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Essays in applied microeconomic theory

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ESSAYS IN APPLIED MICROECONOMIC THEORY

by

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ESSAY I: VALUATION OF THE SHARES OF A LABOR-MANAGED FIRM

I.1. Introduction

The well-known literature on labor-managed firm since Ward (1958) has accused this type of economic organization with several short-comings. For instance, under perfect competition in both output and input markets, if the average product of labor is given by an inverted-U shaped curve and there are no fixed costs, then the firm does not respond to price changes at all. In the case of fixed costs (such as the rental cost of capital or manufacturing overhead), the situation is worse; because, with any inverted-U shaped (or simply, decreasing) curve for average product of labor, as the price of output increases, the labor-managed firm produces less. Furthermore, an increase in the fixed costs leads to an increase in the output. Because of these differences between the profit-maximizing and labor-managed firms, the latter has been said to be perverse.

The source of perversity lies in the objective function assumed for the labor-managed firm (henceforth LMF) where value-added per worker, instead of the total profit, is

maximized. If we restrict ourselves with cases in which the working hours per worker are institutionally fixed, this objective can be achieved only by free adjustment of the number of worker-owners of the LMF. This, on the other hand, necessitates dismissal of some members whenever contraction would increase the value-added per remaining worker. Hence, as it has been pointed out first by Robinson (1967), the LMF model is further confronted with the problem of lack of a mechanism by which those who will depart from the firm will be given the incentive to do so.

This problem has been solved by using complete compensation schemes (Bonin (1984)). In that model, the number of the worker-members of the LMF is fixed, but, in low demand states, some members are temporarily laid-off and fully compensated for any income losses they will incur due to their working elsewhere. This way, members will not have any incentives not to give up their employment in the LMF whenever maximization of value-added per worker requires some of them do so. Moreover, within a range up to the point where all members are employed in the LMF, production varies positively with the product price, so, in this sense, perversity disappears.

On the other hand, perversity, or rigidity of output, does not completely disappear with compensation schemes. In

sufficiently high demand states, the LMF again does not respond to price signals, being constrained by the fixed number of its members (see Bonin (1984)). Differences between the profit-maximizing firm (henceforth PMF) and the LMF survive also in more complicated frameworks. In Spinnewyn and Svejnar (1987) model, in which new members can be admitted into the LMF, the Ward optimum is a lowerbound and the PMF optimum is an upperbound for the LMF employment which depends crucially on the initial membership size.

It has been suggested that the existence of a market for partnership shares of an LMF would cure any perversities (Sertel (1982, 1987)). Shares will be freely transferable in this market, and new workers are admitted into the LMF if they pay a price to obtain the shares of the enterprise, which then compensates for the utility, or income, loss of the existing workers. This way, the labor input of the LMF is determined such that not only the perversities disappear, but also the employment policy of the LMF becomes identical to its twin PMF.

This brings us directly into the property rights issue. In this context, there are at least two views held about how to define an LMF. Jensen and Meckling (1979) argue that the property rights of a labor-managed firm per se must be not transferable, otherwise we would be back to the traditional

PMF. The other view would be to invoke the definition of a workers' cooperative: a firm controlled and operated by the people who work in it (as cited, for example, by Ellerman (1984)). This second definition is general enough to allow for an LMF with transferable property rights. In reality, organizations have been observed to exist which are owned, controlled, and operated by a majority of people who work in it and whose shares are transferable (see Berman (1982)). Regardless of whether they can constitute, from a theoretical point of view, a different class of organization than the traditional firm even though their shares are transferable, they have been characterized and distinguished by workers' ownership and control. From a formal and analytical point of view, workers' ownership and control still implies maximization of income per worker as the objective function of the firm regardless of whether the firm sells its shares and the shares are transferable. Then, if the firm sells its shares, this introduces an additional factor whose implications together with maximization of income per worker should be worked out. Therefore, without necessarily disagreeing with the Jensen and Meckling (1979) position, we believe that it is a necessary exercise to go formally through the effects of share-selling on the behavior of a firm which is owned and controlled by its workers.

In this paper, we formulate a model of an LMF which is

embedded in a capitalist economy. The right to work in this firm can only be acquired by buying its shares which are transferable. The distinguishing feature of the model is its explicit intertemporal context and the emphasis on how the LMF's decision about selling shares is made. Under certain restrictive assumptions, we obtain the result that the perversity disappears and the LMF's employment is identical to that of its twin PMF. Furthermore, the present discounted value of surplus --i.e., surplus over wage earnings-- cash flow is the same for both LMF and PMF, making the LMF look like a entrepreneurial firm with N entrepreneurs and giving some support for Jensen and Meckling's position. However, the result about the disappearance of the perversity is obtained by assuming infinite planning horizon for workers, perfect information, and certainty (or uncertainty together with risk-neutrality), and one wonders whether it will survive if we relax these assumptions. Moreover, if hiring wage labor becomes an alternative option in varying the employment, the LMF remains indifferent between issuing more shares and hiring more non-member workers under frictionless functioning of the share market. We also consider some cases where the members' indifference evaporates in favor of either wage-labor-hiring or share-selling.

Finally, the results are compared with some stylized facts about the experience of some real-life share-selling

LMFs, and, we try to see whether this model, abstract and restrictive as it is, can replicate some characteristics of actual LMFs. For example, the model can generate, after simple modifications, an LMF whose employment is a mixture of members and nonmembers (with only the number of nonmembers expanding or contracting in the short-run) or the extinction of the labor-management over the long-run.

I.2. Preliminaries

I.2.1 Basic Assumptions and the Framework

I.2.1.a. Technology. The LMF produces a nonstorable good using capital, labor (of a single homogeneous type), and intermediate goods. Labor and capital may be substitutes or complements, but if they are complements, we will assume that the capital stock of the firm never represents a binding capacity constraint. In any case, we will not have to assume the capital stock to be fixed throughout the planning horizon, any changes in the capital stock will be reflected in the value-added function to be presented below, through the changes in the productivity and/or costs of renting capital. But we will assume that the investment decisions are exogeneously determined, and we will not deal with them explicitly.

Because of institutional constraints, each worker must work a fixed amount of time per production period. Hence output can only be increased by increasing the number of workers in the enterprise. Let us normalize that fixed amount of time by unity, so that labor input is numerically equal to the number of workers in the firm. In the analysis, we will treat the number of workers as if it is perfectly divisible (i.e. no integer problems).

I.2.1.b. Value added. Let L_t be the number of workers in the firm throughout the production period t . Value added per worker, a continuous and twice differentiable function of the number of workers, will be denoted by $v_t(L_t)$. It is calculated by deducting from the sales revenue the non-labor production costs and payments to capital and dividing then the surplus by the number of workers.¹ The subscript t denotes that the value-added function may differ from period to period as a result of, say, shifts in the demand schedule for the output of the firm or in the cost function, changes in the capital stock of the firm, etc. We do not restrict ourselves to perfect competition in the product market, price elasticity of

¹ Also retained earnings are deducted from the revenue, if they are used to finance the (exogeneous) investments, and $v_t(L_t)$ denotes value-added per worker net of per-worker investment spending out of total value-added, i.e. $v_t(L_t) = (R_t(L_t) - C_t(L_t) - I_t)/L_t$, where R_t is the revenue in period t , C_t is the cost of production (including fixed overhead costs and payments to debtholders), and I_t is investments in period t financed from earnings.

the demand for the product of the firm is embodied in the value-added function.

I.2.1.c. Information. We assume that there is no uncertainty about the function $v_t(\cdot)$ and about the alternative income which can be earned by workers outside of the labor managed firm, for all t . Furthermore, we assume that information is perfect and symmetric about the decisions of the LMF. That is, not only the workers in the LMF but also those outside of the LMF know about the decisions made by the LMF at the beginning of the period t as to $L_t, L_{t+1}, \dots$. Hence, not only the inside workers, but also the outside workers can infer $v_t(L_t), v_{t+1}(L_{t+1}), \dots$.

I.2.1.d. Workers. We will assume that all the workers in the economy, i.e. both those in the LMF we are considering and those outside of it, are identical with respect to their preferences and their productivity, and that workers are infinitely-lived and never retire.

In the preferences of the workers, disutility of work will be ignored in conformity with the assumption about working hours. Hence, workers care only about their income. We will mostly deal with certainty and will not need any assumption about the degree of risk-aversion, but in cases where we do have some uncertainty we will assume that the

workers are risk-neutral.

The assumption about the life-span of workers simplifies the analysis to infinite horizon planning on the part of the workers. But, this type of optimization problem is not intended to be an approximation to the finite horizon problem. If workers have a finite life, the work force of the LMF will be a mixture of workers of different ages, hence of different preferences at a given point in time. For sufficiently old workers, the optimization problem cannot be approximated by one with an infinite horizon. The case of infinite life is rather useful as a benchmark one. It may provide us with some insight about the finite life case and suggest directions for future analysis.

I.2.1.e. Timing of events and actions. Time is divided into alternating periods of production and decision. In other words, suppose the production period is a "week." Production begins at "Monday morning" and continues until "Friday evening." During the week, the number of workers in the firm is constant, say L_{t-1} . "Saturday morning," the decision period starts. L_{t-1} workers in the LMF choose (i) a price for the partnership shares, and (ii) the number of workers for the future production periods t , $t+1$, $t+2$, ..., and then they announce these decisions. On "Sunday," the market for partnership shares opens. Based on the information announced

by the LMF, workers outside of the LMF infer the future per-worker income levels which can be earned by joining the LMF, and buy partnership shares, say δL of them. (If the LMF sets the price by taking into account the price elasticity of the demand for its shares, $L_{t-1} + \delta L$ will automatically be equal to L_t chosen by the LMF; otherwise, there will be a tatonnement with recontracting going on during the "Sunday," and the price set by the LMF will be only the initial price for the iterations in the partnership market. During this process, outside demanders of new shares will not join decision-making.) Then, the next Monday, production begins again with $L_{t-1} + \delta L = L_t$ workers being in the firm. During the production period, no new shares are issued by the LMF, nor can some partners exit the firm by selling their shares to the firm (i.e., to those workers who remain), but exchange of shares between members and outside workers on an individual basis is not restricted.

I.2.1.f. Layoffs. We abstract from temporary layoffs of members as an adjustment policy to low-demand states, i.e. members will not be able to work elsewhere without losing their membership right.

I.2.2 Optimization Problem of the Profit-Maximizing Firm

For purposes of comparison, let us state the first-order

condition for the optimization problem of a PMF which uses the same technology, faces the same demand conditions, and makes the same investment decisions as the LMF for all periods. Let w_t denote the competitive wage rate at which the firm can hire labor at period t . Then its single-period profits will be given by $[v_t(L_t) - w_t] L_t$, and the profit maximizing condition is simply

$$v_t'(L_t) L_t + v_t(L_t) = w_t$$

In addition, $L_t v_t'(L_t) + v_t(L_t)$ is decreasing in L_t so that this condition guarantees a maximum.

I.3. The Share-Selling Labor-Managed Firm

I.3.1. Income Stream and Choices of the Individual Workers

With the introduction of transferable partnership shares, which give to their holders both the ownership right and the right to work in the LMF, the expansion of output of LMF will be achieved by selling new shares to nonmember (outside) workers, and the contraction of output will necessitate that the members who will stay in the firm buy back (and destroy) the shares of some other partners, i.e. the shares of those who leave. Hence, in period t , there is an additional source

of income or cost. Suppose, at the beginning of the period t , the number of the members of the LMF is L_{t-1} , and the share price is p_t . If new shares are sold to outside workers in order to increase the employment in the LMF to L_t in period t , the L_{t-1} members of the LMF will share the revenue $p_t(L_t - L_{t-1})$, and the income of each worker who was in the firm in period $t-1$ will be

$$v_t(L_t) + p_t(L_t - L_{t-1})/L_{t-1}$$

whereas for the new entrants the income earned in t will be $v_t(L_t)$ only. If, on the other hand, some of the existing members' shares are bought back by others in the firm to reduce the employment to L_t , those who remain in the firm will share the cost $p_t(L_{t-1} - L_t)$, and the income of each worker who is in the firm in t will be

$$v_t(L_t) - p_t(L_{t-1} - L_t)/L_t.$$

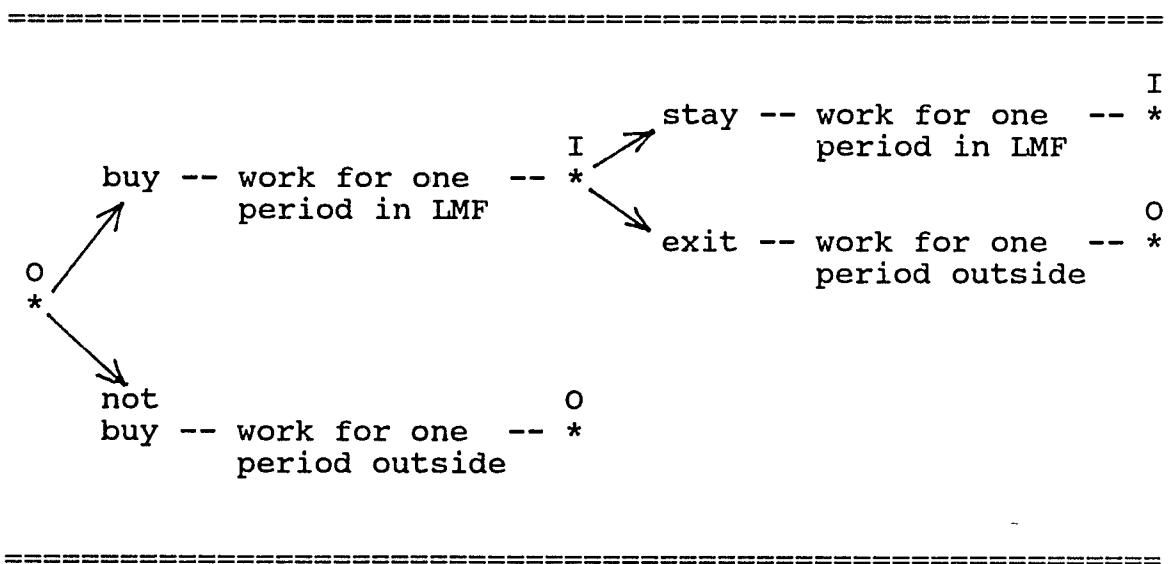
For notational convenience below, define

$$S_t = \begin{cases} p_t(L_t - L_{t-1})/L_{t-1}, & \text{if } L_t > L_{t-1} \\ - p_t(L_{t-1} - L_t)/L_t, & \text{if } L_t < L_{t-1} \end{cases}$$

Now, consider the decisions of an individual worker. Note

that the assumption about infinite life of workers makes them alike with respect to their decisions. In accordance with this assumption, partnership shares are sold for an infinite number of periods. Hence, by purchasing a share, a worker acquires the membership right for the rest of the whole future, including the period t .

The decision tree of an individual worker can be depicted as follows:



In this diagram, the nodes labeled O indicate that the worker has spent the last period outside of the LMF and those labeled I indicate that the worker was a member of LMF in the last period. Let U_t denote the present discounted value of income stream of the worker as a result of his best choice among

buying and not buying a share of the LMF at the beginning of period t , if he is at a O-node, and, similarly, let V_t denote the present discounted value of the income stream of the worker as a result of his best choice among staying in or exiting from the LMF at the beginning of the period t , if he is at a I-node. Both U_t and V_t are evaluated for any given L_t , L_{t+1} , L_{t+2} , ..., and p_t , p_{t+1} , p_{t+2} , ...

First, consider a worker at a O-node and suppose period t is expansionary. Then

$$U_t = \max \{ v_t - p_t + \beta V_{t+1}, w_t + \beta U_{t+1} \} \quad (1)$$

where β is the discount factor, and w_t is the alternative (wage) income which can be earned outside of the firm in period t . In order that the number of members of the LMF can be increased to L_t , the workers outside of the LMF should buy the shares, and that happens if

$$v_t - p_t + \beta V_{t+1} \geq w_t + \beta U_{t+1} \quad (2)$$

Next, consider a worker at a I-node and suppose period t is contractionary. We have

$$V_t = \max \{ v_t + S_t + \beta V_{t+1}, p_t + w_t + \beta U_{t+1} \} \quad (3)$$

where p_t is the price paid by those who stay to those who exit for a share, and S_t is evaluated for $L_t < L_{t-1}$. In order that some members are persuaded to exit and some to stay, both parties must be indifferent between staying and exiting. Hence, the price chosen, say $p_t^\#$, must satisfy²

$$v_t + S_t + \beta V_{t+1} = p_t^\# + w_t + \beta U_{t+1} \quad (4)$$

We can write

$$V_t = v_t + S_t + \beta V_{t+1}, \quad \text{for } L_t < L_{t-1} \quad (5)$$

Now consider a worker at a 0-node, and suppose period t is contractionary. Suppose some of the exiting workers sell their shares to outside workers, instead of to staying members, at a price p' . Then

² In essence, equation (4) embodies the idea of the complete compensation schemes used by, for example, Bonin (1984) and Spinnewyn and Svejnar (1987). Its single-period version leads to

$$v_t + S_t = p_t + w_t = (L_t/L_{t-1}) v_t + [1 - (L_t/L_{t-1})] w_t$$

Miyazaki and Neary (1983) use the same expression for the expected income because in their model L_t/L_{t-1} is the probability that a worker will stay in the LMF, since those who will exit are chosen randomly; and he will either earn v_t , if he is among those who stay in the LMF, or w_t , if he is one of those who work elsewhere. Thus Miyazaki and Neary (1983) do not adopt a complete compensation scheme. Similar remarks apply to Miyazaki (1984a).

$$U_t = \max \{ v_t + S_t + \beta V_{t+1} - p', w_t + \beta U_{t+1} \} \quad (6)$$

assuming that the new workers enter the firm before the cost $p_t^\#(L_{t-1}-L_t)$ is distributed among the members, hence they also incur S_t .³ In order that outside workers buy,

$$v_t + S_t + \beta V_{t+1} - p' \geq w_t + \beta U_{t+1} \quad (7)$$

implying $p' \leq p_t^\#$. Since exiting workers wouldn't sell to outside workers for $p' < p_t^\#$, we either have $p' = p_t^\#$, or no outside workers would buy shares of exiting workers at the beginning of a contractionary period. In any case, we have

$$U_t = w_t + \beta U_{t+1}, \quad \text{for } L_t < L_{t-1} \quad (8)$$

Finally, consider a worker at a I-node, and suppose period t is expansionary. We have

$$V_t = \max \{ v_t + S_t + \beta V_{t+1}, S_t + p^- + w_t + \beta U_{t+1} \} \quad (9)$$

where p^- is the price paid by the outside workers who buy shares from exiting members, and S_t is now evaluated for $L_t >$

³ If an outside worker buys a share from a member after S_t is paid by the staying members to those who exit by selling their shares to the staying, it can still be shown that equation (8) below is valid.

L_{t-1} . It has been assumed that this sell will be made after the revenues from selling new shares have been distributed among L_{t-1} old members.⁴ In order that some members sell their (previously issued) shares to outsiders,

$$v_t + S_t + \beta V_{t+1} \leq S_t + \bar{p} + w_t + \beta U_{t+1} \quad (10)$$

which implies $\bar{p} \geq p_t$, where p_t satisfies (2). Since entering workers wouldn't buy from exiting workers for $\bar{p} > p_t$, we either have $\bar{p} = p_t$, or no outside workers would buy shares of exiting workers in an expansionary period. In any case, we have

$$V_t = v_t + S_t + \beta V_{t+1}, \quad \text{for } L_t > L_{t-1} \quad (11)$$

From the above analysis, a number of conclusions can be drawn. The following subsections are devoted to them.

I.3.2. The Optimization Problem of the Workers in the LMF

By combining (5) and (11), we have

$$V_t = v_t + S_t + \beta V_{t+1}, \quad \text{for all } L_t \quad (12)$$

which implies

⁴ If this sell is made before the revenues from selling new shares have been distributed among L_{t-1} old members, it can still be shown that equation (11) below is valid.

$$V_t = \sum_{k=0}^{\infty} \beta^k (v_{t+k} + S_{t+k}) \quad (13)$$

where the summation runs to infinity (infinity sign is omitted). Since V_t is the present discounted value of the income stream of the member worker at the beginning of the period t , regardless of whether he stays in or exits from the firm in t or any future period, the workers in the LMF will seek to maximize (13), by choosing L_t , L_{t+1} , L_{t+2} , ..., and p_t , p_{t+1} , p_{t+2} , ... at the beginning of period t , given L_{t-1} .

I.3.3. Price of New Shares

Consider again the condition for outside workers buying new shares in an expansionary period:

$$v_t - p_t + \beta V_{t+1} \geq w_t + \beta U_{t+1} \quad (2)$$

Since all workers have been assumed to be alike, both with respect to their preferences and the circumstances they face, the demand for new shares by outside workers will be infinitely elastic at $p_t = p_t^*$; for $p_t < p_t^*$, it will be infinite, and for $p_t > p_t^*$, it will be zero, where p_t^* satisfies

$$v_t - p_t^* + \beta V_{t+1} = w_t + \beta U_{t+1} \quad (14)$$

Since neither V_{t+1} nor U_{t+1} in (2) depend on p_t , the workers in the LMF can increase their income given in (13) by increasing p_t up to p_t^* , for any given $L_t, L_{t+1}, \dots$ and $p_{t+1}, \dots$. In other words, the best choice of the firm is to set $p_t = p_t^*$, making thereby the outside workers indifferent between entering the firm or continuing to work outside, and then to pick the best L_t for each t . This, in turn, because of (1), implies that we can write

$$U_t = w_t + \beta U_{t+1}, \quad \text{for } L_t > L_{t-1} \quad (15)$$

I.3.4. Present Discounted Value of the Income Stream of an Outside Worker

By combining (8) and (15), we have

$$U_t = w_t + \beta U_{t+1}, \quad \text{for all } L_t \quad (16)$$

which implies

$$U_t = \sum_{k=0}^{\infty} \beta^k w_{t+k} \quad (17)$$

which means that, for a worker who was not a member of the LMF in the last period, the present discounted value of his income stream as a result of his best choice among buying and not buying a share of the LMF at the beginning of period t is the

same as the present value of the income stream of a worker who will always work outside of the LMF and earn wage income only, regardless of whether the LMF will expand or contract in any period. Therefore, from the point of view of an outside worker, whether the LMF adopts a policy of selling shares or not does not matter. By using (16), (4) and (14) become

$$v_t + S_t + \beta V_{t+1} = p_t^{\#} + U_t \quad (18)$$

$$v_t - p_t^* + \beta V_{t+1} = U_t \quad (19)$$

meaning the worker will make his decision to exit (enter) the firm as if the alternative is never to enter (exit) again.

I.3.5. The F.O.C. for the Maximization Problem

By substituting

$$S_t = - p_t^{\#} (L_{t-1} - L_t) / L_t$$

into (4) and comparing then (14) and (4), we find

$$p_t^{\#} = p_t^* L_t / L_{t-1} \quad (20)$$

which further leads to the fact that

$$- p_t^{\#} (L_{t-1} - L_t) / L_t = p_t^* (L_t - L_{t-1}) / L_{t-1}$$

implying

$$S_t = p_t^* (L_t - L_{t-1}) / L_{t-1}, \quad \text{for all } L_t \quad (21)$$

or, by substituting for p_t^* from (19) into (21),

$$S_t = (v_t + \beta v_{t+1} - U_t) (L_t - L_{t-1}) / L_{t-1}$$

Substituting this into (12), and rearranging, we obtain

$$V_t = (1/L_{t-1}) [L_t (v_t + \beta v_{t+1}) - (L_t - L_{t-1}) U_t]$$

By leading this equation successively and substituting back into itself, we obtain

$$\begin{aligned} V_t = (1/L_{t-1}) [& L_t v_t + \beta L_{t+1} v_{t+1} + \beta^2 L_{t+2} v_{t+2} + \dots \\ & - (L_t - L_{t-1}) U_t - \beta (L_{t+1} - L_t) U_{t+1} \\ & - \beta^2 (L_{t+2} - L_{t+1}) U_{t+2} - \dots] \end{aligned}$$

or, rearranging, and noting from (16) that $U_t - \beta U_{t+1} = w_t$,

$$\begin{aligned} V_t = U_t + (1/L_{t-1}) [& L_t (v_t - w_t) + \beta L_{t+1} (v_{t+1} - w_{t+1}) \\ & + \beta^2 L_{t+1} (v_{t+1} - w_{t+1}) + \dots] \end{aligned}$$

or, simply,

$$V_t = U_t + (1/L_{t-1}) \sum_{k=0} \beta^k \pi_{t+k} \quad (22)$$

where $\pi_t = L_t(v_t - w_t)$ represents nothing but the profits of an entrepreneurial firm in period t which produces the same quantities as the LMF. We immediately learn two things. First, (22) implies

$$L_{t-1} (V_t - U_t) = \sum_{k=0} \beta^k \pi_{t+k} \quad (23)$$

That is, the discounted sum of payoffs excess of wage earnings to all workers who are in the firm at the beginning of period t is equal to the discounted sum of profits of an entrepreneurial enterprise producing the same quantities as the LMF (see below for more comments on this point). Second, (22) shows that maximizing V_t is the same thing as maximizing the profits of an entrepreneurial enterprise. With L_{t-1} given, the first-order condition for the LMF is found by differentiating (22) with respect to L_t and equating to zero:

$$v_t(L_t) + L_t v_t'(L_t) - w_t = 0 \quad (24)$$

Needless to say, this is nothing but the first-order condition for the profit-maximizing enterprise. Hence, under the assumptions of Section I.2, a labor-managed firm which sells (buys back) its shares in order to expand (contract) its

output will have the same outcome as the profit-maximizing firm, if it is embedded in a capitalistic economy. It follows that the existence of conditions making such a share-selling behavior possible will overcome the perversities found in the labor-managed enterprise.⁵

I.3.6. Total Value of the Shares

Solving (14) for p_t^* and multiplying by L_t we obtain

$$p_t^* L_t = L_t(v_t - w_t) + \beta L_t(V_{t+1} - U_{t+1})$$

or, using (23) for period $t+1$,

$$p_t^* L_t = \pi_t + \beta \sum_{k=1} \beta^k \pi_{t+k} = \sum_{k=0} \beta^k \pi_{t+k} \quad (25)$$

The left-hand side is the value of the outstanding stock of the firm's shares during period t , whereas the right-hand side gives the present discounted value of the profits of a PMF which produces the same quantities as the LMF.

I.3.7. Characterization of Share Prices

Now it is straightforward to characterize the share prices of

⁵ By rewriting (24) as

$$v_t(L_t) - w_t = -L_t v_t'(L_t) \quad (F.1)$$

we obtain the equilibrium condition for Sertel's (1987) partnership market.

the LMF:

1. Choose $L_t, L_{t+1}, \dots$.
2. Calculate $\sum_{k=0} \beta^k \pi_{t+k}$ at $L_t, L_{t+1}, \dots$.
3. If $L_t > L_{t-1}$, calculate the selling price of new shares as

$$p_t^* = (\sum_{k=0} \beta^k \pi_{t+k}) / L_t$$

(This follows from (25).)

4. If $L_t < L_{t-1}$, calculate the buying price of old shares as

$$p_t^\# = (\sum_{k=0} \beta^k \pi_{t+k}) / L_{t-1}$$

(This follows from (25) and (20).)

I.3.8. Present Discounted Value of the Income Stream of the Workers in the LMF in Excess of Wage Earnings

Using (25), we can write (22) as

$$V_t = U_t + p_t^* L_t / L_{t-1} \quad (26)$$

Since $U_t = \sum_{k=0} \beta^k w_{t+k}$, it follows from (26) that the difference between the discounted sum of alternative (or wage) earnings of a worker outside of the firm and that of the earnings which will accrue in periods $t, t+1, \dots$ to a worker in the firm is $p_t^* L_t / L_{t-1}$. This, in turn, is equal to $p_t^\#$, the buy-back price (see (20)), hence

$$V_t = U_t + p_t^\#$$

This shows again that the function of the buying-back price of shares is to compensate an existing worker for the lifetime income loss.

In addition to the above, one can make some other interesting comments about the difference $V_t - U_t$. Consider (22) again:

$$V_t = U_t + (1/L_{t-1}) \sum_{k=0} \beta^k \pi_{t+k} \quad (22)$$

That is, assuming $\sum_{k=0} \beta^k \pi_{t+k} > 0$, each worker in the LMF will enjoy, from t on, a positive payoff excess of wage earnings once he enters the firm before period t . On the other hand, when entering the firm, the price of a share leaves each worker indifferent between joining the LMF or working outside, and, for such a worker, the discounted sum of future earnings in the firm less the share price is equal to the discounted sum of wages (Equation (19)). So, the positive payoff excess of wages which accrues to the worker at (and beyond) period t if he joined the firm before period t must be compensating for some loss which occurred some time between period t and the period the worker joined the firm, and since (22) is valid for any period except the one in which the worker has joined the firm, that loss must have occurred in the period when the worker has joined the firm. Suppose the worker entered the LMF at the beginning of period $t-1$, then, by writing Equation (14)

for $t-1$ and rearranging, we have

$$V_{t-1} - P_{t-1} = W_{t-1} - \beta (V_t - U_t) < W_{t-1}$$

where the left-hand side is the earnings of the worker in period $t-1$ who joined the firm at the beginning of period $t-1$. These earnings are less than w_{t-1} by $\beta (V_t - U_t)$. It follows that $V_t - U_t$, the positive payoff excess of wage earnings, is the present value of the loss in period $t-1$. Of course, we may look upon this loss also as follows: The worker borrows p_{t-1} at $t-1$ in order to buy the partnership share, so, the positive excess earnings in periods $t, t+1, \dots$ will be used to pay out the debt.

This is not true only for the founding members of the LMF. Only for them does $V_t - U_t$ represent a net gain over wages as a reward to their 'entrepreneurial' undertaking. (Any set-up costs financed through borrowing are reflected in the value-added function. The net gain is the return for the costs financed through workers' investments.) For L_0 workers who jointly opened the firm before period 1, we can write equation (23) as

$$L_0 (V_1 - \sum_{k=0} \beta^k w_{1+k}) = \sum_{k=0} \beta^k \pi_{1+k}$$

In other words, the founding members share the discounted sum of the profits which will accrue to a capitalistic entrepreneur who sets up the twin of the LMF and decides to produce the same quantities as the LMF will do. An implication of this expression is that, for any given production plan $L_1, L_2, \dots$, V_1 will be higher if L_0 , the number of initial members, is lower, hence there will be an incentive to set up the firm with as few initial members as possible.

An alternative to the above expression can be written as follows. First, note that (25) implies

$$\begin{aligned} p_t L_t &= \pi_t + \beta p_{t+1} L_{t+1} \\ &= L_t (v_t - w_t) + \beta p_{t+1} L_{t+1} \end{aligned} \quad (27)$$

Now, in each period t , the cash flow to any worker in the LMF who has been a member in period $t-1$ will be $v_t + p_t(L_t - L_{t-1})/L_{t-1}$. By substituting from (27) for p_t , we can write this as

$$v_t + (v_t - w_t + \beta p_{t+1} L_{t+1}/L_t) (L_t - L_{t-1})/L_{t-1}$$

or, upon rearranging,

$$w_t + \pi_t/L_{t-1} + \beta (p_{t+1} L_{t+1}/L_t) (L_t - L_{t-1})/L_{t-1}$$

Hence, the cash flow excess over wage earnings shared by L_{t-1} members in period t is

$$\pi_t + \beta (p_{t+1}L_{t+1}/L_t) (L_t - L_{t-1}) \quad (28)$$

Among these L_{t-1} workers, each of the L_0 founding members will receive the excess over wage earnings

$$(L_0/L_{t-1}) [\pi_t + \beta (p_{t+1}L_{t+1}/L_t) (L_t - L_{t-1})]$$

in period t . Now, it may be shown⁶ easily that, as expected,

$$\begin{aligned} &^6 \text{ By using (27) with } \pi_t = L_t (v_t(L_t) - w_t), \text{ we can rewrite} \\ &(L_0/L_{t-1}) [\pi_t + \beta (p_{t+1}L_{t+1}/L_t) (L_t - L_{t-1})] \end{aligned} \quad (F.2)$$

as

$$(L_0/L_{t-1}) [p_t L_t + \beta p_{t+1} L_{t+1} [(L_t - L_{t-1})/L_t - 1]]$$

or as

$$L_0 [p_t L_t/L_{t-1} - \beta p_{t+1} L_{t+1}/L_t] \quad (F.3)$$

From (26), we obtain

$$V_t - \beta V_{t+1} = p_t L_t/L_{t-1} - \beta p_{t+1} L_{t+1}/L_t + w_t$$

Noting that $\sum_{t=1} \beta^{t-1} (V_t - \beta V_{t+1}) = V_1$, this implies

$$V_1 = \sum_{t=1} \beta^{t-1} [p_t L_t/L_{t-1} - \beta p_{t+1} L_{t+1}/L_t] + \sum_{k=0} \beta^k w_{1+k}$$

and, by the equality of (F.2) and (F.3),

$$\begin{aligned} \sum_{t=1} \beta^{t-1} (L_0/L_{t-1}) [\pi_t + \beta (p_{t+1}L_{t+1}/L_t) (L_t - L_{t-1})] \\ = L_0 (V_1 - \sum_{k=0} \beta^k w_{1+k}) \end{aligned}$$

It follows from (23) that

$$\begin{aligned} \sum_{t=1} \beta^{t-1} (L_0/L_{t-1}) [\pi_t + \beta(p_{t+1}L_{t+1}/L_t) (L_t - L_{t-1})] \\ = \sum_{k=0} \beta^k \pi_{1+k} \end{aligned}$$

The left-hand side is the discounted sum of earnings excess over wages which accrues to founding members, and right-hand side is the discounted sum of the profits of the capitalistic twin of the LMF throughout its entire existence. This equation also shows that the cash flow to the founding members is not equal to π_t period by period. For example, in period 1, the members obtain

$$\pi_1 + \beta (p_2 L_2/L_1) (L_1 - L_0) > \pi_1 \quad \text{for } L_1 > L_0$$

implying that, for some other period(s),

$$(L_0/L_{t-1}) [\pi_t + \beta (p_{t+1}L_{t+1}/L_t) (L_t - L_{t-1})] < \pi_t$$

Let us conclude this subsection by noting that all the remarks made here apply for any plan $L_t, L_{t+1}, \dots$ of the LMF --and of its twin PMF constrained to produce the same quantities, whenever a comparison is made--, i.e. the production levels need not be optimal. In other words, the results of this subsection require only that the price equations (4) and (14)

$$\begin{aligned} \sum_{t=1} \beta^{t-1} (L_0/L_{t-1}) [\pi_t + \beta (p_{t+1}L_{t+1}/L_t) (L_t - L_{t-1})] \\ = \sum_{k=0} \beta^k \pi_{1+k} \end{aligned}$$

hold.

I.4. Role of Price-Setting Behavior

Now we would like to stress a simple but crucial point. In Section I.3, the price of the shares is set by the workers in the LMF. The effect of the change in L_{t-1} on the price which the outside workers are willing to pay --let us denote it by p_t^D -- is not necessarily negligible; moreover, under the assumed information structure, the effect of the change in L_{t-1} on p_t^D is completely predictable by the LMF. Under these circumstances, it is not plausible to give up the assumption that the LMF behaves as a price-setter in the market for partnership shares. Still, considering the consequences of the alternative assumption may be fruitful. The alternative would of course be that the price of the shares is taken both by the outside workers and the workers in the in the firm as given⁷ and determined by the interaction of supply and demand in the market for shares. Of course, this means that the price will be determined as a limit of the convergent tatonnement

⁷ Sertel (1982) calls the left-hand side of (F.1) -- written in utility terms-- the demand price of the shares, and the right-hand side the supply price. A terminological issue arises since these may imply that the share price is taken both by the outside workers and the workers in the firm as given and determined by the interaction of supply and demand in the market for shares.

iterations in the market during "Sunday," and, throughout these iterations, both the LMF and individual workers outside of the LMF will be allowed to bid the price up or down until the share market comes to equilibrium, but the common practice in modelling such situations has been to assume that suppliers and demanders are price-takers and that the price is determined instantaneously in the market. Below, we will pursue such an analysis to make the following point: Price-taking behavior on the part of the workers in the firm would result in path-dependent equilibria, and consequently the behavior of the LMF would deviate from that of the PMF.

For the purposes of this section, we may allow ourselves to be restricted by the fact that period t is an expansionary one, i.e. $L_t > L_{t-1}$. By substituting $S_t = p_t(L_t - L_{t-1})/L_{t-1}$ into (12), we have

$$V_t = v_t + p_t(L_t - L_{t-1})/L_{t-1} + \beta V_{t+1} \quad (29)$$

Define

$$\phi_{t+1}(L_t) = \max_{\{L_{t+1}, L_{t+2}, \dots\}} V_{t+1}$$

given $p_{t+1}, p_{t+2}, \dots$. Assume $\phi_{t+1}(L_t)$ and $\phi_{t+1}'(L_t)$ are continuous in L_t and $\phi_{t+1}'(L_t)$ is decreasing in L_t . Then, if

the workers in the firm take the share price p_t as given, the first-order condition for the maximization of (29) will be

$$v_t'(L_t) + p_t/L_{t-1} + \beta \phi_{t+1}'(L_t) = 0 \quad (30)$$

This condition says that, although $\phi_{t+1}'(L_t)$ is independent of L_{t-1} , L_t , the optimum new number of workers for the workers in the firm at a given share price, is not independent of L_{t-1} , the number of workers in the firm. Rearranging (30), we obtain

$$p_t^S = - L_{t-1} [v_t'(L_t) + \beta \phi_{t+1}'(L_t)]$$

where p_t^S is defined as the supply price of shares.⁸ On the other hand, the demand price will be given by

$$p_t^D = v_t(L_t) + \beta \phi_{t+1}(L_t) - \sum_{k=0} \beta^k w_{t+k} \quad (31)$$

⁸ The following point may be worth emphasizing: Since (30) is a first-order condition, we have the elementary fact that

$$p_t^S/L_{t-1} > - [v_t'(z) + \beta \phi_{t+1}'(z)], \quad \text{for all } z < L_t$$

Hence, by integrating with respect to z from L_{t-1} to L_t ,

$$p_t^S(L_t - L_{t-1})/L_{t-1} > v_t(L_{t-1}) + \beta \phi_{t+1}(L_{t-1}) - [v_t(L_t) + \beta \phi_{t+1}(L_t)]$$

i.e., the revenue made by selling the shares at the supply price more than compensates for the decrease in the present discounted value of the future earnings of the L_{t-1} workers in the LMF, which is caused by the increase in the labor input.

or, by (2) with the equality holding (because competition among demanders would push the price up to the highest which the demanders will be willing to pay for the shares). Now it follows from (30) and (31) that the number of workers in period t , L_t , which satisfies the equilibrium condition $p_t^S = p_t^D$ will not be independent of L_{t-1} . But if that is the case, then, in general, L_t will not satisfy (24), either, simply because (24) is a condition which is supposed to be satisfied by L_t for any L_{t-1} .⁹

I.5. Qualitative Differences Between LMF and PMF

In comparing the LMF with the PMF, we assumed so far that, for a worker, there is no difference between working in either one of these two types of firms except for the income obtained by the worker and the effort spent during the decision-making process in the LMF.¹⁰ Both firms are the same not only in

⁹ In addition, it can be shown that we have lower L_t for higher L_{t-1} , with, of course, $L_t > L_{t-1}$, i.e. the LMF expands output to a lesser extent if there is a larger number of members at the beginning of period t .

¹⁰ In the PMF as represented in this paper, the decisions are made by the entrepreneur who takes also the whole profit. In the LMF, the decision-making function is performed by the member/workers. This oversimplified world can capture neither the multi-level decision structure of a modern firm nor the effects of division of labor and specialization in decision-making activity. But the PMF model, constructed at this level of simplicity, has played its well-known role in the economic literature, and the attempt to compare it with an LMF having

quantitative respects such as technology, market demand which they are subject to, cost structure, capital stock, number of working hours per period per worker, etc., but also in qualitative conditions of work. This leads to the fact that the price a worker is willing to pay for a partnership share is exactly equal to the difference between the discounted income earned in and out of the firm, and this, in turn, is one the reasons behind the identical optimal production levels in these two firms. It follows that any qualitative difference in the working conditions in these two firms for which the worker will be willing to pay a (positive or negative) premium will let the output of the LMF deviate from that of the PMF. For example, if involvement in the decision-making activity has disutility¹¹ for the worker, he will be willing to pay less for a share than the difference in incomes in LMF and PMF, i.e. a negative premium.

a corresponding decision-making structure is justified.

¹¹ Under certainty, the decision-making sessions do not need to be repeated. Everything can be and is decided at $t=0$, and simply executed later on. Hence, there is no disutility associated with decision-making efforts for members who join the firm in later periods. But in the case of the perfectly competitive LMF, where uncertainty does not imply any quantity constraints the firm may face, our model has a direct uncertainty application thanks to the risk-neutrality. All we have to do is replace the output price in $v_t(\cdot)$ by expected output price, and also w_t by expected w_t , for all $t > 0$. Then the model has a direct application in the case of uncertainty, and decision-making sessions have to be repeated whenever expectations are changed.

Specifically, let the premium be denoted by b_t . In the presence of such a premium, price equation (19) for $L_t > L_{t-1}$ will look like

$$\begin{aligned} p_t^* &= v_t + \beta V_{t+1} - U_t + b_t \\ &= v_t + \beta V_{t+1} - (U_t - b_t) \end{aligned} \quad (32)$$

and equation (18) for $L_t > L_{t-1}$ becomes

$$v_t + S_t + \beta V_{t+1} + b_t = p_t^{\#} + U_t$$

or

$$v_t + S_t + \beta V_{t+1} = (U_t - b_t) + p_t^{\#}$$

The equation $p_t^* L_t / L_{t-1} = p_t^{\#}$ still holds, and the objective for a worker in the LMF is again the maximization of

$$V_t = v_t + S_t + \beta V_{t+1}, \quad \text{for all } L_t$$

Upon substitution of from (32), we obtain

$$\begin{aligned} V_t = (1/L_{t-1}) \{ \sum_{k=0}^{\infty} \beta^k [L_{t+k} v_{t+k} \\ - (L_{t+k} - L_{t+k-1}) (U_{t+k} - b_{t+k})] \} \end{aligned}$$

Hence, the first-order condition for the LMF becomes

$$v_t(L_t) + L_t v_t'(L_t) = w_t - (b_t - \beta b_{t+1})$$

Suppose b_{t+k} have the same sign for all $k \geq 0$. Except in periods for which $|b_{t+1}| > (1/\beta) |b_t|$, we have $w_t - (b_t - \beta b_{t+1}) > (<) w_t$ according to whether $b_{t+k} < (>) 0$. It follows that, for negative premiums, the LMF produces less than the PMF, and, for positive premiums, it is the LMF which produces more. To sum up: Qualitative differences which lead to violation of equations (18) and (19) while leaving everything else the same invalidate the result about the equivalent production levels between the LMF and PMF. But perversity remains cured.

I.6. Share-Selling Versus Wage-Labor Hiring

I.6.1. Optimization Problem Reconsidered

So far, we have assumed implicitly that the only means available to the LMF for increasing its labor force is to sell its shares so that its entire labor force consists of only member workers. But, except for normative principles, there is nothing which would prevent the members from hiring wage-workers without giving them the partnership right, since it is embedded in a capitalist economy and it may be the case that it has access to a labor market. If it turns out to be optimal, say in period t , the LMF may turn into a PMF owned by L_{t-1} entrepreneurs hiring $L_t - L_{t-1}$ wage workers. In reality,

it has been observed that the labor-managed firms' labor force consists of a mixture of members and hired wage workers (see Berman (1982)). So then, the question is: Under what conditions, if at all, is it optimal to hire wage workers? Also, is share-selling suboptimal to hiring wage-workers under those conditions?

To analyze these questions, let h_t be the number of workers to be hired during period t , and m_t be the number of partners in the LMF. So, $L_t = m_t + h_t$. Naturally, the revenue made from selling partnership shares at the beginning of period t will be distributed among m_{t-1} members who are in the firm then, and the costs of hiring wage workers will accrue to all members, i.e. to m_t , who are in the firm during period t . Also, note that the total value-added in period t will be

$$L_t v_t(L_t) = (m_t + h_t) v_t(m_t + h_t)$$

Under these circumstances, the revenue (cost) of selling new (buying old) shares is given by

$$S_t = \begin{cases} p_t(m_t - m_{t-1}) / m_{t-1}, & \text{for } m_t > m_{t-1} \\ - p_t(m_{t-1} - m_t) / m_t, & \text{for } m_t < m_{t-1} \end{cases}$$

and v_t in Equations (1) - (11) of Section I.3.1 must be

replaced by

$$(m_t + h_t) v_t(m_t + h_t)/m_t - h_t w_t/m_t$$

but the analysis of Section I.3.1 remains valid after these changes, so that we have, for $m_t > m_{t-1}$,

$$\begin{aligned} p_t^* &= (m_t + h_t) v_t(m_t + h_t)/m_t - h_t w_t/m_t + \beta V_{t+1} \\ &\quad - w_t - \beta U_{t+1} \end{aligned} \quad (33)$$

and, for $m_t < m_{t-1}$,

$$\begin{aligned} p_t^\# + w_t + \beta U_{t+1} &= (m_t + h_t) v_t(m_t + h_t)/m_t \\ &\quad - p_t^\#(m_{t-1} - m_t)/m_t - h_t w_t/m_t \\ &\quad + \beta V_{t+1} \end{aligned} \quad (34)$$

with

$$U_t = w_t + \beta U_{t+1} = \sum_{k=0} \beta^k w_{t+k}$$

and the objective to be maximized for m_{t-1} workers during the decision-making stage at the beginning of period t becomes

$$V_t = (m_t + h_t) v_t(m_t + h_t)/m_t - h_t w_t/m_t + S_t + \beta V_{t+1} \quad (35)$$

From (33) and (34) we see that

$$p_t^{\#} = p_t^* m_t / m_{t-1}$$

implying

$$S_t = p_t^*(m_t - m_{t-1}) / m_{t-1} = - p_t^{\#}(m_{t-1} - m_t) / m_t$$

so that (35) can be written as

$$\begin{aligned} V_t = & (m_t + h_t) v_t(m_t + h_t)/m_t - h_t w_t/m_t \\ & + p_t^*(m_t - m_{t-1})/m_{t-1} + \beta V_{t+1}, \quad \text{for all } m_t \end{aligned}$$

By substituting for p_t^* from (33),

$$\begin{aligned} V_t = (1/m_{t-1}) [& (m_t + h_t) v_t(m_t + h_t) - h_t w_t + \beta m_t V_{t+1} \\ & - (m_t - m_{t-1}) U_t] \end{aligned}$$

Upon leading this equation successively and substituting back into itself,

$$\begin{aligned} V_t = (1/m_{t-1}) [& (m_t + h_t) v_t(m_t + h_t) - h_t w_t \\ & + \beta (m_{t+1} + h_{t+1}) v_{t+1}(m_{t+1} + h_{t+1}) \\ & - \beta h_{t+1} w_{t+1} + \dots - (m_t - m_{t-1}) U_t \\ & - \beta (m_{t+1} - m_t) U_{t+1} - \dots] \end{aligned}$$

or, using $U_t - \beta U_{t+1} = w_t$, and rearranging,

$$V_t = U_t + (1/m_{t-1}) \sum_{k=0} \beta^k (m_{t+k} + h_{t+k}) \\ \times [v_{t+k}(m_{t+k} + h_{t+k}) - w_{t+k}]$$

Again we see that the aggregate excess of lifetime earnings over wages is equal to the discounted profits of an entrepreneurial firm producing the same quantities as the LMF. Since $m_{t-1} < L_{t-1}$, it also follows that $V_t - U_t$ for the members of a LMF whose labor force consists of both members and nonmembers is higher than $V_t - U_t$ for the workers in a LMF whose labor force consists of members only, provided that the two LMFs produce the same quantities.

Let us consider now the optimal choice of membership and wage-worker employment by the m_{t-1} members of the LMF. The first-order condition for m_t is

$$v_t(m_t + h_t) + (m_t + h_t) v_t'(m_t + h_t) = w_t \quad (36)$$

Hence, $m_t + h_t$, the labor force of the LMF, satisfies the condition (24) and it is determined again as being equal to the employment level of the PMF. But the above condition is at the same time precisely the first-order condition for h_t , i.e. we don't have another condition to determine how the optimal level of L_t is split between m_t and h_t . So, m_{t-1} members of the LMF are indifferent between selling $L_t - m_{t-1}$ shares, or hiring

wage workers of the same number, or selling $m_t - m_{t-1}$ shares and hiring $L_t - m_t$ workers at the same time. In other words, suppose there are two different labor managed firms, both embedded in a capitalistic economy. In one of them, the members refuse to hire wage laborers, do not sell their shares either, hence the production of this firm reflects the usual perversity. In the other, the labor force of the firm is a mixture of members and wage-laborers, and the output is expanded or contracted by hiring and firing nonmember workers. Now, suppose that, in order to overcome the perversity of the first LMF, we suggest that the first LMF sell its shares. But by making this suggestion we are only suggesting that the members of the first LMF adopt an organizational change which would leave them indifferent between what we are suggesting they adopt and what they could have done anyway but were refusing to do, namely, to hire nonmember workers. Hence, under the ideal conditions of our framework, share-selling as a device for curing the perversities of LMF seems to put the theory of labor management in an awkward situation.

Naturally, the neutrality of the existing members of the LMF between share-selling and wage-labor hiring disappears if anything which would violate the ideal conditions assumed above is introduced into our framework. A case in which wage-labor-hiring is superior to share-selling is when the outside workers will pay negative premiums introduced in Section I.5.

Under the presence of such premiums, the first-order condition for m_t will be

$$\begin{aligned} v_t(m_t + h_t) + (m_t + h_t) v_t'(m_t + h_t) \\ = w_t - (b_t - \beta b_{t+1}) > w_t \end{aligned} \quad (37)$$

provided that $|b_t| > \beta |b_{t+1}|$, whereas the condition for h_t is still (36), so that, if $m_t + h_t$ satisfies (37), we will have

$$v_t(m_t + h_t) + (m_t + h_t) v_t'(m_t + h_t) > w_t$$

meaning that workers in the LMF will be better off by increasing h_t and decreasing m_t by the same amount. But since the right-hand sides of the last two expressions are independent from either of m_t and h_t , m_t will be decreased to the lowest possible. This not only means that m_{t-1} members will decide not to sell any shares, but also it will be found to be optimal to reduce the number of the members by buying the shares of the exiting members according to the pricing rule (34). But the lowest possible number of members is of course one, so, under the presence of negative premiums, it is optimal for all m_{t-1} members that one of the members will buy the shares of all the others and hires wage workers. In other words, the LMF will turn into a PMF. (However, it may take time for this to happen because the payments to exiting for

their shares may be constrained by cash problems.) In some sense, this case generalizes the one studied by Ben-Ner (1984).

Needless to say, the opposite of the above case is the one when the outside workers will pay positive premiums. Then, share-selling will be superior to wage-labor-hiring, and the LMF will always adjust its labor force by increasing or decreasing the number of its members, never hiring nonmember workers.

I.6.2. Some Comparisons with Reality

Berman (1982) gives an account of the experience of the plywood manufacturing cooperatives in the Pacific north-western United States which typically have been selling their shares which are transferable. Our model assumes idealized conditions, hence cannot be asked to explain reality. Still, we think it may be fruitful to compare the model here with the reality of these plywood cooperatives (henceforth PCs). Comparisons in two respects seem to be meaningful: (i) Wage-labor hiring in PCs, (ii) Life cycle and extinction of the cooperative character of the PCs through share-selling.

I.6.2.1. Wage-Labor Hiring behavior of the PCs

PCs have employed non-member workers paid at going wage rates. Some of these employees are managers, or workers with

particular skills not possessed by any member, so, our model with single homogeneous labor does not capture these cases. Some other non-members, on the other hand, are employed temporarily during seasonal and/or cyclical peak demands. Berman notes that

"An important reason for membership in a worker cooperative is assurance of continuous employment, over business cycle variations as well as over the year. Cooperatives therefore wish to limit membership to the number that can reasonably be assured of remunerative employment in low demand periods. High demand is then met by hiring additional workers. Members afraid of unemployment in the cyclical plywood industry may tend to estimate too conservatively the basic labour need determining membership. A margin of non-member workers may then be maintained as a cushion against unemployment on a fairly permanent basis, when upward membership adjustment would be the theoretically appropriate response." (pp. 83-4) ¹²

Our model is one with certainty, but we can discuss the case of uncertainty about employment outside of the LMF by assuming risk-neutral workers and by replacing U_t in the pricing equations by its expected value EU_t . Let u_t be the probability of unemployment in period t , then expected income to be earned in period t outside of the LMF will be $(1 - u_t) w_t$, hence $EU_t = \sum_{k=0} \beta^k (1 - u_{t+k}) w_{t+k} < U_t$. The pricing equations then become

¹² Berman (1982) also points out the importance of adjustment through changing the working hours. Our framework does not capture this since we work with fixed work hours for each worker. However, as the quoted passage shows, there are limits for adjustment through working hours. So, even if we had flexible hours in our model, the discussion which follows would be justified.

$$p_t^* = (L_t v_t - h_t w_t)/m_t + \beta V_{t+1} - EU_t$$

and

$$(L_t v_t - h_t w_t)/m_t + S_t + \beta V_{t+1} = p_t^\# + EU_t$$

or

$$p_t^* = (L_t v_t - h_t w_t)/m_t + \beta V_{t+1} - U_t + (U_t - EU_t)$$

and

$$(L_t v_t - h_t w_t)/m_t + S_t + \beta V_{t+1} + (U_t - EU_t) = p_t^\# + U_t$$

so that the effect of positive probability of unemployment acts like a positive premium. Workers would pay this premium when entering the LMF since employment is certain in the LMF, and they would ask for a premium when they exit to compensate for the expected loss in the income due to the possibility of unemployment. Since other things in the model are not affected by these modifications, our analysis in Section I.6.1 is still valid, according to which the LMF must change the employment only by adjusting its membership size and not hire wage-laborers; during expansions it should increase the number of members by selling new shares and, during contractions, pay the compensation and let them exit.

But the quotation above shows that it just doesn't happen that way. Without necessarily intending to explain why actual PC's didn't behave that way, the difference between the model and the reality suggests that there are some adjustment costs. Costs associated with changing the number of wage workers may

be lower than those associated with changing the membership size. In particular, the LMF may face cash constraints and have difficulties in paying the fair price of the shares of the exiting workers, and costs of removing these constraints may be prohibitive.

The existence of adjustment costs of various types is further observed in the following:

"There is no market mechanism for selling cooperative shares (resales are handled on an individual basis) ... A new share issue would also incur the expense and delay of security registration procedures. Furthermore, member admission procedures which require individual evaluation and probationary periods for prospective new partners would be severely taxed." (Berman (1982), pp. 84-5)

These observations mean that there are substantial adjustment costs in increasing the membership size, as opposed to increasing h_t . Taken together, the observations on unemployment possibility and limits on upward adjustment of membership size roughly fit a model like the one described by the following relations:

$$\begin{aligned} \max V_t &= (L_t v_t - h_t w_t)/m_t + S_t + \beta V_{t+1} \\ (L_t v_t - h_t w_t)/m_t + S_t + \beta V_{t+1} + (U_t - EU_t) &= p_t^{\#} + U_t \\ m_s &\leq m_{s-1}, \quad s = t, \dots, \infty \end{aligned}$$

where m_{t-1} is given. By successive substitutions for V_{t+1} , V_{t+2} , ..., we obtain

$$V_t = (1/m_{t-1}) \sum_{s=t} \beta^{s-t} [L_s v_s(L_s) - h_s w_s + Dm_s(b_s - U_s)]$$

where $Dm_s = m_s - m_{s-1}$ and $b_s = U_s - EU_s > 0$. The maximization will be performed over h_s , Dm_s , $s = t, \dots, \infty$. The partial derivatives of V_t with respect to h_s and Dm_s are

$$\begin{aligned} \delta V_t / \delta h_s &= (1/m_{t-1}) \beta^{s-t} (L_s v_s'(L_s) + v_s - w_s) \\ \delta V_t / \delta Dm_s &= (1/m_{t-1}) \beta^{s-t} \{ b_s - U_s \\ &\quad + \sum_{k=0} \beta^k [L_{s+k} v_{s+k}'(L_{s+k}) + v_{s+k}] \} \end{aligned}$$

Choosing h_s to satisfy $L_s v_s' + v_s - w_s = 0$ leads to

$$\delta V_t / \delta Dm_s = (1/m_{t-1}) \beta^{s-t} b_s > 0$$

Since the constraints on m_s are rewritten as $Dm_s^2 \leq 0$, $s = t, \dots, \infty$, we will have $m_s = m_{s-1} = m_{t-1}$, $s = t, \dots, \infty$. That is, adjustment of employment is always performed by changing the number of hired workers whereas the membership size is kept constant.

I.6.2.2. Stylized Life-Cycle of the PCs

PCs have been organized by workers either (a.1) as an original company, or (a.2) by taking over a previously operating but failing plywood plant. They end their life either (b.1) due to capital shortages, or (b.2) being sold when "a substantial

number of shareholders approach retirement age simultaneously", or (b.3) losing their cooperative character "through the company buying up and retiring individual shares, until the longest lived few remain as owners of a company relying on hired labor". Since we have taken investment decisions exogeneously and prescribed the same investment and hence borrowing pattern to both the LMF and its twin PMF, there is nothing we can say about (b.1). Although our workers never retire, some account of (b.2) can be made by simulating retirement as a quit resulting from some qualitative difference between working in the LMF and working elsewhere such that the workers are willing to accept a negative premium (Section I.5) paid for their shares. Then a situation similar to that described at the end of Section I.6.1 can arise if a large number of members find themselves with this preference and arrange a large scale sale of shares to an outside investor avoiding transactions costs.¹³ Lastly, some account of the scenario (a.2) + (b.3) can be provided.

¹³ An outside investor will buy the LMF by paying each member of the LMF a price p_t such that

$$v_t + \beta V_{t+1} + b_t = p_t + \sum_{k=0} \beta^k w_{t+k}$$

and will replace them by wage laborers if

$$L_t(v_t - w_t - p_t) + \beta \sum_{k=0} \beta^k \pi_{t+k+1} > 0$$

Since $L_t(V_{t+1} - \sum_{k=0} \beta^k w_{t+k+1}) = \sum_{k=0} \beta^k \pi_{t+k+1}$ (see (23) Section I.3.5), the above condition is equivalent to $b_t < 0$.

(a.2) Suppose $\pi_1 < 0$, and the PMF goes bankrupt due to some restrictions on rescheduling of its liabilities, but $\sum_{k=1} \beta^k \pi_k > 0$, so that shares of a potential LMF are evaluated positively. Since the company has not yet been reorganized as an LMF, there does not exist a market for membership shares, but "the initial organizing group, [a] promoter, or failing-plantwork force" may provide the necessary market mechanism, and the PMF can be saved by reorganizing it as an LMF by using the payments made by joining workers for shares. This possibility is reflected -- see (28) in Section I.3.8 -- by the fact that, for the LMF, the surplus cash flow in period 1 is

$$\pi_1 + \beta (p_2 L_2 / L_1) (L_1 - L_0) = \pi_1 + \beta p_2 L_2$$

(interpreting $L_0 = 0$ as the number of worker/owners in the PMF) and can be positive, even if we do not take contributions from the wage bill into account.

(b.3) Consider a LMF whose labor force is a mixture of members and hired workers and which adjusts its employment by changing the number of nonmember workers only. Suppose this can be accounted for by the difference between the adjustment costs associated with changing the number of members and that of the hired workers, but producer cooperatives employing a mixture of members and hired workers have also been observed to buy

out shares of selling members and, by replacing them with non-members, to lose their cooperative character throughout time. How can we explain this? In our model, such a result can be mimicked by recognizing that there are actually two different prices for shares.

Suppose there are two types of member/workers in the LMF of numbers m^1_{t-1} and m^2_{t-1} with $m^1_{t-1} + m^2_{t-1} = m_{t-1}$, at the end of the period $t-1$, just before the decisions for the next period are made. Let Dm^1_t and Dm^2_t denote the changes in the number of these workers made by selling or buying back shares at the beginning of period t , so, $m_t = m_{t-1} + Dm^1_t + Dm^2_t$, and let

$$Y_t = (L_t v_t(L_t) - h_t w_t)/m_t + \beta V_{t+1}$$

The two types are the same in their productivity but differentiated in their preferences for working in the LMF or outside. Hence, for the contraction of both types of members, the pricing equation for the shares of Type 1 workers will be

$$Y_t + (p^1_t Dm^1_t + p^2_t Dm^2_t)/m_t + b^1_t = p^1_t + U_t$$

and for those of Type 2 workers,

$$Y_t + (p_t^1 Dm_t^1 + p_t^2 Dm_t^2)/m_t + b_t^2 = p_t^2 + U_t$$

Solving these two equations simultaneously for p_t^1 and p_t^2 ,

$$p_t^1 = (1/m_{t-1}) [(m_t - Dm_t^2) b_t^1 + Dm_t^2 b_t^2 + m_t(Y_t - U_t)]$$

$$p_t^2 = (1/m_{t-1}) [Dm_t^1 b_t^1 + (m_t - Dm_t^1) b_t^2 + m_t(Y_t - U_t)]$$

The workers remaining in the firm will seek to maximize

$$V_t = Y_t + (p_t^1 Dm_t^1 + p_t^2 Dm_t^2)/m_t$$

Substituting for prices,

$$V_t = (1/m_{t-1}) [L_t v_t(L_t) - h_t w_t + Dm_t^1 b_t + Dm_t^2 b_t^2 - (Dm_t^1 + Dm_t^2) U_t + \beta m_t V_{t+1}]$$

By successive substitutions for V_{t+1} , V_{t+2} , ..., the above equation can be written as

$$V_t = (1/m_{t-1}) \sum_{s=t}^{\infty} \beta^{s-t} [L_s v_s(L_s) - h_s w_s + Dm_s^1 b_s^1 + Dm_s^2 b_s^2 - (Dm_s^1 + Dm_s^2) U_s]$$

where the maximization is performed over h_s , Dm_s^1 , Dm_s^2 , $s = t, \dots, \infty$. Suppose there are prohibitive costs associated with

upward adjustment of membership. Instead of representing them by explicit functions, let us impose the constraints

$$0 \geq Dm^1_s \quad \text{and} \quad 0 \geq Dm^2_s, \quad s = t, \dots, \infty$$

and work with V_t as differentiable within this domain, using the left-hand derivatives with respect to Dm^1_s and Dm^2_s at $Dm^1_s = 0$ and/or $Dm^2_s = 0$. Moreover, we have the constraints

$$Dm^1_s \geq -m_{1s-1} \quad \text{and} \quad Dm^2_s \geq -m^2_{s-1}, \quad s = t, \dots, \infty$$

The partial derivatives of V_t are

$$\begin{aligned} \delta V_t / \delta h_s &= (1/m_{t-1}) \beta^{s-t} (L_s v_s' (L_s) + v_s - w_s) \\ \delta V_t / \delta Dm^1_s &= (1/m_{t-1}) \beta^{s-t} \{ b^1_s - U_s \\ &\quad + \sum_{k=0} \beta^k [L_{s+k} v_{s+k}' (L_{s+k}) + v_{s+k}] \} \\ \delta V_t / \delta Dm^2_s &= (1/m_{t-1}) \beta^{s-t} \{ b^2_s - U_s \\ &\quad + \sum_{k=0} \beta^k [L_{s+k} v_{s+k}' (L_{s+k}) + v_{s+k}] \} \end{aligned}$$

Let $b^1_s < 0$ and $b^2_s > 0$, for all $s \geq t$. Under these restrictions, it is tedious but straightforward to check that the solution characterized by

$$\begin{aligned} L_s v_s' (L_s) + v_s - w_s &= 0 \\ Dm^1_s &= -m^1_{s-1} \end{aligned}$$

$$Dm^2_s = 0, \quad s = t, \dots, \infty$$

satisfies the necessary Kuhn-Tucker conditions for a maximum (recall that $U_s = \sum_{k=0} \beta^k w_{s+k}$). With strictly concave $L v_s(L)$, this is also the unique solution since the constraints are convex. What it says is, in words, in each period t , keep m^2_{t-1} constant, buy the shares of all m^1_{t-1} members, and replace them by hiring non-member workers paying them at the wage rate w_t , as in the case analyzed in Section I.6.1.

Now suppose that, at the end of each production period and before the decisions for the following period are made, i.e., in terms of our time analogy, at the Friday evening, there occurs a shift in the preferences of the workers such that, when the decision period begins the next Saturday, we have

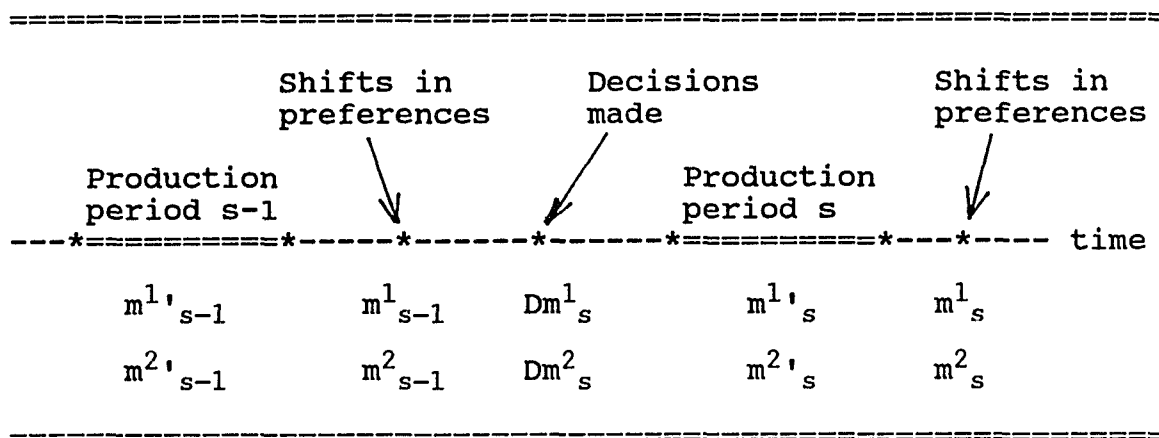
$$\begin{aligned} m^1_s &= m^1'_s + \alpha m^2'_s \\ m^2_s &= (1 - \alpha) m^2'_s, \quad s = t, \dots, \infty \end{aligned}$$

where $0 < \alpha < 1$, $m^1'_s$ is the number of Type 1 workers during the production period s , and $m^2'_s$ is the number of Type 2 workers during the production period s , so

$$m^1'_s = m^1_{s-1} + Dm^1_s$$

and

$$m^2_s = m^2_{s-1} + Dm^2_s, \quad s = t, \dots, \infty$$



The membership size will then decay at the rate α since the above solution implies $m_s = (1-\alpha)m_{s-1}$ because $m_s = m^1_s + m^2_s = 0 + m^2_{s-1} = (1-\alpha)m^2_{s-2} = (1-\alpha)m_{s-1}$ for all s , and the number of members who quit prior to the beginning of period s will be αm_{s-1} . After sufficient time passes to allow for a large number of such quits, the LMF will have turned into a PMF. Needless to say this is only an example showing how we can account for the extinction of labor-management even in a model with infinitely lived workers and certainty.¹⁴

¹⁴ Miyazaki (1984b) uses a model without explicit share-selling but risk-averse workers and uncertainty to explain the life-cycle of LMFs.

ESSAY II: PRICE UNCERTAINTY, HETEROGENEOUS WORKERS, AND EFFECTS OF AN INTERNAL FUTURES MARKET IN A LABOR-MANAGED ENTERPRISE

II.1. Introduction

This paper discusses some implications of the heterogeneity of workers in a labor-managed enterprise. The specific source of the heterogeneity is that the workers have different degrees of risk aversion. In order to be able to concentrate on the direct implications of this type of heterogeneity, workers are assumed to be homogeneous otherwise. The source of uncertainty is the fact that the price of the product of the enterprise is random, and the production decisions must be made before the price is realized and observed.

By the definition of the labor-managed enterprise, the workers are also the partners, i.e. they both own the firm and work in it. The set of all income figures which accrue to different workers as a result of this ownership is in effect a distribution of the total output produced among the workers. Suppose there exists a futures market in the firm in which workers can exchange their rights on the total output of the

firm. Given the distribution is set before the price of output is realized, there will in general be active trading in the futures market. In this paper, we consider a variety of distributions and examine the effects of the trading in the futures market. For all of these distributions, technological constraints, by assumption, will prevent each worker from choosing the number of hours worked individually. Given the chosen output level of the firm, each worker's labor input in hours is determined.

First, we characterize Pareto-optimal distributions of the total output among the workers. Without a futures market, there are many total output levels which correspond to Pareto-optimal distributions whereas there is a unique output level corresponding to all Pareto-optimal distributions if there exists a futures market. Moreover, this unique output level is one which would satisfy the optimality condition for the output of a firm owned by workers having homogeneous preferences such that the workers in the heterogeneous-worker firm are on the average as risk-averse as those in the homogenous-worker firm.

Noting that neither the implementation of nor the choice of one from among Pareto-optimal distributions is an easy matter in practical situations, we consider other distributions as well. We define two simple alternative setups

--organizational modes-- in which the workers choose their individually preferred quantities of output and working hours, and compare the output of the firm and the expected utility of the workers in these two different organizations of the firm.

The first mode, "the fixed-share firm", is defined by the rule that, whatever the total output might be, it will be distributed equally among the workers. Then each worker will prefer a different quantity to be produced. Since the output decision involves a unique quantity, the output in this firm is indeterminate in the sense that some decision-making mechanism not modelled in this paper is needed in order to determine the output level. (Below, we refer to this as the "decision-making problem".) The existence of a futures market does not make this problem disappear because the chosen total output level of the firm has an effect on the price in the futures market. This effect should be taken into account when each worker is deciding about the quantity of output which would maximize his utility when it coincided with the actual output of the enterprise. Consequently, each worker will again prefer a different quantity, and the indeterminacy of the output decision prevails. It is shown easily that the recognition of the relation between the futures price and the output level has a positive effect on the preferred output levels for all workers.

The second organizational mode, "the variable-share firm", allows for the shares of workers in the output of the firm to vary across workers --even though, again, everyone of them must work the same number of hours per period production period-- such that each worker chooses the number of units of output he will have as his share form the total output. The total output of the firm is then determined by a Nash type of equilibrium. Hence, in this case the indeterminacy of the output disappears, there is no need to add some decision-making process to this firm. We then compare the expected utility of the workers in both organizational modes. Since we don't have any device by which the output in the first type of firm is determined, we make comparisons over a range of output levels.

The basic result is one which is intuitively expected: Workers prefer either one of the firms depending on their degree of risk-aversion and the output of the first-type firm. Hence, with heterogeneous workers, we cannot dismiss anyone of these organizational modes due to considerations of efficiency. We provide the formalization of this idea under certain assumptions such as about the utility functions, risk-aversion parameters, etc.

The paper is organized as follows. Section II.2 introduces the basic setup. Section II.3 is devoted to the

equilibrium in the futures market. Section II.4 deals with the effects of the futures market on the Pareto-optimal distributions of the output of the firm among its workers. Section II.5 defines the first-type firm, examines the decision making problem, i.e. the indeterminacy of the output and the source of conflict between workers, in more detail. Section II.6 introduces the second-type firm. Section II.7 compares both firms with homogeneous-worker firms. In Section II.8, we have our central comparisons between both types of heterogeneous-worker firms. Section II.9 concludes.

II.2. Preliminaries

Let there be N workers in the enterprise who both own the firm and supply the labor. We will consider the firm under conditions such that neither the entry of a new partner nor the exit of a present partner is possible. Let Q denote the output of the firm, p the (random) price of the output net of average (=marginal) cost, and s_j , where $j = 1, 2, \dots, N$, the share of the j -th worker/partner in the firm ($0 < s_j < 1$ for all j , and $\sum_j s_j = 1$). Alternatively, define q_j as the number of units of output which worker j has a claim to, so $\sum_j q_j = Q$ and $q_j = s_j Q$. Then the income of the j -th worker will be given by

$$Y_j = s_j (p Q) = p s_j Q = p q_j \quad (1)$$

Time is divided into discrete periods. At the beginning of each period, the partners jointly decide on an output level Q , and stick to it during the period. At the end of the period, the production of Q is completed and the price of the product is observed in the market. This price cannot be anticipated with certainty at the time of the production decision, nor does the output level of the firm have any noticeable effect on the price, but the total output can be sold without constraint at the going price. In other words, the firm is a competitive one facing price uncertainty. Hence the decisions must be based on the density distribution of the price.

The workers are assumed to have the following expected utility function:

$$U_j(Y_j, L_j) = E Y_j - (\delta_j/2) \text{var}(Y_j) - h(L_j) \quad (2)$$

where U_j is the expected utility --the expectations operator in front of U_j is omitted for notational simplicity--, E is the expectations operator conditioned upon the information available at the time when the production decision is made, δ_j is the constant of absolute risk aversion of the j -th worker,

var denotes variance, and L_j is the labor which will be spent by worker j during the current production period. (The choice of utility function as one with a constant absolute --as opposed to relative-- risk aversion coefficient may be somewhat responsible for some of our results, we will return to this below.) The function $h(\cdot)$ represents the disutility of work and is assumed to be strictly convex (the full list of its formal properties is: $h(0) = 0$, $h(L) > 0$ for $L > 0$, and $h'(L), h''(L) > 0$ for all non-negative L). Let the production technology of the firm be linear in the amount of labor supplied, and assume away any capacity constraints. For simplicity, total labor which must be supplied in order to produce Q is equated to Q by proper choice of the units, and we have $\sum_j L_j = Q$.

Finally, we have some assumptions about the technology. Due to considerations of efficiency, or high costs of production scheduling, or complementarity among L_j , number of hours which must be worked by the j -th worker is uniquely determined by the quantity to be produced. In other words, there are technological constraints such that the output level Q necessitates a certain L_j besides a certain total quantity of labor. In particular, we will assume that

$$L_j = Q / N \tag{3}$$

i.e. everyone must work by an equal amount.

II.3. Futures Trading and Equilibrium in the Futures Market

Suppose the workers make the decision to produce Q with the distribution $(q_1, q_2, \dots, q_N)$, $\sum_j q_j = Q$. Substituting (1) and (3) into (2), the utility of the j -th worker will be given by

$$U_j = q_j E p - (\delta_j/2) q_j^2 \text{ var } p - h(Q/N) \quad (4)$$

Let there exist a future market in which shares are traded among workers. That is, we assume that there exist within the firm a futures market for claims to one-period output of the firm.

Let π be the price of a claim to one unit of output in this market, and assume that N is large enough so that effect on π of the trades of a single worker are small enough to be neglected. Let z_j denote the number of claims worker j sells or buys in the futures market ($z_j > 0$ if j sells). The income of j will be given by

$$Y_j = z_j \pi + (q_j - z_j) p = z_j (\pi - p) + q_j p \quad (5)$$

Then, given the production decision Q , the objective of j is to z_j such that

$$U_j = z_j(\pi - E_p) + q_j E_p - (\delta_j/2)(q_j - z_j)^2 \text{var } p - h(Q/N)$$

is maximized. Given π , the current price in the futures market, and Q , the planned output level by the firm, z_j will satisfy the following first-order condition:

$$\pi - E_p + \delta_j (q_j - z_j) \text{var } p = 0 \quad (6)$$

From (6), optimal hedge of j is given as

$$z_j = q_j - (E_p - \pi) / (\delta_j \text{var } p) \quad (7)$$

The equilibrium condition for the futures market is $\sum_j z_j = 0$. Summing (7) over j and equating to zero yields

$$Q = [(E_p - \pi) / \text{var } p] \sum_j (1/\delta_j) \quad (8)$$

which can also be written as

$$E_p - [\text{var } p / \sum_j (1/\delta_j)] Q = \pi \quad (9)$$

Equation (9) gives the price (π) - output combinations which

will keep the futures market in equilibrium. It is readily seen that higher output levels have a negative effect π . When choosing their preferred output levels, rationally behaving workers will take this effect into account.

Equation (7) gives the optimal z_j from the individual worker's point of view for any given (π, Q) combination, not necessarily for one which satisfies the futures market equilibrium. Let us now impose the equilibrium condition for the futures market. Substituting (9) into (7) and simplifying, optimal z_j is found as

$$z_j = q_j - (Q/N) \theta_j \quad (10)$$

where

$$\theta_j = (1/\delta_j) / (1/\delta) \quad (11)$$

with

$$1/\delta = (1/N) \sum_i (1/\delta_i) \quad (12)$$

Here, δ , the harmonic mean of the coefficients of absolute risk-aversion, is a measure of average risk-aversion, and θ_j measures the risk-aversion of j relative to other workers (if

j is more risk-averse than the average, i.e. if $\delta_j > \delta$, then $\theta_j < 1$, and $\delta_j < \delta$ implies $\theta_j > 1$). Equation (10) shows that, as the share of j gets larger (larger q_j) and/or he tends to be more risk-averse in comparison to the average (low θ_j), he tends to sell ($z_j > 0$) at the equilibrium of the futures market.

From (10) we see that optimal z_j at the futures market equilibrium is zero for all j --i.e. there is no active trading in the futures market-- if

$$q_j = (Q/N) \theta_j, \quad j = 1, 2, \dots, N$$

Given Q , this is the only distribution for which there is no active trading in the futures market. Below, we will refer to this distribution as the "no-trade" distribution. For all other distributions, there is active trading in the futures market, even if q_j belong to a Pareto-optimal distribution. It is apparent that the no-trade distribution is not unique because Q is not determined from the above equation, or the no-trade distribution is unique for a given value of the "scale factor" Q .

Some final comparison may shed light upon the role of the absolute risk-aversion utility function used in this paper.

Suppose that the output of the firm, Q , is equal to $(E p / \text{var } p) \Sigma_i (1/\delta_i)$, and, consequently, $\pi = 0$. With this futures price, short-sellers' income is always, i.e. for all realizations of p , less than the income of the buyers of shares, as can be seen from (5). Yet there is trade going on in the futures market, since optimal z_j under the equilibrium of the futures market is independent of π (see Equation (10)). The reason can be understood by considering the role of the z_j . For each given level of Q , the z_j is adjusted to maximize the utility component which is dependent only on income, i.e. $E Y_j - (1/2) \delta_j \text{var } Y_j$. Let q_j be the number of units of output claimed by j . Without the futures market but with variable shares, $Y_j = q_j p$, and $E Y_j - (1/2) \delta_j \text{var } Y_j$ will be maximized at $q_j = E p / (\delta_j \text{var } p)$, once the output decision Q is given. With the futures market but $\pi = 0$, $Y_j = (q_j - z_j) p$, and

$$z_j = q_j - E p / (\delta_j \text{var } p)$$

as it can also be seen by setting $\pi = 0$ in (7). Then z_j in the above expression corresponds to the difference between the actual q_j and the optimal q_j without the futures market -- optimal in the sense of maximizing the income-dependent utility component, or maximizing total utility subject to given Q . If $q_j > E p / (\delta_j \text{var } p)$, worker j will get rid of the difference even though, with $\pi = 0$, this exchange does not pay

him anything, so workers with $z_i < 0$ get it freely. The reason for this result is that, in the assumed utility function, excess of claims to output act like a notorious commodity. Each unit of excess claims embodies in itself some amount of price variance which must be avoided because of its disutility.

II.4. Pareto Optimal Allocations and the Futures Market

In this section, it will be convenient to adopt the notation $U_j(q)$ to denote $U_j(q_j, Q/N)$ where q is the vector $(q_1, q_2, \dots, q_N)$. Recall that $\sum_j q_j = Q$.

II.4.1 Necessary and Sufficient Conditions for a Pareto-optimal Distribution

A Pareto-optimal distribution is defined as a vector $q = (q_1, q_2, \dots, q_N)$ such that $U_i(q)$ can be increased for any i by switching to another distribution, but only at the cost of decreasing $U_j(q)$ at least for some j other than i . Hence, Pareto-optimal distributions can be seen as follows. Let a distribution $(q_1, \dots, q_N)$ be the solution to the problem

$$\begin{aligned}
& \text{maximize } U_i(q) & (P) \\
& \text{subject to} \\
& U_j(q) = \underline{U}_j, \quad j = 1, \dots, i-1, i+1, \dots, N
\end{aligned}$$

for any i . Here, $\underline{U}_j$ are arbitrary and different sets of $\underline{U}_j$ will lead to different solutions q . Pareto-optimal distributions form a subset of the solutions to the above problem such that $\max U_i$ can be increased only by decreasing $\underline{U}_j$ for some j . Therefore, to characterize the distributions which are Pareto-optimal, we can first consider the distributions which solve this problem, and then characterize among these distributions those which are such that decreasing $\underline{U}_j$ for any j increases $\underline{U}_i = \max U_i$. Throughout the rest of the Section, we can choose $i = 1$ without loss of generality.

The first-order conditions necessary for the solutions of the above problem can be expressed in two ways. Either, totally differentiating U_i with respect to q^j , for all i and j , we obtain

$$\begin{aligned}
dU_1 &= U_{11} dq_1 + U_{12} dq_2 + \dots + U_{1N} dq_N \\
dU_2 &= U_{21} dq_1 + U_{22} dq_2 + \dots + U_{2N} dq_N \\
&\dots \\
dU_N &= U_{N1} dq_1 + U_{N2} dq_2 + \dots + U_{NN} dq_N
\end{aligned} \tag{13}$$

where $U_{ik} = \delta U_i / \delta q_k$, and are evaluated at any distribution $(q_1, \dots, q_N)$ which satisfies $U_j = \underline{U}_j$, $j = 2, \dots, N$. Since we want to keep $U_j = \underline{U}_j$, we set $dU_2 = \dots = dU_N = 0$ in the above equations. Solving the last $N-1$ of them for $dq_2, \dots, dq_N$ yields

$$\begin{vmatrix} dq_2 \\ dq_3 \\ \cdot \\ dq_N \end{vmatrix} = \begin{vmatrix} U_{22} & U_{23} & \dots & U_{2N} \\ U_{32} & U_{32} & \dots & U_{3N} \\ \cdot & \cdot & \cdot & \cdot \\ U_{N2} & U_{N3} & \dots & U_{NN} \end{vmatrix}^{-1} \begin{vmatrix} -U_{21} \\ -U_{31} \\ \cdot \\ -U_{N1} \end{vmatrix} dq_1 \quad (14)$$

Substituting $dq_2, \dots, dq_N$ in

$$dU_1 = U_{11} dq_1 + U_{12} dq_2 + \dots + U_{1N} dq_N$$

yields

$$dU_1 = U_{11} - (U_{12} \dots U_{1N}) \begin{vmatrix} U_{22} & \dots & U_{2N} \\ \cdot & \cdot & \cdot \\ U_{N2} & \dots & U_{NN} \end{vmatrix}^{-1} \begin{vmatrix} U_{21} \\ \cdot \\ U_{N1} \end{vmatrix} dq_1 \quad (15)$$

The first-order condition for a maximum is then

$$U_{11} - (U_{12} \dots U_{1N}) \begin{vmatrix} U_{22} & \dots & U_{2N} \\ . & & . \\ U_{N2} & \dots & U_{NN} \end{vmatrix}^{-1} \begin{vmatrix} U_{21} \\ . \\ U_{N1} \end{vmatrix} = 0 \quad (16)$$

It can be verified that this condition implies

$$\det \begin{vmatrix} U_{11} & U_{12} & \dots & U_{1N} \\ U_{21} & U_{22} & \dots & U_{2N} \\ . & . & & . \\ U_{N1} & U_{N2} & \dots & U_{NN} \end{vmatrix} = 0$$

In fact, this has to be the case in order that at least some of the solutions to Problem (P) are Pareto-optimal. The reason can easily be seen as follows. Rewriting Equations (13) in matrix notation,

$$\begin{vmatrix} dU_1 \\ dU_2 \\ . \\ dU_N \end{vmatrix} = \begin{vmatrix} U_{11} & U_{12} & \dots & U_{1N} \\ U_{21} & U_{22} & \dots & U_{2N} \\ . & . & & . \\ U_{N1} & U_{N2} & \dots & U_{NN} \end{vmatrix} \begin{vmatrix} dq_1 \\ dq_2 \\ . \\ dq_N \end{vmatrix} \quad (17)$$

Now, if the matrix in the above equation has an inverse, then, at the given distribution $(q_1, q_2, \dots, q_N)$, one can find a vector of variations in the distribution, $(dq_1, dq_2, \dots,$

dq_N), for any vector of variations in the utilities of workers, $(dU_1, dU_2, \dots, dU_N)$. But this means that, in particular, one can realize a vector of variations in the utilities such that $dU_j > 0$ for at least one j whereas $dU_j \geq 0$ for other j --i.e. the utility of at least one worker can be increased without decreasing the those of the others-- by picking a vector of variations in the distribution such that

$$\begin{vmatrix} dq_1 \\ dq_2 \\ . \\ dq_N \end{vmatrix} = \begin{vmatrix} U_{11} & U_{12} & \dots & U_{1N} \\ U_{21} & U_{22} & \dots & U_{2N} \\ . & . & . & . \\ U_{N1} & U_{N2} & \dots & U_{NN} \end{vmatrix}^{-1} \begin{vmatrix} dU_1 \\ dU_2 \\ . \\ dU_N \end{vmatrix}$$

Hence $(q_1, q_2, \dots, q_N)$ can not be Pareto-optimal if the inverse of matrix of elements U_{ij} exists at $(q_1, q_2, \dots, q_N)$, i.e. if the determinant of the matrix is non-zero. It follows that Pareto-optimal distributions must satisfy the condition

$$\det (U_{ij}) = 0 \quad (18)$$

Another way of obtaining the first-order condition for a maximum would be, of course, to use the Lagrange-multiplier technique. Define the Lagrangian for the Problem (P)

$$Z = U_1 - \sum_{j=2}^N \phi_j (U_j - \underline{U}_j) \quad (19)$$

where ϕ_j are the Lagrange multipliers. Taking the derivatives of Z with respect to q_k and setting them equal to zero, we obtain

$$U_{1k} - \sum_{j=2}^N \phi_j U_{jk} = 0, \quad k = 1, \dots, N \quad (20)$$

Using the last $N-1$ of these equations to solve for the Lagrange multipliers and eliminating them from the first of these equations results again in (16).

Using (15) and (14), one can express the second-order condition for a maximum in a simple way. In order that (16) corresponds to a maximum,

$$U_{11} - (U_{12} \dots U_{1N}) \begin{vmatrix} U_{22} & \dots & U_{2N} \\ \cdot & & \cdot \\ U_{N2} & \dots & U_{NN} \end{vmatrix}^{-1} \begin{vmatrix} U_{21} \\ \cdot \\ U_{N1} \end{vmatrix}$$

must be decreasing in q_1 , as q_j , $j = 2, \dots, N$, change according to (14).

Finally, in order that the solutions to Problem (P) are Pareto-optimal, $\underline{U}_1 = \max U_1$ should be decreasing in $\underline{U}_j$ for all $j = 2, \dots, N$. Using the Lagrange multipliers, we can check whether this holds. According to our definition of Lagrange

multipliers in (19),

$$\delta \underline{U}_1 / \delta \underline{U}_j = \phi_j, \quad j = 2, \dots, N$$

If $\phi_j < 0$ for all $j > 2$, $\underline{U}_1$ can be increased only by decreasing $\underline{U}_j$ for some $j > 2$, and Pareto-optimal distributions can be characterized by examining the distributions which solve Problem (P).

II.4.2. Pareto-optimal Distributions under the Absence of a Futures Market

If there does not exist a futures market, then we have from (4)

$$U_{ii} = E p - \delta_i q_i \text{ var } p - (1/N) h'(Q/N)$$

$$U_{ij} = - (1/N) h'(Q/N)$$

For convenience, we will define

$$a_i = E p - \delta_i q_i \text{ var } p \tag{21}$$

and

$$b = (1/N) h'(Q/N) \tag{22}$$

PROPOSITION 1. In the absence of a futures market, a distribution $(q_1, \dots, q_N)$ is Pareto-optimal if and only if it satisfies

$$1 - (1/N) h'(Q/N) \sum [1/(E p - \delta_i q_i \text{ var } p)] = 0 \quad (23)$$

where a_j and b are given by (21) and (22), respectively.

Proof. The plan of the proof is as follows. First, we show that $a_j > b$ for all j . Then, Equ. (23) will be shown to be the first-order condition for a maximum of U_1 . Third, it will be shown that the second-order condition holds at a distribution which satisfies (23). Finally, the Pareto-optimality property of this distribution will be shown.

The matrix in (17) is of the form

$$\begin{vmatrix} a_1 - b & -b & -b & \dots & -b \\ -b & a_2 - b & -b & \dots & -b \\ -b & -b & a_3 - b & \dots & -b \\ \cdot & \cdot & \cdot & & \cdot \\ -b & -b & -b & \dots & a_N - b \end{vmatrix} \quad (24)$$

where a_j and b are given by (21) and (22), respectively. It is seen that, for Pareto-optimal distributions, we can not have

$a_j \leq b$ for any j , because any column of the above matrix which contains non-positive elements only implies that the distribution is not Pareto-optimal. Let $J = \{1, 2, \dots, N\}$, $J^+ = \{j \in J \mid a_j > b\}$ and $J^- = \{j \in J \mid a_j \leq b\}$. Pick

$$dq_j = 0 \quad \text{for } j \in J^+$$

$$dq_j \leq 0 \quad \text{for } j \in J^- \quad \text{with} \quad dq_j < 0 \text{ at least for some } j$$

Then

$$dU_i = -b \sum_{j \in J^-} dq_j, \quad \text{for } i \in J^+$$

$$dU_i = \sum_{j \in J^-} (\delta_{ij} a_j - b) dq_j, \quad \text{for } i \in J^-$$

where δ_{ij} is the Kronecker delta, and one sees that $dU_i \geq 0$ for all i with $dU_i > 0$ at least for some i . So, any distribution for which $a_j \leq b$ for some j can not be Pareto-optimal. Hence we have $a_j > b$ for all j at any Pareto-optimal distribution, and since $b > 0$, $a_j > 0$ for all j in some neighborhood of a Pareto-optimal distribution.

Now, let us derive the first-order condition for a maximum. Using the definitions of a_j and b , the last $N-1$ of Equ.s (13) can be written as

$$\begin{vmatrix} -b \\ -b \\ . \\ -b \end{vmatrix} dq_1 + \begin{vmatrix} a_2-b & -b & \dots & -b \\ -b & a_3-b & \dots & -b \\ . & . & & . \\ -b & -b & \dots & a_N-b \end{vmatrix} \begin{vmatrix} dq_2 \\ dq_3 \\ . \\ dq_N \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ . \\ 0 \end{vmatrix}$$

Since we have $a_j > 0$ for all j in the neighborhood of a Pareto-optimal distribution, we can rewrite the above equation as

$$\begin{vmatrix} -b/a_2 \\ -b/a_3 \\ . \\ -b/a_N \end{vmatrix} dq_1 + \begin{vmatrix} 1-b/a_2 & -b/a_2 & \dots & -b/a_2 \\ -b/a_3 & 1-b/a_3 & \dots & -b/a_3 \\ . & . & & . \\ -b/a_N & -b/a_N & \dots & 1-b/a_N \end{vmatrix} \begin{vmatrix} dq_2 \\ dq_3 \\ . \\ dq_N \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ . \\ 0 \end{vmatrix}$$

One can verify that the inverse of the matrix in the in the above equation is

$$(1/B) \begin{vmatrix} B + b/a_2 & b/a_2 & \dots & b/a_2 \\ b/a_3 & B + b/a_3 & \dots & b/a_3 \\ . & . & & . \\ b/a_N & b/a_N & \dots & B + b/a_N \end{vmatrix}$$

where $B = 1 - \sum_{j=2} (b/a_j)$. Hence, solving the above equations for $dq_2, \dots, dq_N$, we obtain

$$\begin{vmatrix} dq_2 \\ dq_3 \\ . \\ dq_N \end{vmatrix} = (1/B) \begin{vmatrix} (b/a_2) [B + \Sigma_{j=2} (b/a_j)] \\ (b/a_3) [B + \Sigma_{j=2} (b/a_j)] \\ \dots \\ (b/a_N) [B + \Sigma_{j=2} (b/a_j)] \end{vmatrix} = (1/B) \begin{vmatrix} b/a_2 \\ b/a_3 \\ . \\ b/a_N \end{vmatrix} dq_1$$

(25)

Now consider the first of Equ.s (13). Using the definitions of a_j and b , we have

$$dU_1 = (a_1 - b) dq_1 - b \Sigma_{j=2} dq_j$$

Substituting for dq_j ,

$$dU_1 = (a_1 - b) dq_1 - (b/B) \Sigma_{j=2} (b/a_j) dq_1$$

Using the definition of B and rearranging,

$$dU_1 = (a_1/B) [1 - \Sigma_{j=1} (b/a_j)] dq_1$$

It follows that the first-order condition for a maximum of U_1 is

$$1 - \Sigma_{j=1} (b/a_j) = 0 \quad (26)$$

which, upon substituting for a_j and b , results in (23).¹

Now let us show that the second-order condition is satisfied as well. (26) implies that

$$1 - \sum_{j=2} (b/a_j) = b/a_1 > 0$$

¹ Another way of deriving (23) is as follows. Consider condition (18). In the absence of a futures market, the determinant in (18) will be that of the matrix given by (24). Since $a_j > 0$ for all j , this determinant will be zero only if the determinant

$$\begin{vmatrix} 1 - b/a_1 & -b/a_1 & -b/a_1 & \dots & -b/a_1 \\ -b/a_2 & 1 - b/a_2 & -b/a_2 & \dots & -b/a_2 \\ -b/a_3 & -b/a_3 & 1 - b/a_3 & \dots & -b/a_3 \\ \dots & & & & \\ -b/a_N & -b/a_N & -b/a_N & \dots & 1 - b/a_N \end{vmatrix}$$

is zero. By adding rows 2, 3, ..., N to the first row, the last determinant can be written as

$$\begin{vmatrix} 1 - \sum b/a_j & 1 - \sum b/a_j & 1 - \sum b/a_j & \dots & 1 - \sum b/a_j \\ -b/a_2 & 1 - b/a_2 & -b/a_2 & \dots & -b/a_2 \\ -b/a_3 & -b/a_3 & 1 - b/a_3 & \dots & -b/a_3 \\ \dots & & & & \\ -b/a_N & -b/a_N & -b/a_N & \dots & 1 - b/a_N \end{vmatrix}$$

Subtracting the first column from all the other columns, we obtain

$$\begin{vmatrix} 1 - \sum b/a_j & 0 & 0 & \dots & 0 \\ -b/a_2 & 1 & 0 & \dots & 0 \\ -b/a_3 & 0 & 1 & \dots & 0 \\ \dots & & & & \\ -b/a_N & 0 & 0 & \dots & 1 \end{vmatrix}$$

It follows that

$$\det = 1 - \sum (b/a_j)$$

Hence we can write the condition (18) as

$$1 - \sum (b/a_j) = 0$$

Hence, around a distribution which satisfies (26) we have $B > 0$. So we see from (25) that q_j will increase for all $j = 2, \dots, N$ as q_1 increases. This, in turn, means that a_j will decrease and b/a_j will increase for all $j = 1, 2, \dots, N$ as q_1 increases. It follows that $1 - \sum_{j=1} (b/a_j)$ is decreasing in q_1 . Let q_1^P be the first component of the distribution vector q which satisfies (26) and $U_j(q) = \underline{U}_j$, for all $j > 1$. We have $1 - \sum_{j=1} (b/a_j) > 0$ for $q_1 < q_1^P$ and $1 - \sum_{j=1} (b/a_j) < 0$ for $q_1 > q_1^P$. Consequently, $dU_1/dq_1 > 0$ for $q_1 < q_1^P$ and $dU_1/dq_1 < 0$ for $q_1 > q_1^P$.

Finally, check the Pareto-optimality property of this maximum of U_1 . Lagrange multipliers for this problem satisfy in general Equ.s (20) the last $N-1$ of which in the absence of a futures market take the form

$$\begin{vmatrix} -b & a_2-b & -b & \dots & -b \\ -b & -b & a_3-b & \dots & -b \\ . & - & . & . & . \\ -b & -b & -b & \dots & a_N-b \end{vmatrix} \begin{vmatrix} \phi_2 \\ \phi_3 \\ . \\ \phi_N \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ . \\ 0 \end{vmatrix}$$

Solving for ϕ_j , we find

$$\phi_j = -b / (B a_j), \quad \text{for all } j = 2, \dots, N$$

or, using the fact that $B = b/a_1$ at the maximum of U_1 ,

$$\phi_j = -a_1 / a_j < 0, \text{ for all } j = 2, \dots, N$$

The sign of the multipliers shows that $\underline{U}_1$ can increase only if $\underline{U}_j$ decreases for any $j = 2, \dots, N$. QED

PROPOSITION 2. Under the absence of a futures market, the no-trade distribution $q_j = (Q/N) \theta_j$ with Q satisfying

$$E p - (Q/N) \delta \text{ var } p - h'(Q/N) = 0$$

is Pareto-optimal.

Proof. Consider again the condition (23) in Proposition (1), which implies that any distribution satisfying it is Pareto-optimal:

$$1 - (1/N) h'(Q/N) \sum [1/(E p - \delta_i q_i \text{ var } p)] = 0 \quad (23)$$

Substituting $q_j = (Q/N) \theta_j$ and noting that $\delta_j \theta_j = \delta$,

$$1 - (1/N) h'(Q/N) \sum [1/(E p - (Q/N) \delta \text{ var } p)] = 0$$

or

$$1 - h'(Q/N) [1/(E_p - (Q/N) \delta \text{ var } p)] = 0$$

which, upon multiplying through by $(E_p - (Q/N) \delta \text{ var } p)$, becomes the same as equation (23) in the above Proposition. QED

The following proposition formalizes some property of the Pareto-optimal distributions under the absence of a futures market which could be expected intuitively.

PROPOSITION 3. Under the absence of a futures market, the total output varies as the firm switches from one Pareto-optimal distribution to another.

Proof. Consider the condition (23). In general, it is not possible for $q_1, q_2, \dots, q_N$ to satisfy both (23) and $\sum q_j = \text{constant}$, at the same time. Hence, as we switch from one Pareto-optimal distribution $(q_1, \dots, q_N)$ to another, Q must change. Formally, totally differentiating the last equation with respect to Q and q_j , and multiplying through by N^2 , we obtain

$$\begin{aligned} & h''(Q/N) \sum [1/(E_p - \delta_i q_i \text{ var } p)] dQ \\ & + N h'(Q/N) \sum [(\delta_i \text{ var } p dq_i)/(E_p - \delta_i q_i \text{ var } p)^2] = 0 \end{aligned}$$

Substituting $dq_N = dQ - \sum^{N-1} dq_j$, and rearranging,

$$\begin{aligned} & \{ h''(Q/N) \Sigma [1/(Ep - \delta_i q_i \text{ var } p)] + (\delta_N \text{ var } p)/(Ep - \delta_N q_N \text{ var } p)^2 \} dQ \\ & + N h'(Q/N) \sum^{N-1} [(\delta_i \text{ var } p)/(Ep - \delta_i q_i \text{ var } p)^2 \\ & - (\delta_N \text{ var } p)/(Ep - \delta_N q_N \text{ var } p)^2] dq_i = 0 \quad (27) \end{aligned}$$

Since $h''(.) > 0$, and $Ep - \delta_i q_i \text{ var } p > 0$, for all i , the factor which multiplies dQ is positive. So, in order that $dQ = 0$ for any variation $(dq_1, \dots, dq_N)$, we must have, at any Pareto-optimal distribution $(q_1, \dots, q_N)$,

$$(\delta_i \text{ var } p)/(Ep - \delta_i q_i \text{ var } p)^2 = (\delta_N \text{ var } p)/(Ep - \delta_N q_N \text{ var } p)^2$$

for all $i = 1, \dots, N-1$, which does not hold in general. For example, pick the no-trade distribution which is Pareto-optimal by the previous Proposition, and let $dq_1 > 0$, $dq_2 = \dots = dq_{N-1} = 0$. The second term in (27) reduces to

$$\begin{aligned} & N h'(Q/N) [(\delta_1 \text{ var } p) / (Ep - (Q/N) \delta \text{ var } p)^2 \\ & - (\delta_N \text{ var } p) / (Ep - (Q/N) \delta \text{ var } p)^2] dq_1 \end{aligned}$$

and we have

$$\text{sign } dQ = - \text{sign } (\delta_1 - \delta_N)$$

Since we can have $\delta_1 >, =, < \delta_N$, dQ can be positive, zero, or negative. QED

II.4.3. Pareto-optimal Distributions under the Presence of a Futures Market

Substituting (5) into (2), we obtain,

$$U_j = q_j E p + z_j (\pi - E p) - (1/2) \delta_j (q_j - z_j)^2 \text{ var } p - h(Q/N)$$

Then, substituting (9) and (10) into the above equation, using the definition of θ_j , and rearranging,

$$\begin{aligned} U_j = & q_j (E p - (Q/N) \theta_j \delta_j \text{ var } p) \\ & + (1/2) (Q/N)^2 \theta_j^2 \delta_j \text{ var } p - h(Q/N) \end{aligned}$$

This last equation gives the utility of a worker in the case when there exists a futures market and the workers are aware of the trading possibilities in the futures market at the time when they decide on the distribution. Therefore, we have

$$\begin{aligned} U_{ii} = & E p - (Q/N) \theta_i \delta_i \text{ var } p - (q_i/N) \theta_i \delta_i \text{ var } p \\ & + (1/N) (Q/N) \theta_i^2 \delta_i \text{ var } p - (1/N) h'(Q/N) \end{aligned}$$

$$\begin{aligned}
U_{ij} &= - (q_i/N) \theta_i \delta_i \text{ var } p \\
&\quad + (1/N) (Q/N) \theta_i^2 \delta_i \text{ var } p - (1/N) h'(Q/N) \\
&= - (1/N) [q_i - (Q/N)\theta_i] \theta_i \delta_i \text{ var } p - (1/N) h'(Q/N)
\end{aligned}$$

Recall that, by definition of θ_j (Equ. (11)), $\theta_i \delta_i$ is constant across workers, and by (12), $\theta_i \delta_i = \delta$, where δ is the harmonic mean of δ_j . For convenience, let us define

$$a = E p - (Q/N) \delta \text{ var } p \quad (28)$$

and

$$b_i = (1/N) [q_i - (Q/N)\theta_i] \delta \text{ var } p + (1/N) h'(Q/N) \quad (29)$$

Throughout the rest of this section, we would like to restrict our discussion by the following assumptions:

ASSUMPTION 1. Let $\underline{\theta} = \max \{ \theta_j \}$ and $\bar{Q}/N = (E p / \delta \text{ var } p)$. (Note that at $\bar{Q}$, the price of the futures, π , is zero, and futures are traded freely.) We assume $h'(\bar{Q}/N) > \underline{\theta} E p$.

ASSUMPTION 2. Either $b_j > 0$ or $b_j < 0$, for all j , at $Q = Q^*$, where Q^* satisfies Equ. (30) given in Proposition (4) below.

Now we can state the main result of this section.

PROPOSITION 4. With the above assumptions, a distribution is Pareto-optimal under the existence of the futures market if and only if it satisfies

$$\sum_j q_j = Q^*$$

where Q^* satisfies

$$E p - (Q^*/N) \delta \text{ var } p - h'(Q^*/N) = 0 \quad (30)$$

Proof. We begin by showing that, under Assumption 1 we made above, $b_j > 0$ for all j at $Q = Q^*$. The rest of the plan of the proof is very similar to the proof of Proposition (1). First, we show that $a > b_j$ for all j . Then, Equ. (30) will be shown to be the first-order condition for a maximum of U_1 . Third, it will be shown that the second-order condition holds at a distribution which satisfies (30). Finally, the Pareto-optimality property of this distribution will be shown.

Since

$$b_j = (1/N) [q_j - (Q^*/N) \theta_j] \delta \text{ var } p + (1/N) h'(Q^*/N)$$

at $Q = Q^*$, we can write

$$b_j \geq (1/N) [- (Q^*/N) \otimes \delta \text{ var } p + h'(Q^*/N)]$$

or, using the definition of Q^* ,

$$b_j \geq (1/N) [- \otimes E p + h'(Q^*/N)] > 0$$

where the sign of the right-hand side of the above equation follows from our assumption above. It follows that b_j are positive for all j at $Q = Q^*$.

The matrix in (17) is of the form

$$\begin{vmatrix} a - b_1 & -b_1 & -b_1 & \dots & -b_1 \\ -b_2 & a - b_2 & -b_2 & \dots & -b_2 \\ -b_3 & -b_3 & a - b_3 & \dots & -b_3 \\ \dots & & & & \\ -b_N & -b_N & -b_N & \dots & a - b_N \end{vmatrix} \quad (31)$$

where a and b_j are given by (28) and (29), respectively. It is seen that, for Pareto-optimal distributions, we can not have $a \leq b_j$ for any j , because any column of the above matrix which contains non-positive elements only implies that the distribution is not Pareto-optimal. Let $J = \{1, 2, \dots, N\}$, $J+$

$= \{j \in J \mid a > b_j\}$ and $J^- = \{j \in J \mid a \leq b_j\}$. Pick

$$dq_j = 0 \quad \text{for } j \in J^+$$

$$dq_j \leq 0 \quad \text{for } j \in J^- \quad \text{with} \quad dq_j < 0 \quad \text{at least for some } j$$

Then

$$dU_i = - \sum_{j \in J^-} b_j dq_j, \quad \text{for } i \in J^+$$

$$dU_i = \sum_{j \in J^-} (\delta_{ij} a - b_j) dq_j, \quad \text{for } i \in J^-$$

where δ_{ij} is the Kronecker delta, and one sees that $dU_i \geq 0$ for all i with $dU_i > 0$ at least for some i . So, any distribution for which $a \leq b_j$ for some j can not be Pareto-optimal. Hence we have $a > b_j$ for all j at any Pareto-optimal distribution, and since $b_j > 0$, $a > 0$ in some neighborhood of a Pareto-optimal distribution.

Now, let us derive the first-order condition for a maximum. Using the definitions of a and b_j , the last $N-1$ of Equ.s (13) can be written as

$$\begin{vmatrix} -b_2 \\ -b_3 \\ . \\ -b_N \end{vmatrix} dq_1 + \begin{vmatrix} a-b_2 & -b_2 & \dots & -b_2 \\ -b_3 & a-b_3 & \dots & -b_3 \\ . & . & & . \\ -b_N & -b_N & \dots & a-b_N \end{vmatrix} \begin{vmatrix} dq_2 \\ dq_3 \\ . \\ dq_N \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ . \\ 0 \end{vmatrix}$$

One can verify that the inverse of the matrix in the above equation is

$$(1/aB) \begin{vmatrix} B' + b_2 & b_2 & \dots & b_2 \\ b_3 & B' + b_3 & \dots & b_3 \\ . & . & & . \\ b_N & b_N & \dots & B' + b_N \end{vmatrix}$$

where $B' = a - \sum_{j=2} b_j$. Hence, solving the above equations for $dq_2, \dots, dq_N$, we obtain

$$\begin{vmatrix} dq_2 \\ dq_3 \\ . \\ dq_N \end{vmatrix} = (1/aB) \begin{vmatrix} b_2 (B' + \sum_{j=2} b_j) \\ b_3 (B' + \sum_{j=2} b_j) \\ \dots \\ b_N (B' + \sum_{j=2} b_j) \end{vmatrix} = (1/B) \begin{vmatrix} b_2 \\ b_3 \\ . \\ b_N \end{vmatrix} dq_1 \quad (32)$$

Now consider the first of Equ.s (13). Using the definitions of a and b_j , we have

$$dU_1 = (a - b_1) dq_1 - b_1 \sum_{j=2} dq_j$$

Substituting for dq_j ,

$$dU_1 = (a - b_1) dq_1 - (b_1/B') \sum_{j=2} b_j dq_1$$

Using the definition of B' and rearranging,

$$dU_1 = (a/B') [a - \sum_{j=1} b_j] dq_1$$

It follows that the first-order condition for a maximum of U_1 is

$$a - \sum_{j=1} b_j = 0 \quad (33)$$

From the definition of b_j ,

$$\sum b_j = (1/N) \delta \text{ var } p \sum [q_j - (Q/N)\theta_j] + h'(Q/N)$$

By definition, $\sum \theta_j = N$, implying that the first term in the above equation is zero because

$$\begin{aligned} \sum [q_j - (Q/N)\theta_j] &= \sum q_j - (Q/N) \sum \theta_j \\ &= Q - (Q/N) N = 0 \end{aligned}$$

Hence,

$$\sum b_j = h'(Q/N)$$

Using this and the definition of a (Equ. (28)) in Equ. (33), we have

$$E_p - (Q/N) \delta \text{ var } p - h'(Q/N) = 0$$

which is Equation (30) given in the Proposition.²

Now let us show that the second-order condition is satisfied as well. First, note that, from (32) and the definition of B' ,

$$dQ = \sum_{j=1} dq_j = (a/B') dq_1$$

Hence Q increases or decreases according to the sign of B' as

² Another way of deriving (33) is as follows. Consider the condition (18). With the presence of futures market, the determinant in (18) will be that of the matrix given by (31). By adding rows 2, 3, ..., N to the first row, this determinant can be written as

$$\begin{vmatrix} a - \sum b_j & a - \sum b_j & a - \sum b_j & \dots & a - \sum b_j \\ -b_2 & a - b_2 & -b_2 & \dots & -b_2 \\ -b_3 & -b_3 & a - b_3 & \dots & -b_3 \\ \dots & & & & \\ -b_N & -b_N & -b_N & \dots & a - b_N \end{vmatrix}$$

Now subtract the first column from all the other columns. We obtain

$$\begin{vmatrix} a - \sum b_j & 0 & 0 & \dots & 0 \\ -b_2 & a & 0 & \dots & 0 \\ -b_3 & 0 & a & \dots & 0 \\ \dots & & & & \\ -b_N & 0 & 0 & \dots & a \end{vmatrix}$$

It follows that

$$\det = a^{N-1} (a - \sum b_j)$$

and condition (18) becomes $a - \sum b_j = 0$ since $a = 0$ does not correspond to Pareto-optimality.

q_1 increases. (33) implies that, at the maximum of U_1 ,

$$B' = a - \sum_{j=2} b_j = b_1$$

Hence, around a distribution which satisfies (33) we have $B' > 0$ or < 0 according to the sign of b_1 . Suppose $b_1 > 0$. Then $B' > 0$, and Q increases as q_1 increases. Since $a - \sum_{j=1} b_j$ is decreasing in Q --because of $h(\cdot)$ being a convex function--, we have $a - \sum_{j=1} b_j > 0$ for $q_1 < q_1^P$ and $a - \sum_{j=1} b_j < 0$ for $q_1 > q_1^P$ where q_1^P is the first component of the distribution vector q which satisfies (33) and $U_j(q) = \underline{U}_j$, for all $j > 1$. It follows from the signs of B' and $a - \sum_{j=1} b_j$ that $dU_1/dq_1 > 0$ for $q_1 < q_1^P$ and $dU_1/dq_1 < 0$ for $q_1 > q_1^P$. Suppose now that $b_1 < 0$. Then $B' < 0$, and Q decreases as q_1 increases. Since, again, $a - \sum_{j=1} b_j$ is decreasing in Q , we have $a - \sum_{j=1} b_j < 0$ for $q_1 < q_1^P$ and $a - \sum_{j=1} b_j > 0$ for $q_1 > q_1^P$. It follows from the signs of B' and $a - \sum_{j=1} b_j$ again that $dU_1/dq_1 > 0$ for $q_1 < q_1^P$ and $dU_1/dq_1 < 0$ for $q_1 > q_1^P$. Consequently, the second-order condition for a maximum is satisfied.

Finally, check the Pareto-optimality property of this maximum of U_1 . Lagrange multipliers for this problem satisfy in general Equ.s (20) the last $N-1$ of which in the presence of a futures market take the form

$$\begin{vmatrix} -b_1 \\ -b_1 \\ . \\ -b_1 \end{vmatrix} - \begin{vmatrix} a-b_2 & -b_3 & \dots & -b_N \\ -b_2 & a-b_3 & \dots & -b_N \\ . & . & & . \\ -b_2 & -b_3 & \dots & a-b_N \end{vmatrix} \begin{vmatrix} \phi_2 \\ \phi_3 \\ . \\ \phi_N \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ . \\ 0 \end{vmatrix}$$

One can verify that the inverse of the matrix in the in the above equation is

$$(1/aB) \begin{vmatrix} B' + b_2 & b_3 & \dots & b_N \\ b_2 & B' + b_3 & \dots & b_N \\ . & . & & . \\ b_2 & b_3 & \dots & B' + b_N \end{vmatrix}$$

where, again, $B' = a - \sum_{j=2} b_j$. Solving for ϕ_j , we find

$$\phi_j = -b_1/B', \quad j = 2, \dots, N$$

Since $B' = a - \sum_{j=2} b_j = b_1$ at the maximum of U_1 ,

$$\phi_j = -1 < 0, \quad \text{for all } j = 2, \dots, N$$

The sign of the multipliers shows that $\underline{U}_1$ can increase only if $\underline{U}_j$ decreases for any $j = 2, \dots, N$. QED

Proposition (4) above is interesting because of two

reasons: First, the firm with a futures market is in contrast with the firm without a futures market in that whereas Pareto-optimal distributions correspond to a multiplicity of total output levels in the latter, they correspond to a unique level in the former. Second, this unique output level depends on the risk-aversion characteristics of the workers in the firm only through an average measure of risk-aversion in the firm, which, as will be discussed below, makes it possible to talk about a "homogeneity-equivalent" of this heterogeneous-worker firm. In fact, Equ. (30) is nothing but the first-order condition for a firm whose members are homogeneous in their preferences and hence prefer the same output level (see Equ. (37) in Section II.5 and end of Section II.7). In other words, a heterogeneous-worker firm with a futures market operating at the Pareto-optimal output level is identical to a homogeneous-worker firm whose members' coefficient of absolute risk-aversion, δ , is the harmonic average of the coefficients of absolute risk-aversion, δ_j , of the members of the heterogeneous-worker firm.

PROPOSITION 5. Under the presence of a futures market, the no-trade distribution with Q satisfying

$$E p - (Q/N) \delta \text{ var } p - h'(Q/N) = 0 \quad (34)$$

is Pareto-optimal.

Proof. Follows immediately from the previous Proposition (4) which states that any distribution satisfying (34) is Pareto-optimal. (Note Assumption (1) is irrelevant here because a distribution satisfying (34) is Pareto-optimal regardless of whether there are other Pareto-optimal distributions which do not satisfy (34). Note also that the no-trade distribution satisfies Assumption (2) because, at the no-trade distribution, $b_j = (1/N) h'(Q/N) > 0$ for any Q .) QED

PROPOSITION 6. Let the total output level Q^F correspond to some distribution which is Pareto-optimal with a futures market, and Q^0 to some distribution which is Pareto-optimal under the absence of a futures market. We have Q^0 less than, equal to, or greater than Q^F .

Proof. The possibility that $Q^0 = Q^F$ follows from Propositions (2) and (5) which state that the no-trade distribution with $Q = Q^*$ satisfying (34) is Pareto-optimal with and without a futures market. The possibilities $Q^0 < Q^F$ and $Q^0 > Q^F$ follow from the fact that $Q^0 = Q^F$ at least for once, together with Proposition (3) which states that switching from any Pareto-optimal distribution to another may lead to a decrease or an increase in Q under the absence of a futures market, because, taken together, they imply that switching from the no-trade distribution to another Pareto-optimal distribution in the

firm without a futures market can lead to some $Q^0 < Q^* = Q^F$, or to $Q^0 > Q^* = Q^F$ (an example is provided at the end of the proof of Proposition (3)). QED

Before closing the Section, we would like to stress that, the Assumptions made above serve to prove the results of this Section under a somewhat restricted set of conditions. Their presence does not necessarily mean that the results will not survive after relaxing these assumptions. After relaxing Assumption 1, for example, additional Pareto-optimal distributions may arise but those described in Proposition (4) will not disappear. Effects of relaxing Assumption (2) are less clear; however, they will be limited to the necessity of a more careful specification of the set of Pareto-optimal distributions described in Proposition (4). Even if distributions with $b_j = 0$ for some j at $Q = Q^*$ turn out not to be Pareto-optimal, this would necessitate to modify Proposition (4) only to exclude such distributions. After the exclusion of such distributions, there will still exist infinitely many other distributions described by Proposition (4).

In the following sections we will consider distributions which, in general, will not be Pareto-optimal. The interest in these distributions arises because we don't have a satisfactory story for how the workers will bargain over which

Pareto-optimal distribution to choose, or how a Pareto-optimal distribution will be attained, to begin with. On the other hand, a fixed egalitarian rule of distribution (Section II.5) is very easy to implement, or a non-cooperative game between workers where everybody is deciding on his share (or his q_j) is easy to visualize (Section II.6).

II.5. The Fixed-Share Firm and Definition of the Decision-Making Problem

We define the fixed-share firm as an organizational mode of the labor-managed enterprise such that the distribution of income (output) takes place according to the rule $q_j = L_j$, i.e. one who works more has the proportionately greater share. Actually, at this step, what is fixed across j is only the ratio q_j/L_j .

Now one can think of two extreme cases. Either (1) the technology imposes no restrictions on the distribution of L_j over j , or (2) due to considerations of efficiency, or high costs of production scheduling, or complementarity among L_j , number of hours which must be worked by the j -th worker is uniquely determined by the quantity to be produced. It will be clear that, in the absence of such technological constraints,

there are no problems in decision making of the type mentioned in the Introduction (as long as s_j can be picked by j without any restriction). Formally, using the definition (1), we can rewrite the utility (2) of j as

$$U_j = q_j E p - (\delta_j/2) q_j^2 \text{ var } p - h(q_j) \quad (35)$$

Given the information available at the time of decision making about the distribution of the price, i.e. given the knowledge about $E p$ and $\text{var } p$, worker j 's expected utility is maximized at $q_j = \hat{q}_j$ which satisfies the first order condition

$$E p - \delta_j \hat{q}_j \text{ var } p - h'(\hat{q}_j) = 0 \quad (36)$$

By the properties of the function $h(\cdot)$, there is a unique $\hat{q}_j$ satisfying (36). So, worker j prefers to work $\hat{L}_j = \hat{q}_j$ hours in the present production period, and has (is issued by the firm) a claim to $\hat{q}_j$ units of the final output which will be simply $\sum_j \hat{q}_j$, the sum of the claims to output over all workers. Hence the output is also determined.

Consider now the presence of technological constraints such that the output level Q necessitates a certain L_j besides a certain total quantity of labor. Let, in particular, $L_j = Q/N$, i.e. everyone must work by an equal amount (henceforth,

the term fixed-share firm will refer to the one with this constraint). Together with the more-work-greater-share rule, this implies that $s_j = 1 / N$ (i.e., shares in the strict sense of the term also become fixed across workers), and worker j will prefer the enterprise produce a quantity $Q = Q_j$ which maximizes his utility given by (35) with $q_j = Q_j/N$. Then Q_j satisfies the first-order condition

$$E p - \delta_j (Q_j/N) \text{ var } p - h'(Q_j/N) = 0 \quad (37)$$

Now the question is: given N different preferred quantities Q_j , what will be the actual production decision, say Q^x , of the enterprise? In other words, how will the Q_j 's be translated into Q^x ? That question requires a decision making theory which we don't want to deal with here. Instead, we will consider the implications of the existence of a futures market for a fixed-share firm. According to Section II.3, in general, there should be active trading in the futures market. With fixed-shares, we have $q_j = Q / N$, and hence, the income of j will be given by

$$Y_j = z_j \pi + (Q/N - z_j) p = z_j (\pi - p) + (Q/N) p \quad (38)$$

Given Q and π , the optimal hedge of worker j is

$$z_j = Q/N - (E_p - \pi) / (\delta_j \text{ var } p) \quad (39)$$

Equation (39) gives the optimal z_j from the individual worker's point of view for any given (π, Q) combination, not necessarily for one which satisfies the futures market equilibrium. Substituting (39) into (38) and simplifying, the income of j for any realization of p is found as

$$Y_j = (Q/N) \pi + [(E_p - \pi) / (\delta_j \text{ var } p)] (p - \pi) \quad (40)$$

At the equilibrium condition for the futures market, optimal z_j is found as

$$z_j = (Q/N) (1 - \theta_j) \quad (41)$$

where, again, $\theta_j = (1/\delta_j) / [\Sigma_i (1/\delta_i) / N]$. Equation (41) shows that, in the fixed-share firm, if $\theta_j < 1$, we have $z_j > 0$ at the equilibrium of the futures market. In other words, if $1/\delta_j$ is lower than the average of $1/\delta_i$, i.e. if worker j is more risk-averse than the average, he will sell in the futures market. Conversely, if worker j is less risk-averse than the average ($\theta_j > 1$), he will buy ($z_j < 0$). Using (9) and (41) in (38), we have

$$Y_j = (Q/N) [\theta_j p + (1 - \theta_j) \pi] \quad (42)$$

.

If, at the end of the period, p happens to be equal to π , everybody has the same income. For p above π , short-sellers have lower income than others, and for p below π , short-sellers have higher income and enjoy the benefits of their positive hedge. It should be noticed once again that equations (39) and (40) correspond to worker j 's equilibrium choice z_j for any price (π) / output combination, futures market not necessarily being in equilibrium; whereas (41) and (42) correspond to (39) and (40) plus the equilibrium in the futures market. Furthermore, all four of the equations (39), (40), (41), and (42) are valid with worker j having chosen his hedge z_j , but not necessarily his labor time $L_j = Q / N$, optimally under the given π . In other words, (39), (40), (41), and (42) correspond to optimal choice of z_j , given any Q (chosen by j and all the other workers).

Let us now consider the preferred output level by worker j . Letting z_j , Q , and π vary, the total differential of worker j 's expected utility will be

$$\begin{aligned} dU_j = & [\pi - E p + \delta_j (Q/N - z_j) \text{ var } p] dz_j \\ & + (1/N) [E p - \delta_j (Q/N - z_j) \text{ var } p - h'(Q/N)] dQ \\ & + z_j d\pi \end{aligned}$$

As a result of the optimal choice of z_j by worker j , (6) will

be satisfied. Then, the first term in the above expression will drop, the second term will simplify, and we will have

$$dU_j = (1/N) [\pi - h'(Q/N)] dQ + z_j d\pi \quad (43)$$

Recall that workers are aware of the relation between π and Q via the futures market equilibrium condition, i.e. of (9). Hence, it will be recognized by worker j that³

$$d\pi = - [\text{var } p / \Sigma_i (1/\delta_i)] dQ \quad (44)$$

and z_j will be picked according to (41). Using all (9), (41), and (44) in (43) gives

$$\begin{aligned} dU_j = (1/N) \{ E_p - [\text{var } p / \Sigma_j (1/\delta_j)] Q - h'(Q/N) \} dQ \\ - (Q/N) (1 - \theta_j) [\text{var } p / \Sigma_i (1/\delta_i)] dQ \end{aligned}$$

or, since, using the definition of θ_j as $(1/\delta_j)/[\Sigma_i (1/\delta_i)/N]$, we have $\delta_j \theta_j / N = 1 / \Sigma_i (1/\delta_i)$,

³ On the other hand, if the workers ignore (9), they have to assume $d\pi = 0$ for a given π , and the best thing to happen to worker j is that Q increases until the expression in front of dQ in (43) drops to zero and $\pi - h'(Q/N) = 0$ is satisfied. So, the preferred output is independent of j , that is, the problem of indeterminacy of the output decision is resolved by the existence of the futures market thanks to the homogeneity of workers with respect to the disutility of work.

$$dU_j/dQ = (1/N) [E_p - (2-\theta_j) \theta_j \delta_j (Q/N) \text{ var } p - h'(Q/N)]$$

Now, assuming that the right-hand side of this expression is decreasing⁴ in Q , the optimal choice of Q will be given by the condition

$$E_p - (2 - \theta_j) \theta_j \delta_j (Q/N) \text{ var } p - h'(Q/N) = 0 \quad (45)$$

$\theta_j \delta_j$ is independent of j , but $(2 - \theta_j)$ is not. Hence the optimal choice of Q will differ across workers, and we have again the same kind of decision making problem as we had in the case of the firm without the futures market. This will be true even if the workers cannot discover (9) precisely but conjecture commonly that $E_p - \beta Q = \pi$, $\beta > 0$.

The following proposition is about the effect of introduction of the futures market on the preferred output levels.

PROPOSITION 7. If workers perceive (9) correctly, the preferred output level in the fixed-share firm with the futures market is higher than that in the firm without the futures market for all j except $\theta_j = 1$.

⁴ A sufficient condition would be that $h'(L)$ is convex, assuring the assumed property even with $-(2-\theta_j)\theta_j \delta_j > 0$.

Proof. Comparing equations (37) and (45), we see that Q in (45) for each j will be greater than Q_j in (37) if $(2 - \theta_j) \theta_j < 1$. But this is always the case since it is equivalent to $(1 - \theta_j)^2 > 0$ unless $\theta_j = 1$. QED

But this effect does not change the order of preferred output levels in the fixed-share firm with and without the futures market:

PROPOSITION 8. Let $\{\delta_1, \dots, \delta_N\}$ be fixed and $\delta = 1 / (1/N) \sum_j (1/\delta_j)$. Denote by Q_j' the preferred output level by j in the fixed-share firm without the futures market and by Q_j'' that with the futures market. Then, $Q_i' < Q_j'$ if and only if $Q_i'' < Q_j''$.

Proof. By the definition of θ_j , we have $\delta_i > \delta_j$ if and only if $\theta_i < \theta_j$. Then, from (37) we see that $Q_i' < Q_j'$ if and only if $\theta_i < \theta_j$. Totally differentiating (45) with respect to Q and θ , we obtain

$$\frac{d(Q_j/N)/d\theta_j}{(2 - \theta_j) \delta \text{ var } p + h''(Q_j/N)} = \frac{(Q_j/N) \delta \text{ var } p}{(2 - \theta_j) \delta \text{ var } p + h''(Q_j/N)} > 0$$

because of the second-order condition guaranteeing

$$(2 - \theta_j) \delta \text{ var } p + h''(Q_j/N) > 0.$$

But the above inequality implies that $Q_i'' < Q_j''$ if and only if $\theta_i < \theta_j$, from which the proposition follows. QED

We note that, taken together, these propositions imply that the output level preferred by the average risk-averse worker --i.e. the worker having $\delta_j = \delta$ -- which is the same in both firms --i.e. with and without the futures market-- is between the lowest preferred output level in the firm with the futures market and the highest preferred output level in the firm without the futures market, i.e., we have $\min \{ Q_j'' \} < Q_a < \max \{ Q_j' \}$, where $\delta_a = \delta$.

Some further comparisons between the firms with and without the futures market can be interesting. First, let $\delta = 1/(1/N)\Sigma_j(1/\delta_j) > 0$ and consider a worker who is extremely risk-averse, i.e. $\delta_j \rightarrow \infty$. Then in the firm without the futures market, this worker will prefer $Q_j \rightarrow 0$, but in that with the futures market he will prefer a positive output level, as we can see by substituting $\theta_j = 0$ and $\theta_j \delta_j = \delta$ in (45). Second, note that, with all $\delta_j > 0$, it is possible that $\theta_j = 2$ for some j (whereas we still have $\delta > 0$ on the average). But this implies that these workers will prefer an output level such that $E_p = h'(Q/N)$, which means, these workers will behave

under the existence of the futures market like risk-neutral workers by nature.

Third, consider the effect of the existence of a risk-neutral worker among the partners. Let all but the N -th worker be risk-averse. Since risk-neutrality implies $\delta_N = 0$, we will have $1/\delta = (1/N)\sum_j(1/\delta_j) = \infty$, hence $\delta = 1 / (1/N)\sum_j(1/\delta_j) = 0$, implying $\theta_j = 0$ for all j (except N for which $\theta_N = N$), and $\theta_j\delta_j = 0$ for all j (including N), regardless of the specific values of δ_j . It follows from (45) that all workers will prefer a Q such that $E_p = h'(Q/N)$. In other words, the presence of a risk-neutral worker makes the indeterminacy of the output decision disappear and makes the firm choose an output level which is identical to that of a homogeneous-worker firm with risk-neutral workers. However, this equilibrium of the firm is not necessarily stable in the sense that the worker/partners may prefer now to sell their labor elsewhere: All workers except the N -th one have $\theta_j = 0$, hence, $z_j = (Q/N)$, i.e. they sell their total claim to output to the N -th worker, so $Y_j = (Q/N) \pi$ and variance of the income is totally eliminated. However, the futures price is now each worker's earnings per unit time, and $E_p = h'(Q/N)$ implies that $\pi < h'(Q/N)$. It follows that, if the wage rate paid outside of the firm is equal to or larger than π , the partners would be better off by selling their labor in an auction market to the

point where the marginal disutility of labor is compensated by the marginal utility of income.

Finally, note that, although the distribution in the fixed-share firm is not Pareto-optimal in general, there is a (unique) Pareto-optimal distribution in the fixed-share firm with and without a futures market according to Propositions (1) and (4).

II.6. The Variable-Share Firm

We define the variable-share firm as one where workers are allowed to pick s_j (or q_j) freely, i.e. independent of the hours worked, L_j . Again, there are technological constraints $L_j = Q/N$. The term variable-share refers to the fact that q_j/L_j can now vary across j . Of course, this is also the case for Pareto-optimal distributions; the difference between the Pareto-optimal distributions and the variable-share firm is that the distribution in the variable-share firm will result from the choices of workers each of them acting on his behalf, i.e., the variable-share firm we have a noncooperative solution.

By letting q_j/L_j vary, as many degrees of freedom are

introduced as are necessary to make the decision problem discussed above disappear. There exist a unique output level on which the workers can agree. Let $Q_{-j} = Q - q_j$. Given Q_{-j} , worker j 's problem is to choose a q_j such that his utility

$$U_j = q_j E_p - (\delta_j/2) q_j^2 \text{ var } p - h((Q_{-j} + q_j)/N)$$

is maximized. The chosen q_j^* must satisfy

$$E_p - \delta_j q_j^* \text{ var } p - (1/N) h'((Q_{-j} + q_j^*)/N) = 0 \quad (46)$$

Rewriting this equation as

$$q_j^* = [E_p - (1/N) h'((Q_{-j} + q_j^*)/N)] / (\delta_j \text{ var } p) \quad (47)$$

summing over j , and rearranging, one obtains

$$(\text{var } p / \sum_i (1/\delta_i)) Q^* + (1/N) h'(Q^*/N) = E_p \quad (48)$$

where $Q^* = \sum_i q_i^*$.

PROPOSITION 9. Assume $h'(0) < E_p$. Then, for the variable-share firm, there exist a unique Nash equilibrium $\{q_1^*, q_2^*, \dots, q_N^*\}$ satisfying (46), (48), and $q_j^* > 0$ for all j .

Proof. Nash property is obvious from Q_j being taken as given in (46). By convexity of $h(\cdot)$ and since $h'(0) < E_p$, a positive Q^* is uniquely determined from (48), implying $(1/N)h'(Q^*/N) > E_p$. This, in turn, implies that q_j^* determined from (47) is positive and unique. QED

So, there is no disagreement among the partners about the output to be produced: It will be Q^* . However, this does not necessarily assure the desirability of the variable-share firm as opposed to the fixed-share firm. As Proposition (12) below will demonstrate, workers' preferences of the organizational mode depends on their degree of risk-aversion.

PROPOSITION 10. At the Nash equilibrium, there is no active trading in the futures market.

Proof. Solving for $(1/N) h'(Q^*/N)$ from (48), substituting into (47), rearranging, and using the definition of θ_j yields

$$q_j^* = (Q^*/N) \theta_j$$

for all j , which is the no-trade distribution. QED

II.7. Homogeneity-Equivalents

It will be useful to compare the equilibrium of a variable-

share firm with heterogeneous workers with that of a variable-share firm with homogeneous workers.

Let $Q^{\wedge}$ denote the output produced by a variable-share firm with homogeneous workers. The first-order condition for this firm is obtained simply by substituting $\delta_j = \delta$ for all j in equation (48):

$$(\text{var } p / (N/\delta)) Q^{\wedge} + (1/N) h'(Q^{\wedge}/N) = E p \quad (49)$$

Now we have

PROPOSITION 11. $Q^{\wedge} = (>, <) Q^*$ if and only if $1/\delta = (>, <) (1/N) \sum_j (1/\delta_j)$.

Proof. Follows from the comparison of Equations (48) and (49).

QED

In other words, the variable-share firm with heterogeneous workers will produce the same (a higher, a lower) quantity of output if its workers are on the average as risk-averse as (less risk-averse than, more risk-averse than) the workers of the firm with homogeneous workers. The relevant average is the harmonic mean of δ_j as seen from the right-hand side of the inequality given in the above Proposition. The firm with homogeneous workers having the constant of absolute risk

aversion $\delta = 1 / (1/N) \sum_j (1/\delta_j)$ can be termed the "homogeneity-equivalent" of the firm of Section II.6.

The fixed-share firm does not have a homogeneity-equivalent since we did not establish a unique output level for it⁵. On the other hand, it is easily seen that, if the homogeneity-equivalent of the variable-share firm were to be reorganized into the fixed-share mode, its output would drop. This is seen by substituting $Q_j = \bar{Q}$ and $\delta_j = \delta = 1 / (1/N) \sum_j (1/\delta_j)$, for all j , in Equation (37). We obtain

$$E_p - [\text{var } p / (1/\delta)] (\bar{Q}/N) - h'(\bar{Q}/N) = 0 \quad (50)$$

Comparing (50) with (49) we see that $\bar{Q} < Q^*$.

Finally, we have seen that the total output level corresponding to any Pareto-optimal distribution in the firm with a futures market depends on the average risk-aversion of the workers in the firm. Hence, the heterogeneous-worker firm choosing Pareto-optimal distributions has a homogeneity-equivalent. Comparing (30) with (37), we see that the Pareto-optimal distributions satisfy the first-order condition for a homogeneous worker firm with $\delta_j = \delta$ for all j .

⁵ However, if workers ignore (9), the fixed-share firm has the same homogeneity-equivalent as the variable-share firm, and it produces a lower level of output which is equal to $\bar{Q}$ in (50) below.

II.8. Comparison of Two Heterogeneous-Worker Firms

In this section, we will compare fixed- and variable-share firms in terms of the utility they yield to the workers. In the case of homogeneous workers, one expects intuitively that the fixed-share firm will yield higher utility, as confirmed by Proposition (12) below, and so, all workers will prefer the fixed-share organization. However, if the firms' partner-worker body consists of heterogeneous workers, then preferences between two types of organizational modes will differ across workers. Two crucial factors which determine a workers' preference of the fixed-share firm over the variable-share one are his degree of risk-aversion and the output level in the fixed-share firm. (Recall that the output level of the variable-share firm is determined but the one for the fixed-share firm is indeterminate in this model.) As the output level changes, a worker's preference may change.

To prepare for Proposition (12) below, which formalizes these points, let us express the utility levels of a worker in two different firms. Consider first the fixed-share firm with an internal futures market where the workers recognize the effect of output on the futures price. By substituting Equations (9) and (41) into (5) and then (5) into (2), and rearranging, the expected utility of worker j can be written as

$$U_j^F = (Q/N) [E p - (1/2) (Q/N)^2 (2 - \theta_j) \theta_j \delta_j \text{var } p] - h(Q/N)$$

Note that $\theta_j \delta_j = 1 / (1/N) \sum_j (1/\delta_j)$. Let us fix the workers of the firm, hence all δ_j , so $1/(1/N) \sum_j (1/\delta_j) = \delta$ will be a constant. Also, simplify notation by $Q/N = q$ (in other words, henceforth, q does not denote a distribution vector anymore). The expected utility can now be expressed as

$$U^F(q, \theta_j) = q E p - (1/2) q^2 (2 - \theta_j) \delta \text{var } p - h(q) \quad (51)$$

Consider now the variable-share firm. The expected utility for j is

$$U_j^V = q_j E p - (1/2) q_j^2 \delta_j \text{var } p - h(q^*) \quad (52)$$

where q_j satisfies (46) and $q^* = Q^*/N$ is given by (48). By solving (46) for q_j , substituting for $h'(q^*)/N$ from (48), and rearranging, one obtains $q_j = q^* \theta_j$. Substituting this into (52) above, and using the definition $\theta_j \delta_j = \delta$ again,

$$U^V(\theta_j) = q^* \theta_j E p - (1/2) q^{*2} \theta_j \delta \text{var } p - h(q^*) \quad (53)$$

Next, define

$$\theta(q) = \frac{q^2 \delta \text{ var } p - q E p + h(q) - h(q^*)}{(1/2) (q^2 + q^{*2}) \delta \text{ var } p - q^* E p} \quad (54)$$

We can now state:

PROPOSITION 12. Let $\{\delta_1, \dots, \delta_N\}$ be fixed and $\delta = 1 / (1/N) \sum_j (1/\delta_j)$. (a) At any given q chosen by the fixed-share firm, $U^V(\theta_j) > (=, <) U^F(q, \theta_j)$ if and only if $\theta_j > (=, <) \theta(q)$ for $q < q_0$ where

$$(1/2) (q_0^2 + q^{*2}) \delta \text{ var } p - q^* E p = 0$$

and $q_0 > q^*$.

(b) $\theta(q) > (=, <) 1$ as $U^F(q, 1) > (=, <) U^V(1)$ for $q < q_0$.

(c.1) $\theta(q^*) = 1$.

(c.2) If $U^V(1) > 0$, there exist some q_1 satisfying $\theta(q_1) = 1$ such that $0 < q_1 < q^*$.

Proof. (a) From (49) we have

$$q^* \delta \text{ var } p + (1/N) h'(q^*) - E p = 0$$

which can be written as

$$\begin{aligned} & (1/2) (q^2 + q^{*2}) \delta \text{ var } p - q^* E p \\ & + (1/2) (q^{*2} - q^2) \delta \text{ var } p + (1/N) h'(q^*) = 0 \end{aligned}$$

It follows from the definition of q_0 that

$$(1/2) (q^{*2} - q^2) \delta \text{ var } p + (1/N) h'(q^*) = 0$$

hence, $q_0 > q^*$. Moreover, it is clear that

$$(1/2) (q^{*2} - q^2) \delta \text{ var } p + (1/N) h'(q^*) > 0$$

for $q < q_0$ and hence

$$(1/2) (q^2 + q^{*2}) \delta \text{ var } p - q^* E p < 0 \quad (55)$$

for $q < q_0$. Comparing (51), (53) and (4), and using (55) proves Part (a).

(b) Follows directly from (54) and (55).

(c.1) Follows from Part (b) and the fact that $U^V(1) = U^F(q^*, 1)$, as can be verified using (51) and (53).

(c.2) $U^F(q, 1)$ is strictly concave in q , and, as we know from the comparison of (50) and (49), has its maximum at some $q^- < q^*$. Combining this with the fact that $U^V(1) = U^F(q^*, 1)$, it follows that $U^F(q^-, 1) > U^V(1)$, and from Part (b), that $\theta(q^-)$

> 1 . On the other hand, $U^F(0, 1) = 0 < U^V(1)$, and again from Part (b), we have $\theta(0) < 1$. It follows that $\theta(q_1) = 1$ such that $0 < q_1 < q^*$. QED

A number of remarks about this proposition follow. First, note that it embodies the inequality $U^F(q^*, 1) > U^V(1)$ which is nothing but the formal expression of the intuitively expected fact that in homogeneous-worker firms the fixed-share firm will be preferred to variable-share firm. Second, the Proposition formally expresses the fact that this statement can not be made in the case of heterogeneous-worker firms. For each level of output Q ($= Nq$) chosen by the fixed-share firm, some workers who are sufficiently less risk-averse --i.e. those with $\theta_j > \theta(q)$ -- will prefer the variable-share firm and those who are sufficiently more risk-averse --those with $\theta_j < \theta(q)$ -- will prefer the fixed-share firm. Furthermore, the critical value of the risk-aversion parameter $\theta(q)$ varies as output of the fixed-share firm varies, and it does not necessarily correspond to δ , the average. Third, but in particular, by combining Parts (a) and (c.1) of the Proposition, we find that, if the production level in the fixed-share firm is chosen to be Q^* , that of the variable-share firm, then $\theta(q^*)$ does correspond to δ , i.e. workers who are more risk-averse than average ($\theta_j < 1$) will prefer the fixed-share type of organizational mode and those who are less

risk averse than average ($\theta_j > 1$) will prefer the variable-share firm. Finally, we learn that there exist some range of output for output levels smaller than that of the variable-share firm, such that the critical value of the risk-aversion parameter is larger than one, implying that some workers who are less risk-averse than average --i.e. those with $1 < \theta_j < \theta(q)$ -- also prefer the fixed-share firm for some $Q < Q^*$. In all this, it is interesting to note that a worker's preference of organizational mode does not depend on his constant of absolute risk-aversion, but it depends on only his degree of risk-aversion relative to those of other workers in the firm.

The main message of this Proposition seems to be that the superiority of one of these firms cannot be established without modeling the decision-making process explicitly. The elements of this model as it is are not sufficient to establish a strict preference of variable-share over fixed-share firm, or vice versa. In particular, simply because we didn't model explicitly the decision making process in the fixed-share firm and the associated disutilities with it are not taken into account, the fact that there is no indeterminacy of the output decision in the variable-share firm does not play a role in the choice among both firms. On the other hand, it is not necessarily implied that explicitly modeling the decision-making process will lead to a strict choice. Introducing a decision-making process will determine an

output level for the fixed-share firm, but it is an open question whether at this output level all workers will prefer either one of these organizational modes, and whether the effect of the disutility associated with the decision-making process will only decrease the number of workers who would prefer the fixed-share firm at that output level.

II.9. Conclusions

The analysis of the futures market equilibrium shows that, in the case of a labor-managed firm facing price uncertainty, where the workers are heterogeneous with respect to risk-aversion, the workers will hold either excess or deficient claims to the output of the firm, whatever the distribution of total output among workers, except for a particular distribution. Hence, at the beginning of the production period, i.e. after the distribution is chosen and the output decision is taken, there will be active trading in the futures market. This implies that a futures market is likely to emerge within the firm naturally. Throughout the analysis, we compared situations with and without the futures market.

The presence of a futures market has an interesting effect on the characterization of the Pareto-optimal distributions. In the absence of a future market, a Pareto-

optimal distribution may correspond to one out of a many different total output levels. With a futures market, there is only one output level corresponding to all Pareto-optimal distributions. In addition, the heterogeneous-worker firm producing this output level has a definite homogeneous-worker counterpart.

In Section II.5, we examined a firm in which the distribution is determined by the rule that the output of the firm is distributed equally among the workers. In this "fixed-share" firm, with or without the futures market, each worker will in general prefer a different total output level be produced. When workers recognize the effect of the output of the firm through the equilibrium in the futures market on the futures price, each of them will prefer a higher total output than the one under the absence of the futures market. As a result of the lack of a decision-making mechanism, the output level of the fixed-share firm has been left as indeterminate and/or exogeneously determined throughout the paper.

We then defined an alternative organizational mode for the labor-managed firm where workers choose both the total output of the firm and their shares in the output freely in a noncooperative game. Comparing the output level of this "variable-share" firm with heterogeneous workers with the output of a fixed-share firm with homogeneous workers who have

the same degree of risk aversion as the average worker in the variable-share firm, we see that the variable-share firm produces more. The same is true for a comparison of the variable-share firm with a firm which has an internal futures market and is producing the Pareto-optimal output level, because the latter and a fixed-share firm with homogeneous workers produce the same quantity if the workers in the heterogeneous-worker firm have on the average the same degree of risk aversion as the workers in the homogeneous-worker firm. This, i.e. the fact that the variable-share firm produces more, is because, when choosing their shares optimally in the Nash game, workers expect a negligible increase in the disutility of labor but a strong decrease in the disutility of risk. Consequently, they are stuck with high levels of output in the Nash equilibrium. Therefore, fixed-share organization of labor firms or Pareto-optimal distributions seem to impede the output levels.

On the other hand, one could think that Nash equilibria are maybe not that plausible, because they cost the workers high disutility so that they may revert to fixed-share schemes. But this is precisely the concern of this paper: In the case of heterogeneous workers, the preference between two types of firms depends on, first, the output of the fixed-share firm, and second, the degree of absolute risk-aversion of a worker relative to other workers in the firm. It follows

that either one of the firms will be preferable for some workers, and the other will be preferred by the rest. In particular, there is a critical risk-aversion parameter which varies with the output of the fixed-share firm, and workers who are less risk-averse than indicated by that parameter will prefer the variable-share firm, and those more risk-averse than to that critical degree will prefer the fixed-share firm.

ESSAY III: THE SIMPLE PRICE DYNAMICS OF PORTFOLIO INSURANCE AND PROGRAM TRADING

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III.1. Introduction

The purpose of this paper is to present a simple model of stock market price adjustment and to discuss several aspects of the precipitous fall of equity prices in October 1987 in light of this model. The 508 point decline in the Dow Jones Industrial average on October 19, which easily surpassed the stock market crash of 1929, was an enormous shock. In the ensuing debate as to its causes, attention rapidly focussed on the role played by the stock index futures markets. In the past, futures markets often have been accused of causing an increase cash market price volatility.¹ Stock index futures have been viewed as increasing price swings particularly on

¹ Despite considerable theoretical and empirical evidence to the contrary, the view of futures trading as excessively speculative tends to persist and has been influential in shaping American regulation of futures markets. See Peck (1985) for a discussion of the history of futures trading and regulation. See Kamara (1982) for a survey of the literature on price volatility effects of futures trading.

the day that futures contracts matures.² Since it did not come on a futures expiry date, the collapse of October 1987 shifted the focus onto issues other than the actions of index arbitrage trading at maturity. In particular, the trading technique known as portfolio insurance had become widespread in 1986 and 1987 and was viewed by some as a likely cause of the stock market fall.

The collapse has brought under consideration a number of potentially far-reaching regulatory changes. Some of the measures suggested include: increased margins on stock index futures contracts, price limits, suppression of trading through automated trade execution techniques, trading halts, and banning some or all trading of stock index futures and options. Furthermore, it has been suggested that the entire framework for financial regulation be reformulated to place responsibility for regulating equity markets and stock index futures markets in a single body. Clearly in order to choose what, if any, regulatory changes are required there is a need to better understand the forces which combined to produce such an unprecedented increase in the rate of change in equity prices.

Our approach to these issues is to consider a model which

² This issue has been discussed from various perspectives by Edwards (1987), Grossman (1987), MacKinlay and Ramaswamy (1987), and Stoll (1987).

explicitly incorporates the interactions of the strategies of stock market and futures market participants. While the analysis is theoretical, it is thoroughly grounded in the actual trading techniques employed in the market. We postulate an explicit model of price adjustment in both the stock market and in the stock index futures market. In this framework we are able to distinguish price instability from normal, but perhaps fluctuating, price movements that lead to an eventual equilibrium. One distinctive feature of the model is that we are able to disentangle the separate influences of program trading and portfolio insurance.

The results of the analysis below are: (1) Portfolio insurance can combine with program trading to increase the magnitude of fluctuations around an equilibrium or possibly to render the equilibrium unstable, (2) if portfolio insurance is absent or slight, program trading increases the speed of convergence to an equilibrium, and (3) there are self-correcting features to the system which tend to reduce the amount of portfolio insurance trading and to limit the degree of price volatility, and (4) the payoffs to investors and market makers change as the system parameters move from stable to unstable regions, creating an incentive for these traders to alter their behavior and exacerbating the instability of the system.

The paper is organized as follows. Section III.2 presents the model. In section III.3 we analyze the price dynamics. Section III.4 is devoted to interpretation and policy implications.

III.2. The Model

Our approach is a classical analysis of the stability of equilibrium applied to a stock market economy.³ As such the model is designed to describe short-term price fluctuations around an equilibrium path. Price adjustments take place continuously. Investor expectations are fulfilled in equilibrium; however, since trading takes place outside equilibrium, not all profit opportunities are fully exploited.

There are two markets-- the market for equity shares (stocks) and the market for futures contracts on equity shares (futures). P_S and P_F are the prices of stocks and futures respectively. There are four types of participants in the market-- investors, portfolio insurers, program traders, and

³ The standard techniques of stability analysis are described in Arrow and Hahn (1971), ch.s 4-5.

market makers. We now discuss each of these types in turn.⁴

III.2.1 Investors

We include as investors any of the traditional market participants whose dominant concern is to buy when prices are low relative to their expectations of future earnings. Their behavior is summarized by the equations:

$$D_S^R = -\beta_S (P_S - P^*) + \epsilon_S$$

$$D_F^R = -\beta_F (P_F - P^*) + \epsilon_F$$

Here D_S^R and D_F^R are the flow demands for purchases of stocks and futures respectively. Negative flows represent sales. The β 's are positive constants. P^* is a constant that represents the investors' expectation both of the price of stock and of the price of futures. ϵ_S and ϵ_F are disturbance terms.

⁴ Our model is similar to the model of price adjustment of Beja and Goldman (1980) in that both allow prices to adjust continuously and trading to take place in disequilibrium. Consequently, the extensive discussion in Beja and Goldman (1980) justifying these features apply as well to our model. Notable differences between our analysis and that of Beja and Goldman (1980) are that: we have simultaneous trading in stocks and stock index futures, and we represent index arbitrage and portfolio insurance, trading techniques that have become widespread only recently. Another important difference is discussed below.

The interpretation of the first equation is that investor stock demand is composed of two parts. One part is a speculative demand. If the price of stocks is below the expected price, the investor buys shares. If above the expected price, he sells. The greater the deviation from the expected price the greater the purchases or sales. The second part is the disturbance term. It represents demand induced by factors outside of our model. It is a source of shocks to our system which allows us to initiate a disequilibrium in order to study subsequent price adjustment. The second equation has an analogous interpretation in terms of futures demand.

Investor expectations are taken as given. It will be seen below that $P_S = P_F = P^*$ is the equilibrium in the model so that investor expectations are fulfilled in equilibrium. There is no feed-back from price fluctuations into the formulation of expectations. In principle, it might seem preferable to allow for such feed-back. However, this would raise a number of delicate issues of modelling expectations formations. Given this, our view is that taking expectations as exogenous is preferable for the purposes of clearly understanding short-term disequilibrium price adjustments. Once one takes expectations as given exogenous, making the further assumption that they are constant is largely a matter of expositional convenience. Allowing them to move along an deterministic time path would complicate the analysis but

would not substantially change the conclusions.⁵

III.2.2. Portfolio Insurance

Portfolio insurers pursue a strategy which is aimed at setting a floor on the performance of a stock portfolio while allowing the portfolio to benefit in the event that stock prices rise. The traditional method of achieving a profit profile like this would be to buy a put option on a share so that the holder would be sure of selling the share at no worse than the exercise price on the option. The disadvantages of this approach for a portfolio of shares are that: (1) to maintain a constant performance floor, the shareholder would have to pay the premiums on an unending sequence of puts with fixed expiration dates and (2) insuring a performance floor for individual stocks is a costly way of assuring a performance

⁵ One may think of our investors as "value investors." Our formulation of investor behavior is similar to Beja and Goldman's "fundamental demand" (see Beja and Goldman (1980)). In addition, they posit a "trendist" demand. This results from investors' attempts to profit from disequilibrium price changes by forecasting price trends. The dynamic character of their model follows from the adaptive expectations approach they adopt. We could allow for this type of behavior in our model. The effect would be to complicate the analysis but would not change the main qualitative conclusions. Since our purpose is to concentrate on the interaction of portfolio insurance and program trading, we do not adopt this formulation. We should note, however, that in a model with "trendist" investment demand, price instability could arise from that source even in the absence of portfolio insurance and program trading.

floor for a portfolio since it does not take into account the self-insuring properties of a diversified portfolio. Portfolio insurance is a strategy which overcomes these two disadvantages.

The key to portfolio insurance is to notice that the standard theory of option pricing describes a hedging strategy which allows the writer of an option to off-set his risk by trading stocks. By trading stocks in a way that emulates the behavior of a put option, it is possible to reproduce the cash flows that would accrue to the holder of a put option. For example, the holder of a hundred shares whose price is 100 who wants to assure a maximum loss of 10 % would trade so as to imitate a put option on a hundred shares with an exercise price of 90. Depending upon price volatility, the time horizon, and other factors this may require selling 10 shares. Should the price fall to 98 the hedge would increase by some amount. In the limit as the price would fall to 0, the hedge would approach 100 shares and the total loss on the position would be 10. If, however, instead of falling the price had risen above 100, the hedge would have been reduced.

From this description we see that portfolio insurance avoids one of the disadvantages of buying puts; namely, no

premium is paid.⁶ In principle, the dynamic hedging strategy described above can be implemented by trading individual stocks. However, in order to overcome the second disadvantage it is necessary to have available a means of trading portfolios directly. The advent of stock index futures made such trading possible at a relatively low transaction cost. This was the basis for the sharp growth in the use of portfolio insurance from 1981 to 1987. However, should it become possible to trade portfolios of stocks in the stock markets themselves, the portfolio insurance technique could be applied there as well.

Mathematically, we represent the behavior of portfolio insurers by the equation:

$$D_F^I = \phi(P_S) dP_S/dt$$

where D_F^I are the insurers' purchases per unit time of stock index futures, $\phi(P_S)$ is a non-negative function of P_S , and dP_S/dt is the instantaneous change of stock prices. Notice according to this equation, insurers buy stock index futures when stock prices are rising and sell stock index futures when stock prices are falling. The function $\phi(.)$ varies with the level of stock prices according to a nonlinear relation

⁶ However, in an efficient market the expected loss on a synthetic put equals the corresponding premium on a real put.

implied by the Black-Scholes (see Black and Scholes (1973)) or other option pricing models.⁷ Its basic properties are:

$$\phi(0) = 0$$

$$\phi(P) > 0 \quad \text{for all } P \text{ such that } 0 < P < \infty$$

$$\phi(P) \rightarrow 0 \quad \text{as } P \rightarrow \infty$$

$$\phi'(P) > 0 \quad \text{for all } 0 < P < P_M$$

$$\phi'(P) < 0 \quad \text{for all } P > P_M$$

That is, starting from very low prices, the size of hedging purchases rises until a maximum is attained after which the rate of purchases declines, and approaches zero asymptotically.

The shape of $\phi(\cdot)$ can be justified as follows. Suppose the portfolio insurers' strategy is to replicate a put option for each share of stock held, and let h denote the number of shares held in a hedge portfolio replicating the appropriate put. ($h > 0$ represents a long position.) Then,

$$h = \delta P_p / \delta P_s$$

⁷ A variety of portfolio insurance trading strategies is described in Bookstaber and Langsam (1988). Portfolio insurance as we model it is a special case of dynamic strategies which Brennan and Solanki (1981) show are optimal for a certain class of risk preferences.

P_p is the price of the put. That is, the required hedge is the delta of the appropriate put. It follows that the hedge is short. Any change in P_S will require an adjustment in h such that

$$dh = (\delta h / \delta P_S) dP_S$$

or

$$dh/dt = (\delta h / \delta P_S) dP_S/dt$$

neglecting the direct effects on h of the change in time. It follows then that the change of the hedge per unit change of the stock price is the gamma of the appropriate put. From the theory of option pricing we know that the gamma of a put has the properties that we have assumed for $\phi(\cdot)$ (see Cox and Rubinstein (1985), p. 227). Abstracting from basis risk, the replicating portfolio can be implemented through futures sales, which results in the demand for futures given in the text. We emphasize that D_F^I is the flow demand for futures which gives the change in the hedge position.⁸

⁸ Our formulation assumes that portfolio insurers follow an option emulation strategy. The principal alternative to this is constant proportion portfolio insurance, which is described by Perold (1986), Black and Jones (1987), and Perold and Sharpe (1988). Let N_S be the number of shares held at any time in the insurer's portfolio. With continuous adjustments and suppressing direct time effects, CPPI leads to

The flow demand of stock insurers for stocks is taken to be zero:

$$D_S^I = 0.$$

This assumption is a useful simplification, which, historically at least, may have been realistic. As has been noted already, portfolio insurance type strategies can be implemented in share markets as well. We will discuss below how our conclusions might be altered if we were to change our specification to reflect this.

III.2.3. Program Trading

Program trading is a general term for a variety of strategies which arbitrage stocks and stock index futures. It is called

$$dN_S/dt = [N_0 (m-1) (P_S^{m-2}) / (P_0^{m-1})] dP_S/dt$$

where N_0 and P_0 refer to an initial point and m is a constant greater than unity. Note that this gives portfolio insurers flow demands that are proportional to dP_S/dt as in our text with the term in brackets corresponding to $\phi(\cdot)$. However the term in brackets is monotone in P_S . In particular, for $m > 2$, the number of shares purchased per unit change in price tends to infinity with the price level. Thus a CPPI rule implies different behavior than that assumed for $\phi(\cdot)$ in the text. This would imply different price dynamics as well, a point that we note below. If a CPPI rule were modified to limit the degree of leverage and the degree of short selling, the modified rule would behave much like option emulation portfolio insurance and would result in price dynamics similar to those we obtain.

program trading since the calculation of the precise amounts of the instruments bought and sold require sophisticated calculations carried out on computers. Generally, program traders buy futures and sell stocks when the difference between index of stock prices and futures price exceeds a positive margin and do the converse (buy stocks, sell futures) when the price spread falls below a negative margin. The larger the discrepancy between the stock index and the futures, the greater the amount of arbitrage trading that will be attracted. Thus the arbitrageurs' behavior may be represented by the following equations:

$$D_F^A = \tau (P_S - P_F)$$

$$D_S^A = - \tau (P_S - P_F)$$

where D_F^A and D_S^A are the flow demands of arbitrageurs for futures and stocks, respectively, and τ is a positive constant. According to this formulation, arbitrageurs buy (sell) futures and sell (buy) stocks whenever the price of stocks exceeds (falls below) the price of futures. Notice that these investment rules generate positive profits over any continuous path of prices terminating at $P_S = P_F$. We will see that this condition holds in equilibrium so that these decisions are consistent with optimizing behavior when the system converges to equilibrium.

III.2.4. Market Making

Our model is closed by the addition of market makers who set prices and stand ready to transact with other agents at those prices. Thus market makers absorb the net demand of other agents. The consequence of the activities of market makers is that the demands of other agents are fulfilled with transactions at prices that those agents take as given when they formulate their demands.

A basic characteristic of market making behavior is that the market maker will raise prices if he is a net seller and will lower if he is a net buyer. We represent the behavior of the market makers according to the following price adjustment rules:

$$dP_F/dt = \alpha_F (D_F^I + D_F^A + D_F^R)$$

$$dP_S/dt = \alpha_S (D_S^I + D_S^A + D_S^R)$$

where the dP/dt is the time derivative of P and the α 's are positive constants and represent the speeds of adjustment of prices. Market makers who behave according to these rules on average make no speculative gains or losses and simply earn

their profits from the bid/ask spread.⁹ Our assumption that prices adjust approximately continuously is supported both by empirical observation and by the theoretical recognition that instantaneous price adjustment is unlikely because of the demands it would place on calculation and information dissemination. See Beja and Hakansson (1977). In particular, Zabel (1981) shows that rules similar to these result from the

⁹ We do not explicitly model a positive bid/ask spread in our formulation. We can demonstrate that the price adjustment rules imply zero speculative profits for market makers as follows. Let $z(t)$ be the aggregate share sales by market makers. The price adjustment rule for stocks states $dP_S(t)/dt = \alpha_S z(t)$. Now the cash flows generated by the market makers' trades from 0 to T are

$$\begin{aligned} \int_0^T P_S(v) z(v) dv \\ &= (1/\alpha_S) \int_0^T P_S(v) [dP_S(v)/dv] dv \\ &= (1/\alpha_S) \int d[(1/2)P_S(v)^2] \\ &= (1/2\alpha_S) [P_S(T)^2 - P_S(0)^2] \end{aligned}$$

Consequently, if we consider any period (0, T) over which prices follow a complete cycle so that $P_S(0) = P_S(T)$, market makers receive zero profits ignoring profits from commissions or the bid/ask spread. This would be true in the particular case of a temporary deviation from the expected price, i.e. $P_S(0) = P_S(T) = P^*$. Note also that over such a period there would have been zero net change in the market makers' stock holding, since

$$0 = P_S(T) - P_S(0) = \alpha_S \int_0^T z(v) dv.$$

Similarly, deviations of the futures price from P^* will result in zero trading profits for futures market makers.

optimal management of inventories of a specialist who faces stationary flow demands.

The α 's can be interpreted as indices of the supply of market makers' services or of market liquidity. An increase in the supply of market makers will be reflected as a decrease in the amount of price adjustment required to process a given order imbalance, ie. as a fall in α . This is generally consistent with an inventory costs model of market making (see Stoll (1989)).

III.3. Price Dynamics

Combining the demand functions and price adjustment rules of the previous section we obtain a system of differential equations for prices:

$$dP_F/dt = \alpha_F [\phi(P_S) dP_S/dt + \tau P_S - (\tau + \beta_F) P_F + \beta_F P^*] \quad (1)$$

$$dP_S/dt = \alpha_S [-(\tau + \beta_S) P_S + \tau P_F + \beta_S P^*] \quad (2)$$

If we use equation (2) to substitute for dP_S/dt in (1), we obtain

$$dP_F/dt = \alpha_F [\theta(P_S) - \mu(P_S) P_F] \quad (3)$$

where

$$\Theta(P_S) = \tau P_S + \beta_F P^* - \alpha_S \phi(P_S) [(\tau + \beta_S) P_S - \beta_S P^*]$$

and

$$\mu(P_S) = \tau + \beta_F - \tau \alpha_S \phi(P_S)$$

Equations (2) and (3) constitute the fundamental system for studying the adjustment of prices to equilibrium values within our model.

By setting $dP_S/dt = dP_F/dt = 0$ in equations (1) and (2), we immediately see that the system has a unique equilibrium at (P^*, P^*) . That is, the equilibrium of the system coincides with the investors' expectation for stock prices which is as one would expect in this model. The more interesting issue is the behavior of the system out of equilibrium.

The standard tests for the local stability are the Routh-Hurwitz conditions on the Jacobian matrix:

$$J = \begin{vmatrix} \delta(dP_F/dt) / \delta P_F & \delta(dP_F/dt) / \delta P_S \\ \delta(dP_S/dt) / \delta P_F & \delta(dP_S/dt) / \delta P_S \end{vmatrix}$$

where the partial derivatives are evaluated at the equilibrium (P^*, P^*) . The necessary and sufficient conditions for local stability are then,

$$\text{tr } J < 0 \quad (4)$$

$$\det J > 0 \quad (5)$$

where tr denotes trace and $\det$ denotes determinant. Taking the partial derivatives of equations (2) and (3) we see the elements of J are:

$$\delta(dP_F/dt)/\delta P_F = -\alpha_F [\tau + \beta_F - \tau \alpha_S \phi(P_S)] \quad (6)$$

$$\begin{aligned} \delta(dP_F/dt)/\delta P_S &= \alpha_F \{ \tau - \alpha_S (\tau + \beta_S) \phi(P_S) \\ &\quad + \alpha_S \phi'(P_S) [\tau P_F - (\tau + \beta_S) P_S + \beta_S P^*] \} \end{aligned} \quad (7)$$

$$\delta(dP_S/dt)/\delta P_F = \alpha_S \tau \quad (8)$$

$$\delta(dP_S/dt)/\delta P_S = -\alpha_S (\tau + \beta_S) \quad (9)$$

Evaluating at the equilibrium and using equation (2) with $dP_S/dt = 0$, equation (7) reduces to

$$\delta(dP_F/dt)/\delta P_S = \alpha_F [\tau - \alpha_S (\tau + \beta_S) \phi(P^*)] \quad (10)$$

Using expressions (6), (8), (9), and (10) in J , we see,

$$\begin{aligned}\det J &= \alpha_F [\tau + \beta_F - \tau \alpha_S \phi(P^*)] \alpha_S (\tau + \beta_S) \\ &\quad - \alpha_S \tau \alpha_F [\tau - \alpha_S (\tau + \beta_S) \phi(P^*)] \\ &= \alpha_F \alpha_S [(\tau + \beta_F)(\tau + \beta_S) - \tau^2] > 0\end{aligned}$$

Consequently, (5) is always satisfied, and the stability of the system depends entirely on (4). (Moreover, as the positive sign of $\det J$ shows, the equilibrium will not be a saddle point that is having the property of being stable when approached from certain directions but not from others.) Now, using the results from above, we have

$$\operatorname{tr} J = -\alpha_F [\tau + \beta_F - \tau \alpha_S \phi(P^*)] - \alpha_S (\tau + \beta_S)$$

or

$$\operatorname{tr} J = \tau \alpha_S \alpha_F [\phi(P^*) - \phi^*]$$

where

$$\phi^* = (\tau + \beta_S)/(\tau \alpha_F) + (\tau + \beta_F)/(\tau \alpha_S) > 0$$

Now we can have $\operatorname{tr} J$ either positive or negative. If $\phi(P^*) < \phi^*$, we have $\operatorname{tr} J < 0$ and the system is stable within the

neighborhood of the equilibrium; whereas if $\phi(P^*) > \phi^*$, the system is locally unstable. Consequently, we see that if the level of portfolio insurance is high relative to the other parameters of the system, the result will be local instability.

We now study the dynamic behavior of the prices and the global stability of the system. By setting $dP_S/dt = 0$ in equation (2) we see the stock price is unchanging for P_F and P_S combinations such that

$$P_F = [(\tau + \beta_S)/\tau] P_S - (\beta_S P^*)/\tau \quad (11)$$

Geometrically this describes a straight line in (P_S, P_F) space with slope greater than unity. Above this line the stock price is rising; below it is falling.

Similarly, by setting $dP_F/dt = 0$ in equation (3) we see the futures price is unchanging if

$$P_F = \Gamma(P_S) = \Theta(P_S) / \mu(P_S) \quad (12)$$

where $\Theta(P_S)$ and $\mu(P_S)$ are as defined in equation (3). Again, this equation can be used to define regions of price change in (P_S, P_F) space. If $\mu(P_S)$ is positive the futures price will be

rising below the curve in (12), while if $\mu(P_S)$ negative the opposite will hold.

It is useful to consider the behavior of the system in three cases.

III.3.1. Case (1): Stable equilibrium, monotonic price behavior

Suppose that the function $\phi(P_S)$ is relatively flat so that its value at the equilibrium, $\phi(P^*)$, is quite small. In that case, equation (12) will be close to a straight line with positive slope less than unity. The price dynamics will then be as depicted in figure (1). Here, the (P_S, P_F) plane has been divided into four regions depending on the direction of price change. For example, I is the region where stock price is rising and futures price is falling.

It is clear from this diagram that the equilibrium is globally stable, that is, prices converge to the equilibrium for any possible initial value of (P_S, P_F) . For example suppose prices were initially in region IV. Both prices are below equilibrium values and are rising. Should prices move to the boundary of region I, the rate of change of the futures price drops to zero, and prices remain in IV. Conversely, a

movement to the border of region III would result in a zero rate of change of the stock price, and the movement ceases again to continue out of IV. Similarly, once in region II the prices would stay in it until attaining equilibrium. From an initial position in either I or III prices would eventually enter either region II or IV. We see then if the parameters of the system are such as depicted in figure (1), prices converge to equilibrium.

Furthermore, we see from figure (1) that in this case convergence takes a particularly simple form. If initially both prices are below their equilibrium values they will remain so until equilibrium is attained. That is, prices cannot overshoot. Conversely, if both prices are initially too high, they remain so until reaching equilibrium. If one price is too high and the other too low (eg. $P_F < P^* < P_S$) then at most one of the prices will overshoot at most one time. When price adjustment is of this form we say the system is monotonically convergent.

III.3.2. Case (2): Stable equilibrium, monotonic or oscillatory price behavior

Consider now a case with a higher $\phi(P^*)$ value such that

$$(\tau + \beta_F)/(\tau \alpha_S) < \phi(P^*) < \phi^*.$$

Because of the lefthand-side inequality we will have $\mu(P^*) < 0$, and the system will look like as in figure (2). The righthand -side inequality states that the equilibrium is stable. Now the prices may converge to the equilibrium monotonically or with oscillations (exhibiting spirals around the point (P^*, P^*)). But monotonic convergence in this case is slightly different than that in Case (1): If we start in figure (2) from a point H where both prices are too high, then it is possible that prices decrease so that they reach a point at which $P_S > P_F = P^*$. But then the equilibrium cannot be reached before P_S undershoots the equilibrium at least once: First both prices fall in the region IV, then stock price falls and futures price rises in VI and then they both begin to increase in II. Looking at the whole sequence beginning at point H, we see that it is possible that both prices undershoot the equilibrium before converging to it, even if the convergence is monotonic. For sufficiently higher levels of portfolio insurance, convergence will be oscillatory.

It is also worth considering the effect of relatively small changes in $\phi(P^*)$ on the oscillatory type of behavior separately. Restricting ourselves with the linear system, i.e. with the price movements taking place in a close neighborhood of the equilibrium, we can easily determine that, for larger

$\phi(P^*)$, the amplitude of the oscillations fades out less quickly, and the frequency of the oscillations becomes higher.¹⁰ In other words, as ϕ increases, the convergence to the equilibrium is retarded whereas the number of fluctuations per unit time around the equilibrium is increased (see figure (3)). Needless to say, this can be seen as an increased degree of volatility, although the equilibrium is stable in the technical sense of the word.

For expositional convenience, we postpone the discussion of the global stability of this case until subsection III.3.4.

III.3.3. Case (3): Unstable equilibrium

We now assume $\phi(P^*)$ is larger than ϕ^* so that the system is

¹⁰ For the linear system, the oscillatory convergent behavior of the stock price as a response to some initial disturbance is described by

$$P_S(t) = P^* + (A_1 \cos wt + A_2 \sin wt) \exp(-rt)$$

where A_1 and A_2 are arbitrary constants determined by the initial conditions,

$$r = (1/2) \tau \alpha_S \alpha_F [\phi^* - \phi(P^*)] > 0$$

is the damping coefficient, and

$$w = \sqrt{\det J - r^2}$$

is the (angular) frequency. Clearly, as $\phi(P^*)$ increases and approaches ϕ^* from below, r decreases and w increases-- no matter what the initial conditions are.

locally unstable. We will see that even in this case there are limits to the extent of instability. Specifically, prices will tend toward a regular, cyclical path.

Now, since $\phi^* > (\tau + \beta_F) / \tau \alpha_S$, we will definitely have $\phi(P^*) > (\tau + \beta_F) / \tau \alpha_S$, and hence $\mu(P^*) < 0$, and the system will be again as in figure (2),¹¹ except that the middle branch of the curve $\Gamma(P_S)$ will be rotated clockwise towards the line $dP_S/dt = 0$. Note that the rightmost branch of the curve (12) approaches the line $(\tau P_S + \beta_F P^*) / (\tau + \beta_F)$ asymptotically as $P_S \rightarrow \infty$.

Now consider figure (4). If we pick a point A on the leftmost branch of $\Gamma(P_S)$, a typical trajectory of prices passing through A will be like ABCD.¹² (Since the equilibrium is not stable, the trajectory cannot end up at the equilibrium instead of passing through C. Nor can the trajectory cross the rightmost branch of $\Gamma(P_S)$ because dP_F/dt

¹¹ We have drawn figure (2) on the assumption that all branches of the curve given by equation (12) are positively sloped everywhere. But the results of this section do not depend on the assumption that the curve (12) has nonnegative slope everywhere (see also footnote (14) below).

¹² It can be shown that no trajectory on the (P_S, P_F) plane can have a vertical or horizontal asymptote.

= 0 on it.) Assume that, at the point D, P_S is positive.¹³ Then, after the prices pass through the point D, they will both increase, but they will not be able to cross the leftmost branch of $\Gamma(P_S)$ --since $dP_F/dt = 0$ on it--, neither will they cross the line OB at a point lying higher than B --since that would mean for the trajectory to cut itself. It follows that we will end up at a point like B' which lies below B.

Next consider a trajectory beginning from Q which is close to the equilibrium O. Since the equilibrium is unstable, the prices must diverge from it along the trajectory through Q, but they cannot do so monotonically because that would mean to cut ABCD. Hence the trajectory through Q will be similar to ABCD, except that Q' lies above Q because of the instability of O.

So, considering the trajectories passing through points on the line segment OB, we see that some of them are contracting and some of them are expanding. One immediately realizes that there may be some other which neither contracts nor expands because its next point of intersection --such as B'-- with OB coincides with its previous point of intersection with OB, i.e. the trajectory is a closed curve. In fact, such

¹³ This simplifying assumption provides a condition which is sufficient but not necessary for the results of this section. See footnote (14).

a trajectory can be shown to exist. Pick a point Y on OB , define y as the distance $|OY|$; let Y' be the point at which a trajectory intersects with OB for a second time after it has passed through Y , and define δ as the distance $|YY'|$. Let δ associated with y be denoted by $\delta(y)$. The function $\delta(y)$ will be continuous in y because prices are defined everywhere on the positive quadrant of the (P_S, P_F) plane, and there passes a trajectory through every pair of prices. Furthermore, let δ be positive (negative) if Y' lies above (below) Y . For example, $\delta(q) > 0$ and $\delta(b) < 0$, where $q = |OQ|$ and $b = |OB|$. Now since $\delta(y)$ is continuous, there must be some y between q and b such that $\delta(y) = 0$, which means that Y coincides with Y' , and hence the trajectory is closed. Moreover, we can have $\delta(y) = 0$ on QB for an odd number of times, i.e. a multiplicity of closed trajectories is possible.

Note that if $\delta(y)$ is positive (negative), the trajectory passing through y is an expanding (contracting) spiral. Hence if $\delta(y)$ cuts the $y = 0$ axis from above at $y = y^*$, we have $\delta(y) > 0$ for $y < y^*$ and $\delta(y) < 0$ for $y > y^*$, i.e. the trajectory through $y < y^*$ will be expanding and the one through $y > y^*$ will be contracting, hence they both will approach the closed trajectory T passing through y^* (T will be stable). On the other hand, if $\delta(y)$ cuts the $y = 0$ axis from below, the converse will be true and trajectories around T will diverge from T (T will be unstable). Therefore, closed trajectories

are limiting cases to others.

It is clear from figure (4) that the motion of P_S (and that of P_F) is in the form of cycles around its equilibrium P^* . Expanding (contracting) trajectories on figure (3) correspond to price cycles with gradually increasing (decreasing) amplitudes, whereas closed trajectories mean prices will exhibit cycles of constant amplitude. Finally, if the closed trajectory is stable, so will the constant-amplitude cycle of P_S (and of P_F), i.e. if the initial deviation of the price from its equilibrium is small (large), the motion will develop in cycles with increasing (decreasing) amplitudes so that ultimately a steady state is reached when the price cycle neither blows up or fades out, and is referred to as a (stable) "limit cycle."

Note that, since $\delta(q) > 0$ and $\delta(b) < 0$, where $q < b$, there is a y at which $\delta(y)$ cuts the $y = 0$ axis from above, i.e. stable limit cycles exist for P_S and P_F . Moreover, if the limit cycle is unique, it must be stable.¹⁴

¹⁴ In this section, the existence and stability of limit cycles have been shown under the assumptions that (i) the curve in equation (12) is positively sloped everywhere, and (ii) among the trajectories which cut the leftmost branch of the curve (12), there exists one which cuts the line $dP_S/dt = 0$ for $P_S < P^*$ at a positive P_S , such as at point D in figure (3). These are sufficient but not necessary conditions and served only to simplify the discussion. For example, one can convert the system of first-order differential equations (2)

The existence of at least one stable limit cycle for the system means that there will tend to be bounds on the observed fluctuations of stock and futures prices. Thus in a sense there are self-correcting mechanisms in a system with portfolio insurance and program trading. The key to assuring this is that $\phi(P_S)$ approaches zero either as P_S approaches zero or rises to very high levels. Of course, it should be recognized that regular fluctuations between high and low prices may be perceived as instability. However, we have seen that the price behavior cannot be explosively unstable.

and (3) into a single second-order equation

$$d^2P_S/dt^2 + (\text{tr } J) dP_S/dt + (\det J) P_S = (\det J) P^*$$

or, by defining $x = P_S - P^*$ and $\sigma = \det J$,

$$d^2x/dt^2 + f(x) dx/dt + \sigma x = 0$$

where $f(x) = \tau \alpha_S \alpha_F [\phi^* - \phi(x + P^*)]$. Then, by extending $\phi(\cdot)$ with $\phi(P_S) = 0$ for $P_S < 0$, and following the method of Lienard and defining $y = dx/dt + F(x)$, where $F(x) =$

$$\int_0^x f(s) ds, \text{ one obtains}$$

$$y dy + \sigma x dx = F(x) dy$$

By integrating this equation along a trajectory between appropriately chosen points, and making use of some elementary facts about such integrals (see, for example, Bogoliubov and Mitropolsky (1961)), it is tedious but straightforward to show that, without imposing the restrictions (i) and (ii) above on curve (12) (hence on ϕ), there exists a closed trajectory on the x - y plane, hence a limit cycle for P_S . (Note that, since our $f(x)$ is not symmetric with respect to the $x = 0$ axis, and $F(x)$ is not odd, Levinson-Smith Theorem (see Levinson and Smith (1942)) is not applicable and the limit cycle can, but in general will not be unique.)

III.3.4. Global stability of all cases with a stable equilibrium

Case (2) above, of course, is not the only intermediate case one can think of, nor did we discuss the global stability of this case at all. One can classify all cases with a stable equilibrium in two groups: Either $\phi(P_S) < \phi^*$ for all P_S --such as in Case (1), or in some subcases of Case (2) above--, or $\phi(P_S) > \phi^*$ for some P_S , but not for P^* --such as in some other subcases of Case (2). One can show that local stability is always accompanied by global stability if $\phi(P_S) < \phi^*$ for all P_S . Otherwise we have two possibilities:¹⁵ Either there exists an even number of limit cycles, or there does not exist any limit cycle and the equilibrium is globally stable. If limit cycles exist, they are alternately stable and unstable, and the one which is closest to the equilibrium is unstable; in other words, if we begin at a point lying within the first closed trajectory in the (P_S, P_F) plane, we will converge to (P^*, P^*) .

Our findings may be summarized as follows: The equilibrium can be locally stable or unstable, depending on the relative magnitude of the stock insurance with respect to

¹⁵ Again, all these can be shown by methods such as the one described in footnote (14).

the other parameters of the system. If the system is locally stable, it either is globally stable (i.e. converges to the equilibrium from all points) or it has a stable limit cycle (i.e. from some initial points the prices converge to regular, repeated price fluctuations). If the system is locally unstable, it always has at least one stable limit cycle.

III.4. Interpretation

We have introduced a simple model of disequilibrium price dynamics in a stock market and a related stock index futures market. We have used this model to give an analytical characterization of price stability and instability in terms of the behavior of investors, portfolio insurers, program traders, and, implicitly through our price adjustment functions, market makers.

From the perspective of our model, we see that portfolio insurance and program trading in combination can exaggerate the amount of price change that would result from a given shock to investor demands. For in the absence of both ($\tau = \phi = 0$), the system would fall within case (1) of the previous section. The system would be globally stable and monotonically convergent. Once prices would be too high, they would fall smoothly toward equilibrium. Similarly, the rise

from excessively low prices also would be smooth. When portfolio insurance and program trading are large relative to investor demand (τ and ϕ large relative to β_S and β_F), the system may still be stable, but there can be overshooting. That is, if after a shock to investor demand prices are too high, the futures price would fall beyond the equilibrium level, carrying stock prices with it. Eventually the direction would reverse, and prices would overshoot to above the equilibrium. The process would continue until the amounts of over and undershooting would become small and the equilibrium approached. This kind of oscillatory pattern is likely to be perceived as heightened price volatility even if the system is stable. For very high levels of portfolio insurance and program trading, the price fluctuations would be so great as to render the system unstable.

Both portfolio insurance and program trading must be active in our model for the system to be unstable. The reason is that in their absence the only other trading is that of traditional investors whose trading forces prices in the direction of their expectations which change relatively infrequently. Had we allowed for trend-following demand as in Beja and Goldman (1980), we would have had a second possible source of instability.

There is an important difference in the ways that

portfolio insurance and program trading operate. In the absence of portfolio insurance, the system is stable for all values of the other parameters. In this case greater amounts of program trading (increases in τ) only serve to speed the convergence to equilibrium prices. That is, without portfolio insurance, program trading is stability-enhancing. However, once portfolio insurance would be sufficiently large, increases in τ become destabilizing.¹⁶

While our discussion has highlighted the role of portfolio insurance and program trading in determining price volatility, our model shows that the behavior of market makers plays a critical role as well. The more willing are market makers to absorb temporary imbalances in the demands of other traders the slower will be the speeds of price adjustment (ie. the lower will be α_S and α_F). Since the critical level of portfolio insurance (ϕ^*) is decreasing in α_S and α_F , we see that a fall in the level of participation of market makers can change the system from stable to unstable.

It is important to realize that these conclusions are based on our study of the local stability of the model, that is, on the behavior of prices close to their equilibrium

¹⁶ Formally, if $\phi(P^*) > (\alpha_F + \alpha_S) / (\alpha_F - \alpha_S)$, the coefficient of τ in the expression for $\text{tr } J$ is positive. Then for τ large, $\text{tr } J$ is positive, and the system is unstable.

values. While we have found that portfolio insurance can combine with program trading to induce overshooting and, possibly, local instability, we have seen that there is at least one powerful factor that tends to limit the degree of instability of the model. Specifically, as prices would range far from their equilibrium values, the amount of portfolio insurance trading would tend to diminish. This is because, at very high prices, the optimal hedges are effectively zero, while at very low prices, the insurers are essentially fully hedged. In these regions then, the price movement would be reversed eventually in the direction of the equilibrium.¹⁷

Our analysis assumed that the parameters of the model were constant over time. In fact, the behavior of agents and thus our behavioral parameters can change over time thereby changing the sorts of price dynamics we expect to observe. One possible cause of parameter changes follows from the fact that we have a disequilibrium model, implying that in general there must be some unexploited profit opportunity. Thus if the system is stable, the investors, who buy whenever prices are below the equilibrium level and sell when the opposite is true, realize positive profits. Consequently, for a stable

¹⁷ Formally, this was seen as a limit cycle. We stress that this result depends crucially on our assumption that ϕ tends to zero at very low and very high prices as is the case for option emulation type programs. To the extent that portfolio insurance does not tend to diminish at high prices, as in constant proportion portfolio insurance with no restriction on leverage, this result will not hold.

system there is a competitive incentive for greater investor participation which tends to reinforce the stability of the system. This incentive will be reflected in the levels of value investment (ie. in relatively elevated levels of the parameters β_S and β_F).¹⁸ On the other hand, if the system were unstable, prices will tend to cycle around equilibrium levels and there is no certainty that investors make positive profits. If they make losses then they are unbounded. Thus we see that the system may have the knife-edge property that, beyond a certain level, portfolio insurance combined with program trading tends to discourage value investors (depressing β_S and β_F) thus exacerbating the instability of the system for a given level of portfolio insurance. Similarly, it was seen that the behavior of market makers implied by our price adjustment rules results in zero profits on average if the system is stable.¹⁹ However, if prices diverge from equilibrium levels, these agents make losses. Consequently, if other parameter changes rendered the system unstable, market makers would have an incentive to reduce their participation thus increasing the α 's and exacerbating the instability.

¹⁸ If a sufficiently large group of investors anticipate that prices will converge to equilibrium levels the β 's will be arbitrarily large and the system will converge immediately. In this sense, there would be no problem of instability if the system were always in a state of rational expectations equilibrium.

¹⁹ See footnote (9).

Our framework is consistent with a wide variety of price patterns depending upon the parameters of the model including instability converging toward limit cycles, oscillatory stability and monotonically convergent stability. Consequently, only when combined with hypotheses about the parameters does the model yield empirical predictions. We now consider the implications of what are in our view several interesting hypotheses. We emphasize that the question of the validity of these hypotheses is quite distinct from that of the validity of the model. The reader who takes exception to our hypotheses may apply alternative hypotheses to our framework and obtain different predictions.

The most basic hypothesis is that since its introduction in the early 1980's and until late 1987 portfolio insurance grew relative to traditional stock market investment and market making. According to the model, the price volatility following a shock of given magnitude should have been increasing during this time as portfolio insurance increased.

A second hypothesis is that the participation of market makers will change over time due to changes in the regulations or because of changing credit conditions. The prediction of the model is that periods of reduced participation (ie. increased α_S and α_F) will tend to be associated with periods of heightened price volatility.

The third hypothesis is that parameter changes were tending to increase price volatility in 1987. Then the prediction is that at a certain point we should tend to see a reduction of investor and market-maker participation and thus a sharp increase in price volatility. This is an implication of the knife-edge property discussed above.

Finally, our model predicts that as portfolios under insurance programs approach either a fully invested position or a fully covered position, prices will have a tendency to reverse. This is the limit cycle property of the model.

The evidence from the stock and stock index futures markets, in particular the data assembled in the several studies of the crash, potentially can be used to test these hypotheses as well as others based on our model. We do not do formal testing here. However, it does appear to us that the evidence from this period is at least broadly consistent with the predictions given above.

As was observed by the Brady commission (see Brady et. al. (1988), p. 9) the popularity of portfolio insurance strategies had grown in 1986 and 1987 up to October. The volatility of daily changes of the major American stock market indices increased moderately during 1986 and the first nine

months of 1987 (see Edwards (1988)). This is consistent with the first hypothesis.

The most striking fact about the stock market was the enormous increase in volatility in October. This increase might have been due to some sharp increase in the variance of the underlying stochastic process (eg., macroeconomic variables) driving the market. This explanation may be correct; however, until one has an account of these factors, it is essentially ad hoc. With our model, we predict that a sharp increase in volatility should be seen at the transition point between stable and unstable parameter regions. Thus our model suggests an account for the primary fact that emerged from the crash. Note that, if this explanation is correct, by implication the volatility in 1987 was associated with instability and not mere oscillatory but stable price behavior.

The trading records of October 12-20 shed additional light on our hypotheses. (See Gammill and Marsh (1988), Tables 1 and 2.) During this time portfolio insurers were heavy net sellers of stocks and stock index futures. Furthermore, until October 20 program traders' ("arbitragers and substituters") purchases of futures were roughly equal to their sales of stocks as suggested by our formulation. More significantly, while portfolio insurers and others increased their volume

from the 16th to the 19th and 20th, that of exchange market-makers declined. Thus during this time there may have been an increase in the α 's, thus exacerbating price volatility (consistent with the second hypothesis above). Furthermore, there is considerable evidence that many speculatively oriented trades anticipated the behavior of portfolio insurers (consistent with the knife-edge hypothesis). The rebound in the market on the 20th may have been encouraged by the anticipation that most portfolio insurance programs were approaching full cover so that selling would abate, consistent with the limit cycle property.²⁰

While our model appears to be broadly consistent with some of the principal facts that have emerged from the study of stock price volatility in October 1987 and its aftermath, we emphasize that this constitutes neither a formal test of the model nor a complete account of the data. Potentially, our model could be used as the framework for empirical

²⁰ Some of the evidence emerging from the crash would appear to be superficially inconsistent with our model. In particular, on October 20 it was observed that some portfolio insurers were selling futures while reported prices were rising. In part, this may be due to the time lags between decisions to trade and trade reporting. More importantly, on that day and on the previous afternoon the continuity of the market was disrupted, violating one of the premises of portfolio insurance strategies and of our model. As explained by Rubinstein (1988) uncertainty about the cost of transacting in these market conditions led some portfolio insurers to suspend their sales temporarily, in effect taking a view that the reported price did not reflect the behavior of the true, but unobservable, price.

analysis of price relations. However, the estimation of the parameters of the model must await future research.

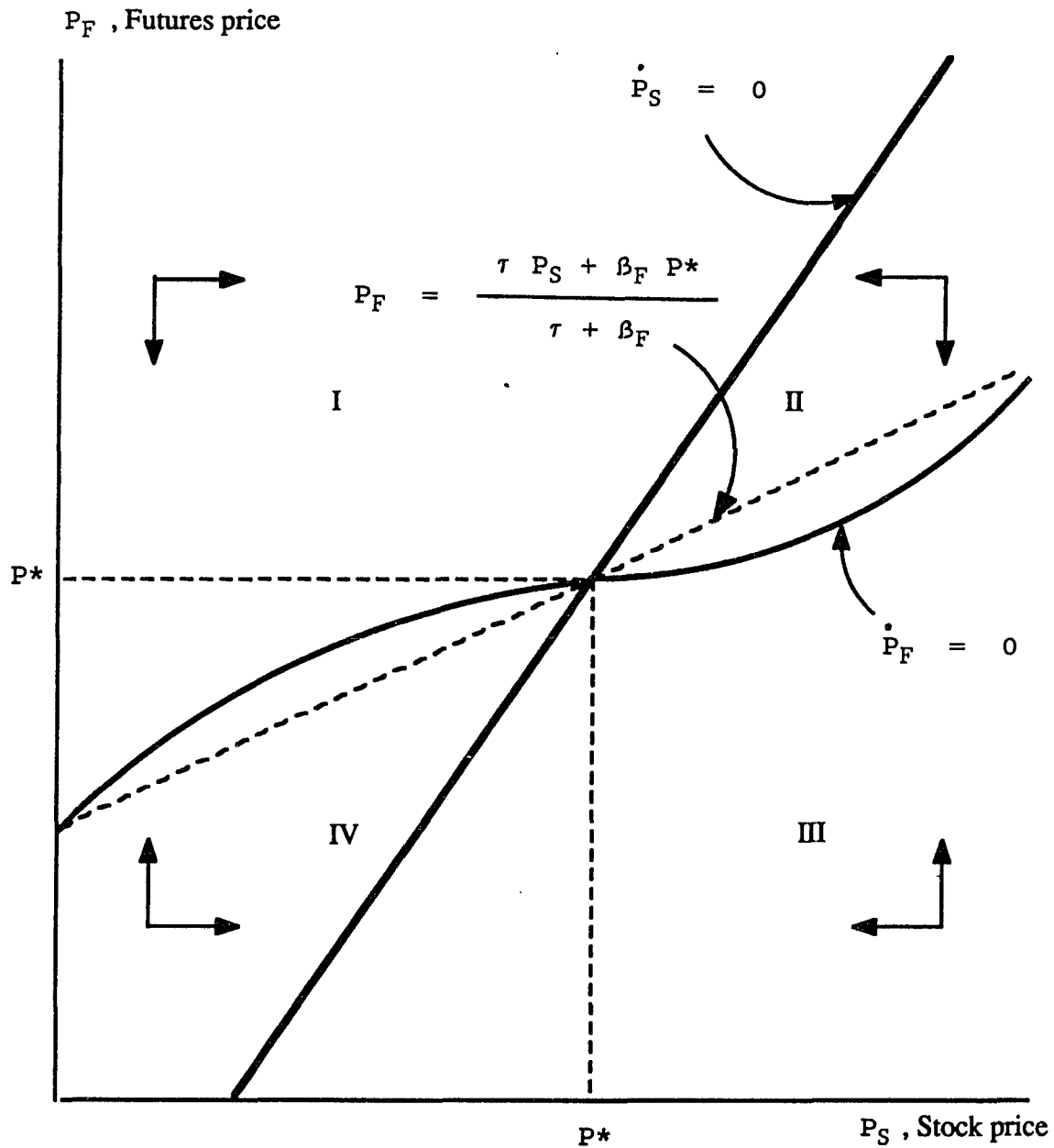
Our results should not be misconstrued as demonstrating that the stock index futures market destabilizes the stock market. Instability occurs in our system when the amount of portfolio insurance is large relative to the amount of expectations-based investment. Thus it is a by-product of a trading strategy. Our model analyzes this strategy in the context of futures markets. However, it is clear that, were portfolio insurance to take place in the share market, we still would find oscillations and instability for sufficiently high levels of portfolio insurance.²¹ Such a respecification of the model would be a more accurate reflection of reality should there emerge in the stock market a low-cost means of transacting portfolios (as opposed to individual shares).

Our positive economic analysis does not in itself allow us to recommend how, if at all, the regulation of stock and futures markets should be changed. However, it does support the view that high levels of portfolio insurance could have

²¹ Kim and Markowitz (1988) carry out simulations of a stock market without futures trading in which some agents follow a constant proportional portfolio insurance strategy. When the proportion of agents following this strategy is sufficiently great, the system is unstable.

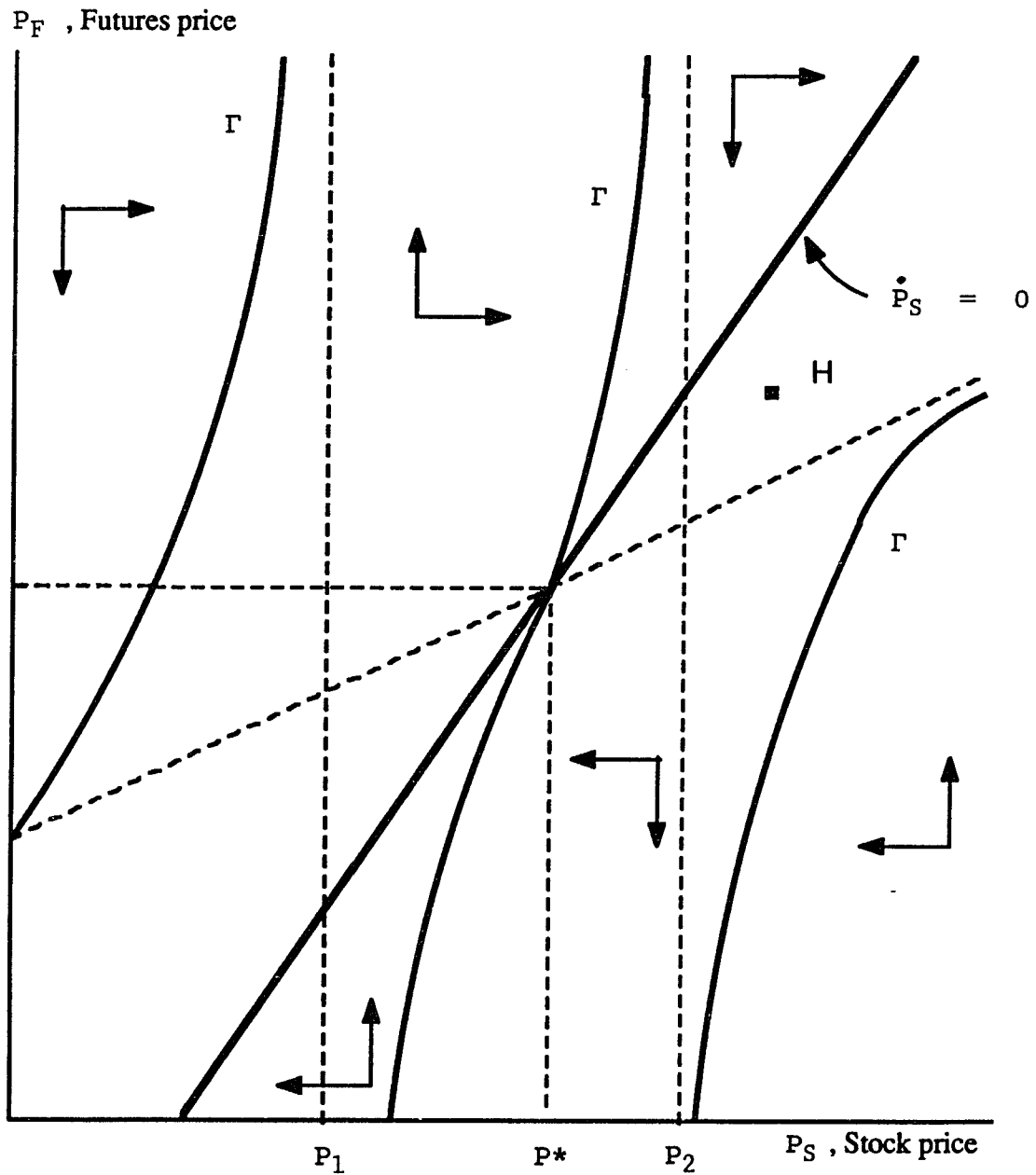
contributed to the extreme price volatility in October 1987.²² Furthermore, policy actions which curb portfolio insurance might reduce price volatility, to the extent that other sources of instability are absent. However, our analysis has also shown that, for volatile but stable systems, there is an incentive for greater investor participation which would tend to reduce volatility without government intervention. Furthermore, one should recognize that dynamic hedging strategies are facilitated by low transactions costs which also serve to facilitate traditional hedging and investment. In light of this, we are fearful that efficiency reducing measures, such as raising margin requirements or suppressing automated trading, may prove to be remedies which are worse than the disease that they are meant to treat.

²² Of course, until one would estimate the parameters of the model one cannot conclude that portfolio insurance was sufficiently high in October 1987 to have been the principal contributing factor to observed volatility.



Phase diagram for case (1):
 Monotonic price behavior with stable equilibrium
 ($\dot{P}$ denotes dP/dt)

Figure (2)



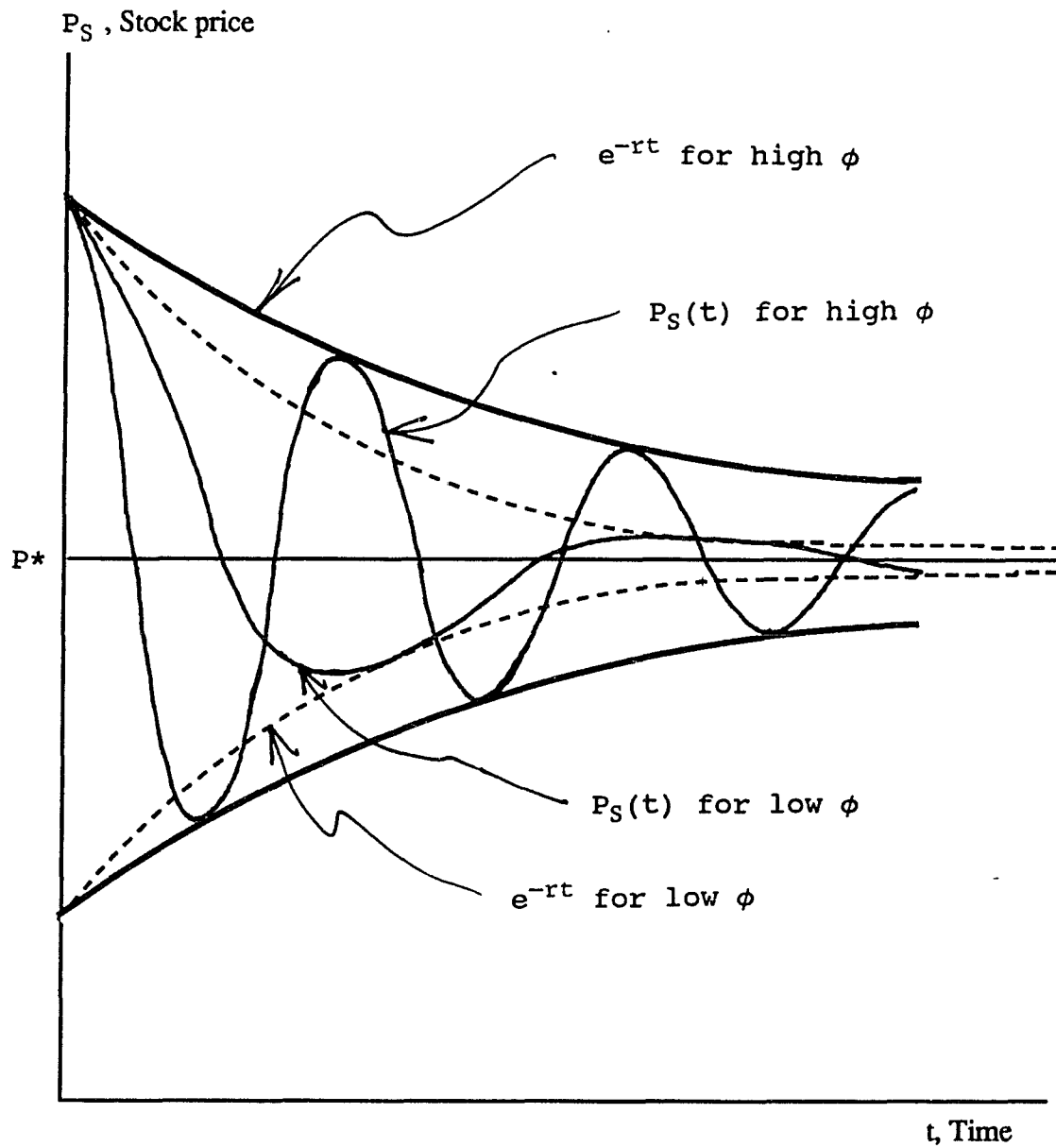
Phase diagram for case (2):

Monotonic or oscillatory price behavior with stable equilibrium

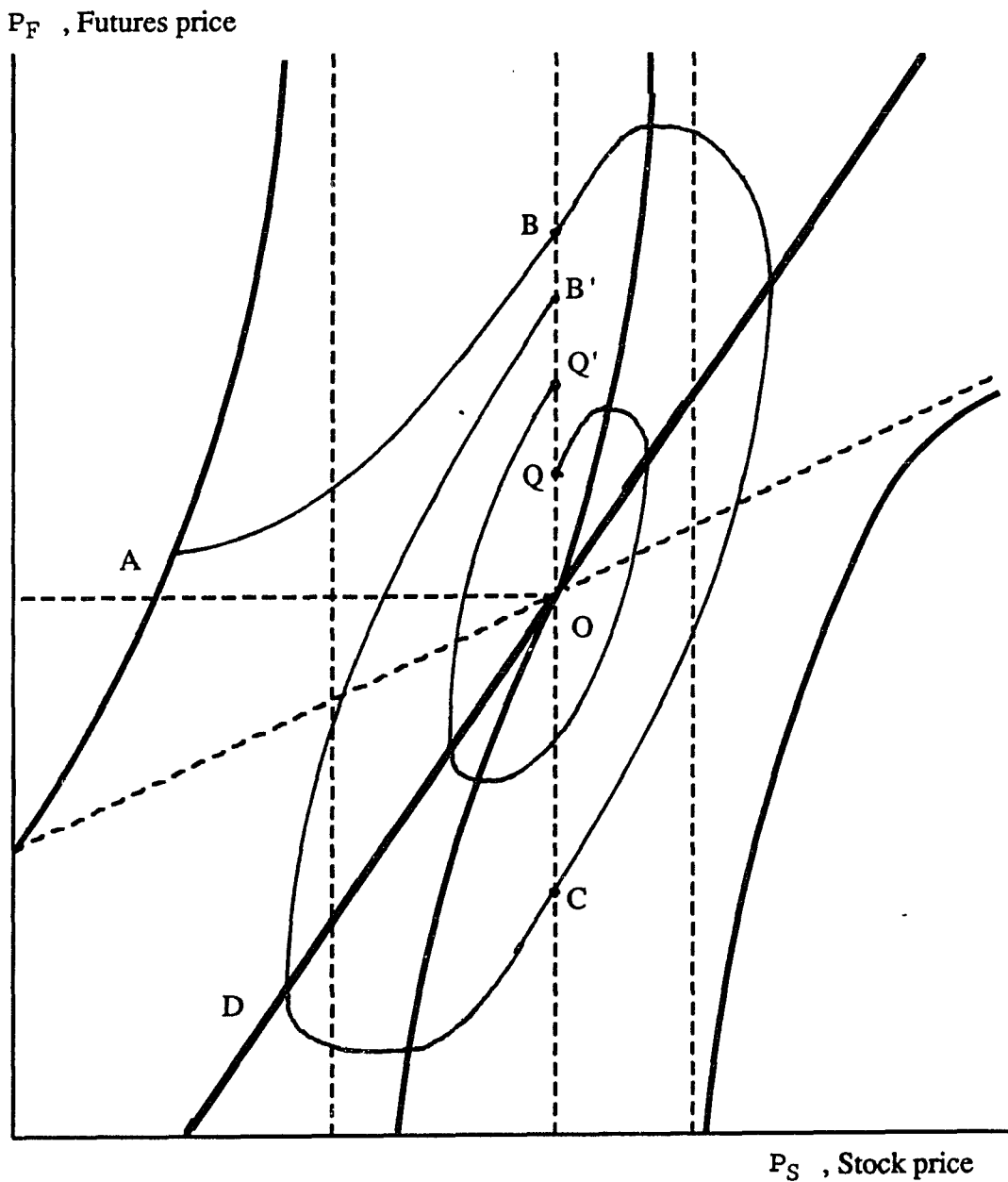
$$\mu(P_1) = \mu(P_2) = 0$$

$$P_F = \Gamma(P_S) \text{ corresponds to } dP_F/dt = 0$$

Figure (3)



Effect of increasing ϕ on the oscillatory convergent price behavior of the linear system in case (2).



Typical price trajectories with unstable equilibrium [case (3)]

ESSAY IV: OPTIMAL WORKER PORTFOLIOS: PENSIONS, ESOPs AND ANTI-ESOPs

(written in coauthorship with Prof. Jeffrey S. Zax; Department of Economics, City University of New York)

IV.1. Introduction

Employer-provided pensions are central to the American system of retirement security. In 1986, 89% of employees in medium- and large-size firms participated in some employer-sponsored retirement plan (Bureau of Labor Statistics (1987)). In 1983, employer-sponsored pensions covered over 51 million civilian workers, or 52.1% of the civilian labor force in the United States (Andrews (1985)). In 1984, covered workers participated in over 795,000 different pension plans (Andrews (1985)).

Since 1974, these plans have been regulated by the Employee Retirement Income Security Act of 1974 (ERISA) and subsequent accompanying legislation. Many of the subsequent statutes regulate employee stock ownership plans (ESOPs), the most celebrated recent innovation in pension plans. In response, the number of ESOPs has increased from less than 1,000 in 1974 to nearly 10,000 (The ESOP Association (1988b)).

From an economic perspective, ERISA and its subsequent legislation are peculiar. They require all pension portfolios to be, at best, second-best. All pension types must choose portfolios without regard to the correlations between asset returns and worker wage flows. Furthermore, all pension types are subject, at least nominally, to restrictions on the types of asset holdings. In consequence, no plan corresponds to the unrestricted maximization of expected lifetime worker utility.

This paper explores the effects of these restrictions. It compares the lifetime utilities expected from the major pension varieties. Section IV.2 describes these pensions. Section IV.3 presents a dynamic stochastic programming model of portfolio choice. In this model, stock returns and wage flows are stochastic. The model solves for optimal pension portfolios, given pension regulations, and optimal worker private portfolios, given pension portfolios.

Section IV.4 compares the conventional pension with the ESOP. If the worker is unconstrained in capital markets, the two yield identical expected utilities because the worker's private portfolio compensates for the imbalances in each pension portfolio. However, if the worker is constrained in capital markets, the private portfolio cannot compensate completely. Simulations encompassing 390,625 different combinations of values for expected excess returns, variances

and covariances indicate that the ESOP is generally inferior to the conventional pension if the worker's stock holdings can be only zero or nonnegative.

Section IV.5 compares the ESOP and the conventional pension to a hypothetical alternative, the anti-ESOP. The anti-ESOP is defined analogously to the ESOP, but with investment restrictions which ensure greater diversification. The anti-ESOP proves superior to the ESOP in most circumstances, and a more attractive alternative to the conventional pension.

Section IV.6 evaluates a second policy alternative; requiring pension plans to evaluate the correlations between assets and worker wage flows when choosing portfolios. Under this requirement, the conventional pension would achieve a first best solution. If 'integration', in this sense, of the conventional pension is impracticable, integration of the ESOP or anti-ESOP would improve their performance. However, the integrated ESOP remains inferior to the integrated anti-ESOP.

These results suggest three policy implications. First, in the absence of substantial and improbable incentive effects, the ESOP's performance does not warrant either the attention or the tax advantages it has received. Second, if regulations are to restrict pensions to limited ranges of

assets, they should emphasize those whose returns have negative rather than positive correlations with wage flows. Third, explicit recognition of these correlations would generally improve all pension plans.

IV.2. Pension Plans

Federal statutes define a variety of pension plans which qualify as tax exempt fringe benefits. The various plans differ along many dimensions. This section describes the variations along the two most important dimensions; the extent to which a plan can hold assets linked to the sponsoring employer and the extent to which a plan can intelligently diversify.

The most important restriction on asset quantities is that which identifies the ESOP. ERISA defines the ESOP as "a stock bonus plan ... which is designed to invest primarily in qualifying employee securities" (paragraph 407(d)(6)(A)). 'Primarily' is not defined, but probably indicates that the plan must hold at least 50% of its assets as employer securities or employer real property (The ESOP Association (1988b)).

For practical purposes, this restriction distinguishes

the ESOP by preventing it from allocating significant portions of its portfolio to other assets. More than half of all ESOPs, 53%, hold only employer securities. An additional 28% hold at least 75% of assets as employer securities (The ESOP Association (1988a)).

Nominally, ERISA also distinguishes the conventional pension from the ESOP by restricting conventional pension holdings of employer securities and real property to 10% of its portfolio (section 407). However, from the worker's perspective, the distinctive characteristic of employer securities is the presumably positive correlation between their returns and the worker's wage flow. ERISA does not restrict conventional pension holdings of other assets whose returns may behave similarly to those of stock in the employer; as examples, stocks of other companies in the same industry and region, or companies that are either suppliers to or customers of the employer.

In practice, then, conventional pensions can, like ESOPs, devote an unrestricted proportion of their portfolios to assets whose returns correlate positively with the wage. For practical purposes, the ESOP is therefore a special case of the conventional pension. For the purposes of the analysis below, the ESOP is simply the conventional pension restricted to only holdings of the employer's stock.

ERISA stipulates that, within the applicable limits on portfolio shares, pension plans must consider the effect of any investment on the expected return and variance of returns to the entire portfolio (Hutchinson and Cole (1980), pgs. 51-52). However, ERISA implies that pension plans cannot consider the correlations between returns to assets held in the plan portfolio and the worker's wage flow (subparagraph 404(a)(1)(A)). Relevant case law makes this prohibition explicit for both traditional pension plans and ESOPs.¹

As examples, pension plans cannot justify asset choice on the grounds that asset returns are negatively correlated with the probability that a contributing worker will experience a wage reduction, demotion or layoff. In effect, pension plans must assign the 'juridical' value of zero to the correlations between returns to assets in pension portfolio and the worker's wage flow, regardless of their actual values.

Actual values are unlikely to be zero, particularly if the pension portfolio contains employer securities. If they are not, the extent of portfolio diversification, as it appears to the pension plan, will differ from the extent of

¹ Blankenship vs. Boyle, (Federal Supplement, Vol 329, 1971, pgs. 1089-1115), Danaher Corp. vs. Chicago Pneumatic Tool Co., (Federal Supplement, Vol. 635, 1986, pgs. 246-250), and Withers vs. Teachers' Retirement System of the City of New York, (Federal Supplement, Vol. 447, 1978, pgs. 1248-1261) are representative.

diversification as evaluated by the worker. The true 'aggregate' risk of the conventional pension may be greater than or less than the 'apparent' risk, depending on the relative importance of stocks whose returns correlate negatively with the worker wage flow. The true aggregate risk of the ESOP portfolio will always be greater than the apparent risk.

The differences between apparent and true pension risk will ordinarily cause the optimal portfolio from the perspective of the pension plan and from the perspective of the worker to differ. From the perspective of the pension plan, the expected utility of the ESOP must be no greater than, and probably less than, that of the conventional pension.² However, the worker may prefer the ESOP in some circumstances, if, as examples, the stochastic properties of asset returns are particularly unfavorable to the conventional pension, or the optimal ESOP leaves the worker with wealth sufficient for a superior private portfolio.

The next section presents a model which formalizes this discussion. Section IV.4 analyzes the circumstances under which the ESOP yields higher expected utility than the conventional pension.

² This assertion implicitly assumes that the composition of pension portfolios does not affect worker productivity. Section IV.7 considers this issue.

IV.3. The theoretical model

The continuous-time model of Merton (1969, 1971), provides the basis for exploring the properties of these pension plans. In the application here, there are only three assets; stocks of companies 1 and 2, and a riskless security. A geometric Brownian motion process represents the rates of return for stocks:

$$dP_i/P_i = \mu_i dt + \sigma_i dz_i$$

where $i=1,2$ identifies the stock, P_i is the price of stock i , μ_i is the expected instantaneous rate of return on stock i , σ_i is the instantaneous variance of the rate of return on stock i , and dz_i are independently and identically distributed with $N(0,dt)$. The rate of return for the riskless security is

$$dP_3/P_3 = r dt$$

where P_3 is the price of the riskless security, and r its rate of return.

The work period runs from 0 to T . Retirement runs from T to T^* . Both T and T^* are known. The presence and absence of a wage flow from the employer to the worker, dw , is the principal distinction between work and retirement. The wage

flow evolves according to the stochastic process

$$dw = \mu_w dt + \sigma_w dz_w$$

where μ_w is the expected instantaneous wage, σ_w captures the uncertainty associated with changes in the wage's real value, and dz_w is a Wiener process.³

Stocks 1 and 2 are distinguished by the correlations between their excess returns and the wage flow. Stock 2, the employer security in this model, has excess returns which correlate positively with the wage flow.⁴ Stock 1, the principal opportunity for diversification in this model, has excess returns which correlate negatively with the wage flow.

C_t represents the worker's consumption flow at time t . $U(C_t)$ represents the worker's instantaneous utility function. The worker maximizes expected lifetime utility

³ The wage flow also includes employer contributions to the pension plan. This model does not permit a meaningful distinction between wages and employer contributions. This model implicitly assumes that human capital is fixed. It also assumes that stock prices will be distributed lognormally and the wage will be distributed normally, for the sake of tractability.

⁴ This correlation should ordinarily be less than one, because μ_2 should be the return to the employer's capital. Many circumstances, including exogenous competitive conditions in labor and capital markets, would create differences between the returns to labor and to capital in the same company.

$$E \left[\int_0^{T^*} U(C_t) \exp\{-\delta t\} dt \right]$$

where δ is worker's rate of time preference. This maximization is subject to a wealth constraint

$$W = \sum_{i=1}^3 (X_i^P + X_i^W) P_i,$$

an instantaneous budget constraint during the work period

$$dW = \sum_{i=1}^3 (X_i^P + X_i^W) dP_i + dS - C_t dt,$$

and an instantaneous budget constraint during retirement

$$dW = \sum_{i=1}^3 (X_i^P + X_i^W) dP_i - C_t dt$$

where W represents wealth, dW the instantaneous change in wealth, X_i^P the pension holdings of asset i , and X_i^W the worker holdings of asset i .⁵

⁵ Merton (1969, 1971) discusses such constraints. This model permits the worker to liquidate pension assets and consume the proceeds during the work period. This ensures that the model is tractable. It is also a necessary condition for a first-best solution. However, actual pension plans prohibit or penalize such liquidations. The treatment here can be justified by allowing workers to borrow against their future pensions. In the simulations below, the expected wage is sufficiently high that, in most cases, it should be greater than consumption during the work period. As all pension plans restrict early liquidations similarly, this simplification should not bias comparisons in favor or against any particular

The pension plan chooses X_i^P to maximize the worker's objective function, subject to the constraints on pension investments.⁶ The worker then chooses X_i^W conditional on X_i^P . Both the pension plan and worker make these choices based on the correct values for expected excess returns to each stock, $\alpha_i = \mu_i - r$. They also both employ the correct values for the variance-covariance matrix of returns to the two stocks

$$V = \begin{vmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{vmatrix}$$

where the covariance between excess returns to stocks, $\sigma_{12} = R_{12}\sigma_1\sigma_2$ with $R_{12}dt = E(dz_1, dz_2)$ and $-1 < R_{12} < 1$. However, in general, X_i^W and the worker's actual expected utility will differ from the values anticipated by the pension plan because the pension plan misconstrues the remaining stochastic properties of the prices of the stocks held by the pension plan and by the worker.

First, the actual covariance matrix between the prices of

plan.

⁶ Formally, ERISA requires the pension plan to manage its portfolio so as to maximize the worker's utility in the retirement period only. This requirement takes the flow of pension contributions as exogenous. Presumably, the employer adopts the objective of maximizing the worker's expected utility for a given compensation expenditure in determining pension contributions.

the stocks held by the pension plan and the prices of those held by the worker, V_{pw} , is equal to V because the pension and the worker hold the same stocks.⁷ In practice, the pension plan chooses x_i^p without any knowledge of x_i^w . This practice is equivalent to the assumption that V_{pw} is a zero matrix.

Second, the actual covariances between excess returns to stocks 1 and 2 and the wage flow are $\sigma_{1w} < 0$ and $\sigma_{2w} > 0$ respectively, where $\sigma_{iw} dt = E(\sigma_i \sigma_w dz_i dz_w) = \sigma_i \sigma_w R_{iw} dt$. Current pension regulations require the pension plan to invest as if these values were both zero. The worker employs the actual values in the evaluation of the utility expected from the pension portfolio.

With these restrictions, the pension plan's value function for the retirement period,

$$J^p(W, t) = \max_{x_i^p, x_i^w, C_t} E \left[\int_T^{T^*} U(C_t) \exp(-\delta t) dt \right]$$

must satisfy the optimality condition

⁷ A more general model would allow asset prices, expected excess returns, variances and covariances to vary between the pension and private portfolios, in order to capture differences in tax treatment, transaction costs, etc. It might also allow asset prices to vary by pension plan to reflect the ESOPs unique tax privileges (The ESOP Association (1988b)). The simplifications adopted here reduce the model's complexity without interfering unduly with its interpretation.

$$-J_t = \max \{ U(C_t) \exp(-\delta t) + J_W ((X^P + X^W)' \alpha + rW - C_t) \\ + (1/2) J_{WW} (X^P' V X^P + X^W' V X^W) \}$$

where X^P is the two-element column vector of $X_i^P P_i$, $i=1,2$, X^W is the two-element column vector of $X_i^W P_i$, $i=1,2$, and α is the two-element column vector of excess returns α_i , $i=1,2$. The worker's value function for the retirement period,

$$J^W(W, t) = \max_{X_i^W, C_t} E \left[\int_0^T U(C_t) \exp(-\delta t) dt \right]$$

must satisfy the optimality condition

$$-J_t = \max \{ U(C_t) \exp(-\delta t) + J_W ((X^P + X^W)' \alpha + rW - C_t) \\ + (1/2) J_{WW} (X^P + X^W)' V (X^P + X^W) \} .$$

If the instantaneous utility function is from the constant -absolute-risk-aversion (CARA) family, $U(C_t) = -\exp(-\tau C_t)/\tau$, with τ the coefficient of absolute risk aversion, the pension plan's solution is $X^P = V^{-1} \alpha$. The worker's solution, conditional on the pension plan's choice of X^P , is $X^W = V^{-1} (\alpha - V X^P)$.⁸ The worker's value function depends on X^P and X^W

⁸ The CARA utility function implies that X^P and X^W are independent of wealth, and that the total value of risky assets is fixed. The assumption that pension plans know the

only through D , where

$$D = -r\tau(X^P + X^W)' \alpha + (1/2)r^2\tau^2((X^P + X^W)'V(X^P + X^W)) \quad (1)$$

and the value function is decreasing in D .

The solution for the working period is analogous. The pension plan's value function,

$$I^P(W, t) = \max_{X_1^P, X_1^W, C_t} E \left[\int_0^T U(C_t) \exp\{-\delta t\} dt + J^P(W, T) \right]$$

must satisfy the optimality condition

$$\begin{aligned} -I_t = \max \{ & U(C_t) \exp\{-\delta t\} + I_W((X^P + X^W)' \alpha + \mu_w + rW - C_t) \\ & + (1/2) I_{WW} (X^P' V X^P + X^W' V X^W + 2(X^P + X^W)' q^P + \sigma_w^2) \} \end{aligned}$$

where q^P is, under current law, the two-element zero column vector. The worker's value function for the work period,

$$I^W(W, t) = \max_{X_1^P, X_1^W, C_t} E \left[\int_0^T U(C_t) \exp\{-\delta t\} dt + J^W(W, T) \right]$$

must satisfy the optimality condition

worker's utility function is not trivial. Employers are often unable to predict employee preferences with regard to the allocation of additional income (Brosnan (1975), Nealey (1964), Nealey and Goodale (1967)).

$$-I_t = \max \{ U(C_t) \exp\{-\delta t\} + I_W((X^P + X^W)' \alpha + \mu_w + rW - C_t) \\ + (1/2) I_{WW}((X^P + X^W)' V (X^P + X^W) + 2(X^P + X^W)' q^w + \sigma_w^2) \}$$

where q^w is the two-element column vector of σ_{iW} , $i=1,2$.

During the work period, the pension plan's solution is $X^P = V^{-1} (\alpha - q^P)$. The worker's solution, conditional on the pension plan's choice of X^P , is $X^W = V^{-1} (\alpha - q^W - V X^P)$. The worker's value function during the work period depends on X^P and X^W only through Q , where

$$Q = -r\tau(X^P + X^W)' \alpha + (1/2) r^2 \tau^2 ((X^P + X^W)' V (X^P + X^W) \\ + 2(X^P + X^W)' q^w)$$

and the value function is decreasing in Q .

At time zero, the worker's expected utility is $I^W(W,0)$. $I^W(W,0)$ is a negative function of the sum $\theta Q + \phi D$, where $\theta = \exp(rT) - 1$ and $\phi = 1 - \exp\{-r(T^*-T)\}$. $I^W(W,0)$ depends on X^P and X^W only through this sum. Therefore, pension plan i yields greater expected utility than pension plan j if $I_i^W(W,0) > I_j^W(W,0)$, or if $\pi_i < \pi_j$, where $\pi_i = \theta Q_i + \phi D_i$, $\pi_j = \theta Q_j + \phi D_j$, and the subscripts i and j identify the two plans.

The assumptions that the variance-covariance matrix of stock returns for the pension plan is equal to that for the private portfolio, and equal to the covariance matrix between holdings in the pension and private portfolios, imply that expected utility is invariant to the allocation of asset holdings between pension and private portfolios. Equations (1) and (2) demonstrate that $\theta Q + \phi D$ depends on pension and private stock holdings only through their sum, $x^p + x^w$. Therefore, two pension plans with the same 'aggregate portfolios', $x_p + x_w$, are equivalent.⁹

IV.4. Conventional pensions and ESOPs

The theoretical model of section IV.3 provides a convenient framework in which to compare the conventional pension and the ESOP. In its terms, the conventional pension is represented simply by the assumption that q^p is the zero vector. The ESOP is represented by this assumption and the restriction that $x_1^p = 0$.

⁹ The comparison between two pension plans is also independent of μ_w , σ_w and wealth. However, it is not independent of T , T^* or r . The simulations adopt fixed values for these parameters in order to focus on the effects of diversification on optimal pension choice.

IV.4.1. Comparisons with no worker capital constraints

Let C represent asset holdings under a conventional pension, E asset holdings under an ESOP. The subscripts 1 and 2 indicate holdings of stocks in company 1 and 2, respectively. The superscript p indicates holdings in the portfolio of either pension plan. The superscript w indicates private holdings of the worker. The subscript R indicates pension and private holdings following retirement.

With these definitions, optimal asset holdings during work and retirement under the conventional pension are:

$$C_1^P = d^{-1} [(\sigma_2^2 \alpha_1 - \sigma_{12} \alpha_2) / \tau r]$$

$$C_2^P = d^{-1} [(\sigma_1^2 \alpha_2 - \sigma_{12} \alpha_1) / \tau r]$$

$$C_1^W = d^{-1} [\sigma_{12} \sigma_{2w} - \sigma_2^2 \sigma_{1w}]$$

$$C_2^W = d^{-1} [\sigma_{12} \sigma_{1w} - \sigma_1^2 \sigma_{2w}]$$

$$C_{1R}^P = d^{-1} [(\sigma_2^2 \alpha_1 - \sigma_{12} \alpha_2) / \tau r]$$

$$C_{2R}^P = d^{-1} [(\sigma_1^2 \alpha_2 - \sigma_{12} \alpha_1) / \tau r]$$

$$C_{1R}^W = 0$$

$$C_{2R}^W = 0$$

where $d = (\sigma_1^2 \sigma_2^2 - \sigma_{12}^2)$, the determinant of V.

The conventional pension portfolio is unaffected by the

worker's retirement; $C_{1R}^P = C_{1P}$ and $C_{2R}^P = C_2^P$. For the purposes of investment, the difference between the work period and retirement is the presence and absence of correlations between wage flows and asset returns. As the conventional pension ignores these correlations, its portfolio is identical in both periods.

In contrast, the worker's private portfolio changes dramatically between the two periods. During the work period, the worker attempts to adjust the aggregate portfolio through private holdings, in order to compensate for the imbalances in the conventional pension portfolio caused by the conventional pension's disregard for the correlations between asset returns and wage flows. During retirement, wage flows cease, so this disregard is no longer material. Therefore, in retirement, the conventional pension portfolio is optimal by itself. Private retirement holdings are zero, $C_{1R}^W = C_{2R}^W = 0$.

With the same definitions, optimal stock holdings under the ESOP are:

$$E_1^P = 0$$

$$E_2^P = \alpha_2 / \tau r \sigma_2^2$$

$$\begin{aligned} E_1^W &= d^{-1} [(\sigma_2^2 \alpha_1 - \sigma_{12} \alpha_2) / \tau r + (\sigma_{12} \sigma_{2w} - \sigma_2^2 \sigma_{1w})] \\ &= C_1^P + C_1^W \end{aligned}$$

$$\begin{aligned}
E_2^w &= d^{-1} [(\sigma_1^2 \alpha_2 - \sigma_{12} \alpha_1) / \tau r + (\sigma_{12} \sigma_{1w} - \sigma_1^2 \sigma_{2w}) \\
&\quad - (1 - R_{12}^2) \sigma_1^2 \alpha_2 / \tau r] \\
&= C_2^P + C_2^w - E_2^P \\
E_{1R}^P &= 0 \\
E_{2R}^P &= \alpha_2 / \tau r \sigma_2^2 = E_2^P \\
E_{1R}^w &= d^{-1} [(\sigma_2^2 \alpha_1 - \sigma_{12} \alpha_2) / \tau r] = C_1^P \\
E_{2R}^w &= d^{-1} [(\sigma_1^2 \alpha_2 - \sigma_{12} \alpha_1) / \tau r - (1 - R_{12}^2) \sigma_1^2 \alpha_2 / \tau r] \\
&= C_2^P - E_2^P
\end{aligned}$$

where $d = \sigma_1^2 \sigma_2^2 - \sigma_{12}^2 = (1 - R_{12}^2) \sigma_1^2 \sigma_2^2$. Therefore,

$$\frac{\alpha_2}{\tau r \sigma_2^2} = \frac{(1 - R_{12}^2) \sigma_1^2 \alpha_2 / \tau r}{\sigma_1^2 \sigma_2^2 - \sigma_{12}^2}$$

This yields the equalities $E_2^w = C_2^P + C_2^w - E_2^P$ and

$$E_{2R}^w = C_2^P - E_2^P.$$

The solutions for optimal stock holdings demonstrate that, with no restrictions on worker portfolios, aggregate portfolios for the conventional pension and the ESOP are identical:

$$\begin{aligned}
C_1^P + C_1^w &= E_1^P + E_1^w \\
C_2^P + C_2^w &= E_2^P + E_2^w
\end{aligned}$$

$$C_{1R}^P + C_{1R}^W = E_{1R}^P + E_{1R}^W$$

$$C_{2R}^P + C_{2R}^W = E_{2R}^P + E_{2R}^W.$$

In other words, if workers face no restrictions in capital markets, all pension plans yield the same expected utility. Furthermore, workers attain the same expected utility under all plans as they would if pension plans did not exist; if they held all assets privately. If employers impose pension plans on workers who face no capital constraints, these workers simply configure their private portfolios so as to ensure that the aggregate of pension and private portfolios is identical to the private portfolio they would choose in the absence of pension plans.

IV.4.2. Comparisons with worker capital constraints

The presence of capital constraints for individual workers is, however, one of the fundamental justifications for the existence of pension plans. If these constraints are not binding, then the case for any type of employer-provided pension is weak.¹⁰ If they are, the model here provides some

¹⁰ Of course, differences in the tax treatment of funds invested in employer-provided pensions and in private portfolios are also responsible for the existence of these pensions. With these differences, both private and pension portfolios would ordinarily be necessary to maximize worker expected utility. To the extent that the 'artificial'

analytical insights into the comparative performance of the conventional pension and the ESOP. Simulations of the model develop these insights further.

IV.4.2.a. No worker stock holdings

The scenario that various authors invoke in their promotion of pension plans is that in which capital constraints prevent workers from holding any assets at all. This case is also the simplest, analytically. Analytical examples and more general simulations demonstrate that, while constraints of this severity certainly justify the existence of conventional pensions, they are unlikely to create circumstances in which ESOPs are useful.

With this constraint, $C_1^w = E_1^w = C_2^w = E_2^w = 0$. The special case where asset variances and expected asset returns are equal, $\alpha_1 = \alpha_2 = \alpha$ and $\sigma_1 = \sigma_2 = \sigma$, emphasizes the role of diversification, as represented by the correlations among

incentives created by tax policy are responsible for the existence of pension plans, the case for them should again be reexamined. These plans are probably inefficient, both because they restrict worker mobility and because they impose the same asset portfolio on all workers, regardless of differences in tastes. Unless tax incentives serve some larger purpose, their distortionary effects on labor market mobility and asset accumulation may be good reasons to abolish them.

asset returns and wages, in the determination of the superior pension plan. In this special case, analytical solutions for C_1^P , E_1^P , C_2^P , and E_2^P allow explicit comparisons of the expected utility from conventional pensions and ESOPs.

With equal returns and variances, the value of $I(W,0)$ for the conventional pension, I_C , will depend on

$$\pi_C = \frac{\alpha^2}{\sigma^2} \frac{1}{1+R_{12}} [\theta \beta (R_{1w} + R_{2w}) - \theta - \phi]$$

where $\beta = r \tau \sigma \sigma_w / \alpha > 0$. The value for the ESOP, I_E , will depend on

$$\pi_E = \frac{\alpha^2}{\sigma^2} [\theta \beta R_{2w} - (1/2) \theta - (1/2) \phi]$$

If $R_{12} \geq 0$, $\pi_C - \pi_E < 0$ and $I_C - I_E > 0$. With equal variances and expected returns, the conventional pension yields higher expected utility than the ESOP if the correlation between expected returns to assets 1 and 2 is nonnegative, regardless of the values for R_{1w} and R_{2w} . This result is striking, because the amount of diversification in the conventional pension portfolio is least when R_{12} is positive.

The conventional pension is superior to the ESOP in this case because, in effect, high values of R_{12} protect workers from the pension plan's ignorance. When stocks 1 and 2 offer relatively few opportunities for diversification, conventional pension plans prudently choose relatively small holdings of both.¹¹ These small holdings have the additional unintended effect of protecting workers from unfavorable values of R_{1w} and R_{2w} .

The remaining cases, where $R_{12} < 0$, reinforce this intuition. If $R_{12} < 0$ and

$$(\theta + \phi) - 2 \theta \beta R_{2w} \geq 0$$

we have still $\pi_C - \pi_E < 0$, the case is equivalent to $R_{12} \geq 0$. It remains to consider the case in which the last condition does not hold, i.e. if

$$\theta + \phi - 2 \theta \beta R_{2w} < 0.$$

With this condition, $\pi_C - \pi_E < 0$ if

$$0 > R_{12} > \frac{(\theta + \phi) - 2 \theta \beta R_{1w}}{(\theta + \phi) - 2 \theta \beta R_{2w}} = \Gamma$$

¹¹ In this special case, holdings of both stocks 1 and 2 are equal to $\alpha / [(1+R_{12}) r \tau \sigma^2]$, a negative function of R_{12} .

This condition is certainly true if $\Gamma < -1$. Therefore, a sufficient condition for $\pi_C - \pi_E < 0$ when $R_{12} < 0$ is

$$(\theta + \phi)/\theta\beta > R_{1w} + R_{2w},$$

where $(\theta + \phi)/\theta\beta \geq 0$. In other words, the conventional pension is superior to the ESOP with negative values for R_{12} if the 'aggregate' correlation between stocks and wages, as indexed by $R_{1w} + R_{2w}$, is not too great.

With equal variances and expected returns, the ESOP is superior to the conventional pension only if the necessary (but not sufficient) condition $0 > \Gamma > -1$ (or $R_{1w} + R_{2w} > (\theta + \phi)/\theta\beta$ with $2R_{2w} > (\theta + \phi)/\theta\beta$) is satisfied. Conventional pensions choose large holdings of assets 1 and 2 when R_{12} is negative in order to take advantage of the opportunities for diversification between the two. If, however, the aggregate correlation between stocks and wages is sufficiently high, large stock holdings create unacceptable levels of risk in the worker's 'effective' portfolio; pension assets and wages. In other words, negative values for R_{12} can 'seduce' conventional pensions into overinvesting in stocks 1 and 2, from the worker's point of view.

These comparisons are illustrative only. Analytical

comparisons do not yield simple intuitions if the expectations and variances of excess returns to stocks 1 and 2 differ. Simulations provide a more comprehensive description of the effects of parameter values on the relative attractiveness of conventional pensions and ESOPs.

The simulations below allow five different values for each of eight parameters. α_1 and α_2 each take on values of .001, .002, .003, .004 and .005. σ_1 and σ_2 each take on values of .02, .03, .04, .05 and .06. For fixed values of σ_1 and σ_2 , σ_{12} takes on the values $-.76\sigma_1\sigma_2$, $-.38\sigma_1\sigma_2$, 0, $.38\sigma_1\sigma_2$ and $.76\sigma_1\sigma_2$.¹² σ_H takes on values of \$500, \$600, \$700, \$800 and \$900. For given values of σ_H and σ_1 , σ_{1H} takes on values of $-.19\sigma_1\sigma_H$, $-.38\sigma_1\sigma_H$, $-.57\sigma_1\sigma_H$, $-.76\sigma_1\sigma_H$ and $-.95\sigma_1\sigma_H$. For given values of σ_H and σ_2 , σ_{2H} takes on values of $.19\sigma_2\sigma_H$, $.38\sigma_2\sigma_H$, $.57\sigma_2\sigma_H$, $.76\sigma_2\sigma_H$ and $.95\sigma_2\sigma_H$.¹³

These ranges represent reasonable values for expectations and variances of expected monthly returns.¹⁴ The simulation

¹² This specification ensures that $-1 < R_{12} < 1$.

¹³ These specifications ensure that $-1 < R_{1H} < 0$ and $0 < R_{2H} < 1$.

¹⁴ The simulations also set the following values for fixed parameters: $T = 240$, (a career of 20 years), and $T^* = 360$ (10 years of retirement). $\alpha_H = \$3500$, $r = \delta = .002$ and $\tau = .02$. This value for τ yields reasonable values for asset holdings.

compares the expected utility from the conventional pension and the ESOP for a total of $5^8 = 390,625$ combinations of parameter values.

Table 1 describes the comparison between the conventional pension and the ESOP in this simulation when workers hold no private portfolios. The comparison is unfavorable for the ESOP.

Table 1

Characteristics of parameter combinations favoring the conventional pension and the ESOP with no worker holdings

	Superior plan:	
	Conventional pension	ESOP
Number of parameter combinations	364099	26526
Percent of parameter combinations	93.2%	6.79%
Average value of:		
α_1	.00307	.00199
σ_1/α_1	17.4	18.7
α_2	.00296	.00354
σ_2/α_2	29.7	11.7
R_{12}	-.0457	.627
Average conventional pension holdings:		
stock 1	\$129242	-\$21580
stock 2	\$119654	\$110033
Average ESOP holdings of stock 2	\$72478	\$90595

First, these simulations completely contradict the common assertion that ESOPs will provide workers with greater stakes

in the companies for which they work. In both parameter combinations which favor the conventional pension and those which favor the ESOP, average ESOP holdings of stock 2 are substantially less than average conventional pension holdings of stock 2.¹⁵ The ESOP cannot optimally hold as much stock 2 as can the conventional pension because it has no other asset against which to diversify the risks associated with this stock.

Second, the ESOP is superior to the conventional pension for only 26,526 parameter combinations, or 6.79% of all combinations. Furthermore, these cases are extreme. On average, they have exceptionally high expected returns to stock 2, exceptionally low expected returns to stock 1 and exceptionally little diversification between stocks 1 and 2. In addition, the coefficient of variation for stock 1, σ_1/α_1 , indicates that the returns to stock 1 must be exceptionally uncertain.¹⁶

Average stock holdings in these combinations suggest that the ESOP is only superior to the conventional pension in cases

¹⁵ Table 1 and all following tables present averages of the optimal stock holdings during the working period, for the sake of brevity. Comparisons between optimal stock holdings during retirement yield the same conclusions.

¹⁶ Table 1 omits average values for R1w and R2w for the sake of brevity. While their signs are crucial, their magnitudes do not play an important role in this comparison.

where stock 1 is so unattractive that the optimal conventional pension portfolio, in effect, sells stock 1 short. Additional analyses demonstrate that the ESOP is even inferior to the conventional pension in the majority of the cases in which short selling is optimal.

In contrast, the conventional pension yields greater expected utility over the vast majority of parameter combinations. Furthermore, average stock holdings indicate that in these cases, the ESOP cannot begin to approximate the conventional pension portfolio. However, in the cases where the ESOP is superior, the conventional pension portfolio is fairly similar to that of the ESOP.

IV.4.2.b. Non-negative worker stock holdings

The ESOP is redundant if workers face no capital constraints. Though ESOP proponents invoke the circumstance where workers are constrained to hold no private assets as the justification for the ESOP, it is apparently unnecessary in this case, as well. The only case in which the ESOP can claim a distinctive role is that in which workers are constrained to hold nonnegative quantities of stocks 1 and 2.

Table 2 presents comparisons under this constraint. Here,

the worker must accept zero stock holdings if the optimal private portfolio would dictate negative holdings. The resulting comparisons are much more favorable to the ESOP than are those of Table 1.

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Table 2

Characteristics of parameter combinations favoring the conventional pension and the ESOP with nonnegative worker holdings

	Combinations for which the superior plan is the		
	Conventional pension	ESOP	the two are equivalent
Number of parameter combinations	131303	143697	115625
Percent of parameter combinations	33.6%	36.8%	29.6%
Average value of:			
α_1	.00319	.00283	.00300
σ_1/α_1	16.1	20.2	18.3
α_2	.00290	.00309	.00300
σ_2/α_2	19.5	17.2	18.3
R_{12}	.558	-.336	-.216
Average conventional pension holdings:			
pension, stock 1	\$74832	\$151225	\$129110
worker, stock 1	\$11713	\$10518	\$14778
pension, stock 2	\$50872	\$173116	\$129110
worker, stock 2	0	0	\$2806
Average ESOP holdings:			
worker, stock 1	\$63006	\$158049	\$143887
ESOP, stock 2	\$71231	\$75972	\$73708
worker, stock 2	0	\$83592	\$58207

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For 115,625 parameter combinations, or 29.6%, the ESOP

and the conventional pension yield identical expected utilities. For these combinations, the optimal worker private portfolios under either pension plan do not require short-selling. Therefore, the worker is able to augment either pension plan so as to construct the optimal aggregate portfolio. This is demonstrated by the equality between average total holdings of stocks 1 and 2 for the combinations in this category.

For 131,303 parameter combinations, or 33.6%, the conventional pension yields greater expected utility than the ESOP. For these combinations, relatively high values for R_{12} force the conventional pension to hold relatively small amounts of both stocks, particularly stock 2. However, the ESOP overinvests in stock 2. The wealth constraint and the prohibition against worker short-selling prevent the worker from compensating for this error in the worker's private portfolio.

For the plurality of parameter combinations, 143,697, or 36.8%, the ESOP is superior to the conventional pension. For these combinations, relatively low values of R_{12} and slightly favorable characteristics for stock 2 encourage the conventional pension to overinvest in stock 2. The ESOP also holds large amounts of stock 2, relative to its average holdings in the combinations of other categories. But because

it does not diversify, it holds far less than the conventional pension. The worker can therefore successfully augment the ESOP portfolio through the worker's private holdings.

Table 2 demonstrates that, when workers are constrained to hold nonnegative stock quantities, more parameter combinations yield higher expected utility under the ESOP than under the conventional pension. However, this demonstration also reveals the weakness underlying popular arguments in favor of the ESOP. For the parameter combinations which yield higher expected utility under the ESOP, ESOP holdings are, on average, only 23.9% of aggregate worker assets. Far from being the worker's only source of saving, the ESOP is successful in precisely those cases where its own holdings are sufficiently small that they do not materially restrict the allocation of aggregate holdings. In these cases, the ESOP permits workers to allocate the the bulk of aggregate holdings themselves, with full regard for the correlations between the two stocks, and between the two stocks and wages.

IV.5. Policy alternative 1: the anti-ESOP

These simulations suggest that the ESOP is a poor alternative to the conventional pension. They also suggest, however, that the conventional pension itself is flawed. In simulations

with no private holdings and in those with nonnegative private holdings, a few parameter combinations yield positive values for π_C , π_E or both. π takes the value of zero if all pension and private stock holdings are zero. Therefore, workers would enjoy greater expected utility for some parameter combinations if they had 'undiversified employment' -- no assets at all -- rather than one of these pension plans. ¹⁷

Both the ESOP and the conventional pension suffer because they disregard the positive value for $R2w$. As this disregard is statutory, an alternative pension plan which simply avoided investment in stock 2 might be attractive. In the model here, the hypothetical alternative would be, like the ESOP, restricted to investments in only one asset. But the 'anti-ESOP' asset would be stock 1, incidentally providing the anti-ESOP with diversification in regard to worker wage flows. Compared to the conventional pension and the ESOP, the anti-ESOP would be 'conservative' in the sense that the actual diversification for the anti-ESOP would always exceed the apparent diversification. ¹⁸

¹⁷ If these combinations are common, the 47.5% of the civilian labor force who are not covered by pensions (Andrews (1985)) may be blessed.

¹⁸ The anti-ESOP might be most representative of the spirit of the ERISA regulation which restricts pension holdings of employer securities. It might also relieve pension trustees of potential conflicts between their responsibilities to the pension beneficiary and to the beneficiary's employer (Ressel (1986)). The anti-ESOP has

The unconstrained stock holdings for the anti-ESOP are:

$$\begin{aligned}
 A_1^P &= \alpha_1 / \tau r \sigma_1^2 \\
 A_2^P &= 0 \\
 A_1^W &= d^{-1} [(\sigma_2^2 \alpha_1 - \sigma_{12} \alpha_2) / \tau r + (\sigma_{12} \sigma_{2w} - \sigma_2^2 \sigma_{1w}) \\
 &\quad - (1 - R_{12}^2) \sigma_2^2 \alpha_1 / \tau r] \\
 &= C_1^P + C_1^W - A_1^P \\
 A_2^W &= d^{-1} [(\sigma_1^2 \alpha_2 - \sigma_{12} \alpha_1) / \tau r + (\sigma_{12} \sigma_{1w} - \sigma_1^2 \sigma_{2w})] \\
 &= C_2^P + C_2^W \\
 A_{1R}^P &= \alpha_1 / \tau r \sigma_1^2 = A_1^P \\
 A_{2R}^P &= 0 \\
 A_{1R}^W &= d^{-1} [(\sigma_2^2 \alpha_1 - \sigma_{12} \alpha_2) / \tau r - (1 - R_{12}^2) \sigma_2^2 \alpha_1 / \tau r] \\
 &= C_1^P - A_1^P \\
 A_{2R}^W &= d^{-1} [(\sigma_1^2 \alpha_2 - \sigma_{12} \alpha_1) / \tau r] = C_2^P
 \end{aligned}$$

Comparison with the unconstrained conventional pension and ESOP holdings confirms that, if workers are free of capital constraints, all three pension plans are equivalent;

properties which are analogous to those of the implicit contract, which insures employees against potential lay-off. In effect, this insurance 'policy' is an asset whose returns correlate negatively with the wage flow, by construction. However, the positive correlation between company and own-worker risks must limit the diversification possible in any such risk-sharing agreement. In a economy with multiple firms, 'explicit contracts' in which two firms whose risks correlate negatively insure each other's work force against unemployment would surely dominate those in which a firm 'insures' its own.

$$C_1^P + C_1^W = E_1^P + E_1^W = A_1^P + A_1^W$$

$$C_2^P + C_2^W = E_2^P + E_2^W = A_2^P + A_2^W$$

$$C_{1R}^P + C_{1R}^W = E_{1R}^P + E_{1R}^W = A_{1R}^P + A_{1R}^W$$

$$C_{2R}^P + C_{2R}^W = E_{2R}^P + E_{2R}^W = A_{2R}^P + A_{2R}^W.$$

However, simulations suggest that the anti-ESOP may be attractive when workers are subject to capital constraints. The anti-ESOP is always superior to undiversified employment, with either no or nonnegative worker stock holdings.

IV.5.1. The anti-ESOP with no worker stock holdings

If workers are constrained to hold no private portfolios, and if the excess returns to stocks 1 and 2 have identical expectations and variances, then π_A determines I_A , the expected utility of the anti-ESOP, where

$$\pi_A = \frac{\alpha^2}{\sigma^2} [\theta \beta R_{1w} - (1/2)\theta - (1/2)\phi]$$

In this special case, the conventional pension is superior to the anti-ESOP, $I_C - I_A > 0$, if $\pi_C - \pi_A < 0$, i.e. if

$$R_{12} < \frac{(\theta + \phi) - 2\theta\beta R_{2w}}{(\theta + \phi) - 2\theta\beta R_{1w}} = \Omega$$

Ω takes the sign of the numerator, $(\theta+\phi)-2\theta\beta R_{2w}$, because $R_{1w} < 0$.

If $2R_{2w} < (\theta+\phi)/\theta\beta$, the correlation between returns to the employer's stock and the wage flow is sufficiently small that it does not reduce the actual diversification in the conventional pension portfolio excessively. In this case, $\Omega > 0$; all negative and some positive values for R_{12} favor the conventional pension. The range of R_{12} values for which the conventional pension is superior is larger for lower values of R_{2w} ; the pension yields greater expected utility if the worker suffers less from correlations between the returns to stock 2 and worker wage flows.

However, $\Omega < 1$ for even small values of R_{2w} (because $\Omega \geq 1$ requires $R_{2w} \leq R_{1w}$, which is impossible). This implies that, regardless of how favorable is R_{2w} , the anti-ESOP is superior to the conventional pension for sufficiently large values of R_{12} .

This comparison suggests that the anti-ESOP is a more attractive alternative to the conventional plan than is the ESOP. With $2R_{2w} < (\theta+\phi)/\theta\beta$ the conventional pension is uniformly superior to the ESOP, but inferior to the anti-ESOP for some values of R_{12} .

With $2R_{2w} > (\theta + \phi)/\theta\beta$ the contrast between the anti-ESOP and the ESOP is sharper. With this condition, the ESOP is almost certainly inferior to the conventional pension, but the anti-ESOP is, at minimum, superior to the conventional pension for all nonnegative values of R_{12} because $\Omega < 0$. In a subset of the cases where $2R_{2w} > (\theta + \phi)/\theta\beta$ and $R_{1w} + R_{2w} > (\theta + \phi)/\theta\beta$, the ESOP is superior to the conventional pension. However, these two conditions also guarantee that $\Omega < -1$; the anti-ESOP is always --i.e. in all of the cases where these two conditions hold-- superior to the conventional pension because the aggregate diversification in the conventional pension plan is small.

The contrasting performances of the ESOP and anti-ESOP in comparison to the conventional pension anticipate the direct comparison between the two. We have $I_A < I_E$ if $\pi_A - \pi_E > 0$ which requires $R_{2w} < R_{1w}$. The anti-ESOP is unambiguously superior to the ESOP because $R_{1w} < 0 < R_{2w}$.

IV.5.2. Simulations for the anti-ESOP and the ESOP

Table 3 presents simulated comparisons between the anti-ESOP and ESOP when expectations and variances of excess returns vary and the worker holds no stocks. The anti-ESOP is

superior to the ESOP for nearly two-thirds of all parameter combinations. The ESOP yields higher expected utility in only those combinations for which returns to stock 1 are unusually low and risky, while returns to stock 2 are unusually high and certain.

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Table 3
Characteristics of parameter combinations favoring the
anti-ESOP and the ESOP with no worker holdings

	Superior plan:	
	anti-ESOP	ESOP
Number of parameter combinations	248640	141985
Percent of parameter combinations	63.7%	36.3%
Average value of:		
α_1	.00342	.00227
σ_1/α_1	13.7	26.3
α_2	.00296	.00354
σ_2/α_2	23.5	9.05

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Table 4 compares the anti-ESOP to the ESOP if workers are constrained to nonnegative stock holdings. In general, workers can more successfully correct the imbalances of the anti-ESOP portfolio than those of the ESOP portfolio through their private holdings. Therefore, these comparisons are more unfavorable to the ESOP than are those of Table 3.

The anti-ESOP is superior to the ESOP in slightly more than half of the parameter combinations. In these

combinations, the ESOP over-invests in stock 2; average worker holdings of stock 2 under the ESOP are identically zero, suggesting that, in at least some of these cases, the worker would prefer to sell stock 2 short. In contrast, the anti-ESOP is conservative; it under-invests in stock 1. On average, workers have positive holdings of stock 1 in their private portfolios.

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Table 4

Characteristics of parameter combinations favoring the anti-ESOP and the ESOP with nonnegative worker holdings

	Combinations for which the superior plan is the		
	anti-ESOP	ESOP	the two are equivalent
Number of parameter combinations	196096	41118	153411
Percent of parameter combinations	50.2%	10.5%	39.3%
Average value of:			
α_1	.00313	.00226	.00303
σ_1/α_1	16.9	26.4	17.8
α_2	.00276	.00406	.00302
σ_2/α_2	20.6	7.98	18.1
R_{12}	.321	.621	-.573
Average anti-ESOP holdings:			
anti-ESOP, stock 1	\$80664	\$36521	\$74784
worker, stock 1	\$10472	\$4.53	\$137054
worker, stock 2	\$31451	\$10384	\$196714
Average ESOP holdings:			
worker, stock 1	\$76148	\$4631	\$211837
ESOP, stock 2	\$57139	\$149607	\$74544
worker, stock 2	0	0	\$122170

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The anti-ESOP is superior to the ESOP in slightly more

than half of the parameter combinations. In these combinations, the ESOP over-invests in stock 2; average worker holdings of stock 2 under the ESOP are identically zero, suggesting that, in at least some of these cases, the worker would prefer to sell stock 2 short. In contrast, the anti-ESOP is conservative; it under-invests in stock 1. On average, workers have positive holdings of stock 1 in their private portfolios.

The ESOP is superior to the anti-ESOP in barely 10% of the parameter combinations. In these combinations, stock 1 is an extremely unattractive investment. Its average expected returns are barely half those of stock 2, its coefficient of variation is three times as large and it offers relatively little diversification in conjunction with stock 2. The ESOP still over-invests in stock 2, but this mistake is not sufficient to reduce its expected utility to the level attained by the anti-ESOP.

In sum, these comparisons demonstrate the value of diversification between the pension portfolio and the worker's wage flow. If stock 1 offers anything like reasonable returns, risks and diversification, the anti-ESOP yields greater expected utility than the ESOP.

IV.5.3. Simulations for the anti-ESOP and the conventional pension

Table 5

Characteristics of parameter combinations favoring the anti-ESOP and the conventional pension with no worker holdings

	Superior plan:	
	anti-ESOP	conventional pension
Number of parameter combinations	47583	343042
Percent of parameter combinations	12.2%	87.8%
Average value of:		
α_1	.00285	.00302
σ_1/α_1	19.8	18.1
α_2	.00238	.00309
σ_2/α_2	24.7	17.4
R_{12}	.471	-.0653

Simulations also confirm that it offers a more attractive alternative to the conventional pension. Table 5 reports that, if workers cannot hold private portfolios, the anti-ESOP is superior to the conventional pension for 47583 parameter combinations, or nearly twice as many as those for which the ESOP is superior to the conventional pension under the same constraint. Furthermore, the combinations which favor the anti-ESOP are less extreme than those which favor the ESOP. While, on average, stock 2 is more attractive than stock 1 in these combinations, the contrast is not dramatic. The distinguishing characteristic of these combinations is that stocks 1 and 2 offer relatively little diversification.

Table 6

Characteristics of parameter combinations favoring the anti-ESOP and the conventional pension with nonnegative worker holdings

	Combinations for which the superior plan is the		
	anti-ESOP	conventional pension	the two are equivalent
Number of parameter combinations	264030	89095	37500
Percent of parameter combinations	67.6%	22.8%	9.60%
Average value of:			
α_1	.00295	.00316	.00300
σ_1/α_1	18.8	16.7	18.3
α_2	.00299	.00304	.00300
σ_2/α_2	18.1	18.8	18.3
R_{12}	-.111	.609	-.665
R_{1w}	-.555	-.540	-.744
R_{2w}	.622	.530	.301
Average anti-ESOP holdings:			
anti-ESOP, stock 1	\$69462	\$86293	\$73708
worker, stock 1	\$57808	\$6797	\$192281
worker, stock 2	\$100349	\$51907	\$253182
Average conventional pension holdings:			
pension, stock 1	\$115599	\$76247	\$244530
worker, stock 1	\$11147	\$11338	\$21459
pension, stock 2	\$115718	\$75892	\$244530
worker, stock 2	0	0	\$8652

If workers can hold nonnegative amounts of stocks 1 and 2 in their private portfolios, the anti-ESOP has an advantage. If stocks 1 and 2 offer any opportunities for diversification, the anti-ESOP underinvests in stock 1 because it does not hold stock 2. The worker can then balance the anti-ESOP portfolio with additional holdings of stock 1, as well as stock 2, in the worker's private portfolio. In contrast, the conventional

pension may choose portfolio allocations between stocks 1 and 2 which the worker cannot correct through private holdings, given the wealth constraint.

The simulation results in Table 6 confirm that, with nonnegative worker holdings, the anti-ESOP is convincingly superior to the conventional pension. It yields higher expected utility for slightly more than two-thirds of all parameter combinations. In these combinations, the conventional pension overinvests in stock 2.

The anti-ESOP and the conventional pension yield identical levels of expected utility in 9.6% of the combinations. In these combinations, average values for R_{1w} and R_{2w} indicate that excess returns to both stocks have relatively low correlations with the worker wage flow. Therefore, the conventional pension portfolio fortuitously provides adequate levels of aggregate diversification.

The conventional pension is superior to the anti-ESOP in only 22.8% of all combinations. In them, relatively high values of R_{12} prevent the conventional pension from severely over-investing in stock 2. The anti-ESOP, ignoring the relatively small values for R_{2w} , over-invests in stock 1. In consequence, the conventional pension yields greater expected utility.

IV.6. Policy alternative 2: integrated pensions

The anti-ESOP, with its emphasis on diversification, is one appealing policy alternative. Regulatory change may be a second alternative. If statutes allowed pension plans to 'integrate' pension risks with worker wage risks -- to explicitly evaluate the correlations between asset excess returns and worker wage flows in the determination of pension portfolios -- both the ESOP and the conventional pension would improve.¹⁹

IV.6.1. The integrated conventional pension

The integrated conventional pension would be a first-best solution to the problem of maximizing expected utility. 'Integration' would consist of replacing the juridical values of zero for the covariances between excess stock returns and worker wage flows with their actual values, σ_{1w} and σ_{2w} . In the notation of section IV.4, with the additional superscript i indicating integrated pension plans, the optimal stock

¹⁹ This statutory change raises additional issues. The worker faces nonzero correlations between asset excess returns and wage flows. However, the retiree has no wage flows. For them, these correlations are zero. If pension plans were legally required to evaluate these correlations when choosing a portfolio, the optimal portfolio for the worker and the retiree would differ. ERISA currently requires pension plans to treat the worker and the retiree identically.

holdings for the integrated conventional pension are:

$$\begin{aligned}
 C_1^{pi} &= C_1^P + C_1^W \\
 C_2^{pi} &= C_2^P + C_2^W \\
 C_{1R}^{pi} &= C_1^P \\
 C_{2R}^{pi} &= C_2^P \\
 C_1^{wi} &= C_2^{wi} = C_{1R}^{wi} = C_{2R}^{wi} = 0
 \end{aligned}$$

The aggregate portfolio of the integrated conventional pension is identical to that of the conventional pension in the absence of worker capital constraints. It is also identical to the portfolio the unconstrained worker would choose in the absence of pension plans.

However, the integrated conventional pension achieves this identity regardless of worker capital constraints, because its maximization problem is identical to that of the worker. Worker capital constraints are irrelevant in this case, because the optimal worker stock holdings are zero. In contrast, the conventional pension achieves the first-best solution only if the worker's private portfolio can compensate for the imbalances in the conventional pension portfolio caused by incorrect juridical covariances.²⁰

²⁰ The first-best property of the integrated conventional pension depends on the equality of expected stock returns, variances and covariances between stocks held by the pension

IV.6.2. The integrated ESOP

If regulatory change were to legalize the integrated conventional pension, the ESOP would have no further role. However, integration in the conventional pension may be impracticable. In the general case, it would require the evaluation of many additional covariances. In contrast, the integration of the ESOP requires the evaluation of only one. The integrated ESOP may therefore be a practical policy alternative where the integrated conventional pension is not.

Intuitively, integration should improve the ESOP, because integration allows the ESOP to address its obvious weakness. Analytically, the unconstrained optimal stock holdings under the integrated ESOP are;

$$\begin{aligned}
 E_1^{pi} &= E_{1R}^{pi} = 0 \\
 E_2^{pi} &= d(\alpha_2 / r r \sigma_2^2 - \sigma_{2w} / \sigma_2^2) \\
 E_{2R}^{pi} &= E_2^p \\
 E_1^{wi} &= C_1^p + C_1^w \\
 E_2^{wi} &= C_2^p + C_2^w - E_2^{pi}
 \end{aligned}$$

and stocks held by the worker. If these parameters differ across pension and worker portfolios, the worker's optimal stock holdings would ordinarily be nonzero. A first-best solution would require a more sophisticated notion of 'integration'; the pension would have to consider not only the stochastic characteristics of the worker's wage flow, but of the worker's private portfolio as well.

$$E_{1R}^{wi} = C_1^P$$

$$E_{2R}^{wi} = C_2^P - E_2^P$$

Again, the aggregate portfolio of the integrated ESOP is identical to that of the ESOP and those of all other pension plans. With unconstrained worker holdings the integrated ESOP is unnecessary. Any pension plan achieves the first-best.

Integration improves ESOP performance if worker suffers capital constraints, The integrated ESOP and the ESOP have the same retirement portfolios. Therefore, the comparison between I_E and I_E^i , where I_E^i represents $I(W,0)$ for the integrated ESOP, depends only on the term in each for expected utility during the work period.

With no worker stock holdings, the difference between these two terms is unambiguous. Let Q_E and Q_E^i represent Q for the ESOP and the integrated ESOP, respectively. Then $Q_E^i - Q_E = r^2 \tau^2 \sigma_w^2 / 2\sigma_2^2 < 0$, the integrated ESOP is always superior to the ESOP.

In simulations with no worker stock holdings, the integrated ESOP also compares more favorably to the conventional pension and the anti-ESOP. However, the difference is only marginal. Table 7 reports that the

integrated ESOP is superior to the conventional pension for approximately 5000 more parameter combinations than is the ESOP. It enjoys a similar advantage with respect to the anti-ESOP. However, this advantage is less than 1.3% of all parameter combinations. With no employee stock holding, the integrated ESOP is superior to the ESOP, but remains inferior to the conventional pension and the anti-ESOP.

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Table 7

Comparative performance of the integrated ESOP and the ESOP with no worker holdings

compared to the:	Parameter combinations in which the		integrated	
	ESOP is superior		ESOP is superior	
	Number	%	Number	%
Conventional pension	26576	6.80%	31598	8.09%
Anti-ESOP	141985	36.3 %	146585	37.5 %

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Nonnegative worker stock holdings improve the performance of the integrated ESOP, just as they do that of the ESOP, because the worker can again compensate for the imbalances in the integrated ESOP portfolio. With nonnegative worker holdings, the integrated ESOP is superior to the ESOP for 280,530, or 71.8% of all parameter combinations. The two are equivalent for all other combinations.

In general, nonnegative worker holdings also improve the

performance of the conventional pension and the anti-ESOP.

Table 8

Comparative performance of the integrated ESOP and the ESOP with nonnegative worker holdings

Parameter combinations in which the								
comp- pared to the:	ESOP is superior		ESOP is equivalent		inte- grated ESOP is superior		inte- grated ESOP is equivalent	
	Number	%	Number	%	Number	%	Number	%
Conven- tional pension	143676	36.8%	115625	29.6%	265979	68.1%	7410	1.90%
Anti- ESOP	41118	10.5%	153411	39.3%	136496	34.9%	75525	19.3%

Nevertheless, the integrated ESOP compares more favorably to these two plans than does the ESOP. Table 8 reports that the integrated ESOP is superior to the conventional pension for nearly twice as many parameter combinations as is the ESOP. It is superior to the anti-ESOP for more than three times as many combinations. The integrated ESOP is as equivalent or superior to the conventional pension and the anti-ESOP for more combinations than is the ESOP.

IV.6.3. The integrated anti-ESOP

These comparisons suggest that, if the integrated conventional

pension is impracticable, the integrated ESOP would be a useful alternative. However, integration also improves the anti-ESOP. Optimal stock holdings for the integrated anti-ESOP are

$$A_1^{pi} = d(\alpha_1 / r r \sigma_1^2 - \sigma_{1w} / \sigma_1^2)$$

$$A_{1R}^{pi} = A_1^p$$

$$A_2^{pi} = A_{2R}^{pi} = 0$$

$$A_1^{wi} = C_1^p + C_1^w - A_1^{pi}$$

$$A_2^{wi} = C_2^p + C_2^w$$

$$A_{1R}^{wi} = C_1^p - A_1^p$$

$$A_{2R}^{wi} = C_2^p$$

With no constraints on worker holdings, the integrated anti-ESOP is also a first-best. With no worker holdings, the integrated anti-ESOP is unambiguously superior to the anti-ESOP. $Q_A^i - Q_A = r^2 \tau^2 \sigma_{1w}^2 / 2 \sigma_1^2 < 0$, where Q_A^i and Q_A represent the terms evaluating expected working period utility for the integrated anti-ESOP and the anti-ESOP, respectively.

With no worker holdings, the integrated anti-ESOP is also as frequently superior to the integrated ESOP as is the anti-ESOP to the ESOP. In this case, the anti-ESOP yields higher expected utility than the ESOP for 248,640, or 63.7% of all parameter combinations. The integrated anti-ESOP yields

higher expected utility than the integrated ESOP for 248,480, or 63.6% of all parameter combinations.

It is only in the case with nonnegative worker holdings that integration significantly reduces the relative advantage of the anti-ESOP with respect to the ESOP. In this case, the anti-ESOP is superior to the ESOP for 196,096 parameter combinations (50.2%) and equivalent for 153,411 combinations (39.3%). With the same constraint, the integrated anti-ESOP is superior to the integrated ESOP for 95,479 combinations (24.4%) and equivalent for 231,219 combinations (59.2%).

IV.7. Evaluation

In the relatively few circumstances which favor the ESOP over other pension plans, it performs as its advocates predict. If the worker is constrained to hold no stocks, the ESOP would be the worker's sole asset in the 5.78% of parameter combinations for which the ESOP is superior to both the conventional pension and the anti-ESOP. The integrated ESOP would be the worker's sole asset in the 36.3% of combinations for which the integrated ESOP is superior to the integrated anti-ESOP.

Of course, in these cases, other assets are prohibited by definition. However, if the worker's stock holdings must be

nonnegative, the ESOP constitutes the bulk of aggregate assets where it yields the highest expected utility. With this constraint, the ESOP is superior to both the conventional pension and the anti-ESOP for 3.94% of all parameter combinations. For these combinations, the ESOP portfolio constitutes 98.1% of the aggregate portfolio during the working period. For the 16.4% of combinations where the integrated ESOP is superior to the integrated anti-ESOP, the integrated ESOP portfolio constitutes 84.2% of the aggregate working period portfolio.

Instead, the ESOP contradicts its advocates predictions because it is superior in so few circumstances. Furthermore, these circumstances are extreme. On average, the expected return to stock 2 is 1.82 times that of stock 1, and the coefficient of variation for stock 2 is only 35.6% of that of stock 1 in the parameter combinations for which the ESOP yields the highest expected utility with no employee holdings. The same statistics are 1.71 and 37.4% in the parameter combinations for which the ESOP yields the highest expected utility with nonnegative employee holdings, 1.73 and 34.4% in the parameter combinations for which the integrated ESOP is superior to the integrated anti-ESOP with no employee holdings, and 1.53 and 54.3% in the parameter combinations for which the integrated ESOP is superior to the integrated anti-ESOP with nonnegative employee holdings, ESOP advocates

might suggest that these parameter combinations are not as extreme as they appear. The purported 'incentive effects' of ESOPs are frequently cited as their principal virtue. A worker covered by an ESOP has an incentive to be more productive in order to increase the value of the ESOP portfolio, precisely because the worker's assets are limited to stock of the employer.

In effect, α_2 may be a positive function of the amount or share of stock 2 in the worker's portfolios. Workers may not be able to take advantage of this relationship individually, because it presumably requires all workers to cooperate both in portfolio choice and productivity. An ESOP might still suffer from free rider problems with regard to productivity. However, it would internalize the externality arising from the effect of one worker's holdings of employer stock on returns to the employer stock held by another.

Actual ESOP incentive effects are unknown. Tentative evidence suggests that profits are no lower, and perhaps higher in companies with ESOPs. However, the differences are much smaller than those which ESOPs require in order to dominate the conventional pension and the anti-ESOP in the simulations above. Furthermore, any actual increase in profits under ESOPs is at least as likely to derive from reduced capital costs, conferred by favorable ESOP tax

treatment.

In addition, the arguments which predict that profitability will be higher at firms with ESOPs are entirely static and partial equilibrium. Dynamic and general equilibrium considerations suggest that increased expected returns due to productivity increases, if they occur, may be only temporary. In general, market forces should work to erode them.

As examples, if ESOPs truly increase profitability, many or all companies will eventually have to adopt them in order to survive. As more companies establish ESOPs, the competitive advantage associated with them will vanish, as will the differential excess returns. Similarly, if the stocks of companies with ESOPs offer higher returns, outside investors will drive expected excess returns down by bidding stock prices up.²¹

In sum, the ESOP is likely to be an unsatisfactory vehicle for worker savings. The conventional pension, though markedly better, can also be unsatisfactory if, on net, the correlation between the return to the pension portfolio and

²¹ Of course, the economy as a whole benefits if ESOPs increase productivity and all firms adopt them. However, there is no guarantee that the distribution of these benefits will yield increased expected utility to the worker.

the wage flow is high. The imperfections of both suggest that further reform of worker savings plans is possible.

The optimal policy, if the purpose is to maximize workers' expected lifetime utility, is to enable and encourage the provision of unrestricted pensions. The simplest way to attain this goal would be to replace employer-sponsored pensions with private portfolios. If employer-sponsored pensions are to persist, they should evaluate their portfolios at true values for R_{1w} and R_{2w} , rather than zero.

However, if the costs of assessing the correlations between wage flows and returns to other assets are too high, practicable savings plans must remain in the realm of the second-best. In the class of second-best plans, the anti-ESOP is an attractive alternative. The simulations of section IV.5 suggest that the anti-ESOP would maximize expected worker utility in a broad range of plausible circumstances.

REFERENCES

References for Essay I

Ben-Ner, Avner (1984) "On the Stability of the Cooperative Type of Organization" Journal of Comparative Economics, Vol.8, pp. 247-260.

Berman, Katrina (1982) "The United States of America: a Cooperative Model for Worker Management" in Frank H. Stephen (ed.) The Performance of Labor Managed Firms, St. Martin's Press, New York.

Bonin, John (1984) "Membership and Employment in an Egalitarian Cooperative" Economica, Vol. 51, pp.295-305.

Ellerman, David (1984) "Workers' Cooperatives: The Question of Legal Structure" in Robert Jackall and Henry M. Levin (ed.s) Worker Cooperatives in America, University of California Press, Berkeley.

Jensen, Michael and Meckling, William (1979) "Rights and Production Functions: An Application to Labor-managed Firms and Codetermination" Journal of Business, Vol. 52, pp. 469-506.

Miyazaki, Hajime (1984a) "Internal Bargaining, Labor Contracts, and a Marshallian Theory of the Firm" American Economic Review, Vol. 74, pp. 381-393.

Miyazaki, Hajime (1984b) "On Success and Dissolution of the Labor-Managed Firm in the Capitalist Economy" Journal of Political Economy, Vol. 92, pp. 909-931.

Miyazaki, Hajime and Neary, Hugh (1983) "The Illyrian Firm Revisited" Bell Journal of Economics, Vol. 14, pp. 259-270.

Robinson, Joan (1967) "The Soviet Collective Farm as a Producer Cooperative: Comment" American Economic Review, Vol. 57, pp. 222-223.

Sertel, Murat (1982) Workers and Incentives, North-Holland, Amsterdam.

Sertel, Murat (1987) "Workers' Enterprises Are Not Perverse" European Economic Review, Vol. 31, pp. 1619-1625.

Spinnewyn, Frans and Svejnar, Jan (1987) "Optimal Membership, Employment and Income Distribution in Unionized, Participatory and Labor-Managed Firm", unpublished.

Ward, Benjamin (1958) "The Firm in Illyria: Market Syndicalism" American Economic Review, Vol. 49, pp. 566-589.

References for Essay III

Arrow, K. and Hahn, F. (1971) General Competitive Analysis. Holden-Day, San Francisco.

Beja, A., and Goldman, M. B. (1980) "On the Dynamic Behavior of Prices in Disequilibrium," Journal of Finance, pp. 235-248.

Beja, A., and Hakansson, N. (1977) "Dynamic Market Processes and the Rewards to Up-to-Date Information," Journal of Finance, 32.

Black, F. and Jones, R. (1987), "Simplifying Portfolio Insurance," The Journal of Portfolio Management, Fall, pp. 48-51.

Black, F., and Scholes, M. (1973) "The Pricing of Options and Corporate Liabilities," Journal of Political Economy, 81, pp. 637-654.

Bogoliubov, N. N., and Mitropolsky, Y. A. (1961) Asymptotic Methods in the Theory of Non-linear Oscillations, Gordon and Breach, New York.

Bookstaber, R., and Langsam, J. (1988) "Portfolio Insurance Trading Rules," Journal of Futures Markets, 8, pp. 15-31.

Brady, N. et.al. (1988) Report of the Presidential Task Force on Market Mechanisms, Washington.

Brennan, M.J. and Solanki, R. (1981) "Optimal Portfolio Insurance," Journal of Financial and Quantitative Analysis, 16, 279-300.

Cox, J.C. and Rubinstein, M. (1985) Options Markets, Prentice-Hall, New York.

Edwards, F. R. (1987) "Financial Futures and Cash Market Volatility," Columbia Futures Center, WP159.

Edwards, F. R. (1988) "Policies to Curb Stock Market Volatility," Columbia Futures Center, WP176.

Gammill, J.F. and Marsh, T.A. (1988) "Trading Activity and Price Behavior in the Stock and Stock Index Futures Markets in October 1987," The Journal of Economic Perspectives, 2, pp. 25-44.

Grossman, S. J. (1987) "An Analysis of the Implications for Stock and Futures Price Volatility and Dynamic Hedging Strategies," Columbia Futures Center, WP158.

Kamara, A. (1982) "Issues in Futures Markets: A Survey," Journal of Futures Markets, 2, pp. 261-294.

Kim, G. and Markowitz, H., (1988) "Investment Rules, Margin and Market Volatility," Baruch College, working paper.

Levinson, N. and Smith, O. K. (1942) "A General Equation for Relaxation Oscillations," Duke Mathematical Journal, IX, pp. 382-403.

MacKinlay, C. and Ramaswamy, K. (1987) "Program Trading and the Behavior of Stock Index Futures Prices," Columbia Futures Center, WP161.

Peck, A. E. (1985) "The Economic Role of Traditional Commodity Futures Markets," in A. E. Peck (ed.) Futures Markets: Their Economic Role, American Enterprise Institute.

Perold, A.F. (1986) "Constant Proportion Portfolio Insurance," working paper, Harvard Business School.

Perold, A.F. and Sharpe W.F. (1988) "Dynamic Strategies for Asset Allocation," Financial Analysts Journal, January-February, pp. 16-27.

Rubinstein, M. (1988) "Portfolio Insurance and the Market Crash," Financial Analysts Journal, January-February, pp. 38-47.

Stoll, H. R. (1987) "Index Futures, Program Trading, and Stock Market Procedures," Columbia Futures Center, WP160.

Stoll, H. R. (1989) "Inferring the Components of the Bid-Ask Spread," Journal of Finance, pp. 115-134.

Zabel, E. (1981) "Competitive Price Adjustment without Market Clearing," Econometrica, 49, 1201-1222.

References for Essay IV

Andrews, Emily S. (1985) The Changing Profile of Pensions in

America, The Employee Benefit Research Institute, Washington, D.C..

Brosnan, Peter (1975) "The Ability to Predict Workers' Preferences: Further Evidence", Human Relations, Vol. 28, No. 6, August, pgs. 519-541.

Bureau of Labor Statistics (1987) Employee Benefits in Medium and Large Firms, 1986, U.S. Department of Labor, Bulletin 2281, U.S. Government Printing Office, Washington, D.C..

The ESOP Association (1988a) ESOP Survey, 1988, Washington, D.C..

The ESOP Association (1988b) How the ESOP Really Works, Washington, D.C..

Federal Supplement, various issues, West Publishing Co., St. Paul, Minnesota.

Hutchinson, James D. and Charles G. Cole (1980) "Legal Standards Governing the Investment of Private Pension Capital", in Should Pension Assets Be Managed For Social/Political Purposes? Employee Benefit Research Institute, Washington, D.C., pgs. 28-93.

Merton, Robert (1969) "Lifetime Portfolio Selection Under Uncertainty: The Continuous-Time Case" Review of Economics and Statistics, Vol. L, August, pp. 247-257.

Merton, Robert (1971) "Optimum Consumption and Portfolio Rules in a Continuous-Time Model" Journal of Economic Theory, Vol. 3, pp.239-246.

Nealey, Stanley M. (1964) "Determining Worker Preferences Among Employee Benefit Programs", Journal of Applied Psychology, Vol. 48, No. 1, pgs. 7-12.

Nealey, Stanley M. and James G. Goodale (1967) "Worker Preferences Among Time-Off Benefits and Pay", Journal of Applied Psychology, Vol. 51, No. 4, pgs. 357-361.

Ressel, Susan S. (1986) "Tender is the Right: Responsibility for Tendering ESOP Shares Under ERISA", Virginia Tax Review, Vol. 6, No. 2, Fall, pgs. 561-591.