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ELEMENTARY PATTERNS FOR LARGE CONFORMAL ARRAYS

City University of New York

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ELEMENTARY PATTERNS FOR LARGE CONFORMAL ARRAYS

by

KANG SOO LEE

A dissertation submitted to the Graduate Faculty
in Engineering in partial fulfillment of the
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1980

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Abstract

ELEMENTARY PATTERNS FOR LARGE CONFORMAL ARRAYS

by

Kang Soo Lee

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A conformal array under consideration has large number of dipole or slot columns circular symmetrically distributed along the circumference of a conducting cylinder. A dipole or slot column is consisted of infinite number of array elements in the cylinder axial direction. Because of this three dimensional array geometry, beam scan in elevation as well as in azimuth can be carried out. To investigate the beam scan characteristics of these antenna arrays flush-mounted on cylindrical surfaces, the elementary patterns of these arrays are derived.

For a conformal dipole array, the separation between the dipole elements and the conducting surface is within a quarter wavelength and the space between them is filled with a dielectric material, the dielec-

tric constant of which is given as ϵ . The special case when ϵ is equal to ϵ_0 , the dielectric constant of the free space, is also studied(Chapter 2). In general, however, the dipole elements can be considered as the printed dipoles on a dielectric board which is clad on a conducting cylinder(Chapter 3). For a conformal slot array, the radiating elements are the open waveguides at the surface of a conducting cylinder which is covered with a thin dielectric sheet(Chapter 4). A dipole or a slot element is activated through a feedline or a waveguide excited by a voltage generator or the waveguide incident field, respectively.

Applying the Watson transform method to the near field calculation around a cylinder circumference, Green's functions that account for the mutual coupling effects between dipole or slot columns can be evaluated. In Chapter 2, the advantage of this complex plane analysis over a direct harmonic series approach is purely its computational efficiency when the radius of an array is large. Additionally, as the dissertation progresses, the complex plane analysis provides physical insight on underlying array behaviors. By investigating the locations of the Watson transform poles on the complex plane and their shifts as functions of various array parameters, the fundamental characteristics of these arrays can be

disclosed.

For a conformal dipole array in Chapter 2, the elementary pattern cut on the horizontal plane has a narrow beamwidth compared to the one on the vertical plane determined by the activated column and the cylinder axis. Capitalizing on the broad elevation beamwidth and alleviating the shortcoming of the narrow azimuthal beamwidth with the circular symmetric array geometry, a wide angle beam scan in elevation as well as in azimuth can be carried out.

In Chapter 3 where array elements are the printed dipoles on a dielectric sheet, the Watson transform poles can be found with a perturbation method. Because of mode couplings, as the beam is scanned away from the horizontal plane to the axial direction, new TE poles which include a real pole are emerging in addition to TM poles. Due to a resonance inside the dielectric sheet launched by the emerging real pole, the elementary pattern cut at an elevation angle undergoes significant changes from the horizontal one as the angle is increasing, which ultimately limits the elevation scan of this dipole array.

For the slot arrays in Chapter 4, the roles of TE and TM poles are interchanged from those of Chapter 3. A resonance launched by a real pole results in a dip for the elementary pattern cut on the horizontal plane. As

the beam is scanned away, the change in the dip angle can be predicted from the shift of the real pole on the complex plane. For a wide angle elevation scan, because of an additional real pole, multiple resonances take place with accompanying overall changes on an elementary pattern cut. Therefore, the elevation scan must be limited to the onset angle of the multiple resonances.

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Chapter 1. Introduction

1.1 Background.

Research and development on circular conformal arrays have been motivated for various reasons. The needs for wide angle beam scans and the requirements of natural radome protections for antenna arrays are among the major reasons. Because of its symmetric nature in azimuth, a circular array can maintain a constant beam pattern on the horizontal plane through a full 360 degree azimuthal scan, which greatly improves the azimuthal resolution of a radar antenna. A dielectric clad on the conducting surface of a circular array can protect the array without a cumbersome radome structure. The need for a conformal array also arises when the shape of an array is dictated by the contour of a vehicle. An antenna array on the surface of a flying object must be flush mounted on the body not to disrupt a smooth aerodynamic flow. In many instances, the body or a part of it can be idealized with a cylindrical structure for the analysis to be tractable.

For beam coverages both in elevation and in azimuth, three dimensional antenna arrays flush mounted on cylindrical structures are considered. For these arrays, in addition to the beam patterns on the horizontal plane, the changes in the patterns as the beams are lifted away from

the horizontal plane to the cylinder axial direction are also of great concern.

Theoretical analysis of conformal arrays has been concentrated on the slot arrays of open waveguide elements on conducting cylindrical surfaces by extending the concepts and the methods of planar phased arrays. Modal approaches capitalizing on the periodic properties of these arrays have been dominantly employed as analysis tools. For two dimensional cylindrical arrays of axial slits on large conducting cylinders, the elementary patterns have been derived using asymptotic integrations (Ref.1) or eigenexcitation methods(Ref.2). The elementary patterns for three dimensional cylindrical arrays of waveguide elements have been also presented(Ref.3). When these arrays are covered with dielectric sheets for radome protection, the presence of the dielectrics has significant impact on the elementary patterns. For a two dimensional cylindrical array of axial slits with a dielectric cover, the resonance phenomenon inside the dielectric sheet has been investigated(Ref.4). Using direct harmonic series calculations, the elementary patterns for dielectrically clad three dimensional cylindrical arrays of waveguide elements have been derived(Ref.5).

With the advance of printed circuit techniques for antenna arrays, microstrip antennas have increasing applications. A paper thin low cost and light weight

microstrip printed circuit board can be conformally wrapped around high velocity flying vehicles. Such an array does not disrupt the aerodynamic flow, neither it interfere inwardly with the mechanical structure. There are a few studies on such arrays (Ref.6) (Ref.7). The radiating elements of these printed circuit arrays are microstrip patches which are useful in many applications. However, these microstrip patch elements have disadvantages since they have narrow bandwidths and, because of feedline structures, it is difficult to apply them to three dimensional array environments. Antenna arrays of printed dipoles can escape these disadvantages, because a dipole element has a wider bandwidth and a dipole array can be easily printed in a three dimensional configuration. While planar dipole arrays without dielectric covers have received wide attention (Ref.8) (Ref.9) (Ref.10), theoretical analysis on non-planar, conformal dipole arrays is few inspite of their availability as printed circuits and increasing importance for wide-band spread-spectrum applications. Only an experimental study on conical arrays of printed dipole elements has been reported (Ref.11).

In this dissertation, in order to find the beam scan characteristics of conformal dipole arrays flush-mounted on conducting cylinders, the elementary patterns for these arrays are derived. The study is carried out strictly through a complex plane analysis instead of a

direct harmonic series approach. With the same method, the elementary patterns for three dimensional conformal slot arrays with dielectric covers are also derived. This complex plane analysis can establish a one-to-one correspondence between the poles on the complex plane and the important characteristics of an elementary pattern. By investigating the pole transitions on the complex plane, an overall pattern oscillation can be predicted for an increased elevation scan and, as a result, a clear cut elevation scan limitation can be found. The direct harmonic series approach taken in Ref.5 does not provide such crucial informations in addition to the computational inefficiency.

1.2 Statement of the problems.

The elementary patterns for the following conformal arrays are derived.

1. In Chapter 2, a conformal array has large number of dipole columns circular symmetrically distributed along the circumference of a conducting cylinder which is infinite in the axial direction (Fig.2-1). In the same direction, each dipole column is an infinite collinear array which is separated from the cylinder surface by the distance of a quarter wavelength. A dipole element is a cylindrical dipole

with a small radius and is excited at its center through a feedline whose characteristic admittance is matched to the broadside radiation.

2. In Chapter 3, the array configuration is exactly the same as in Chapter 2 except the fact that the space between the dipole columns and the conducting cylinder is filled with a dielectric, the dielectric constant of which is varied from $2\epsilon_0$ to $3\epsilon_0$ (Fig.3-1). This is a simplified situation of an actual printed dipole array.
3. In Chapter 4, a conformal array has large number of slot columns circular symmetrically distributed on the surface of a conducting cylinder covered with a thin dielectric sheet (Fig.4-1). The dimension of the array in the axial direction is infinite. A slot is an open waveguide with a narrow width such that the aperture field has no variation in the circumferential direction. Inside a waveguide, only the dominant mode is allowed to propagate.

1.3 Elementary pattern

Due to a linear periodicity in the cylinder axial direction, the elements of a column have the same elementary pattern. On a horizontal plane, because of the circular symmetry of an array, the elementary patterns of all other columns can be generated by azimuthal rotations of a reference column elementary pattern. As a result, the derivation of a single elementary pattern can provide sufficient information on the performance of an array.

The elementary pattern is the far field radiation pattern of a whole array when a single element is activated. The excitation is carried out with a voltage generator through a feedline for a dipole array or a waveguide for a slot array. Voltage generators for all other elements are short-circuited. For a maximum power transfer, it is assumed that the internal admittance of a generator is matched to the characteristic admittance of the feedline or the waveguide. For the elements whose generators are turned off, the feedlines or waveguides are match-terminated to the sides of generators, which eliminates reflected waves and, as a result, simplifies the analysis. In other words, the elementary pattern can be defined as the far field radiation pattern of an array when a single element is excited in the environment of other match-terminated elements. It should not be confused with an isolated element pattern which is the radiation pattern of a single element in the absence of other elements. Due to strong mutual coupling effects between elements, the elementary pattern has a different shape from the isolated element pattern. The latter one is a smooth curve with a broad beamwidth while the elementary pattern is a distorted one with ripples, a narrow beamwidth and, occasionally, pattern dips.

1.4 Separation of the elementary pattern using linear and circumferential periodicities.

The derivation of the elementary pattern directly using its definition is extremely complex, because all the scatterings from the infinite number of elements of the array caused by the activation of a single element must be taken into account. To circumvent this difficulty, linearly and circumferentially periodic properties of the array in the axial and circumferential directions are exploited in the derivation of the elementary pattern.

All the elements of a single column have a same elementary pattern $f(\phi, \theta)$ because of the linear periodicity of the array in the axial direction. To find an elementary pattern cut $f(\phi, \theta')$ at a given elevation angle θ' , all the elements of the single column are excited with voltage generators of the same magnitude, the phases of which are linearly varying in z -direction to result in an in-phase buildup of the individual elementary patterns to the direction of the given elevation angle θ' . Then, the overall radiation pattern of the column to the direction of θ' is given as a product of the elementary pattern cut $f(\phi, \theta')$ and the array factor of the column. This will be clarified by considering the radiation pattern of a column which has a finite but a large number of elements to justify a same elementary pattern $f'(\phi, \theta)$ for most of the elements except a few at the ends of the column. The radiation pattern of the column $f_c'(\phi, \theta)$

is given by the following equation.

$$f'_c(\phi, \theta) = f'(\phi, \theta) \sum_{m=-M}^M A_m e^{i(2d \cdot m \cdot k_z \sin \theta + \phi_m)} \quad (1)$$

where $2M+1$ is the total number of elements and $2d$ is the distance between succeeding elements in the axial direction. A_m and ϕ_m are the amplitude and the phase of m -th voltage generator respectively. In the above equation, the pattern of the column $f'_c(\phi, \theta)$ is given as a product of the elementary pattern $f'(\phi, \theta)$ and an array factor which is represented by a summation. For a given elevation angle θ' , by making A_m 's for all generators to be equal and linearly varying the phases of generators in z -direction to satisfy the equation $\phi_m = -2d \cdot m \cdot k_z \sin \theta'$, $f'_c(\phi, \theta')$ is given as the product of $f'(\phi, \theta')$ and a constant. By the linear phase variation, the array factor can be maintained as a constant regardless of the elevation angle θ' .

Similarly, for the infinite arrays under consideration, with the linear phase variation in z -direction excitation, the elementary pattern cut $f(\phi, \theta')$ at a given elevation angle θ' can be found by normalizing the overall radiation pattern of a single column at the elevation angle with the maximum radiation of the column on the horizontal plane when θ' is equal to zero. The array factor of the column which is a constant due to the linear phase excitation in

z-direction is automatically eliminated in the normalization. Deriving the elementary pattern cuts at various elevation angles, the elementary pattern of the array $f(\phi, \theta)$ can be synthesized.

To avoid the asymmetry in azimuth incurred by a single column excitation, a phase sequence excitation method (Ref.12) is applied in the cylinder circumferential direction. In the following discussion of the phase sequence excitation method, only dipole arrays are considered. For slot arrays, the same argument holds once the dipole current is replaced with a tangential E-field at a slot as the source of a far field radiation.

To find the elementary pattern cut at a given elevation angle, dipole currents of the activated column as well as the remaining columns where the generators at the ends of feedlines are turned off must be known. Instead of finding directly all these column current distributions of the array caused by a single dipole column excitation, the method of phase sequence excitation capitalizing on the circumferential periodicity of the array is adopted. The location of l -th dipole column is $(\rho', \frac{2\pi}{N}l)$ where N is the total number of dipole columns and the azimuthal angle $\frac{2\pi}{N}l$ is an angular distance between the l -th column and a reference column at $(\rho', 0)$. For p -th phase sequence excitation applied circumferentially to N number of dipole columns, the phase difference between generators

driving Q -th column dipole feedlines and the feedlines of the reference column dipoles is $\exp(i\frac{2\pi}{N}Qp)$. When p is varied, the total number of independent phase sequences is N . The net result of N number of phase sequence excitations is exactly same as the single reference column excitation in the environment of other match-terminated dipole columns as Eq. (2) indicates.

$$\frac{1}{N} \sum_{p=0}^{N-1} \exp(i\frac{2\pi}{N}Qp) = \begin{cases} 1 & (Q = 0) \\ 0 & (Q \neq 0) \end{cases} \quad (2)$$

Eq. (2) is the superposition of N number of phase sequence excitations applied to the generators of Q -th dipole column. After N number of phase sequences are applied to the dipole columns in a circumferential direction, only the reference column ($Q=0$) is activated with a voltage source of unit amplitude. However, for each phase sequence separately, a circular symmetry holds for response dipole currents. For a given phase sequence, dipole currents of all columns have a same magnitude while the phases of the currents are following the applied phase sequence. The dipole current of Q -th column $I^Q(p)$ is uniquely determined by the current of the reference column $I(p)$ for p -th phase sequence excitation.

$$I^Q(p) = I(p) \exp(i\frac{2\pi}{N}Qp) \quad (3)$$

Superposing the resultant currents for N number of phase sequence excitations, the desired l -th column dipole current I^l for the single reference column excitation can be found.

$$I^l = \sum_{p=0}^{N-1} I^l(p) = \sum_{p=0}^{N-1} I(p) \exp(i\frac{2\pi}{N}lp) \quad (4)$$

N number of I^l 's for $0 \leq l \leq N-1$ can be found simultaneously using Fast Fourier Transform algorithm(FFT) once N number of response currents $I(p)$'s for $0 \leq p \leq N-1$ are known. With the column current distribution I^l 's, the elementary pattern cut at a given elevation angle can be found. In the following chapters, the derivation of response currents $I(p)$'s under various array circumstances will be discussed.

The elementary patterns for conformal dipole arrays with or without dielectric boards will be derived in chapter 2 and 3 respectively. In chapter 4, elementary patterns for three dimensional conformal slot arrays with dielectric covers will be found.

Chapter 2. Elementary patterns for conformal dipole arrays.

2.1 Introduction.

Planar dipole arrays with or without ground planes have been discussed by several authors (Refs.8,9 and 10). Variations of a dipole input impedance in a E- or H-plane scan have been investigated using various approximations on element dipole currents. As shown by these studies, due to a steep variation of the dipole input impedance at a H-plane scan, a wide angle beam coverage in the H-plane is hard to realize. On the other hand, the change in a dipole input impedance in a E-plane scan is moderate. To capitalize on E-plane scan characteristics as well as to circumvent the impedance mismatch of a planar dipole array in the H-plane scan, a dipole array mounted circular symmetrically over a perfectly conducting cylinder is considered. This array can maintain a constant beam pattern on a horizontal plane through a full 360 degree azimuthal scanning in addition to a wide angle beam coverage in elevation.

The geometry of the array is shown in Fig.(2-1). The array has a large number of dipole columns located circular symmetrically along the circumference of a conducting cylinder. Each dipole column is an infinite collinear array

of axial dipole elements located a quarter wavelength away from the cylinder surface. Each dipole element is excited at its center gap with a feedline of a given characteristic admittance driven by a voltage source. To investigate the overall beam scan characteristics of this array, the elementary pattern of this array is derived.

In the following section, the problem to solve a response current for a given phase sequence excitation is mathematically formulated. In section 2.3, near zone calculations for Green's functions will be discussed. In section 2.4, an active dipole input impedance is found as a function of an excited phase sequence. In section 2.5, elementary patterns are derived. In the final section, concluding remarks are presented.

2.2 Numerical formulation

As discussed in section 1.3, to find an elementary pattern cut at an elevation angle, N number of phase sequences are applied to the dipole columns in a circumferential direction while, in an axial direction, the phases of the sequences are linearly varied to scan a beam to the given elevation angle.

Adopting a five current terms method(Ref.10), the unknown response current $I(p)$ of the reference column for

p-th phase sequence excitation is first represented with a series of five known current terms with the same number of unknown constants.

$$I(p) = \sum_{q=1}^5 c_q(p) i_q \quad (1)$$

where

$$i_1 = \sin k_0 z - \sin k_0 h$$

$$i_2 = \cos k_0 z - \cos k_0 h$$

$$i_3 = \cos \frac{1}{2} k_0 z - \cos \frac{1}{2} k_0 h$$

$$i_4 = \sin k_0 z - \frac{z}{h} \sin k_0 h$$

$$i_5 = \sin \frac{1}{2} k_0 z - \frac{z}{h} \sin \frac{1}{2} k_0 h$$

The coordinate of a dipole center gap is $z=0$ and $2h$ is a dipole length. The five current terms method is effective for the representation of a dipole current which has a slope discontinuity at a dipole center gap due to a narrow gap voltage excitation. Because of the slope discontinuity, many terms are needed with a more general Fourier expansion. The five unknown constants $c_q(p)$'s are determined by imposing a boundary condition at a dipole surface on a resultant E-field due to the assumed dipole currents.

E- and H-fields caused by a dipole column located at (ρ', ϕ') in the neighborhood of a conducting cylinder can be derived from a single z-component electric Hertzian potential

Π_z^e since the source current is parallel to the cylinder axis. Π_z^e satisfies the following equation and boundary conditions at the cylinder surface and at infinity.

$$(\nabla^2 + k_0^2) \Pi_z^e = -\frac{1}{\omega \epsilon_0} I_z(z) \delta(\rho - \rho') \delta(\phi - \phi') \quad (2)$$

where $I_z(z)$ is the current of a dipole column at (ρ', ϕ') . E- and H-fields are derived from Π_z^e using following equations (Ref.13).

$$\begin{aligned} E_\rho &= \frac{\partial^2 \Pi_z^e}{\partial \rho \partial z} & H_\rho &= -i\omega \epsilon_0 \frac{1}{\rho} \frac{\partial \Pi_z^e}{\partial \phi} \\ E_\phi &= \frac{1}{\rho} \frac{\partial^2 \Pi_z^e}{\partial \phi \partial z} & H_\phi &= i\omega \epsilon_0 \frac{\partial \Pi_z^e}{\partial \rho} \\ E_z &= \left(\frac{\partial^2}{\partial z^2} + k_0^2 \right) \Pi_z^e & H_z &= 0 \end{aligned} \quad (3)$$

Π_z^e which satisfies Eq.(2) and boundary conditions can be represented with an infinite series (Ref.13).

$$\Pi_z^e(\rho, \phi, z; \rho', \phi') = \sum_{n=-\infty}^{\infty} G_n(\rho, \phi; \rho', \phi') e^{i\beta_n z} \frac{\pi}{d} \int_{-h}^h I_z(z') e^{-i\beta_n z'} dz' \quad (4)$$

The dipole current $I_z(z)$ of the column is periodic in the axial direction, the phase of which is varied to scan the beam of the column to the elevation angle θ from the horizontal plane. $\beta_n = k_0 \sin \theta + \frac{2n\pi}{2d}$ where $2d$ is a z -direction

dipole center-to-center distance. G_n is two dimensional spatial harmonic Green's function.

$$G_n(\rho, \phi; \rho', \phi') = \frac{-1}{8\pi\omega\epsilon_0} \sum_{m=-\infty}^{\infty} e^{im(\phi-\phi')} \left(\frac{J_m(\sqrt{\rho^2-\rho_n^2}\rho)}{J_m(\sqrt{\rho^2-\rho_n^2}\rho_n)} - \frac{J_m(\sqrt{\rho'^2-\rho_n^2}\rho)}{H_m^{(1)}(\sqrt{\rho'^2-\rho_n^2}\rho_n)} \right) H_m^{(1)}(\sqrt{\rho'^2-\rho_n^2}\rho) \quad (5)$$

($\rho < \rho'$)

a is the radius of the cylinder and, J_m and $H_m^{(1)}$ are the first and the third kind Bessel functions of m -th order. When $\rho > \rho'$, ρ and ρ' in Eq. (5) must be interchanged.

The total electric Hertzian potential $\Pi_z^{\text{et}}(\rho, \phi, z; p)$ for p -th phase sequence excitation can be found by superposing N number of partial electric Hertzian potential Π_z^{e} 's of Eq. (4) arising from each dipole column. From Eqs. (1), (4) and Eq. (3) of chapter 1,

$$\Pi_z^{\text{et}}(\rho, \phi, z; p) = \sum_{q=1}^5 c_q(p) \sum_{\ell=0}^{N-1} \Pi_q(\rho, \phi, z; \frac{2\pi}{N}\ell) \exp(i\frac{2\pi}{N}\ell p) \quad (6)$$

Π_q in Eq. (6) represents Π_z^{e} of Eq. (4) when $I_z(z)$ is substituted with i_q of Eq. (1).

From a E-field boundary condition at a dipole surface, E_z in Eq. (3) must satisfy the following equation when a dipole center gap is excited with a voltage source of magnitude of V .

$$\left(\frac{\partial^2}{\partial z^2} + k_0^2 \right) \Pi_z^{\text{et}} = E_z = -V \delta(z) \quad (7)$$

Solving Eq.(7), the z -dependence of Π_z^{et} is found as

$$\Pi_z^{et}(z) = \frac{1}{k_0} (A \cos k_0 z + B \sin k_0 z - \frac{V}{2} \sin k_0 |z|) \quad (8)$$

A and B are unknown constants. Matching Eqs.(6) and (8) at seven sampling points at a dipole surface, the five unknown constants plus A and B are determined as a function of the dipole center gap voltage V. As a result, a dipole input admittance $Y(p)$ for p -th phase sequence excitation is known. With a given feedline characteristic admittance Y_g , the response dipole current $I(p)$ for p -th phase sequence excitation can be derived. In Eq.(6), a phase factor is the only function which is dependent on the excited phase sequence p . Once Π_q 's for $0 \leq q \leq N-1$ are found, N number of Π_z^{et} 's for $0 \leq p \leq N-1$ can be generated simultaneously using FFT algorithm. To calculate the total electric Hertzian potential Π_z^{et} , various summations in Eqs.(4), (5) and (6) must be carried out, which will be discussed in the following section.

2.3 Green's function in the near zone.

The first summation to be considered is the one in Eq.(5), which is a harmonic series summation for n-th spatial harmonic Green's function $G_n(\rho, \phi; \rho', \phi')$. G_n accounts for the mutual coupling effect between a field column at (ρ, ϕ) and a source column at (ρ', ϕ') which has a linear phase variation of $\beta_n(\phi' = \phi_0 \sin \theta + \frac{2n\pi}{2a})$ in the cylinder axial direction. The convergence of the series is slow when the radius of the cylinder is large and the field point (ρ, ϕ) is located in a near zone. Even for a far field calculation, approximately as many terms as twice the arguments of various Hankel functions are needed. As the field point is moving towards a near zone, more harmonic terms should be included for the series to converge. Since the array under consideration has many dipole columns ($N=200$) around the cylinder and the Green's function is evaluated in a near zone for the analysis of mutual coupling effects, alternate computation schemes instead of the direct summation of the harmonic series are needed.

When the scattering of an incident wave by a conducting cylinder or the radiation from an aperture on a cylinder is investigated, the far field radiation pattern in the shadow region of the cylinder has been calculated using the Watson transform method(Ref.14). It turns out that the same method can be applied to a near zone problem under con-

sideration, where the Watson transform method can cover almost the whole angular region around the circumference of the conducting cylinder (Appendix 1). Only in the neighborhood of the source column which can be called the lit region of the source column, the residue series of the Watson transform has a slow convergence. In this region where $|\phi - \phi'|$ is small, G_n is calculated with a geometric optics approximation.

In the Watson transform method, by introducing a complex ν -plane for the orders of Hankel functions, the harmonic series in Eq.(5) is transformed into a residue series (Appendix 1).

$$G_n(\rho, \phi; \rho', \phi') = \frac{-1}{8\omega\epsilon_0} \sum_m \frac{H_n^2(\sqrt{\rho_0^2 - \beta_n^2}a)}{\frac{\partial H_n(\sqrt{\rho_0^2 - \beta_n^2}a)}{\partial \nu}} H_{\nu}(\sqrt{\rho_0^2 - \beta_n^2}\rho) H'_{\nu}(\sqrt{\rho_0^2 - \beta_n^2}\rho') \frac{\cos[\nu(\pi - |\phi - \phi'|)]}{\sin \nu \pi} \quad (9)$$

where ν_m satisfies the following equation.

$$H'_{\nu}(\sqrt{\rho_0^2 - \beta_n^2}a) = 0. \quad (10)$$

Eq.(10) has infinite number of zeroes in the complex ν -plane the locations of which, for a Hankel function with a large argument, are known as a function of $\sqrt{\rho_0^2 - \beta_n^2}a$ (Ref.15, Appendix 3). The Hankel functions of complex orders and their derivatives are evaluated using Langer's representation of Hankel functions (Refs.14,15,16,17) (Appendix 2). The advantage of the residue series over the harmonic series is its fast convergence

due to an angular factor $\cos \nu_m(\pi - |\phi - \phi'|) / \sin \nu_m \pi$ in Eq. (9).

For ν_m 's in the upper half-plane, it can be approximated as

$$\frac{\cos[\nu_m(\pi - |\phi - \phi'|)]}{\sin \nu_m \pi} \sim -i e^{i \nu_m |\phi - \phi'|} \quad (11)$$

As m of ν_m is increased, the location of ν_m is shifting to an upward direction on the complex-plane (Fig.2-2). Accordingly, the angular factor undergoes an exponential decrease as m is increased and the rate of the decrease is proportional to $|\phi - \phi'|$ which is the angular distance between the source and the field column. The other factor in Eq. (9) which is consisted of Hankel functions and the derivative of a Hankel function with respect to its order,

$$g(\nu_m, a, \rho, \rho') = \frac{H_{\nu_m}^2(\sqrt{\rho_0^2 - \beta_n^2} a)}{\left. \frac{\partial H_{\nu}(\sqrt{\rho_0^2 - \beta_n^2} a)}{\partial \nu} \right|_{\nu=\nu_m}} H_{\nu_m}'(\sqrt{\rho_0^2 - \beta_n^2} \rho) H_{\nu_m}'(\sqrt{\rho_0^2 - \beta_n^2} \rho') \quad (12)$$

does not alter much the convergence property of the residue series when the ratios ρ/a and ρ'/a are close to one, which is the case under consideration since the radius of the cylinder a is large and, ρ' and ρ , the distances of the source and the field columns from the cylinder axis are the same and given as $\rho' = \rho = a + \lambda_0/4$.

To investigate the variation in the magnitude of $g(\nu_m, a, \rho, \rho')$ as m is increased and to find the effect of this change on the overall convergence property of the residue

series in Eq. (9), various approximation formulas for Hankel functions and their derivatives derived in Appendix 2 are used. Using the relationship between Bessel and modified Bessel functions, $I_{\pm\frac{1}{3}}(ix) = e^{\pm i\frac{\pi}{2}} J_{\pm\frac{1}{3}}(x)$, from Eq. (16) of Appendix 2,

$$\begin{aligned} H_{\nu}'(z) &= H_{\nu}'(\nu \operatorname{sech} \alpha) \approx 2 \left(\frac{\alpha - \tanh \alpha}{3 \tanh \alpha} \right)^{\frac{1}{2}} \left[e^{i\frac{\pi}{3}} I_{-\frac{1}{3}}(iR) - e^{i\frac{\pi}{3}} I_{\frac{1}{3}}(iR) \right] \\ &\approx -2i \left(\frac{\alpha - \tanh \alpha}{3 \tanh \alpha} \right)^{\frac{1}{2}} \left[J_{-\frac{1}{3}}(R) + J_{\frac{1}{3}}(R) \right] \quad (13) \end{aligned}$$

where $\alpha = \cosh^{-1} \frac{\nu}{z}$ and $R = -i\nu(\alpha - \tanh \alpha)$. When $\nu = \nu_m$, for $H_{\nu}'(z)$ to be zero,

$$J_{-\frac{1}{3}}(R_m) = -J_{\frac{1}{3}}(R_m) \quad (14)$$

$R_m = -i\nu_m(\alpha_m - \tanh \alpha_m)$ where $\alpha_m = \cosh^{-1} \frac{\nu_m}{z}$. R_m is a positive real number as discussed in Appendix 3. In a similar way,

$$\begin{aligned} H_{\nu}^2(z) &= H_{\nu}^2(\nu \operatorname{sech} \alpha) \approx 2 \left(\frac{\alpha - \tanh \alpha}{3 \tanh \alpha} \right)^{\frac{1}{2}} \left[e^{i\frac{\pi}{3}} I_{-\frac{1}{3}}(iR) - e^{i\frac{\pi}{3}} I_{\frac{1}{3}}(iR) \right] \\ &\approx 2 \left(\frac{\alpha - \tanh \alpha}{3 \tanh \alpha} \right)^{\frac{1}{2}} \left[e^{i\frac{\pi}{6}} J_{-\frac{1}{3}}(R) - e^{i\frac{\pi}{6}} J_{\frac{1}{3}}(R) \right] \quad (15) \end{aligned}$$

From Eqs. (14) and (15), at $\nu = \nu_m$,

$$H_{\nu_m}^2(z) = H_{\nu_m}^2(\nu_m \operatorname{sech} \alpha_m) \approx -2\sqrt{3} \left(\frac{\alpha_m - \tanh \alpha_m}{3 \tanh \alpha_m} \right)^{\frac{1}{2}} J_{\frac{1}{3}}(R_m) \quad (16)$$

From Eq. (18) of Appendix 2, at $\nu = \nu_m$,

$$\begin{aligned} \left. \frac{\partial H_\nu^1(z)}{\partial \nu} \right|_{\nu=\nu_m} &= \left. \frac{\partial H_\nu^1(\nu \operatorname{sech} \alpha)}{\partial \nu} \right|_{\nu=\nu_m} \approx 2 \left(\frac{\alpha_m - \tanh \alpha_m}{3 \tanh \alpha_m} \right)^{\frac{1}{2}} \left[e^{-i\frac{\pi}{3}} J_{\frac{2}{3}}(i R_m) - e^{i\frac{\pi}{3}} J_{-\frac{2}{3}}(i R_m) \right] \\ &\approx 2 \left(\frac{\alpha_m - \tanh \alpha_m}{3 \tanh \alpha_m} \right)^{\frac{1}{2}} \alpha_m \left[J_{\frac{2}{3}}(R_m) - J_{-\frac{2}{3}}(R_m) \right] \\ &\approx 2 \left(\frac{\alpha_m - \tanh \alpha_m}{3 \tanh \alpha_m} \right)^{\frac{1}{2}} \alpha_m \frac{\sqrt{3}}{\pi R_m J_{\frac{2}{3}}(R_m)} \quad (17) \end{aligned}$$

In the derivation of Eq. (17), the Wronskian relationship of a Bessel function (Ref. 18) and Eq. (14) are used. From Eqs. (16) and (17), for $z = \sqrt{k_0^2 - \beta_n^2} a$,

$$\frac{H_{\nu_m}^2(\sqrt{k_0^2 - \beta_n^2} a)}{\left. \frac{\partial H_\nu^1(\sqrt{k_0^2 - \beta_n^2} a)}{\partial \nu} \right|_{\nu=\nu_m}} = - \frac{\pi R_m J_{\frac{2}{3}}^2(R_m)}{\alpha_m} \quad (18)$$

To evaluate $H_{\nu_m}^1(\sqrt{k_0^2 - \beta_n^2} \rho)$ when ν_m is a zero of $H_{\nu_m}^1(\sqrt{k_0^2 - \beta_n^2} a)$, using Eq. (13),

$$H_{\nu_m}^1(\sqrt{k_0^2 - \beta_n^2} \rho) \approx -2i \left(\frac{\alpha'_m - \tanh \alpha'_m}{3 \tanh \alpha'_m} \right)^{\frac{1}{2}} \left[J_{-\frac{1}{3}}(R'_m) + J_{\frac{1}{3}}(R'_m) \right] \quad (19)$$

where $\alpha'_m = \cosh^{-1} \frac{\nu_m}{\sqrt{k_0^2 - \beta_n^2} \rho}$ and $R'_m = -i \nu_m (\alpha'_m - \tanh \alpha'_m)$. For a large value of $|R'_m|$, from an asymptotic expansion of a Bessel function with a large argument (Ref. 18),

$$H_{\nu_m}^1(\sqrt{k_0^2 - \beta_n^2} \rho) \approx -2i \left(\frac{\alpha'_m - \tanh \alpha'_m}{3 \tanh \alpha'_m} \right)^{\frac{1}{2}} \sqrt{\frac{3}{2\pi R'_m}} \left[e^{i(R'_m - \frac{\pi}{4})} + e^{-i(R'_m - \frac{\pi}{4})} \right] \quad (20)$$

R'_m can be given as

$$\begin{aligned}
 R'_m &= -i(\alpha'_m - \tanh \alpha'_m) \psi_m \\
 &= R_m - i\psi_m [\alpha'_m - \alpha_m - (\tanh \alpha'_m - \tanh \alpha_m)] \\
 &= R_m + i\psi_m \psi_m
 \end{aligned} \tag{21}$$

where $\psi_m = \cosh^{-1} \frac{\psi_m}{\sqrt{\beta_0^2 - \beta_n^2} a} - \cosh^{-1} \frac{\psi_m}{\sqrt{\beta_0^2 - \beta_n^2} \rho} + \sqrt{1 - \frac{(\beta_0^2 - \beta_n^2) \rho^2}{\psi_m^2}} - \sqrt{1 - \frac{(\beta_0^2 - \beta_n^2) a^2}{\psi_m^2}}$

Since $\psi_m \psi_m$ has a positive real part on the upper half-plane, Eq.(20) can be simplified as

$$H_{\psi_m}^1(\sqrt{\beta_0^2 - \beta_n^2} \rho) \approx -2i \left(\frac{\alpha'_m - \tanh \alpha'_m}{3 \tanh \alpha'_m} \right)^{\frac{1}{2}} \sqrt{\frac{3}{2\pi R'_m}} e^{-i(R_m - \frac{\pi}{4})} e^{\psi_m \psi_m} \tag{22}$$

From Eqs.(12), (18) and (22), for $\rho = \rho'$,

$$g(\psi_m, a, \rho', \rho) \approx \frac{2 R_m J_{\frac{1}{2}}^2(R_m)}{\alpha_m R'_m} \left(\frac{\alpha'_m - \tanh \alpha'_m}{\tanh \alpha'_m} \right) e^{-iZ(R_m - \frac{\pi}{4})} e^{2\psi_m \psi_m} \tag{23}$$

For a large value of m such that $|\psi_m| \gg \sqrt{\beta_0^2 - \beta_n^2} \rho'$, ψ_m can be approximated as

$$\psi_m \approx \ln(\rho'/a) \tag{24}$$

(Ref.18). For the convergence of the residue series in Eq.(9),

therefore, the exponential in the angular factor of Eq.(11) must decay in a faster rate than the exponential increase in $g(\nu_m, a, \rho', \rho)$. From Eqs.(11), (23) and (24), for the convergence of the residue series,

$$2X_m \ln(\rho' / a) - Y_m |\phi - \phi'| < 0 \quad (25)$$

where X_m and Y_m are the real and imaginary parts of ν_m . The asymptotic values of X_m and Y_m for a large m are given as

$$\begin{aligned} X_m &\approx \frac{\frac{\pi^2}{2} (m + \frac{1}{4})}{\left\{ \ln \frac{(m + \frac{1}{4}) \pi}{e \sqrt{k_0^2 - \beta_n^2} a} \right\}^2} \\ Y_m &\approx \frac{\pi (m + \frac{1}{4})}{\ln \frac{(m + \frac{1}{4}) \pi}{e \sqrt{k_0^2 - \beta_n^2} a}} \end{aligned} \quad (26)$$

(Ref.20). Since the ratio Y_m/X_m can be made as large as we want by increasing m , Eq.(25) can be satisfied for an arbitrary small value of the angular distance $|\phi - \phi'|$. Therefore, theoretically at least, the residue series converges in the whole angular region around the circumference of the cylinder except at $\phi = \phi'$. Practically, however, when only a fixed number of terms are used to evaluate the residue series, $|\phi - \phi'|$ in the angular factor must be large enough to more than compensate the exponential increase in $|g(\nu_m, a, \rho', \rho)|$ and to cause a

fast convergence of the residue series. As a result, the neighbourhood region of the source column which can be called a lit region is excluded in the application of the residue series. In the lit region of the source column, a geometric optics approximation can be used. As the source column is moving away from the cylinder, ψ_m in Eq.(24) is increasing and the angular region where the residue series with a given number of terms can be applied is reduced. On the other hand, the lit region is increasing as the geometric optics shows. In a sense, these two methods, the Watson transform and the geometric optics compensate each other to evaluate the Green's function G_n in the whole angular region around the cylinder circumference.

In the problem under consideration, since ρ'/a is close to one, excluding a narrow angular sector around the source column, the Watson transform method can be applied in a wide angular region along the cylinder circumference. When the source column goes to infinity, $\alpha'_m = \cosh^{-1} \frac{\psi_m}{\sqrt{k_0^2 - \beta_n^2} \rho'}$, $\approx i \frac{\pi}{2}$ and $\tanh \alpha'_m \approx i \frac{\sqrt{k_0^2 - \beta_n^2}}{\psi_m} \rho'$. R'_m in this case is given as

$$\begin{aligned} R'_m &= -i \psi_m (\alpha'_m - \tanh \alpha'_m) \\ &\approx \frac{i\pi}{2} \psi_m - \sqrt{k_0^2 - \beta_n^2} \rho' \end{aligned} \quad (27)$$

When the field point is still in the neighbour of the cylin-

der ($\rho/a \approx 1$), the region of the convergence for a residue series is given from Eqs.(11), (20) and (27) as

$$|\phi - \phi'| - \frac{\pi}{2} > 0 \quad (28)$$

The angular region around the circumference of a cylinder where the residue series converges is reduced to the half of the whole region. As far as the source column is located at a finite distance from the cylinder, the region where the Watson transform can be applied for an actual calculation is wider than that specified by Eq.(28), which is the lower limit.

In practice, the application of the Watson transform method for the calculation of a spatial harmonic Green's function G_n is needed only for the fundamental ($n=0$) or, the fundamental and the first harmonic ($n=-1$) when $k_0 > |\beta_n|$. For higher harmonics, $k_0 < |\beta_n|$ as far as $\lambda_0/2 < 2d < \lambda_0$ and the arguments of Hankel functions are imaginary in Eq.(5). From the relationship between a Hankel and a modified Bessel function $H_\nu^{(1)}(ix) = -i \frac{2}{\pi} e^{-i\frac{\nu\pi}{2}} K_\nu(x)$ (Ref.18), Eq.(10) takes the following form when $k_0 < |\beta_n|$.

$$K_\nu(\sqrt{\beta_n^2 - k_0^2} a) = 0 \quad (29)$$

Zeros of $K_\nu(x)$ when x is positive real are purely imagin-

ary (Ref.20). The locations of zeroes at the imaginary axis of the complex ν - plane can be found from Eq.(9) of Appendix 3 where, for an imaginary z , the real part of ν_m goes to zero. Because the magnitude of the lowest zero ν_1 is approximately same as the argument $\sqrt{\beta_h^2 - k_0^2} a$, the angular factor in Eq.(11) is rapidly decaying as $|\phi - \phi'|$ is increased. As a result, $G_n(\rho, \phi; \rho', \phi')$ in such a case has significance only in the region near the source column where the geometric optics can be used to approximate the Green's functions. For the calculations of harmonic Green's functions when the arguments of Hankel functions are imaginary, the geometric optics approximation is exclusively used.

Deep in the shadow of the cylinder which is caused by the source column, only a single residue term due to ν_1 in Eq.(9) is sufficient for the calculation of G_n . However, to preserve accuracy near the shadow boundary, 15 residue terms are used for Eq.(9).

For the calculation of the Green's function G_n in the narrow angular sector around the source column, G_n is first decomposed into a free space part G_{nf} and a scattered part G_{ns} which can be identified as the first and the second term in Eq.(5). For the free space part G_{nf} , using the addition theorem for Bessel functions, a closed form expression can be obtained (Ref.13).

$$\begin{aligned}
 G_{nf}(\rho, \phi; \rho', \phi') &= \frac{-1}{8\pi\omega\xi_0} \sum_{m=-\infty}^{\infty} \exp(im(\phi - \phi')) J_m(\sqrt{\rho_0^2 - \beta_n^2}\rho) H_m'(\sqrt{\rho_0^2 - \beta_n^2}\rho') \\
 &= \frac{-1}{8\pi\omega\xi_0} H_0'(\sqrt{\rho_0^2 - \beta_n^2}|\vec{\rho} - \vec{\rho}'|) \quad (30)
 \end{aligned}$$

where $|\vec{\rho} - \vec{\rho}'| (= \sqrt{\rho^2 + \rho'^2 - 2\rho\rho'\cos(\phi - \phi')})$ is the distance between the source and the field column. As Eq.(30) shows, G_{nf} is the field in the free space caused by a line source with a z-direction phase variation of β_n . In a special case when the free space Green's function is evaluated at the same dipole surface, whose current is the source for the field, a logarithmic singularity is encountered in the calculation of the Green's function $G_{nf}(\rho', \phi'; \rho', \phi')$. This singularity can be removed using a thick dipole assumption, according to which a dipole has a small but finite radius r_0 and the current of the dipole is symmetrically distributed on the surface (Ref.10). The free space Green's function is given in this special case as

$$\begin{aligned}
 G_{nf} &= \frac{-1}{8\pi\omega\xi_0} \frac{1}{2\pi} \int_0^{2\pi} H_0'(\sqrt{\rho_0^2 - \beta_n^2} 2r_0 \sin\frac{\theta}{2}) d\theta \\
 &= \frac{-1}{8\pi\omega\xi_0} \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} J_m(\sqrt{\rho_0^2 - \beta_n^2} r_0) H_m'(\sqrt{\rho_0^2 - \beta_n^2} r_0) \int_0^{2\pi} e^{im\theta} d\theta \\
 &= \frac{-1}{8\pi\omega\xi_0} J_0(\sqrt{\rho_0^2 - \beta_n^2} r_0) H_0'(\sqrt{\rho_0^2 - \beta_n^2} r_0) \quad (31)
 \end{aligned}$$

The scattered part G_{ns} , which corresponds to a reflected field from the cylinder surface for an incident cylindrical wave, is given in a closed form expression using a geometric optics approximation.

$$G_{ns}(\rho, \phi; \rho', \phi') = \frac{1}{8\pi\omega\xi_0} \sum_{m=-\infty}^{\infty} \exp(im(\phi - \phi')) \frac{J_m(\sqrt{k_0^2 - \beta_n^2}a)}{H_m'(\sqrt{k_0^2 - \beta_n^2}a)} H_m'(\sqrt{k_0^2 - \beta_n^2}\rho) H_m'(\sqrt{k_0^2 - \beta_n^2}\rho')$$

$$\approx \frac{1}{8\pi\omega\xi_0} \frac{H_0'(\sqrt{k_0^2 - \beta_n^2}s')}{H_0'(\sqrt{k_0^2 - \beta_n^2}s_0)} H_0'(\sqrt{k_0^2 - \beta_n^2}s) \quad (32)$$

where s' is the distance between the source and a reflecting point at the cylinder surface. s_0 and s are the distances to the reflecting and the field points from the caustic point of a reflected wave. s_0 is given for a cylinder of radius a as

$$s_0 = \frac{a \cdot s' \cos\psi}{2s' + a \cdot \cos\psi} \quad (33)$$

(Ref.21). ψ is the angle of an incident ray from a cylinder normal. For a small ψ , Eq.(32) yields a good approximation for the harmonic series, especially when the source and the field points are close to the cylinder surface. However, for a large ψ , the discrepancy between the two is severe. The geometric optics approximation alone does not give any information on the range of the incident angle ψ , within which the method can be applied safely. Some estimation on this problem can be provided by the

method of stationary phase integration(Ref.13). The approximation of G_{ns} with this method results in a closed form expression, to which Eq.(32) is reduced when both the source and the field points are far from the cylinder. For the method of stationary phase integration to be justified for the evaluation of G_{ns} , however, there must be placed a restriction on the angle of incidence ψ . From this condition, an estimation on the range of ψ can be made.

To apply the method to approximate G_{ns} , the harmonic series summation in Eq.(32) is first transformed to a contour integral over the real axis of the complex ν -plane in the same way as in the derivation of Eq.(3) in Appendix 1.

$$G_{ns}(\rho, \phi; \rho', \phi') = \frac{1}{16\pi\omega\epsilon_0} (I_{s1} + I_{s2})$$

$$\text{where } I_{s1} = \int_{c_1} H'_\nu(\sqrt{k_0^2 - \beta_n^2}\rho) H'_\nu(\sqrt{k_0^2 - \beta_n^2}\rho') \frac{\cos\nu(|\phi - \phi'| - \pi)}{\sin\nu\pi} d\nu$$

$$I_{s2} = \int_{c_1} \frac{H_\nu^2(\sqrt{k_0^2 - \beta_n^2}a)}{H'_\nu(\sqrt{k_0^2 - \beta_n^2}a)} H'_\nu(\sqrt{k_0^2 - \beta_n^2}\rho) H'_\nu(\sqrt{k_0^2 - \beta_n^2}\rho') \frac{\cos\nu(|\phi - \phi'| - \pi)}{\sin\nu\pi} d\nu \quad (34)$$

c_1 is the same contour used in Fig.(2-2). Since the contour passes over the real axis, the angular factor in Eq.(34) can be expanded as

$$\frac{\cos\nu(|\phi - \phi'| - \pi)}{\sin\nu\pi} = -i \left[e^{i\nu|\phi - \phi'|} + e^{-i\nu(|\phi - \phi'| - 2\pi)} \right] \sum_{j=0}^{\infty} e^{i2\pi\nu j} \quad (35)$$

To carry out the integration in Eq.(34) by the method of stationary phase, the various Hankel functions must be replaced with exponential functions. The representations for Hankel functions of large complex orders derived in Appendix 2 are not appropriate for this purpose, because the stationary points ν_s 's can be small even though the arguments are large. Hankel functions with large arguments can be represented with the following formulas which are derived from the Sommerfeld integral representation of Bessel functions(Ref.13).

$$H_\nu^1(x) \approx \sqrt{\frac{2}{\pi\sqrt{x^2-\nu^2}}} e^{i(\sqrt{x^2-\nu^2}-\nu\cos^{-1}\frac{\nu}{x}-\frac{\pi}{4})} \quad (\nu < x)$$

$$H_\nu^1(x) \approx -i\sqrt{\frac{2}{\pi\sqrt{\nu^2-x^2}}} e^{(\nu\cosh^{-1}\frac{\nu}{x}-\sqrt{\nu^2-x^2})} \quad (\nu > x) \quad (36)$$

The asymptotic forms for $H_\nu^2(x)$ can be obtained by changing the sign in front of i .

When the Hankel functions and the angular factor in Eq.(34) are substituted with Eqs.(35) and (36), stationary points can be found only for the first term of the angular factor, $-i\exp(i\nu|\phi-\phi'|)$. The other terms in Eq.(35) are irrelevant as far as the stationary phase integration is concerned. When $\rho=\rho'$, the stationary point ν_{s1} for the integral I_{s1} is given as $\nu_{s1} = \sqrt{\rho^2 - \beta_n^2} \rho \cos \frac{|\phi-\phi'|}{2}$. The integration I_{s2} has two stationary points ν_{s2} and ν_{s2}' where ν_{s2} is smaller than the argument $\sqrt{\rho_0^2 - \beta_n^2} a$ while ν_{s2}' is larger than

that. As Eq.(36) shows, the ratio of $H_v^{(2)}(\sqrt{\rho_o^2 - \beta_n^2} a) / H_v^{(1)}(\sqrt{\rho_o^2 - \beta_n^2} a)$ is close to -1 when the order v is larger than the argument. As a result, $\nu_{s2}' = \nu_{s1}$ and the contribution from the stationary point ν_{s2}' nullifies I_{s1} . Therefore, G_{ns} can be evaluated entirely with the contribution to I_{s2} from the stationary point ν_{s2} . For $\rho = \rho'$, ν_{s2} is given as

$$\nu_{s2} = \frac{\sqrt{\rho_o^2 - \beta_n^2} \rho a \sin \frac{|\phi - \phi'|}{2}}{\sqrt{\rho^2 + a^2 - 2\rho a \cos \frac{|\phi - \phi'|}{2}}} = \sqrt{\rho_o^2 - \beta_n^2} a \sin \psi \quad (37)$$

and G_{ns} is approximated as

$$G_{ns} \approx \frac{1}{8\pi\omega\epsilon_o} \sqrt{\frac{2S_o}{\pi\sqrt{\rho_o^2 - \beta_n^2} S S'}} e^{i(\sqrt{\rho_o^2 - \beta_n^2} 2S' - \frac{\pi}{4})} \quad (38)$$

where

$$s' = \sqrt{\rho^2 + a^2 - 2\rho a \cos \frac{|\phi - \phi'|}{2}}$$

$$s_o = \frac{\alpha S' (\rho \cos \frac{|\phi - \phi'|}{2} - a)}{2S'^2 + \alpha (\rho \cos \frac{|\phi - \phi'|}{2} - a)}$$

and $s = s' + s_o$. It can be easily noticed that Eq.(32) is equivalent to Eq.(38) when the arguments of Hankel functions are large. For the cases under consideration where the source and the field point are close to the cylinder, Eq.(32) gives a better approximation of G_{ns} than Eq.(38). However, from Eqs. (36) and (37), it is disclosed that a limitation should be placed upon the incident angle ψ for these methods to be justified. The stationary point ν_{s2} in Eq.(37) is increasing

with ψ and equal to the argument of Hankel functions, $H_{\nu}^{(2)}(\sqrt{\beta_0^2 - \beta_a^2}a)$ for $\psi = \pi/2$. When the order ν is close to the argument x , the representations of a Hankel function in Eq. (36) which are instrumental for the method of stationary phase integration to be successful are not valid any more. As a result, the application of this method should be limited to an angular sector around the source column where the angle 2ψ between the incident ray from the source to the cylinder and the reflected ray from the cylinder surface to the field point is far less than π . In other words, the region where the stationary phase integration can be actually applicable is much narrower than the lit region of the geometric optics. An estimation on the angular width of this region can be obtained from the approximation formulas derived in Appendix 2, since, at the boundary of the region, the stationary point ν_{s2} in Eq. (37) is large enough to justify their applications. When ν/z is real and less than one, $\alpha = \cosh^{-1}(\nu/z) = i\cos^{-1}(\nu/z)$ and $R = -i\nu$. $(\alpha - \tanh\alpha) = -\nu \cdot \sqrt{(z/\nu)^2 - 1} \cdot \cos^{-1}(\nu/z)$. R is a negative real number as we can see in Fig. (2-3) where a point between zero and one on the real axis of ν/z -plane is mapped to a point on the negative imaginary axis on the $(\alpha - \tanh\alpha)$ -

$$H_{\nu}^{(1)}(z) \approx -2i \left(\frac{\alpha - \tanh\alpha}{3 \tanh\alpha} \right)^{\frac{1}{2}} \left\{ J_{-\frac{1}{3}} \left[e^{i\pi} \nu \sqrt{\left(\frac{z}{\nu}\right)^2 - 1} - \cos^{-1}\left(\frac{\nu}{z}\right) \right] + J_{\frac{1}{3}} \left[e^{i\pi} \nu \sqrt{\left(\frac{z}{\nu}\right)^2 - 1} - \cos^{-1}\left(\frac{\nu}{z}\right) \right] \right\}$$

$$H_{\nu}^{(2)}(z) \approx -2i \left(\frac{\alpha - \tanh\alpha}{3 \tanh\alpha} \right)^{\frac{1}{2}} \left\{ J_{-\frac{1}{3}} \left[e^{i\pi} \nu \sqrt{\left(\frac{z}{\nu}\right)^2 - 1} - \cos^{-1}\left(\frac{\nu}{z}\right) \right] + J_{\frac{1}{3}} \left[e^{i\pi} \nu \sqrt{\left(\frac{z}{\nu}\right)^2 - 1} - \cos^{-1}\left(\frac{\nu}{z}\right) \right] \right\}$$

$$\sim -2i \left(\frac{\alpha - \tanh \alpha}{3 \tanh \alpha} \right)^{\frac{1}{2}} \left\{ e^{-\frac{i\pi}{3}} J_{-\frac{1}{3}} \left[\nu \left(\sqrt{\frac{z}{k_0^2}} - 1 - \cos \frac{\nu}{2} \right) \right] + e^{\frac{i\pi}{3}} J_{\frac{1}{3}} \left[\nu \left(\sqrt{\frac{z}{k_0^2}} - 1 - \cos \frac{\nu}{2} \right) \right] \right\} \quad (39)$$

The choice of $e^{i\pi}$ for -1 in the above equation is to make $H'_\nu(z)$ be analytic in the first quadrant of ν/z -plane as mentioned in Appendix 2. Eq. (39) is reduced to Eq. (36) if the argument of Bessel functions is large enough compared to the magnitude of the order to permit the asymptotic expansion of Bessel functions. Therefore, for the method of stationary phase integration or the geometric optics approximation to be successful, the argument of Bessel function at the stationary point ν_{s2} must satisfy the following relationship.

$$\nu_{s2} \left[\sqrt{(k_0^2 - \beta_n^2) \alpha^2 / \nu_{s2}^2 - 1} - \cos^{-1}(\nu_{s2} / \sqrt{k_0^2 - \beta_n^2} \alpha) \right] \gg \frac{1}{3} \quad (40)$$

As the incident angle ψ in Eq. (37) is increasing, the argument is decreased until the above relation is violated and the Bessel functions in Eq. (39) can not be represented with the asymptotic expansion of Eq. (36) and, thereby, Eqs. (32) and (38) are not valid any more. The maximum incident angle is determined from Eqs. (37) and (40). The exact magnitude of the argument depends on how much error can be tolerated. The percentage error of the asymptotic expansion can be found by comparing it with the ascending series of Bessel functions. If a five percent error can be tolerated, the incident angle ψ is allowed to increase until the argument

is reduced to one. When the argument is 1.5, the percentage error of the asymptotic expansion is within one percent. For an argument larger than two, it is an excellent representation for the Hankel functions. However, in this case, since the maximum incident angle is reduced, the angular width of the sector around the source column where the stationary phase integration or the geometric optics approximation can be applicable is narrowed and, as a result, more residue terms must be included in Eq.(9) to guarantee a smooth transition in the value of the Green's function G_n right outside the angular sector.

2.4 Active dipole input impedance.

The electric Hertzian potential $\Pi_z^e(\rho, \phi, z; \rho', \phi')$ of Eq.(4) caused by a source column located at (ρ', ϕ') is found by superposing various spatial harmonic Green's functions, each of which can be evaluated using the methods discussed in the preceding section. The number of spatial harmonics which must be included in the summation for Π_z^e depends on the angular distance between the source and the field column $|\phi - \phi'|$. Once again as in the preceding section, the total angular region around the cylinder circumference is divided into a narrow angular sector in the neighborhood of the source column and the remaining region. In the latter region, the fundamental Green's function G_0 or, the fundamental and the first harmonic G_{-1} can replace the infinite summation in Eq.(4). As mentioned earlier, for a higher harmonic, the argument of the Hankel function $H_\nu'(\sqrt{\beta_0^2 - \beta_n^2} a)$ is imaginary and the zeroes of the function are located on the imaginary axis of the complex ν -plane. The lowest zero has a magnitude which is slightly larger than the argument. For these imaginary zeroes of large magnitudes, the angular factor in Eq.(11) undergoes a rapid decay as $|\phi - \phi'|$ is increasing. Accordingly, outside the neighborhood of the source column, the amplitude of a harmonic Green's function is negligible compared to $|G_0|$.

As we discussed in the preceding section, in the neighborhood of the source column, the Green's functions are evaluated exclusively with the geometric optics approximations. Unless the distance between the source column and the field column is close, i.e. less than a half-wavelength, the electric Hertzian potential Π_z^e can be calculated with four or five harmonics plus the fundamental in this region. Many terms of harmonics are needed in the special case when the electric Hertzian potential Π_z^e is evaluated at the dipole surface of the same column, the current of which is the source for Π_z^e . The number of harmonics required depends on a dipole radius r_0 . Since r_0 is small, the amplitude of the free space Green's function $|G_{nf}|$ of Eq.(31) is not decaying rapidly as $|n|$ is increased. For $r_0=0.007\lambda_0$, Π_z^e begins to converge when $|n|$ is larger than 30. Therefore, approximately more than sixty harmonic terms are needed in this case. Because G_{nf} has a closed form expression given by Eq.(31), calculations for all these harmonic terms do not pose a serious computational problem.

Finally, the partial electric Hertzian potentials caused by N number of dipole columns must be summed to yield the total electric Hertzian potential $\Pi_z^{et}(\rho, \phi, z; p)$ in Eq.(6) at the dipole surface of the reference column. Since a partial electric Hertzian potential due to a dipole column deep in the shadow of the cylinder caused by the reference column can be negligible compared to the one from the dipole

column in the lit region,,only a third of N partial electric Hertzian potentials are actually summed for the total electric Hertzian potential Π_z^{et} . This step completes the calculation of Π_z^{et} for the p -th phase sequence excitation.

Matching Eqs.(6) and (8) at seven sampling points of a reference column dipole surface and inverting a 7×7 matrix, a self-consistent dipole current $I(p)$ of the column caused by a p -th phase sequence excitation is known as a function of V , the dipole center gap voltage. As a result, the active dipole input admittance $Y(p)$ for a p -th phase sequence excitation can be found. The dipole currents of other columns are determined from Eq.(3) of chapter 1. The only term in Eq.(6) which is dependent on an imposed phase sequence is the phase factor $\exp(i\frac{2\pi d}{N}p)$. Once N number of electric Hertzian potentials Π_z^e 's due to individual dipole columns are known, $Y(p)$'s when p is varied from 0 to $N-1$ can be derived without consuming excessive computing time. Since the dipole current of the reference column is the same whether the phase sequence is applied to a clockwise or a counter clockwise direction, $I(p)=I(N-p)$ and computations can be reduced to half.

The active dipole input admittance $Y(p)$ is a rapidly varying function of p . Active dipole input impedance curves are plotted as a function of an outgoing azimuthal beam direction ϕ from a cylinder normal. The

angle ϕ is determined by the excited phase sequence p using a planar approximation locally at the circumference of a large cylinder.

$$\phi = \sin^{-1} \left(\frac{p}{d_c \cdot N \sqrt{1 - \sin^2 \theta}} \right) \quad (41)$$

where d_c is the distance between dipole columns.

In the following discussions, the total number of dipole columns N is fixed as 200 and the length of a dipole $2h$ as a half-wavelength. The distance from a dipole element to the cylinder surface is a quarter-wavelength and the radius of a dipole is taken as $0.007\lambda_0$. The variation of an active dipole input impedance as a function of the azimuthal beam direction ϕ is shown in Figs. (2-4), (2-5) and (2-6). The elevation beam angle θ for these figures is equal to zero. An inter-column space d_c as well as a z -direction dipole center-to-center distance $2d$ is $0.6\lambda_0$ in Fig. (2-4) while, in Fig. (2-5), $d_c = 2d = 0.7\lambda_0$. In Fig. (2-6), $d_c = 0.8\lambda_0$ and $2d = 0.7\lambda_0$. The shapes of these curves are similar to those of planar arrays except that their variations in the circular cases are less abrupt than in the planar cases (Ref. 8). The increase of an impedance curve is pronounced at an azimuthal angle ϕ which corresponds to the grating lobe angle of a planar phased array with the same inter-column and z -direction dipole center-to-

center distance. The grating lobe angle $\phi_{g.l.}$ of a planar array in a H-plane scan is given as

$$\phi_{g.l.} = \sin^{-1} \left(\frac{1 - d_c \sqrt{1 - \sin^2 \theta}}{d_c \sqrt{1 - \sin^2 \theta}} \right) \quad (42)$$

While the reactance of an active dipole input impedance has a clear maximum point at the azimuthal beam direction ϕ which is almost same as $\phi_{g.l.}$ of a corresponding planar array, a resistance curve shows a steady increase around this azimuthal angle. Also, it is noticed that the variation in the reactance curve is small compared to that of the resistance curve. Because of a severe impedance mismatch at the azimuthal angle where the reactance has a maximum, a corresponding elementary pattern suffers a steep decrease in a radiated power at the angle and, beyond this angle, a power radiation is negligible. As a result, the beamwidth of an elementary pattern is approximately determined by the maximum reactance angle. The change of an impedance curve as well as this angle as the distance between dipole columns, d_c , is increased from $0.6\lambda_0$ to $0.8\lambda_0$ is shown in Figs. (2-4), (2-5) and (2-6).

When the elevation angle θ is increased from zero, the change in the active impedance curve is shown in Fig. 2-7 where $d_c = 0.67\lambda_0$ and $2d = 0.72\lambda_0$. As the beam is lifted away from the horizontal plane, the azimuthal angle of the maximum reactance which is almost same as the grating lobe angle $\phi_{g.l.}$

of a comparable planar array in Eq.(42) is shifting to the direction of $\phi = \pi/2$. For $\theta = 50^\circ$ or 60° , ϕ_{gl} is a complex number and a grating lobe can not be found in a visible region. As a result, the variations in the impedance curves as a function of an azimuthal beam angle are smoother. As θ is further increased, ϕ_{gl} is moving away from the visible region and the active impedance is almost a constant.

While the impedance curves discussed so far corresponds to those of a H-plane scan in a planar array, the curves shown in Fig.2-8 are comparable to an E-plane scan characteristics in a planar array. In Fig.2-8, with the same array geometry as in Fig.2-7, the variation in the active impedance is plotted as a function of an elevation beam angle θ when the azimuthal beam direction ϕ is fixed as zero. As the impedance variation in an azimuthal scan is close to those in a H-plane scan of a planar array, Fig.2-8 shows a similar characteristics as in the E-plane scan of a comparable planar array with an almost constant resistance curve, which permits a relatively wide beam scanning in an elevation direction.

2-5. Elementary patterns.

Since a dipole element is excited through a feedline which, in turn, is driven by a constant voltage source at the other side of the line, a dipole center gap voltage is not under a direct control. In other words, the excitation is a free excitation instead of a forced excitation where a constant voltage source is directly applied to a dipole center gap. The free excitation describes more adequately an actual array situation.

For a p -th phase sequence excitation, a feedline current and the voltage of a dipole center gap are determined by the active dipole input admittance $Y(p)$ and a feedline characteristic admittance Y_g . To maximize a power radiation to the broadside direction of the array, the real part of $Y(p)$, when $p=0$ and an elevation beam angle θ is also equal to zero, is taken as the value of Y_g while the imaginary part of $Y(p)$ for these p and θ is eliminated using a matching network (A broadside matching). Also, it is assumed that the source admittance of a voltage generator is matched to this value of Y_g with a compensation network. When the amplitudes of constant voltage sources driving the dipole feedlines are $1/N$, the dipole current of Q -th column for a p -th phase sequence excitation is given by the following equation. When B_0 is the imaginary part of $Y(p)$ for $p=0$ and $\theta=0^\circ$,

$$I^p(p) = e^{i\frac{2\pi p}{N}} \frac{Y_g}{N} \frac{Y(p)}{Y_g + Y(p) - iB_0} \frac{\sum_{q=1}^5 C_q(p) i_q(z)}{\sum_{q=1}^5 C_q(p) i_q(0)} \quad (43)$$

where C_q 's and i_q 's are defined by Eq.(1). $I^p(p)$ is a function of z , the coordinate of a dipole axis with $z=0$ as a dipole center. The amplitude of a feedline current for a p -th phase sequence excitation is given as $Y_g Y(p) / (N(Y_g + Y(p) - iB_0))$ from Eq.(43). By superposing $I^p(p)$ for $0 \leq p \leq N-1$, the desired l -th column dipole current I^l for the single reference column excitation by a unit amplitude voltage source is given from Eq.(4) of Chapter 1.

$$I^l = \frac{Y_g}{N} \sum_{p=0}^{N-1} e^{i\frac{2\pi pl}{N}} \frac{Y(p)}{Y_g + Y(p) - iB_0} \frac{\sum_{q=1}^5 C_q(p) i_q(z)}{\sum_{q=1}^5 C_q(p) i_q(0)} \quad (44)$$

A far field radiation pattern in the direction of an elevation beam angle θ can be found by superposing the fields from N number of dipole columns with the source currents given by Eq.(44). The far field at a constant elevation angle θ caused by the single reference column can be derived from the electric Hertzian potential $\Pi_z^e(\rho, \phi, z; \rho', \phi')$ of Eq.(4) with $\phi' = 0$. In this case, only the fundamental Green's function G_0 is needed in Eq.(4), since it is the only term which propagates to that direction. With a possible exception of G_{-1} when $|\beta_{-1}| < k_0$, all harmonic terms undergo exponential decays in the radial direction of a cylinder and,

as a result, have vanishing values in the far zone. The electric Hertzian potential in the far zone $\Pi_{zf}^e(\rho, \phi, z; \rho', \phi')$ is given from Eqs. (4) and (5) as

$$\begin{aligned}\Pi_{zf}^e(\rho, \phi, z; \rho', \phi') &= G_0(\rho, \phi; \rho', \phi') \exp(i\beta_0 z) \frac{\pi}{d} \int_{-h}^h I_z(z') \exp(-i\beta_0 z') dz' \\ &= \frac{-1}{8\pi\omega\epsilon_0} \sum_{m=-\infty}^{\infty} e^{i\lambda_m(\phi-\phi')} \left\{ J_m'(\sqrt{k_0^2 - \beta_0^2} \rho) - \frac{J_m(\sqrt{k_0^2 - \beta_0^2} a)}{H_m'(\sqrt{k_0^2 - \beta_0^2} a)} H_m'(\sqrt{k_0^2 - \beta_0^2} \rho) \right\} \cdot \\ &\quad H_m'(\sqrt{k_0^2 - \beta_0^2} \rho) e^{i\beta_0 z} \frac{\pi}{d} \int_{-h}^h I_z(z') \exp(-i\beta_0 z') dz' \quad (45)\end{aligned}$$

Using Eqs. (30) and (32), Π_{zf}^e in the lit region of the far zone can be approximated as

$$\begin{aligned}\Pi_{zf}^e(\rho, \phi, z; \rho', \phi') &\approx \frac{-1}{8\pi\omega\epsilon_0} \left\{ H_0'(\sqrt{k_0^2 - \beta_0^2} |\vec{\rho} - \vec{\rho}'|) - \frac{H_0'(\sqrt{k_0^2 - \beta_0^2} s)}{H_0'(\sqrt{k_0^2 - \beta_0^2} s)} H_0'(\sqrt{k_0^2 - \beta_0^2} s) \right\} \cdot \\ &\quad \exp(i\beta_0 z) \frac{\pi}{d} \int_{-h}^h I_z(z') \exp(-i\beta_0 z') dz' \quad (46)\end{aligned}$$

where, for a far field, $s = s_0 + \rho - a \cos \psi$, $|\vec{\rho} - \vec{\rho}'| = \rho - \rho'$, $\cos(\phi - \phi')$ and the incidence or the reflection angle from a cylinder normal ψ is the solution of the following equation.

$$\sin(2\psi - |\phi - \phi'|) = \frac{a}{\rho} \sin \psi \quad (47)$$

Since both $|\rho - \rho'|$ and s are large when a field point is in the far zone, using an asymptotic expansion for a first kind Hankel function of zero order, Eq. (46) can be further

simplified as

$$\begin{aligned} \Pi_{zf}^e(\rho, \phi, z; \rho', \phi') \approx & \frac{-1}{8\pi\omega\varepsilon_0} \sqrt{\frac{2}{\pi\sqrt{\beta_0^2 - \beta_z^2}}} e^{i(\sqrt{\beta_0^2 - \beta_z^2}\rho + \beta_z z - \frac{\pi}{4})} \left\{ e^{-i\sqrt{\beta_0^2 - \beta_z^2}\rho' \cos|\phi - \phi'|} \right. \\ & \left. - \frac{H_0'(\sqrt{\beta_0^2 - \beta_z^2} s')}{H_0(\sqrt{\beta_0^2 - \beta_z^2} s_0)} e^{i\sqrt{\beta_0^2 - \beta_z^2}(s_0 - a \cos\psi)} \right\} \frac{\pi}{d} \int_{-h}^h I_z(z') e^{-i\beta_0 z'} dz'. \end{aligned} \quad (48)$$

The width of the angular region $|\phi - \phi'|$ where Eq. (48) is valid is determined by the method of a stationary phase integration as in a near field calculation of section 2-3. The stationary point ν_{s2} is the solution of the following equation.

$$\frac{\pi}{2} - |\phi - \phi'| = 2 \cos^{-1} \frac{\nu_{s2}}{\sqrt{\beta_0^2 - \beta_z^2} a} - \cos^{-1} \frac{\nu_{s2}}{\sqrt{\beta_0^2 - \beta_z^2} \rho'} \quad (49)$$

When the ratio a/ρ' is close to one, ν_{s2} is given approximately as

$$\nu_{s2} \approx \sqrt{\beta_0^2 - \beta_z^2} a \sin|\phi - \phi'| \quad (50)$$

Comparing Eq. (50) to Eq. (37), for a given angular distance $|\phi - \phi'|$, the stationary point in a far field calculation is smaller than the one in a near field calculation. As mentioned in section 2-3, the approximation formula for a Hankel function in Eq. (36) that is instrumental for the application of a stationary phase integration method is not valid when the order and the argument of the Hankel function are close,

which, in effect, restricts the valid angular region for a geometric optics approximation. Since the increase of a stationary point with $|\phi - \phi'|$ is slower in a far field calculation than in the case of a near field, Eq. (48) can cover a much larger angular region than Eq. (32). The exact angular width of the region where Eq. (48) can be safely used is determined from Eqs. (40) and (49). As we can see from Eqs. (40) and (50), the angular width $|\phi - \phi'|$ can be extended to nearly the whole lit region in the far zone ($|\phi - \phi'| = \pi/2$) when both a and ρ' are large and the ratio a/ρ' is close to one. In the shadow region of the cylinder, Π_{zf}^e can be evaluated with the Watson transform method as in section 2-3.

From Eqs. (3) and (48), in the lit region of the far zone, the ϕ -component of a H-field, H_ϕ in the direction of an elevation angle θ which is predetermined by the z -direction phase variation of a column current is given as

$$H_\phi(r, \phi - \phi', \theta) = i\omega\epsilon_0 \frac{\partial \Pi_{zf}^e(\rho, \phi, z; \rho', \phi')}{\partial \rho} \\ \sim \frac{1}{4} \sqrt{\frac{2}{\pi k_0 r}} e^{i(k_0 r - \frac{\pi}{4})} g(\phi - \phi', \theta) F(I_z(z)) \quad (51)$$

where

$$g(\phi - \phi', \theta) = e^{-i k_0 \rho' \cos \alpha \theta \cdot \cos(\phi - \phi')} \frac{H_0' (k_0 \cos \theta \cdot s')}{H_0' (k_0 \cos \theta \cdot s)} e^{i k_0 \cos \theta \cdot (s_0 - a \cos \psi)}$$

$$F(I_z(z)) = \frac{\pi}{d} \int_{-h}^h I_z(z') e^{-i k_0 \sin \theta z'} dz'$$

and $r = \sqrt{\rho^2 + z^2}$. From H_ϕ , the radiated power in the direction of

(ϕ, θ) is given by $Z_0 |H_\phi|^2$ where Z_0 is $\sqrt{\mu_0/\epsilon_0}$. Because of the circular symmetry, once $H_\phi(r, \phi, \theta)$ for the reference dipole column is known, H_ϕ for a l -th column can be obtained simply by replacing ϕ with $\phi - \frac{2\pi l}{N}$ where $\frac{2\pi l}{N}$ is the angular distance between the reference and l -th dipole column.

The total ϕ -component H-field, H_ϕ^t caused by N number of dipole columns can be found by a superposition.

$$\begin{aligned}
 H_\phi^t(r, \phi, \theta) &= \sum_{l=0}^{N-1} H_\phi(r, \phi - \frac{2\pi l}{N}, \theta) \\
 &\approx \frac{1}{4} \sqrt{\frac{2}{\pi k_0 r}} \exp\{i(k_0 r - \frac{\pi}{4})\} \sum_{l=0}^{N-1} g(\phi - \frac{2\pi l}{N}, \theta) F(I^l)
 \end{aligned}
 \tag{52}$$

The N number of $F(I^l)$ for $0 \leq l \leq N-1$ can be found simultaneously using a FFT algorithm after the five known current terms i_q 's in Eq.(44) are first Fourier integrated. When $H_\phi^t(r, \phi, \theta)$ is normalized by H_ϕ^t on the horizontal plane for $\phi = 0$, the elementary pattern cut at the given elevation angle θ is found.

The elementary pattern cuts for the arrays whose active dipole input impedance curves as a function of an azimuthal or an elevation scan are shown by Figs.(2-4) to (2-8) are plotted in Figs.(2-9) to (2-13). In Figs.(2-9), (2-10) and (2-11), the elementary pattern cuts on the horizontal plane are given when the distance between dipole columns d_c is varied from $0.6\lambda_0$ to $0.8\lambda_0$. Comparing these

figures with the corresponding input impedance curves represented by Figs. (2-4), (2-5) and (2-6), it turns out that the maximum reactance angle in an azimuthal scanning approximately coincides with the half-power point of a pattern on the horizontal plane. As a result, it can be concluded that the azimuthal beamwidth of an elementary pattern cut is determined by the maximum reactance angle in an azimuthal scanning. Beyond this angle, the power radiation is steadily reduced and negligible, which can be explained by an impedance mismatch caused by the large magnitude of an active dipole input resistance beyond the maximum reactance angle even though the active dipole input reactance is reduced rapidly. As a distance between dipole columns d_c is increased from $0.6\lambda_0$ to $0.8\lambda_0$, the maximum reactance angle in an azimuthal scanning is shifted towards a broadside direction. Accordingly, the beamwidth on the horizontal plane is narrowed as Figs. (2-9), (2-10) and (2-11) show.

For elementary pattern cuts at various elevation angles, the same relation between an azimuthal beamwidth and the maximum reactance angle in an azimuthal scanning can be observed from Figs. (2-7) and (2-12). As the elevation angle θ is increased, the maximum reactance angle is shifting towards an endfire direction in Fig. 2-7b and, the corresponding azimuthal beamwidth of an elementary pattern is increased in Fig. (2-12). For elevation angles over 40° , the

reactance curves in Fig.2-7b shows a slow increase without a clear maximum reactance angle while the variations in the resistance curves in Fig.2-7a are small. As a result, the corresponding elementary pattern cuts beyond the elevation angle show no clear cut beamwidths in Fig.(2-12). Their amplitudes are only gradually reduced.

Another important aspect of a conformal dipole array is disclosed by Fig.(2-13) which shows the elementary pattern cut on the $\phi = 0$ plane as a function of the elevation angle θ . It is the same array whose elementary pattern cuts at constant elevation angles are plotted in Fig.(2-12) as a function of the azimuthal angle ϕ . The corresponding active dipole input impedance curves are shown in Fig.(2-8). As discussed in the previous section, the resistance is almost constant when the elevation beam angle is varied from 0 to $\pi/2$. While the reactance curve has a significant change, its magnitude is small compared to the value of the resistance and, the variation is rather gradual, which is quite different from the active dipole input impedance curves when the azimuthal beam direction ϕ is increased from 0 to $\pi/2$ with a fixed elevation beam angle θ . The resulting elementary pattern cut on the $\phi = 0$ plane shows a moderate reduction in amplitude as the elevation angle θ is increased. In Fig.(2-13), the half-power point is about $\theta = 50^\circ$. Because of the smooth variation of the active dipole input impedance in an elevation scan, the elevation beamwidth is quite broader than the azi-

muthal beamwidth when a distance between dipole columns d_c is equal to a z-direction dipole center-to-center distance $2d$.

2-6 Conclusion.

An elementary pattern cut at a constant elevation angle θ is characterized with ripples at broadside and a narrow azimuthal beamwidth. The beamwidth can be predicted approximately by the maximum reactance angle of an active dipole input impedance curve in an azimuthal scanning. The discrepancy of this angle with the grating lobe angle ϕ_{g1} in a H-plane scan for a comparable planar array is small. Beyond the maximum reactance angle, a pattern decay is severe and only a negligible amount of power is radiated. In spite of a rapid decrease in an active dipole input reactance after the maximum reactance angle, a pattern dip at the angle can not be detected. This can be explained by a steady increase in an active dipole input resistance before and after the maximum reactance angle which is large enough to override the effect of the reactance reduction after the angle. Also, the maintenance of this high value of the dipole input resistance after the maximum reactance angle which is shown by a nearly flat resistance curve is responsible for the poor radiation beyond the angle.

The absence of a pattern dip as well as a negligible power radiation after the maximum reactance angle are quite different characteristics of a conformal dipole arrays compared to those of conformal slot arrays which are discussed as special cases of dielectrically covered conformal slot arrays in Chapter 4. For the case of a conformal slot array,

a pattern dip can be found at an azimuthal angle which is just inside the grating lobe angle of a comparable planar array and, after the dip angle, a considerable power radiation can be still observed. Since the maximum reactance angle determines an azimuthal beamwidth, by controlling the position of the angle with an adjustment of the space between dipole columns, the beamwidth of an array can be varied.

As for an elementary pattern cut on a constant azimuthal angle ϕ , it shows a smooth power pattern as a function of the elevation angle θ , because there is no rapid variation of an active dipole input impedance in an elevation scan comparable to the one encountered in an azimuthal scan. No significant input impedance changes are noticed at an elevation grating lobe angle. Even though a dipole input reactance curve undergoes a gradual increase with the elevation angle θ , its value is small compared to a dipole input resistance which is almost flat over a wide range of the elevation angle. Because of its large magnitude, the active dipole input resistance is the dominant factor in determining an elementary pattern cut at a constant azimuthal angle. As expected, the resultant elementary pattern cut shows a gradual decrease in power as the elevation angle θ is increased. When a z-direction dipole center-to-center distance $2d$ is equal to an inter-column space between dipole columns d_c , a beamwidth in elevation is broader than in azimuth.

It is meaningful at this stage to look for the underlying physical reasons behind the difference between an azimuthal and an elevation beam scan characteristics of an active dipole input impedance. In an azimuthal scanning, the rapid increase in an active dipole input impedance at the maximum reactance angle can be understood as a result to maintain a self-consistent dipole current. At this scan angle, besides a main wave front to the desired azimuthal direction, a parasitic wave front which is called a grating lobe in a planar array is generated and propagates along the surface of an array into an endfire direction which makes an obtuse angle with the main beam. Because of this additional wave front, an in-phase build-up for E- and H-fields takes place at the array surface resulting in strong field intensities in the near zone of the array for a given dipole current. Since a dipole center gap voltage is finite and given as V in Eq.(7), to satisfy an E-field boundary condition at a dipole surface, especially at its center gap, the dipole current at the maximum reactance angle in an azimuthal scanning must be small. As the main beam is further scanned away from the broadside, the parasitic wave is separated away from the array surface emerging into the free space and, propagates into a direction which maintains a constant azimuthal angle with the main beam direction, i.e. $\pi/2$ plus the maximum reactance angle. As the angle of this additional beam from the array surface is increasing, the tight in-phase condition for a

field build-up is loosened to result in weaker field intensities at the array surface, which, in turn, incurs the reduction of an active dipole input impedance for the self-consistency of a dipole current. When the main beam scan angle is equal to $\pi/2$, the roles for the main beam and the parasitic beam are completely reversed from the case of an azimuthal scan at the maximum reactance angle. Therefore, at these two azimuthal scan angles, the active dipole input impedance is the same as the various curves indicate. In a sense, the variation of the active dipole input impedance as a function of an azimuthal scan angle is the consequence of mutual coupling effects between dipole elements, which is especially strong when the in-phase condition for a field build-up at the array surface is satisfied.

The difference in the scan characteristic of an active dipole input impedance between an elevation and an azimuthal scan can be explained in terms of a weak or a strong mutual coupling effect between dipole elements. When we consider the field caused by a single dipole element, to the broadside direction of the dipole, the field is decreasing proportionally to the inverse of a distance from the source, while, in the axial direction of the dipole, the field decay is inversely proportional to the square of the distance. Consequently, in the axial direction, the mutual coupling effect between two dipole elements is sharply reduced when the

distance is large. Even though the in-phase field build-up condition is met at the elevation grating lobe angle, since the field from individual element is weak in that direction, it does not yield a large active dipole input impedance as in an azimuthal scanning.

Finally, based on an elementary pattern derived for a conformal dipole array, the scan characteristic of the whole array can be predicted, because the overall pattern can be superposed with elementary patterns of individual elements. On a horizontal plane, the beamwidth of an elementary pattern is narrow as in the case of planar dipole array. For a planar array, it is a major problem, since the whole array pattern in a given azimuthal angle is directly proportional to the elementary pattern at the angle because of a linear superposition and, as a result, the effective beam scan range in azimuth is limited to the beamwidth of an elementary pattern. However, for a conformal array, the beamwidth of an elementary pattern is not so critical as for a planar array, because, using the circular symmetry, for any desired azimuthal beam angle, a portion of the array which faces the beam direction at broadside can be found and activated with appropriate phases to compensate phase differences from individual elements on a conformal surface to the given direction. In other words, the array structure itself is capitalized to alleviate the shortcomings caused by the narrow beamwidth of an elementary pattern. For an elevation scan,

because of a linear superposition, the characteristic of a conformal array is similar to that of a planar array. Consequently, the scan range in elevation of a conformal array is determined by the elevation beamwidth of the elementary pattern. Since the array has a broad elevation beamwidth for its elementary pattern, a fairly broad scan range in the direction can be obtained. As a result, it can be concluded that wide angle beam coverages in elevation as well as in azimuth can be achieved with the arrays under consideration.

Chapter 3. Elementary patterns for conformal dipole arrays mounted on dielectrically-clad conducting cylinders.

3.1 Introduction.

With increasing availability of printed circuit technologies for antenna arrays, low cost dipole elements can be printed on a dielectric board whose profile is paper-thin. With this dielectric board, the surface of a flying vehicle can be wrapped around. Such an array does not alter the aerodynamic property of the vehicle and, inside the vehicle, can provide enough room for the installation of other instruments. To use such printed dipoles instead of the free space dipole elements discussed in the preceding chapter, it is necessary to investigate the impacts on the azimuthal and elevation scan characteristics incurred by the presence of a dielectric board between the dipole elements and the conducting surface of a cylinder. The investigation can be carried out by deriving the elementary pattern of the array.

The geometry of the array which is shown in Fig.(3-1) is the same as in Fig.(2-1) except that the space between dipole elements and the cylindrical surface is filled with a dielectric layer. A dipole element is assumed to be cylindrical with a small radius r_0 and the half of it is embedded in the dielectric board, which simplifies the actual situation where flat dipoles are printed at the surface of a dielectric board. Since

other configurations of the array are the same as in the preceding chapter, the various methods used in Chapter 2 in order to derive the elementary pattern are also exploited in this chapter. The major difference, however, is the Green's function for the field caused by a dipole column on a dielectric surface, which greatly alters the elevation scan characteristic of the array discussed in the preceding chapter.

In the following section, a mathematical expression for the Green's function in the presence of a dielectric layer will be derived. In section 3, the Green's function in the near zone will be evaluated with a Watson transform method, the poles of which are traced in the complex ν -plane with a perturbation technique. It varies conceptually the dielectric constant of the board ϵ from ϵ_0 gradually to its final value in order to follow pole transition trajectories. Also, the launching of a resonance inside the dielectric layer will be explained by pole locations in the complex ν -plane. In section 4, the active dipole input impedance as a function of an azimuthal scan direction and the impacts on this impedance caused by the resonance will be discussed. In section 5, elementary pattern cuts at various elevation angles are derived. In the final section, concluding remarks will be presented.

3.2 Numerical formulation.

The activation of an array with N number of phase sequences in azimuth, which also have a linear phase variation in elevation, in order to derive an elementary pattern cut at a given elevation angle specified by the z-direction phase variation is exactly same as in the preceding chapter. Also, the expansion of an unknown dipole current with five current terms method described by Eq.(1) of Chapter 2 is adopted here. However, at the intersection between a dielectric layer and the free space, the boundary conditions for tangential E- and H-fields can not be satisfied with a single z-component electric Hertzian potential Π_z^e as in Chapter 2. To satisfy the boundary conditions at a dielectric surface, a z-component magnetic Hertzian potential Π_z^m is needed. From these Hertzian potentials, various field components can be derived (Ref.13).

$$\begin{aligned}
 E_\rho &= \frac{\partial^2 \Pi_z^e}{\partial \rho \partial z^2} + i\omega\mu_0 \frac{1}{\rho} \frac{\partial \Pi_z^m}{\partial \phi} & H_\rho &= -i\omega\varepsilon \frac{1}{\rho} \frac{\partial \Pi_z^e}{\partial \phi} + \frac{\partial^2 \Pi_z^m}{\partial \rho \partial z} \\
 E_\phi &= \frac{1}{\rho} \frac{\partial^2 \Pi_z^e}{\partial \phi \partial z} - i\omega\mu_0 \frac{\partial \Pi_z^m}{\partial \rho} & H_\phi &= i\omega\varepsilon \frac{\partial \Pi_z^e}{\partial \rho} + \frac{1}{\rho} \frac{\partial^2 \Pi_z^m}{\partial \phi \partial z} \\
 E_z &= (\rho^2 + \frac{\partial^2}{\partial z^2}) \Pi_z^e & H_z &= (\rho^2 + \frac{\partial^2}{\partial z^2}) \Pi_z^m
 \end{aligned}
 \tag{1}$$

($\rho < \rho'$)

where ε and ρ are the dielectric constant and the wave number inside the dielectric board. Also, Π_z^e and Π_z^m repre-

sent the electric and the magnetic Hertzian potential inside the layer. Comparing Eq. (1) to Eq. (3) in Chapter 2, it is noticed that the z-direction H-field is no longer equal to zero. In other words, when there is no dielectric material, only TM modes propagate in the axial direction of the cylinder. In the presence of a dielectric layer, TE modes from the magnetic Hertzian potential Π_z^m are generated in addition to TM modes. Since the electric and the magnetic Hertzian potential have a common denominator, mode couplings between TM and TE waves are taking place.

The electric Hertzian potential Π_z^e caused by a single dipole column at the surface of the dielectric board has the same form as in Eq. (4) of Chapter 2. However, the two dimensional spatial harmonic Green's function G_n is given by a much more complicated equation, which reduces to Eq. (5) in Chapter 2 when $\xi = \xi_0$.

$$G_n(\rho, \phi; \rho', \phi') = \frac{i(2\pi)^{-2}}{\omega \epsilon \sqrt{k^2 - \beta_n^2} \rho} \sum_{m=-\infty}^{\infty} \frac{e^{im(\phi - \phi')} Z(\rho) R}{\left\{ \chi\left(\frac{\xi}{2}\right) H_m'\left(\sqrt{k^2 - \beta_n^2} a\right) - T\left(\frac{\xi}{2}\right) H_m^2\left(\sqrt{k^2 - \beta_n^2} a\right) \right\} R - \frac{(m\beta_n)^2}{4(k^2 - \beta_n^2)} \left(\frac{\rho^2 - \beta_n^2}{\rho_n^2 - \beta_n^2}\right) Z(\rho') W} \quad (\rho \leq \rho') \quad (2)$$

where

$$X(s) = H_m^2(\sqrt{k^2 - \beta_n^2} \rho') - S \sqrt{\frac{k^2 - \beta_n^2}{k^2 - \beta_n^2}} \frac{H_m'(\sqrt{k^2 - \beta_n^2} \rho')}{H_m'(\sqrt{k^2 - \beta_n^2} \rho')} H_m^2(\sqrt{k^2 - \beta_n^2} s) \quad (2a)$$

$$T(s) = H_m'(\sqrt{k^2 - \beta_n^2} \rho') - S \sqrt{\frac{k^2 - \beta_n^2}{k^2 - \beta_n^2}} \frac{H_m'(\sqrt{k^2 - \beta_n^2} \rho')}{H_m'(\sqrt{k^2 - \beta_n^2} \rho')} H_m'(\sqrt{k^2 - \beta_n^2} s) \quad (2b)$$

$$Z(s) = H_m^1(\sqrt{k^2 - \beta_n^2} a) H_m^2(\sqrt{k^2 - \beta_n^2} s) - H_m^1(\sqrt{k^2 - \beta_n^2} s) H_m^2(\sqrt{k^2 - \beta_n^2} a) \quad (2c)$$

$$W = H_m^{1'}(\sqrt{k^2 - \beta_n^2} a) H_m^2(\sqrt{k^2 - \beta_n^2} \rho') - H_m^1(\sqrt{k^2 - \beta_n^2} \rho') H_m^{2'}(\sqrt{k^2 - \beta_n^2} a) \quad (2d)$$

$$R = X(1) H_m^{1'}(\sqrt{k^2 - \beta_n^2} a) - T(1) H_m^{2'}(\sqrt{k^2 - \beta_n^2} a) \quad (2e)$$

$H_m^{1'}$ and $H_m^{2'}$ are the derivatives of Hankel functions with respect to their arguments. The equivalence of the above equation and Eq. (5) in Chapter 2 can be readily shown when $\xi = \xi_0$ ($k = k_0$). In such a case, $T(1) = 0$ and $X(1) = \{H_m^1(\sqrt{k_0^2 - \beta_n^2} \rho') H_m^{2'}(\sqrt{k_0^2 - \beta_n^2} \rho') - H_m^{1'}(\sqrt{k_0^2 - \beta_n^2} \rho') H_m^2(\sqrt{k_0^2 - \beta_n^2} \rho')\} / H_m^1(\sqrt{k_0^2 - \beta_n^2} \rho')$. Using a Wronskian relationship of Hankel functions, $X(1)$ is further reduced as $X(1) = -4i / \{ \pi \sqrt{k_0^2 - \beta_n^2} \rho' H_m^1(\sqrt{k_0^2 - \beta_n^2} \rho') \}$. With these values of T and X , G_n in Eq. (2) is reduced to Eq. (5) in Chapter 2.

To find the response dipole current $I(p)$ for a p -th phase sequence excitation, the point matching of the total electric Hertzian potentials given by Eqs. (6) and (8) in Chapt. 2 at a dipole surface of the reference column is carried out as in the preceding chapter. The free space wave number k_0 in Eq. (8) of Chapt. 2 is replaced by k , the wave number inside the dielectric layer since the matching is taken placed at the dipole surface to the side of the dielectric board using the Green's function in Eq. (2) which is valid inside the layer. Because the z -component E -field is continuous at the intersection, it does not matter at which side the matching is made.

In the following section, the evaluation of the Green's function in the near zone will be discussed.

3.3 Green's function in the near zone.

For the same reasons described in the preceding chapter, it is difficult to evaluate G_n directly using the harmonic series summation given in Eq.(2). The difficulty is more severe in this case, because some of the Hankel functions have larger arguments than in Chapter 2 due to the increased value of the wave number k in the dielectric layer. Since the number of terms needed for the convergence is roughly proportional to the arguments of the Hankel functions, more harmonic terms than in Chapter 2 must be included if a computation is carried out directly using the series.

To circumvent the difficulty caused by the convergence of the harmonic series, alternate computation schemes are employed as in Chapter 2. In the majority of the angular region around the cylinder except in the immediate neighborhood of the source column, a Watson transform method can be used. In the narrow angular sector containing the source, by replacing the cylindrical surface with a planar surface, a planar approximation can be applied to evaluate G_n in that region.

For the Watson transform of the harmonic series, exactly same steps can be taken as in Appendix 1 using the same contours. In a similar way as in Appendix 1, it can be shown that the contour integral on the infinite semi-circle C_3 goes to zero. Therefore, the slowly converging harmonic

series is transformed into a fast converging residue series which is the summation of the residues at poles on the upper half-plane. The remaining problem is to find pole locations in the complex γ -plane.

In the preceding chapter, the poles of a Green's function G_n in the complex γ -plane are known directly from its equation as the zeroes of $H'_y(\sqrt{\epsilon_0^2 - \epsilon_n^2} a)$, which are well known functions of the argument $\sqrt{\epsilon_0^2 - \epsilon_n^2} a$. The situation is quite different for the Green's function G_n in Eq.(2) which has a much more complicated form. It is not a simple task to find zeroes for the denominator of G_n on the two dimensional complex γ -plane. Among the infinite number of zeroes, the zeroes we are looking for are the lowest ones on the upper half-plane which are close to the real axis and, consequently, have significant contributions to the residue series. It is time consuming simply to scan the whole upper half-plane for these zeroes. Instead of the direct searching, with a perturbation technique, the zero locations can be found in an efficient way. With this technique, the dielectric constant of the layer is conceptually varied from the free space value ϵ_0 gradually to its final value and, in the process, the trajectories for the poles of G_n are closely traced in the complex γ -plane. When $\epsilon = \epsilon_0$, as already discussed in the preceding section, the poles of G_n are zeroes of $H'_y(\sqrt{\epsilon_0^2 - \epsilon_n^2} a)$, the locations of which are well known on the complex γ -plane.

As in the preceding chapter, the Watson transform method is applied to calculate mainly the fundamental Green's function G_0 or, G_0 and the first harmonic G_{-1} when $|\beta_{-1}| < k_0$ such that the arguments of Hankel functions for G_{-1} are real. For a higher harmonic, because of the rapid decay of its amplitude with an angular distance $|\phi - \phi'|$, it has importance only in a limited angular region around the source column where the harmonic Green's function can be evaluated by a planar approximation avoiding a rather complicated procedure to find the pole locations in the complex ν -plane. Therefore, in the following discussion on the Watson transform method, the Green's function G_n is limited to the fundamental Green's function G_0 that is also the most important one in determining an electric Hertzian potential Π_z^e .

The poles of G_0 which originate from the zeroes of $H_0^{(1)}(\sqrt{k_0^2 - \beta_0^2}a)$ and make trajectories on the complex ν -plane as $\mathcal{E}$ is increased from $\mathcal{E}_0$, are denoted with ν_m 's where ν_1 is the lowest pole on the upper half-plane and, with m , the distance of a pole from the real axis is increasing. Once $\mathcal{E}$ is deviated from $\mathcal{E}_0$ by an infinitesimal amount, in addition to the small shifts of ν_m 's from their original locations, a new set of poles are emerging on the complex ν -plane from the factor R in the denominator of Eq. (2), because the cancellation of R 's in the numerator and in the denominator of G_0 when $\mathcal{E} = \mathcal{E}_0$ ($k = k_0$) does not hold any more. From Eqs. (2a), (2b) and (2e), when $\mathcal{E}/\mathcal{E}_0$ is close to one, $T(\mathcal{E}/\mathcal{E}_0) \approx 0$ and $X(\mathcal{E}/\mathcal{E}_0) \approx -4i / \{ \sqrt{k_0^2 - \beta_0^2} H_0^{(1)}(\sqrt{k_0^2 - \beta_0^2}a) \}$

as shown in the preceding section. Therefore, the factor R is vanishing at the zeroes of the derivative of a Hankel function $H'_y(\sqrt{k_0^2 - \beta_0^2}a)$ which are also well known as the zeroes of the Hankel function itself, $H'_y(\sqrt{k_0^2 - \beta_0^2}a)$ (Ref.15). The emergence of these new poles, which are denoted by ν'_m 's hereafter, can be interpreted as the result of mode couplings between TM and TE modes because, when $\xi = \xi_0$, E- and H-fields can be superposed only with the TM modes generated from an electric Hertzian potential π_z^e while, for $\xi \neq \xi_0$, TE modes from a magnetic Hertzian potential π_z^m must be included to satisfy boundary conditions at a dielectric surface. Since the two Hertzian potentials are coupled, they have a common denominator which includes the new poles ν'_m 's in addition to the ν_m 's when ξ is increased from ξ_0 . However, even though the dielectric constant of the layer ξ is not equal to ξ_0 , there is an important exception in which case G_0 has only one set of poles, ν_m 's and the TM modes generated from a single scalar function π_z^e are sufficient to expand all field components that satisfy the boundary conditions at the intersection of the dielectric and the free space. This is the case when $\beta_0 (=k_0 \sin \theta)$ is equal to zero. In other words, the beam direction of the dipole column is horizontal. In this exceptional case, as Eq.(2) shows, the cancellation of R 's in the numerator and in the denominator of G_0 is always possible, thereby suppressing the emergence of another set of

poles from the factor R . As a result, for a fixed value of ξ which is not equal to ξ_0 , new poles can also be generated when the beam of the dipole column is lifted away from the horizontal plane to the axial direction. The full physical implications caused by the emerging new poles when θ is increased from zero will be discussed later.

The original locations of $\mathcal{V}_m$'s and $\mathcal{V}_m'$'s on the complex $\mathcal{V}$ -plane for $\xi = \xi_0$ are such that they are placed almost on a straight line that cuts the real axis around $\sqrt{\epsilon_0^2 - \beta_0^2} a$ and, starting from $\mathcal{V}_1'$ as the lowest pole, the two set of poles are alternating along the line as m is increased. The pole transitions on the complex $\mathcal{V}$ -plane for a fundamental Green's function G_0 when ξ is increased from ξ_0 to $2.5\xi_0$ are illustrated in Fig.(3-2). The elevation beam angle $\theta = 30^\circ$ and $\sqrt{\epsilon_0^2 - \beta_0^2} a = 120.28$. Both set of poles except the lowest one $\mathcal{V}_1'$, have small shifts from their initial locations and their trajectories can be easily traced. The general trend in these pole transitions is downward shifts toward the real axis, which implies a slower wave decay along the cylinder circumference as the function of an angular distance from the source $|\phi - \phi'|$ in the presence of a dielectric layer. The lowest pole $\mathcal{V}_1'$ undergoes a large amount of shift on the complex $\mathcal{V}$ -plane increasing its real part while decreasing its imaginary part. The downward shift of $\mathcal{V}_1'$ to the real axis is so rapid that, for a small variation of ξ from ξ_0 , its location is virtually on the real axis.

This lowest pole is the most important one, since the wave component arising from the residue of this pole does not decay as it propagates along the cylinder circumference due to the absence of the imaginary part in ν'_1 . Because of its large transition on the complex ν -plane, it consumes a considerable computing time to trace ν'_1 . Therefore, it is desirable to locate an approximate position for this pole on the complex plane, from which the exact one can be reached within a few iterative procedures. This can be done using a planar approximation. The approximate location for ν'_1 is predicted from a real pole of a Green's function in a planar approximation which is in a more simple form than G_0 and whose pole location on the real axis can be found easily. When the radius of the cylinder is large, the correspondence between these two poles is obvious from the fact that, for a large and real order ν such as ν'_1 , the Hankel functions in G_0 can be represented with the approximation formulas given by Eq. (36) in Chapter 2 and, once the Hankel functions are replaced with these formulas, G_0 has the same denominator as the fundamental Green's function in a planar approximation. The planar Green's function will be further discussed in connection with the evaluation of G_0 in the lit region of the source column.

For a harmonic Green's function G_n with $|n| \geq 1$, the arguments of Hankel functions and their derivatives are mostly imaginary since the absolute value of $\beta_n (= k_0 \sin \theta + \frac{2\pi n}{2d})$

is larger than $\frac{1}{2}$ when $0.5\lambda_0 \leq 2d \leq \lambda_0$ and $\xi_0 \leq \xi \leq 3\xi_0$. These are the ranges of $2d$ and ξ considered in this chapter. In such cases, as discussed in Chapter 2, the initial pole locations for $\xi = \xi_0$ are on the imaginary axis and, starting with ν'_1 as the lowest pole, ν_m 's and ν'_m 's which are zeros of $H'_m(\sqrt{k_0^2 - \beta_m^2}a)$ and $H''_m(\sqrt{k_0^2 - \beta_m^2}a)$ respectively are alternately taking place on the imaginary axis to the upward direction when m is increased. For most harmonics, as ξ is varied from ξ_0 to its final value, the arguments of Hankel functions and their derivatives remain imaginary and, no significant changes in the pole locations on the imaginary axis can be perceived. Since even the lowest pole ν'_1 has a large imaginary value roughly equal to $\sqrt{k_0^2 - \beta_1^2}a$, the waves arising from the residues of these poles undergo rapid decays along the circumference of the cylinder as the angular factor of the Watson transform, $\exp[i\nu|\phi - \phi|]$ indicates, when ν is replaced with any of these imaginary poles. Therefore, as mentioned earlier, the importance of these harmonics is limited to the narrow angular sector in the neighborhood of the source column where $|\phi - \phi'|$ is small and, in this angular region, G_n 's can be evaluated with planar approximations. For the first harmonic G_{-1} , however, it is quite possible that all the arguments of Hankel functions and their derivatives are real, which is the case when the elevation beam angle is large enough to make the absolute value of $\beta_{-1} (= k_0 \sin \theta - \frac{2\pi}{2d})$ be less than $\frac{1}{2}k_0$. In such a case, the evaluation of G_{-1} can be carried

out in the same way as for the fundamental Green's function G_0 . The only difference here is that G_{-1} has two sets of poles $\mathcal{V}_m$'s and $\mathcal{V}_m'$'s for $\mathcal{E} \neq \mathcal{E}_0$, even when the elevation angle θ is equal to zero. The factor R 's in the numerator and the denominator can not be cancelled in this case because $\beta_{-1} \neq 0$. For the first harmonic G_{-1} , it can also happen that, as $\mathcal{E}(\rho)$ is increased from $\mathcal{E}_0(\rho)$, some of the arguments of Hankel functions and their derivatives, i.e. $\sqrt{\rho^2 - \beta_1^2}a$ and $\sqrt{\rho^2 - \beta_1^2}\rho$, which are initially imaginary can be changed to real values. In such an instance, the lowest pole $\mathcal{V}_1'$ on the imaginary axis is rapidly shifted towards the real axis. For higher poles, no significant changes are noticed at this time. The amount of the downward shift for $\mathcal{V}_1'$ depends on the magnitudes of other fixed arguments of Hankel functions and their derivatives when $\mathcal{E}$ is varied, i.e. $\sqrt{\rho_0^2 - \beta_1^2}a$ and $\sqrt{\rho_0^2 - \beta_1^2}\rho$. If these fixed arguments are large imaginary numbers, the impact of the variable arguments, $\sqrt{\rho^2 - \beta_1^2}a$ and $\sqrt{\rho^2 - \beta_1^2}\rho$ on the lowest pole $\mathcal{V}_1'$ is slight and, the downward shift of $\mathcal{V}_1'$ can be suppressed. However, when the fixed arguments are small imaginary numbers, the decay of $\mathcal{V}_1'$ towards the real axis is rapid. Once $\mathcal{V}_1'$ reaches the real axis, for a further increase in $\mathcal{E}$, it is moving away from the origin on the positive real axis. As the lowest pole $\mathcal{V}_1'$ of the fundamental Green's function G_0 , the behavior of this most important pole for G_{-1} can be completely predicted by investigating the planar first harmonic Green's

function.

Once the locations of poles ν_m 's and ν_m' 's on the complex ν -plane are found, the residue series for G_n can be evaluated.

$$G_n(\rho, \phi; \rho', \phi') = \frac{-i(2\pi)^{-1}}{\omega \epsilon \sqrt{k^2 - \beta_n^2} \rho} \sum_{\nu_m, \nu_m'} \frac{\cos[\nu(\phi - \phi' - \pi)]}{\sin \nu \pi} \text{Res}(\nu)$$

where

$$\text{Res}(\nu) = \frac{Z(\rho) R}{\frac{\partial}{\partial \nu} \left[X\left(\frac{\rho}{\epsilon}\right) H'_\nu(\sqrt{k^2 - \beta_n^2} a) - T\left(\frac{\rho}{\epsilon}\right) H''_\nu(\sqrt{k^2 - \beta_n^2} a) \right] R - \left(\frac{\nu \beta_n}{k \sqrt{k^2 - \beta_n^2} \rho} \right) \left(\frac{k^2 - \beta_n^2}{\epsilon^2 - \beta_n^2} \right) Z(\rho) W]} \quad (3)$$

X, T, Z, W and R are the same functions as defined in Eqs. (2a) to (2e) when m , the integer order of the Hankel functions is replaced with a complex order ν . The evaluation of various Hankel functions and their derivatives with respect to the orders as well as to the arguments in Eq. (3) can be carried out using the approximation formulas given in Appendix 2.

The variation of ϵ from ϵ_0 to the own value of the dielectric layer in the perturbation method described above is purely an expediency for an efficient discovery of poles on the complex ν -plane. What actually varied in the operation of an array is the elevation beam angle θ , when the dielectric constant of the layer ϵ is fixed. Since the characteristics of an array are determined by the pole locations on the complex ν -plane, it is interesting to see the pole transitions as a function of the elevation beam angle θ . Not only it helps

to expose the elevation scan characteristics of an array, the pole tracing on the complex $\mathcal{V}$ -plane by a gradual increase of the elevation angle Θ is actually another way to find poles on the plane, which supplements the ξ -perturbation. If it is necessary to find the poles for various elevation angles with close intervals, one of the efficient ways to discover them is to apply first the ξ -perturbation to locate the poles for a given elevation angle and, from these locations, using a Θ -perturbation, to trace the poles for all other elevation angles.

With a fixed value of ξ as $2.5\xi_0$, the pole transitions for a fundamental Green's function G_0 when the elevation beam angle Θ is increased from zero to thirty degrees are plotted in Fig.(3-3). The radius of the cylinder $a = 138.89\lambda_0/(2\pi)$. The array is the same one whose poles, for the fixed angle $\Theta=30^\circ$, are traced in Fig.(3-2) when ξ is varied from ξ_0 to $2.5\xi_0$. As discussed earlier, when Θ is equal to zero ($\beta_0 = 0$), due to the cancellation of R 's in the numerator and the denominator of G_0 , the only poles are $\mathcal{V}_m$'s, the zeros of the factor $\{X(\xi_0/\xi)H_0^1(\sqrt{\xi^2-\beta_0^2}a) - T(\xi_0/\xi)H_0^2(\sqrt{\xi^2-\beta_0^2}a)\}$ which can be traced from the zeros of $H_0^1(\sqrt{\xi^2-\beta_0^2}a)$ on the complex $\mathcal{V}$ -plane using the ξ -perturbation. As Θ is increased from zero by an infinitesimal amount, the other set of poles $\mathcal{V}_m'$'s are emerging from zeros of R , because the factor R can not be eliminated due to a non-zero value of β_0 . The emergence of these new poles does not incur a discontinuity in the value

of G_0 in Eq.(3) because of the presence of the same factor R in the numerator of G_0 . Therefore, the residues arising from γ'_m 's are gradually increased from zero as γ'_m 's are shifting away from the zeros of R on the complex γ -plane, when the beam of the array is lifted away from a horizontal plane. The most important pole in determining the array characteristic is the real pole γ'_1 . For all other poles, because of their imaginary values, the angular factor in Eq.(3) is decaying along the cylinder circumference as Eq.(11) of Chapt. 2 indicates. From the real pole, however, a standing wave is generated inside the dielectric layer. The intensity of the standing wave is getting stronger due to a growth in the residue of this pole as the elevation angle θ is increasing from zero. Because the amplitude of the standing wave does not decrease along the cylinder circumference, in an actual array environment where N number of dipole columns are mounted circularly symmetrically on a dielectric layer, mutual couplings between dipole columns can be strong. For a certain phase sequence excitation to the dipole columns, the effect of mutual couplings can be extremely severe because of an interaction between the phase sequence and the standing wave. In such a circumstance, an in-phase field build-up condition at an array surface is satisfied and, for a given amount of dipole current, the field intensity at the surface is very strong. In other words, by an interference between the phase sequence and the standing wave, a resonance inside the dielec-

tric layer can be launched. At the resonance, the active input impedance of a dipole is maximum to prevent the intense field build-up at the surface for the self-consistency of a dipole current. The phase sequence at the launch of a resonance is determined by the magnitude of the real pole $\mathcal{V}'_1$. A full discussion on the resonance will be given in the following section.

As for the first harmonic Green's function G_{-1} , a similar change in the location of the lowest pole $\mathcal{V}'_1$ as encountered in an $\mathcal{E}$ -perturbation discussed earlier can happen, when θ is increased from zero. It is desirable to prevent a large amount of downward shift of $\mathcal{V}'_1$ in the operating range of the elevation scan, because it causes a rapid variation in the active dipole input impedance as the beam is scanned away from the horizontal plane. Since dipole feedlines are matched to the active dipole input impedance when the radiation is broadside, in such an instance, a severe impedance mismatch can be incurred even for a small increase in θ from zero. One of the ways to suppress this wide ranging pole transition is to make $2d$, the z -direction dipole center-to-center distance be close to $\lambda_0/2$, then the change in $\beta_{-1} (= k_0 \sin \theta - \frac{2\pi}{2d})$ is less sensitive to the increase in θ and, as a result, the downward shift of $\mathcal{V}'_1$ is slowed. When either the elevation scan angle θ is large or the z -direction dipole center-to-center distance is close to λ_0 , $\mathcal{V}'_1$ of G_{-1} can be dropped down to the real axis and be purely real. In such a

case, in addition to the resonance launched by ν'_1 of G_0 , another resonance caused by ν'_1 of G_{-1} is taking place inside the dielectric layer. Because of the interactions between these two real poles from G_0 and G_{-1} , the resonance phenomena are quite complicated compared to a single resonance.

As the above discussion shows, the resonances inside the dielectric layer are the products of real poles which, in turn, come into existence only when the beam is scanned away from the horizontal plane. When the beam direction is horizontal, there are no resonances due to the absence of real poles. Therefore, as will be illustrated later, an elevation pattern cut at a constant angle θ which is not equal to zero shows significant differences from the pattern on the horizontal plane.

The region of convergence for a residue series in Eq.(3) is similar to that of Eq.(9) in Chapter 2. The dominant term which determines the width of the angular region where the residue series can be applied to evaluate a Green's function $G_n(\rho, \phi; \rho, \phi')$ is the angular factor, $\cos[(\phi - \phi' - \pi)] / \sin \nu \pi$. The remaining term $\text{Res}(\nu)$ does not have much impact on the convergence of the series as far as the source and the field points are close to the cylinder surface, because the structure of the term is such that, with the exception of the ratio $H'_1(\sqrt{k^2 - \nu^2} \rho) / H'_1(\sqrt{k^2 - \nu^2} \rho')$, the first and the second kind of Hankel functions, or their derivatives appear in pairs. Since the magnitudes of their arguments are nearly

same when the source and the field points are close to the cylinder, for such pairs, the net effects are the cancellations in their exponential terms. The exponential cancellation between the first and the second Hankel function with the same large argument can be shown from Eqs.(15) and (20) of Chapter 2. Using an asymptotic expansion of a Bessel function with a large argument, Eq.(15) of Chapt. 2 can be further simplified as

$$H_y^2(z) = H_y^2(\nu \operatorname{sech} \alpha) \approx 2i \left(\frac{\alpha - \tanh \alpha}{3 \tanh \alpha} \right)^{\frac{1}{2}} \sqrt{\frac{3}{2\pi R}} e^{i(R - \frac{\pi}{4})} \quad (4)$$

where $\alpha = \cosh^{-1}(\nu/z)$ and $R = -i\nu \cdot (\alpha - \tanh \alpha)$. For the same

$$\arg H_y^1(z) = H_y^1(\nu \operatorname{sech} \alpha) \approx -2i \left(\frac{\alpha - \tanh \alpha}{3 \tanh \alpha} \right)^{\frac{1}{2}} \sqrt{\frac{3}{2\pi R}} \left[e^{i(R - \frac{\pi}{4})} + e^{i(R - \frac{\pi}{4})} \right] \quad (5)$$

$$H_y^1(z) = H_y^1(\nu \operatorname{sech} \alpha) \approx -2i \left(\frac{\alpha - \tanh \alpha}{3 \tanh \alpha} \right)^{\frac{1}{2}} \sqrt{\frac{3}{2\pi R}} \left[e^{i(R - \frac{\pi}{4})} + e^{i(R - \frac{\pi}{4})} \right] \quad (5)$$

For a higher pole ν_m or ν'_m with a large value of m , as Eq. (26) in Chapter 2 indicates, the phase of ν_m or ν'_m is close to $i\pi/2$ for $\xi = \xi_0$ and remains the same in spite of the pole transitions when ξ is varied from ξ_0 . For such a pole, Fig. (2-3) shows that R is on the upper half-plane and, as a result, the product of $H_y^1(z)$ and $H_y^2(z)$ results in the cancellation of their exponential terms since the first exponential in Eq.(5) can be ignored. The exponential cancellation still holds when the product is between $H_y^1(z)$ and $H_y^2(z)$ or vice versa, because a differentiation does not change the exponen-

tial term of a Hankel function. Since there arise no significant exponential terms from $\text{Res}(\gamma_m)$ when a/ρ' is close to one and m is large, the region of convergence for $G_n(\rho, \phi; \rho', \phi')$ is mainly determined by the angular factor, $\cos[\gamma_m(|\phi - \phi'| - \pi)] / \sin \gamma_m \pi$. As a result, if many residue terms are included, the series converges in the whole angular region except at $\phi = \phi'$ as in Chapter 2. However, to exploit the advantage of a residue series over a harmonic series, it is necessary to exclude the direct neighborhood of the source column where $|\phi - \phi'|$ is small in order to provide a fast decrease in the angular factor as m is increasing. Then, outside the excluded region, G_n can be evaluated with a few number of residue terms.

To cover the excluded region of a residue series in the neighborhood of the source column, the curved surfaces of the cylinder and the dielectric layer are replaced with planar surfaces while maintaining the thickness of the layer to be the same as in the original problem and, in this simplified circumstance, a Green's function G_n is approximated. Fig.(3-4) illustrates the geometry in a planar approximation, where $x=0$ plane represents the interface between the dielectric and the free space and $x=-(\rho' - a)$ plane the conducting surface. The coordinate of the source is the origin. A planar Green's function $G_{np}(\rho', \phi; \rho', \phi')$ at a field point on the surface of the dielectric layer which is located at the dis-

tance of $\rho|\phi-\phi'|$ from the source is represented with an infinite integral.

$$G_{np}(\rho', \phi : \rho, \phi') = -\frac{(2\pi)^2}{\omega \xi} \int_{-\infty}^{\infty} \frac{(1-V) R_p e^{i\lambda|\phi-\phi'| \rho' k_y} d k_y}{\left[U(\xi_0/\xi) + VU(-\xi_0/\xi) \right] R_p + \left(\frac{k_y \beta_n}{-k} \right) \left(\frac{k^2 - k_0^2}{k_0^2 - \beta_n^2} \right) (1-V^2)} \quad (6)$$

where

$$U(s) = \sqrt{k^2 - \beta_n^2 - k_y^2} + S \frac{k^2 - \beta_n^2}{k_0^2 - \beta_n^2} \sqrt{k_0^2 - \beta_n^2 - k_y^2} \quad (6a)$$

$$V = e^{i2\sqrt{k^2 - \beta_n^2 - k_y^2}(\rho' - a)} \quad (6b)$$

$$R_p = U(1) - V \cdot U(-1) \quad (6c)$$

It is interesting to compare Eq. (6) to a contour integral representation of $G_n(\rho', \phi : \rho, \phi')$ in Eq. (2). The contour used is C_1 in Fig. (2-2). This representation is discussed as an intermediate step in the derivation of a residue series in Appendix 1. From Eq. (2) and, Eq. (3) of Appendix 1,

$$G_n(\rho', \phi : \rho, \phi') = \frac{-(2\pi)^2}{\omega \xi \sqrt{k^2 - \beta_n^2} \rho} \int_{C_1} \frac{Z(\rho') R [\cos \nu(\phi - \phi' - \pi) / \sin \nu \pi] d\nu}{\left\{ X\left(\frac{\xi}{\xi}\right) H_\nu^1(\sqrt{k^2 - \beta_n^2} a) - T\left(\frac{\xi}{\xi}\right) H_\nu^2(\sqrt{k^2 - \beta_n^2} a) \right\} R \left(\frac{\gamma \beta_n}{2\sqrt{k^2 - \beta_n^2} \rho} \right)^2 \left(\frac{k^2 - k_0^2}{k_0^2 - \beta_n^2} \right) Z(\rho') W} \quad (7)$$

X, T, Z, W and R are the same functions as in Eqs. (2a) to (2e) when the real order m is replaced with a complex order ν .

As discussed already, in addition to the integer poles arising from the angular factor, there exists another real pole $\mathcal{V}'_1$ in the integrand of Eq. (7) whose location on the real axis is between $\sqrt{\beta_n^2 - \beta_n^2} a$ and $\sqrt{\beta_n^2 - \beta_n^2} a$. Unless the order of Hankel functions is close to any of these limits, it is justified to use the approximation formulas of Eq. (36) in Chapter 2 to evaluate the various Hankel functions and their derivatives in Eq. (7), because, for these values of $\mathcal{V}$, the relationship between the order and the argument of a Hankel function for the application of these formulas can be satisfied. Once the approximation formulas are applied to the integrand of Eq. (7) and the relation between $\mathcal{V}$ and k_y , $\mathcal{V}/\rho' = k_y$ is used, except the angular factor, it is reduced to a nearly same function as the integrand of Eq. (6) for large values of a and ρ' . The reason for the difference in the angular factors of Eqs. (6) and (7) will be discussed later. Since the integrands in both equations are approximately same except for the angular factors, the integrand of Eq. (6) also has a real pole k_{yp} as expected, whose relation with the corresponding pole $\mathcal{V}'_1$ is approximately given as $\mathcal{V}/\rho' \approx k_{yp}$. Actually, $-\mathcal{V}'_1$ and $-k_{yp}$ are also poles on the real axis due to the symmetry of an integrand with respect to the origin on a complex plane. When both $\sqrt{\beta_n^2 - \beta_n^2} a$ and $\sqrt{\beta_n^2 - \beta_n^2} a$ are imaginary, which is the cases for most of harmonics, the poles, $\mathcal{V}'_1$ and k_{yp} are found on the imaginary axis between the two arguments. If $\sqrt{\beta_n^2 - \beta_n^2} a$ is imaginary and $\sqrt{\beta_n^2 - \beta_n^2} a$ is real, which can occur for the first harmonic Green's func-

tion G_{-1} , both $\mathcal{V}'_1$ and k_{yp} can be found either on the imaginary or on the real axis between those two arguments. Therefore, for the fundamental as well as for a harmonic Green's function, the location of the lowest pole $\mathcal{V}'_1$ which is also the most important one in determining the function can be predicted by the pole k_{yp} from the planar approximation, the location of which can be found easily from the denominator of the integrand in Eq.(6). This has been capitalized in an ξ -perturbation where, because of the large transition of the lowest pole $\mathcal{V}'_1$, it is more efficient to trace it from the approximate location $\beta'k_{yp}$ instead of a direct application of the method. In the same way as in Eq.(7), when β_0 is equal to zero, the factor R_p 's both in the numerator and in the denominator of the integrand in Eq.(6) is cancelled to result in the elimination of the real pole k_{yp} arising from the factor R_p .

When the poles k_{yp} and $-k_{yp}$ are on the real axis, the contour for the integration in Eq.(6) must be defined carefully to avoid these poles. Otherwise, the integration is mathematically indeterminate. A contour that gives a physically acceptable result is the one which is detoured above the negative pole $-k_{yp}$ and under the positive pole k_{yp} . The contour as well as the various branch cuts on a complex k_y -plane caused by the square roots in the integrand of Eq.(6) is shown in Fig.(3-5). The choice of the contour can be justi-

fied from the following consideration. If the ϵ of the dielectric layer is assumed to be slightly changed to cause the layer to be lossy, the resultant variation in the pole location is an upward shift for k_{yp} and a downward shift for $-k_{yp}$, which places both poles slightly off the real axis of the complex k_y -plane. These pole shifts make the residue term arising from the positive pole k_{yp} be infinitesimally decreasing along the surface of the dielectric layer as the distance from the source $|\phi - \phi|_p$ is increased in Eq.(6), which is consistent for a wave propagation in a lossy media. As ϵ of the dielectric layer is varied by a small amount to make the layer to be lossy, the Green's function G_{np} in Eq.(6) must be continuous. If the contour in Fig.(3-5) is drawn otherwise, a discontinuity in the Green's function arises when the poles are crossing over the contour due to a small variation of ϵ to make the layer to be slightly lossy. The various branch cuts in Fig.(3-5) are chosen to make the square roots $\sqrt{k_0^2 - \beta_n^2 - k_y^2}$ and $\sqrt{k^2 - \beta_n^2 - k_y^2}$ be $i\sqrt{\beta_n^2 + k_y^2 - k_0^2}$ and $i\sqrt{\beta_n^2 + k_y^2 - k^2}$ when the insides of the square roots are negative, which is also consistent for a wave propagation if the variation in time is taken as $\exp(-i\omega t)$.

The actual evaluation of the integral in Eq.(6) can be carried out using various approximations. First, for a higher harmonic planar Green's function G_{np} with a large value of $|n|$, $|\beta_n|$ is large compared to k_0 or k . Consequently, the exponential V in Eq.(6b) is small according to the branch-

cuts chosen above and, can be neglected. Also, ignoring the term which is proportional to β_n^2 in the denominator, the planar Green's function G_{np} with a large $|n|$ can be approximated as

$$G_{np}(\rho', \phi; \rho, \phi) \approx -\frac{1}{\omega \xi (2\pi)^2} \int_{-\infty}^{\infty} \frac{e^{\lambda|\phi - \phi'| \rho' k_y} d k_y}{\sqrt{\rho_k^2 - \rho_n^2 k_y^2} + \frac{\xi_0}{\xi} \frac{\rho_k^2 - \rho_n^2}{k_0^2 - \beta_n^2} \sqrt{\rho_k^2 - \beta_n^2 k_y^2}} \quad (8)$$

Since the square roots in the denominator are almost same when $|\beta_n| \gg k_0$ or k , Eq. (8) can be further simplified as

$$\begin{aligned} G_{np}(\rho', \phi; \rho, \phi) &\approx -\frac{1}{\omega \xi (2\pi)^2 \left[1 + \frac{\xi_0}{\xi} \frac{\rho_k^2 - \beta_n^2}{k_0^2 - \beta_n^2}\right]} \int_{-\infty}^{\infty} \frac{e^{\lambda|\phi - \phi'| \rho' k_y} d k_y}{\sqrt{\rho_k^2 - \beta_n^2 k_y^2}} \\ &\approx \frac{2\lambda K_0(\sqrt{\beta_n^2 - k_0^2} |\phi - \phi'| \rho')}{\omega \xi (2\pi)^2 \left[1 + \frac{\xi_0}{\xi} \frac{\rho_k^2 - \beta_n^2}{k_0^2 - \beta_n^2}\right]} \quad (9) \end{aligned}$$

where K_0 is a modified Bessel function of zero order.

For the fundamental and lower harmonic planar Green's functions, it is more convenient to represent G_{np} of Eq. (6) in the following form.

$$G_{np}(\rho', \phi; \rho, \phi) = -\frac{2(2\pi)^{-2}}{\omega \xi} \int_0^{\infty} \frac{(1-V) R_p \cos(|\phi - \phi'| \rho' k_y) d k_y}{\left[U\left(\frac{\xi_0}{\xi}\right) + VU\left(-\frac{\xi_0}{\xi}\right)\right] R_p + \left(\frac{\rho_k^2 - \beta_n^2}{k_0^2 - \beta_n^2}\right)^2 (1-V)} \quad (10)$$

The total integral interval in Eq. (10) is divided into two.

In the first interval, the integration is carried out numerically. Since the real pole k_{yp} belongs to this interval, a special care must be taken in the neighbourhood of this pole. To have an accurate result, it is better for the contour to pass under the pole maintaining some distance from the pole, since inaccuracy can occur in the immediate proximity to the pole. The second interval extends from a constant k'_y , the end of the first interval to infinity. k'_y is determined such that the exponential V in Eq. (6b) can be ignored for a k_y larger than k'_y in order to cross out V in the entire second interval. However, since a large value of k'_y can cause an excessively long computing time for the numerical integration in the first interval, an adequate adjustment for the magnitude of k'_y is necessary. In the second interval, since k_y is larger than $|\beta_n^2 - k_y^2|$ and $|\beta_0^2 - \beta_n^2|$, the following series expansions for the square roots can be used to approximate the denominator and the numerator of the integrand in Eq. (6).

$$\sqrt{\beta_n^2 - \beta_0^2 - k_y^2} \approx i k_y \left\{ 1 + \frac{\beta_n^2 - k_y^2}{2 k_y^2} - \frac{1}{8} \left(\frac{\beta_n^2 - k_y^2}{k_y^2} \right)^2 + \frac{1}{16} \left(\frac{\beta_n^2 - k_y^2}{k_y^2} \right)^3 + \dots \right\} \quad (11)$$

and it is also same for $\sqrt{\beta_0^2 - \beta_n^2 - k_y^2}$. Ignoring the terms which are proportional to the negative powers of k_y in the denominator and in the numerator, the integral in the second interval I_2 can be simplified as

$$\begin{aligned}
I_2 &\approx i \frac{2(2\pi)^{-2}}{\omega \varepsilon} \int_{k_y'}^{\infty} \frac{k_y \cos(|\phi - \phi'| \rho' k_y)}{A k_y^2 - B} dk_y \\
&\approx i \frac{(2\pi)^{-2}}{\omega \varepsilon A} \left[\int_{k_y'}^{\infty} \frac{\cos(|\phi - \phi'| \rho' k_y)}{k_y + \sqrt{\frac{B}{A}}} dk_y + \int_{k_y'}^{\infty} \frac{\cos(|\phi - \phi'| \rho' k_y)}{k_y + \sqrt{\frac{B}{A}}} dk_y \right]
\end{aligned}
\tag{12}$$

where

$$\begin{aligned}
A &= \left(1 + \frac{\varepsilon_0}{\varepsilon} \frac{-k^2 - \beta_n^2}{-k_0^2 - \beta_n^2}\right) + \left(\frac{\beta_n^2}{k}\right) \left(\frac{k^2 - k_0^2}{-k_0^2 - \beta_n^2}\right)^2 \frac{\beta_n^2 - k_0^2}{k^2 + k_0^2 - 2\beta_n^2} \\
B &= \frac{\beta_n^2 - k^2}{2(-k^2 + k_0^2 - 2\beta_n^2)} \left\{ (\beta_n^2 - k^2) \left(1 + \frac{3\varepsilon_0}{\varepsilon}\right) + (\beta_n^2 - k_0^2) \left(3 + \frac{\varepsilon_0}{\varepsilon}\right) \right\}
\end{aligned}$$

The evaluation of the integrals in Eq.(12) can be readily carried out using the asymptotic expansions for the integrals given in Ref.18.

The planar Green's function $G_{np}(\rho', \phi; \rho, \phi)$ which is approximated with the methods described above is not quite close to the original Green's function $G_n(\rho, \phi; \rho', \phi)$ when these two functions are compared at a shadow boundary. The discrepancy is due to the effects of rotating waves around the cylinder, which can not be accounted for by a planar approximation. These rotating waves are also responsible for the difference in the angular factors of G_{np} and G_n , which is mentioned earlier. The effects of rotating waves arising from all other poles except γ_1' can be ignored because of the severe decays during the waves are circling the cylinder. What

is actually accounted for by the approximation for the phase factor in Eq. (11) of Chapter 2 is a single direct wave from the source to a field point as the path-length of the term $-i \exp(i\gamma_m |\phi - \phi'|)$ indicates. All the rotating wave terms in the phase factor that have longer path-lengths are neglected, which can be justified when the poles have significant imaginary parts as in Chapter 2. However, since the wave arising from the pole γ'_1 encircles the cylinder unabated using the dielectric layer as a propagation media, the effects of the rotating waves from this pole can not be ignored. In other words, the approximation of G_n with G_{np} is not complete unless the effects of these rotating waves are fully taken into account, when both G_n and G_{np} have poles on the real axis. The additional term to be included can be found by decomposing the angular factor for the real pole γ'_1 in the following form.

$$\frac{\cos \gamma'_1 (|\phi - \phi'| - \pi)}{\sin \gamma'_1 \pi} = -i \left[\frac{e^{i\gamma'_1 |\phi - \phi'|}}{1 - e^{i2\pi\gamma'_1}} + \frac{e^{i\gamma'_1 (2\pi - |\phi - \phi'|)}}{1 - e^{i2\pi\gamma'_1}} \right] \quad (13)$$

When the dielectric layer is assumed to be slightly lossy such that γ'_1 is located right above the real axis, the right-hand side of Eq. (13) can be expanded as

$$\frac{\cos \gamma'_1 (|\phi - \phi'| - \pi)}{\sin \gamma'_1 \pi} = -i \left[e^{i\gamma'_1 |\phi - \phi'|} \sum_{m=0}^{\infty} e^{i\gamma'_1 (2\pi m)} + e^{i\gamma'_1 (2\pi - |\phi - \phi'|)} \sum_{m=0}^{\infty} e^{i\gamma'_1 (2\pi m)} \right] \quad (14)$$

As the path-length in the above equation indicate, the first term represents a direct wave from the source to a field point and other infinite number of rotating waves around the cylinder in the same direction. The second term accounts for the wave which reaches the field point from the opposite direction around the cylinder and all the other waves encircling the cylinder in that direction. Because only the direct wave term, $-i\exp(i\gamma_1'|\phi-\phi'|)$ in Eq. (14) is taken care of by the planar approximation, the additional term to be included which fully accounts for the rotating waves is the product between the summation of the remaining terms in Eq. (14) and the residue term arising from γ_1' in Eq. (3).

Therefore, a Green's function G_n in the neighborhood of the source column can be approximated with the following equation.

$$G_n(\rho', \phi; \rho, \phi') \approx G_{np}(\rho', \phi; \rho, \phi') + \frac{2(2\pi)^{-1}}{\omega \xi \sqrt{\epsilon^2 - \beta_n^2} \rho} \frac{\cos \gamma_1' |\phi - \phi'|}{1 - e^{-i2\pi \gamma_1'}} \text{Res}(\gamma_1') \quad (15)$$

At the shadow boundaries, a Green's function G_n approximated with the above equation has a good agreement with the result from the residue series calculation in Eq. (3). For a smooth transition of G_n at the shadow boundaries, 8 residue terms each for γ_m 's and γ_m' 's are included in the series.

3.4 Active dipole input impedance.

Superposing the fundamental and the various spatial harmonic Green's functions, the electric Hertzian potential $\Pi_z^e(\rho, \phi, z; \rho', \phi')$ caused by a source column at (ρ', ϕ') can be found from Eq. (4) of Chapter 2. The calculation of Π_z^e is carried out in the same way as in Chapter 2. The number of harmonic terms which are required in the series for $\Pi_z^e(\rho, \phi, z; \rho', \phi')$ depends on the angular distance between the source and the field point, $|\phi - \phi'|$. As in Chapter 2, in the shadow region of the cylinder caused by the source column, Π_z^e can be approximated with the fundamental or, the fundamental and the first harmonic Green's function when the lowest pole of G_{-1} is close to or on the real axis of a complex ν -plane. Higher harmonics can be neglected because of the severe wave decays along the cylinder circumference as the large imaginary poles of these functions indicate. In the lit region of the source, four or five harmonic terms are enough for Π_z^e when the distance between the source and the field point is not too close, since, as we can see in Eq. (9), the modified Bessel function K_0 is decreasing exponentially when the argument of the function is increased with $|n|$ for a given value of the distance between the source and the field point, $|\phi - \phi'| \rho'$. In the special case when the electric Hertzian potential is calculated at the surface of the source column itself, the distance $|\phi - \phi'| \rho'$ is small and, a large number of harmonic terms must be inclu-

ded for the convergence of the series. Π_z^e in this case can be approximated with an electric Hertzian potential at a distance of r_0 , the radius of a dipole from an equivalent line source which replaces the surface current of a thick dipole for the simplification of the calculation. When r_0 is equal to $0.007\lambda_0$, about sixty harmonic terms are needed in the series for Π_z^e . Since a closed form expression is available in Eq.(9), the calculations of these large number of harmonics do not pose serious computational problem.

In an actual array environment where N number of dipole columns are circular symmetrically mounted on the dielectric layer, the total electric Hertzian potential $\Pi_z^{et}(\rho, \phi, z; p)$ at a dipole surface of a reference column for a p-th phase sequence excitation can be found from Eq.(6) of Chapter 2. In this calculation, the field point of an individual electric Hertzian potential $\Pi_z^e(\rho, \phi, z; \rho', \phi')$ is a point at a dipole surface of the reference column at $\rho = \rho'$ and $\phi = 0$ while the source is a dipole column whose azimuthal coordinate ϕ' is given as $\frac{2\pi}{N}l$ where l is an integer between 0 and N-1 inclusively. Therefore, the superposition for Π_z^{et} is with respect to l when it is varied from 0 to N-1.

As in Chapter 2, by matching this total electric Hertzian potential with another form of Π_z^{et} in Eq.(8) of Chapter 2 which is derived from a tangential E-field boundary condition at a dipole surface, a self-consistent dipole current $I(p)$ is found as a function of the dipole center gap

voltage V . As a result, the active dipole input admittance $Y(p)$ for a p -th phase sequence excitation is known. The computation of $Y(p)$'s when p is varied from 0 to $N-1$ can be carried out in an efficient way using a FFT algorithm as in Chapter 2. Implicitly, $Y(p)$ is also a function of the elevation beam angle θ which is determined by the z -direction voltage phase variation of the N number of phase sequence excitations.

Active dipole input impedances are plotted as a function of the azimuthal scan direction ϕ which depends on the excited phase sequence p as in Eq.(41) of Chapter 2. In the following discussion, the total number of dipole columns N , a dipole length $2h$ and the radius of a dipole r , are fixed as $N=200$, $2h=0.5\lambda_0$ and $r=0.007\lambda_0$, where λ_0 is a wavelength in the free space. Also, the thickness of the dielectric layer is adjusted such that when $\xi=2\xi_0$, $\rho'-a=\lambda/4$ where λ is a wavelength in the dielectric layer. Therefore, in terms of the wavelength in the free space, $\rho'-a=0.1767767\lambda_0$. On the other hand, a distance between dipole columns d_c , a z -direction dipole center-to-center distance $2d$, the elevation beam scan direction θ and the dielectric constant of the layer ξ are taken as variables to investigate the impacts of these parameters upon the characteristics of an array. In Fig.(3-6), $\theta=0^\circ$, $d_c=2d=0.7\lambda_0$ and, the only variable is the dielectric constant of the layer ξ . The figure shows the va-

riations of the impedance curves for $\xi=2\xi_0$, $2.5\xi_0$ and $3\xi_0$. As in chapter 2, a maximum impedance angle which corresponds to the grating lobe angle of a comparable planar array can be found for each curve. In the absence of the dielectric layer ($\xi=\xi_0$), the grating lobe angle of a comparable planar array ($d_c=0.7\lambda_0$ and $\theta=0^\circ$), $\phi_{g,1}=25.377^\circ$ from Eq.(42) of Chapter 2. It is illustrated in the figure that, as ξ is increased, the maximum impedance angle is shifting away from $\phi_{g,1}$ towards the broadside direction ($\phi=0^\circ$). This is expected, because, to the side of the dielectric, the inter-column distance d_c which is fixed as $0.7\lambda_0$ in the free space, is increasing with ξ as $d_c=0.7\sqrt{\xi/\xi_0}\lambda$ when it is measured by the wavelength in the dielectric λ and, for this increased value of d_c , as Eq.(42) of Chapter 2 shows, an in-phase field build-up condition inside the dielectric can be reached at a smaller azimuthal beam scan angle than $\phi_{g,1}$ in the free space. In addition to the shift of a maximum impedance angle towards the broadside direction, Fig.(3-6) also shows that the magnitude of the impedance at this angle is increasing with ξ , which can be explained by a strong mutual coupling effect between dipole columns for an increased value of ξ . As illustrated by the downward shifts of $\mathcal{V}_m$ poles in the complex $\mathcal{V}$ -plane in Fig.(3-2), for a large value of ξ , the waves arising from these poles undergo less decays along the cylinder circumference and, once a field build-up condition is satisfied by the application of an appropriate phase sequence, the field intensity at the array sur-

face is strong. As discussed in detail in Chapter 2, to counter this intense field build-up at the array surface in order to result in a self-consistent dipole current, the magnitude of the impedance at this azimuthal scan angle must be large. The shift of a maximum impedance angle by adjusting an inter-column distance d_c is shown in Fig.(3-7), where $\theta=0^\circ$, $\xi=2.5\xi_0$ and $2d=0.7\lambda_0$. As in chapter 2, a maximum impedance angle is moving towards the broadside direction with an increased separation between dipole columns. The impacts of these active dipole input impedance characteristics on the elementary pattern cuts on the horizontal plane will be discussed in the following section.

When the elevation angle θ is increased from zero, the maximum impedance angle at $\theta=0^\circ$ is shifting away from the broadside direction in the same way as the grating lobe angle $\phi_{g.l.}$ of Eq.(42) in Chapter 2 is varied and, in addition, high impedance peaks which are extremely narrow in ϕ are observed. This is illustrated in Fig.(3-8) where $\xi=2\xi_0$, $d_c=0.7\lambda_0$, $2d=0.55\lambda_0$ and θ is a variable. These impedance peaks which are missing for $\theta=0^\circ$ can be explained from the resonances inside the dielectric layer caused by the emerging real pole $\mathcal{V}'_1$ when θ is increased from zero. To illustrate how a resonance inside the dielectric layer can be launched from an interference between a standing wave arising from a real pole $\mathcal{V}'_1$ and an appropriate phase sequence excitation, $G_n(\rho, \phi; \rho', \phi')$ in Eq.(3) for $n=0$ is decomposed into a standing wave term and

the remaining term which is responsible for all other pole.

$$G_0(\rho, \phi; \rho', \phi') = \frac{-i(2\pi)^{-1}}{\omega \epsilon \sqrt{k^2 - \beta_0^2} \rho'} \left[\frac{\cos \mathcal{V}'_1 (|\phi - \phi'| - \pi)}{\sin \mathcal{V}'_1 \pi} \text{Res}(\mathcal{V}'_1) + \sum'_{\mathcal{V}_n, \mathcal{V}'_n} -i \cdot \exp(i\mathcal{V}|\phi - \phi'|) \cdot \text{Res}(\mathcal{V}) \right] \quad (16)$$

The summation in the above equation includes all other residue terms except the one at $\mathcal{V}'_1$ which is taken out of it and, since all the poles in the summation have significant imaginary parts, the angular factor is approximated according to Eq. (11) of Chapter 2. Substituting G_0 in Eq. (4) of Chapter 2 with the above equation, an electric Hertzian potential due to a dipole column at (ρ', ϕ') is given as

$$\begin{aligned} \Pi_z^e(\rho, \phi, z; \rho', \phi') &= \frac{-i(2\pi)^{-1}}{\omega \epsilon \sqrt{k^2 - \beta_0^2} \rho'} \left[\frac{\cos \mathcal{V}'_1 (|\phi - \phi'| - \pi)}{\sin \mathcal{V}'_1 \pi} \text{Res}(\mathcal{V}'_1) + \right. \\ &\quad \left. \sum'_{\mathcal{V}_n, \mathcal{V}'_n} -i e^{i\mathcal{V}|\phi - \phi'|} \text{Res}(\mathcal{V}) \right] e^{i\beta_0 z} \frac{\pi}{d} \int_{-h}^h I_z(z') e^{-i\beta_0 z'} dz' \\ &\quad + \sum_{n=-\infty}^{\infty} G_n(\rho, \phi; \rho', \phi') e^{i\beta_n z} \frac{\pi}{d} \int_{-h}^h I_z(z') e^{-i\beta_n z'} dz' \quad (17) \end{aligned}$$

The summation with respect to n does not include G_0 which is taken out of it. The total Hertzian potential at a dipole surface of the reference column at $(\rho', 0)$ can be superposed with N number of individual Hertzian potentials which can be obtained from the above equation by substituting ϕ' with

$\frac{2\pi}{N}l$, where the integer l is varied from 0 to $N-1$. $\Pi_q(\rho, \phi, z; \rho')$, $\frac{2\pi}{N}l$ in Eq. (6) of Chapter 2 is found by replacing $I_z(z')$ with a known current term i_q of Eq. (1) in Chapter 2. Therefore, from Eq. (6) of Chapter 2 and the above equation, the total electric Hertzian potential at a dipole surface of the reference column is given as

$$\begin{aligned} \Pi_z^{\text{et}}(\rho', 0, z; p) = & \sum_{q=1}^5 C_q(p) \left[\frac{-i(2\pi)^{-1}}{\omega \epsilon_0 \sqrt{k^2 - \beta_0^2} \rho} \left\{ S_0(\gamma'_1, p) + \sum_{l=0}^{N-1} \sum_{\gamma'_n} -i e^{i \frac{2\pi}{N}(p+\gamma')l} \text{Res}(\gamma) \right\} \right. \\ & \times e^{i\beta_0 z} \frac{\pi}{d} \int_{-h}^h i_q(z') e^{-i\beta_0 z'} dz' \\ & \left. + \sum_{n=-\infty}^{\infty} \left\{ \sum_{l=0}^{N-1} G_n(\rho', 0; \rho', \frac{2\pi}{N}l) e^{i \frac{2\pi}{N}lp} \right\} e^{i\beta_n z} \frac{\pi}{d} \int_{-h}^h i_q(z') e^{-i\beta_n z'} dz' \right] \end{aligned}$$

where

$$S_0(\gamma'_1, p) = \sum_{l=0}^{N-1} \frac{\cos \gamma'_1 (\frac{2\pi}{N}l - \pi)}{\sin \gamma'_1 \pi} \text{Res}(\gamma'_1) e^{i \frac{2\pi}{N}lp} \quad (18)$$

$S_0(\gamma'_1, p)$ is the summation of the standing waves arising from N number of dipole columns at a dipole surface of the reference column, when the dipole columns are activated with a p -th phase sequence excitation. The standing waves are caused by the real pole γ'_1 of G_0 . If the lowest pole γ'_1 of G_{-1} is also real, another standing wave summation $S_{-1}(\gamma'_1, p)$ from this pole can be included in addition to $S_0(\gamma'_1, p)$ in the above equation. To investigate the inter-

ference between the standing waves and the excited phase sequence p , $S_o(\gamma'_1, p)$ of Eq.(18) is given in the following form.

$$S_o(\gamma'_1, p) = \frac{1}{2} \frac{\text{Res}(\gamma'_1)}{\sin \gamma'_1 \pi} \left\{ e^{-j\gamma'_1 \pi} \sum_{l=0}^{N-1} e^{j(\gamma'_1 + p) \frac{2\pi}{N} l} + e^{j\gamma'_1 \pi} \sum_{l=0}^{N-1} e^{j(p - \gamma'_1) \frac{2\pi}{N} l} \right\} \quad (19)$$

If the phase sequence p is close to γ'_1 or $N - \gamma'_1$, one of the summations in the above equation becomes an in-phase superposition and the magnitude of $S_o(\gamma'_1, p)$ is large. For such a value of p , S_o is the dominant term for π_z^{et} in Eq.(18). When π_z^{et} in Eq.(18) is matched to $\pi_z^{\text{et}}(z)$ of Eq.(8) in Chapter 2 at seven sampling points of a reference column dipole surface and the inversion of a resulting 7×7 matrix is carried out to find the unknown constants, $C_q(p)$'s, A and B , the solutions for these constants for a given value of V , the dipole center gap voltage have small magnitudes because of the large determinant of the matrix caused by an increased $S_o(\gamma'_1, p)$ for such a p . The resultant dipole current $I(p)$ in Eq.(1) of Chapter 2 is small and, the active dipole input impedance curve shows an impedance peak at an azimuthal scan angle corresponding to this phase sequence given by Eq.(41) of Chapter 2. In other words, when an appropriate phase sequence is applied to the dipole columns, the path-length difference between various standing waves arising

from N number of dipole columns can be eliminated and, as a result, all the standing waves can be superposed in-phase to build up a strong field intensity at the array surface. As Eq. (19) indicates, the path-length difference between various standing waves can be compensated with the application of an appropriate phase sequence either by directly cancelling the phase difference when p is close to $\mathcal{V}'_1$ or by increasing the total phase difference to 2π when p is close to $N-\mathcal{V}'_1$. In either way, from the interference between the phase sequence and the standing waves, an extremely strong field intensity is produced inside the dielectric layer for a given finite value of a dipole current and, as a result, a resonance is launched at this phase sequence. In order to satisfy an E-field boundary condition at a dipole surface, especially at the center with its given voltage V , a dipole current must be small and an impedance peak is resulted in at the resonance.

The relation between a real pole $\mathcal{V}'_1$ and a resonance angle where an impedance peak occurs in an azimuthal scanning is further clarified in Figs. (3-8) to (3-11). In these figures, an inter-column distance d_c is fixed as $0.7\lambda_0$. Since the total number of dipole columns N is 200, the radius of an array ρ' is given as $\rho' = (0.7 \times 200) / (2\pi) \lambda_0$. As mentioned earlier, $\rho'a = 0.25/\sqrt{2} \lambda_0$, from which a cylinder radius is determined as $a = 22.1049153 \lambda_0$. For these values of ρ' and a , the real pole $\mathcal{V}'_1$ of G_N is calculated from Eq. (2). As discussed in

the previous section, the approximate location of the real pole $\mathcal{V}'_1$ of G_n can be predicted from k_{yp} , the pole of G_{np} in Eq. (6) as $\mathcal{V}'_1 \approx k_{yp} \rho'$, which is utilized to expedite the discovery of the exact location for the real pole. In Table 1, the real pole $\mathcal{V}'_1$ of the fundamental Green's function is tabulated as a function of ξ and θ . The corresponding planar pole k_{yp} of G_{op} and the predicted value for $\mathcal{V}'_1$, $k_{yp} \rho'$ are also shown. It is noticed that the predicted one from the planar approximation is close to the exact value of $\mathcal{V}'_1$. From the real pole, the phase sequence p_r at the launch of a resonance is found as the closest integer to $\mathcal{V}'_1$ or $N - \mathcal{V}'_1$ and, the expected resonance angle ϕ_r is known using Eq. (41) of Chapter 2 from the p_r .

Table 1. The real pole $\mathcal{V}'_1(k_{yp})$ of $G_o(G_{op})$ and an expected resonance angle ϕ_r .

ξ	θ	k_{yp} ($1/\lambda_o$)	$\rho' k_{yp}$	$\mathcal{V}'_1$	ϕ_r
$2\xi_o$	15°	6.9937472	155.832522	155.718612	18.99°
	30°	6.4565818	143.863568	143.786972	27.51°
$2.5\xi_o$	15°	7.6422678	170.282658	169.792069	12.82°
	30°	7.1539647	159.402437	158.974159	19.76°

In Table 2, k_{yp} of the first harmonic G_{-1p} is tabulated when a z-direction dipole center-to-center distance $2d=0.55\lambda_0$. For this value of $2d$, $\beta_{-1} = k_0(\sin\theta - 1/0.55)$. Until $\theta=30^\circ$, k_{yp} is imaginary and ρk_{yp} , the approximate value for ψ'_1 of G_{-1} is an imaginary number with a large magnitude. Because a real pole does not exist, no resonance can arise from G_{-1} . The exact value of ψ'_1 is not pursued since only a planar approximation is enough to evaluate G_{-1} in this case.

Table 2. k_{yp} of the first harmonic G_{-1p} when $2d=0.55\lambda_0$

ξ	θ	k_{yp} ($1/\lambda_0$)	ρk_{yp}
$2\xi_0$	0°	i 8.885387	i 197.981456
	15°	i 6.666269	i 148.535769
	30°	i 4.128047	i 91.9798808
$2.5\xi_0$	0°	i 8.334168	i 185.69936
	15°	i 5.911631	i 131.721152
	30°	i 2.747551	i 61.2200932

When a z-direction dipole center-to-center distance $2d$ is increased to $0.7\lambda_0$ with $\beta_{-1} = k_0(\sin\theta - 1/0.7)$, the lowest pole of G_{-1} is tabulated in Table 3. For this value of $2d$, it is shown in the table that, for an increased value of θ , ψ'_1 which

is way up on the imaginary axis when $\theta=0^\circ$ is dropped down to the real axis and a resonance can be launched due to this pole from G_{-1} .

Table 3. The lowest pole $\nu'_1(k_{yp})$ of $G_{-1}(G_{-1p})$ and an expected resonance angle when $2d=0.7\lambda_0$.

ξ	θ	k_{yp} ($1/\lambda_0$)	ρk_{yp}	ν'_1	ϕ_r
$2\xi_0$	0°	i 5.3861949	i 120.01353	i 119.47821	
	15°	i 1.5690995	i 34.962192	i 33.97072	
	30°	4.1853302	93.256239	93.39530	$50.09^\circ, 61.95^\circ$
2.5 ξ_0	0°	i 4.4180746	i 98.442178	i 97.60709	
	15°	2.6513546	59.076666	59.50435	26.34°
	30°	5.1969944	115.79783	115.6509	$43.85^\circ, 73.09^\circ$

When $\xi=2\xi_0$ and $2d=0.55\lambda_0$ as in Fig.(3-8), it is shown in Table 1 and 2 that there exists only a single real pole from G_0 up until $\theta=30^\circ$. When $\theta=15^\circ$, the phase sequence at a resonance $p_r=156$ or 44. For $p_r=156$, Eq.(41) of Chapter 2 does not yield a real ϕ_r . However, these two phase sequences give a physically identical resonance angle, because the excitation of $p_r=156$ is the same as the excitation of $p_r=44$ in the reverse direction. Therefore, the resonance angle

corresponding to $p_r=156$ is $-\phi_r$ when ϕ_r is the resonance angle for $p_r=44$. The same relations for ϕ_r and p_r hold for the remaining cases in Table 1. The expected resonance angle ϕ_r for $\xi=2\xi_0$ and $2d=0.55\lambda_0$ in Table 1 shows a good agreement with the actual resonance angle in Fig. (3-8) for both $\theta=15^\circ$ and 30° . In the figure, the peak angle of the real part is in a slight discrepancy with that of the imaginary part. A dip right off a peak in Fig. (3-8a) for the real part of an impedance is not actually a dip for the total impedance, because the imaginary part is not small at this angle.

When a z-direction dipole center-to-center distance $2d$ is increased to $0.7\lambda_0$ while ξ is maintained as $2\xi_0$, the impedance curve in an azimuthal scan is plotted in Fig. (3-9). The resonance angle ϕ_r in Table 1 is not altered since G_0 is independent of $2d$. However, because of the variation in β_{-1} $[=k_0(\sin\theta - \frac{1}{2d})]$, the lowest pole ν'_1 of G_{-1} is significantly changed. This is clear when Table 2 is compared with Table 3 where, as θ is increased, the downward shift of ν'_1 is more rapid than in Table 2. In fact, at $\theta=30^\circ$, ν'_1 of G_{-1} is on the real axis providing a resonance condition while ν'_1 in Table 2 is still way up from the real axis. In Fig. (3-9) when $\theta=15^\circ$, a resonance can be found at the predicted angle ϕ_r in Table 1. Also, it is noticed in the figure that the impedance curve near the broadside scan ($\phi=0^\circ$) has a significant change when θ is increased from 0° to 15° . This can be explained in terms of the transition of the lowest pole ν'_1 from G_{-1}

which undergoes a large amount of shift on the imaginary axis for this variation in θ in Table 3. When $\theta=30^\circ$, there arise multiple resonances due to the real poles from G_0 and G_{-1} in Table 1 and 3. For the real pole from G_0 , the agreement between the expected resonance angle ϕ_r and the actual peak angle of the impedance curve in Fig.(3-9) is as good as in Fig.(3-8). However, for the pole V'_1 from G_{-1} in Table 3, a further explanation is needed. For this pole, there exist two expected resonance angles, because both phase sequences $p_{r1}(=93)$ and $p_{r2}(=N-p_{r1})$ give two different real expected resonance angles ϕ_{r1} and ϕ_{r2} from Eq. (41) of Chapter 2. What actually happens in this case is that, when the azimuthal scan angle $\phi=\phi_{r1}$, another wave front is formed to the direction of $-\phi_{r2}$ and as ϕ is further increased to ϕ_{r2} , the direction of the secondary beam is shifted to $-\phi_{r1}$ maintaining a constant angle between the primary and the secondary beams. Therefore, the equivalence of the two phase sequences p_{r1} and p_{r2} except the reversal in the beam directions holds also in this case. Because of the interaction between these two phase sequences, p_{r1} and p_{r2} which are close to each other, the resulting resonances are not simple as the previous one. Since they are close in ϕ , the two resonances are combined in this case as the wider impedance peaks in Fig.(3-9) indicate. Especially, the double peaks in Fig.(3-9b) for the imaginary part are roughly in accordance with the expected

resonance angles ϕ_{r1} and ϕ_{r2} .

When ξ is increased to $2.5\xi_0$, the dipole input impedance curve for $2d=0.55\lambda_0$ is given in Fig.(3-10). The lowest pole locations for this case can be found in Table 1 and 2. For $\theta=15^\circ$ or 30° , only a single resonance due to the real pole from G_0 arises. The actual peak impedance angles in the figure have excellent accords with the resonance angles predicted from the pole locations. Even though the lowest pole from G_{-1} does not cause a resonance because of its location on the imaginary axis, its fast downward shift in Table 2 results in significant changes in the impedance curve near the broadside when θ is increased from 15° to 30° . When a z-direction dipole center-to-center distance $2d$ is increased to $0.7\lambda_0$ while maintaining ξ as $2.5\xi_0$, the lowest pole of G_{-1} originally located on the imaginary axis with a large magnitude is dropped down to the real axis as θ is increased to 15° in Table 3 to result in multiple resonances in Fig.(3-11). For both $\theta=15^\circ$ and $\theta=30^\circ$, except the excellent agreements between ϕ_r 's in Table 1 and the actual peak impedance angles in Fig.(3-11), the resonance angles predicted from the real pole of G_{-1} in Table 3 are wide from other peak impedance angles in the figure. Because of the multiple resonances, the resultant impedance curves are fairly complicated.

3.5 Elementary patterns.

Once the active dipole input impedances $Y(p)$ for $0 \leq p \leq N-1$ are known, the dipole currents of all columns for a single reference column excitation are derived exactly in the same way as in Chapter 2. As discussed there, in order to maximize the broadside radiation, the characteristic admittance Y_g of a feedline is matched to the real part of $Y(p)$ when both p and θ are equal to zero while the imaginary part of $Y(p)$ for these p and θ is neutralized with a compensation network. The dipole current of a l -th column is given by Eq. (44) of Chapter 2.

The radiation pattern of the whole array at the elevation angle θ can be found by superposing the far fields in that direction caused by N number of columns with these dipole currents. The far field of a single dipole column located at (ρ', ϕ') and scanned to the direction of θ can be derived from the following electric Hertzian potential Π_{zf}^e .

$$\Pi_{zf}^e(\rho, \phi, z; \rho', \phi') = G_{of}(\rho, \phi; \rho', \phi') e^{i\beta_0 z} \frac{\pi}{d} \int_{-h}^h I_z(z') e^{-i\beta_0 z'} dz' \quad (20)$$

where the fundamental Green's function in the far zone, G_{of} is derived from the following Green's function $G_o(\rho, \phi; \rho', \phi')$ in the region $\rho \geq \rho'$ by increasing ρ to infinity.

$$G_o(\rho, \phi; \rho', \phi') = \frac{i(2\pi)^{-2}}{\omega \epsilon_0 (\rho^2 - \rho'^2)} \frac{\rho^2 - \beta_0^2}{\rho'^2 - \beta_0^2} \sum_{m=-\infty}^{\infty} \frac{H_m^1(\sqrt{\rho^2 - \beta_0^2} \rho)}{H_m^1(\sqrt{\rho'^2 - \beta_0^2} \rho')} e^{i\lambda m |\phi - \phi'|} Z(\rho') R \times$$

$$\frac{1}{\left[X\left(\frac{\xi_0}{2}\right) H_m^1\left(\sqrt{k^2 - \beta_0^2} a\right) - T\left(\frac{\xi_0}{2}\right) H_m^2\left(\sqrt{k^2 - \beta_0^2} a\right) \right] R - \left(\frac{m \beta_0}{k \sqrt{k^2 - \beta_0^2} \rho} \right)^2 \left(\frac{k^2 - \beta_0^2}{k_0^2 - \beta_0^2} \right)^2 Z(\rho') W} \quad (21)$$

$$(\rho \geq \rho')$$

Z, R, X, T and W are the same functions defined in Eqs. (2a) to (2e). By approximating the Hankel function $H_m^1(\sqrt{k_0^2 - \beta_0^2} \rho)$ with an asymptotic expansion as ρ goes to infinity and transforming the summation in Eq. (21) into a contour integral above the real axis as outlined in Appendix 1,

$$G_{0\pm}(\rho, \phi; \rho', \phi') = \frac{-(2\pi)^{-2}}{\omega \xi \sqrt{k^2 - \beta_0^2} \rho'} \left(\frac{k^2 - \beta_0^2}{k_0^2 - \beta_0^2} \right) \sqrt{\frac{2}{\pi \sqrt{k_0^2 - \beta_0^2} \rho}} e^{i(\sqrt{k_0^2 - \beta_0^2} \rho - \frac{\pi}{4})} \times$$

$$\int_{C_1} \frac{Z(\rho') R e^{-i\frac{\lambda}{2}\nu} / H_m^1(\sqrt{k_0^2 - \beta_0^2} \rho') \frac{\cos \nu(|\phi - \phi'| - \pi)}{\sin \nu \pi}}{\left[X\left(\frac{\xi_0}{2}\right) H_m^1\left(\sqrt{k^2 - \beta_0^2} a\right) - T\left(\frac{\xi_0}{2}\right) H_m^2\left(\sqrt{k^2 - \beta_0^2} a\right) \right] R - \left(\frac{\nu \beta_0}{k \sqrt{k^2 - \beta_0^2} \rho} \right)^2 \left(\frac{k^2 - \beta_0^2}{k_0^2 - \beta_0^2} \right)^2 Z(\rho') W} d\nu \quad (22)$$

G_{0f} in the lit region of the source column can be evaluated with the method of a stationary phase integration. For this purpose, the angular factor of Eq. (22) is expanded into a series given by Eq. (35) in Chapter 2 and only the first term, $-i \exp(i\nu|\phi - \phi'|)$ is preserved since the stationary points can be found only for this term. When the Hankel functions in Eq. (22) are approximated with Eq. (36) of Chapter 2, G_{0f} in the lit region of the source column is given as

$$G_{0f}(\rho, \phi; \rho', \phi') = -\frac{(2\pi)^{-2}}{\omega \xi} \frac{k^2 - \beta_0^2}{k_0^2 - \beta_0^2} \cdot \frac{e^{i\sqrt{k_0^2 - \beta_0^2} \rho}}{(\sqrt{k_0^2 - \beta_0^2} \rho)^{\frac{1}{2}}} \int_{C_1} F \cdot d\nu \quad (23)$$

The integrand,

$$F = \frac{\sqrt{(k_0^2 - \beta_0^2)\rho'^2 - \gamma^2}^{\frac{1}{2}} [D_1(1) - D_2(1)e^{i2D}] [1 - e^{i2D}] e^{i\psi}}{[D_1(\frac{\xi_0}{\epsilon}) + D_2(\frac{\xi_0}{\epsilon})e^{i2D}] [D_1(1) - D_2(1)e^{i2D}] + (\frac{\gamma\beta_0}{k})^2 (\frac{\beta_0^2 - k_0^2}{k_0^2 - \beta_0^2}) (1 - e^{i4D})}$$

$$\text{where } D_1(s) = \sqrt{(k^2 - \beta_0^2)\rho'^2 - \gamma^2} + s \frac{\beta_0^2 - k_0^2}{k_0^2 - \beta_0^2} \sqrt{(k_0^2 - \beta_0^2)\rho'^2 - \gamma^2}$$

$$D_2(s) = \sqrt{(k^2 - \beta_0^2)\rho'^2 - \gamma^2} - s \frac{\beta_0^2 - k_0^2}{k_0^2 - \beta_0^2} \sqrt{(k_0^2 - \beta_0^2)\rho'^2 - \gamma^2}$$

$$D = \sqrt{(k^2 - \beta_0^2)\rho'^2 - \gamma^2} - \sqrt{(k^2 - \beta_0^2)\alpha^2 - \gamma^2} - \gamma \cos^{-1} \frac{\gamma}{\sqrt{k^2 - \beta_0^2}\rho'} + \gamma \cos^{-1} \frac{\gamma}{\sqrt{k^2 - \beta_0^2}a}$$

$$\psi = \sqrt{(k_0^2 - \beta_0^2)\rho'^2 - \gamma^2} + \gamma \cos^{-1} \frac{\gamma}{\sqrt{k_0^2 - \beta_0^2}\rho'} + \gamma (|\phi - \phi'| - \frac{\pi}{2})$$

To evaluate G_{of} with stationary phase integrations, the integrand F is first expanded into an ascending series of e^{i2nD} as

$$F = \sum_{n=0}^{\infty} f_n e^{i(2nD + \psi)} \quad (24)$$

Substituting Eq. (24) into Eq. (23), a stationary phase integration is carried out for each term separately.

$$G_{of}(\rho, \phi; \rho', \phi') = -\frac{(2\pi)^{-2}}{\omega \epsilon} \frac{k^2 - \beta_0^2}{k_0^2 - \beta_0^2} \frac{e^{i\sqrt{k^2 - \beta_0^2}\rho}}{(\sqrt{k^2 - \beta_0^2}\rho)^{\frac{1}{2}}} \sum_{n=0}^{\infty} \int_{C_1}^{\rho} e^{i(2nD + \psi)} d\gamma \quad (25)$$

The stationary point γ_{s_0} for the first integral ($n=0$) can be found from the equation $\frac{\partial \psi}{\partial \gamma} \Big|_{\gamma_{s_0}} = 0$.

$$\nu_{s0} = \sqrt{k_o^2 - \beta_o^2} \rho' \sin|\phi - \phi'| \quad (26)$$

Therefore, the first integration I_{f0} is given as

$$\begin{aligned} I_{f0} &= \int_{c_1}^{f_0} e^{i\psi} d\nu \\ &= \sqrt{2\pi \sqrt{k_o^2 - \beta_o^2} \rho' \cos|\phi - \phi'|} e^{i\left(-\sqrt{k_o^2 - \beta_o^2} \rho' \cos|\phi - \phi'| - \frac{\pi}{4}\right)} \int_0^1 \Big]_{\nu=\nu_{s0}} \quad (27) \end{aligned}$$

From Eqs. (25) and (27), the first integration I_{f0} can be identified as the direct wave from the source column to a field point because of the path-length, $\sqrt{k_o^2 - \beta_o^2}(\rho - \rho' \cos|\phi - \phi'|)$. This is illustrated in Fig. (3-12). For a n-th integration I_{fn} , the stationary point ν_{sn} is the solution of the following equation.

$$\begin{aligned} \left. \frac{\partial (2nD + \psi)}{\partial \nu} \right|_{\nu=\nu_{sn}} &= \cos^{-1} \frac{\nu_{sn}}{\sqrt{k_o^2 - \beta_o^2} \rho'} + |\phi - \phi'| - \frac{\pi}{2} \\ &+ 2n \cdot \left(\cos^{-1} \frac{\nu_{sn}}{\sqrt{k_o^2 - \beta_o^2} a} - \cos^{-1} \frac{\nu_{sn}}{\sqrt{k_o^2 - \beta_o^2} b} \right) = 0 \quad (28) \end{aligned}$$

Carrying out a stationary phase integration near the point ν_{sn} ,

$$\begin{aligned} I_{fn} &= \int_{c_1}^{f_n} e^{i(2nD + \psi)} d\nu \\ &= \sqrt{2\pi} \left[\frac{\partial^2}{\partial \nu^2} (-2nD - \psi) \right]^{-\frac{1}{2}}_{\nu=\nu_{sn}} e^{i\left(-\frac{\pi}{4} - T_{1n} + 2n \cdot T_{2n}\right)} \int_n^1 \Big]_{\nu=\nu_{sn}} \quad (29) \end{aligned}$$

where $T_{1n} = \sqrt{(\rho_0^2 - \beta_0^2)\rho'^2 - \nu_{sn}^2}$

$$T_{2n} = \sqrt{(\rho_0^2 - \beta_0^2)\rho'^2 - \nu_{sn}^2} - \sqrt{(\rho_0^2 - \beta_0^2)\alpha^2 - \nu_{sn}^2}$$

Using Eq.(28), T_{2n} can be transformed into the following form.

$$\begin{aligned} T_{2n} &= (T_{2n}^2)^{\frac{1}{2}} \\ &= \sqrt{\rho_0^2 - \beta_0^2} \sqrt{\alpha^2 + \rho'^2 - 2\alpha\rho' \left(\frac{\nu_{sn}^2}{(\rho_0^2 - \beta_0^2)\alpha\rho'} + \sqrt{1 - \frac{\nu_{sn}^2}{(\rho_0^2 - \beta_0^2)\alpha^2}} \sqrt{1 - \frac{\nu_{sn}^2}{(\rho_0^2 - \beta_0^2)\rho'^2}} \right)} \\ &= \sqrt{\rho_0^2 - \beta_0^2} \sqrt{\alpha^2 + \rho'^2 - 2\alpha\rho' \cos \left(\cos^{-1} \frac{\nu_{sn}}{\sqrt{\rho_0^2 - \beta_0^2}\alpha} - \cos^{-1} \frac{\nu_{sn}}{\sqrt{\rho_0^2 - \beta_0^2}\rho'} \right)} \\ &= \sqrt{\rho_0^2 - \beta_0^2} \sqrt{\alpha^2 + \rho'^2 - 2\alpha\rho' \cos \left\{ \frac{1}{2n} (|\phi - \phi'| - \frac{\pi}{2} + \cos^{-1} \frac{\nu_{sn}}{\sqrt{\rho_0^2 - \beta_0^2}\rho'}) \right\}} \quad (30) \end{aligned}$$

If we define δ_n as

$$\delta_n = \sin^{-1} \frac{\nu_{sn}}{\sqrt{\rho_0^2 - \beta_0^2}\rho'} \quad (31)$$

then $T_{2n} = \sqrt{\rho_0^2 - \beta_0^2} \sqrt{\alpha^2 + \rho'^2 - 2\alpha\rho' \cos \left\{ \frac{1}{2n} (|\phi - \phi'| - \delta_n) \right\}} \quad (32)$

Also, using δ_n ,

$$T_{1n} = \sqrt{\rho_0^2 - \beta_0^2}\rho' \cos \delta_n \quad (33)$$

As shown in Fig.(3-12B), by identifying the angle δ_n as the angle between a transmitted ray to the direction of ϕ at the dielectric and free space interface and a cylinder normal at that point, T_{1n} and T_{2n} are physically clear. As shown in the figure, T_{2n} is a path-length of the ray between the cylinder and the interface. As Eq.(29) indicates, the ray which is represented by I_{fn} has propagated inside the dielectric layer undergoing n number of reflections before it radiates to the direction of ϕ . A part of the ray is transmitted whenever it impinges upon the interface, but the transmitted ray does not radiate to the direction of ϕ until the path-length in the dielectric is increased to $2 \cdot n \cdot T_{2n}$. From the figure, the path-length of the ray from its emergence into the free space to a field point at (ρ, ϕ) is given by $\sqrt{k_0^2 - \beta_0^2} \rho - T_{1n}$. Therefore, the total path-length of the ray is $\sqrt{k_0^2 - \beta_0^2} \rho - T_{1n} + 2nT_{2n}$, which is confirmed from Eqs.(25), (29), (32) and (33).

As a result, the Green's function in the far zone in Eq.(25) can be considered as a summation of the direct ray from the source to a field point and various other rays, each of which has different number of reflections from the cylinder surface before it emerges into the free space to radiate to the given direction ϕ . The paths of these rays inside the dielectric layer are all different as the emerging angle into the free space δ_n as well as the stationary point ν_{sn} of a ray are different from those of another ray.

This is unlike a planar situation where, for a given azimuthal angle ϕ , there exist only a single ray path inside the dielectric layer and whenever the ray impinges upon the interface, a part of it is transmitted and radiated to the direction ϕ . The number of terms which are needed in Eq.(25) for an accurate evaluation of G_{of} depends on the dielectric constant ξ , because it determines the ratio of a transmission to a reflection at the interface. For $\xi=2.5\xi_0$, it is found that a summation up to the sixth term is enough, because for a ray represented by I_{fn} in Eq.(29) with n larger than six, there remains a negligible amount of power to be radiated to the direction of ϕ at its n -th transmission into the free space after it has lost most of the power during the previous $n-1$ times transmissions at the interface, all of which radiate to different directions from ϕ .

Finally, in connection with the stationary phase integrations of Eq.(25), there remains one more subject to be discussed, which is the effect of a pole cross-over by the modification of the original contour C_1 to the steepest descent path through a stationary point. The possibility of a pole cross-over in this case is slight because a stationary point $\nu_{sn}(=\sqrt{k_0^2-\beta_0^2}\rho' \sin\delta_n)$ is not close to the real pole ν'_1 of G_{of} , which is located between $\sqrt{k_0^2-\beta_0^2}\rho'$ and $\sqrt{k_0^2-\beta_0^2}\rho'$ as the position of k_{yp} in Fig.(3-5) indicates. Even if a pole cross-over takes place, it does not have much impact upon the far field Green's function G_{of} , because the residue ari-

sing from this pole is negligibly small in the case of a far field calculation, which is quite different from the case of a near field calculation where the real pole γ_1' is the dominant factor determining a near field Green's function G_0 as discussed in detail in Section (3-3). Comparing the two Green's function G_0 's in Eq. (2) for $\rho \leq \rho'$ and in Eq. (21) for $\rho \geq \rho'$, it is noticed that the main difference is the presence of an additional term $H_m^1(\sqrt{\rho_2^2 - \rho_1^2} \rho) / H_m^1(\sqrt{\rho_2^2 - \rho_1^2} \rho')$ in Eq. (21). In the calculation of the residue term at the real pole, the denominator of the additional term, $H_y^1(\sqrt{\rho_2^2 - \rho_1^2} \rho) \big|_{\rho=\gamma_1'}$, has a large magnitude, because the order γ_1' is greater than the argument in this case and, as Eq. (36) of Chapter 2 indicates, the function is exponentially increasing with the order, which, in turn, proportional to the radius of the array ρ' . Therefore, for a large value of ρ' , the residue arising from the real pole does not have any noticeable impact on the far field. In other words, the wave from the residue at the real pole propagates along the dielectric layer while, outside the layer, it is decaying.

In the shadow region of the cylinder caused by the source column, a far field Green's function G_{of} in Eq. (22) can be evaluated with a Watson transform method as a near field Green's function G_0 in section 3.3. However, the major interest is the lit region Green's function, because it is this function which is mainly used in the derivation of the elementary pattern for a whole array. The elementary pattern

has a significant amplitude only in the lit region of the reference column and, once the dipole currents of the columns near the reference column are known, the elementary pattern in this region can be superposed using the lit region Green's function. The contributions from the other columns to the elementary pattern in the lit region of the reference column can be neglected because the currents of these columns are small.

From Eqs. (20), (25) and (29), the electric Hertzian potential Π_{zf}^e in the far zone is given as

$$\Pi_{zf}^e(\rho, \phi, z; \rho', \phi') = -\frac{(2\pi)^{-2}}{\omega \epsilon} \frac{k_o^2 - \beta_o^2}{k_o^2 - \beta_o^2} \frac{e^{i\sqrt{k_o^2 - \beta_o^2} \rho + \beta_o z}}{(\sqrt{k_o^2 - \beta_o^2} \rho)^{1/2}} \left(\sum_{n=0}^{\infty} I_{fn} \right) F(I_z(z'))$$

$$\text{where } F(I_z(z')) = \frac{\pi}{d} \int_{-h}^h I_z(z') e^{-i\beta_o z'} dz' \quad (34)$$

In the lit region of the source column, the ϕ -component H-field, H_ϕ , in the direction of θ , which is predetermined by the z-direction phase variation of the source column current, is derived from Π_{zf}^e .

$$\begin{aligned} H_\phi(r, \phi - \phi', \theta) &= i\omega \epsilon_o \frac{\partial \Pi_{zf}^e(\rho, \phi, z; \rho', \phi')}{\partial \rho} \\ &= \frac{e^{i k_o r}}{\sqrt{k_o r}} g_d(\phi - \phi'; \theta) F(I_z(z')) \end{aligned} \quad (35)$$

$$\text{where } g_d(\phi - \phi'; \theta) = \frac{1}{2\pi} \frac{\epsilon_o}{\epsilon} \frac{k_o^2 - \beta_o^2}{k_o^2 - \beta_o^2} \sum_{n=0}^{\infty} I_{fn} \quad \text{and } r = \sqrt{\rho^2 + z^2}.$$

Using the circular symmetry of the array, the total ϕ -component H-field, H_{ϕ}^t due to N number of dipole columns can be found by a superposition.

$$\begin{aligned}
 H_{\phi}^t(r, \phi, \theta) &= \sum_{\ell=0}^{N-1} H_{\phi}(r, \phi - \frac{2\pi}{N}\ell, \theta) \\
 &= \frac{e^{ik_0 r}}{\sqrt{k_0 r}} \sum_{\ell=0}^{N-1} g_d(\phi - \frac{2\pi}{N}\ell : \theta) F(I^{\ell}) \quad (36)
 \end{aligned}$$

where I^{ℓ} is the dipole current of ℓ -th column for a single reference column excitation. In addition to these TM modes in the z-direction derived from Π_{zf}^e , there also exist TE modes from a magnetic Hertzian potential Π_{zf}^m . Π_{zf}^m in the lit region of the source column can be obtained in the same way as Π_{zf}^e using the method of a stationary phase integration. However, compared to the TM modes in the far zone, the fields of TE modes are so weak that the addition of TE modes does not result in any noticeable changes to the total far field. When $H_{\phi}^t(r, \phi, \theta)$ is normalized by $H_{\phi}^t(r, \phi, 0)$ on the horizontal plane, the elementary pattern cut at the given elevation angle θ is found.

The elementary patterns are derived for the arrays whose active dipole input impedance variations for several parameters are plotted in Figs.(3-6) to (3-11). Figs.(3-13) to (3-19) show these patterns. For the arrays, whose active dipole input impedance curves are compared in Fig.(3-6) as

a function of ξ , the elementary patterns on the horizontal plane are given in Fig.(3-13) for $\xi=3\xi_0$ and, in Figs.(3-17) and (3-19) for $\xi=2\xi_0$ and $2.5\xi_0$, respectively. In the latter two figures, the elementary pattern on the horizontal plane is represented by the curve for the elevation scan angle $\theta=0^\circ$. Comparing these figures, it is clear that the elementary pattern on the horizontal plane is mainly characterized by the dipole input impedance curve of the array. A reduction in beamwidth as well as a sharp beam cutoff off broadside for an increased ξ are the results of the input impedance variations shown in Fig.(3-6). When ξ is varied from $2\xi_0$ to $3\xi_0$, the rapid increase of the peak at the maximum impedance angle and the shift of this angle toward broadside are responsible for the sharp beam cutoff off broadside and the reduction in beamwidth respectively. The correspondence between the elementary pattern cut on the horizontal plane and the active dipole input impedance curve for $\theta=0^\circ$ of an array is further confirmed by deriving the elementary patterns for the arrays whose active dipole input impedance variations as a function of an inter-column distance d_c have been compared in Fig.(3-7). The elementary patterns for $d_c=0.6\lambda$ and 0.8λ are given in Figs.(3-14) and (3-15) respectively, and the pattern for $d_c=0.7\lambda$ is represented by the curve for $\theta=0^\circ$ in Fig.(3-19). Comparing these figures with the dipole input impedance curves given in Fig.(3-7), it is shown that the beamwidth of an elementary

pattern on the horizontal plane is roughly twice the maximum impedance angle, because this angle corresponds approximately to the half-power point of the elementary pattern. The beamwidth on the horizontal plane as well as the maximum impedance angle in azimuth can be varied by adjusting an inter-column distance d_c .

Due to a resonance inside the dielectric layer when the beam is lifted away from the horizontal plane towards the axial direction, the elementary pattern cut at an elevation angle θ undergoes significant changes from the pattern on the horizontal plane as θ is increasing. Comparing the active dipole input impedance curves in Figs.(3-8) to (3-11) with the corresponding elementary patterns in Figs.(3-16) to (3-19), the impact of a resonance upon an elementary pattern can be known. When θ is increased from zero, the major changes for the impedance curves in Figs.(3-8) and (3-10) in the cases of a single resonance are the shift of the maximum impedance angle at $\theta=0^\circ$ from the broadside towards the endfire direction and the emergence of a sharp resonance peak. For the corresponding elementary patterns in Figs.(3-16) and (3-18), the resultant modifications from the pattern cut on the horizontal plane are the increase in the beamwidth due to the shift of the maximum impedance angle and various ripples on the pattern caused by the emerging resonance peak. As θ is increasing, the ripple effect is more severe because of a stronger resonance inside the dielectric layer and, what is implied

by a beamwidth in this case, is actually the width of the envelope of an elementary pattern cut rather than the pattern cut itself. Unless the width of a resonance peak in azimuth is wide, a pattern dip at the resonance angle ϕ_r which can be expected in a planar array is not materialized in the case of a cylindrical array. For a planar array, the radiation to a given azimuthal angle ϕ is caused by a single phase sequence p as given by Eq. (41) of Chapter 2. Because of the extremely small dipole currents at a resonance phase sequence p_r , the elementary pattern cut of a planar array is expected to have a sharp dip at the resonance angle ϕ_r . However, for a cylindrical array, it is shown in the preceding section that the radiation to a given azimuthal angle ϕ is a superposition of the direct ray from the source and other various rays, each of which has a different ray-path inside the dielectric layer because of the curvatures of cylindrical surfaces before it radiates to the given angle ϕ . To the radiation in the direction of a resonance angle ϕ_r , the contribution from the direct wave at the corresponding phase sequence p_r is negligible. However, the other rays which also contribute to the radiation at that direction are caused by the phase sequences close to but other than the resonance phase sequence p_r . Unless the azimuthal width of the impedance peak is wide, the response dipole currents $I(p)$'s for such nearby phase sequences have considerable magnitudes and the resultant various rays signi-

ficantly contribute to the radiation at ϕ_r to avoid a pattern dip. An exceptional case of a wide impedance peak in azimuth can be found in Fig.(3-9) for $\theta=30^\circ$, especially in its imaginary part. This wide resonance peak is a result of a multiple resonance which has been discussed in the preceding section. The corresponding elementary pattern in Fig.(3-17) shows a broad dip at those directions. It is also noticed in Fig.(3-17) that the broadside pattern for the elevation angle $\theta=15^\circ$ shows a substantial decrease from the pattern on the horizontal plane. The corresponding input impedance curve in Fig.(3-9) has a rapid increase near $\phi=0^\circ$, the reason for which has been discussed in the preceding section as the result of a fast decay of the lowest pole from the first harmonic G_{-1} on the imaginary axis towards the real axis for a small increase in θ . To avoid this substantial pattern decrease at broadside which is deleterious in an elevation scan, a z-direction dipole center-to-center distance $2d$ must be reduced in order to suppress the rapid downward shift of γ'_1 from G_{-1} . By decreasing $2d$ from $0.7\lambda_0$ to $0.55\lambda_0$, the pattern reduction at broadside for $\theta=15^\circ$ is alleviated as in Fig.(3-16). For the cases of multiple resonances caused by an increased z-direction dipole center-to-center distance $2d$, the elementary patterns in Figs.(3-17) and (3-19) as well as their corresponding dipole input impedance curves in Figs.(3-9) and (3-11) show more complicated characteristics

than for the cases of a single resonance. In spite of more severe ripple effects, the discussion on the width of an elementary pattern envelope in the cases of a single resonance still valid in the cases of multiple resonances.

The column current distribution of the array is significantly varied when the beam is scanned away from the horizontal plane. When Θ is equal to zero, the dipole current of a column is rapidly and monotonously decreasing as the angular distance between the column and the activated column (the reference column) is increased. As the beam is lifted away from the horizontal plane, however, the column current distribution of the array becomes oscillatory in its magnitude, which is considered as the result of a standing wave inside the dielectric layer due to the emerging real pole γ_1' from G_0 when Θ is varied from zero and, this standing wave is reinforced by the phase sequence at the time of a resonance. Such an oscillatory current distribution of the array is considered to be the cause of excessive ripples on the elementary pattern cuts when Θ is increased from zero.

3.6 Conclusion.

The elementary pattern on the horizontal plane is characterized by ripples at broadside and narrow beamwidth. The beamwidth is approximately determined by the azimuthal beam scan angle where the maximum dipole input impedance occurs. Beyond this maximum impedance angle, there's no significant power radiation. As the dielectric constant of the layer is increasing, the magnitude of the maximum impedance is enhanced and the azimuthal angle where it occurs is shifting towards the broadside direction. Accordingly, the beamwidth is reduced, while the pattern shows strong ripple effects at broadside and a clear beam cutoff off broadside. Also, the beamwidth of the elementary pattern on the horizontal plane can be varied by adjusting the distance between dipole columns.

The elementary pattern cut at an elevation angle θ shows significant variations from the horizontal pattern as θ is increasing. Excessive ripples due to the resonances inside the dielectric layer can be seen for an increased θ . Considering the wide variations of an elementary pattern cut as a function of the elevation angle, a pattern deterioration for a shaped beam on the horizontal plane is expected as the beam is scanned away from the horizontal plane. Because of the resonance inside the dielectric layer, it is hard to achieve a wide angle beam scanning in elevation.

Chapter 4. Elementary patterns for dielectrically-clad conformal arrays of waveguide elements.

4.1 Introduction.

Because of their importance for airborne applications, dielectrically-clad cylindrical slot arrays have been investigated by several authors. For such an array, a dielectric layer provides a natural radome. For two dimensional circular arrays, where the radiating elements are infinite axial slits symmetrically distributed on a cylinder circumference, elementary patterns have been derived (Ref. 4). The analysis of the mutual coupling effects between axial slits is carried out with a modal approach. While this study provides excellent physical insights for an azimuthal scanning, for the elevation as well as azimuthal scan of a cylindrical array, the investigations of three dimensional arrays are required. A study for dielectrically covered three dimensional cylindrical arrays has been presented (Ref. 5). The analysis of mutual coupling effects between elements is carried out by the direct summations of harmonic series using various approximations based on the large magnitude of a cylinder radius which is close to $100\lambda_0$. Even though such a study provides numerical results, the underlying basic array behaviors can not be disclosed with such an approach.

In this chapter, through a complex plane analysis, the important characteristics of dielectrically-clad cylindrical arrays of waveguide elements can be derived. These features are instrumental for the designs of these arrays. The pattern dips of an elementary pattern cut at a given elevation angle can be predicted by the pole locations of a Green's function on the complex plane without actually solving a whole problem. The relation between a pattern dip and a resonance is more clear-cut than in the preceding chapter. Also, the effects of a multiple resonance inside the dielectric layer caused by a wide angle elevation scan have significant impacts upon an elementary pattern cut, which is one of the most interesting aspects of these arrays. Since a multiple resonance is launched by multiple poles on the real axis when the lowest pole of the first harmonic Green's function drops down to the real axis of a complex plane, the suppression of a multiple resonance which is important for a wide angle scan in elevation can be realized by restricting the downward shift of the pole. In such a way, a close correspondence between the poles of the Green's functions and the important features of the elementary pattern is established for these arrays through a complex plane analysis. Since the resonant poles on the complex plane can be discovered easily using a planar approximation as an intermediate step as described in the preceding chapter, the shifts of these poles on the complex plane as a function

of various array parameters can be easily traced, which is, in array designs, a clear advantage of the complex plane analysis over the direct harmonic series approach taken in Ref. 5 in addition to the computational efficiency of a residue series over a harmonic series in an actual calculation.

The geometry of a dielectrically-clad conformal array of waveguide elements is shown in Fig.(4-1). The array has a large number of slot columns circular symmetrically distributed along a cylinder circumference, each of which is consisted of infinite number of slot elements in the axial direction. Each element is an axial slot narrow in azimuth and the fields at an aperture are assumed to be constant in the circumferential direction. A slot is an open end of a waveguide which allows only a single mode propagation. It turns out in the following discussions that the denominators of Green's functions are the same as those in Chapter 3. Therefore, the pole locations on a complex plane are identical when the cylinder radius and the thickness of the dielectric layer are equal to those in Chapter 3. However, because of the differences in the numerators of Green's functions, beam scan characteristics between the conformal dipole and the slot arrays show significant discrepancies.

In the following section, a mathematical formulation will be derived in order to find the aperture field of an element in an array environment when an incident wave is

excited from the other end of a waveguide. In section 4.3, Green's functions in the near zone will be evaluated. In section 4.4, the variation in the reflection coefficient of the dominant mode inside a waveguide will be investigated as a function of the azimuthal scan angle ϕ as well as the elevation beam angle θ . Also, the impacts of the resonances inside the dielectric layer upon the reflection coefficient will be discussed. In section 4.5, elementary pattern cuts will be derived and, in the final section, conclusions will be presented.

4.2 Numerical formulation.

In the same way as in the preceding chapters, in order to find the elementary pattern cut at a given elevation angle θ , N number of phase sequence excitations, each of which has the same linear phase variation in the z -direction determined by the angle θ , are applied to the slot columns in the circumferential direction as the incident waves of the waveguides. Instead of the dipole currents as the sources of the far field in chapters 2 and 3, the tangential E-fields at the apertures uniquely determine the far field. Since the excitation is a free excitation as an incident wave at the other end of a waveguide instead of a forced excitation at an aperture, the tangential E-field at a slot surface is not under a direct control and, accordingly, is not known in advance. Therefore, the solution of the aperture field in an array environment is virtually tantamount to the solution of the whole problem.

For a given phase sequence excitation, in order to find the tangential E-field at a slot surface, the field inside a waveguide is expanded with the mode functions of the waveguide using the same number of unknown constants as the amplitudes, which is actually the reflection coefficient at the waveguide-dielectric interface. From this E-field, the tangential H-field inside the waveguide can be directly derived. On the other hand, outside the waveguide, the total

H-field at an aperture of the reference column can be found by superposing the partial H-fields caused by the assumed tangential E-fields at the apertures of individual slot columns as the sources. By matching these two H-fields at a slot surface of the reference column, the reflection coefficients of the assumed tangential E-field at an aperture for a given phase sequence excitation can be found. In other words, to find the tangential E-field at a slot surface for a given phase sequence excitation as the incident waves of the waveguides, what is actually matched at an aperture surface are the two expressions for a tangential H-field at a slot of the reference column derived from the assumed tangential E-field. In the superposition of the total H-field outside a waveguide of the reference column, a clear definition is necessary for the partial H-fields at the aperture caused by the tangential E-fields of individual slot columns as sources. As implied by the term, superposition, the partial tangential H-field at a reference column aperture surface caused by l -th column must be calculated in the environment where the aperture E-fields of all other columns including the reference column are equal to zero. In other words, there exists only a single slot column in the calculation of a partial H-field at the location of a reference column aperture, which is actually a conducting surface instead of an aperture when the source column is other than the reference column.

As the above discussion shows, the calculation of a partial H-field at the conducting surface in place of a reference column aperture is greatly simplified, because the apertures of all other columns except the source column are closed. Under such a circumstance, the calculation for the fields can be carried out using Green's functions as in the preceding chapters. The various field components caused by a single slot column both inside the dielectric layer and in the free space can be derived from z-component electric and magnetic Hertzian potentials, Π_z^e and Π_z^m as in Eq.(1) of Chapter 3. It is assumed in the following derivation that the only tangential E-field at an aperture is the ϕ -component E-field, E_ϕ since the width of an axial slot in azimuth, $2b$ is narrow compared to its length in the z-direction, $2h$. Accordingly, the field matching at an aperture surface is carried out with respect to the z-component H-field, H_z which can be derived from the magnetic Hertzian potential Π_z^m as in Eq.(1) of Chapter 3. The magnetic Hertzian potential $\Pi_z^m(a, \phi, z; a, \phi')$ at a field point on the cylinder surface caused by a slot column whose center in azimuth is located at (a, ϕ') is given by the following equation.

$$\Pi_z^m(a, \phi, z; a, \phi') = \sum_{n=-\infty}^{\infty} G_n(a, \phi; a, \phi') e^{i\beta_n z} \frac{\pi}{d} \int_{-h}^h E_\phi(z') e^{-i\beta_n z'} dz' \quad (1)$$

where E_ϕ is the tangential E-field at a slot and a is the

radius of the cylinder. $2b$ and $2h$ are the widths of a slot in azimuth and in the z -direction respectively. The two dimensional spatial harmonic Green's function G_n is given as

$$G_n(a, \phi; a, \phi') = \frac{j(2\pi)^{-2}}{\omega\mu_0\sqrt{k_o^2 - \beta_n^2}a} \sum_{m=-\infty}^{\infty} e^{jm|\phi - \phi'|} \left[2b \cdot \sin(mb/a) / (mb/a) \right] \cdot$$

$$\frac{\left[X\left(\frac{\xi}{2}\right) H_m'\left(\sqrt{k_o^2 - \beta_n^2}a\right) - T\left(\frac{\xi}{2}\right) H_m^2\left(\sqrt{k_o^2 - \beta_n^2}a\right) \right] \left[X(1) H_m'\left(\sqrt{k_o^2 - \beta_n^2}a\right) - T(1) H_m^2\left(\sqrt{k_o^2 - \beta_n^2}a\right) \right] - \left(\frac{m\beta_n}{k_o\sqrt{k_o^2 - \beta_n^2}} \right)^2 \left(\frac{k_o^2 - \beta_n^2}{k_o^2 - \beta_n^2} \right)^2 Z(\rho')^2}{\left[X\left(\frac{\xi}{2}\right) H_m'\left(\sqrt{k_o^2 - \beta_n^2}a\right) - T\left(\frac{\xi}{2}\right) H_m^2\left(\sqrt{k_o^2 - \beta_n^2}a\right) \right] R - \left(\frac{m\beta_n}{k_o\sqrt{k_o^2 - \beta_n^2}} \right)^2 \left(\frac{k_o^2 - \beta_n^2}{k_o^2 - \beta_n^2} \right)^2 Z(\rho') W}$$

(2)

where $\rho = \rho'$ is the dielectric surface and, X, T, Z, W and R are the same functions as defined in Eqs. (2a) to (2e) of Chapter 3. In the derivations of Eqs. (1) and (2), the surface integral over an aperture can be separated into the azimuthal and the z -direction integration since the aperture field $E_\phi(z)$ is dependent on the z -coordinate only. The factor $2b \cdot \sin(mb/a) / (mb/a)$ in Eq. (2) is the azimuthal aperture factor. G_n in the above equation has the same denominator as the Green's function in Eq. (2) of Chapter 3, while the numerators are different. For a p -th phase sequence excitation, the total magnetic Hertzian potential $\Pi_z^{mt}(a, \phi, z; p)$ on the cylinder surface can be superposed with the partial magnetic Hertzian potentials, $\Pi_z^m(a, \phi, z; a, \frac{2\pi}{N}l)$ for $0 \leq l \leq N-1$ due to the individual slot columns.

$$\begin{aligned}
 \Pi_z^{\text{mt}}(a, \phi, z; p) &= \sum_{l=0}^{N-1} \Pi_z^{\text{m}}(a, \phi, z; a, \frac{2\pi l}{N}) e^{i \frac{2\pi}{N} l p} \\
 &= \sum_{l=0}^{N-1} e^{i \frac{2\pi}{N} l p} \left[\sum_{n=-\infty}^{\infty} G_n(a, \phi; a, \frac{2\pi l}{N}) e^{i \beta_n z} \frac{\pi}{d} \int_{-h}^h E_{\phi}(z') \tilde{e}^{-i \beta_n z'} dz' \right]
 \end{aligned} \quad (3)$$

The expansion of the unknown aperture surface field E_{ϕ} is carried out with TE modes of the waveguide. When the incident wave has a unit magnitude, the aperture field E_{ϕ} for a p -th phase sequence excitation can be expanded into a series.

$$E_{\phi} = \sin \frac{\pi(z+h)}{2h} + \sum_{q=1}^{\infty} \Gamma_q(p) \sin \frac{q\pi(z+h)}{2h} \quad (4)$$

($|z| \leq h$)

where $\Gamma_q(p)$'s are unknown reflection coefficients at the waveguide-dielectric interface. From Eqs. (3) and (4), the z -component H-field, $H_z(a, \phi, z; p)$ on the cylinder surface due to the N number of slot columns activated by a p -th phase sequence can be derived as

$$\begin{aligned}
 H_z(a, \phi, z; p) &= \left(\rho_k^2 + \frac{\partial^2}{\partial z^2} \right) \Pi_z^{\text{mt}}(a, \phi, z; p) \\
 &= \frac{\pi}{dh} \sum_{l=0}^{N-1} e^{i \frac{2\pi}{N} l p} \left\{ \sum_{n=-\infty}^{\infty} G_n(a, \phi; a, \frac{2\pi l}{N}) e^{i \beta_n z} \left(\rho_k^2 - \beta_n^2 \right) f_{n1} \right\} \\
 &\quad + \sum_{q=1}^{\infty} \Gamma_q(p) \frac{q\pi}{dh} \left\{ \sum_{l=0}^{N-1} e^{i \frac{2\pi}{N} l p} \left\{ \sum_{n=-\infty}^{\infty} G_n(a, \phi; a, \frac{2\pi l}{N}) e^{i \beta_n z} \left(\rho_k^2 - \beta_n^2 \right) f_{nq} \right\} \right\}
 \end{aligned} \quad (5)$$

where

$$f_{nq} = \begin{cases} \frac{\cos \beta_n h}{\left(\frac{q\pi}{2h}\right)^2 - \beta_n^2} & \text{for a odd } q \\ i \frac{\sin \beta_n h}{\left(\frac{q\pi}{2h}\right)^2 - \beta_n^2} & \text{for a even } q \end{cases}$$

On the other hand, inside a waveguide, H_z can be derived from Eq. (4) as

$$H_z(z;p) = \frac{1}{\omega \mu_0} \left[\sqrt{\frac{\rho^2 - \left(\frac{q\pi}{2h}\right)^2}{k_0^2}} \sin \frac{\pi(z+h)}{2h} - \sum_{q=1}^{\infty} \Gamma_q(p) \sqrt{\frac{\rho^2 - \left(\frac{q\pi}{2h}\right)^2}{k_0^2}} \sin \frac{q\pi(z+h)}{2h} \right] \quad (6)$$

(|z| ≤ h)

In order to solve the unknown reflection coefficients $\Gamma_q(p)$'s, the two expressions for H_z in Eqs. (5) and (6) are matched at an aperture surface of the reference column using the Galerkin method (Ref. 22). The mode functions of the waveguide are chosen as the weighting functions in this method. The products of a weighting function and H_z 's both in Eqs. (5) and (6) are integrated over a reference column aperture surface and the results of these integrations are made to be equal. By repeating the same procedure for all the weighting functions which are included in the expansion of H_z in Eq. (6), a matrix equation for the unknown reflection coefficient, $\Gamma_q(p)$'s is generated. The series for H_z in Eq. (6) must be truncated in order to derive the matrix equation. The number of terms required for an accurate expansion of the aperture field de-

pends on the length of an aperture in the z -direction as well as the elevation beam angle θ . For the arrays considered in this chapter, the length of an aperture is less than λ_0 and the elevation beam scan is carried out only up until $\theta=30^\circ$. Under such circumstances, it is found that a five term expansion of H_z is enough, because the power carried by $\Gamma_q(p)$'s for $q>5$ is negligibly small. The equation for $\Gamma_q(p)$'s for $1 \leq q \leq 5$ is given as

$$\begin{aligned} \sum_{q=1}^5 \Gamma_q(p) & \left\{ \frac{rq\pi^3}{dh^2} \sum_{l=0}^{N-1} e^{i\frac{2\pi l}{N}p} \left\{ \sum_{n=-\infty}^{\infty} \overline{G}_n(a, 0:a, \frac{2\pi l}{N}) (k^2 - \beta_n^2) f_{nr}^* f_{nq} \right\} \right. \\ & \quad \left. + \frac{2bh}{\omega\mu_0} \sqrt{k_0^2 - \left(\frac{r\pi}{2h}\right)^2} \delta_{qr} \right\} \\ & = -\frac{r\pi^3}{dh^2} \sum_{l=0}^{N-1} e^{i\frac{2\pi l}{N}p} \left\{ \sum_{n=-\infty}^{\infty} \overline{G}_n(a, 0:a, \frac{2\pi l}{N}) (k^2 - \beta_n^2) f_{nr}^* f_{n1} \right\} \\ & \quad + \frac{2bh}{\omega\mu_0} \sqrt{k_0^2 - \left(\frac{r\pi}{2h}\right)^2} \delta_{r1} \end{aligned} \quad (r=1, 2, \dots, 5) \quad (7)$$

In the above equation, f_{ij}^* is the complex conjugate of f_{ij} and, δ_{ij} is equal to one when $i=j$ and is zero when $i \neq j$. $\overline{G}_n(a, \phi:a, \phi')$ is the result of an azimuthal integration for $G_n(a, \phi:a, \phi')$ at an aperture surface, the center of which in azimuth is equal to ϕ . Therefore, $\overline{G}_n(a, \phi:a, \phi')$ is equal to $G_n(a, \phi:a, \phi')$ in Eq. (2) when the azimuthal aperture factor,

$2b \cdot \sin(mb/a)/(mb/a)$ is replaced with its square. Through an inversion of a 5×5 matrix, the unknown reflection coefficient $\Gamma_q(p)$'s are solved. As a result, the aperture field of the reference column for a p -th phase sequence excitation, $E_\phi(z,p)$ is determined as in Eq. (4). Repeating the same procedure for N number of phase sequence excitations using a FFT algorithm and superposing the resultant aperture field, $E_\phi(z,p)$ for each column separately, the aperture E -fields of all columns for a single reference column excitation can be found. The waveguides of all non-excited slot columns are match-terminated to the sides of the generators at this time. The aperture E -field of Q -th slot column is given as

$$E_\phi^Q(z) = \frac{1}{N} \sum_{p=0}^{N-1} E_\phi^Q(z,p) = \frac{1}{N} \sum_{p=0}^{N-1} E_\phi(z,p) \exp(i \frac{2\pi}{N} Qp) \quad (8)$$

From these aperture fields, the far field pattern at the elevation angle Θ can be derived in the similar way as in Chapter 3.

In the following section, the evaluation of an azimuthally integrated Green's function $\overline{G}_n(a,\phi;a,\phi)$ will be discussed.

4.3 Green's function in the near zone.

As discussed in the preceding section, the function $\bar{G}_n(a, \phi; a, \phi')$ is the azimuthal integration of $G_n(a, \phi; a, \phi')$ of Eq. (2) at an aperture surface whose center in azimuth is located at ϕ . More precisely, the aperture is a closed one, a conducting surface in place of an open end of a waveguide except when $\phi = \phi'$, because, from the definition of a Green's function, G_n must be evaluated in the environment where there exists only a single column as the source whose center in azimuth is located at ϕ' and all other slot columns are closed.

The azimuthally integrated Green's function $\bar{G}_n(a, \phi; a, \phi')$ can be calculated in the same way as in the previous chapters. In other words, in most of the angular region of a cylinder circumference, $\bar{G}_n$ is evaluated with a Watson transform method while a planar approximation is taken in the narrow angular sector around the source column where the residue series of a Watson transform has a slow convergence because the angular distance $|\phi - \phi'|$ is small. Since the harmonic Green's functions have importance in the narrow angular sector around the source column, the calculations of these harmonics are carried out exclusively using the planar approximation. In the remaining region, only the fundamental Green's function $\bar{G}_0$ or, $\bar{G}_0$ and the first harmonic $\bar{G}_{-1}$ have signifi-

cance because of the exponential decays of the harmonics as the angular distance $|\phi - \phi'|$ is increasing. The importance of the first harmonic in this region arises when the lowest pole ν'_1 of $\bar{G}_{-1}$ is close to or on the real axis of a complex plane due to a wide angle scanning in elevation.

The Watson transform of a harmonic series for $\bar{G}_n(a, \phi; a, \phi')$ into a residue series is carried out exactly in the same way as in the previous chapters. Furthermore, since $\bar{G}_n$ has the same denominator as G_n in Eq. (2) of Chapter 3, the pole locations on a complex plane are also same if the cylinder radius and the thickness of the dielectric layer are equal to those in Chapter 3. However, because of the difference in the numerators of these two Green's functions, the beam scan characteristics as well as the elementary pattern of a slot array are not identical to those of a dipole array. The difference in the beam scan characteristics between these two kinds of arrays can be predicted by comparing the emerging poles for the fundamental Green's function G_0 in Eq. (2) with those for the corresponding Green's function in Eq. (2) of Chapter 3 when the elevation beam angle is increased from zero. When θ is equal to zero, because of the vanishing value of $\beta_0 (= k_0 \sin \theta)$, the same factors in the numerator and the denominator of the fundamental Green's function are cancelled. The way of cancellation for G_0 of Chapter 3 is opposite to the case of G_0 in the present chapter. In the preceding

chapter, the cancelled factor is R , the zeroes of which are the emerging poles when the beam is lifted away from the horizontal plane, and the poles of the Green's function for $\theta = 0^\circ$ are the zeroes of the factor, $[X(\xi_0/\xi)H_y^1(\sqrt{k^2 - \xi_0^2}a) - T(\xi_0/\xi) \cdot H_y^2(\sqrt{k^2 - \xi_0^2}a)]$ while, in the present case, the situation is the reversed one. In other words, the poles of G_0 for a slot column are zeroes of R when θ is equal to zero and the emerging poles as θ is varied from zero are generated from the zeroes of the factor, $[X(\xi_0/\xi)H_y^1(\sqrt{k^2 - \xi_0^2}a) - T(\xi_0/\xi)H_y^2(\sqrt{k^2 - \xi_0^2}a)]$. This reversal between the already existing poles and the emerging poles when θ is infinitesimally varied from zero results in profound changes in the beam scan characteristics as well as the elementary pattern of a slot array from those of a dipole array, because the real pole γ_1' which is instrumental for the launching of a resonance inside the dielectric layer is contained in the factor R . Because of the absence of the real pole γ_1' due to the elimination of the factor R , a resonance can not be launched in the case of a dipole array regardless of the azimuthal beam scan direction ϕ as far as the beam of the array stays on the horizontal plane. When the beam is lifted away from the horizontal plane, the effects of a resonance are gradually appearing on the dipole input impedance curve as well as on the elementary pattern of the array since, because of the presence of the factor R in the numerator of G_0 , the amplitude of the resonant wave inside the dielectric is slowly increasing from zero.

For a slot array, however, since the factor R is the denominator of G_0 when θ is equal to zero, a resonance can be launched from the first by an appropriate azimuthal scan direction ϕ which can greatly reinforce the standing wave inside the dielectric layer caused by the real pole ν'_1 . As will be shown in the following section, the magnitude of the reflection coefficient Γ_1 for the dominant mode of a waveguide is close to one when a resonance arises inside the dielectric layer, and most of the incident power inside a waveguide is reflected at a waveguide-dielectric interface. Consequently, the elementary pattern cut on the horizontal plane shows a pattern dip in the direction of the azimuthal scan angle ϕ_r at which the resonance is launched. As in the preceding chapter, the resonance angle ϕ_r can be determined by the real pole ν'_1 . As the elevation scan angle θ is increased from zero, the major change in the elementary pattern cut is the shift of the dip angle ϕ_r . The change in ϕ_r can be predicted by tracing the real pole on the real axis of a complex plane as a function of the elevation scan angle θ . The impacts on the elementary pattern cut caused by the emerging poles from the factor, $[X(\xi_0/\xi)H_y^1(\sqrt{k^2 - \beta_0^2}a) - T(\xi_0/\xi)H_y^2(\sqrt{k^2 - \beta_0^2}a)]$ are minor since no real pole is contained in this factor and the residues of these poles do not grow fast because of the presence of the factor also in the numerator of a Green's function in Eq.(2). The real change in the elementary pattern cut takes

place when multiple resonances are launched inside the dielectric layer because of a wide angle scanning in elevation. At multiple resonances, in addition to a real pole from G_0 , the lowest pole from the first harmonic G_{-1} , whose initial location is on the imaginary axis way up from the real axis when θ is equal to zero, is brought down to the real axis due to an increased elevation scan angle θ . Because of the interactions between these two real poles, the pattern changes on the launching of the multiple resonances are more complex than a simple addition of another pattern dip to the existing one which is responsible for a single resonance. The elevation scan angle θ at the launching of the multiple resonances can be easily predicted by investigating the downward shift of the lowest pole ν'_1 from G_{-1} using a planar approximation as an intermediate step. In such ways, the important characteristics of the elementary pattern are completely determined by the resonant poles of the fundamental and the first harmonic Green's function.

The azimuthally integrated Green's function $\overline{G}_n(a, \phi; a, \phi')$ can be represented with the following residue series of the Watson transform.

$$\overline{G}_n(a, \phi; a, \phi') = \frac{-i(2\pi)^{-1}}{\omega \mu_0 \sqrt{k^2 - k_n^2} a} \sum_{\nu_n, \nu'_n} \frac{\cos \nu (|\phi - \phi'| - \pi)}{\sin \nu \pi} \left[\frac{2b \sin(\frac{\nu b}{a})}{(\frac{\nu b}{a})} \right]^2 \text{Res}(\nu)$$

(9)

where

$$\text{Res}(\nu) = \frac{\left\{X\left(\frac{\xi_0}{\epsilon}\right)H'_1\left(\sqrt{k^2-\beta_n^2}a\right)-T\left(\frac{\xi_0}{\epsilon}\right)H'_2\left(\sqrt{k^2-\beta_n^2}a\right)\right\}\left\{X(0)H'_1\left(\sqrt{k^2-\beta_n^2}a\right)-T(0)H'_2\left(\sqrt{k^2-\beta_n^2}a\right)\right\}-\left(\frac{\nu\beta_n}{k\sqrt{k^2-\beta_n^2}}\right)^2\left(\frac{k^2-k_0^2}{k_0^2-\beta_n^2}\right)^2Z(\rho')^2}{\frac{\partial}{\partial\nu}\left[\left\{X\left(\frac{\xi_0}{\epsilon}\right)H'_1\left(\sqrt{k^2-\beta_n^2}a\right)-T\left(\frac{\xi_0}{\epsilon}\right)H'_2\left(\sqrt{k^2-\beta_n^2}a\right)\right\}R-\left(\frac{\nu\beta_n}{k\sqrt{k^2-\beta_n^2}}\right)^2\left(\frac{k^2-k_0^2}{k_0^2-\beta_n^2}\right)^2Z(\rho')W\right]}$$

In the neighborhood of the source column where the residue series of Eq. (9) has a slow convergence due to a small angular distance $|\phi-\phi'|$, $\bar{G}_n(a, \phi; a, \phi')$ can be evaluated with a planar approximation, in which the cylindrical surfaces are replaced with planar ones. In a planar situation, the aperture integrated Green's function $\bar{G}_{np}$ is given by the following equation.

$$\bar{G}_{np}(a, \phi; a, \phi') = \frac{2(2\pi)^{-2}}{\omega\mu_0} \int_0^\infty \left(2b \frac{\sin k_y b}{k_y b}\right)^2 \frac{\cos(|\phi-\phi'|a k_y)}{\sqrt{k^2-\beta_n^2-k_y^2}} \cdot$$

$$\frac{\left[u\left(\frac{\xi_0}{\epsilon}\right)+V u\left(-\frac{\xi_0}{\epsilon}\right)\right]\left[u(1)+V u(-1)\right]+\left(\frac{k_y \beta_n}{k}\right)^2\left(\frac{k^2-k_0^2}{k_0^2-\beta_n^2}\right)(1-2V+V^2)}{\left[u\left(\frac{\xi_0}{\epsilon}\right)+V u\left(-\frac{\xi_0}{\epsilon}\right)\right]R_P+\left(\frac{k_y \beta_n}{k}\right)^2\left(\frac{k^2-k_0^2}{k_0^2-\beta_n^2}\right)(1-V^2)} dk_y$$

(10)

where V, U and R_p are the same functions as in Eqs. (6a) to (6c) of Chapter 3. Comparing the above equation with Eq. (10) of Chapter 3, except the azimuthal aperture factor and $\sqrt{k^2-\beta_n^2-k_y^2}$ in the above equation, the denominators of the integrands in the two equations are exactly the same. Therefore, the integration contour in the above equation must be defined carefully

to avoid the same pole, k_{yp} of Eq.(10) of Chapter 3 on the real axis. The modification of the contour around the real pole as well as various branch cuts in a numerical integration are made in the same way as in Chapter 3, which are indicated in Fig.(3-5). Also, as in Chapter 3, the discovery of the real pole or the lowest pole of G_n in Eq.(2) can be expedited using the relationship between the real pole k_{yp} in Eq.(10) and V'_1 given as $V'_1 \approx \sigma' k_{yp}$.

For lower harmonics as well as the fundamental ($n=0$), the integration of Eq.(10) is carried out numerically. In a numerical integration, because of the azimuthal aperture factor $[2b \sin k_y b / (k_y b)]^2$, the contribution to the integral beyond the interval $0 \leq k_y \leq \pi/b$ is negligible and, therefore, the infinite integration can be approximated with the finite integral in this interval.

For a higher harmonic with a large value of $|n|$, the integrand in Eq.(10) is greatly simplified since the exponential V can be crossed out because of its small magnitude compared with other terms. Also, since the magnitude of $\bar{G}_{np}(a, \phi; a, \phi')$ is exponentially decaying as the angular distance $|\phi - \phi'|$ is increased, a higher harmonic is important mostly for the case when $\phi = \phi'$. A field point in this case is located at the surface of the source aperture. When $\phi = \phi'$, $\bar{G}_{np}$ in Eq.(10) with a large value of $|n|$ can be reduced to the following equation.

$$\bar{G}_{np}(a, \phi; a, \phi) = \frac{(2\pi)^{-2}}{\omega\mu_0} \int_{-\infty}^{\infty} (2b \frac{\sin k_y b}{k_y b})^2 \frac{dk_y}{\sqrt{\rho_k^2 - \beta_n^2 - \rho_y^2}} \quad (11)$$

Since the major contribution to the integral comes from the interval, $|k_y| \leq \pi/b$ because of the azimuthal aperture factor, the square root for a large $|n|$ in the above equation can be approximated as

$$\sqrt{\rho_k^2 - \beta_n^2 - \rho_y^2} \approx i\sqrt{\beta_n^2 - \rho_k^2} \left(1 + \frac{\rho_y^2}{2(\beta_n^2 - \rho_k^2)}\right) \quad (12)$$

From Eqs. (11) and (12), $\bar{G}_{np}$ is given in the following form.

$$\begin{aligned} \bar{G}_{np}(a, \phi; a, \phi) &\approx \frac{8b^3 \sqrt{\beta_n^2 - \rho_k^2}}{i\omega\mu_0(2\pi)^2} \int_{-\infty}^{\infty} \frac{\sin^2 x}{x^2} \frac{dx}{g^2 + x^2} \\ &\approx \frac{8b^3 \sqrt{\beta_n^2 - \rho_k^2}}{i\omega\mu_0(2\pi)^2} \left[\int_{-\infty}^{\infty} \frac{\sin 2x}{x(x^2 + g^2)} dx - \int_{-\infty}^{\infty} \frac{1 - \cos 2x}{(x^2 + g^2)^2} dx \right] \\ &\approx \frac{8b^3 \pi \sqrt{\beta_n^2 - \rho_k^2}}{i\omega\mu_0(2\pi)^2 g^2} \left[1 - \frac{1 - e^{-2g}}{2g} \right] \quad (13) \end{aligned}$$

where $g = \sqrt{2(\beta_n^2 - \rho_k^2)}b$. In the above derivation, the infinite integrals on the real axis are evaluated using the residue theorem on a complex plane. The discrepancy between the approximation of Eq. (13) and the direct numerical integration in Eq. (10) is decreasing as $|n|$ is increasing. Even at $|n|=4$, the discrepancy between the two is small. For a larger $|n|$,

Eq.(13) yields an excellent result.

As explained in the derivation of Eq.(15) of Chapter 3, the effects of rotating waves around the cylinder must be added to $\bar{G}_{np}$ to approximate $\bar{G}_n$, because these waves can not be taken into account in the planar approximation. From Eq. (9), by subtracting from the residue of the real pole $\mathcal{V}'_1$ the direct wave term that is already included in the planar approximation, the term corresponding to the rotating waves can be found. The rotating waves due to higher poles are ignored.

$$\bar{G}_n(a, \phi; a, \phi') = \bar{G}_{np}(a, \phi; a, \phi') + \frac{2(2\pi)^{-1} \cos \mathcal{V}'_1 |\phi - \phi'|}{\omega \mu \sqrt{p_k^2 - p_h^2} a} \frac{1}{1 - e^{-i2\mathcal{V}'_1 \pi}} \text{Res}(\mathcal{V}'_1) \quad (14)$$

The correction term in the above equation is necessary only when the lowest pole $\mathcal{V}'_1$ of $\bar{G}_n$ is a real pole. Therefore, except for $\bar{G}_0$ or, $\bar{G}_0$ and $\bar{G}_{-1}$, $\bar{G}_n$ can be evaluated with $\bar{G}_{np}$ without the correction term.

At shadow boundaries, $\bar{G}_0$ in the above equation has a good agreement with the result from Eq.(9). For a smooth transition of $\bar{G}_0$ at the shadow boundaries, sixteen residue terms are included in the calculation of Eq.(9).

4.4 Reflection coefficient of the dominant waveguide mode.

Using the approximation method for the calculation of a Green's function, $\bar{G}_n(a, \phi; a, \phi')$ discussed in the preceding section, the summation with respect to n in Eq. (7) can be carried out. In this summation of Green's functions, higher harmonics are necessary only in the narrow angular sector around the reference slot column where the angular distance between the reference and the source column $|\phi - \phi'| (= \frac{2\pi}{N} l)$ is small. Outside this region, G_0 or, G_0 and G_{-1} can account for the infinite summation with respect to n . The harmonics are important if $|\phi - \phi'|$ is equal to zero, which is the case when the reference and the source column are identical. In this special case, thirty harmonic terms ($|n| \leq 15$) are included in the summation. Since these higher harmonics can be evaluated using the approximation formula given in Eq. (13), the summation of these large number of harmonics does not present a computational difficulty.

Once these summations of Green's functions for $0 \leq q \leq N-1$ are known, using FFT algorithm, the summations with respect to the slot columns can be carried out simultaneously for $0 \leq p \leq N-1$. As a result, the reflection coefficients $\Gamma_q(p)$'s for $0 \leq p \leq N-1$ can be derived in an efficient way.

The variation of the reflection coefficient $\Gamma_1(p)$ for the dominant mode is of special interest since $|\Gamma_1(p)|^2$ is

proportional to the reflected power at the waveguide-dielectric interface for a p -th phase sequence excitation when only the dominant mode is allowed to propagate inside a waveguide. The variation of $\Gamma_1(p)$ as a function of the azimuthal scan angle ϕ is plotted in Figs. (4-2) to (4-5). The azimuthal scan angle ϕ is determined from the imposed phase sequence p as in Eq. (41) of Chapter 2. In these figures, the total number of slot columns N , the thickness of the dielectric layer $\rho'a$ and the width of a slot $2b$ are fixed as $N=200$, $\rho'a=0.15\lambda_0$ and $2b=0.15\lambda_0$ where λ_0 is a wavelength in the free space. A slot center-to-center distance in azimuth d_c , a slot center-to-center distance in the z -direction $2d$, the length of a slot $2h$, the elevation beam direction θ and the dielectric constant of the layer ϵ are variables in the following discussion. In Fig. (4-2), the variation in Γ_1 is plotted when $d_c=0.6\lambda_0$, $2d=0.7\lambda_0$, $2h=0.6\lambda_0$, $\theta=0^\circ$ and ϵ is varied from ϵ_0 to $2.5\epsilon_0$. For $\epsilon \geq 1.5\epsilon_0$, the maximum value of $|\Gamma_1|$ is close to one. At a corresponding azimuthal scan angle where this happens, the incident power of a waveguide is almost totally reflected at a waveguide-dielectric interface causing a pattern null in the same direction. This near total reflection can be explained by the launching of a resonance inside the dielectric layer and the azimuthal scan angle where this occurs can be predicted from the real pole ν'_1 of a Green's function. The mechanism for the launching of a resonance is similar to those cases in Chapter 3. In other words, for an appropriate phase sequence excitation,

the standing waves inside the layer arising from N number of slot columns can be added in phase. As Eq.(19) of Chapter 3 indicates, the phase sequence at the launching of a resonance is the closest integer either to the real pole ν'_1 or $N-\nu'_1$. At this phase sequence, for a given finite aperture E-field, an intense H-field is built up at the surface of an array. Therefore, at an aperture surface, the ratio of $|H_z/E_\phi|$ is extremely large at a resonance. Ignoring higher modes inside a waveguide, the ratio, $|H_z/E_\phi|$ at an aperture surface can be approximated in terms of the modal admittance for the dominant mode of the waveguide. In such a case, from Eqs.(4) and (5),

$$|H_z/E_\phi| = \sqrt{\frac{\epsilon_0}{\mu_0}} \sqrt{1 - \left(\frac{1}{4h}\right)^2} \left| \frac{1 - \Gamma_1(p)}{1 + \Gamma_1(p)} \right| \quad (15)$$

The only way the ratio $|H_z/E_\phi|$ has a large magnitude at a resonance launching phase sequence p_r is that the magnitude of $\Gamma_1(p_r)$ is close to one and its phase is 180 degree. As a result, at a resonance, most of the incident power inside a waveguide is reflected at a waveguide-dielectric interface. A pattern dip is resulted in to the direction of ϕ_r , because as Eq.(4) indicates, with this value of $\Gamma_1(p_r)$, the aperture E-field is too weak to radiate enough power to the direction of ϕ_r .

For the array whose reflection coefficient Γ_1 is plotted in Fig.(4-2), the real pole $\nu'_1(k_{yp})$ of the Green's

function $G_o(G_{op})$ in Eq. (2) (Eq. (10)) as well as the expected resonance angle ϕ_r in an azimuthal scan, which is determined from p_r according to Eq. (41) of Chapter 2, are given in Table 1 for the array parameters mentioned earlier.

Table 1. The real pole $\nu'_1(k_{yp})$ of $G_o(G_{op})$ and the expected resonance scan angle ϕ_r .

ξ	$k_{yp} (1/\lambda_0)$	$k_{yp} \rho'$	ν'_1	ϕ_r
$1.5\xi_0$	6.55287	126.13385	$127.28617 + i0.38169$	37.47°
$2 \xi_0$	6.96429	134.05282	$134.32327 + i0.00306$	33.37°
$2.5\xi_0$	7.44161	143.24069	$143.10752 + i0.7 \times 10^{-6}$	28.36°

Comparing Table 1 and the computed results plotted in Fig. (4-2), the theoretical analysis given for a resonance can be confirmed. At an expected resonance angle ϕ_r in an azimuthal scan, the magnitude of Γ_1 is close to one and its phase is 180 degrees indicating a nearly total reflection of the incident wave at a resonance. For $\xi = 1.5\xi_0$, as Table 1 shows, the lowest pole ν'_1 has still a considerable imaginary part and, because of a wave decay around a cylinder circumference due to the imaginary part, the resultant resonance is not strong enough to make the magnitude of the reflection coefficient equal to one. When ξ is increased to $2\xi_0$ or $2.5\xi_0$, the lowest pole ν'_1 is almost real and, as

a result of a strong resonance, $|\Gamma_1|$ is nearly equal to one. In Fig. (4-2A), the magnitude of the reflection coefficient Γ_1 at a resonance is rapidly increasing as ξ is varied from ξ_0 . This is due to the fast downward shift of the lowest pole ν'_1 towards the real axis when ξ is deviated from ξ_0 as in Fig. (3-2). Also, it is noticed in Fig. (4-2) that, as ξ is increasing, the resonance angle ϕ_r is moving towards the broadside direction, which can be expected from the increasing value of the lowest pole ν'_1 on the real axis with ξ as Fig. (3-2) indicates. In this way, the magnitude of Γ_1 at a resonance angle ϕ_r as well as the angle itself can be completely determined from the location of the lowest pole ν'_1 on a complex plane.

The close relation between the resonance angle ϕ_r and the lowest pole ν'_1 is further illustrated in Fig. (4-3) where an azimuthal slot center-to-center distance d_c is varied while other array parameters are fixed as $2d=0.7\lambda_0$, $2h=0.6\lambda_0$, $\theta=0^\circ$ and $\xi=2.5\xi_0$. Table 2 shows the pole location ν'_1 of G_0 for these parameters. In this case, the pole k_{yp} of the corresponding planar Green's function G_{op} is found in Table 1 as $k_{yp}=7.44161/\lambda_0$. Comparing Table 2 and Fig. (4-3), at an expected resonance angle ϕ_r in an azimuthal scan, the magnitude of Γ_1 is close to one and the phase of it is 180 degrees as predicted. When a slot center-to-center distance in azimuth d_c is varied, the impact on the elemen-

tary pattern incurred by the resultant shift in ϕ_r will be discussed later.

Table 2. The lowest pole ν'_1 of G_o and the expected resonance angle ϕ_r in an azimuthal scan.

d_c	$k_{yp} \rho'$	ν'_1	ϕ_r
$0.55 \lambda_o$	131.39698	131.26633	38.85°
$0.6 \lambda_o$	143.24069	143.10752	28.36°
$0.7 \lambda_o$	166.92809	166.79492	13.63°

In Figs.(4-4) and (4-5), by increasing the elevation beam angle Θ from zero towards the cylinder axial direction, the variation of Γ_1 in an elevation scan is plotted. An azimuthal slot center-to-center distance d_c and the dielectric constant of the layer are fixed as $0.6\lambda_o$ and $2.5 \epsilon_o$ in both figures, respectively. A z-direction slot center-to-center distance $2d$ and the length of a slot $2h$ are $0.6\lambda_o$ and $0.55\lambda_o$ in Fig.(4-4) while, in Fig.(4-5), $2d=0.7\lambda_o$ and $2h=0.6\lambda_o$. The real pole $\nu'_1(k_{yp})$ of the fundamental Green's function $G_o(G_{op})$ is given in Table 3 as a function of the elevation beam angle Θ . Both for Figs.(4-4) and (4-5), the resonance angle ϕ_r in Table 3 predicts the azimuthal scan angle where the magnitude of Γ_1 is maximum and its phase is 180 degrees, since G_o is independent of the z-direction slot center-to-center distance $2d$ and the length of a

slot 2h.

Table 3. The real pole $\nu'_1(k_{yp})$ of $G_o(G_{op})$ and the expected resonance angle ϕ_r .

θ	$k_{yp}(1/\lambda_o)$	$k_{yp} \cdot \rho'$	ν'_1	ϕ_r
0°	7.44162	143.24069	143.10752	28.36°
15°	7.26176	139.77863	139.65929	31.17°
25°	6.95173	133.81109	133.71650	37.36°
30°	6.74596	129.85039	129.77287	42.34°

As the elevation beam angle θ is increasing, the first harmonic Green's function G_{-1} has importance since the lowest pole ν'_1 of G_{-1} is rapidly moving down towards the real axis of a complex plane. For the array whose reflection coefficient Γ_1 is shown in Fig.(4-4), the location of the lowest pole ν'_1 from G_{-1} is tabulated in Table 4. As the table shows, the lowest pole ν'_1 for $2d=0.6\lambda_o$ and $2h=0.55\lambda_o$ stays on the imaginary axis up until $\theta=25^\circ$. When θ is further increased to 30° , ν'_1 is almost real.

Table 4. The lowest pole $\nu'_1(k_{yp})$ of $G_{-1}(G_{-1p})$.

θ	$k_{yp}(1/\lambda_o)$	$k_{yp} \cdot \rho'$	ν'_1	ϕ_r
25°	i2.39193	i 46.04132		
30°	1.28186	24.67406	25.93891+i0.00118	

Table 5 shows the lowest pole γ'_1 of G_{-1} when $2d$ and $2h$ are increased to $0.7\lambda_0$ and $0.6\lambda_0$, respectively. The reflection coefficient Γ_1 of this array is shown in Fig. (4-5). For these values of $2d$ and $2h$, the lowest pole γ'_1 of G_{-1} , which is initially located on the imaginary axis, is rapidly dropping down to the real axis even for a slight increase in θ . As Table 5 shows, γ'_1 is almost real at $\theta=15^\circ$.

Table 5. The lowest pole $\gamma'_1(k_{yp})$ of $G_{-1}(G_{-1p})$.

θ	k_{yp}	$k_{yp} \cdot \rho'$	γ'_1	ϕ_r
15°	1.16557	22.43557	23.83117+i0.00119	11.95°

For Fig. (4-4), Table 3 and 4 predict the resonance angle ϕ_r . Up until $\theta=25^\circ$, there is only a single resonance arising from the real pole of G_0 . When θ is increased from zero, the shift of ϕ_r in this figure is accurately predicted from Table 3. When θ is equal to 30° , due to the emerging new real pole γ'_1 from G_{-1} , multiple resonances are taking place inside the dielectric layer. The impact on Γ_1 caused by these multiple resonances is more complex than a single resonance. In Fig. (4-4A), the shift of the already existing peak of Γ_1 when θ is increasing from 25° to 30° is expected from Table 3. However, the emerging new peak of Γ_1 at $\theta=30^\circ$ around the resonance angle ϕ_r predicted from the real pole of G_{-1} in Table

4 has a different shape from the existing one. Not only the emerging new peak is narrow in azimuth, it also has an accompanying dip of $|\Gamma_1|$. As will be discussed later, the impact of this new resonance upon the elementary pattern is quite different from that of the already existing resonance. Therefore, an elementary pattern cut undergoes a significant change when θ is increased from 25° to 30° . Table 3 and 5 predict the resonance angle ϕ_r 's in Fig.(4-5). The only difference between Fig.(4-4) and Fig.(4-5) is that, in Fig.(4-5), multiple resonances can be launched for a smaller elevation angle θ because of the increased value of a z-direction slot center-to-center distance $2d$.

4.5 Elementary patterns.

Eq.(8) determines the aperture E-field of each slot column for a single reference column excitation in the environment of other slot columns match-terminated to the sides of generators. The elementary pattern cut at a given elevation angle θ , which is specified by the z-direction linear phase variation of the reference column excitation, can be found from the aperture E-fields for the N number of slot columns. To superpose the radiation pattern from these aperture fields, first the E- and H-field in the far zone caused by a single isolated slot column must be known. The derivation for the far field of a single slot column can be carried out in the same way as in Section 3.5 of the preceding chapter. As in Chapter 3, this far field can be superposed with the direct ray from the aperture to a field point as well as various other ray components, each of which is reflected at the dielectric-free space interface once or multiply and propagates inside the dielectric layer before it radiates to a given azimuthal direction. All these ray components are found using the method of a stationary phase integration in the evaluation of the far field Green's function, which is discussed in detail in the preceding chapter. In order to find the elementary pattern cut at a given elevation angle θ , the superposition of the N number of aperture E-fields using the derived far field pattern of a single isolated slot column is carried

out in the same way as in Eq.(36) of Chapter 3.

For the arrays, the reflection coefficients of which are plotted in Fig.(4-2), the elementary patterns on the horizontal plane are shown in Figs.(4-6A) to (4-6D) as the dielectric constant of the layer ξ is increased from ξ_0 to $2.5\xi_0$. As expected from the characteristics of the reflection coefficient Γ_1 in Fig.(4-2), the patterns on the horizontal plane show that, for an increased ξ , the dip of a pattern off broadside is getting narrower and deeper, and the corresponding dip angle ϕ_d is shifting towards the broadside direction. In the same way as the magnitude of Γ_1 in Fig.(4-2) is almost equal to one for $\xi > 1.5\xi_0$, a pattern dip in Fig.(4-6) is saturated for the same range of ξ . With ξ , the maximum power beyond a dip angle ϕ_d is also increasing. For $\xi = 2.5\xi_0$, the maximum power beyond ϕ_d is comparable to the broadside power. Another characteristic the elementary patterns in Fig.(4-6) show is the ripples at broadside.

A dip angle ϕ_d is shifted towards the direction of the broadside from a corresponding resonance angle ϕ_r in an azimuthal scan in Table 1 where the magnitude of Γ_1 is close to one and its phase is 180° . The discrepancy between ϕ_d and ϕ_r is due to the fact that the resonance angle ϕ_r is computed from a resonance launching phase sequence p_r using Eq.(41) of Chapter 2 which is based on a planar approximation at a circumference of a large radius cylinder. In this approximation, the effect of the curvature for the cylindrical surface is

ignored. Even for $\xi = \xi_0$, a small discrepancy is found between the dip angle ϕ_d in Fig.(4-6A) and the grating lobe angle ϕ_{gl} for a comparable planar array given by Eq.(42) of Chapter 2. ϕ_{gl} in this case is found as 41.81 degrees which is slightly beyond ϕ_d in Fig.(4-6A). When ξ is not equal to ξ_0 , because of the curvatures of the dielectric-free space interface as well as the conducting surface, the discrepancy between ϕ_d and ϕ_r is increased. Even though ϕ_r from the planar approximation does not yield a precise location for a dip angle ϕ_d , it provides the approximate location for ϕ_d as functions of the elevation angle θ , the dielectric constant of the layer ξ and an azimuthal slot center-to-center distance d_c , which is important in an array design. When θ , ξ and d_c are varied, the close correspondence between ϕ_r and ϕ_d will be illustrated from the following examples.

By increasing an azimuthal slot center-to-center distance d_c from $0.6\lambda_0$ to $0.7\lambda_0$, the elementary patterns on the horizontal plane in Fig.(4-6) are changed to the patterns in Fig.(4-7). the dip angle of a pattern is moved towards to the broadside direction resulting in a narrower beamwidth. For $\xi = \xi_0$, as in Fig.(4-6A), the dip angle ϕ_d in Fig.(4-7A) is located slightly inside the grating lobe angle ϕ_{gl} of a comparable planar array which is determined from Eq.(42) of Chapter 2 as 25.38 degrees. As in Fig.(4-6), when ξ is not equal to ξ_0 , the dip angle ϕ_d is further removed from a corresponding resonance angle ϕ_r to the broadside direction. By comparing Figs.

(4-6D), (4-7E) and (4-8) with Table 2 or Fig.(4-3) where the reflection coefficients for these arrays are plotted, the close relationship between a dip angle ϕ_d and a resonance angle ϕ_r in an azimuthal scan can be seen also in these cases. In Fig.(4-8), by reducing an azimuthal slot center-to-center distance d_c to $0.55\lambda_0$, a significant increase in the beamwidth is achieved. The power beyond the dip angle ϕ_d is considerably reduced in this case. Elementary patterns for cylindrical arrays of waveguide elements with no dielectric covers have been presented in Ref. 3. The elementary patterns derived in the paper show similar characteristics as the patterns plotted in Fig.(4-6A) and (4-7A), which are the special cases of the cylindrical arrays with dielectric covers when the dielectric constants are equal to ϵ_r .

When the elevation angle is increased from zero to thirty degrees, the elementary pattern cut at the angle θ is plotted in Figs.(4-9A) to (4-9D). The reflection coefficient Γ_1 for this array is given in Fig.(4-4). Also, Table 3 and 4 provide the information on the resonance angle ϕ_r 's. The array parameters are given as $\xi=2.5\xi_0$, $2d=0.6\lambda_0$, $2h=0.55\lambda_0$ and $d_c=0.6\lambda_0$. Until the elevation angle θ reaches thirty degrees in Fig.(4-9D), the only change in the pattern cut for the increase in θ which has significance is the shift of the dip angle ϕ_d from the broadside to the endfire direction. Even though the exact dip angle ϕ_d is located to the direction of the broadside from the resonance angle ϕ_r , this is essen-

tially what Table 3 indicates. A fundamental change in the pattern cut arises when the elevation angle θ is increased to thirty degrees. For this elevation angle, multiple resonances are launched inside the dielectric layer as the lowest pole γ_1' from G_{-1} becomes purely real as in Table 4. This is also indicated in Fig.(4-4A) where the reflection coefficient Γ_1 is plotted. As shown in the figure, the effect of this emerging real pole from G_{-1} on the reflection coefficient Γ_1 is quite different from that of the already existing pole γ_1' from G_0 . This new real pole results in a peak and a dip which are narrow in azimuth compared to the already existing peak of $|\Gamma_1|$. Because of the narrow azimuthal width, the new peak of $|\Gamma_1|$ does not cause a pattern dip at the corresponding azimuthal direction as in the case of the already existing peak. As discussed in Section 3.5, because of the curvatures of the dielectric-free space interface and the conducting surface, the radiation to a given azimuthal direction ϕ is composed of the direct ray from the apertures and various other ray components, each of which has a different ray-path in the dielectric layer before it radiates to the given direction ϕ . The phase sequences which are responsible for these ray components are spread around the phase sequence corresponding to the direct ray. Therefore, a pattern dip can be materialized only when the peak of $|\Gamma_1|$ has a considerable azimuthal width around the resonance scan angle ϕ_r as in the case of the already existing peak. Instead of producing an additional pattern dip, the resonance

caused by the emerging new real pole from G_{-1} brings about a fundamental change on the overall pattern cut which is characterized by the excessive pattern oscillations in Fig.(4-9D). When multiple resonances are launched inside the dielectric layer, the aperture E-fields for the slot columns are rapidly changed from those of a single resonance. For a single resonance, the aperture E-field of a slot column is steadily decreasing as the angular distance between the column and the reference column is increasing. Therefore, only the aperture E-fields of those slot columns close to the reference column have significant contributions to the far field. In the case of multiple resonances, a slot column far from the reference column can have a considerable aperture E-field. Actually, except in the neighbourhood of the reference column, the distribution of aperture E-fields among remaining slot columns is oscillatory in the magnitude of the field. This wide distribution of the aperture E-field among the slot columns of an array can be explained from Eq.(8), where the aperture E-field of Q -th column is superposed with the product of a phase sequence term, $Q^{i\frac{2\pi}{N}Qp}$ and the response aperture field of the reference column $E_\phi(z,p)$ for a p -th phase sequence excitation. As far as $E_\phi(z,p)$ is a slowly varying function of p , the aperture E-field of Q -th column is small when Q is large. However, when $E_\phi(z,p)$ has discontinuities with respect to p , $|E_\phi^Q|$ can have a significant magnitude even when the column is far from the reference column with a large Q . Since the aperture E-field is related to the

reflection coefficient Γ_1 through Eq.(4), the variation of $|\Gamma_1|$ in Fig.(4-4A) has a direct impact on the reference column aperture E-field $E_\phi(z,p)$. Because of the dip and the peak for $|\Gamma_1|$ both narrow in azimuth caused by the emerging real pole from G_{-1} , $E_\phi(z,p)$ has also discontinuities around the new resonance launching phase sequence p_r . As a result, a wide distribution of the aperture E-fields among the slot columns can be made. This wide and oscillatory aperture E-field distribution of the array is responsible for the rapid change of a pattern cut at the launching of the multiple resonances. The elevation angle θ where this overall pattern change takes place depends on a z-direction slot center-to-center distance $2d$. When $2d$ is increased from $0.6\lambda_0$ to $0.7\lambda_0$ and $2h$ from $0.55\lambda_0$ to $0.6\lambda_0$, the elementary pattern cut at $\theta=15^\circ$ in Fig.(4-10) already shows the effects of the multiple resonances. As Table 5 and Fig.(4-5) indicate, the lowest pole γ'_1 from G_{-1} is almost real at the elevation angle.

4.6 Conclusion.

An elementary pattern cut for a conformal slot array of waveguide elements is characterized with ripples at broadside and a pattern dip off broadside. The pattern dip determines the beamwidth of an elementary pattern and, consequently, the angular width of the array sector to be activated to scan the beam to a given azimuthal direction is roughly twice the dip angle ϕ_d .

Through a complex plane analysis, a close relationship can be established between the pattern dip and a resonance inside the dielectric layer launched by the real pole γ_1' of the fundamental Green's function G_0 . At a resonance, the magnitude and the phase of the reflection coefficient γ_1' for the dominant waveguide mode are close to one and 180 degrees, respectively. The approximate location for the dip angle ϕ_d on a pattern cut can be predicted from the real pole γ_1' of G_0 on a complex plane. The variation in ϕ_d as functions of the dielectric constant of the layer ξ , an azimuthal slot center-to-center distance d_c and the elevation angle θ can be easily investigated by tracing the shift of γ_1' on the real axis as the result of the changes in these array parameters.

The pattern cut on the horizontal plane shows a narrower and deeper pattern dip as the dielectric constant of the layer ξ is increasing from ξ_0 . At the same time, the

pattern dip is moving towards the broadside direction, which results in a narrower beamwidth in azimuth. Accordingly, the angular width of an active array sector is reduced diminishing the effective aperture area in an azimuthal scan. For a fixed value of the dielectric constant ξ , by decreasing an azimuthal slot center-to-center distance d_c , the beamwidth of an elementary pattern cut can be made wider, even though it is considerably narrower than the beamwidth when $\xi = \xi_0$ as we can see in Fig. (4-8) where $d_c = 0.55\lambda_0$.

When the elevation angle Θ is increased from zero, the pattern dip is shifting towards the endfire direction. For an increased elevation angle, when the lowest pole from the first harmonic Green's function G_{-1} drops down from the imaginary axis to the real axis of a complex plane, multiple resonances are launched inside the dielectric layer. The new resonance caused by the real pole from G_{-1} has different characteristics compared to the one arising from the real pole of G_0 and, accordingly, its impacts on an elementary pattern cut are also different. Instead of adding a new pattern dip, it causes an overall pattern change on the elementary pattern cut making it excessively oscillatory. In an actual array operation, because of this rapid pattern change, it is difficult to maintain a narrow shaped beam pattern beyond this elevation angle where the multiple resonances are taking place. As a result, the elevation scan range of an array is limited to this elevation scan angle at the launch

ching of the multiple resonances. This angle is critically dependent on a z-direction slot center-to-center distance $2d$ and can be easily determined by tracing the lowest pole γ'_1 from G_{-1} on the imaginary axis. A reduction in $2d$ suppresses the downward shift of this pole on the imaginary axis and, as a result, a wider elevation scan range can be obtained. Incidentally, the elevation angle at the launching of the multiple resonances is far below from the grating lobe angle in an elevation scan.

Chapter 5. Summary and possible extension of the dissertation.

5.1 Summary

The complex plane analysis developed in the dissertation not only provides an efficient computational tool for the evaluation of mutual coupling effects between array elements on a cylindrical surface, but also makes possible physical interpretations which are essential for the exposure of basic array behaviors. With a direct harmonic series approach, however, besides its computational inefficiency, mathematical complexity obscures the array fundamentals. Since it is possible through the complex plane analysis to directly relate the poles on the complex plane to important physical phenomena which can jeopardize an array operation, this method can be also utilized for an appropriate array design. In other words, by investigating the pole transitions on the complex plane when array parameters are varied, the beam scan characteristic of an array can be predicted.

The computed results indicate that, in general, for conformal dipole or slot arrays, the major characteristics of an elementary pattern cut at a constant elevation angle are the ripples at broadside and a narrow azimuthal beamwidth. The beamwidth is

approximately determined by the maximum active dipole input impedance angle for a dipole array and by the maximum reflection angle for a slot array in the azimuthal scan. From the azimuthal beamwidth of an elementary pattern cut, the angular width of the active array sector which is actually excited can be estimated. For an efficient array operation as well as for the suppression of sidelobes, only a portion of the whole array which is roughly same as the beamwidth of an elementary pattern cut is needed to send the beam to a given direction. Since the active array sector occupies only a limited portion of a whole array due to the narrow azimuthal beamwidth, simultaneous multiple beam operations in azimuth are considered as a possibility. In the presence of a dielectric sheet, the elevation scan of an array is limited by a resonance phenomenon inside the layer.

For a conformal dipole arrays, the elementary pattern cuts indicate that no significant radiation takes place beyond the maximum impedance angle in the azimuthal scan, which is different from the cases of conformal slot arrays. For a conformal dipole array with no dielectric sheet between the elements and the conducting cylindrical surface, a wide angle beam coverage can be realized capitalizing the broad beamwidth in elevation and the circular symmetric array geometry

in azimuth. Once a dielectric sheet is inserted between the elements and the cylinder, the azimuthal beamwidth is decreased as the dielectric constant of the layer is increasing. However, the most significant changes in the elementary pattern cut at an elevation angle Θ are arising when Θ is increased from zero. This is due to the resonance phenomenon inside the dielectric layer launched by an emerging real pole on the complex plane as the result of the beam lift-off from the horizontal plane. Because of accompanying excessive ripples on the elementary pattern cut, the elevation scan of such an array is limited.

For a conformal slot array, a close correspondence between the Watson transform poles on a complex plane and an elementary pattern cut can be established. A real pole which exists even when the beam direction is horizontal can launch a resonance inside the dielectric sheet and, as a result, the elementary pattern cut on the horizontal plane has a dip. The variation in the dip angle as a function of various array parameters can be easily found by investigating the shift of the real pole on the complex plane when these parameters are changed. When the elevation angle Θ is increasing, the shift in the dip angle for the elementary pattern cut at Θ can be accurately predicted from the transition of the real pole. For an increased elevation angle, however, a pole

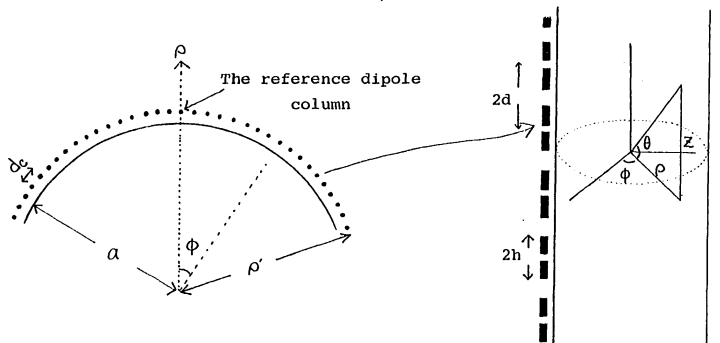
initially located on the imaginary axis of the complex plane is decayed to the real axis launching multiple resonances inside the dielectric sheet. Since the multiple resonances bring about overall changes on an elementary pattern cut, the elevation scan of the array must be limited up to the elevation angle at the onset of the multiple resonances. Therefore, a clear cut elevation scan range can be found through the pole consideration. This range is narrow than the one allowed from the grating lobe consideration in the elevation scan.

5.2 Possible extension of the dissertation.

Since the Green's function approach for the evaluation of the mutual coupling effects between dipole or slot columns is essentially an element by element analysis instead of a modal analysis, the same complex plane method can be applied to investigate more general conformal arrays where the distribution of dipole or slot columns is not necessarily circular symmetric. A typical example of such asymmetric conformal arrays is the case when the array sector of dipole or slot columns is only a portion of the whole cylinder circumference. For these more general arrays, the advantage of the phase sequence excitation method must be renounced and, instead, a direct matrix inversion should be carried out. However, as far as the evaluation of the mutual coupling effects between dipole or slot columns are concerned, the complex plane analysis for the calculation of Green's functions can be used without any modifications. These calculations determine the elements of the matrix.

More practical array environments are those of finite arrays where, instead of infinite dipole or slot columns, the arrays have finite number of elements. To study these finite conformal arrays, the mutual couplings between dipole or slot elements instead

of the couplings between dipole or slot columns must be calculated. With a slight modification of the complex plane analysis, these calculations can be carried out. The Hertzian potential which accounts for the mutual coupling between two elements can be found by changing the summation of various space harmonics in the axial direction to an integration since an infinite column is replaced with a single array element. The propagation constant in the axial direction $\beta_n (-k_0 + \frac{2\pi n}{2d})$ is now continuous and the integration is carried out with respect to β_n . Because the Watson transform poles on the complex plane are functions of β_n , the integration can be evaluated by applying the stationary phase integration method to the residue terms of the Watson transform. Preliminary calculations using this method show good agreements with direct integrations for the cases of small radius cylinders.



(a) Dipole columns around the circumference of a conducting cylinder.

(b) A dipole column.

Fig.2-1 The geometry of an array.

(The number of total dipole column $N = 200$. The distance between dipole elements and the cylinder $\rho' - a = \lambda_0/4$. A dipole length $2h = \lambda_0/2$. The radius of a dipole $r_0 = 0.007\lambda_0$. N , $\rho' - a$, $2h$ and r_0 are constants. A distance between dipole columns d_c , a z -direction dipole center-to-center distance $2d$ and an elevation beam angle θ are variables.)

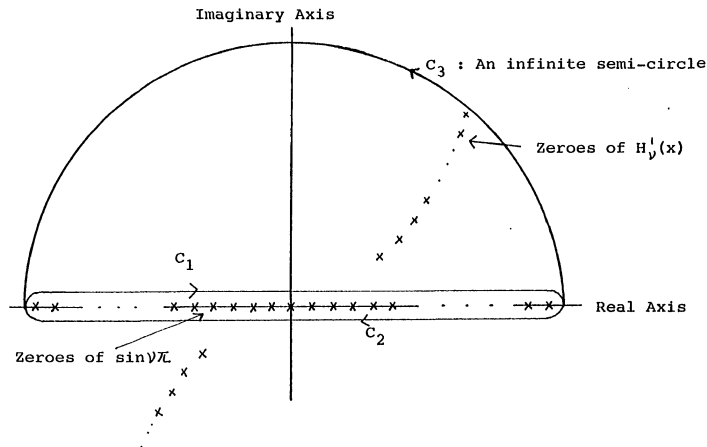


Fig.2-2 A complex ν - plane.

(In the Watson transform method, the original integration on contours C_1 and C_2 is replaced by the one on contours C_1 and C_3 to apply a residue theorem at the zeroes of $H_\nu'(x)$ on the upper half-plane)

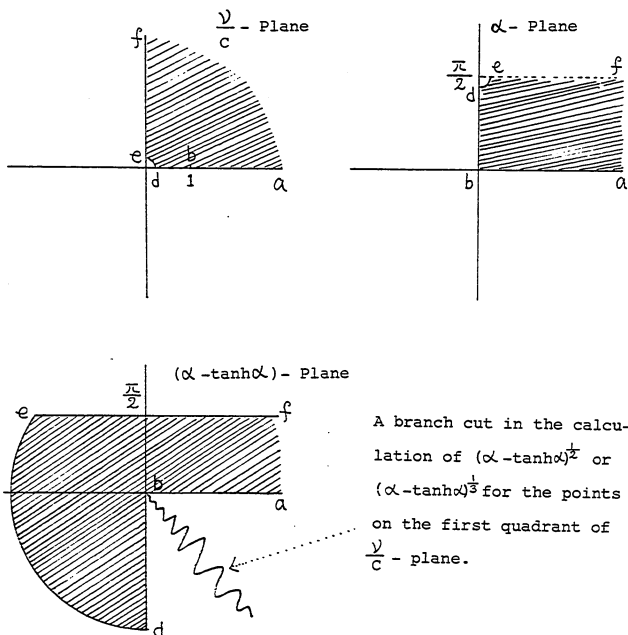


Fig. 2-3 A mapping of the first quadrant on the $\frac{v}{c}$ -plane into the $(\alpha - \tanh \alpha)$ -plane according to a relationship $\alpha = \cosh^{-1} \frac{v}{c}$, which is used in the approximation of a Hankel function $H_v^1(c) = H_v^1(v) \operatorname{sech} \alpha$ by Eq. (16) of Appendix 2.

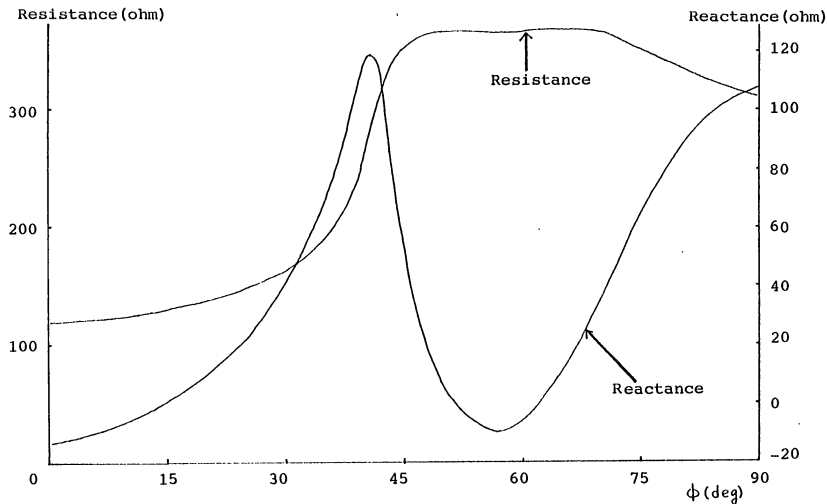


Fig.2-4 Active dipole input impedance versus an azimuthal beam direction ϕ (The distance between dipole elements and the cylinder $\rho'-a=\lambda_0/4$. A dipole length $2h=\lambda_0/2$. A space between dipole columns $d_c=0.6\lambda_0$. A z-direction dipole center-to-center distance $2d=0.6\lambda_0$. An elevation beam direction $\theta=0$)

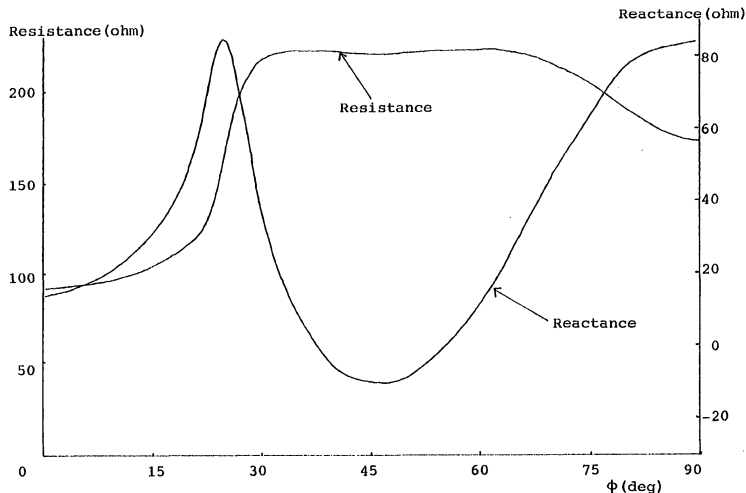


Fig.2-5 Active dipole input impedance versus an azimuthal beam direction ϕ

(The distance between dipole elements and the cylinder $\rho'-a=\lambda_0/4$. A dipole length $2h=\lambda_0/2$. A space between dipole columns $d_c=0.7\lambda_0$. A z-direction dipole center-to-center distance $2d=0.7\lambda_0$. An elevation beam direction $\theta=0$)

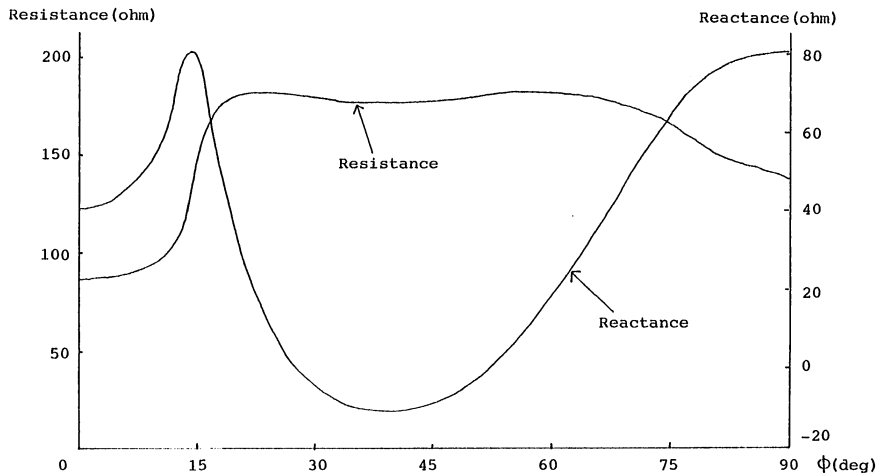


Fig.2-6 Active dipole input impedance versus an azimuthal beam direction ϕ (The distance between dipole elements and the cylinder $\rho' - a = \lambda_0/4$. A dipole length $2h = \lambda_0/2$. A space between dipole columns $d_c = 0.8\lambda_0$. A z-direction dipole center-to-center distance $2d = 0.7\lambda_0$. An elevation beam direction $\theta = 0$)

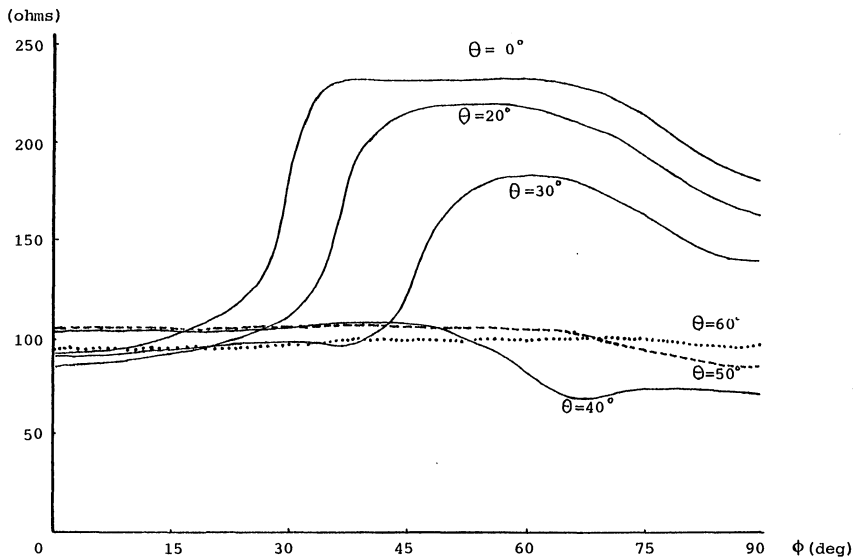


Fig.2-7a Active dipole input resistance versus an azimuthal beam direction ϕ . (The distance between dipole elements and the cylinder $\rho'-a=\lambda/4$. A dipole length $2h=\lambda/2$. A space between dipole columns $d_c=0.67\lambda_0$. A z-direction dipole center-to-center distance $2d=0.72\lambda_0$)

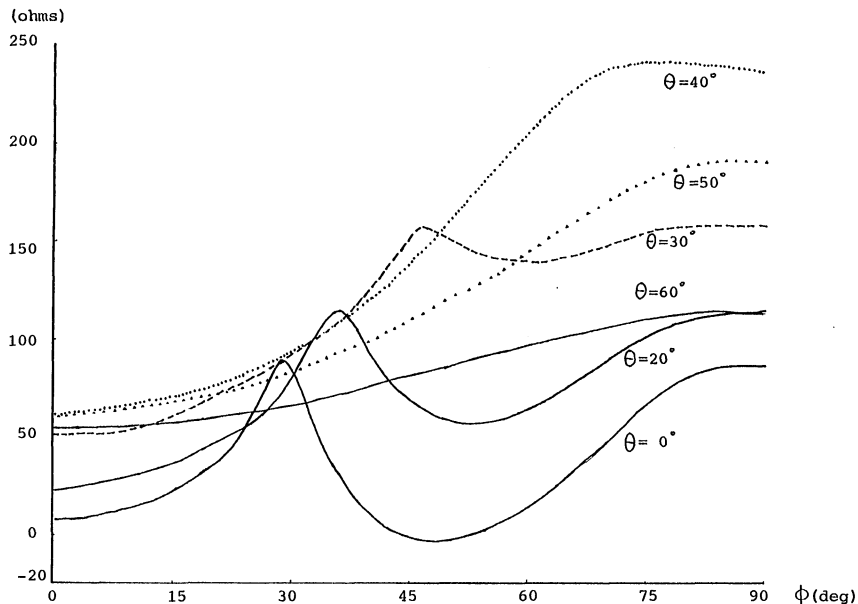


Fig.2-7b Active dipole input reactance versus an azimuthal beam direction ϕ .
 (The distance between dipole elements and the cylinder $\rho' - a = \lambda_0/4$. A dipole length $2h = \lambda_0/2$. A space between dipole columns $d_c' = 0.67\lambda_0$. A z-direction dipole center-to-center distance $2d = 0.72\lambda_0$)

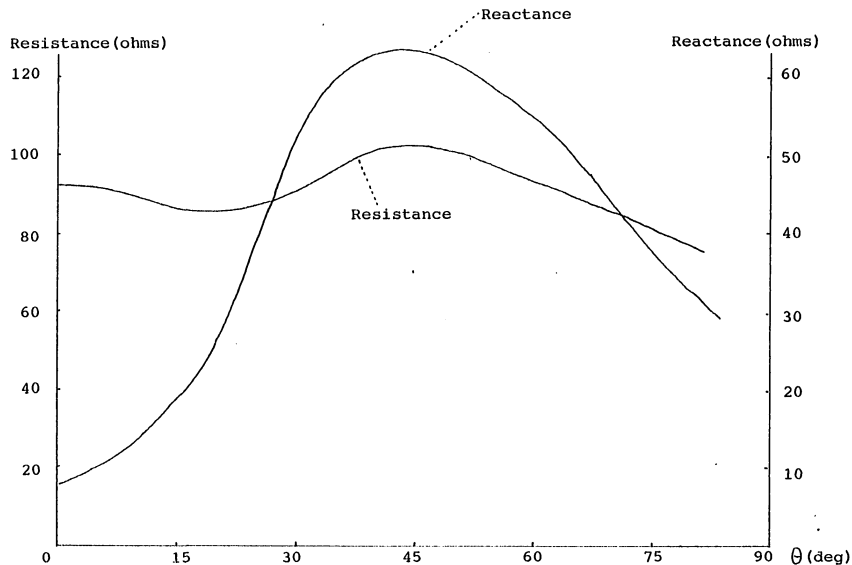


Fig.2-8 Active dipole input impedance versus an elevation scan angle θ .

(The distance between dipole elements and the cylinder $\rho-a=\lambda_0/4$. A dipole length $2h=\lambda_0/2$. A space between dipole columns $d_c=0.67\lambda_0$. A z-direction dipole center-to-center distance $2d=0.72\lambda_0$. An azimuthal beam direction $\phi=0^\circ$)

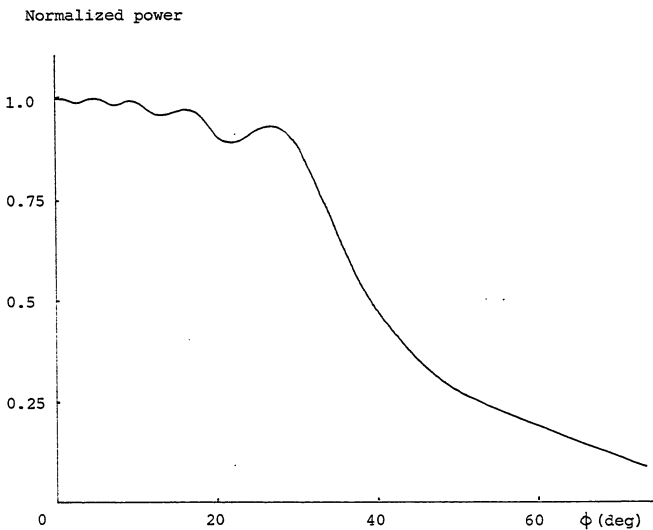


Fig.2-9 Elementary pattern on the horizontal plane.

(The distance between dipole elements and the cylinder $\rho'-a=\lambda_0/4$.
 A dipole length $2h=\lambda_0/2$. A space between dipole columns $d_c=0.6\lambda_0$.
 A z-direction dipole center-to-center distance $2d=0.6\lambda_0$)

Normalized power

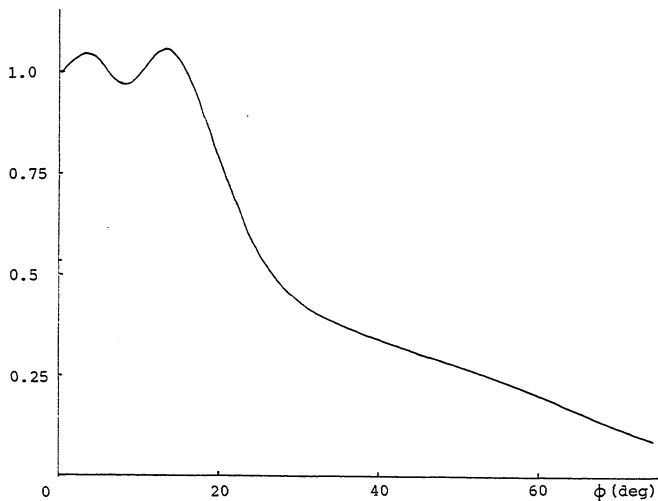


Fig.2-10 Elementary pattern on the horizontal plane.

(The distance between dipole elements and the cylinder $\rho - a = \lambda_0/4$.

A dipole length $2h = 0.5\lambda_0$. A space between dipole columns $d_c =$

$0.7\lambda_0$. A z-direction dipole center-to-center distance $2d = 0.7\lambda_0$)

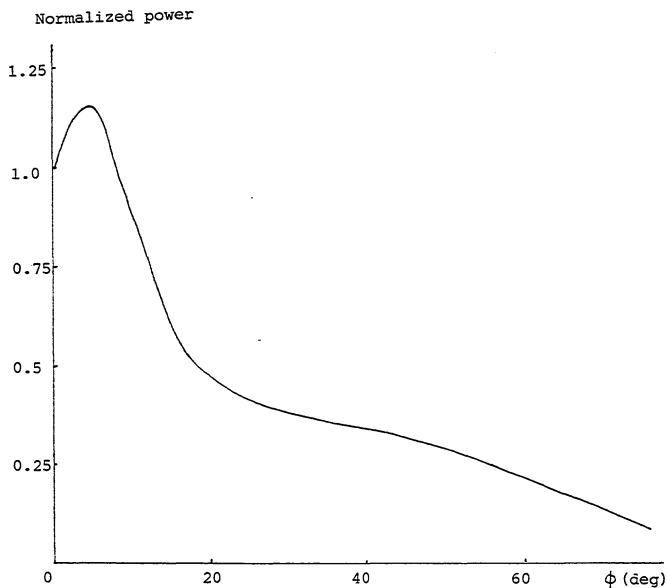


Fig.2-11 Elementary pattern on the horizontal plane.

(The distance between dipole elements and the cylinder $\rho'-a=\lambda_0/4$.

A dipole length $2h=0.5\lambda_0$. A space between dipole columns $d_c=0.8\lambda_0$.

A z-direction dipole center-to-center distance $2d=0.7\lambda_0$.)

Normalized power

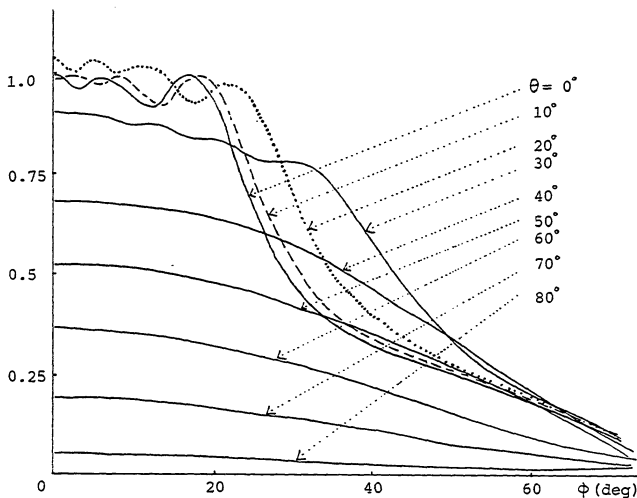


Fig.2-12 Elementary patterns at various elevation angles.

(The distance between dipole elements and the cylinder $\rho' - a = \lambda/4$.

A dipole length $2h = 0.5\lambda_0$. A space between dipole columns $d_c =$

$0.67\lambda_0$. A z-direction dipole center-to-center distance $2d = 0.72\lambda_0$.)

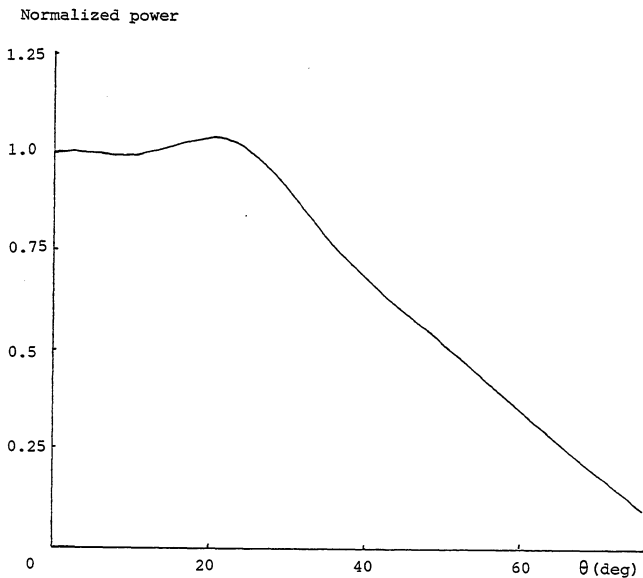
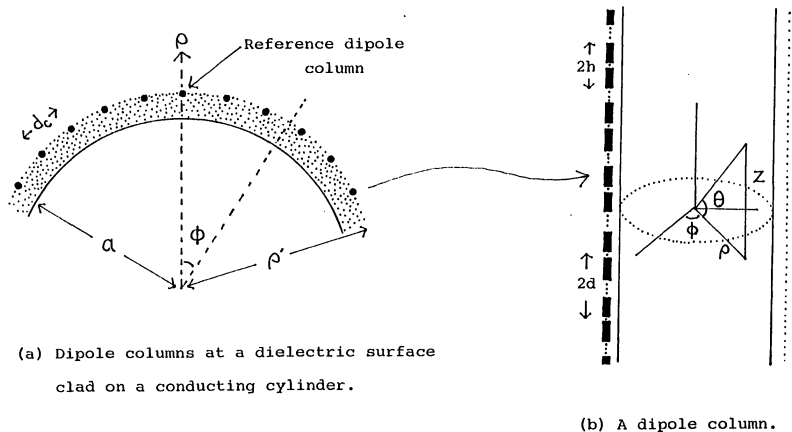


Fig.2-13 Elementary pattern on the $\phi = 0$ plane as a function of an elevation angle θ .

(The distance between dipole elements and the cylinder $\rho' - a = \lambda/4$.
 A dipole length $2h = 0.5\lambda$. A space between dipole columns $d_c = 0.67\lambda$. A z-direction dipole center-to-center distance $2d = 0.72\lambda$)



(a) Dipole columns at a dielectric surface clad on a conducting cylinder.

(b) A dipole column.

Fig.(3-1). The geometry of an array.

(The number of total dipole columns $N=200$, the thickness of the dielectric layer $\rho'-a=0.1768\lambda_0$, a dipole length $2h=0.5\lambda_0$, and the radius of a dipole $r_0=0.007\lambda_0$, where λ_0 is a wavelength in the free space. A distance between dipole columns d_c , a z-direction dipole center-to-center distance $2d$, the elevation beam scan direction θ and the dielectric constant of the layer ξ are variables.)

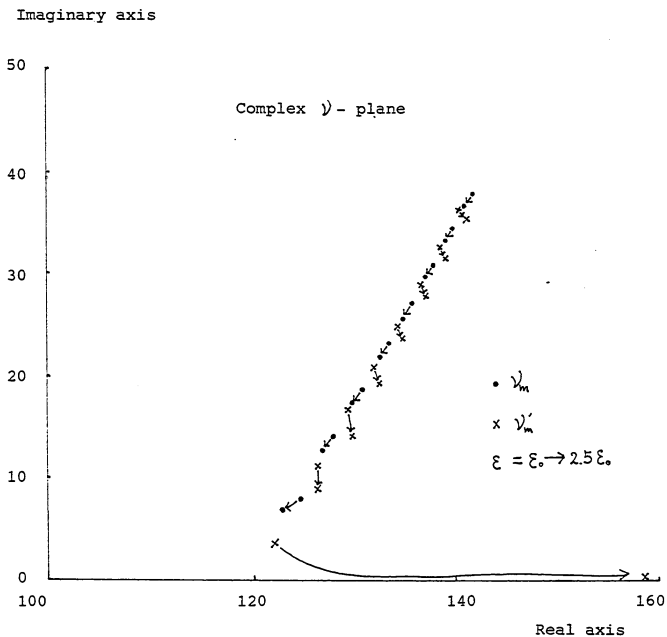


Fig. (3-2). Transitions of first 16 poles on a complex γ -plane as ξ is varied from ξ_0 to $2.5 \xi_0$.

(Elevation beam angle $\theta=30^\circ$, $\sqrt{\rho_k^2 - \xi_0^2} a = 120.28$)

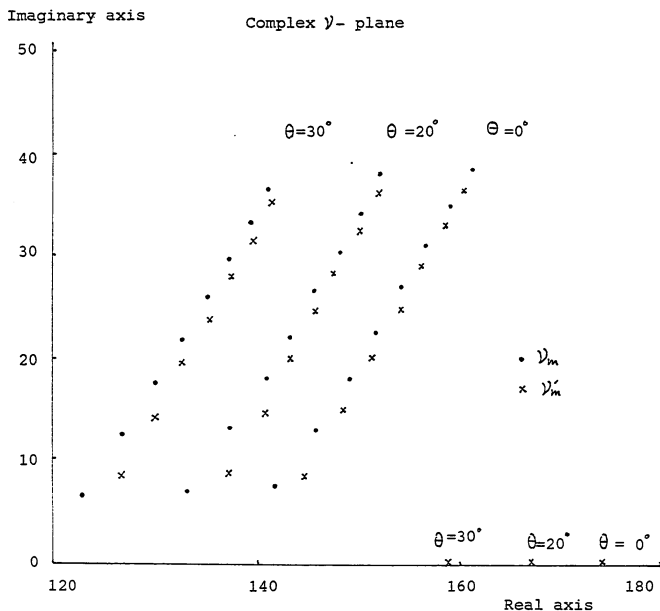


Fig.(3-3). Transitions of first 16 poles in a complex γ -plane as the elevation beam angle is increased from 0° to 30° . (The dielectric constant of the layer $\epsilon=2.5\epsilon_0$, $R_{0a}=138.89$)

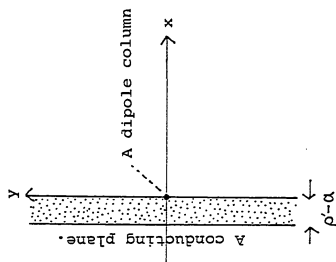
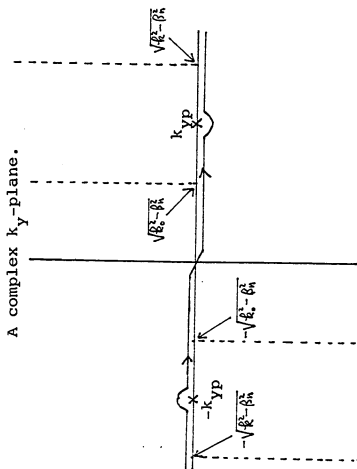


Fig. (3-4). The geometry of a planar approximation near a source column.



A complex k_y -plane.

Fig. (3-5). A contour for the integral in Eq. (6) of Chapter 3 and various branch cuts.

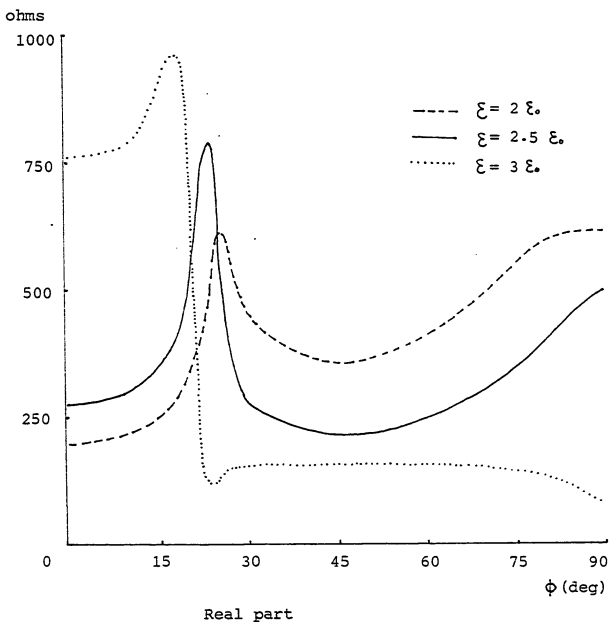


Fig.(3-6a). Active dipole input impedance as a function of the azimuthal beam scan direction. (The elevation beam angle $\theta=0^\circ$, an inter-column space $d_c=0.7\lambda_0$ and a z-direction dipole center-to-center distance $2d=0.7\lambda_0$)

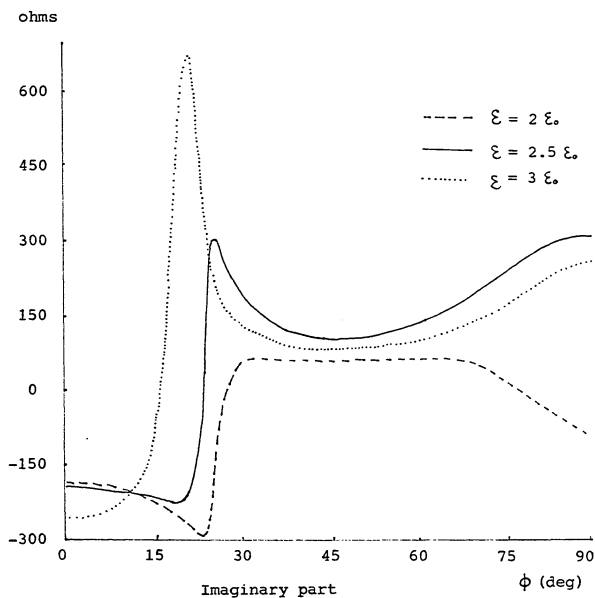


Fig. (3-6b) Active dipole input impedance as a function of the azimuthal beam scan direction. (The elevation beam angle $\theta=0^\circ$, an inter-column space $d_c=0.7\lambda_0$, and a z-direction dipole center-to-center distance $2d=0.7\lambda_0$)

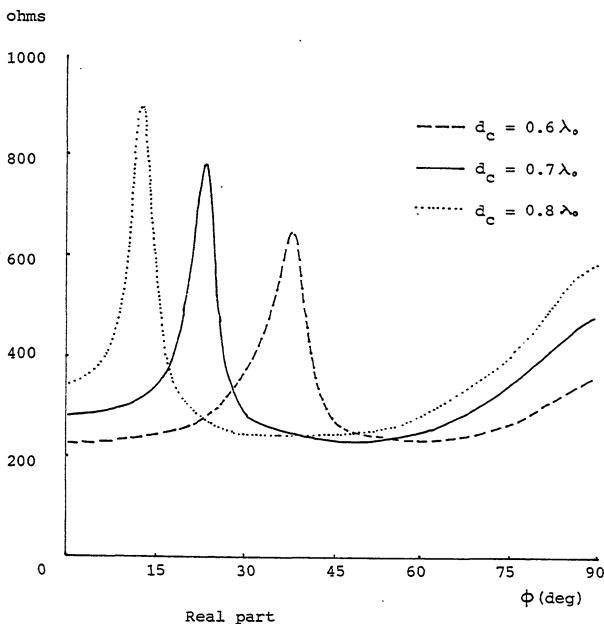


Fig.(3-7a). Active dipole input impedance as a function of the azimuthal beam scan direction. (The elevation beam scan angle $\theta=0^\circ$, the dielectric constant of the layer $\xi=2.5\xi_0$ and a z-direction dipole center-to-center distance $2d=0.7\lambda_0$)

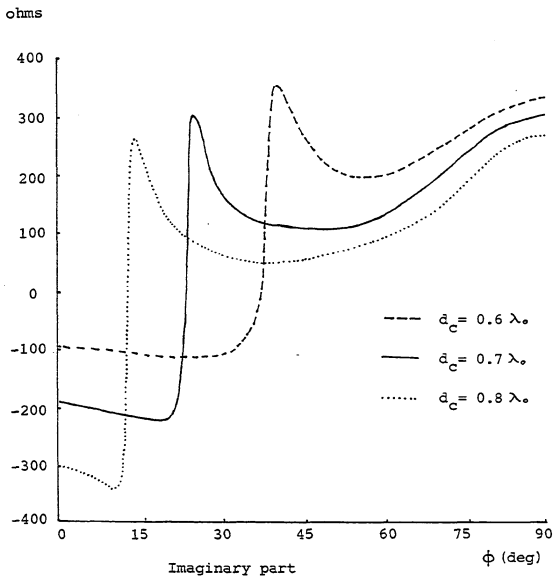


Fig. (3-7b). Active dipole input impedance as a function of the azimuthal beam scan direction. (The elevation beam scan angle $\theta=0^\circ$, the dielectric constant of the layer $\epsilon=2.5\epsilon_0$ and a z-direction dipole center-to-center distance $2d=0.7\lambda_0$.)

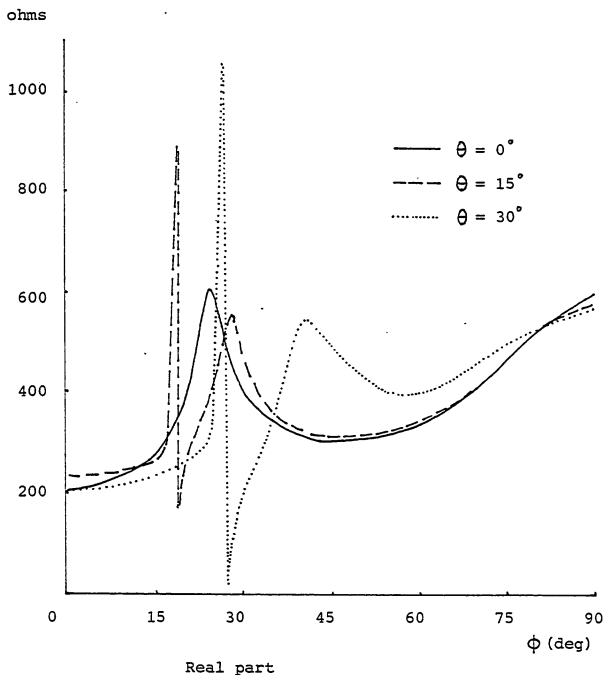


Fig.(3-8a). Active dipole input impedance as a function of the azimuthal beam scan direction. (The dielectric constant of the layer $\xi=2\xi_0$, an inter-column space $d_c=0.7\lambda_0$ and a z-direction dipole center-to-center distance $2d=0.55\lambda_0$.)

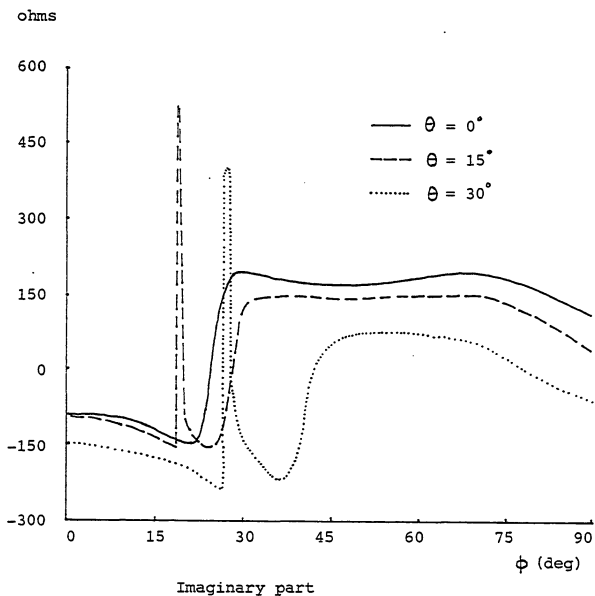


Fig. (3-8b). Active dipole input impedance as a function of the azimuthal beam scan direction. (The dielectric constant of the layer $\epsilon = 2\epsilon_0$, a distance between dipole columns $d_c = 0.7\lambda_0$ and a z-direction dipole center-to-center distance $2d = 0.55\lambda_0$.)

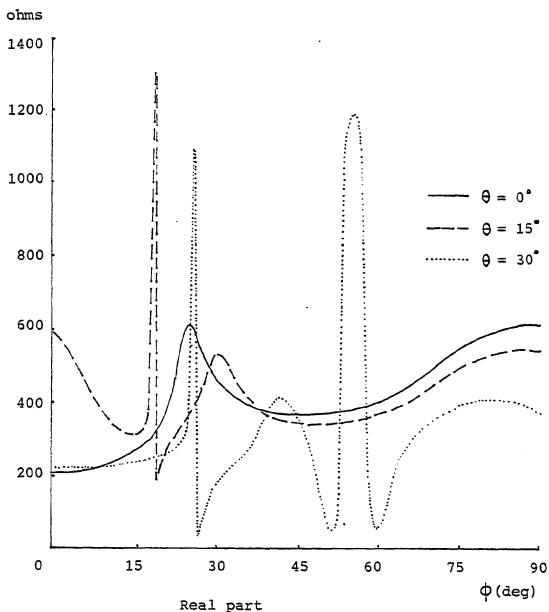


Fig.(3-9a). Active dipole input impedance as a function of the azimuthal beam scan direction. (The dielectric constant of the layer $\xi=2\epsilon$, an inter-column space $d_c=0.7\lambda$, and a z-direction dipole center-to-center distance $2d=0.7\lambda$.)

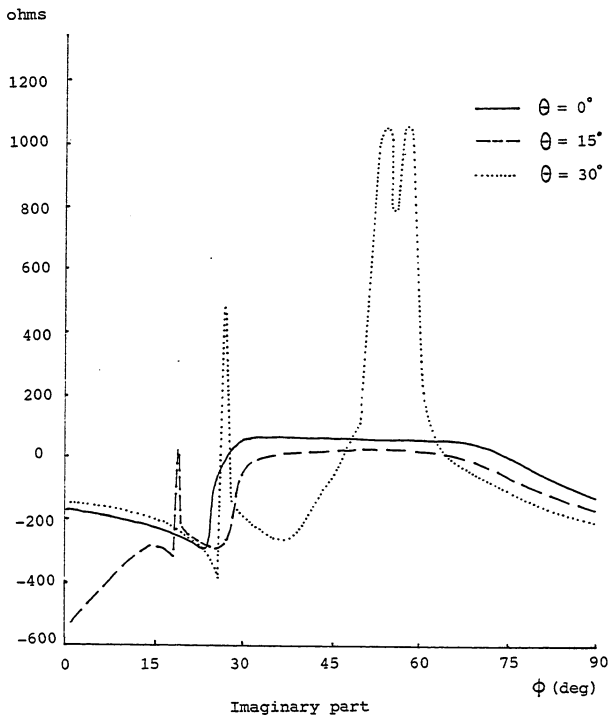


Fig.(3-9b). Active dipole input impedance as a function of the azimuthal beam scan direction. (The dielectric constant of the layer $\xi = 2\xi_0$, an inter-column distance $d_c = 0.7\lambda_0$ and a z-direction dipole center-to-center distance $2d = 0.7\lambda_0$.)

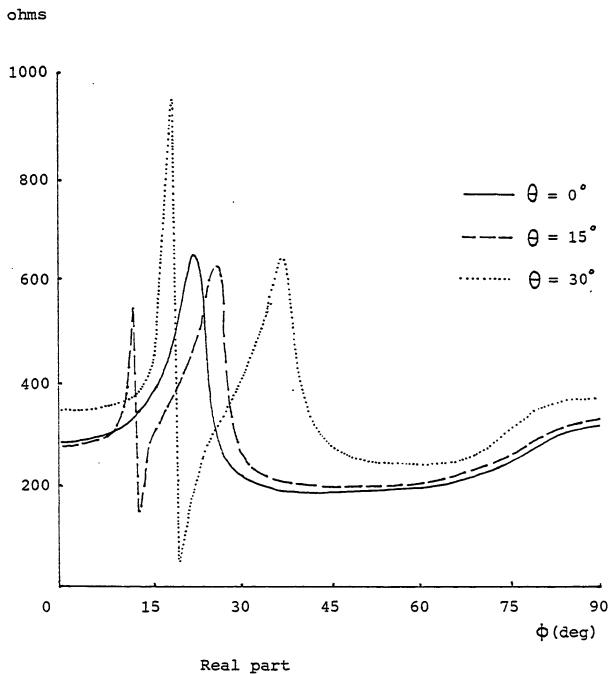


Fig.(3-10a). Active dipole input impedance as a function of the azimuthal beam scan direction. (The dielectric constant of the layer $\epsilon = 2.5\epsilon_0$, an inter-column space $d_c = 0.7\lambda_0$ and a z-direction dipole center-to-center distance $2d = 0.55\lambda_0$)

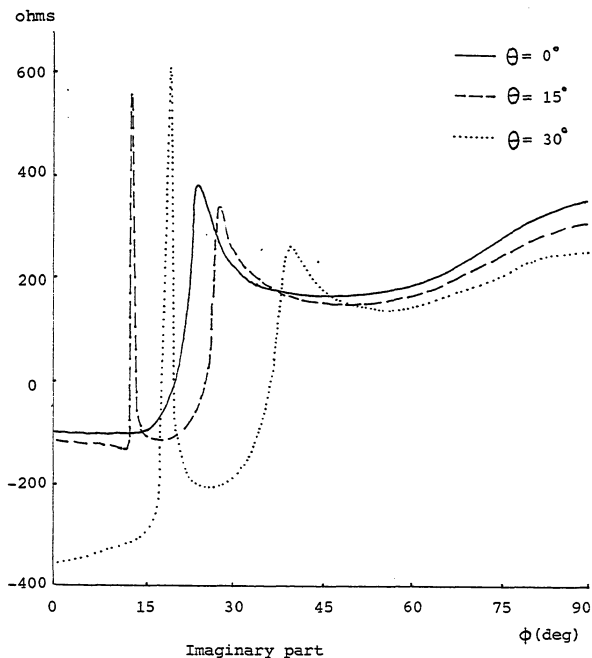


Fig.(3-10b). Active dipole input impedance as a function of the azimuthal beam scan direction. (The dielectric constant of the layer $\xi = 2.5\xi_0$, an inter-column distance $d_c = 0.7\lambda_0$ and a z-direction dipole center-to-center distance $2d = 0.55\lambda_0$)

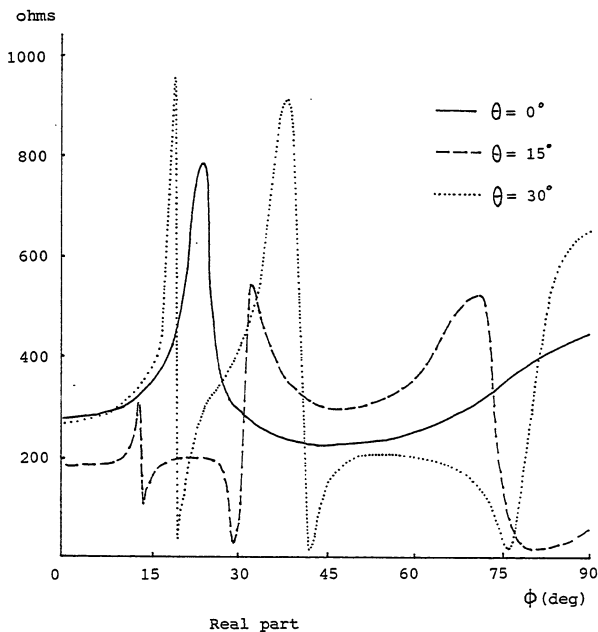


Fig.(3-11a). Active dipole input impedance as a function of the azimuthal beam scan direction. (The dielectric constant of the layer $\xi=2.5\xi_0$, an inter-column space $d_c=0.7\lambda_0$ and a z-direction dipole center-to-center distance $2d=0.7\lambda_0$)

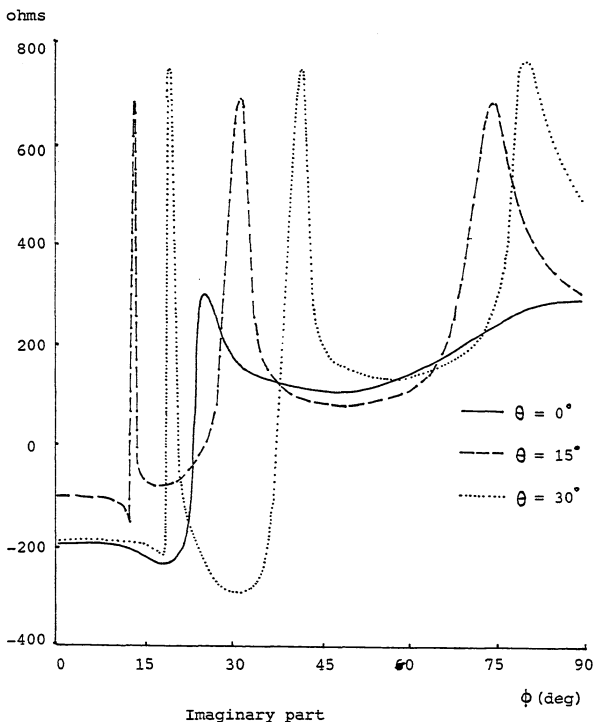
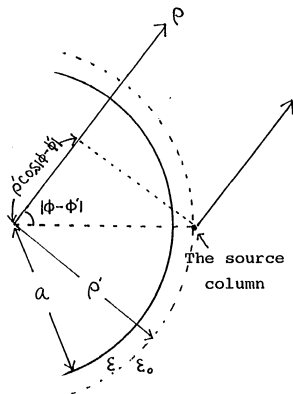
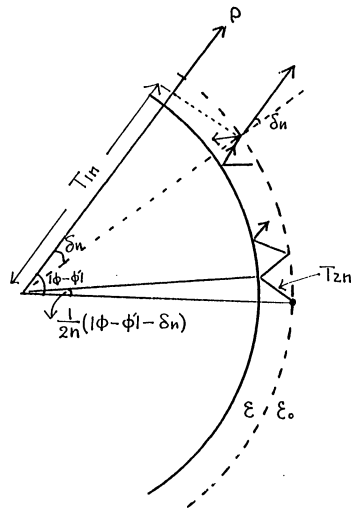


Fig.(3-11b). Active dipole input impedance as a function of the azimuthal beam scan direction. (The dielectric constant of the layer $\epsilon = 2.5\epsilon_0$, an inter-column space $d_c = 0.7\lambda_0$ and a z-direction dipole center-to-center distance $2d = 0.7\lambda_0$.)



(A) The direct wave from the source to a field point in the far zone.

Fig.(3-12). The path-lengths of various ray components radiating to the direction of ϕ .



(B) A ray radiated to the direction of ϕ after n -number of reflections from the cylinder surface during the propagation inside the dielectric layer.

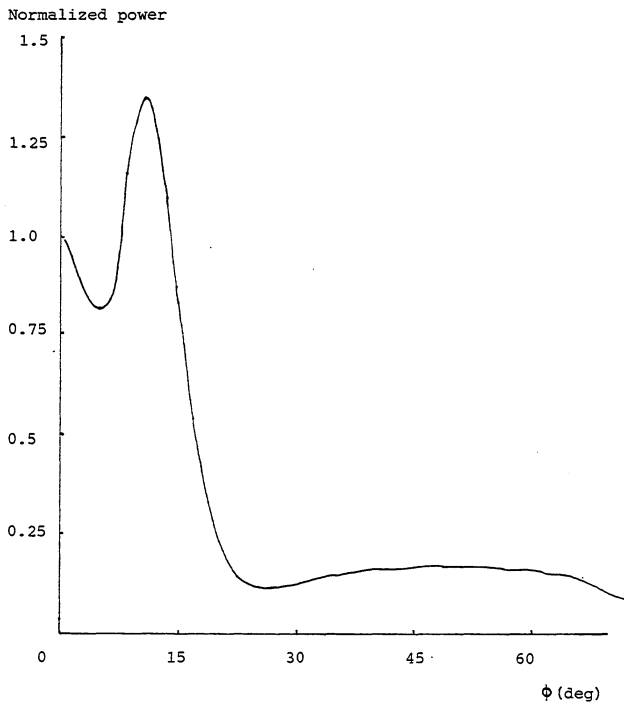


Fig.(3-13). Elementary pattern on the horizontal plane.
 (The dielectric constant of the layer $\xi=3\epsilon_0$,
 an inter-column distance $d_c=0.7\lambda_0$ and a z-
 direction dipole center-to-center distance
 $2d=0.7\lambda_0$.)

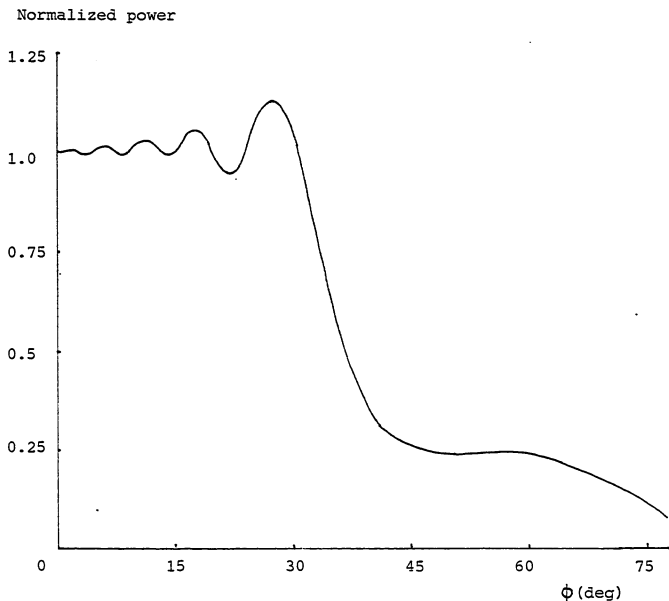


Fig.(3-14). Elementary pattern on the horizontal plane.

(The dielectric constant of the layer $\xi=2.5\xi_0$,
 an inter-column space $d_c=0.6\lambda_0$ and a z-
 direction dipole center-to-center distance
 $2d=0.7\lambda_0$.)

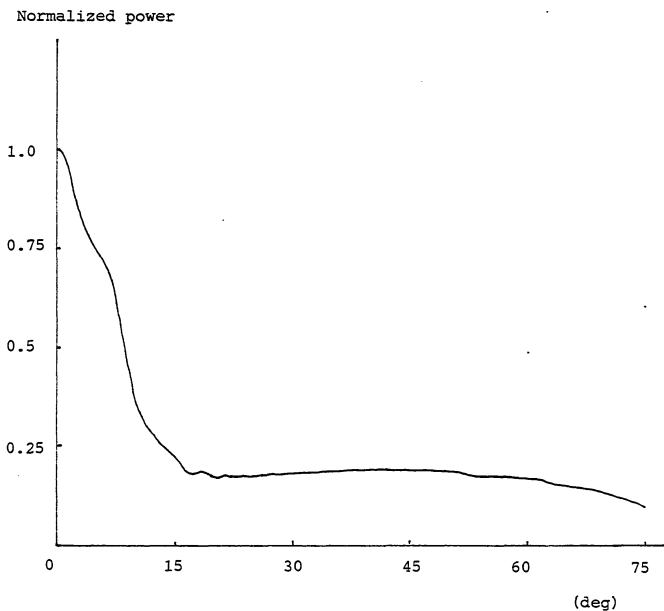


Fig. (3-15). Elementary pattern on the horizontal plane.
 (The dielectric constant of the layer $\xi = 2.5\xi_0$,
 an inter-column space $d_c = 0.8\lambda$, and a z-
 direction dipole center-to-center distance
 $2d = 0.7\lambda_0$)

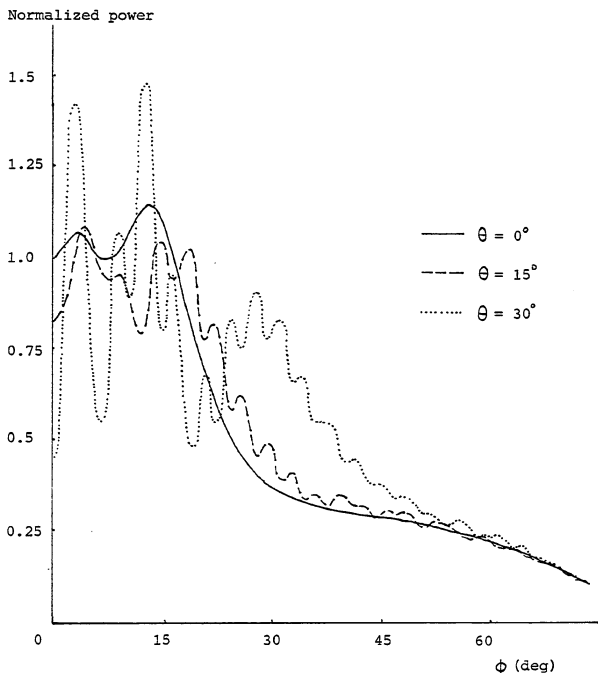


Fig.(3-16) Elementary pattern cuts at various elevation angles.
 (The dielectric constant of the layer $\epsilon = 2\epsilon_0$, an
 inter-column space $d_c = 0.7\lambda_0$ and a z-direction
 dipole center-to-center distance $2d = 0.55\lambda_0$.)

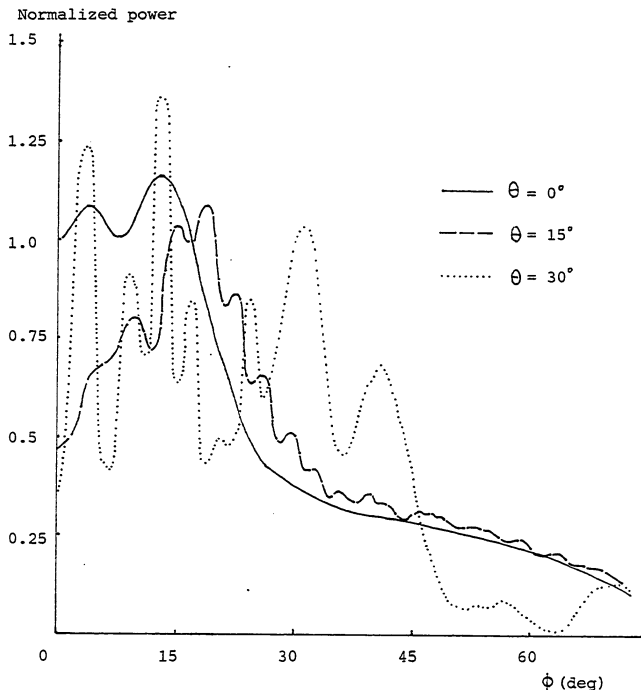


Fig.(3-17). Elementary pattern cuts at various elevation angles. (The dielectric constant of the layer $\xi=2\xi_0$, an inter-column space $d_c=0.7\lambda_0$ and a z-direction dipole center-to-center distance $2d=0.7\lambda_0$.)

Normalized power

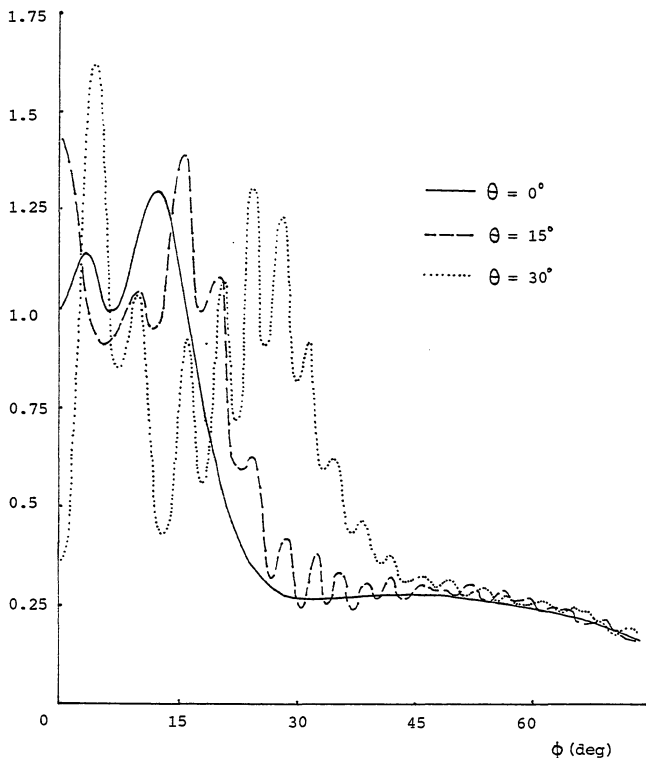


Fig.(3-18). Elementary pattern cuts at various elevation angles. (The dielectric constant of the layer $\xi=2.5\xi_0$, an inter-column space $d_c=0.7\lambda_0$ and a z-direction dipole center-to-center distance $2d=0.55\lambda_0$.)

Normalized power

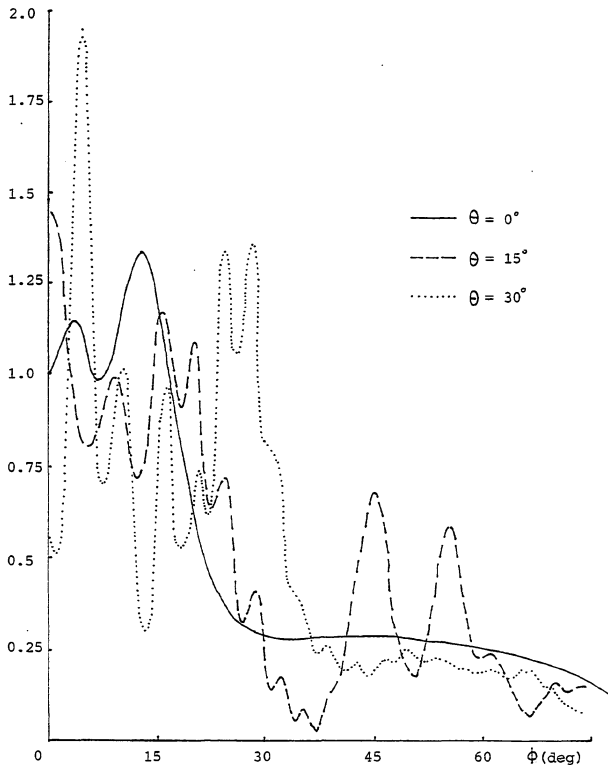


Fig.(3-19). Elementary pattern cuts at various elevation angles.
 (The dielectric constant of the layer $\epsilon=2.5\epsilon_0$, an inter-column distance $d_c=0.7\lambda_0$, and a z-direction dipole center-to-center distance $2d=0.7\lambda_0$.)

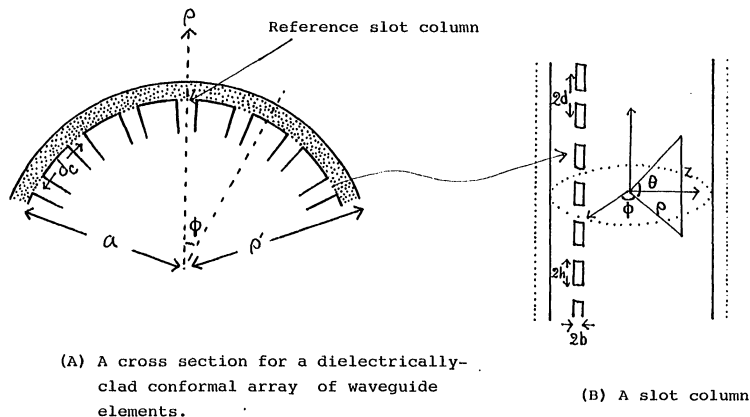


Fig.(4-1). The geometry of an array.

(The number of total slot columns $N=200$, the thickness of the dielectric layer $\rho'-a = 0.15\lambda_0$ and the width of a slot $2b=0.15\lambda_0$ where λ_0 is a wavelength in the free space. A distance between slot columns d_c , a z-direction slot center-to-center distance $2d$, the length of a slot $2h$, the elevation beam scan angle θ and the dielectric constant of the layer ϵ are variables)

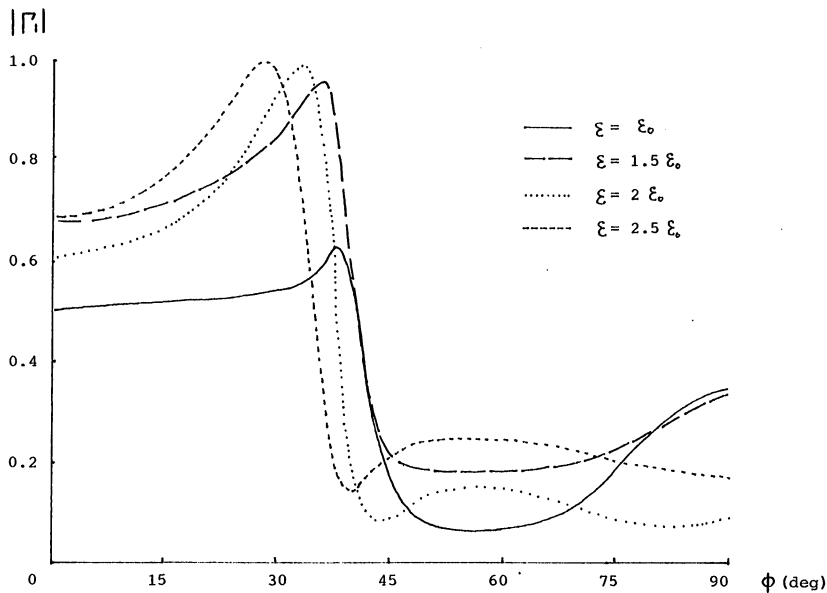


Fig.(4-2A) The magnitude of the reflection coefficient Γ_1 in an azimuthal scan.
 (A distance between slot columns $d_c = 0.6\lambda_0$, a z-direction slot center-to-center distance $2d = 0.7\lambda_0$, the length of a slot $2h = 0.6\lambda_0$ and the elevation beam angle $\theta = 0^\circ$)

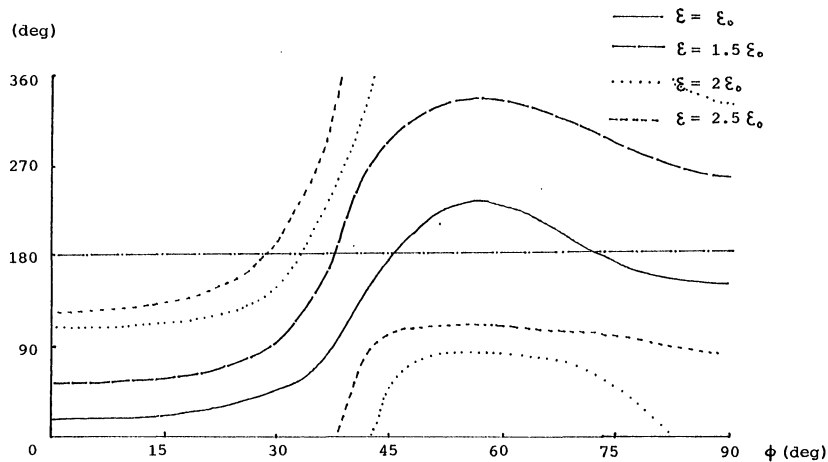


Fig. (4-2B) The phase of the reflection coefficient Γ_1 in an azimuthal scan.
 (A distance between slot columns $d_c = 0.6\lambda_0$, a z-direction slot center-to-center distance $2d = 0.7\lambda_0$, the length of a slot $2h = 0.6\lambda_0$ and the elevation beam angle $\theta = 0^\circ$)

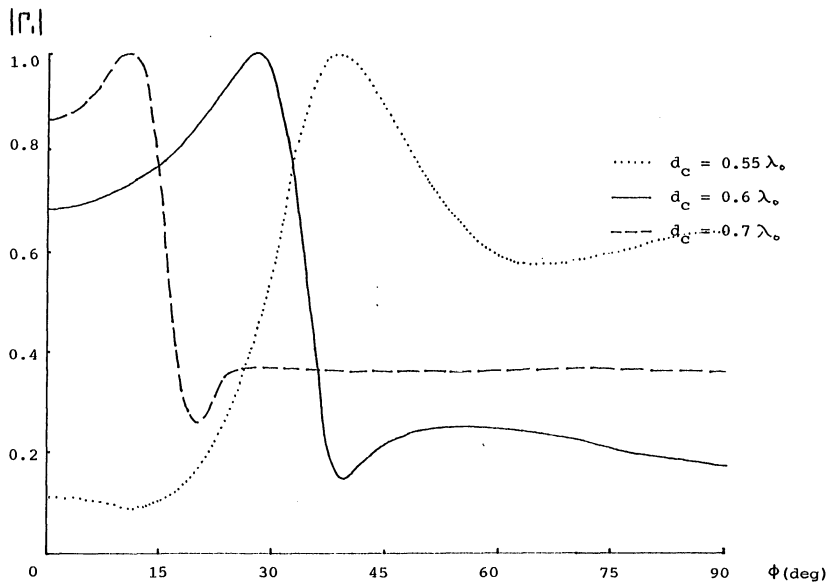


Fig.(4-3A) The magnitude of the reflection coefficient Γ_1 in an azimuthal scan.
 (A z-direction slot center-to-center distance $2d=0.7\lambda_0$, the length of a slot $2h=0.6\lambda_0$,
 the dielectric constant of the layer $\epsilon=2.5\epsilon_0$ and the elevation beam angle $\theta=0^\circ$)

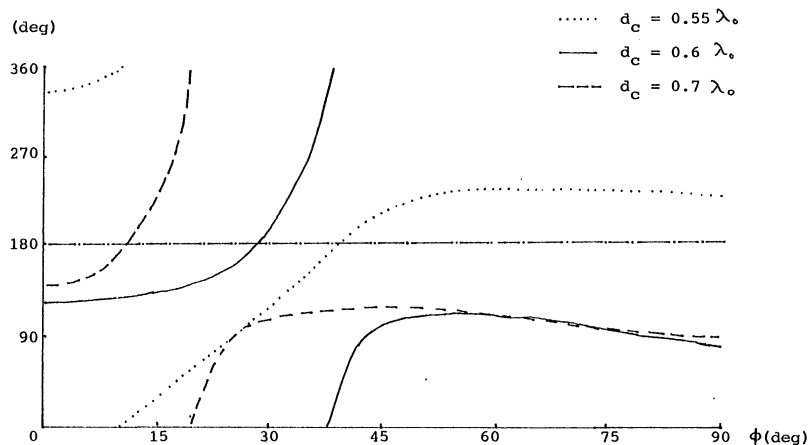


Fig.(4-3B) The phase of the reflection coefficient Γ_1 in an azimuthal scan.

(A z-direction slot center-to-center distance $2d=0.7\lambda_0$, the length of a slot $2h=0.6\lambda_0$, the dielectric constant of the layer $\epsilon=2.5\epsilon_0$, and the elevation beam angle $\theta=0^\circ$)

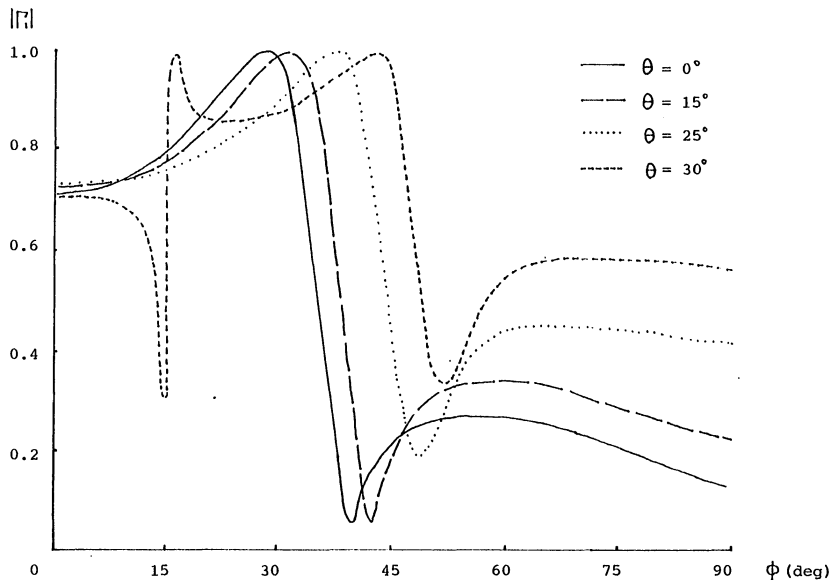


Fig.(4-4A) The magnitude of the reflection coefficient $|\Gamma_1|$ in an azimuthal scan.
 (A z-direction slot center-to-center distance $2d=0.6\lambda_0$, the length of a slot $2h=0.55\lambda_0$,
 a distance between slot columns $d_c=0.6\lambda_0$ and the dielectric constant of the layer $\xi=2.5\xi_0$)

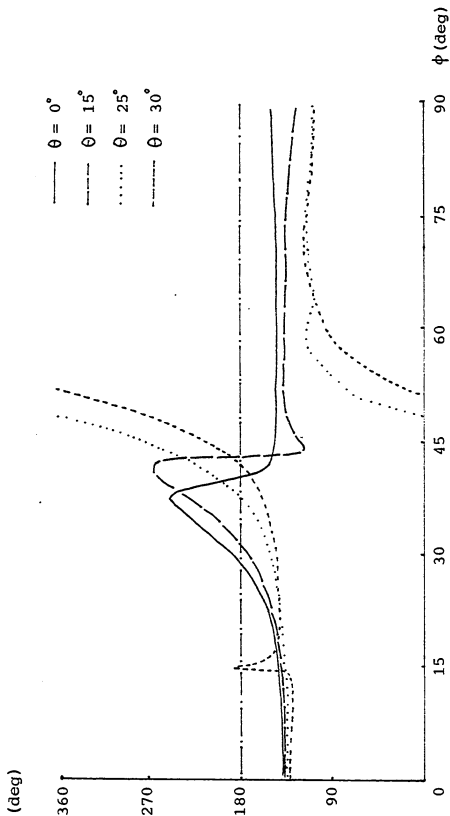


Fig. (4-4B) The phase of the reflection coefficient Γ_1 in an azimuthal scan.
 (A z-direction slot center-to-center distance $2d=0.6\lambda_0$, the length of a slot $2h=0.55\lambda_0$, a distance between slot columns $d_c=0.6\lambda_0$ and the dielectric constant of the layer $\epsilon=2.5\epsilon_0$.)

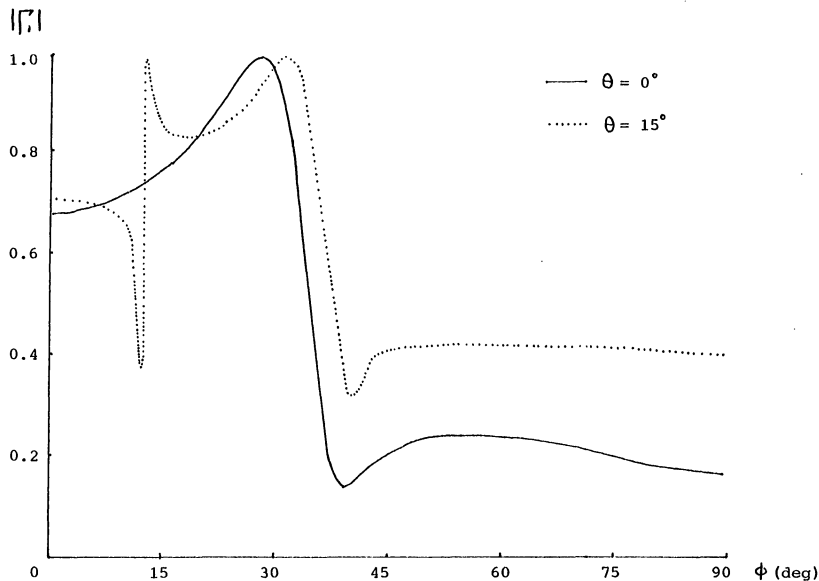


Fig.(4-5A) The magnitude of the reflection coefficient $|\Gamma|$ in an azimuthal scan.
 (A z-direction slot center-to-center distance $2d = 0.7\lambda_0$, the length of a slot $2h = 0.6\lambda_0$, a distance between slot columns $d_c = 0.6\lambda_0$ and the dielectric constant of the layer $\xi = 2.5\xi_0$)

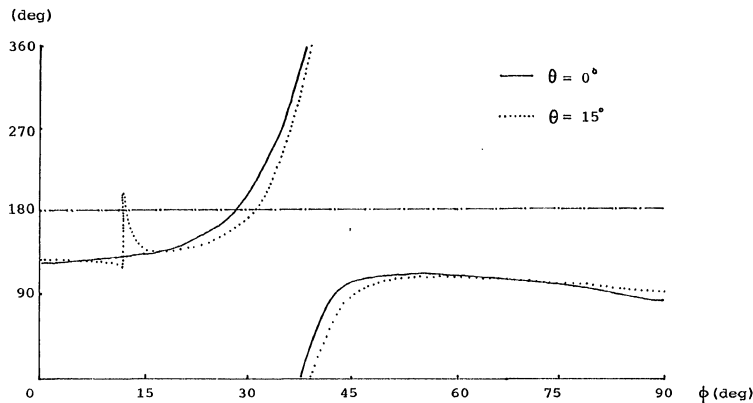


Fig.(4-5B) The phase of the reflection coefficient Γ_1 in an azimuthal scan. (A z-direction slot center-to-center distance $2d=0.7\lambda_0$, the length of a slot $2h=0.6\lambda_0$, a distance between slot columns $d_c = 0.6\lambda_0$ and the dielectric constant of the layer $\xi = 2.5\xi_0$.)

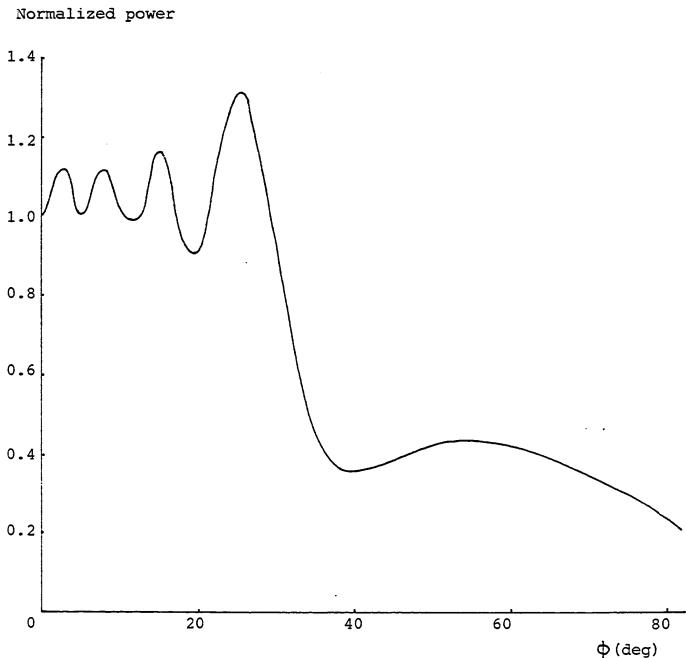


Fig. (4-6A). The elementary pattern on the horizontal plane.

(The dielectric constant of the layer $\xi = \xi_0$, a z-direction slot center-to-center distance $2d = 0.7\lambda_0$, the length of a slot $2h = 0.6\lambda_0$, an azimuthal slot center-to-center distance $d_c = 0.6\lambda_0$)

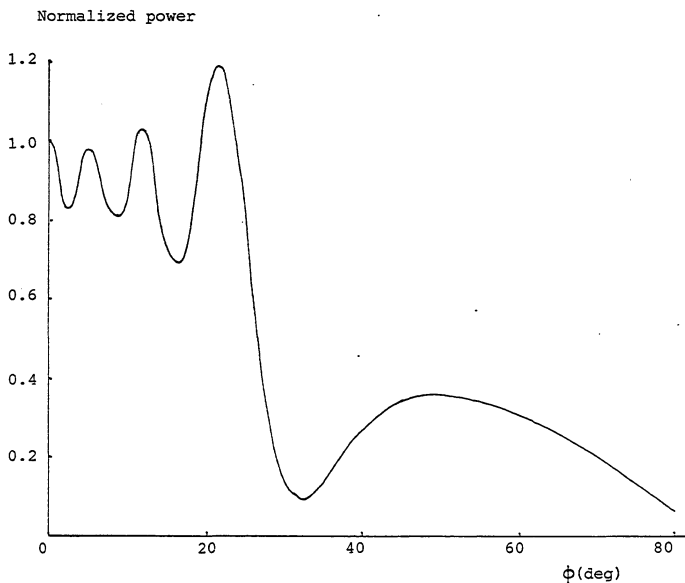


Fig. (4-6B). The elementary pattern on the horizontal plane.

(The dielectric constant of the layer $\xi=1.5\xi_0$, a z-direction slot center-to-center distance $2d=0.7\lambda_0$, the length of a slot $2h=0.6\lambda_0$ and an azimuthal slot center-to-center distance $d_c=0.6\lambda_0$)

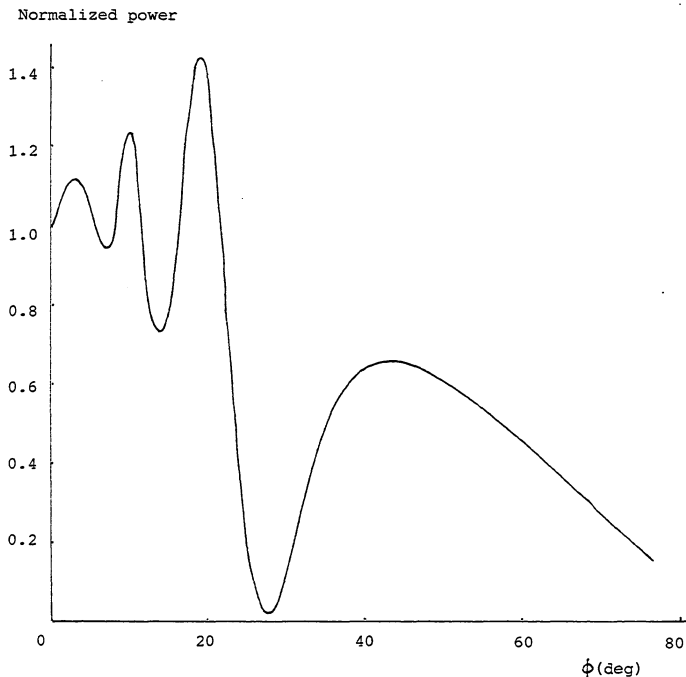


Fig. (4-6C). The elementary pattern on the horizontal plane.

(The dielectric constant of the layer $\xi=2\xi_0$, a z-direction slot center-to-center distance $2d=0.7\lambda_0$, the length of a slot $2h=0.6\lambda_0$, and an azimuthal slot center-to-center distance $d_c=0.6\lambda_0$)

Normalized power

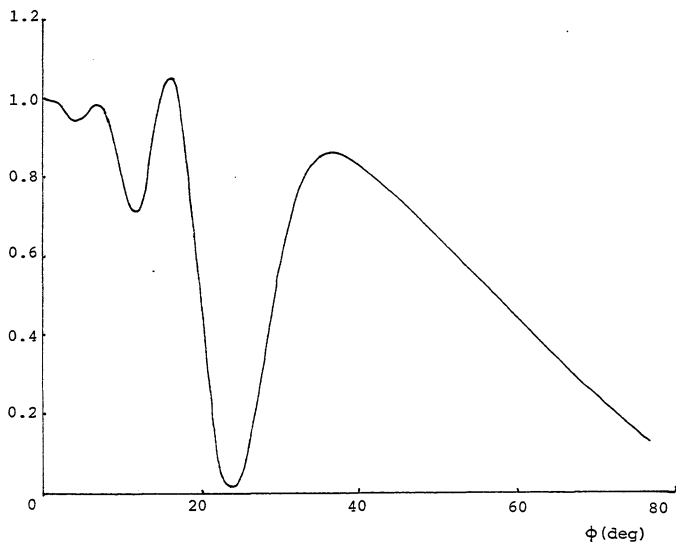


Fig.(4-6D). The elementary pattern on the horizontal plane.
 (The dielectric constant of the layer $\xi=2.5\xi_0$, a z-direction slot center-to-center distance $2d=0.7\lambda_0$, the length of a slot $2h=0.6\lambda_0$, and an azimuthal slot center-to-center distance $d_c=0.6\lambda_0$)

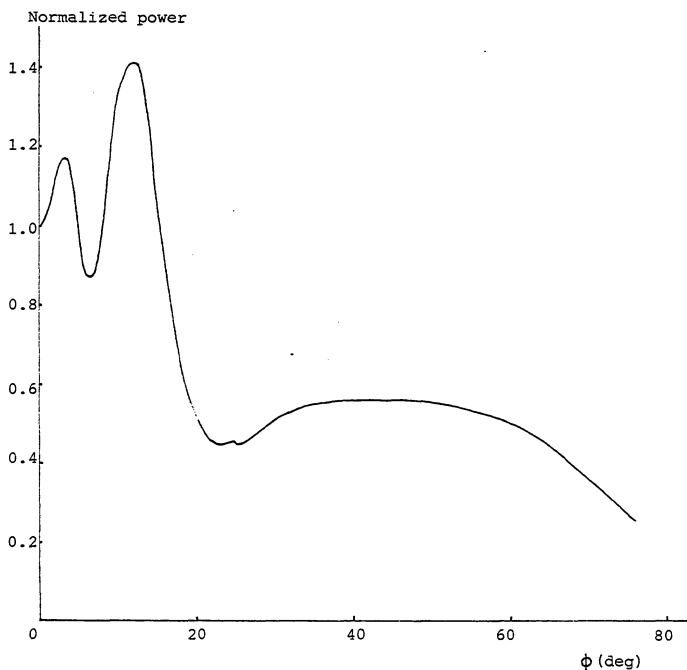


Fig.(4-7A). The elementary pattern on the horizontal plane.

(The dielectric constant of the layer $\xi = \xi_0$, a z-direction slot center-to-center distance $2d = 0.7\lambda_0$, the length of a slot $2h = 0.6\lambda_0$ and an azimuthal slot center-to-center distance $d_c = 0.7\lambda_0$)

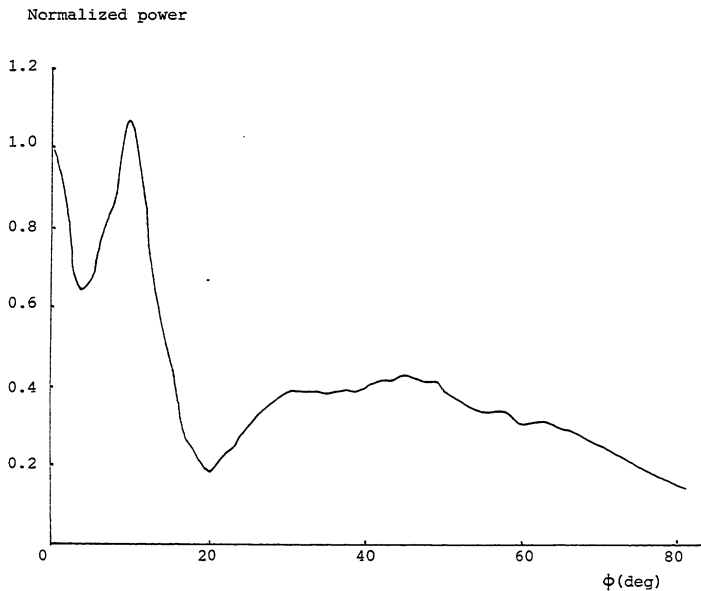


Fig. (4-7B). The elementary pattern on the horizontal plane.

(The dielectric constant of the layer $\xi = 1.25\xi_0$, a z-direction slot center-to-center distance $2d = 0.7\lambda_0$, the length of a slot $2h = 0.6\lambda_0$ and an azimuthal slot center-to-center distance $d_c = 0.7\lambda_0$)

Normalized power

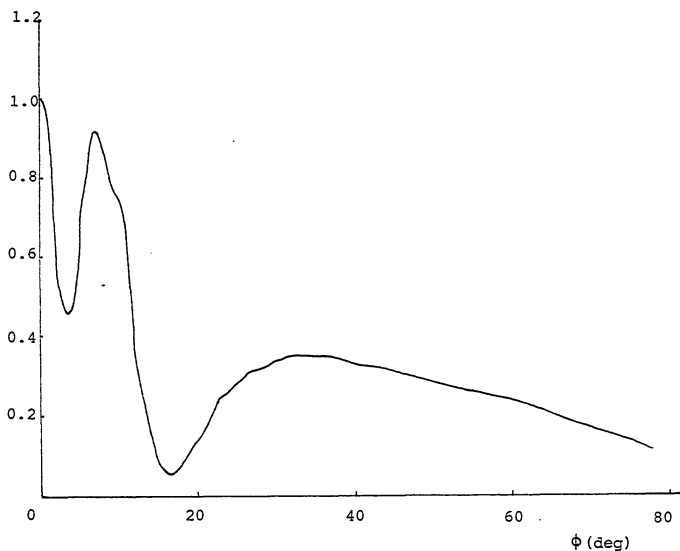


Fig.(4-7C). The elementary pattern on the horizontal plane.
 (The dielectric constant of the layer $\xi=1.5\xi_0$, a z-direction slot center-to-center distance $2d=0.7\lambda_0$, the length of a slot $2h=0.6\lambda_0$, and an azimuthal slot center-to-center distance $d_c=0.7\lambda_0$)

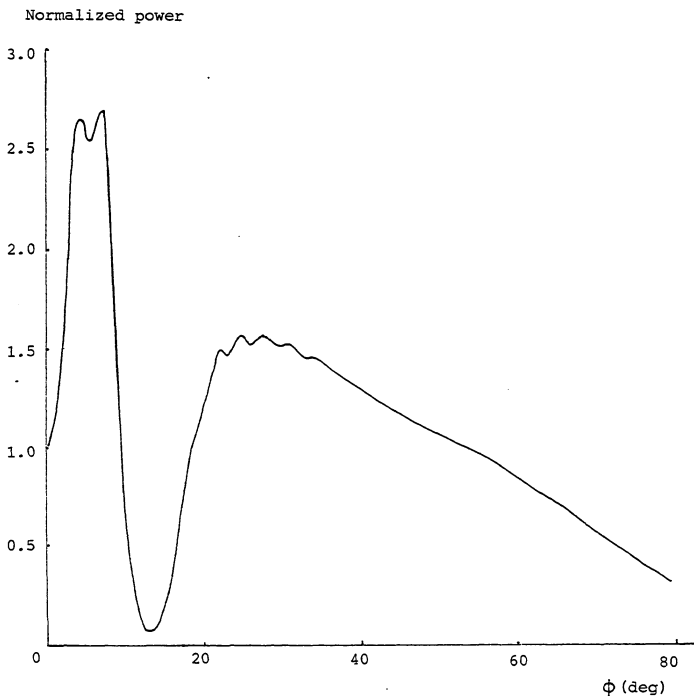


Fig. (4-7D). The elementary pattern on the horizontal plane.

(The dielectric constant of the layer $\xi=2\xi_0$, a z-direction slot center-to-center distance $2d=0.7\lambda_0$, the length of a slot $2h=0.6\lambda_0$, and an azimuthal slot center-to-center distance $d_c=0.7\lambda_0$.)

Normalized power

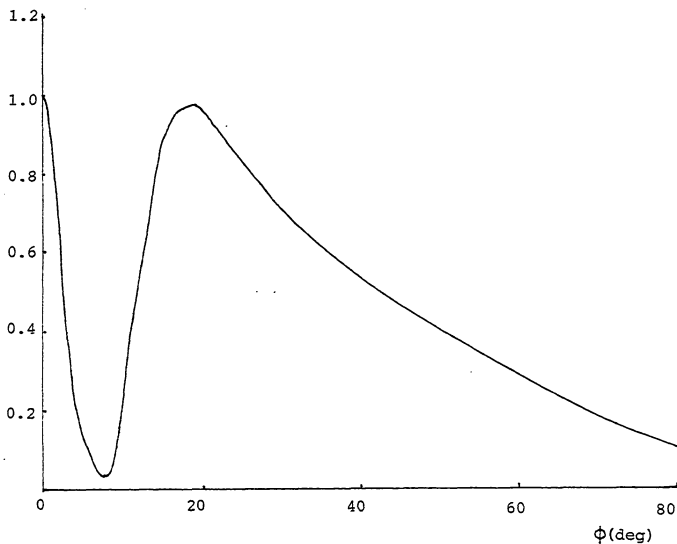


Fig. (4-7E). The elementary pattern on the horizontal plane.

(The dielectric constant of the layer $\xi=2.5\xi_0$, a z-direction slot center-to-center distance $2d=0.7\lambda_0$, the length of a slot $2h=0.6\lambda_0$ and an azimuthal slot center-to-center distance $d_c=0.7\lambda_0$.)

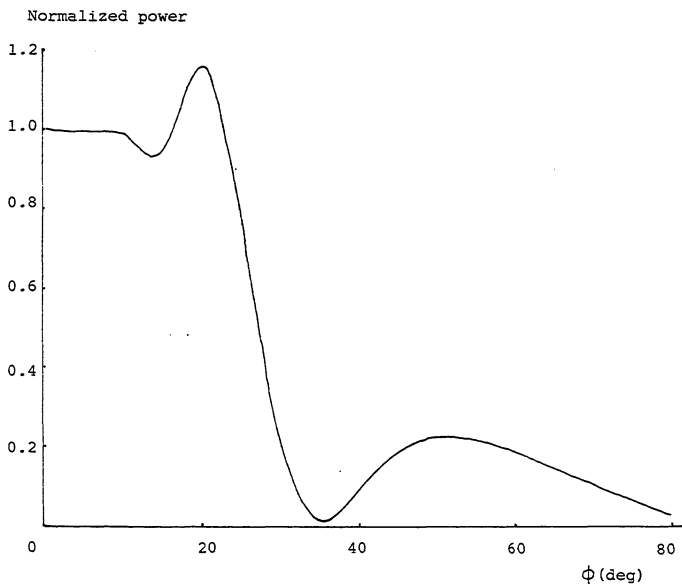


Fig.(4-8). The elementary pattern on the horizontal plane.

(The dielectric constant of the layer $\xi=2.5\xi_0$, a z-direction slot center-to-center distance $2d=0.7\lambda_0$, the length of a slot $2h=0.6\lambda_0$ and an azimuthal slot center-to-center distance $d_c=0.55\lambda_0$.)

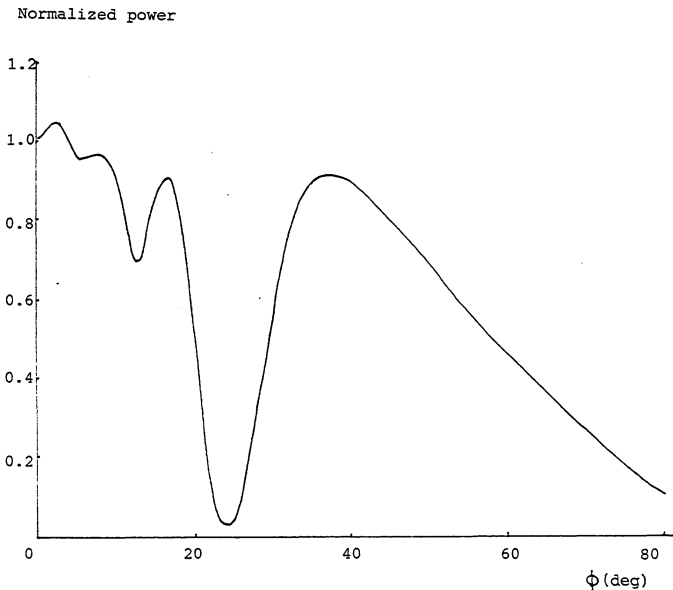


Fig.(4-9A). The elementary pattern on the horizontal plane.

(The dielectric constant of the layer $\xi = 2.5\xi_0$, a z-direction slot center-to-center distance $2d = 0.6\lambda_0$, the length of a slot $2h = 0.55\lambda_0$ and an azimuthal slot center-to-center distance $d_c = 0.6\lambda_0$)

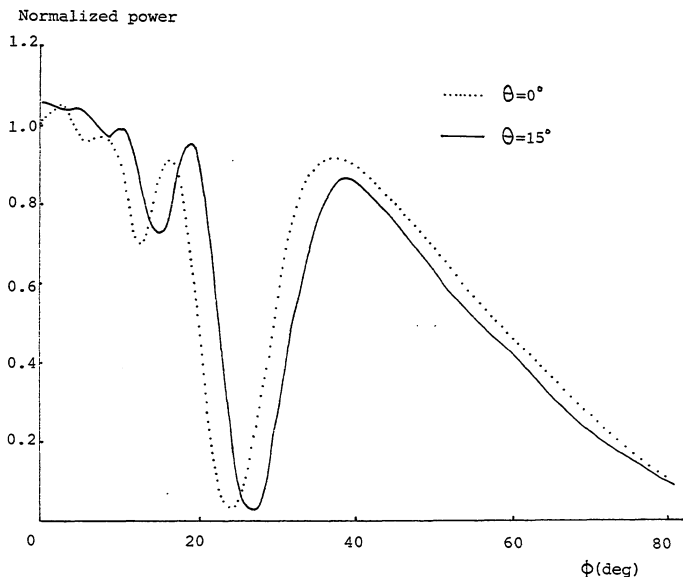


Fig. (4-9B). The elementary pattern cut at an elevation angle θ .
 (The elevation angle $\theta = 15^\circ$, the dielectric constant of the layer $\epsilon = 2.5\epsilon_0$, a z-direction slot center-to-center distance $2d = 0.6\lambda_0$, the length of a slot $2h = 0.55\lambda_0$, and an azimuthal slot center-to-center distance $d_c = 0.6\lambda_0$)

Normalized power

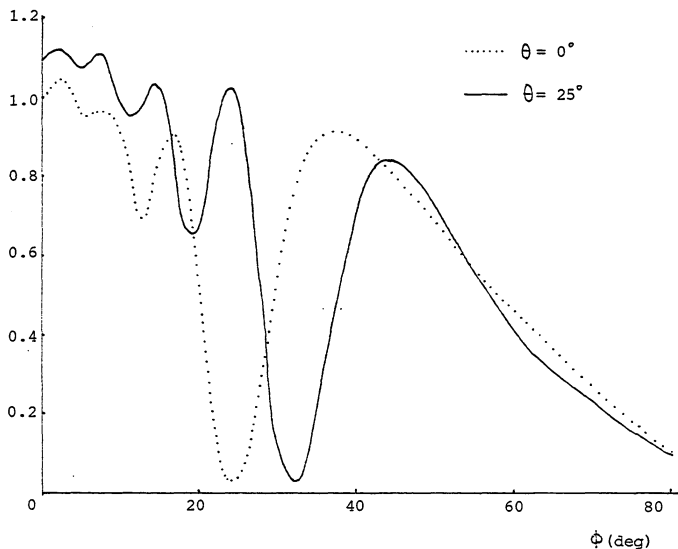


Fig.(4-9C). The elementary pattern cut at an elevation angle θ .

(The elevation angle $\theta=25^\circ$, the dielectric constant of the layer $\xi=2.5\xi_0$, a z-direction slot center-to-center distance $2d=0.6\lambda_0$, the length of a slot $2h=0.55\lambda_0$ and an azimuthal slot center-to-center distance $d_c=0.6\lambda_0$)

Normalized power

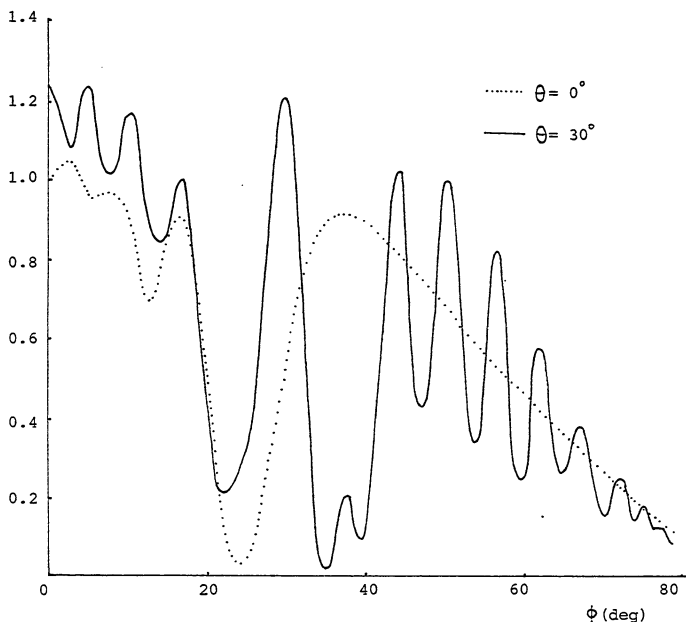


Fig.(4-9D). The elementary pattern cut at an elevation angle θ .
 (The elevation angle $\theta=30^\circ$, the dielectric constant of the layer $\xi=2.5\xi_0$, a z-direction slot center-to-center distance $2d=0.6\lambda_0$, the length of a slot $2h=0.55\lambda_0$, and an azimuthal slot center-to-center distance $d_c=0.6\lambda_0$)

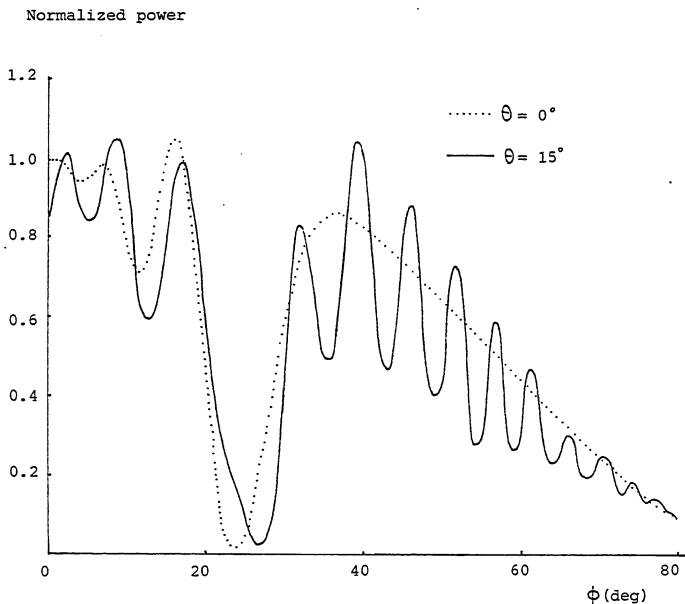


Fig.(4-10). The elementary pattern cut at an elevation angle θ .
 (The elevation angle $\theta=15^\circ$, the dielectric constant of the layer $\xi=2.5\xi_0$, a z-direction slot center-to-center distance $2d=0.7\lambda_0$, the length of a slot $2h=0.6\lambda_0$ and an azimuthal slot center-to-center distance $d_c=0.6\lambda_0$.)

Appendix 1. Watson transform for a near field calculation.

The Eq.(5) of chapter 2 can also be written as

$$G_n(\rho, \phi; \rho', \phi') = -\frac{1}{16\pi\omega\epsilon_0} \sum_{m=-\infty}^{\infty} F(m) \exp(im(\phi - \phi')) \quad (1)$$

where

$$F(m) = \frac{H'_m(\sqrt{\rho_0^2 - \beta_n^2} a) H_m^2(\sqrt{\rho_0^2 - \beta_n^2} \rho) - H_m^2(\sqrt{\rho_0^2 - \beta_n^2} a) H'_m(\sqrt{\rho_0^2 - \beta_n^2} \rho)}{H'_m(\sqrt{\rho_0^2 - \beta_n^2} a)} H'_m(\sqrt{\rho_0^2 - \beta_n^2} \rho) \quad (\rho \leq \rho')$$

Using the residue theorem of complex variables, the harmonic series of Eq.(1) can be transformed into a contour integral where the contour encircles clockwise the real axis of a complex ν -plane (Fig.2-2). The contour C_1 or C_2 passes right over or under the real axis respectively.

$$G_n(\rho, \phi; \rho', \phi') = \frac{1}{16\pi\omega\epsilon_0} \left[\frac{\pi}{2\pi i} \right] \oint_{C_1+C_2} F(\nu) e^{i\nu(\phi-\phi')} \frac{e^{-i\nu\pi}}{\sin\nu\pi} d\nu \quad (2)$$

The contour integral of Eq.(2) is reduced to the harmonic series of Eq.(1) by taking residues at the zeroes of $\sin\nu\pi$ which are integers. Since $H'_{-\nu}(x) = \exp(i\nu\pi) H'_\nu(x)$ and $H^2_{-\nu}(x) = \exp(-i\nu\pi) H^2_\nu(x)$ (Ref.10), $F(\nu)$ is symmetric with respect to the origin of the ν -plane. Accordingly, the integration on contour C_2 can be combined with the one on contour C_1 .

$$G_n(\rho, \phi; \rho', \phi') = \frac{-i}{16\pi\omega\epsilon_0} \int_{C_1} F(\nu) \frac{\cos \nu[(\phi-\phi')-\pi]}{\sin \nu\pi} d\nu \quad (3)$$

Since $H'_\nu(\sqrt{k_0^2 - \beta_n^2} a)$, the denominator of $F(\nu)$, has zeroes on the complex ν -plane, closing the contour C_1 along an infinite semi-circle on the upper half-plane, G_n can be represented with a residue series due to the zeroes of $H'_\nu(\sqrt{k_0^2 - \beta_n^2} a)$, V_n 's and an integral on the contour C_3 .

$$G_n(\rho, \phi; \rho', \phi') = \frac{-1}{8\omega\xi_0} \sum_m \frac{H_\nu^2(\sqrt{k_0^2 - \beta_n^2} a) H'_\nu(\sqrt{k_0^2 - \beta_n^2} \rho)}{\frac{\partial H'_\nu(\sqrt{k_0^2 - \beta_n^2} a)}{\partial \nu}} H'_\nu(\sqrt{k_0^2 - \beta_n^2} \rho) \frac{\cos \nu[(\phi - \phi') - \pi]}{\sin \nu \pi} + I \quad (4)$$

$$\text{where} \quad I = \frac{i}{16\pi\omega\xi_0} \int_{C_3} F(\nu) \frac{\cos \nu[(\phi - \phi') - \pi]}{\sin \nu \pi} d\nu$$

As will be shown in the following discussion, the integral on the infinite semi-circle C_3 goes to zero and the Green's function can be evaluated entirely with the residue series of Eq. (4) which has a fast convergence in the angular region where $|\phi - \phi'|$ is not small.

To investigate the integral on the contour C_3 , the function $F(\nu)$ is further studied as ν goes to infinity on the upper half-plane. Using the following formulas for Hankel functions (Ref.18),

$$H'_\nu(x) = \frac{J_{-\nu}(x) - e^{-i\nu\pi} J_\nu(x)}{i \sin \nu \pi} \quad (5)$$

$$H_\nu^2(x) = \frac{e^{i\nu\pi} J_\nu(x) - J_{-\nu}(x)}{i \sin \nu \pi} \quad (6)$$

$F(\nu)$ is given as

$$F(\nu) = \frac{2 [J_\nu(\sqrt{\rho_0^2 - \beta_n^2} \rho) J_\nu(\sqrt{\rho_0^2 - \beta_n^2} a) - J_\nu(\sqrt{\rho_0^2 - \beta_n^2} a) J_\nu(\sqrt{\rho_0^2 - \beta_n^2} \rho)]}{i \sin \nu \pi [J_\nu(\sqrt{\rho_0^2 - \beta_n^2} a) - e^{-i\nu\pi} J_\nu(\sqrt{\rho_0^2 - \beta_n^2} \rho)]} \cdot$$

$$[J_\nu(\sqrt{\rho_0^2 - \beta_n^2} \rho) - e^{-i\nu\pi} J_\nu(\sqrt{\rho_0^2 - \beta_n^2} a)] \quad (7)$$

As $|\nu|$ goes to infinity, the Bessel function of a fixed argument can be approximated with the first term of the series expansion for the function.

$$J_\nu(x) \approx \frac{(\frac{1}{2}x)^\nu}{\Gamma(\nu+1)} \quad (8)$$

With the following relationship of gamma functions (Ref.18),

$$\Gamma(\nu) \Gamma(-\nu+1) = \frac{\pi}{\sin \nu \pi} \quad (9)$$

from Eqs. (7) and (8), $F(\nu)$ can be approximated by Eq. (10).

$$F(\nu) \approx \frac{2}{i\nu\pi} \left[\left(\frac{\rho}{a}\right)^\nu - \left(\frac{a}{\rho}\right)^\nu \right] \frac{1 - e^{-i\nu\pi} \left(\frac{1}{2}\sqrt{\rho_0^2 - \beta_n^2} \rho\right)^{2\nu} \pi / [\sin \nu \pi \Gamma(\nu) \Gamma(\nu+1)]}{1 - e^{-i\nu\pi} \left(\frac{1}{2}\sqrt{\rho_0^2 - \beta_n^2} a\right)^{2\nu} \pi / [\sin \nu \pi \Gamma(\nu) \Gamma(\nu+1)]} \quad (10)$$

Since $\sin \nu \pi \approx \frac{i e^{-i\nu\pi}}{2}$ on the contour C_3 and

$$\Gamma(\nu) \approx \sqrt{\frac{2\pi}{\nu}} \left(\frac{\nu}{e}\right)^\nu \quad |\arg \nu| < \pi \quad (11)$$

$F(\nu)$ on the infinite semi-circle is reduced to

$$F(\nu) \approx \frac{2}{i\nu\pi} \left[\left(\frac{\rho}{a}\right)^\nu - \left(\frac{a}{\rho}\right)^\nu \right] \frac{1 + i \left[\frac{e\nu\sqrt{\rho_0^2 - \beta_n^2} \rho}{2\nu} \right]^{2\nu}}{1 + i \left[\frac{e\nu\sqrt{\rho_0^2 - \beta_n^2} a}{2\nu} \right]^{2\nu}} \quad (12)$$

$F(\nu)$ can be further simplified by considering it separately in the first and the second quadrants of the complex ν -plane. In the first quadrant,

$$F(\nu) \approx \frac{2}{\lambda \nu \pi} \left[\frac{\rho}{\rho'} \right]^\nu \quad 0 \leq \arg \nu \leq \frac{\pi}{2} \quad (13)$$

In the second quadrant,

$$F(\nu) \approx -\frac{2}{\lambda \nu \pi} \left[\frac{\rho'}{\rho} \right]^\nu \quad \frac{\pi}{2} \leq \arg \nu \leq \pi \quad (14)$$

Since $\rho' \geq \rho$, in the upper half-plane, $F(\nu)$ is decreasing at least at the rate of ν^{-1} . Approximating the angular factor in the contour integration I of Eq.(4) as $\frac{\cos \nu(\phi - \phi' - \pi)}{\sin \nu \pi} \approx -\lambda e^{i\nu|\phi - \phi'|}$ on the upper half-plane, and replacing ν with $Re^{i\theta}$ where R is a constant, the contour integral I is given by the following equation when $\rho' = \rho$.

$$I = \frac{1}{8\pi^2 \omega \epsilon_0} \left[\int_0^{\frac{\pi}{2}} e^{-R|\phi - \phi'|(\sin \theta - i \cos \theta)} d\theta - \int_{\frac{\pi}{2}}^{\pi} e^{-R|\phi - \phi'|(\sin \theta - i \cos \theta)} d\theta \right] \quad (15)$$

As R goes to infinity, the value of I is equal to zero.

Appendix 2. Representations for Hankel functions of large complex orders.

The solution for the differential equation,

$$\frac{d^2 u}{dz^2} = \left\{ \frac{1-z^2}{z^2} \nu^2 - \frac{1}{4z^2} \right\} u \quad (1)$$

is a linear combination of $z^{\frac{1}{2}} J_{\nu}(\nu z)$ and $z^{\frac{1}{2}} Y_{\nu}(\nu z)$ where J_{ν} or Y_{ν} is the Bessel function of the first or second kind. With the following transformations from z to ζ and from u to w

$$\zeta \left(\frac{d\zeta}{dz} \right)^2 = \frac{1-z^2}{z^2} \quad (2)$$

$$w = \left(\frac{d\zeta}{dz} \right)^{\frac{1}{2}} u \quad (3)$$

Eq.(1) is given as

$$\frac{d^2 w}{d\zeta^2} = (\nu^2 \zeta + f(\zeta)) w \quad (4)$$

$$f(\zeta) = -\frac{z^2}{4z^2} + z^{\frac{1}{2}} \frac{d^2}{d\zeta^2} (z^{-\frac{1}{2}})$$

where $z = \frac{dz}{d\zeta}$. Rigorous discussions on the uniform asymptotic expansion of w for a large value of $|\nu|$ can be found in Refs.16,17. The first term of the asymptotic expansion

is the solution of the following differential equation.

$$\frac{d^2 w'}{d\zeta^2} = \nu^2 \zeta w' \quad (5)$$

For the large value of $|\nu|$, the solution of Eq.(5) yields a fairly accurate result for w of Eq.(4).

The solutions for Eq.(5) are Airy functions (Ref.18), or equivalently, linear combinations of $\sqrt{\zeta} I_{\frac{1}{3}}(\frac{2}{3}\nu\zeta^{\frac{3}{2}})$ and $\sqrt{\zeta} I_{-\frac{1}{3}}(\frac{2}{3}\nu\zeta^{\frac{3}{2}})$.

$$w' = a \zeta^{\frac{1}{3}} I_{\frac{1}{3}}(\frac{2}{3}\nu\zeta^{\frac{3}{2}}) + b \zeta^{\frac{1}{3}} I_{-\frac{1}{3}}(\frac{2}{3}\nu\zeta^{\frac{3}{2}}) \quad (6)$$

where a and b are constants and $I_{\nu}(x)$ is a modified Bessel function which satisfies the following differential equation.

$$(x \frac{d^2}{dx^2} + x \frac{d}{dx} - (x + \nu^2)) I_{\nu}(x) = 0 \quad (7)$$

ζ is given as an explicit function of z by solving Eq.(2).

$$\frac{2}{3}\nu\zeta^{\frac{3}{2}} = \int_z^1 \frac{\sqrt{1-z'^2}}{z'} dz' = \ln\left(\frac{1+\sqrt{1-z^2}}{z}\right) - \sqrt{1-z^2} \quad (8)$$

The branches in Eq.(8) are chosen such that ζ is real when z is positive. From Eqs.(2), (3), (6) and (8), $u(z)$ for a large value of $|\nu|$ is given as

$$u(z) \approx \left[3z \frac{\ln \frac{1+\sqrt{1-z^2}}{z} - \sqrt{1-z^2}}{2\sqrt{1-z^2}} \right]^{\frac{1}{2}} \left\{ a I_{\frac{1}{3}} \left[\nu \left(\ln \frac{1+\sqrt{1-z^2}}{z} - \sqrt{1-z^2} \right) \right] + b I_{-\frac{1}{3}} \left[\nu \left(\ln \frac{1+\sqrt{1-z^2}}{z} - \sqrt{1-z^2} \right) \right] \right\} \quad (9)$$

Since $u(z)$ is a linear combination of $z^{\frac{1}{2}} J_{\nu}(\nu z)$ and $z^{\frac{1}{2}} Y_{\nu}(\nu z)$ and, $H'_{\nu}(x) = J_{\nu}(x) + i Y_{\nu}(x)$, from Eq. (9),

$$H'_{\nu}(\nu z) \approx \left[3 \frac{\ln \frac{1+\sqrt{1-z^2}}{z} - \sqrt{1-z^2}}{2\sqrt{1-z^2}} \right]^{\frac{1}{2}} \left\{ a I_{\frac{1}{3}} \left[\nu \left(\ln \frac{1+\sqrt{1-z^2}}{z} - \sqrt{1-z^2} \right) \right] + b I_{-\frac{1}{3}} \left[\nu \left(\ln \frac{1+\sqrt{1-z^2}}{z} - \sqrt{1-z^2} \right) \right] \right\} \quad (10)$$

a and b are determined by comparing expansions of Hankel and modified Hankel functions for large arguments in Eq. (10) as z goes to infinity with a fixed value of ν . For such a value of z , the argument of modified Hankel functions

$$\nu \left(\ln \frac{1+\sqrt{1-z^2}}{z} - \sqrt{1-z^2} \right) \approx e^{i\frac{3\pi}{2}} \left(z - \frac{\pi}{2} \right) \nu \quad (11)$$

in the chosen branches of Eq. (8). Using the expansion of a Hankel function for a large argument, from Eqs. (10) and (11),

$$\begin{aligned} & \sqrt{\frac{2}{\pi z \nu}} e^{i(\nu z - \frac{1}{2}\nu\pi - \frac{1}{4}\pi)} \\ &= \sqrt{\frac{3}{2}} e^{i\frac{\pi}{2}} \left\{ a I_{\frac{1}{3}} \left[e^{i\frac{3\pi}{2}} \left(z - \frac{\pi}{2} \right) \nu \right] + b I_{-\frac{1}{3}} \left[e^{i\frac{3\pi}{2}} \left(z - \frac{\pi}{2} \right) \nu \right] \right\} \\ &= \sqrt{\frac{3}{2}} \left\{ -a J_{\frac{1}{3}} \left[\left(z - \frac{\pi}{2} \right) \nu \right] + b J_{-\frac{1}{3}} \left[\left(z - \frac{\pi}{2} \right) \nu \right] \right\} \quad (12) \end{aligned}$$

where the relationship $I_{\pm\frac{1}{3}}(e^{i\frac{3\pi}{2}} x) = e^{\pm i\frac{\pi}{2}} J_{\mp\frac{1}{3}}(x)$ is used.

For a large $(z - \frac{\pi}{2})\nu$,

$$J_{\frac{1}{3}}[(z - \frac{\pi}{2})v] \approx \frac{1}{\sqrt{2\pi z v}} \left(e^{i(zv - \frac{\pi}{2}v - \frac{5}{12}\pi)} + e^{-i(zv - \frac{\pi}{2}v - \frac{5}{12}\pi)} \right) \quad (13)$$

$$J_{\frac{1}{3}}[(z - \frac{\pi}{2})v] \approx \frac{1}{\sqrt{2\pi z v}} \left(e^{i(zv - \frac{\pi}{2}v - \frac{\pi}{12})} + e^{-i(zv - \frac{\pi}{2}v - \frac{\pi}{12})} \right) \quad (14)$$

When above approximations are inserted in Eq.(12), to satisfy the equation, a and b are determined as $a = -\frac{2}{3}e^{i\frac{\pi}{3}}$ and $b = \frac{2}{3}e^{-i\frac{\pi}{3}}$. Finally,

$$H_v'(vz) \approx 2 \left[\frac{\ln \frac{1+\sqrt{1-z^2}}{z} - \sqrt{1-z^2}}{3\sqrt{1-z^2}} \right]^{\frac{1}{2}} \left\{ e^{-i\frac{\pi}{3}} I_{-\frac{1}{3}} \left[v \left(\frac{\ln \frac{1+\sqrt{1-z^2}}{z} - \sqrt{1-z^2}}{z} \right) \right] - e^{i\frac{\pi}{3}} I_{\frac{1}{3}} \left[v \left(\frac{\ln \frac{1+\sqrt{1-z^2}}{z} - \sqrt{1-z^2}}{z} \right) \right] \right\} \quad (15)$$

Also, $H_v^2(vz)$ can be found by interchanging i and $-i$ in Eq.(15).

By substituting z with $\text{sech} \alpha$ in Eq.(15), a more convenient form of the Hankel function in actual calculations can be derived (Ref.19).

$$H_v'(v \text{sech} \alpha) \approx 2 \left(\frac{\alpha - \tanh \alpha}{3 \tanh \alpha} \right)^{\frac{1}{2}} \left\{ e^{-i\frac{\pi}{3}} I_{-\frac{1}{3}} [v(\alpha - \tanh \alpha)] - e^{i\frac{\pi}{3}} I_{\frac{1}{3}} [v(\alpha - \tanh \alpha)] \right\} \quad (16)$$

The approximation formulas for $H_v'(v \text{sech} \alpha)$, $\frac{\partial H_v'(v \text{sech} \alpha)}{\partial v}$ and $\frac{\partial H_v'(v \text{sech} \alpha)}{\partial v}$, can be found from Eq.(16). $H_v'(v \text{sech} \alpha)$ is the derivative of the Hankel function with respect to its argument. In taking the derivatives with respect to the argument as well as the order, it must be noted that the argument $v \text{sech} \alpha$ is a given constant c and, as a

whole, is independent of the order ν . α is determined by the given argument c with the relationship $\alpha = \cosh^{-1} \frac{\nu}{c}$. From Eq. (16), the following approximation formulas are derived.

$$H'_\nu(\nu \operatorname{sech} \alpha) \approx 2 \cosh \alpha \left(\frac{\alpha - \tanh \alpha}{3 \tanh \alpha} \right)^{\frac{1}{2}} \left\{ \tanh \alpha \left[e^{i\frac{\pi}{3}} I_{-\frac{2}{3}}(r) - e^{-i\frac{\pi}{3}} I_{\frac{2}{3}}(r) \right] - \frac{3 \tanh \alpha - 3 \alpha \operatorname{sech}^2 \alpha - 2 \tanh^3 \alpha}{6 \nu \tanh^2 \alpha (\alpha - \tanh \alpha)} \left[e^{i\frac{\pi}{3}} I_{-\frac{1}{3}}(r) - e^{-i\frac{\pi}{3}} I_{\frac{1}{3}}(r) \right] \right\} \quad (17)$$

$$\frac{\partial H'_\nu(\nu \operatorname{sech} \alpha)}{\partial \nu} \approx 2 \left(\frac{\alpha - \tanh \alpha}{3 \tanh \alpha} \right)^{\frac{1}{2}} \left\{ \alpha \left[e^{-i\frac{\pi}{3}} I_{\frac{2}{3}}(r) - e^{i\frac{\pi}{3}} I_{-\frac{2}{3}}(r) \right] + \left[\frac{\cosh \alpha}{3 \cosh^2 \alpha} + \frac{\alpha - \sinh \alpha \cdot \cosh \alpha}{6 \cosh^2 \alpha (\alpha \cosh \alpha - \sinh \alpha)} \right] \left[e^{i\frac{\pi}{3}} I_{\frac{1}{3}}(r) - e^{-i\frac{\pi}{3}} I_{-\frac{1}{3}}(r) \right] \right\} \quad (18)$$

$$\begin{aligned} \frac{\partial H'_\nu(\nu \operatorname{sech} \alpha)}{\partial \nu} &\approx \frac{2}{c} \left(\frac{\alpha - \tanh \alpha}{3 \tanh \alpha} \right)^{\frac{1}{2}} \frac{\cosh \alpha \sinh \alpha - \alpha}{2 \sinh^2 \alpha} \left[e^{i\frac{\pi}{3}} I_{-\frac{2}{3}}(r) - e^{-i\frac{\pi}{3}} I_{\frac{2}{3}}(r) \right] \\ &+ \frac{2}{c^2} \left(\frac{\alpha - \tanh \alpha}{3 \tanh \alpha} \right)^{\frac{1}{2}} \left\{ c^2 \alpha \sinh \alpha + \frac{1}{12 \sinh \alpha (\alpha \cosh \alpha - \sinh \alpha)} \right. \\ &\left. \left[-\frac{2 \cosh \alpha}{3} - \frac{\alpha}{\sinh \alpha} + \frac{5(\alpha - \sinh \alpha \cosh \alpha)}{3(\alpha \cosh \alpha - \sinh \alpha)} + \frac{5 \cosh \alpha}{4 \sinh^4 \alpha} \right] \right. \\ &\left. \left[e^{i\frac{\pi}{3}} I_{\frac{1}{3}}(r) - e^{-i\frac{\pi}{3}} I_{-\frac{1}{3}}(r) \right] \right\} \quad (19) \end{aligned}$$

In the above equations, the argument of modified Bessel functions $r = \nu (\alpha - \tanh \alpha)$. The corresponding formulas for $H_\nu^{(2)}(\nu \operatorname{sech} \alpha)$, $\frac{\partial H_\nu^{(2)}(\nu \operatorname{sech} \alpha)}{\partial \nu}$ and $\frac{\partial H_\nu^{(2)}(\nu \operatorname{sech} \alpha)}{\partial \nu}$ can be generated simply by interchanging i with $-i$ in the above equations. The calculations of $I_{\pm \frac{1}{3}}(r)$ and $I_{\pm \frac{2}{3}}(r)$ can be carried out using ascending series when $|r|$ is small. For a large $|r|$, the modified Bessel functions are first transformed to first kind Bessel functions using the relationship $I_{\pm \frac{1}{3}}(r) = e^{\pm i \frac{\pi}{6}} J_{\pm \frac{1}{3}}(-ir)$ and $I_{\pm \frac{2}{3}}(r) = e^{\pm i \frac{\pi}{3}} J_{\pm \frac{2}{3}}(-ir)$. The latter functions can be evaluated with an asymptotic formula since the orders of these functions are small compared with their arguments (Ref.18).

For a given values of ν and c , to calculate $H_\nu^{(1)}(c)$, the complex constant α of Eq.(16), which is given as $\alpha = \cosh^{-1} \frac{\nu}{c}$, must be found. A mapping from $\frac{\nu}{c}$ -plane to α -plane which is consistent with the branch cuts specified in the discussion of Eq.(8) is plotted in Fig.(2-3). Also, a mapping from α -plane to $(\alpha - \tanh \alpha)$ -plane is shown in the figure. A point in the first quadrant of the $\frac{\nu}{c}$ -plane, can be mapped to a point in the third quadrant of $(\alpha - \tanh \alpha)$ -plane. In the computation of Eq.(16), when a square root or a cube root of $(\alpha - \tanh \alpha)$ is taken, for the points which belong to the first quadrant of $\frac{\nu}{c}$ -plane, a branch cut must be drawn in the fourth quadrant of the $(\alpha - \tanh \alpha)$ -plane instead of the negative real axis which is common for a

digital computer in the calculation for a square root or a logarithm for a complex number. Otherwise, $H'_y(c)$ is not analytic in the first quadrant of the $\frac{y}{c}$ -plane. Throughout the applications of Eqs. (16), (17), (18) and (19), the branches must be chosen carefully. This is especially the case when c is a complex number.

Appendix 3. Zeroes of $H'_\nu(z)$ in a complex ν - plane.

In Eq.(16) of Appendix 2, an approximation formula for the Hankel function of a large complex order is given as

$$H'_\nu(z) = H'_\nu(\nu \operatorname{sech} \alpha) \approx 2 \left(\frac{\alpha - \tanh \alpha}{3 \tanh \alpha} \right)^{\frac{1}{2}} \left\{ e^{-i\frac{\pi}{3}} I_{-\frac{1}{3}}[\nu(\alpha - \tanh \alpha)] - e^{i\frac{\pi}{3}} I_{\frac{1}{3}}[\nu(\alpha - \tanh \alpha)] \right\} \quad (1)$$

From Eq.(1), for a given value of z , the zeroes of $H'_\nu(z)$, ν_m 's in a complex ν -plane must satisfy the following equation.

$$e^{-i\frac{\pi}{3}} I_{-\frac{1}{3}}[\nu_m(\alpha_m - \tanh \alpha_m)] - e^{i\frac{\pi}{3}} I_{\frac{1}{3}}[\nu_m(\alpha_m - \tanh \alpha_m)] = 0 \quad (2)$$

Using the relationship $I_{\pm\frac{1}{3}}(ix) = e^{\pm\frac{i\pi}{6}} J_{\pm\frac{1}{3}}(x)$, Eq.(2) can be transformed to the following form.

$$J_{-\frac{1}{3}}(R_m) + J_{\frac{1}{3}}(R_m) = 0 \quad (3)$$

where $R_m = -i\nu_m(\alpha_m - \tanh \alpha_m)$. The zeroes of Eq.(3), R_m 's are well known and can be found, for instance, from Ref.18 where the roots of Airy function are tabulated. From these roots, R_m 's are given using the relationship $A_1(-x) = J_{-\frac{1}{3}}(2/3 \cdot x^{\frac{3}{2}}) + J_{\frac{1}{3}}(2/3 x^{\frac{3}{2}})$. As a result, the remaining problem is to solve the following simultaneous equations and to find

corresponding ν_m 's for all R_m 's with a given value of z .

$$R_m = -i\nu_m(\alpha_m - \tanh\alpha_m) \quad (4)$$

$$\nu_m = z \cosh \alpha_m \quad (5)$$

To get an explicit expression for ν_m as a function of R_m and z , $(\alpha_m - \tanh\alpha_m)$ in Eq.(4) is first expanded into an ascending series of α_m . Eliminating ν_m both in Eqs.(4) and (5),

$$\alpha_m^3 \left(1 + \frac{\alpha_m^2}{10} + \frac{\alpha_m^4}{280} + \frac{\alpha_m^6}{15120} + \dots\right) = i \frac{3R_m}{z} = \delta_m^3 \quad (6)$$

To find α_m as a function of δ_m , α_m is represented with an ascending series of δ_m using unknown constants c_i 's.

$$\alpha_m = \delta_m (1 + c_1 \delta_m^2 + c_2 \delta_m^4 + c_3 \delta_m^6 + c_4 \delta_m^8 \dots) \quad (7)$$

From Eqs.(6) and (7), the unknown constants can be determined and α_m is given as

$$\alpha_m = \delta_m \left(1 - \frac{1}{30} \delta_m^2 + \frac{41}{12600} \delta_m^4 - \frac{97}{226800} \delta_m^6 + \dots\right) \quad (8)$$

Expanding $\cosh\alpha_m$ in Eq.(5) into an ascending series of α_m and using Eq.(8), ν_m is given as a function of $\delta_m (= e^{i\pi/4} \frac{3R_m}{z})^{1/3}$.

$$\begin{aligned}
\psi_m = z + \frac{T_m}{4} z^{\frac{1}{3}} - \frac{T_m^2}{240} z^{\frac{2}{3}} + \frac{T_m^3}{2800} z^{\frac{1}{3}} - \frac{281 T_m^4}{18144000} z^{\frac{5}{3}} + \dots \\
+ i \sin \frac{\pi}{3} \left\{ \frac{T_m}{2} z^{\frac{1}{3}} + \frac{T_m^2}{120} z^{\frac{2}{3}} - \frac{281 T_m^4}{9072000} z^{\frac{5}{3}} + \dots \right\} \quad (9)
\end{aligned}$$

where $T_m = (3R_m)^{2/3}$. A similar method can be applied to find the zeroes of $H'_y(z)$, ψ'_m 's in the complex ψ -plane.

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