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SIX ESSAYS ON THE GOVERNMENT REVENUES AND WELFARE LOSSES FROM MONEY CREATION

by

PETER M. SPERLING

A dissertation submitted to the Graduate Faculty in Economics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York

1978

This manuscript has been read and accepted for the Graduate Faculty in Economics in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

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Abstract

SIX ESSAYS ON THE GOVERNMENT REVENUES
AND WELFARE LOSSES FROM MONEY CREATION

by

Peter M. Sperling

Advisor: Professor Alvin Marty

Since 1956, a number of models have been developed aimed at defining and estimating the government revenues and associated welfare losses from money creation. This dissertation consists of six essays which examine various issues that have evolved out of the development of these models.

The first three essays deal with alternative definitions of the government revenues and welfare losses from money creation under various assumptions concerning the growth rate of the economy. The concepts of the average and marginal tax costs of money creation are derived and compared for the various cases. It is shown that, in general, Martin Bailey's indictment of "inflationary finance" as an inefficient source of revenue above relatively modest money growth rates holds up under a variety of assumptions.

The next two essays examine the "inflation tax" in the context of more general taxation models. It is shown that there is one definition of the revenues and welfare costs from money creation that is consistent with a variety of assumptions concerning the institutional constraints on the government's budgetary and monetary processes. It is also shown that the procedures followed in the earlier essays are only approximations to the economically correct way of analyzing the "inflation tax."

The dissertation concludes with an essay that examines some alternative approaches to analyzing the costs from money creation under conditions of uncertainty.

Foreword

> "It is common to speak as though, when a government pays its way by inflation the people of the country avoid taxation. We have seen that this is not so. What is raised by printing notes is just as much taken from the public as is a beer duty or an income tax. What a government spends the public pay for. There is no such thing as an uncovered deficit. But in some countries it seems possible to please and content the public, for a time at least, by giving them, in return for the taxes they pay, finely engraved acknowledgements on watermarked paper. The income tax receipts, which we in England receive from the Surveyor, we throw into the wastepaper basket; in Germany they call them bank notes and put them into their pocket-books; in France they are termed rentes and are locked up in the family safe."[1]

It is not hard to imagine the above paragraph as being part of one of the recent contributions to what has somewhat loosely been called the "theory of inflationary finance". However, this paragraph is in fact from Chapter 2 of Keynes' A Tract on Monetary Reform, the first edition of which was published fifty-five years ago. The recognition that government can acquire real resources, and in the process impose costs on its citizens, through the process of money creation is an economic insight that has been with us for many years.

This thesis consists of six essays on the government revenues and associated welfare costs from money creation. Although the essays touch on a number of issues, one or two themes turn up in most of the essays: (1) what are the economically correct ways to define the government revenues and welfare costs from money creation? and (2) does Martin Bailey's indictment of inflationary finance as a source of government revenue[2] hold up under a variety of different assumptions?

Essay 1 develops Bailey's original model and then extends the model to incorporate induced and/or autonomous growth in real aggregate output. At the end of the chapter I briefly sketch an extension of the model which incorporates time and savings deposits in addition to currency and demand deposits.

Essay 2 develops a somewhat different way of viewing the government's procedures in moving from one money growth rate to another. Essay 3 switches gears by introducing explicitly another way of defining the revenues and costs from money creation.

Essay 4 investigates the revenues and costs from money creation within a more complete model of the government's budgetary process. This analysis provides a more direct rationale for particular measures of the revenues and costs from money creation.

Essay 5 analyzes inflationary finance within the context of explicit models of optimal commodity taxation. These sorts of models trace back to the work of Frank Ramsey in 1927[3] and have been elaborated upon by a number of economists in recent years.

Finally, Essay 6 concludes the thesis by considering two models which attempt to analyze the impact upon money holders when uncertainty about the rate of price change is introduced into the analysis.

I should point out here that throughout the essays I do not consider the host of socio-political factors that lead governments to pursue inflationary policies. The thrust of these essays is on the efficiency properties of inflationary finance. In many real world situations, such efficiency concerns are minor compared with explicit policy goals that the government wishes, or feels constrained, to pursue and which involve, for various reasons, resort to the printing press.

That this thesis has finally been completed is due almost entirely to the efforts of two individuals. I wish to thank my thesis advisor, Professor Alvin L. Marty, for all the help and time he has given me. And I wish to thank my wife, Paula, for her continual encouragement and support throughout this effort, particularly when the press of other business made me think that I would never get time to finish this project.

Footnotes to Foreword

1
John Maynard Keynes, A Tract on Monetary Reform, (London: The Macmillan Press, Ltd.) 1971, pp. 52-53.

2
Martin J. Bailey, "The Welfare Cost of Inflationary Finance," Journal of Political Economy, April, 1956.

3
Frank Ramsey, "A Contribution to the Theory of Taxation," Economic Journal, March, 1927.

Table of Contents

List of Illustrations

ESSAY 1: Basic Models of the Revenues and Welfare Costs of Money Creation

The literature on the revenues and costs of money creation has grown very rapidly since Martin Bailey's pioneer work in this area.[1] The present essay summarizes and generalizes much of this work.

Revenues from Money Creation

Governments can obtain revenue -- which may then be used to hire or purchase real resources -- in three basic ways: taxation; borrowing from the public; and creating new money. In what follows I shall assume that the flow of taxes and government borrowing is held constant, so that increased government expenditures must necessarily be financed by the creation of new money. Assume also -- just for the time being -- a static (non-growing) economy in which money consists solely of non-interest bearing government currency.

Suppose that, to finance a larger flow of government spending, the government decides to increase the money supply at a new, higher constant rate. Eventually, after all adjustments by the public have been made, the rate of inflation will equal the rate of monetary growth and the money rate of interest will be at a new, higher level.

Since the money rate will have risen, households and firms will have chosen to hold a smaller quantity of real cash balances. In addition, both aggregate money income and prices will be increasing at the same rate, leaving aggregate real income unchanged. However, to preserve the smaller quantity of real balances from shrinking further because of the continuing higher-than-previous rate of inflation, households and firms must reduce their rate of spending out of their flow of money incomes in order to continuously increase their nominal money holdings, thus keeping their real cash holdings intact. It is as if the flow of real household disposable income and real net business cash flow had fallen. This will curtail real spending on consumer and newly-produced capital goods, thus releasing the resources used in their production. Government can then hire or purchase these resources, paying for them with freshly minted currency.

Algebraically, government revenue from money creation in the above example -- in real terms -- is $\dot{M}/P$, where M is the nominal money supply (here, currency), P is the price level, and -- a convention I shall use throughout this paper -- a dotted variable represents a time derivative, i.e., $\dot{M}=dM/dt$. Since $\dot{M}=(\dot{M}/M)M$, real government revenue from money creation can be expressed as $(\dot{M}/M)\ (M/P) = \rho\ (M/P)$, where ρ is the proportional rate of growth of the money supply.

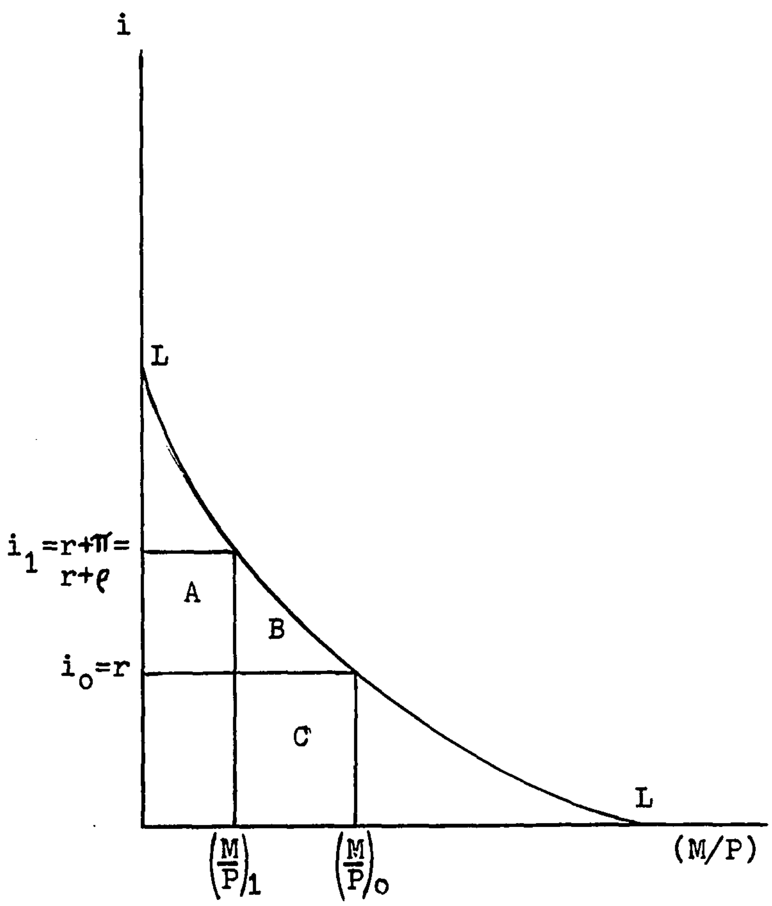
Costs of Money Creation

In discussing the costs of money creation -- and, therefore, the costs associated with prices rising more rapidly, or falling less rapidly, than would otherwise be the case -- writers on current economic conditions usually focus on unanticipated changes in prices. These costs generally fall under two headings: the redistribution of income away from creditors and to debtors; and the uncertainty generated by not knowing, with a reasonable degree of accuracy, the future course of price movements. However, until the final essay, I shall be dealing with alternative equilibrium positions, thus ruling out the above two factors (which are associated with disequilibrium states). I shall assume that expectations are adjusted immediately and accurately to changes in the growth rate of the money supply and that all contracts denominated in money terms calling for payment of money interest have, in effect, variable interest rates.[2] (As will be mentioned later, this assumption will not necessarily apply to time and savings accounts at financial institutions. Also, I shall assume that currency and demand deposits yield no explicit money interest payments.)

Given these assumptions, the total losses from money creation consist of the implicit tax paid by money holders, which is equal to the value of the real resources that must now be devoted to more frequent money transactions, lost leisure time, and the greater reliance on barter arrangements. These latter costs are often referred to as, for example, "bookkeeping costs," "trips to the bank," or "search and information costs."[3] When it is recognized that the demand function for real money balances may be thought of as a marginal productivity function of real balances,[4] the costs of money creation can be illustrated diagrammatically in a straightforward manner.

Figure 1 shows the demand for real balances (the LL curve) as a function of the money rate of interest. For illustrative purposes, I continue to assume a static economy with all money being non-interest bearing currency. With a constant money supply, the money rate of interest would equal the real rate -- $i_o = r$ -- and the quantity of real balances held would equal $(M/P)_o$. If the government now increases the money supply at a constant rate, ρ $(=\dot{M}/M)$, the rate of price change -- $\Pi = \dot{P}/P$ -- would come to equal ρ and the money rate of interest would rise to

Figure 1: Stationary Economy



$i_1 = r + \Pi = r + \rho$. Note that I am assuming that r, the real rate, remains constant, i.e., there is no real balance effect. I shall justify this assumption in later essays, although various monetary growth models point to different conclusions for the case of a growing economy.[5]

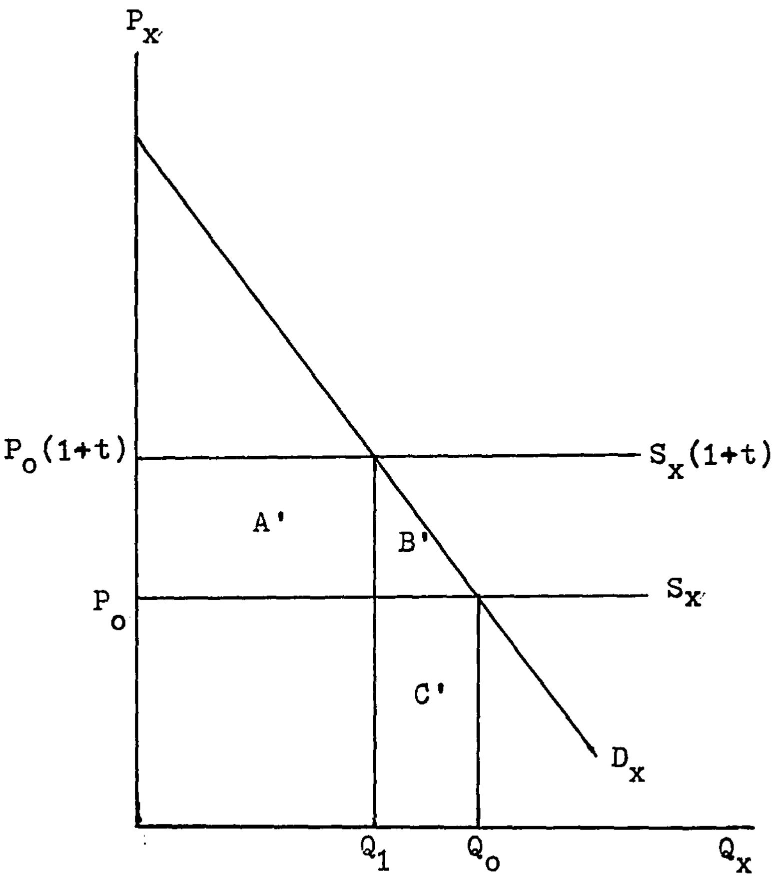
At a money rate equal to i_1, the equilibrium quantity of real money will be $(M/P)_1$. Government revenue, in real terms, is $\rho(M/P)$, which is the rectangle A in figure 1. The total costs (or burden) of money creation consist of the implicit tax on money holders, rectangle A, plus the loss in the total product of real balances suffered by money holders as a consequence of holding a smaller stock of real money. With the demand function viewed as a marginal productivity curve of real balances, this additional loss is represented by the area B + C. The <u>deadweight loss</u> from money creation (or <u>net</u> burden) is, then, B + C, since area A represents simply a transfer of resources from money holders to the government.

Note that this analysis of the (net) welfare loss from money creation is quite similar to the analysis of the welfare effects of an excise tax on a consumer good. However, the two analyses are not exactly identical. Figure 2 illustrates the case of an excise tax on a consumer good in which the supply curve is infinitely elastic, so that there is no loss in producers' surplus (as was the case for money creation). $S_X(1 + t)$ is the supply curve after the excise tax is imposed (at a proportional rate t), and the product price rises to $P_o(1 + t)$. The loss in consumers' surplus is A' + B', while government revenue is A', making the deadweight loss equal to B'.

For the case of an excise tax on an ordinary consumer good, the deadweight loss does not include the area C' in figure 2, while the deadweight loss from money creation does include the corresponding area C in figure 1. The difference is that part of the welfare cost of money creation is the loss in the total product of real balances, measured by the entire area under the demand curve from the initial to final stocks of real money. The loss in consumers' surplus, on the other hand, measures only the loss of the excess in money value of the quantity purchased over and above what consumers actually spend to buy the good at the higher prices.

There is, of course, a case in which the two analyses would yield identical results. If the consumer good had a pre-tax horizontal supply curve at a zero price, and if the demand for real balances was a function of Π rather than i (say because r was zero), then the measure of the deadweight loss from money creation for

Figure 2: Excise Tax



the stationary economy would correspond exactly to that for the ordinary consumer good. (This would also be true for the induced-growth case developed below.) In general, however, there will be some difference in the measures of the net burden in the two cases.

What is "Money?"

Up to this point, I have assumed that "money" consists of currency which yields no explicit money interest. From this point on in this essay I shall also assume that "money" includes non-interest bearing demand deposits. As far as money holders are concerned, though, the concept of money -- for analyzing the welfare losses from money creation -- might well include other assets. In particular, if there exist time and savings accounts at commercial banks and/or thrift institutions which do not yield the full competitive money rate of interest, then these deposits should also be classified as money.

I shall continue to assume that bonds have variable interest rates, so that these assets will not be treated as money. Note, however, that from the government's point of view -- i.e., in terms of the direct revenue from money creation -- nominal receipts are simply the change per unit time in the stock of high-powered money, where high-powered money (or the monetary base) consists of currency in the hands of the non-bank public plus commercial bank reserves.

For most of the remainder of this essay, I shall assume that money consists only of non-interest bearing currency and demand deposits. At the end of this essay I shall show how incorporating savings and time deposits affects the analysis of the revenue and costs of money creation.

The Demand for Money

Instead of using a general functional form of the money demand function, I shall use the specific form utilized by Cagan in his seminal work on hyperinflation[6] and widely used by others. Let

M = nominal money supply,

P = price level,

N = population,

Q = aggregate real income or output,

$q = Q/N$,

i = money rate of interest,

$m = M/PN$,

a, b, k = constants

The per capita demand for real money balances is assumed to be

$$m = q^b k e^{-ai},$$

where "e" is the base of natural logarithms. This particular function has the property that the per capita income elasticity for real cash balances, $\eta = (\partial m/\partial q)/(q/m)$, is constant and equal to b.

Assume also that the aggregate demand for real balances can be written as

$$M/P = Nq^{b}ke^{-ai}.$$

When b = 1, which would seem to be the appropriate assumption for the analysis of steady-state growth paths, the aggregate demand for real balances as a proportion of real aggregate income is

$$M/PQ = ke^{-ai},$$

a form of this function often used in the literature.[8]

Recall that the government's receipts from money creation are equal to the change in high-powered money. Denote this term by $\dot{H}$ (= dH/dt). The change in the money supply following a change in the stock of high-powered money is $\dot{M} = h\dot{H}$, where h is usually referred to as the money multiplier for the narrowly defined money supply. Let G = real governement revenue from money creation. Then $G = (1/h)\ (\dot{M}/P) = h'\rho\ (M/P)$, where $h' = (1/h)$ and ρ is, as before, $(\dot{M}/M)$. Given the Cagan-type demand function specified above, we have

$$G = h'\rho Nq^{b}ke^{-ai}.$$

Government maximizes real receipts from money creation when $\partial G/\partial \rho = 0$, which means that $\rho = (1/a)$ to maximize revenue. (See Appendix A. When $\rho = (1/a)$, the elasticity of demand for real balances with respect to ρ equals unity.) Note that there is a conflict between maximizing revenue and maximizing efficiency. Efficiency -- in the Paretian sense -- requires that the social opportunity cost of holding real balances be equal to the social marginal cost of producing real balances. The social marginal cost is zero, so the social opportunity cost -- here the money rate of interest -- should also equal zero (i.e., money holders should be satiated with real balances for Pareto optimality).

In a stationary economy, satiation would require a continually decreasing nominal money stock, the rate of decline being equal to the real interest rate (thus causing prices to fall at a rate equal to the real rate and the money rate to equal zero). Government would certainly not be obtaining any revenue. In fact, government would be providing subsidies to money holders. Alternatively, if the government were maximizing its receipts from money creation (or, for that matter, if it were obtaining <u>any</u> amount of revenue at all), the money interest rate would necessarily be above zero. Similar reasoning applies to the case of a growing economy.[9]

Relationships Between Money, Prices and Growth

Some important relationships that are used in the remainder of this essay are discussed below and in Appendix B.

Begin with the equation of exchange,

$$MV = PQ$$

where V = income velocity and the other terms are as defined previously. It follows that

$$\dot{M}/M + \dot{V}/V = \dot{P}/P + \dot{Q}/Q.$$

Using the identity $(\dot{q}/q) = (\dot{Q}/Q) - (\dot{N}/N)$ and the definition $\eta = (\partial \ \text{Ln}M/PN) \ / \ (\partial \text{Ln} \ q)$, the following expression can be derived:

$$\rho = \Pi + \ell + b \ (\lambda - \ell),$$

where $\ell = \dot{N}/N$, $\lambda = \dot{Q}/Q$ and ρ, Π, and b have the same meaning as before. Rearranging terms,

$$\Pi = \rho - \ell(1\text{-}b) - b\lambda$$

or, letting $(1\text{-}b) = \beta$

$$\Pi = \rho - \beta\ell - b\lambda.$$

This expression is relevant because, on my assumption that r, the real rate, is constant, $i = r + \Pi = r + \rho - \beta\ell - b\lambda$. For the cases where either $b = 1$ (so that $\beta = 0$) or $\lambda = \ell$, $\Pi = \rho - \lambda$.

In what follows I shall assume that ℓ and b are constant, although ℓ might be zero and b might differ from unity.

As for λ, it may be either autonomous or induced by the government from the revenue derived from money creation or both. Any autonomous growth will be assumed constant and not affected in any way by the process of money creation.

The Stationary Economy

Figure 1 above illustrated geometrically the revenues and costs of money creation for a stationary economy. In this case $\ell = \lambda = 0$. Thus, when $\rho = 0$, $i = r$, and when $\rho > 0$, $\Pi = \rho$, so $i = r + \rho = r + \Pi$. The ratio of the net (or deadweight) welfare loss to real government revenue, W/G, is

$$W/G = \frac{e^{a\rho}-(1+a\rho)}{ah'\rho} + \frac{r\ (e^{a\rho} - 1)}{h'\rho}.$$

(See Appendix C.) Bailey termed this ratio the cost of collecting the government's revenue, and I shall refer to it as the average tax cost of money creation. In most of his paper, Bailey made the demand for real balances a function of Π rather than i -- a reasonable assumption since he was examining hyperinflations (so that by far the largest portion of the opportunity cost of holding real balances consisted of the inflation rate). Implicitly, this amounts to assuming that $r = 0$. In that case, with $\rho = \Pi$,

$$W/G = \frac{e^{a\pi} - (1 + a\Pi)}{ah'\Pi},$$

which is exactly the result derived by Bailey. Based on this formula, Bailey concluded that revenue appropriation through money creation became inefficient when $\rho > 18\%$, using parameters favorable to inflationary finance.

In a recent paper, Tower[10] suggested that the appropriate measure was not the average tax cost of money creation but rather the ratio of the marginal increment in welfare loss to the marginal increment in government revenue, $(\partial W/\partial \rho)/(\partial G/\partial \rho)$.[11] We may call this concept the marginal tax cost of money creation. For the stationary economy, the marginal tax cost is

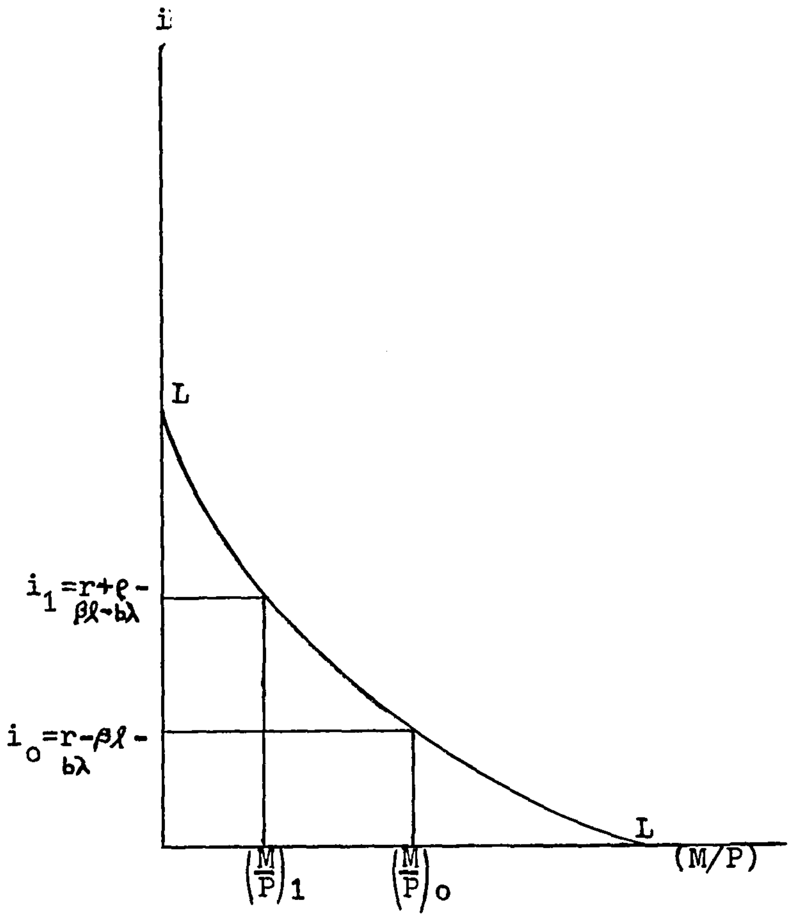
$$MTC = \frac{a\ (r + \rho)}{h'\ (1-a\rho)}.$$

Just as Bailey underestimated the true average tax cost of money creation, by ignoring the term $r(e^{a\rho} - 1)/h'\ \rho$, the marginal tax cost would also be underestimated if r were assumed to be zero.

Autonomous Growth

Now assume that the economy is growing at some fixed rate, λ, that is totally unaffected by the rate of money creation. This case is illustrated in figure 3. With a constant nominal money stock, the rate of deflation would be $(\beta\ell + b\lambda)$ which, if $b = 1$ or $\lambda = \ell$, would be the more commonly used $\Pi = -\lambda$. The money rate of interest, then, would be $(r - \beta\ell - b\lambda)$. With positive nominal money growth, the money rate would rise to $(r + \rho - \beta\ell - b\lambda)$. The average

Figure 3: Autonomous Growth



tax cost of money creation for this case is

$$W/G = \frac{e^{a\rho} - (1 + a\rho)}{ah'\rho} + \frac{(r - \beta\ell - b\lambda)\ (e^{a\rho} - 1)}{h'\rho} .$$

(See Appendix D.)

A comparison of the average tax costs for the autonomously growing economy and the stationary economy indicates that, for identical real interest rates and money growth rates (and with other parameters also identical), the autonomously growing economy will have a lower average tax cost of money creation. (In effect, the existence of autonomous growth is favorable to "inflationary finance.") But the case against money creation as a source of revenue still remains intact.[12] This can be most easily seen by realizing that Bailey's original conclusions were based on calculations using only the first term on the right-hand side of the above expression.

Moreover, an interesting theorem, first put forth by Marty,[13] follows directly. When $b = 1$ and $r = \lambda$ (in effect, equivalent to making the demand for real balances a function of Π in the stationary economy[14]), the average tax cost becomes

$$WG = \frac{e^{a\rho} - (1 + a\rho)}{ah'\rho} .$$

This expression is identical to the average cost for the stationary economy when money demand is a function of Π. It says that, where the per capita income elasticity of demand for real balances is unity and the economy is growing along the "golden rule" growth path, the average tax cost of money creation is completely independent of the rate of autonomous real growth. The average tax cost depends solely on the rate of monetary growth and the parameters a and h'.

The marginal tax cost for the autonomously growing economy is

$$MTC = \frac{a(r + \rho - \beta\ell - b\lambda)}{h'(1 - a\rho)} .$$

Clearly, autonomous growth lowers the marginal tax cost of money creation in the autonomously growing economy relative to the stationary economy. (Thus, Tower is wrong in asserting that, based on marginal tax cost, growth weakens the case for money creation. See Tower, 1971, p. 856.) Note that, for $b = 1$ and $r = \lambda$, the marginal tax cost of money creation for the autonomously growing economy is

$$MTC = \frac{a\rho}{h'(1 - a\rho)}$$

which, like the average tax cost under the same assumptions, is completely independent of λ. However, the marginal tax cost here is less than the marginal cost for the stationary economy.

Induced Growth

Assume now that growth occurs only because of government investment which is financed by money creation. The implicit assumption here is that there is either a pool of unemployed labor that can be hired to work with the newly-produced capital goods or that the labor supply is infinitely elastic at some given real wage (á la Malthus). λ now represents the rate of induced growth, which is a function of the rate of money creation.[15]

Figure 4 illustrates the case of induced growth. When the nominal money stock is constant, no growth occurs and i = r. When positive money growth exists, i rises to $(r + \rho - \beta\ell - b\lambda)$. The average tax cost of money creation is

$$W/G = \frac{e^{a(\rho-\beta\ell-b\lambda)} - [1+a(\rho-\beta\ell-b\lambda)]}{ah'\rho} + \frac{r(e^{a(\rho-\beta\ell-b\lambda)}-1)}{h'\rho}$$

(See Appendix E.) Marty, who was the first to examine this case,[16] assumed that r = 0 and b = 1. Under those assumptions,

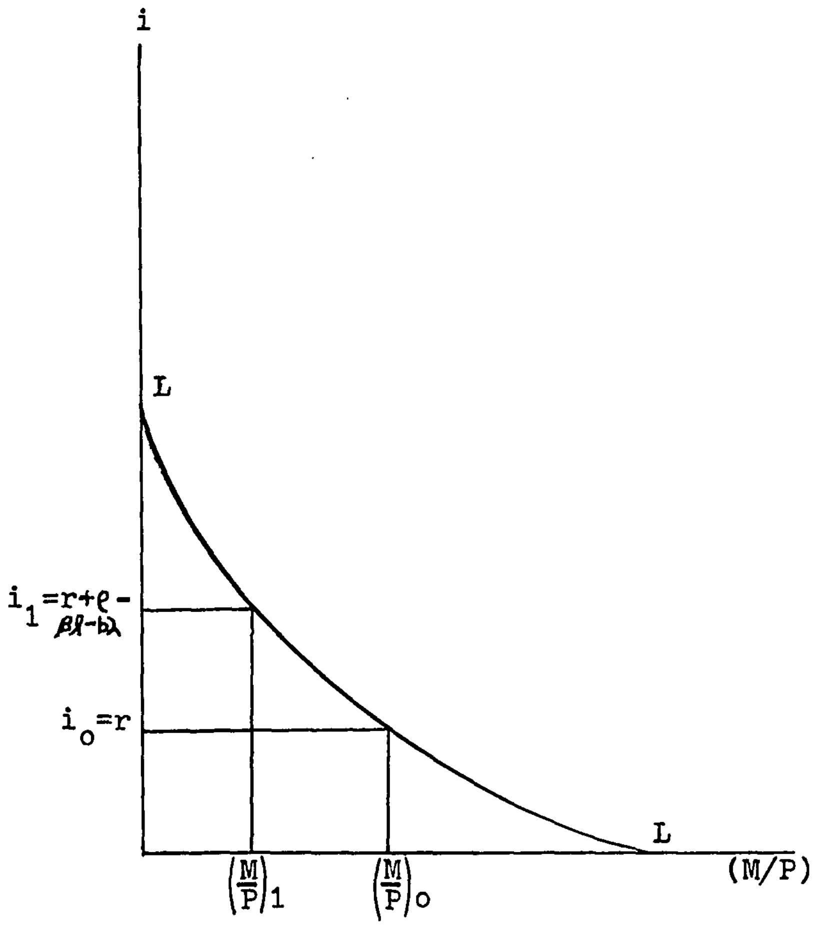
$$W/G = \frac{e^{a(\rho - \lambda)} - [1 + a(\rho - \lambda)]}{ah'\rho}$$

The marginal tax cost of money creation for the induced growth economy is somewhat more complicated than the previous two cases, since λ is now a function of ρ. The marginal tax cost is

$$MTC = \frac{a(r+\rho-\beta\ell-b\lambda) \quad [1-b(d\lambda/d\rho)]}{h'[1-a\rho(1-b(d\lambda/d\rho))]}.$$

When the government is maximizing its real receipts from money creation, and thus maximizing the rate of economic

Figure 4: Induced Growth



growth, $(d\lambda/d\rho) = 0$ and the marginal cost becomes

$$MC = \frac{a(r + \rho - \beta\ell - b\lambda)}{h'(1 - a\rho)}$$

which is, not surprisingly, identical to the expression derived for the autonomously growing economy, since when the government is maximizing growth λ can be treated as a constant.

Autonomous and Induced Growth

This case allows for both of the prior two types of real economic growth. Let λ' = rate of autonomous growth and λ'' = rate of induced growth. When $\rho = 0$, the government obtains no revenue, there is no induced growth, and $i = r - \beta\ell - b\lambda'$ -- the same as in the autonomous growth case when $\rho = 0$. For $\rho > 0$, some induced growth occurs and $i = r + \rho - \beta\ell - b(\lambda' + \lambda'')$. See figure 5.

The average tax cost of money creation is

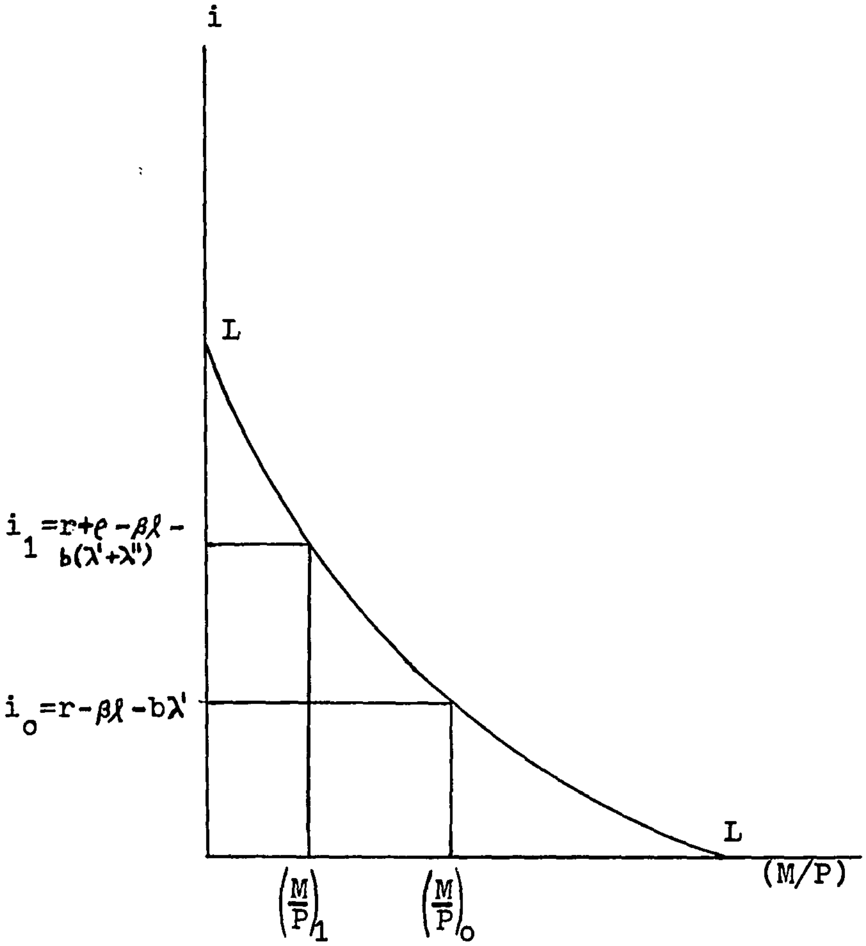
$$W/G = \frac{e^{a(\rho-b\lambda'')} - [1+a(\rho-b\lambda'')]}{ah'\rho} + \frac{(r-\beta\ell-b\lambda')(e^{a(\rho-b\lambda'')}-1)}{h'\rho} .$$

The expression for the marginal tax cost is again somewhat complicated because of the dependence of λ'' on ρ. It is

$$MTC = \frac{a(r+\rho-\beta\ell-b[\lambda'+\lambda''])\ (1-b[d\lambda''/d\rho])}{h'[1-a\rho(1-b(d\lambda''/d\rho))]}.$$

Note that here, too, if the government is maximizing revenue and induced growth, the expression for the marginal cost becomes identical to that for the case of autonomous growth above.

Figure 5: Autonomous and Induced Growth



Defining Money more Broadly

To conclude this essay, I shall examine the case of a stationary economy when "money" includes other assets besides non-interest bearing currency and demand deposits. In particular, let there be some other assets -- such as deposits at mutual savings banks and savings and loan associations -- which do not bear the full competitive money rate of interest. This may be because of reserves, upon which no interest is earned, that must be held against these deposits or because of legal ceilings on interest rates that these institutions can pay on their deposits. (The same principles discussed here could be applied to interest-bearing demand deposits.)

It is now necessary to clearly distinguish between the money rate of interest and the opportunity cost of holding real money balances. In all of the cases examined up to now these two concepts were identical. They are still identical for currency and demand deposits, i.e., the opportunity cost of holding these deposits is still the money rate of interest. For other types of money, however, the opportunity cost is $\gamma i, \gamma<1$. If, for example, thrift institutions earned no interest on their required reserves but paid the highest possible money interest rate given that constraint, γ would equal the average required reserve ratio for these deposits.

The opportunity cost of holding total real money balances is, then, a weighted average of the above two costs (i and γi, the weights being the proportion of total money held in each form of asset -- which propor proportions can (and are likely to) change as money interest rates change:

$$\text{Opportunity Cost} = zi + (1-z)\ \gamma i$$

where z = proportion of total money held in the form of currency and demand deposits, (1-z) = proportion of total money held in the form of thrift deposits, and z = f (i).

Rearranging terms,

$$\text{Opportunity Cost} = i[z\ (1-\gamma)\ +\ \gamma].$$

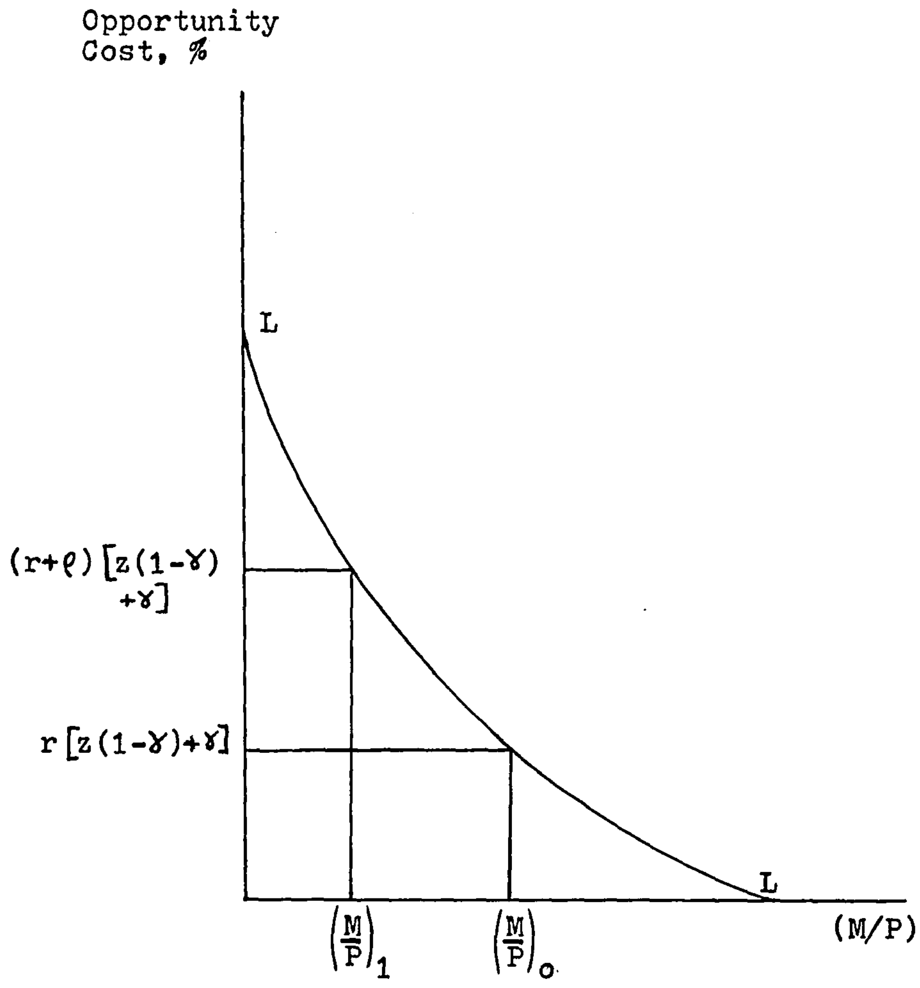
This case is illustrated in figure 6.

The average tax cost of money creation under these assumptions is

$$WG = \frac{e^{a\rho[z(1-\gamma)+\gamma]} - (1+a\rho[z(1-\gamma)\ +\ \gamma])}{ah'\rho} + \frac{r[z(1-\gamma)\ +\gamma]\ (e^{a\ [z(1-\gamma\ +\gamma]}-1)}{h'\rho}.$$

(See Appendix G.) Note that when all money is non-interest bearing currency and demand deposits, z = 1 and the average cost becomes identical to the expression for the stationary economy derived earlier. The formula for the marginal tax cost is a complicated one and is relegated to Appendix G.

Figure 6: Stationary Economy,
Broadly Defined Money



Footnotes to Essay 1

1
Martin J. Bailey, "The Welfare Cost of Inflationary Finance," Journal of Political Economy, April, 1965, pp. 93-110.

2
This corresponds to the type of flexible wage and price model developed in, e.g., Martin J. Bailey, National Income and the Price Level (New York: McGraw-Hill Book Company), 1971, Second Edition, Chapter 4.

3
Good discussions of exactly how these costs manifested themselves during actual hyperinflations can be found in Bailey, op. cit., pp. 100-101, and Otto Friedrich, "Inflation," T.V. Guide, August 10, 1974, pp. 10-12.

4
Bailey, op. cit., pp. 101-102.

5
See, for example, Miguel Sidrauski, "Inflation and Economic Growth," Journal of Political Economy, December, 1967; David Levhari and Donald Patinkin, "The Role of Money in a Simple Growth Model," American Economic Review, September, 1968; and Jerome Stein, Money and Capacity Growth (New York: Columbia University Press), 1971, Chapters 1-2.

6
Philip Cagan, "The Monetary Dynamics of Hyperinflation," in Milton Friedman (ed.), Studies in the Quantity Theory of Money (Chicago: University of Chicago Press), 1956.

7
Alvin L. Marty, "Growth, Satiety and the Tax Revenue from Money Creation," Journal of Political Economy, September/ October, 1973, p. 1147.

8
See, for example, Marty, ibid; Charles D. Cathcart, "Monetary Dynamics, Growth, and the Efficiency of Inflationary Finance," Journal of Money, Credit and Banking, May, 1974; and Alvin L. Marty, "Growth and the Welfare Cost of Inflationary Finance," Journal of Political Economy, February, 1967.

9
This conflict is discussed fully in Marty, 1973, pp. 1147-1151.

10
Edward Tower, "More on the Welfare Cost of Inflationary Finance," Journal of Money, Credit and Banking, November 1971.

11
After the first three essays of this thesis were written, I became aware of a very neat and simple way in which to calculate the marginal tax cost of money creation. See Alvin L. Marty, "A Note on the Welfare Cost of Money Creation," Journal of Monetary Economics, January, 1976.

12
See Marty, 1973, pp. 1140-1142.

13
Marty, 1973, pp. 1140-1141.

14
Marty, 1973, p. 1142.

15
Robert A. Mundell was the first to set forth the basic model of an induced-growth economy. See his "Growth, Stability, and Inflationary Finance," Journal of Political Economy, April, 1965.

16
Marty, 1967.

ESSAY 2: Variation on a Theme

In the previous essay I worked through the analysis of the gains and losses from money creation under what might be termed traditional assumptions. This chapter explores the same analysis under somewhat different assumptions.

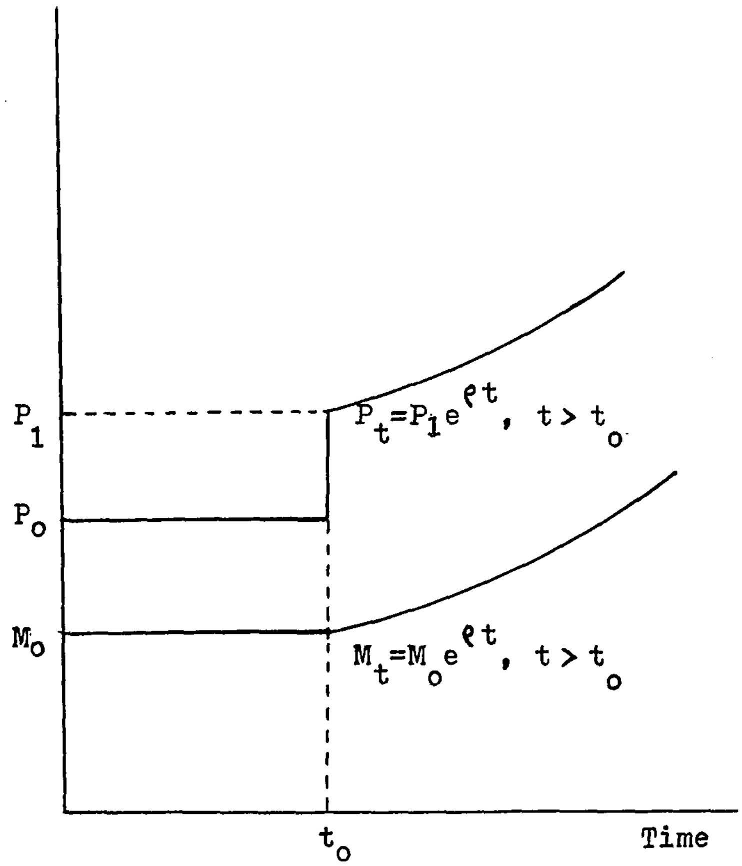
Inflationary Expectations

Up to now I have assumed that expectations of future inflation are correctly anticipated. In fact, however, it seems quite probable that expectations may be incorrect, at least for some period of time, when the traditional assumptions of the last chapter are employed.

The problem of incorrect inflationary anticipations arises in the transition period from one constant rate of nominal money growth to some new, also constant, rate of monetary expansion. In the standard macroeconomic model, such as that developed by Bailey,[1] the time paths of the money stock and the price level -- and thus real money balances -- are illustrated in figure 7.

Suppose, for simplicity, that we are examining a stationary economy in which the money stock and price level have been at constant levels for some time. At time $t=t_0$,

Figure 7: Money Supply and
Price Level



the government announces that the money supply will henceforth increase at a proportional rate $\rho > 0$. There is an immediate "once-and-for-all" rise in the price level to $P=P_1$, and from then on both money and the price level increase at the same rate ($\rho = \Pi$). As a result, there is a "once-and-for-all" decline in the stock of real money balances.

The difficulty with inflationary expectations occurs at $t=t_o$. The instantaneous increase in the price level from P_o to P_1 could be viewed as an extremely short period of extremely rapid inflation. Indeed, it is only after a lengthy period of time that the average rate of price change per unit time, measured with $\underline{P_o}$ as the base level, approaches ρ. During the time periods immediately following t_o, the average rate of price change per unit time can be significantly greater than ρ. (Again, it must be emphasized that P_o is being treated as the base level of prices for this comparison.)

Thus, economic units may, for a while, overestimate the future rate of inflation. This, in turn, through its impact on nominal wage rates and money rates of interest, can lead to disequilibrium in the markets for goods and labor services. While the government may wish to obtain revenue from money creation, it may also desire to minimize the economic disruptions that may arise until the expected rate of inflation comes to equal the rate of money growth.

Constancy of the Real Interest Rate

In the first essay, I assumed that the real rate of interest was constant and unaffected by the gains and losses from money creation (and the disposition of the revenue). This assumption has made the analysis much simpler by allowing me to focus on the impact of an inflationary premium on the money interest rate and not be concerned with the possible impact on the money rate of a simultaneous change in the real rate.

It may well be, though, that changes in the rate of money growth have the effect of changing the real rate, and it may also well be that the government does not wish the real rate to change. To take one simple example, suppose in an autonomously growing neoclassical world the economy is moving along the "golden rule" growth path (where the real rate equals the rate of growth of aggregate output). If a higher rate of money growth were to reduce the real rate, then the result would be an inefficient growth path. That is, consumption per head could be permanently increased by a permanent reduction in the capital-labor ratio and thus a permanent rise in the real rate.

A change in the real rate following a change in the money growth rate might be brought about in two different ways. In the first place, a change in the rate of money growth which changes the stock of real money balances may be viewed as a change in net real private wealth. If household or business saving is dependent upon net real wealth, a change in real money balances would affect the real rate by changing the savings rate (and thus the ratio of investment to income). The simplest example of this sort of effect is the "Pigou effect" in the stationary economy.[2]

Secondly, changes in the rate of money growth might be viewed as having an effect on real disposable income. This sort of effect has usually been treated in the literature on neoclassical monetary growth models.[3]

To show this second type of impact, suppose that real disposable income is defined as the flow of commodities produced per unit time <u>plus</u> the increment to real cash balances (the capital gain or loss on nominal money holdings):

(1) $$Y_d = Y + d\,(M/P)dt,$$

where Y_d = real disposable income

Y = real output of goods

$d(M/P)/dt$ = increment to real money holdings.

Since $d(M/P)/dt = (\rho - \Pi) \cdot (M/P)$, the above expression can be written as

(2) $$Y_d = Y + (\rho - \Pi)\,\frac{M}{P}\,.$$

Assuming a proportional savings function, aggregate savings = $s \cdot Y_d$, where s is the average (and marginal) propensity to save. It is intuitively clear, and can be rigorously shown, that a change in ρ, which changes real balances, will change not only disposable real income but also the flow of physical or material savings and investment. With a linear homogeneous production function, the capital-labor ratio will change and so, therefore, will the real rate. This same result will follow if disposable income is defined to also include the utility yield from real balances (as in Johnson[4]) or the product of the money rate and the stock of real balances (as in Levhari and Patinkin[5]).

Supplementing Money Creation

In order to avoid incorrect price anticipations and changes in the real rate, the government might follow a policy of what Auernheimer[6] would term being "honest." At the time when the government announces and changes the growth rate of the nominal money supply, it should -- under this policy -- also "buy" or "sell" the increment in real money balances desired by money holders.

As discussed with reference to figure 7, the reduction in real balances in the standard macroeconomic model occurs because of a once-and-for-all rise in the price level. In turn, this arises because money holders expect the opportunity cost of holding money to rise as the rate of monetary growth increases, and the attempt to reduce nominal money holdings by a burst of spending on goods drives up the price level.

Under the "honest" government policy, if the change was to a higher rate of money growth, the government would exchange goods that it owns for the nominal money balances that economic units wish to dispose of at the initial price level (P_o in figure 7). Alternatively, if the change was to a lower rate of money growth, the government would supply nominal money balances in exchange for goods from the private sector. A policy such as this has two important consequences. First, the replacement of money balances with a stock of goods (or vice versa) presumably leaves real private net wealth unchanged, so there ought to be no wealth effects to impinge upon the real rate of interest. Second, since the exchange of money for goods occurs at the initial price level, there will be no once-and-for-all change in the price level: in figure 7, there would be no discontinuity at t_o in the curve showing the time path of the price level. Thus there would be less likelihood of incorrect price expectations being formed.

Reformulating the Demand for Money Function

In the previous essay, the gains and losses from money creation were both flow variables. Now, however, part of the gain or loss will consist of a stock, and our analysis must include the sum of stocks and flows. To do this, the demand function for real balances employed in the first chapter must be transformed appropriately.

The function used previously was:

(3) $$\left(\frac{M}{P}\right)_t = N_t q_t^b k e^{-ai},$$

where before the time subscripts were ignored because there was no need for them. We shall deal here only with the case of autonomous growth; the stationary economy case can be derived simply by assuming that the autonomous growth rate is zero, while the induced growth case is left as an exercise for the interested reader.

I assume that

(4) $$N_t = N_o e^{\ell t}$$

and

(5) $$q_t = q_o e^{(\lambda-\ell)t}.$$

(All variables are as defined in the first essay.)

Substituting (4) and (5) in (3) yields:

$$(6) \quad \left(\frac{M}{P}\right)_t = N_o q_o^{\,b} k e^{[\ell+b(\lambda-\ell)]t} e^{-ai} .$$

Finally, letting $A = N_o q_o^{\,b} k$ and $\gamma = \ell + b\,(\lambda-\ell)$,

$$(7) \quad \left(\frac{M}{P}\right)_t = A\, e^{\gamma t} e^{-ai} .$$

Maximizing Government Revenue

I assume that all flow variables are discounted by the real interest rate (assumed constant, at least partly because the government is being "honest"). This assumption is used because I am comparing the real revenues and welfare losses from money creation. Real government receipts are:

$$(8) \quad g = \int_{t=o}^{\infty} \rho \left(\frac{M}{P}\right)_t e^{-rt} + \left[\left(\frac{M}{P}\right)_t - \left(\frac{M}{P}\right)_o\right].$$

$\left(\frac{M}{P}\right)_t$ is the stock of real money balances after adjustments by money holders have been made, while $\left(\frac{M}{P}\right)_o$ is the initial stock of real balances. The second (bracketed) term on the right-hand side of (8) is the government "sale" or "purchase" of real balances; it will be negative if the government is increasing the steady-state rate of monetary growth, positive for the reverse case.

Equation (8) can be written as:

$$(9) \quad g = \rho A\, e^{-ai_t} \int_{t=o}^{\infty} e^{(\gamma-r)t}\, dt + \left[A e^{\gamma t} e^{-ai_t} - A e^{\gamma t} e^{-ai_o}\right],$$

where i_t and i_o are the money interest rates corresponding to $(\frac{M}{P})_t$ and $(\frac{M}{P})_o$. For simplicity, I assume that money consists solely of non-interest bearing currency.

When solved (assuming $r>\gamma$ for convergence), (9) becomes:

(10) $$g = \frac{A}{(r-\gamma)}[\rho e^{-a(r+\rho-\beta\ell-b\lambda)} + (r-\gamma)e^{-a(r+\rho-\beta\ell-b\lambda)} - (r-\gamma)e^{-a(r-\beta\ell-b\lambda)}].$$

(See Appendix H.)

To maximize government receipts, find $\partial g/\partial\rho$ and set it equal to zero, which gives:

(11) $$\rho = \frac{1}{a} - (r - \gamma).$$

(See Appendix I.)

In general, the revenue maximizing rate of monetary growth is <u>not</u> independent of the rate of autonomous growth, as was the case under our previous assumptions. (Recall that, in the last chapter, the revenue-maximizing ρ was s just $\frac{1}{a}$.) Indeed, the faster the rate of autonomous growth, the greater will be the rate of monetary growth which maximizes government receipts.

The only two cases in which the revenue-maximizing ρ would be unaffected by growth would be, first, if $\lambda=\ell=o$ (the stationary economy), so that $\rho=\frac{1}{a} - r$. In this case the "honest" government would set a revenue-maximizing ρ

lower than was the case last chapter (lower by 100r%). Second, if b = 1 and $r = \lambda$ (the "golden rule" growth path), then, to maximize revenue, $\rho = \frac{1}{a}$, the identical result obtained last chapter.

The Average Tax Cost of Money Creation

The average tax cost of money creation for the autonomously growing economy in which the government is "honest" is:

$$(12) \quad \frac{w}{g} = \frac{e^{a\rho} - (1 + a\rho)}{a\rho + a(r-\gamma) - a(r-\gamma)e^{a\rho}} + \frac{2(r-\gamma)\ (e^{a\rho} - 1)}{\rho + (r-\gamma) - (r-\gamma)e^{a\rho}} .$$

(See Appendix J.)

The numerator of each expression on the right-hand side of (12) is positive (recall that $r > \gamma$). Since $e^{a\rho} > 1$. for $\rho > o$, $(r-\gamma) - (r-\gamma)e^{a\rho} < o$. It follows that the denominators of each term on the right-hand side of (12) are less than the corresponding term for the autonomous growth case derived in the last essay, and thus the average tax cost of money creation is greater than before at any given ρ. Measured in terms of the average tax cost, an "honest" government policy weakens the case for revenue appropriation through money creation. (To make a the comparison, note that $\gamma = \beta\ell + b\lambda$ so that $(r-\gamma) = (r-\beta\ell - b\lambda)$.) The same holds true for comparisons between stationary economies, when $\gamma = o$.

There is one case in which the average cost is the same under the assumptions of the last and present chapters. Suppose b=1 and r=λ. Then

$$\text{(13)} \qquad \frac{w}{g} = \frac{e^{a\rho} - (1 + a\rho)}{a\rho},$$

the same result as before. (This is true for the stationary economy also, since then r = λ = o, and we would obtain Bailey's formula.)

The Marginal Tax Cost of Money Creation

The marginal tax cost of money creation for the "honest" government-autonomous growth case is:

$$\text{(14)} \qquad MTC = \frac{a(r+\rho-\beta\ell-b\lambda)}{1-a\rho - a(r-\gamma)} + \frac{a(r-\gamma)}{1-a\rho - a(r-\gamma)}.$$

(See Appendix K.)

Comparison of the above expression with the corresponding expression in Essay 1 clearly shows that, at any given ρ, the marginal tax cost is higher for the "honest" government. This will also be the case for a comparison between the stationary economies under the two assumptions. Again, "honest" government weakens the case for revenue appropriation through money creation.

Here, too, the one case where the marginal tax cost of money creation will be identical to that derived in the last essay is when b = 1 and r = λ (λ≥o).

Conclusion

Potential problems exist for a government intent upon appropriating real resources through money creation. These include the possibility of economic dislocation because of incorrect price anticipations and the possibility of an undesired change in the real rate of interest. The government can minimize these problems by exchanging money balances for goods, or vice versa, at the initial price level when it changes the steady-state rate of monetary growth. However, by doing this it increases -- except under very special assumptions -- both the average and marginal tax costs of money creation compared with a situation in which this policy is not followed.

Footnotes to Essay 2

1
Martin Bailey, National Income and the Price Level (New York: McGraw-Hill Book Company), 1971, Chapter 4.

2
See A. C. Pigou, "The Classical Stationary State," Economic Journal, December, 1943, pp. 343-351. See also Don Patinkin, "Price Flexibility and Full Employment," American Economic Review, September, 1948, pp. 543-564.

3
A concise summary may be found in Robert Solow, Growth Theory: An Exposition (New York: Oxford University Press), 1970, Chapter 4.

4
Harry Johnson, Essays in Monetary Economics (Cambridge, Mass.: Harvard University Press), 1967, Chapter 4.

5
David Levhari and Don Patinkin, "The Role of Money in a Simple Growth Model," American Economic Review, September, 1968.

6
Leonardo Auernheimer, "The Honest Government's Guide to the Revenue from the Creation of Money," Journal of Political Economy, May/June, 1974, pp. 598-606. See also Charles D. Cathcart, "Monetary Dynamics, Growth, and the Efficiency of Inflationary Finance," Journal of Money, Credit and Banking, May, 1974, pp. 169-190.

ESSAY 3: Another Variation

The past two essays have examined a number of permutations of Martin Bailey's original model. One thing that was common throughout those essays was that the definition of the real revenue from money creation -- or, as in Essay 2, a major part of that definition -- was $\rho(M/P)$. Moreover, in evaluating the welfare losses from money creation, $\rho = 0$ was taken as the base at which no welfare costs were incurred. In this essay I shall consider alternative ways of defining the revenues and costs from money creation.

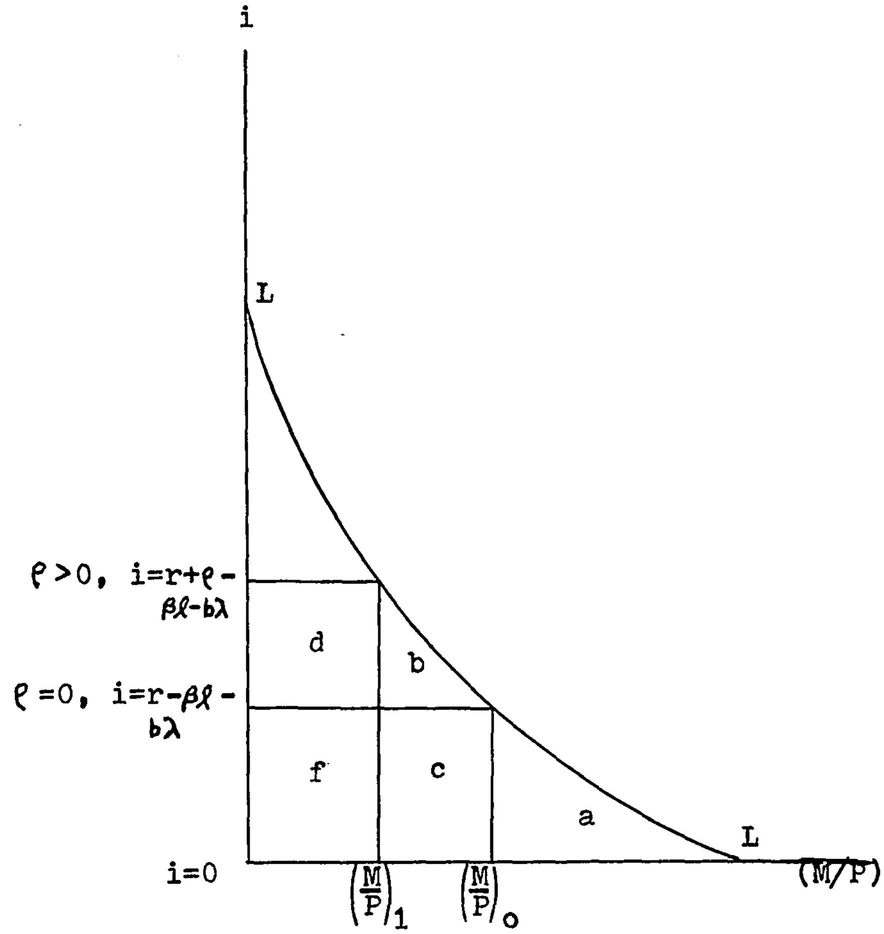
Redefining Government Revenue

The approach taken here views the revenue from money creation as the seignorage that accrues to the government as a consequence of its monopoly of the issuance of interest-free cash.[1] (If commercial bank demand deposits are included as part of the money supply, and if a legal ceiling of zero is placed on the rate of interest paid on demand deposits, then some of this seignorage will accrue to the commercial banking system.) That is, suppose we view the government (or a part of it) as a monopoly bank. Since the marginal cost of producing currency is approximately zero, the monopoly bank could earn net revenues of $i \cdot M$ per time period, these earnings being a result of loans made in the form of currency payouts at the going rate of interest. In real terms, net revenues would be $i(M/P)$. Thus, $i(M/P)$ may be viewed as an alternative way of defining the real revenue flow from money creation. (The nominal capital value of the monopoly bank would be $(iM)/i=M$, and the real value would be (M/P).)

There is another way of rationalizing this new view of the government revenues from money creation. By not having to pay interest on its currency, the government does not have to divert part of its flow of tax receipts (from whatever source) to interest payments. That is, the government can purchase or hire real resources, the real value of which is $i(M/P)$, instead of having to make transfer payments of this same real value. A given level of overall tax receipts can purchase or hire a larger amount of real resources than if interest had to be paid on the government's outstanding stock of currency.[2]

The difference between the two definitions of real government revenue from money creation is illustrated in figure 8 (for the case of an autonomously growing economy). Before, real government revenue was equal, in whole or in part, to area d. Now, real government revenue is equal to area (d + f).

Figure 8: Two Definitions of
Government Revenue



Maximizing Government Revenue

Using the real money demand function employed in the last two essays, we now have

(1) $g = iNq^{b}ke^{-ai}$.

Taking $\partial g/\partial i$ and setting it equal to zero yields the revenue-maximizing $i=1/a$. Because $i=r+\rho-\beta\ell-b\lambda$, the revenue-maximizing $\rho=\frac{1}{a}-(r-\beta\ell-b\lambda)$. This compares with a revenue-maximizing $\rho = 1/a$ derived in Essay 1. Since, in general, $r>(\beta\ell+b\lambda)$, government revenue is maximized at a lower ρ (and i) than was the case in the first Essay. (Or, for the stationary economy, in general, $r > o$, so that $((1/a)-r<(1/a).)$

However, the revenue-maximizing ρ derived here is identical to that derived in Essay 2, where the flow portion of real government revenue was defined as $\rho(M/P)$. Thus, g is maximized here at the same money rate of interest as in that chapter. This result could lead one to conclude that the approach taken by Auernheimer and the one taken here are essentially identical.[3]

However, I believe that the economics of the two cases are quite different; the basic concepts of what constitutes government revenue are not at all similar. Auernheimer essentially views the revenue from money creation in the traditional way, while the approach taken in the present essay constitutes a fundamental shift in the perception of what is the revenue from printing money.

Redefining the Welfare Loss from Money Creation

Along with the government revenue, the welfare loss from money creation may also be redefined. The point here is that, if the government paid the going money rate of interest on currency (or if commercial banks had to pay the going rate on demand deposits), money holders would hold the satiety level of real balances since the opportunity cost of holding money would be zero. However, because the government actually pays no interest at all on its outstanding currency, there is in fact a positive opportunity cost incurred by money holders. Thus, the welfare loss from money creation should now be measured with $i=0$ taken as a base, rather than $\rho=0$, which was the base used in the first two essays. In figure 8, the welfare loss is now area (a+b+c), where before it was only area (b+c).

An interesting result follows from these new definitions of g and w. In the first two essays, the government appropriated real resources, and money holders suffered a welfare loss, only when $\rho>0$. Now, however, revenues and welfare losses can exist even if $\rho \leq 0$. Of course, there is a limit to how rapidly the money supply may decline -- there is always the constraint that $i \geq 0$.

For example, for the case of the stationary economy, $\rho=-r$ is the maximum feasible rate of decline of the money stock. Note that, in the special cases where $r=0$ in the stationary economy or where $r=(\beta\ell+b\lambda)$ in the autonomously growing economy, $i=0$ when $\rho=0$ and the values of g and w under the new definitions become identical to the corresponding values under the old definitions used in Essay 1.

The Average Tax Cost of Money Creation

The average tax cost of money creation for the autonomously growing economy under the new definitions is

$$(2)\quad \frac{w}{g} = \frac{e^{a(r+\rho-\beta\ell-b\lambda)}-1}{a(r+\rho-\beta\ell-b\lambda)} -1.$$

(See Appendix L.)

I have not, in general, been able to compare the average cost derived above with that derived in Essay 1. However, there is one case where the two expressions can be compared. When $b=1$ and $r=\lambda$ (the "golden rule" growth path), the average cost in (2) becomes

$$(3)\quad \frac{w}{g} = \frac{e^{a\rho}-1}{a\rho} -1$$

When the terms for the average cost derived in Essay 1 are rearranged, we get the same expression in equation (3) above. This is not surprising since, when b=1 and $r=\lambda$, $(r-\beta\ell-b\lambda)=0$ and, thus, when $\rho=0$, $i=0$. $\rho<0$ is ruled out because the money rate of interest would be negative for any rate of decline of the money stock. Under these conditions, and as described earlier, the base for calculating w and g under both the old and new definitions is $\rho=0$.

For the stationary economy, $\ell=\lambda=0$, so

$$(4) \qquad \frac{w}{g} = \frac{e^{a(r+\rho)}-1}{a(r+\rho)} - 1.$$

When r=0, this becomes

$$(5) \qquad \frac{w}{g} = \frac{e^{a\rho} - 1}{a\rho} - 1.$$

which, when terms are rearranged, yields the same result derived in Essay 1 (the Bailey case). This is analogous to equation (3) above: when r=0 in the stationary economy ρ is constrained to be ≥ 0, i.e., the base for calculating both w and g is $\rho=0$.

The Marginal Tax Cost of Money Creation

For the autonomously growing economy, the marginal tax cost of money creation under the new definitions is

$$(6) \qquad MC = \frac{a(r+\rho-\beta\ell-b\lambda)}{1-a\rho-a(r-\beta\ell-b\lambda)} .$$

(See Appendix M.)

The expression derived for the marginal tax cost in the first chapter (assuming all money was currency) was

$$(7) \qquad MTC = \frac{a(r+\rho-\beta\ell-b\lambda)}{1-a\rho}$$

Since we may generally assume (as we did in Essay 2) that $r > (\beta\ell+b\lambda)$, so that $(r-\beta\ell-b\lambda) > 0$, it follows that, at any given ρ, the marginal tax cost under the new definitions is greater than the marginal tax cost under the old definitions.

In the stationary economy case,

$$(8) \qquad MTC = \frac{a(r+\rho)}{1-a(r+\rho)} \quad .$$

Since the corresponding expression derived in Essay 1 was

$$(9) \qquad MTC = \frac{a(r+\rho)}{1-a\rho} \quad ,$$

the marginal tax cost under the new assumptions is again greater than the marginal tax cost under the old assumptions.

Conclusions

Alternative definitions of the revenues and welfare losses from money creation have been proposed. At least as measured by the marginal tax cost of money creation, the new definitions imply costs greater than those derived in Essay 1. Thus, Bailey's indictment of money creation as a relatively inefficient source of government revenue at fairly modest rates of money growth continues to hold.

Footnotes to Essay 3

1 See E. S. Phelps, Inflation Policy and Unemployment Theory (New York: W. W. Norton and Company), 1972, and H. G. Johnson, "Appendix: A Note on Seignorage and the Social Saving from Substituting Credit for Commodity Money," in R. A. Mundell and A. K. Swoboda, eds., Monetary Problems of the International Economy (Chicago: University of Chicago Press), 1969.

2 Phelps puts it this way: "--- the proceeds of the inflation tax are metaphorical in the sense that the tax does not produce a visible flow of currency into the treasury like that produced by ordinary taxes or a currency printing press in the treasury basement. However, --- the proceeds of the tax as conceived here are measured in terms of the 'saving' in other (non-metaphorical) tax revenues." E. S. Phelps, "Inflation in the Theory of Public Finance," Swedish Journal of Economics, 1973, p. 68, fn. 2.

3 This seems to be the view held by A. L. Marty. See his "Real Cash Balances and an Optimal Tax Structure," Mimeo., 1976.

ESSAY 4: The "Inflation Tax" in the Context of a More Explicit Macroeconomic Model

The models of the past three essays, while differing in a number of ways, do have one thing in common. They all essentially ignore the impact of changes in the rates of monetary growth and price change on broad macroeconomic variables, such as the savings (investment) - income and capital-labor ratios. That is, implicit in these models are the assumptions that the level of real "ordinary" taxes -- taxes not associated with money creation -- do not change and that real disposable income and real private wealth excluding real balances are unaffected by changes in ρ and Π. Even the "honest government" model of Auernheimer does not really deal adequately with these issues.[1]

In this essay, I shall examine the implications for the government revenues and welfare losses from money creation of models which explicitly take account of the potential impact upon disposable income and wealth of changes in ρ and Π. The seminal work in this area is due to Phelps,[2] but I shall work with a variation of his model developed by Marty.[3] The reason for examining models of the sort discussed below is that

> "--- it is logically necessary to define the inflation tax within a context in which both wealth and income effects on the demand for consumption are held constant. In line with the modern treatment of differential taxation, we ask how the inflation tax can substitute for other taxes when income and wealth effects are held constant so that the time path of broad global variables such as net government expenditure and the ratio of investment to income are unaffected by the move to an alternative rate of inflation."[4]

The Phelps-Marty Model

Following Marty's version of the Phelps model, I shall examine certain properties of a model economy in alternative steady states. Because of the assumptions I shall make about policies pursued by the government, the main differences between alternative steady states will be in the rates of growth of the nominal money supply (ρ) and the rates of price change (Π). Of course, the latter difference implies that the nominal rate of interest (i) and the equilibrium stock of real per capita money balances will also differ in the alternative steady states. For simplicity, money (M) is assumed to consist solely of non-interest bearing currency and, importantly, government interest bearing bonds are assumed to be viewed by the private sector as a part of net private wealth. (The interest rate on these bonds is assumed to be perfectly variable, i.e., changing instantaneously and automatically with changes in the rate of price change.) The government is assumed not to own any factors of production.

Aggregate real disposable income is defined as

$$(1) \quad \frac{Yd}{P} = \frac{Y}{P} + i\,\frac{B^*}{P} + \frac{V}{P} - \frac{T}{P} - \frac{\Pi D^*}{P}$$

where Yd/P= real disposable aggregate income,

Y/P = real aggregate output,

B^*/P= real value of privately held government bonds (so that $i \cdot B^*/P$) is the real value of the interest income on these bonds received by the private sector),

V/P = real government transfer payments to the private sector, <u>excluding</u> interest payments,

T/P = real "ordinary" taxes, i.e., all taxes <u>except</u> the tax from money creation,

D^*/P= real value of the stock of government debt held by the private sector, i.e., $(M/P)+(B^*/P)$.

The final term on the right-hand side of (1) represents the capital gain ($\Pi<0$) or loss ($\Pi>0$) on the stock of privately held government debt. Capital gains or losses would appear to be logical inclusions in the definition of disposable income within the context of a model which compares alternative steady states (i.e., alternative long-run equilibrium growth paths). Note also that (D^*/P) is the real stock of government debt that is privately held. In this model, the Treasury sells bonds to the private sector to cover any excess of its expenditures over its receipts. The Central Bank then monetizes a portion of this debt by purchasing some of it from the private sector in exchange for cash. This is the mechanism by which monetary growth occurs in this model. The total stock of government bonds, (B/P), is then equal to $(M/P) + (B^*/P)$, with the Central Bank owning $(B/P) - (B^*/P)$ -- all outstanding currency in the hands of the private sector is "backed" by bonds held by the Central Bank. In this way, there is a clear dichotomy between monetary and fiscal policy on the part of the government.

To determine the government's real deficit, it is useful to consolidate the receipts and expenditures of both the Treasury and the Central Bank, assuming that all of the interest earnings on the Central Bank's holdings of government bonds (paid by the Treasury) are returned to the Treasury as a gift. (This assumption follows the actual practice of the Federal Reserve System in the United States.) The real deficit, (X/P), is then

$$(2) \quad (X/P) = (E/P) + i\cdot (B^*/P) + (V/P) - (T/P) = (\dot{B}/P),$$

where (E/P) = real government purchases of goods[5] and/or services and $(\dot{B}/P) = (dB/dt)/P$ is the increment in the stock of real government bonds outstanding (in the hands of both the private sector and the Central Bank).

Assuming that $(E/P) = h \cdot (Y/P)$, h a constant with $0<h<1$, substituting for (E/P) in (2) and then substituting that expression in (1) yields

$$(3) \quad (Yd/P) = (Y/P)\cdot (1-h) + (\dot{B}/P) - \Pi \cdot (D^*/P).$$

But $(\dot{B}/P) = (\dot{D}^*/P)$, as implied in the discussion above, so substituting in (3) gives

$$(4) \quad (Yd/P) = (Y/P) \cdot (1-h) + (\dot{D}^*/D^*)\cdot(D^*/P) - \Pi.(D^*/P)$$

Let $G = (D^*/L)$, where L = population. Then

$$(\dot{G}/G) = (\dot{D}^*/D^*) - (\dot{L}/L) = (\dot{D}^*/D^*) -\lambda, \text{ or}$$

$$(\dot{D}^*/D^*) = (\dot{G}/G)+\lambda,$$

where λ is the rate of growth of aggregate output and $\lambda = (\dot{L}/L)$ either because no technical change is occurring or, if it is occurring, then $(\dot{L}/L)$ is viewed as the rate of growth of labor measured in efficiency units (and then all other per capita variables would also be measured in efficiency units of labor). Substituting for $(\dot{D}^*/D^*)$ in (4) yields

$$(5) \quad (Yd/P) = (Y/P)\cdot(1-h) + ((\dot{G}/G) + \lambda - \Pi)\cdot(D^*/P).$$

Now, along a steady-state growth path, $(\dot{G}/G)$ must equal Π because all nominal per capita variables must change at the same rate as the rate of price change in order to keep all real per capita variables unchanged (which, by definition, is a property of a steady-state growth path). Thus, along any steady state growth path,

$$(6) \quad (Yd/P) = (Y/P)\cdot(1-h) + \lambda\cdot (D^*/P).$$

Assume that the government wishes to reduce Π from, say, Π_o to Π_1 $(\Pi_1<\Pi_o)$. This means that the rate of growth of the money supply will fall from ρ_o to ρ_1 $(\rho_1<\rho_o)$. We now wish to know what policies the government must follow if real disposable income and real private wealth are to remain constant when ρ and Π change. Such policies will result in an unchanged savings (investment) - income ratio and, therefore, an unchanged capital-labor ratio and real rate of interest. Thus, all real variables will remain unchanged except, as we shall see, the real stocks of money and privately-held government bonds. (Moreover, depending upon the assumptions one might wish to make, there will be no economic forces pushing this model economy to a different growth rate of real aggregate output.[6])

At the new lower Π, the nominal interest rate will be lower than before, implying that the demand for real money balances will be greater than before. If the Central Bank makes an open market purchase of government bonds for cash at the initial price level, money holders will be able to increase their real money holdings to the level they desire without engaging in any actions which would cause a "blip" in the price level (see Essay 1). In this respect, the mechanism of the present model is similar to that in Auernheimer's "honest government" model (see Chapter 2). The end result is that real private wealth has remained unchanged, since bonds have been exchanged for cash of equal real value.

Aside from unchanged private wealth, however, real disposable income will also remain unchanged. From (6), with λ unchanged and (D*/P) unchanged (because (B*/P) falls by the same amount that (M/P) rises), (Yd/P) remains constant. Thus, with both real disposable income and real private wealth unchanged, the savings (investment)-income ratio doesn't vary,[7] and neither does the capital-labor ratio or the real rate of interest. Here, then, is an explicit policy rationale for (a) treating the real rate as a constant and (b) avoiding the once-and-for-all "blip" in the price level.

One question remains: as a result of these policy considerations, what happens to the level of real "ordinary" taxes? Referring back to equation (3), we see that, when Π falls, the real deficit $(\dot{B}/P)$ must be reduced by an amount equal to the reduced depreciation on the stock of privately held government debt (D^*/P) if (Yd/P) is to remain constant. Must real taxes be increased to accomplish this? (I assume that h and (V/P) are taken as parameters in the model.)

Equation (2) can be rewritten as

$$(7) \quad (T/P) = -(\dot{D}^*/P) + i\cdot((D^*/P) - (M/P) + (E/P) + (V/P),$$

where $(\dot{D}^*/P) = (\dot{B}/P)$ and the second term on the right-hand side of (7) equals $i\cdot(B^*/P)$. Since

$$(8) \quad (\dot{D}^*/P) = (\dot{D}^*/D^*)\cdot(D^*/P) = (D^*/P)\cdot(\Pi+\lambda)$$

because $(\dot{D}^*/D^*) = ((\dot{G}/G) + \lambda)$ and $(\dot{G}/G) = \Pi$ (see above), (7) can be expressed as

$$(9) \quad (T/P) = -(D^*/P)\cdot(\Pi+\lambda) + i\cdot(D^*/P) - i\cdot(M/P) + (E/P) + (V/P).$$

Since r and λ remain unchanged as ρ changes, thanks to pólicies pursued by the government, and since these policies result in (D^*/P) remaining unchanged, the derivative of real "ordinary" taxes with respect to ρ is

$$(10) \quad \partial(T/P)/\partial\rho = -(D^*/P) + (D^*/P) - (M/P) - i\cdot(\partial(M/P)/\partial i),$$

because $\partial\Pi = \partial i = \partial\rho(i=(r+\Pi) = (r+\rho-\lambda)$, r and λ constant). Real "ordinary" taxes can remain unchanged when ρ changes if

$$(11) \quad -\frac{M}{P} - i(\partial(M/P)/\partial i) = 0$$
$$= 1 + (i/(M/P))/(\partial(M/P)/\partial i),$$

i.e., if the elasticity of demand for real money balances <u>with respect to the nominal rate of interest in unity</u>. That is, the maximum degree to which money creation can substitute for real "ordinary" taxes -- in the sense that real "ordinary" taxes can remain unchanged -- is where the above elasticity is unity, implying that the government's revenue from money creation in this model is $i\cdot(M/P)$.

In the last essay, I considered the case where the government's revenue from money creation was simply defined as $i\cdot(M/P)$. It was shown there that Bailey's indictment of money creation as a source of government revenue continued to hold. Thus, the same conclusion can be derived here, although it is derived within the context of a model that takes explicit account of other sources of government revenue and the government's overall budget.[8]

Modifying the Phelps-Marty Model: I

The model developed above rigorously derives certain results that were simply assumed in previous chapters. However, as will be shown below, the results of the Phelps-Marty model can be sensitive to certain assumptions employed by these authors. In this section, I analyze the case of a model economy in which government bonds do not exist. This assumption is not made just to provide us with an intellectual exercise. "Inflationary finance" is probably much more relevant to underdeveloped economies than advanced ones, and these former economies are the ones that are unlikely to have well-developed financial institutions and markets. This lack of financial markets and institutions, plus a host of other socio-economic factors, implies that government bonds (at least those that might be sold to an underdeveloped country's own citizens) are likely to play an insignificant role in such a government's budgetary procedures. This would appear to be adequate justification for examining a model in which government bonds do not exist as a viable financial instrument.

In the present modification of the Phelps-Marty model, I assume that the government owns some proportion, σ $(0 < \sigma < 1)$, of the economy's real capital stock (K), with σ being constant along a given steady-state growth path. This assumption is necessary (as will be shown below) if the government is to be able to eliminate real income and wealth effects when it changes ρ and Π. In contrast, in the simplest sort of monetary growth model -- such as that briefly sketched in Essay 2 -- there are no government bonds and the government owns no real capital. It consists, essentially, of a costless printing press dispensing cash from the rooftops of tall buildings. (Note, also, that in this sort of model there is no real dichotomy at all between monetary and fiscal policy.) Under such conditions, it is impossible for the government to engage in policies which hold real disposable income and real wealth unchanged as the steady-state rates of monetary growth and price change are varied. Consequently, there is no way to avoid a change in the real interest rate, or a "blip" in the price level.

I shall also assume, in this modification, that all earnings on the government's capital stock are returned to the private sector as a gift (equal per capita amounts).[9] The definition of real disposable income then becomes

$$(12)\quad (Yd/P) = (Y/P) + (\overline{V}/P) - (T/P) - \Pi\cdot(M/P).$$

In (12), $\overline{V}$ represents those transfers from the government to the private sector <u>excluding</u> the government's gifts of its capital earnings. Those earnings enter into disposable income in the term (Y/P) since the real income of the private sector consists, in part, of

$$MP_K \cdot (1-\sigma)\cdot K + MP_L \cdot L + MP_K \cdot \sigma \cdot K = MP_K \cdot K + MP_L \cdot L = (Y/P),$$

where MP_K and MP_L are the marginal products of capital and labor, respectively, L is the labor supply, and K is the real capital stock.

The government's real budget deficit is

$$(13) \quad (X/P) = (E/P) + \bar{V}/P) - (T/P) = (\dot{M}_1/P),$$

where $\dot{M}_1$ is that increment in the money supply over and above the cash issued by the Central Bank in exchange for newly produced capital goods. This latter amount I shall call $\dot{M}_2$. It follows that $\dot{M}_2 = \sigma P\dot{K}$, or $(\dot{M}_2/P) = (\sigma\dot{K})$, and $(\dot{M}/P)$, the increment in the <u>total</u> real stock of money, equals $(\dot{M}_1/P) + (\dot{M}_2/P) = (\dot{M}_1/P) + (\sigma\dot{K})$. In this model, then, <u>some</u> of the money stock is backed by real capital owned by the Central Bank. But the remainder of the money stock -- $(M_1/P) = (M/P) - (\sigma K)$ -- represents the net real indebtedness of the government to the private sector.

Substituting (13) in (12) gives (assuming E/P) = $h \cdot (Y/P)$)

$$(14) \quad (Yd/P) = (Y/P)\cdot(1-h) + (\dot{M}_1/P) - \Pi\cdot(M/P).$$

From above, we know that $(\dot{M}_1/P) = (\dot{M/P}) - (\sigma\dot{K})$. But $(\dot{M/P}) = \rho\cdot(M/P)$ and, since $\lambda = (\dot{K}/K)$, $\dot{K}=(\lambda K)$ and $(\dot{M}_1/P) = \rho\cdot(M/P) - (\lambda\sigma K)$. Also, since $(M_1/P) = (M/P) - (\sigma K)$, $\sigma K = (M/P) - (M_1/P)$ and $(\dot{M}_1/P) = \rho\cdot(M/P) - \lambda\cdot(M/P) - \lambda\cdot(M_1/P)$. Since $(\rho-\lambda) = \Pi$, this latter expression becomes

$$(15) \quad (\dot{M}_1/P) = \Pi\cdot(M/P) - \lambda\cdot(M_1/P).$$

Substituting (15) into (14) produces

$$(16) \quad (Yd/P) = (Y/P)\cdot(1-h) - \lambda(M_1/P).$$

Suppose, as was the case earlier, that the government wishes to reduce the steady-state rate of price change. At the lower Π, money holders will demand a larger stock of real balances. Government can provide these larger balances by making an open market purchase of part of the private sector's real capital stock at the initial price level. In this way, real private wealth remains unchanged (money replaces physical capital), money holders get the increased cash holdings they desire, and a "blip" in the price level is avoided. In addition, since nothing in this procedure changes (M_1/P), equation (16) indicates that real disposable income will also remain unchanged. But with both real disposable income and real private wealth unchanged, the savings (investment)-income and, thus, the capital-labor ratio and the real interest rate will remain unchanged.

What happens to the level of real "ordinary" taxes? Rearranging (13) and substituting into (15) gives

$$(17)\quad (T/P) = -\Pi^{\cdot}(M/P) - \lambda\cdot(M_1/P)+(E/P)+(\overline{V}/P).$$

The derivative of (17) with respect to ρ is

$$(18)\quad \partial(T/P)/\partial\rho = -(M/P) - \Pi\cdot(\partial(M/P)/\partial\Pi)$$

since $\partial\Pi = \partial\rho$. (18) = 0 when the elasticity of demand for real money balances with respect to Π equals unity. That is, in this variation of the Phelps-Marty model, the implicit definition of the revenue from money creation is $\Pi^{\cdot}(M/P)$.

Why the difference in the implicit definitions of the revenue from money creation in the two models considered so far? The answer lies in two assumptions made in the present model: (1) that government purchases a constant fraction of newly-produced capital goods in exchange for freshly minted cash; and (2) that the government returns its capital earnings to the private sector as gifts. Since $(\dot{M}_2/P) = (\sigma\dot{K})$ and $(M_2/P) = (\sigma K)$, $(\dot{M}_2/M_2) = (\dot{K}/K) = \lambda$, which means that, even if there was no real deficit in the government's budget $((\dot{M}_1/P) = 0)$, the money supply would be increasing at a rate which kept the price level stable. (Throughout this chapter I assume that the income elasticity of demand for real per capita cash balances in unity.) And, since the earnings on the government's capital are transferred back to the private sector, money creation provides the government with zero revenue unless its rate of change is greater than $(\sigma\dot{K})$, i.e., the government acquires real usable resources through money creation only when $\Pi>0$.

The average tax cost of money creation for an autonomously growing economy (or for an economy experiencing induced growth where the induced growth rate doesn't change), using the demand function employed in the first three essays and a definition of government revenue of $\Pi\cdot(M/P)$, is (See Appendix N)

$$(19)\quad \frac{w}{g} = \frac{e^{a\pi} - (1+a\Pi)}{a\Pi} + \frac{r(e^{a\pi} - 1)}{\Pi} .$$

The Bailey case was an average cost of money creation that equalled the first term on the right-hand side of (19) (see Essay 1). Thus, with the definition of the revenue from money creation implicit in the present model, Bailey's indictment of "inflationary finance" continues to hold.

The marginal tax cost of money creation in this model is (see Appendix O)

$$(20)\quad MTC = \frac{a\ (r + \Pi)}{1 - a\Pi} .$$

(20) is identical to the marginal cost of money creation for a stationary economy assuming that the definition of the revenue from money creation is $\rho\cdot(M/P)$ (see Essay 1).

Modifying the Phelps-Marty Model: II

While the model developed in Modification I above is logically valid, it is perhaps more appealing, from an empirical point of view,[10] to derive the implications of the model on the assumption that the government keeps the earnings on its portion of the capital stock. That is, I now assume that these earnings become another source of net government receipts. This assumption means that the definition of real disposable income now becomes

$$(21)\quad (Yd/P)=(Y/P)-\sigma\cdot r\cdot K+(V/P)-(T/P)-\Pi\cdot(M/P),$$

where $(\sigma\cdot r\cdot K)$ is the earnings on real capital $(r\cdot K)$ that accrue to the government from its ownership of a proportion (σ) of the capital stock.

The government's consolidated real deficit is

$$(22)\quad (X/P) = (E/P)+(V/P)-(T/P)-\sigma\cdot r\cdot K = (\dot{M}_1/P).$$

Assuming that $(E/P) = h\cdot(Y/P)$, and substituting (22) into (21) gives

$$(23)\quad (Yd/P) = (Y/P)\cdot(1-h) + (\dot{M}_1/P) - \Pi\cdot(M/P).$$

But this is the identical definition derived above in equation (14), where it was assumed that government returned its capital earnings to the private sector.

Thus, when the government wishes to move from one steady-state rate of monetary growth to some lower steady-state rate, real private wealth and real disposable income will remain unchanged if the government makes a once-and-for-all purchase of part of the private sector's capital stock.

As for the real government deficit, (22) can be rewritten as

$$(24) \quad (T/P) = -i\cdot(M/P)+(\lambda+r)\cdot(M_1/P)+(E/P)+(V/P),$$

making use of (15) above. The derivative of (24) with respect to ρ is

$$(25) \quad \partial\,(T/P)/\partial\rho = -(M/P) - i\cdot\,(\partial(M/P)/\partial i),$$

since $\partial i = \partial\rho$. (25) implies that no change in real "ordinary" taxes is required when ρ changes if the elasticity of demand for real money balances <u>with respect to i equals unity</u>. Thus, the implicit definition of the government revenue from money creation in this variation of the Phelps-Marty model is $i\cdot(M/P)$ -- the same definition derived in the original version of the model.

The reason for this result clearly lies in the one change in assumptions from Modification I, namely, that the government in this case <u>keeps</u> its capital earnings. This means that the government earns a return of r when $(\dot{M}_1/M_1)=0$ and $(\dot{M}_2/M_2) = \lambda$, i.e., when the money supply is growing just fast enough to keep prices stable. But government revenues of $r\cdot(M/P)$ when the price level is stable implies that revenues are $i\cdot(M/P)$, since $i=r$ when $\Pi=0$. Put differently, we know from the model used in Modification I that, if the government's capital earnings are returned to the private sector, money creation revenues are $\dot{\Pi}(M/P)$. In the present model, the government gets an extra $r\cdot(M/P)$, so its total revenues from money creation are $i\cdot(M/P)$.

Modifying the Phelps-Marty Model: III

Relatively little time will be spent on this final modification, since its implications are intuitively obvious. This is the case where government bonds do exist, but where they are not considered to be part of net wealth by the private sector. Again, this assumption is not made just as an intellectual exercise. Macroeconomic models incorporating this assumption are not new,[11] and there is a growing body of empirical literature which, at least in part, has been aimed at testing the hypothesis that bondholders act as if their holdings are not part of their net wealth.[12] Since the empirical findings on this matter are still not conclusive one way or another, and since I have already examined a model in which bonds are a part of net private wealth, it now seems appropriate to at least briefly consider a model embodying the opposite assumption.

The reason why little time has to be spent on this modification is that it is analytically equivalent to Modification II (or I, depending upon what one wishes to assume about the disposition of the government's capital earnings). That is, if the private sector fully discounts the future taxes implicit in bond-financed government deficits, the existence of these bonds -- and any interest earnings or capital gains or losses on them -- can have no effect on economic behavior, and the model reduces to one in which these bonds simply do not exist. Note that it is necessary to assume that the government does own part of the capital stock. This is because, when ρ and Π are changed to some new steady-state values, an exchange of cash for another asset must be made which leaves real wealth unchanged. If cash were exchanged for government bonds, real wealth would change, since an asset which is part of the net private wealth (cash) has been exchanged for another asset which is not (government bonds).

Since this modification of the Phelps-Marty model reduces to one in which bonds do not exist, the implicit definition of the government revenue from money creation is either $\Pi\cdot(M/P)$ or $i\cdot(M/P)$, depending upon the assumption regarding the disposition of the government's capital earnings.

Conclusion

I have examined several models which explicitly take account of the potential income and wealth effects of a change in government policy which alters the steady-state rates of monetary growth and price change. As long as these models are in the spirit of the "inflation tax" analysis, i.e., as long as they assume that the government keeps any earnings on assets it owns, then the implied definition of the revenue from money creation is $i\cdot(M/P)$, a case which was analyzed in Essay 3. Even in the case where we examined a model not in this spirit (Modification I), Bailey's indictment against "inflationary finance" continued to hold.

Footnotes to Essay 4

1
See Essay 2 of this thesis, where it was pointed out that real disposable income changes in the Auernheimer model, as ρ changes. For other shortcomings of the Auernheimer model, see Alvin L. Marty, "Real Cash Balances and an Optimal Tax Structure," Mimeo., 1976.

2
Edmund S. Phelps, "Inflation in the Theory of Public Finance," Swedish Journal of Economics, March, 1973.

3
Marty, op. cit.

4
Marty, op. cit.

5
These goods cannot be capital goods, since the model assumes that the government owns no productive factors. This assumption is modified in subsequent sections of this chapter.

6
The growth rate of real aggregate output would change if the growth rate of the labor supply were an endogenous variable dependent upon, e.g., real per capita income (which is a function of the capital-labor ratio).

7
I assume that real saving (consumption) is a function of real disposable income and real net private wealth.

8
This model is given a geometric treatment in Marty, op. cit., pages 8-11.

9
The model employed here is equivalent to Model A in Edwin Burmeister and Edmund Phelps, "Money, Public Debt, Inflation and Real Interest," Journal of Money, Credit and Banking, May, 1971. Note that the assumption that the government returns its capital earnings to the private sector is not really "right," since we are asking about the maximum degree to which money creation can substitute for real "ordinary" taxes. Returning the government's capital earnings to the private sector

9 increases the distortionary "ordinary" tax burden on the private sector.

10 Not only empirically appealing. As the previous footnote makes clear, this assumption is in the spirit of the analysis of the revenue from money creation -- the assumption that the government returns its capital earnings to the private sector is not in this spirit.

11 See, for example, Martin J. Bailey, National Income and the Price Level, Second Edition (New York: McGraw-Hill Book Company), 1971, Chapter 9; and Robert J. Barro, "Are Government Bonds Net Wealth?" Journal of Political Economy, November/December, 1974.

12 See, for example, J. Ernest Tanner, "Empirical Evidence on the Short-Run Real Balance Effect in Canada," Journal of Money, Credit and Banking, November, 1970; Levis A. Kochin, "Are Future Taxes Anticipated by Consumers?" Journal of Money, Credit and Banking, August, 1974; Laurence H. Meyer, "Wealth Effects and the Effectiveness of Monetary and Fiscal Policies," Journal of Money, Credit and Banking, November, 1974; and Jess B. Yawitz and Laurence H. Meyer, "An Empirical Investigation of the Extent of Tax Discounting," Journal of Money, Credit and Banking, May, 1976.

ESSAY 5: The "Inflation Tax" in the Context of an Optimal Tax Model

The discussion of the government revenues and welfare losses from money creation throughout the first four essays has lacked one important element -- no attempt has been made to integrate money creation into a general model of optimal taxation. This essay attempts to remedy this deficiency. I shall first set out a model of optimal commodity taxation based on the work of Dixit[1] and Sandmo.[2] Next, I shall show how Marty[3] ties together this optimal taxation model with the monetary literature on the "inflation tax." I then indicate how the previous results can be varied somewhat by modifying the way in which real money balances enter the utility function. The essay concludes with some remarks about the reasonableness of the approach taken in the previous essays, given the results obtained in the present essay.

The Sandmo-Dixit Model of Optimal Commodity Taxation

Assume that society's preferences can be represented by the social utility function[4]

(1) $U = U(X_o, X_1, ---, X_n)$,

where the X_i (i = o, 1, - - -, n) are the commodities in existence in this model economy. ("Commodities," in this model, means not only final goods but factor inputs as well.) Commodity prices are

$$(2) \quad P_i = p_i = t_i, \; i = o, 1, ---, n,$$

where p_i = the producer price and t_i = the tax per unit of the i^{th} commodity. The p_i are assumed to be constant in this analysis.[5] The household's budget constraint is

$$(3) \quad \sum_{i=o}^{n} P_i X_i = 0$$

and the government's tax revenue constraint is

$$(4) \quad \sum_{i=o}^{n} t_i X_i = T.$$

Following Sadmo, I assume that one of the commodities is nontaxable -- for convenience, let it be commodity O, and think of it as labor services. This assumption is made because: (a) it is logically impossible to tax <u>every</u> commodity at the same proportional rate $\Theta_i = (t_i/P_i)$[6]; and (b) in the real world, there will always be some commodities which are, in fact, not taxed. Moreover, the nontaxable commodity will be taken as the numeraire, so that $P_i = p_i = 1$.

Utility maximization by the household yields the first order conditions $U_i = \lambda P_i$, λ= marginal utility of income, which imply the demand functions $X_i = X_i\ (P_i, - - -, P_n)$, recalling that $P_o = 1$. We can then write the indirect utility function $V = U(X_o(P_1, - - -, P_n), - - -, X_n (P_1, - - -, P_n))$. Now, take the partial derivative

$$(5)\ \frac{\partial V}{\partial P_1} = \frac{\partial U}{\partial X_o}\frac{\partial X_o}{\partial P_1} + \frac{\partial U}{\partial X_1}\ \frac{\partial X_1}{\partial P_1} + - - - + \frac{\partial U}{\partial X_n}\ \frac{\partial Xn}{\partial P_1}.$$

Substituting the first order utility maximizing conditions in (5) gives

$$(6)\ \frac{\partial V}{\partial P_1} + \lambda P_o \frac{\partial Xo}{\partial P_1} + \lambda P_1\ \frac{\partial X_1}{\partial P_1} + - - - + \lambda P_n\ \frac{\partial Xn}{\partial P_1}\ .$$

Taking the total derivative of the household's buget constraint with respect to P_1 yields

$$(7)\quad \frac{\partial X_o}{\partial P_1} + X_1 + P_1\ \frac{\partial X_1}{\partial P_1} + P_n\ \frac{\partial X_n}{\partial P_1} = \quad 0.$$

Thus, (6) can be written as

$$(8)\ \frac{\partial V}{\partial P_1} \quad = \lambda\ \left\{ \frac{d\ \left(\sum_{i=o}^{n} P_i X_i\right.}{dP_1} \quad - \quad X_1 \right\} \quad = \quad -\lambda X_1$$

since the first term in the brackets equals zero. Similarly, $(\partial V/\partial P_2) \ = \ -\lambda X_2$ and, more generally,

$$(9)\ \frac{\partial V}{\partial P_i} \quad = \quad -\lambda X_i, \quad i\text{-}1, - - -, n.$$

$((\partial V/\partial P_o) = 0$ since $P_o = 1.)$ Moreover, since the p_i are assumed to be constant, $(\partial V/\partial P_i) = (\partial V/\partial p_i)$, i=1, - - -, n.

In this model, it is assumed that the government's goal is to maximize social utility subject to its tax revenue constraint. That is, government maximizes the function

$$(10)\quad L = U(X_o(P_1, ---, P_n), ---, X_n\ (P_1, ---, P_n)) - \mu\ (t_1X_1 + --- + t_nX_n - T).$$

Taking $(\partial L/\partial P_1)$ gives

$$(11)\quad \frac{\partial L}{\partial P_1} = \frac{\partial V}{\partial P_1} - \mu(t_1\frac{\partial X_1}{\partial P_1} + X_1 + t_2\frac{\partial X_2}{\partial P_1} + --- + t_n\frac{\partial X_n}{\partial P_1}) = 0.$$

Substituting in (11) for $(\partial V/\partial P_1)$ from (9) and rearranging terms produces

$$(12)\quad -\lambda X_1 = \mu X_1 + \mu(t_1\frac{\partial X_1}{\partial P_1} + t_2\frac{\partial X_2}{\partial P_1} + --- + t_n\frac{\partial X_n}{\partial P_1})$$

or

$$(13)\quad t_1\frac{\partial X_1}{\partial P_1} + t_2\frac{\partial X_2}{\partial P_1} + --- + t_n\frac{\partial X_n}{\partial P_1} =$$

$$-\frac{(\lambda+\mu)}{\mu}X_1 = -vX_1$$

More generally, (13) can be written as

$$(14)\quad \sum_{i=1}^{n} t_i\frac{\partial X_i}{\partial P_k} = -vX_k,\ k=1, ---, n.$$

Earlier, we had found the total derivative of the household's budget constraint with respect to P_1. In more general notation, this derivative is

$$(15) \quad \sum_{i=1}^{n} P_i \frac{\partial X_1}{\partial P_k} + X_k + \frac{\partial X_o}{\partial P_k} = 0, \; K=1, ---, \; n.$$

Substituting for X_k from (15) into (14) gives

$$(16) \quad \sum_{i=i}^{n} t_i \frac{\partial X_i}{\partial P_k} = v \left\{ \frac{\partial X_o}{\partial P_k} + \sum_{i=1}^{n} P_i \frac{\partial X_i}{\partial P_k} \right\}, \; K=1, ---, \; n,$$

or, substituting for t_i $(=\theta_i P_i)$ in (16) and rearranging terms,

$$(17) \quad \sum_{i=1}^{n} P_i (\theta_i - v) \frac{\partial X_i}{\partial P_k} = v \frac{\partial X_o}{\partial P_k}, \quad K=1, - - -, \; n.$$

The Inflation Tax in the Optimal Commodity Taxation Model

Dixit[7] has shown that satisfying (17) (actually (14) -- but (17) is derived from (14)) amounts to equating the ratio of the marginal increment in welfare loss to the marginal increment in government revenue -- what we have been calling, in the context of the tax on money creation, the marginal tax cost -- for each taxed commodity.[8] Thus, it might appear that the procedures used in the previous essays (with respect to the marginal tax cost of money creation) are consistent with a general model of optimal commodity taxation. But consider the following sequence: start with the marginal costs of commodity taxation for all taxable commodities <u>except</u> real balances being equal to one another. Now introduce money creation and start increasing its rate to bring its marginal tax cost into equality with the marginal tax cost of other taxed commodities. However, as the opportunity cost (the "price") of money changes, there will, in general, be shifts in the demands for the other commodities. This will change both the marginal welfare losses and marginal government revenues from these other commodities, and we can also expect the marginal tax costs to change in indeterminate ways. Thus, in general, the implicit optimizing procedure underlying the calculation of an optimal rate of money growth (which goes back at least to Bailey) in the previous essays is, strictly speaking, incorrect. That procedure, in effect, is akin to a partial equilibrium type of analysis.

Marty[9] has investigated the circumstances under which the procedures employed in the previous essays <u>are</u> consistent with the model derived above. Those circumstances require the imposition of certain additional assumptions. The first case arises if it is assumed that $(\partial X_i/\partial P_k) = 0$ for all i, k, $i \neq k$, i, k = 1, - - -, n. That is, assume that all cross elasticities are zero within the set of <u>taxable</u> goods. Under this assumption, equation

(14) can be rewritten as

$$(18) \quad t_i \frac{\partial X_i}{\partial P_i} = - vX_i, \qquad i=1, - - -, n,$$

or, since $(\partial X_i/\partial P_i) = (\partial X_i/\partial t_i)$,

$$(19) \quad \frac{t_i}{X_i} \frac{\partial X_i}{\partial t_i} = -v.$$

All tax elasticities are brought into equality at the common value v. But, as mentioned previously with reference to equations (14) and (17), that is equivalent to equating the marginal tax costs for each taxed commodity. This time, however, the marginal costs of each taxed commodity (including real balances) will be independent of one another because there are no cross elastic shifts. If it is assumed that all taxable commodities except real balances are initially optimally taxed, then it is perfectly proper to adjust the rate of money creation until its marginal tax cost is brought into alignment with the marginal tax costs of the other taxed commodities. As this process occurs, nothing will happen to the marginal costs of the remaining taxed commodities because of the absence of any cross elastic effects.

Another interesting case occurs if it is assumed that $(\partial X_o/\partial P_k) = 0$, $K = 1, - - -, n$. That is, assume that labor is supplied completely inelastically. Then the right-hand side of (17) becomes zero and it immediately follows that $\Theta_i = v$, i.e., a uniform proportional tax (=v) on each taxed commodity is optimal. Now

$$\Theta_i = \Theta = v = \frac{\lambda+\mu}{\mu} = \frac{t_i}{p_i+t_i},$$

so

$$\mu t_i = (\lambda+\mu) \quad (p_i + t_i).$$

For the case of real balances, p_i is (approximately) zero, and thus $\mu t = (\lambda+\mu)t$, implying that $\lambda t = 0$. But $\lambda>0$ since λ = marginal utility of income and non-satiation is assumed, so t <u>must</u> equal zero. That is, in this particular case, the optimal tax on real balances is zero so that the optimal quantity of real balances is at the satiety level. In effect, the assumption of a completely inelastic labor supply (more generally, a completely inelastic supply of, or demand for, non-taxable commodities) logically implies that real balances must <u>become</u> another non-taxable commodity.

One Modification of the Optimal Commodity Taxation Model

If there is one thing that is bothersome about extending the optimal tax model to include an "inflation tax," it is the assumption that real balances enter the individual's (or household's or society's) utility function on par with all other commodities. Except for certain types of individuals (like the fictional Silas Marner or religious ascetics) who are usually thought to be unrepresentative, most individuals are generally viewed as not receiving utility (or disutility) directly from the possession of real balances. Thus, the previous analysis can be modified by writing the utility function as

$$(20) \quad U=U\,(X_1,\ ---,\ X_m,\ Y-Z-f\,(X_1,\ ---,\ X_m, X_n)\,),$$

where X_1, ---, X_m are the quantities of the various "ordinary" final commodities, X_n is the quantity of real money balances, Y is total time (24 hours per day, or 168 hours per week, etc.), Z is time spent at work (hours of labor services), and the function $f(\cdot)$ represents transactions time (time that is necessary to acquire ordinary final commodities for consumption). Thus, the expression $Y-Z-f\,(X_1,\ ---,\ X_m, X_n)$ represents the quantity of leisure time, which in this variant of the optimal tax model, is treated as another final commodity. I assume that $f(\cdot)$ is increasing in X_1, ---, X_m and decreasing in X_n. By specifying the utility function in this manner, we explicitly allow for the impact of the quantity of real balances on transactions time and, therefore, on the quantity of leisure.

The difference between the specification of the utility function in (20) and in (1) can be seen most clearly in the expressions for the marginal utilities for the various commodities:

$$(21) \quad U_i = \frac{\partial U}{\partial X_i} - \frac{\partial U}{\partial \ell} \frac{\partial f}{\partial X_i} < \frac{\partial U}{\partial X_i}, \quad i=1, ---, m;$$

$$(22) \quad U_\ell = \frac{\partial U}{\partial \ell} > 0;$$

$$(23) \quad U_n = -\frac{\partial U}{\partial \ell} \frac{\partial f}{\partial X_n} > 0.$$

In (21) - (23), ℓ represents leisure time. These expressions indicate that the marginal utilities of "ordinary" non-leisure commodities are reduced by virtue of the increased transactions time (reduced leisure) which is necessitated when there is an increment in the amount of "ordinary" non-leisure commodities consumed. Also, the marginal utility of real balances is positive because of the savings in transactions time (increased leisure) generated from an increment in real balances.

The individual is assumed to maximize the Lagrangian expression

$$(24) \quad L^* = U(X_1, ---, X_m, Y-Z-f(X_1, ---, X_m, X_n)) - \lambda \sum_{i=1}^{n} P_i X_i.$$

The first order conditions yield the demand functions $X_i = X_i(P_1, ---, P_m, P_n)$ and $\ell = \ell\ (P_1, ---, P_m, P_n)$, similar to those derived in the original version of the optimal tax model. Indeed, if one were then to follow all the mathematical steps as was done in the previous analysis, one would wind up with equation (17) above. Thus, while the specification of the utility function has been altered to explicitly incorporate the role of real money balances, the algebra of the model has not changed at all.

While the algebra of this variant of the optimal tax model is the same as that of the original model, the economic interpretation is different. Suppose, for example, we assume that all cross elasticities among the taxable goods that can be zero, are zero. I put it this way because there are some cross partial derivatives, namely $(\partial X_i / \partial P_n)$, $i = 1, - - -, m$, which cannot initially be assumed to be zero. If those cross partial derivatives were initially assumed to be zero, it would mean that real balances were initially at the satiety level. Put differently, it would mean that, from the very beginning, real balances were untaxed, i.e., that they had become another non-taxable commodity. If this is not the case, then the procedure followed in the previous chapters is at best an approximation to the correct procedure. This is because, assuming that all taxable commodities except real balances were taxed at optimal levels to start with, raising the tax on real balances from zero to some positive level would affect the marginal tax costs of all the other taxed commodities. Thus, the first special case discussed with respect to the original optimal tax model cannot really be applied here unless one assumes that real balances are a non-taxable commodity, in which case the tax on real balances is trivially zero. Note, however, that there appears to be no <u>necessary</u> economic reason why real balances <u>should</u> be treated as non-taxable commodity.

The same sort of reasoning just developed applies also to the second special case discussed in connection with the original optimal tax model. Assuming that leisure is a non-taxable commodity, not <u>every</u> $(\partial \ell/\partial X_i)$, i=1,---,m,n can be assumed to be initially zero unless real balances are a non-taxable commodity to start with, in which case the satiety level of real balances would prevail and it would be legitimate to assume $(\partial \ell/\partial X_n) = 0$. Again, there is no necessary economic reason why this should be so.

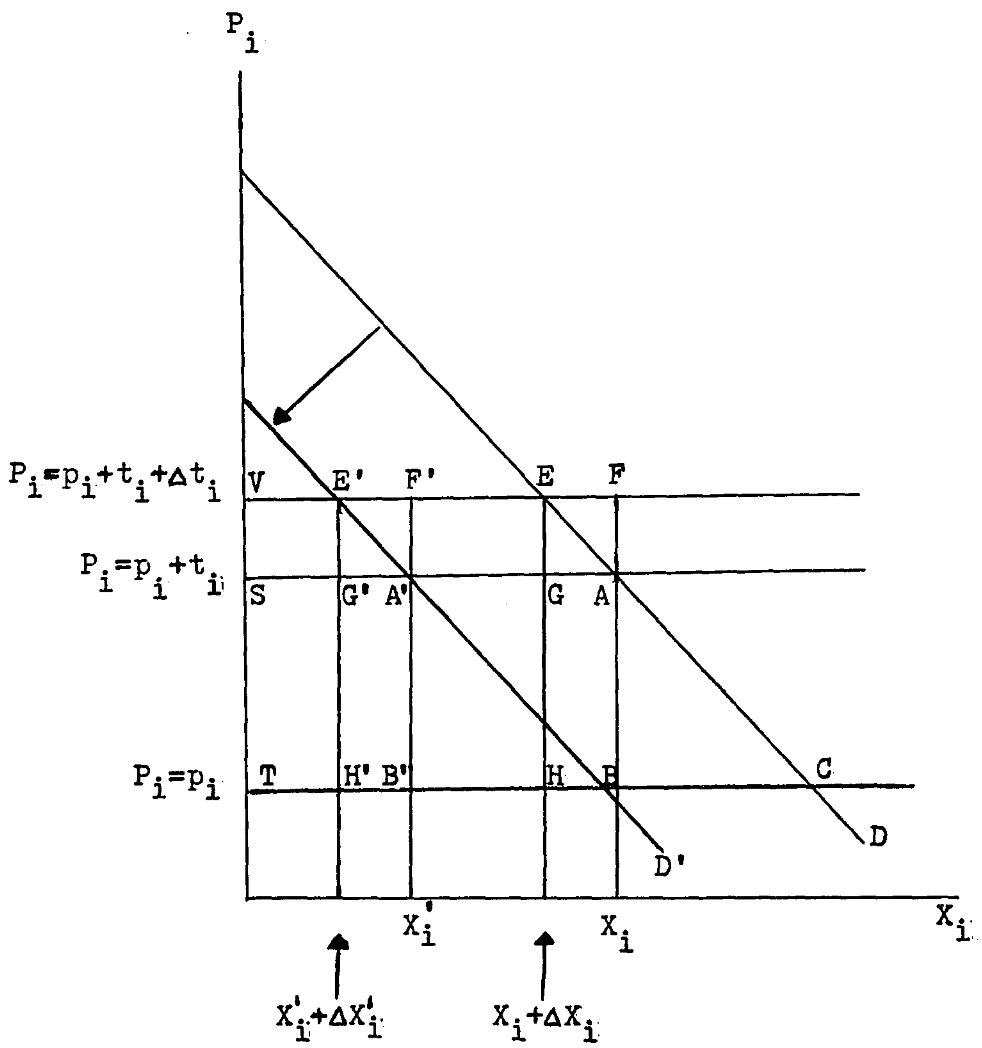
<u>The Inflation Tax and the Optimal Tax Model</u>

The above analysis clearly demonstrates that the procedures followed in earlier essays, whereby the marginal tax cost of money creation was equated to some assumed pre-existing marginal tax cost of all other taxed commodities, lacks a rigorous theoretical foundation. (Indeed, it indicates that equating the average tax cost of money creation to the average cost of taxation for all other taxed commodities is simply incorrect.) Even if one were to reject the modification made to the optimal tax model and return to the original version, the assumption that would permit the analyses of the previous essays to be consistent with the optimal tax model -- all cross elasticities equal to zero within the taxed sector -- is unlikely to be realistic in the (long-run) real world. The question then becomes, does Bailey's original indictment of inflationary finance as a source of government revenue, which has so far held up under a variety of assumptions, continue to hold up?

I am not sure that a definitive answer to this question can be given, but perhaps some inferences relevant to the model just developed can be drawn from a consideration of figure 9. I assume that the demand curve, D, is for an "ordinary" taxed commodity. Assume also that the tax on real balances is zero to begin with and that all other taxes are at their optimal levels, given that the tax on real balances is initially zero. The price per unit of X_i is $P_i = P_i + t_i$. The marginal cost of the tax can be represented geometrically as

$$(25)\quad MTC = \frac{\text{area EABH}}{\text{area VEGA} - \text{area GABH}} .$$ [10]

Figure 9: The Impact on the Marginal Tax Cost of an "Ordinary" Commodity of a Shift in Demand



Now let the rate of money creation increase, thus causing an increase in the tax rate on real money balances (the money rate of interest). As this occurs, the demand curve will shift to the left. This is because, as the tax rate on real balances rises, the equilibrium quantity of real balances held declines and, with all other prices given, transactions costs rise. The demand for leisure will shift to the left, but it is reasonable to infer that not <u>all</u> of the increase in transactions time will be offset by reduced leisure. Some of the increased transactions time is also likely to be offset by reduced consumption of "ordinary" final commodities. Put differently, the increase in transactions costs raises the "effective" prices of "ordinary" final commodities. At every nominal or market price for these commodities, the quantities demanded will be less than before the increase in transactions costs. Thus, the demand curves for "ordinary" final commodities will shift to the left.

Suppose that it is reasonable to assume that, in the relevant range of P_i, the demand curve is approximately linear and also that the shift in the curve leaves the new demand curve (D') parallel to the original one. With demand curve D', the marginal cost of the tax on the i^{th} commodity now becomes

$$(26) \quad MTC' = \frac{\text{area } E'A'B'H'}{\text{area } VE'G'S - \text{area } G'A'B'H'}. \qquad 11.$$

However, because of the assumptions of linear and parallel-shifting demand curves (at least within a certain range of prices), area E'A'B'H' = area EABH and area G'A'B'H' = area GABH. Thus, (26) can be rewritten as

$$(27) \quad MTC' = \frac{\text{area } EABH}{\text{area } VE'G'S - \text{area } GABH}$$

Examining figure 9 shows clearly that area VE'G'S < area VEGS. It follows that MTC' > MTC. That is, as the tax rate on real balances starts to rise in an attempt to equate the marginal tax cost of money creation to the marginal tax costs of other taxed commodities, the marginal tax costs of these other commodities also start to rise (not necessarily proportionately). This means that the tax on money creation will wind up being higher than would otherwise be the case since, in effect, there is a larger gap between the marginal tax cost of money creation and the marginal tax costs of other taxed commodities that has to be eliminated.[12]

Conclusion

In this essay I have outlined a simple model of optimal commodity taxation, indicated how the analyses of the previous essays can be made consistent with this model, and showed how the model can be modified in one way to take account of the particular properties of real money balances. The final portion of the chapter appears to lead to the conclusion that the rate of money growth, and thus the rate of price change, should be higher than the rates implied in the previous essays. However, it does not seem to be possible to determine a priori the magnitude of this discrepancy.

Footnotes to Essay 5

1 Avinash K. Dixit, "On the Optimum Structure of Commodity Taxes," American Economic Review, June, 1970.

2 Agnar Sandmo, "A Note on the Structure of Optimal Taxation," American Economic Review, September, 1974. A good brief survey of the literature on optimal taxation, with a good selection of references, has also been provided by Sandmo in his article, "Optimal Taxation, An Introduction to the Literature," Journal of Public Economics, July/August, 1976.

3 Alvin L. Marty, "Inflation, Taxes and the Public Debt," Mimeo., 1977.

4 Alternatively, the utility function can be viewed as that of the "representative" individual or household.

5 However, it has been shown that the results of this analysis continue to hold under less restrictive assumptions. See A. K. Dixit, op. cit., and P. A. Diamond and J. A. Mirrlees, "Optimal Taxation and Public Production," Parts I and II, American Economic Review, March, 1971, and June, 1971.

6 See Sandmo, 1974, op. cit., p. 702.

7 Dixit, op. cit., p. 287. See also Appendix P.

8 Furthermore, an implication of Dixit's analysis mentioned in the previous footnote is that the appropriate definition of the government revenue from money creation is the money rate of interest times the real money stock.

9 Marty, 1977, op. cit.

10 The average tax cost is, initially, area ABC/area SABT.

11 The average tax cost when the demand curve has shifted to D' is Area A'B'C'/area SA'B'T.

12 A comparison of the average tax costs in the two previous footnotes indicates that the average cost of the "ordinary" taxed commodities also rises as the demand curve shifts to the left.

ESSAY 6: The Government Revenues and Welfare Losses from Non-Steady State Inflation

The previous five essays have examined a variety of models concerned with the government revenues and associated welfare losses from money creation. The models have primarily been concerned with two questions: how well does Bailey's original indictment of "inflationary finance" stand up under alternative assumptions?; and, just what is the proper measure of the revenues and welfare losses from money creation? But there has been one common thread to all of the models that have been examined, and that thread has been the comparison of alternative steady-states. That is, the models have compared alternative equilibrium positions on the assumption that economic agents possessed perfect information about all relevant economic variables, including the future values of those variables.

The method of comparing alternative equilibrium states provides useful insights into some of the costs and gains from money creation, and it is doubtless the most appropriate method for analysis of very long-run situations. But in the real world there are likely to be costs and gains during the transition period from one equilibrium state to another, and the transition period itself may well last for decades. Early in the first essay, I mentioned the possibility of costs and/or gains from the income redistributions that are likely to occur during transition periods. In this essay, however, I shall put income distribution questions to one side and instead examine two models which aim at deriving implications for the costs and gains from money creation under conditions of uncertainty.

The Jaffee-Kleiman Model

Building upon earlier work by Okun[1] and Gordon,[1] Dwight Jaffee and Ephraim Kleiman[3] (hereafter, J-K) have developed a model in which each individual (or, as in the last chapter, the "representative" individual) treats inflation as a stochastic process. It is assumed that each individual forms a subjective evaluation of the probability distribution $f(\Pi)$, and that either $f(\Pi)$ is normal or the individual's utility function is quadratic in real wealth. Thus, the relevant parameters of the probability distribution are its mean, E, and its standard deviation, Σ. Further, J-K assume that there are two assets, one real and one financial, with the individual's initial endowment of these assets being R_o and N_o, respectively.[4] J-K also assume that the nominal return on both assets is the same, but that at the end of the period the real value of the financial asset is decreased by inflation while the real value of the real asset is invariant to price changes. (This is not to say that the real value of real assets is immune to any changes, but just that alternative inflation rates, other things constant, have no impact on the real value of the real asset.) Finally, it is assumed that the individual can exchange the nominal asset for the real asset during the period in the amounts ΔN and ΔR.

J-K write the individual's end-of-period real wealth as[5]

$$(1) \quad W = R_o + \Delta R + \frac{N_o + \Delta N}{1 + \Pi}$$

and the individual's wealth constraint as

$$(2) \quad W = \Delta N + \Delta R = 0.$$

In addition to the definition of the individual's end-of-period wealth and wealth constraint, J-K specify a cost function for switching from one asset to the other:

$$(3) \quad C = C(\Delta R),$$

and assume that $C'(\Delta R) > 0$ and $C''(\Delta R) > 0$, i.e., marginal switching or transactions costs are positive and rising.

To illustrate the implications of their model, J-K first assume that Π is non-stochastic. The individual then faces the optimization problem of maximizing his end-of-period wealth <u>net</u> of the costs of converting one asset into another subject to the wealth constraint. If one forms the Lagrangian

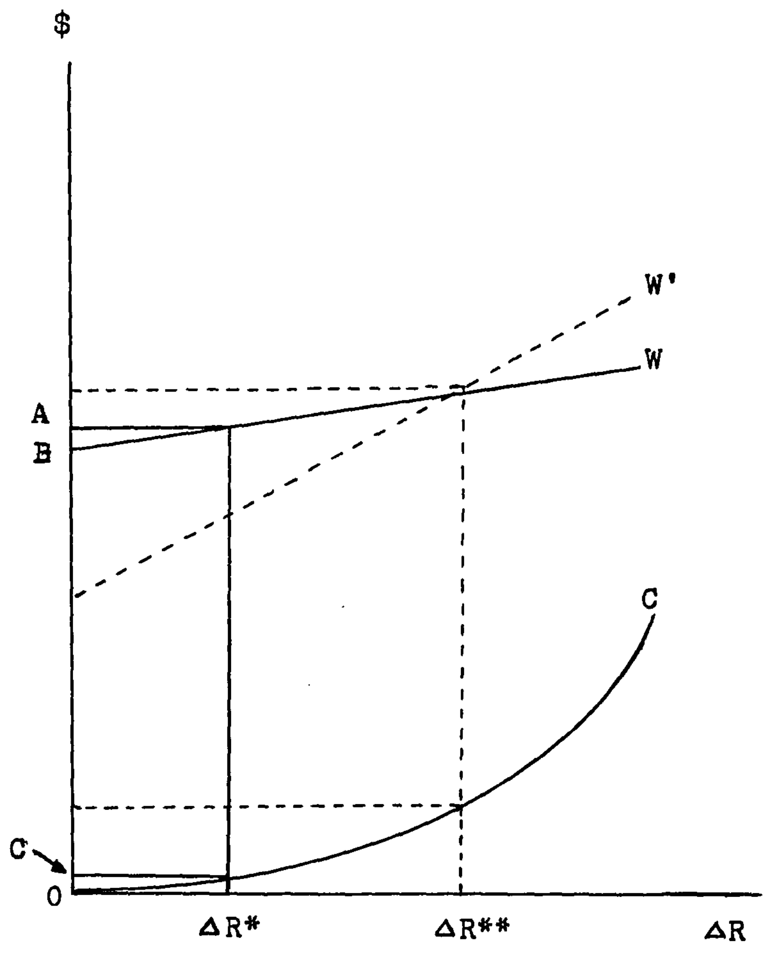
$$(4) \qquad L = R_o + \frac{N_o}{1+\Pi} + \Delta R + \frac{\Delta N}{1+\Pi} - C(\Delta R) + \lambda(\Delta R + \Delta N),$$

one obtains the first order condition

$$(5) \qquad \frac{\Pi}{1+\Pi} = C'(\Delta R).$$

The solution to this optimization problem is illustrated diagrammatically in figure 10. The W line represents the individual's end-of-period wealth. It is a straight line with vertical intercept $R_o + \frac{N_o}{1+\Pi}$ and slope $\frac{\Pi}{1+\Pi}$. The C curve represents the total costs of switching from one asset to another and is rising as a function of ΔR at an increasing rate. The first order condition (5) means that the individual is at his optimum position at that volume of transactions where the slope of the C curve (the marginal transactions cost) equals the slope of the W line. At that point the individual exchanges the nominal asset for the real asset in the amount ΔR^*, his gross end-of-period wealth is OB, and his net end-of-period wealth is

Figure 10: The Jaffee-Kleiman Model



OB-OC, since OC are the costs associated with the conversion of assets having a volume of ΔR^*. Note that if the individual did not attempt to protect himself at all, his end-of-period wealth would be OA $(=R_o + \frac{N_o}{1+\Pi})$.

The effect of an increase in the expected inflation rate is represented in figure 10 by the dashed W' line. As Π rises, the line representing end-of-period wealth gets steeper and its vertical intercept gets smaller (see equation (1)). With an unchanged $C(\Delta R)$ function, the new equilibrium position will involve a greater amount of protection being "purchased," i.e., $\Delta R^{**} > \Delta R^*$.

Now suppose that the inflation rate is a stochastic variable. If the individual is risk averse, or if his utility function is quadratic in real wealth, he will put a higher premium on protection than would otherwise be the case. Going from $\Sigma = 0$ to $\Sigma > 0$ is analogous to going from a lower to a higher expected inflation rate when $\Sigma = 0$. Thus, the more uncertainty that exists about the expected inflation rate (as measured by the standard deviation of the probability function $f(\Pi)$), the more protection will the individual "purchase" and the higher will be transactions costs.[6]

Comments on the J-K Model

There are some puzzling aspects about the J-K model that might render it a less than satisfactory vehicle for analyzing the costs of money creation under conditions of uncertainty. One minor point concerns the assumption that both the nominal and real asset have the same nominal return. This would mean that the nominal asset, e.g., money, earns the going money rate of interest. The opportunity cost of holding money balances would then always be zero and individuals would always hold the satiety level of money balances. However, I do not believe that any fundamental implications of the J-K model would change if they had assumed that the nominal asset had a nominal return of zero.

Perhaps a more important point concerns the cost function $C(\Delta R)$. In a number of places, J-K refer to these costs as transactions costs.[7] Indeed, the authors talk about "costs beyond the well-recognized cash balance tax effect" (emphasis added).[8] On the other hand, in justifying the assumption that $C''(\Delta R) > 0$, J-K say that "- - - the marginal cost might approach infinitely [sic.] if it is interpreted, for example, as the cost of doing without any cash balances,"[9] which seems to imply that they are talking about the lost productivity of real balances as measured by the area under the liquidity preference function. There is no obvious reason why marginal _transactions_ costs should approach infinity as the volume of transactions rises. In fact, it might well be plausible to assume that marginal transactions costs _decline_, perhaps because of volume discounts (as is the case in the American securities industry) or something analogous to declining bloc pricing that has been used by American electric utilities. If marginal costs did decline, the $C(\Delta R)$ curve would be rising at a decreasing rate and, depending upon the exact slope of the W line, it is quite conceivable that one would wind up with a corner solution in which _no_ protection is purchased. But such a result would itself be something of a puzzle: if marginal transactions costs are declining and there is an increase in uncertainty about the expected inflation rate, shouldn't one expect an increase in the amount of protection that is "purchased" by holders of the nominal asset?

(Note that the same potential problem can arise if the transactions cost function is of the form $C=a+b\Delta R$, which seems to me to be another perfectly plausible transactions cost function. Here the marginal cost would be constant (= b) but, as long as $b < \frac{\Pi}{1+\Pi}$, a corner solution would result. And there is another potential problem: suppose $b > \frac{\Pi}{1+\Pi}$. What then is the optimum amount of protection to "purchase"?)

It may be that the above problems suggest themselves because of the manner in which J-K have formulated the individual's optimization problem. As one can see in figure 1, the vertical axis is used to measure both W, a stock, and C, a flow. More formally, J-K maximize end-of-period wealth <u>net</u> of transactions costs. By doing this they subtract a flow variable from a stock variable and it is not at all clear what the resulting net variable (W-C) is.[10]

Finally, it is not clear that J-K's distinction between "cash balance tax effect" costs and transactions costs is a valid one. The area under a liquidity preference function has been interpreted as being the productivity of real balances, which means all the resources saved from the possession of the most liquid of assets (or all the liquidity services provided by that stock of real money). Doesn't this productivity include, at least implicitly, the resource savings that accrue to money holders from <u>not</u> having to switch among assets? If the productivity of real balances <u>does</u> include these resources savings, then the phenomenon of increased transactions costs derived from increased uncertainty does not seem to be a cost that can neatly be separated from "cash balance tax effect" costs in the manner attempted by Jaffee and Kleiman.

The Klein Model

I believe a more satisfactory means of handling non-steady state inflation in the context of the welfare losses and revenues from money creation can be found in a model recently developed by Benjamin Klein.[11] Klein notes that a standard assumption in the monetary economics literature is that real balances produce a flow of services and that this flow is proportional to the stock. Thus, the observed demand for real balances is, under this assumption, a perfectly reasonable proxy for the unobserved demand for the flow of monetary services. But Klein then argues that such an assumption is reasonable only for the most short-term type of macroeconomic models. His point is that the quality of the monetary service flow from a given stock of real balances can vary over time, and this quality change will necessitate a change in the stock of real balances in order to produce the desired flow of monetary services. Put differently, the starting point in any analysis of the demand for real balances must be the demand for monetary services, and one then needs to analyze those factors that translate a given demand for monetary services into a demand for real balances. Below I set out a simplified version of the key features of Klein's analysis.

The relationship between the flow of monetary services and the stock of real balances is given by the representative individual's production function for monetary services[12]

$$(6) \quad S = S((M/P), B),$$

where S is the flow of monetary services and B is a shift variable representing the quality level of the stock of real balances held by the representative individual. For purposes of the diagrammatic illustration below, I shall assume that the production function is of the form

$$(7) \quad S = B_i \cdot (M/P), \quad i = o, 1, ---, n.$$

This particular function means that the marginal services product of real balances, $\partial S/\partial (M/P)$, is constant and equal to the average services product of real balances $S/(M/P)$, at any given B.[13] I assume that $\partial S/\partial B > 0$, i.e., at any given level of real balances, an improvement in the quality (B) of those balances increases the flow of monetary services from the stock.

The particular type of quality that Klein deals with concerns the predictability of future prices. That is, the more unpredictable are prices, the greater is the uncertainty about future price movements and the lower is the quality of the flow of services from a stock of real money balances. Empirically, Klein measures this quality variable by the five-term moving standard deviation from the ten-term moving average of the annual inflation rate (i.e., Klein's measure of quality is conceptually similar to the standard deviation of the inflation rate, Σ, used by Jaffee and Kleiman).

Like any other commodity, the demand for monetary services can be written as a function of the representative individual's real income and the relative price of monetary services:

$$(8) \quad S^D = f((Y/P),\ (P_s/P)),$$

where P_s is the (unobserved) nominal price of monetary services and P is index of commodity prices. Since

$$(9) \quad \left(\frac{M}{P}\right)^{D} = S^D \cdot (S/(M/P)),$$

the derived demand for real balances ($(M/P)^D$) can be written as

$$(10) \quad \left(\frac{M}{P}\right)^{D} = g((Y/P),\ (P_s/P),\ S/(M/P)),$$

where Klein assumes that $\partial(M/P)^D/\partial(Y/P)>0$, $\partial(M/P)^D/\partial(P_s/P)<0$, and $\partial(M/P)^D/\partial(S/(M/P)) < 0$. (The last partial derivative means that, as quality rises and with a given level of demand for monetary services, the demand for real balances declines.)

Before deriving the implications for the demand for real balances when the degree of uncertainty (quality) changes, we note the following: in equilibrium, the representative individual's demand price for a unit of the nominal money stock (which is i, the money rate of interest) is equal to the nominal value of the marginal services product of that stock. Symbolically,

$$(11) \quad i = S_M \cdot P_S ,$$

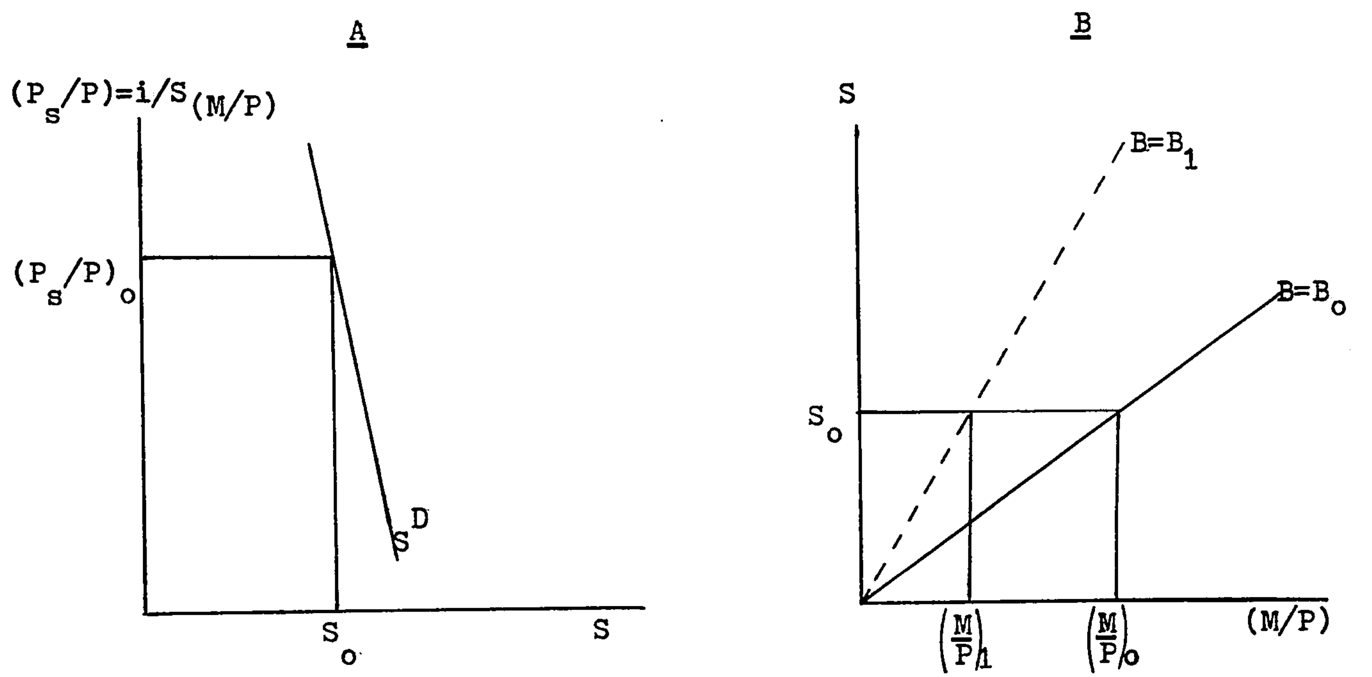
where $S_M = \partial S/\partial M$. But $S_M = S_{(M/P)} \cdot (1/P)$,[14] where $S_{(M/P)} = \partial S/\partial(M/P)$. Thus, equilibrium condition (11) can be rewritten as

$$(12) \quad (P_S/P) = (i/S_{(M/P)}).$$

Klein's model can now be illustrated graphically.[15] Figure 11A shows the representative individual's demand for flows of monetary services as a function of the relative price of monetary services, (P_S/P) ($= i/S_{(M/P)}$ from 12 above). Assume that the relative price of monetary services is $(P_S/P)_O$ to begin with, implying a demand for monetary services equal to S_O.

Figure 11B shows the representative individual's production function for two quality levels, B_O and B_1 ($B_1 > B_O$). Assume that initially $B = B_O$. Then the production function indicates that $(M/P)_O$ of real balances is required to produce the desired flow of monetary services S_O. Since B_O is the marginal services product of real balances, multiplying (P_S/P) by B_O would give the money rate of interest corresponding to the demand for real

Figure 11: The Klein Model



balances $(M/P)_o$, i.e., one point on the derived demand curve for real balances.

Now suppose there is an improvement in the quality level of real balances because the variability of the rate of price change is reduced. Assuming for the moment that the relative price of monetary services is unchanged, the new production function indicates that a smaller quantity of real balances $(M/P)_1$ is necessary to yield the desired flow of services S_o. Thus, for a given money rate of interest, this effect would decrease the demand for real balances.

However, there is another effect present in this analysis. Given i, a rise in B means a decline in the relative price of monetary services, an increase in the demand for those services, and thus an increase in the derived demand for real balances above $(M/P)_1$, along the new (dashed) production function $B = B_1$. At a given i, the net impact on the demand for real balances is a priori indeterminate. This is in contrast to the J-K analysis, where a decrease in uncertainty (improved quality of real balances) implies a smaller amount of switching out of nominal assets and into real assets, i.e., a larger demand for real money balances.

Klein does show[16] that if the demand for real balances is inelastic in the relevant range of i, the net effect will be to produce an increase in the demand for real balances as quality declines. This result appears to be directly contrary to J-K, where increased uncertainty increases the volume of transactions and, by implication, results in a portfolio containing less of the nominal asset and more of the real asset. (Klein's empirical work in his paper indicates that the quality variable enters the demand function for real balances with a positive coefficient, which is consistent with most empirical research showing that the demand for real balances has an interest elasticity of less than unity (in absolute value).)

What can be said of the government revenues and welfare losses from money creation based upon Klein's analysis? At each given i, the demand for real balances will be greater the more uncertain are expectations of future price changes. Thus, a "stop-go" policy by the government, holding the long-term expected rate of price change constant, would increase the demand for real balances and thus increase the base upon which the government's revenues from money creation are derived. On the other hand, as Klein points out,[17] utility is really a function of the level of the flow of monetary services. When government increases the variability of expected inflation through "stop-go" policies, thus increasing uncertainty and reducing the quality level of real balances, the discussion above indicates that the quantity demanded of monetary services will decline at each i. Utility must therefore decline. Thus, the welfare losses from money creation also increase as the degree of uncertainty increases. What is not clear is what happens to some ratio of the welfare losses to government revenues as the degree of uncertainty increases.

Footnotes to Essay 6

1 Arthur Okun, "The Mirage of Steady Inflation," Brookings Papers on Economic Activity, Number 2, 1971.

2 R. A. Gordon, "Steady Anticipated Inflation: Mirage or Oasis," Brookings Papers on Economic Activity, Number 2, 1971.

3 Dwight Jaffee and Ephraim Kleiman, "The Welfare Implications of Uneven Inflation," prepared for the IEA Conference on Inflation, Saltsjobaden, Sweden, (revised June, 1975).

4 Note that N no longer refers to population or labor force as it did in earlier essays.

5 Equation (1) appears to imply that all earnings on the individual's wealth are consumed. I do not believe that any of J-K's results would be qualitatively changed if they had allowed for the reinvestment (savings) of at least some portion of these earnings.

6 J-K also discuss the case where inflation is not uniform across commodities. If the dispersion of price changes across commodities increases then, other things equal, the marginal cost at every ΔR will rise (because of the need to allocate resources among specific real assets and the consequent need for more time, information, etc.) The C curve will be steeper at each ΔR than before and less protection will be "purchased," but the "cash balances tax loss" will be greater.

7 Dwight Jaffee and Ephraim Kleiman, op. cit., pp. 5-6.

8 Ibid., p. 9.

9 Ibid., p. 5.

10
Of course, it is possible to add a flow to a stock, e.g., last period's capital stock plus the current flow of real net investment spending yields this period's capital stock. In the J-K model, however, it might be that transactions costs should be deducted from some measure of gross real income to yield a net real income variable, which variable would then be available for current real consumption or current real saving.

11
Benjamin Klein, "The Demand for Quality-Adjusted Cash Balances: Price Uncertainty in the U.S. Demand for Money Function," Journal of Political Economy, August, 1977.

12
The symbols and terminology I shall use differ in some respects from those of Klein.

13
Klein actually assumes that the marginal services product is declining. However, (8) is consistent with Klein's apparent belief that changes in B change the marginal and average services product in the same direction (Klein, op. cit., pp. 694-695). Also, for a given B, it would appear to be more plausible to assume constant returns in what is essentially a single factor production function. At any rate, the assumption of constant returns does not do violence to Klein's analysis.

14
Klein, op. cit., p. 693.

15
At least, I shall illustrate part of Klein's model. His full model is illustrated in Klein, op. cit., p. 695.

16
Klein, op. cit., p. 697, footnote 7.

17
Klein, op. cit., p. 713.

Appendix A

G = government revenue = $h'\rho N q^{b} k e^{-ai}$, so

$$\frac{\partial G}{\partial \rho} = h'Nq^{b}k \left(e^{-ai} - a\rho e^{-ai} \frac{di}{d\rho}\right).$$

Since $i = r+\rho-\beta\ell-b\lambda$, where β, ℓ, b, and λ are constants (see Appendix B), r = real rate, π = rate of inflation, and ρ = rate of money growth, it follows that

$$\frac{di}{d\rho} = 1 \text{ and } \frac{\partial G}{\partial \rho} = h'Nq^{b}k \left(e^{-ai} - a\rho e^{-ai}\right).$$

For maximum revenue, $\frac{\partial G}{\partial \rho} = 0$ or

$$e^{-ai} = a\rho e^{-ai}$$

so that the revenue-maximizing ρ is:

$$\rho = \frac{1}{a}.$$

Since the demand function for real balances is $m = Nq^{b}ke^{-ai}$,

$$\frac{\partial m}{\partial \rho} \frac{\rho}{m} = -a\rho.$$

Letting $\rho = (1/a)$, the demand elasticity when the government is maximizing revenue is, in absolute value, unity.

Appendix B

$MV=PQ$, $q = \frac{Q}{N}$, so $\frac{\dot{M}}{M} + \frac{\dot{V}}{V} = \frac{\dot{P}}{P} + \frac{\dot{Q}}{Q}$ and $\frac{\dot{q}}{q} = \frac{\dot{Q}}{Q} - \frac{\dot{N}}{N}$.

Therefore, $\frac{\dot{M}}{M} + \frac{\dot{V}}{V} = \frac{\dot{P}}{P} + \frac{\dot{N}}{N} + \frac{\dot{q}}{q}$ or $\frac{\dot{M}}{M} = \frac{\dot{P}}{P} + \frac{\dot{N}}{N} + \frac{\dot{q}}{q} - \frac{\dot{V}}{V}$.

By definition, $\eta = \frac{\partial \ln \frac{M}{PN}}{\partial \ln q} = \frac{\partial \ln M - \partial \ln P - \partial \ln N}{\partial \ln q} = \frac{\frac{\dot{M}}{M} - \frac{\dot{P}}{P} - \frac{\dot{N}}{N}}{\frac{\dot{q}}{q}} = \frac{\frac{\dot{q}}{q} - \frac{\dot{V}}{V}}{\frac{\dot{q}}{q}}$,

so $\eta \frac{\dot{q}}{q} = \frac{\dot{q}}{q} - \frac{\dot{V}}{V}$.

Thus $\frac{\dot{M}}{M} = \frac{\dot{P}}{P} + \frac{\dot{N}}{N} + \eta \frac{\dot{q}}{q} = \frac{\dot{P}}{P} + \frac{\dot{N}}{N} + \eta \left(\frac{\dot{Q}}{Q} - \frac{\dot{N}}{N}\right)$ or

$$\rho = \pi + \ell + b (\lambda - \ell).$$

Rearranging, $\pi = \rho - \ell - b (\lambda - \ell) = \rho - (1 - b) \ell - b\lambda$.

Letting $\beta = 1-b$, $\pi = \rho - \beta\ell - b\lambda$.

When $b = 1$, $\beta = 0$ and $\pi = \rho - \lambda$. The same result follows if $b \neq 1$ but $\lambda = \ell$.

Appendix C

In the stationary economy, the welfare loss is

$$W=Nq^bk\int_0^{r+\rho}e^{-ax}dx-[Nq^bk\int_0^{r}e^{-ax}dx-rNq^bke^{-ar}]-(r+\rho)Nq^bke^{-a(r+\rho)}$$

$$=Nq^bk[\frac{e^{-ar}}{r}-\frac{e^{-a(r+\rho)}}{a}+re^{-ar}-(r+\rho)e^{-a(r+\rho)}].$$

Government receipts are

$$G = h'\rho Nq^bke^{-a(r+\rho)}.$$

Therefore $W/G = \dfrac{e^{a\rho}-[1+a\rho]}{ah'\rho} + \dfrac{r(e^{a\rho}-1)}{h'\rho}$.

$$dW/d\rho = Nq^bka(r+\rho)e^{-a(r+\rho)}, \text{ and}$$

$$dG/d\rho = h'Nq^bk[e^{-a(r+\rho)}-a\rho e^{-a(r+\rho)}].$$

Thus MTC = $(dW/d\rho) / (dG/d\rho) = \dfrac{a(r+\rho)}{h'(1-a\rho)}$.

Appendix D

In the autonomously growing economy, the welfare loss is

$$W=Nq^bk\int_0^{r+\rho-\beta\ell-b\lambda} e^{-ax}dx \quad -[Nq^bk\int_0^{r-\beta\ell-b\lambda} e^{-ax}dx-(r-\beta\ell-b\lambda)Nq^bke^{-a(r-\beta\ell-b\lambda)}$$

$$-(r+\rho-\beta\ell-b\lambda)Nq^bke^{-a(r+\rho-\beta\ell-b\lambda)}$$

$$=Nq^bk[\frac{e}{a}^{-a(r-\beta\ell-b\lambda)} - \frac{e}{a}^{-a(r+\rho-\beta\ell-b\lambda)} + (r-\beta\ell-b\lambda)e^{-a(r-\beta\ell-b\lambda)}$$

$$-(r+\rho-\beta\ell-b\lambda)e^{-a(r+\rho-\beta\ell-b\lambda)}].$$

Government receipts are

$$G = h'\rho Nq^bke^{-a(r+\rho-\beta\ell-b\lambda)}.$$

Therefore $W/G = \dfrac{e^{a\rho}-(1+a\rho)}{ah'\rho} + \dfrac{(r-\beta\ell-b\lambda)\ (e^{a\rho}-1)}{h'\rho}.$

$$dW/d\rho = Nq^bka(r+\rho-\beta\ell-b\lambda)e^{-a(r+\rho-\beta\ell-b\lambda)}, \text{ and}$$

$$dG/d\rho = h'Nq^bk[e^{-a(r+\rho-\beta\ell-b\lambda)}-a\rho e^{-a(r+\rho-\beta\ell-b\lambda)}].$$

Thus $MTC = \dfrac{a(r+\rho-\beta\ell-b\lambda)}{h'(1-a\rho)}$.

Appendix E

In the induced-growth economy, the welfare loss is

$$W = Nq^b k \int_0^{r+\rho-\beta\ell-b\lambda} e^{-ax}dx - [Nq^b k \int_0^r e^{-ax}\, dx - rNq^b k\, e^{-ar}]$$

$$-(r+\rho-\beta\ell-b\lambda)\, Nq^b k\, e^{-a(r+\rho-\beta\ell-b\lambda)}$$

$$= Nq^b k\, [\frac{e^{-ar}}{a} - \frac{e^{-a(r+\rho-\beta\ell-b\lambda)}}{a} + re^{-ar} - (r+\rho-\beta\ell-b\lambda)e^{-a(r+\rho-\beta\ell-b\lambda)}\,].$$

Government receipts are

$$G = h'\rho\, Nq^b k e^{-a(r+\rho-\beta\ell-b\lambda)}.$$

Therefore

$$W/G = \frac{e^{a(\rho-\beta\ell-b\lambda)} - [1+a(\rho-\beta\ell-b\lambda)]}{ah'\rho} \quad \frac{r(e^{a(\rho-\beta\ell-b\lambda)} - 1)}{h'\rho}\, .$$

$$dw/d\rho = Nq^b k\, [a(r+\rho-\beta\ell-b\lambda)\, e^{-a(r+\rho-\beta\ell-b\lambda)} (1-b\frac{d\lambda}{d\rho})],$$ and

$$dG/d\rho = h'Nq^b k\, [e^{-a(r+\rho-\beta\ell-b\lambda)} - a\rho e^{-a(r+\rho-\beta\ell-b\lambda)} (1-b\, \frac{d\lambda}{d\rho})\,].$$

Thus $$MTC = \frac{a(r+\rho-\beta\ell-b\lambda)\quad(1-b\,\frac{d\lambda}{d\rho})}{h'\,[1-a\rho(1-b\,\frac{d\lambda}{d\rho}\,]}\, .$$

Appendix F

For the case of combined autonomous and induced growth, the welfare loss is

$$W = Nq^b k \int_0^{r+\rho-\beta\ell-b(\lambda'+\lambda'')} e^{-ax}\,dx - [Nq^b k \int_0^{r-\beta\ell-b\lambda'} e^{-ax}\,dx - (r-\beta\ell-b\lambda')\, Nq^b k e^{-a(r-\beta\ell-b\lambda')}] - [r+\rho-\beta\ell-b(\lambda'+\lambda'')]\, Nq^b k e^{-a[r+\rho-\beta\ell-b(\lambda'+\lambda'')]}$$

$$= Nq^b k \left[\frac{e^{-a(r-\beta\ell-b\lambda')}}{a} - \frac{e^{-a[r+\rho-\beta\ell-b(\lambda'+\lambda'')]}}{a} + (r-\beta\ell-b\lambda')e^{-a(r-\beta\ell-b\lambda')} - (r+\rho-\beta\ell-b[\lambda'+\lambda''])\, e^{-a[r+\rho-\beta\ell-b(\lambda'+\lambda'')]}\right].$$

Government receipts are

$$G = h'\rho N q^b k e^{-a[r+\rho-\beta\ell-b(\lambda'+\lambda'')]}.$$

Therefore

$$W/G = \frac{e^{a(\rho-b\lambda'')} - [1+a(\rho-b\lambda'')]}{ah'\rho} + \frac{(r-\beta\ell-b\lambda')\,(e^{a(\rho-b\lambda'')}-1)}{h'\rho}$$

$$dW/d\rho = Nq^b k\, [a(r+\rho-\beta\ell-b[\lambda'+\lambda''])\, e^{-a(r+\rho-\beta\ell-b[\lambda'+\lambda''])} (1-b\frac{d\lambda''}{d\rho})],$$ and

$$dG/d\rho = h'Nq^b k\, [e^{-a(r+\rho-\beta\ell-b[\lambda'+\lambda''])} - a\rho e^{-a(r+\rho-\beta\ell-b[\lambda'+\lambda''])} (1-b\frac{d\lambda''}{d\rho})].$$

Thus $$MTC = \frac{a(r+\rho-\beta\ell-b[\lambda'+\lambda''])\ (1-b\frac{d\lambda''}{d\rho})}{h'\ [1-a\rho(1-b\ \frac{d\lambda''}{d\rho})]}.$$

Appendix G

The welfare cost in the stationary economy for broadly defined money is

$$W = Nq^b k \int_0^{(r+\rho)[z(1-\gamma)+\gamma]} e^{-ax}\,dx - Nq^b k \int_0^{r[z(1-\gamma)+\gamma]} e^{-ax}\,dx -$$

$$Nq^b k r[z(1-\gamma)+\gamma] e^{-ar[z(1-\gamma)+\gamma]}$$

$$- (r+\rho)[z(1-\gamma)+\gamma]\, Nq^b k e^{-a(r+\rho)[z(1-\gamma)+\gamma]}$$

$$= Nq^b k \Big[\frac{e^{-ar[z(1-\gamma)+\gamma]}}{a} - \frac{e^{-a(r+\rho)[z(1-\gamma)+\gamma]}}{a}$$

$$+ r[z(1-\gamma)+\gamma]\, e^{-ar[z(1-\gamma)+\gamma]}$$

$$- (r+\rho)[z(1-\gamma)+\gamma] e^{-a(r+\rho)[z(1-\gamma)+\gamma]}\Big].$$

Government receipts are

$$G = h'\rho Nq^b k\, e^{-a(r+\rho)[z(1-\gamma)+\gamma]}$$

Therefore

$$W/G = \frac{e^{a\rho[z(1-\gamma)+\gamma]} - (1 + a\rho(z[1-\gamma]+\gamma))}{ah'\rho} + \frac{r[z(1-\gamma)+\gamma](e^{a\rho[z(1-\gamma)+\gamma]} - 1)}{h'\rho}.$$

$$dW/d\rho = Nq^b k \Big\{ a(r+\rho)[z(1-\gamma)+\gamma]$$

$$e^{-a(r+\rho)[z(1-\gamma)+\gamma]} ([z(1-\gamma)+\gamma] + (r+\rho)(1-\gamma)\frac{dz}{d\rho})$$

$$- \; ar[z(1-\gamma)+\gamma] \, e^{-ar[z(1-\gamma)+\gamma]} \left(r(1-\gamma) \frac{dz}{d\rho}\right\} \text{, and}$$

$$dG/d\rho = h'Nq^{b}k \left\{ e^{\{-a(r+\rho)\;[z(1-\gamma)+\gamma]} \right.$$

$$\left. -a\rho e^{-a(r+\rho)\;[z(1-\gamma)+\gamma]} \left([z(1-\gamma)+\gamma] + (r+\rho)\;(1-\gamma)\frac{dz}{d\rho}\right) \right\}\}$$

Thus

$$MTC = \frac{a(r+\rho)\;[z(1-\gamma)+\gamma]\;\left([z(1-\gamma)+\gamma]+(r+\rho)\;(1-\gamma)\frac{dz}{d\rho}\right) - ar[z(1-\gamma)+\gamma]e^{a\rho}\left(r(1-\gamma)\frac{dz}{d\rho}\right).}{h'\;\left\{1-a\rho\left([z(1-\gamma)+\gamma] + (r+\rho)\;(1-\gamma)\frac{dz}{d\rho}\right)\right\}}$$

Appendix H

$$g = \rho A \cdot e^{-ai_t} \int_{t=o}^{\infty} e^{(\gamma - r)t} dt + Ae^{\gamma t} e^{-ai_t} - Ae^{\gamma t} e^{-ai_o} =$$

$$\frac{A}{(r-\gamma)} \{\rho e^{-ai_t} + (r-\gamma) e^{-ai_t} - (r-\gamma) e^{-ai_o}\},$$ given that

$Ae^{\gamma t} = 1$ at $t=o$.

Since $i_t = r+\rho-\beta\ell-b\lambda$ and $i_o = r-\beta\ell-b\lambda$,

$$g = \frac{A}{(r-\gamma)} \{e^{-a(r+\rho-\beta\ell-b\lambda)} + (r-\gamma) e^{-a(r+\rho-\beta\ell-b\lambda)} - (r-\gamma) e^{-a(r-\beta\ell-b\lambda)}\}.$$

Appendix I

$$g = \frac{A}{(r-\gamma)}\{\rho e^{-a(r+\rho-\beta\ell-b\lambda)} + (r-\gamma)\, e^{-a(r+\rho-\beta\ell-b\lambda)} - (r-\gamma)\, e^{-a(r-\beta\ell-b\lambda)}\}.$$

$$\partial g/_{\partial\rho} = \frac{A}{(r-\gamma)}\ \{e^{-a(r+\rho-\beta\ell-b\lambda)} - a\,\rho\, e^{-a(r+\rho-\beta\ell-b\lambda)} - a(r-\gamma)\, e^{-a(r+\rho-\beta\ell-b\lambda)}\} =$$

$$\frac{A}{(r-\gamma)}\quad e^{-a(r+\rho-\beta\ell-b\lambda)}\{1 - a\rho - a(r-\gamma)\}$$

Setting $\partial g/_{\partial\rho} = o$ gives $1 - a\rho - a(r-\gamma) = o$ or

$$\rho = \frac{1}{a} - (r-\gamma).$$

Appendix J

The present value of the "honest government" welfare loss is the present value of the welfare loss derived in Essay 1 plus the "purchase" or "sale" of real balances desired by money holders:

$$W = \frac{A}{(r-\gamma)} \left\{ \frac{e^{-ai_o}}{a} - \frac{e^{-ai_t}}{a} + i_o e^{-ai_o} - i_t e^{-ai_t} \right\} +$$

$$Ae^{\gamma t} e^{-ai_o} - Ae^{-ait}$$

$$= A\left\{ \frac{e^{-ai_o}}{a} - e^{-ai_t} + i_o e^{-ai_o} - i_t e^{-ai_t} + \frac{(r-\gamma) e^{-ai_o} - (r-\gamma) e^{-ai_t}}{(r-\gamma)} \right\}$$

Thus,

$$W/g = \frac{e^{-ai_o} - e^{-ai_t} + ai_o e^{-ai_o} - ai_t e^{-ai_t} + a(r-\gamma) e^{-ai_o} - a(r-\gamma) e^{-ai_t}}{a e^{-ai_t} + a(r-\gamma) e^{-ai_t} - a(r-\gamma) e^{-ai_o}}$$

$$= \frac{e^{a\rho} - (1+a\rho)}{a\rho + a(r-\gamma) - a(r-\gamma)e^{a\rho}} + \frac{(r-\beta\ell-b\gamma)\ (e^{a\rho}-1)}{+(r-\gamma)-(r-\gamma)e^{a\rho}}$$

$$\frac{(r-\gamma)\ e^{a\rho} - (r-\gamma)}{\rho + (r-\gamma)-(r-\gamma)\ e^{a\rho}} \qquad \text{or}$$

$$W/g = \frac{e^{a\rho} - (1 + a\rho)}{a\rho + a(r-\gamma) - a(r-\gamma)e^{a\rho}} + \frac{2(r-\gamma)\ (e^{a\rho} - 1)}{\rho + (r-\gamma) - (r-\gamma)e^{a\rho}}$$

since $r-\beta\ell-b\lambda = r - \gamma$

Appendix K

$$\partial w/\partial \rho = \frac{A}{(r-\gamma)}\{a(r-\gamma)e^{-a(r+\rho-\beta\ell-b\lambda)}+a(r+\rho-\beta\ell-b\lambda)e^{-a(r+\rho-\beta\ell-b\lambda)}\},$$

$$\partial g/\partial \rho = \frac{A}{(r-\gamma)}\{e^{-a(r+\rho-\beta\ell-b\lambda)}-a\rho e^{-a(r+\rho-\beta\ell-b\lambda)}-a(r-\gamma)e^{-a(r+\rho-\beta\ell-b\lambda)}\},$$

Therefore MTC = $(\partial w/\partial \rho)/(\partial g/\partial \rho) = \dfrac{a(r+\rho-\beta\ell-b\lambda)}{1-a\rho-a(r-\gamma)} + \dfrac{a(r-\gamma)}{1-a\rho-a(r-\gamma)}$.

Appendix L

$$w = Nq^b k\{\int_0^{r+\rho-\beta\ell-b\lambda} e^{-ax}dx \quad -(r+\rho-\beta\ell-b\lambda)e^{-a(r+\rho-\beta\ell-b\lambda)}\} =$$

$$Nq^b k\{-\frac{e^{-a(r+\rho-\beta\ell-b\lambda)}}{a} + \frac{1}{a} - (r+\rho-\beta\ell-b\lambda)e^{-a(r+\rho-\beta\ell-b\lambda)}\}.$$

$$g = i\frac{M}{P} = (r+\rho-\beta\ell-b\lambda)Nq^b ke^{-a(r+\rho-\beta\ell-b\lambda)}.$$

$$\frac{w}{g} = \frac{1-e^{-a(r+\rho-\beta\ell-b\lambda)}-a(r+\rho-\beta\ell-b\lambda)e^{-a(r+\rho-\beta\ell-b\lambda)}}{a(r+\rho-\beta\ell-b\lambda)e^{-a(r+\rho-\beta\ell-b\lambda)}} =$$

$$\frac{e^{a(r+\rho-\beta\ell-b\lambda)}-1-a(r+\rho-\beta\ell-b\lambda)}{a(r+\rho-\beta\ell-b\lambda)} = \frac{e^{a(r+\rho-\beta\ell-b\lambda)}-1}{a(r+\rho-\beta\ell-b\lambda)} - 1.$$

For the stationary economy, $\ell=\lambda=0$ and

$$\frac{w}{g} = \frac{e^{a(r+\rho)}-1}{a(r+\rho)} - 1.$$

Appendix M

In terms of the money rate of interest,

$$w = \frac{Nq^b k}{a}\left(1-e^{-ai}-aie^{-ai}\right), \text{ so}$$

$$\frac{dw}{di} = \frac{Nq^b k}{a}\left(ae^{-ai}-ae^{-ai}+a^2 ie^{-ai}\right) = Nq^b kaie^{-ai}.$$

$$\frac{dg}{di} = Nq^b k\left(e^{-ai}-aie^{-ai}\right).$$

Thus, $MTC = \dfrac{aie^{-ai}}{e^{-ai}-aie^{-ai}} = \dfrac{ai}{1-ai}.$

Since $i = r+\rho-\beta\ell-b\lambda$, $MC = \dfrac{a(r+\rho-\beta\ell-b\lambda)}{1-a(r+\rho-\beta\ell-b\lambda)} = \dfrac{a(r+\rho-\beta\ell-b\lambda)}{1-a\rho-a(r-\beta\ell-b\lambda)}.$

For the stationary economy, with $\ell=\lambda=0$,

$$MTC = \frac{a(r+\rho)}{1-a(r+\rho)}.$$

Appendix N

$$w = Nq^b k \int_o^{r+\pi} e^{-ax} dx - [Nq^b k \int_o^{r} e^{-ax} dx - rNq^b k e^{-ar}]$$

$$-(r+\pi) Nq^b k e^{-a(r+\pi)} .$$

$$w = Nq^b k \; [\frac{e}{a}^{-ar} - \frac{e}{a}^{-a(r+\pi)} + re^{-ar} - (r+\pi) e^{-a(r+\pi)} .$$

$$g = \pi N q^b k e^{-a(r+\pi)} .$$

$$\frac{w}{g} = \frac{e^{-ar} - e^{-a(r+\pi)} + are^{-ar} - a(r+\pi) e^{-a(r+\pi)}}{a\pi e^{-a(r+\pi)}} =$$

$$\frac{e^{a\pi} - (1+a\pi)}{a\pi} + \frac{r(e^{a\pi}-1)}{\pi} .$$

Appendix O

$$\frac{dw}{d\pi} = Nq^b k \; \{e^{-a(r+\pi)} - [e^{-a(r+\pi)} - (r+\pi)ae^{-a(r+\pi)}]\} =$$

$$aNq^b k(r+\pi)e^{-a(r+\pi)}.$$

$$\frac{dg}{d\pi} = Nq^b k \; \{e^{-a(r+\pi)} - a\pi e^{-a(r+\pi)}\}.$$

$$MTC = \frac{dw/d\pi}{dg/d\pi} = \frac{a(r+\pi)e^{-a(r+\pi)}}{e^{-a(r+\pi)} - a\pi \; e^{-a(r+\pi)}} = \frac{a(r+\pi)}{1-a\pi}.$$

Appendix P

Assume all taxable commodities are being taxed at their optimal levels. If the tax on the i^{th} commodity is increased by Δt_i $(=\Delta P_i)$, the change in consumers' surplus is

(P1) $$\Delta CS = \tfrac{1}{2}\Delta X_i \Delta P_i + \Delta P_i (X_i - \Delta X_i) \simeq \Delta P_i X_i .$$

(P2) $$G = \sum_{K=1}^{n} t_K X_K, \quad \frac{\partial G}{\partial t_i} = \sum_{K=1}^{n} t_K \frac{\partial X_K}{\partial t_i} + X_i .$$

(P3) $$\Delta G = \frac{\partial G}{\partial t_i}\Delta P_i = (X_i + \sum_{K=1}^{n} t_K \frac{\partial X_K}{\partial t_i})\ \Delta P_i .$$

(P4) $$\Delta W = \Delta CS - \Delta G = (-\sum_{K=1}^{n} t_K \frac{\partial X_K}{\partial t_i})\ \Delta P_i .$$

But equation (14) shows that $\sum_{K=1}^{n} t_K \frac{\partial X_K}{\partial t_i} = -vX_i$,

so

(P5) $$\frac{\Delta W}{\Delta G} = MC = \frac{VX_i \Delta P_i}{X_i \Delta P_i - VX_i \Delta P_i} = \frac{V}{1-V} .$$

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