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MODELS OF OVERLAPPING GENERATIONS

by Jonathan Sampson

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Abstract:

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The dissertation consists of a set of essays discussing various aspects of overlapping generations models. Following an introductory chapter, the essays are abstracted and introduced separately. The introduction reviews literature related to the basic problem of why money (or an alternative fiat asset) exists. Chapter two considers the role of learning in asset bubbles. In the final chapter, we evaluate the kinds of nonlinear dynamics that can emerge when health capital is included in a simple overlapping generations model.

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Chapter One

Introduction: Welfare Consequences of a Fiat Asset

Why should societies "contrive" to value an intrinsically worthless, albeit storable commodity?

Imagine that there exist N persons at all of T dates, each person n receiving an arbitrary endowment $e_{n,t}$ of a fragile good, e.g., chocolate, at time t. Then society as a whole can do no better than to allow a free financial market; that is, the market rate at which person m can trade a unit of $e_{m,t}$ in exchange for n's promise to a unit of $e_{n,t+1}$ will be such that no individual can gain (utility), except by stealing someone else's chocolate. (Pareto-optimality of a closed, competitive market.) Yet, throughout history, people have traded their excess chocolates for shells, gold, etc. It is this paradox which motivated Paul Samuelson (1958 [**2**]) to create a precise model of aggregate consumption-loan behavior.

One possible rationale for the existence of a fiat store of value is that financial markets are not closed in the conventional sense; this is the avenue explored by Samuelson. Rather than allowing everyone to exist all the time, Samuelson built a more naturalistic model in which agents born at different times enjoyed the same three-period lifecycles. They could trade only with those agents with whom they shared at least two periods of existence. Since his agents were homogeneous, they would only trade with agents of a different generation, and the temporal overlap between trading partners would always be exactly two periods. Samuelson's agents had one more characteristic—they only earned income during the first two periods of life. Therefore, all trade assumed the form of middle-aged agents bribing young agents to support them in their retirement years.

Samuelson showed that there exist at least two stationary equilibria in this setup: (a) the Pareto-optimum, in which agents trade at the "biological" interest rate (equal to zero if the population is constant), and (b) a suboptimum, in which the interest rate is negative. To determine which is the relevant or stable equilibrium, Samuelson considers the case of two-period lived agents, a case in which he knows that no financial market is possible. A mindless execution of the mathematical requirements for equilibrium yields the biological rate of interest—suggesting that some implicit assumption is at work. Moreover, Samuelson determines that, due to the infinitely forward-looking nature of his model, there is no determinate nonstationary equilibrium. He generates a numerical simulation in which the first period's rate of interest is suboptimal, and finds that the economy approaches equilibrium (b). So the only stable interest rate is negative: people are willing to pay extra chocolate to avoid retirement starvation. He concludes that, in the absence of a perfect social security system (i.e., a benevolent dictator), the Pareto-optimum can only be upheld by some market driven *quid pro quo* allowing an agent to buy chocolate today with the promise to give someone else chocolate tomorrow. Hence, the spontaneous social contrivance of money. This was the first model of a one-good world in which agents lack a double coincidence of wants. Later models of dynamic monetary economies (e.g., Townsend's "turnpike" model (1980 [5])) feature agents which are separated spatially, rather than temporally. While such models explain the need for ready cash, they do not explain the value of vaulted gold, nor are they adequate for a world in which long-distance trading partners can communicate electronically. Furthermore, Samuelson's contrivance is not "money" in the ordinary (M1) sense, but rather a fiat store of value, for example, the Polynesian "big wheel", which is passed on from generation to generation. As shown by Tirole (1985 [4]), it is precisely this "chain-letter" effect which gives an intrinsically worthless commodity its value.

Why is it that our *laissez faire* solution is less than Pareto-optimal? Does the explanation spring, as Samuelson hints, from the desperate bargaining position of retirees? Or is it in the structure of the problem? Karl Shell (1971 [3]) was the first to unravel this knot. He considered the classic (two-period lived agents) analog of the Samuelson model, minus any of Samuelson's incentives to save. Agents are endowed with a unit of chocolate in both periods of life, and are neutral as to when they consume their chocolates. That is, agents maximize a utility function of the form c_1+c_2, in which chocolate in the first period of life (c_1) is a perfect substitute for chocolate in the second period of life (c_2). Allow that the state institutes a social security system in which all agents, after generation zero, contribute chocolate in the amount c (to the aged) when young, and receive

c (when aged) from the young of the next generation. Clearly, all agents will be neutral toward this arrangement, except for the initial aged, who gain some free chocolate. This is the very definition of a Pareto-improvement, since one agent (or class of agents) gains without hurting anyone else. The Pareto-optimum is achieved when c equals one. Equivalently, if we wish to spread out this free lunch, we could require a transfer of c^1 from the first generation, c^2 ($> c^1$) from the second, and so forth, which is Pareto-optimal so long as the sequence $\{c^i\}$ approaches unity.

Shell proved that the welfare gains of intergenerational transfers arise as a result of the "double infinity" aspect of the overlapping generations framework. He presents an alternative example which illustrates the point quite clearly. Suppose that we require a bed in an infinitely large hotel. But the hotel is occupied, in the sense that all of its infinite number of beds are taken by a corresponding infinity of clients. Nonetheless, if we can persuade all the clients to move over one bed—we have "created" a free bed!

The welfare implications of the existence of fiat assets carry over into a production economy. It turns out that, if capital is the only way for agents to satisfy their hump-savings requirement, they will overaccumulate capital (from the standpoint of dynamic efficiency). (See Diamond (1965 [1].) The indication is that government must issue an appropriate quantity of real-valued consols to substitute for the interest bearing role played by physical assets.

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Chapter Two

Learning by Survival in Rational Bubbles

Abstract

We study the implications of Bayesian learning for nonstationary stochastic bubbles in both partial and general equilibrium settings. Emphasis is placed on a pure exchange, overlapping generations framework.

Introduction

The past decade has seen a renewal of interest in the classic accounts of economic pathology, most notably bubbles [McKay (1841) [9]] and long-lived depressions and booms [Keynes (1935) [8]]. It now appears that certain patterns of aggregate behavior, not fully ascribed to innovations in the production function, may constitute the real effects of potentially independent expectational phenomena, sometimes called animal spirits. Research in this area has focused on a simple class of general equilibrium, rational expectations (RE) models, generally termed models of overlapping generations (OG), after the naturalistic structure which relates their agents in time. The trajectory of a deterministic OG economy follows one among multiple stationary and nonstationary equilibria, depending on the disposition of the initial aged regarding their own expected consumption; the probabilistic case is much richer insofar as uncertainty can arise in any generation, either *ex nihilo* (the classic bubble burst), or, in the more likely account, extraneously (e.g., Azariadis' Delphic prophecies [(1981) [**1**]])— in either case, only because the agents expect it to. Such equilibria can also be classed as (stochastically) stationary or nonstationary, and may be related to their deterministic counterparts. (See, for example, the work of Azariadis and Guesnerie [(1984) [**2**]] on the relation between binary sunspot equilibria and stationary orbits.) Because they achieve multiple equilibria within a pristine neoclassical framework, OG models represent, in principle, a more radical challenge than models invoking aggregate externalities, increasing returns to scale, etc. Above all, these models operate at the very edge of Occam's razor: they provide a simple rationale for the comprehensive reign of expectations.

As in the case of *ad hoc* RE equilibria, the main theoretical disadvantage of all this indeterminacy is the embarrassing wealth of possibilities it provides. One corrective is to narrow this realm *via* learning, applied with some success to stochastically [Woodford (1987) [**13**]], as well as cyclically [Grandmont (1985) [**7**]] stationary equilibria. We are specifically interested in the process whereby happenstance empirical phenomena mature into full-fledged self-justifying superstitions. What are the intermediate steps, if any, and what are the effects of learning itself? These questions seem all the more relevant for nonstationary equilibria, insofar as periodic "lessons" may teach themselves. The goal of this particular study is to apply Bayesian learning to the general case of nonstationary stochastic bubbles.

In what respect can people learn "to believe in" a bubble? We require, at least, some disequilibrium (or "abnormal return") from the standpoint of our fundamental solution, in conjunction with uncertainty as to whether this disequilibrium embraces the first return to a longer-lasting bubble. If, for example, this uncertainty is resolved after the first (that is, disequilibrial) period, and later periods are characterized either by the fundamental solution or by an ever-expanding bubble, then, by a mere rewording, we can posit the existence of a bubble from the first period onward, with the qualification that our bubble has some chance of bursting in period one. Allowing a competitive return on our bubble, we expect this return to lessen if and when the bubble becomes certain. (After period one, agents are no longer due the differential compensation for uncertainty.) This dampening of returns will be a repeated theme of our project. Our (inverse) meaure of confidence in the continued existence of any stochastic bubble is the perceived risk, *ipso facto* the actual risk, of a burst at a particular time. We learn, that is, adjust this risk, by surviving certain extraneous events, the portents of a possible burst. Following this interpretation, we confine our study to stochastic bubbles with confidence levels increasing over time, that is, decreasing probabilities of a burst, and their corresponding dampening effects.

Section 1 presents several instructive examples of evolving stochastic bubbles in *ad hoc* RE setups; the first, an elaborate version of our one-time dampening effect, applies learning to a simple arbitrage model, while the second involves the Cagan [(1956) [4]] model for hyperinflation. Since the bubble is in this case a price bubble, a "burst" is, in fact, a "renewal" as pertains to the currency. Readers less familiar with the basic models involved, or with their extensions to rational bubbles, are advised to review the fuller explanations in Blanchard and Fischer [(1989) [**3**]].

Section 2 extends Weil's [(1987) [**12**]] results on stochastic bubbles in pure exchange OG economies to include learning about, or "to believe in", the survivability of a political regime. A constraint on the Arrow-Pratt measure for relative risk aversion provides the sufficient condition for dampening effects analogous to those in our instructive cases. The section concludes with an appraisal of the issues and complications left untouched by our simplifying framework.

Section 3 outlines some conspicuous directions for future research.

1.Partial equilibria

a.A one-time dampening effect

In our first example, risk neutral agents arbitrage between a stock and a riskless asset. The stock derives its value from both dividends and capital gains. Allowing a constant one-period discount factor β, we have

$$p_t = \beta(E[p_{t+1} \mid I_t] + d_t),$$

where p_t denotes the price of the stock in period t, E is an expectations operator, I_t denotes the common information set at time t, and d_t denotes dividends known at t to be distributed at the end of the period. Then the fundamental solution,

p_t^*, is equal to the expected discounted flow of dividends, $\sum_{i=0}^{\infty} \beta^{i+1} E[d_{t+i} \mid I_t]$, and any other p_t can be described as

$$p_t = p_t^* + b_t.$$

Here, b is our bubble, and it must follow

$$E[b_{t+1} \mid I_t] = \beta^{-1} b_t.$$

The general process for a bursting bubble is

$$b_{t+1} = \begin{cases} (\beta(1-z_t))^{-1} b_t + e_{t+1}, & \text{with probability } 1 - z_t, \\ e_{t+1}, & \text{with probability } z_t, \end{cases} \tag{1}$$

where $E[e_{t+1} \mid I_t] = 0$. Here z_t, the probability of a burst at time t, is a stochastic variable satisfying $0 < z_t < 1$,and the additional variable e exists mainly to allow the formation of new bubbles after a burst. Either z or e can be correlated with unexpected movements in any variable included in the information set.

We imagine that our bubble can burst for one or both of two independent "reasons", and we decompose z_t in the following manner:

$$z_t = z^* + l_t - z^* l_t,$$

where z^* denotes the constant probability of a "causeless" burst, and l denotes the probability of a burst "caused by" the occurence of a discrete extraneous event, for example, a blue moon. We also suppose that

$$l_t = \phi l_t^*,$$

where ϕ denotes the constant probability of a blue moon occuring, and l^* denotes the probability that a particular blue moon will "cause" a burst.

Allow that, for any bubble, the appearance of a blue moon will either always cause a burst, or else never cause a burst. Furthermore, assume that agents know the *a priori* probability, l^{**}, that their own bubble is blue moon sensitive; but they can only be sure after surviving (or not surviving) a single blue moon. Then the relevant event is not any blue moon, but the first blue moon (FBM). Allowing the period when the FBM occurs to be denoted t_{FBM}, the values of l^* obey:

$$l_t^* = \begin{cases} l^{**}, & \text{for } t < t_{FBM}, \\ 0, & \text{for } t \geq t_{FBM}, \end{cases}$$

given $b_{t_{FBM}} > e_{t_{FBM}}$ (no burst). If the agents survive their FBM, the probability that a blue moon will lead to a burst is adjusted from l^{**} to 0. Implicitly, the return to the bubble must reflect this lowered probability. In this instance, the expected gross return to the bubble, conditional on survival, drops from $(\beta(1-z^*)(1-\phi l^{**}))^{-1}$ to $(\beta(1-z^*))^{-1}$, a net loss of $\phi l^{**} \times 100\%$. The temporal distribution of this dampening effect is identical to the distribution for t_{FBM}, that is

$$\Pr[t = t_{FBM}] = (1-\phi)^{t-1}\phi.$$

In contrast to our simple example, the many blue moons, sunspots, witching hours, etc., which may infest a financial market are hardly Bernoulli processes. Indeed, they may enjoy a considerable complexity of auto- and cross- correlations. The dampening effect of surviving a sunspot must be balanced against the "moonspots" that might follow in its wake. Clearly, we are only scratching the surface.

b.Continual dampening

Our second example is also intended to be indicative, rather than descriptive, of a real world phenomenon.

In its simplest version, the Cagan money demand function can be written

$$p_t = \beta E[p_{t+1} \mid I_t] + (1-\beta)m_t,$$

where, in this case, p_t and m_t denote the natural logarithms of the price level and nominal money supply, repectively, at time t. We define β by

$$\beta \equiv \frac{\alpha}{1+\alpha},$$

where α denotes the absolute value of the constant elasticity of real money demand with respect to expected inflation.

We depart from Cagan in assuming rational, rather than adaptive, expectations. Our fundamental solution is then given by

$$p_t^* = (1-\beta)\sum_{i=0}^{\infty} \beta^i E[m_{t+i} \mid I_t],$$

and all other solutions are described by

$$p_t = p_t^* + b_t,$$

where b must again follow

$$E[b_{t+1} \mid I_t] = \beta^{-1} b_t.$$

Our bursting bubble process is identical to (1), with the exception that the sequence $\{z_t\}$ is now deterministic. This sequence will reflect a systematic pattern of Bayesian adjustments to an underlying risk.

Before proceeding further, we must clarify the meaning of a "burst". It sometimes happens that a monetary authority will recall the currency— to change several letters in its name, perhaps, or to lop off some decimal places. In this example, we interpret such an event as a discrete political sunspot, which, excepting noise (e), eliminates the price bubble. The fundamental solution is temporarily revived, and money, revalued, so as to assure the noninflationary level of real balances. In a more general setting, such a sunspot would have a probability much lower than one of effecting this kind of change. Our example is clearly contrived so as to demonstrate a particular type of learning.

The monetary authority is randomly selected from a binary distribution of "interventionist" and "noninterventionist" authorities. Agents don't know the nature of their own authority, but they start out with an initial *a priori* probability π_0 that it is interventionist; if it is, they face a fixed probability σ that the price bubble will be destroyed in any period. (For noninterventionist authorities, that probability is zero.)

We denote by π_t the posterior probability that the authority is interventionist as of t periods. Then the joint probability of interventionism and bubble survival (in period t) is $(1-\sigma)\pi_t$, while the total probability that the bubble survives equals $1-\sigma\pi_t$. According to Bayes theorem, the posterior probability that the authority is interventionist in $t+1$ (conditional on surviving t) is given by

$$\pi_{t+1} = \frac{(1-\sigma)\pi_t}{1-\sigma\pi_t}. \tag{2}$$

Since

$$z_t = \sigma\pi_t,$$

we can substitute z for π in (2) to get

$$z_{t+1} = (1-\sigma)(z_t^{-1} - 1)^{-1}.$$

The derivative of any z with respect to its value one period earlier is both positive and increasing (in the range from zero to one). So the sequence $\{z_t\}$ is decreasing toward the limit $z_\infty = 0$.

The expected return of the bubble is subject to continual dampening effects, and assymptotically approaches β^{-1}.

2.General equilibrium

a. A stochastic process.

We now turn to a different kind of sunspot effect, namely, the collapse of a political regime. It is well instantiated that a fiat money is subject to the risk of political default. Our national example is the Confederate dollar. Allow that agents expect a complete devaluation of the currency to follow a regime collapse, and that this expectation is fully self-justifying. Then our confidence in the monetary bubble is linked to the survivability of the regime,and events which strengthen or weaken the regime must be reflected in the return to money. Learning occurs intermittently as political events unfold.

We define a stochastic variable ϵ, i.i.d. with differentiable c.d.f. $G : \mathbf{R} \rightarrow (0,1)$, $G' > 0$, to be a proxy for the level of "social upheaval". Then we posit the existence of a value μ, which is taken to represent the minimal ϵ required to collapse the regime. This threshold is unknown by our economic agents, who face an *a priori* distribution for μ with differentiable c.d.f. $F : \mathbf{R} \rightarrow (0, 1)$, $F' > 0$. Learning occurs as progressively higher levels of ϵ cause agents to rule out possible values of μ.

For each value ϵ_t greater than all preceding ϵ ($\epsilon_t > \epsilon_i \; \forall 0 < i < t$), there is an *a posteriori* distribution F_t such that

$$F_t(x) = \begin{cases} 0, & \text{for all } x \leq \epsilon_t, \\ \frac{F(x)-F(\epsilon_t)}{1-F(\epsilon_t)}, & \text{for all } x > \epsilon_t, \end{cases} \tag{3}$$

given $\mu > \epsilon_t$ (no regime collapse). The maximal ϵ experienced determines the range over which μ cannot exist, given regime survival. Positive probabilities for μ are then defined by F <u>conditional on</u> $\mu > \epsilon_t$. Allowing

$$H_t \equiv 1 - F_t,$$

we can rewrite (3) as

$$H_t(x) = \begin{cases} 1, & \text{for all } x \leq \epsilon_t, \\ \frac{H(x)}{H(\epsilon_t)}, & \text{for all } x > \epsilon_t, \end{cases} \tag{4}$$

given $\mu > \epsilon_t$.

Denote by q the probability of survival (no regime collapse). Then the posterior probability of survival in $t+1$ (conditional on survival in t) is given by

$$q_{t+1} = \int_{-\infty}^{\infty} H_t(\epsilon) dG(\epsilon); \tag{5}$$

i.e., the probability of survival is the chance that ϵ_{t+1} will fall in the "survival range". We can substitute (4) into (5) to decompose q_{t+1} as

$$q_{t+1} = G(\epsilon_t) + \int_{\epsilon_t}^{\infty} \frac{H(\epsilon)}{H(\epsilon_t)} dG(\epsilon).$$

Since $H' < 0$ (follows from $F' > 0$), the term $\frac{H(\epsilon)}{H(\epsilon_t)}$ is less than one for all $\epsilon > \epsilon_t$. Our decomposition shows that q increases assymptotically toward one as the maximal ϵ stochactically approach infinity.

Denote by t_i the period in which a maximal ϵ is experienced for the i^{th} time, and denote by k_i the interval $t_{i+1} - t_i$. Then, given ϵ_{t_i},

$$\Pr_{\epsilon_{t_i}}[k = k_i] = G(\epsilon_{t_i})^{k-1}(1 - G(\epsilon_{t_i}));$$

the occurence of the next maximal ϵ is a Bernoulli process. So

$$\begin{aligned} E(k \mid \epsilon_{t_i}) &= \sum_{j=1}^{\infty} G(\epsilon_{t_i})^{j-1}(1 - G(\epsilon_{t_i}))j \\ &= \frac{G(\epsilon_{t_i})}{1 - G(\epsilon_{t_i})}; \end{aligned}$$

moreover,

$$\lim_{\epsilon_{t_i} \to \infty} E(k \mid \epsilon_{t_i}) = \infty.$$

Therefore, the intervals between Bayesian adjustments to q approach infinity alongside the maximal ϵ. The probability of survival enjoys progressively longer intervals of stability.

b. Aspects of the model

We wish to consider the role of learning in a growing, pure exchange economy. Following Weil, we assume an infinite sequence of overlapping, homogeneous, two-period-lived agents. The rate of population growth is constant at $n \times 100\%$ per period. Each agent maximizes a time-separable, discounted von Neumann-Morgenstern expected utility function with respect to consumption in the two periods of life. The agents receive identical portions of "manna from heaven", or, equivalently, supply labor inelastically over identical career life-cycles. The stream of endowments or earnings is given as e_1 for the first period of life, and e_2 for the second. By assumption, $0 < e_2 < e_1$.

Since the model is designed to supress financial markets, agents' only hope of smoothing their consumption is to pass on the stochastic real monetary bubble, b, to the next generation. Given a constant subjective discount factor β, any possible b must satisfy the recursive relation

$$u'(e_1 - b_t) = \beta(1+n)q_{t+1}\frac{b_{t+1}}{b_t}u'(e_2 + (1+n)b_{t+1}). \tag{6}$$

This relation states that an agent born at t equates expected marginal utilities from the first and second periods of his life (t and $t+1$). u', $u' : \mathbf{R}^+ \rightarrow \mathbf{R}^+$, $u'' < 0$,denotes the marginal utility to any agent of consumption in either period, and q_{t+1} denotes the posterior probability that the bubble will burst, given the t^{th} agent's information set $\{\epsilon_j : 0 \leq j \leq t\}$. (Unless ϵ_t is maximal, $q_{t+1} = q_t$.)

The value of the monetary bubble at $t+1$, conditional on regime survival in t, is known with certainty by agents born at t to be b_{t+1}; to satisfy a like expectation of agents born at $t-1$, we must allow that agents born at t take b_t as given. So agents of generation t apply (6) to determine the value b_{t+1}, given b_t and q_{t+1}. If we allow that $b_{t+1} \mid b_t$ can be written as a differentiable function $B(q_{t+1})$, $B : (0,1) \rightarrow \mathbf{R}^+$,then equation (6) implies

$$\frac{\partial q_{t+1}B(q_{t+1})u'(e_2 + (1+n)B(q_{t+1}))}{\partial q_{t+1}} = 0.$$

$B(q_{t+1})u'(e_2 + (1+n)B(q_{t+1}))$ and q_{t+1} are both positive. So it follows from the product rule that

$$B'(q_{t+1})[u'(e_2+(1+n)B(q_{t+1}))+(1+n)B(q_{t+1})u''(e_2+(1+n)B(q_{t+1}))] < 0. \quad (7)$$

The return on b is dampened by increasing confidence whenever $B'(q_{t+1}) < 0$. Dividing through (7) by $B'(q_{t+1})u'(e_2 + (1 + n)B(q_{t+1}))$, we can say that dampening occurs if

$$1 + \frac{(1+n)B(q_{t+1})u''(e_2 + (1+n)B(q_{t+1}))}{u'(e_2 + (1+n)B(q_{t+1}))} > 0. \quad (8)$$

Allow

$$x_{t+1} \equiv e_2 + (1+n)B(q_{t+1}).$$

Since u'' is negative, and e_2 is positive, it follows that

$$\frac{(1+n)B(q_{t+1})u''(e_2 + (1+n)B(q_{t+1}))}{u'(e_2 + (1+n)B(q_{t+1}))} > \frac{x_{t+1}u''(x_{t+1})}{u'(x_{t+1})}.$$

So a sufficient condition for (8) is

$$1 + \frac{x_{t+1}u''(x_{t+1})}{u'(x_{t+1})} \geq 0. \quad (9)$$

(9) implies that increasing confidence will always dampen returns so long as

$$-\frac{xu''(x)}{u'(x)} \leq 1. \quad (10)$$

A sufficient condition for dampening effects is that the Arrow-Pratt measure for relative risk aversion be less than or equal to one. This criterion is met by such widely used utility functions as the logarithmic and the Cobb-Douglass.

On the "flip side" of our argument is the implication that bubbles will increase with increasing confidence, if the agents are sufficienly risk averse. This apparently perverse effect becomes intuitive when we consider that such agents must be "lured" into our bubble (*via* increasing confidence). Nonetheless, we will restrict our attention to those cases where (10) is inviolable. Interestingly, (10) is also a sufficient condition for the interest elasticity of savings to be positive. So we have effectively eliminated the possibility of a backward-bending offer curve.

c. Possible equilibria.

We seek to proceed deductively, that is, to rule out all but one class of possible equilibria. In doing so, we hope to identify this class with a well-defined tendency or characteristic.

An expanding bubble b, $b_{t+1} > b_t \; \forall t$, is only possible if dampening effects are sufficiently large and continual so as to rule out a bubble that eventually exceeds e_1. (We implicitly invoke Weil to rule out expanding bubbles for stationary q.) The intervals k between dampening effects are themselves stochastic, with range $[1, \infty)$. Moreover, $E(k)$ is increasing (stochastically) with time. Therefore any expanding bubble will grow beyond economic capacity with a positive probability in a finite period of time.

We now consider a bubble which is stationary ($b_t = b_{t+1}$) for any length of time. Call this stationary value M'. To satisfy (6), our bubble must be stationary over an interval during which no adjustments to q occur; call the stationary value of q Q'. Allow that this interval is not identical to the one-period initial interval during which no change in q has <u>ever</u> taken place. Denote by M and Q the values of b and q which hold in the period immediately preceding the stationary interval. Then (6) implies the system:

$$u'(e_1 - M) = \beta(1+n)Q\frac{M'}{M}u'(e_2 + M') \tag{11}$$

$$u'(e_1 - M') = \beta(1+n)Q'u'(e_2 + M').$$

Dividing the upper equation in (11) by the lower equation gives

$$\frac{u'(e_1 - M)}{u'(e_1 - M')} = \frac{M'Q}{MQ'}. \tag{12}$$

Since $Q' > Q$, $M' < M$ (dampening effect in a nonexpanding bubble). $u'' < 0$, therefore $u'(e_1 - m)$, $0 < m < e_1$, is increasing with respect to m. So

$$\frac{u'(e_1 - M)}{u'(e_1 - M')} > 1 > \frac{M'Q}{MQ'},$$

ruling out (12).

The elimination of expanding and stationary bubbles implies $b_{t+1} < b_t \ \forall t$— a monetary bubble must decline. Consider equilibria in which the nominal stock of money is either constant or growing. Such equilibria will invariably manifest permanent hyperinflation, compounded by a random stream of positive price shocks corresponding to the record levels of "social upheaval".

d.Complications

We have emphasized the continuance of a political regime as a natural determinant of monetary confidence. Added complexities may contribute vitally toward a more realistic scenario. For example, inflation may aggravate social conditions, implying that the distribution of our upheaval proxy be positively related to the parameter for inflation. Such an assumption would speed the evolution of our economy: the regime is "tested" more severely as inflation yields civil unrest, which in turn (by the dampening effect) yields a span of inflation. An "outside" observer could say that inflation is self-generated, or, equivalently, that the ϵ are autocorrelated.

We have thus far ignored the abiding issue of monetary policy. In the case of a bubble, policy considerations are complicated by the insight that an unexpected shift in the supply (or nature) of the currency is itself a sunspot effect, potentially as profound as a regime collapse (although not, necessarily, in the same direction). This is in addition to the non-sunspot effects of monetary disturbances, which could pose a substantial source of risk.

3. Concluding remarks

There are many variations on the line of thinking contained in section 2. Presented here are a few of the more obvious:

(a) The existence of bubbles on several intrinsically valueless assets; we are especially thinking of an alternative investment, such as gold, which allows agents to hedge against political default.

(b) The application of learning to a production economy à la Diamond [(1965) [**5**]]. This is the most natural context for a discussion of dynamic efficiency. To what extent might dampening effects alleviate the technical problem of a bubble which overreaches the host economy?

(c) A model with overlapping generations of long-lived agents, something akin to the seminal work by Samuelson [(1958) [**10**]]. Agents could contract loans with each other at the "biological" interest rate, and also buy into an evolving stochastic bubble. This setup would allow us evaluate shifts between the money and financial markets.

We would like to conclude with a note of caution. The indication, given earlier, that learning would "narrow our spectrum" was not entirely truthful. If anything, we have enriched our abundance— although, in giving it form, we have ruled out some distinct possibilities.

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Chapter Three

The Dynamics of Collective Aging: Cycles, Growth, and the Demand for Health

Abstract

Health capital is introduced in a dynamic equilibrium setting. Limit cycles exist on per capita output when aggregate output is fixed. The example is extended to allow for exogenous growth.

I. Introduction

The continued growth of the U.S. health sector prompts the responsible specialist to consider its interaction with the economy as a whole. The competition between investment in life extension versus the future productivity of youth is particularly important *vis-a-vis* the conjecture that our standard of living is declining below some trend. Is this conjecture reasonable, and, if so, does it consist in a self-correcting disequilibrium, a long-term tendency, or the first indication of an expanding orbit? To answer these questions, we require a framework wherein the diverse ends of contemporaneous generations can compete directly for society's resources.

The overlapping generations (OG) framework derives from Samuelson's description of intergenerational consumption-loan behavior in a dynamic general equilibrium setting (1958[**7**]); this work provided one of the first motivations for a fiat store of value. Endogenous cycles were observed as early as Gale (1973[**3**]), and were analyzed further by Grandmont (1985[**4**]), who showed that OG cycles result when intertemporal substitution is overpowered by a life-long income effect. These cycles are characterized by varying consumption accross generations under stationary output, but can be generalized to allow for fluctuations in GNP. (For related examples, see Farmer (1986[**2**]), Reichlin (1986[**6**]).)

Through endogenous mortality—specifically, by allowing the population of retirees to fluctuate—we afford limit cycles in per capita income while positing a stationary total GNP. These cycles are motivated by endogenous fluctuations in the demand for health over time. We specialize the notion of health capital to include only its effects on mortality, bearing in mind that the impact of health on the supply of labor (in efficiency units) is qualitatively no different from other types of human capital. By tying health exclusively to mortality, we also avoid incorporating remedial health care, or any notion of health that is not explicitly preventive. The mortality aspect of health capital was included in the seminal literature on health (Grossman (1972[**5**])), and was later extended by Ehrlich and Chuma (1990[**1**]), in the form of a deterministic stopping problem for long-lived agents, an inappropriate approach for the classic (two-period-lived agents) version of the OG model. We therefore adopt uncertainty into our model, varying the probability of survival to retirement as a function of health investment in youth.

In addition to highlighting periodic phenomena, we hope to evaluate specific attributes of the path of health capital under increasing output. Can we define an overall pattern of collective aging as a response to economic growth? We develop a three-stage scenario in which growth sustains a noncyclical balance between the health and nonhealth sectors, a balance which decays as growth declines.

Section II describes the simplifying assumptions of our model; section III develops the model and presents intended applications; section IV applies the model to obtain results; and section V concludes with a note on countercyclical policy.

II. Physical setup

We posit an infinite sequence of homogeneous, overlapping, two-period-lived agents. Since meaningful financial markets are ruled out by the structure of the model, the only store of value is a stock of fiat money which is passed on from older to younger generations in exchange for a consumption good. (Note that, in the place of money, we could substitute gold, art, consols, or any other asset on which a Tirolian bubble (1985 [8]) might exist.)

Agents supply labor inelastically when young, producing and receiving one unit of output which they divide between direct investment in health capital and the purchase of money from the coexistent older generation. The stock of health capital determines agents' probability of survival to old age. When and if aged, agents consume output of the current young in exchange for money saved.

In order to avoid accidental bequests, we posit the existence of a competitive insurance company, which receives the savings of nonsurvivors and distributes them among survivors of the same generation. Actuarial fairness requires that expected consumption equal savings. The law of large numbers is implicitly invoked.

III. The model

Agents of generation g maximize

$$\pi(h_g)V(0) + (1 - \pi(h_g))V(c_g)$$

subject to the budget constraint

$$(1 - \pi(h_g))c_g = R_g(1 - h_g),$$

where $\pi, \pi : R^+ \rightarrow (0,1), \pi''(x) > 0 > \pi'(x) \, \forall x > 0$, denotes the probability of an early death, h_g, $1 > h_g > 0$, denotes the agent's stock of health capital, $V, V : R \rightarrow R, V'(x) > 0 > V''(x) \, \forall 1 > x \geq 0$, denotes a Von Neumann Morgenstern expected utility function, $c_g, 1 > c_g > 0$, denotes consumption by the agent conditional upon survival to the second period of life, and R_g, $R_g > 0$, is the gross rate of return on money facing generation g in a competitive equilibrium. Agents maximize expected utility subject to the condition that expected consumption (when aged) be equal to the future value of money savings.

π defines the probability that an agent will not reach retirement as a function of his health investment, and is assymptotically equal to zero as health tends toward infinity. We implicitly define some minimum chance of an early death $\pi(1)$ not equal to zero; this value is arbitrary, and can be adjusted to suit the periods involved.

The equilibrium condition

$$1 = (1 - \pi(h_g))c_g + h_{g+1}$$

describes the allocation of per capita supply to the expected consumption of the aged and the health investment of the next generation. This condition, together with agents' maximizing behavior, defines the recursive relation $z(h_g, h_{g+1}) = 0$. z is derived for the general case in Mathematical appendix A. Then the trajectory of health capital is determined by z and h_0.

Our solution concept is a proof by example of the existence of limit cycles, *via* a described set of simulated competitive equilibria invoking prototypical functional forms p and u, satisfying the properties of π and V respectively. This concept enjoys a twin rationale. Firstly, we are more concerned with proving that the phenomenon in question is not entirely pathological, than we are with defining the precise conditions for its existence in a highly simplified structure. Secodnly, the existence of a concrete example greatly facilitates an extended application to the case of exogenous growth.

IV. Results

Allow $p(h_g) \equiv \exp(-2h_g)$ and $u(c_g) \equiv 2c_g - c_g^2$.

Then

$$h_{g+1} = m(h_g) \equiv 1-(1-\exp(-2h_g))[1+\exp(-2h_g)(1-\exp(-2h_g))^{-1}(1-h_g)]^{-1}.$$

There is a dynamically unstable stationary equilibrium in the neighborhood of

$$H^* = m(H^*) \doteq .50484882$$

and a limit cycle in the neighborhood of

$$H^{**} = m(H^{***}) \doteq .886579847,$$

$$H^{***} = m(H^{**}) \doteq .188617280.$$

Specifically, if $H^{**} > h_j > H^*$ for any j, then $H^{**} > h_{j+k+2} > h_{j+k} > H^* > h_{j+k+1} > h_{j+k+3} > H^{***}$ for all k even; while if $h_j > H^{**}$ for any j, then $h_{j+k} > h_{j+k+2} > H^{**}$ and $H^{***} > h_{j+k+3} > h_{j+k+1}$ for all k even.

Since

$$I_g \equiv (2 - \exp(-2h_{g-1}))^{-1},$$

where I_g denotes per capita income when generation g is young, our limit cycle can be described by

$$I^{**} \doteq .760893255,$$

$$I^{***} \doteq .546387071.$$

To allow for increasing productivity, while at the same time retaining u, we require that output follow a growth function that is rigged to remain at or below the unitary level. In addition, we prefer a pattern of growth that accords with the empirically sound Kuznets curve, that is, incremental additions to output should first increase, and then decrease, as a function of output. To satisfy these requirements, we opt for the logistic function

$$F_g = \exp(g)(J + \exp(g))^{-1},$$

where F_g denotes output produced by agents of generation g, and J is a positive constant.

As evinced in the following simulations, we observe three major phases in the trajectory of health demand as a response to economic growth: $h_{g+1} > h_g$ for all g in the "growth dominant" phase; if $h_j > h_{j+1}$ for any j, then $h_{j+k+3} > h_{j+k+1} > h_{j+k+2} > h_{j+k}$ for all k odd in the "mixed" phase; if $h_m > h_{m+1}$ for any m, then $h_{m+n+2} > h_{m+n} > h_{m+n+1} > h_{m+n+3}$ for all n even in the "cycle dominant" phase. See Figure 1.

Allow $h_0 = .05$ as a control and vary J. For $J = 10$,$J = 12.5$, and $J = 15$, respectively:

g	F_g	h_g
0	.090909091	.05
1	.213730272	.145217570
2	.424925658	.215449432
3	.667614375	.415561846
.	.	.
4	.845196805	.372664171
5	.936873931	.568624123
6	.975812038	.397119389
7	.990963582	.619933638
.	.	.
8	.996656590	.379365929
9	.998767423	.654286279
10	.999546207	.352303430

0	.074074074	.05
1	.178619496	.101182560
2	.371513664	.235327900
3	.616394229	.310334425
4	.813705743	.472644367
.	.	.
5	.922318348	.419803566
6	.969946779	.558863810
7	.988729938	.427802207
8	.995824227	.589379861
.	.	.
9	.998459753	.412036459
10	.999432823	.614473151

0	.062500000	.05
1	.153416785	.068363177
2	.330029818	.249175647
.	.	.
3	.572473409	.223662180
4	.784477030	.561614999
5	.908208127	.298886239
6	.964151607	.706267689
.	.	.
7	.986506340	.288018617
8	.994993254	.764126957
9	.998152273	.262134516
10	.999319465	.802011442

Society ages steadily until growth is sufficiently decelerated; at some point, aging follows a pattern of "two steps backward for every three steps forward"; finally, the cyclical nature of health demand is fully manifest, and we degenerate toward the limit cycle.

V. Note on countercyclical policy

For the case of stationary output, a permanently stabilizing policy in the above example takes the form of a one time redistribution from health investment of the young to consumption of the aged. Specifically, we apply a lump-sum tax or subsidy of amount τ on generation 0 such that

$$h_0 + \tau = H^*.$$

See Mathematical appendix B for a proof of the Pareto-optimality of the resulting stationary equilibrium.

Economic growth is a destabilizing factor even in the cycle dominant phase. Broadly speaking, stabilization can be defined by a series of interventions enabling

$$H^* \geq h_{g+1} \geq h_g \, \forall g$$

subject to a feasibility constraint.

Mathematical appendix A

We form the Lagrangian expression

$$\mathcal{L} \equiv \pi(h_g)V(0) + (1 - \pi(h_g))V(c_g) + \lambda((1 - \pi(h_g))c_g - R_g(1 - h_g)).$$

First order conditions for a maximum are:

$$\frac{\partial \mathcal{L}}{\partial h_g} = \pi'(h_g)(V(0) - V(c_g)) + \lambda(R_g - c_g\pi'(h_g)) = 0,$$

$$\frac{\partial \mathcal{L}}{\partial c_g} = (1 - \pi(h_g))(V'(c_g) + \lambda) = 0.$$

So,

$$V'(c_g) = \frac{\pi'(h_g)(V(0) - V(c_g))}{R_g - c_g\pi'(h_g)}. \qquad (1)$$

From the budget constraint,

$$R_g = \frac{(1 - \pi(h_g))c_g}{1 - h_g}.$$

Substitute for R_g in eq. 1 for:

$$V'(c_g) = \frac{\pi'(h_g)(V(0) - V(c_g))}{c_g((1 - \pi(h_g))(1 - h_g)^{-1} - \pi'(h_g))}. \qquad (2)$$

From the equilibrium condition,

$$c_g = \frac{1 - h_{g+1}}{1 - \pi(h_g)}.$$

Substitute for c_g in eq. 2 for:

$$z(h_g, h_{g+1}) = V'\left(\frac{1 - h_{g+1}}{1 - \pi(h_g)}\right) - \frac{\pi'(h_g)(V(0) - V\left(\frac{1-h_{g+1}}{1-\pi(h_g)}\right))}{(1 - h_{g+1})((1 - h_g)^{-1} - \frac{\pi'(h_g)}{1-\pi(h_g)})} = 0. \quad (3)$$

Generalizing to the case of growth, we get:

$$z(h_g, h_{g+1}) = V'\left(\frac{F_{g+1} - h_{g+1}}{1 - \pi(h_g)}\right) - \frac{\pi'(h_g)(V(0) - V\left(\frac{F_{g+1}-h_{g+1}}{1-\pi(h_g)}\right))}{(F_{g+1} - h_{g+1})((F_g - h_g)^{-1} - \frac{\pi'(h_g)}{1-\pi(h_g)})},$$

where F_g denotes output produced by agents of generation g.

Mathematical appendix B

To establish a stationary Pareto-optimum, we maximize expected utility subject to a feasibility constraint, in this case identical to the equilibrium condition. We form the Lagrangian expression

$$\mathcal{L} \equiv \pi(h)V(0) + (1 - \pi(h))V(c) + \lambda(1 - (1 - \pi(h))c - h).$$

First order conditions for a maximum are:

$$\frac{\partial \mathcal{L}}{\partial h} = \pi'(h)(V(0) - V(c)) - \lambda(1 - c\pi'(h)) = 0,$$

$$\frac{\partial \mathcal{L}}{\partial c} = (1 - \pi(h))(V'(c) - \lambda) = 0.$$

So,

$$V'(c) = \frac{\pi\prime(h)(V(0) - V(c))}{1 - c\pi'(h)}. \quad (4)$$

From the feasibility constraint,

$$c = \frac{1-h}{1-\pi(h)}.$$

Substitute for c in eq. 4 for:

$$V'\left(\frac{1-h}{1-\pi(h)}\right) = \frac{\pi'(h)(V(0) - V\left(\frac{1-h}{1-\pi(h)}\right))}{1 - \frac{\pi'(h)(1-h)}{1-\pi(h)}}. \qquad (5)$$

Note that eq. 5 is merely a specialization of eq. 3 to the stationary case. Therefore, any stationary equilibrium is Pareto-optimal.

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