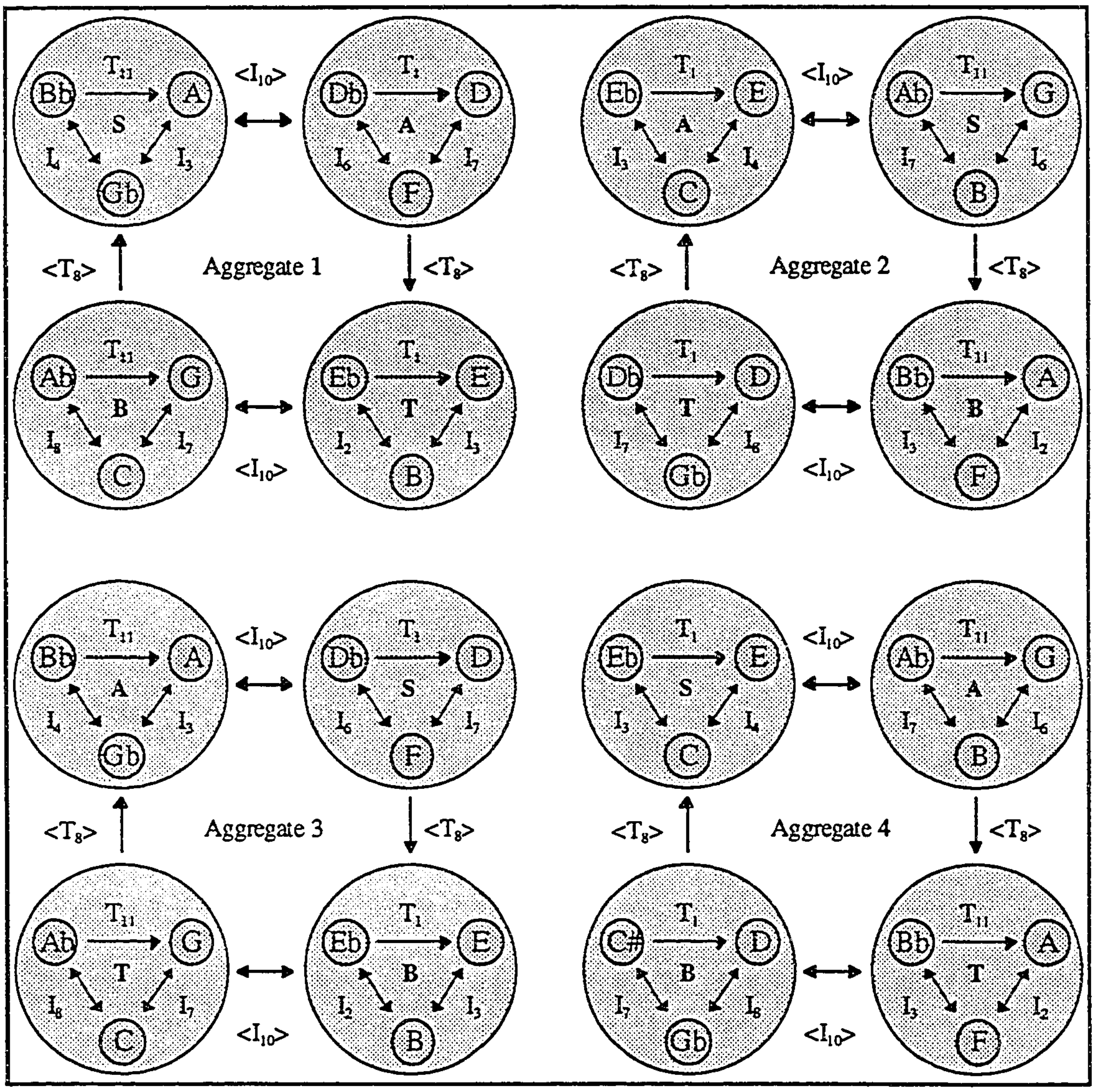
I will take one more brief analytical pass through *Semi-Simple Variations*. Working from the array tetrachords is somewhat abstract despite the clear registral deployment of the lynes. The non-alignment of the tetrachord elements make a linear approach more appealing than a harmonic approach. It is possible to model each of the trichordal aggregates as a supernetwork that captures the relationships among the linear trichords within each registral voice. This is possible by modeling each linear trichord as a 2+1 Klumpenhouwer network nested within a node of a giant 2+2 supernetwork. Example 3.3–26 shows the four trichordal aggregates of the Theme and Variation I modeled this way. The small inner trichordal networks are built around the common [01] dyads of the [015] and [012] trichords that make up the horizontal structure of these rows. As linear models these 2+1 networks always retain their temporal order. The aggregates almost never overlap, and isolating them as individual networks is more satisfactory than isolating the individual tetrachords of previous examples. Of particular significance in this case is the fact that all of the aggregates are strongly isographic, and that they are positively isographic to g2 previously modeling many of the individual tetrachords.

Example 3.3–26: Linear supernetworks modeling the Theme and Variation I



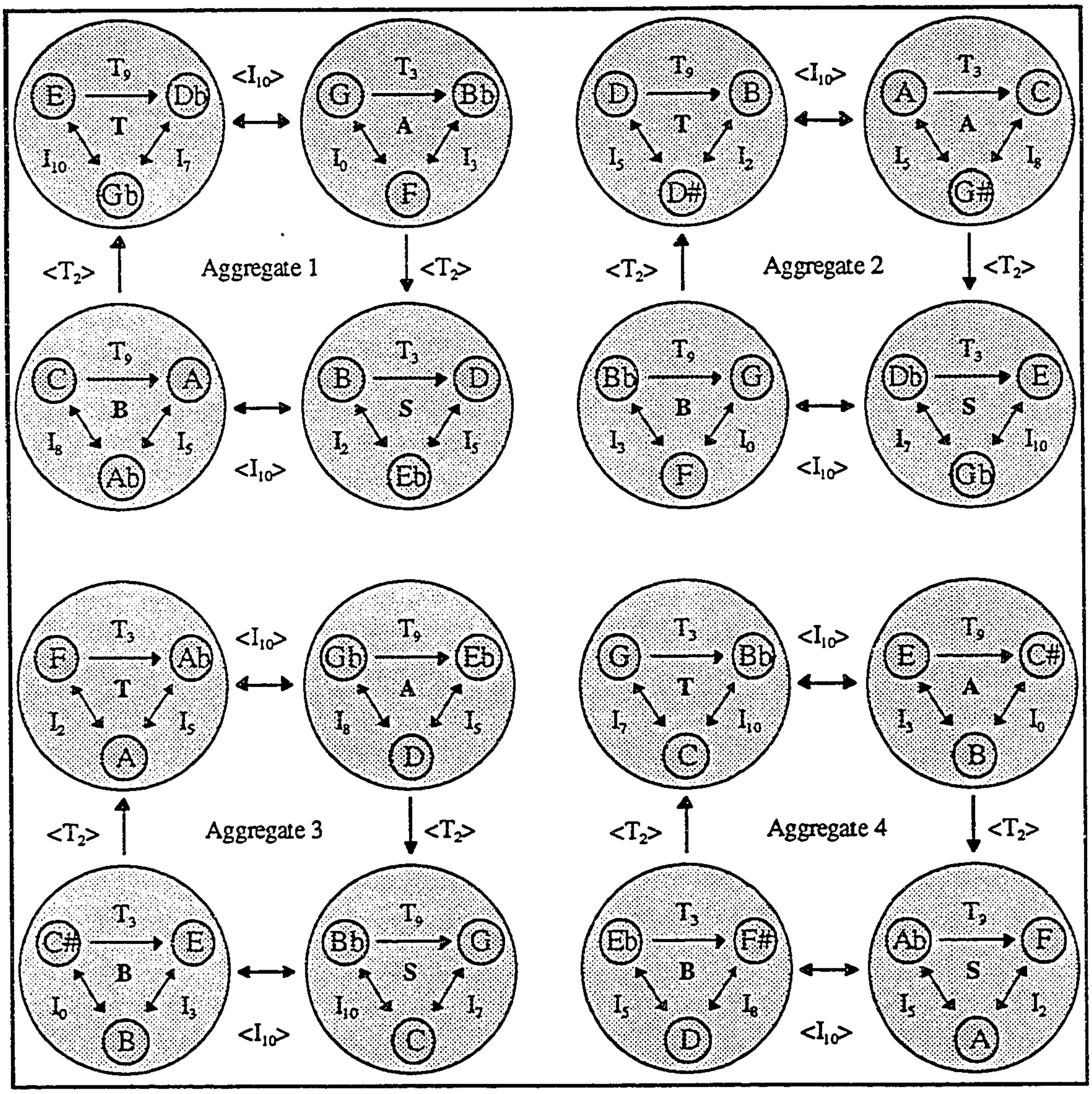
Example 3.3–27 similarly models Variation II. Note that these networks are also constructed around shared [01] dyads though the linear trichords for the Variation II rows are [014] and [015]. More significantly, these supernetworks are positively isographic to g1 modeling earlier tetrachords. Ironically, this is the variation that I modeled exclusively with g2 in Example 3.3–20. Perhaps <<W_6>> relates the aggregates of the Theme and Variation I to those of Variation II.

Example 3.3–27: Linear supernetworks modeling Variation II

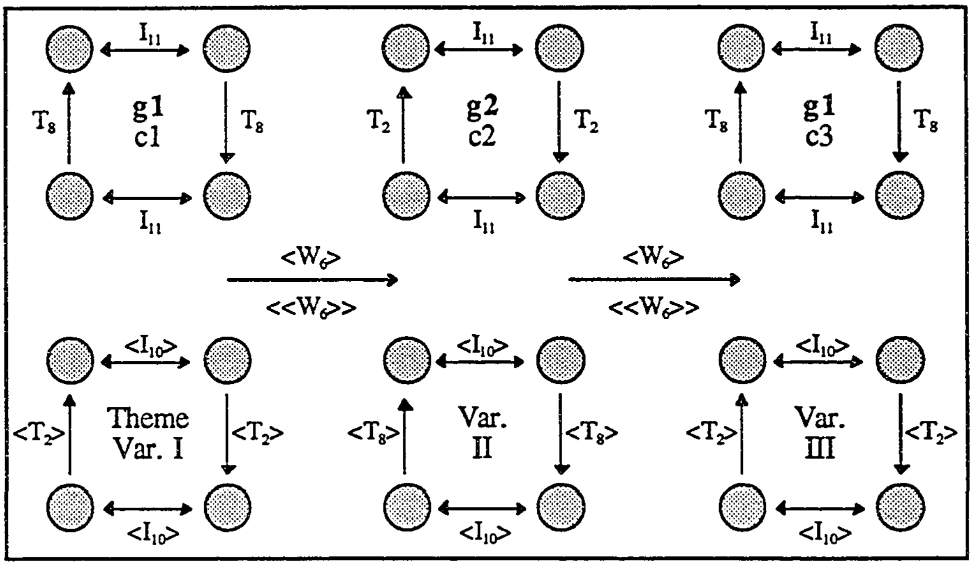


As expected by now, Variation III yields supernetworks similar to the Theme and Variation I. The primary difference is that the inner trichordal networks share [03] dyads rather than [01] as the linear trichords in this variation are [014] and [025]. Particularly interesting is the fact that the {Ab, A} dyad switch does not impact this graphic interpretation. It is also worth noting that $\langle\langle W_6 \rangle\rangle$ again transforms the supernetworks modeling Variation II into those modeling Variation III. The two $\langle\langle W_6 \rangle\rangle$ transformations linking the first three sections of the piece are reminiscent of the two $\langle W_6 \rangle$ transformations generating the trichordal aggregates in the Theme and Variation I. Example 3.3–29 compares these two progressions. It is striking that equivalent transformations generate both the trichordal aggregates within the first section of the piece and the progression of the first three sections themselves.

Example 3.3–28: Linear supernetworks modeling Variation III



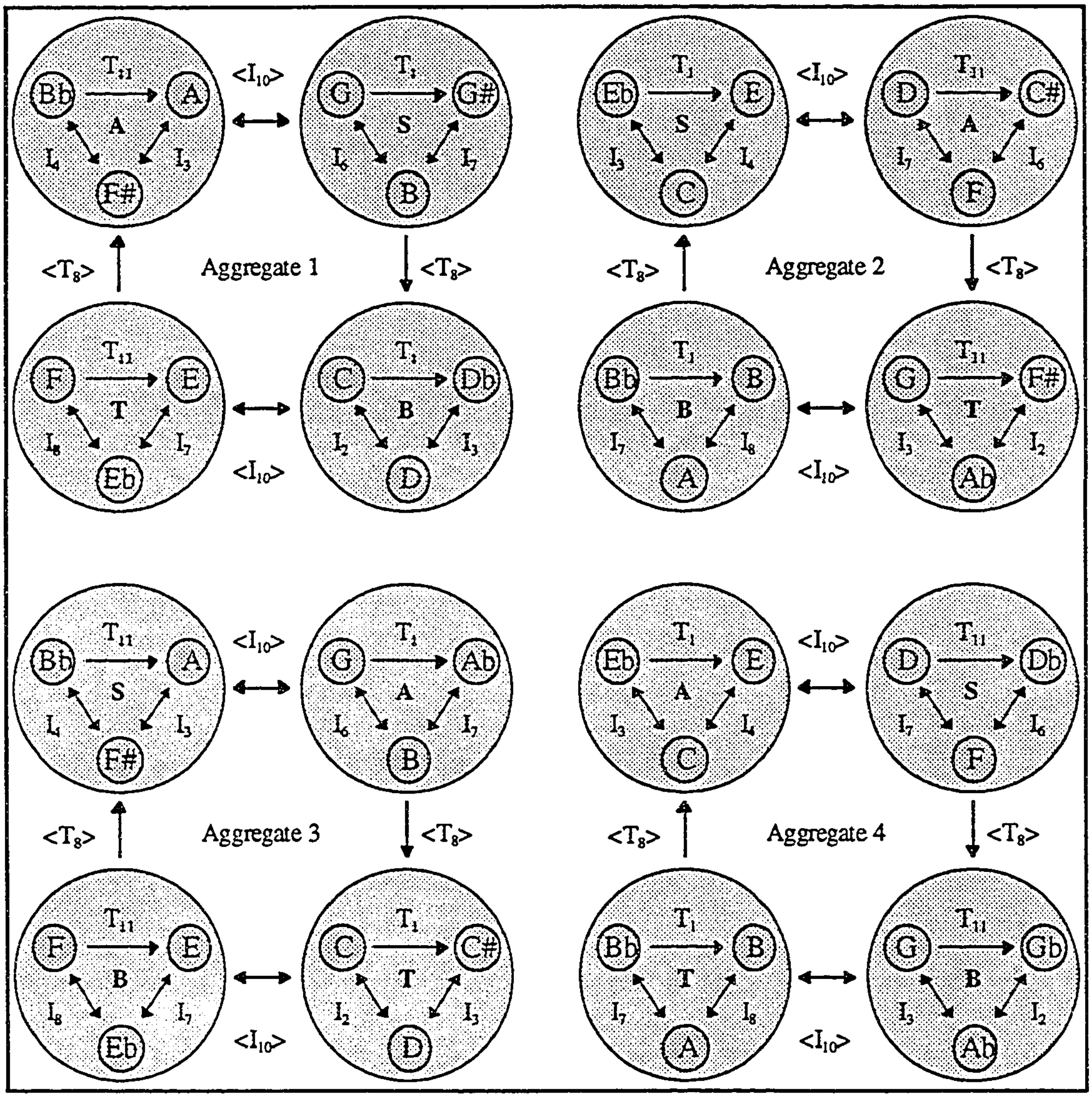
Example 3.3–29: Comparison of thematic aggregate networks and linear supernetworks



It is possible to model Variation IV this way around the shared [05] dyads of the linear [015] and [025] trichords, but the resulting supernetworks add little of interest. I prefer noting at this point that the supernetwork *G* of Example 3.3–23 is positively isographic to the aggregate supernetworks of the above three examples.

Most interesting is Variation V, which returns to the shared [01] dyads, derived from the linear [014] and [012] trichords. Not only is this variation similar to Variation II, but now the supernetworks are strongly isographic. In previous interpretations Variation V was centered around the new I_5 vertical dyads, but now familiar $\langle I_{10} \rangle$ relationships replace them. It is at this point that the most interesting structural feature is revealed: as Klumpenhouwer transformations combine independent operations, $I_{11} + I_{11}$ of the first three sections of the piece become $\langle I_{10} \rangle$, and the $I_5 + I_5$ of Variation V also becomes $\langle I_{10} \rangle$. It is this interpretation that I find most satisfactory in its sense of return and restabilization at the conclusion of the work.

Example 3.3–30: Linear supernetworks modeling Variation V



3.4 Igor Stravinsky: Movements for Piano and Orchestra, I

Stravinsky's growing interest in serialism during the 1950's culminated in his *Movements for Piano and Orchestra*, which he called "the cornerstone" of his later work.[19] The twelve-tone methods used to generate this work are rather complex, as supported by Stravinsky's statements: "I have discovered new...serial combinations in the *Movements for Piano and Orchestra*...and the *Movements* are the most advanced music from the point of view of construction of anything I have composed."[20] The idiosyncratic twelve-tone procedures employed in *Movements* make it difficult to do even a rudimentary twelve-count, much less account for the harmony and voice leading of the musical surface. After a brief overview of Stravinsky's twelve-tone methodology, I will point out a few particularly interesting structural properties of his precompositional material. In turn, I hope to illustrate how these precompositional structures shed some new light on the first movement of this work.[21]

Example 3.4–1 reproduces a portion of Stravinsky's precompositional row charts for *Movements*.[22] The row charts involve independent rotational permutations of the series' discrete hexachords (labeled α and β in his sketches), and their transposition to create twelve "verticals" (producing γ and δ hexachords). The rotations are labeled with Roman numerals I through V. The γ and δ hexachords are independent transpositions of these rotations in order to begin on the starting notes of the original unrotated α and β hexachords; in other words, all the γ hexachords are transposed to begin with Eb, and all

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[19] Igor Stravinsky and Robert Craft, *Themes and Episodes* (New York: Knopf, 1966): 23.

[20] Stravinsky and Craft, *Memories and Commentaries* (New York: Doubleday, 1960): 100.

[21] For a recent discussion of this topic with a different focus, see Douglas Rust's "Stravinsky's Twelve-Tone Loom: Composition and Precomposition in *Movements*," *Music Theory Spectrum* 16.1 (1994): 62–76.

[22] Joseph Straus, "Stravinsky's Evolving Serial Practice: A Sketch-Based Study," paper presented at the Society for Music Theory Annual Conference (Cincinnati, 1991), Example 10.

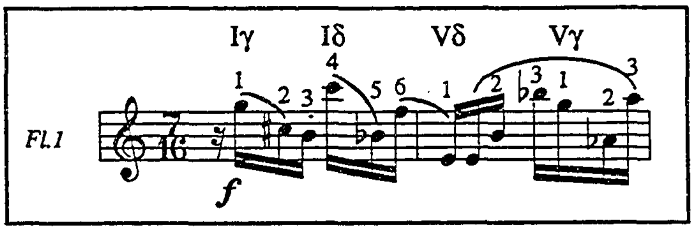
the δ hexachords are transposed to begin with C. Lastly, the diagonal lines trace the serial migration of each original pitch-class.

Example 3.4–1: Stravinsky's row charts

Though the precompositional function of the γ and δ hexachords is the creation of verticals, Stravinsky uses them primarily for melodic material in this movement. Substantial portions of the musical surface are patched together from fragments of these four hexachords and their related forms. This free use of precompositional material results in music related in only an oblique way to the original series. As Stravinsky boasted: "No theorist could determine the spelling of the note order in, for example, the flute solo near the beginning...simply by knowing the original order, no matter how unique the combinatorial properties of this particular series."[23] Example 3.4–2 illustrates this problem using the first two measures of the infamous flute solo.

[23] Stravinsky and Craft, *Memories and Commentaries*, 100.

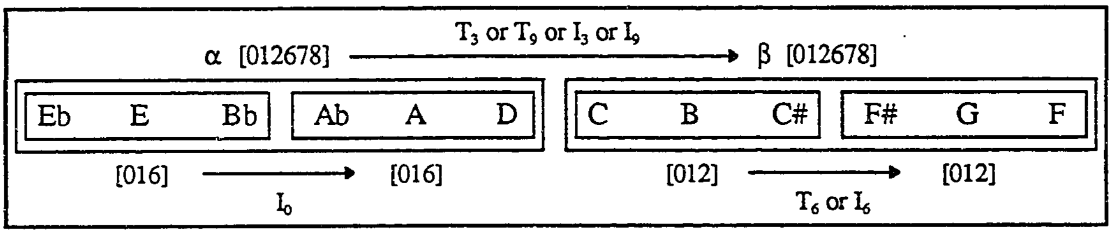
Example 3.4–2: Flute solo with Stravinsky's analytical markings, mm. 13–14



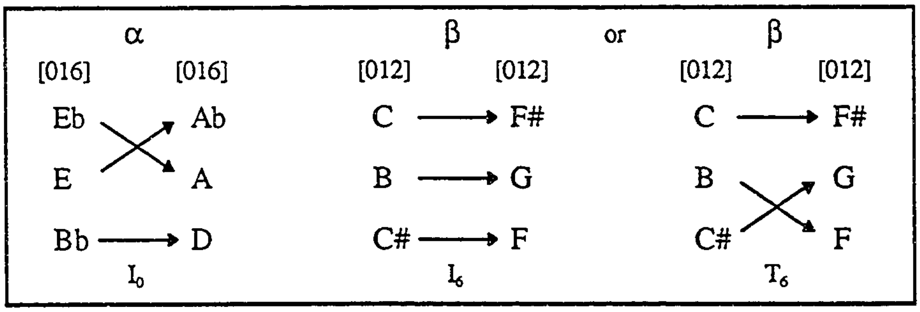
The Roman numerals I and V represent the first and fifth rotations of the γ and δ hexachords, while the Arabic numerals refer to serial position. Stravinsky uses two different rotations of each hexachord, as well as various sets of serial positions. Additionally, the fragments are jointly transposed at T_4 for reasons of instrumentation. Stravinsky's original sketch of the melody includes the Ab below middle C; the T_4 operation brings the melody up into the flute's range. Note that this series of hexachordal fragments does not create a twelve-tone aggregate, nor do any six adjacent pitch-classes form a hexachord of the same set class as the original hexachords 6–7 [012678]. In fact, the latter holds true throughout the entire flute solo.

Although Stravinsky was correct in his boast regarding the cryptic serial procedures in *Movements*, the tools to explore this work's musical surface are nonetheless found in the structure of the series itself. Example 3.4–3 demonstrates that each hexachord is derived from a different trichord: the first, α, from 3–5 [016] and the second, β, from 3–1 [012]. This example also lists the traditional operations transforming the first trichord of each pair into the second, as well as the four possible transformations between the two hexachords. Example 3.4–4 shows the mappings the traditional trichordal transformations yield, with the trichords arranged in serial order. Example 3.4–5 is the first presentation of the series in the movement (measures 1–2) and Example 3.4–6 rearranges the pitch classes of the serial example in the registral order of their first deployment in the work. A couple of the traditional operations satisfactorily capture different aspects of this progression. The I_0 transformation generating the α hexachord coincides with Stravinsky's use of register, while the I_6 transformation generating the β hexachord coincides with its serial order. The alternative T_6 transformation generating the β hexachord coincides with neither order nor register, but a case can be made for its significance using Klumpenhouwer networks. These networks also suggest transformations linking all four trichords, which by definition is not possible using traditional operations, thereby unifying this transformational interpretation of the row into a more cohesive whole.

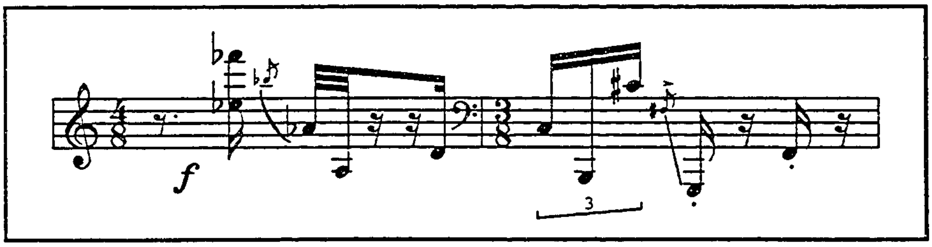
Example 3.4–3: Trichordal derivation of the original series



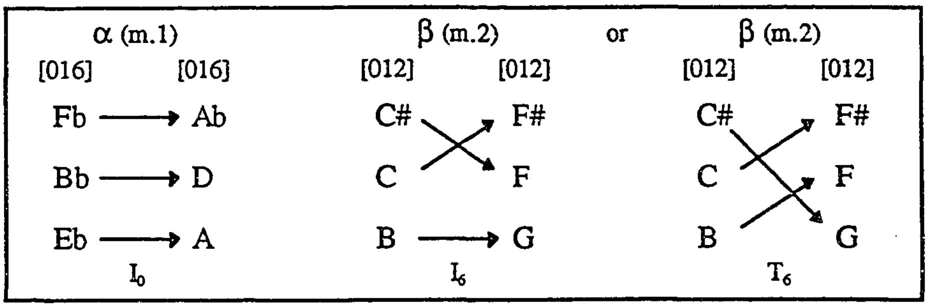
Example 3.4–4: Mappings arranged in serial order



Example 3.4–5: Stravinsky, Movements for Piano and Orchestra, I, mm. 1–2 (piano)

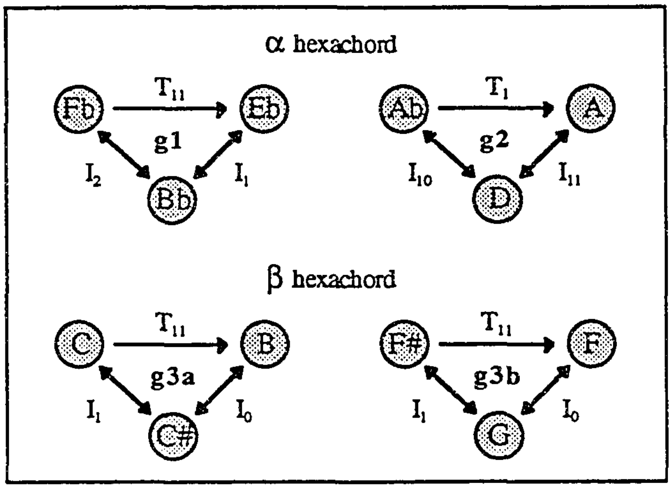


Example 3.4–6: Mappings arranged in registral order

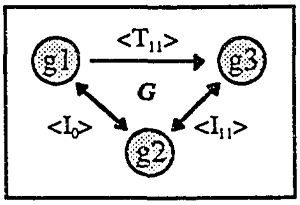


Example 3.4–7 models the α hexachord as the two trichordal networks g1 and g2, and the β hexachord as networks g3a and g3b. The transformation $<I_0>$ generating g2 from g1 yields the same mappings as the traditional operation I_0 ($I_0 = I_0/I_0 = I_0 + I_0 = <I_0>$), while the transformation $<T_0>$ generating g3b from g3a yields the same mappings as the traditional operation T_6 ($T_6 = T_6/T_6 = T_6 + T_6 = <T_0>$). The strong isography between networks g3a and g3b gives greater significance to the T_6 transformation than the mappings in previous examples. Furthermore, the shared interval-class 1 in these Klumpenhouwer networks suggest the $<I_{11}>$ transformation linking the two hexachords—not possible within the boundaries of traditional operations—through the adjacent [016] and [012] trichords. Example 3.4–8 presents recursive supernetwork *G* incorporating the complete row. This recursive structure is present whenever Stravinsky uses the row in its entirety, as in the opening of the work (refer back to Example 3.4–5), or at other critical junctures such as the first and second endings (measures 23–26 and 29–30 respectively), the start of the second large formal section (measures 31–33), and the conclusion of the movement (measure 42), to name a few.

Example 3.4–7: Klumpenhouwer networks modeling all four discrete trichords

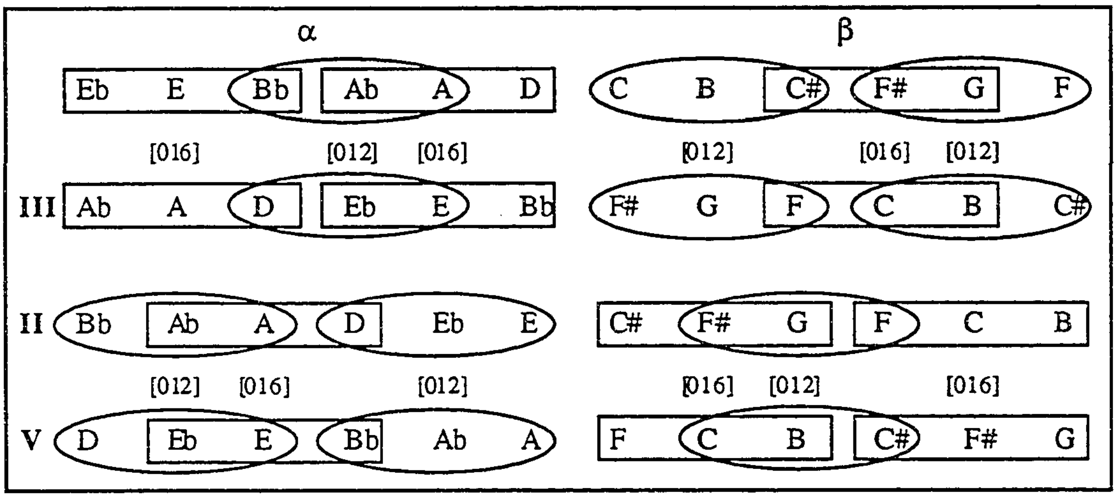


Example 3.4–8: Supernetwork G modeling the complete row



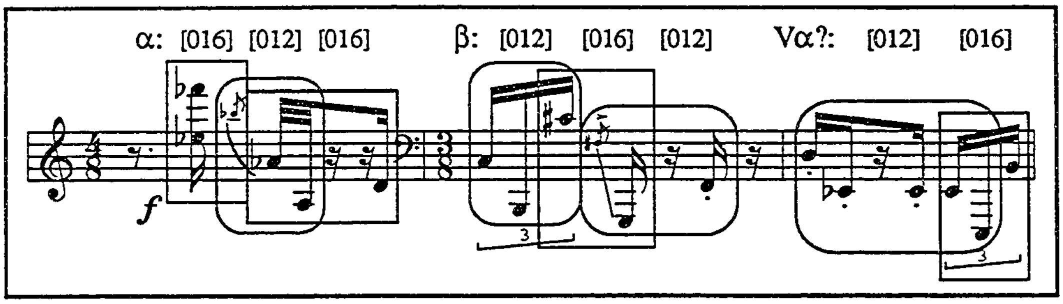
Further exploration of the rotating α and β hexachords reveals more complex patterns involving their imbricated trichords. Example 3.4–9 shows that each hexachord contains its complement's generating trichord—at the same serial location—in a manner that suggests progressive trichordal motion. More explicitly, α uses [012] to link the discrete [016] trichords, and β conversely uses [016] to link the discrete [012] trichords. Rectangles enclose the [016] trichords, while ellipses enclose the [012] trichords in the example. The prominence of these two generating trichords is further emphasized by Stravinsky's methods of rotational permutation. Example 3.4–9 also illustrates that when dealing with a derived hexachord, the third rotation merely reverses the location of the generating trichords, thus maintaining the derivation. Stravinsky's subtle structuring of the hexachords (particularly the specific placement of the two previously mentioned imbricated trichords) significantly allows Rotations II and V to be derived from the complementary hexachord's generating trichord, as also shown in this example. The serial position, <345>, of the intervening trichords remains the same for rotation III, and shifts to position <234> for rotations II and V. It is thus clear, owing to Stravinsky's precompositional choices, that the trichordal subsets of the series, particularly [016] and [012], are destined to be of the utmost importance on the musical surface.

Example 3.4–9: Hexachord rotation and imbricated trichords



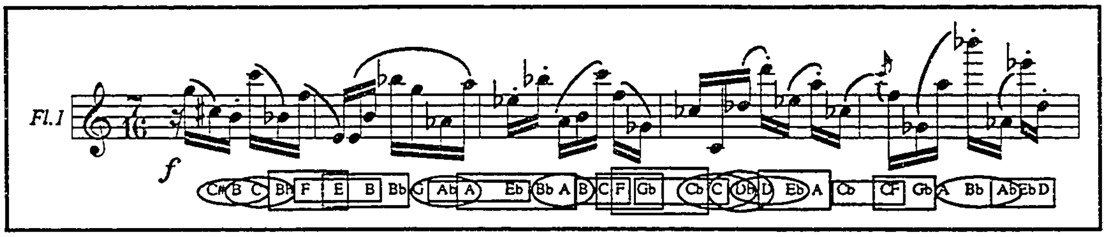
Indeed, much of the musical surface can be understood in terms of local progressions involving representatives of set-classes [016] and [012]. Example 3.4–10, like Example 3.4–5, presents the opening of the movement, but now with the addition of measure 3. The interlocking progression of trichords highlighted in the example will be present each time the complete hexachords are sounded, which amounts to a total duration of approximately half the movement. More significantly, even when Stravinsky uses fragments of these hexachords, chains of the two generating trichords are present. As shown in the example, the first four pitch classes of Vα sound in measure 3; the hexachord is never completed, but the overlapping trichords are still present. This progressive structure is a significant unifying factor in the fragmentary sections of the work, such as the previously mentioned flute solo.

Example 3.4–10: Stravinsky, Movements, I, mm. 1–3 (piano)



Example 3.4–11 reproduces the entire flute melody spanning measures 13–17. Ignoring the initial G (to which I will return later), the solo unfolds an overlapping series of these two primary trichords. In fact, as shown in the example, there is not a single note that is not involved in such a progression, despite the total absence of the original hexachord. Once again, ellipses enclose [012] trichords, while rectangles enclose [016] trichords. In addition, transpositions of this line create equivalent series of trichords in the piano part in measures 7–10 and 18–21.[24]

Example 3.4–11: Stravinsky, Movements, I, mm. 13–17 (flute solo)

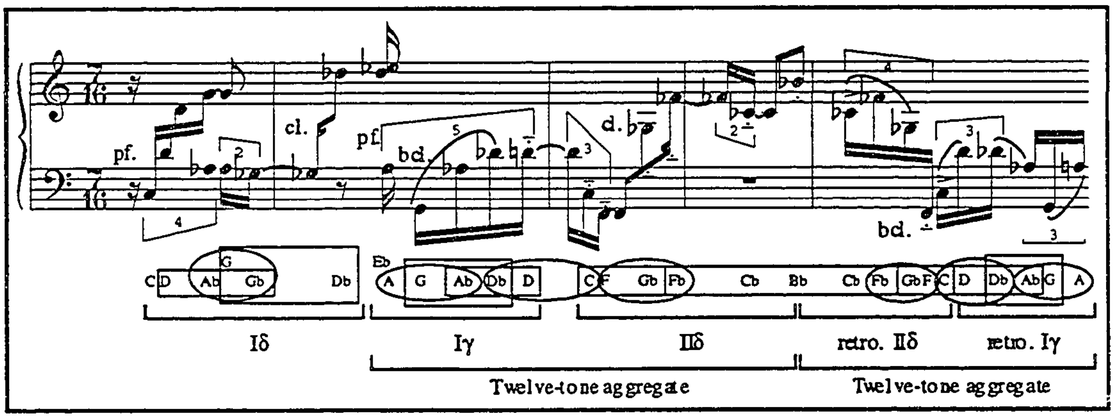


The flute's accompaniment has quite different, and slightly less obscure, precompositional origins than the solo. Stravinsky uses complete hexachord forms as follows: Iδ, Iγ, IIδ, retrograde IIδ, retrograde Iγ. Example 3.4–12 reduces the piano and clarinet accompaniment to two staves, and highlights the serial structure. Once again

[24] These transpositional relationships are discussed in greater detail in Martin Boykan, "Neoclassicism in Late Stravinsky," *Perspectives of New Music* 1.2 (1963): 155–169; William Walden, "Stravinsky's *Movements for Piano and Orchestra*: The Relationship of Formal Structure, Serial Technique, and Orchestration," *Journal of the Canadian Association of University Schools of Music* 9.2 (1979): 73–95; and Rust, "Stravinsky's Twelve-Tone Loom."

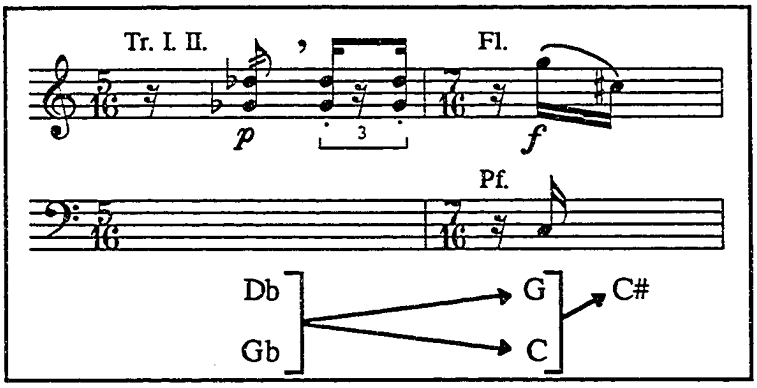
ignoring the initiating note, C in this case, two series of these trichordal progressions unfold, broken by the Eb initiating the Iγ hexachord (the only other pitch-class not encased in a grouping). The extensive second series, created by the two aggregate-forming hexachords Iγ/IIδ and their subsequent retrograde, includes trichordal links between the discrete hexachords which are impossible in the original series. As shown earlier in Example 3.4–10, the [016] and [012] trichords do not overlap the hexachordal boundaries.

Example 3.4–12: Stravinsky, Movements, I, mm. 13–17 (accompaniment)



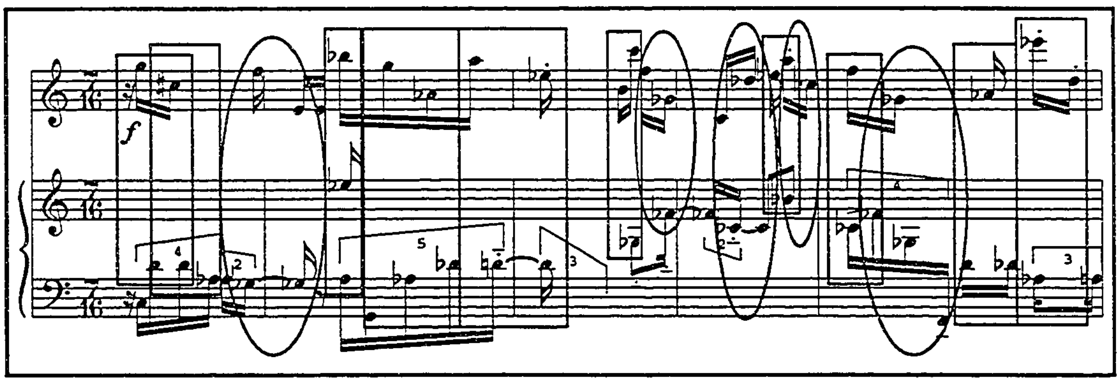
A trumpet fifth, {Gb, Db} in measure 12, punctuates the end of the previous piano solo, and signals the start of the flute solo. As [016] pervades the musical surface, this isolated trumpet gesture—which provides two notes of a [016] trichord—creates the expectation that either pitch-classes G or C will complete the trichord. As shown in Example 3.4–13, these two pitch-classes are in fact the initiating notes of the flute solo and its accompaniment, which immediately follow the trumpet gesture in measure 13. In this manner each of the starting notes provides a link to the preceding material by completing a previous chain of [012] and [016] trichords. As also shown in Example 3.4–13, the initiating {C, G} fifth between the piano and the flute may in turn set up the expectation for the flute's second pitch, C#.

Example 3.4–13: Stravinsky, Movements, I, mm. 12–13 (trumpet gesture)



Several additional manifestations of [012] and [016] trichords are heard in the interplay between the flute solo and its accompaniment. Some of these are illustrated in Example 3.4–14, including the Eb accompaniment note excluded from the linear progression back in Example 3.4–12. A few non-participatory notes are omitted in order to maintain at least a small degree of clarity. This example demonstrates that the referential trichords are not purely horizontal phenomena. With the inclusion of the C and G initiating notes, and the aforementioned Eb, every single pitch event of measures 13–17 participates in some form of this harmonic progression, be it horizontally, vertically, or in a combination of the two dimensions.

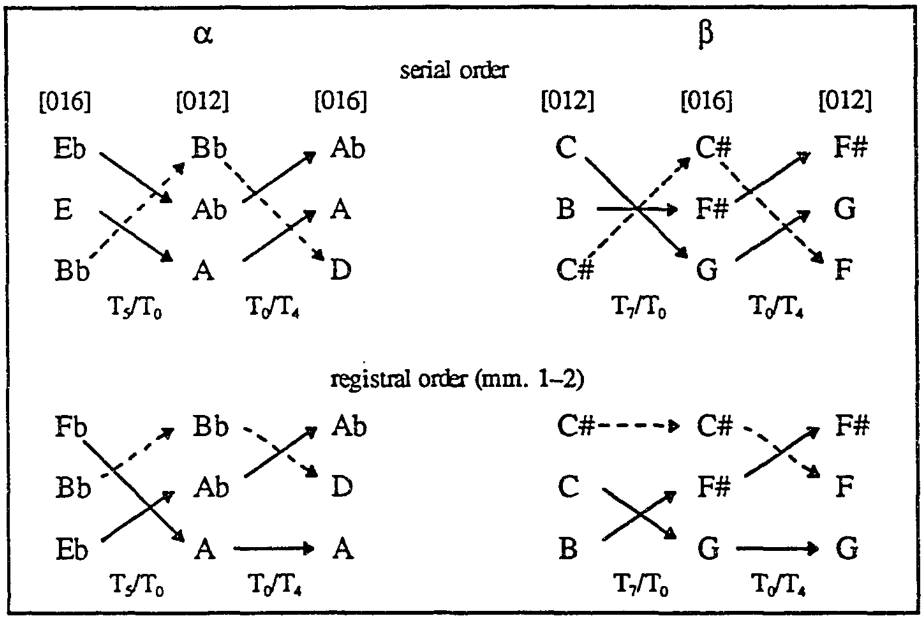
Example 3.4–14: Stravinsky, Movements, I, mm. 13–17 (excerpt)



Thus far my examples merely point out the substantial presence of the two trichords in question (sometimes owing to, and sometimes despite the absence of, the original row).

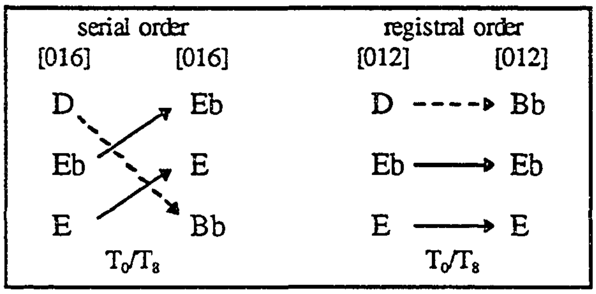
Now I would like to examine some significant voice leading motions between these two prevalent sets. Example 3.4–15 outlines the progressive trichordal motion within each hexachord as a series of dual transpositions. It is possible to interpret α as two dual transformations $\langle I_0/T_0, T_0/I_0 \rangle$ with the resulting mappings concurring with the traditional I_0 operation of the row supernetwork. The reasons I do not choose this path are: first, the progressive structure of the β hexachord does not agree with the T_6 operation of the supernetwork; and secondly, the use of pure dual transpositions is more compatible with the extremely surface-level approach suggested by the imbricated trichords. Example 3.4–15 includes the trichords arranged in both serial and registral order for comparison with previous examples. The voice-leading model is best described as a series of dual transpositions in which the values are most often: n or n + x = 0 and x = ±4 or ±5. The bulk of harmonic activity in the first movement can be described by the dual transpositions resulting from these values (for example, T_5/T_0, T_0/T_4, T_7/T_0, and T_0/T_8).

Example 3.4–15: Dual transpositions mapping the progressive trichords



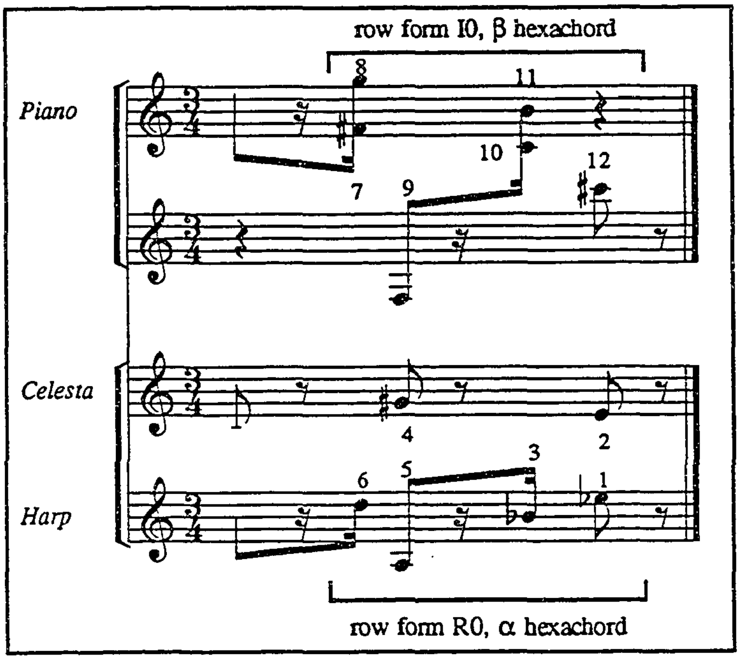
The initial series—as shown in Example 3.4–15—includes three of the four significant x-values mentioned above: ±5, and +4, generating transformations T_5/T_0, T_7/T_0, and T_0/T_4. Example 3.4–16 presents the other frequently occurring x-value, x = -4, in this case specifically the transformation T_0/T_8. Although this transformation is not included in the original series, these particular mappings are drawn from measure 3, previously shown in Example 3.4–10.

Example 3.4–16: T_0/T_8 mapping (x = -4), m. 3

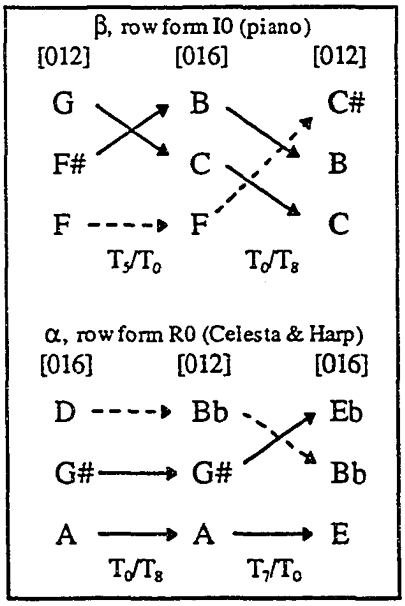


Example 3.4–17 presents the first movement's final cadence, which rhythmically articulates these common-tone progressions. The piano performs the β hexachord, completing a statement of the row-form I0, while the harp and celeste perform the α hexachord, completing the row-form R0. The dyad plus singleton deployment of these instrumental pairs highlights the common-tone voice leading of the trichordal progressions. As shown in Example 3.4–18 below the musical example, the dyads map onto the dyads around the singletons, and, conversely, the singletons map onto the singletons around the dyads. The dyads are related at -5 in the piano hexachord and at +5 in the celeste/harp hexachord, while the singletons in both hexachords are related at -4.

Example 3.4–17: Stravinsky, Movements, I, final cadence, m. 42c



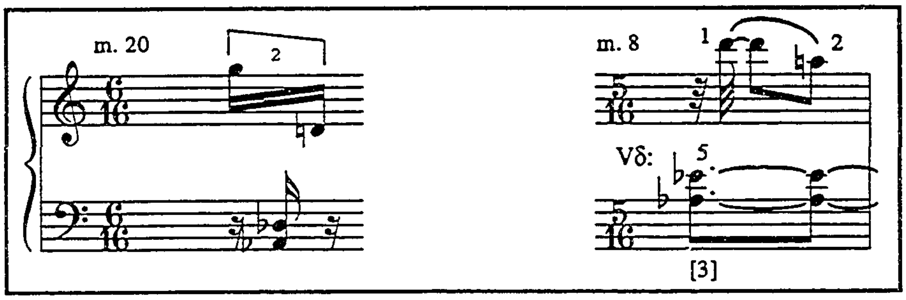
Example 3.4–18: Mappings for Example 3.4–17



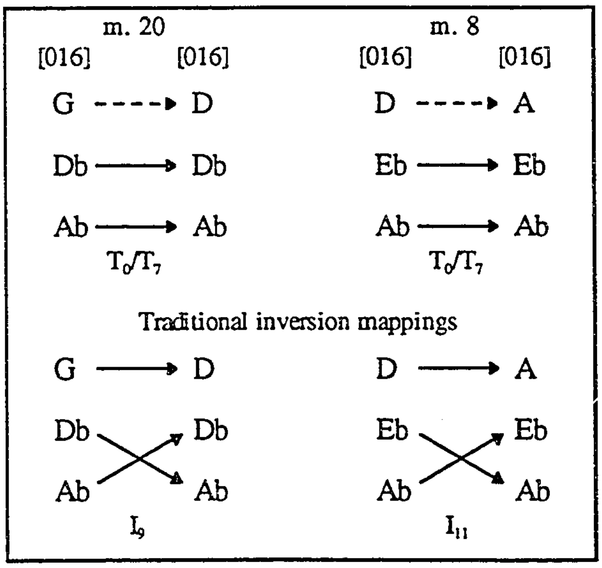
In Example 3.4–19 similar harmonic motion is accomplished by a single melodic voice moving against a fixed dyad. In both brief examples the piano performs a 4–9 [0167] tetrachord in which a fixed dyad participates in a progression of two [016] trichords.

Significantly, the x-value is again -5, yielding the transformation T_0/T_7 shown in Example 3.4–20 below the example. The more traditional I_n operations between these trichords (I_9 and I_{11}) involve flipping the dyads onto themselves, as shown in the bottom mappings. The T_0/T_7 transformation is not only more consistent with my voice-leading paradigm, but it also better reflects the melody versus accompaniment texture created by the fixed dyads.

Example 3.4–19: Stravinsky, Movements, I, mm. 20 and 8 (piano)



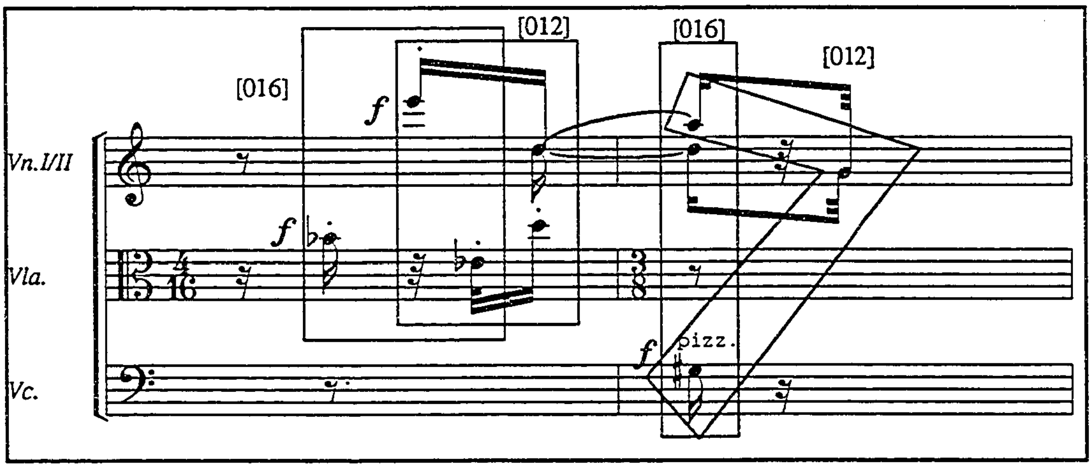
Example 3.4–20: Mappings for Example 3.4–19



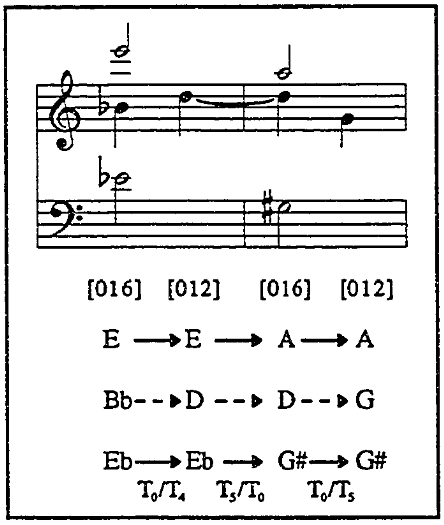
Example 3.4–21, and its reduction Example 3.4–22, present a registrally explicit instance of these harmonic progressions; Example 3.4–22 also summarizes the specific mappings. In this string outburst of measures 5–6, trichords 1 and 3 are pitch-class identical to the discrete trichords of the original hexachord α, but the mediating trichord is

different owing to rotation and retrograde operations. The hexachord in this case is the retrograde of IIIα. In spite of this change, the important intervallic motions remain consistent. The tied D in the second violins highlights the common tone between trichords 2 and 3, and the progression is further emphasized by the [016] simultaneity on the downbeat of measure 6. As shown in these examples, the progression continues beyond the hexachordal boundaries to include the violins' G, which initiates a statement of the β hexachord.

Example 3.4–21: Stravinsky, Movements, I, mm. 5–6 (strings)



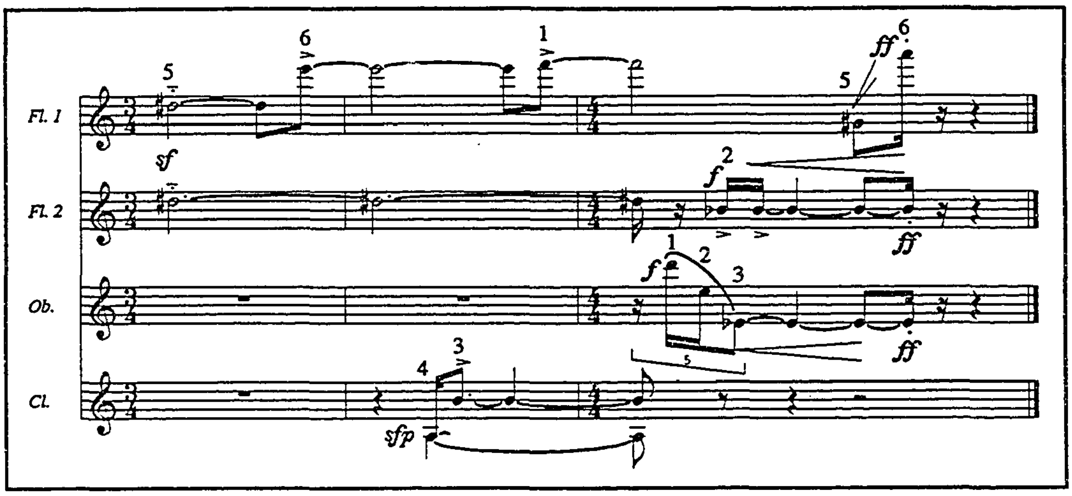
Example 3.4–22: Reduction of Example 3.4–21



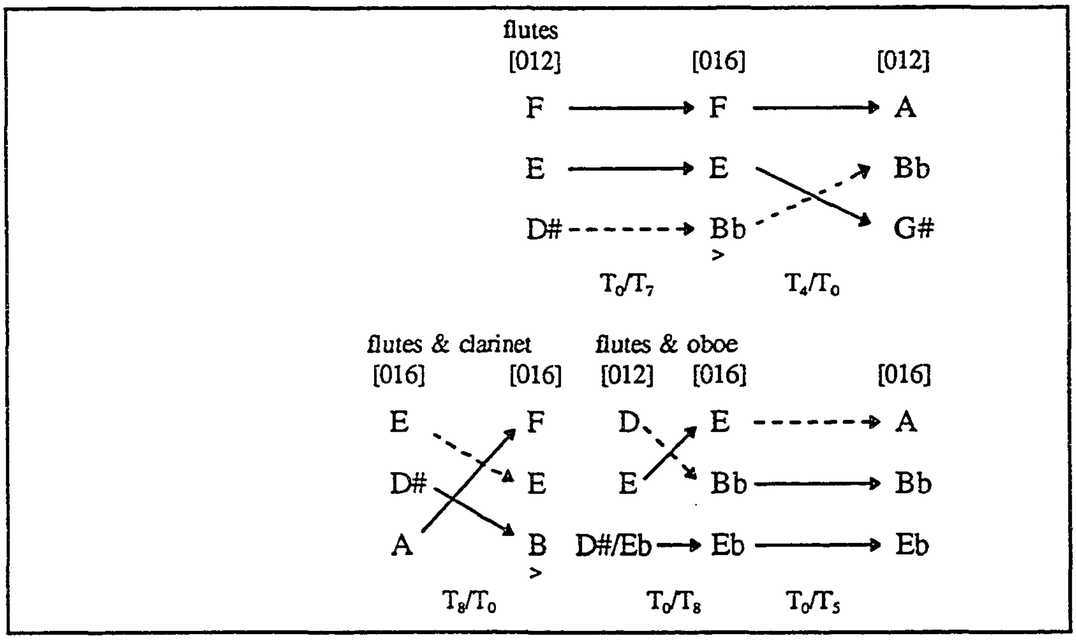
A more intricate manifestation of these trichords is heard in the first interlude or postlude, measures 43–45, shown in Example 3.4–23. Consistent with one's expectations by this point, the serial structure of this passage is very intricate. The RI9 form of the α hexachord generates the first two measures of the example plus the second flute's Bb in the third measure. As one can deduce from the Arabic numerals (which are Stravinsky's own serial markings), in addition to the retrograde-inversion and T_9 operations, the hexachord is reordered <5, 6, 4, 3, 1, 2>. The R9 form of the β hexachord, in serial order, generates the remaining notes in the oboe and flutes. Notably, the T_9 operation renders this β hexachord pitch-class identical to the original α hexachord. Furthermore, these two hexachord forms do not create a twelve-tone aggregate, but instead they share four common tones.

The mappings, shown in Example 3.4–24, focus on instrumental timbres to highlight the rich web of dual transpositions embedded in this cadential progression. The flutes' two [012] trichords—the first created by <5, 6, 1> of α and the second by <4, 5, 6> of β—are linked by a [016] trichord which is emphasized by similar articulation (indicated by the accent under the chord), as well as temporal order. Note that the Bb common to both hexachords is the link between trichords 2 and 3. The progression incorporating the clarinets and flutes takes place entirely within the α hexachord, but similarly uses articulation to associate the members of the second trichord. The two mappings could be integrated by branching the mappings from the first trichord of the flute/clarinet progression to both its second trichord and the first trichord of the flute progression, as the two mappings occur simultaneously. Note that the A maps onto the F in the flute/clarinet mappings, and the F in turn maps back onto the A in the flutes, thereby relating the two representatives of the common-tone A.

Example 3.4–23: Stravinsky, Movements, I, Interlude 1, mm. 43–46



Example 3.4–24: Mappings for Example 3.4–23

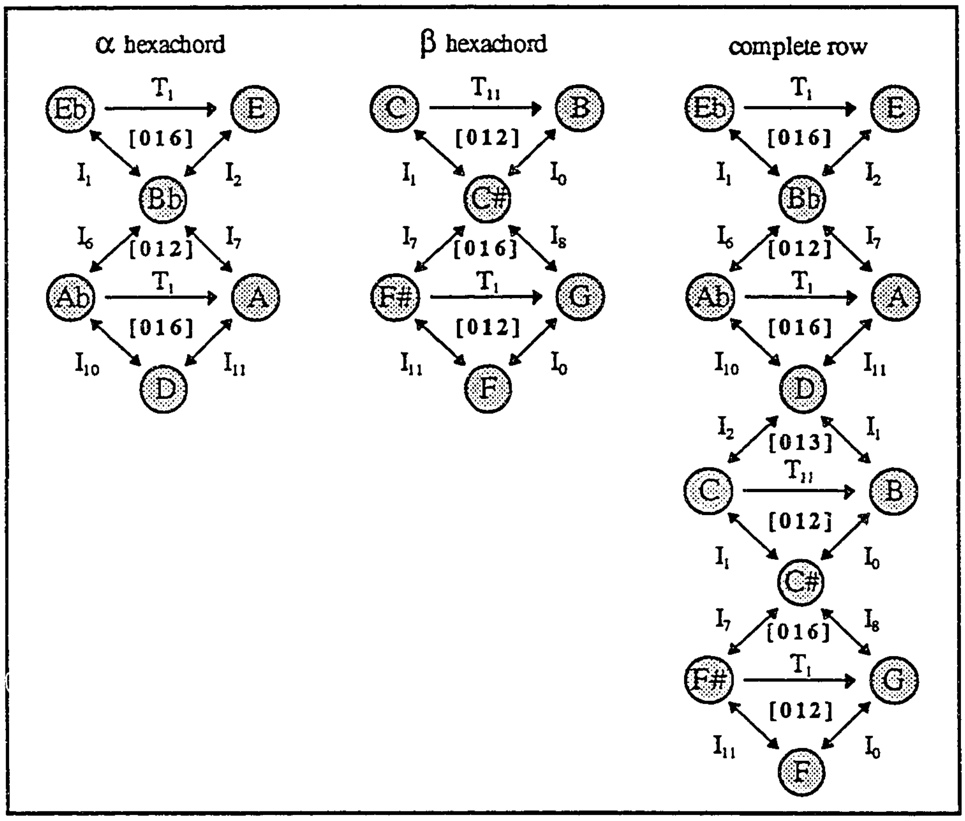


The third set of mappings in Example 3.4–24 interpret the final cadence in the flutes and oboe along similar lines. The D#/Eb shared by both hexachords is one of the links between all three trichords, while the common-tone Bb is again highlighted between trichords 2 and 3. The path of the remaining common-tone E points out the latent potential for many-to-one and one-to-many mappings inherent in this transformation. In this context

E maps onto A, as F had previously (therefore many-to-one), and E had also previously mapped onto G# (therefore one-to-many). Significantly, these three examples include all four important voice-leading x-values, ±4 and ±5 (yielding the four transformations T_8/T_0, T_4/T_0, T_0/T_5, and T_0/T_7), making this example a good musical summary of the movement's transformational processes.

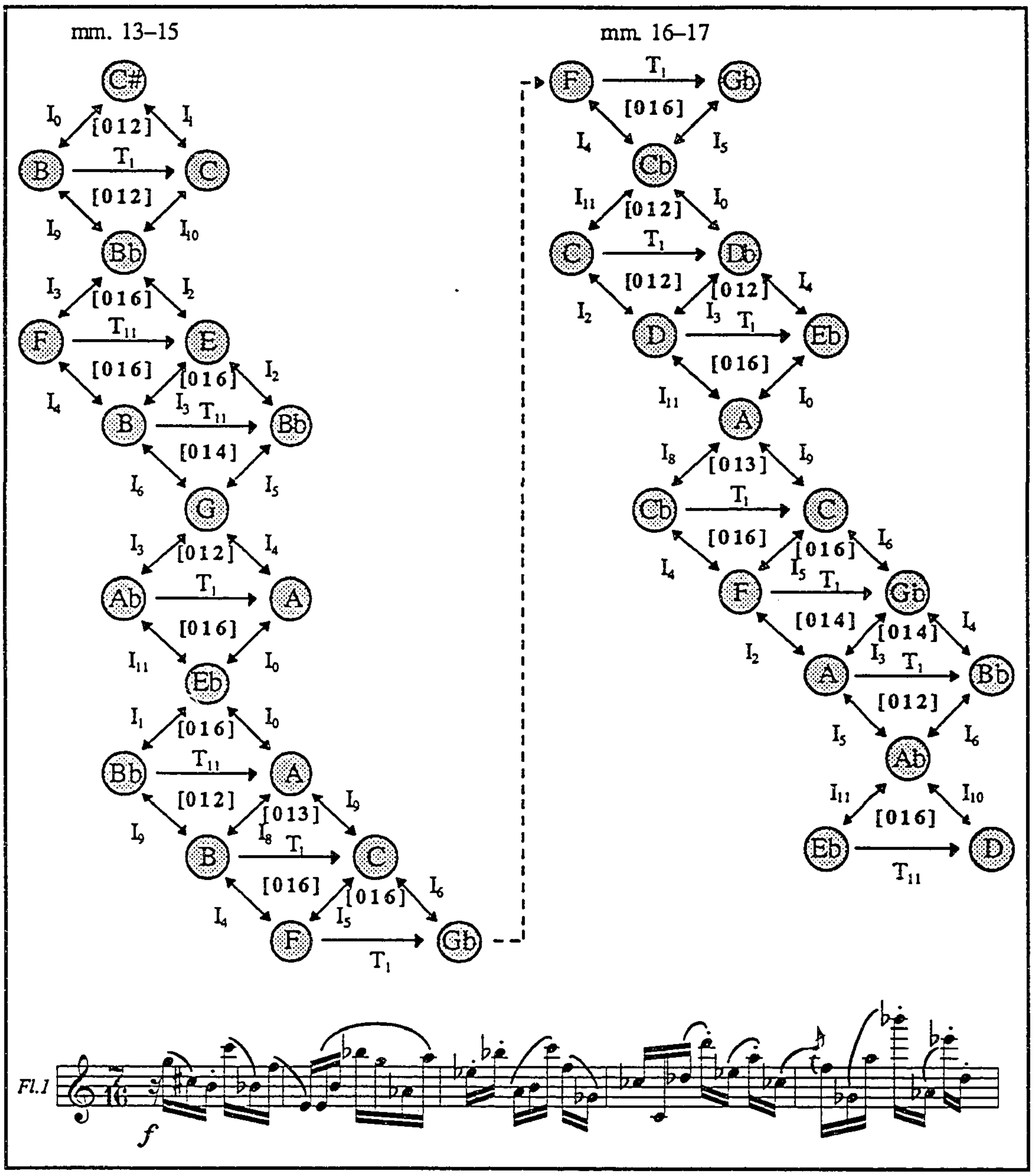
Almost every pitch-event in the first of Stravinsky's *Movements* partakes in some form of this trichordal progression, either vertically or horizontally, or in both dimensions. Horizontally these progressions express and emphasize properties built into the original hexachords, whether or not they are explicitly present. Vertically they range in explicitness from the registral and serial clarity of Example 3.4–21, to the rich timbral complexity of Example 3.4–23. These examples suggest a musical surface that is much more aurally unified than the fragmented serial processes and/or traditional twelve-tone analytical methods imply. Taking this interpretation one step further, it is possible to imagine the imbricated trichordal progressions as extensive Klumpenhouwer chains. Example 3.4–25 illustrates such an interpretation. The chaining in the α and β hexachords does not produce positively or negatively isographic networks, but rather a hybrid with the top portion of the networks negatively isographic and the bottom two thirds positively isographic. Nonetheless, the structural similarities between the two hexachordal networks are self evident and it is the chaining itself that becomes motivic. The far right of the example links the two hexachords, via a 3–2 [013] trichord, in a longer chain encompassing the complete row. While any series of imbricated trichords could generate Klumpenhouwer chains, the significance in this case is the consistency of interval-class 1 as the shared dyads, as well as the dominance of the referential trichords [016] and [012].

Example 3.4–25: Klumpenhouwer chains modeling the original series



These Klumpenhouwer chains are not only significant in all portions of the movement incorporating the literal hexachords, but also in the more serially fragmented sections of the work such as the flute solo. Example 3.4–26 interprets the flute solo as one of these Klumpenhouwer chains. I urge the reader to keep in mind that the hexachord is completely absent from this melody, yet the chain is composed of the same building blocks as the hexachordal and twelve-tone chains. I include the solo again (previously shown in Example 3.4–11) below the chain for convenient reference. It is possible to interpret all of the dual transformations in the previous examples as similar Klumpenhouwer chains. In fact, a model might be constructed to encompass all of these chains and their interactions, both horizontal and vertical, though such a model is beyond the scope of this dissertation as it would necessitate a three-dimensional medium.

Example 3.4–26: Klumpenhouwer chains modeling the flute solo



Postlude

In this dissertation I introduced, explained, expanded, and demonstrated by analytical example a transformational model of atonal voice leading. The most critical component of this model is the concept of dual transformations: T_n/T_{n+x}, I_n/I_{n+x}, W_n/W_{n+x}, and their hybrid counterparts. Inspired by the transformational work of Forte, Straus, Lewin, and Klumpenhouwer, dual transformations generalize and expand upon the increasingly common partitional approach to pitch-class set mappings. Isographic and recursive Klumpenhouwer networks are the other essential ingredients in my model, and I hope that my development of Klumpenhouwer classes, chains, *double-emploi*, and $<W_n>$ will encourage other musicians to further apply and refine this already elegant graphic technique. It is my incorporation of these two, often contradictory, models—dual transformations and Klumpenhouwer networks—into pseudo-levels that is the most original, and perhaps most problematic, aspect of my methodology. The different levels represent voice-leading interpretations at various distances from the concrete musical surface, but I am suggesting peaceful coexistence rather than hierarchical subsumption among them.

As the case in all scholarly endeavors, I consider the development of this voice-leading model a work-in-progress despite reaching the end of this dissertation. The two most fascinating topics that I would like to further develop are three-dimensional networks and real-time transformations. Three-dimensional networks, both physical and computer-generated, could more explicitly capture the molecular metaphor. The potential applications of three-dimensional models range from ball-and-stick kits for the theory classroom, to computer-rendered snapshots of possible network structures in highly polyphonic music, for example, the DNA-like chains shown at the end of my analysis of Stravinsky's *Movements for Piano and Orchestra*. Multimedia computer applications could also be put to work generating real-time transformations. The transformational approach shifts analytical emphasis from static relationships to dynamic processes, thereby making the temporal dimension of music increasingly important. Listening to a performance while viewing a real-time graphic interpretation of its transformational processes could potentially reveal some interesting features of a musical work. Obviously, the success rate of such a technique would vary greatly depending upon the particular work in question; for example, Webern's Op. 5, No. 2, might be much more suited to a real-time analysis than Op. 5, No. 3, for reasons of tempo alone.

It is my intention that the nine analytical essays comprising chapters 2 and 3 of this study serve two fundamental purposes. First, they are meant to demonstrate the analytical viability of my transformational voice-leading model in the context of a diverse repertoire of non-tonal twentieth-century music. Second, and more importantly, I hope that they offer new insights into each of the individual musical works. In the "Prelude" to this dissertation I suggested that my analytical work seeks a balance between the extremes on two related, but non-identical, continua: exclusivity⇔promiscuity and concrete⇔abstract. I leave it to the reader to determine my degree of success in meeting this objective. In closing, I hope that I have offered at least a plausible twentieth-century interpretation of Zarlino's intelligible passage from one sound to the next.

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