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# An empirical study of a simultaneous-equation approach to models of covariance


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# AN EMPIRICAL STUDY OF A SIMULTANEOUS-EQUATION APPROACH TO MODELS OF COVARIANCE

by

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Abstract

AN EMPIRICAL STUDY OF
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In portfolio analysis, since the number of covariance grows rapidly as the number of securities under consideration increases, some models of covariance have been proposed to reduce the required amount of covariance input. In this study, another type of model of covariance is proposed using a simultaneous-equation approach to improve the prediction of future covariances or correlation coefficients between security returns.

Simultaneous-equation approach is the most fundamental way to incorporate industry interaction in a model of covariance. Three forms of models of covariance with industry interaction were experimented with in this study: a stock index-based Multi-Index Econometric Model, an economic factor-oriented Multi-Factor Input-Output Model and an index/factor-mixed Market Model with Input-Output Analysis. These three covariance

models with industry interaction were compared to other models of covariance with respect to their ability to estimate the dependence structure of security returns.

The evaluation of the dependence structure of security returns predicted by alternative models of covariance is based on MSE criterion and its decomposition. The Theil Inequality Coefficient measures whether a model of covariance can predict better than simple historical extrapolation.

It is found that as long as the choice of underlying factors in a security return-generating equation is reasonably good, more complicated models of covariance typically performed better than simpler models in tracking the historical dependence structure of security returns. However, without the Bayesian coefficient adjustment, simpler models of covariance outperformed more complicated models in predicting the future correlation structure of security returns. But the Bayesian approach enables the Multi-Index Econometric Models to surpass all the other models of covariance in predicting the future correlation structure of security returns in the more stable, second forecasting period.

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While the afore-mentioned individuals have aided in the writing of this dissertation, all opinions and any errors contained herein are my own responsibility.

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# CHAPTER I

# INTRODUCTION

Since Markowitz introduced portfolio theory in 1952, researchers have devoted an enormous effort to simplify his original full covariance model in order to reduce the input data required in portfolio analysis. The input reduction could lead to more practical application of portfolio theory. In fact, Markowitz realized this and proposed the first model of covariance in his 1959 book, Portfolio Selection: Efficient Diversification of Investments.[1] Since then, several models of covariance such as the single-index model, the multi-index model and even the average or constant model, have been proposed to extend his original idea as applied to the portfolio selection problem.

The single-index model assumes that security returns are correlated only through one common effect called the market index. However, in addition to the market index, other factors such as industry effects may also play an important role in the security return-generating process. One version of the multi-index model of covariance incorporates both market and industry effects into the security equation for the purpose of improving the prediction of the dependence structure of security returns. Of course, the more complicated multi-index model increases the input requirements of the portfolio selection problem.

Unfortunately, in the literature the multi-index model has been claimed to be inferior to the single-index model in predicting future correlation structure of security returns. Because the more complicated models of covariance fail to reach their goal in terms of the trade-off between simplification and predicting power, it leads researchers to propose even simpler models. The so called constant or average model assumes a common mean correlation coefficient in the whole group or subgroups of securities. The average models have been demonstrated to outperform other more complicated models of covariance in predicting future correlation structure of share prices.

However, the importance of industry interrelationship has long been recognized in the literature. The interaction between industry returns may result from the interdependence among various industries and the unclear classification of industry groups. It is possible that the underperformance of more complicated multi-index model may be due to the failure of recognizing the importance of industry interrelationship.

The purpose of this study is to recognize the existence of industry interrelationship and solve the problem with a simultaneous-equation approach. Through a system of simultaneous equations, the effect of industry interaction is entered directly into the return-generating function in the hope of increasing the predicting power of the model of covariance. In essence, the proposed models of covariance with industry interaction are an extension of the simpler multi-index model.

The simultaneous equation system in the model of covariance will also provide a prediction on the industry index or output. As a result, the industry covariance can be determined endogenously in the model. Three forms of covariance models with industry interaction will be examined in this study: a stock index-based model (the Multi-Index Econometric Model), an economic factor- based model (the Multi-Factor Input-Output Model) and an index/factor-mixed model (the Market Model with Input-Output Analysis).

Among the proposed covariance models with industry interaction, the Multi-Index Econometric Model differs from the covariance form of multi-index model introduced by Cohen and Pogue (1967). Instead of recognizing industry interrelationship in the security return-generating equations, the covariance form of multi-index model only considers the industry covariance in estimating the covariance between security returns. The industry covariance is exogenously determined in the covariance form of the multi-index model. The Multi-Index Econometric Model also differs from the diagonal form of the multi-index model used in the Cohen & Pogue's study. Although the diagonal form of the multi-index model predicts the industry index from the market index in industry return- generating function, it fails to recognize the interaction among industry indices in calculating the covariance between security returns.

In fact, it can be shown that the diagonal form of Cohen & Pogue's multi-index model is a special case of the general multi-index model without industry interaction.[2]

In the proposed multi-index model with industry interaction, not only are the industry interrelationships recognized, but the industry covariance is also generated from the industry prediction. The industry block in the multi-index model with industry interaction is a simultaneous-equation econometric model with industry indices as dependent variables and the market index as a predetermined variable. It is used to generate industry returns which, in turn, appear in the security return-generating equations. The only exogenous variable in the model is the market index. Figure 1 shows the relationship between the industry and the security blocks in the whole equation system. It also displays the detailed procedure of estimation for the models of covariance with industry interaction.

A by-product of the multi-index model with industry interaction is the prediction of the industry index from the simultaneous-equation econometric model. The prediction of the industry index has become more important since the AMEX introduced industry index options in April 1983. An industry index is a weighted index of security prices, therefore it should be determined simultaneously in the model, instead of being taken exogenously. A simultaneous-equation econometric industry model will be built for the purpose of providing a better prediction of industry indices.

For a long time, stock price indices have been used to capture different effects in the literature of covariance model. In this study, the potentials of economic factors in estimating the covariance structure of security returns will be explored. In fact, Markowitz(1959) has long proposed a single underlying factor, such as the gross national product, in the model of covariance.

In addition, the lack of theoretical foundation in the multi-index model with industry interaction gives incentive to search for an economic theory which incorporates the interindustry analysis in the economic factor-based model of covariance. The Input-Output Model developed by Leontief in the late 1930's is helpful in this aspect. The fundamental purpose of input-output framework is to analyze the interdependence of industries in a nation's economy. It projects industry outputs needed to fulfill the final demand from consumers in different industries. The projection is widely used in strategic production planning. As far as the covariance model is concerned, the Leontief input-output analysis provides a theoretical framework to forecast industry covariance. It will serve as the industry model under the Multi-Factor Input-Output Model of covariance. The input-output analysis can also be combined with the traditional market model to form a model of covariance called the Market Model with Input-Output Analysis. It is hoped that the covariance models with industry interaction will offer a better prediction of security returns and outperform other simpler models of covariance in estimating the dependence structure of security returns.

The evaluation of the covariance models with industry interaction could be based on popular criteria such as the mean squared forecasting error. The dependence structure of security returns can be represented in terms of covariance or correlation coefficient between these securities' returns. The performance of different models of covariance evaluated in different measures may yield different ranking. Nevertheless, both measures have their own usefulness. While covariance is a direct input in the problem of standard portfolio selection, correlation coefficient is widely used and easy to interpret. The evaluation based on both measures may suggest possible reasons of different ranking among alternative models of covariance. It also provides a consistent basis for the comparison with previous studies.

Chapter II reviews relevant studies in the literature. A review of several covariance models is presented in Chapter III. Chapter IV provides some empirical evidence about industry interrelationship. Based on the importance of industry interrelationship, three covariance models with industry interaction are constructed in Chapter V. Chapter VI proposes the methodology for empirical evaluation to be applied in this study. It explains the construction of the data bank and empirical procedure. The criteria for evaluating alternative covariance models are also described in detail in this chapter. Chapter VII is used to describe the construction of the industry models used in the models of covariance with industry interaction. In addition, it provides the prediction of industry and security returns based on the return-generating equations in different models of covariance. Evaluation of the prediction is also made in this chapter to see whether there is any connection between the prediction of industry and security returns and the prediction of the dependence structure of security returns among alternative models of covariance. The major performance evaluation of different models of covariance is done in Chapter VIII. It evaluates their prediction of the dependence structure of security returns on the basis of both covariance and correlation coefficient in both the estimation and the forecasting periods. The final chapter concludes this study and gives some suggestions about future research.

# CHAPTER II

# LITERATURE REVIEW

The first model of covariance was proposed by Markowitz in his 1959 book for the purpose of simplifying the application of the portfolio selection problem. Sharpe (1963) extended the idea with the diagonal model and applied it to the security selection procedure. In essence, the diagonal model is a single-index model in which security returns are assumed to be correlated only through one common factor such as the market index.

The assumption of only a single index in the relationship of security returns was empirically tested by King(1966). He demonstrated the significance of industry index in addition to the market index in stock price behavior by factor analytic technique. Rosenberg(1974) also showed that some extra-market components exist in the covariance of security returns. The introduction of more indices, such as the industry index, into a single-index model forms a multi-index model. The multi-index model represents a more complex covariance model in the process of simplifying portfolio analysis. The issue is whether the complication provides a better estimation of the covariance structure or not.

Cohen and Pogue(1969) studied the problem by empirically evaluating alternative portfolio-selection models. They also attempted to recognize the industry interrelationship in portfolio analysis in terms of the "covariance form" of the multi-index model. Conversely, the "diagonal form" of the multi-index model represents a simple extension of single-index model and assumes away the industry covariance in forming covariance between security returns. Although they showed that multi-index models may perform better than a single-index model over some restricted ranges, they concluded that a single-index model is superior to multi-index models in reproducing the true correlation matrix and generating the efficient frontier.

The problem of industry interrelationship associated with the traditional industry group, used in Cohen and Pogue's study in the multi-index model, was investigated by Elton and Gruber(1973). They constructed orthogonal indices using principal components analysis and employed these indices in the multi-index model. In distinguishing the explanatory power from the predicting power among various covariance models, they found that a multi-index model did a better job of reproducing the historical correlation matrix but did not perform better than the single-index model in predicting the future correlation structure of stock prices. In addition, Elton and Gruber also compared the index model to the average model. The average model assumed that all correlation coefficients equal a common mean correlation. The major conclusion in their study was that the mean model outperformed the index model in estimating the correlation structure by both statistical and economic criteria. The conclusion implied that the more complicated model only picks up more random disturbance. This might surprise some researchers.

Based on the above results, Elton, Gruber and Padberg(1976) and Elton and Gruber(1987) designed some simple criteria for optimal portfolio selection using the average correlation and the average covariance assumptions of the covariance models. In other words, both covariance and correlation coefficient are applicable to the portfolio selection problem. Elton, Gruber and Urich(1978) extended the Elton and Gruber(1973) study by adjusting parameter estimates in the model of covariance. Both Bayesian and Blume approaches to beta adjustment were employed to improve the estimation of the correlation structure of security returns. They found an improvement of performance in the Bayesian approach. It dominated all other models (except the overall average model) under investigation at a statistically significant level in the testing period.

Eun and Resink(1984) applied the estimation of correlation structure to the international capital market. Models of covariance for estimating the international correlation matrix were empirically tested relative to full historical extrapolation. Their results indicated that the national average model of covariance strictly dominates all the other models investigated in terms of forecasting accuracy. So far, the empirical results in the literature lead to the prevailing conclusion that the average model outperforms the stock index model in estimating the correlation structure of security returns in both domestic and international contexts.

However, it is possible that the underperformance of the stock index model may be attributed to the methodology of empirical testing procedure and/or the construction of the model of covariance. In realizing the industry interrelationship or colinearity in the traditional industry classification, Farrell (1974) extended Elton and Gruber's idea of orthogonal index to form homogeneous groups and used the indices of homogeneous groups as inputs to a multi-index model. The homogeneous groups in his study included growth, stable, cyclical and oil stocks. He concluded that his multi-index model based on homogeneous groups outperformed the single-index model. Implied in his results is the better performance of the multi-index model with homogeneous grouping to the one with traditional industry grouping. However, Farrell's results are far from conclusive. Fertuck (1975) followed a procedure similar to Farrell's in forming pseudo-industries. But his results differed from Farrell's; he found that the multi-index model with traditional industry grouping outperformed the one with pseudo-industry indices in most cases.

In this study, another approach is proposed to solve the problem of industry interrelationship in a multi-index model. Essentially, the idea is to recognize the existence of industry interrelationship and introduce the effect of industry interaction directly into the return-generating function in terms of a simultaneous-equation approach. The models of covariance with industry interaction will be evaluated in terms of the prediction of both the covariance and the correlation structures of security returns and compared with the performance of all the other models.

# CHAPTER III

## DESCRIPTION OF SOME BASIC MODELS OF COVARIANCE

As mentioned earlier, enormous inputs are required in a full covariance matrix as the number of securities under consideration increases, hence some covariance models have been suggested to reduce the input requirements. Before another approach to the simplification is introduced, a review of different covariance models that could serve as a theoretical induction to new models is provided in this chapter.

Essentially, two kinds of models of covariance can be found in the literature: the average model and the stock index model. While the average model is the simplest way to predict future covariance or correlation coefficient, it does not allow for individual behavior in the security return-generating process. On the contrary, the stock index model always relates a security's behavior to some sort of index to generate security returns and form the covariance or correlation matrix accordingly. The economic factor model is proposed in this study to represent an alternative to the stock index model. It employs economic factors instead of stock price indices in generating security returns and their associated covariance and correlation matrices. The average, the stock index and the economic factor models can be compared to a full historical model representing pure historical extrapolation.

## III.A. Full Historical Model

The simplest way to estimate the future dependence structure of security returns is to calculate each pairwise covariance or correlation of security returns over a historical period and assume that the historical values of these covariances or correlations are the best estimates of their future values. Since each pairwise covariance has to be directly estimated, there is no reduction of input requirements in the portfolio analysis under this model. The Full Historical Model can be regarded as the most disaggregate of all models under consideration. Therefore, the results for the Full Historical Model could serve as the benchmark against which more aggregate average models, and sophisticated models of covariance are evaluated.

## III.B. Average Models

The average model simplifies the correlation matrix by assuming an average correlation in either all the entries in the matrix or the entries in different subgroups. Therefore, it assumes that historical data contains information about the future average correlation, but not information about pairwise differences from the average correlation. Because the observed pairwise differences from the mean are assumed to be random in the average model, the best estimate of the pairwise differences would be zero.

The most aggregate type of average model is to set every entry in the correlation matrix equal to the average of all historical pairwise correlations. This model is referred to as the Overall Average Model. However, there could be a common mean correlation within and between subgroups of securities. Thus, a more disaggregate type of average model would assume every intra-industry pairwise correlation to be the average of all historical pairwise correlations within the industry and every inter-industry pairwise correlation to be the average of all historical pairwise correlations between securities from two different industries. This model is referred to as the Industry Average Model.

In terms of notations, the Overall Average Model can be written as

$$\rho_{ij} = \bar{\rho} \qquad \text{for all } i \text{ and } j, \text{ and } i \neq j \qquad (1)$$

where $\rho_{ij}$ represents correlation and $\bar{\rho}$ is mean correlation in the group, and the Industry Average Model can be described as follows: For any $i \neq j$,

$$\rho_{ij} = \bar{\rho}_I \qquad \text{if both } i \text{ and } j \text{ are in industry } I.$$

$$= \bar{\rho}_{IJ} \qquad \text{if } i \text{ is in industry } I \text{ and } j \text{ is in industry } J, \text{ where } I \neq J. \qquad (2)$$

The above description of the average models is based on the assumption of average correlation. The average correlation assumption under average models is adopted in all previous empirical studies in estimating the correlation structure of security returns.[3] In the case that covariance measures the dependence structure of security returns, the covariance under the average models may be estimated in two ways. The first way derives the average covariance from the average correlation. In other words, the derived average covariance in a particular group is equal to the average correlation multiplied by the average standard deviations in the group. The second way simply computes the average covariance from the historical covariance matrix directly. The average covariance assumption under the average models can be compared to the derived covariance based on the average correlation assumption. In fact, some simple rules of portfolio selection have been designed in the literature on the basis of both assumptions. However, the comparison between these two assumptions under the average models will provide portfolio managers with some information about choices in the portfolio selection procedure.

## III.C. Stock Index Models

An alternative to direct estimation of the covariance is to assume that the security price comovements are due to their common response to a single index or multiple indices. By assuming a specific behavioral security return-generating model, the implicit covariance from the parameter estimate of the model can be derived. The implied covariance could serve as an estimate of future covariance. The estimated correlation coefficient is derived from the estimated covariance by dividing it by the estimate of respective standard deviations. In this case, historical standard deviation will serve as the estimate of future standard deviation. Once this point is kept in mind, the following discussion in this chapter can be based on the prediction of covariance.

### III.C.a. Single-Index Model

The Single-Index Model assumes that securities move together only because they respond to a common market index. The model can be described as:

$$R_i = \alpha_i + \beta_i R_M + \varepsilon_i \qquad i=1,2,\ldots,n \qquad (3)$$

with the assumptions that;

(i) $E(\varepsilon_i)=0$ for all i

(ii) $Cov(\varepsilon_i, R_M)=0$ for all i

(iii) $Cov(\varepsilon_i, \varepsilon_j)=0$ for all i & j, and $i \neq j$

where $R_i$ and $R_M$ are, respectively, the rates of return on security i and the market index M and $\alpha_i$ and $\beta_i$ are parameters specific to security i. While the first two assumptions follow from a classical linear model, the last assumption is attributed to the set-up of the Single-index Model: since securities are correlated with each other only through their common response to the market index, the security residuals after market effect are assumed to be uncorrelated.

After estimating the parameters in Equation (3), the covariance implied in the Single-Index Model can be calculated as follows:

$$\sigma_{ij} = \beta_i \beta_j \sigma_M^2 \quad \text{for all i \& j, and } i \neq j \qquad (4)$$

While the $\beta$'s in Equation (4) are essentially estimated from past data, variance of the market rate of return is assumed to be known and generated from actual data in order to derive the implicit covariance of the model. However, since beta stability has long been disputed in the literature, beta can be estimated in different ways. In this study, two different estimates of $\beta$ are employed in generating the implied covariances under the Single-Index Model: an unadjusted historical $\beta$ and a Bayesian $\beta$.

The unadjusted betas are the betas obtained from ordinary least-squares regression of security returns on the return on the market index over a historical period. These sample estimates of betas are used to predict future covariance according to Equation (4). The Bayesian estimation of beta adjusts the sample estimate of beta from a historical period toward the best prior estimate. The degree of adjustment depends on the precision of both the sample estimate and the prior distribution. Vasicek(1973) suggested the average value of sample cross-sectional betas as the best prior estimate. Thus, a weighted average of the unadjusted beta and the average beta of the sample securities is used as the Bayesian beta. The weights are sumed to equal one and are determined by both the standard error of the security beta and the standard deviation of sample betas. These Bayesian estimates of betas are used to predict future covariance by Equation (4).

### III.C.b. Multi-Index Model

The Multi-Index Model assumes that securities move together not only because of their common response to a market index but also because of their association with some other kinds of index such as the industry index. The model incorporates the industry index to capture additional information. In view of Equation (3), the model implies $Cov(\varepsilon_i, \varepsilon_j)=0$. Instead, the Multi-Index Model is written as:

$$R_i = A_i + B_i R_M + C_i R_I + U_i \qquad i=1,2,\ldots,n_I \ \& \ i \in I \qquad I=1,2,\ldots,m \tag{5}$$

with the assumptions that;

| | | |
|---|---|---|
| (i) | $E(U_i)=0$ | for all i |
| (ii) | $Cov(U_i, R_M)=0$ | for all i |
| (iii) | $Cov(U_i, R_I)=0$ | for all i & I, and $i \in I$ |
| (iv) | $Cov(R_M, R_I)=0$ | for all I |
| (v) | $Cov(R_I, R_J)=0$ | for all I & J, and $I \neq J$ |
| (vi) | $Cov(U_i, U_j)=0$ | for all i & j, and $i \neq j$ |

where $A_i$, $B_i$ and $C_i$ are parameters associated with security i in industry I and $R_I$ is the residual return to industry I's index, constructed to be orthogonal to the return on market index $R_M$.

The Multi-Index Model in Equation (5) assumes that a security return is affected by the market effect and the industry to which the security belongs. While the first three assumptions come from the classical linear model, the fourth assumption is a result of the particular construction of the residual return. The fifth condition implies that there is no interaction between industry indices and the final assumption indicates that security returns are not correlated with each other after removing the market and industry effects.

Given the Multi-Index Model of Equation (5), covariances can be calculated as follows: For any $i \neq j$,

$$\sigma_{ij} = B_i B_j \sigma_M^2$$

between securities in different industries.

$$= B_i B_j \sigma_M^2 + C_i C_j \sigma_I^2 \qquad (6)$$

between securities in the same industry.

where $\sigma_M^2$ and $\sigma_I^2$ represent, respectively, the variances of return on the market index M and the industry index I.

When Compared to Equation (4), the Multi-Index Model and the Single-Index Model with unadjusted beta will produce identical estimates of inter-industry pairwise covariances. This is because, (1) for the orthogonal case, the B estimates from multiple regression are equal to the β's estimates from simple regression;[4] and, (2) no interaction between industry indices under the Multi-Index Model is assumed. However, the estimates for the intra-industry covariances are different. Thus, any difference in performance between these two models is attributed to their differential estimates of intra-industry covariances.

The estimated parameters in the Multi-Index Model can also be adjusted in terms of the Bayesian approach. In this version, the Multi-Index Model with Bayesian Coefficient Adjustment will produce identical estimates of inter-industry pairwise covariances to those in the Single-Index Model with Bayesian beta adjustment due to the reasons mentioned above.

## III.D. Economic Factor Models

In this study, the economic factor model refers to the model of covariance using economic forces as the common factors in the security return-generating process. The underlying economic factors are differentiated from the commonly-used stock price indices in the process of simplifying the estimation of covariance between security returns. The economic factor-based model of covariance also assumes the existence of common economic factors in the security return-generating process. Two forms of economic factor-based models of covariance are proposed: a Single-Factor Model and a Multi-Factor Model.

### III.D.a. Single-Factor Model

The formation of a Single-Factor Model is the same as the Single-Index Model except that instead of a market index, Gross National Product (GNP) is used to represent the single common effect. Securities are assumed to move together only because of their common response to GNP. For a long time, GNP has been considered to be the best indicator representing the strength of the national economy. Thus, it is the natural choice when only one economic factor is to be selected as the common effect to security prices.

However, it is well known that the stock market leads the national economy. Therefore, the "unanticipated" change in GNP is proposed as the single factor affecting the generation of security returns. In estimating the unanticipated change in GNP, the prediction of GNP by ASA-NBER is used to purge off the anticipated part.[5] In other words, the prediction of GNP by ASA-NBER serves as the expectation of general investors for the change in GNP.

In notations, the Single-Factor Model assumes that security returns are generated as follows:

$$R_i = \alpha_i' + \beta_i' R_{GNP} + \varepsilon_i' \qquad i=1,\ldots\ldots,n \qquad (7)$$

with the assumptions that

(i) $E(\varepsilon_i')=0$ for all i

(ii) $Cov(\varepsilon_i', R_{GNP})=0$ for all i

(iii) $Cov(\varepsilon_i', \varepsilon_j')=0$ for all i & j, and $i \neq j$

where $R_{GNP}$ is the rate of unanticipated change in GNP, and $\alpha_i'$ and $\beta_i'$ are parameters specific to security i. The implied covariance in the Single-Factor Model will be;

$$\sigma_{ij} = \beta_i' \beta_j' \sigma^2_{GNP} \quad \text{for all i \& j, and } i \neq j \qquad (8)$$

While the $\beta'$'s in the above equation can be estimated from historical data, they can also be adjusted to reflect the prior information based on the Vasicek (1973) method.

III.D.b. Multi-Factor Model

More than one factor may be assumed to generate security returns. An unscientific extension from the Multi-Index Model to the economic factor-based model of covariance is to include a factor such as GNP to represent the national economy and another factor such as industrial production to represent sector output. It is unscientific because so far no empirical evidence shows that these two factors are the only two important common effects in the security return-generating process. However, it is interesting to know the performance of this simple extension in predicting the future covariance matrix.

The Multi-Factor Model can be expressed as:

$$R_i = A_i' + B_i' R_{GNP} + C_i' R_K + U_i' \qquad i=1,2,\ldots,n \;\&\; i \in K$$
$$K=1,2,\ldots,m \qquad (9)$$

with the assumptions that;

(i) $E(U_i')=0$

(ii) $Cov(U_i', R_{GNP})=0$ for all i

(iii) $Cov(U_i', R_K)=0$ for all i & K, and $i \in K$

(iv) $Cov(R_{GNP}, R_K)=0$ for all K

(v) $Cov(R_K, R_{K'})=0$ for all K & K', and $K \neq K'$

(vi) $Cov(U_i', U_j')=0$ for all i & j, and $i \neq j$

where $A_i'$, $B_i'$ and $C_i'$ are parameters associated with security i in industry K. $R_K$ is the rate of change of industry K's production, which is constructed to be unrelated to the rate of unanticipated change in GNP. Once $B_i'$ and $C_i'$ are estimated, implied covariances can be calculated as follows: For any $i \neq j$,

$$\sigma_{ij} = B_i' B_j' \sigma^2_{GNP}$$

between securities in different industries

$$= B_i' B_j' \sigma^2_{GNP} + C_i' C_j' \sigma^2_K$$

between securities in the same industry

(10)

where $\sigma^2_K$ represents the variance of the change in industry K's production.

In calculating the implicit variance of the Multi-Factor Model, the Bayesian approach can also be applied to adjust the historical coefficient estimates.

# CHAPTER IV

## EMPIRICAL EVIDENCE OF INDUSTRY INTERRELATIONSHIPS

In the models of covariance proposed later, the idea that security returns are related with each other through the industry effect is assumed. The residual covariance between security returns after removing market and industry effects is assumed to be zero.

An industry interaction in the national economy is quite obvious. If the nation's economy is divided into several sectors, it is unavoidable that products from different industries will flow across sectors. That is a product in one industry could be the raw materials for another industry. One industry's output will become another industry's input. Indeed, this is what business about. This real phenomenon is the foundation of the Leontief Input-Output Model. However, the interacting phenomenon among industry outputs may not reflect in the stock price indices formed for different industries.

Thus, it becomes necessary to investigate empirically the degree of industry interrelationship in order to incorporate the industry interaction in the Multi-Index Model reviewed above. Strong interrelationship among industry price indices will support the approach of building a multi-index model with industry interaction.

In terms of stock price index, previous evidence of industry interrelationship provided in the literature is encouraging. In Cohen and Pogue's paper, for the purpose of investigating the assumption of covariance form of a multi-index model, they provided the distribution of correlation coefficients among the ten industry indices used in the study. The distribution indicated a high interindex correlation. Two-thirds of the correlation coefficients ranged between .8 and 1. They also provided the distribution of correlation coefficients of the industry index residuals after removing the market effect. Sixty percent of the absolute value of residual correlation coefficients were greater than .3. The results showed evidence of a strong interrelation among industry indices even though the number of industries investigated in their study was not large.

The most comprehensive examination of residual industry effects so far was done by Livingston (1977). He used a security's monthly rate of return for the period between January 1966 and June 1970 to investigate the industry movements of 734 common stocks. These securities represented 100 different industries. After claiming the superiority of regression analysis over factor analysis in providing the estimates of after-removal-of-market covariances, he presented the residual industry effect using company data. Of the cross-industry residuals, 8% were significantly different from zero at the 5% level of significance. Of the within-industry correlations, 20% were significantly different from zero. The results indicated that both cross-industry and within-industry residual comovements existed and that within-industry comovement was much larger than cross-industry comovement.

In this study, the degree of industry interrelationship is investigated by examining the correlation matrix of industry indices. The industry indices of stock prices are taken from the Security Price Index Record, 1984 Edition, published by Standard and Poor's Corporation. Industry grouping is based on

Standard and Poor's industry classification system. All the available data of industry indices between January 1974 and December 1983 are used. Sixty-one industries are examined. Almost all the Standard and Poor's 500 companies are represented except those that belong to the miscellaneous category.

Because of data availability considerations at the company level, which will be encountered later in model building, the degree of industry interrelationship is examined using two different investment horizons of industry price indices: monthly and quarterly. Both monthly and quarterly rates of return are calculated by taking the first difference of industry price indices then dividing by the beginning price index.

The correlation matrix of industry returns can then be easily obtained. Figure 2 shows the frequency distributions of correlation coefficients between both monthly and quarterly industry returns. As shown, both frequency distributions are skewed to the right with almost all the correlation coefficients located in the positive section. However, quarterly figure tends to be more dispersed. While the mode of monthly correlation coefficients is located in the range of .5 to .6, the quarterly correlation coefficients are centered between .7 and .8. Figure 1 gives us a preliminary indication of the significance of industry price comovements.

In addition, t-statistic can be used to test the hypothesis of zero correlation between industry price indices. The hypothesis test of linear relationship can be stated as follows:

Null Hypothesis $H_0 : \rho = 0$

which means the true correlation between industry price indices is zero.

Alternative Hypothesis $H_1 : \rho \neq 0$

which means the true correlation between industry price indices is not zero.

Then, $t=(r-\rho)/S_r$ has t distribution with n-2 degrees of freedom, where r is the sample correlation coefficient, n is the sample size, and $S_r$ represents the standard error of the sample correlation coefficients and is given by $S_r = \sqrt{(1-r^2)/(n-2)}$ .[6] To reject the null hypothesis that there is zero correlation between industry price indices, the absolute value of r must be greater than 0.1793 for monthly correlation coefficients and 0.3121 for quarterly correlation coefficients at the 5% level of significance. Of the 1,830 ((61x60)/2) monthly correlation coefficients, 1,794 calculated monthly correlation coefficients have the value greater than 0.1793 and thus are significantly different from zero at the 5% level of significance. This is 98% of the total number of monthly correlation coefficients. At the same 5% level of significance, 92.8% (1699 out of 1830) of the quarterly correlation coefficients between industry indices are significantly different from zero. This result indicates a high significance of industry interrelationship.

However, the large percentage of significant correlation coefficients may have to do with the industry price indices which can be regarded as a portfolio of securities that tend to move with the market index at the same time. In order to find out the real degree of industry interrelationship, the correlation matrix of the residual industry price indices after removing market effect should also be investigated. The S & P 500 price index is used as the market index. After regressing each industry return to the S & P 500 market return, the difference between the historical industry return and the calculated industry return from the regression is used as the residual industry return to represent the residual industry effect.

Figure 3 shows the frequency distribution of both monthly and quarterly residual industry returns. As in the previous case, the quarterly figure is more dispersed than the monthly one. But this time, the skewness of the frequency distribution is not so obvious in both monthly and quarterly correlation coefficients of residual industry returns. However, the similar bell-shaped type of frequency distribution allows adoption of some significant industry interrelationship in model building. As a matter of fact, the significance of industry interrelationship is more easily seen by examining the t-statistic hypothesis test.

Of the 1,830 monthly correlation coefficients of residual industry returns, 813, which represents 44.4% of the total number, have an absolute value greater than 0.179. That means 44.4% of the monthly correlation coefficients between residual industry returns are significantly different from zero at the 5% level of significance. With the same confidence level, 24% (440 out of 1,830) of the quarterly correlation coefficients between residual industry returns are significantly different from zero. This result indicates the significance of the residual industry effect. Therefore there exists a sufficient degree of industry interrelationship to encourage building an econometric model based on industry price comovements.

# CHAPTER V

## MODELS OF COVARIANCE WITH INDUSTRY INTERACTION

One of the assumptions in the simple Multi-Index Model and Multi-Factor Model mentioned in Chapter III is that there is no interaction between industry returns. However, as shown in Chapter IV, significant industry interrelationship does exist. Thus, the more complex model of covariance should incorporate the industry interaction to improve the model's performance of covariance and correlation estimations. One way to adopt the industry interaction is to employ a simultaneous-equation approach in the industry model. As a block in the whole equation system, the industry equations would provide necessary inputs for another block of equations in which both market and industry factors influence security returns. From the whole equation system's point of view, the simultaneous-equation model is block recursive. Three forms of covariance model with industry interaction will be proposed in this section: a stock index-based Multi-Index Econometric Model, an economic factor-based Multi-Factor Input-Output Model, and an index/factor-mixed Market Model with Input-Output Analysis.

The following discussion in this chapter is based on the prediction of covariance. As mentioned before, the estimated correlation coefficient can be generated by dividing the estimated covariance by the respective standard deviations. In general, historical standard deviation will serve as the estimate of future standard deviation.

V.A. Multi-Index Econometric Model

As long as the system of equations provides all the necessary estimates, the covariance between security returns can be calculated from the multi-index model with industry interaction. In other words, an adequate system of simultaneous equations should be built to generate the required inputs for the covariance matrix under the Multi-Index Econometric Model.

Among the equation systems within this model, the variables used in the security return-generating equation do not pose much problem. The security equation would include one exogenous variable, the market index, and one endogenous variable, the industry index, on the right-hand side of the equation to account for the influences of the market and the industry to which the security belongs. However, the building of the simultaneous-equation industry model is more an art than a science. The structure of the industry model will affect the final prediction performance of the covariance matrix. In some cases, the under-performance of the covariance estimation under the Multi-Index Econometric Model could be attributed to the failure of building a better industry model. Nevertheless, it is understood that the industry model should reflect the market effect and the industry interaction. Hence the form of industry model could be as follows:

$$R_I = a_I + b_I R_M + c_I R_J + d_I R_{Iv} + e_I \qquad I=1,2,\ldots,m \;\&\; I \neq J \qquad (11)$$

with the assumptions that;

(i) $E(e_I)=0$ for all I

(ii) $Cov(e_I, R_M)=0$ for all I

(iii) $Cov(e_I, R_{IV})=0$ for all I

where $a_I$ , $b_I$ , $c_I$ and $d_I$ are parameters associated with industry I, $R_J$ is the return on another endogenous industry variable which significantly affects industry I, and $R_{IV}$ is the return on the exogenous variable which also significantly affects the industry under consideration. The market index is exogenous in the model. Once all m industry equations are formed, the model becomes an equation system. These equations are simultaneous because the dependent variable in one equation appears in another equation as an independent variable. Through simultaneous equations, the industry interaction is built into the industry model.

Once industry returns are predicted from the simultaneous-equation industry model, they can be used recursively to predict security returns as follows:

$$R_i = a_i + b_i R_M + c_i R_I + e_i \qquad i=1,2,\ldots,n \qquad (12)$$

with the assumptions that;

(i) $E(e_i)=0$ for all i

(ii) $Cov(e_i, R_M)=0$ for all i

(iii) $Cov(e_i, R_I)=0$ for all i & I, and $i \in I$

(iv) $Cov(e_i, e_j)=0$ for all i & j, and $i \neq j$

where $R_I$ is the return on the industry to which security i belongs. It is generated from the simultaneous-equation industry model described above.

Taking Equations (11) and (12) together, the model as a whole assumes that security return is affected by both market and industry effects with the industry interaction being built in the prediction of industry return. Given the multi-index model with industry interaction, covariance between security returns can be calculated as follows: For any $i \neq j$,

$$\sigma_{ij} = b_i b_j \sigma_M^2 + b_i c_j \sigma_{MJ} + b_j c_i \sigma_{MI} + c_i c_j \sigma_{IJ}$$

between securities in different industries

$$= b_i b_j \sigma_M^2 + b_i c_j \sigma_{MI} + b_j c_i \sigma_{MI} + c_i c_j \sigma_I^2 \qquad (13)$$

between securities in the same industry

where $\sigma_{IJ}$ and $\sigma_{MI}$ represent, respectively, the covariance between industry indices I and J and the covariance between market index M and industry index I. From Equation (13), the input requirement in portfolio analysis is increased. But since the number of industries is much less than the number of companies, the input to the multi-index model with industry interaction is still greatly reduced from the full covariance model.

V.B. Multi-Factor Input-Output Model

One of the drawbacks of the Multi-Index Econometric Model described above is the lack of economic theory to support the industry block even though the industry interrelationship is empirically justified. The influence of market effect on the industry index can be justified when the industry index is treated as a portfolio. However, without a theoretical foundation of industry interaction, the introduction of other industry indices in an industry equation becomes somewhat arbitrary. Fortunately, the Input-Output Model invented by Leontief (1936) provides the necessary framework. The Input-Output Model will replace the econometric model of industry price index as a simultaneous-equation system. The industry output is the desired solution of the equation system. It also serves as the industry factor in the security return-generating function.

An Input-Output Model is a simultaneous-equation system which is constructed from observed data for a particular economic area such as a nation. The national economic activity can be divided into a number of producing sectors by industries. Interindustry flows of products are quite common in the national economy. One industry's products could be another industry's raw materials. Thus, one industry's demand for inputs from other industries in a given period will be related to the outputs supplied by another industry over the same period. Besides, all the industry's demands from one particular industry plus the final demand from consumers for the industry's product constitute the total output of the industry. Therefore, one industry's demand for inputs from a particular industry in a given period will depend on the total output of the industry over the same period. In the input-output framework, a set of technical coefficients is used to represent the ratios of interindustry flows to total outputs in different industries. The so-called technical coefficient, or input-output coefficient, can be expressed as follows:

$$c_{IJ} = \frac{x_{IJ}}{F_J} \tag{14}$$

where $c_{IJ}$ represents the technical coefficient from industry I to industry J, $x_{IJ}$ is the flow of input from industry I to industry J, and $F_J$ is the total output of industry J.

In the above equation, the technical coefficient measures the assumed fixed relationship between an industry's output and its inputs. Thus, economies of scale and alternate methods of production are ignored.[7] In a Leontief Input-Output system, production is assumed to be constant returns to scale. In this case, the expansion path representing input combinations that are used for various levels of output will be a straight line. The homogeneous production function of degree one is a major assumption in the Leontief interindustry model.

Based on the above description, the Input-Output Model can be constructed as follows:

$$
\begin{aligned}
F_1 &= c_{11}F_1 + c_{12}F_2 + \cdots + c_{1m}F_m + Y_1 \\
F_2 &= c_{21}F_1 + c_{22}F_2 + \cdots + c_{2m}F_m + Y_2 \\
&\vdots \\
F_m &= c_{m1}F_1 + c_{m2}F_2 + \cdots + c_{mm}F_m + Y_m
\end{aligned}
\tag{15}
$$

where $Y_I$ denotes the final demand from consumers for industry I's product. In each equation in (15), an industry's output is equal to the sum of its input to all the industries and consumers' final demand. The system of industry equations explicitly presents the dependence of interindustry flow on the total outputs of each industry. Once all the technical coefficients ($c_{IJ}$) are exogenously determined, the predictions in the final demands of every industries will provide the output from each of the industries necessary to supply these final demands. This type of input-output analysis can be solved through standard simultaneous-equation system. Equation system (15) can be arranged as follows:

$$\begin{aligned}
(1-c_{11})F_1 &- c_{12}F_2 - \cdots - c_{1m}F_m = Y_1 \\
-c_{21}F_1 &-(1-c_{22})F_2 - \cdots - c_{2m}F_m = Y_2 \\
&\vdots \\
-c_{m1}F_1 &- c_{m2}F_2 - \cdots -(1-c_{mm})F_m = Y_m
\end{aligned} \tag{16}$$

In matrix terms, define

$$C = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1m} \\ c_{21} & c_{22} & \cdots & c_{2m} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ c_{m1} & c_{m2} & & c_{mm} \end{bmatrix} \qquad F = \begin{bmatrix} F_1 \\ F_2 \\ \cdot \\ \cdot \\ \cdot \\ F_m \end{bmatrix} \qquad Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ \cdot \\ Y_m \end{bmatrix}$$

and let $I$ be the $m \times m$ identity matrix. Then the system of simultaneous equations can be expressed as:

$$(I - C)F = Y \tag{17}$$

Depending on whether $(I-C)$ is singular, the system may have a unique solution. In other words, if the determinant $|I-C| \neq 0$, then the inverse $(I-C)^{-1}$ can be found and the unique solution is given by;

$$F = ( I - C )^{-1} Y \tag{18}$$

where the matrix C is known as the matrix of technical, or input-output coefficients, and $(I-C)^{-1}$ is often referred to as the Leontief inverse. In this way, industry's outputs are predicted by a matrix of fixed technical coefficients and a vector of given final demands as described in equation (18).

The determination of technical coefficients is a nontrivial issue in the literature of input-output analysis. Two problems concerning technical coefficients are: (1) availability of historical technical coefficients and (2) stability of these coefficients. The table of technical coefficients in the national economy released by the U. S. government is generally lagged by several years. So far, only the historical data in 1947, 1958, 1963, 1967, 1972 and 1977 are constructed and made available.

One simple way to forecast future technical coefficients is to use a matrix of historical coefficients such as the one in 1972 or 1977. It is clear that techniques of production will and do change over time for various reasons such as changing technology, changing prices and changing economies of scale. Technical coefficients in an input-output table may not remain stable over time. Both survey and nonsurvey methods have been proposed in the literature to adjust historical technical coefficients.[8] Nevertheless, due to the problem of data availability, the technical coefficients are assumed to be stable over a segment of the investigation period in this study.

Once the matrix of technical coefficients (C) is determined, industry outputs (F) will be solved by Equation (18) with a given vector of industry's final demands (Y). Time series of projected industry outputs are generated and their associated growth rates in industry outputs can be calculated. The predicted growth rates in industry outputs will be used as the industry factor in the security return- generating function under the Multi-Factor Input-Output Model. The Multi-Factor Input-Output Model is an extension of the Single-Factor Model described in Chapter III. Thus, the security return-generating function could be expressed under this model as follows:

$$R_i = a_i' + b_i' R_{GNP} + c_i' R_F + e_i' \qquad i=1,2,\ldots\ldots,n \qquad (19)$$

with the assumptions that;

(i) $E(e_i')=0$ for all i

(ii) $Cov(e_i', R_{GNP})=0$ for all i

(iii) $Cov(e_i', R_F)=0$ for all i & F, and $i \in F$

(iv) $Cov(e_i', e_j')=0$ for all i & j, and $i \neq j$

where $R_F$ is the change in the outputs of industry F to which security i belongs. $R_F$ is generated from the projected industry outputs in the Input-Output Model.

The Multi-Factor Input-Output Model as a whole assumes that security return is affected by both national economy and industry outputs with industry interaction being built in the Leontief Input-Output Model. Given the multi-factor model with industry interaction, covariance between security returns can be calculated as follows: For any $i \neq j$ and $F \neq F'$,

$$\sigma_{ij} = b_i' b_j' \sigma^2_{GNP} + b_i' c_j' \sigma_{GNP,F'} + b_j' c_i' \sigma_{GNP,F} + c_i' c_j' \sigma_{FF'}$$

between securities in different industries

$$= b_i' b_j' \sigma^2_{GNP} + b_i' c_j' \sigma_{GNP,F} + b_j' c_i' \sigma_{GNP,F} + c_i' c_j' \sigma^2_F \qquad (20)$$

between securities in the same industry

where $\sigma^2_{GNP}$ and $\sigma^2_F$ represent, respectively, the variance of unanticipated change in the nation's gross national product and the change in industry outputs. $\sigma_{FF'}$ is the covariance between the changes in the output of industry F and industry F' and $\sigma_{GNP,F}$ indicates the covariance between the unanticipated change in the gross national product and the change in the output of industry F.

V.C. Market Model with Input-Output Analysis

For a long time, researchers have considered the market effect as the major force in the security return-generating process. The single market index equation gains its popularity as the Market Model. Thus, an index/factor-mixed covariance model with industry interaction could combine the Market Model with the Leontief Input-Output Analysis.

In the Market Model with Input-Output Analysis, the industry block will remain the same as in the Multi-Factor Input-Output Model. The security return in this covariance model is generated as follows:

$$R_i = a_i'' + b_i'' R_M + c_i'' R_F + e_i'' \qquad i=1,2,\ldots\ldots,n \qquad (21)$$

with the assumptions that;

| | | |
|---|---|---|
| (i) | $E(e_i'')=0$ | for all i |
| (ii) | $Cov(e_i'', R_M)=0$ | for all i |
| (iii) | $Cov(e_i'', R_F)=0$ | for all i & F, and $i \in F$ |
| (iv) | $Cov(e_i'', e_j'')=0$ | for all i & j, and $i \neq j$ |

Based on the Input-Output Model and the above security return-generating equation, the covariance implied in the covariance model can be expressed as follows: For any $i \neq j$ and $F \neq F'$,

$$\sigma_{ij} = b_i'' b_j'' \sigma_M^2 + b_i'' c_j'' \sigma_{MF'} + b_j'' c_i'' \sigma_{MF} + c_i'' c_j'' \sigma_{FF'}$$

between securities in different industries

$$= b_i'' b_j'' \sigma_M^2 + b_i'' c_j'' \sigma_{MF} + b_j'' c_i'' \sigma_{MF} + c_i'' c_j'' \sigma_F^2 \qquad (22)$$

between securities in the same industry

# CHAPTER VI

# METHODOLOGY OF EMPIRICAL EVALUATION

Based upon the covariance models with industry interaction as proposed in the previous chapter and some simpler models of covariance discussed in Chapter III, the alternative models of covariance to be evaluated are listed in Table 1.

## VI.A. Construction of Data Bank and Empirical Procedure

In evaluating the performance of the different models of covariance in Table 1 for estimating the dependence structure of security returns, two sets of investigation periods will be chosen to observe whether consistent results are maintained among these models of covariance in different periods. In the first set, 1971-1975 is used as the estimation period and 1976-1980 as the forecasting period. The second set will use 1976-1980 as the estimation period and 1981-1985 as the forecasting period.

In the stock index models, six broad industry groups based on Standard & Poor's industry classification system are adopted. They include Capital Goods(CAP), Consumer Goods(CON), Financial(FIN), Oil(OIL), Transportation(TRA), and Utility(UTI) groups. The number of companies included in these six groups represents more than 83% (416/500) of the S & P's 500 companies. Stock price indices in these six industry groups are readily available in the Security Price Index Record published by Standard & Poor's Corp. Also from the same source is the S & P 500 index which will serve as the market index in stock index models and the mixed model with industry interaction.

In the economic factor models, the Standard Industrial Classification adopted by the Bureau of Economic Analysis is followed due to data availability difficulties. Seven-sector aggregation is employed to define industries as: Agriculture(AGR), Mining(MIN), Construction(CNT), Manufacturing(MFT), Transportation and Trade(TAT), Services(SER), and Other(OTH). The total of these seven sectors forms the nation's economy. The U. S. national input-output tables are used to obtain the historical technical coefficients in 1972 and 1977. Most of the quarterly economic data come from the Citibank Economic Database.

Two versions of the industry average model are adopted in the empirical test: IA1 and IA2. IA1 is the Industry Average Model based on the S & P industry classification system; whereas IA2 represents the version using the Standard Industrial Classification system. As indicated in Chapter III, two forms of average models (average correlation assumption and average covariance assumption) are used when the covariance structure of security returns under the average models is estimated.

The sample size in this study includes 100 companies. All the 100 companies are sampled from the 500 companies composed of the S & P 500 index. Sample companies were chosen to spread evenly over all the industry groups. However, the sample companies only come from five sectors in the second industry group due to the problem of locating S & P 500's companies in the Agriculture and the Other sectors. Return data of each company was retrieved from the CRSP tape.

For the 100 sample companies, the full covariance matrix of security returns includes 4,950 entries of different covariance. Models of covariance in this study are designed to predict the full 4,950 covariances. Different models of covariance will have a different number of estimates required to generate the full covariance matrix. As far as the input reduction in portfolio analysis is concerned, the following table provides a clear comparison about the number of required estimates among the models of covariance evaluated in this study:

| Covariance Model | Number of Required Estimates |
|---|---|
| FH | 4950 |
| OA | 1 |
| IA1 | 21 |
| IA2 | 15 |
| SI | 101 |
| MI | 207 |
| SF | 101 |
| MF | 206 |
| MIE | 228 |
| MFID | 221 |
| MMID | 221 |

As the above table indicates, compared to the full covariance matrix, the input requirements in simplified models are reduced by a great amount. In fact, as long as the number of industries remains relatively small, the input requirements in portfolio analysis will be reduced quite rapidly as the number of security under consideration grows larger. As the model of covariance gets more complicated, the input data will increase slightly more. The main concern here is whether the more complicated model of covariance will offer a more accurate prediction in the dependence structure of security returns, or just pick up more random errors of estimation. However, both the predicting power of different models of covariance and their ability to reproduce a historical covariance matrix will be examined in this study.

All equations under different covariance models are assumed to be in linear form. Variables in the equations are measured quarterly in terms of rate of return or change. Ordinary least-squares technique is employed to estimate parameters in security return-generating equations. However, the parameters in the simultaneous equations of the industry econometric model under the Multi-Index Econometric Model were estimated by the three-stage least-squares method in order to solve the problems of inconsistency and inefficiency encountered in ordinary least-squares estimation.

In each model, parameters estimated in the estimation period will be used to generate the prediction of covariance matrix in the forecasting period. If the Bayesian approach is used in a model of covariance, its estimated parameters will be adjusted accordingly. Otherwise, parameter estimates are assumed to be stable during the investigation period. The Bayesian approach of coefficient adjustment in security return-generating equation is extensively investigated in this study because the problem of "beta" stability is widely recognized in the literature.

## VI.B. Criteria for Evaluating Alternative Covariance Models

The most popular method of evaluating the performance of different forecasting models is perhaps the mean squared forecasting error(MSE). Because each model of covariance is designed to predict the future covariance or correlation, the MSE will be applied to measure its performance. The MSE measures the degree of forecasting errors by the calculation of:

$$MSE = \sum_{i=1}^{n} ( F_i - A_i )^2 / n \qquad (23)$$

where $( F_i , A_i )$ is the paired forecasting and actual values in the i-th entry of the n-entries covariance matrix. Essentially, the MSE assumes a quadratic loss function of forecasting results. In order to obtain a performance measure which has the same dimension as the predicted variable, the square root of the MSE, the root mean squared forecasting error(RMSE), will be calculated.

The MSE can be decomposed into three terms, each represents a particular source of forecasting error:[9]

$$MSE = E(F-A)^2 = (\bar{F}-\bar{A})^2 + (\sigma_F - \sigma_A)^2 + 2(1-r)\sigma_F \sigma_A \qquad (24)$$

where $(\bar{F}, \bar{A})$ and $(\sigma_F, \sigma_A)$ are, respectively, the means and the standard deviations of the predicted and the actual covariances or correlations, and r represents the correlation coefficient between the predicted and the actual covariances or correlations. The first term of Equation (24) measures the error due to a biased prediction; the second term measures the error due to unequal variation; and the third term measures the error due to an imperfect correlation. By dividing the MSE in both sides of Equation (24), we obtain three proportions which add to unity as follows:

$$1 = (\bar{F}-\bar{A})^2/MSE + (\sigma_F-\sigma_A)^2/MSE + 2(1-r)\sigma_F\sigma_A/MSE$$
$$1 = U_m + U_v + U_c \qquad (25)$$

where $U_m$, $U_v$ and $U_c$ represent the bias, variance and covariance proportions respectively. These proportions measure the relative importance of the error components.

An alternative decomposition can be characterized by the following three terms:[10]

$$MSE = E(F-A)^2 = (\bar{F}-\bar{A})^2 + (1-\delta)^2\sigma_F^2 + (1-r^2)\sigma_A^2 \qquad (26)$$

where $\delta$ is the regression coefficient of actual covariances on predicted covariances. The first term of Equation (26) still measures the error due to a biased prediction as in Equation (24). The second term measures the error due to an inefficient prediction. It represents the model's tendency to systematically overpredict or underpredict covariances or correlations. The last term measures the error due to random disturbance in prediction. By dividing the MSE in both sides of Equation (26), we also could obtain the proportions describing the relative importance of these error components as follows:

$$1 = (\bar{F}-\bar{A})^2/MSE + (1-\delta)^2\sigma_F^2/MSE + (1-r^2)\sigma_A^2/MSE$$

$$1 = U_m + U_i + U_e \qquad (27)$$

where $U_m$, $U_i$ and $U_e$ are the bias, inefficiency and residual proportions respectively.

In order to test the statistical significance of the dominance of one model of covariance over a second model under the MSE criterion, an ordinary two-tailed t-test is applied to the mean of the difference, $D_i$, between the squared forecasting errors of these two models: $D_i = (F_{i1}-A_i)^2 - (F_{i2}-A_i)^2$, where $F_{i1}$ and $F_{i2}$, respectively, refer to the predicted values of Model 1 and Model 2 for the i-th entry of the covariance or correlation matrix, and $A_i$

refers to the actual value of the entry. The hypothesis test can be described as follows:[11]

Null Hypothesis $H_0 : \mu = 0$

which means the true mean difference is zero, i.e., the two models have equal predicting power.

Alternative Hypothesis $H_1 : \mu \neq 0$

which means the true mean difference is not zero, i.e., the two models have different predicting power.

The t-statistic is calculated by $t = \frac{\bar{D} - \mu}{S_D / \sqrt{n}}$ where $\bar{D}$ and $S_D$, respectively, are the mean and the standard deviation of the differences $D_i$, and n is the number of entry in the covariance or correlation matrix. Model 1 is judged to dominate Model 2 if the mean of the differences is negative and significantly different from zero at the 5 percent level of significance.

To evaluate the performance of each model of covariance relative to the benchmark (i.e., the Full Historical Model), the Theil Inequality Coefficient (TIC) is computed for each of the covariance models:[12]

$$TIC = \sqrt{\sum_{i=1}^{n}(F_i - A_i)^2 \Big/ \sum_{i=1}^{n}(H_i - A_i)^2} \qquad (28)$$

where $H_i$ is the i-th predicted value of the covariance or correlation matrix under the Full Historical Model. From Equation (28), it is clear that TIC=1 if the prediction is as accurate as the one provided by historical extrapolation, and TIC=0 in the event of perfect prediction. A value of TIC less than 1 implies that the prediction is better than historical extrapolation.

# CHAPTER VII

# PREDICTION OF INDUSTRY AND SECURITY RETURNS

The prediction of industry and security returns is not the main concern in this study. It is the by-product on the way to predict the covariance between security returns. Industry model and security equations are built for the purpose of providing the necessary parameter estimates for covariance and correlation predictions. These parameter estimates can also be used to derive the prediction of industry and security returns. It is interesting and helpful to know the relationship between the predicting power of industry and security returns and that of covariance or correlation structure under different models of covariance.

The prediction of security return can be considered as the expected security return over the prediction period. The expected return of each security is one of the necessary inputs to the portfolio selection problem.

With the data of expected return of each security available, this study's results can be more easily applied to the portfolio selection problem.

In recent years, the prediction of industry return has become more important because of the introduction of industry index options in the AMEX. In the case of stock index models of covariance, an industry index is a weighted average of security prices; therefore, it should be determined simultaneously in the model. Under the economic factor model of covariance, the use of predicted industry growth makes the prediction of the dependence structure of security returns more realistic.

The prediction of industry and security returns under different models of covariance is investigated in this chapter. Their performance of covariance and correlation predictions will be evaluated in the next chapter. After reviewing these two predictions, some ideas about their relationship in different models of covariance can be formed. If their prediction powers of security returns and dependence structure are closely related among alternative models of covariance, that would give a helpful suggestion in the process of improving the performance of covariance or correlation prediction. If not, they may give some indications about the underlying reasons for different performance among various models of covariance which, in turn, also suggest possible ways of improvement.

VII.A. Building a Simultaneous-Equation Industry Model

In Chapter V, the theoretical framework used to generate the covariance and correlation matrices under the models of covariance with industry interaction was proposed. Because of the block recursive nature of the equation system under the models of covariance with industry interaction, the block of industry equations can be built first and then the block of security equations follows. The predicted result of industry equations in the industry block will serve as inputs in the security block. A diagram of the equation system including industry and security blocks has been provided in Figure 1. While there exists simultaneity in industry equations due to the introduction of industry interaction, the security equations are mutually independent by assumption.

For security equations, there is little problem in choosing appropriate variables. Under the Multi-Index Econometric Model, each security's return is a function of the return of the market index and the index return of the particular industry to which the security belongs. For example, AMR's return is affected by the S & P 500's return and the return of the Transportation index. Under the Multi-Factor Input-Output Model, each security's performance is a function of the nation's economy and the output of the industry to which the security belongs. In this case, AMR's return depends on the unanticipated growth rate in Gross National Product and the predicted rate of change in the output of the Trade and Transportation group. Under the Market Model with Input-Output Analysis, each security's performance is assumed to be a function of the market effect and the output of the industry to which the security belongs. Thus, AMR's return is influenced by S & P 500's return and the predicted rate of growth in the output of the Trade and Transportation group. Remember that different industry classification systems exist under stock index models and economic factor models of covariance due to the problem of data availability.

The construction of industry equations is not as straightforward as that of security equations. From an empirical viewpoint, to some extent the building of the industry model necessitates much more technical efforts. Model building has been considered as an art rather than a science. This argument is more apparent when an equation system instead of a single equation is encountered. The point now is to build a reasonable industry model with consideration of the interaction between different industries.

The introduction of industry interaction can be naturally adapted in terms of simultaneous equations in the industry block of an equation system. In the industry block, industry returns are mutually influenced and simultaneously determined. Because of the simultaneity in the determination of industry returns, an industry will have both direct and indirect impacts on other industries. The building of simultaneous equations in the industry return-generating process is the most fundamental way to take industry interaction into account in estimating the dependence structure of security returns. In this way, the parameter estimates used to derive the covariance between security returns will reflect the interdependent nature of various industries.

## VII.A.a. Industry Equations under the Multi-Index Econometric Model

The lack of a complete theoretical foundation about the interrelationship among industry stock indices makes the building of an industry model vulnerable. The interaction between industry stock indices may result from the relationship of industry inputs and outputs, the arbitrary classification of industry groups and the product diversification in many companies. In general, a company is classified into a particular industry on the basis of its primary product. But, the company may diversify its business in different product lines or even in different industries. The difficulty in industry classification is one of the reasons for the interrelationship among industry stock indices. In addition, security returns could be correlated as a result of the industry interaction.

From an empirical perspective, the interrelationship between industry stock indices can be verified by calculating their correlation coefficients as demonstrated in Chapter IV. As far as the industry group chosen to build the industry model is concerned, a similar correlation matrix can be formed as a basis for determining the industry interaction in the industry model. In fact, these correlation coefficients provide an empirical reference in forming the individual equation in the industry model with industry interaction.

Table 2 presents the correlation coefficients between industry returns and between residual industry returns in different sample periods. Industry groups under the stock index models include Capital Goods, Consumer Goods, Financial, Oil, Transportation, and Utility. Residual industry returns are the residuals after removing the market effect represented by the S & P 500 stock market index. Different sample periods are chosen to measure the stability of the industry interrelationship over time. Mean and standard deviation of correlation coefficients are calculated to indicate the average degree and variation of industry interrelationship among the selected sample periods. The r** in the table represents the critical correlation coefficient that is significant at the 95% confidence interval with different sizes of time period in the sample.

Table 2a gives a very consistent picture of industry interrelationship. Nearly every entry is significant at the 95% confidence interval and most have a very high correlation. However, the high correlation between industry returns essentially results from the common correlation of industry indices to the market index. Based on this reason, the correlation coefficients between industry returns after removing the market effect are focused on in Table 2b.

Table 2b provides a more realistic picture on which the choice of industry interdependence is based. Among these correlation coefficients, those which are significant in terms of both degree and stability can be selected as the potential variables in the industry equations. For example, the relationship between Capital Goods and Utility, Consumer Goods and Oil, and Financial and Utility should not be ignored in building the industry model.

However, in forming each industry's equation in the industry model, the importance of the correlation coefficient between industry returns should not be overemphasized because of the different concept between correlation and regression (or dependence) and the consideration of simultaneity among industry equations in the model. The choice of variables in each industry equation should be appealing and fit common sense. The behavior of industry equations should be expected to persist over the different segments of sample period. Even though these constraints are not easily maintained simultaneously, it is important to keep them in mind and build a reasonable industry model within these constraints. The correlation coefficients of the related industries finally selected are indicated by "*" in Table 2.

For each equation in the industry model, it is implicitly assumed that an industry's return is directly dependent on another industry's return and the performance of an economic variable. Only one other industry is chosen to simplify the process of model building and reflect its primary influence on the particular industry under consideration. However, any secondary or indirect effect from other industries on the particular industry under consideration will be picked up through the simultaneity among industry equations in the industry model. In addition to the market effect and the related industry effect, one other economic variable is introduced in each equation to solve the identification problem in the simultaneous-equation econometric model. In the meantime, it is hoped that the inclusion of these variables will improve the performance of predicting the dependence structure of security returns under the Multi-Index Econometric Model. Listed below is the industry model in functional form:

CAP = f ( SP, FIN, DLEAD )

CON = f ( SP, TRA, IPC )

FIN = f ( SP, UTI, INT )

OIL = f ( SP, CON, GASP )

TRA = f ( SP, UTI, IPDT )

UTI = f ( SP, CAP, INT )

f(·) indicates "a function of". All the functions are assumed to be linear. SP represents the return of S & P 500 market index. CAP, CON, FIN, OIL, TRA and UTI are the six S & P industry groups: capital goods, consumer goods, financial, oil, transportations and utilities. DLEAD, IPC, INT, GASP and IPDT represent leading indicator, industrial production of consumer goods, interest rate, gasoline price and industrial production of transportation equipment respectively. Appendix B presents the three-stage least-squares estimates of the industry model in the two sets of sample periods.

In view of these two estimates, the second period yields a more appealing result that fits common sense on the basis of regression sign, t statistic (in the parentheses under the associated coefficients), standard error of the regression(SEE) and the Durbin-Watson(DW) statistic. In this period, most of t statistics are significant at 95% confidence interval (i.e., t>2.0). Most of DW statistics are around 2, indicating a small serial correlation in the regression residuals. Furthermore, regression coefficients present correct signs in the equation. For example, interest rates affect negatively the stock returns in the financial group because the rise in interest rates indicates an increase of operating costs in the financial group which hurts the earnings and prices of stocks in the group. Thus it may be expected that the industry equations in the second period will produce a more reasonable prediction of industry returns and dependence structure of security returns under the Multi-Index Econometric

Model.

As the results in the second period indicate, all industry returns are positively related to the market return and the beta coefficients range from 0.75 for the financial group to 1.71 for the transportation group. Industry returns are interrelated in the industry equations. The return of the utility group has a large negative, but significant, impact on the return of the transportation group and a moderate positive impact on the return of the financial group. In a moderate but significant degree, the return of the financial group affects inversely the return of capital goods. Similar negative impact happens to the return of the transportation group on the return of consumer goods and the return of capital goods on the return of the utility group. The return of consumer goods has only a minor impact on the return of the oil group.

Economic variables affect industry returns in different ways. Gasoline price affects positively and significantly the return of the oil group. Interest rate has a negative impact on both the financial and utility groups. As may be anticipated, increased industrial production of the transportation and the consumer goods helps the return of the transportation group and consumer goods respectively in a positive way. The leading indicator also generates a positive and significant impact on the return of capital goods.

### VII.A.b. Interindustry Analysis under the Multi-Factor Input-Output Model

In input-output analysis, the fundamental information concerns the flow of products from each producing industry to each of the consuming industries. This basic information is contained in an input-output transactions table. In the table, the columns describe the industrial and other value- added inputs required by a particular industry to produce its output. The rows indicate the distribution of a particular industry's output to different industries as inputs and to consumers as final demand.

Basically, Leontief's input-output model is developed from the transactions table. In Chapter IV, the mathematical derivation of the input-output model from the transactions table was presented. In essence, the matrix of input-output coefficients and the Leontief inverse are generated from the information in the transactions table. The Leontief inverse represents total requirements as distinguished from the direct requirements in the matrix of input-output coefficients. Given the final demands from different industries, the total outputs in each industry can be solved from the Leontief inverse by standard simultaneous-equation mathematics. Therefore, under the Multi-Factor Input-Output Model, the industry block is a system of simultaneous equations with the coefficients in the Leontief inverse as the parameter estimates.

The U.S. national input-output tables have been periodically compiled by the Bureau of Economic Analysis of the U.S. Department of Commerce since the late 1950s as a part of the national accounts. The original tables were developed at various levels of aggregation. In this study, the 7-sector aggregation of input-output tables was chosen. The investigation period is segmented into 1971 to 1975, 1976 to 1980 and 1981 to 1985; thus, the most appropriate choices are the 1967, 1972 and 1977 tables. However, industry classification and definition change over time. For the purpose of comparison, the input-output tables should conform with each other as closely as possible as to the industry classification and other conventions. The 1967, 1972 and 1977 tables used in this study are adapted from Miller and Blair (1985). The input-output coefficients and the Leontief inverse in 1972 and 1977 are presented in Appendix C.

Based on the Leontief inverse, the simultaneous-equation industry block under the Multi-Factor Input-Output Model can be formed, using equation (18). But without the predicted final demands by industry, there is no way to solve the simultaneous equations for the predicted industry outputs over the forecasting period. Unfortunately, the information of final demands by industry is not publicly available. However, since the focus is on the predicted "rate of change" in industry outputs over time, the national income by industry available in Citibase may serve as a surrogate for the final demands by industry.

Inspection of the input-output coefficients or the coefficients in the Leontief inverse in 1972 and 1977 reveals a general pattern. In most cases, the coefficients increase over time. The trend reflects the fact that the use of inputs has increased over time. This fact is more obvious in using Mining, Construction, Manufacturing or Trade and Transportation as inputs.

The above fact can also be seen more clearly in Figure 4. Figures 4a and 4b show the stability of input-output coefficients and the coefficients in the Leontief inverse on the basis of the 1972 and 1977 tables. The horizontal axis is used to measure the size of coefficients in 1972 and the vertical axis measures the size of coefficients in 1977. If all coefficients remained unchanged over the period from 1972 to 1977, then all the points would fall along a 45-degree line. On the other hand, if the coefficients increase over time, the points will fall above the 45-degree line. Similarly, if the coefficients have decreased over time, then the points will fall below the 45-degree line. Figure 4 shows that most of the points fall above the 45-degree line. It confirms the increasing use of inputs (especially, Mining, Construction, Manufacturing, and Trade and Transportation) in productive processes over the 1972-1977 period.

The stability problem of input-output data raises the question of how useful these coefficients are. Research studies in this field show that in most sectors structural change was very gradual.[13] This supports the contention that input-output coefficients may remain useful for a number of years, even though they may appear to be out of date in the year in which they were constructed. Therefore, in this study, 1972 coefficients are used to form the simultaneous industry equations and to predict industry outputs for the forecasting period from 1976 to 1980. Similarly, 1977 coefficients are used to predict industry outputs and thereby calculate their predicted rates of change for the forecasting period from 1981 to 1985. These two years (1972 and 1977) are the closest year to the forecasting period in which input-output tables are constructed and publicly available.

## VII.B. Predicting Industry Returns

As we mentioned in the beginning of this chapter, the industry model is built for the purpose of providing the parameter estimates for covariance and correlation predictions. The prediction evaluation of industry returns may help to detect the possible reasons for weakness in a model of covariance even though the covariance and correlation estimations discussed in the next chapter are not based on the prediction of industry returns.

Furthermore, the prediction of industry returns assumes knowledge of exogenous variables. For example, the levels of market index and other economic variables are assumed to be available in the prediction period. Based on this assumption, once the industry model is built, the prediction of industry returns can be generated.

Among the models of covariance investigated in this study, only the Multi-Index Model, the Multi-Factor Model and the models of covariance with industry interaction have the ability to predict industry returns. All the other models of covariance are simpler in nature because industry effect is not recognized explicitly as a force in the security return-generating function.

In the case of Multi-Index and Multi-Factor Models of covariance, industry returns are predicted through the orthogonal procedure by a simple regression equation in which the market effect or the effect of the national economy is the only factor. Therefore, the prediction of industry returns in these two models are relatively straightforward. With the parameter estimates such as beta coefficients and time series of actual market returns and growth rates in the national economy available, the prediction of industry returns can be generated by simple multiplication and addition.

The story of the models of covariance with industry interaction is entirely different. The industry block of these models is a system of simultaneous equations. For the Multi-Index Econometric Model, the popular Gauss-Siedel algorithm is used to solve the system of industry equations. The Gauss-Siedel method solves the entire system iteratively, substituting the required values of endogenous variables from the last iteration. For the Multi-Factor Input-Output Model and the Market Model with Input-Output Analysis, the standard procedure of solving simultaneous equations can be applied in terms of matrix inversion to solve the system of industry equations.

The root mean squared prediction errors (RMSE) and the Theil's Inequality Coefficient (TIC) of industry returns are provided in Table 3 to show the predicting power of the industry model. Since different industry classification systems are used for stock-index oriented and economic-factor oriented models of covariance, these two types of model are compared separately and their performance measures of industry return predictions are presented in different tables.

In Table 3a, the Multi-Index Model (MI) is compared with the Multi-Index Econometric Model (MIE). In this table, the results in the estimation period show the tracking ability of each industry model while the performance in the forecasting period indicates their forecasting power. The first panel shows the performance of these two models in the estimation period. In the first estimation period, among the six industry groups, only two of them performed better under the MIE model. The results in this period do not give a consistent pattern of the tracking ability between the MI and the MIE models. However, the picture becomes much clearer in the second estimation period. In all these six industry groups, the MIE model consistently performed better than the MI model in reproducing historical industry returns in the second estimation period. Besides, almost all the prediction errors of industry returns in the second estimation period are smaller than those in the first estimation period under both the MI and the MIE models.

The underlying reasons can be easily seen from the regression statistics of the system of industry equations as provided in Appendix B and discussed in the first section of this chapter. Even when the same functional industry equations exist under the MIE model, the regression results of the industry equation system tend to be more significant in the second estimation period. In general, these significant results are shown in terms of the t statistics, the standard errors of the regression (SEE) and the Durbin-Watson statistics (DW) and carried over to the prediction of industry returns.

The predicting power of the industry equations under both the MI and the MIE in the forecasting period is presented in the second panel of Table 3a. The forecasting pattern in both periods is widely dispersed. But all the industry equations under both models of covariance performed better than simple historical extrapolation in both forecasting periods as can be seen from the all less-than-one TIC's. The industry equations under the MIE model performed better in two industries in the first forecasting period and better in three industries in the second forecasting period. The consistent predicting ability of industry returns under the MIE model in the second period does not persist in the forecasting period. Therefore, it is possible that the prediction of dependence structure of security returns being analyzed in the next section may not give us a consistent result between the MI and the MIE models in the forecasting period.

Table 3b presents a comparison of industry return prediction among the economic factor oriented models of covariance. The models investigated in this table include the Multi-Factor Model (MF), the Multi-Factor Input-Output Model (MFIO) and the Market Model with Input-Output Analysis (MMIO). The results in this table should be interpreted carefully. Under the MFIO or the MMIO model, they are not prediction errors of regression equations. Instead, they are derived from the solution of a mathematical simultaneous-equation system with known coefficients. The solution is unique because the number of unknown variables is equal to the number of equations in the system. However, in an econometric simultaneous-equation model, the equations in the model are usually overidentified. In the case of overidentification, even though the parameter estimates of the structural-form coefficients can be obtained, there is no guarantee that the number of equations is equal to the number of predetermined variables in the system. Thus, an iterative technique is usually applied to solve the econometric simultaneous-equation model.

Since the prediction errors of industry equations under the MFIO or the MMIO model are derived from the solution of a mathematical simultaneous-equation system, there is only one result in the period of 1976-1980 regardless of whether this period is classified as estimation or forecasting period. But it may yield a different conclusion when compared to different results. The prediction performance of industry equations under the MMIO or MFIO model is quite consistent. No matter whether compared to the estimation or forecasting result of industry returns in the period of 1976-1980 under the MF model, all the industry groups except for mining (MIN) under the MFIO or MMIO performed better in predicting industry returns. The results under the models of covariance with input-output analysis are even more overwhelming in the period of 1981-1985. However, in the period of 1971-1975, most of their predictions of industry returns are dominated by the MF model.

Because the 1967 technical coefficients are used as the prediction in the 1971-1975 period, 1972 coefficients for the 1976-1980 period, and 1977 coefficients for the 1981-1985 period, it may be generalized that the technology and industry environment are more stable since 1970's. The superior performance of the MMIO or MFIO model over the MF model with respect to industry return prediction in the later years may be expected to reflect on the prediction of the dependence structure of security returns in those years as reveiled in the next chapter.

## VII.C. Predicting Security Returns

In the models of covariance with multiple effects in the security return-generating function, the predicted industry returns are used as the industry effect along with the market effect to generate security returns. The prediction of security returns is by no means the objective of this study. In fact, as a different research area, security returns should be predicted on the basis of some better constructed valuation models. Here, some simple security return-generating equations are used as intermediaries for the purpose of simplifying the estimation of the dependence structure of security returns. Therefore, the prediction of security returns is not necessarily closely related to the prediction of the dependence structure of these security returns. Nevertheless, it might provide some indications about the performance of different covariance models. It could also help to locate the underlying reasons for different results produced by various models of covariance.

The RMSE's of predicted security returns under different models of covariance are computed for each of 100 sample companies in the two sets of investigation periods and presented in Tables 4, 5, 6 and 7. Average models are not included in these tables because, by the nature of these models, they do not lead to the prediction of security returns. For each security, the model with the lowest RMSE is indicated by " * ". That means the security return-generating equation in this model of covariance has the best predicting power of security return for the particular security. The best performing security return prediction are compiled for each model of covariance with the summary percentage provided in Table 8.

According to Table 8, the performance of security return prediction among alternative models of covariance is very obvious. Regardless of what period used; estimation, forecasting, first, or second period, the security return-generating equation under the Multi-Index Model (MI) always performed best in predicting security returns. By and large, more than half of the securities were predicted more accurately by the equation in the MI model. Next to the MI model, the stock index related models of covariance with industry interaction such as MEI and MMIO likewise substantially outperformed in security return prediction.

The superior performance of the security equation in the MI model demonstrates its reproduction and forecasting powers in predicting security returns. It even overwhelms the better tracking records of industry return prediction under the MIE model as discussed in the previous subsection. That means the predicting power of an industry equation may not be transferred to a security equation in which the industry factor serves as one of the effects.

The major reason for the MI model's superior performance probably results from the construction of security return-generating equations under this model. Conceptally, the security equation of the MI model uses "actual" industry return information while the MIE model adopts the "predicted" industry return generated in the industry simultaneous-equation model. No matter how accurate the industry return prediction, some prediction errors are unavoidable over time. Furthermore, it may be easier to estimate the impact on "actual" return than on "predicted" return. Of course, in reality, it is more reasonable to use predicted data in making forecasts because actual data is not available beforehand. But the comparison is still helpful because ex-post evaluation is being conducted and the prediction may also be affected by other factors such as the estimated degree of influence and the market factor in addition to the industry return itself.

Will the above fact have a great impact on the prediction of the dependence structure of security returns? It is possible, but it is not necessarily always true because the performance of different models of covariance is affected by the estimated degrees of influence from both market and industry factors and the prediction of market and industry return variations rather than the straight prediction of industry returns. As demonstrated later in the performance evaluation of covariance and correlation predictions, the results of industry and security return predictions are only partially reflected in the performance of alternative models of covariance.

# CHAPTER VIII

## PERFORMANCE EVALUATION OF ALTERNATIVE MODELS OF COVARIANCE

The main purpose of the industry model and security equations discussed in the previous chapter is to provide the necessary parameter estimates for covariance or correlation prediction. For the models of covariance with industry interaction, as long as all the parameter estimates are obtained from the regression results of the industry model and security equations, Equations (13), (20) and (22) will give the covariance estimation. In order to evaluate the prediction of dependence structure under the models of covariance with industry interaction, it is best to compare these models' results with all the other models.

Two investigation periods have been chosen to see whether consistent results are maintained among alternative models of covariance in different periods.

Both periods include a 10-year time span. The first sample period goes from 1971 to 1980; whereas the second one covers 1976 to 1985. In each sample period, the first five years are considered as the estimation period. It is used to generate the required parameter and variable estimates for the covariance or correlation predictions in the next five years (called the forecasting period.)

In addition to evaluating the performance of different models of covariance in the forecasting period, it is also interesting to see their performance in the estimation period using the parameters estimated in the same period. While the performance in the forecasting period measures the model's power to forecast future covariances or correlations, the result in the estimation period will indicate the model's ability to reproduce history.

Thus, several considerations are relevant in the performance evaluation of alternative models of covariance. The literature is not clear about the assumption of average models of covariance. A constant correlation is usually assumed in all the companies or in the subgroup of companies in the literature. Constant correlation can lead to some simple rules of security selection in portfolio analysis.[14] However, covariance is the direct input in the standard portfolio selection model. The assumption of constant covariance can also provide the required solution in the selection of an optimum portfolio.[15] Of course, the choice of different assumptions in the average models depends on a decision maker's choice regarding the use of a portfolio selection model. But, as far as the performance evaluation of different models of covariance is concerned, it will be shown whether different assumptions of the average model create a different ranking among all the available models of covariance. The result may affect a portfolio manager's decision about the appropriate assumption used in portfolio analysis.

In fact, under the constant correlation assumption in the average models, the covariance can be derived to be constant by multiplying the constant correlation by respective standard deviations. Standard deviations are also assumed to be constant in the group under estimation. On the other hand, the correlation coefficient between security returns under more sophisticated models of covariance can also be generated by dividing the predicted covariance by respective standard deviations. For these models, historical standard deviations will serve as the estimates of predicted standard deviations. All these assumptions and calculations boil down to the prediction of both covariance and correlation coefficient.

The performance evaluation in terms of both covariance and correlation coefficient may indicate reasons for different ranking among alternative models of covariance. For example, the different ranking may result from the assumption of average models, the parameter estimates of regression equation or the estimation of standard deviations. However, the evaluation of both covariance and correlation predictions has its own usefulness. It provides portfolio managers an opportunity of choice and comparison.

## VIII.A. Performance in the Estimation Period

The evaluation of different covariance models' performance in the estimation period enables understanding of these models' ability to reproduce a historical covariance or correlation matrix. The ability to reproduce history is the starting point in building a more sophisticated model. More complex models of covariance usually generate parameter estimates of regression equations and other statistics such as market and industry variances from the estimation period. If all these historical statistics and data are used to form the covariance or correlation coefficient implied by these models in the same period, they should be able to reproduce the history well enough.

Tracking is a simple way to test a behavioral model. A behavioral model must be able to track the historical record well enough in order to predict the future. If the model fails to reproduce history, it is quite possible that the model cannot perform well in predicting the future. However, it does not mean that a model with a good tracking record will always have high predicting power in the future. Tracking ability is just the first step in building a better model.

## VIII.A.a. Evaluation Based on Covariance

Covariance is the direct input in standard portfolio selection problem. Its use as an instrument in evaluating the performance of alternative models of covariance is a natural choice. Probably, that is also the reason why the model to simplify the procedure in portfolio analysis is called a model of "covariance". The use of covariance as the instrument also avoids the possible bias introduced by the estimation of standard deviation when correlation coefficient is used instead.

Table 9 provides the performance measures and decomposition information from reproducing historical covariance matrices in the two estimation periods for all the models of covariance under investigation. These models are arranged in descending order according to their performance from best to worst in terms of RMSE. The best model has the smallest RMSE and stands at the top of the ranking. All differences in the MSE are statistically significant at the 5% level of significance, except the group of models enclosed in brackets.

Comparing the two panels in Table 9, it can be seen that the performance of covariance reproduction among different models is fairly consistent in the two estimation periods. In both cases, the Multi-Index Econometric Model (MIE) performs the best. Average models of covariance stand in the middle and pure economic factor models of covariance perform the worst in reproducing the history.

In terms of RMSE, all the models of covariance produce smaller error in the second estimation period than in the first estimation period. In other words, all the models of covariance track the historical record better in the more recent years. This fact indicates that the pattern of covariance between security returns tends to be more stable in recent years. Therefore, it may be expected that most of these covariance models will also have less prediction error in the second forecasting period as compared to the error in the first forecasting period. This assertion will be verified later in this chapter. Due to the stability of covariance pattern in more recent years, it is also probable that the ranking among different models of covariance created in the second forecasting period is more reliable. The result of prediction in the two forecasting periods will be shown later in this chapter.

From the decomposition of the MSE, it can be seen that, in each model of covariance, except the overall average and pure economic factor models, most of the prediction error is due to difference in co-variation or due to random disturbance in prediction. One version of overall average model (OA-CV) assumes a constant covariance in all off-diagonal entries of the covariance matrix. The standard deviation of these entries is, therefore, zero. The zero standard deviation results in the zero figure in the covariance and the efficiency components of the MSE. The large proportion of bias and variance components under the pure economic factor oriented models (SF, MF & MFIO) is the major reason why these models cannot perform better than other models. Can much be expected from a model that mispredicts the average figure a lot? Of course not.

The inferior performance of pure economic factor models of covariance is attributable to the choice of economic factor in the security return-generating function. Although gross national product is widely considered as the indicator of prosperity in the national economy, it is by no means a good variable to simplify the procedure of portfolio analysis.[16] Probably, that is why the use or discussion of gross national product is seldom seen in the literature of capital market and portfolio theories. The appropriateness of gross national product in the model of covariance can be easily seen when replaced by a stock market index such as S & P 500 in the security return-generating process. In this case, SF and MFIO will become SI and MMIO. The table shows the improvement from this substitution in terms of both the amount of RMSE and the ranking among different models of covariance. In fact, the better performance of stock index oriented models of covariance demonstrates the above argument.

The most important result in Table 9 is the superior performance of the models of covariance with industry interaction such as MIE and MMIO over simpler models. If the pure economic factor models are ignored due to the argument mentioned above, the consistent performance of the models of covariance with industry interaction confirms the conclusion prevailing in the literature: the more complicated model does a better job in reproducing the historical covariance structure of security returns.[7] In general, stock index models dominate average models and multi-index models perform better than single-index model in reproducing the history. Finally, the models of covariance with industry interaction track historical record better than other simpler models. However, the inferior performance of pure economic factor models adds reservations to the above general conclusion. The more complicated models can do a better job in reproducing the historical covariance structure of security returns only when reasonable indices or factors are chosen as the common effects in the return-generating process.

Compared to other stock index and economic factor models, the reason why the models of covariance with industry interaction can perform better in the estimation periods is not very clear as far as the decomposition of MSE is concerned. Roughly speaking, the superior performance may be attributable to the smaller proportions of prediction error representing bias and variance. In other words, the estimates under these models tend to be more BLUE (Best Linear Unbiased Estimator) than those under other stock index and economic factor models. Therefore, most of their errors result from random disturbance in prediction. The interpretation is more apparent when the models with industry interaction are compared with the pure economic factor models. However, this explanation should not be applied to the comparison with average models, especially the Overall Average Model, due to the special assumption of average models. The average model assumes a common mean in the group. This assumption purports to reduce the bias component in the prediction error.

The different assumption of constant covariance or correlation under the average models does not change their ranking in the performance evaluation of covariance prediction in the estimation periods. But, the assumption of constant covariance does improve the performance and reduce the prediction error of covariance prediction even though the improvement is not significant at the 95% confidence interval. The impact of this different assumption on the covariance prediction is consistent in the two estimation periods. The reason for this improvement can be seen more clearly from the error components of MSE. In most cases, the use of the covariance assumption in the covariance prediction reduces both the bias and the variance proportions of MSE. In other words, the estimate of respective standard deviations contributes some errors in the covariance prediction in the two estimation periods.

VIII.A.b. Evaluation Based on Correlation Coefficient

For a long time, correlation coefficient has been used in the literature to evaluate the performance of alternative models of covariance. While the original reason for its use is untraceable, the easy interpretation of its range is of clear value. Since correlation coefficient is equal to covariance divided by respective standard deviations, the estimate of these standard deviations may result in different ranking among alternative models of covariance when these two different statistics are used as an instrument in evaluating the performance of predicting the dependence structure of security returns.

The evaluation based on correlation coefficient has its own usefulness. First of all, some simple rules of portfolio selection have been proposed in the literature on the basis of correlation coefficient. These rules are ready to use. Secondly, most studies evaluating the performance of different models of covariance estimate the correlation structure of security returns. Several conclusions about the correlation prediction have been drawn. Results of evaluation on the same basis as previous studies can provide a more legitimate comparison with their conclusions. Finally, the comparison of the performance of these two statistics using the same data set may reveal the underlying reasons of different conclusions, if any.

Table 10 gives the performance measures of estimating the historical correlation coefficient between security returns among different models of covariance in the two estimation periods. The results in the two estimation periods are still consistent with each other in some degrees. In particular, when all the new models of covariance proposed in this study are ignored, the conclusion prevailing in the literature is still held. That is, more complicated models did a better job in reproducing the historical correlation structure of security returns.

Once the proposed models of covariance including the models with industry interaction are introduced, the pattern becomes a little less obvious. Compared to the result based on covariance in Table 9, the pure economic factor models still perform the worst and the average models still stand in the middle. But the ranking among the stock index models changes a little bit. The Multi-Index Econometric Model (MIE) still performed the best in the second estimation period; but in the first estimation period, it fell behind the simple Multi-Index Model (MI). The Market Model with Input-Output Analysis (MMIO) beats the Single-Index Model (SI) in both periods; but it was beaten by the MI in both periods. In general, if the pure economic factor models are excluded, it may still be concluded that more complicated models performed better in reproducing the historical correlation structure of security returns in the second period.

The RMSE's in the second estimation period are not consistently lower than those in the first estimation period any more. The amount of RMSE under the stock index oriented models was increased in the second period. But all the other models still produced less prediction error in the later period. Here, it can be seen that the estimate of standard deviation changes the consistent pattern of covariance estimation under the stock index oriented models of covariance.

## VIII.B. Performance in the Forecasting Period

The main purpose of building a model of covariance is to predict future covariances or correlation coefficients between security returns. The tracking power of alternative models of covariance has been examined in the previous section for the purpose of verifying their potential predicting ability in future periods. However, a model with high tracking power may not yield a good performance in predicting the future. The problem is the stability of a model.

A model's stability per se is a purely empirical issue. Theoretically, It may be justified that a more complicated model should be able to produce better result over time. But, once an empirical test is performed, a series of problems arise. Data on each variable must be available and as accurate as possible. Testing periods should be chosen to avoid some other major disturbances. They should also represent typical samples in the population of time periods. Parameter estimation is also critical. An econometric method should be applied to ensure that the estimator is best and unbiased. If the model is complicated, it may not be easy to obtain a feasible and cost-beneficial estimation technique. In addition, because of an unstable universe, the best estimator may not give the best prediction. All these problems have nothing to do with the construction of a theoretical model; but they may contribute empirically to the stability of the model's performance. Therefore, the empirical results should be interpreted carefully.

As far as the models of covariance are concerned, the above empirical problem is attacked in several ways. Two investigation periods are chosen to examine the consistency of different covariance models. A sophisticated econometric method is used to estimate the parameters in the models of covariance with industry interaction. The potential benefits of the Bayesian approach are also investigated in its improvement on the parameter estimates, and thereby the prediction of future covariance and correlation matrices. The reproduction ability that is analyzed in the last section gives an assurance that more complicated models are well constructed. However, the performance of these models in the forecasting period is of major concern. These models' performance in the forecasting period tells their predicting power in generating the future dependence structure of security returns.

VIII.B.a. Evaluation Based on Covariance

The prediction of future covariance between security returns should be based on the information available at the beginning of the forecasting period. In the case of average models, the use of historical data is quite obvious. A common covariance is assumed in the group on the basis of the historical covariance matrix in the estimation period. The common covariance then serves as the prediction in the forecasting period. For other models of covariance, such as stock index models, economic factor models and models with industry interaction, both parameter estimates and variable prediction should also come from historical information. Parameters are estimated using the data in the estimation period. Variable statistics such as the variances of market return and industry return are also formed on the basis of historical data.

However, sometimes the actual data in the forecasting period can be used to form these variable statistics in order to detect significant disturbances in the estimation periods. The assumption of perfect information for these variables helps to avoid the biased prediction on these variables in the process of generating a future covariance matrix. In this case, the performance of covariance prediction is solely attributed to the parameter estimates in the security return-generating function. Even though actual data are not the basis of our tables and figures, yet this information has been used to detect the instability of market and industry variances in the first investigation period.(See footnote 18)

Based on all historical information, Table 11 presents the performance measures and decomposition results of covariance prediction produced by different models of covariance in the two forecasting periods under investigation. The two panels in this table fail to give consistent results. As a matter of fact, the results in these two panels are completely different. In the first forecasting period, the pure economic factor models performed best, followed by the average models and then the stock index oriented models. But, in the second forecasting period, the stock index oriented models stand at the top, the average models follow next and the pure economic factor models are the worst. The pure economic factor models performed even worse than the benchmark full historical model in the second forecasting period.

Compared to the performance results in the estimation period in Table 9, the ranking of alternative covariance models in the second forecasting period is more or less justifiable. However, their results in the first forecasting period is really peculiar. The order generated in terms of reproduction ability and forecasting power among different models of covariance in the first investigation period is nearly reverse. A model's tracking record gives no indication at all about its forecasting power in this case! In fact, this peculiar inference results from the instability of market volatility during the period from 1971 to 1980. The volatility in the stock market in the estimation period

1971-1975 was greatly influenced by the Arab oil embargo in 1973. Thus, the historical variances of market, industry and security returns are a biased prediction of these various variances in the forecasting period 1976-1980. The instability of these variable statistics contributes to the peculiar ranking in the first forecasting period. If perfect information about these variable statistics were available, the substitution of actual data for the historical biased prediction would yield a more reasonable result.[18]

As indicated in the evaluation of the models' performance in the estimation period, the consistently lower RMSE in different models of covariance in the second period provides us an indication of a more reliable ranking in the second investigation period. This phenomenon also happens to all the average and the stock index oriented models of covariance in the second forecasting period. In addition, most of these models perform better than the benchmark full historical model because their TIC's are all less than one.

In view of the decomposition of the MSE, the consistently larger bias proportion (UM) in the first forecasting period should be the major reason why this period produced a greater amount of RMSE among all the models of covariance. This phenomenon also happens to the pure economic factor models in the second forecasting period. Most of the prediction errors generated by different covariance models in the second forecasting period are still attributed to co-variation (UC) or random disturbance (UE) in prediction.

The different assumption of constant covariance or correlation in the average models still cannot give them a breakthrough in the ranking. The constant covariance assumption may have a better result than the constant correlation assumption in some cases even though the improvement is not significant at the 95% confidence interval in most cases. But they all gathered in the middle of the ranking in both investigation periods. Therefore, the conclusion of the superiority of the average model over the index model as appears in the literature cannot be applied to the case where covariance is used as the instrument of evaluation.

Based on the results in the forecasting period, in general, it can be concluded that the stock index models predicted the future covariance structure better than the average models and the multi-index model performed better than the single-index model among the stock index models. But the more significant result is that the more sophisticated models of covariance with industry interaction cannot beat the multi-index model in this case. In other words, the more complicated models just pick up random errors in predicting future covariance.

The failure of the models of covariance with industry interaction may be attributed to the construction of industry equations. As discussed in the previous chapter, the inability of the industry equations under the Multi-Index Econometric Model to predict industry returns better is carried over to the prediction of security returns. This inability also reflects on the prediction of future covariance between security returns. The use of the Leontief input-output analysis in the market model gives a better result, but still falls a little behind the Multi-Index Model. This result is also partially reflected on the prediction of security returns. In fact, this may be the reason that causes the superior performance of the Multi-index Model in the second forecasting period.

The way to improve the forecasting performance of the models of covariance with industry interaction is by adjusting coefficients in industry and security equations. In the case of the input- output industry model, a so-called RAS approach can be used to reflect the changing techniques of production and adjust the technical coefficients. The parameter estimates in security return-generating equation may also be adjusted by the Bayesian approach. Table 12 provides the ranking of covariance prediction with Bayesian coefficient adjustment among different models of covariance in the forecasting period.

As shown in Tables 11 and 12, while the average models are not affected by Bayesian coefficient adjustment due to the setup of these models, the ranking among all the models of covariance is changed. In most cases, the Bayesian approach reduces the prediction error of covariance prediction. But this result is not guaranteed. The ranking of the models of covariance with industry interaction in fact is improved through the use of Bayesian coefficient adjustment. Nevertheless, in view of the amount of RMSE, the more complicated models of covariance with Bayesian coefficient adjustment still cannot outperform the simple Multi-Index Model in predicting future covariance structure of security returns. But the result will be different if a correlation coefficient is used to represent the dependence structure of security returns, as will be shown next.

VIII.B.b. Evaluation Based on Correlation Coefficient

The prediction of future correlation coefficient between security returns is also based on historical information. Average models assume a constant correlation coefficient in the group. The predicted constant correlation in the forecasting period is computed on the basis of the historical correlation matrix in the estimation period. In the cases of the stock index and the economic factor oriented models, first of all, the predicted covariance matrix is generated using the procedure described in the previous subsection. The predicted covariance between two security returns is then divided by the historical standard deviations of these two security returns to obtain their predicted correlation coefficient in the forecasting period.

Table 13 provides the performance measures and decomposition results of the correlation coefficient predicted by different models of covariance in the two forecasting periods. In both panels of this table, the Overall Average Model (OA) stands at the top of the ranking. All the pure economic factor models are located at the bottom and lag behind the Full Historical Model (FH). In other words, in predicting future correlation structure of security returns, the simplest OA model performed the best and the pure economic factor models performed the worst, even worse than simple historical extrapolation. All the models except for the pure economic factor models predicted better than simple historical extrapolation because they all had TIC less than one.

In terms of the amount of RMSE, all the models produced smaller RMSE's in the second forecasting period. That means all the models are able to predict more accurately in the second forecasting period. This may reflect the fact that the second investigation period is more stable. The fact can also be confirmed by looking at the relative proportions of MSE decomposition. For all these models of covariance, the proportion representing bias component (UM) is smaller in the second forecasting period than the one in the first forecasting period. Most of their errors result from random disturbance in prediction (UE). In other words, in the second forecasting period, all the models produced a more unbiased estimate so that they can predict a future correlation structure of security returns more accurately.

It is more interesting to examine the results in the first forecasting period. In this panel, by and large, the Overall Average Model (OA) stands at the top, followed by the Industry Average Model (IA), then the Single-Index Model (SI), the Multi-Index Model (MI) and finally the models of covariance with industry interaction. All these models performed better than the Full Historical Model (FH). The three pure economic factor models still lag behind the FH model because of the inappropriateness of Gross National Product in the model of covariance as discussed in the earlier part of this chapter. If the pure economic factor models were ignored, there would be a result consistent with the prevailing conclusion in the literature: simpler models of covariance performed better than more complicated models in predicting future correlation structure of security returns. That means the more complicated models just pick up random errors in predicting future correlation coefficient.

Nevertheless, the above conclusion is not so obvious in the second forecasting period. The dominance of the OA model is not significant any more. In the second forecasting period, even though the OA still stands at the top, yet the ranking among the IA and the stock index oriented models does not follow a pattern as expected. Their ranking in fact is just the opposite. Therefore, the conclusion that a simpler model of covariance performs better in predicting future correlation structure of security returns is not universal. The conclusion may be greatly dependent upon the investigation period chosen under study. It may also depend on the method used to estimate the parameters in the return-generating function. As shown next, the underperformance of the models of covariance with industry interaction can also be improved by the application of Bayesian adjustment to parameter estimates.

The parameters estimated for the security return-generating equation can be adjusted by the Bayesian approach described in Chapter III. Bayesian adjustment to parameter estimates had been demonstrated in the literature to be useful in improving the power of stock index models of covariance in predicting future correlation structure of security returns. Table 14 presents information similar to Table 13 except that Bayesian coefficient adjustment has been applied in this table. Of course, the average models are not affected by the application of Bayesian approach, but the ranking of different covariance models will be changed.

In the first forecasting period, the prediction errors generated by most of the stock index oriented models are reduced if the Bayesian approach is applied to adjust their parameter estimates. But, even the most sophisticated models of covariance with industry interaction still cannot outperform the OA model in this period. However, as far as the first forecasting period is concerned, the result is consistent with the one in the literature that Overall Average Model is a preferred method of forecasting future correlation coefficients in comparison to the more sophisticated models of covariance with Bayesian coefficient adjustment.

The result in the second forecasting period gives encouragement concerning the models of covariance with industry interaction proposed in this study. With Bayesian coefficient adjustment, the Multi-Index Econometric Model outperformed the Overall Average Model in predicting future correlation structure of security returns even though the improvement is not significant at the 95% confidence interval. Probably the second investigation period is so stable that the reproduction ability of the better models of covariance is able to be consistent with the predicting power of these models. If the results in the second estimation and forecasting periods in Table 10 and Table 14 are compared, it can be seen that, except for the OA model, the ranking of all the other models of covariance in these two tables matches perfectly with each other. Therefore, the ranking of forecasting performance among different models of covariance may be affected by the use of Bayesian coefficient adjustment and the stability of the dependence structure of security returns in the investigation period.

### VIII.B.c. A Synthetic Evaluation

Alternative models of covariance may be better evaluated by taking both covariance and correlation coefficient into account. Therefore, it is more convenient to present all the performance measures of different models of covariance in a graph. The Theil Inequality Coefficient (TIC) provides a consistent basis for this purpose. Figures 5, 7 and 9 show the performance of forecasting both the covariance and the correlation structures of security returns in both periods among alternative models of covariance. Figures 6, 8 and 10 present similar performance with Bayesian coefficient adjustment.

In Figures 5, 6, 7 and 8, the vertical axis represents the TIC of a model of covariance. The horizontal axis indicates the time period (first or second) and the prediction measure (covariance or correlation coefficient). The order of these four measures is arranged on the horizontal axis in a figure to show the changing ranking pattern of the prediction measure in different time periods among alternative models of covariance. For example, the vertical regions A and C in Figures 5 and 6 show whether the performance of alternative models of covariance is consistent between the first and the second forecasting periods in terms of covariance and correlation coefficient. Regions A and C in Figures 7 and 8 show whether their performance is consistent between covariance and correlation coefficient in both forecasting periods. The more cross-over lines, the less consistent among different models of covariance. If these lines are parallel with one another, the performance of different models is consistent in the sense of different periods or different measures. A model with line segments below the horizontal straight line representing the FH model performed better than simple historical extrapolation.

In Figures 5 and 6, many lines cross one another. That means, regardless of whether covariance or correlation coefficient is used as the measure, a consistent result in the two investigation periods is not generally realized. It reflects the instability of the dependence structure of security returns over the whole testing period. But there are some exceptions. For example, without Bayesian coefficient adjustment, the OM model consistently performed the best on the basis of correlation coefficient. Under the same condition, simpler models of covariance tend to perform better in predicting future correlation structure of security returns as shown in Figure 5.

In Figures 7 and 8, many lines still cross one another in region A, but there are more parallel lines in region C. It indicates that the dependence structure of security returns is more stable in the second period and, thereby, the performance of different models of covariance is more consistent in the second period no matter which measure is used for the evaluation. More complicated models of covariance may perform better with Bayesian coefficient adjustment, as shown in Figure 8. This is especially true of the MIE model, which dominated all the other models under investigation in predicting the dependence structure of security returns in the more stable second period.

Figures 9 and 10 present the performance measures of different covariance models with and without Bayesian coefficient adjustment in different ways. They focus on the models' performance in comparison to simple historical extrapolation. Horizontal and vertical lines represent the TIC in the first and the second periods, respectively. Two symbols, * and +, in the graph, are used to indicate the covariance and the correlation prediction under different models of covariance. Consider the FH model as the origin. The points located in quadrant III perform better than simple historical extrapolation in both periods; whereas the points in quadrant I do worse in both periods. Quadrant II (IV) shows the models that perform better in the first (second) but worse in the second (first) period relative to the FH model. A 45-degree line can be drawn in these two figures to indicate the models' relative performance in the two periods in comparison to simple historical extrapolation. A model located above the 45-degree line performs better in the first period than it does in the second period relative to the FH model.

As can be seen in Figures 9 and 10, most points are clustered in the third quadrant and above the 45-degree line. That means most models of covariance performed better than simple historical extrapolation in both periods and had more predicting power in the first period relative to the FH model regardless of whether covariance or correlation coefficient is used as the evaluation measure. The pure economic factor models are outliers. Based on correlation coefficient, they performed worse than simple historical extrapolation in both periods. If covariance is used as the measure, they performed far better in the first period than they did in the second period. The more complicated models of covariance such as MIE, MMIO, and MI predicted the correlation structure of security returns better in the second period relative to the performance of the FH model because they are located under the 45-degree line.

In sum, most models of covariance performed better than simple historical extrapolation no matter which measure is chosen, which period is tested, or whether the Bayesian coefficient adjustment is used. Without Bayesian coefficient adjustment, a result consistent with the conclusion in the literature is obtained, when the correlation coefficient is used to represent the dependence structure of security returns. That is, simpler models of covariance performed better than complicated models in predicting the future correlation structure of security returns. But when the Bayesian approach is applied to adjust the parameter estimates, the more sophisticated models of covariance with industry interaction exceeded simpler models in a more stable investigation period.

# CHAPTER IX

## CONCLUSION AND SUGGESTIONS

In portfolio analysis, it has been learned that the means and variances of security returns are required in generating efficient portfolios. Since the number of covariance grows rapidly as the number of securities under consideration increases, some models of covariance have been proposed to reduce the requirement of covariance input. Among them, average models and stock index models are widely known. In this study, in addition to experimenting with economic factor models, another type of model of covariance is proposed using a simultaneous-equation approach to improve the prediction of future covariances or correlation coefficients between security returns.

It has been shown in this study that significant interrelationships exist among industry returns after removing the market effect. Thus it would be beneficial to incorporate the industry interrelationship into a model of covariance. Based on this reason, the Multi-Index Econometric Model was built to capture the market effect and the industry effect with industry interaction. The phenomenon of industry interaction in the national economy establishes the basis to build the more theory-oriented Multi-Factor Input-Output Model. A mixed model of covariance with industry interaction is also proposed to combine the Market Model with the Leontief Input-Output Analysis. These three covariance models with industry interaction were compared to other models of covariance with respect to their ability to estimate the dependence structure of security returns.

The dependence structure of security returns may be represented in terms of either covariances or correlation coefficients. While covariances are direct inputs to the standard portfolio selection problem, correlation coefficients are widely used and easy to interpret. Both measures were investigated in this study. Since the correlation coefficient is equal to covariance divided by respective standard deviations, the estimate of standard deviations may affect the ranking of performance among different models of covariance. The choice between these two measures largely depends upon the portfolio manager's objective. However, the possible different results based on different measures should be recognized in the application of a covariance model.

The evaluation of the dependence structure of security returns predicted by alternative models of covariance is based on the MSE criterion. The MSE can be decomposed into different components of prediction error. The decomposition of the MSE enables one to understand the sources of prediction error and suggests possible ways of improvement for the proposed covariance models. All models of covariance are also compared to a full historical model to see whether they can predict better than simple historical extrapolation. This ability is best described in terms of the Theil Inequality Coefficient.

Measures of fit in the estimation period show how well different models of covariance track the historical dependence structure of security returns. In terms of covariance, the Multi-Index Econometric Model performed the best in both estimation periods. Except for the pure economic factor models of covariance, more complex models reproduced the historical covariance structure of security returns better than simpler models did. In other words, the models of covariance with industry interaction performed better than the multi-index model which, in turn, dominated the single-index model and the average models performed the worst. The superior fit of the models of covariance with industry interaction demonstrates their ability to replicate the past, which provides a good foundation for the prediction of the future covariance structure of security returns.

If correlation coefficient is used to represent the dependence structure of security returns, the ranking of fit in the estimation period among different models of covariance changes a little bit. While the multi-index model, the single-index model and the average models are still ranked in the same descending order, the dominance of the models of covariance with industry interaction is more apparent in the later and more stable estimation period. In the earlier estimation period, the models of covariance with industry interaction are dominated by the simple multi-index model. The pure economic factor models of covariance still performed the worst among all the models investigated in the case of reproducing the historical correlation structure of security returns.

In general, as long as the choice of underlying factors in security return-generating equation is reasonably good, more complicated models of covariance typically perform better than simpler models in reproducing the historical dependence structure of security returns. However, as demonstrated in this study, the ranking of historical fit among different models of covariance is affected by the choice of underlying factors, the use of covariance or correlation coefficient as the instrument in performance evaluation, and the stability of security's interdependent structure in the investigation period.

The forecasting power of different models of covariance is the major concern in this study. More complicated models of covariance are built for the purpose of improving their forecasting power in generating the future dependence structure of security returns. In terms of covariance, the better tracking record of the models of covariance with industry interaction does not carry over to the forecasting period. Their performance may be better in a more stable investigation period. But it is possible that the more complicated models of covariance still cannot exceed the simpler multi-index model in predicting the future covariance structure of security returns. The picture looks better if the parameter estimates of a security return-generating equation are adjusted using the Bayesian approach. Nevertheless, the Bayesian coefficient adjustment does not guarantee the improvement of all the models' forecasting ability. In most cases, the Bayesian approach reduces the forecasting error; but it may also give an opposite result.

When correlation coefficient is employed as the measure representing the dependence structure of security returns, some interesting results are generated. Consistent with the conclusion in the literature, simpler models of covariance outperformed more complicated models in predicting the future correlation structure of security returns. The result is more apparent in the earlier forecasting period. In both forecasting periods, the Overall Average Model dominated all the other models of covariance. However, among all the other models of covariance, more complicated models performed better than other simpler models in the more stable second forecasting period. In both forecasting periods, the pure economic factor models performed the worst and even worse than simple historical extrapolation.

Another important result which is consistent with the conclusion in the literature in the case of correlation coefficient is the improvement of forecasting power in different models of covariance when the Bayesian coefficient adjustment is employed. The Bayesian approach also enables the Multi-Index Econometric Model to surpass all the other models of covariance (including the Overall Average Model) in predicting the future correlation structure of security returns in the more stable, second forecasting period. Thus, it can be seen that, in predicting the future dependence structure of security returns, the use of Bayesian approach, the stability of security interdependence in the investigation period, the decision of covariance or correlation coefficient and the choice of underlying factors are all important.

In sum, a new approach to modelling covariance is introduced in the hope of providing a better prediction of the dependence structure of security returns. When the objective and prediction procedure are determined appropriately, the simultaneous-equation approach to the model of covariance yields the anticipated result. However, the result is very sensitive to the various factors mentioned above. Models of covariance with industry interaction can still be improved. For example, a better industry model should be able to be built to improve the prediction of industry returns and the dependence structure of security returns. Some techniques can also be applied to adjust the coefficient estimates in the industry model. In the case of Leontief Input-Output Model, a so-called RAS approach can be applied to adjust the technical coefficients in the model and, thereby, reflect the changing techniques of production over time.

The simultaneous-equation approach to the model of covariance can be applied to predict the future dependence structure of security returns in an international context. More and more empirical evidence supports the integration of international capital markets. It becomes more common for individual and institutional investors to hold securities issued in different countries to obtain the potential gains from international portfolio diversification. Gross national products also become more related due to the large volume of international trade. The characteristics of country interrelationship among national capital markets and trading patterns gives an incentive to apply the simultaneous-equation approach to an international model of covariance. As a matter of fact, in Project LINK, different country or regional econometric models have been linked together through the use of a trade matrix to reflect the trading interrelationship among these countries or regions. A similar procedure could also be applied to the model of covariance with national interaction. This extension should provide an interesting and promising topic for future research.

FOOTNOTES:

1. See Markowitz(1959, p.100)

2. For the proof, see Elton and Gruber (1984, pp.154-155).

3. The only exception is the Elton and Gruber(1987) study. In this theoretical study, they derived a portfolio selection procedure based on the assumption of constant covariance under the average model of covariance.

4. See Johnston(1984, p.82) for the proof of the equality of regression coefficients in the orthogonal case.

5. The ASA-NBER survey is a consensus forecasting system. As indicated in Su and Su (1975 & 1983), the ASA-NBER predicts most aggregate variables such as GNP relatively better than econometric forecasting services in the short-term forecasting horizon.

6. See Morrison(1976, pp.102-103) for a detailed discussion of the distribution of correlation coefficient.

7. See Manne and Markowitz(1963).

8. For example, see Miller and Blair(1985, p.289).

9. For a detailed discussion of the decomposition of the MSE, see Theil(1971, p.29).

10. For a detailed discussion of the decomposition of the MSE, see Mincer and Zarnowitz(1969, p.11). An excellent empirical application of the decomposition can be found in Su and Su(1975).

11. The discussion of hypothesis testing on the mean can be found in Mood, Graybill and Boes(1974, pp.428-431).

12. For a detailed discussion of the Inequality Coefficient, see, Theil(1971, p.28).

13. For example, see Anne P. Carter, Structural Change in The American Economy, Cambridge: Harvard Press, 1976.

14. See Elton, Gruber and Padberg(1976) for the simple rules under the assumption of constant correlation.

15. In the case of constant covariance, reference can be made to Elton and Gruber(1987).

16. The argument that the stock market leads the national economy is not an issue here. Introduction of this argument was attempted in the performance evaluation, the ranking created by this experiment is the same as the one reported here.

17. For example, Elton and Gruber(1973) reported the same result in terms of correlation coefficient.

18. A ranking with actual market and industry variances moves most of the stock index oriented models to a higher position. If the Bayesian coefficient adjustment is applied to all the stock index and economic factor oriented models, the MMIO model would perform the best among different models with RMSE equal to 60.97 in the first forecasting period.

Table 1: Models of Covariance Evaluated in This Study

A. Old Models:
   a. Benchmark Model:
      Full Historical Model (FH)
   b. Average Models:*
      Overall Average Model (OA)
      Industry Average Model** (IA1 & IA2)
   c. Stock Index Models:
      Single-Index Model (SI)
      Multi-Index Model (MI)

B. New Models:
   d. Economic Factor Models:
      Single-Factor Model (SF)
      Multi-Factor Model (MF)
   c. Models of Covariance with Industry Interaction:
      Multi-Index Econometric Model (MIE)
      Multi-Factor Input-Output Model (MFIO)
      Market Model with Input-Output Analysis (MMIO)

* For average models, if covariance is used as the performance measure, both the overall average and the industry average models will have two versions. One version predicts covariance directly (OA-CV, IA1-CV and IA2-CV); the other version derives covariance from correlation coefficient (OA-CC, IA1-CC and IA2-CC).

** Industry average model has two variations: IA1 is the model based on the S & P industry classification system and IA2 is the one based on the Standard Industrial Classification system.

Table 2: Industry Correlation Coefficients in Different Sample Periods

a. Between Industry Returns

| | | Sample Periods | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1-60 | 1-20 | 21-40 | 41-60 | 1-40 | 21-60 | 13-52 | Mean | St.Dev. |
| | r12 | .8406 | .9230 | .8450 | .8081 | .8849 | .7941 | .8575 | .8506 | .0439 |
| * | r13 | .7911 | .8040 | .7733 | .7943 | .7926 | .7819 | .7679 | .7893 | .0097 |
| | r14 | .5920 | .5035 | .6705 | .5871 | .6096 | .6015 | .6373 | .6002 | .0516 |
| | r15 | .8191 | .7458 | .7858 | .9152 | .7640 | .8557 | .8687 | .8220 | .0612 |
| * | r16 | .6216 | .7062 | .4639 | .6905 | .6189 | .5661 | .6187 | .6123 | .0809 |
| | r23 | .8077 | .7740 | .8918 | .8229 | .8085 | .8426 | .8188 | .8238 | .0365 |
| * | r24 | .3717 | .4245 | .6134 | .2749 | .4721 | .3291 | .4120 | .4140 | .1092 |
| * | r25 | .6981 | .7271 | .6425 | .8317 | .5711 | .6996 | .7186 | .7127 | .0598 |
| | r26 | .7036 | .7045 | .6674 | .7073 | .6893 | .6944 | .6722 | .6912 | .0160 |
| | r34 | .4952 | .6364 | .6427 | .3090 | .6352 | .4018 | .5470 | .5239 | .1301 |
| | r35 | .7060 | .7328 | .6137 | .7709 | .6770 | .6907 | .6974 | .6984 | .0486 |
| * | r36 | .7993 | .8231 | .7737 | .8131 | .8065 | .7862 | .8165 | .8026 | .0176 |
| | r45 | .5028 | .2972 | .6594 | .5522 | .4810 | .5919 | .6404 | .5321 | .1229 |
| | r46 | .5517 | .7357 | .5622 | .4171 | .6728 | .4138 | .5774 | .5615 | .1195 |
| * | r56 | .4782 | .5139 | .2710 | .6947 | .4185 | .4581 | .4864 | .4744 | .1258 |
| | r** | ±.254 | ±.444 | ±.444 | ±.444 | ±.312 | ±.312 | ±.312 | | |

b. Between Residual Industry Returns

| | | Sample Periods | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1-60 | 1-20 | 21-40 | 41-60 | 1-40 | 21-60 | 13-52 | Mean | St.Dev. |
| | r12 | -.357 | -.122 | -.396 | -.365 | -.251 | -.383 | -.282 | -.308 | .098 |
| * | r13 | -.235 | -.228 | -.462 | -.171 | -.298 | -.290 | -.343 | -.290 | .094 |
| | r14 | -.083 | -.235 | -.305 | .037 | -.221 | -.056 | -.174 | -.148 | .119 |
| | r15 | .268 | .215 | .263 | .065 | .262 | .212 | .333 | .231 | .084 |
| * | r16 | -.557 | -.587 | -.586 | -.428 | -.594 | -.515 | -.525 | -.542 | .059 |
| | r23 | .042 | -.363 | .320 | .263 | -.125 | .304 | .132 | .082 | .253 |
| * | r24 | -.746 | -.830 | -.598 | -.724 | -.777 | -.715 | -.791 | -.740 | .074 |
| * | r25 | -.219 | .125 | -.430 | -.086 | -.171 | -.315 | -.313 | -.201 | .182 |
| | r26 | -.008 | -.490 | .211 | -.005 | -.172 | .186 | -.016 | -.042 | .237 |
| | r34 | -.154 | .244 | -.196 | -.496 | .104 | -.423 | -.143 | -.152 | .263 |
| | r35 | .019 | .298 | -.281 | -.230 | .082 | -.246 | -.146 | -.072 | .213 |
| * | r36 | .428 | .415 | .564 | .424 | .445 | .481 | .529 | .469 | .058 |
| | r45 | -.034 | -.307 | .139 | -.076 | -.062 | .085 | .129 | -.018 | .156 |
| | r46 | .126 | .497 | .115 | -.130 | .318 | -.088 | .136 | .139 | .218 |
| * | r56 | -.368 | -.210 | -.486 | -.306 | -.349 | -.450 | -.402 | -.367 | .092 |
| | r** | ±.254 | ±.444 | ±.444 | ±.444 | ±.312 | ±.312 | ±.312 | | |

* : the correlation coefficients of the related industries that are finally selected in the industry equations.

r** : the critical correlation coefficient that is significant at 95% confidence interval.

Note: Subscripts 1, 2, 3, 4, 5 and 6 represent industries CAP, CON, FIN, OIL, TRA and UTI respectively. Sample period indicates the quarters covered in the sample. For example, 1-60 means the sample covers the first quarter to the 60th quarter, i.e., the first quarter in 1971 to the fourth quarter in 1985. Mean and standard deviation represent these two statistics over the seven selected sample periods in this table.

Table 3: Industry Return Prediction

a. S & P Industries:

| | MI | | MIE | | MI | | MIE | |
|---|---|---|---|---|---|---|---|---|
| | RMSE | TIC | RMSE | TIC | RMSE | TIC | RMSE | TIC |
| 1. Estimation Period: | | | | | | | | |
| | 1971-1975 | | | | 1976-1980 | | | |
| CAP | 2.396* | - | 2.692 | - | 2.769 | - | 2.214* | - |
| CON | 3.234* | - | 3.259 | - | 2.485 | - | 2.292* | - |
| FIN | 6.248* | - | 6.249 | - | 4.159 | - | 3.075* | - |
| OIL | 6.908 | - | 6.868* | - | 4.652 | - | 3.610* | - |
| TRA | 6.812 | - | 6.710* | - | 6.394 | - | 4.704* | - |
| UTI | 5.813* | - | 6.840 | - | 5.155 | - | 3.724* | - |
| 2. Forecasting Period: | | | | | | | | |
| | 1976-1980 | | | | 1981-1985 | | | |
| CAP | 2.897* | .1806 | 3.113 | .1959 | 2.984 | .1538 | 2.783* | .1391 |
| CON | 3.414* | .2096 | 3.556 | .2166 | 5.122 | .3012 | 4.848* | .2845 |
| FIN | 4.214 | .2393 | 4.044* | .2311 | 6.016* | .2799 | 6.148 | .2817 |
| OIL | 6.067* | .4453 | 6.606 | .4899 | 8.992* | .4998 | 10.820 | .5706 |
| TRA | 7.052* | .4185 | 8.043 | .4843 | 4.125* | .1970 | 5.368 | .2489 |
| UTI | 5.412 | .4193 | 4.904* | .3476 | 4.155 | .3478 | 4.077* | .3323 |

b. SIC Industries:

| | MF | | MFIO & MMIO | | MF | | MFIO & MMIO | |
|---|---|---|---|---|---|---|---|---|
| | RMSE | TIC | RMSE | TIC | RMSE | TIC | RMSE | TIC |
| 1. Estimation Period: | | | | | | | | |
| | 1971-1975 | | | | 1976-1980 | | | |
| MIN | 7.458 | - | 6.137* | - | 6.939* | - | 17.760 | - |
| CNT | 1.951* | - | 4.033 | - | 2.408 | - | 1.941* | - |
| MFT | 2.220* | - | 2.709 | - | 2.179 | - | 1.698* | - |
| TAT | 1.157* | - | 1.966 | - | 1.441 | - | .897* | - |
| SER | .523* | - | 1.023 | - | .740 | - | .447* | - |
| 2. Forecasting Period: | | | | | | | | |
| | 1976-1980 | | | | 1981-1985 | | | |
| MIN | 8.901* | .5906 | 17.760 | .7225 | 7.409 | .6119 | 3.871* | .4893 |
| CNT | 3.243 | .5239 | 1.941* | .2322 | 2.370 | .4316 | .747* | .1613 |
| MFT | 2.290 | .3587 | 1.698* | .2333 | 2.837 | .5375 | .774* | .1512 |
| TAT | 1.442 | .2656 | .838* | .1387 | 1.405 | .3184 | .320* | .0791 |
| SER | 1.154 | .2028 | .447* | .0684 | 1.179 | .2134 | .443* | .0970 |

* : indicates better prediction in the period.

Table 4: RMSE of Security Return in the First Estimation Period (1971-1975)

| | SI | MI | MIE | SF | MF | MFIO | MMIO |
|---|---|---|---|---|---|---|---|
| AMR | 21.199 | 16.767* | 19.695 | 28.303 | 28.079 | 28.139 | 21.159 |
| AMT | 8.346 | 8.302 | 8.071 | 12.396 | 12.258 | 12.178 | 7.612* |
| AET | 15.059 | 10.792* | 15.049 | 18.385 | 18.344 | 18.178 | 14.743 |
| AMX | 14.101 | 13.780 | 14.094 | 15.528 | 14.991 | 15.326 | 13.384* |
| AMB | 6.842 | 6.230* | 6.275 | 10.858 | 9.657 | 10.454 | 6.584 |
| ABC | 16.411 | 16.130 | 15.756 | 27.317 | 26.812 | 26.948 | 15.599* |
| AC | 9.254 | 9.047 | 8.091* | 10.871 | 10.230 | 10.529 | 9.063 |
| ACY | 9.865 | 9.560 | 8.647* | 12.868 | 11.316 | 12.707 | 9.698 |
| AEP | 8.194 | 5.017* | 8.186 | 11.826 | 11.823 | 11.417 | 7.512 |
| AMI | 19.247 | 19.165 | 18.415 | 29.228 | 29.228 | 28.763 | 18.371* |
| AST | 17.591 | 16.333 | 15.488* | 22.991 | 20.759 | 22.705 | 17.306 |
| AR | 11.780 | 11.211* | 11.638 | 14.453 | 13.593 | 14.436 | 11.550 |
| ARC | 12.652 | 7.244* | 12.615 | 12.506 | 12.361 | 12.503 | 12.564 |
| AUD | 19.263 | 19.009 | 17.665* | 29.978 | 29.914 | 29.835 | 18.959 |
| BT | 8.972 | 7.324* | 8.962 | 13.736 | 13.733 | 13.735 | 8.963 |
| BRY | 5.542 | 5.456 | 4.988* | 14.757 | 13.569 | 14.446 | 5.233 |
| BNL | 11.170 | 9.830* | 10.713 | 17.712 | 17.679 | 17.088 | 9.979 |
| BS | 10.007 | 9.614 | 9.627 | 13.229 | 11.018 | 12.574 | 9.549* |
| BMY | 13.895 | 13.493 | 12.866* | 17.609 | 15.006 | 17.029 | 13.496 |
| BNI | 18.318 | 15.450* | 17.296 | 18.325 | 17.928 | 17.777 | 17.738 |
| BGH | 7.796 | 7.649 | 7.794 | 12.967 | 11.931 | 12.099 | 7.057* |
| CBS | 12.287 | 10.602* | 11.878 | 18.285 | 17.184 | 18.285 | 12.279 |
| CAT | 8.660 | 8.192 | 7.734* | 13.546 | 11.519 | 13.299 | 8.578 |
| CHL | 7.942 | 6.244* | 7.771 | 13.125 | 13.116 | 13.110 | 7.933 |
| CHV | 8.952 | 5.446* | 8.337 | 10.278 | 10.166 | 10.211 | 8.919 |
| CCI | 11.666 | 8.448* | 11.260 | 16.297 | 16.061 | 16.248 | 11.623 |
| KO | 8.482 | 8.203 | 8.191 | 18.755 | 16.504 | 18.207 | 8.031* |
| CG | 9.128 | 8.160* | 8.613 | 12.538 | 12.527 | 12.533 | 9.127 |
| CWE | 6.335 | 4.147* | 6.187 | 9.450 | 9.433 | 9.391 | 6.216 |
| CSC | 28.120 | 26.929* | 27.631 | 37.152 | 35.936 | 36.756 | 27.742 |
| ED | 17.785 | 12.059* | 17.562 | 21.923 | 21.800 | 21.923 | 17.782 |
| CNF | 17.867 | 13.766* | 17.865 | 22.590 | 21.796 | 21.643 | 17.167 |
| CNG | 5.769 | 3.784* | 5.466 | 8.044 | 8.044 | 8.007 | 5.703 |
| CIC | 11.026 | 9.612* | 10.784 | 12.499 | 12.418 | 12.314 | 10.757 |
| DAL | 12.057 | 11.075* | 12.001 | 18.064 | 17.307 | 17.868 | 11.971 |
| DIS | 15.451 | 14.248* | 14.844 | 33.057 | 32.777 | 33.054 | 15.408 |
| DD | 8.931 | 8.813 | 8.149* | 13.247 | 13.007 | 13.235 | 8.863 |
| EAF | 10.999 | 10.975 | 10.999 | 18.487 | 17.816 | 18.482 | 10.915* |
| XON | 7.711 | 3.908* | 7.686 | 9.423 | 9.108 | 9.173 | 7.699 |
| FJQ | 20.182 | 20.043 | 19.077* | 32.050 | 29.966 | 31.885 | 20.116 |
| FIN | 18.773 | 18.680 | 18.546 | 23.473 | 22.897 | 22.525 | 17.545* |
| I | 11.389 | 9.994* | 11.388 | 17.650 | 17.604 | 17.627 | 11.371 |
| F | 10.803 | 8.110* | 10.057 | 15.087 | 14.988 | 14.909 | 10.470 |
| FWC | 22.908 | 20.799 | 22.878 | 25.615 | 24.696 | 23.977 | 19.748* |
| GE | 6.630 | 6.208* | 6.453 | 14.830 | 12.856 | 14.716 | 6.626 |
| GRL | 18.324 | 15.286* | 17.369 | 33.592 | 30.841 | 33.361 | 18.308 |
| GM | 8.095 | 6.889* | 7.774 | 14.092 | 13.589 | 14.077 | 8.073 |
| GSX | 13.287 | 13.011 | 12.253* | 20.721 | 17.145 | 20.690 | 13.269 |
| GT | 9.927 | 7.636* | 9.915 | 14.637 | 13.892 | 14.620 | 9.922 |
| GAP | 16.544 | 15.924* | 16.542 | 18.749 | 18.704 | 18.690 | 16.508 |

Table 4: RMSE of Security Return in the First Estimation Pericd (1971-1975) (Continued)

| | SI | MI | MIE | SF | MF | MFIO | MMIO |
|---|---|---|---|---|---|---|---|
| HAL | 13.031 | 13.031 | 12.982* | 14.463 | 14.380 | 14.267 | 13.004 |
| HCA | 23.451 | 23.210 | 22.021* | 37.067 | 37.057 | 36.349 | 22.044 |
| N | 9.412 | 9.208* | 9.375 | 12.619 | 12.495 | 12.611 | 9.357 |
| IR | 14.573 | 13.999 | 13.911* | 16.009 | 13.659 | 15.888 | 14.560 |
| IBM | 8.599 | 7.652* | 8.416 | 12.008 | 11.865 | 11.703 | 8.485 |
| HR | 8.682 | 8.514 | 7.755* | 13.082 | 12.036 | 12.966 | 8.530 |
| IGL | 17.816 | 17.394 | 17.201* | 17.614 | 17.480 | 17.596 | 17.696 |
| INI | 9.976 | 9.685 | 9.440* | 10.848 | 10.760 | 10.812 | 9.918 |
| KM | 10.736 | 10.441 | 10.002 | 17.496 | 17.400 | 16.559 | 9.523* |
| K | 8.494 | 8.140 | 8.054* | 11.336 | 10.507 | 11.222 | 8.489 |
| LLX | 16.256 | 12.876* | 16.248 | 17.773 | 17.323 | 17.740 | 15.936 |
| MCA | 19.781 | 19.780 | 18.997* | 25.725 | 25.497 | 25.584 | 19.447 |
| MZ | 17.196 | 16.055* | 17.177 | 24.558 | 24.110 | 23.537 | 16.416 |
| MCD | 11.207 | 11.110 | 11.090 | 24.935 | 24.917 | 24.417 | 10.522* |
| MHP | 13.771 | 12.584* | 12.824 | 22.796 | 20.456 | 22.740 | 13.766 |
| MOB | 8.579 | 6.315* | 8.132 | 11.818 | 11.756 | 11.812 | 8.508 |
| MTC | 10.111 | 9.768* | 10.093 | 15.509 | 14.553 | 14.893 | 9.873 |
| NWA | 13.813 | 10.700* | 13.359 | 23.921 | 23.679 | 23.193 | 13.097 |
| DR | 8.102 | 7.949* | 8.017 | 9.502 | 9.445 | 9.497 | 8.097 |
| NEM | 10.852 | 9.578* | 10.787 | 13.256 | 13.219 | 13.135 | 10.168 |
| OEC | 6.250 | 5.104* | 6.211 | 10.082 | 10.079 | 9.960 | 5.999 |
| OI | 11.869 | 10.510* | 11.858 | 14.597 | 14.082 | 14.597 | 11.848 |
| PCG | 7.674 | 5.702* | 7.614 | 8.929 | 8.904 | 8.656 | 7.349 |
| PLT | 6.404 | 6.092* | 6.389 | 9.640 | 9.627 | 9.639 | 6.403 |
| PEL | 14.001 | 13.350* | 13.996 | 16.017 | 16.003 | 15.651 | 13.543 |
| JCP | 10.537 | 9.466* | 10.155 | 18.952 | 18.191 | 18.945 | 10.517 |
| PD | 9.189 | 8.373* | 9.105 | 13.359 | 13.321 | 13.178 | 9.188 |
| P | 17.468 | 8.039* | 17.106 | 16.763 | 16.582 | 16.746 | 17.112 |
| PG | 7.789 | 6.901* | 7.740 | 10.860 | 9.592 | 10.817 | 7.788 |
| REV | 7.795 | 7.717 | 7.585 | 14.924 | 12.902 | 14.478 | 7.443* |
| RAD | 24.462 | 24.461 | 24.193* | 41.097 | 41.091 | 41.006 | 24.440 |
| RD | 8.439 | 8.439 | 8.345 | 12.784 | 11.925 | 12.069 | 7.774* |
| SLB | 13.995 | 13.972* | 13.976 | 15.582 | 15.520 | 15.565 | 13.988 |
| S | 6.054 | 4.369* | 6.021 | 14.873 | 14.828 | 14.747 | 5.973 |
| SNT | 10.815 | 9.204* | 10.804 | 13.430 | 13.429 | 13.408 | 10.779 |
| SQD | 9.297 | 9.252 | 9.281 | 17.263 | 16.564 | 17.262 | 9.236* |
| SUN | 9.858 | 8.130* | 9.790 | 9.028 | 9.001 | 8.964 | 9.858 |
| TFB | 20.445 | 18.927* | 20.404 | 34.720 | 34.718 | 34.527 | 19.947 |
| TX | 5.603 | 4.644* | 5.602 | 8.884 | 8.758 | 8.613 | 5.427 |
| TA | 11.755 | 11.500* | 11.755 | 19.083 | 19.083 | 19.080 | 11.755 |
| TIC | 10.817 | 7.300* | 10.716 | 17.701 | 17.630 | 17.556 | 10.490 |
| UAL | 18.002 | 10.650* | 17.748 | 23.782 | 22.507 | 21.520 | 15.687 |
| X | 11.616 | 10.289* | 11.036 | 13.866 | 11.838 | 12.695 | 10.830 |
| UCL | 13.497 | 7.823* | 13.496 | 13.092 | 12.746 | 13.072 | 13.293 |
| USH | 18.308 | 15.087* | 17.511 | 28.257 | 27.952 | 28.138 | 18.048 |
| WLA | 10.257 | 10.038 | 8.917* | 16.090 | 15.045 | 15.260 | 9.578 |
| WX | 10.833 | 10.726 | 10.651* | 20.269 | 18.506 | 19.986 | 10.789 |
| WHR | 9.299 | 8.176 | 7.034* | 21.776 | 18.811 | 21.350 | 8.941 |
| WMB | 17.063 | 17.016 | 16.948 | 16.179 | 15.977 | 16.132 | 16.753* |
| Z | 15.629 | 15.514 | 15.626 | 22.383 | 22.382 | 21.698 | 15.051* |
| # of * | 0 | 61 | 22 | 0 | 0 | 0 | 17 |

* : indicates the best prediction.

Table 5: RMSE of Security Return in the Second Estimation Period (1976-1980)

| | SI | MI | MIE | SF | MF | MFIO | MMIO |
|---|---|---|---|---|---|---|---|
| AMR | 16.121 | 15.026* | 15.221 | 17.444 | 17.397 | 17.416 | 16.115 |
| AMT | 12.765 | 12.727 | 12.453 | 12.363 | 12.123* | 12.338 | 12.351 |
| AET | 8.756 | 7.240* | 8.006 | 10.369 | 9.434 | 10.082 | 8.754 |
| AMX | 12.413 | 12.300 | 11.994* | 14.136 | 14.121 | 13.986 | 12.396 |
| AMB | 6.740 | 6.398 | 4.998* | 9.134 | 7.920 | 8.436 | 6.545 |
| ABC | 14.529 | 14.498 | 14.364 | 15.141 | 14.716 | 14.802 | 14.247* |
| AC | 6.495 | 5.966 | 4.805* | 8.264 | 8.250 | 8.216 | 6.441 |
| ACY | 9.164 | 8.788* | 8.805 | 10.994 | 10.994 | 10.941 | 9.107 |
| AEP | 6.489 | 3.342* | 4.457 | 6.722 | 6.238 | 6.398 | 6.080 |
| AMI | 12.159 | 11.141* | 12.157 | 14.173 | 14.122 | 14.135 | 11.144 |
| AST | 12.125 | 12.111 | 11.943 | 16.589 | 16.200 | 16.572 | 11.178* |
| AR | 22.034 | 21.984 | 21.980 | 23.267 | 21.134* | 23.126 | 21.604 |
| ARC | 8.622 | 7.053* | 7.811 | 9.208 | 9.208 | 8.867 | 8.524 |
| AUD | 6.658 | 6.546 | 6.325* | 9.609 | 8.662 | 8.743 | 6.469 |
| BT | 7.707 | 6.493* | 7.490 | 10.600 | 8.876 | 9.419 | 7.649 |
| BRY | 6.952 | 6.484 | 5.847* | 8.369 | 7.770 | 7.741 | 6.470 |
| BNL | 9.682 | 5.616* | 8.062 | 13.569 | 12.155 | 12.898 | 9.616 |
| BS | 10.807 | 9.936* | 10.494 | 15.165 | 15.158 | 15.129 | 10.328 |
| BMY | 7.382 | 7.310 | 7.072* | 8.796 | 8.628 | 8.099 | 7.337 |
| BNI | 14.054 | 11.015* | 13.326 | 16.588 | 16.557 | 16.378 | 13.996 |
| BGH | 11.458 | 11.201* | 11.457 | 12.798 | 12.716 | 12.727 | 11.306 |
| CBS | 6.524 | 6.512 | 6.491 | 7.315 | 6.799 | 6.715 | 6.412* |
| CAT | 5.706 | 4.694* | 5.522 | 8.038 | 7.969 | 7.905 | 5.328 |
| CHL | 6.686 | 5.280* | 6.529 | 9.125 | 8.316 | 8.715 | 6.685 |
| CHV | 8.858 | 6.575* | 8.288 | 9.771 | 9.712 | 9.764 | 8.827 |
| CCI | 8.526 | 7.105* | 8.455 | 12.141 | 11.342 | 11.826 | 8.388 |
| KO | 5.039 | 4.716 | 5.014 | 5.401 | 5.278 | 5.160 | 4.301* |
| CG | 8.369 | 8.208* | 8.340 | 8.629 | 8.588 | 8.522 | 8.366 |
| CWE | 7.309 | 4.295* | 4.983 | 7.614 | 7.158 | 7.329 | 7.020 |
| CSC | 18.088 | 17.699 | 17.624* | 20.966 | 20.436 | 20.298 | 18.020 |
| ED | 6.790 | 4.136* | 5.571 | 7.377 | 6.452 | 6.690 | 6.157 |
| CNF | 9.107 | 9.092 | 8.983* | 14.746 | 14.740 | 14.725 | 9.022 |
| CNG | 9.653 | 8.492* | 9.652 | 10.043 | 9.767 | 9.907 | 9.649 |
| CIC | 6.838 | 6.183* | 6.287 | 8.582 | 6.852 | 7.861 | 6.674 |
| DAL | 10.474 | 9.280* | 9.529 | 13.010 | 12.993 | 12.992 | 10.085 |
| DIS | 10.861 | 10.857 | 10.727 | 12.405 | 12.397 | 12.397 | 9.711* |
| DD | 6.771 | 6.120* | 6.754 | 10.551 | 10.478 | 9.804 | 6.735 |
| EAF | 12.913 | 11.487 | 12.431 | 10.813 | 10.781 | 10.712* | 12.598 |
| XON | 6.265 | 3.779* | 5.891 | 7.914 | 7.914 | 7.698 | 6.176 |
| FJQ | 32.390 | 32.057 | 30.745 | 37.353 | 36.840 | 36.834 | 29.214* |
| FIN | 24.053 | 21.391 | 19.986* | 32.149 | 30.291 | 30.464 | 23.877 |
| I | 9.957 | 6.712* | 9.620 | 12.258 | 11.558 | 12.072 | 9.957 |
| F | 11.584 | 11.032 | 11.389 | 11.955 | 11.604 | 11.821 | 10.756* |
| FWC | 14.489 | 14.231 | 13.172* | 16.459 | 16.419 | 16.179 | 14.473 |
| GE | 6.147 | 6.146 | 6.115 | 8.110 | 7.934 | 8.107 | 5.729* |
| GRL | 13.959 | 12.785 | 12.465* | 18.633 | 18.524 | 18.511 | 13.803 |
| GM | 8.333 | 7.697* | 8.183 | 10.145 | 10.130 | 10.107 | 8.238 |
| GSX | 11.031 | 8.778* | 9.784 | 12.293 | 12.282 | 11.910 | 10.881 |
| GT | 5.805 | 5.761 | 5.563* | 8.498 | 8.200 | 8.246 | 5.791 |
| GAP | 15.968 | 15.960* | 15.967 | 19.877 | 19.795 | 19.761 | 15.964 |

Table 5: RMSE of Security Return in the Second Estimation Period (1976-1980) (Continued)

| | SI | MI | MIE | SF | MF | MFIO | MMIO |
|---|---|---|---|---|---|---|---|
| HAL | 10.165 | 10.165 | 10.142 | 12.688 | 12.680 | 12.224 | 10.133* |
| HCA | 8.761 | 8.438 | 8.304* | 11.125 | 10.635 | 10.601 | 8.710 |
| N | 11.852 | 10.939* | 11.428 | 13.472 | 13.073 | 13.442 | 11.830 |
| IR | 8.151 | 7.995 | 7.834 | 11.116 | 11.096 | 11.041 | 7.672* |
| IBM | 6.142 | 4.091* | 5.455 | 7.726 | 7.396 | 7.706 | 5.968 |
| HR | 11.956 | 10.562* | 11.616 | 13.857 | 13.657 | 13.320 | 11.813 |
| IGL | 12.817 | 12.648 | 11.525* | 14.711 | 14.711 | 14.629 | 12.729 |
| INI | 11.167 | 11.165 | 11.049* | 13.100 | 12.205 | 12.603 | 11.155 |
| KM | 9.112 | 8.099 | 8.077* | 11.299 | 11.029 | 11.298 | 9.096 |
| K | 9.340 | 9.293 | 8.923 | 10.836 | 10.429 | 10.095 | 8.800* |
| LLX | 10.773 | 10.676 | 10.295 | 14.308 | 13.224 | 12.883 | 10.500* |
| MCA | 13.880 | 13.844 | 13.299 | 12.865 | 12.668 | 12.424* | 13.704 |
| MZ | 11.735 | 11.117 | 10.051* | 15.091 | 14.970 | 15.077 | 11.700 |
| MCD | 8.805 | 7.947* | 8.746 | 10.428 | 9.942 | 10.382 | 8.333 |
| MHP | 5.613 | 5.595 | 5.281 | 8.196 | 8.183 | 8.070 | 5.196* |
| MOB | 8.886 | 6.915* | 7.701 | 9.468 | 9.454 | 9.347 | 8.869 |
| MTC | 9.755 | 9.693 | 9.738 | 13.924 | 13.869 | 13.807 | 9.627* |
| NWA | 9.437 | 9.371 | 9.372 | 11.466 | 11.465 | 11.421 | 9.218* |
| DR | 12.480 | 12.425 | 12.356 | 15.768 | 15.768 | 15.765 | 12.078* |
| NEM | 13.339 | 12.448* | 13.211 | 15.801 | 15.666 | 15.548 | 13.329 |
| OEC | 7.190 | 3.805* | 4.984 | 7.706 | 7.036 | 7.332 | 6.837 |
| OI | 8.479 | 6.288* | 8.337 | 10.341 | 9.980 | 10.329 | 8.335 |
| PCG | 5.942 | 2.619* | 3.742 | 5.422 | 4.906 | 5.104 | 5.297 |
| PLT | 8.695 | 5.003* | 6.763 | 9.101 | 8.085 | 8.535 | 8.136 |
| PEL | 9.959 | 9.160* | 9.927 | 11.540 | 10.912 | 11.313 | 9.956 |
| JCP | 7.703 | 7.160* | 7.474 | 9.977 | 9.747 | 9.805 | 7.382 |
| PD | 12.313 | 12.030 | 11.906* | 15.258 | 15.091 | 15.254 | 12.194 |
| P | 8.873 | 8.284 | 8.009* | 10.594 | 10.592 | 10.518 | 8.854 |
| PG | 6.389 | 5.925* | 6.274 | 7.151 | 7.149 | 6.864 | 6.385 |
| REV | 7.871 | 7.077* | 7.868 | 9.496 | 9.416 | 9.090 | 7.842 |
| RAD | 8.258 | 6.660* | 8.247 | 13.813 | 13.624 | 13.810 | 7.892 |
| RD | 4.934 | 4.855* | 4.918 | 7.486 | 7.080 | 7.170 | 4.901 |
| SLB | 8.271 | 8.115 | 8.102* | 8.646 | 8.642 | 8.214 | 8.265 |
| S | 6.560 | 6.296* | 6.321 | 8.915 | 8.559 | 8.915 | 6.437 |
| SNT | 9.946 | 9.929 | 9.718* | 10.594 | 10.243 | 10.422 | 9.805 |
| SQD | 6.489 | 6.101* | 6.172 | 12.618 | 12.578 | 12.197 | 6.460 |
| SUN | 7.368 | 6.817* | 6.956 | 9.297 | 8.611 | 8.128 | 6.897 |
| TFB | 12.251 | 12.169 | 11.811* | 13.416 | 13.116 | 12.920 | 12.188 |
| TX | 9.181 | 6.726* | 8.641 | 9.486 | 8.935 | 9.458 | 9.153 |
| TA | 9.323 | 8.928* | 9.322 | 13.294 | 12.532 | 12.986 | 9.110 |
| TIC | 6.790 | 6.480* | 6.649 | 9.167 | 8.790 | 9.064 | 6.644 |
| UAL | 15.438 | 14.851 | 14.827* | 15.227 | 15.216 | 15.224 | 15.119 |
| X | 10.228 | 9.207* | 9.408 | 13.920 | 13.880 | 13.719 | 10.049 |
| UCL | 11.025 | 10.376 | 10.073* | 12.238 | 12.049 | 11.172 | 10.926 |
| USH | 10.736 | 7.963* | 10.561 | 12.162 | 11.713 | 12.161 | 10.618 |
| WLA | 7.271 | 7.033 | 7.202 | 7.796 | 7.796 | 7.689 | 6.768* |
| WX | 10.259 | 10.008 | 10.011 | 11.587 | 11.092 | 11.550 | 9.296* |
| WHR | 7.617 | 6.823* | 7.084 | 11.158 | 10.562 | 10.660 | 7.611 |
| WMB | 13.549 | 13.540 | 13.506 | 15.624 | 15.009 | 14.731 | 13.291* |
| Z | 9.483 | 9.478 | 9.090* | 12.398 | 12.396 | 12.341 | 9.481 |
| # of * | 0 | 52 | 25 | 0 | 2 | 2 | 19 |

* : indicates the best prediction.

Table 6: RMSE of Security Return in the First Forecasting Period (1976-1980)

| | SI | MI | MIE | SF | MF | MFIO | MMIO |
|---|---|---|---|---|---|---|---|
| AMR | 17.714 | 20.692 | 22.121 | 18.151 | 18.979 | 18.195 | 17.664* |
| AMT | 14.984 | 14.979 | 14.594 | 15.299 | 15.156 | 14.744 | 13.928* |
| AET | 9.116 | 8.381* | 8.943 | 11.116 | 11.607 | 11.470 | 9.302 |
| AMX | 12.714 | 13.509 | 12.653* | 17.107 | 19.220 | 23.026 | 27.827 |
| AMB | 7.267 | 7.889 | 7.083* | 10.240 | 9.671 | 10.347 | 7.413 |
| ABC | 18.152 | 17.174 | 16.620 | 15.506* | 18.251 | 16.715 | 18.826 |
| AC | 6.570 | 6.321* | 7.352 | 8.323 | 9.041 | 8.563 | 6.890 |
| ACY | 9.228 | 8.860 | 8.662* | 11.883 | 13.027 | 12.100 | 9.618 |
| AEP | 7.102 | 3.874* | 6.797 | 7.232 | 7.417 | 8.879 | 8.832 |
| AMI | 19.961 | 20.253 | 21.606 | 23.490 | 23.334 | 20.678 | 15.698* |
| AST | 13.439* | 15.206 | 16.214 | 19.818 | 23.347 | 20.897 | 15.076 |
| AR | 23.784 | 23.915 | 23.830 | 25.784 | 28.415 | 25.318 | 23.741* |
| ARC | 9.189 | 7.825* | 9.444 | 10.841 | 11.302 | 10.854 | 9.289 |
| AUD | 12.868 | 13.772 | 16.184 | 11.403* | 12.906 | 12.276 | 13.607 |
| BT | 9.558 | 8.830* | 9.613 | 13.178 | 13.283 | 13.146 | 9.406 |
| BRY | 10.126 | 9.476 | 10.351 | 8.901 | 8.907 | 7.984* | 9.619 |
| BNL | 9.816 | 7.388* | 8.814 | 14.515 | 14.943 | 15.654 | 11.219 |
| BS | 12.531 | 11.449* | 13.011 | 15.476 | 16.678 | 15.428 | 12.842 |
| BMY | 7.639 | 7.494* | 8.634 | 10.007 | 11.649 | 9.902 | 8.313 |
| BNI | 16.998 | 12.289* | 19.830 | 18.716 | 19.617 | 19.165 | 17.553 |
| BGH | 13.046 | 12.870* | 13.061 | 13.517 | 14.624 | 13.262 | 13.200 |
| CBS | 10.408 | 8.756* | 11.500 | 9.481 | 14.275 | 9.524 | 10.506 |
| CAT | 6.999 | 6.380* | 8.765 | 8.470 | 11.344 | 8.273 | 7.124 |
| CHL | 7.534 | 6.617* | 7.732 | 10.530 | 10.718 | 10.693 | 7.696 |
| CHV | 9.920 | 7.233* | 10.937 | 13.045 | 13.534 | 13.280 | 9.751 |
| CCI | 10.574 | 9.255* | 10.272 | 12.868 | 11.818 | 12.503 | 10.276 |
| KO | 12.543 | 11.246 | 12.994 | 5.923* | 11.344 | 7.390 | 13.088 |
| CG | 9.841 | 9.390* | 10.987 | 9.982 | 10.352 | 10.079 | 9.873 |
| CWE | 7.911 | 5.429* | 6.765 | 8.083 | 8.651 | 8.595 | 8.484 |
| CSC | 19.787* | 21.460 | 21.807 | 23.092 | 33.177 | 24.571 | 21.886 |
| ED | 8.877 | 7.823* | 8.371 | 7.873 | 9.530 | 7.871 | 8.901 |
| CNF | 9.865* | 15.382 | 9.879 | 15.745 | 17.160 | 16.682 | 10.930 |
| CNG | 10.583 | 9.321* | 11.151 | 11.361 | 11.344 | 11.160 | 10.345 |
| CIC | 7.267 | 6.581 | 6.330* | 9.482 | 11.428 | 10.769 | 8.741 |
| DAL | 10.770 | 10.354* | 10.382 | 13.900 | 15.013 | 13.745 | 10.553 |
| DIS | 21.331 | 18.743 | 22.327 | 14.048* | 15.142 | 14.141 | 21.410 |
| DD | 7.036* | 7.953 | 7.398 | 10.678 | 10.827 | 10.544 | 7.315 |
| EAF | 16.413 | 16.488 | 16.413 | 13.810* | 15.350 | 13.779 | 16.553 |
| XON | 6.325 | 4.789* | 6.537 | 10.447 | 11.225 | 10.492 | 6.299 |
| FJQ | 33.390 | 33.132* | 35.418 | 40.756 | 43.619 | 41.723 | 34.115 |
| FIN | 28.665 | 28.181 | 28.139* | 35.246 | 35.760 | 35.095 | 28.840 |
| I | 11.756 | 10.199* | 11.757 | 14.750 | 15.624 | 15.092 | 12.064 |
| F | 12.719 | 11.298* | 11.394 | 13.535 | 13.634 | 13.525 | 12.420 |
| FWC | 15.827 | 18.624 | 15.582* | 21.452 | 22.436 | 23.127 | 22.103 |
| GE | 7.800 | 8.742 | 8.069 | 8.460 | 12.838 | 8.763 | 7.750* |
| GRL | 17.860* | 20.075 | 20.182 | 22.971 | 26.584 | 23.934 | 18.119 |
| GM | 8.928 | 7.832* | 8.366 | 10.175 | 10.664 | 10.167 | 8.900 |
| GSX | 11.747 | 10.706* | 14.029 | 13.479 | 17.766 | 13.462 | 11.648 |
| GT | 6.441 | 7.375 | 6.357* | 9.629 | 9.092 | 9.549 | 6.437 |
| GAP | 16.810* | 17.702 | 16.840 | 20.072 | 19.986 | 20.240 | 16.929 |

Table 6: RMSE of Security Return in the First Forecasting Period (1976-1980) (Continued)

| | SI | MI | MIE | SF | MF | MFIO | MMIO |
|---|---|---|---|---|---|---|---|
| HAL | 10.440 | 10.435* | 10.803 | 13.355 | 13.127 | 14.422 | 10.630 |
| HCA | 16.554 | 18.333 | 18.649 | 12.184* | 12.579 | 13.807 | 17.146 |
| N | 11.855 | 11.188* | 12.059 | 13.609 | 14.104 | 13.764 | 12.733 |
| IR | 9.446 | 9.380* | 10.113 | 11.288 | 14.093 | 11.126 | 9.483 |
| IBM | 6.295 | 4.940* | 5.720 | 7.972 | 8.748 | 8.448 | 6.715 |
| HR | 12.059* | 13.175 | 13.723 | 17.407 | 16.675 | 17.321 | 12.312 |
| IGL | 14.005* | 15.859 | 14.375 | 14.979 | 15.251 | 15.080 | 14.181 |
| INI | 11.986* | 12.373 | 13.414 | 13.820 | 13.942 | 13.887 | 12.001 |
| KM | 13.381 | 11.630* | 13.593 | 14.381 | 14.163 | 16.439 | 15.036 |
| K | 10.371 | 11.133 | 10.341 | 12.133 | 12.296 | 11.610 | 10.253* |
| LLX | 11.867* | 11.970 | 11.985 | 16.269 | 15.969 | 16.450 | 12.447 |
| MCA | 19.179 | 19.082 | 21.379 | 14.087 | 13.553* | 15.603 | 19.875 |
| MZ | 12.342 | 11.907* | 12.377 | 16.438 | 16.782 | 16.852 | 12.648 |
| MCD | 20.188 | 19.052 | 20.012 | 14.319 | 14.083* | 15.863 | 20.524 |
| MHP | 9.097* | 10.732 | 10.956 | 11.981 | 14.824 | 12.325 | 9.267 |
| MOB | 10.793 | 8.614* | 11.573 | 11.858 | 12.057 | 11.801 | 10.615 |
| MTC | 12.199 | 12.189 | 12.260 | 14.807 | 16.303 | 14.298 | 12.037* |
| NWA | 13.785 | 18.163 | 14.527 | 11.749* | 12.437 | 12.699 | 13.981 |
| DR | 13.854 | 14.404 | 13.832 | 16.218 | 16.232 | 16.189 | 13.805* |
| NEM | 13.839 | 12.899* | 13.975 | 17.035 | 17.324 | 20.120 | 24.376 |
| OEC | 7.465 | 5.003* | 6.868 | 8.228 | 8.408 | 8.850 | 8.175 |
| OI | 8.620 | 12.022 | 8.634 | 10.415 | 11.915 | 10.415 | 8.588* |
| PCG | 6.407 | 3.054* | 7.200 | 5.948 | 6.092 | 6.307 | 6.988 |
| PLT | 9.042 | 7.643* | 9.324 | 9.975 | 9.957 | 9.971 | 9.041 |
| PEL | 11.255 | 10.478* | 11.150 | 13.343 | 13.159 | 12.963 | 10.780 |
| JCP | 11.644 | 9.016* | 12.046 | 10.702 | 11.355 | 10.706 | 11.698 |
| PD | 12.516 | 12.247* | 12.599 | 16.023 | 16.418 | 19.405 | 12.591 |
| P | 9.103* | 14.916 | 10.232 | 14.124 | 14.858 | 14.207 | 9.798 |
| PG | 8.341 | 6.555* | 8.474 | 7.959 | 9.044 | 7.642 | 8.320 |
| REV | 8.730 | 8.150* | 9.106 | 10.145 | 11.398 | 10.060 | 9.030 |
| RAD | 19.316 | 19.401 | 20.142 | 14.767* | 14.925 | 15.352 | 19.287 |
| RD | 5.628 | 5.603 | 5.505* | 8.591 | 8.841 | 9.455 | 6.722 |
| SLB | 8.463 | 8.395 | 8.358* | 8.758 | 9.093 | 8.938 | 8.463 |
| S | 9.486 | 7.506* | 9.642 | 9.397 | 9.107 | 9.701 | 9.522 |
| SNT | 11.064 | 11.645 | 10.863 | 13.185 | 13.302 | 12.893 | 10.675* |
| SQD | 6.624* | 7.071 | 6.846 | 12.898 | 13.328 | 12.878 | 6.707 |
| SUN | 12.474 | 9.785* | 12.657 | 13.844 | 13.886 | 13.891 | 12.473 |
| TFB | 20.352 | 18.084 | 20.565 | 13.846* | 14.016 | 15.359 | 21.167 |
| TX | 10.514 | 8.951* | 10.535 | 11.451 | 12.110 | 12.178 | 10.950 |
| TA | 9.969 | 9.741* | 9.971 | 15.098 | 15.075 | 15.192 | 9.991 |
| TIC | 7.954 | 8.780 | 7.794* | 9.596 | 10.162 | 9.716 | 7.801 |
| UAL | 18.687 | 24.267 | 19.611 | 17.310* | 19.944 | 19.410 | 19.687 |
| X | 13.328 | 12.054* | 14.284 | 15.025 | 16.812 | 14.269 | 13.213 |
| UCL | 12.941 | 11.040* | 12.963 | 14.490 | 14.721 | 14.547 | 13.006 |
| USH | 14.761 | 13.658 | 17.327 | 12.477 | 15.483 | 12.380* | 14.525 |
| WLA | 11.255 | 12.762 | 12.776 | 8.624* | 10.053 | 9.494 | 11.930 |
| WX | 12.664* | 12.768 | 13.021 | 14.093 | 18.354 | 15.371 | 13.156 |
| WHR | 12.248 | 9.406* | 13.736 | 12.282 | 12.966 | 11.510 | 12.090 |
| WMB | 14.527 | 14.805 | 14.083* | 17.565 | 17.512 | 17.831 | 15.362 |
| Z | 10.316 | 10.139* | 10.316 | 12.614 | 12.617 | 13.503 | 10.857 |
| # of * | 14 | 50 | 11 | 11 | 2 | 2 | 10 |

* : indicates the best prediction.

Table 7: RMSE of Security Return in the Second Forecasting Period (1981-1985)

| | SI | MI | MIE | SF | MF | MFIO | MMIO |
|---|---|---|---|---|---|---|---|
| AMR | 29.132* | 29.320 | 29.643 | 30.015 | 29.958 | 29.910 | 29.158 |
| AMT | 16.929 | 17.001 | 16.254 | 18.609 | 19.637 | 18.318 | 15.738* |
| AET | 9.726 | 6.610* | 10.165 | 10.739 | 13.176 | 11.731 | 9.737 |
| AMX | 19.897* | 20.468 | 20.282 | 21.941 | 21.952 | 22.234 | 20.016 |
| AMB | 9.087 | 9.237 | 10.492 | 8.992* | 11.393 | 10.400 | 9.338 |
| ABC | 15.643 | 15.014* | 15.778 | 18.174 | 18.421 | 17.934 | 16.967 |
| AC | 10.834 | 10.058 | 9.764* | 13.066 | 13.142 | 13.157 | 10.758 |
| ACY | 6.722* | 7.533 | 7.701 | 10.045 | 10.027 | 9.886 | 6.930 |
| AEP | 7.021 | 4.140* | 6.417 | 6.466 | 5.773 | 5.560 | 5.355 |
| AMI | 18.829 | 25.313 | 18.732 | 21.335 | 22.590 | 22.225 | 15.065* |
| AST | 10.692 | 10.817 | 10.767 | 15.466 | 15.531 | 15.289 | 9.718* |
| AR | 19.358 | 19.119 | 18.863 | 21.238 | 22.479 | 20.836 | 18.309* |
| ARC | 11.496 | 8.220* | 12.930 | 13.103 | 13.106 | 13.501 | 11.937 |
| AUD | 11.404 | 11.313* | 11.712 | 15.502 | 16.967 | 16.318 | 11.754 |
| BT | 7.993 | 6.905* | 7.928 | 13.223 | 16.229 | 15.319 | 8.022 |
| BRY | 10.030 | 8.738* | 9.523 | 10.463 | 10.496 | 10.035 | 9.668 |
| BNL | 12.145 | 17.792 | 11.037* | 15.844 | 17.868 | 16.463 | 11.789 |
| BS | 14.726 | 13.719* | 14.782 | 15.560 | 15.584 | 15.720 | 14.354 |
| BMY | 7.865 | 7.409* | 8.921 | 9.896 | 9.708 | 9.886 | 7.997 |
| BNI | 12.153 | 10.042* | 11.632 | 16.762 | 16.827 | 17.029 | 12.584 |
| BGH | 10.267 | 10.218* | 10.268 | 12.825 | 13.612 | 12.265 | 11.180 |
| CBS | 14.717 | 14.472* | 14.692 | 17.833 | 17.859 | 17.734 | 15.073 |
| CAT | 10.673 | 9.263* | 10.349 | 13.160 | 13.073 | 13.585 | 9.860 |
| CHL | 11.791 | 8.107* | 11.581 | 15.687 | 16.446 | 16.029 | 11.795 |
| CHV | 12.282 | 8.606* | 13.087 | 13.707 | 13.840 | 13.753 | 11.949 |
| CCI | 9.961 | 6.685* | 9.798 | 13.549 | 13.329 | 12.711 | 11.406 |
| KO | 9.811 | 8.735* | 9.961 | 11.403 | 12.134 | 12.390 | 11.483 |
| CG | 9.915* | 9.995 | 9.923 | 10.400 | 10.446 | 10.656 | 9.959 |
| CWE | 8.207 | 6.483 | 6.845 | 7.364 | 6.618 | 6.131* | 6.350 |
| CSC | 15.539 | 15.567 | 15.623 | 18.977 | 21.112 | 21.128 | 15.523* |
| ED | 6.312 | 4.284* | 5.052 | 5.414 | 6.459 | 5.704 | 5.176 |
| CNF | 16.557 | 16.535* | 17.332 | 17.560 | 17.587 | 17.535 | 16.774 |
| CNG | 9.492 | 9.378* | 9.515 | 10.767 | 11.483 | 11.077 | 9.569 |
| CIC | 9.526 | 6.654* | 9.208 | 11.545 | 12.215 | 11.272 | 8.832 |
| DAL | 20.067* | 20.317 | 20.225 | 20.386 | 20.476 | 20.491 | 20.350 |
| DIS | 15.665* | 15.843 | 15.735 | 19.616 | 19.404 | 19.432 | 19.184 |
| DD | 7.868* | 8.955 | 7.940 | 11.772 | 12.049 | 12.801 | 7.996 |
| EAF | 19.175 | 18.017* | 19.232 | 26.555 | 26.603 | 26.593 | 19.596 |
| XON | 5.895* | 7.560 | 8.172 | 7.221 | 7.223 | 7.586 | 6.288 |
| FJQ | 20.872 | 21.578 | 19.375* | 23.105 | 24.403 | 23.676 | 21.953 |
| FIN | 25.676* | 31.435 | 25.927 | 37.261 | 44.591 | 41.994 | 26.846 |
| I | 10.121 | 11.512 | 10.952 | 14.151 | 17.533 | 15.572 | 10.074* |
| F | 18.519 | 16.203* | 18.366 | 21.520 | 22.377 | 22.352 | 20.791 |
| FWC | 17.822 | 17.629 | 17.002* | 19.718 | 20.353 | 21.529 | 18.301 |
| GE | 7.392* | 7.403 | 7.457 | 11.128 | 12.213 | 11.221 | 8.745 |
| GRL | 21.851 | 22.348 | 22.385 | 24.767 | 24.801 | 25.234 | 21.256* |
| GM | 11.024 | 9.935* | 11.062 | 13.024 | 12.920 | 12.883 | 11.365 |
| GSX | 8.402 | 9.900 | 9.746 | 13.454 | 13.412 | 14.456 | 8.014* |
| GT | 13.806 | 13.020* | 14.123 | 15.486 | 14.376 | 14.609 | 14.023 |
| GAP | 19.727 | 19.132 | 19.702 | 18.896 | 18.223 | 17.979* | 19.601 |

Table 7: RMSE of Security Return in the Second Forecasting Period (1981-1985) (Continued)

| | SI | MI | MIE | SF | MF | MFIO | MMIO |
|---|---|---|---|---|---|---|---|
| HAL | 16.801 | 16.767 | 16.615* | 20.310 | 20.527 | 22.019 | 17.398 |
| HCA | 15.725 | 17.364 | 15.017* | 18.464 | 21.480 | 21.118 | 15.088 |
| N | 11.518 | 9.352* | 11.264 | 16.235 | 16.224 | 16.226 | 11.424 |
| IR | 8.236 | 7.966 | 7.915 | 10.159 | 10.072 | 10.488 | 7.281* |
| IBM | 8.562 | 8.042* | 8.672 | 12.428 | 13.968 | 12.149 | 9.434 |
| HR | 20.845 | 20.709 | 19.405* | 24.601 | 25.728 | 26.229 | 19.821 |
| IGL | 12.736 | 12.636 | 11.697* | 14.465 | 14.462 | 14.988 | 12.064 |
| INI | 13.706 | 13.676 | 13.183* | 15.463 | 18.812 | 17.478 | 14.011 |
| KM | 13.836 | 10.785* | 13.415 | 14.384 | 15.305 | 14.329 | 13.648 |
| K | 12.575 | 13.690 | 11.539 | 12.199 | 11.560 | 11.225* | 11.546 |
| LLX | 15.776 | 14.288 | 17.027 | 13.733* | 16.203 | 16.611 | 16.888 |
| MCA | 12.922 | 12.197* | 13.215 | 14.803 | 14.349 | 14.675 | 13.991 |
| MZ | 14.328 | 10.835* | 14.922 | 17.823 | 18.357 | 17.681 | 14.550 |
| MCD | 10.668 | 5.931* | 10.293 | 13.257 | 14.528 | 13.668 | 12.249 |
| MHP | 9.797 | 9.852 | 9.480* | 13.951 | 13.642 | 13.771 | 10.493 |
| MOB | 10.910 | 8.216* | 13.064 | 12.319 | 12.282 | 12.813 | 11.223 |
| MTC | 10.430* | 10.814 | 10.539 | 12.697 | 12.725 | 12.787 | 10.849 |
| NWA | 18.401 | 18.330* | 18.589 | 19.828 | 19.833 | 19.991 | 18.708 |
| DR | 7.600 | 8.236 | 7.615 | 8.686 | 8.692 | 8.678 | 7.434* |
| NEM | 15.681 | 14.408* | 15.586 | 18.971 | 18.684 | 19.051 | 15.738 |
| OEC | 8.450 | 6.715* | 7.548 | 8.927 | 7.969 | 7.688 | 6.745 |
| OI | 7.476* | 16.198 | 8.034 | 9.817 | 10.601 | 9.787 | 7.631 |
| PCG | 8.110 | 6.438* | 7.453 | 7.883 | 7.615 | 7.144 | 6.961 |
| PLT | 8.665 | 7.345* | 8.568 | 9.042 | 10.862 | 9.768 | 8.937 |
| PEL | 11.643 | 13.098 | 11.409* | 13.843 | 17.507 | 15.798 | 11.884 |
| JCP | 14.330 | 9.678* | 13.992 | 13.974 | 14.866 | 14.898 | 15.537 |
| PD | 17.532 | 17.097* | 17.361 | 20.057 | 19.511 | 20.064 | 17.407 |
| P | 12.540 | 9.429* | 14.394 | 12.740 | 12.743 | 12.998 | 12.311 |
| PG | 9.306 | 7.384* | 9.005 | 11.334 | 11.228 | 10.375 | 9.155 |
| REV | 9.946 | 11.798 | 9.982 | 11.045 | 10.840 | 10.890 | 9.936* |
| RAD | 13.384 | 11.139* | 13.404 | 16.095 | 16.196 | 16.097 | 13.606 |
| RD | 10.374 | 9.051* | 10.569 | 11.758 | 11.503 | 11.694 | 10.481 |
| SLB | 16.273 | 15.407* | 15.794 | 17.711 | 17.858 | 19.237 | 16.514 |
| S | 13.611 | 10.997* | 13.351 | 16.831 | 17.765 | 16.817 | 14.275 |
| SNT | 13.970 | 14.098 | 13.430 | 14.386 | 16.603 | 15.738 | 12.859* |
| SQD | 9.537 | 9.845 | 10.417 | 14.098 | 14.087 | 14.660 | 9.478* |
| SUN | 17.676 | 13.713* | 18.245 | 18.486 | 18.868 | 19.553 | 18.647 |
| TFB | 7.207 | 7.076* | 7.239 | 8.324 | 9.332 | 9.567 | 7.116 |
| TX | 10.694 | 8.199* | 12.435 | 11.567 | 12.153 | 11.453 | 10.431 |
| TA | 9.237 | 8.777* | 9.244 | 13.863 | 17.193 | 15.378 | 9.180 |
| TIC | 10.354 | 8.281* | 10.297 | 11.543 | 11.689 | 11.480 | 11.045 |
| UAL | 22.986 | 22.986* | 23.535 | 26.089 | 26.076 | 26.130 | 23.554 |
| X | 16.453 | 16.084* | 17.467 | 16.469 | 16.352 | 16.930 | 16.344 |
| UCL | 20.082 | 16.191* | 21.150 | 19.573 | 19.898 | 21.382 | 20.745 |
| USH | 12.664 | 15.608 | 12.774 | 14.923 | 17.030 | 15.010 | 12.392* |
| WLA | 13.746 | 12.697* | 14.101 | 16.025 | 16.049 | 16.807 | 15.414 |
| WX | 8.017 | 8.691 | 8.006* | 13.436 | 15.236 | 13.778 | 10.365 |
| WHR | 13.267 | 9.114* | 12.410 | 14.637 | 14.144 | 13.910 | 13.394 |
| WMB | 13.085 | 12.972 | 12.794* | 15.471 | 17.181 | 17.625 | 14.727 |
| Z | 10.415 | 10.751 | 9.923* | 13.784 | 13.715 | 13.410 | 10.482 |
| # of * | 12 | 55 | 24 | 2 | 0 | 3 | 14 |

* : indicates the best prediction.

Table 8: Summary of Security Return Prediction

| | SI | MI | MIE | SF | MF | MFIO | MMIO | Total |
|---|---|---|---|---|---|---|---|---|
| 1st Estimation Period | 0 | 61 | 22 | 0 | 0 | 0 | 17 | 100 |
| 2nd Estimation Period | 0 | 52 | 25 | 0 | 2 | 2 | 19 | 100 |
| 1st Forecasting Period | 14 | 50 | 11 | 11 | 2 | 2 | 10 | 100 |
| 2nd Forecasting Period | 12 | 55 | 14 | 2 | 0 | 3 | 14 | 100 |

Table 9: Performance Measures of Covariance in the Estimation Period

| | Performance Measures | | Decomposition of MSE | | | | |
|---|---|---|---|---|---|---|---|
| | RMSE | MSE* | UM | UV | UC | UI | UE |
| A. 1971-1975 | | | | | | | |
| [MIE | 58.44 | 3415.7 | 0.0272 | 0.1910 | 0.7818 | 0.0668 | 0.9060 |
| MI] | 58.47 | 3419.1 | 0.0210 | 0.2090 | 0.7700 | 0.0783 | 0.9007 |
| MMIO | 59.41 | 3529.8 | 0.0321 | 0.2068 | 0.7611 | 0.0763 | 0.8916 |
| SI | 60.43 | 3652.3 | 0.0347 | 0.2113 | 0.7540 | 0.0776 | 0.8877 |
| [OA-CV | 153.65 | 23609.2 | 0 | 1.0000 | 0 | 0 | 1.0000 |
| QA-CC] | 153.90 | 23685.9 | 0.0032 | 0.9668 | 0 | 0 | 0.9668 |
| [IA1-CV | 155.57 | 24203.7 | 0.0266 | 0.3067 | 0.6667 | 0.0461 | 0.9273 |
| IA1-CC] | 156.10 | 24366.0 | 0.0349 | 0.3393 | 0.6258 | 0.0385 | 0.9266 |
| [IA2-CV | 166.22 | 27628.9 | 0.0410 | 0.2910 | 0.6689 | 0.1078 | 0.8512 |
| IA2-CC] | 166.63 | 27764.9 | 0.0576 | 0.3206 | 0.6218 | 0.0944 | 0.8480 |
| MF | 225.17 | 50702.3 | 0.5370 | 0.3549 | 0.1081 | 0.0008 | 0.4622 |
| MFIO | 229.41 | 52628.4 | 0.5560 | 0.3725 | 0.0715 | 0.0001 | 0.4439 |
| SF | 229.89 | 52849.4 | 0.5549 | 0.3957 | 0.0494 | 0.0001 | 0.4450 |
| B. 1976-1980 | | | | | | | |
| MIE | 32.79 | 1075.5 | 0.0432 | 0.2091 | 0.7477 | 0.0095 | 0.9473 |
| [MMIO | 33.47 | 1120.5 | 0.0352 | 0.2083 | 0.7565 | 0.0066 | 0.9582 |
| MI] | 33.60 | 1129.1 | 0.0250 | 0.2030 | 0.7720 | 0.0045 | 0.9705 |
| SI | 34.69 | 1203.6 | 0.0390 | 0.2237 | 0.7373 | 0.0066 | 0.9544 |
| [IA1-CV | 48.79 | 2380.0 | 0.0157 | 0.2602 | 0.7241 | 0.0481 | 0.9362 |
| IA1-CC | 48.82 | 2383.3 | 0.0131 | 0.2430 | 0.7441 | 0.0540 | 0.9330 |
| OA-CV | 49.10 | 2411.2 | 0 | 1.0000 | 0 | 0 | 1.0000 |
| QA-CC] | 49.11 | 2411.4 | 0 | 1.0000 | 0 | 0 | 1.0000 |
| [IA2-CV | 52.37 | 2742.4 | 0.0331 | 0.3351 | 0.6318 | 0.0909 | 0.8750 |
| IA2-CC] | 52.43 | 2748.4 | 0.0391 | 0.3369 | 0.6240 | 0.0066 | 0.9543 |
| MFIO | 71.51 | 5113.8 | 0.5479 | 0.3122 | 0.1399 | 0.0001 | 0.4520 |
| MF | 72.53 | 5260.7 | 0.5677 | 0.3092 | 0.1231 | 0.0022 | 0.4301 |
| SF | 73.79 | 5445.0 | 0.5750 | 0.3263 | 0.0987 | 0.0023 | 0.4227 |

* All differences in the MSE are statistically significant at the 5% level of significance, except the groups of models enclosed in brackets.

Table 10: Performance Measures of Correlation Coefficient in the Estimation Period

| | Performance Measures | | Decomposition of MSE | | | | |
|---|---|---|---|---|---|---|---|
| | RMSE | MSE* | UM | UV | UC | UI | UE |
| A. 1971-1975 | | | | | | | |
| MI | 0.1553 | 0.0241 | 0.0125 | 0.1083 | 0.8792 | 0.0045 | 0.9830 |
| [MIE | 0.1608 | 0.0258 | 0.0222 | 0.1075 | 0.8703 | 0.0067 | 0.9711 |
| [MMIO | 0.1620 | 0.0262 | 0.0260 | 0.1118 | 0.8622 | 0.0062 | 0.9678 |
| SI] | 0.1637 | 0.0268 | 0.0280 | 0.1104 | 0.8616 | 0.0073 | 0.9647 |
| OA | 0.2232 | 0.0498 | 0 | 1.0000 | 0 | 0 | 1.0000 |
| IA1 | 0.2707 | 0.0733 | 0.0569 | 0.0903 | 0.8528 | 0.2638 | 0.6793 |
| IA2 | 0.2874 | 0.0826 | 0.0914 | 0.0782 | 0.8304 | 0.3092 | 0.5994 |
| MF | 0.5079 | 0.2579 | 0.8111 | 0.1027 | 0.0862 | 0.0018 | 0.1871 |
| [MFIO | 0.5269 | 0.2776 | 0.8263 | 0.1218 | 0.0519 | 0.0001 | 0.1736 |
| SF] | 0.5275 | 0.2783 | 0.8259 | 0.1368 | 0.0373 | 0.0004 | 0.1737 |
| B. 1976-1980 | | | | | | | |
| MIE | 0.1724 | 0.0297 | 0.0495 | 0.2373 | 0.7132 | 0.0001 | 0.9504 |
| MI | 0.1778 | 0.0316 | 0.0245 | 0.2555 | 0.7200 | 0.0004 | 0.9751 |
| MMIO | 0.1819 | 0.0331 | 0.0412 | 0.2701 | 0.6887 | 0.0002 | 0.9586 |
| SI | 0.1868 | 0.0349 | 0.0438 | 0.2829 | 0.6733 | 0.0008 | 0.9553 |
| OA | 0.2117 | 0.0448 | 0 | 1.0000 | 0 | 0 | 1.0000 |
| IA1 | 0.2372 | 0.0563 | 0.0360 | 0.1778 | 0.7862 | 0.1712 | 0.7928 |
| IA2 | 0.2442 | 0.0596 | 0.0647 | 0.1900 | 0.7453 | 0.1839 | 0.7514 |
| MFIO | 0.3819 | 0.1459 | 0.7207 | 0.1575 | 0.1218 | 0.0001 | 0.2792 |
| MF | 0.3928 | 0.1543 | 0.7358 | 0.1492 | 0.1150 | 0.0001 | 0.2641 |
| SF | 0.4022 | 0.1618 | 0.7423 | 0.1675 | 0.0902 | 0.0006 | 0.2571 |

* All differences in the MSE are statistically significant at the 5% level of significance, except the groups of models enclosed in brackets.

Table 11: Performance Measures of Covariance in the Forecasting Period

| | Performance Measures | | Decomposition of MSE | | | | |
|---|---|---|---|---|---|---|---|
| | RMSE* | TIC | UM | UV | UC | UI | UE |
| A. 1976-1980 | | | | | | | |
| MF | 71.56 | 0.3860 | 0.4945 | 0.1709 | 0.3346 | 0.0400 | 0.4655 |
| SF | 75.36 | 0.4065 | 0.5630 | 0.2825 | 0.1545 | 0.0126 | 0.4244 |
| MFIO | 75.83 | 0.4090 | 0.5524 | 0.2185 | 0.2291 | 0.0284 | 0.4192 |
| IA2-CC | 106.75 | 0.5758 | 0.4895 | 0.0091 | 0.5014 | 0.2990 | 0.2115 |
| IA1-CC | 111.94 | 0.6039 | 0.5835 | 0.0148 | 0.4017 | 0.2306 | 0.1859 |
| IA2-CV | 113.81 | 0.6139 | 0.5068 | 0.0174 | 0.4758 | 0.3071 | 0.1861 |
| OA-CC | 116.73 | 0.6297 | 0.8235 | 0.1765 | 0 | 0 | 0.1765 |
| IA1-CV | 117.19 | 0.6322 | 0.5807 | 0.0246 | 0.3947 | 0.2496 | 0.1697 |
| OA-CV | 124.71 | 0.6727 | 0.8450 | 0.1550 | 0 | 0 | 0.1550 |
| SI | 159.28 | 0.8592 | 0.4215 | 0.2323 | 0.3462 | 0.4914 | 0.0871 |
| MMIO | 160.26 | 0.8645 | 0.4214 | 0.2340 | 0.3446 | 0.4925 | 0.0861 |
| [MI | 161.59 | 0.8717 | 0.4319 | 0.2319 | 0.3362 | 0.4838 | 0.0843 |
| MIE] | 161.79 | 0.8728 | 0.4214 | 0.2385 | 0.3401 | 0.4944 | 0.0842 |
| FH | 185.38 | 1.0000 | 0.3827 | 0.3181 | 0.2992 | 0.5543 | 0.0630 |
| B. 1981-1985 | | | | | | | |
| MI | 55.56 | 0.9051 | 0.0545 | 0.2269 | 0.7186 | 0.0124 | 0.9331 |
| MMIO | 55.91 | 0.9109 | 0.0622 | 0.2263 | 0.7115 | 0.0127 | 0.9251 |
| [MIE | 56.34 | 0.9178 | 0.0660 | 0.2182 | 0.7158 | 0.0154 | 0.9186 |
| SI] | 56.38 | 0.9186 | 0.0663 | 0.2419 | 0.6918 | 0.0110 | 0.9227 |
| [IA1-CC | 59.71 | 0.9727 | 0.0492 | 0.3511 | 0.5997 | 0.0150 | 0.9358 |
| IA1-CV] | 60.01 | 0.9777 | 0.0525 | 0.3639 | 0.5836 | 0.0144 | 0.9331 |
| [OA-CV | 60.91 | 0.9923 | 0.0160 | 0.9840 | 0 | 0 | 0.9840 |
| OA-CC | 60.96 | 0.9932 | 0.0178 | 0.9822 | 0 | 0 | 0.9822 |
| FH] | 61.38 | 1.0000 | 0.0156 | 0.0339 | 0.9505 | 0.1678 | 0.8166 |
| [IA2-CV | 65.21 | 1.0624 | 0.0695 | 0.4075 | 0.5230 | 0.0726 | 0.8579 |
| IA2-CC] | 65.39 | 1.0653 | 0.0760 | 0.4075 | 0.5165 | 0.0707 | 0.8533 |
| MFIO | 85.47 | 1.3926 | 0.5024 | 0.3599 | 0.1377 | 0.0019 | 0.4957 |
| MF | 86.34 | 1.4067 | 0.5207 | 0.3578 | 0.1215 | 0.0001 | 0.4792 |
| SF | 87.45 | 1.4249 | 0.5290 | 0.3738 | 0.0972 | 0.0001 | 0.4709 |

* All differences in the MSE are statistically significant at the 5% level of significance, except the groups of models enclosed in brackets.

Table 12: Performance Measures of Covariance
in the Forecasting Period
(with Bayesian Coefficient Adjustment)

| | Performance Measures | | Decomposition of MSE | | | | |
|---|---|---|---|---|---|---|---|
| | RMSE* | TIC | UM | UV | UC | UI | UE |
| A. 1976-1980 | | | | | | | |
| MF | 71.76 | 0.3871 | 0.5197 | 0.2637 | 0.2166 | 0.0146 | 0.4658 |
| MFIO | 74.68 | 0.4029 | 0.5675 | 0.3882 | 0.0445 | 0.0005 | 0.4321 |
| SF | 74.81 | 0.4035 | 0.5692 | 0.4066 | 0.0243 | 0.0001 | 0.4308 |
| IA2-CC | 106.75 | 0.5758 | 0.4895 | 0.0091 | 0.5014 | 0.2990 | 0.2115 |
| IA1-CC | 111.94 | 0.6039 | 0.5835 | 0.0148 | 0.4017 | 0.2306 | 0.1859 |
| IA2-CV | 113.81 | 0.6139 | 0.5068 | 0.0174 | 0.4758 | 0.3071 | 0.1861 |
| OA-CC | 116.73 | 0.6297 | 0.8235 | 0.1765 | 0 | 0 | 0.1765 |
| IA1-CV | 117.19 | 0.6322 | 0.5807 | 0.0246 | 0.3947 | 0.2496 | 0.1697 |
| OA-CV | 124.71 | 0.6727 | 0.8450 | 0.1550 | 0 | 0 | 0.1550 |
| MMIO | 128.25 | 0.6918 | 0.5346 | 0.0911 | 0.3743 | 0.3309 | 0.1346 |
| SI | 128.51 | 0.6932 | 0.5330 | 0.0933 | 0.3738 | 0.3332 | 0.1338 |
| MI | 131.72 | 0.7105 | 0.5303 | 0.1043 | 0.3655 | 0.3430 | 0.1268 |
| MIE | 136.78 | 0.7378 | 0.4851 | 0.0930 | 0.4221 | 0.3878 | 0.1272 |
| FH | 185.38 | 1.0000 | 0.3827 | 0.3181 | 0.2992 | 0.5543 | 0.0630 |
| B. 1981-1985 | | | | | | | |
| MIE | 56.54 | 0.9211 | 0.0935 | 0.4890 | 0.4175 | 0.0180 | 0.8885 |
| MI | 57.89 | 0.9432 | 0.0837 | 0.5401 | 0.3762 | 0.0158 | 0.9005 |
| MMIO | 58.67 | 0.9559 | 0.0893 | 0.5382 | 0.3725 | 0.0110 | 0.8997 |
| SI | 59.17 | 0.9640 | 0.0915 | 0.5609 | 0.3476 | 0.0122 | 0.8963 |
| IA1-CC | 59.71 | 0.9727 | 0.0492 | 0.3511 | 0.5997 | 0.0150 | 0.9358 |
| IA1-CV | 60.01 | 0.9777 | 0.0525 | 0.3639 | 0.5836 | 0.0144 | 0.9331 |
| OA-CV | 60.91 | 0.9923 | 0.0160 | 0.9840 | 0 | 0 | 0.9840 |
| OA-CC | 60.96 | 0.9932 | 0.0178 | 0.9822 | 0 | 0 | 0.9822 |
| FH | 61.38 | 1.0000 | 0.0156 | 0.0339 | 0.9505 | 0.1678 | 0.8166 |
| IA2-CV | 65.21 | 1.0624 | 0.0695 | 0.4075 | 0.5230 | 0.0726 | 0.8579 |
| IA2-CC | 65.39 | 1.0653 | 0.0760 | 0.4075 | 0.5165 | 0.0707 | 0.8533 |
| MFIO | 85.34 | 1.3903 | 0.5022 | 0.4477 | 0.0501 | 0.0006 | 0.4972 |
| MF | 87.09 | 1.4190 | 0.5253 | 0.4334 | 0.0413 | 0.0054 | 0.4693 |
| SF | 87.50 | 1.4256 | 0.5299 | 0.4452 | 0.0249 | 0.0176 | 0.4525 |

* All differences in the MSE are statistically significant at the 5% level of significance, except the groups of models enclosed in brackets.

Table 13: Performance Measures of Correlation Coefficient in the Forecasting Period

| | Performance Measures | | Decomposition of MSE | | | | |
|---|---|---|---|---|---|---|---|
| | RMSE* | TIC | UM | UV | UC | UI | UE |
| A. 1976-1980 | | | | | | | |
| OA | 0.2482 | 0.8278 | 0.2726 | 0.7274 | 0 | 0 | 0.7274 |
| IA2 | 0.2649 | 0.8838 | 0.0261 | 0.0674 | 0.9065 | 0.3376 | 0.6363 |
| IA1 | 0.2661 | 0.8876 | 0.0600 | 0.0690 | 0.8710 | 0.3080 | 0.6320 |
| SI | 0.2749 | 0.9169 | 0.1383 | 0.0242 | 0.8375 | 0.2767 | 0.5850 |
| [MMIO | 0.2759 | 0.9204 | 0.1408 | 0.0238 | 0.8354 | 0.2782 | 0.5810 |
| MI | 0.2760 | 0.9207 | 0.1657 | 0.0205 | 0.8138 | 0.2594 | 0.5749 |
| MIE] | 0.2763 | 0.9216 | 0.1466 | 0.0221 | 0.8313 | 0.2752 | 0.5782 |
| FH | 0.2998 | 1.0000 | 0.1872 | 0.0014 | 0.8114 | 0.3404 | 0.4724 |
| MF | 0.3907 | 1.3033 | 0.7032 | 0.1496 | 0.1472 | 0.0078 | 0.2890 |
| [SF | 0.4088 | 1.3635 | 0.7317 | 0.2017 | 0.0666 | 0.0013 | 0.2670 |
| MFIO] | 0.4092 | 1.3648 | 0.7285 | 0.1773 | 0.0942 | 0.0047 | 0.2668 |
| B. 1981-1985 | | | | | | | |
| [OA | 0.2262 | 0.8514 | 0.0242 | 0.9758 | 0 | 0 | 0.9758 |
| MI | 0.2283 | 0.8592 | 0.0011 | 0.1983 | 0.8006 | 0.0930 | 0.9059 |
| MMIO | 0.2283 | 0.8594 | 0.0001 | 0.2170 | 0.7829 | 0.0885 | 0.9114 |
| MIE] | 0.2285 | 0.8603 | 0.0002 | 0.1756 | 0.8242 | 0.1016 | 0.8982 |
| SI | 0.2312 | 0.8704 | 0.0003 | 0.2310 | 0.7687 | 0.0966 | 0.9031 |
| IA1 | 0.2437 | 0.9173 | 0.0015 | 0.2105 | 0.7880 | 0.1610 | 0.8375 |
| IA2 | 0.2550 | 0.9599 | 0.0110 | 0.2149 | 0.7741 | 0.2251 | 0.7639 |
| FH | 0.2657 | 1.0000 | 0.0175 | 0.0019 | 0.9806 | 0.3262 | 0.6563 |
| MFIO | 0.3680 | 1.3853 | 0.6164 | 0.1971 | 0.1856 | 0.0162 | 0.3674 |
| MF | 0.3749 | 1.4114 | 0.6473 | 0.1902 | 0.1625 | 0.0052 | 0.3475 |
| SF | 0.3824 | 1.4397 | 0.6622 | 0.2127 | 0.1251 | 0.0022 | 0.3356 |

* All differences in the MSE are statistically significant at the 5% level of significance, except the groups of models enclosed in brackets.

Table 14: Performance Measures of Correlation Coefficient in the Forecasting Period (with Bayesian Coefficient Adjustment)

| | Performance Measures | | Decomposition of MSE | | | | |
|---|---|---|---|---|---|---|---|
| | RMSE* | TIC | UM | UV | UC | UI | UE |
| A. 1976-1980 | | | | | | | |
| OA | 0.2482 | 0.8278 | 0.2726 | 0.7274 | 0 | 0 | 0.7274 |
| [MMIO | 0.2612 | 0.8712 | 0.1581 | 0.0964 | 0.7455 | 0.1895 | 0.6524 |
| SI] | 0.2614 | 0.8720 | 0.1577 | 0.0920 | 0.7503 | 0.1915 | 0.6508 |
| [MI | 0.2626 | 0.8760 | 0.1735 | 0.0703 | 0.7561 | 0.1885 | 0.6380 |
| IA2 | 0.2649 | 0.8838 | 0.0261 | 0.0674 | 0.9065 | 0.3376 | 0.6363 |
| IA1] | 0.2661 | 0.8876 | 0.0600 | 0.0690 | 0.8710 | 0.3080 | 0.6320 |
| FH | 0.2998 | 1.0000 | 0.1872 | 0.0014 | 0.6114 | 0.3404 | 0.4724 |
| MIE | 0.3334 | 1.1121 | 0.1707 | 0.0026 | 0.8267 | 0.4275 | 0.4018 |
| MF | 0.3934 | 1.3123 | 0.7103 | 0.1824 | 0.1073 | 0.0032 | 0.2865 |
| MFIO | 0.4081 | 1.3612 | 0.7311 | 0.2423 | 0.0266 | 0.0001 | 0.2683 |
| SF | 0.4084 | 1.3621 | 0.7320 | 0.2518 | 0.0162 | 0.0001 | 0.2679 |
| B. 1981-1985 | | | | | | | |
| [MIE | 0.2247 | 0.8458 | 0.0001 | 0.2581 | 0.7418 | 0.0648 | 0.9351 |
| [[OA | 0.2262 | 0.8514 | 0.0242 | 0.9758 | 0 | 0 | 0.9758 |
| [MI] | 0.2283 | 0.8594 | 0.0012 | 0.2836 | 0.7152 | 0.0724 | 0.9264 |
| MMIO] | 0.2303 | 0.8671 | 0.0001 | 0.3189 | 0.6810 | 0.0759 | 0.9240 |
| SI | 0.2326 | 0.8755 | 0.0001 | 0.3293 | 0.6706 | 0.0859 | 0.9140 |
| IA1 | 0.2437 | 0.9173 | 0.0015 | 0.2105 | 0.7880 | 0.1610 | 0.8375 |
| IA2 | 0.2550 | 0.9599 | 0.0110 | 0.2149 | 0.7741 | 0.2251 | 0.7639 |
| FH | 0.2657 | 1.0000 | 0.0175 | 0.0019 | 0.9806 | 0.3262 | 0.6563 |
| MFIO | 0.3647 | 1.3730 | 0.6197 | 0.2891 | 0.0912 | 0.0051 | 0.3752 |
| MF | 0.3780 | 1.4229 | 0.6560 | 0.2731 | 0.0709 | 0.0001 | 0.3439 |
| SF | 0.3815 | 1.4362 | 0.6635 | 0.2925 | 0.0440 | 0.0025 | 0.3340 |

* All differences in the MSE are statistically significant at the 5% level of significance, except the groups of models enclosed in brackets.

Figure 1: Flowchart of Performance Evaluation for the Models of Covariance with Industry Interaction

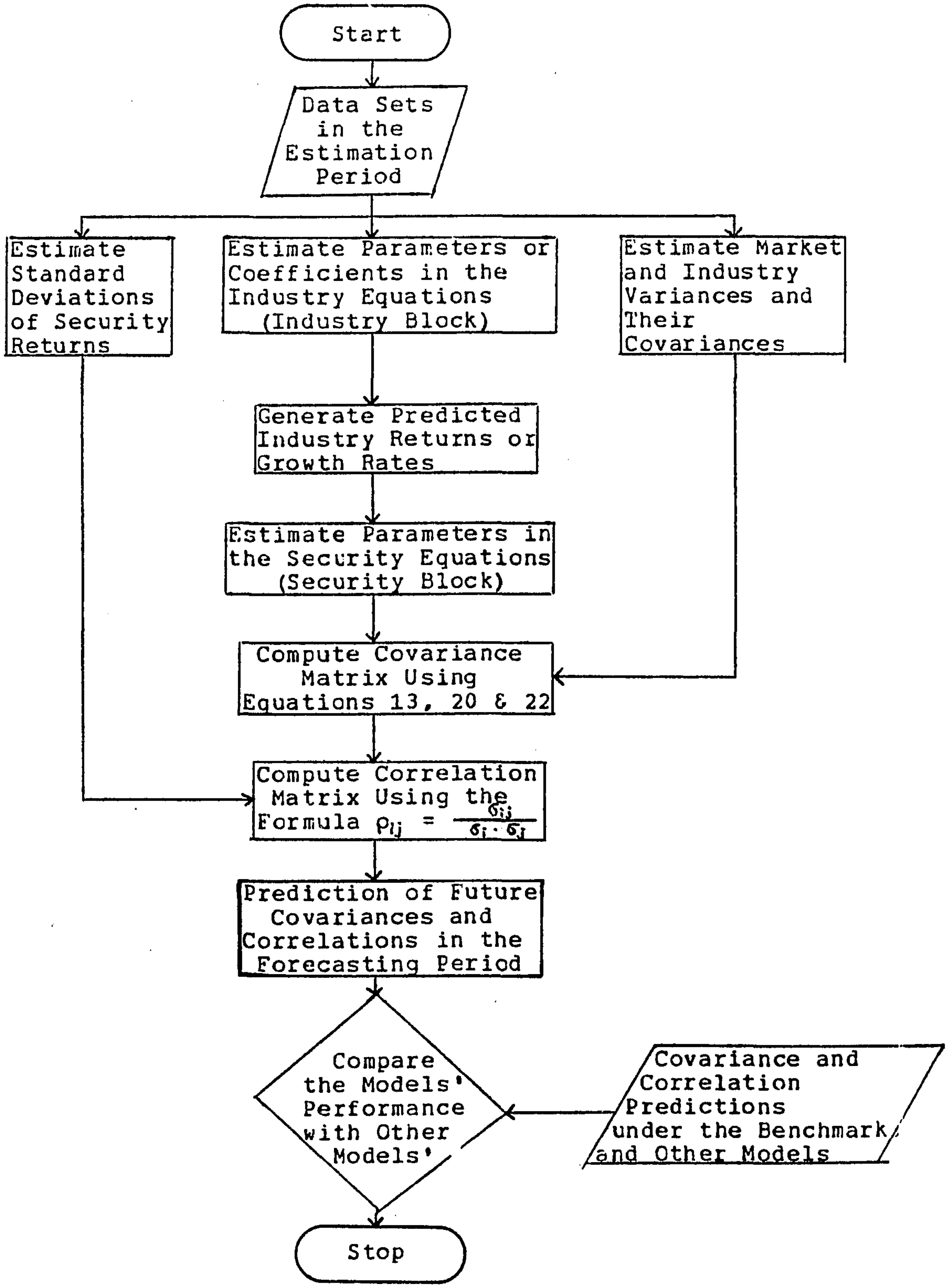


Figure 2a: Frequency Distribution of r

between Monthly Industry Returns

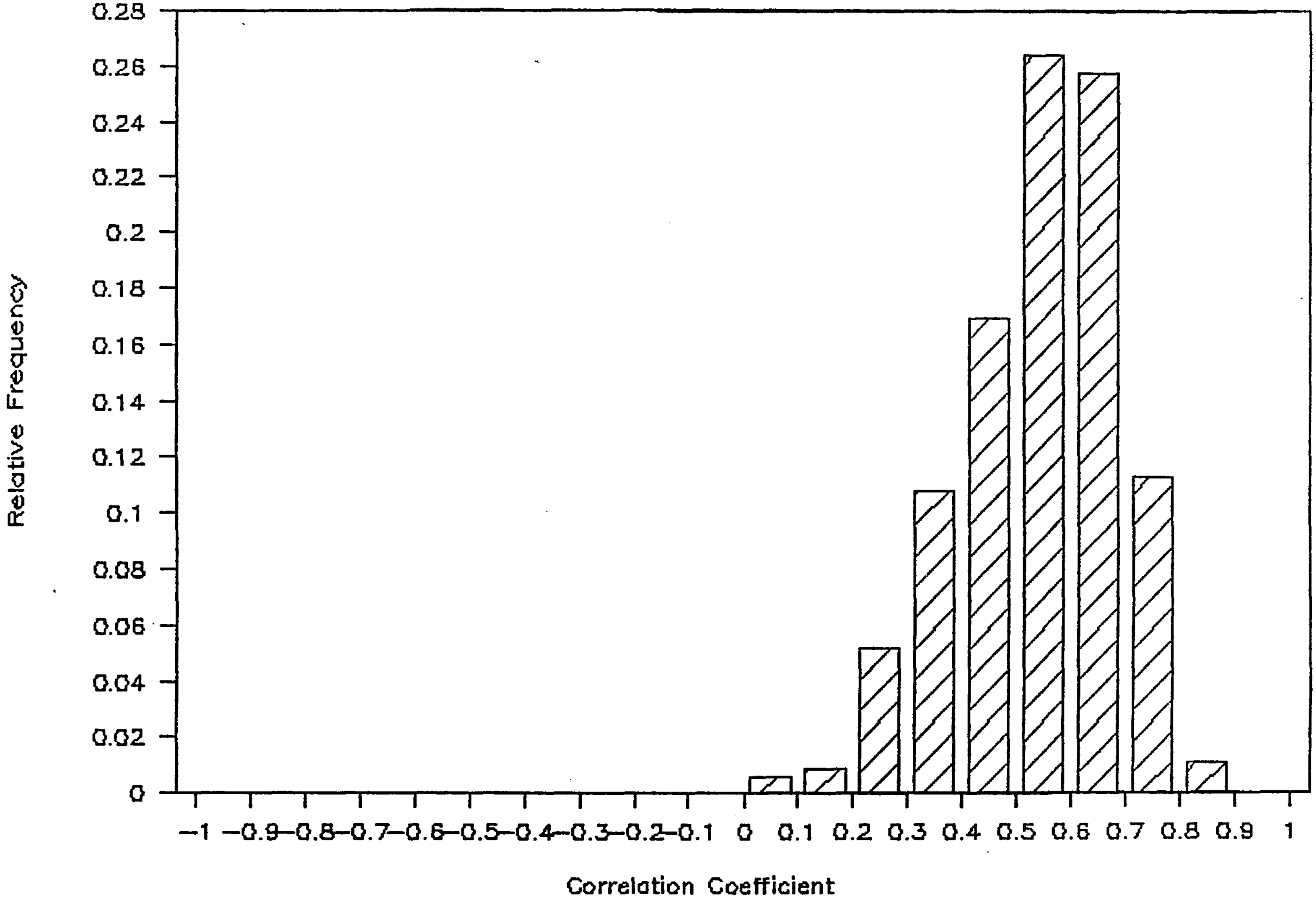
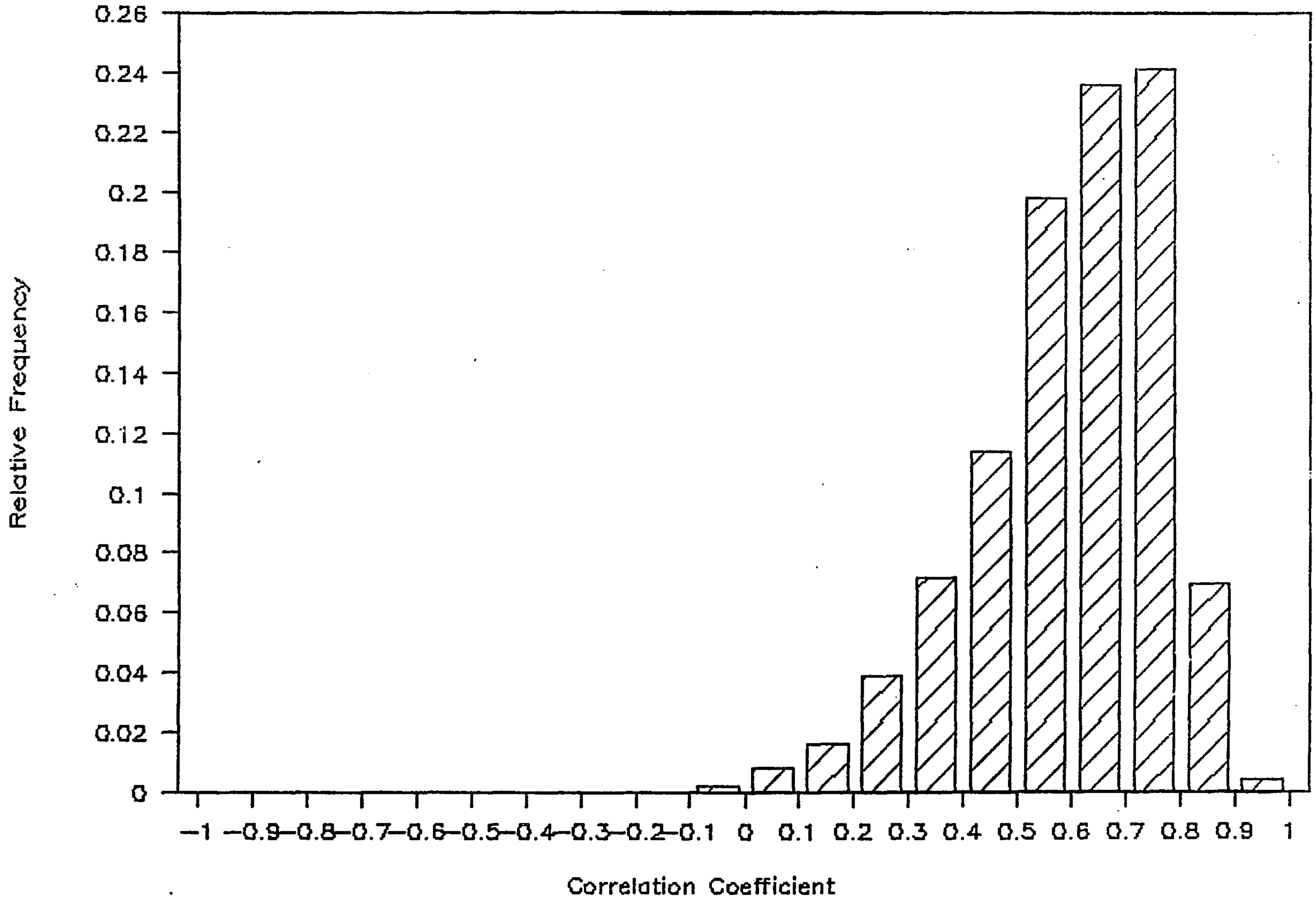


Figure 2b: Frequency Distribution of r

between Quarterly Industry Returns



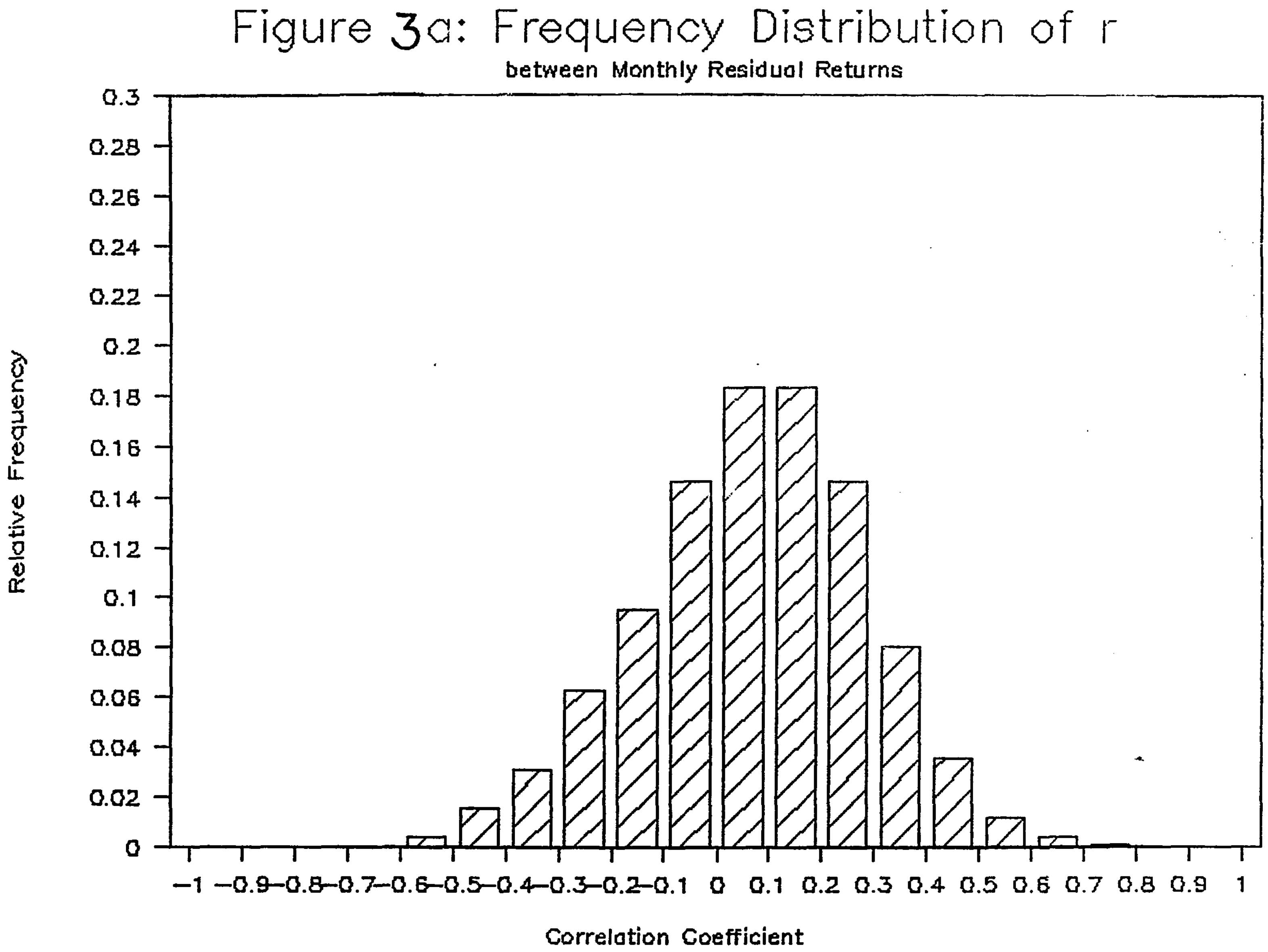


Figure 3a: Frequency Distribution of r
between Monthly Residual Returns
Relative Frequency
0.3
0.28
0.26
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-1 -0.9 -0.8 -0.7 -0.6 -0.5 -0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Correlation Coefficient

# Figure 3b: Frequency Distribution of r

between Quarterly Residual Returns

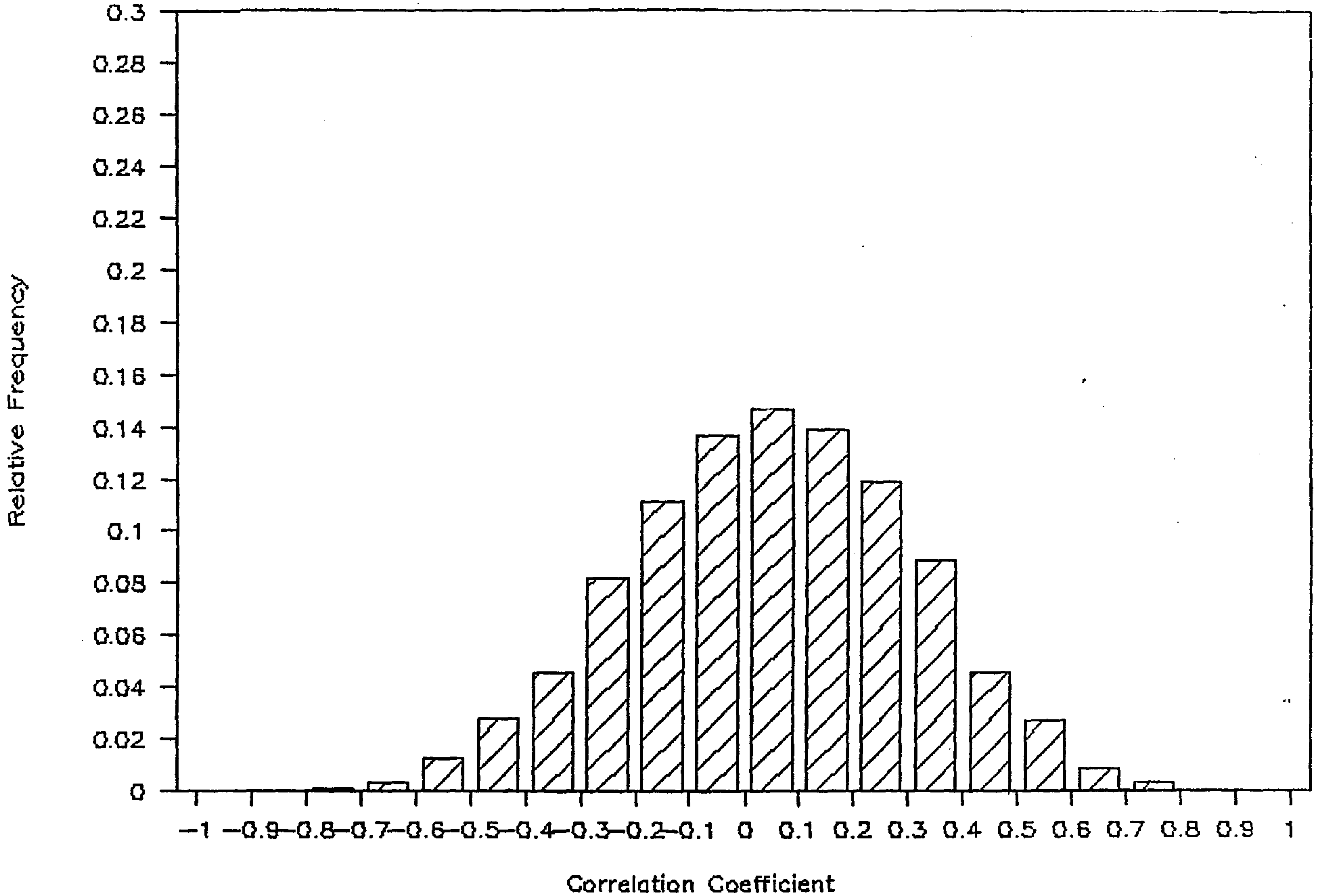
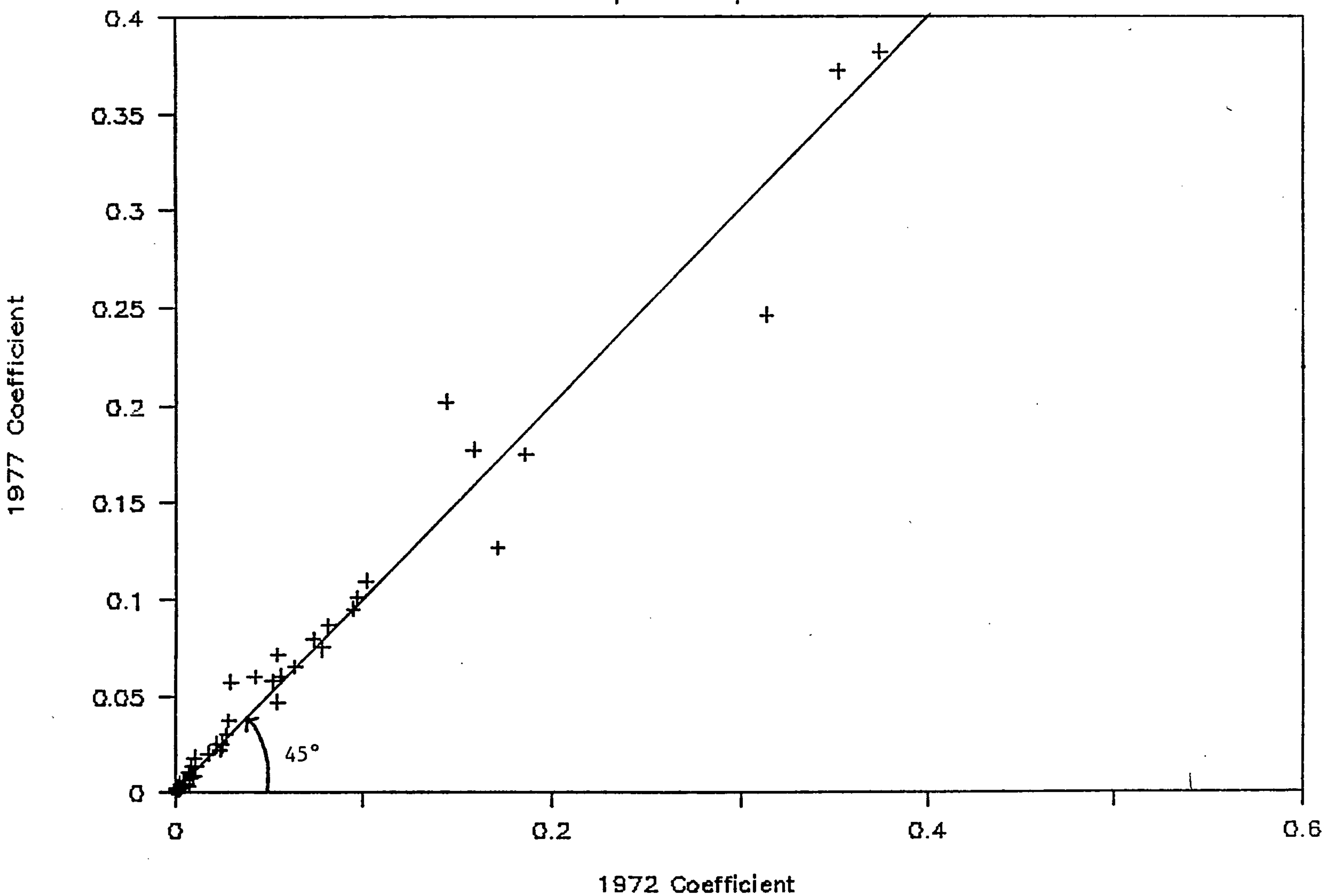
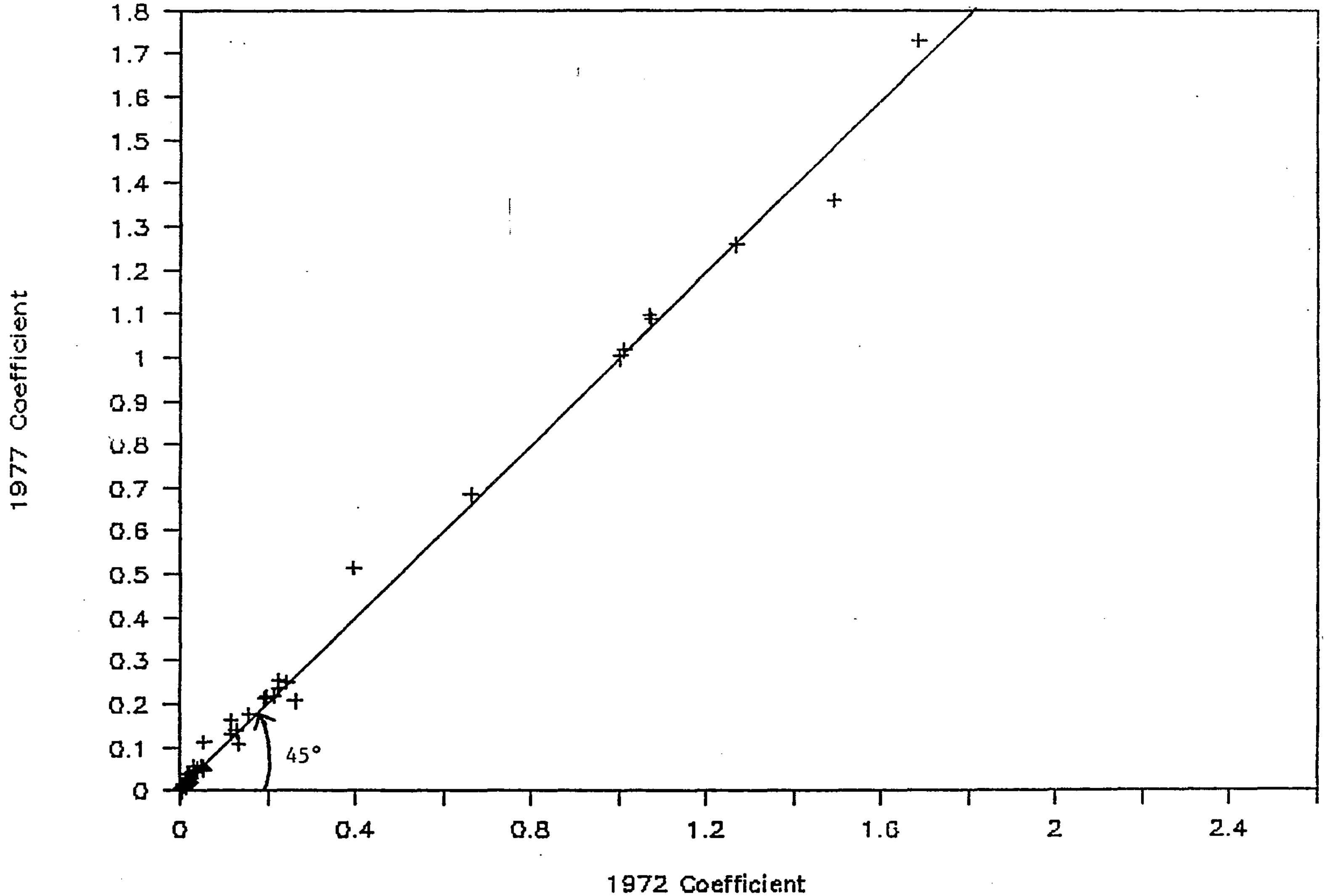


Figure 4a: Stability of
Input — Output Coefficients

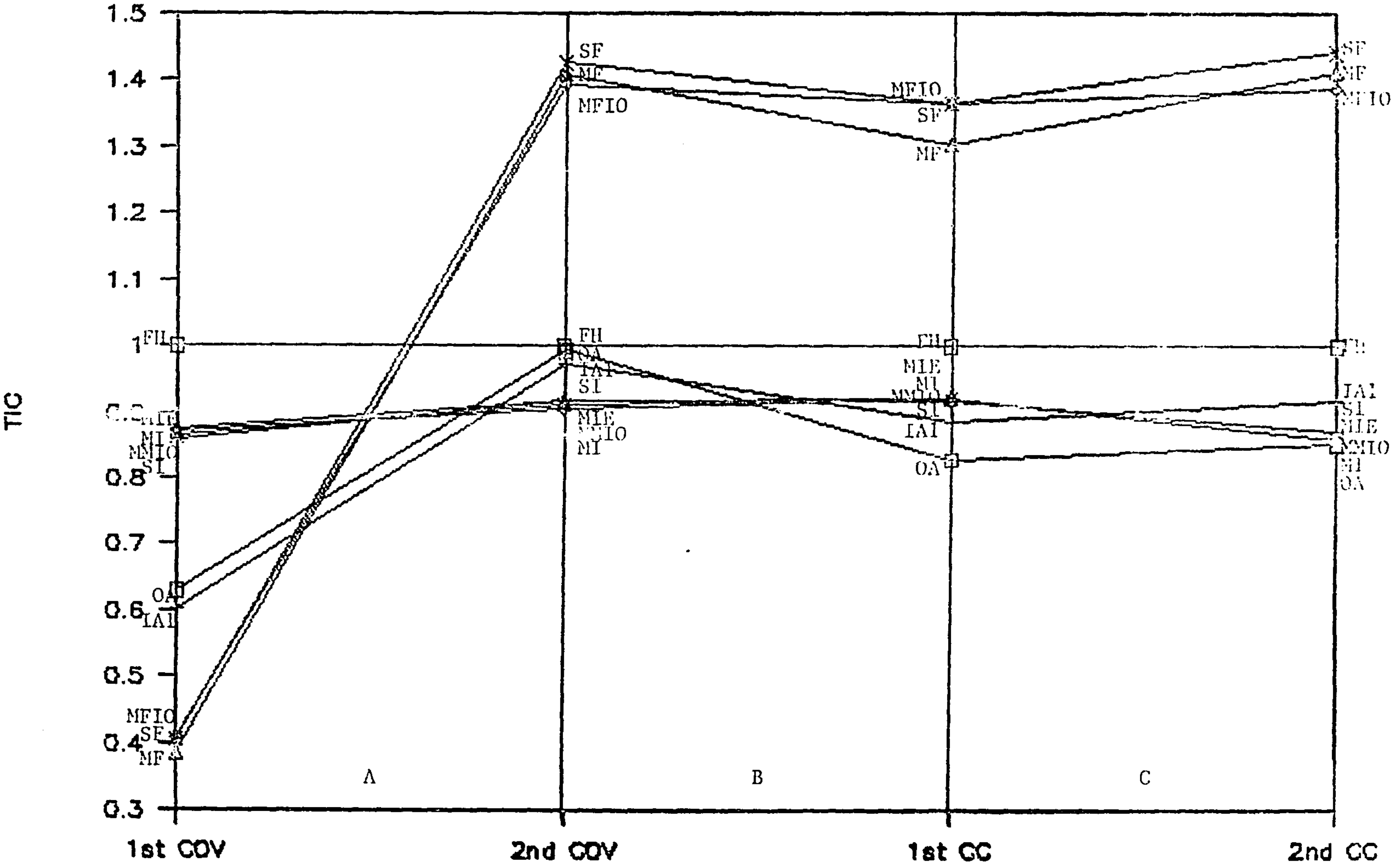


# Figure 4b: Stability of Coefficients

in the Leontief Inverse

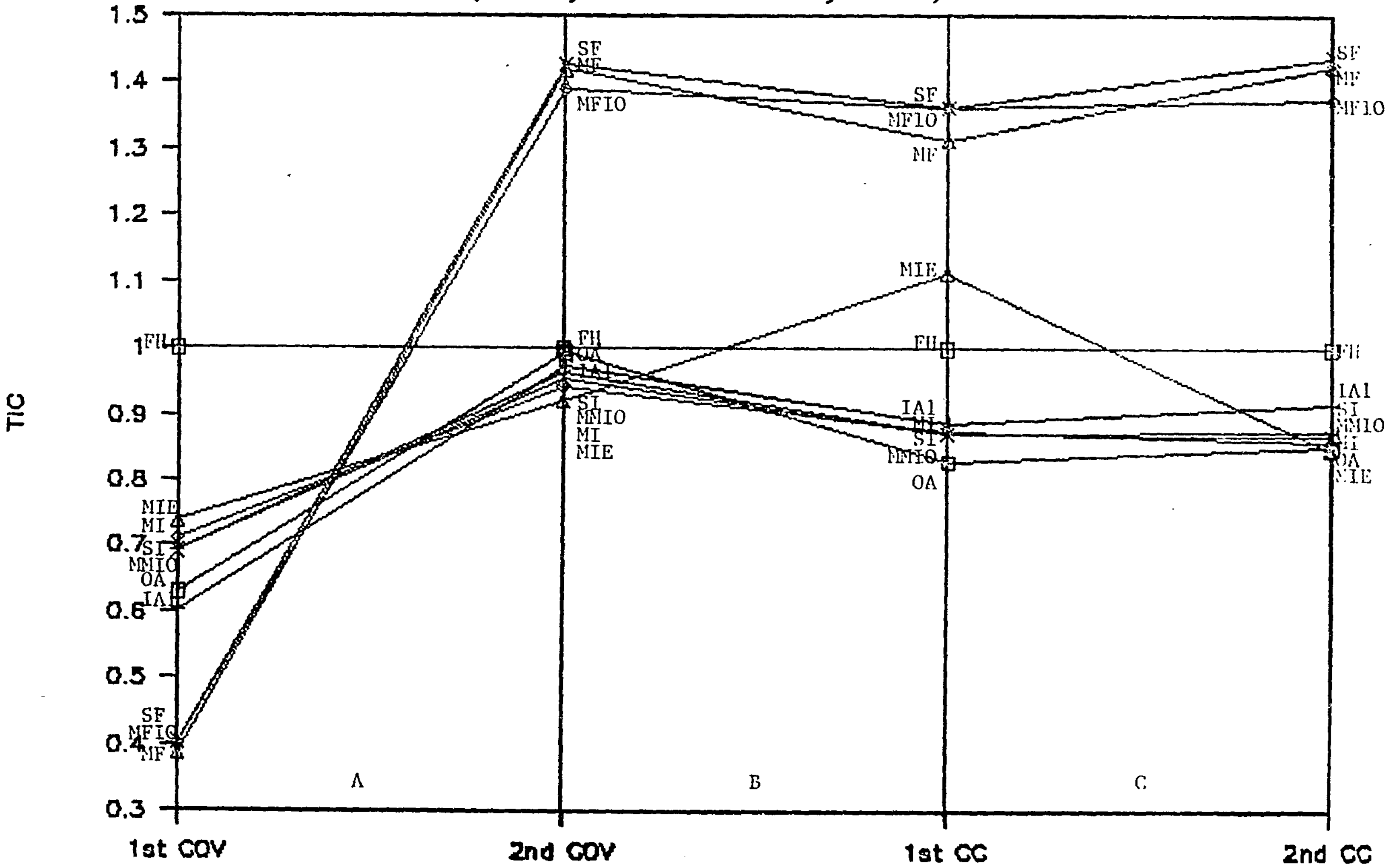


# Figure 5 : Performance of Alternative Models of Covariance

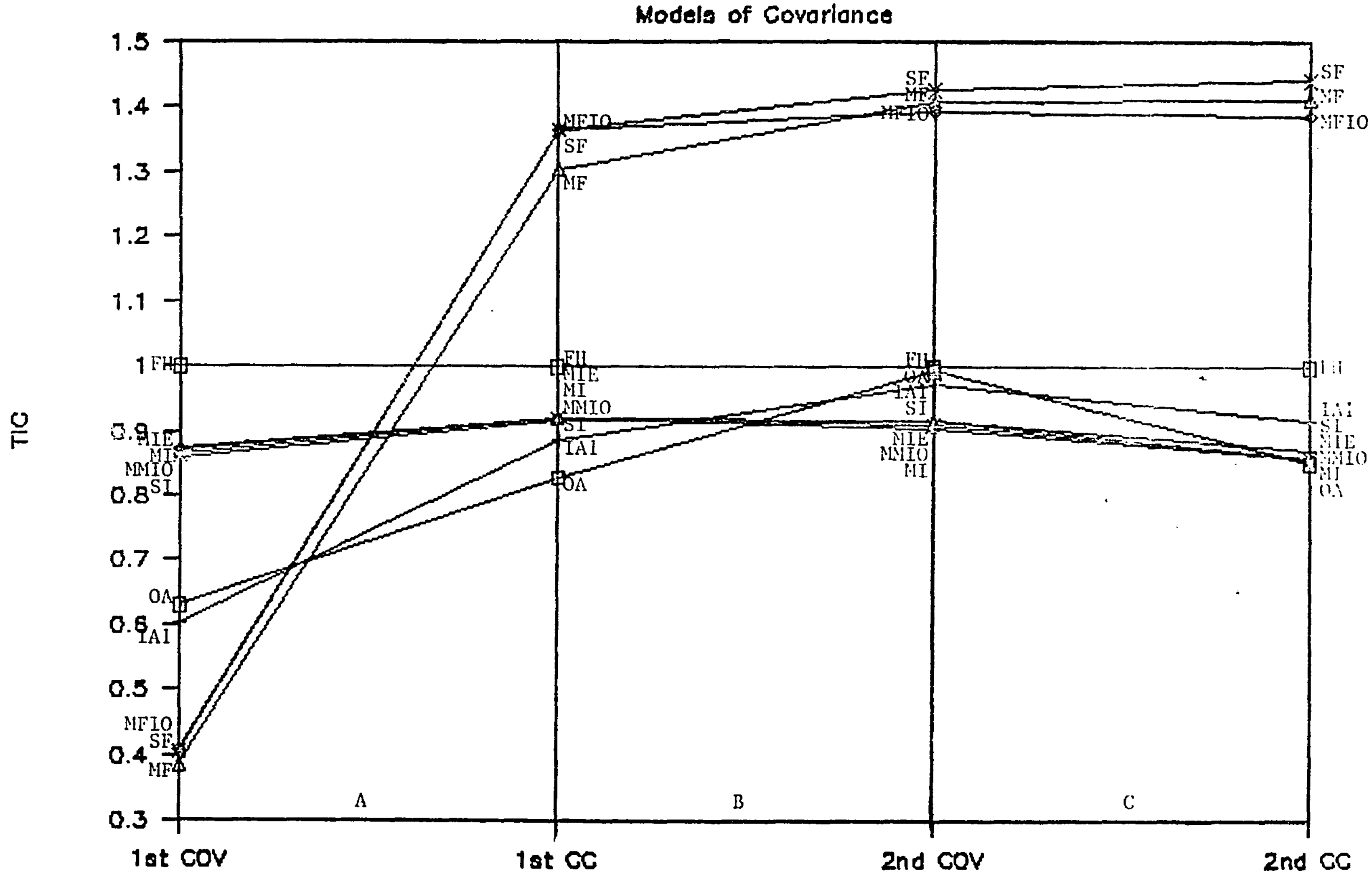


# Figure 6: Performance Evaluation

**(with Bayesian Coefficient Adjustment)**



# Figure 7 : Performance of Alternative



# Figure 8: Performance Evaluation

(with Bayesian Coefficient Adjustment)

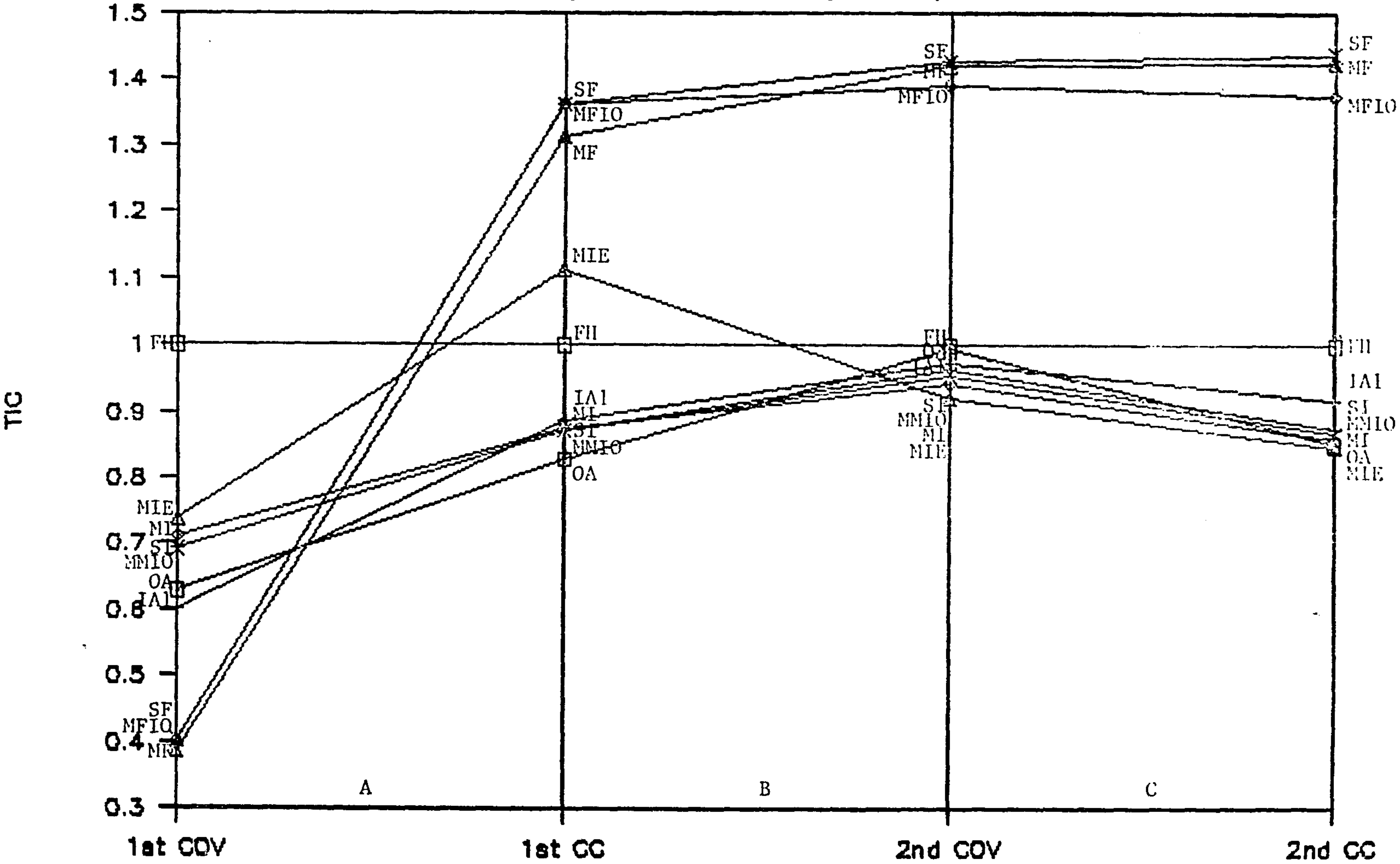
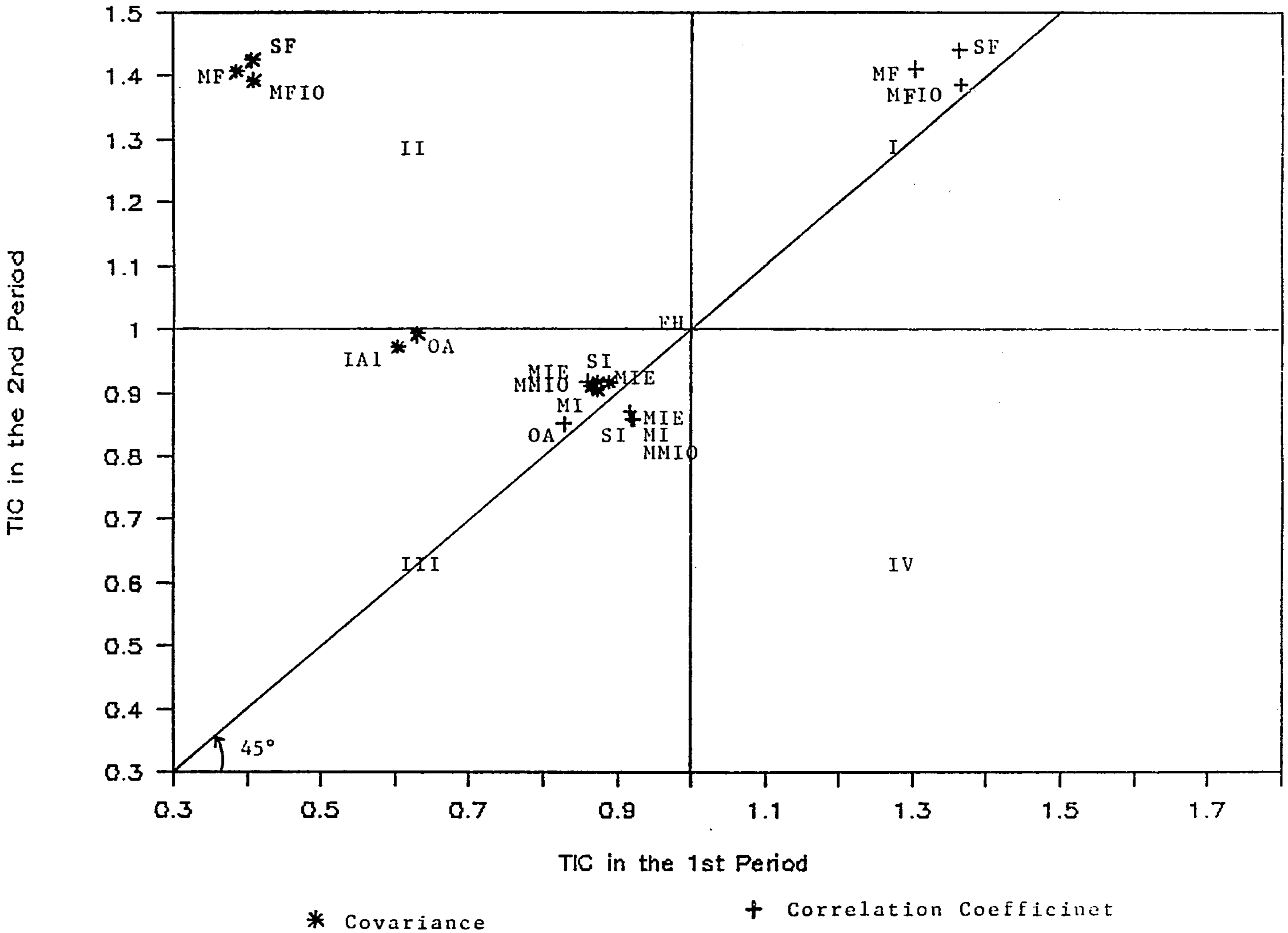
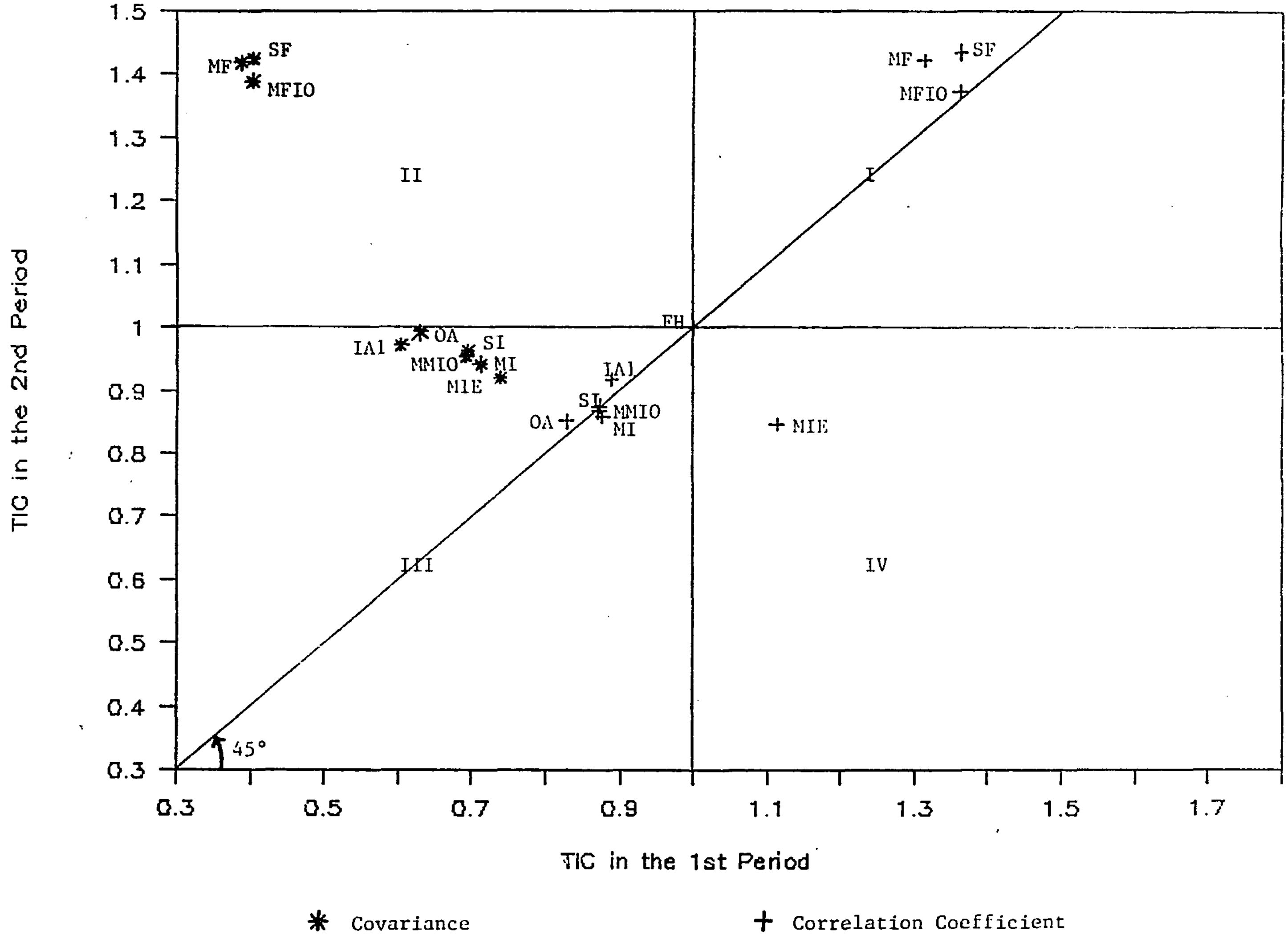


Figure 9: Performance of Alternative Models of Covariance



## Figure 10: Performance Evaluation

(with Bayesian Coefficient Adjustment)



Appendix A: List of 100 Sample Companies

| | S & P Industry | SIC Industry | Ticker Symbol | Company Name |
|---|---|---|---|---|
| 1 | TRA | TAT | AMR | A M R CORP DEL |
| 2 | CAP | MFT | AMT | ACME CLEVELAND CORP |
| 3 | FIN | SER | AET | AETNA LIFE & CAS CO |
| 4 | CAP | MIN | AMX | AMAX INC |
| 5 | CON | MFT | AMB | AMERICAN BRANDS INC |
| 6 | CON | SER | ABC | AMERICAN BROADCASTING COS INC |
| 7 | CON | MFT | AC | AMERICAN CAN CO |
| 8 | CAP | MFT | ACY | AMERICAN CYANAMID CO |
| 9 | UTI | SER | AEP | AMERICAN ELEC PWR INC |
| 10 | CON | SER | AMI | AMERICAN MED INTL INC |
| 11 | CAP | MFT | AST | AMERICAN STD INC |
| 12 | CAP | MIN | AR | ASARCO INC |
| 13 | OIL | MFT | ARC | ATLANTIC RICHFIELD CO |
| 14 | CAP | SER | AUD | AUTOMATIC DATA PROCESSING INC |
| 15 | FIN | SER | BT | BANKERS TR N Y CORP |
| 16 | CON | MFT | BRY | BEATRICE CO |
| 17 | FIN | SER | BNL | BENEFICIAL CORP |
| 18 | CAP | MFT | BS | BETHLEHEM STL CORP |
| 19 | CON | MFT | BMY | BRISTOL MYERS CO |
| 20 | TRA | TAT | BNI | BURLINGTON NORTHN INC |
| 21 | CAP | MFT | BGH | BURROUGHS CORP |
| 22 | CON | SER | CBS | C B S INC |
| 23 | CAP | MFT | CAT | CATERPILLAR TRACTOR CO |
| 24 | FIN | SER | CHL | CHEMICAL N Y CORP |
| 25 | OIL | MFT | CHV | CHEVRON CORPORATION |
| 26 | FIN | SER | CCI | CITICORP |
| 27 | CON | MFT | KO | COCA COLA CO |
| 28 | UTI | SER | CG | COLUMBIA GAS SYS INC |
| 29 | UTI | SER | CWE | COMMONWEALTH EDISON CO |
| 30 | CAP | SER | CSC | COMPUTER SCIENCES CORP |
| 31 | UTI | SER | ED | CONSOLIDATED EDISON CO N Y INC |
| 32 | TRA | TAT | CNF | CONSOLIDATED FREIGHTWAYS INC |
| 33 | UTI | SER | CNG | CONSOLIDATED NAT GAS CO |
| 34 | FIN | SER | CIC | CONTINENTAL CORP |
| 35 | TRA | TAT | DAL | DELTA AIR LINES INC DEL |
| 36 | CON | SER | DIS | DISNEY WALT PRODTNS |
| 37 | CAP | MFT | DD | DU PONT E I DE NEMOURS & CO |
| 38 | TRA | TAT | EAF | EMERY AIR FGHT CORP |
| 39 | OIL | MFT | XON | EXXON CORP |
| 40 | CAP | MFT | FJQ | FEDDERS CORP |
| 41 | FIN | SER | FIN | FINANCIAL CORP AMER |
| 42 | FIN | SER | I | FIRST INTST BANCORP |
| 43 | CON | MFT | F | FORD MTR CO DEL |
| 44 | CAP | CNT | FWC | FOSTER WHEELER CORP |
| 45 | CAP | MFT | GE | GENERAL ELEC CO |
| 46 | CAP | MFT | GRL | GENERAL INSTR CORP |
| 47 | CON | MFT | GM | GENERAL MTRS CORP |
| 48 | CAP | MFT | GSX | GENERAL SIGNAL CORP |
| 49 | CON | MFT | GT | GOODYEAR TIRE & RUBR CO |
| 50 | CON | TAT | GAP | GREAT ATLANTIC & PAC TEA INC |

Appendix A: List of 100 Sample Companies (Continued)

| | S & P Industry | SIC Industry | Ticker Symbol | Company Name | |
|---|---|---|---|---|---|
| 51 | CAP | CNT | HAL | HALLIBURTON CO | |
| 52 | CON | SER | HCA | HOSPITAL CORP AMER | |
| 53 | CAP | MIN | N | INCO LTD | |
| 54 | CAP | MFT | IR | INGERSOLL RAND CO | |
| 55 | CAP | MFT | IBM | INTERNATIONAL BUSINESS MACHS | |
| 56 | CAP | MFT | HR | INTERNATIONAL HARVESTER CO | |
| 57 | CAP | MFT | IGL | INTERNATIONAL MINERALS & CHEM | |
| 58 | UTI | SER | INI | INTERNORTH INC | |
| 59 | CON | TAT | KM | K MART CORP | |
| 60 | CON | MFT | K | KELLOGG CO | |
| 61 | OIL | MFT | LLX | LOUISIANA LD & EXPL CO | |
| 62 | CON | SER | MCA | M C A INC | |
| 63 | CON | TAT | MZ | MACY R H & CO INC | |
| 64 | CON | TAT | MCD | MC DONALDS CORP | |
| 65 | CON | MFT | MHP | MC GRAW HILL INC | |
| 66 | OIL | MFT | MOB | MOBIL CORP | |
| 67 | CAP | MFT | MTC | MONSANTO CO | |
| 68 | TRA | TAT | NWA | NWA INC | |
| 69 | CON | MFT | DR | NATIONAL DISTILLERS & CHEM CORP | |
| 70 | CAP | MIN | NEM | NEWMONT MNG CORP | |
| 71 | UTI | SER | OEC | OHIO EDISON CO | |
| 72 | CON | MFT | OI | OWENS ILL INC | |
| 73 | UTI | SER | PCG | PACIFIC GAS & ELEC CO | |
| 74 | UTI | SER | PLT | PACIFIC LTG CORP | |
| 75 | UTI | SER | PEL | PANHANDLE EASTN CORP | |
| 76 | CON | TAT | JCP | PENNEY J C INC | |
| 77 | CAP | MIN | PD | PHELPS DODGE CORP | |
| 78 | OIL | MFT | P | PHILLIPS PETE CO | |
| 79 | CON | MFT | PG | PROCTER & GAMBLE CO | |
| 80 | CON | MFT | REV | REVLON INC | |
| 81 | CON | TAT | RAD | RITE AID CORP | |
| 82 | OIL | MFT | RD | ROYAL DUTCH PETE | ADR |
| 83 | CAP | CNT | SLB | SCHLUMBERGER LTD | |
| 84 | CON | TAT | S | SEARS ROEBUCK & CO | |
| 85 | UTI | SER | SNT | SONAT INC | |
| 86 | CAP | MFT | SQD | SQUARE D CO | |
| 87 | OIL | MFT | SUN | SUN INC | |
| 88 | CON | SER | TFB | TAFT BROADCASTING CO | |
| 89 | OIL | MFT | TX | TEXACO INC | |
| 90 | FIN | SER | TA | TRANSAMERICA CORP | |
| 91 | FIN | SER | TIC | TRAVELERS CORP | |
| 92 | TRA | TAT | UAL | U A L INC | |
| 93 | CAP | MFT | X | UNITED STS STL CORP | |
| 94 | OIL | MFT | UCL | UNOCAL CORP | |
| 95 | FIN | SER | USH | USLIFE CORP | |
| 96 | CON | MFT | WLA | WARNER LAMBERT CO | |
| 97 | CAP | MFT | WX | WESTINGHOUSE ELEC CORP | |
| 98 | CON | MFT | WHR | WHIRLPOOL CORP | |
| 99 | CAP | MFT | WMB | WILLIAMS COS | |
| 100 | CON | TAT | Z | WOOLWORTH F W CO | |

## Appendix B: Industry Equations with Three-Stage Least-Squares Estimation

### a. First Estimation Period (1971-1975):

(1) $CAP = -0.7936 + 1.4950*SP - 0.4260*FIN - 0.1236*DLEAD$
(-0.6289) (2.2290) (-0.7545) (0.4090)
SEE = 3.340 DW = 1.59

(2) $CON = 0.2447 + 1.2923*SP - 0.1299*TRA - 0.2107*IPC$
(0.1641) (4.8991) (-0.4052) (-0.7176)
SEE = 3.457 DW = 1.84

(3) $FIN = 0.3569 + 1.4033*SP - 0.3829*UTI - 0.0491*INT$
(0.1156) (1.4918) (-0.3416) (-0.5475)
SEE = 7.449 DW = 2.46

(4) $OIL = 0.8734 + 2.4709*SP - 1.5894*CON - 0.0599*GASP$
(0.3091) (5.1361) (-4.0210) (-0.4036)
SEE = 3.827 DW = 2.41

(5) $TRA = -1.0867 + 0.5001*SP + 0.3210*UTI - 0.0970*IPDT$
(-0.3169) (0.5216) (0.2903) (-0.3793)
SEE = 7.699 DW = 1.30

(6) $UTI = 2.4763 + 0.3161*SP + 0.4682*CAP - 0.0972*INT$
(0.9112) (0.3695) (0.5369) (-1.1924)
SEE = 7.749 DW = 1.91

### b. Second Estimation Period (1976-1980):

(1) $CAP = -0.9057 + 1.4753*SP - 0.3127*FIN + 0.6105*DLEAD$
(-1.4654) (9.2961) (-2.6994) (3.6225)
SEE = 2.118 DW = 2.40

(2) $CON = -0.9270 + 1.4144*SP - 0.3446*TRA + 0.3729*IPC$
(1.1605) (7.6910) (-2.2941) (1.811)
SEE = 2.361 DW = 1.68

(3) $FIN = 0.1544 + 0.7504*SP + 0.6396*UTI - 0.0338*INT$
(0.1068) (1.5031) (0.6351) (-0.1985)
SEE = 3.517 DW = 2.09

(4) $OIL = -1.1560 + 1.1176*SP - 0.1669*CON + 0.6908*GASP$
(-0.7130) (1.6342) (-0.2514) (4.0477)
SEE = 3.385 DW = 1.95

(5) $TRA = 2.3996 + 1.7086*SP - 0.9768*UTI + 0.1853*IPDT$
(1.3791) (5.1756) (-2.3652) (0.5823)
SEE = 5.936 DW = 1.60

(6) $UTI = 0.6863 + 0.9182*SP - 0.3615*CAP - 0.1392*INT$
(0.5613) (1.4003) (-0.6672) (-2.6898)
SEE = 3.511 DW = 1.66

Appendix C: U.S. National Input-Output Tables

a. 1972

Input-Output Coefficients

| Sector | AGR | MIN | CNT | MFT | TAT | SER | OTH |
|---|---|---|---|---|---|---|---|
| AGR | .3141 | .0003 | .0028 | .0542 | .0006 | .0048 | .0001 |
| MIN | .0019 | .0543 | .0090 | .0294 | .0002 | .0100 | .0020 |
| CNT | .0069 | .0282 | .0003 | .0043 | .0107 | .0269 | .0171 |
| MFT | .1435 | .0943 | .3521 | .3745 | .0422 | .0813 | .0080 |
| TAT | .0515 | .0234 | .1014 | .0634 | .0556 | .0219 | .0072 |
| SER | .0967 | .1710 | .0731 | .0781 | .1583 | .1858 | .0244 |
| OTH | .0022 | .0071 | .0028 | .0077 | .0076 | .0094 | .0008 |

Leontief Inverse

| Sector | AGR | MIN | CNT | MFT | TAT | SER | OTH |
|---|---|---|---|---|---|---|---|
| AGR | 1.4944 | .0204 | .0549 | .1349 | .0118 | .0247 | .0030 |
| MIN | .0181 | 1.0676 | .0313 | .0552 | .0065 | .0199 | .0037 |
| CNT | .0205 | .0391 | 1.0113 | .0172 | .0185 | .0364 | .0185 |
| MFT | .3960 | .2255 | .6227 | 1.6847 | .1160 | .1973 | .0303 |
| TAT | .1163 | .0531 | .1587 | .1287 | 1.0748 | .0485 | .0128 |
| SER | .2437 | .2625 | .1948 | .2163 | .2249 | 1.2673 | .0382 |
| OTH | .0098 | .0124 | .0111 | .0168 | .0113 | .0142 | 1.0015 |

b. 1977

Input-Output Coefficients

| Sector | AGR | MIN | CNT | MFT | TAT | SER | OTH |
|---|---|---|---|---|---|---|---|
| AGR | .2463 | .0004 | .0035 | .0470 | .0014 | .0044 | .0007 |
| MIN | .0021 | .0713 | .0091 | .0573 | .0007 | .0183 | .0049 |
| CNT | .0107 | .0375 | .0011 | .0064 | .0141 | .0297 | .0202 |
| MFT | .2020 | .0952 | .3720 | .3819 | .0601 | .0862 | .0134 |
| TAT | .0576 | .0221 | .1096 | .0651 | .0603 | .0253 | .0092 |
| SER | .1007 | .1265 | .0792 | .0750 | .1772 | .1749 | .0253 |
| OTH | .0027 | .0037 | .0032 | .0077 | .0079 | .0088 | .0020 |

Leontief Inverse

| Sector | AGR | MIN | CNT | MFT | TAT | SER | OTH |
|---|---|---|---|---|---|---|---|
| AGR | 1.3608 | .0170 | .0491 | .1097 | .0139 | .0213 | .0042 |
| MIN | .0404 | 1.0963 | .0567 | .1119 | .0163 | .0389 | .0092 |
| CNT | .0289 | .0500 | 1.0172 | .0253 | .0250 | .0415 | .0224 |
| MFT | .5162 | .2391 | .6841 | 1.7307 | .1637 | .2191 | .0456 |
| TAT | .1303 | .0549 | .1762 | .1384 | 1.0867 | .0562 | .0172 |
| SER | .2503 | .2088 | .2127 | .2206 | .2553 | 1.2570 | .0426 |
| OTH | .0112 | .0084 | .0121 | .0171 | .0123 | .0136 | 1.0030 |

* Adapted from Miller & Blair(1985), pp. 424-425

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