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The pure economic theory of default risk for sovereign debt

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THE PURE ECONOMIC THEORY OF DEFAULT RISK FOR SOVEREIGN DEBT.

by

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ABSTRACT

THE PURE ECONOMIC THEORY OF DEFAULT RISK FOR SOVEREIGN DEBT.

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Leonid Golubchin

Adviser: Professor Salih Neftci.

This paper presents a model of sovereign country default spreads, which takes into account stochastic movements of risk-free rates and production function. Default occurs at the first time when the value of objective function with default is larger than the value of objective function with borrowing and debt service. First, the easier and intuitive model is presented - the discrete one, and some numerical results on spreads are given. Then the general continuous time model is developed and an explicit formula for the probability of default is presented. This theory predicts positive correlations between risk-free interest rates and risk spreads which are reported for Brady bonds.

This thesis is dedicated to my wife Irina whose belief in my ability to complete it got me through the times I was not sure I could, to my children Vera and Isaac for their patience and encouragement, to my sister Lana and our parents whose unlimited love and support I felt every day.

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CONTENTS.

Introduction.

Recently a new kind of Sovereign Debt Obligations were introduced to capital markets. They were created through the exchange of existing commercial bank loans to foreign entities for new obligations in connection with debt restructuring under a plan introduced by former US Secretary of Treasury Nicholas F. Brady (the "Brady Plan"). These debt obligations, customarily referred to as "Brady bonds", are actively traded in the over-the-counter secondary market. Restructuring arrangements have included, among other things, reducing and rescheduling interest and principal payments by negotiating new or amended credit agreements or converting outstanding principal and unpaid interest to Brady bonds, and obtaining new credit to finance interest payments. Brady Plan debt restructurings totaling more than $80 billion have been implemented to date in Argentina, Bolivia, Costa Rica, Mexico, Nigeria, the Philippines, Uruguay and Venezuela with the largest proportion of Brady bonds having been issued to date by Argentina, Mexico and Venezuela. Brazil has announced plans to issue Brady bonds in respect of approximately $44 billion of bank debt.

Dollar-denominated, Brady bonds, which may be fixed rate par bonds or floating rate discount bonds, are generally collateralized in full as to principal due at maturity by US Treasury zero-coupon obligations which have the same maturity as the Brady bonds. Interest payments on these Brady bonds generally collateralized by cash or securities in an amount that, in the case of fixed rate bonds, is equal to at least one year of rolling interest payments or, in the case of floating rate bonds, initially is equal to at least one year's rolling interest payments based on the applicable interest rate at that time and is adjusted at regular intervals thereafter. Certain Brady bonds are entitled to "value recovery payments" in certain circumstances, which in effect constitute supplemental interest payments but generally are not collateralized.

There is a history of defaults with respect to commercial bank loans by public and private entities of countries issuing Brady bonds. Expropriation, nationalization, confiscatory taxation, political, economic or social instability or other similar developments, such as military coups, have occurred in the past in these countries. Issuers of Sovereign Debt Obligations or governmental authorities that control the repayment of Sovereign Debt Obligations may be unable or unwilling to repay principal or pay interest on the instruments when due, or may request that debt be rescheduled or that holders of debt extend further loans. Therefore, investments in Brady bonds are viewed as speculative and rated as "junk bonds". Brady bonds are often viewed as having three or four valuation components: i) the collateralized repayment of principal at final maturity; ii) the collateralized interest payments; iii) the uncollateralized interest payments; iv) any uncollateralized repayment of principal at maturity. These uncollateralized amounts constitute the "residual risk". In the event of default with respect to Collateralized Brady bonds as a result of which the payment obligations of the issuer are accelerated, the US Treasury zero-coupon obligations held as a collateral for the payment of principal will not be distributed to investors, nor will such obligations be sold and proceeds distributed. The collateral will be held by the collateral agent to the scheduled maturity of the defaulted Brady bonds, which will continue to be outstanding, at which time the face amount of the collateral will be equal to the principal payments which would have then been due on the Brady bonds in the normal course. Designed as "exit" instruments Brady bonds will not be subject to future new money requests or restructuring, therefore they represent a good object for studying the risk of repudiation by sovereign countries.

While the theory of default risk on corporate debt has been developed to some satisfactory degree, it is not obvious how to deal with the default risk for Sovereign Debt Obligations. This work makes an attempt to explain the risk of repudiation by sovereign countries using purely economic reasons such as stochastic movements of risk-free interest rates and production function. The first part of the paper is a theoretical approach to risk spreads where it is argued that if the risk-free rate becomes unexpectedly high then the *best strategy* for the country is to default. If the theory is right then the movements in the risk spread must be positively correlated with the movements in interest rates. These correlations for Brady bonds are reported in the second part of this work.

Literature Review.

The current literature pertaining to the risk of lending to sovereign countries can be grouped into three major areas of research. The first comes from the theory of finance where sophisticated models for security pricing are developed. The next is a theoretical and empirical analysis of the nature of credit relations between developed and developing countries. The third part overviews the international lending in historical perspective.

While it is absolutely necessary to distinguish between the reasons for bankruptcy of an individual economic agent and default by government, it is still worthwhile to consider the basic model of pricing corporate bond and examine its possibilities and limitations in applications to sovereign debt. The original work which tackles the problem of pricing bond when there is a significant probability of default risk is Merton [1974]. He has shown that the risk structure of interest rates can be modeled in a contingent claims valuation framework. The following is a brief summary of the Merton model. First, a process for the total value of the firm V is postulated. Merton valuation model assumes that it follows a Geometric Brownian motion. Second, explicit assumptions about how and when bankruptcy occurs are made. The firm is assumed to have a single debt issue outstanding, which is a zero-coupon bond which matures T years from now. If r is a default-free interest rate, than the price of a one dollar default-free bond is $P(r, t) = e^{-rt}$. While r is assumed to be deterministic in original Merton model it should not be interpreted as such. Also, let y be the yield on bond subject to default risk and F be the face value of debt. Then the bond price is $B = Fe^{-yT}$. When the bond is due, the firm's assets are worth V_T. If $V_T \leq F$, the bond is in default and the bondholder receives the assets. If $V_T > F$, the bond is paid off and the bondholder receives the F. In this manner, the bond is equivalent to a call option on the asset of the firm. Then the Black and Scholes [1973] theory of option pricing applied. As an insurance against default, the bondholder purchase a European put option on the assets of the firm. The exercise price is F, and the put price p(V, F, T). If $V_T < F$, the firm defaults on the bond, but bondholder exercises the put, bringing a payoff of V_T from the bond and $F\text{-}V_T$ from the put for a total payoff of F. If $V_T \geq F$, the bond is paid off and the put expires, worthless yielding a payoff of F from the bond and zero from the put for a total payoff of F. Since the portfolio containing the risky bond and the put is riskless, its current value must be FP(r, T), so that B + p(V, F, T) = FP(r, T). Then the value of corporate bond is equal to an equivalent riskless bond minus the value of a put option on the value of firm with strike price equal to the face value of the debt. The holder of a risky bond thus accepts the risk of the firm's assets and receives compensation in the form of the put premium.

The approach described above has been used and extended by many authors, e.g. Black and Cox [1976], Ingersoll [1977 a and b], Brennan and Schwartz [1977], Geske [1977 and 1979] and others. Recent developments in the theory concern the way of modeling the occurrence of default. The idea is that in reality default occurs whenever the firm is unable to meet any payment on any of the debt issues outstanding, or when it fails to meet some criterion stipulated in the covenants that form part of some of these debt contracts, and each of these events may happen at any time. A very general approach to a valuation model for corporate fixed-income securities in which default may occur at any instant is developed in Nielsen, Saa-Requejo and Santa-Clara [1993]. They compute the yield spread for zero-coupon bonds as a function of maturity, and decompose the price differential between riskless and risky bond (to be referred to as the term structure of default spreads) explicitly into a product of the probability of default and the present value of the loss suffered by bondholders in case of default. Specifically, they assume the instantaneous interest rate follows some diffusion process, therefore the price process of any riskless bond follows a diffusion process. It is also assumed that the market value of the asset of the firm also follows some diffusion process. The default is triggered simultaneously for all debt issues the first time the value of the asset reaches a critical level, called the default barrier. This critical level changes over time according to a compound diffusion process which implies a correlation between the default boundary and the interest rate. This model was shown to produce a variety of shapes for the term structure of default spreads. For reasonable parameter values the spreads obtained were consistent with spreads observed in practice.

There were few attempts to adapt option pricing methodologies to the problem of pricing sovereign debt, e.g. models used by Genotte, Kharas and Sadeq [1987], Dixit and Bartolini [1991], and Cohen [1993]. These models are based on the assumption that sovereign debtors are analogous to corporations and that creditors receive a share of the debtor's GDP up to the amount they are owed, so that GDP becomes the sovereign debtor's analog of a corporation's earnings (Stiglitz and Weiss [1981]). Recently this approach to sovereign debt valuation was implemented by Claessens and van Wijnbergen [1993] to Mexico' Brady deal. The main issue for option pricing is to specify the underlying stochastic process driving Mexico's repayments to the commercial banks. The following procedure for deriving the stochastic process for the net amount of financing available each period to service foreign commercial bank debt has been used. First, they model nonoil exports, import requirements, and net scheduled capital in- or outflows consistent with the constraints imposed by domestic economic and political factors to obtain the expected nonoil, noninterest current account. Second, the nonoil, noninterest current account is adjusted for debt service to more senior claim holders, for foreign direct investment flows and for capital account transactions such as reserve accumulation. Third, oil earnings are added to the flows. In accordance with large share of oil in total exports and the very high variance of oil prices, the behavior of oil exports introduces the stochastic element in the net amount of financing available to service commercial bank debt. With this framework the option pricing model can be introduced. Each period Mexico will pay to its creditors as much as available financing allows, but never more than its contractual obligations in the period:

$$R'_t = min[R_t, FX_t],$$

where R'_t is the actual repayment, R_t is the contractual obligation, and FX_t is the resources available to service commercial debt, all in period t. This equation can be rearranged as:

$$R'_t = R_t - max[0, R_t-FX_t].$$

But $max[0, R_t-FX_t]$ equals the value of a put option written on FX_t with a strike price of R_t. Thus, uncertain repayment can be represented by a certain repayment R_t minus a put. Then the current value of this stream is:

$$V(R'_t) = exp(-rt)R_t - P(R_t, FX_t, r, t, \sigma),$$

where r is the (continuously compounding) interest rate and $P(R_t, FX_t, r, t, \sigma)$ is the current value of a put written on FX_t with exercise price R_t, interest rate r, maturity t, and standard deviation σ. Then the current value of a loan with a series of contractual obligations R_t coming due over time (where R_t can be different for each period depending on the terms on the loan) is the sum of the current values of a series of these claims over the maturity of the contract.

All these models for pricing sovereign debt have one important deficiency. In case of an individual economic agent in a national economy, bankruptcy usually reflects negative worth. Bankruptcy laws provide an institutional framework defining this condition, and creditors are compensated to the extent that assets allow. Domestic bankruptcy laws prohibit an agent to maintain full control of his assets while shedding his liabilities. Therefore traditional concepts of solvency and liquidity are of little help in understanding problems of sovereign debt. Countries can not be sold, restructured, or be taken over. Unless the governments of private creditors are willing to coerce debtor governments into servicing debt, there is no explicit mechanism deterring a government from repudiating its external debt. Different approaches were developed in order to explain the existence of private loans to foreign government. Eaton and Gersovitz [1981] argue that a country's ability to pay depends on the national wealth, which is considerably larger than the debt outstanding, and therefore it is a willingness to pay that matters. In this case default and bankruptcy are possible strategies in financial transactions. They consider a multistaged game-theoretic model with two participants - the borrower and the lender. There is an endogenous penalty imposed by the lender such that in case of default the borrower is to be excluded from future loans. If the game is finite then the loan would not be repaid at the last period, then the lender knowning this would not lend money in the previous period and so on by backward induction. Therefore no money would be lent in the finite game models. Hence the game must be infinite, and it is only when the future always holds some possibility of transfers in *both* directions, the penalty becomes effective. Some support for this model is found in Jorgensen and Sachs [1991] where the history of international capital movements is characterized as intermittent bond defaults to be a normal cyclical occurrence for Latin America by the late nineteenth century. As a rule those defaults were quickly followed by settlements in order to get back to usual business of shifting capital from Europe, especially Britain, to the periphery.

Since sovereign loans are owed by the governments of countries, repayment is not constrained by net worth of the country, but by that component of net worth that the government can (or is willing to) appropriate, because raising taxes could push the economy towards the subsistence wage level, social uprisings and political coups. This line of research is discussed in Eaton, Gersovitz and Stiglitz [1986]. Another aspect of debt-servicing is large private outflows of capital from developing countries (Cumby and Levich [1987]), because capital flight might be an important source of potential funds for credit-constrained debtor countries. The formal demonstration that countries with a large inherited debt may run into debt-servicing difficulties if the private sector is exporting capital when debt must be repaid by taxing wage income could be found in Charrette [1991]. As Bulow and Rogoff [1990] point out, high interest rates abroad, combined with low commodity prices and misguided economic policies discourage HIC's (Highly Indebted Country) citizens from making domestic investments. They argue, that if one aggregates the government and private sectors, some Latin American HICs may even be net creditors.

Some issues of international borrowing and lending can be clarified by investigation of earlier historical experience. Eichengreen and Lindert [1991] have identified several recurrent periods of debt crisis over a historical time, with the 1820s, 1880s, 1900 - 1914, and the 1920s serving as prominent examples. All these "waves" share similar characteristics from which lessons can be drawn. Each past lending wave has always combined with an economic or political shock to form a debt crisis, and nothing in the longer historical trend suggests any future immunity. One of the most striking feature of individual "country risk" is that it does not depend on the past performances. Several generations of history nominate Bolivia, Brazil, Chile, Peru, Romania, Turkey and Uruguay among others as consistent defaulters. Argentina was only one of the Latin American countries in the 1930s that maintained full servicing on its national debt; Bolivia, Chile, Colombia and Peru defaulted. According to calculations of Jorgensen and Sachs [1991] Argentina paid out 1.25 times in present value terms for the moneys it borrowed abroad. Compare it to Bolivia - 0.54, Chile - 0.56, Colombia - 0.85 and Peru - 0.52. Nevertheless Argentina did not get any preferences in terms or rates in subsequent borrowing. Cardoso and Dornbusch [1991] make a similar point that from the 1930s to the 1960s Brazil, the defaulter, had no more trouble borrowing than the faithful repayer Argentina. Again, the same point is made by Fishlow [1991] about Argentina and Brazil in the late nineteenth century. One possible explanation for this lies in creditors' willingness to lend even to countries with a bad history of default if their governments adopt monetary and fiscal policies consistent with sustained economic growth.

PART A

THE THEORY OF SOVEREIGN DEBT RISK.

DISCRETE MODEL OF BORROWING WITH DEFAULT.

Section I. ***Theoretical Setting.***

The following is a theoretical approach to the term structure of default risk spreads in the world with uncertainty. The main idea of this section is to show how to construct utility-maximizing willingness-to-pay model for a debtor country with an option to default, and define the optimal strategy for consumption and investment at any time. The formal setting is similar to Eaton and Gersovitz [1981] and Cohen and Sachs [1986] but economic meaning is different from the both. Let y_t be a net output in period t. It is not storable. There is no population growth and capital depreciates fully. Define:

c_t - consumption at time t

b_t - borrowing at time t

k_t - capital to invest into production at time t

p_t - debt service payment at time t

r_t - external default-free rate at time t (e.g. Treasury rate)

Also let q_t be a penalty imposed for defaulting, e.g. the embargo on future borrowing and cutoff of aid or retaliatory interference by the creditors or their governments with commodity trade.

The borrower's objective function to maximize is

$$\mathbf{E}[\sum_{t=0}^{\infty} \beta^t u(c_t-q_t)]; \qquad 0<\beta<1$$

with the standard Inada condition on utility function $u(\bullet)$:

$u'(0) = \infty, \quad u'(\infty) = 0,$

$u' > 0, \quad u'' < 0;$

and subject to identity:

$c_t + k_{t+1} + (1 + i_{t-1})b_{t-1} = y_t(k_t, b_t,)$

where i_t is the interest rate at which b_t is contracted and the debt service obligation at time t is $(1 + i_{t-1})b_{t-1}$. In case of debt payments falls short of debt obligation the country is no longer allowed to borrow and suffers from imposed penalty. Hence the debtor will choose either $p_t = 0$ or $p_t = (1 + i_{t-1})b_{t-1}$.

Define the value of objective function at time t given a decision to default in t as:

$$v_t^d = max_{\{c_t\}} E[\sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_\tau - q_\tau)];$$

and the value of objective function at time t given a decision not to default in t as:

$$v_t^{nd} = sup_{\{c_t\}} \{u(y_t(k_t, b_t) - (k_{t+1} + (1 + i_{t-1})b_{t-1})) + \beta E max[v_{t+1}^d, v_{t+1}^{nd}];$$

Default is optimal in time t if and only if $v_t^d > v_t^{nd}$. Define probability λ_t of default in period t as anticipated at time $t-1$ as $\mathbf{Pr}[v_t^d > v_t^{nd}]$. Lenders are assumed to be risk-neutral and they either lend to the country at i_t or, alternatively, invest at default-free rate r_t. Therefore, lenders will only make loans which guarantee them the expected rate of return at least as high as the market interest rate, i.e.

$$(1 - \lambda_t)(1 + i_t)b_t = (1 + r_t)b_t$$

or

$$(1 + i_t) = (1 + r_t)/(1 - \lambda_t)$$

The sequence $\{ \lambda_t \}_{t=0}^{\infty}$ is called a term structure of risk spreads. The purpose of this work is to characterize the procedure of finding this sequence of risk spreads. Now the dynamic problem might be written as:

$$v_t = sup_{\{c_t\}} \{u(y_t(k_t, b_t,) - (k_{t+1} + (1 + i_{t-1})b_{t-1})) + \beta E max[\lambda_{t+1} v_{t+1}^{d} + (1 - \lambda_{t+1}) v_{t+1}^{nd}]\};$$

Note, that if the penalty is very severe (e.g. output in time $t+1$ would be lost for the debtor) then the probability of default $\lambda_{t+1} = 0$ and lenders would provide loans at default-free rate r_t. On the other hand, the absence of penalty guarantees default, $\lambda_{t+1} = 1$ and contracted rate i_t would go to infinity.

In order to simplify the problem make some specific assumptions. Some of these assumptions will be relaxed later in a more general continuous-time setting. First, the only penalty for default is the exclusion from world financial markets, so the country will be in the state of financial autarky. Indeed, as it is pointed out by Jorgensen and Sachs [1991], the defaults during interwar period were greatly disruptive to capital inflows to Latin America, with some bonds evading permanent settlement for decades. It appears that penalties for choosing to default in the 1930 were severe. Access to international capital markets was closed for decades and capital flow did not reappear until the 1970s. Then, the production function is assumed to be Cobb-Douglas one: $y_t(k_t, b_t) = Ax_t^{\alpha}$, where x_t is an input. The last, the utility function is assumed to be a natural logarithm. The problem now is digressed to:

$$v_t = sup_{\{c_t\}} \{ln(A(k_t + b_t)^\alpha - (k_{t+1} + (1 + i_{t-1})b_{t-1})) + \beta \mathbf{E} max[\lambda_{t+1} v_{t+1}^d + (1 - \lambda_{t+1}) v_{t+1}^{nd}]\};$$

where:

$$v_{t+1}^d = \mathbf{E}[\sum_{\tau=t+1}^{\infty} \beta^{\tau-(t+1)} ln(Ak_\tau^\alpha - k_{\tau+1})],$$

which means that the optimal path for a country is to default and stay in financial autarky thereafter;

and

$$v_{t+1}^{nd} = \mathbf{E}[\sum_{\tau=t+1}^{\infty} \beta^{\tau-(t+1)} ln(A(k_\tau + b_\tau)^\alpha - (k_{\tau+1} + (1 + i_{\tau-1})b_{\tau-1}))],$$

which means that it is not optimal for country to default right now and decision is postponed for the next period.

Assume no uncertainty for a while (shall introduce it later). Then the optimal path for a country in financial autarky which maximizes the objective function in this setting, i.e.

$$v_{t+1}^d = \sum_{\tau=t+1}^{\infty} \beta^{\tau-(t+1)} ln(Ak_\tau^\alpha - k_{\tau+1}),$$

is well known (e.g. Stokey and Lucas [1989]) and given by:

$$k_t = \alpha\beta A k_{t-1}^\alpha$$

The next step is to find the optimal path for capital, borrowing and investment in a whole which maximizes the following objective function:

$$v_{t+1}^{nd} = \sum_{\tau=t+1}^{\infty} \beta^{\tau-(t+1)} ln(A(k_\tau + b_\tau)^\alpha - (k_{\tau+1} + (1 + i_{\tau-1})b_{\tau-1})).$$

Theorem: Under specified utility and production functions the optimal decision rule for capital, borrowing and investment in a whole are given by:

$$k_t = (((\alpha\beta)^{-1} - (1 + i_t))/i_t)\alpha\beta A(i_{t-1}k_{t-1} / ((\alpha\beta)^{-1} - (1 + i_{t-1})))^{\alpha}\times\Theta_t;$$
$$b_t = (((\alpha\beta)^{-1} - 1)/i_t)\alpha\beta A(i_{t-1}b_{t-1} / ((\alpha\beta)^{-1} - 1))^{\alpha}\times\Theta_t;$$
$$k_t + b_t = \alpha\beta A(k_{t-1} + b_{t-1})^{\alpha}\times\Theta_t;$$

where:

$$\Theta_t = (i_{t+1} - i_{t-1})/(i_{t+1} - i_t + \beta(1 + i_{t+1})(i_t - i_{t-1})).$$

Proof:

The first order conditions are:

$$\partial v / \partial k_{t+1} = 0 \Rightarrow$$
$$\alpha\beta A(k_{t+1} + b_{t+1})^{\alpha-1}[A(k_t + b_t)^{\alpha} - (k_{t+1} + ((1 + i_{t-1})b_{t-1})] = A(k_{t+1} + b_{t+1})^{\alpha} - (k_{t+2} + ((1 + i_t)b_t); \quad (*)$$

$$\partial v / \partial b_{t+1} = 0 \Rightarrow$$
$$\alpha\beta A(k_{t+1} + b_{t+1})^{\alpha-1}[A(k_{t+2} + b_{t+2})^{\alpha} - (k_{t+3} + ((1 + i_{t+1})b_{t+1})] = \beta(1 + i_{t+1})[A(k_{t+1} + b_{t+1})^{\alpha} - (k_{t+2} + ((1 + i_t)b_t)]; \quad (**)$$

Dividing (*) by (**):

$$A(k_{t+2} + b_{t+2})^{\alpha} - (k_{t+3} + ((1 + i_{t+1})b_{t+1}) = \beta(1 + i_{t+1})[A(k_t + b_t)^{\alpha} - (k_{t+1} + ((1 + i_{t-1})b_{t-1})].$$

Blackwell's sufficient conditions for a contraction are easily verified in this case (Stokey and Lucas[1989], p. 54-55), hence there exists an optimal path and steady state where:

$$k_{t-1} = k_t = k$$

and

$$b_{t-1} = b_t = b.$$

Assuming, the steady state is already reached,

$$A(k + b)^{\alpha} - (k + ((1 + i_{t+1})b) = \beta(1 + i_{t+1})[A(k + b)^{\alpha} - (k + ((1 + i_{t-1})b)];$$

or

$$A(k + b)^{\alpha} = k + wb;$$

where:

$$w = (1 + i_{t+1})(1 - \beta(1 + i_{t-1})) / (1 - \beta(1 + i_{t+1})).$$

Substituting this expression into (*):

$$\alpha\beta(k + wb) = (k + b)(w - (1 + i_t)) / (w - (1 + i_{t-1})) = (k + b)\ \Theta_t^{-1};$$

This gives the third expression.

More algebraic manipulations bring the first and the second expressions. ♦

Note that, if an interest rate is a constant through time, the optimal path for investment collapses to the optimal path for capital in autarky. In the absence of uncertainty the optimal path is determined and depends on short-term rates.

There are two ways to introduce uncertainty in this model explicitly. One way is to specify random shocks on technology $y_t(k_t, b_t, t) = z(t)Ax_t^{\alpha}$, where $z(t)$ are random technological shocks. Another way to introduce uncertainty is to apply a stochastic process on a short-term rate. It means that the source of randomness is a stochastic interest rate in international capital market, which is tied to US Treasury yields. This second way is adopted here. Therefore, all probabilities of default are due to stochastic behavior of interest rates. In a more general continuous time setting both kinds of uncertainty are considered.

Section II. ***Building a Trinomial Tree.***

This section shortly describes how to build a trinomial tree, upon which the objective functions will be computed.

The n-year spot interest rate is the interest rate on a pure investment without any intermediate payments lasting for n years. The zero-coupon yield curve is a curve showing the relationship between spot rates (i.e. zero-coupon yields) and maturity. Future interest rates are the rates of interest implied by current spot rates for periods of times in the future. In the reported statistics only the yields to maturity on available bonds are reported. Salomon Brothers regularly publishes an index of the annual yields to maturity on par US Treasury bonds. This is called the par yield curve. Given a complete range of yields across maturities for these bonds one can determine the underlying term structure of interest rates, i.e. to find the zero-coupon yield curve.

The procedure of translating the par-yield curve into the zero-coupon yield curve is the following:

For a par bond of arbitrary maturity k the price is

$$P_k = \sum_{j=1}^{k} y_k(1 + y_k)^{-j} + (1 + y_k)^{-k} = 1;$$

here $y_1,\dots,y_n$ are n par yields, given;

$k = 1, 2, \dots, n.$

On the other hand all payments (coupons and principal) might be viewed as a set of zero bonds with the same price computed as

$$P_k = \sum_{i=1}^{k} y_k(1 + z_i)^{-j} + (1 + y_k)^{-k};$$

here $z_1,\ldots, z_n$ are n zeroes.

If one knows $z_1,\ldots, z_{k-1}$ then it is easy to find z_k. Note that $z_1 = y_1$. This iterative algorithm for translating a par-yield curve into a zero-coupon yield curve is implemented by using the computer program. These estimated yields for a set of zero-coupon Treasury maturities together with volatility (assumed to be const for simplicity) is an input for a tree. The model used in this work is similar to one described in Black, Derman and Toy [1990]. The short-term rate r is described as a random variable: given its value today it will be uncertain tomorrow and more uncertain the next day. At each time in future, the uncertainty on r is characterized by a lognormal probability distribution specified by mean $\mu(r,t)$ and a standard deviation σ. Formally:

$$d\ln r = \mu(r,t)dt + \sigma dz;$$

where:

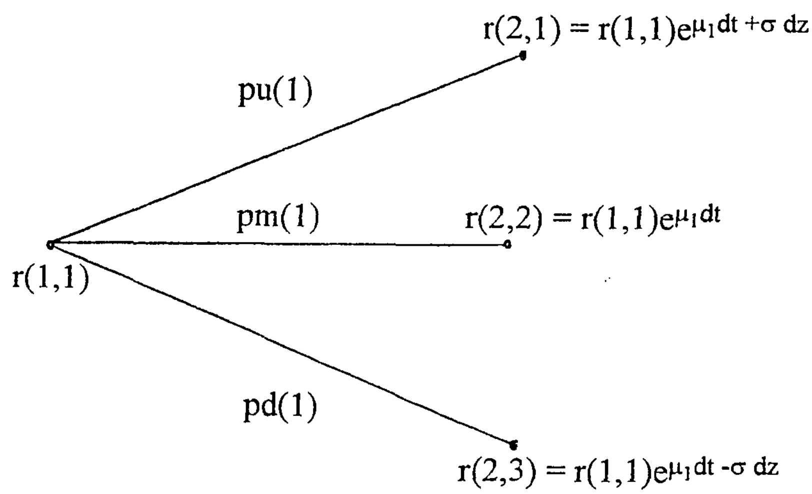
$\mu(r,t)$ is a non stochastic drift, the expected growth rate of the mean;

σ is a volatility, i.e. the standard deviation of proportional change in r, assumed to be const;

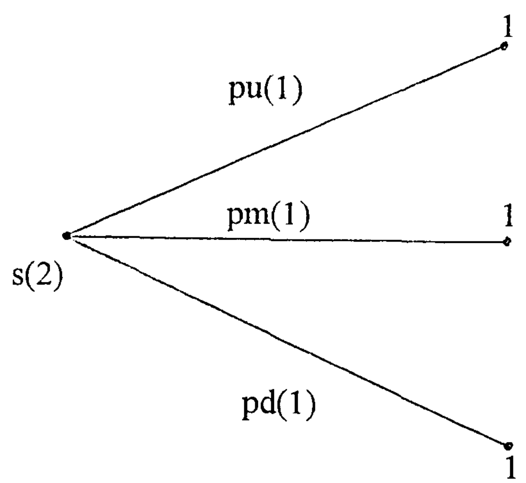
dz is a standard Wiener process, describes the normal distribution of random proportional change in r;

The one step tree is constructed as following (Pic. 1) (as a matter of fact, two trees are built simultaneously: a short-rate tree and a value tree branching out into the future):

tree of rates



tree of zero-coupon prices



Pic. 1

Here:

pu(1) - probability that rate will go up;

pm(1) - probability that rate will stay the same;

pd(1) - probability that rate will go down;

s(2) is a price of a two-period zero-coupon bond.

The problem, of course, is to find these probabilities on every step, then all probabilities and rates, associated with every node, will be known. To find three unknowns one needs to tie them in a system of three well-defined equations.

i. pu(1) + pm(1) + pd(1) = 1; (sum of probabilities is always equal to one).

In one period from now this two-period zero-coupon bond will be worth s(2)(1 + r(1,1)). But two periods from now it will mature and will be worth exactly its face value, no matter what the short rate r(2,j) will be. Therefore:

ii. S(2) = [pu(1)/(1 + r(2,1)) + pm(1)/(1 + r(2,2)) + pd(1)/(1 + r(2,3))] / (1 + r(1,1)).

For the third equation use the general definition of variance:

$$\mathbf{Var}X = \mathbf{E}[X^2] - (\mathbf{E}[X])^2;$$

Trinomial trees have a number of advantages in comparison to binomial trees. One of them in a better convergence. Another is in a possibility to establish the relation between d*ln*r and time step dt. Hull and White [1990 and 1991] is a good source for numerical procedures for building one-factor interest-rate models including trinomial trees. In particular, they have shown that the relation

$$d\,ln\,r = \sqrt{3}\,\sigma dz$$

ensures the very stable tree. Now, substitute d*ln*r for X:

$$\mathbf{Var}[d\,ln\,r] = 3\sigma^2 dt;$$

$$\mathbf{E}[d\,ln\,r] = \mu dt \;\Rightarrow\; (\mathbf{E}[d\,ln\,r])^2 = (\mu dt)^2;$$

$$\mathbf{E}[(d\,ln\,r)^2] = pu(\mu dt + \sqrt{3}\,\sigma dz)^2 + pm(\mu dt)^2 + pd(\mu dt - \sqrt{3}\,\sigma dz)^2 .$$

After simplification the third equation is:

iii. $pu(3\sigma + 2\mu\sqrt{3}\,dz) + pd(3\sigma - 2\mu\sqrt{3}\,dz) = \sigma.$

This general procedure described above is used to construct a trinomial tree which reflects "different states of the world" where every node has a possible value of short-term rate with the associated probability.

Section III. ***Expected Optimal Path.***

The expected optimal path for capital, borrowing and investment in a whole is constructed by implementation of the derived optimal decision rules for capital, borrowing and investment in a whole for every possible "state of the world":

$$k_t = (((\alpha\beta)^{-1} - (1 + r_t))/r_t)\alpha\beta A(r_{t-1}k_{t-1} / ((\alpha\beta)^{-1} - (1 + r_{t-1})))^{\alpha}\times\Theta_t;$$

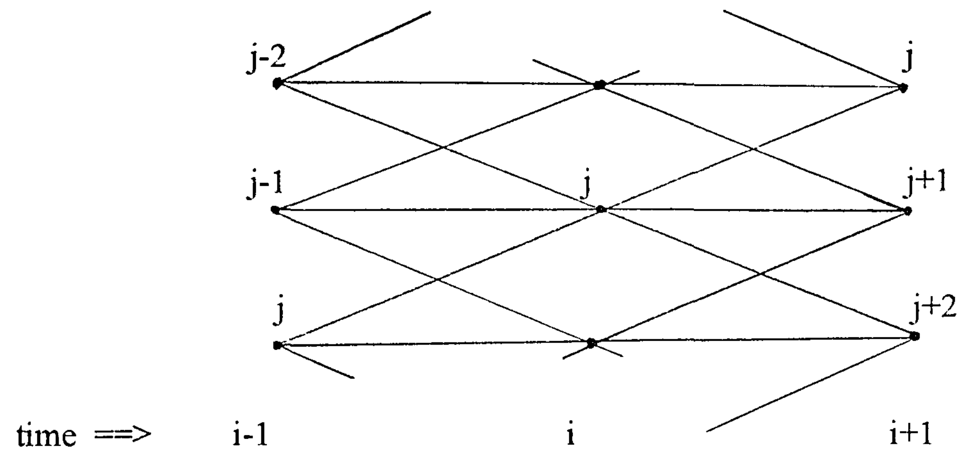
$$b_t = (((\alpha\beta)^{-1} - 1)/r_t)\alpha\beta A(r_{t-1}b_{t-1} / ((\alpha\beta)^{-1} - 1))^{\alpha}\times\Theta_t;$$

$$k_t + b_t = \alpha\beta A(k_{t-1} + b_{t-1})^{\alpha}\times\Theta_t;$$

where:

$$\Theta_t = (r_{t+1} - r_{t-1})/(r_{t+1} - r_t + \beta(1 + r_{t+1})(r_t - r_{t-1})).$$

Note, that all r_t's are known, since they are obtained from the tree. Therefore k_t, b_t, and $k_t + b_t$ can be easily computed for every node. The contractual rate i is substituted here by default-free rate r. This is justified, since ratios and differences in rates are playing major role. Assuming, that k_{i-1} and b_{i-1} are known (the initial conditions k_o and b_o are given), the borrower could find himself in any node j in time i (Pic. 2). At every node the rate r(i,j) and the probability to be in this node p(i,j) are known. To appear in node (i,j) is possible only from nodes (i-1,j-2), (i-1,j-1) or (i-1,j) with rates and probabilities to be there also known. Hence, the probability to come in node (i,j) from one of those nodes could be computed. Also conditional probabilities to go to nodes (i+1,j), (i+1,j+1) or (i+1,j+2) subject to be at node (i,j) could be computed.



Pic. 2.

So, to find an optimal path for capital, borrowing and investment in a whole is a two-step procedure. First, compute all possible optimal paths for capital, borrowing and investment in a whole in node (i,j) (in general there are nine paths) and find the expected (weighted average) optimal path for k(i,j), b(i,j) and k(i,j) + b(i,j). Then compute expected optimal path as a weighted average sum over all nodes:

$$k(i) = \sum_{j=1}^{2i-1} k(i,j)p(i,j);$$

and

$$b(i) = \sum_{j=1}^{2i-1} b(i,j)p(i,j).$$

This expected optimal path for capital, borrowing and investment in a whole could be viewed as a precommited path, since a country borrows in advance and pays a floating rate which tied to short-term risk-free rate.

Section IV. ***Risk Spreads.***

In order to find risk spreads one needs to calculate the value function with debt at every node and compare it with the value function in financial autarky. The whole procedure is performed backward. This is possible because the problem is finite horizon. The boundary condition is such that the value function beyond horizon is set to zero (this is justified since the utility function is bounded): $v_{T+1} = 0$. At any time i and for every node j the both value functions are computed. If at some particular node the value function with debt is less than the value function in financial autarky then the *default* is the *optimal strategy*. The sum of probabilities over those nodes where it is optimal to default is exactly the probability of defaulting in time i. To find the expected optimal path and risk spreads a computer program was written, which consists of two parts: the first part uses the described algorithm to build a trinomial tree; the second part utilize the algorithm to produce the expected optimal path and risk spreads.

The representative numerical results of implementation of the model developed above are the following. Imaging the following par yield curve:

time to maturity	**rate**
3m	0.03000
6m	0.03025
1y	0.03050
2y	0.03087
3y	0.03106
5y	0.03080
7y	0.03054
10y	0.03025
30y	0.03000

The volatility $\sigma = 15\%$

The production function is $y_t(k_t, b_t) = Ax_t^{\alpha}$;

where:

$A = 0.725$ and $\alpha = .87$.

The rate of time preference $\beta = .955$.

Then the risk spreads over seven years:

year	λ
0	0.0000
1	0.0000
2	0.0097
3	0.0000
4	0.0281
5	0.0148
6	0.0350
7	0.0389

One needs to be reminded that these numbers reflect only the pure economic risk, without so called "political considerations". Actual spreads are generally from 6% to 8%, not from 1% to 4%.

CONTINUOUS MODEL OF BORROWING WITH DEFAULT.

Section I. ***Optimal Growth in Autarky.***

In this section the analysis of optimal economic growth under uncertainty is developed, by deriving the intertemporal conditions that are satisfied on the optimal path that would be chosen by a central planner when no borrowing is permitted. As a result of this analysis the *optimal saving* policy function is derived. This policy function describes the optimal allocation of the output between consumption and investment. Similar model can be found in Malliaris and Brock [1982], without the rate of time preference discounting though.

Given:

$k(t)$ is a capital stock at time t, k_0 is initial capital stock;

$f(k)$ is a production function;

$s(k)$ is a marginal propensity to save;

$c(k) = (1 - s(k))f(k)$ is a consumption.

The equation of capital accumulation is

$$dk = s(k)f(k)dt + \sigma(k)dz.$$

Here dz is a standard Wiener process defined on some probability space (Ω, F, P) and $(dz)^2 = dt$.

Consider the social planner's problem:

$$max_{\{s\}}\ E_0[\int_0^{\infty} e^{-\beta\tau}\, u[(1 - s(k))f(k)]d\tau]$$

subject to

$$dk = s(k)f(k)dt + \sigma(k)dz$$

here u(•) is a strictly concave, von Neumann-Morgenstern utility function with standard Inada conditions:

$$u'(0) = \infty,$$

$$u'(\infty) = 0,$$

$$u' > 0,\ \ u'' < 0.$$

A standard technique for such a problem is *Bellman's Principle of Optimality* (Bellman [1957, p.83]) according to which "an optimal policy has the property that, whatever the initial state and control are, the remaining decision must constitute an optimal policy with regard to the state resulting from the first decision".

Let

$$J(k(t), t, T) = max_{\{s\}}\ E_t \int_t^{T} e^{-\beta\tau}\, u[(1 - s(k))f(k)]d\tau.$$

Following Malliaris and Brock [1982] write

$$W(k(t), t, T) = e^{\beta t}J(k(t), t, T)$$

and also

$$J_t = \frac{d}{dt}\{e^{-\beta t}W\} = -\beta e^{-\beta t}W + e^{-\beta t}W_t .$$

Then

$$W(k(t), t, T) = e^{\beta t} max_{\{s\}}\ \mathbf{E}_t \int_t^T e^{-\beta\tau}\, u[(1 - s(k))f(k)]d\tau =$$

$$e^{\beta t} max_{\{s\}}\ \mathbf{E}_t \left[\int_t^{t+dt} e^{-\beta\tau}\, u[(1 - s(k))f(k)]d\tau + \int_{t+dt}^T e^{-\beta\tau}\, u[(1 - s(k))f(k)]d\tau\right] =$$

$$e^{\beta t} max_{\{s\}}\ \mathbf{E}_t \int_t^{t+dt} e^{-\beta\tau}\, u[(1 - s(k))f(k)]d\tau +$$

$$e^{\beta t} max_{\{s\}}\ \mathbf{E}_t \int_{t+dt}^T e^{-\beta\tau}\, u[(1 - s(k))f(k)]d\tau =$$

$$e^{\beta t} max_{\{s\}}\ \mathbf{E}_t\left[\int_t^{t+dt} e^{-\beta\tau}\, u[(1 - s(k))f(k)]d\tau + e^{\beta t}J(k(t+dt), t+dt, T)\right] =$$

/ using Mean Value theorem for integrals /

$$max_{\{s\}}\ \mathbf{E}_t[u[(1 - s(k))f(k)]dt + e^{\beta t}J(k(t+dt), t+dt, T)] =$$

/ expanding into Taylor's series /

$$max_{\{s\}}\ \mathbf{E}_t[u[(1 - s(k))f(k)]dt + e^{\beta t}\{J(k(t), t, T) + J_k dk + J_t dt + \frac{1}{2}J_{kk}(dk)^2 +$$

$$\frac{1}{2}J_{kt}(dk)(dt) + \frac{1}{2}J_{tt}(dt)^2\}] =$$

$$max_{\{s\}}\ \mathbf{E}_t[u[(1 - s(k))f(k)]dt + W(k(t), t, T) + W_k dk + e^{\beta t}J_t dt +$$

$$\frac{1}{2}W_{kk}(dk)^2 + \frac{1}{2}W_{kt}(dk)(dt) + \frac{1}{2}W_{tt}(dt)^2] =$$

$$max_{\{s\}}\ \mathbf{E}_t[u[(1 - s(k))f(k)]dt + W(k(t), t, T) + W_k(s(k)f(k)dt + \sigma(k)dz) +$$

$$e^{\beta t}J_t dt + \frac{1}{2}W_{kk}(s(k)f(k)dt + \sigma(k)dz)^2 + \frac{1}{2}W_{kt}(s(k)f(k)dt + \sigma(k)dz)(dt) +$$

$$\frac{1}{2}W_{tt}(dt)^2] =$$

$$max_{\{s\}}\ \mathbf{E}_t[u[(1 - s(k))f(k)]dt + W(k(t), t, T) + W_k(s(k)f(k)dt + \sigma(k)dz) +$$

$$(-\beta W + W_t)dt + \frac{1}{2}W_{kk}(\sigma(k))^2 dt].$$

Pass $\mathbf{E}_t$ through the parenthesis, note that $\mathbf{E}_t[W_k\sigma(k)dz] = 0$, and after dividing by dt we get the following equation:

$$\beta W - W_t = max_{\{s\}}\{u[(1 - s(k))f(k)] + W_k s(k)f(k) + \frac{1}{2}W_{kk}(\sigma(k))^2\}$$

which is known as the *Hamilton-Jacobi-Bellman* equation of stochastic control theory.

The first order condition to be satisfied by the optimal policy s^* is

$$0 = u'[(1 - s^*)f(k)]f(k) - f(k)W_k\,,$$

which becomes

$$u'[(1 - s^*)f(k)] = e^{\beta t}J_k .$$

Note that u' means $\frac{du}{d(1-s)}$. To solve for the optimal policy s^*, in principle, one solves the first order condition as a function of t, T, and J_k, and then substitutes this solution into the *Hamilton-Jacobi-Bellman* equation for J. Once the *Hamilton-Jacobi-Bellman* equation is solved then its solution is substituted back into the first order condition to determine s^* as a function of k , t and T. The non linearity of the *Hamilton-Jacobi-Bellman* equation causes difficulties in finding a closed form solution. The way to overcome this difficulty in a given problem is by letting $T \to \propto$. In this case the partial differential equation is reduced to an ordinary differential equation. Suppose that optimal policy exists and is not a function of t as $T \to \propto$. Note, that in this case $W_t = 0$. Then

$$W(k(t), t) = e^{\beta t}J(k(t), t) = max_{\{s\}} \int_0^\infty e^{-\beta(\tau-t)} u[(1 - s^*(k))f(k)]d\tau =$$

$$\int_0^\infty e^{-\beta\xi} u[(1 - s^*(k))f(k)]d\xi = W(k) = C.$$

C here is a bliss level of utility associated with the maximum steady state consumption. Now the *Hamilton-Jacobi-Bellman* equation is converted into the ordinary differential equation of the second order for W(k):

$$u[(1 - s^*(k))f(k)] - \beta W(k) + s^*(k)f(k)W_k + \frac{1}{2}(\sigma(k))^2W_{kk} = 0.$$

Next, differentiate the first order condition with respect to capital k:

$$W_{kk} = u''[(1 - s^*(k))f(k)] ((1 - s^*(k))f'(k) - s^{*\prime}(k)f(k)).$$

Substitution of these expressions for W_k and W_{kk} into the *Hamilton-Jacobi-Bellman* equation gives:

$$u[(1 - s^*(k))f(k)] - \beta C + s^*(k)f(k)u'[(1 - s^*(k))f(k)] +$$

$$\frac{1}{2}(\sigma(k))^2 u''[(1 - s^*(k))f(k)] ((1 - s^*(k))f'(k) - s^{*\prime}(k)f(k)) = 0$$

or, suppressing notation for some dependent variables

$$\frac{1}{2}(\sigma(k))^2 u'' + s^*(k)f(k)u' + u = \beta C.$$

It is important to note that in case of no uncertainty, that is $\sigma = 0$, this equation collapses to

$$s^*(k)f(k)u' + u = \beta C$$

which is commonly written as:

$$s^*(k)f(k) = \frac{\beta C - u}{u'}$$

and known as "*Ramsey's Rule*" for optimal savings.

Section II. ***Optimal Growth with Borrowing without Default.***

In this section the problem is to determine the *optimal saving* policy function under uncertainty when borrowing is permitted. In addition to all variables defined in *section I* assume that there exists a riskless "short-term" rate r(t). Also assume that no default is possible in this section (e.g. penalty for default is restrictively high) therefore the contracted rate for borrowing b(r) is a riskless "short-term" rate r(t). Of course, borrowing itself is a function of interest rate. The consumption at time t is

$$c = (1 - s(k))f(k+b) - p\ ,$$

where $p = (1 + r)b(r)$ is a debt service payment.

The equation of capital accumulation:

$$dk = s(k)f(k+b)dt + \sigma_1(k)dz_1.$$

The evolution of interest rate:

$$dr = \mu(r)dt + \sigma_2(r)dz_2.$$

Here dz_1 and dz_2 are standard Wiener processes such that $(dz_1)^2 = (dz_2)^2 = dt$ and $(dz_1)(dz_2) = \rho dt$, where ρ is a correlation coefficient between these two processes.

The social planner's problem is

$$max_{\{s\}}\ \mathbf{E}_0\left[\int_0^{\infty} e^{-\beta\tau}\, u[(1 - s)f(k+b) - (1+r)b]d\tau\right]$$

subject to

$$dk = s(k+b)f(k+b)dt + \sigma_1(k)dz_1.$$

$$dr = \mu(r)dt + \sigma_2(r)dz_2.$$

The procedure to deal with this problem is similar to one developed in *section I.*

Let

$$J((k+b)(t), t, T) = \max_{\{s\}} \mathbf{E}_t \int_t^T e^{-\beta\tau} u[(1 - s)f(k+b) - (1+r)b]d\tau$$

and

$$W((k+b)(t), t, T) = e^{\beta t} J((k+b)(t), t, T).$$

Then

$$W(k+b, t, T) = e^{\beta t} \max_{\{s\}} \mathbf{E}_t \int_t^T e^{-\beta\tau} u[(1 - s)f(k+b) - (1+r)b]d\tau =$$

$$e^{\beta t} \max_{\{s\}} \mathbf{E}_t \int_t^{t+dt} e^{-\beta\tau} u[(1 - s)f(k+b) - (1+r)b]d\tau +$$

$$e^{\beta t} \max_{\{s\}} \mathbf{E}_t \int_{t+dt}^T e^{-\beta\tau} u[(1 - s)f(k+b) - (1+r)b]d\tau =$$

$$e^{\beta t} \max_{\{s\}} \mathbf{E}_t[\int_t^{t+dt} e^{-\beta\tau} u[(1 - s)f(k+b) - (1+r)b]d\tau +$$

$$e^{\beta t} J((k+b)(t+dt), t+dt, T)] =$$

$$\max_{\{s\}} \mathbf{E}_t[u[(1 - s)f(k+b) - (1+r)b]dt + e^{\beta t} J((k+b)(t+dt), t+dt, T)] =$$

$$\max_{\{s\}} \mathbf{E}_t[u[(1 - s)f(k+b) - (1+r)b]dt + e^{\beta t}\{J(k+b, t, T) + J_{k+b}d(k+b) +$$

$$J_t dt + \frac{1}{2}J_{(k+b)(k+b)}(d(k+b))^2 + \frac{1}{2}J_{(k+b)t}(d(k+b))(dt) + \frac{1}{2}J_{tt}(dt)^2\}] =$$

$$\max_{\{s\}} \mathbf{E}_t[u[(1 - s)f(k+b) - (1+r)b]dt + W(k+b, t, T) + W_{k+b}d(k+b) +$$

$$e^{\beta t}J_t dt + \frac{1}{2}W_{(k+b)(k+b)}(d(k+b))^2 + \frac{1}{2}W_{(k+b)t}(d(k+b))(dt) + \frac{1}{2}W_{tt}(dt)^2\}] =$$

$$\max_{\{s\}} \mathbf{E}_t[u[(1 - s)f(k+b) - (1+r)b]dt + W(k+b, t, T) + W_{k+b}(dk+db) +$$

$$(-\beta W + W_t)dt + \frac{1}{2}W_{(k+b)(k+b)}(dk+db))^2] =$$

/ *Using Ito's lemma for* db(r) /

$$\max_{\{s\}} \mathbf{E}_t[u[(1 - s)f(k+b) - (1+r)b]dt + W(k+b, t, T) +$$

$$W_{k+b}(sf(k+b)dt + \sigma_1(k)dz_1 + [b_t + b_r\mu(r) + \frac{1}{2}b_{rr}(\sigma_2(r))^2]dt +$$

$$b_r\sigma_2(r)dz_2 + (-\beta W + W_t)dt + \frac{1}{2}W_{(k+b)(k+b)}(sf(k+b)dt + \sigma_1(k)dz_1 +$$

$$[b_t + b_r\mu(r) + \frac{1}{2}b_{rr}(\sigma_2(r))^2]dt + b_r\sigma_2(r)dz_2))^2] =$$

$$max_{\{s\}}\ \mathbf{E}_t[u[(1 - s)f(k+b) - (1+r)b]dt + W(k+b, t, T) + W_{k+b}(sf(k+b)dt +$$

$$\sigma_1(k)dz_1+[b_t + b_r\mu(r) + \frac{1}{2}b_{rr}(\sigma_2(r))^2]dt + b_r\sigma_2(r)dz_2 + (-\beta W + W_t)dt +$$

$$\frac{1}{2}W_{(k+b)(k+b)}(\sigma_1{}^2 + 2b_r\sigma_1\sigma_2 + b_r{}^2\sigma_2{}^2)dt.$$

Pass $\mathbf{E}_t$ through the parenthesis, note that $\mathbf{E}_t[W_{k+b}(\sigma_1 dz_1 + \sigma_2 dz_2)] = 0$, and after dividing by dt we get the following *Hamilton-Jacobi-Bellman* equation:

$$\beta W - W_t = max_{\{s\}}\{u[(1 - s)f(k+b) - (1+r)b] + (sf(k+b) + b_t +$$

$$b_r\mu(r) + \frac{1}{2}b_{rr}(\sigma_2{}^2)W_{k+b} + \frac{1}{2}(\sigma_1{}^2 + 2b_r\sigma_1\sigma_2 + b_r{}^2\sigma_2{}^2)W_{(k+b)(k+b)}\}.$$

The first order condition to be satisfied by the optimal policy s^{**} is

$$0 = u'[(1 - s^{**})f(k+b) - (1+r)b]f(k+b) - f(k+b)W_{k+b},$$

which becomes

$$u'[(1 - s^{**})f(k+b) - (1+r)b] = e^{\beta t}J_{k+b}.$$

Here u' means $\frac{du}{d(1-s)}$.

Differentiate the first order condition with respect to (k+b):

$$W_{(k+b)(k+b)} = u''[(1 - s^{**})f(k+b) - (1+r)b] \times$$

$$\{(1 - s^{**})f'(k+b) - s^{**\prime}f(k+b) - (1+r)\}.$$

If an optimal policy exists and is not a function of t as $T \to \propto$ then

$$W(k+b, t) = e^{\beta t}J(k+b, t) = max_{\{s\}} \int_0^{\infty} e^{-\beta(\tau-t)} u[(1 - s^{**})f(k+b) - (1+r)b]d\tau =$$

$$\int_0^{\infty} e^{-\beta\xi} u[(1 - s^{**})f(k+b) - (1+r)b]d\xi = W(k+b) = C;$$

where C is a bliss level of utility associated with the maximum steady state consumption. Therefore, the *Hamilton-Jacobi-Bellman* equation is converted into the ordinary differential equation of the second order for $W(k+b)$:

$$u[(1 - s^{**})f(k+b) - (1+r)b] - \beta W(k+b) + (s^{**}f(k+b) + b_t + b_r\mu(r) +$$

$$\frac{1}{2}b_{rr}(\sigma_2^2))W_{k+b} + \frac{1}{2}(\sigma_1^2 + 2b_r\sigma_1\sigma_2 + b_r^2\sigma_2^2)W_{(k+b)(k+b)} = 0.$$

Substitution for W_{k+b} and $W_{(k+b)(k+b)}$ into the *Hamilton-Jacobi-Bellman* equation gives:

$$u[(1 - s^{**})f(k+b) - (1+r)b] - \beta C +$$

$$(s^{**}f(k+b) + b_t + b_r\mu(r) + \frac{1}{2}b_{rr}(\sigma_2{}^2)) \times u'[(1 - s^{**})f(k+b) - (1+r)b] +$$

$$\frac{1}{2}(\sigma_1{}^2+2b_r\sigma_1\sigma_2+b_r{}^2\sigma_2{}^2) \times \{(1 - s^{**})f'(k+b) - s^{**\prime}f(k+b) -$$

$$(1+r)\} \times u''[(1 - s^{**})f(k+b) - (1+r)b] = 0.$$

In order to find the optimal consumption function it is necessary to characterize the borrowing function $b(r)$. While it explicitly depends only on interest, it is not independent of initial capital and chosen optimally. It is also precommmitted and so does not depends on time ($b_t = 0$). This argument about "precommitment" is plausible because the debtor generally borrows capital in advance at time zero and then continuously invests it and continuously makes debt service payments. It is obvious that if the initial rate is high or the country under consideration has abundance of capital, then $b = 0$ and $s^{**} = s^*$.

Section III. ***Optimal Growth with Borrowing under Risk of Debt Repudiation.***

In this section the results of two previous sections are incorporated and the possibility of default is introduced. At any instance the social planner considers the two possible strategies: continue to borrow and service debt or default payment on previous debt which means to be punished by being cutting off from capital markets forever. Since two optimal saving policies s^{**} and s^{*} are already known, the expected value of objective function can be computed in both cases. If the value of objective function with default is larger than the value of objective function without it, then default is the *best strategy*. The value of the objective function with borrowing randomly depends on the short-term interest rate (floating rate bonds). Therefore the probability to default in the next instance is the probability that the value of objective function with repudiation is larger than the value of objective function without it and directly depends on random behavior of production function and interest rates. It is worthwile to note, that the decision not to default is made just for the next instance, while the decision to default is accepted only once. If this theory is right, then risk spreads must be positively correlated with short-term interest rates and the farther away the time to maturity the higher the correlation should be.

Define the value of objective function at time t given a decision to default as:

$$V^d(t) = \mathbf{E}_t[\int_t^{\infty} e^{-\beta\tau}\, u[(1 - s^*)f(k+b^t)]d\tau];$$

where b^t is an amount of borrowing resources at time t which is never repaid back.

Define the value of objective function at time t given a decision not to default as:

$$V^{nd}(t) = \mathbf{E}_t[u[(1 - s^{**})f(k+b) - (1+r)b] + \mathbf{E}_t max\{V^d(t+dt), V^{nd}(t+dt)\}].$$

Default is optimal in time $t+dt$ if and only if $V^d(t+dt) > V^{nd}(t+dt)$. Define the probability $\lambda(t)$ of default at any time t as $Pr[V^d(t) > V^{nd}(t)]$. This probability represents the risk spread over the risk-free rate, since the expected rate of return $i(t)$ for risk-neutral lenders must be at least as high as the market risk-free rate, i.e.

$$(1 - \lambda(t))\,(1 + i(t))\,b(t) = (1 + r(t))\,b(t)$$

or

$$(1 + i(t)) = (1 + r(t)) / (1 - \lambda(t)).$$

The problem of finding this probability is a little bit evolved. The following analysis outlines the approach.

$$u[(1 - s^*)f(k+b^t)] - u[(1 - s^{**})f(k+b) - (1+r)b] =$$

/ expanding into Taylor's series /

$$u'[(1 - s^*)f(k+b^t)]((1 - s^*)f(k+b^t) - ((1 - s^{**})f(k+b) - (1+r)b)).$$

Note also that always $u'[(1 - s^*)f(k+b^t)] > 0$ because of the Inada condition and does not depend on time. The last expression helps to find the probability:

$$\Pr[V^d(t) > V^{nd}(t)] =$$

$$\Pr[\int_t^\infty e^{-\beta\tau} u[(1 - s^*)f(k+b^t)]d\tau > \int_t^\infty e^{-\beta\tau} u[(1 - s^{**})f(k+b) - (1+r)b]d\tau] =$$

$$\Pr[\int_t^\infty e^{-\beta\tau} (u[(1 - s^*)f(k+b^t)] - u[(1 - s^{**})f(k+b) - (1+r)b])d\tau > 0] =$$

$$\Pr[\int_t^\infty e^{-\beta\tau} (u'[(1 - s^*)f(k+b^t)]((1 - s^*)f(k+b^t) -$$

$$((1 - s^{**})f(k+b) - (1+r)b))d\tau > 0] =$$

$$\Pr[\int_t^\infty e^{-\beta\tau} ((1 - s^*)f(k+b^t) - ((1 - s^{**})f(k+b) - (1+r)b))d\tau > 0] =$$

$$\Pr[\int_t^\infty e^{-\beta\tau} rbd\tau > \int_t^\infty e^{-\beta\tau} ((1 - s^{**})f(k+b) - b - (1 - s^*)f(k+b^t))d\tau].$$

Recall that b is precommitted and the optimal consumption strategy does not depend on time, hence

$$\int_t^\infty e^{-\beta\tau}((1 - s^{**})f(k+b) - b - (1 - s^{*})f(k+b^t))d\tau =$$

$$\frac{1}{\beta}e^{-\beta t}((1 - s^{**})f(k+b) - b - (1 - s^{*})f(k+b^t))$$

and

$$\int_t^\infty e^{-\beta\tau}\, rbd\tau = b\int_t^\infty e^{-\beta\tau}\, rd\tau.$$

In general $dr = \mu(r)dt + \sigma_2(r)dz_2$, but in order to be able to handle the problem assume that r is a Brownian motion, i.e. $r(\tau) = \sigma_2 z_2(\tau)$, $\sigma_2 =$ *const*. One can point out that r here could became negative. This problem could be remedied easily by assigning proper functions μ and σ; or even easier by assigning $r = 0$ to all states where r would be negative otherwise. This restriction does not change the result.

Call $X(t) = \int_t^\infty e^{-\beta\tau}\sigma_2 z_2(\tau)d\tau$.

Taking this integral by parts:

$$X(t) = -\frac{1}{\beta}\sigma_2 e^{-\beta\tau}z_2(\tau)\Big]_t^\infty - \left(-\frac{1}{\beta}\sigma_2\int_t^\infty e^{-\beta\tau}dz_2(\tau)\right) =$$

$$\frac{1}{\beta}\sigma_2 e^{-\beta t}z_2(t) + \frac{1}{\beta}\sigma_2\int_t^\infty e^{-\beta\tau}dz_2(\tau).$$

Since X(t) consists of two terms, it is necessary to analyze both:
For the first term, it follows from

$$z_2(t) \sim N(0, t),$$

that

$$\frac{1}{\beta}\sigma_2 e^{-\beta t} z_2(t) \sim N(0, (\frac{1}{\beta}\sigma_2)^2 e^{-2\beta t}\, t).$$

For the second term:

$$\mathbf{E}_t[\frac{1}{\beta}\sigma_2 \int_t^\infty e^{-\beta\tau} dz_2(\tau)] = 0.$$

$$\mathbf{E}_t[\frac{1}{\beta}\sigma_2 \int_t^\infty e^{-\beta\tau} dz_2(\tau)]^2 = (\frac{1}{\beta}\sigma_2)^2 \int_t^\infty e^{-2\beta\tau} d\tau = (\frac{1}{\beta}\sigma_2)^2 e^{-2\beta t}\frac{1}{2\beta}.$$

Therefore,

$$\frac{1}{\beta}\sigma_2 \int_t^\infty e^{-\beta\tau} dz_2(\tau) \sim N(0, (\frac{1}{\beta}\sigma_2)^2 e^{-2\beta t}\frac{1}{2\beta}).$$

To show that $\frac{1}{\beta}\sigma_2 e^{-\beta t} z_2(t)$ and $\frac{1}{\beta}\sigma_2 \int_t^\infty e^{-\beta\tau} dz_2(\tau)$ are independent recall the property of independence of increments for the Brownian motion and observe that the first term is a non stochastic function of the Brownian motion at time t while the second term is a "sum" of such future increments. This intuitive argument can be put on the firm mathematical foundation, but this is out of scope of the task. More on Markov properties of the Brownian motion are in Karatzas and Shreve [1991]. In the case of independence the variance of the sum is equal the sum of variances. Therefore,

$$X(t) \sim N(0, (\frac{1}{\beta}\sigma_2)^2 e^{-2\beta t}(t + \frac{1}{2\beta})),$$

or

$$(\beta/\sigma_2)e^{\beta t}X(t) \sim N(0, (t + \frac{1}{2\beta})).$$

This means that the process $(\beta/\sigma_2)e^{\beta t}X(t)$ is a shifted Brownian motion $\boldsymbol{B}(t + \frac{1}{2\beta})$ with shift $\frac{1}{2\beta}$, since it is a continuous process, a martingale, and is distributed normally with variance $(t + \frac{1}{2\beta})$.

At this point pick up the analysis of the probabilities:

$$\Pr[\int_t^\infty e^{-\beta\tau}\, rbd\tau > \int_t^\infty e^{-\beta\tau}\,((1 - s^{**})f(k+b) - b - (1 - s^{*})f(k+b^t))d\tau] =$$

$$\Pr[bX(t) > \frac{1}{\beta}e^{-\beta t}((1 - s^{**})f(k+b) - b - (1 - s^{*})f(k+b^t))] =$$

$$\Pr[(\beta/\sigma_2)e^{\beta t}X(t) > (1/b\sigma_2)((1 - s^{**})f(k+b) - b - (1 - s^{*})f(k+b^t))] =$$

$$\Pr[\boldsymbol{B}(t + \frac{1}{2\beta}) > (1/b\sigma_2)((1 - s^{**})f(k+b) - b - (1 - s^{*})f(k+b^{t}))] =$$

$$\Pr[\boldsymbol{B}(t + \frac{1}{2\beta}) > A] = \frac{1}{\sqrt{2\pi}} \int_{\frac{A}{\sqrt{t+\frac{1}{2\beta}}}}^{\infty} e^{-y^2/2} dy,$$

where

$$A = (1/b\sigma_2)((1 - s^{**})f(k+b) - b - (1 - s^{*})f(k+b^{t})).$$

Here is a main result of this section - the probability of default at any time t:

$$\Pr[V^{d}(t) > V^{nd}(t)] = \frac{1}{\sqrt{2\pi}} \int_{\frac{A}{\sqrt{t+\frac{1}{2\beta}}}}^{\infty} e^{-y^2/2} dy.$$

When analyzing the result two different cases should be considered: A is negative and A is positive.

1. $A < 0$ means $(1 - s^{**})f(k+b) - b < (1 - s^{*})f(k+b^{t})$

This situation must be rare since the highest probability of default is when $t = 0$ and the higher A in absolute value the closer the probability to one, therefore no lending will occur.

2. To allow initial lending ($t = 0$) the condition

$$(1 - s^{**})f(k+b) - b > (1 - s^{*})f(k+b^{t})$$

should be satisfied, i.e. $A > 0$. Then the probability of default depends on the size of debt b. As expected, the larger debt the higher is the probability of default. It is also depends on volatility σ_2 of short-term rate, again the larger the volatility the higher is the probability of default. It is also worth to note, that as time increases so does the probability of default. The highest probability of default in this case is $\frac{1}{2}$ when $t \rightarrow \propto$.

PART B

EMPIRICAL STUDY OF CORRELATION BETWEEN YIELDS OF US TREASURY BONDS AND BRADY BONDS.

The risk of a country to default could be considered consisting of two parts: the pure economic risk and everything else including political and international developments. If the pure economic risk depends on risk-free rate, than the correlation between prices of US Treasury bonds and sovereign debt must be positive. Another implication of the theory developed in the part A is that since the pure economic risk is growing with time the correlation between prices of US Treasury bonds and sovereign debt will also grow with time to maturity. To test these conjectures different kinds of Brady bonds from different countries were used. The data for each country consists of blended yield, stripped yield, blended spread, stripped spread and nearest Treasury yield. The series used are collected in the *Appendix*. Blended yield is an actual yield which investors receive. The difference between blended yield and Treasury yield of the same maturity is a blended spread. Since the value of Brady bonds is enhanced through the provision of principal and interest guarantees and collateral it is possible to "strip" them from the total cash flows to leave only cash flows subject to sovereign risk. What remains is a stripped yield and the difference between stripped yield and Treasury yield of the same maturity is a stripped spread. These average stripped spreads are reported in the **Table 1**. It was already noted in introduction that these spreads do not depend on the past performance. Therefore the difference in spreads (for bonds of the same maturity) for different countries could be explained by the difference in production functions, since expected fluctuations of the riskless interest rate are the same for all countries. The results on correlations are in the **Table 2**. These results show high correlations for all bonds and for all countries except Venezuela. The most important and clear indicator is the correlation between Treasury yield and stripped spread. It shows also that correlations grow with time to maturity, e.g. Argentina: correlation is higher for 30 year bond in comparison with 7 year bond. Results for Venezuela could be explained by special provision for this country bond which tied the oil export and payments on these instruments. The scattered plots of stripped yields versus Treasury yeilds are in *Appendix* also.

Table 1.

COUNTRY	*TYPE OF BOND*	*NEAREST TREASURY BOND*	*AVERAGE STRIPPED SPREAD (%)*
ARGENTINA	Floating-rate bonds due 2005 (FRB)	7 y	7.439
ARGENTINA	Discount bonds due 2023	30 y	7.764
ARGENTINA	Par bonds due 2023	30 y	7.788
BRAZIL	Interest due and unpaid bond due 2001 (IDU)	7 y	9.966
MEXICO	Floating-rate bonds due 2008 (Aztec)	30 y	6.573
MEXICO	Par bonds due 2019	30 y	5.231
MEXICO	Discount bonds due 2019	30 y	6.092
NIGERIA	Par bonds due 2020	30 y	15.791
PHILIPPINES	Par bonds due 2017	30 y	6.232
VENEZUELA	Front-loaded interest reduction bonds due 2007 (FLIRB).	10 y	8.931
VENEZUELA	Par bonds due 2020	30 y	7.423
VENEZUELA	Floating rate debt conversion bonds due 2007 (DCB)	10 y	8.512

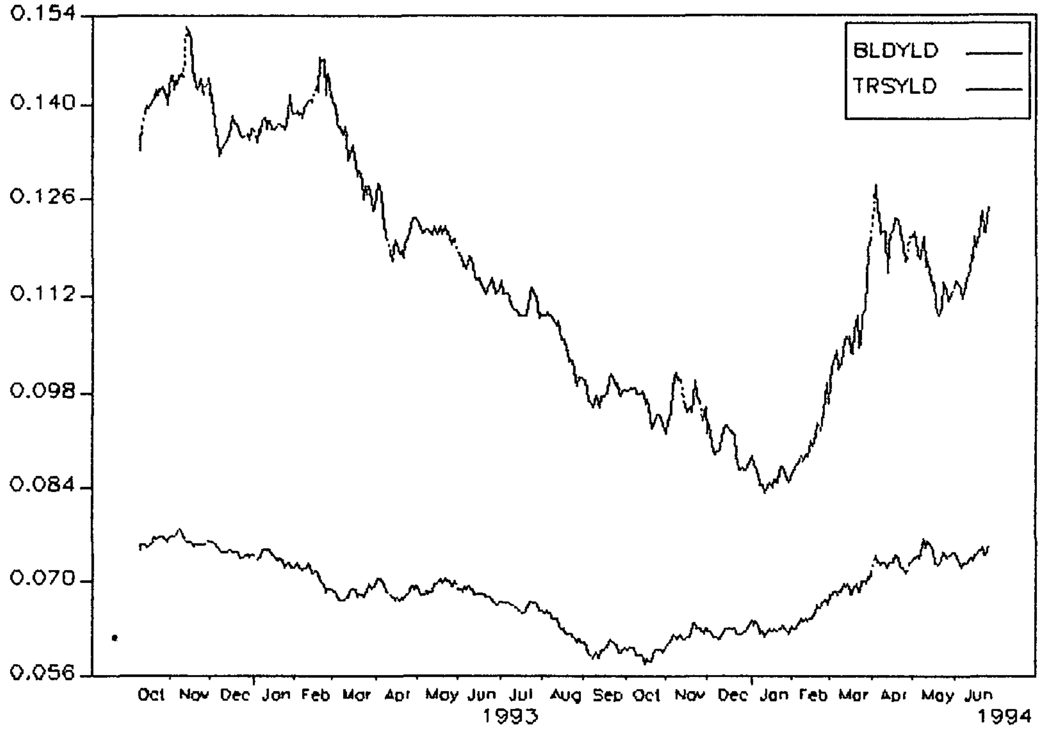
Table 2.

COUNTRY	*TYPE OF BOND*	*NEAREST TREASURY BOND*	*CORRELATION BETWEEN TREASURY YIELD AND*			
			BLENDED YIELD	*STRIPPED YIELD*	*BLENDED SPREAD*	*STRIPPED SPREAD*
ARGENTINA	Floating-rate bonds due 2005 (FRB)	7 y	0.45511	0.45511	0.32139	0.32746
ARGENTINA	Discount bonds due 2023	30 y	0.82236	0.74436	0.68561	0.60562
ARGENTINA	Par bonds due 2023	30 y	0.82353	0.74630	0.64570	0.60101
BRAZIL	Interest due and unpaid bond due 2001 (IDU)	7 y	0.57138	0.57138	0.45679	0.45316
MEXICO	Floating-rate bonds due 2008 (Aztec)	30 y	0.86365	0.73250	0.65714	0.50781
MEXICO	Par bonds due 2019	30 y	0.88064	0.76000	0.57228	0.53766
MEXICO	Discount bonds due 2019	30 y	0.89622	0.80811	0.72272	0.65248
NIGERIA	Par bonds due 2020	30 y	0.84384	0.67481	0.67051	0.54989
PHILIPPINES	Par bonds due 2017	30 y	0.91000	0.83211	0.69246	0.65244
VENEZUELA	Front-loaded interest reduction bonds due 2007 (FLIRB).	10 y	0.26965	0.27161	-0.12278	-0.10689
VENEZUELA	Par bonds due 2020	30 y	0.43029	0.01184	-0.28152	-0.33574
VENEZUELA	Floating rate debt conversion bonds due 2007 (DCB)	10 y	0.27817	0.27817	-0.11255	-0.11230

APPENDIX.

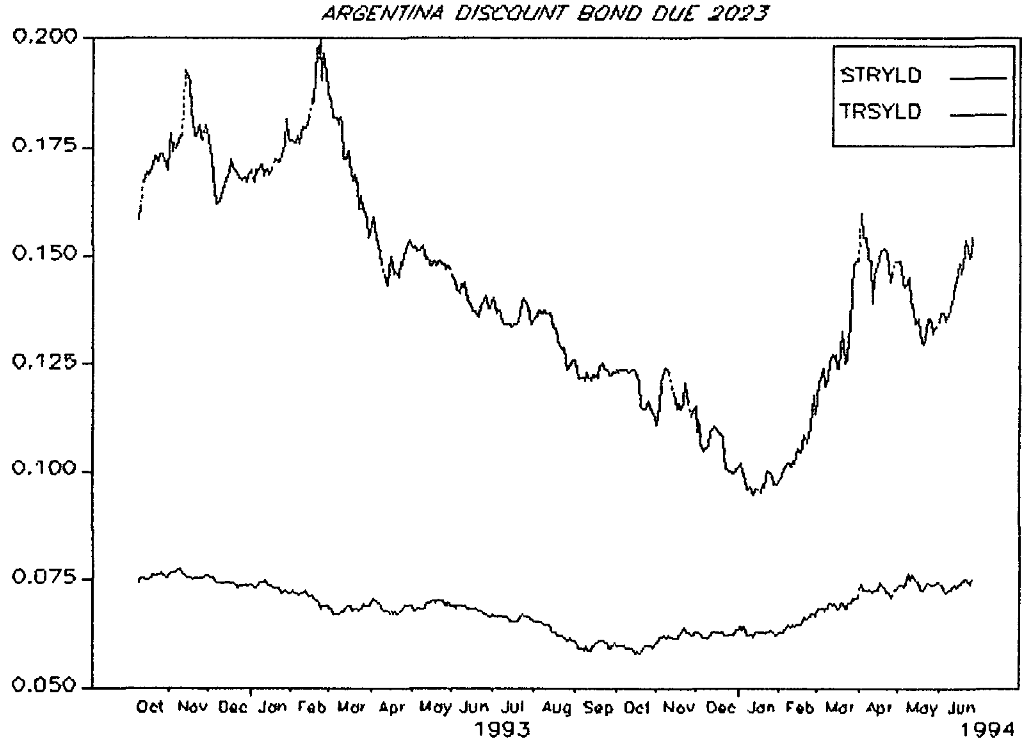
BLENDED YIELD vs 30y TREASURY BOND YIELD

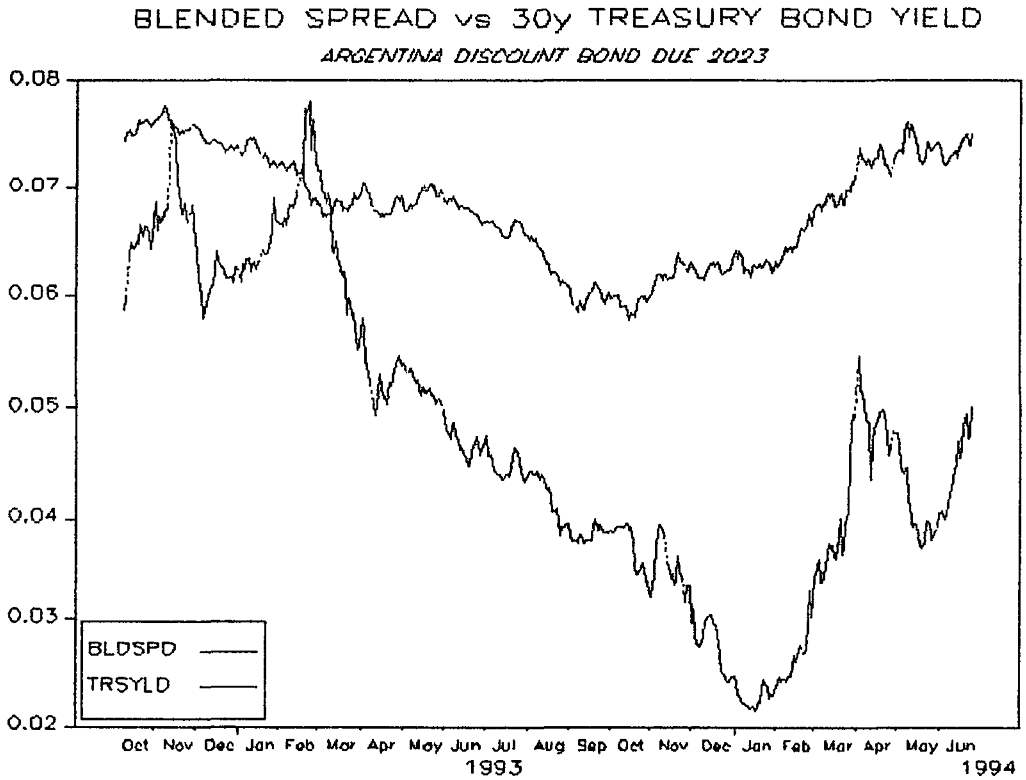
ARGENTINA DISCOUNT BOND DUE 2023

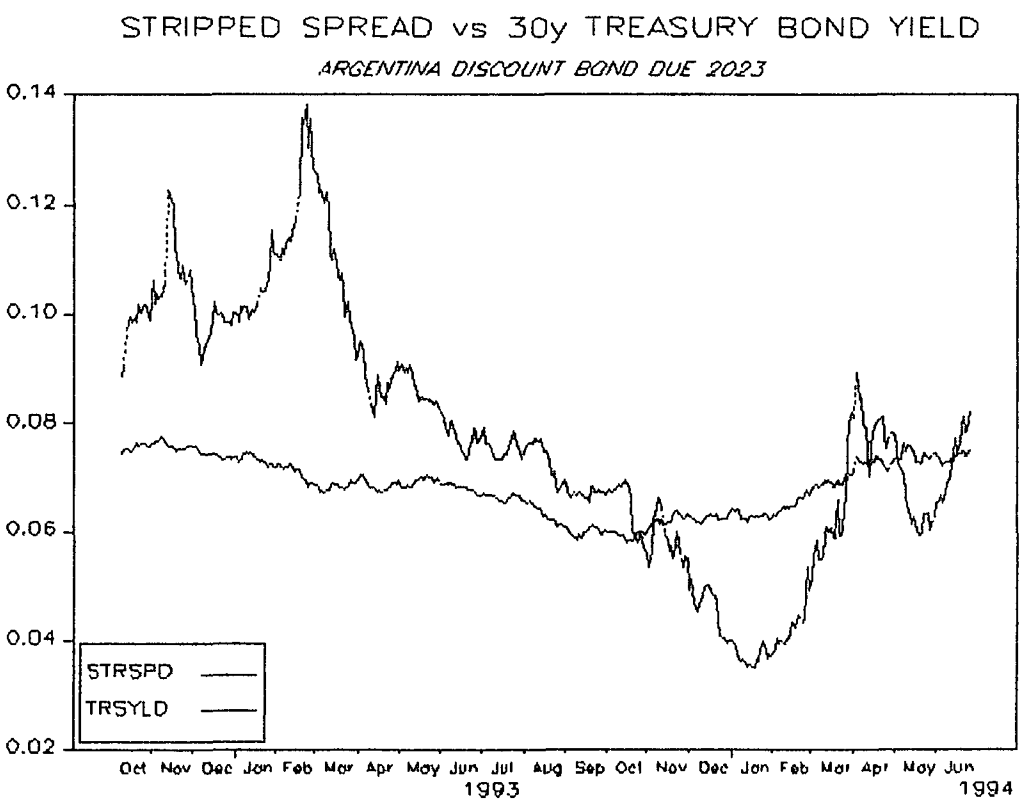


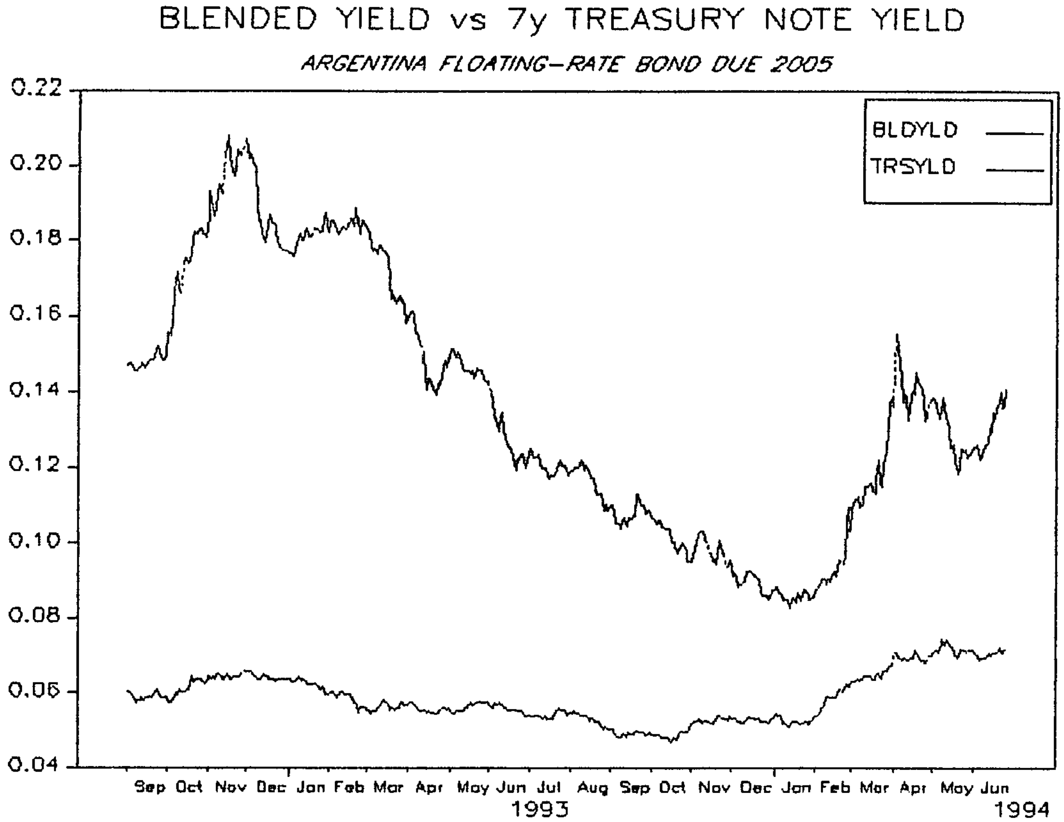
STRIPPED YIELD vs 30y TREASURY BOND YIELD

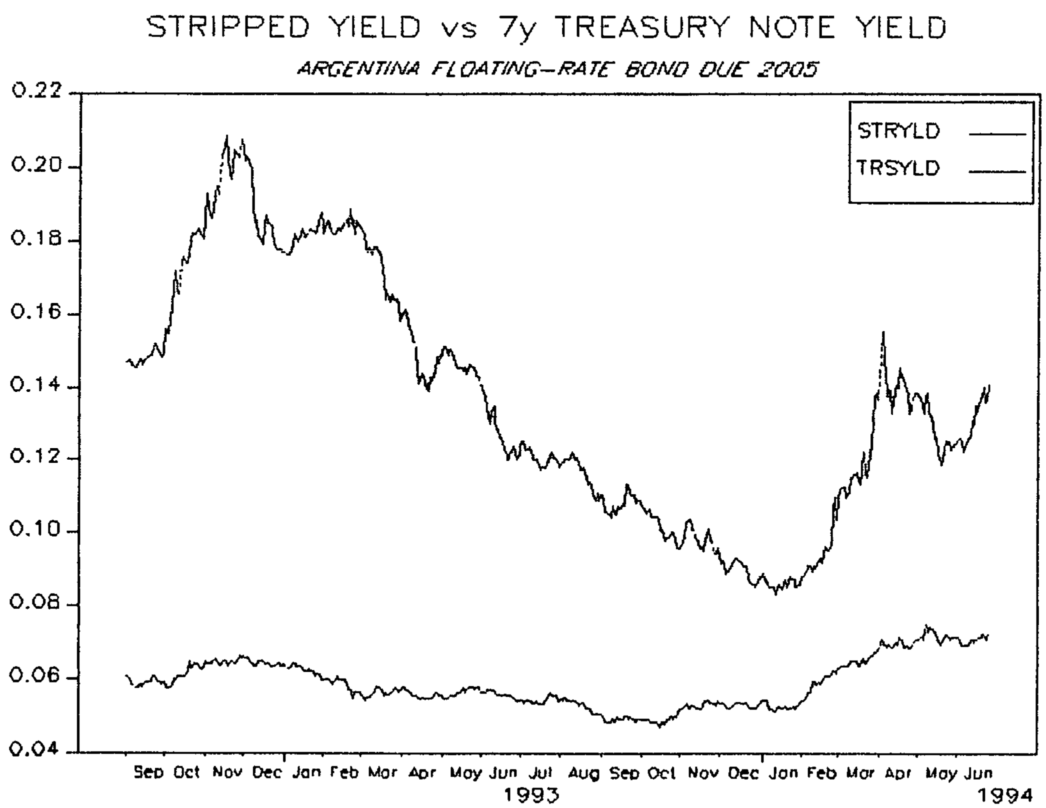
ARGENTINA DISCOUNT BOND DUE 2023




BLENDED SPREAD vs 30y TREASURY BOND YIELD
ARGENTINA DISCOUNT BOND DUE 2023
0.08
0.07
0.06
0.05
0.04
0.03
0.02
BLDSPD
TRSYLD
Oct Nov Dec Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan Feb Mar Apr May Jun
1993
1994

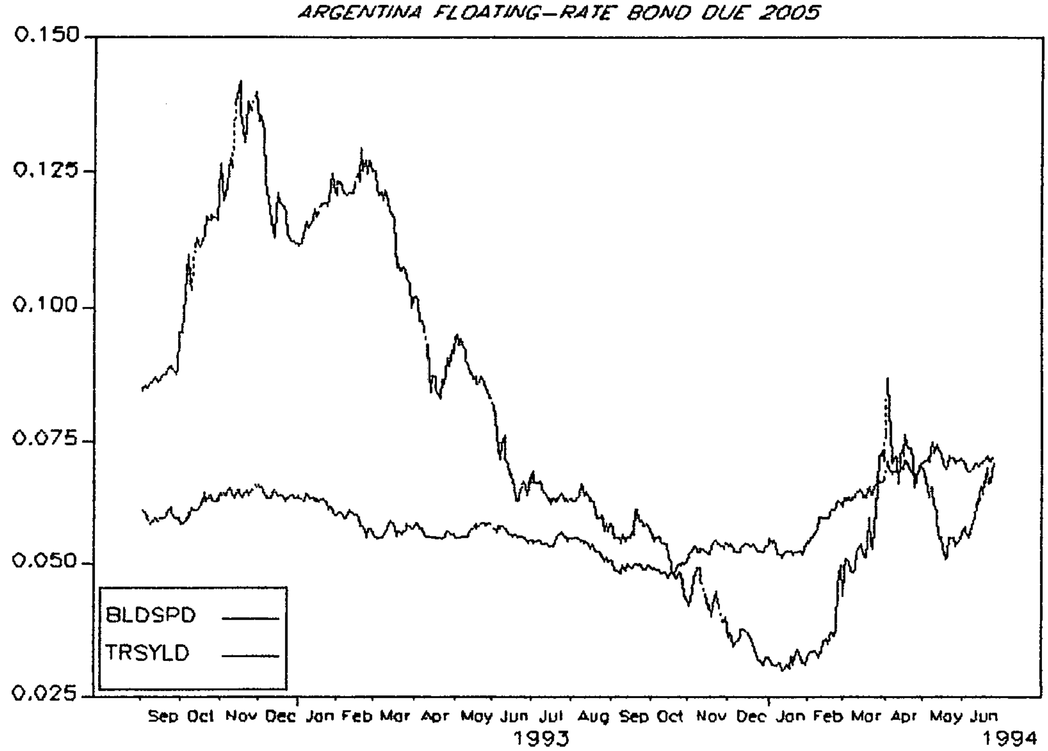

STRIPPED SPREAD vs 30y TREASURY BOND YIELD
ARGENTINA DISCOUNT BOND DUE 2023
0.14
0.12
0.10
0.08
0.06
0.04
0.02
STRSPD
TRSYLD
Oct Nov Dec Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan Feb Mar Apr May Jun
1993
1994


BLENDED YIELD vs 7y TREASURY NOTE YIELD
ARGENTINA FLOATING-RATE BOND DUE 2005
BLDYLD
TRSYLD
0.22
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
Sep Oct Nov Dec Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan Feb Mar Apr May Jun
1993
1994


STRIPPED YIELD vs 7y TREASURY NOTE YIELD
ARGENTINA FLOATING-RATE BOND DUE 2005
STRYLD
TRSYLD
0.22
0.20
0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
Sep Oct Nov Dec Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan Feb Mar Apr May Jun
1993
1994

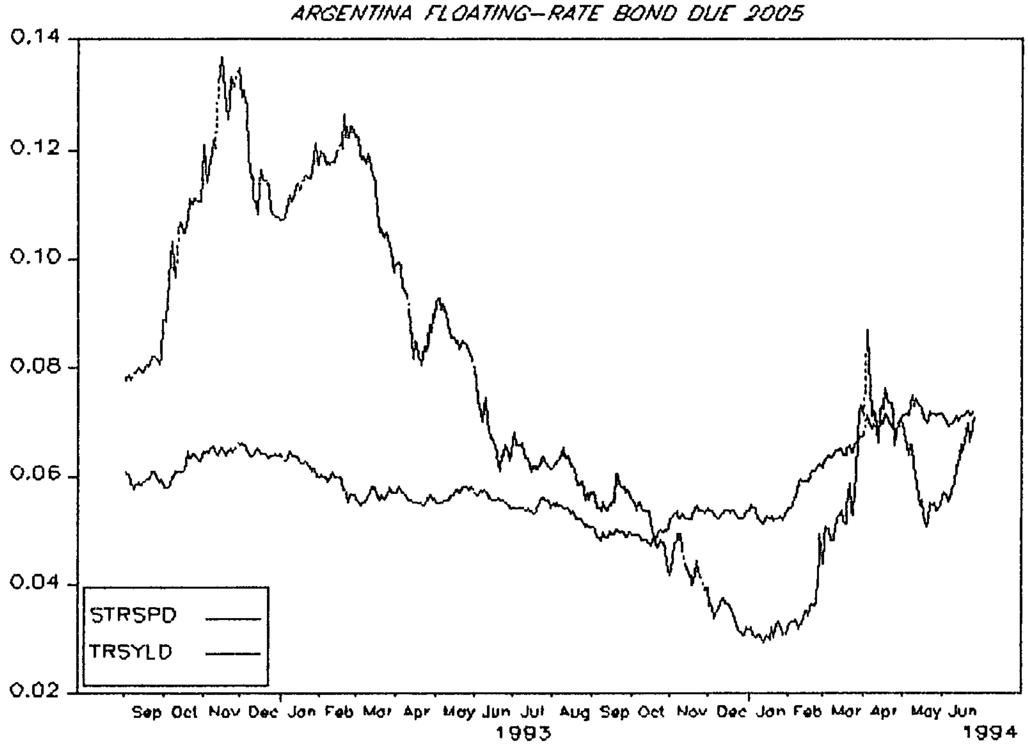
BLENDED SPREAD vs 7y TREASURY NOTE YIELD

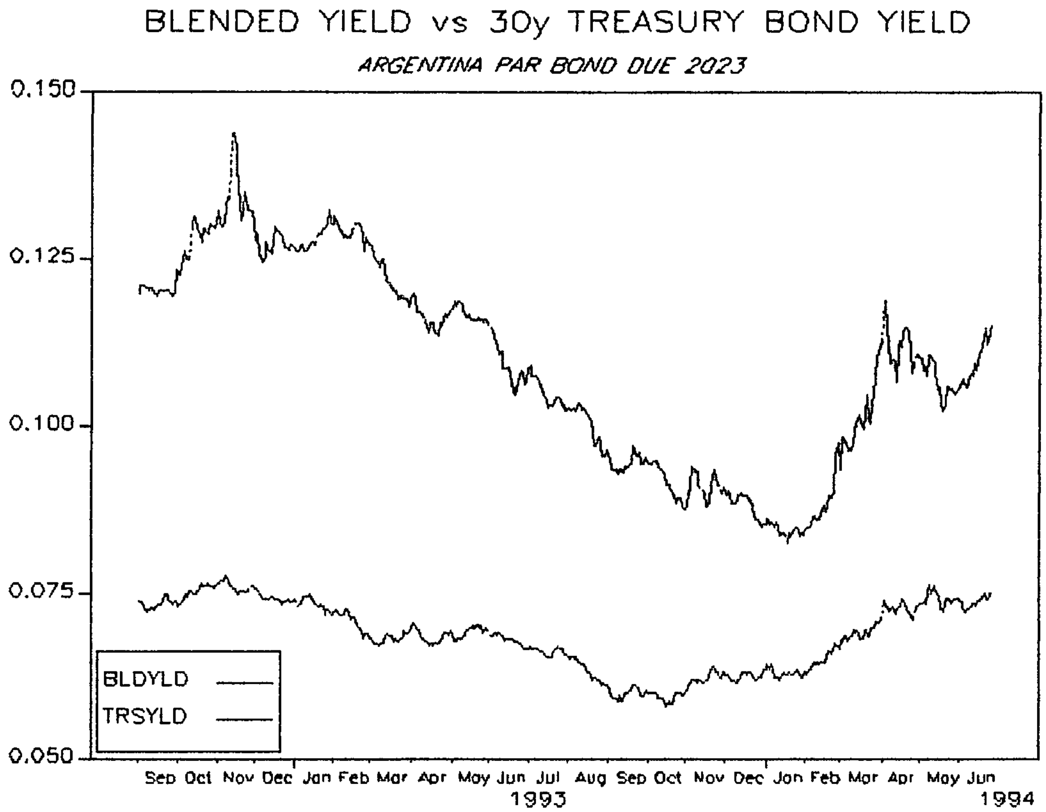
ARGENTINA FLOATING-RATE BOND DUE 2005

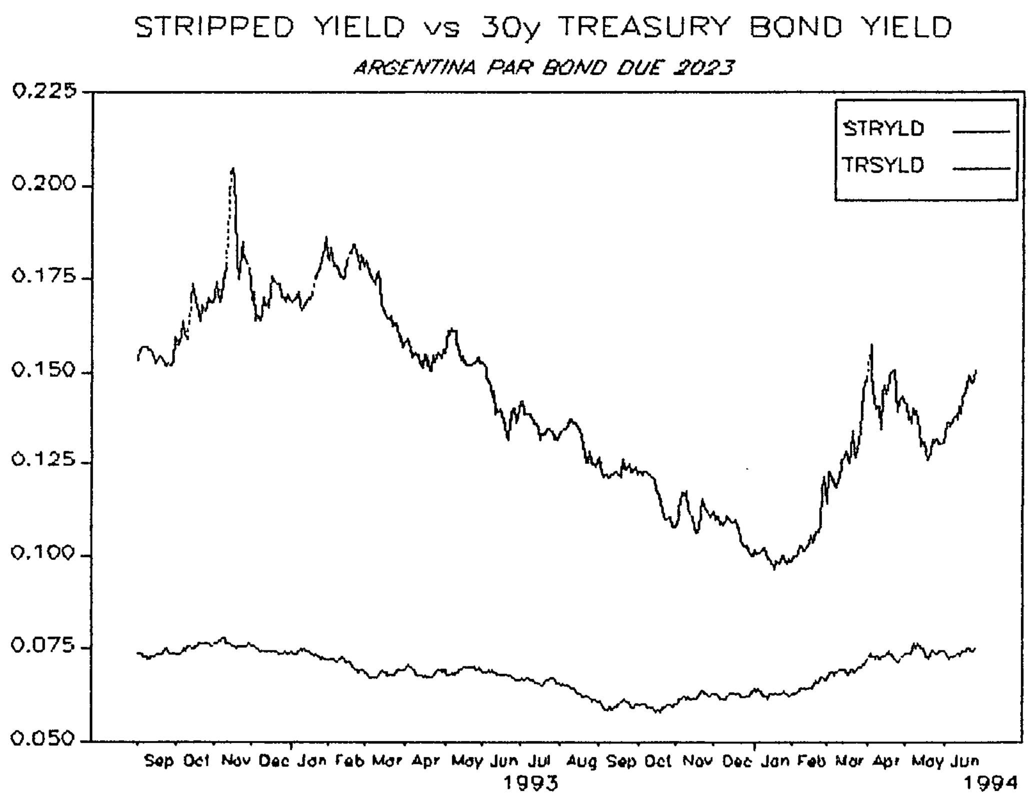


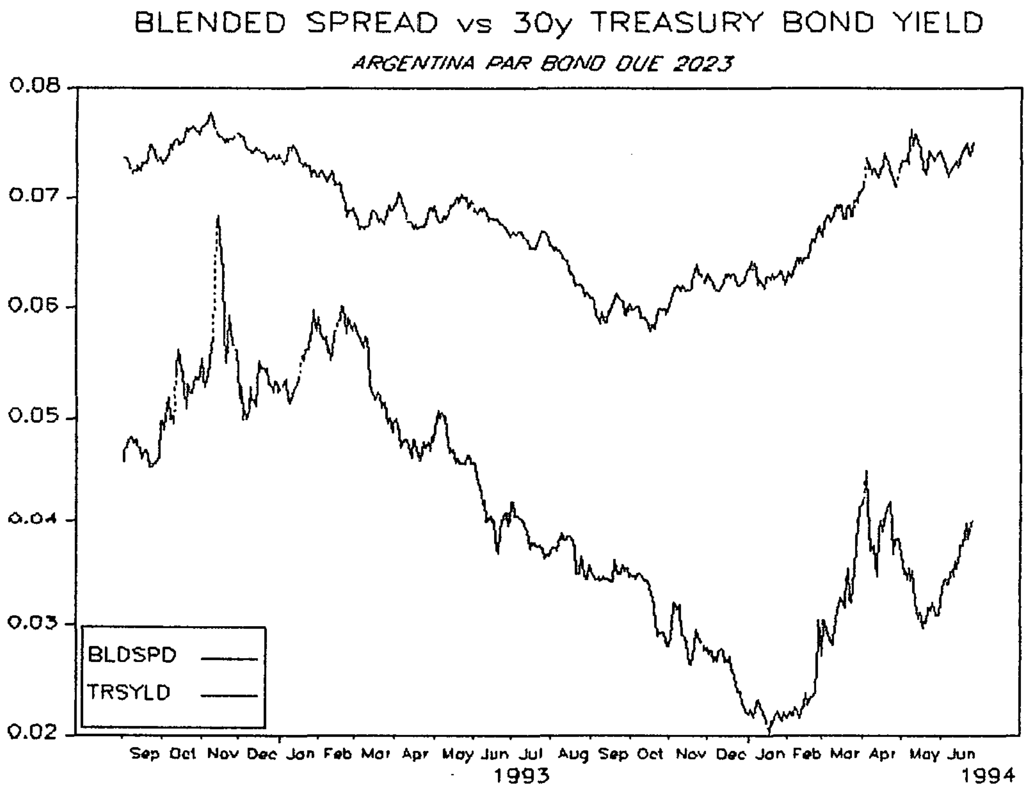
STRIPPED SPREAD vs 7y TREASURY NOTE YIELD

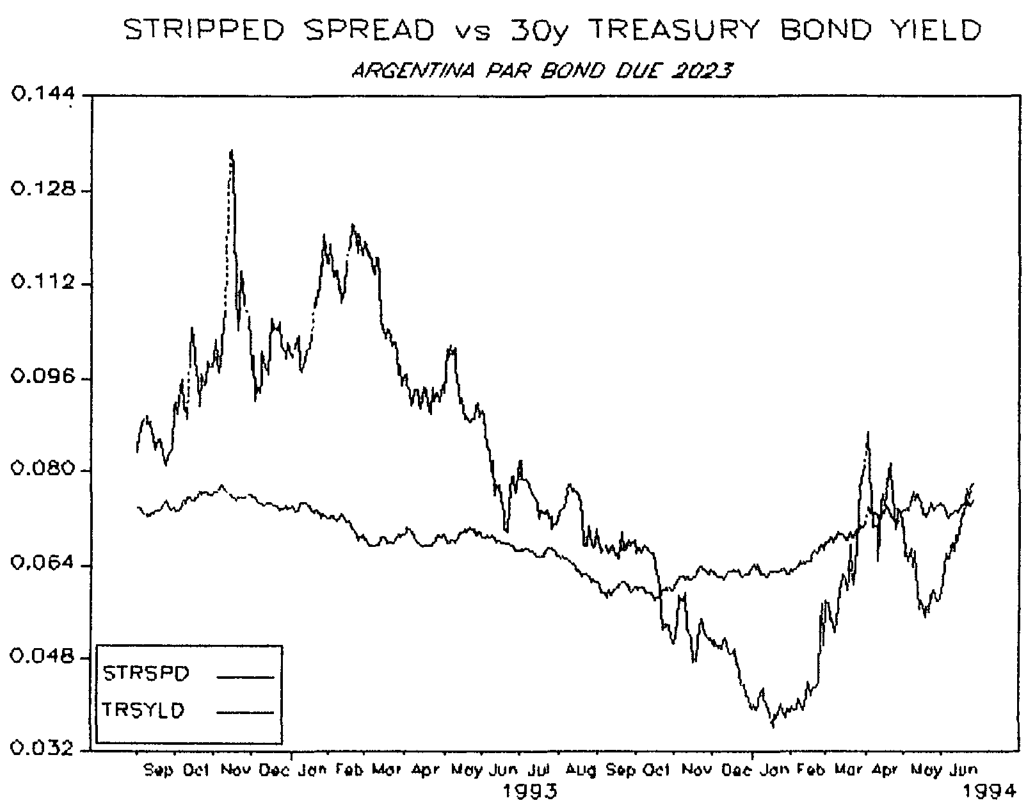
ARGENTINA FLOATING-RATE BOND DUE 2005




BLENDED YIELD vs 30y TREASURY BOND YIELD
ARGENTINA PAR BOND DUE 2023
0.150
0.125
0.100
0.075
0.050
BLDYLD
TRSYLD
Sep Oct Nov Dec Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan Feb Mar Apr May Jun
1993
1994

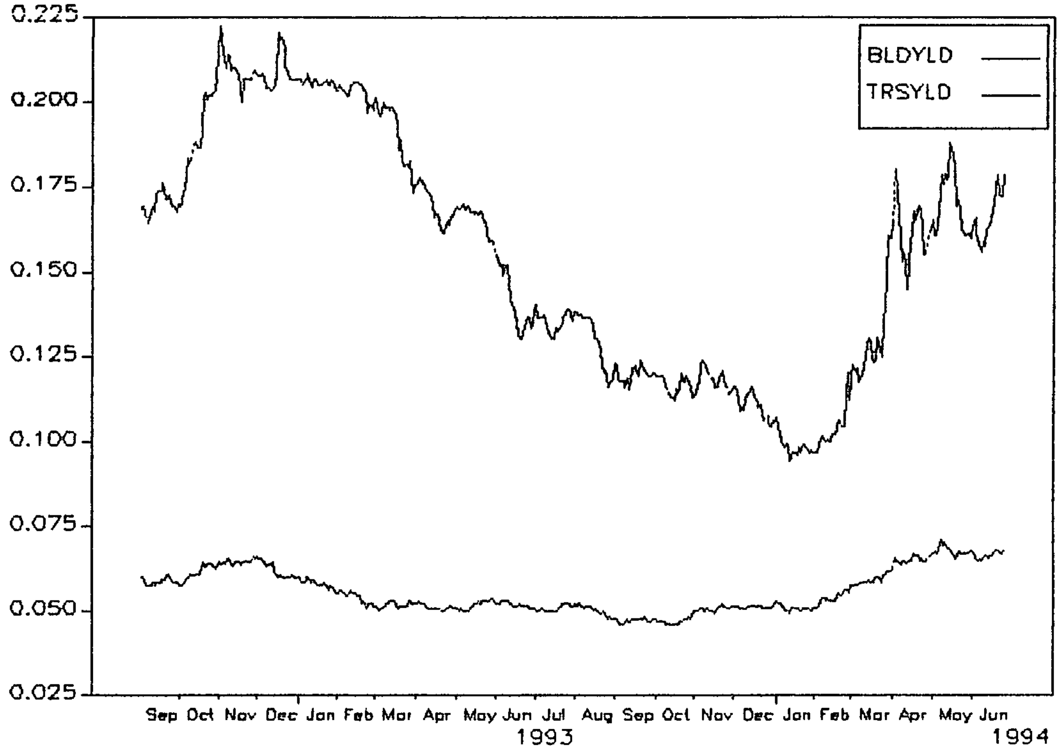

STRIPPED YIELD vs 30y TREASURY BOND YIELD
ARGENTINA PAR BOND DUE 2023
0.225
0.200
0.175
0.150
0.125
0.100
0.075
0.050
STRYLD
TRSYLD
Sep Oct Nov Dec Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan Feb Mar Apr May Jun
1993
1994


BLENDED SPREAD vs 30y TREASURY BOND YIELD
ARGENTINA PAR BOND DUE 2023
0.08
0.07
0.06
0.05
0.04
0.03
0.02
BLDSPD
TRSYLD
Sep Oct Nov Dec Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan Feb Mar Apr May Jun
1993
1994


STRIPPED SPREAD vs 30y TREASURY BOND YIELD
ARGENTINA PAR BOND DUE 2023
0.144
0.128
0.112
0.096
0.080
0.064
0.048
0.032
STRSPD
TRSYLD
Sep Oct Nov Dec Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec Jan Feb Mar Apr May Jun
1993
1994

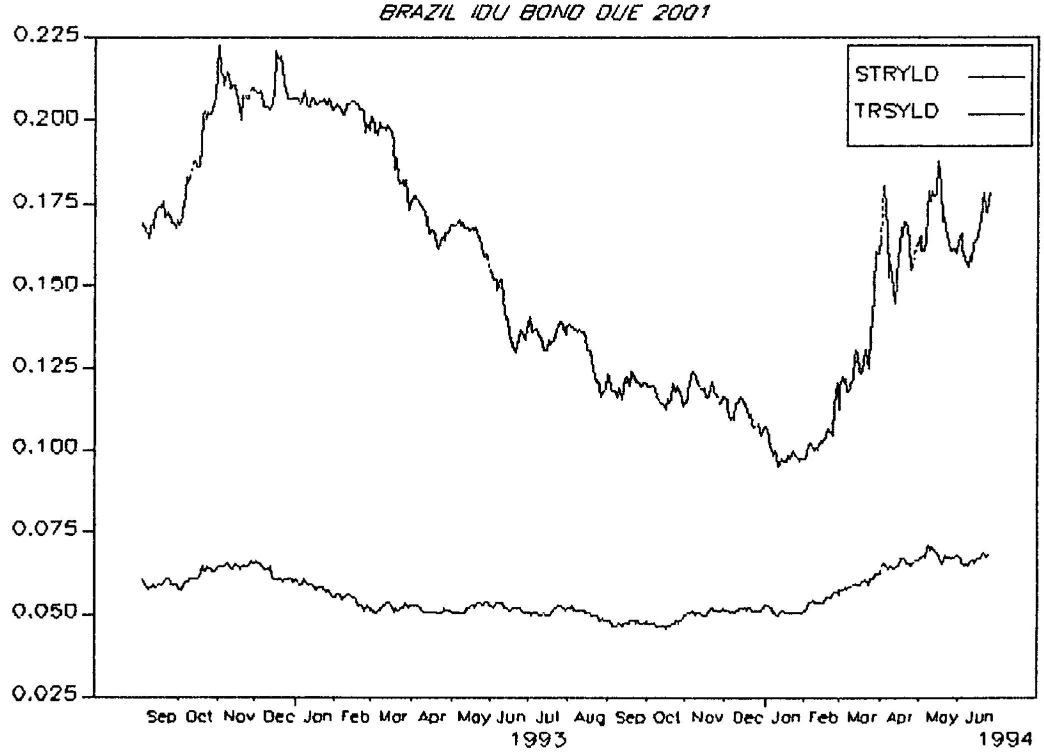
BLENDED YIELD vs 7y TREASURY NOTE YIELD

BRAZIL IDU BOND DUE 2001



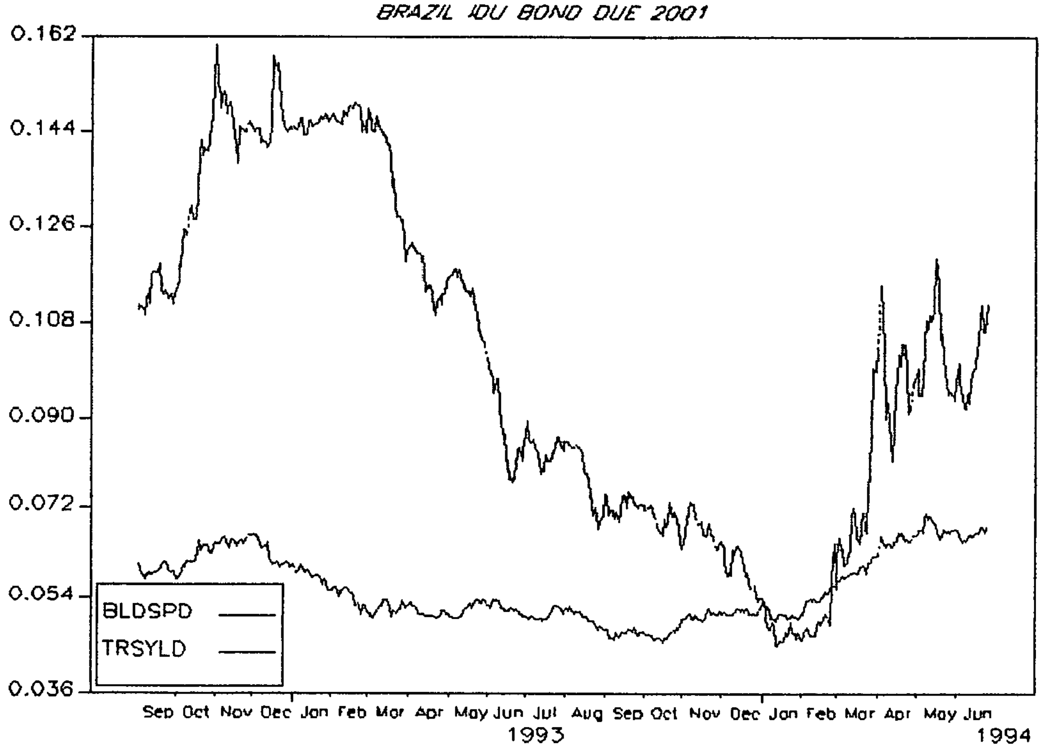
STRIPPED YIELD vs 7y TREASURY NOTE YIELD

BRAZIL IDU BOND DUE 2001



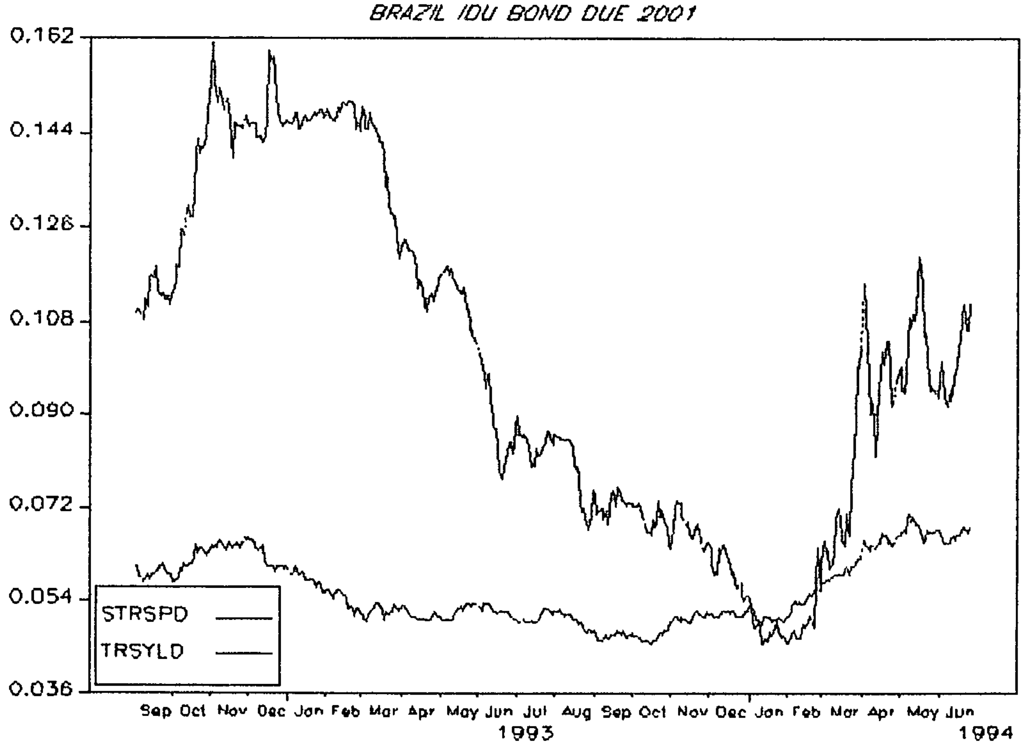
BLENDED SPREAD vs 7y TREASURY NOTE YIELD

BRAZIL IDU BOND DUE 2001



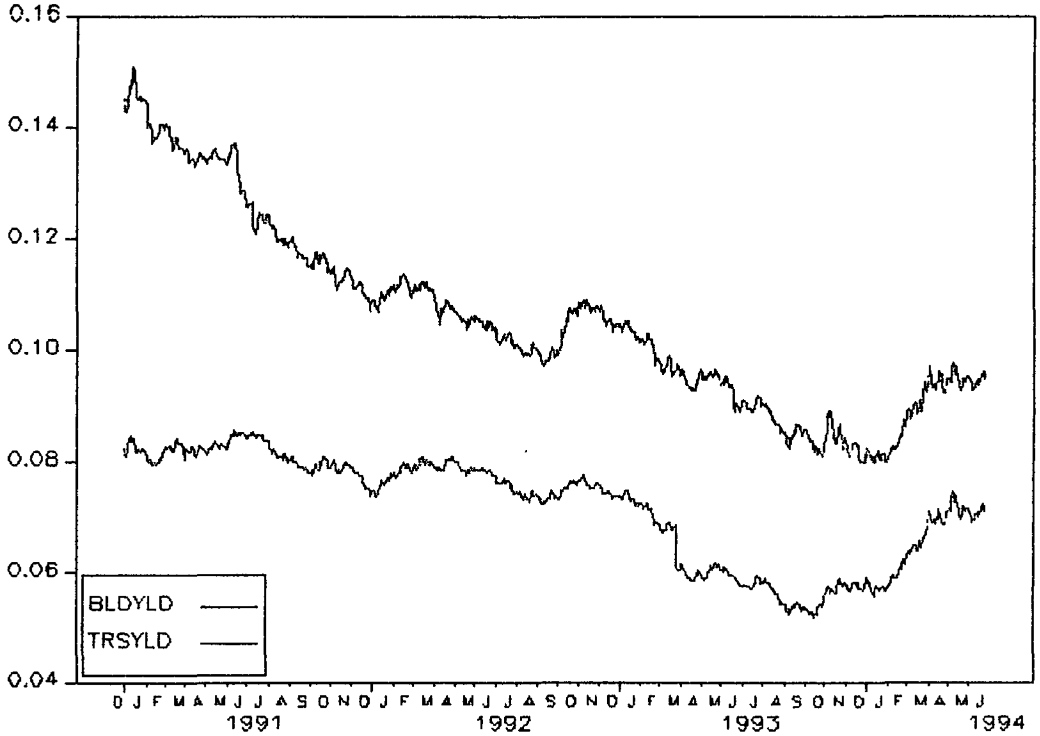
STRIPPED SPREAD vs 7y TREASURY NOTE YIELD

BRAZIL IDU BOND DUE 2001



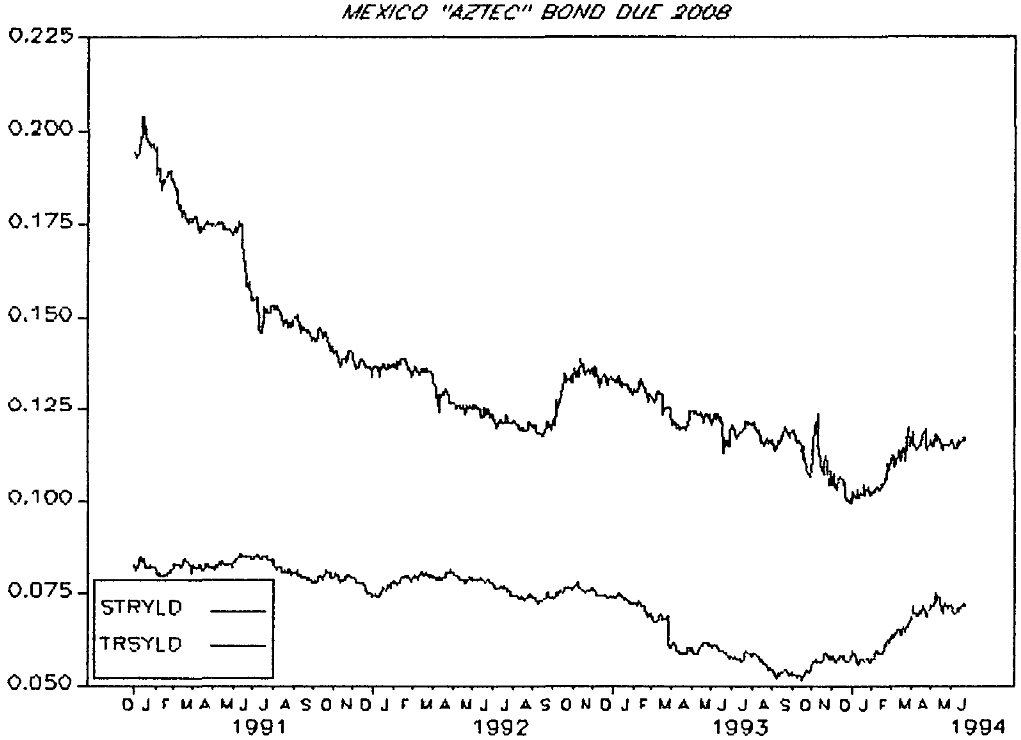
BLENDED YIELD vs 30y TREASURY BOND YIELD

MEXICO "AZTEC" BOND DUE 2008



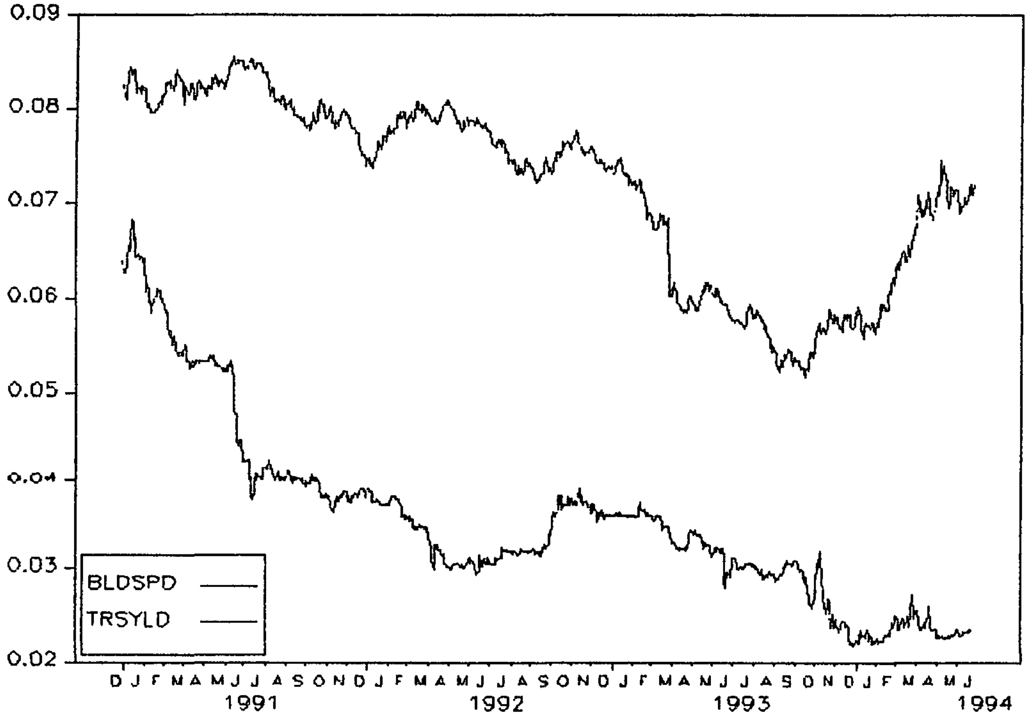
STRIPPED YIELD vs 30y TREASURY BOND YIELD

MEXICO "AZTEC" BOND DUE 2008



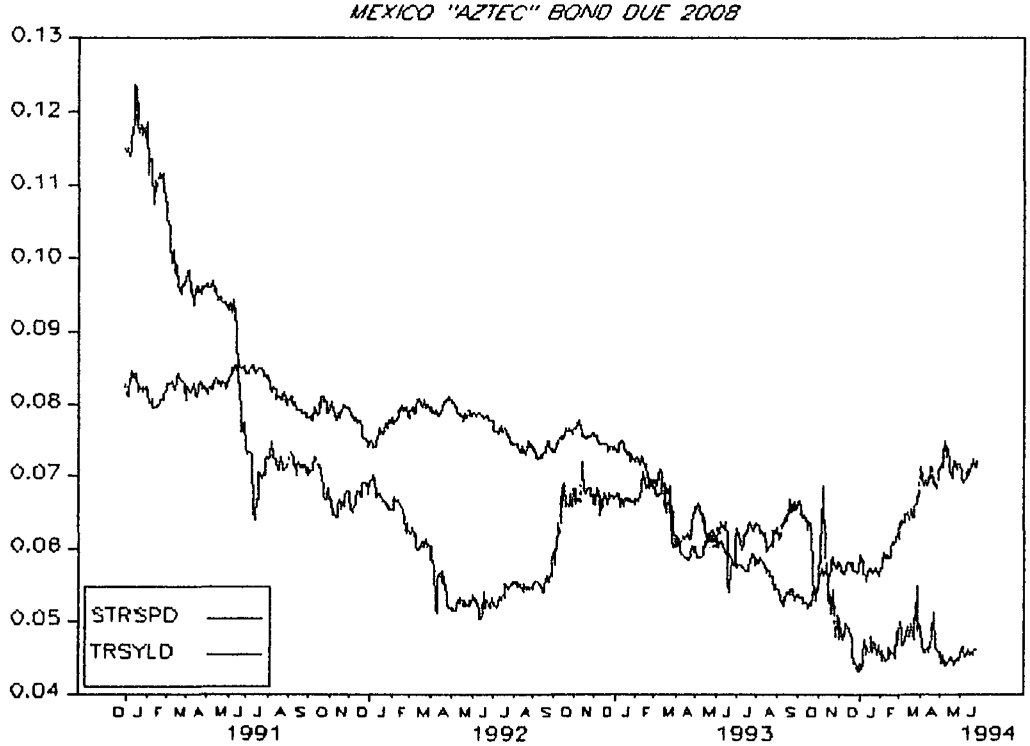
BLENDED SPREAD vs 30y TREASURY BOND YIELD

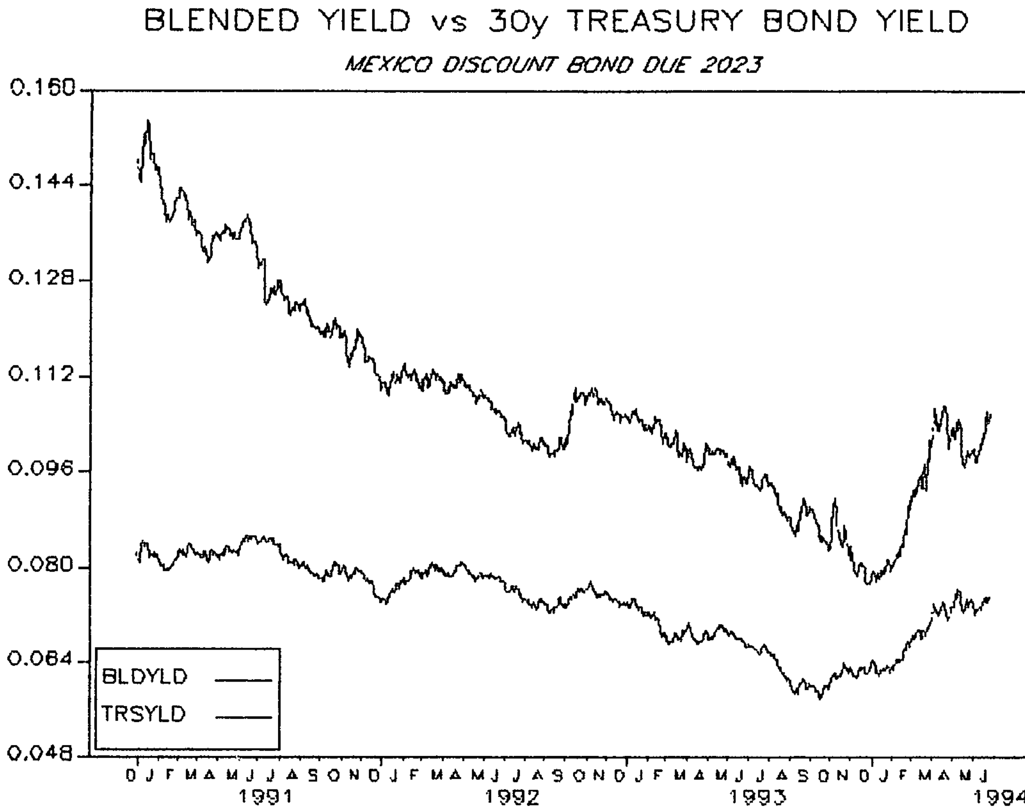
MEXICO "AZTEC" BOND DUE 2008

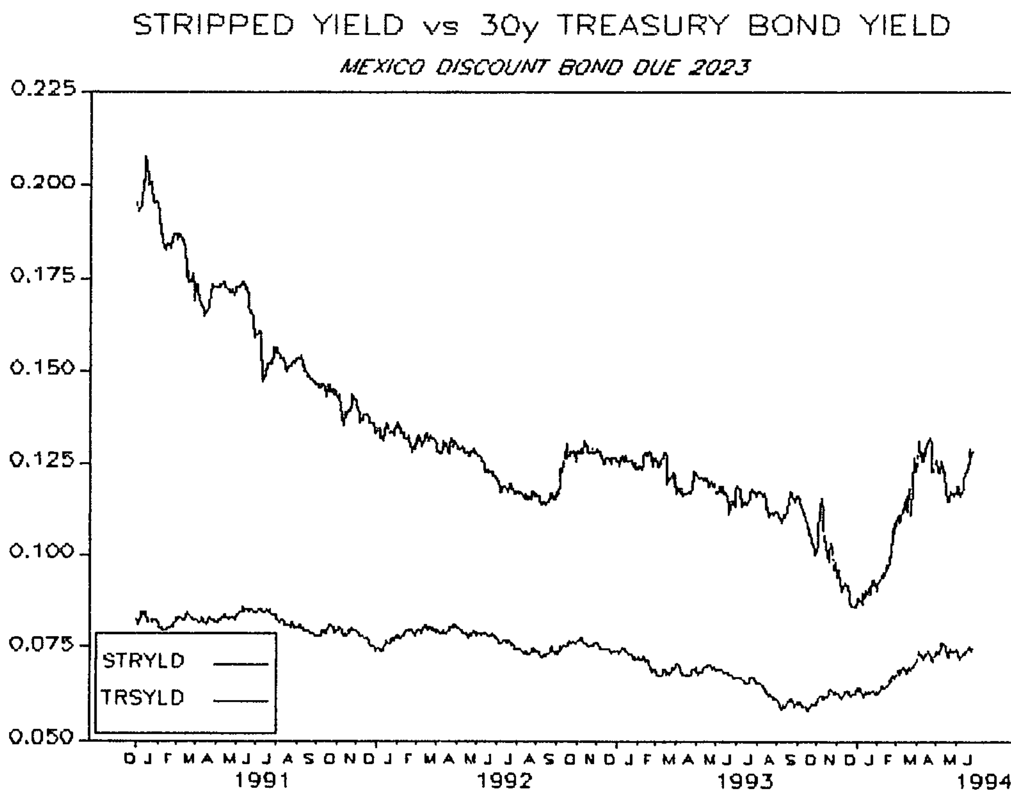


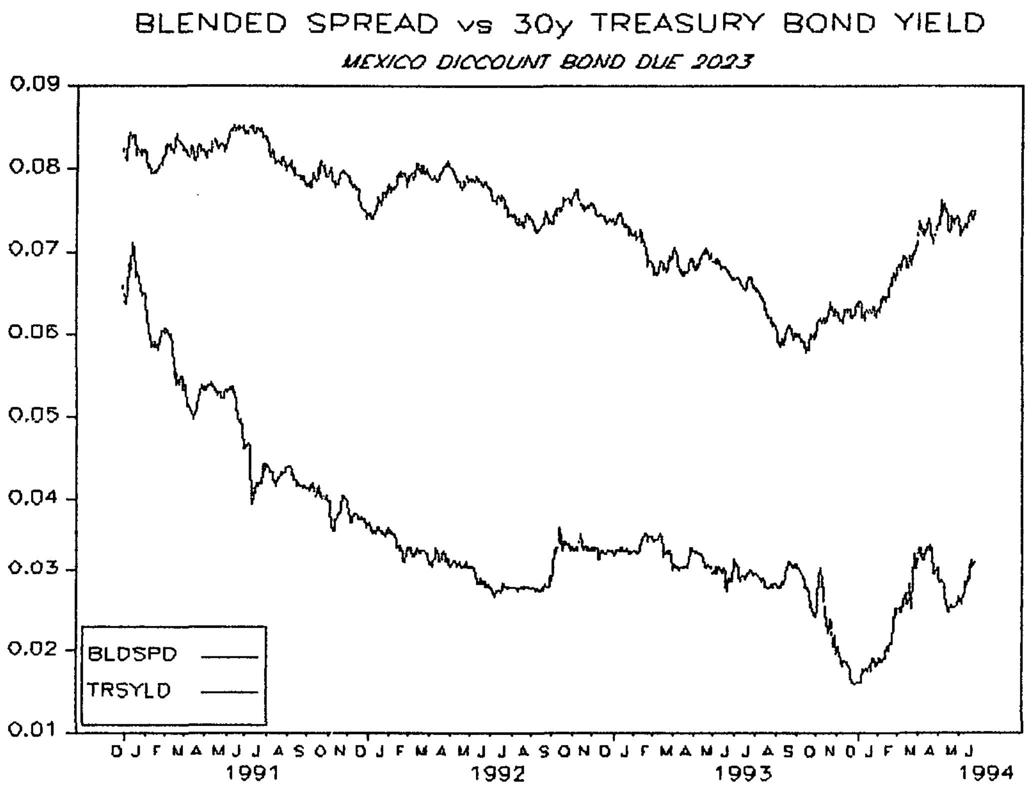
STRIPPED SPREAD vs 30y TREASURY BOND YIELD

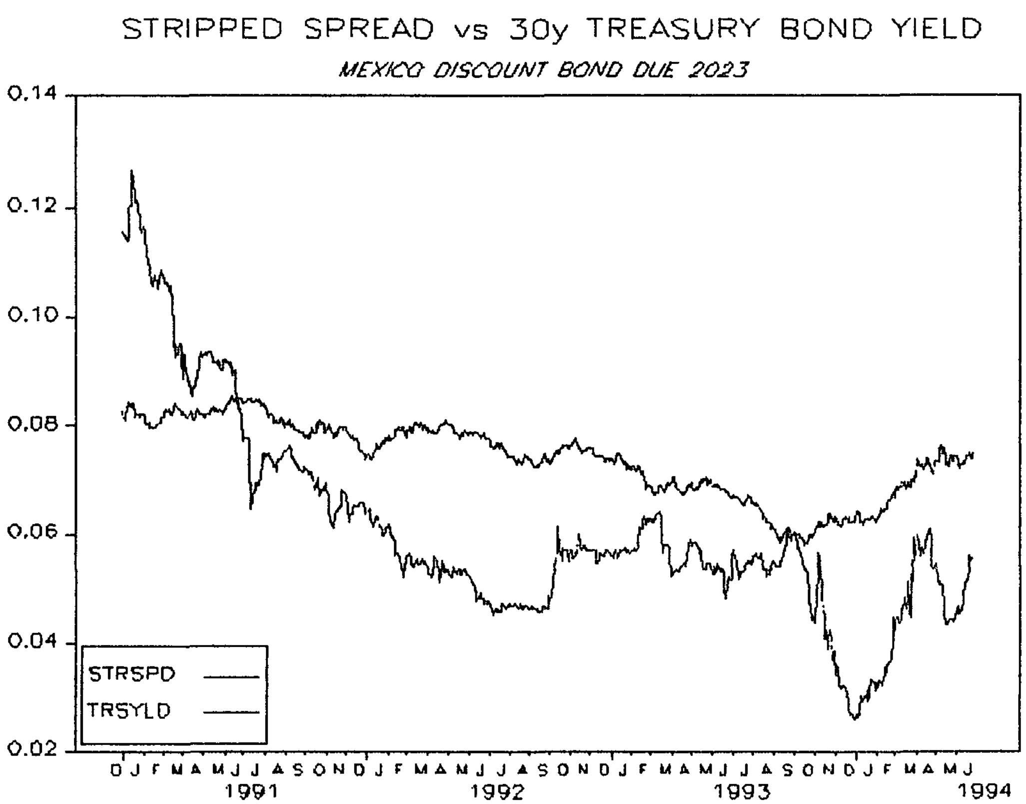
MEXICO "AZTEC" BOND DUE 2008




BLENDED YIELD vs 30y TREASURY BOND YIELD
MEXICO DISCOUNT BOND DUE 2023
0.160
0.144
0.128
0.112
0.096
0.080
0.064
0.048
BLDYLD
TRSYLD
D J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J
1991
1992
1993
1994

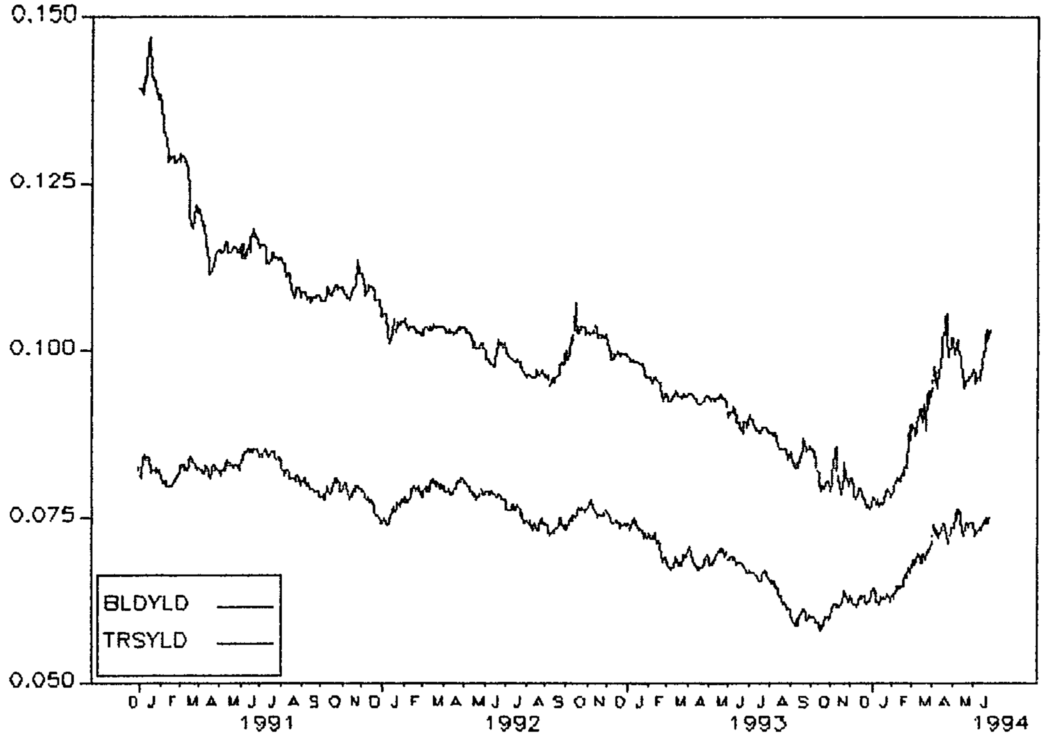

STRIPPED YIELD vs 30y TREASURY BOND YIELD
MEXICO DISCOUNT BOND DUE 2023
0.225
0.200
0.175
0.150
0.125
0.100
0.075
0.050
STRYLD
TRSYLD
D J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J
1991
1992
1993
1994


BLENDED SPREAD vs 30y TREASURY BOND YIELD
MEXICO DICCOUNT BOND DUE 2023
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
BLDSPD
TRSYLD
D J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J
1991
1992
1993
1994


STRIPPED SPREAD vs 30y TREASURY BOND YIELD
MEXICO DISCOUNT BOND DUE 2023
0.14
0.12
0.10
0.08
0.06
0.04
0.02
STRSPD
TRSYLD
D J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J
1991
1992
1993
1994

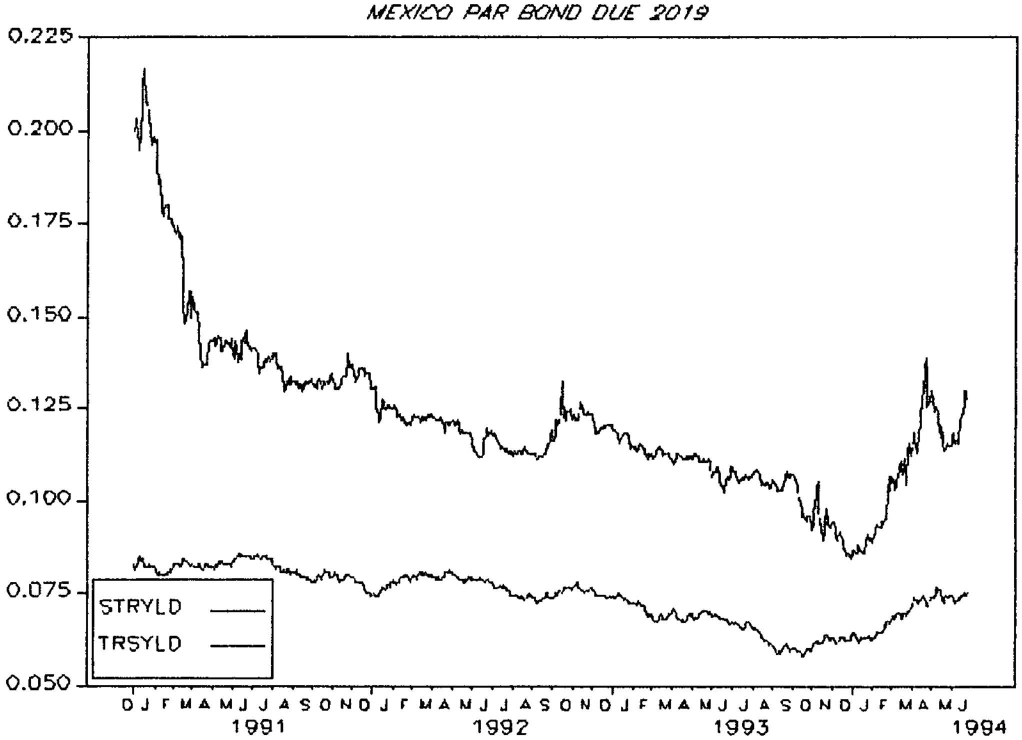
BLENDED YIELD vs 30y TREASURY BOND YIELD

MEXICO PAR BOND DUE 2019



STRIPPED YIELD vs 30y TREASURY BOND YIELD

MEXICO PAR BOND DUE 2019

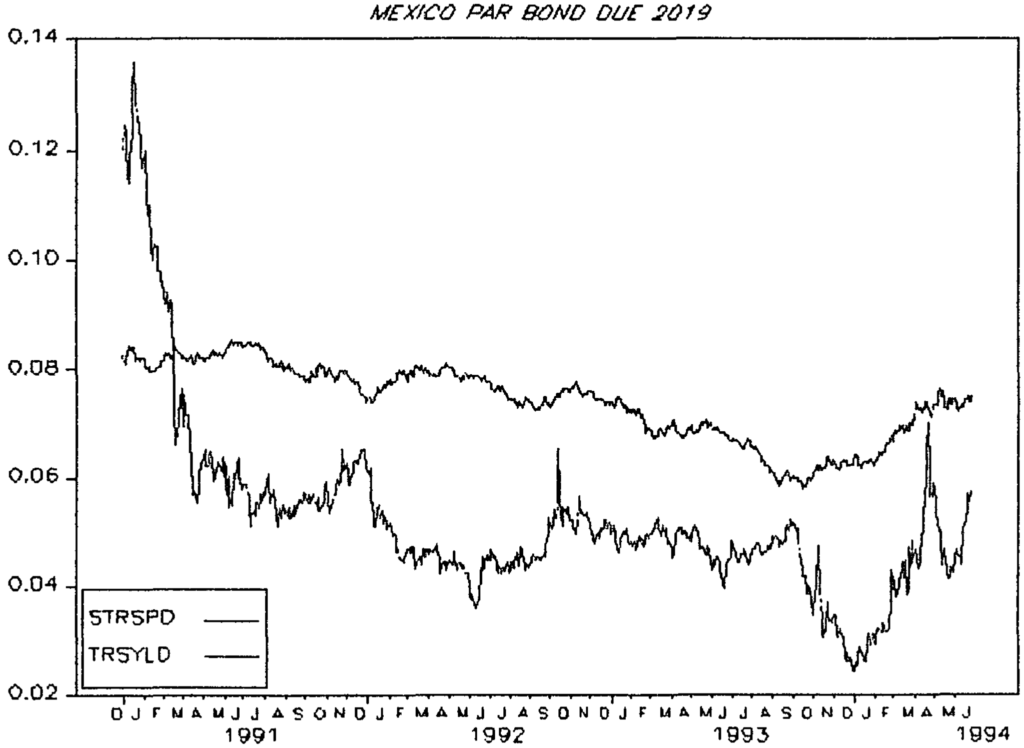


BLENDED SPREAD vs 30y TREASURY BOND YIELD

MEXICO PAR BOND DUE 2019

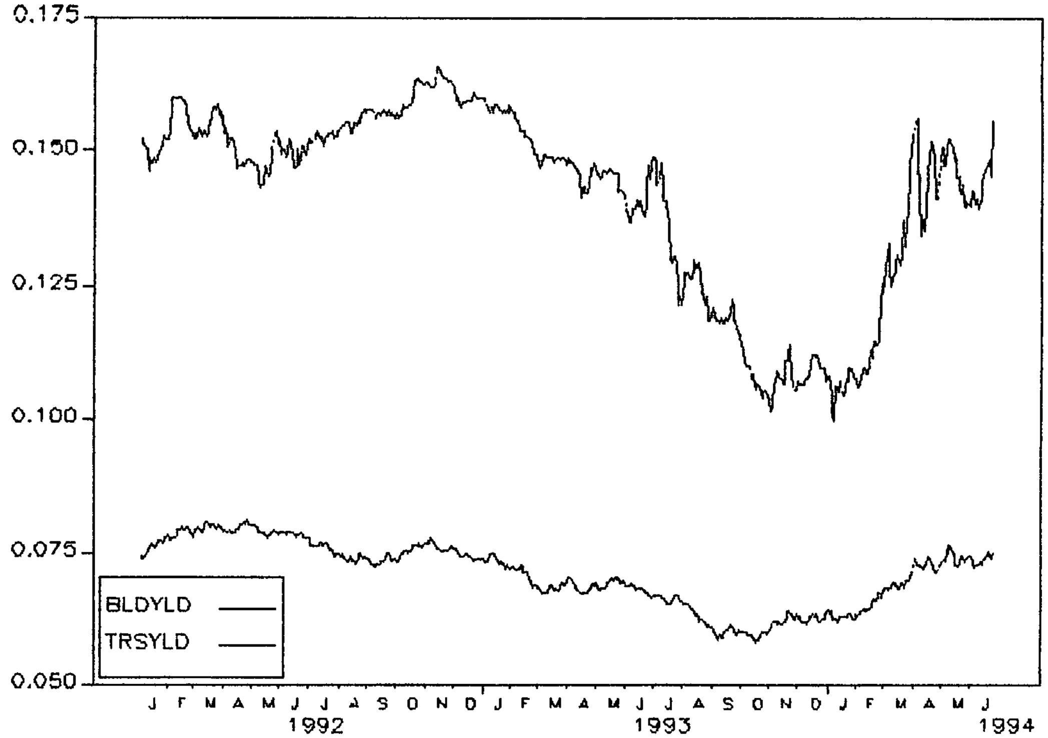
STRIPPED SPREAD vs 30y TREASURY BOND YIELD

MEXICO PAR BOND DUE 2019



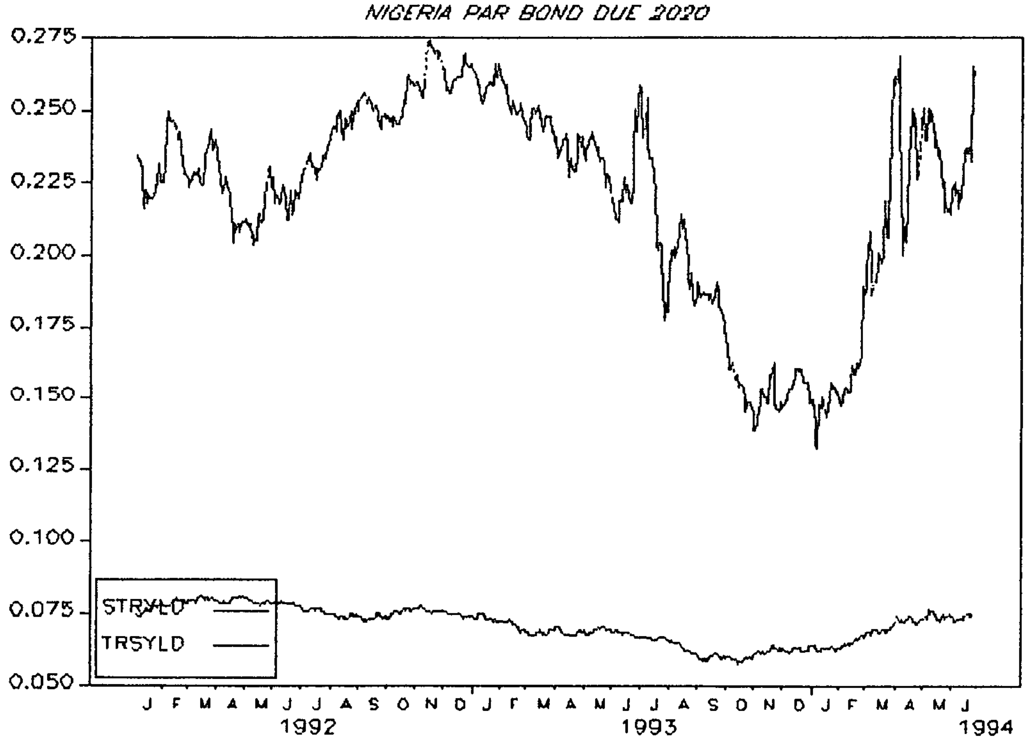
BLENDED YIELD vs 30y TREASURY BOND YIELD

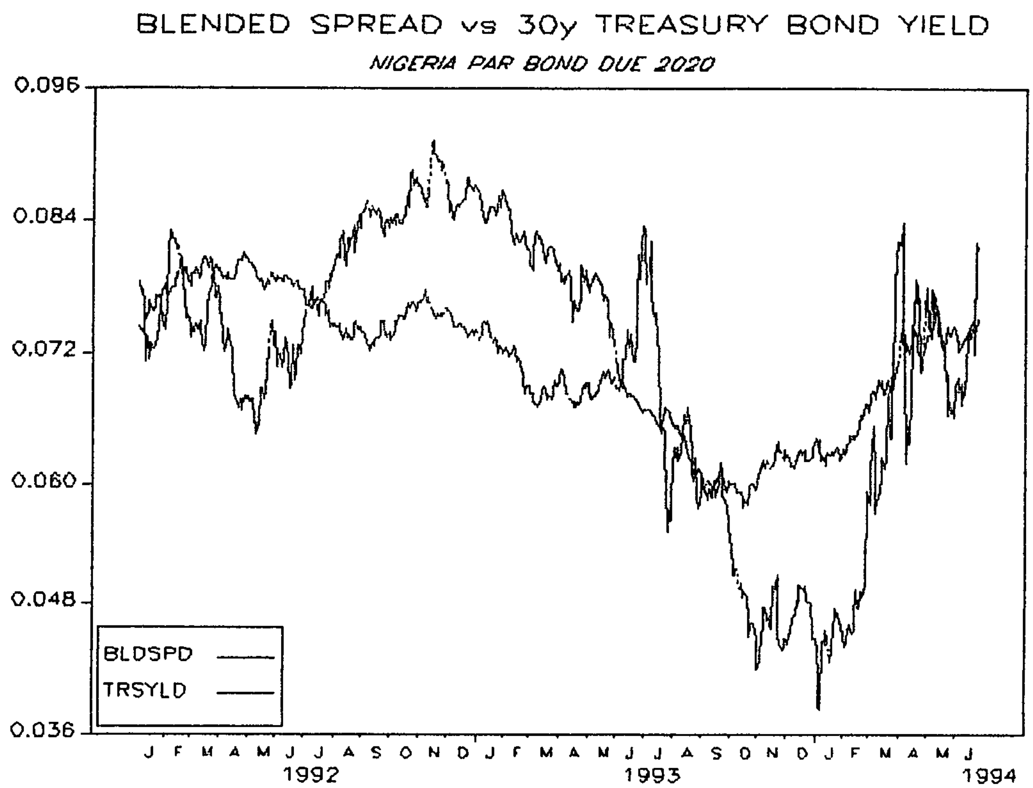
NIGERIA PAR BOND DUE 2020

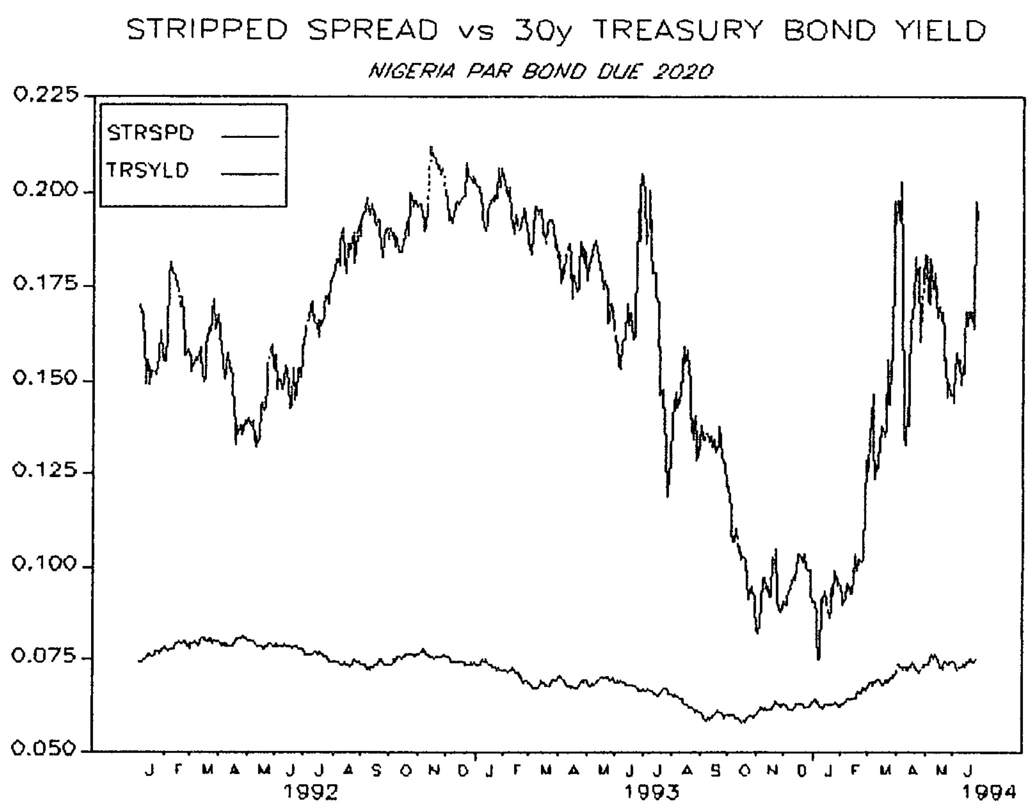


STRIPPED YIELD vs 30y TREASURY BOND YIELD

NIGERIA PAR BOND DUE 2020

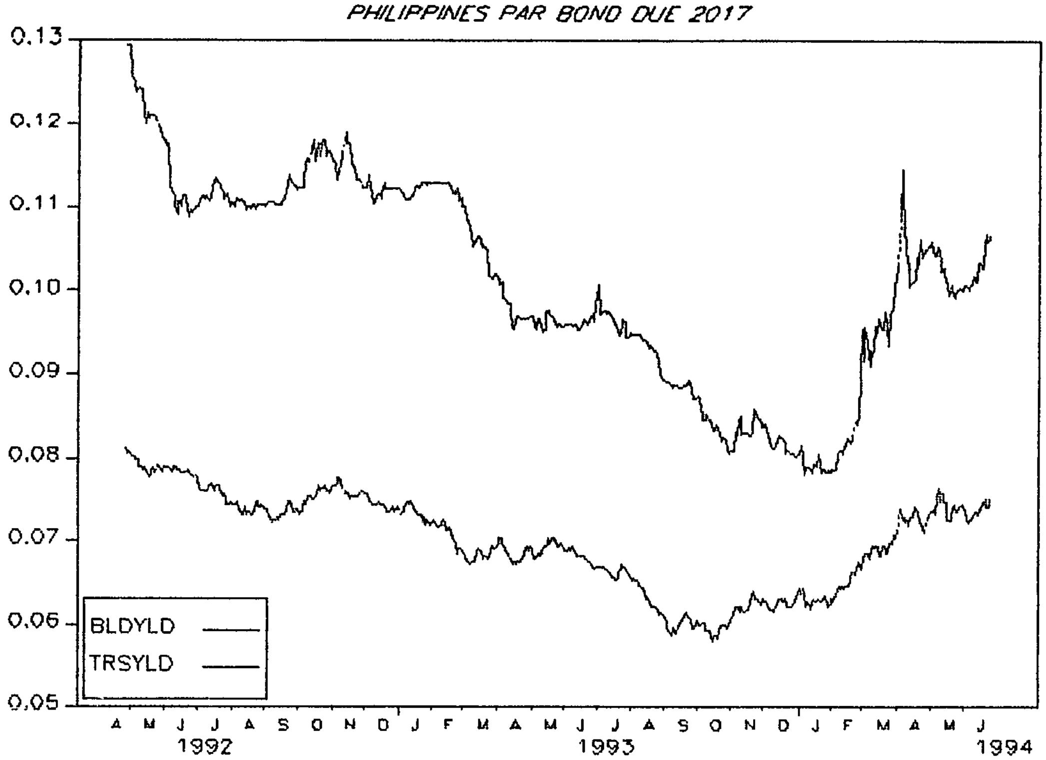



BLENDED SPREAD vs 30y TREASURY BOND YIELD
NIGERIA PAR BOND DUE 2020
0.096
0.084
0.072
0.060
0.048
0.036
BLDSPD
TRSYLD
J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J
1992
1993
1994


STRIPPED SPREAD vs 30y TREASURY BOND YIELD
NIGERIA PAR BOND DUE 2020
0.225
0.200
0.175
0.150
0.125
0.100
0.075
0.050
STRSPD
TRSYLD
J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J
1992
1993
1994

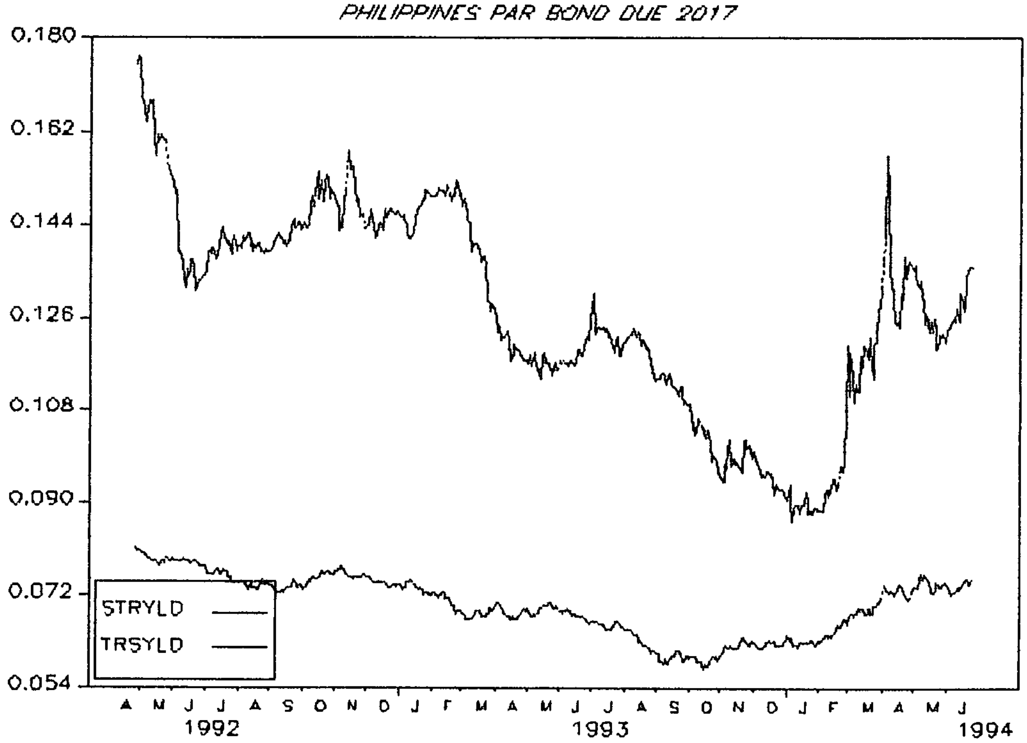
BLENDED YIELD vs 30y TREASURY BOND YIELD

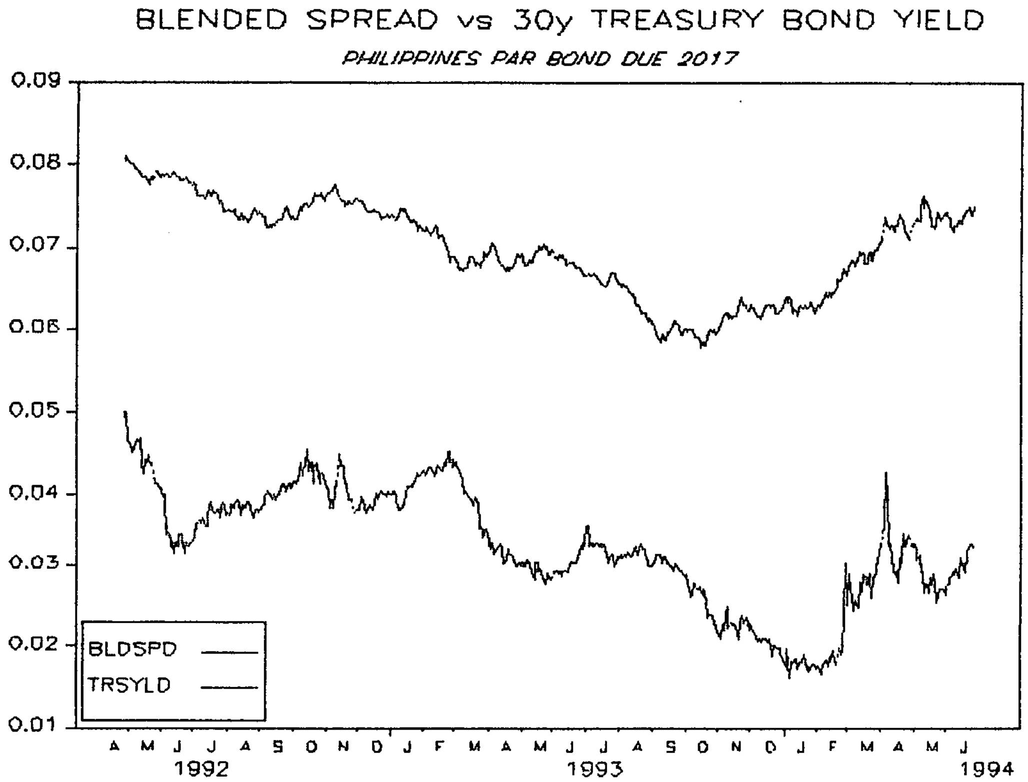
PHILIPPINES PAR BOND DUE 2017

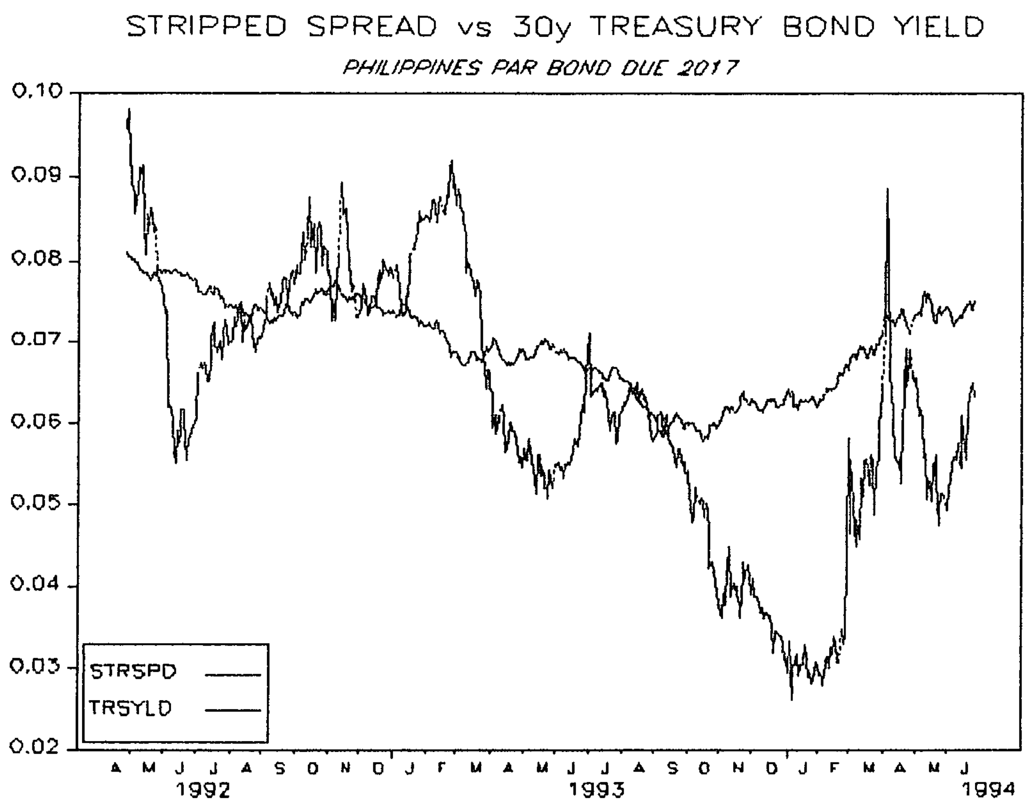


STRIPPED YIELD vs 30y TREASURY BOND YIELD

PHILIPPINES PAR BOND DUE 2017

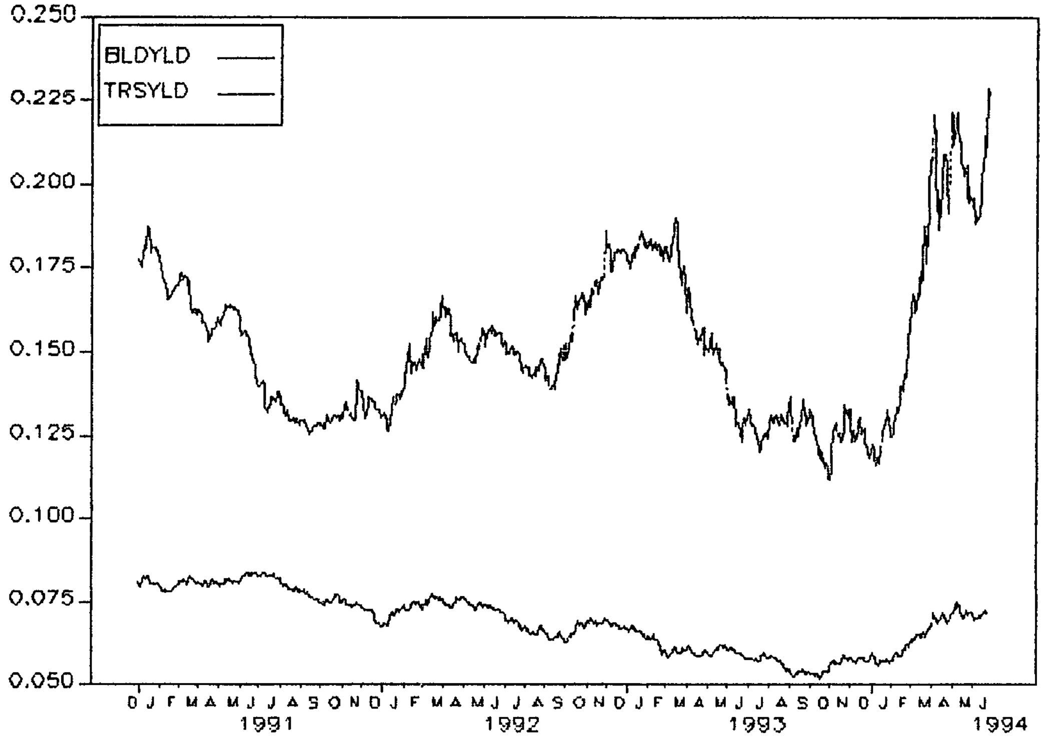



BLENDED SPREAD vs 30y TREASURY BOND YIELD
PHILIPPINES PAR BOND DUE 2017
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
BLDSPD
TRSYLD
A M J J A S O N D J F M A M J J A S O N D J F M A M J
1992
1993
1994


STRIPPED SPREAD vs 30y TREASURY BOND YIELD
PHILIPPINES PAR BOND DUE 2017
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
STRSPD
TRSYLD
A M J J A S O N D J F M A M J J A S O N D J F M A M J
1992
1993
1994

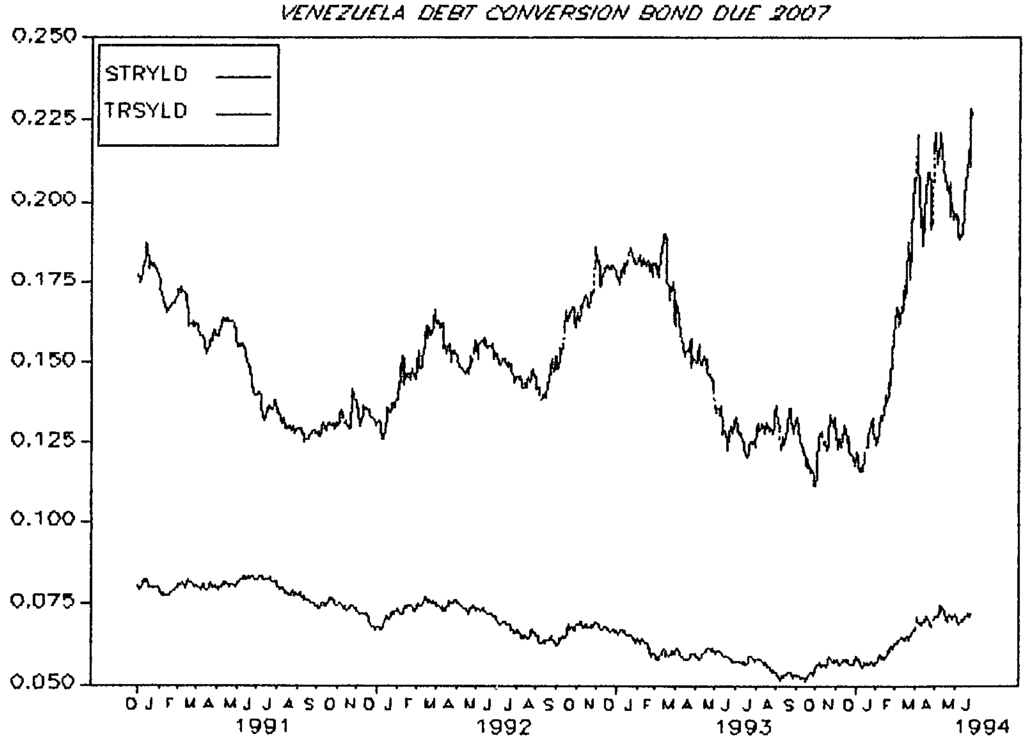
BLENDED YIELD vs 10y TREASURY BOND YIELD

VENEZUELA DEBT CONVERSION BOND DUE 2007



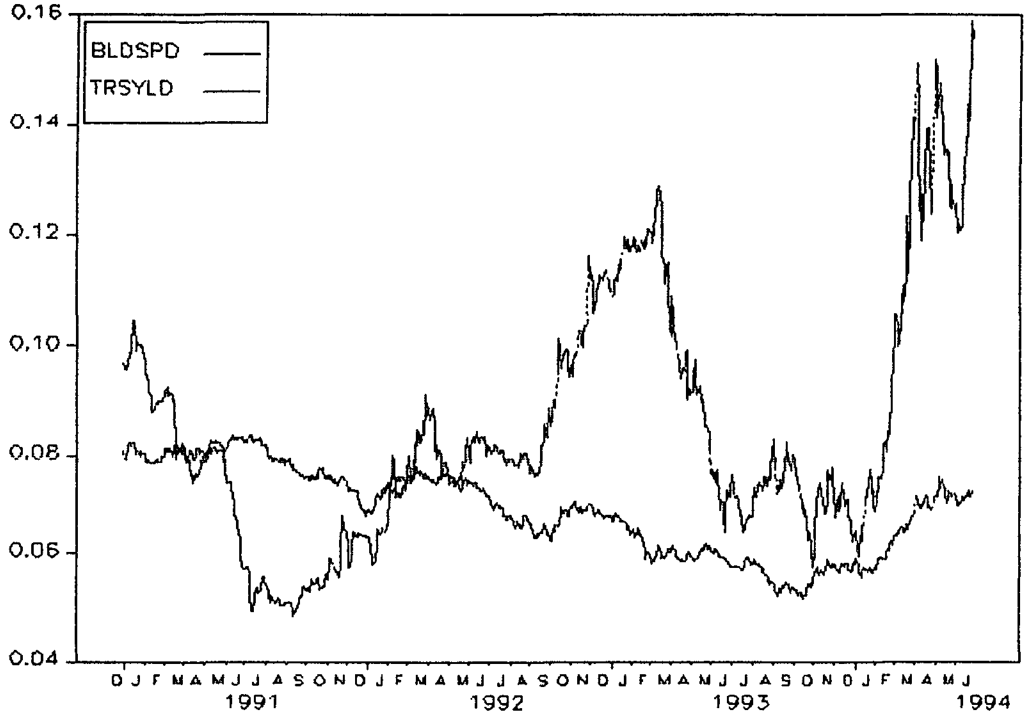
STRIPPED YIELD vs 10y TREASURY BOND YIELD

VENEZUELA DEBT CONVERSION BOND DUE 2007



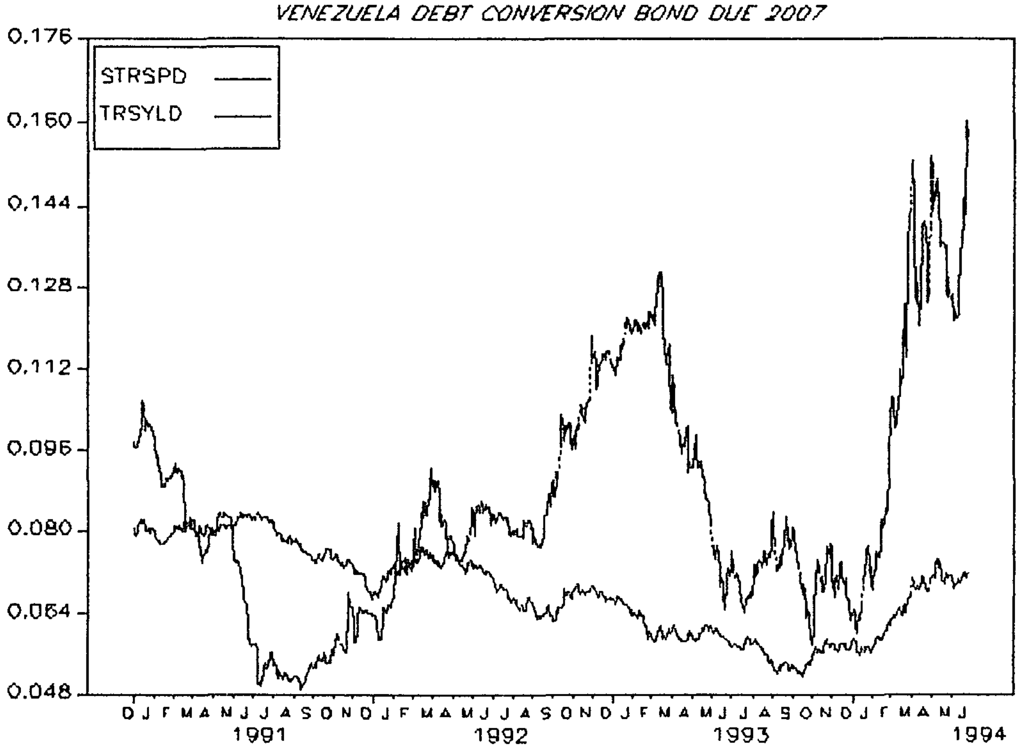
BLENDED SPREAD vs 10y TREASURY BOND YIELD

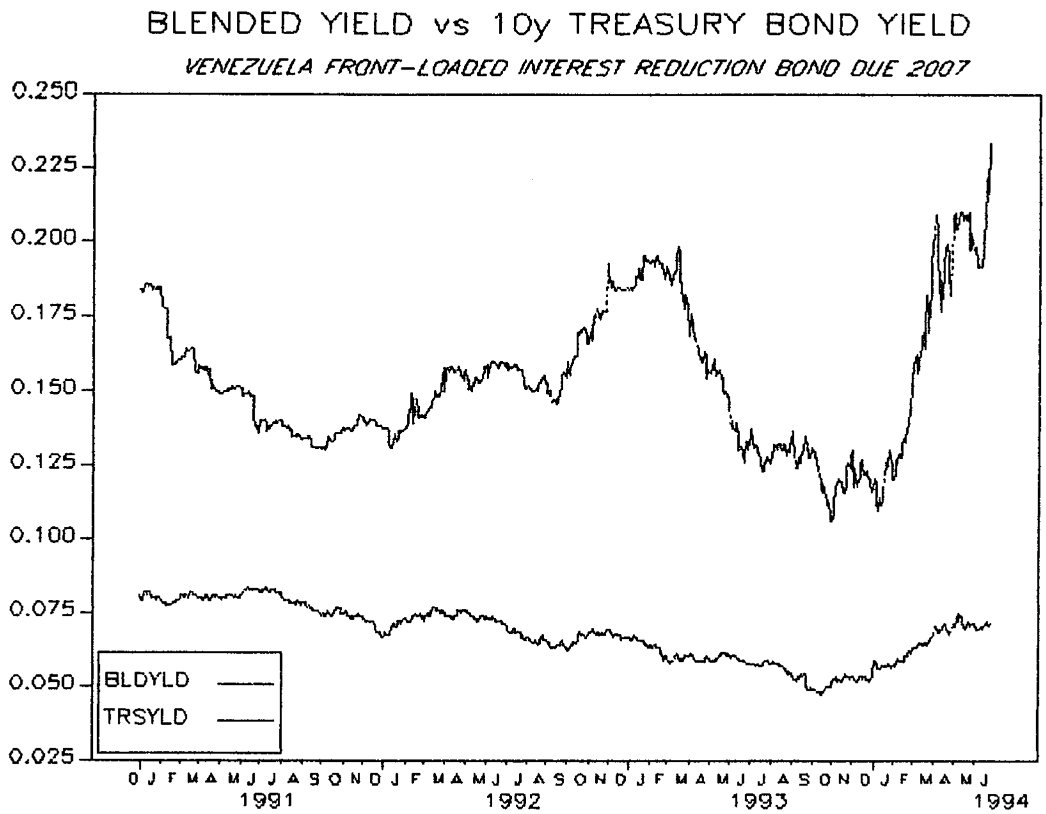
VENEZUELA DEBT CONVERSION BOND DUE 2007

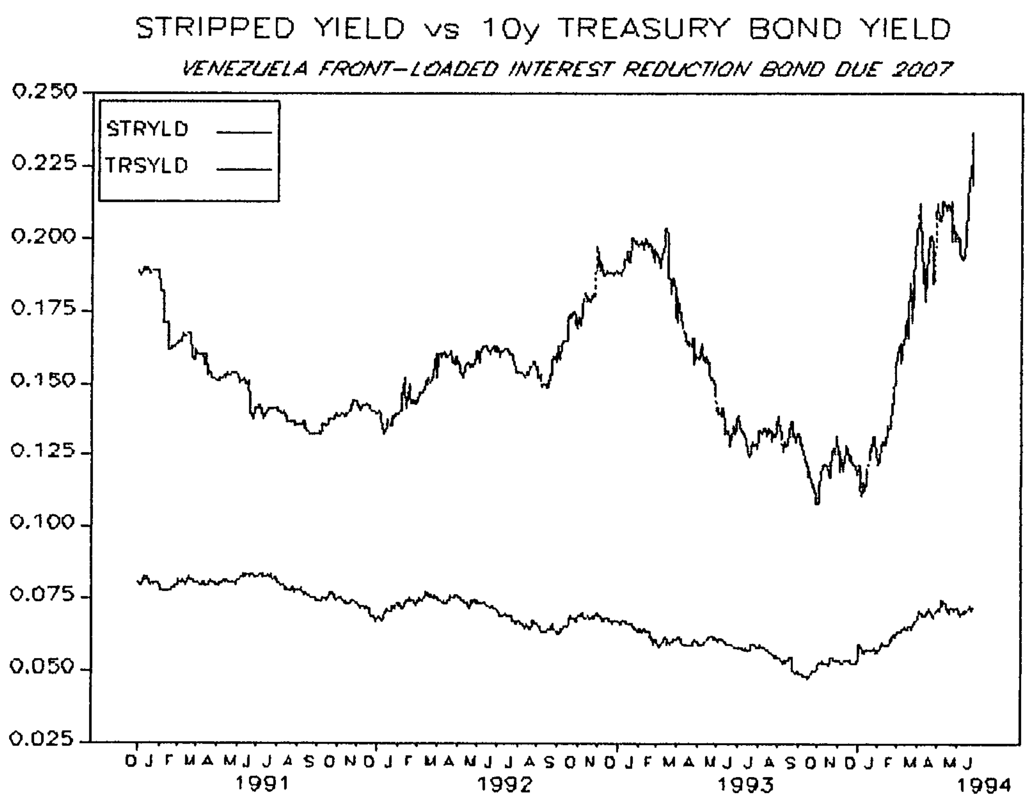


STRIPPED SPREAD vs 10y TREASURY BOND YIELD

VENEZUELA DEBT CONVERSION BOND DUE 2007

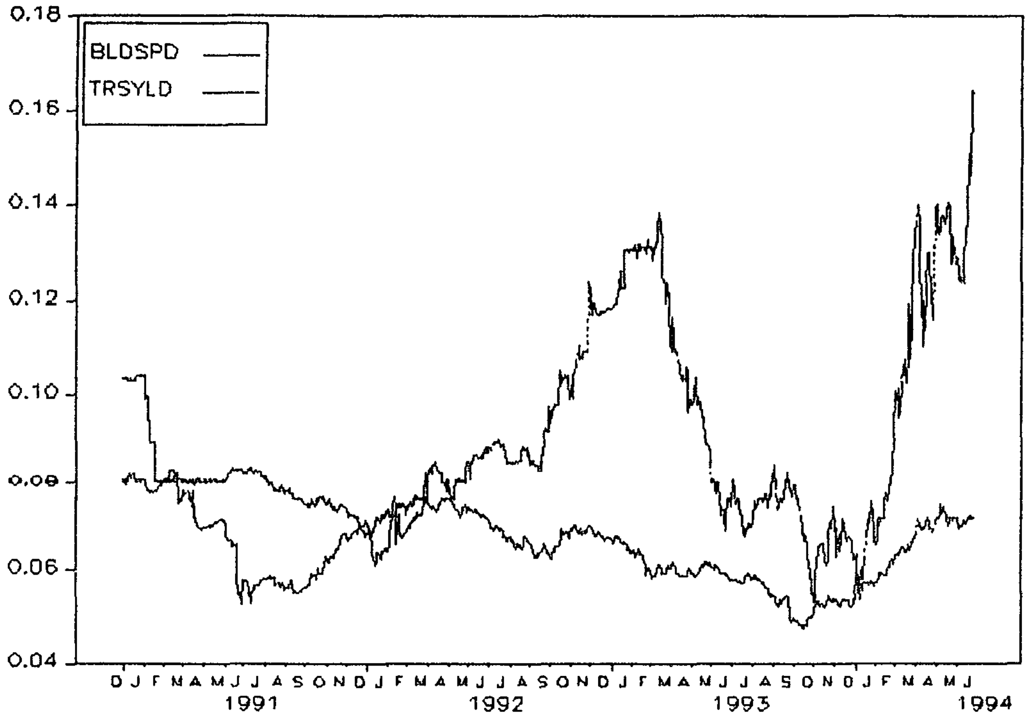



BLENDED YIELD vs 10y TREASURY BOND YIELD
VENEZUELA FRONT-LOADED INTEREST REDUCTION BOND DUE 2007
0.250
0.225
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025
BLDYLD
TRSYLD
O J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J
1991
1992
1993
1994


STRIPPED YIELD vs 10y TREASURY BOND YIELD
VENEZUELA FRONT-LOADED INTEREST REDUCTION BOND DUE 2007
0.250
0.225
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025
STRYLD
TRSYLD
O J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J J A S O N D J F M A M J
1991
1992
1993
1994

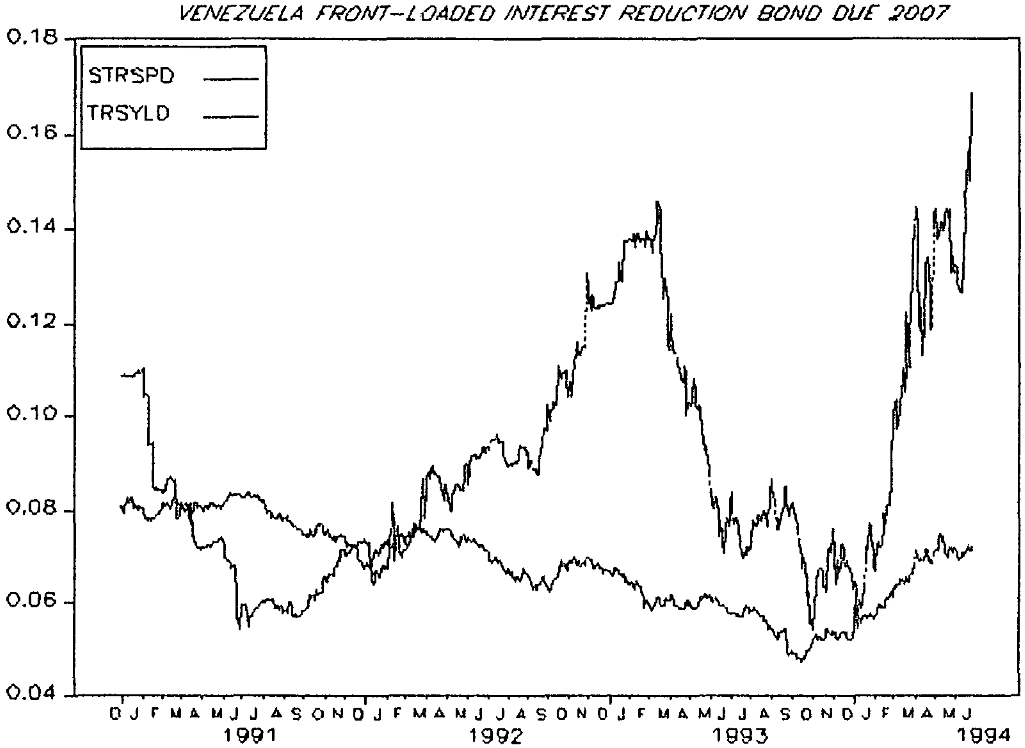
BLENDED SPREAD vs 10y TREASURY BOND YIELD

VENEZUELA FRONT-LOADED INTEREST REDUCTION BOND DUE 2007



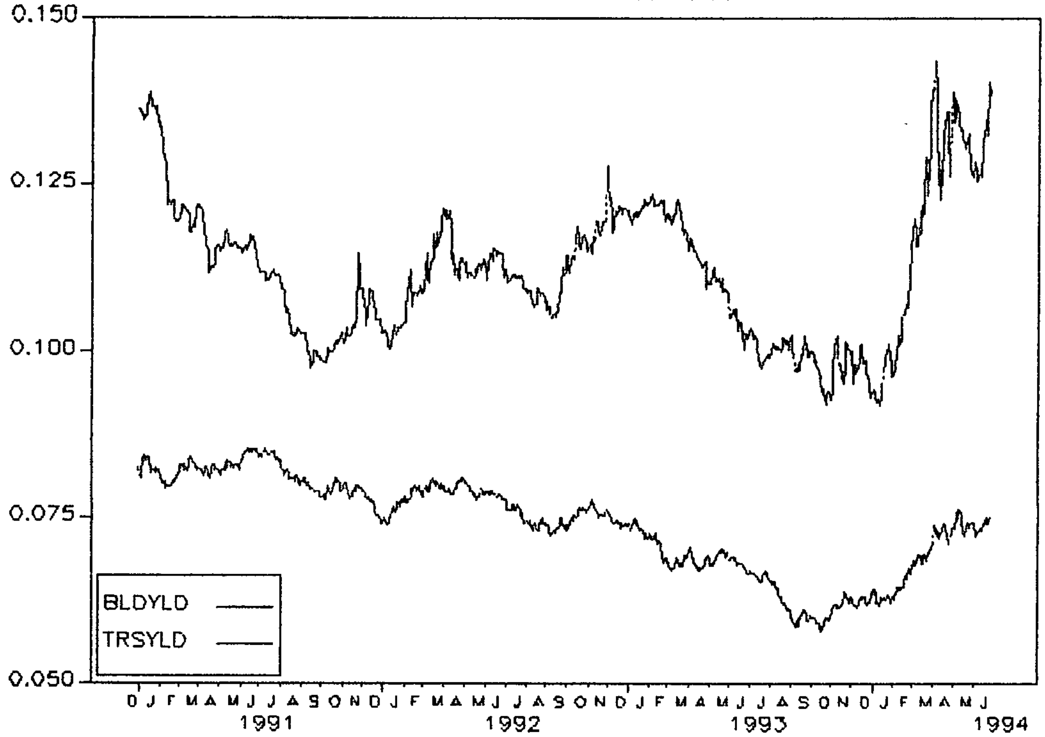
STRIPPED SPREAD vs 10y TREASURY BOND YIELD

VENEZUELA FRONT-LOADED INTEREST REDUCTION BOND DUE 2007



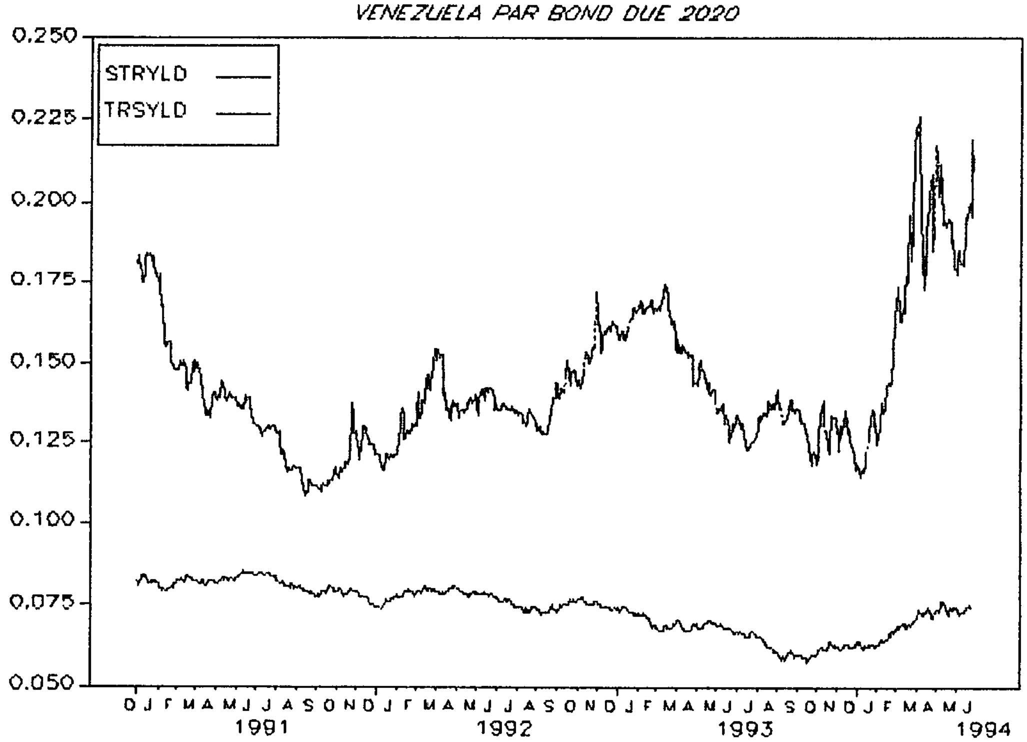
BLENDED YIELD vs 30y TREASURY BOND YIELD

VENEZUELA PAR BOND DUE 2020



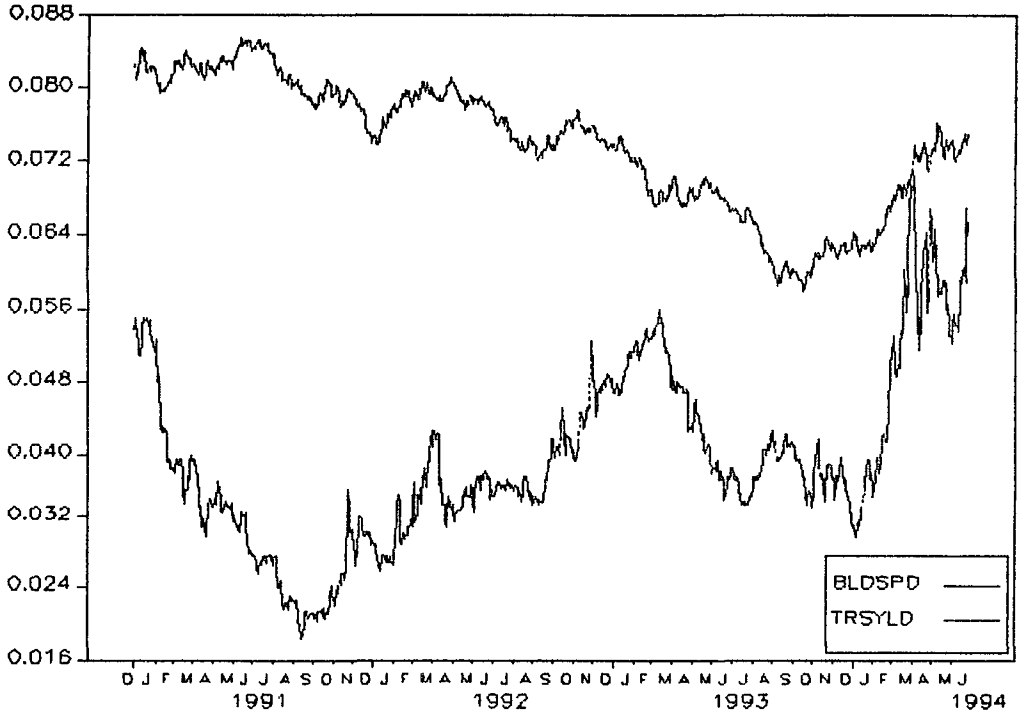
STRIPPED YIELD vs 30y TREASURY BOND YIELD

VENEZUELA PAR BOND DUE 2020



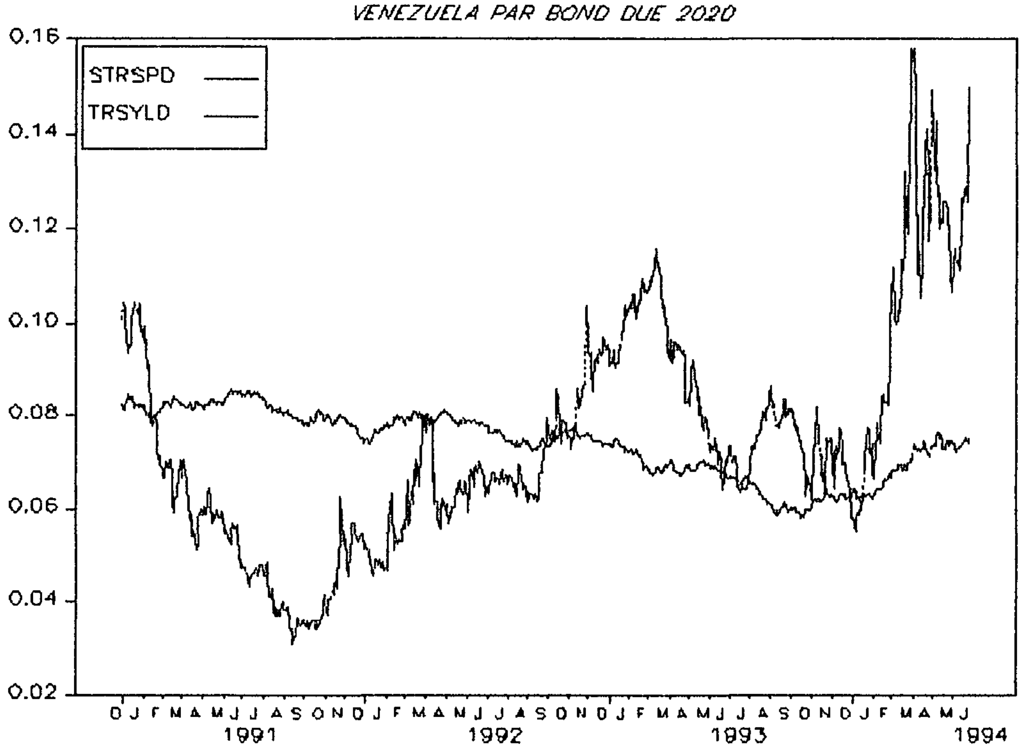
BLENDED SPREAD vs 30y TREASURY BOND YIELD

VENEZUELA PAR BOND DUE 2020



STRIPPED SPREAD vs 30y TREASURY BOND YIELD

VENEZUELA PAR BOND DUE 2020



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