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BERENSON, Mark L., 1941-
A MONTE CARLO STUDY OF THE POWER OF SEVERAL
k SAMPLE TESTS FOR ORDERED ALTERNATIVES.

The City University of New York, Ph.D., 1974
Business Administration

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A MONTE CARLO STUDY OF THE POWER OF SEVERAL
k SAMPLE TESTS FOR ORDERED ALTERNATIVES

by

MARK L. BERENSON, M.B.A.

A dissertation submitted to the Graduate
Faculty in Business in partial fulfillment
of the requirements for the degree
of Doctor of Philosophy, The City Uni-
versity of New York.

1974

This manuscript has been read and accepted for the Graduate Faculty in Business Administration in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

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ACKNOWLEDGEMENTS

I would like to take this opportunity to express my appreciation to the following individuals who have contributed to this dissertation. First of all, I wish to express my gratitude to my advisor, Dr. Edward Wolf, for his continuous guidance and encouragement on this research effort. Our many long and fruitful discussions were invaluable. Also, my grateful appreciation to the other members of the Supervisory Committee--Dr. Sidney Lirtzman and Dr. Leon Schiffman, for their helpful suggestions, with a special note of thanks to Dr. Manus Rabinowitz for his time and assistance. In addition, I wish to thank some of the members of the Department of Statistics Educational Computer Center at Baruch College--Ms. Pam Anderson, Mr. L. James Latour, Mr. Ray Rauth--for their assistance, with a special note of thanks to Ms. Shiela Frankel. Finally, I would like to express my appreciation and dedicate this dissertation to my wife Rhoda, for her continuous encouragement and advice, and my children, Kathy and Lori, for their patience and understanding.

Mark L. Berenson

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CHAPTER I

INTRODUCTION

1.1 THE PROBLEM: SELECTING AN APPROPRIATE TEST

The selection of an appropriate experimental design for any statistical research endeavor is of fundamental importance. Two basic designs widely used in the behavioral sciences and in marketing research are the completely randomized design (one-way analysis of variance) and the randomized block design (two-way analysis of variance with one observation per cell). Once the appropriate model or design is selected, however, the researcher is often still confronted with the problem of choosing an appropriate statistical test from among several known test procedures which are applicable to the endeavor. If it is desired to test the null hypothesis of the form

$$H_0: \mu_j = \quad \forall \quad j = 1, 2, \dots, k \quad (1.1.1)$$

against the general alternative of the form

$$H_a: \mu_j \neq \quad (1.1.2)$$

the most widely applied parametric procedures are the One and Two Way F Tests while the most widely used nonparametric procedures are the Kruskal-Wallis Test (1952) and the Friedman Test (1937). If, however, it is desired to test against ordered alternatives of the form

$$H_a: \quad \mathcal{M}_1 \leq \mathcal{M}_2 \leq \dots \leq \mathcal{M}_k \quad (1.1.3)$$

(with at least one inequality strict), little is known as to which procedures are most useful.¹

Through the use of Monte Carlo procedures (which permit an examination of the simulated powers of several k sample statistical tests, both parametric and nonparametric, under a variety of conditions), the purpose of this dissertation then is to investigate the problem of test selection when the experimental design is of the completely randomized or randomized block format. In particular, it is the intent of this research effort to focus on those k sample tests with specified alternatives of the form (1.1.3) in order to provide the researcher with statistical techniques which, when used under appropriate conditions, will prove to be a valuable aid in research.

1.2 SIGNIFICANCE OF THE RESEARCH

It should be noted that some work has been done on the comparison of the asymptotic power functions of the related test procedures as a criterion for selecting between these procedures (for example, see Bartholomew (1961A), Hollander (1967), or Puri (1965)). According to Noether (1955), the theoretical concept was developed by Pitman and appears in the literature as Pitman efficiency or asymptotic relative efficiency (A.R.E.). The concept of asymptotic relative efficiency

¹For a discussion of appropriate criteria for test selection see section 1.5.

is defined as the ratio of the power of one test to that of another under the same level of significance, the same alternative, and for $n \rightarrow \infty$. On the other hand, a more practical concept of "power efficiency" is defined to be the sample size necessary for a weaker test to achieve the same power as the superior test at the same level of significance and for the same alternative (Siegel (1956), pp. 20-21).²

Little has been done, however, to compare the powers of various tests when n is small (Gilbert (1972)). Exact mathematical solutions are often intractable if not impossible. Hence, this dissertation will attempt to make a contribution in this area by comparing the empirical small sample powers of the various tests under a variety of conditions. This study then is intended to fill a void in current knowledge by providing the behavioral scientist and marketing researcher with those techniques most appropriate for analyzing certain types of problems when the researcher can hypothesize or assume the existence of certain conditions pertinent to a particular research endeavor.

1.3 INVESTIGATIONS

The following will be empirically investigated:

1. A comparison of the powers of k sample procedures under two extreme forms of (1.1.3)--that for a strictly linear trend alternative

²For further discussion pertaining to these properties see Appendix A4: "Various Concepts of Power Efficiency."

$$\mu_{.1} < \mu_{.2} < \dots < \mu_{.k} \quad (1.3.1)$$

(with all means equally spaced) versus that for an end gap alternative

$$\mu_{.1} = \dots = \mu_{.k-1} < \mu_{.k} \quad (1.3.2)$$

(with all but one of the means equal).³

2. A comparison of the powers of the k sample tests as the shift parameter, defined in Donaldson (1968, p. 661) as

$$\Delta = \sqrt{\frac{n \sum_{j=1}^k (\mu_{.j} - \bar{\mu})^2}{k \sigma_e^2}}, \quad (1.3.3)$$

approaches zero (the true null hypothesis).

3. A comparison of the powers of nonparametric versus parametric tests under various parent distributions as well as an examination of the robustness of the parametric procedures under violations in normality.

4. A comparison of the powers of the k sample tests as α , n and/or k are altered.

5. The effect on power of the allocation of a fixed number of test objects $N=nk$, over various combinations of n and k.

³Due to the exorbitant amount of computer time required, it was necessary to limit the scope of this dissertation to only two levels of (1.1.3). Thus, the aforementioned two extreme forms of (1.1.3) were selected so that empirical upper (lower) and lower (upper) bounds of power could be obtained for tests designed against trend (gap) when the ordered location alternative is true. In addition, such a selection provides the basis for a comparison with asymptotic results--all of which consider these two extreme forms. However, research will be proposed to investigate the small sample powers of the various test procedures under two intermediate forms of the ordered location alternative--balanced gap and unbalanced gap.

1.4 OBJECTIVES

The following are the objectives of this research effort:

1. To demonstrate to the researcher the utility of k sample tests with ordered alternatives by showing that when the alternative can be specified these techniques are significantly more powerful than the best known k sample tests with unrestricted alternatives under a variety of conditions.
2. To determine through the use of Monte Carlo procedures which k sample tests with ordered alternatives are most useful to the behavioral scientist or marketing researcher under various conditions.
3. To show that some nonparametric k sample procedures, which are quicker and easier to apply and make fewer and less stringent assumptions than their parametric counterparts, are almost as powerful as the parametric tests when the assumptions of the latter are met, and are more powerful when the assumptions of normality and/or homoscedasticity are violated.
4. To show the researcher the effects on power of sample size (n) and numbers of sample groups (k) in the formulation of an experimental design model.
5. To examine the robustness of the parametric tests to violations in the assumption of normality.
6. To demonstrate, through specific behavioral and marketing research applications the value of the obtained results.
7. To point out areas requiring further research.

1.5 CRITERIA FOR SELECTING AN APPROPRIATE TEST

While power is a most important criterion for selecting among alternative procedures, there are other valid criteria which must also be considered. For example, by examining the power of various tests, the researcher will be able to analyze the trade-off between selecting a test with less power for a given sample size or increasing the power of that test by increasing the sample size. Sampling, however, involves costs--monetary as well as time and effort. Moreover, these costs will be different if the observations are being precisely measured as opposed to merely categorized or ranked. Another selection criterion to be considered by the researcher is the computational ease (usually attributed to the nonparametric methods) of the procedure. As this is related to time and effort it too has a cost. Other important selection criteria tied in with the latter are the "comprehendability" of the procedure and the availability of tabled critical values for the test. Last but not least, the selection of the test is also dependent upon the assumptions of the test and the degree of generality desired in the conclusions. If the researcher wants to avoid the more stringent assumptions of the parametric methods a nonparametric procedure should be employed. A similar statement can be made if the conclusions desired are to be more general. While all of these criteria can be evaluated when deciding upon which test to select for a specific research endeavor, they all revolve around the power functions of the

various tests.⁴

1.6 THE PROCEDURE

This dissertation has been divided into seven chapters:

- I. Introduction
- II. Development of k Sample Procedures
- III. k Sample Techniques for a Completely Randomized Design
- IV. k Sample Techniques for a Randomized Block Design
- V. Empirical Small Sample Power Comparisons
- VI. Applications
- VII. Reviewing the Importance of k Sample Tests for Ordered Alternatives.

In Chapter I the basic purpose and focus of this dissertation has been stated along with its importance for behavioral and marketing research. In addition to power various other criteria for choosing among k sample tests were listed.

Chapter II will present the historical development of both parametric and nonparametric k sample procedures, and will demonstrate the need for techniques which are appropriate against ordered alternatives.

⁴The above discussion has assumed that the data for a particular research investigation could be measured in at least the strength of an interval scale and, thus, be amenable to analysis by both parametric and nonparametric methods. However, as data collected in behavioral or marketing research are often not of the strength of an interval scale, the more powerful nonparametric procedures will be an important tool for the researcher. Several articles on both scaling and nonparametric techniques have appeared in the behavioral and marketing research literature (see, for example, Green and Rao (1970), Miller (1956) and Morris (1969)).

Chapter III will describe and examine the nineteen k sample tests for a completely randomized design that have been selected in this research effort (see Table 1.1). Of these nineteen tests, six are parametric and thirteen are nonparametric procedures.

Chapter IV will describe and examine the nineteen k sample techniques for a randomized block design that have been considered in this study (see Table 1.1). Of these nineteen, six are parametric and thirteen are nonparametric procedures.

Chapter V will analyze, in detail, the empirical small sample powers of the aforementioned k sample tests. These powers will be obtained by using Monte Carlo procedures. This will be accomplished under several conditions--changes in the alternatives considered; changes in the underlying distributions considered; changes in the shift parameters used; changes in the number of observations obtained; changes in the number of samples selected; and changes in the levels of significance chosen.

Chapter VI will present applications utilizing some of the k sample techniques described in the dissertation to solving problems in various fields of endeavor--including those in both behavioral and marketing research. References will be cited from the literature which could have employed one of the k sample techniques against ordered alternatives in order to strengthen their findings.

Chapter VII will present a brief summary of the study,

draw major conclusions from the findings, list the accomplishments and limitations, and offer recommendations for future research.

TABLE 1.1
A LISTING OF k SAMPLE PROCEDURES

<u>PROCEDURES FOR A COMPLETELY RANDOM DESIGN:</u>	<u>CODE NAME^a</u>
The One-Way F Test	FTEST
Bartholomew's $\bar{E}^2$ Test	BARTH
One-Way One-Tail Regression Test	LRCON
One-Way Linear-2 Contrast	DEREG
One-Way Linear 2-4 Contrast	QEREG
Abelson and Tukey One-Way Optimal Contrast	ATMAX
Kruskal-Wallis Test	HTEST
Chacko's Test	CHAKO
Jonckheere's One-Way Test	JONKE
Puri's Expected Normal Scores Test	PURIS
Johnson-Mehrotra Expected Normal Scores Test	JMENS
Fisher and Yates One-Way Test	FYENS
Bell and Doksum One-Way Test	BDRNS
A One-Sided k Sample Kolmogorov-Smirnov Test	CONKS
Cox and Stuart's Weighted Sign Test	COXS1
Cox and Stuart's Unweighted Sign Test	COXS2
Cox and Stuart's Best Unweighted Sign Test	COXS3
Mosteller's k Sample Slippage Test	MOSSL
Conover's First k Sample Slippage Test	CONSL
 <u>PROCEDURES FOR A RANDOMIZED BLOCK DESIGN:</u>	 <u>CODE NAME^a</u>
The Two-Way F Test	F2WAY
Hollander's Parametric Test	HPARA
Two-Way One-Tail Regression Test	LRCN2
Two-Way Linear-2 Contrast	DERG2
Two-Way Linear 2-4 Contrast	QERG2
Abelson and Tukey Two-Way Optimal Contrast	ATMX2
Friedman's Test	FRIED
Shorack's Test	SHORT
Jonckheere's Two-Way Test	JONK2
Page's L Test	PAGET
A Modified Hollander Rank Test	HOLLT
A Modified Doksum Rank Test	DOKST
Pirie's Sign Test	PSIGN
Pirie-Tukey Rank Sum Test	PTRST
Pirie and Hollander's Expected Normal Scores Test .	PHENS
Fisher and Yates Two-Way Test	FYNS2
Bell and Doksum Two-Way Test	BDNS2
A General Aligned Rank Order Test	SMSAT
Sen's Aligned Rank Order Test	SENAT

^aCode names are used as headings in computerized tables.

CHAPTER II

DEVELOPMENT OF k SAMPLE PROCEDURES

2.1 k SAMPLE PARAMETRIC PROCEDURES FOR GENERAL ALTERNATIVES

Whenever a completely randomized design or a randomized block design has been employed in a research endeavor and certain assumptions and conditions have been met, analysis of variance procedures (the Snedecor F Tests) are used to test whether there are any significant differences among a set of k means from k groups. Under the basic assumption of the F test--randomness, independence of observations, normality, homoscedasticity, and additivity of effects--and under the condition that the data collected constitute an interval or ratio scale, a null hypothesis is set up which states that there are no real or significant differences between the means of the k groups ($\mu_1 = \mu_2 = \dots = \mu_k$) and that any observed differences are due to chance or random variation only. A problem arises, though, when the researcher wants to reject the null hypothesis. The alternate hypothesis can only state that the means are not equal ($\mu_j \neq$, $j = 1, \dots, k$) but it does not state which mean or means (other than the extreme means) are not equal to the other means. Thus it is unknown whether one mean is significantly different from all the rest, or if all the means are significantly different from each other. In fact, there are

$k(k-1)/2$ potential significant differences which could exist if the null hypothesis is to be rejected.

Use of A Posteriori Procedures

To alleviate this situation, a set of techniques called a posteriori multiple comparisons have been developed. Such techniques determine, a posteriori, which mean or means differ from which others. A posteriori multiple comparisons consist of multiple range tests and the Multiple S Test. There are five such techniques that are widely used. The multiple range tests are: the Tukey HSD Test, the Student-Newman-Keuls Test, the Duncan New Multiple Range Test, and the Link-Wallace Short Cut method. The Multiple S Test was developed by Scheffe.

Traditionally, the researcher has utilized one of the above tests whenever the F Test determined that the null hypothesis be rejected. Moreover, the use of an a posteriori technique shows that the researcher was not concerned, a priori, with any particular difference or differences among the k means.

Use of A Priori Procedures

On the other hand, it is frequently the case that the researcher is interested--a priori to obtaining data and performing the F Test--in finding specific differences between the means of the k groups. When such differences are asserted in advance of experimentation the researcher has utilized one of the a priori multiple comparison techniques developed for this purpose. The most widely used a priori multiple comparison

technique developed for this purpose is the method of orthogonal contrasts. In such circumstances, the researcher would, a priori, formulate subordinate hypotheses about some or all of the k means. After performing the F Test and rejecting the main null hypothesis that all the k means are equal, the researcher would then test the subordinate null hypotheses by using an a priori multiple comparison technique.

2.2 k SAMPLE NONPARAMETRIC PROCEDURES FOR GENERAL ALTERNATIVES

When the experimental design is of the completely randomized or randomized block format several nonparametric procedures have been developed to test the null hypothesis of no differences between the k sample groups. These nonparametric procedures make fewer and more general assumptions than their parametric counterpart (the F Test). For the former model the most widely used technique is the Kruskal-Wallis Test (1952) while for the latter model, the Friedman Test (1937) is most frequently employed. Two other types of k sample nonparametric procedures for generalized alternatives, however, have been utilized in an effort to improve upon the power of the above tests. These are the Fisher-Yates procedures (1949) and the Bell and Doksum procedures (1965) which are based, respectively, on expected normal scores and random normal scores. It should be noted that both the Fisher-Yates techniques and the Bell and Doksum techniques are designed not only for the completely randomized model, but for the randomized block model as well. (All of the above methods have been included in this dissertation so that

their empirical powers may be compared against those from tests with more restrictive alternatives.)

Nonparametric techniques, in general, have been used extensively by behavioral scientists and marketing researchers who wish to avoid the more rigorous assumptions of the parametric F Test. Moreover, the nonparametric techniques are particularly valuable when the observed data have not been measured on an interval or ratio scale but merely are ranked or categorized. The aforementioned tests, for example, require that the data collected constitute an ordinal scale. Thus, nonparametric techniques have played a vital role in both behavioral and marketing research since the data often do not meet the measurement conditions required of the F Test.¹ On the other hand, when the data collected constitute an interval or ratio scale but the experimenter desires more generality in his conclusions (thereby avoiding the rigorous assumptions underlying the parametric procedures), a powerful nonparametric methodology was developed by Hodges and Lehmann (1962) for problems specific to the randomized block format. Hodges and Lehmann showed that large gains in power over the Friedman Test could be made with a nonparametric conditional approach which

¹It is for this reason that Page (1963, p. 225) finds that the parametric "F Test of the contribution of linear regression to the treatment sum of squares" in the randomized block design, as described in such well-known texts as Edwards (1960, pp. 239-240), and Cochran and Cox (1957, pp. 108-109), is frequently not a very useful method for analyzing either behavioral or marketing research data.

took into account the interblock comparisons absent in Friedman's intra-block procedure. Their approach was based on a combined ranking of all observations after an intra-block "alignment". This paper, along with subsequent papers by Lehmann (1963A, 1963B) stimulated independent investigations by both Mehra and Sarangi (1967) and Sen (1968) who developed identical aligned rank order tests for generalized alternatives.

2.3 NEED FOR k SAMPLE TESTS FOR ORDERED ALTERNATIVES

Unfortunately, however, most of the k sample nonparametric techniques designed to test against general alternatives have a major drawback when compared to their parametric counterpart, the F Test. When the data collected only constitute a nominal or ordinal level of measurement, these nonparametric tests are useful only if it is desired to detect differences in the k groups. Similar to the F Test, a rejection of the null hypothesis merely states that at least two of the k groups differ from each other but what other differences exist, if any, among the $k(k-1)/2$ possible group comparisons are generally not discernable for most of these nonparametric tests. The a priori and a posteriori multiple comparison procedures previously mentioned can only be applied to data measured on an interval scale or ratio scale. On the other hand, although several nonparametric a posteriori multiple comparison procedures have been developed (for example, Steel (1961), Nemenyi (1963), Dunn (1964), Wilcoxon and Wilcox (1964), Sherman (1965),

McDonald and Thompson (1967)), little has been accomplished (aside from the work of R.G.D. Steel (1960)) toward the development of nonparametric a priori multiple comparison techniques.² Moreover, these a posteriori procedures are only rank sum procedures--designed specifically for that class of nonparametric tests whose data constitute an ordinal level of measurement (the Kruskal-Wallis Test or Friedman Test, for example) rather than for those classes of nonparametric tests whose data either are categorical or are transformed to normal scores.

However, since it is often the desire of the researcher to specify--in advance--the direction of the differences expected among the k groups, it thus becomes necessary to develop both specific nonparametric and parametric procedures to accomplish this. In the former case it would fill, in part, a void in the knowledge; in the latter, it would facilitate data analysis through the use of a direct approach--thereby avoiding the indirect, "result-guided" methods employed (F Test with either an a priori or a posteriori multiple comparison procedure.)

²Steel derived a multiple comparison rank sum test for the simultaneous comparison of all pairs of groups when the design is of the completely randomized type. While the theoretical approach treats k samples, Steel only presents tabled critical values for $k = 3$, a rather restrictive condition to be of much practical use.

Development of Parametric Procedures

Aside from the standard Regression F Test (particularly useful for trend analysis), two types of parametric procedures have been developed to handle the problem of testing k groups with ordered alternatives. Bartholomew, in four monumental papers (1959A, 1959B, 1961A, 1961B) developed a likelihood ratio test approach for handling the problem under a variety of conditions--partial or complete ordering, ordering with unknown direction, and ordering when some of the k variances are unequal. Unfortunately, his papers were only concerned with problems relating to a completely randomized design. However, Bartholomew's work influenced several others--including Chacko (1963) and Hogg (1965) who constructed test procedures which are variations of the concepts he developed for the completely randomized design. Hollander (1967) and Shorack (1967) then independently extended Hogg's work to that of a randomized block design by developing likelihood ratio test procedures.

The second type of parametric procedure to be developed, "optimal" contrasts, is applicable to the k sample problem with ordered alternatives for both design situations--completely randomized or randomized block. The "optimal" contrast approach was investigated by Abelson and Tukey (1963) and subsequently extended by Schaafsma and Smid (1966).

A recent and most informative article by Proctor (1971) compared the two parametric procedures and concluded that the contrast approach not only appears to be easier for computation,

but the generated results appear to be more comprehensive as well.³

Development of Nonparametric Procedures

The earliest nonparametric k sample tests against ordered alternatives were developed, independently, by Jonckheere (1954A) and Terpstra (1952) for the completely randomized design and by Jonckheere (1954B) for the randomized block model.⁴ These techniques, based on Kendall's coefficient of rank correlation, tau, preceded Bartholomew's parametric work by five

³Proctor's article was intended for a social science audience. He had been stimulated by the work of Leik and Gove (1969) who were attempting to develop index numbers for "asymmetric monotonic relationships in sociology." He found that one of the Leik and Gove proposals appeared to be equivalent to the maximum likelihood criterion offered by Bartholomew. It is interesting to point out that Proctor chose, as an example for comparing the two types of parametric procedures, data measured on a 5 point scale. It could be argued that such data are more appropriately analyzed by using a nonparametric procedure rather than having to make the assumption of interval level measurement with something so gross as a 5 point scale. It was to avoid the stringent assumptions and measurement conditions of the parametric procedures that led to the development of nonparametric methods.

⁴Mosteller (1948) developed a k Sample Slippage Test in 1948 which investigates the problem whereby one of the k sample groups "slipped" to the right. Since that time several slippage techniques have appeared in the literature--including one presented by Conover (1968). Only these two tests, however, have been included in this study (in order to examine their empirical powers particularly against the end gap alternative) since it is believed that a Monte Carlo evaluation of this category of procedures is more appropriately the subject of another investigation.

years. Moreover, it is interesting to note that at around the same time period that both Jonckheere and Terpstra were separately developing their rank tests, Cox and Stuart (1955) began investigating "some quick sign tests for trend in location and dispersion" based also, in part, on Kendall's coefficient of rank correlation. Unlike the above mentioned rank procedures, however, the simple sign tests of Cox and Stuart were developed only for problems requiring a completely randomized design. Besides the standard Regression F Test, these nonparametric counterparts, then, represented the existing methodology for the direct handling of k sample problems with ordered alternatives until 1960, when Bartholomew's likelihood ratio procedures and Abelson and Tukey's "optimal" contrast procedure spurred new interest in this field.

Since 1960 several k sample nonparametric tests against ordered alternatives have been developed. Among the techniques proposed for a one-way analysis (completely randomized design) are those offered by Chacko (1963), Puri (1965), Conover (1967) and Johnson and Mehrotra (1971). Among the techniques proposed for a two-way analysis (randomized block design) are those offered by Lubin (1961), Page (1963), Hollander (1967), Doksum (1967), Shorack (1967), Sen (1968), Puri and Sen (1968), Pirie (1970) and Pirie and Hollander (1972). Many of these procedures are examined in this dissertation.

CHAPTER III

k SAMPLE TECHNIQUES FOR A COMPLETELY RANDOMIZED DESIGN

In this chapter, nineteen statistical procedures which are appropriate for a completely randomized design are described.¹ As shown in Table 3.1 (p. 21), these procedures may be classified according to family as well as type of alternative--general (1.1.2) or ordered (1.1.3).

Of the six parametric procedures presented, all but the One-Way F Test are designed against ordered location alternatives. While the One-Way F Test is a procedure to be utilized when there is no reason for ordering the alternative, it is presented here for purposes of comparison with the other parametric and nonparametric methods. Similarly, three of the thirteen nonparametric procedures--the Kruskal-Wallis Test, the Fisher and Yates One-Way Test, and the Bell and Doksum One-Way Test--are included for purposes of comparison not only among themselves, but also against the other nonparametric procedures as well as with the One-Way F Test.

¹In the following chapter statistical procedures which are appropriate for a randomized block design are presented. For a discussion of these two types of design, see Appendix A1.

TABLE 3.1
CLASSIFICATION OF ONE-WAY PROCEDURES^a
BY FAMILY & BY ALTERNATIVE

PARAMETRIC METHODS		NONPARAMETRIC METHODS							
		SIGN TESTS		RANK TESTS		SCORES TESTS		OTHERS	
GENERAL	ORDERED	GENERAL	ORDERED	GENERAL	ORDERED	GENERAL	ORDERED	GENERAL	ORDERED
FTEST	BARTH		COXS1	HTEST	JONKE	FYENS	PURIS		CONKS
	ATMAX		COXS2		CHAKO	BDRNS	JMENS		CONSL
	LRCON		COXS3						MOSSL
	DEREG								
	QEREG								

^aThe names of the procedures represented by the coded values in this table are given in Table 1.1.

PARAMETRIC PROCEDURES

3.1 THE ONE-WAY F TEST

The oldest and most widely known procedure for testing for differences between k group means when the design format is of the completely randomized type is the One-Way F Test. The objective is to test a null hypothesis of equality among k means against the general alternative that at least one of the means differs from at least one other. A computed F ratio is compared to a critical value obtained from the F distribution for a chosen level of significance and appropriate degrees of freedom, and the null hypothesis is rejected if the computed F equals or exceeds the critical F. For an appropriate set of data having equal sample size n in each of k groups, the F statistic is computed from:

$$F = \frac{n \sum_{j=1}^k (\bar{X}_{.j} - \bar{X}_{..})^2 / k - 1}{\sum_{j=1}^k \sum_{i=1}^n (X_{ij} - \bar{X}_{.j})^2 / N - k} \quad \text{d.f. } F \text{ with } (k-1) \text{ and } (N-k) \text{ d.f.} \quad (3.1.1)$$

In this dissertation it was decided to make use of the relationship between the F distribution and the incomplete beta function in order to more readily obtain critical values needed for several other test procedures rather than to just tabulate critical F values needed here. This relationship is expressed by Kendall and Stuart (1958) who show that $1 - \mathcal{X} = (1 + \frac{V_1 F}{V_2})^{-1}$ where $V_1 = k - 1$ and $V_2 = N - k$, the degrees of freedom for the numerator and denominator, respectively, in the

F ratio. Thus,

$$1-x = \frac{1}{1 + \frac{V_1 F}{V_2}} = \frac{1}{\frac{V_2 + V_1 F}{V_2}} = \frac{V_2}{V_2 + V_1 F}.$$

Generally, when $p = \frac{V_1}{2}$ and $q = \frac{V_2}{2}$, the tabled value $I_x(p, q)$

is given by:

$$I_x(p, q) = \frac{\beta_x(p, q)}{\beta(p, q)} = \frac{\int_0^x x^{p-1} (1-x)^{q-1} dx}{\int_0^1 x^{p-1} (1-x)^{q-1} dx} \quad \text{where the}$$

denominator can also be written as $\frac{\Gamma(p) \Gamma(q)}{\Gamma(p+q)}$.

When $p < q$, then $I_x(p, q) = 1 - I_{1-x}(q, p)$. Now $x = \frac{V_1 F}{V_2 + V_1 F}$ and the probability of obtaining such an F or larger under the null hypothesis is simply $I_{1-x}(q, p) = I_{1-x}(\frac{V_2}{2}, \frac{V_1}{2})$. Hence, by this procedure a computed F statistic may be transformed into an incomplete beta statistic with the resulting probability compared against the established level of significance, α , to determine whether or not the null hypothesis is rejected.²

3.2 BARTHOLOMEW'S E^2 TEST

Whereas the F Test is a parametric procedure for testing the equality of k means against general alternative

($H_a: \mu_j \neq \mu_k, j = 1, 2, \dots, k$), Bartholomew (1961A) developed a parametric procedure applicable to a completely randomized design format for testing against ordered alternatives

($H_a: \mu_1 \leq \mu_2 \leq \dots \leq \mu_k$ with at least one inequality strict).

Since it is known that "one-sided" tests are more powerful than "two-sided" procedures (say $k = 2$, for example), such tests are

²See K. Pearson (1956) for tables of the incomplete beta function.

useful when it is desirable to specify the direction of the alternative. Hence, Bartholomew sought a more powerful modification of the more general F Test. In addition, he expected that when the assumptions of the parametric methods are met the $\bar{E}^2$ Test would be more powerful than its nonparametric competitors and, at the same time, be more useful than the One-Way One-Tail Regression Test (see section 3.3) under situations (often occurring in behavioral experimentation) where the treatment levels are qualitative or in ranks so that it is difficult to find a suitable level of measurement.³ As expected, the $\bar{E}^2$ Test has good asymptotic properties when compared to its competitors.⁴

Essentially, Bartholomew uses the likelihood ratio principle to develop the $\bar{E}^2$ Test which is sensitive to an ordered alternative.⁵ The test statistic for the case of equal size samples, n , is given by:

$$\bar{E}^2 = \frac{n \sum_{j=1}^k (\hat{\mu}_{.j} - \bar{X}_{..})^2}{\sum_{j=1}^k \sum_{i=1}^n (X_{ij} - \bar{X}_{..})^2} \quad (3.2.1)$$

where $\sum_{j=1}^k \sum_{i=1}^n X_{ij} / N = \bar{X}_{..}$, $N = nk$, and $\hat{\mu}_{.1}, \hat{\mu}_{.2}, \dots, \hat{\mu}_{.k}$ are

³The assumptions of the parametric test are: normality, homoscedasticity, additivity, independent and random selection of observations, and a measurement level at least the strength of an interval scale.

⁴See Bartholomew (1961A and 1961B).

⁵A basic description of likelihood ratio tests is found in such well known texts as Hoel (1966, pp. 220-228) or Brunk (1960, pp. 248-256).

the maximum likelihood estimates of $\mu_1, \mu_2, \dots, \mu_k$. Bartholomew (1961A) shows the sample means $\bar{X}_1, \bar{X}_2, \dots, \bar{X}_k$ are the maximum likelihood estimators of their respective parameters $\mu_1, \mu_2, \dots, \mu_k$, provided that $\bar{X}_1 \leq \bar{X}_2 \leq \dots \leq \bar{X}_k$. When the sample means are in the same order as that stated by the alternative hypothesis, $\bar{E}^2$ is simply the ratio of the between sum of squares to the total sum of squares as obtained for developing the F Test (see 3.1.1). However, if any consecutive pair of sample means are not in the order stated by the alternative hypothesis an "amalgamation process" described by Brunk (1958) is used to find estimates $\hat{\mu}_j$ (where $j = 1, 2, \dots, k$).

Assuming the sample means are in order up to $\bar{X}_j$ (i.e., $\bar{X}_1 \leq \bar{X}_2 \leq \dots \leq \bar{X}_j$), if $\bar{X}_j > \bar{X}_{j+1}$, the procedure is to form the average of the two sample means and use these as the two "temporary" estimates. Hence,

$$\hat{m}_j = \hat{m}_{j+1} = \frac{n \bar{X}_j + n \bar{X}_{j+1}}{2n} = \frac{\bar{X}_j + \bar{X}_{j+1}}{2}.$$

Now compare these with the previous term, $\bar{X}_{j-1}$. If $\bar{X}_{j-1} \leq \hat{m}_j = \hat{m}_{j+1}$, the order is correct. However, if $\bar{X}_{j-1} > \hat{m}_j = \hat{m}_{j+1}$, take the average of all three terms and form the three "temporary" estimates

$$\hat{m}'_{j-1} = \hat{m}'_j = \hat{m}'_{j+1} = \frac{n \bar{X}_{j-1} + n \bar{X}_j + n \bar{X}_{j+1}}{3n} = \frac{\bar{X}_{j-1} + \bar{X}_j + \bar{X}_{j+1}}{3}$$

which then again must be compared to the preceding term, $\bar{X}_{j-2}$, for order and the process repeated. When all the means or "temporary" estimates through m_{j+1} are finally in order according to the specified alternative, the succeeding term, $\bar{X}_{j+2}$, must be compared to m_{j+1} . If $m_{j+1} \leq \bar{X}_{j+2}$, the order is

correct. However, if $m_{j+1} > \bar{X}_{j+2}$, the latter must be averaged together with all the "temporary" estimates equivalent to m_{j+1} and a new "temporary" estimate formed.

The amalgamation process continues until all k terms have been examined and "adjusted" wherever required. This ordered sequence of unadjusted means and "temporary" estimates then becomes the estimators, $\hat{\mu}_j$, of μ_j (where $j = 1, 2, \dots, k$) to be used in obtaining the test statistic $\bar{E}^2$ in (3.2.1).

The null hypothesis of identical population means may be rejected in favor of the ordered alternative if the probability of obtaining a value of $\bar{E}^2$ equal to or exceeding the observed test statistic is less than or equal to α , the stated level of significance. That is, reject H_0 if $P\{\bar{E}^2 \geq \bar{E}_{obs}^2\} \leq \alpha$. This probability is computed from:

$$P\{\bar{E}^2 \geq \bar{E}_{obs}^2\} = \sum_{\ell=2}^k p(\ell, k) I_{1-\bar{E}^2} \left(\frac{N-\ell}{2}, \frac{\ell-1}{2} \right) \quad (3.2.2)$$

where $I_{1-\bar{E}^2} \left(\frac{N-\ell}{2}, \frac{\ell-1}{2} \right)$ is the incomplete beta function, with parameters $\frac{N-\ell}{2}$ and $\frac{\ell-1}{2}$, and $p(\ell, k)$ are the "combining probabilities" which can be computed from the following recurrence relation derived by Miles (1959) for the case of equal size samples, n , in each of the groups:

$$p(\ell, k) = [p(\ell-1, k-1) + (k-1)p(\ell, k-1)]/k \quad (3.2.3)$$

where $2 \leq \ell \leq k-1$; $p(1, k) = \frac{1}{k}$; and $p(k, k) = \frac{1}{k!}$. Now $p(\ell, k)$ may be defined as the probability that the k terms are reduced to ℓ distinct by an amalgamation process in order to have the estimates in the same sequence as stated by the

alternative hypothesis. Values of $p(l,k)$ developed from the recurrence relation (3.2.3) are given in Table A53 for $k = 3 (1) 7$.⁶

3.3 ONE-WAY ONE-TAIL REGRESSION TEST

The One-Way One-Tail Regression Test is a parametric procedure which is used in testing the null hypothesis that all k means are equal ($\mu_j = \mu$ where $j = 1, 2, \dots, k$) against the directed alternative that the k means are monotonically increasing ($\mu_1 \leq \mu_2 \leq \dots \leq \mu_k$ with at least one inequality strict), and in particular, are increasing in a linear manner.

Two approaches for utilizing the One-Way One-Tail Regression Test are developed below. Both of these approaches, yielding identical results, are often found in the literature under the topic of trend analysis. The first of these is a least squares linear regression approach as described by Banks (1965) for the case of k groups each containing a sample of equal size n . The second is a linear contrast approach as described in Ferguson (1966) for the case of k samples of equal size n . Other than the usual assumptions required for a parametric analysis of variance procedure, this linear contrast approach usually requires the additional assumption that the k levels of the treatment variable are equally spaced.

For the least squares linear regression approach, let

⁶While the $\bar{E}^2$ statistic itself does not require equal sample sizes, n , the recurrence relation used to test the significance of $\bar{E}^2$ does depend on this assumption.

W_j be the value of the j^{th} level of the treatment variable (where $j = 1, 2, \dots, k$). Moreover, let $\bar{X}_{.j}$ be the average "observed value" over the n observations pertaining to W_j . The objective is to form the least squares linear regression equation $\hat{\bar{X}}_{.j} = a + bW$ using the k pairs of observed values $(W_j, \bar{X}_{.j})$. The slope, b , and intercept, a , are found by:

$$b = \frac{\sum_{j=1}^k (W_j \bar{X}_{.j}) - \bar{W} \sum_{j=1}^k \bar{X}_{.j}}{\sum_{j=1}^k W_j^2 - \bar{W} \sum_{j=1}^k W_j} \quad \text{where } \bar{W} = \frac{\sum_{j=1}^k W_j}{k};$$

and

$$a = \frac{\sum_{j=1}^k \bar{X}_{.j}}{k} - b\bar{W}.$$

Now the treatment sum of squares in the One-Way F Test is decomposed into two parts: that due to linear regression and that due to deviations from linear regression. The sum of squares due to linear regression, having one degree of freedom, is given by:

$$SS_{LR} = nb^2 \left[\sum_{j=1}^k W_j^2 - \frac{\left(\sum_{j=1}^k W_j \right)^2}{k} \right], \quad (3.3.1)$$

while the sum of squares of deviations from linear regression having $k-2$ degrees of freedom, is given by:

$$SS_{NLR} = n \sum_{j=1}^k (\hat{\bar{X}}_{.j} - \bar{X}_{.j})^2 = n \sum_{j=1}^k (\bar{X}_{.j} - \bar{X}_{..})^2 - SS_{LR}. \quad (3.3.2)$$

The test statistic for a One-Way Regression F Test is thus determined from:

$$RF' = \frac{nb^2 \left[\sum_{j=1}^k W_j^2 - \frac{\left(\sum_{j=1}^k W_j \right)^2}{k} \right]}{\sum_{j=1}^k \sum_{i=1}^n (X_{ij} - \bar{X}_{.j})^2 / (N-k)} \stackrel{d}{=} F \text{ with } 1 \text{ and } (N-k) \text{ d.f.} \quad (3.3.3)$$

The alternative approach to obtain the RF' statistic utilizes the concept of orthogonal linear contrasts. Under the assumption of equally spaced treatment levels, the calculations required to obtain SS_{LR} in order to calculate the RF' statistic can be greatly facilitated by using a set of coefficients of orthogonal linear contrasts, C_j , as replacements for the W_j , the values of the k treatment levels (where $j = 1, 2, \dots, k$).⁷ The C_j coefficients to be selected depend on k , the number of treatment levels. The following table gives coefficients for $k = 3$ (1) 7.

TABLE 3.2
COEFFICIENTS OF LINEAR CONTRASTS
($k = 3$ (1) 7)

k	C_j Coefficients
3	-1 0 1
4	-3 -1 1 3
5	-2 -1 0 1 2
6	-5 -3 -1 1 3 5
7	-3 -2 -1 0 1 2 3

As described in Hicks (1964), let $\mathcal{C}$ be a contrast defined by: $\mathcal{C} = \sum_{j=1}^k C_j X_{.j}$

where $\sum_{j=1}^k C_j = 0$, and provided that there are n observations in each of the k groups. The sum of squares for such

⁷Abelson and Tukey (1963) argue, however, that it is frequently the case where the levels of W_j are only qualitative or in ranks, and there is no sufficient reason to assume these levels equidistant apart.

a contrast is given by:

$$SS_c = \frac{C^2}{n \sum_{j=1}^k C_j^2} = \frac{\left(\sum_{j=1}^k C_j X_{.j}\right)^2}{n \sum_{j=1}^k C_j^2} \quad (3.3.4)$$

and it has one degree of freedom. This term, however, is equivalent to SS_{LR} defined in (3.3.1) above. Hence,

$$RF' = \frac{\left(\sum_{j=1}^k C_j X_{.j}\right)^2 / n \sum_{j=1}^k C_j^2}{\sum_{j=1}^k \sum_{i=1}^n (X_{ij} - \bar{X}_{.j})^2 / N - k} \stackrel{d}{=} F \text{ with } 1 \text{ and } (N-k) \text{ d.f. .} \quad (3.3.5)$$

If developed (as above) through analysis of variance procedures an interesting problem arises when attempting to utilize the RF' test statistic in the context of this dissertation. It must be noted that the RF' statistic based on a regression approach (3.3.3) is not predicting the direction of the trend (slope)--only whether or not a significant regression coefficient (slope) exists. Similarly, based on a linear contrast approach the RF' statistic (3.3.5) is not predicting, a priori, the direction of the differences between the groups examined by the contrast--only whether or not there are significant differences between these groups. However, it might be argued that when the experimenter is concerned with linear trend analysis there are occasions when it is desirable to predict the direction of that trend. Unfortunately, both Banks and Ferguson fail to point out that the RF' statistic in (3.3.3) and (3.3.5) does not test for an a priori specified direction in trend.

To resolve this dilemma and obtain a one-tail regression

procedure, the well-known relationship between the F distribution and Gosset's t (when $k = 2$) is used.⁸ Since the F Test is a test against general alternatives (see section 3.1), the test statistic F (with 1 and V degrees of freedom) for a specified level of significance, α , is identical to the square of the two-tail test statistic t (with V degrees of freedom) for a total level of significance, α , such that α is divided equally into both tails of the t distribution. That is, $F_{\alpha}(1, V) \equiv t_{\frac{\alpha}{2}}^2(V)$. Now a one-tail t test can be employed against the specified trend alternative. Thus, the One-Way One-Tail Regression Test with statistic RF takes the form:

$$RF = \frac{\sqrt{n} b \left[\sum_{j=1}^k W_j^2 - \frac{\left(\sum_{j=1}^k W_j \right)^2}{k} \right]^{\frac{1}{2}}}{\left(\sum_{j=1}^k \sum_{i=1}^n (X_{ij} - \bar{X}_{.j})^2 / N - k \right)^{\frac{1}{2}}} \stackrel{d}{=} t \quad \text{with } (N-k) \text{ d.f.}, \quad (3.3.6)$$

or

$$RF = \frac{\sum_{j=1}^k C_j X_{.j} / \left(n \sum_{j=1}^k C_j^2 \right)^{\frac{1}{2}}}{\left(\sum_{j=1}^k \sum_{i=1}^n (X_{ij} - \bar{X}_{.j})^2 / N - k \right)^{\frac{1}{2}}} \stackrel{d}{=} t \quad \text{with } (N-k) \text{ d.f.}, \quad (3.3.7)$$

depending on whether a regression approach or a contrast approach, respectively, is used. Now the null hypothesis may be rejected at the α level of significance if $RF \geq t_{\alpha}[(N-k)] \text{ d.f.}$, where t is the upper α level of Gosset's t distribution.⁹

⁸See Li (1964, pp. 188-190).

⁹For ease of computation in this dissertation the statistic RF' was calculated using the contrast approach and was compared to the F distribution with 1 and $N-k$ degrees of freedom (transformed to the incomplete beta function) with a " 2α " level of significance. However, the null hypothesis could only be rejected if the numerator of the test statistic was found to be positive--prior to squaring--since a positive numerator implies a positive slope. This computational method was also used for several other procedures to be described in this and the following chapter (see sections 3.4, 3.5, 3.6, 4.2, 4.3, 4.4, 4.5 and 4.6).

3.4 ONE-WAY LINEAR-2 CONTRAST

The One-Way Linear-2 Contrast procedure was discussed by Abelson and Tukey (1963) in their endeavor to improve upon the power of the One-Way One-Tail Regression Test presented above. The procedure is particularly useful when the k levels of the treatment variable W_j are either qualitative or ordinal, but not necessarily equidistant apart.

The One-Way Linear-2 Contrast is a parametric procedure which is completely analagous to (and at times identical to) the linear contrast approach to the One-Way One-Tail Regression Test.¹⁰ The distinguishing factor is that in the Linear-2 Contrast the extreme coefficients, C_1 and C_k , of the linear contrast method are doubled. The following is a table of coefficients, d_j , for the Linear-2 Contrast for $k = 3 (1) 7$.

TABLE 3.3
COEFFICIENTS OF LINEAR-2 CONTRASTS
($k = 3 (1) 7$)

k	d_j Coefficients					
3	-2	0	2			
4	-6	-1	1	6		
5	-4	-1	0	1	4	
6	-10	-3	-1	1	3	10
7	-6	-2	-1	0	1	2 6

¹⁰The results of the two procedures are identical when $k = 3$.

The test statistic for the One-Way Linear-2 Contrast

is:

$$DL = \frac{\sum_{j=1}^k d_j \bar{X}_{.j} / (n \sum_{j=1}^k d_j^2)^{1/2}}{\left(\sum_{j=1}^k \sum_{i=1}^n (X_{ij} - \bar{X}_{.j})^2 / N - k \right)^{1/2}} \stackrel{d}{=} t \text{ with } (N-k) \text{ d.f.}, \quad (3.4.1)$$

where $\sum_{j=1}^k d_j = 0$ and provided that the k samples are of equal size n . The null hypothesis may be rejected at the α level of significance if $DL \geq t_{\alpha}[(N-k)] \text{ d.f.}$, the upper α tail of the t distribution.

3.5 ONE-WAY LINEAR 2-4 CONTRAST

Analogous to the One-Way One-Tail Regression Test and the One-Way Linear-2 Contrast, the One-Way Linear 2-4 Contrast procedure was also described by Abelson and Tukey (1963) in an effort to improve upon the power of a linear contrast approach.¹¹ The distinction between this method and the aforementioned methods is in the selection of coefficients for the contrast. Using the linear coefficients, C_j , as a base, the coefficients, q_j , of the Linear 2-4 Contrast procedure are obtained by quadrupling the extreme coefficients, C_1 and C_k , and doubling the "next to extreme" coefficients, C_2 and C_{k-1} . The following table presents the q_j coefficients of the Linear 2-4 Contrast for $k = 3$ (1) 7.

¹¹All three procedures are identical when $k = 3$. However, the Linear-2 Contrast and the Linear 2-4 Contrast are identical when $k = 4$ and 5 as well.

TABLE 3.4
COEFFICIENTS OF LINEAR 2-4 CONTRASTS
(k = 3 (1) 7)

k	q _j Coefficients							
3	-4	0	4					
4	-12	-2	2	12				
5	-8	-2	0	2	8			
6	-20	-6	-1	1	6	20		
7	-12	-4	-1	0	1	4	12	

The test statistic for the One-Way Linear 2-4 Contrast is:

$$QD = \frac{\sum_{j=1}^k q_j \bar{X}_{.j} / (n \sum_{j=1}^k q_j^2)^{1/2}}{\left(\sum_{j=1}^k \sum_{i=1}^n (X_{ij} - \bar{X}_{.j})^2 / N - k \right)^{1/2}} \stackrel{d}{=} t \text{ with } (N-k) \text{ d.f. ,} \quad (3.5.1)$$

where $\sum_{j=1}^k q_j = 0$ and provided that the k samples are of equal size n. The null hypothesis is rejected if $QD \geq t_{\alpha} [(N-k) \text{ d.f.}]$ for a specified level of α .

3.6 ABELSON AND TUKEY ONE-WAY OPTIMAL CONTRAST

Abelson and Tukey (1963) developed the parametric One-Way Optimal Contrast for testing the hypothesis that all k means are equal ($\mu_j =$ where $j = 1, 2, \dots, k$) against the ordered alternative that the k means are monotonically increasing ($\mu_{.1} \leq \mu_{.2} \leq \dots \leq \mu_{.k}$ with at least one inequality strict.) A requirement for applying this technique is that all k samples

be of equal size n .¹²

The objective of Abelson and Tukey was to determine a set of coefficients, M_j , of a "maximin" or "optimal" contrast in order to test the above hypothesis under the conditions that the k levels of the treatment variable, W_j , are either qualitative or are only ranks, and, for which it is not desirous to assume that the levels are equally spaced (i.e., not all $D_{j,j+1} =$, where $D_{j,j+1} \stackrel{\text{def}}{=} W_{j+1} - W_j$ for all j , where $j = 1, 2, \dots, k-1$).¹³ Using a geometric approach, the "optimal" coefficients, M_j , are derived from:

$$M_j = \left\{ (j-1) \left[1 - ((j-1)/k) \right] \right\}^{1/2} - \left\{ j \left(1 - (j/k) \right) \right\}^{1/2}. \quad (3.6.1)$$

Since the coefficients to be used are dependent upon k , the number of groups, the following is a table of M_j coefficients for $k = 3 (1) 7$.

TABLE 3.5
COEFFICIENTS OF OPTIMAL CONTRASTS
($k = 3 (1) 7$)

k	M_j Coefficients							
3	-.816	0	.816					
4	-.866	-.134	.134	.866				
5	-.894	-.201	0	.201	.894			
6	-.913	-.242	-.070	.070	.242	.913		
7	-.926	-.269	-.114	0	.114	.269	.926	

¹²Schaafsma and Smid (1966) have extended the work of Abelson and Tukey for the case where the sample sizes in the k groups were not equal.

¹³The authors cite references to their previous work in psychological scaling and attitude testing.

The test statistic for the Abelson and Tukey One-Way Optimal Contrast is:

$$AT = \frac{\sum_{j=1}^k M_j \bar{X}_{.j} / (n \sum_{j=1}^k M_j^2)^{1/2}}{\left(\sum_{j=1}^k \sum_{i=1}^n (X_{ij} - \bar{X}_{.j})^2 / (N-k) \right)^{1/2}} \triangleq t \quad \text{with } (N-k) \text{ d.f.}, \quad (3.6.2)$$

where $\sum_{j=1}^k M_j = 0$ and provided that there are equal size samples in each of the k groups. The null hypothesis may be rejected at the α level of significance if $AT \geq t_{\alpha} [(N-k)] \text{ d.f.}$, the upper α tail of Gosset's t distribution having $N-k$ degrees of freedom.

NONPARAMETRIC PROCEDURES

3.7 KRUSKAL-WALLIS TEST

Kruskal and Wallis (1952) devised a k sample extension to the Mann-Whitney U Test for two independent samples. The Kruskal-Wallis Test then is a nonparametric analogue of the One-Way F Test just as the Mann-Whitney procedure is a nonparametric analogue of the t Test and has similar asymptotic properties (see Bradley (1968)).

The procedure is useful when the observed data have only achieved an ordinal level of measurement or when the researcher does not wish to make the rigorous assumptions necessary for the parametric One-Way F Test. Other than an ordinal level of measurement the only assumptions needed for the Kruskal-Wallis Test are that the k samples are independent and the observations in each sample are randomly drawn from continuous

parent populations. The facts that the procedure is easy to understand and use are its two major assets. The null hypothesis of equality among the k distribution functions may be tested against the general alternative that at least one of the k populations differs from at least one other population. To perform the test (for equal sample sizes n), the original observations, X_{ij} , must be replaced by their corresponding ranks, R_{ij} , such that a rank of one is given to the smallest X_{ij} and rank of N is given to the largest. The test statistic H for the Kruskal-Wallis Test is then given by:

$$H = \frac{12}{N(N+1)n} \left[\sum_{j=1}^k R_j^2 \right] - 3(N+1) \stackrel{d}{\approx} \chi^2 \text{ with } (k-1) \text{ d.f.} \quad (3.7.1)$$

where $R_j^2 = \left(\sum_{i=1}^n R_{ij} \right)^2$ and R_{ij} is the rank given to the i^{th} value in the j^{th} sample. The null hypothesis may be rejected if $H \geq \chi_{\alpha}^2 [(k-1) \text{ d.f.}]$ for a particular α level of significance. This "large sample" approximation may be used whenever the k samples of size n equal or exceed five (for k of four or more).¹⁴

In this dissertation, however, it was decided to use the relationship between the χ^2 distribution and the incomplete gamma function in order to more easily obtain the critical values necessary for several other test procedures rather than to just tabulate the critical χ^2 values required here.

¹⁴When $k=3$ and $n=5$, the critical values should be obtained from the null distribution of the test statistic. These values are tabled (see Owen (1962)) and a randomization procedure employed to test the null hypothesis at the nominal 5% and 1% levels of significance.

Kendall and Stuart (1958) express the following relationship between χ^2 with V degrees of freedom and the incomplete gamma function as given in Pearson's tables (1957):

The probability of equaling or exceeding some given χ^2 with V degree of freedom can be found from:

$$P\{H_{obs} \geq \chi^2(V) \text{ d.f.}\} = 1 - I(u, p)$$

where $I(u, p)$ are the tabled values based on the ratio of the incomplete gamma function to the gamma function with

$$u = \frac{\chi^2/2}{\sqrt{V/2}} \quad \text{and} \quad p = \frac{V}{2} - 1.$$

Thus, by this method a computed H statistic may be transformed into an incomplete gamma statistic with the resulting probability compared against the specified α level of significance in order to determine whether or not to reject the null hypothesis.

3.8 CHACKO'S TEST

Whereas the Kruskal-Wallis Test is a nonparametric procedure for testing the equality among the k distribution functions against the general alternative ($F_j(X) \neq$ where $j = 1, 2, \dots, k$), Chacko (1963) developed a nonparametric procedure applicable to a completely randomized design format for testing against ordered alternatives ($F_1(X) \geq F_2(X) \geq \dots \geq F_k(X)$ with a least one inequality strict.) In essence, Chacko's work extended that of Bartholomew (see section 3.2). A likelihood ratio approach was used to develop the test statistic:

$$\bar{H} = \frac{12n}{N(N+1)} \sum_{j=1}^k \left[\hat{R}_{\cdot j} - \frac{N+1}{2} \right]^2 \quad (3.8.1)$$

where $N = nk$; $\bar{R}_j = \sum_{i=1}^n R_{ij}/n$ is the average rank for the j^{th} sample; and $\hat{\bar{R}}_j$ is the obtained "estimate" of the average rank for the j^{th} sample after all k rank means have been replaced by a set of nondecreasing rank means or pooled rank means--the "estimates" obtained via the amalgamation process described in section 3.2.

The null hypothesis is rejected for large enough values of the statistic $\bar{H}$ for a given level of significance α (that is, if $\bar{H} \geq \bar{H}_{cr}(\alpha)$). As n gets large (five or more) the significance of $\bar{H}$ is determined by

$$\alpha' = \sum_{\ell=2}^{k-1} p(\ell, k) P\{\chi_{\ell-1}^2 \geq \bar{H}\} \quad (3.8.2)$$

where $\chi_{\ell-1}^2$ is a random variate from the χ^2 distribution with $\ell - 1$ degrees of freedom and $p(\ell, k)$ are the "combining probabilities" given in (3.2.3) and in Table A53 for $k = 3$ (1) 7. The null hypothesis is rejected if $\alpha' \leq \alpha$. Thus, the χ^2 statistic (or its transformation to incomplete gamma) yields the critical region for an approximate test of the null hypothesis if n , the size of each sample, is large enough.

In comparison to the Kruskal-Wallis Test on which it is based, Chacko's Test was designed specifically for ordered alternatives and is a more powerful tool when the researcher believes an ordering exists among the k sample groups. Both tests make the same assumptions that the k samples are independently drawn from continuous parent populations and that each value in a particular sample is randomly selected.

Moreover, the observed values need only be in ranks. If the observed values are measurements, they may be replaced by the appropriate ranks over the entire set of data. Further, Chacko shows that his $\bar{H}$ Test statistic relative to $\bar{E}^2$ has the same good asymptotic properties as does the Kruskal-Wallis Test relative to the One-Way F Test.

It is interesting to show that when the k sample groups are in the order specified by the alternative hypothesis, the test statistic $\bar{H}$ is identical to the Kruskal-Wallis statistic H (3.7.1) but the critical regions of the two procedures are different--making Chacko's Test the more powerful method.

Recall that

$$H = \frac{12}{N(N+1)n} \left[\sum_{j=1}^k R_{.j}^2 \right] - 3(N+1) \quad (3.7.1)$$

where the k samples are of equal size n . Now

$$\bar{H} = \frac{12n}{N(N+1)} \sum_{j=1}^k \left[\hat{R}_{.j} - \frac{N+1}{2} \right]^2 \quad (3.8.1)$$

When the k samples are in order, however, $\hat{R}_{.j} \equiv \bar{R}_{.j}$ for all $j = 1, 2, \dots, k$, since the amalgamation process is not used.

Rewriting $\hat{R}_{.j}$ as $\frac{R_{.j}}{n}$ and then expanding,

$$\bar{H} = \frac{12n}{N(N+1)} \sum_{j=1}^k \left[\frac{R_{.j}^2}{n^2} - \frac{2(N+1)R_{.j}}{2n} + \frac{(N+1)^2}{4} \right]$$

and

$$\bar{H} = \frac{12n \sum_{j=1}^k R_{.j}^2}{N(N+1)n^2} - \frac{12n(N+1) \sum_{j=1}^k R_{.j}}{N(N+1)n} + \left(\frac{12n(N+1)^2}{4N(N+1)} \right) k.$$

But in the second term $\sum_{j=1}^k R_{.j}$ can be replaced by $N(N+1)/2$ so that

$$\bar{H} = \frac{12}{N(N+1)n} \left[\sum_{j=1}^k R_{.j}^2 \right] - 6(N+1) + 3(N+1).$$

Thus,

$$\bar{H} = \frac{12}{N(N+1)n} \left[\sum_{j=1}^k R_{.j}^2 \right] - 3(N+1)$$

and $\bar{H} = H$, thereby proving that the two test statistics are identical under conditions that the k groups are monotonically increasing in the direction of the specified alternative. However, to show that the critical regions of the two test statistics are not the same and that $\bar{H}$ is the more powerful test against the ordered alternative, the null distributions of $\bar{H}$ and H will be generated for the case where $k = 3$ and $n = 2$.

There are $N! = 720$ possible arrangements of the ranks 1 to 6 into a matrix of $n \times k = 2 \times 3$. Note that 48 of these arrangements have ranks 1 and 2 in the first column. Moreover, there are 48 more arrangements with ranks 1 and 2 in the second column as well as third column, a total of 144 arrangements. However, there exist 144 arrangements for each pair of ranks (1,3), (1,4), (1,5) and (1,6) as well. The total, therefore, is 720 arrangements. The list of all possible matrices and rank sums where elements 1 and 2 appear in the first column are given below. Each rank sum in Table 3.6 could be achieved in eight ways as previously described.

	<u>1 3 5</u>	<u>1 3 6</u>	<u>1 4 5</u>	<u>1 4 6</u>	<u>2 3 5</u>	<u>2 3 6</u>	<u>2 4 5</u>	<u>2 4 6</u>
	<u>2 4 6</u>	<u>2 4 5</u>	<u>2 3 6</u>	<u>2 3 5</u>	<u>1 4 6</u>	<u>1 4 5</u>	<u>1 3 6</u>	<u>1 3 5</u>
RANK SUMS	3 7 11	3 7 11	3 7 11	3 7 11	3 7 11	3 7 11	3 7 11	3 7 11
	<u>1 3 4</u>	<u>1 3 6</u>	<u>1 5 4</u>	<u>1 5 6</u>	<u>2 3 4</u>	<u>2 3 6</u>	<u>2 5 4</u>	<u>2 5 6</u>
	<u>2 5 6</u>	<u>2 5 4</u>	<u>2 3 6</u>	<u>2 3 4</u>	<u>1 5 6</u>	<u>1 5 4</u>	<u>1 3 6</u>	<u>1 3 4</u>
RANK SUMS	3 8 10	3 8 10	3 8 10	3 8 10	3 8 10	3 8 10	3 8 10	3 8 10
	<u>1 3 4</u>	<u>1 3 5</u>	<u>1 6 4</u>	<u>1 6 5</u>	<u>2 3 4</u>	<u>2 3 5</u>	<u>2 6 4</u>	<u>2 6 5</u>
	<u>2 6 5</u>	<u>2 6 4</u>	<u>2 3 5</u>	<u>2 3 4</u>	<u>1 6 5</u>	<u>1 6 4</u>	<u>1 3 5</u>	<u>1 3 4</u>
RANK SUMS	3 9 9	3 9 9	3 9 9	3 9 9	3 9 9	3 9 9	3 9 9	3 9 9
	<u>1 4 3</u>	<u>1 4 6</u>	<u>1 5 3</u>	<u>1 5 6</u>	<u>2 4 3</u>	<u>2 4 6</u>	<u>2 5 3</u>	<u>2 5 6</u>
	<u>2 5 6</u>	<u>2 5 3</u>	<u>2 4 6</u>	<u>2 4 3</u>	<u>1 5 6</u>	<u>1 5 3</u>	<u>1 4 6</u>	<u>1 4 3</u>
RANK SUMS	3 9 9	3 9 9	3 9 9	3 9 9	3 9 9	3 9 9	3 9 9	3 9 9
	<u>1 4 3</u>	<u>1 4 5</u>	<u>1 6 3</u>	<u>1 6 5</u>	<u>2 4 3</u>	<u>2 4 5</u>	<u>2 6 3</u>	<u>2 6 5</u>
	<u>2 6 5</u>	<u>2 6 3</u>	<u>2 4 5</u>	<u>2 4 3</u>	<u>1 6 5</u>	<u>1 6 3</u>	<u>1 4 5</u>	<u>1 4 3</u>
RANK SUMS	3 10 8	3 10 8	3 10 8	3 10 8	3 10 8	3 10 8	3 10 8	3 10 8
	<u>1 5 3</u>	<u>1 5 4</u>	<u>1 6 3</u>	<u>1 6 4</u>	<u>2 5 3</u>	<u>2 5 4</u>	<u>2 6 3</u>	<u>2 6 4</u>
	<u>2 6 4</u>	<u>2 6 3</u>	<u>2 5 4</u>	<u>2 5 3</u>	<u>1 6 4</u>	<u>1 6 3</u>	<u>1 5 4</u>	<u>1 5 3</u>
RANK SUMS	3 11 7	3 11 7	3 11 7	3 11 7	3 11 7	3 11 7	3 11 7	3 11 7

TABLE 3.6
TABLE OF RANK SUMS AND ALL POSSIBLE VALUES OF H STATISTIC

RANK SUMS						$\sum_{j=1}^k R_{.j}^2$	$H = \frac{12}{84} \left[\sum_{j=1}^k R_{.j}^2 \right] - 21$
3 7 11	3 11 7	7 3 11	7 11 3	11 3 7	11 7 3	179	4.571
3 8 10	3 10 8	8 3 10	8 10 3	10 3 8	10 8 3	173	3.714
3 9 9	3 9 9	9 3 9	9 9 3	9 3 9	9 9 3	171	3.429
4 6 11	4 11 6	11 4 6	11 6 4	6 4 11	6 11 4	173	3.714
4 7 10	4 10 7	10 4 7	10 7 4	7 4 10	7 10 4	165	2.571
4 8 9	4 9 8	9 4 8	9 8 4	8 4 9	8 9 4	161	2.000
5 5 11	5 5 11	11 5 5	11 5 5	5 11 5	5 11 5	171	3.429
5 7 9	7 5 9	9 7 5	9 5 7	5 9 7	7 9 5	155	1.143
5 8 8	5 8 8	8 5 8	8 5 8	8 8 5	8 8 5	153	0.857
6 5 10	6 10 5	5 6 10	5 10 6	10 6 5	10 5 6	161	2.000
6 6 9	6 6 9	9 6 6	9 6 6	6 9 6	6 9 6	153	0.857
6 8 7	6 7 8	8 7 6	8 6 7	7 8 6	7 6 8	149	0.286
7 5 9	7 9 5	5 9 7	5 7 9	9 5 7	9 7 5	155	1.143
7 6 8	7 8 6	6 8 7	6 7 8	8 6 7	8 7 6	149	0.286
7 7 7	7 7 7	7 7 7	7 7 7	7 7 7	7 7 7	147	0.000

Now the null distribution of H , where $n = 2$ and $k = 3$ is presented below.¹⁵

TABLE 3.7
NULL DISTRIBUTION OF H
($n = 2, k = 3$)

H	$P_r(H)$	$P_r(H_{OBS} \geq H)$	$P_r(H_{OBS} \geq \chi^2_{2d.f.} \cong H)$
0.000	1/15	1.0000	1.00
0.286	2/15	0.9333	0.86
0.857	2/15	0.8000	0.65
1.143	2/15	0.6667	0.57
2.000	2/15	0.5333	0.37
2.571	1/15	0.4000	0.30
3.429	2/15	0.3333	0.28
3.714	2/15	0.2000	0.16
4.571	1/15	0.0667	0.11

For Chacko's $\bar{H}$ Test the 720 possible arrangements of the mean ranks (or rank sums) are obtained by combining groups until an ordering is achieved. Each of the following "combined" rank sums arise in 8 ways as previously shown.

¹⁵A comparison of the exact null distribution of H for $n = 2$ and $k = 3$ against that approximated by χ^2 with 2 d.f. shows why χ^2 is not a good approximation for H at small sample sizes.

TABLE 3.8
"COMBINED" RANK SUMS

3	7	11	3	9	9	5	5	11	7	7	7	7	7	7	7	7	7
3	8	10	3	9	9	5.5	5.5	10	7	7	7	6.5	6.5	8	7	7	7
3	9	9	3	9	9	6	6	9	7	7	7	6	6	9	7	7	7
4	6	11	4	8.5	8.5	7	7	7	7	7	7	5	5	11	6	7.5	7.5
4	7	10	4	8.5	8.5	7	7	7	7	7	7	5.5	5.5	10	7	7	7
4	8	9	4	8.5	8.5	6.5	6.5	8	7	7	7	6	6	9	7	7	7
5	5	11	5	5	11	7	7	7	7	7	7	5	8	8	5	8	8
5	7	9	6	6	9	7	7	7	7	7	7	5	8	8	7	7	7
5	8	8	5	8	8	6.5	6.5	8	6.5	6.5	8	7	7	7	7	7	7
5.5	5.5	10	6	7.5	7.5	5	6	10	5	8	8	7	7	7	7	7	7
6	6	9	6	6	9	7	7	7	7	7	7	6	7.5	7.5	6	7.5	7.5
6	7.5	7.5	6	7	8	7	7	7	7	7	7	7	7	7	6.5	6.5	8
6	6	9	7	7	7	5	8	8	5	7	9	7	7	7	7	7	7
6.5	6.5	8	7	7	7	6	7.5	7.5	6	7	8	7	7	7	7	7	7
7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7

From the above the null distribution of the $\bar{H}$ statistic is formed and presented in Tables 3.9 and 3.10 below.¹⁶

TABLE 3.9
CALCULATION OF $\bar{H}$
(n = 2, k = 3)

RANK SUMS	$\sum_{j=1}^k R_{.j}^2$	$\bar{H} = \frac{12}{84} \left[\sum_{j=1}^k R_{.j}^2 \right] - 21$
7 7 7	147	0.000
6.5 6.5 8	148.5	0.214
6 7 8	149	0.286
6 7.5 7.5	148.5	0.214
6 6 9	153	0.857
5.5 5.5 10	160.5	1.929
5 5 11	171	3.429
5 7 9	155	1.143
5 8 8	153	0.857
5 6 10	161	2.000
4 6 11	173	3.714
4 7 10	165	2.571
4 8 9	161	2.000
4 8.5 8.5	160.5	1.929
3 7 11	179	4.571
3 8 10	173	3.714
3 9 9	171	3.429

¹⁶A comparison of the exact null distributions of $\bar{H}$ for n = 2 and k = 3 against that approximated by χ^2 shows the latter not to be a good approximation for $\bar{H}$ at such small sample sizes.

TABLE 3.10
NULL DISTRIBUTION OF $\bar{H}$
($n = 2, k = 3$)

$\bar{H}$	$P_r(\bar{H})$	$P_r(\bar{H}_{obs} \geq \bar{H})$	$\sum_{\ell=2}^k p(\ell, k) P_r\left\{\bar{H}_{obs} \geq \chi^2_{(\ell-1) d.f.} \cong \bar{H}\right\}$
0.000	40/90	1.0000	$1/2(1.00) + 1/6(1.00) = 0.667$
0.214	12/90	0.5556	$1/2(0.65) + 1/6(0.90) = 0.475$
0.286	2/90	0.4222	$1/2(0.59) + 1/6(0.86) = 0.438$
0.857	14/90	0.4000	$1/2(0.35) + 1/6(0.65) = 0.283$
1.143	2/90	0.2444	$1/2(0.29) + 1/6(0.57) = 0.240$
1.929	6/90	0.2222	$1/2(0.17) + 1/6(0.39) = 0.150$
2.000	2/90	0.1556	$1/2(0.16) + 1/6(0.37) = 0.142$
2.571	1/90	0.1333	$1/2(0.11) + 1/6(0.30) = 0.105$
3.429	8/90	0.1222	$1/2(0.07) + 1/6(0.28) = 0.082$
3.714	2/90	0.0333	$1/2(0.06) + 1/6(0.16) = 0.057$
4.571	1/90	0.0111	$1/2(0.03) + 1/6(0.11) = 0.035$

Identical test statistics of H and $\bar{H}$ can now be compared to show that the critical regions are different and that $\bar{H}$ yields the more powerful test against ordered alternatives. If H and $\bar{H}$ are calculated to be 3.714, for example, the probability of finding such a value of H or larger under the null hypothesis is .20 (with the Kruskal-Wallis Test) while the probability of finding such a value of $\bar{H}$ or larger under the null hypothesis is .0333 (using Chacko's Test). An analogous argument could therefore be made for the One-Way F Test (section 3.1) in comparison to Bartholomew's $\bar{E}^2$ Test (section 3.2).

3.9 JONCKHEERE'S ONE-WAY TEST

Development

Jonckheere (1954A) developed a nonparametric k sample test against ordered alternatives for a completely randomized design. Jonckheere's work was stimulated by the lack of procedures available for testing when the researcher is able to state, on the basis of experience or theory, the expected rank order of magnitude of the effects of different experimental treatments. Among the analysis of variance procedures available, the parametric F Test and its nonparametric counterpart, the Kruskal-Wallis Test, were designed to test against more general alternatives. On the other hand, the parametric One-Way One-Tail Regression Test, which was designed to test against ordered alternatives, was found to be inappropriate in certain situations (Jonckheere (1954A, p. 134)):

"A further aspect of many experiments is that there appears to be no suitable metric by which the different treatments to be investigated may be characterized. All that an experimenter is able to assert, particularly in psychology, is that they may be ranked in some order, such as increasing 'stress'. When this is the case it is no longer possible to use any form of regression analysis, since the independent variate is not suitably quantified."

The test is developed as follows:

Select k random samples of size $n_1, n_2, \dots, n_k$, from populations with respective continuous cumulative distribution functions $F_1(X), F_2(X), \dots, F_k(X)$. Let X_{ij} be the i^{th} value

in the j^{th} sample drawn from a population with c.d.f. $F_j(X)$ and let $X_{\lambda m}$ be the λ^{th} value in the m^{th} sample drawn from a population with c.d.f. $F_m(X)$.

Consider

$$p_{ij, \lambda m} = \begin{cases} 1 & \text{if } X_{ij} < X_{\lambda m} \\ 0 & \text{if } X_{ij} \geq X_{\lambda m} \end{cases} \quad (3.9.1)$$

where $j = 1, 2, \dots, k-1$; $m = 1 + j$; $i = 1, 2, \dots, n_j$; and $\lambda = 1, 2, \dots, n_m$. Now define

$$P_{jm} = \sum_{i=1}^{n_j} \sum_{\lambda=1}^{n_m} p_{ij, \lambda m} ,$$

and let

$$S = 2 \sum_{j=1}^{k-1} \sum_{m=1+j}^k P_{jm} - \sum_{j=1}^{k-1} \sum_{m=1+j}^k n_j n_m . \quad (3.9.2)$$

S is the test statistic to be used for testing the null hypothesis that the k samples are selected from populations with identical c.d.f.s ($F_j(X) = F_m(X)$ for all $j = 1, 2, \dots, k$) against the alternative of ordered c.d.f.s ($F_1(X) \geq F_2(X) \geq \dots \geq F_k(X)$). In this dissertation, however, it was assumed that all k samples would be of equal size n . Hence, it is easy to show that (3.9.2) can be reduced to:

$$S = 2 \sum_{j=1}^{k-1} \sum_{m=1+j}^k P_{jm} - \frac{k(k-1)n^2}{2} . \quad (3.9.3)$$

The second term on the right becomes a constant when the samples are of equal size n since

$$\sum_{j=1}^{k-1} \sum_{m=1+j}^k n_j n_m = \sum_{j=1}^{k-1} \sum_{m=1+j}^k n^2 = \binom{k}{2} n^2 = \frac{k(k-1)n^2}{2} .$$

Jonckheere has tabulated the null distribution of S for extremely small samples. As the total sample size gets larger ($N = nk \geq 15$ say), however, Jonckheere states that

$$S \stackrel{d}{=} N\left(\mu_s = 0, \sigma_s^2 = \frac{1}{18} \left[N^2(2N+3) - \sum_{j=1}^k n_j^2(2n_j+3) \right] \right).$$

Hence, using a continuity correction as suggested by Jonckheere, the standard normal variate Z can be employed

$$Z = \frac{S-1}{\sqrt{\frac{1}{18} \left[N^2(2N+3) - \sum_{j=1}^k n_j^2(2n_j+3) \right]}} \stackrel{d}{=} N(0,1). \quad (3.9.4)$$

When all k samples are of equal size n (3.9.4) reduces to:

$$Z = \frac{S-1}{\sqrt{\frac{1}{18} \left[N^2(2N+3) - k(n^2)(2n+3) \right]}} \stackrel{d}{=} N(0,1). \quad (3.9.5)$$

The null hypothesis may be rejected if $Z \geq Z_\alpha$, the upper tail critical value under the normal distribution for a specified level of significance, α . However, using (3.9.5), tables of approximate critical S values have also been developed for $n = 5$ (1) 20 and $k = 3$ (1) 7 and for the 5% and 1% levels of significance. These are found in Table A54.

Related Tests

It is interesting to note that several tests related to that of Jonckheere can be found in the literature. While Jonckheere is usually given credit for developing the above test it should be noted that Terpstra (1952) independently derived the same procedure and considered its asymptotic properties. Moreover, Ferguson (1966, pp. 365-367), without reference to Jonckheere, simply described the Jonckheere

procedure under the title "monotonic trend test for independent samples." In addition, Puri (1965) used Jonckheere's One-Way Test as a basis for developing a procedure employing normal scores and showed that under a specific set of conditions, the test format reduced back to the Jonckheere procedure (see section 3.10). Finally, Bartholomew (1961A) studied the asymptotic power functions of his $\bar{E}^2$ Test with the One-Way F Test and Jonckheere's Test, and then used the latter as a basis for describing a parametric "scores" test (a more powerful procedure having the more rigorous assumptions) which would be equivalent to the One-Way One-Tail Regression Test on the assumption that the k levels of the treatment variable W_j (where $j = 1, 2, \dots, k$) are equally spaced.¹⁷ Such a "scores" test would be expected to have more power than Jonckheere's Test since it takes into account the actual differences rather than just the signs of the differences.

The "scores" test to which Bartholomew refers is developed as follows from section 3.9. Instead of

$$p_{ij,lm} = \begin{cases} 1 & \text{if } x_{ij} < x_{lm} \\ 0 & \text{if } x_{ij} \geq x_{lm} \end{cases} \text{ as in (3.9.1.),} \quad \text{let}$$

¹⁷The asymptotic power functions (for $k = 3(1)4(4)12$) under the true ordered alternative show the $\bar{E}^2$ Test to always be preferred to the F Test. Moreover, Jonckheere's Test is slightly preferred to the $\bar{E}^2$ Test under an equally spaced alternative ($\mu_1 < \mu_2 < \dots < \mu_k$) but Jonckheere's procedure is even worse than the F Test as k gets larger under a gap alternative ($\mu_1 = \dots = \mu_{k-1} < \mu_k$). It must be noted, however, that Jonckheere's Test has fewer and less stringent assumptions than the others and is easier to perform.

$$p'_{ij, \ell m} = (X_{\ell m} - X_{ij}) \quad (3.9.6)$$

where $j = 1, 2, \dots, k-1$; $m = 1 + j$; $i = 1, 2, \dots, n_j$; and $\ell = 1, 2, \dots, n_m$. Now define

$$p'_{jm} = \sum_{i=1}^{n_j} \sum_{\ell=1}^{n_m} p'_{ij, \ell m} = \sum_{i=1}^{n_j} \sum_{\ell=1}^{n_m} (X_{\ell m} - X_{ij}) = n_j X_{\cdot m} - n_m X_{\cdot j} \quad (3.9.7)$$

Substituting in (3.9.2), the "scores" test statistic, S' , is given by:

$$S' = 2 \sum_{j=1}^{k-1} \sum_{m=1+j}^k p'_{jm} - \sum_{j=1}^{k-1} \sum_{m=1+j}^k n_j n_m$$

so that

$$S' = 2 \sum_{j=1}^{k-1} \sum_{m=1+j}^k (n_j X_{\cdot m} - n_m X_{\cdot j}) - \sum_{j=1}^{k-1} \sum_{m=1+j}^k n_j n_m \quad (3.9.8)$$

However, since $X_{\cdot m} = n_m \bar{X}_{\cdot m}$ and $X_{\cdot j} = n_j \bar{X}_{\cdot j}$, the "scores" test statistic may be written as:

$$S' = 2 \sum_{j=1}^{k-1} \sum_{m=1+j}^k (n_j n_m \bar{X}_{\cdot m} - n_j n_m \bar{X}_{\cdot j}) - \sum_{j=1}^{k-1} \sum_{m=1+j}^k n_j n_m$$

and

$$S' = 2 \sum_{j=1}^{k-1} \sum_{m=1+j}^k [n_j n_m (\bar{X}_{\cdot m} - \bar{X}_{\cdot j})] - \sum_{j=1}^{k-1} \sum_{m=1+j}^k n_j n_m ,$$

so that finally

$$S' = 2 \sum_{j=1}^{k-1} \sum_{m=1+j}^k \left[n_j n_m (\bar{X}_{\cdot m} - \bar{X}_{\cdot j}) - \frac{n_j n_m}{2} \right] \quad (3.9.9)^{18}$$

¹⁸It should be noted that (3.9.9) differs from the result given by Bartholomew (1961A, p. 329). Bartholomew's version has a typing error since the left bracket must include $n_j n_m$ as proved in (3.9.9).

For the situation in which all samples are of equal size n , (3.9.9) reduces to:

$$\begin{aligned}
 S' &= 2 \sum_{j=1}^{k-1} \sum_{m=1+j}^k \left[n^2 (\bar{X}_{.m} - \bar{X}_{.j}) - \frac{n^2}{2} \right] \\
 S' &= 2n^2 \sum_{j=1}^{k-1} \sum_{m=1+j}^k (\bar{X}_{.m} - \bar{X}_{.j}) - \sum_{j=1}^{k-1} \sum_{m=1+j}^k n^2 \\
 S' &= 2n^2 \sum_{j=1}^{k-1} \sum_{m=1+j}^k (\bar{X}_{.m} - \bar{X}_{.j}) - \frac{k(k-1)n^2}{2} .
 \end{aligned} \tag{3.9.10}$$

Note that the double summation in (3.9.10) is over all positive combinations of pairwise differences in sample means. If $k = 3$, then $\sum_{j=1}^2 \sum_{m=2}^3 (\bar{X}_{.m} - \bar{X}_{.j}) = 2\bar{X}_{.3} - 2\bar{X}_{.1}$ and

$$S' = 4n^2 (\bar{X}_{.3} - \bar{X}_{.1}) - 3n^2 . \tag{3.9.11}$$

If $k = 4$, then $\sum_{j=1}^3 \sum_{m=2}^4 (\bar{X}_{.m} - \bar{X}_{.j}) = 3\bar{X}_{.4} + \bar{X}_{.3} - \bar{X}_{.2} - 3\bar{X}_{.1}$ and

$$S' = 2n^2 [(3\bar{X}_{.4} + \bar{X}_{.3} - \bar{X}_{.2} - 3\bar{X}_{.1}) - 3] . \tag{3.9.12}$$

It is therefore apparent that the sum of terms in the parentheses of (3.9.11) and (3.9.12) are the linear contrasts described for the One-Way One-Tail Regression Test (see section 3.3).

In general, for any k ,

$$\sum_{j=1}^{k-1} \sum_{m=1+j}^k (\bar{X}_{.m} - \bar{X}_{.j}) = \sum_{j=1}^k (c'_j \bar{X}_{.j}) \quad \text{where}$$

$$c'_j = [(1-k) + 2(j-1)] = (2j - k - 1) \tag{3.9.13}$$

for all $j = 1, 2, \dots, k$.¹⁹

Substituting (3.9.13) into (3.9.10),

$$S' = 2n^2 \sum_{j=1}^k (2j-k-1) \bar{X}_{.j} - \frac{k(k-1)n^2}{2} . \quad (3.9.14)$$

Now Bartholomew (1961A, p. 329) has stated that S' given in (3.9.9) can also be written as

$$S' = 2 \sum_{j=1}^k b_j n_j \bar{X}_{.j} - K \quad (3.9.15)$$

where K is a constant; $b_1 = -\sum_{r=2}^k n_r$; $b_k = \sum_{r=1}^{k-1} n_r$;

and $b_q = \sum_{r=1}^{q-1} n_r - \sum_{r=q+1}^k n_r$ for $q = 2, 3, \dots, k-1$.

For the case where all k samples are of the same size n it is easy to show that S' , written as in (3.9.15), is equivalent to that obtained in (3.9.14). Moreover, it then must follow that since the b_j coefficients are in an arithmetic progression (and are only a constant multiple of the C_j coefficients), S' written as (3.9.15) or (3.9.14) is the equivalent of the numerator term (i.e., the sums of squares of the contrast) of the RF' statistic given in (3.3.5).

If $n_j = n$ for all $j = 1, 2, \dots, k$, then S' from (3.9.15) becomes

$$S' = 2n \sum_{j=1}^k b_j \bar{X}_{.j} - K . \quad (3.9.16)$$

¹⁹If k is odd, $C'_j = 2 C_j$; if k is even, $C'_j = C_j$, where C_j are the linear contrast coefficients presented in section 3.3 under the assumption of equally spaced intervals between the k levels of the treatment variable. A constant multiplier of the coefficients has no effect on the results of the test procedure.

Now the b_j coefficients given in (3.9.15) have the form

$$((1-k)n + 2(j-1)n) = (2j - k - 1) n \quad (3.9.17)$$

for all $j = 1, 2, \dots, k$. (Comparing the b_j coefficients in (3.9.17) to the C'_j coefficients in (3.9.13), it is noted that the b_j are a multiple of n of the C'_j). Substituting this expression for the b_j into (3.9.16), after pulling out the constant n , yields

$$S' = 2n^2 \sum_{j=1}^k (2j - k - 1) \bar{X}_{.j} - K \quad (3.9.18)$$

so that the constant term, K , must be $\frac{k(k-1)}{2} n^2$ from (3.9.14).

3.10 PURI'S EXPECTED NORMAL SCORES TEST

Puri (1965) proposed a nonparametric procedure against ordered alternatives which is based primarily on the work of Jonckheere (1954A). His objective was to obtain a more powerful test and yet not have to make the stringent assumptions associated with a parametric procedure.²⁰ The test is developed as follows:

Let X_{ij} be the i^{th} observation in the j^{th} sample, $j = 1, 2, \dots, k$ and $i = 1, 2, \dots, n_j$.²¹ Now consider the k samples

²⁰Puri compares the asymptotic power of his U^* Test to that of Jonckheere (section 3.9), Bartholomew (section 3.2) and the One-Way F Test (section 3.1). The results show Puri's Test to be superior to its competitors when the true alternative is an equally spaced linear trend but inferior to E^2 when the true alternative is an end gap.

²¹While, in general, for this procedure it is not necessary to have k equal sized samples, this restriction has been used in this dissertation.

in pairs--there being $\binom{k}{2} = k(k-1)/2$ pairs of samples.

$$\text{Let } \xi_v^{(j,m)} = \begin{cases} 1 & \text{if the } v^{\text{th}} \text{ smallest observation in the} \\ & \text{combined } j^{\text{th}} \text{ and } m^{\text{th}} \text{ samples is from the} \\ & j^{\text{th}} \text{ sample;} \\ 0 & \text{otherwise.} \end{cases} \quad (3.10.1)$$

$$\text{and let } \eta_v^{(j,m)} = \begin{cases} -1 & \text{if the } v^{\text{th}} \text{ smallest observation in the} \\ & \text{combined } j^{\text{th}} \text{ and } m^{\text{th}} \text{ samples is from the} \\ & m^{\text{th}} \text{ sample;} \\ 0 & \text{otherwise.} \end{cases} \quad (3.10.2)$$

$$\text{Now consider } h_{jm} = T_{jm}^{(j)} + T_{jm}^{(m)} \quad (3.10.3)$$

$$\text{where } T_{jm}^{(j)} = \sum_{v=1}^{n_j+n_m} E_v^{(j,m)} \xi_v^{(j,m)} / n_j \quad \text{and} \quad T_{jm}^{(m)} = \sum_{v=1}^{n_j+n_m} E_v^{(j,m)} \eta_v^{(j,m)} / n_m$$

where the $E_v^{(j,m)}$ (with $v = 1, 2, \dots, n_j+n_m; j < m$) are "constants" having certain restrictions to be specified below.

Puri then establishes the test statistic U of the form

$$U = \sum_{j=1}^{k-1} \sum_{m=1+j}^k n_j n_m h_{jm} \quad (3.10.4)$$

for testing the null hypothesis that each of the samples are independently drawn from the continuous populations having identical c.d.f.s ($F_j(X) = F_k(X)$ for all $j = 1, 2, \dots, k$) against the ordered alternative that $F_1(X) \geq F_2(X) \geq \dots \geq F_k(X)$ (with at least one inequality strict).

Substituting for h_{jm} , U may be rewritten as:

$$U = \sum_{j=1}^{k-1} \sum_{m=1+j}^k n_j n_m \left[\frac{\sum_{v=1}^{n_j+n_m} E_v^{(j,m)} \xi_v^{(j,m)}}{n_j} + \frac{\sum_{v=1}^{n_j+n_m} E_v^{(j,m)} \eta_v^{(j,m)}}{n_m} \right] \quad (3.10.5)$$

Now if the "constants" $E_v^{(j,m)}$ are replaced by $V/(n_j + n_m)$ in (3.10.5) it may be noted that $U = -S/2$, where S is the test

statistic from Jonckheere's One-Way Test (see (3.9.2) and (3.9.3)). In particular, with all k samples of equal size n , the relationship between the two tests can be examined from (3.9.3) and

$$U = \sum_{j=1}^{k-1} \sum_{m=1+j}^k n_j^2 \left[\sum_{v=1}^{2n} \left(\frac{v}{2n^2} \right) \xi_v^{(j,m)} + \sum_{v=1}^{2n} \left(\frac{v}{2n^2} \right) \eta_v^{(j,m)} \right]$$

$$U = \frac{1}{2} \sum_{j=1}^{k-1} \sum_{m=1+j}^k \left[\sum_{v=1}^{2n} (v \xi_v^{(j,m)}) + \sum_{v=1}^{2n} (v \eta_v^{(j,m)}) \right] \quad (3.10.6)$$

However, if the "constants" $E_v^{(j,m)}$ are replaced by expected normal scores (see Appendix A2), Puri proposes a test with statistic U^* of the form: (3.10.7)

$$U^* = \sum_{j=1}^{k-1} \sum_{m=1+j}^k n_j n_m \left[\frac{\sum_{v=1}^{n_j+n_m} e_{v|n_j+n_m} \xi_v^{(j,m)}}{n_j} + \frac{\sum_{v=1}^{n_j+n_m} e_{v|n_j+n_m} \eta_v^{(j,m)}}{n_m} \right]$$

where $e_{v|n_j+n_m}$ is the expected value of the v^{th} smallest observation in the combined $n_j + n_m$ samples out of a "total" sample of N observations from a standardized normal distribution.

With all k samples of equal size n , the test statistic in (3.10.7) may be rewritten as:

$$U^* = n \sum_{j=1}^{k-1} \sum_{m=1+j}^k \left[\sum_{v=1}^{2n} e_{v|2n} \xi_v^{(j,m)} + \sum_{v=1}^{2n} e_{v|2n} \eta_v^{(j,m)} \right] \quad (3.10.8)$$

Upon careful examination of (3.10.8), the test statistic U^* can be put more simply as

$$U^* = n \sum_{j=1}^{k-1} \sum_{m=1+j}^k (e_{.j} - e_{.m}) \quad (3.10.9)$$

where $e_{.j} = \sum_{i=1}^n e_{ij}$ is the sum of the expected normal scores in the j^{th} sample and $e_{.m} = \sum_{i=1}^n e_{im}$ is the sum of the expected normal scores in the m^{th} sample. Thus, the test statistic U^* is readily obtained by first finding the "totals" for each sample and then summing over the $k(k-1)/2$ possible pairwise differences in sample "totals" (for $j < m$). The resulting value is then multiplied by n to yield U^* .

Note that under the alternative hypothesis the k samples are expected to be monotonically increasing and U^* should be a "very negative" number. Thus, the null hypothesis could be tested using (3.10.9) and a rejection made at some specified level of significance provided that $U^* \leq U_{CR}^*(\alpha)$, the α percent critical value from the null distribution of U^* . Since no tables exist for $U_{CR}^*(\alpha)$, the null distribution of U^* may be generated exactly by a randomization method or empirically via the computer. For $k = 3$ and $n = 2$, say, it is not difficult to list the 90 distinct arrangements out of all 720 possible permutations of $N = 6$ expected normal scores, and then obtain the exact null distribution of U^* by calculating the test statistic in each of these 90 instances. From the cumulative (null) distribution function the following "critical values" are presented along with the probability that any observed test statistic will be less than or equal to the "critical value".

U^*	$P_r (U_{OBS}^* \leq U^*)$
- 9.36	0.1111
-11.00	0.0889
-11.76	0.0667
-11.92	0.0556
-13.52	0.0333
-15.28	0.0111

As N gets larger it becomes tedious to obtain the exact null distribution and Monte Carlo methods may be used to obtain the empirical null distribution. This is accomplished by computing three replicates of 400 "histories" of the test statistic U^* under the null hypothesis for matrices of size $n \times k$ where $n = 5(1)10(5)20$ and $k = 3(1)7$. All the above empirical null distributions of U^* are tested for normality (using the Modified Shapiro-Wilk Test described in Appendix A3) and the results show no reason to reject that the empirical null distribution of U^* is normal--regardless of n and k . Thus, for each combination of n and k , the three null distributions are combined into one empirical distribution of U^* containing 1200 histories. Since $U^* \stackrel{d}{=} N(\mu_{U^*}, \sigma_{U^*}^2)$ and symmetrical about zero, a table of critical values $U^*(\alpha)$ can be obtained for each n and k by multiplying the Z value for a given α (one-tail) by the square root of an unbiased estimate of $\sigma_{U^*}^2$.²² The generated critical values $U^*(\alpha) = Z \hat{\sigma}_{U^*}$ are

²²The pooled variance of the three independent replicates of 400 observations of U^* is an unbiased estimator of $\sigma_{U^*}^2$. Since the variance is calculated for each replicate,

presented in Table A55 for $n = 5(1)10(5)20$, $k = 3(1)7$, and $\alpha = .01$ and $.05$. The null hypothesis may be rejected if the absolute value of the observed test statistic equals or exceeds the critical value for a given level of significance (provided U_{OBS}^* is negative). That is, reject if $|U_{OBS}^*| \geq U^*(\alpha)$ and $U_{OBS}^* < 0$.

3.11 JOHNSON-MEHROTRA EXPECTED NORMAL SCORES TEST

Johnson and Mehrotra (1971) developed a nonparametric test against ordered alternatives as a competitor to Puri's Test described in the previous section.²³ In essence, however, the Johnson-Mehrotra procedure is a contrast approach which utilizes the optimal contrasts developed by Abelson and Tukey (1963) as "weights" for the means of the expected normal scores in each sample.²⁴ Their test statistic, JM, is

22 (Cont'd) the three are combined as shown below based on the well-known statistical theorem that the variance of a sum is equal to the sum of the separate variances:

$$\hat{\sigma}_{U^*} = \left[\left(\sum_{i=1}^H (U_{i1}^* - \bar{U}_{.1})^2 + \sum_{i=1}^H (U_{i2}^* - \bar{U}_{.2})^2 + \sum_{i=1}^H (U_{i3}^* - \bar{U}_{.3})^2 \right) / (3H-3) \right]^{1/2}$$

where H is 400--the number of histories in each of the 3 replicates, U_{ij}^* is the i th observation of the test statistic in the j th replicate, and $\bar{U}_{.j} = \sum_{i=1}^H U_{ij}^* / H$ is the mean of the H test statistics in the j th replicate.

²³ Johnson and Mehrotra also proposed a nonparametric procedure against ordered alternatives which is an extension of Chacko's Test (section 3.8) in which the ranks of the observations are replaced by their corresponding expected normal order statistics.

²⁴ Johnson and Mehrotra compare the asymptotic power of their test to that of Puri (section 3.10), E^2 (section 3.2) and the One-Way F Test (section 3.1). They find their test to be equivalent to Puri's for $k = 3$ --regardless of the type of ordered alternative. However, for $k > 3$, they observe their procedure to be the best of the four against the end gap (cont'd)

given by:

$$JM = \sum_{j=1}^k M_j(\bar{e}_{.j}) \quad (3.11.1)$$

where M_j are the Abelson and Tukey "optimal contrasts":

$$M_j = \left\{ (j-1) \left[1 - ((j-1)/k) \right] \right\}^{\frac{1}{2}} - \left\{ j (1 - (j/k)) \right\}^{\frac{1}{2}} . \quad (3.6.1)$$

and $\bar{e}_{.j} = \sum_{i=1}^n e_{ij}/n$ is the mean of the expected normal scores in the j^{th} sample, $j = 1, 2, \dots, k$.

Thus, for a given α , the null hypothesis that the k samples are independently drawn from continuous parent populations with identical c.d.f.s ($F_j(X) \equiv$ for all $j = 1, 2, \dots, k$) can be tested against the ordered alternative that

$$F_1(X) \geq F_2(X) \geq \dots \geq F_k(X) \text{ (with at least one inequality strict).}^{25}$$

As in the previous section, no tables exist for the α percent critical values from the null distribution of JM .

²⁴(Cont'd) alternative but asymptotically inferior to both Puri's and Bartholomew's procedures against the trend alternative. However, if one examines their tables (1971, p. 21) it is observed that they are incorrect. That is, their test is superior to the E^2 Test against trend.

²⁵Some of the techniques examined in this dissertation require samples of equal size and thus, this restriction is needed for comparison purposes. The Johnson-Mehrotra Test, however, does not require the k samples to be of equal size because of the Schaafsma and Smid (1966) extension of the work of Abelson and Tukey.

Thus, the null distribution must either be generated exactly through randomization methods or empirically through Monte Carlo methods. Again, for $k = 3$ and $n = 2$, it is not hard to list the 90 distinct arrangements out of all 720 permutations of $N = 6$ expected normal scores. The exact null distribution of JM could then be gotten by calculating the test statistic for each of these 90 distinct matrices. From the cumulative (null) distribution the following "critical values" are presented along with the probability that any randomly observed test statistic will equal or exceed the "critical value":²⁶

JM	$P_r(JM_{OBS} \geq JM)$
0.997	0.1111
1.167	0.0889
1.248	0.0667
1.264	0.0556
1.434	0.0333
1.621	0.0111

When N gets large and it becomes unwieldy to obtain the exact null distribution for JM through randomization methods, Monte Carlo procedures are used to generate the empirical null distribution. This is accomplished by computing three replicates of 400 "histories" of the test statistic JM under the null hypothesis for matrices of size $n \times k$ where $n = 5(1)10(5)20$

²⁶A comparison of these results with those obtained from Puri's Test statistic U^* verifies Johnson and Mehrotra's statement that the two tests are identical whenever $k = 3$.

and $k = 3(1)7$. All (120) of the generated null distributions are tested for normality (see Appendix A3 for discussion of Modified Shapiro-Wilk Test) and, in almost all instances there is no evidence to reject the hypothesis that the empirical null distribution of JM is normal. Thus, for each combination of n and k the three replicates are combined into one empirical null distribution of JM containing 1200 histories. Since $JM \stackrel{d}{=} N(\mu_{JM}, \sigma_{JM}^2)$ and symmetrical about zero, a table of critical values can be gotten for each n and k by multiplying the Z value for a given α (one-tail) by the square root of an unbiased estimate of σ_{JM}^2 .²⁷ The generated critical values $JM(\alpha) = Z \hat{\sigma}_{JM}$ are presented in Table A56 for $n = 5(1)10(5)20$, $k = 3(1)7$, and $\alpha = .01$ and $.05$. The null hypothesis may then be rejected if the observed value of JM equals or exceeds the critical value for a given α .

²⁷The pooled variance of the three independent replicates of 400 observations of JM is an unbiased estimator of σ_{JM}^2 . Since the variance is computed for each replicate, the three are combined as shown below (see Footnote 22 in section 3.10):

$$\hat{\sigma}_{JM} = \sqrt{\frac{\sum_{j=1}^3 \sum_{i=1}^H (JM_{ij} - \bar{JM}_{\cdot j})^2}{3(H-1)}}$$

where H is 400--the number of histories in each of the three replications, JM_{ij} is the i th observation of the test statistic in the j th replicate and $\bar{JM}_{\cdot j} = \sum_{i=1}^H JM_{ij} / H$ is the mean of the H test statistics in the j th replicate.

3.12 FISHER AND YATES ONE-WAY TEST

In 1938, Fisher and Yates (1949) described a test procedure for data which either constitute only an ordinal level of measurement or data for which the researcher would rather draw conclusions only from the order of the observed values. The procedure consists of replacing the observed data with the corresponding expected normal scores (see Appendix A2) and then performing the analysis of variance as described in section 3.1, testing the null hypothesis that the k groups are identical against the general alternative that at least two of the groups differ from each other. The test statistic FY is computed from:

$$FY = \frac{n \sum_{j=1}^k (\bar{e}_{.j} - \bar{e}_{..})^2 / k-1}{\sum_{j=1}^k \sum_{i=1}^n (e_{ij} - \bar{e}_{.j})^2 / N-k} \stackrel{d}{=} F \text{ with } (k-1) \text{ and } (N-k) \text{ d.f.}, \quad (3.12.1)$$

where R_{ij} is the rank (from smallest (1) to largest (N)) corresponding to the i^{th} observation in the j^{th} sample and e_{ij} is the expected value of the observation having rank R_{ij} in a random sample from a standardized normal distribution with N observations. The null hypothesis may be rejected if $FY \geq F_{\alpha} [(k-1), (N-k)] \text{ d.f.}$, the critical F ratio for a specified level of significance.²⁸

It must be noted here that much discussion has been raised as to whether or not the F distribution is actually

²⁸ It is recalled from section 3.1 that F is transformed into an incomplete beta statistic to facilitate the computations.

appropriate for this kind of procedure since the values used are not random normal variates but rather expected normal variates. Among the advocates of the procedure given by Fisher and Yates are E.S. Pearson and H.O. Hartley (1954) and Harter (1961). Opposition to the procedure, however, comes from Kendall and Smith (1939), Bell and Doksum (1965) and Bradley (1968). Preliminary work in this dissertation, however, shows no significant discrepancies between the F distribution and the empirical null distribution for the test statistic FY. Hence, the F distribution is used here as the approximate null distribution of FY, as proposed by Fisher and Yates. That is:

$$FY \stackrel{d}{\approx} F \text{ with } (k-1) \text{ and } (N-k) \text{ d.f.}$$

$$\text{since } \lim_{N \rightarrow \infty} FY \stackrel{d}{=} F \text{ with } (k-1) \text{ and } (\infty) \text{ d.f. .}$$

3.13 BELL AND DOKSUM ONE-WAY TEST

Similar to Fisher and Yates, Bell and Doksum (1965) also describe a test procedure for data which constitute only an ordinal level of measurement or data which constitute a higher level of measurement but for which the researcher would rather draw conclusions only from the order of the observed values. The distinguishing feature between the two methods, however, is the "form" in which the data are selected. Whereas Fisher and Yates replace the observed data by the corresponding expected normal scores (i.e., expected normal order statistics), Bell and Doksum replace the observed data by the

corresponding random normal order statistics--a method initially suggested by A.S.C. Ehrenberg (1951). Bell and Doksum did not believe the F distribution to be appropriate for the Fisher-Yates Test and felt that a randomization procedure was necessary for obtaining the critical values for the FY statistic of the previous section. Thus, they developed their nonparametric procedure for testing the null hypothesis that the k groups are identical against the general alternative that at least two of the k groups differ from each other. Such a test was found to have the same good A.R.E. properties (>1.0 relative to the One-Way F Test) as that given by Fisher and Yates. The test statistic BD is given by:

$$BD = n \sum_{j=1}^k (\bar{Z}_{.j} - \bar{Z}_{..})^2 \triangleq \chi^2 \text{ with } (k-1) \text{ d.f. ,} \quad (3.13.1)$$

where Z_{ij} is the random normal order statistic corresponding to the i^{th} observation in the j^{th} sample, $\bar{Z}_{.j} = \sum_{i=1}^n Z_{ij} / n$, and $\bar{Z}_{..} = \sum_{j=1}^k \sum_{i=1}^n Z_{ij} / N$.

It is noted that the BD Test statistic resembles the sum of squares term in the numerator of test procedures which take on the F distribution (see sections 3.1 and 3.12). It is unnecessary to use an analysis of variance procedure here, however, since the denominator of such a procedure represents the pooled variance estimator of the unknown common variance --which does not exist here since the random sample of standardized normal order statistics has a known population variance of one imposed by the test procedure.

Considerable discussion has been raised over the employment of random normal order statistics as replacements for the original observed data. While the Bell and Doksum Test is shown to have good A.R.E. properties (as $N \rightarrow \infty$), Jogdeo (1966) shows that the "superimposed noise" of the additional (random normal) sample may create undesirable properties for this test when the sample size is not very large. Moreover, Jogdeo makes the rather interesting but intuitively distressing point that with such a procedure, which depends on both the ranks of the original data and the particular sample of random normal order statistics selected to replace the original data, two statisticians may arrive at different conclusions even though they are given the same original set of data to analyze! This can occur because of the introduction of "noise"--the fact that the first random normal sample may differ from the second.²⁹

3.14 A ONE-SIDED k SAMPLE KOLMOGOROV-SMIRNOV TEST

Conover (1967) and Wolf (1967) independently extended the One-Sided Two Sample Kolmogorov-Smirnov Test to that of k samples by deriving the null distribution for the test

²⁹Conover (1971) states that while there does not appear to be any empirical study as to how this "additional unwarranted variation" would effect the BD statistic, he believes that this "noise factor" should be small when the number of observations is reasonably large. Thus, it seems that further empirical research is needed to examine this property.

statistic

$$D_{*}^{+}(k,n) = \sup_{x; j < k} [F_j^{*}(x) - F_{j+1}^{*}(x)] \quad (3.14.1)$$

where $F_j^{*}(x)$ and $F_{j+1}^{*}(x)$ are the empirical cumulative distribution functions of the j^{th} and $j + 1^{\text{th}}$ samples. The procedure involves testing the null hypothesis that each of the k independent samples of equal size n are randomly selected from continuous populations with identical c.d.f.s against the alternative that the j^{th} population tends to have smaller values than the $j + 1^{\text{th}}$ population, for at least one value of j (where $j = 1, 2, \dots, k-1$). That is,

H_0 : $F_j(x) = F_{j+1}(x)$ where $F_j(x)$ is the c.d.f. for the j^{th} population ($j = 1, 2, \dots, k$);

H_a : $F_1(x) \geq F_2(x) \geq \dots \geq F_k(x)$ with at least one inequality strict.

The test consists of first ordering all n observations (from the smallest to the largest) within each sample so as to obtain the empirical c.d.f.s $F_j^{*}(x)$ of all j samples ($j = 1, 2, \dots, k$). Secondly, a count is made of the number of times out of n that an observation from $F_j^{*}(x)$ is exceeded by observations from $F_{j+1}^{*}(x)$ and the largest of these values is recorded. Such comparisons are made consecutively for all adjacent pairs of empirical distribution functions (1 vs. 2, 2 vs. 3, ..., j vs. $j+1$, ..., $k-1$ vs. k) and the statistic $D_{*}^{+}(k, n)$ is defined as the maximum of these recorded values. That is, $D_{*}^{+}(k, n)$ may be considered as the largest vertical distance attained by $F_j^{*}(x)$ over $F_{j+1}^{*}(x)$ for all adjacent

pairs of samples.

Since $D_{*}^{+}(k, n)$ is discrete, ranging from $0/n$ to n/n , its exact null distribution may be obtained. Conover derived the exact distribution of $D_{*}^{+}(k, n)$ based on the expression:

$$P_r \left\{ D_{*}^{+}(k, n) < \frac{c}{n} \right\} = \left| B^{k \times k} \right|, \quad (3.14.2)$$

where $\left| B^{k \times k} \right|$ is the determinant of the $k \times k$ matrix B having elements b_{ij} (where $b_{ij} = 0$ only if $n - (i-j)c' < 0$ or $b_{ij} = n!/(n-(i-j)c')!$ if $n - (i-j)c' \geq 0$, and c' is defined as the smallest integer $\geq c$).

Wolf and Naus (1973) give tables of critical c values for $k = 3(1)10$ and $n = 5(1)20$ (see Table A57). These critical values are the smallest integers such that

$$P_r \left\{ D_{*}^{+}(k, n) \geq \frac{c}{n} \right\} \leq \alpha. \quad (3.14.3)$$

In addition, Wolf and Naus give the exact probabilities associated with the critical c values. On the other hand, Conover (1971) has also tabled "critical values" for $k = 2(1)10$ and $n = 3(1)10(2)20(5)50$. This table, however, should be used with caution since Conover's "critical values" must be exceeded to cause rejection of the null hypothesis.

It should be noted that rather than being sensitive only to differences in location, the Kolmogorov-Smirnov type test is designed to detect all kinds of differences (location, dispersion, skewness and kurtosis) that may exist between

$F_j^*(X)$ and $F_{j+1}^*(X)$. Hence, the test may not be as sensitive to detecting ordered differences in location as other procedures described in this chapter but may, in fact, be even more sensitive if ordered location differences are accompanied by ordered differences in either dispersion, skewness, or kurtosis.³⁰ Aside from the sensitivity aspect (with ordered location alternatives) the other disadvantage of the test procedure is that equal size samples are required for each of the k groups.

3.15 COX AND STUART'S WEIGHTED SIGN TEST

Cox and Stuart (1955) developed three sign tests against trend alternatives. Their objective was to present a group of procedures which were both easy to understand and quick to compute. The best of their three procedures is the Weighted Sign Test which Cox and Stuart claim has an A.R.E. of 0.86 when compared to the Regression Test presented in section 3.3. Their two other procedures, both "equally-weighted" tests, will be discussed in sections 3.16 and 3.17. The methodology and assumptions given below are, however, common to all three tests.

Methodology Underlying the Three Cox and Stuart Sign Tests

Given a set of observations $Y_1, Y_2, \dots, Y_N$ (where $N = nk$) listed in the sequence in which they are obtained

³⁰See Conover (1971, p. 317) and Wolf and Naus (1973, Section 3).

(Y_1 is selected first and Y_N last), it is desired to test a null hypothesis of "randomness" against, say, the alternative that an upward trend exists in the sequence. This could be detected if the observations drawn later in the sequence are larger than those drawn earlier. On the other hand, if no trend is present in the observed set of data, it is as likely that an observation obtained later in the sequence exceeds one selected earlier as it is that one selected earlier exceeds one drawn later. This can be expressed in the form of a null and alternative hypothesis as follows:

H_0 : $P(Y_E < Y_L) = P(Y_E > Y_L)$ where E represents "early" and L represents "late."

H_a : $P(Y_E < Y_L) > P(Y_E > Y_L)$.

That is, either no trend exists or there is an upward trend.

Assumptions Required for the Adaptation of the Three Tests

In order to utilize the Cox and Stuart procedures as an appropriate set of tests against ordered location alternatives in a completely randomized design, it is necessary to assume that a sequence of N observations X_{ij} can be readily formed by "stringing" together the k sample groups according to the direction predicted by the alternative hypothesis.

Thus, the N observations would appear as follows:

$$\left\{ \begin{array}{l} Y_1, \quad Y_2, \quad Y_3, \dots, Y_n, \quad Y_{n+1}, \quad Y_{n+2}, \quad Y_{n+3}, \dots, Y_{2n}, \dots, Y_{(k-1)n+1}, \\ X_{11}, \quad X_{21}, \quad X_{31}, \dots, X_{n1}, \quad X_{12}, \quad X_{22}, \quad X_{32}, \dots, X_{n2}, \dots, X_{1k}, \end{array} \right\}$$

$$\left\{ \begin{array}{l} Y_{(k-1)n+2}, \quad Y_{(k-1)n+3}, \dots, Y_N \\ X_{2k}, \quad X_{3k}, \dots, X_{nk} \end{array} \right\}.$$

Moreover, under a true ordered alternative the observations on the right side of the above sequence would be significantly larger than the observations on the left side, while under a true null hypothesis there is just as much chance for observations on the right side being larger than those on the left side of the sequence as the reverse situation.

Assumptions Underlying the Three Cox and Stuart Sign Tests

While Cox and Stuart developed their procedures based on observations drawn from continuous distributions (so as to avoid the problem of ties (1955, p. 81)), Conover (1971) claims that the observed sequence of data should at least follow an ordinal scale of measurement. The only other assumptions are that the observations are randomly drawn (assigned to a treatment group) and that whether or not a particular observation is larger or smaller than the observation on the "other side" of the sequence that it is being compared to is independent of the outcome for any other pair.

Development of the Three Cox and Stuart Sign Tests

Let $N' = N$ if N is an even number. However, define $N' = N-1$ if N is an odd number. In the latter case, the middle observation in the sequence is discarded so that an even number of observations N' may be studied. Further, let $m = \frac{N'}{2} + 1$. Now define for $l < m$ the score

$$\delta_{lm} = \begin{cases} +1 & \text{if } Y_l < Y_m \\ 0 & \text{if } Y_l > Y_m \end{cases} \quad (3.15.1)$$

All three tests are concerned only with the comparison of

independent pairs of observations. Thus, no observation is compared with more than one other observation and, therefore, the total number of independent comparisons cannot exceed $\frac{N'}{2}$. Furthermore, all three tests are nonparametric since they are all based on $\delta_{\ell m}$ which, under the null hypothesis, is a Bernoullian variate (0-1) with probabilities .5 and .5, no matter what population the X_{ij} are drawn from.

Cox and Stuart's Weighted Sign Test

Based on (3.15.1) let

$$\delta_{\ell, N'-\ell+1} = \begin{cases} +1 & \text{if } Y_{\ell} < Y_{N'-\ell+1} \\ 0 & \text{if } Y_{\ell} > Y_{N'-\ell+1} \end{cases} \quad (3.15.2)$$

WS, the Cox and Stuart Weighted Sign Test statistic, is now defined as:

$$WS = \sum_{\ell=1}^{\frac{N'}{2}} (N' - 2\ell + 1) \delta_{\ell, N'-\ell+1} \quad (3.15.3)$$

where the weights, $N' - 2\ell + 1$, are the distances between the two observations being compared.

Under the null hypothesis, the test statistic WS is approximately normally distributed with $\mu = \frac{N'^2}{8}$ and $\sigma^2 = \frac{N'(N'^2-1)}{24}$.

Hence,

$$Z_{ws} = \frac{WS - \frac{N'^2}{8}}{\sqrt{\frac{N'(N'^2-1)}{24}}} \stackrel{d}{=} N(0, 1) \quad (3.15.4)$$

While the WS statistic approaches normality as $N' \rightarrow \infty$, preliminary work in this dissertation has shown that the approach to normality is quite rapid, even for small $N = nk$. The empirical null distribution is developed for the WS statistic

for $n = 5(1)10(5)20$ and $k = 3(1)7$, and, using the Modified Shapiro-Wilk Test for normality (see Appendix A3) it is determined that only when $n=5$ and $k=3$ is the normal curve not a good fit for the standardized WS statistic. In this case $N' = 14$ and there are only $\frac{N'}{2} = 7$ independent comparisons. Hence, the empirical standardized critical values are obtained for the WS Test statistic based on 1200 observations from the empirical null distribution. Adopting a randomized procedure to obtain the nominal levels of significance when $N' = 14$, the critical value at the 5% level of α is 1.73 for 81.8% of the generated test statistics and 1.64 for 18.2% of these histories. At the 1% level, the critical values are 2.29 for 44.4% of the histories and 2.20 for 55.6% of the generated statistics.

3.16 COX AND STUART'S UNWEIGHTED SIGN TEST

In addition to their Weighted Sign Test, Cox and Stuart devised two "unweighted" (or rather, "equally weighted") sign tests which enjoy the properties of simplicity, ease in calculation, and speed in calculation. The first of these procedures, the Unweighted Sign Test, is developed as follows:

As in section 3.15, let $N' = N$ if N is even and let $N' = N-1$ if N is odd (discarding the middle value of the sequence.) Based on (3.15.1) let

$$\delta_{\ell, \frac{N'}{2} + \ell} = \begin{cases} +1 & \text{if } Y_{\ell} < Y_{\frac{N'}{2} + \ell} \\ 0 & \text{if } Y_{\ell} > Y_{\frac{N'}{2} + \ell} \end{cases} \quad (3.16.1)$$

The Cox and Stuart Unweighted Sign Test statistic US is now defined as:

$$US = \sum_{\ell=1}^{\frac{N'}{2}} \delta_{\ell, \frac{N'}{2} + \ell} \quad (3.16.2)$$

Hence, US is clearly a binomial random variable whose values range from zero to $\frac{N'}{2}$. Under the null hypothesis, the probability is .5 that the Bernoullian variable $\delta_{\lambda, \frac{N'}{2} + \ell}$ takes the value +1 (when $Y_\lambda < Y_{\frac{N'}{2} + \ell}$). Thus, to find the probability of observing a particular value (US_v) of US or larger is found by:

$$P_r(US \geq US_v) = \sum_{v=US_v}^{\frac{N'}{2}} \binom{\frac{N'}{2}}{US_v} (.5)^{\frac{N'}{2}} \quad (3.16.3)$$

and the null hypothesis is rejected at the chosen α level of significance if $P_r(US \geq US_v) \leq \alpha$.

Since the binomial distribution is discrete and yields exact probabilities, a randomization procedure is used so that the empirical power function of the test at the nominal 5% and 1% levels of significance could be compared more readily to other procedures presented in this chapter.

When $\frac{N'}{2}$ exceeds twenty, a normal approximation to the binomial is utilized and the test statistic evaluated as follows--employing a correction for continuity:

$$Z_{us} \cong \frac{(|US_v - (.5)(\frac{N'}{2})| - .5)}{\sqrt{(.5)(.5)(\frac{N'}{2})}} = \frac{(|US_v - \frac{N'}{4}| - .5)}{\sqrt{\frac{N'}{8}}} \quad (3.16.4)$$

3.17 COX AND STUART'S BEST UNWEIGHTED SIGN TEST

Cox and Stuart show that the A.R.E. of their Unweighted Sign Test is 0.78 when compared to the Regression Test of section 3.3. Moreover, the A.R.E. of US when compared to WS is 0.91. In this section Cox and Stuart's Best Unweighted Sign

Test is presented. This test is shown to have A.R.E.s of 0.83 and 0.96, when respectively compared to the Regression Test and the Weighted Sign Test (Cox and Stuart (1955), p. 85).

The objective of this procedure is to compare the observations in the first third of the sequence with the observations in the last third of the sequence while disregarding the observations in the middle third of the sequence. It is interesting to note that "throwing away" data in the middle of the sequence leads to an increase in power efficiency over the US Test because the observations compared are now further apart and more likely to show a difference between the left side and right side of the sequence listed in section 3.15 when the ordered location alternative is actually true.

The test is developed as follows:

Let T equal the largest integer in $\frac{N}{3}$. Based on (3.15.1) let

$$\delta_{\ell, N-T+\ell} = \begin{cases} +1 & \text{if } Y_{\ell} < Y_{N-T+\ell} \\ 0 & \text{if } Y_{\ell} > Y_{N-T+\ell} \end{cases} . \quad (3.17.1)$$

BUS, the Cox and Stuart Best Unweighted Sign Test statistic, is thus determined by:

$$BUS = \sum_{\ell=1}^T \delta_{\ell, N-T+\ell} \quad (3.17.2)$$

which is also a binomial random variable. To find the probability of a particular value (BUS_v) of BUS or larger, compute

$$P_r (BUS \geq BUS_v) = \sum_{v=BUS_v}^T \binom{T}{BUS_v} (.5)^T \quad (3.17.3)$$

and the null hypothesis is rejected at the chosen α level of significance if $P_r(BUS \geq BUS_v) \leq \alpha$.

As in the US Test, a randomization procedure on α is needed so that the empirical power of the BUS Test can be compared with other tests in this chapter.

Moreover, for $T > 20$, a normal approximation to the binomial, corrected for continuity, is employed when computing the BUS Test statistic. Hence,

$$Z_{BUS} \cong \frac{(|BUS_v - (.5)T| - .5)}{\sqrt{(.25)T}} \quad (3.17.4)$$

3.18 MOSTELLER'S k SAMPLE SLIPPAGE TEST

Mosteller (1948) devised a test for determining whether one of the k groups has "slipped to the right" of the others under the null hypothesis that the observations in the k sample groups are randomly drawn from identically distributed continuous populations. Mosteller coined the phrase "the problem of the greatest one" to describe the statistical question connected with a slippage test--the ability to select the best from among several (k) processes under investigation--and he believed that this problem was of considerable practical importance (1948, p. 58). Mosteller's k Sample Slippage Test is presented in this dissertation because it attempts to deal with a particular k sample location problem--the gap alternative.

Mosteller's Slippage Test is nonparametric. It has as

its chief advantages the fact that it is both quick and easy to use. On the other hand, however, Mosteller did not believe that the technique would be very powerful. An additional drawback to the procedure is that samples of equal size are necessary for each of the k groups. The test is developed as follows:

It is desired to test the null hypothesis that all k samples come from populations with the same location parameter against the alternative that one of the populations from which the samples are selected has a larger location parameter than the others. The test consists of finding the sample group with the largest observation and, in that group, determining how many of the n observations are larger than all the observations in the $k-1$ other sample groups. Letting this value be r , the null hypothesis may be rejected in favor of the ordered (end gap) alternative if $r \geq r_{\text{crit}}$. If, however, for some chosen α , $r < r_{\text{crit}}$, the null hypothesis is not rejected. Alternatively, one may compute $P_r(r)$, the probability that the sample group containing the largest of the $N = nk$ observations has r or more observations which exceed all the observations in the remaining $k-1$ sample groups. $P_r(r)$ is given by:

$$P_r(r) = \frac{k \binom{N-r}{n-r}}{\binom{N}{n}} = \frac{k (N-r)! n!}{N! (n-r)!} . \quad (3.18.1)$$

$P_r(r)$ may then be compared against α , the chosen level of significance, and the null hypothesis rejected if $P_r(r) \leq \alpha$.

It should be noted here that since r is a discrete random variable ranging from 1 to n , $P_r(r)$ yield exact probabilities. Thus, to determine the empirical power functions of the test using the nominal 5% and 1% levels of significance, a randomization procedure is employed. In addition, while Mosteller presented critical r values with their respective probabilities for various sample sizes, n , and $k = 3(1)6$, these have been extended in this dissertation so as to include all sample sizes, n , from 5(1)20 with the number of sample groups, k , from 3(1)7. This is presented in Table A58.

3.19 CONOVER'S FIRST k SAMPLE SLIPPAGE TEST

Conover (1968) presented two k sample slippage tests--the first of which is given here. This nonparametric procedure also has as its major features ease in understanding and ease in computation.³¹ Its major disadvantage is that each of the k samples must be of equal size n . As in Mosteller's Test, it is assumed that the observations in the k sample groups are randomly drawn from identically distributed continuous populations and that the data constitute at least an ordinal level of measurement. The null hypothesis to be tested is that the k samples come from populations having identical distribution functions against the alternative that at least

³¹A major advantage to this procedure is that it may be used as an a posteriori multiple comparison test once the null hypothesis is rejected in order to determine how the k populations from which the samples are selected may be grouped.

one of the populations has "slipped to the right" of at least one of the others. The test consists of first finding the largest observation in each of the k samples and then ranking the k samples according to their extreme values--the sample with the largest extreme having rank 1 and the sample with the smallest extreme having rank k . The procedure is then to count the number of observations in the sample containing the largest extreme value that exceeds the smallest extreme value. This count yields the test statistic $M(1,k)$ which may be compared to a table of critical values, M_{CR} , and the null hypothesis rejected if $M(1,k) \geq M_{CR}$ for a specified α level of significance.

Since the test statistic $M(1,k)$ is discrete and ranges from 1 to n its critical values, M_{CR} , are obtained based on exact probabilities. Hence, it becomes necessary to employ a randomization procedure in order to compare the empirical power functions (at nominal 5% and 1% levels of significance) of Conover's First k Sample Slippage Test with those of other procedures presented in this chapter for the completely randomized design.

Whereas Conover presents critical values for a variety of sample sizes (1968, pp. 617-618), Table A59 extends these to include values of n from 11(2)19--heretofore unpublished. These critical values and their associated probabilities are derived from the null distribution of the test statistic $M(1,k)$ based on the following equation:

$$P_r \left\{ M(1, k) \geq M_{CR} \right\} = \sum_{\beta=0}^{k-2} \frac{(n-1)_{(M_{CR}-1)} (k-1)_{(k-1)} (-1)^{k-2-\beta}}{(n(k-\beta)-1)_{(M_{CR}-1)} (k-1-\beta) \beta! (k-2-\beta)!} \quad (3.19.1)$$

where any term $A_{(i)}$, say, represents the falling factorial such that $A_{(i)} = A(A-1)(A-2)\dots(A-i+1)$.

Thus, (3.19.1) can be rewritten as:

$$P_r \left\{ M(1, k) \geq M_{CR} \right\} = \sum_{\beta=0}^{k-2} \frac{(n-1)! (k-1)! (-1)^{k-2-\beta} (n(k-\beta) - M_{CR})!}{(n - M_{CR})! (n(k-\beta) - 1)! (k-1-\beta) \beta! (k-2-\beta)!} \quad (3.19.2)$$

These then are the probabilities of obtaining a test statistic equal to or larger than M_{CR} .

CHAPTER IV

k SAMPLE TECHNIQUES FOR A RANDOMIZED BLOCK DESIGN

In the previous chapter statistical procedures appropriate for a completely randomized design were described. In the present chapter, however, nineteen statistical techniques which are appropriate for a randomized block design with one observation per cell will be presented.¹ As shown in Table 4.1, these procedures may be categorized according to family as well as type of alternative--general (1.1.2) or ordered (1.1.3).

Of the six parametric procedures presented in Table 4.1 (p. 83), all but the Two-Way F Test are tests designed against ordered location alternatives. Similarly, among the thirteen nonparametric techniques presented, all but Friedman's Test, the Fisher and Yates Two-Way Test, the Bell and Doksum Two-Way Test, and a General Aligned Rank Order Test are tests against ordered alternatives. These procedures are included here, however, for purposes of comparison amongst themselves as well as with tests designed against ordered location alternatives.

¹For a discussion of these two types of experimental design models, see Appendix A1.

TABLE 4.1
CLASSIFICATION OF TWO-WAY PROCEDURES^a
BY FAMILY & BY ALTERNATIVE

PARAMETRIC METHODS		NONPARAMETRIC METHODS							
		SIGN TESTS		RANK TESTS		SCORES TESTS		ALIGNED TESTS	
GENERAL	ORDERED	GENERAL	ORDERED	GENERAL	ORDERED	GENERAL	ORDERED	GENERAL	ORDERED
F2WAY	ATMX2	PSIGN		FRIED	JONK2	FYNS2	PHENS	SMSAT	SENAT
	LRCN2				PAGET	BDNS2			
	DERG2				HOLLT				
	QERG2				DOKST				
	HPARA				SHORT				
					PTRST				

^aThe name of the procedures represented by the coded values in this table are given in Table 1.1.

PARAMETRIC PROCEDURES

4.1 THE TWO-WAY F TEST

When the experimental design layout is randomized blocks with one observation per cell the most frequently used procedure for testing for differences between k group means is the Two-Way F Test. The test statistic for this procedure is computed from:

$$F = \frac{n \sum_{j=1}^k (\bar{x}_{.j} - \bar{x}_{..})^2 / k - 1}{\sum_{i=1}^n \sum_{j=1}^k (x_{ij} - \bar{x}_{i.} - \bar{x}_{.j} + \bar{x}_{..})^2 / (n-1)(k-1)} \stackrel{d}{=} F \text{ with } (k-1) \text{ and } (n-1)(k-1)$$

(4.1.1)

The computed F ratio is then compared with a critical F ratio for a stated level of significance and appropriate degrees of freedom and the null hypothesis of no differences among the means may be rejected in favor of the general alternative that at least one of them differs from one other if

$$F \geq F_{\alpha}[(k-1), (n-1)(k-1)] \text{ d.f., the critical F.}$$

A comparison of the denominators of (3.1.1) and (4.1.1) clearly distinguishes the One-Way F Test from the Two-Way F Test.

4.2 HOLLANDER'S PARAMETRIC TEST

Hollander (1967) developed a parametric approach for testing that there are no differences in treatment effects against the alternative that the treatment effects are monotonically increasing. The test is described as follows:

$$\text{Let } X_{ij} = b_i + (j-1) \phi + E_{ij} \quad (4.2.1)$$

where b_i is the "nuisance parameter" for the i^{th} block, $\emptyset$ is the treatment effect, and the E_{ij} are independently and identically distributed according to $N(0, \sigma^2)$, the normal distribution with mean of zero and variance of σ^2 .

The likelihood ratio statistic t for testing $\emptyset = 0$ is now defined by:

$$t = \frac{\hat{\emptyset}}{\hat{\sigma}_{\hat{\emptyset}}} \triangleq t \quad \text{with } (n-1)(k-1) \text{ d.f.} \quad (4.2.2)$$

where $\hat{\emptyset}$ is the least squares estimate of $\emptyset$ given by

$$\hat{\emptyset} = 6 \sum_{i=1}^n \sum_{j=1}^k (2j-k-1) X_{ij} / N(k^2-1), \quad (4.2.3)$$

and $\hat{\sigma}_{\hat{\emptyset}}$, the estimate of the standard deviation of $\hat{\emptyset}$, is given by

$$\hat{\sigma}_{\hat{\emptyset}} = \sqrt{12 S^2 / N(k^2-1)}, \quad (4.2.4)$$

where

$$S^2 = \sum_{i=1}^n \sum_{j=1}^k (X_{ij} - \hat{b}_i - (j-1)\hat{\emptyset})^2 / (n(k-1)-1) \quad (4.2.5)$$

and

$$\hat{b}_i = \left(\sum_{j=1}^k X_{ij} / k \right) - (k-1) \frac{\hat{\emptyset}}{2}, \quad (4.2.6)$$

the estimate of the "nuisance parameter" in the i^{th} block.

The null hypothesis may be rejected at some chosen level of significance, α , if $t \geq t_{\alpha} [(n-1)(k-1)]$ d.f., the upper α value of the t distribution with $(n-1)(k-1)$ degrees of freedom.

While Hollander's Parametric Test appears to be somewhat more cumbersome than some of the other approaches

described in this chapter, it must be pointed out, however, that Hollander has found this test to have good A.R.E. properties.

4.3 TWO-WAY ONE-TAIL REGRESSION TEST

The Two-Way One-Tail Regression Test is analogous to its (one-way) parametric counterpart presented in section 3.3 for the situation where the experimental design is of the completely randomized type. Again, two approaches may be used to determine the test statistic--a linear regression approach or a linear contrast approach. The test statistic RF, developed under the former, is given by:

$$RF = \frac{\sqrt{n} b \left[\sum_{j=1}^k W_j^2 - \frac{\left(\sum_{j=1}^k W_j \right)^2}{k} \right]^{\frac{1}{2}}}{\left[\sum_{i=1}^n \sum_{j=1}^k (X_{ij} - \bar{X}_{i.} - \bar{X}_{.j} + \bar{X}_{..})^2 / (n-1)(k-1) \right]^{\frac{1}{2}}} \stackrel{d}{=} t \quad \text{with } (n-1)(k-1) \text{ d.f.} \quad (4.3.1)$$

whereas under the linear contrast approach

$$RF = \frac{\sum_{j=1}^k C_j X_{.j} / \left(n \sum_{j=1}^k C_j^2 \right)^{\frac{1}{2}}}{\left[\sum_{i=1}^n \sum_{j=1}^k (X_{ij} - \bar{X}_{i.} - \bar{X}_{.j} + \bar{X}_{..})^2 / (n-1)(k-1) \right]^{\frac{1}{2}}} \stackrel{d}{=} t \quad \text{with } (n-1)(k-1) \text{ d.f.} \quad (4.3.2)$$

A table of C_j coefficients for $k = 3(1)7$ is given in section 3.3. It is noted that the numerators are the same as was presented in (3.3.6) and (3.3.7) while the denominator (without the radicand) is the same as that in the Two-Way F Test (see (4.1.1)).

The null hypothesis of equality among the k means may be rejected in favor of the specified alternative

$$\mu_{.1} \leq \mu_{.2} \leq \cdots \leq \mu_{.k} \quad (\text{with at least one inequality strict})$$

if $RF \geq t_{\alpha} [(n-1)(k-1)] \text{ d.f.}$, the upper α value of the t distribution with $(n-1)(k-1)$ degrees of freedom.

4.4 TWO-WAY LINEAR-2 CONTRAST

The Two-Way Linear-2 Contrast is similar to its one-way parametric counterpart presented in section 3.4. The test statistic DL may be computed by:

$$DL = \frac{\sum_{j=1}^k d_j X_{.j} / (n \sum_{j=1}^k d_j^2)^{\frac{1}{2}}}{\left[\sum_{i=1}^n \sum_{j=1}^k (x_{ij} - \bar{X}_{i.} - \bar{X}_{.j} + \bar{X}_{..})^2 / (n-1)(k-1) \right]^{\frac{1}{2}}} \stackrel{d}{=} t \text{ with } (n-1)(k-1) \text{ d.f.} \quad (4.4.1)$$

and a table of d_j coefficients for $k = 3(1)7$ is presented in section 3.4.

The null hypothesis is rejected at the α level of significance if $DL \geq t_{\alpha} [(n-1)(k-1)] \text{ d.f.}$, the upper α value of the t distribution having $(n-1)(k-1)$ degrees of freedom.

4.5 TWO-WAY LINEAR 2-4 CONTRAST

Except for the selection of coefficients for the contrast, the Two-Way Linear 2-4 Contrast procedure is otherwise similar to the Regression Test and the Linear-2 Contrast procedures described in the previous two sections. In fact, when $k = 3$, all three procedures yield identical results and when $k = 4$ or 5 , the Linear-2 Contrast is equivalent to the Linear 2-4 Contrast given here.

A discussion of the Linear 2-4 Contrast approach is presented in section 3.5 along with a table of coefficients, q_i , for $k = 3(1)7$. The test statistic QD is given by:

$$QD = \frac{\sum_{j=1}^k q_j X_{.j} / (n \sum_{j=1}^k q_j^2)^{\frac{1}{2}}}{\left[\sum_{i=1}^n \sum_{j=1}^k (X_{ij} - \bar{X}_{i.} - \bar{X}_{.j} + \bar{X}_{..})^2 / (n-1)(k-1) \right]^{\frac{1}{2}}} \stackrel{d}{=} t \text{ with } (n-1)(k-1) \text{ d.f.} \quad (4.5.1)$$

and the null hypothesis rejected if $QD \geq t_{\alpha} [(n-1)(k-1)] \text{ d.f.}$ for a specified level of significance.

4.6 ABELSON AND TUKEY TWO-WAY OPTIMAL CONTRAST

The "maximin" or "optimal" contrast coefficients, M_j , developed by Abelson and Tukey (1963) from

$$M_j = \left\{ (j-1) \left[1 - ((j-1)/k) \right] \right\}^{\frac{1}{2}} - \left\{ j(1 - (j/k)) \right\}^{\frac{1}{2}} \quad (3.6.1)$$

can also be used when the experimental design format is randomized block. A discussion of the procedure as applied to a completely randomized design is presented in section 3.6 along with a table of M_j coefficients for $k = 3(1)7$.

The test statistic for the Abelson and Tukey Two-Way Optimal Contrast is:

$$AT = \frac{\sum_{j=1}^k M_j X_{.j} / (n \sum_{j=1}^k M_j^2)^{\frac{1}{2}}}{\left[\sum_{i=1}^n \sum_{j=1}^k (X_{ij} - \bar{X}_{i.} - \bar{X}_{.j} + \bar{X}_{..})^2 / (n-1)(k-1) \right]^{\frac{1}{2}}} \stackrel{d}{=} t \text{ with } (n-1)(k-1) \text{ d.f.} \quad (4.6.1)$$

and the null hypothesis of equality among the k means can be rejected in favor of the alternative that the k means are monotonically increasing ($\mu_1 \leq \mu_2 \leq \dots \leq \mu_k$ with at least one inequality strict) if $AT \geq t_{\alpha} [(n-1)(k-1)] \text{ d.f.}$, the upper α value of the t distribution with $(n-1)(k-1)$ degrees of freedom.

NONPARAMETRIC PROCEDURES

4.7 FRIEDMAN'S TEST

Friedman (1937) developed a nonparametric competitor of the Two-Way F Test which would be useful for testing against general alternatives when the data collected are only ordinal or when the investigator does not desire to make the more stringent assumptions underlying the parametric procedure.²

The test is developed as follows:

Let X_{ij} be the observed value under the j^{th} treatment in the i^{th} block (where $j = 1, 2, \dots, k$ and $i = 1, 2, \dots, n$). In each of the n independent blocks replace the k observations by their corresponding ranks (such that the rank of one is given to the smallest observation in the block and the rank of k is given to the largest.) Thus, R_{ij} is the rank (from 1 to k) associated with the j^{th} treatment in the i^{th} block. Under the null hypothesis, each ranking within a block is equally likely. Thus, there are $k!$ possible rankings within a particular block and $(k!)^n$ possible arrangements of ranks over all n blocks. It would be possible, for small n and k , to list all possible arrangements under the null hypothesis, compute the value of the test statistic in each case, and then form the exact null distribution of the test statistic in order to find its critical value and the exact probability associated

²Van Elteren and Noether (1959) have determined the A.R.E. of Friedman's Test with respect to the F Test under conditions appropriate to the latter. They report that $E_{T,F} = 3k/\pi(k+1)$ which, therefore, is dependent upon the number of treatment groups k .

with the critical value. As n and/or k become large, however, such a procedure becomes tedious and Monte Carlo methods may be employed to obtain the empirical null distribution of the test statistic. Writing in 1937, however, Friedman chose to use the Central Limit Theorem as the basis upon which to devise a test statistic as n , the number of blocks, becomes larger.

Recall that for the Two-Way F Test the null hypothesis can be written as:

$$H_0: \mu_j = \text{for all } j = 1, 2, \dots, k. \quad (1.1.1)$$

Hence, using the Friedman Test, if the k treatment conditions are really equal, there would be no real differences among the average ranks for each treatment taken over all n blocks. The average rank in the i^{th} block, $\bar{R}_{i.}$, is found by:

$$\bar{R}_{i.} = \sum_{j=1}^k R_{ij} / k = k(k+1)/2k = (k+1)/2. \quad (4.7.1)$$

Also, the variance in the i^{th} block, $\sigma_{\bar{R}_{i.}}^2$, is obtained from:

$$\sigma_{\bar{R}_{i.}}^2 = \frac{(k^2-1)}{12}. \quad (4.7.2)$$

Now the total mean rank over all n independent blocks is found from:

$$\sum_{i=1}^n \bar{R}_{i.} = n\bar{R}_{i.} = n(k+1)/2 \quad (4.7.3)$$

while the total variance over all n blocks, $\sigma_{\bar{R}_{..}}^2$, is given by:

$$\sum_{i=1}^n \sigma_{\bar{R}_{i.}}^2 = n\sigma_{\bar{R}_{i.}}^2 = n(k^2-1)/12. \quad (4.7.4)$$

Furthermore, consider the test statistic R_j , say, where $R_j = \sum_{i=1}^n R_{ij}$, the sum of the ranks under the j^{th} treatment. Using the Central Limit Theorem for large N ,

$$Z_j = \frac{R_j - n \bar{R}_{.j}}{\sqrt{n \sigma_{\bar{R}_{.j}}^2}} \stackrel{d}{\approx} N(0,1). \quad (4.7.5)$$

Now it is well known that $\sum_{j=1}^k Z_j^2 \stackrel{d}{=} \chi^2$ with k d.f. (4.7.6) if the Z_j are independent. However, only $k-1$ of the R_j are independent since $\sum_{j=1}^k R_j = R_{..}$ is fixed by

$$R_{..} = \sum_{i=1}^n \sum_{j=1}^k R_{ij} = \frac{n k (k+1)}{2} = \frac{N(k+1)}{2}. \quad (4.7.7)$$

Thus, Friedman defines the test statistic T as:

$$T = \frac{k-1}{k} \sum_{j=1}^k Z_j^2 \quad (4.7.8)$$

$$\text{where } \lim_{n \rightarrow \infty} T \stackrel{d}{=} \chi^2 \text{ with } (k-1) \text{ d.f.} \quad (4.7.9)$$

Much work, however, has been done to show the relevant range of n and k for which the test statistic $T \stackrel{d}{=} \chi^2$ with $(k-1)$ d.f..³ It was determined by Friedman (1940) that χ^2 gives a fairly good approximation if $n > 5$ when $k = 3$ and for any n if $k > 3$. In this dissertation, however, a randomization procedure is used to determine the nominal 5% and 1% levels of significance from the exact null distribution of T as developed by Owen (1962) for $k = 3$ and $n = 5(5)15$ and for $k = 4$ with $n = 5$. The critical values are given in Table A60. For all other

³See Kendall and Smith (1939), Friedman (1940) and Owen (1962).

combinations of n and k , the χ^2 distribution is used for testing T .

The computational form of T is obtained by substituting the z_j^2 in (4.7.8) and expanding. Hence,

$$T = \frac{k-1}{k} \sum_{j=1}^k \left[\frac{(R_{.j} - n \bar{R}_{i.})^2}{n \sigma_{\bar{R}_{i.}}^2} \right] = \left(\frac{k-1}{k} \right) \frac{\sum_{j=1}^k (R_{.j} - \frac{n(k+1)}{2})^2}{\frac{n(k^2-1)}{12}} \quad (4.7.10)$$

Thus,

$$T = \frac{12}{N(k+1)} \left[\sum_{j=1}^k R_{.j}^2 - n(k+1) \sum_{j=1}^k R_{.j} + \frac{k n^2 (k+1)^2}{4} \right] \quad (4.7.11)$$

but $\sum_{j=1}^k R_{.j} = N(k+1)/2$ so that

$$T = \frac{12}{N(k+1)} \left[\sum_{j=1}^k R_{.j}^2 - \frac{n(k+1)^2 N}{2} + \frac{n N (k+1)^2}{4} \right],$$

$$T = \frac{12}{N(k+1)} \left[\sum_{j=1}^k R_{.j}^2 - \frac{n N (k+1)^2}{4} \right]$$

and, finally,

$$T = \frac{12}{N(k+1)} \sum_{j=1}^k R_{.j}^2 - 3n(k+1) \quad (4.7.12)$$

Thus, (4.7.12) is used for testing the null hypothesis and a rejection is made if T equals or exceeds the critical value for some specified level of significance.

4.8 SHORACK'S TEST

Whereas Friedman's Test is a nonparametric procedure

for testing against general alternatives, Shorack (1967) developed a nonparametric modification to this procedure which is appropriate for testing against ordered alternatives. In essence, Shorack's work extended that of Bartholomew (section 3.2) and Chacko (section 3.8). A likelihood ratio approach is used to develop the test statistic:

$$\bar{T} = \frac{12n}{k(k+1)} \sum_{j=1}^k \left[\hat{\bar{R}}_{.j} - \left(\frac{k+1}{2} \right) \right]^2 \quad (4.8.1)$$

where $\bar{R}_{.j} = \sum_{i=1}^n R_{ij}/n$ is the average rank over all n blocks under the j^{th} treatment and $\hat{\bar{R}}_{.j}$ is the obtained "estimate" of the average rank for the j^{th} treatment after all k rank means have been replaced by a set of ordered rank means or pooled rank means--the "estimates" obtained using the amalgamation process described in section 3.2. Hence, once the data in each of the n independent blocks are replaced by their corresponding ranks the null hypothesis may be tested against the ordered alternative using (4.8.1) and a rejection made if the computed value $\bar{T}$ equals or exceeds $\bar{T}(\alpha)$, the critical value of the null distribution of $\bar{T}$ for a specified α level of significance. Shorack shows, however, that as n gets large the significance of $\bar{T}$ is determined by:

$$\alpha' = \sum_{l=2}^k p(l, k) P_{\chi^2_{l-1}} \{ \chi^2_{l-1} \geq \bar{T} \} \quad (4.8.2)$$

where χ^2_{l-1} is a random variable from the χ^2 distribution with $l-1$ degrees of freedom and $p(l, k)$ are the "combining probabilities" given in (3.2.3) and Table A53 for $k = 3(1)7$. The null

hypothesis is rejected if $\alpha' \leq \alpha$. Hence, the χ^2 distribution (or its transformation to incomplete gamma) yields the critical region for an approximate test of the null hypothesis if n , the size of each sample, is large enough.

In comparison to Friedman's Test, the procedure given by Shorack is a more powerful tool when the researcher believes an ordering among the k treatments exists. It is easy to show that when the k treatments are in the order specified by the alternative hypothesis, $\bar{T}$ is computationally identical to Friedman's statistic T (4.7.12) but the critical regions are different--enabling Shorack's procedure to be the more powerful. It is recalled that:

$$T = \binom{k-1}{k} \frac{\sum_{j=1}^k (R_{.j} - \frac{n(k+1)}{2})^2}{\frac{n(k^2-1)}{12}} \quad (4.7.10)$$

which can be rewritten as:

$$T = \frac{12}{N(k+1)} \sum_{j=1}^k \left(R_{.j} - \frac{n(k+1)}{2} \right)^2 . \quad (4.8.3)$$

Now

$$\bar{T} = \frac{12n}{k(k+1)} \sum_{j=1}^k \left(\bar{R}_{.j} - \frac{(k+1)}{2} \right)^2 \quad (4.8.4)$$

since $\hat{\bar{R}}_{.j} = \bar{R}_{.j}$, if the treatments are in correct order.

Upon expanding, (4.8.3) and (4.8.4) are identical. The computational form of T is:

$$T = \frac{12}{N(k+1)} \sum_{j=1}^k R_{.j}^2 - 3n(k+1) . \quad (4.7.12)$$

When expanding (4.8.4),

$$\bar{T} = \frac{12n}{k(k+1)} \left[\sum_{j=1}^k \bar{R}_{.j}^2 - (k+1) \sum_{j=1}^k \bar{R}_{.j} + \sum_{j=1}^k \frac{(k+1)^2}{4} \right] \quad (4.8.5)$$

and since $\sum_{j=1}^k \bar{R}_{.j} = \frac{k(k+1)}{2}$,

$$\bar{T} = \frac{12n}{k(k+1)} \left[\sum_{j=1}^k \bar{R}_{.j}^2 - \frac{k(k+1)^2}{2} + \frac{k(k+1)^2}{4} \right] \quad (4.8.6)$$

so that

$$\bar{T} = \frac{12}{k(k+1)} \sum_{j=1}^k \bar{R}_{.j}^2 - 3n(k+1) . \quad (4.8.7)$$

Replacing $\sum_{j=1}^k \bar{R}_{.j}^2$ by $\sum_{j=1}^k \left[\frac{R_{.j}}{n} \right]^2$ and factoring out the denominator which is constant,

$$\bar{T} = \frac{12}{N(k+1)} \sum_{j=1}^k R_{.j}^2 - 3n(k+1) \quad (4.8.8)$$

and the identity with (4.7.12) is proved.⁴ Furthermore, by analogy to the comparison made between the critical regions of Chacko's Test and the Kruskal-Wallis Test (see section 3.8), it can be stated that Shorack's Test is the more powerful technique under a directed alternative.

⁴In general, however, for computational purposes, the following form of the test statistic $\bar{T}$ is most appropriate:

$$\bar{T} = \frac{12n}{k(k+1)} \sum_{j=1}^k \hat{\bar{R}}_{.j}^2 - 3n(k+1) . \quad (4.8.9)$$

4.9 JONCKHEERE'S TWO-WAY TEST

Development

In addition to the k sample nonparametric procedure against ordered alternatives for a completely randomized design (see section 3.9), Jonckheere (1954B) also developed a similar test when the design format is randomized block. At that time, Jonckheere's Two-Way Test was the chief competitor of Friedman's Test which is useful against more general alternatives. Thus, Jonckheere successfully attempted to fill a void in the knowledge by providing a two-way nonparametric technique against ordered alternatives. Jonckheere believed that such a procedure would be particularly appropriate in both educational and social psychology and cited some examples in these fields.

The test is developed below under the restriction that one observation is selected for each of k treatment conditions (categories) within each of n blocks.⁵

Let the k treatment conditions be considered as ranked in order of increasing value. Let X_{ij} represent the observation in the i^{th} block under the j^{th} ordered treatment condition where $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, k$. (The k observations in the i^{th} block ($i = 1, 2, \dots, n$) may be replaced by

⁵Jonckheere describes a more general procedure in which k' observations in each block are described by k categories ($k' \geq k$) or treatment conditions. The equality of k and k' is a necessary restriction in this dissertation for comparison purposes.

their corresponding ranks such that the smallest observed value in the block has a rank of one while the largest has a rank of k).

$$p_{ij,im} = \begin{cases} 1 & \text{if } X_{ij} < X_{im} \\ 0 & \text{if } X_{ij} \geq X_{im} \end{cases} \quad (4.9.1)$$

where X_{im} is the observed value in the i^{th} block under the m^{th} treatment condition ($m = 1 + j$).

Now define

$$P_i = \sum_{j=1}^{k-1} \sum_{m=1+j}^k p_{ij,im}, \quad (4.9.2)$$

and let

$$P = \sum_{i=1}^n P_i. \quad (4.9.3)$$

P is the test statistic to be used for testing the null hypothesis that the k samples are selected from populations having identical c.d.f.s ($F_j(X) = F_k(X)$ for all $j = 1, 2, \dots, k$) against the ordered alternative $F_1(X) \geq F_2(X) \geq \dots \geq F_k(X)$ (with at least one inequality strict). Jonckheere states that as either n or k get large, $P \stackrel{d}{\approx} N(\mu_P, \sigma_P)$

where

$$\mu_P = \frac{n k (k-1)}{4} = \frac{N(k-1)}{4}$$

and

$$\sigma_P = \sqrt{\left(\frac{n k}{72}\right) (k-1)(2k+5)} = \sqrt{\frac{N(k-1)(2k+5)}{72}}$$

Transforming the test statistic P into the standardized form Z and using the suggested correction for continuity, the test format is given by:

$$Z = \frac{\left| P - \mu_P \right| - \frac{1}{2}}{\sigma_P} = \frac{\left| P - \frac{N(k-1)}{4} \right| - \frac{1}{2}}{\sqrt{\frac{N(k-1)(2k+5)}{72}}}. \quad (4.9.4)$$

The null hypothesis may be rejected if $Z \geq Z_{1-\alpha}$, the upper tail critical value under the normal distribution for a specified α level of significance, provided that $P > \mu_p$.

A problem arises, however, when n and k are small. No tables of critical P values exist. To find the relevant range of n and k for the test statistic Z in (4.9.4) Monte Carlo methods are employed. Three replications of 400 histories of the test statistic P under the null hypothesis are generated for matrices of size $n \times k$ where $n = 5(1)10(5)20$ and $k = 3(1)7$, and a Modified Shapiro-Wilk Test for normality (see Appendix A3) is performed on each of these null distributions. The results indicate that when $k = 3$ the Z test of (4.9.4) is inappropriate for all samples considered here ($n = 5-20$). In addition, when $k = 4$, the Z test is appropriate only when $n \geq 10$. Thus, for matrices of size $n = 5(5)20 \times k = 3(1)4$, the empirical null distribution of Z_p , the standardized P , is formed by combining the three replicates for each $n \times k$ matrix into one distribution with 1200 observations. The generated critical values are presented in Table A61 for $\alpha = .01$ and $.05$. The null hypothesis may be rejected if the computed value of the test statistic from (4.9.4) equals or exceeds $Z_p(\alpha)$.⁶

⁶Since P is discrete and its critical values under the null distribution have exact probabilities, a randomization procedure is needed to obtain the empirical powers of the test at the nominal 5% and 1% levels of significance so that comparisons with other procedures can be made.

Related Tests

It is interesting to examine the similarity between the P statistic given by (4.9.3) and the S statistic for Jonckheere's One-Way Test defined by (3.9.3). Using (4.9.2), S_i may thus be defined by $S_i = 2 P_i - \frac{k(k-1)}{2}$ (4.9.5)

so that

$$S = \sum_{i=1}^n S_i = 2P - \frac{n k(k-1)}{2} = 2P - \frac{N(k-1)}{2}. \quad (4.9.6)$$

Jonckheere, however, states that it is more convenient to use the P statistic in a randomized block situation. On the other hand, Ferguson (1966), without reference to Jonckheere, uses the S statistic to describe the same test procedure developed by Jonckheere. Ferguson transforms the test statistic S to a standardized deviate Z and, provided $S > 0$, tests the null hypothesis ($F_j(X) = \text{for all } j = 1, 2, \dots, k$) against the ordered alternative ($F_1(X) \geq F_2(X) \geq \dots \geq F_k(X)$ with at least one inequality strict) by using

$$Z = \frac{|S| - 1}{\sqrt{\sigma_s^2}} \stackrel{d}{\cong} N(0, 1) \quad (4.9.7)$$

corrected for continuity, where $S = \sum_{i=1}^n S_i$ defined in (4.9.6), and $\sigma_s^2 = \sum_{i=1}^n \sigma_{s_i}^2 = n \sigma_{s_i}^2$ where $\sigma_{s_i}^2 = \frac{k(k-1)(2k+5)}{18}$.⁷ Thus, the identity of Ferguson's Test (4.9.7) with that of Jonckheere (4.9.4) is easily shown:

⁷This is based on the theorem that the variance of a sum of independent samples is equal to the sum of the separate variances.

<u>JONCKHEERE'S Z</u>	VS.	<u>FERGUSON'S Z</u>
(1) $\frac{\left p - \frac{N(k-1)}{4} \right - \frac{1}{2}}{\sqrt{\frac{N(k-1)(2k+5)}{72}}}$	$\stackrel{?}{=}$	$\frac{ S - 1}{\sqrt{\frac{N(k-1)(2k+5)}{18}}}$
(2) $\frac{\sqrt{\frac{N(k-1)(2k+5)}{18}}}{\sqrt{\frac{N(k-1)(2k+5)}{72}}}$	$\stackrel{?}{=}$	$\frac{ S - 1}{\left p - \frac{N(k-1)}{4} \right - \frac{1}{2}}$
(3) 2	$\stackrel{?}{=}$	$\frac{ S - 1}{\left p - \frac{N(k-1)}{4} \right - \frac{1}{2}}$
(4) $\left 2p - \frac{N(k-1)}{2} \right - 1$	$\stackrel{?}{=}$	$ S - 1$

and

$$(5) \quad 2p - \frac{N(k-1)}{2} \quad \equiv \quad S \quad \text{as defined by (4.9.6).}$$

4.10 PAGE'S L TEST

Development

Page (1963) devised a nonparametric procedure for testing against ordered alternatives which is useful when the data collected are only ordinal in form or when it is desired not to make the stringent assumptions necessary for a parametric procedure. In essence, the test consists of replacing the k

observations in each of the n independent blocks by their corresponding ranks such that the rank of one is given to the smallest element in a block and k is given to the largest. The sum $R_{.j} = \sum_{i=1}^n R_{ij}$ is then obtained (over all n blocks) for the j^{th} treatment (where $j = 1, 2, \dots, k$). Now the null hypothesis of no differences between the treatments (groups) can be tested against the ordered alternative that differences among treatments are monotonically increasing. Page defines the test statistic L by:

$$L = \sum_{j=1}^k (\hat{R}_j^* R_{.j}) \quad (4.10.1)$$

where $\hat{R}_j^*$ is the hypothetical ranking (one to smallest and k to largest) given to the j^{th} treatment under the ordered alternative. The null hypothesis may be rejected if $L \geq L(\alpha)$, the critical value under the null distribution of L for a specified α level of significance. In this dissertation approximate critical L values have been developed for $n = 5(1)20$ and $k = 3(1)7$ for the 5% and 1% levels of significance. These are found in Table A62. The values are based on the following formula given by Page:

$$L(\alpha) = \frac{n(k^3 - k)}{12} \left[\frac{Z_\alpha}{\sqrt{n(k-1)}} + \frac{3(k+1)}{(k-1)} \right] \quad (4.10.2)$$

where Z_α is the upper α tail of the normal distribution.⁸

⁸As suggested by Page, all fractional values of $L(\alpha)$ in (4.10.2) are rounded up to the next integer.

Related Tests

Page's L Test is, in a sense, a nonparametric extension of Friedman's Test (section 4.7). However, it may be recalled that Jonckheere's Two-Way Test (section 4.9) is likewise such an extension and, therefore, is analogous to the Page L Test. Interestingly, Jonckheere's Test statistic P is related to Kendall's $\hat{\tau}$ while Page's Test statistic L is shown to be related to Spearman's ρ . Two major advantages of Page's Test, however, over Jonckheere's Test are its distinct simplicity and its computational ease.

4.11 A MODIFIED HOLLANDER RANK TEST

Hollander (1967) developed a k sample Wilcoxon Signed-Ranks type test for ordered alternatives. Hollander's test statistic, Y , is shown to be neither distribution-free for finite n nor asymptotically distribution-free. Thus, Hollander utilizes a consistent estimate of the null variance of Y in order to define a procedure which is asymptotically distribution-free. However, upon examining Hollander's Rank Test in this dissertation, the calculation of this consistent estimate is found to be so tedious that no researcher would select this test for practical usage. A modification of Hollander's Rank Test is, therefore, suggested here so that the researcher may take advantage of this highly powerful and simple technique and, at the same time, avoid the tedious calculation described by Hollander. Hollander's Rank Test and the suggested modification are presented below.

Let X_{ij} be the value in the i^{th} block ($i = 1, 2, \dots, n$) under the j^{th} treatment ($j = 1, 2, \dots, k$). Now for the u^{th} and v^{th} treatments (where $u < v$) define $Y_{uv}^{(i)}$ as $|X_{iu} - X_{iv}|$ and let $R_{uv}^{(i)}$ be the rank of $Y_{uv}^{(i)}$ in the ranking from least to greatest ($i = 1, 2, \dots, n$). In addition, let $T_{uv} = \sum_{i=1}^n R_{uv}^{(i)} \psi_{uv}^{(i)}$ where $\psi_{uv}^{(i)} = \begin{cases} 1 & \text{if } X_{iu} < X_{iv} \\ 0 & \text{otherwise} \end{cases}$.

The test statistic Y is now given by:

$$Y = \sum_{u < v} T_{uv} \quad (4.11.1)$$

It should be noted that while each of the $k(k-1)/2$ individual Wilcoxon Signed-Rank statistics T_{uv} (where $u < v$) is distribution-free with mean $\mathcal{M}_{T_{uv}} = \frac{n(n+1)}{4}$ and variance $\sigma_{T_{uv}}^2 = \frac{n(n+1)(2n+1)}{24}$, the test statistic Y is not distribution-free since Hollander (1966) shows that the null correlation coefficient $\rho^n(F)$ between T_{uv} and T_{uw} ($u \neq v$, $u \neq w$, $v \neq w$) depends on the particular c.d.f., "F" (if the sample--block--size n exceeds one), as does the null variance of Y . Thus, the test statistic Y has mean

$$E_o(Y) = \mathcal{M}_Y = \frac{k(k-1)n(n+1)}{8} = \frac{N(k-1)(n+1)}{8}, \quad (4.11.2)$$

and variance

$$\sigma_o^2(Y) = \frac{n(n+1)(2n+1)k(k-1)(3+2(k-2)\rho^n(F))}{144} \quad (4.11.3)$$

and Hollander proves

$$\rho^n(F) = \frac{[(24\lambda(F)-6)n^2 + (48\mathcal{M}(F)-72\lambda(F)+7)n + (48\lambda(F)-48\mathcal{M}(F)+1)]}{(n+1)(2n+1)} \quad (4.11.4)$$

where $\mathcal{U}(F) \leq .3089$ for all F and where, according to Lehmann (1964), $6/24 < \lambda(F) \leq 7/24$ for all F .

The following table gives values for $\lambda(F)$ and $\mathcal{U}(F)$ for the rectangular, exponential, and normal distributions.

TABLE 4.2
VALUES FOR $\lambda(F)$ AND $\mathcal{U}(F)$

F	Rectangular	Exponential	Normal	"Upper Bounds"
$\lambda(F)$	.2909	.2894	.2902	$7/24 = .29167$
$\mathcal{U}(F)$	.3083	.3056	.3075	.3089

The table below presents, for various n , values of $\mathcal{J}_o^n(F)$ for the rectangular, exponential, and normal distributions, as well as the "upper bounds" solution, $\mathcal{J}_u^n$, obtained by replacing $\mathcal{U}(F)$ by .3089 and $\lambda(F)$ by .29167 in (4.11.4).

TABLE 4.3
VALUES FOR $\mathcal{J}_o^n(F)$

F	n						
	5	10	15	20	25	50	∞
Rectangular	.439	.462	.471	.476	.479	.484	.490
Exponential	.424	.446	.454	.458	.461	.467	.472
Normal	.434	.456	.464	.469	.471	.477	.483
"Upper Bounds"	.444	.470	.479	.484	.487	.493	.500

To obtain $\mathcal{J}_o^n(F)$ as $n \rightarrow \infty$, let $\mathcal{J}^*(F) = \lim_{n \rightarrow \infty} \mathcal{J}_o^n(F)$.
 From (4.11.4) $\mathcal{J}^*(F) = 12 \lambda(F) - 3$ (4.11.5)
 and therefore, $0 < \mathcal{J}^*(F) \leq \frac{1}{2}$ based on Lehmann's proof for the

bounds on $\lambda(F)$. However, in order that the procedure based on Y be asymptotically distribution-free a consistent estimate of $\mathcal{P}^*(F)$ or, equivalently $\lambda(F)$, is required (see (4.11.4) and (4.11.5)). Lehmann suggested estimating $\lambda(F)$ by the proportion (over all sextuples $(\alpha, \beta, \gamma; u, v, w)$) of cases in which the following event occurs:

$$\left\{ \begin{array}{l} X_{\alpha u} < X_{\alpha v} + X_{\beta u} - X_{\beta v} \\ X_{\alpha u} < X_{\alpha w} + X_{\gamma u} - X_{\gamma w} \end{array} \right\} . \quad (4.11.6)$$

As he admits, such a computation is "tedious" and in practice a small subset of the original number of sextuples is used.⁹ Hollander (1967) therefore suggests using $\hat{\lambda}_1$ to estimate $\lambda(F)$ where $\hat{\lambda}_1$ is the proportion (over a subset of the total number of (α, β, γ) 3-tuples) of cases in which the following event occurs:¹⁰

$$\left\{ \begin{array}{l} X_{\alpha k} < X_{\alpha 1} + X_{\beta 1} - X_{\beta k} \\ X_{\alpha k} < X_{\alpha 2} + X_{\gamma 1} - X_{\gamma k} \end{array} \right\} . \quad (4.11.7)$$

⁹To say that such a computation is "tedious" is an understatement. With (α, β, γ) distinct and (u, v, w) distinct there are $n(n-1)(n-2)k(k-1)(k-2)$ possible arrangements of (4.11.6). Thus, the smallest matrix considered in this dissertation ($n = 5$ and $k = 3$) requires 360 arrangements while the largest ($n = 20$ and $k = 6$) requires 820,800 arrangements.

¹⁰To examine all arrangements of (4.11.7) is still computationally tedious and this prohibits any practical usage of Hollander's Test. There are $n(n-1)(n-2)$ such arrangements--60 when $n = 5$ and 6,840 when $n = 20$. Hollander, however, suggests using only a subset of this total but does not actually specify any particular number.

Hollander now proposes the following test of the null hypothesis $(F_j(X) \equiv \text{for all } j = 1, 2, \dots, k)$ against the ordered alternative $(F_1(X) \geq F_2(X) \geq \dots \geq F_k(X)$ with at least one inequality strict):

$$Z \cong \frac{Y - E_o(Y)}{\hat{\sigma}_o(Y)} = \frac{Y - \frac{N(k-1)(n+1)}{8}}{\sqrt{[n(n+1)(2n+1)(k-1)(3+2(k-2)\hat{\rho}_n^2)]/144}} \quad (4.11.8)$$

where $\hat{\rho}_n = \begin{cases} \min[\rho_u^n, \hat{\rho}_1] & \text{if } > 0 \\ 0 & \text{otherwise} \end{cases} ;$

ρ_u^n is an "upper bounds" solution for $\rho_o^n(F)$; and $\hat{\rho}_1$ is a consistent estimate of $\rho^*(F)$ in (4.11.5) and is given by:

$$\hat{\rho}_1 = 12\hat{\lambda}_1 - 3. \quad (4.11.9)$$

The null hypothesis may be rejected at some specified level of significance α if the computed value of Z equals or exceeds the upper α tail critical value under the normal distribution. Now Hollander has stated that when the ordered alternative is really true the system of inequalities in (4.11.7) would tend to be satisfied less frequently than when the null hypothesis is true. Hence, as the estimate of $\rho_o^n(F)$ approaches zero, the null variance of Y , $\sigma_o^2(Y)$, given by (4.11.3) becomes smaller so that the power of the Z test given by (4.11.8) is increased.

Since Hollander does not specify how large a subset of the 3-tuples to use in (4.11.7) only $n-2$ of them are considered here to obtain an estimate of $\hat{\lambda}_1$ and, subsequently, $\hat{\rho}_n$. Thus, for convenience, the events considered here are of the following

form:

$$\begin{aligned}
 & \left\{ \begin{aligned} X_{1,k} &< X_{11} + X_{21} - X_{2,k} \\ X_{1,k} &< X_{12} + X_{31} - X_{3,k} \end{aligned} \right\} \\
 & \left\{ \begin{aligned} X_{2,k} &< X_{21} + X_{31} - X_{3,k} \\ X_{2,k} &< X_{22} + X_{41} - X_{4,k} \end{aligned} \right\} \\
 & \vdots \\
 & \left\{ \begin{aligned} X_{n-2,k} &< X_{n-2,1} + X_{n-1,1} - X_{n-1,k} \\ X_{n-2,k} &< X_{n-2,2} + X_{n1} - X_{n,k} \end{aligned} \right\} .
 \end{aligned} \tag{4.11.10}$$

This small subset of the total number of 3-tuples is used in an effort to obtain consistent estimates and, at the same time, make the procedure computationally practical. However, a Monte Carlo study of normality, performed by generating three replications of 400 histories of Hollander's standardized test statistic Z under the null hypothesis for matrices of size $n \times k$ (where $n = 5(1)10(5)20$ and $k = 3(1)7$, shows that these Z values could not be considered as coming from a normal distribution as k increased--regardless of n .¹¹ However, to take larger subsets of (4.11.7) would diminish the utility of the test from a practical stand point. Thus, to resolve this dilemma the following modification of Hollander's procedure is hereby suggested:

The "upper bound" solution, ρ_u^n , should be used as an estimate of $\rho_o^n(F)$ --regardless of n or k . Substituting in (4.11.3) and in (4.11.8) yields a distribution-free test. Moreover, the entire estimating process using sextuples or 3-tuples is eliminated--thereby making the test computationally feasible

¹¹The Modified Shapiro-Wilk Test is used (see Appendix A3).

and simple for the researcher. Finally, the modified version of Hollander's Test yields Z values that, under the null hypothesis, may be considered as coming from parent normal populations.¹² The only adverse effect of such a modification, however, is that the power of the test is diminished somewhat because the "upper bounds" solution, $\mathcal{J}_u^n$, used as an estimate of $\mathcal{J}_0^n(F)$, will always cause $\hat{\sigma}_0(Y)$ in (4.11.8) to be larger than that obtained by either Lehmann's approach or Hollander's approach. On the other hand, since Wilcoxon type procedures are known to be highly powerful, a slightly more conservative modification of Hollander's Rank Test should present no real problem--especially since the modified procedure is both practical and simple and not merely a mathematical exercise.¹³

4.12 A MODIFIED DOKSUM RANK TEST

Working independently of Hollander (1967), Doksum (1967) developed a nonparametric procedure for testing against ordered alternatives that is highly related to Hollander's Rank Test. Similar to Hollander's procedure, Doksum's Rank Test is computationally tedious, and therefore, infeasible for practical usage.¹⁴ Thus, a modification of the procedure is suggested

¹²Using three replicates of 400 histories each the Modified Shapiro-Wilk Test fails to reject a hypothesis of normality for all values of n and k considered.

¹³While Hollander's Rank Test, in its original form, is computationally impractical, it is shown to have good asymptotic power properties when compared to Jonckheere's Test (section 4.9) and Page's L Test (section 4.10).

¹⁴Doksum's Rank Test requires the same consistent estimate of $\mathcal{J}_0^n(F)$ as does Hollander's Rank Test in order to make the procedure asymptotically distribution-free.

for usage in this dissertation. Doksum's Rank Test and the proposed modification are presented below:

Let X_{ij} be the value in the i^{th} block ($i = 1, 2, \dots, n$) under the j^{th} treatment ($j = 1, 2, \dots, k$). Now for the u^{th} and v^{th} treatments (where $u < v$) define $Y_{uv}^{(i)}$ as $|X_{iu} - X_{iv}|$ and let $R_{uv}^{(i)}$ be the rank of $Y_{uv}^{(i)}$ in the ranking from least to greatest ($i = 1, 2, \dots, n$). Moreover, let $T_{uv} = \sum_{i=1}^n R_{uv}^{(i)} \gamma_{uv}^{(i)}$ where

$$\gamma_{uv}^{(i)} = \begin{cases} 1 & \text{if } X_{iu} < X_{iv} \\ 0 & \text{otherwise} \end{cases}.$$

Now consider $D_{uv} = (T_{uv} - \sum_{i=1}^n \gamma_{uv}^{(i)}) / \binom{n}{2}$. The Doksum Rank Test statistic Y^* is given by:

$$Y^* = \sum_{u < v} (\bar{D}_u - \bar{D}_v) \quad (4.12.1)$$

where $\bar{D}_u = \sum_{j=1}^k D_{uj} / k$ when $j \neq u$ (since $D_{uu} = 0$), and where $D_{vu} = 1 - D_{uv}$. The test statistic Y^* has mean $E_0(Y^*) = \mu_{Y^*} = 0$ and variance $\sigma_o^2(Y^*) = \frac{k(k^2-1)V}{6}$ where V is given by:

$$V = \frac{\{2n-1 + (k-2)[24(n-2)\lambda(F) + 13 - 6n]\}}{3kn(n-1)} \quad (4.12.2)$$

Now if an estimate of the null variance Y^* is obtained by replacing $\lambda(F)$ in V by a consistent estimate, say $\hat{\lambda}_1$, as proposed by Lehmann (1964) or Hollander (1967) the following test may be considered:

$$Z \cong \frac{Y^* - E_0(Y^*)}{\hat{\sigma}_o(Y^*)} = \frac{(Y^*)[18n(n-1)]^{\frac{1}{2}}}{((k^2-1)\{2n-1 + (k-2)[24(n-2)\hat{\lambda}_1 + 13 - 6n]\})^{\frac{1}{2}}} \quad (4.12.3)$$

Unfortunately, it may be recalled from the discussion in the previous section that the computation of the consistent estimate

is so tedious that the test is rendered useless from a practical standpoint unless a small subset of inequalities are selected to obtain an estimate of $\lambda(F)$. However, as with Hollander's Rank Test, for the small subset of inequalities chosen $(n-2)$ to estimate $\lambda(F)$, Doksum's procedure also fails to meet the criteria of normality under the Modified Shapiro-Wilk Test (see Appendix A3) as k increases--regardless of n .¹⁵ The problem is easily resolved, however, by adapting the modification of Doksum's Rank Test that is hereby proposed:

Replace $\lambda(F)$ in (4.12.2) by its upper bound of $7/24$ as proved by Lehmann (1964). This will make $\hat{\sigma}_0^2(Y^*)$ slightly larger than it really is when the alternative is true and, therefore, decrease the power of the test somewhat. Doksum, however, claims that the test based on Y^* is, asymptotically, slightly more powerful than Hollander's Rank Test described in the previous section. Hence, the modification--while making the procedure slightly more conservative in terms of power--makes the test usable from a practical standpoint. However, when compared with the modified version of Hollander's procedure, the extra steps required to obtain Y^* makes the test slightly more cumbersome to perform than that of Hollander. Nevertheless, an asymptotically powerful test of the null hypothesis $(F_j(X) \equiv \text{for all } j = 1, 2, \dots, k)$ against the ordered alternative $(F_1(X) \geq F_2(X) \geq \dots \geq F_k(X)$ with at least one

¹⁵See the discussion in section 4.11.

inequality strict) can be made at some specified level of significance α by computing the modified Doksum statistic

$$Z = \frac{Y^*}{\left[\frac{(k^2-1)\{2n-1+(k-2)(n-1)\}}{18n(n-1)} \right]^{\frac{1}{2}}} = \frac{Y^*}{\left[\frac{(k^2-1)(N-k+1)}{18n(n-1)} \right]^{\frac{1}{2}}} \quad (4.12.4)$$

and rejecting the null hypothesis if the computed value equals or exceeds the upper α tail critical value under the normal distribution.

4.13 PIRIE'S SIGN TEST

Pirie (1970) defines a nonparametric procedure with test statistic J as follows:

$$J = \sum_{i=1}^n \eta_i \quad (4.13.1)$$

where $\eta_i = \begin{cases} 1 & \text{if } X_{i1} \leq X_{i2} \leq \dots \leq X_{ik} \\ 0 & \text{otherwise} \end{cases}$

Moreover, Pirie claims that such a test has a very small A.R.E. when compared to Page's L Test (section 4.10) regardless of what continuous parent populations the sample data are taken from. It is apparent, however, that such a procedure, regardless of its asymptotic inefficiencies, is a simple "Sign Test" with the advantages of being easiest to understand and quickest to compute. In addition, it is easy to show that the null distribution of J is binomial. Critical values are therefore obtained at the 5% and 1% levels of significance and tabled for $n = 5(1)20$ and $k = 3(1)7$ (see Table A63).

Pirie's Sign Test may be described as follows:

Let X_{ij} be the observed value under the j^{th} treatment

in the i^{th} block ($i = 1, 2, \dots, n$ and $j = 1, 2, \dots, k$). A Sign Test is desired for testing the null hypothesis of equality among the k treatment effects against the alternative that the treatment effects are monotonically increasing. To obtain such a test, examine the observed order sequence in each of the n independent blocks. In the i^{th} block give a value of "one" (to η_i) if the block is in the sequence specified by the alternative hypothesis and give a value of "zero" otherwise. Note that there are $k!$ permutations or sequences of numerical values that are possible under the null hypothesis and all of these are equally likely to occur. Only one such sequence, however, is monotonically increasing as specified by the ordered alternative. Thus, under the null hypothesis, the probability of assigning a value of "one" to η_i is $\frac{1}{k!}$. This holds for each of the n independent blocks, yielding the binomial distribution for the test statistic J .¹⁶ Hence, the probability of obtaining a particular value of the test statistic, J_{OBS} , or larger, is gotten by:

$$P_r \{ J \geq J_{\text{OBS}} \} = \sum_{J=J_{\text{OBS}}}^n \binom{n}{J} \left(\frac{1}{k!} \right)^J \left(1 - \frac{1}{k!} \right)^{n-J}, \quad (4.13.2)^{17}$$

¹⁶ J takes on all integer values from 0 to n ; the probability of "success" ($\eta_i = 1$) is constant from block to block; and each of the n blocks are independent.

¹⁷As k gets larger, a Poisson approximation to the binomial distribution may be used so that

$$P_r \{ J \geq J_{\text{OBS}} \} = \sum_{J=J_{\text{OBS}}}^n \frac{e^{(-\frac{n}{k!})} \left(\frac{n}{k!} \right)^J}{J!} \quad \text{where} \quad \mu = \frac{n}{k!}.$$

and the null hypothesis may be rejected at an α level of significance if $P\{J \geq J_{obs}\} \leq \alpha$.¹⁸

4.14 PIRIE-TUKEY RANK SUM TEST

Pirie (1970) describes a nonparametric procedure against ordered alternatives which is related to a proposal made by Tukey (1957). Pirie shows that this test has the same, poor asymptotic power properties as does the Sign Test defined in the previous section. The Pirie-Tukey Rank Sum Test is developed as follows:

As in the Pirie Sign Test, let

$$\eta_i = \begin{cases} 1 & \text{if } x_{i1} \leq x_{i2} \leq \dots \leq x_{ik} \\ 0 & \text{otherwise} \end{cases}$$

where $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, k$. Now consider

$$S_i = \min_{1 \leq j \leq k-1} \{x_{i,j+1} - x_{ij}\}.$$

Furthermore, let R_i be the rank of S_i such that a value of one is given to the smallest and n to the largest. The test statistic J^* may thus be defined as:

$$J^* = \sum_{i=1}^n R_i \eta_i \quad (4.14.1)$$

and the null hypothesis of equality among the k treatment effects may be rejected, for a chosen level of significance, in favor of the alternative that the treatment effects are monotonically increasing if $J^* \geq J^*(\alpha)$ --the upper critical value. However, since no tables of critical values for $J^*(\alpha)$

¹⁸In order to compare this test with others discussed in this chapter, a randomization procedure is used to obtain nominal α levels of 5% and 1%.

exist Monte Carlo methods are used in this dissertation to generate the empirical null distribution of J^* .¹⁹ The empirical values, $J^*(\alpha)$, along with their probabilities, are presented in Table A64 for $n = 5(1)10(5)20$ and $k = 3(1)7$.

It should be noted that while the Pirie Sign Test only requires an ordinal level of measurement within each of the n blocks, the Pirie-Tukey Rank Sum Test requires that the observations in each block be at least the strength of an interval scale. This requirement is necessary in order to obtain for the i^{th} block ($i = 1, 2, \dots, n$) the value of S_i by comparing the observations under all $k-1$ adjacent pairs of treatments in the block and finding the minimum difference between these adjacent pairs of treatments. It therefore appears that the more stringent requirement on the level of measurement coupled with the extra steps involved in obtaining R_i tend to diminish the value of the Pirie-Tukey Rank Sum Test in light of Pirie's indication that the asymptotic power of the procedure is equivalent to the more simple Pirie Sign Test (which itself is not very powerful).

4.15 PIRIE AND HOLLANDER'S EXPECTED NORMAL SCORES TEST

Pirie and Hollander (1972) recently developed a non-parametric procedure based on expected normal scores for testing against ordered alternatives. Their test is an extension of Page's L Test (see section 4.10)--the difference being that Page uses the ranks of the observations in each block

¹⁹The empirical distribution of J^* is based on 1200 histories for each combination of n and k .

while Pirie and Hollander replace the ranks with their corresponding expected normal scores. Thus, to perform the test, in the i^{th} block ($i = 1, 2, \dots, n$) replace the ranks R_{ij} of the original observations X_{ij} (where $j = 1, 2, \dots, k$) by their respective expected normal scores e_{ij} (where e_{ij} is the expected value of the observation having rank R_{ij} --the j^{th} smallest value in the i^{th} block--in a random sample of k observations from a standardized normal distribution). The sum of these scores, $e_{.j} = \sum_{i=1}^n e_{ij}$, is then determined (over all n independent blocks) for each of the k treatment levels. Now the null hypothesis of no differences in treatment effects can be tested against the ordered alternative that the treatment effects are monotonically increasing by using the test statistic W defined by:

$$W = \sum_{j=1}^k (\hat{R}_j^* e_{.j}) \quad (4.15.1)$$

where $\hat{R}_j^*$ is the hypothetical ranking (one to the smallest and k to the largest) given to the j^{th} treatment under the ordered alternative. The null hypothesis may be rejected if $W \geq W(\alpha)$, the critical value under the null distribution of W for a chosen α level of significance. Tables of critical values are given by Pirie and Hollander for $n = 1(1)10$ with $k = 4$, and $n = 1(1)5$ with $k = 5$ --for α of .01, .05, and .10. However, they also demonstrate that a normal approximation to the exact distribution is quite good even for small n and/or k . In this dissertation then, the following statistic is used to test against

ordered alternatives:

$$Z = \frac{W - E(W)}{[\text{Var}(W)]^{\frac{1}{2}}} \stackrel{d}{=} N(0, 1) \quad (4.15.2)$$

where $E(W) = 0$ and $\text{Var}(W) = \frac{N(k+1)}{12} \sum_{j=1}^k e_{ij}^2$.

The null hypothesis is rejected if $Z \geq Z_{\alpha}$, the upper α tail of the normal distribution.

It should be mentioned here that Pirie and Hollander highly recommend this nonparametric procedure for testing against ordered alternatives since it is easy to calculate (provided tables of expected normal scores such as Harter's (1961) are available) and has an A.R.E. at least as good as Page's L Test (section 4.10) under a variety of distributions.

4.16 FISHER AND YATES TWO-WAY TEST

When dealing with a randomized block design Fisher and Yates (1949) suggested a method for evaluating data by the conventional analysis of variance (see section 4.1) after the original observations in each block are transformed into expected normal scores (the analogous procedure for a completely randomized design--along with the merits and demerits of the test--is discussed in section 3.12).²⁰ Thus, whether the original data

²⁰Recall that the Friedman Test (section 4.7) utilizes the ranks R_{ij} , where $j = 1, 2, \dots, k$, in each block. The Friedman Test appeared in the literature in 1937 while the Fisher and Yates procedure was published in 1938. It appears that Fisher and Yates were either unaware of Friedman's work or unconcerned--if not unconvinced--of the merits of employing ranks if a (Cont'd)

in each block constitute a measurement or a rank, these values are replaced by the corresponding expected normal scores, e_{ij} , which is defined as the expected value of the observation having a rank R_{ij} in a random sample of k (the number of elements in a block) observations from a standardized normal distribution.

The test statistic to be used is:

$$FY = \frac{n \sum_{j=1}^k (\bar{e}_{.j} - \bar{e}_{..})^2 / k - 1}{\sum_{i=1}^n \sum_{j=1}^k (e_{ij} - \bar{e}_{i.} - \bar{e}_{.j} + \bar{e}_{..})^2 / (n-1)(k-1)} \stackrel{d}{\approx} F \text{ with } (k-1) \text{ and } (n-1)(k-1) \text{ d.f.} \quad (4.16.1)$$

and the null hypothesis that the k groups are identical may be rejected in favor of the general alternative that at least two of the groups differ from each other if the test statistic $FY \geq F_{\alpha} [(k-1), (n-1)(k-1)]$ d.f., the critical F .²¹

4.17 BELL AND DOKSUM TWO-WAY TEST

The Bell and Doksum Two-Way Test (1965) is a nonparametric analogue of both the Friedman Test (section 4.7) and the Fisher and Yates Two-Way Test (section 4.16). Whereas Friedman's procedure deals with the ranks of the k observations in each block and the Fisher-Yates approach utilizes the expected normal order statistics for the k observations in each

²⁰ (Cont'd) better test, based on expected normal scores, could be had. On the other hand, in 1952, Kruskal and Wallis derived a rank test for the completely randomized design that is analogous to Friedman's Test for the randomized block model. Perhaps Kruskal and Wallis believed that a less cumbersome and less restrictive test--if not a better test--was needed.

²¹ To facilitate computations, the incomplete beta function is used as discussed in section 3.1.

block, the Bell and Doksum Two-Way Test uses a random sample of normal order statistics for the k observations in each block. The major distinction between the Bell and Doksum Two-Way Test and the Bell and Doksum One-Way Test previously discussed (see section 3.13) is in the assignment of the random sample of normal order statistics as replacements for the original set of observations. It is recalled that the One-Way Test requires the replacement of each of the N ordered original observations by a corresponding observation from a sample of N random normal order statistics. When the design format is randomized block, however, the Two-Way Test requires the replacement of each of the k ordered original observations in each block by a corresponding observation drawn from a separate sample (one for each block) of k random normal order statistics (reference is made to section 3.13 for a full discussion of the test--including its merits and demerits).

The test statistic used for testing the null hypothesis of k identical populations against the general alternative that at least two of them differ from each other is given by:

$$BD = n \sum_{j=1}^k (\bar{Z}_{.j} - \bar{Z}_{..})^2 \stackrel{d}{=} \chi^2 \text{ with } (k-1) \text{ d.f.} \quad (4.17.1)$$

where $\bar{Z}_{.j} = \sum_{i=1}^n Z_{ij} / n$ for $j = 1, 2, \dots, k$

and $\bar{Z}_{..} = \sum_{i=1}^n \sum_{j=1}^k Z_{ij} / N$.

The null hypothesis is rejected if the test statistic BD equals or exceeds the critical value from the χ^2 distribution having $k-1$ degrees of freedom associated with a particular α level

of significance.²²

4.18 A GENERAL ALIGNED RANK ORDER TEST

Working independently, Mehra and Sarangi (1967) and Sen (1968) developed the same nonparametric procedure (General Aligned Rank Order Test) for testing against general alternatives. This procedure is based on the concept of "ranking after alignment" proposed by Hodges and Lehmann (1962) in an effort to improve upon the relative efficiency of some of the rank tests used in a randomized block design (for example, see the Friedman Test presented in section 4.7).²³ Alignment refers to the removal of the block effects from the observations in a randomized block design by subtracting from each observation in a particular block some measure of location for that block such as the block mean, median, trimmed mean, or Winsorized mean. Thus, in order to use the General Aligned Rank Order Test the level of measurement attained for the data must be higher than an ordinal scale.

The argument expounded by Hodges and Lehmann for a combined ranking of all observations after alignment in an

²²To facilitate computations the incomplete gamma statistic is used as described in section 3.7.

²³Mehra and Sarangi show that the A.R.E. of the General Aligned Rank Order Test with respect to Friedman's Test is 1.5 when $k = 2$ and decreases to 1 as $k \rightarrow \infty$ when the underlying distribution is normal. Furthermore, the A.R.E. of the test with respect to the F Test under conditions of normality is $3/\pi$ when $k = 2$, reaches a maximum of .9662 when $k = 3$, and then decreases to $3/\pi$ as $k \rightarrow \infty$.

effort to increase relative efficiency was based on the fact that intra-block rank tests (such as Friedman's Test), which contain separate and independent rankings within each block, fail to take into account any inter-block comparisons and thus, are less efficient than tests which utilize the information contained in inter-block comparisons. However, the key difference in the data manipulation between Friedman's Test and the General Aligned Rank Order Test is that in the former, the observations in each block are ranked separately from (smallest) one to (largest) k while in the latter, a joint ranking of all N observations is made after the k observations in each block are aligned. It is interesting to recall here that this combined ranking of all N observations is identical to the data manipulation method used in the Kruskal-Wallis Test (see section 3.7)--a test for a completely randomized design. However, the General Aligned Rank Order Test has already taken into account the intra-block comparisons when the joint ranking is made. Mehra and Sarangi point out that alignment is necessary prior to ranking because no worthwhile information would be gleaned from a joint ranking before alignment if there exist unequal or unknown block effects. By removing these block effects first the ranks do contain relevant information--assuming the marginal distributions of all aligned observations under the null hypothesis are identical.

The General Aligned Rank Order Test is developed below for the case of one observation per cell. It should be noted,

however, that the procedure, as proposed by Hodges and Lehmann and extended by Mehra and Sarangi and Sen, is more general and may have more than one observation per cell with possible differences from cell to cell.

Let X_{ij} be the observation in the i^{th} block ($i = 1, 2, \dots, n$) under the j^{th} treatment ($j = 1, 2, \dots, k$). Hence, $\bar{X}_i = \sum_{j=1}^k X_{ij}/k$ is the mean of the observations in the i^{th} block. Now let a_{ij} be the observation in the i^{th} block under the j^{th} treatment after alignment. Thus, $a_{ij} = X_{ij} - \bar{X}_i$ for all i and j . Consider R_{ij} as the rank corresponding to a_{ij} in the combined ranking of all N aligned observations such that a rank of one is given to the smallest aligned observation and a rank of N is given to the largest. Now let $R_{.j} = \sum_{i=1}^n R_{ij}$, the sum of the ranks under the j^{th} treatment ($j = 1, 2, \dots, k$). The proposed test statistic A , adopted from Mehra and Sarangi (1967, equation 2.6) takes the form:

$$A = \left[\frac{k-1}{k \sum_{i=1}^n T_i^2} \right] \left[\sum_{j=1}^k \left(R_{.j} - \frac{n(N+1)}{2} \right)^2 \right] = \chi^2 \text{ with } (k-1) \text{ d.f. ,} \quad (4.18.1)$$

where $T_i^2 = \frac{\sum_{j=1}^k (R_{ij} - \bar{R}_{i.})^2}{k}$ and $\bar{R}_{i.} = \sum_{j=1}^k R_{ij}/k$,

and the null hypothesis of no differences in treatment effects ($F_j(X) \equiv$ for all $j = 1, 2, \dots, k$) may be rejected in favor of the general alternative ($F_j(X) \neq$ for all $j = 1, 2, \dots, k$) if $A \geq \chi_{\alpha}^2 (k-1) \text{ d.f.}$ for a particular α level of significance.

It is convenient to express the test statistic A given

in (4.18.1) in a form more conducive to practical computation. Substituting for $T_{i.}^2$ in (4.18.1) and expanding,

$$A = \frac{k-1}{\sum_{i=1}^n \sum_{j=1}^k (R_{ij} - \bar{R}_{i.})^2} \left[\sum_{j=1}^k R_{.j}^2 - \frac{2n(N+1)}{2} \sum_{j=1}^k R_{.j} + \sum_{j=1}^k \left(\frac{n^2(N+1)^2}{4} \right) \right] \quad (4.18.2)$$

But $\sum_{j=1}^k R_{.j} = N(N+1)/2$, so that

$$A = \frac{k-1}{\sum_{i=1}^n \sum_{j=1}^k (R_{ij} - \bar{R}_{i.})^2} \left[\sum_{j=1}^k R_{.j}^2 - \frac{nN(N+1)^2}{2} + \frac{kn^2(N+1)^2}{4} \right] \quad (4.18.3)$$

Replacing N by kn in the same term,

$$A = \frac{k-1}{\sum_{i=1}^n \sum_{j=1}^k (R_{ij} - \bar{R}_{i.})^2} \left[\sum_{j=1}^k R_{.j}^2 - \frac{kn^2(N+1)^2}{2} + \frac{kn^2(N+1)^2}{4} \right] \quad (4.18.4)$$

Finally,

$$A = \frac{k-1}{\sum_{i=1}^n \sum_{j=1}^k (R_{ij} - \bar{R}_{i.})^2} \left[\sum_{j=1}^k R_{.j}^2 - \frac{kn^2(N+1)^2}{4} \right] \quad (4.18.5)$$

It can now be shown through expansion that the form of the S statistic as given by Sen (1968, equation 3.5) is identical to (4.18.5) above. The S statistic takes the form:

$$S = \frac{n}{\sigma_p^2} \sum_{j=1}^k \left(\frac{R_{.j}}{n} - \frac{R_{..}}{N} \right)^2 \stackrel{d}{=} \chi^2 \text{ with } (k-1) \text{ d.f. ,} \quad (4.18.6)$$

where
$$\sigma_p^2 = \frac{\sum_{i=1}^n \sum_{j=1}^k (R_{ij} - \bar{R}_{i.})^2}{n(k-1)}$$

and
$$R_{..} = \sum_{i=1}^n \sum_{j=1}^k R_{ij} = N(N+1)/2 .$$

Substituting for σ_p^2 and $R_{..}$ in (4.18.6) and expanding,

$$S = \frac{n^2(k-1)}{\sum_{i=1}^n \sum_{j=1}^k (R_{ij} - \bar{R}_{i.})^2} \left[\frac{\sum_{j=1}^k R_{.j}^2}{n^2} - \frac{2(N+1)}{2n} \sum_{j=1}^k R_{.j} + \sum_{j=1}^k \left(\frac{N+1}{2} \right)^2 \right] \quad (4.18.7)$$

But $\sum_{j=1}^k R_{.j} = N(N+1)/2$ so that

$$S = \frac{n^2(k-1)}{\sum_{i=1}^n \sum_{j=1}^k (R_{ij} - \bar{R}_{i.})^2} \left[\frac{\sum_{j=1}^k R_{.j}^2}{n^2} - \frac{N(N+1)^2}{2n} + \frac{k(N+1)^2}{4} \right] \quad (4.18.8)$$

Replacing N by kn in the same term,

$$S = \frac{n^2(k-1)}{\sum_{i=1}^n \sum_{j=1}^k (R_{ij} - \bar{R}_{i.})^2} \left[\frac{\sum_{j=1}^k R_{.j}^2}{n^2} - \frac{k(N+1)^2}{2} + \frac{k(N+1)^2}{4} \right] \quad (4.18.9)$$

so that

$$S = \frac{n^2(k-1)}{\sum_{i=1}^n \sum_{j=1}^k (R_{ij} - \bar{R}_{i.})^2} \left[\frac{\sum_{j=1}^k R_{.j}^2}{n^2} - \frac{k(N+1)^2}{4} \right] \quad (4.18.10)$$

and

$$S = \frac{k-1}{\sum_{i=1}^n \sum_{j=1}^k (R_{ij} - \bar{R}_{i.})^2} \left[\sum_{j=1}^k R_{.j}^2 - \frac{k n^2 (N+1)^2}{4} \right] = A. \quad (4.18.5)$$

4.19 SEN'S ALIGNED RANK ORDER TEST

Sen (1968) not only developed a General Aligned Rank Order Test but also constructed a test against ordered alternatives as well. Thus, in order to test the null hypothesis of no differences in treatment effects ($F_j(X) \equiv$ for all

$j = 1, 2, \dots, k$) against the ordered alternative that $F_1(X) \geq F_2(X) \geq \dots \geq F_k(X)$ (with at least one inequality strict), Sen proposes the statistic T^* given by:

$$T^* = \frac{\sqrt{12n} \sum_{j=1}^k \left[\left(j - \frac{1}{2}(k+1) \right) \frac{R_{.j}}{n} \right]}{\sqrt{\frac{k(k^2-1)}{n(k-1)} \sum_{i=1}^n \sum_{j=1}^k (R_{ij} - \bar{R}_{i.})^2}} \stackrel{d}{=} N(0, 1), \quad (4.19.1)$$

where R_{ij} is the rank corresponding to the observation in the i^{th} block under the j^{th} treatment after alignment in the joint ranking of all N aligned observations such that a rank of one is given to the smallest and a rank of N is given to the largest. Also, $R_{.j} = \sum_{i=1}^n R_{ij}$, the sum of the ranks under the j^{th} treatment ($j = 1, 2, \dots, k$), and $\bar{R}_{i.} = \sum_{j=1}^k R_{ij} / k$.

For a specified level of significance, α , the null hypothesis may be rejected if the test statistic T^* equals or exceeds the upper α percent critical value under the normal distribution. However, the test statistic given in (4.19.1) is rather cumbersome for practical usage. For computational purposes then, the following form of T^* is developed by expansion.

Since (k^2-1) can be replaced by the product $(k-1)(k+1)$, (4.19.1) may be rewritten as:

$$T^* = \frac{\sqrt{12n} \sum_{j=1}^k \left[\left(j - \frac{1}{2}(k+1) \right) \frac{R_{.j}}{n} \right]}{\left[\frac{k(k+1)}{n} \sum_{i=1}^n \sum_{j=1}^k (R_{ij} - \bar{R}_{i.})^2 \right]^{\frac{1}{2}}} \quad (4.19.2)$$

Note, however, that the n is cancelled so that (4.19.2) becomes

$$T^* = \frac{\sum_{j=1}^k \left(j - \frac{1}{2}(k+1) \right) R_{.j}}{\left[\frac{k(k+1)}{12} \sum_{i=1}^n \sum_{j=1}^k (R_{ij} - \bar{R}_{i.})^2 \right]^{\frac{1}{2}}} \quad (4.19.3)$$

and, by expanding the numerator,

$$T^* = \frac{\sum_{j=1}^k j R_{.j} - \frac{1}{2}(k+1) \sum_{j=1}^k R_{.j}}{\left[\frac{k(k+1)}{12} \sum_{i=1}^n \sum_{j=1}^k (R_{ij} - \bar{R}_{i.})^2 \right]^{\frac{1}{2}}} \quad (4.19.4)$$

However, $\sum_{j=1}^k R_{.j} = N(N+1)/2$, so that the following computational form for T^* is given by:

$$T^* = \frac{\sum_{j=1}^k j R_{.j} - \frac{(N)(N+1)(k+1)}{4}}{\left[\frac{k(k+1)}{12} \sum_{i=1}^n \sum_{j=1}^k (R_{ij} - \bar{R}_{i.})^2 \right]^{\frac{1}{2}}} \quad (4.19.5)$$

CHAPTER V

EMPIRICAL SMALL SAMPLE POWER COMPARISONS

5.1 INTRODUCTION

In this chapter an investigation is made of the empirical small sample powers of various k sample tests attained via Monte Carlo methods. The objective of this analysis is to provide the researcher with valuable information as to the usefulness of the various k sample tests under a variety of conditions. To accomplish this, the chapter is divided into the following four sections:

Section 5.2 describes in detail the development and implementation of the Monte Carlo study.

Section 5.3 presents a preliminary comparison of the various one-way and two-way tests. Power is considered along with other criteria in order to select a set of k sample procedures worthy of a broader investigation.

Section 5.4 presents a detailed analysis of the powers of the selected one-way and two-way tests.

Section 5.5 discusses the choice of appropriate one-way and two-way tests.

5.2 DEVELOPMENT AND IMPLEMENTATION OF MONTE CARLO STUDY

Monte Carlo procedures are employed in order to obtain the empirical small sample powers of the various tests under

a variety of conditions. These controlled conditions include: (1) the kinds of ordered location alternatives chosen; (2) the types of underlying populations considered; (3) the changes in the shift parameters used; (4) the changes in the number of observations obtained; (5) the changes in the number of samples selected; and (6) the changes in the levels of significance chosen. A listing of the imposed set of conditions is given in Table 5.1.

TABLE 5.1
CONTROLLED CONDITIONS USED IN MONTE CARLO POWER STUDY

CONDITION	LEVELS	DESCRIPTION
Ordered Location Alternatives	2	1 - Trend: $\mu_{.1} < \mu_{.2} < \dots < \mu_{.k}$ (with equal spacing between means) 2 - End Gap: $\mu_{.1} = \dots = \mu_{.k-1} < \mu_{.k}$
Distributions	3	1 - Rectangular: $f(x) = \frac{1}{b-a}, \quad a < x < b$ 2 - Exponential: $f(x) = \lambda e^{-\lambda x}, \quad 0 < x < \infty$ 3 - Normal: $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty$
Shift Parameter	4	$\Delta = \sqrt{\frac{n \sum_{j=1}^k (\mu_{.j} - \bar{\mu})^2}{k \sigma_e^2}} = O(.5)1.5$
Sample Size	4	$n = 5(5)20$
Number of Groups	4	$k = 3(1)6$
Level of Significance	2	$\alpha = .01, .05$

The selection of the particular set of conditions outlined in Table 5.1 is useful for several reasons:¹

1. Examining the small sample empirical powers of the tests under the two extreme ordered location alternatives (trend and end gap) will provide information as to the sensitivity of a test in detecting the specific kind of ordered alternative.

2. Examining the small sample empirical powers of the tests over the range of Δ ($\Delta = 0(.5)1.5$) will provide information as to which procedures are more likely to detect small departures from the null hypothesis.²

3. Examining the small sample empirical powers of various tests under the three parent distributions (rectangular, exponential and normal) will provide information as to the power of k sample tests as well as to the robustness of the parametric tests with respect to violations in the assumption of normality (see Appendix A7).

The main features of the computerizational aspects of the Monte Carlo study are depicted in Figure 5.1. From this

¹In addition to the major reasons presented here it might also be of interest to the researcher to empirically demonstrate the relationships between power and α , n, k, and the effect on power caused by the allocation of fixed N observations into various combinations of n and k. These are discussed in Appendix A8.

²Larger values of Δ are not selected because of the rapid convergence of the powers of some of the procedures to their limits.

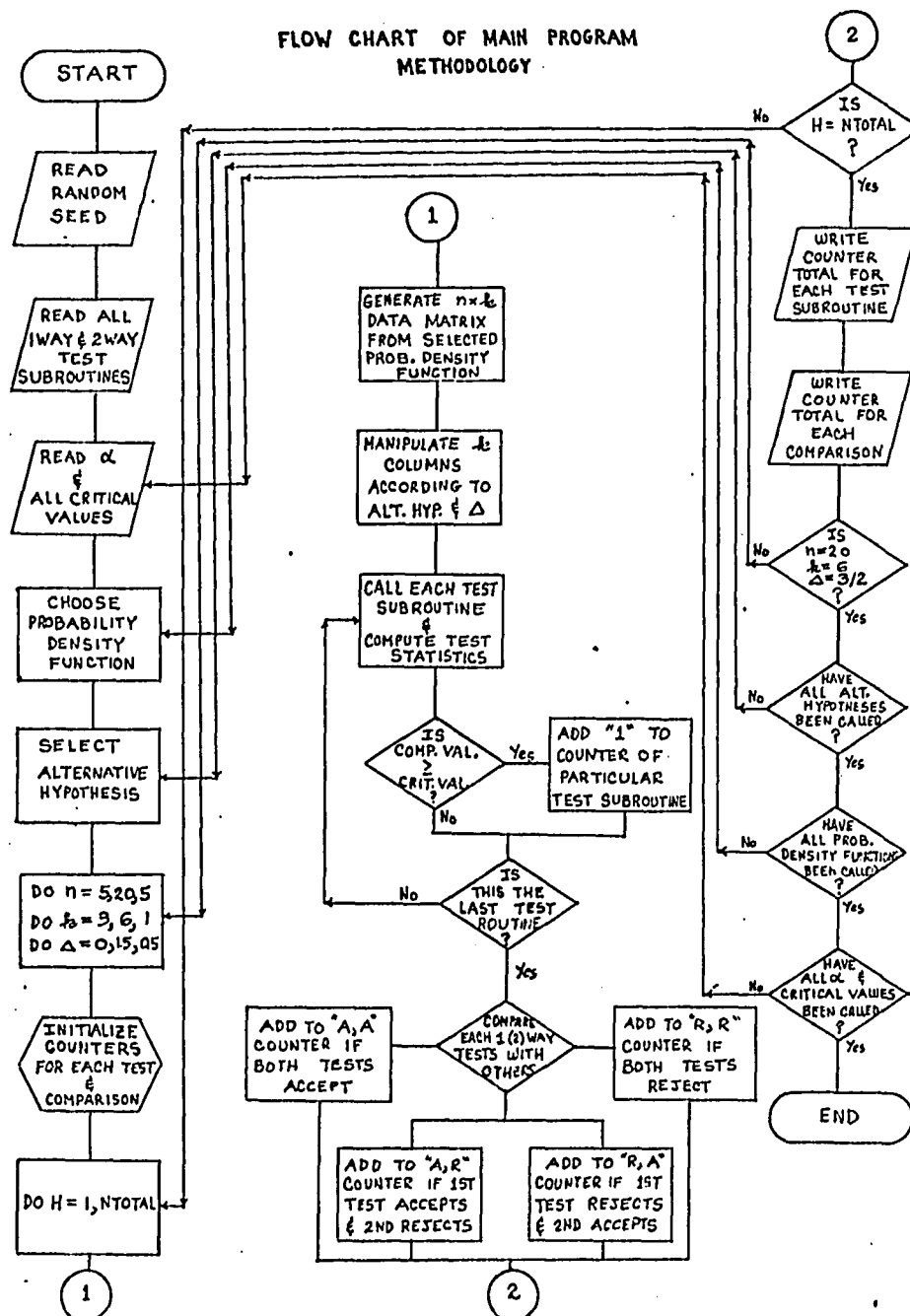


FIGURE 5.1: MONTE CARLO STUDY

it is important to discuss how data are generated and manipulated (according to Δ).

Generation of Data

In order to examine the effect that violating the assumption of normality has on the powers of several tests under true ordered location alternatives, it is necessary to insure that the variances within the k sample groups are equal.³ To achieve this, all k samples in the Monte Carlo study are generated from the same parent population (rectangular, exponential, or normal) and then the particular samples shifted by the addition of a constant to every observation in the samples--depending on whether the alternative is trend or gap. A schematic representation of the shift in the location parameter when $k = 3$ is presented in Figure 5.2 (p. 131) for the trend alternative. In general, under the trend alternative when $k = 3$, the samples from the aforementioned parent distributions are considered to be generated from density functions having the following forms and means (see Table 5.2, p. 132).

It should be noted that the variances of the three rectangular, exponential, and normal populations are each equal to $(b-a)^2/12$, $1/\lambda^2$, and σ^2 , respectively, maintaining the property of homoscedasticity. Similarly, this property is

³Violations in the assumption of homoscedasticity in addition to that of normality is a separate problem to be considered as a proposed research project. For further discussion see Appendix A5. Preliminary investigations here appear to back up the claims made by Conover (1968), and Donaldson (1968), among others, as to how certain k sample tests are affected by such violations.

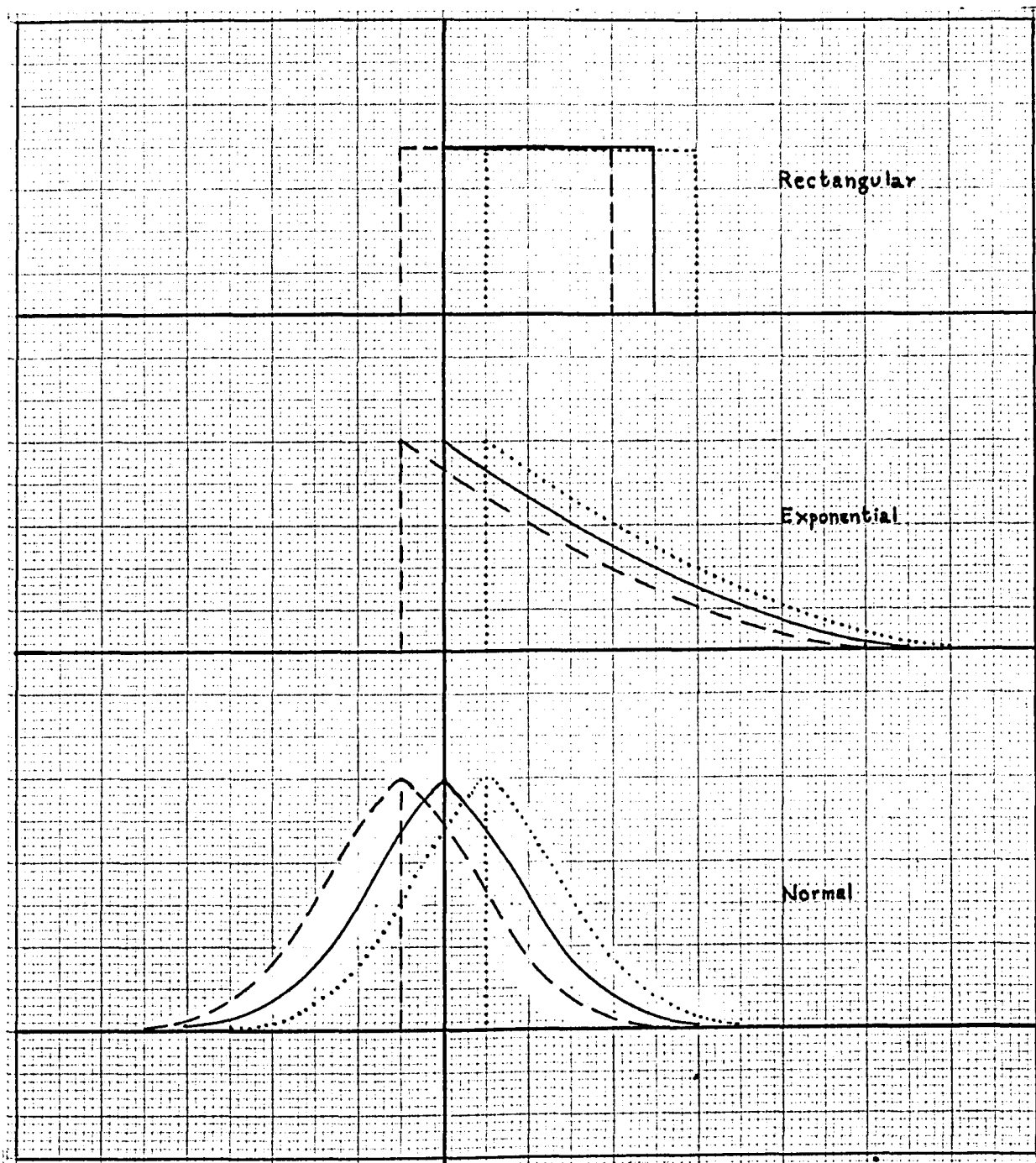


FIGURE 5.2: SCHEMATIC REPRESENTATION OF SHIFT IN LOCATION
PARAMETER ($k = 3$)

TABLE 5.2
GENERATION OF DATA UNDER TREND ALTERNATIVE (k = 3)

DISTRIBUTION		FIRST GROUP	SECOND GROUP	THIRD GROUP
Rectangular	f_j	$\frac{1}{b-a}$ where $a-\theta < x < b-\theta$	$\frac{1}{b-a}$ where $a < x < b$	$\frac{1}{b-a}$ where $a+\theta < x < b+\theta$
	μ_j	$\frac{a+b}{2} - \theta$	$\frac{a+b}{2}$	$\frac{a+b}{2} + \theta$
Exponential	f_j	$\lambda e^{-\lambda(x+\theta)}$ where $-\theta < x < \infty$	$\lambda e^{-\lambda x}$ where $0 < x < \infty$	$\lambda e^{-\lambda(x-\theta)}$ where $\theta < x < \infty$
	μ_j	$\frac{1}{\lambda} - \theta$	$\frac{1}{\lambda}$	$\frac{1}{\lambda} + \theta$
Normal	f_j	$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu+\theta)^2}{2\sigma^2}}$ where $-\infty < x < \infty$	$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ where $-\infty < x < \infty$	$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu-\theta)^2}{2\sigma^2}}$ where $-\infty < x < \infty$
	μ_j	$\mu - \theta$	μ	$\mu + \theta$

maintained under the gap alternative where, for any $k > 2$, the samples from the parent populations are considered to be selected from density functions having the following forms and means:

TABLE 5.3
GENERATION OF DATA UNDER END GAP ALTERNATIVE

DISTRIBUTION	FIRST GROUP, ..., $k-1^{\text{th}}$ GROUP	k^{th} GROUP
Rectangular f_j μ_{ij}	$\frac{1}{b-a}$ where $a-\theta < x < b-\theta$ $\frac{a+b}{2} - \theta$	$\frac{1}{b-a}$ where $a+(k-1)\theta < x < b+(k-1)\theta$ $\frac{a+b}{2} + (k-1)\theta$
Exponential f_j μ_{ij}	$\lambda e^{-\lambda(x+\theta)}$ where $-\theta < x < \infty$ $\frac{1}{\lambda} - \theta$	$\lambda e^{-\lambda(x-(k-1)\theta)}$ where $(k-1)\theta < x < \infty$ $\frac{1}{\lambda} + (k-1)\theta$
Normal f_j μ_{ij}	$\frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu+\theta)^2}{2\sigma^2}}$ where $-\infty < x < \infty$ $\mu - \theta$	$\frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu-(k-1)\theta)^2}{2\sigma^2}}$ where $-\infty < x < \infty$ $\mu + (k-1)\theta$

Specific to this dissertation, however, the rectangular distribution is taken over the unit interval ($a = 0$ to $b = 1$); the exponential distribution uses $\lambda = \frac{1}{2}$; and the normal distribution generated is the standardized distribution.

Establishing the Shift Parameters

Dixon and Massey (1969, pp. 277-279) and Donaldson (1968)

define the parameter Δ for an alternative to the null F statistic to be:

$$\Delta = \sqrt{\frac{n \sum_{j=1}^k (\mu_j - \bar{\mu})^2}{k \sigma_e^2}} \quad (5.2.1)$$

Donaldson argues that while his results are based on equal differences between the k means, as long as the assumption of homoscedasticity holds the mean square error is independent of the location of the k means and Δ proportional to $\sum_{j=1}^k (\mu_j - \bar{\mu})^2$.⁴ The results would thus be valid for variable differences between successive means.⁵ While this may be true for determining the powers of tests against general alternatives--such as the One-Way F Test and Two-Way F Test--it does, however, become necessary to account for the kinds of ordered differences between successive means when examining the powers of tests designed against ordered alternatives.⁶ Since it is the concern here to consider two kinds of ordered alternatives--trend (wherein the μ_j are equally spaced) and gap (wherein only the last mean, μ_k , differs)--the differences (distances) between

⁴Donaldson (1968, p. 673) states that under a violation of homoscedasticity, the mean square error is the simple average (when $n_j =$) of the k sample variances and is an unbiased estimate of σ_e^2 , the average of the k population variances (i.e., $\sigma_e^2 = \frac{\sigma_1^2 + \sigma_2^2 + \dots + \sigma_k^2}{k}$). This becomes important for the research outlined in Appendix A5.

⁵See example in Dixon and Massey (1969, p. 278).

⁶See tables of the power functions in Bartholomew (1961A), Puri (1965), and Johnson-Mehrotra (1971), for example, which show powers of various tests under the trend and gap alternatives.

the means for given values of Δ , n , and k must be calculated for comparison purposes.⁷ This is necessary since Δ , defined in (5.2.1), is a function of n , k , and these distances.

Under the trend alternative, define the successive (equally spaced) means to be θ units apart. Moreover, let $d = (k-1)\theta$, the total distance from smallest to largest mean. Finally, consider the value of the mean of means, $\bar{\mu}$, as zero. Under these conditions, the following is true:

$$k = 3: \mu_1 = -\theta \quad \mu_2 = 0 \quad \mu_3 = \theta$$

$$k = 4: \mu_1 = -\frac{3}{2}\theta \quad \mu_2 = -\frac{1}{2}\theta \quad \mu_3 = \frac{1}{2}\theta \quad \mu_4 = \frac{3}{2}\theta$$

$$k = 5: \mu_1 = -2\theta \quad \mu_2 = -\theta \quad \mu_3 = 0 \quad \mu_4 = \theta \quad \mu_5 = 2\theta$$

$$k = 6: \mu_1 = -\frac{5}{2}\theta \quad \mu_2 = -\frac{3}{2}\theta \quad \mu_3 = -\frac{1}{2}\theta \quad \mu_4 = \frac{1}{2}\theta \quad \mu_5 = \frac{3}{2}\theta \quad \mu_6 = \frac{5}{2}\theta$$

For each value of k the coefficients sum to zero. From (5.2.1) it is seen that when $\Delta = 0$ (the null hypothesis is true) $\mu_j = \bar{\mu}$ for all $j = 1, 2, \dots, k$. For particular $\Delta > 0$ (the alternative hypothesis is true), the distances θ between successive means can be readily determined for all levels of k , n , and σ_e^2 (the latter depending on the underlying distribution).⁸ The results are presented in Table 5.4 (p. 136).⁹ In addition, Table 5.5

⁷See Appendix A8 for a discussion of some power relationships which require these distances.

⁸For the Monte Carlo study consider $\sigma_e^2 = 1/12$, 4, and 1, respectively, for the rectangular, exponential, and normal distribution.

⁹For example, when $n = 5$, $k = 3$, and $\Delta = 0.5$, the distance θ between successive means under the trend alternative is obtained from (5.2.1) as follows:

$$0.5 = \sqrt{\frac{5 [(-\theta-0)^2 + (0-0)^2 + (\theta-0)^2]}{3\sigma_e^2}}, \text{ whereby solving}$$

for θ yields: $\theta = .2739 \sigma_e$.

TABLE 5.4
DISTANCES θ BETWEEN SUCCESSIVE MEANS UNDER TREND ALTERNATIVE
FOR VARIOUS LEVELS OF Δ , n , AND k (in standard deviation units)

SHIFT PARAMETER		$\Delta = 0.5$				$\Delta = 1.0$				$\Delta = 1.5$			
n	k	3	4	5	6	3	4	5	6	3	4	5	6
5		.2739	.2000	.1581	.1309	.5477	.4000	.3162	.2619	.8216	.6000	.4743	.3928
10		.1937	.1414	.1118	.0926	.3873	.2828	.2236	.1852	.5810	.4242	.3354	.2777
15		.1581	.1155	.0913	.0756	.3162	.2310	.1826	.1512	.4743	.3464	.2739	.2268
20		.1369	.1000	.0791	.0655	.2739	.2000	.1581	.1309	.4108	.3000	.2372	.1964

TABLE 5.5
TOTAL DISTANCES d FROM SMALLEST TO LARGEST MEAN UNDER TREND
ALTERNATIVE FOR VARIOUS LEVELS OF Δ , n , and k (in standard deviation units)

SHIFT PARAMETER		$\Delta = 0.5$				$\Delta = 1.0$				$\Delta = 1.5$			
n	k	3	4	5	6	3	4	5	6	3	4	5	6
5		.5477	.6000	.6324	.6550	1.0954	1.2000	1.2648	1.3090	1.6432	1.8000	1.8971	1.9640
10		.3873	.4242	.4472	.4630	.7746	.8484	.8944	.9260	1.1620	1.2726	1.3416	1.3890
15		.3162	.3464	.3652	.3780	.6324	.6928	.7304	.7560	.9486	1.0392	1.0956	1.1340
20		.2739	.3000	.3164	.3270	.5478	.6000	.6324	.6550	.8216	.9000	.9488	.9820

(p. 136) presents the total distances, d , from smallest to largest mean over all values of n , k , and Δ . From Tables 5.4 and 5.5 one may determine which combinations of n , k , and Δ have means the same distance apart and comparisons between these combinations can be made (see Appendix A8).

When developing the distance values under the gap alternative it must be recalled that only the k^{th} mean, $\mu_{.k}$, differs from the others. Defining θ' as the distance between the $(k-1)$ smallest means and $\bar{\mu}$, and letting $\bar{\mu} = 0$ and $d = k\theta'$, the following is true:

$$k = 3: \quad \mu_{.1} = -\theta' \quad \mu_{.2} = -\theta' \quad \mu_{.3} = 2\theta'$$

$$k = 4: \quad \mu_{.1} = -\theta' \quad \mu_{.2} = -\theta' \quad \mu_{.3} = -\theta' \quad \mu_{.4} = 3\theta'$$

$$k = 5: \quad \mu_{.1} = -\theta' \quad \mu_{.2} = -\theta' \quad \mu_{.3} = -\theta' \quad \mu_{.4} = -\theta' \quad \mu_{.5} = 4\theta'$$

$$k = 6: \quad \mu_{.1} = -\theta' \quad \mu_{.2} = -\theta' \quad \mu_{.3} = -\theta' \quad \mu_{.4} = -\theta' \quad \mu_{.5} = -\theta' \quad \mu_{.6} = 5\theta'$$

For all combinations of n , k , and Δ , Table 5.6 (p. 138) gives the θ' values computed from (5.2.1) and Table 5.7 (p. 138) presents the distances from the $(k-1)$ means to the k^{th} mean under the gap alternative.

5.3 PRELIMINARY ANALYSIS

Because of the magnitude of this undertaking it is obviously necessary to reduce the number of test procedures from thirty-eight to a more manageable set. To achieve this, a preliminary Monte Carlo investigation of all k sample tests is accomplished under the normal and exponential distributions for each condition-combination of n , k , and ordered alternative--for α of 5% and for small values of Δ . Three hundred histories

TABLE 5.6
VALUES OF θ' DISTANCES UNDER GAP ALTERNATIVE
(in standard deviation units)

SHIFT PARAMETER		$\Delta = 0.5$				$\Delta = 1.0$				$\Delta = 1.5$			
n	k	3	4	5	6	3	4	5	6	3	4	5	6
5		.1581	.1291	.1118	.1000	.3162	.2582	.2236	.2000	.4743	.3873	.3354	.3000
10		.1118	.0913	.0791	.0707	.2236	.1826	.1582	.1414	.3354	.2739	.2373	.2121
15		.0913	.0745	.0645	.0577	.1826	.1490	.1290	.1154	.2739	.2235	.1935	.1731
20		.0791	.0645	.0559	.0500	.1582	.1290	.1118	.1000	.2373	.1935	.1677	.1500

TABLE 5.7
TOTAL DISTANCE d BETWEEN $(k-1)$ MEANS AND μ_k UNDER THE GAP ALTERNATIVE
(in standard deviation units)

SHIFT PARAMETER		$\Delta = 0.5$				$\Delta = 1.0$				$\Delta = 1.5$			
n	k	3	4	5	6	3	4	5	6	3	4	5	6
5		.4743	.5164	.5590	.6000	.9486	1.0328	1.1180	1.2000	1.4229	1.5492	1.6770	1.8000
10		.3354	.3652	.3955	.4242	.6708	.7304	.7910	.8484	1.0062	1.0956	1.1865	1.2726
15		.2739	.2980	.3225	.3462	.5478	.5960	.6450	.6924	.8217	.8940	.9675	1.0386
20		.2373	.2580	.2795	.3000	.4746	.5160	.5590	.6000	.7119	.7740	.8385	.9000

for each such combination are obtained. The objective is to determine a set of "good tests"--ones which more readily detect small departures from the null hypothesis--for inclusion in a more detailed Monte Carlo investigation. The results, presented in Tables A1 through A8, show the small sample empirical efficiencies of each test (using the F Test and "best test" as a base for both the one-way and two-way procedures) under all imposed conditions of the true alternative.¹⁰ Moreover, Tables A9 through A16 show the power ratios of each test relative to the F and "best tests" for each $k = 3(1)6$ and for each $n = 5(5)20$.

To compare the various tests, a ranking of the power ratios is made for each alternative and distribution combination for the "extremes"-- $n = 5$ and 20 regardless of k , and $k = 3$ and 6 regardless of n . Summaries of these results are presented in Tables 5.8 (p. 140) and 5.9 (p. 141) for the one-way and two-way procedures respectively.

Thus, one can evaluate the ordinal position of each test: (1) when the particular distribution-alternative combination can be specified; (2) when only the alternative can be specified (without knowledge of the distribution); or (3) when only the distribution can be specified (without knowledge of the kind of ordered alternative.)

¹⁰A discussion of various concepts or "power efficiency"--including that of small sample empirical efficiencies or power ratios--is presented in Appendix A4.

TABLE 5.8

A COMPARATIVE RANKING OF THE POWERS OF NINETEEN ONE-WAY TESTS
 UNDER A VARIETY OF CONDITIONS*
 (RANK 1 = BEST, RANK 19 = WORST)

TEST	EXPONENTIAL			NORMAL			"COMBINED"		
	TREND	GAP	OVERALL	TREND	GAP	OVERALL	TREND	GAP	OVERALL
FTEST	17	17	17	14	13	13	15.5	15	16
HTEST	15	16	16	16	17	17	15.5	17	17
CONKS	16	13	14	17	14	16	17	13	15
JONKE	3	4	3	8	9	9	4	8	6
BARTH	10	6	10	7	5	7	10	6	9.5
CHAKO	4	3	4	9	7	8	6	3.5	3
ATMAX	11	6	11	6	1	4	10	2	8
LRCON	7	10	7	1	6	5	3	9.5	7
DEREG	8.5	8.5	8.5	4	3	3	5	7	5
QEREG	8.5	8.5	8.5	5	2	2	7	5	4
JMENS	2	1	1.5	3	4	1	2	1	1
PURIS	1	2	1.5	2	8	6	1	3.5	2
FYENS	13	12	12	13	12	12	13	12	12
BDRNS	14	15	15	15	15	15	14	15	14
COXS1	5	6	5	10	10	10	8	9.5	9.5
COXS2	12	14	13	12	16	14	12	15	13
COXS3	6	11	6	11	11	11	10	11	11
CONSL	18	18.5	18	18	18	18	18	18	18
MOSSL	19	18.5	19	19	19	19	19	19	19

*Ranks obtained from Tables A1 - A4 and A9 - A12.

TABLE 5.9

A COMPARATIVE RANKING OF THE POWERS OF NINETEEN TWO-WAY TESTS
 UNDER A VARIETY OF CONDITIONS*
 (RANK 1 = BEST, RANK 19 = WORST)

TEST	EXPONENTIAL			NORMAL			"COMBINED"		
	TREND	GAP	OVERALL	TREND	GAP	OVERALL	TREND	GAP	OVERALL
F2WAY	19	18	19	13	13	13	16.5	15	15
PAGET	3	4	3	9.5	10	9	6	8.5	6
FRIED	17.5	15	16	18	18	18	18	16.5	18
PHENS	2	2	2	9.5	11	10	3	7	5
JONK2	7	12	11	12	12	12	12	12	12
ATMX2	12	8.5	12	5	1	4	11	3	8.5
LRCN2	8	7	7	1	4	2	2	4	2
DERG2	10	8.5	10	2	3	1	4	5	3
QERG2	11	5	8	3	2	3	9	1.5	4
FYNS2	14	13	13	14	14	14	13	13	13
BDNS2	17.5	16.5	17	19	19	19	19	19	19
PSIGN	13	14	14	17	16.5	17	14.5	14	14
PTRST	15	19	18	15	15	15	14.5	18	17
HPARA	9	11	9	4	5	5	8	10	11
HOLLT	4.5	6	5	8	8	8	6	8.5	7
DOKST	6	10	6	6.5	7	7	6	11	8.5
SHORT	4.5	3	4	11	9	11	10	6	10
SMSAT	16	16.5	15	16	16.5	16	16.5	16.5	16
SENAT	1	1	1	6.5	6	6	1	1.5	1

*Ranks obtained from Tables A5 - A8 and A13 - A16.

It is important to note that when replacing the ratios by their ordinal values information is lost since it appears that in most instances ten of the nineteen one-way tests and twelve of the nineteen two-way tests have fairly good power when compared to the best test.¹¹ In many cases the differences in the power ratios are quite small. From this rank structure, however, one is able to see at a glance the better test procedures for the various conditions imposed.

Criteria for Test Selection

It should be recalled that four types of one-way and two-way test procedures are examined in this dissertation: parametric and nonparametric general tests; parametric and nonparametric ordered tests. In reducing the number of tests for further study, the following criteria, in order of importance, are established: (1) overall ranking in power; (2) cumbersome-ness in practical application; (3) generality of assumptions required for a test; and (4) representation of different kinds of tests for study.

¹¹From Kendall and Stuart (1958, p. 232), the variance of the ratio of two independent variables, P_1 and P_2 , is given by:

$$\text{Var}(P_1/P_2) = \left\{ \frac{E(P_1)}{E(P_2)} \right\}^2 \left\{ \frac{\text{Var}(P_1)}{(E(P_1))^2} + \frac{\text{Var}(P_2)}{(E(P_2))^2} \right\} .$$

This may be used to establish confidence interval estimates for the true power ratios. For example, if one procedure has 50% power while another has 60%, the power ratio of the former with respect to the latter is $50\%/60\% = .833$. With 300 histories (as in Tables A1 - A8), the 95% confidence limits for the true power ratio are .709 and .958. With 1,200 histories (as in Tables A9 - A16), the limits are .771 and .895.

Selection of One-Way Tests

From the rankings of one-way tests in Table 5.8, the following procedures are noted to be comparatively weaker in power relative to others presented herein and are excluded from further consideration:¹²

A One-Sided k Sample Kolmogorov-Smirnov Test

Conover's First k Sample Slippage Test

Mosteller's k Sample Slippage Test

Cox and Stuart's Unweighted Sign Test

Cox and Stuart's Best Unweighted Sign Test

Fisher and Yates One-Way Test

Bell and Doksum One-Way Test

Moreover, the following test is eliminated from further consideration because it is only average in terms of power and simplicity:

Cox and Stuart's Weighted Sign Test.

¹²The Kolmogorov-Smirnov type test is designed for detecting all kinds of ordered differences--not just ordered location differences. Furthermore, the Fisher and Yates Test and the Bell and Doksum Test are both designed against more generalized alternatives. Hence, these tests are expected to be weaker in power under true ordered location alternatives. On the other hand, the slippage tests are "sign type" tests designed for use against the end gap alternative. However, the results show these tests to be extremely poor against trend and only slightly better as "quick and dirty" methods for testing against gap (which they were designed for). Nevertheless, it is interesting to note that based on other preliminary research, Mosteller's Test appears to be one of the best procedures against the rectangular-gap combination and not too bad against normal-gap and exponential-gap if k gets large and/or n gets large. More work is needed to examine the unique properties of this test. In addition, Conover's First Test for Slippage, according to the inventor, has good power under violations of homoscedasticity. Other preliminary work here has borne this out and further research is proposed to investigate the effects on the powers of all the procedures to violations not only in normality but homoscedasticity as well.

Furthermore, the following two contrast procedures are eliminated from further consideration because of their similarity to other tests included herein (see sections 3.3 through 3.6) and because of the rather arbitrary nature by which their contrasts are formed (see criticism by Page (1963)):

One-Way Linear-2 Contrast

One-Way Linear 2-4 Contrast.

Thus, while it appears that ten one-way procedures have reasonably good overall power, in accordance with the aforementioned criteria for test selection (i.e., power, cumbersome-ness, generality, and similarity), seven "good tests" are chosen for further study along with the One-Way F Test (a parametric standard for general alternatives) and the Kruskal-Wallis Test (a nonparametric standard for generalized alternatives).

The reduced set of nine one-way tests are presented in Table 5.10.

TABLE 5.10
REDUCED SET OF ONE-WAY PROCEDURES

TEST	TYPE	ALTERNATIVE DESIGNED FOR
One-Way F Test	Parametric	General
Bartholomew's $\bar{E}^2$ Test	Parametric	Ordered
Abelson & Tukey Optimal Contrast	Parametric	Ordered
One-Tail Regression Test	Parametric	Ordered
Kruskal-Wallis Test	Nonparametric (ranks)	General
Chacko's Test	Nonparametric (ranks)	Ordered
Jonckheere's Test	Nonparametric (ranks)	Ordered
Puri's Test	Nonparametric (normal scores)	Ordered
Johnson-Mehrotra Test	Nonparametric (normal scores)	Ordered

Selection of Two-Way Tests

From the rankings of the two-way tests in Table 5.9 (p. 141), the following procedures are noted to be comparatively weaker in power relative to others presented herein and are excluded from further consideration:

Pirie Sign Test

Pirie-Tukey Rank Sum Test

Fisher and Yates Two-Way Test

Bell and Doksum Two-Way Test

A General Aligned Rank Order Test.

Moreover, the following test is eliminated from further consideration because it is only average in power and simplicity:

Jonckheere's Two-Way Test.

In addition, the following test is eliminated because of its cumbersomeness and lack of generality:

Hollander's Parametric Test.

Furthermore, the following tests are eliminated because of their similarity to other tests included herein (see sections 4.3 through 4.6 and 4.11 and 4.12):

Two-Way Linear-2 Contrast

Two-Way Linear 2-4 Contrast

A Modified Doksum Rank Test.

Based on the aforementioned criteria for test selection, the following reduced set of seven "good" two-way procedures are chosen for further analysis along with the Two-Way F Test (a parametric standard for generalized alternatives) and the

Friedman Test (a nonparametric standard for generalized alternatives).

TABLE 5.11
REDUCED SET OF TWO-WAY PROCEDURES

TEST	TYPE	ALTERNATIVE DESIGNED FOR
Two-Way F Test	Parametric	General
Abelson & Tukey Optimal Contrast	Parametric	Ordered
One-Tail Regression Test	Parametric	Ordered
Friedman's Test	Nonparametric (ranks)	General
Shorack's Test	Nonparametric (ranks)	Ordered
Hollander's Test (Modified)	Nonparametric (ranks)	Ordered
Page's L Test	Nonparametric (ranks)	Ordered
Pirie-Hollander Test	Nonparametric (normal scores)	Ordered
Sen's Aligned Rank Order Test	Nonparametric (ranking after alignment)	Ordered

5.4 DETAILED ANALYSIS

The reduced set of one-way and two-way tests are then subjected to a detailed Monte Carlo study (see section 5.2) under all condition-combinations-- n , k , distribution and ordered alternative--for various levels of Δ and α . Whenever $n \leq 10$, 800 histories of each test statistic are generated under the null hypothesis ($\Delta = 0$) while 400 histories are obtained when $\Delta = 0.5(0.5)1.5$.¹³ For $n \geq 15$, however, half of the above

¹³When $\Delta = 0$ a check on the computer program is obtained through a comparison of the generated levels of significance with the expected levels. This is presented in Appendix A6. In addition a detailed discussion on the robustness of parametric tests to violations in normality is given in Appendix A7.

number of histories are obtained.

Presentation of Results

Tables A17 through A40 present the generated powers of each one-way and two-way procedure over the entire set of imposed conditions. In addition, Tables A41 through A52 give, for each condition, the power ratio of each test relative to the "best test". In order to enhance some of the findings, various types of charts are constructed from the obtained data. As an example, Figure 5.3 (p. 148) depicts the differences in power between the nine one-way procedures over all values of Δ for $n = 5$, $k = 3$ and for $n = 5$, $k = 6$ under the rectangular distribution and trend alternative. A representative selection of various kinds of charts, however, is given in Figures A1 through A10.¹⁴

¹⁴Figures A1 through A6 may be used to depict, for samples of size five and for each combination of k , distribution, and alternative, the powers of the F Test against that of a highly ranked parametric ordered procedure and nonparametric ordered procedure over all values of Δ (for both the one-way and two-way tests), using a level of significance of 5%. In addition, to demonstrate the relationship between power and number of groups, Figure A7 depicts, for each alternative, for each k , and for samples of size five under the exponential distribution, the power of each one-way test over all values of Δ . Similarly, to demonstrate the relationship between power and sample size, Figure A8 depicts, for each alternative, for each n , and for $k = 3$ under the exponential distribution, the power of each two-way test over all values of Δ . Furthermore, to get an idea of the differences in the power of a test under the trend versus the gap alternative, Figures A9 and A10 depict, for each Δ , distribution, and $k = 3(3)6$, the power of the One-Way F versus the Jonckheere procedure over all values of n . It should be noted that each figure referred to above contains only a sampling of all possible condition-combinations that could have been plotted.

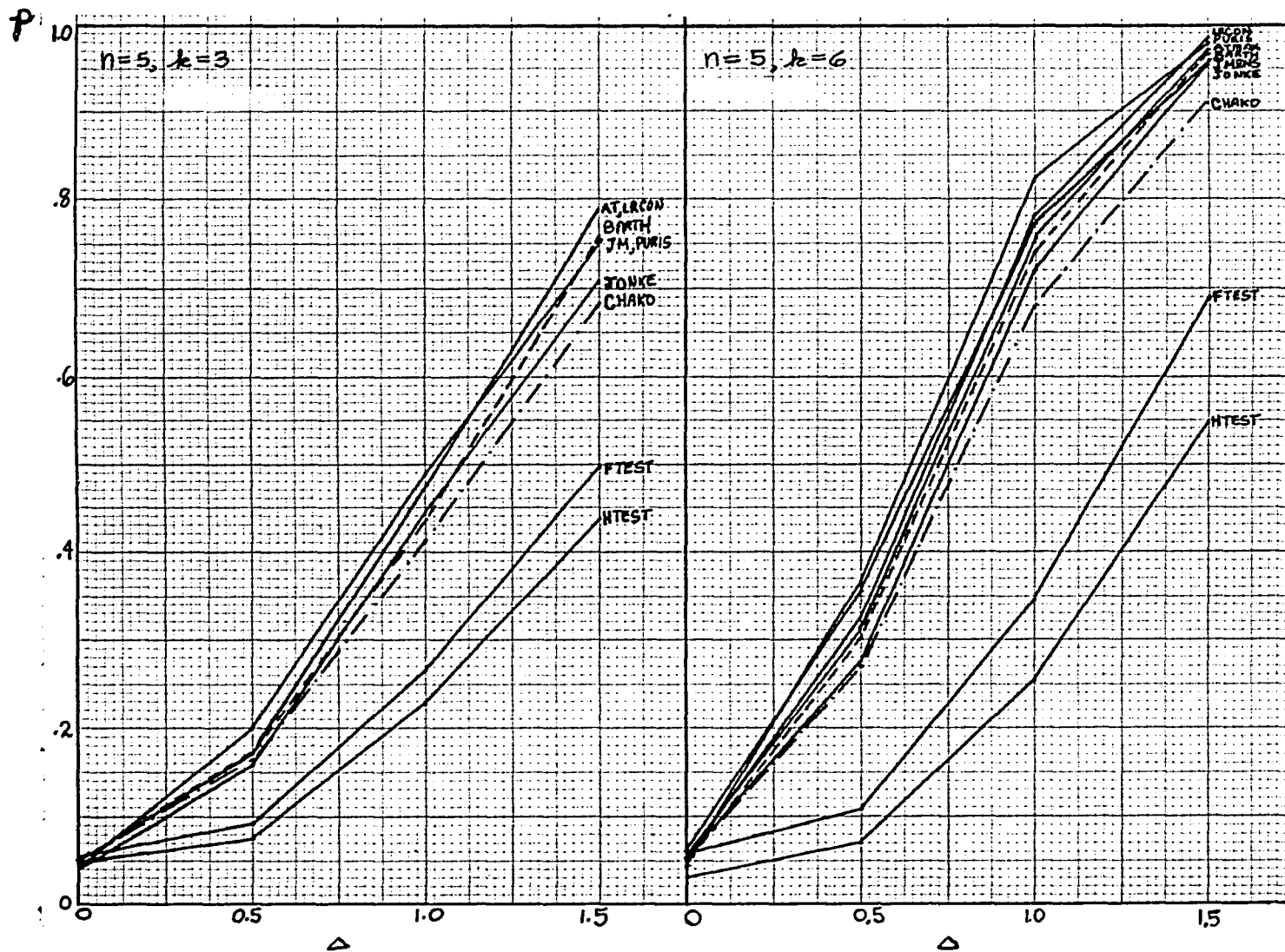


FIGURE 5.3: POWERS OF NINE ONE-WAY TESTS UNDER RECTANGULAR DISTRIBUTION AND TREND ALTERNATIVE ($n = 5$ & $k = 3$ versus $n = 5$ & $k = 6$)

Analysis of Power

Effects of Δ

From Tables A17 through A40 and Figures A1 through A6 one may observe that the powers for many tests converge to their limits quite rapidly as Δ increases--particularly under the trend alternative. What is more interesting to observe, however, is the phenomenon of interaction between tests and Δ . That is, some tests are more powerful than others in detecting small departures from the null hypothesis but are not as powerful as these others when Δ increases. As examples, Table 5.12 (p. 150) compares the powers of the Two-Way F Test versus Page's L Test against the gap alternative under the rectangular distribution while Table 5.13 (p. 150) compares the powers of Shorack's Test versus Sen's Aligned Rank Order Test against the gap alternative under the exponential distribution. Thus, it is seen that the Page and Sen procedures both exhibit a desirable property--the ability to detect small departures from the null.¹⁵

Overall Findings

Based on Tables A17 through A40, Tables 5.14 and 5.15 (p. 151, 152) present a comparative ranking of the powers of the various one-way and two-way procedures (over all n , k ,

¹⁵This empirical finding is observed asymptotically as well. Moreover, Puri (1965) draws a similar conclusion when he states that his test is always preferred to the One-Way F Test even though his table shows that as Δ increases the latter becomes more powerful under the end gap alternative.

TABLE 5.12
POWER OF TWO-WAY F TEST VS.
PAGE'S L TEST UNDER RECTANGULAR-GAP^a
($n = 10$, $k = 5$, $\alpha = .05$)

TEST	Δ			
	0.0	0.5	1.0	1.5
F	.057	.105	.325	.712
PAGE	.047	.180	.337	.577

^aBased on 400 Trials.

TABLE 5.13
POWER OF SHORACK'S TEST VS. SEN'S
ALIGNED RANK ORDER TEST UNDER EXPONENTIAL GAP^a
($n = 20$, $k = 4$, $\alpha = .05$)

TEST	Δ			
	0.0	0.5	1.0	1.5
SHORACK	.047	.260	.755	.960
SEN	.045	.325	.670	.915

^aBased on 200 Trials.

TABLE 5.14

A COMPARATIVE RANKING OF THE POWERS OF NINE ONE-WAY PROCEDURES^{a,b}
 [Significantly Most Powerful Tests are Indicated by Brackets]
 (Rank 1 = Highest Power, Rank 9 = Lowest Power)

TEST	RECTANGULAR		EXPONENTIAL		NORMAL		OVERALL	OVERALL	OVERALL
	TREND	GAP	TREND	GAP	TREND	GAP	TREND	GAP	
FTEST	8	8	9	9	8	8	8	8.5	8
HTEST	9	9	8	7	9	9	9	8.5	9
JONKE	5	7	[2]	4	5	7	4.5	6	6.5
BARTH	6	[2]	7	3	6	[1]	7	1	4
CHAKO	7	5	4	[1]	7	[3]	6	2.5	4
ATMAX	4	[3]	6	6	[3]	[2]	4.5	4	4
LRCON	3	6	5	8	[1]	5	3	7	6.5
JMENS	[2]	[1]	[3]	2	[4]	4	2	2.5	1
PURIS	[1]	[4]	[1]	5	[2]	6	1	5	2

^aRanks are based on "mean power ranks" over all n , k , and Δ where $\alpha = .05$.

^bTests which are significantly most powerful based on a Friedman-type a posteriori multiple comparison procedure due to Nemenyi (1963).

TABLE 5.15
A COMPARATIVE RANKING OF THE POWERS OF NINE TWO-WAY PROCEDURES^{a,b}
[Significantly Most Powerful Tests are Indicated by Brackets]
(Rank 1 = Highest Power, Rank 9 = Lowest Power)

TEST	RECTANGULAR		EXPONENTIAL		NORMAL		OVERALL	OVERALL	OVERALL
	TREND	GAP	TREND	GAP	TREND	GAP	TREND	GAP	
F2WAY	8	8	9	9	8	8	8	8	8
PAGET	6	7	3	6	6	7	6	7	7
FRIED	9	9	8	8	9	9	9	9	9
PHENS	5	6	2	5	5	6	5	5.5	6
ATMX2	[2]	[1]	7	3	[2]	[1]	3.5	1	1.5
LRCN2	[1]	[2]	6	4	[1]	[2]	1.5	3	3
HOLLT	3	5	4	7	[4]	5	3.5	5.5	4.5
SHORT	7	3	5	[1]	7	3	7	2	4.5
SENAT	4	4	[1]	[2]	[3]	4	1.5	4	1.5

^aRanks are based on "mean power ranks" over all n , k , and Δ where $\alpha = .05$.

^bTests which are significantly most powerful based on a Friedman-type a posteriori multiple comparison procedure due to Nemenyi (1963).

and Δ) for each distribution-alternative combination.¹⁶ In addition, Tables 5.14 and 5.15 indicate, respectively, the significantly most powerful one-way and two-way tests (under each distribution-alternative condition-combination) based on a Friedman-type a posteriori multiple comparison procedure developed by Nemenyi (1963).¹⁷ The purpose of these tables then is to provide the researcher with a comparative summary as to how each test ranks in terms of power, over a variety of conditions and which tests are significantly most powerful from a statistical standpoint.

Moreover, based on Tables A41 through A52, Tables 5.16 (p. 154) and 5.17 (p. 155) present, for the one-way and two-way procedures respectively, the mean and standard deviation

¹⁶Differences between the rank orders presented in Tables 5.14 and 5.15 versus those given in Tables 5.8 and 5.9 are attributable to differences in the range of the Δ considered and to sampling error.

¹⁷In essence, when n is large, two groups (treatments) may be considered as significantly different from each other if

$$|\bar{R}_{.j} - \bar{R}_{.l}| \geq q_{k,\infty}^{\alpha} \left[\frac{k(k+1)}{12n} \right]^{\frac{1}{2}},$$

where

$$\bar{R}_{.j} = \sum_{i=1}^n R_{ij}/n = R_{.j}/n; \quad \bar{R}_{.l} = \sum_{i=1}^n R_{il}/n = R_{.l}/n;$$

$$j = 1, 2, \dots, k, \text{ and } l = 1, 2, \dots, k \text{ such that } j \neq l;$$

and the constant $q_{k,\infty}^{\alpha}$ is the upper α percentile point of the studentized range distribution. See Miller (1966) or Hollander and Wolfe (1973) for a detailed discussion of the Friedman-type simultaneous rank procedures.

TABLE 5.16
MEANS & STANDARD DEVIATIONS OF POWER RATIOS OF NINE ONE-WAY PROCEDURES^a

DIST.	ALT.	FTEST	HTEST	JONKE	BARTH	CHAKO	ATMAX	LRCON	JMENS	PURIS
R	T	$\bar{x}$ 0.5093	0.4353	0.8462	0.8429	0.7893	0.8796	0.8915	0.9763	0.9981
		S 0.1632	0.1523	0.0928	0.1021	0.1054	0.0989	0.0935	0.0256	0.0069
R	G	$\bar{x}$ 0.5924	0.4981	0.7282	0.9292	0.8511	0.8801	0.8228	0.9582	0.8898
		S 0.1685	0.1577	0.0833	0.1027	0.0943	0.0755	0.0921	0.0544	0.0845
E	T	$\bar{x}$ 0.4898	0.6223	0.9577	0.7374	0.9007	0.7688	0.7810	0.9827	0.9963
		S 0.2249	0.2286	0.0541	0.1836	0.0825	0.1715	0.1661	0.0199	0.0074
E	G	$\bar{x}$ 0.5746	0.7331	0.8205	0.8053	0.9940	0.7648	0.7225	0.8772	0.8226
		S 0.1832	0.1801	0.0944	0.1233	0.0289	0.1072	0.1186	0.0750	0.1017
N	T	$\bar{x}$ 0.5523	0.5057	0.9478	0.9251	0.8817	0.9637	0.9853	0.9631	0.9874
		S 0.1560	0.1621	0.0438	0.0637	0.0730	0.0371	0.0208	0.0297	0.0186
N	G	$\bar{x}$ 0.6332	0.5568	0.7939	0.9699	0.9354	0.9310	0.8698	0.9145	0.8499
		S 0.1520	0.1510	0.0785	0.0664	0.0659	0.0410	0.0687	0.0575	0.0833

^aData extracted from Tables A41 through A46.

TABLE 5.17
MEANS & STANDARD DEVIATIONS OF POWER RATIOS OF NINE TWO-WAY PROCEDURES^a

DIST.	ALT.	F2WAY	PAGET	FRIED	PHENS	ATMX2	LRCN2	HOLLT	SHORT	SENAT
R	T	$\bar{X}$ 0.5417 S 0.1379	0.8400 0.0858	0.3978 0.1138	0.8570 0.0912	0.9714 0.0402	0.9927 0.0190	0.9414 0.0366	0.7803 0.0779	0.9440 0.0348
R	G	$\bar{X}$ 0.6609 S 0.1680	0.7309 0.0819	0.4764 0.1234	0.7498 0.0828	0.9959 0.0172	0.9294 0.0604	0.8006 0.1047	0.8551 0.1050	0.8342 0.0846
E	T	$\bar{X}$ 0.4894 S 0.2126	0.9203 0.0601	0.5527 0.2084	0.9283 0.0587	0.7985 0.1444	0.8125 0.1384	0.8761 0.0981	0.8615 0.0878	0.9999 0.0008
E	G	$\bar{X}$ 0.6081 S 0.1775	0.7959 0.0709	0.6238 0.1846	0.7970 0.0699	0.8330 0.1019	0.7808 0.1108	0.7211 0.0883	0.9498 0.0892	0.9099 0.0890
N	T	$\bar{X}$ 0.5477 S 0.1529	0.8464 0.0823	0.4143 0.1410	0.8547 0.0841	0.9674 0.0354	0.9933 0.0179	0.9553 0.0396	0.7921 0.0887	0.9605 0.0355
N	G	$\bar{X}$ 0.6690 S 0.1686	0.7616 0.0552	0.4829 0.1540	0.7709 0.0554	0.9965 0.0186	0.9304 0.0649	0.8167 0.1083	0.8724 0.0993	0.8613 0.0825

^aData extracted from Tables A47 through A52.

of the power ratios of each test relative to the "best test" (over all n , k , and Δ) for each distribution-alternative combination. The purpose of these tables is to indicate to the researcher both the average relative power of each test (with respect to the "best test") over a variety of conditions and the variability so that, for a particular distribution-alternative combination, each test may be compared against a "theoretical standard (best test)" having mean of 1.0 and standard deviation of 0.0.

From the aforementioned tables, the following characteristics of each test are noted:

One-Way Procedures:

JMENS: 1. It has a slightly higher mean power ratio against trend than against gap.

2. It is a powerful test under all observed conditions.

3. It is a powerful test against trend (regardless of distribution) and rectangular-gap.

4. It is excellent for detecting small departures from the null ($\Delta = 0$).

PURIS: 1. It has a higher mean power ratio against trend than against end gap.

2. It is a most powerful test against trend (regardless of distribution) and rectangular-gap.

3. It is excellent for detecting small departures from the null ($\Delta = 0$).

ATMAX: 1. Its mean power ratios are similar for trend and end gap.

2. It is a most powerful test against both normal-trend and normal-gap as well as rectangular-gap.

BARTH: 1. It has a higher mean power ratio against end gap than against trend.

2. Its powers converge to their limits very rapidly as Δ increases.

3. It is a most powerful test against rectangular-gap and normal-gap.

CHAKO: 1. It has a higher mean power ratio against end gap than against trend.

2. It is the most powerful test against exponential-gap and is a most powerful test against normal-gap.

LRCON: 1. It has a higher mean power ratio against trend than against end gap.

2. It is a most powerful test against normal-trend.

JONKE: 1. It has a higher mean power ratio against trend than against end gap.

2. It is a most powerful test against exponential-trend.

FTEST: 1. Designed against more generalized alternatives, this test has a higher mean power ratio against end gap than against trend.

2. It is poor relative to other procedures under

asymmetrical departures from normality.

3. Its powers converge to their limits rapidly as Δ increases.

HTEST: 1. Designed against more generalized alternatives, this test has a higher mean ratio power against end gap than against trend.

2. It is poor relative to other procedures under normality or symmetrical departures therefrom.

Two-Way Procedures:

SENAT: 1. It has a higher mean power ratio against trend than against end gap.

2. It is a powerful test under all observed conditions.

3. It is the most powerful test against exponential-trend and a most powerful test against exponential-gap and normal-trend.

4. It is excellent for detecting small departures from the null ($\Delta = 0$).

ATMX2: 1. It has a slightly higher mean power ratio against end gap than against trend.

2. It is a most powerful test against both trend and gap under the rectangular and normal distributions.

LRCN2: 1. It has a higher mean power ratio against trend than against end gap.

2. It is a most powerful test against both trend and gap under the rectangular and normal distributions.

HOLLT: 1. It has a higher mean power ratio against trend than against end gap.

2. It is a most powerful test against normal-trend.

SHORT: 1. It has a higher mean power ratio against end gap than against trend.

2. It is a most powerful test against exponential-gap.

PHENS: 1. It has a higher mean power ratio against trend than against end gap.

2. It is good for detecting small departures from the null ($\Delta = 0$).

PAGET: 1. It has a higher mean power ratio against trend than against end gap.

2. It is good for detecting small departures from the null ($\Delta = 0$).

F2WAY: 1. Designed against more generalized alternatives, this test has a higher mean power ratio against end gap than against trend.

2. It is poor relative to other procedures under asymmetrical departures from normality.

3. Its powers converge to their limits rapidly as Δ increases.

FRIED: 1. Designed against more generalized alternatives, this test has a higher mean power ratio against end gap than against trend.

2. It is poor relative to other procedures under normality or symmetrical departures therefrom.

Summary of Findings

From the results it is clearly evident that some tests against ordered alternatives are better than others under some conditions but no "best test" exists for all conditions considered. First of all, some tests (Page's L Test under rectangular-gap and Sen's Test under exponential-gap, for example) are more sensitive to detecting small differences in the shift parameter and yet converge to their limiting powers more slowly than do other tests. Secondly, it is evident that some tests (Puri's procedure for a one-way analysis and Sen's procedure for a two-way analysis) are more sensitive to detecting linear trend alternatives while other tests (Chacko's procedure for a one-way analysis and Shorack's procedure for a two-way analysis) are more sensitive to detecting gap alternatives. Finally, while no uniformly most powerful test exists under all conditions considered, if a choice had to be made based on power alone it is clear that the Johnson-Mehrotra Expected Normal Scores Test and Sen's Aligned Rank Order Test stand out, among the nonparametric procedures evaluated, as the "somewhat most powerful tests" against all ordered location alternatives.

On the other hand, while some parametric procedures are seen to be very powerful under certain conditions (i.e., normality or symmetrical departures therefrom), they tend to be weaker than the better nonparametric tests under asymmetrical

departures from normality. Recall, further, from the discussion on robustness (Appendix A7) that the powers of the parametric tests under departures from normality are likely to be overstated. Even so, the parametric tests should not be chosen when the researcher does not believe his data are coming from normal or symmetrical populations unless the appropriate "normalizing" transformations are employed. For example, it is noted that the One-Way and Two-Way One-Tail Regression Tests, as expected, are the best tests against linear trend when the underlying distribution is normal and are among the best tests against trend under a rectangular distribution as well. However, when the distribution is exponential, the tests rate poorly when compared to their nonparametric counterparts. Furthermore, while Bartholomew's $\bar{E}^2$ Test is excellent against gap alternative among the one-way procedures under the normal or rectangular distribution, it is significantly inferior to Chacko's nonparametric procedure under the exponential distribution. Similarly, among the two-way procedures, the Abelson and Tukey Optimal Contrast Test is much inferior when compared to Shorack's Test under the exponential distribution but it is the best test against gap when the underlying distribution is normal or rectangular. From the above discussion it is seen that some nonparametric tests are almost as powerful as their best parametric counterparts when the assumptions of the parametric tests are met and are more powerful when the normality assumption is violated.

5.5 SELECTION OF APPROPRIATE ONE-WAY AND TWO-WAY PROCEDURES

From Tables 5.14 through 5.17 and the discussion thereon, it is recalled that based on power alone, the Johnson-Mehrotra and Sen procedures appear to be the best overall nonparametric tests against ordered location alternatives. Furthermore, it is stated that when the researcher does not want to make the rigorous assumptions required of the parametric procedures (or desires more general conclusions) they should not be employed. However, a third factor for test selection--that pertaining to the "cumbersomeness" (or its complement, "simplicity") of the test procedure--must also be considered by the researcher. Based on the descriptions of the various tests given in Chapters III and IV as well as from the computer subroutines developed for them, subjective "cumbersomeness" rankings could be made by the researcher and compared to the overall rankings given in Tables 5.14 and 5.15. While it should be noted that these "cumbersomeness" rankings may differ, depending on the mathematical background of the researcher, one such rating is presented in Tables 5.18 and 5.19 (p. 163) for the one-way and two-way procedures, respectively, and a comparison is made with the overall rankings of each test from Tables 5.14 and 5.15.

It appears from Tables 5.18 and 5.19 that an inverse relationship exists between the simplicity of a test and its power. It should, however, be apparent that when it is possible to order the alternative hypothesis, these ordered procedures are highly superior--in terms of power--to the more general

TABLE 5.18

A COMPARISON OF "POWER" VS. "CUMBERSOMENESS"
RANKINGS ON ONE-WAY PROCEDURES

ONE-WAY TEST	CUMBERSOMENESS (Least = 1) (Most = 9)	OVERALL POWER (Best = 1) (Worst = 9)
Kruskal-Wallis	1	9
Jonckheere	2	6.5
Puri	3	2
Johnson & Mehrotra	4	1
F	5	8
One-Tail Regression	6	6.5
Abelson & Tukey Contrast	7	4
Chacko	8	4
Bartholomew's $\bar{E}^2$	9	4

TABLE 5.19

A COMPARISON OF "POWER" VS. "CUMBERSOMENESS"
RANKINGS ON TWO-WAY PROCEDURES

TWO-WAY TEST	CUMBERSOMENESS (Least = 1) (Most = 9)	OVERALL POWER (Best = 1) (Worst = 9)
Page's L	1	7
Friedman	2	9
Pirie-Hollander	3	6
Hollander	4	4.5
Sen's Aligned Rank Order	5	1.5
F	6	8
One-Tail Regression	7	3
Abelson & Tukey Contrast	8	1.5
Shorack	9	4.5

tests. Therefore, when the alternative can be ordered and yet it is either inappropriate or undesirable to make the rigorous assumptions required of the parametric methods, the researcher may combine power, simplicity and generality by selecting either the Johnson-Mehrotra procedure or the Puri procedure (both of which are based on the expected normal scores) from among the one-way tests. The former is good against trend and is best against gap; the latter is best against trend but only fairly good against gap. From among the two-way procedures the researcher should consider the Sen Aligned Rank Order Test, which is very good against both types of ordered location alternatives, provided that the observed data constitute a higher level of measurement than an ordinal scale (see sections 4.18 and 4.19). On the other hand, when the level of measurement is ordinal, the researcher should consider the Page L Test (based on ranks) which is very simple to use and yet has fairly good power against ordered location alternatives.¹⁸ If, however, it is believed that the assumptions of the parametric methods are met, the researcher should consider the Abelson and Tukey Optimal Contrast Test (from among the one-way and two-way procedures) for the best combination of power and simplicity.

¹⁸It is recalled that all the tests designed against ordered alternatives that were selected for a more detailed investigation (based on the preliminary findings in section 5.3) have reasonably good power when the alternative is truly ordered.

CHAPTER VI

APPLICATIONS

6.1 INTRODUCTION

A scanning of all articles appearing in both the Journal of Advertising Research and Journal of Marketing Research over the period 1971 through 1972 reveals that almost thirty percent either use or could have used some of the k sample techniques described in this dissertation. During this period, as expected, in not one instance were either the k sample parametric or nonparametric techniques designed against ordered location alternatives employed. Thus, there appears to be a strong need for making the researcher aware of these methods so that they may be added to his toolkit and used to strengthen future research efforts. Nowhere is this more evident than from the following statement by Peter J. McClure (1971) about a statistical procedure devised some thirty-four years earlier:

"Many marketing researchers have not been aware of the Friedman two-way analysis of variance by ranks. However, it has potential for use in situations common to many consumer surveys in which sets of ordinal data are generated by each respondent."

If there is any truth to McClure's remark then it is certain that marketing researchers are even less familiar with most of the procedures described in this dissertation--particularly

those designed against ordered alternatives.¹

This chapter, then, will demonstrate, through specific applications in both marketing and behavioral research, the usefulness of some of the k sample tests against ordered location alternatives as compared to the more generalized k sample tests. In addition, several references in the literature will be made to studies that either employ or could have employed these kinds of techniques.

6.2 APPLICATION FOR A COMPLETELY RANDOMIZED MODEL:

CONSUMER DISSATISFACTION REVISITED

Among the studies more conducive to analysis by the one-way k sample techniques presented in this dissertation is the recent research effort by Rolph E. Anderson (1973) on measuring consumer dissatisfaction. In this interesting study, four psychological theories are considered in determining the effects of disconfirmed expectations on perceived product performance. These four theories are:

1. Assimilation (Cognitive Dissonance)--any discrepancy between ones expectation of product performance and perception of actual performance will be minimized by an adjustment of the

¹McClure's statement can be contested in light of all the basic level statistics texts written during the past decade which describe the Friedman procedure--not to mention the often referenced book by Siegel(1956) or the useful work of Banks (1965) which it is felt should be part of every marketing researcher's library. Most of the procedures described in this dissertation, however, are not found in the behavioral or marketing research literature. Hence, the need exists for providing the researcher with the more useful methods.

perception so that it is more consistent with the expectation.

2. Contrast--any discrepancy between ones expectation of product performance and perception of actual performance will be magnified by an adjustment of the perception so that it is less consistent with the expectation.

3. Generalized negativity--any discrepancy between ones expectation of product performance and perception of actual performance causes a negative adjustment of the perception.

4. Assimilation-contrast--any discrepancy between ones expectation of product performance and perception of actual performance will be minimized provided that the discrepancy is plausible or magnified if the discrepancy is unrealistic.

Anderson's work sought to differentiate which of the above theories is correct. Such a determination would be important in the selection of appropriate advertising copy. To accomplish this, Anderson randomly assigned 144 students to one of five different levels of persuasive product information (expectation) or no product information (control). The six groups of students (twenty-four in each group) were then asked to evaluate identical, unmarked, ballpoint pens selling at retail for one dollar. A "product rating scale" was employed and three dependent variables obtained--(1) an "unweighted" mean of product characteristics; (2) a "combined-characteristics" rating; and (3) an estimate of the ballpoint pen's worth. A separate one-way analysis of variance for the three dependent measures was performed and each rejected the null hypothesis

that product perceptions are not significantly different for various levels of expectations. Subsequently, three separate Regression F Tests were performed and each showed highly significant deviations from linearity for the relationship between expectations and perceived product performance. Thus, Anderson concludes that assimilation-contrast theory is most appropriate.

In order to demonstrate the applicability of some of the k sample techniques to this kind of problem the original data are needed. Unfortunately, however, such information has not been provided. It therefore becomes necessary to consider a replication of this study since any well designed experiment should be capable of reproduction. Herein lies a major problem--a careful examination into the methodology of this research effort casts many doubts as to its validity. It is, therefore, not altogether infeasible that the aforementioned conclusion is reached by Anderson only because of improper experimental design and analysis! In particular, the following features of the design and analysis in increasing order of importance are found questionable:

1. The choice of the rating scale used.
2. The need for, and choice of, three dependent variables.
3. The experimental design model selected.
4. The employment of the Regression F Tests.
5. The "convenience transformation" made on the data.

Firstly, it is not clear why a "modified logarithmic" product rating scale is employed rather than one with groups of equal intervals? Moreover, it is not clear why the limits of the scale are so absurdly low (\$.04) and high (\$64.00). It would be rather hard to convince anyone astute enough to purchase and test a pen of this quality (the subjects of the study) that its value is so low or so high.

Secondly, one may seriously question the need for, and choice of, three dependent variables. The concept of an "unweighted" arithmetic mean (the first dependent variable) across all product characteristics is absurd. Even if it could be assumed that the list of fifteen characteristics of the product are mutually exclusive and collectively exhaustive, it is unreasonable to attach equal weights ($1/15$) to these characteristics to obtain such an average since some of the features are vastly more important than others. Perhaps a factor analysis of the fifteen characteristics would be more appropriate prior to obtaining the mean. Additionally, the concept of a subjectively "weighted" mean (the second dependent variable), as considered by Anderson, seems inappropriate because each subject will perceive the "combined-characteristics" rating in terms of the features he deems most important. Perhaps a more objectively "weighted" mean could be constructed--one in which each of the fifteen characteristics has attached to it a relative weight, developed, say, from a pilot study. Furthermore, the ambiguity of the question: "Exactly how much do you think

this ballpoint pen is worth?" puts doubt in the results obtained for the third dependent variable. Anderson desired to elicit a response reflecting an estimate of the pen's price. Perhaps he should have asked: "Exactly how much do you think this pen sells for at Macy's?" An examination of the mean responses given by Anderson reveals, in every situation, that the third dependent variable (what's it worth) is larger than the second dependent variable (what does it cost). Would such a result be expected--especially during an inflationary economy? Intuitively, it seems the reverse might be true. It is, therefore, highly suspected that a Hawthorne effect is present here. Subjects, in trying to please, may have put down a larger amount for the pen's worth than they really believed. Lastly, the need for three dependent measures is questioned here. Would it not be better to have but one meaningful measure?

Since Anderson chose three dependent variables, the third question to be raised is whether the experimental design used is appropriate? It seems inappropriate to use three completely randomized designs and study, separately, the differences in mean responses. Perhaps the design could have been set up as a two factor-factorial such that the six expectations and the three measurements are the main effects. Such a design would permit an examination of differences in mean response (in dollars and cents) over increasing levels of product expectation as well as over different measuring devices.

Moreover, a test for the interaction between measurement and expectation could be made. Thus, in one design, with eight observations per cell, Anderson's entire experiment could have been run.

Furthermore, Anderson runs three separate Regression F Tests in order to examine the various psychological theories. It may be recalled from section 3.3, however, that such tests usually assume equal spacing between treatment conditions. It is not apparent from Anderson's results that this assumption is met. Perhaps several different "persuasive product information statements" could be tested and then only those that appear to have equal spacing adopted in order to utilize the regression procedure.

The fifth and final major criticism of Anderson's work is based on the employment of a "convenience transformation" to facilitate the analysis. Results obtained in dollars and cents on a "modified logarithmic" scale were transformed to equidistant integers. Not only is it improper to perform the analysis of variance on these integers but, even more importantly, such transformed data erroneously equates differences such as between \$.04 and \$.06 with differences such as between \$48.00 and \$64.00. The net effect of this transformation causes such a distortion in the means that it is possible that one group may have a higher average on the transformed integer scale and yet have a lower average on the original data!

In order to "partially replicate" Anderson's study,

mainly for the purpose of demonstrating the applicability of the k sample procedures, seventy-five students were randomly assigned to five different levels of persuasive product information. These five levels (presented in Figure 6.1) are described as follows:

L1 substantially understates the product's features.

L2 slightly understates the product's features.

L3 describes the product accurately.

L4 slightly overstates the product's features.

L5 substantially overstates the product's features.

Evaluation of identical, unmarked ballpoint pens selling at \$.39 each were obtained in group administration from seventy-five students enrolled in an undergraduate statistics course. Unlike Anderson, subjects were told they could not keep the pen they were evaluating but that a random drawing would be held at the end of the experiment and the selected subjects would then receive a pen of the same quality as they rated, whether or not it turned out to be one of the expensive ones. Subjects were permitted to inspect and test the product for the same length of time and then record their reactions on a product rating scale given in dollars and cents (see Figure 6.2). The subjects rated the ballpoint pen on fifteen visual features and performance characteristics, as well as on an overall basis. Most important, however, subjects estimated the ballpoint pen's price and this response was desired for the dependent variable of this study.

The null hypothesis to be tested is that product perceptions (in terms of estimated monetary value) are not significantly different for various levels of expectations. The alternative hypothesis is that product perceptions (in terms of estimated monetary value) will vary directly with the level of expectations--assimilation theory.

After conducting this "pilot investigation" it became apparent through discussion with the subjects that the store name--Macy's--caused an upward bias in their responses. The objective was to obtain an estimate of the price of a pen at a large department store--not a thrift shop nor a specialty shop. The subjects, however, mainly coming from a lower economic stratum, associated the particular store selected as one carrying higher quality and higher priced goods. While an examination of the data showed no reason to reject the null hypothesis, the five following questions arise:

1. Are there sufficient differences between the various levels of persuasive product information shown in Figure 6.1? A variety of advertising copy needs to be tested here.

2. Did the "store naming" so bias the subjects' expectations (L_1 and L_2) that they could not perceive the product as really poor? Names such as Korvette's, Klein's, or Alexander's might be tried.

3. It is possible that a fifth psychological "theory" (not mentioned by Anderson)--that of "generalized positivity"--is evolving here? That is, within a narrow range of the

L5	This light-weight, retractable, and refillable* pen is made from a durable metallic compound and assembled under precision control to insure quality. It is perfectly balanced for unmatched writing ease and is mechanically guaranteed to work forever. In addition, the construction of this pen allows for efficient use with straight edges, triangles, templates, or lettering guides. *Refills can be obtained individually or in packs of five.
L4	This light-weight, retractable, and refillable* pen is made from a durable metallic compound and assembled for comfort and ease of handling and writing. Moreover, the construction of this pen allows for not only high quality writing but also for efficient use with straight edges, triangles, templates, or lettering guides. *Refills can be obtained individually or in packs of five.
L3	This light-weight, retractable, and refillable* pen is made from a metallic compound and assembled for comfort and ease of handling and writing. It is designed to provide high quality writing at a low enough price to be recommended for student use. *Refills can be obtained individually or in packs of five.
L2	This light-weight, retractable pen is designed to emulate more expensive models. While this pen is not meant for extremely rough handling, it is designed to provide, under most situations, a fairly high quality of writing at a low price.
L1	This light-weight, retractable pen is designed to emulate more expensive models. While this pen is not meant for rough handling, nor usage under very high or very low temperatures, it is designed to provide, under most situations, a fairly high quality of writing at the lowest possible price.

FIGURE 6.1: FIVE LEVELS OF PERSUASIVE PRODUCT INFORMATION

On this pen, each of these characteristics is about like the ones of pens costing between \$ & \$.

PRODUCT RATING SCALE

	GENERAL APPEARANCE	WRITING QUALITY	CONSTRUCTION	RETRACTOR BUTTON	INK CARTRIDGE	POCKET CLIP	INK FLOW	GENERAL FEEL	WRITING PERFORMANCE	INK QUALITY	DURABILITY	STYLE	QUALITY CONTROL	WEIGHT	SHAPE	COMBINED CHARACTERISTICS
\$.05																
.10																
.15																
.20																
.25																
.30																
.35																
.40																
.45																
.50																
.60																
.70																
.80																
.90																
\$ 1.00																
1.25																
1.50																
1.75																
2.00																
2.50																
3.00																
3.50																
4.00																

* Exactly how much do you think this pen sells for at Macy's? \$

FIGURE 6.2: PRODUCT RATING SCALE FOR \$.39 BALLPOINT PEN

accurate product description, might not consumers react positively to slight over or understatement of the product? A refinement of the study will provide more insight here.

4. Is it not possible that the subjects are forming product expectations which the advertisement was not designed to elicit? If the advertising copy does not create the image or expectation it was designed to achieve there is no reason to suspect assimilation theory is not plausible. The preliminary work here indicates a perfect correlation between expectation and perception--but not in the same order as the five advertisements were designed to elicit. Hence, more copy testing is required here.

5. What effect does advertising have on consumers once they have become accustomed to the product? In other words, suppose the product is given to the subjects for inspection prior to the advertisement. How will one's perceptions be affected by the a posteriori expectation?

While the above comments reflect the ways in which the pilot study can be improved upon and expanded, it might still be of interest to examine the combined characteristic rating (which Anderson calls the second dependent variable), in spite of its aforementioned limitations, in order to: (1) determine whether there is any consistency between these results and that given by Anderson; and (2) to demonstrate how some of the better k sample tests against ordered alternatives may be applied. The data for this pilot study are presented in Table 6.1.

TABLE 6.1
 "COMBINED CHARACTERISTICS" PRICE RATINGS OF PENS UNDER
 FIVE LEVELS OF PERSUASIVE PRODUCT INFORMATION
 (Data in Ordered Array)

L1	L2	L3	L4	L5
\$.20	\$.16	\$.28	\$.15	\$.20
.25	.19	.30	.24	.30
.25	.28	.39	.40	.50
.30	.39	.40	.42	.56
.35	.48	.40	.60	.70
.45	.50	.44	.70	.75
.60	.80	.45	.70	.79
.65	.95	.60	.80	.90
.80	1.00	.70	1.00	.95
.90	1.00	1.25	1.25	.99
1.00	1.00	1.25	1.25	1.00
1.00	1.15	1.25	1.37	1.50
1.19	1.25	1.25	1.50	2.00
1.50	1.45	1.25	2.50	2.15
1.50	1.75	1.89	2.50	2.50
TOTALS: \$10.94	\$12.35	\$12.10	\$15.38	\$15.79
MEANS: \$.73	\$.82	\$.81	\$ 1.03	\$ 1.05

Prior to using a parametric technique it is appropriate to test for homogeneity of variance. From Table 6.2 it is seen that neither Bartlett's Test, Cochran's Test, nor Hartley's Test detect any violations of homoscedasticity at the 5% level. However, the computed values of these test statistics are large

enough to cause apprehension for several reasons. First of all, it is widely believed that the tests for homogeneity of variance are not very powerful. Moreover, the five means appear to be proportional to their standard deviations--suggesting a logarithmic transformation. Furthermore, a "combined-characteristic" rating is likely to be a logarithmic function of individual characteristics (rather than linear). For these reasons, then, the researcher may be inclined to use a logarithmic transformation of the data (see Table 6.3, p. 180) in order to perform the usual parametric procedure for testing against general alternatives--the One-Way F Test. The results appear in Table 6.4, p. 180.

The computed F value, 0.61, is insignificant. The probability of obtaining such an F or larger is .6572. Since the null hypothesis cannot be rejected, all that can be said is that differences between the levels of persuasive product information are due only to chance. No a posteriori comparisons can be made.

If the researcher had desired to employ a more simplified procedure, the original data could have been transformed to ranks and the Kruskal-Wallis Test used for testing the null hypothesis against general alternatives (that at least two of the groups differ from each other). The transformation to ranks is given in Table 6.5, p. 181.

TABLE 6.2
TESTING FOR HOMOGENEITY OF VARIANCE

LEVELS	$\bar{X}_{.j}$	s_j^2	s_j	$\bar{X}_{.j}/s_j^2$	$\bar{X}_{.j}/s_j$	$\text{LOG } s_j^2$
L1	10.94	.1964	.443	55.70	24.70	-0.70686
L2	12.35	.2284	.478	54.07	25.84	-0.64130
L3	12.10	.2508	.501	48.25	24.16	-0.60067
L4	15.38	.5271	.726	29.18	21.18	-0.27811
L5	15.79	.4688	.685	33.68	23.06	-0.32901

TEST	COMPUTED STATISTIC	CRITICAL VALUE ($\alpha = .05$)
BARTLETT	$\chi_B^2 = \frac{(2.3026)(n-1) \left[k \log \left(\frac{\sum_{j=1}^k S_j^2 / k}{k} \right) - \sum_{j=1}^k \log S_j^2 \right]}{\left(1 + \frac{(k+1)}{3k(n-1)} \right)}$	$\chi_{\alpha(4)}^2 = 9.488$
COCHRAN	$\chi_B^2 = 5.536$ $C_F = \text{Largest } S_j^2 / \sum_{j=1}^k S_j^2 = .3153$	$C_{\alpha}(k=5, v=16) = .3645$ $C_{\alpha}(k=5, v=14) = ?$ $C_{\alpha}(k=5, v=10) = .4118$
HARTLEY	$H_F = \text{Largest } S_j^2 / \text{Smallest } S_j^2 = 2.68$	$H_{\alpha}(15, 15) = 4.37$ $H_{\alpha}(14, 14) = ?$ $H_{\alpha}(12, 12) = 5.30$

TABLE 6.3
DATA TRANSFORMED TO LOGARITHMS

L1	L2	L3	L4	L5
-0.699	-0.796	-0.553	-0.824	-0.699
-0.602	-0.721	-0.523	-0.620	-0.523
-0.602	-0.553	-0.409	-0.398	-0.301
-0.523	-0.409	-0.398	-0.377	-0.252
-0.456	-0.319	-0.398	-0.222	-0.155
-0.347	-0.301	-0.357	-0.155	-0.125
-0.222	-0.097	-0.347	-0.155	-0.102
-0.187	-0.022	-0.222	-0.097	-0.046
-0.097	0.000	-0.155	0.000	-0.022
-0.046	0.000	0.097	0.097	-0.004
0.000	0.000	0.097	0.097	0.000
0.000	0.061	0.097	0.137	0.176
0.076	0.097	0.097	0.176	0.301
0.176	0.161	0.097	0.398	0.332
0.176	0.243	0.276	0.398	0.398
TOTALS:	-3.353	-2.656	-1.545	-1.022
MEANS:	-0.224	-0.177	-0.173	-0.103

TABLE 6.4
ANOVA FOR COMPLETELY RANDOMIZED ANALYSIS
(Coded Data)

SOURCE	df	SUM OF SQUARES	MEAN SQR.
Between Information Levels	4	23.3906	5.8477
Within Information Levels	70	672.9062	9.6129
Totals	74	696.2968	--

TABLE 6.5
DATA AFTER TRANSFORMATION TO RANKS

	L1	L2	L3	L4	L5
	4.5	2	9.5	1	4.5
	7.5	3	12	6	12
	7.5	9.5	15.5	18	25.5
	12	15.5	18	20	27
	14	24	18	29	33.5
	22.5	25.5	21	33.5	36
	29	39	22.5	33.5	37
	31	43.5	29	39	41.5
	39	49	33.5	49	43.5
	41.5	49	58.5	58.5	45
	49	49	58.5	58.5	49
	49	53	58.5	63	66.5
	54	58.5	58.5	66.5	71
	66.5	64	58.5	74	72
	66.5	69	70	74	74
RANK SUMS:	493.5	553.5	541.5	623.5	638

The H statistic is computed from (3.7.1) as follows:

$$H = \frac{12}{N(N+1)n} \left[\sum_{j=1}^k R_{.j}^2 \right] - 3(N+1) \quad (3.7.1)$$

so that $H = \frac{12}{(75)(76)(15)} (1,638,923) - 3(76) = 2.0243.$

Comparing H to the χ^2 distribution with 4 degrees of freedom, the probability of equalling or exceeding a value of 2.0243 is .7395. Here again, there is no evidence of a difference

among price estimates under the five persuasive levels of product information.

If, however, the researcher desires to test against the ordered alternative that price estimates significantly increase over increasing levels of product expectations (assimilation theory), either the Johnson-Mehrotra or Puri Expected Normal Scores Tests, or the Abelson and Tukey Optimal Contrast could be employed.

To perform the Johnson-Mehrotra Test, the original data are first transformed into expected normal scores (see Appendix A2) as shown in Table 6.6.

TABLE 6.6
DATA TRANSFORMED TO EXPECTED NORMAL SCORES

	L1	L2	L3	L4	L5
	-1.61	-2.03	-1.17	-2.40	-1.61
	-1.32	-1.82	-1.02	-1.44	-1.02
	-1.32	-1.17	- .84	- .73	- .43
	-1.02	- .84	- .73	- .64	- .38
	- .91	- .48	- .73	- .30	- .15
	- .54	- .43	- .60	- .15	- .07
	- .30	.03	- .54	- .15	- .03
	- .24	.18	- .30	.03	.11
	.03	.38	- .15	.38	.18
	.11	.38	.75	.75	.24
	.38	.38	.75	.75	.38
	.38	.52	.75	.96	1.17
	1.02	.75	.75	1.17	1.55
	1.17	1.02	.75	2.03	1.67
	1.17	1.36	1.44	2.03	2.03
TOTALS:	-3.00	-1.77	-0.89	2.29	3.64
MEANS:	-0.200	-0.118	-0.059	0.153	0.243

After obtaining the means, $\bar{e}_{.j}$, for each of the k groups, the statistic JM is calculated from (3.11.1) using the optimal contrast coefficients, M_j , given in (3.6.1) and tabled in section 3.6. Thus,

$$JM = \sum_{j=1}^k M_j (\bar{e}_{.j}) \quad (3.11.1)$$

so that $JM = (-.894)(-0.200) + (-.201)(-0.118) + (0)(-0.059) + (.201)(0.153) + (.894)(0.243)$

and $JM = .4504$.

Comparing this value to the developed empirical critical value presented in Table A56 for $k = 5$ and $n = 15$, the null hypothesis cannot be rejected at the 5% level. However, to determine the size of the test,² a transformation to a standardized normal deviate is obtained by using the standard deviations given in Table A56 for $k = 5$ and $n = 15$. Thus, $z = \frac{JM}{\hat{\sigma}_{JM}} = \frac{.4504}{.3373} = 1.34$, and the probability of observing such a z (or JM of .4504) or larger is .0901--significant at the 10% level.

To perform Puri's Test, however, one must calculate all $k(k-1)/2$ possible pairwise differences in the totals of the expected normal scores (given in Table 6.6). The result is then multiplied by n and the U^* statistic attained.

²The size of the test refers to the lowest level of significance that a test could still be rejected at. From another point of view, it is the probability of obtaining a particular result or larger purely by chance.

Thus,

$$U^* = n \sum_{j=1}^{k-1} \sum_{m=1+j}^k (e_{.j} - e_{.m}) \quad (3.10.9)$$

so that

$$U^* = 15 [(-3.00 - (-1.77)) + (-3.00 - (-0.89)) + \dots + (2.29 - 3.64)]$$

and

$$U^* = 15 [-39.26] = -588.90.$$

Comparing this value to the empirical critical value given in Table A55, the null hypothesis cannot be rejected at the 5% level. To determine the size of the test, a standardized normal variate is obtained by using the standard deviation developed in Table A55 as follows:

$$Z = \frac{|U^*|}{\hat{\sigma}_{U^*}} = \frac{588.90}{366.42} = 1.61,$$

and the probability of observing such a Z (or $|U^*|$ of 588.90) or larger is .0537--almost significant at the 5% level.

Finally, if the researcher had desired to utilize a parametric procedure designed for testing against ordered alternatives, the Abelson and Tukey One-Way Optimal Contrast can be used on the logarithmic data in Table 6.3. The test statistic AT is computed from (3.6.2) where M_j are the optimal contrast coefficients from (3.6.1) and $X_{.j}$ are the treatment totals:

$$AT = \frac{\sum_{j=1}^k M_j X_{.j} / (n \sum_{j=1}^k M_j^2)^{\frac{1}{2}}}{\left(\sum_{j=1}^k \sum_{i=1}^n (X_{ij} - \bar{X}_{.j})^2 / (N - k) \right)^{\frac{1}{2}}} \quad (3.6.2)$$

$$\text{Hence, } AT = \frac{23.0723 / \sqrt{25.188}}{\sqrt{9.6129}} = 1.483.$$

The probability of obtaining an AT value as large or larger is .1457 and the null hypothesis cannot be rejected.

A summary of the computed sizes of each test is given in Table 6.7.

TABLE 6.7
SUMMARY OF THE COMPUTED SIZES OF FIVE TESTS
($k=5$, $n=15$, Parametric Assumptions are Met)

TEST	SIZE OF TEST
Puri's Test	.0537
Johnson-Mehrotra Test	.0901
Abelson and Tukey Optimal Contrast	.1457
F Test	.6572
Kruskal-Wallis Test	.7395

A comparison of the empirical powers of these five procedures, presented in Table A27 (for the normal-trend combination when $k = 5$, $n = 15$, and Δ small), indicates that the order of the above results would not be unexpected. Hence, the power of techniques designed for detecting possible ordered alternatives is clearly demonstrated. It is noted that even though the assumptions of the parametric model are met (after a laborious logarithmic transformation --if not done by computer), the two nonparametric procedures are the ones that are detecting possible monotonically increasing price estimates over the increasing levels of persuasive product information at the 10% level of significance. Thus, while the possible emergence of assimilation

theory is noted from this "pilot study", the researcher would want to make the refinements indicated earlier and then reconduct the study using a larger sample prior to drawing any broad conclusions.

6.3 APPLICATION FOR A RANDOMIZED BLOCK MODEL:

VISUAL PATTERN-RECOGNITION.

An interesting application using the two-way procedures is based on a behavioral investigation conducted by Levine, et al (1973) on visual pattern-recognition. The practical ramifications of this research effort appear to lie in the field of advertising.³

In this study eight subjects were each presented with a single numerical digit (0-9) for a variable on-time (4,8,12,16 or 20 msec) immediately followed (no delay time) by the presentation of a randomly selected masking noise field which remained on for a constant 500 msec prior to extinction. A computer program also randomly generated the order in which the numerical digits appeared as well as the length of on-time until ten trials of each digit-on-time combination were observed by each subject.

³For example, a recent article by Bither and Wright (1973) examines the relationship between a receiver's self-confidence and his attitudinal acceptance of a television advertisement and determines that it is dependent on the level of potential distraction found in the commercial. The physiological study conducted by Levine, et al (1973), however, appears to have a more direct bearing on the kinds of symbols and/or letters to be used in the advertising copy design itself--particularly on billboards--in order to achieve maximum recognition.

In order to demonstrate how some of the two-way procedures described in this dissertation can be applied, a randomized block design with one observation per cell (percentage of correct responses out of 100 trials per on-time level) is used. The null hypothesis of no differences in mean percent recall over the three highest on-time levels can be tested against the ordered alternative that mean percent recall significantly increases over these increasing on-time levels.⁴ The data for the eight subjects (blocks) are given in Table 6.8.

TABLE 6.8
MEAN PERCENT CORRECT RESPONSE
(FOR ALL DIGITS) BY SUBJECT

SUBJECT	ON-TIME (msec)		
	12	16	20
I	55	68	67
II	78	83	84
III	34	53	54
IV	56	67	65
V	79	78	85
VI	20	29	30
VII	68	88	92
VIII	59	58	72

⁴It was obvious that percent mean correct response increased with increasing on-times up to 12 msec. For purposes of comparing the various k sample techniques, only the three largest on-time levels are considered and the data, extracted from Levine's study, are presented in Table 6.8.

Since the observed data are percent recall scores an arcsin root transformation is required before any parametric procedure can be used. The transformation data appear in Table 6.9.

TABLE 6.9
DATA AFTER ARCSIN ROOT TRANSFORMATION

SUBJECT	ON-TIME (msec)		
	12	16	20
I	47.9	55.6	54.9
II	62.0	65.6	66.4
III	35.7	46.7	47.3
IV	48.4	54.9	53.7
V	62.7	62.0	67.2
VI	26.6	32.6	33.2
VII	55.6	69.7	73.6
VIII	50.2	49.6	58.0

The standard parametric procedure, the Two-Way F Test, is usually employed at this point to test the null hypothesis of no differences among mean percent recall over the on-time levels against the general alternative that at least one of the mean percent recalls differ from another. Obtaining the necessary sums, squares, and squares of sums, the following ANOVA table presents the results (p.189). The computed F value, 14.44, is so highly significant that the null hypothesis could have even been rejected at an α below .00043.

TABLE 6.10
ANOVA FOR RANDOMIZED BLOCK ANALYSIS

SOURCE	df	SUM OF SQUARES	MEAN SQUARES
Between On-Times	2	284.44	142.22
Between Subjects	7	3063.69	437.67
Error	14	137.87	9.85
TOTALS	23	3486.00	---

Once the null hypothesis is rejected, however, a posteriori procedures are needed to determine whether the three means are significantly different from each other or whether only one of them differs from the other two. Thus, the Duncan Multiple Range Test is employed and it is determined that the difference in mean percent recall between the shortest (12 msec) and longest (20 msec) on-times is highly significant, as is the difference in mean percent recall between the shortest and middle (16 msec) on-time levels. However, the difference in mean percent recall between the middle and longest on-time levels is not significant at the 5% level.

If the researcher had not desired to make the normality transformation (and thereby utilize the parametric technique) he could have transformed the data into ranks and employed the Friedman Test against the general alternative that at least one of the groups differs from one another. The transformation to ranks within each block is given in Table 6.11.

TABLE 6.11
DATA AFTER TRANSFORMATION TO RANKS

SUBJECT	ON-TIMES (msec)		
	12	16	20
I	1	3	2
II	1	2	3
III	1	2	3
IV	1	3	2
V	2	1	3
VI	1	2	3
VII	1	2	3
VIII	2	1	3

Obtaining the rank sums, R_j , for each of the k groups, the statistic T is computed from (4.7.12) as follows:

$$T = \frac{12}{N(k+1)} \sum_{j=1}^k R_j^2 - 3n(k+1) \quad (4.7.12)$$

so that $T = \frac{12}{24(4)} (10^2 + 16^2 + 22^2) - 3(8)(4)$

and $T = \frac{840}{8} - 96 = 9.0$.

The computed result is highly significant and the null hypothesis could be rejected at an α of .009.

If, however, the researcher wishes to test against the ordered alternative that mean percent recall significantly increases over increasing on-time levels, either Page's L Test or Sen's Aligned Rank Order Test may be employed. To perform Page's L Test the original data are transformed to ranks within each block as shown in Table 6.9 above (p.188).

The rank sums $R_{.j}$ are multiplied by $\hat{R}_j^*$, the hypothetical rankings given to the k treatments under the ordered alternative. The L statistic is then computed from (4.10.1) as follows:

$$L = \sum_{j=1}^k (\hat{R}_j^* R_{.j}) = (1)(10) + (2)(16) + (3)(22) = 108.$$

The computed result is so highly significant that the null hypothesis could be rejected at an α of .00135.

In order to employ Sen's Aligned Rank Order Test it is necessary to first "align" the observations in each block by subtracting each block observation from its mean. This is accomplished from the original data and presented in Table 6.12.

TABLE 6.12
ALIGNED DATA

SUBJECTS	ON-TIMES (msec)			BLOCK MEANS
	12	16	20	
I	- 8.33	4.67	3.67	63.33
II	- 3.67	1.33	2.33	81.67
III	-13.00	6.00	7.00	47.00
IV	- 6.67	4.33	2.33	62.67
V	- 1.67	-2.67	4.33	80.67
VI	- 6.33	2.67	3.67	26.33
VII	-14.67	5.33	9.33	82.67
VIII	- 4.00	-5.00	9.00	63.00

The next steps are to rank all N observations from smallest

(1) to largest (N) and then obtain the block means, $\bar{R}_{i.}$, and treatment rank sums, $R_{.j}$, as shown in Table 6.13.

TABLE 6.13
RANKED DATA AFTER ALIGNMENT

SUBJECTS	ON-TIMES (msec)			$\bar{R}_{i.}$
	12	16	20	
I	3	19	15.5	12.500
II	8	11	12.5	10.500
III	2	21	22	15.000
IV	4	17.5	12.5	11.333
V	10	9	17.5	12.167
VI	5	14	15.5	11.500
VII	1	20	24	15.000
VIII	7	6	23	12.000
$R_{.j}$	40	117.5	142.5	--

The Sen statistic T^* is computed from (4.19.5) as follows:

$$T^* = \frac{\sum_{j=1}^k j R_{.j} - \frac{N(N+1)(k+1)}{4}}{\sqrt{\frac{k(k+1)}{12} \sum_{i=1}^n \sum_{j=1}^k (R_{ij} - \bar{R}_{i.})^2}} \quad (4.19.5)$$

$$\begin{aligned} \text{so that } T^* &= \frac{(1)(40) + (2)(117.5) + (3)(142.5) - \frac{(24)(25)(4)}{4}}{\sqrt{\frac{3(4)}{12} \sum_{i=1}^n \sum_{j=1}^k (R_{ij} - \bar{R}_{i.})^2}} \\ &= \frac{102.5}{\sqrt{1090.8334}} \end{aligned}$$

$$\text{and } T^* = 3.103.$$

The computed T^* value, 3.103, is so highly significant that the null hypothesis could be rejected at an α of .00096.

Finally, if the researcher had desired to employ a parametric procedure designed against ordered alternatives, the Abelson and Tukey Two-Way Optimal Contrast can be used on the data which had undergone an arcsin root transformation (see Table 6.9). The test statistic AT is computed from (4.6.1) as follows:

$$AT = \frac{\sum_{j=1}^k M_j X_{.j} / (n \sum_{j=1}^k M_j^2)^{\frac{1}{2}}}{\left(\sum_{i=1}^n \sum_{j=1}^k (X_{ij} - \bar{X}_{i.} - \bar{X}_{.j} + \bar{X}_{..})^2 / ((k-1)(n-1)) \right)^{\frac{1}{2}}} \quad (4.6.1)$$

where M_j are the optimal contrast coefficients and $X_{.j}$ are the treatment totals. Thus,

$$AT = \frac{53.2293 / \sqrt{10.67}}{\sqrt{9.85}} = 5.192.$$

The computed AT value, 5.192, is so highly significant that the null hypothesis could be rejected at an α of .00013.

A summary of the computed sizes of each of the five procedures is given in Table 6.14 below.

TABLE 6.14
SUMMARY OF THE COMPUTED SIZES OF FIVE TESTS
($k=3$, $n=8$, Parametric Assumptions are Met)

TEST	SIZE OF TEST
Abelson & Tukey Optimal Contrast	.00013
F-Test	.00043
Sen's Aligned Rank Order Test	.00096
Page's L Test	.00135
Friedman's Test	.009

The researcher should not really be surprised at these results. While the differences in mean percent recall is found to be highly significant under each test, the parametric procedures did so well for two reasons: First of all, the data were transformed so that the assumptions underlying the parametric methods were met; secondly, the results of the F Test and the subsequent a posteriori procedures indicate the gap alternative to be present. A comparison of these five tests under the normal-gap combination when $k = 3$, $n = 5$ and when $k = 3$, $n = 10$ is obtained from Tables A37 and A38, respectively. From these tables it is clear that as Δ increases (the alternative hypothesis is further from the null) beyond the values tabularized that such an ordering would occur.

As to which test is most appropriate for the original set of data, the answer is left to the researcher who is invited to rework these examples starting from the observed data given in Table 6.8. The researcher must question whether or not it is worth the effort of first having to transform the data in order to meet the requirements of the parametric procedures rather than getting more general results immediately from the original data--as could have been done with the non-parametric procedures. Moreover, it might be interesting for the researcher to compare the amount of time and effort needed to employ the F Test (after the data are transformed) along with the a posteriori comparisons versus Page's L Test (on the original data).

6.4 OTHER APPLICATIONS

In addition to the specific applications presented in the previous two sections, an examination of the marketing research literature reveals several other kinds of problems for which the k sample tests against ordered alternatives might have been used to enhance the analysis. Among these are studies concerned with measured market shares (E.E. Anderson, 1972); evaluating consumer attitudes toward brands (McClure and Ryans (1968), McClure (1971)); testing preferences among products or package designs (Abrams (1969), Day (1965), Day (1968), Day (1969), Roper (1969), Taylor (1968)); and examining effects of various advertising campaigns (Domermuth and Cundiff (1967), Hunt (1970), Bither (1972)). Thus it is clear that a knowledge of these k sample techniques will be of value to the researcher.

CHAPTER VII

REVIEWING THE IMPORTANCE OF k SAMPLE TESTS FOR ORDERED ALTERNATIVES

7.1 SUMMARY

This dissertation has been concerned, primarily, with the problem of test selection when the experimental design is of the completely randomized or randomized block format and the alternative could be ordered. To provide the behavioral and marketing researcher with the most useful techniques appropriate to a given research endeavor, a Monte Carlo investigation of the empirical small sample powers of nineteen one-way techniques and nineteen two-way techniques was accomplished under a variety of conditions: sizes of samples, numbers of groups, values of the shift parameter, levels of significance, types of parent distributions, and kinds of ordered alternatives. Based on a comparison of the generated small sample powers, along with other criteria for test selection (e.g., simplicity and generality), some of the techniques were suggested for consideration and both behavioral and marketing research applications utilizing them were demonstrated. A summary listing of the selected set of techniques under particular situations is given in Table 7.1 (p. 197).

TABLE 7.1

SUGGESTED SET OF TESTS AGAINST ORDERED LOCATION ALTERNATIVES

REQUIREMENTS OF PARAMETRIC TESTS MET?	ORDERED ALTERNATIVE HYPOTHESIS	TYPES OF TESTS	
		FOR COMPLETELY RANDOMIZED DESIGN	FOR RANDOMIZED BLOCK DESIGN
NO	Trend: $\mu_{.1} < \mu_{.2} < \dots < \mu_{.k}$	Puri's Test	Sen's Aligned Rank Order ^b Test
	End Gap: $\mu_{.1} = \dots = \mu_{.k-1} < \mu_{.k}$	Johnson & Mehrotra Test	Sen's Aligned Rank Order ^b Test
	$\mu_{.1} \leq \mu_{.2} \leq \dots \leq \mu_{.k}$ ^a	Johnson & Mehrotra Test	Sen's Aligned Rank Order ^b Test
YES	Trend: $\mu_{.1} < \mu_{.2} < \dots < \mu_{.k}$	One-Tail Regression Test	One-Tail Regression Test
	End Gap: $\mu_{.1} = \dots = \mu_{.k-1} < \mu_{.k}$	Abelson & Tukey Optimal Contrast	Abelson & Tukey Optimal Contrast
	$\mu_{.1} \leq \mu_{.2} \leq \dots \leq \mu_{.k}$ ^a	Abelson & Tukey Optimal Contrast	Abelson & Tukey Optimal Contrast

^aAt least one inequality strict.^bWhen data constitute only an ordinal scale Page's L Test should be substituted.

7.2 CONCLUSIONS

The following conclusions are drawn from the findings of this research effort:

1. Based strictly on the concept of power, no "best test" exists for all conditions considered. Some tests (e.g., Puri's Test, One-Tail Regression Test, and Sen's Test) are particularly sensitive to detecting trend alternatives while others (e.g., Bartholomew's $\bar{E}^2$ Test, Chacko's Test, and Shorack's¹ Test) are particularly sensitive to detecting gap alternatives. In addition, some tests are more sensitive to detecting small departures from the null hypothesis when compared to other tests, and yet, are less sensitive than these others in detecting large discrepancies from the null hypothesis.

2. There is, in general, an inverse relationship between a test's power and its simplicity. Depending on the conditions of a particular investigation, the researcher should select, from among the better procedures listed, that test which offers the most power combined with the least amount of effort in order to yield the kinds of conclusions desired.

3. While Donaldson's (1968) statement that there are no differences in the power of the F Test under trend or gap alternatives is borne out, it does not hold, however, when

¹Preliminary research on Jonckheere's One-Way Test under various kinds of gap location alternatives (end, balanced and unbalanced) show significant differences to exist. More research is needed to investigate these properties for the various k sample tests.

examining the powers of tests designed against specific ordered alternatives.

4. An examination of the robustness of parametric procedures to violations of normality verifies Donaldson's claim of conservative α when the underlying populations are skewed (see Appendix A7).

5. An examination of the empirical powers of the various k sample tests reveals that these power functions are distribution-dependent--regardless of the kind of k sample test (see Appendix A7).

6. As is expected based on statistical theory, power is: (1) an increasing function of n --holding other things constant; (2) an increasing function of k --holding other things constant; and (3) an increasing function of Δ --holding other things constant (see Appendix A8).

7. Provided that the smallest and largest of the treatment means are anticipated, when dealing with consumer panels (or subjects) of fixed size N , it is better to design an experiment whereby the $N = nk$ observations can be allocated over fewer k with larger n in each k than vice versa. However, when these means cannot be anticipated, and the researcher is considering whether or not to add more treatment levels, it is better to design the experiment whereby the $N = nk$ observations can be allocated over more k with smaller n than vice versa (see Appendix A8).

7.3 ACCOMPLISHMENTS AND LIMITATIONS

The following objectives (listed in Chapter I) have been accomplished:

1. The need for considering k sample techniques designed for ordered alternatives has been demonstrated by showing that such techniques are significantly more powerful than their unrestricted counterparts under a variety of conditions.
2. The behavioral or marketing researcher has been provided with a set of useful k sample techniques appropriate to specific conditions.
3. A comparison of the small sample powers of the various tests indicates that the better nonparametric procedures are more powerful than their parametric counterparts when the assumption of normality is violated, but the reverse is true when the assumptions of the parametric tests are met.
4. The effects on power of sample size (n) and numbers of sample groups (k) in the formulations of the experimental design model have been demonstrated.
5. The robustness of parametric tests to violations in the assumption of normality has been examined.
6. Through specific behavioral and marketing research applications the value of some of these k sample techniques have been demonstrated.

In addition to accomplishing the stated objectives, the following achievements of this dissertation are noted:

1. Tables of known critical values of two k sample procedures--Mosteller's Test for Slippage and Conover's First Test for Slippage--are extended, as is the table of critical values for the Shapiro-Wilk Test for normality.
2. Tables of critical values for several k sample procedures--Puri's Test, Johnson and Mehrotra's Test, Cox and Stuart's Weighted Sign Test, Pirie-Tukey Rank Sum Test, and Jonckheere's Two-Way Test--are empirically developed.
3. The null distribution for Pirie's Sign Test statistic is derived and a table of its critical values is generated.
4. Modified versions of Hollander's Rank Test and Doksum's Rank Test are developed for practical usage.
5. The applicability of the F distribution for the Fisher-Yates statistic when n and k are small is empirically demonstrated.
6. The poor finite sample power properties of the Bell and Doksum Tests, noted by Jogdeo (1966), is empirically borne out.
7. The power functions of the One (Two) -Way F Tests, presented in Dixon and Massey (1969), are verified empirically.
8. The equivalence of Sen's General Aligned Rank Order Test with that given by Mehra and Sarangi is demonstrated algebraically.
9. The equivalencies of the Chacko versus Kruskal-Wallis statistics, as well as the Shorack versus Friedman statistics, are demonstrated algebraically for the condition

where the alternative is ordered.

10. The identity between Jonckheere's Two-Way Test and that mentioned by Ferguson (1966) is algebraically demonstrated.

11. The properties for a "Scores Test" based on Jonckheere's One-Way Test are described (from comments by Bartholomew (1961A)).

12. An efficient sorting routine, useful for Monte Carlo investigations, is developed (Rabinowitz and Berenson (1973)).

13. A routine useful for generating expected normal scores is developed.

14. A critical analysis of the methodology employed by Anderson (1973) in his "consumer dissatisfaction" study is presented.

However, the following limitations of this dissertation are noted:

1. Because of the magnitude of this research effort the scope had to be limited to the specific values of n , k , Δ , and α earlier described as well as to three underlying parent populations. Moreover, the property of heterogeneity of variance could only be examined preliminarily, as were the differences between balanced and unbalanced gap alternatives. Also, the generated number of histories had to be limited.

2. Each of the k groups were limited to samples of equal size n . While this condition is necessary for a

randomized block design, it is restrictive for a completely randomized design. Nevertheless, since this condition is necessary for some one-way tests (for example, a One-Sided k Sample Kolmogorov-Smirnov Test) it had to be imposed on all tests for comparison purposes.

3. The selection of the specific k sample tests examined in this dissertation is another limitation. While thirty-eight such techniques were chosen, it must be mentioned that this is by no means a collectively exhaustive list. This dissertation has dealt specifically with k sample procedures against location alternatives for a completely randomized design or randomized block design (with one observation per cell). Hence, the following must be noted:

A. There exist in the literature classes of k sample parametric and nonparametric techniques each of which could provide the basis for a detailed research effort. They have, therefore, been excluded so as not to unnecessarily broaden this research topic. Among these are: slippage tests; multiple comparison procedures; tests for ordered dispersion alternatives; and the whole class of k sample techniques against ordered alternatives whose data constitute a nominal level of measurement and thus require categorization into contingency tables.

B. Certain k sample procedures have been intentionally excluded from this dissertation. Among these are: Cochran's Q Test (1950), a k Sample Sign Test (Berenson (1966)), a Normal

Scores Extension of Chacko's Test (Johnson and Mehrotra (1971)), and Puri and Sen's Normal Scores Test (1968). Cochran's Q Test and the k Sample Sign Test were excluded because they were designed specifically for testing against more general alternatives and they are not very powerful nonparametric procedures. While Puri and Sen's Normal Scores Test is an extension of Hollander's Rank Test (ranks are converted into expected normal scores), the procedure was excluded because any gains in power it may have over the former test are more than offset by its cumbersomeness and overall lack of simplicity.² Thus, it would not be a very practical procedure for the behavioral or marketing researcher to use. The Normal Scores Extension of Chacko's Test was excluded for similar reasons.

4. A final but important consideration is the treatment of ties. It should be mentioned here that most of the k sample tests considered in this dissertation make the assumption that the k samples are drawn from underlying continuous distributions. For this reason, all the distributions selected for this investigation are continuous. Ties in continuous data really cannot exist and are observed only because of the grossness in the measuring devices used. Thus, to facilitate the computerization and computations of this research endeavor, it was assumed

²Puri and Sen (1968, p. 972) show that the A.R.E. of their test relative to Hollander's Rank Test is ≥ 1 . In addition, they state that the asymptotic power of their test is essentially similar to that of Sen's Aligned Rank Order Test which is considered in this dissertation.

that observed ties will occur very infrequently and do not affect the empirical power of the test procedure. Following Jonckheere (1954A, p. 143) then, all of the tests selected were programmed so that ties work against detecting true significant differences. Hence, the generated empirical powers are all conservative.

7.4 RECOMMENDATIONS FOR FUTURE RESEARCH

Based on the findings of this dissertation the following are recommended for further research:

1. That a Monte Carlo study of the small sample empirical powers of the various k sample tests against ordered location alternatives be made under violations in homoscedasticity as well as normality for the distributions considered herein (see Appendix A5).

2. That a Monte Carlo study of the small sample powers of k sample tests against ordered location alternatives be ascertained under other common distributions--both symmetrical and skewed.

3. That a detailed investigation be made of the effects on the powers of k sample tests attributed to the various kinds of gap alternatives--end gaps, balanced gaps, an unbalanced gaps.

4. That an empirical investigation be made of the differences in the powers of the Aligned Rank Order Tests caused by the employment of various methods of alignment.

5. That an empirical investigation of the Bell and Doksum Tests be undertaken in order to study the variability

in results attributable to the selection of particular sets of random normal scores.

6. That a system responsive to user needs be devised whereby the subroutines of the better k sample procedures are packaged and easily accessed--depending on the particular input requirements such as type of design, type of distribution, and type of alternative.

7. That a comparison be made, via Monte Carlo procedures, of the differences in the Shapiro-Wilk Test versus the Lilliefors Test for normality under a variety of conditions.

8. That a test for normality be developed which is based on the "extremes of the distribution" rather than on other concepts.

9. That a nonparametric test be developed which is based on the use of expected normal scores--rather than ranks--after alignment. Such a test, intuitively, would be the most powerful k sample test against ordered alternatives in a randomized block design.

10. That the behavioral and marketing research populace be made aware of these kinds of techniques so that they may be used, when appropriate, to enhance their investigations.

11. That a Monte Carlo study of the small sample powers of various k sample tests (against ordered alternatives) which are appropriate for contingency table analysis be undertaken.

12. That a detailed empirical investigation of Mosteller's Test for Slippage be made in order to examine some of its

unique properties.

13. That a detailed investigation of the "consumer dissatisfaction" hypothesis be pursued.

TECHNICAL APPENDICES

APPENDIX A1

A COMPARISON OF TWO EXPERIMENTAL DESIGN MODELS:
THE COMPLETELY RANDOMIZED DESIGN
AND
THE RANDOMIZED BLOCK DESIGN

Two basic experimental design models have been considered in this dissertation--the completely randomized design and the randomized block design with one observation per cell. The major distinction in the format of these two designs is the manner in which the experimental units are randomized. In a randomized block design the experimental units are first grouped into homogeneous blocks and then each of the experimental units within a block are randomly assigned to the treatments. On the other hand, in a completely randomized design all the experimental units are randomly assigned to the treatments without any prior grouping.¹

When the researcher has some knowledge about the experimental material he may attempt to increase the power of his tests by reducing the experimental error through the process of "blocking"--grouping together similar experimental units.² Differences in the outcome within each block of "matched" experimental units may then be construed as due to chance variation or to significant treatment effects.

¹See Li (1964, p. 215).

²See Snedecor (1962, p. 291).

Even though there are definite advantages to using a randomized block design it is frequently both impractical and impossible to do so. The researcher may be ignorant of useful matching variates or unable to find good matches. Thus, if the researcher does not have enough information about the experimental material to warrant blocking, he should avoid it because, as shown below, random matching is unsuitable. A loss of precision results from random blocking in that fewer degrees of freedom are available in the mean square error term for estimating F and, therefore, a larger value of F is required for a given level of significance (see (3.1.1) and (4.1.1) in sections 3.1 and 4.1, respectively). Hence, when the use of the randomized block design is inappropriate, the researcher may use the completely randomized design--one in which the k treatment groups are considered to be independent of each other.

To show that random matching is inefficient consider the relationship between the t distribution and F distribution discussed in section 3.3. When $k = 2$, the square root of the expression for F in (3.1.1) yields a two sided t test for independent samples while the root of the expression for F in (4.1.1) yields a two sided "matched-pairs" t test.³ The former has $2(n-1)$ degrees of freedom when the two samples are of equal size while the latter has $n-1$ degrees of freedom. Thus, rewriting the expressions for F in (3.1.1) and (4.1.1) yields,

³See Li (1964, pp. 142-144 and 108-111).

respectively,

$$t = \frac{\bar{X}_{.1} - \bar{X}_{.2}}{\sqrt{\frac{\sum_{i=1}^n (X_{i1} - \bar{X}_{.1})^2 + \sum_{i=1}^n (X_{i2} - \bar{X}_{.2})^2}{n(n-1)}}} \stackrel{d}{=} t \text{ with } 2(n-1) \text{ d.f.} \quad (\text{A1.1})$$

for the completely randomized design and

$$t = \frac{\bar{X}_{.1} - \bar{X}_{.2}}{\sqrt{\frac{\sum_{i=1}^n [(X_{i1} - X_{i2}) - (\bar{X}_{.1} - \bar{X}_{.2})]^2}{n(n-1)}}} \stackrel{d}{=} t \text{ with } (n-1) \text{ d.f.} \quad (\text{A1.2})$$

for the randomized block design. Expanding the denominator in (A1.2),

$$t = \frac{\bar{X}_{.1} - \bar{X}_{.2}}{\sqrt{\frac{\sum_{i=1}^n (X_{i1} - \bar{X}_{.1})^2 + \sum_{i=1}^n (X_{i2} - \bar{X}_{.2})^2}{n(n-1)} - \frac{2 \sum_{i=1}^n (X_{i1} - \bar{X}_{.1})(X_{i2} - \bar{X}_{.2})}{n(n-1)}}}, \quad (\text{A1.3})$$

which may be written as

$$t = \frac{\bar{X}_{.1} - \bar{X}_{.2}}{\sqrt{\frac{\sum_{i=1}^n (X_{i1} - \bar{X}_{.1})^2 + \sum_{i=1}^n (X_{i2} - \bar{X}_{.2})^2}{n(n-1)} - \frac{2r s_1 s_2}{n(n-1)}}, \quad (\text{A1.4})$$

where s_1 and s_2 are sample standard deviations (not unbiased estimates of σ_1 and σ_2) and r is the correlation between matched pairs. Hence,

$$s_1^2 = \frac{\sum_{i=1}^n (X_{i1} - \bar{X}_{.1})^2}{n}, \quad s_2^2 = \frac{\sum_{i=1}^n (X_{i2} - \bar{X}_{.2})^2}{n},$$

and

$$r = \frac{\sum_{i=1}^n (X_{i1} - \bar{X}_{.1})(X_{i2} - \bar{X}_{.2})}{n s_1 s_2}.$$

If the true correlation, ρ , is zero, on the average $r = 0$ is likely, and hence, the computed value of t is the same for either (A1.1) or (A1.2) since (A1.2) reduces to (A1.1).

This proves that when the matched items are not correlated, as is the case, for example, in random pairing, the researcher would gain nothing by using the random block design and he loses degrees of freedom since (A1.2) has a t distribution with $n-1$ degrees of freedom while (A1.1) is distributed as t with $2(n-1)$ degrees of freedom. In such cases there is less likelihood of detecting existing significant differences than if the completely randomized design is used. On the other hand, if ρ is large (close to unity), r is likely to be large on average. If observations are paired it will reduce the variance (denominator) and a large t value will be computed so that there will be a better chance of detecting existing significant differences. It is clearly evident that in such cases where ρ (and therefore r) is large, pairing (matching) techniques should be utilized.

APPENDIX A2

EXPECTED NORMAL SCORES

For several tests considered in this dissertation, expected normal scores are used to transform to a standardized normal distribution either ordinal type data or data containing a higher level of measurement but for which it is not desirable to assume as coming from underlying normal populations. Thus, the expected normal score for the i^{th} largest observation in a sample of n observations (or, in other words, the expected value of the i^{th} standard normal order statistic) is given by Fisher and Yates (1949) as:

$$e_i = E(X_{i/n}) = \frac{n!}{(i-1)!(n-i)!} \int_{-\infty}^{\infty} Z p^{n-i} q^{i-1} X dX, \quad (\text{A2.1})$$

where Z is the ordinate of the normal curve, and p and q are the probabilities of falling short of, or exceeding X , respectively. According to Harter (1961) Z may be replaced by $\frac{1}{\sqrt{2\pi}} e^{-\frac{X^2}{2}}$, q may be replaced by $\frac{1}{2} - \int_0^X \frac{1}{\sqrt{2\pi}} e^{-\frac{X^2}{2}} dX$, and p may be replaced by $\frac{1}{2} + \int_0^X \frac{1}{\sqrt{2\pi}} e^{-\frac{X^2}{2}} dX$. It should be noted that Harter has published the most extensive set of tables of expected normal scores--giving values for $n = 2(1)100(25)250(50)400$. In addition, Harter provides the recurrence formula

$$E(X_{i/n-1}) = \frac{1}{n} \left\{ i E(X_{i+1/n}) + (n-i) E(X_{i/n}) \right\} \quad (\text{A2.2})$$

for working "downwards" (with no serious accumulation of rounding errors) to $E(X_{i/n-1})$, the i^{th} largest standard normal

order statistic from a sample of size $n-1$, based on a knowledge of $E(X_{i+1/n})$ and $E(X_{i/n})$, the $i+1^{\text{th}}$ largest standard normal order statistic and the i^{th} largest standard normal order statistic, respectively, from a sample of size n . This recurrence formula was used in this dissertation for generating the expected normal order statistics needed for various sample sizes. The EXNS subroutine is presented below.

```

SUBROUTINE EXNS(KQ,LILJ)
DIMENSION A(20,7),E(150),RANK(20,7),PPP(7,7),AA(20,7)
COMMON /COM/ A,E,RANK,PPP,AA
KQLILJ = KQ-LILJ
DO 6 J = 1,KQLILJ
  RJ = J
  RC = KQ+1-J
  NRK = RC
  NRKM1 = NRK-1
  DO 6 I = 1, NRKM1
    RI = I
    E(I) = (RI*E(I+1)+(RC-RI)*E(I))/RC
6  CONTINUE
RETURN
END

```

FIGURE A2.1: EXNS SUBROUTINE FOR GENERATING EXPECTED NORMAL SCORES

APPENDIX A3

A MODIFIED SHAPIRO-WILK TEST FOR NORMALITY

In this dissertation it was decided to test for normality the empirical null distribution of several tests statistics that were generated via Monte Carlo methods. Thus, an appropriate statistical procedure was sought. An extensive empirical sampling study of the sensitivities of nine statistical tests for normality was accomplished by Shapiro, Wilk, and Chen (1968). The nine techniques examined were: the Shapiro-Wilk Test, the $\sqrt{B_1}$ Test, the B_2 Test, the Kolmogorov-Smirnov Test, the Cramer-Von Mises Test, Anderson and Darling's (Weighted Cramer-Von Mises) Test, Durbin's (Modified Kolmogorov-Smirnov) Test, the χ^2 Test, and the David-Hartley-Pearson (Studentized Range) Test.

It was found that the Shapiro-Wilk Test was, in general, the superior technique under a wide variety of conditions. Unfortunately, Shapiro and Wilk (1965) only gave necessary coefficients and critical values based on samples up to size 50.¹ However, Shapiro and Francia (1972) recently developed a modified version of the Shapiro-Wilk Test, using coefficients depending only on expected normal scores (see Appendix A2).

An empirical sampling study comparing the sensitivities of the modified version of the original test in detecting non-normality was conducted for samples of size 35 and 50 under

¹These coefficients required obtaining an approximation to the covariance matrix of the normal order statistics.

a wide variety of conditions and concluded that the two procedures were essentially similar. Thus, it would be expected that the Modified Shapiro-Wilk Test would also be generally superior to the eight aforementioned procedures.

Shapiro and Francia present empirical percentage points (critical values) of the modified test for samples up to size 99. However, in studying the empirical null distributions of several test statistics in this dissertation it was desired to generate larger samples. Since the Modified Shapiro-Wilk Test required coefficients depending only on the expected normal order statistics which have been tabled by Harter (1961) for samples of size $n = 2(1)100(25)250(50)400$, the table of empirical critical values given by Shapiro and Francia have been extended here to include values for samples of size 400 (see Table A3.1 p.217). This was accomplished by generating 1,000 histories of the test statistic and forming the empirical cumulative distribution function as described by Shapiro and Francia. Hence, by extending the table of empirical percentage points of the Modified Shapiro-Wilk Test to samples of size 400, it was then possible to generate the empirical null distributions of several test statistics in this dissertation based on samples of that size in order to test for normality.²

²Larger empirical null distributions could not be tested by the Modified Shapiro-Wilk procedure since Harter tabularizes expected normal scores for samples only up to size 400. While Harter does show a recurrence formula due to Federer for "working upwards", a warning is given to the user because of large rounding errors which set in and are, unfortunately, cumulative.

TABLE A3.1
EMPIRICAL PERCENTAGE POINTS OF MODIFIED SHAPIRO-WILK TEST

n ^a	α										
	.01	.05	.10	.15	.20	.50	.80	.85	.90	.95	.99
50	.935	.953	.963	.968	.971	.981	.987	.988	.990	.991	.994
99	.967	.976	.980	.982	.984	.989	.993	.994	.994	.995	.996
400	.9915	.9932	.9942	.9949	.9954	.9968	.9977	.9979	.9981	.9983	.9988

^aEmpirical percentage points for $n = 50$ and $n = 99$ are given by Shapiro & Francia (1972). Empirical percentage points for $n = 400$ are obtained by using the formula $\alpha(n+1)$ to locate the percentile positioning point in the empirical null cumulative distributions of the test statistic.

The Modified Shapiro-Wilk Test is described as follows: Let e_i represent the expected value of the i^{th} standard normal order statistic ($i = 1, 2, \dots, n$) from a sample of n . Moreover, let S_i represent the i^{th} ordered random observation (i.e., the i^{th} empirical test statistic) from a sample of size n . It is desired to obtain a test that the S_i are a sample from a normal distribution. The test statistic W' of the Modified Shapiro-Wilk Test is given by:

$$W' = \left(\sum_{i=1}^n b_i S_i \right)^2 / \sum_{i=1}^n (S_i - \bar{S})^2 \quad (\text{A3.1})$$

where $\bar{S} = \sum_{i=1}^n S_i / n$ and $b_i = e_i / \left(\sum_{i=1}^n e_i^2 \right)^{1/2}$.

A rejection of the null hypothesis that the sample data can be considered as coming from a normal population would be made at a particular α level of significance if $W' < W(\alpha)$, the α percentile given either in Table A3.1 or in Shapiro and Francia (1972, p. 216) for a given $n < 100$.

It is interesting to note that in their massive study on various tests of normality, Shapiro, Wilk, and Chen (1968) did not investigate a procedure given by Lilliefors (1967) in the very same journal almost two years earlier. A good description of the Lilliefors Test appears in Conover's text (1971) along with critical values. For several data sets of size 400, selected at random from among the various empirical null distributions of test statistics that were generated, both the Modified Shapiro-Wilk Test and a "modified" Lilliefors Test were used to test for normality with the surprising result that the

latter appeared to be more sensitive in detecting departures therefrom.³ Thus, as an outgrowth of this dissertation, a study of the sensitivities of these two procedures in detecting discrepancies from normality will be conducted under a wide variety of conditions.

³The modification on the Lilliefors procedure was that required comparisons were made over equal intervals along the cumulative distribution function rather than over all data points. Such a modification would make the procedure less likely to detect departures from normality but was utilized for convenience.

APPENDIX A4

VARIOUS CONCEPTS OF "POWER EFFICIENCY"

A most important criterion for choosing among statistical procedures appropriate to a particular study is power. The power of a test may be defined as the probability that the test rejects the null hypothesis when in fact it should be rejected. Power then represents the ability of a procedure to detect statistically significant differences when they actually exist. Therefore, one test may be considered as better--more powerful--than another if it is more sensitive to finding these real differences. Thus, for a given alternative, underlying distribution, level of significance, number of treatments, and sample size, the empirical powers of a group of related tests may be computed over a suitable number of replications via Monte Carlo methods in order to determine the better procedures under the set of conditions imposed. Moreover, these empirical powers may be compared to that of a standard procedure under an imposed set of conditions by forming the ratio of the powers (with the standard as the base). Such empirical "power ratios", or "small sample empirical efficiencies" as termed by Gilbert (1972), are appropriate for the kind of Monte Carlo power study undertaken here since sample size is one of the controlled set of imposed conditions. However, it should be stated that in comparing two test procedures a more commonly used concept of power efficiency is that of relative efficiency. Relative

efficiency may be defined as the ratio of the sample sizes for two test procedures required to achieve the same power for a given alternative, underlying distribution, level of significance, and number of treatments.¹ Thus, the distinction between the concepts of "small sample empirical efficiency" and relative efficiency is that the former is the ratio of the powers of two procedures under an imposed set of conditions including that of equal sized samples while the latter is the reciprocal of the ratio of the sample sizes of two procedures under an imposed set of conditions including that of equal power. If power is plotted as a function of sample size the empirical relative efficiencies for pairs of tests are easily obtained by:

(1) selecting a particular power level; (2) reading across until it intersects the power functions; (3) reading perpendicularly to determine the sample sizes of the two compared procedures needed to achieve the chosen level of power; and (4) taking the ratio of these two sample sizes.

The aforementioned concepts of power efficiency are most important from a practical viewpoint since the researcher

¹For an imposed set of conditions, relative efficiency expresses the amount of increase in sample size that is required to make one test procedure as powerful as another. Thus, if procedure I is the more powerful and requires a sample of size n_I to achieve a specified level of power, the relative efficiency of procedure II relative to procedure I is given by (n_I/n_{II}) where n_{II} is the sample size of procedure II necessary to achieve the same specified level of power.

can now utilize the results obtained from the analysis presented in Chapter V as an aid in selecting an appropriate statistical procedure for a given set of conditions. In making this selection, Hoeffding's (1951) comment must be borne in mind that one ought to consider not only the power functions of the various test procedures but also the labor required for performing the tests. Thus, the researcher may use the concept of (relative) power-efficiency to decide whether or not it is worthwhile to employ a computationally simpler but otherwise less powerful technique by taking a larger sample and accruing the costs thereof so that the power of the utilized procedure equals that of a more computationally cumbersome technique not selected.²

As an alternative, from a theoretical standpoint, mathematicians have sought a concept of power efficiency which depends on fewer conditions and therefore may be considered as a "summary measure" of the aforementioned practical concepts. Thus, the concept of "asymptotic relative efficiency" (A.R.E.) was developed by Pitman and may be considered as the limit of the relative efficiency (ratio) as the sample sizes become large while the true alternative hypothesis approaches the null (Noether (1955)). Unfortunately, however, as Mendenhall and Reinmuth (1971, p. 455) argue, the concept is meaningless from a practical standpoint because the researcher is not

²Discussions of this are found in Snedecor (1962, pp. 120-121), Siegel (1956, pp. 20-21), Conover (1971, pp. 85-92), and Bradley (1968, pp. 56-62).

usually concerned with extremely minute departures from the null hypothesis and does not deal with infinitely large samples. On the other hand, it is interesting in the context of this dissertation to examine the relationship between the empirical small sample power efficiencies to the corresponding asymptotic efficiencies so that the researcher may gain insight for selecting between statistical procedures when only the latter are readily available. However, as mentioned by Puri (1965), when the forms of the limiting distributions of two test statistics differ (are not $N(0,1)$) it is not possible to compare the procedures by determining asymptotic relative efficiency. In these situations one may study the large sample power functions of the test procedures in order to aid in the test selection process. Thus, it is also of interest here to examine these asymptotic power functions against the empirical small sample powers obtained via Monte Carlo methods (see Appendix A8).

APPENDIX A5

VIOLATING THE ASSUMPTIONS OF NORMALITY AND HOMOSCEDASTICITY

While further research is needed, it is reported in Chapter V that preliminary investigations appear to give credence to the claim that certain k sample techniques are greatly affected by simultaneous violations of the normality and homoscedasticity assumptions.

For the trend alternative when $k = 3$, the shift in both location and scale parameters may be noted from the samples generated from underlying parent rectangular, exponential, or normal populations having the following densities, means and variances:

TABLE A5.1
SHIFTS IN LOCATION AND SCALE PARAMETERS

DISTRIBUTION	FIRST GROUP	SECOND GROUP	THIRD GROUP
Rectangular: f_j	$\frac{1}{1-2\theta}$ where $0 < x < 1-2\theta$	1 where $0 < x < 1$	$\frac{1}{1+2\theta}$ where $0 < x < 1+2\theta$
μ_{ij}	$\frac{1}{2} - \theta$	$\frac{1}{2}$	$\frac{1}{2} + \theta$
σ_{ij}^2	$\frac{(1-2\theta)^2}{12}$	$\frac{1}{12}$	$\frac{(1+2\theta)^2}{12}$
Exponential: f_j	$(2/1-2\theta)e^{-(2/1-2\theta)x}$ where $0 < x < \infty$	$2e^{-2x}$ where $0 < x < \infty$	$(2/1+2\theta)e^{-(2/1+2\theta)x}$ where $0 < x < \infty$
μ_{ij}	$\frac{1-2\theta}{2}$	$\frac{1}{2}$	$\frac{1+2\theta}{2}$
σ_{ij}^2	$\left(\frac{1-2\theta}{2}\right)^2$	$\frac{1}{2^2}$	$\left(\frac{1+2\theta}{2}\right)^2$
Normal:	f_j	$\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ where $-\infty < x < \infty$	$\frac{1}{\sqrt{2\pi}(\sigma+\theta)} e^{-\frac{(x-\mu-\theta)^2}{2(\sigma+\theta)^2}}$ where $-\infty < x < \infty$
μ_{ij}	$\mu - \theta$	μ	$\mu + \theta$
σ_{ij}^2	$(\sigma - \theta)^2$	σ^2	$(\sigma + \theta)^2$

From the table it is seen that each successive density function has associated with it a larger mean and variance. Hence, such density functions may be programmed so that a detailed Monte Carlo study could be employed to examine the effects of the aforementioned violations.

APPENDIX A6

VERIFICATION OF GENERATED LEVELS OF SIGNIFICANCE

It is necessary to examine the generated levels of significance for two reasons. First of all, under the condition that $\Delta = 0$ (true null hypothesis), a check on the computer program is made by comparing observed levels of significance to expected levels. Secondly, an empirical determination can be made as to how violations in the assumption of normality affect the α levels of parametric tests (robustness). This appendix is concerned with the former. The latter is evaluated in Appendix A7.

The sampling errors associated with α values of 5% and 1% are, respectively, $\pm 0.77\%$ and $\pm 0.35\%$, when 800 histories are obtained and $\pm 1.1\%$ and $\pm 0.50\%$, respectively, when 400 histories are generated. Thus, it would be expected that 99.73% of all generated significance levels would be contained within 3 sampling errors around the chosen α . For α of 5%, the "acceptable" ranges of the generated significance levels are 2.7% to 7.3% or 1.7% to 8.3%--depending on whether 800 or 400 histories are obtained. For α of 1%, the "acceptable" ranges are, respectively, 0.0% to 2.1% or 0.0% to 2.5%. An examination of Tables A17 through A40 reveals that ten violations occur out of 864 generated α values. In particular, two procedures --Friedman's Test and the Pirie-Hollander Expected Normal Scores Test--have an excessive amount of computed α values outside

the acceptable range. The α values obtained for the former are always too low on the occasions of violation while the computed α values for the latter are always too high. The Friedman procedure has violations occurring whenever $n = 5$ and $k = 3$, regardless of distribution. In addition, violations occur under the rectangular distribution for $n = 10$ and $k = 3$ as well as for $n = 5$ and $k = 6$. Recall, however, from section 4.7 that Friedman (1940) had claimed that χ^2 gives a fairly good approximation to his test statistic if $n > 5$ when $k = 3$ and for any n if $k > 3$. It should be noted, however, that a preliminary investigation using the χ^2 approximation for all $n = 5(5)10$ and $k = 3(1)4$ determined that the Friedman Test is not very powerful. In an effort to use a "better approximation" than χ^2 , and perhaps obtain a more powerful test, a randomization procedure is used to determine the nominal 5% and 1% levels of significance from the exact null distribution of the Friedman statistic as developed by Owen (1962) for $k = 3$ and $n = 5(5)15$, and for $k = 4$ and $n = 5$ (see Table A60 for critical values). For other combinations of n and k the χ^2 distribution is used. Thus, it appears that some discrepancies exist between the tabularized values of Owen --especially for values of $n = 5$ and $k = 3$ --and the empirical α levels generated here. Regardless, the Friedman Test is designed for testing against generalized alternatives and there is much evidence to show that under the true trend or gap alternatives the power of this procedure would not increase substantially.¹

¹Examine the power of the Friedman Test over all values of Δ for particular n and k under situations where no violations of the acceptable range occurs.

With respect to the Pirie-Hollander procedure, recall from section 4.15 that the authors (1972) claim a normal approximation to the test statistic is quite good even for small n and/or k . Here too, then, the discrepancies existing between the nominal and generated α levels (on three occasions) can most likely be accounted for due to the (approximation) formula employed.

APPENDIX A7

ROBUSTNESS AND OTHER DISTRIBUTION EFFECTS

A7.1 DISTRIBUTION EFFECTS WHEN $\Delta = 0$: ROBUSTNESS

When $\Delta = 0$, an examination of the generated α levels of the parametric tests under the rectangular and exponential distributions reveal that these parametric procedures appear to be "somewhat non-robust" with respect to such violations in normality.¹ While these α levels tend to be conservative, it is noted that the observed α levels are significantly smaller under the exponential distribution than they are under the rectangular distribution.² Thus it appears that violations in the symmetry of the distribution have a greater effect on the α levels of the parametric tests than do other kinds of departures from normality. Moreover, these results are congruous with those obtained for the F Test by Donaldson who stated that

¹For an interesting discussion of the concept of "robustness" see Bradley (1968, pp. 24-43). A robust test is one which is insensitive to violations in its assumptions so that the distribution of the test statistic and the level of significance are not appreciably altered.

²Under each combination of $n = 5(5)20$ and $k = 3(1)6$, the computed α values of all seven (one-way and two-way) parametric procedures are compared under the rectangular, exponential, and normal distributions. A ranking scheme is used and the sum of ranks obtained over all sixteen n and k combinations for each parametric test. A randomized block design is then formed in which the treatment levels are the three distributions and the blocks are the seven test procedures. The Page L Test is employed to test the hypothesis of no differences among the observed α levels against the ordered alternative that departures from normality, and especially symmetry, will cause a reduction in the α level. The results are significant at the 0.001 level.

the greater the skewness and kurtosis in the distribution, the greater the difference between the nominal and actual α levels. In addition, Donaldson concluded that for small n , the F Test is conservative with respect to α when normality is violated. Finally, for nonnormal distributions Donaldson also observed that the F Test is conservative with respect to β error over a large range of Δ --depending on n and α . Hence, in such situations, power is overstated. It is difficult to verify these latter statements, however, because Donaldson's study considered a wider range of n .

A7.2 DISTRIBUTION EFFECTS

Since the nonparametric procedures are not affected by the normality assumption required of their parametric counterparts it is inappropriate to consider the concept of robustness in such cases. However, upon examination of the powers of the nonparametric tests under the various distributions one may question why it is observed that the powers are dependent upon the particular distribution when other conditions-- n , k , Δ , and ordered alternative--are held constant? As an example, the powers for Jonckheere's One-Way Test and Puri's Test are presented in Table A7.1 (p. 231) under the three distributions for condition-combinations $n = 5$, $k = 6$ versus $n = 15$, $k = 6$ for the trend alternative.

The reason that the empirical powers of the nonparametric tests are distribution dependent is apparently due, however, to the methods by which the data are generated for the Monte Carlo study. It may be recalled from section 5.2 of that chapter that under the trend alternative, say, k samples are randomly drawn

TABLE A7.1

POWER OF JONCKHEERE'S ONE-WAY TEST & PURI'S TEST AGAINST TREND UNDER THREE DISTRIBUTIONS
($\alpha = .05$)

PARAMETERS		JONCKHEERE'S TEST			PURI'S TEST		
n, k	Δ	NORMAL	RECTANGULAR	EXPONENTIAL	NORMAL	RECTANGULAR	EXPONENTIAL
5, 6	0.0	.040	.054	.039	.054	.049	.037
	0.5	.272	.277	.560	.290	.362	.600
	1.0	.710	.720	.940	.725	.822	.940
	1.5	.967	.952	.990	.972	.977	.987
15, 6	0.0	.052	.062	.060	.050	.062	.062
	0.5	.315	.280	.665	.335	.440	.755
	1.0	.765	.735	.960	.815	.865	.985
	1.5	.965	.985	1.000	.975	.995	1.000

from a particular distribution and then a constant is added (or subtracted) to the generated observations in each sample so as to simulate their being drawn from populations with equally spaced means (this occurs, of course, a priori to any transformation to ranks or normal scores--peculiar to nonparametric tests).

It is easy to show, however, that the "overlapping" of populations with equally spaced means differs for the various distributions while other conditions are held constant. For example, from Figure 5.2, there appears to be much more overlap in the shifted normal and rectangular distributions than there is in the shifted exponential distributions--other things being equal. For the trend alternative, with $n = 20$, $k = 3$, and $\Delta = 1.5$, it is seen from Table 5.4 that $\theta = .411\sigma$. Under the (unit) rectangular distribution the amount of overlap between adjacent samples is $\int_0^{1-\theta} dX = \int_0^1 dX = 88.1\%$, while the amount of overlap between extreme samples is $\int_\theta^{1-\theta} dX = 76.2\%$. Under the standardized normal distribution similar results are obtained--83.4% for adjacent samples and 68.2% for the extreme samples. However, under the exponential distribution (with

$\int_0^\infty \frac{1}{2} e^{-\frac{1}{2}(x+\theta)} dX = \int_0^\infty \frac{1}{2} e^{-\frac{1}{2}x} dX = 66.4\%$, while the amount of overlap between the extreme samples is $\int_\theta^\infty \frac{1}{2} e^{-\frac{1}{2}(x+\theta)} dX = 44.0\%$.
 or overlap between the extreme samples is $\int_\theta^\infty \frac{1}{2} e^{-\frac{1}{2}x} dX = 44.0\%$.

As the amount of overlapping area becomes smaller, it is more likely that the observed data will be ordered in accordance with the trend alternative. It is, therefore, not surprising

that the procedures are significantly more powerful under the shifted exponential distributions. It should be noted that both these relationships, overlap and power, hold over all examined values of θ presented in Table 5.4.

APPENDIX A8

VARIOUS POWER RELATIONSHIPS

As indicated in section 5.2, the selection of the particular set of conditions (presented in Table 5.1) for the Monte Carlo study provides the basis for an examination of the major aspects of this dissertation. However, several other relationships involving some of the imposed set of conditions--although known theoretically--can be empirically demonstrated and may be of interest to the researcher. These relationships include:

1. A comparison of the small sample empirical powers of various tests for changes in α .
 2. An examination of the small sample empirical powers of various tests over the range of n ($n = 5(5)20$) and k ($k = 3(1)6$).
 3. The effects on power of the allocation of fixed N observations into various combinations of n and k .
- In addition, a comparison of the small sample empirical powers of various tests to their asymptotic powers can be made.

A8.1 POWER AND CHANGES IN α

Table A8.1 (p. 235) presents a comparative ranking of the powers of nine one-way tests for changes in α . Ranks are shown for both trend and gap alternatives under the normal distribution for $n = 5$, $k = 6$, and $\Delta = 1.0$. As expected, a high correlation is observed in the rank ordering of the tests (under each alternative) for changes in α .

TABLE A8.1
A COMPARATIVE RANKING OF POWER OVER VARIOUS α
($n = 5$, $k = 6$, $\Delta = 1.0$)

TEST	NORMAL TREND		NORMAL GAP	
	$\alpha = .05$	$\alpha = .01$	$\alpha = .05$	$\alpha = .01$
FTEST	8	8	8	8
HTEST	9	9	9	9
JONKE	5	3	7	7
BARTH	6	6	1	1
CHAKO	7	7	2	3
ATMAX	3	4	3	2
LRCON	1	1	5	4
JMENS	4	5	4	5
PURIS	2	2	6	6

A8.2 POWER AND SAMPLE SIZE, NUMBER OF GROUPS, AND THE ALLOCATION OF N

In order to examine the relationships between power and sample size, number of samples, and the allocation of fixed N observations--reference is made to section 5.2 and the discussion based on Tables 5.4 and 5.5 therein.

First of all, the relationship of power as a function of sample size for fixed distances θ between k means is provided by data obtained under the condition-combinations $n = 5$, $k = 4$, $\Delta = .5$ versus $n = 20$, $k = 4$, $\Delta = 1.0$. For these condition-combinations, Table A8.2 (p. 236) presents, as an example, a comparison of the powers of the nine one-way procedures against

trend under the rectangular distribution. Using the Sign Test one may reject the hypothesis of no differences in power in favor of the ordered alternative that power is an increasing function of sample size since the powers of each of the nine procedures are greater as n increases. The probability that this could happen by chance is only 1 out of 512.

TABLE A8.2

COMPARISON OF POWERS OF ONE-WAY TESTS FOR INCREASES IN n^a

TEST	CONDITION-COMBINATIONS	
	$n = 5, k = 4$ $\Delta = .5, \theta = .2$	$n = 20, k = 4$ $\Delta = 1.0, \theta = .2$
FTEST	.110	.355
HTEST	.080	.305
JONKE	.242	.610
BARTH	.215	.600
CHAKO	.200	.560
ATMAX	.242	.605
LRCON	.235	.620
JMENS	.260	.695
PURIS	.282	.740

^a $\alpha = .05$; Rectangular Distribution; Trend Alternative.

Secondly, the relationship of power as a function of k for fixed distances θ between the k means is approximated by data obtained under the condition-combinations $k = 4$ and $\Delta = 1.0$ versus $k = 6$ and $\Delta = 1.5$ for the same sample size.¹ As an

¹From Table 5.4 it is noted that $\theta = .4000$ for the conditions $n = 5, k = 4, \Delta = 1.0$. Moreover, $\theta = .3928$ where $n = 5, k = 6$, and $\Delta = 1.5$.

example, Table A8.3 demonstrates the differences in the powers of the one-way procedures when the population is rectangular and the alternative is trend. Here too, the null hypothesis of no differences in power is rejected in favor of the ordered alternative that power is an increasing function of k for fixed distances θ between each of the k means.

TABLE A8.3
COMPARISON OF POWER OF ONE-WAY TESTS FOR INCREASES IN k^a

TEST	CONDITION-COMBINATIONS	
	$n = 5, k = 4,$ $\Delta = 1.0, \theta = .4000$	$n = 5, k = 6,$ $\Delta = 1.5, \theta = .3928$
FTEST	.252	.688
HTEST	.195	.547
JONKE	.522	.952
BARTH	.522	.962
CHAKO	.472	.907
ATMAX	.555	.970
LRCON	.563	.982
JMENS	.585	.960
PURIS	.597	.977

^a $\alpha = .05$; Rectangular Distribution; Trend Alternative.

Thirdly, the effects on power of the allocation of fixed N observations into various combinations of n and k can be studied from data obtained under conditions $n = 20, k = 3$, and $\Delta = 1.0$ versus $n = 10, k = 6$, and $\Delta = 1.5$ since the distances θ between adjacent means are approximately the same. To accomplish this a three factor-factorial experiment is

designed wherein "power" is a function of test procedures (nine levels), condition-combinations (two levels), and parent distributions (three levels). The experiment, performed separately under the trend alternative for the one-way and two-way procedures, yields identical results for the main effect of interest --the condition-combinations. It is observed from Table A8.4, for example, that power is significantly increased if a total of N observations is allocated over more k with smaller n than vice versa. This result, however, holds provided that the additional equally spaced means are θ standard deviation units apart. Under such conditions, as k increases the total distance ($d = (k-1) \theta$ under the trend alternative) between the smallest and largest mean increases.

What might be even more important is to examine the effects of the allocation of fixed N observations into various combinations of n and k such that the total distance d between the smallest and the largest mean is held constant. From Table 5.5 it is seen that while no d values are identical for combinations of fixed $N = nk$, a comparison under condition-combinations $n = 20$, $k = 3$, and $\Delta = 1.5$ versus $n = 10$, $k = 6$ and $\Delta = 1.0$ might provide useful information (the distance d is .9260 standard units for the latter and .8216 for the former). Running the three factor-factorial experiment described above, for both the one-way and two-way procedures, again yields identical results. This time, however, it is observed that power is significantly increased if, within a fixed distance

TABLE A8.4
THREE FACTOR-FACTORIAL DESIGN LAYOUT
(EXPRESSING POWER AS A FUNCTION OF ONE-WAY TESTS, CONDITION-COMBINATIONS, & PARENT DISTRIBUTION)

$$\text{MODEL: } X_{ijk} = \mu + t_i + C_j + tC_{ij} + d_k + td_{ik} + Cd_{jk} + tCd_{ijk}$$

(where $tCd_{ijk} = E_{ijk}$).

FACTOR	SYMBOL	LEVELS	TYPE OF FACTOR
Test Procedures	t	i = 1, ..., 9	Fixed
Condition-Combinations	C	j = 1, ..., 2	Fixed
Parent Distributions	d	k = 1, ..., 3	Fixed

ANOVA

SOURCE OF VARIATION	d.f.	SUM OF SQUARES	MEAN SQUARES	E.M.S.	F CALCULATED
t_i	8	0.78770	0.09846	$\sigma_e^2 + 6\sigma_t^2$	49.48 (highly significant)
C_j	1	3.71846	3.71846	$\sigma_e^2 + 27\sigma_c^2$	1,868.57 (" ")
d_k	2	0.28609	0.14305	$\sigma_e^2 + 18\sigma_d^2$	71.88 (" ")
tC_{ij}	8	0.02468	0.00309	$\sigma_e^2 + 3\sigma_{tc}^2$	1.55 (not significant)
td_{ik}	16	0.12286	0.00768	$\sigma_e^2 + 2\sigma_{td}^2$	3.86 (highly significant)
Cd_{jk}	2	0.00748	0.00374	$\sigma_e^2 + 9\sigma_{cd}^2$	1.88 (not significant)
Residual = tCd_{ijk} *	16	0.03191	0.00199	σ_e^2	----
TOTAL:	53	4.97919	---	---	----

* Assume $\sigma_{tcd}^2 = 0$. According to Hicks (1964, pp. 162-163), such an assumption is required for a fixed model with no replication in a factorial design because the highest order interaction is confounded with experimental error.

d between the smallest and largest mean, a total of N observations is allocated over fewer k with larger n than vice versa.² This result is of practical importance to marketing research studies involving consumer panels of fixed size.

A8.3 COMPARISON WITH ASYMPTOTIC POWERS

Finally, it is interesting to compare, whenever possible, the asymptotic powers of the various k sample tests to those of the empirically generated small sample powers. In general, there appears to be a high correlation between the two.³ From a practical standpoint, however, it is more worthwhile to examine these empirical small sample powers to determine relative differences between tests since the asymptotic power functions are a theoretical concept (see Appendix A4).

²This result is obtained even though the distance between the six groups is actually larger than the distance between the three groups by .1044 standard deviation units. Thus, if d could have been held constant, even greater differences would have been observed.

³The largest discrepancy between the small sample and asymptotic powers occurs in the Bell and Doksum Tests. Asymptotically they are superior to the F Tests but empirically they are inferior (see sections 3.13 and 4.17 and Tables A1 through A16). Jogdeo (1966) has made mention of these poor finite sample power properties of the Bell and Doksum procedures and research will be proposed to investigate this further.

APPENDIX TABLES

TABLE A1

EMPIRICAL EFFICIENCIES OF ONE-WAY TESTS RELATIVE TO F TEST -- EXPONENTIAL DISTRIBUTION

n	K	FTEST	HTEST	COXK3	JOIKE	GARTH	CHAKO	ATMAX	LRCHN	DEREG	DEREG	JMEYS	PURIS	FYENS	BOBNS	COXS1	COXS2	COXS3	CONS1	MUSL1	BASE	POWER
TREND																						
5	3	1.700	1.563	1.813	3.638	2.500	3.250	3.188	3.188	3.188	3.188	4.000	4.000	1.750	1.375	3.188	1.913	3.000	1.125	0.938	0.053	
5	4	1.000	1.177	1.036	2.143	1.993	2.071	1.679	1.679	1.679	1.679	2.429	2.429	1.321	1.071	2.107	1.079	1.571	1.036	0.714	0.093	
5	5	1.000	0.762	1.143	2.619	1.813	2.571	1.429	2.000	1.524	1.524	2.571	2.905	1.095	0.952	2.048	1.813	2.190	1.193	0.714	0.070	
5	6	1.000	0.789	0.617	2.211	1.316	1.942	1.526	1.737	1.474	1.526	2.842	3.000	1.158	0.842	1.684	1.747	1.842	1.158	1.053	0.063	
10	3	1.000	1.370	1.296	2.481	1.704	2.333	1.778	1.778	1.778	1.778	2.630	2.630	1.593	1.333	2.185	1.222	2.222	0.593	0.556	0.090	
10	4	1.000	1.142	0.819	2.591	1.773	2.273	1.682	1.636	1.727	1.727	2.955	2.864	1.591	1.409	2.091	2.273	1.909	1.136	0.636	0.073	
10	5	1.000	1.000	1.294	3.706	2.176	3.176	2.765	2.588	2.824	2.824	4.471	4.412	1.118	1.647	3.000	1.547	2.588	1.059	0.765	0.057	
10	6	1.000	0.979	0.642	2.318	1.364	1.909	1.500	1.364	1.500	1.500	2.591	2.545	1.045	1.227	1.773	1.545	1.818	0.909	0.455	0.073	
15	3	1.000	1.700	1.600	3.350	2.100	2.750	2.350	2.350	2.350	2.350	4.250	4.250	1.800	1.650	2.350	1.900	2.450	0.800	0.600	0.067	
15	4	1.000	1.240	1.040	2.560	1.760	2.360	1.760	1.800	1.800	1.800	3.080	3.000	1.560	1.280	2.040	1.200	2.040	0.880	0.840	0.083	
15	5	1.000	1.000	0.708	2.458	1.792	2.208	1.500	1.625	1.542	1.542	2.917	2.958	1.333	1.292	2.000	1.292	1.125	0.875	0.708	0.080	
15	6	1.000	1.053	0.875	2.895	1.737	2.526	1.368	1.632	1.421	1.421	3.000	3.421	1.000	1.263	2.263	2.158	1.632	0.947	0.947	0.063	
20	3	1.000	1.560	1.600	3.160	1.720	2.800	2.080	2.080	2.080	2.080	3.640	3.640	1.880	1.720	2.240	1.280	2.160	0.600	0.440	0.083	
20	4	1.000	1.429	1.233	3.190	1.905	3.095	1.905	2.190	1.905	1.905	4.095	4.381	1.905	1.810	2.238	1.905	2.238	0.667	0.762	0.070	
20	5	1.000	1.450	1.150	2.650	2.000	2.600	1.950	2.300	2.050	2.050	3.700	3.600	1.800	1.650	2.550	1.250	2.100	0.850	1.100	0.067	
20	6	1.000	1.174	0.696	2.478	1.783	2.087	1.739	1.783	1.739	1.696	3.435	3.391	1.478	1.217	1.870	1.826	1.739	0.696	0.670	0.077	
GAP																						
5	3	1.000	1.111	1.441	2.337	1.373	2.937	1.481	1.481	1.441	1.481	1.889	1.889	1.259	1.000	1.778	1.296	1.704	1.000	0.593	0.090	
5	4	1.000	0.927	1.571	2.513	2.714	2.786	2.643	2.429	2.714	2.714	3.143	3.071	1.786	1.357	2.786	2.214	2.000	1.357	1.071	0.047	
5	5	1.000	0.591	0.682	1.591	1.591	1.591	1.545	1.545	1.545	1.545	1.818	1.773	1.045	0.909	1.364	1.313	1.136	0.591	0.818	0.073	
5	6	1.000	0.900	1.100	1.550	1.450	1.400	1.800	1.700	1.700	1.700	2.000	1.700	1.400	1.000	1.150	0.750	1.300	0.900	1.300	0.067	
10	3	1.000	1.241	1.414	1.555	1.310	1.793	1.310	1.310	1.310	1.310	1.517	1.517	1.379	1.241	1.448	1.276	1.621	0.724	0.414	0.097	
10	4	1.000	0.941	0.941	2.412	2.529	2.706	2.118	1.941	2.059	2.059	2.706	2.706	1.294	1.294	2.000	1.294	1.824	0.588	0.588	0.057	
10	5	1.000	1.154	0.962	1.515	1.154	1.692	1.308	1.346	1.346	1.346	1.923	1.962	1.231	1.385	1.346	0.808	1.192	0.808	0.346	0.087	
10	6	1.000	1.118	1.353	1.741	2.176	2.412	1.588	1.824	1.598	1.765	2.588	2.471	1.118	1.118	1.647	0.706	1.647	1.176	0.647	0.057	
15	3	1.000	1.200	1.240	2.320	1.640	2.280	1.560	1.560	1.560	1.560	2.560	2.560	1.400	1.440	1.840	1.000	1.720	0.680	0.760	0.083	
15	4	1.000	1.040	1.440	1.800	1.600	1.840	1.400	1.360	1.400	1.400	2.000	2.080	1.160	1.440	1.800	0.920	1.320	0.840	0.720	0.083	
15	5	1.000	1.175	1.176	1.529	1.766	2.118	1.412	1.235	1.235	1.235	1.941	1.765	1.235	1.353	1.412	0.588	0.765	1.294	1.059	0.057	
15	6	1.000	1.333	1.200	2.270	2.133	2.067	2.067	2.267	2.067	2.133	2.733	2.533	1.200	1.333	1.733	1.733	0.933	1.267	1.267	0.050	
20	3	1.000	1.373	1.364	1.783	1.394	1.879	1.394	1.394	1.394	1.394	2.030	2.030	1.364	1.242	1.515	0.636	1.455	0.636	0.515	0.110	
20	4	1.000	0.920	1.200	2.120	1.440	2.120	1.480	1.640	1.480	1.480	2.480	2.520	1.240	1.080	1.320	1.200	1.440	0.640	0.600	0.083	
20	5	1.000	1.000	1.000	1.755	1.409	1.636	1.727	1.545	1.682	1.682	2.545	2.227	1.136	1.000	2.227	0.955	1.500	0.864	0.864	0.073	
20	6	1.000	1.400	1.447	2.867	2.467	2.667	2.067	2.000	2.200	2.067	3.400	3.333	1.667	1.533	2.267	2.200	2.533	0.800	1.600	0.050	

TABLE A2

EMPIRICAL EFFICIENCIES OF ONE-WAY TESTS RELATIVE TO F TEST -- NORMAL DISTRIBUTION

N	K	FTST	HTEST	COX1	JURKE	BARTH	CHAO	ATHAX	LRCON	DURFG	QURFG	JMF'S	PURIS	RYENS	BURNS	COXS1	COXS2	COXS3	CONS1	MOSSL	BASE POWER
TREND																					
5	3	1.000	1.000	0.765	1.863	1.961	1.961	2.039	2.039	2.039	2.039	1.980	1.980	1.078	0.824	1.412	1.314	1.549	0.567	0.549	0.170
5	4	1.000	0.732	0.532	1.447	1.723	1.638	1.553	1.745	1.596	1.596	1.617	1.660	1.106	0.766	1.021	1.255	0.915	0.596	0.511	0.157
5	5	1.000	0.714	0.095	1.335	2.613	2.391	3.143	3.238	3.143	3.143	3.391	3.238	0.905	0.905	2.381	1.352	2.190	0.810	0.952	0.070
5	6	1.000	0.737	1.000	1.135	1.632	1.363	3.211	3.421	3.211	3.211	3.421	3.316	1.211	0.947	2.369	2.158	2.421	0.684	0.737	0.063
10	3	1.000	0.927	0.942	1.636	1.891	1.727	1.855	1.855	1.855	1.855	1.727	1.727	0.964	0.909	1.309	1.073	1.455	0.673	0.436	0.183
10	4	1.000	0.842	0.720	2.760	2.129	2.360	2.680	2.640	2.680	2.680	2.640	2.480	1.000	1.120	2.080	2.160	2.000	0.680	0.840	0.083
10	5	1.000	0.786	0.929	3.036	2.393	2.357	2.679	2.857	2.679	2.679	2.750	2.929	0.929	1.071	1.786	1.300	2.107	0.714	0.536	0.093
10	6	1.000	1.032	0.548	2.000	2.097	2.129	2.065	2.129	2.032	2.032	2.161	2.161	1.065	1.032	1.742	0.806	1.387	0.613	0.452	0.103
15	3	1.000	0.932	0.932	1.824	1.647	1.627	1.784	1.784	1.784	1.784	1.843	1.824	0.941	0.941	1.314	0.706	1.490	0.647	0.471	0.170
15	4	1.000	0.765	0.526	1.553	1.532	1.468	1.617	1.596	1.617	1.617	1.702	1.638	0.851	0.830	1.064	0.650	1.043	0.553	0.426	0.157
15	5	1.000	1.000	0.889	2.303	2.273	2.182	2.212	2.364	2.212	2.212	2.091	2.333	0.970	0.939	1.485	1.391	0.727	0.727	0.303	0.110
15	6	1.000	0.850	0.700	2.703	2.800	2.650	2.950	3.050	3.000	3.000	2.950	3.250	0.950	0.850	2.450	2.000	1.250	0.650	0.800	0.067
20	3	1.000	0.943	0.831	1.712	1.593	1.729	1.644	1.644	1.644	1.644	1.695	1.695	1.017	0.966	1.271	0.559	1.390	0.373	0.271	0.197
20	4	1.000	0.960	0.605	1.721	1.698	1.698	1.814	1.744	1.837	1.837	1.860	1.860	0.977	1.000	1.233	1.349	1.000	0.558	0.349	0.143
20	5	1.000	0.912	0.757	2.000	1.784	1.730	1.973	2.108	1.973	1.973	2.054	2.108	0.865	0.973	1.459	1.354	1.351	0.541	0.432	0.123
20	6	1.000	0.964	0.535	2.179	2.143	2.107	2.214	2.357	2.286	2.214	2.393	2.536	1.000	0.857	1.679	1.750	1.750	0.821	0.643	0.093
GAP																					
5	3	1.000	0.872	1.026	1.615	1.538	1.615	1.769	1.769	1.769	1.769	1.641	1.641	1.000	0.846	1.821	1.179	1.333	0.872	0.872	0.130
5	4	1.000	0.649	0.855	1.486	1.703	1.368	1.649	1.459	1.649	1.649	1.703	1.622	0.892	0.973	1.486	0.838	1.108	0.892	0.649	0.123
5	5	1.000	0.808	0.646	1.692	2.038	2.115	1.530	1.615	1.500	1.500	1.654	1.769	0.962	0.923	1.462	0.762	1.346	0.731	0.692	0.087
5	6	1.000	0.767	0.633	1.533	1.667	1.467	1.500	1.233	1.467	1.467	1.433	1.033	1.233	0.833	1.133	0.867	0.967	1.000	0.800	0.100
10	3	1.000	0.800	0.844	1.867	1.778	1.778	1.849	1.889	1.849	1.889	1.822	1.822	0.867	0.933	1.422	1.156	1.489	0.778	0.667	0.150
10	4	1.000	0.794	1.059	1.324	1.735	1.559	1.676	1.500	1.676	1.676	1.647	1.412	1.000	0.794	1.147	0.882	1.176	0.676	0.735	0.113
10	5	1.000	0.765	1.000	1.353	1.471	1.552	1.559	1.412	1.500	1.500	1.706	1.441	0.912	0.794	1.059	0.647	1.098	0.735	0.523	0.113
10	6	1.000	0.857	0.935	2.048	2.476	2.476	2.571	2.381	2.619	2.714	2.762	2.333	0.905	1.333	1.762	0.619	1.762	1.381	1.238	0.070
15	3	1.000	0.835	0.747	1.582	1.636	1.545	1.691	1.691	1.691	1.691	1.691	1.691	0.927	0.836	1.691	0.564	1.455	0.691	0.382	0.183
15	4	1.000	0.905	0.810	1.310	1.571	1.571	1.571	1.548	1.548	1.548	1.500	1.500	0.976	0.905	1.119	0.619	0.810	0.543	0.524	0.140
15	5	1.000	0.893	0.746	2.036	2.000	2.071	2.107	2.143	2.071	2.071	1.964	1.893	1.000	0.750	1.571	0.750	0.857	0.964	0.500	0.093
15	6	1.000	1.083	1.125	1.958	2.167	2.300	2.208	1.583	2.167	2.298	2.250	1.917	1.042	1.125	1.458	1.125	0.958	0.792	0.417	0.080
20	3	1.000	0.873	0.978	1.587	1.587	1.543	1.717	1.717	1.717	1.717	1.652	1.652	1.065	0.935	1.217	0.587	1.326	0.783	0.587	0.153
20	4	1.000	0.923	1.231	1.885	2.462	2.077	2.423	2.154	2.423	2.423	2.231	2.115	1.000	0.885	1.269	1.192	1.077	0.692	0.923	0.087
20	5	1.000	0.862	0.897	1.655	1.724	1.621	1.724	1.517	1.724	1.724	1.897	1.483	0.966	1.000	1.517	0.621	1.172	0.552	0.517	0.097
20	6	1.000	0.930	1.300	2.150	2.460	2.050	2.500	2.050	2.350	2.400	2.450	2.150	0.950	1.200	1.600	2.050	1.650	0.600	0.950	0.067

TABLE A3

EMPIRICAL EFFICIENCIES OF ONE-WAY TESTS RELATIVE TO "BEST TESTS" -- EXPONENTIAL
DISTRIBUTION

n	K	FTEST	HTEST	COHNS	JUNKE	BARTH	CHAKO	ATMAX	LRCON	DEREG	DEREG	JHENS	PURIS	FYENS	BDNRS	COXS1	COXS2	COXS3	COVSL	MOSSL	BASE POWER
TREND																					
5	3	0.250	0.344	0.453	0.722	0.625	0.813	0.797	0.797	0.797	0.797	1.000	1.000	0.438	0.344	0.797	0.453	0.750	0.281	0.234	0.213
5	4	0.412	0.455	0.420	0.882	0.777	0.853	0.691	0.691	0.691	0.691	1.000	1.000	0.544	0.441	0.868	0.691	0.647	0.426	0.294	0.227
5	5	0.344	0.262	0.363	0.912	0.623	0.885	0.492	0.689	0.525	0.525	0.885	1.000	0.377	0.328	0.705	0.623	0.754	0.410	0.246	0.203
5	6	0.333	0.263	0.211	0.737	0.431	0.614	0.509	0.579	0.491	0.509	0.947	1.000	0.386	0.281	0.561	0.649	0.614	0.386	0.351	0.190
15	3	0.380	0.521	0.473	0.944	0.648	0.887	0.676	0.676	0.676	0.676	1.000	1.000	0.606	0.507	0.831	0.465	0.845	0.225	0.211	0.237
15	4	0.336	0.470	0.277	0.977	0.600	0.767	0.569	0.554	0.585	0.585	1.000	0.969	0.538	0.477	0.708	0.769	0.646	0.385	0.215	0.217
10	5	0.224	0.224	0.249	0.829	0.487	0.711	0.618	0.579	0.632	0.632	1.000	0.987	0.250	0.368	0.671	0.368	0.579	0.237	0.171	0.253
10	6	0.385	0.351	0.263	0.895	0.526	0.737	0.579	0.526	0.579	0.579	1.000	0.982	0.404	0.474	0.684	0.404	0.702	0.351	0.175	0.190
15	3	0.235	0.403	0.376	0.788	0.494	0.694	0.553	0.553	0.553	0.553	1.000	1.000	0.424	0.388	0.553	0.447	0.576	0.188	0.141	0.283
15	4	0.325	0.403	0.338	0.831	0.571	0.766	0.571	0.610	0.584	0.584	1.000	0.974	0.506	0.416	0.662	0.390	0.662	0.286	0.273	0.257
15	5	0.338	0.338	0.239	0.831	0.606	0.746	0.507	0.549	0.521	0.521	0.986	1.000	0.451	0.437	0.676	0.437	0.380	0.296	0.239	0.237
15	6	0.292	0.353	0.262	0.846	0.508	0.738	0.400	0.477	0.415	0.415	0.877	1.000	0.292	0.369	0.662	0.631	0.477	0.277	0.277	0.217
25	3	0.275	0.427	0.440	0.868	0.473	0.769	0.571	0.571	0.571	0.571	1.000	1.000	0.516	0.473	0.515	0.352	0.593	0.165	0.121	0.303
20	4	0.274	0.325	0.263	0.728	0.435	0.707	0.435	0.500	0.435	0.435	0.935	1.000	0.435	0.413	0.511	0.435	0.511	0.152	0.174	0.307
20	5	0.277	0.322	0.311	0.770	0.541	0.703	0.527	0.622	0.554	0.554	1.000	0.973	0.486	0.446	0.687	0.338	0.566	0.230	0.297	0.247
20	6	0.291	0.342	0.253	0.722	0.519	0.608	0.506	0.519	0.506	0.494	1.000	0.987	0.430	0.354	0.544	0.532	0.506	0.203	0.253	0.263
GAP																					
5	3	0.421	0.545	0.727	1.000	0.673	1.000	0.727	0.727	0.727	0.727	0.927	0.927	0.618	0.491	0.873	0.636	0.836	0.491	0.291	0.183
5	4	0.312	0.235	0.500	0.795	0.844	0.946	0.841	0.773	0.864	0.864	1.000	0.977	0.568	0.432	0.886	0.705	0.636	0.432	0.341	0.147
5	5	0.550	0.325	0.375	0.875	0.825	0.875	0.850	0.850	0.850	0.850	1.000	0.975	0.575	0.500	0.750	0.725	0.625	0.325	0.450	0.133
5	6	0.500	0.450	0.550	0.775	0.925	0.700	0.900	0.850	0.850	0.850	1.000	0.850	0.700	0.500	0.575	0.375	0.650	0.450	0.650	0.133
10	3	0.558	0.592	0.784	0.923	0.731	1.000	0.731	0.731	0.731	0.731	0.846	0.846	0.769	0.692	0.808	0.712	0.904	0.404	0.231	0.173
10	4	0.273	0.343	0.349	0.891	0.935	1.000	0.783	0.717	0.761	0.761	1.000	1.000	0.478	0.478	0.739	0.478	0.674	0.217	0.217	0.153
10	5	0.513	0.584	0.437	0.824	0.588	0.863	0.667	0.686	0.686	0.686	0.980	1.000	0.627	0.706	0.686	0.412	0.608	0.412	0.176	0.170
10	6	0.346	0.432	0.573	0.750	0.841	0.732	0.614	0.705	0.614	0.682	1.000	0.955	0.432	0.432	0.636	0.273	0.636	0.455	0.250	0.147
15	3	0.371	0.469	0.484	0.996	0.641	0.891	0.609	0.609	0.609	0.609	1.000	1.000	0.547	0.563	0.719	0.391	0.672	0.266	0.297	0.213
15	4	0.491	0.519	0.642	0.865	0.763	0.885	0.673	0.654	0.673	0.673	0.962	1.000	0.558	0.692	0.865	0.442	0.635	0.404	0.346	0.173
15	5	0.472	0.355	0.546	0.722	0.806	1.000	0.667	0.583	0.583	0.583	0.917	0.833	0.583	0.639	0.667	0.278	0.361	0.611	0.500	0.120
15	6	0.366	0.468	0.439	0.805	0.780	0.756	0.756	0.829	0.756	0.780	1.000	0.927	0.439	0.488	0.634	0.634	0.341	0.463	0.463	0.137
20	3	0.493	0.542	0.672	0.881	0.687	0.925	0.687	0.687	0.687	0.687	1.000	1.000	0.672	0.612	0.746	0.313	0.716	0.313	0.254	0.223
20	4	0.337	0.365	0.476	0.841	0.571	0.841	0.587	0.651	0.587	0.587	0.984	1.000	0.492	0.429	0.524	0.476	0.571	0.254	0.238	0.210
20	5	0.393	0.393	0.393	0.768	0.554	0.643	0.679	0.607	0.661	0.661	1.000	0.875	0.446	0.393	0.875	0.375	0.589	0.339	0.339	0.187
20	6	0.494	0.471	0.431	0.843	0.725	0.784	0.608	0.588	0.647	0.608	1.000	0.980	0.490	0.451	0.667	0.647	0.745	0.235	0.471	0.170

TABLE A4

EMPIRICAL EFFICIENCIES OF ONE-WAY TESTS RELATIVE TO "BEST TESTS" -- NORMAL
DISTRIBUTION

n	K	FTEST	WTEST	COXKS	JOIKE	BARTH	CHAKR	ATMAX	LYCON	DERIG	DEREG	JMENS	PURIS	FYENS	BURNS	COXS1	COXS2	COXS3	COXSL	MOSSL	BASE	POWER
TREND																						
5	3	0.440	0.490	0.375	0.913	0.962	0.962	1.000	1.000	1.000	1.000	0.971	0.971	0.529	0.404	0.692	0.644	0.760	0.279	0.269	0.347	
5	4	0.573	0.472	0.305	0.429	0.988	0.939	0.893	1.000	0.915	0.915	0.927	0.951	0.634	0.439	0.585	0.720	0.524	0.341	0.293	0.273	
5	5	0.275	0.211	0.375	0.915	0.775	0.794	0.935	0.958	0.930	0.930	1.000	0.958	0.268	0.268	0.704	0.577	0.648	0.239	0.282	0.237	
5	6	0.275	0.203	0.275	0.355	1.000	0.899	0.884	0.942	0.884	0.884	0.942	0.913	0.333	0.261	0.652	0.594	0.667	0.188	0.203	0.230	
10	3	0.527	0.470	0.519	0.465	1.000	0.913	0.981	0.981	0.981	0.981	0.913	0.913	0.510	0.481	0.692	0.567	0.769	0.156	0.231	0.347	
10	4	0.362	0.306	0.261	1.000	0.768	0.855	0.771	0.957	0.771	0.771	0.957	0.899	0.362	0.406	0.754	0.783	0.725	0.246	0.304	0.230	
10	5	0.329	0.239	0.375	1.000	0.738	0.776	0.882	0.941	0.882	0.882	0.906	0.965	0.306	0.353	0.588	0.329	0.694	0.235	0.176	0.283	
10	6	0.463	0.478	0.254	0.725	0.970	0.985	0.955	0.985	0.940	0.940	1.000	1.000	0.493	0.478	0.806	0.373	0.642	0.284	0.209	0.223	
15	3	0.543	0.467	0.449	0.789	0.894	0.883	0.968	0.968	0.968	0.968	1.000	0.989	0.511	0.511	0.713	0.383	0.809	0.351	0.255	0.313	
15	4	0.567	0.433	0.350	0.912	0.960	0.862	0.950	0.938	0.950	0.950	1.000	0.962	0.500	0.487	0.625	0.387	0.612	0.325	0.253	0.267	
15	5	0.421	0.423	0.397	0.774	0.962	0.923	0.936	1.000	0.936	0.936	0.845	0.987	0.410	0.397	0.628	0.462	0.308	0.308	0.128	0.260	
15	6	0.398	0.262	0.215	0.892	0.862	0.815	0.908	0.938	0.923	0.923	0.908	1.000	0.292	0.262	0.754	0.615	0.385	0.200	0.246	0.217	
20	3	0.578	0.569	0.440	0.779	0.972	1.000	0.951	0.951	0.951	0.951	0.980	0.980	0.588	0.559	0.735	0.324	0.804	0.216	0.157	0.340	
20	4	0.537	0.462	0.325	0.725	0.912	0.912	0.975	0.938	0.987	0.987	1.000	1.000	0.525	0.537	0.662	0.725	0.537	0.300	0.188	0.267	
20	5	0.474	0.435	0.359	0.949	0.846	0.821	0.936	1.000	0.936	0.936	0.974	1.000	0.410	0.462	0.692	0.500	0.641	0.256	0.205	0.260	
20	6	0.394	0.380	0.211	0.859	0.845	0.831	0.873	0.930	0.901	0.873	0.944	1.000	0.394	0.338	0.662	0.690	0.690	0.324	0.254	0.237	
GAP																						
5	3	0.549	0.477	0.553	0.897	0.845	0.887	0.972	0.972	0.972	0.972	0.901	0.901	0.549	0.465	1.000	0.648	0.732	0.479	0.479	0.237	
5	4	0.567	0.361	0.504	0.873	1.000	0.721	0.768	0.857	0.968	0.968	1.000	0.952	0.524	0.571	0.873	0.492	0.651	0.524	0.381	0.210	
5	5	0.473	0.382	0.400	0.370	0.764	1.000	0.709	0.764	0.709	0.709	0.782	0.836	0.455	0.436	0.691	0.455	0.636	0.345	0.327	0.183	
5	6	0.600	0.460	0.500	0.520	1.000	0.880	0.900	0.740	0.880	0.880	0.860	0.620	0.740	0.500	0.680	0.520	0.580	0.600	0.480	0.167	
10	3	0.577	0.424	0.407	0.390	0.741	0.941	1.000	1.000	1.000	1.000	0.965	0.965	0.459	0.494	0.753	0.512	0.788	0.412	0.353	0.283	
10	4	0.576	0.459	0.610	0.753	1.000	0.898	0.956	0.864	0.966	0.966	0.949	0.814	0.576	0.458	0.661	0.508	0.678	0.390	0.424	0.173	
10	5	0.555	0.448	0.536	0.733	0.862	0.914	0.914	0.828	0.879	0.879	1.000	0.845	0.534	0.466	0.621	0.379	0.638	0.431	0.310	0.193	
10	6	0.367	0.310	0.379	0.741	0.877	0.897	0.931	0.862	0.948	0.983	1.000	0.845	0.323	0.483	0.638	0.224	0.638	0.500	0.448	0.193	
15	3	0.591	0.505	0.430	0.935	0.768	0.914	1.000	1.000	1.000	1.000	1.000	1.000	0.548	0.495	0.645	0.333	0.860	0.409	0.226	0.310	
15	4	0.634	0.575	0.515	0.833	1.000	1.000	1.000	0.985	0.985	0.985	0.955	0.955	0.621	0.576	0.712	0.394	0.515	0.409	0.333	0.220	
15	5	0.447	0.417	0.367	0.750	0.933	0.767	0.983	1.000	0.967	0.967	0.917	0.883	0.467	0.350	0.733	0.350	0.400	0.450	0.233	0.200	
15	6	0.444	0.481	0.510	0.870	0.963	0.887	0.981	0.704	0.963	0.981	1.000	0.852	0.463	0.500	0.648	0.500	0.426	0.352	0.185	0.180	
20	3	0.592	0.506	0.570	0.924	0.924	0.899	1.000	1.000	1.000	1.000	0.962	0.962	0.620	0.544	0.709	0.342	0.772	0.456	0.342	0.263	
20	4	0.445	0.375	0.500	0.756	1.000	0.844	0.984	0.875	0.984	0.984	0.906	0.859	0.406	0.359	0.516	0.484	0.438	0.281	0.375	0.213	
20	5	0.527	0.455	0.473	0.873	0.909	0.855	0.909	0.800	0.909	0.909	1.000	0.782	0.509	0.527	0.800	0.327	0.618	0.291	0.273	0.193	
20	6	0.490	0.360	0.520	0.860	0.960	0.820	1.000	0.820	0.940	0.960	0.980	0.860	0.380	0.480	0.640	0.820	0.660	0.240	0.380	0.167	

TABLE A5

EMPIRICAL EFFICIENCIES OF TWO-WAY TESTS RELATIVE TO F TEST -- EXPONENTIAL DISTRIBUTION

N	K	F2WAY	PATET	FRIED	PHI'S	JONK2	ATMX2	LRCV2	DERG2	QERG2	FYNS2	BUN52	PSIGN	PTRST	HPARA	HOLLI	UCKST	SHORT	SMSAT	SEAT	BASE	POWER
TREND																						
5	3	1.000	2.200	0.850	2.200	1.350	2.450	2.450	2.450	2.450	1.050	0.850	2.150	2.100	2.400	2.500	2.400	2.300	0.800	3.250	0.067	
5	4	1.000	2.111	0.869	2.074	1.519	1.519	1.778	1.519	1.519	1.593	0.926	0.926	1.148	1.741	2.074	1.778	2.333	0.963	2.148	0.090	
5	5	1.000	2.333	0.810	2.476	1.524	1.476	2.000	1.524	1.524	1.286	0.905	1.048	0.905	1.952	2.095	2.333	2.238	0.910	2.619	0.070	
5	6	1.000	2.737	0.737	2.684	2.737	1.474	1.799	1.474	1.421	1.263	0.947	0.947	0.895	1.842	2.263	1.947	2.158	0.737	2.526	0.063	
10	3	1.000	2.034	0.753	2.034	1.759	1.621	1.621	1.621	1.621	0.931	0.966	1.931	1.345	1.483	1.828	1.670	1.862	1.241	2.310	0.097	
10	4	1.000	2.810	1.143	3.048	2.381	1.952	1.810	1.952	1.952	1.476	1.238	1.714	1.810	1.667	2.286	2.143	2.429	0.952	2.905	0.070	
10	5	1.000	3.053	1.105	3.211	2.632	2.577	2.263	2.579	2.579	1.316	1.105	1.789	1.737	2.368	2.789	2.579	2.684	0.842	3.421	0.063	
10	6	1.000	2.040	0.720	1.840	1.640	1.320	1.120	1.280	1.320	1.000	0.800	0.560	0.560	1.240	1.400	1.440	1.560	0.640	1.920	0.083	
15	3	1.000	2.182	0.955	2.182	1.909	2.273	2.273	2.273	2.273	0.955	1.136	2.364	1.818	2.318	2.407	2.364	2.227	1.409	3.273	0.073	
15	4	1.000	2.200	0.960	2.200	2.280	1.840	1.920	1.840	1.840	1.120	0.920	1.200	1.600	1.960	2.040	2.080	2.080	1.160	2.520	0.083	
15	5	1.000	2.571	0.955	2.455	2.136	1.727	1.864	1.682	1.682	1.091	1.045	1.273	0.727	1.818	2.136	2.391	2.318	1.045	2.682	0.073	
15	6	1.000	2.382	1.527	2.941	3.059	1.529	1.824	1.588	1.647	1.706	1.529	0.588	0.529	1.765	2.294	2.235	2.647	1.059	3.235	0.057	
20	3	1.000	2.583	1.458	2.583	2.250	2.125	2.125	2.125	2.125	1.458	0.833	2.458	2.083	2.125	2.458	2.333	2.250	1.500	3.083	0.080	
20	4	1.000	2.800	1.100	2.750	2.500	2.000	2.200	2.000	2.000	1.400	1.250	1.650	1.250	2.150	2.550	2.800	2.700	1.650	3.250	0.067	
20	5	1.000	2.450	1.450	2.550	2.300	1.900	2.150	1.900	1.900	1.850	1.450	1.400	1.250	2.050	2.250	2.250	2.400	1.400	2.750	0.067	
20	6	1.000	2.304	1.174	2.261	2.043	1.652	1.826	1.696	1.696	1.217	1.130	0.609	0.565	1.826	1.783	1.957	2.217	1.130	2.435	0.077	
GAP																						
5	3	1.000	1.819	0.642	1.819	1.273	1.818	1.818	1.818	1.818	0.909	1.071	1.503	1.636	1.909	2.000	1.773	2.227	0.818	2.682	0.073	
5	4	1.000	1.400	0.450	1.800	1.000	1.900	1.950	1.900	1.900	1.250	0.750	0.750	0.900	1.700	1.750	1.600	2.300	0.550	2.000	0.067	
5	5	1.000	1.246	0.757	1.619	1.233	1.571	1.571	1.571	1.571	1.333	1.143	1.095	0.905	1.619	1.476	1.286	1.524	0.619	1.567	0.070	
5	6	1.000	1.529	0.824	1.716	1.412	2.000	1.941	2.000	2.176	1.824	1.118	0.941	0.765	1.765	1.235	1.471	1.588	0.882	1.941	0.057	
10	3	1.000	1.679	0.857	1.679	1.393	1.393	1.393	1.393	1.393	1.107	0.893	1.107	0.929	1.357	1.464	1.357	1.464	1.214	1.857	0.093	
10	4	1.000	1.988	0.938	2.135	1.750	2.000	2.125	2.000	2.070	1.000	1.313	1.625	1.313	2.125	1.688	1.750	1.938	0.750	2.438	0.053	
10	5	1.000	1.789	0.826	1.783	1.261	1.391	1.478	1.435	1.435	1.261	0.913	0.870	0.826	1.565	1.565	1.609	1.565	1.217	1.870	0.077	
10	6	1.000	1.889	1.167	1.722	1.389	1.667	1.667	1.667	1.722	1.167	1.000	0.833	0.833	1.722	1.611	1.333	1.833	0.944	1.989	0.060	
15	3	1.000	1.542	0.875	1.542	1.458	1.625	1.625	1.625	1.625	0.792	1.042	1.167	0.958	1.583	2.042	1.833	1.458	0.917	2.250	0.080	
15	4	1.000	1.913	1.174	1.913	1.877	1.522	1.478	1.478	1.478	1.391	1.043	1.000	1.783	1.565	1.348	1.334	2.000	1.174	2.087	0.077	
15	5	1.000	1.500	1.222	1.611	1.500	1.222	1.278	1.167	1.167	1.222	1.278	1.000	0.778	1.222	1.222	1.111	1.933	1.333	1.500	0.060	
15	6	1.000	2.000	1.318	2.000	1.875	1.938	2.125	2.000	1.938	1.438	1.500	0.688	0.625	2.125	1.688	1.688	1.750	1.188	2.125	0.053	
20	3	1.000	1.300	1.361	1.500	1.250	1.278	1.278	1.278	1.278	1.361	0.861	0.833	0.639	1.278	1.278	1.306	1.472	1.194	1.722	0.120	
20	4	1.000	2.429	1.095	2.571	2.075	1.857	1.952	1.857	1.857	1.190	1.476	1.381	1.429	1.952	2.190	2.190	2.238	1.000	2.619	0.070	
20	5	1.000	1.625	0.792	1.625	1.292	1.417	1.375	1.458	1.458	0.875	0.750	0.750	0.708	1.417	1.500	1.500	1.667	0.958	1.792	0.080	
20	6	1.000	2.667	1.400	2.800	2.333	2.133	2.067	2.067	2.133	1.467	1.533	1.133	1.067	2.000	2.067	2.133	2.267	1.533	2.733	0.050	

TABLE A6

EMPIRICAL EFFICIENCIES OF TWO-WAY TESTS RELATIVE TO F TEST -- NORMAL DISTRIBUTION

N	K	F2WAY	PAGET	TRIPD	PHENS	JUMP2	ATMX2	LRG2	DIRG2	ORIG2	FYUS2	RDHS2	PSIGN	PTST	HPARA	HULLT	DIRST	SHURT	SMSAT	SENAT	BASE	POWER
TREND																						
5	3	1.000	1.315	0.500	1.315	1.017	1.815	1.815	1.815	1.815	0.667	0.685	1.111	1.204	1.704	1.796	1.796	1.426	0.667	1.759	0.180	
5	4	1.000	1.774	0.711	1.747	1.289	1.921	2.000	1.921	1.921	1.184	0.763	0.842	0.947	1.711	1.868	1.789	1.579	0.579	1.316	0.127	
5	5	1.000	3.100	0.650	3.050	2.650	3.050	3.250	3.100	3.100	0.750	0.800	1.400	1.300	3.450	3.100	3.500	2.350	0.650	3.050	0.067	
5	6	1.000	3.110	0.588	3.059	3.059	3.529	3.824	3.706	3.647	1.353	0.882	1.235	1.176	3.824	3.529	3.471	2.941	0.647	3.412	0.057	
10	3	1.000	1.567	2.686	1.667	1.373	2.078	2.078	2.078	2.078	0.922	0.725	1.059	1.157	1.980	1.961	2.020	1.510	0.941	1.902	0.170	
10	4	1.000	1.667	0.633	1.867	1.567	2.133	2.100	2.133	2.133	0.733	0.600	1.000	0.800	2.133	2.100	2.200	1.533	0.733	2.233	0.100	
10	5	1.000	1.948	0.576	2.700	1.697	2.333	2.545	2.364	2.364	0.758	0.485	0.818	0.818	2.545	2.242	2.364	1.848	0.727	2.515	0.110	
10	6	1.000	2.032	1.000	2.085	1.805	2.000	2.129	2.032	2.000	1.194	0.677	0.774	0.742	2.097	2.032	1.968	1.935	0.968	2.032	0.103	
15	3	1.000	1.625	1.063	1.625	1.604	1.896	1.896	1.896	1.896	1.042	0.896	1.208	1.042	1.938	2.021	1.896	1.625	0.896	1.979	0.160	
15	4	1.000	1.919	0.651	1.442	1.419	1.814	1.744	1.837	1.837	0.721	0.791	0.721	1.070	1.721	1.651	1.698	1.419	0.850	1.744	0.143	
15	5	1.000	1.941	0.917	1.874	1.735	2.029	2.353	2.059	2.059	0.971	0.824	1.118	0.794	2.412	2.176	2.176	1.853	0.765	2.206	0.113	
15	6	1.000	2.550	1.050	2.600	2.550	2.900	3.050	2.950	3.000	1.200	0.850	0.650	0.600	3.100	2.950	3.000	2.600	0.800	3.000	0.067	
20	3	1.000	1.277	0.785	1.277	1.031	1.615	1.615	1.615	1.615	0.785	0.554	0.662	0.785	1.554	1.538	1.585	1.185	0.862	1.646	0.217	
20	4	1.000	1.559	0.732	1.707	1.415	2.000	1.927	1.976	1.976	0.756	0.756	0.707	0.707	1.927	1.780	1.805	1.415	0.902	1.805	0.137	
20	5	1.000	2.057	0.771	2.000	1.857	2.143	2.286	2.114	2.114	0.857	0.714	0.657	0.600	2.286	2.085	2.086	1.686	0.914	2.029	0.117	
20	6	1.000	2.214	0.750	2.250	1.714	2.143	2.393	2.357	2.250	0.821	0.679	0.464	0.429	2.393	2.250	2.286	1.607	0.929	2.393	0.093	
GAP																						
5	3	1.000	1.250	0.630	1.250	0.917	1.722	1.722	1.722	1.722	0.806	0.917	1.194	1.389	1.567	1.722	1.611	1.306	0.889	1.806	0.120	
5	4	1.000	1.590	0.621	1.654	1.000	2.172	2.034	2.172	2.172	1.172	0.966	0.931	1.207	1.931	1.862	1.966	1.724	0.586	1.966	0.087	
5	5	1.000	1.538	0.769	1.615	1.267	1.654	1.731	1.654	1.654	1.192	0.846	1.000	0.962	1.615	1.615	1.577	1.731	0.692	1.654	0.087	
5	6	1.000	0.939	0.697	1.000	0.848	1.394	1.182	1.394	1.394	0.909	0.545	0.697	0.667	1.030	0.970	0.848	0.909	0.879	0.970	0.110	
10	3	1.000	1.261	0.537	1.261	0.871	1.826	1.826	1.826	1.826	0.739	0.652	0.761	0.870	1.783	1.761	1.848	1.043	0.804	1.717	0.153	
10	4	1.000	1.109	0.811	1.162	0.802	1.486	1.405	1.514	1.514	0.973	0.730	0.703	1.081	1.405	1.324	1.324	1.351	0.793	1.297	0.123	
10	5	1.000	1.400	0.910	1.500	1.033	1.700	1.600	1.700	1.700	1.100	1.033	0.500	0.633	1.500	1.500	1.533	1.533	0.900	1.567	0.100	
10	6	1.000	1.364	0.636	1.810	1.727	2.545	2.277	2.409	2.409	0.955	0.818	0.545	0.545	2.136	2.000	2.000	2.136	0.773	1.909	0.073	
15	3	1.000	1.269	0.885	1.269	1.231	1.769	1.769	1.769	1.769	0.865	0.750	0.846	0.731	1.769	1.712	1.788	1.269	0.750	1.750	0.173	
15	4	1.000	1.425	0.700	1.450	1.500	1.650	1.575	1.650	1.650	0.800	0.650	0.625	0.775	1.550	1.525	1.525	1.425	0.800	1.475	0.133	
15	5	1.000	1.703	0.586	1.590	1.621	1.966	1.931	1.966	1.966	0.724	0.793	0.621	0.448	1.931	1.931	1.397	1.517	0.793	1.928	0.097	
15	6	1.000	2.091	1.245	2.091	2.409	2.318	1.818	2.273	2.318	1.182	1.273	0.909	0.864	1.773	1.864	2.045	2.182	0.909	2.000	0.073	
20	3	1.000	1.206	0.973	1.208	0.958	1.688	1.688	1.688	1.688	0.979	0.667	0.917	0.938	1.688	1.521	1.542	1.167	0.896	1.521	0.160	
20	4	1.000	1.778	1.000	1.778	1.639	2.272	2.000	2.148	2.148	1.074	0.889	1.037	1.037	2.037	1.889	1.889	1.963	0.926	1.667	0.090	
20	5	1.000	1.593	0.778	1.556	1.444	1.889	1.667	1.889	1.889	0.889	0.889	0.815	0.926	1.667	1.704	1.704	1.593	1.000	1.852	0.090	
20	6	1.000	1.909	0.618	1.773	1.545	2.273	1.909	2.182	2.182	0.818	0.818	0.727	0.727	1.864	1.773	1.773	1.818	0.773	1.955	0.073	

TABLE A7

EMPIRICAL EFFICIENCIES OF TWO-WAY TESTS RELATIVE TO "BEST TESTS" -- EXPONENTIAL
DISTRIBUTION

N	K	F2WAY	PAJFI	FRIED	PHENS	JUNK2	ATMX2	LRCN2	DERG2	GERG2	FYNS2	BDNS2	PSIGM	PTST	HPARA	HGLLT	CKST	SHORT	SMSAT	SEVAT	BASE POWER
TREND																					
5	3	0.373	0.677	0.262	0.677	0.415	0.754	0.754	0.754	0.754	0.323	0.262	0.662	0.646	0.738	0.769	0.738	0.708	0.246	1.000	0.217
5	4	0.429	0.705	0.321	0.889	0.651	0.651	0.767	0.651	0.651	0.683	0.397	0.397	0.492	0.746	0.889	0.762	1.000	0.413	0.921	0.210
5	5	0.372	0.391	0.319	0.345	0.582	0.564	0.764	0.582	0.582	0.491	0.345	0.400	0.345	0.745	0.890	0.764	0.855	0.309	1.000	0.183
5	6	0.365	1.000	0.269	0.981	1.000	0.538	0.654	0.538	0.519	0.462	0.346	0.346	0.327	0.673	0.827	0.712	0.788	0.269	0.923	0.173
10	3	0.433	0.381	0.328	0.881	0.761	0.701	0.701	0.701	0.701	0.403	0.418	0.436	0.582	0.642	0.791	0.731	0.806	0.537	1.000	0.223
10	4	0.323	0.327	0.375	1.000	0.781	0.641	0.594	0.641	0.641	0.484	0.406	0.563	0.594	0.547	0.750	0.703	0.797	0.313	0.953	0.213
10	5	0.292	0.492	0.323	0.938	0.769	0.754	0.662	0.754	0.754	0.385	0.323	0.523	0.508	0.692	0.815	0.754	0.785	0.246	1.000	0.217
10	6	0.490	1.000	0.353	0.902	0.804	0.647	0.549	0.627	0.647	0.490	0.392	0.275	0.275	0.608	0.686	0.706	0.765	0.314	0.941	0.170
15	3	0.306	0.667	0.292	0.667	0.583	0.694	0.694	0.694	0.694	0.292	0.347	0.722	0.556	0.708	0.736	0.722	0.681	0.431	1.000	0.240
15	4	0.397	0.673	0.301	0.873	0.905	0.730	0.762	0.730	0.730	0.444	0.365	0.476	0.635	0.778	0.810	0.825	0.825	0.460	1.000	0.210
15	5	0.373	0.966	0.356	0.915	0.797	0.644	0.695	0.627	0.627	0.407	0.390	0.475	0.271	0.678	0.797	0.780	0.864	0.390	1.000	0.187
15	6	0.309	0.391	0.473	0.959	0.945	0.473	0.564	0.491	0.509	0.527	0.473	0.182	0.164	0.545	0.709	0.691	0.818	0.327	1.000	0.183
20	3	0.324	0.638	0.473	0.838	0.730	0.689	0.689	0.689	0.689	0.473	0.270	0.797	0.676	0.689	0.797	0.757	0.730	0.486	1.000	0.247
20	4	0.330	0.862	0.338	0.908	0.767	0.615	0.677	0.615	0.615	0.431	0.385	0.508	0.385	0.662	0.785	0.862	0.831	0.508	1.000	0.217
20	5	0.354	0.391	0.527	0.927	0.836	0.691	0.782	0.691	0.691	0.673	0.527	0.509	0.455	0.745	0.813	0.818	0.873	0.509	1.000	0.183
20	6	0.411	0.945	0.482	0.929	0.833	0.679	0.750	0.696	0.696	0.500	0.464	0.250	0.232	0.750	0.732	0.804	0.911	0.464	1.000	0.187
GAP																					
5	3	0.273	0.678	0.254	0.678	0.475	0.678	0.678	0.678	0.678	0.339	0.407	0.559	0.610	0.712	0.746	0.661	0.831	0.305	1.000	0.197
5	4	0.435	0.783	0.195	0.783	0.435	0.826	0.848	0.826	0.826	0.543	0.326	0.326	0.391	0.739	0.761	0.696	1.000	0.239	0.870	0.153
5	5	0.600	0.771	0.571	0.971	0.743	0.943	0.943	0.943	0.943	0.860	0.686	0.657	0.543	0.971	0.886	0.771	0.914	0.371	1.000	0.117
5	6	0.459	0.703	0.378	0.784	0.643	0.919	0.892	0.919	1.000	0.838	0.514	0.432	0.351	0.811	0.568	0.676	0.730	0.405	0.892	0.123
10	3	0.536	0.904	0.442	0.904	0.577	0.750	0.750	0.750	0.750	0.596	0.481	0.596	0.500	0.731	0.788	0.731	0.788	0.654	1.000	0.173
10	4	0.410	0.795	0.345	0.872	0.718	0.821	0.872	0.821	0.821	0.410	0.538	0.667	0.538	0.872	0.692	0.718	0.795	0.308	1.000	0.130
10	5	0.535	0.930	0.442	0.953	0.674	0.744	0.791	0.767	0.767	0.674	0.498	0.465	0.442	0.837	0.837	0.860	0.837	0.651	1.000	0.143
10	6	0.529	1.000	0.618	0.912	0.735	0.882	0.882	0.882	0.912	0.618	0.529	0.441	0.441	0.912	0.853	0.971	0.971	0.500	1.000	0.113
15	3	0.444	0.585	0.349	0.685	0.648	0.722	0.722	0.722	0.722	0.352	0.463	0.519	0.426	0.704	0.907	0.815	0.648	0.407	1.000	0.180
15	4	0.479	0.717	0.563	0.717	0.876	0.729	0.708	0.708	0.708	0.667	0.500	0.479	0.854	0.750	0.646	0.525	0.958	0.563	1.000	0.160
15	5	0.545	0.818	0.667	0.879	0.818	0.667	0.697	0.636	0.636	0.667	0.697	0.545	0.424	0.667	0.667	0.806	1.000	0.727	0.818	0.110
15	6	0.471	0.741	0.618	0.941	0.882	0.712	1.000	0.941	0.912	0.676	0.706	0.324	0.294	1.000	0.794	0.794	0.824	0.559	1.000	0.113
20	3	0.581	0.871	0.730	0.871	0.726	0.742	0.742	0.742	0.742	0.790	0.500	0.484	0.371	0.742	0.742	0.758	0.855	0.694	1.000	0.207
20	4	0.357	0.727	0.418	0.982	0.800	0.709	0.745	0.709	0.709	0.455	0.564	0.527	0.545	0.745	0.836	0.836	0.855	0.382	1.000	0.183
20	5	0.558	0.707	0.442	0.907	0.721	0.791	0.767	0.814	0.814	0.488	0.419	0.419	0.395	0.791	0.837	0.837	0.930	0.535	1.000	0.143
20	6	0.357	0.752	0.500	1.000	0.833	0.762	0.738	0.738	0.762	0.524	0.548	0.405	0.381	0.714	0.738	0.762	0.810	0.548	0.976	0.140

TABLE A8

EMPIRICAL EFFICIENCIES OF TWO-WAY TESTS RELATIVE TO "BEST TESTS" -- NORMAL
DISTRIBUTION

N	K	F2WY	PAJET	FRIED	PHENS	JONKZ	ATMX2	LRCN2	DERG2	QERG2	FYNS2	HONS2	PSIGN	PTKST	HPARA	HOLIT	UCKST	SHORT	SMSAT	SEVAT	BASE	POWER
TREND																						
5	3	0.551	0.724	0.276	0.724	0.561	1.000	1.000	1.000	1.000	0.367	0.378	0.612	0.663	0.939	0.990	0.990	0.786	0.367	0.969	0.327	
5	4	0.500	0.387	0.355	0.974	0.645	0.961	1.000	0.961	0.961	0.592	0.382	0.421	0.474	0.855	0.934	0.855	0.789	0.289	0.908	0.253	
5	5	0.256	0.985	0.105	0.871	0.586	0.871	0.929	0.886	0.986	0.214	0.229	0.400	0.371	0.986	0.886	1.000	0.671	0.136	0.871	0.233	
5	6	0.262	0.315	0.154	0.970	0.800	0.923	1.000	0.969	0.954	0.354	0.231	0.323	0.368	1.000	0.923	0.908	0.769	0.169	0.892	0.217	
10	3	0.461	0.802	0.330	0.802	0.661	1.000	1.000	1.000	1.000	0.443	0.349	0.509	0.557	0.953	0.943	0.972	0.726	0.453	0.915	0.353	
10	4	0.444	0.746	0.254	0.836	0.701	0.955	0.940	0.955	0.955	0.328	0.269	0.448	0.358	0.955	0.940	0.985	0.687	0.328	1.000	0.223	
10	5	0.393	0.726	0.226	0.786	0.667	0.917	1.000	0.929	0.929	0.298	0.190	0.321	0.321	1.000	0.881	0.929	0.726	0.286	0.988	0.280	
10	6	0.470	0.955	0.470	0.770	0.848	0.939	1.000	0.955	0.939	0.561	0.318	0.364	0.348	0.985	0.955	0.924	0.909	0.455	0.955	0.220	
15	3	0.475	0.804	0.526	0.874	0.794	0.938	0.938	0.938	0.938	0.515	0.443	0.598	0.515	0.959	1.000	0.938	0.804	0.443	0.979	0.323	
15	4	0.544	0.772	0.334	0.785	0.772	0.987	0.949	1.000	1.000	0.392	0.430	0.392	0.582	0.937	0.893	0.924	0.772	0.468	0.949	0.263	
15	5	0.415	0.805	0.378	0.756	0.720	0.841	0.976	0.854	0.854	0.402	0.341	0.463	0.329	1.000	0.902	0.902	0.768	0.317	0.915	0.273	
15	6	0.323	0.955	0.339	0.939	0.823	0.935	0.984	0.952	0.968	0.387	0.274	0.210	0.194	1.000	0.952	0.968	0.839	0.258	0.968	0.207	
20	3	0.427	0.775	0.477	0.776	0.625	0.981	0.981	0.981	0.981	0.477	0.336	0.402	0.477	0.944	0.935	0.963	0.720	0.523	1.000	0.357	
20	4	0.500	0.829	0.355	0.854	0.707	1.000	0.963	0.988	0.988	0.378	0.378	0.354	0.354	0.963	0.890	0.902	0.707	0.451	0.902	0.273	
20	5	0.438	0.770	0.337	0.875	0.813	0.938	1.000	0.925	0.925	0.375	0.313	0.287	0.262	1.000	0.912	0.912	0.737	0.460	0.887	0.267	
20	6	0.418	0.725	0.313	0.940	0.716	0.896	1.000	0.985	0.940	0.343	0.284	0.194	0.179	1.000	0.940	0.955	0.672	0.398	1.000	0.223	
GAP																						
5	3	0.554	0.672	0.354	0.672	0.504	0.954	0.954	0.954	0.954	0.446	0.508	0.662	0.769	0.723	0.954	0.692	0.723	0.492	1.000	0.217	
5	4	0.450	0.773	0.286	0.762	0.460	1.000	0.937	1.000	1.000	0.540	0.444	0.429	0.556	0.869	0.857	0.905	0.794	0.270	0.905	0.210	
5	5	0.579	0.389	0.444	0.733	0.733	0.956	1.000	0.956	0.956	0.689	0.489	0.578	0.556	0.933	0.933	0.911	1.000	0.400	0.956	0.150	
5	6	0.717	0.574	0.500	0.717	0.607	1.000	0.848	1.000	1.000	0.652	0.391	0.500	0.478	0.739	0.696	0.609	0.652	0.630	0.596	0.153	
10	3	0.541	0.882	0.313	0.682	0.482	0.988	0.988	0.988	0.988	0.400	0.353	0.412	0.471	0.965	0.953	1.000	0.565	0.435	0.929	0.283	
10	4	0.661	0.732	0.515	0.768	0.580	0.982	0.929	1.000	1.000	0.643	0.482	0.464	0.714	0.929	0.875	0.875	0.893	0.464	0.957	0.187	
10	5	0.560	0.824	0.570	0.882	0.608	1.000	0.941	1.000	1.000	0.647	0.678	0.294	0.373	0.882	0.882	0.902	0.902	0.529	0.922	0.170	
10	6	0.373	0.737	0.257	0.714	0.677	1.000	0.875	0.946	0.946	0.375	0.321	0.214	0.214	0.839	0.786	0.786	0.839	0.304	0.750	0.187	
15	3	0.520	0.710	0.475	0.710	0.688	0.989	0.989	0.989	0.989	0.494	0.419	0.473	0.409	0.989	0.957	1.000	0.710	0.419	0.978	0.310	
15	4	0.605	0.864	0.424	0.879	0.900	1.000	0.955	1.000	1.000	0.485	0.394	0.379	0.470	0.939	0.924	0.924	0.864	0.485	0.894	0.220	
15	5	0.509	0.712	0.278	0.860	0.825	1.000	0.982	1.000	1.000	0.368	0.404	0.316	0.228	0.982	0.982	0.965	0.772	0.404	0.930	0.190	
15	6	0.415	0.368	0.434	0.869	1.000	0.962	0.755	0.943	0.962	0.491	0.528	0.377	0.358	0.736	0.774	0.849	0.906	0.377	0.830	0.177	
20	3	0.573	0.715	0.500	0.715	0.568	1.000	1.000	1.000	1.000	0.580	0.375	0.543	0.556	1.000	0.901	0.914	0.691	0.531	0.901	0.270	
20	4	0.450	0.770	0.450	0.800	0.733	1.000	0.900	0.967	0.967	0.483	0.400	0.467	0.467	0.917	0.850	0.850	0.883	0.417	0.750	0.200	
20	5	0.527	0.843	0.412	0.824	0.765	1.000	0.882	1.000	1.000	0.471	0.471	0.431	0.490	0.882	0.902	0.902	0.843	0.529	0.980	0.170	
20	6	0.440	0.840	0.360	0.780	0.680	1.000	0.840	0.960	0.960	0.360	0.360	0.320	0.320	0.820	0.780	0.780	0.800	0.340	0.860	0.167	

TABLE A9

EMPIRICAL EFFICIENCIES OF ONE-WAY TESTS RELATIVE TO F TEST -- $k = 3(1)6$

DIST	ALT	k	FTEST	HTEST	COXK	JUNKF	HATTH	CHAKO	ATHAX	LPCHN	DEREG	DEREG	JMENS	PURIS	FYENS	BORNS	COXS1	COXS2	COXS3	COXSL	MOSSL	BASE POWER
E	T	3	1.701	1.534	1.545	1.071	1.743	2.773	2.250	2.250	2.250	2.250	3.534	3.534	1.750	1.523	2.420	1.530	2.128	0.732	0.602	0.073
		4	1.700	1.229	1.731	2.503	1.833	2.417	1.750	1.833	1.771	1.771	3.083	3.104	1.573	1.365	2.115	1.740	1.917	0.938	0.740	0.080
		5	1.700	1.047	1.749	2.854	1.727	2.598	1.854	2.085	1.927	1.927	3.341	3.402	1.341	1.366	2.354	1.480	1.939	0.988	0.817	0.068
		6	1.700	0.988	0.723	2.470	1.554	2.084	1.542	1.627	1.542	1.542	2.976	3.084	1.181	1.145	1.892	1.723	1.759	0.916	0.819	0.049
N	T	3	1.700	0.754	0.370	1.753	1.767	1.750	1.824	1.824	1.824	1.824	1.806	1.831	1.000	0.912	1.324	0.903	1.468	0.560	0.426	0.180
		4	1.700	0.734	0.573	1.753	1.722	1.716	1.815	1.860	1.833	1.833	1.864	1.833	0.981	0.901	1.253	1.247	1.142	0.546	0.474	0.135
		5	1.700	0.874	0.906	2.521	2.210	2.110	2.412	2.555	2.412	2.412	2.462	2.503	0.916	0.975	1.706	1.210	1.504	0.681	0.513	0.059
		6	1.700	0.718	0.663	2.449	2.551	2.449	2.510	2.633	2.531	2.510	2.633	2.714	1.051	0.929	1.970	1.582	1.663	0.634	0.633	0.082
E	G	3	1.700	1.219	1.377	1.730	1.421	1.702	1.430	1.430	1.430	1.430	1.982	1.982	1.351	1.228	1.632	1.335	1.614	0.754	0.561	0.085
		4	1.700	0.775	1.784	2.140	1.938	2.272	1.790	1.753	1.790	1.790	2.494	2.519	1.321	1.284	1.864	1.309	1.580	0.815	0.716	0.068
		5	1.700	0.977	0.743	1.678	1.414	1.736	1.474	1.425	1.460	1.460	2.057	1.943	1.161	1.161	1.586	0.931	1.172	0.862	0.736	0.073
		6	1.700	1.239	1.769	2.070	2.134	2.070	1.866	1.925	1.866	1.896	2.627	2.448	1.343	1.224	1.657	1.284	1.582	1.030	1.194	0.056
N	G	3	1.700	0.849	0.681	1.459	1.638	1.616	1.762	1.762	1.762	1.762	1.703	1.703	0.962	0.886	1.357	0.843	1.405	0.773	0.605	0.154
		4	1.700	0.813	0.964	1.458	1.813	1.662	1.777	1.626	1.770	1.770	1.727	1.626	0.964	0.892	1.252	0.849	1.029	0.727	0.683	0.116
		5	1.700	0.829	0.889	1.667	1.786	1.821	1.718	1.658	1.692	1.692	1.803	1.632	0.957	0.863	1.385	0.735	1.111	0.744	0.556	0.098
		6	1.700	0.875	1.021	1.726	2.126	1.947	2.126	1.747	2.084	2.126	2.147	1.779	1.053	1.095	1.453	1.126	1.284	0.947	0.832	0.079

TABLE A10

EMPIRICAL EFFICIENCIES OF ONE-WAY TESTS RELATIVE TO F TEST -- $n = 5(5)20$

DIST	ALT	n	FTEST	WTEST	COXKS	JONKE	HARTN	CHAKO	ATMAX	LRCOM	DLREG	QEREG	JMENS	PURIS	FYENS	HDRNS	COXS1	COXS2	COXS3	CONSL	MOSSL	BASE	POWER
E	T	5	1.000	1.036	1.117	2.571	1.937	2.359	1.869	2.060	1.881	1.893	2.857	2.976	1.310	1.048	2.202	1.798	2.060	1.119	0.833	0.070	
		10	1.000	1.135	1.023	2.775	1.727	2.375	1.875	1.795	1.890	1.898	3.057	3.011	1.364	1.386	2.216	1.523	2.114	0.898	0.591	0.073	
		15	1.000	1.237	1.045	2.784	1.841	2.489	1.739	1.864	1.773	1.773	3.284	3.364	1.432	1.364	2.148	1.591	1.795	0.875	0.773	0.073	
		20	1.000	1.404	1.180	2.721	1.843	2.640	1.921	2.079	1.944	1.933	3.708	3.742	1.764	1.596	2.213	1.562	2.056	0.697	0.775	0.074	
N	T	5	1.000	0.819	0.768	2.040	2.217	2.094	2.203	2.312	2.217	2.217	2.263	2.246	1.080	0.833	1.558	1.507	1.551	0.630	0.523	0.115	
		10	1.000	0.905	0.827	2.771	2.079	2.058	2.216	2.259	2.209	2.209	2.194	2.201	0.786	1.007	1.640	1.194	1.669	0.669	0.532	0.116	
		15	1.000	0.874	0.784	1.737	1.701	1.834	1.999	2.020	1.987	1.987	2.000	2.066	0.921	0.894	1.424	0.947	1.152	0.636	0.464	0.126	
		20	1.000	0.934	0.707	1.454	1.754	1.784	1.856	1.492	1.874	1.862	1.934	1.970	0.970	0.753	1.371	1.072	1.341	0.533	0.384	0.139	
E	G	5	1.000	0.992	1.193	1.400	1.747	1.872	1.771	1.711	1.759	1.759	2.108	2.012	1.325	1.036	1.687	1.325	1.506	0.928	0.904	0.069	
		10	1.000	1.135	1.180	1.843	1.663	2.056	1.517	1.539	1.517	1.551	2.067	2.056	1.270	1.270	1.562	1.034	1.539	0.809	0.472	0.074	
		15	1.000	1.183	1.240	1.976	1.732	2.073	1.573	1.561	1.537	1.547	2.293	2.244	1.256	1.402	1.720	1.024	1.256	0.763	0.402	0.068	
		20	1.000	1.179	1.253	2.084	1.577	2.011	1.600	1.589	1.611	1.589	2.484	2.411	1.326	1.189	1.747	1.105	1.632	0.716	0.789	0.079	
N	G	5	1.000	0.773	0.902	1.462	1.712	1.667	1.621	1.530	1.614	1.614	1.614	1.523	1.015	0.894	1.500	0.970	1.189	0.879	0.758	0.110	
		10	1.000	0.779	0.743	1.627	1.797	1.776	1.859	1.746	1.851	1.866	1.896	1.701	0.918	0.925	1.313	0.873	1.351	0.636	0.739	0.116	
		15	1.000	0.913	0.926	1.651	1.772	1.725	1.819	1.718	1.799	1.805	1.779	1.711	0.973	0.886	1.248	0.705	1.081	0.745	0.450	0.124	
		20	1.000	0.884	1.064	1.767	1.742	1.760	2.000	1.818	1.975	1.983	1.967	1.793	1.008	0.983	1.364	0.967	1.289	0.678	0.702	0.101	

TABLE A11

EMPIRICAL EFFICIENCIES OF ONE-WAY TESTS RELATIVE TO "BEST TESTS" -- $k = 3(1)6$

DIST	ALT	K	FTEST	HTEST	CXKS	JTKE	WTH	CHAKN	ATMAX	LRCON	DEREG	DEREG	JMENS	PURIS	FYENS	BORNS	CXK51	CXK52	CXK53	CONSL	MOSSL	BASE	POWER
E	T	3	0.293	0.434	0.437	0.875	0.550	0.785	0.637	0.637	0.637	0.637	1.000	1.000	0.495	0.431	0.685	0.424	0.678	0.209	0.170	0.259	
		4	0.322	0.375	0.337	0.832	0.591	0.777	0.564	0.591	0.570	0.570	0.993	1.000	0.507	0.440	0.681	0.560	0.617	0.302	0.238	0.248	
		5	0.274	0.308	0.307	0.837	0.566	0.763	0.545	0.613	0.566	0.566	0.982	1.000	0.394	0.401	0.692	0.437	0.570	0.290	0.240	0.233	
		6	0.324	0.320	0.234	0.801	0.504	0.676	0.500	0.527	0.500	0.500	0.965	1.000	0.383	0.371	0.613	0.559	0.570	0.297	0.266	0.213	
N	T	3	0.548	0.523	0.477	0.962	0.970	0.964	1.000	1.000	1.000	1.000	0.990	0.987	0.548	0.500	0.726	0.495	0.805	0.337	0.234	0.328	
		4	0.536	0.421	0.321	0.940	0.924	0.921	0.974	0.947	0.983	0.983	1.000	0.983	0.526	0.483	0.672	0.664	0.613	0.315	0.255	0.252	
		5	0.300	0.341	0.354	0.784	0.867	0.826	0.941	0.997	0.941	0.941	0.961	1.000	0.357	0.380	0.666	0.472	0.587	0.266	0.200	0.254	
		6	0.366	0.339	0.244	0.877	0.940	0.902	0.925	0.970	0.932	0.925	0.970	1.000	0.387	0.342	0.733	0.583	0.613	0.256	0.233	0.222	
E	G	3	0.504	0.615	0.675	0.973	0.717	1.000	0.721	0.721	0.721	0.721	1.000	1.000	0.681	0.619	0.823	0.522	0.814	0.381	0.283	0.183	
		4	0.327	0.347	0.510	0.853	0.770	0.702	0.711	0.696	0.711	0.711	0.990	1.000	0.525	0.510	0.740	0.520	0.627	0.324	0.284	0.170	
		5	0.486	0.475	0.454	0.816	0.687	0.844	0.726	0.693	0.709	0.709	1.000	0.944	0.564	0.564	0.771	0.453	0.570	0.419	0.358	0.149	
		6	0.381	0.460	0.463	0.775	0.813	0.745	0.710	0.733	0.710	0.722	1.000	0.932	0.511	0.466	0.631	0.469	0.602	0.392	0.455	0.147	
N	G	3	0.547	0.432	0.501	0.942	0.929	0.917	1.000	1.000	1.000	1.000	0.966	0.966	0.546	0.503	0.770	0.479	0.798	0.439	0.344	0.272	
		4	0.552	0.446	0.532	0.910	1.000	0.917	0.980	0.897	0.976	0.976	0.952	0.897	0.532	0.492	0.690	0.468	0.567	0.401	0.377	0.210	
		5	0.549	0.455	0.488	0.915	0.781	1.000	0.944	0.911	0.930	0.930	0.991	0.897	0.526	0.474	0.761	0.404	0.610	0.408	0.305	0.178	
		6	0.465	0.417	0.475	0.804	0.990	0.907	0.990	0.814	0.971	0.990	1.000	0.828	0.490	0.510	0.676	0.525	0.598	0.441	0.387	0.170	

TABLE A12

EMPIRICAL EFFICIENCIES OF ONE-WAY TESTS RELATIVE TO "BEST TESTS" -- n = 5(5)20

DIST	ALT	n	FTEST	HTEST	COXK5	JONKE	WARTH	CHIKO	ATMAX	LRCON	DEREG	DEREG	JMENS	PURIS	FYENS	BDRIS	COXS1	COXS2	COXS3	CUNSL	MOSSL	BASE POWER
E	T	5	0.336	0.348	0.376	0.854	0.624	0.776	0.628	0.692	0.632	0.636	0.960	1.000	0.440	0.352	0.740	0.634	0.692	0.376	0.280	0.208
		10	0.327	0.372	0.335	0.895	0.565	0.777	0.613	0.587	0.621	0.621	1.000	0.985	0.446	0.454	0.725	0.498	0.691	0.294	0.193	0.224
		15	0.297	0.358	0.311	0.829	0.547	0.740	0.517	0.554	0.527	0.527	0.976	1.000	0.426	0.405	0.639	0.473	0.534	0.260	0.230	0.247
		20	0.267	0.375	0.315	0.781	0.492	0.706	0.514	0.556	0.520	0.517	0.991	1.000	0.471	0.426	0.592	0.417	0.550	0.186	0.207	0.278
N	T	5	0.433	0.354	0.332	0.937	0.756	0.906	0.953	1.000	0.959	0.759	0.981	0.972	0.467	0.361	0.674	0.622	0.671	0.273	0.270	0.266
		10	0.443	0.401	0.365	0.975	0.920	0.711	0.951	1.000	0.978	0.978	0.971	0.975	0.436	0.446	0.726	0.529	0.739	0.296	0.236	0.262
		15	0.494	0.423	0.301	0.762	0.920	0.848	0.958	0.978	0.962	0.962	0.968	1.000	0.446	0.433	0.689	0.458	0.558	0.308	0.224	0.260
		20	0.506	0.474	0.359	0.362	0.891	0.936	0.942	0.960	0.951	0.945	0.982	1.000	0.492	0.486	0.696	0.544	0.681	0.271	0.198	0.274
E	G	5	0.474	0.423	0.566	0.871	0.829	0.877	0.840	0.811	0.834	0.834	1.000	0.954	0.629	0.491	0.800	0.627	0.714	0.440	0.429	0.146
		10	0.514	0.347	0.571	0.891	0.806	0.975	0.734	0.745	0.734	0.750	1.000	0.995	0.614	0.614	0.755	0.500	0.745	0.391	0.228	0.153
		15	0.434	0.516	0.537	0.862	0.755	0.774	0.686	0.681	0.672	0.676	1.000	0.979	0.548	0.612	0.750	0.447	0.548	0.420	0.394	0.157
		20	0.403	0.475	0.504	0.837	0.636	0.809	0.644	0.640	0.648	0.640	1.000	0.970	0.534	0.479	0.703	0.445	0.657	0.288	0.318	0.197
N	G	5	0.504	0.451	0.527	0.854	1.000	0.973	0.947	0.894	0.942	0.942	0.942	0.889	0.593	0.522	0.876	0.566	0.695	0.513	0.442	0.188
		10	0.528	0.421	0.507	0.858	0.949	0.937	0.980	0.921	0.976	0.984	1.000	0.898	0.484	0.488	0.693	0.461	0.713	0.441	0.390	0.212
		15	0.550	0.502	0.454	0.978	0.974	0.948	1.000	0.945	0.989	0.993	0.978	0.941	0.535	0.487	0.686	0.367	0.594	0.410	0.247	0.226
		20	0.500	0.442	0.533	0.880	0.971	0.880	1.000	0.909	0.988	0.992	0.983	0.897	0.504	0.492	0.682	0.433	0.645	0.339	0.351	0.202

TABLE A13

EMPIRICAL EFFICIENCIES OF TWO-WAY TESTS RELATIVE TO F TEST -- $k = 3(1)6$

DIST	ALT	K	F2WAY	PAGET	FRIED	PHEN5	IGNK2	ATM42	LRC42	DERG2	QERG2	FYNS2	BDNS2	PSIGN	PTST	HPAKA	HOLLT	DUKST	SHURT	SMSAT	SEMAT	BASE POWER
E	T	3	1.700	2.242	1.000	2.242	1.832	2.074	2.074	2.074	2.074	1.095	0.947	2.211	1.800	2.032	2.263	2.158	2.137	1.253	2.926	0.079
		4	1.700	2.441	1.011	2.516	2.129	1.806	1.914	1.806	1.806	1.398	1.065	1.333	1.441	1.871	2.215	2.161	2.366	1.161	2.656	0.078
		5	1.700	2.538	1.073	2.553	2.134	1.902	2.061	1.902	1.902	1.378	1.122	1.366	1.134	2.037	2.305	2.229	2.402	1.024	2.854	0.068
		6	1.000	2.447	1.012	2.367	2.286	1.488	1.607	1.500	1.512	1.262	1.071	0.667	0.631	1.643	1.881	1.857	2.095	0.881	2.464	0.070
N	T	3	1.700	1.454	0.752	1.454	1.234	1.835	1.835	1.835	1.835	0.844	0.702	0.986	1.032	1.775	1.807	1.807	1.417	0.839	1.807	0.182
		4	1.700	1.671	0.686	1.724	1.414	1.954	1.928	1.954	1.954	0.849	0.737	0.803	0.888	1.855	1.829	1.849	1.480	0.776	1.875	0.127
		5	1.700	2.137	0.738	2.123	1.811	2.311	2.533	2.328	2.328	0.844	0.697	0.451	0.828	2.582	2.320	2.418	1.885	0.779	2.377	0.102
		6	1.700	2.406	0.865	2.476	2.156	2.530	2.638	2.615	2.573	1.115	0.750	0.740	0.698	2.698	2.552	2.542	2.156	0.865	2.583	0.080
E	G	3	1.700	1.618	0.721	1.618	1.255	1.491	1.491	1.491	1.491	1.082	0.955	1.109	0.982	1.491	1.634	1.527	1.618	1.064	2.064	0.092
		4	1.700	2.025	0.926	2.170	1.644	1.800	1.850	1.787	1.787	1.225	1.137	1.162	1.375	1.813	1.737	1.700	2.125	0.847	2.275	0.067
		5	1.700	1.947	0.930	1.653	1.314	1.407	1.430	1.419	1.419	1.163	1.000	0.919	0.802	1.465	1.453	1.393	1.640	1.023	1.721	0.072
		6	1.700	2.000	1.167	2.030	1.727	1.924	1.939	1.924	1.985	1.470	1.273	0.894	0.818	1.874	1.636	1.773	1.848	1.121	2.152	0.055
N	G	3	1.700	1.247	0.786	1.247	1.011	1.753	1.753	1.753	1.753	0.852	0.736	0.912	0.951	1.731	1.676	1.703	1.192	0.830	1.692	0.152
		4	1.700	1.456	0.774	1.481	1.248	1.835	1.714	1.827	1.827	0.985	0.789	0.797	1.008	1.692	1.617	1.639	1.579	0.752	1.571	0.111
		5	1.700	1.580	0.757	1.589	1.339	1.804	1.732	1.804	1.804	0.973	0.893	0.723	0.732	1.679	1.688	1.679	1.589	0.848	1.723	0.093
		6	1.700	1.616	0.788	1.596	1.545	2.051	1.717	1.990	2.000	0.960	0.828	0.717	0.697	1.626	1.576	1.576	1.667	0.838	1.626	0.083

TABLE A14

EMPIRICAL EFFICIENCIES OF TWO-WAY TESTS RELATIVE TO F TEST -- $n = 5(5)20$

DIST	ALT	n	F2WAY	PAGE2	FRID	PHIPS	JUNK2	ATMX2	LRCM2	DERG2	GERG2	FYNS2	BDNS2	PSIGN	PRST	HPARA	HULLT	DUKST	SHURT	SMSAT	SEKAT	BASE POWER
E	T	5	1.777	2.322	2.828	2.333	1.747	1.713	1.949	1.724	1.713	1.322	0.908	1.241	1.253	1.966	2.218	2.011	2.264	0.839	2.598	0.073
		10	1.777	2.415	2.904	2.447	2.043	1.809	1.667	1.798	1.809	1.149	1.011	1.489	1.319	1.638	2.011	1.934	2.074	0.936	2.564	0.078
		15	1.777	2.470	1.070	2.477	2.307	1.860	1.977	1.860	1.872	1.186	1.128	1.395	1.221	1.977	2.209	2.186	2.291	1.174	2.895	0.072
		20	1.777	2.529	1.299	2.575	2.264	1.720	2.069	1.931	1.931	1.471	1.149	1.540	1.299	2.034	2.253	2.322	2.379	1.414	2.874	0.073
N	T	5	1.777	2.023	0.597	2.700	1.527	2.264	2.357	2.295	2.287	0.922	0.752	1.093	1.140	2.256	2.248	2.279	1.814	0.636	2.194	0.108
		10	1.777	1.736	1.717	1.867	1.579	2.131	2.200	2.145	2.138	0.903	0.634	0.931	0.917	2.166	2.069	2.124	1.683	0.855	2.138	0.121
		15	1.777	1.779	0.923	1.752	1.710	2.041	2.117	2.062	2.069	0.752	0.841	0.966	0.931	2.145	2.076	2.035	1.752	0.641	2.103	0.121
		20	1.777	1.686	1.763	1.592	1.408	1.955	1.959	1.929	1.911	0.799	0.657	0.639	0.669	1.735	1.828	1.958	1.414	0.893	1.888	0.141
E	G	5	1.777	1.612	0.725	1.737	1.225	1.813	1.813	1.913	1.850	1.300	1.025	1.087	1.075	1.750	1.637	1.537	1.925	0.712	2.037	0.067
		10	1.777	1.788	0.929	1.800	1.318	1.565	1.612	1.576	1.588	1.141	1.003	1.082	0.953	1.635	1.565	1.600	1.659	1.071	1.976	0.071
		15	1.777	1.728	1.123	1.753	1.667	1.568	1.605	1.556	1.543	1.185	1.185	0.988	1.086	1.605	1.593	1.494	1.753	1.136	2.012	0.068
		20	1.777	1.917	1.167	1.869	1.615	1.573	1.573	1.573	1.583	1.219	1.073	0.979	0.896	1.573	1.656	1.677	1.813	1.146	2.074	0.080
N	G	5	1.777	1.331	0.677	1.355	0.992	1.726	1.653	1.726	1.726	1.000	0.815	0.960	1.065	1.548	1.532	1.484	1.387	0.774	1.589	0.103
		10	1.777	1.368	0.726	1.378	1.059	1.822	1.726	1.807	1.807	0.919	0.785	0.652	0.822	1.674	1.622	1.659	1.415	0.793	1.600	0.113
		15	1.777	1.545	0.797	1.531	1.566	1.860	1.755	1.853	1.860	0.867	0.811	0.748	0.706	1.741	1.727	1.776	1.503	0.797	1.727	0.118
		20	1.777	1.540	0.911	1.504	1.315	1.952	1.790	1.919	1.919	0.952	0.790	0.887	0.919	1.790	1.685	1.694	1.548	0.903	1.702	0.103

TABLE A15

EMPIRICAL EFFICIENCIES OF TWO-WAY TESTS RELATIVE TO "BEST TESTS" -- $k = 3(1)6$

DIST	ALT	K	F244V	PAGE2	FRIED	PIPER	JUNK2	ATK2	LRCN2	DERG2	CLRG2	FYNS2	BUN22	PSIGN	PIRST	HPARA	MULT	DOKST	SHORT	SMSAT	SENAT	BASE POWER
E	T	3	0.342	0.766	0.342	0.766	0.426	0.709	0.709	0.709	0.709	0.374	0.324	0.755	0.615	0.694	0.773	0.737	0.730	0.428	1.000	0.232
		4	0.377	0.919	0.381	0.947	0.807	0.690	0.721	0.680	0.680	0.526	0.401	0.522	0.543	0.704	0.834	0.814	0.891	0.437	1.000	0.206
		5	0.350	0.919	0.376	0.932	0.748	0.667	0.722	0.667	0.667	0.483	0.393	0.479	0.397	0.714	0.808	0.778	0.842	0.359	1.000	0.195
		6	0.406	0.943	0.411	0.961	0.928	0.604	0.652	0.609	0.614	0.512	0.435	0.271	0.256	0.667	0.763	0.754	0.850	0.357	1.000	0.173
H	T	3	0.545	0.792	0.413	0.792	0.672	1.000	1.000	1.000	1.000	0.460	0.382	0.537	0.563	0.767	0.985	0.985	0.772	0.457	0.985	0.333
		4	0.512	0.859	0.350	0.882	0.724	1.000	0.987	1.000	1.000	0.434	0.377	0.411	0.455	0.949	0.736	0.946	0.758	0.397	0.960	0.248
		5	0.347	0.827	0.284	0.822	0.707	0.895	0.981	0.902	0.902	0.327	0.270	0.368	0.321	1.000	0.898	0.937	0.730	0.302	0.921	0.263
		6	0.371	0.832	0.320	0.872	0.797	0.927	1.000	0.969	0.954	0.413	0.279	0.274	0.259	1.000	0.746	0.742	0.799	0.320	0.958	0.216
E	G	3	0.445	0.744	0.480	0.784	0.608	0.722	0.722	0.722	0.722	0.524	0.463	0.537	0.476	0.722	0.793	0.743	0.784	0.515	1.000	0.189
		4	0.440	0.893	0.407	0.973	0.742	0.791	0.813	0.786	0.786	0.538	0.500	0.511	0.604	0.797	0.764	0.747	0.934	0.390	1.000	0.152
		5	0.581	0.399	0.541	0.765	0.764	0.818	0.831	0.824	0.824	0.676	0.581	0.534	0.466	0.851	0.845	0.811	0.953	0.595	1.000	0.123
		6	0.465	0.930	0.542	0.744	0.903	0.994	0.901	0.894	0.923	0.683	0.592	0.415	0.380	0.880	0.761	0.824	0.859	0.521	1.000	0.118
N	G	3	0.571	0.712	0.448	0.712	0.577	1.000	1.000	1.000	1.000	0.486	0.420	0.520	0.542	0.987	0.956	0.972	0.660	0.473	0.966	0.266
		4	0.545	0.799	0.422	0.817	0.680	1.000	0.934	0.996	0.996	0.537	0.430	0.434	0.549	0.922	0.881	0.893	0.861	0.410	0.857	0.203
		5	0.554	0.876	0.421	0.881	0.743	1.000	0.960	1.000	1.000	0.540	0.495	0.401	0.406	0.931	0.936	0.931	0.881	0.470	0.955	0.168
		6	0.433	0.748	0.384	0.778	0.754	1.000	0.837	0.970	0.975	0.468	0.404	0.350	0.340	0.793	0.768	0.768	0.813	0.409	0.793	0.169

TABLE A16

EMPIRICAL EFFICIENCIES OF TWO-WAY TESTS RELATIVE TO "BEST TESTS" -- n = 5(5)20

DIST	ALT	n	F2WAY	PAGET	FRIED	PHENS	JCNK2	ATMX2	LPCW2	DEK2	QEK2	FYNS2	HONS2	PSIGN	PIRST	HPARA	HOLLT	DOKST	SHORT	SMSAT	SCMAT	BASE	POWER
E	T	5	0.345	0.894	0.312	0.898	0.673	0.659	0.765	0.664	0.659	0.509	0.357	0.478	0.482	0.757	0.854	0.774	0.872	0.323	1.000	0.188	
		10	0.390	0.942	0.353	0.954	0.797	0.705	0.647	0.701	0.705	0.448	0.394	0.581	0.515	0.639	0.784	0.743	0.809	0.365	1.000	0.201	
		15	0.345	0.839	0.369	0.831	0.735	0.643	0.683	0.643	0.647	0.410	0.397	0.482	0.422	0.683	0.763	0.755	0.791	0.406	1.000	0.208	
		20	0.348	0.890	0.452	0.876	0.788	0.668	0.720	0.672	0.672	0.512	0.400	0.536	0.452	0.708	0.784	0.808	0.828	0.492	1.000	0.208	
N	T	5	0.424	0.859	0.253	0.849	0.644	0.761	1.000	0.974	0.970	0.391	0.319	0.464	0.484	0.957	0.954	0.967	0.770	0.270	0.931	0.253	
		10	0.455	0.912	0.326	0.850	0.718	0.967	1.000	0.975	0.972	0.411	0.288	0.423	0.417	0.984	0.940	0.966	0.765	0.389	0.972	0.266	
		15	0.466	0.830	0.421	0.817	0.737	0.952	0.987	0.761	0.965	0.444	0.392	0.450	0.434	1.000	0.968	0.958	0.817	0.392	0.981	0.259	
		20	0.511	0.861	0.390	0.864	0.719	0.973	1.000	0.985	0.976	0.408	0.335	0.326	0.341	0.988	0.734	0.949	0.722	0.456	0.964	0.276	
E	G	5	0.479	0.772	0.347	0.832	0.587	0.868	0.868	0.868	0.884	0.623	0.491	0.521	0.515	0.838	0.784	0.737	0.922	0.341	1.000	0.139	
		10	0.506	0.905	0.470	0.911	0.667	0.792	0.815	0.798	0.804	0.577	0.506	0.548	0.482	0.827	0.792	0.810	0.839	0.542	1.000	0.140	
		15	0.497	0.859	0.554	0.871	0.828	0.779	0.798	0.773	0.767	0.589	0.589	0.491	0.540	0.798	0.791	0.742	0.871	0.564	1.000	0.136	
		20	0.478	0.915	0.557	0.940	0.771	0.751	0.751	0.751	0.756	0.582	0.512	0.468	0.428	0.751	0.791	0.801	0.866	0.547	1.000	0.168	
N	G	5	0.579	0.771	0.393	0.785	0.575	1.000	0.958	1.000	1.000	0.579	0.472	0.556	0.617	0.897	0.888	0.860	0.804	0.449	0.921	0.178	
		10	0.549	0.740	0.398	0.756	0.581	1.000	0.947	0.992	0.992	0.504	0.431	0.358	0.451	0.919	0.890	0.911	0.776	0.435	0.878	0.205	
		15	0.530	0.831	0.427	0.873	0.847	1.000	0.944	0.996	1.000	0.466	0.436	0.402	0.380	0.936	0.929	0.955	0.808	0.429	0.929	0.222	
		20	0.512	0.789	0.467	0.773	0.674	1.000	0.917	0.983	0.983	0.488	0.405	0.455	0.471	0.917	0.864	0.868	0.793	0.463	0.872	0.202	

TABLE A17

EMPIRICAL POWERS OF NINE ONE-WAY TESTS UNDER RECTANGULAR DISTRIBUTION: $n = 5$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
FTEST	T	0.05	0.056	0.092	0.265	0.497	0.060	0.110	0.252	0.582	0.036	0.120	0.327	0.622	0.059	0.107	0.347	0.688
		0.01	0.011	0.027	0.085	0.222	0.016	0.027	0.095	0.272	0.007	0.025	0.107	0.350	0.011	0.017	0.130	0.425
HTEST	T	0.05	0.049	0.075	0.227	0.435	0.046	0.080	0.195	0.440	0.026	0.077	0.250	0.475	0.030	0.070	0.252	0.547
		0.01	0.010	0.025	0.067	0.175	0.001	0.002	0.025	0.072	0.001	0.002	0.037	0.157	0.002	0.0	0.042	0.188
JCNKE	T	0.05	0.040	0.157	0.442	0.705	0.045	0.242	0.522	0.847	0.039	0.265	0.667	0.897	0.054	0.277	0.720	0.952
		0.01	0.007	0.027	0.135	0.345	0.009	0.060	0.210	0.540	0.010	0.085	0.360	0.677	0.006	0.090	0.377	0.762
BARTH	T	0.05	0.052	0.170	0.438	0.757	0.064	0.215	0.522	0.855	0.041	0.265	0.635	0.915	0.050	0.300	0.740	0.962
		0.01	0.015	0.035	0.185	0.402	0.015	0.080	0.242	0.590	0.009	0.100	0.385	0.697	0.007	0.097	0.417	0.790
CHAKO	T	0.05	0.053	0.162	0.412	0.682	0.060	0.200	0.472	0.800	0.040	0.220	0.590	0.850	0.051	0.270	0.680	0.907
		0.01	0.005	0.022	0.110	0.290	0.010	0.052	0.160	0.430	0.004	0.063	0.275	0.555	0.002	0.055	0.310	0.675
ATMAX	T	0.05	0.051	0.170	0.472	0.785	0.049	0.242	0.555	0.892	0.042	0.267	0.665	0.945	0.059	0.302	0.752	0.970
		0.01	0.012	0.035	0.217	0.452	0.012	0.085	0.265	0.652	0.011	0.095	0.410	0.747	0.004	0.102	0.442	0.822
LRCON	T	0.05	0.051	0.170	0.472	0.785	0.051	0.235	0.563	0.890	0.049	0.272	0.692	0.930	0.051	0.315	0.780	0.982
		0.01	0.012	0.035	0.217	0.452	0.015	0.092	0.265	0.647	0.007	0.112	0.427	0.765	0.006	0.095	0.480	0.852
JWENS	T	0.05	0.042	0.200	0.487	0.755	0.054	0.260	0.585	0.895	0.046	0.325	0.710	0.930	0.060	0.352	0.770	0.960
		0.01	0.010	0.030	0.155	0.372	0.012	0.065	0.252	0.580	0.009	0.090	0.407	0.715	0.007	0.125	0.438	0.787
PURIS	T	0.05	0.042	0.200	0.487	0.755	0.049	0.282	0.597	0.900	0.052	0.335	0.742	0.932	0.049	0.362	0.822	0.977
		0.01	0.010	0.030	0.155	0.372	0.010	0.067	0.257	0.610	0.010	0.105	0.435	0.742	0.006	0.115	0.482	0.827
FTEST	G	0.05	0.056	0.100	0.185	0.477	0.060	0.112	0.257	0.542	0.036	0.087	0.255	0.617	0.059	0.090	0.325	0.665
		0.01	0.011	0.032	0.063	0.215	0.016	0.030	0.102	0.260	0.007	0.007	0.085	0.330	0.011	0.015	0.162	0.382
HTEST	G	0.05	0.049	0.095	0.175	0.417	0.046	0.075	0.195	0.392	0.026	0.050	0.180	0.412	0.030	0.045	0.212	0.427
		0.01	0.017	0.025	0.037	0.150	0.001	0.005	0.017	0.050	0.001	0.002	0.015	0.082	0.002	0.002	0.035	0.080
JCNKE	G	0.05	0.040	0.142	0.317	0.552	0.045	0.177	0.347	0.555	0.039	0.182	0.337	0.552	0.054	0.162	0.332	0.550
		0.01	0.007	0.037	0.072	0.177	0.009	0.032	0.095	0.217	0.010	0.035	0.100	0.220	0.006	0.050	0.095	0.235
BARTH	G	0.05	0.052	0.177	0.365	0.703	0.064	0.195	0.455	0.792	0.041	0.217	0.482	0.855	0.050	0.202	0.587	0.870
		0.01	0.015	0.065	0.112	0.357	0.015	0.070	0.185	0.487	0.009	0.047	0.212	0.575	0.007	0.072	0.325	0.682
CHAKO	G	0.05	0.053	0.155	0.330	0.647	0.060	0.185	0.420	0.715	0.040	0.180	0.415	0.762	0.051	0.175	0.480	0.830
		0.01	0.005	0.027	0.082	0.237	0.010	0.042	0.125	0.287	0.004	0.027	0.117	0.367	0.002	0.050	0.177	0.425
ATMAX	G	0.05	0.051	0.182	0.370	0.667	0.049	0.197	0.425	0.712	0.042	0.222	0.467	0.777	0.059	0.192	0.547	0.835
		0.01	0.012	0.060	0.115	0.342	0.012	0.060	0.185	0.442	0.011	0.055	0.172	0.510	0.004	0.090	0.267	0.565
LRCON	G	0.05	0.051	0.182	0.370	0.667	0.051	0.197	0.407	0.680	0.049	0.207	0.425	0.720	0.051	0.177	0.460	0.725
		0.01	0.012	0.060	0.115	0.342	0.015	0.057	0.160	0.407	0.007	0.042	0.155	0.425	0.006	0.063	0.192	0.442
JWENS	G	0.05	0.042	0.195	0.345	0.607	0.054	0.207	0.432	0.640	0.046	0.232	0.447	0.675	0.060	0.235	0.547	0.727
		0.01	0.010	0.037	0.087	0.192	0.012	0.055	0.142	0.265	0.009	0.055	0.155	0.327	0.007	0.077	0.172	0.395
PURIS	G	0.05	0.042	0.195	0.345	0.607	0.049	0.215	0.415	0.610	0.052	0.212	0.412	0.617	0.049	0.190	0.415	0.610
		0.01	0.010	0.037	0.087	0.192	0.010	0.047	0.117	0.242	0.010	0.050	0.122	0.260	0.006	0.050	0.120	0.275

TABLE A18

EMPIRICAL POWERS OF NINE ONE-WAY TESTS UNDER RECTANGULAR DISTRIBUTION: $n = 10$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
FTEST	T	0.05	0.047	0.097	0.320	0.577	0.042	0.105	0.357	0.615	0.051	0.122	0.350	0.695	0.037	0.140	0.392	0.770
		0.01	0.011	0.032	0.122	0.290	0.006	0.032	0.120	0.375	0.007	0.030	0.150	0.457	0.004	0.042	0.155	0.512
HTEST	T	0.05	0.042	0.107	0.265	0.497	0.035	0.085	0.310	0.525	0.041	0.107	0.300	0.595	0.029	0.105	0.322	0.677
		0.01	0.005	0.025	0.090	0.207	0.004	0.025	0.070	0.252	0.005	0.022	0.092	0.360	0.002	0.027	0.112	0.357
JOYKE	T	0.05	0.039	0.182	0.490	0.735	0.054	0.240	0.635	0.860	0.051	0.300	0.647	0.732	0.045	0.315	0.755	0.962
		0.01	0.005	0.065	0.220	0.460	0.007	0.077	0.367	0.630	0.006	0.115	0.367	0.735	0.005	0.112	0.480	0.835
BARTH	T	0.05	0.044	0.188	0.512	0.765	0.047	0.235	0.632	0.857	0.047	0.320	0.625	0.742	0.039	0.315	0.737	0.962
		0.01	0.011	0.052	0.242	0.462	0.006	0.067	0.355	0.620	0.011	0.107	0.367	0.745	0.007	0.100	0.472	0.852
CHAKO	T	0.05	0.040	0.172	0.495	0.727	0.047	0.215	0.582	0.797	0.045	0.310	0.575	0.885	0.041	0.290	0.700	0.938
		0.01	0.005	0.047	0.188	0.380	0.005	0.060	0.290	0.517	0.006	0.092	0.315	0.630	0.004	0.082	0.402	0.762
ATMAX	T	0.05	0.041	0.195	0.545	0.810	0.046	0.242	0.652	0.897	0.041	0.335	0.692	0.947	0.046	0.307	0.757	0.967
		0.01	0.007	0.067	0.255	0.537	0.007	0.072	0.400	0.675	0.007	0.130	0.382	0.807	0.007	0.120	0.515	0.870
LRCCV	T	0.05	0.041	0.195	0.545	0.810	0.045	0.232	0.672	0.897	0.049	0.330	0.692	0.957	0.047	0.322	0.787	0.980
		0.01	0.007	0.067	0.255	0.537	0.006	0.080	0.417	0.717	0.012	0.127	0.422	0.813	0.006	0.125	0.557	0.900
JWENS	T	0.05	0.041	0.222	0.570	0.817	0.045	0.290	0.695	0.910	0.047	0.395	0.745	0.952	0.049	0.432	0.820	0.977
		0.01	0.002	0.063	0.257	0.505	0.009	0.085	0.400	0.667	0.007	0.172	0.447	0.815	0.007	0.167	0.600	0.885
PURIS	T	0.05	0.041	0.222	0.570	0.817	0.050	0.313	0.720	0.917	0.051	0.402	0.765	0.962	0.052	0.455	0.855	0.987
		0.01	0.002	0.063	0.257	0.505	0.007	0.090	0.420	0.710	0.007	0.172	0.485	0.862	0.005	0.192	0.662	0.952
FTEST	G	0.05	0.047	0.087	0.317	0.585	0.042	0.077	0.287	0.630	0.051	0.112	0.350	0.735	0.037	0.120	0.390	0.802
		0.01	0.011	0.030	0.112	0.305	0.006	0.032	0.110	0.377	0.007	0.022	0.142	0.447	0.004	0.040	0.157	0.570
HTEST	G	0.05	0.042	0.077	0.267	0.512	0.035	0.065	0.227	0.532	0.041	0.085	0.280	0.587	0.029	0.095	0.302	0.677
		0.01	0.005	0.017	0.055	0.205	0.004	0.020	0.042	0.240	0.005	0.012	0.087	0.300	0.002	0.022	0.097	0.340
JOYKE	G	0.05	0.039	0.157	0.370	0.632	0.054	0.145	0.350	0.640	0.051	0.182	0.370	0.682	0.045	0.220	0.405	0.650
		0.01	0.005	0.040	0.130	0.300	0.007	0.037	0.142	0.310	0.006	0.045	0.145	0.357	0.005	0.063	0.142	0.357
BARTH	G	0.05	0.044	0.176	0.438	0.750	0.047	0.167	0.477	0.845	0.047	0.215	0.542	0.885	0.039	0.245	0.602	0.945
		0.01	0.011	0.050	0.202	0.475	0.006	0.063	0.222	0.567	0.011	0.057	0.275	0.715	0.007	0.070	0.357	0.807
CHAKO	G	0.05	0.040	0.170	0.410	0.695	0.047	0.160	0.430	0.767	0.045	0.192	0.477	0.830	0.041	0.217	0.525	0.887
		0.01	0.005	0.035	0.167	0.385	0.005	0.042	0.170	0.450	0.006	0.045	0.212	0.552	0.004	0.055	0.265	0.685
ATMAX	G	0.05	0.041	0.157	0.390	0.702	0.046	0.175	0.432	0.755	0.041	0.217	0.510	0.827	0.046	0.242	0.565	0.885
		0.01	0.007	0.042	0.170	0.410	0.007	0.055	0.215	0.447	0.007	0.070	0.237	0.612	0.007	0.077	0.285	0.692
LRCCV	G	0.05	0.041	0.157	0.390	0.702	0.045	0.165	0.412	0.742	0.049	0.192	0.455	0.782	0.047	0.220	0.477	0.802
		0.01	0.007	0.042	0.170	0.410	0.006	0.042	0.200	0.410	0.012	0.063	0.195	0.520	0.006	0.070	0.217	0.545
JWENS	G	0.05	0.041	0.177	0.438	0.702	0.045	0.177	0.442	0.745	0.047	0.277	0.545	0.797	0.049	0.297	0.595	0.870
		0.01	0.002	0.045	0.140	0.337	0.009	0.050	0.180	0.372	0.007	0.082	0.220	0.542	0.007	0.095	0.290	0.627
PURIS	G	0.05	0.041	0.177	0.438	0.702	0.050	0.190	0.427	0.697	0.051	0.252	0.472	0.750	0.052	0.282	0.505	0.762
		0.01	0.002	0.045	0.140	0.337	0.007	0.045	0.170	0.330	0.007	0.063	0.182	0.440	0.005	0.090	0.225	0.472

TABLE A19

EMPIRICAL POWERS OF NINE ONE-WAY TESTS UNDER RECTANGULAR DISTRIBUTION: $n = 15$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
FTEST	T	0.05	0.042	0.120	0.335	0.590	0.065	0.115	0.390	0.670	0.047	0.115	0.415	0.700	0.050	0.120	0.390	0.780
		0.01	0.075	0.335	0.125	0.320	0.012	0.055	0.150	0.420	0.005	0.030	0.205	0.445	0.005	0.050	0.180	0.565
HTEST	T	0.05	0.040	0.120	0.320	0.510	0.050	0.110	0.320	0.605	0.037	0.105	0.375	0.630	0.047	0.110	0.330	0.710
		0.01	0.005	0.020	0.095	0.245	0.010	0.030	0.105	0.340	0.002	0.015	0.180	0.350	0.005	0.030	0.115	0.445
JONKE	T	0.05	0.037	0.190	0.540	0.780	0.047	0.255	0.630	0.870	0.030	0.315	0.710	0.935	0.062	0.280	0.735	0.985
		0.01	0.012	0.365	0.265	0.480	0.037	0.100	0.370	0.645	0.005	0.120	0.440	0.780	0.012	0.110	0.480	0.890
BARTH	T	0.05	0.045	0.205	0.570	0.775	0.062	0.275	0.590	0.890	0.040	0.285	0.700	0.935	0.052	0.280	0.720	0.970
		0.01	0.037	0.355	0.245	0.475	0.015	0.090	0.360	0.650	0.005	0.100	0.420	0.775	0.012	0.090	0.485	0.875
CHAKO	T	0.05	0.040	0.200	0.540	0.730	0.055	0.245	0.525	0.850	0.037	0.255	0.625	0.910	0.047	0.265	0.690	0.960
		0.01	0.010	0.365	0.230	0.405	0.010	0.090	0.310	0.570	0.002	0.085	0.380	0.665	0.012	0.085	0.410	0.810
ATMAX	T	0.05	0.035	0.205	0.595	0.840	0.045	0.280	0.670	0.885	0.035	0.325	0.715	0.950	0.062	0.305	0.735	0.985
		0.01	0.015	0.065	0.280	0.540	0.015	0.105	0.370	0.710	0.005	0.110	0.420	0.790	0.012	0.125	0.500	0.905
LRCON	T	0.05	0.035	0.205	0.595	0.840	0.050	0.300	0.690	0.910	0.035	0.325	0.745	0.960	0.057	0.290	0.765	0.990
		0.01	0.015	0.365	0.280	0.540	0.012	0.090	0.390	0.710	0.005	0.120	0.460	0.815	0.015	0.130	0.545	0.945
JWENS	T	0.05	0.037	0.265	0.675	0.865	0.047	0.365	0.750	0.925	0.027	0.380	0.810	0.970	0.067	0.405	0.860	0.995
		0.01	0.015	0.095	0.345	0.605	0.015	0.165	0.445	0.780	0.002	0.155	0.525	0.825	0.017	0.175	0.590	0.940
PURIS	T	0.05	0.037	0.265	0.675	0.865	0.050	0.355	0.760	0.925	0.027	0.420	0.840	0.980	0.062	0.440	0.865	0.995
		0.01	0.015	0.095	0.345	0.605	0.012	0.160	0.460	0.795	0.005	0.175	0.565	0.865	0.015	0.180	0.645	0.965
FTEST	G	0.05	0.042	0.380	0.260	0.620	0.065	0.090	0.300	0.665	0.047	0.100	0.340	0.685	0.050	0.100	0.410	0.800
		0.01	0.305	0.025	0.085	0.355	0.012	0.030	0.125	0.455	0.005	0.030	0.170	0.475	0.005	0.040	0.215	0.605
HTEST	G	0.05	0.040	0.375	0.220	0.545	0.050	0.090	0.275	0.590	0.037	0.110	0.275	0.635	0.047	0.090	0.370	0.720
		0.01	0.005	0.025	0.075	0.285	0.010	0.025	0.100	0.345	0.002	0.020	0.125	0.335	0.005	0.030	0.140	0.435
JONKE	G	0.05	0.037	0.170	0.360	0.645	0.047	0.140	0.420	0.675	0.030	0.220	0.390	0.650	0.062	0.215	0.375	0.640
		0.01	0.012	0.355	0.125	0.355	0.037	0.045	0.180	0.355	0.005	0.060	0.180	0.375	0.012	0.065	0.135	0.380
BARTH	G	0.05	0.045	0.145	0.420	0.755	0.062	0.175	0.505	0.865	0.040	0.250	0.565	0.855	0.052	0.255	0.645	0.965
		0.01	0.307	0.050	0.180	0.525	0.015	0.060	0.235	0.590	0.005	0.065	0.275	0.695	0.012	0.075	0.365	0.745
CHAKO	G	0.05	0.040	0.135	0.360	0.730	0.055	0.185	0.440	0.820	0.037	0.235	0.520	0.785	0.047	0.245	0.560	0.700
		0.01	0.010	0.045	0.150	0.415	0.010	0.065	0.210	0.500	0.002	0.065	0.235	0.580	0.012	0.065	0.310	0.660
ATMAX	G	0.05	0.035	0.175	0.430	0.715	0.045	0.175	0.465	0.765	0.035	0.235	0.450	0.810	0.062	0.275	0.530	0.840
		0.01	0.015	0.055	0.155	0.455	0.015	0.050	0.225	0.495	0.005	0.075	0.220	0.575	0.012	0.105	0.270	0.605
LRCON	G	0.05	0.035	0.175	0.430	0.715	0.050	0.180	0.450	0.745	0.035	0.200	0.430	0.755	0.057	0.215	0.405	0.745
		0.01	0.015	0.355	0.155	0.455	0.012	0.030	0.200	0.460	0.005	0.070	0.185	0.490	0.015	0.070	0.185	0.500
JWENS	G	0.05	0.037	0.225	0.495	0.735	0.047	0.270	0.565	0.790	0.027	0.275	0.520	0.825	0.067	0.320	0.590	0.850
		0.01	0.015	0.070	0.190	0.455	0.015	0.070	0.290	0.480	0.002	0.100	0.235	0.550	0.017	0.135	0.310	0.610
PURIS	G	0.05	0.037	0.225	0.495	0.735	0.050	0.290	0.505	0.740	0.027	0.240	0.480	0.730	0.062	0.270	0.510	0.745
		0.01	0.015	0.070	0.190	0.455	0.012	0.065	0.245	0.445	0.005	0.090	0.235	0.480	0.015	0.085	0.205	0.500

TABLE A20

EMPIRICAL POWERS OF NINE ONE-WAY TESTS UNDER RECTANGULAR DISTRIBUTION: $n = 20$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
FTEST	T	0.05	0.050	0.090	0.340	0.620	0.052	0.095	0.355	0.665	0.060	0.130	0.350	0.720	0.035	0.115	0.440	0.820
		0.01	0.007	0.035	0.095	0.280	0.012	0.015	0.130	0.420	0.005	0.050	0.160	0.470	0.002	0.030	0.210	0.610
HTEST	T	0.05	0.047	0.080	0.320	0.520	0.045	0.060	0.305	0.595	0.057	0.140	0.305	0.640	0.025	0.085	0.370	0.750
		0.01	0.007	0.030	0.080	0.235	0.012	0.010	0.095	0.295	0.0	0.035	0.140	0.395	0.002	0.025	0.165	0.530
JONKE	T	0.05	0.057	0.225	0.520	0.810	0.057	0.190	0.610	0.880	0.030	0.350	0.610	0.945	0.057	0.315	0.790	0.975
		0.01	0.005	0.070	0.270	0.485	0.015	0.060	0.340	0.655	0.015	0.105	0.375	0.765	0.007	0.150	0.505	0.890
BARTH	T	0.05	0.062	0.210	0.495	0.785	0.042	0.170	0.600	0.870	0.045	0.295	0.620	0.930	0.075	0.315	0.785	0.960
		0.01	0.005	0.070	0.285	0.505	0.007	0.050	0.330	0.650	0.012	0.115	0.375	0.765	0.002	0.135	0.480	0.880
CHAKO	T	0.05	0.062	0.295	0.485	0.715	0.040	0.155	0.560	0.840	0.047	0.285	0.575	0.895	0.077	0.295	0.730	0.950
		0.01	0.005	0.060	0.235	0.435	0.010	0.050	0.285	0.570	0.012	0.110	0.345	0.695	0.0	0.115	0.445	0.835
ATMAX	T	0.05	0.055	0.230	0.550	0.860	0.055	0.210	0.605	0.905	0.030	0.340	0.640	0.970	0.072	0.275	0.795	0.980
		0.01	0.005	0.080	0.315	0.550	0.015	0.075	0.365	0.715	0.010	0.105	0.385	0.815	0.010	0.140	0.525	0.905
LRCON	T	0.05	0.055	0.230	0.550	0.860	0.057	0.205	0.620	0.915	0.032	0.370	0.645	0.965	0.062	0.330	0.825	0.985
		0.01	0.005	0.080	0.315	0.550	0.015	0.075	0.380	0.725	0.015	0.105	0.405	0.825	0.002	0.150	0.545	0.920
JHENS	T	0.05	0.060	0.290	0.610	0.900	0.050	0.290	0.695	0.935	0.037	0.445	0.760	0.985	0.070	0.440	0.890	0.990
		0.01	0.005	0.095	0.370	0.660	0.015	0.080	0.470	0.790	0.010	0.170	0.510	0.885	0.015	0.205	0.725	0.955
PURIS	T	0.05	0.060	0.290	0.610	0.900	0.045	0.300	0.740	0.945	0.032	0.475	0.775	0.980	0.077	0.455	0.935	1.000
		0.01	0.005	0.095	0.370	0.660	0.015	0.100	0.525	0.810	0.010	0.195	0.500	0.935	0.020	0.245	0.755	0.960
FTEST	G	0.05	0.051	0.100	0.280	0.620	0.052	0.135	0.370	0.715	0.060	0.085	0.390	0.730	0.035	0.115	0.380	0.785
		0.01	0.007	0.020	0.120	0.405	0.012	0.045	0.160	0.445	0.005	0.025	0.165	0.515	0.002	0.030	0.180	0.575
HTEST	G	0.05	0.047	0.085	0.275	0.585	0.045	0.110	0.310	0.640	0.057	0.085	0.345	0.680	0.025	0.105	0.340	0.710
		0.01	0.007	0.020	0.095	0.320	0.012	0.030	0.115	0.325	0.0	0.025	0.120	0.390	0.002	0.015	0.150	0.425
JONKE	G	0.05	0.057	0.155	0.445	0.665	0.057	0.205	0.425	0.665	0.030	0.165	0.435	0.685	0.057	0.215	0.420	0.700
		0.01	0.005	0.035	0.160	0.405	0.015	0.060	0.185	0.385	0.015	0.045	0.190	0.430	0.007	0.060	0.200	0.405
BARTH	G	0.05	0.062	0.185	0.495	0.790	0.042	0.235	0.525	0.840	0.045	0.195	0.605	0.890	0.075	0.245	0.625	0.925
		0.01	0.005	0.040	0.200	0.540	0.017	0.065	0.300	0.645	0.012	0.045	0.310	0.705	0.002	0.080	0.355	0.780
CHAKO	G	0.05	0.062	0.175	0.450	0.740	0.040	0.210	0.475	0.790	0.047	0.185	0.550	0.845	0.077	0.235	0.555	0.885
		0.01	0.005	0.030	0.155	0.455	0.010	0.065	0.225	0.555	0.012	0.050	0.255	0.615	0.0	0.060	0.290	0.700
ATMAX	G	0.05	0.055	0.170	0.470	0.735	0.055	0.225	0.485	0.750	0.030	0.200	0.565	0.865	0.072	0.255	0.550	0.865
		0.01	0.005	0.030	0.185	0.455	0.015	0.055	0.255	0.530	0.010	0.050	0.270	0.595	0.010	0.110	0.275	0.625
LRCON	G	0.05	0.055	0.170	0.470	0.735	0.057	0.205	0.465	0.745	0.032	0.180	0.485	0.780	0.062	0.220	0.475	0.770
		0.01	0.005	0.030	0.185	0.455	0.015	0.050	0.215	0.510	0.015	0.050	0.240	0.490	0.002	0.055	0.230	0.550
JHENS	G	0.05	0.060	0.215	0.525	0.770	0.050	0.295	0.570	0.830	0.037	0.285	0.605	0.900	0.070	0.360	0.655	0.895
		0.01	0.005	0.040	0.215	0.470	0.015	0.110	0.300	0.520	0.010	0.085	0.335	0.650	0.015	0.145	0.375	0.705
PURIS	G	0.05	0.060	0.215	0.525	0.770	0.045	0.285	0.550	0.815	0.032	0.230	0.530	0.845	0.077	0.290	0.565	0.805
		0.01	0.005	0.040	0.215	0.470	0.015	0.100	0.290	0.515	0.010	0.085	0.275	0.520	0.020	0.130	0.285	0.585

TABLE A21

EMPIRICAL POWERS OF NINE ONE-WAY TESTS UNDER EXPONENTIAL DISTRIBUTION: $n = 5$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
FTEST	T	0.05 0.01	0.034 0.010	0.142 0.037	0.352 0.127	0.640 0.337	0.035 0.002	0.090 0.030	0.352 0.145	0.690 0.438	0.039 0.007	0.140 0.042	0.410 0.182	0.775 0.522	0.051 0.012	0.127 0.040	0.430 0.180	0.797 0.610
HTEST	T	0.05 0.01	0.044 0.011	0.172 0.057	0.420 0.182	0.627 0.387	0.030 0.002	0.102 0.017	0.390 0.095	0.732 0.320	0.031 0.004	0.152 0.020	0.482 0.160	0.772 0.425	0.050 0.006	0.135 0.015	0.507 0.175	0.820 0.532
JONKE	T	0.05 0.01	0.036 0.009	0.295 0.085	0.662 0.345	0.862 0.602	0.050 0.006	0.380 0.122	0.777 0.480	0.962 0.820	0.050 0.012	0.505 0.225	0.877 0.657	0.980 0.910	0.039 0.009	0.560 0.275	0.940 0.715	0.990 0.970
BARTH	T	0.05 0.01	0.036 0.009	0.237 0.090	0.577 0.285	0.787 0.560	0.037 0.004	0.235 0.070	0.630 0.335	0.897 0.710	0.049 0.004	0.347 0.132	0.730 0.445	0.932 0.825	0.042 0.016	0.330 0.140	0.785 0.517	0.970 0.882
CHAKO	T	0.05 0.01	0.046 0.007	0.300 0.085	0.635 0.292	0.837 0.510	0.050 0.005	0.305 0.070	0.722 0.357	0.922 0.715	0.046 0.009	0.430 0.155	0.820 0.515	0.965 0.817	0.044 0.009	0.482 0.195	0.857 0.595	0.985 0.907
ATMAX	T	0.05 0.01	0.034 0.007	0.260 0.095	0.587 0.313	0.810 0.595	0.037 0.005	0.245 0.082	0.657 0.360	0.917 0.735	0.052 0.007	0.392 0.145	0.752 0.450	0.952 0.820	0.044 0.011	0.340 0.145	0.797 0.537	0.977 0.895
LRCOV	T	0.05 0.01	0.034 0.007	0.260 0.095	0.587 0.313	0.810 0.595	0.045 0.005	0.260 0.080	0.667 0.352	0.920 0.750	0.049 0.007	0.387 0.155	0.767 0.502	0.960 0.855	0.046 0.014	0.385 0.155	0.832 0.552	0.982 0.925
JMENS	T	0.05 0.01	0.040 0.007	0.340 0.100	0.680 0.337	0.837 0.563	0.051 0.006	0.375 0.135	0.772 0.452	0.940 0.777	0.051 0.010	0.502 0.202	0.855 0.595	0.977 0.867	0.036 0.010	0.565 0.262	0.902 0.655	0.982 0.935
PURIS	T	0.05 0.01	0.040 0.007	0.340 0.100	0.680 0.337	0.837 0.563	0.046 0.006	0.372 0.137	0.780 0.480	0.950 0.807	0.050 0.012	0.545 0.255	0.870 0.630	0.975 0.890	0.037 0.010	0.600 0.285	0.940 0.697	0.987 0.952
FTEST	G	0.05 0.01	0.034 0.010	0.110 0.017	0.335 0.160	0.585 0.327	0.035 0.002	0.135 0.037	0.362 0.130	0.722 0.410	0.039 0.007	0.100 0.027	0.410 0.175	0.757 0.505	0.051 0.012	0.115 0.032	0.385 0.167	0.787 0.520
HTEST	G	0.05 0.01	0.044 0.011	0.122 0.067	0.377 0.147	0.627 0.287	0.030 0.002	0.127 0.010	0.325 0.037	0.605 0.095	0.031 0.004	0.110 0.005	0.382 0.045	0.635 0.120	0.050 0.006	0.107 0.022	0.310 0.025	0.630 0.125
JONKE	G	0.05 0.01	0.036 0.009	0.235 0.027	0.445 0.162	0.705 0.282	0.050 0.006	0.252 0.055	0.490 0.175	0.692 0.277	0.050 0.012	0.242 0.060	0.457 0.172	0.625 0.307	0.039 0.009	0.220 0.047	0.427 0.142	0.637 0.270
BARTH	G	0.05 0.01	0.035 0.009	0.202 0.060	0.502 0.250	0.782 0.487	0.037 0.004	0.252 0.082	0.545 0.300	0.875 0.642	0.049 0.004	0.237 0.075	0.605 0.313	0.900 0.725	0.042 0.016	0.245 0.065	0.672 0.335	0.938 0.815
CHAKO	G	0.05 0.01	0.046 0.007	0.257 0.022	0.580 0.217	0.845 0.395	0.050 0.005	0.322 0.063	0.622 0.210	0.905 0.490	0.046 0.009	0.287 0.060	0.652 0.260	0.932 0.592	0.044 0.009	0.265 0.032	0.732 0.242	0.957 0.667
ATMAX	G	0.05 0.01	0.034 0.007	0.210 0.047	0.495 0.235	0.762 0.452	0.037 0.005	0.245 0.085	0.530 0.275	0.815 0.567	0.052 0.007	0.215 0.080	0.520 0.277	0.840 0.645	0.044 0.011	0.235 0.080	0.592 0.327	0.892 0.697
LRCOV	G	0.05 0.01	0.034 0.007	0.210 0.047	0.495 0.235	0.762 0.452	0.045 0.005	0.237 0.082	0.485 0.252	0.810 0.527	0.049 0.007	0.220 0.065	0.475 0.227	0.777 0.560	0.046 0.014	0.217 0.052	0.510 0.222	0.825 0.572
JMENS	G	0.05 0.01	0.040 0.007	0.255 0.025	0.442 0.160	0.688 0.272	0.051 0.006	0.307 0.067	0.510 0.205	0.730 0.317	0.051 0.010	0.272 0.065	0.522 0.202	0.730 0.375	0.036 0.010	0.262 0.065	0.542 0.212	0.785 0.447
PURIS	G	0.05 0.01	0.040 0.007	0.255 0.025	0.442 0.160	0.688 0.272	0.046 0.006	0.280 0.067	0.462 0.170	0.688 0.270	0.050 0.012	0.250 0.063	0.465 0.165	0.635 0.313	0.037 0.010	0.220 0.042	0.438 0.155	0.657 0.297

TABLE A22

EMPIRICAL POWERS OF NINE ONE-WAY TESTS UNDER EXPONENTIAL DISTRIBUTION: $n = 10$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
FTEST	T	0.05	0.047	0.107	0.352	0.625	0.042	0.127	0.415	0.695	0.060	0.135	0.407	0.797	0.064	0.135	0.455	0.855
		0.01	0.016	0.032	0.147	0.380	0.005	0.037	0.172	0.450	0.012	0.035	0.205	0.587	0.014	0.030	0.225	0.670
HTEST	T	0.05	0.044	0.182	0.457	0.735	0.031	0.188	0.522	0.827	0.047	0.182	0.597	0.892	0.047	0.197	0.627	0.945
		0.01	0.006	0.057	0.215	0.482	0.001	0.040	0.220	0.542	0.009	0.050	0.322	0.705	0.006	0.060	0.367	0.802
JOYKE	T	0.05	0.049	0.355	0.742	0.925	0.040	0.497	0.867	0.987	0.060	0.490	0.935	0.997	0.032	0.575	0.942	1.000
		0.01	0.009	0.157	0.472	0.762	0.010	0.220	0.637	0.900	0.019	0.210	0.767	0.962	0.005	0.287	0.815	0.987
BARTH	T	0.05	0.045	0.237	0.567	0.792	0.034	0.267	0.667	0.912	0.064	0.267	0.717	0.957	0.037	0.310	0.760	0.975
		0.01	0.016	0.067	0.305	0.572	0.009	0.087	0.390	0.712	0.012	0.095	0.438	0.837	0.015	0.105	0.527	0.922
CHAKO	T	0.05	0.050	0.327	0.685	0.870	0.040	0.438	0.802	0.965	0.055	0.445	0.887	0.977	0.039	0.497	0.902	1.000
		0.01	0.011	0.125	0.490	0.688	0.009	0.152	0.517	0.835	0.011	0.170	0.650	0.920	0.010	0.205	0.727	0.970
ATMAX	T	0.05	0.042	0.267	0.590	0.827	0.037	0.302	0.690	0.905	0.065	0.295	0.735	0.960	0.041	0.352	0.782	0.982
		0.01	0.015	0.085	0.310	0.615	0.017	0.125	0.435	0.757	0.016	0.097	0.477	0.877	0.011	0.127	0.530	0.942
LRCON	T	0.05	0.042	0.267	0.590	0.827	0.034	0.302	0.695	0.915	0.059	0.280	0.747	0.967	0.034	0.370	0.822	0.980
		0.01	0.015	0.095	0.310	0.615	0.015	0.117	0.425	0.762	0.016	0.102	0.490	0.895	0.010	0.142	0.577	0.945
JYENS	T	0.05	0.050	0.400	0.742	0.917	0.039	0.497	0.852	0.977	0.067	0.542	0.922	0.990	0.040	0.632	0.917	0.997
		0.01	0.006	0.150	0.445	0.710	0.009	0.202	0.590	0.847	0.015	0.250	0.730	0.952	0.006	0.335	0.805	0.982
PURIS	T	0.05	0.050	0.400	0.742	0.917	0.036	0.510	0.857	0.972	0.062	0.567	0.940	0.995	0.040	0.652	0.952	0.997
		0.01	0.006	0.150	0.445	0.710	0.010	0.217	0.582	0.877	0.014	0.255	0.775	0.952	0.006	0.365	0.837	0.982
FTEST	G	0.05	0.047	0.110	0.352	0.688	0.042	0.120	0.387	0.740	0.060	0.110	0.417	0.820	0.064	0.140	0.475	0.872
		0.01	0.016	0.032	0.130	0.417	0.005	0.035	0.160	0.505	0.012	0.032	0.192	0.620	0.014	0.040	0.217	0.657
HTEST	G	0.05	0.044	0.180	0.470	0.805	0.031	0.185	0.547	0.870	0.047	0.170	0.575	0.915	0.047	0.177	0.597	0.920
		0.01	0.006	0.055	0.182	0.487	0.001	0.022	0.170	0.517	0.009	0.037	0.200	0.650	0.006	0.037	0.247	0.645
JOYKE	G	0.05	0.049	0.300	0.642	0.842	0.040	0.257	0.590	0.837	0.060	0.287	0.600	0.835	0.032	0.262	0.605	0.775
		0.01	0.009	0.095	0.282	0.545	0.010	0.100	0.310	0.535	0.019	0.075	0.307	0.565	0.005	0.075	0.255	0.480
BARTH	G	0.05	0.045	0.222	0.495	0.813	0.034	0.225	0.615	0.885	0.064	0.237	0.617	0.932	0.037	0.232	0.682	0.960
		0.01	0.016	0.067	0.240	0.572	0.009	0.067	0.320	0.688	0.012	0.067	0.367	0.792	0.015	0.095	0.450	0.855
CHAKO	G	0.05	0.050	0.320	0.660	0.902	0.040	0.327	0.757	0.955	0.055	0.330	0.805	0.982	0.039	0.370	0.890	0.990
		0.01	0.011	0.115	0.313	0.690	0.009	0.090	0.435	0.807	0.011	0.092	0.482	0.907	0.010	0.092	0.560	0.938
ATMAX	G	0.05	0.042	0.235	0.497	0.782	0.037	0.212	0.577	0.837	0.065	0.232	0.563	0.872	0.041	0.242	0.640	0.895
		0.01	0.015	0.057	0.222	0.500	0.017	0.072	0.282	0.595	0.016	0.065	0.335	0.698	0.011	0.085	0.322	0.707
LRCON	G	0.05	0.042	0.235	0.497	0.782	0.034	0.180	0.522	0.795	0.059	0.212	0.510	0.832	0.034	0.182	0.522	0.827
		0.01	0.015	0.057	0.222	0.500	0.015	0.067	0.267	0.563	0.016	0.065	0.287	0.602	0.010	0.070	0.250	0.572
JYENS	G	0.05	0.050	0.320	0.580	0.805	0.039	0.265	0.615	0.825	0.067	0.307	0.617	0.857	0.040	0.313	0.697	0.880
		0.01	0.006	0.082	0.212	0.465	0.009	0.072	0.295	0.512	0.015	0.087	0.350	0.632	0.006	0.117	0.335	0.615
PURIS	G	0.05	0.050	0.320	0.580	0.805	0.036	0.252	0.557	0.785	0.062	0.292	0.563	0.795	0.040	0.275	0.577	0.765
		0.01	0.006	0.082	0.212	0.465	0.010	0.090	0.252	0.465	0.014	0.087	0.302	0.537	0.006	0.095	0.255	0.465

TABLE A23

EMPIRICAL POWERS OF NINE ONE-WAY TESTS UNDER EXPONENTIAL DISTRIBUTION: $n = 15$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
FTEST	T	0.05	0.055	0.130	0.295	0.640	0.032	0.105	0.490	0.750	0.042	0.130	0.465	0.805	0.040	0.165	0.465	0.905
		0.01	0.005	0.050	0.130	0.410	0.005	0.030	0.270	0.580	0.007	0.035	0.240	0.640	0.002	0.045	0.225	0.730
HTEST	T	0.05	0.052	0.205	0.495	0.865	0.037	0.165	0.605	0.930	0.045	0.200	0.665	0.935	0.040	0.215	0.715	0.975
		0.01	0.012	0.075	0.240	0.630	0.005	0.040	0.390	0.725	0.005	0.055	0.450	0.800	0.007	0.085	0.425	0.895
JONKE	T	0.05	0.067	0.460	0.735	0.995	0.045	0.410	0.905	0.995	0.032	0.505	0.935	1.000	0.060	0.665	0.960	1.000
		0.01	0.022	0.160	0.485	0.880	0.010	0.185	0.700	0.950	0.007	0.250	0.810	0.980	0.010	0.315	0.860	1.000
BARTH	T	0.05	0.050	0.225	0.500	0.845	0.042	0.250	0.685	0.945	0.040	0.225	0.720	0.955	0.027	0.350	0.780	0.990
		0.01	0.010	0.075	0.225	0.575	0.002	0.080	0.470	0.750	0.007	0.115	0.485	0.840	0.005	0.115	0.500	0.960
CHAKU	T	0.05	0.052	0.430	0.685	0.950	0.050	0.385	0.850	0.975	0.045	0.455	0.895	1.000	0.042	0.555	0.910	1.000
		0.01	0.017	0.105	0.410	0.800	0.005	0.125	0.590	0.925	0.012	0.195	0.705	0.950	0.010	0.295	0.790	0.990
ATMAX	T	0.05	0.050	0.280	0.520	0.890	0.040	0.290	0.700	0.960	0.027	0.270	0.770	0.965	0.035	0.375	0.795	0.995
		0.01	0.015	0.075	0.270	0.610	0.037	0.085	0.515	0.775	0.007	0.095	0.540	0.860	0.002	0.140	0.535	0.955
LRCON	T	0.05	0.050	0.280	0.520	0.890	0.052	0.295	0.725	0.960	0.030	0.290	0.800	0.965	0.052	0.410	0.825	0.995
		0.01	0.015	0.075	0.270	0.610	0.037	0.105	0.495	0.775	0.005	0.115	0.545	0.885	0.002	0.150	0.575	0.975
JWENS	T	0.05	0.067	0.520	0.795	0.990	0.045	0.455	0.905	0.990	0.032	0.570	0.955	0.995	0.050	0.705	0.945	1.000
		0.01	0.017	0.220	0.525	0.890	0.010	0.250	0.740	0.960	0.002	0.270	0.780	0.970	0.002	0.410	0.850	1.000
PURIS	T	0.05	0.067	0.520	0.795	0.990	0.052	0.440	0.930	0.995	0.025	0.595	0.945	0.995	0.062	0.755	0.985	1.000
		0.01	0.017	0.220	0.525	0.890	0.007	0.255	0.755	0.965	0.002	0.300	0.840	0.960	0.005	0.415	0.880	1.000
FTEST	G	0.05	0.055	0.125	0.325	0.695	0.032	0.110	0.400	0.805	0.042	0.160	0.455	0.845	0.040	0.125	0.560	0.885
		0.01	0.005	0.035	0.155	0.445	0.005	0.035	0.185	0.540	0.007	0.045	0.205	0.630	0.002	0.025	0.300	0.690
HTEST	G	0.05	0.052	0.215	0.530	0.850	0.037	0.155	0.585	0.925	0.045	0.200	0.670	0.945	0.040	0.190	0.760	0.985
		0.01	0.002	0.050	0.280	0.600	0.005	0.025	0.295	0.740	0.005	0.065	0.325	0.810	0.007	0.040	0.430	0.835
JONKE	G	0.05	0.067	0.265	0.640	0.875	0.045	0.260	0.665	0.925	0.032	0.245	0.650	0.865	0.060	0.335	0.625	0.905
		0.01	0.022	0.105	0.355	0.615	0.010	0.075	0.385	0.720	0.007	0.085	0.315	0.660	0.010	0.120	0.345	0.675
BARTH	G	0.05	0.060	0.215	0.500	0.800	0.042	0.175	0.535	0.905	0.040	0.225	0.665	0.940	0.027	0.240	0.760	0.990
		0.01	0.010	0.070	0.255	0.585	0.002	0.080	0.300	0.745	0.007	0.110	0.430	0.820	0.005	0.075	0.485	0.860
CHAKU	G	0.05	0.052	0.290	0.670	0.940	0.050	0.285	0.785	0.985	0.045	0.335	0.870	0.990	0.042	0.445	0.935	1.000
		0.01	0.017	0.105	0.405	0.775	0.005	0.105	0.505	0.905	0.012	0.120	0.625	0.950	0.010	0.125	0.730	0.990
ATMAX	G	0.05	0.050	0.215	0.495	0.765	0.040	0.175	0.515	0.860	0.027	0.215	0.605	0.845	0.035	0.250	0.680	0.910
		0.01	0.015	0.055	0.245	0.505	0.007	0.065	0.320	0.625	0.007	0.085	0.330	0.695	0.002	0.090	0.415	0.750
LRCON	G	0.05	0.050	0.215	0.495	0.765	0.052	0.175	0.485	0.855	0.030	0.200	0.555	0.805	0.052	0.225	0.580	0.865
		0.01	0.015	0.055	0.245	0.505	0.007	0.060	0.300	0.595	0.005	0.055	0.270	0.605	0.002	0.055	0.290	0.630
JWENS	G	0.05	0.067	0.280	0.640	0.855	0.045	0.285	0.640	0.905	0.032	0.290	0.690	0.875	0.050	0.365	0.735	0.930
		0.01	0.017	0.120	0.345	0.540	0.010	0.090	0.410	0.715	0.002	0.100	0.375	0.640	0.002	0.135	0.435	0.765
PURIS	G	0.05	0.067	0.280	0.640	0.855	0.052	0.285	0.670	0.900	0.025	0.280	0.630	0.830	0.062	0.345	0.610	0.860
		0.01	0.017	0.120	0.345	0.540	0.007	0.095	0.385	0.680	0.002	0.070	0.295	0.585	0.005	0.100	0.315	0.595

TABLE A24

EMPIRICAL POWERS OF NINE ONE-WAY TESTS UNDER EXPONENTIAL DISTRIBUTION: $n = 20$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
FTEST	T	0.05	0.055	0.125	0.315	0.695	0.050	0.105	0.440	0.745	0.050	0.090	0.395	0.800	0.047	0.140	0.485	0.855
		0.01	0.007	0.020	0.185	0.430	0.007	0.035	0.230	0.530	0.007	0.015	0.185	0.655	0.010	0.035	0.280	0.665
HTEST	T	0.05	0.050	0.205	0.560	0.890	0.072	0.175	0.650	0.930	0.045	0.200	0.640	0.930	0.027	0.270	0.755	0.985
		0.01	0.007	0.105	0.340	0.690	0.010	0.060	0.410	0.780	0.010	0.060	0.390	0.845	0.002	0.080	0.550	0.885
JONKE	T	0.05	0.062	0.355	0.795	0.980	0.042	0.455	0.895	0.985	0.047	0.565	0.935	1.000	0.042	0.660	0.965	1.000
		0.01	0.012	0.155	0.520	0.880	0.007	0.170	0.695	0.980	0.010	0.300	0.760	0.980	0.010	0.370	0.890	0.995
BARTH	T	0.05	0.052	0.220	0.545	0.845	0.045	0.245	0.635	0.925	0.047	0.280	0.650	0.950	0.037	0.355	0.795	0.980
		0.01	0.010	0.075	0.275	0.625	0.007	0.070	0.405	0.750	0.012	0.095	0.410	0.825	0.010	0.135	0.540	0.915
CHAKO	T	0.05	0.060	0.320	0.745	0.970	0.065	0.410	0.830	0.985	0.040	0.520	0.890	0.995	0.047	0.565	0.945	1.000
		0.01	0.005	0.160	0.460	0.830	0.015	0.155	0.635	0.930	0.007	0.215	0.665	0.945	0.007	0.305	0.820	0.980
ATMAX	T	0.05	0.060	0.210	0.625	0.870	0.047	0.245	0.695	0.940	0.050	0.300	0.715	0.960	0.042	0.350	0.780	0.985
		0.01	0.010	0.070	0.295	0.650	0.010	0.065	0.435	0.785	0.010	0.100	0.415	0.850	0.012	0.160	0.585	0.900
LRCON	T	0.05	0.060	0.210	0.625	0.870	0.040	0.240	0.700	0.935	0.045	0.335	0.725	0.970	0.042	0.380	0.815	0.985
		0.01	0.010	0.070	0.295	0.650	0.002	0.065	0.445	0.840	0.012	0.115	0.475	0.865	0.010	0.170	0.605	0.940
JMFNS	T	0.05	0.057	0.420	0.830	0.985	0.042	0.515	0.895	0.990	0.047	0.620	0.940	1.000	0.050	0.745	0.980	1.000
		0.01	0.017	0.180	0.605	0.900	0.012	0.265	0.735	0.955	0.010	0.370	0.800	0.970	0.010	0.435	0.885	0.995
PURIS	T	0.05	0.057	0.420	0.830	0.985	0.052	0.535	0.920	0.985	0.052	0.655	0.950	1.000	0.057	0.755	0.990	1.000
		0.01	0.017	0.180	0.605	0.900	0.010	0.255	0.745	0.975	0.010	0.390	0.825	0.980	0.010	0.490	0.915	1.000
FTEST	G	0.05	0.055	0.130	0.425	0.705	0.050	0.145	0.475	0.780	0.050	0.140	0.410	0.840	0.047	0.145	0.460	0.885
		0.01	0.007	0.045	0.185	0.470	0.007	0.050	0.175	0.520	0.007	0.035	0.180	0.660	0.010	0.045	0.235	0.660
HTEST	G	0.05	0.050	0.135	0.660	0.910	0.072	0.210	0.700	0.945	0.045	0.225	0.700	0.960	0.027	0.250	0.740	0.990
		0.01	0.007	0.060	0.385	0.705	0.010	0.075	0.330	0.795	0.010	0.040	0.340	0.855	0.002	0.080	0.430	0.895
JONKE	G	0.05	0.062	0.295	0.700	0.900	0.042	0.310	0.685	0.925	0.047	0.295	0.655	0.930	0.042	0.360	0.690	0.920
		0.01	0.012	0.090	0.455	0.720	0.007	0.110	0.400	0.730	0.010	0.115	0.380	0.690	0.010	0.130	0.395	0.650
BARTH	G	0.05	0.052	0.230	0.540	0.825	0.045	0.245	0.620	0.875	0.047	0.230	0.630	0.960	0.037	0.260	0.710	0.965
		0.01	0.010	0.070	0.320	0.600	0.007	0.085	0.345	0.705	0.012	0.110	0.385	0.835	0.010	0.090	0.455	0.855
CHAKO	G	0.05	0.060	0.305	0.770	0.965	0.065	0.315	0.855	0.985	0.040	0.390	0.925	1.000	0.047	0.420	0.930	1.000
		0.01	0.005	0.095	0.520	0.850	0.015	0.120	0.600	0.915	0.007	0.135	0.615	0.960	0.007	0.165	0.715	0.985
ATMAX	G	0.05	0.060	0.220	0.520	0.790	0.047	0.225	0.530	0.810	0.050	0.235	0.560	0.900	0.042	0.260	0.665	0.910
		0.01	0.010	0.055	0.290	0.520	0.010	0.070	0.300	0.620	0.010	0.100	0.330	0.705	0.012	0.100	0.370	0.680
LRCON	G	0.05	0.060	0.220	0.520	0.790	0.040	0.235	0.510	0.800	0.045	0.210	0.500	0.840	0.042	0.225	0.575	0.840
		0.01	0.010	0.055	0.290	0.520	0.002	0.070	0.290	0.585	0.012	0.085	0.280	0.600	0.010	0.080	0.260	0.585
JMFNS	G	0.05	0.057	0.355	0.680	0.890	0.042	0.370	0.725	0.930	0.047	0.320	0.715	0.955	0.050	0.395	0.810	0.930
		0.01	0.017	0.115	0.435	0.690	0.012	0.135	0.455	0.730	0.010	0.160	0.420	0.755	0.010	0.210	0.500	0.800
PURIS	G	0.05	0.057	0.355	0.680	0.890	0.052	0.355	0.690	0.925	0.052	0.320	0.650	0.890	0.057	0.330	0.695	0.895
		0.01	0.017	0.115	0.435	0.690	0.010	0.135	0.400	0.680	0.010	0.125	0.370	0.635	0.010	0.180	0.355	0.610

TABLE A25

EMPIRICAL POWERS OF NINE ONE-WAY TESTS UNDER NORMAL DISTRIBUTION: $n = 5$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
FTEST	T	0.05	0.042	0.097	0.282	0.542	0.055	0.095	0.287	0.622	0.045	0.150	0.357	0.680	0.059	0.080	0.317	0.725
		0.01	0.010	0.015	0.097	0.255	0.010	0.025	0.087	0.337	0.014	0.057	0.112	0.410	0.020	0.017	0.132	0.452
HTEST	T	0.05	0.045	0.093	0.272	0.502	0.037	0.082	0.220	0.542	0.034	0.127	0.287	0.567	0.040	0.057	0.245	0.635
		0.01	0.006	0.012	0.130	0.237	0.0	0.002	0.025	0.122	0.004	0.020	0.052	0.190	0.004	0.002	0.042	0.232
JONKE	T	0.05	0.042	0.167	0.472	0.745	0.054	0.260	0.600	0.895	0.061	0.330	0.705	0.912	0.040	0.272	0.710	0.967
		0.01	0.007	0.030	0.192	0.405	0.005	0.072	0.240	0.622	0.012	0.110	0.385	0.735	0.007	0.077	0.460	0.837
BARTH	T	0.05	0.050	0.172	0.502	0.767	0.064	0.250	0.563	0.880	0.060	0.292	0.700	0.905	0.040	0.230	0.700	0.962
		0.01	0.014	0.047	0.227	0.460	0.009	0.067	0.267	0.630	0.014	0.122	0.402	0.732	0.011	0.090	0.410	0.840
CHARD	T	0.05	0.055	0.172	0.472	0.732	0.052	0.215	0.520	0.845	0.060	0.272	0.652	0.867	0.041	0.202	0.657	0.945
		0.01	0.006	0.035	0.170	0.360	0.004	0.057	0.200	0.540	0.011	0.097	0.297	0.632	0.007	0.072	0.327	0.747
ATMAX	T	0.05	0.047	0.205	0.525	0.790	0.061	0.247	0.607	0.905	0.064	0.300	0.705	0.917	0.052	0.277	0.720	0.970
		0.01	0.016	0.055	0.260	0.512	0.009	0.085	0.305	0.680	0.015	0.130	0.415	0.777	0.011	0.102	0.430	0.855
LRCON	T	0.05	0.047	0.205	0.525	0.790	0.055	0.255	0.617	0.907	0.061	0.337	0.747	0.935	0.042	0.275	0.740	0.977
		0.01	0.016	0.055	0.260	0.512	0.007	0.092	0.302	0.685	0.015	0.150	0.445	0.785	0.009	0.115	0.482	0.870
JMENS	T	0.05	0.046	0.195	0.492	0.777	0.055	0.257	0.607	0.900	0.066	0.313	0.688	0.902	0.055	0.287	0.717	0.965
		0.01	0.006	0.030	0.200	0.415	0.005	0.075	0.272	0.647	0.012	0.105	0.375	0.720	0.007	0.090	0.420	0.807
PURIS	T	0.05	0.046	0.195	0.492	0.777	0.056	0.255	0.600	0.902	0.070	0.345	0.730	0.930	0.054	0.290	0.725	0.972
		0.01	0.006	0.030	0.200	0.415	0.006	0.077	0.270	0.640	0.011	0.132	0.420	0.767	0.007	0.087	0.462	0.855
FTEST	G	0.05	0.042	0.092	0.287	0.540	0.055	0.095	0.282	0.532	0.045	0.112	0.295	0.647	0.059	0.102	0.385	0.737
		0.01	0.010	0.017	0.097	0.252	0.010	0.025	0.117	0.302	0.014	0.020	0.092	0.347	0.020	0.020	0.147	0.447
HTEST	G	0.05	0.045	0.090	0.240	0.525	0.037	0.080	0.217	0.442	0.034	0.077	0.185	0.510	0.040	0.070	0.282	0.557
		0.01	0.006	0.012	0.082	0.210	0.0	0.010	0.032	0.065	0.004	0.005	0.025	0.100	0.004	0.010	0.037	0.102
JONKE	G	0.05	0.042	0.142	0.382	0.577	0.054	0.175	0.387	0.565	0.061	0.207	0.415	0.520	0.040	0.132	0.407	0.600
		0.01	0.007	0.025	0.087	0.235	0.005	0.037	0.140	0.230	0.012	0.047	0.127	0.235	0.007	0.035	0.127	0.265
BARTH	G	0.05	0.050	0.162	0.425	0.747	0.064	0.190	0.457	0.807	0.060	0.240	0.550	0.845	0.046	0.182	0.657	0.915
		0.01	0.014	0.045	0.185	0.417	0.009	0.052	0.220	0.485	0.014	0.063	0.245	0.592	0.011	0.047	0.345	0.745
CHARD	G	0.05	0.055	0.175	0.422	0.740	0.052	0.192	0.455	0.737	0.060	0.217	0.502	0.772	0.041	0.167	0.592	0.880
		0.01	0.006	0.025	0.127	0.290	0.004	0.035	0.157	0.355	0.011	0.040	0.150	0.438	0.007	0.035	0.232	0.565
ATMAX	G	0.05	0.049	0.162	0.425	0.692	0.061	0.200	0.465	0.752	0.064	0.242	0.510	0.762	0.052	0.160	0.590	0.877
		0.01	0.016	0.040	0.172	0.400	0.009	0.055	0.195	0.427	0.015	0.065	0.215	0.485	0.011	0.057	0.290	0.642
LRCON	G	0.05	0.049	0.162	0.425	0.692	0.055	0.177	0.440	0.725	0.061	0.227	0.462	0.667	0.042	0.130	0.500	0.785
		0.01	0.016	0.040	0.172	0.400	0.007	0.055	0.190	0.395	0.015	0.052	0.177	0.410	0.009	0.042	0.227	0.507
JMENS	G	0.05	0.046	0.157	0.400	0.620	0.055	0.195	0.457	0.672	0.066	0.245	0.487	0.682	0.055	0.160	0.547	0.795
		0.01	0.006	0.027	0.097	0.237	0.005	0.047	0.150	0.280	0.012	0.055	0.170	0.342	0.007	0.055	0.212	0.455
PURIS	G	0.05	0.046	0.157	0.400	0.620	0.056	0.185	0.427	0.615	0.070	0.232	0.438	0.572	0.054	0.142	0.432	0.667
		0.01	0.006	0.027	0.097	0.237	0.006	0.047	0.140	0.247	0.011	0.055	0.157	0.257	0.007	0.040	0.140	0.310

TABLE A26

EMPIRICAL POWERS OF NINE ONE-WAY TESTS UNDER NORMAL DISTRIBUTION: $n = 10$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
FTEST	T	0.05	0.035	0.092	0.265	0.563	0.051	0.117	0.337	0.667	0.064	0.137	0.327	0.712	0.042	0.110	0.380	0.782
		0.01	0.006	0.010	0.092	0.305	0.007	0.015	0.115	0.410	0.007	0.047	0.120	0.447	0.009	0.027	0.188	0.550
HTEST	T	0.05	0.030	0.087	0.237	0.515	0.042	0.117	0.280	0.602	0.063	0.130	0.285	0.677	0.052	0.090	0.332	0.727
		0.01	0.005	0.012	0.077	0.237	0.002	0.010	0.075	0.310	0.004	0.025	0.087	0.370	0.004	0.017	0.142	0.457
JONKE	T	0.05	0.045	0.200	0.500	0.797	0.049	0.240	0.605	0.872	0.049	0.282	0.700	0.950	0.047	0.310	0.762	0.967
		0.01	0.005	0.042	0.207	0.497	0.004	0.092	0.317	0.677	0.015	0.110	0.372	0.775	0.009	0.097	0.460	0.880
BARTH	T	0.05	0.039	0.205	0.480	0.792	0.047	0.242	0.590	0.877	0.056	0.277	0.647	0.938	0.055	0.285	0.742	0.960
		0.01	0.006	0.055	0.200	0.472	0.007	0.090	0.310	0.655	0.011	0.130	0.347	0.757	0.006	0.105	0.442	0.842
CHAKO	T	0.05	0.044	0.200	0.445	0.775	0.046	0.227	0.552	0.847	0.055	0.245	0.620	0.910	0.054	0.245	0.695	0.952
		0.01	0.004	0.042	0.162	0.427	0.007	0.087	0.262	0.587	0.011	0.110	0.297	0.717	0.007	0.080	0.422	0.797
ATMAX	T	0.05	0.040	0.200	0.527	0.807	0.042	0.222	0.645	0.900	0.052	0.290	0.702	0.950	0.055	0.302	0.760	0.965
		0.01	0.006	0.042	0.232	0.545	0.009	0.097	0.340	0.710	0.011	0.127	0.405	0.785	0.005	0.110	0.460	0.860
LRCON	T	0.05	0.040	0.200	0.527	0.807	0.045	0.237	0.657	0.897	0.051	0.305	0.717	0.960	0.056	0.320	0.795	0.972
		0.01	0.006	0.042	0.232	0.545	0.006	0.097	0.360	0.715	0.011	0.117	0.427	0.807	0.005	0.122	0.512	0.960
JWENS	T	0.05	0.042	0.192	0.492	0.802	0.037	0.225	0.617	0.887	0.057	0.297	0.692	0.952	0.056	0.313	0.757	0.967
		0.01	0.005	0.032	0.182	0.475	0.006	0.072	0.280	0.660	0.012	0.110	0.362	0.770	0.006	0.105	0.455	0.855
PURIS	T	0.05	0.042	0.192	0.492	0.802	0.041	0.237	0.632	0.872	0.049	0.317	0.727	0.960	0.057	0.335	0.820	0.970
		0.01	0.006	0.032	0.182	0.475	0.006	0.080	0.307	0.675	0.014	0.122	0.407	0.795	0.010	0.122	0.505	0.900
FTEST	G	0.05	0.035	0.112	0.272	0.655	0.051	0.087	0.320	0.655	0.064	0.080	0.332	0.730	0.042	0.105	0.380	0.772
		0.01	0.006	0.025	0.115	0.365	0.007	0.027	0.110	0.407	0.007	0.020	0.150	0.490	0.009	0.030	0.160	0.555
HTEST	G	0.05	0.030	0.105	0.232	0.595	0.042	0.082	0.265	0.592	0.063	0.067	0.290	0.677	0.052	0.085	0.325	0.702
		0.01	0.005	0.015	0.072	0.270	0.002	0.010	0.052	0.307	0.004	0.007	0.092	0.392	0.004	0.017	0.087	0.405
JONKE	G	0.05	0.045	0.162	0.385	0.685	0.049	0.162	0.422	0.667	0.049	0.192	0.397	0.690	0.047	0.192	0.420	0.705
		0.01	0.005	0.050	0.122	0.355	0.004	0.022	0.162	0.410	0.015	0.055	0.157	0.377	0.009	0.065	0.157	0.400
BARTH	G	0.05	0.039	0.195	0.425	0.782	0.047	0.182	0.532	0.837	0.056	0.190	0.550	0.890	0.055	0.207	0.610	0.922
		0.01	0.006	0.052	0.167	0.510	0.007	0.052	0.250	0.587	0.011	0.060	0.277	0.700	0.006	0.085	0.332	0.795
CHAKO	G	0.05	0.044	0.182	0.402	0.755	0.046	0.180	0.495	0.827	0.055	0.182	0.497	0.872	0.054	0.202	0.572	0.912
		0.01	0.004	0.042	0.137	0.420	0.007	0.040	0.180	0.530	0.011	0.042	0.210	0.622	0.007	0.067	0.295	0.715
ATMAX	G	0.05	0.040	0.198	0.410	0.737	0.042	0.167	0.480	0.780	0.052	0.222	0.482	0.825	0.055	0.220	0.563	0.870
		0.01	0.006	0.067	0.165	0.460	0.009	0.052	0.222	0.502	0.011	0.060	0.237	0.595	0.005	0.092	0.267	0.662
LRCON	G	0.05	0.040	0.198	0.410	0.737	0.045	0.170	0.460	0.755	0.051	0.192	0.462	0.767	0.056	0.200	0.467	0.802
		0.01	0.006	0.067	0.165	0.460	0.006	0.037	0.200	0.470	0.011	0.060	0.190	0.487	0.005	0.067	0.188	0.537
JWENS	G	0.05	0.042	0.177	0.395	0.688	0.037	0.155	0.445	0.742	0.057	0.217	0.485	0.797	0.056	0.235	0.542	0.850
		0.01	0.006	0.047	0.112	0.337	0.006	0.035	0.172	0.417	0.012	0.052	0.202	0.505	0.006	0.085	0.255	0.615
PURIS	G	0.05	0.042	0.177	0.395	0.688	0.041	0.147	0.420	0.710	0.049	0.195	0.427	0.730	0.057	0.217	0.460	0.770
		0.01	0.006	0.047	0.112	0.337	0.006	0.022	0.147	0.367	0.014	0.067	0.160	0.410	0.010	0.070	0.170	0.452

TABLE A27

EMPIRICAL POWERS OF NINE ONE-WAY TESTS UNDER NORMAL DISTRIBUTION: $n = 15$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
FTEST	T	0.05	0.057	0.115	0.270	0.530	0.037	0.105	0.355	0.645	0.055	0.105	0.395	0.775	0.060	0.090	0.395	0.790
		0.01	0.012	0.040	0.120	0.325	0.0	0.020	0.150	0.420	0.010	0.035	0.150	0.540	0.007	0.030	0.190	0.525
HTEST	T	0.05	0.047	0.150	0.250	0.500	0.037	0.105	0.305	0.585	0.052	0.110	0.360	0.745	0.050	0.055	0.325	0.750
		0.01	0.007	0.030	0.065	0.270	0.0	0.020	0.115	0.315	0.002	0.035	0.120	0.455	0.005	0.015	0.170	0.460
JONKE	T	0.05	0.037	0.230	0.530	0.770	0.045	0.255	0.610	0.865	0.050	0.260	0.690	0.975	0.052	0.315	0.765	0.965
		0.01	0.010	0.110	0.190	0.445	0.0	0.110	0.370	0.665	0.012	0.135	0.440	0.845	0.012	0.105	0.505	0.890
BARTH	T	0.05	0.037	0.195	0.495	0.745	0.037	0.240	0.600	0.865	0.057	0.285	0.665	0.960	0.042	0.260	0.735	0.960
		0.01	0.007	0.100	0.190	0.465	0.002	0.085	0.325	0.620	0.012	0.095	0.385	0.815	0.017	0.090	0.455	0.860
CHAKU	T	0.05	0.042	0.195	0.475	0.695	0.032	0.235	0.575	0.835	0.060	0.280	0.620	0.940	0.042	0.255	0.690	0.960
		0.01	0.007	0.090	0.155	0.425	0.0	0.080	0.295	0.575	0.015	0.110	0.370	0.775	0.012	0.070	0.395	0.830
ATMAX	T	0.05	0.040	0.230	0.565	0.805	0.042	0.260	0.590	0.880	0.060	0.260	0.700	0.950	0.060	0.290	0.750	0.970
		0.01	0.010	0.135	0.210	0.490	0.0	0.135	0.385	0.665	0.010	0.140	0.415	0.835	0.010	0.110	0.505	0.880
LACON	T	0.05	0.040	0.230	0.565	0.805	0.047	0.270	0.630	0.875	0.052	0.270	0.725	0.970	0.052	0.320	0.815	0.970
		0.01	0.010	0.105	0.210	0.490	0.002	0.115	0.405	0.690	0.010	0.140	0.455	0.885	0.015	0.090	0.540	0.905
JENNS	T	0.05	0.045	0.240	0.555	0.795	0.045	0.265	0.620	0.865	0.047	0.265	0.680	0.955	0.055	0.290	0.765	0.965
		0.01	0.010	0.135	0.200	0.455	0.0	0.130	0.415	0.665	0.010	0.115	0.370	0.805	0.012	0.115	0.490	0.880
PURIS	T	0.05	0.045	0.240	0.555	0.795	0.055	0.275	0.635	0.875	0.050	0.270	0.710	0.975	0.050	0.335	0.815	0.975
		0.01	0.010	0.105	0.200	0.455	0.002	0.120	0.425	0.695	0.007	0.130	0.405	0.855	0.015	0.100	0.535	0.895
FTEST	G	0.05	0.057	0.085	0.245	0.590	0.037	0.095	0.360	0.655	0.055	0.130	0.385	0.770	0.060	0.095	0.405	0.795
		0.01	0.012	0.025	0.110	0.330	0.0	0.020	0.215	0.415	0.010	0.045	0.175	0.525	0.007	0.015	0.150	0.580
HTEST	G	0.05	0.047	0.080	0.215	0.535	0.037	0.100	0.340	0.605	0.052	0.120	0.370	0.725	0.050	0.090	0.305	0.740
		0.01	0.007	0.025	0.040	0.260	0.0	0.025	0.110	0.305	0.002	0.025	0.120	0.415	0.005	0.015	0.100	0.495
JONKE	G	0.05	0.037	0.210	0.365	0.655	0.045	0.170	0.410	0.670	0.050	0.235	0.475	0.700	0.052	0.190	0.360	0.770
		0.01	0.010	0.045	0.145	0.380	0.0	0.055	0.180	0.380	0.012	0.070	0.185	0.425	0.012	0.055	0.190	0.490
BARTH	G	0.05	0.037	0.180	0.385	0.755	0.037	0.225	0.535	0.825	0.057	0.255	0.590	0.885	0.042	0.215	0.570	0.930
		0.01	0.007	0.055	0.175	0.480	0.002	0.040	0.280	0.585	0.012	0.085	0.335	0.720	0.017	0.045	0.355	0.795
CHAKU	G	0.05	0.042	0.170	0.385	0.725	0.032	0.215	0.520	0.780	0.060	0.225	0.615	0.865	0.042	0.210	0.535	0.930
		0.01	0.007	0.045	0.165	0.410	0.0	0.055	0.245	0.525	0.015	0.060	0.290	0.660	0.012	0.045	0.305	0.745
ATMAX	G	0.05	0.040	0.205	0.375	0.710	0.042	0.200	0.445	0.740	0.060	0.295	0.540	0.800	0.060	0.225	0.510	0.885
		0.01	0.010	0.045	0.150	0.435	0.0	0.040	0.225	0.500	0.010	0.080	0.260	0.620	0.010	0.045	0.295	0.685
LACON	G	0.05	0.047	0.205	0.375	0.710	0.047	0.200	0.445	0.730	0.052	0.255	0.485	0.765	0.052	0.190	0.415	0.825
		0.01	0.010	0.045	0.150	0.435	0.002	0.040	0.210	0.460	0.010	0.075	0.205	0.530	0.015	0.050	0.205	0.580
JENNS	G	0.05	0.045	0.245	0.380	0.705	0.045	0.220	0.430	0.695	0.047	0.275	0.515	0.780	0.055	0.210	0.500	0.875
		0.01	0.010	0.050	0.145	0.410	0.0	0.045	0.230	0.470	0.010	0.065	0.215	0.555	0.012	0.050	0.300	0.635
PURIS	G	0.05	0.045	0.245	0.380	0.705	0.055	0.200	0.420	0.685	0.050	0.250	0.470	0.725	0.050	0.195	0.405	0.785
		0.01	0.010	0.050	0.145	0.410	0.002	0.050	0.200	0.405	0.007	0.060	0.175	0.460	0.015	0.060	0.205	0.510

TABLE A28

EMPIRICAL POWERS OF NINE ONE-WAY TESTS UNDER NORMAL DISTRIBUTION: $n = 20$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
FTEST	T	0.05	0.035	0.095	0.320	0.600	0.060	0.090	0.330	0.675	0.037	0.130	0.370	0.765	0.035	0.145	0.395	0.835
		0.01	0.005	0.025	0.140	0.350	0.015	0.020	0.140	0.420	0.015	0.020	0.150	0.515	0.010	0.035	0.205	0.580
HTEST	T	0.05	0.037	0.095	0.295	0.605	0.045	0.075	0.325	0.645	0.040	0.120	0.345	0.740	0.032	0.110	0.375	0.800
		0.01	0.005	0.030	0.085	0.295	0.007	0.010	0.110	0.350	0.007	0.015	0.125	0.450	0.007	0.030	0.165	0.490
JOYKE	T	0.05	0.050	0.165	0.515	0.780	0.072	0.235	0.555	0.895	0.050	0.240	0.665	0.915	0.037	0.330	0.730	0.975
		0.01	0.010	0.050	0.250	0.550	0.015	0.060	0.335	0.730	0.007	0.100	0.420	0.815	0.002	0.150	0.505	0.895
BARTH	T	0.05	0.060	0.175	0.515	0.795	0.050	0.245	0.520	0.895	0.045	0.210	0.685	0.930	0.025	0.320	0.705	0.975
		0.01	0.012	0.030	0.265	0.550	0.020	0.060	0.335	0.685	0.007	0.105	0.365	0.800	0.005	0.140	0.460	0.895
CHAKO	T	0.05	0.047	0.155	0.495	0.765	0.075	0.225	0.515	0.880	0.037	0.210	0.645	0.905	0.027	0.305	0.690	0.965
		0.01	0.015	0.040	0.210	0.515	0.017	0.050	0.315	0.640	0.010	0.090	0.345	0.765	0.005	0.125	0.440	0.855
ATMAX	T	0.05	0.047	0.170	0.535	0.800	0.060	0.285	0.585	0.910	0.050	0.240	0.675	0.930	0.035	0.340	0.755	0.970
		0.01	0.010	0.050	0.265	0.570	0.015	0.075	0.330	0.725	0.007	0.090	0.430	0.825	0.005	0.140	0.465	0.895
LRCON	T	0.05	0.047	0.170	0.535	0.800	0.065	0.280	0.590	0.910	0.037	0.255	0.705	0.940	0.030	0.355	0.755	0.995
		0.01	0.010	0.050	0.265	0.570	0.015	0.080	0.360	0.755	0.007	0.105	0.430	0.845	0.005	0.150	0.515	0.920
JWENS	T	0.05	0.057	0.170	0.530	0.815	0.060	0.290	0.570	0.900	0.055	0.245	0.695	0.940	0.032	0.355	0.755	0.970
		0.01	0.010	0.055	0.260	0.570	0.010	0.060	0.340	0.730	0.007	0.090	0.440	0.830	0.005	0.155	0.475	0.890
PURIS	T	0.05	0.057	0.170	0.530	0.815	0.062	0.305	0.590	0.910	0.040	0.260	0.705	0.950	0.042	0.385	0.770	0.995
		0.01	0.010	0.055	0.260	0.570	0.012	0.075	0.365	0.765	0.007	0.100	0.445	0.830	0.005	0.175	0.545	0.915
FTEST	G	0.05	0.035	0.060	0.255	0.650	0.060	0.095	0.345	0.665	0.037	0.130	0.375	0.785	0.035	0.125	0.420	0.785
		0.01	0.005	0.020	0.095	0.340	0.015	0.025	0.130	0.405	0.015	0.040	0.140	0.545	0.010	0.040	0.220	0.555
HTEST	G	0.05	0.037	0.055	0.245	0.590	0.045	0.090	0.305	0.620	0.040	0.110	0.310	0.755	0.032	0.115	0.375	0.725
		0.01	0.005	0.010	0.075	0.310	0.007	0.015	0.130	0.310	0.007	0.035	0.105	0.480	0.007	0.030	0.170	0.495
JOYKE	G	0.05	0.050	0.145	0.365	0.700	0.072	0.120	0.470	0.680	0.050	0.200	0.425	0.730	0.037	0.235	0.380	0.740
		0.01	0.010	0.030	0.135	0.435	0.015	0.035	0.225	0.370	0.007	0.045	0.155	0.495	0.002	0.070	0.190	0.425
BARTH	G	0.05	0.060	0.170	0.415	0.810	0.050	0.160	0.530	0.830	0.045	0.175	0.585	0.980	0.025	0.240	0.590	0.950
		0.01	0.012	0.040	0.175	0.570	0.020	0.045	0.280	0.610	0.007	0.080	0.295	0.730	0.005	0.095	0.365	0.780
CHAKO	G	0.05	0.047	0.165	0.405	0.775	0.075	0.150	0.485	0.815	0.037	0.205	0.545	0.870	0.027	0.230	0.585	0.920
		0.01	0.015	0.030	0.165	0.465	0.017	0.030	0.250	0.510	0.010	0.085	0.230	0.690	0.005	0.090	0.340	0.705
ATMAX	G	0.05	0.047	0.165	0.405	0.730	0.060	0.155	0.500	0.780	0.050	0.210	0.540	0.805	0.035	0.285	0.545	0.860
		0.01	0.010	0.035	0.165	0.470	0.015	0.030	0.265	0.520	0.007	0.085	0.245	0.605	0.005	0.095	0.275	0.690
LRCON	G	0.05	0.047	0.165	0.405	0.730	0.065	0.140	0.510	0.720	0.037	0.185	0.455	0.780	0.030	0.250	0.435	0.785
		0.01	0.010	0.035	0.165	0.470	0.015	0.025	0.240	0.435	0.007	0.055	0.175	0.540	0.005	0.070	0.260	0.560
JWENS	G	0.05	0.057	0.180	0.415	0.730	0.060	0.155	0.505	0.750	0.055	0.235	0.540	0.785	0.032	0.310	0.545	0.850
		0.01	0.010	0.035	0.175	0.435	0.010	0.030	0.260	0.455	0.007	0.080	0.240	0.595	0.005	0.095	0.270	0.685
PURIS	G	0.05	0.057	0.180	0.415	0.730	0.062	0.135	0.510	0.710	0.040	0.210	0.460	0.765	0.042	0.260	0.425	0.790
		0.01	0.010	0.035	0.175	0.435	0.012	0.040	0.225	0.410	0.007	0.050	0.170	0.505	0.005	0.070	0.200	0.525

TABLE A29

EMPIRICAL POWERS OF NINE TWO-WAY TESTS UNDER RECTANGULAR DISTRIBUTION: $n = 5$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
F2WAY	T	0.05	0.065	0.082	0.232	0.455	0.062	0.120	0.242	0.520	0.045	0.125	0.300	0.610	0.047	0.110	0.342	0.662
		0.01	0.011	0.020	0.057	0.152	0.014	0.040	0.090	0.220	0.005	0.027	0.102	0.317	0.011	0.030	0.122	0.400
PASFT	T	0.05	0.041	0.115	0.332	0.560	0.050	0.215	0.462	0.767	0.035	0.262	0.610	0.832	0.054	0.267	0.647	0.930
		0.01	0.005	0.032	0.097	0.262	0.009	0.072	0.160	0.407	0.005	0.070	0.277	0.577	0.009	0.087	0.342	0.700
FRIED	T	0.05	0.026	0.060	0.110	0.267	0.040	0.087	0.145	0.355	0.031	0.067	0.202	0.430	0.024	0.072	0.212	0.477
		0.01	0.002	0.0	0.012	0.030	0.012	0.025	0.063	0.122	0.001	0.0	0.017	0.095	0.0	0.005	0.022	0.152
PHENS	T	0.05	0.041	0.115	0.332	0.560	0.050	0.215	0.462	0.767	0.037	0.277	0.627	0.845	0.051	0.272	0.688	0.935
		0.01	0.005	0.032	0.097	0.262	0.011	0.080	0.182	0.438	0.005	0.075	0.300	0.605	0.011	0.097	0.360	0.727
ATMX2	T	0.05	0.056	0.162	0.450	0.760	0.056	0.227	0.542	0.872	0.042	0.272	0.655	0.930	0.056	0.285	0.750	0.967
		0.01	0.011	0.042	0.172	0.422	0.015	0.087	0.265	0.590	0.012	0.097	0.382	0.740	0.007	0.105	0.430	0.810
LRGN2	T	0.05	0.056	0.162	0.450	0.760	0.056	0.237	0.552	0.882	0.046	0.272	0.680	0.942	0.055	0.315	0.785	0.985
		0.01	0.011	0.042	0.172	0.422	0.016	0.087	0.252	0.600	0.007	0.115	0.415	0.745	0.006	0.095	0.445	0.840
HCLLT	T	0.05	0.047	0.155	0.420	0.712	0.059	0.265	0.540	0.850	0.044	0.285	0.662	0.895	0.055	0.275	0.715	0.970
		0.01	0.004	0.025	0.112	0.390	0.011	0.077	0.197	0.492	0.006	0.075	0.335	0.662	0.005	0.087	0.385	0.785
SHORT	T	0.05	0.039	0.122	0.332	0.557	0.059	0.202	0.405	0.700	0.032	0.227	0.567	0.775	0.050	0.237	0.587	0.875
		0.01	0.009	0.035	0.095	0.245	0.010	0.070	0.130	0.365	0.004	0.050	0.212	0.485	0.004	0.070	0.270	0.577
SEVAT	T	0.05	0.054	0.167	0.422	0.712	0.050	0.242	0.505	0.845	0.045	0.275	0.657	0.897	0.049	0.300	0.727	0.960
		0.01	0.005	0.032	0.120	0.287	0.011	0.070	0.192	0.495	0.009	0.092	0.357	0.647	0.007	0.092	0.382	0.750
F2WAY	G	0.05	0.065	0.102	0.190	0.435	0.062	0.095	0.267	0.527	0.045	0.077	0.237	0.597	0.047	0.087	0.313	0.655
		0.01	0.011	0.025	0.075	0.165	0.014	0.022	0.082	0.227	0.005	0.022	0.075	0.297	0.011	0.017	0.125	0.332
PASFT	G	0.05	0.041	0.107	0.255	0.450	0.050	0.182	0.352	0.525	0.035	0.155	0.310	0.492	0.054	0.137	0.307	0.490
		0.01	0.005	0.025	0.085	0.162	0.009	0.032	0.075	0.167	0.005	0.022	0.072	0.177	0.009	0.035	0.105	0.205
FRIED	G	0.05	0.026	0.047	0.112	0.257	0.040	0.060	0.155	0.307	0.031	0.070	0.142	0.332	0.024	0.047	0.180	0.345
		0.01	0.002	0.002	0.007	0.027	0.012	0.012	0.040	0.097	0.001	0.005	0.005	0.040	0.0	0.0	0.017	0.060
PHENS	G	0.05	0.041	0.107	0.255	0.450	0.050	0.180	0.352	0.525	0.037	0.172	0.330	0.510	0.051	0.145	0.322	0.517
		0.01	0.005	0.025	0.085	0.162	0.011	0.035	0.085	0.192	0.005	0.032	0.082	0.210	0.011	0.040	0.117	0.240
ATMX2	G	0.05	0.056	0.182	0.370	0.635	0.056	0.190	0.407	0.715	0.042	0.210	0.445	0.772	0.056	0.207	0.542	0.830
		0.01	0.011	0.057	0.117	0.325	0.015	0.050	0.177	0.407	0.012	0.045	0.177	0.477	0.007	0.072	0.255	0.570
LRGN2	G	0.05	0.056	0.182	0.370	0.635	0.056	0.190	0.387	0.680	0.046	0.190	0.400	0.710	0.055	0.180	0.450	0.735
		0.01	0.011	0.057	0.117	0.325	0.016	0.047	0.162	0.395	0.007	0.035	0.155	0.420	0.006	0.065	0.182	0.425
HCLLT	G	0.05	0.047	0.162	0.327	0.525	0.059	0.180	0.320	0.502	0.044	0.167	0.317	0.447	0.055	0.180	0.295	0.447
		0.01	0.004	0.030	0.067	0.142	0.011	0.030	0.087	0.180	0.006	0.022	0.085	0.137	0.005	0.042	0.092	0.165
SHORT	G	0.05	0.039	0.135	0.290	0.497	0.059	0.188	0.397	0.655	0.032	0.162	0.347	0.675	0.050	0.170	0.472	0.737
		0.01	0.009	0.030	0.090	0.222	0.010	0.035	0.085	0.225	0.004	0.032	0.087	0.275	0.004	0.045	0.145	0.350
SEVAT	G	0.05	0.054	0.142	0.337	0.563	0.050	0.182	0.345	0.557	0.045	0.160	0.340	0.563	0.049	0.165	0.330	0.547
		0.01	0.005	0.027	0.075	0.152	0.011	0.032	0.090	0.202	0.009	0.025	0.085	0.188	0.007	0.050	0.092	0.202

TABLE A30

EMPIRICAL POWERS OF NINE TWO-WAY TESTS UNDER RECTANGULAR DISTRIBUTION: $n = 10$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
F2WAY	T	0.05	0.049	0.105	0.292	0.542	0.049	0.110	0.330	0.605	0.057	0.127	0.342	0.677	0.040	0.137	0.392	0.752
		0.01	0.010	0.032	0.117	0.245	0.007	0.035	0.115	0.327	0.005	0.040	0.152	0.457	0.002	0.042	0.162	0.487
PAGE2	T	0.05	0.032	0.157	0.380	0.627	0.050	0.192	0.547	0.755	0.047	0.287	0.575	0.872	0.041	0.300	0.672	0.927
		0.01	0.005	0.055	0.130	0.322	0.007	0.057	0.245	0.477	0.007	0.090	0.302	0.660	0.004	0.085	0.417	0.762
FRIED	T	0.05	0.025	0.072	0.160	0.337	0.041	0.075	0.260	0.432	0.050	0.090	0.225	0.540	0.035	0.107	0.272	0.567
		0.01	0.006	0.010	0.057	0.132	0.004	0.015	0.065	0.188	0.009	0.025	0.065	0.257	0.002	0.025	0.085	0.300
PHENS	T	0.05	0.032	0.157	0.380	0.627	0.050	0.212	0.580	0.765	0.052	0.310	0.600	0.895	0.032	0.310	0.677	0.938
		0.01	0.005	0.055	0.130	0.322	0.007	0.077	0.260	0.510	0.009	0.095	0.290	0.662	0.004	0.100	0.445	0.792
ATMX2	T	0.05	0.037	0.192	0.537	0.795	0.046	0.242	0.655	0.900	0.045	0.332	0.672	0.950	0.044	0.310	0.770	0.970
		0.01	0.005	0.077	0.250	0.497	0.005	0.085	0.392	0.667	0.010	0.130	0.370	0.787	0.007	0.115	0.505	0.862
LRCN2	T	0.05	0.037	0.192	0.537	0.785	0.046	0.250	0.660	0.900	0.049	0.320	0.695	0.952	0.047	0.325	0.787	0.972
		0.01	0.005	0.077	0.250	0.497	0.005	0.082	0.395	0.695	0.010	0.132	0.407	0.817	0.005	0.125	0.552	0.907
HCLLT	T	0.05	0.032	0.180	0.487	0.757	0.044	0.227	0.637	0.865	0.042	0.295	0.650	0.927	0.045	0.300	0.747	0.965
		0.01	0.006	0.063	0.205	0.442	0.005	0.067	0.310	0.597	0.007	0.105	0.330	0.742	0.006	0.110	0.455	0.845
SHORT	T	0.05	0.027	0.137	0.337	0.570	0.042	0.185	0.490	0.707	0.039	0.270	0.525	0.832	0.039	0.270	0.630	0.897
		0.01	0.004	0.050	0.130	0.317	0.009	0.052	0.220	0.425	0.009	0.075	0.235	0.577	0.004	0.085	0.310	0.697
SEVAT	T	0.05	0.036	0.180	0.492	0.740	0.050	0.240	0.647	0.860	0.047	0.297	0.645	0.915	0.042	0.320	0.762	0.962
		0.01	0.005	0.067	0.190	0.422	0.006	0.067	0.347	0.585	0.006	0.115	0.342	0.720	0.005	0.100	0.485	0.830
F2WAY	G	0.05	0.049	0.095	0.285	0.557	0.049	0.082	0.280	0.610	0.057	0.105	0.325	0.712	0.040	0.120	0.360	0.813
		0.01	0.010	0.025	0.082	0.267	0.007	0.025	0.115	0.365	0.005	0.017	0.137	0.445	0.002	0.040	0.150	0.552
PAGE2	G	0.05	0.032	0.117	0.292	0.512	0.050	0.117	0.297	0.555	0.047	0.180	0.337	0.577	0.041	0.170	0.390	0.607
		0.01	0.005	0.030	0.090	0.220	0.007	0.032	0.085	0.255	0.007	0.055	0.122	0.307	0.004	0.065	0.137	0.320
FRIED	G	0.05	0.025	0.057	0.152	0.317	0.041	0.082	0.180	0.465	0.050	0.082	0.267	0.512	0.035	0.102	0.250	0.602
		0.01	0.006	0.012	0.037	0.145	0.004	0.015	0.057	0.197	0.009	0.015	0.072	0.197	0.002	0.032	0.085	0.267
PHENS	G	0.05	0.032	0.117	0.282	0.512	0.050	0.125	0.313	0.575	0.052	0.200	0.350	0.595	0.032	0.172	0.390	0.612
		0.01	0.005	0.030	0.090	0.220	0.007	0.035	0.097	0.270	0.009	0.057	0.130	0.317	0.004	0.065	0.145	0.345
ATMX2	G	0.05	0.037	0.152	0.400	0.692	0.046	0.182	0.420	0.762	0.045	0.212	0.505	0.830	0.044	0.245	0.547	0.887
		0.01	0.005	0.042	0.157	0.390	0.005	0.057	0.210	0.422	0.010	0.072	0.230	0.595	0.007	0.067	0.272	0.688
LRCN2	G	0.05	0.037	0.152	0.400	0.692	0.046	0.172	0.397	0.742	0.049	0.192	0.447	0.775	0.047	0.227	0.477	0.790
		0.01	0.005	0.042	0.157	0.390	0.005	0.042	0.182	0.410	0.010	0.063	0.185	0.500	0.005	0.065	0.207	0.542
HCLLT	G	0.05	0.032	0.150	0.350	0.610	0.044	0.122	0.317	0.607	0.042	0.167	0.382	0.637	0.045	0.200	0.380	0.587
		0.01	0.006	0.030	0.112	0.260	0.005	0.040	0.112	0.285	0.007	0.037	0.112	0.300	0.006	0.057	0.122	0.302
SHORT	G	0.05	0.027	0.097	0.277	0.535	0.042	0.130	0.360	0.667	0.039	0.195	0.432	0.752	0.039	0.207	0.472	0.837
		0.01	0.004	0.040	0.125	0.277	0.009	0.047	0.125	0.362	0.009	0.055	0.202	0.480	0.004	0.050	0.212	0.557
SEVAT	G	0.05	0.036	0.162	0.367	0.622	0.050	0.152	0.357	0.637	0.047	0.190	0.385	0.672	0.042	0.222	0.417	0.675
		0.01	0.005	0.030	0.105	0.272	0.006	0.035	0.117	0.287	0.006	0.052	0.130	0.330	0.005	0.057	0.132	0.355

TABLE A31

EMPIRICAL POWERS OF NINE TWO-WAY TESTS UNDER RECTANGULAR DISTRIBUTION: $n = 15$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
F2WAY	T	0.05	0.042	0.110	0.325	0.535	0.055	0.110	0.375	0.690	0.040	0.115	0.420	0.690	0.050	0.115	0.385	0.790
		0.01	0.005	0.035	0.115	0.280	0.010	0.045	0.130	0.435	0.007	0.025	0.215	0.450	0.005	0.035	0.195	0.540
PAGEF	T	0.05	0.037	0.125	0.395	0.600	0.045	0.225	0.520	0.775	0.030	0.335	0.655	0.880	0.082	0.260	0.670	0.965
		0.01	0.015	0.050	0.175	0.365	0.005	0.075	0.265	0.545	0.002	0.090	0.360	0.625	0.020	0.085	0.390	0.810
FRIED	T	0.05	0.055	0.115	0.220	0.380	0.045	0.100	0.230	0.545	0.035	0.100	0.325	0.520	0.040	0.085	0.295	0.640
		0.01	0.015	0.025	0.065	0.185	0.0	0.035	0.080	0.275	0.002	0.010	0.100	0.280	0.007	0.035	0.130	0.345
PHENS	T	0.05	0.037	0.125	0.395	0.600	0.047	0.225	0.525	0.790	0.030	0.325	0.670	0.885	0.087	0.285	0.695	0.970
		0.01	0.015	0.050	0.175	0.365	0.007	0.080	0.280	0.575	0.002	0.105	0.370	0.650	0.020	0.110	0.415	0.820
ATX2	T	0.05	0.037	0.225	0.590	0.805	0.047	0.285	0.650	0.890	0.040	0.320	0.715	0.950	0.060	0.300	0.730	0.985
		0.01	0.012	0.050	0.265	0.510	0.007	0.105	0.365	0.715	0.007	0.110	0.430	0.780	0.012	0.115	0.505	0.910
LRCN2	T	0.05	0.037	0.225	0.590	0.805	0.052	0.300	0.690	0.910	0.037	0.330	0.735	0.960	0.060	0.285	0.775	0.985
		0.01	0.012	0.050	0.265	0.510	0.012	0.105	0.375	0.715	0.005	0.120	0.485	0.800	0.015	0.130	0.535	0.935
HCLLT	T	0.05	0.045	0.195	0.535	0.760	0.045	0.270	0.630	0.885	0.035	0.315	0.700	0.920	0.065	0.290	0.715	0.975
		0.01	0.012	0.060	0.215	0.440	0.005	0.090	0.340	0.655	0.002	0.110	0.415	0.740	0.010	0.125	0.495	0.900
SHORT	T	0.05	0.045	0.175	0.365	0.545	0.045	0.220	0.470	0.745	0.037	0.250	0.575	0.825	0.067	0.220	0.655	0.920
		0.01	0.007	0.045	0.145	0.330	0.012	0.075	0.200	0.525	0.005	0.080	0.310	0.540	0.015	0.070	0.350	0.740
SEVAT	T	0.05	0.032	0.195	0.510	0.755	0.047	0.255	0.640	0.870	0.035	0.325	0.710	0.925	0.067	0.285	0.735	0.975
		0.01	0.007	0.065	0.225	0.445	0.007	0.095	0.335	0.660	0.002	0.120	0.440	0.755	0.015	0.115	0.485	0.880
F2WAY	G	0.05	0.042	0.080	0.285	0.620	0.055	0.100	0.320	0.675	0.040	0.120	0.330	0.710	0.050	0.100	0.465	0.805
		0.01	0.005	0.025	0.085	0.320	0.010	0.040	0.115	0.420	0.007	0.025	0.155	0.450	0.005	0.035	0.200	0.605
PAGEF	G	0.05	0.037	0.150	0.270	0.470	0.045	0.140	0.360	0.610	0.030	0.190	0.375	0.610	0.082	0.195	0.320	0.610
		0.01	0.015	0.055	0.115	0.275	0.005	0.025	0.130	0.280	0.002	0.055	0.115	0.310	0.020	0.050	0.090	0.340
FRIED	G	0.05	0.055	0.075	0.175	0.455	0.045	0.110	0.230	0.485	0.035	0.105	0.235	0.560	0.040	0.090	0.270	0.620
		0.01	0.015	0.010	0.060	0.195	0.0	0.030	0.075	0.275	0.002	0.015	0.070	0.250	0.007	0.020	0.115	0.325
PHENS	G	0.05	0.037	0.150	0.270	0.470	0.047	0.155	0.380	0.615	0.030	0.185	0.375	0.610	0.087	0.200	0.350	0.640
		0.01	0.015	0.055	0.115	0.275	0.007	0.025	0.145	0.305	0.002	0.060	0.130	0.325	0.020	0.060	0.120	0.345
ATX2	G	0.05	0.037	0.190	0.410	0.720	0.047	0.175	0.475	0.755	0.040	0.225	0.430	0.820	0.060	0.260	0.530	0.845
		0.01	0.012	0.050	0.175	0.430	0.007	0.040	0.230	0.480	0.007	0.075	0.220	0.580	0.012	0.105	0.270	0.590
LRCN2	G	0.05	0.037	0.190	0.410	0.720	0.052	0.160	0.455	0.740	0.037	0.200	0.435	0.765	0.060	0.225	0.415	0.745
		0.01	0.012	0.050	0.175	0.430	0.012	0.040	0.205	0.455	0.005	0.065	0.195	0.485	0.015	0.065	0.185	0.480
HCLLT	G	0.05	0.045	0.180	0.385	0.630	0.045	0.155	0.415	0.665	0.035	0.215	0.365	0.630	0.065	0.205	0.330	0.605
		0.01	0.012	0.040	0.125	0.290	0.005	0.030	0.190	0.360	0.002	0.065	0.165	0.345	0.010	0.040	0.105	0.310
SHORT	G	0.05	0.045	0.110	0.270	0.560	0.045	0.195	0.410	0.720	0.037	0.180	0.440	0.715	0.067	0.210	0.505	0.855
		0.01	0.007	0.035	0.080	0.295	0.012	0.060	0.160	0.405	0.005	0.085	0.200	0.525	0.015	0.060	0.235	0.565
SEVAT	G	0.05	0.032	0.155	0.380	0.650	0.047	0.145	0.405	0.655	0.035	0.220	0.380	0.660	0.067	0.210	0.365	0.630
		0.01	0.007	0.035	0.140	0.340	0.007	0.045	0.195	0.355	0.002	0.060	0.185	0.370	0.015	0.065	0.135	0.375

TABLE A32

EMPIRICAL POWERS OF NINE TWO-WAY TESTS UNDER RECTANGULAR DISTRIBUTION: $n = 20$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
F2WAY	T	0.05	0.042	0.085	0.315	0.630	0.050	0.080	0.335	0.660	0.057	0.120	0.350	0.725	0.037	0.105	0.420	0.795
		0.01	0.005	0.030	0.120	0.260	0.010	0.020	0.130	0.405	0.010	0.055	0.150	0.485	0.0	0.035	0.200	0.620
PAGE2	T	0.05	0.075	0.190	0.440	0.650	0.045	0.210	0.525	0.775	0.047	0.310	0.595	0.890	0.055	0.270	0.710	0.960
		0.01	0.015	0.035	0.215	0.340	0.012	0.030	0.220	0.535	0.010	0.105	0.295	0.670	0.0	0.130	0.450	0.820
FRIED	T	0.05	0.070	0.070	0.250	0.405	0.052	0.085	0.265	0.485	0.050	0.125	0.240	0.585	0.060	0.085	0.345	0.670
		0.01	0.010	0.005	0.070	0.145	0.007	0.025	0.095	0.260	0.010	0.045	0.080	0.310	0.002	0.025	0.130	0.375
PHENS	T	0.05	0.075	0.190	0.440	0.650	0.045	0.220	0.545	0.800	0.047	0.315	0.595	0.905	0.060	0.280	0.740	0.960
		0.01	0.015	0.035	0.215	0.340	0.012	0.040	0.245	0.555	0.010	0.130	0.285	0.710	0.0	0.140	0.480	0.850
ATMX2	T	0.05	0.047	0.225	0.530	0.850	0.057	0.200	0.605	0.900	0.032	0.320	0.640	0.970	0.072	0.275	0.815	0.975
		0.01	0.002	0.065	0.295	0.575	0.015	0.065	0.340	0.705	0.010	0.105	0.385	0.800	0.007	0.140	0.515	0.890
LRCN2	T	0.05	0.047	0.225	0.530	0.850	0.060	0.225	0.640	0.910	0.032	0.355	0.630	0.960	0.067	0.330	0.830	0.985
		0.01	0.002	0.065	0.295	0.575	0.015	0.060	0.365	0.720	0.015	0.105	0.395	0.820	0.002	0.160	0.535	0.915
HCLLT	T	0.05	0.057	0.205	0.480	0.780	0.052	0.195	0.600	0.880	0.035	0.345	0.615	0.950	0.057	0.300	0.785	0.970
		0.01	0.0	0.050	0.230	0.470	0.012	0.040	0.295	0.665	0.015	0.085	0.335	0.755	0.005	0.140	0.485	0.890
SHORT	T	0.05	0.060	0.150	0.400	0.565	0.045	0.235	0.490	0.725	0.057	0.260	0.520	0.840	0.060	0.265	0.650	0.915
		0.01	0.015	0.015	0.175	0.290	0.007	0.045	0.225	0.460	0.015	0.090	0.240	0.610	0.005	0.110	0.390	0.755
SEVAT	T	0.05	0.055	0.205	0.505	0.800	0.060	0.200	0.600	0.880	0.032	0.355	0.595	0.925	0.062	0.310	0.780	0.965
		0.01	0.0	0.065	0.245	0.455	0.015	0.060	0.325	0.655	0.012	0.105	0.365	0.755	0.002	0.150	0.495	0.895
F2WAY	G	0.05	0.047	0.110	0.265	0.605	0.050	0.125	0.345	0.720	0.057	0.075	0.390	0.730	0.037	0.095	0.375	0.790
		0.01	0.005	0.025	0.110	0.375	0.010	0.035	0.140	0.425	0.010	0.025	0.165	0.490	0.0	0.030	0.180	0.595
PAGE2	G	0.05	0.075	0.120	0.315	0.520	0.045	0.195	0.380	0.650	0.047	0.165	0.400	0.670	0.055	0.220	0.385	0.665
		0.01	0.015	0.030	0.125	0.275	0.012	0.045	0.110	0.290	0.010	0.030	0.155	0.325	0.0	0.065	0.180	0.360
FRIED	G	0.05	0.070	0.080	0.220	0.475	0.052	0.090	0.225	0.465	0.050	0.105	0.315	0.575	0.060	0.120	0.295	0.630
		0.01	0.010	0.020	0.050	0.225	0.007	0.020	0.075	0.210	0.010	0.025	0.100	0.280	0.002	0.010	0.130	0.350
PHENS	G	0.05	0.075	0.120	0.315	0.520	0.045	0.200	0.395	0.665	0.047	0.170	0.410	0.675	0.060	0.230	0.390	0.690
		0.01	0.015	0.030	0.125	0.275	0.012	0.045	0.120	0.310	0.010	0.040	0.175	0.365	0.0	0.070	0.200	0.365
ATMX2	G	0.05	0.047	0.170	0.475	0.745	0.057	0.220	0.490	0.760	0.032	0.195	0.555	0.860	0.072	0.275	0.540	0.875
		0.01	0.002	0.035	0.170	0.460	0.015	0.045	0.240	0.535	0.010	0.055	0.270	0.585	0.007	0.095	0.290	0.620
LRCN2	G	0.05	0.047	0.170	0.475	0.745	0.060	0.210	0.465	0.750	0.032	0.175	0.480	0.775	0.067	0.215	0.480	0.760
		0.01	0.002	0.035	0.170	0.460	0.015	0.040	0.215	0.505	0.015	0.045	0.230	0.505	0.002	0.050	0.235	0.550
HCLLT	G	0.05	0.057	0.150	0.410	0.670	0.052	0.215	0.410	0.685	0.035	0.155	0.415	0.725	0.057	0.210	0.385	0.660
		0.01	0.0	0.035	0.130	0.345	0.012	0.040	0.170	0.385	0.015	0.040	0.185	0.395	0.005	0.050	0.180	0.395
SHORT	G	0.05	0.060	0.140	0.330	0.580	0.045	0.220	0.410	0.735	0.057	0.185	0.545	0.790	0.060	0.250	0.510	0.820
		0.01	0.015	0.030	0.110	0.325	0.007	0.060	0.165	0.415	0.015	0.050	0.240	0.530	0.005	0.070	0.285	0.600
SEVAT	G	0.05	0.055	0.165	0.415	0.675	0.060	0.210	0.430	0.680	0.032	0.180	0.475	0.695	0.062	0.210	0.425	0.705
		0.01	0.0	0.045	0.140	0.340	0.015	0.045	0.190	0.405	0.012	0.040	0.170	0.410	0.002	0.050	0.190	0.395

TABLE A33

EMPIRICAL POWERS OF NINE TWO-WAY TESTS UNDER EXPONENTIAL DISTRIBUTION: $n = 5$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
F2WAY	T	0.05	0.039	0.127	0.317	0.560	0.040	0.080	0.310	0.660	0.042	0.127	0.390	0.740	0.050	0.122	0.390	0.782
		0.01	0.039	0.035	0.115	0.302	0.036	0.027	0.127	0.400	0.006	0.032	0.155	0.460	0.012	0.040	0.155	0.592
PAGE2	T	0.05	0.037	0.232	0.560	0.752	0.045	0.327	0.705	0.910	0.051	0.438	0.840	0.955	0.035	0.512	0.870	0.980
		0.01	0.012	0.082	0.235	0.425	0.007	0.105	0.327	0.660	0.007	0.157	0.490	0.813	0.011	0.212	0.597	0.917
FRIED	T	0.05	0.025	0.087	0.235	0.412	0.035	0.105	0.325	0.595	0.039	0.127	0.392	0.675	0.044	0.132	0.397	0.727
		0.01	0.002	0.002	0.025	0.063	0.009	0.025	0.142	0.367	0.004	0.020	0.097	0.270	0.004	0.015	0.107	0.400
PHENS	T	0.05	0.037	0.232	0.560	0.752	0.045	0.325	0.705	0.910	0.052	0.447	0.850	0.952	0.040	0.522	0.872	0.982
		0.01	0.012	0.082	0.235	0.425	0.009	0.112	0.347	0.685	0.009	0.180	0.510	0.825	0.011	0.225	0.602	0.925
ATMX2	T	0.05	0.035	0.242	0.570	0.797	0.044	0.240	0.660	0.915	0.051	0.365	0.735	0.950	0.045	0.347	0.800	0.975
		0.01	0.007	0.085	0.270	0.522	0.006	0.092	0.335	0.710	0.007	0.137	0.442	0.813	0.012	0.130	0.532	0.880
LRCN2	T	0.05	0.036	0.242	0.570	0.797	0.045	0.255	0.670	0.920	0.052	0.377	0.755	0.957	0.049	0.380	0.837	0.980
		0.01	0.007	0.085	0.270	0.522	0.006	0.080	0.347	0.727	0.007	0.145	0.477	0.852	0.015	0.155	0.545	0.920
HOLLT	T	0.05	0.042	0.255	0.585	0.807	0.059	0.297	0.720	0.932	0.055	0.400	0.795	0.972	0.046	0.427	0.870	0.987
		0.01	0.009	0.070	0.210	0.412	0.006	0.087	0.345	0.742	0.004	0.165	0.515	0.832	0.010	0.175	0.585	0.952
SHORT	T	0.05	0.040	0.230	0.505	0.680	0.051	0.285	0.620	0.855	0.040	0.380	0.760	0.925	0.044	0.430	0.795	0.967
		0.01	0.007	0.072	0.225	0.397	0.006	0.085	0.315	0.605	0.007	0.112	0.422	0.742	0.011	0.162	0.535	0.835
SEVAT	T	0.05	0.036	0.310	0.657	0.850	0.050	0.367	0.762	0.942	0.047	0.492	0.862	0.977	0.037	0.547	0.912	0.987
		0.01	0.010	0.095	0.270	0.487	0.010	0.117	0.442	0.770	0.011	0.217	0.587	0.877	0.010	0.265	0.675	0.957
F2WAY	G	0.05	0.039	0.097	0.310	0.557	0.040	0.122	0.347	0.665	0.042	0.087	0.370	0.740	0.050	0.110	0.357	0.767
		0.01	0.009	0.020	0.132	0.237	0.006	0.030	0.115	0.357	0.006	0.020	0.162	0.455	0.012	0.025	0.155	0.472
PAGE2	G	0.05	0.037	0.188	0.367	0.575	0.045	0.247	0.465	0.635	0.051	0.212	0.410	0.597	0.035	0.217	0.380	0.585
		0.01	0.012	0.030	0.117	0.222	0.007	0.055	0.147	0.217	0.007	0.050	0.140	0.255	0.011	0.045	0.125	0.202
FRIED	G	0.05	0.025	0.042	0.200	0.347	0.035	0.120	0.255	0.450	0.039	0.092	0.287	0.472	0.044	0.092	0.225	0.417
		0.01	0.002	0.0	0.017	0.020	0.009	0.025	0.075	0.132	0.004	0.005	0.045	0.063	0.004	0.022	0.020	0.052
PHENS	G	0.05	0.037	0.188	0.367	0.575	0.045	0.245	0.460	0.635	0.052	0.210	0.415	0.610	0.040	0.215	0.397	0.612
		0.01	0.012	0.030	0.117	0.222	0.009	0.065	0.162	0.252	0.009	0.052	0.165	0.290	0.011	0.047	0.130	0.222
ATMX2	G	0.05	0.035	0.205	0.467	0.740	0.044	0.230	0.522	0.807	0.051	0.220	0.530	0.835	0.041	0.242	0.600	0.897
		0.01	0.007	0.042	0.195	0.422	0.006	0.067	0.265	0.542	0.007	0.067	0.265	0.637	0.012	0.080	0.313	0.692
LRCN2	G	0.05	0.036	0.205	0.467	0.740	0.045	0.227	0.477	0.797	0.052	0.205	0.465	0.790	0.049	0.212	0.495	0.820
		0.01	0.007	0.042	0.195	0.422	0.006	0.060	0.240	0.485	0.007	0.065	0.237	0.542	0.015	0.045	0.212	0.547
HOLLT	G	0.05	0.042	0.215	0.395	0.567	0.059	0.210	0.410	0.547	0.055	0.185	0.367	0.500	0.046	0.195	0.340	0.485
		0.01	0.009	0.022	0.100	0.167	0.006	0.057	0.107	0.182	0.004	0.045	0.102	0.202	0.010	0.037	0.080	0.172
SHORT	G	0.05	0.040	0.192	0.460	0.705	0.051	0.265	0.567	0.850	0.040	0.202	0.575	0.827	0.044	0.255	0.590	0.895
		0.01	0.007	0.029	0.167	0.310	0.005	0.060	0.155	0.310	0.007	0.047	0.157	0.407	0.011	0.035	0.188	0.502
SEVAT	G	0.05	0.036	0.267	0.455	0.702	0.050	0.267	0.497	0.685	0.047	0.255	0.472	0.632	0.037	0.240	0.432	0.647
		0.01	0.010	0.117	0.135	0.237	0.010	0.063	0.162	0.262	0.011	0.065	0.160	0.287	0.010	0.055	0.137	0.252

TABLE A34

EMPIRICAL POWERS OF NINE TWO-WAY TESTS UNDER EXPONENTIAL DISTRIBUTION: $n = 10$

NO. OF GROUPS:			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
F2WAY	T	0.05	0.045	0.100	0.340	0.592	0.051	0.140	0.380	0.670	0.051	0.125	0.395	0.785	0.061	0.132	0.425	0.837
		0.01	0.012	0.037	0.150	0.327	0.006	0.032	0.160	0.412	0.017	0.032	0.195	0.547	0.015	0.035	0.205	0.660
PAGEF	T	0.05	0.046	0.317	0.627	0.805	0.039	0.400	0.767	0.932	0.059	0.417	0.867	0.990	0.040	0.497	0.905	0.995
		0.01	0.011	0.150	0.340	0.585	0.009	0.135	0.480	0.787	0.012	0.175	0.667	0.902	0.007	0.227	0.727	0.965
FRIED	T	0.05	0.036	0.142	0.332	0.575	0.037	0.160	0.410	0.725	0.047	0.167	0.495	0.810	0.052	0.165	0.540	0.900
		0.01	0.007	0.045	0.132	0.325	0.001	0.032	0.152	0.440	0.007	0.045	0.240	0.565	0.004	0.052	0.262	0.688
PHENS	T	0.05	0.046	0.317	0.627	0.805	0.041	0.435	0.772	0.942	0.064	0.430	0.872	0.992	0.037	0.505	0.900	0.995
		0.01	0.011	0.150	0.340	0.585	0.011	0.157	0.492	0.805	0.011	0.177	0.655	0.897	0.006	0.230	0.725	0.965
ATMX2	T	0.05	0.039	0.265	0.580	0.815	0.035	0.302	0.690	0.905	0.065	0.292	0.730	0.960	0.044	0.335	0.777	0.985
		0.01	0.015	0.077	0.310	0.585	0.015	0.115	0.405	0.750	0.016	0.102	0.457	0.862	0.010	0.137	0.517	0.940
LRN2	T	0.05	0.039	0.265	0.580	0.815	0.035	0.302	0.685	0.915	0.061	0.280	0.762	0.967	0.035	0.372	0.807	0.980
		0.01	0.015	0.077	0.310	0.585	0.015	0.117	0.400	0.757	0.015	0.102	0.477	0.882	0.012	0.127	0.580	0.947
HCLLT	T	0.05	0.041	0.295	0.630	0.850	0.040	0.357	0.725	0.938	0.065	0.347	0.835	0.980	0.035	0.395	0.865	0.992
		0.01	0.007	0.110	0.327	0.575	0.010	0.122	0.427	0.747	0.016	0.115	0.555	0.910	0.004	0.147	0.645	0.957
SHORT	T	0.05	0.036	0.250	0.530	0.770	0.042	0.385	0.730	0.912	0.057	0.390	0.817	0.965	0.040	0.442	0.865	0.997
		0.01	0.012	0.115	0.317	0.540	0.006	0.117	0.400	0.710	0.012	0.150	0.545	0.852	0.009	0.182	0.620	0.940
SEVAT	T	0.05	0.044	0.370	0.717	0.910	0.045	0.477	0.852	0.975	0.064	0.490	0.922	0.990	0.037	0.555	0.932	0.997
		0.01	0.010	0.147	0.412	0.712	0.011	0.220	0.587	0.862	0.015	0.197	0.727	0.945	0.006	0.272	0.787	0.980
F2WAY	G	0.05	0.045	0.127	0.327	0.667	0.051	0.125	0.387	0.735	0.051	0.117	0.430	0.810	0.061	0.127	0.462	0.850
		0.01	0.012	0.025	0.110	0.375	0.006	0.032	0.147	0.470	0.017	0.022	0.175	0.582	0.015	0.040	0.227	0.620
PAGEF	G	0.05	0.046	0.255	0.500	0.750	0.039	0.212	0.530	0.742	0.059	0.240	0.555	0.767	0.040	0.255	0.563	0.742
		0.01	0.011	0.075	0.215	0.402	0.009	0.082	0.222	0.415	0.012	0.082	0.250	0.517	0.007	0.070	0.212	0.390
FRIED	G	0.05	0.036	0.132	0.295	0.570	0.037	0.170	0.422	0.702	0.047	0.155	0.460	0.822	0.052	0.155	0.505	0.813
		0.01	0.009	0.025	0.137	0.317	0.001	0.030	0.127	0.367	0.007	0.027	0.145	0.455	0.004	0.035	0.202	0.460
PHENS	G	0.05	0.046	0.255	0.500	0.750	0.041	0.220	0.542	0.760	0.064	0.255	0.567	0.762	0.037	0.247	0.540	0.740
		0.01	0.011	0.075	0.215	0.402	0.011	0.087	0.232	0.422	0.011	0.077	0.242	0.487	0.006	0.070	0.195	0.382
ATMX2	G	0.05	0.039	0.242	0.497	0.780	0.035	0.197	0.577	0.827	0.065	0.240	0.563	0.872	0.044	0.240	0.635	0.895
		0.01	0.015	0.047	0.202	0.495	0.015	0.060	0.282	0.605	0.016	0.067	0.322	0.677	0.010	0.085	0.315	0.692
LRN2	G	0.05	0.039	0.242	0.497	0.780	0.035	0.180	0.527	0.787	0.061	0.217	0.512	0.827	0.035	0.190	0.525	0.820
		0.01	0.015	0.047	0.202	0.495	0.015	0.057	0.257	0.552	0.015	0.065	0.287	0.602	0.012	0.065	0.250	0.572
HCLLT	G	0.05	0.041	0.245	0.487	0.710	0.040	0.190	0.467	0.700	0.065	0.207	0.502	0.730	0.035	0.200	0.460	0.635
		0.01	0.007	0.057	0.172	0.337	0.010	0.052	0.170	0.350	0.016	0.070	0.202	0.365	0.004	0.045	0.152	0.280
SHORT	G	0.05	0.036	0.242	0.470	0.782	0.042	0.275	0.650	0.892	0.057	0.267	0.702	0.945	0.040	0.322	0.817	0.982
		0.01	0.012	0.082	0.247	0.515	0.006	0.095	0.320	0.630	0.012	0.102	0.405	0.815	0.009	0.100	0.435	0.815
SEVAT	G	0.05	0.044	0.315	0.640	0.920	0.045	0.245	0.587	0.842	0.064	0.300	0.587	0.845	0.037	0.277	0.597	0.785
		0.01	0.010	0.107	0.295	0.502	0.011	0.102	0.285	0.515	0.015	0.090	0.397	0.565	0.006	0.082	0.245	0.460

TABLE A35

EMPIRICAL POWERS OF NINE TWO-WAY TESTS UNDER EXPONENTIAL DISTRIBUTION: $n = 15$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
F2WAY	T	0.05	0.055	0.125	0.270	0.620	0.030	0.115	0.460	0.735	0.040	0.120	0.445	0.815	0.040	0.155	0.460	0.905
		0.01	0.007	0.040	0.130	0.385	0.005	0.030	0.250	0.555	0.010	0.030	0.205	0.615	0.002	0.040	0.220	0.710
PAGE2	T	0.05	0.045	0.360	0.620	0.910	0.065	0.335	0.825	0.975	0.040	0.455	0.890	0.990	0.047	0.555	0.910	1.000
		0.01	0.020	0.135	0.345	0.705	0.037	0.110	0.580	0.900	0.037	0.185	0.700	0.935	0.005	0.280	0.790	0.985
FRIED	T	0.05	0.057	0.175	0.360	0.710	0.052	0.140	0.545	0.860	0.065	0.185	0.595	0.895	0.052	0.205	0.635	0.950
		0.01	0.002	0.050	0.185	0.435	0.035	0.045	0.295	0.580	0.005	0.065	0.280	0.735	0.007	0.065	0.340	0.800
PHENS	T	0.05	0.045	0.360	0.620	0.910	0.062	0.330	0.830	0.980	0.035	0.455	0.890	0.990	0.045	0.595	0.930	1.000
		0.01	0.020	0.135	0.345	0.705	0.037	0.120	0.595	0.905	0.035	0.185	0.730	0.950	0.007	0.325	0.790	0.985
ATMX2	T	0.05	0.057	0.260	0.520	0.875	0.040	0.280	0.705	0.965	0.030	0.265	0.775	0.960	0.035	0.385	0.800	0.995
		0.01	0.015	0.100	0.245	0.605	0.035	0.070	0.510	0.760	0.007	0.095	0.510	0.875	0.002	0.140	0.530	0.955
LRCH2	T	0.05	0.057	0.260	0.520	0.875	0.050	0.300	0.705	0.965	0.032	0.290	0.810	0.965	0.047	0.430	0.825	0.990
		0.01	0.015	0.100	0.245	0.605	0.005	0.090	0.520	0.780	0.005	0.110	0.530	0.880	0.002	0.135	0.575	0.975
HCLLT	T	0.05	0.052	0.355	0.580	0.925	0.050	0.335	0.765	0.970	0.025	0.350	0.860	0.985	0.045	0.480	0.890	1.000
		0.01	0.015	0.075	0.290	0.675	0.005	0.115	0.570	0.865	0.005	0.125	0.595	0.915	0.002	0.185	0.680	0.980
SHORT	T	0.05	0.050	0.340	0.560	0.890	0.055	0.305	0.760	0.960	0.050	0.380	0.850	0.990	0.050	0.520	0.880	0.995
		0.01	0.012	0.125	0.270	0.615	0.010	0.105	0.535	0.855	0.007	0.145	0.630	0.920	0.007	0.245	0.710	0.980
SEIAT	T	0.05	0.077	0.460	0.740	0.985	0.047	0.405	0.895	0.995	0.032	0.515	0.930	0.995	0.060	0.630	0.955	1.000
		0.01	0.022	0.160	0.465	0.815	0.007	0.175	0.675	0.945	0.002	0.235	0.765	0.955	0.005	0.320	0.835	0.990
F2WAY	G	0.05	0.055	0.120	0.310	0.690	0.030	0.100	0.385	0.790	0.040	0.155	0.435	0.845	0.040	0.135	0.565	0.875
		0.01	0.007	0.035	0.160	0.450	0.005	0.030	0.170	0.515	0.010	0.045	0.180	0.625	0.002	0.035	0.285	0.665
PAGE2	G	0.05	0.045	0.220	0.480	0.735	0.065	0.200	0.570	0.895	0.040	0.235	0.590	0.805	0.047	0.295	0.575	0.835
		0.01	0.020	0.110	0.290	0.515	0.037	0.060	0.295	0.565	0.007	0.060	0.260	0.555	0.005	0.080	0.325	0.585
FRIED	G	0.05	0.057	0.170	0.415	0.735	0.052	0.155	0.480	0.810	0.065	0.170	0.515	0.905	0.052	0.180	0.680	0.935
		0.01	0.002	0.040	0.160	0.475	0.005	0.025	0.205	0.580	0.005	0.080	0.265	0.635	0.007	0.045	0.305	0.700
PHENS	G	0.05	0.045	0.200	0.480	0.735	0.062	0.210	0.575	0.900	0.035	0.240	0.590	0.805	0.045	0.285	0.565	0.835
		0.01	0.020	0.110	0.290	0.515	0.037	0.065	0.305	0.585	0.005	0.060	0.265	0.555	0.007	0.090	0.320	0.580
ATMX2	G	0.05	0.057	0.235	0.465	0.780	0.040	0.175	0.505	0.855	0.030	0.220	0.605	0.835	0.035	0.255	0.670	0.910
		0.01	0.015	0.050	0.235	0.500	0.035	0.050	0.310	0.620	0.007	0.085	0.330	0.670	0.002	0.100	0.420	0.760
LRCH2	G	0.05	0.057	0.205	0.465	0.780	0.050	0.175	0.480	0.840	0.032	0.185	0.565	0.800	0.047	0.215	0.570	0.855
		0.01	0.015	0.050	0.235	0.500	0.005	0.050	0.270	0.585	0.005	0.050	0.265	0.595	0.002	0.070	0.305	0.625
HCLLT	G	0.05	0.052	0.195	0.485	0.745	0.050	0.175	0.550	0.810	0.025	0.175	0.525	0.735	0.045	0.245	0.510	0.790
		0.01	0.015	0.060	0.220	0.430	0.035	0.050	0.250	0.460	0.005	0.045	0.215	0.445	0.002	0.080	0.245	0.460
SHORT	G	0.05	0.050	0.200	0.515	0.820	0.055	0.240	0.690	0.955	0.050	0.285	0.785	0.970	0.050	0.370	0.890	1.000
		0.01	0.012	0.090	0.265	0.575	0.010	0.095	0.365	0.790	0.007	0.075	0.445	0.870	0.007	0.120	0.565	0.935
SEIAT	G	0.05	0.077	0.255	0.650	0.860	0.047	0.255	0.665	0.930	0.032	0.245	0.650	0.865	0.060	0.350	0.645	0.900
		0.01	0.022	0.085	0.335	0.585	0.007	0.070	0.360	0.700	0.002	0.085	0.365	0.635	0.005	0.115	0.345	0.655

TABLE A36

EMPIRICAL POWERS OF NINE TWO-WAY TESTS UNDER EXPONENTIAL DISTRIBUTION: $n = 20$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
F2WAY	T	0.05	0.055	0.110	0.335	0.685	0.052	0.105	0.455	0.740	0.065	0.085	0.400	0.790	0.035	0.135	0.480	0.855
		0.01	0.012	0.030	0.150	0.405	0.007	0.030	0.215	0.520	0.010	0.020	0.180	0.640	0.010	0.045	0.260	0.695
PAGE2	T	0.05	0.067	0.300	0.685	0.925	0.040	0.385	0.815	0.975	0.040	0.515	0.875	0.980	0.047	0.610	0.940	0.995
		0.01	0.027	0.130	0.425	0.715	0.007	0.180	0.580	0.870	0.010	0.220	0.660	0.955	0.005	0.320	0.820	0.975
FRIED	T	0.05	0.082	0.165	0.445	0.740	0.045	0.185	0.555	0.855	0.042	0.160	0.525	0.890	0.027	0.215	0.700	0.940
		0.01	0.005	0.070	0.200	0.425	0.0	0.060	0.325	0.585	0.015	0.045	0.295	0.765	0.002	0.075	0.450	0.820
PHENS	T	0.05	0.067	0.300	0.685	0.925	0.040	0.420	0.815	0.980	0.040	0.515	0.880	0.980	0.042	0.630	0.945	0.995
		0.01	0.027	0.130	0.425	0.715	0.010	0.200	0.590	0.890	0.012	0.230	0.675	0.960	0.005	0.315	0.835	0.975
ATMX2	T	0.05	0.057	0.220	0.635	0.855	0.042	0.240	0.690	0.945	0.052	0.310	0.710	0.960	0.045	0.345	0.785	0.980
		0.01	0.007	0.075	0.310	0.655	0.010	0.070	0.435	0.795	0.010	0.100	0.415	0.855	0.010	0.155	0.585	0.905
LRN2	T	0.05	0.057	0.220	0.635	0.855	0.045	0.220	0.685	0.935	0.042	0.330	0.730	0.970	0.040	0.380	0.820	0.985
		0.01	0.007	0.075	0.310	0.655	0.005	0.075	0.445	0.825	0.010	0.105	0.475	0.870	0.010	0.165	0.615	0.940
HOLLT	T	0.05	0.055	0.255	0.660	0.905	0.042	0.290	0.740	0.980	0.045	0.420	0.800	0.985	0.035	0.510	0.880	1.000
		0.01	0.015	0.110	0.360	0.710	0.005	0.105	0.535	0.880	0.010	0.110	0.520	0.915	0.010	0.215	0.705	0.970
SHORT	T	0.05	0.072	0.265	0.675	0.895	0.047	0.355	0.785	0.980	0.037	0.435	0.835	0.980	0.055	0.545	0.915	0.995
		0.01	0.022	0.130	0.370	0.630	0.007	0.175	0.540	0.855	0.007	0.175	0.580	0.910	0.005	0.275	0.775	0.970
SEHAT	T	0.05	0.060	0.340	0.780	0.970	0.045	0.455	0.880	0.995	0.047	0.540	0.925	0.995	0.050	0.650	0.955	1.000
		0.01	0.012	0.150	0.510	0.860	0.007	0.165	0.660	0.965	0.010	0.280	0.725	0.970	0.007	0.355	0.870	0.985
F2WAY	G	0.05	0.055	0.115	0.405	0.680	0.052	0.150	0.425	0.775	0.065	0.125	0.430	0.830	0.035	0.155	0.445	0.890
		0.01	0.012	0.045	0.165	0.445	0.007	0.055	0.160	0.510	0.010	0.035	0.170	0.665	0.010	0.060	0.240	0.645
PAGE2	G	0.05	0.067	0.210	0.565	0.815	0.040	0.265	0.615	0.825	0.040	0.280	0.640	0.855	0.047	0.300	0.640	0.890
		0.01	0.027	0.075	0.345	0.525	0.007	0.090	0.290	0.565	0.010	0.115	0.330	0.605	0.005	0.090	0.310	0.595
FRIED	G	0.05	0.082	0.160	0.500	0.780	0.045	0.145	0.565	0.875	0.042	0.165	0.555	0.920	0.027	0.180	0.645	0.955
		0.01	0.005	0.070	0.200	0.465	0.0	0.070	0.205	0.620	0.015	0.060	0.235	0.735	0.002	0.060	0.330	0.790
PHENS	G	0.05	0.067	0.210	0.565	0.815	0.040	0.270	0.615	0.825	0.040	0.270	0.615	0.850	0.042	0.290	0.635	0.875
		0.01	0.027	0.075	0.345	0.525	0.010	0.090	0.300	0.585	0.012	0.115	0.325	0.600	0.005	0.080	0.310	0.570
ATMX2	G	0.05	0.057	0.205	0.515	0.775	0.042	0.245	0.520	0.805	0.052	0.235	0.550	0.905	0.045	0.260	0.655	0.910
		0.01	0.007	0.050	0.300	0.520	0.010	0.080	0.305	0.625	0.010	0.095	0.320	0.705	0.010	0.100	0.385	0.690
LRN2	G	0.05	0.057	0.205	0.515	0.775	0.045	0.220	0.515	0.800	0.042	0.215	0.510	0.840	0.040	0.225	0.575	0.840
		0.01	0.007	0.050	0.300	0.520	0.005	0.070	0.280	0.580	0.010	0.085	0.285	0.590	0.010	0.090	0.270	0.585
HOLLT	G	0.05	0.065	0.200	0.535	0.785	0.042	0.220	0.515	0.795	0.045	0.235	0.545	0.820	0.035	0.240	0.555	0.800
		0.01	0.015	0.060	0.345	0.480	0.005	0.075	0.270	0.510	0.010	0.090	0.220	0.510	0.010	0.075	0.250	0.495
SHORT	G	0.05	0.072	0.200	0.630	0.890	0.047	0.260	0.755	0.960	0.037	0.290	0.825	0.990	0.055	0.405	0.865	1.000
		0.01	0.022	0.075	0.380	0.620	0.007	0.090	0.445	0.825	0.007	0.110	0.495	0.885	0.005	0.125	0.620	0.940
SEHAT	G	0.05	0.060	0.295	0.685	0.895	0.045	0.325	0.670	0.915	0.047	0.305	0.690	0.915	0.050	0.340	0.700	0.910
		0.01	0.012	0.095	0.445	0.685	0.007	0.110	0.400	0.710	0.010	0.110	0.400	0.700	0.007	0.130	0.385	0.660

TABLE A37

EMPIRICAL POWERS OF NINE TWO-WAY TESTS UNDER NORMAL DISTRIBUTION: $n = 5$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
F2WAY	T	0.05	0.050	0.082	0.270	0.510	0.056	0.095	0.235	0.567	0.051	0.132	0.325	0.635	0.061	0.095	0.297	0.712
		0.01	0.005	0.015	0.087	0.200	0.011	0.022	0.077	0.305	0.009	0.052	0.130	0.340	0.014	0.017	0.125	0.420
PAGE2	T	0.05	0.032	0.130	0.367	0.630	0.060	0.230	0.522	0.830	0.071	0.290	0.627	0.887	0.041	0.252	0.665	0.940
		0.01	0.004	0.020	0.147	0.282	0.010	0.063	0.192	0.472	0.005	0.105	0.307	0.622	0.010	0.087	0.390	0.780
FRIED	T	0.05	0.020	0.042	0.157	0.290	0.035	0.067	0.177	0.405	0.035	0.102	0.225	0.465	0.040	0.063	0.195	0.537
		0.01	0.0	0.0	0.017	0.030	0.005	0.007	0.050	0.188	0.001	0.012	0.037	0.122	0.005	0.005	0.032	0.152
PHENS	T	0.05	0.032	0.130	0.367	0.630	0.060	0.230	0.522	0.822	0.074	0.302	0.647	0.885	0.047	0.265	0.675	0.945
		0.01	0.004	0.020	0.147	0.282	0.012	0.067	0.205	0.490	0.005	0.102	0.332	0.655	0.010	0.090	0.400	0.787
ATX2	T	0.05	0.044	0.188	0.510	0.780	0.059	0.255	0.585	0.887	0.062	0.307	0.688	0.912	0.054	0.262	0.710	0.967
		0.01	0.010	0.040	0.237	0.472	0.011	0.087	0.270	0.665	0.015	0.130	0.397	0.770	0.010	0.092	0.420	0.845
LRN2	T	0.05	0.044	0.188	0.510	0.780	0.061	0.260	0.602	0.897	0.061	0.335	0.745	0.930	0.049	0.280	0.737	0.972
		0.01	0.010	0.040	0.237	0.472	0.011	0.080	0.277	0.665	0.012	0.127	0.422	0.780	0.009	0.105	0.465	0.867
HULLT	T	0.05	0.042	0.190	0.467	0.740	0.054	0.272	0.617	0.872	0.061	0.327	0.707	0.912	0.046	0.297	0.700	0.970
		0.01	0.006	0.020	0.150	0.313	0.012	0.052	0.227	0.567	0.007	0.100	0.372	0.710	0.007	0.085	0.432	0.840
SHORT	T	0.05	0.040	0.137	0.357	0.605	0.054	0.205	0.447	0.755	0.056	0.265	0.602	0.845	0.036	0.217	0.597	0.917
		0.01	0.006	0.025	0.142	0.262	0.006	0.042	0.172	0.417	0.006	0.077	0.240	0.517	0.006	0.070	0.285	0.667
SFVAT	T	0.05	0.046	0.182	0.477	0.777	0.057	0.260	0.580	0.895	0.066	0.327	0.702	0.915	0.042	0.285	0.715	0.967
		0.01	0.005	0.012	0.165	0.350	0.009	0.065	0.227	0.605	0.011	0.117	0.367	0.717	0.007	0.087	0.447	0.835
F2WAY	G	0.05	0.050	0.082	0.222	0.505	0.056	0.107	0.252	0.527	0.051	0.085	0.275	0.620	0.061	0.095	0.375	0.705
		0.01	0.005	0.017	0.095	0.190	0.011	0.030	0.097	0.237	0.009	0.015	0.087	0.325	0.014	0.017	0.142	0.447
PAGE2	G	0.05	0.032	0.112	0.275	0.467	0.060	0.145	0.365	0.537	0.071	0.190	0.370	0.502	0.041	0.140	0.387	0.582
		0.01	0.004	0.022	0.077	0.167	0.010	0.035	0.090	0.170	0.005	0.035	0.105	0.172	0.010	0.020	0.107	0.212
FRIED	G	0.05	0.020	0.047	0.132	0.290	0.035	0.075	0.157	0.352	0.035	0.057	0.162	0.395	0.040	0.047	0.220	0.438
		0.01	0.0	0.002	0.017	0.012	0.005	0.022	0.063	0.115	0.001	0.007	0.012	0.060	0.005	0.005	0.022	0.067
PHENS	G	0.05	0.032	0.112	0.275	0.467	0.060	0.145	0.362	0.535	0.074	0.192	0.382	0.517	0.047	0.140	0.397	0.610
		0.01	0.004	0.022	0.077	0.167	0.012	0.042	0.105	0.185	0.005	0.035	0.112	0.195	0.010	0.020	0.125	0.237
ATX2	G	0.05	0.044	0.137	0.390	0.660	0.059	0.177	0.452	0.715	0.062	0.232	0.505	0.765	0.054	0.170	0.580	0.875
		0.01	0.010	0.030	0.127	0.342	0.011	0.050	0.190	0.415	0.015	0.065	0.202	0.467	0.010	0.065	0.315	0.645
LRN2	G	0.05	0.044	0.137	0.390	0.660	0.061	0.165	0.435	0.700	0.061	0.227	0.450	0.660	0.049	0.135	0.482	0.782
		0.01	0.010	0.030	0.127	0.342	0.011	0.052	0.185	0.390	0.012	0.065	0.172	0.390	0.009	0.047	0.230	0.505
HULLT	G	0.05	0.042	0.150	0.347	0.505	0.054	0.165	0.357	0.500	0.061	0.200	0.342	0.422	0.046	0.127	0.335	0.470
		0.01	0.006	0.015	0.080	0.175	0.012	0.032	0.120	0.167	0.007	0.040	0.102	0.160	0.007	0.032	0.090	0.167
SHORT	G	0.05	0.040	0.140	0.315	0.537	0.054	0.162	0.402	0.695	0.056	0.195	0.438	0.695	0.036	0.145	0.527	0.817
		0.01	0.006	0.030	0.087	0.245	0.006	0.032	0.120	0.272	0.006	0.025	0.117	0.275	0.006	0.022	0.197	0.502
SFVAT	G	0.05	0.046	0.157	0.367	0.580	0.057	0.165	0.382	0.555	0.066	0.217	0.395	0.517	0.042	0.127	0.410	0.612
		0.01	0.005	0.012	0.080	0.197	0.009	0.032	0.132	0.207	0.011	0.047	0.127	0.205	0.007	0.030	0.122	0.245

TABLE A38

EMPIRICAL POWERS OF NINE TWO-WAY TESTS UNDER NORMAL DISTRIBUTION: $n = 10$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
F2WAY	T	0.05	0.030	0.090	0.242	0.557	0.051	0.107	0.320	0.655	0.062	0.152	0.313	0.697	0.045	0.125	0.382	0.770
		0.01	0.005	0.020	0.102	0.267	0.010	0.015	0.092	0.390	0.005	0.037	0.110	0.438	0.012	0.035	0.182	0.535
PAGE2	T	0.05	0.036	0.160	0.399	0.695	0.049	0.192	0.510	0.807	0.044	0.235	0.615	0.897	0.044	0.270	0.705	0.950
		0.01	0.004	0.040	0.147	0.377	0.005	0.065	0.220	0.587	0.005	0.112	0.310	0.730	0.012	0.090	0.415	0.815
FRIED	T	0.05	0.027	0.052	0.162	0.365	0.036	0.092	0.222	0.530	0.047	0.107	0.230	0.565	0.035	0.077	0.317	0.660
		0.01	0.0	0.022	0.052	0.140	0.006	0.012	0.057	0.237	0.004	0.017	0.055	0.267	0.005	0.020	0.105	0.342
PHENS	T	0.05	0.036	0.160	0.399	0.695	0.056	0.210	0.530	0.827	0.050	0.250	0.627	0.910	0.040	0.265	0.705	0.952
		0.01	0.004	0.040	0.147	0.377	0.005	0.072	0.232	0.607	0.005	0.107	0.310	0.732	0.011	0.090	0.407	0.810
ATMX2	T	0.05	0.042	0.188	0.522	0.813	0.044	0.232	0.637	0.890	0.057	0.287	0.692	0.945	0.054	0.300	0.755	0.960
		0.01	0.002	0.047	0.205	0.520	0.011	0.100	0.330	0.688	0.010	0.127	0.407	0.777	0.006	0.110	0.457	0.865
LRCN2	T	0.05	0.042	0.188	0.522	0.813	0.042	0.240	0.642	0.895	0.054	0.292	0.715	0.957	0.052	0.327	0.800	0.967
		0.01	0.002	0.047	0.205	0.520	0.007	0.097	0.340	0.690	0.007	0.120	0.438	0.802	0.009	0.125	0.515	0.897
HCLLT	T	0.05	0.051	0.180	0.467	0.790	0.045	0.240	0.610	0.867	0.044	0.285	0.672	0.938	0.041	0.302	0.742	0.965
		0.01	0.005	0.057	0.188	0.462	0.005	0.092	0.287	0.640	0.007	0.102	0.375	0.770	0.010	0.097	0.445	0.875
SHORT	T	0.05	0.031	0.117	0.340	0.632	0.044	0.205	0.490	0.780	0.051	0.207	0.557	0.855	0.050	0.252	0.657	0.930
		0.01	0.004	0.045	0.142	0.355	0.009	0.055	0.190	0.515	0.006	0.087	0.237	0.617	0.006	0.075	0.357	0.722
SEVAT	T	0.05	0.035	0.207	0.487	0.800	0.047	0.237	0.615	0.867	0.054	0.292	0.697	0.940	0.047	0.313	0.755	0.965
		0.01	0.005	0.037	0.177	0.462	0.007	0.082	0.302	0.647	0.012	0.107	0.380	0.770	0.012	0.100	0.457	0.887
F2WAY	G	0.05	0.030	0.107	0.270	0.625	0.051	0.082	0.317	0.647	0.062	0.077	0.340	0.712	0.045	0.102	0.375	0.755
		0.01	0.005	0.030	0.110	0.313	0.010	0.025	0.102	0.385	0.005	0.020	0.122	0.485	0.012	0.030	0.135	0.537
PAGE2	G	0.05	0.036	0.150	0.290	0.575	0.049	0.152	0.362	0.587	0.044	0.165	0.350	0.590	0.044	0.160	0.375	0.637
		0.01	0.004	0.032	0.087	0.262	0.005	0.030	0.120	0.322	0.005	0.040	0.127	0.313	0.012	0.042	0.135	0.335
FRIED	G	0.05	0.027	0.050	0.175	0.387	0.036	0.063	0.202	0.490	0.047	0.052	0.230	0.557	0.035	0.065	0.282	0.615
		0.01	0.0	0.015	0.060	0.205	0.006	0.010	0.065	0.232	0.004	0.007	0.070	0.297	0.005	0.010	0.060	0.310
PHENS	G	0.05	0.036	0.150	0.290	0.575	0.056	0.157	0.382	0.602	0.050	0.175	0.365	0.632	0.040	0.177	0.380	0.650
		0.01	0.004	0.032	0.087	0.262	0.005	0.032	0.135	0.330	0.005	0.040	0.127	0.313	0.011	0.042	0.137	0.347
ATMX2	G	0.05	0.042	0.185	0.415	0.732	0.044	0.177	0.475	0.785	0.057	0.212	0.482	0.820	0.054	0.222	0.560	0.872
		0.01	0.002	0.070	0.147	0.438	0.011	0.052	0.225	0.487	0.010	0.065	0.232	0.565	0.006	0.092	0.275	0.662
LRCN2	G	0.05	0.042	0.185	0.415	0.732	0.042	0.165	0.462	0.767	0.054	0.188	0.445	0.767	0.052	0.215	0.462	0.800
		0.01	0.002	0.070	0.147	0.438	0.007	0.040	0.197	0.475	0.007	0.063	0.192	0.492	0.009	0.060	0.180	0.537
HCLLT	G	0.05	0.051	0.160	0.365	0.652	0.045	0.147	0.415	0.627	0.044	0.190	0.370	0.675	0.041	0.182	0.365	0.615
		0.01	0.005	0.052	0.127	0.307	0.005	0.030	0.142	0.320	0.007	0.042	0.140	0.285	0.010	0.037	0.110	0.300
SHORT	G	0.05	0.031	0.120	0.290	0.587	0.044	0.170	0.425	0.707	0.051	0.157	0.430	0.790	0.050	0.195	0.542	0.877
		0.01	0.004	0.027	0.130	0.340	0.009	0.032	0.150	0.435	0.006	0.027	0.200	0.537	0.006	0.047	0.245	0.630
SEVAT	G	0.05	0.035	0.180	0.380	0.688	0.047	0.160	0.417	0.642	0.054	0.185	0.397	0.692	0.047	0.192	0.417	0.712
		0.01	0.005	0.047	0.117	0.317	0.007	0.030	0.152	0.372	0.012	0.055	0.147	0.362	0.012	0.057	0.145	0.390

TABLE A39

EMPIRICAL POWERS OF NINE TWO-WAY TESTS UNDER NORMAL DISTRIBUTION: $n = 15$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
F2WAY	T	0.05	0.050	0.145	0.285	0.530	0.050	0.120	0.360	0.660	0.065	0.100	0.330	0.775	0.055	0.105	0.385	0.780
		0.01	0.015	0.045	0.105	0.310	0.0	0.025	0.170	0.400	0.012	0.030	0.160	0.495	0.007	0.025	0.185	0.520
PAGEY	T	0.05	0.035	0.175	0.360	0.630	0.037	0.220	0.510	0.790	0.047	0.240	0.595	0.975	0.045	0.290	0.710	0.945
		0.01	0.010	0.065	0.180	0.405	0.007	0.060	0.270	0.540	0.010	0.105	0.335	0.745	0.012	0.070	0.430	0.825
FRIED	T	0.05	0.052	0.090	0.225	0.415	0.047	0.065	0.250	0.485	0.045	0.135	0.240	0.665	0.052	0.090	0.305	0.690
		0.01	0.007	0.020	0.090	0.190	0.002	0.0	0.110	0.250	0.005	0.020	0.085	0.370	0.007	0.0	0.105	0.375
PHENS	T	0.05	0.035	0.175	0.360	0.630	0.037	0.225	0.540	0.800	0.045	0.240	0.600	0.970	0.040	0.295	0.730	0.950
		0.01	0.010	0.065	0.180	0.405	0.007	0.075	0.305	0.585	0.010	0.105	0.350	0.745	0.015	0.075	0.460	0.820
ATMX2	T	0.05	0.042	0.250	0.560	0.830	0.040	0.245	0.585	0.875	0.052	0.265	0.695	0.955	0.057	0.290	0.755	0.975
		0.01	0.010	0.110	0.205	0.495	0.002	0.120	0.395	0.660	0.010	0.115	0.415	0.840	0.012	0.110	0.500	0.875
LRGN2	T	0.05	0.042	0.250	0.560	0.830	0.050	0.265	0.620	0.890	0.047	0.270	0.715	0.970	0.052	0.325	0.805	0.970
		0.01	0.010	0.110	0.205	0.495	0.002	0.115	0.405	0.675	0.010	0.140	0.435	0.890	0.012	0.100	0.530	0.895
HOLLY	T	0.05	0.037	0.230	0.505	0.755	0.050	0.260	0.595	0.860	0.047	0.280	0.670	0.975	0.047	0.325	0.755	0.970
		0.01	0.010	0.090	0.170	0.440	0.002	0.075	0.330	0.620	0.007	0.115	0.430	0.840	0.010	0.085	0.505	0.895
SHORT	T	0.05	0.032	0.150	0.390	0.600	0.052	0.195	0.505	0.735	0.037	0.255	0.540	0.920	0.042	0.240	0.635	0.915
		0.01	0.010	0.035	0.155	0.315	0.005	0.050	0.220	0.475	0.005	0.090	0.290	0.695	0.007	0.060	0.345	0.765
SENAT	T	0.05	0.045	0.245	0.525	0.735	0.052	0.245	0.620	0.860	0.050	0.265	0.685	0.975	0.050	0.330	0.770	0.965
		0.01	0.012	0.100	0.180	0.430	0.0	0.105	0.355	0.635	0.012	0.125	0.400	0.845	0.012	0.095	0.525	0.880
F2WAY	G	0.05	0.050	0.085	0.240	0.590	0.050	0.100	0.350	0.640	0.065	0.125	0.380	0.750	0.055	0.090	0.375	0.785
		0.01	0.015	0.030	0.105	0.315	0.0	0.020	0.180	0.410	0.012	0.045	0.160	0.510	0.007	0.015	0.150	0.565
PAGEY	G	0.05	0.035	0.170	0.260	0.550	0.037	0.160	0.345	0.605	0.047	0.220	0.445	0.675	0.045	0.170	0.370	0.720
		0.01	0.010	0.055	0.115	0.285	0.007	0.030	0.135	0.290	0.010	0.050	0.180	0.375	0.012	0.035	0.150	0.405
FRIED	G	0.05	0.052	0.075	0.170	0.430	0.047	0.075	0.285	0.500	0.045	0.110	0.310	0.615	0.052	0.095	0.235	0.695
		0.01	0.007	0.030	0.065	0.210	0.002	0.020	0.110	0.235	0.005	0.025	0.095	0.300	0.007	0.020	0.075	0.355
PHENS	G	0.05	0.035	0.170	0.260	0.550	0.037	0.155	0.370	0.600	0.045	0.215	0.425	0.680	0.040	0.165	0.370	0.720
		0.01	0.010	0.055	0.115	0.285	0.007	0.035	0.150	0.305	0.010	0.060	0.190	0.405	0.015	0.035	0.155	0.440
ATMX2	G	0.05	0.042	0.220	0.375	0.710	0.040	0.215	0.445	0.735	0.052	0.290	0.550	0.800	0.057	0.220	0.515	0.890
		0.01	0.010	0.045	0.130	0.400	0.002	0.040	0.220	0.495	0.010	0.080	0.245	0.605	0.012	0.040	0.305	0.670
LRGN2	G	0.05	0.042	0.220	0.375	0.710	0.050	0.205	0.445	0.720	0.047	0.260	0.480	0.775	0.052	0.195	0.415	0.820
		0.01	0.010	0.045	0.130	0.400	0.002	0.050	0.215	0.450	0.010	0.080	0.205	0.510	0.012	0.035	0.220	0.575
HOLLY	G	0.05	0.037	0.215	0.375	0.650	0.050	0.175	0.410	0.655	0.047	0.240	0.430	0.665	0.047	0.180	0.365	0.735
		0.01	0.010	0.045	0.135	0.345	0.002	0.030	0.165	0.335	0.007	0.065	0.165	0.365	0.010	0.035	0.185	0.380
SHORT	G	0.05	0.032	0.175	0.270	0.560	0.052	0.165	0.450	0.705	0.037	0.205	0.550	0.785	0.042	0.200	0.495	0.870
		0.01	0.010	0.050	0.105	0.290	0.005	0.035	0.220	0.430	0.005	0.055	0.255	0.580	0.007	0.050	0.215	0.675
SENAT	G	0.05	0.045	0.215	0.370	0.655	0.052	0.165	0.395	0.665	0.050	0.235	0.470	0.715	0.050	0.195	0.360	0.770
		0.01	0.012	0.050	0.130	0.360	0.0	0.060	0.160	0.375	0.012	0.065	0.170	0.415	0.012	0.040	0.190	0.465

TABLE A40

EMPIRICAL POWERS OF NINE TWO-WAY TESTS UNDER NORMAL DISTRIBUTION: $n = 20$

NO. OF GROUPS			3				4				5				6			
NONCENTRALITY PARAMETER			0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5	0.0	0.5	1.0	1.5
TEST	ALT	ALPHA																
F2WAY	T	0.05	0.035	0.095	0.310	0.600	0.050	0.085	0.310	0.680	0.037	0.135	0.365	0.770	0.030	0.130	0.405	0.835
		0.01	0.007	0.015	0.120	0.350	0.010	0.015	0.135	0.370	0.015	0.020	0.135	0.500	0.007	0.030	0.210	0.575
PAGEY	T	0.05	0.062	0.115	0.405	0.690	0.065	0.225	0.500	0.835	0.055	0.220	0.610	0.910	0.030	0.315	0.680	0.960
		0.01	0.010	0.040	0.160	0.445	0.010	0.060	0.240	0.525	0.0	0.080	0.330	0.730	0.005	0.130	0.455	0.840
FRIED	T	0.05	0.055	0.090	0.240	0.470	0.045	0.075	0.255	0.485	0.037	0.120	0.285	0.635	0.032	0.080	0.285	0.695
		0.01	0.010	0.020	0.065	0.245	0.005	0.025	0.090	0.235	0.007	0.030	0.120	0.340	0.002	0.010	0.090	0.385
PHENS	T	0.05	0.062	0.115	0.405	0.690	0.077	0.225	0.500	0.855	0.057	0.210	0.610	0.905	0.030	0.310	0.680	0.955
		0.01	0.010	0.040	0.160	0.445	0.010	0.070	0.240	0.540	0.002	0.080	0.345	0.735	0.005	0.140	0.430	0.840
ATMX2	T	0.05	0.045	0.170	0.530	0.790	0.052	0.275	0.575	0.915	0.047	0.240	0.680	0.945	0.035	0.350	0.765	0.970
		0.01	0.007	0.050	0.260	0.575	0.007	0.065	0.310	0.720	0.007	0.080	0.405	0.820	0.002	0.125	0.475	0.895
LRCN2	T	0.05	0.045	0.170	0.530	0.790	0.062	0.290	0.580	0.900	0.040	0.265	0.700	0.940	0.030	0.385	0.765	0.995
		0.01	0.007	0.050	0.260	0.575	0.015	0.065	0.335	0.745	0.007	0.100	0.430	0.840	0.002	0.145	0.520	0.920
HCLLT	T	0.05	0.052	0.155	0.510	0.760	0.075	0.250	0.570	0.910	0.047	0.235	0.665	0.920	0.032	0.330	0.715	0.990
		0.01	0.005	0.045	0.200	0.540	0.007	0.070	0.300	0.705	0.010	0.090	0.390	0.805	0.005	0.140	0.480	0.905
SHORT	T	0.05	0.042	0.135	0.400	0.660	0.062	0.205	0.470	0.790	0.047	0.175	0.550	0.840	0.022	0.300	0.615	0.940
		0.01	0.010	0.035	0.150	0.375	0.015	0.045	0.235	0.460	0.005	0.075	0.295	0.690	0.002	0.115	0.355	0.785
SENAT	T	0.05	0.057	0.150	0.520	0.790	0.067	0.250	0.545	0.890	0.050	0.235	0.665	0.920	0.032	0.335	0.735	0.980
		0.01	0.010	0.045	0.215	0.550	0.015	0.075	0.320	0.715	0.007	0.090	0.390	0.805	0.002	0.140	0.500	0.895
F2WAY	G	0.05	0.035	0.060	0.280	0.620	0.050	0.105	0.345	0.640	0.037	0.145	0.390	0.780	0.030	0.135	0.415	0.790
		0.01	0.007	0.015	0.090	0.315	0.010	0.015	0.135	0.400	0.015	0.040	0.135	0.530	0.007	0.035	0.215	0.555
PAGEY	G	0.05	0.062	0.130	0.300	0.595	0.065	0.125	0.410	0.595	0.055	0.190	0.365	0.695	0.030	0.230	0.370	0.690
		0.01	0.010	0.035	0.105	0.335	0.010	0.030	0.190	0.320	0.0	0.050	0.115	0.390	0.005	0.065	0.130	0.365
FRIED	G	0.05	0.055	0.040	0.175	0.470	0.045	0.085	0.285	0.520	0.037	0.115	0.270	0.655	0.032	0.100	0.325	0.635
		0.01	0.010	0.010	0.050	0.210	0.005	0.015	0.110	0.240	0.007	0.030	0.075	0.355	0.002	0.030	0.105	0.435
PHENS	G	0.05	0.062	0.130	0.300	0.595	0.077	0.125	0.430	0.595	0.057	0.185	0.355	0.670	0.030	0.240	0.365	0.685
		0.01	0.010	0.035	0.105	0.335	0.010	0.030	0.190	0.335	0.002	0.050	0.110	0.400	0.005	0.065	0.145	0.375
ATMX2	G	0.05	0.045	0.165	0.390	0.735	0.052	0.160	0.495	0.765	0.047	0.220	0.535	0.800	0.035	0.300	0.545	0.855
		0.01	0.007	0.045	0.170	0.475	0.007	0.040	0.260	0.490	0.007	0.090	0.240	0.615	0.002	0.100	0.260	0.695
LRCN2	G	0.05	0.045	0.165	0.390	0.735	0.062	0.135	0.510	0.730	0.040	0.205	0.440	0.770	0.030	0.240	0.445	0.795
		0.01	0.007	0.045	0.170	0.475	0.015	0.030	0.240	0.440	0.007	0.060	0.180	0.545	0.002	0.080	0.205	0.535
HCLLT	G	0.05	0.052	0.160	0.365	0.710	0.075	0.145	0.465	0.650	0.047	0.185	0.390	0.710	0.032	0.230	0.375	0.725
		0.01	0.005	0.030	0.125	0.375	0.007	0.025	0.225	0.335	0.010	0.055	0.155	0.400	0.005	0.080	0.150	0.395
SHORT	G	0.05	0.042	0.100	0.275	0.625	0.062	0.150	0.475	0.705	0.047	0.195	0.440	0.795	0.022	0.235	0.510	0.850
		0.01	0.010	0.020	0.100	0.330	0.015	0.045	0.235	0.435	0.005	0.065	0.185	0.570	0.002	0.075	0.260	0.640
SENAT	G	0.05	0.057	0.155	0.370	0.680	0.067	0.135	0.480	0.685	0.050	0.210	0.420	0.730	0.032	0.240	0.395	0.735
		0.01	0.010	0.035	0.140	0.410	0.015	0.040	0.210	0.335	0.007	0.050	0.135	0.480	0.002	0.070	0.190	0.420

TABLE A41: EFFICIENCIES OF NINE ONE-WAY TESTS RELATIVE TO "BEST TESTS" -- RECTANGULAR-TREND

n	k	Δ	FTEST	HTEST	JONKE	BARTH	CHAKO	ATMAX	LRCON	JMENS	PURIS	BASE POWER
5	3	0.5	0.4625	0.3750	0.7875	0.8500	0.8125	0.8500	0.8500	1.0000	1.0000	0.2000
5	3	1.0	0.5436	0.4667	0.9077	0.8974	0.8462	0.9692	0.9692	1.0000	1.0000	0.4875
5	3	1.5	0.6338	0.5541	0.8981	0.9650	0.8694	1.0000	1.0000	0.9618	0.9618	0.7850
5	4	0.5	0.3894	0.2832	0.8584	0.7611	0.7080	0.8584	0.8319	0.9204	1.0000	0.2825
5	4	1.0	0.4226	0.3264	0.8745	0.8745	0.7908	0.9289	0.9414	0.9791	1.0000	0.5975
5	4	1.5	0.6472	0.4889	0.9417	0.9500	0.8889	0.9917	0.9889	0.9944	1.0000	0.9000
5	5	0.5	0.3582	0.2313	0.7910	0.7910	0.6567	0.7985	0.8134	0.9701	1.0000	0.3350
5	5	1.0	0.4411	0.3367	0.8990	0.8552	0.7946	0.8956	0.9327	0.9562	1.0000	0.7425
5	5	1.5	0.6587	0.5026	0.9497	0.9683	0.8995	1.0000	0.9841	0.9841	0.9868	0.9450
5	6	0.5	0.2966	0.1931	0.7655	0.8276	0.7448	0.8345	0.8690	0.9724	1.0000	0.3625
5	6	1.0	0.4225	0.3070	0.8754	0.8997	0.8267	0.9149	0.9483	0.9362	1.0000	0.8225
5	6	1.5	0.6997	0.5573	0.9695	0.9796	0.9237	0.9873	1.0000	0.9771	0.9949	0.9825
10	3	0.5	0.4382	0.4831	0.8202	0.8427	0.7753	0.8764	0.8764	1.0000	1.0000	0.2225
10	3	1.0	0.5614	0.4649	0.8596	0.8991	0.8684	0.9561	0.9561	1.0000	1.0000	0.5700
10	3	1.5	0.7064	0.6086	0.8991	0.9358	0.8899	0.9908	0.9908	1.0000	1.0000	0.8175
10	4	0.5	0.3360	0.2720	0.7680	0.7520	0.6880	0.7760	0.7440	0.9280	1.0000	0.3125
10	4	1.0	0.4965	0.4306	0.8819	0.8785	0.8090	0.9063	0.9340	0.9653	1.0000	0.7200
10	4	1.5	0.6703	0.5722	0.9373	0.9346	0.8692	0.9782	0.9782	0.9918	1.0000	0.9175
10	5	0.5	0.3043	0.2671	0.7453	0.7950	0.7702	0.8323	0.8199	0.9814	1.0000	0.4025
10	5	1.0	0.4575	0.3922	0.8464	0.8170	0.7516	0.9052	0.9052	0.9739	1.0000	0.7650
10	5	1.5	0.7221	0.6182	0.9688	0.9792	0.9195	0.9844	0.9948	0.9896	1.0000	0.9625
10	6	0.5	0.3077	0.2308	0.6923	0.6923	0.6374	0.6758	0.7088	0.9505	1.0000	0.4550
10	6	1.0	0.4591	0.3772	0.8830	0.8626	0.8187	0.8860	0.9211	0.9591	1.0000	0.8550
10	6	1.5	0.7797	0.6861	0.9747	0.9747	0.9494	0.9797	0.9924	0.9899	1.0000	0.9875
15	3	0.5	0.4528	0.4528	0.7170	0.7736	0.7547	0.7736	0.7736	1.0000	1.0000	0.2650
15	3	1.0	0.4763	0.4741	0.8000	0.8444	0.8000	0.8815	0.8815	1.0000	1.0000	0.6750
15	3	1.5	0.6821	0.5876	0.9017	0.8960	0.8092	0.9711	0.9711	1.0000	1.0000	0.8650
15	4	0.5	0.3151	0.3014	0.6986	0.7534	0.6712	0.7671	0.8219	1.0000	0.9726	0.3650
15	4	1.0	0.5132	0.4211	0.8289	0.7763	0.6908	0.8816	0.9079	0.9868	1.0000	0.7600
15	4	1.5	0.7243	0.6541	0.9405	0.9622	0.9189	0.9568	0.9838	1.0000	1.0000	0.9250
15	5	0.5	0.2738	0.2500	0.7500	0.6786	0.6071	0.7738	0.7738	0.9048	1.0000	0.4200
15	5	1.0	0.4940	0.4464	0.8452	0.8333	0.7440	0.8512	0.8869	0.9643	1.0000	0.8400
15	5	1.5	0.7143	0.6429	0.9541	0.9541	0.9286	0.9694	0.9796	0.9898	1.0000	0.9800
15	6	0.5	0.2727	0.2500	0.6364	0.6364	0.6023	0.6932	0.6591	0.9205	1.0000	0.4400
15	6	1.0	0.4509	0.3815	0.8497	0.8324	0.7977	0.8497	0.8844	0.9942	1.0000	0.8650
15	6	1.5	0.7839	0.7136	0.9899	0.9749	0.9648	0.9899	0.9950	1.0000	1.0000	0.9950
20	3	0.5	0.3103	0.2759	0.7759	0.7241	0.7069	0.7931	0.7931	1.0000	1.0000	0.2900
20	3	1.0	0.5574	0.5246	0.8525	0.8115	0.7951	0.9016	0.9016	1.0000	1.0000	0.6100
20	3	1.5	0.6889	0.5778	0.9000	0.8722	0.7944	0.9556	0.9556	1.0000	1.0000	0.9000
20	4	0.5	0.3167	0.2000	0.6333	0.5667	0.5167	0.7000	0.6833	0.9667	1.0000	0.3000
20	4	1.0	0.4797	0.4122	0.8243	0.8108	0.7568	0.8176	0.8378	0.9392	1.0000	0.7400
20	4	1.5	0.7037	0.6296	0.9312	0.9206	0.8889	0.9577	0.9683	0.9894	1.0000	0.9450
20	5	0.5	0.2737	0.2947	0.7368	0.6211	0.6000	0.7158	0.7789	0.9368	1.0000	0.4750
20	5	1.0	0.4516	0.3935	0.7871	0.8000	0.7419	0.8258	0.8323	0.9806	1.0000	0.7750
20	5	1.5	0.7310	0.6497	0.9594	0.9442	0.9086	0.9848	0.9797	1.0000	0.9949	0.9850
20	6	0.5	0.2527	0.1868	0.6923	0.6923	0.6484	0.6044	0.7253	0.9670	1.0000	0.4550
20	6	1.0	0.4706	0.3957	0.8449	0.8396	0.7807	0.8503	0.8824	0.9519	1.0000	0.9350
20	6	1.5	0.8200	0.7500	0.9750	0.9600	0.9500	0.9800	0.9850	0.9900	1.0000	1.0000

TABLE A42: EFFICIENCIES OF NINE ONE-WAY TESTS RELATIVE TO "BEST TESTS" -- RECTANGULAR-GAP

n	k	Δ	FTEST	HTEST	JONKE	BARTH	CHAKO	ATMAX	LRCON	JMENS	PURIS	BASE POWER
5	3	0.5	0.5128	0.4872	0.7308	0.9103	0.7949	0.9359	0.9359	1.0000	1.0000	0.1950
5	3	1.0	0.5000	0.4730	0.8581	0.9865	0.8919	1.0000	1.0000	0.9324	0.9324	0.3700
5	3	1.5	0.6821	0.5964	0.7893	1.0000	0.9250	0.9536	0.9536	0.8607	0.8607	0.7000
5	4	0.5	0.5233	0.3488	0.8256	0.9070	0.8605	0.9186	0.9186	0.9651	1.0000	0.2150
5	4	1.0	0.5559	0.4286	0.7637	1.0000	0.9231	0.9341	0.8956	0.9505	0.9121	0.4550
5	4	1.5	0.6845	0.4953	0.7003	1.0000	0.9022	0.8991	0.8580	0.8076	0.7697	0.7925
5	5	0.5	0.3763	0.2151	0.7849	0.9355	0.7742	0.9570	0.8925	1.0000	0.9140	0.2325
5	5	1.0	0.5285	0.3731	0.6995	1.0000	0.8601	0.9689	0.8808	0.9275	0.8549	0.4825
5	5	1.5	0.7222	0.4825	0.6462	1.0000	0.8918	0.9094	0.8421	0.8129	0.7222	0.8550
5	6	0.5	0.3810	0.1915	0.6915	0.8617	0.7447	0.8191	0.7553	1.0000	0.8085	0.2350
5	6	1.0	0.5532	0.3617	0.5660	1.0000	0.8170	0.9319	0.7830	0.9319	0.7064	0.5875
5	6	1.5	0.7472	0.4803	0.6180	1.0000	0.9326	0.9382	0.8146	0.8174	0.6854	0.8900
10	3	0.5	0.4930	0.4366	0.8873	0.9577	0.9577	0.8873	0.8873	1.0000	1.0000	0.1775
10	3	1.0	0.7257	0.6114	0.8457	1.0000	0.9371	0.8914	0.8914	1.0000	1.0000	0.4375
10	3	1.5	0.7800	0.6833	0.8433	1.0000	0.9267	0.9367	0.9367	0.9367	0.9367	0.7500
10	4	0.5	0.4079	0.3421	0.7632	0.8816	0.8421	0.9211	0.8684	0.9342	1.0000	0.1900
10	4	1.0	0.6021	0.4764	0.7330	1.0000	0.9005	0.9058	0.8639	0.9267	0.8953	0.4775
10	4	1.5	0.7456	0.6302	0.7574	1.0000	0.9083	0.8935	0.8787	0.8817	0.8254	0.8450
10	5	0.5	0.4054	0.3063	0.6577	0.7748	0.6937	0.7838	0.6937	1.0000	0.9099	0.2775
10	5	1.0	0.6422	0.5138	0.6789	0.9954	0.8761	0.9358	0.8349	1.0000	0.8670	0.5450
10	5	1.5	0.8305	0.6638	0.7712	1.0000	0.9379	0.9350	0.8842	0.9011	0.8475	0.8850
10	6	0.5	0.4034	0.3193	0.7395	0.8235	0.7311	0.8151	0.7395	1.0000	0.9496	0.2975
10	6	1.0	0.6473	0.5021	0.6722	1.0000	0.8714	0.9378	0.7925	0.9876	0.8382	0.6025
10	6	1.5	0.8492	0.7169	0.6878	1.0000	0.9392	0.9365	0.8492	0.9206	0.8069	0.9450
15	3	0.5	0.3556	0.3333	0.7556	0.6444	0.6000	0.7778	0.7778	1.0000	1.0000	0.2250
15	3	1.0	0.5253	0.4444	0.7273	0.8485	0.7273	0.8687	0.8687	1.0000	1.0000	0.4950
15	3	1.5	0.8212	0.7219	0.8543	1.0000	0.9669	0.9470	0.9470	0.9735	0.9735	0.7550
15	4	0.5	0.3333	0.3333	0.5185	0.6481	0.6852	0.6481	0.6667	1.0000	0.9259	0.2700
15	4	1.0	0.5110	0.4867	0.7434	0.8938	0.7788	0.8230	0.7965	1.0000	0.8938	0.5650
15	4	1.5	0.7588	0.6821	0.7803	1.0000	0.9480	0.8844	0.8613	0.9133	0.8555	0.8650
15	5	0.5	0.3636	0.4000	0.8000	0.9091	0.8545	0.8545	0.7273	1.0000	0.8727	0.2750
15	5	1.0	0.6018	0.4867	0.6903	1.0000	0.9204	0.7965	0.7611	0.9204	0.8496	0.5650
15	5	1.5	0.8012	0.7427	0.7602	1.0000	0.9181	0.9474	0.8830	0.9649	0.8538	0.8550
15	6	0.5	0.3125	0.2812	0.6719	0.7969	0.7656	0.8594	0.6719	1.0000	0.8437	0.3200
15	6	1.0	0.6357	0.5736	0.5814	1.0000	0.8682	0.8217	0.6279	0.9147	0.7907	0.6450
15	6	1.5	0.8290	0.7461	0.6632	1.0000	0.9326	0.8705	0.7720	0.8808	0.7720	0.9650
20	3	0.5	0.4551	0.3953	0.7209	0.8605	0.8140	0.7907	0.7907	1.0000	1.0000	0.2150
20	3	1.0	0.5333	0.5238	0.8476	0.9429	0.8571	0.8952	0.8952	1.0000	1.0000	0.5250
20	3	1.5	0.7848	0.7405	0.8418	1.0000	0.9367	0.9304	0.9304	0.9747	0.9747	0.7900
20	4	0.5	0.4576	0.3729	0.6949	0.7966	0.7119	0.7627	0.6949	1.0000	0.9661	0.2950
20	4	1.0	0.6491	0.5439	0.7456	0.9211	0.8333	0.8509	0.8158	1.0000	0.9649	0.5700
20	4	1.5	0.3512	0.7619	0.7917	1.0000	0.9405	0.8929	0.8869	0.9881	0.9702	0.8400
20	5	0.5	0.2982	0.2982	0.5789	0.6842	0.6491	0.7018	0.6316	1.0000	0.8070	0.2850
20	5	1.0	0.6446	0.5702	0.7190	1.0000	0.9091	0.9339	0.8017	1.0000	0.8760	0.6050
20	5	1.5	0.8111	0.7556	0.7611	0.9889	0.9389	0.9611	0.8667	1.0000	0.9389	0.9000
20	5	0.5	0.3194	0.2917	0.5972	0.6806	0.6528	0.7083	0.6111	1.0000	0.8056	0.3600
20	6	1.0	0.5802	0.5191	0.6412	0.9542	0.8473	0.8397	0.7252	1.0000	0.8626	0.6550
20	6	1.5	0.8486	0.7676	0.7568	1.0000	0.9568	0.9351	0.8324	0.9676	0.8703	0.9250

TABLE A43: EFFICIENCIES OF NINE ONE-WAY TESTS RELATIVE TO "BEST TESTS" -- EXPONENTIAL-TREND

n	k	Δ	FTEST	HTEST	JONKE	BARTH	CHAKO	ATMAX	LRCON	JMENS	PURIS	BASE POWER
5	3	0.5	0.4191	0.5074	0.8676	0.6985	0.8824	0.7647	0.7647	1.0000	1.0000	0.3400
5	3	1.0	0.5184	0.6176	0.9743	0.8493	0.9338	0.8640	0.8640	1.0000	1.0000	0.6800
5	3	1.5	0.7420	0.7275	1.0000	0.9130	0.9710	0.9391	0.9391	0.9710	0.9710	0.8625
5	4	0.5	0.2368	0.2597	1.0000	0.6184	0.8026	0.6447	0.6842	0.9868	0.9803	0.3800
5	4	1.0	0.4519	0.5000	0.9968	0.8077	0.9263	0.8429	0.8558	0.9904	1.0000	0.7800
5	4	1.5	0.7169	0.7610	1.0000	0.9325	0.9584	0.9532	0.9558	0.9766	0.9870	0.9625
5	5	0.5	0.2569	0.2798	0.9266	0.6376	0.7890	0.7202	0.7110	0.9220	1.0000	0.5450
5	5	1.0	0.4672	0.5499	1.0000	0.8319	0.9345	0.8575	0.8746	0.9744	0.9915	0.8775
5	5	1.5	0.7908	0.7883	1.0000	0.9515	0.9847	0.9719	0.9796	0.9974	0.9949	0.9800
5	6	0.5	0.2125	0.2250	0.9333	0.5500	0.8042	0.5667	0.6417	0.9417	1.0000	0.6000
5	6	1.0	0.4574	0.5399	1.0000	0.8351	0.9122	0.8484	0.8856	0.9601	1.0000	0.9400
5	6	1.5	0.8056	0.8283	1.0000	0.9798	0.9949	0.9874	0.9924	0.9924	0.9975	0.9900
10	3	0.5	0.2587	0.4562	0.8875	0.5937	0.8157	0.6687	0.6687	1.0000	1.0000	0.4000
10	3	1.0	0.4747	0.6162	1.0000	0.7643	0.9226	0.7946	0.7946	1.0000	1.0000	0.7425
10	3	1.5	0.6757	0.7946	1.0000	0.8568	0.9405	0.8946	0.8946	0.9919	0.9919	0.9250
10	4	0.5	0.2500	0.3676	0.9755	0.5245	0.8578	0.5931	0.5931	0.9755	1.0000	0.5100
10	4	1.0	0.4784	0.6023	1.0000	0.7695	0.9251	0.7954	0.8012	0.9827	0.9885	0.8675
10	4	1.5	0.7038	0.8380	1.0000	0.9241	0.9772	0.9165	0.9266	0.9899	0.9848	0.9875
10	5	0.5	0.2379	0.3216	0.8634	0.4714	0.7841	0.5198	0.4934	0.9559	1.0000	0.5675
10	5	1.0	0.4335	0.6356	0.9947	0.7633	0.9441	0.7819	0.7952	0.9814	1.0000	0.9400
10	5	1.5	0.7995	0.8947	1.0000	0.9599	0.9799	0.9624	0.9699	0.9925	0.9975	0.9975
10	6	0.5	0.2069	0.3027	0.8812	0.4751	0.7625	0.5402	0.5670	0.9693	1.0000	0.6525
10	6	1.0	0.4777	0.6588	0.9895	0.7979	0.9475	0.8215	0.8635	0.9633	1.0000	0.9525
10	6	1.5	0.8550	0.9450	1.0000	0.9750	1.0000	0.9825	0.9800	0.9975	0.9975	1.0000
15	3	0.5	0.2500	0.3942	0.8846	0.4327	0.8269	0.5385	0.5385	1.0000	1.0000	0.5200
15	3	1.0	0.3711	0.6226	0.9245	0.6289	0.8616	0.6541	0.6541	1.0000	1.0000	0.7950
15	3	1.5	0.6432	0.8693	1.0000	0.8492	0.9548	0.8945	0.8945	0.9950	0.9950	0.9950
15	4	0.5	0.2308	0.3626	0.9011	0.5495	0.8462	0.6374	0.6484	1.0000	0.9670	0.4550
15	4	1.0	0.5269	0.6505	0.9731	0.7366	0.9140	0.7527	0.7796	0.9731	1.0000	0.9300
15	4	1.5	0.7538	0.9347	1.0000	0.9497	0.9799	0.9648	0.9648	0.9950	1.0000	0.9950
15	5	0.5	0.2185	0.3361	0.8487	0.3782	0.7647	0.4538	0.4874	0.9580	1.0000	0.5950
15	5	1.0	0.4869	0.6963	0.9791	0.7539	0.9372	0.8063	0.8377	1.0000	0.9895	0.9550
15	5	1.5	0.8050	0.9350	1.0000	0.9550	1.0000	0.9650	0.9650	0.9950	0.9950	1.0000
15	6	0.5	0.2185	0.2848	0.8808	0.4636	0.7351	0.4967	0.5430	0.9338	1.0000	0.7550
15	6	1.0	0.4721	0.7259	0.9746	0.7919	0.9239	0.8071	0.8376	0.9594	1.0000	0.9850
15	6	1.5	0.9050	0.9750	1.0000	0.9900	1.0000	0.9950	0.9950	1.0000	1.0000	1.0000
20	3	0.5	0.2976	0.4881	0.8452	0.5238	0.7619	0.5000	0.5000	1.0000	1.0000	0.4200
20	3	1.0	0.3795	0.6747	0.9578	0.6566	0.8976	0.7530	0.7530	1.0000	1.0000	0.8300
20	3	1.5	0.7056	0.9036	0.9949	0.8579	0.9848	0.8832	0.8832	1.0000	1.0000	0.9850
20	4	0.5	0.1963	0.3271	0.8505	0.4579	0.7664	0.4579	0.4486	0.9626	1.0000	0.5350
20	4	1.0	0.4783	0.7065	0.9728	0.6902	0.9022	0.7554	0.7609	0.9728	1.0000	0.9200
20	4	1.5	0.7525	0.9394	0.9949	0.9343	0.9949	0.9495	0.9444	1.0000	0.9949	0.9900
20	5	0.5	0.1374	0.3053	0.8626	0.4275	0.7939	0.4580	0.5115	0.9466	1.0000	0.6550
20	5	1.0	0.4158	0.6737	0.9842	0.6842	0.9368	0.7526	0.7632	0.9895	1.0000	0.9500
20	5	1.5	0.8000	0.9300	1.0000	0.9500	0.9950	0.9600	0.9700	1.0000	1.0000	1.0000
20	6	0.5	0.1854	0.3576	0.8742	0.4702	0.7483	0.4636	0.5033	0.9868	1.0000	0.7550
20	6	1.0	0.4899	0.7626	0.9747	0.8030	0.9545	0.7879	0.8232	0.9899	1.0000	0.9900
20	6	1.5	0.8550	0.9850	1.0000	0.9800	1.0000	0.9850	0.9850	1.0000	1.0000	1.0000

TABLE A44: EFFICIENCIES OF NINE ONE-WAY TESTS RELATIVE TO "BEST TESTS" -- EXPONENTIAL-GAP

n	k	Δ	FTEST	HTEST	JONKE	BARTH	CHAKO	ATMAX	LRCON	JMENS	PURIS	BASE POWER
5	3	0.5	0.4272	0.4757	0.9126	0.7864	1.0000	0.8155	0.8155	0.9903	0.9903	0.2575
5	3	1.0	0.5776	0.6509	0.7672	0.8664	1.0000	0.8534	0.8534	0.7629	0.7629	0.5800
5	3	1.5	0.5923	0.7426	0.8343	0.9260	1.0000	0.9024	0.9024	0.8136	0.8136	0.8450
5	4	0.5	0.4186	0.3953	0.7829	0.7829	1.0000	0.7597	0.7364	0.9535	0.8682	0.3225
5	4	1.0	0.5823	0.5221	0.7871	0.8755	1.0000	0.8514	0.7791	0.8193	0.7430	0.6225
5	4	1.5	0.7983	0.6685	0.7652	0.9669	1.0000	0.9006	0.8950	0.8066	0.7597	0.9050
5	5	0.5	0.3478	0.3826	0.8435	0.8261	1.0000	0.7478	0.7652	0.9478	0.8696	0.2875
5	5	1.0	0.6284	0.5862	0.7011	0.9272	1.0000	0.7969	0.7280	0.8008	0.7126	0.6525
5	5	1.5	0.8123	0.6810	0.6702	0.9651	1.0000	0.9008	0.8552	0.7828	0.6810	0.9325
5	6	0.5	0.4340	0.4057	0.8302	0.9245	1.0000	0.8868	0.8208	0.9906	0.8302	0.2650
5	6	1.0	0.5256	0.4232	0.5836	0.9181	1.0000	0.8089	0.6962	0.7406	0.5973	0.7325
5	6	1.5	0.8225	0.6580	0.6658	0.9791	1.0000	0.9321	0.8616	0.8198	0.6867	0.9575
10	3	0.5	0.3437	0.5625	0.9375	0.6953	1.0000	0.7344	0.7344	1.0000	1.0000	0.3200
10	3	1.0	0.5341	0.7121	0.9735	0.7500	1.0000	0.7538	0.7538	0.8788	0.8788	0.6600
10	3	1.5	0.7518	0.8920	0.9335	0.9003	1.0000	0.8670	0.8670	0.8920	0.8920	0.9025
10	4	0.5	0.3664	0.5649	0.7863	0.6870	1.0000	0.6489	0.5496	0.8092	0.7710	0.3275
10	4	1.0	0.5116	0.7228	0.7789	0.8119	1.0000	0.7624	0.6898	0.8119	0.7360	0.7575
10	4	1.5	0.7749	0.9110	0.8770	0.9267	1.0000	0.8770	0.8325	0.8639	0.8220	0.9550
10	5	0.5	0.3333	0.5152	0.8712	0.7197	1.0000	0.7045	0.6439	0.9318	0.8864	0.3300
10	5	1.0	0.5186	0.7143	0.7453	0.7671	1.0000	0.6988	0.6335	0.7671	0.6988	0.8050
10	5	1.5	0.8346	0.9313	0.8499	0.9491	1.0000	0.8880	0.8473	0.8728	0.8092	0.9825
10	6	0.5	0.3784	0.4797	0.7095	0.6284	1.0000	0.6554	0.4932	0.8446	0.7432	0.3700
10	6	1.0	0.5337	0.6713	0.6798	0.7669	1.0000	0.7191	0.5871	0.7837	0.6489	0.8900
10	6	1.5	0.8813	0.9293	0.7828	0.9697	1.0000	0.9040	0.8359	0.8889	0.7727	0.9900
15	3	0.5	0.4310	0.7414	0.9138	0.7414	1.0000	0.7414	0.7414	0.9655	0.9655	0.2900
15	3	1.0	0.4851	0.7910	0.9552	0.7463	1.0000	0.7388	0.7388	0.9552	0.9552	0.6700
15	3	1.5	0.7394	0.9043	0.9309	0.8511	1.0000	0.8138	0.8138	0.9096	0.9096	0.9400
15	4	0.5	0.3860	0.5439	0.9123	0.6140	1.0000	0.6140	0.6140	1.0000	1.0000	0.2850
15	4	1.0	0.5096	0.7452	0.8471	0.6815	1.0000	0.6561	0.6178	0.8153	0.7898	0.7850
15	4	1.5	0.8173	0.9391	0.9391	0.9188	1.0000	0.8731	0.8680	0.9188	0.9137	0.9850
15	5	0.5	0.4776	0.5970	0.7313	0.6716	1.0000	0.6418	0.5970	0.8657	0.7761	0.3350
15	5	1.0	0.5230	0.7701	0.7471	0.7644	1.0000	0.6954	0.6379	0.7931	0.7241	0.8700
15	5	1.5	0.8535	0.9545	0.8737	0.9495	1.0000	0.8535	0.8131	0.8838	0.8384	0.9900
15	6	0.5	0.2809	0.4270	0.7528	0.5393	1.0000	0.5618	0.5056	0.8202	0.7753	0.4450
15	6	1.0	0.5989	0.8128	0.6684	0.8128	1.0000	0.7273	0.6203	0.7861	0.6524	0.9350
15	6	1.5	0.8850	0.9850	0.9050	0.9900	1.0000	0.9100	0.8650	0.9300	0.8600	1.0000
20	3	0.5	0.3662	0.5211	0.8028	0.6479	0.8592	0.6197	0.6197	1.0000	1.0000	0.3550
20	3	1.0	0.5519	0.8571	0.9091	0.7013	1.0000	0.6753	0.6753	0.8831	0.8831	0.7700
20	3	1.5	0.7306	0.9430	0.9326	0.8549	1.0000	0.8187	0.8187	0.9223	0.9223	0.9650
20	4	0.5	0.3919	0.5676	0.8378	0.6622	0.8514	0.6081	0.6351	1.0000	0.9595	0.3700
20	4	1.0	0.5556	0.8187	0.8012	0.7251	1.0000	0.6199	0.5965	0.8480	0.8070	0.8550
20	4	1.5	0.7919	0.9594	0.9391	0.8883	1.0000	0.8223	0.8122	0.9442	0.9391	0.9850
20	5	0.5	0.3590	0.5769	0.7564	0.5897	1.0000	0.6026	0.5385	0.8205	0.8205	0.3900
20	5	1.0	0.4432	0.7568	0.7081	0.6811	1.0000	0.6054	0.5405	0.7730	0.7027	0.9250
20	5	1.5	0.8400	0.9600	0.9300	0.9600	1.0000	0.9000	0.8400	0.9550	0.8900	1.0000
20	6	0.5	0.3452	0.5952	0.8571	0.6190	1.0000	0.6190	0.5357	0.9405	0.7857	0.4200
20	6	1.0	0.4946	0.7957	0.7419	0.7634	1.0000	0.7151	0.6183	0.8710	0.7473	0.9300
20	6	1.5	0.8850	0.9900	0.9200	0.9650	1.0000	0.9100	0.8400	0.9300	0.8950	1.0000

TABLE A45: EFFICIENCIES OF NINE ONE-WAY TESTS RELATIVE TO "BEST TESTS" -- NORMAL - TREND

n	k	Δ	FTEST	HTEST	JONKE	BARTH	CHAKO	ATMAX	LRCON	JMENS	PURIS	BASE POWER
5	3	0.5	0.4756	0.4390	0.8171	0.8415	0.8415	1.0000	1.0000	0.9512	0.9512	0.2050
5	3	1.0	0.5381	0.5190	0.9000	0.9571	0.9000	1.0000	1.0000	0.9381	0.9381	0.5250
5	3	1.5	0.6067	0.6361	0.9430	0.9715	0.9272	1.0000	1.0000	0.9842	0.9842	0.7900
5	4	0.5	0.3554	0.3173	1.0000	0.9615	0.8269	0.9519	0.9809	0.9904	0.9803	0.2600
5	4	1.0	0.4656	0.3563	0.9717	0.9109	0.8421	0.9838	1.0000	0.9838	0.9717	0.6175
5	4	1.5	0.6860	0.5978	0.9862	0.9697	0.9311	0.9972	1.0000	0.9917	0.9945	0.9075
5	5	0.5	0.4348	0.3596	0.9565	0.8478	0.7899	0.8696	0.9783	0.9058	1.0000	0.3450
5	5	1.0	0.4783	0.3846	0.9431	0.9365	0.8729	0.9431	1.0000	0.9197	0.9766	0.7475
5	5	1.5	0.7273	0.6070	0.9759	0.9679	0.9278	0.9813	1.0000	0.9652	0.9947	0.9350
5	6	0.5	0.2759	0.1983	0.9397	0.7931	0.6983	0.9569	0.9483	0.9914	1.0000	0.2900
5	6	1.0	0.4291	0.3311	0.9595	0.9459	0.8885	0.9730	1.0000	0.9696	0.9797	0.7400
5	6	1.5	0.7417	0.6496	0.9898	0.9847	0.9668	0.9923	1.0000	0.9872	0.9949	0.9775
10	3	0.5	0.4512	0.4268	0.9756	1.0000	0.9756	0.9756	0.9756	0.9390	0.9390	0.2050
10	3	1.0	0.5024	0.4502	0.9479	0.9100	0.8436	1.0000	1.0000	0.9336	0.9336	0.5275
10	3	1.5	0.6966	0.6378	0.9876	0.9814	0.9598	1.0000	1.0000	0.9938	0.9938	0.8075
10	4	0.5	0.4845	0.4845	0.9897	1.0000	0.9381	0.9175	0.9794	0.9278	0.9794	0.2425
10	4	1.0	0.5133	0.4259	0.9202	0.8973	0.8403	0.9810	1.0000	0.9392	0.9620	0.6575
10	4	1.5	0.7417	0.6694	0.9694	0.9750	0.9417	1.0000	0.9972	0.9861	0.9694	0.9000
10	5	0.5	0.4331	0.4094	0.8898	0.8740	0.7717	0.9134	0.9606	0.9370	1.0000	0.3175
10	5	1.0	0.4502	0.3918	0.9622	0.8900	0.8522	0.9656	0.9863	0.9519	1.0000	0.7275
10	5	1.5	0.7422	0.7057	0.9896	0.9766	0.9479	0.9896	1.0000	0.9922	1.0000	0.9600
10	6	0.5	0.3284	0.2687	0.9254	0.8507	0.7313	0.9030	0.9552	0.9328	1.0000	0.3350
10	6	1.0	0.4534	0.4055	0.9299	0.9055	0.8476	0.9268	0.9695	0.9238	1.0000	0.8200
10	6	1.5	0.8046	0.7481	0.9949	0.9871	0.9794	0.9923	1.0000	0.9949	0.9974	0.9725
15	3	0.5	0.4792	0.6250	0.9583	0.8125	0.8125	0.9583	0.9583	1.0000	1.0000	0.2400
15	3	1.0	0.4779	0.4425	0.9381	0.8761	0.8407	1.0000	1.0000	0.9823	0.9823	0.5650
15	3	1.5	0.6584	0.6211	0.9565	0.9255	0.8634	1.0000	1.0000	0.9876	0.9876	0.8050
15	4	0.5	0.3818	0.3818	0.9273	0.8727	0.8545	0.9455	0.9818	0.9636	1.0000	0.2750
15	4	1.0	0.5591	0.4803	0.9606	0.9449	0.9055	0.9291	0.9921	0.9764	1.0000	0.6350
15	4	1.5	0.7330	0.6648	0.9830	0.9830	0.9489	1.0000	0.9943	0.9830	0.9943	0.8800
15	5	0.5	0.3684	0.3860	0.9123	1.0000	0.9825	0.9123	0.9474	0.9298	0.9474	0.2850
15	5	1.0	0.5448	0.4966	0.9517	0.9172	0.8552	0.9655	1.0000	0.9379	0.9793	0.7250
15	5	1.5	0.7949	0.7641	1.0000	0.9846	0.9641	0.9744	0.9949	0.9795	1.0000	0.9750
15	6	0.5	0.2587	0.1642	0.9403	0.7761	0.7612	0.8657	0.9552	0.8657	1.0000	0.3350
15	6	1.0	0.4847	0.3988	0.9387	0.9018	0.8466	0.9202	1.0000	0.9387	1.0000	0.8150
15	6	1.5	0.8103	0.7692	0.9897	0.9846	0.9846	0.9949	0.9949	0.9897	1.0000	0.9750
20	3	0.5	0.5429	0.5429	0.9429	1.0000	0.8857	0.9714	0.9714	0.9714	0.9714	0.1750
20	3	1.0	0.5961	0.5514	0.9626	0.9626	0.9252	1.0000	1.0000	0.9907	0.9907	0.5350
20	3	1.5	0.7362	0.7423	0.9571	0.9755	0.9387	0.9816	0.9816	1.0000	1.0000	0.8150
20	4	0.5	0.2751	0.2459	0.7705	0.8033	0.7377	0.9344	0.9180	0.9508	1.0000	0.3050
20	4	1.0	0.5593	0.5508	0.9407	0.8814	0.8729	0.9915	1.0000	0.9661	1.0000	0.5900
20	4	1.5	0.7418	0.7388	0.9835	0.9835	0.9670	1.0000	1.0000	0.9890	1.0000	0.9100
20	5	0.5	0.5000	0.4615	0.9231	0.8077	0.8077	0.9231	0.9808	0.9423	1.0000	0.2600
20	5	1.0	0.5248	0.4894	0.9433	0.9716	0.9149	0.9574	1.0000	0.9858	1.0000	0.7050
20	5	1.5	0.8053	0.7789	0.9632	0.9789	0.9526	0.9789	0.9895	0.9895	1.0000	0.9500
20	6	0.5	0.3766	0.2857	0.8571	0.8312	0.7922	0.8831	0.9221	0.9221	1.0000	0.3850
20	6	1.0	0.5130	0.4870	0.9481	0.9156	0.8961	0.9805	0.9805	0.9805	1.0000	0.7700
20	6	1.5	0.8392	0.8040	0.9799	0.9799	0.9698	0.9749	1.0000	0.9749	1.0000	0.9950

TABLE A46: EFFICIENCIES OF NINE ONE-WAY TESTS RELATIVE TO "BEST TESTS" -- NORMAL - GAP

N	k	Δ	FTEST	HTEST	JONKE	BARTH	CHAKO	ATMAX	LRCON	JMENS	PURIS	BASE POWER
5	3	0.5	0.5206	0.5143	0.8143	0.9286	1.0000	0.9286	0.9286	0.9000	0.9000	0.1750
5	3	1.0	0.6765	0.5647	0.9000	1.0000	0.9941	1.0000	1.0000	0.9412	0.9412	0.4250
5	3	1.5	0.7224	0.7023	0.7726	1.0000	0.9900	0.9264	0.9264	0.8294	0.8294	0.7475
5	4	0.5	0.4750	0.4330	0.8750	0.9500	0.9625	1.0000	0.8875	0.9750	0.9250	0.2000
5	4	1.0	0.6075	0.4677	0.8333	0.9839	0.9785	1.0000	0.9462	0.9839	0.9194	0.4650
5	4	1.5	0.6594	0.5480	0.6997	1.0000	0.9133	0.9319	0.8978	0.8328	0.7616	0.8075
5	5	0.5	0.4592	0.3163	0.8469	0.9796	0.8878	0.9286	0.9286	1.0000	0.9490	0.2450
5	5	1.0	0.5364	0.3364	0.7545	1.0000	0.9136	0.9273	0.8409	0.8864	0.7955	0.5500
5	5	1.5	0.7603	0.6036	0.6154	1.0000	0.9142	0.9024	0.7899	0.8077	0.6775	0.8450
5	6	0.5	0.5616	0.3836	0.7260	1.0000	0.9178	0.8767	0.7123	0.8767	0.7808	0.1825
5	6	1.0	0.5856	0.4297	0.6198	1.0000	0.9011	0.8973	0.7605	0.8327	0.6578	0.6575
5	6	1.5	0.8069	0.6093	0.6557	1.0000	0.9617	0.9590	0.8579	0.8689	0.7295	0.9150
10	3	0.5	0.5769	0.5385	0.8333	1.0000	0.9359	0.9615	0.9615	0.9103	0.9103	0.1950
10	3	1.0	0.6412	0.5471	0.9059	1.0000	0.9471	0.9647	0.9647	0.9294	0.9294	0.4250
10	3	1.5	0.8371	0.7604	0.8754	1.0000	0.9649	0.9425	0.9425	0.8786	0.8786	0.7825
10	4	0.5	0.4795	0.4521	0.8904	1.0000	0.9863	0.9178	0.9315	0.8493	0.8082	0.1825
10	4	1.0	0.6039	0.4977	0.7934	1.0000	0.9296	0.9014	0.8638	0.8357	0.7887	0.5325
10	4	1.5	0.7821	0.7075	0.7970	1.0000	0.9881	0.9313	0.9015	0.8866	0.8478	0.8375
10	5	0.5	0.3596	0.3034	0.8652	0.8539	0.8202	1.0000	0.8652	0.9775	0.8764	0.2225
10	5	1.0	0.6045	0.5273	0.7227	1.0000	0.9045	0.8773	0.8409	0.8818	0.7773	0.5500
10	5	1.5	0.8202	0.7612	0.7753	1.0000	0.9803	0.9270	0.8624	0.8961	0.8202	0.8900
10	6	0.5	0.4468	0.3617	0.8191	0.8830	0.8617	0.9362	0.8511	1.0000	0.9259	0.2350
10	6	1.0	0.5230	0.5328	0.6885	1.0000	0.9385	0.9221	0.7664	0.8893	0.7541	0.6100
10	6	1.5	0.8374	0.7615	0.7642	1.0000	0.9892	0.9431	0.8699	0.9214	0.8347	0.9225
15	3	0.5	0.3469	0.3265	0.8571	0.7347	0.6939	0.8367	0.8367	1.0000	1.0000	0.2450
15	3	1.0	0.6364	0.5584	0.9481	1.0000	1.0000	0.9740	0.9740	0.9870	0.9870	0.3850
15	3	1.5	0.7815	0.7086	0.8675	1.0000	0.9603	0.9404	0.9404	0.9338	0.9338	0.7250
15	4	0.5	0.4222	0.4444	0.7556	1.0000	0.9556	0.8889	0.8889	0.9778	0.8889	0.2250
15	4	1.0	0.6729	0.6355	0.7664	1.0000	0.9720	0.8318	0.8318	0.8037	0.7850	0.5350
15	4	1.5	0.7939	0.7333	0.8121	1.0000	0.9455	0.8970	0.8848	0.8424	0.8303	0.8250
15	5	0.5	0.4407	0.4068	0.7966	0.8644	0.7627	1.0000	0.8644	0.9322	0.8475	0.2950
15	5	1.0	0.6260	0.6016	0.8016	0.9593	1.0000	0.8780	0.7886	0.8374	0.7642	0.6150
15	5	1.5	0.8701	0.8192	0.7910	1.0000	0.9774	0.9040	0.8644	0.8814	0.8192	0.8850
15	6	0.5	0.4222	0.4000	0.8444	0.9556	0.9333	1.0000	0.8444	0.9333	0.8667	0.2250
15	6	1.0	0.7105	0.5351	0.6316	1.0000	0.9386	0.8947	0.7281	0.8772	0.7105	0.5700
15	6	1.5	0.9548	0.7957	0.8280	1.0000	1.0000	0.9516	0.8871	0.9409	0.8441	0.9300
20	3	0.5	0.3333	0.3056	0.8056	0.9444	0.9167	0.9167	0.9167	1.0000	1.0000	0.1800
20	3	1.0	0.6145	0.5904	0.8795	1.0000	0.9759	0.9759	0.9759	1.0000	1.0000	0.4150
20	3	1.5	0.8025	0.7284	0.8642	1.0000	0.9568	0.9012	0.9012	0.9012	0.9012	0.8100
20	4	0.5	0.5937	0.5625	0.7500	1.0000	0.9375	0.9688	0.8750	0.9688	0.8438	0.1600
20	4	1.0	0.6509	0.5755	0.8868	1.0000	0.9151	0.9434	0.9623	0.9528	0.9623	0.5300
20	4	1.5	0.8012	0.7470	0.8193	1.0000	0.9819	0.9398	0.8675	0.9036	0.8554	0.8300
20	5	0.5	0.5532	0.4681	0.8511	0.7447	0.8723	0.8936	0.7772	1.0000	0.8936	0.2350
20	5	1.0	0.6410	0.5299	0.7265	1.0000	0.9316	0.9231	0.7778	0.9231	0.7863	0.5850
20	5	1.5	0.8323	0.8580	0.8295	1.0000	0.9886	0.9148	0.8864	0.8920	0.8693	0.8800
20	6	0.5	0.4032	0.3710	0.7581	0.7742	0.7419	0.9194	0.8065	1.0000	0.8387	0.3100
20	6	1.0	0.7119	0.6355	0.6441	1.0000	0.9915	0.9237	0.7373	0.9237	0.7203	0.5900
20	6	1.5	0.8263	0.7632	0.7789	1.0000	0.9684	0.9053	0.8263	0.8947	0.8316	0.9500

TABLE A47: EFFICIENCIES OF NINE TWO-WAY TESTS RELATIVE TO "BEST TESTS" -- RECTANGULAR - TREND

n	k	Δ	F2WAY	PAGET	FRIED	PHENS	ATMX2	LRCN2	HOLLT	SHORT	SENAT	BASE POWER
5	3	0.5	0.4925	0.6866	0.3582	0.6866	0.9701	0.9701	0.9254	0.7313	1.0000	0.1675
5	3	1.0	0.5167	0.7389	0.2444	0.7389	1.0000	1.0000	0.9333	0.7389	0.9389	0.4500
5	3	1.5	0.5987	0.7368	0.3520	0.7368	1.0000	1.0000	0.9375	0.7336	0.9375	0.7600
5	4	0.5	0.4528	0.8113	0.3302	0.8113	0.8585	0.8962	1.0000	0.7642	0.9151	0.2650
5	4	1.0	0.4389	0.8371	0.2624	0.8371	0.9819	1.0000	0.9774	0.7330	0.9140	0.5525
5	4	1.5	0.5892	0.8597	0.4023	0.8697	0.9887	1.0000	0.9632	0.7932	0.9575	0.8825
5	5	0.5	0.4386	0.9211	0.2368	0.9737	0.9561	0.9561	1.0000	0.7982	0.9649	0.2850
5	5	1.0	0.4412	0.8971	0.2978	0.9228	0.9632	1.0000	0.9743	0.8346	0.9669	0.6800
5	5	1.5	0.6472	0.8833	0.4562	0.8966	0.9867	1.0000	0.9496	0.8223	0.9523	0.9425
5	6	0.5	0.3492	0.8492	0.2302	0.8651	0.9048	1.0000	0.8730	0.7540	0.9524	0.3150
5	6	1.0	0.4363	0.8248	0.2707	0.8758	0.9554	1.0000	0.9108	0.7484	0.9268	0.7850
5	6	1.5	0.6726	0.9442	0.4848	0.9492	0.9822	1.0000	0.9848	0.8883	0.9746	0.9850
10	3	0.5	0.5455	0.8182	0.3766	0.8182	1.0000	1.0000	0.9351	0.7143	0.9351	0.1925
10	3	1.0	0.5442	0.7070	0.2977	0.7070	1.0000	1.0000	0.9070	0.6279	0.9163	0.5375
10	3	1.5	0.6911	0.7994	0.4299	0.7994	1.0000	1.0000	0.9650	0.7261	0.9427	0.7850
10	4	0.5	0.4400	0.7700	0.3000	0.8500	0.9700	1.0000	0.9100	0.7400	0.9600	0.2500
10	4	1.0	0.5000	0.8295	0.3939	0.8788	0.9924	1.0000	0.9659	0.7424	0.9811	0.6600
10	4	1.5	0.6722	0.8389	0.4806	0.8500	1.0000	1.0000	0.9611	0.7861	0.9556	0.9000
10	5	0.5	0.3835	0.8647	0.2707	0.9323	1.0000	0.9624	0.8872	0.8120	0.8947	0.3325
10	5	1.0	0.4928	0.8273	0.3237	0.8633	0.9676	1.0000	0.9353	0.7554	0.9281	0.6950
10	5	1.5	0.7113	0.9160	0.5669	0.9396	0.9974	1.0000	0.9738	0.8740	0.9606	0.9525
10	6	0.5	0.4231	0.9231	0.3308	0.9538	0.9538	1.0000	0.9231	0.8308	0.9846	0.3250
10	6	1.0	0.4984	0.8540	0.3460	0.8603	0.9778	1.0000	0.9492	0.8000	0.9683	0.7875
10	6	1.5	0.7738	0.9537	0.5835	0.9640	0.9974	1.0000	0.9923	0.9229	0.9897	0.9725
15	3	0.5	0.4889	0.5556	0.5111	0.5556	1.0000	1.0000	0.8667	0.7778	0.8667	0.2250
15	3	1.0	0.5508	0.6695	0.3729	0.6695	1.0000	1.0000	0.9068	0.6186	0.8644	0.5900
15	3	1.5	0.6646	0.7453	0.4720	0.7453	1.0000	1.0000	0.9441	0.6770	0.9379	0.8050
15	4	0.5	0.3667	0.7500	0.3333	0.7500	0.9500	1.0000	0.9000	0.7333	0.8500	0.3000
15	4	1.0	0.5435	0.7536	0.3333	0.7609	0.9420	1.0000	0.9130	0.6812	0.9275	0.6900
15	4	1.5	0.7582	0.8516	0.5989	0.8681	0.9780	1.0000	0.9725	0.8187	0.9560	0.9100
15	5	0.5	0.3433	1.0000	0.2985	0.9701	0.9552	0.9851	0.9403	0.7463	0.9701	0.3350
15	5	1.0	0.5714	0.8912	0.4422	0.9116	0.9728	1.0000	0.9524	0.7823	0.9660	0.7350
15	5	1.5	0.7183	0.9167	0.5417	0.9219	0.9896	1.0000	0.9583	0.8594	0.9635	0.9600
15	6	0.5	0.3833	0.8667	0.2833	0.9500	1.0000	0.9500	0.9667	0.7333	0.9500	0.3000
15	6	1.0	0.4968	0.8645	0.3806	0.8968	0.9419	1.0000	0.9226	0.8452	0.9484	0.7750
15	6	1.5	0.8020	0.9797	0.6497	0.9848	1.0000	1.0000	0.9898	0.9340	0.9898	0.9850
20	3	0.5	0.3778	0.8444	0.3111	0.8444	1.0000	1.0000	0.9111	0.6667	0.9111	0.2250
20	3	1.0	0.5343	0.8302	0.4717	0.8302	1.0000	1.0000	0.9057	0.7547	0.9528	0.5300
20	3	1.5	0.7412	0.7647	0.4755	0.7647	1.0000	1.0000	0.9176	0.6647	0.9412	0.8500
20	4	0.5	0.3404	0.8936	0.3617	0.9362	0.8511	0.9574	0.8298	1.0000	0.8511	0.2350
20	4	1.0	0.5234	0.8203	0.4141	0.8516	0.9453	1.0000	0.9375	0.7656	0.9375	0.6400
20	4	1.5	0.7253	0.8516	0.5330	0.8791	0.9890	1.0000	0.9670	0.7967	0.9670	0.9100
20	5	0.5	0.3380	0.8732	0.3521	0.8873	0.9014	1.0000	0.9718	0.7324	1.0000	0.3550
20	5	1.0	0.5469	0.9297	0.3750	0.9297	1.0000	0.9844	0.9609	0.8125	0.9297	0.6400
20	5	1.5	0.7474	0.9175	0.6031	0.9330	1.0000	0.9897	0.9794	0.8660	0.9536	0.9700
20	6	0.5	0.3182	0.8182	0.2576	0.8485	0.8333	1.0000	0.9091	0.8030	0.9394	0.3300
20	6	1.0	0.5060	0.8554	0.4157	0.8916	0.9819	1.0000	0.9458	0.7831	0.9398	0.8300
20	6	1.5	0.8071	0.9746	0.6802	0.9746	0.9898	1.0000	0.9848	0.9289	0.9797	0.9850

TABLE A48: EFFICIENCIES OF NINE TWO-WAY TESTS RELATIVE TO "BEST TESTS" -- RECTANGULAR - GAP

n	k	Δ	F2WAY	PAGET	FRIED	PHENS	ATMX2	LRCN2	HOLLT	SHORT	SENAT	BASE POWER
5	3	0.5	0.5616	0.5890	0.2603	0.5890	1.0000	1.0000	0.8904	0.7397	0.7806	0.1825
5	3	1.0	0.5135	0.6892	0.3041	0.6892	1.0000	1.0000	0.8851	0.7838	0.9122	0.3700
5	3	1.5	0.6557	0.7087	0.4055	0.7087	1.0000	1.0000	0.8268	0.7835	0.8858	0.6350
5	4	0.5	0.5000	0.9605	0.3158	0.9474	1.0000	1.0000	0.9474	0.9868	0.9605	0.1900
5	4	1.0	0.6564	0.8650	0.3804	0.8650	1.0000	0.9509	0.7853	0.9755	0.8466	0.4075
5	4	1.5	0.7378	0.7343	0.4301	0.7343	1.0000	0.9510	0.7028	0.9161	0.7797	0.7150
5	5	0.5	0.3690	0.7381	0.3333	0.8214	1.0000	0.9048	0.7976	0.7738	0.7619	0.2100
5	5	1.0	0.5337	0.6966	0.3202	0.7416	1.0000	0.8989	0.7135	0.7809	0.7640	0.4450
5	5	1.5	0.7735	0.6375	0.4304	0.6602	1.0000	0.9191	0.5793	0.8738	0.7282	0.7725
5	6	0.5	0.4217	0.6627	0.2289	0.6988	1.0000	0.8675	0.8675	0.8193	0.7952	0.2075
5	6	1.0	0.5700	0.5668	0.3318	0.5945	1.0000	0.8295	0.5438	0.8710	0.6083	0.5425
5	6	1.5	0.7892	0.5904	0.4157	0.6235	1.0000	0.8855	0.5392	0.8886	0.6596	0.8300
10	3	0.5	0.5846	0.7231	0.3538	0.7231	0.9385	0.9385	0.9231	0.6000	1.0000	0.1625
10	3	1.0	0.7125	0.7062	0.3812	0.7062	1.0000	1.0000	0.8750	0.6937	0.9187	0.4000
10	3	1.5	0.8051	0.7401	0.4585	0.7401	1.0000	1.0000	0.8809	0.7726	0.8989	0.6925
10	4	0.5	0.4521	0.6438	0.4521	0.6849	1.0000	0.9452	0.6712	0.7123	0.8356	0.1825
10	4	1.0	0.6667	0.7083	0.4286	0.7440	1.0000	0.9464	0.7560	0.8571	0.8512	0.4200
10	4	1.5	0.8000	0.7279	0.6098	0.7541	1.0000	0.9738	0.7967	0.8754	0.8361	0.7625
10	5	0.5	0.4941	0.8471	0.3882	0.9412	1.0000	0.9059	0.7882	0.9176	0.8941	0.2125
10	5	1.0	0.6436	0.6683	0.5297	0.6931	1.0000	0.8861	0.7574	0.8564	0.7624	0.5050
10	5	1.5	0.8584	0.6958	0.6175	0.7169	1.0000	0.9337	0.7681	0.9066	0.8102	0.8300
10	6	0.5	0.4898	0.6939	0.4184	0.7041	1.0000	0.9286	0.8163	0.8469	0.9082	0.2450
10	6	1.0	0.6575	0.7123	0.4566	0.7123	1.0000	0.8721	0.6941	0.8630	0.7626	0.5475
10	6	1.5	0.9155	0.6845	0.6789	0.6901	1.0000	0.8901	0.6620	0.9437	0.7606	0.8875
15	3	0.5	0.4211	0.7895	0.3947	0.7895	1.0000	1.0000	0.9474	0.5789	0.8158	0.1900
15	3	1.0	0.6951	0.6585	0.4268	0.6585	1.0000	1.0000	0.9390	0.6585	0.9268	0.4100
15	3	1.5	0.8511	0.6528	0.6319	0.6528	1.0000	1.0000	0.8750	0.7778	0.9028	0.7200
15	4	0.5	0.5128	0.7179	0.5641	0.7949	0.8974	0.8205	0.7949	1.0000	0.7436	0.1950
15	4	1.0	0.6737	0.7579	0.4842	0.8000	1.0000	0.9579	0.8737	0.8632	0.8526	0.4750
15	4	1.5	0.8940	0.8079	0.6424	0.8146	1.0000	0.9801	0.8808	0.9536	0.8675	0.7550
15	5	0.5	0.5333	0.8444	0.4667	0.8222	1.0000	0.8889	0.9556	0.8000	0.9778	0.2250
15	5	1.0	0.7500	0.8523	0.5341	0.8523	0.9773	0.9886	0.8295	1.0000	0.8636	0.4400
15	5	1.5	0.8659	0.7439	0.6829	0.7439	1.0000	0.9329	0.7683	0.8720	0.8049	0.8200
15	6	0.5	0.3846	0.7500	0.3462	0.7692	1.0000	0.8554	0.7885	0.8077	0.8077	0.2600
15	6	1.0	0.8774	0.6038	0.5094	0.6604	1.0000	0.7830	0.6226	0.9528	0.6887	0.5300
15	6	1.5	0.9415	0.7135	0.7251	0.7485	0.9883	0.8713	0.7076	1.0000	0.7368	0.8550
20	3	0.5	0.6471	0.7059	0.4706	0.7059	1.0000	1.0000	0.8824	0.8235	0.9706	0.1700
20	3	1.0	0.5579	0.6632	0.4632	0.6632	1.0000	1.0000	0.8632	0.6947	0.8737	0.4750
20	3	1.5	0.8121	0.6980	0.6376	0.6980	1.0000	1.0000	0.8993	0.7785	0.9060	0.7450
20	4	0.5	0.5682	0.8864	0.4091	0.9091	1.0000	0.9545	0.9773	1.0000	0.9545	0.2200
20	4	1.0	0.7041	0.7755	0.4592	0.8061	1.0000	0.9490	0.8367	0.8367	0.8776	0.4900
20	4	1.5	0.9474	0.8553	0.6118	0.8750	1.0000	0.9868	0.9013	0.9671	0.8947	0.7600
20	5	0.5	0.3846	0.8462	0.5385	0.8718	1.0000	0.8974	0.7949	0.9487	0.9231	0.1950
20	5	1.0	0.7027	0.7207	0.5676	0.7387	1.0000	0.8549	0.7477	0.9820	0.7650	0.5550
20	5	1.5	0.8488	0.7791	0.6686	0.7849	1.0000	0.9012	0.8430	0.9186	0.8081	0.8600
20	6	0.5	0.3455	0.8000	0.4364	0.8364	1.0000	0.7818	0.7636	0.9091	0.7636	0.2750
20	6	1.0	0.6944	0.7130	0.5463	0.7222	1.0000	0.8889	0.7130	0.9444	0.7870	0.5400
20	6	1.5	0.9029	0.7600	0.7200	0.7886	1.0000	0.8686	0.7543	0.9371	0.8057	0.8750

TABLE A49: EFFICIENCIES OF NINE TWO-WAY TESTS RELATIVE TO "BEST TESTS" -- EXPONENTIAL - GAP

n	k	Δ	F2WAY	PAGET	FRIED	PHENS	ATMX2	LRCN2	HOLLT	SHORT	SENAT	BASE POWER
5	3	0.5	0.4113	0.7500	0.2823	0.7500	0.7823	0.7823	0.8226	0.7419	1.0000	0.3100
5	3	1.0	0.4929	0.8517	0.3574	0.8517	0.8669	0.8669	0.8897	0.7681	1.0000	0.6575
5	3	1.5	0.6588	0.8853	0.4853	0.8853	0.9382	0.9382	0.9500	0.8000	1.0000	0.8500
5	4	0.5	0.2177	0.8912	0.2857	0.8844	0.6531	0.6939	0.8095	0.7755	1.0000	0.3675
5	4	1.0	0.4066	0.9246	0.4262	0.9246	0.8656	0.8787	0.9443	0.8131	1.0000	0.7625
5	4	1.5	0.7003	0.9655	0.6313	0.9655	0.9708	0.9761	0.9894	0.9072	1.0000	0.9425
5	5	0.5	0.2589	0.8883	0.2589	0.9086	0.7411	0.7665	0.8122	0.7716	1.0000	0.4925
5	5	1.0	0.4522	0.9739	0.4551	0.9855	0.8522	0.8754	0.9217	0.8812	1.0000	0.8625
5	5	1.5	0.7570	0.9770	0.6905	0.9744	0.9719	0.9795	0.9949	0.9463	1.0000	0.9775
5	6	0.5	0.2237	0.9361	0.2420	0.9543	0.6347	0.6941	0.7808	0.7854	1.0000	0.5475
5	6	1.0	0.4274	0.9534	0.4356	0.9562	0.8767	0.9178	0.9534	0.8712	1.0000	0.9125
5	6	1.5	0.7924	0.9924	0.7367	0.9949	0.9873	0.9924	1.0000	0.9797	1.0000	0.9875
10	3	0.5	0.2703	0.8581	0.3851	0.8581	0.7162	0.7162	0.7973	0.6757	1.0000	0.3700
10	3	1.0	0.4739	0.8746	0.4634	0.8746	0.8084	0.8084	0.8780	0.7387	1.0000	0.7175
10	3	1.5	0.6511	0.8846	0.6319	0.8846	0.8956	0.8956	0.9341	0.8462	1.0000	0.9100
10	4	0.5	0.2932	0.8377	0.3351	0.9110	0.6335	0.6335	0.7487	0.8063	1.0000	0.4775
10	4	1.0	0.4457	0.9003	0.4809	0.9062	0.8094	0.8035	0.8504	0.8563	1.0000	0.8525
10	4	1.5	0.6872	0.9564	0.7436	0.9667	0.9282	0.9385	0.9615	0.9359	1.0000	0.9750
10	5	0.5	0.2551	0.8520	0.3418	0.8776	0.5969	0.5714	0.7092	0.7959	1.0000	0.4900
10	5	1.0	0.4282	0.9404	0.5366	0.9458	0.7913	0.8266	0.9051	0.8862	1.0000	0.9225
10	5	1.5	0.7309	0.9975	0.8161	1.0000	0.9673	0.9748	0.9874	0.9723	0.9975	0.9925
10	6	0.5	0.2387	0.8964	0.2973	0.9099	0.6036	0.6712	0.7117	0.7973	1.0000	0.5550
10	6	1.0	0.4558	0.9705	0.5791	0.9651	0.8338	0.8660	0.9276	0.9276	1.0000	0.9325
10	6	1.5	0.8396	0.9975	0.9023	0.9975	0.9875	0.9825	0.9950	1.0000	1.0000	0.9975
15	3	0.5	0.2717	0.7826	0.3804	0.7826	0.5652	0.5652	0.7717	0.7391	1.0000	0.4600
15	3	1.0	0.3549	0.8378	0.4865	0.8378	0.7027	0.7027	0.7838	0.7568	1.0000	0.7400
15	3	1.5	0.6244	0.9239	0.7208	0.9239	0.8883	0.8883	0.9391	0.9036	1.0000	0.9850
15	4	0.5	0.2840	0.8272	0.3457	0.8148	0.6914	0.7407	0.8272	0.7531	1.0000	0.4050
15	4	1.0	0.5140	0.9218	0.6089	0.9274	0.7877	0.7877	0.8547	0.8492	1.0000	0.8950
15	4	1.5	0.7387	0.9799	0.8643	0.9849	0.9698	0.9698	0.9749	0.9648	1.0000	0.9950
15	5	0.5	0.2330	0.8835	0.3592	0.8835	0.5146	0.5631	0.6796	0.7379	1.0000	0.5150
15	5	1.0	0.4785	0.9570	0.6398	0.9570	0.8333	0.8710	0.9247	0.9140	1.0000	0.9300
15	5	1.5	0.8191	0.9950	0.8995	0.9950	0.9648	0.9698	0.9899	0.9950	1.0000	0.9950
15	6	0.5	0.2460	0.8810	0.3254	0.9444	0.6111	0.6825	0.7619	0.8254	1.0000	0.6300
15	6	1.0	0.4817	0.9529	0.6649	0.9738	0.8377	0.8639	0.9319	0.9215	1.0000	0.9550
15	6	1.5	0.9250	1.0000	0.9500	1.0000	0.9950	0.9900	1.0000	0.9950	1.0000	1.0000
20	3	0.5	0.3235	0.8824	0.4853	0.8824	0.6471	0.6471	0.7500	0.7794	1.0000	0.3400
20	3	1.0	0.4295	0.8782	0.5705	0.8782	0.8141	0.8141	0.8462	0.8654	1.0000	0.7800
20	3	1.5	0.7062	0.9536	0.7629	0.9536	0.8814	0.8814	0.9330	0.9227	1.0000	0.9700
20	4	0.5	0.2308	0.8462	0.4066	0.9231	0.5275	0.4835	0.6374	0.7802	1.0000	0.4550
20	4	1.0	0.5173	0.9261	0.6307	0.9261	0.7841	0.7784	0.8409	0.8920	1.0000	0.8800
20	4	1.5	0.7513	0.9898	0.8680	0.9949	0.9594	0.9492	0.9949	0.9949	1.0000	0.9850
20	5	0.5	0.1574	0.9537	0.2963	0.9537	0.5741	0.6111	0.7778	0.8056	1.0000	0.5400
20	5	1.0	0.4324	0.9459	0.5676	0.9514	0.7676	0.7892	0.8649	0.9027	1.0000	0.9250
20	5	1.5	0.7940	0.9849	0.8945	0.9849	0.9648	0.9749	0.9899	0.9849	1.0000	0.9950
20	6	0.5	0.2077	0.9385	0.3308	0.9692	0.5308	0.5846	0.7846	0.8385	1.0000	0.6500
20	6	1.0	0.5026	0.9843	0.7330	0.9895	0.8220	0.8586	0.9215	0.9581	1.0000	0.9550
20	6	1.5	0.8550	0.9950	0.9400	0.9950	0.9800	0.9850	1.0000	0.9950	1.0000	1.0000

TABLE A50: EFFICIENCIES OF NINE TWO-WAY TESTS RELATIVE TO "BEST TESTS" -- EXPONENTIAL - GAP

n	k	Δ	F2WAY	PAGET	FRIED	PHENS	ATMX2	LRCN2	HOLLT	SHORT	SENAT	BASE POWER
5	3	0.5	0.3545	0.7009	0.1589	0.7009	0.7664	0.7664	0.8037	0.7196	1.0000	0.2675
5	3	1.0	0.6631	0.7861	0.4278	0.7861	1.0000	1.0000	0.8449	0.9840	0.9733	0.4675
5	3	1.5	0.7534	0.7770	0.4696	0.7770	1.0000	1.0000	0.7669	0.9527	0.9493	0.7400
5	4	0.5	0.4579	0.9252	0.4486	0.9159	0.8598	0.8505	0.7850	0.9907	1.0000	0.2675
5	4	1.0	0.6123	0.8194	0.4493	0.8106	0.9207	0.8414	0.7225	1.0000	0.8767	0.5675
5	4	1.5	0.7824	0.7471	0.5294	0.7471	0.9500	0.9382	0.6441	1.0000	0.8059	0.8500
5	5	0.5	0.3431	0.8333	0.3627	0.8235	0.8627	0.8039	0.7255	0.7941	1.0000	0.2550
5	5	1.0	0.6435	0.7130	0.5000	0.7217	0.9217	0.8087	0.6391	1.0000	0.8217	0.5750
5	5	1.5	0.8862	0.7156	0.5659	0.7305	1.0000	0.9461	0.5988	0.9910	0.7575	0.8350
5	6	0.5	0.4314	0.8529	0.3627	0.8431	0.9510	0.8333	0.7647	1.0000	0.9412	0.2550
5	6	1.0	0.5958	0.6333	0.3750	0.6625	1.0000	0.8250	0.5667	0.9833	0.7208	0.6000
5	6	1.5	0.8552	0.6518	0.4652	0.6825	1.0000	0.9136	0.5404	0.9972	0.7214	0.8975
10	3	0.5	0.4048	0.8095	0.4206	0.8095	0.7698	0.7698	0.7778	0.7698	1.0000	0.3150
10	3	1.0	0.5198	0.7937	0.4683	0.7937	0.7897	0.7897	0.7738	0.7460	1.0000	0.6300
10	3	1.5	0.8140	0.9146	0.6951	0.9146	0.9512	0.9512	0.8659	0.9543	1.0000	0.8200
10	4	0.5	0.4545	0.7727	0.6182	0.8000	0.7182	0.6545	0.6909	1.0000	0.8909	0.2750
10	4	1.0	0.5962	0.8154	0.6500	0.8346	0.8885	0.8115	0.7192	1.0000	0.9038	0.6500
10	4	1.5	0.8235	0.8319	0.7871	0.8515	0.9272	0.8824	0.7843	1.0000	0.9445	0.8925
10	5	0.5	0.3917	0.8000	0.5167	0.8500	0.8000	0.7250	0.6917	0.8917	1.0000	0.3000
10	5	1.0	0.6121	0.7900	0.6548	0.8078	0.8007	0.7295	0.7153	1.0000	0.8363	0.7025
10	5	1.5	0.8571	0.8122	0.8704	0.8069	0.9233	0.8757	0.7725	1.0000	0.8942	0.9450
10	6	0.5	0.3953	0.7907	0.4806	0.7674	0.7442	0.5891	0.6202	1.0000	0.8605	0.3225
10	6	1.0	0.5657	0.6881	0.6177	0.6606	0.7768	0.6422	0.5627	1.0000	0.7309	0.8175
10	6	1.5	0.8651	0.7557	0.8270	0.7532	0.9109	0.8346	0.6463	1.0000	0.7990	0.9825
15	3	0.5	0.4706	0.7843	0.6667	0.7843	0.8039	0.8039	0.7647	0.7843	1.0000	0.2550
15	3	1.0	0.4769	0.7385	0.6385	0.7385	0.7154	0.7154	0.7462	0.7923	1.0000	0.6500
15	3	1.5	0.8023	0.8547	0.8547	0.8547	0.9070	0.9070	0.8663	0.9535	1.0000	0.8600
15	4	0.5	0.3922	0.7843	0.6078	0.8235	0.6863	0.6863	0.6863	0.9412	1.0000	0.2550
15	4	1.0	0.5580	0.8261	0.6957	0.8333	0.7319	0.6957	0.7971	1.0000	0.9638	0.6900
15	4	1.5	0.8272	0.9372	0.8482	0.9424	0.8953	0.8796	0.8482	1.0000	0.9733	0.9550
15	5	0.5	0.5439	0.8246	0.5965	0.8421	0.7719	0.6491	0.6140	1.0000	0.8596	0.2850
15	5	1.0	0.5541	0.7516	0.6561	0.7516	0.7707	0.7197	0.6688	1.0000	0.8280	0.7850
15	5	1.5	0.8711	0.8299	0.9330	0.8299	0.8608	0.8247	0.7577	1.0000	0.8918	0.9700
15	6	0.5	0.3649	0.7973	0.4865	0.7703	0.6892	0.5811	0.6622	1.0000	0.9459	0.3700
15	6	1.0	0.6348	0.6461	0.7640	0.6348	0.7528	0.6404	0.5730	1.0000	0.7247	0.8900
15	6	1.5	0.8750	0.8350	0.9350	0.8350	0.9100	0.8550	0.7900	1.0000	0.9000	1.0000
20	3	0.5	0.3873	0.7119	0.5424	0.7119	0.6949	0.6949	0.6780	0.6780	1.0000	0.2950
20	3	1.0	0.5912	0.8248	0.7299	0.8248	0.7518	0.7518	0.7810	0.9197	1.0000	0.6850
20	3	1.5	0.7598	0.9106	0.8715	0.9106	0.8659	0.8659	0.8771	0.9944	1.0000	0.8950
20	4	0.5	0.4615	0.8154	0.4462	0.8308	0.7538	0.6769	0.6769	0.8000	1.0000	0.3250
20	4	1.0	0.5529	0.8146	0.7483	0.8146	0.6887	0.6821	0.6821	1.0000	0.8874	0.7550
20	4	1.5	0.8073	0.8594	0.9115	0.8594	0.8385	0.8333	0.8281	1.0000	0.9531	0.9600
20	5	0.5	0.4098	0.9180	0.5410	0.8852	0.7705	0.7049	0.7705	0.9508	1.0000	0.3050
20	5	1.0	0.5212	0.7758	0.6727	0.7455	0.6667	0.6182	0.6606	1.0000	0.8364	0.8250
20	5	1.5	0.8384	0.8636	0.9293	0.8586	0.9141	0.8485	0.8283	1.0000	0.9242	0.9900
20	6	0.5	0.3827	0.7407	0.4444	0.7160	0.6420	0.5556	0.5926	1.0000	0.8395	0.4050
20	6	1.0	0.5145	0.7399	0.7457	0.7341	0.7572	0.6647	0.6416	1.0000	0.8092	0.8650
20	6	1.5	0.8900	0.8900	0.9550	0.8750	0.9100	0.8400	0.8000	1.0000	0.9100	1.0000

TABLE A51: EFFICIENCIES OF NINE TWO-WAY TESTS RELATIVE TO "BEST TESTS" -- NORMAL - TREND

n	k	Δ	F2WAY	PAGET	FRIED	PHENS	ATMX2	LRCN2	HOLLT	SHORT	SENAT	BASE POWER
5	3	0.5	0.4342	0.6842	0.2237	0.6842	0.9868	0.9868	1.0000	0.7237	0.9605	0.1900
5	3	1.0	0.5294	0.7206	0.3088	0.7206	1.0000	1.0000	0.9167	0.7010	0.9363	0.5100
5	3	1.5	0.6538	0.8077	0.3718	0.8077	1.0000	1.0000	0.9487	0.7756	0.9968	0.7800
5	4	0.5	0.3486	0.8440	0.2477	0.8440	0.9358	0.9541	1.0000	0.7523	0.9541	0.2725
5	4	1.0	0.3806	0.8462	0.2874	0.8462	0.9474	0.9757	1.0000	0.7247	0.9393	0.6175
5	4	1.5	0.6323	0.9248	0.4513	0.9164	0.9889	1.0000	0.9721	0.8412	0.9972	0.8975
5	5	0.5	0.3955	0.8657	0.3060	0.9030	0.9179	1.0000	0.9776	0.7910	0.9776	0.3350
5	5	1.0	0.4362	0.8423	0.3020	0.8691	0.9228	1.0000	0.9497	0.8087	0.9430	0.7450
5	5	1.5	0.6828	0.9543	0.5000	0.9516	0.9812	1.0000	0.9812	0.9086	0.9839	0.9300
5	6	0.5	0.3193	0.8487	0.2101	0.8908	0.8824	0.9412	1.0000	0.7311	0.9580	0.2975
5	6	1.0	0.4034	0.9017	0.2644	0.9153	0.9627	1.0000	0.9492	0.8102	0.9695	0.7375
5	6	1.5	0.7326	0.9666	0.5527	0.9717	0.9949	1.0000	0.9974	0.9434	0.9949	0.9725
10	3	0.5	0.4337	0.7711	0.2530	0.7711	0.9036	0.9036	0.8675	0.5663	1.0000	0.2075
10	3	1.0	0.4641	0.7464	0.3110	0.7464	1.0000	1.0000	0.8947	0.6507	0.9330	0.5225
10	3	1.5	0.6862	0.8554	0.4492	0.8554	1.0000	1.0000	0.9723	0.7785	0.9846	0.8125
10	4	0.5	0.4479	0.8021	0.3854	0.8750	0.9688	1.0000	1.0000	0.8542	0.9896	0.2400
10	4	1.0	0.4981	0.7938	0.3463	0.8249	0.9922	1.0000	0.9494	0.7626	0.9572	0.6425
10	4	1.5	0.7318	0.9022	0.5922	0.9246	0.9944	1.0000	0.9693	0.8715	0.9693	0.8950
10	5	0.5	0.5214	0.8034	0.3675	0.8547	0.9829	1.0000	0.9744	0.7094	1.0000	0.2925
10	5	1.0	0.4371	0.8601	0.3217	0.8776	0.9685	1.0000	0.9406	0.7797	0.9755	0.7150
10	5	1.5	0.7285	0.9373	0.5901	0.9504	0.9869	1.0000	0.9791	0.8930	0.9817	0.9575
10	6	0.5	0.3817	0.8244	0.2366	0.8092	0.9160	1.0000	0.9237	0.7710	0.9542	0.3275
10	6	1.0	0.4781	0.8813	0.3969	0.8813	0.9438	1.0000	0.9281	0.8219	0.9438	0.8000
10	6	1.5	0.7959	0.9819	0.6822	0.9845	0.9922	1.0000	0.9974	0.9612	0.9974	0.9675
15	3	0.5	0.5800	0.7000	0.3600	0.7000	1.0000	1.0000	0.9200	0.6000	0.9800	0.2500
15	3	1.0	0.5089	0.6429	0.4018	0.6429	1.0000	1.0000	0.9018	0.6964	0.9375	0.5600
15	3	1.5	0.6386	0.7590	0.5000	0.7590	1.0000	1.0000	0.9096	0.7229	0.8855	0.8300
15	4	0.5	0.4528	0.8302	0.2453	0.8491	0.9245	1.0000	0.9811	0.7358	0.9245	0.2650
15	4	1.0	0.5806	0.8226	0.4032	0.8710	0.9435	1.0000	0.9597	0.8145	1.0000	0.6200
15	4	1.5	0.7416	0.8876	0.5449	0.8989	0.9831	1.0000	0.9663	0.8258	0.9663	0.8900
15	5	0.5	0.3571	0.8571	0.4821	0.8571	0.9464	0.9643	1.0000	0.9107	0.9464	0.2800
15	5	1.0	0.4615	0.8322	0.3357	0.8392	0.9720	1.0000	0.9371	0.7552	0.9580	0.7150
15	5	1.5	0.7749	1.0000	0.6821	0.9949	0.9795	0.9949	1.0000	0.9436	1.0000	0.9750
15	6	0.5	0.3182	0.8788	0.2727	0.8939	0.8788	0.9848	0.9848	0.7273	1.0000	0.3300
15	6	1.0	0.4733	0.8820	0.3789	0.9068	0.9379	1.0000	0.9379	0.7888	0.9565	0.8050
15	6	1.5	0.8000	0.9692	0.7077	0.9744	1.0000	0.9949	0.9949	0.9385	0.9897	0.9750
20	3	0.5	0.5588	0.6765	0.5294	0.6765	1.0000	1.0000	0.9118	0.7941	0.8824	0.1700
20	3	1.0	0.5849	0.7642	0.4528	0.7642	1.0000	1.0000	0.9623	0.7547	0.9811	0.5300
20	3	1.5	0.7595	0.8734	0.5949	0.8734	1.0000	1.0000	0.9620	0.8354	1.0000	0.7900
20	4	0.5	0.2931	0.7759	0.2586	0.7759	0.9483	1.0000	0.8621	0.7069	0.8621	0.2900
20	4	1.0	0.5345	0.8621	0.4397	0.8621	0.9914	1.0000	0.9828	0.8103	0.9397	0.5800
20	4	1.5	0.7432	0.9126	0.5301	0.9344	1.0000	0.9836	0.9945	0.8634	0.9727	0.9150
20	5	0.5	0.5094	0.8302	0.4528	0.7925	0.9057	1.0000	0.8868	0.6604	0.8868	0.2650
20	5	1.0	0.5214	0.8714	0.4071	0.8714	0.9714	1.0000	0.9500	0.7857	0.9500	0.7000
20	5	1.5	0.8148	0.9630	0.6720	0.9577	1.0000	0.9947	0.9735	0.8889	0.9735	0.9450
20	5	0.5	0.3377	0.8182	0.2078	0.8052	0.9091	1.0000	0.8571	0.7792	0.8701	0.3850
20	6	1.0	0.5294	0.8889	0.3725	0.8889	1.0000	1.0000	0.9346	0.8039	0.9608	0.7650
20	6	1.5	0.8392	0.9648	0.6985	0.9598	0.9749	1.0000	0.9950	0.9447	0.9849	0.9950

TABLE A52: EFFICIENCIES OF NINE TWO-WAY TESTS RELATIVE TO "BEST TESTS" -- NORMAL-GAP

n	k	Δ	F2WAY	PAGET	FRIED	PHENS	ATMX2	LRCN2	HOLLT	SHORT	SENAT	BASE POWER
5	3	0.5	0.5238	0.7143	0.3016	0.7143	0.8730	0.8730	0.9524	0.8889	1.0000	0.1575
5	3	1.0	0.5705	0.7051	0.3397	0.7051	1.0000	1.0000	0.8910	0.8077	0.9423	0.3900
5	3	1.5	0.7652	0.7083	0.4394	0.7083	1.0000	1.0000	0.7652	0.8144	0.8788	0.6600
5	4	0.5	0.6056	0.8169	0.4225	0.8169	1.0000	0.9296	0.9296	0.9155	0.9296	0.1775
5	4	1.0	0.5580	0.8066	0.3481	0.8011	1.0000	0.9613	0.7901	0.8895	0.8453	0.4525
5	4	1.5	0.7373	0.7517	0.4930	0.7483	1.0000	0.9790	0.6993	0.9720	0.7762	0.7150
5	5	0.5	0.3656	0.8172	0.2473	0.8280	1.0000	0.9785	0.8602	0.8387	0.9355	0.2325
5	5	1.0	0.5446	0.7327	0.3218	0.7574	1.0000	0.8911	0.6782	0.8663	0.7822	0.5050
5	5	1.5	0.8135	0.6569	0.5163	0.6765	1.0000	0.8627	0.5523	0.9085	0.6765	0.7650
5	6	0.5	0.5588	0.8235	0.2794	0.8235	1.0000	0.7941	0.7500	0.8529	0.7500	0.1700
5	6	1.0	0.6466	0.6681	0.3793	0.6853	1.0000	0.8319	0.5776	0.9095	0.7069	0.5800
5	6	1.5	0.8057	0.6657	0.5000	0.6971	1.0000	0.8943	0.5371	0.9343	0.7000	0.8750
10	3	0.5	0.5811	0.8108	0.2703	0.8108	1.0000	1.0000	0.8649	0.6486	0.9730	0.1850
10	3	1.0	0.6506	0.6988	0.4217	0.6988	1.0000	1.0000	0.8795	0.6988	0.9157	0.4150
10	3	1.5	0.8532	0.7850	0.5290	0.7850	1.0000	1.0000	0.8908	0.8020	0.9386	0.7325
10	4	0.5	0.4648	0.8592	0.3521	0.8873	1.0000	0.9296	0.8310	0.9577	0.9014	0.1775
10	4	1.0	0.6584	0.7632	0.4263	0.8053	1.0000	0.9737	0.8737	0.8947	0.8789	0.4750
10	4	1.5	0.8248	0.7484	0.6242	0.7675	1.0000	0.9777	0.7994	0.9013	0.8694	0.7850
10	5	0.5	0.3647	0.7765	0.2471	0.8235	1.0000	0.8824	0.8941	0.7412	0.8706	0.2125
10	5	1.0	0.7047	0.7254	0.4767	0.7565	1.0000	0.9223	0.7668	0.8912	0.8238	0.4825
10	5	1.5	0.9689	0.7195	0.6799	0.7713	1.0000	0.9360	0.7378	0.9634	0.8445	0.8200
10	6	0.5	0.4507	0.7191	0.2921	0.7978	1.0000	0.9663	0.8202	0.8764	0.8652	0.2225
10	6	1.0	0.6696	0.6696	0.5045	0.6786	1.0000	0.8259	0.6518	0.9688	0.7455	0.5600
10	6	1.5	0.8653	0.7307	0.7049	0.7450	1.0000	0.9169	0.7049	1.0000	0.8165	0.8725
15	3	0.5	0.3964	0.7727	0.3409	0.7727	1.0000	1.0000	0.9773	0.7955	0.9773	0.2200
15	3	1.0	0.6400	0.6933	0.4533	0.6933	1.0000	1.0000	1.0000	0.7200	0.9867	0.3750
15	3	1.5	0.8310	0.7746	0.6056	0.7746	1.0000	1.0000	0.9155	0.7887	0.9225	0.7100
15	4	0.5	0.4551	0.7442	0.3488	0.7209	1.0000	0.9535	0.8140	0.7674	0.7674	0.2150
15	4	1.0	0.7778	0.7667	0.6333	0.8222	0.9889	0.9889	0.9111	1.0000	0.8778	0.4500
15	4	1.5	0.8707	0.8231	0.6803	0.8163	1.0000	0.9796	0.8912	0.9592	0.9048	0.7350
15	5	0.5	0.4310	0.7586	0.3793	0.7414	1.0000	0.8966	0.8276	0.7069	0.8103	0.2900
15	5	1.0	0.6909	0.8091	0.5636	0.7727	1.0000	0.8727	0.7818	1.0000	0.8545	0.5500
15	5	1.5	0.9375	0.8437	0.7687	0.8500	1.0000	0.9688	0.8312	0.9812	0.8938	0.8000
15	6	0.5	0.4091	0.7727	0.4318	0.8409	1.0000	0.8864	0.8182	0.9091	0.8864	0.2200
15	6	1.0	0.7282	0.7184	0.4563	0.7184	1.0000	0.8058	0.7087	0.9612	0.6990	0.5150
15	6	1.5	0.8820	0.8090	0.7809	0.8090	1.0000	0.9213	0.8258	0.9775	0.8652	0.8900
20	3	0.5	0.3536	0.7879	0.2424	0.7879	1.0000	1.0000	0.9697	0.6061	0.9394	0.1650
20	3	1.0	0.7179	0.7692	0.4487	0.7692	1.0000	1.0000	0.9359	0.7051	0.9487	0.3900
20	3	1.5	0.8435	0.8095	0.6395	0.8095	1.0000	1.0000	0.9660	0.8503	0.9252	0.7350
20	4	0.5	0.6562	0.7813	0.5312	0.7813	1.0000	0.8438	0.9063	0.9375	0.8438	0.1600
20	4	1.0	0.6705	0.8039	0.5588	0.8431	0.9706	1.0000	0.9118	0.9314	0.9412	0.5100
20	4	1.5	0.8366	0.7778	0.6797	0.7778	1.0000	0.9542	0.8497	0.9216	0.8954	0.7650
20	5	0.5	0.6591	0.8636	0.5227	0.8409	1.0000	0.9318	0.8409	0.8864	0.9545	0.2200
20	5	1.0	0.7290	0.6822	0.5047	0.6636	1.0000	0.8224	0.7290	0.8224	0.7850	0.5350
20	5	1.5	0.9750	0.8688	0.8187	0.8375	1.0000	0.9625	0.8875	0.9937	0.9125	0.8000
20	6	0.5	0.4500	0.7667	0.3333	0.8000	1.0000	0.8000	0.7667	0.7833	0.8000	0.3000
20	6	1.0	0.7615	0.6789	0.5963	0.6697	1.0000	0.8165	0.6881	0.9358	0.7248	0.5450
20	6	1.5	0.9240	0.8070	0.7427	0.8012	1.0000	0.9298	0.8480	0.9942	0.8596	0.8550

TABLE A53
VALUES OF $P(l, k)$

l	k				
	3	4	5	6	7
1	.33333	.25000	.20000	.16667	.14286
2	.50000	.45833	.41667	.38056	.35000
3	.16667	.25000	.29167	.31250	.32222
4		.04167	.08333	.11806	.14583
5			.00833	.02083	.03472
6				.00139	.00417
7					.00020

TABLE A54
CRITICAL VALUES OF JONCKHEERE'S
ONE-WAY TEST

α	n	h				
		3	4	5	6	7
.05	5	33	50	70	92	116
	6	42	65	92	120	152
	7	53	82	115	151	190
	8	64	99	140	184	232
	9	76	118	166	219	276
	10	88	138	194	255	322
	11	101	159	223	294	371
	12	115	180	254	335	423
	13	129	203	286	377	476
	14	144	226	319	421	532
	15	159	251	353	466	589
	16	175	276	389	513	649
	17	192	302	426	562	710
.01	18	209	329	463	612	773
	19	226	356	502	663	838
	20	244	384	542	716	905
	5	46	71	99	130	163
	6	59	92	129	170	214
	7	74	115	162	213	268
	8	89	140	197	259	327
	9	106	166	234	308	389
	10	124	194	273	361	455
	11	142	224	315	415	524
	12	162	254	358	473	597
	13	182	286	403	532	672
	14	203	319	450	595	751
	15	225	354	499	659	832
	16	247	389	549	725	916
	17	271	426	601	794	1003
	18	295	464	655	865	1092
	19	319	503	710	937	1184
	20	344	543	766	1012	1279

TABLE A55: CRITICAL VALUES OF PURI'S TEST

n	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$
$\hat{G}_U^*$					
5	30.5160	47.8966	67.5046	90.6709	116.3833
6	40.6024	64.1317	92.7168	121.2136	154.0471
7	50.5946	80.0867	117.3646	151.6702	188.7706
8	61.2364	97.9820	140.8483	187.1712	231.0555
9	76.2663	116.6566	169.2081	225.9467	267.7512
10	88.7450	141.3928	193.8724	252.7549	323.1250
15	159.3658	251.0447	366.4153	477.4890	610.9607
20	247.3721	387.9150	555.2512	721.3635	909.4458
$U^*(.05)$					
5	50.1987	78.7898	111.0450	149.1536	191.4505
6	66.7909	105.4967	152.5191	199.3963	253.4074
7	83.2280	131.7426	193.0647	249.4973	310.5271
8	100.7339	161.1804	231.6954	307.8965	380.0859
9	125.4581	191.9001	278.3467	371.6819	440.4507
10	145.9855	232.5911	318.9199	415.7813	531.5400
15	262.1563	412.9680	602.7529	785.4692	1005.0300
20	406.9270	638.1201	913.3882	1186.6428	1496.0378
$U^*(.01)$					
5	71.1022	111.5990	157.2857	211.2632	271.1729
6	94.6035	149.4270	216.0302	282.4272	358.9297
7	117.8853	186.6020	273.4595	353.3911	439.8352
8	142.6808	228.2981	328.1765	436.1086	538.3591
9	177.7005	271.8098	394.2546	526.4553	623.8606
10	206.7759	329.4451	451.7224	588.9187	752.8809
15	371.3220	584.9338	853.7480	1112.5498	1423.5383
20	576.3767	903.8425	1293.7356	1680.7771	2119.0083

TABLE A56: CRITICAL VALUES OF JOHNSON-MEHROTRA TEST

n	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$
$\hat{\sigma}_{JM}$					
5	0.4983	0.5314	0.5578	0.5783	0.6045
6	0.4604	0.4932	0.5295	0.5361	0.5576
7	0.4215	0.4506	0.4922	0.4936	0.5010
8	0.3906	0.4247	0.4473	0.4661	0.4718
9	0.3844	0.4003	0.4318	0.4446	0.4298
10	0.3623	0.3932	0.4003	0.4087	0.4184
15	0.2892	0.3087	0.3373	0.3406	0.3487
20	0.2525	0.2715	0.2838	0.2911	0.2969
$JM(.05)$					
5	0.8197	0.8741	0.9175	0.9513	0.9945
6	0.7574	0.8112	0.8711	0.8818	0.9173
7	0.6934	0.7413	0.8096	0.8120	0.8241
8	0.6426	0.6986	0.7359	0.7667	0.7762
9	0.6323	0.6585	0.7103	0.7313	0.7070
10	0.5960	0.6468	0.6584	0.6722	0.6882
15	0.4757	0.5078	0.5548	0.5602	0.5737
20	0.4153	0.4467	0.4668	0.4789	0.4884
$JM(.01)$					
5	1.1611	1.2381	1.2996	1.3474	1.4086
6	1.0728	1.1491	1.2338	1.2490	1.2993
7	0.9822	1.0499	1.1467	1.1501	1.1673
8	0.9101	0.9895	1.0423	1.0859	1.0994
9	0.8956	0.9326	1.0061	1.0359	1.0014
10	0.8442	0.9162	0.9326	0.9522	0.9748
15	0.6737	0.7192	0.7859	0.7935	0.8126
20	0.5883	0.6326	0.6612	0.6784	0.6918

TABLE A57: CRITICAL VALUES OF ONE-SIDED k SAMPLE KOLMOGOROV-SMIRNOV TEST
($\alpha = .05$)

n	k									
	3	4	5	6	7	8	9	10	11	12
5	0.07909 4	0.00794 5	0.11692 4	0.01189 5	0.15320 4	0.01583 5	0.18799 4	0.01974 5	0.22134 4	0.02365 5
6	0.14179 4	0.02596 5	0.20705 4	0.03876 5	0.05139 5	0.00433 6	0.06386 5	0.00540 6	0.07616 5	0.00648 6
7	0.05294 4	0.00816 5	0.07866 5	0.01222 6	0.10369 5	0.01626 6	0.12803 5	0.02029 6	0.15171 5	0.02430 6
8	0.08674 5	0.01864 6	0.12808 5	0.02787 6	0.16754 5	0.03702 6	0.20522 5	0.04607 6	0.05505 6	0.00744 7
9	0.12519 5	0.03354 6	0.05002 6	0.00943 7	0.06621 6	0.01256 7	0.08213 6	0.01567 7	0.09778 6	0.01878 7
10	0.05237 6	0.01234 7	0.07784 6	0.01847 7	0.10262 6	0.02456 7	0.12673 6	0.03062 7	0.15019 6	0.03663 7
11	0.07449 6	0.02073 7	0.11026 6	0.03099 7	0.14464 6	0.04113 7	0.05118 7	0.01089 8	0.06111 7	0.01305 8
12	0.09920 6	0.03142 7	0.14616 6	0.04687 7	0.06208 7	0.01567 8	0.07704 7	0.01955 8	0.09176 7	0.02343 8
13	0.12587 6	0.04423 7	0.06583 7	0.01893 8	0.08695 7	0.02517 8	0.10759 7	0.03138 8	0.12776 7	0.03754 8
14	0.05894 7	0.01878 8	0.08750 7	0.02808 8	0.11519 7	0.03729 8	0.14205 7	0.04641 8	0.05545 8	0.01464 9
15	0.07531 7	0.02624 8	0.11146 7	0.03918 8	0.05195 8	0.01527 9	0.06455 8	0.01905 9	0.07698 8	0.02282 9
16	0.09306 7	0.03498 8	0.05215 8	0.01677 9	0.06901 8	0.02230 9	0.08558 8	0.02781 9	0.10185 8	0.03328 9
17	0.11195 7	0.04491 8	0.06685 8	0.02328 9	0.08827 8	0.03093 9	0.10921 8	0.03853 9	0.12967 8	0.04607 9
18	0.05595 8	0.02074 9	0.08311 8	0.03100 9	0.10948 8	0.04116 9	0.05120 9	0.01660 10	0.06115 9	0.01989 10
19	0.06798 8	0.02674 9	0.10075 8	0.03993 9	0.05293 9	0.01838 10	0.06576 9	0.02293 10	0.07842 9	0.02746 10
20	0.08089 8	0.03352 9	0.05000 8	0.05000 9	0.06619 9	0.02448 10	0.08210 9	0.03052 10	0.09775 9	0.03651 10

TABLE A57: (continued)
($\alpha=.01$)

n	h_2									
	3	4	5	6	7	8	9	10	11	12
5	0.07909 4	0.00793 5	0.01189 5	_____	0.01582 5	_____	0.01974 5	_____	0.02365 5	_____
6	0.02596 5	0.00216 6	0.03876 5	0.00324 6	0.05139 5	0.00432 6	0.06385 5	0.00540 6	0.07616 5	0.00648 6
7	0.05294 5	0.00816 6	0.01222 6	0.00087 7	0.01626 6	0.00116 7	0.02029 6	0.00146 7	0.02430 6	0.00175 7
8	0.01864 6	0.00249 7	0.02787 6	0.00373 7	0.03701 6	0.00497 7	0.04607 6	0.00621 7	0.05504 6	0.00744 7
9	0.03354 6	0.00629 7	0.05002 6	0.00943 7	0.01256 7	0.00148 8	0.01567 7	0.00185 8	0.01878 7	0.00222 8
10	0.01234 7	0.00206 8	0.01847 7	0.00308 8	0.02456 7	0.00411 8	0.03061 7	0.00513 8	0.03663 7	0.00616 8
11	0.02073 7	0.00437 8	0.03099 7	0.00654 8	0.04113 7	0.00872 8	0.01089 8	0.00164 9	0.01305 8	0.00196 9
12	0.03142 7	0.00786 8	0.01177 8	0.00224 9	0.01567 8	0.00299 9	0.01955 8	0.00374 9	0.02342 8	0.00448 9
13	0.01265 7	0.00287 8	0.01893 8	0.00431 9	0.02517 8	0.00574 9	0.03138 8	0.00717 9	0.03754 8	0.00860 9
14	0.01878 8	0.00490 9	0.02808 8	0.00734 9	0.03729 8	0.00978 9	0.01221 9	0.00255 10	0.01464 9	0.00306 10
15	0.02624 8	0.00765 9	0.01147 9	0.00275 10	0.01527 9	0.00367 10	0.01905 9	0.00459 10	0.02282 9	0.00550 10
16	0.01120 9	0.00301 10	0.01676 9	0.00452 10	0.02230 9	0.00602 10	0.02781 9	0.00752 10	0.03328 9	0.00902 10
17	0.01556 9	0.00461 10	0.02327 9	0.00691 10	0.03093 9	0.00920 10	0.01149 10	0.00288 11	0.01378 10	0.00345 11
18	0.02074 9	0.00667 10	0.03100 9	0.00999 10	0.01330 10	0.00368 11	0.01660 10	0.00459 11	0.01989 10	0.00551 11
19	0.02674 9	0.00922 10	0.01381 10	0.00415 11	0.01838 10	0.00553 11	0.02293 10	0.00691 11	0.02746 10	0.00628 11
20	0.01230 10	0.00397 11	0.01841 10	0.00595 11	0.02448 10	0.00792 11	0.03051 10	0.00989 11	0.01186 10	0.00334 11

TABLE A58: CRITICAL VALUES OF MOSTELLER'S k SAMPLE SLIPPAGE TEST
($\alpha=.05$)

n	k									
	3	4	5	6	7	8	9	10	11	12
5	0.06593 3	0.01099 4	0.21053 2	0.03509 3	0.16667 2	0.02174 3	0.13793 2	0.01478 3	0.11765 2	0.01069 3
6	0.07353 3	0.01471 4	0.21739 2	0.03953 3	0.17241 2	0.02463 3	0.14286 2	0.01681 3	0.12195 2	0.01219 3
7	0.07895 3	0.01754 4	0.22222 2	0.04274 3	0.17647 2	0.02674 3	0.14634 2	0.01829 3	0.12500 2	0.01330 3
8	0.08301 3	0.01976 4	0.22581 2	0.04516 3	0.17949 2	0.02834 3	0.14893 2	0.01943 3	0.12727 2	0.01414 3
9	0.08615 3	0.02154 4	0.22857 2	0.04706 3	0.18182 2	0.02960 3	0.15094 2	0.02032 3	0.12903 2	0.01481 3
10	0.08867 3	0.02299 4	0.23077 2	0.04858 3	0.18367 2	0.03061 3	0.15254 2	0.02104 3	0.13043 2	0.01534 3
11	0.09073 3	0.02419 4	0.23256 2	0.04983 3	0.18519 2	0.03145 3	0.15385 2	0.02163 3	0.13161 2	0.01579 3
12	0.09244 3	0.02521 4	0.05088 3	0.01018 4	0.18644 2	0.03214 3	0.15493 2	0.02213 3	0.13253 2	0.01617 3
13	0.09388 3	0.02608 4	0.05176 3	0.01056 4	0.18750 2	0.03274 3	0.15583 2	0.02256 3	0.13332 2	0.01648 3
14	0.09512 3	0.02683 4	0.05252 3	0.01090 4	0.18841 2	0.03325 3	0.15666 2	0.02293 3	0.13400 2	0.01675 3
15	0.09619 3	0.02748 4	0.05318 3	0.01120 4	0.18915 2	0.03369 3	0.15731 2	0.02324 3	0.13462 2	0.01699 3
16	0.09713 3	0.02806 4	0.05376 3	0.01146 4	0.18987 2	0.03408 3	0.15791 2	0.02352 3	0.13513 2	0.01720 3
17	0.09796 3	0.02857 4	0.05427 3	0.01169 4	0.19047 2	0.03443 3	0.15844 2	0.02377 3	0.13559 2	0.01739 3
18	0.09869 3	0.02903 4	0.05472 3	0.01190 4	0.19100 2	0.03473 3	0.15889 2	0.02399 3	0.13601 2	0.01755 3
19	0.09935 3	0.02944 4	0.05514 3	0.01208 4	0.19148 2	0.03501 3	0.15929 2	0.02418 3	0.13638 2	0.01770 3
20	0.09994 3	0.02981 4	0.05551 3	0.01225 4	0.19195 2	0.03526 3	0.15968 2	0.02436 3	0.13671 2	0.01783 3

TABLE A58: (continued)
($\alpha=.01$)

n	k									
	3	4	5	6	7	8	9	10	11	12
5	0.01099 4	0.00100 5	0.03509 3	0.00413 4	0.02174 3	0.00198 4	0.01478 3	0.00109 4	0.01069 3	0.00067 4
6	0.01471 4	0.00210 5	0.03953 3	0.00565 4	0.02463 3	0.00274 4	0.01681 3	0.00153 4	0.01219 3	0.00094 4
7	0.01754 4	0.00310 5	0.04274 3	0.00684 4	0.02674 3	0.00334 4	0.01829 3	0.00188 4	0.01330 3	0.00116 4
8	0.01976 4	0.00395 5	0.04516 3	0.00779 4	0.02834 3	0.00383 4	0.01943 3	0.00216 4	0.01414 3	0.00133 4
9	0.02154 4	0.00468 5	0.04706 3	0.00856 4	0.02960 3	0.00423 4	0.02032 3	0.00239 4	0.01481 3	0.00148 4
10	0.02299 4	0.00530 5	0.04858 3	0.00919 4	0.03061 3	0.00456 4	0.02104 3	0.00258 4	0.01534 3	0.00160 4
11	0.02419 4	0.00584 5	0.04983 3	0.00972 4	0.03145 3	0.00484 4	0.02163 3	0.00275 4	0.01579 3	0.00171 4
12	0.02521 4	0.00630 5	0.01018 4	0.00185 5	0.03214 3	0.00508 4	0.02213 3	0.00289 4	0.01617 3	0.00180 4
13	0.02608 4	0.00671 5	0.01056 4	0.00198 5	0.03274 3	0.00528 4	0.02256 3	0.00301 4	0.01648 3	0.00187 4
14	0.02683 4	0.00706 5	0.01090 4	0.00210 5	0.03325 3	0.00546 4	0.02293 3	0.00311 4	0.01675 3	0.00194 4
15	0.02748 4	0.00737 5	0.01120 4	0.00220 5	0.03369 3	0.00561 4	0.02324 3	0.00321 4	0.01699 3	0.00200 4
16	0.02806 4	0.00765 5	0.01146 4	0.00229 5	0.03408 3	0.00575 4	0.02352 3	0.00329 4	0.01720 3	0.00205 4
17	0.02857 4	0.00790 5	0.01169 4	0.00237 5	0.03443 3	0.00588 4	0.02377 3	0.00336 4	0.01739 3	0.00210 4
18	0.02903 4	0.00813 5	0.01190 4	0.00245 5	0.03473 3	0.00599 4	0.02399 3	0.00343 4	0.01755 3	0.00214 4
19	0.02944 4	0.00833 5	0.01208 4	0.00252 5	0.03501 3	0.00609 4	0.02418 3	0.00349 4	0.01770 3	0.00218 4
20	0.02981 4	0.00852 5	0.01225 4	0.00258 5	0.03526 3	0.00618 4	0.02436 3	0.00354 4	0.01783 3	0.00221 4

TABLE A59: CRITICAL VALUES OF CONOVER'S FIRST k SAMPLE SLIPPAGE TEST
($\alpha=.05$)

n	k_2									
	3	4	5	6	7	8	9	10	11	12
5	0.08425 4	0.01487 5	0.11402 4	0.02107 5	0.13908 4	0.02669 5	0.16070 4	0.03184 5	0.17969 4	0.03661 5
6	0.10651 4	0.02820 5	0.14335 4	0.03972 5	0.05005 5	0.00780 6	0.05944 5	0.00946 6	0.06805 5	0.01105 6
7	0.12232 4	0.03886 5	0.05450 5	0.01290 6	0.06844 5	0.01660 6	0.08101 5	0.02008 6	0.09249 5	0.02337 6
8	0.13408 4	0.04733 5	0.06618 5	0.01923 6	0.08287 5	0.02468 6	0.09787 5	0.02979 6	0.11150 5	0.03460 6
9	0.05414 5	0.01725 6	0.07552 5	0.02477 6	0.09438 5	0.03173 6	0.11127 5	0.03822 6	0.12655 5	0.04431 6
10	0.05971 5	0.02061 6	0.08314 5	0.02954 6	0.10373 5	0.03779 6	0.12211 5	0.04545 6	0.05263 6	0.01636 7
11	0.06434 5	0.02352 6	0.08944 5	0.03366 6	0.11145 5	0.04299 6	0.05166 6	0.01734 7	0.05974 6	0.02032 7
12	0.06823 5	0.02604 6	0.09474 5	0.03722 6	0.11793 5	0.04749 6	0.05700 6	0.02042 7	0.06587 6	0.02391 7
13	0.07155 5	0.02824 6	0.09925 5	0.04032 6	0.05140 6	0.01905 7	0.06165 6	0.02319 7	0.07118 6	0.02715 7
14	0.07442 5	0.03017 6	0.10314 5	0.04304 6	0.05483 6	0.02112 7	0.06571 6	0.02569 7	0.07582 6	0.03005 7
15	0.07692 5	0.03187 6	0.10651 5	0.04544 6	0.05785 6	0.02298 7	0.06928 6	0.02795 7	0.07990 6	0.03267 7
16	0.07911 5	0.03339 6	0.10948 5	0.04758 6	0.06053 6	0.02467 7	0.07245 6	0.02998 7	0.08351 6	0.03503 7
17	0.08105 5	0.03475 6	0.11210 5	0.04948 6	0.06291 6	0.02620 7	0.07528 6	0.03182 7	0.08673 6	0.03717 7
18	0.08278 5	0.03597 6	0.05119 6	0.02134 7	0.06506 6	0.02759 7	0.07781 6	0.03350 7	0.08962 6	0.03911 7
19	0.08433 5	0.03707 6	0.05274 6	0.02233 7	0.06700 6	0.02886 7	0.08009 6	0.03502 7	0.09222 6	0.04088 7
20	0.08573 5	0.03807 6	0.05414 6	0.02324 7	0.06875 6	0.03002 7	0.08216 6	0.03642 7	0.09457 6	0.04250 7

TABLE A59: (continued)
($\alpha=0.01$)

n	h									
	3		4		5		6		7	
5	0.01487 5		0.02107 5		0.02669 5		0.03184 5		0.03661 5	
6	0.02820 5	0.00417 6	0.03972 5	0.00604 6	0.05005 5	0.00780 6	0.05944 5	0.00946 6	0.01105 6	
7	0.03886 5	0.00894 6	0.01290 6	0.00167 7	0.01660 6	0.00219 7	0.02008 6	0.00269 7	0.02337 6	0.00317 7
8	0.01336 6	0.00273 7	0.01923 6	0.00400 7	0.02468 6	0.00521 7	0.02979 6	0.00638 7	0.03460 6	0.00751 7
9	0.01725 6	0.00440 7	0.02477 6	0.00644 7	0.03173 6	0.00838 7	0.01025 7	0.00195 8	0.01204 7	0.00231 8
10	0.02061 6	0.00601 7	0.02954 6	0.00878 7	0.01142 7	0.00273 8	0.01394 7	0.00336 8	0.01636 7	0.00398 8
11	0.02352 6	0.00751 7	0.01095 7	0.00299 8	0.01422 7	0.00393 8	0.01734 7	0.00484 8	0.02032 7	0.00572 8
12	0.02604 6	0.00887 7	0.01292 7	0.00390 8	0.01676 7	0.00511 8	0.02042 7	0.00629 8	0.02391 7	0.00743 8
13	0.01010 7	0.00323 8	0.01470 7	0.00476 8	0.01905 7	0.00624 8	0.02319 7	0.00767 8	0.02715 7	0.00906 8
14	0.01121 7	0.00379 8	0.01631 7	0.00558 8	0.02112 7	0.00730 8	0.02569 7	0.00898 8	0.01060 8	0.00329 9
15	0.01222 7	0.00431 8	0.01776 7	0.00634 8	0.02298 7	0.00830 8	0.01019 8	0.00334 9	0.01202 8	0.00397 9
16	0.01313 7	0.00479 8	0.01907 7	0.00705 8	0.02467 7	0.00922 8	0.01132 8	0.00389 9	0.01335 8	0.00462 9
17	0.01396 7	0.00524 8	0.02026 7	0.00770 8	0.01007 8	0.00357 9	0.01236 8	0.00441 9	0.01458 8	0.00524 9
18	0.01471 7	0.00566 8	0.02134 7	0.00831 8	0.01087 8	0.00398 9	0.01333 8	0.00491 9	0.01571 8	0.00583 9
19	0.01540 7	0.00604 8	0.02233 7	0.00888 8	0.01160 8	0.00436 9	0.01423 8	0.00538 9	0.01677 8	0.00638 9
20	0.01603 7	0.00640 8	0.02324 7	0.00940 8	0.01228 8	0.00472 9	0.01506 8	0.00583 9	0.01774 8	0.00691 9

TABLE A60
CRITICAL VALUES OF FRIEDMAN'S TEST

α	n	$k=3$		$k=4$	
.05	5	.0930 5.200	.0390 6.400	.0570 7.320	.0490 7.800
	10	.0660 5.600	.0460 6.200	a	a
	15	.0590 5.733	.0470 6.400	a	a
	5	.0240 7.600	.0085 8.400	.0110 9.720	.0092 9.960
	10	.0120 8.600	.0073 9.600	a	a
	15	.0100 8.933	_____	a	a

a χ^2 used.

TABLE A61

CRITICAL VALUES OF JONCKHEERE'S TWO-WAY TEST

α	n	$k=3$		$k=4$	
.05	5	.05500 1.867	.01833 2.334	.07583 1.670	.04167 1.974
	10	.05167 1.816	.02542 2.146	.06542 1.610	.04542 1.825
	15	.07375 1.617	.04208 1.887	.06833 1.490	.04708 1.665
	20	.05458 1.751	.02792 1.984	.06333 1.594	.04667 1.746
.01	5	.01833 2.334	.00375 2.803	.01833 2.278	.00917 2.581
	10	.01167 2.476	.00417 2.806	.01167 2.470	.00583 2.684
	15	.01333 2.426	.00708 2.696	.01500 2.192	.00875 2.367
	20	.01333 2.218	.00583 2.451	.01042 2.354	.00667 2.506

TABLE A62
CRITICAL VALUES OF PAGE'S L TEST

α	n	k				
		3	4	5	6	7
.05	5	66	136	244	397	603
	6	78	162	291	473	719
	7	91	188	337	549	834
	8	103	214	384	625	950
	9	115	240	430	701	1065
	10	128	266	477	776	1180
	11	140	291	523	852	1295
	12	153	317	569	927	1410
	13	165	343	615	1002	1524
	14	177	368	661	1078	1639
	15	190	394	707	1153	1753
	16	202	419	753	1228	1868
	17	214	445	799	1303	1982
	18	226	471	845	1378	2096
	19	239	496	891	1453	2210
	20	251	522	937	1528	2325
.01	5	68	141	252	409	620
	6	81	167	299	486	738
	7	93	193	346	563	855
	8	106	219	393	640	972
	9	118	246	440	717	1088
	10	131	272	487	793	1205
	11	143	298	534	869	1321
	12	156	324	581	946	1437
	13	168	350	627	1022	1552
	14	181	376	674	1098	1668
	15	193	402	721	1174	1783
	16	206	427	767	1249	1899
	17	218	453	813	1325	2014
	18	230	479	860	1401	2129
	19	243	505	906	1476	2244
	20	255	531	953	1552	2359

TABLE A63: CRITICAL VALUES OF PIRIE'S SIGN TEST
($\alpha=.05$)

n	k									
	3	4	5	6	7	8	9	10	11	12
5	0.19625 2	0.03549 3	0.19168 1	0.01596 2	1.00000 0	0.04098 1	1.00000 0	0.00693 1	1.00000 0	0.00099 1
6	0.06229 3	0.00870 4	0.22536 1	0.02328 2	1.00000 0	0.04897 1	1.00000 0	0.00830 1	1.00000 0	0.00119 1
7	0.09578 3	0.01763 4	0.25764 1	0.03170 2	0.05690 1	0.00142 2	1.00000 0	0.00968 1	1.00000 0	0.00139 1
8	0.13485 3	0.03066 4	0.28857 1	0.04111 2	0.06475 1	0.00188 2	1.00000 0	0.01106 1	1.00000 0	0.00159 1
9	0.17826 3	0.04022 4	0.05142 2	0.00503 3	0.07255 1	0.00240 2	1.00000 0	0.01243 1	1.00000 0	0.00178 1
10	0.06973 4	0.01546 5	0.06254 2	0.00696 3	0.08028 1	0.00299 2	1.00000 0	0.01380 1	1.00000 0	0.00198 1
11	0.09557 4	0.02451 5	0.07438 2	0.00928 3	0.08794 1	0.00363 2	1.00000 0	0.01517 1	1.00000 0	0.00218 1
12	0.12512 4	0.03635 5	0.08686 2	0.01199 3	0.09554 1	0.00434 2	1.00000 0	0.01654 1	1.00000 0	0.00238 1
13	0.05116 5	0.01266 6	0.09990 2	0.01511 3	0.10308 1	0.00510 2	1.00000 0	0.01791 1	1.00000 0	0.00258 1
14	0.06898 5	0.01908 6	0.11344 2	0.01864 3	0.11055 1	0.00591 2	1.00000 0	0.01927 1	1.00000 0	0.00277 1
15	0.08977 5	0.02740 6	0.12742 2	0.02259 3	0.11797 1	0.00678 2	1.00000 0	0.02063 1	1.00000 0	0.00297 1
16	0.11339 5	0.03779 6	0.14177 2	0.02696 3	0.12532 1	0.00771 2	1.00000 0	0.02199 1	1.00000 0	0.00317 1
17	0.05039 6	0.01469 7	0.15644 2	0.03174 3	0.13260 1	0.00869 2	1.00000 0	0.02335 1	1.00000 0	0.00337 1
18	0.06527 6	0.02064 7	0.17138 2	0.03694 3	0.13983 1	0.00972 2	1.00000 0	0.02471 1	1.00000 0	0.00357 1
19	0.08243 6	0.02808 7	0.18654 2	0.04254 3	0.14700 1	0.01081 2	1.00000 0	0.02606 1	1.00000 0	0.00376 1
20	0.10184 6	0.03714 7	0.20187 2	0.04854 3	0.15411 1	0.01194 2	1.00000 0	0.02741 1	1.00000 0	0.00396 1

TABLE A63: (continued)
($\alpha=.01$)

n	k									
	3	4	5	6	7	8	9	10	11	12
5	0.03549 3	0.00334 4	0.01596 2	0.00068 3	0.04098 1	0.00068 2	1.00000 0	0.00693 1	1.00000 0	0.00099 1
6	0.06229 3	0.00870 4	0.02328 2	0.00132 3	0.04897 1	0.00102 2	1.00000 0	0.00830 1	1.00000 0	0.00119 1
7	0.01763 4	0.00200 5	0.03170 2	0.00223 3	0.05690 1	0.00142 2	1.00000 0	0.00968 1	1.00000 0	0.00139 1
8	0.03066 4	0.00461 5	0.04111 2	0.00346 3	0.06475 1	0.00188 2	0.01106 1	0.00005 2	1.00000 0	0.00159 1
9	0.04802 4	0.00895 5	0.05143 2	0.00503 3	0.07255 1	0.00240 2	0.01243 1	0.00007 2	1.00000 0	0.00178 1
10	0.01546 5	0.00244 6	0.06254 2	0.00696 3	0.08028 1	0.00299 2	0.01380 1	0.00009 2	1.00000 0	0.00198 1
11	0.02451 5	0.00461 6	0.07438 2	0.00928 3	0.08794 1	0.00363 2	0.01517 1	0.00011 2	1.00000 0	0.00218 1
12	0.03635 5	0.00793 6	0.01199 3	0.00114 4	0.09554 1	0.00434 2	0.01654 1	0.00013 2	1.00000 0	0.00238 1
13	0.01266 6	0.00240 7	0.01511 3	0.00159 4	0.10308 1	0.00510 2	0.01791 1	0.00015 2	1.00000 0	0.00258 1
14	0.01908 6	0.00411 7	0.01864 3	0.00215 4	0.11055 1	0.00591 2	0.01927 1	0.00017 2	1.00000 0	0.00277 1
15	0.02740 6	0.00661 7	0.02259 3	0.00284 4	0.11797 1	0.00678 2	0.02063 1	0.00020 2	1.00000 0	0.00297 1
16	0.01007 7	0.00215 8	0.02696 3	0.00366 4	0.12532 1	0.00771 2	0.02199 1	0.00023 2	1.00000 0	0.00317 1
17	0.01469 7	0.00347 8	0.03174 3	0.00463 4	0.13260 1	0.00869 2	0.02335 1	0.00026 2	1.00000 0	0.00337 1
18	0.02064 7	0.00534 8	0.03694 3	0.00576 4	0.13983 1	0.00972 2	0.02471 1	0.00029 2	1.00000 0	0.00357 1
19	0.02808 7	0.00789 8	0.04254 3	0.00706 4	0.01081 2	0.00051 3	0.02606 1	0.00032 2	1.00000 0	0.00376 1
20	0.01126 8	0.00284 9	0.04854 3	0.00854 4	0.01194 2	0.00059 3	0.02741 1	0.00036 2	1.00000 0	0.00396 1

TABLE A64: CRITICAL VALUES OF PIRIE-TUKEY RANK SUM TEST
($\alpha=.05$)

n	k									
	3	4	5	6	7	8	9	10	15	20
5	0.05417 7	0.02833 8	0.07833 3	0.03583 4	1.00000 0	0.04917 1	1.00000 0	0.01000 1	1.00000 0	0.00083 1
6	0.05333 9	0.02917 10	0.06750 4	0.03333 5	1.00000 0	0.04667 1	1.00000 0	0.01333 1	1.00000 0	—
7	0.06083 11	0.04333 12	0.06417 5	0.03833 6	0.05083 1	0.03667 2	1.00000 0	0.01167 1	1.00000 0	0.00167 1
8	0.05250 14	0.03417 15	0.07250 5	0.04000 6	0.05833 2	0.03917 3	1.00000 0	0.00750 1	1.00000 0	0.00167 1
9	0.05375 17	0.04125 18	0.05250 7	0.03375 8	0.05125 3	0.04000 4	1.00000 0	0.01375 1	1.00000 0	0.00500 1
10	0.05333 19	0.04000 20	0.06917 7	0.04917 8	0.06417 2	0.04917 3	1.00000 0	0.01500 1	1.00000 0	0.00167 1
15	0.05250 37	0.04667 38	0.05417 12	0.04167 13	0.05083 6	0.04000 7	1.00000 0	0.02583 1	1.00000 0	0.00083 1
20	0.05750 57	0.04917 58	0.05667 21	0.04999 21	0.06833 8	0.04999 8	1.00000 0	0.03000 1	1.00000 0	0.00583 1

TABLE A64: (continued)
($\alpha=.01$)

n	h									
	3	4	5	6	7	8	9	10	11	12
5	0.01583 9	0.00750 10	0.01417 5	0.00667 6	0.02000 3	0.00917 4	1.00000 1	0.00999 1	1.00000 0	0.00083 1
6	0.01167 12	0.00500 13	0.01583 7	0.00999 7	0.01333 4	0.00583 5	0.01333 1	0.00667 2	1.00000 0	—
7	0.01083 16	0.00667 17	0.01167 8	0.00750 9	0.01500 4	0.00833 5	0.01167 1	0.00750 2	1.00000 0	0.00167 1
8	0.01250 18	0.00917 19	0.02167 7	0.00667 8	0.01500 5	0.00833 6	1.00000 0	0.00750 1	1.00000 0	0.00167 1
9	0.01125 26	0.00999 26	0.01125 11	0.00875 12	0.01125 6	0.00750 7	0.01375 2	0.00999 2	1.00000 0	0.00500 1
10	0.01250 25	0.00999 25	0.01083 14	0.00750 15	0.01250 6	0.00583 7	0.01167 2	0.00833 3	1.00000 0	0.00167 1
15	0.01417 48	0.00999 48	0.01167 19	0.00917 20	0.01083 11	0.00833 12	0.01250 5	0.00999 5	1.00000 0	0.00083 1
20	0.01083 71	0.00750 72	0.01083 31	0.00917 32	0.01167 14	0.00917 15	0.01167 9	0.00999 9	1.00000 0	0.00583 1

APPENDIX FIGURES

FIGURE A1: POWERS OF SELECTED ONE-WAY TESTS UNDER RECTANGULAR DISTRIBUTION: $n = 5$

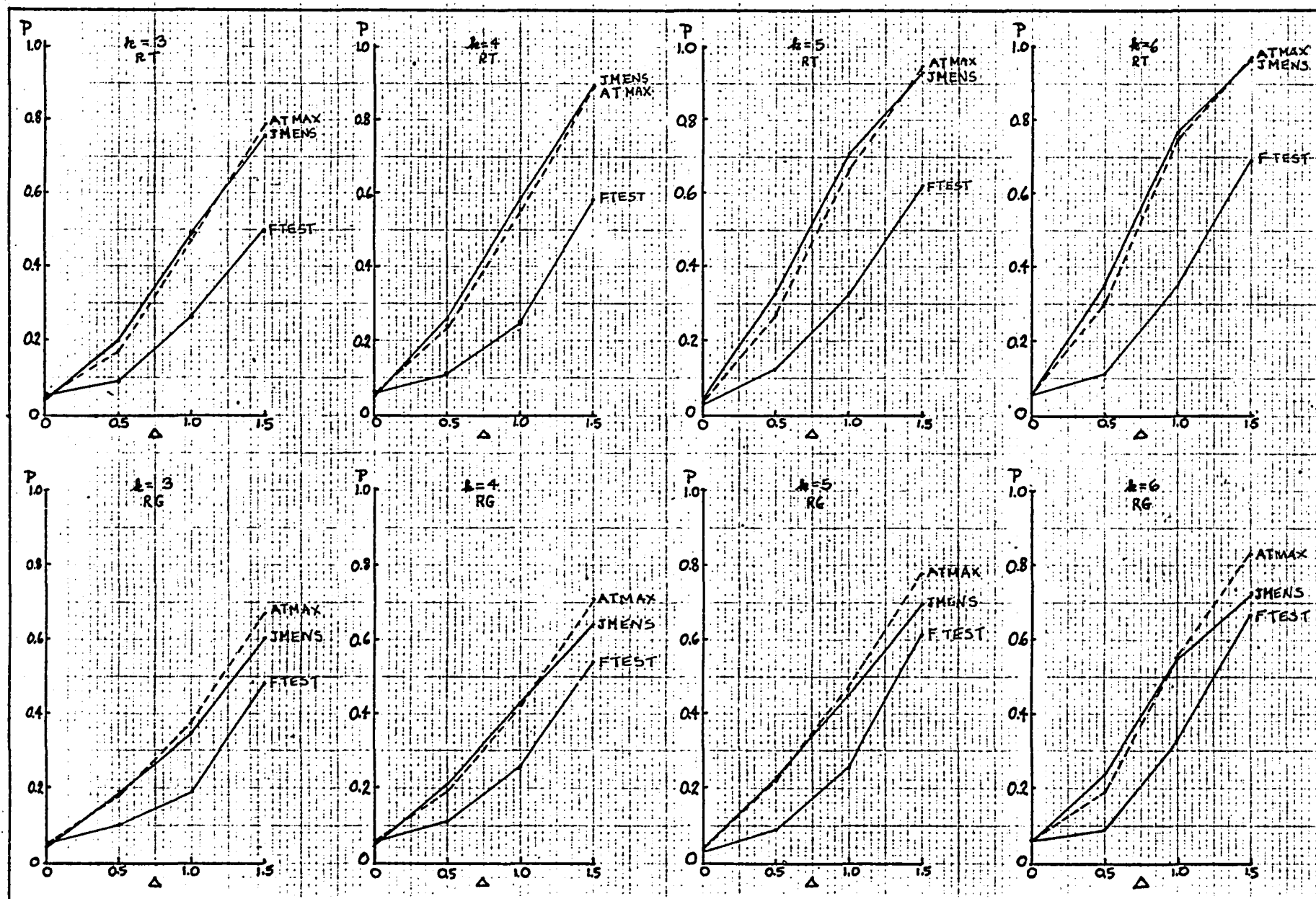


FIGURE A2: POWERS OF SELECTED ONE-WAY TESTS UNDER EXPONENTIAL DISTRIBUTION: $n = 5$

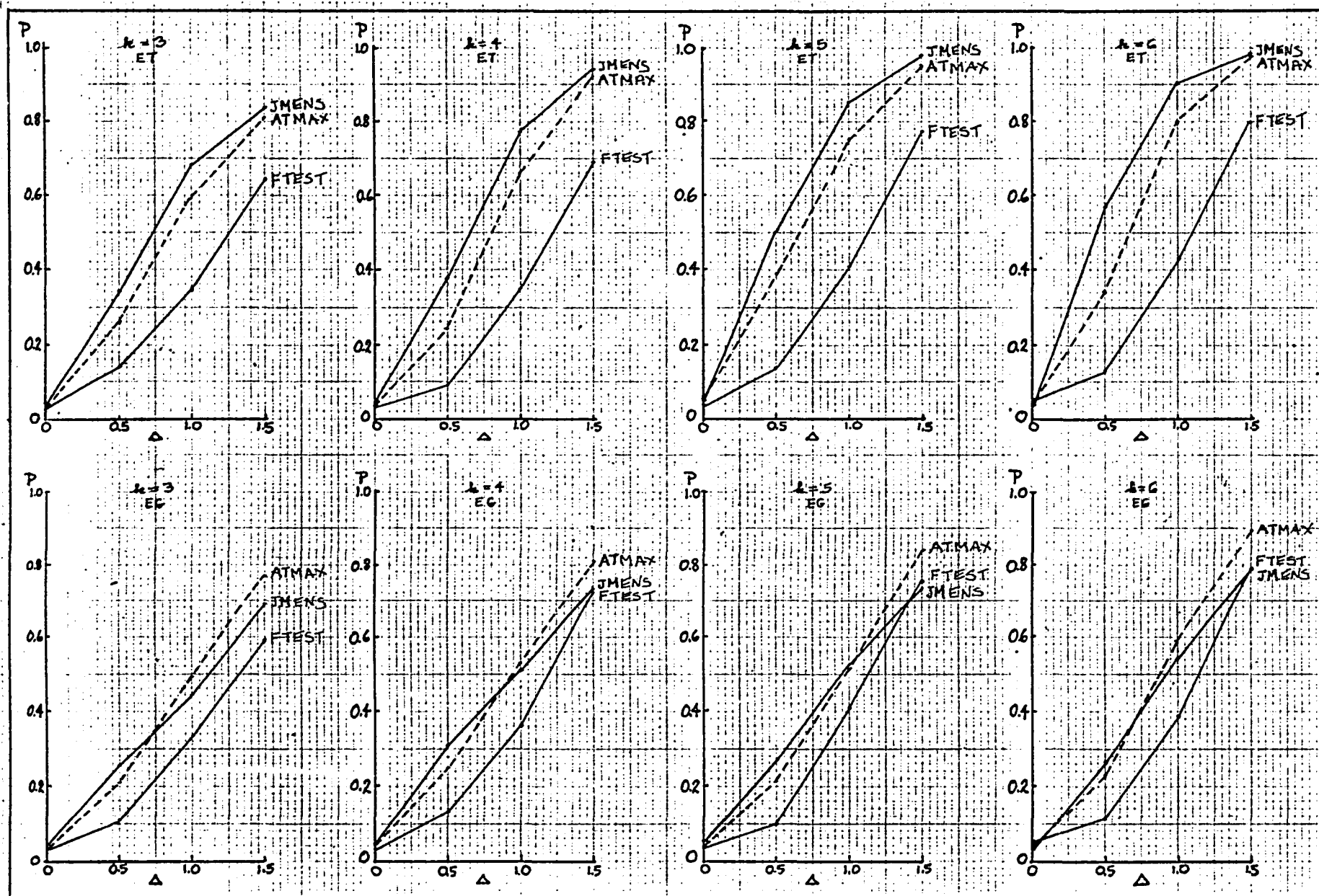


FIGURE A3: POWERS OF SELECTED ONE-WAY TESTS UNDER NORMAL DISTRIBUTION: $n = 5$

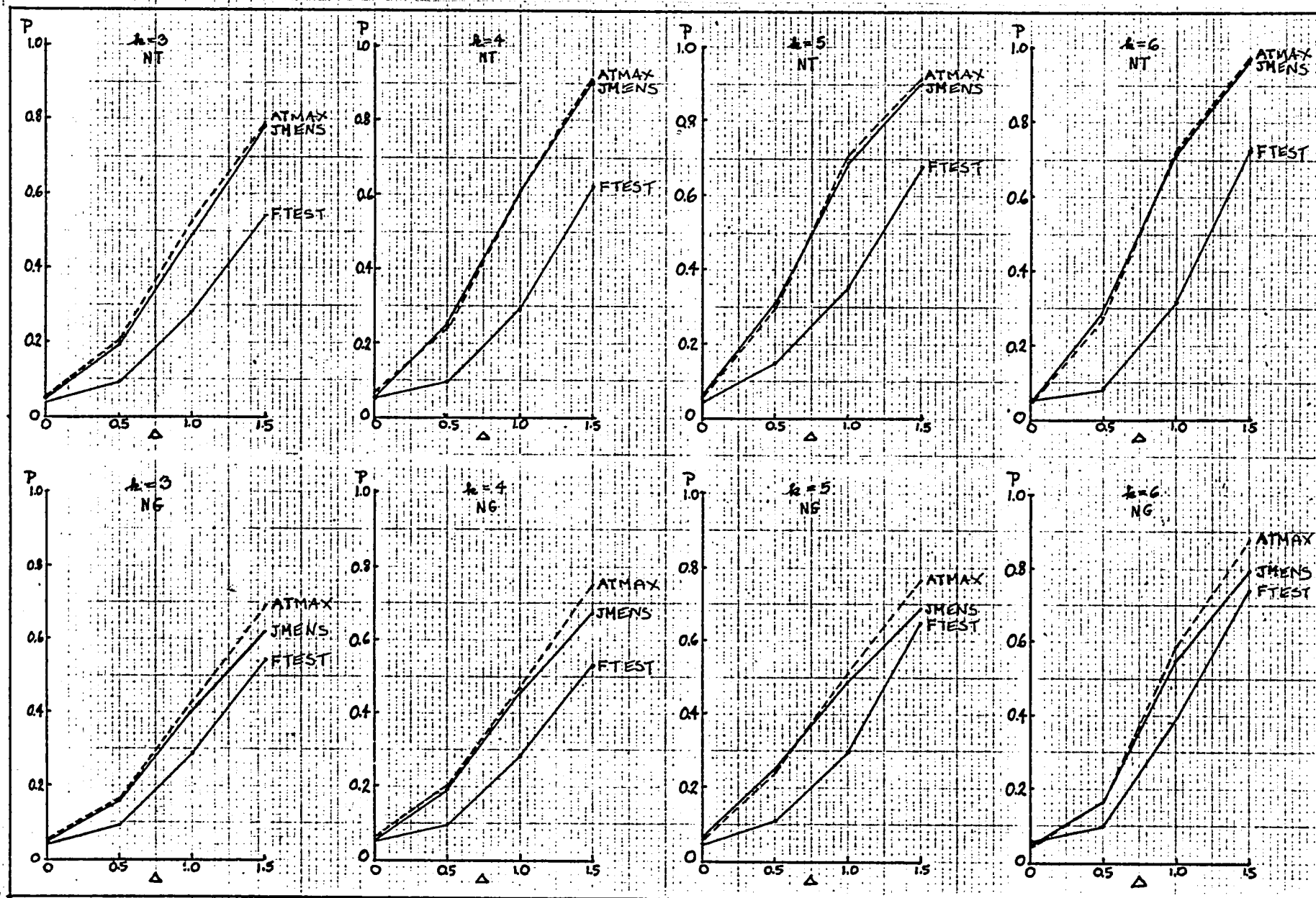


FIGURE A4: POWERS OF SELECTED TWO-WAY TESTS UNDER RECTANGULAR DISTRIBUTION: $n = 5$

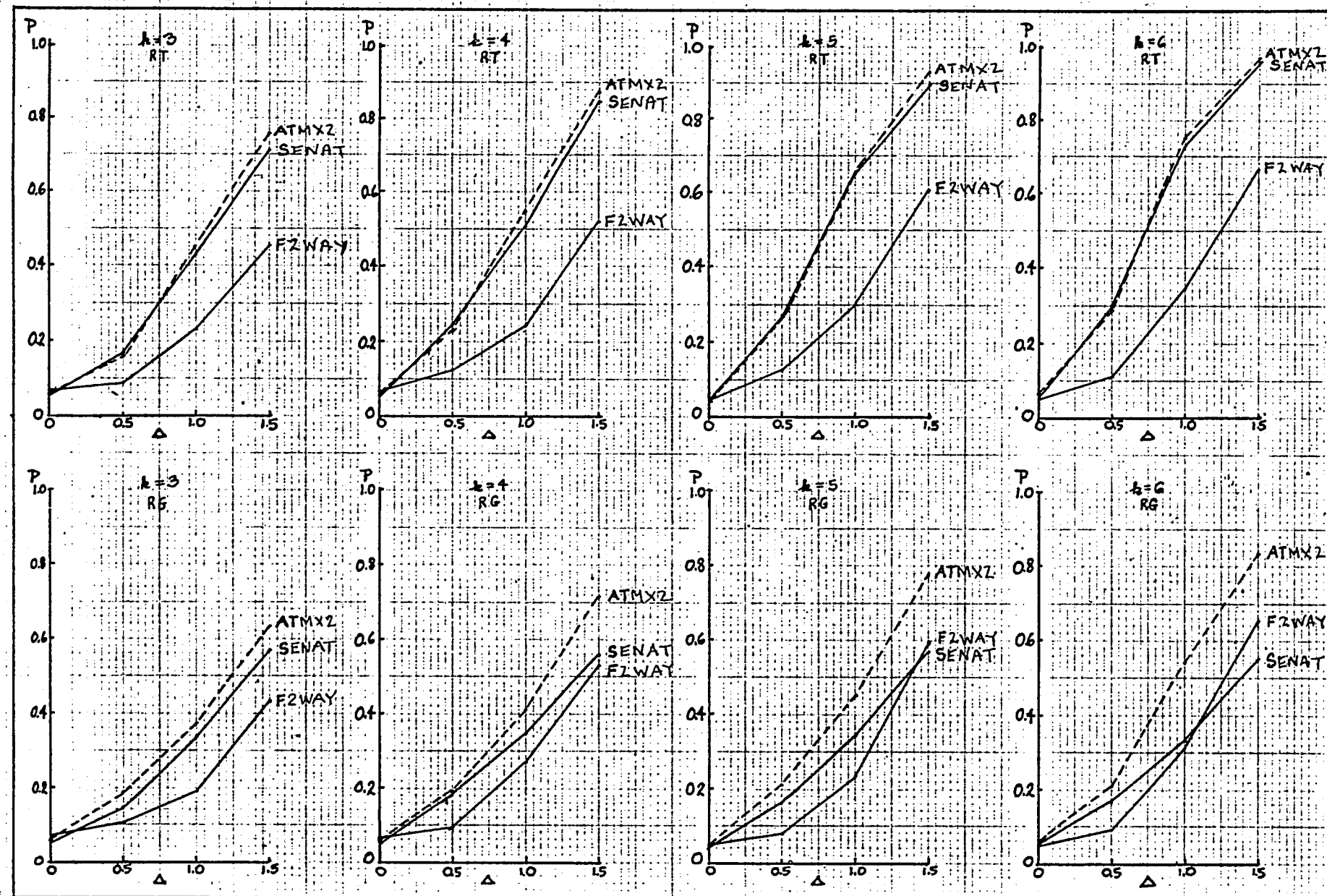


FIGURE A5: POWERS OF SELECTED TWO-WAY TESTS UNDER EXPONENTIAL DISTRIBUTION: $n = 5$

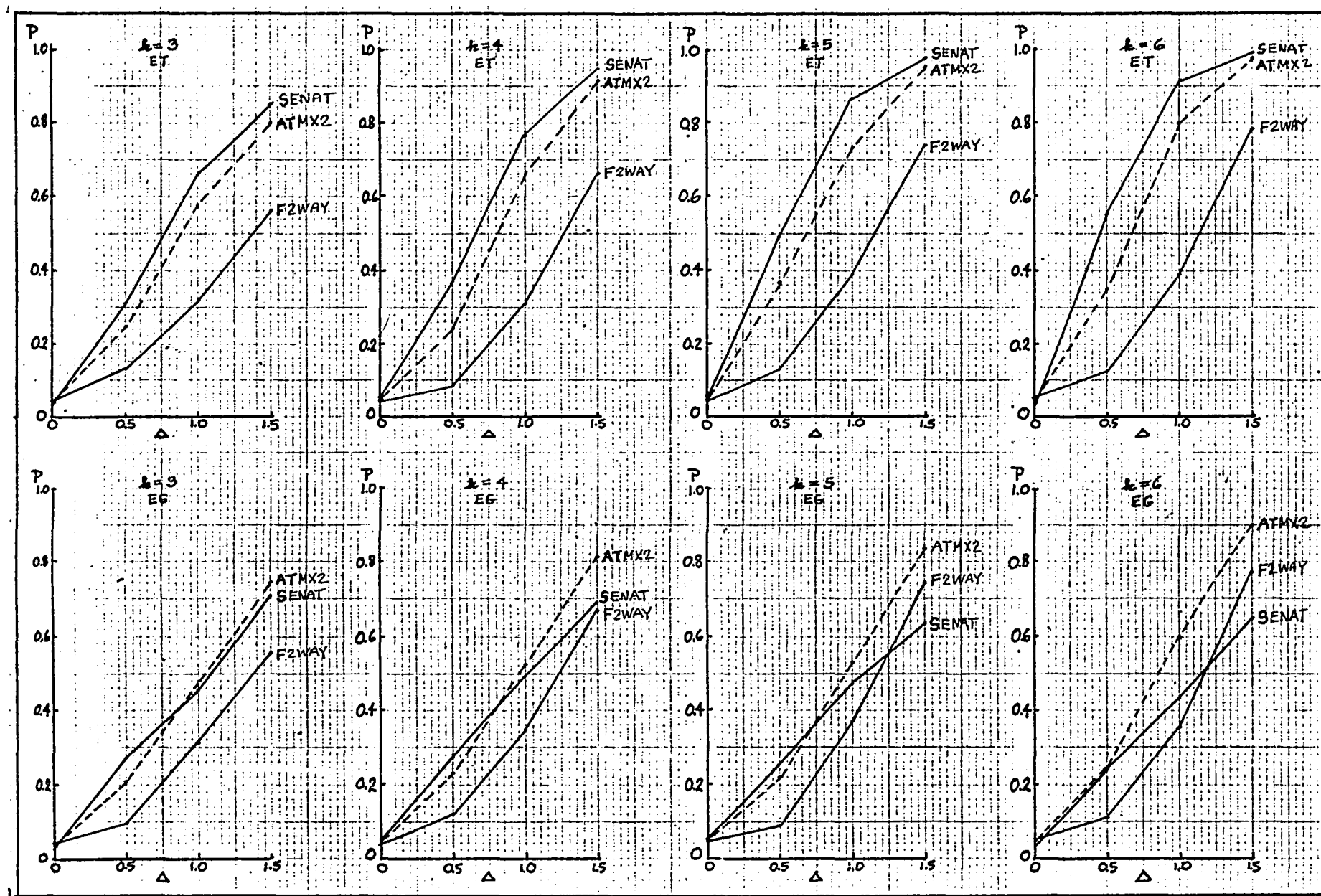


FIGURE A6: POWERS OF SELECTED TWO-WAY TESTS UNDER NORMAL DISTRIBUTION: $n = 5$

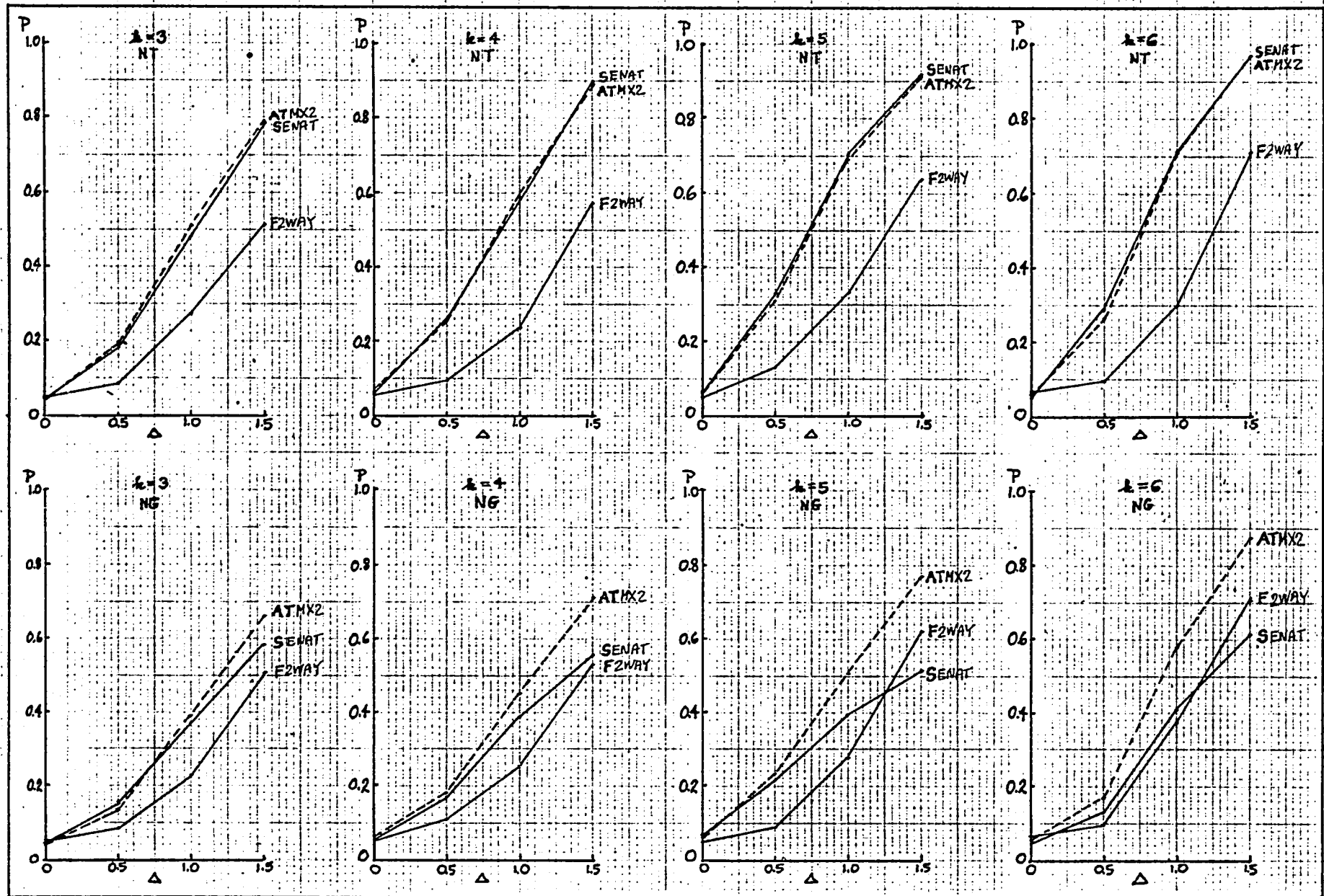


FIGURE A7: POWERS OF NINE ONE-WAY TESTS UNDER EXPONENTIAL
DISTRIBUTION: $n = 5$

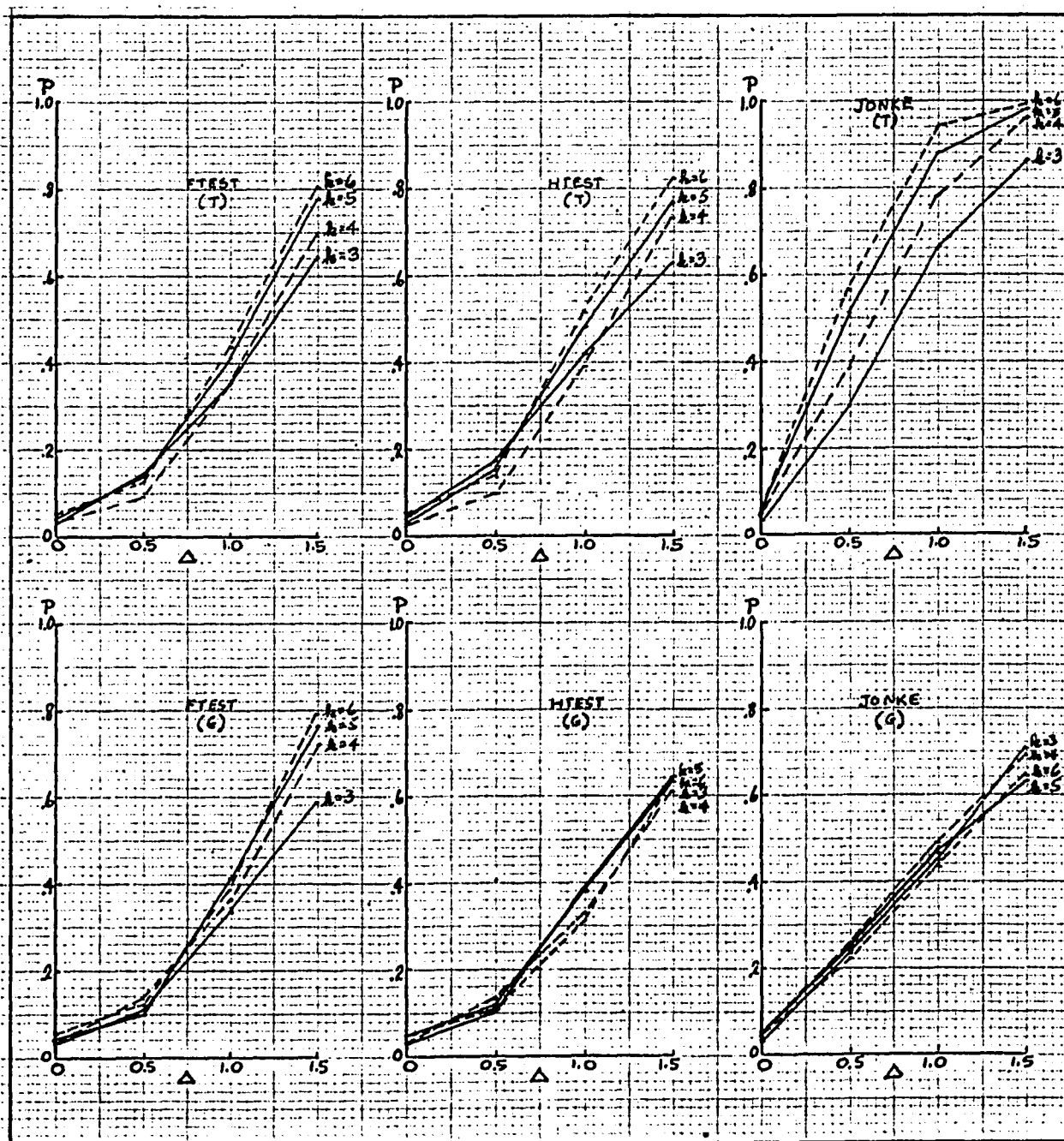


FIGURE A7
(continued)

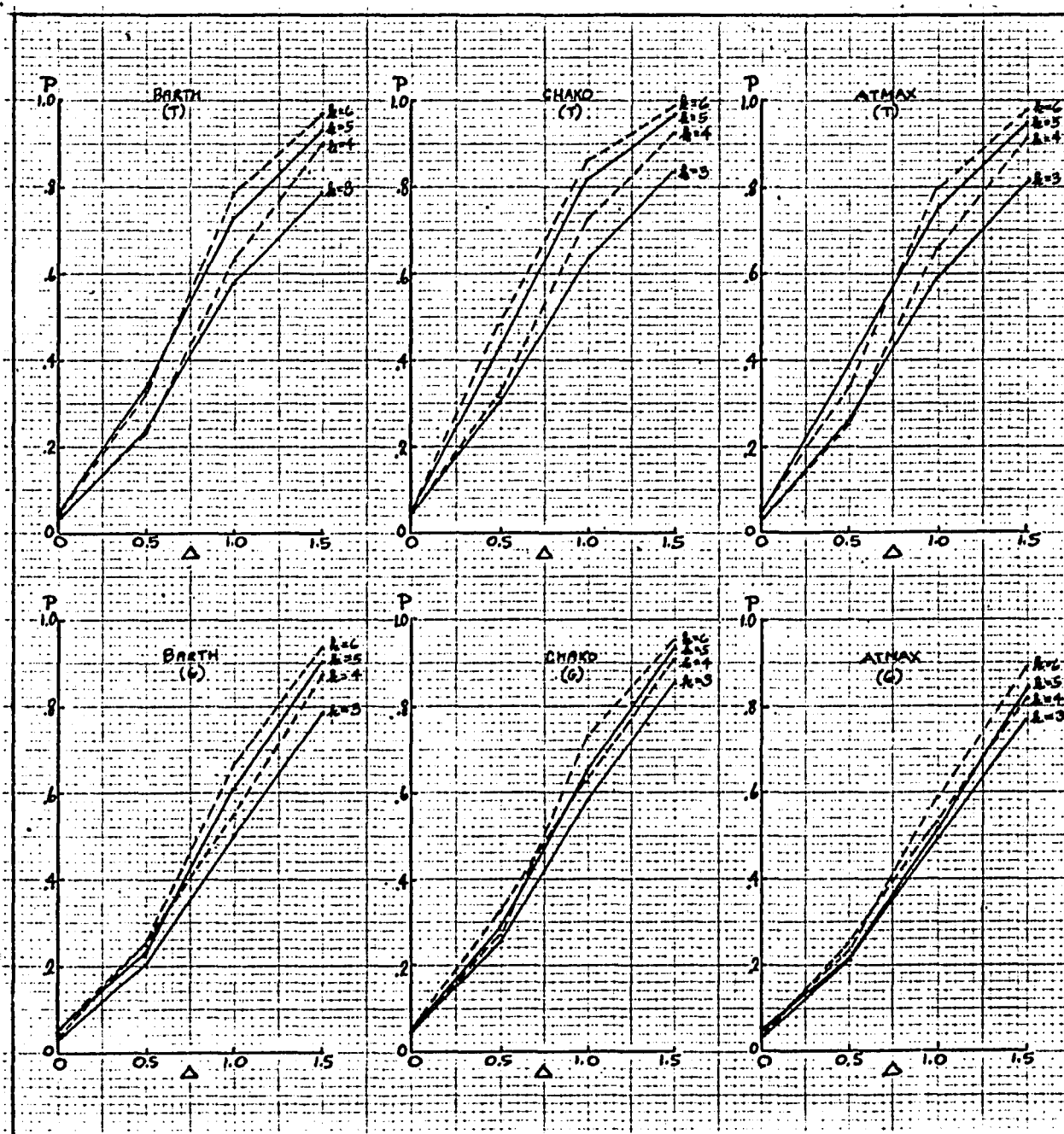


FIGURE A7
(continued)

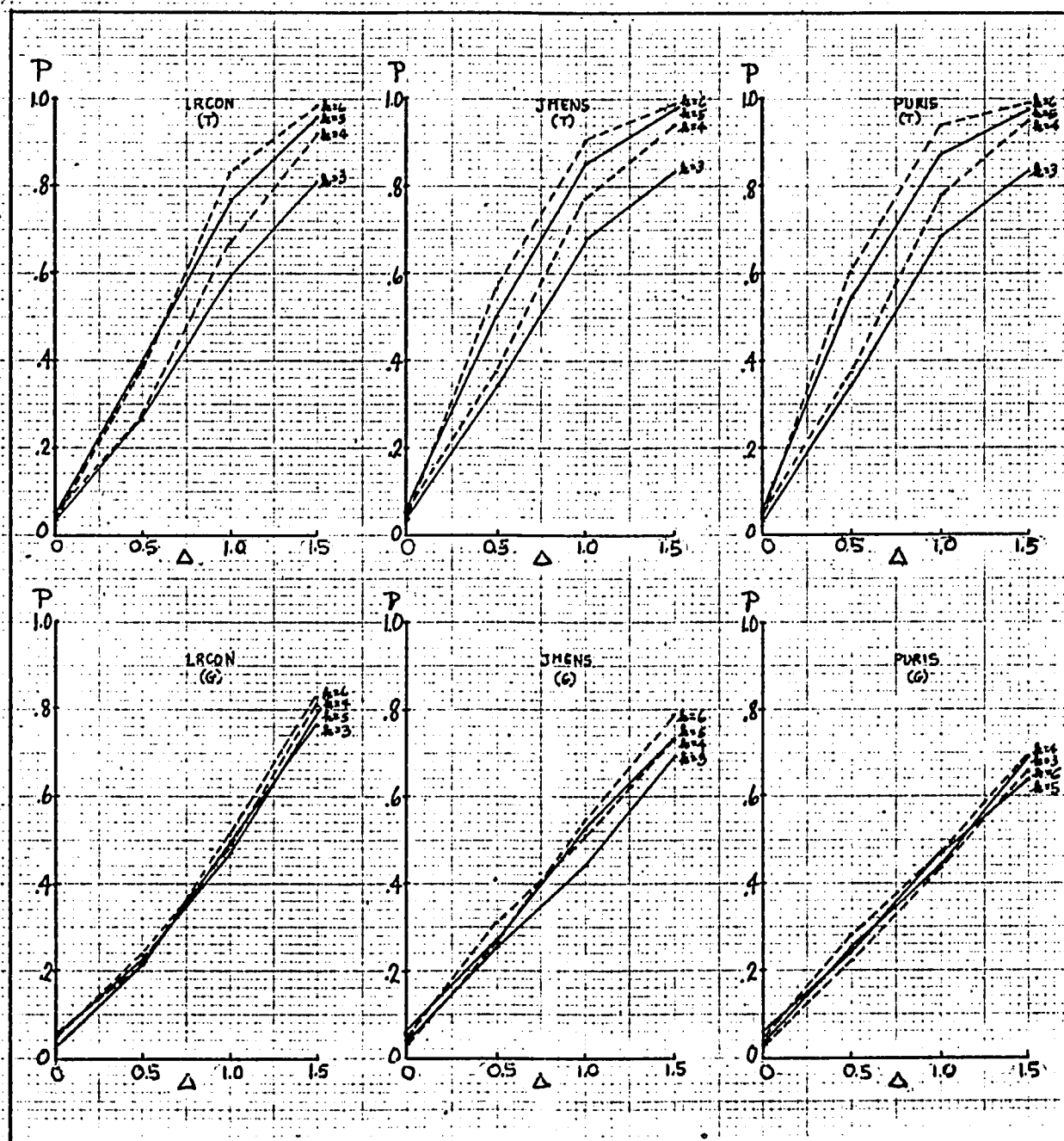


FIGURE A8: POWERS OF NINE TWO-WAY TESTS UNDER EXPONENTIAL DISTRIBUTION: $k = 3$

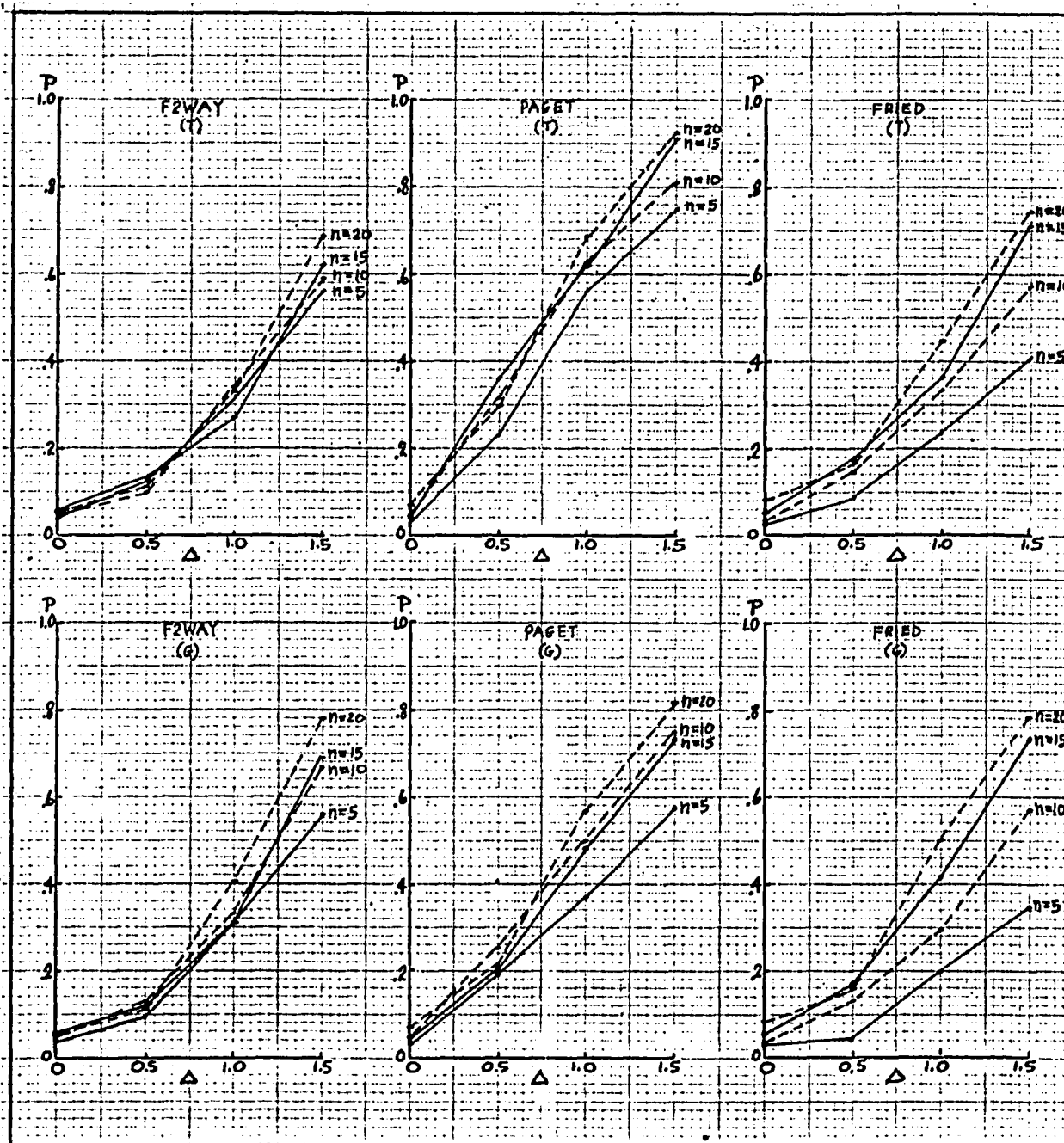


FIGURE A8
(continued)

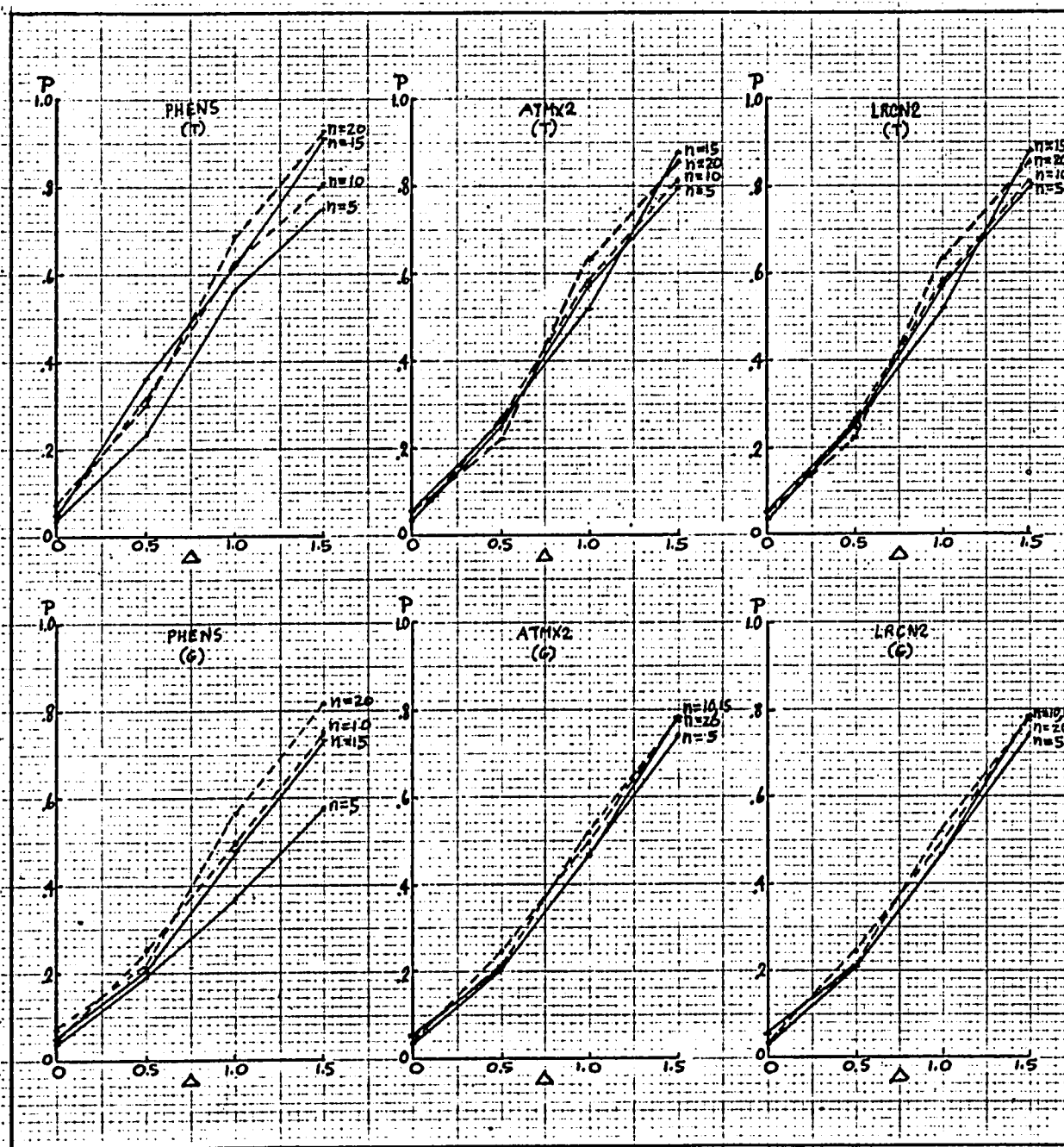


FIGURE A8
(continued)

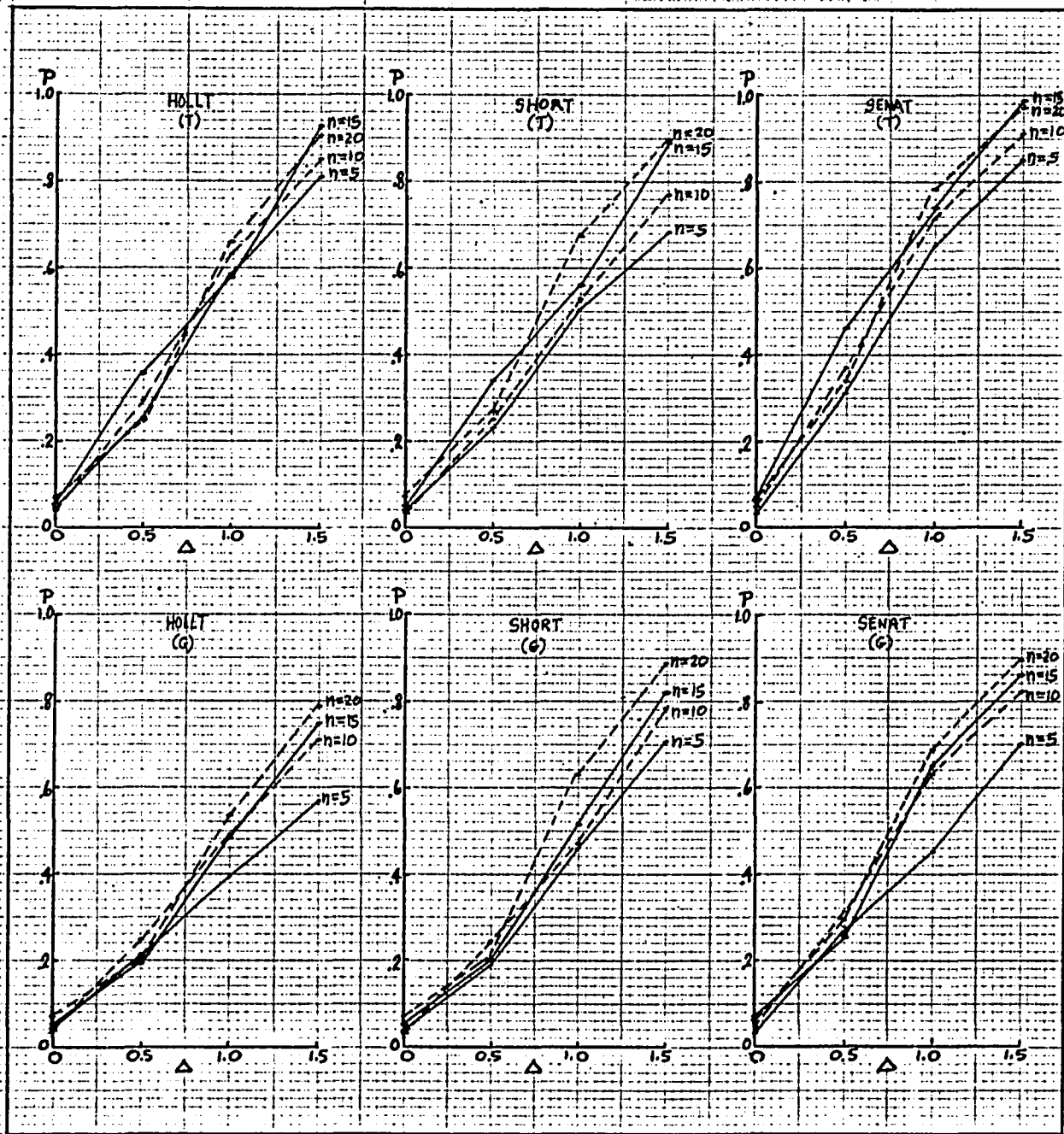


FIGURE A9: A COMPARISON OF POWERS OF THE ONE-WAY F TEST
AGAINST TREND AND END GAP ALTERNATIVES

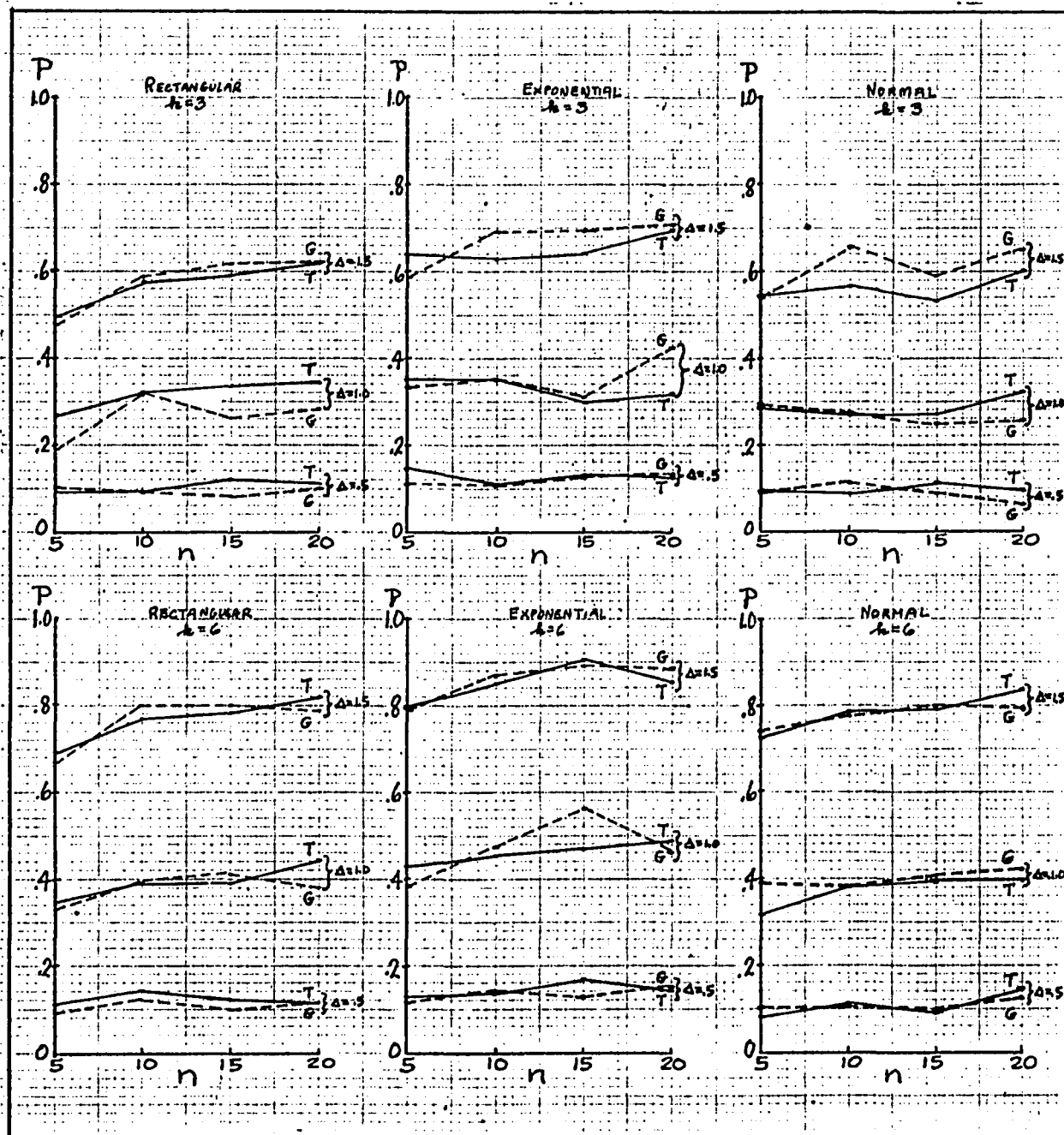
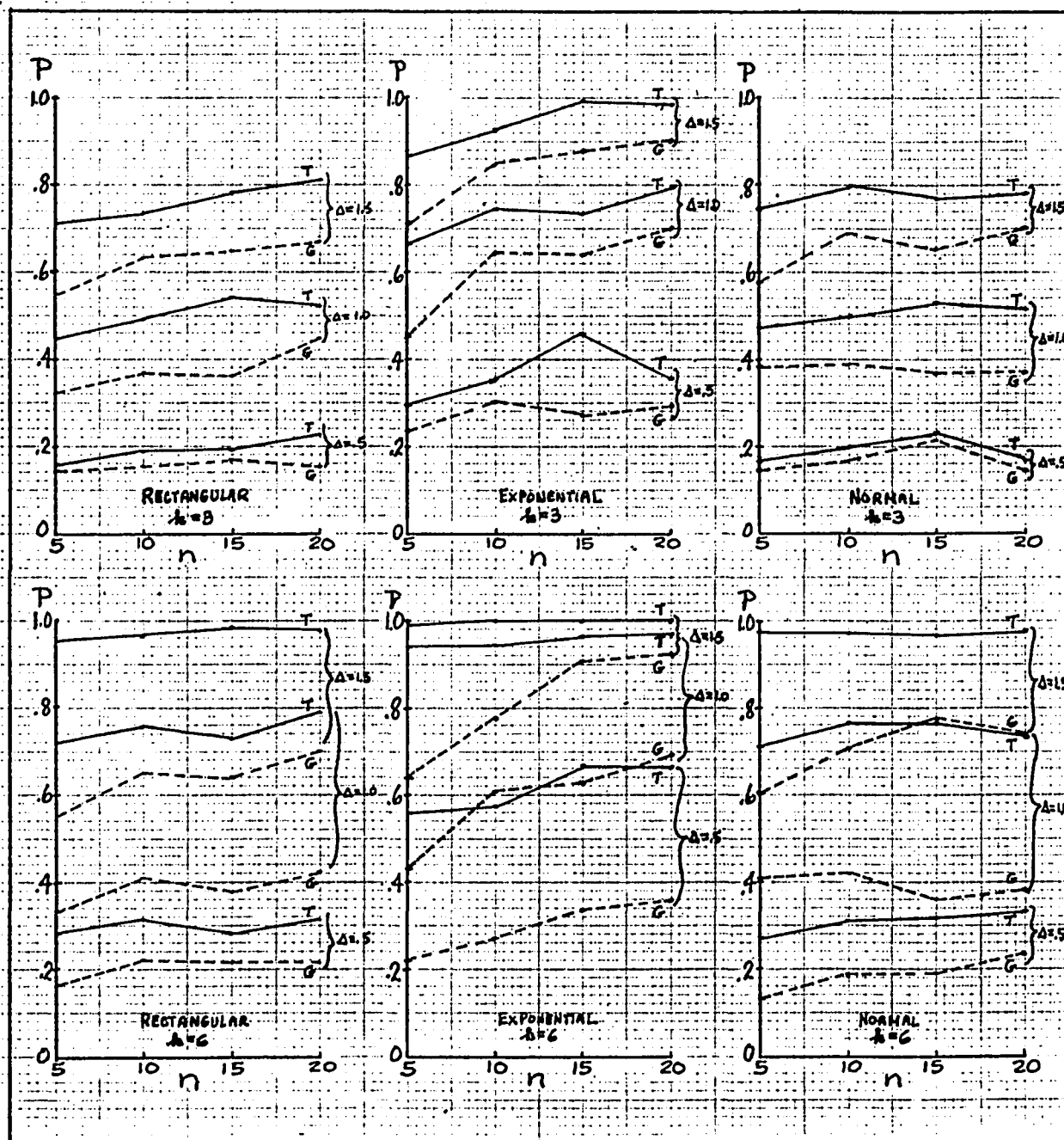


FIGURE A10: A COMPARISON OF THE POWERS OF JONCKHEERE'S
ONE-WAY TEST AGAINST TREND AND GAP ALTERNATIVES
($k = 3$ vs. $k = 6$)



BIBLIOGRAPHY

BIBLIOGRAPHY

- Abelson, R.P., and J.W. Tukey, "Efficient Utilization of Non-Numerical Information in Quantitative Analysis: General Theory and the Case of Simple Order", Annals of Mathematical Statistics (1963), Vol. 34, pp. 1347-1369.
- Abrams, Jack, "Reducing the Risk of New Product Marketing Strategies Testing", Journal of Marketing Research (May, 1969), Vol. 6, No. 2, pp. 216-220.
- Anderson, Decima A., Computer Programming Fortran IV, New York: Meredith Publishing Company, 1966.
- Anderson, Evan E., "Split Plot Design: Measuring Market Share", Journal of Advertising Research (August, 1972), Vol. 12, No. 1, pp. 17-20.
- Anderson, Rolph E., "Consumer Dissatisfaction: The Effect of Disconfirmed Expectancy on Perceived Product Performance", Journal of Marketing Research (February, 1973), Vol. 10, No. 1, pp. 38-44.
- Andrews, F.C., "Asymptotic Behavior of Some Rank Tests for Analysis of Variance", Annals of Mathematical Statistics (1954), Vol. 25, pp. 724-735.
- Banks, Seymour, Experimentation in Marketing, New York: McGraw Hill Book Company, Inc., 1965.
- Bartholomew, D.J., "A Test of Homogeneity for Ordered Alternatives", Biometrika (June, 1959A), Vol. 46, pp. 36-48.
- _____, "A Test of Homogeneity for Ordered Alternatives: II", Biometrika (December, 1959B), Vol. 46, pp. 328-335.
- _____, "Ordered Tests in the Analysis of Variance", Biometrika (December, 1961A), Vol. 48, pp. 325-332.
- _____, "A Test of Homogeneity of Means under Restricted Alternatives (with discussion)", Journal of the Royal Statistical Society, Series B (1961B), Vol. 23, No. 2, pp. 239-272.

- Barton, D.E., and C.L. Mallows, "The Randomization Bases of the Problem of the Amalgamation of Weighted Means", Journal of the Royal Statistical Society, Series B (1961), Vol. 23, No. 2, pp. 423-433.
- Bell, C.B., and K.A. Doksum, "Some New Distribution-free Statistics", Annals of Mathematical Statistics (1965), Vol. 36, pp. 203-214.
- Berenson, Mark L., "'Two-Sample' Nonparametric Tests." Unpublished M.B.A. Thesis, The City College of the City University of New York, February, 1966.
- Bither, Stewart W., "Effects of Distraction & Commitment on the Persuasiveness of Television Advertising", Journal of Marketing Research (February, 1972), Vol. 9, No. 1, pp. 1-5.
- _____ and Peter L. Wright, "The Self-Confidence--Advertising Response Relationship: A Function of Situational Distraction", Journal of Marketing Research (May, 1973), Vol. 10, No. 2, pp. 146-152.
- Bradley, J.V., Distribution-Free Statistical Tests, Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1968.
- Brunk, H.D., An Introduction to Mathematical Statistics, Boston: Ginn & Company, 1960.
- _____, "On the Estimation of Parameters Restricted by Inequalities", Annals of Mathematical Statistics (1958), Vol. 29, pp. 437-454.
- Chacko, V.J., "Testing Homogeneity Against Ordered Alternatives", Annals of Mathematical Statistics (1963), Vol. 34, pp. 945-956.
- _____, "Modified χ^2 for Ordered Alternatives", Sankhya, Series B (1966), Vol. 28, pp. 185-190.
- Chen, Edwin H., "Power of Shapiro-Wilk W Test for Normality in Samples from Contaminated Normal Distributions", Journal of the American Statistical Association (1971), Vol. 66, pp. 760-762.
- Cochran, William G., and Gertrude M. Cox, Experimental Designs, New York: John Wiley and Sons, Inc., 1957.
- Conover, W.J., "A k-Sample Extension of the One-Sided Two-Sample Smirnov Test Statistic", Annals of Mathematical Statistics (1967), Vol. 38, pp. 1726-1730.

- Conover, W.J., "Two k-Sample Slippage Tests", Journal of the American Statistical Association (1968), Vol. 63, pp. 614-626.
- _____, Practical Nonparametric Statistics, New York: John Wiley and Sons, Inc., 1971.
- Cox, D.R., and A. Stuart, "Some Quick Sign Tests for Trends in Location and Dispersion", Biometrika (1955), Vol. 42, pp. 80-95.
- Day, Ralph L., "Systematic Paired Comparisons in Preference Analysis", Journal of Marketing Research (November, 1965), Vol. 2, No. 4, pp. 406-412.
- _____, "Preference Tests and the Management of Product Features", Journal of Marketing (July, 1968), Vol. 32, No. 3, pp. 24-29.
- _____, "Position Bias in Paired Product Tests", Journal of Marketing Research (February, 1969), Vol. 6, No. 1, pp. 98-100.
- Dixon, Wilfrid J., and Frank J. Massey, Jr., Introduction to Statistical Analysis, Third Edition, New York: McGraw-Hill Book Company, Inc., 1969.
- Doksum, K.A., "Robust Procedures for Some Linear Models with One Observation per Cell", Annals of Mathematical Statistics (1967), Vol. 38, pp. 878-883.
- Dommermuth, W.P., and Edward W. Cundiff, "Shopping Goods, Shopping Centers, and Selling Strategies", Journal of Marketing (October, 1967), Vol. 31, No. 4, pp. 32-36.
- Donaldson, Theodore S., "Robustness of the F-Test to Errors of Both Kinds and the Correlation Between the Numerator and Denominator of the F-Ratio", Journal of the American Statistical Association (1968), Vol. 63, pp. 660-676.
- Dunn, O.J., "Multiple Comparisons Using Rank Sums", Technometrics (1964), Vol. 6, pp. 241-252.
- Edwards, Allen L., Experimental Designs in Psychological Research, Revised Edition, New York: Holt, Rinehart and Winston, 1960.
- Ehrenberg, A.S.C., "Note on Normal Transformation of Ranks", British Journal of Psychology (1951), Vol. 4, pp. 133-134.

Ferguson, George A., Statistical Analysis In Psychology, Second Edition, New York: McGraw-Hill Book Company, Inc., 1966.

Fisher, R.A., and F. Yates, Statistical Tables for Biological, Agricultural, and Medical Research, Third Edition, New York: Hafner Publishing Company, 1949.

Friedman, Milton, "The Use of Ranks to Avoid the Assumption of Normality Implicit in the Analysis of Variance", Journal of the American Statistical Association (1937), Vol. 32, pp. 675-701.

_____, "A Comparison of Alternative Tests of Significance for the Problem of m Rankings", Annals of Mathematical Statistics (1940), Vol. 11, pp. 86-92.

Gilbert, R.O., "Monte Carlo Study of Analysis of Variance and Competing Rank Tests for Scheffe's Mixed Model", Journal of the American Statistical Association (1972), Vol. 67, pp. 71-75.

Green, Paul E., and Vithala R. Rao, "Rating Scales and Information Recovery--How Many Scales and Response Categories to Use?", Journal of Marketing (July, 1970), Vol. 34, No. 3, pp. 33-39.

Harter, H. Leon, "Expected Values of Normal Order Statistics", Biometrika (1961), Vol. 48, pp. 151-165.

Hicks, Charles R., Fundamental Concepts in the Design of Experiments, New York: Holt, Rinehart and Winston, Inc., 1964.

Hodges, J.L. Jr., and E.L. Lehmann, "Rank Methods for Combinations of Independent Experiments in the Analysis of Variance", Annals of Mathematical Statistics (1962), Vol. 33, pp. 482-497.

Hoefding, Wassily, "'Optimum' Nonparametric Tests", Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability, Berkeley, Cal.: University of California Press, 1951 (pp. 83-92).

Hoel, Paul G., Introduction to Mathematical Statistics, Third Edition, New York: John Wiley and Sons, Inc., 1966.

Hogg, Robert V., "On Models and Hypotheses with Restricted Alternatives", Journal of the American Statistical Association (1965), Vol. 60, pp. 1153-1162.

- Hollander, Myles, "An Asymptotically Distribution-Free Multiple Comparison Procedure--Treatments vs. Control", Annals of Mathematical Statistics (1966), Vol. 37, pp. 735-738.
- _____, "Rank Tests for Randomized Blocks when the Alternatives have an Apriori Ordering", Annals of Mathematical Statistics (1967), Vol. 38, pp. 867-877.
- _____, and Douglas A. Wolfe, Nonparametric Statistical Methods, New York: John Wiley and Sons, Inc., 1973.
- Hunt, Shelby D., "Post-transaction Communications and Dissonance Reduction", Journal of Marketing (July, 1970), Vol. 34, No. 3, pp. 46-51.
- Jogdeo, Kumar, "On Randomized Rank Score Procedures of Bell and Doksum", Annals of Mathematical Statistics (1966), Vol. 37, pp. 1697-1703.
- Johnson, R.A., and K.G. Mehrotra, "Some c-Sample Nonparametric Tests for Ordered Alternatives", University of Wisconsin Technical Report No. 263 (March, 1971).
- Jonckheere, A.R., "A Distribution-Free k-Sample Test Against Ordered Alternatives", Biometrika (June, 1954A), Vol. 41, pp. 133-145.
- _____, "A Test of Significance for the Relation Between m Rankings and k Ranked Categories", British Journal of Statistical Psychology (1954B), Vol. 7, pp. 93-100.
- Kemphorne, Oscar, The Design and Analysis of Experiments, New York: John Wiley and Sons, Inc., 1952.
- Kendall, M.G., and B. Babington Smith, "The Problem of m Rankings", Annals of Mathematical Statistics (1939), Vol. 10, pp. 275-287.
- _____, and A. Stuart, Advanced Theory of Statistics, Volume I, New York: Hafner Publishing Co., 1958.
- Kruskal, W.H., and W.A. Wallis, "Use of Ranks in One-criterion Variance Analysis", Journal of the American Statistical Association (1952), Vol. 47, pp. 583-621.
- Lehmann, E.L., "Robust Estimation in Analysis of Variance", Annals of Mathematical Statistics (1963A), Vol. 34, pp. 957-966.

Lehmann, E.L., "Asymptotically Nonparametric Inference: An Alternative Approach to Linear Models", Annals of Mathematical Statistics (1963B), Vol. 34, pp. 1494-1506.

_____, "Asymptotically Nonparametric Inference In Some Linear Models with One Observation per Cell", Annals of Mathematical Statistics (1964), Vol. 35, pp. 726-734.

Leik, R.K., and W.R. Gove, "The Conception and Measurement of Asymmetric Monotonic Relationships in Sociology", The American Journal of Sociology (May, 1969), Vol. 74, pp. 696-709.

Levine, D.M., and S. Wachspress, P. McGuire, and M.S. Mayzner, "Visual Information Processing of Numerical Inputs", Bulletin of Psychonomic Society (1973), Vol. 1 (6A), pp. 404-406.

Li, Jerome C.R., Statistical Inference, Volume I, Ann Arbor, Michigan: Edwards Brothers, Inc., 1964.

Lilliefors, H.W., "On the Kolmogorov-Smirnov Test for Normality with Mean and Variance Unknown", Journal of the American Statistical Association (1967), Vol. 62, pp. 399-402.

Lubin, A., "L'utilisation des correlations par rang pour eprouver une tendance dans un ensemble de moyennes", Bulletin du Centre d'Etudes et Recherches Psychotechniques (1961), Vol. 10, pp. 433-444.

Lyerly, S.B., "The Average Spearman Rank Correlation Coefficient", Psychometrika (1952), Vol. 17, pp. 421-428.

McClure, Peter J., and John K. Ryans, Jr., "Difference Between Retailers' and Consumers' Perceptions", Journal of Marketing Research (February, 1968), Vol. 5, No. 1, pp. 35-40.

_____, "Analyzing Image Data Using the Friedman Two-Way Analysis of Variance by Ranks", Journal of Marketing Research (August, 1971), Vol. 8, No. 3, pp. 370-371.

McCracken, Daniel D., A Guide to Fortran IV Programming, New York: John Wiley and Sons, Inc., 1965.

McDonald, B.J., and W.A. Thompson, "Rank Sum Multiple Comparisons in One- and Two-Way Classifications", Biometrika (1967), Vol. 54, pp. 487-498.

- Mehra, K.L., and J. Sarangi, "Asymptotic Efficiency of Rank Tests for Comparative Experiments", Annals of Mathematical Statistics (1967), Vol. 38, pp. 90-107.
- Mendenhall, W., and J.E. Reimuth, Statistics for Management and Economics, Belmont, California: Duxbury Press, 1971.
- Miles, R.E., "The Complete Amalgamation into Blocks, by Weighted Means, of a Finite Set of Real Numbers", Biometrika (1959), Vol. 46, pp. 317-327.
- Miller, George A., "The Magical Number Seven, Plus or Minus Two: Some Limits on Our Capacity for Processing Information", Psychological Review (March, 1956), Vol. 63, pp. 81-97.
- Miller, Rupert G. Jr., Simultaneous Statistical Inference, New York: McGraw-Hill Book Company, Inc., 1966.
- Morris, John, "Nonparametric Statistics on the Computer", Journal of Marketing Research (February, 1969), Vol. 6, No. 1, pp. 86-92.
- Mosteller, F., "A k-Sample Slippage Test for an Extreme Population", Annals of Mathematical Statistics (1948), Vol. 19, pp. 58-65.
- _____, and J.W. Tukey, "Significance Levels for a k-Sample Slippage Test", Annals of Mathematical Statistics (1950), Vol. 21, pp. 120-123.
- Nemenyi, P., "Distribution-Free Multiple Comparisons." Unpublished Ph.D. Dissertation, Princeton University, June, 1963.
- Noether, G.E., "On a Theorem of Pitman", Annals of Mathematical Statistics (1955), Vol. 26, pp. 64-68.
- _____, Elements of Nonparametric Statistics, New York: John Wiley and Sons, Inc., 1967.
- Owen, D.B., Handbook of Statistical Tables, Reading, Massachusetts: Addison-Wesley, Inc., 1962.
- Page, E.B., "Ordered Hypotheses for Multiple Treatments: A Significance Test for Linear Ranks", Journal of the American Statistical Association (1963), Vol. 58, pp. 216-230.
- Pearson, E.S., and H.O. Hartley, Biometrika Tables for Statisticians, Volume I, Cambridge, England: University Press, 1954.

- Pearson, Karl, Tables of the Incomplete Beta-Function, Cambridge, England: University Press, 1956.
- _____, Tables of the Incomplete Gamma-Function, Cambridge, England: University Press, 1957.
- Pirie, W.R., "Distribution-Free Tests For Ordered Alternatives in the Randomized Block Model." Unpublished Ph.D. Dissertation, Florida State University, June, 1970.
- _____, and Myles Hollander, "A Distribution-Free Normal Scores Test for Ordered Alternatives in the Randomized Block Design", Journal of the American Statistical Association (1972), Vol. 67, pp. 855-857.
- Proctor, C.H., "Processing Survey Data with Ordered Alternative Hypotheses", Journal of the American Statistical Association (1971), Vol. 66, pp. 720-724.
- Puri, M.L., "Some Distribution-free k-Sample Rank Tests of Homogeneity Against Ordered Alternatives", Communications on Pure and Applied Mathematics (1965), Vol. 18, pp. 51-63.
- _____, and P.K. Sen, "On Chernoff-Savage Tests for Ordered Alternatives in Randomized Blocks", Annals of Mathematical Statistics (1968), Vol. 39, pp. 967-972.
- Rabinowitz, Manus, and Mark L. Berenson, "A Comparison of Various Methods of Obtaining Random Order Statistics for Monte Carlo Computations", paper accepted for publication, The American Statistician, July, 1973.
- Roper, Burns W., "Sensitivity, Reliability and Consumer Taste Testing: Some 'Rights' and 'Wrongs'", Journal of Marketing Research (February, 1969), Vol. 6, No. 1, pp. 102-106.
- Schaafsma, W., and L.J. Smid, "Most Stringent Somewhat Most Powerful Tests Against Alternatives Restricted by Number of Linear Inequalities", Annals of Mathematical Statistics (1966), Vol. 37, pp. 1161-1172.
- Sen, P.K., "Asymptotically Most Powerful Rank Order Tests for Grouped Data", Annals of Mathematical Statistics (1967), Vol. 38, pp. 1229-1239.
- _____, "On a Class of Aligned Rank Order Tests in Two-Way Layouts", Annals of Mathematical Statistics (1968), Vol. 39, pp. 1115-1124.

- Shapiro, S.S., and M.B. Wilk, "An Analysis of Variance Test for Normality (Complete Samples)", Biometrika (1965), Vol. 52, pp. 591-610.
- _____, _____, and H.J. Chen, "A Comparative Study of Various Tests for Normality", Journal of the American Statistical Association (1968), Vol. 63, pp. 1343-1372.
- _____, and R.S. Francia, "Approximate Analysis of Variance Test for Normality", Journal of the American Statistical Association (1972), Vol. 67, pp. 215-216.
- Sherman, E., "A Note on Multiple Comparisons Using Rank Sums", Technometrics (1965), Vol. 7, pp. 255-256.
- Shorack, G.R., "Testing Ordered Alternatives in Model I Analysis of Variance; Normal Theory and Nonparametric", Annals of Mathematical Statistics (1967), Vol. 38, pp. 1740-1751.
- Siegel, Sidney, Nonparametric Statistics for the Behavioral Sciences, New York: McGraw-Hill Book Company, Inc., 1956.
- Snedecor, George W., Statistical Methods, Fifth Edition, Ames, Iowa: The Iowa State University Press, 1962.
- Steel, Robert G.D., "A Rank Sum Test for Comparing All Pairs of Treatments", Technometrics (1960), Vol. 2, pp. 197-207.
- _____, "Some Rank Sum Multiple Comparison Tests", Biometrics (1961), Vol. 17, pp. 539-551.
- Taylor, James R., "Reevaluation of Preference Distribution Analysis", Journal of Marketing Research (November, 1968), Vol. 5, No. 4, pp. 434-438.
- Terpstra, T.J., "The Asymptotic Normality and Consistency of Kendall's Test Against Trend when Ties are Present in One Ranking", Proceedings, Nederlands Akademie Wetenschappen, Series A (1952), Vol. 55, pp. 329-333.
- Tukey, John, "Sums of Random Partitions of Ranks", Annals of Mathematical Statistics (1957), Vol. 28, pp. 987-992.
- Van Elteren, P.H., and G.E. Noether, "Asymptotic Efficiency of the χ^2 -Test for a Balanced Incomplete Block Design", Biometrika (1959), Vol. 46, pp. 475-477.

Wilcoxon, Frank, and Roberta Wilcox, Some Rapid Approximate Statistical Procedures, Revised Pamphlet, Pearl River, New York: Lederle Laboratories of The American Cyanamid Company, 1964.

Wolf, Edward H., "Tests for Randomness on the Line and a Related k-Sample Test for Homogeneity." Unpublished Ph.D. Dissertation, Rutgers University, The State University of New Jersey, December, 1967.

_____, and Joseph I. Naus, "Tables of Critical Values for a k-Sample Kolmogorov-Smirnov Test Statistic", Journal of the American Statistical Association (1973), Vol. 68, pp. 994-997.