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THE QUALITY ADJUSTED DEMAND FOR PUBLIC ELEMENTARY  
SCHOOL TEACHERS: A LABOR MARKET ANALYSIS

by

STEPHEN CHAIKIND

A dissertation submitted to the Graduate  
Faculty in Economics in partial fulfillment  
of the requirements for the degree of  
Doctor of Philosophy, The City University  
of New York.

1978

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This manuscript has been read and accepted for the Graduate Faculty in Economics in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

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## ABSTRACT

This thesis is an empirical and theoretical approach to the demand for public elementary school teachers. The quality adjusted demand for teachers is derived from an implicit production function for education. The foundations for the model are based on human capital, as well as derived demand theory.

Data from the 1970 Census were aggregated across SMSA's and were used in testing the model. The primary regression technique used was weighted two stage least squares. Many of the more interesting regression results are shown, and the coefficients are interpreted.

The results show that the own price elasticity of demand for public elementary school teachers is significantly negative and inelastic. It is also shown, among some of the other results, that mothers are net, as well as gross, substitutes for teachers in educating their children. Paraprofessionals are also shown to be substitutes for teachers. These conclusions are then discussed in the context of their implications for public policy.

## ACKNOWLEDGMENTS

A thesis is not a work done in solitude and devoid of personal interactions. Over the course of the past years, a number of people have left their mark on my thinking, and I wish to take this opportunity to thank them.

The extraordinary amount of guidance and effort by Professor Michael Grossman, as my principle advisor, can never be repaid. The influence of his comments can be found throughout this thesis. The theoretical and empirical shape of this dissertation would have been vastly different had he not been involved. And his personal encouragement will not be forgotten.

Professor Elliot Zupnick's unusual insights into many diverse areas were important in the analysis of the issues. Professor Herbert Geyer, as the third member of my committee, also contributed useful ideas. Among my colleagues at the Graduate Center, two stand out as having contributed time and thought to the development of this paper: Hyman Sanders and Henry Saffer.

Finally, and with love, to my parents and grandmother, who knew all along that this is what I wanted to do.

## TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT.....	iv
ACKNOWLEDGMENTS.....	v
LIST OF TABLES.....	vii
LIST OF FIGURES.....	viii
CHAPTER	
I. Introduction.....	1
II. Theoretical Aspects of the Demand for Public Elementary School Teachers.....	5
III. The Data Set.....	25
IV. The Empirical Tests.....	37
V. The Supply Equations.....	65
VI. Implications.....	83
APPENDICES	
A.....	95
B.....	101
BIBLIOGRAPHY.....	103

## LIST OF TABLES

	<u>Page</u>
CHAPTER III.	
III.1 Summary Statistics of Public Elementary School Teacher and Occupational Characteristics, Overall SMSA Means or Percentages.....	29
III.2 Summary Statistics of Family, Community and Children Characteristics, Overall SMSA Means or Percentages.....	34
III.3 Supplemental Statistics of all Public Elementary School Teachers, 1969.....	36
CHAPTER IV.	
IV.1 Two Stage Least Squares Estimates of The Demand Equations.....	45
CHAPTER V.	
V.1 Two Stage Least Squares Estimates of The Supply Equations.....	71
V.2 Two Stage Least Squares Estimates of The Labor Force Equations.....	78

LIST OF FIGURES

	<u>Page</u>
CHAPTER IV.	
IV.1 Effect of the Salary Schedule Variable.....	57

CHAPTER I  
INTRODUCTION

Since the education boom in the United States began in the early 1960's as a response to the launch of the Russian satellite Sputnik, educational production has been studied and examined in countless ways. It has been viewed as the result of inputs of schools and families, as the cause for lifetime earnings differentials, and as the impetus to economic growth.<sup>1</sup>

In addition, the general areas of household production to satisfy more basic needs and investments in human capital have spawned specialized research in the areas of health, nutrition, migration, on the job training, child demand and even marriage and religion.<sup>2</sup> The goal of these works is to show how an investment in any of these areas can improve one's standard of living over time. Arising from some of

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<sup>1</sup>See volume by Frederick Mosteller and Daniel P. Moynihan, On Equality of Educational Opportunity (New York: Vintage Books, 1972); Edward F. Denison, Why Growth Rates Differ (Washington, D.C.: The Brookings Institution, 1976); James P. Smith and Finis Welch, "Black-White Male Wage Ratios: 1960-70," American Economic Review 67 (June 1977): 232-338.

<sup>2</sup>See Michael Grossman, The Demand for Health: A Theoretical and Empirical Investigation, Occasional Paper No. 119 (New-York: Columbia University Press, 1972); Larry Sjaastad "The Costs and Returns to Human Migration", Journal of Political Economy 30 (October 1972): 80-93; Gary Becker, Human Capital, 2nd ed. (New York: National Bureau of Economic Research, 1975); Dennis DeTray, "Child Quality and the Demand for Children," in Economics of the Family; Marriage, Children and Human Capital, ed. Theodore W. Schultz (Chicago: The University of Chicago Press, 1973); Robert Willis, "Economic Theory of Fertility Behavior," in Schultz.

these fields of concentration has been a closer examination of the necessity for the various inputs that are used in the production process for these commodities.<sup>3</sup>

Elementary school teachers are an important input into the production of education, as well as into the economy of any nation. Not only do they affect directly the quality of the education children receive, but the foundations laid in these early years of schooling are vital to the future well being of society. It is surprising, however, that for an occupation as important and as vast as that of teachers, so little research into their labor market has been done.

This study, then, is one analytical approach to the labor market for public elementary school teachers. Viewing this market primarily from the demand side, it takes the demand for teachers as being derived from their use as an input into the production of a more basic commodity, quality education. The model and the derivation of the demand for teachers from a hypothetical production function for quality education will be presented first in chapter II.

The purpose of presenting such a model is twofold. First, a unified theoretical framework will be created which will allow a better grasp of some of the important issues in the economics of labor markets. The model will specifically relate to teachers, but can be readily generalized. Second, within this framework, the demand curve for teachers will be

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<sup>3</sup>Lee Benham, "The Labor Market for Registered Nurses: A Three Equation Model," Review of Economics and Statistics, 53 (August 1971): 246-252.

estimated, giving insight into a service market which is primarily female. A hypothetical supply equation is also given; allowing for a simultaneous system to be examined. The model will allow a testing of the direction and magnitude of the labor force response, again primarily from the demand side, to changes in certain environmental, as well as economic, variables. The results of these empirical tests will be given in chapters IV and V. The data set used for the empirical work is documented in chapter III.

The understanding of the relationships of this model for a very specific labor market for public elementary school teachers, as well as of the results, will enable decision makers better insights into their approach to and implementation of public policy. The implications of the model discussed will include an analysis of the significance of the own price elasticity of demand to the development of unions in this public service industry. Second, the question of the effects of substitutes for teachers on teacher demand will be looked at. The specific substitute of interest here will be paraprofessionals. The issue of the use of paraprofessionals is especially current in many fields other than education, and the model will develop a framework for testing the use of paraprofessionals on the demand for any other (professional) public service. Finally, some other policy issues will be briefly focused upon in chapter VI.

Underlying this study is the fact that the inputs into the educational process are adjusted for efficiency. An analysis of what is meant by the efficiency of teachers and

the effect of this on their demand will be made. The whole issue of efficiency is a crucial determinant in the market for teachers, and will be developed in such a way so as to include not only the direct quality of the individual teacher, but also the effects of the child's environment and other socio-economic characteristics on the teacher's and other input's efficiency.

To begin, then, in chapter II, with the theoretical model.

## CHAPTER II

THEORETICAL ASPECTS OF THE DEMAND FOR  
PUBLIC ELEMENTARY SCHOOL TEACHERSA. The Model

Formulated here is a model of the demand for public elementary school teachers which will allow broad theoretical and empirical observations into the nature of their labor market. The foundations for the model are based both on the classical approach to labor markets and on the more recent household production approach.

The determination of the demand for any group of labor can be analyzed either in terms of labor's contribution to productivity, or as being derived from the need for labor as an input into production. Theoretically, the two approaches are tantamount to saying the same thing. Each approach tends to spotlight certain aspects of a given problem, but the resultant demands for labor should be identical. The first approach to demand, stating that firms set the wage equal to the value of the marginal product of labor, in perfect competition, is equivalent to presenting a profit maximizing situation where marginal revenue equals marginal cost. The sum of these individuals firms' demand curves for labor is a good approximation of market demand, although the market demand for the factor will be somewhat less elastic than the ordinary horizontal summation since the demand curve for output is downward sloping at the industry level. As Friedman points out, however, "...to say that wages are equal to the value of the marginal product says relatively

little in and of itself. Its function is rather to suggest what to look for in further analysis. . . . (The value of the marginal product) will depend on the quality and quantity of workers, the quantity of capital they have to work with, the quality of management in organizing their activities, the institutional structure of the markets in which they are hired and the product sold, etc."<sup>1</sup> In addition, the entire marginal productivity approach will depend on clearly identifying the nature of the output, as well as its price.

It is for these reasons and most especially because the output of the educational process is not clearly defined that the demand for teachers is taken as being derived from the educational production function. An educational production function is defined as "...the relationship between school and student inputs and a measure of school output."<sup>2</sup> The model hypothesized here is that this educational output is implicitly produced within a market using certain inputs of goods and time, adjusted for quality, thus enabling the derivation of the desired demand curve. This approach permits the inclusion of some of the more important familial and institutional factors now known to affect the demand for teachers, and suggests the substitutability of certain social variables for purchasable economic ones.

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<sup>1</sup>Milton Friedman, Price Theory, revised ed. (Chicago: Aldine, 1971), p. 123

<sup>2</sup>Samuel Bowles, "Towards an Educational Production Function," in Education and Human Capital, ed. W. Lee Hansen (New York: Columbia University Press, 1970), p. 12.

Let us at first suppose that any educational output is determined by three broad classes of variables: teachers, other school inputs, and parental and student inputs, and is further affected by the efficiency of these variables. Thus, let the hypothesized production function for education within a homogeneous school district be:

$$(1) \quad Q = f(B_T T, B_R R, B_M M),$$

where:

- Q = some measure of the educational output
- T = a physical measure of teacher input
- R = a vector of other school personnel inputs, in physical units
- M = a vector of parents' and students' inputs of time spent in aiding and obtaining child education out of school, while the child is attending school

$B_T, B_R, B_M$  = efficiency parameters of T, R, M.

In addition, the following symbols are used below:

- N = number of children in school district
- $w, r, m$  = prices of respective inputs T, R, and M
- $\eta$  = the price elasticity of demand for education, Q
- $P_Q$  = (shadow) price per unit of education
- $s_T, s_R, s_M$  = shares of costs of inputs in total costs of output
- $\sigma_{ij}$  = elasticity of substitution of inputs  
(i, j = T, R, M)
- C = total cost of producing education
- D = vector of all other shift parameters
- E = percentage change in = dlog operator.

From this production function, we can derive the market demand equation for teachers, assuming, at first, a linearly

homogeneous production function with perfect competition in the factor and output market. The objective of the derivation, presented fully in appendix A, is to minimize cost, given the production function (1) where the cost function is:

$$(2) \quad C = \frac{w}{B_T} B_T T + \frac{r}{B_R} B_R R + \frac{m}{B_M} B_M M,$$

letting  $\frac{w}{B_T}$ ,  $\frac{r}{B_R}$ ,  $\frac{m}{B_M}$  be the effective prices paid to the

inputs. The necessary condition for maximum output is that ratios of the effective wages to the marginal products be equal, or:

$$\frac{\frac{w}{B_T}}{f_T} = \frac{\frac{r}{B_R}}{f_R} = \frac{\frac{m}{B_M}}{f_M} \quad 3$$

We are also able to show that in a competitive equilibrium, the prices of inputs are equal to the value of their marginal products, in effective units, or:

$$P_Q f_T = \frac{w}{B_T}, \quad P_Q f_R = \frac{r}{B_R} \quad \text{and} \quad P_Q f_M = \frac{m}{B_M}.$$

Since output is a function of prices, we have the following set of conditions:

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$$^3 \text{Where } f_T = \frac{\partial Q}{\partial (B_T T)}, \quad f_R = \frac{\partial Q}{\partial (B_R R)} \quad \text{and} \quad f_M = \frac{\partial Q}{\partial (B_M M)},$$

$$\begin{aligned} \text{and} \quad \frac{\frac{w}{B_T}}{\frac{\partial Q}{\partial (B_T T)}} &= \frac{\frac{r}{B_R}}{\frac{\partial Q}{\partial (B_R R)}} = \frac{\frac{m}{B_M}}{\frac{\partial Q}{\partial (B_M M)}} \\ &= \frac{w}{MP_T} = \frac{r}{MP_R} = \frac{m}{MP_M}. \end{aligned}$$

$$(3a) \quad f(B_{TT}, B_{RR}, B_{MM}) = Q = \vartheta(P)$$

$$(3b) \quad \frac{w}{B_T} = P_Q \frac{dQ}{d(B_{TT})}$$

$$(3c) \quad \frac{r}{B_R} = P_Q \frac{dQ}{d(B_{RR})}$$

$$(3d) \quad \frac{m}{B_M} = P_Q \frac{dQ}{d(B_{MM})}$$

By differentiating, setting in elasticity form, and by defining the price elasticity of demand for education to be  $\eta = -\frac{P_Q}{Q} \frac{dQ}{dP_Q}$  and the elasticity of substitution, between say M and T, under constant returns to scale as:

$$\sigma_{TM} = \frac{d \left( \frac{B_{TT}}{B_{MM}} \right)}{\frac{B_{TT}}{B_{MM}}} \bigg/ \frac{d \left( \frac{\frac{m}{B_M}}{\frac{w}{B_T}} \right)}{\frac{\frac{m}{B_M}}{\frac{w}{B_T}}} = \frac{\frac{d \left( \frac{T}{M} \right)}{\frac{T}{M}}}{\frac{d \left( \frac{m}{w} \right)}{\frac{m}{w}}},$$

we have

$$(4) \quad E \left( \frac{T}{M} \right) = s_T (\sigma_{TT} - \eta) Ew - (s_T (\sigma_{TT} - \eta) + 1) EB_T \\ + \sum_{j=R}^M s_j (\sigma_{Tj} - \eta) Ew_j - \sum_{j=R}^M s_j (\sigma_{Tj} - \eta) EB_j \\ + ED \quad . \quad j = R, M.$$

The market demand for teachers is shown to be functionally related to their wages, effectiveness, the price and effectiveness of other inputs and various other shift parameters.<sup>4</sup>

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<sup>4</sup>See appendix A.

At times, it is easier to analyze this model when it is expressed as a two variable case. To do this, define  $V$  as some vector of all the inputs in the model other than teachers, and  $v$  a vector of their wages. Now, equation (4) collapses into the two variable case:

$$(4a) \quad ET = s_T(\sigma_{TT} - \eta)Ew - (s_T(\sigma_{TT} - \eta) + 1)EB_T \\ + s_V(\sigma_{TV} - \eta)Ev - s_V(\sigma_{TV} - \eta)EB_V + ED.$$

Further using the identity  $s_T\sigma_{TT} + s_V\sigma_{TV} = 0$  and  $s_T\sigma_{TT} = -s_V\sigma_{TV}$ , we have;

$$(4b) \quad ET = -(s_T\eta + s_V\sigma_{TV})Ew \\ + ((s_T\eta - 1) + s_V\sigma_{TV})EB_T + s_V(\sigma_{TV} - \eta)Ev \\ - s_V(\sigma_{TV} - \eta)EB_V + ED.$$

Although this formulation is not strictly developed in a household production context, the theory owes much to that general framework.<sup>5</sup> An expanded approach might be noted to see how the model may be compatible to the household theories. This approach takes the stock of child services,  $C$ , as one of the basic commodities going into the household utility function,  $U = U(C, S)$ , with  $S$  being all other goods contributing to the household's standard of living. Child services are then said to be produced according to a production function  $C = C(N, Q)$ , with  $N$  the number of children in the family and  $Q$  their quality. Concurrently, child quality,  $Q$ , is said to be produced with inputs of time and

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<sup>5</sup>Gary Becker, "A Theory of the Allocation of Time," Economic Journal LXXV (September 1965): 493-517.

goods by various members of the household, taking into account their efficiency. The hypothesis presented here might be said to partition  $Q$  into the various components making up child quality, with the one of interest here being the quality of a child's education. Household time and goods, and their quality, similarly go into this production function. From this, we can derive the demand for teachers, which is one of the goods used by the household (embodied in the school district) in producing education.<sup>6</sup>

The efficiency adjusted derived demand methodology used here has been ascribed by some to H. Gregg Lewis, although the mathematical derived demand formulations date at least to Hicks' Theory of Wages. Marshall hints at having mathematically derived his rules for joint demand, as Hicks gives him credit for, but the full proof was never published prior to Hicks. A fuller treatment is in R.G.D. Allen, where the proof for the general equation is given, first for the two variable case and then for the multi-dimensional case. C.E. Ferguson extended and clarified the process to include the technological coefficients.<sup>7</sup>

Recently, a number of empirical works have emerged incorporating this approach into their models. In his analysis of the demand for child services, Dennis DeTray

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<sup>6</sup>See also DeTray, Willis.

<sup>7</sup>DeTray, p. 114; J.R. Hicks, The Theory of Wages (New York: The MacMillian Co., 1957); Alfred Marshall, Principles of Economics, 8th ed. (New York: The MacMillian Co., 1938), note XV, p. 853; R.G.D. Allen, Mathematical Analysis For Economists (New York: St. Martin's Press, 1938), pp. 369-374, 505-509; C.E. Ferguson, The Neoclassical Theory of Production and Distribution, (Cambridge: The University Press, 1969).

derives demand equations of the type used here to treat the substitutability of quality for quantity of children in the household's production of child services.<sup>8</sup> Setting the production function for child services, again, as  $C = C(Q,N)$ , with  $Q$  as the quality and  $N$  as quantity of children in a household, and taking full wealth considerations in account, he derives the demand equation for the percentage change in child quantity as:

$$\frac{\Delta N}{N} = b_0 \frac{\Delta V}{V} - b_1 \frac{\Delta P_N}{P_N} + b_2 \frac{\Delta P_i}{P_i} + b_3 \frac{\Delta w_j}{w_j} + b_4 \frac{\Delta B}{B} + b_5 \frac{\Delta Y}{Y}$$

where  $V$  is non-wage income,  $P_N$  is the price per child,  $P_i$  is the price of  $i$ th market good going into  $N$ ,  $w_j$  is wage rate of  $j$ th household member,  $B$  is husband's efficiency in non-market production and  $Y$  the wife's efficiency in non-market production. The coefficients, when expanded, include terms for the shares of expenditures, income and output elasticities, lifetime wealth terms, and the elasticities of substitution between child services and other goods, as well as between child quality and quantity.

James P. Smith also follows this general methodology to analyze the ways in which income transfer programs affect market work hours supplied by various family members, within a household decision model.<sup>9</sup> His model develops a set of derived demand equations showing the variables affecting the

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<sup>8</sup>DeTray, see especially pp. 94-95, 114-116.

<sup>9</sup>James P. Smith, "On the Labor Supply Effects of Age-Related Income Maintenance Programs", Journal of Human Resources X (Spring 1975): 27.

amount of time required in home production at all ages for each family member. He has, in one case, the percentage change in home time for a family member (inversely related to market work time) dependent on the percentage changes in the family's real wealth, the costs of consumption at a given age relative to lifetime prices, wages of various family members at various ages, and on the members' age.

Again the theoretical answer to the problem at hand depends on the size and sign of the terms within the coefficients--the elasticities of substitution (here between inputs in home production), price elasticities of demand for output, cross elasticities of demand for output, cross elasticities of demand, shares of wealth and costs, etc. A major justification for using this approach lies in this breakdown of the coefficients. After a brief discussion of the educational output term, the model's coefficients will be examined.

#### B. The Variables

Deriving the demand in this fashion allows a way out of the necessity to explicitly define the final product of the educational production process. However, since the demand for teachers is derived from the production function whose final product is variously and synonymously called in this study "education," "quality education" or simply "educational output," and although this output will not in itself be measured, it might be best to elaborate briefly on this educational function.

Indeed, if there is any common ground in the literature on educational production functions, it is the agreement on the disagreements in defining and measuring both the inputs and outputs of the educational process. However, within any community, it is safe to say that the local public good is the "...'quality' of public education parents desire to purchase for their children."<sup>10</sup> The crux of the debate over the meaning of quality education is based on the idea that the output of an educational process is subjective -- it means different things to different people.

The usual measures of the quality of an education are the scores on a set of standardized achievement tests. There are, however, many other measures of "...the output of the educational process, such as...juvenile delinquency rates, post-school income streams, occupational choice or level of education completed."<sup>11</sup> And again, "(s)cholastic achievement...is not the only dimension of school output. Literally hundreds of tests have been devised to measure 'achievement' alone, and this is only one aspect of the effect of schooling on cognitive skills and personality. In addition to economic performance in post-school years, schooling may

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<sup>10</sup>Sherwin Rosen and Joseph R. Antos, "Discrimination in The Labor Market for Teachers," Journal of Econometrics 3 (May 1975): 125.

<sup>11</sup>Eric Hanushek, "Teacher Characteristics and Gains in Student Achievement: Estimation Using Microdata," AER Papers and Proceedings (May 1971), p. 281.

affect an individual's self-concept and his sense of control over his environment."<sup>12</sup>

Thus, the output of the educational process is "multi-dimensional with a vengeance."<sup>13</sup> For the decision makers, though, as Hanushek points out, most interest is shown in the measurable achievement output of schools for public policy purposes.<sup>14</sup> The standard rationale for using achievement scores as the measure of education is that they are "...considered proxies for, or influences on, later economic performance, (and in addition) many socially or individually valued characteristics are themselves functions of scholastic achievement."<sup>15</sup>

Welch takes an approach towards the educational output which also hints at a justification for using a household production function for education. Education, he says, is the "...quality times the quantity of schooling..."<sup>16</sup> He goes on to say that the quality aspect of schooling is the "...partial product of a production process in which inputs by school systems and time in attendance are transformed into education."<sup>17</sup> One additional unit of education can now be taken as proportional to either more intensive inputs by

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<sup>12</sup>Bowles, p. 22.

<sup>13</sup>Bowles, p. 23.

<sup>14</sup>Hanushek, p. 281.

<sup>15</sup>Bowles, p. 21.

<sup>16</sup>Finis Welch, "Measurement of the Quality of Schooling," American Economic Review LVI (May 1966): 380.

school systems or more time in attendance, or both. In addition, with a slight twist, one can say that it is market inputs and time by members of the school district (e.g. parents, administrators, teachers, taxpayers, students) that are transformed into the educational output, or the quality of education.

It is more important, however, having said this about educational output, to examine each of the exogenous variables and their theoretical coefficients in the model (whether in singular, index or vector form), since it is these inputs, and not the output, which appear in the demand equation. The coefficient of the teacher wage variable is the community's own price elasticity of demand for teachers, and is equal to  $s_T(\sigma_{TT} - \eta)$  from (4). This is the only elasticity with an unambiguous sign in the model. All shares of total cost are positive, including  $s_T$ . The price elasticity of demand for output,  $\eta$ , is defined as positive, and the own elasticity of substitution term  $\sigma_{TT}$  is negative. Thus, the wage coefficient must be negative. As the price of teachers increases, for example, all else constant, the cost of education increases, quantity demanded of education decreases and the quantity demanded of teachers falls by  $s_T\eta$ . At the same time, one can see that other inputs are being substituted for teachers on the aggregate (the  $s_T\sigma_{TT}$  in the wage coefficient in equation (4) or the  $s_V\sigma_{TV}$  in equation (4b)), and the demand for teachers also decreases for this reason.

Also of interest here is the  $r$  variable, which is formally defined as a price vector of other school personnel

inputs -- although capital inputs may also be included in this vector without changing the model. One of the assumptions about school inputs is that there is no percentage change in physical capital costs in a cross-sectional study, so that the subset of  $E_r$  (in the equation as part of  $\sum Ew_j$ ) representing possible capital costs is zero. Thus, the remaining subsets of  $E_r$  come to mean school personnel costs; the wages of paraprofessionals, administrators, secretaries, etc., who work in public elementary schools. In particular, the substitutability of paraprofessionals for teachers is an important labor market issue as is the general tradeoff between paraprofessionals and other professional groups. The theory tells us that if  $r$  is the wage of paraprofessionals, the sign of its coefficient,  $s_R(\sigma_{TR} - \eta)$ , depends on the sign and magnitude of  $\sigma_{TR}$ . If teachers and paraprofessionals are net substitutes,  $\sigma_{TR}$  is positive and as the wages of paraprofessionals go up, more teachers will be demanded as they are substituted for paraprofessionals. But since the cost of education increases, fewer teachers will be demanded, and the ultimate change in the demand for teachers will increase or decrease as  $\sigma_{TR}$  is greater or less than  $\eta$ . On the other hand, if paraprofessionals are complements of teachers, the demand for teachers must decrease ( $\sigma_{TR}$  is negative). Similar interpretations to the other wage coefficients of the other school personnel apply. On the average, though, teachers and the bundle of all other inputs must be net substitutes ( $\sigma_{TV}$  in (4b) must be positive).

The model also includes a vector of inputs reflecting the societal situation of all the children in public elementary school, including their parents' background, the child's ability and his neighborhood characteristics. Generally speaking, the output of the educational process is "...dependent on influences either related to expenditure or to the environment in which the school district finds itself."<sup>18</sup> It is easiest in terms of the model to justify the inclusion of the physical units of the inputs upon which direct expenditures are spent, such as teachers, other personnel, books, etc. It is harder, however, to explicitly take into account environmental inputs. The model has been developed in such a way so as to include these environmental and household variables.

In a reexamination of the Equality of Educational Opportunity Survey (commonly called the Coleman Report), David Armor reviews and lists a number of factors comprising the environment of the child.<sup>19</sup> First, he says, there is the environment "...of the child's immediate family. A family's life style - their stress on education, their economic well being, their child rearing practices and other similar factors - is likely to have an important effect upon the

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<sup>18</sup>Herbert Kiesling, "Measuring a Local Government Service: A Study of School Districts in New York State," The Review of Economics and Statistics XLIX (August 1967): 857.

<sup>19</sup>David J. Amor, "School and Family Effects on Black and White Achievement: A Reexamination of the USOE data," in Mosteller and Moynihan, pp. 172177.

child's interest in and ability to carry out academic work....There has been much research in sociology, prior to the Coleman report, which has established the existence of a relationship between school achievement and family socioeconomic status or life style." "Families," adds Hanushek, "obviously have considerable impact on education through...direct (time) involvement in the educational process."<sup>20</sup>

It follows that if family background has an influence on educational production, and the demand for teachers is derived from this production process, then these variables should have their effect on teacher demand tested. In terms of the model, the inclusion of family background can be interpreted as being included by the use of the effectiveness of parents and students term ( $B_M$ ), as well as their price of time (primarily the parents' or mother's) ( $m$ ), and family income terms. From time to time one reads in the press of parents removing their children from school in response to a disagreement with teaching methods, promising to educate the children themselves. Although this is not to imply that parents within a school district are complete substitutes for teachers, it does question as to whether there is any significant substitution between teachers' and parents' time.

One of the measures of parental efficiency as it affects the demand for teachers is the educational attainment of the parents (mother). This educational level can be tested not

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<sup>20</sup>Hanushek, p. 281.

only as the mother's efficiency ( $B_M$ ) in the model, but alternatively as a shadow cost of her time ( $m$ ) or as her potential contribution to full family income.<sup>21</sup> Inclusion of these variables also accounts for the quantity and quality of the time devoted to children within the household, since these have been shown to be positively related to parents' education.<sup>22</sup>

Not only is  $B_M$  a measure of the flow of parental inputs which the child receives while he is in school, but it may also be interpreted as a reflection of the stock of human capital acquired by the child prior to entering elementary school. It has been shown that "... (b) y the time children enter first grade, significant differences in verbal and mathematical competence exist among them."<sup>23</sup> If child ability is a partial measure of the stock of parental inputs, then the efficiency of a student may be interpreted as such in the model.

Another factor presumed to influence the child's education and thus the demand for teachers is the "... neighborhood or community in which the family resides."<sup>24</sup> or variously the "... socioeconomic attributes of the

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<sup>21</sup>Linda Nasif Edwards, "The Economics of Schooling Decisions: Teenage Enrollment Rates," Journal of Human Resources X (Spring 1975): 160.

<sup>22</sup>Arleen Leibowitz, "Home Investments in Children," in Schultz, pp. 434, 437.

<sup>23</sup>Ibid., p. 432

<sup>24</sup>Armour, p. 173.

community."<sup>25</sup> This too can be a stock-flow measure, and can be proxied by different inputs in the model. Family income, of course, can be a strong measure of the neighborhood effect. Race may be a measure of community influence, and may be included as a shift parameter. Or it may be shown as a measure of teacher effectiveness ( $B_T$ ).

The meaning of  $B_T$  and  $B_V$  as measures of efficiency needs clarification. Given a simple, homogeneous, variable proportions production function, under perfect competition, with two inputs of teachers (T) and all else (V), the changes in demand for teachers with respect to changes in effectiveness only are  $ET = ((s_T \eta - 1) + s_V \sigma_{TV}) EB_T$  and  $ET = -s_V (\sigma_{TV} - \eta) EB_V$ . Similarly,  $\frac{w}{B_T}$  and  $\frac{v}{B_V}$  are wages per efficiency unit, and  $\left(\frac{w}{B_T} + \frac{v}{B_V}\right)$  is the effective wage cost.<sup>26</sup>

With this in mind, the question to be answered in reality is what actually is the change in the demand for teachers with any change in efficiency. As teachers become more efficient ( $B_T$  increases), the cost of education  $\left(\frac{w}{B_T} + \frac{v}{B_V}\right)$  goes down and the demand for teachers goes up by  $s_T \eta$  (via the increase in the demand for education) in fixed proportions. The demand for teachers also increases as a result of substituting the relatively cheaper teachers for

<sup>25</sup>Kiesling, p. 357.

<sup>26</sup>Alan Monheit "An Economic Analysis of State Licensing of Nursing Homes," (Ph.D. dissertation, The City University of New York, 1975), note 62, pp. 56-57.

other inputs ( $= s_V \sigma_{TV}$ ) in the two variable case, where T and V must be substitutes. But since the same output can be produced with less of all inputs, the demand for teachers decreases. The net effect of these changes determines whether the final demand for teachers decreases or increases as they become more effective.

The problem of just what affects teachers' effectiveness is difficult. The usual way of defining a teachers' efficiency is by his education, experience and intelligence. However, recent studies have shown that for low achievers, for example, less experienced teachers are more effective than more experienced ones, as measured by certain criteria.<sup>27</sup> It is presumed that teachers' attitudes towards the student population they are working with, as well as their similarity of background vis-a-vis their students, will also affect their effectiveness. Hanushek, for one presents some interesting evidence on teachers' effectiveness: "The teacher characteristics that appear to be important are not the characteristics that are purchased by schools (educational level, years of experience, etc.)." He states that it is "...the score on the verbal facility test and the recentness of education (that) are the most important factors."<sup>28</sup> In addition, there is a third "quasi-teacher

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<sup>27</sup>Anita A. Summers and Barbara L. Wolfe, Efficiency and Equity in Philadelphia Public Schools (Philadelphia: Federal Reserve Bank of Philadelphia, 1975), p. 20.

<sup>28</sup>Eric Hanushek, Education and Race (Lexington, Massachusetts: Lexington Books, 1972), p. 44.

characteristic -- the percentage of time spent on discipline -- that is important.

The significance of the verbal facility measure of teacher efficiency is that it is both a measure of communicative ability and overall intelligence, independent of formal or on-the-job training. The more recent a teacher's educational experience is also shown to positively affect the efficiency of the teacher. This would justify requiring teachers to continue taking courses, even if not enrolled in a formal degree program. It is thus important to include a vector of these parameters in order to see how they might affect the demand for teachers.<sup>29</sup>

The variables included in the model's demand equation form the backbone of the current genre of educational production functions, albeit in a different arrangement. These functions follow a vein similar to one used by Hanushek and Kain.<sup>30</sup> They have:

$$A_{it} = g(F_i(t), P_i(t), I_i, S_i(t)), \text{ where}$$

$A_{it}$  = a vector of educational achievement of  $i$ th student at time  $t$

$F_i(t)$  = vector of individual and family characteristics for  $i$ th student cumulative to  $t$

$P_i(t)$  = vector of student body characteristics (peer influences), i.e., socioeconomic and background characteristics of other students in the school, cumulative to  $t$

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<sup>29</sup>Ibid., pp. 42-47.

<sup>30</sup>Eric Hanushek and John F. Kain "On the Value of 'Equality of Educational Opportunity' As a Guide to Public Policy," in Mosteller and Moynihan, p. 123.

$I_i$  = vector of initial endowments of ith individual

$S_i(t)$  = vector of school inputs relevant to ith student,  
cumulative to t.

These variables, when rearranged, are identical to the model's.<sup>31</sup>

A general shift parameter, D, is included in the current model to enable the insertation of other variables said to affect the demand for teachers. Elaboration of some of these variables, which may include such items as exogenous tax changes or degrees of unionization, will be given in the empirical sections below.

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<sup>31</sup>Also see, for other formulations, U.S. Department of Health, Education and Welfare, Office of Education, Equality of Educational Opportunity, Vol. 1, by James S. Coleman (Washington, D.C.: Government Printing Office, 1966); Bowles; two articles by Hanushek; other articles in Mosteller and Moynihan.

CHAPTER III  
THE DATA SET

A. The Data

The data set used for this study is the 1/100 15% Standard Metropolitan Statistical Area (SMSA) and County Group sample tapes of the 1970 Census of Population.<sup>1</sup> These tapes contained data on two million individuals, one-fifteenth of the 30 million people for whom data was collected. The relevant variables were grouped by the 125 SMSA's as classified by the Census Bureau, and the model was tested using these cross-sectional cases.

Alternatively, school districts would have been an appropriate unit of measurement, since each district may be viewed as an individual market for teachers. The difficulty with using school district data put out by the National Center for Education Statistics, the Office of Education, or with most other sources, however, lies in the fact that the specification of the model requires socio-economic characteristics of teachers, students, parents and neighborhoods, as well as the usual price and quantity data used in labor market studies. In addition, the reduced form of the model requires information about the teachers' background, and about the background of their spouses and families. A cohesive breakdown of all of these types of

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<sup>1</sup>U.S. Bureau of the Census, Public Use Samples of Basic Records From the 1970 Census: Description and Technical Documentation, Washington, D.C. 1972.

variables does not exist in school district form. One can obtain teacher price and quantity data by school district, although not all of the available figures are broken down into individual elementary and secondary units. But very little else is obtainable by school districts, such as figures on public elementary school teachers' marital status, their husbands' income, age, nor on the educational level of the parents of the children they teach, their incomes or the number of siblings in each family.

Thus, these variables are constructed using Standard Metropolitan Statistical Areas as the unit of observation. SMSA's are thought to closely resemble a labor market where a teacher is most likely to seek information about jobs, and individual employers are likely to search for prospective employees (with an eye, of course, on neighboring communities' wages -- the so called "spillover effect"). The 125 SMSA's will give a better variation in the variables than the 50 states as the unit of observation. SMSA's are also individually more homogeneous and collectively allow more logical boundaries, from a labor market point of view, than state lines since an SMSA market can cross state lines. Rural areas are omitted because the interest here is in urban communities.

The 15% sample was chosen since it contains an indication of the nature of the school (public or private) one is attending. The 15% sample also includes a good measure of teacher migration as an additional variable not found in other samples.

Limiting the study to elementary schools allows a way to test a specific education labor market without having to partition the demand for teachers into different specialties, which should be done in secondary studies where there may be a shortage of, say, biology teachers at the same time as a surplus of social studies teachers. And elementary school, in all cases, is compulsory for those of the age usually associated with attending elementary school -- 13 years old or below. Public schools were chosen as a more homogeneous market, omitting the characteristics of those who teach in or attend private or religious schools, who may be responsive to a different set of rules. It allows a closer focus on a specific labor market.

#### B. The Variables

The measure of the quantity of teachers (as the dependent variable) is the number of full time equivalent teachers per public elementary school pupil in the SMSA (the inverse of the student-teacher ratio). This approach corrects for different sized school districts. The number of teachers within each area was taken as the total of all teachers who were at work, for pay, in an elementary school, exclusive of kindergarten teachers, who also worked for a local, state or federal government school (but not in the armed forces). Most of these were local employees. Since teachers work varying schedules in different regions of the country, any teacher who reported her weeks worked within the 40-47 week group or higher was considered full time. This presumes a school vacation of 5 to 12 weeks, although some

teachers reported working 50 to 52 weeks a year. The number of full time teachers plus one half the number of part timers (all others) was set as equal to the number of full time equivalents.

Table III.1, which gives selected teacher summary statistics, shows the overall SMSA means and standard deviations of the values of the observations used in the regressions. There are, on the average, 41 full time equivalent teachers per SMSA, which projects to 4100 in the full population. At .0284 teachers per student (unweighted for SMSA size here), there are 35.2 students per class. This student-teacher ratio is higher than many other published ratios (see table III.3, for example) since it is calculated on the basis of full time equivalents. Many of the part timers are specialists who may be counted in the usual published reports without correcting for full time status, as are other part timers. In addition, many of the specialists appear in the census classifications in an occupational group which is more closely related to their specialty, rather than as an elementary school teacher, further pulling up the calculated student-teacher ratio.

The relevant wage variables used posed a more complicated question. Operationally, the wage upon which communities base their hiring decisions is the annual salary. The wage variable, then, is full time equivalents' wage, a yearly figure including wages, salary and money income before taxes paid to teachers in the given census survey year, 1969. This was computed by taking the sum of all earned wages for

TABLE III.1

Summary Statistics of Public Elementary School Teacher and Occupational Characteristics, Overall SMSA Means or Percentages

<u>Variable</u>	<u>Mean</u>	<u>Standard Deviation</u>
Students in school	1382	1773
Full time equivalent teachers	41.00	62.23
Full time equivalent teachers per student	.0284	.0080
Full time equivalent teachers' wage (annual)	8310	1207
Teachers' education completed	16.08	.40
Percent of teachers female	84.02	8.72
Teachers' age	39.51	3.07
Percent of teachers non-white	10.44	11.75
Highest wage paid to teachers with exactly a bachelor's degree	10527	2827
Teachers' labor force participation rate	71.04	7.75
Percent of teachers under age 40	55.38	11.14
Total babies ever born to working teachers	1.69	.33
Number of children under age 18 in all female teachers' households	1.06	.33
Number of children under age 6 in all female teachers' households	.37	.16
Percent of teachers currently married, spouse present	69.61	10.62
Teachers' spouse total income	9574	1821
Teachers' spouse education completed	16.57	.77
Teachers' total family income	15346	2015

TABLE III.1 (continued)

<u>Variable</u>	<u>Mean</u>	<u>Standard Deviation</u>
Paraprofessional wage per hour	2.18	1.03
Paraprofessionals' education completed	12.22	1.12
Percent of paraprofessionals non-white	28.71	31.94
Administrators' earned wage (annual)	13186	2771
Administrators' education completed	17.32	.65
Secretaries' wage per hour	2.63	1.10
Secretaries' education completed	12.34	.75

all teachers who worked, and dividing it by the number of full time equivalents.

As an alternative, teachers' wage per hour was also calculated as the total earned wage divided by the product of the means of the intervals of hours worked and weeks worked. When this measure of wage was used in the empirical analysis, the results were not significant. One reason that this variable is unreliable is that many teachers reported to the census working more than sixty hours per week, indicating that many perceived at home work as part of their paid work schedule. Weeks worked also varied widely, further making teacher wage per hour a less useful figure than annual wage. Also, as checks on the salary schedule, certain steps on the schedule were calculated. These include the single highest wage paid to a teacher with only a bachelor's degree, the highest wage paid to any teacher, and the average wage paid to teachers with only a bachelor's degree. The average SMSA full time equivalent wage in 1969 was computed to be \$8,310, with a range of from \$5,181 to \$14,091.

The wages, both annual and hourly, for paraprofessionals were constructed similarly, with the difference being that paraprofessionals working 13 to 26 weeks or more were considered as full time. In the census reports, the occupational group called "teacher aides" are referred to as paraprofessionals in this paper. Paraprofessionals have been referred to by different titles in various school districts, such as educational assistants, educational associates or classroom aides, in addition to teacher aides. From accounts

of the job descriptions of paraprofessionals, these teacher aides are paraprofessionals primarily concerned with the teaching, on an individualized basis, of reading and arithmetic in the classroom. The computed values for teacher aides compares favorably with the characteristics of paraprofessionals in a study of elementary schools for the New York City Board of Education. They are mostly female, older, high school graduates with some college background.<sup>2</sup>

All of these aides were working for pay for a public (again local, state or federal) school. It is not possible to determine how they are distributed between elementary and secondary schools, however. It is assumed that across SMSA's, the elementary-secondary distribution is the same, and that this distribution will not affect the wage the community pays its homogeneous group of paraprofessionals. Similar data were collected for public school administrators and secretaries as alternative substitutes for teachers. Summary statistics are also presented in table III.1 for some of these variables.

Certain data were obtained about the parents and families for each child in a public elementary school. For calculations, the parent of each child was counted as one individual. If a family had two children in elementary school, then that parent's characteristics were counted twice

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<sup>2</sup>Henry M. Brickell, Paraprofessional Influence on Student Achievement and Attitudes and Paraprofessional Performance in the Classroom (New York: Institute for Educational Development, 1971).

in obtaining group means. Family data is used as a mean measure of the socio-economic background of the elementary student population. Therefore, average values of these variables as they affect the demand for teachers per student would indicate a slight weight is needed in the direction of those families with more children in school.

For example, if one assumes that there is an educational flow from a mother to each of her children which affects the child in school, then the average educational level of the mother, all else constant, must be attributed to each of her children in school. Or, one wealthy family with one child in school and two poor families each with several children in school would indicate a per family wealth greater in unweighted form than the actual per student background would justify.

Table III.2 presents summary statistics for the family variables in each SMSA. The fathers of these children are slightly more educated than the mothers. The fathers also have the expected much higher labor force participation rate, although over forty percent of these mothers with young children present reported they were in the labor force some amount of time. The 4.01 babies ever born to these mothers might appear somewhat high, but this is caused by the weight in favor of the large families.

SMSA family income was also computed only for families with children in public elementary school, while median income was for all families in the SMSA. All educational levels used in this study for both parents and for the

TABLE III.2

Summary Statistics of Family, Community and Children  
 Characteristics, Overall SMSA Means or Percentages

<u>Variable</u>	<u>Mean</u>	<u>Standard Deviation</u>
Number of students in public school	1382	1773
Total population of SMSA	9784	14421
Percent of children non-white	15.65	12.29
Fathers' education completed	11.35	.77
Mothers' education completed	11.12	.63
Number of children in students' household under age 18	3.40	.18
Number of children in students' household under age 6	.54	.08
Babies ever born in students' household	4.01	.28
Fathers' labor force participation rate	95.85	2.28
Mothers' labor force participation rate	41.42	5.57
Fathers' total income	10139	1464
Income of students' family	11360	1541
Median income of SMSA	10061	1201
Per capita SMSA property tax	128.89	47.40

occupational classes were means of years completed in school, in actual grades (12 equals a high school graduate, 16 a college graduate). Race is always the percentage of non-whites in the area. The data on teachers' spouses and families were counted twice if both husband and wife were teachers.

As an additional data source, the Statistical Abstract of the United States was used to obtain data for 1969 (from the 1970 census) on the total SMSA population, median family income, and per capita SMSA property taxes.<sup>3</sup>

Table III.3 presents a supplementary set of teacher statistics by which to compare the calculated figures. These are not only SMSA variables, but statistics for all public elementary (here including kindergarten) school teachers in the United States. The data were also not corrected for full time status, but is useful in understanding the nature of the market. Note that teacher earnings for the country as a whole are lower than SMSA earnings, while the percentages of females and non-whites, as well as the median age and education levels, are similar to the calculated figures.

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<sup>3</sup>U.S. Bureau of the Census, Statistical Abstract of the United States, 1971, Washington, D.C., 1971.

TABLE III.3

## Supplemental Statistics of all Public Elementary School Teachers, 1969

Variable

Full and part time students per teacher <sup>1</sup>	24.8
Median earnings <sup>2</sup>	7088
Percent female <sup>2</sup>	83.40
Teachers' median age <sup>2</sup>	37.79
Percent non-white <sup>2</sup>	8.80
Median education completed <sup>2</sup>	16.67
1971 enrollment in metropolitan areas	19,799,000

- Sources:
1. U.S. Bureau of the Census, Statistical Abstract of the United States: 1971, Washington, D.C., 1971;
  2. U.S. Bureau of the Census, 1970 Census of Population, Subject Reports, Government Workers, PC (2)-70, Washington, D.C., 1973;
  3. U.S. Bureau of the Census, Statistical Abstract of the United States: 1973, Washington, D.C., 1973.

CHAPTER IV  
THE EMPIRICAL TESTS

A. The Estimation Strategy

The theoretical demand equation (4) presented above was initially estimated using both Ordinary Least Squares and Two Stage Least Square estimation approaches. The two stage least squares approach was chosen because, in any study, there exists the possibility of simultaneous equation bias in testing a single equation model, and the simultaneous estimation procedure will purge this bias.<sup>1</sup> In addition, since two stage least squares is equivalent to estimation by instrumental variables, it reduces any bias that might exist due to measurement error in the wage variable. Further, if there is any elasticity in the supply function, it becomes empirically interesting to observe the two stage results for demand, incorporating the second, or supply equation into the model. For the empirical work, then, a more complete set of structural equations are estimated in order to better test the data. The basic empirical specification of the demand equation is:

$$\begin{aligned} \text{LTS} = & b_0 + b_1 \text{LTFTEW} + b_2 \text{LTEDC} + b_3 \text{LPWPERH} + b_4 \text{PARAED} \\ & + b_5 \text{LTBAHIW} + b_6 \text{LPROPTAX} + b_7 \text{LKID18} + b_8 \text{MED} \\ & + b_9 \text{PRACE} + b_{10} \text{LKID6}. \end{aligned}$$

Variations in the basic formulation are generally semi-log variations of this specification, or include additional

<sup>1</sup>J. Johnston, Econometric Methods, 2nd ed. (New York: McGraw-Hill Book Co., 1972), pp. 341-348.

explanatory variables. These results are shown in the demand equation in table IV.1 in section B of this chapter. The variables used are:

L	= natural log of (before variable name)
TS	= ratio of the number of full time equivalent teachers to students
TFTEW	= wage of full time equivalent teachers, annually
TEDC	= teachers' education level completed
PWPERH	= paraprofessional wage per hour
PARAED	= paraprofessional education completed
TBAHIW	= highest annual wage for teachers with exactly a bachelor's degree
PROPTAX	= annual per capita property tax in SMSA
MED	= mothers' education completed
PRACE	= percent of paraprofessionals non-white
KID6	= number of children under age 6 in household of students
KID18	= number of children under age 18 in household of students
ADMW	= annual wage of school administrators
ADMED	= administrators' education completed
SWPERH	= secretaries' wage per hour
SECED	= secretaries' education completed
CN	= child number per SMSA
TPCTCOL5	= percent of teachers in college five years ago or less
TAGE	= age of teachers
FAGE	= age of fathers
MERW	= earned wage of mother

Since the primary theoretical, empirical and policy issues of this dissertation concern the demand for public elementary school teachers, the supply theory is less developed than that of demand, and follows previous work done in labor markets for women.<sup>2</sup> A more complete supply model is presented in chapter V. The hypothesized supply of public elementary school teachers per student is set to be a function of three broad classes of variables: their own wages ( $W$ ), a vector of teacher family and personal characteristics ( $TFAM_j$ ), and a vector of some community characteristics ( $CCOM_j$ ). The supply of teachers per student is then:

$$TS = f(W, TFAM_j, CCOM_j).$$

Included in the second group are the characteristics of the teachers' spouse and the presence of small children in their households. The third group includes some proxies for the type of area in which the teacher chooses to live, such as its income level or other neighborhood characteristics which may affect the decision to live in the area, and thus affect their supply.

The reduced form of the model includes the exogenous variables from both the demand and supply equations. In the two stage procedure, the endogenous variable used is the teachers' full time equivalent wage ( $TFTEW$ ). The reduced

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<sup>2</sup>See Jacob Mincer, "Labor Force Participation of Married Women," in Aspects of Labor Economics, ed. H. Gregg Lewis (Princeton: Princeton University Press, 1962).

form of the equations has a coefficient of determination of .4327.

The results presented in this and the next chapter were obtained using a weighted two stage least squares estimation procedure.<sup>3</sup> Weighted TSLS was chosen since the regressions were run over the grouped data from each of the 125 SMSA's, and the group size differs substantially over statistical areas. Thus, the residual variances may be heteroscedastic across regions. A weight equal to the square root of the group size will correct the residuals towards homoscedasticity.<sup>4</sup> The weight chosen is the square root of the total population of each SMSA. An alternative weight of the square root of the number of children in each SMSA was also tried, as were unweighted regressions. There was little change in the size of the coefficients or in the approximate level of significance among the two weighted and the unweighted results. Some t values increase and some decrease among the different sets of regressions.

<sup>3</sup>The Regression Analysis Program for Economists was used for the estimates. The  $R^2$  should be interpreted with caution. I forced the  $R^2$  to fall between zero and one by computing it based on the variance in the logarithm of the predicted wage rather than the variance in the logarithm of the actual wage. This procedure was used to get a rough approximation of the equations' "explanatory power." See William J. Raduchel, The Regression Analysis Program for Economists, Reference Guide, Technical Paper Number 10 (Revised), Harvard Institute of Economic Research, Cambridge, 1974.

<sup>4</sup> $\bar{Y}_i = b_0 + b_1 \bar{X}_i + \bar{u}_i$  is a grouped data equation with the bar indicating the grouped variables. The residual variance of the grouped data is:

$$\sigma^2(\bar{e}_i) = \frac{\sigma^2(e_i)}{N_i},$$

There is missing data in 28 (out of 125) SMSA's for paraprofessionals, and in 8 for administrators and secretaries (the substitutes for teachers). Predicted values for these missing variables were ultimately obtained using an instrumental variable technique. Separate sets of regressions were run on all of the complete data to obtain coefficients for the explanatory variables to be used for the prediction of each of these missing variables. In a few cases, an explanatory variable was also missing, and these were deleted in estimating the coefficients. Their means, though, were used in making the actual predictions. Regressions were also tried using a listwise deletion of all cases with missing values, but the OLS results were poor with that sample. The predicted values also yielded better results than assigning to them the values of the parameters of adjacent SMSA's as proxies.

One additional reason for choosing the two stage approach, other than the reduction of the simultaneous bias, is to correct for any measurement error that might exist, particularly in the number of full time equivalent teachers. This variable is used in constructing both the quantity

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<sup>4</sup>(continued) where  $N$  is the group size. A weight of  $\sqrt{N_i}$  gives:

$$\sqrt{N_i} \bar{Y}_i = b_0 + b_1(\sqrt{N_i} \bar{X}_i) + \sqrt{N_i} \bar{u}_i,$$

and

$$\sigma^2(\sqrt{N_i} \bar{e}_i) = \frac{N_i \sigma^2(e_i)}{N_i} = \sigma^2(e_i).$$

Thus, the weight yields homoscedastic residuals. See Chiswick and Chiswick, pp. 161-168; Johnston, pp. 228-238.

variable (TS) and the full time equivalent wage variable (TFTEW). If there is any measurement error in the number of full time equivalent teachers, the OLS estimates of the wage elasticity will be biased either towards minus one, or towards zero if the error in the dependent variable can be consolidated into the equations' measurement error.<sup>5</sup> The two stage results are consistent and do yield more error free results.

<sup>5</sup>Following Aigner, Borjas (p. 4, footnote), Johnston (pp. 281-283), Huang (pp. 158-160):

$$\text{Ln} \left( \frac{T^*}{S} \right) = B \text{Ln} \left( \frac{E}{T^*} \right) + u$$

is the true model, letting  $T^* = Tv$  with  $T^*$  being the true value of  $T$ , and  $v$  being an error term. Substituting:

$$\text{Ln} \left( \frac{Tv}{S} \right) = B \text{Ln} \left( \frac{E}{Tv} \right) + u$$

$$\text{Ln} \left( \frac{T}{S} \right) + \text{ln}v = B \text{Ln} \left( \frac{E}{T} \right) - B \text{ln}v + u$$

$$\text{Ln} \left( \frac{T}{S} \right) = B \text{Ln} \left( \frac{E}{T} \right) + (u - (1 + B) \text{ln}v).$$

We can prove:

$$\text{Plim } \hat{B} = \frac{-\sigma_{\text{ln}v}^2 (1 + B)}{\sigma_{\text{ln} \left( \frac{E}{T^*} \right)}^2 + \sigma_{\text{ln}v}^2} + B$$

$$\text{Plim } \hat{B} = \frac{-\sigma_{\text{ln}v}^2 (1 + B) + B(\sigma_{\text{ln} \left( \frac{E}{T^*} \right)}^2 + \sigma_{\text{ln}v}^2)}{\sigma_{\text{ln} \left( \frac{E}{T^*} \right)}^2 + \sigma_{\text{ln}v}^2}$$

$$\text{Plim } \hat{B} = \frac{-\sigma_{\text{ln}v}^2 + B\sigma_{\text{ln} \left( \frac{E}{T^*} \right)}^2}{\sigma_{\text{ln} \left( \frac{E}{T^*} \right)}^2 + \sigma_{\text{ln}v}^2}$$

$$\text{Plim } \hat{B} = \frac{B}{\frac{\sigma_{\text{ln}v}^2}{\sigma_{\text{ln} \left( \frac{E}{T^*} \right)}^2} + 1} - \frac{1}{\frac{\sigma_{\text{ln} \left( \frac{E}{T^*} \right)}^2}{\sigma_{\text{ln}v}^2} + 1}. \quad (\text{continued})$$

In addition, the two stage least squares procedure may possibly show a higher absolute value of the price elasticity of demand, since a one equation OLS model presumes an infinitely elastic supply. It has been pointed out that ". . .(i)f (a) government employer does face positively sloped labor supply curves, this will bias (the OLS) estimated wage coefficients, in a positive direction. Hence, the (OLS) estimates. . .may understate (in absolute terms) the wage elasticity of demand for public employees."<sup>6</sup>

#### B. Demand

The regression results for a number of alternative specifications of the demand equation are presented in table

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<sup>5</sup>(continued) Thus, the error term is composed of two components:

1. A bias towards zero,  $\frac{B}{\frac{\sigma_{\ln v}^2}{\sigma_{\ln\left(\frac{E}{T^*}\right)}^2} + 1}$ , as  $\sigma_{\ln v}^2$  becomes

larger. This is the usual measurement error if only the independent variable has error (Johnston, p. 281), or if the error in the dependent variable can be merged with the equation's measurement error. (Huang, p. 160).

2. An additive bias of  $\frac{-1}{\frac{\sigma_{\ln\left(\frac{E}{T^*}\right)}^2}{\sigma_{\ln v}^2} + 1}$  since the error in the

dependent variable is equivalent to the error in the independent variable.

<sup>6</sup>Ronald G. Ehrenberg, The Demand for State and Local Government Employees (Lexington, Massachusetts: Lexington Books, 1972), p. 16.

IV.1.<sup>7</sup> The own price elasticity of demand for public elementary school teachers,  $\frac{LTS}{LTFTEW}$ , is shown to be inelastic and in the range of  $-.7000$  to  $-.8000$  in most cases. The wage used, again, is annual full time equivalents' wage. The theoretical coefficient  $s_T (\sigma_{TT} - \eta)$  empirically is of the correct sign and expected magnitude.

The own elasticity of demand must be negative -- it is the only sure sign in the model -- since the share of total cost of teachers ( $s_T$ ) is positive, the price elasticity of demand for education ( $\eta$ ) is defined as positive and the own elasticity of substitution ( $\sigma_{TT}$ ) is negative. This result is consistent with previous hypothesis' on the demand for both education and for public school teachers (both elementary and secondary together), and with previous estimates of the elasticity of demand for teachers, however sparse they are.

Michael Moskow concludes that the demand for teachers should be inelastic in a wide range. From Marshall's rules of derived demand, we know that the demand for teachers is more inelastic (1) the more inelastic the demand for education, (2) the more essential teachers are in the production of education, (3) the smaller the fraction of the

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<sup>7</sup> Both the supply and demand equations are overidentified. It is assumed that the coefficient-standard error ratio (the t value) for the simultaneous system approaches a normal distribution for large samples, and the t value is an approximation of ". . . a degree of statistical significance in a crude test of whether the relevant variable belongs in the equation." See Christ, p. 598, Dhrymes, pp. 272-277.

TABLE IV.1

Two Stage Least Squares Estimates of the Demand Equations  
 Dependent Variable = LTS  
 (Absolute t Values in Parentheses)

Independent Variable	Specification			
	(1)	(2)	(3)	(4)
LTFTEW	-.7662 (2.469)	-.7644 (2.462)	-.7575 (2.400)	-.7668 (2.399)
LTEDC	1.5700 (1.457)	.0962* (1.424)	1.1573 (1.400)	1.5468 (1.414)
LPWPERH	.0813 (1.871)	.0813 (1.871)	.0800 (1.821)	.0807 (1.830)
PARAED	-.0404 (1.869)	-.0405 (1.872)	-.0441 (1.997)	-.0448 (2.002)
LTBAHIW	.3711 (4.065)	.3717 (4.070)	.3873 (4.002)	.3904 (3.986)
LPROPTAX	.1707 (2.153)	.1707 (2.152)	.1716 (2.153)	.1827 (2.206)
LMED	-.5178 (1.177)	-.5140 (1.169)	-.4722 (1.058)	-.4767 (1.060)
PRACE	.0008 (.990)	.0008 (.996)	.0007 (.869)	.0008 (.971)
LKID6	.3498 (1.469)	.3509 (1.473)	.2914 (1.158)	.3116 (1.213)
LKID18	-1.1178 (2.009)	-1.1219 (2.016)	-1.0585 (1.882)	-1.1115 (1.933)
LADMW			-.0651 (.599)	-.0590 (.532)
LADMED			-.0227 (.553)	-.0237 (.570)
LSWPERH				-.0242 (.383)
SECED				-.0134 (.388)
R <sup>2</sup>	.3444	.3438	.3498	.3516

\*Natural form

TABLE IV.1 (continued)

Two Stage Least Squares Estimates of the Demand Equations  
Dependent Variable = LTS  
(Absolute t Values in Parentheses)

Independent Variable	Specification			
	(5)	(6)	(7)	(8)
LTFTEW	-.7847 (2.488)	-1.0399 (2.961)	-.8693 (2.675)	-1.0457 (2.962)
LTEDC	1.5830 (1.463)	1.4928 (1.394)	1.6468 (1.526)	1.528 (1.415)
LPWPERH	.0810 (1.858)	.0828 (1.921)	.0802 (1.848)	.0823 (1.899)
PARAED	-.0403 (1.858)	-.0460 (2.114)	-.0400 (1.843)	-.0452 (2.055)
LTBAHIW	.3769 (4.054)	.4436 (4.389)	.4040 (4.192)	.4471 (4.382)
LPROPTAX	.1800 (2.156)	.2090 (2.543)	.2227 (2.390)	.2230 (2.400)
LMED	-.5141 (1.164)	-.5492 (1.296)	-.4947 (1.124)	-.5376 (1.221)
PRACE	.0008 (1.006)	.0011 (1.353)	.0009 (1.110)	.0011 (1.349)
LKID6	.3509 (1.468)	.3435 (1.453)	.3456 (1.452)	.3427 (1.444)
LKID18	-1.1107 (1.988)	-1.0495 (1.895)	-1.0403 (1.855)	-1.0300 (1.841)
LTPCTCOL5	-.0186 (.370)		-.1040** (1.061)	-.0365** (1.260)
TAGE		.0162 (1.622)		.0144 (1.260)
R <sup>2</sup>	.3542	.3593	.3508	.3599

\*\*Predicted

TABLE IV.1 (continued)

Two Stage Least Squares Estimates of the Demand Equations  
Dependent Variable = LTS  
(Absolute t Values in Parentheses)

Independent Variable	Specification			
	(9)	(10)	(11)	(12)
LTFTEW	-.8780 (2.564)	-.7502 (2.586)	-.7776 (2.612)	-.8969 (2.724)
LTEDC	1.6458 (1.496)	2.1722 (2.171)	2.1828 (2.199)	2.2552 (2.242)
LPWPERH	.0808 (1.830)	.0917 (2.287)	.0889 (2.238)	.0917 (2.285)
PARAED	-.0437 (1.949)	-.0307 (1.498)	-.0261 (1.308)	-.0298 (1.452)
LTBAHIW	.4120 (4.086)	.7359 (6.515)	.7440 (6.594)	.7531 (6.561)
LPROPTAX	.2332 (2.340)	.1976 (2.627)	.2187 (2.569)	.2412 (2.668)
LMED	-.4781 (1.063)	-.6483 (1.583)	-.6277 (1.557)	-.6470 (1.582)
PRACE	.0009 (1.127)	.0019 (2.422)	.0018 (2.422)	.0020 (2.536)
LKID6	.3155 (1.227)	.4368 (1.862)	.4933 (2.246)	.4396 (1.872)
LKID18	-1.0688 (1.841)	-1.5663 (2.957)	-1.4863 (2.854)	-1.5227 (2.859)
LADMW	-.0183 (.153)	.0211 (.207)		.0561 (.511)
LADMED	-.0254 (.609)	-.0496 (1.303)		-.0509 (1.335)
LSWPERH	-.0326 (.509)	-.0325 (.565)		-.0397 (.683)
SECED	-.0143 (.414)	-.0181 (.577)		-.0189 (.601)
LCN		-.1612 (4.956)	-.1537 (4.821)	-.1607 (4.926)
LTPCTCOL5	-.0983** (.912)		-.0705** (.785)	-.8540 (.872)
R <sup>2</sup>	.3536	.4709	.4624	.4746

\*\*Predicted

TABLE IV.1 (continued)

Two Stage Least Squares Estimates of the Demand Equations  
 Dependent Variable = LTS  
 (Absolute t Values in Parentheses)

Independent Variable	Specification	
	(13)	(14)
LTFTEW	-.6141 (1.935)	-.7087 (2.492)
LTEDC	1.4926 (1.401)	2.0963 (2.094)
LPWPERH	.0587 (1.318)	.0890 (2.235)
PARAED	-.0352 (1.634)	-.0260 (1.299)
LTBAHIW	.3391 (3.695)	.7219 (6.474)
LPROPTAX	.1445 (1.817)	.1820 (2.499)
LMED	-.2408 (.525)	-.6488 (1.607)
PRACE	.0005 (.695)	.0017 (2.266)
LKID6	.3846 (1.625)	.5067 (2.286)
LKID18	-1.1469 (2.085)	-1.5354 (2.967)
LCN		-.1582 (4.822)
LFAGE	2.5265 (1.914)	
MERW		.00002 (.326)
R <sup>2</sup>	.3649	.4600

total cost of education that goes to teachers and (4) the more inelastic the supply curve of the other factor.<sup>8</sup> These rules can be illustrated most clearly in the two equation model (4b), where the elasticity of demand is:

$$\frac{E\left(\frac{T}{N}\right)}{E_w} = -(s_T \eta + s_V \sigma_{TV}).$$

We expect, then, a more inelastic demand for teachers as  $\eta$  and  $\sigma_{TV}$  are smaller.  $\frac{E\left(\frac{T}{N}\right)}{E_w}$  is also more inelastic as  $s_T$  is smaller as  $\eta > \sigma_{TV}$ .<sup>9</sup> The supply curve of the other factor is not relevant here, since it is assumed infinitely elastic.

An a priori speculation of an inelastic demand for teachers is given by Moskow when he points out that ". . .1. the teachers' service is essential to the production of education ( $\sigma_{TV}$  is low), 2. the demand for education is highly inelastic ( $\eta$  is low), (and) 3. labor costs are 62 percent or a high percentage of total costs of production ( $s_T$  is high). . ."<sup>10</sup> In this model, the share of teachers' cost in education is much lower than 62 percent, since included in

<sup>8</sup> Marshall (7th edition), pp. 381-388.

<sup>9</sup>  $\frac{E\left(\frac{T}{N}\right)}{E_w} = e_T = -(s_T \eta + (1 - s_T) \sigma_{TV})$

since  $s_T + s_V = 1$ .

$$\frac{de_T}{ds_T} = -\eta + \sigma_{TV}.$$

Thus, as  $s_T$  falls,  $e_T$  also falls if  $-\eta + \sigma_{TV} < 0$  and  $\eta > \sigma_{TV}$ .

<sup>10</sup> Michael H. Moskow, Teachers and Unions (Philadelphia: The University of Pennsylvania, 1966), p. 80.

the total cost is the price of mothers' time in educating the child, as well as other opportunity costs. This lower share of teachers' cost will further the argument for an inelastic demand. On this basis, Moskow concludes that ". . .the demand for teachers will be highly inelastic within a wide range. Theoretically (however), there must be some point above which salaries cannot be increased without affecting employment, and at some extremely low level a school board will begin to hire more teachers."<sup>11</sup> At these levels, the demand for teachers will become highly elastic.

As far as can be determined, no empirical evidence exists specifically concerned with the estimation of wage elasticities of public elementary school teachers. Indeed, says Ronald Ehrenberg, ". . .(t)o date (1972) no estimates of the wage elasticities of demand for public employees have been presented, and thus no quantitative discussion of the strength of the market forces. . .has occurred."<sup>12</sup> In his estimations of a set of demand equations for groups of local government employees, Ehrenberg does estimate the elasticity of demand for educational personnel. Although his model differs from the present one -- he assumes quality to be constant, supply of the educational personnel to be infinitely elastic, and interaction of employment opportunities among all groups of government employees -- the elasticity of demand derived is nevertheless interesting to

<sup>11</sup>Ibid., p. 81.

<sup>12</sup>Ehrenberg, p. 7.

note. This myriad of assumptions and estimation procedures yields a large array of elasticities to choose from. But the underlying results confirm that the wage elasticity of demand for educational personnel is consistently negative and inelastic. The range of elasticities under various OLS estimation procedures is from  $-.132$  to  $-.462$ . His three stage least squares estimates for a linear equation is  $-.425$ , while at the upper limit it is  $-.804$ . These results are not dissimilar to those presented here.

As opposed to the annual measure of wages for teachers, teacher aides are conceived of in many communities as being less than full time, hourly employees. As expected, paraprofessional annual wages appeared less significant in the results than their wages per hour (PWPERH). The cross elasticity of demand for teachers as a result of changes in wages for paraprofessionals was  $.0813$  in the results' equation 1, and of a similar significant magnitude in the other equations. This highly inelastic yet positive coefficient indicates that paraprofessionals are gross substitutes for teachers, as well as net substitutes. With an increase in the hourly wage for teacher aides, there is an increase in the cost for education, lowering the demand for education and teachers, while at the same time a greater percentage of teachers are hired as substitutes for paraprofessionals, and therefore, the demand for teachers increases.  $\sigma_{TR}$  in the coefficient  $+s_R(\sigma_{TR} - \alpha)$  must be

positive, and teachers and paraprofessionals must thus also be net substitutes.

The other occupational groups tested as alternatives to teachers were public school secretaries and administrators. Secretaries are thought to be a substitute for the largely female teacher labor force in determining relative wage values. The attempt at measuring the secretaries' wage cross elasticity proved insignificant, whether the wage variable was wages per hour or annual wages. Specifications 4, 9, 10, and 12 show a few of the regressions where they were included. The inclusion of secretaries, in addition, does not affect the coefficients of the other variables. The educational level of administrators of the public schools does have a weak negative effect on the demand for teachers when LCN is in the model (equations 10, 12). Their wage, whether per hour or annually, though, has no effect on teacher demand.

Teacher education presents a more difficult question. To analyze the meaning of the education coefficient, isolate the wage and education results of two identical equations, where teacher education is in logarithmic form (a) and in natural form (b), below. Tests using a natural teacher education term did not alter the sign or significance of the teachers's education coefficient. We have:

$$(a) \text{ LTS} = -.7662 \text{ LTFTEW} + 1.5700 \text{ LTEDC}$$

(2.469)                      (1.457)

and

$$(b) \text{ LTS} = -.7664 \text{ LTFTEW} + .0962 \text{ TEDC.}$$

(2.462)                      (1.424)

The correlation coefficient between TFTEW and TEDC is .2509, a low figure probably explained by the fact that TFTEW is a constructed variable. Both the LTEDC and the TEDC coefficients are weakly significant.

The theoretical coefficients for specification (a), again, are  $ETS = s_T(\sigma_{TT} - \tau)Ew - (s_T(\sigma_{TT} - \tau) + 1)EB_T$ . If one perceives of the educational level of teachers as a measure of their efficiency ( $B_T$ ), then the model tells us that the coefficient of this "quality" variable should be equal to the negative of the coefficient of the wage term plus one, or:

$$-(s_T(\sigma_{TT} - \tau) + 1) = -(-.7662 + 1) = -.2338.$$

There are a number of ways of explaining the empirically positive sign for the LTEDC coefficient. In the theory, the  $EB_T$  term is an efficiency parameter which corrects nominal units of both teachers and wages into effective units. In reality, (the log of) the educational level of teachers may not be viewed in itself as an efficiency parameter. But rather, teachers' effectiveness in affecting children in school may be a function of the teachers' education, but not equal to it. In this case, though, the variable does not do what it is supposed to do. Alternatively, as recent studies have suggested, as mentioned above, the quality of education a child receives is not affected by the educational level of the teachers at all, but by such things as the recentness of education, the similarity of teachers' background to

students', etc.<sup>13</sup> In order to test this concept, a number of proxies for teacher efficiency, as affected by the students' environment in which teaching takes place, were created and tested in the OLS form of the model in place of TEDC. One proxy for teacher-student background similarity was a ratio of teacher race to child race. Another was a ratio of teachers' family income to student family income. Both proved insignificant. Different specifications were tried for teacher experience, including their age minus the number of children they had had, and the percentage in an SMSA under forty years of age. None were significant in logarithmic or natural form, nor as replacements for or along with teacher education.

Teacher age, in the two stage estimates, has a positive sign and is weakly significant (6,8). The analysis of teacher age can be analogous to that of teacher education. The positive signs simply state that school districts demand older, more highly educated teachers (or at least don't let go of those who are). But, if one perceives of these teachers as more efficient, then the model predicts, holding all else constant, a decline in the demand for the number of teachers with an inelastic demand. The implication is that, even though there is a greater demand for older and more educated teachers, these characteristics alone are not good indicators of teacher effectiveness in educating children. Indeed, age

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<sup>13</sup>Hanuskek, 1971, p. 288.

might be indicative of a decline in teacher efficiency, and then a positive coefficient would be justified.

After a number of tests, one promising measure of teacher efficiency appeared to be the percentage of teachers in college five years ago or less (TPCTCOL5) when it is included in the TSLS regressions as an endogenous variable. Note that this measure is not akin to teacher age, since the percent in college five years ago includes those going for advanced credit. When this variable, as a proxy for teacher efficiency was included in the regressions, in unpredicted form (specification 5), its coefficient was negative but insignificant. It can, however, be included as an endogenous variable, since teachers' wages and continuing education are probably correlated. Also, teachers' efficiency may be a function of a number of other variables, including their education. The coefficients of TPCTCOL5 in this form yield somewhat stronger results (specifications 7, 8, 9).

In many areas, the salary schedule offered to teachers by school boards may be determined largely by forces imposed on the community over which they have little control. The community must respond to these forces out of necessity in their wage, but not quantity, determinations. These forces, such as differentials in cost of living indexes, in working conditions, or in the imposition of union demands, will partly distort the true wage-quantity relationships. For example, a community with a higher cost of living may have to pay a higher salary schedule than one with lower costs, even

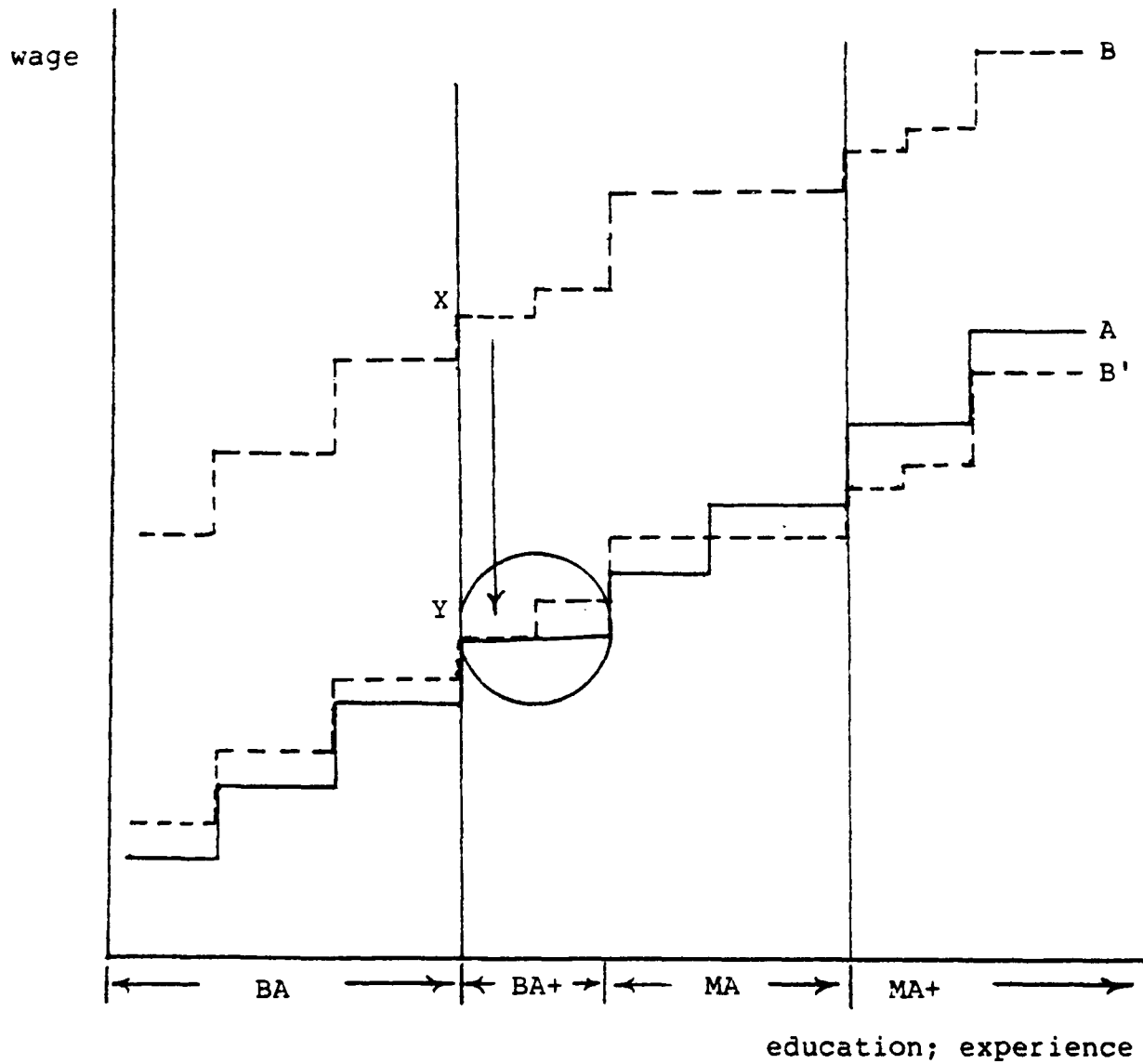
though the higher price paid is not reflected in the teacher-student ratio. To the extent that money wages and the cost of living are correlated, a proxy for cost of living adjustments should be included in the demand equation. In addition, as Moskow points out, relatively wealthy states may ". . .direct their higher expenditures towards raising teacher salaries above the national average instead of reducing class size or increasing other types of expenditures."<sup>14</sup> Or, a union may impose higher salary demands on one community than another, while holding constant (or even increasing) the teacher-student ratio. (This only applies to the portion of union policy which seeks to arbitrarily control employment or class size. The extent that unions' wage increases are met by changes in employment in the district is reflected in the wage coefficients.)

This can be shown with the aid of Figure IV.1. Let line B be a hypothetical salary schedule for a high cost of living community, and A for a lower cost of living area. Each step is given by some experience level within each education level. In order to bring the salary schedule together, so as to compare the effect of the adjusted wage differentials on teacher demand, a number of points on the salary schedule were computed. The one reported is the highest salary paid within a community to a teacher with only a bachelor's degree, no more nor less (TBAHIW). Other points tested to

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<sup>14</sup>Moskow, p. 67.

FIGURE IV.I  
Effect of the Salary Schedule Variable



bring the schedules together were the highest wage paid to any teacher in the school district (which yielded equally good results), and the average wage paid to teachers with only a bachelor's degree. The effect of holding TBAHIW constant, which is represented by points X and Y on Figure IV.1, would be to merge these two points, with line B shifting to B'. Points X and Y now coincide at Y. Now, the mean wage paid to teachers may be compared on the basis of less differentiated salary schedules. Notice, in this case, that holding one point constant can show the higher wage district B actually paying lower adjusted wages at higher levels of teacher education, while remaining higher at lower levels of education. If education and experience are held constant in the regressions, the wage variable will represent the means of those wages within the circle.

The coefficient of LTBAHIW itself is shown to be positive, which means that as the highest wage to teachers with a bachelor's degree goes up, the demand for teachers per student goes up. This may show that higher cost of living areas demand more teachers. Or, as unions gain a higher wage, exogenous to the community's control, they may also restrict class size, empirically allowing a greater (forced) demand for teachers as wages increase. By holding these exogenous forces constant, we can state that the areas subject to them, and having higher wages, might demand more teachers per student because comparable adjusted wages are, in reality, lower.

It can be argued that the size of the community can have some effect on the demand for teachers. The dependent variable used in the equations shown is the natural log of the teacher-student ratio (LTS). The teacher-student ratio was constructed as the number of full time equivalent teachers divided by the number of students.  $TS = \frac{TNFTE}{CN}$ , so that:

$$(6) \ln \left( \frac{TNFTE}{CN} \right) = b_0 + b_1 X_i,$$

or:

$$(7) \ln TNFTE = b_0 + b_1 X_i + b_2 \ln CN$$

are two alternate ways of specifying the model and correcting for different sized SMSA's, using the child number to correct for size ( $X_i$  is a vector of all other independent variables). LTS was chosen as the most appropriate measure of teacher quantity. When form (7) was tested in OLS, it yielded an extraordinarily high coefficient of determination. The size, signs, and levels of significance of the coefficients in the two forms were otherwise of the same magnitude.

Most of the ultimate forms of the regressions omitted  $\ln CN$  as a separate independent variable. However, if the correct dependent variable is LTS, then from (7)

$$\ln TNFTE - \ln CN = b_0 + b_1 X_i + b_2 \ln CN - \ln CN,$$

and

$$(8) \ln \left( \frac{TNFTE}{CN} \right) = b_0 + b_1 X_i + (b_2 - 1) \ln CN.$$

This allows use of an appropriate measure of the dependent variable, teachers per student, while at the same time allowing community size to be held constant and for the

possibility that  $b_2 \neq 1$ . Including LCN also compensates for the possibility that a different specification of the form

$$\ln \left( \frac{\text{TNFTE}}{\text{CN}} \right) = b_0 + b_1 \left( \frac{X_i}{\text{CN}} \right) + (b_2 - 1) \ln \text{CN}$$

exists, so that

$$\ln \left( \frac{\text{TNFTE}}{\text{CN}} \right) = b_0 + b_1 X_i + (b_2 - 2) \ln \text{CN}.$$

The theoretical interpretation of the coefficient is ambiguous upon what one presumes about the true nature of the other exogenous variables. Empirically, though, (specifications 10, 11, 12, 14), LCN is significantly negative, showing that as a community's size increases, the number of teachers per student will decrease. The community size variable may also be picking up a bundle of omitted characteristics of the community, which may be more appropriately tested in and of themselves, if they were known.

The most important of the community variables is that of the educational level of mothers of children in public elementary school. MED can be interpreted in a number of ways within the context of the model. In both the logarithmic and natural form (LMED and MED), the sign of the coefficient is negative and mildly significant. Mother's education can be viewed as a measure of the efficiency of her contribution to her children's education outside of school. The theoretical coefficient of this variable is  $-s_M(\sigma_{TM} - \tau)$ . Since this is empirically negative, it tells us that the elasticity of this substitution between mothers' time and teachers must be positive, and teachers and mothers are

substitutes in educating children. This finding is contrary to the idea that more highly educated mothers want more teachers for their children, not less. It is not teachers these mothers (parents) want more of, though, but education for their children. It is true that as mothers' education increases, the family's income increases and more teachers per student are demanded. When income is held constant, as well as the wage schedule of the teachers, an increase in mothers' education shows that the substitution effect of this improvement in her efficiency overtakes the output effect. As mothers' education increases, the price of education falls (in effective units) and the demand for education and teachers increases in fixed proportions. However, the effect of mothers' time being a substitute for teachers' time overtakes this fixed proportions output effect; and the demand for teachers, *ceteris paribus*, falls. In symbols,

$$\sigma_{TM} > \eta .$$

This idea has been noted before. Christopher Jencks, in describing his own tests with the Equality of Educational Opportunity data, stated that ". . .it looks as though districts which spent money on small classes had some other (unmeasured) attribute (education-conscious parents?) that caused the children to overachieve." (authors' parentheses) ". . .(B)ut when I compare otherwise similar schools in the same district, those with small classes did not outscore those with large ones."<sup>15</sup> One can postulate that the

<sup>15</sup>Christopher S. Jencks, "The Quality of the Data Collected by the Equality of Educational Opportunity Survey," in Mosteller and Moynihan, p. 83.

educationally conscious parents are the more educated ones, and that when income and all else is held constant in "otherwise similar schools" (other than parents' education levels), children in large classes -- with lower teacher-student ratios -- may outachieve those in small classes. This means that a greater percentage of the education of children in large classes must be occurring outside of the schools (with the help of the more educated mother?). Thus, when income is held constant, an increase in mothers' education may be related to larger class size but better educated children.

As noted above, mothers' education has also been shown to be positively related to child ability, or alternatively, as a measure of the child's stock of ability prior to his start of school. If the sign of the child ability coefficient is negative, children's ability is apparently also a substitute for teachers in their education. A district with more able children should demand fewer teachers per child.

Alternative ways of viewing parental effectiveness in educating their children may be from the number of children in each household. The direction of the change in the demand for teachers as the number of children in a household under the age of six increases is positive, while the number of children from six to eighteen in the household has a negative effect on the demand for teachers. Many small children in a household can seriously inhibit a mother from working as long

as she might like with her other children in elementary school. In addition, her energies at any one time might be diverted among many other activities, causing a fall in her effectiveness in educating her school age children, and thus more teachers are needed. The hypothesis that mothers are substitutes for teachers holds. But, as the children in the household get older, and as there are more of them, the effectiveness of mothers may again rise, and the demand for teachers again goes down. Not only is there less stress from early childhood care, but the mother now has helpers in educating the elementary school children in the family -- the older children. To the extent that there is a flow of knowledge from older to younger children, older siblings may be a substitute for teachers as well.

There are a number of ways to identify community income and its effect on that community's demand for teachers. Many of these values tried, such as mean family income, median family income of all families in the SMSA, or the total income of the fathers of the school's children gave insignificant results, probably due to strong collinearity with groups of other variables. The property tax per capita, to the extent that it reflects community wealth, is positively related to the demand for teachers, as expected. But property tax may also be a reflection of a community's desire for public education (a taste parameter), and should be positively related to the demand for teachers apart from community wealth.

Fathers' age and education are thought to be good proxies for income, with the income positively related to these characteristics. Specification 13 includes fathers' age. It can be seen that although the sign of its coefficient is positive and significant, it is highly collinear with mothers' and paraprofessionals' education, and thus not much faith is put in this as a measure of the income elasticity of demand for teachers. Fathers' education is weakly positively correlated, and doesn't appear to be much collinear with any of the other variables.

Paraprofessional race is positively correlated to child race. It is used as a proxy for neighborhood race. The positive sign shows that as the percentage of adults in the community becomes more non-white, the demand for teachers increases.

CHAPTER V  
THE SUPPLY EQUATION

A. A Brief Model

The traditional labor-leisure analysis from which the supply theory follows states that within any given household, there is a tradeoff between work time and leisure time, not only for one individual, but also among individuals within the household. Further, non-market work time can be alternatively viewed not as leisure, but as time spent within the household producing more basic commodities, one of which is leisure. Leisure, though, does not have to be completely time intensive.

In order to better explain the labor market for public elementary school teachers, a very basic model of teachers' labor supply is presented, being derived in an exactly analogous way to the methodology of the demand model. This model assumes a two adult household (teacher and spouse) who are presented with two choices for the use of time: market work time or non-market time (which will be called leisure). The household attempts to maximize its utility subject to full income, where the utility function is  $U = U(X, B_t, B_{sp})$  and full income is  $S = w_t t + w_{sp} sp + P_x X$ . It can be shown that the demand within the household for leisure time by the teacher, the inverse of the supply of work, is:

$$(9) \quad E_t = b_0 + b_1 Ew + b_2 Ew_{sp} + b_3 EB_t + b_4 EB_{sp} + b_5 EV \\ + b_6 EH$$

where:

- $t$  = teachers' leisure time  
 $sp$  = spouse leisure time  
 $X$  = commodities  
 $w, w_{sp}$  = market wages of teachers and spouses, respectively  
 $B_t, B_{sp}$  = some vector of efficiency parameters affecting teacher and spouse non-market time  
 $V$  = non-labor income  
 $H$  = a vector of other supply shift parameters.

Since the purpose here is to show the labor market response to changes in teacher and spouse wages, the price of commodities is assumed constant. The EH term will include any variable which may shift the demand for leisure time which is not included in the other shift parameters.

The theoretical coefficients include the relationships between elasticities of substitution for teacher and spouse leisure time (as well as own elasticity of substitution), the income elasticity of demand (defined as positive), and the shares of full income going to each of the inputs.<sup>1</sup> The signs of the coefficients depend on these relative values, as they

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<sup>1</sup>Although the derivation is not appropriate here, this specification might be expanded to show the theoretical coefficients. We have:

$$\begin{aligned}
 Et = & (s_t \sigma_{tt} + (1 - s_t - s_v - \frac{w_{sp}}{S} \eta) \eta_t) Ew \\
 & + ((1 - s_{sp} - s_v - \frac{w}{S}) \eta_t + s_{sp} \sigma_{tsp}) Ew_{sp} \\
 & + (-s_t (\sigma_{tt} - \eta_t) - 1) EB_t - s_{sp} (\sigma_{tsp} - \eta_t) EB_{sp} \\
 & + s_v \eta_t EV + EH,
 \end{aligned}$$

do in demand theory. Unlike the demand theory, in this case, however, the sign of the teacher wage coefficient,  $w$ , need not be negative. It may be negative or positive depending on the relative strengths of the opposite signs of the income and own substitution effects.

As mentioned, there are three broad groups of factors said to affect teacher supply. The first, teacher wage, is shown in the model as the  $Ew$  term. The characteristics of the teachers' spouses can be explained either in the  $Ew_{sp}$  or  $EB_{sp}$  terms. The neighborhood characteristics are either taken into account in the shift parameter or in some of the efficiency measures.

#### B. The Supply Results

The characteristics of the market for public elementary school teachers show that it is one which is primarily female (84 percent), and one in which nearly 70 percent of all teachers are married with spouse present. Therefore, some of the relationships observed in the market for married women may be presumed to hold. Within this market, it has been

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<sup>1</sup>(continued) where:

$\sigma_{tt}$  = the elasticity of substitution in consumption between teachers' and  $i$ 's leisure time, with  $i = t, sp$

$\eta_t$  = the full income elasticity of demand for  $t$ , leisure (assumed positive)

$\Omega$  = total time

$s_j$  = the share of  $j$  in full income, with  $j = t, sp, V$ .

For a complete proof, see Saffer.

shown that the labor force participation rate, as a measure of supply, is directly related to their own wage, and inversely related to their husbands' income and the number of small children present in the household.<sup>2</sup> However, it should be pointed out that a relatively large percentage of the teachers in the labor force are men, and an even larger percentage are not married. The presence of these large numbers of married men and/or single teachers in the sample, whose behavior patterns may be different from those of married women with regard to aggregate spouse income or the presence of small children in the household, may make a priori speculation about certain coefficients somewhat risky. In addition, the households of teachers are usually ones in which both adults are highly educated and may not conform to traditional labor-leisure substitution assumptions.

One approach to this problem would be to isolate only the women teachers, a tack taken in Benham's study of registered nurses, where he discards the 1.5 percent of the observations who are male nurses.<sup>3</sup> To do this for 16 percent of the labor force, in this case, would clearly be inappropriate. To compensate for the 30.4 percent of the teachers who are not married, in part (although some of these are not women), an exogenous variable on marital status is included in the regressions. It should also be reiterated

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<sup>2</sup>See Mincer.

<sup>3</sup>Benham, p. 247.

here that the quantity variable is the number of full time equivalent teachers per student, a variable that can be expected to yield different results than the labor force participation rate measure of quantity. A brief analysis of the model using the labor force participation rate as an alternative quantity parameter will be given in section C below.

The basic operational form of the supply equation, then, is:

$$(10) \text{ LTS} = c_0 + c_1 \text{LTFTEW} + c_2 \text{LASPTOTINC} + c_3 \text{LCURMAR} \\ + c_4 \text{LASPED} + c_5 \text{LAFTKID6} + c_6 \text{LTMABB} + c_7 \text{MED} \\ + c_8 \text{LFAGE} + c_9 \text{TAGE}.$$

The new variables used here are:

- ASPTOTINC = the total income of the spouses of all teachers in the occupational group in public schools
- CURMAR = the percentage of all teachers currently married
- ASPED = the educational level of all spouses
- AFTKID6 = the number of children under age 6 in the household of all female teachers
- TMABB = the total number of babies born to all working teachers
- TPCTF = percent of teachers female
- ABSF = percent of students with father not present
- AFTKID18 = the number of children in all female teachers' households under age 18
- TELF = ratio of employed teachers to their labor force.

A number of different specifications were estimated for supply using two stage least squares, and the more

illustrative ones are presented in table V.1. For most variables, the  $t$  values are over one.

Since we are dealing with the number of teachers per student as the supply variable, the positively sloped but inelastic supply elasticity is expected. A supply elasticity of, say, .5 says that for each one percentage increase in annual salary, there will be a one half of a percent increase in the number of teachers supplied per student. A one percent increase in wage in a community with 250,000 students and 10,000 teachers, and a student-teacher ratio of .04 will mean an increase in this ratio to .0402 and 50 more teachers in the supply, as meant here.

While the quantity variable is taken as teachers actually working per student, in full time equivalents, the correct personal characteristics to use appears to be those for all teachers, whether employed, unemployed or not in the labor force. The supply of teachers, in many cases, is affected by the distribution of young children in the trained teachers' household. As the number of children under six increases, the supply of teachers should be expected to decrease, since more teachers are staying at home with the children.

The relevant child variable, then, is taken to be the number of children under the age of six in the households of all female teachers. The presence of children for female teachers was used since small children will primarily affect their work time, and will have a lesser effect on the work

TABLE V.1

Two Stage Least Squares Estimates of the Supply Equations  
 Dependent Variable = LTS  
 (Absolute t Values in Parentheses)

Independent Variable	Specification			
	(1)	(2)	(3)	(4)
LTFTEW	.4763 (1.572)	.4681 (1.526)	.5304 (1.774)	.6127 (1.805)
LCURMAR	-.0649 (1.368)	-.0683 (1.424)	-.0654 (1.379)	-.0715 (1.489)
MED	.0476 (1.089)	.0496 (1.122)		.0426 (.966)
LFAGE	5.0286 (3.652)	4.9587 (3.559)	4.5787 (3.483)	5.1738 (3.729)
LASPTOTINC	.0700 (.517)	.0906 (.648)	.1085 (.829)	.1539 (.933)
LAFTKID6	-.1003 (1.498)	-.0650 (.954)	-.0920 (1.382)	-.0909 (1.341)
LTMABB	.2515 (1.714)		.2558 (1.743)	.2488 (1.695)
TAGE	-.0211 (2.330)	-.0185 (2.004)	-.0216 (2.375)	
LAFTKID18		.0221 (.262)		
LCN				-.0331 (.894)
R <sup>2</sup>	.1967	.1769	.1885	.2023

TABLE V.1 (continued)

Two Stage Least Squares Estimates of the Supply Equations  
 Dependent Variable = LTS  
 (Absolute t Values in Parentheses)

Independent Variable	Specification			
	(5)	(6)	(7)	(8)
LTFTEW	.6002 (2.111)	.5587 (1.861)	.4307 (1.392)	.4690 (1.529)
LCURMAR	-.0728 (1.644)	-.0748 (1.597)	-.0667 (1.403)	-.0676 (1.421)
MED	.0781 (1.884)	.1125 (2.163)	.0425 (.960)	
FAGE	3.8949 (2.967)	4.7594 (3.501)	5.1934 (3.718)	4.8332 (3.593)
LASPTOTINC	.1527 (1.193)	-.0168 (.121)	.0271 (.184)	.0523 (.361)
LASPED			.4871 (.748)	.5827 (.906)
LAFTKID6	-.0684 (1.087)	-.0911 (1.381)	-.1031 (1.535)	-.0964 (1.443)
LTMABB	.1512 (1.088)	.2543 (1.762)	.2599 (1.764)	.2654 (1.102)
TAGE	-.0249 (2.918)	-.0202 (2.255)	-.0198 (2.138)	-.0199 (2.149)
TELF	.0110 (4.255)			
ABSF		.0188 (2.216)		
R <sup>2</sup>	.3060	.2296	.2006	.1942

TABLE V.1 (continued)

Independent Variable	Specification			
	(9)	(10)	(11)	(12)
LTFTEW	.5675 (1.643)	.5568 (1.917)	.5341 (1.729)	.3889 (1.291)
LCURMAR	-.0734 (1.524)	-.0746 (1.678)	-.0754 (1.603)	-.0649 (1.476)
MED	.0375 (.839)	.0732 (1.743)	.1081 (2.015)	.0659 (1.592)
FAGE	5.3402 (3.794)	4.0531 (3.044)	4.8474 (3.496)	3.7852 (2.910)
LASPTOTINC	.1111 (.636)	.1119 (.805)	-.0349 (.235)	.1210 (.949)
LASPED	.4899 (.752)	.4614 (.759)	.2359 (.361)	
LAFTKID6	-.0937 (1.377)	-.0711 (1.126)	-.0928 (1.398)	-.0685 (1.102)
LTMABB	.2573 (1.444)	.1593 (1.141)	.2583 (1.778)	.1580 (1.150)
TAGE	-.0239 (2.312)	-.0236 (2.712)	-.0195 (2.139)	-.0244 (2.901)
TELF		.0110 (4.239)		.0113 (4.423)
ABSF			.0182 (2.015)	
LCN	-.0332 (.896)			
LTEDC				2.0959 (1.947)
R <sup>2</sup>	.2062	.3095	.2305	.3283

time of the small number of married male teachers with children under six present. In order to hold the number of children over the age of six constant, the number of babies ever born to all female working teachers and the number of children under the age of eighteen for all female teachers and for all working female teachers were tested. The former was chosen as the best measure. The coefficient of LAF TKID6 has a negative sign, while LTMABB is positive, as expected.

From equation (9), a young child in the household might be assumed to decrease the efficiency of the teacher within the household in her use of non-market time. Even though this would predict a substitution of husband's time for wife's in non-market activity (which may, in fact, occur with additional father's work in the household) and a reduction in the demand for some non-market goods at the relatively higher prices (or lower full income), the wife can produce less of the same quantity of leisure or non-market goods with the same amount of time. Thus, she will probably increase non-market time and decrease work time. As her children get older, however, teachers' labor supply will again increase.

A number of measures of the incomes of teachers' spouses were tried, including total earned wages and spouse total income. The first proved insignificant, and spouse total income was weakly significant and positive in a few of the specifications (5, 4, table V.1). This latter result, which runs counter to the notion that a husband's time is a strong substitute for his wife's, can be explained in terms of the

substitution effect in the  $E_{w_{sp}}$  coefficient. Although the income effect is positive, teachers and their husbands may be complements in the use of leisure time. As the wage of one increases, both husband and wife will attempt to work more, as  $\sigma_{tsp}$  is negative, possibly causing  $t$  to fall and work time to rise, as  $E_{w_{sp}}$  is positive. This may be a characteristic of households where both primary adults are highly educated. As one works more in a professional, "self-fulfilling" position, the other also has a strong desire to expand his or her capacity. Since leisure is usually enjoyed jointly, some market work may be complementary.

Other teacher characteristics are also shown to affect their supply. As more and more teachers are married, with a spouse present, the number supplied per student decreases. As the student population of an SMSA increases, the number of teachers supplied also decreases. The spouse's education has an insignificant relationship with the supply. When teachers' education was included (specification 12), one sees that areas with more educated teachers have a greater supply of teachers per student. This may be interpreted as showing that teachers who have invested more in their training will want to continue working under conditions where others might not, in order to capture the returns to this advanced training.

One of the primary assumptions of the demand model has been that the socio-economic characteristics of the community can affect that community's demand for teachers. Similarly,

it follows that community characteristics have some effect on the number of teachers living there, directly and indirectly, and thus on the supply of teachers. A number of characteristics were included in the regressions to test this. The age of the fathers of children in school, as noted in chapter IV, may be a proxy for community income. As community income increases, so does the supply of teachers, either of their own accord, or as a result of their husbands' moving to higher income areas. Other tests of income, including median SMSA income, family income and total income of the children's fathers were tried. All proved insignificant.

The educational level of the mothers of the children in school is included as one more measure of community educational level, since many teachers may want to live among a more educated peer group, or one in which there are more cultural advantages than elsewhere. Mothers' education could proxy this, although the size of the community, as measured by the absolute number of students (LCN), may be a deterrent to teachers living there. The positive relationship between mothers' education and the supply of teachers may also indicate that teachers prefer to work where there is a greater probability of their students being more able. Specification 5 (and 10) shows that there is a strong positive correlation between the supply of teachers per student and the ratio of the number of teachers employed to the labor force (TELF). TELF is the inverse of the

unemployment rate. As more and more teachers become unemployed, fewer and fewer are working per student. Even though the occupational job situation is becoming worse, this may indicate that many teachers are still seeking jobs while fewer remain in the schools. Some teachers who are working, seeing the deteriorating conditions in the market (more layoffs, larger class sizes, etc.), may also leave the market when other alternatives arise. The number of teachers per student is used, in addition, as a proxy for other supply measures. As the unemployment rate increases, the (proxy for) supply decreases. For example, if many teachers are laid off, those who were formerly employed are now seeking jobs, but at the same time, their supply will fall, indicating a net discouraged worker effect.

### C. The Teacher Labor Force Participation Rate

The number of teachers per student, while being the appropriate quantity variable here, is not the usual quantity measure used in many supply studies. In order to see the compatibility of this set of supply equations with others, they were regressed on the teachers' labor force participation rate (TLFPR) as the dependent variable. These specifications are shown in table V.2, and are identical to a number of the specifications in table V.1, except, of course, for the TLFPR dependent variable term.

The labor force participation rate for teachers differs from the number of full time equivalents per student as a

TABLE V.2

Two Stage Least Squares Estimates of the  
Labor Force Equations  
Dependent Variable = LTLFPR  
(Absolute t Values in Parentheses)

Independent Variable	Specification		
	(1)	(2)	(3)
LTFTEW	.1799 (1.526)	.2794 (1.962)	.2685 (1.898)
LCURMAR	-.0296 (1.605)	-.0367 (1.664)	-.0287 (1.567)
MED	-.0283 (1.665)	-.0275 (1.617)	-.0279 (1.663)
LFAGE	.5029 (.938)	.6412 (1.174)	.5517 (1.012)
LASPTOTINC	-.0570 (1.080)	-.0734 (1.353)	-.0704 (1.307)
LAFTKID6	-.0165 (.633)	-.0139 (.534)	
LTMABB	.0223 (.391)	.0343 (.595)	.0588 (.995)
TAGE	.0021 (.593)	-.0036 (.959)	-.0038 (1.047)
TPCTF		.0018 (1.240)	.0016 (1.113)
LATKID6			-.0374 (1.429)
R <sup>2</sup>	.0823	.0944	.1080

TABLE V.2 (continued)

Two Stage Least Squares Estimates of the  
Labor Force Equations  
Dependent Variable = LTLFPR  
(Absolute t Values in Parentheses)

Independent Variable	Specification	
	(4)	(5)
LTFTEW	.1795 (1.537)	.2817 (1.975)
LCURMAR	-.0277 (1.509)	-.0297 (1.608)
MED	-.0287 (1.714)	-.0256 (1.485)
LFAGE	.4203 (.789)	.5522 (.987)
LASPTOTINC	-.0555 (1.063)	-.0534 (.883)
LAFTKID6		-.0132 (.505)
LTMABB	.04966 (.847)	.0290 (.498)
TAGE	-.0025 (.726)	-.0039 (1.033)
TPCTF		.0015 (1.004)
LATKID6	-.0410 (1.574)	
LASPED		-.1974 (.755)
R <sup>2</sup>	.0984	.0989

supply measure as it includes those working and those unemployed and seeking work. It is also somewhat independent of the number of students in the area in that it reflects the desire of teachers to work per se. It not only includes the actual supply, but it includes the potential supply of teachers.

The own price elasticity of demand is still positive and mildly significant. The percentage change in the teachers' labor force participation rate is much more inelastic to changes in wage than that of the number of teachers per student. The major difference between the two approaches to supply seems to be in the personal characteristic variables of the teachers: their husbands' income and the number of young children in the household.

While the LAFTKID6 coefficient still has a negative sign, it appears in this specification as insignificant. Although the supply of teachers per student might significantly fall when female teachers have babies, the labor force might not. Female teachers on maternity leave might classify themselves as still in the labor force (with a job but not at work, for example, or as unemployed) while not actually working. The census data is such that it is left for the respondent to more generally answer these questions from their own judgment.

However, when the number of children under the age of six in all teachers' (male and female) households is substituted for the equivalent female variable, it again

appears as mildly significant. This could be interpreted as showing the effect of male teachers with young children present leaving the (relatively low paying) teacher labor force and seeking other work when confronted with a baby to support. Further, the fact that these men with young children did not affect the number of teachers per student in the previous regressions indicates that it is the unemployed male teachers who increasingly leave the labor force when their children are born. The percentage of teachers who are female has a weakly positive effect on TLFPR, while it has no effect on the number of teachers per student. This may be another indication that the sex composition of the teacher population has some bearing on the labor force participation rate, while having little on the teacher per student supply measure.

The only sign changes with the new dependent variable are in the LASPTOTINC and MED terms. It was argued above that as the spouses' income increased, and as husbands are complements (or weak substitutes) to wives in leisure, the supply of teachers would increase as spouse income increased in terms of the physical number per student. However, in terms of the labor force participation rate, as spouse income increases, the supply of teachers decreases. The implication here is that something is happening to the teachers who were unemployed as their spouses' incomes increase. As this income goes up, many of those who were unemployed may be able to leave their search for a job and the labor force. A

number, however, may also be entering the labor force (as complements to the time of their spouse), or some seeking jobs will now accept one at a less than previously expected wage rate or package of working conditions. The net effect of these two movements would be to increase the supply of teachers per student while diminishing the labor force participation rate. A similar analysis may be made for the change in the MED sign. It may be also true that while more teachers supply themselves to teach children of better abilities in more educated communities, there also might be more alternative employment for teachers in these areas, inducing a reduction in the labor force participation rate.

The coefficient of determination in the LTLFPR equations are consistently lower than in the LTS ones. It appears that the number of teachers per student is the correct specification of the quantity variable based on the model.

CHAPTER VI  
IMPLICATIONS

There are many implications of the model and its findings as to how it can be used to aid communities in decision making. Some of these implications were pointed out above, such as the idea given in the chapter on demand that a teacher's education and experience level might not be the best method of scheduling salary increases on the basis of efficiency.

In addition, the traditional methods used to predict the demand for teachers in physical units views the demand as dependent on the projected birth rates and on teacher attrition rates, to give the number of teachers needed in the future at present class size. The model presented in this study suggests that this method will not give a clear picture of changing economic, social and environmental conditions as they affect the demand for teachers per child. Conceivably, as Gross National Product increases, even with a declining birth rate, more teachers per student will be demanded, and the absolute demand for teachers might not fall at all. Or as the school age population becomes more non-white, more teachers would be needed.

A further implication of the model lies in the debate over whether schools should continue to be primarily financed by local property taxes. There is a rising sentiment within communities, and within governments, opposing ever increasing property taxes for education. Increasingly greater segments

of communities are less willing to pay larger and larger burdens of the costs of financing education in the form of these taxes. The equity of financing schools in this manner is also being questioned. The significant and positive relationship of the property tax variable to the demand for teachers may be useful in analyzing certain aspects of this debate.

Finally, presented below are two brief additional examples, of unions and of paraprofessionals, of how the model may be used as a guide in formulating policy questions and decisions.

#### A. Unions

For a long period of time, until the early 1960's, teacher organizations did not operate as collective bargaining agents for their members to any degree, but were more or less fraternal groups. The feeling among teachers that they are professionals, working in an industry where a large percentage of them are women who are not career minded and are secondary wage earners in a family, and the illegality of strikes by public employees all combined to make union militancy difficult to obtain.<sup>1</sup>

The more militant of the two major national unions, the American Federation of Teachers (AFT), had just over 70,000 members in 1962, an increase of only 20,000 in the ten previous years. By 1968, the membership increased more than

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<sup>1</sup>Moskow, p. 195.

twofold to 165,000.<sup>2</sup> Even more teachers were being covered by the AFT's negotiated contracts. From 1965 to 1967, statutes were passed by the legislatures of nine states which required school boards to engage in collective bargaining with teacher unions, where only one such law was passed previously.<sup>3</sup> Strikes were increasingly popular by 1968. Due to the growth in these years, teacher unions were responsible for teacher contracts in 12 percent of the school districts with 1,000 or more pupils and for 40 percent of the teachers in these districts in the United States by 1967. The National Education Association (NEA) represented about one million of the over two million teachers and administrators, while the AFT had its strength around the large Eastern and Midwestern cities.<sup>4</sup>

Several important reasons are given for this rise in teacher unions. The ". . . pattern of growth and development of collective bargaining in the public schools in recent years (can be explained by) (t)he rapidly deteriorating economic status of teachers, particularly in our large urban areas; widespread inactment and agitation for favorable

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<sup>2</sup>Harry Cohany and Lucretia M. Dewey, "Union Membership Among Government Employees," Monthly Labor Review, 93 (July 1970): 17.

<sup>3</sup>Michael Moskow and Robert E. Doherty, chapter in Teacher Unions and Associations: A Comparative Study, ed. Albert Blum (Urbana, Illinois: University of Illinois Press, 1969), p. 245.

<sup>4</sup>Herschel Kasper, "The Effects of Collective Bargaining on Public School Teachers' Salaries," Industrial and Labor Relations Review, 24 (October 1970): 59-60.

collective bargaining legislation; increased financial and organizational support from organized labor; the impact of the civil rights movement on the attitudes of teachers and the public towards civil disobedience; and a change in the composition of the nations' teaching force in the direction of increasing percentages of men, secondary teachers and number of teachers concentrated in large urban areas. . ."<sup>5</sup> Unrestful and unruly students, as well as the realization by parents of the essentiality of education for their children, have also caused teacher organizations to develop at a faster pace.

In addition, the empirical finding reported above that the price elasticity of demand for teachers is now less than one can explain some of this growth in unions among teachers. It is not known what this elasticity was prior to 1969, but the strong increase in awareness of the importance of education in the decade of the 1960's and the desire by parents for a strong education for their children at almost any cost, as well as the above mentioned current trends in teaching as a career, suggests that the elasticity has just recently become inelastic. (This is a question for future research, as is speculation that the elasticity in the 1970's, with some de-emphasis on formal education, might be increasing.) The leadership of the labor unions bargaining for the teachers, or those wishing to organize teachers into

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<sup>5</sup>William Moore, "Collective Negotiations and Teachers: A Behavioral Analysis," Industrial and Labor Relations Review. 24 (January 1971): 256.

collective units can be guided by this result that the price elasticity of demand for public elementary school teachers is inelastic over the relevant range.

It is well known that unions facing an inelastic demand for their members' services are more effective in increasing wages with a lower loss in employment than ones facing an elastic demand. The interpretation of the low elasticity of demand follows Marshall's rules of derived demand given in chapter IV. In general, the ". . . more inelastic the demand for union labor, the smaller the effect of a given wage increase on employment and therefore the larger the probable influence of a union on relative wages."<sup>6</sup>

Although an inelastic demand means there will be a relatively less steep decline in employment with a given wage increase, larger wage increases will lead to a larger absolute loss in employment and membership. Teacher unions, in particular, may attempt to impose class size restrictions on school districts, in addition to higher wage demands, to keep employment up. The empirically inelastic demand isolates the wage-quantity relationship for teachers, while holding this attempted class size restriction constant. If teacher unions cannot manipulate the employment level of their members (via constraint on the class size), they must do so by moderating their wage demands. "The elasticity of demand for union labor will limit the wage gains of a union,"

<sup>6</sup>Albert Rees, The Economics of Trade Unions (Chicago: Phoenix Books, 1962), p. 70. Also, for a complete review of union wage options, see Rees.

says Albert Rees, "only if the union is concerned about the employment of its members. . . . A union will ordinarily be less concerned about loss of employment among its members where they have good alternative employment opportunities."<sup>7</sup> Public elementary school teachers may have less alternatives in occupations in many parts of the country than many other workers, and one would therefore expect the teacher unions to have concern over the size of wage demands and, indeed, be sometimes conservative in pressing for collective bargaining agreements.

There has been a debate in the literature on whether teacher unions do in fact raise salary levels within a school district. There are conflicting results on both sides. Papers by Herschel Kasper and by David Lipsky and John Drotning conclude that there is little or no effect on salaries in unionized areas. One by Robert Baird and John Landon shows a significant salary advantage in districts with some type of collective bargaining, with the advantage being in the area of up to \$250 in annual wage differentials.<sup>8</sup>

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<sup>7</sup> Ibid, p. 73.

<sup>8</sup> Papers by Kasper, op. cit., and David B. Lipsky and John Drotning, "The Influence of Collective Bargaining on Teachers' Salaries in New York State," Industrial and Labor Relations Review, 27 (October 1973) show little or no effect of unions on salaries. One by Robert N. Baird and John H. Landon, "The Effects of Collective Bargaining on Public School Salaries, Comment," Industrial and Labor Relations Review, 25 (April 1972) shows that there are significant salary advantages in districts with some type of collective bargaining agreement.

The low elasticity of demand for teachers found here may be a sign to unions that new attempts to organize the majority of teachers still not covered by a collective agreement may be more successful than they might have been in the past. Once the decision to organize is made, a union can determine what wage-employment policy it initially wants to follow. One such policy might be to organize all of the workers within the school, and set a wage policy which will maximize the total gain in income in the incoming union.

The model can be used to show the relative wage increases afforded to each group of professional workers in such a situation. A primary purpose of this test is to show the relative strengths between the elasticities of substitution between a substitute for the two professional groups of workers in the school and these professionals, even with highly restrictive assumptions. Assume three inputs into the educational process of the child: teachers (T), paraprofessionals (R), and mothers (M). Also, assume a union which enters the industry with the two groups of professional workers --teachers and paraprofessionals. The union wishes to maximize union income, U, where  $U = (w - w')T + (r - r')R$ , with  $w'$  and  $r'$  the respective wages before the union becomes effective,  $w$  and  $r$  after. By maximizing U with respect to  $w$  and  $r$ , and by solving the system for  $(w - w')$  and  $(r - r')$ , we can show that this wage policy, all else constant, will yield the relationship:

$$(11) \frac{E_w}{E_r} = \frac{(1 - s_M) \sigma_{TR} + s_M \sigma_{RM}}{(1 - s_M) \sigma_{TR} + s_M \sigma_{TM}}$$

(See appendix B for proof.)

This says that the relative increase in wages between the two groups of school personnel obtained by the union policy will depend only on the elasticity of substitution between mothers' time and teachers or paraprofessionals ( $\sigma_{TM}$  and  $\sigma_{RM}$ ). If  $\sigma_{RM} > \sigma_{TM}$ , mothers are better substitutes for paraprofessionals than for teachers in educating their children, and the percentage change in salary for teachers will be greater for teachers than for paraprofessionals. Fewer teachers will be laid off with equivalent wage increase demands by unions.

This was tested using the census data. A model,  $LTFTEW = b_1 LPWPERH$  was regressed in OLS with no intercept. The coefficient  $b_1$  represents the right hand side of equation (11). The empirical  $b_1$  equals 8.19 with a t value of 13.69. This implies that mothers are a better substitute for paraprofessionals than for teachers, and that unions should raise teacher salaries 8 percent for every 1 percent increase in paraprofessional salaries' to maximize new union income. These are relative changes; the actual percentage changes have to be determined exogenously.

### B. Paraprofessionals

The relationship between teachers and paraprofessionals can be further elaborated. Paraprofessionals are those

teacher aides who work directly under the supervision of the classroom teacher. They are "non-certified (teacher certification) personnel who directly aid the teacher and/or work with pupils under the supervision of the teacher."<sup>9</sup> They are employed in many schools, both urban and suburban, and are classified as teacher aides, teacher helpers, teacher assistants or sub-professionals, as well as paraprofessionals.

A major issue among economists is in the general area of the use of paraprofessionals as an aid to professional workers, and how these substitutes might affect the market for the professionals. The two major education unions have shown concern over the use of paraprofessionals in the schools, both as they might affect the demand for teachers and as targets for organization. Paraprofessionals were first viewed as a method to cut costs and reduce class size by school boards. Any potential use of paraprofessionals as scabs during strikes would weaken union power, especially if they are strong substitutes for teachers. To offset this potential threat, the AFT and the NEA have been actively courting paraprofessionals, with the AFT again being the more aggressive of the two. Thus, the alleged conflict between teachers and paraprofessionals has been eased by each supporting the other's demands under the umbrella of the same union. Union leaders also point out that paraprofessionals

<sup>9</sup>Paul C. Shank and Wayne R. M. Elroy, The Paraprofessionals (Midland, Michigan: Pendall Publishing Co., 1970), p. ix.

have been shown to be effective ". . .in working with inner city children and interpreting the school to the community," and emphasize that ". . .the responsibility of paraprofessionals is to assist teachers 'by performing duties which are assigned and directed by those teachers, without infringing on the professional duties reserved for certified teachers.'"<sup>10</sup>

It was shown that in the case of a revenue maximizing union, paraprofessionals are much stronger substitutes for educational personnel other than teachers (i.e., mothers). It was also pointed out that they were substitutes for teachers. The coefficient of the wage per hour of paraprofessionals is  $s_R(\sigma_{TR} - \eta)$  and empirically found to be in the neighborhood of .08. Thus,  $\sigma_{TR}$  must be positive, and paraprofessionals are thus net substitutes for teachers.

The size of  $\sigma_{TR}$ , given  $(\sigma_{TR} - \eta)$  is positive, is dependent upon the elasticity of demand for output, since the share of the cost of paraprofessionals in educating children is a small part of the total cost and is even smaller when the total cost includes the cost of parents' time. If the elasticity of demand for education is assumed to be about 1, then  $\sigma_{TR}$  is at least 1, and greater than that to yield a coefficient of .08. If  $\eta$  is quite low (the elasticity of demand for output is inelastic), then  $\sigma_{TR}$  will be lower.

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<sup>10</sup> National School Public Relations Association, Paraprofessionals in Schools, Washington, D.C., 1972, p. 46.

In the multi-variable model (4), the coefficient of the teacher wage term is:

$$(s_T \sigma_{TT} - s_T \eta) = (-s_M \sigma_{TM} - s_R \sigma_{TR} - s_T \eta) = -.8.$$

Since  $s_T$  is a high part of the cost of education, and may be assumed to be about 1 (but not equal to 1, of course),  $\sigma_{TR}$  can again be postulated to be smaller the smaller  $\eta$  if  $\sigma_{TM}$  is larger. The 8 percent decrease in the demand for teachers per student as the wages of teachers go up by 10 percent that this predicts may be explained not by a massive substitution of paraprofessionals for teachers, but by the relatively greater weighted elasticity of demand for education term ( $s_T \eta$ ) and by mothers substituting themselves for teachers. It was shown above that mothers were substitutes for teachers. The relative strength of the substitution of paraprofessionals for teachers may not be of great importance yet in affecting the demand for teachers.

However, as the share of total cost attributed to paraprofessionals grows, paraprofessionals might be an increasingly important cause in the change in the demand for teachers. As of 1972, almost 300,000 paraprofessionals work with public school pupils. Leon Keyserling has projected that ultimately there will be one paraprofessional for every two classroom teachers.<sup>11</sup> This, again, is speculation. Further work must be done on the market for paraprofessionals.

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<sup>11</sup>Ibid, p. 1.

To the extent that paraprofessional education is a measure of their effectiveness, the results also show that as paraprofessionals become more efficient, the demand for teachers falls. This also confirms the substitutability of teacher aides for teachers. Again, the magnitude of the change is relatively small, at .04 percent fewer teachers for each one additional year of paraprofessional education. Most paraprofessionals in the inner cities have a high school degree, while those in the suburbs have had some college. The potential for increasing the educational level of paraprofessionals in these areas by one year or more seems improbable.

## APPENDIX A

This appendix presents the derivation of the market demand for teachers equation given in chapter II. The derivation for the multi-variable case assumes perfect competition in the factor and output markets and a linearly homogeneous production function.

Using the symbols given above, and following R. G. D. Allen (pp. 505-509), the production function is given as:

$$Q = f(B_T T, B_R R, B_M M)$$

and the cost function as:

$$C = \frac{w}{B_T} B_T T + \frac{r}{B_R} B_R R + \frac{m}{B_M} B_M M.$$

We wish to derive the demand for teachers in physical units (T). However, to allow for analytical ease in deriving the demand for an input in the usual way, let

$$B_T T = T^*$$

$$B_R R = R^*$$

$$B_M M = M^*$$

$$\frac{w}{B_T} = w^*$$

$$\frac{r}{B_R} = r^*$$

$$\frac{m}{B_M} = m^*,$$

and

$$Q = f(T^*, R^*, M^*)$$

$$C = w^* T^* + r^* R^* + m^* M^*.$$

These variables will be reconverted at the end of the proof.

Now the objective is to minimize C with respect to a given Q, which is identical to maximizing Q, given C. To

minimize cost, let the lagrangian multiplier be  $\lambda$ , and the lagrangian function

$$L = w^*T^* + r^*R^* + m^*M^* + \lambda(Q - f(T^*, R^*, M^*)).$$

The first order condition is:

$$\frac{\partial L}{\partial T} = w^* - \lambda f_{T^*} = 0$$

$$\frac{\partial L}{\partial R} = r^* - \lambda f_{R^*} = 0$$

$$\frac{\partial L}{\partial M} = m^* - \lambda f_{M^*} = 0$$

$$\frac{\partial L}{\partial \lambda} = Q - f(T^*, R^*, M^*) = 0$$

where

$$f_{T^*} = \frac{\partial Q}{\partial T^*} = \frac{\partial Q}{\partial (B_T T)}.$$

Alternatively, this necessary condition for minimum cost may be written as

$$\frac{w^*}{f_{T^*}} = \frac{r^*}{f_{R^*}} = \frac{m^*}{f_{M^*}} = \lambda.$$

We can also show that under the above assumptions, marginal cost (MC) equals average cost (AC) at all outputs,  $Q$ . Making use of Euler's Theorem,  $Q = T^*f_{T^*} + R^*f_{R^*} + M^*f_{M^*}$ , and  $w^* = \lambda f_{T^*}$ ,  $r^* = \lambda f_{R^*}$ ,  $m^* = \lambda f_{M^*}$ , we have average cost:

$$\begin{aligned} AC &= \frac{C}{Q} = \frac{w^*T^* + r^*R^* + m^*M^*}{Q} \\ &= \frac{\lambda f_{T^*}T^* + \lambda f_{R^*}R^* + \lambda f_{M^*}M^*}{Q} \\ &= \lambda \left( \frac{f_{T^*}T^* + f_{R^*}R^* + f_{M^*}M^*}{Q} \right) \\ &= \lambda \left( \frac{Q}{Q} \right) = \lambda. \end{aligned}$$

Similarly, when  $C = w^*T^* + r^*R^* + m^*M^*$ , marginal cost is:

$$\begin{aligned} MC &= \frac{dC}{dQ} = w^* \frac{dT^*}{dQ} + r^* \frac{dR^*}{dQ} + m^* \frac{dM^*}{dQ} \\ &= \lambda \left( f_{T^*} \frac{dT^*}{dQ} + f_{R^*} \frac{dR^*}{dQ} + f_{M^*} \frac{dM^*}{dQ} \right). \end{aligned}$$

We know that  $f_{T^*} \frac{dT^*}{dQ} + f_{R^*} \frac{dR^*}{dQ} + f_{M^*} \frac{dM^*}{dQ} = 1$ , so  $MC = \lambda$ . Thus,  $MC = AC = \lambda$ , and are independent of output.

If the competitive price in equilibrium for output,  $P_Q$ , is given, we know

$$P_Q = AC = MC = \lambda,$$

and

$$P_Q = \frac{w^*}{f_{T^*}} = \frac{r^*}{f_{R^*}} = \frac{m^*}{f_{M^*}}$$

or

$$P_Q f_{T^*} = w^*, \quad P_Q f_{R^*} = r^*, \quad P_Q f_{M^*} = m^*,$$

where wages are equal to the value of the marginal product in effective units. We also know that market demand for education is a function of its price --  $Q = \phi(P_Q)$ , and the price elasticity of demand for education is  $= -\frac{P_Q}{Q} \frac{dQ}{dP_Q}$ .

In summary, we have the conditions:

- (1)  $f(T^*, R^*, M^*) = Q = \phi(P_Q)$
- (2)  $P_Q f_{T^*} = w^*$
- (3)  $P_Q f_{R^*} = r^*$
- (4)  $P_Q f_{M^*} = m^*$ .

Now, to determine the elasticity of demand for teachers, we differentiate these equations with respect to  $w^*$ , the price of the input teachers,  $T^*$ .

$$(1') \quad f_{T^*} \frac{dT^*}{dw^*} + f_{R^*} \frac{dR^*}{dw^*} + f_{M^*} \frac{dM^*}{dw^*} = -\eta_{P_Q} \frac{dP_Q}{dw^*}$$

$$(2') \quad f_{T^*} \frac{dP_Q}{dw^*} + P_Q \left( f_{T^*T^*} \frac{dT^*}{dw^*} + f_{T^*R^*} \frac{dR^*}{dw^*} + f_{T^*M^*} \frac{dM^*}{dw^*} \right) = \frac{dw^*}{dw^*} = 1$$

$$(3') \quad f_{R^*} \frac{dP_Q}{dw^*} + P_Q \left( f_{T^*R^*} \frac{dT^*}{dw^*} + f_{R^*R^*} \frac{dR^*}{dw^*} + f_{M^*R^*} \frac{dM^*}{dw^*} \right) = 0$$

$$(4') \quad f_{M^*} \frac{\partial P_Q}{\partial w^*} + P_Q \left( f_{T^*M^*} \frac{\partial T^*}{\partial w^*} + f_{R^*M^*} \frac{\partial R^*}{\partial w^*} + f_{M^*M^*} \frac{\partial M^*}{\partial w^*} \right) = 0.$$

Rearranging (1')-(4'), we have:

$$(1'') \quad Q \lambda \left( \frac{1}{P_Q} \frac{\partial P_Q}{\partial w^*} \right) + f_{T^*} \frac{\partial T^*}{\partial w^*} + f_{R^*} \frac{\partial R^*}{\partial w^*} + f_{M^*} \frac{\partial M^*}{\partial w^*} = 0$$

$$(2'') \quad f_{T^*} \left( \frac{1}{P_Q} \frac{\partial P_Q}{\partial w^*} \right) + f_{T^*T^*} \frac{\partial T^*}{\partial w^*} + f_{T^*R^*} \frac{\partial R^*}{\partial w^*} + f_{T^*M^*} \frac{\partial M^*}{\partial w^*} = \frac{1}{P_Q}$$

$$(3'') \quad f_{R^*} \left( \frac{1}{P_Q} \frac{\partial P_Q}{\partial w^*} \right) + f_{T^*R^*} \frac{\partial T^*}{\partial w^*} + f_{R^*R^*} \frac{\partial R^*}{\partial w^*} + f_{R^*M^*} \frac{\partial M^*}{\partial w^*} = 0$$

$$(4'') \quad f_{M^*} \left( \frac{1}{P_Q} \frac{\partial P_Q}{\partial w^*} \right) + f_{T^*M^*} \frac{\partial T^*}{\partial w^*} + f_{R^*M^*} \frac{\partial R^*}{\partial w^*} + f_{M^*M^*} \frac{\partial M^*}{\partial w^*} = 0.$$

This system, in determinant notation, is:

$$\begin{vmatrix} Q \lambda & f_{T^*} & f_{R^*} & f_{M^*} \\ f_{T^*} & f_{T^*T^*} & f_{T^*R^*} & f_{T^*M^*} \\ f_{R^*} & f_{T^*R^*} & f_{R^*R^*} & f_{R^*M^*} \\ f_{M^*} & f_{T^*M^*} & f_{R^*M^*} & f_{M^*M^*} \end{vmatrix} \begin{vmatrix} \frac{1}{P_Q} \frac{\partial P_Q}{\partial w^*} \\ \frac{\partial T^*}{\partial w^*} \\ \frac{\partial R^*}{\partial w^*} \\ \frac{\partial M^*}{\partial w^*} \end{vmatrix} = \begin{vmatrix} 0 \\ \frac{1}{P_Q} \\ 0 \\ 0 \end{vmatrix}.$$

Let  $F$  denote the left hand determinant,  $F_{ij}$  the cofactor of  $a_{ij}$ . It can be shown that the partial elasticities of substitution are:

$$(5) \quad \sigma_{T^*T^*} = \frac{Q}{T^*T^*} \frac{F_{T^*T^*}}{F}$$

$$(6) \quad \sigma_{T^*R^*} = \frac{Q}{T^*R^*} \frac{F_{T^*R^*}}{F}$$

$$(7) \quad \sigma_{T^*M^*} = \frac{Q}{T^*M^*} \frac{F_{T^*M^*}}{F}.$$

Also, the shares of total cost going to the inputs are:

$$(8) \quad s_{T^*} = \frac{T^*w^*}{P_Q Q} = \frac{T^*f_{T^*}}{Q}$$

$$(9) \quad s_{R^*} = \frac{R^*f_{R^*}}{Q}$$

$$(10) \quad s_{M^*} = \frac{M^*f_{M^*}}{Q}.$$

Using (5)-(10), we have:

$$(11) \quad s_{T^*} \sigma_{T^*T^*} = \frac{f_{T^*F} F_{T^*T^*}}{T^*F}$$

$$(12) \quad s_{R^*} \sigma_{T^*R^*} = \frac{f_{R^*F} F_{T^*R^*}}{T^*F}$$

$$(13) \quad s_{M^*} \sigma_{T^*M^*} = \frac{f_{M^*F} F_{T^*R^*}}{T^*F}$$

Now, using Cramer's Rule to solve for, say,  $\frac{dT^*}{dW^*}$ , we have:

$$\frac{dT^*}{dW^*} = \frac{\begin{vmatrix} Q\eta & 0 & f_{R^*} & f_{M^*} \\ f_{T^*} & \frac{1}{P} & f_{T^*R^*} & f_{M^*M^*} \\ f_{R^*} & 0 & f_{R^*R^*} & f_{R^*M^*} \\ f_{M^*} & 0 & f_{R^*M^*} & f_{M^*M^*} \end{vmatrix}}{|F|} = \frac{\frac{1}{P} (F_{T^*T^*} + Q\eta F_{0T^*T^*})}{|F|}$$

where  $F_{0T^*T^*} = -\frac{T^*T^*F}{Q^2}$  (Allen, p. 482). Manipulating, and using (11)-(13),

$$\frac{dT^*}{dW^*} = \frac{\frac{1}{P} \frac{F_{T^*T^*} f_{T^*}}{f_{T^*}} \left( \frac{F_{T^*T^*} f_{T^*}}{F_{T^*T^*}} - \frac{Q\eta}{F_{T^*T^*}} \frac{T^*T^*F}{Q^2} f_{T^*} \right)}{|F|} = \frac{T^*}{W^*} s_{T^*} (\sigma_{T^*T^*} - \eta).$$

Putting in elasticity form:

$$\frac{dT^*}{dW^*} \frac{W^*}{T^*} = s_{T^*} (\sigma_{T^*T^*} - \eta)$$

$$\frac{ET^*}{Ew^*} = s_{T^*} (\sigma_{T^*T^*} - \eta).$$

We can also solve for:

$$\frac{ET^*}{Er^*} = s_{R^*} (\sigma_{T^*R^*} - \eta)$$

$$\frac{ET^*}{Em^*} = s_{M^*} (\sigma_{T^*M^*} - \eta).$$

So, then,  $ET^* = s_{T^*} (\sigma_{T^*T^*} - \eta) Ew^* + s_{R^*} (\sigma_{T^*R^*} - \eta) Er^* + s_{M^*} (\sigma_{T^*M^*} - \eta) Em^*$ .

Since  $s_{T^*} = \frac{T^*w^*}{P_QQ} = \frac{B_T T^w}{P_QQ} = s_T$  and  $\sigma_{T^*T^*} = \sigma_{TT}$  etc.,

and  $ET^* = E(B_T T) = \frac{d(B_T T)}{B_T T} = \frac{TdB_T + B_T dT}{B_T T} = EB_T + ET,$

$$E(B_T T) = s_T(\sigma_{TT} - \eta)E\left(\frac{w}{B_T}\right) + s_R(\sigma_{TR} - \eta)E\left(\frac{r}{B_R}\right) \\ + s_M(\sigma_{TM} - \eta)E\left(\frac{m}{B_M}\right), \text{ or}$$

$$ET = s_T(\sigma_{TT} - \eta)Ew - (s_T(\sigma_{TT} - \eta) + 1)EB_T + s_R(\sigma_{TR} - \eta)Er \\ - s_R(\sigma_{TR} - \eta)EB_R + s_M(\sigma_{TM} - \eta)Em - s_M(\sigma_{TM} - \eta)EB_M.$$

Including the vector of shift parameters,  $D$ , this is the multivariate model used in the text.

## APPENDIX B

An empirical test was performed in chapter VI to give a rough approximation of the relative strengths of the elasticities of substitution between mothers and teachers, and mothers and paraprofessionals. The test was performed by assuming two inputs, T and R, to be in an industry where an industrial union comes in and attempts to maximize new union income. A third input is also available in this production process, M, but is not unionized. The relative prices of these inputs are w, r, and m after unionization, w', r', and m' before.

That is to say, the unions attempt to maximize  $U = (w - w')T + (r - r')R$ . We then can show:

$$\frac{dU}{dw} = T + (w - w') \frac{dT}{dw} + (r - r') \frac{dR}{dw} = 0 \quad \text{and}$$

$$\frac{dU}{dr} = R + (w - w') \frac{dT}{dr} + (r - r') \frac{dR}{dr} = 0$$

are first order conditions for a maximum. Note both employments T and R are functions of both w and r. In elasticity form, we have (multiplying the first equation by  $\frac{w}{T}$  and the second by  $\frac{r}{R}$ ):

$$(w - w') \frac{ET}{Ew} + (r - r') \frac{ER}{Ew} \frac{R}{T} = -w$$

$$(w - w') \frac{ET}{Er} \frac{T}{R} + (r - r') \frac{ER}{Er} = -r.$$

In matrix form:

$$\begin{vmatrix} \frac{ET}{Ew} & \frac{ER}{Ew} & \frac{R}{T} \\ \frac{ET}{Er} & \frac{T}{R} & \frac{ER}{Er} \end{vmatrix} \begin{vmatrix} (w - w') \\ (r - r') \end{vmatrix} = \begin{vmatrix} -w \\ -r \end{vmatrix}.$$

Solving for  $(w - w')$  and  $(r - r')$  by Cramer's Rule:

$$(w - w') = \frac{-wER}{Er} + \frac{rEB}{Ew} \frac{R}{T} \frac{1}{|D|}$$

$$(r - r') = \frac{-rET}{Ew} + \frac{wET}{Er} \frac{T}{R}, \text{ with } |D| = \text{left hand determinant.}$$

And also:

$$\frac{(w - w')}{w} = \frac{-\frac{ER}{Er} + \frac{r}{w} \frac{ER}{Ew} \frac{R}{T} \frac{P_{OQ}}{P_{OQ}}}{\frac{(r - r')}{r} = \frac{-\frac{ET}{Ew} + \frac{w}{r} \frac{ET}{Er} \frac{T}{R} \frac{P_{OQ}}{P_{OQ}}}$$

Since, holding output constant, we know that

$$\frac{ET}{Ew} = -s_R \sigma_{TR} - s_M \sigma_{TM}$$

$$\frac{ET}{Er} = s_R \sigma_{TR}$$

$$\frac{ER}{Ew} = s_T \sigma_{TR}$$

$$\frac{ER}{Er} = -s_T \sigma_{TR} - s_M \sigma_{RM}.$$

Manipulating and substituting:

$$\frac{(w - w')}{w} \frac{(r - r')}{r} = \frac{\frac{s_R s_T \sigma_{TR} + s_T \sigma_{TR} + s_M \sigma_{RM}}{s_T}}{\frac{s_T s_R \sigma_{TR} + s_R \sigma_{TR} + s_M \sigma_{TM}}{s_R}}$$

$$\frac{Ew}{Er} = \frac{(1 - s_M) \sigma_{TR} + s_M \sigma_{RM}}{(1 - s_M) \sigma_{TR} + s_M \sigma_{TM}}$$

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