Order Number 9510647

Monetary policy in interdependent economies: An application of trigger mechanisms in noncooperative repeated games

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City University of New York, 1994

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MONETARY POLICY IN INTERDEPENDENT ECONOMIES

---- An Application of Trigger Mechanisms
in Noncooperative Repeated Games

by

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A dissertation submitted to the Graduate Faculty in Economics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York

1994

This manuscript has been read and accepted for the Graduate Faculty in Economics in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

September 6, 1994
date

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THE CITY UNIVERSITY OF NEW YORK

To my family

for their love and support

Abstract

Monetary Policy in Interdependent Economies

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Adviser: Professor Salih Neftci

Game theory has been applied in Macroeconomics and Monetary Economics since the last decade.

In an open economy, cooperative games usually yield efficient outcomes, and noncooperative games usually yield inefficient outcomes because the externalities between countries are not internalized when the policymaker in each country changes the monetary policies. In fact, the non-cooperative policymakers can achieve the same efficient outcomes by using the trigger mechanism in the repeated games. Research has shown that the trigger mechanism did not work at all in games with finite horizon. However recent developments show that it can be employed in finitely repeated games if there are multiple Nash equilibria in one-shot game.

Mattew Canzoneri and Dale Henderson achieved one Nash equilibrium in their book, but they did not obtain two Nash equilibria by different monetary policies in the one-shot games.

Anticipated changes in money supply have real effects on the capital and the output, and the nonneutrality of the money supply is the famous Tobin effect.

In my dissertation, under the assumptions that prices are fully flexible and expectations are formed rationally, I obtained two Nash equilibria by the anticipated permanent increase in money supply and the anticipated increase in the rate of growth of money supply, and one is better than the other. So, policymaker's good behavior in early periods can be rewarded with good Nash solution in the later periods and bad behavior will be punished by ending up with the bad Nash solution in the later periods. With this strategies, a Pareto optimum which all policymakers prefer to the Nash equilibrium in one-shot games can be attained as an equilibrium of the noncooperative repeated game.

ACKNOWLEDGMENTS

I am grateful to all Professors in Macroeconomics at the Graduate Center, especially to Prof. Neftci. With his assistance, I developed expertise in macro dynamics and game theory, and applied it in my dissertation.

I would like to acknowledge Prof. Marty for his guidance while I studied New Keynesian theory and the rigorous training he imparted.

I thank Prof. Thurston and Prof. Asikoglu for introducing me to the important theories in modern Macroeconomics and Monetary Economics.

Finally, I am grateful to Prof. Grossman for his guidance in selecting my field of specialization and his assistance in obtaining financial support that enabled me to complete my work.

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1. Introduction and Summary

A long-standing discussion in monetary policy concerns whether the monetary authority should use the money supply or interest rates to control the outputs. In an important article, Poole (1970) argued that without uncertainty, a monetary policymaker is indifferent between those instruments, because he faces the same trade-off between target variables no matter whether he uses the money supply or the interest rate as his instrument. Poole analyzed a monetary policymaker in a closed economy with fixed fiscal policy, but in fact his logic can be applied in the games between monetary policymakers.

Henderson and Zhu (1990) found that in the open economy once the home policymaker has chosen one of the instruments, the foreign policymaker is indifferent between those two instruments since the reason is the same as that is in Poole's paper. Thus, if each policymaker can use either the money supply or the interest rate, there are four possible equilibria. But their further research has shown that the utility of each ploicymaker depends crucially on the instruments chosen by his opponent and so if the foreign policymaker uses his money supply, the home policymaker will face a steeper inflation-unemployment trade-off and therefore he can achieve a lower inflation rate, but neither policymaker's utility can be higher than it is when both of the policymakers use money supply as their instruments. This is the reason why the policymakers should choose the money supply as the instrument in the open economies.

According to game theory, cooperative games yield efficient outcomes, and noncooperative games yield inefficient outcomes because the externalities between countries are not internalized when the policymaker in each country changes the monetary policies. But the problem in any cooperative game is that each country's policymaker has an incentive to cheat on other countries's policymakers. Therefore, implicit in any cooperative game structure is the ability of a policymaker to commit to binding agreements. But to the sovereign policyamker, it may be impossible to make such commitment, since the commitment requires the policymakers to submit to a higher authority (maybe the third country) and to suffer a loss of sovereignty when they switch from noncooperation to cooperation. In fact, the noncooperative players can achieve the same efficient outcomes that would result from cooperation by using the trigger mechanisms to punish bad behavior. Many results have shown that the trigger mechanism did not work at all in the games with finite horizon. Recent developments have shown that the trigger mechanisms can be employed in finitely repeated games if the one-shot game has multiple solutions, one of which is better than the other. The finitely repeated games are more practical and more realistic than the infinitely repeated games, but the crucial problem is whether multisolutions can be found.

Matthew B. Canzoneri used the Cobb-Douglas production function and assumed that the capital was constant and that each policymaker minimized his loss function with respect to his money supply when he has a Nash player. One Nash equilibrium was achieved in chapter 2, and in chapter 4, he mentioned that he could create two Nash equilibria by incomplete information and complete information. Obviously the cause of those Nash equilibria is not on the money supply side.

When the expectations are formed rationally,the anticipated changes in money stock have real effects, and the source of nonneutrality of anticipated money changes is the famous Tobin effect. The anticipated changes in money supply affect the anticipated rate of inflation; and then affect the capital stock and the output. The anticipated permanent increase in money supply and the anticipated increase in the growth rate of money supply should result in two different steady states which have different capital stocks, because faster growth of money is associated with higher capital stock and output in the steady state, and also because higher inflation leads savers to shift their portfolios in favor of capital. Those alternative monetary policies are studied by Russell Boyer and Rober Hodrick (1982).According to their research, monetary policies are announced at one time and implemented at some future time and the announcement of the policy causes an initial jump in the price level and so an inflation between announcement and implementation,and then the whole economy converges to two different equilibria under the different monetary policies.

In the open economies, those two monetary policies give rise to two Nash equilibria in one-shot games.

The outline of my paper is as follows:

Chapter 2 presents a recursive time model employed by Fischer (1979). I used the backward and forward method to obtain the present price and capital functions under rational expectations, and then I set up the loss functions and showed the two different Nash equilibria under two different monetary policies in the one-shot games. Besides those, I also described two leader-follower game solutions. One is the familiar Stackelberg solution , and the other is fixed-exchange-rate solution suggested by Canzoneri and Gray (1985). By using the same loss functions,I demonstrated that one or both of the home and foreign countries can be better off in the leader-follower solution than in the Nash equilibria. Finally the Pareto-optimal was achieved if both policymakers can commit themselves in the one-shot games.

Chapter 3 presents the trigger mechanisms by which noncooperative players can achieve the same efficient outcomes that would result from the cooperation.There are two Nash equilibria in chapter 2 and one is better than the other for both noncooperative players, so there are two possible outcomes in the last period of the games with finite horizon. Player's good behavior in early periods can be rewarded with the good Nash solution in the later periods and bad behavior will be punished by ending up with the bad Nash solution in the later periods. If they do want to cheat, then when is the best time for them to cheat ? If nobody wants to cheat, then when should they switch to the Nash solution?

2. One-shot Games and Two Nash Equilibria

2.1 Description of the Model

Suppose that there are only two countries in the world economy, the home country and the foreign country. Each country produces one good. If there is nc any productivity disturbance,their outputs are the same size when measured in the same good. In Fischer's recursive time model, all variables are expressed in terms of logrithms. Home country and foreign country use the same model and variables with asterisks are foreign country variables. For simplicity, sometimes time subscripts are omitted.

Cobb-Douglas production function is used in the model and the employment is assumed to be a constant.

$$Y = C\ N^{1-\alpha 1}\ K^{\alpha 1}$$

$$\ln Y = \ln C + (1-\alpha_1)\ \ln N + \alpha_1 \ln K$$

$$y_t = \alpha_o + \alpha_1 k_t + x' \qquad (2.1.1)$$

$$y_t^* = \alpha_o + \alpha_1 k_t^* + x' \qquad (2.1.2)$$

Production functions are increasing function of capital and decreasing function of a world production disturbance ($x' > 0$). The coefficients of the production functions ($0 < \alpha_1 < 1$) is the same in both countries. x' is identically and independently distributed with a zero mean.

The logarithms of the real return to capital in period $t, \hat{r}$, can be derived from the following relationship,

$$\frac{\partial Y}{\partial K} = C\ \alpha_1\ K^{\alpha 1 - 1}\ N^{1-\alpha 1}$$

$$\hat{r} = \ln\left(\frac{\partial Y}{\partial K}\right) = \ln(C\alpha^1) + (1-\alpha^1)\ln N + (\alpha_1 - 1)\ln K$$

$$= \ln(C\alpha_1) + (1-\alpha_1)\ln N - (1-\alpha_1)\ln K$$

So $\hat{r}_t = \alpha - (1-\alpha_1)\, k_t$ (2.1.3)

$\hat{r}_t^* = \alpha - (1-\alpha^1)\, k_t^*$ (2.1.4)

[where $\alpha = \ln C\,\alpha_1 + (1-\alpha_1)\ln N$].

In assets markets, the demand for capital is

$$k_{t+1} = \beta_o + \beta_1\, E(\hat{r}_{t+1}|t) + \beta_2\, E[(p_{t+1}|t) - p_t] + y_t \qquad (2.1.5)$$

$$k_{t+1}^* = \beta_o + \beta_1\, E(\hat{r}_{t+1}^*|t) + \beta_2\, E[(p_{t+1}^*|t) - p_t^*] + y_t^* \qquad (2.1.6)$$

The demand for capital is assumed proportional to current labour income, which is the worker's total income, and labour income is proportional to total income ---- hence the coefficients on y and y^* are unit in (2.1.5) and (2.1.6). Also capital demand is assumed to be an increasing function of the expected real return on capital and of the expected rate of inflation. In this model, the expected rate of inflation represents the portfolio shifts toward captial as the expected real rate of return on the holding of money falls.

The demand for real balabces is proportional to income and negatively related to the expected real rate of return on capital and to the expected inflation.

$$m_t - p_t = \tau_o - \tau_1\, E(\hat{r}_{t+1}|t) - \tau_2\, E[(p_{t+1}|t) - p_t] + y_t \qquad (2.1.7)$$

$$m_t^* - p_t^* = \tau_o - \tau_1\, E(\hat{r}_{t+1}^*|t) - \tau_2\, E[(p_{t+1}^*|t) - p_t^*] + y_t^* \qquad (2.1.8)$$

He also assumed that the expected real reture on capital had a greater influence on the demand for capital than on the demand for real balances relative to the influence of the expected rate of inflation on the respective asset demands. Specially , he assumed that β_1 and $\beta_2 > 0$, τ_1 and $\tau_2 > 0$ and $(\beta_1/\beta_2) \geq (\tau_1/\tau_2)$.

In order to know the current capital stock, let's start from (2.1.3) and (2.1.4).

$$E(\hat{r}_{t+1}|t) = \alpha - (1 - \alpha_1)E(k_{t+1}|t)$$

$$= \alpha - (1 - \alpha_1)\, k_{t+1} \qquad (2.1.9)$$

then substituting (2.1.1) and (2.1.9) into (2.1.5)

$$k_{t+1} = \beta_o + \beta_1[\alpha - (1 - \alpha_1)k_{t+1}) + \beta_2\, E[(p_{t+1}|t) - p_t]$$

$$+ \alpha_o + \alpha_1 k_t + x'$$

So $\phi_{-1}k_{t+1} = (\alpha_o + \beta_o + \alpha\beta_1) + \beta_2\, E[(p_{t+1}|t) - p_t] + \alpha_1 k_t + x'$ (2.1.10)

$$\phi_{-1}k_{t+1}^* = (\alpha_o + \beta_o + \alpha\beta_1) + \beta_2\, E[(p_{t+1}^*|t) - p_t^*] + \alpha_1 k_t^* + x' \qquad (2.1.11)$$

Where $\phi = [1 + \beta_1(1 - \alpha_1)]^{-1}$ and $0 < \phi < 1$

Given k_{t+1} and k_{t+1}^* , tomorrow's capitla stock is greater, the higher the expected rate of inflation----- reflecting the prfolio shift to capital.

We can solve (2.1.10) and (2.1.11) to obtain

$$k_{t+1} = \phi(\alpha_o + \beta_o + \alpha\beta_1) + \beta_2\phi\sum_1^\infty (\alpha_1\phi)^{i-1}\, E[(p_{t-i+1}|t-i) - p_{t-i}]$$

$$+\phi x' \qquad (2.1.12)$$

$$k_{t+1}^* = \phi(\alpha_o + \beta_o + \alpha\beta_1) + \beta_2\phi\sum_1^\infty (\alpha_1\phi)^{i-1}\, E[(p_{t-i+1}^*|t-i) - p_{t-i}^*]$$

$$+\phi x' \qquad (2.1.13)$$

Since $0 < \alpha_1\phi < 1$, the capital stock is finite if past anticipated rates of inflation were finite.

From equations (2.1.7) and (2.1.8), it is apparent that the current price level must be a function of the nominal money stock, the expected price level and the capital stock. But the capital stock is from equations (2.1.12) and (2.1.13), a function of past expected rate of inflation. So I used the backward and forward method to solve for the current price level as a function of current and expected future valves of money stocks and current capital level as well.

$$p_t = b_o + b_1 m_t + b_2 \sum_1^\infty (b_2)^i E(m_{t+i}|t) - b_3 k_t + \tau_1(1-\alpha_1)(1+\phi)x' \qquad (2.1.18)$$

$$p_t^* = b_o + b_1 m_t^* + b_2 \sum_1^\infty (b_2)^i E(m_{t+i}^*|t) - b_3 k_t^* + \tau_1(1-\alpha_1)(1+\phi)x' \qquad (2.1.19)$$

Those two equations are specified in the Appendix A.

As usual, CPIs (consumer price indexes) (q_t , q_t^*) are defined as weighted averages of the prices of home and foreign goods:

$$q_t = (1 - \beta) p_t + \beta(e_t - p_t) = p_t + \beta z_t \qquad (2.1.20)$$

$$q_t^* = \beta(p_t - e_t) + (1 - \beta) p_t^* = p - \beta z_t \qquad (2.1.21)$$

Here $0 < \beta < 1$, and it is the average propensity to import. The nominal exchange rate e is the same in both countries and real exchange rate z is

$$z = e + p^* - p \qquad (2.1.22)$$

Expected real interest rates are :

$$r_t = i_t - E(q_{t+1}|t) + q_t \qquad (2.1.23)$$

$$r_t^* = i_t^* - E(q_{t+1}^*|t) + q_t^* \qquad (2.1.24)$$

Here $[E(q_{t+1}|t) - q_t]$ can be used as the rate of inflation, and i_t and i_t^* are nominal interest rates on home currency and foreign currency bonds and where the subscript $E(q_{t+1}|t)$ indicates the expected value of a variable from tomorrow conditional on today's available information.

Residents in home country and foreign country regard bonds of both countries as perfect substitutes, so they will hold positive amount of both counties's bonds when the following equation holds:

$$i_t = i_t^* + E(e_{t+1}|t) - e_t \qquad (2.1.25)$$

Since each country only produces one good ,so the market equilibrium conditions for the two goods are:

$$y = \delta z + (1-\beta)\epsilon y + \epsilon y^* - (1-\beta)\nu r - \beta\nu r^* + u \qquad (2.1.26)$$

$$y^* = -\delta z + \beta\epsilon y + (1-\beta)\epsilon y^* - \beta\nu r - (1-\beta)\nu r^* + u \qquad (2.1.27)$$

The demands for two goods in two countries increase with both outputs. Residents of each country increase spending by the same fraction ($0 < \epsilon < 1$) of increase in their outputs. The marginal propensity to import ($0 < \beta < 1$) is equal to the average propensity to import in each country.

But the demands for two goods decrease with expected real interest rates. Redidents of each country decreas spending by the same amount (ν) for each percentage increase in expected real interest rate. In the above two equations, u is the positive demand disturbance.

Substracting (2.1.26) from (2.1.25) and rearranging yields the condition that the difference between the excess demand for the home good and the excess for the foreign good must be equal to zero.

$$- [1- (1 - 2\beta)\epsilon] (y- y^{*}) +2\delta z - (1-2\beta)\nu(r-r^{*})+2u =0 \qquad (2.1.28)$$

Substracting (2.1.24) from (2.1.23) and using (2.1.20), (2.1.21),(2.1.22),(2.1.25) yield the condition that real interest differential in favour of the home country must equal a constant times the expected rate of real depreciation of home currency:

$$r_t - r_t^{*} = (1 - 2\beta)E[(z_{t+1}|t) -z_t] \qquad (2.1.29)$$

Eliminating (y - y*) by (2.1.1) and (2.1.2) and (2.1.29) we can obtain an expression for the real exchange rate:

$$-[1-(1-2\beta)\epsilon]\tau_1(k_t -k_t^{*}) +2\delta z_t -(1-2\beta)^2[E(z_{t+1}|t) -z_t] +2u=0 \qquad (2.1.30)$$

$$z_t =\Xi\tau\alpha_1(k_t -k_t^{*}) +\zeta\tau E(z_{t+1}|t) -2\tau u$$

$$\Xi = 1-(1-2\beta)\epsilon$$

$$\zeta = (1-2\beta)^2\nu$$

$$\tau = 1/[2\delta +(1-2\beta)^2\nu]$$

Since $0 < \beta < 1$ and $0 < \epsilon < 1$, Ξ must be positive.

We assume that there are no speculative bubbles. That means that we can impose the transversality condition on this model $\mathrm{Lim}_{t\to\infty}\ \zeta\tau E(z_{t+1}|t) = 0$ since $0 < \zeta\tau < 1$ ($\delta > 0$).

Under this assumption, the reduced form for real exchange rate is $z_t = \Xi\tau\alpha_1(k_t - k_t^*) - 2\tau u$ (2.1.31)

So the reduced forms for CPIs should be like this

$$q_t = p_t + \beta z_t = p_t + \beta[\Xi\tau\alpha_1(k_t - k_t^*) - 2\tau u]$$

$$= p_t + \beta\Xi\tau\alpha_1(k_t - k_t^*) - 2\beta\tau u \qquad (2.1.32)$$

$$q_t^* = p_t + \beta\Xi\tau\alpha_1(k_t^* - k_t) - 2\beta\tau u \qquad (2.1.33)$$

2.2 Explanation of the model

In section 2.1, we obtained equations of capital stock and current price :

$$p_t = b_o + b_1 m_t + b_2 \sum_1^{\infty} (b_2)^i E(m_{t+i}|t) - b_3 k_t + \tau_1 (1-\alpha_1)(1+\phi) x' \quad (2.1.18)$$

$$p_t^* = b_o + b_1 m_t^* + b_2 \sum_1^{\infty} (b_2)^i E(m_{t+i}^*|t) - b_3 k_t^* + \tau_1 (1-\alpha_1)(1+\phi) x' \quad (2.1.19)$$

$$k_{t+1} = \phi(\alpha_o + \beta_o + \alpha\beta_1) + \beta_2 \phi \sum_1^{\infty} (\alpha_1 \phi)^{i-1} E[(p_{t-i+1}|t-i) - p_{t-i}] + \phi x' \quad (2.1.12)$$

$$k_{t+1}^* = \phi(\alpha_o + \beta_o + \alpha\beta_1) + \beta_2 \phi \sum_1^{\infty} (\alpha_1 \phi)^{i-1} E[(p_{t-i+1}^*|t-i) - p_{t-i}^*] + \phi x' \quad (2.1.13)$$

Suppose that the monetary authorities announce in period (t-1) that they'll increase the money supply in future periods.Equations (2.1.18) and (2.1.19) show that the announcement of a future increase in the money supply itself increases the price level today. Inflation takes place in advance of the increase in money stock. This is because people look forward. The authorities know that in the period before the money stock is increased, people will anticipate inflation and attempt to reduce their real money balances. It is this anticipated inflation in period (t-1) that causes the higher capital accumulation in period t.

The anticipated increases in money stock raise the price level in period t (proof is in Appendix B).

An increase in home money supply raises the home capital stock and also home CPI amd lowers the foreign CPI, as known in equations (2.1.32) and (2.1.33), an increase in foreign money supply has the same effects.

So if only the home money supply increases, the price of home output rises and capital accumulation increases, and if the price of foreign output and capital stock remains unchanged, then real exchange rate rises, as shown in equation (2.1.31).

The market equilibrium conditions for the two goods gives us the equation (2.1.28). This equation shows that the difference between the excess demand for home and foreign output must be zero and that the difference depends negatively on the difference between home and foreign outputs, positively on the real exchange rate, and --with $\beta < 1/2$ -- negatively on the real interest differential in favor of the home country.

From the Cobb-Doyglas production function, we know that the increased capital stock in home country will raise the home output. It makes the difference between home and foreign demands negative [equation (2.1.28)becomes negative]. In order for this difference to return to zero, the real exchange rate must rise,since

$$-[1-(1-2\beta)\epsilon]\tau_1(k_t - k_t^*) + 2\delta z_t + (1-2\beta)^2 z_t + 2u = 0.$$

At the same time, the real interest differential falls, because the home capital stock increases and the foreign capital stock remains unchanged and the increased home capital stock pulls down the home real interest rate.

From equation (2.1.34), we know that an increase in the home money supply raises its capital stock and also raises the real exchange rate and the nominal exchange rate [equations (2.1.31) and (2.1.34)]. Equal changes in the home and foreign money supply leave the real and nominal exchange rates unchanged [from (2.1.31) and (2.1.34)].

Equal increase in both money supply cause equal increase in both outputs and both prices. Since outputs change by the same amount, equal changes in the two real interest rates reequilibrate the good markets, and the real exchange rates are unchanged. Because the real exchange rate is unchanged and each country's product price measured in its own currency changes by the same amount, the nominal exchange rate also is unchanged.

2.3 Loss functions of the Policymakers

The loss function in the book by Cazoneri and Henderson is the sum of squared deviations of employments from the full employment and squared changes of CPIs, and they are:

$$L = \tfrac{1}{2}\ [\ \sigma(n)^2 \ + \eta(q_t - q_{t-1})^2\ ]$$
$$L^* = \tfrac{1}{2}\ [\ \sigma(n^*)^2 \ + \eta(q_{t*} - q_{t-1*})^2\ ]$$

In my paper, I assume that the capital stock is a variable and the employment is a constant. Because money is nonneutural, the changes of money supply have effects on capital accumulation and CPIs. Thus I use the changes of capital accumulation in stead of the deviations of employment from full employment. My loss functions are

$$L = \tfrac{1}{2}\ [\sigma(k_t - k_{t-1})^2 + \eta(q_t - q_{t-1})^2\ ]$$
$$L^* = \tfrac{1}{2}\ [\sigma(k_t^* - k_{t-1}^*)^2 + \eta(q_t^* - q_{t-1}^*)^2\ ]$$

The losses of the policymakers rise with squared difference between the capital stock in period t and the capital stock in period (t-1), and the losses also rise with squared changes in CPIs.

The ratio of the loss from a CPI changes to the loss from changes of capital stocks (σ / η) is the same in the two countries. In this loss function, we can see that CPIs are the same thing as inflations.

If there are no any disturbances ($x'=0$, $u = 0$), then two countries'money supply is constant ($m = m^* = 0$), so it does not yield any changes of capital stock ($k_t-k_{t-1}=k_t^*-k_{t-1}^*=0$) in different periods and also zero inflation ($q_t - q_{t-1} = q_t^* - q_{t-1}^* =0$).and therefore zero loss for both countries. But, if there are disturbances, there will be policy conflicts. Here x' is the world productivity disturbance and u is the demand disturbance. Since the nature of the policy conflict depends on the type of disturbance, in my paper only one disturbance is analyzed---a world productivity disturbance. Different losses result from different monetary policies. Two monetary policies are applied in both countries in my paper ---- the anticipated permanent increase in money supply and the anticipated increase in rate of growth in money supply.

2.4 A World Production Disturbance

Suppose that there is a world productivity ($x' > 0$, $u = 0$). This disturbance is a symmetric one because it affects both countries in the same way.

Because two countries have the same positive productivity disturbance, so their outputs increase, and if their money supplies remain the same, and the prices will decrease.

Because today's capital stock is the summation of past anticipated inflation rate, today's productivity disturbance does not have any influences on today's capital stock, but it does have influences on today's prices and the CPIs.

Both countries' outputs increase by the same amount, so the difference between the excess demands for home and foreign goods remains equal to zero [from (2.1.28)].

In order to combat deflation, each policymaker has an incentive to increase the money supply, given the money supply of other policymaker. It can lead to a loss by increasing inflation a little and it can gain by raising capital stock in next period.

When the home policymaker increases its money supply, it results in real depreciation of the home currency and limits foreign inflation. If foreign country does not increase the money supply, it will have appreciation of its currency, and will be less competitive and worse off in the open economy.

Just because of this reason, both home and foreign countries should increase their money supplies.

After the productivity disturbance happens, the policymakers can choose different monetary policies to reach several possible equilibria. Let's begin with considering the Nash noncooperative equilibria.

2.4.1 The Nash Noncooperative Equilibrium 1

Suppose that in period (t-1), authorities of both countries anticipate that there will be a positive world productivity disturbance in period t and deflation will happen in period. In order to combat this disturbance, monetary authorities will change their money supply in period t with a one-step increase. That means that there is an anticipated permanent increase in money supply. If the policymakers in both countries are Nash players, each of them will minimize his loss function with respect to his money supply, taking other policymaker's money supply as given. Both of them assume that the value of the disturbance is given. Their loss functions are:

$$L_m = \tfrac{1}{2} \left[\sigma(k_t - k_{t-1})^2 + \eta(q_t - q_{t-1})^2 \right]$$

$$L^*_m = \tfrac{1}{2} \left[\sigma(k_t^* - k_{t-1*})^2 + \eta(q_t^* - q_{t-1*})^2 \right]$$

so $$L_m = \tfrac{1}{2} \left[(\Delta m)^2 + (\Delta m - 2\theta' \Delta m^* - x'')^2 \right] \qquad (2.4.1.1)$$

$$L_{m*} = \tfrac{1}{2} \left[(\Delta m^*)^2 + (\Delta m^* - 2\theta' \Delta m - x'')^2 \right] \qquad (2.4.1.2)$$

(details are in Appendix C).

Before period t, money supply is a constant. In period (t-1), people know that there will be an change in money supply in period t, so the diference of money stock between period (t-1) and period t is $\Delta m = m_t - m_{t-1}$.

The partial derivatives of L with respect to m and of L^* with respect to m^* are

$$\frac{\partial L}{\partial m} = 2(\Delta m) - 2\theta'(\Delta m^*) - x''$$

$$\frac{\partial L^*}{\partial m^*} = 2(\Delta m^*) - 2\theta'(\Delta m) - x''$$

Then setting those partial derivatives equal to zero, we can obtain two first-order conditions and they are:

$$\Delta m = \theta' \ (\Delta m^*) + \ x''/2 \qquad (2.4.1.3)$$

$$\Delta m^* = \theta' \ (\Delta m) + \ x''/2 \qquad (2.4.1.4)$$

After the home country increases the money supply, taking the money supply of the foreign country as given, it increases inflation both by increasing the price of home output and by causing an depreciation of the home currency. Increasing the home money supply also increases the home capital stock. The home policymaker increases his country's money supply to the point where the gains from further increase in capital are just offset by the losses from higher inflation. The foreign country's policymaker is following the same policy. Both of them can increase their money supply until they reach the point at which neither policymaker can lower his loss by increasing his money supply further, given the value of the money stock of the other country.

Now setting $m = m^*$ in equation (2.4.1.3) yields the money supply of Nash equilibriun 1, and substituting these money supplies into (2.4.1.1) and (2.4.1.2) yields the Nash losses:

$$m^{N1} = m^{*N1} = \frac{x'''}{2\,(1-\theta')} \qquad (2.4.1.5)$$

$$L^{N1} = L^{N1*} = x'''^{2} l^{N1} = \frac{x'''}{4\,(1-\theta')^{2}} \qquad (2.4.1.6)$$

2.2 Nash Noncooperative Equilibrium 2 and Other Equilibria

When both of policymakers are Nash players, they could change the rate of growth in money supply μ and μ^* instead of increasing money supply with one-step increase. Their loss functions are:

$$L_\mu = \tfrac{1}{2} [\sigma(k_t - k_{t-1})^2 + \eta(q_t - q_{t-1})^2]$$

$$L_{\mu*} = \tfrac{1}{2} [\sigma(k_t^* - k_{t-1*})^2 + \eta(q_t^* - q_{t-1}^*)^2]$$

By substituting some equations related to capital stock and price level into these loss functions, we can have their loss:

$$L_\mu = \tfrac{1}{2} [(\mu)^2 + (\mu - 2\theta\mu^* - x)^2] \quad (2.4.2.1)$$

$$L_{\mu*} = \tfrac{1}{2} [(\mu^*)^2 + (\mu^* - 2\theta\mu - x)^2] \quad (2.4.2.2)$$

(details are in Appendix C)

The partial derivatives of L_μ with respect to μ and of L_μ^* with respect to μ^* are:

$$\frac{\partial L}{\partial \mu} = 2\mu - 2\theta\mu^* - x$$

$$\frac{\partial L^*}{\partial \mu^*} = 2\mu^* - 2\theta\mu - x$$

Then setting these two partial derivatives equal to zero to obtains the two first-order conditions

$$\mu = \theta\mu^* + x/2 \quad (2.4.2.3)$$

$$\mu^* = \theta\mu + x/2 \quad (2.4.2.4)$$

Now if we set $\mu=\mu^*$, then the above equations can reach the money supply of Nash equilibrium 2 and the Nash losses are:

$$\mu^{N2} = \mu^{N2*} = \frac{x}{2(1-\theta)} \qquad (2.4.2.5)$$

$$L_\mu^{N2} = L_\mu^{N2} = x^2 \, l^{N2} = \frac{x^2}{4(1-\theta)^2} \qquad (2.4.2.6)$$

Because $x < x'''$,and also $\theta' > \theta$, so $L^{N2}=L^{N2*} < L^{N1}=L^{N1*}$. That means that the losses in Nash equilibrium 2 are less than these in Nash equilibrium 1.

What is the economic explanation behind this ?

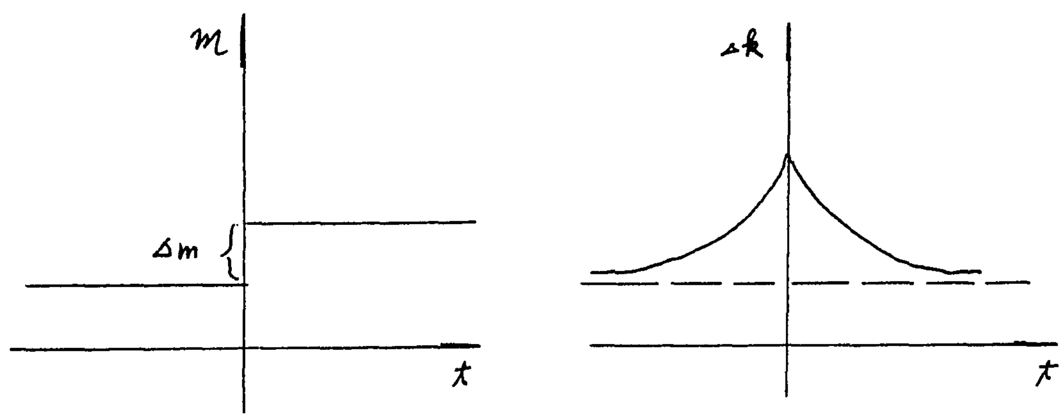
Let's look at the price function of the home country

$$p_t = b_0 + b_1 m_t + b_2 \sum_1^\infty (b_2)^i E(m_{t+1}|t) - b_3 k_t - \tau_1(1-\alpha_1)(1+\phi)x'$$

Under expectation, people look forward and price goes up immediately after the announcement of a permanent increase in money supply. So it creates inflation between the announcement and the implementation, and capital stock will rise after the announcement.

But after the implementation of the monetary policy, money supply will stay at the same level because the growth rate of money supply will be zero.The inflation rate reaches a maximum between period (t-1) and period t, and declines thererafter. The maximum change in capital accumulation caused by the increase in money stock occurs between period (t-1) and period t. It is shown in fig.1 But on the other hand, after the policymakers announce the increase in the rate of growth of money supply, the anticipated rate of increase in money stock in every period will cause inflation. The anticipated inflation leads to the increases in capital accumulation. This means that capital stock increases steadly after implementation of the policy, and also the capital stock approaches the steady state as time passes.

Fig.1 Effects of an anticipated permanent increase in money supply on capital stock.



If the expectation is rationally formed and if the price is fully flexible, then after the announcement of the monetary policy, there is an immediate jump in the current price.

In section 2.1, the formula of the price is:

$$p_t = b_o + b_1 m_t + b_2 \sum_1^{\infty} (b_2)^i E(m_{t+i}|t) - b_3 k_t - \tau_1(1-\alpha_1)(1+\phi)x' \quad (2.1.18)$$

At two Nash equilibria, this immediate jump can be measured by the formulas:

$$p_{t-1} - p_{t-2} = (1 - b_3 \beta_2 \phi)\Delta m \qquad (2.4.2.7a)$$

$$p_{t-1} - p_{t-2} = (1 + b_1 - b_3 \beta_2 \phi)\mu \qquad (2.4.2.7b)$$

(details are in Appendix B and C).

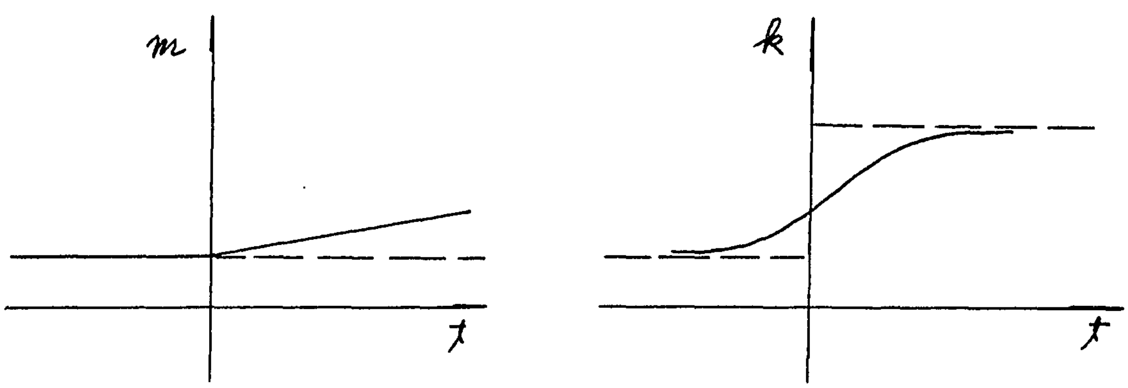
At the two Nash equilibria, the money supplies are

$$\Delta m = \frac{x'''}{2(1-\theta')} \quad \text{and} \quad \mu = \frac{x}{2(1-\theta)} \quad \text{respectively.}$$

It is easy to show that (2.4.2.7a) > (2.4.2.7b) (details are in Appendix D) .

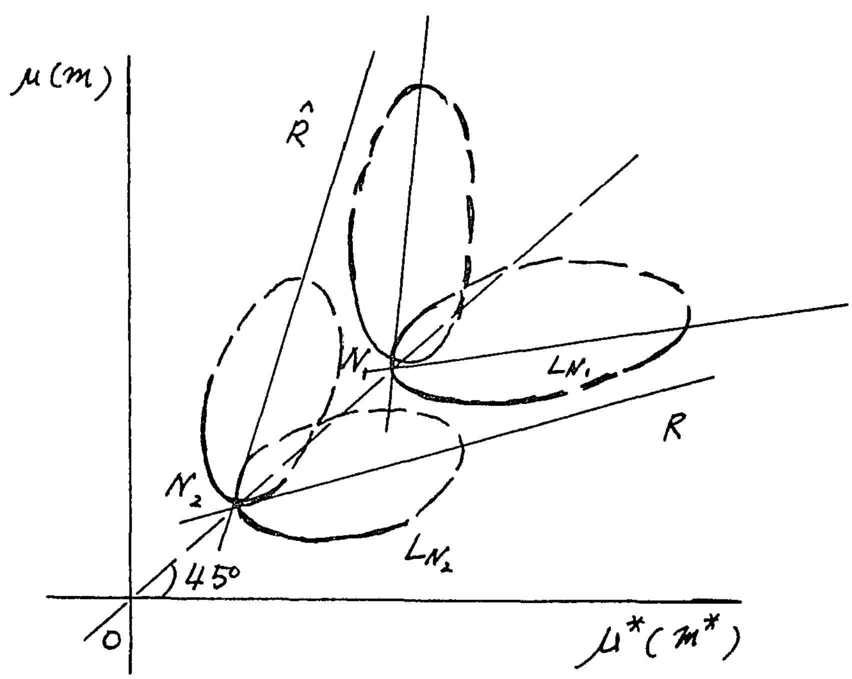
This means that CPIs in Nash 2 are higher than CPIs in Nash 1. However, just because the higher jump of price causes the higher capital accumulation in Nash 2 in future periods than that in Nash 1, so in Nash 2 the gain from higher capital stock dominates the loss from the higher CPI even more than in Nash 1. This is the reason why $L^{N2} = L^{N2*} < L^{N1} = L^{N1*}$.

Fig.2 Effects of an anticipated increase in rate of money supply on capital stock



Equations (2.4.1.3),(2.4.1.4),(2.4.2.3) and (2.4.2.4) are obtained from the first-order conditions, and represent four linear equations. After we plot them, we can have four straight lines in Fig.3. Those four equations are also the reaction functions of the policymakers in both countries under the different monetary policies. Those four loss functions have the shapes of ellipse. At Nash noncooperative equilibria, home country and foreign country have the same money supplies, so their loss functions intersect at points of $\mu = \mu^*$ and $m = m^*$, respectively.

Fig.3 Two Nash noncooperative equilibria under a world productivity disturbance



A region of improvement for both policymakers in Nash equilibrium 2 is towards northeast.

The partial derivatives of each policymaker's loss function with respect to the other policymaker's instrument are

$$\frac{\partial L}{\partial \mu^*} = -2\,\Theta(\mu - 2\Theta\mu^* - x) \qquad (2.4.2.8)$$

$$\frac{\partial L^*}{\partial \mu} = -2\,\Theta(\mu^* - 2\Theta\mu - x) \qquad (2.4.2.9)$$

Substitute (2.4.2.5) into (2.4.2.8) and (2.4.2.9) to obtain these partial derivatives evaluated at Nash 2:

$$\frac{\partial L}{\partial \mu^*} \Big| N_2 = \frac{\partial L^*}{\partial \mu} \Big| N_2 = + \frac{2\Theta x}{2(1-\Theta)} \qquad (2.4.2.10)$$

At Nash equilibrium, the increase in the home money supply has no effect on home losses, because the gain from increase in capital stock is just offset by the loss from the increase in home CPI inflation. But let's consider a small increase in the home country's money supply accompanied by an increase of the same size in the foreign country's money supply. The increase of the same size in the foreign money supply raises home loss by preventing home currency from depreciating in real terms, and it limits the increase in home CPI inflation and lowers capital stock, and finally in the home country the loss from lower capital stock cannot domenate the gain from CPI inflation. In the same way, equal increases in the home and the foreign money supplies increase the foreign loss.

Stackelberg Leadership and Commitment

In Stackelberg leadership equilibrium, one policymaker commits himself to deliver a particular money supply, and he is called the Stackelberg leader. The other policymaker does the best he can for himself , given the leader's money supply and he is called the Stackelberg follower. The leader chooses his money supply, taking into account how the follower will react to his choice.

If the home country is the leader, he minimizes his loss subject to the reaction of the foreign country.

$$\frac{\partial L}{\partial \mu} + \left(\frac{\partial L}{\partial \mu^*} \right) \left(\frac{\partial \mu^*}{\partial \mu} \right)\Big|_{\hat{R}} = 0 \qquad (2.4.2.11)$$

$$\frac{\partial L^*}{\partial \mu^*} = 0 \qquad (2.4.2.12)$$

From (2.4.2.11) we have

$$\frac{\partial \mu^*}{\partial \mu} = \frac{2\mu - 2\theta\mu^* - x}{2\theta(\mu - 2\theta\mu^* - x)}$$

Equations (2.4.2.3) and (2.4.2.4) are reaction functions in Nash equilibrium, so the value of $\frac{\partial \mu^*}{\partial \mu}$ should be on the reaction function,

$$\frac{\partial \mu^*}{\partial \mu} = \frac{2\mu - 2\theta\mu^* - x}{2\theta(\mu - 2\theta\mu^* - x)} = \theta = \frac{\partial \mu^*}{\partial \mu}\Big|_{\hat{R}} \qquad (2.4.2.13)$$

$$\mu^* = \theta\mu + x/2 \qquad (2.4.2.14)$$

and then

$$\mu^s = \frac{(1 + \theta - \theta^2 - 2\theta^3)x}{2(1 - 2\theta^2 + 2\theta^4)} \qquad (2.4.2.15)$$

$$\mu^{*s} = \frac{(1+\theta)x - 2\theta^2(1+\theta)x}{2(1 - 2\theta^2 + 2\theta^4)} \qquad (2.4.2.16)$$

The loss in the Stackelberg equilibrium is

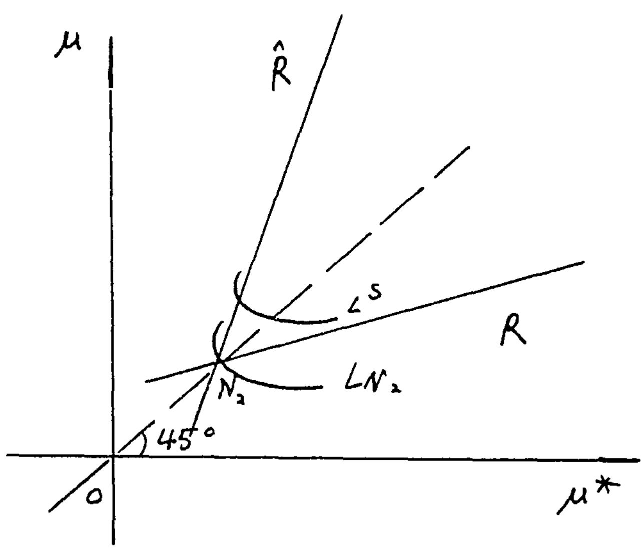
$$L_\mu^{s} = x^2\, l^{s} = \frac{(1 + \theta - \theta^2 - 2\theta^3)^2\, x^2}{4(\, 1 - 2\theta^2 + 2\theta^4\,)^2} \qquad (2.4.2.17)$$

$$L_\mu^{*s} = \frac{(1 + \theta)^2\, x^2}{4(\, 1 - 2\theta^2 + 2\theta^4\,)^2}$$

and also we can show that

$$l^{s}/l^{N2} = \frac{(\, 1 - 2\theta^2 - \theta^3 + 2\theta^4\,)^2}{(\, 1 - 2\theta^2 + 2\theta^4\,)^2} < 1 \qquad (\, 2.4.2.18\,)$$

Fig.4 A world productivity disturbance:
Adding the Stackelberg equilibrium



When they are Nash players, they have no incentive to increase their money supplies more because each of them has minimized his loss, taking the other's money supply as given. The situation changes when the home country acts as a

Stackelberg eader. The home policymaker considers the fact that if he increases more than in Nash equilibrium, the foreign Policymaker will increase more as well.

In Stackelberg leadership equilibrium, the increase in the foreign money supply is less than the increase in home money supply and the foreign policymaker's loss is reduced less than the country's loss.

Because the foreign policymaker follows the home policymaker,so if he raises his money supply by more than the money supply in Nash 2, he can both raise capital stock above the Nash level and keep inflation above the Nash level. Because his capital gain domenates the CPI loss, so his loss is lower than that in Nash 2. When the Stackelberg leader acts in his own interest, he helps the other policymaker at the same time.

Fixed-Exchange-Rate Leadership and Commitment

In the fixed-exchange-rate leadership, both policymakers agree that the home policymaker commits to deliver a particular money supply and the foreign policymaker commits to adjust his money supply so as to fix the real or nominal exchange rate given the value of the home country money supply.

In this equilibrium, the home policymaker minimizes his loss according to the reaction function. The fixed-exchange-rate equilibrium achieves at the point χ where the home loss ellipse L is tangent to R^*_x.

The money supplies are given by two conditions

$$\frac{\partial L}{\partial \mu} + \left(\frac{\partial L}{\partial \mu^*} \right) \left(\frac{\partial \mu^*}{\partial \mu} \right) \Big|_{R^*_x} = 0 \qquad (2.4.2.19)$$

$$\begin{aligned} z &= \Xi\tau\alpha_1 \ (k - k^*) \\ &= \Xi\tau\alpha_1 \ (\mu - \mu^*) \end{aligned} \qquad (2.4.2.20)$$

also $\partial L/\partial\mu = 2\mu - 2\theta\mu^* - x$ and $\partial L/\partial\mu^* = -2\theta(\mu - 2\theta\mu^* - x)$

so $$\partial\mu/\partial\mu^* = \frac{-\ \partial L/\partial\mu}{\partial L/\partial\mu^*} = \frac{-\ (2\mu - 2\theta\mu^* - x)}{-2\theta(\mu - 2\theta\mu^* - x)} = 1 = (\partial\mu/\partial\mu^*)|\ R^*_x$$

$$(2.4.2.21)$$

Since $\mu=\mu^*$, then we have

$$\mu^x = \mu^{*x} = \frac{(1-2\theta)\ x}{2(1-2\theta+2\theta^2)} \qquad (2.4.2.22)$$

$$L^x_\mu = L^{*x}_\mu = x^2\ l^x = \frac{x^2}{4(1-2\theta+2\theta^2)} \qquad (2.4.2.23)$$

also $$l^x/l^{N2} = \frac{1 - 2\theta + \theta^2}{1 - 2\theta + 2\theta^2} < 1 \qquad (2.4.2.24)$$

$$l^x/l^s = \frac{1 - 4\theta^2 + 8\theta^4 - 8\theta^6 + 4\theta^8}{1 - 3\theta^2 + 7\theta^4 - 2\theta^5 - 10\theta^6 + 8\theta^8} < 1 \qquad (2.4.2.24)'$$

Since L^x and L^{*x} are tangent at χ , the fixed-exchange-rate outcome is Pareto efficient. In this equilibrium, the home policymaker takes account of the fact that if he increases money supply more than in the Nash equilibrium 2, the foreign policymaker will also increase money supply more. Thus, the both policymakers have an incentive to increase more, thereby making both of them better off.

The policyamkers in both countries increase more in the fixed-exchange-rate leadership equilibrium than in the Stackelberg equilibrium. When the home policymaker is a fixed-exchange-rate leader, he realizes that the foreign policymaker will increase the foreign money supply above the Nash level by the same rate as the home money supply. The foreign policymaker must behave in this way in order to keep the exchange rate fied. When the home policymaker is a Stackelberg leader, he realizes that the foreign policymaker will increase the foreign money supply above the Nash level by smaller rate than the home money supply for the reason given before.

The Efficient Equilibrium amd Cooperation

If the two policymakers can commit themselves, they can achieve one of the Pareto-efficient equilibria. We refer to Pareto-efficient equilibrium as efficient equilibrium. If both policymakers commit themselves to an efficient equilibrium, then they are cooperative players.

If both policymakers are cooperative, they'll act as a single policymaker and minimize a weighted sum of their losses to find the efficient equilibrium, where each loss function has a weight of 1/2. Their money supplies are decided by the following first-order conditios

$$\partial L/\partial \mu + \partial L^{*}/\partial \mu = 0 \qquad (2.4.2.25)$$

$$\partial L/\partial \mu^{*} + \partial L^{*}/\partial \mu^{*} = 0 \qquad (2.4.2.26)$$

The two conditions are symmetric, and the productivity disturbance affects them in the same way. Therefore, they will set $\mu = \mu^{*}$. Since from equations (2.4.2.17) and (2.4.2.18),

$(\partial L/\partial \mu) = -(\partial L^{*}/\partial \mu) = -(\partial L^{*}/\partial \mu^{*}) = (\partial L/\partial \mu^{*})$ and then we have $(\partial L^{*}/\partial \mu) = -(\partial L/\partial \mu^{*})$ and then the efficient money supplies can be obtained from the two conditions

$$\partial L/\partial \mu^{*} + \partial L/\partial \mu = 0 \qquad (2.4.2.27)$$

$$\mu = \mu^{*} \qquad (2.4.2.28)$$

and $\mu^{E} = \mu^{*E} = \dfrac{(1-2\Theta)x}{2(1-2\Theta+2\Theta^{2})}$

After that we can obtain the efficient losses:

$\mu^{E} = \mu^{*E} = \mu^{x} = \mu^{*x}$

$L^{E} = L^{*E} = L^{x}{}_{\mu} = L^{*x}{}_{\mu} = x^{2}\ 1^{x}$

In the one-shot game the efficient equilibrium can be obtained only if both policymakers can commit themselves. If one of them doesn't commit himself, the efficient equilibrium cannot be achieved. For example, if home country belivies that the foreign policymaker will play μ^{*E}, he will be tempted to cheat by playing μ^F, and he minimizes his loss by the money supply at point of μ^F, because $L_\mu^F < L_\mu^E$. The proof is as follows:

At point E, the foreign money supply is $\mu^{*E} = \frac{(1-2\theta)x}{2(1-2\theta+2\theta^2)}$

If home country cheats at E, he will choose his money supply according to the following first-order condition:

$$\partial L/\partial\mu = \mu + \mu - 2\theta\mu^* - x = 0$$

so home money supply is:

$$\mu^F = \theta\mu^* - \tfrac{1}{2}x = \theta\left[\frac{(1-2\theta)x}{2(1-2\theta+2\theta^2)}\right] + \tfrac{1}{2}x = \frac{(1-\theta)x}{2(1-2\theta+2\theta^2)} \qquad (2.4.2.29)$$

and his loss is

$$L^F = \tfrac{1}{2}x^2\left\{\tfrac{1}{2} + \frac{2\theta^2(1-2\theta)^2}{[1+(1-2\theta)^2]^2} + \frac{2\theta(1-2\theta)}{1+(1-2\theta)^2}\right\}$$

$$= x^2 l^F = x^2\frac{(1-\theta)^2}{4(1-2\theta+2\theta^2)} = x^2 l^F$$

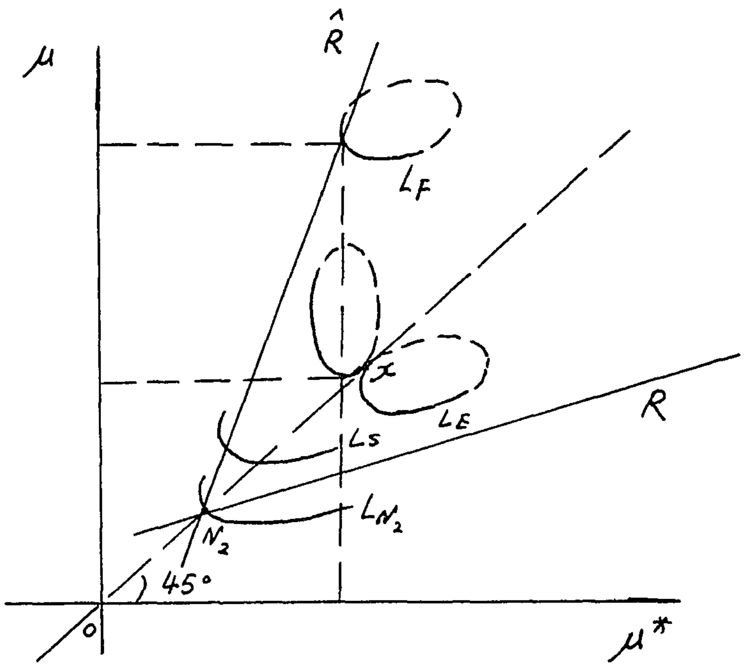
and

$$l^F/l^E = (1-\theta)^2 < 1 \qquad (0 < \theta < \tfrac{1}{2})$$

so $l^F < l^E$.

The home policymaker's money supply is higher at F than at E, and his loss is lower than the foreign country's loss. The foreign policymaker knows that it is likely that the home country will not play μ^E and therefore he will not play μ^{*E} in the absence of a binding agreement.

Fig.5 A world productivity disturbance: Adding the efficient equilibrium



2.5 Summary

In this chapter , the loss of the policymaker of each country depends not only on the money supply of his own country but also on the money supply of the other country.This is the external effects. At the Nash equilibrium of one-shot game, both countries reach a Pareto inferior outcome. In a fixed-exchange-rate leadership, they obtain a Pareto optimal outcome.

From the above models, we know that leadership results in better outcome for both policymakers than that at the Nash equilibria. The ranking of loss at those equilibria is as follows:

$$L(\text{efficiency}) < L(\text{leadership}) < L(\text{Nash2}) < L(\text{Nash1})$$

In the one-shot game,the noncooperative behavior usually leads to an inefficient outcome, because the one-shot game is static.

Imagine that the same game is played not just once, but repeatedly over an infinite time-horizon or over a finite time-horizon. Each country can observe the past actions of the other country, and can use this information about the history of the game in making its current decision. The outcomes will be better in repeated games than that in one-shot games, because repeated games are dynamic. The theory of repeated games will be applied to the analysis of monetary policy in interdependent economies in chapter 3.

3. Trigger Mechanism in Repeated Games

At Nash equilibrium, each country minimizes the loss given the other country's policy and Nash solution is found by solving the simultaneous equations

$$\frac{\partial L}{\partial \mu} = 0 \quad \text{and} \quad \frac{\partial L^*}{\partial \mu^*} = 0 \tag{3.1}$$

$$\left(\text{or} \quad \frac{\partial L}{\partial m} = 0 \quad \text{and} \quad \frac{\partial L^*}{\partial m^*} = 0 \right)$$

At Pareto optimum, no country can be better off without the other country being worse off, because different country's interests conflict. It means that if any country were to change his money supply in order to decrease his loss, this would increase other country's loss.

$$\left.\frac{\partial L}{\partial \mu}\right|_{\mu^p} < 0 \quad \text{and} \quad \left.\frac{\partial L^*}{\partial \mu}\right|_{\mu^p} > 0$$

$$\text{or} \quad \left.\frac{\partial L^*}{\partial \mu^*}\right|_{\mu^{*p}} < 0 \quad \text{and} \quad \left.\frac{\partial L}{\partial \mu^*}\right|_{\mu^{*p}} > 0 \tag{3.2}$$

also at Pareto optimum, $(\omega_1 L + \omega_2 L^*)$ is minimized with some non-zero constants ω_1 and ω_2 and the first order condition is:

$$\begin{cases} \omega_1 \dfrac{\partial L}{\partial \mu} + \omega_2 \dfrac{\partial L^*}{\partial \mu} = 0 \\ \omega_1 \dfrac{\partial L}{\partial \mu^*} + \omega_2 \dfrac{\partial L^*}{\partial \mu^*} = 0 \end{cases} \tag{3.2)'$$

Obviously, (3.1) and (3.2)'cannot be satisfied simultaneously, and this means that Nash equilibrium is not always the Pareto optimum.

Since at Pareto optimum, $\frac{\partial L}{\partial \mu} < 0$ and $\frac{\partial L^*}{\partial \mu^*} < 0$, each country pursues its own interest and wants to minimize its loss, so in fact no country plays the Pareto optimally and efficiently, and eventually they drive the whole system down to the Pareto inefficiency.

If two countries are cooperative, they will fully exploit the gains from the cooperation. If, however, they are noncooperative, each country's pursuit of his self interest usually results in a Pareto inferior equilibrium. Usually this is the outcome in static games.

In dynamic games, the distinction between cooperative-game equilibrium and non-cooperative-game equilibrium is blurred. In a dynamic noncooperative game, there may be equilibrium which is Pareto optimum. This Pareto optimum can be obtained as an equilibrium in the repeated games if both countries apply some strategies of "threatening" to "punish" the country who deviates from the Pareto optimum.

According to Trigger mechanism, if time horizon is infinitive and if the Nash equilibrium is unique, players choose the Pareto optimum in period 1 and continue to choose Pareto optimum as long as it has been chosen in all past periods. If any country deviates from the Pareto optimum and cheats on other country, then the Nash equilibrium will be chosen in rest periods of the game.

Let's denote a Pareto optimum by (a_h^P, a_f^P) and unique Nash by (a_h^N, a_f^N). Here a--action, h--home country, f--foreign country,P--Pareto optimum, N--Nash equilibrium. If the home country continues forever to play a_h^P and the foreign country follows always, then the discounted total loss is

$$\sum_1^\infty d^{t-1}L(a_h^P,a_f^P)= [1/(1-d)]L(a_h^P,a_f^P) \qquad (3.3)$$

But at the Pareto optimum, the home country will choose some policy a_h" which is different from a_h^P in period 1 to minimize his loss given that the foreign country is playing the Pareto optimum. Since the foreign country could not respond to home country's deviation until next period.So in period 2, the foreign country will play a_f^N to punish the home country. By deviation from the Pareto optimum in period 1, home country gains lower loss, but he must trade the lower loss off against higher loss in the future. Thus the discounted total loss of the home country is:

$$L(a_h",a_f^P) +\sum_2^\infty d^{t-1}L(a_h^N,a_f^N)=L(a_h",a_f^P)+d/(1-d)L(a_h^N,a_f^N) \qquad (3.4)$$

The home country will not deviate from the playing Pareto optimum if the gain does not exceed the loss.It means that

(3.3) > (3.4) and $$d \geq \frac{L(a_h^P,a_f^P) - L(a_h^N,a_f^N)}{L(a_h",a_f^P) - L(a_h^N,a_f^N)}$$

If the discounted rate is satisfied with this inequality, the the Pareto optimum can be sustained as an equilibrium. The repeated playing of a Pareto optimum is a dynamic Nash equilibrium.

Suppose that the world productivity disturbance lasts for some periods and then a new productivity disturbance comes along to start another game. In the periods of the same disturbance, the home country and the foreign country repeat the one-shot game which is described in chapter 2. Each player can monitor the behavior of his opponent, each player can affect future play with his own behavior. Bad behavior will be punished and good behavior will be rewarded in the future.

Trigger mechanism can be applied in repeated games and the applications are the main contents of section 3.1 and 3.2.

3.1 Trigger Mechanism in Infinitely Repeated Games

In this section we assume that the world productivity disturbance lasts for ever. If one-shot game is repeated infinitely, and if future loss is discounted properly, then trigger mechanism always supports efficient outcomes.

Suppose that the home and the foreign policymakers agree that they play E(efficient equilibrium) every period, if either of them cheats on his opponent after their agreement, then both of them play Nash solution 2 in next period.

If the home policymaker decides to play F (he deviates from the efficient equilibrium) in the same period, then his loss in that period will be smaller than l^E, and the temptation is:

Temptation = $l^E - l^F$

If the home policymaker cheats on the foreign policymaker during this period, his loss in next period will be l^{N2}, which is larger than l^E. The reward for playing E this period is:

Reward = $d\,(l^{N2} - l^E)$

d -- discount factor (reward accrues next period)

If $d(l^{N2} - l^E) > (l^E - l^F)$, then no incentive for the home policymaker to cheat on his opponent. If this inequality holds always, both of them will play E in every period. If d is not small [$d > \frac{(l^E - l^F)}{(l^{N2} - l^E)}$] , and future is not heavily discounted, or if the temptation to cheat is not great, then the trigger mechnaism will enforce both policymakers to stay in efficient equilibrium.

Let's consider the Nash strategy S_1:

(1) begin playing E

(2) continue playing E unless one of them cheats, and if one of them cheats, then switch to Nash solution 2 for the rest periods of the game.

If both of them adopt S_1, then

$$L(S_1) = \sum_1^{\infty} d^{t-1}\, l^E = l^E/(1-d) \quad (0 < d < 1) \qquad (3.1.1)$$

If the home policymaker cheats in period 1, the home loss will be:

$$L = l^F + \sum_2^{\infty} d^{t-1}\, l^{N2} = l^F + d l^{N2}/(1-d) \qquad (3.1.2)$$

and

$$L - L(S_1) = l^F + \frac{d\, l^{N2}}{1-d} - \frac{l^E}{1-d}$$

$$= l^F - l^E + \frac{d\,(l^{N2} - l^E)}{1-d} \qquad (3.1.3)$$

If (3.1.3) is positive, the home policymaker would not want to cheat in period 1. According to this simarity, the home policymaker would not cheat in period 2, period 3, and so on.

If (3.1.3) is positive, and if the foreign policymaker adopts S_1 , and then the home policymaker has to adopts S_1 too. So S_1 is indeed a Nash strategy.

Both of them must choose S_1, otherwise either one of them will be worse off. In the mathematics, their behaviors satisfy the conditions described in (3.2). So we can say that trigger mechanism in infinitely repeated games can achieve efficient outcomes. So it is not necessary to force sovereign policymakers to cooperate.

Nash strategies allow the possibility of retaliation and so cooperative behavior can be enforced and cooperative equilibrium can be achieved in noncooperative games.

When the discounted rate $d < \frac{(1^E - 1^F)}{(1^{N2} - 1^E)}$, it means that the future loss is discounted very heavily, and that a threat in Nash strategy is not enough to dissuade any country from deviation from the Pareto optimial equilibrium, then the trigger mechanism fails. But there is a number of equilibria in dynamic games.

The repeated playing of a Pareto optimum is a dynamic Nash equilibrium. And if the trigger mechanism fails, the repeated playing the static Nash equilibrium is also an equilibrium of the repeated games.

3.2 Trigger Mechanism in Finitely repeated Games

The trigger mechanism which is described in section 3.1 fails when there is only one Nash equilibrium in one-shot game and also the game is repeated only a *finite* number of times.

Suppose that the home and the foreign policymakers promise to play E in every period. In the last period T, if any of them cheats, then he will not be punished, since the game will be over after period T. So any promise of playing E in the last period will not be credible and both of them must play Nash solution in period T. So the outcome of playin Nash in period T is predetermined. Any promise of playing E in period (T-1) is not credible either and Nash solution must also be played in period (T-1). By the same reason, Nash solution must be played in (T-2),(T-3),...... and so on. The reason that the trigger mechanism in section 3.1 fails is because any good ehavior in a current period can not be rewarded in the future period.

Friedman (1985) suggested a more transparent way to make his trigger mechanism work in finitely repeated games. If the one-shot game has a second Nash solution, there are two possible outcomes in the last period of the game and if one of the Nash solution is better for both players, then good behavior in early period can be rewarded with the good Nash solution in later periods.

In this section, we will consider the one-shot game with two Nash solutions---- Nash solution 1 and Nash solution 2 ---- and the latter is better than the former for both players.

According to Friedman's suggestion, the Nash strategy S_2 is that: (1) begin playing E

(2) continue playing E unless one of them cheats, if either of them cheats, then both of switch to the Nash solution 1 for the rest periods of the game

(3) if nobody cheats by period t^*, then switch to the Nash solution 2 for the rest of the game. So the noncooperative players will play E in all but the last $(T - t^*)$ periods.

S_2 is Nash strategy for the finitely repeated game. If the foreign policymaker adopts S_2, the the home policymaker has to adopt S_2 in response. If the home policymaker quits before t^*,the foreign policymaker will play the Nash solution 1 for the rest periods of the game. If the home policymaker quits in or after period t^*, the foreign policymaker will play the Nash solution 2, so we know that there is no any chance for the home policymaker to return back to E in period later than t^* since the strategy of the foreign policymaker is unaffected by the behavior of the home policymaker.

They start with the E equilibrium, and if no country cheats, then they switch to Nash 2 and the total loss is

$$\sum^{t^*}_1 d^t\ l^E + \sum^T_{t^*+1} d^t\ l^{N2} \qquad (3.2.1)$$

Because from period t^* on, the foreign country will play Nash solution 2, so if the home country wants to quit and he has to quit before period t^* ($t^0 < t^*$) and then his total loss will be :

$$\sum^{t^*-1}_1 d^t\ l^E + d^t l^F + \sum^T_{t^*+1} d^t\ l^{N1} \qquad (3.2.2)$$

.43.

No alternative will deliver a lower loss than the best alternative under the equation (3.2.1).

In order to show this, we can compare (3.2.1) with (3.2.2),

$$\sum^{t^*}_1 d^t\ l^E + \sum^T_{t^*+1} d^t\ l^{N2} < \sum^{t^*-1}_1 d^t\ l^E + d^t l^F + \sum^T_{t^*+1} d^t l^{N1}$$

The inequality is equivalent to

$$d > \frac{l^E - l^F}{l^{N1} - l^F - d^{t^*-to}(l^{N2} - l^E) - d^{T-t^*}(l^{N1} - l^{N2})} = P$$

Suppose T and t^* is fixed, then the right side is the increasing function of t^o, because

$$\frac{\partial P}{\partial t^o} = \frac{-(l^E - l^F)[d^{t^*-to}(l^{N2} - l^E)\log d + d^{T-t^*}(l^{N2} - l^{N1})\log d]}{[\ l^{N1} - l^F - d^{t^*-to}(l^{N2} - l^E) - d^{T-t^*}(l^{N1} - l^{N2})]^2}$$

(3.2.3)

Since $\log(d) < 0$, $\frac{\partial P}{\partial t^o} > 0$, when $0 \le t^0 \le t^*$.

So if the home country wants to quit, he should quit in period t^*, and the inequality (3.2.3) becomes

$$d \geq \frac{l^{E} - l^{F}}{l^{N1} - l^{F} - (l^{N2} - l^{E}) - d^{T-t^{*}} (l^{N1} - l^{N2})} \qquad (3.2.4)$$

The smallest integer value of $T-t^{*}$ which satisfies this inequality is 1. When the discounted rate is calculated by (3.2.4), the optimal time for home country to quit is in the period t^{*} and if no country quits in or before period t^{*}, the best time for them to switch to Nash 2 is in the period ($T - 1$).

4. Conclusions and Comments

In my paper, I used $\hat{r}_t$ as a rent of capital (marginal product of capital) and $r_t = i_t - [E(q_{t+1}|t) - q_t]$ as the real interest. The rent of capital is the market price of the capital and the real interest rate is the supply price. If the price of existing unit of capital exceeds the price of a newly produced unit of output, then investors will demand newly produced output. Real interest rate is not always equal to rent of capital. In the international trading, real interest rate is used in obtaining equilibrium condition in the goods market.

In my paper, I used Fischer's recursive time model and obtained price and capital stock formulas by backward and forward method. I preferred recursive time model to continuous time model, since I think that if the domestic price is fully flexible and expectation is rationally formed, after the announcement of increases of money supply, the current price has an immediate jump, just because in mathematics, the function of price has a discontinuation at this point. How much is the jump ? It depends on the summation of expected future money stock.

The repeated noncooperative game in my paper is the simplest kind of dynamic games with complete information. It can be generalized by adding a random element, or by changing the structure of the games over time. Let's return to chapter 2. Suppose the world productivity disturbance changes over time. In any given period, policymakers know that th game will be repeated in future periods because of a new productivity disturbance, but they do not know the size of the disturbance that will occur then. The structure of the game changes from period to period. The game which is actually played in any particular time period is drawn according to a known probability distribution of productivity disturbance (the repeated game of chapter 2 is a degenerate stochastic game).The dynamics of this kind of games are determined by country's monetary policy and by the probability distribution function of productivity distribution which determines the selection of each period's game. Each country attempts to minimize the expected present value of all future losses. At Nash equilibrium, each country's expected discounted losses is at a minimum given the monetary policy of other country. If we have the probability distribtution of the world productivity disturbance, then it is easier to use the continuous time model.

In the repeated noncooperative games of chapter 2, home country and foreign country know each other's loss functions, because they have complete information, but in fact , every country does know his own loss function and his loss function is the same in all periods, and he has little knowledge of other's loss function. Also each country does know, in period t, what the strategy was played in the period (t-1) by other country. Each country's behavior in period t depends on the information commonly available to him and his opponent-----their actions which both of them realized in period (t-1). The situation in repeated game with incomplete information will be different.

Suppose in period (t-1), there is no any productivity disturbance to both countries, and there will be a world productivity disturbance in period t. Foreign country only knows that home country could apply several different monetary policies and his beliefs or his estimation of home country's policies is a kind of probability,

$P_F\ (K_H|u)$, and $\sum_{KF=1}^{Mf} P_F\ (K_H|u) = 1$

u -- the state and it is the pair of policies which will be applied by both countries when there is produtivity disturbance.

K_H -- different monetary policies by home country could apply

M_H -- the whole monetary policy set of home country and $K_H \in M_H$

To each home country's policy, foreign country has a relevant policy to respond to that, and it denotes as $a(K_F\ ,\ K_H)$ [$a(K_F\ ,\ K_H)$ is from payoff matrix $A(K_F\ ,\ K_H)$] and also this action' probability is $Q_F(k_F \mid u)$, and $\sum_{Kf=1}^{MF} Q_F(K_F|u) = 1$.

$L_F\ (Q_F, P_F | u)$

$$= \sum_{KF \epsilon MF} \sum_{KH \epsilon MH} Q_F(K_F\ |u)\ P_F(K_H|u)\ a(K_F\ ,\ K_H)$$

$L_H\ (Q_H, P_H | u)$

$$= \sum_{KF \epsilon MF} \sum_{KH \epsilon MH} Q_H(K_H\ |u)\ P_H(K_F|u)\ a(K_F\ ,\ K_H)$$

This one-shot payoff, conditional on the state u, depends on the foreign country's Q_F and his beliefs about his oponent's action's P_F.

The transition mechanism for foreign country is denoted by G_F.(G_H is for home country)

G_F is a $M_F \times M_H$ rows and columns matrix and then the expected payoff is:

$\sum_{t=0}^{\infty} d^t\ G_F^t\ L_F\ (\ Q_f\ ,\ P_F\ )$ and $\sum_{t=0}^{\infty} d^t\ G_H^t\ L_H\ (\ Q_H\ , P_H\ )$

then they maximize or minimize those two summations to obtain the best policy which they apply in this kind of games.

This kind of games is defined as a stationary two-person incomplete information supergame and the existence of equilibrium is proved by James Friedman.

Appendix (A)

BACKWARD AND FORWARD SOLUTIONS FOR CURRENT PRICE LEVEL AND CAPITAL STOCK

In chapter 2, we have the following equations for the home country,

$$y_t = \alpha_o + \alpha_1 k_t \quad (2.1.1)$$

$$\hat{r}_t = \alpha - (1 - \alpha_1) k_t \quad (2.1.3)$$

$$k_{t+1} = \beta_o + \beta_1 E(\hat{r}_{t+1}|t) + \beta_2\{ E(p_{t+1}|t) - p_t \} + y_t \quad (2.1.5)$$

$$m_t - p_t = \tau_o - \tau_1 E(\hat{r}_{t+1}|t) - \tau_2\{E(p_{t+1}|t) - p_t\} + y_t \quad (2.1.7)$$

and also

$$E(\hat{r}_{t+1}|t) = \alpha - (1 - \alpha_1) k_{t+1} \quad (2.1.9)$$

$$k_{t+1} = \phi(\alpha_o + \beta_o + \alpha\beta_1) + \phi\beta_2\{E(p_{t+1} - p_t) + \alpha_1\phi k_t + \phi x' \quad (2.1.10)$$

then substitute (2.1.1), (2.1.9) into (2.1.7)

$$m_t - p_t - \tau_2 p_t = \tau_o - \tau_1\alpha + \alpha_o - \tau_2 E(p_{t+1}|t) + \alpha_1 k_t + \tau_1(1 - \alpha_1) k_{t+1} + (1 + \phi) x' \quad (2.1.14)$$

tnen substitute (2.1.10) into (2.1.14)

$$m_t - (1 + \tau_2) p_t = \tau_o - \tau_1\alpha + \alpha_o + \tau_1(1 - \alpha_1)\phi(\alpha_o + \beta_o + \alpha\beta_1) + \tau(1 - \alpha_1)\phi\beta_2 E(p_{t+1}|t) - \tau_1(1 - \alpha_1)\phi\beta_2 p_t + \tau_1(1 - \alpha_1)\alpha_1\phi k_t - \tau_2 E(p_{t+1}|t) + \alpha_1 k_t + \tau_1(1 - \alpha_1)(1 + \phi) x'$$

so

$$p_t = b_o + b_1 m_t + b_2 E(p_{t+1}|t) - b_3 k_t + \tau_1(1 - \alpha_1)(1 + \phi) x' \quad (2.1.15)$$

Where

$$b_o = \frac{\tau_1\alpha - \tau_o - \alpha_o - \tau_1(1 - \alpha_1)\phi(\alpha_o + \beta_o + \alpha\beta_1)}{1 + \tau_2 - \phi\beta_2\tau_1(1 - \alpha_1)}$$

$$b_1 = \frac{1}{1 + \tau_1 - \phi \beta_2 \tau_1 (1 - \alpha_1)} \qquad 0<b_1<1$$

$$b_2 = \frac{\tau_2 - \tau_1 (1 - \alpha_1) \phi \beta_2}{1 + \tau_2 - \phi \beta_2 \tau_1 (1 - \alpha_1)} \qquad 0<b_2<1$$

$$b_3 = \frac{\alpha_1 \tau_1 (1 - \alpha_1) \phi + \alpha_1}{1 + \tau_2 - \phi \beta_2 \tau_1 (1 - \alpha_1)}$$

[because $\beta_1/\beta_2 \geq \tau_1/\tau_2$, if $\tau_2 > \tau_1 \beta_2 (1 - \alpha_1) - \tau_2 \beta_1 (1 - \alpha_1)$,

then, $\tau_2 - \tau_1 (1 - \alpha_1) \phi \beta_2 > 0$]

Now, we are ready to use the backward and forward method to solve for the price level.

A solution for current p_t is a function :

$$p_t = \sum_1^\infty a_i m_{t-i} + b m_t + \sum_1^\infty c_i E(m_{t+i} | t) \qquad (2.1.16)$$

The coefficients a_i, b, c_i satisfies equation (2.1.15).

Using equation (2.1.16) once and taking expectations on both sides,conditional or information available at time t:

$$E(p_{t+1} | t) = \sum_1^\infty a_i m_{t-i+1} + bE(m_{t+1} | t) + \sum_1^\infty c_i E(m_{t+i+1} | t) \qquad (2.1.17)$$

Replacing (2.1.17) into (2.1.15) and identifing term by term with (2.1.16), we can obtain that

$$p_t = b_o + b_1 m_t + b_2 \{ \sum_1^\infty a_i m_{t-i+1} + bE(m_{t+1} | t) + \sum_1^\infty c_i E(m_{t+i+1} | t) \}$$

$$- b_3 k_t + \tau_1 (1 - \alpha_1)(1 + \phi) x'$$

$$= \sum_1^\infty a_i m_{t-i} + b m_t + \sum_1^\infty c_i E(m_{t+i} | t)$$

When $i=1,\ b_1 + b_2 a_1 = bm_t$

$i=2,\ b_2 a_2 = a_1$

$i=3,\ b_2 a_3 = a_2$

....

so, $$a_1 = \frac{b - b_1}{b_2}$$

$$a_2 = \frac{b - b_1}{b_2} \cdot \frac{1}{b_2} = a_1 \cdot \frac{1}{b_2}$$

$$a_3 = a_2 \cdot \frac{1}{b_2}$$

....

$$a_{i+1} = a_i \cdot \frac{1}{b_2} \qquad i=1,2,.......$$

When $i=0,\ b_2 b = c_1$

$i=1,\ b_2 c_1 = c_2$ so $c_2 = bb_2^2$,

....

$c_{i+1} = b_2 c_i$ $\qquad i=1,2,......$

In the current price, the backward part is

$$p_t^{(B)} = \sum_1^\infty a_i m_{t-i} = \sum_1^\infty a_1 \ (1/\ b_2\ )^{i-1} m_{t-i}$$

Because $0 < b_2 < 1$, so $1/b_2 > 1$, and $p_t^{(B)}$ goes to the infinitive when i goes to the infinitive.

There two reasons that we can ignore $p_t^{(B)}$. First, stationarity is required in the model. Secondary, in the backward solution, the price $p_t^{(B)}$ does not move in response to current changes in money. We only choose forward solution, and we can set all a_i=0, then b_i=b, and

$$p_t = b_o m_t + b_1 \sum_1^{\infty} (b_2)^i E(m_{t+i}|t) - b_3 k_t + \tau_1 (1 - \alpha_1)(1 + \phi) x' \qquad (2.1.18)$$

By the same way, we can have the current price function of foreign country,

$$p_t^* = b_o m_t^* + b_1 \sum_1^{\infty} (b_2)^i E(m_{t+i}^*|t) - b_3 k_t^* + \tau_1 (1 - \alpha_1)(1 + \phi) x' \qquad (2.1.19)$$

APPENDIX (B)

PRICE INCREASES WITH THE INCREASES OF MONEY SUPPLY

Current price levels of home country and foreign country are the equations (2.18) and (2.19),

$$p_t = b_o m_t + b_1 \sum_1^\infty (b_2)^i E(m_{t+i}|t) - b_3 k_t + \tau_1 (1 - \alpha_1)(1 + \phi) x' \quad (2.1.18)$$

$$p_t^* = b_o m_t^* + b_1 \sum_1^\infty (b_2)^i E(m_{t+i}^*|t) - b_3 k_t^* + \tau_1 (1 - \alpha_1)(1 + \phi) x' \quad (2.1.19)$$

with b_o, b_1, b_2 and b_3 in Appendix (A).

After the anticipated permanent money supply increases, the increment of the money supply between period (t-1) and period t is Δm, so equation (2.1.18) becomes:

$$p_t = b_o + b_1 (m_{t-1} + \Delta m) + b_1 \sum_1^\infty (b_2)^i \{E(m_{t-1} + \Delta m|t)\}$$

$$-b_3 \{\frac{\alpha_o + \beta_o + \alpha \beta_1}{1 + \beta_1 (1 - \alpha_1)} + \beta_2 \phi \Delta m\} + \tau_1 (1 - \alpha_1)(1 + \phi) x'$$

$$= b_o + b_1 m_{t-1} + b_1 \sum_1^\infty (b_2)^i m_{t-1} - b_3 k_{t-1} + b_1 \{b_2 / (1 - b_2)\} \Delta m$$

$$-b_3 \beta_2 \phi \Delta m + \tau_1 (1 - \alpha_1)(1 + \phi) x' \quad (2.1.18)'$$

$$p_{t-1} = b_o + b_1 m_{t-1} + b_1 \sum_1^\infty (b_2)^i m_{t-1} - b_3 k_{t-1}$$

$$+ \tau_1 (1 - \alpha_1)(1 + \phi) x' \quad (2.1.18)''$$

$$p_t - p_{t-1} = b_2 \, m - b_3 \beta_2 \phi \Delta m + b_1 \, m = (1 - b_3 \beta_2 \, \phi) \Delta m$$

As long as $(1 - b_3 \beta_2 \, \phi) > 0$, the price increases with the increase of money supply.

When the rate of growth of money supply increases, equation(2.1.18) becomes

$$p_t = b_o + b_1(m_{t-1} + \mu) + b_1\sum_1^\infty (b_2)^i \{E(m_{t-1} + i\mu|t)\}$$

$$-b_3\{\frac{\alpha_o + \beta_o + \alpha\beta_1}{1+\beta_1(1 - \alpha_1)} + \beta_2\phi\ \mu\} + \tau_1(1 - \alpha_1)(1 + \phi)x'$$

$$= b_o + b_1 m_{t-1} + b_1\sum_1^\infty (b_2)^i m_{t-1} - b_3 k_{t-1}$$

$$+b_1\{\mu + [b_2/(1 - b_2) + b_1 b_2/(1-b_2)^2]\}$$

$$-b_3\beta_2\phi\mu + \tau_1(1 - \alpha_1)(1 + \phi)x'$$

p_{t-1} is the same as in (2.18)",so $p_t - p_{t-1} = (1+b_1 - b_3\beta_2\phi)>0$

APPENDIX (C)

LOSS FUNCTIONS UNDER DIFFERENT ANTICIPATED MONETARY POLICIES

Loss functions under the anticipated permanent increases in money supply are:

$$L_m = \tfrac{1}{2}[\sigma(k_t - k_{t-1})^2 + \eta(q_t - q_{t-1})^2]$$

$$L_m^* = \tfrac{1}{2}[\sigma(k_t^* - k_{t-1}^*)^2 + \eta(q_t^* - q_{t-1}^*)^2]$$

$$\begin{aligned} L_m = \tfrac{1}{2}\Big\{ &\sigma\{ v + \beta_2\phi\textstyle\sum_1^\infty (\alpha_1\phi)^{i-1} [E(p_{t-i+1}|t-i) - p_{t-i}] \\ &- v - \beta_2\phi\textstyle\sum_1^\infty (\alpha_1\phi)^{i-1} [E(p_{t-i}|t-i-1) - p_{t-i-1}]\}^2 \\ &+ \eta\{b_o + b_1 m_t + b_1\textstyle\sum_1^\infty (b_2)^i E(m_{t+i}) - b_3 k_t + \beta\Xi\tau\alpha_1(k_t - k_t^*) + x'' \\ &- b_o - b_1 m_{t-1} - b_1\textstyle\sum_1^\infty (b_2)^i E(m_{t+i-1}) + b_3 k_{t-1} - \beta\Xi\tau\alpha_1(k_{t-1} - k_{t-1}^*)\}2\Big\} \end{aligned}$$

Where $x''=\tau_1(1 - \alpha_1)(1 + \phi)x'$, and $v=\dfrac{\alpha_o+\beta_o+\alpha\beta_1}{1+\beta_1(1-\alpha_1)}$

Before period t, money supply is constant, and in period (t-1), authorities announce that they'll increase money supply by Δm ammount and increase once but for ever.

Set $E(p_t|t-1) - p_{t-1} \approx E(m_t|t-1) - m_{t-1} = \Delta m$

After the announcement, people predict that the money supply in the future will be $E(m_{t+i}|t)=m_{t-1} + \Delta m$.

Before and up to period (t-1), people only think that the money supply in future is a constant, and it is $E(m_{t+i-1}|t-1)=m_{t-1}$.

So the loss function becomes:

$$\begin{aligned} L_m = \tfrac{1}{2}\Big\{ &\sigma[\beta_2\phi(\Delta m)]^2 + \eta[b_1\Delta m + b_1\textstyle\sum(b_2)^i(\Delta m) - b_3(\Delta k) - \beta\beta_2\phi\Xi\tau\alpha_1(\Delta m - \Delta m^*) \\ &- x''']^2\Big\} \end{aligned}$$

$$=\frac{1}{2}[(\Delta m)^2 +(\Delta m -2\theta'\Delta m^* -x''')^2]$$

$$\sqrt{\sigma}=\frac{1}{\beta_2\phi} < 1, \quad \phi'=1-b_3\beta_2\phi \ , \ \rho= \beta\beta_2\phi\Xi\tau\alpha_1$$

$\phi'+\rho = 1-b_3\beta_2\phi + \beta\beta_2\phi\Xi\tau\alpha_1$, $\sqrt{\eta}= 1/(\phi'+\rho)$, $x'''=\sqrt{\eta}\ x''$,

and $0< \theta'=\frac{1}{2}\sqrt{\eta}\rho<\frac{1}{2}$

So, $L_m =\frac{1}{2} [(\Delta m)^2 +(\Delta m -2\theta'\Delta m^* -x''')^2]$

$L_m^* =\frac{1}{2} [(\Delta m^*)^2 +(\Delta m^* -2\theta'\Delta m -x''')^2]$

The loss functions under the anticipated increase in the rate of growth of money supply are:

$$L_\mu = \frac{1}{2}[\sigma(k_t -k_{t-1})^2 + \eta(q_t -q_{t-1})^2]$$

$$L_\mu^* = \frac{1}{2}[\sigma(k_t^* -k_{t-1}^*)^2 + \eta(q_t^* -q_{t-1}^*)^2]$$

then substitute the equations related with capital stock and relative price into the loss functions:

$$\begin{aligned} L_\mu =\frac{1}{2}\Big\{ & \sigma'\{\beta_2\phi E[(p_t|t-1)-p_{t-1}]\}^2 \\ & + \eta'\{b_1(m_t -m_{t-1})+b_1\textstyle\sum_1^\infty(b_2)^i[E(m_{t+i}|t)-E(m_{t+i-1}|t-1)] \\ & -b_3(k_t-k_{t-1}) -\beta\beta_2\phi\Xi\tau\alpha_1 [(k_t -k_{t-1})-(k_t^* -k_{t-1}^*)]\}\Big\} \end{aligned}$$

$[E(p_t|t-1) -p_{t-1}]$ is the anticipated inflation in period (t-1). Under the rational expectation , the expected rate of inflation will equal to the rate of growth of money supply , and in my model it is equall to μ.

After period t, the anticipated money supply is

$$\mu=\log\frac{M_t-M_{t-1}}{M_t} \approx \log M_t -\log M_{t-1} =m_t -m_{t-1}$$

Because at the beginning of period(t-1), people do not expect that money supply will increase ,so $E(m_{t+i-1}|t-1)=m_{t-1}$.

After the announcement of increase in money supply ,then in period t+i,the expected money stock will be $E(m_{t+i}|t)$ and the difference between the expected money stock in period (t+1)[it is expectation in period conditional on the information in period t] and the expected money stock in period (t-1) [it is the expectation in period (t-1) conditional on the information in period (t-1)] is:

$$E(m_{t+i}|t) - E(m_{t+i-1}|t-1) = m_t + i\mu - m_{t-1} = \mu + i\mu$$

So, $$L_\mu = \tfrac{1}{2}\{ \sigma'(B_2\phi\mu)^2 + \eta'[b_1\mu + b_1\textstyle\sum_1^\infty (b_2)^i(\mu+i\mu) - b_3(B_2\phi\mu) - BB_2\Xi\tau\alpha_1\phi(\mu-\mu^*) + x'']^2\}$$

$$= \tfrac{1}{2}\{ \sigma'(B_2\phi\mu)^2 + \eta'[b_1\mu + b_1\mu(1+b_2/b_1) - b_3(B_2\phi\mu) - BB_2\Xi\tau\alpha_1\phi(\mu-\mu^*) + x'']^2\}$$

$$= \tfrac{1}{2}\{ \sigma'(B_2\phi\mu)^2 + \eta'[(1+b_1)\mu - b_3(B_2\phi\mu) - BB_2\Xi\tau\alpha_1\phi(\mu-\mu^*) + x'']^2\}$$

$\phi'' = 1+b_1 + b_3B_2\phi$, $\rho = BB_2\Xi\tau\alpha_1\phi$,

$\rho = BB_2\Xi\tau\alpha_1\phi$, $\phi'' + \rho = 1+b_1 + b_3B_2\phi + BB_2\Xi\tau\alpha_1\phi$

$\sqrt{\sigma} = \sqrt{\sigma'} = 1/B_2\phi < 1$, $x = \sqrt{\eta'}\ x''$

$\sqrt{\eta'} = 1/(\phi'' + \rho)$, $0 < \theta = \tfrac{1}{2}\sqrt{\eta'}\rho < \tfrac{1}{2}$

So $$L_\mu = \tfrac{1}{2}[\mu^2 + (\mu - 2\theta\mu^* - x)^2]$$

$$L_{\mu^*} = \tfrac{1}{2}[\mu^{*2} + (\mu^* - 2\theta\mu - x)^2]$$

APPENDIX (D)

JUMPS IN CURRENT PRICE UNDER TWO DIFFERNT ANTICIPATED MONEY SUPPLIES

In section 2.4.2,

$$p_{t-1} - p_{t-2} = (1 - b_3\beta_2\phi)\Delta m \qquad (2.4.2.7a)$$

$$p_{t-1} - p_{t-2} = (1 + b_1 - b_3\beta_2\phi)\mu \qquad (2.4.2.7b)$$

At the two Nash equilibria, the money supplies are

$$\Delta m = \frac{x'''}{1 - \theta'} \quad \text{and} \quad \Delta\mu = \frac{x}{1 - \theta}$$

and substitute them into the following inequality in order to show that the difference in (2.4.2.7b) is larger than that in (2.4.2.7a).

$$|(1 + b_1 - b_3\beta_2\phi)\mu| > |(1 - b_3\beta_2\phi)\Delta m| \qquad (2.4.2.7c)$$

(2.4.2.7c) is equivelent to the following inequality,

$$\frac{1 + b_1 - b_3\beta_2\phi}{1 - b_3\beta_2\phi} > \left(\frac{1-\theta}{1-\theta'}\right)\left(\frac{x'''}{x}\right),$$

the right side of this inequality is

$$\left(\frac{1-\theta}{1-\theta'}\right)\left(\frac{x'''}{x}\right) = \frac{(1 - \frac{1}{2}\sqrt{\eta'}\rho)}{(1 - \frac{1}{2}\sqrt{\eta}\,\rho)} \cdot \frac{\sqrt{\eta}\,x''}{\sqrt{\eta'}\,x''}$$

$$= \frac{1-[\rho/2(\rho+\phi'')]}{1-[\rho/2(\rho+\phi')]} \cdot \frac{1/(\rho+\phi')}{1/(\rho+\phi'')} = \frac{\rho + 2\phi''}{\rho + \phi'}$$

so (2.4.2.7c) is equivalent to

$$\frac{1 + b_1 - b_3\beta_2\phi}{1 - b_3\beta_2\phi} > \frac{\rho + 2+2b_1 - 2b_3\beta_2\phi 2}{\rho + 2 - 2b_3\beta_2\phi}$$

then just because $b_1\rho > 0$, ao it is easy to show that

$$(1+b_1 - b_3\beta_2\phi)(\rho+2-2b_3\beta_2\phi) > (\rho+2+2b_1 - 2b_3\beta_2\phi 2)(1-b_3\beta_2\phi)$$

So the inequality is correct.

Reference:

1. **Blanchard, Olivier J (1978)**
" Backward and Forward Solutions for Economies with Rational Expectations " unpublished working paper, Harvard University

2. **Blanchard, Olivier J and Fischer, Stanley (1989)**
" Lectures on Macroeconomics "
Cambridge, Massachusetts: MIT press

3. **Boyer, Russell S and Hodrick, Robert J (1982)**
" The Dynamic Adjustment Path for Perfectly Foreseen Changes in Monetary Policy "
Journal of Monetary Economics Vol.9 pp.185-201

4. **Canzoneri, Matthew B and Henderson, Dale W (1991)**
" Monetary Policy in Interdependent Economies ---- A Game-Theoretic Approach "
Cambridge, Massachusetts: MIT press

5. **Dornbusch, Rudiger (1976)**
" Expectations and Exchange Rate Dynamics "
Journal of Political Economy Vol.84 No.6 pp.1161-1176

6. **Fischer, Stanley (1979)**
" Anticipations and the Nonneutrality of Money "
Journal of Political Economy Vol.87 No.2 pp.225-252

7. **Friedman, Benjamin M and Haln, Frank H (1990)**
" Handbook of Monetary Economics " Vol.1
Amsterdam: Elsevier Science Publishers B.V

8. **Friedman, James (1985)**
" Cooperative Equilibria in Finite Horizon Noncooperative Supergames "
Journal of Economic Theory Vol.35 pp.390-398

9. **Friedman, James (1990)**
" Game Theory with Applications to Economies "
New York: Oxford University Press

10. **Henderson, Dale W and Zhu, Ming (1990)**
" Uncertainty and the Choice of Instruments in A Two-Country Monetary Policy Game "
Open Economics Review Vol.1 pp.39-66

11. **Jones,R.W and Kenen, P.B (1981)**
" Handbook of International Economics " Vol.2
Amsterdam: Elsevier Science Publishers B.V

12. **McMillan, John (1986)**
" Game Theory in International Economics "
New York: Harwood Academic Publishers

13. **Rogoff, Kenneth (1987)**
" Reputational Constraints on Monetary Policy "
Carnegie-Rochester Conference Series on Public Policy (26) pp.141-182

14. **Sargent, T.J and Wallace, N (1973)**
" Rational Expectations and the Dynamics of Hyperinflation "
International Economic Review
Vol.14 No.2 pp.328-350

15. **Sargent, T.J and Wallace, N (1973)**
" The Stability of Models of Money and Growth with Perfect Foresight "
Econometrica Vol.41 No.6 pp.1043-1048

16. **Stokey, Nancy and Lucas, Robert E. Jr (1989)**
" Recursive Methods in Economic Dynamics "
Cambridge,Massachusetts: Harvard University Press

17. **Wilson, Charles A (1979)**
" Anticipated Shocks and Exchange Rate Dynamics "
Journal of Political Economy Vol.87 No.6 pp.639-647