

“Optimal” Inflation Under Dollarization

by

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A dissertation submitted to the Graduate Faculty in Economics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York

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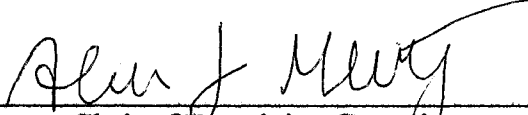
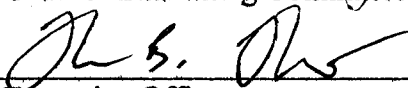
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Abstract

“Optimal” Inflation Under Dollarization

by

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This paper analyzes the effects of dollarization, where a country (C) uses money produced by another country (M). We derive a general formula to determine the “optimal” rate of inflation, which maximizes M’s welfare but does not take into account any loss to C. We show how this inflation varies with relative income size and output growth. Estimates of the “optimal” inflation rate are made for some countries. We also analyze a contract, under which M shares a fraction of total potential seignorage with C to induce M to inflate at a rate which leads to no seignorage accruing to M from C. Finally, we provide estimates of the present discounted value of the seignorage accruing from M to C. Despite this loss, we estimate the net welfare gain to C’s household when dollarization results in a lower rate of inflation.

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Finally, I dedicate this work to my son, Jun-no-suke Kurasawa, who was born just a few months before an oral defense of the dissertation. I am looking forward to seeing my grown-up son reading my work someday (and spotting my mistakes).

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1. Introduction

There is a resurgence of interest in a dollarization regime in which one country, C, uses money produced by another country, M¹. In this dollarization regime, M uses inflation to raise seigniorage from C. The seigniorage takes the form of a balance of payment deficit that M runs with C².

However, such seigniorage is produced at the cost of an externality - the welfare loss imposed by inflation on C that M does not take into account. Assume M has no fiscal constraint so that any seigniorage from M's households is returned to them by the authorities. Then, M equates the marginal seigniorage from C's households to the marginal cost to households in M caused by the loss of the services of real balances due to inflation. The resulting "optimal" inflation maximizes the welfare of M's households but ignores the disutility caused by inflation to C's households. Our analysis makes use of a general formula to determine this "optimal" inflation, which allows us to feed in alternative demand functions for money and takes into account the relative income size of M to C. We then turn to the analysis of a growth of income in M and / or C and its effect on M's "optimal" inflation. We also provide some empirical estimates of the "optimal" inflation rate and the seigniorage M receives from C at this rate of inflation. Can the

¹ See Antinolfi and Keister (2001) and Change (2000) for a review of the literature.

² A variant of this analysis occurs when not only M can raise seigniorage from C but various regions in C compete for seigniorage from a weak central bank in C. The resulting Nash equilibrium leads to an undesirably high rate of inflation since each region ignores the external effects produced by inflation on competing regions. For example, see the interesting papers by Aizenman (1992) and Cooper and Kempf (2001).

provisions of a contract between M and C in which any potential seignorage accruing to M is shared with C induce M not to take any seignorage from C? We derive a sharing parameter of such a contract.

Finally, we provide estimates of the present discounted values of the cost to C's households of the seignorage accruing from C to M. However, dollarization by C may well result in the gain to C's households of lower inflation. Estimates of the present discounted value of the net gain to C's households are provided.

2. The Model

In our model, M is populated with N_m households and C, N_c households. Each household in M produces y_m units of output and y_c units in C. Output per household may differ between M and C but does not vary with the rate of inflation³.

Initially, assume that output per household and the number of households are unchanged over time (this assumption is later relaxed). The money-income ratio is $f(\rho)$ in M and $g(\rho)$ in C, where ρ is the anticipated rate of inflation. $f(\rho)$ and $g(\rho)$ are functions of ρ alone since the income elasticity of demand for money is assumed unity (the analysis is a steady state one). A household's real money demand function is $m_m = f(\rho)y_m$ in M and $m_c = g(\rho)y_c$ in C⁴. Aggregating over households,

$$\begin{aligned} M_m &= \sum^{N_m} m_m \\ &= N_m f(\rho) y_m \\ &= Y_m f(\rho) \end{aligned} \tag{1}$$

$$\begin{aligned} M_c &= \sum^{N_c} m_c \\ &= N_c g(\rho) y_c \\ &= Y_c g(\rho) \end{aligned} \tag{2}$$

³ In this assumption, our analysis differs from that of Cooper and Kempf (2001). Cooper and Kempf use an overlapping generations model in which higher inflation reduces output directly. This requires, as they note, that substitution effect dominate income effect on the labor supply.

⁴ These money demand functions can be derived from a general equilibrium model using explicit utility functions. For example, see Chapters 2 and 3 of Walsh (1998).

$Y_m = N_m y_m$ and $Y_c = N_c y_c$ are aggregate output. The welfare cost to M's households is

$$\begin{aligned} W_m &= \int_{\rho=0}^{\pi} M_m d\rho - \pi M_m \\ &= \int_{\rho=0}^{\pi} Y_m f(\rho) d\rho - \pi Y_m f(\pi)^5 \end{aligned} \quad (3)$$

which is the area under M's real money demand function between the quantities held at a point of no money growth and at an inflation rate π (π is an actually prevailing value of ρ). We assume M has no fiscal constraint so that any seigniorage from M's households is returned to them. Since there is no growth, the growth rate of money σ is equal to π . The seigniorage M receives from C is

$$S_c = \pi M_c = \pi Y_c g(\pi) \quad (4)$$

The excess of the seigniorage from C over M's welfare cost is M's net gain from inflating at π :

$$\begin{aligned} G_m &= S_c - W_m \\ &= \pi Y_c g(\pi) - [\int_{\rho=0}^{\pi} Y_m f(\rho) d\rho - \pi Y_m f(\pi)] \end{aligned} \quad (5)$$

M sets the inflation rate to maximize this net gain to M's households. The first-order condition for this maximization is

$$M_c^* (1 - \eta_c^*) = M_m^* \eta_m^* \quad (6)$$

where

$$\eta_m^* = - \pi^* f'(\pi^*) / f(\pi^*)$$

⁵ Equation (3) is the net consumers' surplus after seigniorage is returned to households. Lucas (2000) shows a measure of the consumers' surplus is a good approximation to the welfare cost of inflation measured by the compensating variation which is derived from explicit utility maximization.

⁶ This is a generalization of the formula for the ratio of the marginal welfare loss to the marginal revenue in a single economy when the maximization is taken with respect to the money rate of interest: $dW / dS = \eta_i / (1 - \eta_i)$, where η_i is the elasticity of demand of real balance with respect to the money rate of interest. See Marty (1976). Here, the rate of price change is taken as an approximation.

$$\eta_c^* = -\pi^* g'(\pi^*) / g(\pi^*)^7$$

η_m^* and η_c^* are the elasticity of the real money demand with respect to the common inflation rate under dollarization. M_m^* , M_c^* , η_m^* and η_c^* are all evaluated at π^* , which is “optimal” from the limited point of view of M’s households. At this “optimal” rate, our formula (6) equates the marginal seigniorage from C to the marginal welfare loss to M. The formula can be used to feed in alternative demand functions (this is done later) and to determine how M’s “optimal” inflation varies with alternative output levels.

To see how π^* varies with output levels, let $R = Y_c / Y_m$ and $h(\pi^*, R) = Rg(\pi^*)(1 - \eta_c^*) - f(\pi^*)\eta_m$. $\partial h(\pi^*, R) / \partial \pi^* < 0$ from the second-order condition and $\partial h(\pi^*, R) / \partial R = g(\pi^*)(1 - \eta_c^*) > 0$ given that $f(\pi^*)$, $g(\pi^*)$ and η_m^* are all positive⁸. Then,

$$\partial \pi^* / \partial R = - [\partial h(\pi^*, R) / \partial R] / [\partial h(\pi^*, R) / \partial \pi^*] > 0 \quad (7)$$

This result makes intuitive sense. The larger is C relative to M, the higher is M’s “optimal” inflation. Moreover, in the absence of growth, dollarization always induces M to inflate ($\pi^* = 0$ only if $R = 0$).

At π^* , M receives $S_c^* = \pi^* M_c^* = \pi^* Y_c g(\pi^*)$ from C. Take the ratio of S_c^* to Y_c and differentiate this ratio with respect to R (π^* is an increasing function of R). Then,

$$\partial (S_c^* / Y_c) / \partial R = [g(\pi^*) + \pi^* g'(\pi^*)](\partial \pi^* / \partial R) > 0^9 \quad (8)$$

This shows the larger is R, the greater proportion of C’s income is appropriated by M as seigniorage.

⁷ Note that the elasticities are independent of aggregate output.

⁸ If $f(\pi^*)$, $g(\pi^*)$ and η_m^* are all positive, η_c^* is less than unity from (6). This may be seen as follows. If no welfare loss is imposed by inflation on M’s households, M will set π^* where the seigniorage is maximized at $\eta_c = 1$. The fact that inflation produces a loss to M’s households implies that η_c is less than 1.

⁹ The first-order condition implies $g(\pi^*) + \pi^* g'(\pi^*) = g(\pi^*)(1 - \eta_c^*) > 0$.

Now, we apply the formula (6) to alternative demand functions. Suppose that $f(\rho)$

and $g(\rho)$ are linear in both countries:

$$f(\rho) = a - \alpha\rho \quad a > 0 \text{ and } \alpha > 0 \quad (9)$$

$$g(\rho) = b - \beta\rho \quad b > 0 \text{ and } \beta > 0 \quad (10)$$

From (6),

$$\pi^* = bY_c / (\alpha Y_m + 2\beta Y_c) \quad (11)$$

Alternatively, suppose that $f(\rho)$ and $g(\rho)$ are both semi-log:

$$f(\rho) = a\text{Exp}[-\alpha\rho] \quad a > 0 \text{ and } \alpha > 0 \quad (12)$$

$$g(\rho) = b\text{Exp}[-\beta\rho] \quad b > 0 \text{ and } \beta > 0 \quad (13)$$

π^* satisfies the following condition:

$$Y_c b \text{Exp}[-\beta\pi^*](1 - \beta\pi^*) = Y_m a \text{Exp}[-\alpha\pi^*]\alpha\pi^* \quad (14)$$

A closed-form solution is not generally possible. However, suppose that the semi-elasticities are equal ($\alpha = \beta$). Then, a closed-form solution is obtained:

$$\pi^* = bY_c / \alpha(aY_m + bY_c) \quad (15)$$

If we further assume that M and C have the same semi-log function ($\alpha = \beta$ and $a = b$),

then (15) reduces to

$$\pi^* = Y_c / \alpha(Y_m + Y_c) \quad (15)'$$

3. Growth in Output

We now relax the assumption that output per household and the number of households are unchanged over time. Output per household grows at a constant rate: g_m in M and g_c in C. A household's real money demand at time t is

$$\begin{aligned} m_{m,t} &= f(\rho)y_{m,t} \\ &= f(\rho)y_{m,0}(1 + g_m)^t \end{aligned} \quad (16)$$

$$\begin{aligned} m_{c,t} &= g(\rho)y_{c,t} \\ &= g(\rho)y_{c,0}(1 + g_c)^t \end{aligned} \quad (17)$$

$y_{m,0}$ and $y_{c,0}$ are initial conditions. The number of households also grows at a constant rate: n_m in M and n_c in C. The aggregate real money demand function is

$$\begin{aligned} M_{m,t} &= \sum^{N_{m,t}} m_{m,t} \\ &= N_{m,t}f(\rho)y_{m,t} \\ &= N_{m,0}(1 + n_m)^t f(\rho)y_{m,0}(1 + g_m)^t \\ &= Y_{m,0}(1 + \lambda_m)^t f(\rho) \end{aligned} \quad (18)$$

$$\begin{aligned} M_{c,t} &= \sum^{N_{c,t}} m_{c,t} \\ &= N_{c,t}g(\rho)y_{c,t} \\ &= N_{c,0}(1 + n_c)^t g(\rho)y_{c,0}(1 + g_c)^t \\ &= Y_{c,0}(1 + \lambda_c)^t g(\rho) \end{aligned} \quad (19)$$

$(1 + \lambda_m) = (1 + g_m)(1 + n_m)$ and $(1 + \lambda_c) = (1 + g_c)(1 + n_c)$. $Y_{m,0} = N_{m,0}y_{m,0}$ and $Y_{c,0} = N_{c,0}y_{c,0}$ are initial conditions.

Initially, suppose M and C grow at the same rate: $\lambda = \lambda_m = \lambda_c$. Denote σ_t as the growth rate of money at time t. Then, the welfare cost of inflation to M is¹⁰

$$\begin{aligned} W_{m,t} &= [\int_{\rho=-\lambda}^{\pi} Y_{m,t} f(\rho) d\rho - \sigma_t Y_{m,t} f(\pi_t)] \\ &= [\int_{\rho=-\lambda}^{\pi} Y_{m,0} f(\rho) d\rho - \sigma_t Y_{m,0} f(\pi_t)] (1 + \lambda)^t \end{aligned} \quad (20)$$

The seigniorage from C is

$$S_{c,t} = \sigma_t M_{c,t} = \sigma_t Y_{c,0} g(\pi) (1 + \lambda)^t \quad (21)$$

M's net gain is

$$\begin{aligned} G_{m,t} &= S_{c,t} - W_{m,t} \\ &= \{ \sigma_t Y_{c,0} g(\pi_t) - [\int_{\rho=-\lambda}^{\pi} Y_{m,0} f(\rho) d\rho - \sigma_t Y_{m,0} f(\pi_t)] \} (1 + \lambda)^t \end{aligned} \quad (22)$$

(22) implies that π_t and σ_t are constant if M maximizes (22). Since the money-income ratio is constant for a given rate of inflation, the following relationship holds at each point of time:

$$\sigma = \pi + \lambda \quad (23)$$

Using (23), rewrite (22):

$$\begin{aligned} G_{m,t} &= S_{c,t} - W_{m,t} \\ &= \{ (\pi + \lambda) Y_{c,0} g(\pi) \\ &\quad - [\int_{\rho=-\lambda}^{\pi} Y_{m,0} f(\rho) d\rho - (\pi + \lambda) Y_{m,0} f(\pi)] \} (1 + \lambda)^t \end{aligned} \quad (22)'$$

Maximizing (22)' with respect to π yields

$$\begin{aligned} M_{c,0}^* (1 - \eta_c^*) + \lambda (\partial M_{c,0}^* / \partial \pi^*) \\ = M_{m,0}^* \eta_m^* - \lambda (\partial M_{m,0}^* / \partial \pi^*) \end{aligned} \quad (24)$$

π^* satisfies this condition.

¹⁰ In all cases, the lower limit of integration is taken at a point where money growth is zero. In the no growth case, this is zero inflation; in the growth case, it is at a rate of deflation equal to output growth.

If M and C grows, π^* is a function of λ . How does growth influence “optimal” inflation? (24) implies

$$\partial\pi^* / \partial\lambda = - [\partial h(\pi^*, \lambda) / \partial\lambda] / [\partial h(\pi^*, \lambda) / \partial\pi^*] < 0^{11} \quad (25)$$

That is, the higher is aggregate output growth (both growth rates are equal), the lower is the “optimal” inflation rate. Note output growth has a level effect on π^* ; π^* is constant since the relative income size of C to M remains constant over time.

As in the no growth case, we apply (24) to the linear and semi-log functions. If $f(\rho)$ and $g(\rho)$ are both linear, from (24),

$$\begin{aligned} \pi^* &= bY_{c,0} / (\alpha Y_{m,0} + 2\beta Y_{c,0}) \\ &\quad - \lambda[(\alpha Y_{m,0} + \beta Y_{c,0}) / (\alpha Y_{m,0} + 2\beta Y_{c,0})] \end{aligned} \quad (26)$$

If $Y_{c,0}$ is sufficiently small relative to $Y_{m,0}$, (26) is approximated by

$$\pi^* = bY_{c,0} / (\alpha Y_{m,0} + 2\beta Y_{c,0}) - \lambda \quad (26)'$$

Alternatively, for the semi-log functions, π^* satisfies

$$\begin{aligned} Y_{c,0}b\text{Exp}[-\beta\pi^*](1 - \beta\pi^*) - \lambda\beta b\text{Exp}[-\beta\pi^*]Y_{c,0} \\ = Y_{m,0}a\text{Exp}[-\alpha\pi^*]\alpha\pi^* + \lambda\alpha a\text{Exp}[-\alpha\pi^*]Y_{m,0} \end{aligned} \quad (27)$$

Assuming that $\alpha = \beta$,

$$\pi^* = bY_{c,0} / \alpha(aY_{m,0} + bY_{c,0}) - \lambda \quad (28)$$

If we further assume that M and C have the same semi-log function ($\alpha = \beta$ and $a = b$),

$$\pi^* = Y_{c,0} / \alpha(Y_{m,0} + Y_{c,0}) - \lambda \quad (28)'$$

We later use this expression for empirical estimates.

¹¹ To prove this, let $h(\pi^*, \lambda) = M_{c,0}^*(1 - \eta_c^*) - M_{m,0}^*\eta_m^* + \lambda[(\partial M_{m,0}^* / \partial\pi^*) + (\partial M_{c,0}^* / \partial\pi^*)]$. $\partial h(\pi^*, \lambda) / \partial\pi^* < 0$ from the second-order condition and $\partial h(\pi^*, \lambda) / \partial\lambda = (\partial M_{m,0}^* / \partial\pi^*) + (\partial M_{c,0}^* / \partial\pi^*) < 0$. Totally differentiating $h(\pi^*, \lambda)$ yields (24).

Comparing the growth cases (26)' and (28) with the no growth cases (11) and (15), we see that inflation is lower than in the no growth case by the growth rate of output. Note, however, the larger is λ , the greater proportion of C's income is appropriated by M as seigniorage. For the linear functions,

$$S_{c,t}^* / Y_{c,t} = [bY_{c,0} / (\alpha Y_{m,0} + 2\beta Y_{c,0})](b - \beta\pi^*) \quad (29)$$

Similarly, for the semi-log functions,

$$\begin{aligned} S_{c,t}^* / Y_{c,t} \\ = [bY_{c,0} / \alpha(aY_{m,0} + bY_{c,0})](b\text{Exp}[-\beta\pi^*]) \end{aligned} \quad (29)'$$

Since $\partial\pi^* / \partial\lambda < 0$, $\partial(S_{c,t}^* / Y_{c,t}) / \partial\lambda > 0$ for both functions. This result that the proportion of C's income accruing to M as seigniorage increases despite a fall in inflation may appear counterintuitive. We provide a geometric explanation for this result, using Figure 1.

Figure 1 shows C's money-income ratio $M_{c,t} / Y_{c,t} = g(\rho)$. Suppose that π_0^* is imposed by M on C in the absence of growth. The seigniorage accruing to M as a fraction of C's income ($S_{c,t}^* / Y_{c,t}$) is then area A + B. If M and C both grow at λ , the inflation rate is lower to $\pi_0^* - \lambda$ and the seigniorage is area B + C + D + E. Note area A = area C since the inflation rate falls by the output growth rate λ . Thus, the seigniorage accruing to M is larger by area D + E in the growth case than in the no growth case.

Now, consider the more general case where aggregate output growth differs between M and C ($\lambda_m \neq \lambda_c$). The growth rate of total output (or $Y_{m,t} + Y_{c,t}$) at time t is

$$\begin{aligned} \lambda_t = \{ \lambda_m + \lambda_c(Y_{c,0}/Y_{m,0})[(1 + \lambda_c) / (1 + \lambda_m)]^t \} \\ / \{ 1 + (Y_{c,0}/Y_{m,0}) \} [(1 + \lambda_c) / (1 + \lambda_m)]^t \} \end{aligned} \quad (30)$$

Growth rises through time and approaches the rate of the fastest growing country. If M grows fastest,

$$\lim_{t \rightarrow \infty} \lambda_t = \lambda_m \quad (30)'$$

Equivalently, for C

$$\lim_{t \rightarrow \infty} \lambda_t = \lambda_c \quad (30)''$$

Suppose that M grows faster than C. Any revenue from C becomes increasingly unimportant. In the limit, the welfare of M's households is maximized by holding money supply constant and allowing prices to fall at the growth rate of M's output:

$$\lim_{t \rightarrow \infty} \pi_t^* = -\lambda_m \quad (31)$$

At this point, no seigniorage accrues to M from C. The elasticity of demand for money with respect to the inflation rate is zero.

On the other hand, suppose that C grows faster than M. Any welfare cost to M's households becomes increasingly insignificant as compared to the seigniorage from C. In the limit, M simply maximizes the seigniorage from C by setting C's elasticity of money demand at unity. For the linear function,

$$\lim_{t \rightarrow \infty} \pi_t^* = b / 2\beta - \lambda_c / 2 \quad (32)$$

Similarly, for the semi-log function

$$\lim_{t \rightarrow \infty} \pi_t^* = 1/\beta - \lambda_c \quad (32)'$$

(32) and (32)' show inflation is reduced by C's growth.

It may be of interest to analyze a special case in which growth occurs only in C and in which M issues money in a way that insulates M's households from the effects of inflation¹². Then, M sets π to maximize the seigniorage from C:

$$S_{c,t} = \sigma_t M_{c,t} = (\pi_t + \lambda_t) Y_{c,0} g(\pi_t) (1 + \lambda_c)^t \quad (33)$$

The first-order condition is

$$M_{c,t}^* (1 - \eta_c^*) + \lambda_c (\partial M_{c,t}^* / \partial \pi_t^*) = 0 \quad (34)$$

Note that σ_t and π_t are constant over time.

If $g(\rho)$ is linear, from (34)

$$\pi^* = b / 2\beta - \lambda_c / 2 \quad (35)$$

This solution is identical to the asymptotic case (32). "optimal" inflation is constant and lowered by C's growth.

Similarly, for the semi-log functions,

$$\pi^* = 1/\beta - \lambda_c \quad (36)$$

This is also identical to (32)'.

¹² This case was originally analyzed by Mundell (1973), using linear demand functions. He assumes that the growth of money in M is issued as a pro-rata-share of original money holdings of M's households (so-called Weldon money) so that cash balances effectively pay an interest rate equal to the rate of inflation. He distinguishes between income-widening growth due to an increase in the number of households and the growth of income per household. In the latter case, his analysis purports to show that the rate of inflation increases without limit. As our analysis shows, his conclusions are erroneous since the rate of inflation is lowered by the growth rate of output and time-invariant whether due to an increase in the number of households or income per household.

4. Empirical Estimates

Figure 2 plots (28)', showing how the "optimal" rate of inflation varies with the relative share of C's aggregate output $Y_c / (Y_m + Y_c)$ with $\lambda = 0.02$. The semi-elasticity takes three different values: 5, 7 and 9.

Note that "optimal" inflation is very low if C is small relative to M. Take Ecuador and El Salvador, which currently dollarize with the United States. 2001's relative share is 0.0018 for Ecuador and 0.0014 for El Salvador¹³. For a larger country, Argentina, its relative share is 0.0257. Assuming the common values of the semi-elasticity (5, 7 and 9 for $\alpha = \beta$) and of the growth rate (0.02 for λ), Table 1 shows that estimated π^* 's are generally small. For Argentina, the estimates range from -1.8% to -1.5%. They are negligibly small for Ecuador and El Salvador.

It may be amusing to analyze the case in which the United Kingdom "dollarizes" with the European Monetary Union (EMU). Although unlikely, suppose that the European Central Bank (ECB) sets the inflation rate to maximize the welfare of the current member countries at a cost of UK households. Table 2 shows the "optimal" inflation rate that would be set by the ECB, given that the relative share of UK GDP is 0.1907 in 2001. Again, we assume the common values of the semi-elasticity (5, 7 and 9 for $\alpha = \beta$) and of the growth rate (0.02 for λ). All estimated π^* 's are somewhat larger than in the previous cases, ranging from 0.1% to 1.8%

¹³ Output figures are GDP from *World Development Indicator Online* (<http://publications.worldbank.org/WDI/>).

At these “optimal” rates of inflation, what proportion of C’s income accrues to M as seigniorage? Assuming $\alpha = \beta$ and $a = b$, (29)’ reduces to

$$S_{c,t}^* / Y_{c,t} = (a / \alpha)[Y_{c,0} / (Y_{m,0} + Y_{c,0})] \text{Exp}[-\alpha\pi^*] \quad (29)''$$

Table 3 shows estimates of the proportion of C’s income accruing to M as seigniorage, using different sets of the parametric values (0.35 for $a = b$, 5, 7 and 9 for $\alpha = \beta$ and 0.02 for λ)¹⁴. π^* ’s are computed from (28)’ (the figures are given in Tables 1 and 2). For Argentina, Ecuador and El Salvador, the estimates are all negligibly small and below 0.2%. For the United Kingdom, the estimates are somewhat larger although they do not exceed 1.3%.

¹⁴ We follow Lucas (2000) for the parametric values of the semi-log function.

5. A Sharing Arrangement

It has been proposed that M share potential seignorage with C to induce M not to take any seignorage from C¹⁵. In this section, we analyze such a sharing arrangement.

Initially, we assume the absence of growth.

Under this contract, M potentially is compelled to pay back a fraction of total seignorage. Total seignorage is the sum of seignorage from the households in M and C:

$$\begin{aligned} S &= S_m + S_c \\ &= \pi[Y_m f(\pi) + Y_c g(\pi)] \end{aligned} \quad (37)$$

M receives S_c from C and pays back ϕS to C, where ϕ denotes the sharing fraction. Then, M's net receipt of the seignorage is $S_c - \phi S$, which is also M's balance of payment deficit.

M's net gain is

$$\begin{aligned} G_m &= S_c - \phi S - W_m \\ &= \pi Y_c g(\pi) - \phi \pi [Y_m f(\pi) + Y_c g(\pi)] \\ &\quad - \left[\int_{\rho=0}^{\pi} Y_m f(\rho) d\rho - \pi Y_m f(\pi) \right] \end{aligned} \quad (38)$$

Given ϕ , M maximizes (38) with respect to π . Then,

$$(1 - \phi)M_c^*(1 - \eta_c^*) - \phi M_m^*(1 - \eta_m^*) = M_m^* \eta_m^* \quad (39)$$

(39) shows that the "optimal" rate of inflation is now a function of ϕ and implies

$$\partial \pi^* / \partial \phi = - [\partial \hat{h}(\pi^*, \phi) / \partial \phi] / [\partial \hat{h}(\pi^*, \phi) / \partial \pi^*] < 0^{16} \quad (40)$$

¹⁵ Senator Connie Mack introduced such proposals, the International Monetary Stability Act (S.1879) and its revised version (S. 2101), in the US senate.

That is, π^* is a decreasing function of ϕ under the sharing arrangement.

To make inflation zero, ϕ^{**} of the total potential seigniorage must be paid back to C:

$$\phi^{**} = \frac{[M_c^{**}(1 - \eta_c^{**}) - M_m^{**}\eta_m^{**}]}{[M_m^{**}(1 - \eta_m^{**}) + M_c^{**}(1 - \eta_c^{**})]} \quad (41)$$

M_m^{**} , M_c^{**} , η_m^{**} and η_c^{**} are all evaluated at zero inflation. For M_m^{**} and M_c^{**} to exist, real money demands must be satiated at finite quantities so that $\eta_m^{**} = \eta_c^{**} = 0$ ¹⁷.

Then, (41) becomes

$$\phi^{**} = M_c^{**} / (M_m^{**} + M_c^{**}) \quad (42)$$

This ratio determines the sharing parameter ϕ^{**} that is potentially paid back to C to induce M not to inflate. Of course, no seigniorage accrues to M at zero inflation.

If we assume that M and C have the same money demand function (or equivalently, $f(\rho) = g(\rho)$), then (42) reduces to

$$\phi^{**} = Y_c / (Y_m + Y_c) \quad (42)'$$

In this case, ϕ^{**} is determined by the relative share of C's aggregate output¹⁸.

We now extend this analysis of the sharing arrangement to the case where M and C grow at the same rate λ . At time t , M's net gain is

¹⁶ Let $\hat{h}(\pi^*, \phi) = (1 - \phi)M_c^*(1 - \eta_c^*) - \phi M_m^*(1 - \eta_m^*) - M_m^*\eta_m^*$. $\partial \hat{h}(\pi^*, \phi) / \partial \pi^* < 0$ from the second-order condition and $\partial \hat{h}(\pi^*, \phi) / \partial \phi = -M_c^*(1 - \eta_c^*) - M_m^*(1 - \eta_m^*) < 0$ since $0 < \phi < 1$. Then, (40) results.

¹⁷ For any demand function that is satiated at a finite quantity, the price elasticity of demand is zero at a point of satiation. Let the price elasticity of demand as $\varepsilon = (\partial Q^d / \partial P)(P/Q^d)$. Then, $\varepsilon = 0$ if Q^d is non-zero at $P = 0$ (and $\partial Q^d / \partial P$ is finite).

¹⁸ (42)' is identical to what Cooper and Kempf (2001) find in an overlapping generations model and we are indebted to their analysis. They assume that representative agents are identical in both countries, which implies that $f(\rho) = g(\rho)$. Our analysis requires only that ϕ^{**} is determined by M_m^{**} and M_c^{**} .

$$\begin{aligned}
G_{m,t} &= S_{c,t} - \phi S_t - W_{m,t} \\
&= \{(\pi + \lambda)Y_{c,0}g(\pi) - \phi(\pi + \lambda)[Y_{m,0}f(\pi) + Y_{c,0}g(\pi)] \\
&\quad - [\int_{\rho=-\lambda}^{\pi} Y_{m,0}f(\rho)d\rho - \pi Y_{m,0}f(\pi)]\}(1 + \lambda)^t \tag{43}
\end{aligned}$$

Maximizing (43) with respect to π yields:

$$\begin{aligned}
(1 - \phi)M_{c,0}^*[1 - (\lambda + \pi^*)\pi^{*-1}\eta_c^*] \\
- \phi M_{m,0}^*[1 - (\lambda + \pi^*)\pi^{*-1}\eta_m^*] \\
= M_{m,0}^*(\lambda + \pi^*)\pi^{*-1}\eta_m^* \tag{44}
\end{aligned}$$

In the growth case, the sharing parameter ϕ^{**} is set so that M is forced to deflate at the rate of aggregate output:

$$\phi^{**} = M_{c,0}^{**} / (M_{m,0}^{**} + M_{c,0}^{**}) \tag{45}$$

$M_{c,0}^{**}$ and $M_{m,0}^{**}$ are evaluated at $\pi^* = -\lambda$. This sharing parameter leads to no seigniorage accruing to M from C . Note (45) is a generalization of (42). In both cases, M_c^{**} and M_m^{**} are evaluated at a point of no money growth; in the no growth case, $\pi^* = 0$; in the growth case, $\pi^* = -\lambda$.

If M and C have the same money demand function, (45) reduces to

$$\phi^{**} = Y_{c,0} / (Y_{m,0} + Y_{c,0}) \tag{45}'$$

This is identical to (42)'.

Note the values of the sharing parameter would be very small for Argentina, Ecuador and El Salvador under the contract with the United States. Using the relative shares of GDP in 2001, (42)' and (45)' indicate the United States would be compelled to

pay back 2.5% of total potential seigniorage to Argentina and 0.1% to Ecuador and El Salvador¹⁹.

¹⁹ GDP is from *World Development Indicator Online*.

6. A Cost-Benefit Analysis of Dollarization

Finally, we provide estimates of the present discounted value (P.D.V.) of the cost to C's households of the seigniorage from C to M as well as the net welfare gain to C's household when dollarization results in permanently lower inflation (if only it could last!).

At the inflation rate π^* imposed by M on C, the seigniorage is

$$\begin{aligned} S^*_{c,t} &= \sigma^* M_{c,t} \\ &= \sigma^* Y_{c,t} g(\pi^*) \\ &= (\pi^* + \lambda_c) Y_{c,0} g(\pi^*) (1 + \lambda_c)^t \end{aligned} \quad (46)$$

Assuming real interest rate r and output growth rate λ_c are constant, the P.D.V. at $t = 0$ of the seigniorage is,

$$\begin{aligned} PDVS_{c,0}^* &= \sum_{t=0}^{\infty} (1+r)^{-t} S^*_{c,t} \\ &= (\pi^* + \lambda_c) Y_{c,0} g(\pi^*) [(1+r)/(r - \lambda_c)]^{20} \end{aligned} \quad (46)'$$

where $r > \lambda_c$ (the steady state is efficient).

Assuming M is the United States, Table 4 provides 2001's estimates for Argentina, Ecuador and El Salvador of $PDVS_{c,0}^*$ as a fraction of C's current GDP ($PDVS_{c,0}^* / Y_{c,0}$). We assume the semi-log function $g(\rho) = b \text{Exp}[-\beta\rho]$ with $b = 0.35$ and let β take three different values: 5, 7 and 9. π^* 's are computed from (28)' under the

²⁰ See Schmitt-Grohe and Uribe (1999) for an analysis of how the present discounted value of the seigniorage is measured if C grows.

assumption that M and C have the same money demand functions ($f(\rho) = g(\rho)$) and grow at the same rate $\lambda = 0.02$; the figures are given in Table 1. In order to give these formulas a run, we assume $r = 0.03$. Using the alternative values of the semi-elasticity β , the estimates for Argentina range from 12.0% to 20.0% of GDP. The estimates are negligibly small for Ecuador (0.9% - 1.4%) and El Salvador (0.7% - 1.1%)²¹.

Dollarization by C, on the other hand, may well result in lower inflation. This is a welfare gain to C's households. Suppose C's government inflates at π_c before dollarization. Initially, assume C's government returns any seigniorage taken from C's households to them (we later make a more realistic assumption that C's government does not return any seigniorage). The welfare cost of inflation to C's households before dollarization is

$$\begin{aligned} L_{c,t} &= \int_{\rho=-\lambda_c}^{\pi_c} M_{c,t} d\rho - \sigma_c M_{c,t} \\ &= \int_{\rho=-\lambda_c}^{\pi_c} Y_{c,t} g(\rho) d\rho - (\pi_c + \lambda_c) Y_{c,t} g(\pi_c) \end{aligned} \quad (47)$$

Suppose dollarization permanently reduces C's inflation rate from π_c to π^* , where π^* is imposed on C by M under dollarization. At π^* , the cost of inflation to C's households, excluding the seigniorage from C to M, is

$$\begin{aligned} L_{c,t}^* &= \int_{\rho=-\lambda_c}^{\pi^*} M_{c,t} d\rho - \sigma^* M_{c,t} \\ &= \int_{\rho=-\lambda_c}^{\pi^*} Y_{c,t} g(\rho) d\rho - (\pi^* + \lambda_c) Y_{c,t} g(\pi_c) \end{aligned} \quad (47)'$$

If dollarization results in permanently lower inflation, the cost is reduced from (47) to (47)'. This reduction is the welfare gain to C's households from permanent reduction in inflation. This gain is

²¹ Velde and Veracierto (1999) estimate the present discounted value of the lost seigniorage as a fraction of current GDP for Argentina and Cnahg (2000), for Argentina, Brazil and Mexico from the central banks' balance sheets.

$$\begin{aligned}
G_{c,t}^* &= L_{c,t} - L_{c,t}^* \\
&= \left[\int_{\rho=\lambda_c}^{\pi_c} Y_{c,t} g(\rho) d\rho - (\pi_c + \lambda_c) Y_{c,t} g(\pi_c) \right] \\
&\quad - \left[\int_{\rho=\lambda_c}^{\pi^*} Y_{c,t} g(\rho) d\rho - (\pi^* + \lambda_c) Y_{c,t} g(\pi_c) \right] \quad (48)
\end{aligned}$$

where $\pi_c > \pi^*$. The P.D.V. of this gain is the capital sum whose yield at the real rate of interest shows the gain per annum in utility to C's households from permanent reduction in inflation (equivalently, it represents the consumption that C's households would be willing to give up per year if π_c is lowered to π^*). The P.D.V. is

$$\begin{aligned}
&PDVG_{c,0}^* \\
&= \sum_{t=0}^{\infty} (1+r)^{-t} G_{c,t}^* \\
&= \left\{ \left[\int_{\rho=\lambda_c}^{\pi_c} g(\rho) d\rho - (\pi_c + \lambda_c) g(\pi_c) \right] \right. \\
&\quad \left. - \left[\int_{\rho=\lambda_c}^{\pi^*} g(\rho) d\rho - (\pi^* + \lambda_c) g(\pi_c) \right] \right\} \\
&\quad Y_{c,0} [(1+r) / (r - \lambda_c)] \quad (48)'
\end{aligned}$$

Table 5 provides 2001's estimates for Argentina, Ecuador and El Salvador of $PDVG_{c,0}^*$ as a fraction of C's current GDP ($PDVG_{c,0}^* / Y_{c,0}$). We use a 10-year average annual percentage growth rate of implicit GDP deflator prior to dollarization for π_c (1981-1990 for Argentina, 1990-1999 for Ecuador and 1991-2000 for El Salvador; the figures are given in Table 5)^{22,23}. We assume $g(\rho) = b \text{Exp}[-\beta\rho]$ with $b = 0.35$ and $\beta = 5, 7$ and 9. π^* 's are computed from (28)' under the assumption that $f(\rho) = g(\rho)$ and $\lambda_m = \lambda_c = 0.02$ (the figures are given in Table 1) $r = 0.03$. Using the alternative values of the semi-elasticity β , the estimates for Argentina range from 479.4% to 796.6% of GDP. Using the

²² The Argentine Congress passed the convertibility law in March 1991 and established the convertibility at the rate of 10,000 australes per U.S. dollar in April 1991. Ecuador and El Salvador replaced their national currencies in September 2000 and January 2001, respectively.

²³ Inflation figures are from *World Development Report*

same semi-elasticity values, the estimates range from 415.3% to 463.5% for Ecuador and from 64.7% to 99.7% for El Salvador.

For C's households, the net welfare gain from dollarization is any excess of the gain from lower inflation over the seigniorage paid by C to M. The P.D.V. of this net gain is (48)' - (46)':

$$PDVNG_{c,0}^* = PDVG_{c,0}^* - PDVS_{c,0}^* \quad (49)$$

Table 6 gives 2001's estimates for Argentina, Ecuador and El Salvador of $PDVNG_{c,0}^*$ as a fraction of C's current GDP ($PDVNG_{c,0}^* / Y_{c,0}$). The estimates indicate the P.D.V. of this net gain is positive and significantly large. As before, we assume the alternative values of the semi-elasticity (5, 7 and 9). The net gain ranges from 467.4% to 776.6% of GDP for Argentina, from 414.5% to 462.1% for Ecuador and from 63.6% to 99.0% for El Salvador.

Now, more realistically, assume C's government inflates at π_c before dollarization and does not return any seigniorage taken from C's households to them. This is more realistic because many dollarizing countries run fiscal deficits and inflate on the wrong side of the Laffer curve. Under this assumption, the welfare cost of inflation to C's households before dollarization is

$$\begin{aligned} L_{c,t}' &= \int_{\rho=\lambda_c}^{\pi_c} M_{c,t} d\rho \\ &= \int_{\rho=\lambda_c}^{\pi_c} Y_{c,t} g(\rho) d\rho \end{aligned} \quad (50)$$

Dollarization permanently reduces the inflation rate from π_c to π^* . At π^* , the cost to C's households, excluding the seigniorage from C to M, is

$$\begin{aligned} L_{c,t}^* &= \int_{\rho=\lambda_c}^{\pi^*} M_{c,t} d\rho - \sigma_c M_{c,t} \\ &= \int_{\rho=\lambda_c}^{\pi^*} Y_{c,t} g(\rho) d\rho - (\pi^* + \lambda_c) Y_{c,t} g(\pi_c) \end{aligned} \quad (50)'$$

Thus, the gain from permanently lower inflation is (50) – (50)′:

$$\begin{aligned}
 G_{c,t}^{*'} &= L_{c,t}' - L_{c,t}^{*'} \\
 &= \int_{\rho=-\lambda_c}^{\pi_c} Y_{c,t} g(\rho) d\rho \\
 &\quad - \left[\int_{\rho=-\lambda_c}^{\pi^*} Y_{c,t} g(\rho) d\rho - (\pi^* + \lambda_c) Y_{c,t} g(\pi_c) \right] \quad (51)
 \end{aligned}$$

The P.D.V. of (51) is

$$\begin{aligned}
 PDVG_{c,0}^{*'} &= \sum_{t=0}^{\infty} (1+r)^{-t} G_{c,t}^{*'} \\
 &= \left[\int_{\rho=-\lambda_c}^{\pi_c} g(\rho) d\rho - \left[\int_{\rho=-\lambda_c}^{\pi^*} g(\rho) d\rho - (\pi^* + \lambda_c) g(\pi_c) \right] \right] \\
 &\quad Y_{c,0} [(1+r) / (r - \lambda_c)] \quad (51)'
 \end{aligned}$$

Table 7 provides 2001's estimates for Argentina, Ecuador and El Salvador of $PDVG_{c,0}^{*'}$ as a fraction of C's current GDP ($PDVG_{c,0}^{*'}/Y_{c,0}$) under the assumption C's government does not return any signiorage to C's households before dollarization. As before, we use a 10-year average annual percentage growth rate of implicit GDP deflator prior to dollarization for π_c and assume $g(\rho) = b \text{Exp}[-\beta\rho]$ with $b = 0.35$ and $\beta = 5, 7$ and 9. π^* 's are computed from (28)' under the assumption that $f(\rho) = g(\rho)$ and $\lambda_m = \lambda_c = 0.02$ (the figures are given in Table 1). $r = 0.03$. Using the alternative values of the semi-elasticity, the estimates range from 479.4% to 796.6% of GDP for Argentina, from 465.3% to 684.0% for Ecuador and from 273.8% to 298.8% for El Salvador.

For C's households, the P.D.V. of this net welfare gain is (51)' - (46)′:

$$PDVNG_{c,0}^{*'} = PDVG_{c,0}^{*'} - PDVS_{c,0}^* \quad (52)$$

Table 8 gives 2001's estimates for Argentina, Ecuador and El Salvador of $PDVNG_{c,0}^{*'}$ as a fraction of C's current GDP ($PDVNG_{c,0}^{*'}/Y_{c,0}$) under the assumption C's

government does not return any signiorage to C's households before dollarization. The estimates indicate the P.D.V. of this net welfare gain from dollarization is larger than in the previous case. Assuming the alternative values of the semi-elasticity (5, 7 and 9), the net gain ranges from 467.4% to 776.6% of GDP for Argentina, from 464.5% to 682.6% for Ecuador and from 273% to 297.7% for El Salvador.

7. Conclusions

When C dollarizes with M, inflation can be set by M which maximizes solely welfare from the point of view of M's households. The analysis is conducted under the reasonable assumption that any seigniorage initially accruing to M's government is returned to M's households²⁴.

We also analyze a contract under which M shares a fraction of seigniorage from C that results in a zero rate of inflation. Empirical estimates are also made of the P.D.V of the gain to C's households when dollarization permanently reduces inflation. These estimates are made under two alternative assumptions. Firstly, any revenue accruing C's authorities is returned to C's households and, secondly, the more realistic assumption that it is not returned. Here the P.D.V. of the gain to C's households from permanently lower inflation is significant (if only it could last)²⁵.

The limitations of our analysis should be borne in mind. Under dollarization, a country gives up an independent monetary policy – one aspect of this is the loss by the central bank of the lender of last resort function. Moreover, tying the exchange rate to the dollar may result in an appreciated real exchange rate when a competitor devalues (Argentina versus Brazil). Finally, our analysis assumes inflation is anticipated. This

²⁴ If M is the United States, this assumption is reasonable. Seigniorage in the United States is an insignificant fraction of its income. For example, King and Plosser (1985) reports seigniorage equals 0.3% of GNP in the 1952-1982 period.

²⁵ It was the fiscal need for revenue in a country of low tax morality like Argentina and competition among provinces in Argentina that led the central bank to inflate at rates that were on the wrong side of the Laffer curve.

assumption put to one side the effect of variable and less than fully anticipated inflation which, amongst other things, may lead to a confusion of absolute and relative prices²⁶.

²⁶ See, for example, "Costs of Inflation" by Driffel, Mizos and Ulph (1990), Chapter 19 in *Handbook of Monetary Economics*, vol. 2.

Table 1. The “Optimal” Rate of Inflation π^* for Ecuador, El Salvador and Argentina

$\alpha = \beta$	5	7	9
Argentina	-0.0149	-0.0163	-0.0171
Ecuador	-0.0196	-0.0197	-0.0198
El Salvador	-0.0197	-0.0198	-0.0198

The optimal rate of inflation π^* is computed from (28)' under the assumption that M and C have the same semi-log demand function ($f(\rho) = a\text{Exp}(-\alpha\rho)$ and $g(\rho) = b\text{Exp}(-\beta\rho)$, where $a = b$ and $\alpha = \beta$) and grow at the same rate $\lambda = 0.02$. M is the United States. The relative share is $Y_{c,t} / (Y_{m,t} + Y_{c,t})$, where $Y_{c,t}$ and $Y_{m,t}$ are C's and M's GDP in 2001.

Table 2. The “Optimal” Rate of Inflation π^* for the United Kingdom

$\alpha = \beta$	5	7	9
UK	0.0181	0.0072	0.0012

The optimal rate of inflation π^* is computed from (28)' under the assumption that M and C have the same semi-log demand function ($f(\rho) = a\text{Exp}(-\alpha\rho)$ and $g(\rho) = b\text{Exp}(-\beta\rho)$, where $a = b$ and $\alpha = \beta$) and grow at the same rate $\lambda = 0.02$. M is the twelve member countries of the EMU. The relative share is $Y_{c,t} / (Y_{m,t} + Y_{c,t})$, where $Y_{c,t}$ and $Y_{m,t}$ are C's and M's GDP in 2001.

Table 3. The Proportion of C's Income Accruing to M as Seigniorage at the "Optimal" Inflation Rate ($S_{c,t}^* / Y_{c,t}$)

$\alpha = \beta$	5	7	9
Argentina	0.0019	0.0014	0.0012
Ecuador	0.0001	0.0001	0.0001
El Salvador	0.0001	0.0001	0.0001
UK	0.0122	0.0091	0.0073

The optimal rate of inflation π^* is computed from (28)' under the assumption that M and C have the same semi-log demand function ($f(\rho) = a\text{Exp}(-\alpha\rho)$ and $g(\rho) = b\text{Exp}(-\beta\rho)$, where $a = b$ and $\alpha = \beta$) and grow at the same rate $\lambda = 0.02$. M is the United States for Argentina, Ecuador and El Salvador, and the twelve member countries of the EMU for the United Kingdom. The relative share is $Y_{c,t} / (Y_{m,t} + Y_{c,t})$, where $Y_{c,t}$ and $Y_{m,t}$ are C's and M's GDP in 2001. Figures of π^* are given in Tables 1 and 2. Using $a = b = 0.35$, $S_{c,t}^* / Y_{c,t}$ is computed from (29)'.

Table 4. The Present Discounted Value of the Seigniorage Accruing to M as a Fraction of C's Current GDP at the "Optimal" Inflation Rate ($PDVS_{c,0}^* / Y_{c,0}$) in 2001

β	5	7	9
Argentina	0.1996	0.1484	0.1201
Ecuador	0.0143	0.0106	0.0086
El Salvador	0.0111	0.0083	0.0067

The optimal rate of inflation π^* is computed from (28)' under the assumption that M and C have the same semi-log demand function ($f(\rho) = a\text{Exp}(-\alpha\rho)$ and $g(\rho) = b\text{Exp}(-\beta\rho)$, where $a = b$ and $\alpha = \beta$) and grow at the same rate $\lambda = 0.02$. M is the United States. The relative share is $Y_{c,t} / (Y_{m,t} + Y_{c,t})$, where $Y_{c,t}$ and $Y_{m,t}$ are C's and M's GDP in 2001. Figures of π^* are given in Table 1. Using $b = 0.35$ and $r = 0.03$, $PDVS_{c,0}^* / Y_{c,t}$ is computed from (46)'.

Table 5. The Present Discounted Value of the Welfare Gain to C's Households from Permanently Lower Inflation as a Fraction of C's Current GDP ($PDVG_{c,0}^* / Y_{c,0}$) in 2001

β	5	7	9
Argentina ($\pi_c = 3.952$)	7.9657	5.9220	4.7940
Ecuador ($\pi_c = 0.371$)	4.6350	4.4902	4.1534
El Salvador ($\pi_c = 0.074$)	0.6474	0.8373	0.9966

π_c is a 10-year average annual percentage growth rate of implicit GDP deflator prior to dollarization (1981-1990 for Argentina, 1990-1999 for Ecuador and 1991-2000 for El Salvador). The optimal rate of inflation π^* is computed from (28)' under the assumption that M and C have the same semi-log demand function ($f(\rho) = a\text{Exp}(-\alpha\rho)$ and $g(\rho) = b\text{Exp}(-\beta\rho)$, where $a = b$ and $\alpha = \beta$) and grow at the same rate $\lambda = 0.02$. M is the United States. The relative share is $Y_{c,t} / (Y_{m,t} + Y_{c,t})$, where $Y_{c,t}$ and $Y_{m,t}$ are C's and M's GDP in 2001. Figures of π^* are given in Table 1. Using $b = 0.35$ and $r = 0.03$, $PDVG_{c,0}^* / Y_{c,t}$ is computed from (48)'.

Table 6. The Present Discounted Value of the Net Welfare Gain to C's Households of Dollarization as a Fraction of C's Current GDP ($PDVNG_{c,0}^* / Y_{c,0}$) in 2001

β	5	7	9
Argentina ($\pi_c = 3.172$)	7.7661	5.7736	4.6738
Ecuador ($\pi_c = 0.475$)	4.6206	4.4795	4.1448
El Salvador ($\pi_c = 0.155$)	0.6362	0.8290	0.9899

π_c is a 10-year average annual percentage growth rate of implicit GDP deflator prior to dollarization (1981-1990 for Argentina, 1990-1999 for Ecuador and 1991-2000 for El Salvador). The optimal rate of inflation π^* is computed from (28)' under the assumption that M and C have the same semi-log demand function ($f(\rho) = a\text{Exp}(-\alpha\rho)$ and $g(\rho) = b\text{Exp}(-\beta\rho)$, where $a = b$ and $\alpha = \beta$) and grow at the same rate $\lambda = 0.02$. M is the United States. The relative share is $Y_{c,t} / (Y_{m,t} + Y_{c,t})$, where $Y_{c,t}$ and $Y_{m,t}$ are C's and M's GDP in 2001. Figures of π^* are given in Table 1. Using $b = 0.35$ and $r = 0.03$, $PDVNG_{c,0}^* / Y_{c,0}$ is computed from (49).

Table 7. The Present Discounted Value of the Welfare Gain to C's Households from Permanently Lower Inflation as a Fraction of C's Current GDP ($PDVG_{c,0}^* / Y_{c,0}$) in 2001 under the Assumption that C's Government Does Not Return Any Seigniorage to C's Households before Dollarization

β	5	7	9
Argentina ($\pi_c = 3.952$)	7.9657	5.9220	4.7940
Ecuador ($\pi_c = 0.371$)	6.8402	5.5402	4.6534
El Salvador ($\pi_c = 0.074$)	2.9881	2.8560	2.7376

π_c is a 10-year average annual percentage growth rate of implicit GDP deflator prior to dollarization (1981-1990 for Argentina, 1990-1999 for Ecuador and 1991-2000 for El Salvador). The optimal rate of inflation π^* is computed from (28)' under the assumption that M and C have the same semi-log demand function ($f(\rho) = a\text{Exp}(-\alpha\rho)$ and $g(\rho) = b\text{Exp}(-\beta\rho)$, where $a = b$ and $\alpha = \beta$) and grow at the same rate $\lambda = 0.02$. M is the United States. The relative share is $Y_{c,t} / (Y_{m,t} + Y_{c,t})$, where $Y_{c,t}$ and $Y_{m,t}$ are C's and M's GDP in 2001. Figures of π^* are given in Table 1. Using $b = 0.35$ and $r = 0.03$, $PDVG_{c,0}^* / Y_{c,t}$ is computed from (51).

Table 8. The Present Discounted Value of the Net Welfare Gain to C's Households of Dollarization as a Fraction of C's Current GDP ($PDVNG_{c,0}^* / Y_{c,0}$) in 2001 under the Assumption that C's Government Does Not Return Any Seigniorage to C's Households before Dollarization

β	5	7	9
Argentina ($\pi_c = 3.172$)	7.7661	5.7736	4.6738
Ecuador ($\pi_c = 0.475$)	6.8259	5.5296	4.6448
El Salvador ($\pi_c = 0.155$)	2.9769	2.8477	2.7309

π_c is a 10-year average annual percentage growth rate of implicit GDP deflator prior to dollarization (1981-1990 for Argentina, 1990-1999 for Ecuador and 1991-2000 for El Salvador). The optimal rate of inflation π^* is computed from (28)' under the assumption that M and C have the same semi-log demand function ($f(\rho) = a\text{Exp}(-\alpha\rho)$ and $g(\rho) = b\text{Exp}(-\beta\rho)$, where $a = b$ and $\alpha = \beta$) and grow at the same rate $\lambda = 0.02$. M is the United States. The relative share is $Y_{c,t} / (Y_{m,t} + Y_{c,t})$, where $Y_{c,t}$ and $Y_{m,t}$ are C's and M's GDP in 2001. Figures of π^* are given in Table 1. Using $b = 0.35$ and $r = 0.03$, $PDVNG_{c,0}^* / Y_{c,t}$ is computed from (52).

Figure 1. Growth and Seigniorage

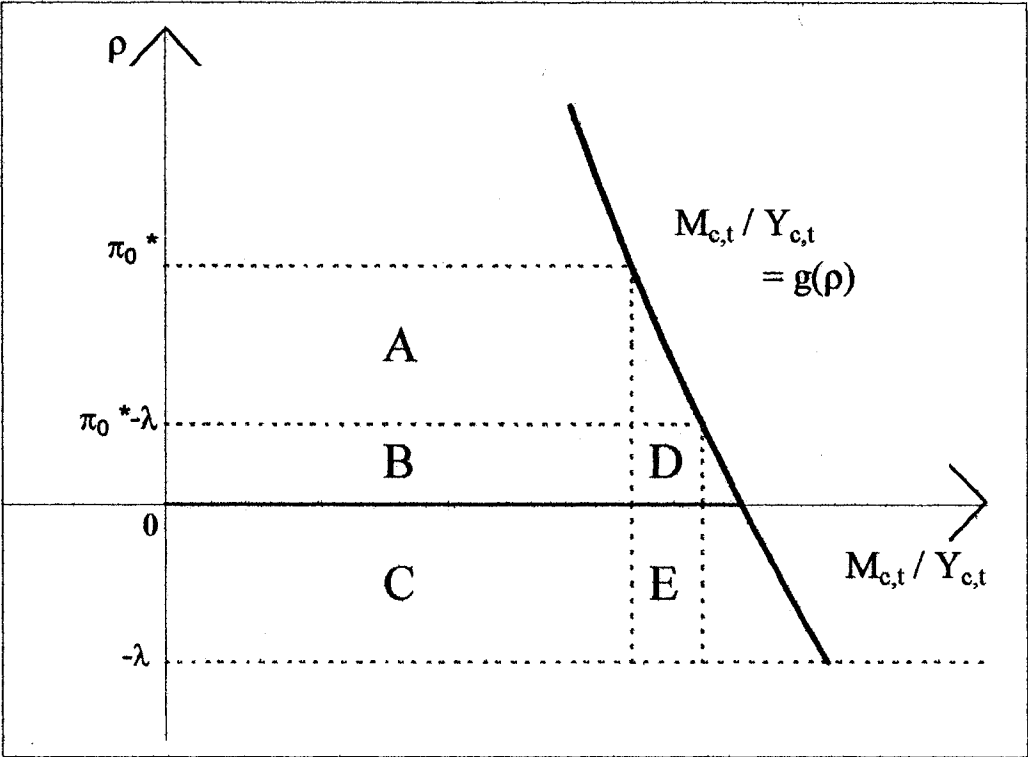
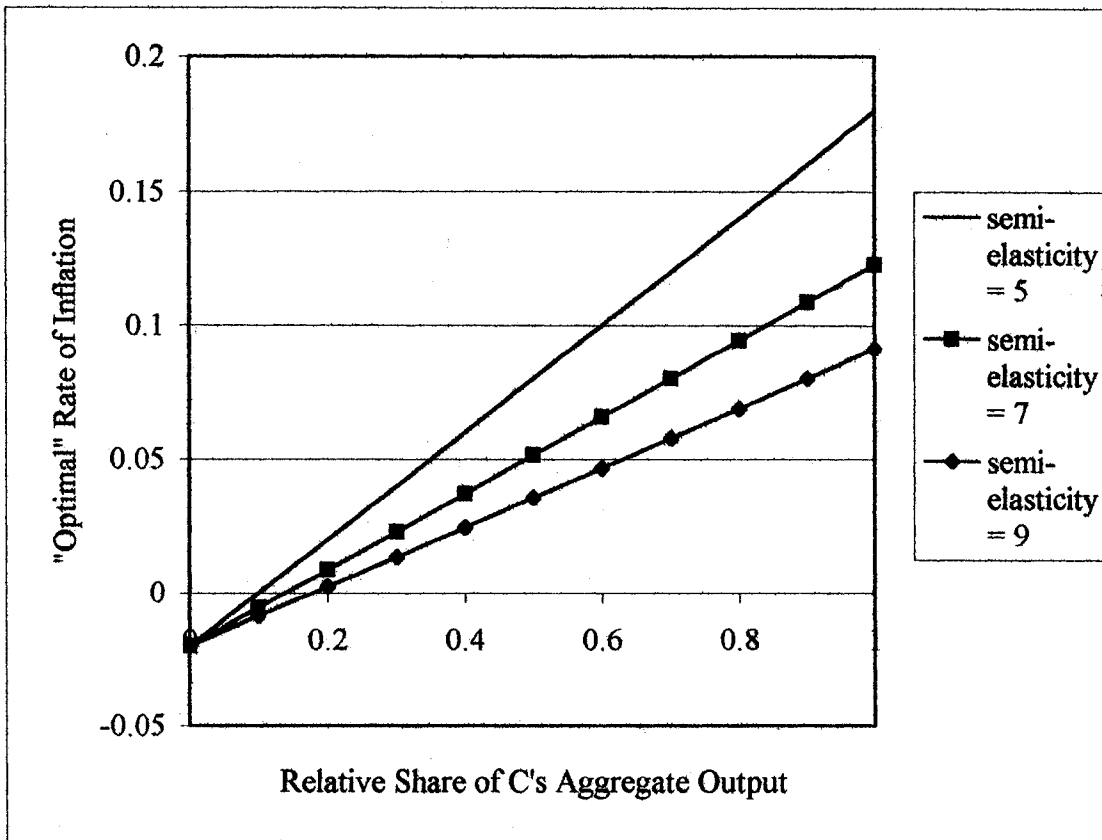


Figure 2. "Optimal" Rate of Inflation π^*



The optimal rate of inflation π^* is computed from (28)' under the assumption that M and C have the same semi-log demand function ($f(\rho) = a\text{Exp}(-\alpha\rho)$ and $g(\rho) = b\text{Exp}(-\beta\rho)$, where $a = b$ and $\alpha = \beta$) and grow at the same rate $\lambda = 0.02$. The relative share is $Y_{c,t} / (Y_{m,t} + Y_{c,t})$, where $Y_{c,t}$ and $Y_{m,t}$ are C's and M's income at time t.

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