Spatial Data Sets and Map Projections: An Analysis of Distortion

BY

Karen A. Mulcahy

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Date

Keith C. Clarke

1/28/99
Date

Frederick C. Shaw

Dr. Sean Ahearn

Dr. Stanley Gedzelman

Dr. Victor Goldsmith

Dr. Scott Loomer

The City University of New York

ABSTRACT

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Karen A. Mulcahy

Advisor: Dr. Keith C. Clarke

The depiction of our planet in digital form is at the core of regional and global research efforts. When the size of a study area is minute compared to the size of the earth, an insignificant amount of distortion is introduced during map projection transformation; however, when the area of interest is broad, significant distortion occurs. When raster spatial data sets are transformed by map projections, the actual data values may change significantly, reducing the quality of the data set.

This study explores the changes occurring within regional or global raster data sets as a result of map projection transformations. The methodology consists of tracking pixel loss and pixel duplication separately through the application of new metrics called ***PD*** and ***PL*** created for this purpose. The spatial extent of this analysis is continental and greater. The world map projections examined are primarily the equal area pseudocylindricals. Azimuthal projections were employed for examination of continental data sets. The Mollweide and Sinusoidal projections are of particular interest as the component projections of the Goode Homolosine projection in use by the US Geological Survey for release of global data sets.

The pseudocylindrical projections show complex behaviors of ***PL*** and ***PD*** with both pixel loss and duplication occurring simultaneously. Based on the ***PL*** and ***PD*** results there is an inverse relationship between angular distortion and pixel distortion on the pseudocylindrical projections. In addition, the examination of pixel value changes using ***PL*** and ***PD*** indicate that distortion patterns on the azimuthal projections are influenced heavily by the angular changes occurring in the graticule. The transformation of raster spatial data sets of large extent by map projections is often an invertible process with current technologies and algorithms.

This research challenges basic cartographic theory regarding map projection distortion. Further research will be needed to define the full set of changes and basic map projection guidelines will need to be updated to include the recommendations from this research. Cartographic education, spatial data users, data producers and spatial data standards are all impacted by these results.

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CHAPTER 1

INTRODUCTION

Representing the three dimensional surface of the Earth as a two dimensional planar map inevitably leads to a distorted portrayal of the Earth, whether it is on the physical surface of a paper map or the virtual map in a digital data base. The transformations of locations on the earth's surface to locations on a map involves map projection. However, even in cartography, map projection use and especially the nature and distribution of map projection distortion are poorly understood. Recent GIS use has generated a larger pool of users of global data in need of an understanding of map projections, and of their propagation effect on GIS error. The purpose of this research is to identify and quantify the amount of uncertainty that is being introduced into regional and global spatial databases via the map projection transformation; to develop map projection distortion metrics; and to recommend visualization methods for communicating these measures to world map users.

Many shared and distributed data sets of regional or global extent have a complex lineage with regards to map projection transformations. For example, a vast data set derived from satellite observation is often constructed by mosaicing individual images together. The same process occurs when paper map sheets are converted to digital form and joined together. As the spatial extent of the data set expands, a map projection will be chosen as the best for the larger spatial area. Several iterations of data set alterations may occur and with each a new map projection transformation may occur. Furthermore, each time a raster data set is transformed from one map projection to another, the data values for the final data set are determined by resampling and interpolation. Some data values, or pixels, are duplicated and other pixels are eliminated. The values of the remaining pixels may also be altered in a method determined by the type of spatial resampling method used. For each map projection transformation performed the data set will undergo further, potentially irreversible, change.

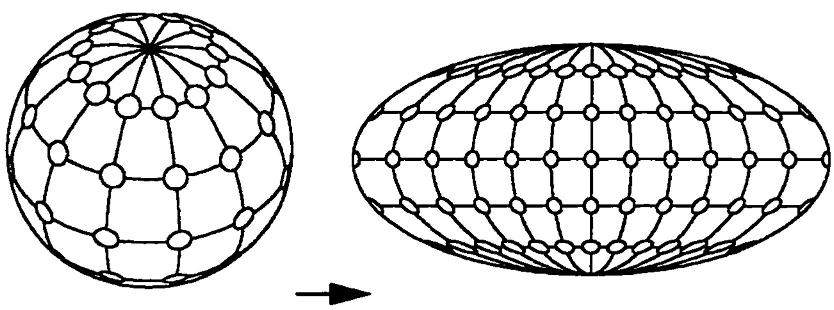
The distortion created by map projection transformations dictates the accuracy of any derived spatial data set. Willmott, Rowe and Philpot (1985) conducted a sensitivity analysis to compare interpolation in the spherical and planar realms, finding local temperature errors in the latitudes greater than 60^{o} were as large as 5^{o} to 10^{o}C. Is is theoretically possible to avoid the distortion introduced by Euclidean interpolation on map projections by performing interpolation on the sphere or by other methods that avoid the use of world map projections. Recently the development of alternative referencing systems such as polygonal tessellations of the globe and optimized map projections for those tessellations (Mulcahy 1997) show promise. Nevertheless, the current status of implemented technology necessitates the use of map projections to transform from the globe to a planar map. Little quantification has been performed to measure the effects of map projection transformations, the impact of resampling during map projection transformation, and the consequences map projection distortion on the further use of spatial data sets.

1.1 What is a Map Projection?

Maling (1992) defined a map projection as:

any systematic arrangement of meridians and parallels portraying the curved surface of the sphere or spheroid upon a plane.

Fig. 1. – From the Spheroidal Earth to a Planar Map



Snyder (1989) wrote:

> *A map projection is a systematic representation of all or part of the surface of a round body, especially the Earth, on a plane.* (See Fig. 1.) *This usually includes lines delineating meridians and parallels, as required by some definitions of a map projection, but it may not depending upon the purpose of the map.*

Maling's definition, and others that include statements resembling, "*any systematic arrangement of meridians and parallels,*" originate from the days of manual cartography when a change in map projection meant a very slow, painstaking and expensive manual process of first creating an arrangement of meridians and parallels, the graticule, and then transferring map features in relation to this established graticule. John Snyder's definition updates this phrase to a "*systematic representation of all or part of the surface of a round body on a plane.*" We employ map projections and coordinate systems for nearly all planar representations of data or paper maps, because the current state of technology for handling spatially referenced data depends upon map projection transformations.

The essential elements of a map projection are the *systematic means* for transforming features *from a surface to a plane*. The default referencing method for the earth is geographic latitude and longitude. The systematic relationship between latitude and longitude coordinates and projected planar coordinates is described functionally as:

$$x = f_1(\varphi,\lambda)$$
$$y = f_2(\varphi,\lambda)$$
(Equation 1)

Where φ is latitude, λ is longitude; and x and y are coordinate eastings and northings respectively, and the functions, f_1 and f_2, may be expressing one of literally hundreds of different map projections. These range from simple geometric constructions to extremely complex mathematical functions.

The process of transforming the spheroidal surface of the Earth to a planar surface inescapably leads to a distorted depiction of the Earth's spatial properties, so a fundamental problem is to understand how the Earth is being misrepresented. Complicating the problem is the fact that there are hundreds of map projections that have been developed since the birth of solutions to this problem in the time of Ptolemy (approximately A.D. 150).

1.2 Map Projection Classification Systems

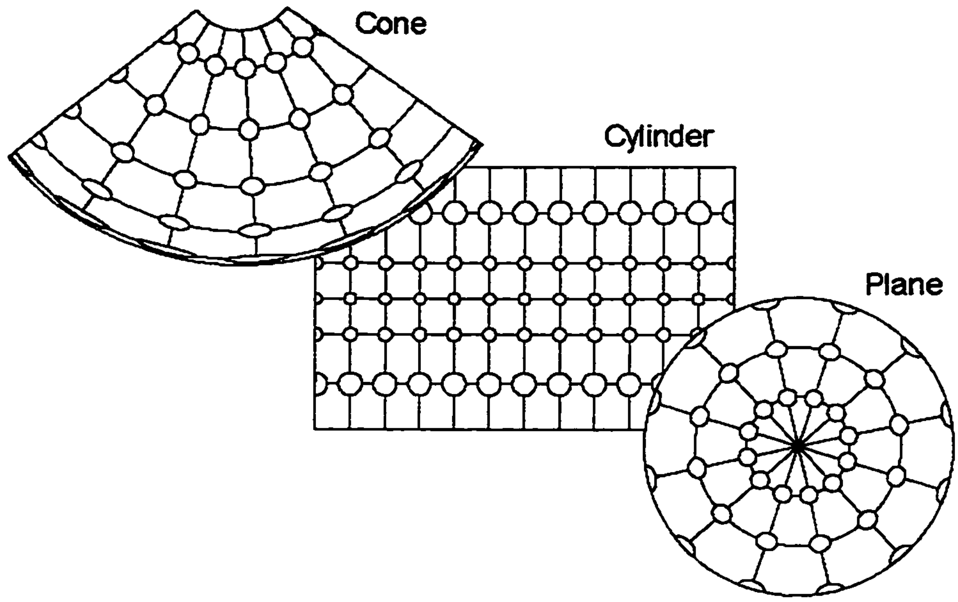
Projections are classified by a number of different systems. The most common of these methods are classification by geometric model, geometric properties, and parametric classification. In addition, aspect and the properties of the graticule are employed to classify or assist in description of a projection. Classification systems provide a means for organizing the hundreds of existing projections.

The classification systems based upon the geometric model and geometric properties .also provide a descriptive vocabulary useful for immediately grasping the essential properties of a projection. Canters and Decleir use this descriptive labeling for all of the world map projection in their text. Although descriptive their terms sometimes increase the confusion that occurs with multiple names representing a single projection. A table composed of the equivalents used by ESRI (1994) and in the ARC/INFO software (Canters and Decleir (1989) and Snyder (1987, 1993), and in this document are included in "APPENDIX B. MAP PROJECTION NAME EQUIVALENTS". A very different approach to classifying map projections are Tobler's four group parametric classification (1962) and Maling's extension of Tobler's system (1992).

1.2.1 Geometric Model and Aspect

The majority of the hundreds of map projections in existence can be assigned to one of three basic categories based upon an imagined construction surface. The surface are the cylinder, illustrated by the Mercator projection; a cone, illustrated by the Lambert Conformal Conic projection, and the plane, illustrated by the Azimuthal Equidistant projection. See Figure 2. One can imagine a cone wrapping around a globe with the cone

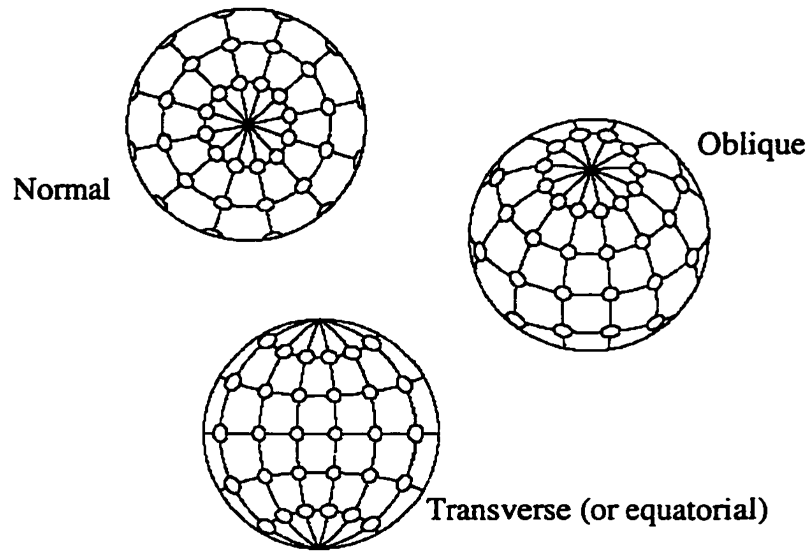
Figure 2. -- Projections of Three Developable Surfaces: Cone, Cylinder, and Plane



over the north pole. If the features from the globe were transferred to the cone, it could then be unwrapped from the globe to become a two dimensional map without any further distortion of the map features. A projection such as the one labeled Cone in Figure 2 would result. In reality, map projections are generally mathematical functions but the concept of the geometric model is very useful for categorizing projections and is an excellent learning device (Mulcahy and Clarke, 1995).

Aspect, while not strictly a classification system is used to describe the manner in which the projection surface is oriented with respect to the graticule. The aspect of a projection is most often changed to assist in obtaining the best possible distortion characteristics for a particular map. Although Wray (1974) describes seven possible aspects of a map projection there are three basic aspects that can describe the full set adequately these include: the normal, transverse, and oblique aspects (Figure 3). The

Figure 3. -- Map Projection Aspects: Normal, Transverse and Oblique



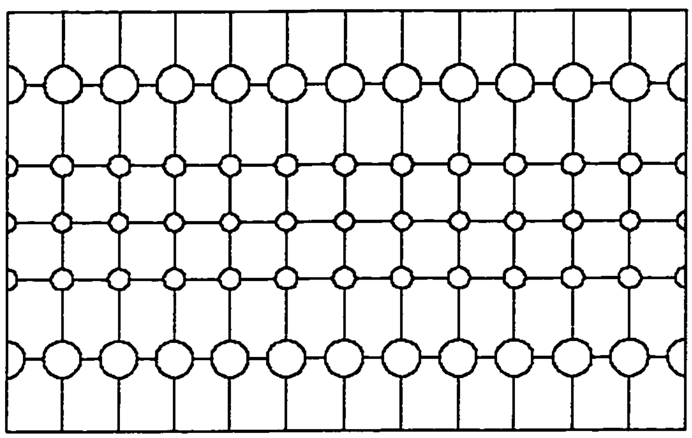
normal aspect is the geometrically simplest for each of the geometric models. For an azimuthal projection in the normal aspect the projection is centered upon the pole. In the transverse aspect the azimuthal projection is centered upon the equator and is often called the equatorial aspect. Finally, an oblique aspect is a projection centered anywhere except for the poles and the equator. The choice of aspect primarily affects the appearance of the graticule. The distortion patterns as measured by traditional methods remain the same. As a classification system the geometric model concept is not sufficient in itself to uniquely identify all possible projections.

1.2.2 Geometric Properties

The geometric properties classification system is frequently used in conjunction with a description of the geometric model system to more fully describe a projection. The focus is upon which of the geometric properties, conformality, equal area or equidistance, are retained in the final projection. Like the geometric model concept this system also has the drawback of not being able to fully classify all projections. For example there is a large group of compromise projections that are not true in any one of the properties of equal area or conformality but may have other special properties.

Conformality is a characteristic of map projections where the local angles or shapes are correct. A map is said to be conformal when scale deformation is the same in all directions from a particular point. Tissot's indicatrix is an excellent visualization of the map projection properties of conformality and equal area. For a conformal projection the indicatrix circles are variously sized but remain as circular symbols (Figure 4). Scale distortion varies across the map but at any particular location scale is distorted the same amount in all directions.

Figure 4. -- The Mercator, a Conformal Map Projection



Conformal projections, such as the Universal Transverse Mercator (UTM), are used for nearly all large scale mapping throughout the world. The Space Oblique Mercator projection is a nearly conformal projection designed for continuous mapping of satellite imagery (Snyder 1987).

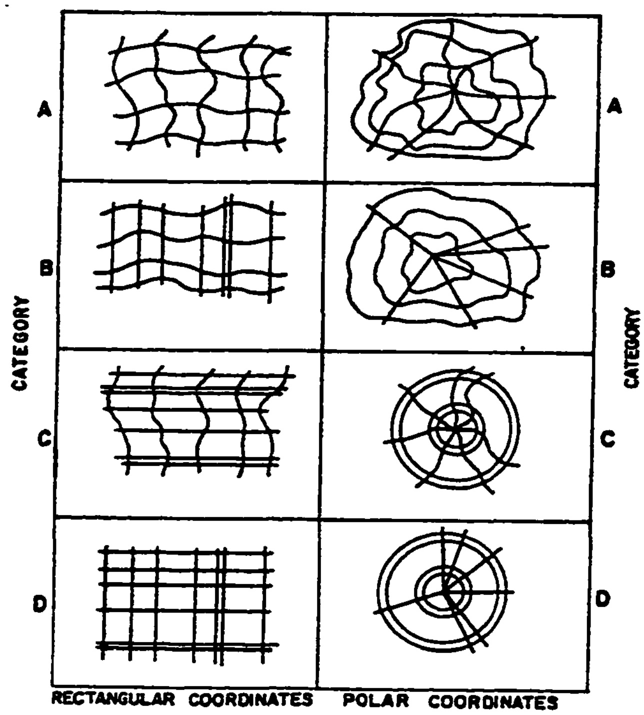
For many projects concerned with studying the entire globe or large portions of it, the calculation of area is desired. Equal area projections are recommended because they ensure that a region on the surface of the earth is represented by the same amount of area on a map. A map is described as equal area when the scale distortions balance one another, such as on the sinusoidal projection (Figure 6). The shape of the Tissot indicatrix changes throughout the map but the area of each ellipse remains true relative to the area it represents on the globe.

1.2.3 Groups, Classes and Series

One of the primary drawbacks of using the geometric model or geometric properties as a classification system is that these systems are not "collectively exhaustive and mutually exclusive" (Maling, 1992). Maling described seven classes including the polyconic, pseudocylindrical, pseudoazimuthal, pseudoconical, cylindrical, azimuthal and conical classes. His system expands upon the basic set of geometric models and relates these classes to the form of the analytical function that describes the projection. To understand Maling's system of groups, classes, and series, one must turn first to Tobler's parametric classification system (1962).

Figure 5. --Tobler's Four Group Classification

Source: Tobler 1962.



Category	Rectangular coordinates	Polar coordinates
A	$x = f_1(\varphi, \lambda)$, $y = f_2(\varphi, \lambda)$	$r = f_1(\rho, \lambda)$, $\theta = f_2(\rho, \lambda)$
B	$x = f_1(\lambda)$, $y = f_2(\varphi, \lambda)$	$r = f_1(\lambda)$, $\theta = f_2(\rho, \lambda)$
C	$x = f_1(\varphi, \lambda)$, $y = f_2(\varphi)$	$r = f_1(\varphi, \lambda)$, $\theta = f_2(\varphi)$
D	$x = f_1(\lambda)$, $y = f_2(\varphi)$	$r = f_1(\lambda)$, $\theta = f_2(\varphi)$

Tobler (1962) described a parametric classification system that placed all map projections into one of four groups based upon the functional relationships (Figure 5). His goal was to solve the problem of organizing an infinite set of projections "into a comprehensible and useful finite number of all-inclusive and preferably non-overlapping classes" (Tobler, 1962). Schematic drawings indicate possible forms of lines of latitude and longitude. In group A both parallels and meridians may be variably spaced and curved. For Group B the meridians may be variably spaced and straight while parallels may be variably spaced and curved. Group C is similar to B but in reverse. In Group D both parallels and meridians may be variably spaced.

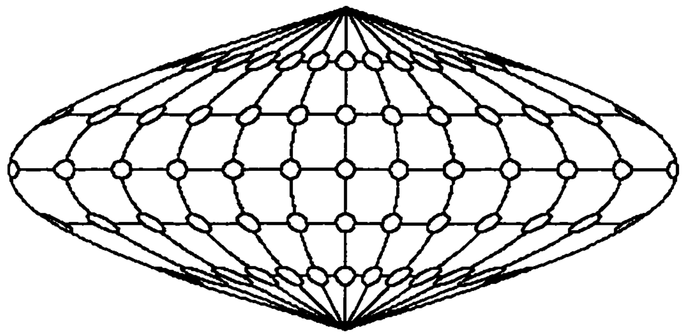
Maling (1992) takes Tobler's four groups and subdivides into seven classes as seen in Table 13. He then breaks these into series based upon the separation of the parallels decreasing away from the equator, remaining equidistant, or increasing with distance from the equator.

TABLE 13. Classified List of Some Principal Map Projections - Maling

Group	Class	Series		
		Decreasing separation	Equidistant spacing of parallels	Increasing separation
D	Cylindrical $x = f_1(\lambda)$ $y = f_2(\varphi)$	Lambert cylindrical equal-area	Plate Carrée	Mercator
D	Azimuthal $r = f_1(\lambda)$	Stereographic Gnomonic Minimum-error (Airy)	Azimuthal equidistant	Lambert azimuthal equal area
D	Conical $r = f_1(\lambda)$	Lambert equal-area conic	Ptolemy equidistant conic	Lambert conformal conic
C	Pseudocylindrical $x = f_1(\varphi, \lambda)$	Mollweide Pseudocylindrical equal-area Parabolic	Sinusoidal Polyhedric	
C	Pseudoazimuthal $r = f_1(\varphi)$ $\theta = f(\varphi, \lambda)$			Equal-area pseudoazimuthal
C	Pseudoconical		Bonne	
A	Polyconic $x = f_1(\varphi, \lambda)$ $y = f_2(\varphi, \lambda)$	Hammer-Aitoff	Simple polyconic Aitoffs Winkel-Tripel	

Canters and Decleir (1989) expand upon the significance of the variation in parallel spacing. They note that projections with small or no angular distortion tend to have increased parallel spacing while projections with little or no area distortion tend to have decreased parallel spacing.

Figure 6. --Sinusoidal projection



$x = f(\phi, \lambda)$

$y = g(\phi)$

1.2.4 Projection Class and Property for Global Data

Global data, specifically global data to be used for studying global land surface and near surface changes require the use of an equal area projection (Steinwand et. al., 1995). Several of the pseudocylindrical and polyconic world map projections are particularly important during the selection of projections suitable for global data sets. The study by Steinwand et. al. (1995) recommended two projections, the Wagner IV and Wagner VII as minimal error choices for use with world data sets. There were two pseudocylindrical interrupted projections also suggested, the interrupted Mollweide and the Goode's Homolosine which is a combination of the Sinusoidal and the Mollweide projections. All of the projections recommended for world data sets share the common geometric property of equivalence.

1.3 Mathematical Projections

There are three basic methods of relating the Cartesian coordinates (x,y) to geographic coordinates of (λ,φ). Maling (1991) included:

- Analytical transformation,
- Direct or grid-on-grid transformation,
- Polynomial transformation.

1.3.1 Analytical Transformation

The transformation of geographical coordinates to Cartesian coordinates by the analytical method most closely approximates traditional cartographic methods of "locating and plotting points from their geographical coordinates" (Maling, 1991). Frequently the digitized coordinates of point locations from a particular map or database (x',y') are converted to geographical coordinates, an inverse (or reverse) solution. The conversion from geographical coordinates (λ,φ) to plane coordinates (x,y) is referred to as the forward solution. Since many GIS studies use several data sets, often from different sources, these data may have been in a variety of different map projections and would need to be converted to a common projection, or to geographic coordinates. ESRI (1994) refers to projecting data, after the initial georeferencing process to a particular projection, as a "secondary projection". Maling (1991) refers to this common projection as the "GIS Framework." He summarized the transformation process as:

$$(x',y') \rightarrow (\varphi,\lambda) \rightarrow (x,y)$$

<inverse solution> <forward solution>

An example of an analytical, point-to-point, transformation for a projection suitable for a world map or data base is the following forward solutions for the Sinusoidal projection (Snyder 1987).

For λ = longitude east of Greenwich, λ_0 = longitude east of the central meridian, R = radius of the sphere, and φ = geographic latitude.

$$x = R(\lambda - \lambda_o)\cos\phi$$ (Equation 2)

$$y = R\phi$$ (Equation 3)

In addition to the formula as stated above, formulas exist for the reverse projection, for the forward and reverse projections using an ellipsoid model of the earth, for changes in the aspect of the projection, and for calculation of point based distortion measurements.

The primary advantages of the analytical method are that it is accurate, "rigorous and it is independent of the size of the area to be mapped." (Maling 1991). A disadvantage of the analytical method is that the transformation can be slow when applied to large data files. A further disadvantage, specific to the case of raster data, is the fact that this method produces complex error. (Maling 1991) As a result, it is generally not the one employed in software implementations, where the polynomial transformation is used instead.

1.3.2 Direct Transformation

A direct transformation by the method of grid-on-grid is a technique used for regridding a map or for plotting a second grid onto a map. It does not require the inverse solution, but instead relates the rectangular coordinates of identical points on two different projections. It is commonly used by analytical stereoplotters with conventional aerial photography and for applying geometrical corrections to satellite imagery. The two major kinds of transformations are the "linear conformal, similarity or Helmert transformation, expressed in the general form" (Maling 1991) in Equation 4; and the affine transformation expressed in Equation 5.

$$X = A + Cx + Dy$$
$$Y = B - Dx + Cy \qquad \text{(Equation 4)}$$

$$X = A + Cx + Dy$$
$$Y = B - Ex + Fy \qquad \text{(Equation 5)}$$

These equations have in common the transformation of known coordinates (x,y) in one system to the (X,Y) coordinates of a second system. In the first case four coefficient A-D are sufficient but in the second, affine transformation six coefficients (A-F) are required to account for scale corrections in each direction (Maling 1991). This affine transformation is also referred to as the six parameter affine. See Clarke (1995) for a derivation and worked example.

1.3.3 Polynomial Transformation

A numerical method of relating geographical and Cartesian coordinates is through the construction of "polynomial expressions to fit the data and use the resulting coefficients to transform the coordinates of the remaining points of map detail" (Maling 1991). The order or degree of the polynomial defines the number or required control points and the amount of computation that is required. Snyder (1985) describes the calculation of the coefficients by both the development of a Taylor series and by a least squares approach. He recommends the least squares approach over the Taylor series as it avoids many of the complex derivations and differentiations.

Snyder (1985) cautions that "If the region of the map is continent-sized, a polynomial of sufficient accuracy will usually contain an excessive number of terms and will not save computer time." Although he was describing these calculations several generations ago in terms of the evolution of computational capabilities; the large areas and large data sets used for regional and global studies make the comment still valid.

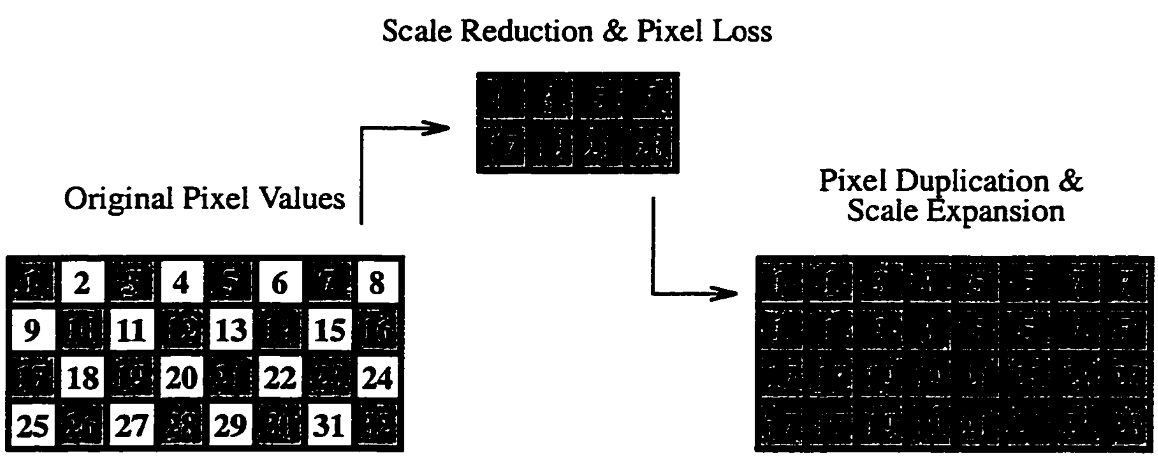
1.4 Geometric Transformation of Raster Data

The type of data structure used to represent spatial data is significant. A data structure is generally described as the logical organization of information. For cartographic information two data structures are common - vector and raster. A vector data structure describes space in terms of point, line and area features in a manner that is very similar to an analog map. The data can be stored with great precision and, when projected via the use of analytical point-to-point transformations, is generally able to retain all of the geographic information contained in the data set.

Data sets used for regional and global environmental studies are often derived from satellite imagery and these data are stored in a raster structure. The raster data structure is essentially a matrix with each element in the matrix being described as a pixel. A pixel is the "two-dimensional picture element that is the smallest nondivisible element of a digital image" (ANSI 1995).

Geometric transformation of a raster data structure is performed in a backwards fashion. Projection or reprojection, "of raster data sets involves calculation of a geometric transformation model, which is used to warp images in the original projection space to the selected projection space" (Steinwand, et al., 1995). First, an output matrix of empty cells is created in the desired geometry, then the original pixel values are used, together with a resampling method, to determine the output pixel value. The resampling methods most commonly used are nearest neighbor, bilinear interpolation, and cubic convolution. These will be explored further in a later chapter.

Figure 7. --Pixel Loss and Duplication During Transformation



The problems related to transforming raster data by map projections are two fold. First, there are the changes in area, angles or distances which are dependent on the characteristics of the chosen map projection. These alterations can be evaluated with traditional map projection distortion analysis. Secondly, there are pixel or grid cell value changes that occur. Pixel values may be duplicated in areas of scale enlargement and eliminated in areas of scale reduction. As seen in Figure 7 the original set of 32 pixels was reduced to eight as a result of scale reduction but during a subsequent projection transformation the scale expanded and the reduced set of pixels were duplicated to fill the available space. The idealized example in Figure 7 highlights the problem of permanent data loss. Pixels may also be shifted geometrically, or the value of the pixel may be altered. These changes in the data set represent a reduction in the quality of the data and they are generally not reversible. However, the errors are almost exclusively systematic, and theoretically can be measured using analytic methods. Pixel loss and pixel duplication changes are the focus of this analysis.

1.5 Spatial Data Quality and Map Projections

The quality of spatial, or geospatial information, is often defined by fitness for use based upon distinguishing characteristics of a data set and its individual elements and attributes. The U.S. federal government recognizes five characteristics of spatial data quality in the Spatial Data Transfer Standard (SDTS) also known as FIPS 173, while the International Cartographic Association (ICA) recognizes two additional characteristics. The basic five characteristics described in SDTS include lineage, positional accuracy, attribute accuracy, logical consistency and completeness; and the additional two recognized by the ICA are temporal information and semantic accuracy. SDTS is now being evaluated by the American National Standards Institute (ANSI) as a full ANSI standard (1997), and requires that a report on data quality accompany any transfer of spatial data. Furthermore, if a spatial variation of data quality is known, this information must also accompany the data transfer as a quality overlay. Such is invariably the case when map projection has impacted raster data.

STDS describes a quality overlay as a "collection of points, lines, and areas organized to represent quality information for another set of map information. An overlay describing lineage may be termed a source data index; a positional accuracy overlay may be termed a reliability diagram." (ANSI, Logical Specifications, 1997)

Of the five characteristics of spatial data quality recognized by the U.S. Federal Government, at least three need to be addressed in relation to map projection transformations. These include lineage, positional accuracy and attribute accuracy.

The first element of spatial data quality is its lineage, or history. "The lineage portion of a quality report must include a description of the source material from which the data were derived and the methods of derivation, including all transformations involved in producing the final digital files...." (ANSI 1997). The issue of preserving a record of map projection transformations is covered under the following section:

> *The lineage portion must describe the mathematical transformations of coordinates used in each step from the source material to the final product. The locations of any registration points for coordinate transformations must be given. The methods used to make coordinate transformations must be documented. To fulfill this standard, it is acceptable to make reference to separate documentation for the coordinate transformation algorithm used, but the specific parameters applied must be described for the particular case. Documentation of a transformation algorithm must include the nature of computational steps taken to avoid loss of digits through round off and must include a set of sample computations including*

numerical values of coefficients to confirm equivalence of transformations.

Full disclosure of all projection transformations and the parameters used would go a long way towards building an understanding of the alterations that may have occurred with a global data set.

Many researchers may use off-the-shelf software and so the following note is included in the ANSI draft Standard. "The documentation of a transformation algorithm must be available on request by a user obtaining digital data even if that user is not licensed to use the particular software" (ANSI, Logical Specifications, 1997). This may cause greater scrutiny of what previously were software 'black boxes' and provide detailed information to data producers and providers.

Positional accuracy is the second element of data quality. Positional accuracy must be reported not only in terms of a spatial registration standard but must also consider "the effects on the quality of all transformations performed on the data...." (Morrison 1995). Interestingly the SDTS defines a transformation as a "computational process of converting a position from one coordinate system to another". There is no specific mention of the map projection that may, or may not, be a part of converting between coordinate systems. During the resampling process pixels may shift up to one half the size of the pixel. As the pixel represents a specific area of the earth, one or more transformations may impact positional accuracy.

The SDTS positional accuracy report "must consider the quality of the final product after all transformations" (ANSI 1997). The report on positional accuracy only refers to the state of the data at the time of transfer so any incremental changes that may have occurred may not necessarily be reported. The methods available for describing positional accuracy are listed as: deductive estimate, internal evidence, and comparison to source or an independent source of higher authority. Positional accuracy is not specifically addressed in this research.

The third element of spatial data quality related to map projection transformations is attribute accuracy. During the projection of raster data sets various changes take place that may alter the pixel values including data loss, repositioning, and duplication. The standard states that: "Accuracy assessment for measures on a continuous scale must be performed using procedures similar to those used for positional accuracy (providing a numerical estimate of expected discrepancies)." In addition, "The report of a test of attribute accuracy must include the date of the test and the dates of the materials used.... Spatial variations in attribute accuracy may be reported in a quality overlay (ANSI 1997)".

The challenge with regards to spatial data quality is to determine how to characterize map projection distortion. Angular distortion, distance alterations and area changes are the result of map projection transformations and these are a reflection of the geometric properties of a map or data set. These changes are primarily the result of predictable and measurable analytical transformations. If the calculation of a land cover classes in a vector data set is required and an equal area map projection is employed, there will be no significant error in overall area. However, when an non-equal area projection is employed in the same case, this area measurement will be in error. Further, even if a raster land cover data set is projected into an equal area projection and a sum of the pixels is performed, the various classes may contain an error component due to pixel loss or duplication.

So when is map projection distortion just a geometric characteristic of the map space and when is it error? Where and when should levels of uncertainty be evaluated and applied to map projection related distortions? van der Wel et. al. (1994) include predictable map projection scale based changes in their discussion of positional accuracy. In terms of locating map projection distortion within the context of the five category data quality model used in the U.S., only positional accuracy comes close to describing the geometric changes.

Overall, the determination of the changes that occur as a result of map projection transformations on attribute accuracy are not well known, so it is unlikely that these changes are being recorded under the current standards. In addition to the quality statements, current spatial data standards require specific information regarding spatial referencing including map projections. Standards may vary in the manner in which spatial referencing information is recorded. There are significant differences between the following two U.S. standards. The Federal Geographic Data Committee content standards for digital geospatial metadata (FGDC 1994) include a section for spatial reference information. The standard provides a means for describing horizontal referencing that includes geographic, planar (including 21 map projections and an additional "other projection category"), and local referencing systems; as well as for the geodetic model used. By comparison the SDTS provides a three level approach of relating coordinates to latitude and longitude.

- Level 1 describes a relationship with latitude and longitude known through the use of one of the preferred external reference systems including Universal Transverse Mercator/Universal Polar Stereographic (UTM/UPS) Grid Systems, and State Plane Coordinate Systems (metric)
- Level 2 describes a relationship with latitude and longitude known through the specification of an external reference system and its projection and parameters

- Level 3 has an unspecified relationship with latitude and longitude (ANSI 1997)

1.6 Summary

In the current two dimensional computing environment, global data sets suffer from degradation due to map projection transformations. These data sets are degraded because it is impossible to flatten a large part of a spherical surface to a planar surface without introducing distortion and data alterations. Point-to-point vector based transformation map projections are almost completely invertible. Raster based data sets of large spatial extent are not invertible without error.

In a raster data set the basic map projection distortion of scale, angle and area changes are magnified by changes in pixel values, by the loss of pixels and the duplication of existing pixels. The magnitude and distribution of pixel based changes are not well known, nor are the effects of multiple re-projections of global raster data sets.

Further complicating this problem is a new generation of users that are either not well trained in the use of projections during a geo-science education, or have no background with spatial mapping concepts and map projections at all. Moreover, current software implementations force the use of a two dimensional model of global data and mandate the use of a limited set of implemented map projections.

Spatial data standards encourage full disclosure of projection transformations but since the nature and distribution of map distortion are poorly understood, it is unlikely that these changes are being recorded under the current standards.

The challenge then is to expand our understanding of map projection distortion to the realm of global raster data sets. New metrics developed for and presented in this research identify and quantify the amount of uncertainty that is being introduced into regional and global spatial databases via map projection transformations. Combining this information with existing guidelines will contribute to a rule base for preferred map projection use as a means of minimizing data quality loss.

CHAPTER 2

GLOBAL SPATIAL DATA AND WORLD MAP PROJECTIONS

The most common map projection used for the distribution of world data sets is - no projection at all, instead geographic coordinates are employed. Several world map projections are used however. The choice of a projection may be determined for an entire agency, such as the USGS EROS Data Center's choice of the Goode Homolosine. At other times the choice of a distribution format may reflect an individual's choice or limitations in current software implementations. The reasoning leading to the choice of a particular map projection is rarely provided in data set documentation, and in no case is information provided regarding the relationship between data quality and the projection selected.

The current status of map projection use for regional and world data sets is examined in this chapter through existing global data sets and through the implementation of world map projections by the market leaders in geographic information system (GIS) and image processing (IP) software. Several global mapping initiatives are also examined to underscore the importance and prevalence of global mapping.

2.1 Thematic Content and Data Structure of Global Data Sets

Thematically, global data ranges from characteristics of the Earth and the sea floor to the atmosphere. Anything regarding the Earth that can be mapped and that is useful to researchers is finding its way into the public domain, often in a variety of resolutions and formats. The data riches available today are remarkable compared to only a few years ago.

The rapid increase in the number of data sets available may explain the lack of any published inventory of the thematic content or structure of global data. To begin to address this lack of information a survey was sent to 46 attendees of the First Specialist Meeting for the National Center for Geographic Information and Analysis, Research Initiative 15: Multiple Roles for GIS in US Global Change Research. Of these attendees only a relatively small number were actually involved in data creation activities and overall there were 16 responses. The questionnaire and tabular summary of the responses are APPENDIX A. GLOBAL DATA SURVEY. Although the responses were small in number they confirm the information being obtained from other sources. First, there is some confusion regarding map projections in general. In two cases the researchers answering the survey provided significant detail regarding the thematic content and overall structure of data sets but failed to properly identify the map projection in use. Another response asked for further information regarding proper use of map projections for continents and countries. Geographic coordinates were the most common referencing system employed. Finally, when data was provided regarding the history of data sets, there is evidence of multiple reprojection of data and the combination of materials from a variety of source map projections.

The best means for locating global data sets are the on-line search engines designed for the task. Even the number of search engines for global data has exploded. At the USGS Global Land Information System (1998) the list of related sites has twenty five links to other search engines and information systems. The undertaking of a survey of existing and planned global data sets is a separate research topic in itself.

Only a small set of global data is presented here for testing and discussion purposes. Included is a limited set of global data that provides a glimpse at the amazing variety one may encounter. Variations occur in thematic content, data structure, metadata content, availability, data developers and providers, source materials, map projections, coordinate systems, history, resolution and distribution means and formats.

Thematic content in these data sets covers everything from the sea floor to the atmosphere and includes both natural and anthropogenic features. The theme of elevation for example can be accessed through global data sets with multiple resolutions. The coarsest resolution is represented by ETOPO5 (NGDC 1993) with data values at five minute intervals and at a higher resolution is GTOPO30 (USGS-EDC 1998a) at 30 arc second resolution. The theme of hydrography is exemplified by the HYDRO1k (USGS-EDC 1998b). A hydrographic data series was derived from the GTOPO30 that includes, aspect, flow direction, flow accumulations, slope, CTI (a wetness indicator), stream lines and drainage basins.

Surface cover features including vegetation, soils and general land cover are represented by the Global Land Characteristics data set, Global 1km AVHRR data, Matthews Global Vegetation and Zobler's World Soils. Two population related data sets include the Global Demography Project and the Nighttime Lights of the World. An increasing number of global data sets are being produced to describe the biogeochemical aspects of the Earth surface and near surface. The Carbon Dioxide Emission Estimate from Fossil Fuel Burning falls into this category. Ecosystems are represented by the Desertification Atlas and by the Major World Ecosystem Complexes data sets. All of the

Table 14: Selected Global Data Sets

Title & Originator	Theme	Geographic Extent & resolution	Spatial Data Structure	Spatial Reference/Map Projection
DCW (Vmap Level 0)[a] NIMA	Drainage, Airports Populated places, Shorelines, Transportation (roads, railways), Political Boundaries	Global 1:1 million	Vector	Geographic coordinates
ETOPO5[b] NGDC	Elevation	Global 5 minute	Raster	Geographic coordinates
GTOPO30[c] USGS EDC	Elevation	Global/ 30 arc second	Raster	Geographic coordinates
HYDRO1k[d] USGS EROS	Hydrologic (derived from GTOPO30)	Global by Continent 1km	Raster Vector	Lambert Azimuthal Equal Area
Global Hydrographic[e]	Hydrographic (18 files land to ice runoff)	Global 1 degree	Raster	Geographic Coordinates
Global Land Characteristics[f] USGS & UNL	Land Cover	Global 1km	Raster	Global - Goode Homolosine Regional - Lambert AEA
Global 1km AVHRR USGS EROS Data Center	AVHRR Composites and vegetation index	Global 1km	Raster	Goode Homolosine
Matthews Global Vegetation Data Set[g] E. Matthews	Vegetation (includes vegetation type, cultivation intensity, albedo)	Global 1 degree	Raster	Geographic
Zobler World Soils data set[h] L. Zobler	Soils (including soil classification, texture and slope)	Global 1 degree	Raster	Geographic
Global Demography Project[i] NCGIA	Population	Global 5 minute	Raster Vector	Geographic
Nighttime Lights of the World[j] C. D. Elvidge	Land Cover of Human Settlements	73N to 60S 1km	Raster	Goode Homolosine

Table 14: Selected Global Data Sets

Title & Originator	Theme	Geographic Extent & resolution	Spatial Data Structure	Spatial Reference/Map Projection
CO_2 Emission Estimates from Fossil Fuel Burning[k] ORNL-A. L. Brenkert	Atmospheric Emissions	Global 1 degree	Raster	Geographic
Desertification Atlas[l] UNEP/GRID	Terrestrial Ecosystems	Regional and Global to 72N and 57S	Raster	Van der Grinten
Major World Ecosystem Complexes[m] Olson et. al - ORNL	Terrestrial Ecosystems	Global 0.5 degree	Raster	Geographic

a. <URL http://164.214.2.59/publications/vmap0.html>
b. <URL http://edcwww.cr.usgs.gov/Webglis/glisbin/guide.pl/glis/hyper/guide/etopo5>
c. <URL http://edcwww.cr.usgs.gov/landdaac/gtopo30/gtopo30.html>
d. <URL http://edcwww.cr.usgs.gov/landdaac/gtopo30/hydro/index.html>
e. <URL http://edcwww.cr.usgs.gov/Webglis/glisbin/guide.pl/glis/hyper/guide/gghydro>
f. <URL http://edcwww.cr.usgs.gov/landdaac/glcc/glcc.html>
g. <URL http://edcwww.cr.usgs.gov/Webglis/glisbin/guide.pl/glis/hyper/guide/gvds>
h. <URL http://edcwww.cr.usgs.gov/Webglis/glisbin/guide.pl/glis/hyper/guide/world_soil>
i. <URL http://www.ciesin.org/datasets/gpw/globldem.doc.html>
j. <URL http://www.ngdc.noaa.gov/dmsp/dmsp.html>
k. <URL http://cdiac.esd.ornl.gov/ndps/ndp058a.html>
l. <URL http://www.unep.org/unep/eia/ein/grid/>
m. <URL http://edcwww.cr.usgs.gov/Webglis/glisbin/guide.pl/glis/hyper/guide/worldeco>

data sets listed in Table 14 are global in extent, or nearly so. The resolution varies from three arc seconds to one degree. This study is primarily interested in raster data but in some cases, such as in the HYDRO1k data set, both vector and raster layers are released as a set.

The spatial referencing system most commonly used for distribution of global data is geographic coordinates but a variety of map projections have been employed. The Goode Homolosine and the Lambert Azimuthal Equal Area projections were recommend by

Steinwand et. al. (1995) and are in use by the USGS EDC. Other projections include the stereographic projection for polar data, the Van der Grinten and even the Mercator projection. Some of these projection choices are significantly better than others. The use of equal area projections with minimal distortion characteristics is preferable for most global data sets. It is difficult to ascertain why choices such as the stereographic or the Mercator were made. Hints can sometimes be found in the choice of map projections implemented within commercial software and in the guidelines provided by software manufacturers.

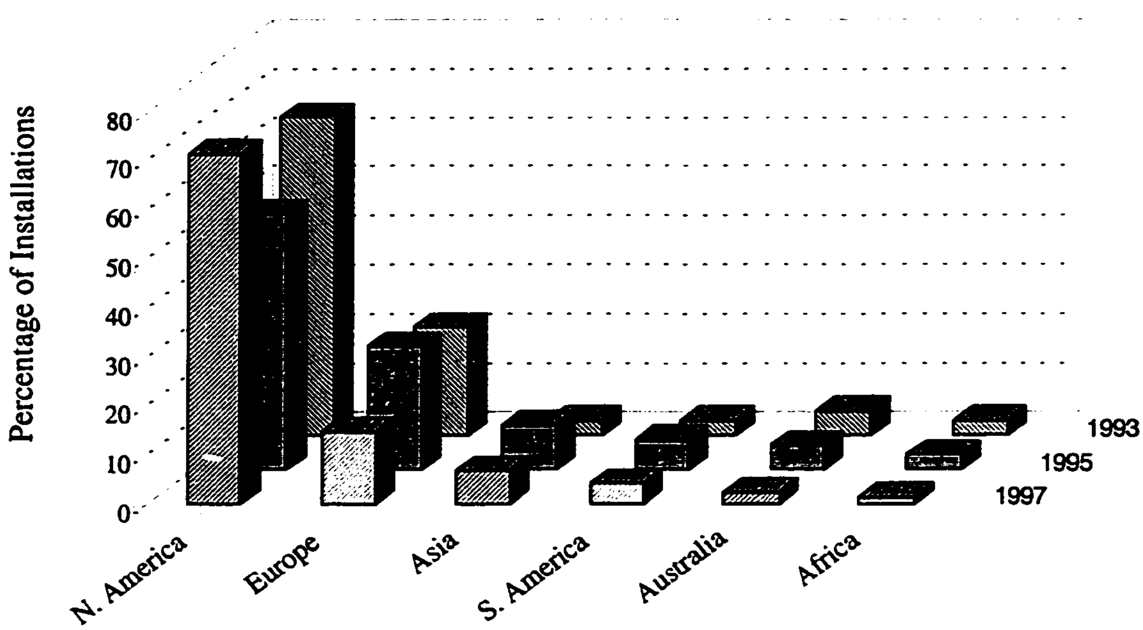
2.2 Implementation of Global Map Projections

Ideally the selection of a world map projection should be based upon sound guidelines and numerical evaluation to determine a minimal error solution. In reality, both the data distribution format and a user's software implementation may have undue influence on the choice of a projection. Four software packages by the three market leaders in GIS and image processing (IP) software packages are examined with regards to world map projections.

An international survey of GIS and Image Processing software packages by UNEP/ GRID Sioux Falls (1997) concluded that the software leaders in terms of licensed installations world wide are ESRI with 200,000, IDRISI with 20,000 and ERDAS with 15,000. The number of installations world-wide is increasing at a rapid pace. ESRI for example, reported 7,900 installations in 1993, 130,000 in 1995, and 200,000 in 1997. The overall percentages of software implementations by continent are relatively stable. (See

Figure 8) The UNEP/GRID survey does not indicate what percentage of these software implementations are oriented towards regional and global applications. The big three companies all produce software packages that are suitable for data sets of very large extent.

Figure 8. --Number of GIS and IP Installations by Continent



In terms of functionality, changing map projections was reported as one of the top five GIS functions and geometric rectification as one of the top five functions for IP. In IP software geometric rectification functions are closely related to the functions for changing map projections. Table 15 summarizes the implementation of map projections, data structures and resampling methodologies in the software of the three market leaders ESRI, IDRISI, and ERDAS.

All of the software packages in Table 15 provide map projection transformation capabilities. An exception is ESRI's ArcView software which requires an external module to actually convert data. Its inherent functionality is limited to an on-the-fly projection for display purposes only. The number of world projections vary by software implementation with the most extensive implementation in ESRI's ArcInfo software.

Table 15: Implementation of Map Projections by the GIS and IP Market Leaders

Software	Data Model	Resampling[a]	Hemispheric & World Map Projections	# Equal Area World Projections
ESRI ARCINFO 7.04	vector	N/A	**Equal Area** - Craster Parabolic, Cylindrical, Flat Polar Quartic, Mollweide, Sinusoidal, Lambert Azimuthal, Hammer-Aitoff, Eckert IV, EckertVI **Conformal** - Mercator, Stereographic **Equidistant** - Azimuthal Equidistant, Plate Carrée **Compromise** - Miller, Robinson	7
	raster	NN BL CC		
ARCVIEW 3.0a[b]	vector raster	N/A	**Equal Area** - Cylindrical, Sinusoidal, Lambert Azimuthal, Peters, Behrmann **Conformal** - Mercator, Stereographic **Equidistant** - Azimuthal Equidistant **Compromise** - Miller, Robinson	4
ERDAS IMAGINE 8.1	raster	NN BL CC	**Equal Area** - Sinusoidal, Lambert Azimuthal **Conformal** - Stereographic **Equidistant** - Azimuthal Equidistant, Plate Carrée **Compromise** - Miller, Van der Grinten	1
	vector	N/A		
Clark Labs IDRISI	raster	NN BL CC	**Equal Area** - Hammer-Aitoff, Lambert Azimuthal, **Conformal** - Mercator, Stereographic **Equidistant** - Plate Carrée	1

a. NN - nearest neighbor resampling, BL - bilinear interpolation, CC - cubic convolution.

b. ArcView does not actually convert data, it can only change the visualization of data input in decimal degrees.

Table 15 lists only those map projections appropriate for hemispheric or world data sets. ARCINFO has the most extensive implementation of map projections transformations with 16 possible projections for a hemisphere or the world. ARCVIEW has 12 available, IMAGINE has 7, and IDRISI only 5. If this set of projections is examined in light of what is appropriate for global mapping, particularly global change and environmental mapping that requires an equal area map projection, the count of projections diminishes rapidly. ARCINFO has 7, ARCVIEW has 4, and IMAGINE and IDRISI only have only one equal area world projection each.This is a highly restrictive set of choices for global mapping.

If this set of projections is compared with those projections recommended by Steinwand et. al. (1995) the picture is even bleaker. None of the recommended global equal area projections is implemented in any of the market leader software packages. Two of the recommended projections, the interrupted Mollweide and the Goode Homolosine are problematic because none of these software packages can explicitly handle the interruptions in the projections. ARCINFO can manipulate the individual projections that together comprise the interrupted Mollweide and the Goode Homolosine but they must be pasted together without an explicitly defined referencing system. The two other recommended world map projections, the Wagner IV and Wagner VII are not interrupted projections and could be implemented within the existing software data model.

2.3 Global Mapping Initiatives

Global mapping initiatives take on several forms and their objectives range from basic scientific inquiry in the International Geosphere Biosphere Program (IGBP) to data set coordination and distribution in the Global Map initiative. Intermediate between the two are the United Nations Environment Programme/Global Resource Information Database (UNEP/GRID) centers that focus upon data set development and the Committee on Earth Observation Satellites (CEOS) Task Team on Global Mapping which focuses upon improved use of map projections and the development of alternative referencing systems. These various global initiatives often work cooperatively to develop global data.

2.3.1 Global Map

Global Map is a data development and distribution effort and is defined in the Rules of International Steering Committee for Global Mapping as "a group of global geographic data sets of known and verified quality, with consistent specifications which will be open to the public. Global Map is considered a common asset of mankind, and will be distributed worldwide at marginal cost" (ISCGM 1996). The concept and the establishment of an international body to pursue this data base development effort was proposed by the Ministry of Construction of Japan in 1992. In 1994 the Geographical Survey Institute of Japan proposed the first draft Map Specifications (ISCGM 1998).

Global Map takes advantage of existing global data sets and aims to improve the reliability and accuracy of this data over time. The base data sets include GTOPO30, the Global Land Cover Characteristic Database and VMAP Level 0. From these three data sets a total of nine thematic layers will be made available (Table 16). The product is global in extent at a nominal scale of 1:1 million.

Table 16: Data Sources for Global Map

Source Data Sets	Thematic Content	Structure
GTOPO30[a] (USGS, EROS Data Center)	Elevation (DEM)	Raster
Global Land Cover Characteristics Database[b] (Joint - USGS, University of Nebraska, EC)	Land Cover Vegetation	Raster
VMAP Level 0[c] (NIMA)	Drainage Airports Populated places Shorelines Transportation (roads, railways) Political Boundaries	Vector

a. <URL http://edcwww.cr.usgs.gov/landdaac/gtopo30/gtopo30.html>

b. <URL http://edcwww.cr.usgs.gov/landdaac/glcc/glcc.html>

c. <URL http://164.214.2.59/publications/vmap0.html>

The Global Map data base as an integrated product is due for distribution in the year 2000 on CD-ROM and through internet access. Version 2 specification are currently available on-line (ISCGM 1996). The individual data sets are also currently available. The Global Map will employ a variably sized tiling system and geographic coordinates. Tile size varies from 5° by 5° close to the equator, to 5° by 90° close to the poles.This is the same tiling system currently employed for the VMAP Level 0 product.

The Global Map concept is intended to mimic the infrastructure of a national mapping agency at the Global level. The Global Map participants believe that a global agency can provide an infrastructure to assist with implementing global agreements and conventions for environmental protection, the mitigation of national disasters, and encourage economic growth (ISCGM 1998).

2.3.2 International Geosphere-Biosphere Program

The International Geosphere-Biosphere Programme (IGBP) is focused on acquiring basic scientific knowledge about the interactive processes of biology and chemistry of the Earth as they relate to Global Change. It is an interdisciplinary scientific activity established and sponsored by the International Council of Scientific Unions (ICSU) in 1986, and the IGBP Secretariat was established at the Royal Swedish Academy of Sciences in 1987. The goal of the programme is:

> *To describe and understand the interactive physical, chemical and biological processes that regulate the total Earth system, the unique environment that it provides for life, the changes that are occurring in this system, and the manner in which they are influenced by human actions.*
> (IGBP 1998)

Table 17: IGBP Program Elements

Research Question	Core Projects
How is the chemistry of the global atmosphere regulated and what is the role of biological processes in producing and consuming trace gases?	International Global Atmospheric Chemistry Project (IGAC)

Table 17: IGBP Program Elements

Research Question	Core Projects
How will global changes affect terrestrial ecosystems?	Global Change and Terrestrial Ecosystems (GCTE) Land-Use and Land-Cover Change (LUCC)
How does vegetation interact with physical processes of the hydrological cycle?	Biospheric Aspects of the Hydrological Cycle (BAHC)
How will changes in land-use, sea level and climate alter coastal ecosystems, and what are the wider consequences?	Land-Ocean Interactions in the Coastal Zone (LOICZ)
How do ocean biogeochemical processes influence and respond to climate change?	Joint Global Ocean Flux Study (JGOFS) Global Ocean Ecosystem Dynamics (GLOBEC) project
What significant climate and environmental changes have occurred in the past and what were their causes?	Past Global Changes (PAGES).
Framework Activities	IGBP Data and Information System (IGBP-DIS) Global Analysis, Interpretation and Modeling (GAIM) Global Change System for Analysis, Research and Training (START)

During the first five years of the IGBP detailed plans were developed for its eleven components. The expression of these components consists of eight broadly discipline oriented projects. The topics covered include atmospheric science, terrestrial ecology, oceanography, hydrology and links between the natural and the social sciences. In addition, there are three framework activities on data, modeling, and regional research. The basic research questions and core projects are listed in Table 17. Although focused mainly on basic scientific inquiry the IGBP has been instrumental in data development through the core projects

Currently there are several data sets available produced by the IGBP or in collaborative efforts with other organizations such as the USGS and CEOS. Map projection use by the IGBP varies with some data available in latitude/longitude coordinates. The GLOBE30 topographic data web page provides graphics in latitude/longitude coordinates, and the Goode Homolosine and the Winkle Tripel projections. They do not indicate current access or metadata information on the GLOBE30 home page. No overall summary appears to be available for an overview of data formats for IGBP data.

2.3.3 UNEP Global Resources Information Database

The United Nations Environment Programme/Global Resource Information Database (UNEP/GRID) is dedicated to making environmental data and information readily accessible to environmental analysts. It consists of a network of cooperating centers (see Figure 9) whose mission is to provide timely and reliable geo-referenced environmental data and information to help address environmental issues at global, regional and national levels. (UNEP/GRID 1997). The long term objectives of GRID are:

1. to enhance availability and open exchange of global and regional environmental geo-referenced data sets,
2. to provide UN and intergovernmental bodies with access to improved environmental data management technologies, and
3. to enable all countries in the world to make use of GRID-compatible technology for national environmental assessment and management. (UNEP/GRID 1997)

Figure 9. --World-wide UNEP/GRID Centers

Source:<URL:http://grid2.cr.usgs.gov/adrs.html>

GRID maintains a distributed global archive of geo-referenced and tabular data and provides both data and support to GIS users world-wide. The GRID data may be regional or global and themes include: population, topography, climate, soils, vegetation, biodiversity, protected area and wilderness.

Each GRID center has a distinct regional and topical focus. UNEP/GRID Sioux Falls, for example, cites its specific mission being, "To assist UNEP and its partners by contributing environmental data and information, as well as methodological techniques for handling such data, to enhance the scientific basis for decision making and help advance sustainable development initiatives" (UNEP/GRID Sioux Falls 1998). In addition to global data sets of vegetation, soils, climate, topography, and population, the Sioux Falls site also has regional data in the form of an Africa Population Distribution Database and an Asian Population Database.

The use of map projections at UNEP/GRID Sioux Falls is similar to that found overall in regional and global data sets. The most frequent distribution format is latitude and longitude. Global population is also being released as dBase V files and at least one data set is being released in the Mollweide projection.

2.3.4 CEOS Task Team for Global Mapping

The Committee on Earth Observation Satellites (CEOS) is an international organization composed of representatives of nearly all major space organization (NASA, NOAA, ESA, RSA, NASDA) and major international and scientific programs (IGBP, FAO, GCOS, WCRP, etc.) are CEOS affiliates. The Task Team for Global Mapping is a special task force within CEOS and endeavors to:

- investigate the needs and directions for modern global mapping
- facilitate the harmonization and standardization of global mapping technology
- chart a path for improved spatial and temporal global mapping within the next decade.

The Task Team for Global Mapping is focusing on the need to be able to manage and distribute global digital data with particular attention on the problem of map projections, and avoiding them whenever possible. They recognize that now is a period of transition in terms of global digital mapping. The two dimensional flat map projection paradigm served well in the past but the current challenges "have arisen as a result of the use of an inappropriate traditional paradigm within the new digital media" (CEOS 1996).

The Task Team for Global Mapping prefers to develop alternative paradigms including, an alternative mapping schema based upon spherical tessellations of the globe, and on-demand mapping schemes that will "allow data to be retained and distributed within their native spatial and temporal domains" (CEOS 1996). Still they do recognize that map projections are currently in use and will continue to be so. Therefore they intend to provide advice and assistance in the following areas:

- standardization of algorithms for map projection transforms
- increased awareness and standardization in the use of various datum standards internationally
- advice regarding the appropriateness of different projections for different applications and regions of interest
- advice and assistance in the measurement and specification of angular, areal, and spatial distortions within map projections
- recommendations concerning requirements for complete and efficient descriptions of projections and grids within data descriptors (e.g. within headers, metadata, etc.)

Furthermore they recognize that the issue of resampling needs attention with regards to standardization of algorithms, recommendations for appropriate use of methods as well as the effects of resampling on the statistical characteristics of data.

2.4 Summary

The range of map projections currently in use by the global data community is very small. Data producers most frequently use geographic coordinates when distributing data. Projections employed include at the least, the Goode Homolosine, Van Der Grinten, Mollweide, Lambert Azimuthal Equal Area, Stereographic and the Mercator. The choice of a map projection or coordinate system should be based upon sound guidelines and numerical analysis.

The study by Steinwand et. al. (1995) provides specific minimal error choices for regional and world map projections but these are implemented primarily by the USGS EDC who initiated the study. In reality the choice of a map projection appears to be driven in part by a deficit in software implementation of map projections. Only the projection recommended for regional data sets, the Lambert Azimuthal Equal Area is implemented in software packages by all three software market leaders. None of the recommended world map projections are implemented by any of these companies. Furthermore, none of the three software companies have incorporated measurement of map projection distortion in their software.

The various world mapping initiatives cover a broad range of activities from basic research to standards development and data set production. The activities of these initiatives are marked by a high degree of cooperation and joint efforts towards data development. Along with the software vendors, these global mapping activities greatly influence the choice of map projections in use by researchers through distribution formats and through standards activities. Of particular interest in the CEOS Task Team on Global Mapping which addresses the issue of map projection use directly and plans to release a book on global mapping and data referencing.

How can a data producer or data user be expected to make a reasoned best choice of projection? Cartographically literate researchers are aware, at least in a general sense that the choice of a projection is significant. Anecdotal evidence indicate that 'map projection anxiety' is common as researchers struggle with the selection of a projection but are not confident of their decision due to a lack of objective evidence. For less cartographically trained individuals or organizations, confusion reigns. The ARCVIEW discussion group for example exhibits several questions regarding coordinate systems, map projections, and datum transformations weekly. This research will help to address these issues of map projection selection guidelines and implementation of global map projections.

CHAPTER 3

MEASUREMENT AND ANALYSIS OF MAP PROJECTION DISTORTION

The cartographic treatment of map projection distortion includes both distortion analysis and the map projection selection processes. A significant component of the projection selection process is the display of map projection distortion, which will be examined separately in Chapter 4. The analysis of map projection distortion includes a variety of methods including analysis by Tissot's theorem, shape measures, statistical analysis, minimal error approaches and the recently developed methods of grid squares and checkerboards.

Additional background included here are a discussion of alternatives to the use of planar map projections through procedures that are adapted to the spherical domain, hierarchal referencing systems, and tessellations of geographic space. Finally, the special problems associated with the resampling of raster data are described and the three most common resampling methodologies are described.

3.1 Choosing a Map Projection

According to Jankowski and Nyerges, when choosing a map projection, "few people, even few cartographers, commonly know which projection is good for what purpose and the trade-offs involved" (1989). Snyder sums up the traditional approach to the choice of a map projection as being based upon four primary factors: the purpose of the map, the shape and size of the region being mapped, and whether the particular map is part of an established series or is to stand alone (1994). In addition, the choice of projection for a world map has subjective criteria such as "the relative importance of land versus water portions, of polar versus equatorial region, and of straight parallels versus curved parallels" (Steinwand, et. al., 1995).

Maling (1992) takes a slightly different tack, beginning his discussion of choosing a suitable map projection by discussing the subject of map projections in relation to Geographic Information Systems. Three areas are discussed; determining the best means for presenting results; taking into consideration the need to combine data from different map projections and coordinate systems into a common system; and the use of quantitative measurements to check distortion and possibly to apply corrections. The latter is a natural addition to GIS capabilities but it has been completely overlooked in system development to date. Despite the fact that a computer is able to reference spatial data with great precision, significant discrepancies between data sets result from data that was originally created by different processes and mapped on different projections.

Maling (1992) continues his discussion of choosing a suitable map projection with the statement that:

> *The amount of distortion which is likely to be encountered in a conventional map depends upon the location, size and shape of the area to be mapped.... The three variables - location, size and shape - usually determine the choice of origin, aspect and class of a suitable projection.*

There is ample literature to assist the researcher in choosing a map projection based upon these traditional criteria developed primarily during the era of manual cartography. For the most part these same recommendations are being recycled for digital data. Map Projections (ESRI 1994), presents Snyder's 1987 list of recommended projections with only minor additions and deletions. These changes are based upon the availability of projections in the ESRI ArcInfo software and not on a new understanding of projections in a digital environment. The challenge then is to expand this literature to the realm of digital data sets. Maling (1991; 1992) touched on this problem in a discussion of changes in the cartographic meaning of zero dimension in a digital environment and in his discussion of a GIS framework. A GIS framework is described as a common map projection framework to which all data layers must relate. Unfortunately, he did not go on to explain how the selection of a common map projection may be affected, if at all, by a variety of source map projections.

3.1.1 Traditional Selection Methods

An example of the traditional method for choosing a map projection is described by Bugayevskiy and Snyder as a process of locating the best solution to several groups of factors. The factors are organized in three groups.

Group 1 factors include the region being mapped, including the geographical position, the dimensions, and the shape of the region as well as the adjacent regions that may also need to be represented.

Group 2 factors are related to the characterization of the map as well as its use. These include: the purpose of the map and any special features such as scale, contents, uses (such as cartometric or navigation), accuracy requirements, the display of the final product (wall map, atlas, temporal map), whether the map is a single product or is part of a series, relative placement of geographic regions to one another and assorted others.

Group 3 factors characterize the map projection to be used. They includes the type of distortion, requirements for minimizing distortion and the acceptable levels of various types and distributions of distortion, appearance of the graticule, and manner of representing the poles. Finally, the general visual perception of the projection including the relative spherical appearance may also be factors here.

These three groups of factors are basically a restatement and expansion of the basic factors listed by Snyder in his paper with Steinwand and Hutchinson (Steinwand et. al. 1995). These guidelines in turn were actually stated and implemented in a prior work by Snyder alone. Snyder (1987) provides an outline that consists of several levels, each narrowing the choice of projection (Figure 10). At the highest level the region to be mapped is selected; world, hemisphere and continent, ocean or smaller region. Then within each section various map projection characteristics are considered; equal area, conformal etc., finally the shape and location are considered in some cases. This approach leans toward an 'expert system' with its branched, knowledge based structure.

Figure 10. -- Snyder's Suggested Projections (partial)

Region Mapped

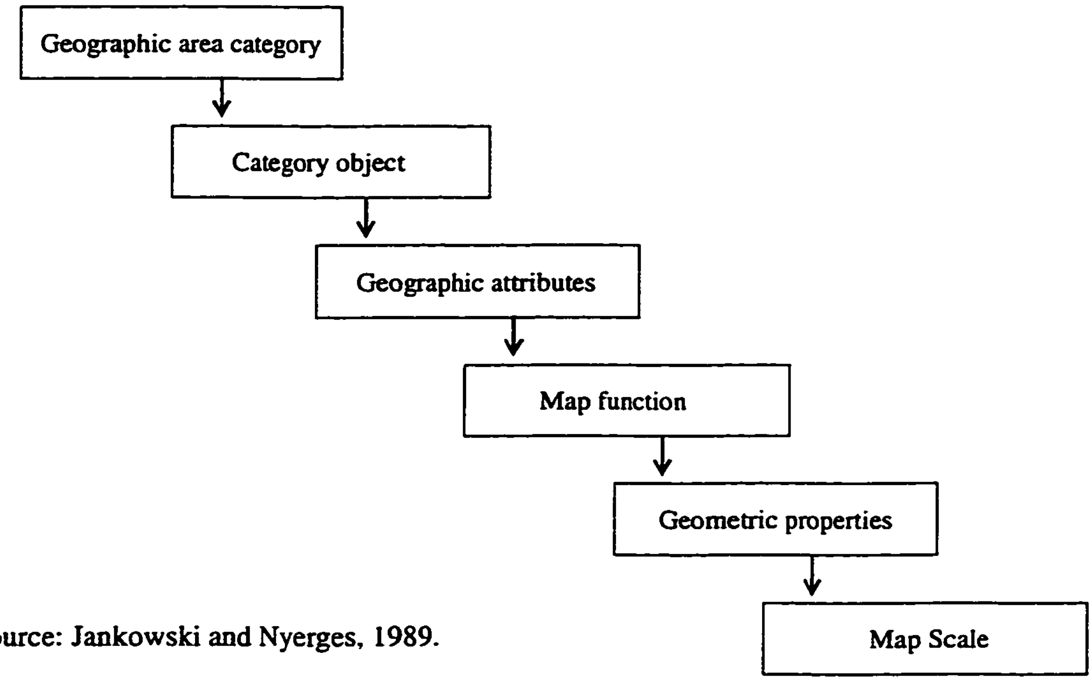
1. **World**
   - A. **Conformal** (gross area distortion)
     - (1) Constant scale along Equator
       - Mercator
     - (2) Constant scale along meridian
       - Transverse Mercator
     - (3) Constant scale along oblique great circle
       - Oblique Mercator
     - (4) Entire Earth shown
       - Lagrange
       - August
       - Eisenlohr
   - B. **Equal-Area**
     - (1) Standard without interruption
       - Hammer
       - Mollweide
       - Eckert IV or VI
       - McBryde-Thomas variation
       - Boggs Eumorphic
       - Sinusoidal
       - misc. pseudocylindricals
     - (2) Interrupted for land or ocean
       - Any of the above except Hammer
       - Goode Homolosine
     - (3) Oblique aspect to group continents
       - Briesemeister
       - Oblique Mollweide
   - etc.

Source: Snyder, 1987, pp. 34.

3.1.2 Automated Selection Methods

An attempt to create an automated knowledge based system is that by Jankoswski and Nyerges (1987). They use a top down hierarchy as illustrated in Figure 11. The

Figure 11. -- Hierarchy of information in the projection selection process



Source: Jankowski and Nyerges, 1989.

categories of geographic area may be any from world, continent or ocean to region. Geographic attributes may include location and directional extent. Geometric attributes refer to the distortion on the final projection whether in size, direction, distance, scale. The categories listed here are a restatement of the same basic guidelines that are used within a traditional map projection selection analysis.

Rather than using an expert system approach, Bugayevskiy and Snyder (1995) tackle the problem of automated selection by means of an equation consisting of a number of terms that relate to the factors described previously. All of the factors that are important for a given application are identified, sorted in order of importance and a weighting is assigned to each.

Figure 12. -- Factors for Computerized Selection

$$\varepsilon_1 = a/b - 1$$
$$\varepsilon_2 = ab - 1$$
$$\varepsilon_3 = \{[(a-1)^2 + (b-1)^2]/2\}^{1/2}$$
$$\varepsilon_4 = k_{mid}/k_{midmax} - 1$$
$$\varepsilon_5 = (\Delta d)/(\Delta d_{max}) - 1$$
$$\varepsilon_6 = (\Delta k_M)/(\Delta k_{Mmax}) - 1$$
$$\varepsilon_7 = (\Delta k_p)/(\Delta k_{pmax}) - 1$$
$$\varepsilon_8 = C_T/C_{Tmax} - 1$$
$$\varepsilon_9 = (\Delta \varepsilon)/(\Delta \varepsilon_{max}) - 1$$

and others.

Where a and b are maximum and minimum local scale factors, k_{mid} is the mean curvature on the map of a parallel or meridian, Δd characterizes the derivation of a rhumb line from a straight line, Δk_M is the difference between the curvature of a meridian at a specific point on a projection and its true value, Δk_P is the same for a parallel, C_T characterizes the degree of conformality, and $\Delta\varepsilon$ is the difference between the declination of an angle at a particular point on a projection and its true value. Each of these terms is evaluated via a sum of squares of the variable with appropriate weighting for each.

Unfortunately neither of these approaches have been implemented in commercial software. An analysis, even if limited only to traditional map projection distortion measurements, would go a long way towards supporting GIS users who are inexperienced and insufficiently trained in distortion analysis and map projection selection.

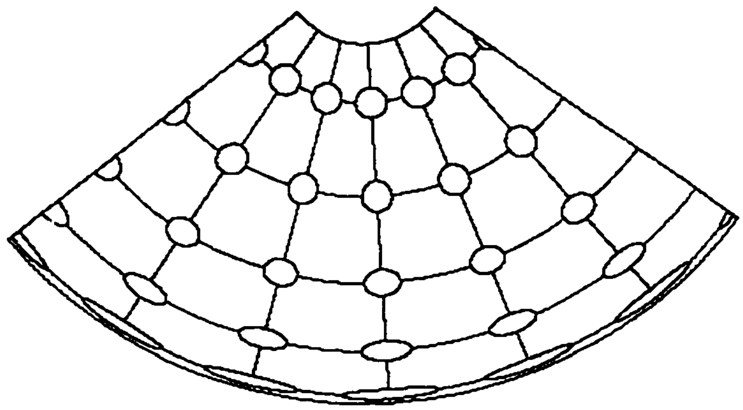
3.2 Map Projection Distortion Analysis Methodologies

3.2.1 Tissot's Theorem

Nicholas August Tissot, a French mathematician was a giant among the many contributors of the application of mathematical principals to map projection science during the 19th century. In 1881 he published his *Memoire sur la representation des surfaces et les projections des cartes geographiques.* In it Tissot "proposed an analysis of distortion that has had a major impact on the work of many twentieth-century writers on map projections (Snyder 1993). Laskowski (1989) commented that for "over 100 years the Indicatrix proved very useful in revealing the inherent distortion characteristics of the projection transformation." It is still an effective point based means for evaluation and display of map projection distortion.

The visual expression of Tissot's theorem is the Indicatrix, a cartographic symbol that consists of greatly enlarged representations of small circles on the Earth's surface being projected to ellipses on the planar map (Figure 13). The axes of Tissot's ellipse of distortion represent the two principal directions. The maximum and minimum particular scales, usually referred to as *a* and *b*, occur in these directions. These particular scales may be evaluated if the scale factors, *h* and *k* along the meridian and parallel at that point and

Figure 13. --Albert Equal Area Conic with Tissot's Indicatrix



the angle made by the intersection of the meridian and parallel on the map at this point are known. Further, the maximum angular deformation ω, and the measure of area distortion, *p* can be determined (Maling 1992).

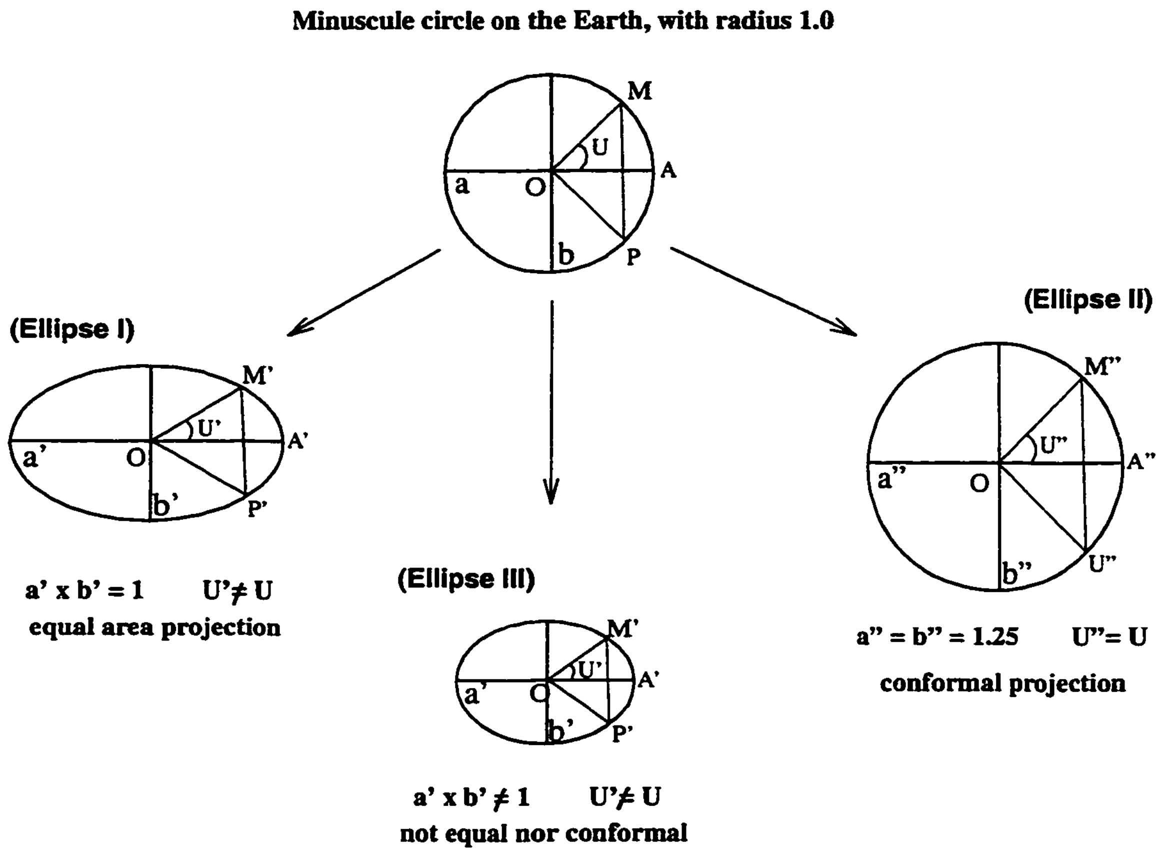
Maling (1992), describes Tissot's Theorem and the principal directions:

> *Whatever the system of projection there are, at every point on one of the surfaces and, if angles are not preserved, there are only two of them, such that the directions which correspond to them on the one surface also intersect on another at right angles.*

Tissot's Theorem describes two orthogonal directions associated with nearly each point on the spheroidal surface which also exist at right angles on a flat map. These directions are called the *principal directions* and serve as major and minor axes in Tissot's graphical device known the Indicatrix, or Ellipse of Distortion (Robinson et. al. 1995, and Snyder 1987).

The Indicatrix is the projection of an infinitely small circle on the surface of the earth to an infinitely small ellipse on the map projection plane (Figure 14). This ellipse describes the map projection characteristics locally near the infinitely small ellipse. In actual distortion displays the ellipse is enlarged for visibility, but the definition of an infinitely small ellipse allows the two surfaces to be related by a 2-dimensional affine transformation; and hence the rules of projective Euclidean geometry apply.

Figure 14. --Tissot's Ellipse of Distortion



Tissot's uses the infinitely small circle on the Earth to represent the distortion properties at that point on a corresponding, projected map. See Figure 14. The unit circle has radius 1.0, so a = b = 1.0. The axes marked as a and b represent the principal directions, the two directions that occur on both the Earth and on the projected map at right angles.

The result of first transformation of the unit circle is shown in Figure 14, Ellipse I. The axes of the ellipse have changed such that a' x b' = 1.0. The area represented by the original circle on the Earth and the projected ellipse on the map are the same. This defines the characteristic of the map projection as equal area. To calculate the amount of angular distortion that has occurred the point of maximum distortion is identified as *M* and the angles as *MOA* = *U* and *M'OA'* = *U'*. Maximum angular distortion is equal to *U* - *U'*.

In Figure 14 Ellipse II, the original unit circle has been projected and the angle *U* = *U'* and a" = b" = 1.25. The scale has increased proportionally in both principal directions, a and b, and therefore the angle *U* remains the same. This identifies the projection as having the property of conformality. In the third case as seen in Ellipse III, a' x b' does not equal 1 as in the equal area case nor does the angle U' equal U as in the conformation case. This projection does not strictly maintain either property but such a projection may be used in developing an overall minimum error projection.

Piotr Laskowski (1989), describes an algebraic approach for computing parameters of the Tissot Indicatrix. Based upon the modern theory of the algebraic eigenvalue problem he uses the Singular Value Decomposition (SVD) to determine the parameters of the Indicatrix.

Laskowski's presentation of distortion theory is quite extensive. Divided into two parts, the first presents the traditional concept of the Indicatrix from the non-traditional perspective, as a "study of the variation of the symmetric, positive definite quadratic form." The second part of his presentation describes his new approach that is independent of the traditional Tissot's distortion theory. "It is based upon a single theorem of linear algebra, with some preliminary uses of differential geometry." Laskowski reports that his results are numerically identical to the results of the Tissot's traditional method.

The use of bidimensional regression was examined by Tobler in a 1977 discussion paper and republished in Tobler 1994. Bidimensional regression is an extension of regression analysis where both the dependent and independent variables are two-dimensional. It is useful in any of a number of situations where the resemblance of two plane figures are to be evaluated.

Tobler describes the curvilinear case of this method where:

> *...the regression coefficients constitute a spatially varying, but coordinate invariant, second-order tensor field defined by the matrix of partial derivatives of the transformation.*

He goes on to describe this method as essentially equivalent to Tissot's Indicatrix. Similar to Laskowski's method, the eigenvectors and eigenvalues of the coefficient matrix are important to analyzing distortion. A system of coordinates that are orthogonal in both the original and compared image is developed based upon Tissot's Theorem.

Tobler's is the only example of attempts to apply Tissot's concept to evaluate differences between two different map projections. By definition, Tissot's Theorem refers to transformations between the globe and the projection plane. Later, Steinwand, Hutchinson, and Snyder specifically mention the development of alternative methods for examining distortion between two projected maps. They base the need for new metrics, at least partially on the premise that Tissot's Theorem was developed only to compare latitude and longitude coordinates to projected map coordinates. This use by Tobler seems more consistent with the original derivation of Tissot's Theorem which specifies the use of an infinitesimally small circle on the Earth. This allows a comparison treatment of the Earth coordinates with map coordinates by assuming the comparison of a pair of two dimensional planar surfaces.

3.2.2 Selection by Minimal Error

One of the leading mathematicians of all time, Carl Friedrick Gauss (1777 - 1855), figures prominently in the history of map projections and map projection distortion theory. He engaged in an extensive study of the transformation of one surface to another (Snyder 1993). At the age of 18 Gauss invented the concept of least squares which was to be used extensively in the development and evaluation of map projections.

Gauss' distortion theory is explored as applied to cartography in a general case by Richardus and Adler (1972), and by Canters and Decleir (1989) for the spherical assumption. The Gaussian fundamental quantities were used exclusively by Pearson (1990) without further reference to Tissot's Theorem. Maling (1992) provided a concise development of the concept, more suitable for a beginner to distortion theory. He describes an evaluation of scale along the meridian, h, along the parallel, k, and along any arc. These scales are referred to as *particular scales* and he defines them as:

the relation between an infinitesimal linear distance in any direction at any *point on a map projection and the corresponding linear distance of the globe.*

The concept of least squares was invented by Gauss and developed by Gauss and Legendre in the first two decades of the 19th century. Snyder (1994) explains the concept in his article exploring the practicality of minimal error map projections.

The concept of minimum error is closely tied to that of least squares,.... This principle states that the best value for a quantity, given a set of measurements of that quantity, is the value for which the sum of the squares of deviations of these measurements from this value is least. For a minimum-error map projections, the sum of the squares of the deviations of all the actual scale values from the state scale is made a minimum according to a prescribed definition.

Numerous minimum error world map projections have been developed since the appearance of the concept. Eisenlohr presented a minimum error conformal map projection in 1870. Behrman presented a cylindrical equal area map projection in 1910. Snyder (1994) reports numerous attempts to develop world maps that are neither conformal nor equal area since the 1980's.

According to Snyder, there is a greater justification for the development of regional minimum error maps than for world maps. This is based upon Chebyshev's theory from 1856. He theorized that a "conformal mapped region bounded by a line of constant scale has the least overall error, or is minimal error (Snyder 1994)." Chebyshev's theorem was proven later and when applied to an area that is circular the minimal error choice of projection is an azimuthal projection where distortion increases radially from the center of the projection.

This principal has also been applied to equal area projections, although there is no analytical proof for the equal area case. Snyder (1994) applied it to the development of oval and rectangular regions as well as to Alaska through the formulas developed by John Dyer. Snyder also applied it to the choice of equal area continental land-use maps for use by the U.S. Geological Survey's EROS Data Center.

The two measurements used for the analysis include:

"(1) the maximum and minimum values of the scale factor over a map, and

(2) the root-mean-square error or RMSE of the scale factor (Snyder 1994)."

Specifically Snyder's method is to take scale errors from small and equal portions of the map, square them, add up the squares, divide by the number of measurements, and find the square root of the quotient for the root-mean-square error for the scale factors on a map (Snyder 1994).

3.2.3 Grid Squares

The development of the grid squares method was prompted by a need to understand the distortion introduced when reprojecting raster data sets of large areal extent (Steinwand, et. al. 1995). They report Tissot's Indicatrix as being limited to providing a means for calculating the distortion introduced when transforming features from the sphere to a particular projection. Global Research studies often use digital data that has already been transformed by a map projection. Data that has already been transformed must be projected back to geographic coordinates, and then into the chosen map projection. The reverse may also occur if a region is extracted from a global data set and reprocessed for optimal distortion characteristics.

A raster data structure is composed of discrete cell or pixel values which represent areas. During the transformation between geometries, area compression may result in a reduction of image resolution where fewer values, or pixels, represent the same geographic area. Conversely an enlargement in geographic area means that a given number of discrete values will now represent a larger area. Resolution does not increase along with the increasing local scale. Of particular importance is the fact that when data values are lost, as in the case of area reduction, the data is gone and it cannot be recovered, see Figure 7.

The grid squares themselves can be used to quantify the scale changes resulting in area distortion although it does not have the mathematical basis that the Tissot method carries for evaluating angular distortion. The grid squares method consists of a vector based structure of three by three grids that are located at the intersections of the graticule of a map in the same manner at Tissot's indicatrix (Figure 23). Once transformed to a new map projection the area of the grid squares can be compared between the two maps.

3.2.4 Checkerboard Method

Steinwand, et. al. (1995) developed a method for quantifying data degradation due to map projection and resampling transformations. A grid of alternating 10 X 10 black-and-white squares was created in the Plate Carrée and projected to the Lambert Azimuthal Equal-Area projection. The Plate Carrée projection is similar to recording data as geographic coordinates. Where x, y are determined by:

$$x = R(\lambda - \lambda_o)$$
$$y = R\phi$$
(Equation 6)

If R, the radius at the scale of the map is set to 1, and λ_o, the longitude east of Greenwich is set to 0, then the resulting projection is defined by:

$$x = \lambda$$
$$y = \phi$$
(Equation 7)

and the Plate Carrée projection is then essentially equivalent to the geographic coordinates of latitude and longitude when treated as rectangular coordinates. In the current two dimensional data structure implementation in GIS software the visual appearance of geographic coordinates and the Plate Carrée projection will be identical. The Plate Carrée can be used to model data sets that contain data values by degrees of latitude and longitude as they are transformed to a map projection. The majority of global spatial data sets are distributed in geographic coordinates.

The checkerboard image squares were counted as a means of calculating changes due to the image warping of the raster structure during the projection transformation. These results were then compared to the distortion evaluated from the grid squares method which evaluates area changes due to the projection transformation alone.

Table 18: Area Percent Change by Latitude and Longitude

Latitude/Longitude	Percent of Original Area Used	
	Projection change alone	Projection change and discrete image pixels
20°N 160°W	94%	81%

Table 18: Area Percent Change by Latitude and Longitude

Latitude/Longitude	Percent of Original Area Used	
	Projection change alone	Projection change and discrete image pixels
40°N 160°W	77%	66%
60°N 100°W	50%	41%
60°N 60°W	50%	41%
80°N 60°W	17%	Not countable

Table 18 contains the results published in the Steinwand, et. al. (1995) article. The results of this analysis refer to a transformation from a Plate Carrée projection to a Lambert Azimuthal projection. What do they actually mean in terms of data quality, data loss, or pixel duplication? These numbers refer only to a single state change. Raster data stored in a Plate Carrée or geographic latitude longitude coordinates has already been drastically altered by a projection transformation. Meaningful results of area and data changes should be evaluated with regards to an initial or early state of the development of the data set.

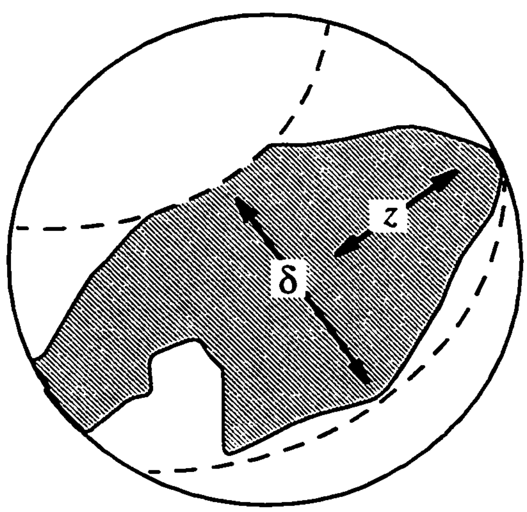
3.2.5 Shape Measures

Overwhelmingly, the most often used shape measure is the concept of maximum angular distortion previously discussed in relation to Tissot's Theorem. This is often mapped onto a projected graticule as isolines or shown as a combination of graticule and basic outline map as Tissot's Indicatrix. Other shape measures that have been used include those associated with locating a minimum error map projection, or in the case of Young's Rule, a shape measure which guides our choice of map projection class.

In conjunction with a statistical comparison of the distortion on equal area map projections, Kimerling, Overton, and White (1995) chose to use a normalized shape measure. They describe their measure as "the normalized magnitude of a vector formed by the lengths of the orthogonal *a* and *b* axes in the Tissot Indicatrix," for equal area projections.

When a choice is to be made between the classes of map projections - cylindrical, conical, azimuthal, Young's Rule can be applied. The principle is based upon the shape of an area to be mapped. It was first stated by Young in 1920 and was independently derived and expanded by Ginzburg and Salmanova in 1957 (Maling, 1992).

Figure 15. –Young's Rule



The imaginary country in Figure 15 illustrates the rule. The country to be mapped has a maximum radial distance, z from the boundary of the country to its center. The country is also bounded by arcs of small circles that may be considered as parallels of latitude, δ.

Young reported that for very compact or nearly circular countries where $z/\delta < 1.41$, an azimuthal projection should be used. The azimuthal projections have a distortion pattern where angular and area distortion increases in all directions from the center of the projection outward. At a value greater that this critical value he recommended the use of a conical or cylindrical projection. Ginzburg and Salmanova extended this work through a study of particular scale in a range of $0^{\circ} < z < 25^{\circ}$ and $0^{\circ} < \delta < 35^{\circ}$. Through this they were able to recommend cylindrical projections by characteristics, $z/\delta = 1.41$ - conformal projections were to be used, for $z/\delta = 1.73$, equidistant, and $z/\delta = 2.00$ equal-area projections were to be employed. More recently in a study by Kimerling, Overton, and White (1995), Young's Rule guided the selection of the Lambert azimuthal equal area projection for the truncated icosahedron hexagon and pentagon faces for the global tessellation under study.

Tissot's theorem has provided an excellent means for determining map projection distortion characteristics. Minimal error techniques have also been successfully applied. The primary drawbacks to these methods are that they are applied to the earth-to-projection transformation only. The have not been applied to the type of transformations that may be occurring between two projections. Further, these techniques are measuring error at a point. Actual raster data sets that are to be mosaicked together and reprojected represent significant areas of the earth's surface. The grid squares method attempts to address the projection-to-projection transformation as well as the problem of measuring distortion that is occurring due to a data set of some areal extent. Unfortunately the grid square can only provide area based information. The method cannot address the specific issue of pixel loss or duplication. The second method developed by Steinwand et. al. (1995), the checkerboard method, does address distortion at the level of the pixel and it also addresses the issue of projecting data sets that represent significant amounts of area. However, this methods does not provide detailed information on the actual degree of pixel loss or duplication. The implementation of the checkerboard method is lacking in flexibility being limited to the examination of the Plate Carree-to-other projection transformation only.

The checkerboard method was however a catalyst for the development of the pixel duplication and pixel loss metrics for this research. The calculation of these metrics are based upon 10 x 10 pixel grids, similar to the checkerboard method, but with unique values from 1-100. The grids can be used to measure pixel loss and duplication independently. In addition this method addresses the projection of actual raster data sets, and overcomes the limitation of only providing analysis for the Plate Carree projection transformation.

3.3 Alternatives to Using World Map Projections

3.3.1 Spherical Analysis

Although most geographical analysis has been performed on a two dimensional plane the possibility of performing analysis on the sphere has received attention from a variety of different fields. Robert Raskin, in his review of spatial analysis on the sphere (1994), lists geology, astronomy, meteorology, statistics, operations research, and computer science, in addition to geography, as fields of study interested in using spherical analysis.

Raskin reports on three conceptual modeling approaches. The first is via a projection where a conventional map projection transformation is performed, analysis is performed on the plane, and the results are projected back to the sphere. The second approach is termed intrinsic. In this model the sphere surface is "considered an intrinsic space in its own right," and analysis is performed in non-Euclidean space. The third model is described as embedding. This model treats the sphere as a subset of an overall three dimensional space. Analysis is performed in this space with a constraint imposed to limit the solutions to the surface of the sphere.

Raskin reports that he found only one example of working GIS that operates in the spherical domain. In 1994, he reports that the Hipparchus GIS was the one working spherical based GIS. Raskin reports that Spherekit was developed at the National Center for Geographic Information and Analysis at the University of California, Santa Barbara by a group of developers. Spherekit is described as an integrated toolkit for spatial interpolation and comparison of spatial interpolation algorithms. It permits interpolation over continental or global scales and computations are based upon spherical distances and orientations (Raskin 1996).

Robeson (1997) reviews and examines several methods for spatial interpolation on the sphere. The methods evaluated are grouped into three classes, distance weighting, functional minimization, and tessellation. A method from each of these classes is evaluated using both a hypothetical mathematical surface as well as global topography to simulate smooth and nonsmooth surfaces. For smooth surfaces a thin-plate spline provides a low interpolation error and are an excellent choice. The choices for nonsmooth surfaces such as the global topography include, distance-weighting, C^0 surface patches, and interpolating thin-plate splines to produce low interpolation error.

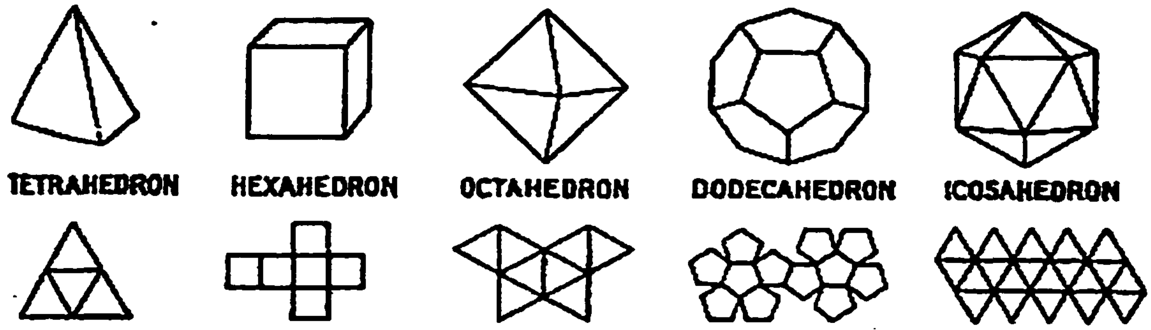
An additional representation of data for spherical analysis is through the use of spherical harmonics. Climatology data, topography and even world population have been represented with spherical harmonics (Raskin, 1994). Other methods being investigated to represent and manage global data in the geographic literature are suggestions for tessellations, polyhedral globes, and hierarchical referencing of the Earth as a basis for global analysis.

3.3.2 Hierarchal Referencing & Tessellated Globes

The management of spatial data sets of large extent is possible, and is under development, through the use of various referencing schema. Several researchers from diverse points of view have been exploring this possibility. The CEOS group (1997) recommends, "Further support for investigation into alternative gridding schemes, such as spherical tessellation, which might allow near-distortion-free aggregation of global data." Raskin (1994) defines a tessellation of the sphere as "a spatial decomposition that is exhaustive and mutually exclusive (except perhaps along boundaries)" and goes on to indicate that these tessellations may be classified as either polyhedral or empirical.

The most common spatial form selected by these investigators is of one of the five Platonic solids. See Figure 16 for examples of both the solids themselves and the flattened forms of the shapes. The tetrahedron has 4 equilateral triangles, the hexahedron, or cube, has 6 squares, the octahedron has 8 triangles, the dodecahedron has 12 pentagons, and the

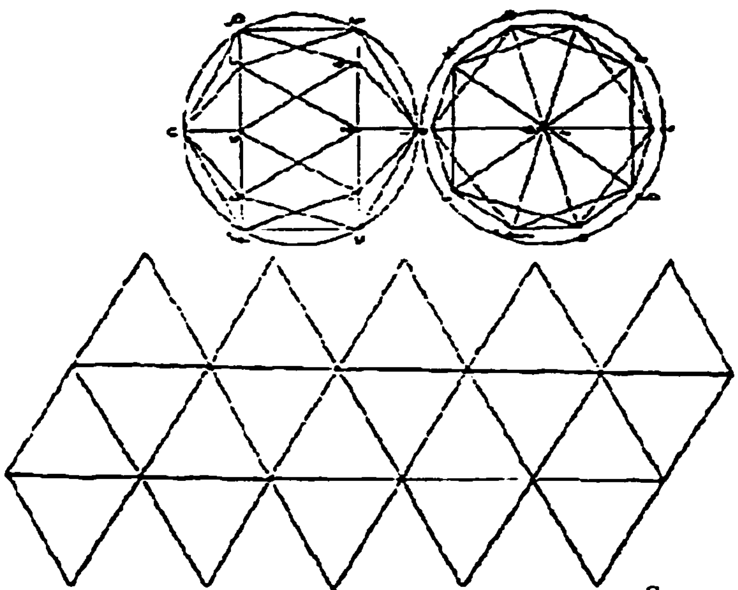
Figure 16. --The Five Platonic Bodies



Source: Fisher and Miller

icosahedron has 20 equilateral triangles. This is far from a new concept however. See Figure 17. The artist Albrecht Dürer considered several of the Platonic solids for the purpose of creating a world globe in 1538. The use of global tessellations and hierarchal data structures for managing data about the surface of the Earth map provide a means for more accurate and richer spatial representations.

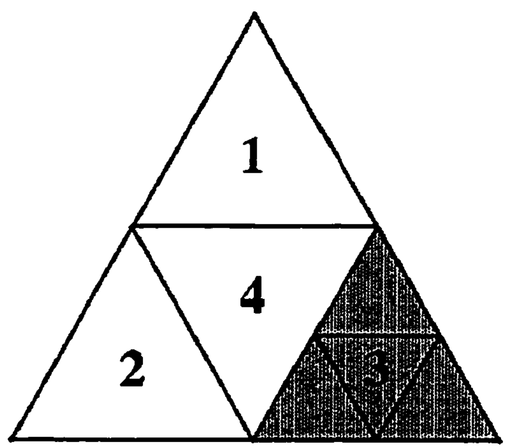
Figure 17. --Icosahedron. Globe Sketches by Dürer, 1538



Source: Fisher and Miller

A series of publications by Dutton (1983, 1988, 1989, 1996) have focused upon his development and implementation of a polyhedral based hierarchical data structure. Dutton needed an efficient way of modeling global terrain data including its assembly and management. He found that the traditional models used for modeling terrain data, gridded terrain data and triangular irregular networks, fell short of ideal for global data. He cites poor documentation, and variable accuracy through the gridded surface, both in horizontal and vertical dimensions, as just two of the problems he encountered.

Figure 18. --The Pattern of Subdivision used for QTM Facets

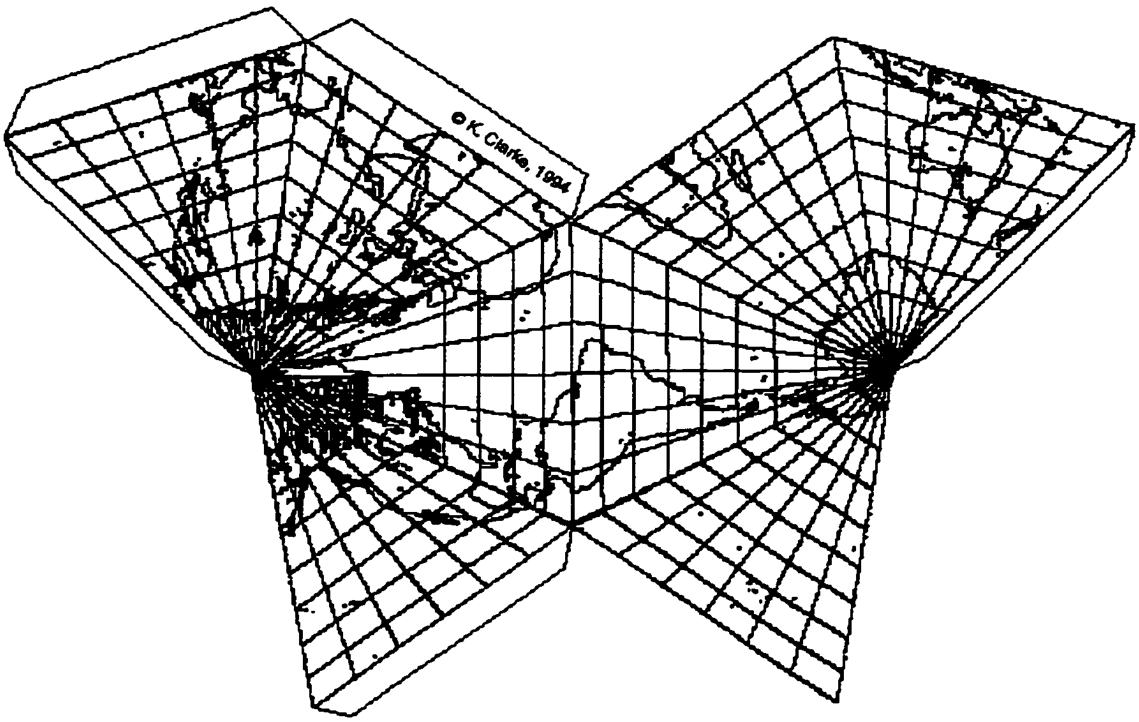


Dutton's data structure is known as the quaternary triangular mesh (QTM) and he describes its characteristics as storing elevation alone explicitly, being global in scope, using hierarchical sampling, having an economic data structure and verifiable accuracy. QTM is based upon subdivision of the octahedron (Figure 18). Each triangular facet of the original octahedron is subdivided into four triangles. Each of these four new triangles is then subdivided into four triangles. Dutton has continued examining this QTM structure to include information regarding both qualitative and quantitative aspects of the features being encoded.

The implementation of a hierarchical data structure to take advantage of data compression and fast access was the objective of Goodchild and Shiren (1992). They define three criteria for the development of this data structure; at any level of the hierarchy the cells should be of equal size and likewise, they should be of the same shape at any level of the hierarchy, and the data structure must correctly describe the relationships between neighboring cells. They propose a system similar to Dutton's. The data structuring also begins with the use of an octahedron and subdivides each of the eight triangular faces recursively using the same pattern seen in Figure 18. Ultimately these researchers would like to apply this data structure to a global GIS. They report a partial success in terms of their three criteria as the data structure does accurately incorporate relationships, but it only provides an approximate solution in terms of maintaining recursively divided cells of equal area and shape.

Lugo (1994) implements a data structure based on triangular quadtrees and two dimensional run length encoding to represent global relief. Based on Dutton's QTM system, his objective was "to store a digital data set in a triangular hierarchical structure and link it directly onto a a three dimensional data structure." He calls his data structure the TQS, or triangulated quadtree sequencing. Clarke (1995) developed the Modified Interrupted Collignon, also known as Clarke's Butterfly (Figure 19) to support this mapping effort. Lugo successfully linked his Triangulated Quadtree Structure to a portion of the ETOPO5 global data set to the Modified Interrupted Collignon projection.

Figure 19. -- Interrupted Collignon or Clarke's Butterfly



A polyhedral based solution was also reached by a group of researchers with a very different objective. Kimerling, Overton and White (1992) needed to develop a statistically sound sampling program for global ecological data. They described a number of criteria including; a randomly positioned sampling grid, an equal area map projection to allow for equal probabilities for sampling locations, compact areas, regions that did not align with cultural or physical features, and an ability to densify the sampling grid in a hierarchical pattern. A sampling grid was devised by the group based upon the Lambert Azimuthal Equal Area map projection on the faces of a 20 sided truncated icosahedron.

The determination of how to map the Earth onto a polyhedral globe has usually been approached by using a gnomonic map projection, recentered for each face of the figure. The characteristics of the gnomonic projection introduce significant area and scale distortion (Snyder 1987). There are several projections that have been developed specifically for the purpose of mapping onto polyhedral globes. In 1965 Lee presented the conformal projection of the sphere on a regular tetrahedron. It was regarded as interesting novelty at first due to its attractive presentation of repetitions of the sphere. One critic sarcastically suggested that it would be "suitable as a design for a curtain" (Lee 1973). Today polyhedral based projections are receiving much more interest and the conformal tetrahedral projection would be a suitable choice when the preservation of shapes is desired.

If an equal area projection is required there are two choices available. Snyder (1992) presents a modified Lambert Azimuthal Equal Area projection that was derived from an approach by Irving Fisher. Although the Snyder projection may be applied to any polyhedral globe, it provides the best (lowest) distortion characteristics for the 12 sided dodecahedron and the 20 sided truncated icosahedron (Figure 16). He reports the angular deformation as not exceeding 3.75 degrees for the truncated icosahedron. Clarke and Mulcahy (1995) describe the development of Clarke's Butterfly, a near equal area projection that was motivated by the implementation of a global relief data structure based upon Dutton's QTM system.

A referencing system that would support the spherical nature of the earth's surface would be ideal in permitting data storage without data alterations due to the map projection process as would analysis being performed in the spherical domain. Meanwhile, as long as software and data structures do not provide the ideal environment for global data, and as long as we employ data that was developed using a planar surface, there is a need to understand the data alterations introduced by map projection transformation.

3.4 Resampling of Raster Data

Resampling is a process that is used to determine pixel values when geometrically transforming a raster image. During this process the values to be assigned to the new image are derived from the original image by one of several methods. Resampling is a very common process in image processing that occurs whenever the location or resolution of output pixels is to be different than the input pixels. These processes include geometric correction of sensor data, during georectification procedures, a reduction in spatial resolution, and most important in this context, during map projection transformations.

Currently there are three standard resampling methods widely available; nearest neighbor, bilinear interpolation, and cubic convolution. Software implementations have settled pretty comfortably with using one of these three methods. Nevertheless, these standard methods all have fairly serious drawbacks from the introduction of spatial error in the case of nearest neighbor resampling to changes in the pixel values for the other two methods. The original choice of a resampling method was heavily influenced by computational cost considerations (Colwell 1983). Advances in both hardware and software may now allow for the implementation of more computationally intense methodologies and related research into digital image compression techniques may prove to be a source of new ideas.

3.4.1 Nearest Neighbor Resampling

Nearest neighbor resampling is the simplest method for determining a output pixel value. It is also the only resampling method available for categorical data. The nearest neighbor method assigns the intensity value to an output pixel (*i,j*) the value of the input pixel(*k,l*) that is spatially closest. This pixel is found by:

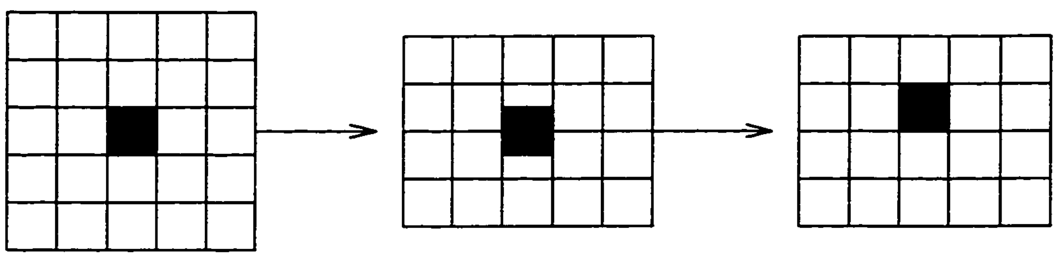
$$k = \text{integer part of } (x + 0.5)$$
$$j = \text{integer part of } (y + 0.5)$$

such that

$$Intensity_{i,j} = Intensity_{k,l} \qquad \text{(Equation 8)}$$

The values of the original matrix are preserved and no new classes are created. The main disadvantage of the nearest neighbor resampling method is that it creates a spatial shift in the data. The output pixel may be shifted spatially up to one half of the size of a

Figure 20. --Pixel Shift Due to Nearest Neighbor Resampling



pixel. See Figure 20 from left to right; the original data, the new geometric space with offset pixel, and the final output space with the pixel shifted 1/2 pixel. Further, comparisons between digital images is difficult due to misregistration because the pixels have shifted spatially (Colwell 1983). The visual appearance of the image after nearest neighbor resampling is very blocky.

3.4.2 Bilinear Interpolation

The bilinear interpolation technique uses a distance weighted average of the four closest pixels from the original matrix and assigns that value to the output pixel. The bilinear method is more computationally intensive than the nearest neighbor method. The result of interpolator is a smoother image with 1/4 of the mean squared resampling error as compared to the nearest neighbor interpolator (Shlien 1979). The disadvantage of the bilinear interpolator over nearest neighbor resampling is that the digital values have been changed and therefore the spectral characteristics of the original image have changed and cannot be recovered. The bilinear interpolation causes a blurred appearance.

3.4.3 Cubic Convolution

By modeling the image locally by a polynomial surface resampling accuracy can be improved. A cubic solution uses the 16 nearest neighbors to the output pixel. Cubic convolution avoids the spatial shift of values that is found in a nearest neighbor solution and produces less image degradation than the bilinear interpolation.

In nearest neighbor resampling the new pixel value is derived from the nearest pixel in the original matrix. This method has the advantage of computational simplicity, retention of original values, and the disadvantage of spatial offset of pixels in the output matrix. Bilinear interpolation is the distance-weighted average of the nearest four neighboring pixels and a two dimensional equivalent of linear interpolation. This interpolator has the advantage of creating a smoother image with a modest increase in computational intensity although the results are a bit too smooth and may create a smeared appearance. Cubic convolution creates a pixel value by evaluating a block of 16 pixels surrounding the original DN. This method is more computationally intense. Both the bilinear and cubic methods change the original pixel value making them unsuitable for categorical data (Lillesand 1994, Campbell 1987). For the purposes of this research the nearest neighbor resampling interpolation method will be employed. The measurement of pixel loss and pixel duplication depends upon counting unique pixel values. The only method that will retain pixel values is nearest neighbor.

CHAPTER 4

CARTOGRAPHIC DISPLAY OF MAP PROJECTION DISTORTION

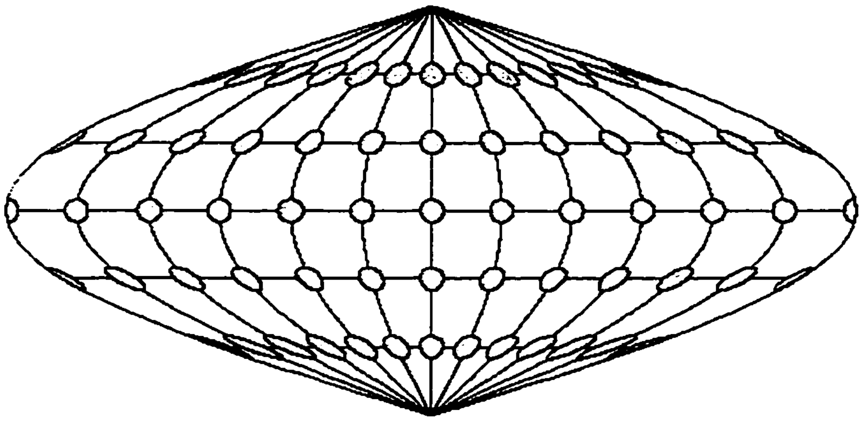
The visualization of map projection distortion is critical to an understanding of map projections. In this context the visualizations have been separated into two general headings. Analytical methods refer to visualizations that are related to the map projection selection process by presenting results from analytical evaluation of map projection distortion. Exploratory visualization methods are used primarily to provide a general understanding of map projection distortion by exposing the characteristics of map projection distortion, often in a learning environment. Analytical and exploratory visualization relate to the concept of MacEachren and Kraak's goals of map use, in the map use cube (1997).

4.1 Analytical Visualization Methods

4.1.1 Tissot Indicatrix

The oldest and most elegant means for depicting map projection distortion is the use of Tissot's indicatrix. The indicatrix is a visualization of the mathematical basis for Tissot's Theorem whereby an infinitely small circle on the earth is projected onto a two dimensional map projection. The result is an ellipse with its axes used to describe scale changes visually. In Figure 21 the ellipses are placed at the intersections of a 30^{o} graticule. The changes in shape relate to angular change. The Sinusoidal projection has the geometric property of

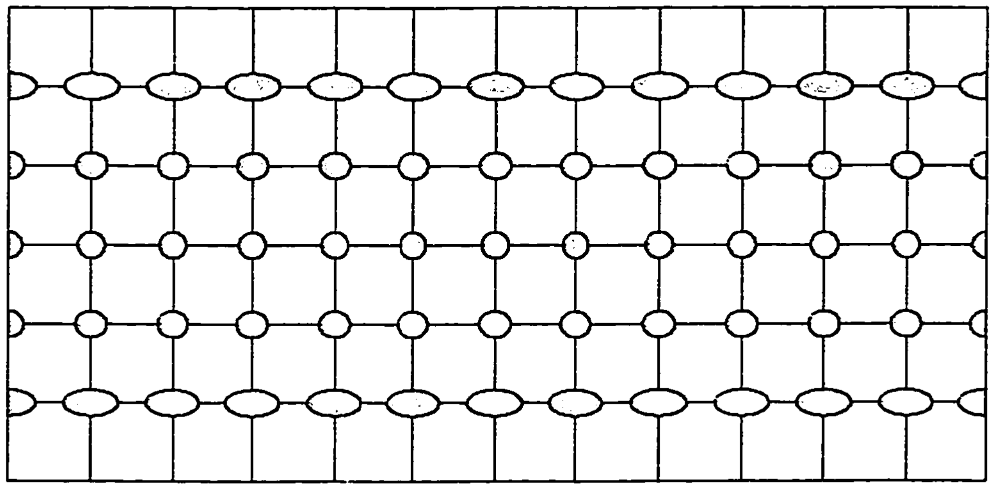
Figure 21. --Tissot's Indicatrix Visualization on a Sinusoidal Projection



equal area. The ellipses indicate this by retaining the same area although increasingly distorted with increased distance from the center of the projection. The orientation of the ellipses provide detail and the direction of maximum angular deformation.

Recommendations provided by van der Wal et. al (1994) for the depiction of positional accuracy include: color (under specific conditions), size, and saturation. It is interesting to note that visual variables employed in this over 100 year old method include size, shape, and orientation and do not require the use of other visual variables to be effective. The visual display of the Tissot's indicatrix provides a very effective means for communication of the type, amount, direction, and overall spatial distribution of map projection distortion throughout the map. The drawback to Tissot's method is that it is limited in application to forward transformations from the globe to a map projection only. Brainerd and Pang (1997) also cite the fact that the indicatrix obscures underlying data. This negative comment may indicate a lack of understanding of basic cartographic

Figure 22. --Tissot's Indicatrix on the Plate Carrée Projection

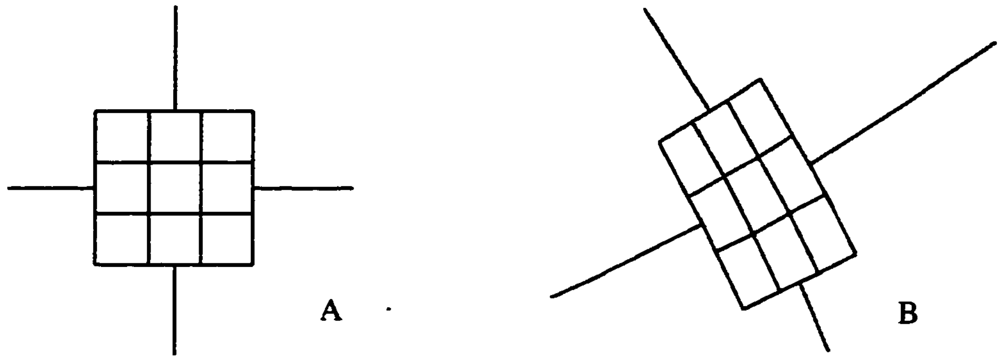


principles. The indicatrix is frequently used with only a graticule and perhaps shorelines. In this way the full spatial distribution of distortion is emphasized and the pattern of distortion is easily grasped.

4.1.2 Grid Squares

In terms of visual variables, the grid squares method developed and presented by Steinwand et. al (1995) is similar to Tissot's indicatrix. It appears as a raster style indicator exchanging squares for the circles of Tissot. The type, amount, direction, and overall spatial distribution of map projection distortion throughout the map is communicated effectively by this method.The grid squares method consists of the creation of sets of three by three vector grids that are centered on the intersection of parallels and meridians. The original grid square shown in Figure 23A was formed on a Plate Carrée projection and then projected Figure 23B. The advantage of using the grid squares over Tissot's indicatrix is that this methods was developed specifically for examining distortion between different

Figure 23. --Grid Squares



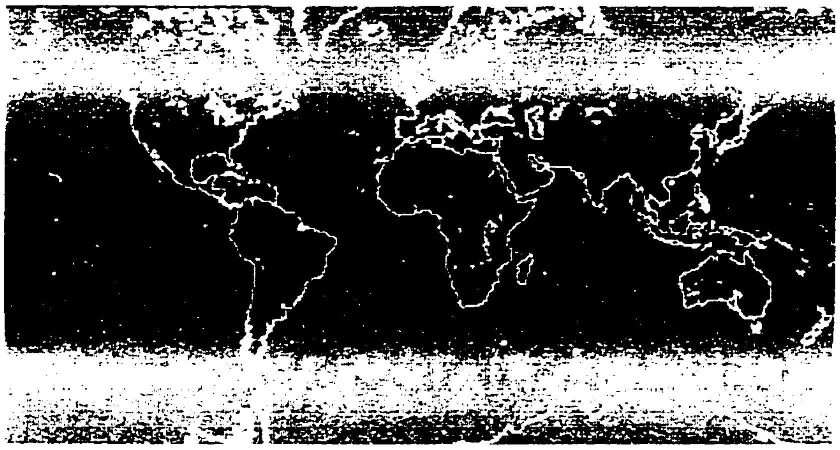
projections and is not limited solely to the forward projection. The visual effectiveness of the square for angular distortion and direction of maximum angular deformation may be somewhat less effective than for Tissot's indicatrix. The grid squares method however, being composed of squares, does have the advantage of an identification with a raster data structure.

4.1.3 Color Method

The method of presenting map projection distortion via a continuous color display shows promise for providing effective displays of distortion in raster data sets. A variety of measures can be mapped using this particular display method. The visual variables that may be used include hue, saturation, and intensity. These can be used in a univariate display using a single hue and varying intensity or a more complex multivariate display may be produced using a combination of HSI (hue, saturation, intensity) or RGB (red, green, blue).

The distortion display presented here was previously developed by Clarke (Clarke and Mulcahy 1995). Shown here in grayscale, the color method records the east-west scale distortion along the parallels, the north-south scale distortion along the meridians and the angle of grid convergence. These values are mapped onto red, green, and blue (Figure 24).

Figure 24. --Color Method Depicted on a Mercator projection



The Mercator projection has increasing distortion away from the equator. This method was also used to depict locational error by Clarke and Teague (1998) for a comparison of two vector maps and employed versions of RGB and HSI.

This method takes advantage of a raster data structure to provide a visually continuous representation of the magnitude of distortion and is superb at providing a snapshot of the overall distortion pattern. It effectively communicates the spatial distribution and degree of deformation and has the advantage of being able to display more that one variable at the same time. This method may also show an advantage for researchers having grown accustomed to viewing global data sets in similar color displays but who may not have a background in cartographic principles and visualization methods such as Tissot's indicatrix.

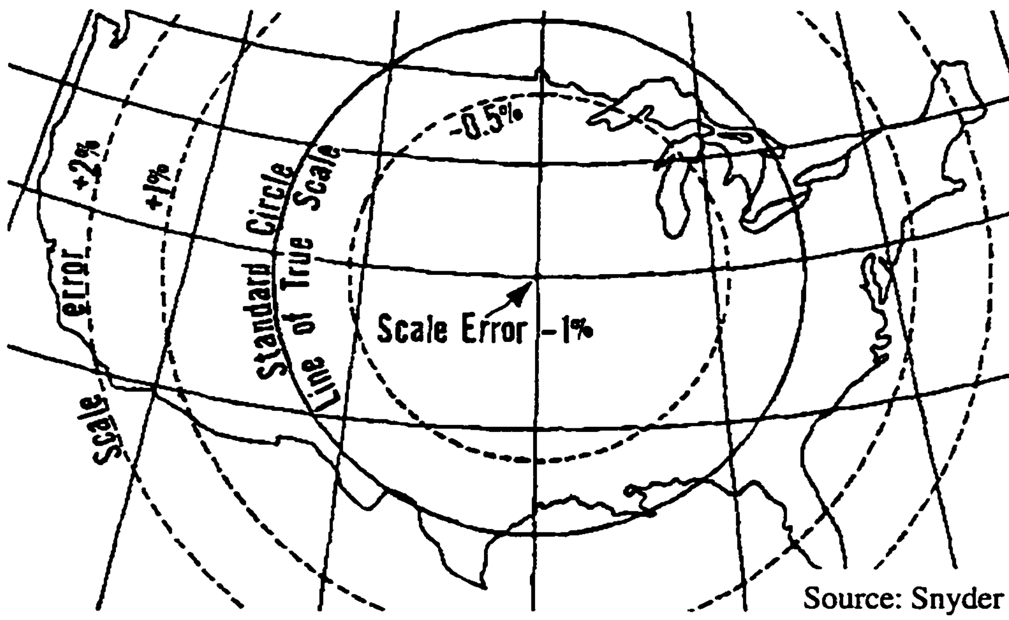
4.1.4 Surface Representations

Another possibility for the display of map projection distortion is through the use of a three dimensional surface where the surface represents the magnitude of angular or area distortion or for pixel value, loss or duplication changes. Surfaces provide a univariate display of data. Clarke and Teague (1998) use this method to create a series of small multiples with error mapped separately for east-west, north-south, and overall magnitude of positional error. There is also the possibility of adding another visual variable such as hue as an overlay to the surface variable. For example, the angular distortion present in a Mollweide projection would produce very large elevation at the extremes of the projection with minimal elevation in the center of the projection (it may even be necessary to invert the surface because the highly elevated edges may obscure the interior). A surface visualization may also be presented in plan view as in reviewing results for continental data.

4.1.5 Isolines

Isolines are an effective visualization for displaying the amount and distribution of scale change or angular distortion. This method has the advantage of showing actual numerical values. The use of isolines, lines representing equality with respect to a given variable, are the most common distortion display after Tissot's indicatrix. Isolines are used, often with shading, to locate values of areal exaggeration or angular deformation. Numerous examples can be found in Snyder (1989), Maling (1992), and in Canters and Decleir (1989).

Figure 25. -- Scale Error Expressed as Isolines



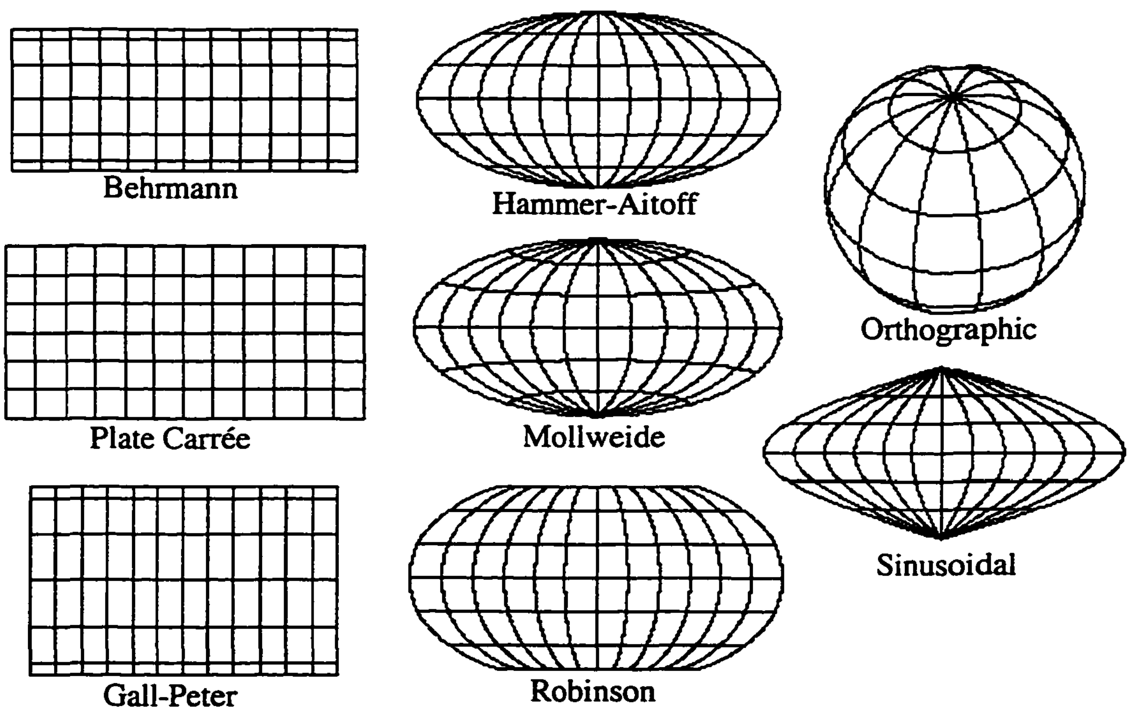
Source: Snyder (1987)

Isolines also provide an overall representation of the spatial distribution of distortion and can be particularly helpful during the selection of a map projection for a particular region. In Maling (1992) the relative merit of the Azimuthal equal-area projection and Bonne's projection are compared by overlaying isolines of maximum angular distortion (ω) and an approximate coastline of the North American continent. Another example in Maling (1992) compares the isolines for particular scales on the normal aspect of the Mercator, Transverse Mercator and the Stereographic projection for a conformal map of Latin America. Isolines are used extensively by Maling (1992) and almost exclusively by Canters and Decleir (1989) in their directory of world map projections.

4.1.6 The Graticule

Projections are evaluated sometimes simply based upon the appearance of the graticule of latitude and longitude. The graticule, sometimes referred to as a grid (particularly in large scale mapping), has a long history of being used as an analytical tool for understanding projection distortion and for identifying unknown projections. This type

Figure 26. -- Graticules of Selected Projections



of visual inspection is useful for comparing different projections and the different aspects of a particular projection. No matter what the aspect chosen for a particular projection, the pattern of distortion for the projection remains the same when measuring with traditional map projection distortion methods. Robinson, et. al (1995) list the following nine visual characteristics. See Figure 26 for the graticules from an assortment of variously shaped graticules.

1. Parallels are parallel.
2. Parallels when shown at a constant interval are spaced equally on meridians.
3. Meridians and great circles on a globe appear as straight lines when viewed orthogonally.
4. Meridians converge toward the poles and diverge toward the equator.
5. Meridians when shown at a constant interval are equally spaced on the parallels, but their spacing decreases from the equator to the pole.
6. When both are shown with the same intervals, meridians and parallels are equally-spaced at or near the equator.
7. When both are shown with the same intervals, meridians at 60^{o} latitude are half as far apart as parallels.
8. Parallels and meridians always intersect at right angles.
9. The surface area bounded by any two parallels and two meridians (a given distance apart) is the same anywhere between the same parallels.

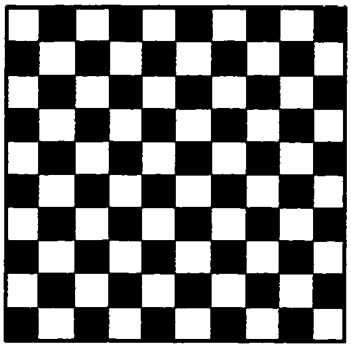
The argument presented by Robinson et. al (1995) is that the evaluation of many projections can be accomplished effectively by comparing the graticule on the map to that of a reference globe. Both Tobler (1962) and Maling (1992) include the appearance of the graticule as a fundamental organizing concept of their classification systems. See Figure 5 and Figure 13. Canters and Decleir (1989) also use separation between the parallels as a direct indicator of distortion characteristics.

4.2 Exploratory Visualization Methods

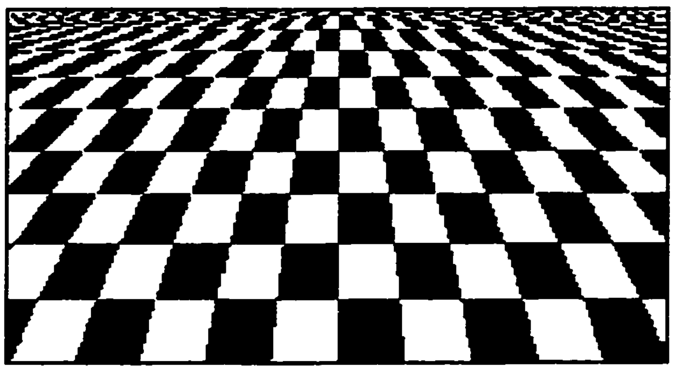
4.2.1 Checkerboard

The checkerboard method was previously described an as analytical measurement tool but it may also be employed as an effective visualization method. When used for visualizing map projection distortion, a vector data set replaces the raster data structure.

Figure 27. -- Checkerboard visualization method



10 by 10 checkerboard grid



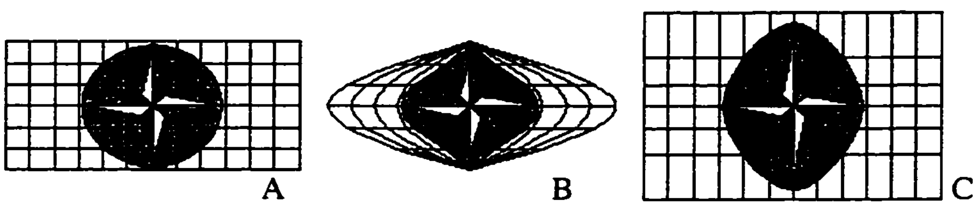
A checkerboard in an orthographic projection, converging towards the north pole. Along the lower edge there are 10 grid cells but towards the pole they become indistinguishable and

The visual variables of shape, pattern and alterations in either hue, saturation or value are employed in this visualization. The data set is developed with alternating colored pixels and then projected. The checkerboard method provides a means for exploring various projections. The pattern formed by the alternating hues or values is particularly effective in exposing the spatial distribution of distortion. Visualization of the alterations in shapes in the vector depiction provides an insight into the limitations of the raster structure in terms of geometrical alterations. A raster data structure does not respond to geometric alteration in the same manner as vector data and the checkerboard assists in communicating the differences between them. The checkerboard seen projected in Figure 27 lower right is a fragment from a global vector based grid data set. The checkerboard data set seen in Figure 27 was originally created in a Plate Carrée projection and so there were originally 360 cells representing 89° to 90° north latitude and these converge to an uncountable, indistinguishable region near the pole.

4.2.2 Familiar Shape

The use of a familiar shape can be very instructive in conveying the concepts of map projection distortion. A face, or regular shape is created in geographic space and the resultant distortions are immediately recognizable to anyone, cartographic training not required. This method, though excellent as a learning device is not amenable to

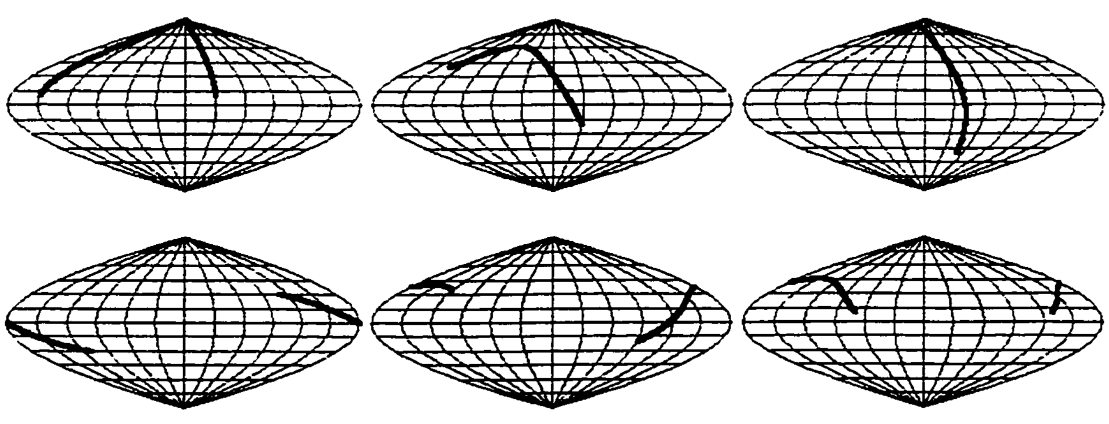
Figure 28. --Familiar Shape



quantification. As seen in Figure 28, the use of a familiar shape can illustrate some basic changes which occur in map projections. A north arrow was inscribed in a circle against the graticule of the Plate Carrée projection (A). This same symbol appears in the Sinusoidal projection (B) and the Miller Cylindrical projection (C).

This symbol was created and projected using the ARC/INFO geographic information system software package. Two of the many other examples of the use of familiar shapes can be found in Robinson et al (1995) where the profile of a human head is projected; and the current Hammond International Atlas of the World (1994) where a cartoon face is created from circles, a triangle and a line. The use of a familiar figure encourages comparison of the changes occurring in size and shape. A very different example of the use of a familiar shape is the series of great circle arcs on the sinusoidal projection. A great circle is the shortest distance between two points on the globe. These are shown in Robinson et. al. (1995) courtesy of W. R. Tobler and can also be recreated from a prepared script in the MicroCAM for Windows (Loomer, 1994) cartographic software package (Figure 29).

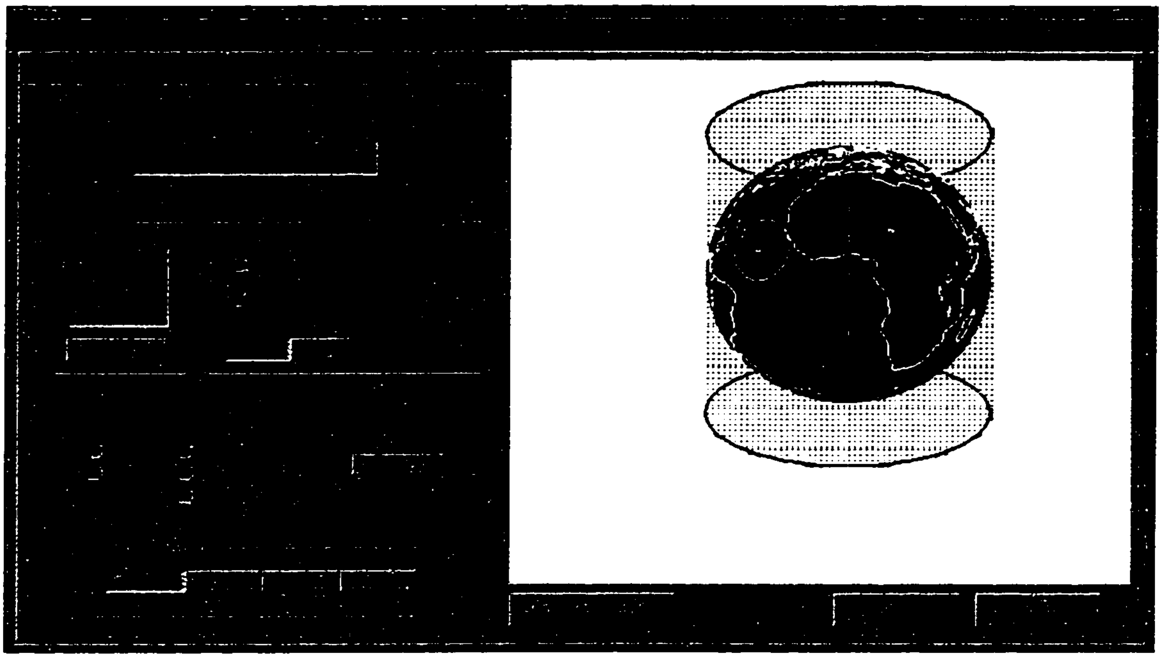
Figure 29. --Great Circle Arcs on Sinusoidal Projection



4.2.3 Floating Ring

The floating ring visualization tool provides a method for interactive visualization of map projection distortion based upon the concept of a familiar shape and the color method of portraying distortion. This contribution to cartographic visualization comes from

Figure 30. --The Globe Window with the graticule, coastline, and cylinder

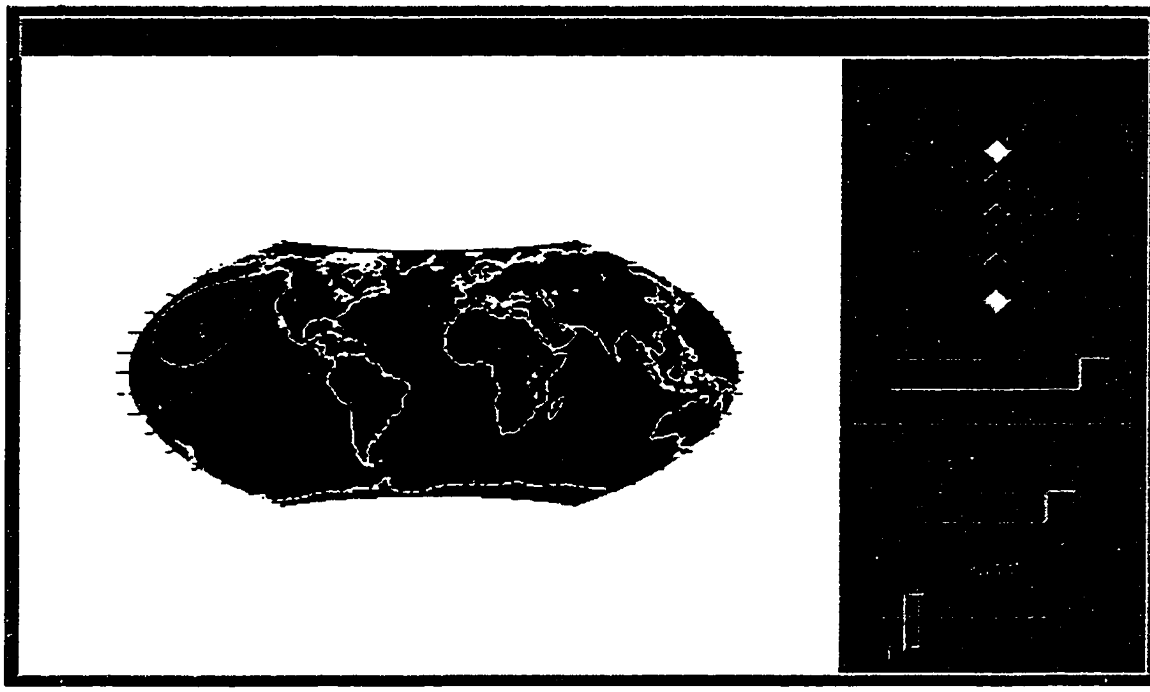


Source: http://www.cse.ucsc.edu/research/slvg/map.html

a computer science department indicating the interest in map projections outside the traditional cartographic and geographic realms. It is intended as a tool for map users in a wide range of applications and disciplines and is implemented in the MapViz Interactive Map Projection System.

In the tool a ring floats freely upon the surface of the globe like a contact lens (Figure 30). The ring can be scaled as well as moved over the surface of the globe. The user is able to interactively manipulate the ring to permit real-time viewing of the shape

Figure 31. –The Wagner VII projection in the MapViz projection window



Source: http://www.cse.ucsc.edu/research/slvg/map.html

distortion. Color is also used to enhance the visual cues. In Figure 31 the floating ring provides a visualization of both area and angular distortion using a combination of a familiar shape - a circle, and the color method. The ring can be filled with a continuous color display that indicates either area or angular distortion. The greatest advantage of the method is that it permits intuitive user interaction. The major drawback of the floating ring is that even with the use of multiple rings, this method cannot show distortion throughout the entire map at one time. The ring is also much more effective when the circle is filled with continuous color rather than as an outline.

CHAPTER 5

METHODOLOGY

This study explores the changes that will affect the quality of a regional or global raster data set as a result of map projection transformations. Little is known about this process of data alteration so a set of basic hypotheses, developed from an examination of cartographic literature, are listed. The changes that occur within a data set depend upon many variables such as the map projections chosen, the resampling method used, data structure, etc. A full range of the variables involved in the map projection process are explored and the condition chosen for this research are presented. The methodology of this research consists of tracking pixel loss and pixel duplication separately throughout the map at ten degree intervals. The metrics used, *PD* and *PL* were created specifically for this purpose. These metrics are used to examine map projections suitable for thematic global and continental data sets. The analysis is performed in the ARCINFO (ESRI, 1996) software environment.

5.1 Study Parameters and Limiting Conditions

Variables that may exist during the projection of data sets include: scale change, including areas of expansion and contraction and angular change; the creation of singular points; coordinate systems and units of measurement; pixel location; pixel size; pixel value changes; pixel duplication; pixel loss; resampling methodology, data structure, software implementations of map projections; the choice of projection, and the spatial extent of the data.

The geometric changes that occur during point-to-point transformations from geographic coordinates to specific projections are well documented in the map projection literature. The traditional analysis of map projection distortion examines changes in scale that result in distance, area and angular distortion. Map projections that retain the property of equivalence, or equal area projections will be the primary focus of this study because there is an emphasis on the calculation and use of area in global and regional studies and models. Other map projections will only be for comparison with the equal area projections.

During transformation by map projection coordinate systems, datums, and measurement units may be altered at the same time. The coordinate system or choice of measurement unit does not affect the quality of the data. Using a particular datum as the reference model may cause changes in digital data sets at larger scales but the relatively large minimum pixel size and the broad spatial extent of the data in this study eliminate datum changes as a significant factor and they will not be explored. A spherical model of the earth will be employed for these regional and world map projections. The consistent use of a fixed level of precision and meters as the unit of measurement will be employed and pixel size is held constant.

Pixel locations, and the features they represent may appear to move as a result of the nearest neighbor resampling method. The output pixel may be shifted spatially up to one half of the size of a pixel. This positional shift may be avoided through the use of bilinear interpolation or cubic convolution methods although these resampling methods are unsuitable for categorical data (Lillesand and Kiefer 1994). Spatial shifts of pixels due to nearest neighbor resampling will not be explicitly measured in this study. Although the presentation of individual grids permit a visualization of the shifting of grid cells relative to one another. The nearest neighbor method will be used to resample the raster grids in the context of the loss and duplication of pixels.

An additional goal of this study is to make use of existing data sets and software as much as is possible so that the practical experiences and limitations of data producers and users can be duplicated and examined. The problem of non-standard use of algorithms has been identified by the CEOS Global Mapping Task team (1997) as a problem, but one that does not fall within the scope of this project. The CEOS team also identified "inconsistencies, inadequacies, and ambiguities in the description of map projections" in the metadata that accompanies spatial data sets (CEOS 1997). The primary software employed in this study was the ARCINFO 7.0.4 software (ESRI 1996). ARCINFO is overwhelmingly the market leader in the study by UNEP GRID, Sioux Falls (1997). ESRI's ARCVIEW GIS is also employed, primarily for the creation of visualizations.

The choice of map projection transformations to be explored is based upon those used currently for the creation, distribution and analysis of regional and global data sets by scientists in the USA. Of particular interest are the projections implemented by the USGS EROS Data Center for the release of the Global Land Characterization and other data sets (1997). The projections used are those recommended by Steinwand et. al. (1995), the Goode Homolosine and the Lambert Azimuthal Equal Area.

Vector data sets are not evaluated in this study because the vector data structure is flexible with regards to the scale changes that are induced during point-to-point transformation of the cells of vector data - coordinate pairs. As long as the coordinate pairs are sufficiently close, but not too close together, vector data flows effortlessly from projection to projection. There can be difficulties with regards to specific points or edges of a data set, but in general areas can expand and contract and lines bend and straighten. The resultant data is for the most part invertible. Raster data is not so flexible. The rigid structure of a raster data set forces data to be located at discrete, regular intervals. Lines stair step, areas expand by the application of cartographic license whereby existing values are simply duplicated to fill the increased space. Likewise, data is tossed out when there are areas of scale reduction. The data structure employed in this study is the raster structure, called a GRID in ARCINFO terminology.

The spatial extent of the data examined in this study is regional (consisting of North America, or Antarctica for example) and global. Smaller spatial regions will be employed only in modeling the development of a global data set from satellite scenes or digitized maps in the form of 10km x 10km grids. The small source data sets that are used during this study are 10 x 10 pixels and referred to as 10 x 10 grids.

5.2 Hypotheses

The goal of understanding and quantifying the changes in spatial data sets are addressed by testing the following hypotheses using the pixel loss and duplication measures developed for this research.

The transformation of raster spatial data sets of large extent by map projections is an invertible process with current technologies and algorithms. The data is irrevocably changed.

The amount, and distribution of data change will depend upon the map projections employed and the distortion characteristics of those projections.

To understand the distortion characteristics of various projections the evaluation of pixel changes should be from a source of higher authority and not evaluated from the typical distribution format of a latitude and longitude grid.

5.3 Analysis of P_L and P_D Pixel Measures

The evaluation of pixel loss and duplication is organized into a series of ten by ten pixel raster grids. Each grid has pixels numbered sequentially from 1 to 100. (See Figure 32) The cell size employed is one kilometer, making each grid represent a 100 kilometer

Figure 32. --Structure of Ten-by-ten Raster Grids

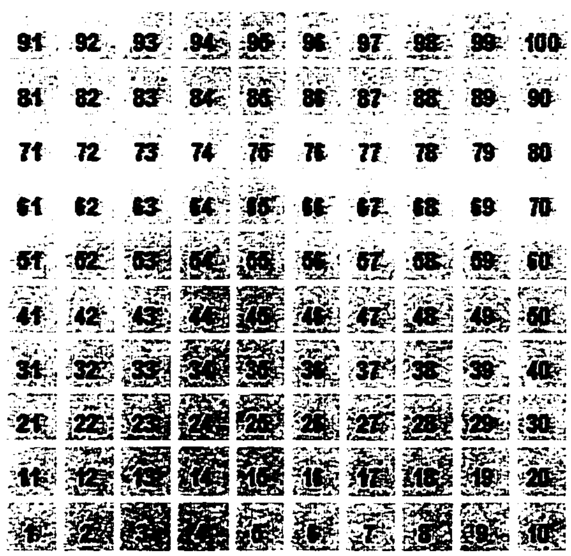
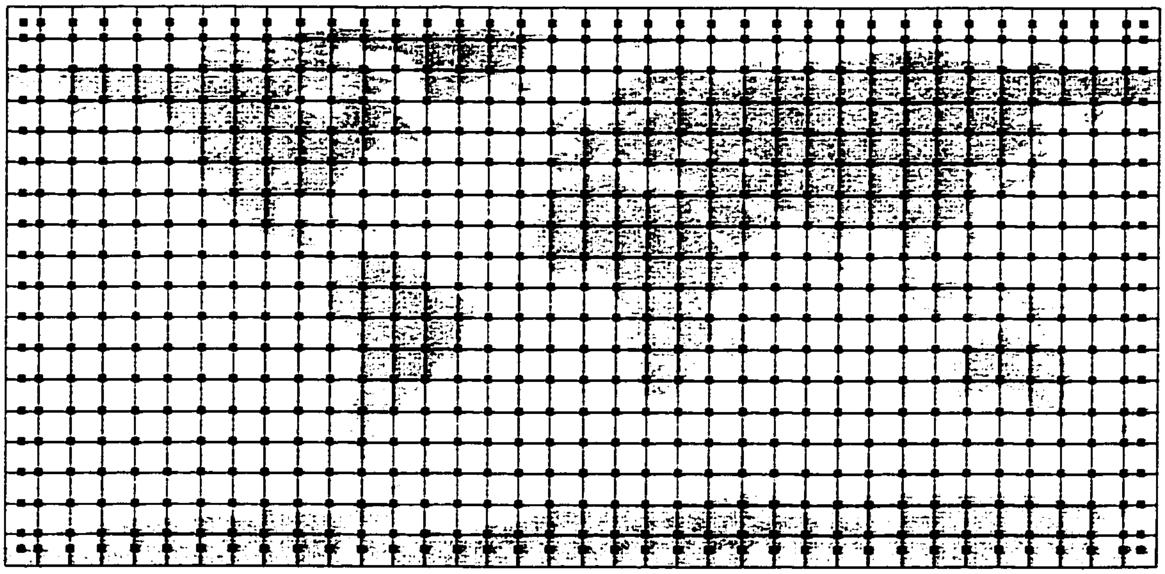


Figure 33. --Layout of Ten-by-ten Grids on the Plate Carrée Projection



area. The grids must be evaluated with sufficient frequency to describe the spatial pattern of pixel changes throughout the data set and to provide information for subsequent visualizations. Key features of the spatial arrangement include a ten degree spacing between the grids with the exception of the borders of the map. See Figure 33 which depicts the distribution of grids on the Plate Carrée. On all four borders the spacing of the grids were adjusted five degrees towards the center of the map to avoid boundary effects that would prevent the creation and projection of the grids. Additional features of the spatial arrangement include a full set of measurements along the equator and central meridian of the map.

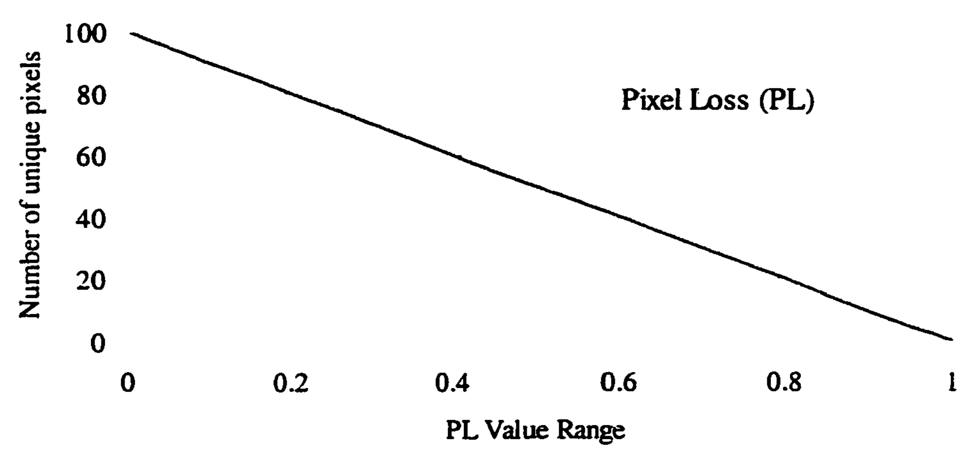
5.3.1 Pixel Loss (P_L) and Duplication (P_D) Measures

When it occurs, the loss of pixels is an invertible consequence of the transformation of raster data sets of large spatial extent. P_L provides a direct measure of pixel loss. For each control grid of 10 x 10 pixels that is transformed by a map projection, P_L is equal to one minus the number of unique pixel values of the projected grid divided by the number of pixels in the initial grid. See equation 9.

$$P_L = 1 - \frac{P_{unique}}{P_{initial}} \qquad \text{(Equation 9)}$$

If no pixel values have been discarded during the transformation P_{unique} will be equal to P_{inital}. P_L evaluates to 0, (see Figure 34) and if pixels have been lost P_L represents the proportion of lost pixels. For this analysis the P_{inital} value equals 100.

Figure 34. --Behavior of Pixel Measure P_L

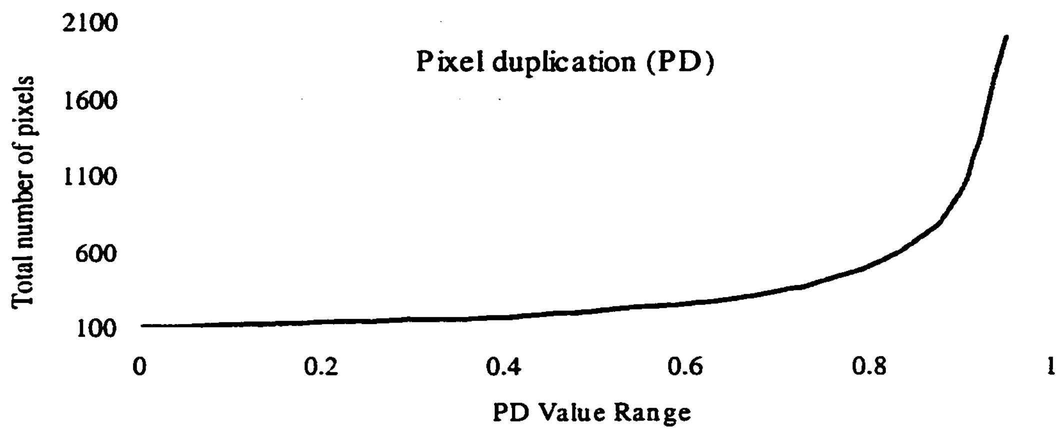


The second pixel measure *PD*, tracks the number of pixels duplicated during the map projection transformation process. *PD* is equal to one minus the number of unique pixel values present in the grid prior to projection divided by the total number of pixels after projection.

$$PD = 1 - \frac{P_{unique}}{P_{total}} \qquad \text{(Equation 10)}$$

PD evaluates to 0 if there has been no duplication of pixel values. See Figure 35. When duplication of pixels has occurred, *PD* evaluates to greater than 0. The resulting value indicates the pixel values that have been duplicated during transformation.

Figure 35. --Behavior of Pixel Measure ***PD***



PD and *PL* were developed for this research. The essential notion of projecting a small grid was inspired by the work of Steinwand et. al. (1995). Their 10 x 10 checkerboard created in the Plate Carrée projection evolved in this work to a 10 x 10 grid with 100 unique values that can be assigned any projection. Once projected these grids provide separate, though not independent measurements of pixel loss and pixel duplication. To fully understand the alterations occurring within data sets separate measures are essential. *PD* and *PL* were adjusted to range from 0.0 - 1.0 with low values indicating a low degree of data alteration and high values a greater degree of data change. Simply put, low is good, high is bad. At lower values *PD* and *PL* are comparable. For example, a value of 0.01 for both *PD* and *PL* indicates a duplication and a loss of one pixel respectively. At higher values, *PD* and *PL* are less comparable. The two metrics vary in functional form and are not independent of one another. A high *PD* value associated with a low *PL* indicates a large increase in area. A high *PD* with a high *PL* indicates a balancing of the pixel loss and duplication effects and the maintenance of area.

5.3.2 Evaluation of *PD* and *PL*

Testing of map projection induced data changes using the *PD* and *PL* measures was arranged by grouping similar transformations. Series 1 examines the transformation from the most common distribution format, geographic coordinates to various world map projections. Series 2 consists of examining the changes that occur when transforming from source materials such as individual satellite scenes or individual large scale map sheets. Series 3 examines the transformation from source materials to the Goode Homolosine projection because it is a currently being employed for distribution of land based data. Finally, Series 4 examines the response of *PD* and *PL* on azimuthal projections. Table 19 provides a summary of the all four series and the projection transformations associated with them.

Conceptually, the Series 2, Series 3, and Series 4 transformations evaluate from source materials to a world map projection. To mimic a map data source-to-projection transformation, the stereographic projection is employed. Often map projections for large scale data are conformal, such as the space oblique mercator, the Lambert conformal conic or the universal transverse Mercator. The area represented by the ten-by-ten grids is relatively small, only $100km^2$, so any number of projections could have been employed. The ten-by-ten grid is centered in the area of zero distortion for the projection. This assumption was tested comparing the Lambert azimuthal equal area projection to the Stereographic and the exact same results were produced. Each of the 703 10 x 10 grids is evaluated with a unique set of projection parameters that place the ten-by-ten grid at the center of the projection space.

5.3.3 Description of Software Implementation

The first step in evaluating *PD* and *PL* is to establish the original projection space for each grid. Evaluation of Series 2 - 4 transformations are accomplished in ARCINFO by defining a unique coordinate space for grid in the stereographic projection. The ten-by-ten pixel grid was defined with a pixel size of 1000 meters, and the pixel values are numbered sequentially from 1 to 100. See Figure 32.

Each ten-by-ten grid is centered upon its assigned coordinates with the origin of the projection at the center of the grid using the PROJECTDEFINE command. See the fragment of ArcInfo AML code below.

```
PROJECTDEFINE GRID GRID%GRIDNUM%
     PROJECTION STEREOGRAPHIC
     UNITS METERS
```

```
DATUM NONE
SPEROID SPHERE
1                    (Type 1 parameters)
0                    (Spheroid default to 6,370,997m)
%XCOORD%             (Longitude center of projection)
%YCOORD%             (Latitude center of projection)
0                    (False easting)
0                    (False northing)
```

Where GRID%GRIDNUM% is the unique grid identifier, numbered sequentially west to east from #1 at -90, -180 to #703 at 90, 180 (see Figure 33 for layout of the grids) and %XCOORD% and %YCOORD% are the x, y coordinates assigned to that individual grid.

The series 1 transformations use a similar approach to establish the original projection coordinate system. The primary difference is that there is one single original projection definition. Each grid is located by its assigned coordinates within a Plate Carrée projection. See Table 19.

TABLE 19. Evaluation Series for Analysis

Series	Extent	Source projection	Secondary projection
1	global	Plate Carrée	Craster Parabolic Flat Polar Quartic Miller Mollweide Sinusoid Eckert IV Eckert VI Hammer
2	global	Stereographic - with unique parameters for each grid	Craster Parabolic Flat Polar Quartic Miller Mollweide Sinusoid Eckert IV Eckert VI Hammer Plate Carrée

TABLE 19. Evaluation Series for Analysis

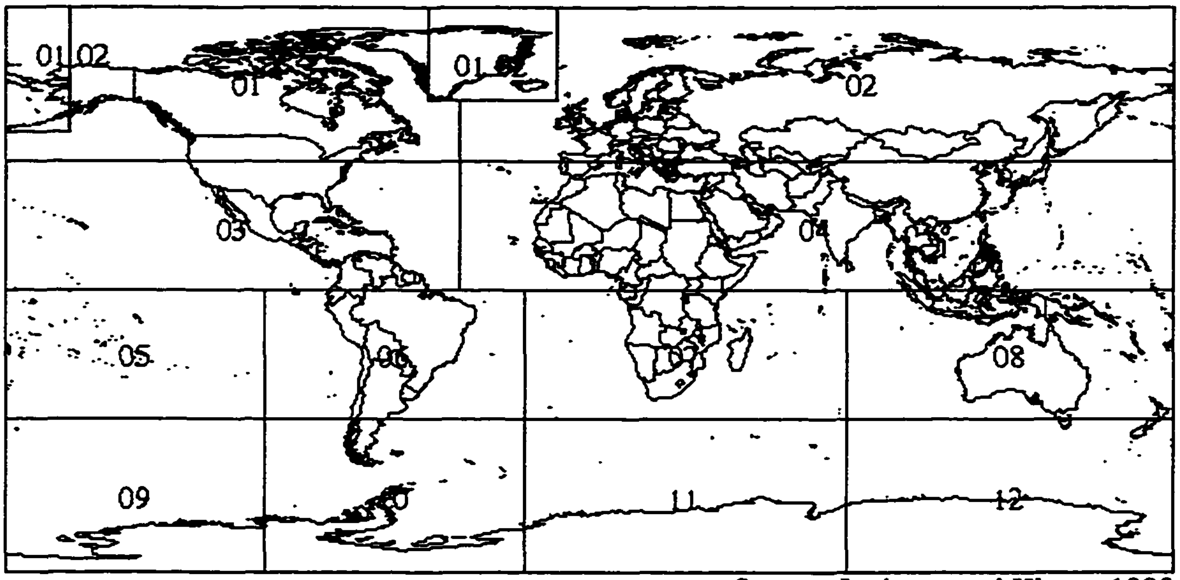
Series	Extent	Source projection	Secondary projection
3	global	Stereographic - with unique parameters for each grid	segments 1, 2, 9-12 - Mollweide segments 3-8 Sinusoid
4	continental	Stereographic - with unique parameters for each grid	Azimuthal - Lambert Azimuthal Equal Area Stereographic Azimuthal Equidistant

Once the original coordinate system and projection has been defined the next step is to project each grid to a secondary projection. For the Series 1 and 2 transformations, the grid is then projected to a world map projection that has the equator and central meridian as its origin. The cell size is constrained to the be same as the original one kilometer cells. The nearest neighbor resampling methodology is used to maintain the original cell values. Below is a sample projection sequence for the Mollweide.

```
PROJECT GRID GRID%GRIDNUM% GRID%GRIDNUM%P # NEAREST 1000
     OUTPUT
     PROJECTION MOLLWEIDE
     UNITS METERS
     DATUM NONE
     SPHEROID SPHERE
     PARAMETERS
     0 0 0.000          (Longitude of projection center)
     END
```

Series 3 transformations consist of transforming from the Stereographic projection to the twelve separate map segments that comprise the Goode Homolosine projection. The ARCINFO software cannot explicitly support any interrupted projection. A work-around was developed by Lethoce and Klaver (1998), that simulates the Interrupted Goode Homolosine. Goode's projection is actually composed of 12 separate map segments. See Figure 36 for a view of the segments on a Plate Carrée projection. Each segment is

Figure 36. --Segments of the Goode Homolosine



Source: Lethoce and Klaver 1998.

projected into either the Mollweide projection, Figure 36 sections 1, 2, 9-12; or the Sinusoid projection, Figure 36 sections 3-8. A unique set of projection parameters are defined for each section. The contents of one projection file for section 1 is listed below.

```
OUTPUT
PROJECTION MOLLWEIDE
UNITS METERS
DATUM NONE
SPHEROID SPHERE
XSHIFT -11119487.42847
YSHIFT -336410.83237
PARAMETERS
-100 0 0.000          (Longitude of projection center)
END
```

In the projection file the central meridian is 100W which cuts through the center of North America. A coordinate offset is defined by the `XSHIFT` and `YSHIFT` subcommands in the parameters for the Mollweide projections and by false eastings and northings in the definition of the Sinusoidal projection. The purpose of applying these specifications to the final coordinate system permit the individual segments to eventually be reassembled using the MAPJOIN command. The total coordinate space extends from -20,015,500, -8,673,500 to 20,015,500, 8,673,500. For this analysis the 12 projection files were applied to the appropriate grids and the resultant *PD* and *PL* value files were appended and sorted for display.

CHAPTER 6

RESULTS OF PIXEL ANALYSES: GLOBAL

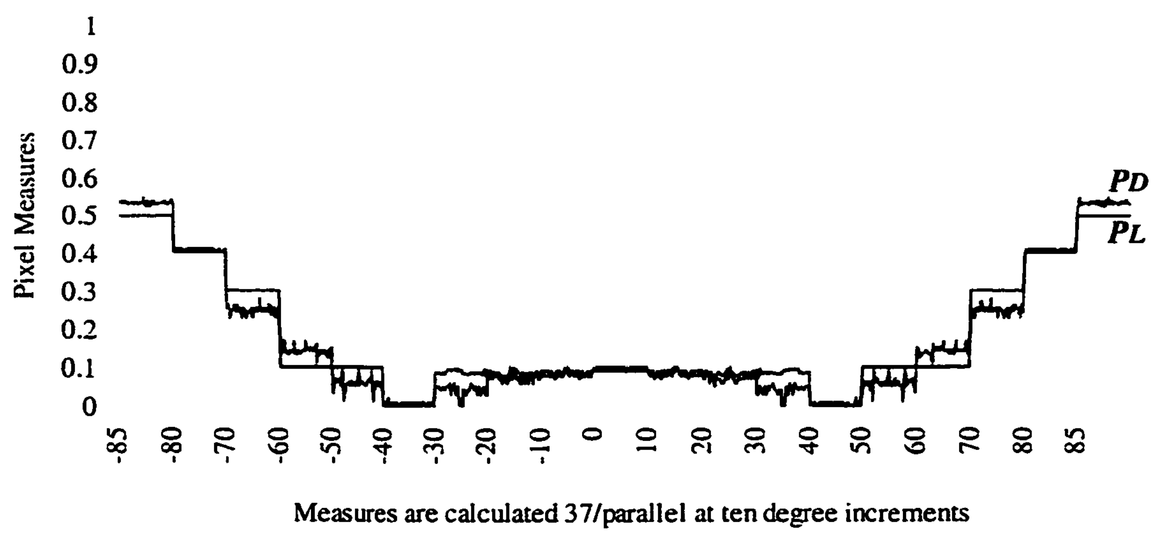
6.1 Introduction

Detailed information is available for the investigation of pixel value changes in this study through the use of the newly developed measures of P_L and P_D. The values of P_L and P_D can be mapped directly onto a map of the world or they can be shown as linear graphs. The bulk of the analysis here will be through the examination of the graphs themselves which permit a detailed examination of the pixel based distortion characteristics while maintaining some of the geographic context. Maps of the values of P_L and P_D are also included to illustrate the spatial distribution of the distortion. The P_L and P_D graph is examined first and then the three series of P_L and P_D measurements. The focus of this chapter is global in extent and the projections examined are primarily the equal area pseudocylindricals. Other global projections are examined for comparison purposes. The Mollweide and Sinusoidal projections are of particular interest because they are the component projections of the Goode Homolosine projection. The Goode projection is in use by the US Geological Survey (1997) for release of global data sets.

6.2 Examination of *PL* and *PD* Graphs

The pixel measures *PL* and *PD* were described previously as a means to track both the loss of pixels and the duplication of existing pixels simultaneously. For each map projection transformation examined, a set of 703 measurements is made. Figure 33 shows the layout of the measurement locations. The values are calculated from the lower left of the map across each parallel sequentially until the upper right of the map is reached. On the graphs shown here, the extreme left of the graph contains data near the south pole and it continues, often in a stair-step pattern, to the values near the north pole on the right. In Figure 37 the stair-step appearance is partially due to the groups of 37 measurements per parallel examined. The evaluation occurs every ten degrees with the exception of the five degree separation at 85 degrees north and south. The pattern of distortion would appear

Figure 37. --Graph of *PL* and *PD* for the Mollweide Projection



more continuous, with a smaller distance between measurements. The values along the x-axis indicate the latitudes from 85 degrees south to 85 degrees north. Note that the latitude is indicated at the starting point of a group of 37 measurements along a parallel that extends in a step to the right. All of the graphs in this chapter employ the same line symbols with the black line indicating pixel loss (***PL***) and the gray line indicating pixel duplication (***PD***).

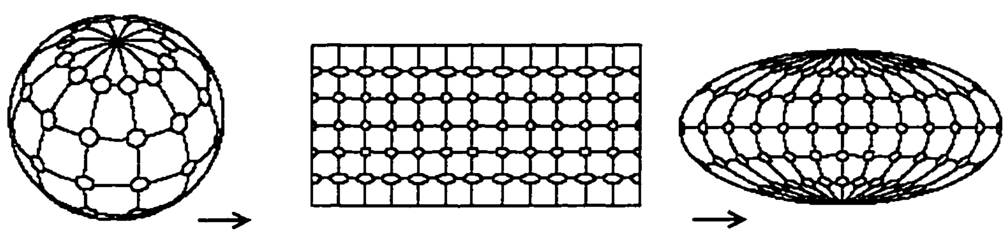
For the Mollweide projection illustrated in Figure 37 both *PL* and *PD* values are above zero throughout most of the graph, which indicates that both pixel loss and duplication are occurring simultaneously throughout the map. This result is counter intuitive to the assumption that pixels should be either duplicated to help 'fill in' areas of scale enlargement, or be eliminated in areas of scale reduction as described in Steinwand et. al (1995). The Mollweide projection shows complex behaviors of *PL* and *PD* with both pixel loss and duplication occurring simultaneously. Furthermore, the *PL* and *PD* take on very different characteristics. For latitudes greater than 40 degrees north and south, pixel loss is consistent along the parallels, indicated by the flat steps, while pixel duplication is more variable. The cause of the flat steps of consistent pixel loss compared to variable step of pixel duplication will be explored in more detail in the discussion of Figure 41.

The information seen in Figure 37 has not been explored previously by traditional point based methods of map projection distortion analysis. This area based measurement of distortion can describe some basic characteristics of the projection. For example, the Mollweide is a pseudocylindrical equal-area projection. The property of equal area is evident by the balancing of pixel loss and pixel duplication. Note the drop in *PL* and *PD* to near zero along the 40th parallel north and south in Figure 37. In the Mollweide scale is true along latitudes 40°44' north and south (Snyder 1987). Another feature of the Mollweide is the overall increase in traditionally measured angular distortion with increasing distance from the equator north and south. Likewise, both *PL* and *PD* show an increase with greater distance north and south from the equator in Figure 37.

6.3 Results of PL and PD Series 1

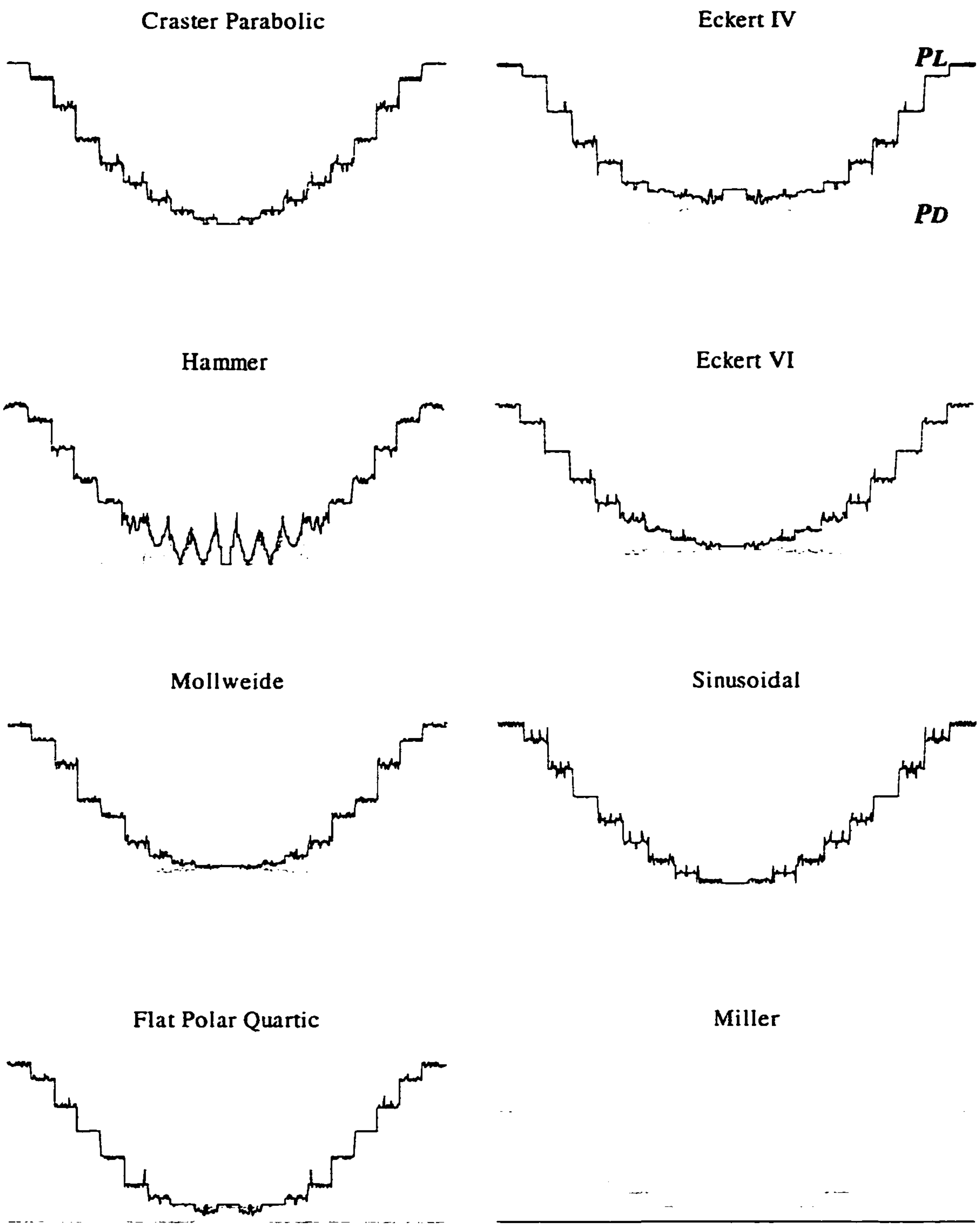
The Series 1 analysis of *PL* and *PD* was undertaken to examine the projection transformation from the Plate Carrée projection to various other projections as described in Steinwand et. al (1995). Figure 39 shows a collection of *PL* and *PD* graphs for the Plate Carrée to the other projections. Most of the projections are equal-area pseudocylindrical projections with the graphs taking on a 'smiley face' appearance and showing extreme distortion values, particularly pixel loss, in the higher latitudes. This is due in part to the fact that data sets have undergone two projection transformations. In Figure 38 the Plate

Figure 38. --Globe to Plate Carrée to Pseudocylindrical Projection Sequence



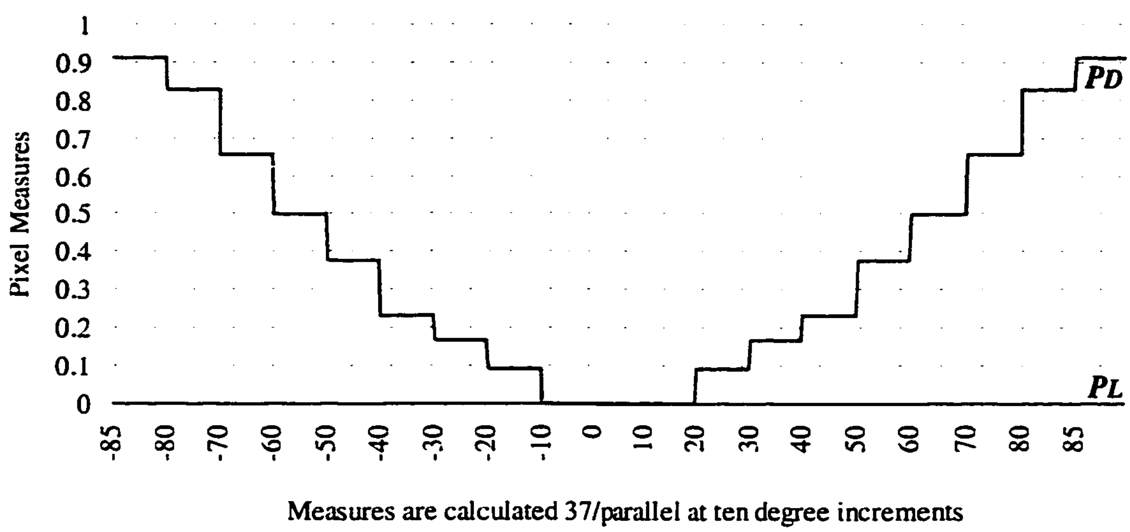
Carrée projection is shown in the center with Tissot's Indicatrix. The Tissot indicatrix indicates the type and amount of scale distortion that is occurring when projecting from the spherical surface of the earth to a planar map. The globe-like view on the left shows

Figure 39. --PD and PL for Series 1 Transformations



Tissot's Indicatrix as consistently sized circles. The Plate Carrée has a true representation of the circles only along the equator, the line of zero distortion for the Plate Carrée. The higher latitudes show a greater degree of both area expansion and angular changes. Data in the Plate Carrée projection (or in geographic coordinates in a regular raster grid), will exhibit large amounts of data duplication, particularly in the high latitudes, infinite at 90° north and south. The pattern of pixel duplication for the Plate Carrée shown in Figure 40

Figure 40. --Earth Surface to Plate Carrée Distortion Response



takes on the same smiley shape as the set of projections shown in Figure 39. The value of 0.91 for ***PD*** at 85° north and south indicates that the original 100 pixels has increased to approximately 1,111 pixels. There is no pixel loss as indicated by a flat zero line for ***PL***. In Figure 39 the distortion characteristics introduced by transformation to the Plate Carrée are superimposed upon the characteristics of the secondary projection being examined. The extreme pixel loss shown on these graphs is largely a reversal of the pixel duplication that occurs during the globe to Plate Carrée transformation. Although it is possible to interpret the graphs, it is impossible to separate the two phases of distortion that have been introduced. In order to meaningfully evaluate the distortion that is being introduced into a data set, data should be compared to an independent source of higher authority.

The examples presented by Steinwand et. al (1995) describe evaluation of distortion between the Plate Carrée and other projections. They developed the grid squares and checkerboard methods to evaluate these projection-to-projection changes. The analysis here however, indicates that this is not a valid means for determining the alterations that occur with regards to global spatial data sets. The first step in evaluating and understanding pixel based changes must be to explore a single transformation. The series 2-4 transformations in this study model the first, single transformation, from source materials that represent features on the earth's surface to a map projection. This method is analogous to comparing to a source of higher authority.

6.4 Results of PL and PD Series 2

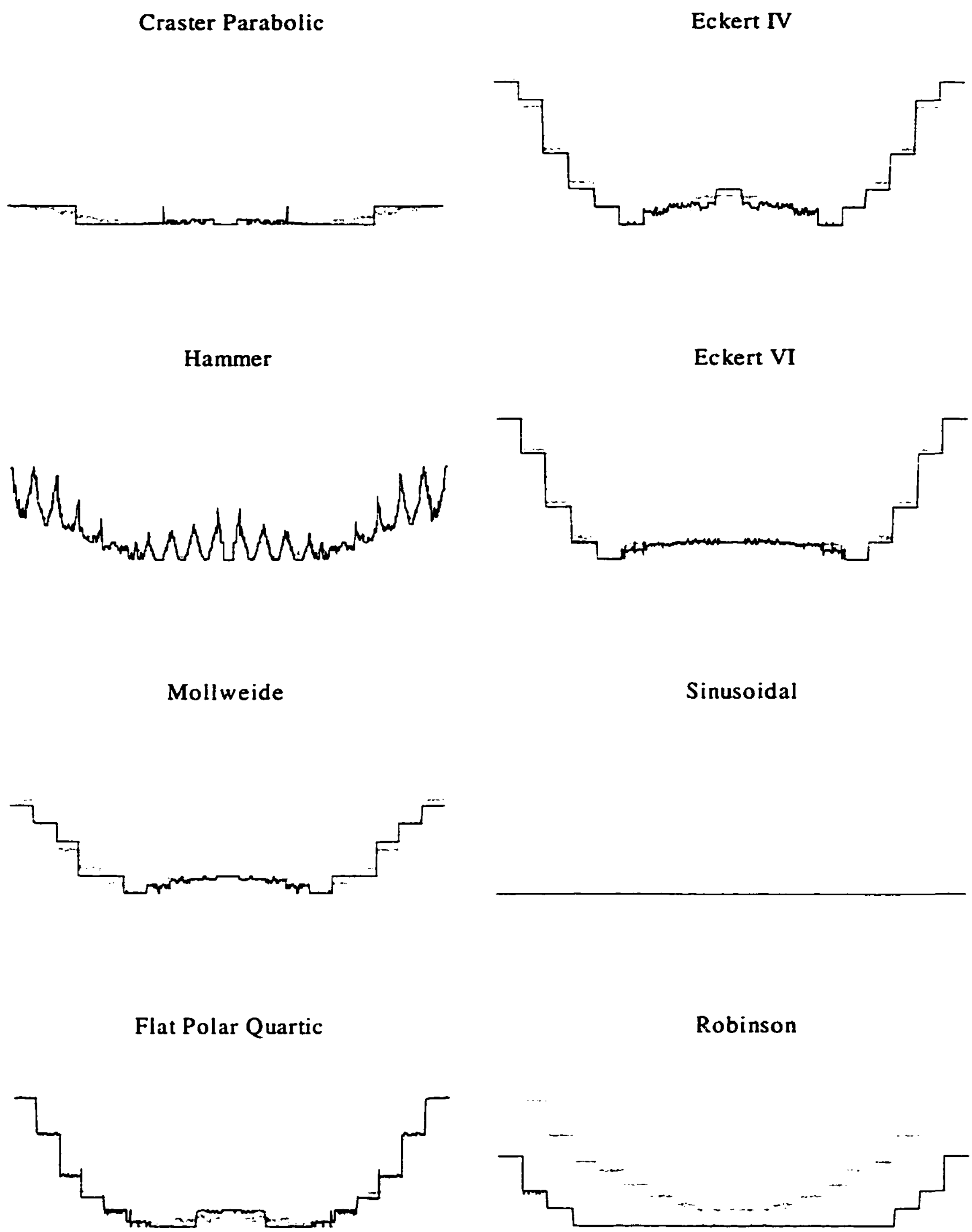
The results from the Series 2 transformations model a transformation from a source of higher authority to a single map projection. The results are therefore expressing only the changes from a single transformation and are more amenable to interpretation. One overall characteristic immediately obvious when examining world equal area projections in Figure 41 is the close correspondence between data loss and data duplication. In at least two cases it is difficult to even distinguish between ***PL*** and ***PD***. The Hammer projection for example has a very complex distortion pattern of saw tooth edges where P_L and P_D overlap. Another extreme case the Sinusoidal projection, has a very simple pattern. Both P_L and P_D evaluate to zero for the entire set of 703 measurements.

The close correspondence between values of P_L and P_D is interesting to note, and there are other significant questions. Why do projections show very different quantities and patterns of distortion? Why do P_L and P_D behave differently even when nearly balanced in amount? Is there a relationship between distortion as measured by traditional methods that would predict the best projection for a particular? Is there a "best" projection for global equal area data?

All of these map projections are very similar in that they exhibit the property of equivalence (with the exception of the Robinson which is shown for comparison). Of the projections shown in Figure 41, the Hammer projection is defined as a polyconic and the others are defined as pseudocylindricals. There is a family relationship exhibited by the two Eckert projections with Eckert VI showing lower values of P_L and P_D. The Craster Parabolic projection fits logically between the distortion characteristics of the Sinusoidal and Mollweide projections as might be expected. It was originally defined as being between these two projections in construction (Canters and Decleir 1989).

In Figure 41 the behavior of P_L is much more variable than P_D in all cases with the exception of the Sinusoid and the Hammer projections. The Sinusoidal projection is unique in that it exhibits no pixel loss or duplication. This projection is examined more closely by viewing individual grids in the context of exploring the Goode Homolosine (Figure 47 and Figure 48). In addition to the unusual zero distortion on the Sinusoid, specific details emerge that have no ready explanation. Ignoring the Sinusoid for the moment, all of the

Figure 41. --PD and PL for Series 2 Transformations

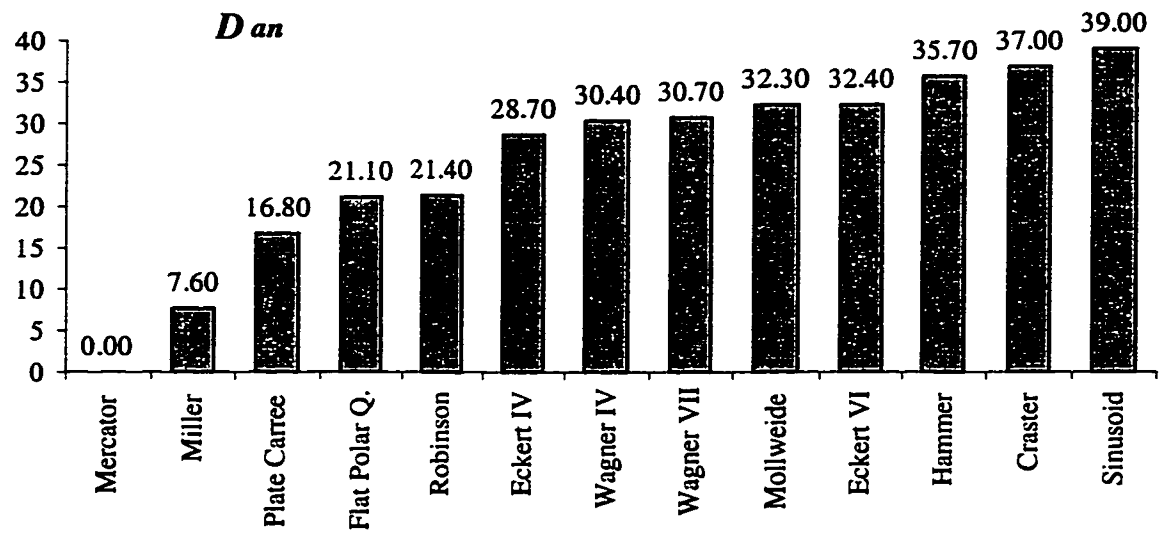


other projections with the exception of the Hammer, are classified as pseudocylindrical in construction. Both the overall all distribution of ***PL*** and ***PD*** and the differences in behavior of the two measures are similar on these projections. Throughout much of the graph, ***PL*** takes the form of flat line segments.This indicate a constant level of pixel loss along the parallels. By contrast, ***PD*** is frequently jagged. See for example the 85th parallel north and south in Figure 37. ***PL*** holds steady at 0.5 indicating a 50% loss of unique pixel values. By comparison, ***PD*** takes on values from 0.5238 to 0.5454 indicating that the total number of pixels varies from 105 to 100. Other than observing the general characteristics of ***PL*** and ***PD*** and linking them to a class of projections, the mechanism that causes the more variable ***PL*** versus the flatter ***PD*** results on the pseudocylindrical projections is unknown.

The Hammer projection, although described as polyconic by Canters and Decleir (1989) was actually derived from the Lambert Azimuthal Equal Area projection (Snyder 1987). The saw-tooth pattern of the closely aligned ***PL*** and ***PD*** values on the Hammer projection are very similar to those seen on the azimuthal projections examined in Chapter 7, Figure 49 and Figure 50.

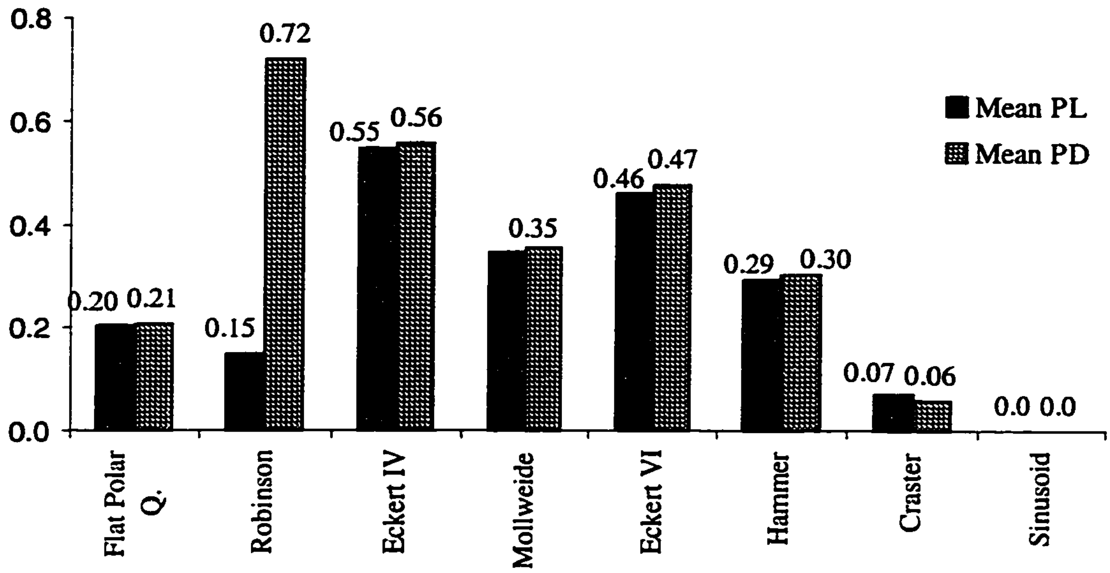
Canters and Decleir (1989) provide six distortion measures for each of the 68 world map projections in their directory. Figure 42 shows the angular distortion values for all of the projections shown in Figure 41 as well as several cylindrical projections and the Wagner IV and VII projections recommended by Steinwand et. al. (1995). The measure employed, D_{an} is the mean angular deformation calclulared by averaging the maximum angular deformation throughout the map. The results of the ***PL*** and ***PD*** evaluations shown in Figure 41 indicate that the Sinusoidal and Craster-Parabolic projections have the lowest degree of pixel distortion and this corresponds to the highest levels of angular distortion.

Figure 42. --Angular Distortion for Projections by Canters and Decleir



Mean values for *PL* and *PD* are illustrated in Figure 43 and confirm that, as might be expected for the equal area projections, pixel loss and duplication are closely associated. To maintain the property of equal area the loss and duplication of pixels must be balanced. The exact relationship between low measures of pixel distortion and high values of angular distortion as measured by traditional methods is less clear. The map projections in Figure 43 are in the same order as in Figure 42, from low to high angular distortion. The

Figure 43. --Mean PD and PL Values for Series 2 Transformations



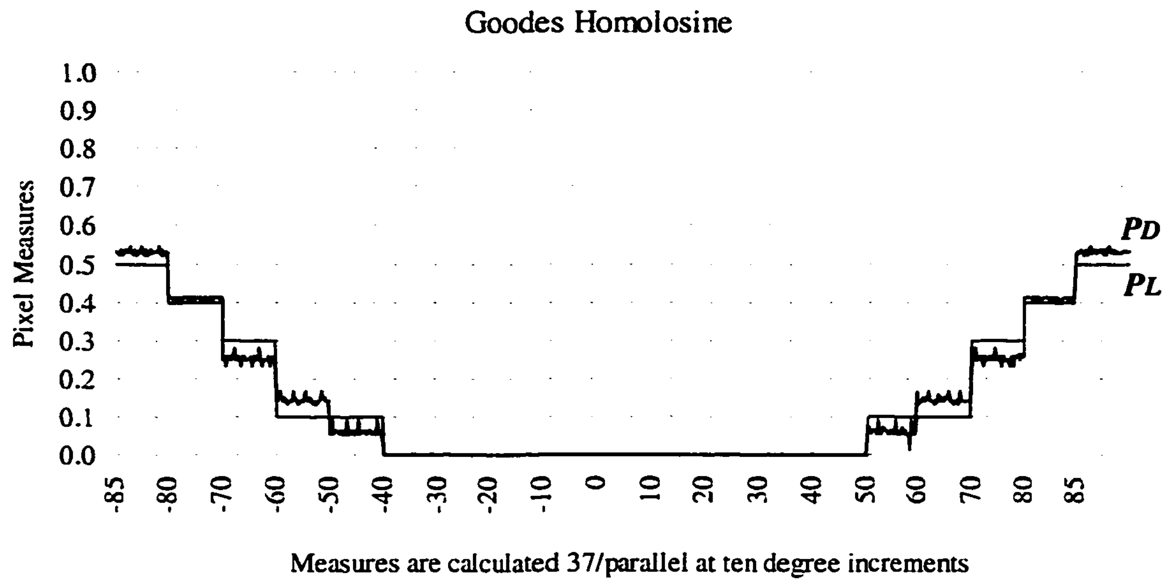
relationship of high angular distortion and low pixel count change holds fairly well for the Sinusoid, Craster, Hammer, Eckert VI, Mollweide, and Eckert IV but not for the Flat Polar Quartic projection.

The relationship of low pixel distortion and high angular distortion is not consistent with traditional map projection selection guidelines. During the selection of an equal area world map projection the projection with the lowest angular distortion would be recommended as the minimal error choice. At least for raster data sets, a low angular distortion choice would lead to increased changes in the pixel values.

6.5 Results of PL and PD Series 3

The Goode Homolosine projection is composed of twelve independently projected segments and employs both the Sinusoidal and Mollweide projections. Figure 44 graphs the results of the separate calculations of distortion characteristics for each map segment and the agglomeration of the data values into a continuous graph. Comparison of Figure 44 with

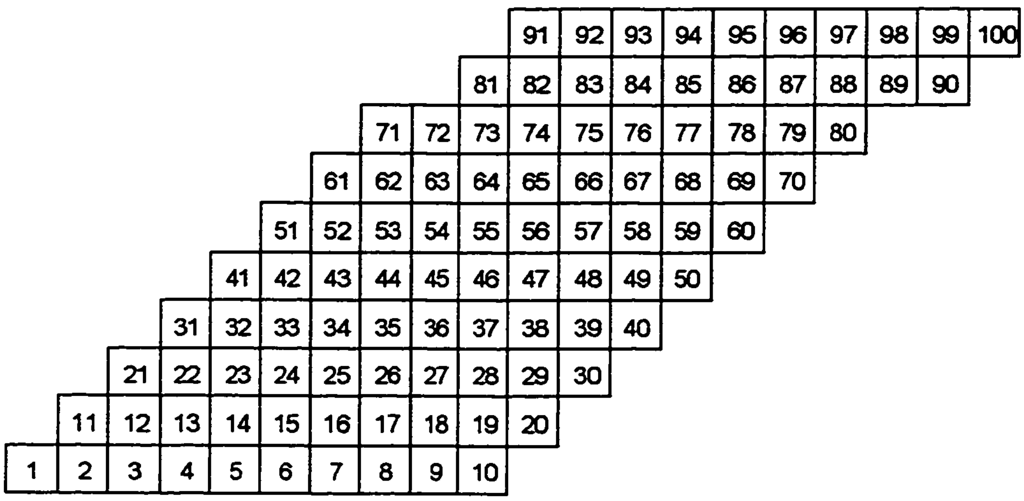
Figure 44. --PD and PL for the Series 3 Transformations



the Sinusoidal and Mollweide graphs in Figure 43 is straightforward. The zero level of distortion of the Sinusoidal projection extends through the sections of the map that are composed of the Sinusoid projection. The stair step appearance of the Mollweide projection is evident above 40 north and south at the latitudes in which the Mollweide is employed.

Currently the USGS EDC currently uses the Goode Homolosine projection for release of the Global Land Characteristics (USGS 1997) data set and others. An alternative to this interrupted projection may be the Sinusoid projection even with its extreme angular shearing at high latitudes. Figure 45 consists of the ten-by-ten grid #490 after projection to

Figure 45. --Grid #490 at 40N, 100W after Transformation to Sinusoidal Projection



the Sinusoidal. A remarkable characteristic of the Sinusoidal projection is the maintenance of all pixels without duplication even with a large amount of geometric reorganization of the grid shape.

To verify that there are no changes in pixel values when projecting to the Sinusoidal projection a sample of five grids was selected for closer examination. These grids span the range of deformation that may be expected when transforming the entire globe to the

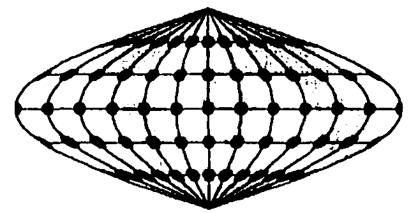
Sinusoidal projection. See Figure 47 and Figure 48. The northwest quadrant of the globe was used to select the grids. The Mollweide projection was chosen for comparison with the Sinusoidal projection.

Moving from south to north, the first two grids are 351 and 334. The results from projecting grid 334 are exactly the same as for 351 which is shown in Figure 47. At 10W, 0N, grid 351 is very close to the origin of the projection at 0,0. Grid 334 is also centered upon the equator but at 175W longitude. The location along the parallel is irrelevant. These grids show ~10% pixel loss and duplication. The Mollweide grid has been compressed east-west by one column and expanded north-south by one row. Grid 334 in the Sinusoid projection has not been altered at all.

Grid 490 at 100W, 40N is significantly altered geometrically but only one pixel has been lost on the Mollweide and no pixels values are altered on the Sinusoid case.
This is close to the parallel of optimum distortion characteristics for the Mollweide by traditional methods of evaluation. Both cases of 490 show significant geometric alterations in terms of the overall grid. There is a shift to the east for both towards the direction of the central meridian.

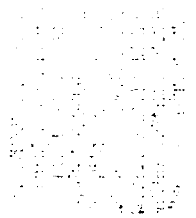
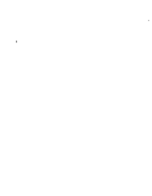
The next two sets of grids occur along the 80th parallel. The first, grid 647 in Figure 47 is located at close to the central meridian at 10W and 631 in Figure 48 is located at 170W. Grid 647 shows minimal rotation towards the central meridian but shows significant overall shape changes in the Mollweide case. It has been compressed north-south from ten to six rows and expanded east-west from ten to seventeen columns. After projection the grid has only 60 unique values remaining from the original 100 and those 60 values are spread over 102 of the resultant pixels.

Figure 46. --Northwest Quadrant



669	670	671	672	673	674	675	676	677	678	679	680	681	682	683	684
632	633	634	635	636	637	638	639	640	641	642	643	644	645	646	647
595	596	597	598	599	600	601	602	603	604	605	606	607	608	609	610
558	559	560	561	562	563	564	565	566	567	568	569	570	571	572	573
521	522	523	524	525	526	527	528	529	530	531	532	533	534	535	536
484	485	486	487	488	489	490	491	492	493	494	495	496	497	498	499
447	448	449	450	451	452	453	454	455	456	457	458	459	460	461	462
410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425
373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388
336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351

Figure 47. --Comparison of ten-by-ten grids #351, #490, and #647

	Mollweide	Sinusoid
#351 10W, 0N	 P_L = 0.1 P_D = 0.09	 P_L = 0 P_D = 0
#490 100W, 40N	 P_L = 0.01 P_D = 0	 P_L = 0 P_D = 0
#647 10W, 80N	 P_L = 0.4 P_D = 0.41	 P_L = 0 P_D = 0

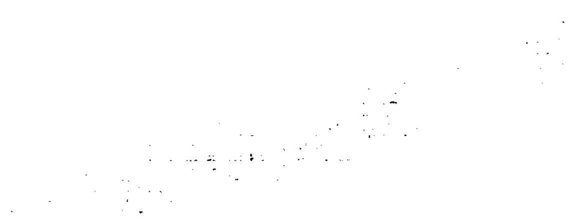
Grid 631 in Figure 48 shows extreme geometric distortion for both the Mollweide and Sinusoid cases. Similar to grid 490, the Mollweide has PL and PD values ~0.4, along with north-south compression and east-west expansion, but with a greater rotation towards the central meridian. The Sinusoidal case of grid 631 appears similar in terms of these characteristics but has a very significant difference. There are no pixel value changes on the Sinusoid case.

Figure 48. --Comparison of ten-by-ten grids #647 at 170W, 80N

Mollweide

P_L = 0.4
P_D = 0.411

Sinusoid



P_L = 0
P_D = 0

6.6 Summary: Global Level

The pixel measures ***PL*** and ***PD*** can be used as an effective means to track both the loss of pixels and the duplication of existing pixels simultaneously. Graphed, ***PL*** and ***PD*** can identify distortion characteristics by latitude and identify the presence or absence of the of equal area property (Figure 41). To be an effective tool for understanding and predicting the degree of pixel alteration in a data set ***PL*** and ***PD*** distortion characteristics introduced by transformation to the Plate Carrée must not be superimposed upon the characteristics of a secondary projection (Figure 39 and Figure 40). In order to meaningfully evaluate the distortion that is being introduced into a data set, data should be compared to a source of higher authority.

The nature of pixel based changes appears to be related, at least, partially to the functional nature of the projection. A number of the pseudocylindrical projections show complex behaviors of *PL* and *PD* with both pixel loss and duplication occurring simultaneously. The exact mechanism that leads to steady ***PL*** values versus variable ***PD*** values on the pseudocylindrical projections is unknown. This phenomena, as well as overall distortion patterns may be closely related to the general functional nature of the projections. The Hammer projection (Figure 41) and the azimuthal projections (Figure 49 and Figure 50) show very similar saw-toothed distortion responses that are completely different from the pseudocylindrical projections.

There is also a relationship between angular distortion and pixel distortion even if the exact relationship is not yet well defined (Figure 42 and Figure 43). More importantly the relationship of low pixel distortion and high angular distortion is not consistent with traditional map projection selection guidelines. During the selection of an equal area world map projection the projection with the lowest angular distortion would be preferred. When raster data sets are being transformed by one of the pseudocylindrical equal area map projections the low angular distortion choice may lead to increased changes in the pixel values. This fact may indicate that the USGS currently uses a less than ideal projection for release of the Global Land Characteristics (USGS 1997) data set and others. A zero pixel altering alternative to this interrupted projection may be the Sinusoid, even with its extreme angular shearing at high latitudes.

CHAPTER 7

RESULTS OF PIXEL ANALYSES: CONTINENTAL

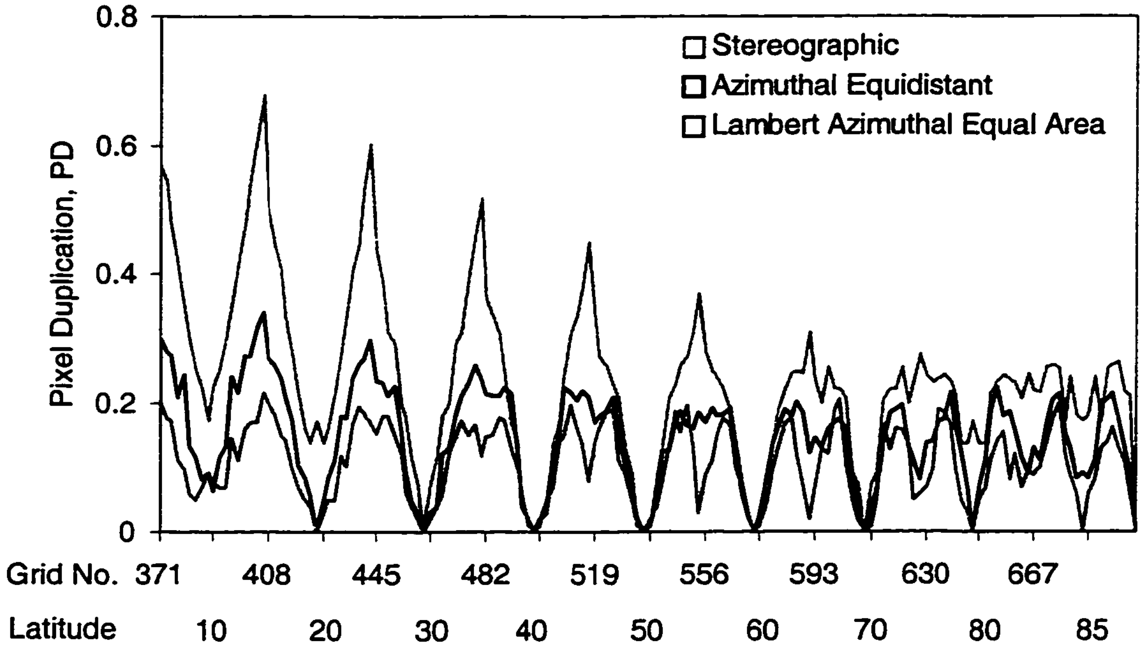
Unlike the previous chapter, the focus of this analysis is restricted to the extent of a single continent. The Lambert Azimuthal Equal Area projection is of particular interest because Steinwand et. al. (1995) recommended its use and it has been implemented nationally for the distribution of raster data. The goals of the continental based analysis are to explore the patterns of pixel distortion over a continent. The North American continent is examined first by comparing the results from three azimuthal projections. The South American and African continents are then examined using the Lambert Azimuthal Equal Area.

Like the global analysis there are many basic questions to be answered. What are the general patterns of P_L and P_D for the projections? Are the patterns of P_L and P_D for these projections circular as predicted by standard map projection analysis? Do the distortion patterns vary by the type of projection properties retained in the chosen projection? Do the distortion patterns vary by aspect? A close examination of the azimuthal projections may provide basic information to assist in understanding the relationship of pixel changes and map projection transformations.

7.1 Three Azimuthal Projections

The grids used for this analysis are a subset of the same 703 grid used for the global analysis. For each parallel of latitude eighteen grids are examined. Figure 49 compares the pixel duplication for three azimuthal projections. The three shown exhibit the properties of conformality, equivalence and equidistance The grid numbers shown on the x-axis indicate

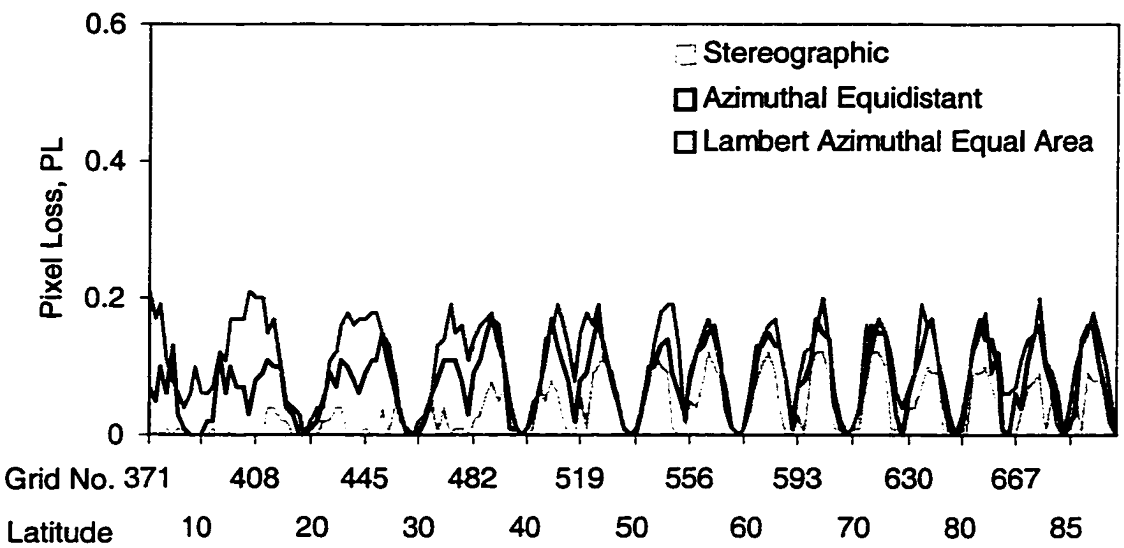
Figure 49. --Comparison of Pixel Duplication for Three Azimuthal Projections



the most westward grid of a west to east series along a parallel of latitude. The center of the projection is 100W, 50N, and is represented by grid number 527. This coordinate pair was defined by Steinwand et. al. (1995) as the optimal center for minimal error characteristics for the North American continent. The valleys in Figure 49 represent the evaluation of ***PD*** along the central meridian of the projection at 100W. The peaks occur towards the extreme west and east of the projection center.

All three projections exhibit similar pattern of distortion, although the Stereographic projection clearly exhibits the highest amount of pixel duplication. There is

Figure 50. --Comparison of Pixel Loss for Three Azimuthal Projections

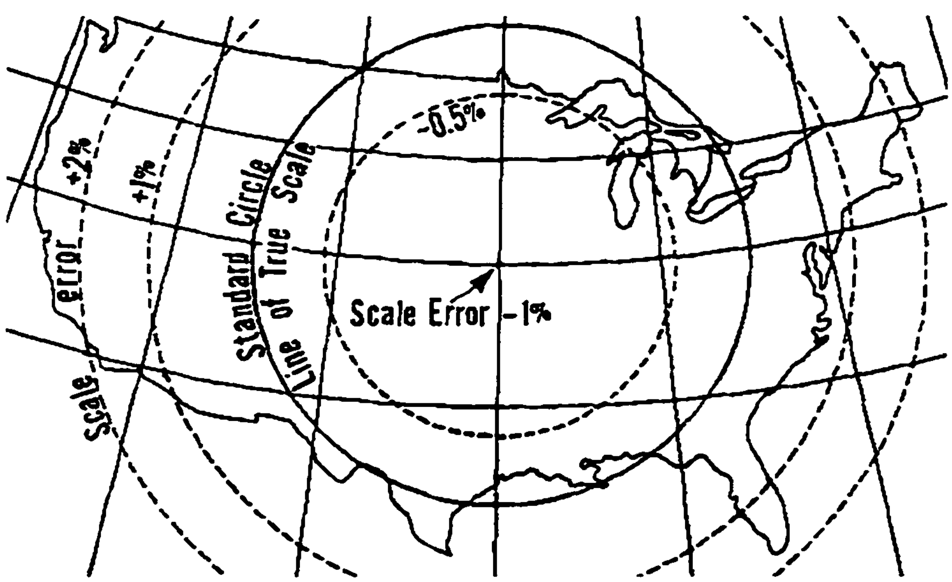


low or no duplication of pixels at the tangent point and along the central meridian. Distortion is symmetrical along the central meridian and less symmetrical along the parallel of tangency. There is a trend among all three projections for greater values of ***PD*** towards the southern extreme and lower values of ***PD*** at high northern latitudes. The Lambert Azimuthal Equal Area comes the closest to exhibiting a symmetrical pattern of pixel duplication.

In Figure 50 the same three projections are compared with regards to pixel loss. The primary difference between the pixel duplication and pixel loss is the lower values of ***PL*** for the Stereographic and the Azimuthal Equidistant projections. The degree of pixel loss is approximately the same as pixel duplication for the Lambert Azimuthal Equal Area projection. This is to be expected for an equal area projection. A distinctive feature of both ***PL*** and ***PD*** is a drop in distortion values on the outside edges of the projection. These are indicated by the secondary valleys at grid values above the numbered grids.

Map projection distortion on an azimuthal projection as measured by traditional point based methods exhibits a pattern of distortion that consists of concentric circles around the tangent point or line.The amount of distortion increases with distance from the tangent point or line. The spatial patterns of both ***PL*** and ***PD*** exhibit a dramatic departure from the circular distortion patterns. See Figure 52 through Figure 54 for distortion patterns

Figure 51. -- Concentric Circles of Distortion on the Stereographic



Source: Snyder (1987)

on the Lambert Azimuthal Equal Area, Azimuthal Equidistant and Stereographic projections. The patterns exhibited by P_L and P_D bear little resemblance to the ideal distortion patters for an azimuthal projection.

Figure 52A depicts the pattern of P_L and Figure 52B the pattern of P_D. These graphics were created by interpolating the measured values of P_L and P_D. The set of points used for the interpolation are relatively sparse and therefore the surfaces produced contain artifacts. A greater density of measurements would produce a smoother surface but the essential spatial structure of the distortion pattern is evident.

Instead of the expected circular pattern the distortion pattern in Figure 52 is generally triangular with a vertical strip that has no pixel loss or duplication. The Lambert Azimuthal projection has the property of equivalence and this is evident in Figure 52 by the fact that both parts A. Pixel Loss, and part B. Pixel Duplication, have nearly identical distortion patterns.

The distortion on the Azimuthal Equidistant projection as seen in Figure 53 also shows a triangular pattern of distortion for both P_L and P_D. Overall the values for P_L in Figure 53A are lower than for P_D in Figure 53B. This was clear on the graphs shown in Figure 49 and Figure 50. The Azimuthal Equidistant projection expands the outer edges of the projection space to retain the property of equal distance from the center of the projection outwards in all directions.

The Azimuthal Equidistant and the Stereographic projection have similar distortion characteristics with regards to P_L and P_D. The Stereographic projection has a greater range of values however. As seen in Figure 54A and Figure 54B, the overall pixel loss is less than

Figure 52. --Pixel Distortion on the Lambert Azimuthal for North America

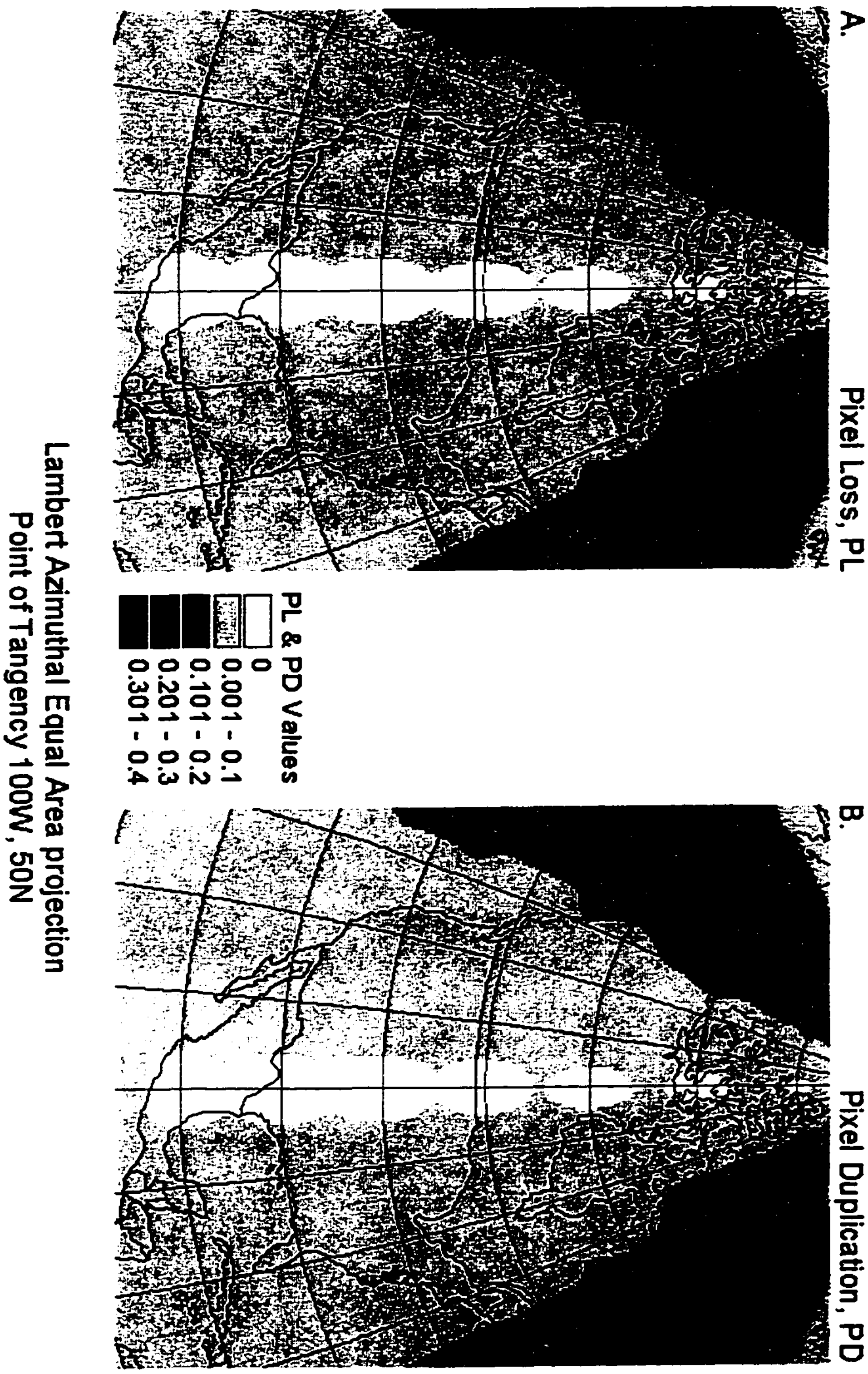


Figure 53. --Pixel Distortion on the Azimuthal Equidistant for North America

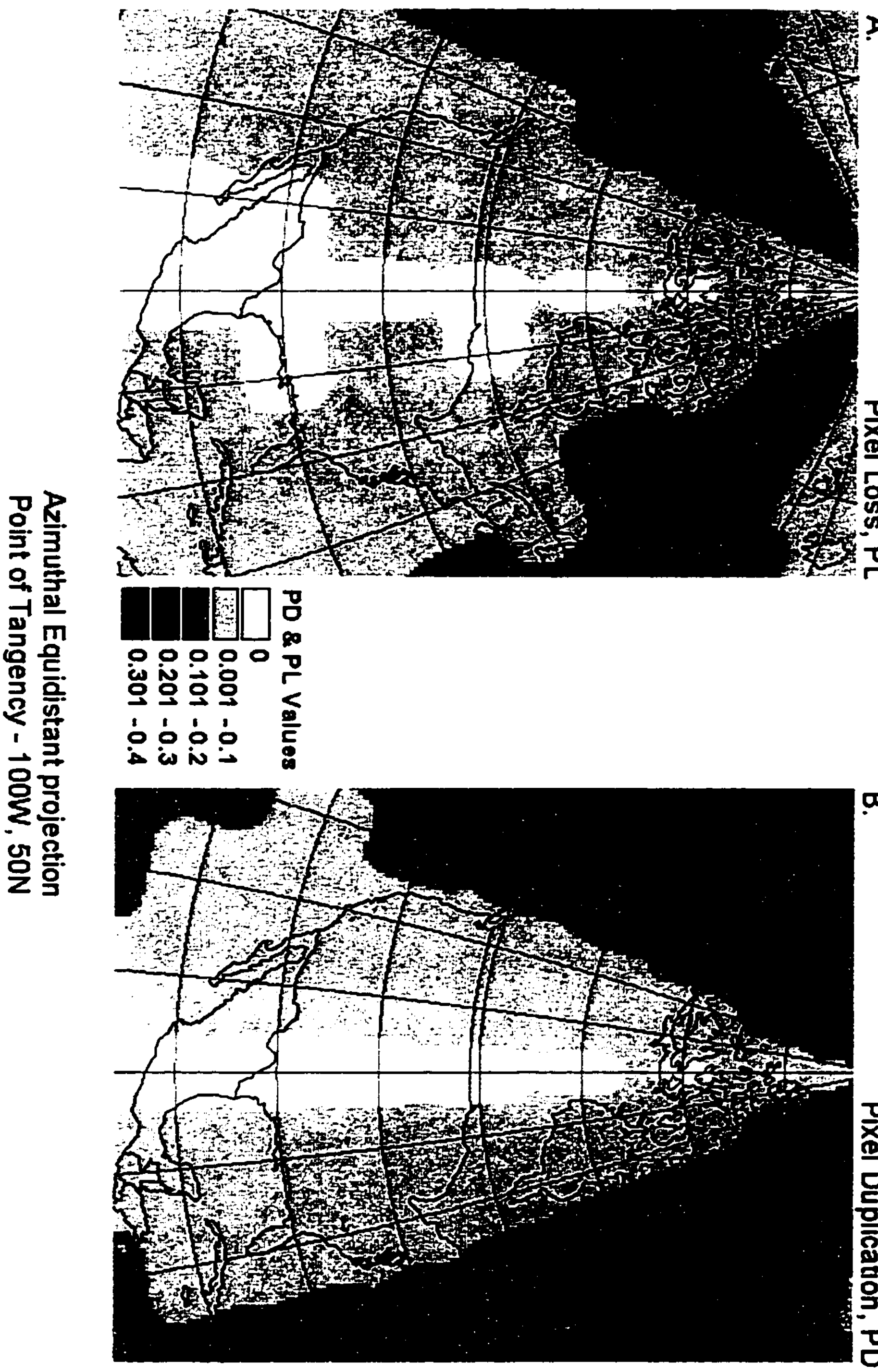
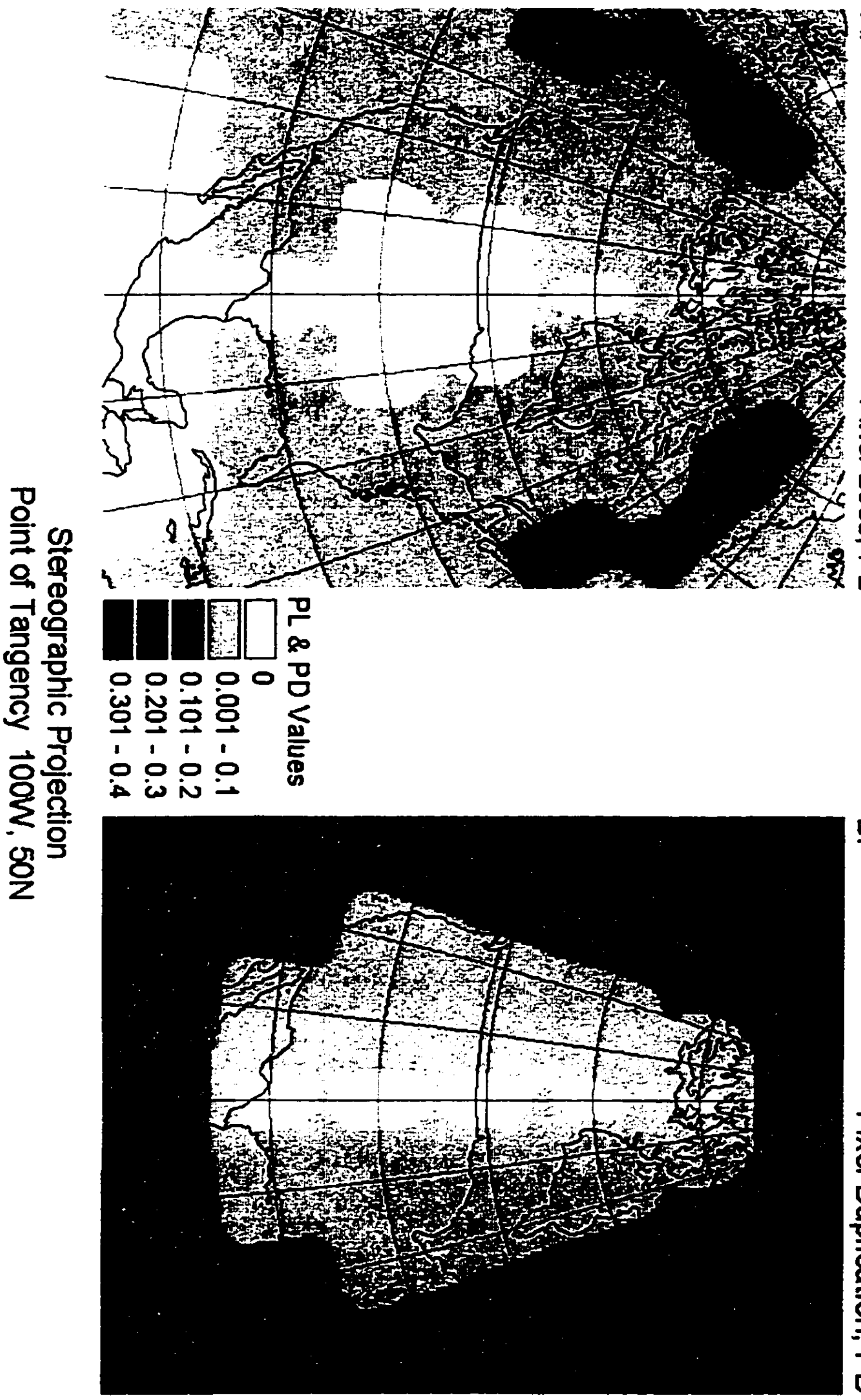


Figure 54. --Pixel Distortion on the Stereographic for North America



the Azimuthal Equidistant but the overall pixel duplication is much greater. Figure 54B also has the least amount of anomalous values on the extremes of the map. All of the other projections have dips in the values of ***PL*** and ***PD*** in these areas.

7.2 Lambert Azimuthal - North America and Africa

A closer examination of these distortion patterns for the individual grids was made. Figure 55 depicts the grids that result from projecting each 10km x 10km grid from the center of a stereographic projection to the Lambert Azimuthal Equal Area projection centered upon 100W, 50W. The grids are spaced ten degrees apart and therefore represent longitude 120W - 80W and latitudes 40N - 60N. The sizes of the grids in Figure 55, shown with the graticule for the Lambert projection, are not proportional to the ten degree graticule shown.The grids are shown with the graticule to explore the relationship of the spatial reorganization of the grids to the angular changes in the graticule.

The direction and degree of reorganization of the grids is related to the angular changes in the graticule. The meridian passing through the center of the projection at 100W remains at a right angle to the parallels crossing it. The grids, created originally in alignment with the graticule, are not altered. The pixels do not appear to shift and no alteration of the pixel values takes place. The values of ***PL*** and ***PD*** are zero for grids numbered 564, 527, and 490. See Table 20. The grids to the east and west of the central meridian are rotated in amount and direction following the graticule. Patterns formed by the pixels of the grids are nearly symmetrical on either side of the central meridian.

Table 20: Selected *PL* and *PD* Values, LAEA - North America

Grid ID	*PL*	*PD*
489	0.01	0.01
490	0.00	0.00
491	0.01	0.01
525	0.05	0.05
526	0.01	0.01
527	0.00	0.00
528	0.01	0.02
529	0.04	0.04
563	0.01	0.02
564	0.00	0.00
565	0.01	0.01

The values of *PL* and *PD* for grids 563, 526, 489, 565, 528 and 491 is 0.01 with two exceptions of values of 0.02. Changes at the level of 0.01 indicate that a single pixel value has been lost or duplicated. Although the individual pixels are not labeled, grid 489 in Figure 55 has lost the pixel value of #6 and the #96 pixel has been duplicated. The location of a lost pixel value or duplicated value in these six grids is random. It is interesting to note that - at least when measured by these 10km by 10km grids - pixel loss and duplication are not equal in two of the six cases. Grid 528 for example has a total of 101 pixels with 99 unique values. It has actually shown an increase in area.

The grids further to the east and west, numbers 525 and 529, show a greater rotation in general orientation of the grids and higher *PL* and *PD* values. The *PL* and *PD* values are 0.05 and 0.04 respectively. Grid 529 still has a total of 100 pixels but there are only 96 unique values and four of the existing pixels values are duplicated. Moving even further away from the center of the projection, the degree of rotation of the pixels from the original

Figure 55. –Grid Distortion on the Lambert Azimuthal for North America

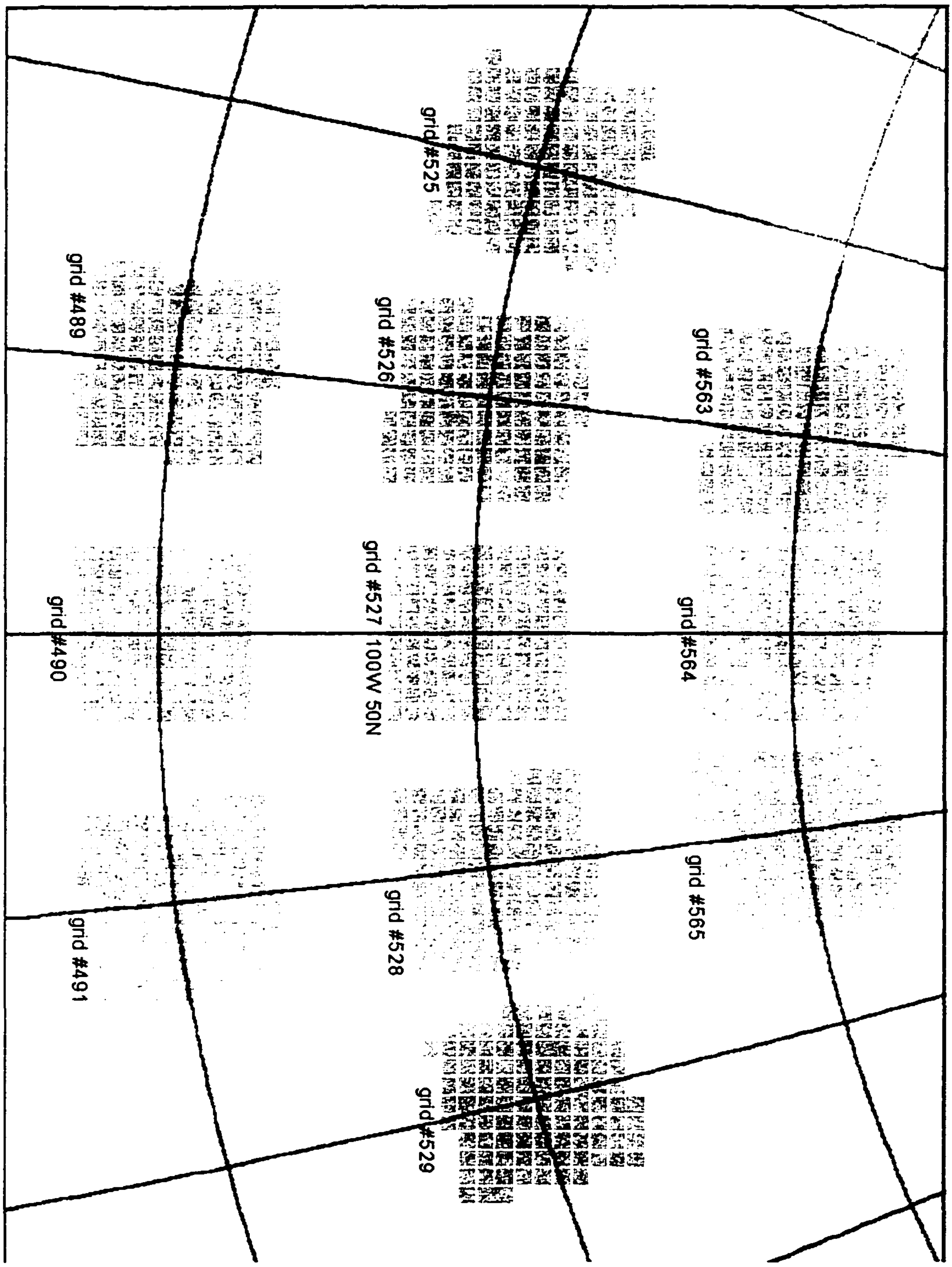
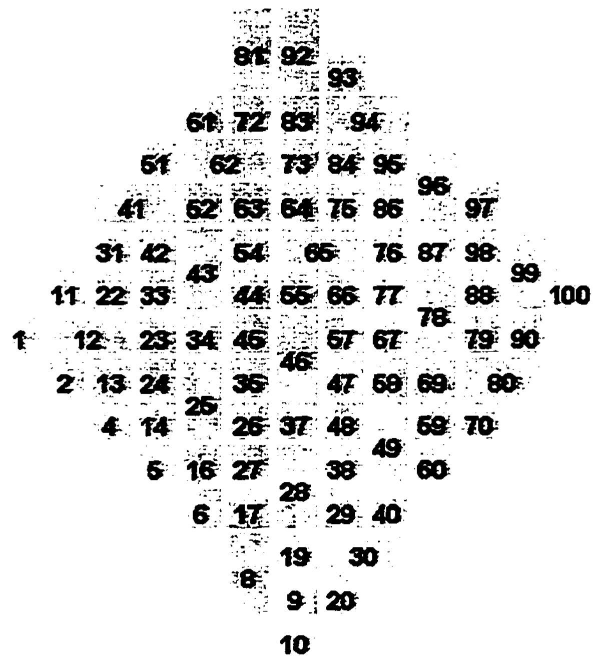


Figure 56. --Distortion of Grid 447



square shape increases. See Figure 56. Grid 447 is located at 160W, 30N and has now been rotated 45 degrees east towards the central meridian from the original. The ***PL*** and ***PD*** values for grid 447 are both 0.18.

As mentioned previously, traditional map projection guidelines predict a regular, circular pattern of distortion on all of the azimuthal projections. Further they predict that this distortion is constant regardless of the aspect of the projection. Clearly distortion is not circular on the azimuthal projections for North America, but are distortion patterns invariant of aspect? In Figure 57 and Figure 58 the African continent is examined. Again, the parameters recommended by Steinwand et. al. (1995) for the Lambert Azimuthal Equal Area projection are employed.

The patterns of distortion for the African continent in Figure 57 vary considerably from those for North American in Figure 52. The point of tangency of this aspect is at 20E, 5N. The pixel duplication over Africa (Figure 57B) is greater than for pixel loss in Figure

57A. A closer view of these changes appear in Figure 58 where five grids are examined. The degree of rotation is less for these grids and must be related to the lesser amount of angular change in the graticule close to the equator. These grids exhibit greater pixel duplication than pixel loss (Table 21). In four of these five grids pixel duplication exceeds pixel loss between 2% and 6%. The reason for this apparent increase in area is unclear.

Table 21: Selected *PL* and *PD* Values, LAEA - Africa

Grid ID	*PL*	*PD*
244	0.00	0.06
394	0.03	0.05
388	0.03	0.05
466	0.00	0.03
462	0.02	0.02

Finally, the last continent examined is South America. The point of tangency for this aspect is 60W, 15S (Figure 59). Instead of being shifted slightly north of the equator, this case is 15S. The pattern of distortion is different again, although it is more similar to the North American case than the African case. The pattern of distortion begins to approach the triangular shape as in Figure 52A and B but is less well defined. Similar to the North American case, the angular changes in the graticule appear closely related to the changes in the pixel values.

7.3 Summary: Continental Level

An examination of the spatial patterns of continental scale pixel loss and pixel duplication on the azimuthal projections provides a significant amount of basic information. The most fundamental information exposed indicates that point based

Figure 57. --Map of *PL* and *PD* for the African Continent - Lambert Azimuthal

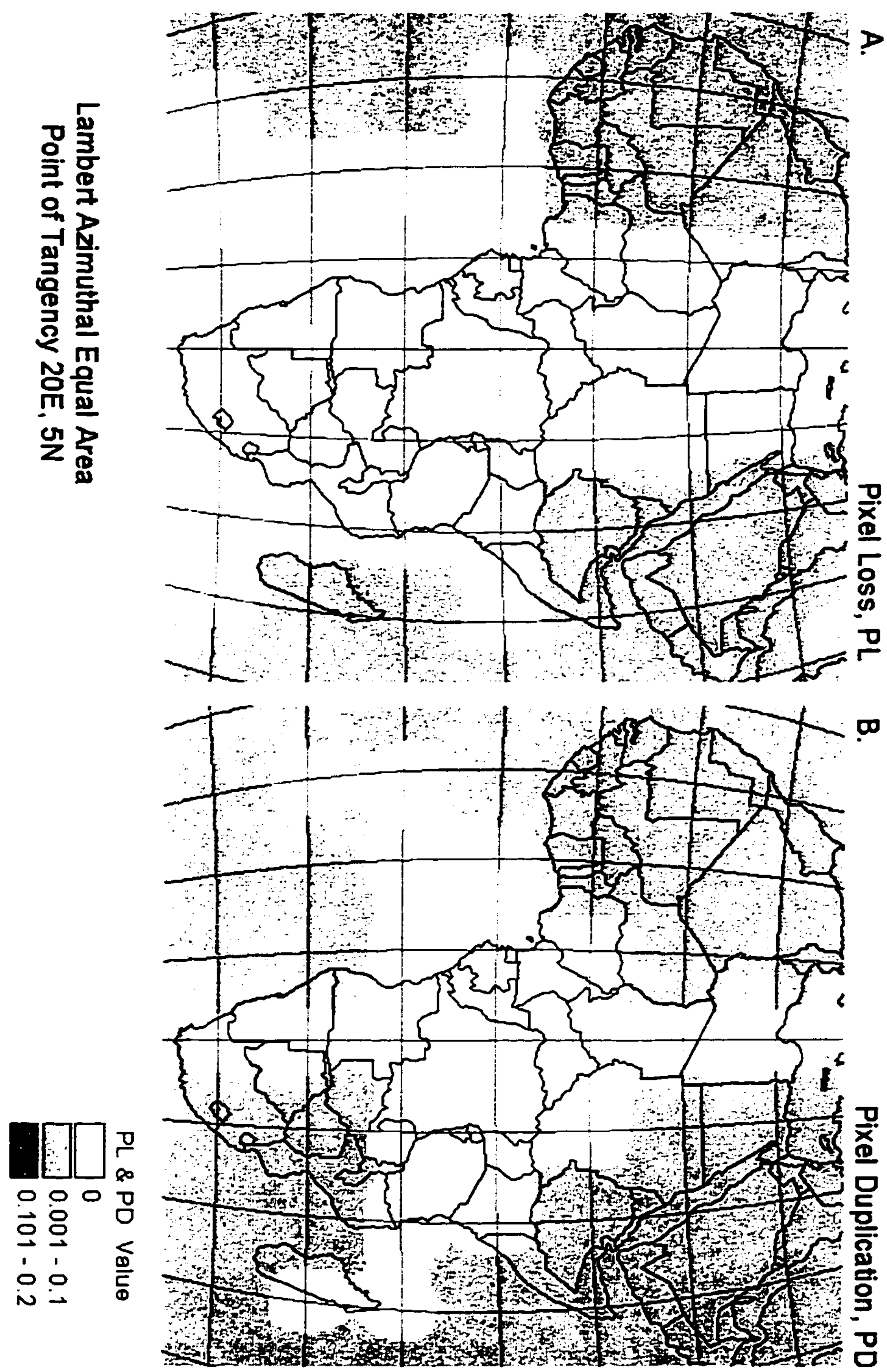


Figure 58. --Grid Distortion for the African Continent - Lambert Azimuthal

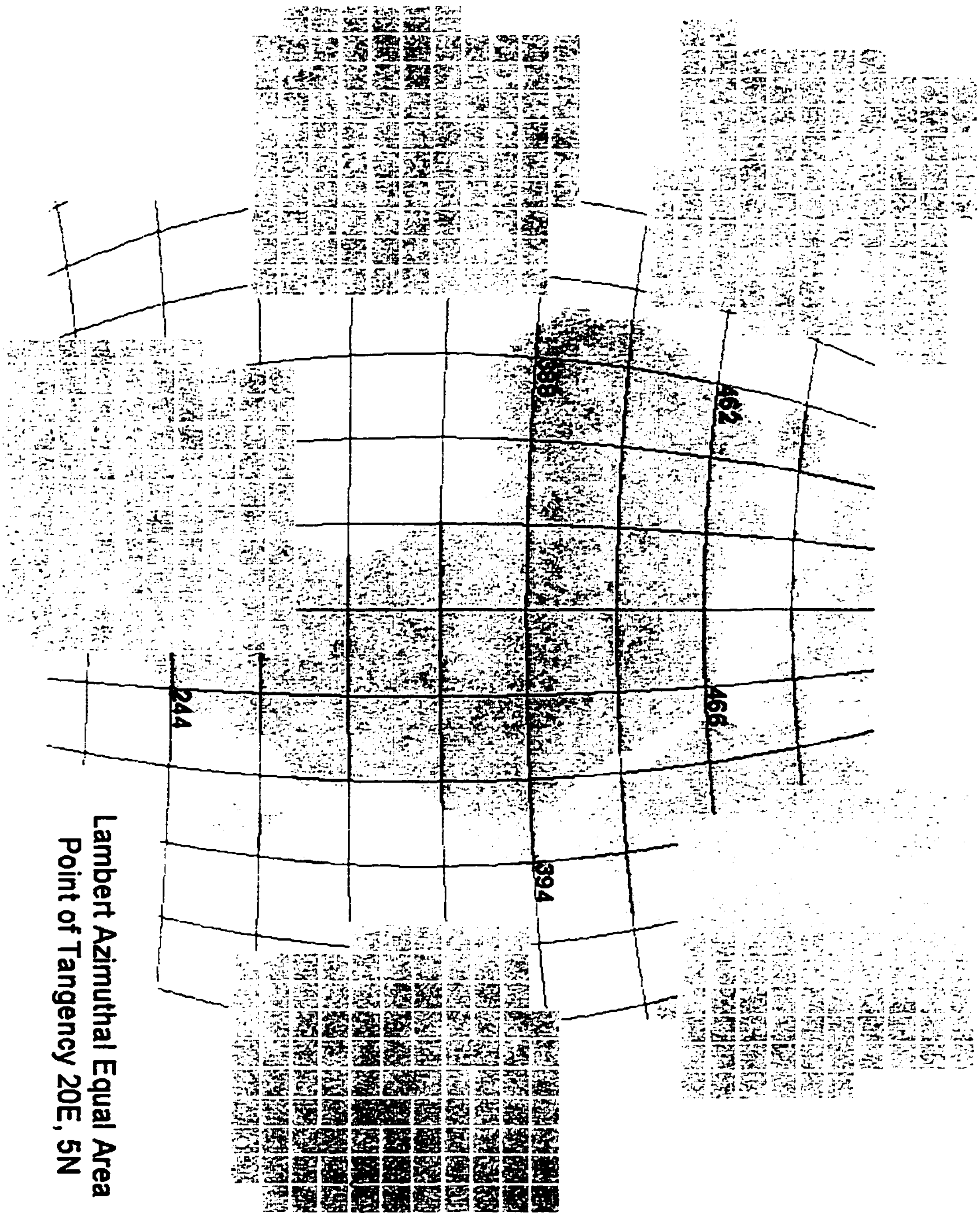
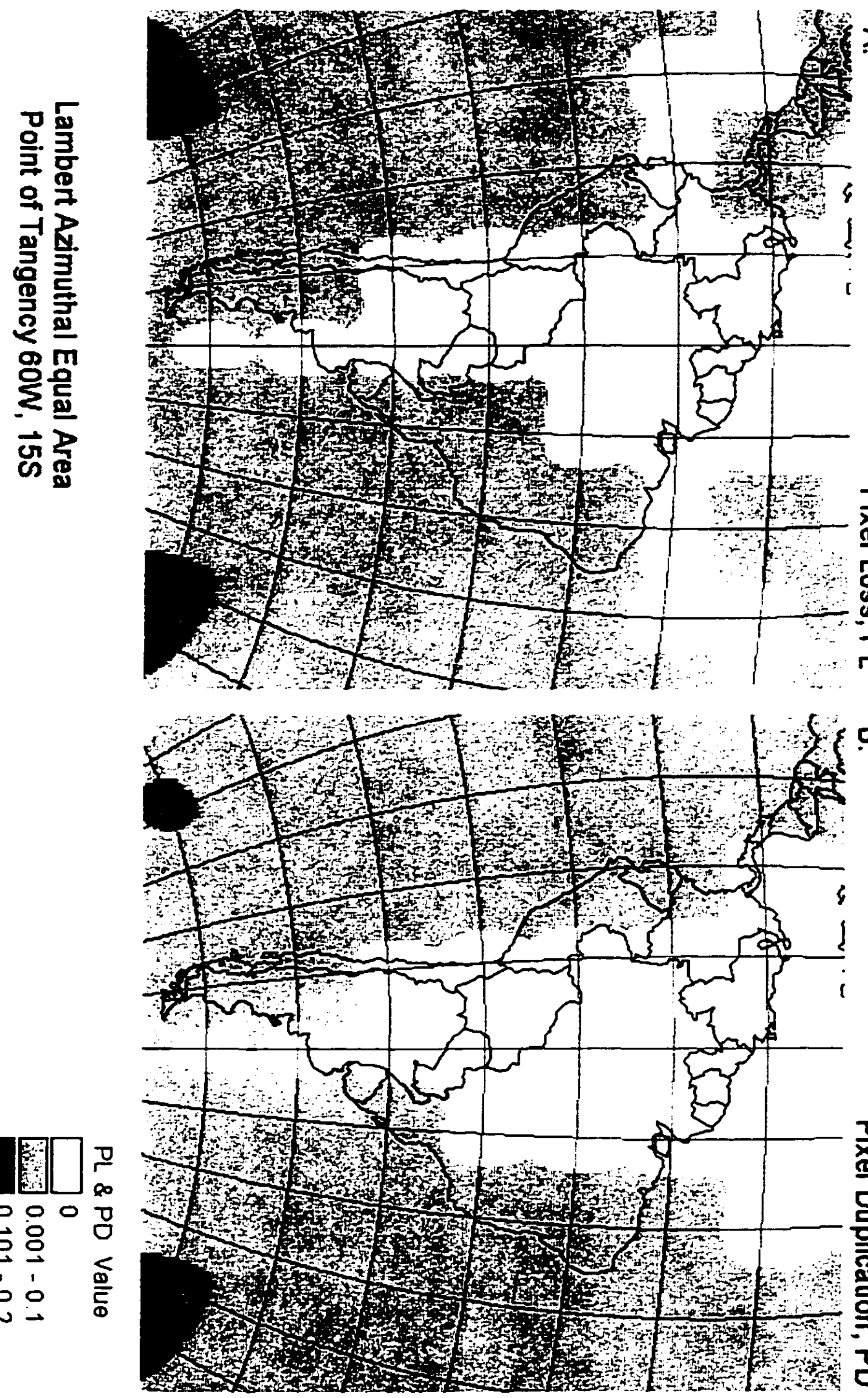


Figure 59. --Map of *PL* and *PD* for the S. American Continent - Lambert Azimuthal



evaluation of map projection distortion, such as with the application of Tissot's theorem does not correspond closely with the area based method of ***PL*** and ***PD***. Instead of regular circular patterns of distortion increasing radially from a tangent point (Figure 51) - distortion patters are more irregular (Figure 52 - Figure 54). These patterns exist in the point measurements before interpolation for visualization. A more dense sampling may refine the patterns, but the essential spatial distributions would remain the same. Distortion patterns may be triangular or ill defined in terms of a regular geometric pattern.

Traditional point based methods also predict an invariant pattern of distortion. All of the azimuthal projections are expected to produce circular patterns of distortion that are invariant with respect to the aspect of the projection. Examination of pixel value changes using ***PL*** and ***PD*** indicate that distortion patterns are influenced heavily by the changes occurring in the graticule (Figure 52, Figure 57 and Figure 59). Unlike the pseudocylindrical projections, higher angular distortion on the azimuthal appears linked to greater ***PL*** and ***PD*** distortion.

Similar to traditional distortion measures, distortion measured by ***PL*** and ***PD*** vary by projection. The Lambert Azimuthal Equal Area, the Azimuthal Equidistant and the Stereographic projections exhibit different distortion patterns that are related to changes in local scale (Figure 52 - Figure 54). Finally, the concept of the equal area projection and pixel distortion is unclear. The Lambert Azimuthal Equal Area projection appears to not be quite equal area in the near equatorial case of Africa. This challenges the very basic notion of using the equal area projection to maintain true area representation. The reasons for this apparent increase in area may be related to the fact that discrete units of area are being projected rather than a continuous data set.

CHAPTER 8

CONCLUSIONS

This research has focused on understanding the changes that occur as a result of map projection transformations. The hypotheses presented in Chapter 5 are directed toward a basic understanding of the nature of pixel based distortion and the creation of guidelines for choosing a projection for continental and global data sets. The results of the pixel analysis are grouped by these hypotheses. The majority of the results are discussed under the first hypothesis that examines the fundamental nature of pixel based changes. Following the summary of the research results indications for continued research are presented. Finally, the implications of this research to the larger spatial data community are explored.

8.1 Summary of Results

The amount and distribution of data change depends upon the map projections employed and the distortion characteristics of those projections. The assumptions behind this hypothesis are that pixel value loss or duplication would closely follow the results of traditional point based measurements of distortion. It was assumed that pixel loss would occur in areas of scale reduction and pixel duplication in areas of scale enlargement. The new metrics for tracking pixel based change, P_L and P_D, were used as an effective means to track both the loss and the duplication of pixel values simultaneously. When graphed, P_L

and ***PD*** can identify distortion characteristics by latitude and identify the presence or absence of the equal area property (Figure 41). These graphs also provide a means for grouping the projections into similar distortion classes. The visualization of ***PL*** and ***PD*** as a continuous surface is very effective at depicting the amount and spatial distribution of distortion.

A number of surprising results were exposed by the application of ***PL*** and ***PD***. The pseudocylindrical projections show complex behaviors of ***PL*** and ***PD*** with both pixel loss and duplication occurring simultaneously (Figure 37, Figure 41 and Figure 44). Given that equal area world map projections retain the same relative area as on the globe, pixel based changes were not expected to be extensive. Instead there are extensive changes that increase with latitude (Figure 41 and Figure 44) but that also occur at a level of approximately 10% data value change at the equator and low latitudes.

There are two low distortion options available, the Craster Parabolic and the Sinusoid (Figure 41). These projections have very high angular distortion and are very similar in appearance and the amount and spatial distribution of angular distortion. The Sinusoid is the obvious choice if the primary decision factor is to maintain data values. Even the Goode Homolosine (Figure 44) which was designed for its low, traditionally measured, distortion properties does not perform well when measured by ***PL*** and ***PD***.

The inverse relationship between angular distortion and pixel distortion was unexpected (Figure 42 and Figure 43). The relatively small number of projections examined does not permit drawing a final conclusion regarding this relationship on the specific case of world equal area projections. In addition, on the azimuthal projections the examination of pixel value changes using ***PL*** and ***PD*** indicate that distortion patterns are influenced heavily by the angular changes occurring in the graticule (Figure 52, Figure 57, and Figure 59). In these figures, the North American, African and South American continents are examined. Unlike the pseudocylindrical projections, higher angular distortion on the azimuthal is linked to greater P_L and P_D.

The implications are these two findings are significant. First the inverse relationship of low pixel distortion and high angular distortion differs from traditional map projection selection guidelines. All other factors being equal, the equal area world map projection with the lowest distortion characteristics measured by traditional methods would be chosen. When thematic raster data sets are being transformed by one of the pseudocylindrical equal area map projections and employing nearest neighbor resampling, the low angular distortion choice may lead to increased changes in the pixel values. The correct choice for maintaining the veracity of the data is the Sinusoid with its high angular distortion and zero pixel value changes.

The concept of the equal area projection and pixel distortion requires further study. The Lambert Azimuthal Equal Area projection appears to not be quite equal area in the near equatorial case of Africa (Figure 58) and the pseudocylindrical projections appear to balance higher levels of pixel duplication at one latitude with higher pixel loss at others. It is more likely that this is an artifact of the software implementation.

Similar to traditional distortion measures, distortion measured by P_L and P_D vary by projection. The Lambert Azimuthal Equal Area, the Azimuthal Equidistant and the Stereographic projections exhibit different distortion patterns that are related to changes in local scale. However, a fundamental rule of map projection distortion does not hold true for pixel based distortion. Instead of regular increasing radial patterns of distortion from a tangent point - distortion patters are more irregular. Distortion patterns may be triangular or ill defined in terms of a regular geometric shape. Further traditional point based methods also predict an invariant pattern of distortion. All of the azimuthal projections are expected to produce circular patterns of distortion that are invariant with respect to the aspect of the projection. As seen on the three oblique aspects of the North American case (Figure 52), the African case (Figure 57), and the South American case (Figure 59) distortion patterns do vary with aspect.

To understand the distortion characteristics of various projections the evaluation of pixel changes should be from a source of higher authority and not evaluated from the typical distribution format of a latitude and longitude grid. To be an effective tool for understanding and predicting the degree of pixel alteration in a data set, the pixel distortion characteristics introduced by transformation to the Plate Carrée or geographic coordinates must not be superimposed upon the characteristics of a secondary projection. In order to meaningfully evaluate the distortion that is being introduced into a data set, data should be compared to a source of higher authority. In this research the control data in the form of 10 by 10 pixel grids serve as the means for tracking pixel distortion in a manner as close to a source of higher authority as possible.

The transformation of raster spatial data sets of large extent by map projections is often an invertible process with current technologies and algorithms. Once data values are lost in the resampling process the data set can never revert to its original state. A zero pixel altering alternative to this interrupted projection may be the Sinusoid, even with its extreme angular shearing at high latitudes.

8.2 Study Limitations and Directions for Future Study

The results of this examination of pixel values changes and map projection transformation of continental and global data sets are striking but they are preliminary. The set of limiting conditions that were defined for this work were appropriate to the study objectives. The results however, indicate that the pixel value changes are both more extensive and more complex than anticipated.

The limiting conditions that shaped the structure of this study fall into two general areas, software implementation and data issues. The selection of ARCINFO as the software to be used during this study fulfilled the objective of working with the actual environment of software users but placed limits on the range of projections that were examined. The subset of projections of interest were those in use or that were recommended for use with continental and global data. ARCINFO could not evaluate the Wagner projections that had been recommended by Steinwand et. al. (1995). A larger set of projections may be required to determine the exact nature of the relationships between individual projections, or projection classes and pixel based distortion. The software implementation also did not support the direct detailed comparison of pixel distortion and traditional measurements of angular and area distortion.

The data and resampling metrics used for this analysis was restricted to the ten-by-ten, one kilometer pixel grids. The resampling method employed is nearest neighbor. This arrangement made detailed examination of the amount and type of changes occurring within raster data sets. Steinwand et. al. have also indicated that pixel distortion appears to shift to higher latitudes with a decrease in pixel size. Pixel size was held constant in this study to permit comparison between projections.

Using the pixel measures developed for this research a larger set of world and regional map projections should be studied than are currently implemented in commercial GIS or IP software. The relationships between map projection distortion as measured by traditional point based methods and by area based pixel counting methods should be defined. A selection of actual discrete data sets need to be evaluated with the nearest neighbor resampling method and continuous data sets with a selection of different resampling methods so that the P_L and P_D results can be related to the alterations in actual data sets.

The distortion introduced into these raster data sets varies by scale, as predicted, but also by the amount of angular change occurring in the data, and by angular changes of the graticule. Pixel distortion characteristics also vary by class of projection but not necessarily following the same patterns traditionally based distortion analysis provides. Pseudocylindricals, cylindricals, azimuthal and conic projections should all be evaluated for appropriateness of use for global and continental data.

The distortion that is introduced into raster data sets will also depend upon the characteristics of the data set itself. A raster data set that is composed of larger homogenous attribute areas will retain more of the original data characteristics than a data set that is more highly variable. A series of real world data sets of varying degrees of attribute homogeneity must be examined to determine the significance of data loss for a particular projection sequence.

8.3 Implications for the Spatial Data Community

There's more here than meets the eye -- may be the best description of the results obtained from the pixel level analysis of map projection transformation. The existence, amount, character and distribution of pixel level distortion are greater than expected. As a result, the implications for the larger spatial data community are greater. Everything from basic cartographic principles of map projections to spatial data standards will be affected.

At a fundamental level this research challenges the basic cartographic theory regarding map projections. The patterns of distortion contradict basic cartographic theory and selection guidelines. Further research will be needed to define the full set of changes. In the meantime, guidelines should be updated to reflect the current state of technology and basic map projection guidelines need to be updated to include the recommendations from this research. Cartographic education will feel the impact as well. Map projections have never been an easy subject to teach and now this work indicates that it is even more complex. The simple first approach of teaching geometric models and geometric properties will have to have to include information regarding the more complex patterns of distortion exposed in this research. Data users in particular need more extensive guidelines and tools for the analysis and selection of map projections. Digital spatial data and spatial data software is being used by an increasing number of researchers, often outside the traditional geoscience realm.

Data producers are the only ones to have control over the construction of a data set from initial data collection to formatting and release. Currently the USGS is releasing data in the Goode Homolosine projection. The Goode Homolosine is an interrupted projection that is not supported in current software implementations and is not the low distortion alternative to be used if data veracity is of primary concern. The USGS currently uses the Goode Homolosine projection for release of the Global Land Characteristics (USGS 1997) data set and others. The use of the Sinusoidal projection would eliminate the problem of coping with an unsupported interrupted projection as well as providing a zero pixel change alternative.

The projection chosen to release the same data as continental scale data is the Lambert Azimuthal Equal Area. This choice is also questionable based upon the results of this work. The negation of the fundamental concept of invariant distortion patterns on the azimuthal projections invalidate the general application of this projection to all continents. Further research may provide other alternatives but in the meantime, the Sinusoidal projection may be applied with an appropriate central meridian. This would provide a zero pixel change alternative and would also avoid the extreme shearing that occurs near the edges of a world sinusoidal map.

Software developers are a key element in many data producer and data user activities and need to improve the state of software implementations with regards to map projections and resampling techniques. A wider selection of projections needs to be offered in all of the GIS and IP commercial software. The example of the ARCVIEW software by ESRI that does not support map projection transformations or simultaneous display of data in different coordinate systems should not be repeated. Software vendors should also implement tools for analysis and display of map projection distortion. The formulas for traditional point based projection transformations have been available compiled into a convenient form since Snyder's 1987 publication at least. The ***PL*** and ***PD*** method could also be implemented. This would go a long way towards supporting researchers and other data users who try very hard to make solid choices regarding map projection use and transformation but who have no solid measure of the implications of their choices.

The Spatial Data Transfer Standard already calls for a data quality overlay to indicate the degree of attribute data change (ANSI 1997). There are currently no guidelines encouraging the use of quality overlays in relation to map projection transformation. Traditional map projection distortion characteristics are considered a known quantity. The results of this research indicate that pixel attributes do vary when transformed by a projection transformation and resampling. A quality overlay that tracks the changes for a data set should be produced and included with the data.

In the current state of technology in GIS and IP our computing environment is still treating the surface of the earth as a flat plane. Global and continental data sets are irreversibly changed during the map projection transformation. While the vector data structure is very flexible with regards to projection transformations, the raster data structure is not. Whenever possible vector data should be projected to accommodate raster data. For raster data, the analysis of pixel based changes in global and continental data sets has been advanced by the invention and application of the ***PL*** and ***PD*** metrics. Although our knowledge of this phenomena is incomplete, this research has provided a wealth of information regarding both the magnitude, distribution and complexity of pixel value changes.

APPENDIX A. GLOBAL DATA SURVEY

TABLE 22. Map Projection Questionnaire Responses

Data set name	Contributor	Thematic content	Data Set Purpose	Used alone or with other Data Sets?	Extent	Source	Resolution	Map Projection	Processing History
California Gap Analysis Land Cover/ Vegetation Layer	David Stoms Department of Geography, UCSB. stoms@geog.ucsb.edu	Land cover	Regional Conservation Planning	Either	Regional - CA except for offshore islands	Landsat Thematic Mapper, larger scale maps, field survey, aerial photos	100 meters	Albers using a Clarke 1866 Spheroid	Most of the Landsat TM imagery was obtained in UTM zone 11 projection, georectified and terrain corrected. The imagery was projected into the Albers Equal Area to conform to the state data center standard projection. The larger scale maps were also reprojected from their native projection into Albers. Source maps varied in their projections.
CASA-Biosphere model	Christopher Potter. Ecosystem Science an technology Branch. NASA-Ames Research Center * Mail Stop 242-4. Moffett Field, CA 94035. cpotter@mail.arc.nasa.gov	Global/ regional ecosystem production and biogenic trace gases	Biogeochemical studies	Part of global/ regional modeling system	Global and regional U.S. and Brazil	See references provided.	Global - 0.5 or 1.0 degree resolution. Also regional U.S. and Brazil at 8km	Equal Area. decimal degree. or km.	See references provided.

TABLE 22. Map Projection Questionnaire Responses

Data set name	Contributor	Thematic content	Data Set Purpose	Used alone or with other Data Sets?	Extent	Source	Resolution	Map Projection	Processing History
NCEP Eta Model forecast of NA	Jim Washburne, GLOBE Soil Moisture Scientist, Dept. of Hydrology & Water Resources, Harshbarger Bldg. #11, Rm 308 or PO Box 210011, UofA, Tucson, AZ 85721-0011, jwash@hwr.arizona.edu	2d- surface fluxes, (meteorology)	compare with surface network	with CART-ARM obs. data	Regional, central OK and KS. (about 250 x 350km)	NCEP	49km	NCEP awip212 grid	Displays using ferret, which requires a regular grid in lat-long, so I really distort things. But this is just the qualitative aspect of my work so I live with it.
ISCCP	Rong Fu, PAS #81, Rm 576, PO Box 210081, Tucson, AZ 85721-0081, fu@air.atmo.arizona.edu	clouds and atmospheric temperature and humidity	Understand the cloud formation processes	yes, (i think he means data set used alone.	Global	Langley DAAC, NASA	2.5 degree	latitude and longitude	Request info on how to obtain data set.
Global Land Cover Test Sites	Kenneth McGwire, DRI Biological Sciences Center, 7010 Dandini Blvd., Reno, NV 89512, kenm@maxey.unr.edu	Land cover and elevation Regional test sites comprising a global network where site data is available	Testing of algorithm for deriving land cover properties from satellite data. Validation of global land cover data products. Site specific research activities.	Multiple data types are packaged together in the product and users will compare this data with other field and satellite based data sets.	Regional 175km x 175km scenes from Landsat and 500km x 500km scenes for AVHRR. Coverage for DEM and land cover map varies.	Data is available through EROS Data Center to federally funded earth science researchers for non-commercial use	Landsat TM at 30m, MSS at 80m, AVHRR at 1000m. DEM and land cover data varies by site.	UTM, zone varies by site.	Landsat from EDC archive or foreign ground stations. AVHRR from EDC and extracted from processing flow for IGBP global 1km land cover classification. Land cover and DEM varies by site.

TABLE 22. Map Projection Questionnaire Responses

Data set name	Contributor	Thematic content	Data Set Purpose	Used alone or with other Data Sets?	Extent	Source	Resolution	Map Projection	Processing History
Coastline of the US	Millington Lockwood, NOAA/ National Ocean Service. 1315 East-West Hwy. Sta. 6222, Silver Spring, MD 20910 millington.lockwood@noaa.gov	Continuous line representing connection of points on the surface of the earth where land and water meet. Stated interms of tidal datum reference	Maritime boundary, ownership, registration, nautical charting, topographic mapping, digital elevation modeling. Global Climate Change.	Either; often it is used with bathymetric data or topographic mapping. It is essentially the dividing line between "dry" and "wet"	Regional: coastal US include Great Lakes	photogrammetry or plane table surveys of the coast of the US conducted by the US Coast & Geodetic Survey over the last 150 or so years.	Source photography is nominally 1:40,000 - 1:60,000 depending upon the altitude of the aircraft and camera type.	Most source is converted/ transformed to Mercator projection and uses the NAD83/ WGS84 horizontal data. .	Cont. and plotted as a "T-Sheet" at 1:10,000 - 1:20,000 scale. The final product that the national shoreline is portrayed (the Nautical Chart) is normally either 1:40,000 or 1:80,000. Smaller scale may be used if larger scale is unavailable.
Bathymetry of the US exclusive Economic Zone		Depth of the ocean, estuarine, and Great Lakes waters of the US. From the shoreline to the 200 mile limit of the US EEZ	Safe navigation, environmental monitoring, hydrodynamic modeling, environmental prediction, habitat identification, marine geomorphology, seismic surveying, seismic data reduction.	Either; often it is used with shoreline and land elevation or topographic mapping. Bathymetric data are essentially the offshore elevation model of the submarine portions of the earth.	Coastal US include Great Lakes. To the 200 mile limit of the US EEZ	Surveys of the coastal waters of the US by the C&GS since ~1835. Data are collected from approximately 15,000 individual field surveys "Or smooth sheets".	Scale and resolution of these filed sheets varies with the geographic area, depth of water, likelihood of encountering a navigational hazard. See processing.	Most source is converted/ transformed to Mercator projection and sues the NAD83/ WGS84	Data are maintained as hard copy archive. Approximately 50% of the more recent surveys are digitized and reside at the National Geophysical Data Center in Boulder, CO Hompage that describes the process of surveying for bathymetry see http:// wave.nos.noaa.gov/ocs/

APPENDIX B. MAP PROJECTION NAME EQUIVALENTS

The map projections employed in this study are frequently referred to by different common names and descriptions by different sources. In Table 23 Projection Name refers to the usage in this study. ARCINFO keywords are in capital letters as well as alternatives provided in software documentation. Canters and Decleir (1989) provide descriptive titles for the projections as well as alternative common names in their directory of world map projections. The Snyder reference is to his history of map projections (1993) and provides the first and other alternatives terms used in his reference.

TABLE 23. Map Projection Name Equivalents

Projection Name	ARCINFO Projection command name and alternative names	Canters & Decleir (1989)	Snyder (1993)
Craster Parabolic	CRASTER_PARABOLIC Craster parabolic Putnins (P_4)	Pseudocylindrical equal-area projection with parabolic meridians Craster's parabolic equal area and Putnins P4	Craster parabolic Putnins (P_4)
Eckert IV	ECKERTVI	Pseudocylindrical equal-area projection with elliptical meridians and pole line	Eckert IV
Eckert VI	ECKERTVI	Pseudocylindrical equal-area projection with sinusoidal meridians and pole line	Eckert VI
Flat Polar Quartic	FLAT_POLAR_QUARTIC McBryde-Thomas Flat-Polar Quartic	Pseudocylindrical equal-area projection with quartic meridians and pole line	McBryde-Thomas no. 4 - flat-polar quartic
Hammer	HAMMER_AITOFF	Polyconic equal-area projection	Hammer, Hammer-Aitoff
Mercator	MERCATOR	Cylindrical conformal projection	Mercator
Miller	MILLER Miller Cylindrical	Cylindrical Projection Miller 1	Miller Cylindrical

TABLE 23. Map Projection Name Equivalents

Projection Name	ARCINFO Projection command name and alternative names	Canters & Decleir (1989)	Snyder (1993)
Mollweide	MOLLWEIDE Babinet, Elliptical, Homolographic, and Homalographic	Pseudocylindrical equal-area projection with elliptical meridians Homolographic	Mollweide, Homolographic
Plate Carrée	EQUIRECTANGULAR Simple Cylindrical, Equidistant Cylindrical, Rectangular or Plate Carrée	Cylindrical equidistant projection Plate Carrée Simple Cylindrical	Equirectangular, Plate Carrée (if parallel of true scale is the equator)
Robinson	ROBINSON Orthophanic	Pseudocylindrical projection with pole line Robinson	Robinson
Sinusoid	SINUSOIDAL Sanson-Flamsteed	Pseudocylindrical equal-area projection with Sinusoidal Meridians Sanson Sanson-Flamsteed	Sinusoidal Sanson-Flamsteed
Wagner IV	Not Implemented	Pseudocylindrical equal-area projection with elliptical meridians and pole line Wagner IV, Putnins P2', Werenskiold III projection	Wagner IV - Putnins (P'_2)
Wagner VII	Not Implemented	Polyconic equal-area projection with pole line Hammer-Wagner	Wagner VII Wagner - Hammer

APPENDIX C: GOODE HOMOLOSINE PARAMETERS

Appendix C consists of the component projection files used within the ARCINFO software to project the 12 regions that comprise the Goode Homolosine projection from Lethcoe and Klaver (1998).

dd.r01

```
input
projection GEOGRAPHIC
units DD
parameters
output
projection MOLLWEIDE
units METERS
spheroid SPHERE
xshift -11119487.42847
yshift   -336410.83237
parameters
-100 00 00
end
```

dd.r02

```
input
projection GEOGRAPHIC
units DD
parameters
output
projection MOLLWEIDE
units METERS
spheroid SPHERE
xshift 3335846.22854
yshift -336410.83237
parameters
30 00 00
end
```

dd.r03

```
input
projection GEOGRAPHIC
units DD
parameters
```

```
output
projection SINUSOIDAL
units METERS
parameters
6370997.0
-100 00 00
-11119487.42847
        0.0
end
```

dd.r04

```
input
projection GEOGRAPHIC
units DD
parameters
output
projection SINUSOIDAL
units METERS
parameters
6370997.0
30 00 00
3335846.22854
      0.0
end
```

dd.r05

```
input
projection GEOGRAPHIC
units DD
parameters
output
projection SINUSOIDAL
units METERS
parameters
6370997.0
-160 00 00
-17791179.88555
        0.0
end
```

dd.r06

```
input
projection GEOGRAPHIC
units DD
parameters
output
projection SINUSOIDAL
units METERS
```

```
parameters
6370997.0
-60 00 00
-6671692.45708
       0.0
end
                                                       dd.r07
input
projection GEOGRAPHIC
units DD
parameters
output
projection SINUSOIDAL
units METERS
parameters
6370997.0
20 00 00
2223897.48569
      0.0
end
                                                       dd.r08
input
projection GEOGRAPHIC
units DD
parameters
output
projection SINUSOIDAL
units METERS
parameters
6370997.0
140 00 00
15567282.39985
       0.0
end
                                                       dd.r09
input
projection GEOGRAPHIC
units DD
parameters
output
projection MOLLWEIDE
units METERS
spheroid SPHERE
xshift -17791179.88555
yshift    336410.83237
```

```
parameters
-160 00 00
end
```

dd.r10

```
input
projection GEOGRAPHIC
units DD
parameters
output
projection MOLLWEIDE
units METERS
spheroid SPHERE
xshift -6671692.45708
yshift   336410.83237
parameters
-60 00 00
end
```

dd.r11

```
input
projection GEOGRAPHIC
units DD
parameters
output
projection MOLLWEIDE
units METERS
spheroid SPHERE
xshift 2223897.48569
yshift  336410.83237
parameters
20 00 00
end
```

dd.r12

```
input
projection GEOGRAPHIC
units DD
parameters
output
projection MOLLWEIDE
units METERS
spheroid SPHERE
xshift 15567282.39985
yshift   336410.83237
parameters
140 00 00
end
```

APPENDIX D: AML CODE - SOURCE TO PROJECTION

```
/***************************************************************************
/***************************************************************************
/* source2proj.aml Program to calculate pixel counting measures from
/* "source" to specified projection
/* Karen Mulcahy 12/97, East Carolina University
/* Source path and coverage name are the source materials for the
/* identification of the real world locations  of the grids.
/*
/* USAGE: &RUN source2proj <source path> <destination path> <covername>
<projection file>
/***************************************************************************
/***************************************************************************

&ARGS PATHSOURCE PATHDEST COVNAME PROJFILE
&ECHO &ON

/******************************* Prep work *******************************
/* create workspace, watchfile, copy vector coverage, select the grid
numbers
/* and create the text files for input to gridshIFt etc.
/*
  CW %PATHDEST%
  &WATCH %PATHDEST%/%PATHDEST%.wa
  COPY %PATHSOURCE%/%COVNAME% %PATHDEST%/%COVNAME%
  WORKSPACE %PATHDEST%
  ADDXY %COVNAME%
  TABLES
    SELECT %COVNAME%.PAT
    RESELECT %COVNAME%-ID > 0
    SORT %COVNAME%-ID
    UNLOAD gridno.txt %COVNAME%-ID
    UNLOAD xcoord.txt X-COORD
    UNLOAD ycoord.txt  Y-COORD
  QUIT
/****************************** End prep work ***************************

/*************************** Open files ******************************
&SV gridnumber := [open gridno.txt openstat -r]
   &IF %openstat% NE 0 &THEN
       &RETURN &ERROR Error opening file gridno.txt
&SV pvfile := [open pixelvalues.txt openstat -w]
   &IF %openstat% NE 0 &THEN
       &RETURN &ERROR Error opening file pixelvalues.txt
&SV xcoords := [open xcoord.txt openstat -r]
   &IF %openstat% NE 0 &THEN
       &RETURN &ERROR Error opening file xcoord.txt
&SV ycoords := [open ycoord.txt openstat -r]
   &IF %openstat% NE 0 &THEN
       &RETURN &ERROR Error opening file ycoord.txt
```

```
/**************************** End Open files ******************************
/*
/*
/****************************Start read files ****************************
/* Set initial values for gridnumber, x and y coordinates
&SV index = 1
&SV GRIDNUM := [read %gridnumber% readstat]
   &IF %readstat% NE 0 &THEN
      &RETURN &ERROR Error reading file %gridnumber%
&SV XCOORD := [read %xcoords% readstatx]
   &IF %readstatx% NE 0 &THEN
      &RETURN &ERROR Error reading file %xcoords%
&SV YCOORD := [read %ycoords% readstaty]
   &IF %readstaty% NE 0 &THEN
      &RETURN &ERROR Error reading file %ycoords%
/**************************** End Read files ******************************
/*
/*
/****************** Start Do Loop to create values **********************
&DO &UNTIL %INDEX% eq 704
  /*********************** IF THEN ELSE BLOCK ***************************
   &IF %INDEX% = %GRIDNUM%  &THEN &DO
      /**********************Shift and project grids *******************
       /* the original ten by ten pixel grid is shIFted to map space
       COPY ../%PATHSOURCE%/TEN_TEN GRID%GRIDNUM%
       PROJECTDEFINE GRID GRID%GRIDNUM%
         PROJECTION STEREOGRAPHIC
         UNITS METERS
         PARAMETERS
         1
         0
         %XCOORD%
         %YCOORD%
         0
         0
/*
      PROJECT GRID GRID%GRIDNUM% GRID%GRIDNUM%P # NEAREST 1000 projfiles/
%PROJFILE%
/*
      /*****************End shift and project  grids*******************
      /*
      /********************** Calculate values **************************
       &IF [exists GRID%GRIDNUM%P -VAT] &THEN &DO
            /* Get number of unique values for a grid
           &SV UNVAL = [listunique GRID%GRIDNUM%P  -vat value temp]
            /* Assume single transformation so total no. of pixels = 100.
           &SV INITIAL = 100
            /* Convert to points and use the describe command to set
            /* varibles and collect info.
           GRIDPOINT GRID%GRIDNUM%P GRID%GRIDNUM%Pnt VALUE
           &DESCRIBE GRID%GRIDNUM%Pnt
           &SV TOTAL = %DSC$POINTS%
```

```
            /* Get number of unique values for a grid
          &SV UNVAL = [listunique GRID%GRIDNUM%P   -vat value temp]
             /* Calculate PD  1 - (# of unique values/total # of pixels)
          &SV PD = [calc 1 - [calc %UNVAL%.0 / %TOTAL%]]
              /* Calculate PL 1 - (# unique values/original # of pixels)
          &SV PL = [calc 1.0 - [calc %UNVAL% / %INITIAL%]]
        /********************* End Calculate values *******************
        /*
        /********************* Write the values ***********************
          &SV writestatpl = [write %pvfile% %GRIDNUM%,%PL%,%PD%]
        /********************* End Write Values ***********************
        /********************* Clean up normal case *******************
          KILL GRID%GRIDNUM%Pnt
          KILL GRID%GRIDNUM%P
          KILL GRID%GRIDNUM%
      &END
      &ELSE
      /****************** Clean up missing grid case ******************
        /* create null value record for missing grids
        &DO
           &SV writestatpl = [write %pvfile% %GRIDNUM%,,]
           KILL GRID%GRIDNUM%Pnt
           KILL GRID%GRIDNUM%P
           KILL GRID%GRIDNUM%
        &END
       /**************** End Write null Values *************************
      &END
    /********************************** ELSE ***************************
      &ELSE
    /************************ Write null values *********************
      &DO
         &SV writestatpl = [write %pvfile% %GRIDNUM%,,]
         KILL GRID%GRIDNUM%Pnt
         KILL GRID%GRIDNUM%P
         KILL GRID%GRIDNUM%
      &END
    /******************* End Write null Values **********************

/********************** Increment to the next record *******************
   &SV INDEX = %INDEX% + 1
   &SV GRIDNUM := [read %gridnumber% readstat]
      &IF %readstat% NE 0 &THEN
      &RETURN &ERROR Error reading file %gridnumber%
   &SV XCOORD := [read %xcoords% readstatx]
      &IF %readstatx% NE 0 &THEN
      &RETURN &ERROR Error reading file %xcoords%
   &SV YCOORD := [read %ycoords% readstaty]
      &IF %readstaty% NE 0 &THEN
      &RETURN &ERROR Error reading file %ycoords%
/****************** End Increment to the next record ******************
&END
/******************* End Do Loop to create values  *********************
```

```
/*
/************************* Close open files ****************************
&SV closestat :=[close %gridnumber%]
&SV closestatpv :=[close %pvfile%]
&SV closestatpv :=[close %xcoords%]
&SV closestatpv :=[close %ycoords%]
/*************************End Close open files *************************
/*
/*
&RETURN
```

APPENDIX E: RESULTS OF MOLLWEIDE TRANSFORMATION

TABLE 24. Tablular Results from Source to Mollweide Transformation

Grid ID	x-coord	y-coord	PL	PD
1	-175	-85	0.5	0.53271028
2	-170	-85	0.5	0.537037037
3	-160	-85	0.5	0.53271028
4	-150	-85	0.5	0.537037037
5	-140	-85	0.5	0.53271028
6	-130	-85	0.5	0.528301887
7	-120	-85	0.5	0.53271028
8	-110	-85	0.5	0.53271028
9	-100	-85	0.5	0.53271028
10	-90	-85	0.5	0.528301887
11	-80	-85	0.5	0.53271028
12	-70	-85	0.5	0.53271028
13	-60	-85	0.5	0.528301887
14	-50	-85	0.5	0.53271028
15	-40	-85	0.5	0.53271028
16	-30	-85	0.5	0.53271028
17	-20	-85	0.5	0.545454545
18	-10	-85	0.5	0.528301887
19	0	-85	0.5	0.523809524
20	10	-85	0.5	0.537037037
21	20	-85	0.5	0.523809524
22	30	-85	0.5	0.528301887
23	40	-85	0.5	0.537037037
24	50	-85	0.5	0.53271028
25	60	-85	0.5	0.528301887
26	70	-85	0.5	0.528301887
27	80	-85	0.5	0.528301887
28	90	-85	0.5	0.53271028
29	100	-85	0.5	0.53271028
30	110	-85	0.5	0.53271028
31	120	-85	0.5	0.537037037
32	130	-85	0.5	0.53271028
33	140	-85	0.5	0.528301887
34	150	-85	0.5	0.528301887
35	160	-85	0.5	0.53271028
36	170	-85	0.5	0.541284404
37	175	-85	0.5	0.53271028
38	-175	-80	0.4	0.405940594
39	-170	-80	0.4	0.411764706
40	-160	-80	0.4	0.411764706
41	-150	-80	0.4	0.405940594
42	-140	-80	0.4	0.411764706
43	-130	-80	0.4	0.405940594
44	-120	-80	0.4	0.411764706
45	-110	-80	0.4	0.405940594
46	-100	-80	0.4	0.405940594
47	-90	-80	0.4	0.411764706
48	-80	-80	0.4	0.411764706
49	-70	-80	0.4	0.411764706
50	-60	-80	0.4	0.4
51	-50	-80	0.4	0.411764706
52	-40	-80	0.4	0.405940594
53	-30	-80	0.4	0.411764706
54	-20	-80	0.4	0.405940594
55	-10	-80	0.4	0.411764706
56	0	-80	0.4	0.411764706
57	10	-80	0.4	0.411764706
58	20	-80	0.4	0.405940594
59	30	-80	0.4	0.411764706
60	40	-80	0.4	0.405940594
61	50	-80	0.4	0.411764706
62	60	-80	0.4	0.411764706
63	70	-80	0.4	0.411764706
64	80	-80	0.4	0.405940594
65	90	-80	0.4	0.411764706
66	100	-80	0.4	0.405940594

Grid ID	x-coord	y-coord	PL	PD	Grid ID	x-coord	y-coord	PL	PD
67	110	-80	0.4	0.405940594	104	110	-70	0.3	0.263157895
68	120	-80	0.4	0.405940594	105	120	-70	0.3	0.255319149
69	130	-80	0.4	0.411764706	106	130	-70	0.3	0.239130435
70	140	-80	0.4	0.411764706	107	140	-70	0.3	0.255319149
71	150	-80	0.4	0.405940594	108	150	-70	0.3	0.255319149
72	160	-80	0.4	0.405940594	109	160	-70	0.3	0.230769231
73	170	-80	0.4	0.411764706	110	170	-70	0.3	0.247311828
74	175	-80	0.4	0.411764706	111	175	-70	0.3	0.255319149
75	-175	-70	0.3	0.255319149	112	-175	-60	0.1	0.142857143
76	-170	-70	0.3	0.263157895	113	-170	-60	0.1	0.142857143
77	-160	-70	0.3	0.230769231	114	-160	-60	0.1	0.134615385
78	-150	-70	0.3	0.247311828	115	-150	-60	0.1	0.166666667
79	-140	-70	0.3	0.255319149	116	-140	-60	0.1	0.142857143
80	-130	-70	0.3	0.263157895	117	-130	-60	0.1	0.134615385
81	-120	-70	0.3	0.247311828	118	-120	-60	0.1	0.134615385
82	-110	-70	0.3	0.263157895	119	-110	-60	0.1	0.142857143
83	-100	-70	0.3	0.255319149	120	-100	-60	0.1	0.142857143
84	-90	-70	0.3	0.255319149	121	-90	-60	0.1	0.142857143
85	-80	-70	0.3	0.230769231	122	-80	-60	0.1	0.166666667
86	-70	-70	0.3	0.255319149	123	-70	-60	0.1	0.134615385
87	-60	-70	0.3	0.247311828	124	-60	-60	0.1	0.134615385
88	-50	-70	0.3	0.230769231	125	-50	-60	0.1	0.142857143
89	-40	-70	0.3	0.247311828	126	-40	-60	0.1	0.134615385
90	-30	-70	0.3	0.247311828	127	-30	-60	0.1	0.142857143
91	-20	-70	0.3	0.247311828	128	-20	-60	0.1	0.142857143
92	-10	-70	0.3	0.255319149	129	-10	-60	0.1	0.142857143
93	0	-70	0.3	0.230769231	130	0	-60	0.1	0.166666667
94	10	-70	0.3	0.255319149	131	10	-60	0.1	0.150943396
95	20	-70	0.3	0.255319149	132	20	-60	0.1	0.150943396
96	30	-70	0.3	0.247311828	133	30	-60	0.1	0.142857143
97	40	-70	0.3	0.255319149	134	40	-60	0.1	0.150943396
98	50	-70	0.3	0.278350515	135	50	-60	0.1	0.142857143
99	60	-70	0.3	0.247311828	136	60	-60	0.1	0.142857143
100	70	-70	0.3	0.247311828	137	70	-60	0.1	0.158878505
101	80	-70	0.3	0.255319149	138	80	-60	0.1	0.108910891
102	90	-70	0.3	0.247311828	139	90	-60	0.1	0.134615385
103	100	-70	0.3	0.247311828	140	100	-60	0.1	0.134615385

Grid ID	x-coord	y-coord	PL	PD
141	110	-60	0.1	0.142857143
142	120	-60	0.1	0.134615385
143	130	-60	0.1	0.142857143
144	140	-60	0.1	0.134615385
145	150	-60	0.1	0.126213592
146	160	-60	0.1	0.134615385
147	170	-60	0.1	0.142857143
148	175	-60	0.1	0.150943396
149	-175	-50	0.1	0.032258065
150	-170	-50	0.1	0.0625
151	-160	-50	0.1	0.072164948
152	-150	-50	0.1	0.0625
153	-140	-50	0.1	0.0625
154	-130	-50	0.1	0.052631579
155	-120	-50	0.1	0.072164948
156	-110	-50	0.1	0.090909091
157	-100	-50	0.1	0.010989011
158	-90	-50	0.1	0.052631579
159	-80	-50	0.1	0.052631579
160	-70	-50	0.1	0.0625
161	-60	-50	0.1	0.052631579
162	-50	-50	0.1	0.0625
163	-40	-50	0.1	0.0625
164	-30	-50	0.1	0.052631579
165	-20	-50	0.1	0.052631579
166	-10	-50	0.1	0.052631579
167	0	-50	0.1	0.090909091
168	10	-50	0.1	0.052631579
169	20	-50	0.1	0.0625
170	30	-50	0.1	0.0625
171	40	-50	0.1	0.0625
172	50	-50	0.1	0.052631579
173	60	-50	0.1	0.052631579
174	70	-50	0.1	0.0625
175	80	-50	0.1	0.072164948
176	90	-50	0.1	0.0625
177	100	-50	0.1	0.090909091
178	110	-50	0.1	0.010989011
179	120	-50	0.1	0.042553191
180	130	-50	0.1	0.042553191
181	140	-50	0.1	0.0625
182	150	-50	0.1	0.052631579
183	160	-50	0.1	0.0625
184	170	-50	0.1	0.052631579
185	175	-50	0.1	0.032258065
186	-175	-40	0.01	0
187	-170	-40	0	0
188	-160	-40	0.01	0
189	-150	-40	0	0
190	-140	-40	0.02	0
191	-130	-40	0	0
192	-120	-40	0	0
193	-110	-40	0.01	0
194	-100	-40	0	0
195	-90	-40	0	0
196	-80	-40	0.01	0
197	-70	-40	0	0
198	-60	-40	0	0
199	-50	-40	0.01	0
200	-40	-40	0	0
201	-30	-40	0.01	0
202	-20	-40	0	0
203	-10	-40	0.01	0
204	0	-40	0	0
205	10	-40	0.01	0
206	20	-40	0	0
207	30	-40	0.01	0
208	40	-40	0.01	0
209	50	-40	0.01	0
210	60	-40	0	0
211	70	-40	0	0
212	80	-40	0.01	0
213	90	-40	0	0
214	100	-40	0.01	0

Grid ID	x-coord	y-coord	PL	PD
215	110	-40	0.01	0
216	120	-40	0	0
217	130	-40	0	0
218	140	-40	0	0
219	150	-40	0.01	0
220	160	-40	0	0
221	170	-40	0.01	0
222	175	-40	0	0
223	-175	-30	0.05	0.086538462
224	-170	-30	0.05	0.086538462
225	-160	-30	0.05	0.086538462
226	-150	-30	0.04	0.085714286
227	-140	-30	0.04	0.085714286
228	-130	-30	0.04	0.085714286
229	-120	-30	0.05	0.086538462
230	-110	-30	0.04	0.085714286
231	-100	-30	0.05	0.095238095
232	-90	-30	0.05	0.095238095
233	-80	-30	0.06	0.096153846
234	-70	-30	0.03	0.093457944
235	-60	-30	0.05	0.095238095
236	-50	-30	0.04	0.094339623
237	-40	-30	0.06	0.096153846
238	-30	-30	0.03	0.093457944
239	-20	-30	0.05	0.095238095
240	-10	-30	0	0.090909091
241	0	-30	0	0.090909091
242	10	-30	0	0.082568807
243	20	-30	0.04	0.085714286
244	30	-30	0.04	0.085714286
245	40	-30	0.03	0.08490566
246	50	-30	0.04	0.085714286
247	60	-30	0.04	0.085714286
248	70	-30	0.06	0.087378641
249	80	-30	0.05	0.086538462
250	90	-30	0.05	0.086538462
251	100	-30	0.04	0.085714286

Grid ID	x-coord	y-coord	PL	PD
252	110	-30	0.05	0.086538462
253	120	-30	0.03	0.08490566
254	130	-30	0.05	0.086538462
255	140	-30	0.03	0.08490566
256	150	-30	0.04	0.085714286
257	160	-30	0.05	0.095238095
258	170	-30	0.03	0.076190476
259	175	-30	0.03	0.093457944
260	-175	-20	0.07	0.088235294
261	-170	-20	0.07	0.088235294
262	-160	-20	0.09	0.080808081
263	-150	-20	0.07	0.079207921
264	-140	-20	0.08	0.089108911
265	-130	-20	0.08	0.089108911
266	-120	-20	0.08	0.08
267	-110	-20	0.08	0.089108911
268	-100	-20	0.07	0.088235294
269	-90	-20	0.07	0.079207921
270	-80	-20	0.06	0.078431373
271	-70	-20	0.07	0.079207921
272	-60	-20	0.07	0.079207921
273	-50	-20	0.06	0.078431373
274	-40	-20	0.07	0.088235294
275	-30	-20	0.09	0.099009901
276	-20	-20	0.07	0.088235294
277	-10	-20	0.07	0.079207921
278	0	-20	0.1	0.090909091
279	10	-20	0.08	0.089108911
280	20	-20	0.07	0.097087379
281	30	-20	0.07	0.079207921
282	40	-20	0.06	0.078431373
283	50	-20	0.06	0.078431373
284	60	-20	0.07	0.088235294
285	70	-20	0.07	0.088235294
286	80	-20	0.06	0.078431373
287	90	-20	0.07	0.079207921
288	100	-20	0.06	0.087378641

Grid ID	x-coord	y-coord	PL	PD
289	110	-20	0.08	0.08
290	120	-20	0.08	0.089108911
291	130	-20	0.06	0.078431373
292	140	-20	0.08	0.089108911
293	150	-20	0.08	0.08
294	160	-20	0.09	0.080808081
295	170	-20	0.07	0.088235294
296	175	-20	0.07	0.079207921
297	-175	-10	0.09	0.09
298	-170	-10	0.08	0.08
299	-160	-10	0.09	0.080808081
300	-150	-10	0.09	0.080808081
301	-140	-10	0.09	0.080808081
302	-130	-10	0.09	0.080808081
303	-120	-10	0.08	0.08
304	-110	-10	0.09	0.080808081
305	-100	-10	0.08	0.08
306	-90	-10	0.07	0.079207921
307	-80	-10	0.09	0.080808081
308	-70	-10	0.08	0.08
309	-60	-10	0.09	0.080808081
310	-50	-10	0.09	0.080808081
311	-40	-10	0.08	0.08
312	-30	-10	0.08	0.08
313	-20	-10	0.1	0.090909091
314	-10	-10	0.1	0.090909091
315	0	-10	0.1	0.090909091
316	10	-10	0.1	0.090909091
317	20	-10	0.09	0.080808081
318	30	-10	0.09	0.09
319	40	-10	0.09	0.09
320	50	-10	0.08	0.08
321	60	-10	0.09	0.080808081
322	70	-10	0.09	0.09
323	80	-10	0.09	0.080808081
324	90	-10	0.07	0.079207921
325	100	-10	0.08	0.08
326	110	-10	0.09	0.080808081
327	120	-10	0.08	0.08
328	130	-10	0.08	0.08
329	140	-10	0.09	0.080808081
330	150	-10	0.08	0.08
331	160	-10	0.09	0.080808081
332	170	-10	0.08	0.08
333	175	-10	0.09	0.09
334	-175	0	0.1	0.090909091
335	-170	0	0.1	0.090909091
336	-160	0	0.1	0.090909091
337	-150	0	0.1	0.090909091
338	-140	0	0.1	0.090909091
339	-130	0	0.1	0.090909091
340	-120	0	0.1	0.090909091
341	-110	0	0.1	0.090909091
342	-100	0	0.1	0.090909091
343	-90	0	0.1	0.090909091
344	-80	0	0.1	0.090909091
345	-70	0	0.1	0.090909091
346	-60	0	0.1	0.090909091
347	-50	0	0.1	0.090909091
348	-40	0	0.1	0.090909091
349	-30	0	0.1	0.090909091
350	-20	0	0.1	0.090909091
351	-10	0	0.1	0.090909091
352	0	0	0.1	0.090909091
353	10	0	0.1	0.090909091
354	20	0	0.1	0.090909091
355	30	0	0.1	0.090909091
356	40	0	0.1	0.090909091
357	50	0	0.1	0.090909091
358	60	0	0.1	0.090909091
359	70	0	0.1	0.090909091
360	80	0	0.1	0.090909091
361	90	0	0.1	0.090909091
362	100	0	0.1	0.090909091

Grid ID	x-coord	y-coord	PL	PD
363	110	0	0.1	0.090909091
364	120	0	0.1	0.090909091
365	130	0	0.1	0.090909091
366	140	0	0.1	0.090909091
367	150	0	0.1	0.090909091
368	160	0	0.1	0.090909091
369	170	0	0.1	0.090909091
370	175	0	0.1	0.090909091
371	-175	10	0.09	0.09
372	-170	10	0.08	0.08
373	-160	10	0.09	0.080808081
374	-150	10	0.08	0.08
375	-140	10	0.09	0.080808081
376	-130	10	0.07	0.079207921
377	-120	10	0.08	0.08
378	-110	10	0.09	0.080808081
379	-100	10	0.08	0.08
380	-90	10	0.07	0.079207921
381	-80	10	0.09	0.080808081
382	-70	10	0.09	0.09
383	-60	10	0.09	0.080808081
384	-50	10	0.08	0.08
385	-40	10	0.09	0.09
386	-30	10	0.09	0.09
387	-20	10	0.09	0.080808081
388	-10	10	0.1	0.090909091
389	0	10	0.1	0.090909091
390	10	10	0.1	0.090909091
391	20	10	0.1	0.090909091
392	30	10	0.09	0.09
393	40	10	0.08	0.08
394	50	10	0.09	0.080808081
395	60	10	0.09	0.080808081
396	70	10	0.08	0.08
397	80	10	0.09	0.080808081
398	90	10	0.07	0.079207921
399	100	10	0.08	0.08

Grid ID	x-coord	y-coord	PL	PD
400	110	10	0.09	0.080808081
401	120	10	0.08	0.08
402	130	10	0.09	0.080808081
403	140	10	0.09	0.080808081
404	150	10	0.09	0.080808081
405	160	10	0.09	0.080808081
406	170	10	0.08	0.089108911
407	175	10	0.09	0.09
408	-175	20	0.07	0.079207921
409	-170	20	0.07	0.088235294
410	-160	20	0.09	0.080808081
411	-150	20	0.08	0.08
412	-140	20	0.08	0.089108911
413	-130	20	0.06	0.078431373
414	-120	20	0.08	0.089108911
415	-110	20	0.08	0.08
416	-100	20	0.05	0.077669903
417	-90	20	0.07	0.079207921
418	-80	20	0.07	0.088235294
419	-70	20	0.07	0.088235294
420	-60	20	0.06	0.078431373
421	-50	20	0.06	0.078431373
422	-40	20	0.06	0.078431373
423	-30	20	0.07	0.079207921
424	-20	20	0.07	0.097087379
425	-10	20	0.07	0.079207921
426	0	20	0.1	0.090909091
427	10	20	0.08	0.089108911
428	20	20	0.07	0.088235294
429	30	20	0.08	0.089108911
430	40	20	0.06	0.078431373
431	50	20	0.06	0.078431373
432	60	20	0.07	0.079207921
433	70	20	0.07	0.079207921
434	80	20	0.07	0.088235294
435	90	20	0.07	0.079207921
436	100	20	0.07	0.088235294

Grid ID	x-coord	y-coord	PL	PD
437	110	20	0.08	0.08
438	120	20	0.08	0.08
439	130	20	0.07	0.079207921
440	140	20	0.08	0.089108911
441	150	20	0.07	0.079207921
442	160	20	0.09	0.080808081
443	170	20	0.07	0.088235294
444	175	20	0.07	0.088235294
445	-175	30	0.03	0.093457944
446	-170	30	0.03	0.076190476
447	-160	30	0.05	0.095238095
448	-150	30	0.04	0.085714286
449	-140	30	0.03	0.08490566
450	-130	30	0.05	0.086538462
451	-120	30	0.03	0.08490566
452	-110	30	0.05	0.086538462
453	-100	30	0.04	0.085714286
454	-90	30	0.05	0.086538462
455	-80	30	0.05	0.086538462
456	-70	30	0.06	0.087378641
457	-60	30	0.04	0.085714286
458	-50	30	0.04	0.085714286
459	-40	30	0.03	0.08490566
460	-30	30	0.04	0.085714286
461	-20	30	0.04	0.085714286
462	-10	30	0	0.082568807
463	0	30	0	0.090909091
464	10	30	0	0.090909091
465	20	30	0.05	0.095238095
466	30	30	0.03	0.093457944
467	40	30	0.06	0.096153846
468	50	30	0.04	0.094339623
469	60	30	0.05	0.095238095
470	70	30	0.03	0.093457944
471	80	30	0.06	0.096153846
472	90	30	0.05	0.095238095
473	100	30	0.04	0.094339623

Grid ID	x-coord	y-coord	PL	PD
474	110	30	0.04	0.085714286
475	120	30	0.05	0.086538462
476	130	30	0.04	0.085714286
477	140	30	0.04	0.085714286
478	150	30	0.04	0.085714286
479	160	30	0.05	0.086538462
480	170	30	0.05	0.086538462
481	175	30	0.05	0.086538462
482	-175	40	0	0
483	-170	40	0.01	0
484	-160	40	0	0
485	-150	40	0.01	0
486	-140	40	0	0
487	-130	40	0	0
488	-120	40	0	0
489	-110	40	0.01	0
490	-100	40	0.01	0
491	-90	40	0	0
492	-80	40	0.01	0
493	-70	40	0	0
494	-60	40	0	0
495	-50	40	0.01	0
496	-40	40	0.01	0
497	-30	40	0.01	0
498	-20	40	0	0
499	-10	40	0.01	0
500	0	40	0	0
501	10	40	0.01	0
502	20	40	0	0
503	30	40	0.01	0
504	40	40	0	0
505	50	40	0.01	0
506	60	40	0	0
507	70	40	0	0
508	80	40	0.01	0
509	90	40	0	0
510	100	40	0	0

Grid ID	x-coord	y-coord	PL	PD
511	110	40	0	0
512	120	40	0	0
513	130	40	0	0
514	140	40	0.02	0
515	150	40	0	0
516	160	40	0.01	0
517	170	40	0	0
518	175	40	0.01	0
519	-175	50	0.1	0.042553191
520	-170	50	0.1	0.052631579
521	-160	50	0.1	0.0625
522	-150	50	0.1	0.052631579
523	-140	50	0.1	0.0625
524	-130	50	0.1	0.042553191
525	-120	50	0.1	0.042553191
526	-110	50	0.1	0.010989011
527	-100	50	0.1	0.090909091
528	-90	50	0.1	0.0625
529	-80	50	0.1	0.072164948
530	-70	50	0.1	0.0625
531	-60	50	0.1	0.052631579
532	-50	50	0.1	0.052631579
533	-40	50	0.1	0.0625
534	-30	50	0.1	0.0625
535	-20	50	0.1	0.0625
536	-10	50	0.1	0.052631579
537	0	50	0.1	0.090909091
538	10	50	0.1	0.052631579
539	20	50	0.1	0.052631579
540	30	50	0.1	0.052631579
541	40	50	0.1	0.072164948
542	50	50	0.1	0.0625
543	60	50	0.1	0.052631579
544	70	50	0.1	0.0625
545	80	50	0.1	0.052631579
546	90	50	0.1	0.052631579
547	100	50	0.1	0.010989011

Grid ID	x-coord	y-coord	PL	PD
548	110	50	0.1	0.090909091
549	120	50	0.1	0.072164948
550	130	50	0.1	0.052631579
551	140	50	0.1	0.0625
552	150	50	0.1	0.0625
553	160	50	0.1	0.072164948
554	170	50	0.1	0.0625
555	175	50	0.1	0.032258065
556	-175	60	0.1	0.150943396
557	-170	60	0.1	0.142857143
558	-160	60	0.1	0.134615385
559	-150	60	0.1	0.126213592
560	-140	60	0.1	0.134615385
561	-130	60	0.1	0.134615385
562	-120	60	0.1	0.142857143
563	-110	60	0.1	0.142857143
564	-100	60	0.1	0.134615385
565	-90	60	0.1	0.134615385
566	-80	60	0.1	0.108910891
567	-70	60	0.1	0.158878505
568	-60	60	0.1	0.142857143
569	-50	60	0.1	0.142857143
570	-40	60	0.1	0.150943396
571	-30	60	0.1	0.142857143
572	-20	60	0.1	0.150943396
573	-10	60	0.1	0.150943396
574	0	60	0.1	0.166666667
575	10	60	0.1	0.142857143
576	20	60	0.1	0.142857143
577	30	60	0.1	0.142857143
578	40	60	0.1	0.134615385
579	50	60	0.1	0.142857143
580	60	60	0.1	0.142857143
581	70	60	0.1	0.134615385
582	80	60	0.1	0.166666667
583	90	60	0.1	0.142857143
584	100	60	0.1	0.142857143

Grid ID	x-coord	y-coord	PL	PD
585	110	60	0.1	0.142857143
586	120	60	0.1	0.142857143
587	130	60	0.1	0.142857143
588	140	60	0.1	0.142857143
589	150	60	0.1	0.166666667
590	160	60	0.1	0.134615385
591	170	60	0.1	0.142857143
592	175	60	0.1	0.142857143
593	-175	70	0.3	0.255319149
594	-170	70	0.3	0.247311828
595	-160	70	0.3	0.230769231
596	-150	70	0.3	0.255319149
597	-140	70	0.3	0.255319149
598	-130	70	0.3	0.247311828
599	-120	70	0.3	0.255319149
600	-110	70	0.3	0.270833333
601	-100	70	0.3	0.255319149
602	-90	70	0.3	0.247311828
603	-80	70	0.3	0.255319149
604	-70	70	0.3	0.247311828
605	-60	70	0.3	0.247311828
606	-50	70	0.3	0.278350515
607	-40	70	0.3	0.247311828
608	-30	70	0.3	0.247311828
609	-20	70	0.3	0.255319149
610	-10	70	0.3	0.255319149
611	0	70	0.3	0.230769231
612	10	70	0.3	0.255319149
613	20	70	0.3	0.247311828
614	30	70	0.3	0.247311828
615	40	70	0.3	0.247311828
616	50	70	0.3	0.230769231
617	60	70	0.3	0.247311828
618	70	70	0.3	0.255319149
619	80	70	0.3	0.230769231
620	90	70	0.3	0.255319149
621	100	70	0.3	0.255319149

Grid ID	x-coord	y-coord	PL	PD
622	110	70	0.3	0.263157895
623	120	70	0.3	0.247311828
624	130	70	0.3	0.263157895
625	140	70	0.3	0.255319149
626	150	70	0.3	0.255319149
627	160	70	0.3	0.230769231
628	170	70	0.3	0.263157895
629	175	70	0.3	0.247311828
630	-175	80	0.4	0.411764706
631	-170	80	0.4	0.411764706
632	-160	80	0.4	0.405940594
633	-150	80	0.4	0.405940594
634	-140	80	0.4	0.411764706
635	-130	80	0.4	0.411764706
636	-120	80	0.4	0.405940594
637	-110	80	0.4	0.411764706
638	-100	80	0.4	0.405940594
639	-90	80	0.4	0.411764706
640	-80	80	0.4	0.405940594
641	-70	80	0.4	0.411764706
642	-60	80	0.4	0.411764706
643	-50	80	0.4	0.411764706
644	-40	80	0.4	0.411764706
645	-30	80	0.4	0.411764706
646	-20	80	0.4	0.4
647	-10	80	0.4	0.411764706
648	0	80	0.4	0.411764706
649	10	80	0.4	0.411764706
650	20	80	0.4	0.4
651	30	80	0.4	0.411764706
652	40	80	0.4	0.405940594
653	50	80	0.4	0.405940594
654	60	80	0.4	0.4
655	70	80	0.4	0.411764706
656	80	80	0.4	0.411764706
657	90	80	0.4	0.411764706
658	100	80	0.4	0.405940594

Grid ID	x-coord	y-coord	PL	PD
659	110	80	0.4	0.411764706
660	120	80	0.4	0.405940594
661	130	80	0.4	0.405940594
662	140	80	0.4	0.411764706
663	150	80	0.4	0.405940594
664	160	80	0.4	0.411764706
665	170	80	0.4	0.411764706
666	175	80	0.4	0.405940594
667	-175	85	0.5	0.528301887
668	-170	85	0.5	0.545454545
669	-160	85	0.5	0.528301887
670	-150	85	0.5	0.53271028
671	-140	85	0.5	0.528301887
672	-130	85	0.5	0.53271028
673	-120	85	0.5	0.53271028
674	-110	85	0.5	0.53271028
675	-100	85	0.5	0.528301887
676	-90	85	0.5	0.537037037
677	-80	85	0.5	0.53271028
678	-70	85	0.5	0.537037037
679	-60	85	0.5	0.528301887
680	-50	85	0.5	0.53271028
681	-40	85	0.5	0.537037037
682	-30	85	0.5	0.53271028
683	-20	85	0.5	0.523809524
684	-10	85	0.5	0.537037037
685	0	85	0.5	0.523809524
686	10	85	0.5	0.537037037
687	20	85	0.5	0.545454545
688	30	85	0.5	0.53271028
689	40	85	0.5	0.528301887
690	50	85	0.5	0.537037037
691	60	85	0.5	0.528301887
692	70	85	0.5	0.53271028
693	80	85	0.5	0.53271028
694	90	85	0.5	0.528301887
695	100	85	0.5	0.528301887
696	110	85	0.5	0.53271028
697	120	85	0.5	0.537037037
698	130	85	0.5	0.53271028
699	140	85	0.5	0.53271028
700	150	85	0.5	0.541284404
701	160	85	0.5	0.53271028
702	170	85	0.5	0.528301887
703	175	85	0.5	0.53271028

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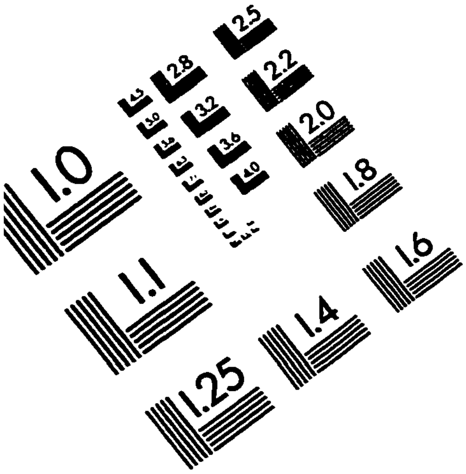
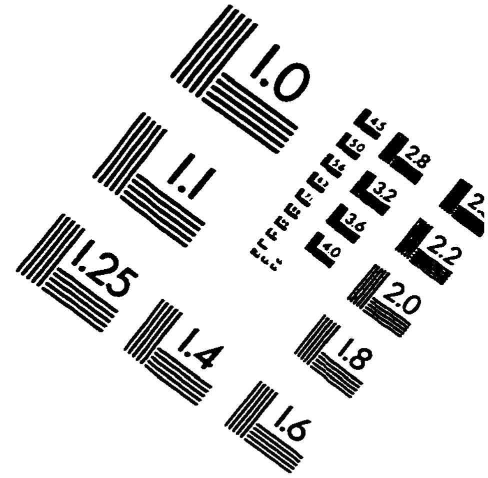
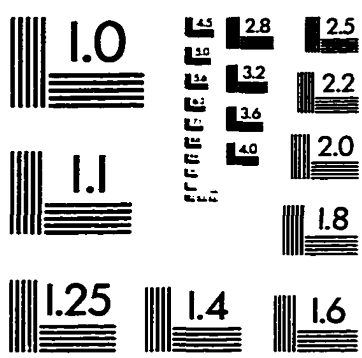
IMAGE EVALUATION TEST TARGET (QA-3)

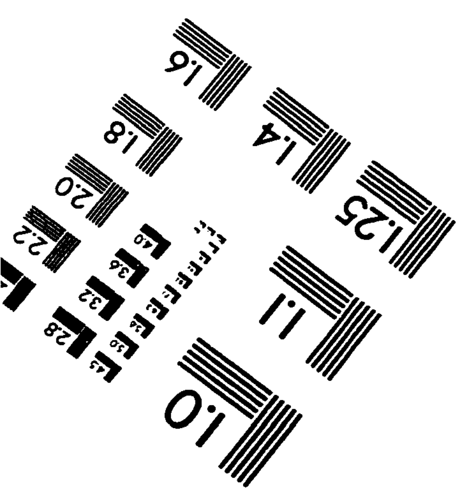


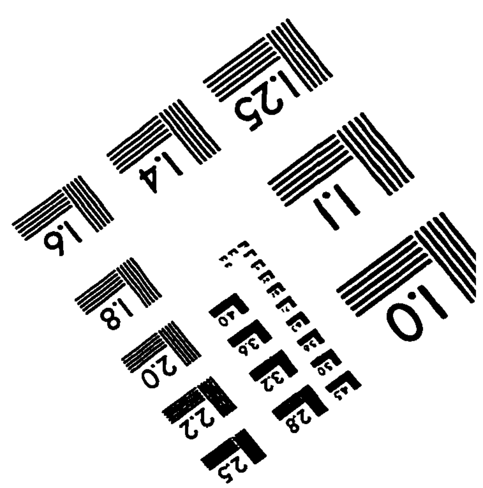




150mm

6"





APPLIED IMAGE . Inc
1653 East Main Street
Rochester, NY 14609 USA
Phone: 716/482-0300
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