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Forecasting with Bayesian vector autoregressions: An application to post-liberalization Turkey, 1980–1991

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FORECASTING WITH BAYESIAN VECTOR AUTOREGRESSIONS: AN APPLICATION TO POST-LIBERALIZATION TURKEY: 1980-1991

by

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A dissertation submitted to the Graduate Faculty in Economics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

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Introduction

Large scale simultaneous-equation econometric models have been used for both forecasting and policy-analysis purposes. The way these models are constructed and the use of their outcome have been criticized by several leading authors in economics since the 1960s [Lucas (1976), Lucas and Sargent (1979), Sims (1980)]. Objections raised against these modeling techniques can be summarized in two categories:

i. The doctrines on which the models is based are fundamentally wrong:

> Keynesian macroeconometric models cannot provide reliable guidance in the formulation of monetary, fiscal, or other types of policy. . . . [With] their lack of a sound theoretical or econometric basis, . . . there is no hope that minor or even major modification of these models will lead to significant improvement in their reliability.
> —Lucas and Sargent (1979)

ii. Even though they perform forecasting and policy-analysis functions, the imposed restrictions on these models are "incredible."

Besides these criticisms, the poor forecasting performance of traditional models has led to the development of different modeling methods for economic time series.

It is possible to summarize the basic features of economic aggregates without relying upon any economic theory. Box-Jenkins modeling techniques, known as ARIMA representation, have long been used in the literature to get stylized macroeconomic facts [see Blanchard and Fisher (1989), Ch. 1]. These models also have a good record of forecast performance compared to other large-scale models such as Wharton-EFU, Klein, etc. [Granger and Newbold (1986), p. 289]. While the univariate ARIMA models capture the important features of the series, the same methods can be used to form multivariate models with a large number of exclusionary restrictions.

Sims (1980) proposed the Vector Autoregressive (VAR) model as an alternative modeling technique. A VAR model is constructed by selecting a group of variables and allowing all them to interact with their own and each other's current and past values. Unconditional forecasts, i.e. forecasts assuming no specific path for any of the variables contained in the model, are made by taking into consideration the impact of each variable on its own future values and the future values of the other variables.

The main drawback of the VAR specification is that the number of free parameters in the model decreases exponentially with the number of variables in the system. For example, given monthly data from post-liberalization Turkey, a model with eight variables and twelve lags would have very few degrees of freedom. With so many coefficients, the model fits the data perfectly. In fact, the result is overfitting of the data, implying a large mean square error of out-of-sample forecasts. In practice, a solution to the overfitting problem is to reduce the number of parameters to be estimated or to impose a priori restrictions on the parameters estimated. Reducing the number of variables or assuming shorter lags as a solution to the overfitting problem amounts to what the VAR modeling was proposed against–namely, imposing exclusionary restrictions on the parameters of the model.

Litterman(1980) suggests the Bayesian Vector Autoregression (BVAR) approach as a solution to the overparameterization problem of the VAR technique. The BVAR model imposes "fuzzy" restrictions on the coefficients, using a well-defined prior for the coefficient distribution [Litterman (1979); Doan, Litterman, and Sims (1984)]. The forecast history of this technique is quite encouraging. [Granger and Newbold (1986); Hoehn, Gruben, and Fomby (1984); Kinal and Ratner (1983); Litterman (1980, 1984a. 1984b, 1985); Newbold (1984); Sims (1989); Todd (1984)].

In this study, the time-series properties of nine macroeconomic variables from the Turkish economy are examined and different forecasting techniques are considered. Recent developments in the Turkish economy and brief summary of statistics of the major aggregates are presented in Section 1.2 of this chapter. Section 1.3 investigates the nonstationarity of the underlying series. In sections 1.4 and 1.5, a univariate model is constructed and the forecast statistics are defined. In Chapter 2, VAR modeling is applied to variables in question. Impulse responses, forecast results, and forecast performance from the VAR application can be found in the same chapter. Chapter 3 examines the Kalman filter technique from a Bayesian perspective and looks at the details of BVAR modeling. Finally, an application of BVAR and determination of prior parameters, along with a forecast performance of the model, can be found in Chapter 4.

1.2. The Recent Developments in Turkish Economy

Following a period of economic, financial, and political difficulties in the late 1970s, Turkish government implemented a comprehensive economic program beginning January 1980. The program aimed for a basic reorientation of economic policy away from direct governmental intervention and control toward greater reliance on market forces as a means of promoting economic growth and better resource allocation. Although there have been many deviations from the program, subsequent Turkish governments have committed themselves to the economic policies introduced in 1980.

The main objective of this section is to give an overview of the characteristics of the economy during the post-liberalization period. The main indicators for each year between 1981 and 1990 are given in Tables 1.1 and 1.2.

Unlike many other countries with similar problems, Turkey succeeded to reach above-the-average growth rates for the past ten years. During the first three years of the program (1981-1983), economic growth averaged 4%. In December 1983, the first civilian government after the military coup was installed. This government was in strong favor of the liberalization program introduced in 1980. During its first term, 1984-1987, more radical changes in economic policy took effect, and the economy reached a remarkable 6.7 % average annual growth. During that government's second term, 1988-1990, the economy didn't grow as fast as the previous period, it witnessed 4.9 % growth per year. It is this impressive growth of the economy that made Turkey the "most quoted example" of international finance organizations.

Table 1.1 Main Economic Indicators of Turkey *(Annual Percentage Change)*

	1981	1982	1983	1984	1985
GNP	4.1	4.5	3.3	5.9	5.1
CPI	38.8	23.1	31.2	48.0	45.5
WPI	37.7	27.2	29.5	49.2	42.5
RM	53.4	52.6	37.2	46.2	55.8
M2	84.9	56.7	29.8	58.0	55.5
M2Y	84.9	56.7	29.8	66.1	61.2
RIR	7.6	6.3	−8.9	9.0	15.3
REXPEN	13.1	12.7	0.7	−9.8	2.5
RREVE	26.1	0.4	6.5	−26.9	15.3
EXC	46.3	45.9	39.2	63.0	42.5
TERK	−5.9	−12.9	−5.8	−5.6	0.4
TM	12.9	−1.0	4.4	16.5	5.5
TX	61.6	22.2	−0.3	24.5	11.6
TDEF	−15.4	−26.8	13.3	3.3	−6.6
TED	0.7	2.8	2.9	8.7	17.3

GNP: Real Gross National Product (1968 prices)
RM: Reserve Money
M2: Money Supply
M2Y: M2 + Foreign Exchange Deposits
RIR: Real Interest Rate
CPI: Consumer Price Index
WPI: Wholesale Price Index
REXPEN: Real Consolidated Budget Expenditures
RREVE: Real Consolidated Budget Revenues
EXC: Average Exchange Rate (TL/USD)
TERK: Trade Weighted Exchange Rate (Negative change indicates depreciation)
TX: Total Exports (F.B.O)
TM: Total Imports (C.I.F)
TDEF: Trade Deficit (TM minus TX)
TED: Total External Debt

For data sources and definitions, see Appendix I.

Table 1.1 (cont.)

	1986	1987	1988	1989	1990
GNP	8.1	7.5	3.6	1.9	9.2
CPI	34.9	38.5	71.1	65.7	61.8
WPI	33.3	34.4	64.4	66.4	52.8
RM	33.5	37.6	84.3	61.6	42.6
M2	43.8	44.2	53.6	73.3	51.8
M2Y	49.7	56.3	59.4	67.1	53.8
RIR	9.5	−12.9	−2.9	4.4	1.0
REXPEN	16.2	13.6	1.6	7.6	17.1
RREVE	17.5	4.7	5.1	7.9	17.3
EXC	28.8	27.8	66.1	49.3	23.0
TERK	−8.0	−5.3	−4.2	9.3	11.5
TM	−2.1	25.3	3.1	10.2	41.2
TX	−6.3	36.7	14.5	−0.3	11.5
TDEF	7.8	2.0	−28.0	56.0	124.2
TED	23.2	26.5	1.2	2.5	17.5

GNP: Gross National Product
RM: Reserve Money
M2: Money Supply
M2Y: M2 + Foreign Exchange Deposits
RIR: Real Interest Rate
CPI: Consumer Price Index
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REXPEN: Real Consolidated Budget Expenditures
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TDEF: Trade Deficit (TM minus TX)
TED: Total External Debt

For data sources and definitions, see Appendix I.

Table 1.2 Main Economic Indicators of Turkey *(Percentage of GNP)*

	1981	1982	1983	1984	1985
RM	9.6	11.0	11.4	10.5	10.8
M2	26.1	30.7	30.1	29.9	30.7
M2Y	26.1	30.7	30.1	31.4	33.5
EXPEN	22.1	23.9	24.2	20.1	19.4
REVE	22.3	21.2	22.3	15.2	16.6
PSBR	2.8	3.6	1.1	7.0	4.6
TM	15.0	16.3	17.9	21.4	21.2
TX	7.9	10.6	11.1	14.2	14.9
TDEF	7.1	5.7	6.8	7.2	6.3
TED	32.2	36.2	39.2	43.7	48.3

RM: Reserve Money
M2: Money Supply
M2Y: M2 + Foreign Exchange Deposits
EXPEN: Consolidated Budget Expenditures
REVE: Consolidated Budget Revenues
PSBR: Public Sector Borrowing Requirements
TX: Total Exports (F.O.B)
TM: Total Imports (C.I.F)
TDEF: Trade Deficit
TED: Total External Debt

For data sources and definitions, see Appendix I.

Table 1.2 (cont.)

	1986	1987	1988	1989	1990
RM	10.2	9.4	10.1	9.6	8.1
M2	31.2	30.2	27.0	27.7	24.9
M2Y	35.4	37.2	34.5	34.0	29.4
EXPEN	21.1	21.9	21.3	22.7	23.7
REVE	18.3	17.2	17.5	18.6	19.7
PSBR	6.0	9.6	6.3	7.0	9.4
TM	18.9	20.3	20.2	19.6	20.2
TX	12.7	14.9	16.5	14.5	11.8
TDEF	6.2	5.4	3.8	5.2	8.5
TED	54.1	58.7	57.5	51.9	44.5

RM: Reserve Money
M2: Money Supply
M2Y: M2 + Foreign Exchange Deposits
EXPEN: Consolidated Budget Expenditures
REVE: Consolidated Budget Revenues
PSBR: Public Sector Borrowing Requirements
TX: Total Exports (F.O.B)
TM: Total Imports (C.I.F)
TDEF: Trade Deficit
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For data sources and definitions, see Appendix I.

The policy objectives of the stabilization program included, among others, convertibility of Turkish lira, flexible exhange-rate policy, export promotion, and import liberalization, as means of improving the balance-of-payments situation. In 1980, the Turkish Lira was devaluated seven times, a total of 144 %, leading a 28.5 % real exchange rate depreciation in that year. After May 1981, the exchange rate was adjusted daily. Until 1986, nominal exchange-rate devaluations were above the domestic inflation rate. Between 1981 and 1986, TL devaluation averaged 44.3 % per year while inflation was 36 percent. Consequently, real exchange-rate depreciation was 6.3 % per year in this period. After 1986, the real depreciation of the TL slowed down and the trend reversed. In 1989 and 1990, the real exchange-rate appreciation was 10.4 %.

The break in the steady pattern of real depreciations can be attributed to several factors. Although it promotes export growth in the economy, real depreciation policy has adverse effects on other variables. First, it negatively affects the government's budget balance, increasing the burden of external debt. This may result in loss of credibility in financial markets unless there are other positive developments. Second, a constant depreciation of the currency in high-inflation economies causes economic agents to anticipate further depreciations and hence to hold money balances in terms of foreign currencies. The second phenomenon especially is evident in the case of Turkey. After the liberalization of the financial system in 1984, residents were allowed to open foreign-exchange deposit accounts. The share of foreign-exchange deposit accounts in total deposits gradually increased from 6 % in

1984 to 31.6 % in 1988. Although the real appreciation of the TL in 1989 and 1990 made these deposits less attractive, there was no significant change in the ratio. It declined slightly, to 27 %, in 1989 and increased again to 29.3 % in 1990. This currency substitution has been one of the main concerns of monetary authorities.

In financial markets, high nominal rates combined with slowing of TL depreciation made external borrowing more attractive for the private sector and reduced the pressure on domestic credit markets. This in turn helped the public sector to finance its deficit without relying on the central bank sources. (For a detailed evaluation of the financial system in Turkey after 1980, see Akyüz (1990) and the references therein).

The most noteworthy development since the liberalization of the economy has been the fast growth of total exports. In spite of unfavorable conditions in the world economy, total exports increased from $2.26 billion in 1979 to $12.96 billion in 1990. Also, the ratio of total exports to GNP increased from 2.4 % to 11.8 %. The success was mainly the result of export-promoting policies which included tax rebates, credit subsidies, and foreign-exchange allocations allowing duty-free import of intermediate goods and raw materials. Among other things, this export boom helped the country to regain its international creditworthiness [Baysan and Blitzer (1990), Şenses (1990)].

The import-liberalization measures were taken slowly–perhaps out of the worry that the balance-of-payments situation might get worse. Nevertheless, tariffs and duties were lowered and quotas were eliminated [Baysan and Blitzer (1990)]. Consequently, total imports increased from $5.1 billion in 1979 to $22.3 billion in 1990. The ratio of total imports to GNP also rose from 15.0 % in 1981 to 20.2 % in 1990. As a result, especially after 1984, the trade deficit-to-GNP ratio slowly declined to 3.8 % in 1988 but rose again in 1989 and 1990 to 5.2 % and 8.5 % –the same incidentally as it had been in 1980.

On the negative side of liberalization results, the most alarming is the change in external debt. During the early years of the program (1981-1983), annual percentage increase in external debt averaged 2.1. During the 1984-1987 the same figure jumped up to 16.2% . Even though there was a slowdown in 1988 and 1989, growth of external debt was 17.4 % in 1990. The Turkish debt-to-GNP ratio, which averaged above 50 % for the past five years, is well above the average of developing countries and only slightly better than in the "problem" countries. [Celasun and Rodrik (1990)].

Since the early days of the Republic, the public sector has played a leading role in Turkey. The 1980 liberalization program targeted major improvements in fiscal management, in accordance with the stabilization, output recovery, and liberalized finance system objectives of the overall program. These improvements included, but were not restricted to, the reformation of the tax system, reduction of the burden of state enterprises on the budget, and restriction of government expenditures. Indeed, during the first three years of the program (1981–1983), real consolidated budget expenditures and revenues increased an average of 8.8 % and 11.0 % per year respectively. Consequently, the ratio of public-sector borrowing requirements to GNP ratio went down from 10.5 % in 1980 to an average of 3.0 percent between 1981 and 1983. This was mainly due to a reduction of deficits of state economic enterprises, by means of constant increases in their prices, and a steady decline in real wages and salaries.

After 1983, the government expanded the number and the volume of extra-budgetary funds in order to increase the flexibility of the central government. Even with this exclusion, the relation of consolidated budget to GNP did not change. In fact, it increased slightly. Between 1984 and 1990, the previously declining trend in PSBR-GNP ratio was reversed. This ratio between 1984 and 1987 averaged 6.8 %. After 1988, the ratio has increased more rapidly than 1985-1987, reaching 9.4 % in 1990. In financing its deficit, the government shifted from central bank sources to domestic borrowing through financial markets at high interest rates. This, of course, has increased the credit squeeze on the private sector [Celasun (1990), Akyüz (1990)].

On the price side, there seems to be an accelerating structural inflation. Exchange depreciation, state-enterprice price increases, high real-interest rates, and an increasing real-budget deficit are among the major contributing factors to high levels of inflation. Especially since 1987, the inflation rate has accelerated and the growth rate declined significantly in the industrial sector. Even though the growth rate of the GNP was 9.2 % in 1990, the recovery seems to be temporary. Early estimates show that the growth in 1991 was much lower.

In June 1991, the government adopted an expansionary fiscal policy as a preparation for coming elections but the ruling party did not gain enough seats to form a new government. The price adjustments in the public sector have been delayed for months. It is expected at this time (November 1991) that Turkey's real budget deficit will increase and the inflation rate will reach record levels during the last quarter of 1991 and early 1992.

In order to construct a forecasting model capable of capturing the post-liberalization characteristics of the Turkish economy, nine fundamental macroeconomic variables have been selected. The Central Bank manufacturing industrial production index (MIPI) is taken as a proxy for total production in the economy. To represent foreign trade, Total Exports (TX) and Total Imports (TM) are chosen. For the public sector, real consolidated Budget Expenditures (REXPEN) and Revenues (RREVE) are taken. The Money Supply (M2), Average Wholesale-Price Index (P), Average Exchange Rate between the TL and U.S. dollar (EXC), and the Job Seekers Index (ISSIZI) are the other variables. The realized level or growth rates of these variables are presented in Figures 1.1 through 1.9. The description of the data set and data sources are given in Appendix I.

FIG.1.1 Monthly Money Growth

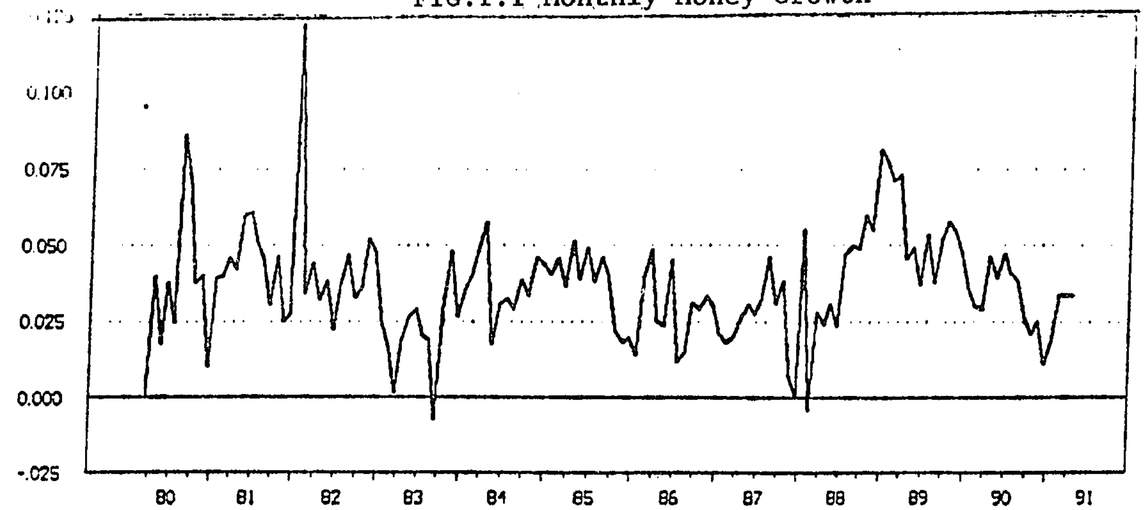


Figure 1 2 Monthly Inflation 1980:03--1991:04
(seasonally adj.)

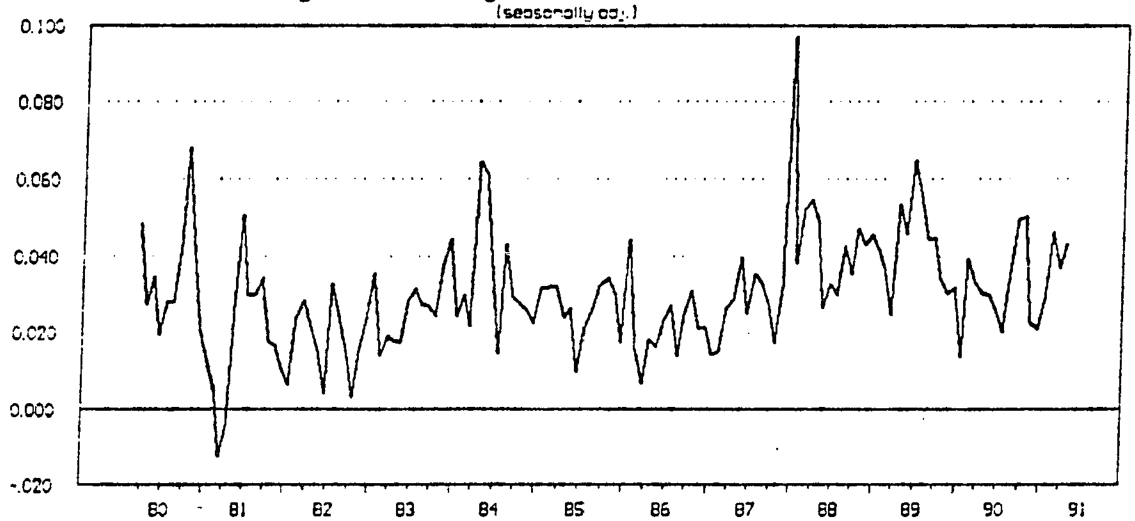


Figure 1 3 Monthly Devaluation 1980:03--1991:04
(seasonally adj.)

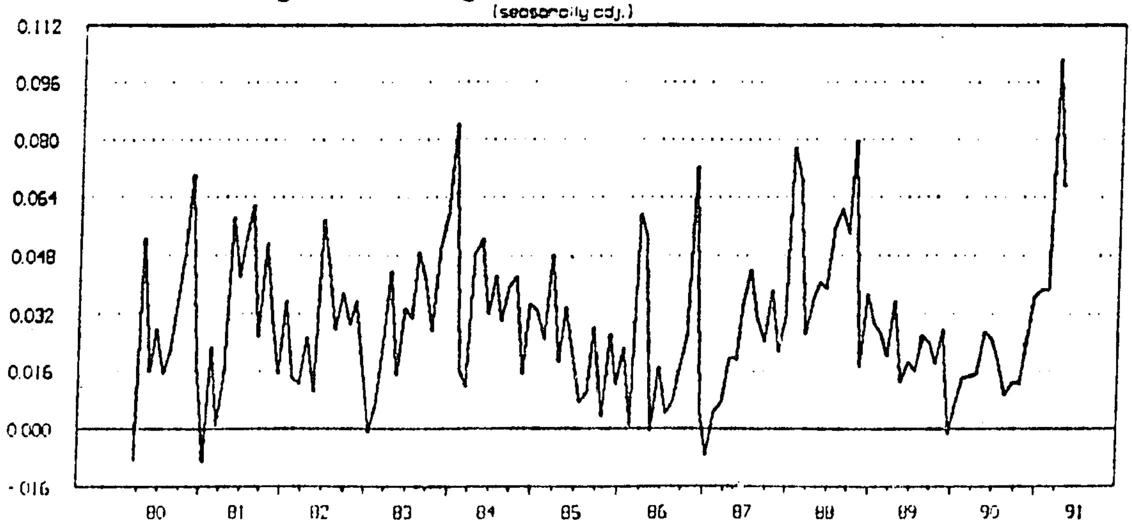


Figure I 4 Total Imports 1980:03--1991:04
(seasonally adj.)

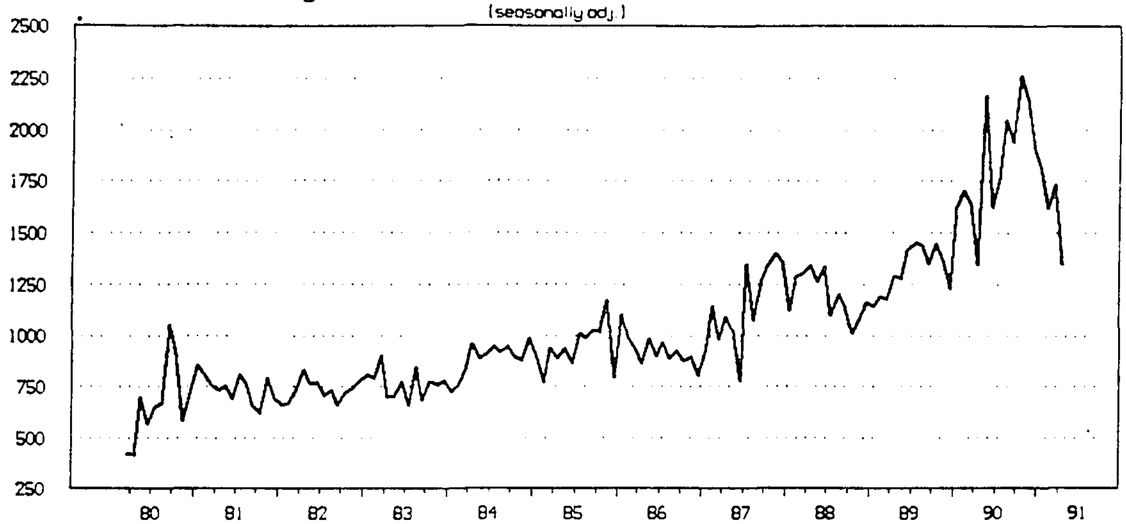


Figure I 5 Total Exports 1980:03--1991:04
(seasonally adj.)

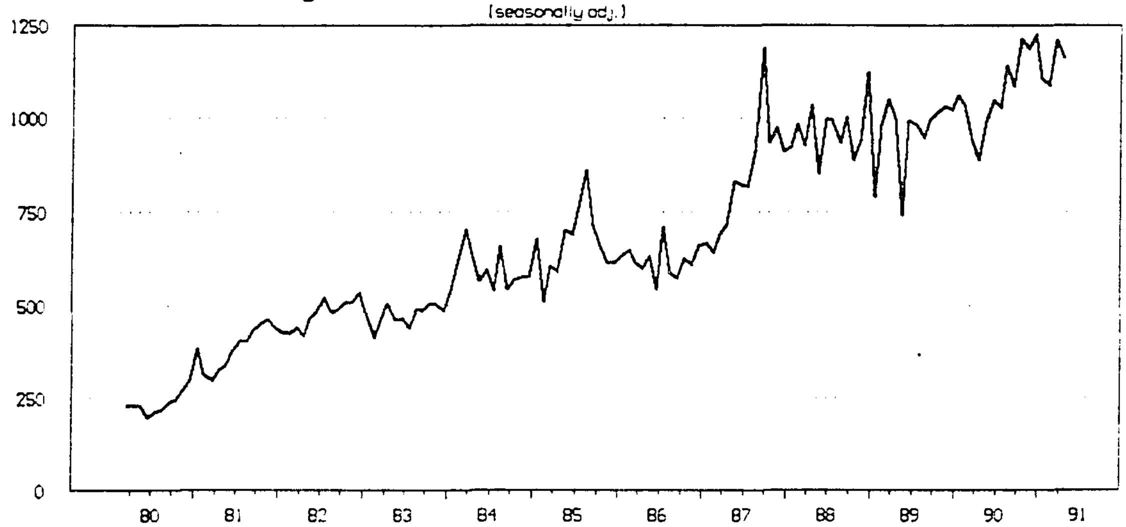


Figure I 6 MIPI 1980:03--1991:04
(seasonally adj.)

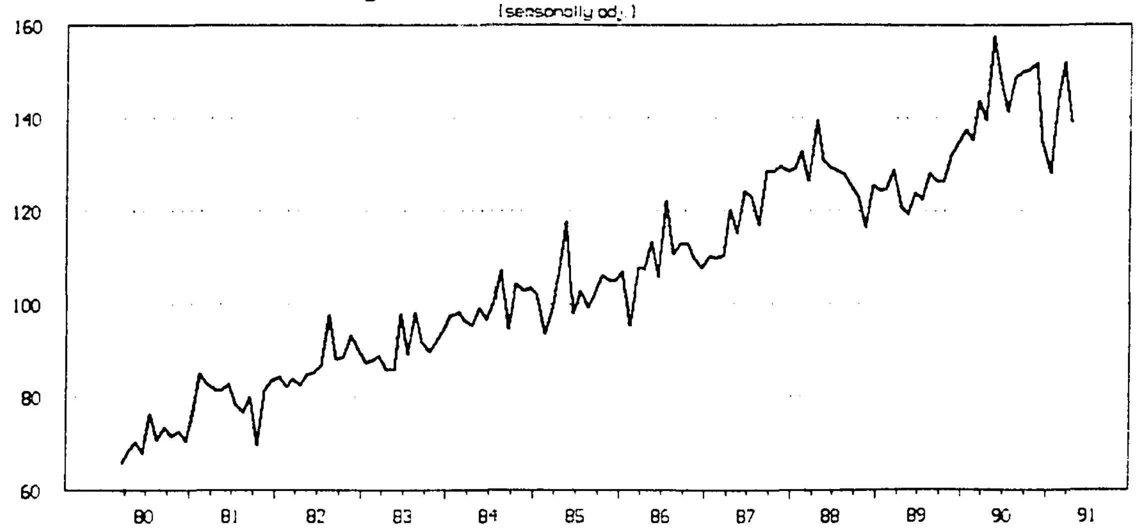


Figure 1.7 Real Expenditures 1980:03--1991:04
(seasonally adj.)

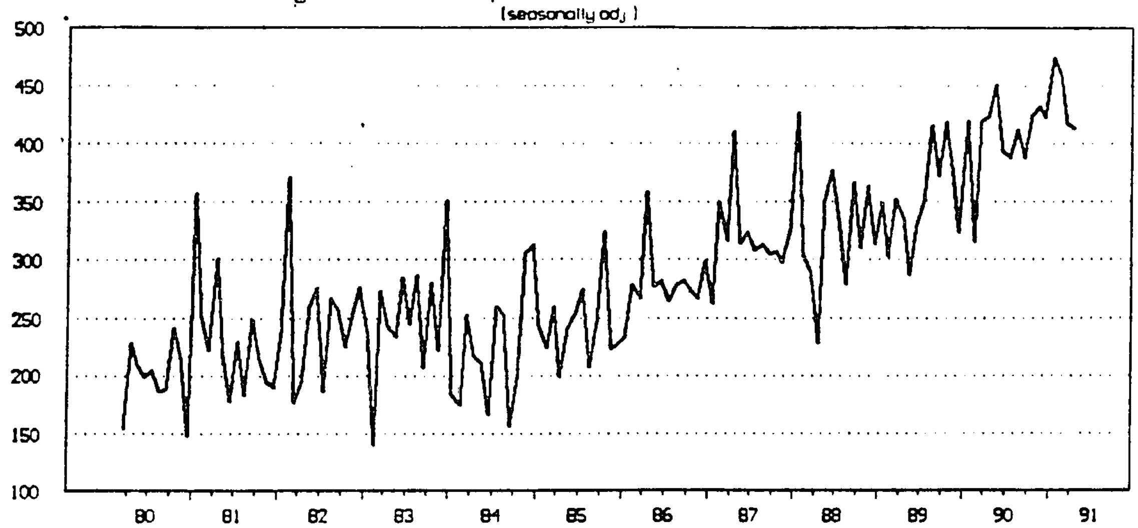


Figure 1.8 Real Revenues 1980:03--1991:04
(seasonally adj.)

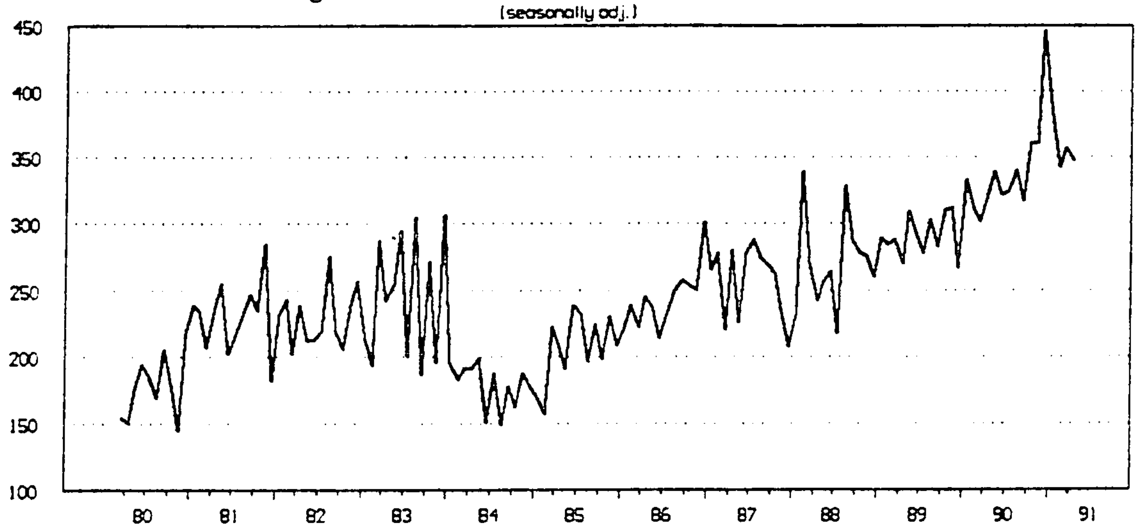
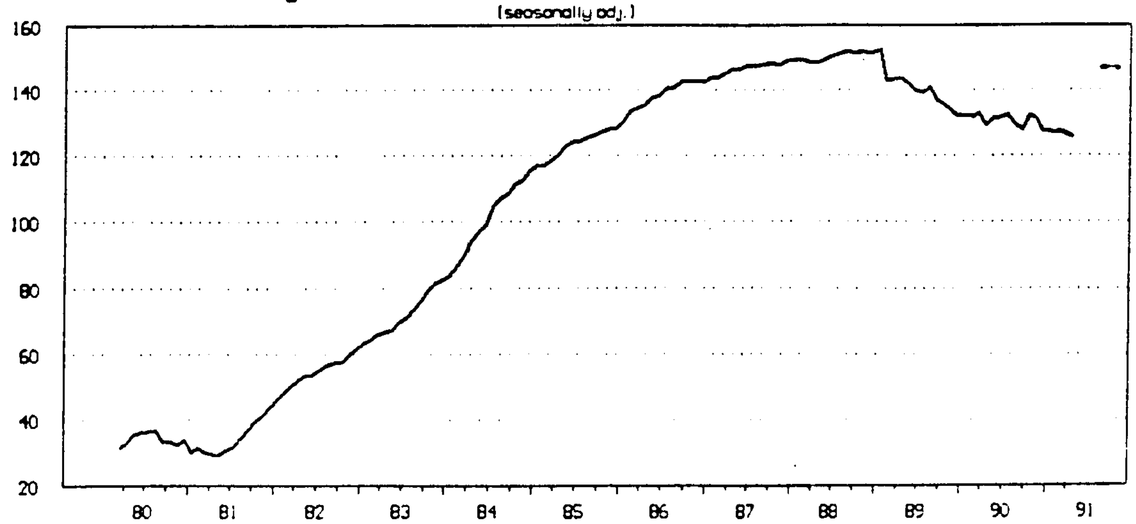


Figure 1.9 Job Seekers Index 1980:03--1991:04
(seasonally adj.)



1.3. Investigation of Unit-Root Nonstationarity of the Series

One of the basic concerns in time-series economics is to understand the underlying stochastic properties of the series. If the stochastic properties of the series change over time, the series is called *nonstationary*. On the other hand, the characteristics of the stochastic process may be invariant with respect to time. In this case the series is called *stationary* [Harvey (1990), p. 23]. Sometimes a simple inspection of the correlograms (plots of the autocorrelation function of the series) will give us some idea of whether the series is covariance-stationary. The autocorrelation function for a covariance-stationary series drops off as p, the number of lags, becomes large. This is not the case for a series which is not covariance-stationary. [Box and Jenkins (1976) Ch. 2].

Nonstationarity of a series leads to spurious results, alters the understanding of an economy, and consequently affects forecasts [Engle and Granger (1987), Granger and Newbold (1974)]. Working with economic aggregates in particular may lead researchers to believe that there are 'significant relations' between the variables, since most macroeconomic time series are trending. As shown by Granger and Newbold (1974), the presence of common trends can present false evidence about the relationship between variables of interest.

Consider the following first-order autoregressive model

$$X_t = \phi X_{t-1} + \eta_t \tag{1.1}$$

where η_t is zero mean white noise. Stationarity requires that $|\phi| < 1.0$. If ϕ is greater than one in absolute value, the series is explosive, i.e. nonstationary.

Sometimes a differenced series, (ΔX_t), will be stationary even though the level of the series, (X_t), is not:

$$\Delta X_t = \phi \Delta X_{t-1} + \eta_t \tag{1.2}$$

$$X_t = (1+\phi)X_{t-1} - \phi X_{t-2} + \eta_t \tag{1.3}$$

where ϕ is less than one. Nonstationarity of Equation (1.3) comes from the fact that it contains a *unit root*. Using the lag operator L, (1.3) can be written as

$$(1-(1+\phi)L + \phi L^2)X_t = \eta_t \tag{1.4}$$

$$(1-\phi L)(1-L)X_t = \eta_t$$

The roots of the polynomial in Equation (1.4) are $1/\phi$ and 1.0. The first root is outside the unit circle while the second one is on the unit circle. Since the unit root corresponds to the first difference operator, only differencing would cancel it out, resulting in a stationary series.

If differencing is required for stationarity, as in Equation (1.3), the series is called *difference stationary process* and denoted as $X_t \sim (d)$. On the other hand, if a series is a *trend stationary process,* a simple detrending is enough to obtain a stationary series. Suppose that the model consists of a deterministic trend, a constant, *and* a stationary process:

$$X_t = \phi_0 + \phi_1 t + \phi_2 X_{t-1} + \eta_t. \tag{1.5}$$

In this case, detrending X_t is enough to obtain stationarity as long as ϕ_2 is less than one in absolute value. Note that if $\phi_2 \geq 1.0$ the detrended series will still be nonstationary. Harvey (1989 p. 291) strongly advises against detrending, noting that "a deterministic trend is an exception rather than the rule in economic time series."

One example of a stochastic nonstationary time series recently in common usage in econometric modeling is the *random walk*

$$X_t = X_{t-1} + \eta_t. \tag{1.6}$$

The random walk process simply says that the current observation is equal to the previous observation and a random disturbance term. The use of the random walk hypothesis in studies dealing with financial and commodity markets has a long history [Fama (1965), Samuelson (1965)]. The study by Hall (1978) showed that there was enough evidence to support the random walk hypothesis for consumption expenditure. This study was influential on further research on the random walk hypothesis as applied to aggregate time series [Campell and Mankiw (1987a) (1987b), Gardner and Kimbrough (1989), Nelson and Plosser (1982), Phillips (1987)]. As will be seen in later chapters, Doan et al.(1984) incorporated this hypothesis on VAR modeling as a Bayesian prior.

In a random walk process, a detrended series will still be nonstationary, as can be seen from Equation (1.5). Only differencing will help to obtain a stationary series. In addition to its statistical importance, the random walk model carries some implications about the series. If the series are following random walk, the effect of a temporary shock will be permanent, and the shocks affecting one variable will have economy-wide alterations. This can be shown easily for Equation (1.6). Using the lag operator L, Equation (1.6) can be written as

$$(1 - L)X_t = \eta_t$$

$$X_t = \frac{\eta_t}{(1 - L)}$$

$$X_t = \sum_{i=0}^{\infty} \eta_{t-i} = \eta_t + \eta_{t-1} + \eta_{t-2} + \dots$$

$$E_{(t+i)}X_{(t+i)} = \eta_{(t+i-1)} + \eta_{(t+i-2)} + \eta_{(t+i-3)} + \eta_{(t+i-4)} + \dots$$

For example, suppose that the level of total imports follows a simple random walk. An unexpected shock to the level of total imports in any given period will remain forever, since the future levels of total imports are simply the sum of the shocks in their past. Besides, the shock will have a permanent effect on all other variables which are functions of the level of total imports. Thus, for a time series containing a unit root it is not meaningful to talk about some 'long-term mean' or 'steady-state level' of the series.

If the underlying process of a series is not random walk, however, the effect of a shock will be only temporary, and the series will tend to revert back to some long-run average (or trend) following the shock. For example, the expected value of X_{t+i} in Equation (1.5) at time $t + i$ is

$$E_{t+i}X_{t+i} = \phi_0 + \phi_1(t + i) + \phi_2 X_{t+i-1}$$

Here, the effect of the shock at time t will die out quickly, since ϕ_2 is less than one in absolute value.

For the U.S. and other Western market economies there have been several studies to investigate whether major economic variables are random walks or trend-reverting. The consensus among researchers is that macroeconomic variables are difference stationary. Most of these studies employ the *unit root test* proposed by Dickey and Fuller (1979, 1981). Extensions to the Dickey-Fuller test have recently been suggested by Said and Dickey (1984, 1985), Phillips and Perron (1988), Perron(1988), and Schwert (1987). For a critical approach to the unit root issue, see Sims (1988), and Sims and Uhlig (1988).

Early applications of the unit root test employed the assumption that time series are generated by a pure autoregressive process. Schwert (1987) shows that if the underlying process contains a moving average component, use of Dickey-Fuller critical values would reject the hypothesis of nonstationary too often. Said and Dickey (1984), on the other hand, show that an ARIMA$(p, 1, q)$ process can be approximated by an ARIMA$(l, 0, 0)$. Therefore, the unit root test based upon a higher order of AR approximation will produce the same results as Dickey-Fuller.

Following Said and Dickey, the equation below is estimated to implement the Dickey-Fuller test,

$$\Delta X_{i,t} = \phi_0 + \phi_1 t + \phi_2 X_{i,t-1} + \sum_{k=3}^{p} \phi_k \Delta X_{i,t+k-4} + \eta_{it} \tag{1.7}$$

where X_i refers to series (in natural logs, so the differences of the series are growth rates), Δ is the difference operator, and η_{it} is a white noise process.

The test statistics, Φ_1, Φ_2 and Φ_3, are for the following hypothesis

1) $H_0 : \phi_2 = 0$

2) $H_0 : \phi_0 = \phi_2 = 0$

3) $H_0 : \phi_0 = \phi_1 = \phi_2 = 0$

with the alternative in each case being the stationarity of the $X_{i,t}$ series (i.e. $\phi_2 < 0$).

To ensure that η_{it} is white noise, the lag lenght, p, is selected from the inspection of the corresponding correlogram of the series. These correlograms are presented in Figures 1.10 through 1.27. The autocorrelation and partial autocorrelation functions of log-differenced series indicate that all the series contain moving average components in addition to autoregressive parts. Hence, it is decided to use a high-order autoregressive representation (with $p = 15$) to approximate the ARIMA$(p, 1, q)$ process. The results of the tests are given in Table 1.3.

If the calculated test statistics are greater than the critical values, as tabulated by Dickey and Fuller (1981), we reject the null hypothesis of a unit root. From the test results in Table 1.3, we conclude that the series are not stationary. That is, we can not reject the random walk hypothesis for all of the variables. Thus, these series can be described as difference stationary.

Phillips (1987) shows that the Dickey-Fuller tests are affected by autocorrelations in the errors from Equation (1.7). An informal inspection of the correlograms of the errors did not indicate any autocorrelation, so there was no need to refer to modified Dickey-Fuller tests.

Table 1.3. Results of the unit root tests

Test Stat.	REXPEN	M2	AP	EXC	TM
Φ_1	−1.45	−2.99	−0.99	−2.32	−3.56
Φ_2	2.39	4.55	3.87	6.31	6.44
Φ_3	3.64	5.47	6.61	2.69	5.72

Test Stat.	TX	MIPI	ISSIZI	RREVE
Φ_1	−2.62	−3.28	−0.28	−1.62
Φ_2	4.28	5.61	1.00	1.52
Φ_3	4.42	6.62	0.69	1.78

Table 1.4 The critical values for the test statistics* (n=100)

Test Stat.	0.01	0.025	0.05	0.10
Φ_1	−4.04	−3.73	−3.45	−3.15
Φ_2	8.730	7.440	6.490	5.470
Φ_3	6.500	5.599	4.880	4.160

* The critical values for the statistics are from Fuller (1976, p. 373) and Dickey and Fuller (1981, p. 1063, Table V-VI). All variables are estimated in natural logs.

Figure 1.10 AC function of Log Dif. REXPEN

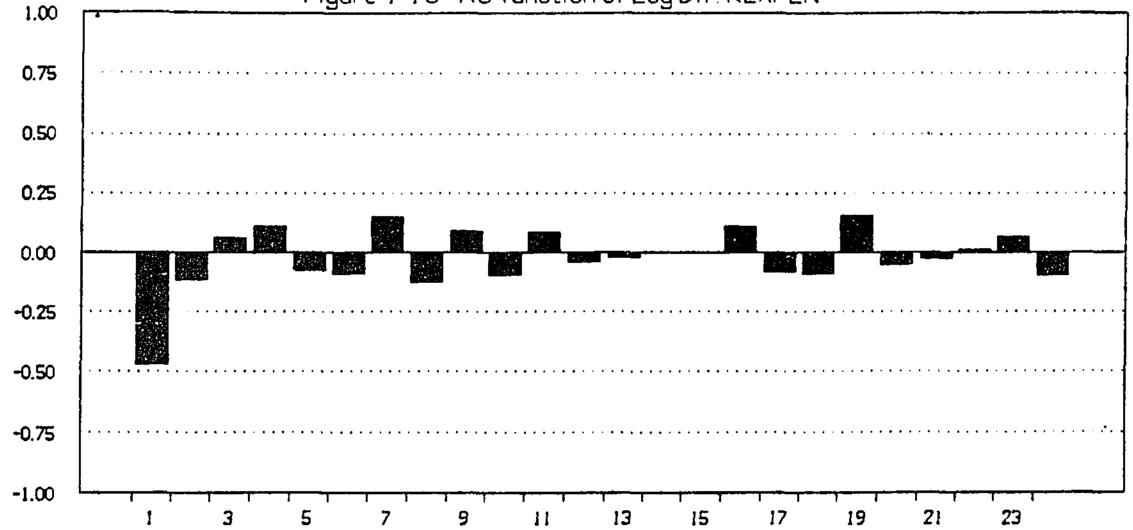


Figure 1.11 AC function of Log Dif. M2

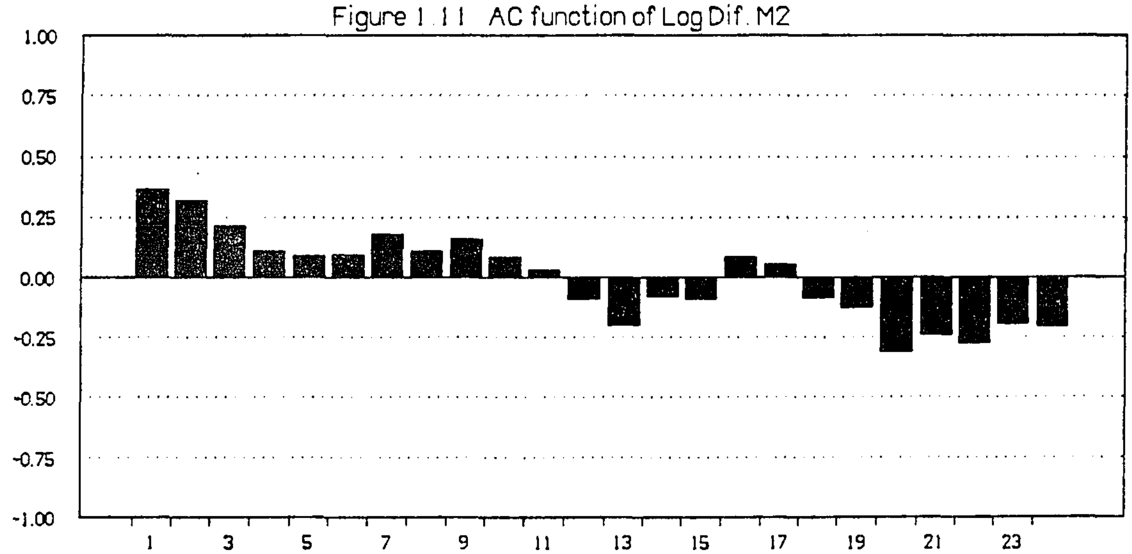


Figure 1.12 AC function of Log Dif. P

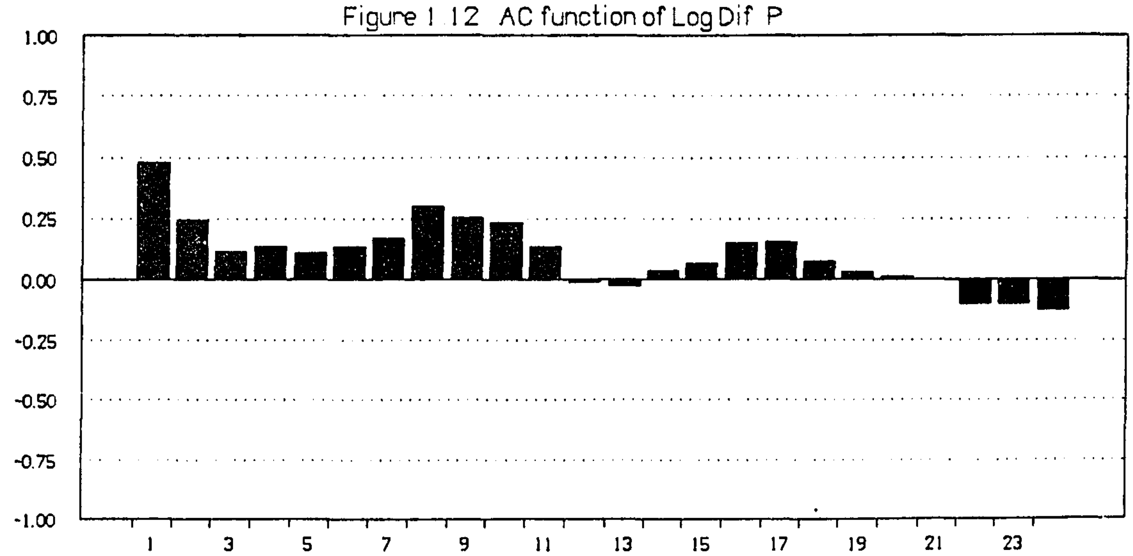


Figure 1.13 AC function of Log Dif. EXC

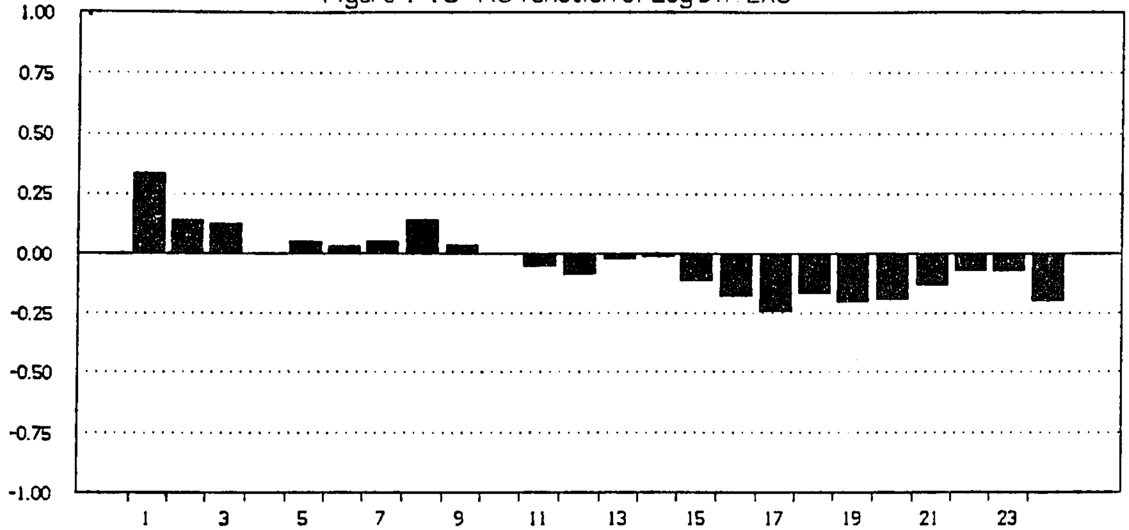


Figure 1.14 AC function of Log Dif. TM

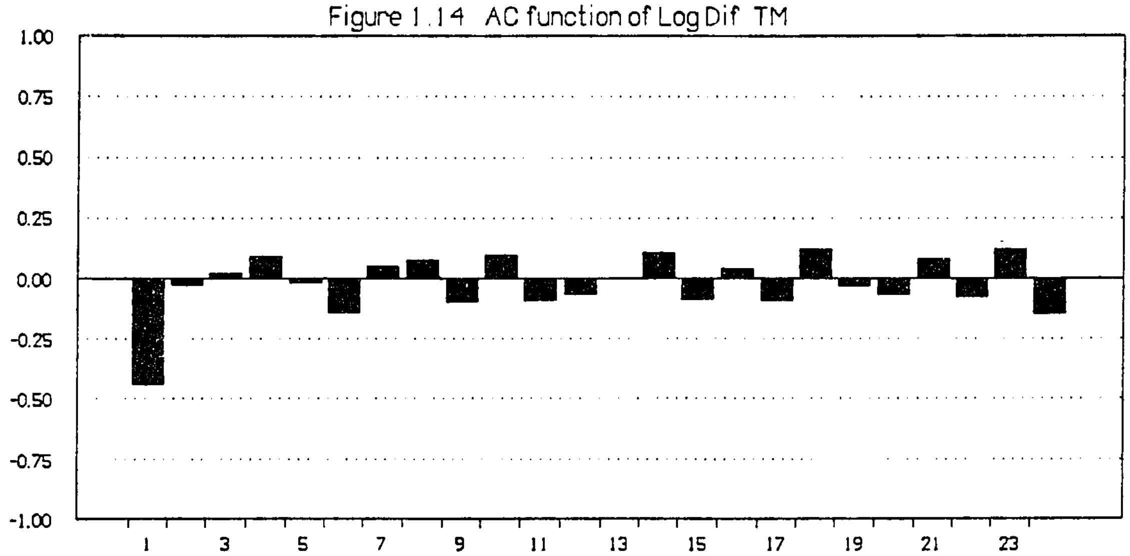


Figure 1.15 AC function of Log Dif. TX

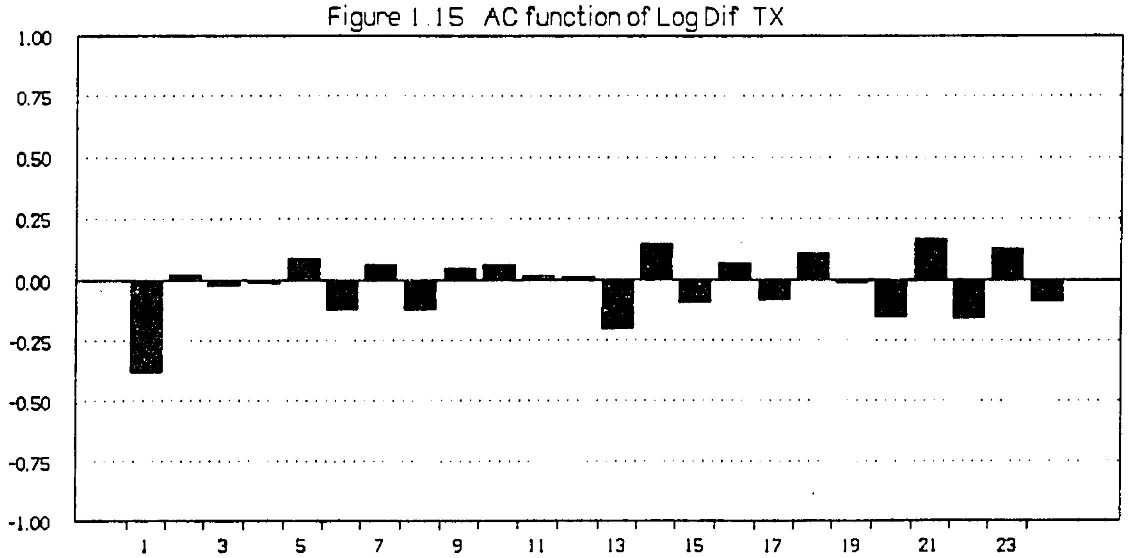


Figure 1.16 AC function of Log Dif MIPI

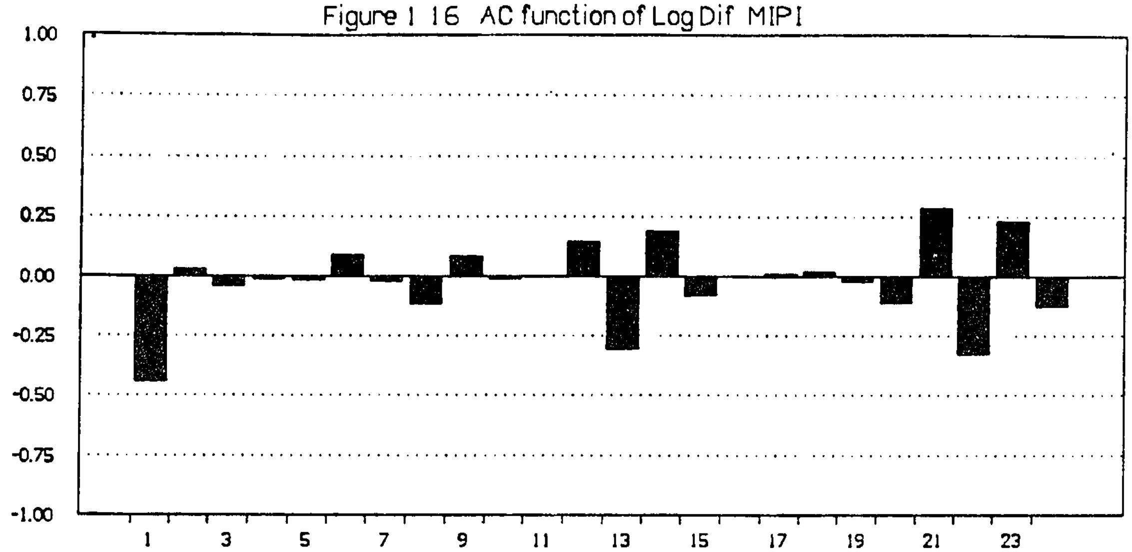


Figure 1.17 AC function of Log Dif. ISSIZI

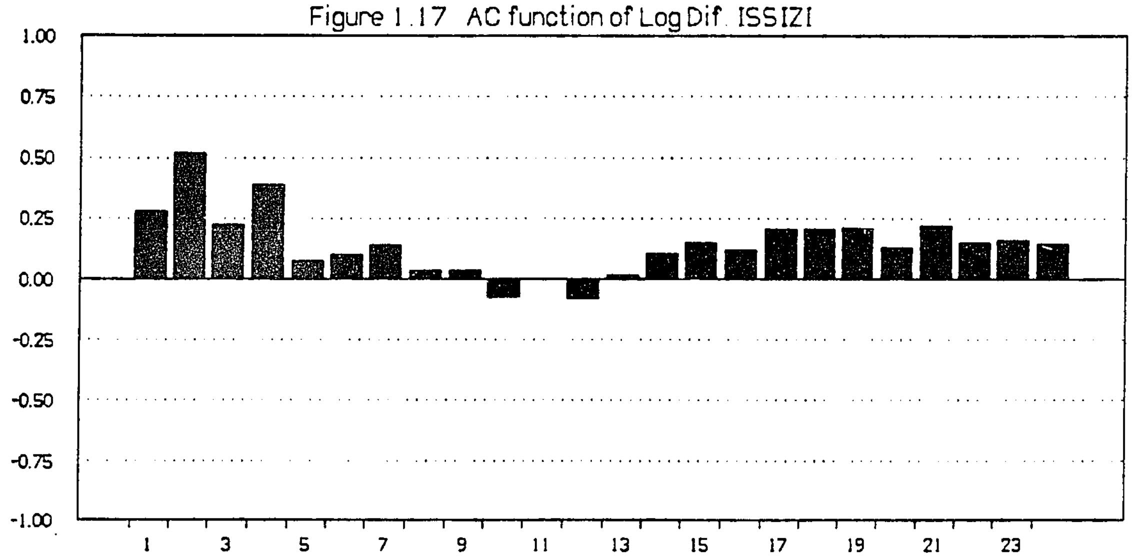


Figure 1.18 AC function of Log Dif RREVE

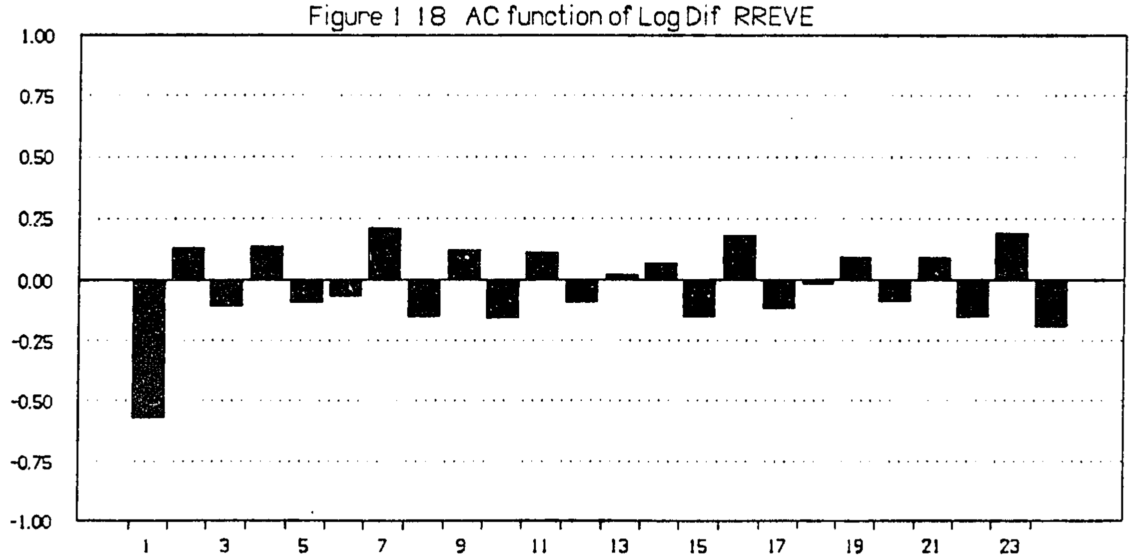


Figure 1.19 PAC function of Log Dif REXPEN

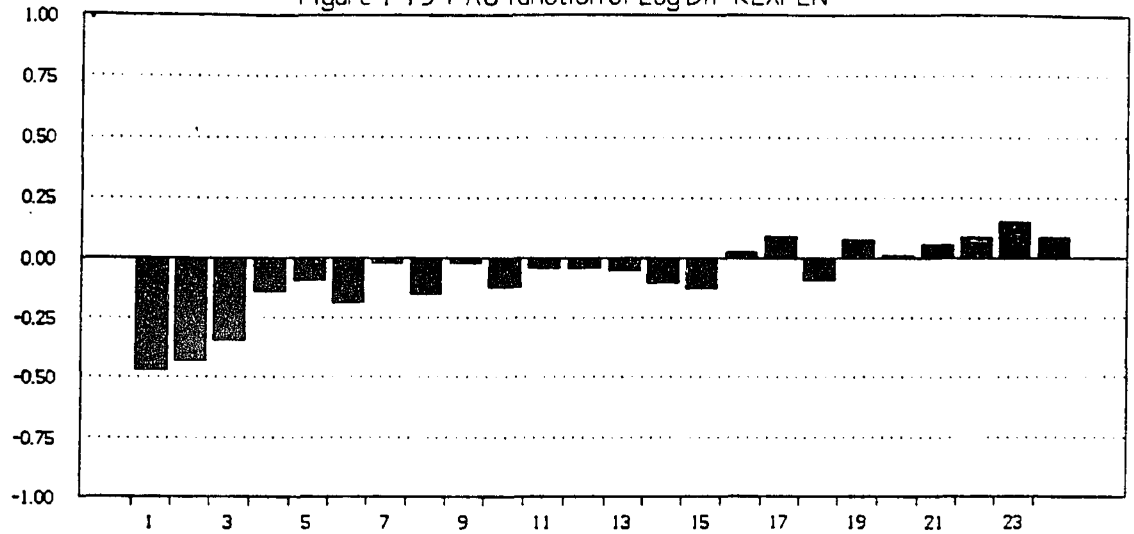


Figure 1.20 PAC function of Log Dif. M2

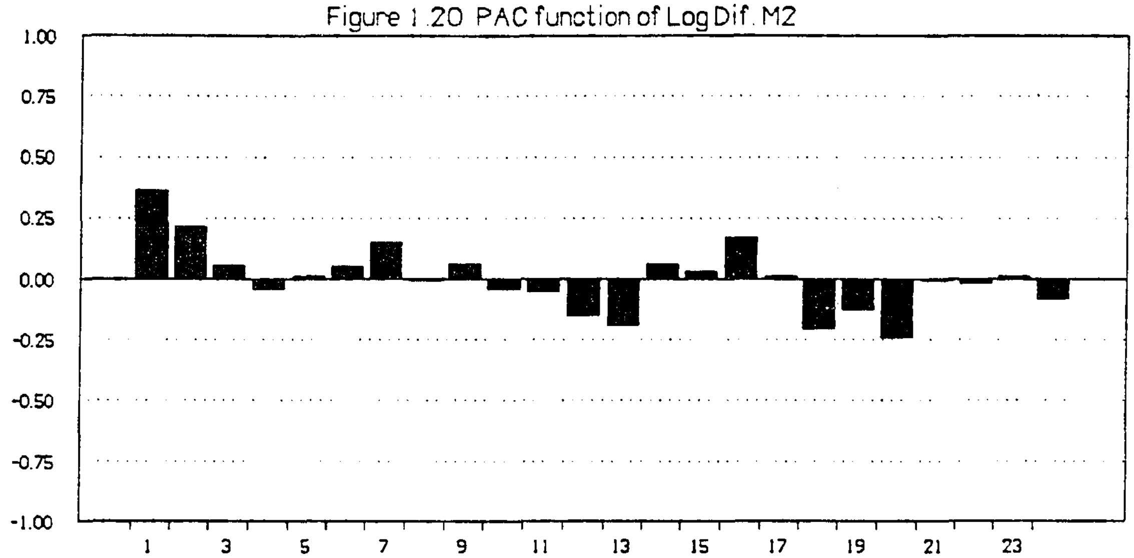


Figure 1.21 PAC function of Log Dif P

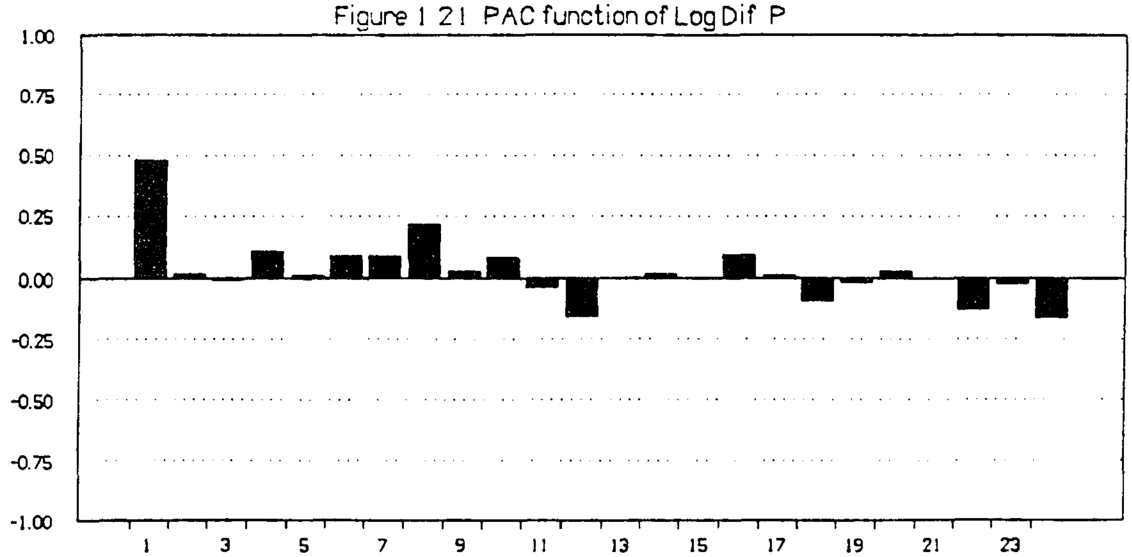


Figure 1 22 PAC function of Log Dif EXC

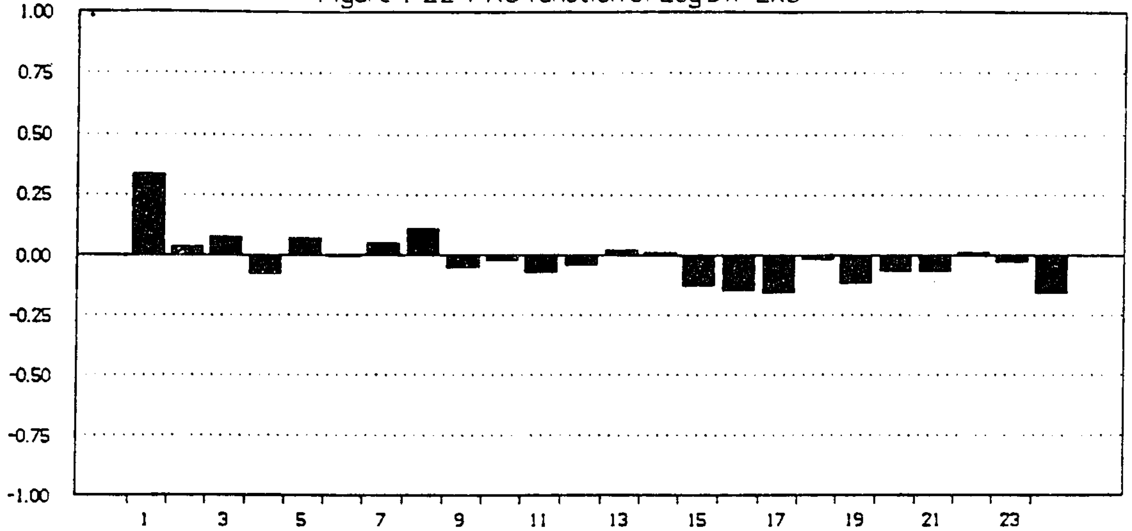


Figure 1 23 PAC function of Log Dif TM

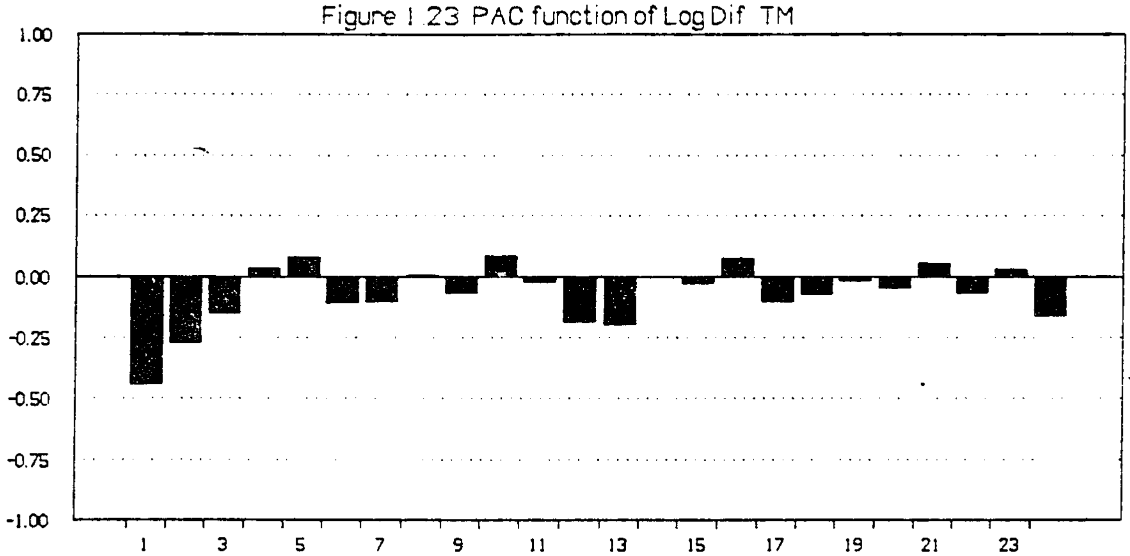


Figure 1 24 PAC function of Log Dif TX

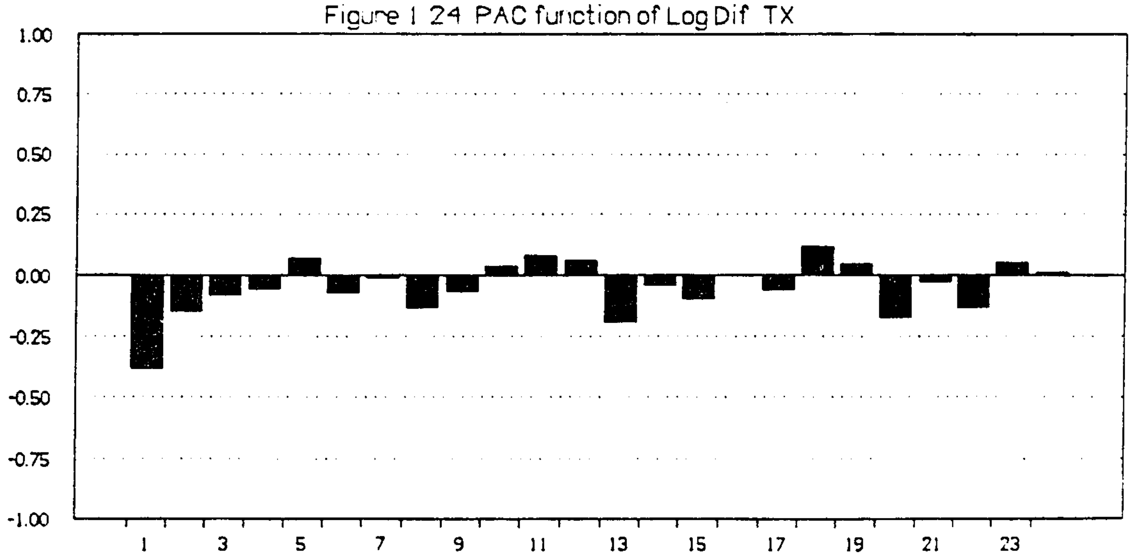


Figure 1.25 PAC function of Log Dif MIPI

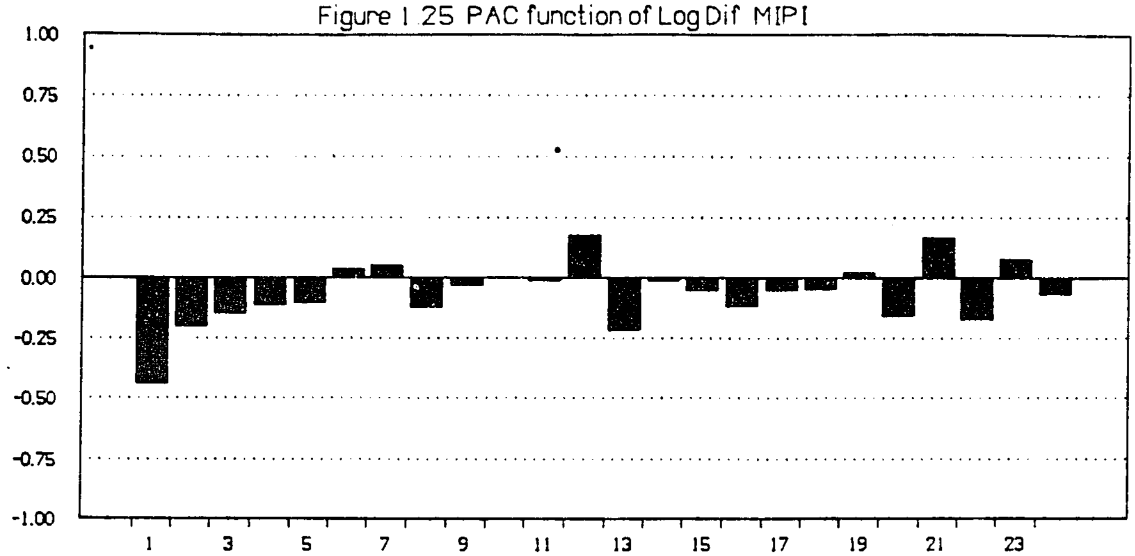


Figure 1.26 PAC function of Log Dif ISSIZI

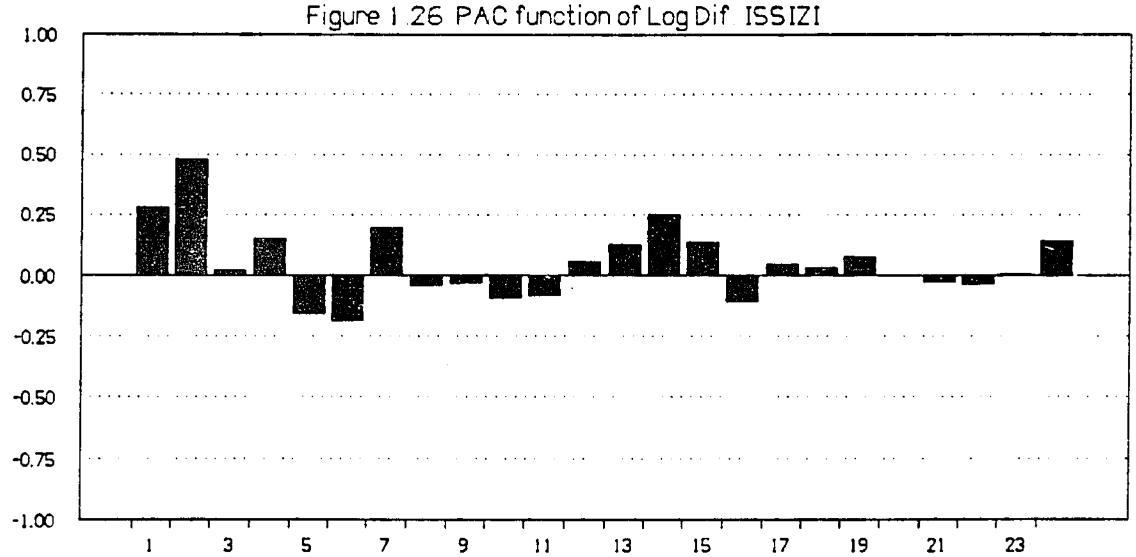
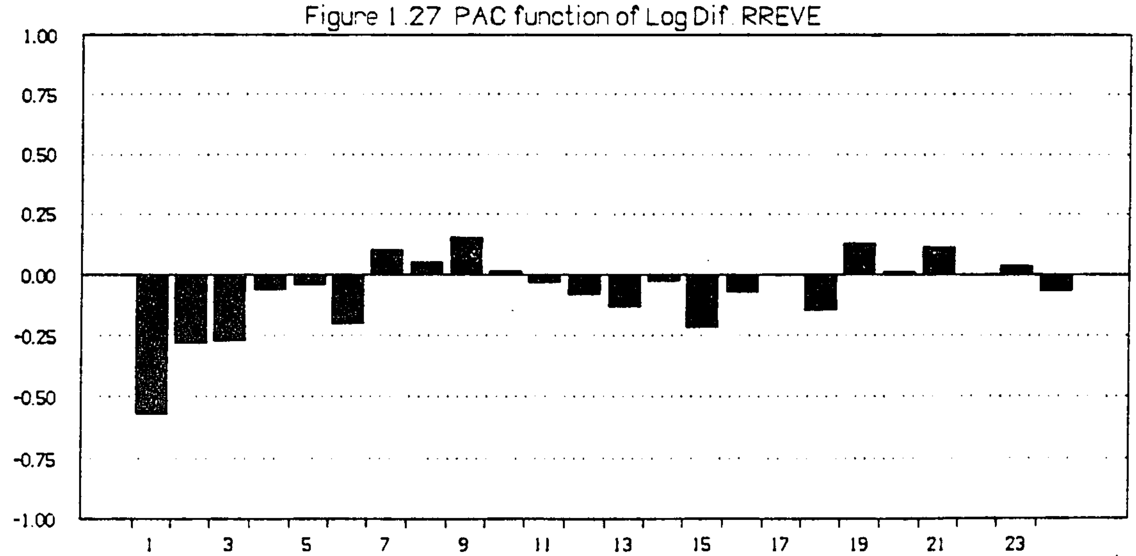


Figure 1.27 PAC function of Log Dif RREVE



1.4. The Univariate Model as a Benchmark

In order to define the gain in terms of forecast performance from the application of the VAR and the BVAR models, each variable is estimated with the following univariate, fixed-coefficient autoregression

$$X_t = C + \sum_{p=1}^{m} \alpha_p X_{t-p} + \sum_{i=1}^{s-1} \beta_i S_{it} + u_t \tag{1.8}$$

where X_t is the variable to be forecasted, C is a constant, the S_{it}'s and β_i's are the seasonal dummies and their respective coefficients, and m is the number of lags while s is the number of seasons. It is assumed that u_t is zero mean white noise with constant variance σ_u^2.

Throughout this study, seasonally unadjusted series are used deliberately for several reasons. In a forecasting model search like this one, the use of seasonally adjusted data may cause one to lose some degree of information about the nature of the series, depending on the way in which the data is adjusted [Harvey (1989), p. 308]. Furthermore, production of seasonally adjusted series by government agencies will build autocorrelations into the series [Greene (1990), p. 429].

Since all the variables are seasonally unadjusted raw data, seasonal dummies were included in each equation.

Rather than applying the same lag length, as is done in some other studies [see Doan, Litterman, and Sims (1984)], we preferred to find an appropriate lag length for each equation using certain statistical procedures. Several statistics may be considered for determining the maximum lag length. Two of these procedures are *Akaike Information Criterion (AIC)* [Akaike (1973)] and *Schwarz Criterion (SC)*

[Schwarz (1978)]:

$$AIC(k) = \frac{(RSS + 2k\sigma^2)}{T} \tag{1.10}$$

$$SC(k) = \frac{(RSS + k(logT)\sigma^2)}{T} \tag{1.11}$$

where k is the number of regressors, RSS is the residual sum of the squares and σ^2 is the variance of the estimated regression. It is known that AIC is inconsistent, tending to choose an overparameterized model. However, this can be avoided by comparing the results of each criterion. Note that SC puts a heavier penalty on longer lengths than AIC.

These procedures were employed in the following fashion: First, a maximum possible lag length was chosen, call it $MAXLAG$, then each equation was estimated with $k = 1, 2 \ldots, MAXLAG$. Finally, the one that gave the smallest AIC was selected as an appropriate lag length for that equation and the corresponding SC was checked to avoid unnecessarily long lag lengths. To make direct comparison possible, the regressions were run over the same time interval (April 1980-April 1991). For M2, P, ISSIZI, and EXC, very flat minimum values of AIC were observed, and 12 lags were chosen. For other variables, different lag lengths and corresponding AIC values are given in Table 1.5

Table 1.5 *AIC* Values for Different Lags in Univariate Model

Lag	REXPEN	TM	TX	MIPI	RREVE
1	8.87	2.27	1.63	0.51	3.96
2	8.05	1.73	1.40	0.45	3.32
3	7.06	1.69	1.38*	0.42	3.13
4	6.27	1.66*	1.39	0.42	2.98
5	6.25	1.68	1.39	0.42	3.02
6	6.35	1.69	1.42	0.42	3.07
9	6.20	1.77	1.43	0.42	3.09
12	5.42*	1.83	1.49	0.43	2.91*
13	5.51	1.85	1.45	0.40	2.94
14	5.59	1.83	1.43	0.39*	2.98
15	5.65	1.86	1.45	0.40	3.03

All entries are multiplied by 100. Variables are estimated in natural logs. (*) indicates the Selected Lag Length.

Having specified the final form of the univariate model, each equation was estimated with the ordinary least squares (OLS) method. Since our primary focus is forecasting, regression coefficients and their standard errors are not among the results given in Table 1.6. Because of the presence of lagged dependent variables, F statistics are not distributed as F in this case. For the same reason, Durbin-Watson test statistics are not valid, being biased toward a finding of no autocorrelation [Nerlove and Wallis (1966)]. Both statistics are included to make comparison with other specifications.

In all estimates, monthly observations of the variables were used beginning in March 1979. All variables are in natural logs. The units of the variables are millions of current U.S. dollars for TX and TM, billions of current TL for M2, billions of 1984 TL for REXPEN and RREVE, and 1984=100 for MIPI, ISSIZI and P. EXC is defined as TL/(1 U.S. Dollar). Other details of the data set and data sources are given in Appendix I.

Table 1.6 Univariate Model 1980:03-1990:06

Regression	$\overline{R}^2$	$D-W$	Q	F	$\hat{\sigma}$
REXPEN	0.69	1.801	25.15	13.71	0.221
M2	0.99	1.720	39.95	32679.87	0.019
P	0.99	1.758	41.96	43371.13	0.013
EXC	0.99	1.299	75.39	15363.99	0.022
TM	0.83	1.902	28.94	42.85	0.145
TX	0.95	1.998	37.08	163.21	0.112
MIPI	0.94	1.994	30.84	77.54	0.057
ISSIZI	0.99	1.912	27.60	3676.52	0.026
RREVE	0.58	1.795	26.58	9.04	0.180

Unconditional forecasts of all the variables were calculated from the estimated model, according to Equation (1.8). Forecasts were generated through December 1992. Figures 1.28 through 1.36 plot the monthly forecasted values of each variable along with 95 % confidence intervals.

In order to give a general picture of the forecasts, quarterly level and growth rate forecasts are given in Tables 1.7 and 1.8

As to the forecast results, a small decline in trade deficit (compared to year 1990) and a substantial increase in budget deficit are expected. For the year 1991, total exports are forecasted at \$14.2 billion while total imports are projected to be \$19.6 billions indicating a \$5.4 billion trade deficit. The real budget deficit is predicted to be 1,390.6 billion TL.

Money supply (M2), Average Wholesale Price Index (P), and Nominal Exchange Rate (EXC) are forecasted to increase above the average in 1991. Finally, if Manufacturing Industrial Production Index (MIPI) is taken as a proxy to Gross National Product (GNP), the univariate model forecast indicates an economic slowdown for the year 1991 from 15.4 % realized growth in 1990 to 0.3 % decline in 1991.

Table 1.7 Unconditional Forecast Results (UNIV)

Variable	1991:2	1991:3	1991:4	1992:1
REXPEN	1,273.3 (±348.0)	1,312.3 (±524.0)	1,852.5 (±754.8)	1,267.5 (±524.2)
M2	82,918.5 (±4,304.0)	92,941.2 (±10.013.8)	104,216.0 (±15,705.0)	115,964.0 (±22,254.0)
P	2,097.4 (±79.5)	2,299.3 (±152.8)	2,646.3 (±228.8)	3,088.0 (±339.9)
EXC	4,046.2 (±229.6)	4,389.3 (±424.4)	4,767.6 (±584.3)	5,307.8 (±731.0)
TM	4,419.1 (±850.8)	4,675.5 (±1,441.9)	5,506.6 (±1,951.2)	4,431.6 (±1,720.0)
TX	3,255.8 (±597.1)	3,187.3 (±936.4)	4,362.1 (±1,563.9)	3,523.5 (±1,436.4)
MIPI	136.4 (±9.7)	143.5 (±16.8)	156.4 (±19.8)	146.5 (±19.6)
ISSIZI	124.6 (±4.9)	120.1 (±14.9)	119.2 (±23.5)	122.5 (±32.0)
RREVE	1,010.7 (±202.9)	1,010.6 (±317.8)	1,186.9 (±413.3)	961.6 (±360.0)

1991:x stands for the x quarter of 1991. REXPEN, RREVE, and M2 are in billions of TL. TM and TX are in millions of USD. ISSIZI, P, and MIPI 1984=100. EXC is TL/USD. REXPEN, RREVE, TX, and TM are quarterly sums. M2, EXC, and P are as of end of period. MIPI and ISSIZI are quarterly averages. 95 % confidence intervals are in parentheses

Table 1.8 Unconditional Forecast Results (UNIV) (Growth Rates)

Variable	1991:2	1991:3	1991:4	1992:1
REXPEN	5.3	11.5	20.9	9.3
M2	20.6	35.2	51.6	11.3
P	30.5	43.0	64.6	16.7
EXC	40.9	52.9	66.0	11.4
TM	−9.3	−14.0	−24.4	−9.9
TX	18.6	11.5	0.0	4.3
MIPI	−2.7	0.0	−1.0	0.0
ISSIZI	−5.3	−7.4	−8.2	−3.8
RREVE	4.4	3.3	−7.4	−3.7

M2, P, and EXC show percentage change form the end of previous year. Others are percentage change from the same period of previous year.

Figure 1.28 FORECASTED REXPEN

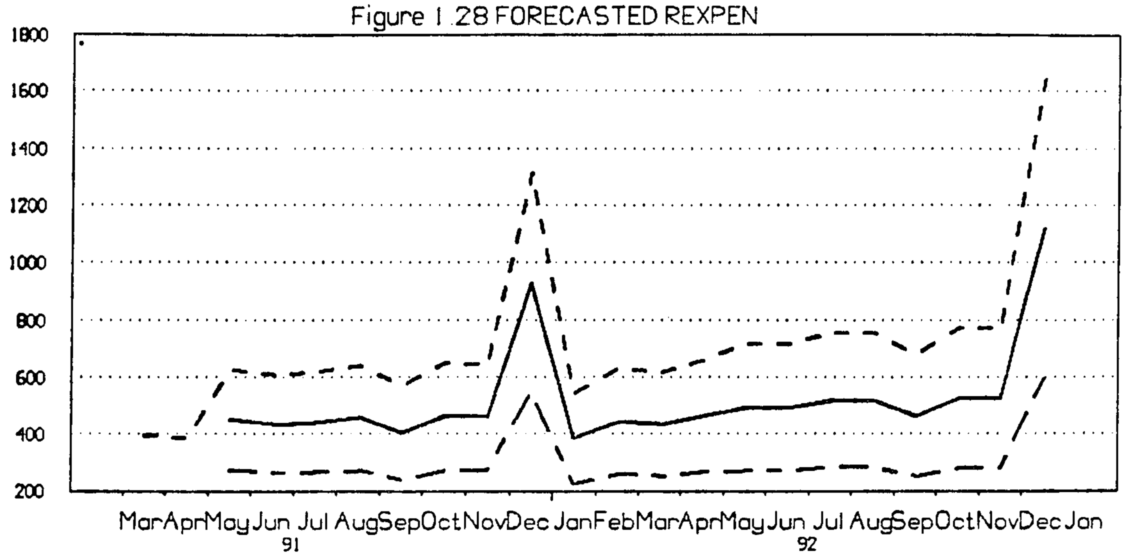


Figure 1.29 FORECASTED M2

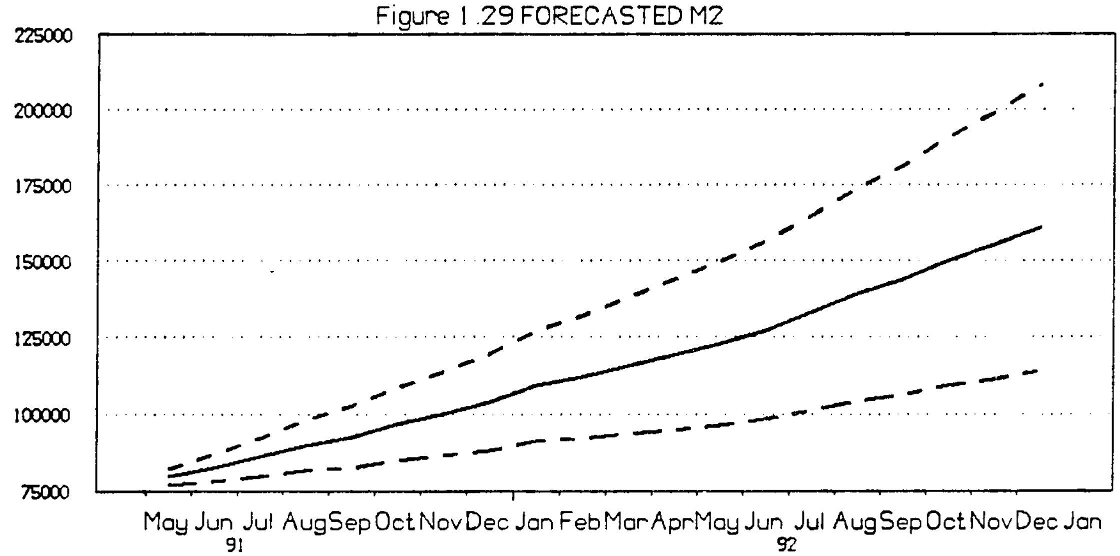


Figure 1.30 FORECASTED P

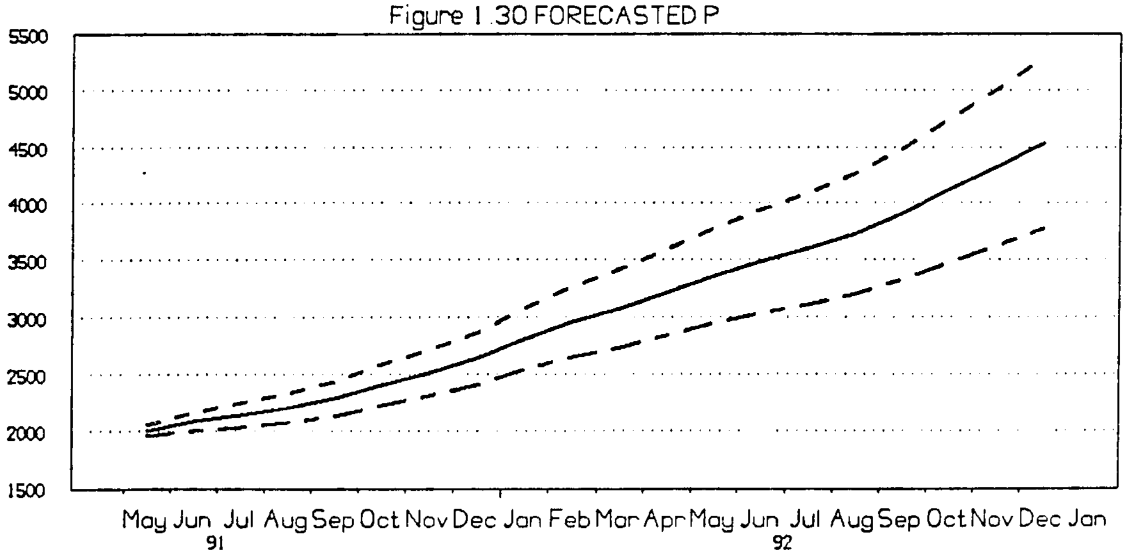


Figure 1 31 FORECASTED EXC

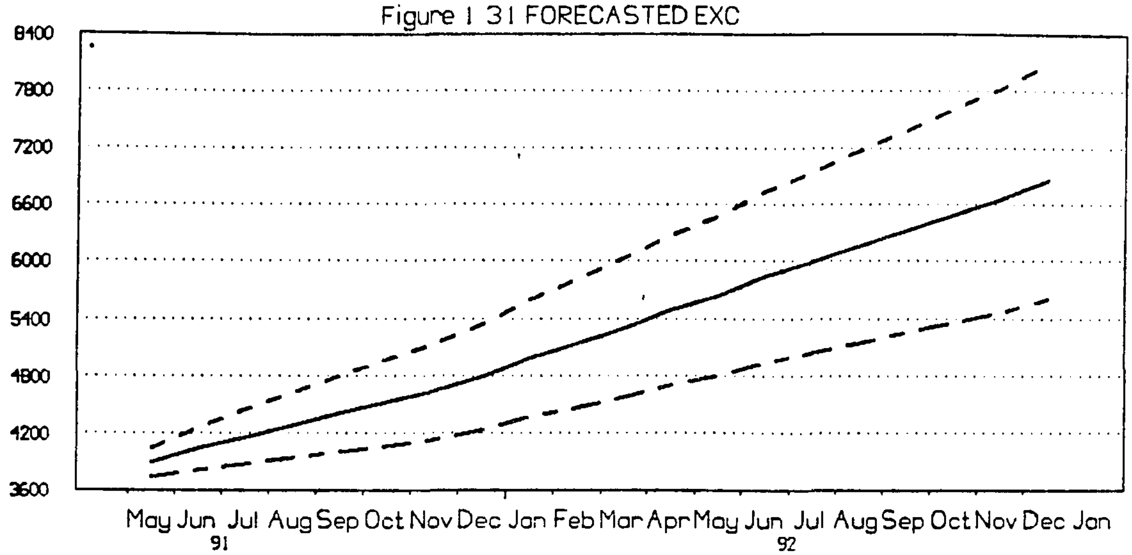


Figure 1 32 FORECASTED TM

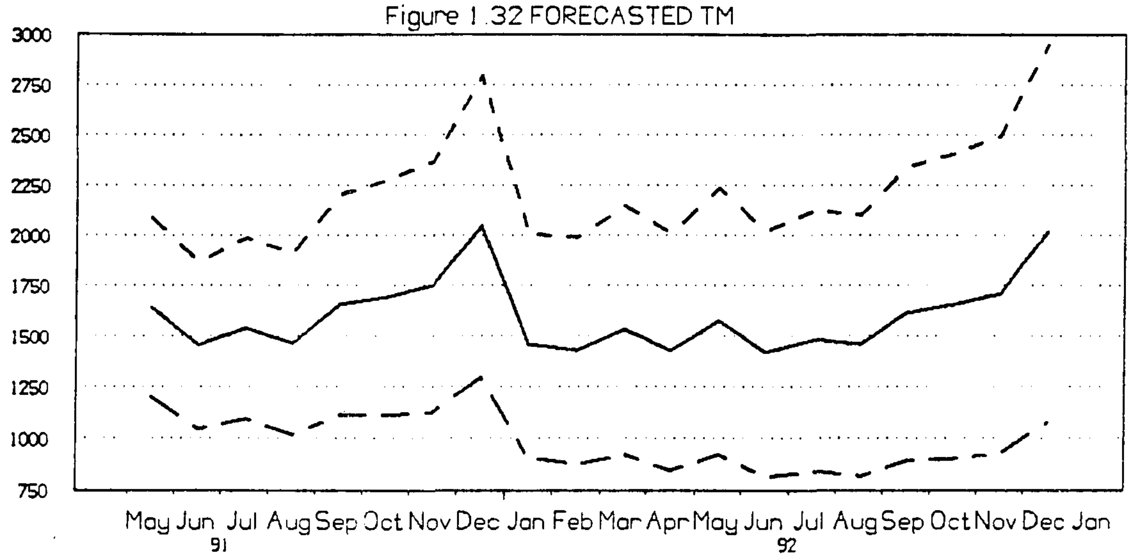


Figure 1 33 FORECASTED TX

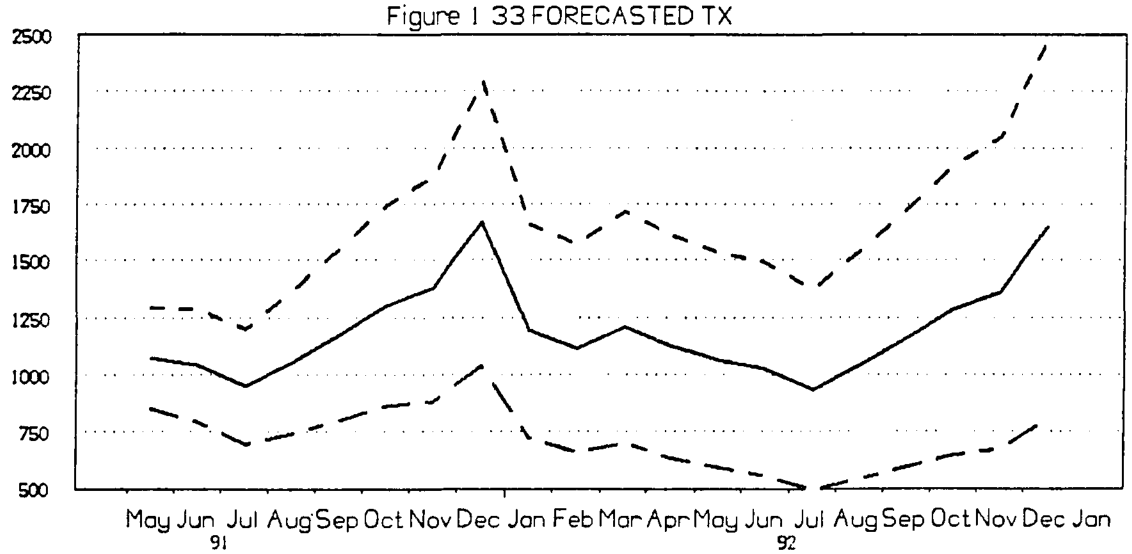


Figure 1 34 FORECASTED MIPI

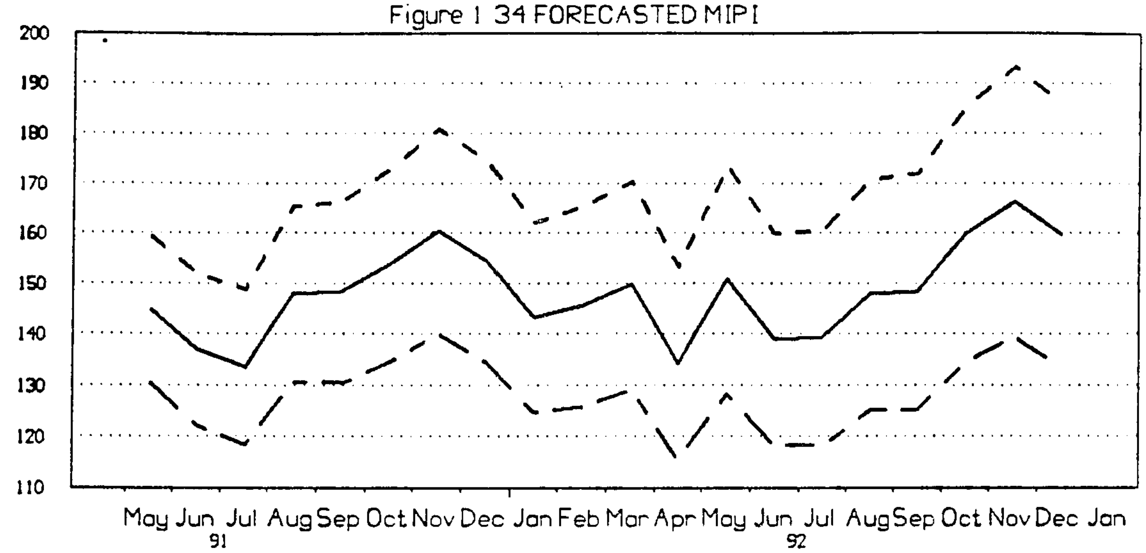


Figure 1 35 FORECASTED ISSIZI

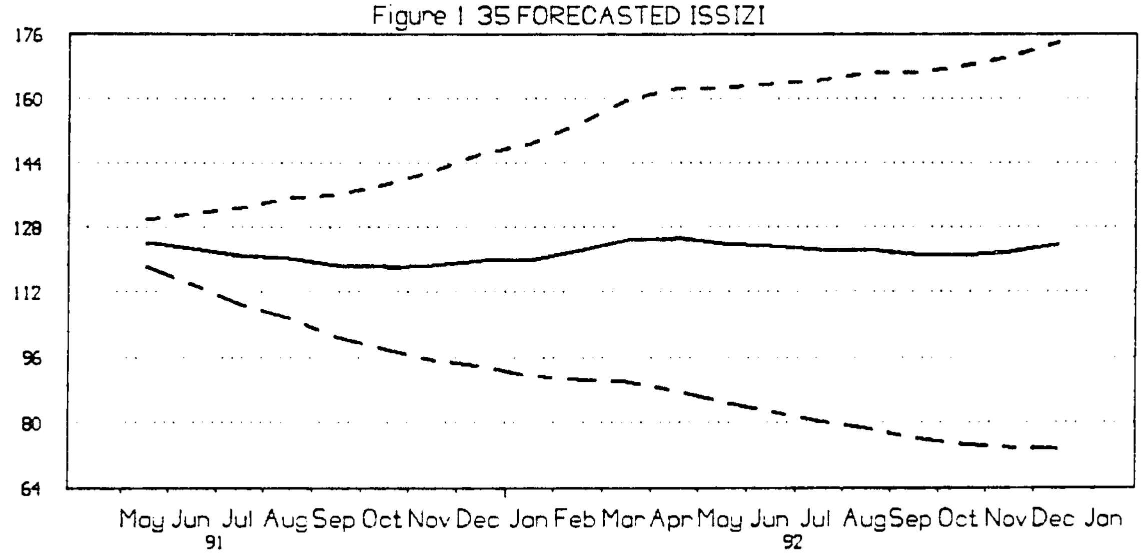
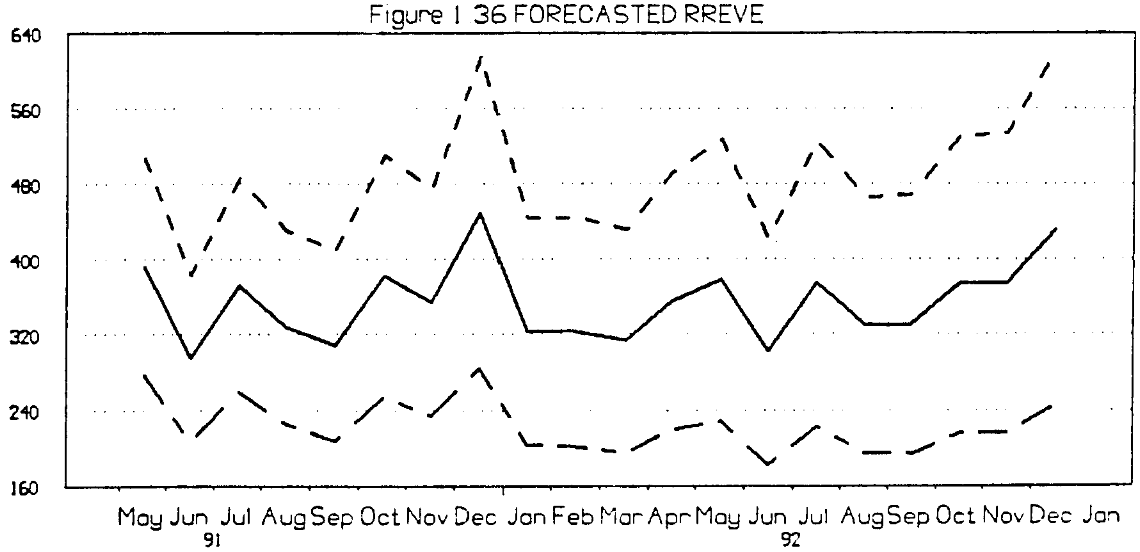


Figure 1 36 FORECASTED RREVE



1.5 Forecast Performance Statistics

The main purpose of generating the univariate model is to compare its forecast performance to other models' results in order to see overall improvement in forecast performance, and to define the gain, if any, from the application of other models to the same set of data. There are several statistics proposed to serve this purpose [Theil (1961) and Fair (1984)]. Among these, root mean squared error ($RMSE$), mean error (ME), mean absolute error (MAE), and Theil U (TU) statistics and some combination of them will be used throughout this study. These measures are defined in terms of the k-step ahead forecasts made at time t, $\hat{x}_{t+k}$, and in terms of the actual value at time $t+k$, x_{t+k}.

Let l be the number of elements in the projection period, then

$$ME = \frac{1}{l} \sum_{k=1}^{l} (x_{t+k} - \hat{x}_{t+k}) \tag{1.12}$$

$$MAE = \frac{1}{l} \sum_{k=1}^{l} |x_{t+k} - \hat{x}_{t+k}| \tag{1.13}$$

$$RMSE = \sqrt{\frac{1}{l} \sum_{k=1}^{l} (x_{t+k} - \hat{x}_{t+k})^2} \tag{1.14}$$

$$TU = \sqrt{\frac{\frac{1}{l} \sum_{k=1}^{l} (x_{t+k} - \hat{x}_{t+k})^2}{\frac{1}{l} \sum_{k=1}^{l} (x_{k+l})^2}} \tag{1.15}$$

Note that the first three quntities are unit-bounded while TU has the advantage of being a unit-free statistic. The procedure for generating these performance statistics, reported in Table 1.9, is the following:

First, our data set was divided into two subperiods, from 1980:03 to 1990:06 as the estimation period, and from 1990:06 to 1991:04 as the projection period. The coefficients of the model, Equation (1.2), were estimated, using the data in the estimation period. Next, a series of forecasts of the next k values of X was calculated by feeding forward forecasts of early periods. The coefficients of the model were re-estimated using an additional observation of the sample, and a new set of k forecasts was generated. The procedure of re-estimating forecasts and coefficients was carried out for each observation in the projection period. Forecast errors generated in this way were used to form the performance statistics are given in Table 1.9.

From the definitions of the statistics, it is clear that if the Mean Error has the same value as Mean Absolute Error the model consistently forecasts either low or high. If the Root Mean Squared Error is several times higher than MAE, the indication is that there are few large errors in forecasting period.

For each equation, *average Theil U* (ATU_i) is calculated using the formula

$$ATU_i = \frac{1}{l}\sum_{k=1}^{l} TU_{ki} \tag{1.16}$$

where i refers to the equation and k is the forecast step. This measure will be used to compare the forecast performance of different models for i'th variable.

To have an overall measure of the forecast performance of a model, the following *'figure of merit'* (FOM) is defined:

$$FOM = \frac{1}{W}\sum_{i=1}^{n} w_i ATU_i \tag{1.17}$$

where w_i is the weight assigned to variable i, and W is the sum of the weights. Using this definition, model a is preferred to model b if $FOM_a < FOM_b$.

Note that since the model is estimated in natural logs of the variables, reported mean errors in Table 1.6 actually show the percentage deviations from realized level value of the variable.

Table 1.9 Univariate Model Forecast Performance 1990:07-1991:04

Variable	Steps	*ME*	*MAE*	*RMSE*	*TU*
REXPEN	1	−0.022	0.079	0.092	0.36
	3	−0.003	0.085	0.100	0.32
	6	−0.031	0.093	0.113	0.41
M2	1	−0.005	0.009	0.011	0.35
	3	−0.024	0.029	0.030	0.35
	6	−0.066	0.066	0.069	0.41
P	1	−0.001	0.012	0.014	0.36
	3	−0.002	0.029	0.033	0.28
	6	−0.025	0.025	0.027	0.12
EXC	1	0.009	0.024	0.035	0.70
	3	0.017	0.055	0.073	0.57
	6	0.022	0.073	0.086	0.41
TM	1	−0.011	0.154	0.185	0.84
	3	−0.031	0.174	0.217	0.64
	6	−0.057	0.147	0.183	0.48
TX	1	0.025	0.063	0.077	0.38
	3	0.054	0.093	0.100	0.32
	6	0.082	0.082	0.094	0.31
MIPI	1	−0.013	0.068	0.081	0.66
	3	−0.016	0.078	0.090	0.57
	6	−0.070	0.075	0.093	0.79
ISSIZI	1	−0.007	0.020	0.026	1.48
	3	−0.022	0.032	0.042	2.05
	6	−0.068	0.068	0.082	2.63
RREVE	1	0.048	0.109	0.122	0.60
	3	0.059	0.127	0.138	0.54
	6	0.107	0.149	0.156	0.66

CHAPTER 2

In this chapter, we are going to examine whether the application of VAR modeling to the same data set as described in Chapter 1 yields substantial gains in terms of smaller forecast errors. After the estimation of the model different uses of the VAR will be presented.

The technical details of the VAR model are given in Section 1. Section 2 shows the estimation of the VAR model for our data set. The forecast results and forecast performance are presented in Section 3. Finally, Section 4 gives the moving average representation of the system and the impulse response function, i.e. the dynamic response of the system to an innovation in any one of its components.

2.1 The VAR Modeling

In a multivariate time series model, the interactions among the variables are used to forecast each individual variable. The model building techniques employed for this purpose can be classified into two groups. The first group consists of what are called "structural macroeconometric models." In these models, the specific relationships among variables are based on economic theory. The second group of models are known as "atheoretical models" [Cooley and LeRoy (1985), Fair (1984)]. In atheoretical models, the forecaster lets the data, rather than some economic theory, specify the dynamic structure of the system. Multivariate ARIMA(p,d,q) fashioned by Box and Jenkins (1974), is one example of the latter modeling. Following the conventional notation, an ARIMA model specified as $X_t \sim$ ARIMA(p,d,q) refers to a process generated by

$$A(L)(1-L)^d X_t = B(L)\epsilon_t$$

where ϵ_t is a zero-mean white noise, $A(L)$ and $B(L)$ are the polynomials in the lag operator of orders p and q, and d is an integer.

Even though there are several procedures used to determine the ordering of a univariate ARIMA process, it is difficult to identify p and q in a multivariate case. To avoid the identification and estimation problems of multivariate ARIMA, time series are usually estimated as pure VAR(p) models.

From the viewpoint of structural econometric models, VAR modeling may be interpreted as a reduced form of some unknown underlying structural system of equations [Cooley and LeRoy (1985)].

For the data set we have, consider the following mth order vector autoregression representation:

$$X_t = A(L)X_{t-1} + C + u_t \tag{2.1}$$

where X_t is a 1xn vector of variables, A is an nxnm matrix of coefficients, C is the constants, and u_t is an nx1 vector of white noise disturbance terms.

$$u_t \sim N(0, \Omega_t) \tag{2.2}$$

where Ω_t is the orthogonal variance-covariance matrix of disturbances. According to Equation (2.1), ith component of the data vector X at time t is determined by

$$X_{(it)} = \sum_{j=1}^{n}\sum_{p=1}^{m} X_{(j,t-p)}\alpha^i_{(j,p)} + \alpha^i_{(j+1,s)} + u_{it} \tag{2.3}$$

Once the lag length is chosen—with either AIC or SC technique mentioned in Chapter One—the system can be estimated by OLS. Since the right-hand side variables in Equation (2.1) are all predetermined, we do not need to worry about simultaneous-equations bias [Piendyck and Rubinfeld (1991), p. 291 and 355]. Besides, since each equation has the same right-hand side variables, the model can be estimated separately, resulting in asymptotically efficient estimates.

The system of equations implied in (2.1) can also be written as a classical recursive system. From Equation (2.2) it can be seen that $E(u_t u_t') = \Omega_t$. Because Ω is a symmetric and positive definite matrix, a nonsingular matrix P can be found such that

$$\Omega = PP' \tag{2.4}$$

and

$$P = V\Lambda^{1/2} \tag{2.5}$$

where V is the matrix of eigenvectors and Λ is the eigenvalues. [Johnston (1984) p. 153]. Defining the lower triangular matrix as D and $D = P^{-1}$, premultiplying Equation (2.1) by D results in

$$DX_t = DA(L)X_{t-1} + DC + Du_t \tag{2.6}$$

or

$$DX_t = A^* X_{t-1} + DC + \epsilon_t \tag{2.7}$$

where $A^* = DA(L)$. It can be verified that $E(\epsilon_t \epsilon_t') = I$ and $E(\epsilon_t) = 0$

It can be seen from Equation (2.7) that the equation for X_{it} contains contemporaneous values of X_{jt} , $j < i$, and the error term ϵ_{it}. Ignoring the constant terms in (2.7) for simplicity [Hakkio and Morris (1984)],

$$d_{11}X_{1t} = \sum_{p=1}^{m}(\alpha^{*1}_{1,p}X_{1,t-p} + \alpha^{*1}_{2,p}X_{2,t-p} + \cdots + \alpha^{*1}_{n,p}X_{n,t-p}) + \epsilon_{1t} \tag{2.8}$$

$$d_{ii}X_{it} = -\sum_{j=1}^{i-1}(d_{ij}X_{jt}) + \sum_{p=1}^{m}(\alpha^{*i}_{1,p}X_{1,t-p} + \alpha^{*i}_{2,p}X_{2,t-p} + \cdots + \alpha^{*i}_{n,p}X_{n,t-p}) + \epsilon_{it} \tag{2.9}$$

where d_{ij} is the (ij) element of D, α^{*i}_{jp} is the corresponding element in A^*, and $i = 2, 3, \ldots, n$ in Equation (2.9).

Even though both ϵ_{it} and u_{it} are called the innovation in X_{it} they are not equal to each other unless D is an identity matrix. From Equation (2.3), u_{it} is the prediction error when the information set contains only the lagged values of the data vector X_{it} . It is clear that the ordering of the variables does not affect the error vector u_t . On the other hand, the error vector ϵ_t is not invariant to the ordering of the variables, since different orderings lead to different D matrices. Note that when the residuals are close to being uncorrelated, the ordering will not change the results obtained using vector ϵ_t .

If the data vector X_t is stationary, i.e. the roots of the characteristic equation $|I - A(L)L| = 0$ lie outside the unit circle [Box and Jenkins (1976)], the corresponding VMA representation of (2.7) is

$$DX_t = A^* X_{t-1} + DC + \epsilon_t \tag{2.7}$$

$$D[\ I - A^* L]\, X_t = DC + \epsilon_t \tag{2.10}$$

Ignoring the constant part in (2.10), we can write

$$\begin{aligned} X_t &= M(L)\epsilon_t \\ M(L) &= [\ I - A^* L]^{-1} D^{-1} \end{aligned} \tag{2.11}$$

According to equation (2.11), X_{it} contains its own contemporaneous error term as well the error term for X_{jt}, $j < i$.

$$X_{1t} = d^*_{11}\epsilon_{1t} \;-\; \sum_{p=1}^{+\infty}(m_{11p}\epsilon_{1,(t-p)} \;+\; m_{12p}\epsilon_{2,(t-p)} + \cdots + m_{1np}\epsilon_{n,(t-p)})$$

$$X_{it} = \sum_{j=1}^{i}(d^*_{ij}\epsilon_{jt}) \;-\; \sum_{p=1}^{+\infty}(m_{i1p}\epsilon_{1,(t-p)} \;+\; m_{i2p}\epsilon_{2,(t-p)} + \cdots + m_{inp}\epsilon_{n,(t-p)}) \tag{2.12}$$

where d_{ij} is the (ij) element of D^{-1} and $j = 2, 3, \ldots, n$ in the second equation of (2.12). Note that the variance of the error term ϵ_{it} is normalized to equal one and the contemporaneous covariance terms are zero by construction.

By selecting the relevant variables and an appropriate lag length, Equation (2.3) can be estimated by OLS. After the estimation, there are several uses of the VAR model: forecasting, testing hypothesis, innovation accounting, and impulse responses.

The optimal linear forecast of X_{t+1} given the information at time t is the linear OLS projection of X_{t+1} on $X_t, X_{t-1}, \ldots$

$$E_t(X_{t+1}|\ I_t) = A(L)X_t$$

$$X_{t+1} = E_t(X_{t+1}|\ I_t) + u_{t+1} \tag{2.13}$$

In general, the k step ahead forecast of X_t can be written as

$$E_t(X_{t+k}|\ I_t) = A(L)^k X_t \tag{2.14}$$

The moving average representation of the model, laid out in equations (2.10) through (2.12), can be used to see the response of the system to a unit standard error shock in ϵ_t . The shock is maintained only during one period, and is called "impulse." Of course, we do not expect the innovations to different variables to occur independently. Since the system is linear, the response to a combination of innovations is the sum of the responses [Litterman (1979) p. 82].

In some cases one may want to test whether the lags of one set of variables do not enter into the equations for the remaining variables. This amounts to putting restrictions on the coefficients matrix $A(L)$. The test, which is a generalized version of Granger-Sims casuality tests [Granger (1969, 1988), Sims (1972), Granger and Newbold (1986) p. 259], can be carried out by estimating the restricted and unrestricted model and comparing the two covariance matrices of residuals [RATS Manual 14-178].

Another common use of the VAR model, popularized by Sims (1980, 1982, 1986) is known as "variance decomposition." One can see the proportion of the total forecast variance of one component of X_{t+k} caused by the shocks to the MA representation of another variable. For a technical discussion, see Judge, et al. (1988 p. 771–775 and 1985 Ch. 16). On recent applications of VAR, see Blanchard (1988), Bryant et al. (1988), Mocan (1990), and Runkle (1987) and the comments by Sims, Blanchard, and Watson on Runkle (1987).

2.2 Estimation of the VAR

In Chapter 1, nine major macroeconomic variables were chosen to represent the Turkish economy. Unfortunately, we are not able to examine all nine variables using VAR. In VAR modeling, the degrees of freedom get smaller with increasing lags. We have to trade off between having enough lags and having a sufficient number of free parameters.

Because we are working with monthly data, any lag length less than 12 would fail to capture the important dynamics between variables. For instance, many economic agents—either government or private—base their current decision on a comparison to the one made a year earlier [Malinvaud (1984)]. For this reason, it has been decided to keep the number of lags at a minimum 12 and to reduce the number of variables from nine to five. The variables chosen are: exchange rate (EXC), total imports (TM), real government expenditures (REXPEN), money supply (M2), and price index (P). The details of the data and the data sources are given in Appendix I.

With five variables, Equation (2.3) takes the following form:

$$X_{(it)} = \sum_{j=1}^{i} \sum_{p=1}^{m} X_{j,(t-p)} \alpha^{i}_{j,(p)} + \alpha^{i}_{j+1,(s)} + u_{it} \tag{2.3}$$

where $i = 1, \ldots, 5$; s is the number of seasons ($s = 1, \ldots, 12$); and m is the number of lags.

After the number of variables is determined, the number of lags is determined in following fashion. First, model (2.1) is estimated with 12 lags ($m = 12$). Later, the same model is tested as restriction of the models with longer lags. We failed to reject the null hypothesis that there is no difference between two models with different lags. For example, take modified χ^2, proposed by Sims (1980 fn. 18), we obtain $\chi^2(100) = 93.87$ for 16 lags and $\chi^2(50) = 46.80$ for 14 lags. The corresponding significance levels are about 0.65 and 0.66. Consequently, it is decided to estimate the model with 12 lags.

The results of estimation are given in Table 2.1. Due to high collinearity, standard errors of the coefficents are not among the results in Table 2.1. Also individual coefficients, $\alpha^i_{j,p}$, are not reported to save space. All variables are in natural logs.

Table 2.1 Regression results for the VAR model 1980:03-1991:04

Equation	$\overline{R}^2$	$D-W$	Q	$\hat{\sigma}$
EXC	0.99	1.54	26.79	0.017
TM	0.90	2.16	33.70	0.110
REXPEN	0.76	1.89	29.17	0.187
M2	0.99	1.87	20.89	0.014
P	0.99	2.00	31.16	0.0121

Both $D-W$ and Q statistics are not valid for testing purposes, since each equation includes the lagged values of the dependent variable. These statistics are reported to make comparisons with other models.

Theoretically, if the model is specified correctly, the errors from each equation should be white noise with zero autocorrelation. Following Plosser and Schwert (1977), and Box and Jenkins (1976), it can be shown that the differencing a white noise series creates a first-order moving average process with an MA parameter equal to 1.0, so that the first order correlation coefficient is -0.50.

Estimated autocorrelation coefficients of the errors are not different from zero. The estimated first-order autocorrelation coefficients, ρ_{i1}, for the first differences of the errors from the model are:

$$\rho_{11} = -0.32, \quad \rho_{21} = -0.59, \quad \rho_{31} = -0.48, \quad \rho_{41} = -0.46, \quad \rho_{51} = -0.47.$$

The higher order coefficients can be found in Figures 2.1 through 2.5. With a sample size of 134, the 95 % confidence interval is approximately ± 0.17. From the inspection of the correlograms and estimated autocorrelation coefficients, we conclude that the errors are white noise.

Forecast results and forecast performance from the estimated model above are presented in the next section. The last section examines the dynamics of the model with MA representation.

Figure 2 1 AC Function of 1 DIF of ERROR1

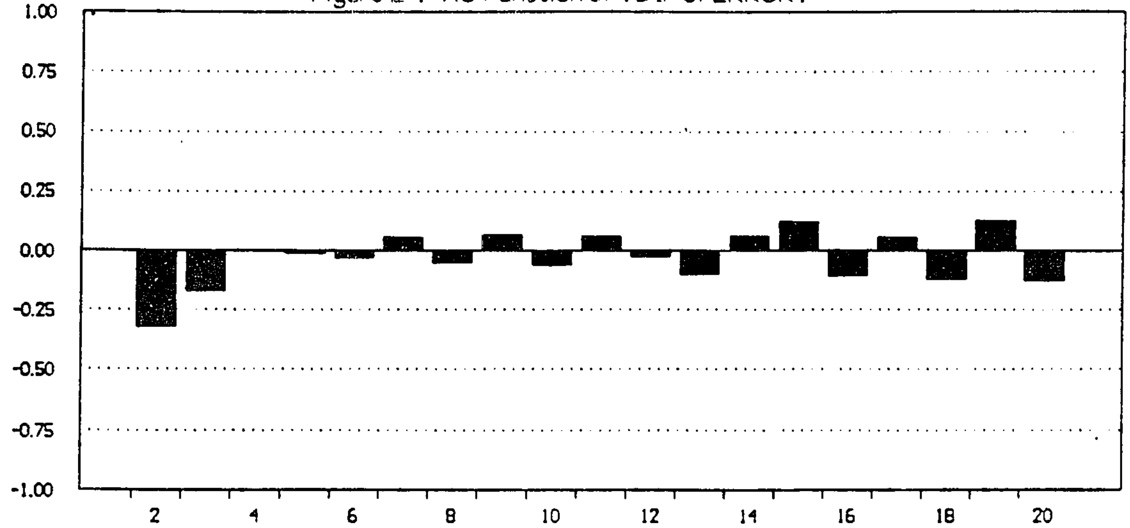


Figure 2 2 AC Function of 1 DIF of ERROR2

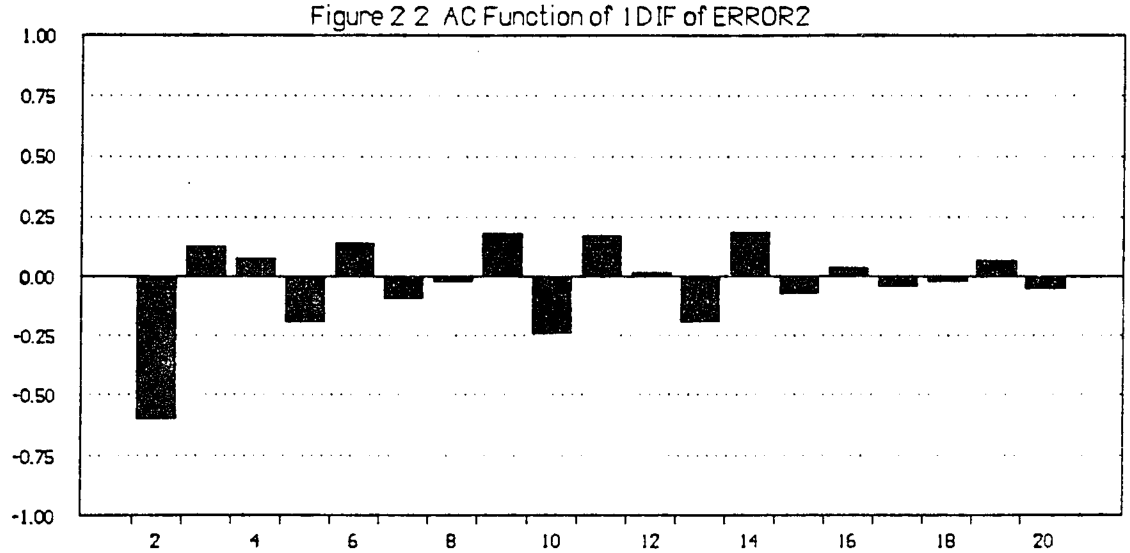
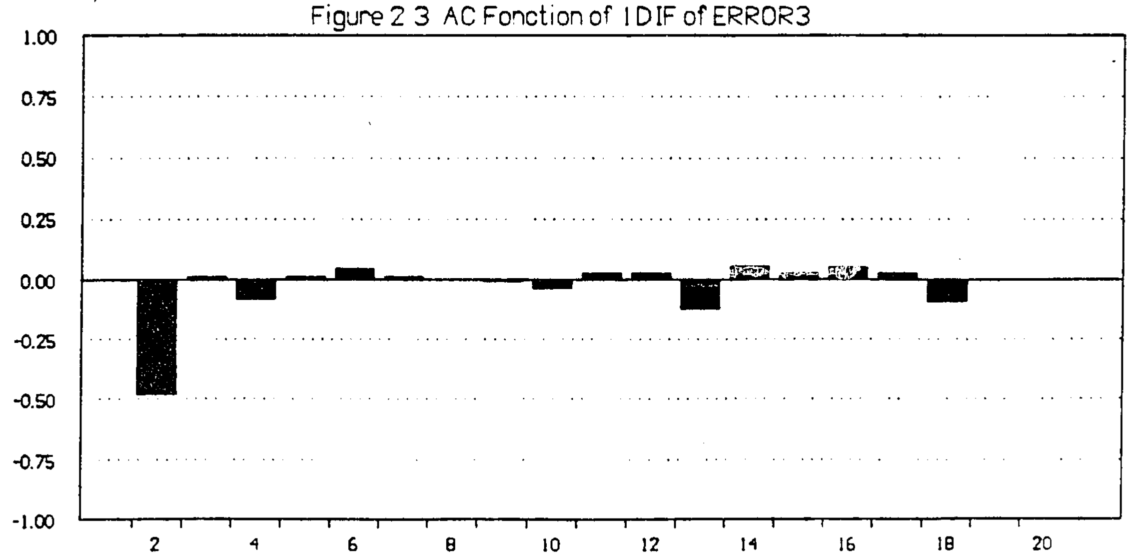


Figure 2 3 AC Fonction of 1 DIF of ERROR3



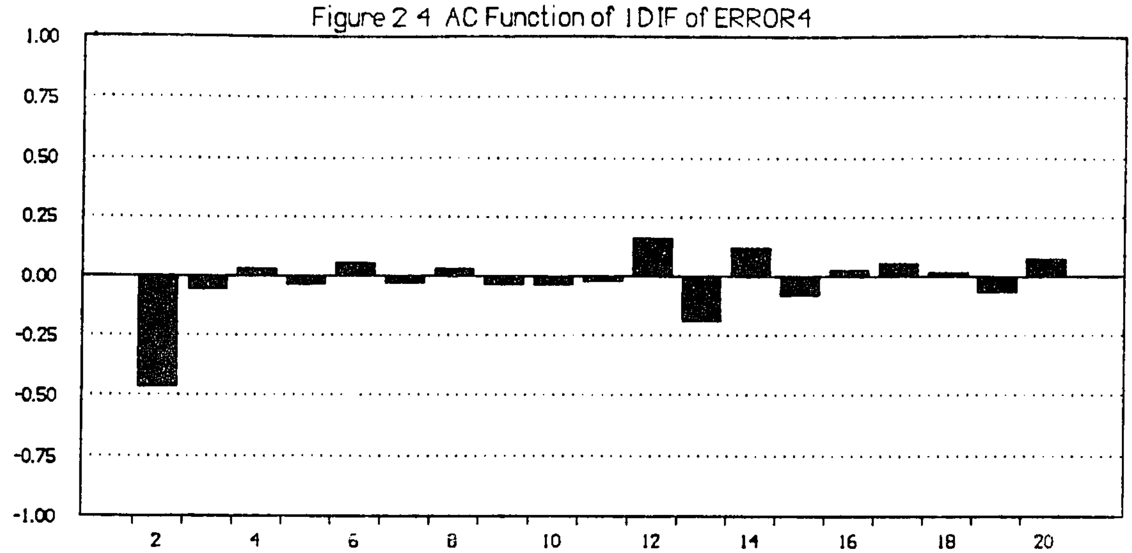


Figure 2 4 AC Function of 1 DIF of ERROR4
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
2
4
6
8
10
12
14
16
18
20

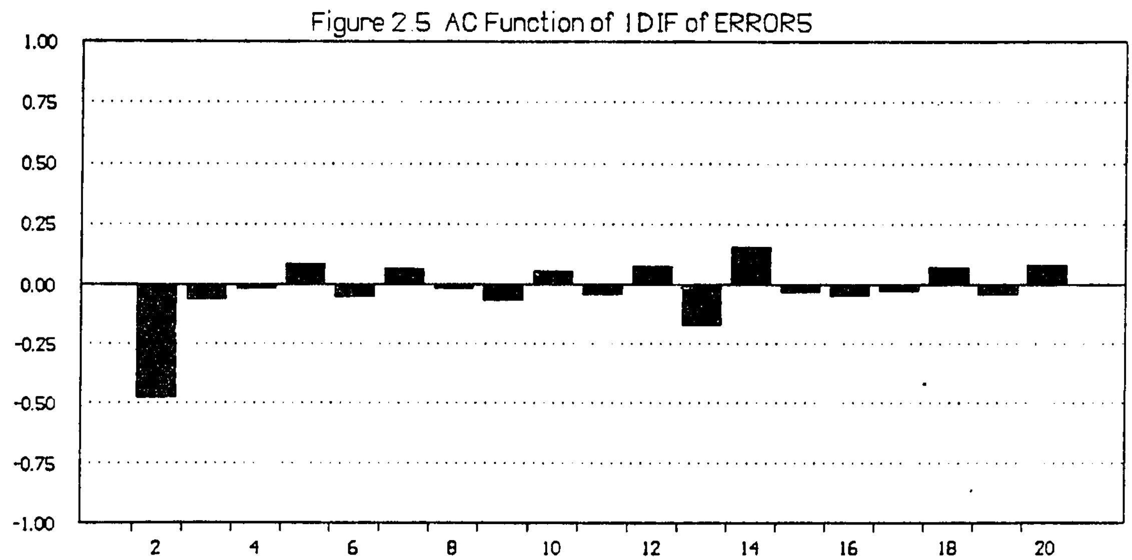


Figure 2.5 AC Function of 1 DIF of ERROR5
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
2
4
6
8
10
12
14
16
18
20

2.3 Forecasting with VAR

The optimal linear forecast of each variable can easily be calculated using the estimated model in Equation (2.2). Conditional expectation of X_{t+k} at time t–conditional on all the information available at time t–is called the unconditional k step ahead forecast of X_t. If we assign a certain future path for one of the variables in the system and make forecasts conditional on this specific path, the new forecast is called a conditional forecast of X_t.

According to the Lucas critique [Lucas (1976)], neither structural nor VAR models can be used for conditional forecasts. The reason for this is that if we assign certain values for the policy variables in the model, the coefficients in the $A(L)$ matrix will not remain constant, as they are assumed to do. Nevertheless, this critique might not be that important under some conditions [Sims (1982)]. For example, if the specified path for the policy variable is similar to the past realized values of the variable, we expect little change in the $A(L)$ matrix. Also, the response from economic agents to changes in policy may occur with lag. In this case, even a conditional forecast will be valid in the short-run.

In this section, only the unconditional forecasts of the variables are generated. Figures 2.6 through 2.10 give the monthly forecasts with a 95 % confidence interval. Compared to the univariate model of Chapter 1, significant improvements in terms of narrower confidence intervals can be seen for each variable.

As in Chapter 1, the period 1990:06-1991:04 is defined as "in-sample forecast period." Forecast performance statistics, reported in Table 2.2, are calculated following the same procedure described in Chapter 1 (p. 40). Since all the variables are in natural logs, the errors are percentage deviations from the realized level values of the variables.

Quarterly forecasts of the variables, along with 95 % confidence intervals are presented in Table 2.3. Forecasts of quarterly *average* growth rates of Price Index, Money Supply, and Exchange Rate are presented in Table 2.4. Since the growth rates are calculated as "log difference" of the level of related variable, they should be interpreted accordingly. In low inflation economies, common practice of inflation calculation using price indexes (as $\pi_t = \frac{(P_t - P_{t-12})}{(P_{t-12})}$, where π is yearly inflation and P is the monthly price index) may not give significantly different results if one uses just log differences (as $\pi_t = logP_t - \log P_{t-12}$). On the other hand, in high inflation economies like Turkey, the results will be different depending on the way π_t is calculated. The same applies to the calculation of growth rates of other variables. "Average monthly growth" below refers to the monthly change in the log of the variables. "Twelve-month growth" is calculated as $(1 - \pi_{yt}) = (1 + \pi_{mt})^{12}$ where π_{yt} is twelve-month growth and π_{mt} is the average monthly growth.

According to our forecasts, average monthly inflation rate for the third quarter of 1991 is 3.06 %, which implies a 43.6 % 12-month inflation. For the last quarter of the same year, average monthly, and 12-month inflation rates are 4.15 and 62.9 % respectively. For the first quarter of 1992, 12-month inflation forecast is 70.1 % with 4.53 % average monthly inflation. Given the history of price indexes in Turkey, we can expect higher inflation rates for consumer price indexes.

The Money Supply (M2) forecast for the year 1991 indicates a 73.4 % 12-month increase. For 1992, the same forecast is 54.9 %. 1991 forecast shows a significant increase in average monthly growth of the money supply: from 4.6 % for the second quarter to 6.0 %t for the last quarter.

Total Imports (TM) are forecasted to decrease 9.7 % in 1991 from $ 22,302.2 million realized value in 1990 to $ 20,133.5. Total imports in the last two quarters of 1990 were subject to an unexpected shock due to the Persian Gulf war and the sudden increase it brought in oil prices. Besides, the GNP growth rate was already above average. Given these two facts, a decrease in TM during the year 1991 should not be suprising.

Central bank exchange rate (EXC) is forecasted to increase 71 % in 1991, with an average monthly growth of 4.57 %. This above-average increase in EXC might be another factor contributing to the lower level forecast for total imports.

Finally, the forecast of real government expenditures (REXC)–consolidated budget expenditures at 1984 prices–shows an 11.3 % increase in 1991 and a 14.3 % increase for 1992.

Table 2.2. VAR Forecast Performance 1990:06 1991:04

Variable	Steps	*ME*	*MAE*	*RMSE*	*TU*
REXPEN	1	−.02	0.14	0.17	0.85
	3	−.02	0.11	0.14	0.44
	6	−.04	0.13	0.17	0.61
M2	1	0.00	0.02	0.02	0.56
	3	−.00	0.02	0.02	0.21
	6	−.02	0.045	0.04	0.23
P	1	0.01	0.02	0.03	0.62
	3	0.07	0.07	0.09	0.75
	6	0.16	0.16	0.18	0.79
EXC	1	0.01	0.03	0.04	0.78
	3	0.05	0.06	0.08	0.60
	6	0.12	0.12	0.13	0.60
TM	1	−.02	0.14	0.17	0.79
	3	−.06	0.19	0.22	0.65
	6	−.18	0.19	0.25	0.66

Table 4.3 Unconditional Forecast Results (VAR)

Variable	1991:2	1991:3	1991:4	1992:1
REXPEN	1,376.4 (±258.2)	1,161.6 (±338.9)	1,776.6 (±554.4)	1,259.2 (±405.1)
M2	85,507.5 (±1,885.0)	99,584.8 (±4,336.2)	119,385.0 (±5,939.0)	134,399.0 (±7,547.0)
P	2,021.1 (±43.7)	2,214.9 (±80.7)	2,508.3 (±129.8)	2,873.4 (±196.6)
EXC	4,096.5 (±112.7)	4,444.2 (±200.4)	4,909.9 (±334.8)	5,191.4 (±461.3)
TM	4,561.7 (±467.1)	4,677.0 (±750.2)	5,978.1 (±1,059.0)	5,643.6 (±1,039)

1991:x stands for the x quarter of 1991. REXPEN (Real Government Expenditures) and M2 (Money Supply) are in Billions of TL. P (Average wholesale Price Index): 1984=100. TM (Total Imports) are in millions of US Dollars. EXC is TL for 1 US Dollar. M2, P, and EXC are as end of period; REXPEN and TM are quarterly sums. 95 % confidence intervals are in parentheses.

Table 4.4 Unconditional Forecast Results (VAR) (Growth Rates)

Variable	1991:2	1991:3	1991:4	1992:1
M2	13.91	15.24	18.13	11.85
P	11.75	9.16	12.44	13.59
EXC	14.96	8.15	9.96	5.58

Quarterly Average Growth Rates. Each growth rate is calculated as a log difference of the forecasted level.

Figure 2.6 FORECASTED EXC

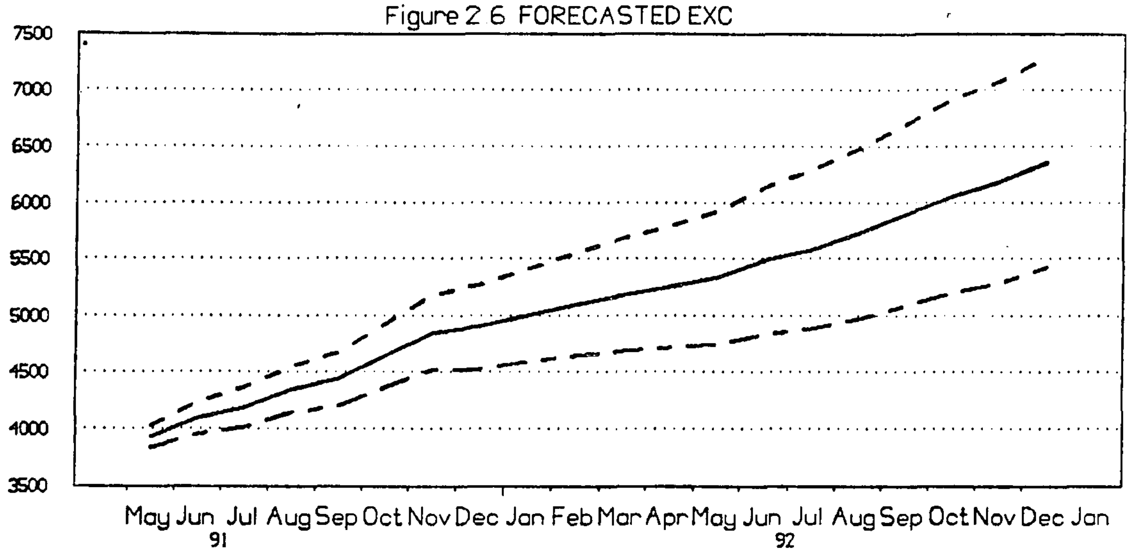


Figure 2.7 FORECASTED TM

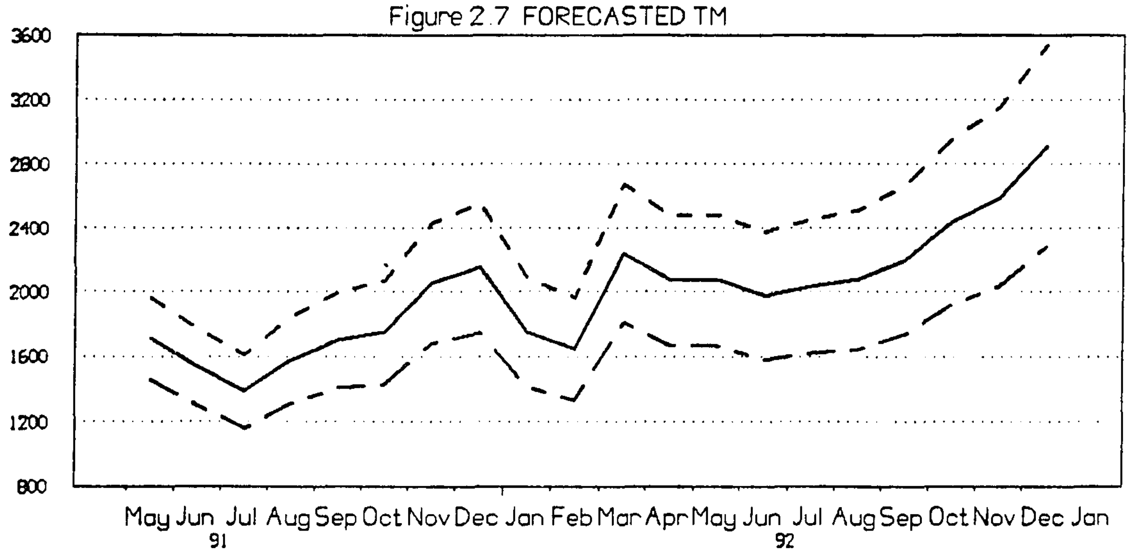


Figure 2.8 FORECASTED REXPEN

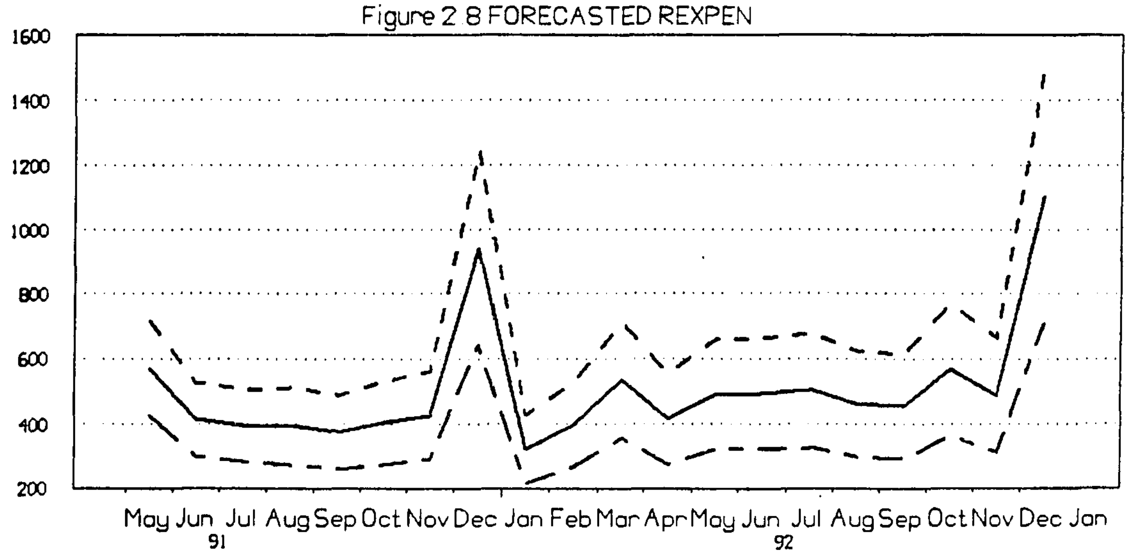


Figure 2.9 FORECASTED M2

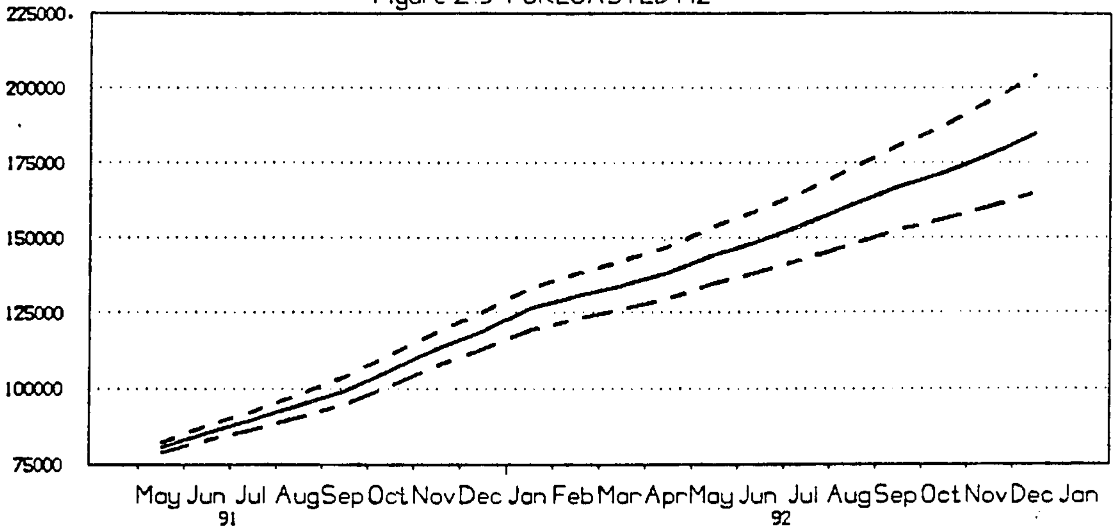
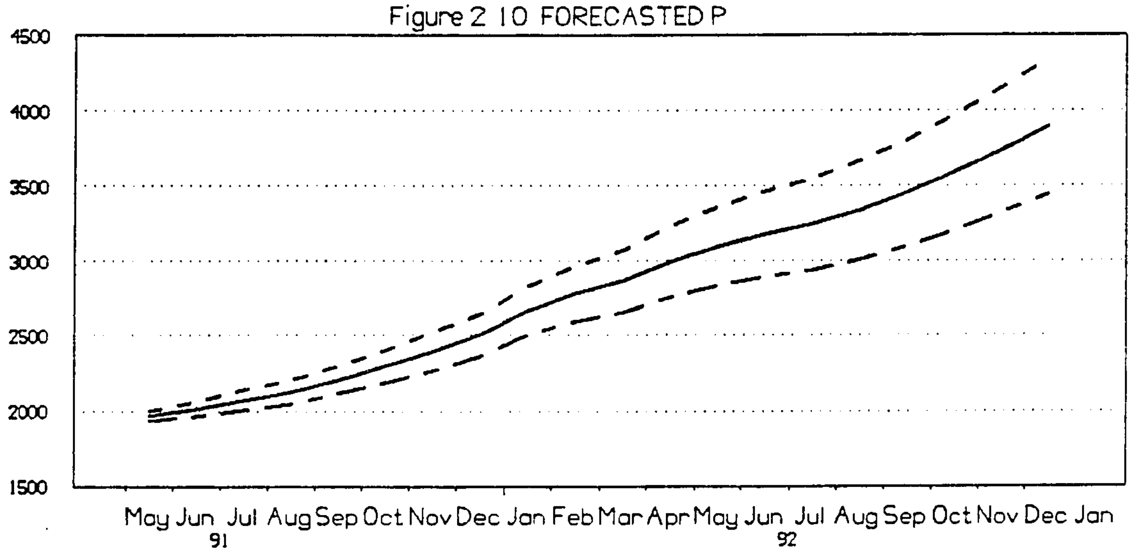


Figure 2.10 FORECASTED P



2.4 Impulse Response Functions

Moving average represantation (MAR) of a general VAR model was given in Equations 2.12 in Section 2.1. For the model we are interested in, MAR representation can be written as

$$\begin{aligned}
X_{1t} &= m_{100} + \sum_{p=0}^{\infty}(m_{11p}\epsilon_{1,t-p}) \\
&\quad + \sum_{p=1}^{\infty}(m_{i2p}\epsilon_{2,t-p} + m_{23p}\epsilon_{3,t-p} + m_{24p}\epsilon_{4,t-p} + m_{25p}\epsilon_{5,t-p}) \\
X_{2t} &= m_{200} + \sum_{p=0}^{\infty}(m_{21p}\epsilon_{1,t-p} + m_{22p}\epsilon_{2,t-p}) \\
&\quad + \sum_{p=1}^{\infty}(m_{23p}\epsilon_{3,t-p} + m_{24p}\epsilon_{4,t-p} + m_{25p}\epsilon_{5,t-p}) \\
X_{3t} &= m_{300} + \sum_{p=0}^{\infty}(m_{31p}\epsilon_{1,t-p} + m_{32p}\epsilon_{2,t-p} + m_{33p}\epsilon_{3,t-p}) \\
&\quad + \sum_{p=1}^{\infty}(m_{34p}\epsilon_{4,t-p} + m_{35p}\epsilon_{5,t-p}) \\
X_{4t} &= m_{400} + \sum_{p=0}^{\infty}(m_{41p}\epsilon_{1,t-p} + m_{42p}\epsilon_{2,t-p} + m_{43p}\epsilon_{3,t-p} + m_{44p}\epsilon_{4,t-p}) \\
&\quad + \sum_{p=1}^{\infty} m_{45p}\epsilon_{5,t-p} \\
X_{5t} &= m_{500} + \sum_{p=0}^{\infty}(m_{11p}\epsilon_{1,t-p} + + m_{52p}\epsilon_{2,t-p} + m_{53p}\epsilon_{3,t-p} + m_{54p}\epsilon_{4,t-p} \\
&\quad + m_{55p}\epsilon_{5,t-p})
\end{aligned} \tag{2.15}$$

As discussed in Section 2.1. different orderings of the variables will give different error vectors ϵ_t. The ordering chosen is (EXC, TM, REXPEN, M2, P). From Section 2.1 we know that current innovation of any given variable enters only its own equation and the equations of variables following. For example a current total imports innovation enters only the TM, REXPEN, M2 and P equations while the current innovation in average price index enters only its own equation. Similarly, current innovation in REXPEN enters only the REXPEN, M2, and P equations.

The impulse response function can be interpreted as the response of the variables to one standard shock to each of the variable. This can be seen by rewriting the first equation in (2.15) for successive t

$$X_{1t} = m_{100} + m_{110}\epsilon_{1t} + m_{111}\epsilon_{1,t-1} + +m_{112}\epsilon_{1,t-2} + \dots$$

$$X_{1,t+1} = m_{100} + m_{110}\epsilon_{1,t+1} + m_{111}\epsilon_{1,t} + +m_{112}\epsilon_{1,t-1} + \dots$$

$$X_{1,t+2} = m_{100} + m_{110}\epsilon_{1,t+2} + m_{111}\epsilon_{1,t+1} + +m_{112}\epsilon_{1,t} + \dots$$

It is easy to see that m_{11k} is the effect of a unit increase in ϵ_{1t} on $X_{1,t+k}$, i.e. $\frac{(dX_{1,t+k})}{(d\epsilon_{1t})}$. Note that a unit increase in ϵ_{1t} equals one standard deviation change in u_{1t} of Equation (2.1) by construction.

Even though the main purpose of this study is to compare forecasting performances of different models, impulse responses can be used to examine the effect of some policy variables on others.

In different studies [Saraçoğlu (1985), Kumcu (1988)], it is argued that the real depreciation of domestic currency in Turkey increases output *and reduces* inflation. The argument in these studies is supported by some empirical findings using methods similar to those employed in this study.

As can be seen from Figure 2.15, we have found some evidence to suggest the contrary. One standard-deviation shock to the nominal exchange rate, that is a 1.2 % increase in its level, will have positive, permanent effect on price level. The effect is 0.25 % inflation in the first month, reaching 1 % monthly inflation after one year.

As would be expected from the results of the random walk, an unexpected shock has a permanent effect on the log level of EXC.

From the covariance-correlation coefficient matrix of the error vector $\hat{u}_t$, it is determined that shocks do not occur independently to each variable. Instead of reporting the impulse responses of each variable separately, producing pages of output, we decided to report the response of the system to a most probable combined shock of the variables. The estimated covariance-correlation matrix indicated that an unexpected shock to price level and exchange rate in the same time period was most likely. Figures 2.16 through 2.20 show the response of each variable to a combined shock to EXC and P, i.e. the response of each variable to a unit standard deviation shock to EXC *and* P.

From the inspection of the figures, it can be seen that an innovation in EXC and P will have permanent, positive effect on EXC, M2, and P. Since the variables are estimated in natural logs, responses are percentage changes in the level of each variable. The combined shock will lead insignificant mixed changes in real variables REXPEN and TM.

Fig. 2.1.1 RESPONSE OF EXC TO A SHOCK IN EXC

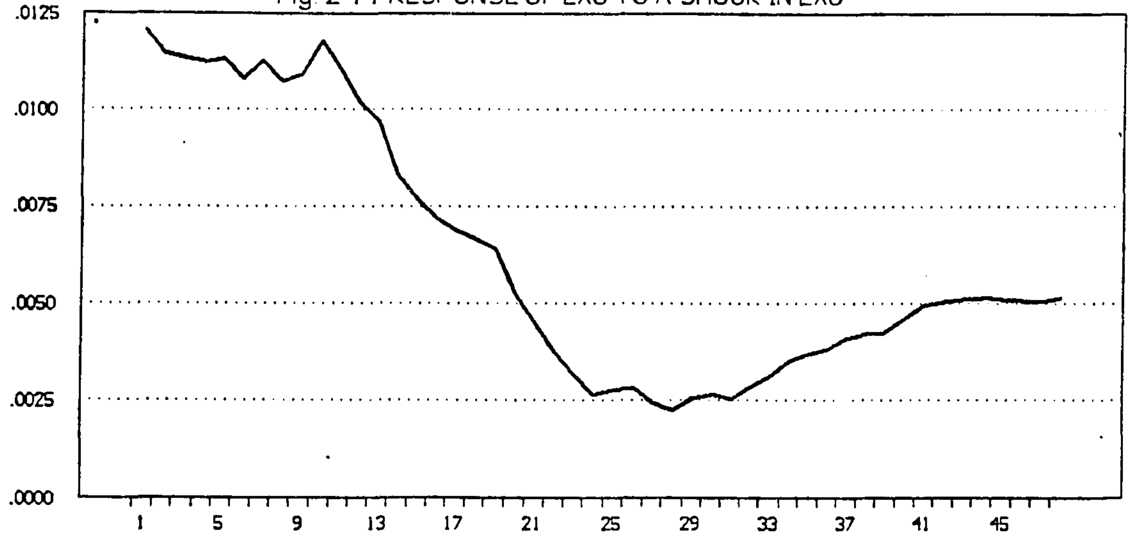


Fig 2.1.2 RESPONSE OF TM TO A SHOCK IN EXC

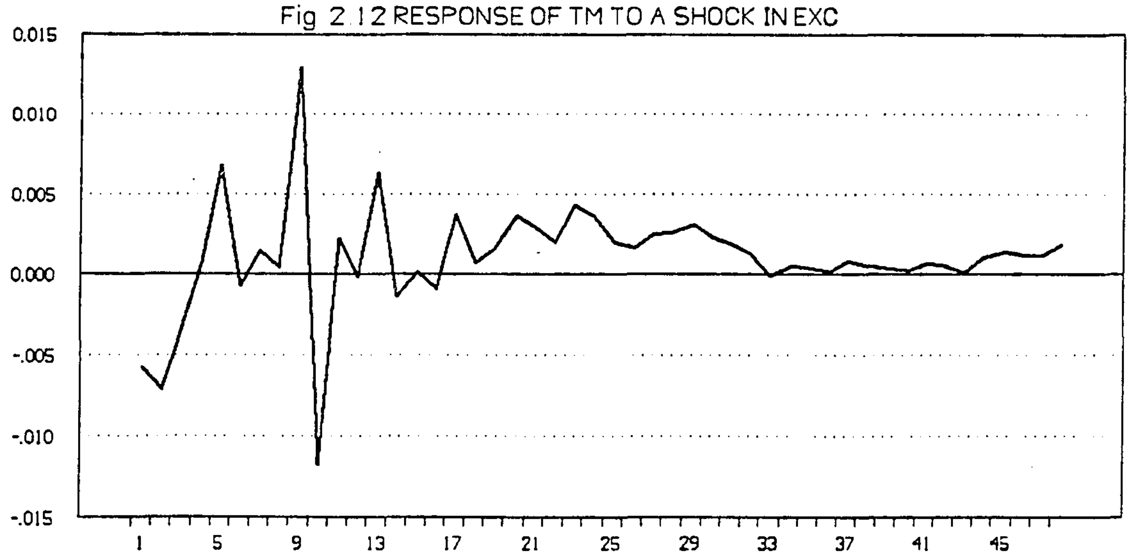


Fig 2.1.3 RESPONSE OF REXPEN TO A SHOCK IN EXC

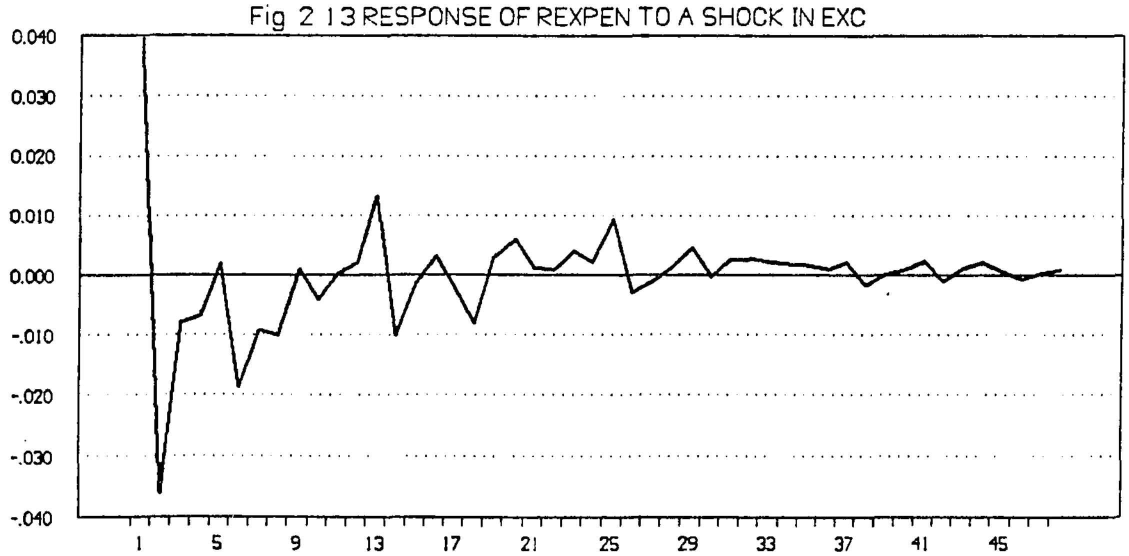


Fig 2 14 RESPONSE OF M2 TO A SHOCK IN EXC

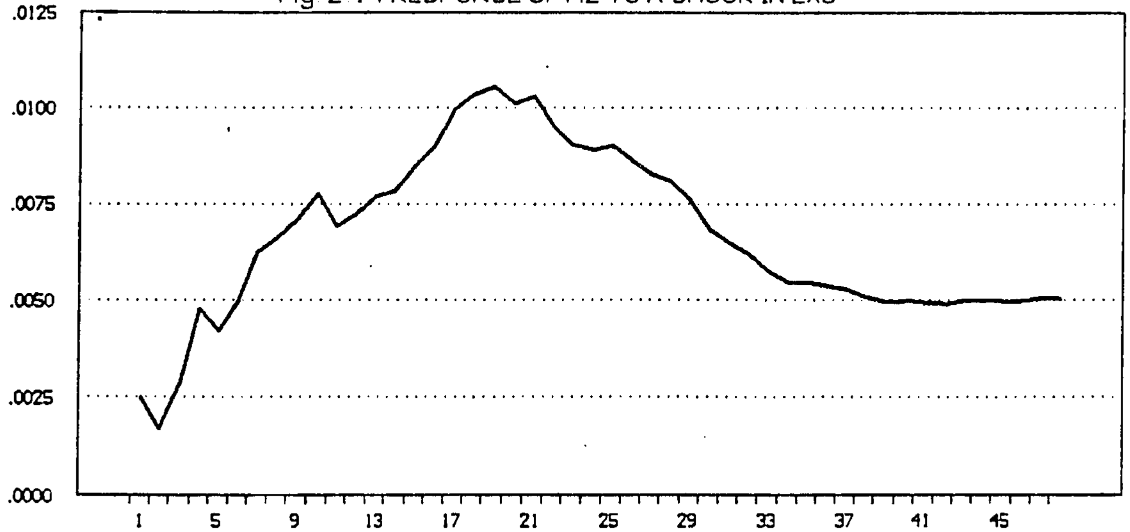


Fig 2 15 RESPONSE OF P TO A SHOCK IN EXC

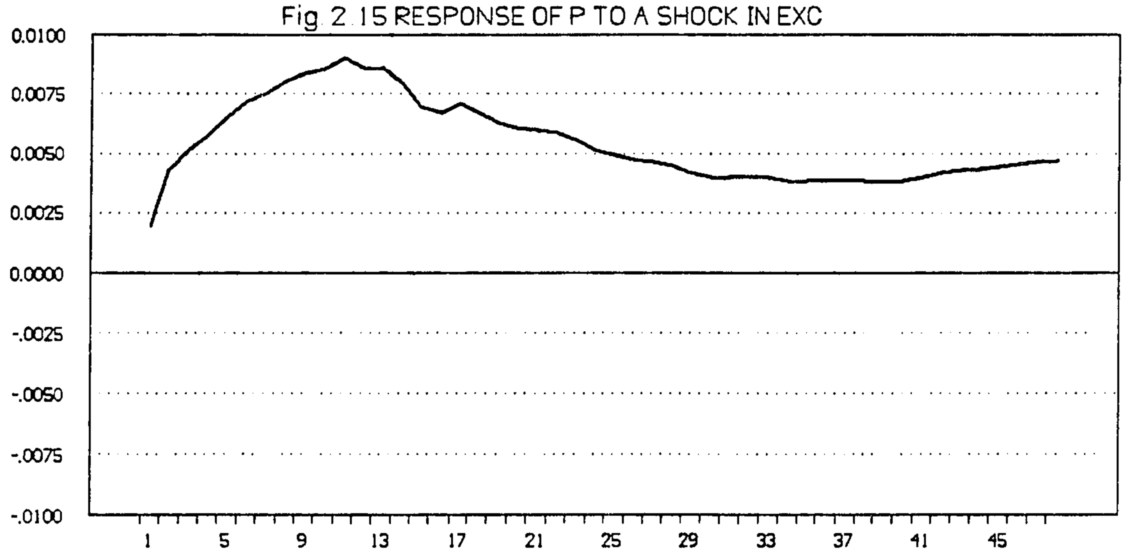


Fig 2.16 RESPONSE OF EXC TO SHOCKS IN EXC AND P

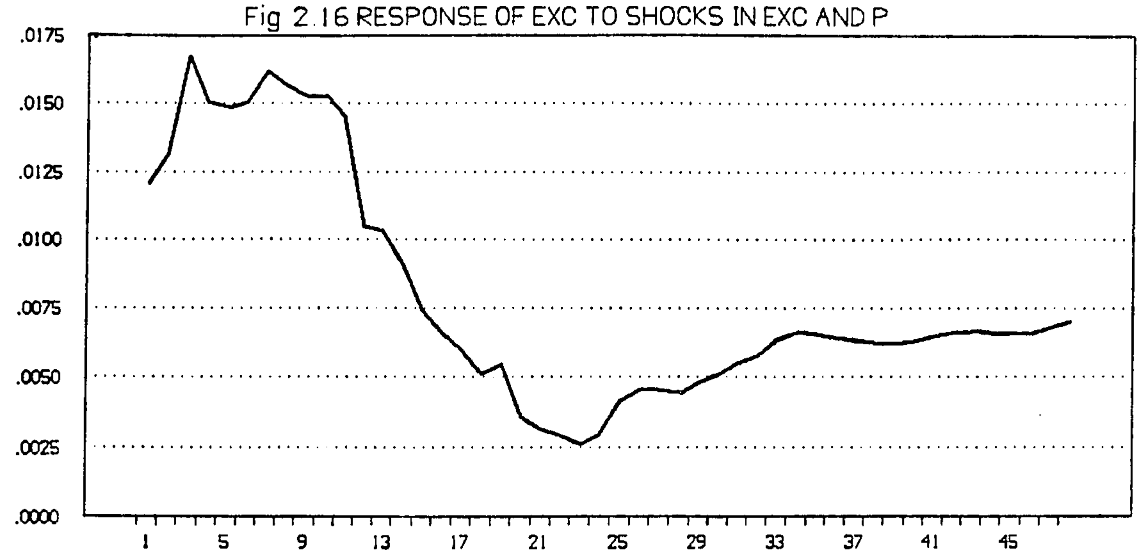


Fig 2.17 RESPONSE OF TM TO SHOCKS IN EXC AND P

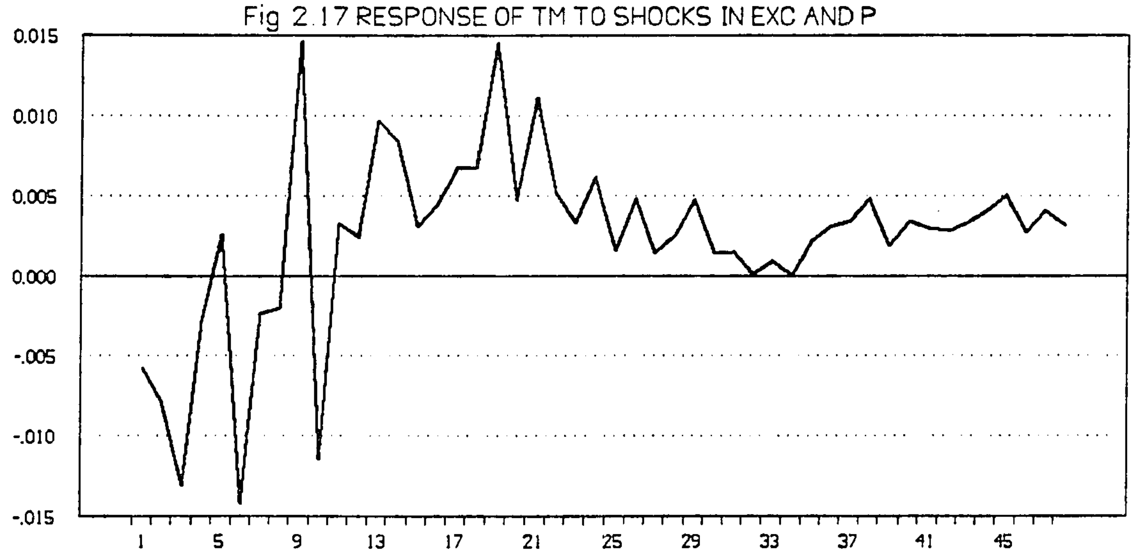


Fig 2.18 RESPONSE OF REXPEN TO SHOCKS IN EXC AND P

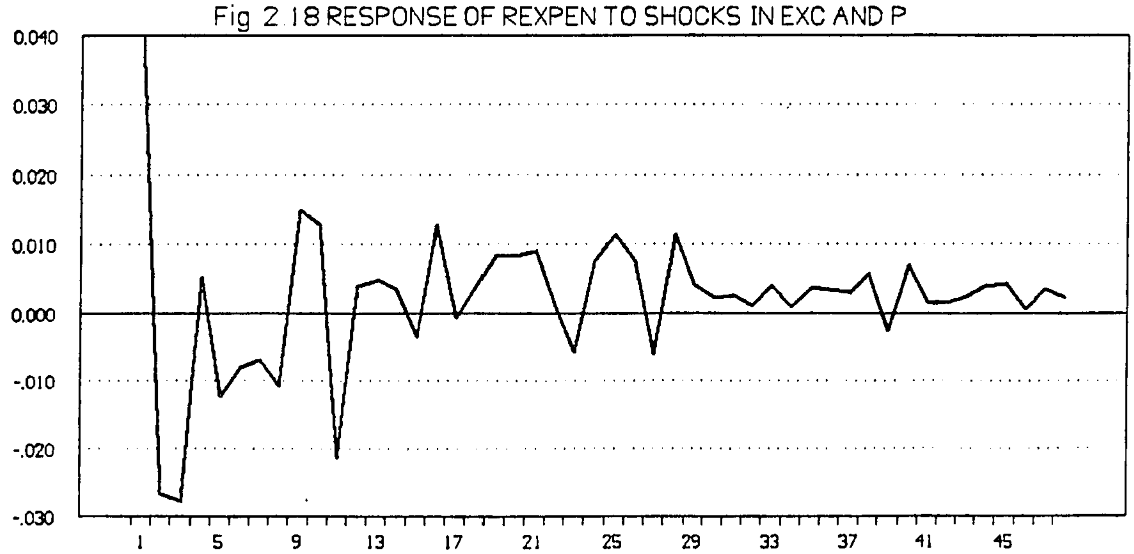


Fig 2.19 RESPONSE OF M2 TO SHOCKS IN EXC AND P

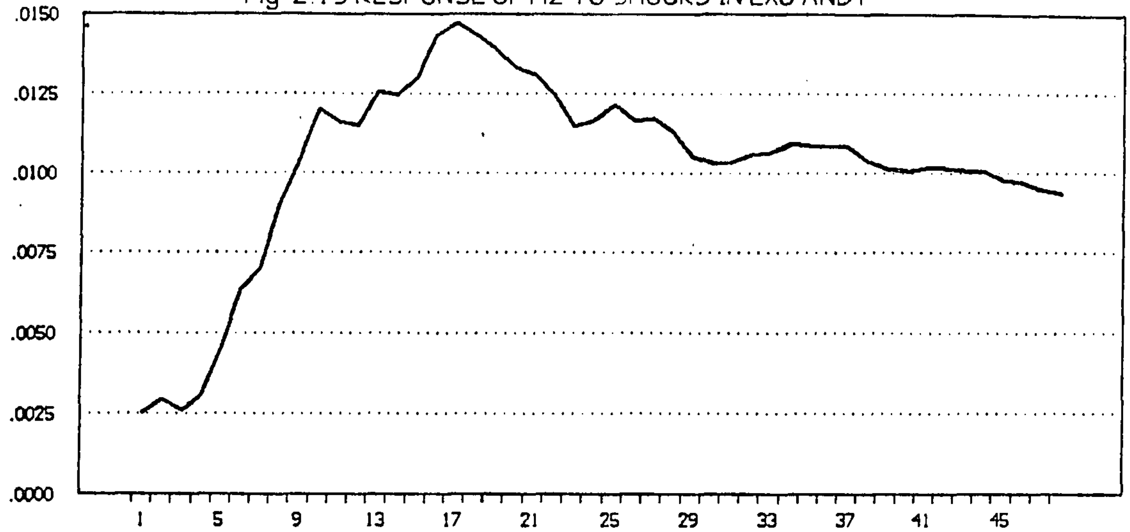
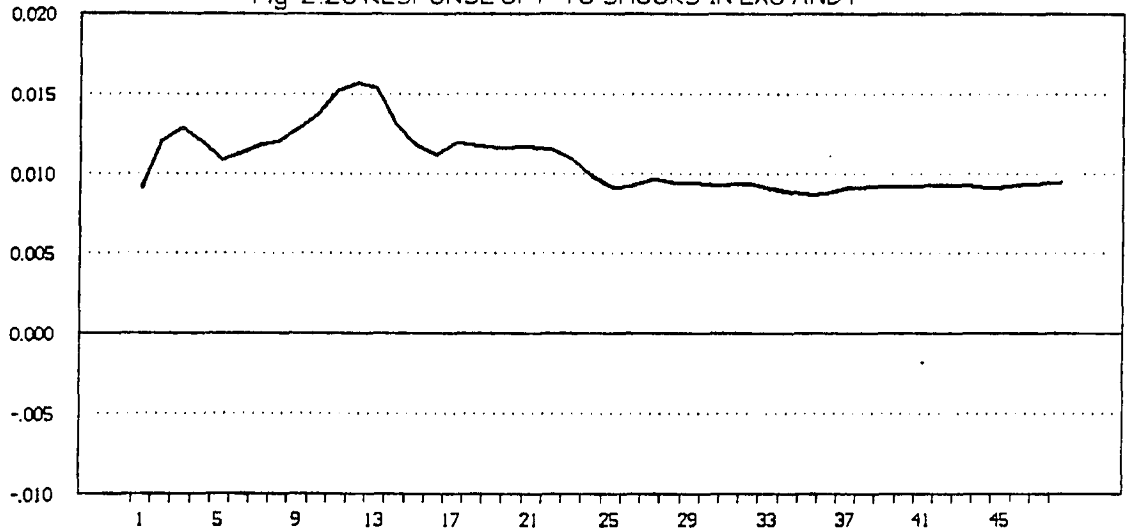


Fig 2.20 RESPONSE OF P TO SHOCKS IN EXC AND P



CHAPTER 3

In this chapter, the Bayesian approach to VAR is presented in a *state space modeling* setting. Estimation procedure with maximum likelihood is derived, and faciliated by the *Kalman filter* technique.

Technical aspects of the Kalman filter are discussed in Section 1. Section 2 describes the model we are interested in and defines the particular likelihood function. The application of the Kalman filter to this specific problem is given in Appendix II. The details of the covariance matrix of the system are presented in Section 3.

3.1. The Kalman Filter

The Kalman filter is a recursive algorithm for estimating and evaluating dynamic linear models. Since the filter is commonly employed by engineers, most of the published work on it is in engineering journals (including the orginal work by Kalman (1960)). Starting in the early 70s, the use of the Kalman filter appeared in some other areas as well. In 1973 the National Bureau of Economic Research published a special issue of *Annals of Economic Social Measurement* on time-varying parameters. Some of the articles in the issue show how to apply the Kalman filter on time-varying regression coefficient models [Belsley (1973), Cooley and Presscott (1973), Sarris (1973)]. More recently it became a subject matter of textbooks dealing with forecasting economic time series [Aoki (1987), Harvey (1989)].

In this section, the technical aspects of the filter and its implementation will be examined. For the purposes of this study, we prefer to present it as a Bayesian inference problem [see Meinhold and Singpurwalla (1983)].

Let X_t be the data vector, observed at time $t, t-1, ...1$. Assume that X_t depends on another unobservable vector A_t called the *state vector*. The relationship between the vectors can be represented as

$$X_t = Z_t A_t + u_t \tag{3.1}$$

where Z_t is a known quantity. This equation is called an *observation* or *data-generating equation*. The observation error, u_t, is assumed to be white noise with a known variance Ω_t

$$u_t \sim N(0, \ \Omega_t)$$

The following *state transition* or *system equation* specifies how the state vector A_t evolves with time:

$$A_t = D_t A_{t-1} + \eta_t \tag{3.2}$$

where D_t is a known quantity and the transition error η_t is assumed to be white noise with known variance M_t

$$\eta_t \sim N(0,\ M_t)$$

In addition to the above specifications, it is assumed that η_t and u_t are not correlated. Our goal is to make an inference about the *state of the nature*,A_t, given the data set X_t.

Consider the well known Bayes's theorem

$$Pr(A_t \mid X_t) \ \propto \ Pr(X_t \mid A_t \ ,X_{t-1}) * Pr(A_t|X_{t-1}) \tag{3.3}$$

where the term on the left side is the posterior distribution for A_t at time t, and the first and second terms are the likelihood and the prior distribution for A, respectively. At time $t-1$, the posterior distribution of A_{t-1} can be stated as

$$(A_{t-1} \mid X_{t-1}) \ \sim \ N(\hat{A}_{t-1},\ \Sigma_{t-1}) \tag{3.4}$$

It is worth noting at this point that the recursive procedure starts off at time zero selecting the appropriate values of A_0 and Σ_0; the importance of this will be explained later.

Before observing X_t, the best choice for A_t is given by Equation (3.2). It follows that the prior distribution of A_t is

$$(A_t|X_{t-1}) \ \sim \ N(D_t\hat{A}_{t-1},\ \ R_t = D_t\Sigma_{t-1}D_t' + M_t) \tag{3.5}$$

where D_t' denotes the transpose of D_t. Here, we used the relation $X \sim N(a, V) \Rightarrow DX \sim N(Da, DVD')$.

After observing X_t, we want to calculate the posterior of A_t, which requires the calculation of the likelihood $Pr(X_t|A_t, X_{t-1})$. Let e_t be the error in predicting X_t at time $t-1$.

$$e_t = X_t - \hat{X}_t = X_t - Z_t D_t \hat{A}_{t-1} \tag{3.6}$$

Observing X_t is equivalent to observing e_t as all other quantities in (3.6) are known. So we can write the likelihood as $Pr(e_t|A_t,\ X_{t-1})$. Also, using Equation (3.1) we can show that

$$e_t = Z_t(A_t - D_t\hat{A}_{t-1}) + u_t$$

It follows that since v_t is normally distributed with zero mean and the variance Ω_t the likelihood can be described as

$$(e_t|A_t,\ X_{t-1}) \sim N(Z_t(A_t - D_t\hat{A}_{t-1}),\ \ \Omega_t) \tag{3.7}$$

Finally, using the Bayes's theorem again we obtain the posterior of A_t

$$Pr(A_t|X_t, X_{t-1}) = \frac{Pr(e_t|A_t, X_{t-1}) * Pr(A_t|X_{t-1})}{\int_{allA_t} P(e_t, A_T|X_{t-1})dA_t} \tag{3.8}$$

Instead of dealing with Equation (3.8) to obtain the posterior distribution of A_t, we are going to use a standard theorem concerning multivariate normal distributions.

Let Y_1 and Y_2 have a bivariate normal distribution with means μ_1 and μ_2, and a covariance matrix σ_{ij}

$$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \sim N\left[\begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix}, \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}\right] \tag{3.9}$$

From (3.9), the conditional distribution of Y_1, given Y_2, is

$$(Y_1|Y_2 = y_2) \sim N(\mu_1 + \sigma_{12}\sigma_{22}^{-1}(Y_2 - \mu_2),\ \sigma_{11} - \sigma_{12}\sigma_{22}^{-1}\sigma_{21}) \tag{3.10}$$

where $\sigma_{12}\sigma_{22}^{-1}$ is known as the *coefficient of the least squares regression* of Y_1 on y_2 [Meinhold and Singpurwalla (1983)]. Notice that whenever (3.10) holds, (3.9) will hold and vice versa.

For our case, let Y_1 correspond to e_t and Y_2 correspond to A_t. This implies that

$$\mu_2 \Leftrightarrow D_t\hat{A}_{t-1}$$

$$\sigma_{22} \Leftrightarrow R_t$$

$$\mu_1 + \sigma_{12}R_t^{-1}(A_t - D_t\hat{A}_{t-1}) \Leftrightarrow Z_t(A_t - D_t\hat{A}_{t-1})$$

$$\sigma_{12} \Leftrightarrow Z_tR_t$$

$$\sigma_{11} - \sigma_{12}\sigma_{22}^{-1}\sigma_{21} = \sigma_{11} - Z_tR_tZ_t' \Leftrightarrow \Omega_t$$

$$\sigma_{11} \Leftrightarrow \Omega_t + Z_tR_tZ_t'$$

Then it is easy to show that (through (3.5)) the joint distribution of A_t and e_t, given X_{t-1}, can be described as [Granger and Newbold (1986), p. 299]

$$\left[\begin{pmatrix} A_t \\ e_t \end{pmatrix} |X_{t-1}\right] \sim N\left[\begin{pmatrix} D_t\hat{A}_{t-1} \\ 0 \end{pmatrix}, \begin{pmatrix} R_t & R_tZ_t' \\ Z_tR_t & \Omega_t + Z_tR_tZ_t' \end{pmatrix}\right] \tag{3.11}$$

The posterior distribution of A_t can now be written, using Equations (3.9), (3.10), and (3.11) as

$$(A_t|e_t, X_{t-1}) \sim N[D_t\hat{A}_{t-1} + R_tZ_t'(\Omega_t + Z_tR_tZ_t')^{-1}e_t,\ R_t - R_tZ_t'(\Omega + Z_tR_tZ_t')^{-1}Z_tR_t] \tag{3.12}$$

For a detailed discussion of the Kalman filter, see Anderson and Moore (1979), Harvey (1989, p. 104) and Aoki (1987, Ch. 7).

Note that the mean of posterior distribution (3.12) consists of two quantities: $D_t\hat{A}_{t-1}$, which is the mean of the prior distribution of A_t (from (3.5)), and a multiple of the one step ahead prediction error e_t. The multiplier here is the *Kalman Gain*. One way to view the Kalman filter is to think of it as an updating procedure consisting of forming a prior guess about the state of the nature and adding a correction to it. The correction comes from the innovation at each point in time.

Our posterior has the updating equation

$$\hat{A}_t = D_t\hat{A}_{t-1} + K_t e_t \tag{3.13}$$

and the variance Σ_t

$$\Sigma_t = R_t - K_t Z_t R_t \tag{3.14}$$

where K_t is the *Kalman gain*

$$K_t = R_t Z_t'(\Omega + Z_t R_t Z_t')^{-1}$$

Note that R_t, the variance of error in the prediction of A_t, is defined in (3.5) as

$$R_t = (D_t \Sigma_{t-1} D_t' + M_t).$$

3.2. The Bayesian Vector Autoregression (BVAR) Model

In earlier chapters a univariate and a vector autoregressive (VAR) model were used to generate unconditional forecasts of the variables we are investigating. In this section a Bayesian vector autoregression (BVAR) model is applied to the same set of variables and a corresponding likelihood function is derived. The model defined here bears a close resemblance to the model developed by Doan et al. (1984).

The data vector X_t can be represented as an unrestricted, time-varying, p'th order VAR with the following observation equation

$$X_t = A_t(L)X_{t-1} + C_t + u_t \tag{3.15}$$

where $A_t(L)$ is an $(n\text{x}np)$ matrix of polynomials of order p in the lag operator L and u_t is a zero-mean $(n\text{x}1)$ vector of disturbances independent of X_s, $s < t$

$$u_t \sim N(0,\ \Omega_t)$$

Let α_t^i be the vector of coefficients for the i'th equation of the n variable VAR. Then the i'th equation X_{it} is determined according to

$$X_{(it)} = \sum_{j=1}^{n}\sum_{p=1}^{m} X_{(j,t-p)}\alpha^i_{(t,j,p)} + \alpha^i_{(t,j+1,1)} + \sum_{s=2}^{S-1} \alpha^i_{(t,j+s,s)} D_{st} + u_{it} \tag{3.16}$$

where D_{st}s are seasonal dummies and u_{it} is the white noise equation disturbance.

For the coefficient vector α_t^i at time t, we assume a normal distribution. Then the posterior of α_t conditional on the information at time t is

$$\alpha_t \sim N(\hat{\alpha}_t,\ \Sigma_t) \tag{3.17}$$

At first glance, Equations (3.15) and (3.16) look the same as the VAR representation in Equation (2.1), but a closer examination reveals the difference in the specification of the coefficient vector.

The main weakness of VAR modeling, as noted earlier, is that the number of free parameters increases very fast with the number of variables in the system. One has to trade off the sufficient number of lags to capture the dynamics of the system against the sufficient degrees of freedom. In the case of a limited data set, one is forced to choose a smaller number of lags than he wishes to employ. This shortcoming of the VAR model forces the econometrician to do exactly what the VAR modeling attempts to avoid: to impose exclusionary restrictions on the model.

One solution to exclusionary restrictions is to impose "fuzzy" restrictions on the coefficients rather than to exclude some of them. In the literature, there are several examples of this kind of estimation techniquee. The ridge estimation, proposed by Hoerl and Kennard (1970) [see Judge et al. (1985, p. 913–922)], the Stein rule (Stein 1974) [see Judge et al. (1989, p. 836–838)], and the Shiller's smoothnes prior [Shiller (1973)] are some of the well-known procedures.

These Bayesian approaches to the overparameterization problem are incorporated into VAR modeling by Litterman (1979), and later developed by Doan, Litterman, and Sims (1984), and the economists associated with the FRB of Minneapolis. Instead of reducing the number of coefficients in the system, the BVAR modeling assumes that some of the coefficients have less influence. This is done by defining a *normal prior distribution* for the coefficient vector.

With the heavy use of priors, the BVAR seems close to traditional model-building practice in the sense that both use prior beliefs to reduce overfitting. However, the way these prior beliefs are incorporated into the models, as well as the sources of the priors, are different in the BVAR model than in a structural model. While economic theory is the main source of implicit priors in traditional model building, the BVAR relies upon statistics in the determination of priors. Besides, the BVAR priors are not set with complete confidence or complete ignorance, i.e either setting coefficients to zero or letting data determine them no matter what the researcher believes, as is done with simultaneous equations models [Todd (1984), Litterman (1984a)].

In BVAR modeling, the prior distribution, being a probabilistic statement, explicitly shows the modeler's belief regarding the coefficients and the possible values they can take. The specification also lets the data override the prior guess. In other words, the BVAR solution to the overfitting problem is to "guess" the influence of the coefficients, and to revise this guess at each point of time.

For (3.17), we assume that at time zero

$$\alpha_0^i \sim N(a,\ \Sigma_0 = f(\pi)) \tag{3.18}$$

Here the initial prior vector α_0^i has the mean a and the covariance matrix Σ_0, which is a function of the prior parameters π. Definition of the elements of the prior parameter vector π and the specific form of the covariance matrix Σ_0 are given in Section 3.3. Any change in the prior vector π leading to smaller (larger) variances of the coefficients is called *tightening (loosening)*.

Since we have hundreds of coefficients in the model, it is impossible to determine the prior distribution parameter by parameter. Here, we are adopting the convention developed by Doan et al. (1984) and Sims (1989).

The best guess we have for the coefficient values, i.e. for the vector a, comes from the *random walk hypothesis*. According to the test results from Chapter 1, the logs of most of the variables follow a random walk. Consequently, we assign a mean of 1.0 for the coefficient of the first own lag in each equation. The mean of the prior distribution for all other coefficients is assumed to be zero.

For the deterministic part of each equation (constant and seasonal dummies) we are going to assume noninformative (flat) priors. Since the variables are in logarithmic form, the constant part in each equation represents the *drift*, i.e. percentage increase per period in the variable. We let the data determine the drift by setting the variance for the constant to a very large number. As the prior is tightened around its mean, each equation will take the form of a *random walk with drift*

$$\log X_t = \log X_{t-1} + C \tag{3.19}$$

The determination of the variances for each coefficient, i.e the complete specification of Σ_0, is given in Section 3.4.

We need to define the transition equation to complete the specification of the model. The coefficient vector α_t^i is assumed to change over time according to the following *transition equation*

$$\alpha_t^i = \pi_1 \alpha_{(t-1)}^i \; + \; (1 - \pi_1) a \; + \; \eta_t \tag{3.20}$$

where η_t is normally distributed with zero mean and the covariance matrix proportional to prior covariance matrix Σ_0

$$\eta_t \sim N(0,\ \pi_2 \Sigma_0) \tag{3.21}$$

In the transition equation (3.20), when we set π_1 to 1.0, the model for the coefficients is a simple random walk. π_2, which is another prior parameter, determines the time variation in the coefficient vector. When we set this parameter to zero, no time variation is allowed in the model.

After these specifications, the conditional distribution of α_{t+1} is

$$\alpha_{t+1} \sim N(\pi_1 \alpha_t + (1 - \pi_1)a, \pi_2 \Sigma_0) \tag{3.22}$$

and the conditional distribution of X_{t+1} is

$$X_{t+1} \sim N(Z_t \alpha_{t+1}, \Omega_{t+1}) \tag{3.23}$$

where Z_{t-1} is the right hand side variables in observation equation (3.15).

After the joint normal distribution has been specified for α_t (3.17), α_{t+1} (3.23), and X_{t+1} (3.24), conditional on data up to time t, the Kalman filter is applied to each equation recursively to obtain the optimal estimator of the state vector α_t^i based on the full information available at that time. Using these specifications, the condititonal distribution of X_{t+1} can be re-expressed as

$$X_{t+1} | X_t, X_{t-1}, \ldots \sim N[Z_t(\pi_1 \alpha_t + (1 - \pi_1)a), S_t]. \tag{3.24}$$

The derivation of S_t is given in Appendix II.

Let s_{it} be the variance of the one step ahead forecast of the i'th component of X_{t+1} (i'th column of S_t) and $e_{(i,t+1)}$ be the prediction error. Then the sample log likelihood is the sum over t of the terms given by

$$-(1/2)(\log s_{it}^2 \; - (e_{i,t+1}/s_{it}^2)). \qquad (3.25)$$

As is shown in Doan, Litterman, and Sims (1984), the Kalman filter will produce the same series of coefficient estimates if Σ_0, Ω_t, and $\pi_2\Sigma_0$ are all multiplied by the same constant. From this, the following pseudo-likelihood function for equation i can be derived

$$(-1/2)\left[\sum \log s_{it}^2 - T\log\left(\left(\sum e_{it}/s_{it}^2\right)T\right)\right] \qquad (3.26)$$

where T is the number of observations. Equation (3.26) will guide us to find the optimum prior setting for the present model. The derivation of the Kalman filter–coefficient update equation, Kalman gain, etc.–for the model we are interested in can be found in Appendix II.

3.3. The Covariance Matrix Σ_0

The covariance matrix Σ_0, which defines the prior variances on the coefficient vector α_{it}^i, is assumed to be determined as a function of the vector of ***prior parameters***, π, in (3.18). In this section, the characteristics of this matrix are laid out.

In a BVAR model, the number of coefficients (including one constant term in each equation) is $n(1+np)$, n and p being the number of variables and the number of lags, respectively. It would be unrealistic to try to determine a prior variance for each coefficient in the system. We are going to employ a procedure which makes this cumbersome deliberation unnecessary. It defines the full set of prior variances once we choose some key features. It is very similar to the one described in Doan et al. (1984).

In this procedure, the covariance matrix Σ_0 is determined in two stages. First, we divide the coefficients of a given equation into three groups. In each group, the coefficient variances are assumed to take a specific form as a function of the prior parameter vector π. At the second stage, a set of weights, f_{ij}, is defined to reflect our prior knowledge about the effect of variable j in equation i.

For any equation in the system, coefficients can be classified as

i. constant(s),

ii. coefficients of own lags,

iii. coefficients of cross lags.

For the coefficients of the deterministic part of each equation, i.e the constant and seasonal dummies, it is assumed that the variance is given by

$$\sigma^{2i}_{\alpha(t,j+s,s)} = \pi_3 * \pi_4 \quad s = 1, 2, \ldots, 12 \tag{3.27}$$

For the coefficients of current and past values of the variable the equation forecasts, $\alpha^i_{t,i,p}$, $p = 1, 2, \ldots, m$; m being the number of the lags in the equation, the variance is assumed to be

$$\sigma^{2i}_{\alpha(t,i,p)} = \frac{\pi_4 * \pi_5 * f_{ii}}{p * \pi_6} \tag{3.28}$$

For the lags of other variables in equation i, the variance of the coefficient $\alpha^i_{t,j,p}$, $p = 1, 2, \ldots, m$ and $i \neq j$ is given by

$$\sigma^{2i}_{\alpha(t,j,p)} = \frac{\pi_4 * \pi_7 * f_{ij}}{p * \pi_6} \tag{3.29}$$

Regardless of the lag structure we are imposing on the system, setting the variance (3.29) to a small value implies that we are putting high probability on small coefficients of cross lags. This might be the case if our prior belief is that the equation behaves more like a univariate model. In some other cases we may have a prior belief that only some of the variables have less (or no) influence in the equation. The weights set, f_{ij}, enables us to discriminate the effect of certain variables in the equation. This is done simply by setting f_{ij} to a number to show the effect of variable j in equation i.

The value of f_{ij} gets closer to zero as we believe that the value of the relevant coefficients is closer to zero. So for the variables that we believe behave like a random walk, f_{ij} is specified close to zero. On the other hand, if we expect the variable forecasted to have significant interaction with the right-hand side variables, f_{ij} is set to a value further from zero.

In our model, for all of the variables but REXPEN and RREVE, f_{ii} is assumed to be 1.0. For REXPEN and RREVE, the value of the weight is $f_{ii} = 4.0$. Since we expect the variables EXC, RREVE, and REXPEN to behave like a univariate model, f_{ij} is set to 0.025 for these variables. For all other variables, $f_{ij} = 1.0$.

The elements of prior vector π and their corresponding roles are given on table 3.1.

Table 3.1. Prior parameters

π_i	Role
π_1	Rate of coefficient decay
π_2	Tightness on time variation
π_3	Relative tightness on constant and seasonality
π_4	Overall tightness
π_5	Relative tightness on own lags
π_6	Rate of tightness decay with increasing lag
π_7	Relative tightness on lags of other variables

CHAPTER 4

This chapter reports forecasting results from the application of the Bayesian VAR to the same data set we discussed in Chapter 1.

Section 1 considers a preliminary search for priors. In Section 2, final specification and likelihood maximizing values of the priors are given. Section 3 reports the forecast performance of the model in terms of out-of-sample forecast errors and compares it to other models' performance. Finally, unconditional forecast results, together with the historical values of the series, are presented in Section 4.

4.1. Preliminary Search for Prior Distribution

Prior to a search for the likelihood-maximizing values of vector π and weights f_{ij} , the system is estimated with a much simpler prior setting in the spirit of BVAR. As in (3.18) the prior is defined as a multivariate normal distribution for the coefficient vector α^i

$$\alpha^i_{(j,p)} \sim N(\alpha_0,\ \sigma^{2i}_{(i,j,p)}) \tag{4.1}$$

where $\sigma^2_{(i,j,p)}$ refers to the variance of the prior distribution for lag p of variable j in equation i for all i, j, and p.

In this part we assume that there is *no time variation* in the coefficients of the system ($\pi_2 = 0.0$). As earlier, the mean of the prior distribution for all coefficients is assumed to be zero with the exception of the first own lag in each equation for the reasons discussed earlier. For the deterministic part of each equation (constant and seasonal dummies), flat priors are assumed.

The variance for each coefficient, i.e. the elements of the Σ_0 matrix, is assumed to be determined by the following function

$$\sigma_{(i,j,p)} = \frac{\pi_4 f(i,j)}{p} \tag{4.2}$$

where π_4 is the overall tightness, $p = 1, 2, \ldots, m$; m being the number of lags in the equation, and $f(i,j)$ is the tightness on variable j in equation i.

For the reasons given in Chapter 2, the number of lags p is set to 14 to capture the dynamics between the variables. It can be argued that a longer lag length with a decay would be more appropriate. Unfortunately the system we are estimating has severe data restrictions. Monthly data is not available for some of the variables before 1979:01. Furthermore, we intentionally avoided including the time period before 1980:2 in our sample, as the liberalization measures were introduced to the economy starting that month. On the other hand, any shorter lag length might have failed to pick up important interactions between the variables.

Since the overall tightness controls the degree of tightness for all coefficients in the system, it is preferable to hold π_4 at a lower value and determine the tightness with changes in the function $f(i,j)$. For this reason, the overall tightness is set to 0.05, and several experiments are carried out with changing values of $f(ij)$.

First the system is estimated assuming $f(i,j) = 1$ for all i and j for the sample period 1980:03 - 1990:06. Later, holding other specifications constant, the same system is estimated with the following $f(i,j)$ function

$$f(i,j) = \begin{cases} 1 & \text{if } i = j \\ 0.5 & \text{otherwise} \end{cases}$$

After several experiments, it became clear that some of the equations need to be treated as univariate autoregressions. Of course, it is almost impossible to determine the full specification of the function $f(i,j)$. The way we determined this function is somewhat arbitrary, and it is not the "best" form in the sense of minimizing some cost functions such as out-of-sample forecast errors. Informally, TUs are used to check the forecast performance of different settings.

Final form of the $f(i,j)$ function in the preliminary search is the following $PMAT$ matrix, in which element i,j is $f(i,j)$.

$$PMAT_{i,j} = \begin{bmatrix} 2.00 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 \\ 1.00 & 1.00 & 0.20 & 0.20 & 0.20 & 0.20 & 0.20 & 0.20 & 0.20 \\ 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ 1.00 & 1.00 & 1.00 & 0.05 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ 0.50 & 0.50 & 0.50 & 0.50 & 1.50 & 0.50 & 0.50 & 0.50 & 0.50 \\ 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 \\ 0.50 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 0.50 & 0.50 \\ 0.10 & 0.10 & 0.10 & 0.10 & 0.10 & 0.10 & 0.10 & 1.00 & 0.10 \\ 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 0.05 & 2.00 \end{bmatrix}$$

With this specification, the model is re-estimated for the sample period 1980:03-1990:06 using the Kalman filter. Forecast performance statistics, calculated for the projection period 1990:07–1991:04 as in Chapter 1 and Chapter 2, are given in Table 4.1. The comparison of the forecast performance of the preliminary BVAR model (PBVAR) to the forecast performance of the univariate model estimated in Chapter 1 shows that there are significant improvements in terms of out-of-sample forecast errors. For M2, P, EXC, TM, TX, and ISSIZI we observe smaller TU values, while there is a deterioration for REXPEN, MIPI, and RREVE. Forecast results, with 95 % confidence intervals, are given in Tables 4.2 and 4.3.

What we have learned from the preliminary search is that a system with this many variables and a limited data set requires a higher tightness fixed through either the overall tightness or other parameters in the prior set. Furthermore, some of the variables should be treated more like a univariate autoregression. In addition to our expectations about the variables, the $PMAT$ matrix may shed some light on variable interactions in the model.

Table 4.1 BVAR Forecast Performance 1990:06 1991:04 (Preliminary)

Variable	Steps	*ME*	*MAE*	*RMSE*	*TU*
REXPEN	1	−0.001	0.085	0.11	0.43
	3	0.003	0.110	0.133	0.43
	6	−0.014	0.148	0.164	0.59
M2	1	−0.003	0.007	0.009	0.29
	3	−0.014	0.019	0.020	0.24
	6	−0.039	0.039	0.042	0.25
P	1	0.002	0.012	0.014	0.34
	3	0.011	0.031	0.035	0.29
	6	0.005	0.022	0.024	0.11
EXC	1	0.010	0.021	0.031	0.62
	3	0.024	0.051	0.072	0.57
	6	0.031	0.052	0.069	0.33
TM	1	−0.000	0.140	0.168	0.77
	3	−0.029	0.193	0.224	0.62
	6	−0.098	0.163	0.215	0.57
TX	1	−0.014	0.076	0.100	0.49
	3	−0.017	0.081	0.098	0.31
	6	−0.034	0.051	0.058	0.19
MIPI	1	−0.042	0.078	0.099	0.82
	3	−0.068	0.090	0.117	0.74
	6	−0.126	0.126	0.147	1.25
ISSIZI	1	0.000	0.018	0.023	1.29
	3	0.001	0.022	0.028	1.38
	6	−0.015	0.029	0.034	1.08
RREVE	1	−0.000	0.101	0.124	0.62
	3	−0.003	0.117	0.140	0.55
	6	0.008	0.141	0.147	0.56

Table 4.2 Unconditional Forecast Results (PBVAR)

Variable	1991:2	1991:3	1991:4	1992:1
REXPEN	1,242.3 (±332.0)	1,280.4 (±502.9)	1,844.5 (±728.1)	1,242.7 (±491.3)
M2	83,564.8 (±3,376.6)	97,017.2 (±6,114.8)	114,392.0 (±8,443.0)	136,317.0 (±11,229.0)
P	2,111.4 (±62.6)	2,442.2 (±126.6)	2,993.6 (±220.7)	3,702.0 (±369.5)
EXC	4,227.1 (±174.2)	4,917.1 (±345.3)	5,703.3 (±575.9)	6,735.9 (±894.7)
TM	4,397.3 (±684.2)	4,890.9 (±1,147.8)	6,112.7 (±1,469.3)	5,242.2 (±1,277.8)
TX	3,392.3 (±411.3)	3,504.4 (±740.2)	5,010.4 (±1,111.3)	4,219.4 (±968.1)
MIPI	134.6 (±12.7)	139.9 (±13.6)	154.9 (±15.6)	143.5 (±14.7)
ISSIZI	123.7 (±6.4)	116.6 (±11.3)	112.5 (±15.3)	111.0 (±17.7)
RREVE	1,025.9 (±202.7)	1,063.1 (±313.2)	1,257.2 (±377.2)	1,048.4 (±337.7)

1991:x stands for the x quarter of 1991. REXPEN, RREVE, and M2 are in billions of TL. TM and TX are in millions of U.S. dollars. ISSIZI, P, and MIPI 1984=100.0. EXC is TL for 1 U.S. dollar. REXPEN, RREVE, TX, and TM are quarterly sums. M2, EXC, and P are as of period end. MIPI and ISSIZI are quarterly averages. 95 % confidence intervals are in parentheses

Table 4.3 Unconditional Forecast Results (PBVAR) *(Growth Rates)*

Variable	1991:2	1991:3	1991:4	1992:1
REXPEN	2.8	8.8	20.4	7.2
M2	21.6	41.1	66.4	19.2
P	31.3	51.9	86.2	23.7
EXC	47.2	71.3	98.6	18.1
TM	−9.8	−10.0	−16.1	6.6
TX	23.6	22.6	14.9	24.5
MIPI	−4.0	−2.3	−2.0	1.1
ISSIZI	−6.0	−10.0	−13.4	−12.8
RREVE	6.0	8.6	−2.0	5.0

M2, P, and EXC are percentage change over the end of previous year. Others are percentage change compared to the same period of previous year.

4.2 Determination of the Priors

In addition to the univariate and VAR models estimated before, a preliminary BVAR model was used in the previous section to generate a set of forecasts and forecast performance measures. In this section, the determination of prior vector π for the final BVAR model is reported.

Unlike the PBVAR specification, it is now assumed that the parameters of the system *randomly change over time* according to Equation (3.20). Consequently, the application of the Kalman filter is required to estimate the updated parameters. The "optimum" prior vector is searched as a function of *out-of-sample* and *in-sample* forecasting errors. Concentrating only on in-sample forecast errors might result in overfitting the data. As a signal-extraction problem, this would lead us to pick up excess noise, and the final hyperparameters would be contaminated by accidental patterns in the data [Litterman (1984), Todd (1984)]. Nevertheless, we preferred to include in-sample forecast minimization criteria, since we are concerned not only about forecasts but also about how well the model explains the data.

In order to capture the variable interaction in the system, a set of weights, f_{ij}, was defined in Chapter 3. In the first section of this chapter, our preliminary search for priors led us to the matrix $PMAT$. In this stage of the search, we keep the same setting of weights for each variable as described in the $PMAT$ matrix of the previous section.

We took the parameter setting of the π vector in the previous section as our starting point. The parameter setting of priors is

$\pi_1 = 1.00, \quad \pi_3 = 10000.0, \quad \pi_5 = 0.05, \quad \pi_6 = 1.00, \quad \pi_7 = 0.0025$

we started to minimize the quasi-likelihood function. At the same time, the *ATU* for each equation was checked to see the direction of the change in the forecast performance. After several experiments along this line, the search led us to the values $\pi_4 = 0.10$ and $\pi_2 = 10^{-8}$. There was no significant change in forecast performance in terms of *ATU* in the neighborhood of the final specification of priors. The final values of the priors are

$\pi_1 = 1.0; \; \pi_2 = 10^{-8}; \; \pi_3 = 10^4; \; \pi_4 = 0.098; \; \pi_5 = 0.05; \; \pi_6 = 1.00; \; \pi_7 = 0.0025.$

In the final specification, the amount of parameter variation may seem suprisingly small. It should be noted that in a model as large as this one in terms of coefficients on the right side of each equation even a small parameter drift implies large standard errors of forecasts. We do know from the first chapter that the series we are interested in do not imply such high standard errors in short-horizon forecasts. Besides, it is known that naïve models or simple random walk models in general perform as well in forecasting these series. This characteristic of the data is also inconsistent with large parameter drifts. Of course, if we had been looking more than just point projections and defining our criteria other than by forecast errors, it would have been a necessity to allow for a time variation through π_8. This could have led us to obtain a large amount of time variation in estimated parameters.

Here, the basic weakness of the Kalman filter used to generate updated coefficients and forecasts is the assumption that the innovations to coefficients are uncorrelated with the errors in regression. This assumption implies that the movements in coefficients cannot represent any structure or policy change. Consequently, the small parameter change implied by the final specification cannot be interpreted as "no structural change" or "no policy change" during the estimation period. In order to test whether there was a structural change or not, some other statistical methods can be used [see Harvey (1989 p. 397)]. However, this is beyond the scope of this study.

Even though the VAR and the Bayesian VAR forecasting techniques are widely used in developed countries, there are very few published applications of these models to Turkey [Kumcu et al. (1987), Kumcu (1988), Conway (1990), also see comments in Neftçi (1991)]. We expect that the performance of the models can be improved after seeing the results of other research on the same subject. In this respect, Final BVAR results may be seen as a report to other researchers on our beliefs about priors.

4.3 Forecast Performance of the BVAR

Throughout this study, certain forecast performance statistics have been generated to compare the forecast performance of different models. These measures are defined in Chapter 1 in terms of the k-step-ahead forecasts made at time t, $\hat{x}_{t+k}$, and the actual value at time $t + k$, x_{t+k}.

$$ME = \frac{1}{l}\sum_{k=1}^{l}(x_{t+k} - \hat{x}_{t+k}) \tag{1.6}$$

$$MAE = \frac{1}{l}\sum_{k=1}^{l}|x_{t+k} - \hat{x}_{t+k}| \tag{1.7}$$

$$RMSE = \sqrt{\frac{1}{l}\sum_{k=1}^{l}(x_{t+k} - \hat{x}_{t+k})^2} \tag{1.8}$$

$$TU = \sqrt{\frac{\frac{1}{l}\sum_{k=1}^{l}(x_{t+k} - \hat{x}_{t+k})^2}{\frac{1}{l}\sum_{k=1}^{l}(x_{k+l})^2}} \tag{1.9}$$

where l is the number of elements in the projection period.

Statistics reported in Table 4.5 are calculated as follows: To make the direct comparison possible, the data set is divided into two parts, from 1980:03 to 1990:06 as an estimation period and from 1990:07 to 1991:04 as the projection period. The coefficients of the model are estimated by applying the Kalman filter, using the data in the estimation period. Next, a series of forecasts of the next k values of X is calculated by feeding forward forecasts of early periods as if they were realized values. The coefficients of the model then are re-estimated using an additional observation of the sample and a new set of k forecasts is generated. The procedure of re-estimating forecasts and coefficients is carried out for each observation in the projection period.

When we compare the final BVAR model to the earlier models, we do not see a uniform result for all of the variables. For TX, TM and ISSIZI, we observe significant improvement in terms of smaller TU. For REXPEN, M2, P, and EXC, the preliminary BVAR model performed better. For MIPI and RREVE, there were no gains moving from the univariate to the multivariate models.

Table 4.4 *Average Theil's U*s from Different Models

Variable	*UNIV*	*VAR*	*PBVAR*	*BVAR*
REXPEN	2.03	0.92	1.94	2.93
M2	0.37	0.25	0.24	0.28
P	0.18	0.82	0.17	0.26
EXC	0.46	0.78	0.44	0.48
TM	0.71	0.94	0.82	0.64
TX	0.43	--	0.28	0.23
MIPI	0.65	--	0.99	0.76
ISSIZI	2.58	--	1.21	1.09
RREVE	0.89	--	1.07	1.21
FOM	0.920	--	0.798	0.875

The last row of Table 4.4 shows the overall performance of each model. The figure of merit (FOM), defined in Chapter 1, indicates that the preliminary BVAR performs best, whereas the performance of the univariate model is the worst.

Tables 4.1 and 4.5 show that one step ahead average forecast errors are less than 1 % for almost all of the variables in the preliminary BVAR and the final BVAR. Especially in longer-horizon forecasts, the performance of the final BVAR is much better than that of the other models. Note that since the model is estimated in natural logs, reported mean errors show the percentage deviations from the realized level values of the variables.

Table 4.5 FBVAR Forecast Performance 1990:06 1991:04

Variable	Steps	*ME*	*MAE*	*RMSE*	*TU*
REXPEN	1	0.059	0.123	0.152	0.59
	3	0.092	0.138	0.165	0.54
	6	0.121	0.145	0.201	0.73
M2	1	−0.003	0.008	0.009	0.29
	3	−0.015	0.020	0.021	0.25
	6	−0.047	0.047	0.051	0.30
P	1	0.004	0.013	0.015	0.37
	3	0.015	0.034	0.038	0.32
	6	0.024	0.047	0.051	0.18
EXC	1	0.013	0.024	0.035	0.70
	3	0.025	0.057	0.076	0.60
	6	0.037	0.075	0.095	0.45
TM	1	0.010	0.149	0.178	0.81
	3	0.029	0.171	0.202	0.60
	6	−0.002	0.170	0.181	0.48
TX	1	−0.003	0.065	0.082	0.40
	3	0.006	0.075	0.087	0.28
	6	−0.009	0.043	0.048	0.16
MIPI	1	−0.018	0.078	0.092	0.76
	3	−0.017	0.082	0.097	0.61
	6	−0.075	0.081	0.100	0.63
ISSIZI	1	0.002	0.018	0.022	1.24
	3	0.008	0.024	0.029	1.44
	6	−0.003	0.024	0.033	1.05
RREVE	1	0.051	0.129	0.148	0.74
	3	0.096	0.157	0.176	0.69
	6	0.163	0.183	0.227	0.87

4.4. Unconditional Forecast Results

After the estimation of hyperparameters of the system, unconditional forecasts of the series are easily estimated for the period of 1991:05-1992:12. Throughout the forecasting, the most recent estimates of randomly varying coefficients are used. At each step, forecast values of each variable are used to calculate the "updated" values of these coefficients. Since the forecasts are nonlinear functions of the parameters, our forecasts are unconditional [Doan et. al. (1984)]. The summary of the forecasts as quarterly values is presented in Table 4.6. Table 4.7 lists the forecasted percentage change of each variable.

Before reporting the results of the forecasts we would like to make few remarks about the current economic and political situation in Turkey. Foreseeing the coming early general elections, the Turkish government implemented an 'electionary fiscal and monetary policy' starting in 1991:06. This policy included an expansionary government spending program financed by internal and external borrowing and monetary expansion. According to the election results, the current government, which had been in power for the past eight years could not win enough seats in the parliament to form a new government. These changes have altered expectations in the economy. For this reason, interpretations of the forecasts should be made with caution.

According to the monthly forecast results, prices in the economy, P, and EXC, as well as money supply M2, will increase at a rate above the average in the near future. Real deficit (REXPEN–RREVE) will worsen due to a slight increase in

REXPEN and a fall in RREVE. Similar to the earlier forecasts, the model predicts a turning point in total imports (11.7 % decrease in 1991). Considering the forecasted 68 % increase in EXC and almost zero growth in MIPI, forecasted decline in total imports should not be surprising. As far as we know, other published forecasts and the State Planning Organization Program Forecast [UTAF, Main Economic Indicators, May 1991, p. 39] do not indicate any decline in total imports. This expected decline in total imports, together with a 14 % increase in total exports, signals a decrease in the foreign trade deficit.

For REXPEN (Real Government Expenditures), the 1991 forecast is 5,120.2 billion TL, which is a 4.3 % increase from the previous year. On the other hand, RREVE (Real Government Revenues) is expected to decline 2.1 %. (from 4,095.2 billion TL in 1990 to 4,010.6 billion TL in 1991). This real increase in the fiscal deficit indicates a debt increase of the public sector, and consequently a credit squeeze for the private sector.

Money Supply, M2, and the Average Wholesale Price Index, P, are forecasted to increase 64 % and 59 % respectively in 1991. The exchange rate between the TL and the U.S. dollar is also expected to increase 68 %.

The Manufacturing Industrial Production Index (MIPI) forecast for 1991 shows a turning point from the realized value of the previous year. Contrary to the previous high growth rates of production, the 1991 forecast indicates a 1.1 % decline. Nevertheless, the Registered Job Seekers Index ISSIZI is forecasted to decline a 6.3 % in 1991.

The overall picture drawn from the forecast results of different models is one of higher inflation, higher devaluation, a worsening fiscal deficit, and a slowdown in production in 1991. Even though external trade forecast indicates an improvement in the balance of payments, the projected deficit ($ 4.9 billion) is well above the average deficit of the last five years.

Table 4.6 Unconditional Forecast Results (FBVAR)

Variable	1991:2	1991:3	1991:4	1992:1
REXPEN	1,181.8 (±358.8)	1,167.0 (±548.7)	1,611.8 (±788.7)	1,035.4 (±520.2)
M2	83,553.3 (±3,042.2)	97,145.5 (±5,749.1)	112,656.0 (±8,719.9)	129,177.0 (±11,972.3)
P	2,060.3 (±59.5)	2,245.0 (±113.4)	2,554.9 (±186.9)	2,919.8 (±290.6)
EXC	4,052.6 (±228.1)	4,415.1 (±397.3)	4,814.8 (±553.7)	5,369.5 (±730.6)
TM	4,227.1 (±928.8)	4,699.4 (±1,878.8)	5,840.2 (±2,494.3)	4,900.9 (±2,127.9)
TX	3,300.9 (±515.1)	3,355.4 (±1,101.7)	4,733.0 (±1,736.6)	3,939.5 (±1,516.2)
MIPI	131.9 (±16.3)	140.8 (±21.0)	159.2 (±24.8)	148.8 (±23.5)
ISSIZI	124.7 (±5.8)	120.3 (±10.1)	119.4 (±14.4)	121.2 (±18.3)
RREVE	968.3 (±212.2)	957.9 (±351.2)	1,085.5 (±427.2)	869.0 (±358.1)

1991:x stands for the x quarter of 1991. REXPEN, RREVE, and M2 are in billions of TL. TM and TX are in millions of US dollars. ISSIZI, P, and MIPI 1984=100.0 EXC is TL for 1 US dollar. REXPEN, RREVE, TX, and TM are quarterly sums. M2, EXC, and P are as end of period. MIPI and ISSIZI are quarterly averages. 95 percent confidence intervals are in paranthesis

Table 4.7 Unconditional Forecast Results (FBVAR) *(Growth Rates)*

Variable	1991:2	1991:3	1991:4	1992:1
REXPEN	−2.2	−0.8	5.2	−10.0
M2	21.5	41.3	63.9	14.7
P	28.1	39.6	58.9	14.3
EXC	41.2	53.8	67.7	11.5
TM	−13.2	−13.5	−19.8	−0.3
TX	20.2	17.4	8.5	16.6
MIPI	−5.9	−2.1	0.8	4.8
ISSIZI	−5.2	−7.1	−8.1	−4.9
RREVE	0.0	−2.1	−15.3	13.0

M2, P, and EXC are percentage changes over the end of previous year. Others are percentage changes compared to the same period of previous year.

Figure 4.1 FORECAST OF REXPEN

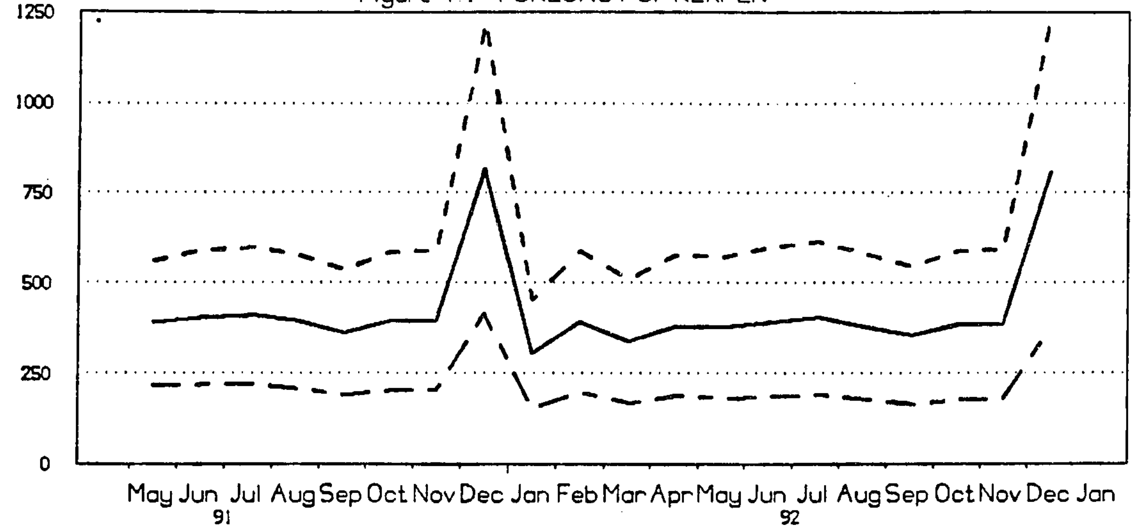


Figure 4.2 FORECAST OF M2

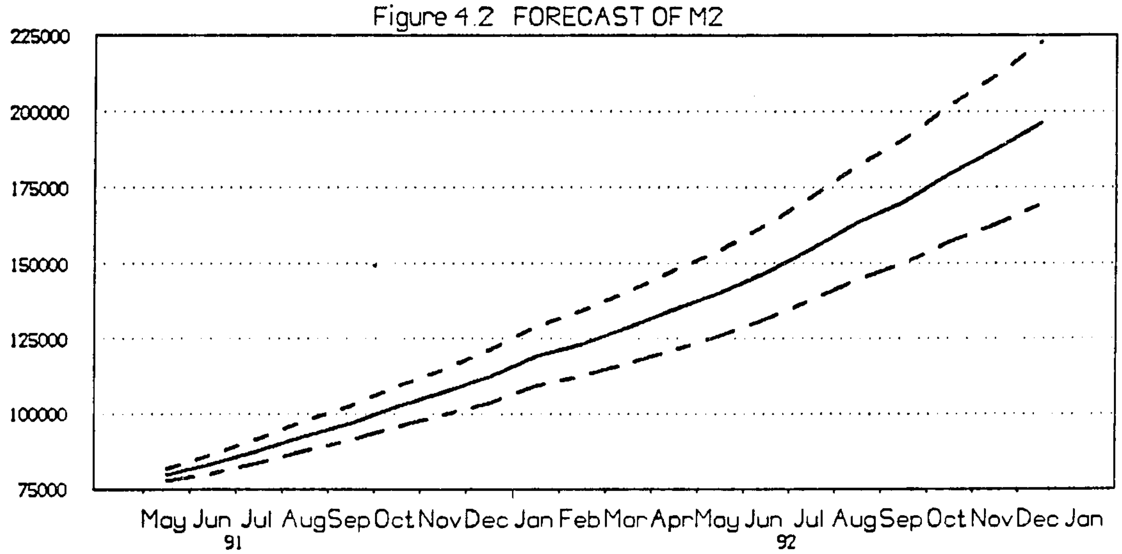


Figure 4.3 FORECAST OF P

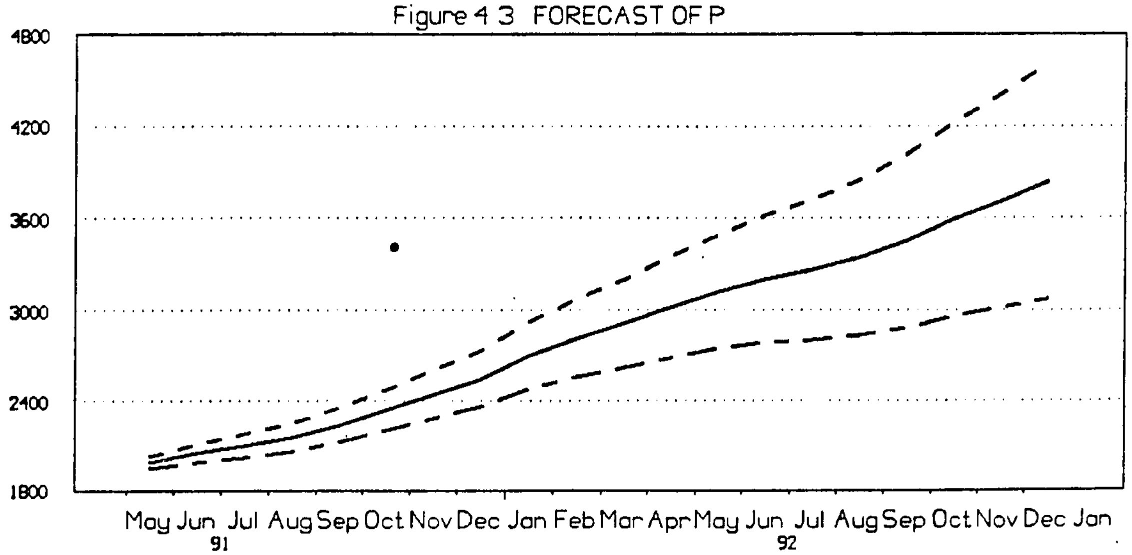


Figure 4.4 FORECAST OF EXC

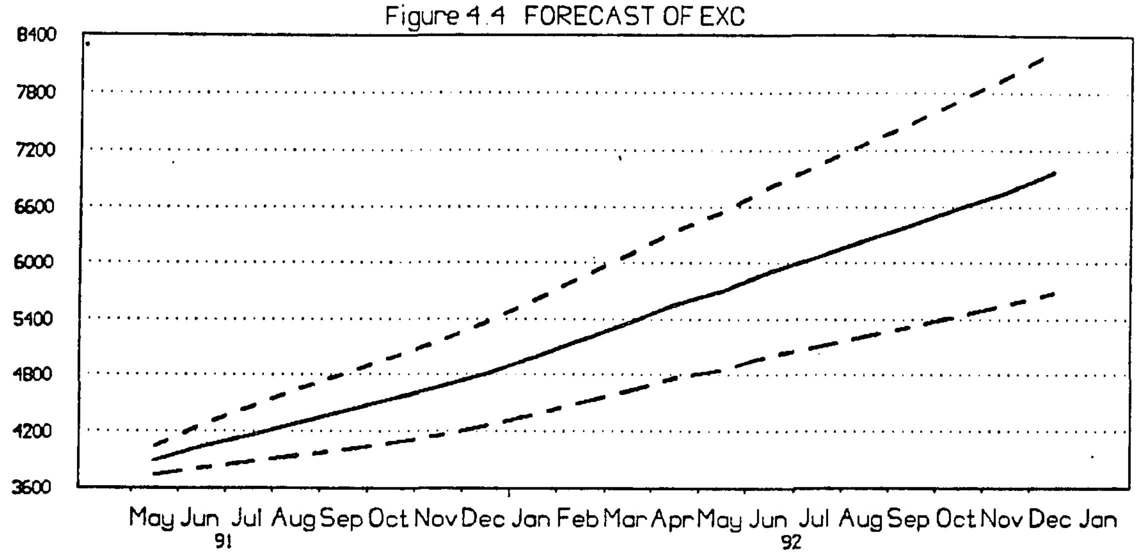


Figure 4.5 FORECAST OF TM

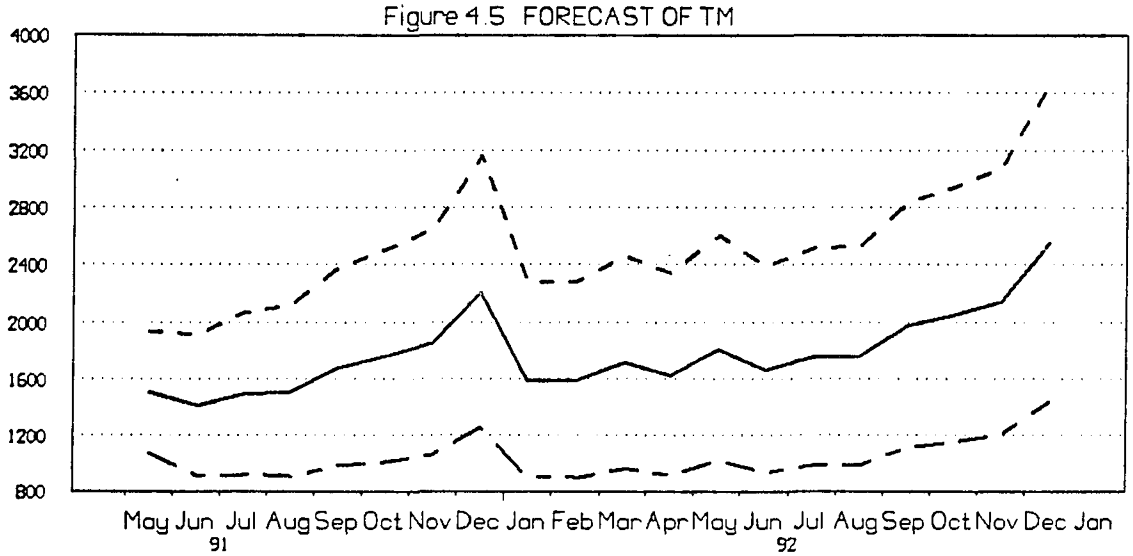


Figure 4.6 FORECAST OF TX

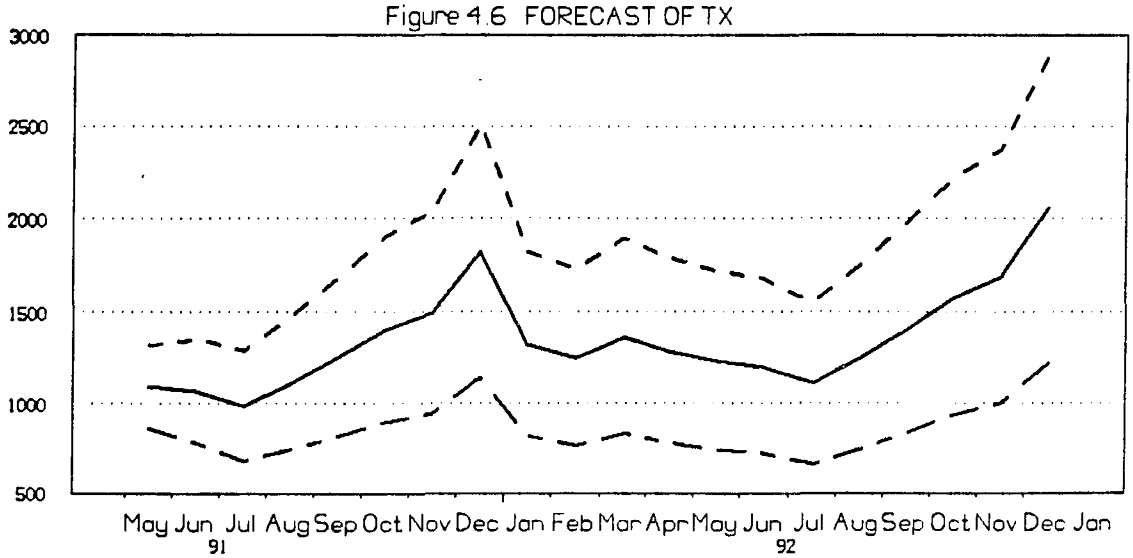


Figure 4.7 FORECAST OF MIPI

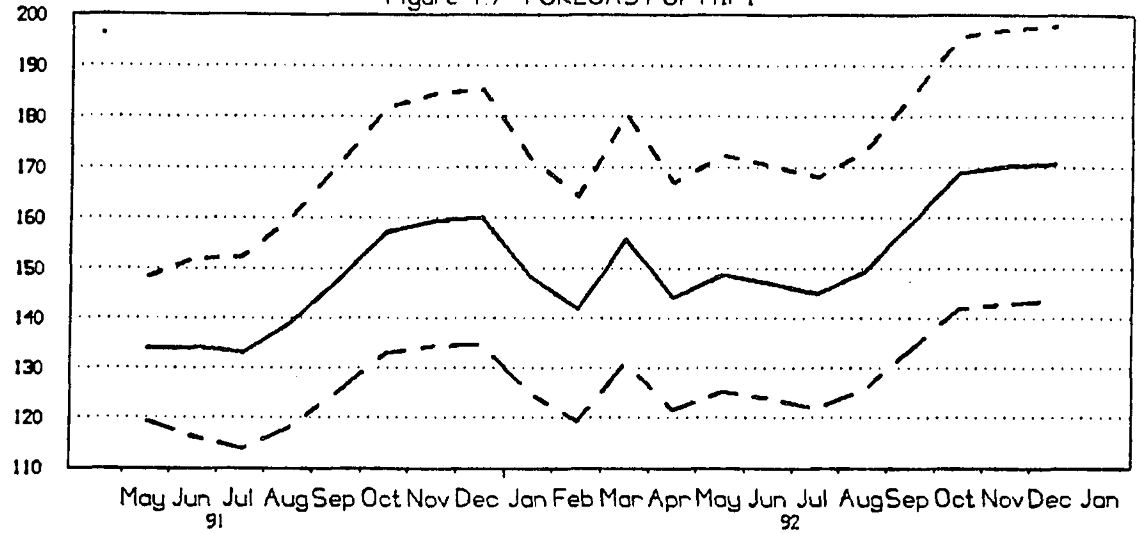


Figure 4.8 FORECAST OF ISSIZI

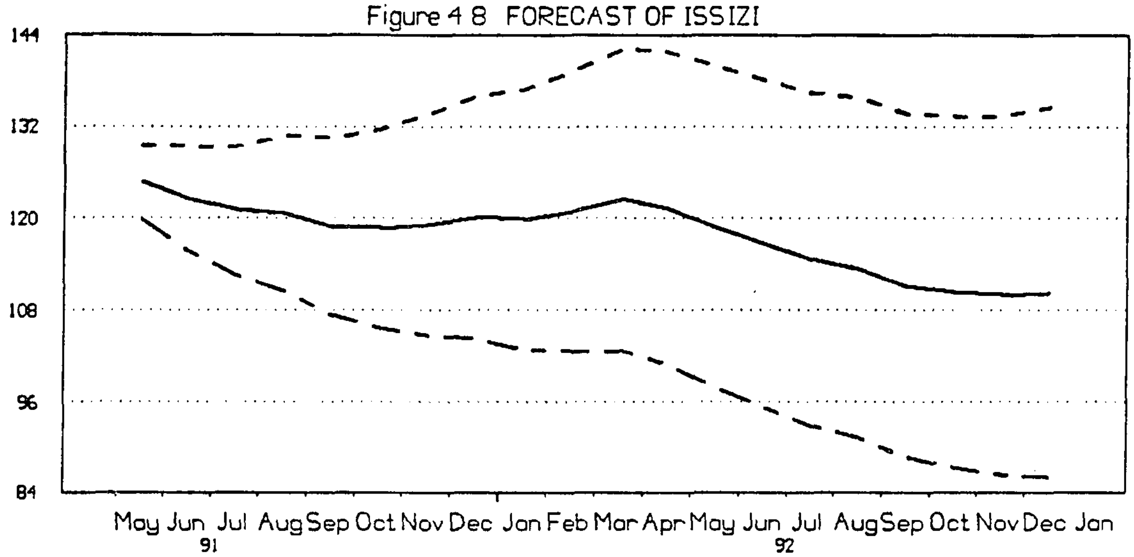
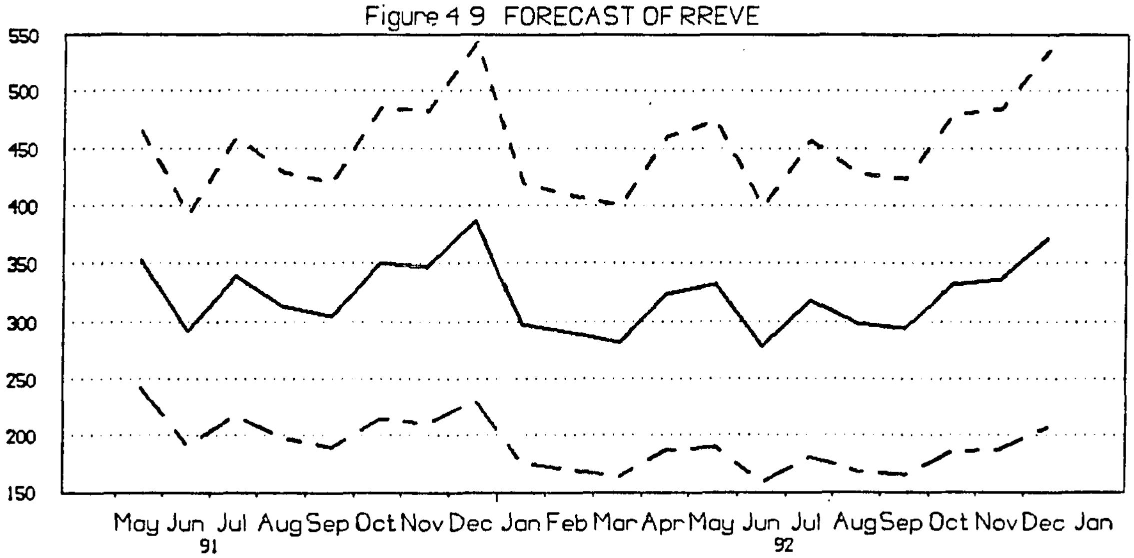


Figure 4.9 FORECAST OF RREVE



Conclusion

In this study, we have attempted to evaluate the performance of different time series models in forecasting major macroeconomic variables in the Turkish economy.

After a brief review of the recent developments in the Turkish economy, time series properties of the selected variables were examined. As a benchmark of comparison a univariate model was defined and forecasts were generated for each variable. Different forecast statistics were calculated using these forecasts. A VAR model and two BVAR models were applied to the same data set and the forecast and forecast statistics were generated in subsequent chapters.

Overall, our results support the idea that there are substantial gains moving from simple univarite models to multivariate models in terms of smaller out-of-sample forecast errors.

In particular, the use of the BVAR model enabled us to incorporate explicit probabilistic theory into the modeling exercise. It also allowed us to assume a time variation in its coefficients, which is more realistic and essential to generate good forecasts for some of the variables.

Two main findings have emerged from the analysis. First, with regard to the out-of-sample forecasts the BVAR model appeared to have more accurate forecasts over long horizons for most of the variables than the univariate and the simple VAR model. Our results point to the superiority of the BVAR model in capturing the important long-run interactions between the variables.

Second, there were minor deteriorations in forecast performances after moving from the simple BVAR to the final BVAR model. It suggests that more accurate forecasts could be obtained further improving the specification of the BVAR model. That will be the object of our future research.

APPENDIX I

Data

In this appendix, the data used in the study is summarized and the data sources are given. All data are monthly unless otherwise noted.

M2

M2 Money Supply. M1+Time deposits-Public Time Deposits. Billion TL. Monthly Statistical Bulletin of the Central Bank of Turkey, June 1991, p.71. No seasonal adjustment. M2 figures used in section 1.2 are taken from the Quarterly Bulletin of the Central Bank of Turkey, 1991:01 p.266. No seasonal adjustment.

M2Y

M2 Money Supply + Foreign Exchange Deposits. Billion TL. Quarterly Bulletin of the Central Bank of Turkey, 1991:01 p.266 (Yearly)

RM

Reserve Money. Currency Issued + Required Reserves + Free Reserves + Other Items. Billions TL. CBMSB, 1991:06 p.38. No seasonal adjustment.

P

Average Wholesale Price Index. 1983:12=100.0 Average of Undersecreteriat of Tresuary and Foreign Trade Wholesale Price Index (30 %), The State Institute of Statistics Wholesale Price Index(40 %), and Istanbul Chamber of Commerce

Wholesale Price Index (30 %). Taken as a proxy to GNP deflator. Monthly Statistical Bulletin of the Central Bank of Turkey, June 1991, p. 87. No seasonal adjustment.

WPI

Wholesale Price Index (P) 12–Month Inflation. See P. WPI=[P(t)-P(t-12)/P(t-12)]*100.0. Yearly figures are 12–month averages. No seasonal adjustment.

CPI

Consumer Price Index 12–Month Inflation. CPIinf(t) = [CPI(t)-CPI(t-12)/CPI(t-12)]*100.0 1979:01–1986:12 period is taken from CPI78 inflation. 1987:01–1990:12 period figures are calculated as 40% CPI78 plus 60% CPI87 inflation. 1991:01–1991:08 period is the same with CPI87 inflation. No seasonal adjustment.

CPI78

The State Institute of Statistics (SIS) Consumer Price Index, 1978–1979=100.0. The series has not been published since 1990:12. CBMSB, 1991:06 p. 77. No seasonal adjustment.

CPI87

The State Institute of Statistics (SIS) Consumer Price Index, 1987=100.0 1986:01–1991:08. CBMSB, 1991:06., p. 77. No seasonal adjustment.

EXC

The Central Bank Exchange Rate. Monthly average of daily exchange rate of US dollar. Yearly figures refer to 12–month average of that year. Monthly Statistical Bulletin of the Central Bank of Turkey, June 1991, p. 86. No seasonal adjustment.

MIPI

Central Bank Manufacturing Industrial Production Index. Quarterly Bulletin of the Central Bank of Turkey, 1991–II, p. 64. No seasonal adjustment.

REVE

Consolidated Budget Revenues. Billions TL. The State Planning Organization Main Economic Indicators, May 1991, p.70. RREVE(t) = [REVE(t)/P(t)] * 100.0

EXPEN

Consolidated Budget Expenditures. Billions TL. The State Planning Organization Main Economic Indicators, May 1991, p.70. REXPEN(t) = [EXPEN(t)/P(t)]*100.0

TX

Total Exports (f.o.b). Million US dollars. The State Planning Organization Main Economic Indicators, May 1991, p. 48. No seasonal adjustment.

TM

Total Imports (c.i.f). Million US dollars. The State Planning Organization Main Economic Indicators, May 1991, p. 39. No seasonal adjustment.

ISSIZI

Job Seekers Index, 1984:06=100.0 The State Institute of Statistics. No seasonal adjustment.

GNP

Gross National Product at Market Prices, The State Planning Organization Main Economic Indicators, May 1991, p. 5. Growth Rates are calculated using GNP at 1968 prices. No seasonal adjustment. (Yearly).

RIR

Real Interest Rate. CBMSB, 1991:06, p.86. No seasonal adjustment. Calculated as 'RIR = [(1.0+R(t)) / (1.0+P(t+12)]-1.0 where R(t) is the highest after-tax annual interest revenue. Yearly figures are 12–month averages.

TERK

Trade Weighted Real Effective Exchange Rate. CBMSB, 1991:06, p.84. Yearly figures indicate the twelve month averages. No seasonal adjustment.

TED

Total External Debt. 1980–1986 figures are taken from *World Debt Tables 1988*, Washington, D.C.: World Bank p. 390. 1987–1990 figures are from *Main Economic Indicators 1991:05*, p.107 Undersecretariat of Treasury and Foreign Trade: Ankara

PSBR/GNP

The Ratio of Public Sector Borrowing Requirements to GNP. 1980–1987 figures are from. 1988–1991 figures are from *Main Economic Indicators 1991:05*, p.69. Undersecretariat of Treasury and Foreign Trade: Ankara

Table A.1 Summary Statistics of the Data Set *(Level)* 1979:01–1991:04

Variable	Mean	STD. Error	Minimum	Maximum
REXPEN	281.8	119.88	88.50	780.6
M2	14,192.0	19,615.0	335.8	77,320
P	368.7	465.1	14.26	1923.6
EXC	833.1	918.3	25.00	3794.5
TM	984.9	416.1	272	2452.4
TX	640.9	319.2	146.3	1,663.1
RREVE	244.3	60.9	117.0	477.73
MIPI	103.81	24.8	58.2	166.0
ISSIZI	96.8	47.1	20.1	152.2

REXPEN, RREVE, and M2 are in Billions of TL. For P, MIPI, and ISSIZI; 1984=100. TX and TM are in millions of US Dollars. EXC is TL for 1 US Dollar.

Table A.2 Summary Statistics of the Data Set *(Growth Rates)* 1979:02 – 1991:04

Variable	Mean	STD. Error	Minimum	Maximum
REXPEN	0.34	50.62	−101.0	130.4
M2	3.70	2.2	−2.3	12.2
P	3.34	2.36	−1.07	21.8
EXC	3.42	4.79	−0.60	49.00
TM	1.10	23.30	−58.22	66.6
TX	1.13	17.17	−66.48	38.7
RREVE	0.13	24.00	−81.6	51.9
MIPI	0.40	8.51	−22.00	18.6
ISSIZI	1.1	4.78	−0.60	50.1

Average monthly growth rates as percentages. Calculated as the log differences of the realized levels of the variables.

APPENDIX II

Application of the Kalman Filter

In this appendix, the coefficient update equation and the Kalman gain are derived for the model in section 3.2.

Given the observation equation (3.15) and the transition equation (3.20)

$$X_t = A_t(L)X_{(t-1)} \; + \; C_t \; + u_t \tag{A2.1}$$

$$\alpha_t = \pi_1 \alpha_{(t-1)} \; + \; (1 - \pi_1) * a \; + \; \eta_t \tag{A2.2}$$

let $\hat{\alpha}_{t+1}$ be the 'best' estimator of α_{t+1} using all the information available at time $t+1$

$$\hat{\alpha}_{t+1} \;\; = \;\; E(\alpha_{t+1} \; | I_{t+1}). \tag{A2.3}$$

Our information set I_{t+1} consists of all the observations available at time $t+1$ of data vector X.

$$I_{t+1} = (X_{t+1}, X_t, X_{t-1}, \ldots, X_0). \tag{A2.4}$$

Let e_{t+1} denote the one step ahead error in predicting X_{t+1} from the point t. Observing X_{t+1} is equivalent to observing e_{t+1}, so the information set (A2.4) can be written as

$$I_{t+1} = (e_{t+1}, X_t, X_{t-1}, \ldots, X_0) \tag{A2.5}$$

since e_{t+1} and I_t are orthogonal by assumption,

$$\hat{\alpha}_{t+1} \;\; = \;\; E(\alpha_{t+1} | I_t) \; + \; E(\alpha_{t+1} | e_{t+1}). \tag{A2.6}$$

We know that the expected value of α_{t+1} conditional on the information set at time t can be found by the equation (A2.2). Then defining $(\beta_{t+1} e_{t+1})$ as the expected value of α_{t+1} conditional on one step ahead prediction error e_{t+1}, (A2.6) can be rewritten as

$$\hat{\alpha}_{t+1} = \pi_1\hat{\alpha}_t + (1-\pi_1)a + \beta_{t+1}e_{t+1} \qquad (A2.7)$$

where β_{t+1} is the coefficient which minimizes $E(\alpha_{t+1}-\beta_{t+1}e_{t+1})^2$. We can calculate $\hat{\beta}_{t+1}$ in the following fashion

$$\hat{\beta}_{t+1} = \frac{E(e_{t+1}\alpha_{t+1})}{E(e_{t+1}^2)}. \qquad (A2.8)$$

Let Z_{t-1} be the list of right hand side variables in Equation (A2.1). Then

$$\begin{aligned}
e_{t+1} &= [(Z_t\alpha_{t+1} + u_{t+1}) - (Z_t\hat{\alpha}_{t+1})] \\
&= Z_t\alpha_{t+1} + u_{t+1} - Z_t(\pi_1\hat{\alpha}_t + (1-\pi_1)a) \\
&= Z_t(\pi_1\alpha_t + (1-\pi_1)a + \eta_{t+1}) + u_{t+1} - Z_t(\pi_1\hat{\alpha}_t + (1-\pi_1)a) \\
&= Z_t\pi_1(\alpha_t - \hat{\alpha}_t) + Z_t\eta_{t+1} + u_{t+1} \\
E(e_{t+1}^2) &= Z_t\pi_1^2\Sigma_t Z_t' + Z_t\pi_2\Sigma_0 Z_t' + \Omega_t \\
&= Z_t(\pi_1^2\Sigma_t + \pi_2\Sigma_0)Z_t' + \Omega_t \\
&= S_t
\end{aligned} \qquad (A2.9)$$

where Σ_t is the variance-covariance matrix at time t and $\pi_2\Sigma_0$ the variance of the state error η_t. The variance of the one step ahead forecast of the i'th component of X_{t+1} using the data through time t is the i'th column of S_t described above. We are going to call this variance as s_{it}^2 (see equation (3.25)). For the covariance between e_{t+1} and α_{t+1}:

$$E(e_{t+1}\alpha_{t+1}) = E[(Z_t\alpha_{t+1} + u_{t+1} - Z_t\hat{\alpha}_{t+1})\alpha_{t+1}]$$
$$= Z_t(\pi_1^2\Sigma_t \ + \ \pi_2\Sigma_0). \qquad (A2.10)$$

Substituting Equations (A2.9) and (A2.8) into Equation (A2.7) for β_{t+1}, the coefficient update equation (3.31) is

$$\hat{\alpha}_{t+1} \ = \ \pi_1\hat{\alpha}_t \ + \ (1-\pi_1)a \ + \ Z_t(\pi_1^2\Sigma_t + \pi_2\Sigma_0)S_t^{-1}e_{t+1}. \qquad (A2.11)$$

From the last part of (A2.11) the *Kalman Gain* at time t is

$$K_{t+1} \ = \ Z_t(\pi_1^2\Sigma_t + \pi_2\Sigma_0)S_t^{-1}. \qquad (A2.12)$$

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