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# MATH FICTIONS: ELEMENTARY MATHEMATICS EDUCATION AND THE MODERN PEDAGOGICAL PARADIGM

by

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A dissertation submitted to the Graduate Faculty in Sociology in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York

1997

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TABLE OF CONTENTS

CHAPTER ONE: CRISES AND REFORMS 1

1.1. Math Fictions 10

1.2. Education, Authority, and Culture 15

1.3. Child-Centered Classrooms 24

1.4. Progressive Methods 31

1.5. Math Constructions 36

1.6. The New New Math 40

1.7. Inherent Tensions 44

CHAPTER TWO: THEORETICAL TOOLS 52

2.1. The Field of Education 55

2.2. The Didactic Game 72

2.3. The Problem of Context 88

2.4. The Production of Reasoning Subjects 94

CHAPTER THREE: FICTIONALIZING ARITHMETIC 106

3.1. On Arithmetic Textbooks 110

3.2. Arithmetic Textbooks and the Curriculum Field 118

3.3. The Child vs. Arithmetic 127

3.4. Arithmetic for the Child 142

3.5. Arithmetic Textbooks in the Era of Efficiency 152

3.6. The Mystery of Problem Solving 164

3.7. Incidentalism and Other Discontents 183

continued

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3.8. Banality and Fatality in Arithmetic Teaching 191

3.9. The Death of the Arithmetic Textbook 196

CHAPTER FOUR: THE NEW MATH 208

4.1. Sputnik and its Aftermath 210

4.2. Anti-Didactical Inversions 215

4.3. The Fiction of Embodiments 227

4.4. The Revolution that Did Not Happen 236

CHAPTER FIVE: THE NEW NEW MATH 250

5.1. The Need for Reform 252

5.2. Risk, Excellence, and Educational Goals 263

5.3. Skills, Standards, Opportunities, and Assessments 280

5.4. The New Didactical Transposition 294

5.5. Towards New Didactic and Assessment Contracts 314

5.6. On Skills and Other Dilemmas 329

CHAPTER SIX: CLASSROOM PERFORMANCES 334

6.1. New York City, El Barrio, PS XYZ 340

6.2. Tricks 358

6.3. Clues and Hints 370

6.4. Thirty-Two Circles and Two Steps 380

6.5. Cubes and Coins 389

6.6. Good Reasons to Share Your Pizza 397

continued

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6.7. Circus Performances 402

CHAPTER SEVEN: THE PERFORMANCE OF ASSESSMENT 406

7.1. Ninety-Eight Dollar Bills 410

7.2. Make Up Your Own Problem 417

7.3. Funny Jobs 422

7.4. Short Cuts 435

7.5. Thirty-Three Ways of Un-Learning Division 449

7.6. A Pizza Party 460

CHAPTER EIGHT: MATH FICTIONS 469

APPENDIX 494

LIST OF ABBREVIATIONS 498

BIBLIOGRAPHY 499

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# CHAPTER ONE: CRISES AND REFORMS

> It is hoped that the near future will bring reforms in mathematical teaching in this country. We are in sad need of them. From nearly all our colleges and universities comes the complaint of the inefficient preparation on the part of the students applying for admission; from the high school comes the same doleful cry. Errors in mathematical instruction are committed at the very beginning, in the study of arithmetic (Cajori 1890).
>
> [C]hildren are not able to use the arithmetic they are taught in dealing with the simple personal quantitative problems of the adult world into which they are presently to enter; indeed they are not even sensitive to the quantitative aspects of their own daily lives. The poverty of our teaching has been exposed, too, in the arithmetical deficiencies of adults who have been subjected to eight or more years of school arithmetic (Brownell 1941).
>
> We are at risk of becoming a divided nation in which knowledge of mathematics supports a productive, technologically powerful elite while a dependent, semiliterate majority, disproportionately Hispanic and Black, find economic and political power beyond its reach. Unless corrected, innumeracy and illiteracy will drive America apart (Mathematical Sciences Education Board 1989).

In the United States, the teaching and learning of mathematics has suffered from chronic diseases at least since the turn of the century.[1] Observers have lamented the 'tragedy' of students' errors (Myers 1925); the sad fact that 'Johnny cannot add' (Kline 1973); the persistence of gender, ethnic, and race differences in mathematical achievement;[2] widespread 'math anxiety' among adolescent girls (Tobias 1980);[3] a generalized lack of 'problem solving

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[1]See, for example, Cajori 1890; Young 1900; Rice 1894-1904; Smith 1909, 1923; Judd 1928; Reeve 1936; Brueckner 1939.

[2]Cocking and Mestre 1988; Cuevas and Driscoll 1993; Secada, Fennema, and Adajian 1995.

[3]'Math anxiety' is "a slight discomfort with mathematics ... [which] can develop into a full-fledged syndrome of anxiety and avoidance" (Tobias 1980, p.24). Similarly, 'mathophobia' refers to an irrational fear of mathematics whose main symptom is a fixation upon isolated rules, in contrast to an understanding of mathematics as a collection of interrelated concepts (Resek and Rupley 1980).

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skills' (Dossey *et al.* 1993)[4] embarrassingly low scores obtained by American students in international mathematics tests (McKnight *et al.* 1987; National Education Goals Panel 1994);[5] 'underachievement' and 'underrepresentation' in mathematics and sciences of women and minorities (Oakes and RAND Corporation 1990, Oaxaca and Reynolds 1990); shortages of US-born 'hard' scientists;[6] and 'innumeracy' among the general population, with all the social, political, and economic consequences that this disgrace supposedly entails (Paulos 1988, Steen 1990).[7]

To confront these crises, a series of reforms and counterreforms have periodically been tried, each attempting to solve the problems caused by its predecessor. Yet, judging from the cyclical nature of the crises, none of

---

[4]Studies conducted, since the early 1970s, by the National Assessment of Educational Progress indicate that students lack the ability to solve 'non-routine,' 'multi-step' word problems (Carpenter *et al.* 1975, 1981; Lindquist *et al.* 1983; Kouba *et al.* 1988; Dossey *et al.* 1988).

[5]In the Second International Assessment of Educational Progress (IAEP 1991), American 13 year-olds were outperformed by students from Korea, Switzerland, and Taiwan in all areas of mathematics, and by students from Hungary, Korea, and Taiwan in three out of four areas of science (The National Education Goals Panel 1994).

[6]In 1989, only 45 percent of the 861 doctorates granted in mathematics went to American students.

[7]"Innumeracy, an inability to deal comfortably with the fundamental notions of number and chance, plagues far too many otherwise knowledgeable citizens" (Paulos 1988; p. 3). "...ignorance of basic quantitative tools is endemic in American society and is approaching epidemic levels among many subcultures of the American mosaic." (Steen 1990; p. 211).

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these interventions seems to have dealt satisfactorily with the question of how to teach mathematics so that "everybody can count."[8] And, in the meantime, for an overwhelming number of children, mathematics continues to be that part of their school experience they hate and fear the most, and the discipline in which they are most likely to fail.[9]

In addition to these seemingly perennial problems, today the perception is widespread that the United States is at risk of becoming divided along economic and racial lines by knowledge of mathematics. In response to this situation, the field is undergoing a radical transformation. This nationwide 'mathematical empowerment' movement, which is often referred to as the 'new new math,' consists of a series of more or less concerted interventions in the curriculum standards for school mathematics, the structure of math lessons, the technologies for math instruction, and the instruments for assessing students' mathematical performances.

But, what happened to the 'new math'? In 1957, the flight of the Russian satellite Sputnik I stimulated American educators to a stronger emphasis on sciences and mathematics. An unprecedented stress was put on the need for

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[8]*Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (Mathematical Sciences Education Board 1989); "Perhaps nobody will count" (Effros, *Education Week*, 4/12/89); and "Will everybody count?" (Steen 1990).

[9]"The problem is the fact that only a small percentage of our students survive their school mathematics experience" (Usiskin 1989).

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students to discover the underlying structures of these disciplines. In mathematics, new language and content were introduced, including set theory, non-metric geometry, and non-base ten number systems. *Arithmetic* textbooks were replaced by *mathematics* textbooks written in the language of sets. And classrooms were stacked with Cuisinaire rods, Dienes blocks, and other parafernalia.

Yet, researchers soon found out that the new math revolution did not really happen. The reformers' recommendations hardly entered the classroom doors.[10] When they did, teachers' implementation of the proposed innovations banalized the original intentions.[11] And, once again, the 'grammar of schooling' remained intact (Tyack and Cuban 1995). Other critics underline the negative effects of the new math, as demonstrated in the widespread symptoms of sterilization in students' minds due to 'premature formalization' (Kline 1973), and the decrease in national test scores (NACOME 1975, Carpenter *et al.* 1975). In the light of reports on the decline of students' mathematical proficiency, and with a concern for standards of minimal competence, the 1970s witnessed a return to basic skills and 'applied mathematics' courses.

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[10]Price *et al.* 1977; Suydam and Osborne 1977; Weiss 1978; Stake and Easley 1978.

[11]Baruk 1973; Skemp 1976; Brousseau 1986; Resnick and Osmanson 1987.

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But it soon became apparent that there was no 'good old math' to recycle. In 1980, the National Council of Teachers of Mathematics (NCTM) recommended that problem solving be the focus of school mathematics for the years to come (NCTM 1980). And, by the end of the 1980s, experts in the field of math education began to insist that, rather than transmitting ready-made concepts and procedures, the teacher's role was to create the conditions for students to construct 'their own' mathematical knowledge. For this to happen, it was deemed necessary that teachers acquire expertise not only in mathematics and pedagogy, but also in the didactics of mathematics. 'Didactics of mathematics,' a term which is common in the German and French-speaking world, refers to the discipline dedicated to studying the processes of teaching and learning mathematics, as well as to the development of models and materials for improving these processes.[12]

However, in the United States, developing a cadre of well-trained teachers is generally seen as a technically difficult, politically unattractive, and financially demanding enterprise, whose effects may only materialize in the long run (Cohen 1995). An alternative point of attack--one which has been proposed as a magic bullet for educational troubles at least since the times of the 'teaching machine' (Skinner 1958)--is the 'substitution' of

[12]Brousseau 1986, Artigue 1989, Balacheff 1990, Biehler *et al.* 1994.

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deficient teachers, dry textbooks, and pre-industrial manipulatives, with new, upgraded cognitive technologies.[13]

Also, since the early 1980s, experts in the field began to work towards new modalities of evaluation and assessment of school mathematics. Given that 'testing drives instruction'--or that 'what you test is what you get'--a radical renewal of the examination system is seen by many math educators as the *sine qua non* condition for change.[14]

In the light of the failure of past reforms, a growing consensus is emerging within the math education community as well as in the circles of private and public funding agencies, that in order to be effective and long lasting, change must be 'systemic.' Thus, the current new new math reform aims at operating simultaneously on four fronts, i.e. demarcating a new text of knowledge, modifying the craft of teaching, upgrading the technologies for learning, and transforming the examination machine.

Will it work this time? Let us first point to a grammatical subtlety: it makes a difference whether one uses the term "'new new' math" or "new 'new math.'" "'New new' math" emphasizes the idea of newness and, after all, who can be against renewal? Recall Hannah Arendt's commentary, in "The Crisis in Education" (1954), on American society and its enthusiasm for the new. Instead, the variation "new 'new

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[13]Papert 1980; Pea 1987; Hoyles and Sutherland 1989.

[14]Resnick *et al.* 1992; Romberg 1995; NCTM 1993, 1995.

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math'" links the present reform to its 1960s predecesor and, in doing so, anticipates its failure.[15] The expression 'new new math' carries with it these two connotations. Rather than choosing between the two positions defining this battlefield, I suggest that the failure of each reform highlights the illusory nature of the belief in the autonomy of the system of mathematics teaching vis à vis its surrounding environment.

This lack of autonomy is particularly striking in inner city public schools attended by poor and minority students. In these classrooms, teaching and learning is subject to severe constraints including high teacher-student ratios; tracking by ability level; limited access to material resources; and the pressure from state and citywide testing, school-wide unit or chapter tests, and district-wide pacing calendars. To this list of 'external' constraints, two 'internal' constraints must be added: the limitations of classroom time and the deficient quality of teachers' knowledge of mathematics and its didactics. Not only do these constraints act as limiting factors vis à vis the promises of each reform but, to the extent that these are underestimated, they result in an array of unintended,

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[15]For example, the term 'new 'new math' has been chosen by 'some angry parents' from the most affluent districts in Palo Alto, California, in reference to "the curriculum-reform effort of the 1960s that is widely viewed by the public as a failed experiment" (Peter West, "The New 'New Math'?", *Education Week*, 5/10/95). On the use of the expression 'new new math,' see Hayden and Rudolph 1984; Herold 1992.

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negative effects.

Ever since the detection of the first signs of failure in mathematics among certain segments of the school population--e.g. Black and Hispanic children, girls, students of lower socioeconomic status, children attending rural schools, non-English speakers, and students whose parents are illiterate and/or have recently immigrated from underdeveloped countries--study after study has provided a new explanation and yet another solution to the problem.

Consider, as a hypothetical scenario, the diagnostic statement: "Maria cannot do fractions." In the first two decades of this century, it was argued that Maria was not motivated, or did not have enough opportunities for drill and practice. At that time, it was also proposed that, before considering Maria's difficulties with fractions, survey data needed to be gathered on exactly which kinds of fractions were most commonly used in her hometown.

Around the 1930s and 1940s, many critics lamented that the method of teaching in Maria's classroom did not emphasize the meaning of fractions, and the word problems she was given for the application of fractions were far from interesting and engaging. In the 1960s and 1970s, it was noted that Maria needed to be placed in an environment equiped with the appropriate concrete materials for the exploration and discovery of the rules governing fractions. Throughout these decades, an accusatory finger would be

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placed on the incompetence of Maria's teacher, the poverty of the textbooks and workbooks in use in her classroom, or both. More recently, the thrust has been put in making available at Maria's school the latest advances in computer software for the learning of fractions.

Today, it is recommended that for Maria to learn fractions, her classroom ought to operate like a community of discourse where, together with her classmates and teacher, she will engage in the activity of arguing about and presenting different solutions for worthwhile problems involving fractions. In the course of these inquiries, Maria and her classmates will construct the concept of fraction and understand its multiple uses inside as well as outside of the math classroom. Considering that, up to the present, successive changes in the modality of pedagogical transmission and periodic infusions of technological cornucopia have proven insufficient to help Maria learn her fractions, the question remains 'Will it work this time?'

Maria's difficulties with fractions have also been attributed to the fact that she is a girl, lives in poverty, has a Hispanic background, speaks Spanish at home, and her mother never went to school. In the last decades, a growing attention has been paid to the difficulties entailed by learning mathematics in a second language.[16] There has also

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[16]Austin and Howson 1979; Dawe 1983; Cummins 1979; Cuevas 1984; Cocking and Mestre 1988.

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been an increasing awareness of the masculinist character of most mathematics texts (Burton 1986, Walkerdine 1989), the subtle mechanisms whereby girls are excluded from classroom talk (Gore and Roumagoux 1983), and the even subtler logic whereby girls' mathematical performances are systematically interpreted as 'rule-following,' rather than as 'real understanding' (Walkerdine 1988). Yet, the various efforts at making math texts more girl-friendly and culturally sensitive, creating opportunities for Hispanic mothers to become more involved in their children's math lessons, and working towards a pedagogy in which all voices are included, may not be sufficient to help Maria learn her fractions.[17]

So, what really solves this problem? Or, more generally, can we ever answer satisfactorily the question as to why so many students fail in school mathematics? Although this thesis may be read as an attempt to answer this question, my intent is rather to contribute a redefinition of the problematic at stake.

## 1.1 Math Fictions

As a point of departure, I consider two assertions.

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[17]Of course, the case of Maria and the learning of fractions is presented here only as a hypothetical example. Although any other topic of elementary school math could have been chosen, it must be noted that, together with long division and percentage, fractions is one of the topics which has proven most difficult for teachers to teach and, consequently, for students to learn. For the treatment of the area of fractions in the math education literature, see Streefland 1991, 1991a; Ball 1993 (representational contexts). For a review of research on initial fraction concepts, see Pitkethly and Hunting 1996.

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First, although it continues to be a topic of debate, the dominant conception of mathematics is still that of a formal discipline, an abstract science, a de-contextualized knowledge, a subjectless and noiseless language, a monological discourse, and a value-free field of inquiry.[18] And second, in most societies, cultures, and school systems in the West, mathematics occupies a central place as a status symbol, a highly valued form of academic and cultural capital, and an almost inescapable educational filter. It is my contention that, to the extent that these two conditions are maintained, an effective teaching of mathematics will be in tension with the three tenets which constitute the modern pedagogical paradigm: child-centeredness, progressivism, and constructivism.

Within the boundaries of this threefold paradigm, the child has been placed at the center of the classroom, turning the teacher into a guide or a provider of rich learning environments (child-centeredness). A relationship of continuity has been posited between school and society, whereby the former is to mirror the latter (progressivism). And, finally, it has been proposed that for authentic and lasting learning to occur, students are to become active

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[18]For critical discussions on the pedagogical and didactical obstacles which result from the peculiar epistemological status of mathematics, see Piaget 1941, 1970, 1972; Bruner 1960, 1966; Dienes 1966, 1971; Freudenthal 1973, 1983; Baruk 1973, 1985; Chevallard 1985, 1990; Brousseau 1986, 1988; Burton 1986; Mellin-Olsen 1987; Walkerdine 1988; Mason 1989; Chaiklin and Lave 1993; Skovmose 1993; Noss 1994.

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constructors of knowledge, rather than passive receptors of it (constructivism). *When pushed to the limit*--or, in their most deformed versions--child-centeredness, progressivism, and constructivism result in a peculiar set of prescriptions to the teacher: omit whatever will not engage the children (reductionism); teach something only when it proves necessary for the accomplishment of the project or activity at hand (incidentalism); and restrain from explicit teaching, to allow students to construct knowledge by themselves (anti-didacticism).

The modern pedagogical paradigm rests on three highly problematic assumptions: that the power/knowledge relationship between teachers and students may be made symmetrical; that classroom practices are not reproductive--i.e. passing on knowledge from one generation to the next, but productive--i.e. creating the conditions for students to construct 'their own' knowledge; and that the child as constituted by the pedagogical gaze--i.e. 'the learning subject' or 'the student'--may be equated to actual children.

Instead, I will assume throughout this work that, first, the relationship between teachers and students is always asymmetrical--i.e. the teacher is the one supposed to know, and the one who anticipates what the students will

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know;[19] second, teaching mathematics consists in introducing students to a pre-constructed body of knowledge, even when this process takes the form of a guided reinvention;[20] and, third, a distinction ought to be kept between the child as the subject of pedagogical discourses and real children.[21]

The tension between the teaching of mathematics and the modern pedagogical paradigm translates, for teachers and students, into real and difficult dilemmas (Lampert 1990, Ball 1993). Yet, the normal functioning of the system of mathematics teaching requires that this tension be misrecognized.[22] In other words, although an effective teaching of mathematics may require the suspension of one or another of the main assumptions underlying this paradigm, ways will be found to maintain the coherence of the overall ideological framework. What results is a series of 'math fictions,' that is, a set of discursive practices which help maintain the illusion that mathematics can, in fact, be taught and learned within the constraints of the dominant paradigm.

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[19]Dewey 1938; Arendt 1954; Bernstein 1975, 1990; Bourdieu and Passeron 1977; Foucault 1979; Chevallard 1985; Brousseau 1986; Delpit 1986, 1995; Walkerdine 1988.

[20]Freudenthal 1973, 1983; Vygotsky 1978; Brousseau 1986; Lampert 1990; Ball 1993.

[21]Walkerdine 1984, 1988; Jenks 1995.

[22]'Misrecognition' (*meconnaisance*) is "the process whereby power relations are perceived not for what they objectively are but in a form which renders them legitimate in the eyes of the beholder" (Bourdieu and Passeron 1977, translator's note).

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These fictions include:

1) the belief in the autonomy of the 'didactic system' of math teaching vis à vis its surrounding environment;

2) the trust in the possibility of making school mathematics continuous with out-of-school mathematics--e.g. with the mathematics of the mathematicians, 'street' or 'everyday' mathematics, 'ethnomathematics,' or 'multicultural' mathematics; 3) the hope that the student/teacher relationship may be made symmetrical--by implicitly underplaying the element which accounts for the difference between: i.e. knowledge of subject matter;

4) the belief in the transparency of written texts, concrete materials, visual imagery, and other forms of representation;

5) the faith in the possibility of turning math classrooms into 'microcosms of mathematical culture;'

6) the hope of accessing the mind of the student while in operation thus obtaining authentic evidence of their thinking process and level of understanding; and

7) the trust in the possibility of making room for the entrance of the child's subjectivity into the process of learning mathematics.

It must be noted that these fictions did not develop in unison. Their emergence is shaped by the pedagogical discourse which dominates the field of math education in each historical period. Moreover, each reform may be

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characterized by the emphasis it places on one or another of these fictions. For example, many of the projects at work during the new math era relied on the fiction of the transparency of manipulatives,[23] that is, on the hope that if children were made to play with concrete materials especially designed for that purpose, sooner or later they would 'discover' the mathematical structures embodied therein (Dienes 1963, 1966). Today, some advocates of the new new math reform sustain the fiction of a classroom modelled as a microcosm of mathematical culture, that is, the expectation that children will acquire 'mathematical power' as they participate, with their teacher and classmates, in a 'community of discourse' in the context of which alternative conjectures and problem-solving strategies are negotiated.[24]

## 1.2 Education, Authority, and Culture

Concerns about the sorry state of education have been expressed frequently in the form "Why Johnny can't ...?" (Flesch 1955, 1981; Kline 1973). As suggested by Arendt, much more is involved in this complaint, or in the oft-asked question "why the scholastic standards of the average

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[23]The use of the term 'manipulatives' denotes roughly the same thing as 'physical' or 'practical' apparatuses. It refers to Dienes multibase arithmetic blocks, Cuisinaire rods, and other concrete materials used as an aid in the teaching and learning of mathematics (Dienes 1963, 1966; Resnick and Ford 1981; Pimm 1995).

[24]NCTM 1989, 1991; Richards 1991; Cobb, Yackel, and Wood 1992; Borasi 1992; Cobb and Bauersfeld 1995; Yackel and Cobb 1996.

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American school lag so very far behind the average standards in actually all the countries of Europe" (1954, p. 179). As she notes, although educational crises are generally worldwide phenomena, in the United States these crises tend to acquire particular severity. Historically, American schools have been made the repository of the unfulfilled promises and deepest fears of the larger social project, e.g. realizing the American Dream, homogenizing the melting pot, fighting the Red Menace, solving economic ills, or 'empowering' us all. No wonder educational crises are perceived as so severe.

Arendt argues that crises in education are linked, on the one hand, to crises in culture--that is, in the selection of a tradition, the definition of our relationship to the past, and the decision as to which (or whose) knowledge is worth transmitting--and, on the other hand, to crises in authority--more specifically, in the modalities of relationship between grown-ups and children. In her view, at the root of the crisis in education lies our collective denial of the truth that, as a mediating practice between the old and the new, education must operate with

> a concept of authority and an attitude toward the past which are appropriate to it, but have no general validity and must not claim a general validity in the world of grown-ups (ibid. p. 195).

In line with her claim that education belongs to the 'pre-political' sphere, Arendt suggests that conservatism, understood as conservation, is inherent to the task of

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educating. As she puts it, "for the sake of what is new and revolutionary in every child,

> education must be conservative; it must preserve this newness and introduce it as a new thing into an old world, which, however revolutionary its actions may be, is always, from the standpoint of the next generation, superannuated and close to destruction (ibid. p. 192-3).

Yet, *contra* Arendt, the common sense in education is that we must be radical. If in no other country but in the United States "have the education problems of a mass society become so acute," argues Arendt, "nowhere else have the most modern theories in the realm of pedagogy been so uncritically and slavishly accepted" (p. 179). These theories recommend that we displace the authority of the teacher and the textbook and bring the child to the center of the pedagogical stage; update curricula in response to present-day social needs and integrate all knowledge "to eradicate the fatal disconnection of subjects which kills the vitality of our modern curriculum" (Whitehead 1929); upgrade teaching materials by taking advantage of the developments in computer technology; and prepare our students in the art of living, rather than instruct them into what the world is like.

In matters of learning, the new pedagogical credo includes the proposition that we can know and understand only what we have done ourselves--"I hear and I forget. I

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see and I remember. I do and I understand."[25] As Arendt notes, *when brought to the extreme*, this substitution of doing for learning leads to a peculiar demand: that the less acquainted are teachers with their subject matter, the lesser the risk of their transmitting dead knowledge or 'inert ideas'[26] to the students. Due to the influence of pragmatism and of modern educational theories, pedagogy has evolved into a science of teaching in general, "wholly emancipated from the actual material to be taught" (Arendt, ibid., p. 182).

It is as if, in order to make the teacher/student relationship symmetrical, knowledge--that is, the term which accounts for the power difference between the other two terms-- has to be underestimated. Hence, in many teacher education programs across the United States, prospective teachers take courses in mathematics and pedagogy, but are rarely given the *didactical tools* they need to play an effective mediating role between children's 'pre-mathematical' constructions (Freudenthal 1971) and the 'pre-constructed' field of mathematics (Chevallard 1985). In the same stroke, with the reduction of the didactical

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[25]This Chinese proverb was adopted as part of the title of a teacher's guide to a new math's project in Great Britain (*Nuffield Mathematics*). A discussion of the assumptions underlining this project, in the context of a critique of child-centeredness may be found in Walkerdine (1984).

[26]"In training a child to activity of thought, above all things we must beware of what I will call 'inert ideas'--that is to say, ideas that are merely received into the mind without being utilised, or tested, or thrown into flesh combinations" (Whitehead 1929, pp.1-2).

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relationship to a dyad, the most legitimate source of the teacher's authority--that is, the one based on her knowing and knowing what the students are supposed to know--may no longer be effective.

In *Pedagogy of the Oppressed* (1970), Brazilian educator Paulo Freire strongly criticizes this very conception of teaching as the transmission of knowledge from those who know to those who do not know. In this 'banking' approach to teaching and learning, education

> becomes an act of depositing, in which the students are the depositories and the teacher is the depositor. Instead of communicating, the teacher issues communiques and makes deposits which the students patiently receive, memorize, and repeat (p. 58).

To this pedagogical model, which is typical of most school systems in capitalist societies, Freire opposes an alternative, 'problem-posing' approach. In this 'dialogic' practice which, as Freire advocates, may be used in adult education projects as an instrument for liberation, the teacher is no longer as a conveyor of knowledge, but a mediator or facilitator of students' learning.[27]

Freire's work has been central to the development of 'critical' or 'liberation' pedagogy not only in Brazil but also in the rest of the Third World (Freire and Betto 1985;

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[27]"Through dialogue, the teacher-of-the-students and the students-of-the-teacher cease to exist and a new term emerges: teacher-student with students-teachers"... "Here, no one teaches another, nor is anyone self-taught. Men teach each other, mediated by the world, by the cognizable objects which in banking education are 'owned' by the teacher" (ibid. p. 67).

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Horton and Freire 1990). In the United States, it influenced a new generation of progressive educators, i.e. 'radical' pedagogues (Giroux 1979, 1981, 1983; McLaren 1986; Gore 1993; McLaren and Leonard 1993). Moreover, Freire's writings contributed to the emergence of a critical perspective in mathematics education. Within this framework, researchers worldwide consider questions such as how to teach mathematics so as to 'empower' students to decode the quantitative reifications of the social and economic world (Frankenstein 1983); how 'monological' math classrooms can transformed into 'dialogical' communities (Skovmose 1993, 1994; Noddings 1993); what may be done to connect the culture of mathematics with the culture of the students supposed to learn it (D'Ambrosio 1985, Gerdes 1985, Mellin-Olsen 1987, Bishop 1988, Joseph 1991, Zazlavsky 1993); and, given that in capitalist educational systems, mathematics education is used as a vehicle for introducing students to instrumental reason and techno-scientific knowledge, how a theory of mathematics learning may nevertheless be articulated with a critical theory of society (Skovmose 1985, 1988; Keitel 1987).[28]

Yet, critics have pointed out that, when implemented in specific segments of the *elementary school* system, some of

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[28]"Can we afford to present mathematics to the student only as an instrument of rational understanding, of elucidation and objectivity, and withhold from him or her the fact that it is also used as an instrument of power?" (Keitel 1987).

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the prescriptions of radical pedagogy may produce unintended negative effects (Delpit 1986, 1995; Ellsworth 1989; Walkerdine 1990; Cazden 1995). For example, while acknowledging the obvious shortcomings of an instruction which aims solely at the development of basic skills, Lisa Delpit worries that progressive and child-centered holistic models such as the 'whole language' approach to reading, the 'process' method for teaching writing, or 'mathematics as problem solving,' may *de facto* result in the withdrawal from explicit, knowledge-based teaching. In her view, the implicit prescription not to teach may be particularly harmful to Black students and students from other minority groups, for whom school is the only opportunity to acquire the tools of the dominant culture.

Delpit also argues that when teachers do not make explicit the rules of the 'culture of power' to those children who do not participate in it, "what it feels like to people who are old enough to judge is that secrets are being kept, that time is being wasted, that the teacher is abdicating his or her duty to teach" (1995, p. 31). When students understand authority in the same way as their teacher,

> it is mutually agreed upon that they are to obey her no matter how indirect, soft-spoken, or unassuming she may be. Her indirectness and soft-spokenness may indeed be... an attempt to reduce the implication of overt power in order to establish a more egalitarian and nonauthoritarian classroom atmosphere (p.36).

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But, if they "operate under another notion of authority.... then there is trouble" (ibid.).

Linking the crisis in education with the crisis in culture and authority, Arendt anticipates not only Delpit's critique of progressivism, but also many of the central debates in the field of mathematics education. I propose that at the core of these debates is the misrecognized tension between an effective teaching of elementary school mathematics and a full-fledged realization of the modern pedagogical paradigm. As I described it above, this paradigm relies on three discourses: childcenteredness both in the selection and organization of subject-matter and in the mode of inculcation of knowledge; progressivism in the conception of the aims of education as well as in the relationship between school and society; and constructivism in the underlying theory of cognition and learning.

Historically, the emergence of this three-fold paradigm is linked to an extensive literature, whose roots may be found in the works of G. Stanley Hall; John Dewey and William Kilpatrick; and Jean Piaget.[29] Yet, this paradigm is far from internally consistent. In fact, making these discourses compatible has been a central preoccupation of education experts and school reformers at least since the turn of the century. For example, George Counts, in his 1932

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[29]Hall 1883, 1904; Dewey 1902, 1910, 1916, 1938; Kilpatrick 1918, 1936; Piaget 1952, 1954, 1972.

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pamphlet addressed to teachers entitled *Dare the Schools Build a New Social Order?,* bluntly stated: "If Progressive education is to be genuinely progressive... [it] cannot place its trust on a child-centered school."[30] Holding on to a progressive educational agenda, early critics of child-centeredness expressed their concerns for what they perceived as an excessively individualist definition of educational aims; a sentimentalist view of children, linked to a mythical belief in their innate development, native interests, and personal needs;[31] and a lack of concern for broader social change.[32]

During the 1960s' new math reform, some progressive educators argued that, for mathematics to be 'honestly' taught, it could not rely exclusively on a progressivist approach to curriculum and a child-centered view of pedagogical practice (Bruner 1960, 1962, 1966). The teaching of elementary school math as a structured discipline was seen as requiring the suspension of the principles of

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[30]Similar views have been voiced by Boyd Bode (1938) and John Dewey (1933, 1938).

[31]William Kilpatrick, one of the main defenders of child-centeredness in education and the inventor of the 'project method,' is a leading representative of what Cremin (1961) calls 'sentimentalism' in education, that is, the placing of the child's motivation as the crucial element in education.

[32]The limits of progressivism--its tendecy towards individualism, voluntarism, and reformism, as well as its complicity with the 'efficiency' and the 'testing' movements--have been discussed by Katz (1968, 1971, 1987), Feinberg (1975), Spring (1976, 1988), Gonzalez (1982), and Westbrook (1991). See, also, the work of those critics who discuss the mutations suffered by the ideas of Dewey, in the hands of those who attempted to turn his educational project into classroom realities (Kliebard 1987, Zilversmit 1993).

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utility and social significance, as well as a moratorium on an exclusive concern with the interests of the child.

In recent years, some math education researchers are considering the possibility that, for a constructivist pedagogy to be genuinely constructivist, it cannot place its thrust on a curriculum designed exclusively along progressivist lines. For example, word or 'story' problems ought to be not only engaging and socially significant, but they must also leave enough 'mathematical residue' (Hiebert *et al.* 1996). Nor can an effective pedagogy rely on a fully child-centered classroom. For example, some 'telling' may be necessary if the aim is to lead students to the right mathematical constructions.[33]

## 1.3 Child-Centered Classrooms

Since the turn of the century, the move from teacher- and curriculum-centered toward child-centered practices has been a constant trend in the United States and abroad. As discussed by Larry Cuban (1993), two versions of student-centered instruction have emerged. There was, on the one hand, a practical, 'a-theoretical' version which operated in rural, one-room schools, due to the precarious conditions existing in those settings. This pedagogical practice was characterized by lack of materials, isolation

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[33]Davis, Maher, and Noddings 1990; Lampert 1990, 1990a; Ball 1993; Cobb and Bauersfeld 1995.

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from inspection, and group spirit. Children helped each other, and they learned content which was drawn from the life of the surrounding community. On the other hand, there was a more 'theoretical' version, strongly inspired by the work of progressivist educators John Dewey and William Kilpatrick, which was tried in small, mostly private schools.[34]

Education historians identify the move towards child-centeredness with the challenge to the humanist and mental discipline approach to curriculum-making by G. Stanley Hall and his followers in the 'child study' movement (Kliebard 1987). From a more critical stance, child-centeredness has been described as a mode of regulation belonging to the series of transformations of the mechanisms through which, since the 19th century, modern industrialized nations exercise control over their populations (Foucault 1979, Donzelot 1979). British psychologist and cultural critic Valerie Walkerdine (1984) argues that the dream of a pedagogy which would set children free, was already present in the early progressive movement of the 1920s and 1930s, and was central to the radical pedagogy approaches which emerged in the 1970s.

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[34]Examples of these experimental schools were those conducted in the 1860s, by Sheldon in the Oswego State Normal and Training School, in New York (Cremin 1961, Kliebard 1987); by Parker--"the father of Progressivism"--with his pioneering work as a Superintendent of School in Quincy, Massachusetts, since the 1870s (Patridge 1886, Cremin 1961), and by Dewey in his Laboratory School, between 1896 and 1903 (Dewey 1916, 1936, Cremin 1961, Kliebard 1987, Westbrook 1991).

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According to Basil Bernstein (1975), these transformations have taken the form of a shift from a 'visible' to an 'invisible' pedagogy. The basis of visible pedagogy is a teacher- and curriculum-centered model in which power relations are hierarchical and explicit and classroom discipline is harsh. In contrast, invisible pedagogy is based on a child-centered model in which power relations are less obviously hierarchical and classroom discipline is apparently more relaxed.

What does the child-centered classroom look like? The teacher has abandoned the center of the stage; textbooks are no longer followed *a la lettre*; the furniture is movable--i.e. no more desks bolted to the floor--and the use of time and space is more flexible, to accommodate the unpredictable and spontaneous expression of students' interests. In the child-centered classroom, learning is accomplished through activity and play. Work is pathologized. Anything that resembles hard, routine work is interpreted as rote-learning and is thus discouraged (Walkerdine 1984, 1988).

Just like the substitution of doing for learning, substituting play for work is a highly problematic move. As Arendt put it,

> The very thing that should prepare the child for the world of adults, the gradually acquired habit of work and of not-playing, is done away in favor of the autonomy of the world of childhood (1954, p. 183).

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Similarly, in his prison notebooks, written between 1929 and 1935, Italian communist leader, philosopher, and cultural critic Antonio Gramsci notes that in education

> one is dealing with children in whom one has to inculcate certain habits of diligence, precision, poise (even physical poise), ability to concentrate on specific subjects, which cannot be acquired without the mechanical repetition of disciplined and methodical acts (1971, p. 37).[35]

As for the curriculum, Bernstein (1971, 1975) argues that invisible pedagogy blurs the distinction between different fields of knowledge, as well as between the inside and the outside of school (i.e. 'weak classification'). All school disciplines are interconnected, and there is an emphasis on 'real life' applications. In the United States, by the first decades of this century, advocates of child-centeredness had established an alliance with supporters of another trend in curriculum-making: social efficientism (Kliebard 1987). For the teaching of arithmetic in the early grades, this happy marriage was epithomized by 'playing store,' an activity seen as the best suited both to children's immediate interests, and to their needs as future shopkeepers and consumers. Whether the guiding principle was 'child interest' or 'social utility,' both positions

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[35]Gramsci's views on education have puzzled educators for many decades. Many explanations have been tried for the perceived inconsistency between Gramsci's 'radical' views on politics and culture and his 'conservative' views on education. The latter are expressed, for example, in his stressing of the importance of discipline, the need to acquaint the working classes with knowledge of dominant culture (including Greek, Latin, logic, and algebra); and the limiting effects of a curriculum centered on students' immediate concerns, needs, and interests (Gramsci 1971).

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supported the reductionist policies in curriculum-making which pervaded the 'cult of efficiency' era of the 1910s to the 1930s (Callahan 1962).[36]

Aside from changes in the spacio-temporal organization of classroom activities and in the classification of the knowledge-to-be-taught, the child-centered model radically redefines the examination machine. In invisible pedagogies, students are encouraged to make their personal experiences public, and this creates the opportunity for constant monitoring. Bernstein (1971, 1975) describes this as a shift towards a 'weak framing' of the student/teacher relationship. Within the present reform in US math education, while students' subjectivities are constituted by means of confessional practices such as the keeping of daily logs in 'math journals' (Borasi and Rose 1989), their acquisition of knowledge is regulated through the enforcement of rational argument in the form of written and oral mathematical communications.[37]

Interestingly enough, education historian Arthur Zilversmit (1993) underlines the constancy in the nationwide sale statistics for movable classroom furniture for the period from 1930 to 1960 (particularly in the South). As he sees it, this may be taken as an indicator of the low

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[36]The 'elimination of non-essentials' resulted primarily from the four reports published by the National Education Association Committee on the Economy of Time in Education (NSSE 1915, 1917, 1918, 1919).

[37]NCTM 1993, 1995; Cobb and Bauersfeld 1995; Yackel and Cobb 1996.

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incidence of child-centeredness in real classrooms. Along the same lines, in his study of classroom practices during the period from 1890 to 1980, Cuban (1993) highlights the similarities between how schoolteachers taught in the past and how they teach today.

Social researchers have noted the uneven impact of child-centeredness in present-day classrooms. For example, in a study of more than twenty school districts in Chicago (1981), Susan Stodolsky and her colleagues found that teacher-centered classrooms--i.e. recitation, seat work, and small group instruction--were more frequent in the 'lower' than in the 'upper socioeconomic schools.' In general, teachers working in suburban, middle-class classrooms attended by white, English-speaking students seem more likely to convert to child-centeredness than their counterparts working in inner city classrooms which serve a population of non-white, lower class, and often non-English speaking students.

Critics have suggested that this phenomenon should not be seen as a failure on the teachers' part to accommodate their practices to the prescriptions of modern pedagogy. Instead, this resistance to reform ought to be interpreted as a response to the constraints affecting the daily pedagogical work in these settings--e.g. high student/teacher ratio; lack of space, time, and material resources; and the pressure of standardized testing (Cuban

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1993, Tyack and Cuban 1995).

Moreover, other educators have proposed that if the underlying grammar of these lessons remains intact--or if, at best, child-centeredness appears in these settings in a deformed or hybridized version--this ought to be understood as teachers' conscious accommodation to their students' specific needs, as well as to their class-, ethnic-, or culturally-specific expectations about power, knowledge, language, and authority (Delpit 1986, 1995; Cazden 1995).[38]

Stodolsky and her colleagues (1981) also found that child-centeredness was a function of 'subject matter.' As opposed to social studies, the teaching of mathematics was more likely to occur in a teacher-centered than in a child-centered modality of instruction; and this differential impact seemed to cut across socio-economic differences. The persistence of teacher-centered math lessons is usually linked to teachers' own experiences as former students--i.e. teachers teach the way they themselves have been taught--as well as to their assumptions about mathematics--e.g. the nature of mathematics is such that its rules and procedures cannot be discovered and, therefore, must be told.

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[38]For example, as discussed by Cazden (1995), the effectiveness of compensatory reading programs such as 'Reading Recovery,' resides precisely in the teachers' purposeful reliance on a hybrid pedagogical model. In other words, this program incorporates elements of a visible pedagogy, teacher-centered approach, to its overall child-centered pedagogy.

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I suggest that, together with these personal and epistemological factors, we consider yet another factor which is specific to the didactics of mathematics, or better put, to the lack thereof. Guided by their wisdom of practice, conscious of their lack of appropriate tools and models for facilitating more open-ended tasks, and unaware of the existence of alternative, more child-centered models of *efficacy* (Smith 1996),[39] teachers may be reluctant to abandon the center of the stage to their students and abide to the prescription not to tell.

## 1.4 Progressive Methods

Since its birth, earlier in this century, in the educational theories and experimental schools of John Dewey and his many disciples, progressivism has been taken generally as synonymous with modern American education.[40]

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[39]As for the efficacy of different modalities of transmission, this is a topic of heated controversy. Although there is a substative body of 'process-outcome' studies indicating the comparative superiority of teacher-centered models (e.g. Stallings and Kasowitz's 'Follow Through' Classroom Evaluation research; the Brophy and Evertson studies; the studies conducted by Good and Grouws, and those conducted by Gage and his collaborators at Stanford University), the idea that child-centered classroom environments are more conducive to meaningful learning continues to dominate (cf. *American Educator*, Fall 1996).

[40]The literature on progressivism is vast and continues to grow. Since its review goes beyond the purpose of this section, I limit myself to mentioning the work of Cremin (1961, 1988), Katz (1968, 1971, 1987), Tyack (1974), Cuban (1993), and Zilversmit (1993). For Cuban, the first three historians coincide in distinguishing between administrative progressives (e.g. Strayer and Cubberley, who used a 'scientific management' approach to organize their school systems), social reformers (e.g. Counts, Childs, Beatty, who advocated the use of schools as instruments for national regeneration), and pedagogical progressives (e.g. Parker and Kilpatrick, concerned with setting the child as the center in the teaching and learning

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The agenda of educational progressivism overlaps with that of child-centeredness in many respects, including the respect for the right of children to develop as unique personalities, and the placing of activity as the root of all education and interest as the motive of all school work. Progressive educators argue that the relationship between students and teacher which is most conducive to learning is that in which the teacher abandons any claims of power, and restrains from imposing her authority on the students. The function of schooling is defined as moving beyond the transmission of knowledge, encompassing other tasks related to the physical and mental well-being of children. And the classroom is envisioned as a small scale social world, where students and teachers jointly learn to exercise their rights and duties *qua* responsible citizens in the larger society (*Progressive Education* 1920).

Perhaps the most characteristic feature of educational progressivism is its radical attempt at redefining the relationship between school and the world/society/community/everyday life. This redefinition aims at filling the gap between the school and its surrounding environment, which is inherent in traditional, formalist pedagogical models. Within the progressivist

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process). Despite their substantial differences, they all draw upon the ideas of John Dewey.

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framework, education is generally seen as akin to life rather than as a preparation for it.

By the late 1910s, it was proposed that school activities be organized around 'projects' or 'whole-hearted purposeful activities' (Kilpatrick 1918, 1934; NSSE 1920; Hotchkiss 1924). For Kilpatrick, the inventor of the 'project method,' whatever activity children decide spontaneously to undertake could be considered a worthwhile project.[41] The project method translated into 'incidental teaching,' a practice whereby traditional school disciplines were encountered only in so far as these were relevant to such endeavors. For example, a boy working on a 'raising corn project' would learn about botany and chemistry as a result of his immediate needs regarding the purpose-at-hand. His relationship to knowledge would thus be of a practical rather than a theoretical kind.[42]

The project method led to statements such as

> *[w]e find arithmetic in many little pieces scattered along the path of life.* These we shall teach as we need them. Some students, particularly the most mathematically inclined, will, from time to time, put the pieces together and form wholes more or less complete. Later some will specialize in the subject (Kilpatrick 1918, p. 357, emphasis added).

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[41]Dewey (1933) criticized this laissez-faire approach and, in particular, the fallacious attempt at minimizing the teacher's leadership role in curriculum matters. He tells the story of "a young child who, on arriving at school, said to the teacher: 'Do we have to do to-day what we want to do?'" (p. 274).

[42]This example is given by Byod Bode, in the context of his critique of the project method (1927, p. 149).

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As sociologist Boyd Bode critically noted, while various definitions of 'project' were proposed which would emphasize the organization of the material, the aims of the activity, or the child's attitude towards it, underneath all these lurked the problematic purpose of making "the schoolroom continuous with the rest of life" (1927, p. 147).

Some educators considered that the project method was too unstructured, and arguments were presented for a certain amount of compulsion and directiveness, particularly in the acquisition of basic number skills (e.g. Bagley, in TCR 1921; Charters 1922; NSSE 1934; Horn, in NSSE 1951). For example, in the *10th NCTM Yearbook*, Burdette Buckingham remarked:

> Doubtless, if rich number experiences could be carried far enough it would be unnecessary to resort to formal teaching and learning of number facts. Unfortunately, however, there is a definite job here to be done. The best activity program is likely to be relatively unsystematic. *There comes a time when it is necessary to pick up the loose ends* (1935, p. 56, emphasis added).

If knowledge of number for its own sake was outside the range of children's 'natural' interests, how could the contents of school arithmetics be covered by incidental teaching? Kilpatrick himself did not hide his discomfort regarding this matter. There are certain minimum essentials, such as reading and number work, which are

> so useful for future progress in school and life, both immediate and remote, that we should use compulsion if need be to get them; so important that if they are not got otherwise there would eventually come a time when we should, if need be,

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> drop practically everything else and compel the learning of them" (1925, pp. 365-6).

In the *10th NCTM Yearbook* (1935), Paul Hanna summarized the controversy around the 'activity program' in terms of three different viewpoints. First, those who considered that the students' 'functional' experiences are adequate for school arithmetic. As they saw it, it is in the context of these projects or activities, that the 'truly important' arithmetical skills will be mastered. Second, those who insisted that the goal of arithmetic teaching is the students' systematic mastery of basic arithmetic skills, to be obtained by systematic drill and practice. And, third, those--including the majority of the members of the NCTM's Committee on Arithmetic--who recognized the two goals, and sought for ways of integrating them. Hanna insisted that, in the course of these activities and projects, it would be up to the "intelligent teacher," to decide--according to her perception of the students' level of understanding--what needs to be skipped, what is worth stressing, and what may require further practice.

In those innovative programs which followed the project approach, groups of students engaged in the solution of student- or community-generated mathematics problems (Whipple 1921, Hotchkiss 1924, Hanna 1935). In the meantime, in most classrooms throughout the nation, the learning of basic number facts was complemented with endless lists of textbook-generated word problems--'projects,' or

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'activities'--for the application of the fundamental arithmetical operations. Yet, study after study found that, despite these attempts at 'clothing' or 'camouflaging' mathematics in child-like micro-narratives, students continued to respond mechanically to these materials thus producing mountains of horrible errors.[43]

## 1.5 Math Constructions

In mathematics education, 'constructivism' denotes a spectrum of different positions, all of which originate in the work of Swiss psychologist Jean Piaget (1950, 1952, 1954, 1970, 1972). 'Radical' constructivists believe that knowledge is not passively received, but actively built up by the knowing subject, and that, far from the discovery of an ontological reality, the function of cognition is the organization of the experiential world.[44] 'Weak' or 'moderate' constructivists subscribe only to the first premise.[45] Finally, those who 'sociologize' this model by locating the learning process not in the solitary individual, but in communities of teachers and students engaged in the ongoing negotiation of meanings, norms, and procedures, refer to themselves as 'social'

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[43]Osburn 1922, 1925; Myers 1925; Hydle and Clapp 1926; Buswell 1927; Wheat 1929, 1951; Brownell 1931.

[44]von Glasersfeld 1990, 1991; Davis, Maher, and Noddings 1990; Steffe 1991.

[45]Schoenfeld 1987, Kilpatrick 1987, Goldin 1990.

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constructivists.[46]

During the 1960s' new math era, as Piaget's genetic epistemology became the most influencial theory of learning, the child *vs*. math dichotomy reached--albeit temporarily--a happy resolution. A way was found, at least in theory, to place children and their spontaneous interests at the center of the classroom while keeping the structure of mathematics intact. What made this move possible was a key assumption underlying Piaget's theory, namely, that the genesis and structure of the child's intelligence are homologous to the genesis and structure of mathematics. This homology 'reconciliated' the child with mathematics and, therefore, eliminated the need to 'camouflage' the latter in seductive narratives to make it more palatable to the former. At last mathematics could be presented *in vivo*, stripped of all didactical deformations!

As in the child-centered and the progressivist models, at the center of the new math's classroom was the individual child engaged in 'manipulative play' and conceived as a proto-mathematician. The teacher, acting as a guide or facilitator, was displaced to the background. This approach, which was also referred to as 'discovery learning,' assumes that the process of learning is more akin to the endeavors of an individual scientist producing new knowledge, than it

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[46]Bauersfeld 1988, 1992; Cobb *et al*. 1992; Cobb and Bauersfeld 1995; Yackel and Cobb 1996.

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is to the reproductive routines whereby ready-made concepts and procedures are transmitted from teachers to students.

This double continuity which links children's 'pre-mathematics' with school mathematics, and school mathematics with the mathematics of mathematicians, was a central feature of the 1960s' new math reform. As Dutch mathematics educator Hans Freudenthal strongly argued (1973), this fiction lead to a harmful 'anti-didactical inversion,' that is, it blinded reformers to the need for classroom teachers to be equipped with the necessary didactical tools for mediating between these three rather distinct modalities of knowing mathematics.

The failure of the new math reform forced math educators to rethink their models. And, shortly after the 1970s 'back to the basics' interregnum, the new new math project promised to deliver all that had been left out of the picture: the proactive role of the teacher; the effects of linguistic, cultural, and gender differences on math learning, the classroom as a community; and the rhetorical dimensions of mathematics.

Within the current reform scenario, most math educators agree on the need to transform 'school mathematics' into 'inquiry mathematics.'[47] Conceiving of teachers and students as scientists engaged in 'mathematical communications,' social constructivists define a productive classroom

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[47]NCTM 1991, Richards 1991, Borasi 1992, Cobb *et al.* 1995.

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atmosphere as one where conditions are created for both parts to explain, clarify, refute and revise their mathematical assumptions and conjectures.

Just like advocates of progressivism and of child-centeredness, social constructivists are concerned with the fact that school mathematics often turns into a self-contained activity which is unrelated to students' out-of-school activities. Combining a radical constructivist view on learning with a symbolic interactionist view of the classroom, they suggest as a remedy for this alienation, a restructuring of classroom discourse along the lines of 'communities of mathematical validators,' with standards of argumentation defining what are to be taken as acceptable accounts of taken-as-shared mathematical objects (Bauersfeld 1992; Lo and Wheatley 1994; McClain, Cobb, and Whitenack 1995; Yackel and Cobb 1996).

Yet, both within and outside of the constructivist paradigm, critics have noted the risk of conflating two distinct phenomena: the process whereby school mathematics is taught and learned in classroom contexts, and the process whereby new mathematical knowledge is generated and validated within the scientific community.[48] In the words of German mathematics educator Christine Keitel (1987),

> w[hereas the mathematician inspects the *terra incognita* of a concrete problem area from the

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[48]Chevallard 1985; Brousseau 1986, 1988; Keitel 1987; Bernstein 1990, 1996; Lampert 1990, 1990a; Chaiklin and Lave 1993; Ball 1993.

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> *terra firma* of his or her science, the child moves on towards mathematics from the little island of his experience and perception, knowing but a few steps, and with the destination unknown.

Moreover, as constructivist teacher and teacher educator Deborah Ball put it (1993), while discussing the dilemmas faced by classroom teachers as they implement the recommendations of present-day reformers,

> constructing a classroom pedagogy on the discipline of mathematics would be in some ways inappropriate, even irresponsible. Mathematicians focus on a small range of problems, working out their ideas largely alone. Teachers, in contrast, are charged with helping *all* students learn mathematics, in the same room at the same time. The required curriculum must be covered and skills developed.... Moreover, certain aspects of the discipline would be unattractive to replicate in mathematics classrooms. For instance, the competitiveness among research mathematicians... is hardly a desirable model for an elementary classroom (p.6).

Is it possible to design an effective pedagogy of mathematics while following *à la lettre* the prescriptions of child-centeredness and progressivism? How feasible is a constructivist approach to the teaching of mathematics? These and other similar questions point our attention to the inherent tension between an effective teaching of mathematics and the modern pedagogical paradigm.

## 1.6 The New New Math

The new new math reform began in the mid 1980s from a consensus among the political, business, and scientific communities, the math education research community, and the

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community of teachers of mathematics, on the need for a radical transformation in the practices of teaching, learning, and assessing this school discipline.

The spirit of the reform is best encapsulated by two mottos: "Everybody Counts" (Mathematical Sciences Education Board 1989) and "Problem-solving, communication, reasoning, and connections" (NCTM 1991). In the *Professional Standards* (1991), the NCTM defines 'mathematical power' as

> the ability to explore, conjecture, and reason logically; to solve nonroutine problems; to communicate about and through mathematics; and to connect ideas within mathematics and between mathematics and other intellectual activity. Mathematical power also involves the development of personal self-confidence and a disposition to seek, evaluate, and use quantitative and spacial information in solving problems and in making decisions (p. 1).

The math classroom is to operate like "a community of people collaborating to make sense of mathematical ideas." The teacher is to 'orchestrate' the discourse of the classroom by means of questions such as 'How did you solve this problem?,' 'Who did it in a different way?' and 'Can you paraphrase what so and so just explained?' Computation skills are not to be taught and assessed in isolation as in the old drill and practice approach but, instead, in the course of engaging in 'worthwhile mathematical tasks' (NCTM 1991). An unprecedented stress is put on the realism, relevance, authenticity, and open-endedness of 'story' problems. Finally, another central tenet of this reform is the ongoing monitoring of students' learning by means of

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whole-class discussions, clinical interviews, math journals, portfolios, and 'performance' assessments (NCTM 1989, 1991, 1993, 1995).[49]

The math education researcher community--and the NCTM as well--anticipates the difficulties teachers may face as they implement these recommendations. When in the course of 'math investigations,' teachers act only as guides, given the limitations of classroom time, students may end up not constructing anything;[50] whole-class discussions may last forever unless teachers take a proactive role in picking up what is worth pursuing, and leaving aside what should be discarded;[51] and some students may be resistant to participating in this new modality of classroom discourse.[52] That is, accustomed to the old, math- and teacher-centered model, where what mattered were right answers, some of students may be unwilling and/or unable to engage in math

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[49]As compared to the traditional multiple-choice test, the newly-emerging 'performance assessments' in mathematics (also referred to as 'authentic assessments') solicit from students, in addition to final answers to the problems presented, a more or less detailed account of the strategies they have followed to arrive at their solutions (Wiggins 1991; Leder 1992; Lesh and Lamon 1992; Romberg 1992, 1995; NCTM 1995).

[50]Arsac, Balacheff, and Mante 1992; Lampert 1990; Ball 1993; Simon 1995.

[51]Lampert 1990; NCTM 1991; McClain, Cobb, and Whitenack 1995; Cobb *et al.* 1995.

[52]Lampert 1990. The *Professional Standards* anticipate that some students may be resistant to participating in this new modality of classroom practice "without relying on teacher's authority and without becoming hostile or defensive," and "without bursting in, frantically, weaving hands, or showing impatience" (p.58), and a variety of techniques are suggested for teachers to face these obstacles.

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conversations for the sake of explaining and understanding.

As a matter of fact, the math-classroom-as-a-community-of-inquiry implicitly relies on the subtle imposition of the *will to understand*. One of the rules of this new 'didactic game' (Brousseau 1986) is that, in order to show that you understood a certain mathematical notion, procedure, or solution, you must be able to provide your own explanation, or paraphrase somebody else's. And, of course, in order to paraphrase, you must listen to others and be willing to make sense of what they say. If, in the old model, teachers would impose discipline with measures extrinsic to the subject matter--e.g. extra homework, detention, or low grades--in the new model, they may do so with suggestions such as: 'If you don't pay attention, you won't be able to understand so and so's explanation.'

It has been noted also that, when left alone, students tend to produce rather idiosyncratic mathematical constructions (e.g. erroneous concepts, awkward forms of notation, time-consuming algorithms, or primitive solution procedures). Unless teachers are equipped with the appropriate didactical tools--and feel entitled to use them--for mediating between students' inventions, and the notions, conventions, and procedures of mathematics, the desired aim of incorporating the students into the community of mathematically literate people will hardly be achieved,

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or only be achieved with great difficulty.[53]

## 1.7 Inherent Tensions

Dichotomies such as mathematics *vs.* the child, the computational *vs.* the social value of mathematics, children's inventions *vs.* mathematical conventions are central to old and new discursive practices of elementary school mathematics. These dichotomies encapsulate some of the tensions, contradictions, and dilemmas faced by teachers and math educators, as they accommodate an effective teaching of mathematics to a full-fledged child-centered, progressivist, and constructivist paradigm. But, wherein lays the source of these inherent tensions?

First, whereas the child-centered pedagogical model relies on notions of subjectivity, activity, interest, and play, mathematics has been constituted as a subjectless discourse. That is, the addressee of mathematical statements is always a non-subject.[54] Although, in the early school years, children are engaged in the manipulation of concrete materials, it is expected that they will undergo an adaptive process whereby their actions will become increasingly decentered, interiorized, depersonalized, and independent of physical or contextual constraints (Piaget 1941, 1950, 1954). That is, in order for the implicit schemes of action

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[53]Lampert 1990; Ball 1990, 1993.

[54]Baruk 1985, Chevallard 1985, Walkerdine 1988, Rotman 1993.

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to become explicit logico-mathematical patterns of thought--i.e. 'reflective abstraction' (Piaget 1972)--the child's subjectivity is to be sacrificed.[55] The embodied learner becomes the disembodied epistemological subject of rational discourse. Yet, as Walkerdine suggests (1988), while this process necessarily entails the repression of the imaginary, this repression may be worthy of its price, i.e. the mastery of a rather powerful discourse.

Second, primarily concerned with the social significance of schooling, progressive math educators have often been at pains in justifying a place in the curriculum for those topics which were seen as irrelevant to the immediate demands of social and economic life. Also, in the light of their advocacy of pedagogical practices centered around purposeful projects or activities, the question as to how to conciliate this approach with the need for securing students' mastery of computation skills returned again and again. After all, a progressivist educational agenda could not abandon to the randomness of incidental learning, a school subject which, since the late 19th century, has occupied a central place in the constitution of an educated citizenry.

Third, as we recall, constructivist math educators see the teacher's role as facilitating rich environments for students to construct their own knowledge. It is proposed

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[55]Chevallard 1985, Brousseau 1986, Walkerdine 1988.

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that, as students engage in mathematically relevant investigations and mathematically-rich talk, sooner or later, their pre-mathematical concepts and procedures will approximate the concepts and procedures of mathematics. As math educators embark in the task of translating their constructivist theory of learning into a constructivist theory of teaching, many are the obstacles to be faced.[56] For, at stake are nothing less than the daily routines whereby--in limited segments of time, with ill-prepared teachers, and under the pressure of standardized testing--about thirty students are to be guided towards becoming legitimate practicioners of a subject matter which, over the centuries, has developed into an artifact made up of sophisticated, efficient, and elegant structures, procedures, and notations.

Yet, since mathematics ought to be taught within the boundaries of the modern pedagogical paradigm, these inherent tensions are for the most part misrecognized. As a result, researchers, didacticians, test-makers, and teachers sustain a series of math fictions which, discursively as well as practically, fill in the gaps which these tensions generate. These fictions are, of course, fictional constructs. Yet, they can be said to have material effects. How are we to access them?

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[56]Lampert 1990; Simon and Schifter 1991; Simon 1995; Cobb *et al.* 1995.

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In Chapter Two, I delineate the theoretical tools I have chosen to carry on the proposed analysis. These include ways of thinking about the field of education in relation to the complex set of social, political, and economic forces affecting its structure and functioning (Gramsci, Althusser, Bourdieu and Passeron, and Bernstein); a discussion of the 'didactic system' of mathematics teaching in light of the internal and external constraints affecting it, as well as a model for the relationship between school mathematics and scientific mathematics (Chevallard); and a framework for analyzing the complex dynamics which unfolds as teacher and students engage in the 'didactic game' of school mathematics (Brousseau and Walkerdine).

To explore these fictions I consider three objects. First, the prefaces of a sample of arithmetic textbooks in circulation during the period from the time when, in most school districts, the provision of free textbooks was made mandatory, to the Sputnik crisis as the result of which arithmetic textbooks metamorphosed into texts in modern mathematics (Chapter Three). Next, a set of official documents and other founding texts from both the new math and the new new math reforms (Chapters Four and Five). And, finally, a series of classroom events which I gathered in the course of a one-and-a-half year long fieldwork in the fourth and fifth grades of a public, bilingual school in El Barrio (East Harlem), New York City (Chapters Six and

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Seven).

My analysis of the introductions of a sample of arithmetic textbooks in circulation between 1882 and 1957, highlights the uneasy relationship between arithmetic and the emerging child-centered paradigm. Central to the accommodation of arithmetic textbooks to the demands of a radically transformed curriculum field were such difficult questions as: How may one make a textbook interesting for the young readers while, at the same time, remaining faithful to the structure of arithmetic? In their attempts at making arithmetic palatable to 'the child,' textbook authors shaped the singular microworld of textbook arithmetic. As they 'fictionalized' arithmetic, these authors unknowingly revealed the fictional character of the claim that school arithmetic may be made continuous with the discipline from which it originates.

Next, I explore the new math reform throughout some of its founding documents. Situating these texts within the political conjuncture of the 1960s, I explore the fictions engendered by the reform, explore wherein resided their appeal, discuss some of their effects, and present the various oppositional stances against it. Then I contrast the 1960s 'revolution in school mathematics' with the new new math reform currently underway. Situating this 'systemic' intervention in the neoconservative 1980s' and 1990s' political scenario, I trace the fictions at work in the new

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new math's founding documents, consider their singular seductive power, and I anticipate some of their possible classroom effects.

Finally, I discuss a series of classroom episodes gathered in the course of my fieldwork in an inner city, bilingual, public, elementary school. In particular, I look at how a group of fourth and fifth grade students attending this school, interpret, solve, and account for their solutions of 'word' and 'story problems.' I analyze their errors, confusions, and misunderstandings; their resistance to participating in math talk; and the often unexpected responses they offer to their teachers when oral or written accounts of their procedures and understandings are elicited both, in the context of regular math lessons and in preparation for performance--or 'show your work'--assessments. These findings illustrate the obstacles which surround the shift in mathematics education towards a more 'invisible' pedagogical model, in settings in which teaching and learning practices operate under severe constraints. The ways in which students read the new story problems and account for their problem-solving strategies, expose the limits and constraints of the ongoing reform, its unintended consequences, and its perverse effects.

The choice of a minority-segregated, bilingual school located in a low-income area of New York City, rather than a school attended by middle class, White, English-speaking

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children, is motivated by my interest in exploring the effects of the new new math in a site where these may be most severe. One may argue that, rather than representing an instanciation of the modern pedagogical paradigm, this is a school where this paradigm appears, if at all, in its most distorted or hybrid version. One may also object that, since this school is not a 'demonstration site' of the reform, whatever happens in it may be seen as a shameful remainder of an old regime about to be superseded.

I propose that, in classrooms attended by low income and minority children, precisely *because* the tenets of the modern pedagogical paradigm appear in a distorted form--affected as they are by complex and severe constraints--the tension will appear with more strength and thus produce the most damaging effects. For, in the case of children from the dominant social groups, this tension is mitigated by the fact that they come to school already endowed with the prerequisites to operate in childcentered, progressive and constructivist environments.

Also, the spirit of the reform has made itself apparent in this school at least in two ways. The teachers in this school have been made aware of the NCTM Standards--for example, through school-wide staff development meetings, district-wide math liaison sessions, occasional reading of teachers' journals, and the media in general--and, with varying degrees of enthusiasm and commitment, are have

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aligned their practices to the new recommendations. Also, since the early 1990s, the New York City Board of Education began administering two new examinations--the CAT-5 and the PAM tests--which reflect the new new math's philosophy regarding assessment.

Thus, whatever may be said about math teaching and learning in this school contributes to our understanding of the dystopian effects which result from the overlapping of old and new regimes of elementary school mathematics.

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# CHAPTER TWO: THEORETICAL TOOLS

> ...it is no accident that the moment of transition from ruthless methods of imposition to more subtle methods is doubtless the most favourable moment for bringing to light the objective truth of that imposition.... If 'there is no science but of the hidden,' it is clear why sociology is allied with the historical forces which, in every epoch, oblige the truth of power relations to come into the open, if only by forcing them to mask themselves yet further (Bourdieu and Passeron, Foreword to *Reproduction*).

This thesis centers around a series of math fictions which, since the turn of the century, have defined the discursive practices of elementary mathematics education in the United States. These fictions include the autonomy of the 'didactic system' of math teaching vis à vis its surrounding environment; the continuity of school- and out-of-school mathematics; a student/teacher relationship defined as a dyad (with exclusion of the third term, i.e. knowledge); the transparency of mathematical texts and the neutrality of visual and other forms of representation; a classroom modelled as a microcosm of mathematical culture; the accessibility to the student's mind; and the entry of the child's subjectivity into the process of learning mathematics.

I have chosen to explore these math fictions as they materialized in a sample of arithmetics textbooks in circulation during the period from 1882 to 1957, in the founding documents of both the 1960s' new math and the current new new math reforms, and in the daily, fourth- and fifth-grade math lessons in a public, bilingual, elementary

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school in El Barrio (East Harlem), New York City. This choice is guided by my belief that telling the story from one angle--e.g. a historical inquiry into old-time arithmetic textbooks, a discourse analysis of the reforms' official documents, or an ethnographic exploration of daily classroom practices--may situate us either in the fatality of absolute social and economic determinations, or in the banality of an educational system conceived of as autonomous from external constraints.

Education may be thought of as a paramount mechanism of social regulation, a machine for the production of reasoning and reasonable subjects, an assembly line which reproduces a skilled and disciplined labor force, or an apparatus for the production and reproduction of those knowledges and dispositions which legitimate the dominant social order. One may also look at education as a field in which

> different agents and institutions constantly struggle, according to the regularities and the rules constitutive of this space of play (and, in given conjunctures, over those rules themselves), with various degrees of strength and therefore diverse probabilities of success, to appropriate the specific products at stake in the game (Bourdieu and Wacquant 1992, p. 102).

Bourdieu's dictum that "those who dominate in a given field are in a position to make it function to their advantage but they must always contend with the resistance, the claim, the contention, 'political' or otherwise, of the dominated" (ibid.), suggests the need to examine the ways in which students and teachers actively participate in the daily work

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of pedagogical inculcation. By keeping an attentive eye on the factors which constrain the educational system 'from outside' as well as on the unpredictable logic and unexpected subtleties of classroom practices, we may understand how the didactic game is being played out and, eventually, contribute to changing the rules of the game as it is presently being played.

To discuss the processes whereby the teaching of mathematics produces reasoning subjects, requires a reflection on a series of interconnected problematics. These include: the structure and functioning of the educational system in relation to capital and the State; the 'didactic system' and its internal and external constraints; the 'didactic contract' binding students and teachers in the math classroom; the constitution of school mathematics as a result of the didactical deformation of scientific mathematics ('the didactical transposition of mathematical knowledge'); and alternative models of mathematical thinking, learning and understanding.

In linking the machinery of social regulation with the individual mind, this sequence addresses, albeit indirectly, a central problem in contemporary social theory: the macro/micro dilemma. In particular, sociologists of education still struggle to establish meaningful links between social reproduction and everyday life in schools (e.g. Willis 1977, 1981; Wexler 1987; McLaren 1986, 1989;

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Morrow and Torres 1995). This apparently unbridgeable gap may be seen as symptomatic of two crucial limitations in contemporary educational thought--i.e. the lack of theorizing about the relationship between education and the State, and a blindness to the often contradictory ways in which, in modern disciplinary societies, educational institutions are implicated in the production and reproduction of subjectivity and difference. To 'correct' these pitfalls may require a reflection on the constitution of sociology of education as an academic discipline, in relation to the State and to the various public spheres in which the discursive practices of education exercise their disciplinary power--from the elementary school classroom to the university's lecture hall.

Although these omissions deprive educational research of some of its potential for critical thought, they allow practitioners to maintain a thrust in the autonomy of the educational system vis à vis its broader socio-political context. In this way, this perspective can be made compatible with voluntaristic beliefs in the liberatory power of schooling as well as with populist illusions as to the possibility of effectively incorporating 'subjugated' knowledges into mainstream curricula.

## 2.1 The Field of Education

In their examination of contemporary theories of

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education, Karabel and Halsey (1977) discuss structural and technological functionalism, human capital theory, methodological empiricism, conflict theories, and the new sociology of education. The authors argue that these different ways of thinking about education and society are shaped, on the one hand, by macro-level events such as the Cold War, the technological determinism of the Sputnik era, the War on Poverty, the apparent lack of impact of compensatory education programs, the May 68 events in France, the political assaults against economic inequality and the new social movements in the 1960s and early 1970s, the crisis of the welfare state, the debates around deskilling and the dual labor market; and, on the other hand, by such micro-level events as changes in funding policies for educational research.

Consensus, liberal, and structural-functionalist theorists conceive of education as a set of institutional mechanisms whereby each generation assures the transmission of the culture inherited from the past to the next generation. As critics have pointed out, these models assume that the different forms of pedagogical action at work in a given time and place collaborate, more or less harmoniously, in reproducing a cultural capital which is considered the jointly owned property of the whole 'society' (Bourdieu and Passeron 1977).

On the ocassion of his Presidential Address to the

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American Economic Association, Theodore Schultz (1961) proposed that the process of acquiring skills and knowledge through education not be viewed as a form of consumption but instead as a form of productive investment. As he argued, investment in human capital increases individual productivity and, in doing so, lays the technical base of the type of labor force necessary for the nation's rapid economic growth.[1]

Emerging around the 1970s, and with a focus on the linkages between economy and schooling (or capital and education) 'conflict,' neo-Marxist, and 'political economy' perspectives of education conceive of educational institutions as systems put in place in order to satisfy the needs of capital and, thus, meet the interests of the dominant classes. Best represented by Samuel Bowles and Herbert Gintis' *Schooling in Capitalist America* (1976), these models rely on a macro-political conception of power. As Bowles and Gintis see it, those pieces of knowledge and forms of thought which reinforce a worldview consistent with the ideology of the dominant classes, materialize in overt and 'hidden' school curricula and circulate throughout the

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[1]As described by Karabel and Halsey (1977), "[t]he theory of human capital was consonant with the forms of technological functionalism which attracted many sociologists in the 1950s. Both theories stress the technical function of education and emphasize the efficient use of human resources. A concern with eliminating waste also supports the liberal notion of equality of opportunity. For the technological functionalist there is the enemy of ascription and for the human capitalist the blight of under-investment" (p. 13).

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different levels of the educational system. As many critics have remarked, correspondence theories of education shape a picture of schooling which offers little room for imagining an effective resistance to the systems' meritocratic ideologies and practices (Willis 1977, 1981; Giroux 1983).

Originating in Great Britain around the 1960s and 1970s, the new sociology of education, is forwarded as a critique of the effacement of agency in structuralist theories of ideology. Researchers working within this perspective look at the effects of the 'hidden' curriculum in everyday schooling practices (Young 1971; Apple 1979, 1982).[2] The audience of new sociologists of education is composed, for the most part, of teachers, and they tend to be somehow voluntaristic in their tactical recommendations.

The models described so far do not sufficiently theorize the relationship between educational discourses and practices and the capitalist State. Thus, it seems worth considering the work of Antonio Gramsci, Louis Althusser, and Pierre Bourdieu. While Gramsci sees schooling as implicated in the process of reproduction of the hegemonic social and economic order, Althusser and Bourdieu conceive of schools as involved in a systematic exercise of symbolic

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[2]For Young (1971), the sociology of education is, in fact, the sociology of knowledge. Similarly, Apple (1979) argues that the central aim of the sociology of education is the ideology-critique of the curriculum field.

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violence, always misrecognized as it refashions itself in new clothing ("the famous new methods!" Althusser 1971).

The relevance of Gramsci's theory of hegemony lays in its consideration of the hegemonic, economic, and repressive functions of educational institutions. By defining the State as

> the entire complex of practical and theoretical activities with which the ruling class not only justifies and maintains its dominance, but manages to win the active consent of those over whom it rules (*Prison Notebooks* [1929-1935] 1971, p. 244),

Gramsci envisions a state which encompasses political and civil society (i.e. "hegemony protected by the armour of coercion"). Among the State's functions, Gramsci underlines the educational one:

> the school as a positive educative function, and the courts as a repressive and negative educative function, are the most important state activities in this sense (ibid. p. 258).

In so far as it participates, through its 'organic' intellectuals, in the production and distribution of knowledge, the State functions also as a 'scientist.' According to Poulantzas (1973), the 'scientist state,' which results from the separation of intellectual and manual labor, further reproduces and intensifies this division due to its monopoly over the production, distribution, and use of legitimate knowledge.[3]

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[3]As discussed by Sohn-Rethel (1978), the roots of the intellectual/manual labor divide are to be located in the labor process in which, with the distinction concrete/abstract labor, a simple process of abstraction becomes a process of domination.

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In *Ideology and Ideological State Apparatuses* (1971), Althusser analyses how the State accomplishes its repressive and ideological functions. Repressive state apparatuses (RSA)--government, administration, courts, prisons, police, army, etc--function primordially by repression and only secondarily by ideology. Instead, ideological state apparatuses (ISA)--education, family, church, law, media, political parties, unions, etc.--function primarily by ideology and secondarily by repression. Although the ISA function more independently than the RSA, what provides both set of apparatuses with unity and direction is the ideology of the ruling classes.

Althusser follows Gramsci in underlining the centrality of educational institutions in capitalist social formations. As he sees it, in modern societies, the combination of school and family has displaced the church and family couple as the main ideological agent (1971, p. 154). As the only institution which enjoys the obligatory audience of most children during extensive periods of time, schools are called upon to perform the most important hegemonic role among all the ideological apparatuses (ibid. p. 156).

For Althusser, education plays a central role in the process of social reproduction by providing the necessary skills for the labor process as well as the ideologies which help sustain the social division of labor. At the same time,

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the school constitutes the ideal site for the transformation of individual children into learning subjects. This is a particular instance of the mechanism of "interpellation," which Althusser defines as the process whereby ideology transforms individuals into subjects (ibid. p. 174). Notwithstanding the severe criticisms to which it has been made the object--e.g. its foreclosure of agency, which is typical of structuralist models--Althusser's writings still provide a useful framework for thinking about the relation between power and knowledge--a problematic which also constitutes the core of Pierre Bourdieu's work.

The power/knowledge link is conceptualized by Bourdieu as a relation between the political instruments which legitimate a social order as a space of hierarchically organized arrangements and social distinctions, and the instruments of knowledge which legitimate a symbolic order as a space of hierarchically organized dichotomies (e.g. low/high, concrete/abstract, body/mind, female/male). Central to Bourdieu's conceptual apparatus is the notion of 'habitus.' A socially constituted system of cognitive and motivating structures, the habitus produces individual and collective practices which tend to reproduce the regularities immanent in the objective conditions of production of their generative principle ([1972] 1989).

These processes of inculcation occur in all the institutionalized spheres of social life, including the

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family, the school, and the workplace. In *Distinction: A Social Critique of Judgement* (1984), Bourdieu argues that, in each of these spheres, the required competencies are constituted by usage itself, and that these sites operate as systems of exchange which, "by their positive or negative sanctions, evaluate performance, reinforcing what is acceptable, discouraging what is not, condemning valueless dispositions to extinction" (p. 85).

For Bourdieu, class and capital define the agents' locations and trajectories in the social space. Assuming the general convertibility of different forms of capital--i.e. economic, symbolic, cultural, linguistic, and academic--Bourdieu's overall model of the social is subsumed under a general theory of the economy of practices ([1980] 1990). Perhaps Bourdieu' and Passeron's main contribution to the sociology of education is their elaboration of the complex relationship between reproduction, schooling, and 'cultural capital.' As they define it, the latter refers to "the instruments for the appropriation of symbolic wealth socially designated as worth of being sought and possessed" (1977, p. 487).

While power/knowledge relations are constitutive of schooling in fundamental ways, this connection often operates in an obscure and mystified fashion. Bourdieu and Passeron see this as the *sine qua non* condition for the rhetorical and material force of schools' meritocratic

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ideology. The notion of 'misrecognition' (*meconnaisance*)--i.e. "the process whereby power relations are perceived not for what they objectively are but in a form which renders them legitimate in the eyes of the beholder" (1977, p. xiii)--helps understand the systematic exercise of 'symbolic violence' which is intrinsic to all pedagogic work.

Central to this argument is the fact that, in capitalist social formations, the cognitive and motivating structures--i.e. dispositions, abilities, and interests--which best predispose children to superior school performances are a prerogative of the dominant classes. However, pedagogical discourses and educational practices rely on the assumption that all children enter the classroom doors equally endowed with these prerequisites. Thus,

> [t]he educational system reproduces all the more perfectly the structure of the distribution of cultural capital among classes in that the culture which it transmits is closer to the dominant culture and that the mode of inculcation to which it has recourse is less removed from the mode of inculcation practice in the family (Bourdieu 1973, p. 493).

This process affects general abilities such as the disposition to appreciate works of art, understand and follow directions when these are not explicitely stated, or construct rhetorically powerful personal narratives or elaborated logico-mathematical explanations. Insofar as these abilities are assumed to be already there and are thus excluded from explicit pedagogic work, the school further participates in the reproduction of social inequality. As

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Bourdieu and Passeron suggest, success in school depends heavily on the education previously accomplished in the earliest years of life:

> we know that through all the skill-learning processes of everyday life, and particularly through the acquisition of the mother tongue or the manipulation of kinship terms and relationships, logical dispositions are mastered at their practical state. These dispositions, more or less complex, more or less elaborated symbolically, depending on the group or class, predispose children unequally towards the mastery of the operations implied as much in a mathematical demonstration as in decoding a work of art (1977, p. 43).

Bourdieu and Passeron propose that "[a]ll pedagogic action (PA) is, objectively, symbolic violence insofar as it is the imposition of a cultural arbitrary by an arbitrary power" (p.5), and that "[p]edagogic action necessarily implies, as a social condition of its exercise, pedagogic authority (PAu) and the relative autonomy of the agency commissioned to exercise it" (ibid. pp. 11-2). Their view of pedagogic authority as intrinsic to pedagogic action, coupled with their belief in the non-reducibility of pedagogic communication to a mere relation of communication, allows Bourdieu and Passeron to consider 'soft' approaches and 'non-directive' teaching with a skeptical eye.[4] For,

> if one reduces the relation of pedagogic communication to a pure and simple relation of communication, one is unable to understand the social conditions of its specifically symbolic and

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[4]Cf. Althusser's modern methods, Bernstein's invisible pedagogy, and Durkheim's dictum "There is no liberal education."

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> specifically pedagogic efficacy which lie precisely in the concealment of the fact that it is not a simple relation of communication (ibid. p. 23).

Within this logic, pedagogic communications are best seen as relationships between two partners--student and teacher--who are always already situated in asymmetrical relations of power and knowledge.

The proposed model does a good job in accounting for school failure among children of the dominated social sectors. This process is described as one of unconscious self-elimination, in that

> [t]he negative predispositions towards the school which result in the self-elimination of most children from the most culturally unfavored classes and sections of a class--such as self-depreciation, devaluation of the school and its sanctions, or a resigned attitude to failure and exclusion--must be understood as an anticipation, based upon the unconscious estimation of the objective probabilities of success possessed by the whole category, of the sanctions objectively reserved by the school for those classes or sections of a class deprived of cultural capital (Bourdieu 1973, p. 495).

Yet, as suggestively described by Paul Willis in his oftquoted ethnographic study of a group of working class high school boys in Hammertown (England)--*Learning to Labor: How Working Class Kids get Working Class Jobs* (1977)--failing in school and staying in the working class may be understood as a conscious choice which results from young people's ability to 'see through' the school's meritocractic promises. As Willis forcefully argues, if in doing so, students offer their own contribution to the machine of

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social reproduction, this process is not simply one of unconscious or blind choices.

The strength of Willis' argument lays in his highlighting the complex chain of mediations through which schooling participates in the constitution of class, gender, and race identities and differences. In his study, working class male students embrace their identity as future manual laborers by identifying with their fathers' shopfloor culture, and consciously devaluing intellectual labor--i.e. doing school--as feminine. In this gendered way, the logic of reproduction finds its ally in the 'free' will of the dominated.

In *The Ignorant Schoolmaster* ([1987] 1991) Jacques Ranciere discusses the circular logic of most reproduction arguments. By underlining the two propositions which constitute the 'perfect circle' of reproduction, he ironizes about the 'Bourdieu effect': working class youth are excluded from the university because they are unaware of the true reasons for which they are excluded (Bourdieu and Passeron [1964] 1977), and their ignorance of the true reasons for which they are excluded is a structural effect produced by the very existence of the system that excluded them.

As several of Bourdieu's critics have suggested, within the boundaries of his model, there appears to be no way out of the effects of misrecognition which conceal the school's

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systematic exercise of symbolic violence. While sharing the view of education as implicated in the work of cultural, social and economic reproduction, American radical pedagogues have strongly criticized Bourdieu's line of thought. For example, while acknowledging the value of Bourdieu's contributions, Giroux sees them as "trapped in a notion of power and domination that is one-sided and over-determined" (1983, p. 90).[5] Yet, for the most part, Bourdieu's ideas have been misread as forwarding a structural-functionalist, deterministic, reproductivist, or 'culture of poverty' model, and have been too quickly dismissed in favor of 'resistance,' 'radical,' and 'border' pedagogies which see the public sphere of schooling as a contested terrain of multiple social forces.[6]

Together with the work of Bourdieu and Passeron in France, and more strongly influenced by Durkheim's work on education than his French counterparts, Basil Bernstein's sociolinguistic theory--forwarded in his *Class, Codes, and Control* (1971, 1973, 1975, 1990)--attempts also at breaching the macro/micro gap. In his view,

> (e)ducation is a class-allocatory device, socially creating, maintaining and reproducing non-specialized and specialized skills, and

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[5]Furthermore, the notion of habitus "constitutes a conceptual straightjacket that provides no room for modification or escape" (1983).

[6]For discussions on the reception of Bourdieu's work outside of France, see: Bourdieu and Wacquant 1992, and Calhoun *et al* 1993.

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> relevance to the mode of production (1975, p. 185).

Although Bernstein rejects Bowles' and Gintis' argument on the correspondence between the dispositions acquired in school and those required by production, in his model, the determinant of school failure or success is also social class.

What makes Bernstein's work especially relevant for understanding the subtle mechanisms through which education participates in reproducing inequality, is his proposition that the effects of class on education are mediated by linguistic codes--elaborated/restricted--meaning systems--universalistic/ particularistic--and family socialization styles-- personal/positional (1975). As opposed to restricted codes, elaborated codes are forms of expression which are less tied to specific social contexts and thus tend to promote universalistic meanings. Along the same lines as Bourdieu and Passeron, Bernstein notes that schooling practices assume that all children are equally capable of accessing elaborated codes, universalistic meaning systems, and personal socialization styles, when, in fact, these are class-specific endowments.

As we discussed in Chapter One, modern educational practices are increasingly moving towards a less positional, more personalized form of communication, that is, a shift from visible to invisible pedagogies (Bernstein 1975). In his latter work (1990, 1996), Bernstein distinguishes

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between a 'discourse of instruction,' which creates specialized skills and their relationship to one another, and a 'discourse of regulation,' which creates order, relations, and identity. Although, as he argues, the regulatory discourse dominates over the instructional discourse--or, put differently, the discourse of instruction is always embedded in the discourse of regulation--this dominance may be more or less apparent depending on the pedagogical model at work.

Whereas visible pedagogy relies on the 'teacher initiation-student response-teacher evaluation' or 'IRE' routine (Mehan 1979, Cazden 1988), invisible pedagogy operates on the 'funnel' or 'elicitation pattern of interaction' ('EPI') (Bauersfeld 1992). The IRE is a modality of classroom discourse in which the teacher is supposed to know all the answers. Thus, her questions are 'pseudoquestions' in that, rather than motivated by a genuine search for information, these questions serve an evaluative and regulatory function. Pseudo-questioning is generally combined with 'double questioning'--i.e. repeating the question--a technique employed by teachers when the students deliver the wrong answers (Mehan 1979, Cazden 1988, Walkerdine 1988).

Instead, the EPI creates the fiction of a genuine, two-way communication between students and teachers. This modality of classroom discourse responds to the following

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structure: The teacher proposes an ambiguous, seemingly open-ended task. The students offer different answers and solutions, which the teacher evaluates. This phase corresponds to the claim that students are to be encouraged to make their own spontaneous discoveries, interpretations, and solutions. Yet, it so happens that, if the students' contributions are too divergent, through a funnel-like sequence of questions and answers, the teacher guides the students toward the correct answer. Inscribed in the overall regulative discourse of the classroom, the EPI appears as the best compromise resolution, albeit a precarious one, of the tension between two contradictory pressures: to elicit a clear-cut body of knowledge and to preserve the atmosphere of a liberal and child-centered classroom.

With their emphasis on subjectivity and the constant displacement of the teacher's authority, invisible pedagogies increasingly rely on personal as opposed to positional socialization styles. Since lower class children arrive to schools with 'restricted' linguistic codes, and are used at home to a positional socialization mode, often times, and despite their teachers' good intentions, this puts them in a disadvantaged position with respect to children from middle and upper classes.

For Bernstein, the question of *context* is fundamental to education in that formal education always entails de-contextualizing and re-contextualizing processes. As he

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explains, the procedures and performances by means of which children offer and receive communications, are acquired by them in informal everyday family and peer group experiences (i. e. 'primary contextualization,' 1975). Formal education abstracts from, and refocuses these procedures and performances in a process which Bernstein defines as 'secondary contextualization' or 're-contextualization' (ibid.) Although, as Bernstein argues, it is not possible to distinguish analytically between de- and re-contextualizing,

> empirically the *form* de-contextualizing takes may make very different demands on different groups of children, which affects the extent to which children acquire the performances required by the recontextualizing code (1975, p. 32).

I suggest that Bernstein's approach is particularly relevant for our matter at hand, in that, from all school subjects, mathematics is the one whose mastery demands the most radical de-contextualizations.

Together with the work of Bourdieu and Passeron and that of other 'reproduction' theorists (e.g. Bowles and Gintis 1977), Bernstein's contribution to the sociology of education was dismissed during the 1980s, particularly by 'resistance' theorists and 'radical' pedagogues.[7] Yet, in the last few years, the field of education has witnessed a critical revisiting of his work (Delpit 1988, Walkerdine 1988, Atkinson *et al.*, 1995, Sadovnik 1995). Despite their

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[7]See, for example, Giroux 1981, 1983; Aronowitz and Giroux 1985; McLaren 1986, 1989; Gore 1993.

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deterministic temptations, especially in their earlier work, theorists like Bourdieu and Bernstein offer a fruitful conceptual apparatus for understanding the mechanisms through which schools participate in the processes whereby social differences are produced and reproduced.

## 2.2. The Didactic Game

The power of these and other theoretical tools may be best appreciated when applied to a specific problematic. This is the case with the contributions to the field of mathematics education of French didacticians Yves Chevallard (1985, 1990) and Guy Brousseau (1986, 1988).

Informed by the work of Gramsci, Althusser, Bourdieu, Foucault, and Lacan, in *La Transposition Didactique: Du Savoir Savant au Savoir Enseigne* (1985), Chevallard takes as his object of inquiry the 'didactic system' of mathematics teaching. As he describes it, the didactic system is a relatively autonomous 'techno-cultural object,' affected by internal and external constraints. Whereas the external constraints stem from the relationship between the didactic system and the surrounding environment, the internal constraints are those conditions--e.g. the limitations of classroom time and of the teacher's epistemology--which directly affect the 'didactical relation,' that is, the threefold relationship linking teacher, students, and the 'knowledge-to-be-taught.'

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The didactic system's immediate environment is constituted, in the first place, by the system of teaching. Apart from the classroom teachers, this system is also made up of the group of experts who contribute to shaping the politics of mathematics education. Chevallard gives to this group the parodic name of 'noosphere,' i. e. "the members of the mathematics education intelligentsia" (1990, p.6). Aside from participating in the production of pedagogic discourses, noospherians must protect, within acceptable limits, the relative autonomy of the didactic system of math teaching. Yet, the survival of the didactic system presupposes that it be made compatible with its surrounding environment. The latter requires that the didactic system responds to the demands accompanying and justifying the social project of which it is the actualization. Yet, continues Chevallard, the very openness of the didactic system corresponds, to the closure of its consciousness. That is, the system must operate *as if* it where an autonomous entity. Paradoxically, its best response consists in ignoring all external demands.

Along the same lines, yet with a focus on the micro-level of the classroom, in *Fondements et Methods de la Didactique des Mathematiques* (1986), Brousseau examines the 'didactic contract.' The didactic contract is an agreement, implicitely signed by teacher and students at the beginning of each academic year, around a carefully constructed

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knowledge domain, commonly referred to as 'the curriculum.' Designating this knowledge as the stake in the shared project of teaching and learning, the didactic contract places teacher and students in the same site--i.e. the classroom--to play the 'didactic game.'

In its simplest form, the didactic contract establishes that the teacher is obliged to teach and the pupils are obliged to learn. On the one hand, the teacher is supposed to create sufficient conditions for the students' appropriation of knowledge, and to recognize this appropriation when it has taken place; on the other hand, the students are supposed to satisfy these conditions (i.e attempt to appropriate this knowledge). In particular, the teacher has to assure that the students' prior knowledge, as well as the conditions she offers give students the possibility for new acquisitions.

The didactic contract subjects teachers and students to a double-bind: in order to be obeyed, the contract must be broken. And, yet, the didactic relation has to continue by all means. As Brousseau argues, in line with a constructivist theory of learning, the teacher must be willing not to intervene, whatever happens, in order to give full responsibility to the students, and the students must accept the responsibility of solving a problem whose solution they have not been directly taught. Of course, it is impossible to anticipate in what ways the contract will

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be broken, and how will students and teachers get out of the crisis. Yet, meaningful learning is precisely what may result from the resolution of these crises.

In Chevallard's view (1985), a focus on the teacher-student relationship has obscured, more conspicuously in educational circles in the United States, the most transparent of all didactic facts: i.e. the didactical transposition of knowledge. 'Didactical transposition' refers to the passage from scientific knowledge (*savoir savant*) to knowledge-to-be-taught (*savoir enseigne*). As Chevallard further elaborates, in order for the teaching of any fragment of knowledge to be possible at all, it has to undergo a series of deformations. For teachers and students, the recognition of the didactical transposition destroys their happy participation in the didactic game. But, in fact, there can be neither teacher nor teaching without this fiction. As a result, the knowledge produced by the didactical transposition--i.e. school mathematics--will be a knowledge exiled from origins, that is, cut off from its historical roots in the scientific field--i.e. the mathematics of mathematicians.

Brousseau (1986) argues that the 'axiomatic' mode of presentation appears as the best suited teaching approach in that it allows to define the objects of study on the basis of the notions previously presented, thus providing a way for teacher and students to accumulate, in the least amount

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for teacher and students to accumulate, in the least amount of time, the maximum of knowledge which is closest to 'official' mathematics. Yet, he continues, this mode of presentation, which is typical of most mathematics textbooks, eliminates all traces of the process whereby mathematical knowledge is produced (e.g. the difficulties and questions which gave origin to the notions being taught, as well as the many conflicts within the discipline). In other words, the axiomatic presentation masks the true functioning of mathematics as a field of inquiry.

The didactical transposition is not just a school phenomenon; it is also part of the rhetorical functioning of the scientific community. For, before communicating the new knowledge they have produced, mathematicians must de-personalize, de-contextualize, and de-temporalize their findings. This reification is an indispensable condition, so that readers are not forced to retrace the route travelled by the inventor and, instead, may utilize these results to produce new ones, demonstrate their falsehood, or verify their truth.[8]

The student's role in the didactical transposition is, to an extent, similar to that of the mathematician. A good reproduction by the students of the activity of doing mathematics requires that they formulate, prove, and

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[8]Richards (1991), refers to this as the passage from 'research mathematics' to 'journal mathematics.'

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ability to apply these concepts, models, and theories to the solution of problems; and that they learn to communicate their findings to others.

The teacher's role is the reverse of that of the researcher and the student. She must first produce a re-contextualization and a re-personalization of the knowledge-to-be-taught. Yet, she must also re-decontextualize and re-depersonalize the knowledges 'produced' or 'constructed' in the classroom. In this way, students will be given the opportunity to identify the knowledges they have acquired--e.g 'school mathematics'--with the knowledged which are part of the scientific community. Thus, if the math classroom may be made to operate like a proto-scientific community, this is only a simulation, just as the mathematics presented in an axiomatic form is not akin to the real mathematics.

In Chevallard's view (1985), far from constituting a once-and-for-all accomplishment, the didactical transposition of mathematical knowledge is a process which entails ongoing efforts to maintain the equilibrium of the system of math teaching in the face of conflicting demands. Ordinarily, he suggests, the knowledge taught lives very well enclosed in itself, protected by "the closure of the didactic consciousness." In fact, the didactic functioning reveals a real capacity for producing knowledge for the

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purposes of self-consumption.[9] If the didactic system is able to feed itself with teachable knowledge, why is it, wonders Chevallard, that in certain moments in history, this 'pleasant economy' has to open itself up to assets which are not its own?

As he explains, the external constraints on the didactical system derive from the inevitable 'aging' (*enviellisement*) of school knowledge. On the one hand, school mathematics becomes 'biologically' old when its distance from the knowledge legitimated by the community of mathematicians increases to intolerable levels. Certain portions of school mathematics become obsolete. On the other hand, school mathematics becomes 'morally' old when it approaches too much the mathematical practices of parents and the common people. The result of this process of banalization is that the legitimacy of the school as the specialized transmission agency is questioned.

A delicate balance ought to be kept here. For, any attempt at decreasing the distance between 'real' math and school math may alienate parents and, similarly, correcting the moral aging of school mathematics, may further delegitimize it in the eyes of the experts. Noospherians may be called upon to re-establish the lost equilibrium. When mathematicians worry about the inauthenticity of school

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[9] A clear example of this phenomenon are 'the cases of factorization' taught in high school algebra, which do not correspond to any step in the constitution of algebraic theory (Chevallard 1985).

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mathematics, and the public at large worries about its inadequacy, the didactic system becomes doubly suspicious. And, in most cases, to re-establish the equilibrium, a new influx of knowledge appears as the best solution.

Chevallard 'applies' his model to the case of the 1960s' new math reform.[10] As a result of this reform, the entire mathematics curriculum--from the elementary to the high school level and college levels--was reorganized around the theory of sets. In the same stroke, the four arithmetical operations, a banalized technique, were replaced by 'operators,' entities endowed with a higher mathematical value. Yet, new problems soon emerged, as parents complained about the esoteric nature of the new concepts, teachers found these too difficult to teach, and the concepts themselves became banalized.[11]

Chevallard considers the process of aging from another point of view. When students do not learn, it is as if the knowledge would not pass through--i.e. knowledge has become too old for the children. Yet, since the students cannot be changed, we opt for changing the content, with the hope that renewing knowledge may help re-establish the students' appetite for learning. (The old problem of motivation!). Now, Chevallard asks, why would a modification in the

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[10]We may recall that, far from a product of American education, the new math or 'modern mathematics' was an almost worldwide phenomenon.

[11]E.g., the omnipresence of Venn diagrams, and the overuse and misuse of the language of sets (Baruk 1973).

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knowledge-to-be-taught eliminate learning difficulties? In fact, it does not. Instead, what takes place is a process whereby students' errors are re-classified. The rearranging of the text of knowledge brings with it a supposedly more accurate re-ordering of the pathologies of learning (cf. also Baruk 1973). Thus, within the conceptual boundaries of each reform circle, students' errors are interpreted as transgressions of the new system's laws, when these are not attributed to some kind of confusion.[12]

This analysis seems to indicate that, beneath all transformations in the text of school mathematics, the deep structure of the didactic relation remains intact. Far from precluding a closer inspection of the didactic game, this skeptical perspective suggests the need to examine how it continues to operate, despite well-intended efforts at reforming it.

Modern, child-centered, and constructivist conceptions of math teaching require that the teacher produce adaptations on the part of the students by means of carefully chosen problems or 'a-didactic situations' (Brousseau 1986). As Brousseau explains, the teacher offers to the students an a-didactic situation which, hopefully, will induce in them the most independent and fruitful activity. For that purpose, she will communicate or abstain

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[12]On the classification of children's errors in mathematics and a critique of circularity in the concept of confusion, see Baruk 1985.

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to communicate information, questions, hints, and techniques of learning. In this way, the teacher is centraly implicated as a didactic mediator in the system of interactions of the student and the given problem.

To this broader relationship--student, teacher, a-didactic situation (i.e. a problem-to-be-solved)--Brousseau gives the name of 'didactic situation.' The students know that the problem has been chosen in order for them to acquire a certain knowledge. But they must realize that this knowledge is entirely justified by the internal logic of the situation, and they should be able to construct it with no appeal to didactic reasons. This is precisely what will allow them later to put that knowledge at work in other contexts, outside of the didactic situation.[13]

Thus, although students' activities are determined by the milieu (or context) in which these activities are embedded--i.e. the school, the classroom, the math lesson--their responses to the task at hand should not be motivated by the obligations linked to the didactic contract. Instead, these are to result from the specific conditions and constraints of the a-didactic situation. In the course of the students' progress, the didactic representation of the milieu would approach 'reality,' and the relations of the learning subject with the milieu will increasingly be devoid

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[13]i.e. The old problem of 'transfer,' nowadays referred to as the problem of 'learning in context!'

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of didactic camouflages.[14] The final a-didactic situation of reference--which is what characterizes scientific knowledge--operates as an ideal towards which both are converging. For that to happen, it is part of the teacher's role to present students with camouflaged situations and, at the same time, help them strip these situations from their didactic clothing.

In its complex dynamics between didactic and a-didactic situations, the didactic game presents students and teacher with a menu of possible moves. The initial scene in the game, and the *sine qua non* for a fruitful didactic process, is what Brousseau calls 'the devolution of the problem' (1986). The teacher provides the students with a problem requiring a numerical solution--or a proof asking for a demonstration--and the students accept the responsibility for obtaining the desired result. As I see it, this seemingly unproblematic gesture opens up, for the students, a whole spectrum of subject positions. They can either accept or reject this 'gift,' joyfully read it or strategically scheme it, easily decode it or barely decipher it, recognize its familiar face or treat it as one more piece of irrelevant school matter, creatively reinterpret it or take it at face value, fully enact it or refuse to inhabit it, and consider it metacognitively or mechanically

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[14]For example, compare the dressed-up character of the word problems given to students in the early grades with the complex problem solving tasks which are presented in college algebra.

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solve it and move on to the next one.

It goes without saying that the best scenario results from the joint performance whereby the teacher has truly abdicated her responsibility for solving the problem, and the students have fully accepted it. And here comes the trick, which Brousseau calls the 'paradox of the devolution of the situation.' Although the giving of this gift occurs in the context of a didactic situation, it is implicit in the didactic contract that the students ought to be willing to act upon it out of an intrinsic desire to learn. For, in the long run, it does not pay for them to "try to get away with it," by accepting the suggestions which the teacher, under the pressure of two constraints--the limitations of the 'official,' or institutional teaching time, and her own epistemology of mathematics (Arsac, Balacheff, and Mante 1992)--may feel tempted to provide. That is, students will learn if and only if they are willing (and/or able) to take the risk not to learn--for example, by consciously refusing the teacher's help, usually presented in the form of suggestions or 'hints'--and if and only if the teacher is willing (and/or able) to risk that her students may not solve the problem--or construct the proof--in the expected time (or perhaps never at all).

Further elaborating on his metaphor of classroom practices as theatrical performances, Brousseau gives special names to the various pathological resolutions--or,

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as I call them, 'dystopian effects'--of the double bind linking students and teachers in the didactic game: the 'Topaz,' 'Jourdain,' and 'Dienes' effects, the paradox of the adaptation to the situation, and the paradox of the comedian.

Inspired by a scene from Marcel Pagnol, the 'Topaz' effect results from the following scenario: The student makes a gross orthographic error. The teacher cannot face it and, eventually, ends up telling the student, albeit indirectly, how to write the phrase. The 'Jourdain' effect (from Moliere's *The Bourgeois Gentilhomme*) results from a situation whereby the teacher 'finds' evidence that the student has learned what she has taught when, in fact, the evidence does not point in that direction.[15]

With the term 'Dienes' effect, Brousseau alludes to a manipulative designed by Zoltan Dienes--a key figure in the new math reform--which consists of a set of wooden cubes and bars designed to facilitate students' learning of various mathematical structures in the course of their play-like activities. In line with the prescriptions of a child-centered pedagogy, the teacher's role is limited to choosing the materials, explaining the 'rules of the game,' and

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[15]This effect was typical of the structuralist didactics of the 1960s: while the student would only see an example or a concrete object, the teacher would see a structure; or, at another level, whereas mathematicians recognize the teacher's use of math structures, the teacher is just using it as a didactical device (e.g. Venn diagrams).

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encouraging the students to play. This didactics frees her from any responsibility for teaching--she just has to wait and see, letting the students play, manipulate, think, and discover for themselves.

Brousseau underlines the circularity implicit in this game: the rules to play with the Dienes blocks are precisely those which have to be learned. If the teacher teaches the rule, the game becomes a mere exercise; if she doesn't, the game becomes one of 'guessing the rule.' As Brousseau critically notes, the Dienes effect results from a systematic confusion between the structure of the situation (the game), the structure of the task (e. g. addition), the intellectual process, and knowledge itself (the structures of mathematics).

Modern pedagogies assume that knowing derives, to a significant degree, from the fact that the students have acquired it by adapting themselves to a didactic situation; and, also, that for every piece of knowledge-to-be-taught there is a family of situations susceptible to give it an accurate representation. Yet, in many cases, these situations are not totally accessible, efficacious, or appropriate for the students' developmental stage. Thus, the teacher confronts 'the paradox of the adaptation to the situation.' That is, she must choose between teaching a formal knowledge in a meaningless way, or teaching a more or less false knowledge which later on the students will have

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to rectify.

Both the students who have been taught a portion of knowledge formally but with no meaning, and those who have been taught this same portion of knowledge in a meaningful, albeit distorted fashion, will see all kinds of contradictions in what they have learned. Here is the paradox: in order to learn, to a certain extent, students must refuse to understand; for, in trying to understand, they may run the risk of not learning. As suggested by Chevallard, a way out of this paradox requires we abandon the belief in the linear, cumulative, progressive character of didactic time, a belief which operates as one of the central psycho-pedagogic and didactic fictions.[16]

Finally, there is the 'the paradox of the comedian,' inspired by the work of Diderot. As Brousseau reminds us, the knowledge-to-be-taught is not the teacher's production. It is a cultural artifact, cited or recited. Its reproduction is more comparable to a theater piece replayed for the students than to a joint journey of discovery. If the students experience the pedagogic work of inculcation as a discovery, it is because their teacher, necessarily an actor, knows in advance what she will teach, and how the students will respond to her teaching. It is this very

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[16]Borrowing from Freud--via Lacan--the notion of the *apres coup*, Chevallard suggests an alternative, non-linear model of didactic time (1985).

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knowledge which allows her to create the illusion of discovery. As Brousseau presents it, this is more than just a metaphor: the teacher is a performer--with or without a text--engaged in making her students 'experience' the reproduction of knowledge as their own production.

Does the teacher have to reinvent each time her text as in the *commedia del arte*, wonders Brousseau, or must she stay faithful to an already rehearsed text? It is best that the teacher be ready to improvise, leave questions unanswered, and utilize students' questions in the course of the lesson. For, by introducing herself the questions and answers, she deprives her students of the possibility of agency. Yet, for an effective didactics to operate, the knowledge-to-be-learned has to be determined in advance, which means that this 'freedom' is nothing but the actor's game. In these classroom performances, the students are invited to be actors as well and, to a greater or lesser extent, both parts are restricted to a text which they simply cannot evade.

As discussed in the previous chapter, progressive, child-centered, constructivist pedagogical models often seem to go as far as to postulate the need for the teacher to be ignorant of the knowledge she is transmitting, so as to effect this passage in the most meaningful and convincing way. Apart from adressing the illusory nature of this postulate, Brousseau proposes a theory of didactical

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situations, which, acknowledging the inevitability of paradoxes, dilemmas, and dystopian effects, nevertheless contributes to making teachers aware of them and, thus, prepares them better for responding in which ways may foster meaningful learning.[17]

## 2.3 The Problem of Context

The concern with the anticipation and prevention of 'pathological' modalities of teaching and learning which we have just tracked in the heirs of French structuralist thinking, is also at the center of the two theoretical paradigms which continue to dominate the noosphere in the Anglo-Saxon world: cognitive science and clinical psychology. But what preoccupies experts working in these fields is not so much the operations of the didactic system understood as a contract, a relation, a game, or a performance, but the development of cognitive abilities in *individual* children. Thus, while French didacticians continue their work towards the development of a didactics of mathematics as a specialized field,[18] their American

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[17]Cf. also Artigue's 'didactic engineering' (1989, 1995) and Arsac, Balacheff, and Mante (1992). Within the French *didactique des mathematiques*, 'didactical engineering' aims at preparing classroom teachers to predict which meanings learners will construct, guess which didactical facts will appear in the classroom, and try to prevent students from constructing unintended conceptions.

[18]Similar developments have been taking place in the German didaktik of mathematics (e.g. Biehler *et al.* 1994, Klafki 1995, Menck 1995), and in the Dutch 'didactical phenomenology,' and more recently 'realistic mathematics education' (Freudenthal 1973, 1983;

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counterparts search for ways of 'applying' in the classroom constructivist theories of cognition and learning.

In the last two decades, constructivism has moved beyond the confines of cognitive psyschology, becoming almost common sense in the circles of math education. Just as, in general educational matters, there seems to be no choice but to be progressivist and support child-centered classroom practices, in mathematics education, one ought to be a constructivist.

Where does constructivism come from? Since the 1950s, the dominant position in cognitive psychology has been occupied by the constructivist conceptual apparatus produced by Jean Piaget.[19] As Piaget explains, in its efforts of adaptation to an environment which is the source of constant disequilibrium, living organisms develop increasingly higher-order responses, in a dynamic process of 'accommodations' and 'assimilations.' The formal operational stage of development which defines human intelligence, is the result 'reflective abstraction,' that is, a process whereby human beings construct increasingly general and abstract systems of knowledge (1972).

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Streefland 1991; Gravemeijer 1994; van den Heuvel Panhuizen 1996).

[19]Piaget 1941, 1948, 1950, 1954, 1970, 1972. As Piaget imagines it, in the last instance, the world stands on the feet of evolutionary biology. As he put it, although we "can never get back to the point where we can say 'Here is the beginning of logical structures,'" a regressive analysis will inevitably lead us into biology (1972, p. 19).

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Piaget posits an homology between the history of mathematics and the development of the mind (of reason and of logic). In fact, for Piaget, mathematics is reasoning. Logico-mathematical structures are the structures of rational thought. Therefore, within the conceptual boundaries of this rational cognitivism, success in mathematics is taken as an indicator of success in reasoning. As for the child's acquisition of number, undoubtedly the most central question in the didactics of elementary school mathematics, Piaget's investigations suggest that it is children's synthesis of two logical notions--i.e. classification and ordering--which constitutes the necessary and sufficient conditions for their understanding of number ([1941] 1952).

Although Piaget was never interested in issues of instruction, his theories have been used worldwide to build models for the teaching of mathematics.[20] Yet, pedagogues who have applied Piaget to the classroom have underrrated the depth of his work, by limiting their attention to the form of activities--e.g. one-to-one correspondences, equivalences, conservation, and so on--thus, neglecting the content aspect of these activities (Mellin-Olsen 1987). Mellin-Olsen takes advantage of a seldom-quoted remark by Piaget (1972a) in which he expresses his skepticism about

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[20]Duckworth 1987; Furth and Wachs 1974; Kamii and Devries 1978; Furth 1981; Ginsburg 1983; Kamii 1985, 1989, Fosnot 1989.

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the universality of the notion of 'formal operational stage':

> [i]t seemed that the ability to think ahead and use assumptions as a conducting means for reasoning, cannot be described generally. This kind of logic was more likely to relate to a particular profession, working experience, culture, and so on (p.21).

In general, Piaget's theory of the development of the concept of number in the child has been interpreted as an assertion that the term 'number' has a unitary meaning, so that it is possible to specify the necessary and sufficient conditions for having the concept of number, irrespective of the cultural practices and contexts in which numbers are used. As noted by Solomon (1989), although current psychological theories are concerned with explaining how children come to know numbers, the question of *what* children actually know when they know about numbers remains largely unexplored.

As for students' errors in school mathematics, psychologists tend to interpret these pathologies as delays in cognitive development. What results from this 'deficit' model (Bernstein 1975), is a system of classifications which indicates which attitudes and performances are to be taken as 'normal' and which ones are to be taken as 'pathological' responses (Walkerdine 1988). The recommendations flowing from deficit theories are limited to referrals to compensatory instruction (e.g. math labs), changes in pedagogical methods and in the teacher-student relationship

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(e.g. emphasis on rules, more explicit instruction, or use of manipulatives to facilitate abstraction), and/or changes within the family (e.g. mothers are invited to become more involved in their children's schooling).

This deficit approach has been challenged by empirical evidence which indicates that low-achieving students do not necessarily show delays in their cognitive development.[21] This research has demonstrated that school underachievement is linked to a complex set of social, cultural, and linguistic factors. Studies focusing on national, class, race and gender-based differences in mathematical abilities emphasize the effects of culture and language in the transformation of children's mathematical concepts.[22]

In recent years, learning has begun to be understood not as the individual's acquisition of knowledge, but as a process of apprenticeship whereby social actors become legitimate members in 'communities of practice' (Lave and Wenger 1991). This broader concept of learning applies equally to trades, languages, and games. Within this perspective, learning is explored as it occurs not only in classrooms, but also in supermarkets, streets, workplaces,

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[21]Bernstein 1975; Labov 1972; Ginsburg and Russell 1981; Ginsburg and Allardice 1984; Cocking and Mestre 1988.

[22]Luria 1971, Ginsburg 1983, Mellin-Olsen 1987, Solomon 1989, Walkerdine 1989, Watson 1990.

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and other social settings.[23]

What unifies researchers working within all these new perspectives is their attention to the role of *context* in mathematics learning. A growing consensus seems to have emerged --among 'mainstream' cognitive scientists and advocates of 'cognitive apprenticepship' models; radical, weak, and social constructivist math educators; and 'situated cognition' theorists, 'activity theory' researchers, and poststructuralist critics--that 'understanding number' is a question of mastering social practices, that is, of knowing how and when to use numbers appropriately in different contexts.[24] Yet, differences lurk underneath this apparent consensus. 'Context' has different meanings in each of these theoretical models (Chaiklin and Lave 1993). Moreover, simply 'adding context' to the existing individualist and rationalist models of learning and cognition will not do (Walkerdine 1988).

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[23]Lave, Murtaugh and de la Rocha (1984) studied the implicit math knowledge of middle class adults shoppers in Californian supermarkets; Carraher, Schliemann, and Carraher (1988) explored the oral mathematics of child vendors in Recife, Brazil; and Scribner and Stevens (1989) conducted experimental studies of the relationship between school mathematics and work mathematics.

[24]In the last thirty years, there have been significant changes in the approach to mathematics and mathematics education. The fruitfulness of these efforts is due, in part, to the crossing of disciplinary boundaries so that the discussion of math and math education and its social and cultural context touches on issues of labor (Sohn-Rethel 1978, Restivo 1985, Watson 1990), language (Pimm 1987, Mellin-Olsen 1987), everyday cultural practices (Rogoff and Lave 1984, Saxe 1988, Solomon 1989, Lave 1988), play (Winter 1991), critical literacy and community organizing (Frankenstein 1983, Moses *et. al.* 1989), and the interface of gender, power and subjectivity (Walkerdine 1989, 1990).

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During the early 1980s, Walkerdine and her collaborators have conducted a series of investigations about girls and mathematics (Walden and Walkerdine 1982, 1985). The interest of this work, whose relevance goes beyond a specific concern with gender, justifies a thorough discussion.

## 2.4. The Production of Reasoning Subjects

Walkerdine (1984, 1988, 1990) critically appropriates Bernstein's theory of visible and invisible pedagogies. Bernstein (1975) argues that invisible pedagogy relies on four presuppositions: a central role of the mother as a crucial preparing agent in socialization; a particular concept of time which presupposes a long educational life and implicit sequencing rules based on theories of the child's inner development; a concept of space according to which every space, not just the desk and the classroom, may become a space for learning; and a modality of social control through interpersonal communication which is based on elaborated, as opposed to restricted codes.

Central to Walkerdine's concern is the role that working-class and middle-class mothers are expected to play in their daughters' math performances (1988). As in child-centered models, the learning activities which are supposed to be part of children's everyday life result from the transformation of women's domestic labor into playful

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teaching. It is at this point that class differences enter the scene: in working-class homes there is less time and space available for transforming domestic routines into learning opportunities. While middle-class mothers can relax and create environments for their children to explore, discover, and learn, working-class mothers can barely keep up with their daily chores.[25]

Following Foucault, Walkerdine suggests that the different contexts in which learning takes place--i. e., homes, classrooms, playgrounds, streets--are best seen as specific social settings with different configurations of knowledge and power relations (parent/child, teacher/students, boys/girls, etc.). Far from constituting simple cases of knowledge transfer, the transformation of non-mathematical practices--e.g. cooking, gardening, shopping, and dealing with money--into mathematical practices (and vice versa) entails a change both in language and in the position of the learners with respect to this language.

As Walkerdine reminds us (1988), while in the 1970s British and American developmental psychologists were occupied with "grafting context into Piaget's genetic

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[25]Within the invisible pedagogy model, working class mothers are seen as lacking: they don't undertand children's needs, they help them the wrong way, they buy the wrong books and the wrong toys. The whole discourse of parental involvement assumes that teachers must teach mothers how to help their children. The target is invariably black and white working class parents. There is no sense of listening to and learning from these parents; they are already defined as wrong and reactionary.

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epistemology," continental social theorists were preoccupied with how to rework Marx's theory of ideology to account for the production of subjectivity. The writings of Althusser, Barthes, Lacan, Derrida, Deleuze, Guattari, and Foucault shaped new ways of problematizing the relationship of the material world to the social world, in which a central place is occupied by discourse and language. This shift is generally referred to as the 'linguistic turn' in the human sciences. In cognitive psychology, the linguistic turn resulted in a critical stance towards Piaget's work. For, the Piagetian system of thought assumes the primacy of individual action, thought, and logic, over language, instruction, culture, and other social relations. Simultaneously, there has been, in the Anglo-Saxon world, a growing--albeit belated--appropriation of the work published in the 1920s and 1930s by Russian psychologists Lev Vygotsky and Alexander Luria.[26]

But Walkerdine does not adopt a Vygotskian perspective. For, as she notes, drawing upon Saussure's linguistics, both the Piagetian and the Vygotskian psychological models conceive of the relationship of signifier to signified as one of representation, where the signified arises extra-discursively, that is, from the general coordination of actions. Instead, Walkerdine put forward the notion of

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[26]Vygotsky 1978; Luria 1976; Leont'ev 1978, 1981; Wersch 1981, 1991; Daniels 1993; Van der Veer and Valsiner 1994.

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'relations of signification'. This term refers to the complex phenomenon whereby words, objects, and actions are articulated in the contexts of specific discursive practices.[27]

Relations of signification occur within practices--e.g. in school, at home, in the experimental laboratory, etc. The subject-position of children, mothers, teachers, or adults in general, in these practices has an effect on how they 'make sense' of those significations. As Walkerdine reminds us, Saussure not only asserted the primacy of the signified over the signifier, but he also underlined the latter's arbitrary nature. Given this arbitrariness, the relevant question is

> [h]ow do children come to read the myriad of arbitrary signifiers-the words, gestures, objects, etc.-with which they are surrounded, such that their arbitrariness is banished and they appear to have that meaning which is conventional? (Walkerdine, 1988, p.3)

A suggestive example of the complexities surrounding the transformation of non-mathematical practices into school mathematics, may be found in Walkerdine's discussion of 'the shopping game' (1988). In this didactic situation, six and seven-year old children are given ten 1 pence pieces of plastic money and a pack of cards with pictures of items to be bought, accompanied by their price in pence. Children

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[27]This notion is proposed as an attempt "to understand the phenomenon of word/object/action, not as a relation of signification of an object by a word, a signified by a signifier, but as a complex, a relation of signification within practice itself" (1988, p. 15).

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take turns in picking up a card, decide on what to buy, and record the change from 10 pence as a written subtraction. Walkerdine argues that playing this game demands from children a delicate discursive shift. They must move from a 'real' life practice in which calculations are done mentally and as part of a purchase activity, into a classroom practice where calculations are performed with pencil and paper, and the final product is the calculation itself, rather than a purchase. At some point during the game, the teacher reacts to a students' confusion:

> I know what you did, you spent two pence there didn't you and that left you with eight pence and so the next time when your toy cost eight pence you thought you had nothing left, but it says start with ten pence every time doesn't it so you can work out again ten pence... (p. 157).

As we see, the teacher made explicit the fictional nature of the didactical situation. With her belated hint, she aimed at assuring the students' acquisition of the subtraction algorithm which, after all, was the ultimate goal of the activity. Walkerdine notes that the 'slow' learners--for the most part the girls--quickly accepted the solicitation to pretend that the activity was, indeed, one of shopping, and they got seduced by the fantasy of being rich (owning 10 pence). On the contrary, the 'fast' learners--for the most part the boys--were able to see through the a-didactical 'shopping game' scenario, the teacher's didactical intention to help the students learn to subtract. Rather unexpectedly, in this particular lesson,

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playing the shopping game as a contextualized and personalized classroom practice turned into an obstacle for some of the children to master the decontextualized and depersonalized discourse of mathematics.

As in Brousseau's and in Chevallard's work, Walkerdine's theorizing of the practices which produce mathematically literate subjects, depicts mathematics as a de-personalized and a de-contextualized discourse. This does not mean that doing school mathematics is a practice outside of a context, or one without a subject. If the addressee of a mathematical statement is always a non-subject, and if school mathematics takes the form of a de-contextualized discourse, this is because the transformations whereby the subject is repressed and the context is eliminated, play a central role in the functioning of the didactical transposition of mathematical knowledge.

The discursive practices of school mathematics *do* interpellate individual children as learning subjects; and this interpellation takes the form of contradictory solicitations. And the discursive practices of school mathematics *do* have a context, namely, the context of the school, which for children is, after all, very much their everyday life. The relative success with which children respond to the demands of de-personalization and de-contextualization is a measure of their potential for the mastery of mathematical reason.

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In so far as the ultimate goal is for students to master what, for many centuries, has been constituted as a subjectless and a decontextualized discourse, a central condition for mathematics teaching to be effective is that, notwithstanding the necessary didactical personalizations and contextualizations, it provides opportunities for students to acquire the ability to 'mathematize' reality and, eventually, to 'mathematize mathematics' (Freudenthal 1973, 1983; Gravemeijer 1995).

In the Dutch didactics of mathematics, *mathematization* is defined as the goal of mathematics education at all levels--from the early grades to the college years. Freudenthal (1973, 1983) regards children's mathematization as the activity whereby they move back and forth between the real world and the world of symbols. In line with Freudenthal, Adri Treffers (1987) elaborates on the distinction between 'horizontal' and 'vertical mathematization.' In horizontal mathematization one mathematizes real life contexts, that is, one translates the world of reality--i.e. a real life situation or problem--to the world of numbers--i.e. a mathematical model for that situation. In vertical mathematization one mathematizes mathematics; that is, operating within the world of numbers and other mathematical entities, one reflects and acts upon one's own activity of mathematizing, moving towards higher levels of de-contextualization.

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Cognitive scientists and learning psychologists still struggle to draw a clear boundary between 'procedural' and 'conceptual' understanding (Hiebert 1986). In mathematics, conceptual understanding is usually defined as a connected web, that is, as knowledge rich in relationships. Instead, procedural understanding is defined as the formal language or symbol representation system, as well as the algorithms or rules for completing mathematical tasks. These two modes of knowing--i.e. 'knowing how' and 'knowing why'--have also been referred to as 'conceptual understanding' and 'successful action' (Piaget 1978), 'relational' and 'instrumental understanding' (Skemp 1972), or 'semantics' and 'syntax' (Resnick 1982).

In 1923, enumerating his fourteen theses of educational measurement, William McCall made the following prognostic,

> In a still invisible future it may be possible to secure a 'movie' of a pupil's mental machinery when in operation and thus secure the desired information but for the present it is necessary to measure the product produced and, if desired, infer the inner condition of the pupil (p.5).

Although the clear window to students' minds has yet to be found, the desire to answer the question "What goes on in children's heads when they perform mathematical tasks?" (Resnick and Ford 1981) has not been abandoned. According to Resnick and Ford (1981), old learning theories--e.g. Edward Thorndike's connectionism--were good in process, but bad in content: the models of the mind produced by these theories were like 'opaque movies'--the mind was conceived of as a

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blackbox, but the model was useful for describing changes. The new learning theories are like 'transparent snapshots,' that is, they are good in content, but bad in process. The emphasis is on the static mental structures rather than on students' mathematical *performances*. In their view, what instructional theory needs

> is 'transparent movies.' Instructional processes cannot be greatly clarified and improved unless we can *both* look into the mind *and* chart changes in capability. Only then can we develop theories about how to present information and organize exercises in ways that will help students to acquire and strengthen mathematical skill and knowledge (p. 245).

Are we at last getting closer to the inner workings of the students' minds? The noosphere is divided on this question. For many, the future look promising, as artificial intelligence models are throwing new light on human thinking processes. Since the whole educational enterprise is predicated on the possibility of accessing the students' inner mental world--i.e. what is taught must be assessed--a complex chain of investigations have been set in place. Students investigate mathematical problems, teachers investigate students' problem solving strategies, and researchers investigate teachers' and students' activities as well as their own theories in action. If all this goes well, students will acquire mathematical power, teachers will assess the effects of their lessons, and researchers will further understand the whole picture, refine their learning theories, develop new curriculum and new materials,

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and device better teacher training courses.

Yet, this assessment chain is not protected against misunderstandings, misleadings, and misrecognitions. Students may mislead their teachers by simulating a level of understanding they do not really have (Bennett 1995); teachers may be satisfied in being mislead by these simulations (Brousseau 'Jordain effect'), and researchers may have to face the limits of the performance assessments and other instruments for gathering evidence of real understanding that they have devised and put at work.

Real understanding seems to be a very elusive state of mind. it is as if the more we become conscious of the need to really understand, the more we realize our limitations. In the words of a classroom teacher,

> I really would like to know what they [the students] are thinking while I'm talking to them. This never used to be a worry for me. Now, I realize that students are black boxes to me" (Thompson and Thompson 1995).

On the basis of observations, clinical interviews, and oral and written assessments, students' mathematical performances are generally interpreted along the lines of the distinction between 'rule-following' and 'real understanding.' Walkerdine elaborates on this dichotomy, as she considers the production of gender difference in mathematical ability. She notes that, whereas girls' math performances are usually interpreted by teachers as resulting from their obedient compliance with rules, the

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math performances of boys are usually read as evidence of their real understanding of the matter in question. Beyond Walkerdine's critical perspective on gender and knowledge, we see in her work a powerful attempt at deconstructing current conceptions of understanding.

Yet, nobody in this century has gone as far in this direction as Ludwig Wittgenstein. As his work seems to suggest, what if *all* mathematical practices are, in fact, rule following? In *Philosophical Investigations* (1953) and *Remarks on the Foundations of Mathematics* (1967), Wittgenstein proposes that meaning has to do with use, and that the way people use concepts is by following the rules they have learned as members of their language community. Thus, what may be said about our ways of counting and calculating is the same as what may be said about our language: not that it is true, but "that it is usable, and above all, it is used" (1967, #4). Within this framework, the very idea of an internal, private process of 'real' understanding becomes, to say the least, problematic. In other words, it may be nothing but a fiction and perhaps the most pervasive one, as it sustains the whole endeavor of teaching and learning.

French didacticians such as Chevallard and Brousseau have set for themselves the task of unveiling the truths which practitioners within and outside the didactic system--teachers and 'noospherians'--are invested in denying. As we

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have seen, the work of Walkerdine points in the same direction. Although the theoretical framework just presented relies heavily on these works, I remain skeptical as to the possibility of finding that vantage point outside the system from where all fictions can be named, all lies denounced, all secrets discovered, and the truth revealed. Instead, I suggest that, regardless of our theoretical perspective or our specific location in the academic field, we are all engaged in the process of creating, protecting, and reproducing these and other fictions.

The temptation to let oneself be seduced by one or another fiction may be linked to that constructive mood which is peculiar to the practice of education research. If, in education, we are made to feel the obligation of being optimistic, I give this statement a Gramscian turn, and invite an attitude which combines "a pessimism of the intellect with an optimism of the will." In other words, that we keep a vigilant eye on the seductive power of all math fictions--and the truth effects of the tools we choose for their deconstruction.

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# CHAPTER THREE: FICTIONALIZING ARITHMETIC

> How many cows can you buy for 2952 dollars, at the rate of 24 dollars each? (Brooks, *The Normal Elementary Arithmetic*, 1888)
>
> The total immigration into the United States during the last three decades was in round numbers: 5,247,000 (1881-1890), 3,688,000 (1891-1900), and 8,795,000 (1901-1910). What per cent of the total immigration of the 30 years entered during the first decade? (Byrnes, Richman, and Roberts, *The Pupil's Arithmetic*, 1922)
>
> When Tom was 11 years old, he averaged $1.50 a week in earnings.... In some of our factories children as young as 8 years old have been found sewing buttons on clothing. Perhaps you can find a book about 'child labor' in an encyclopedia. Find out all you can about it and explain to the class why child labor is a bad thing (McMurry and Benson, *Social Arithmetic*, 1927)
>
> The first telephone conversation in this country took place in 1876. Today, on the average, there are 148,082,000 daily telephone conversations in the United States. To the nearest million, what would this number be?... Do you see now that to find out about our world we need to understand and use numbers? (Carpenter and Pepper, *Making Arithmetic Clear*, 1957)

The cyclical repetition of remedies for the sorry state of elementary mathematics education in the United States invites an examination of old-time practices of teaching and learning this school discipline. Since the rhetorical gymnastics by which each new reform predicts its success includes a careful enumeration of its differences vis à vis previous interventions, it makes sense to address the question of repetition and difference from an historical point of view. This retrospective considers as its specific object the arithmetic textbook. The rationale behind this choice is that the tensions, controversies, contradictions, and dilemmas around this cultural artifact prefigure central issues in contemporary mathematics education.

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A complete history of arithmetic textbooks is still to be written, a strange omission, considering the central role of this instructional tool in the classroom.[1] Experts in the field find it astonishing that these "important and intriguing artifacts of a literate, industrial society," which have been "standard schoolroom fixtures for as long as most living citizens of this country can remember," have not been made a topic of inquiry by educational researchers (Woodward, Eliot and Nagel 1988, p.1).

Following Chevallard (1985), I suggest that this omission is best linked to the marginal status which US educational research allows for the knowledge or subject matter element of the didactic relation. Thinking on education tends to focus primarily on teacher-student interaction, within a loosely defined environment, setting, or context. I have identified this reduction as one of the math fictions operating in the field of math education: the triadic didactic relationship--students, teacher and mathematics--is reduced to a dyad.

While texbooks have been mostly ignored by mainstream educational researchers, a critical gaze has been placed on them by scholars working in neo-Marxist, cultural studies,

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[1]The National Society for the Study of Education (NSSE) produced two documents on textbooks, one in 1931--*The Textbook in American Education*--and one in 1990--*Textbooks and Schooling in the United States*. From the preface to the latter, we learn that not much has happened in the field since the 1930s, aside from the work edited by Cronbach (1955), and the volume co-edited by Cole and Sticht (1981), the last one resulting from a conference at the Library of Congress.

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and radical pedagogy fields. For example, Michael Apple considers the question "What do the social and economic relations within the publishing industry have to do with schools, with the politics of knowledge distribution in education?" (1985, p.8). This interrogation sets up the background for his discussion of the ways in which textbooks, as cultural commodities, have responded to the economic and ideological pressures exercised upon them.

Along the same lines, others critics have explored the ideological effects of a particular kind of textbook: the reader (Luke 1988; de Castell, Luke, and Luke 1989). Assuming that these texts articulate the authorized version of society's valid knowledge, these authors examine changes in the readers' mode of discourse and material quality, as well as in the institutional practices circumscribing them. Yet, since these studies focus on those portions of school knowledge "whose codification systems are primarily linguistic" (de Castell, Luke, and Luke 1989, p.viii), mathematics textbooks are excluded from consideration.

Arithmetic textbooks are disciplinary texts. As opposed to literary texts, disciplinary texts strive for linguistic transparency and for unambiguous presentation of factual data. While this fiction of transparency operates in all disciplinary texts, it is particularly strong in the case of arithmetic textbooks, due to the characteristics of this subject matter.

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As discussed in Chapter Two, school mathematics results from an alchemic process--i.e. didactical transposition--whereby mathematics is made into mathematics-to-be-taught (Chevallard 1985). This transposition involves a complex dynamics of personalizations and re-contextualizations (Chevallard 1985; Brousseau 1986, 1988; Christiansen, Howson, and Otte 1986; Walkerdine 1988; Dowling 1991). As a result of these processes, school mathematics is filled with all sorts of didactical 'camouflages' (Freudenthal 1973, Baruk 1973, 1985; Adda 1976). Yet, to a significant extent it preserves the high 'discursive saturation' of the field of knowledge which gave it birth.[2]

Arithmetic textbooks are pedagogical texts. As Luke explains (1988) in his analysis of the structure and function of readers in postwar literacy instruction in British Columbia, a textbook's pedagogical intent finds its expression in the various rhetorical and material devices employed by its author to elicit from the reader a specific kind of cognitive and behavioral responses (p.206).

Yet, like any other text, arithmetic books are situated texts, that is, they are re-written with each reading. As

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[2]Bernstein (1990) defines the degree of 'discursive saturation' of a field of knowledge as the extent to which it minimizes its reliance upon non-discursive elements. Mathematics is a clear case of high discursive saturation, in that it is an activity which is highly organized within language and, thus, produces generalized or decontextualized utteraces. Paul Dowling makes skillful use of Bernstein's framework in his analysis of two sets of mathematics textbooks currently in circulation in British schools (in Ernest 1994).

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suggestively put by Horace Mann in 1845,

> Text books contain a much greater proportion of isolated facts and of abstract principles, than of relations and application. This is the circumstance which gives pertinence and significance to their distinctive appellation--text books. They are books containing texts. *These texts the teacher is to expound. Each of them should be the foundation of a discourse, or a series of discourses. This is teaching* (cited in Caldwell and Courtis 1925, emphasis added).

Given that knowledge is not *in* the text, a clear a distinction must be kept between the text *per se* and the text as used in in the classroom. As the static counterpart to the dynamics of classroom activity, the textbook is both an object of this activity and a mediating artifact or "a catalyst in the reaction between the concept and the student's cognitive activity" (Keitel 1982, p.261).

## 3.1 On Arithmetic Textbooks

The few available studies of arithmetic textbooks offer a view of cumulative progress. It is proudly noted that, throughout its history, American textbooks have incorporated illustrations, diagrams, and photographic reproductions, and that this has enhanced their appearance, making them more palatable to their ultimate addressee, i.e. the learning child.[3] As described in the 30th Yearbook of the National

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[3]The addressee is always the generic 'Johnny.' In *Textbooks are Indispensable!*, by the American Textbook Publishers Institute (195?), we read: "What are Johnny's textbooks made of? Paper and ink? Words and pictures? Of these, yes--but not of these alone. Within the covers, behind the words and pictures lies the idea, the real substance of the textbook and the one that can spur Johnny's growth through all his years of schooling" (1956?, p. 31) 31).

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Society for the Study of Education (NSSE), which was dedicated to textbooks,

> The new education, with its emphasis on the child-centered school, has forced authors to give careful consideration to making the textbook interesting to children. In fact, the modern textbook hasn't a chance unless it can make reasonable claims that nothing has been overlooked in the way of motivation" (Schorling and Edmonson 1931, p. 47).

In most accounts of the history of textbooks, the omissions, errors, and trivializations found in most series are also acknowledged. However, it is generally noted that authors have taken care of these deficiencies, by responding to the suggestions of experts in the field. In a review of more than a hundred and fifty arithmetic textbooks in circulation between 1900 and 1957, Dooley (1960) found that, although writers tended to incorporate new recommendations within five years after these were made public, this was only when these recommendations were "clear, concise, and exact." The new was reflected in the textbooks only after reaching their authors in a 'popularized' form (Johnsen 1993, p.49).

Arithmetic textbooks have also been made the object of complaint. It is fair to say that textbooks have competed only with teachers in being blamed for the deficient quality of students' performances. The uneasiness caused by these little books, which is felt even by those who praise them, may have less to do with the slow pace at which authors respond to expert advice than with the many constraints

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under which arithmetic textbook-making operates.[4]

It suffices to inaugurate this review in the early 1880s. Around this time, the first rumors were heard of a 'crisis' in mathematics education; these were followed by anxious calls for changing content, methods, and materials; different ideas emerged on how to effect these changes, and these resulted in a sequence of more or less successful reforms. Whether timidly characterized as evolutions or vociferously announced as revolutions, these interventions aimed at establishing a new didactic contract for the arithmetic classroom, redefining the didactical transposition of mathematical knowledge, or both.

From the early 1880s to the mid 1950s, the arithmetic textbook went through an entire life-cycle. From the 1880s to the 1910s, it emerged as an indispensable character in the classroom stage. Printed in small-size font and generally non-graded, these textbooks were authored by well-known figures in the local mathematics education community, when they were not adaptations from British texts.

Between roughly 1911 and 1929, arithmetic textbooks experienced a peak in the number of volumes in circulation and in the variety of series available. The first graded

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[4]In his introduction to *Textbooks in American Society* (Cole and Sticht 1990), Daniel Boorstin points our attention to "the disproportion between the glamour and prestige of the authorship of textbooks and the significance of textbooks for civilization" (p.ix). If one considers the fact that textbook authors "are seldom the greatest thinkers of the subject," and that "in a free society textbooks are books chosen for us by somebody else," this may shed some light to this uneasiness.

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texts appeared, and some began to include 'diagnostic tests,' 'self-testing drills,' 'progress charts,' suggestions for classroom activities, small doses of 'pictorial and diagrammatical aids to understanding,' and practice material for students at different 'levels of ability.' Although many of these books still carried the signature of one or two leading figures in the field, some were now co-authored by school administrators and/or classroom teachers.

From the early 1930s to the end of the Second World War, arithmetic textbooks continued to change their face. Illustrations and all the new features listed above became a must, together with an increase in line spacing and font size, which made them easier to read. The books became more standardized, and the number of authors per textbook increased.

Finally, from the mid 1940s to the late 1950s, arithmetic textbooks--now with three or four authors, and often with no preface or introduction--continued to incorporate newly available printing technologies. If this gesture did not improve the quality of the material contained therein, it helped convey, in words and in images, a celebratory image of American culture and its post-war economic and technological progress. Yet, this sophisticated appearance and optimistic tone did nothing to prevent their imminent decline and final death.

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A word of caution is necessary here. It is not that textbooks suddenly disappeared. If the present review ends in 1957, it is because since then, with the birth of the new math, there will be no more *arithmetic* textbooks. In fact, 'arithmetic' as such will disappear from the curricula. For, the term

> 'elementary school mathematics' (instead of arithmetic) gives nominal recognition to the view that mathematics is a unified field of ideas beginning with primitive number classifications and extending to highly complex abstractions (Scott 1966, p.3)

What appears as an epistemological matter was, in fact, a decisive political move, strongly motivated by the Sputnik crisis.[5] During the new math era, the newly released 'elementary mathematics' series focused on sets, structures, and logic. Although during the 1970s' 'back to the basics' interregnum textbooks returned to their traditional emphasis on computation skills and word problems, they kept 'mathematics' in their title.

The period from 1882 to 1957 is demarcated by two State interventions in the matter of school texts. In 1882, that is, thirty years after the passing of the compulsory education law, and in the context of further regulating the process of schooling, Massachusetts became the first state

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[5]This sudden 'upgrading' of school knowledge was the strategy designed in order to offer American youngsters a better chance to compete with students around the world, and especially with their Soviet counterparts, who have been learning 'the real thing' all along. As an emergency solution to what was defined as 'a national security problem,' the new math attempted the most radical didactical transposition of mathematical knowledge to this date.

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to require of its school districts the provision of free textbooks for all students. This decree signaled the State's recognition of the textbook as a key component of classroom practices.[6] Almost eighty years later, the 1958 National Defense Education Act declared as a question of national security "the fullest development of the mental resources and technical skills" of American youngsters. This resolution resulted in the allocation of significant amounts of federal funds for updating curricula, and designing new materials for the teaching of foreign languages, sciences, and mathematics. In the decades enclosed by these two regulations--one aimed at 'equality,' the other at 'excellence'--textbooks operated as an indispensable instructional tool, accompanying teachers and students in their daily routines.

As suggested above, a decontextualized text provides a distorted picture of the text-in-use-in-the-classroom. Yet, since during the period in question, textbooks operated as a sort of bible for classroom recitations, they do offer a proxy access to the daily practices of arithmetic teaching and learning. As described by Cuban (1993),

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[6]This decision did happen without controversy. See, for example, in the proceedings from the Annual Meeting at the American Institute of Instruction, a speech on "Free Text-Books," delivered by Thomas Emerson, the Superintendent of Public Schools in Newton (Mass), dated in July 1887 (Emerson 1887). Ian Westbury notes that this policy implied regulation at the level of price and physical quality of the texts, but it involved no constraints on their content. This policy has survived to the present (Elliot and Woodward 1990).

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> With a largely untrained corps of teachers expected to teach a variety of subjects and skills it comes as no surprise that textbooks flourished. By the 1880s textbooks had already become the teacher's primary tool and the student's main source of knowledge. Published courses of study also determined for teachers was had to be taught and when. These syllabi were often stuffed with page listings from textbooks for each subject" (p.31).

That is, during this period, the expression 'textbooks define curriculum' came closest to the truth.[7]

The production of a textbook has always been a demanding enterprise. While 'curriculum-builders' have had a stake in deciding on the place of mathematics in an expanding curriculum which catered to a school population increasing both in raw numbers and differences in 'inborn abilities,' textbook authors have had as big a stake in selling their commodity to bureaucratically-minded school administrators, 'incompetent,' 'ill-prepared' and 'overburdened' teachers, and an always too skeptical public.

Together with other cultural commodities, textbooks are thrown into an increasingly diversified market (Apple 1988, Elliot and Woodward 1990, Altbach *et al.* 1991). In this competitive game, authors employ a variety of strategies, rhetorical and otherwise, to enhance their product. This situation, which goes back to the mid-18th century, when the

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[7]To a significant extent, this situation has remained unchanged to this day. Andrew Porter (1981) discusses the relevance of textbooks for classroom practices: "Over 75 percent of the teachers in a survey sponsored by the National Advisory Committee on Mathematics Education (NACOME) reported using a single textbook predominantly in the classroom, and 53 percent of the teachers reported that they followed the texts closely" (p. 19).

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first arithmetic texts were written and published in the United States, was acknowledged in 1879 by Edward Olney, the author of *Practical Arithmetic*: "The time has gone by when a single spelling-book, a single Reader, and a single Arithmetic will be accepted throughout the country."

As child-centeredness became the dominant pedagogical discourse, textbook authors have had to respond to even more conflicting demands. Without sacrificing scholarly content, their books had to be affordable and, yet, engaging and attractive; general, so as to reach wide markets, but respectful of individual, cultural, and regional differences; and 'readable,' especially since many of their users were recently migrated, non English-speaking children.

Yet, textbooks have never been just commodities; much more has been expected from them. Through their prefaces, these peculiar artifacts speak to us on the 'whats' and the 'hows' of arithmetic teaching. As stated in 1931, the textbook author

> should formulate a philosophy of arithmetic to guide him (sic) in his selection of content. For example, he must decide what position he will take on such issues as *skill versus understanding, formal drill versus incidental drill in projects, and computation versus social-economic problems* (Shorling and Edmonson 1931, p.49, emphasis added).

Aside from these polemical issues, textbook authors shed light on the question of why teach arithmetic at all. As a matter of fact, for the period under study, the 'whys' will be just as important as the 'whats' and the 'hows.' For a

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question was to be settled before any didactic contract was established, and any didactical transposition effected, "a question joyous of sound to many a boy and a girl wearied with obvious futilities--why should we teach arithmetic at all?" (Carson 1913, p.65).

As British mathematician and teacher George St. Carson insisted, this question may be asked not "in whimsical or revolutionary spirit," but in search of an agreement on "the motives which determine the appropriation of so many valuable hours in the life of a child to this one subject" (ibid. p.65). Although the answer to this question may be quite obvious today, we may recall that, ever since the rise of compulsory education, mathematics has had to defend its place in the curriculum, in the face of serious attacks.[8] As Carson himself vehemently put it,

> Unless fitting answers are ready, our work will stand condemned, the degradation of our subject to the domain of purely immediate utility will surely follow, as also the loss of that higher mental training which is so essential to the formation of an efficient citizen.

## 3.2 Arithmetic Textbooks and the Curriculum Field

The answers to Carson's question may help us identify the different actors who, during this period, struggled to define the curriculum for American schools. These

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[8]Among the defenses of mathematics as a school subject, see McLellan and Ames 1897, Judd 1928, Smith 1928, Buckingham 1930, and Reeve 1936.

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confrontations have been studied by many education historians, including Herbert Kliebard (1987), who organizes the curriculum field around four 'interest groups'--the humanists, the developmentalists, the social efficientists, and the social meliorists.[9]

What was the context of this debate? In the late 19th century, railroads and printed media helped transform the United States from a set of isolated rural communities into a modern, urban, and industrialized nation. Although, as Kliebard explains, these transformations had begun long before, it was only in the 1890s that an awareness of these changes brought about a sudden crisis. In the last four decades of the 19th century, the arrival of about fourteen million immigrants doubled the American population.[10]

Together with the factors previously listed, this demographic change generated "a psychic tension" (Kliebard) around what kind of America was in the making. In particular, these transformations had an impact on the

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[9]A similar classification was provided by Raymond Williams, in his discussion of the different ideological trends in 19th century British education (1961). Williams' three-fold classification--old humanists, industrial trainers, and public educators--is later adopted by Paul Ernest, in his analysis of the contemporary field of mathematics education in England (1991). In Ernest's scheme, there are five the interest groups struggling to impose their agenda; that is, to the three groups identified by Williams, he adds the technological pragmatists and the progressive educators.

[10]As for the educational system's expansion, Kliebard notes, in particular, the massive influx of students into secondary schools beginning around 1890. From 6 and 7% (in 1890) to more than 11% of 14 to 17-year olds (in 1900) was attending secondary school, to about 1/3 in 1920. By 1930, it reaches over 51 percent of that population.

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curriculum, that is, on "the set of emphases and omissions" (Williams 1961), through which that knowledge which is designed as worth transmitting to different classes of students is selected and thus reproduced.

Until the turn of the century, the hegemonic position in curriculum matters was held by humanists such as Charles W. Eliot, William T. Harris, Jacob W. A. Young, and David E. Smith. Humanists saw themselves as guardians of an ancient tradition, tied to the power of reason and the finest elements of the Western cultural heritage--a tradition which began with Plato and continued, with Kant, throughout the European Enlightenment. From this perspective, mathematics--which, together with grammar, literature and art, geography and history, constituted "the five windows of the soul"[11]--had its place in the school secured, not only for its contribution to the 'discipline and furniture of the mind,' but, also, due to its immutability and universality, the beauty of its abstractions, the purity and perfection of its

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[11]This expression belongs to William Harris. As defined in the *Report of the Committee of Fifteen on Elementary Education* (1895), chaired by Harris, "Side by side with language study is the study of mathematics in the schools, claiming the second place in importance of all studies. It has been pointed out that mathematics concerns the laws of time and space--their structural form, so to speak--and hence that it formulates the logical conditions of all matter both in rest and in motion. Be this as it may, the high position of mathematics as the science of all quantity is universally acknowledged. The elementary branch of mathematics is arithmetic, and this is studied in the primary and grammar schools from six to eight years, or even longer. The relation of arithmetic to the whole field of mathematics has been stated.... to be that of the final step in a process of calculation, in which results are stated numerically" (NEA 1895, p. 17).

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forms, and the certainty of its truths. In 1928, David E. Smith--a leading figure in the mathematics education community, and the author of many arithmetic textbook series in circulation during the first quarter of this century--thus elaborates on the value of arithmetics:

> So men may smile at the suggestion that mathematics is an *epic of ideas*, but only because they have not learned to read it in its tongue.... [Th]e most hopeful symbol we could give to attract the attention of a world much older than our own, and probably more refined, would be the figure of the theorem of Pythagoras, the squares on the three sides of a right angle (1928, pp. 15, 17).

In matters of learning, humanists ascribed to the doctrine of mental discipline. This doctrine, which may be traced back to the 18th century, assumed the mind to be composed of separate faculties--including understanding, abstraction, judgement, and reasoning-- each of which needs to be thoroughly exercised, as if it were a muscle. Edward Brooks--who, aside from being a mathematics educator was also a prolific turn-of-the-century textbook author[12]--argued that arithmetic was to be taught both as a 'mental' and as a 'written' discipline, as both contribute to strengthening the mind's reasoning power.[13]

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[12]Brooks was president of the Normal Department of the National Teachers' Association in 1876; he was Superintendent of schools in Philadelphia (1891), and a member of the NEA Committee of Fifteen. His first mathematics text appeared in 1859, and his texts were still in the market in the early 1920s (Bidwell and Clason 1970).

[13]Whereas the object of 'mental' or 'intellectual' arithmetic is discipline in analysis, that of 'oral,' 'written,' or 'practical' arithmetics is skill in practical calculation (Brooks 1879, 1880, 1901). About the former, Brooks says, "When properly taught, it gives quickness of perception, keenness of insight, toughness of mental fiber, and an intellectual power and grasp,

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Mental disciplinarians assumed the mind to be capable of transfer. That is, that knowledge acquired in one formal discipline--e.g. Latin, Greek, or mathematics--affects the learning of other disciplines. The dismissal of this doctrine is usually tied to William James' *Principles of Psychology* (1890) and to Edward Thorndike and Robert Woodworth's empirical demonstration of the lack of transfer in learning (1901). Although, at the turn of the century, humanists lost their hegemony in the curriculum field, they remained a boundary-keeping voice, defending the place of mathematics in the school, and expressing their concern for the deformations suffered by it in its didactical transpositions.

Against humanists, G. Stanley Hall and his disciples in the newly emerging 'child study' movement argued that the curriculum should be defined in terms of the child's natural development. Tied to romantic ideas about childhood, was developmentalists' trust in the power of psychology to determine the appropriate knowledge-to-be-taught at each developmental stage. Hall's seminal paper, "The Content of Children's Minds," written in 1883, informed educators on what children did *not* know. The bad news was that "teachers assumed too much about the content of children's minds--that a lot of Boston's children did not know what a cow was or a

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that can be acquired by no other elementary branch of study" (1879, p. 10)

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hill or an island" (Kliebard 1987).[14]

The impact of Hall's ideas was spectacular: the paramount criteria for deciding on the value of any a concept, procedure, or problem were the natural and evolving needs and interests of the child. With the twin assumption that children are naturally inclined to play and how early on a natural curiosity and interest for money, 'playing store' soon became the favorite activity for introducing young children into the world of numbers. Yet, leading specialists in the field objected that, despite all these studies of children, not much was really known about what children *did* know, understand, and enjoy. As vehemently expressed by Guy Buswell, Professor of Educational Psychology, and the author of many textbook series in circulation during the 1930s and 1940s,

> Just why all children should be interested in grocery stores has never been pointed out. Certainly such an interest is not native to the child, and one may harbor the suspicion that such an interest is the result of the group cooperation in the activity rather than interest in the grocery store (1927, p.90).

In any case, for developmentalists, the place of arithmetic in the elementary classroom was far from secured. For what could hold less elective affinity than the logic of numbers and the natural interests of the child?

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[14]Hall argued that, with urbanization and the consequences this processes had on children's experiences, schools could no longer assume that children brought with them the same concepts as in the farm days.

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As efficientism and social meliorism joined the field, other concerns emerged. Within this camp, the priority of Joseph Rice, Franklin Bobbitt, David Snedden, and others was not to so much the nurture of growing children, as the management of overcrowded and tax-burdening public school systems. In fact, from the early 1910s onwards, 'efficiency' and 'scientific management' became the ultimate buzzwords in educational discourse (Callahan 1962). Guiding these authors' social engineering views was the sense that a rapidly progressing society needed a much greater specialization of skills. This translated into a further differentiation of school curricula. And, for optimizing educational outputs, the secret resided in saving 'seat time' through the elimination of 'waste' (Bobbitt 1912).[15] In 1913, two years after the publication of Taylor's *Principles of Scientific Management,* Bobbitt declared,

> Within the past decade we have come to see that it is possible to set up definite standards for the various educational products. The ability to add at a speed of 65 combinations per minute, with an accuracy of 94 per cent is as definite a specification as can be set up for any aspect of the work of the steel plant.

Finally, social meliorists and progressive educators such as Lester Ward (1883, 1893), George Counts (1930, 1932) and Boyd Bode (1927, 1938), conceived of schools as major institutional forces in the path towards justice and social

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[15]Bobbitt (1912) argued that students had to be taught only what they would later use, and that, therefore, educators needed to institute a process of scientific management leading to the anticipation of students' occupations in adult life.

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change. For them, in order to have social value, school curricula had to address pressing problems such as corruption, vice, inequalities, and abuse of power, so that the newer generations would be best prepared for dealing with them. In this way, schools would not simply react to social change but help create new visions. In particular, social meliorism resulted in attempts at defining 'the social value of arithmetic.' Within this perspective, arithmetic problems were seen as more than occasions for the exercise of calculating powers and problem solving skills. In fact, some of the 'social arithmetic' textbook series circulating during the late 1920s reminded children that solving arithmetic problems would help them understand, sometimes reformulate, and eventually solve the 'big problems' of their times (McMurry and Benson 1926, 1927).

Different interpretations of the term *social* were at stake in social meliorism and social efficientism. Whereas the social *value* principle translated into attempts at 'socializing' or bringing social issues into arithmetic, the social *utility* principle invited a reduction of arithmetic to whatever was seen as useful in business, home, and daily life. By the end of the 1930s, social efficientists 'won' out over social meliorists, imposing their utilitarian and vocationalist approach to educational matters and, thus, sharing the dominant position with Hall's disciples and other developmentalists. An implicit alliance soon emerged

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between these two groups. This alliance resulted in the almost obligatory recourse to 'playing store,' as the arithmetically meaningful activity best suited to children's immediate interests, as well as to their future needs as shopkeepers and consumers.

Although this is only a simplified version of the story, it proves useful, to a certain extent, for 'classifying' arithmetic textbooks. If the four contending groups did not always offer "clear, concise and exact" recommendations, they did confront textbook authors, albeit implicitly, with challenging questions. The ways in which textbook authors accommodated to this embattled curriculum field may be best appreciated in the justifications they provide, in prefaces, forewords, and introductions, for the selection and organization of the word problems in their texts.

Interestingly enough, these micronarratives often depicted similar kinds of situations--e.g. buying farm houses; selling cows or trees; sharing apples; earning, saving, and spending huge or insignificant amounts of money; selling newspapers or cutting lawns; dividing a fortune among many daughters and sons; and counting the pages left before finishing a book. Yet, the placing of these problems in the text, and the justifications given for such choice, point towards four different narratives. First, a section on

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"Multiplication and Division Problems," indicates an organization based on the arithmetical operations involved; second, a chapter entitled "On a Farm Near Town" reads like story-telling; third, sections such as "Bank Accounts" or "The Arithmetic of the Home," announce that problems therein are classified according to well-defined applications; and, fourth, a story like "Making Bread," tells us that problems are embedded in broader social problematics.

As for the textbooks selected for this review, I have chosen those which, read against each other, as well as against the work of leading experts in the field, highlight three broader themes: the increasing psychologization and vocationalization of arithmetic; the uneasy relationship between arithmetic and the emerging child-centered paradigm; and the maskings and unmaskings of the didactical transposition of mathematical knowledge.

### 3.3. The Child vs. Arithmetic

The trajectory of arithmetic textbooks begins far back into the Renaissance with *Treviso Arithmetic*, published in the Venetian Republic in 1478, and considered the earliest known printed mathematics books in the West.[16] For our

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[16]The *Treviso Arithmetic* is an "algorism," that is, a treatise explaining how to use Hindu-Arabic numerals, and "intended for self study and relevant to commercial and reckoning needs." Yet, as Frank Swetz notes it (1987), the fact that it was written in the Venetian dialect, and not in Latin--"the language of scholarly instruction and education"--testifies to "its egalitarian mission of communicating knowledge to a broad audience."

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purposes, it suffices to begin with two 19th century American 'arithmetics'-- Thomas Dilworth's *Schoolmaster's Assistant*, dated 1743, and Warren Colburn's *First Lessons in Intellectual Arithmetics*, first published in 1821. Whereas Dilworth's text has been described as "a Pandora's box of disconnected rules, and a text which completely ignored "the existence of reasoning powers in the mind of the learner" (Cajori 1890), Colburn's has been praised for lifting "the mind from the slavery of dull routine to the freedom of independent thought."[17]

Dilworth's and Colburn's texts are as different as two artifacts of the same sort can be. This can be illustrated by the following excerpts, both taken from their beginning sections.[18]

> Q: What is Arithmetic?
> A. Arithmetic is the Art or Science of computing by Numbers, either Whole or in Fractions.
> Q. What is Number?
> A. Number is one or more Quantities, answering to the Question, how Many?... (Dilworth)

> 1. How many thumbs have you on your right hand? how many on your left? how many on both together?
> 2. How many hands have you?

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[17]As Edward Brooks continued, "Through the influence of this little book, arithmetic was transformed form a dry collection of mechanical processes into a subject full of life and interest" (*Philosophy of Arithmetic*, p.iii).

[18]As for the authors' view of the relationship between arithmetic and the child, consider the following two statements: "I have thrown the Subject of the following Pages into a Catechetical form, that they may be the more instructive; for Children can better judge of the Force of an Answer, than follow Reason thro' a Chain of Consequences" (Dilworth). "...the questions are asked in such a manner, that, if the instructor pursues the method explained in the Key, it will be almost impossible for the pupil to perform any example without understanding the reason of it" (Colburn)

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> 3. If you have two nuts in one hand and one another, how many have you in both? ... (Colburn)

It is against the background of these two opposing models--one centered on the logic of arithmetic, the other on the mind of the child--that the material which constitutes the object of this study, i.e. the prefaces and introductions of arithmetics textbooks in circulation between the late 1880s and the mid 1950s, will evolve.

In 1888, Edward Brooks published a new version of his *Normal Elementary Arithmetic,* which first appeared in 1865. The front page included the motto "The highest science is the greatest simplicity" and, as the title page announced, the text embraced "A Course of Easy and Progressive Exercises in Elementary Written Arithmetic designed for Primary Schools, and Primary Classes in Common Schools, Graded Schools, Model Schools, etc."

In the introduction, Brooks explains that the "Normal Method" differs from the German-born method popular at the time--the "Grube Method"--in crucial ways. The latter instructs students in all four arithmetical operations simultaneously; it does not go beyond the teaching of the first ten numbers in the first year of school; and it aims at students' *comprehension of number.* On the contrary, the Normal Method covers a larger scope of topics besides the fundamental operations with small numbers; and its aim is *skill in the operations with number.*

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The distinction between comprehension of number and skill in its use will soon become a central topic in the mathematics education field (e.g. mechanical drill vs. meaningful understanding, and rule-following vs. real understanding). Underneath these distinctions lurks the problematic assumption that following a rule 'blindly' is an accomplishment of a different--and lower--level than following a rule 'with understanding.'[19] Moreover, it is commonly feared that one form of acquisition may preclude the other, as Truman Safford noted, in his preface to Eaton's *The New Arithmetic* ([1885] 1901):

> The golden mean between doing everything by mechanical rules, but accurately, and thinking out every process with the possibility of mistakes at some point, is hard to hit; and the successful teacher is he or she who avoids both extremes (p.?).[20]

Back to Brooks' preface. He mentions the three distinctive features of his text: simplicity, gradation, and the inclusion of practical problems. As to simplicity,

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[19]"We are trying to get hold of the mental process of understanding which seems to be hidden behind those coarser and therefore more readily visible accomplishments. But we do not succeed; or, rather, it does not get as far as a real attempt. For even supposing I had found something that happened in all those cases of understanding,--why should it be the understanding?" (Wittgenstein 1953, # 153).

[20]There appears to be something mysterious and rather deep about the notions of meaning and understanding. Questions such as 'How do we measure real understanding? (NSSE *45th Yearbook*, 1946) and 'What is the meaning of meaning? (NCTM *21st Yearbook*, 1953), testify to an awareness of their elusive nature within the mathematics education community.

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Brooks explains that

> Great care has been taken to make definitions, explanations, solutions, rules, etc. so simple that the youngest pupils can easily understand them. In this, however, the scientific character of the subject has not been sacrificed; for it should ever be remembered that *the highest science is the greatest simplicity;* and conversely, *the highest simplicity is the highest science.*

Concerning gradation, he stresses that "great pains has been taken to avoid those sudden transitions from the easy to the difficult for which elementary works are so often criticized." Finally, as to practical problems, he concedes that

> Arithmetics have been criticized for the abstract and unpractical character of their problems. To avoid this error, I have given a large number of practical problems, drawn from the actual events of life.

Among these are "Historical, Geographical, and Biographical problems; problems on the battles of the Revolution; farmers', merchants' etc. problems," all of which "not only will add interest to the study of arithmetic, but will present much valuable information to the pupil."[21]

The common distinction at the time was between 'intellectual' and 'practical' arithmetic. In his *Key and Manual to the Normal Union Arithmetic and Also Methods of Teaching,* dated 1878, Brooks asserts that "all Arithmetic should be practical." Yet, in his view, a concern for the

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[21]Interestingly enough, the section on practical problems in Brooks' textbook includes typical, rather unrealistic scenarios such as "If a man walk[s] 112.1184 miles in 9.16 days, how many miles does he walk each day? Ans. 12.24 miles"

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practical is not to affect the discipline's integrity. After all, as he states in the first chapter, "Arithmetic is the logic of numbers, and hence its truths and principles should be derived by logical processes" (p.5). Two principles are, thus, made to operate, i.e. scientific and practical. Whereas the former means "such an arrangement as the logical development of the subject suggests," the latter is "the adaptation to the wants of pupils in pursuing the study." In fact, as Brooks continues,

> One of the most important and difficult things in the preparation of a text-book of this kind, is the adaptation to the many different classes of pupils who are to study it.

Brooks comments also that "until within a few years, American text-books and American instruction almost completely ignored the principles of the science, making Arithmetic to consist entirely in the solution of problems." As he lamented, this is "a great error, and one most pernicious to mental discipline." As a good mental disciplinarian, Brooks ascribed to the view that "Mental strength is the result of mental labor," which meant that the science of number is to be made simple enough for students to grasp, yet not so simple as to spare them from exercising their mental muscle.

As opposed to the introduction to *Key and Manual*, the preface to the *Normal Elementary Arithmetic* does not criticize prevailing trends in textbook-making. Instead, in the new text, Brooks shows his willingness to adapt to the

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changing times. And these are clearly different from the 1860s, when the mental discipline doctrine was still unchallenged. Yet, in his promise that the "scientific character" of the subject matter has not been "sacrificed," he points to the seriousness of what was at stake: preventing the scientificity of the whole enterprise from degrading into didactical nonsense.

This polemical stance brings us to the core of the debate at the turn of the century. It is at this time that arithmetic--"the science of numbers and the art of calculation" (Brooks 1878)--first confronted the young child, its developing mind, and its surrounding environment. In the decades to come, developmentalists would put forward a didactics which would go as far as to sacrifice the structure of the discipline in the *interest* of the child. Within this perspective, the changing needs and idiosyncratic interests of the child were seen as demanding a didactical transposition--otherwise, the subject matter *in vivo* would do violence to the child's natural development.

For Brooks, it is not interest but rather mental discipline that linked arithmetic and the child. An elective affinity is assumed between these two 'entities' such that, it is precisely the former's very difficulty that helps build the latter's mental power. The dismissal of this doctrine will require a redefinition of this link.

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Around the late 1880s, the first cries began to be heard about the damage done to children by their 'excessive' exposure to arithmetic. Accusations became harsher: the subject was too abstract, too difficult, and too stressful. As recommended in 1887 by the Boston School Board,

> All exercises [...] should be confined to small numbers, and to simple subjects and processes, the main purpose throughout being to secure thoroughlessness, accuracy, and a reasonable degree of facility on plain ordinary ciphering.... In 'practical problems' and in examples illustrative of arithmetical principles, all exercises are to be avoided in which a fairly intelligent and attentive child of the age concerned would find considerable difficulty in making the statement which is preliminary to the performance of the properly arithmetical operations (cited in Cajori 1890, pp. 111-2).

Along the same lines, in 1895, the *Report of the Committee of Fifteen on Elementary Studies* decreed a reduction in the number of hours devoted to arithmetic, and the exclusion of anything but incidental number work during the first year of schooling.[22]

As the century closed, a new pedagogical model began to emerge: the child-centered school. This was an environment conceived less as a gymnasium for exercising the 'mental muscle,' than as a garden for the cultivation of children's

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[22]Although the 1895 Report, signed by the Committee's president William T. Harris, is considered a humanist education manifesto, in its 'reductionist' gesture, it announces already the weakening of the mental discipline doctrine.

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physical and mental abilities.[23] More fundamentally, it was believed that regulating children by overt discipline was not the most effective technique for keeping an orderly and productive learning environment. Techniques predicated on the basis of children's thinking capability, subjectivity, and freedom to play were now seen as more conducive towards learning.[24]

Yet, among all school disciplines, arithmetic was the most resistant to these winds of change. What was an arithmetic lesson like in those days? Joseph Rice, a young pediatrician who abandoned medicine to dedicate himself to education, is perhaps in a good position to respond. In

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[23]This new perspective gave rise to numerous experiments in progressive schooling, from Colonel Parker's Quincy School, to Dewey's Laboratory School. See *The Quincy Methods Illustrated*, by Lelia E. Patridge, 1886; and Dewey's writings on the Laboratory School, which operated between 1896 and 1904, at the University of Chicago. E.g. "My Pedagogic Creed" (1897), in the *Early Works*, and "The Dewey School: Introduction" (1936), in the *Later Works*.

[24]The transformation in the discursive practices of school discipline from overt to covert was a slow and complex process, which entailed a radical redefinition of the notion of children's school work. Consider, for example, the following two excerpts, the first from Jas Hughes' *How to Keep Order* (1893), and the second from William Bagley's *School Discipline* (1916), "Order is positive, not negative. It is the conscious working out of definite aims in productive activity... order is work systematized.... It is only when the disciplining agencies work from within outwards, that this powers become progressive, and productive to their fullest extent" (p. 17). "It is a paradox of the well-disciplined school that discipline is conspicuous by its absence.... The well-disciplined school is 'dominated by the fashion of good order.' Some call it interest of pupils in their work, other call it absorption on problems that appeal to them, or spirit of cooperation, or sympathy." The most sophisticated and influential analysis of this process can be found in Michel Foucault's *Discipline and Punish* (1979). See, along the same lines, Chris Jenks, "Decoding Childhood," (Atkinson *et al* 1995). Jenks discusses changes in the regulation of childhood, in a process he describes as the transition from the Dionysian to the Apollonean child.

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1892, Rice took a surveying tour throughout the nation's schools, observing 1,200 teachers in 36 cities.[25] Included in the events he collected was a geometry lesson in which children took turns standing up and describing the geometrical figure each of them had been given in a little flag. As Rice commented (1893), the procedure was done at such high speed that, in a few minutes, about seventy children got to recite.

A detailed description of the teacher-centered, 'recitation' model for written arithmetic may also be found in Edward Brooks' *Key and Manual* (1878). This model involved the following steps: preparation of the blackboard (a few pupils are asked by the teacher to come to the board; they divide it with vertical lines, and stay still, waiting for the teacher to proceed); assignment of problems (in the lower grades, the same arithmetic problem is given to all pupils; in the upper grades, different problems may be assigned); writing the problem; working the problem (during this process, "neatness and exactness" are a must); pupils' explanation of the problems (this is to be done "in full and natural tone of voice," and standing "in an erect and easy attitude with the pointer in one hand and the other hanging down by the side" p. 30); criticisms (the other pupils are invited to comment on the solutions: they may correct errors

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[25]As described by Cuban (1993), Rice found "untrained teachers, unimaginative methods, and textbook-bound instruction" in all, except for a few classrooms.

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and suggest improvements); finally, teacher's explanation of the problem (the teacher is to make sure that difficult problems are left unexplained for a week or two "to test pupils' strength" for, pupils must assure "by severe labor" that the problem "is beyond their powers" p. 32). While the textbook is to be followed *a la lettre*, it is recommended that, through the presentation of "new matter," the teacher awakens in students "the desire of going beyond the limits of the present text-book" (p. 32).

In the decades to come, the emerging child-centered paradigm--one which is predicated on the educative value of play--put this recitation model into question. Yet, although number and play will be made to coexist in classrooms and textbooks, this process would not take place without resistance. Again and again, arithmetic--especially with its tiresome 'multiplication tables' and other 'number facts'--will bring back the specter of work through the classroom's back door.[26]

It is in this context that our second textbook came to life. Addressed to first grade teachers, *Primary Arithmetic* was written by William Speer--an Assistant Superintendent of Schools in Chicago--and was first published in 1896 by Ginn

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[26]As Walkerdine suggests (1988), this can be read as a sort of return of the repressed sign: since its inception, in the mid 19th century, compulsory education was designed, among other functions, as an alternative to child's labor. See, also, Zelizer 1985, and Katz 1986.

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and Company.[27] Together with Colburn's text, this text is considered the most radical contribution to 19th century arithmetic teaching (Bidwell and Clason 1970). As opposed to most didactics at the time, which were based on counting, Speer's 'ratio' method focuses on the comparison of quantities.[28] The first step does not consist, as in Brooks's text, in asking children to count from one to ten, but in inviting them to explore their surrounding environment, to search for objects of different shapes--an activity designed primarily as a form of *sense training*. As the author states it, when children engage in activities such as seeing, hearing, touching, cutting, drawing, and so on, they are inducing "judgements of relative magnitude."

Children taught a la Speer are set free to explore the world around them, with minimal directives from the teacher. It is important, recommends Speer, to

> put things before the pupil and leave him free to see and to tell all he can before interfering with his action by questioning or direction.

The opposition between the logic of the discipline and the mind of the child is hinted at clearly: "The questioning may be logical, but the learner connects only that which he himself relates." In this approach, "the child is not required to build out of parts a whole which he has never

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[27]Soon to become one of the main publishing houses in the United States textbook industry.

[28]In this respect, Speer follows Herbert Spencer, for whom comparison is the basis of human thought (Bidwell and Clason 1970).

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seen...." As we see, Speer has no remorse in sacrificing the structure of arithmetics.

A third term mediates between the opposing pair child-arithmetic: interest. It is assumed that, just as in real life children are naturally interested in exploring the world around them--by touching and handling things--in school, they should also be given the opportunity to do so. Moreover,

> Interest will lead the child to control himself, but repression from without induces dullness, indifference, and antagonism.

Interest is asked to play as demanding a job as that played by discipline in the classical model. Yet, this depends on another term: freedom. When students are given the freedom to express themselves, this not only contributes to their normal development, but provides valuable information to educators about their natural needs and interests. Free work reveals the pupil and makes it possible to meet his needs.[29] With Speer, key notions of the child-centered model such as interest, self-control, self-activity, freedom, and 'the whole child' are made central to the didactics of arithmetic.

Speer was not the only didactician working along these

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[29]A similar view can be found decades later in Dewey's *Experience and Education* (1938). In reference to an "increase in outward freedom," he argues that "without its existence it is practically impossible for a teacher to gain knowledge of the individuals with whom he is concerned. Enforced quiet and acquiescence prevent pupils from disclosing their real natures" (p. 62).

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lines. In 1897, James A. McLellan, a Canadian high-school inspector and principal of a school of pedagogy in Ontario, and A. F. Ames published their *Public School Arithmetic* (MacMillan Company), a text whose indebtedness to McLellan and Dewey's *Psychology of Number* (1895) is made clear in the title page.[30]

McLellan and Dewey emphasize the shortcomings of the two common methods for teaching arithmetic, i.e. "blind manipulation of symbols" and "physical operations with things." In their view, both are "vitiated by the same psychological error," namely, ignorance of the fact that number "arises in and through *the activity of the mind dealing with objects*."

Along these lines--and like the text by Speer--*Public School Arithmetic* favors an activity-based approach. Yet, in McLellan and Ames' text, the focus is on measuring rather than comparing. Moreover, I suggest that the notion of activity operating in McLellan and Ames' text is of a different kind than that at work in Speer's. As the following paragraph suggests, it is closer to human labor than to children's play:

> what *activity* is represented by the pound, by the bushel, by the foot? What is the connection between the decimal system and the ten fingers of the hands? What activity does the dollar stand

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[30]In turn, the authors of *The Psychology of Number* acknowledge also the influence of William James's *Principles of Psychology*.

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for? (emphasis added).[31]

The authors of *Public School Arithmetic* dedicate some space in their preface to comment on the fierce reactions against arithmetic as a school subject. Given that, in their view, this is due to "irrational" teaching methods, far from removing the subject or reducing the time assigned for its study, what is needed is an improvement in its teaching.

Three narratives of the child's acquisition of the notion of number have been offered thus far. In Brooks' world, number results from the universal and compulsory counting routine. In Speer's universe, number is the natural product of the child's free activity of comparing objects. And, for McLellan and Ames, number emerges from the social activity of measuring quantities.[32]

We will also encounter the 'measurement' or 'ratio' approach to arithmetic in *Myers' Arithmetic,* published in 1908 by Scott, Foresman and Company.[33] Myers, Professor of Mathematics and Astronomy College of Education at the

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[31]For McLellan and Dewey (1895), number is not a property of things but "a mode of measuring value." It "does not belong to things in themselves, but arises in the economical adaptation of things to some use or purpose..." In other words, number "is not (psychologically) got *from* things, it is put *into* them."

[32]This approach is strikingly similar to the socio-historical perspective developed in Russia during the 1930s by Lev Vygotsky, which later became the "activity theory." As noted by Wertsch (1991), in many occasions, Dewey underlined the limited potential of American psychology to account for social phenomena due to its focus on the individual organism (1991). See, also, Wertsch 1981.

[33]Many decades later, Scott, Foresman and Company will become one of the most prolific textbook-publishing houses in the United States.

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University of Chicago, tells us that "the arrangement of number work for the grades must be in accordance with the natural unfolding of the child's mind." This means that "measurement is the most direct and certain route to correct notions of number." As Myers continues,

> there can be no contradiction between the demands of the child's mental development and the logical requirements of the subject.

It is only "when logic is construed to mean the procedure of adult mind that the demands of logic become mischievous in the elementary school." Fortunately, he adds, "recent work of educational experts has proved that the best nurture for the child *as a child* remains always the best nurture he could have had."

3.4 Arithmetic for the Child

> We halt often between the logical and the psychological order, failing to appreciate that evolution gives place to each. The logical order is the order of proof, of demonstration; the psychological order is the order or discovery, of learning. Children do not learn logically; they come later to see logical relations in what they have learned. The well-equipped teacher knows both logic and psychology. He (sic) is prepared to guide the pupil in his natural course of learning, and also to point out to him the structure of relationship of what he has learned. Text book writers the world over have been slow to see this distinction; but with but few exceptions, the best American text books, which control so powerfully all school processes, are in advance of those most in use in Europe. The logical order is so simple, so coherent, and so attractive, that it seems a pity to surrender it for the less trim and less precise order of development; but this will have to be done if teaching efficiency according to evolution is to be had (Butler 1900, pp. 313-4).

With these words, Nicholas Murray Butler addressed, in

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Chicago, the Department of Superintendency of the National Education Association.[34]

In his *Complete Graded Arithmetic* (Fourth Grade), published in 1901, George Atwood acknowledges the existing pressure for arithmetic to defend its position "as one of the most prominent subjects in our schools." Yet, rather than locating the source of negative reactions against arithmetic in "irrational" teaching methods, Atwood puts the blame on the "character" of the textbooks used. It is rather self-evident that

> the use of a book which is unpedagogical in plan, arrangement, and character gives meager results, while the use of a book which is in harmony with the laws of mental development and mental growth gives results which are conmensurate with the time and energy devoted to it.

Atwood is committed to preparing "a text which is entirely adapted to the needs and gradually developing powers of the child." How does this translate into the "character" of his text? The author of the *Complete Graded Arithmetic* confesses to have made no attempt at presenting "any phase of what may be termed the 'science of numbers.'" He continues,

> Formal definitions, principles, and rules are omitted. There are few explanations and directions on how to perform any type of work. A child is confident and self-reliant because he is conscious of his own power. He must therefore be trained to rely upon himself rather than to follow a printed rule or the direction of the teacher.

Many textbook series of that time were designed, like

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[34]Butler's speech was entitled "Status of Education at the Close of the Century."

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Atwood's, with the growing and autonomous child in mind. However, books such as *The Felmley and Shutts Arithmetics* (Book I)--written in 1904 by David Felmley and George Shutts[35] anticipated a newly emerging approach to curriculum matters: social efficientism.

In this text, the activities of measuring, cutting, and so on, are chosen also as the road for introducing children into arithmetic. The authors--the first, President of the Illinois State Normal University, and the second, Institute Conductor and Teacher of Mathematics at the State Normal School in Wisconsin--recommend that

> Much work with the ruler, scissors, blocks and splints is required, because mathematical results worked out in this way mean more to the pupil, and are more easily remembered; and because, being a form of *manual training*, it helps the pupil to gain control of himself physically. Every child should have at hand a twelve-inch ruler as an accessory to the work (emphasis added).[36]

In their view, abstract processes--"a set of instruments which have no particular value in themselves"--have a subordinated function vis a vis concrete problems, which, "if well chosen, represent wants of society." Felmley and Shutts bring their instrumentalist approach further home by reminding us that

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[35]The Felmley and Shutts series was published by McNally and Company.

[36]The first lesson, entitled "Relations of Simple Numbers," begins with:
1. Make a straight line with the ruler.
2. Make 5 such lines.
3. Make 2 dots. With the ruler draw a line from one dot to another.
4. Make one of the lines which you have drawn, longer.

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> No one ever adds a column of figures for the sake of the addition, but he may use *the instrument addition* to find the value of several pieces of property.... The divorcing of pure arithmetic from its true function is equivalent to teaching the construction of a machine instead of training for skill in its use... (emphasis added).

This preface includes the almost obligatory complaint that "pupils cannot add or perform the other operations of arithmetic with accuracy and facility." For Felmley and Shutts, what gets in the way of this accomplishment is an excessive focus on children's thinking. Their view resonates with Safford's hard-to-hit "golden mean": too much understanding may prevent skillful mastery of arithmetical operations. Felmley and Shutts propose that, given that thought is a "by-product" of arithmetic teaching,

> Even in the development of the rules by which the processes are performed, only so much of theory should be introduced as will bring the pupil most quickly, and at the same time most surely and lastingly, to a fair standard of skill.

Their lessons are structured as a sequence of commands,[37] a format which suggests a classroom modelled more like a factory, than like a playground. For Felmley and Shutts, the overall aim of arithmetic teaching is manual

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[37]While the commanding nature of the text is hereby apparent--cf. #63--in more or less visible ways, teacher-initiated communications always have the form of orders or commands. This has been elaborated by critics of child-centeredness (e.g. Walkerdine). In relation to what he calls the "order-word assemblage" inherent in the compulsory education machine, French philosopher Gilles Deleuze comments: "When the schoolmistress instructs her students on a rule of grammar or arithmetic, she is not informing them, any more than she is informing herself when she questions a student. She does not so much instruct as 'insign,' give orders or commands. A teacher's commands are not external or additional to what he or she teaches us..." ([1980] 1987, p.75).

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training rather than sense training (Speer) or the disciplining of the mind (Brooks). how did this factory-like approach fit within the prescriptions of the child-centered model? The answer to this question may require a closer look at the context surrounding American schools in the first decade of this century.

While, in his address to the 1902 NEA Convention, Dewey made a call for schools to widen their functions and become "social centers,"[38] our reporter Joseph Rice tested six thousand children throughout the country, and found their arithmetic skills alarmingly deficient. He had already conducted similar tests in 1893, and the news about the overwhelmingly low achievement in arithmetic of third-grade children--released in the widely circulating journal *The Forum*--caused heated controversies among educators and the public at large. In Rice's view, these low performances were to be blamed on an *inefficient* approach to teaching and to school administration.[39] This situation was seen as aggravated by the incidence of 'backwardness' in the school population, particularly among non-white and immigrant

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[38]As he argued (1902), this would morally uplift the quality of urban living by replacing brothels, saloons, and dance halls as centers of recreation. Dewey saw education as a "means of bringing people and their ideas and beliefs together, in such ways as will lessen friction and instability, and introduce deeper sympathy and wider understanding."

[39]This view will be better articulated years later, in his *Scientific Management in Education* (1912).

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students.[40] To make matters worse, in the face of a rising industrial competition with Prussia, American business leaders called impatiently for more vocational training, while making public their concern for the negative effects of a dysfunctional school system.[41]

Authors like Felmley and Shutts were attuned to their times. They did not insist that "mental strength is the result of mental labor," nor did they indulge in a romantic celebration of children's natural proclivities, or consider it worth the effort to illuminate the social and historical roots of arithmetical processes. Instead, they took seriously that the purpose of arithmetic teaching--and the function of a textbook--was to train children's minds and bodies for their future occupations, as well as for the skillful management of their daily affairs. Their text thus anticipated the soon-to-become-dominant social efficiency approach to the teaching of arithmetic.

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[40]Leonard Ayres demonstrated the presence of "laggards"--retarded children--in American schools. Ayres was one of the main proponents of the school-as-factory model, that is, the application of business and industrial values and procedures to the practices of schooling (Ayres 1909; cf. also Callahan 1962).

[41]In his address to the 1905 NEA convention, Frank Vanderlip, the Vice-President of the National City Bank of New York, said: "I have had a somewhat unusual opportunity to study the underlying causes of the economic success of Germany, and I am firmly convinced that the explanation of that progress can be encompassed in a single word--schoolmaster. He is the great corner-stone of Germany's remarkable commercial and industrial success" (cited in Callahan 1962, p. 12). As to the teaching of mathematics in Prussia, Jacob Young (1900) had already stated that the roots of Prussian superiority were to be found, not in the students' or teachers' mental make up, but, instead, in the fact that the Prussian system was a centralized one, they had a cadre of well-trained, high status teachers, and they operated with outstanding methods of instruction.

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In matters of method, before the 1910s and 1920s, that is, before Thorndike's connectionism became the dominant theory of learning,[42] the increasingly pervasive social efficiency logic had to be made compatible with the still prevalent, though already under siege, doctrine of mental discipline. *The Southworth-Stone Arithmetics: A Rational Method,* published also in 1904, by Benjamin H. Sanborn and Company, is an example of this linkage.

Its authors were not classroom teachers. (Gordon A. Southworth was a School Superintendent in Somerville, Massachusetts, and John C. Stone was an Associate Mathematics Professor at the State Normal School in Michigan). Yet, in a gesture which would soon become typical, they claimed that their series "have been prepared not by theorists to exploit their peculiar notions, but by teachers of long and successful experience." Anticipating the 'reductionist' tendencies of the 1910s, Southworth and Stone acknowledged that various "subjects heretofore treated in arithmetic have been omitted as non-essential or beyond the legitimate work of the elementary school." Yet, as they continued, enough material could be found in their textbook for meeting "the demands of business" and for furnishing "the requisite mental discipline."

It is worth contrasting Southworth and Stone's preface

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[42]Thorndike 1911, 1913, 1921, and 1922.

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with David E. Smith's introduction to his *Grammar School Arithmetic,* also published in 1904, by Ginn and Company. In it, Smith claimed to have selected the arithmetic problems so as

> to touch the *actual life* of this country at this time; to give correct ideas of the *business customs* of the day; to embody mathematical principles in *interesting and instructive* groups of problems;

As he continued, this care for the social and economic applications of arithmetic went hand in hand with another concern, namely,

> to touch the *genuine interest* of the pupils in the *story of our national resources and industries* rather than to dwell upon the technicalities of minor trades in which they have no immediate or prospective concern;

"And," he added,

> to come in contact with *human life* rather than with those phases of science which are quite as foreign to the interests of boys and girls as are *the mere abstract problems of numbers* (all emphases added).

Embedded in these claims are our four paradigms in curriculum-making. In the construction of this text, arithmetic problems have been selected to embody mathematical principles, inform about current business costums, depict a broader social reality, and engage the learning child. From one angle, Smith's struggle can be read as that of an old-time humanist, attempting to accommodate some of the demands of his adversaries. It is tempting to compare Smith's statement as to the degrading of the subject

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in the process of making it interesting for children (1909), with

> the child's interest in the quantitative side of life should be the highest immediate aim of the teacher of mathematics in the grades (Smith and McMurry 1903).

As Smith and McMurry continued, the "nature of the child, together with the needs of society, should constitute the main standard on selecting subject matter." These back and forths between subject matter and child are best read as symptomatic of the uneasy relationship between the teaching of arithmetic and the newly emerging child-centered paradigm.

Other textbook authors were less inclined than Smith to open up their books to "the story of our national resources and industries." For example, in the preface to his *Practical Arithmetic: Oral and Written* (1907), Frank Hall confesses that no effort has been made "to follow the lead of some recently published arithmetics and make the book an encyclopedia of geographical, historical, and statistical number facts." As he continues, while the correlation of arithmetic with other school disciplines may be helpful or desirable, "this should be incidental." Instead, the emphasis is put on "accuracy in the mechanical processes employed in the solution of the simple, everyday problems of the home, the shop, and the farm."

Along the same lines, yet targeting a more restricted reader--i.e. the industrial school student--James Baldwin's

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*Industrial Primary Arithmetic* (1902) published also by Ginn and Company, stands as an early example of the tendency towards a psychologization and vocationalization of school arithmetic. Presented as "an illustration of the natural method of teaching numbers," the author claims to have organized his text so that "the pupil is constantly led to make discoveries for himself." Baldwin does not care much for abstract thought or explanations--"The pupil is still too young to understand the science of arithmetic." If theoretical matters are to be avoided, what goes without saying ought be excluded also: "Do not waste time in pretending to teach that which you know the pupils already understand." This concern for redundancy is most apparent in the first lesson, which begins with the number 2, "for every child of normal intelligence knows all about One that it is necessary for him to know at this stage."

If Baldwin's motto is "Let every exercise, every question, and every problem be business-like and practical," this is coupled with "Let every lesson be a voyage of discovery." Are these two logics compatible? The extent to which this compatibility could be made real in the arithmetic classroom will be a crucial issue in the decades to come. Baldwin's preface suggests that, at least rhetorically, a concern for the useful and the practical did not have to make instruction mechanical or constraining to the child.

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## 3.5 Arithmetic Textbooks in the Era of Efficiency

During the 1910s, the "cult of efficiency" in American education reached its peak (Callahan 1962). The decade was marked by two pieces of State legislation in the area of vocational education: the 1914 Smith-Lever Bill, involving support for agricultural education, and the 1917 Smith-Hughes Act, which legislated federal aid for the training of teachers in trade and industry. An Immigration Act was passed in 1917, which mandated a literacy (and numeracy) test for sixteen-year-old children. Yet, these rites of passage had long been in place, as can be gathered from the recollections of Pauline Notkoff, a Jewish immigrant from Poland interviewed in 1895,

> They asked us questions. 'How much is two and one? How much is two and two?' But the next young girl also from our city, went and they asked her, 'How do you wash stairs, from the top or from the bottom?' She says, 'I don't go to America to wash stairs.'[43]

While these and similar measures were taken to limit the entrance of those who, as it was feared, would further overburden American schoolteachers with special linguistic and cultural needs, in matters of school curricula other forms of regulation were in the making. The decade of the 1910s brought with it multiple and systematic efforts at eliminating 'nonessentials' from school arithmetic. In great part, these reductions resulted from the four Reports

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[43]This testimony is part of the exhibition at the Ellis Island Immigration Museum, in New York City.

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released by the NEA Committee on the Economy of Time in Education (NSSE 1915, 1917, 1918, 1919). These recommendations were so "clear, concise, and exact," that their translation into textbooks occurred almost immediately.

The first of these Reports (NSSE 1915), stated that non-essentials were to be eliminated from arithmetic curricula, and that the criteria for the inclusion of any topic had to be "whatever is comprehensive by children of that age, and whatever responds to the social needs common to ordinary American children." Among the victims of these eliminationist efforts were traditional topics such as apothecaries' weight, cube root, certain cases of percentage, compound and complex fractions of more than two digits, troy weight, unreal fractions, foreign money, and the metric system. The Report further stated that

> the time saved through the omission of these topics should be wisely devoted to the study of the social, economic and arithmetical issues involved in such facts as saving and loaning money, taxation, public expenditure, banking, borrowing, building and loan associations, investments, bonds and stocks, tax levies, insurance, profits, public utilities and the like.

In the coming decades, thanks to the generous federal funding, within the genre of 'industrial arithmetic,' there would be books for every taste. This is best illustrated by Mary L. Gardner and Cleo Murtland's *Industrial Arithmetic for Girls' Trade Schools*, first published in 1910, by D. C.

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Heath and Company.[44]

We may open this text as one of its prospective readers, the tens of thousands of recently immigrated girls like Pauline Notkoff, most of them working in the hundreds of sweatshops in New York City and other major urban centers. As we do so, we learn that the authors of this 'Trade School Arithmetic' assumed the girls to be already skilled in arithmetical processes. Thus, rather than insisting on drills, they made their text consist entirely of 'live' problems. This category included not only purely trade problems but also industrial, civic and historical problems, the latter ones

> given to show the connection between the past and present economic conditions, particularly those related to the trades in which women and girls are engaged.

These problems were intended "to add to the worker's interest in *earning* money, an interest in the wise *use* of money," and to "direct the attention of trade pupils to some undesirable social conditions by means of such problems as those based on sweatshop work" (emphasis added).

Although this approach seems to anticipate the 'social arithmetic' of the 1920s, a difference must be emphasized. 'City arithmetics' such as Gardner and Murtland's, together

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[44]D. C. Heath and Company would become one of the most powerful corporations in the field. Many contemporary textbook series and other teaching materials carry its trademark.

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with their 'rural' equivalents,[45] *did* include 'social problems,' and these were not manufactured as excuses for embodying arithmetical processes, in the manner of 19th century texts. Yet, neither were these problems designed to elicit in students a critical viewpoint on the situations depicted. Consider, for example, the following problem, included in the section "Industrial and Civic Problems" (subsection "Children who are Breadwinners"):

> There were in 1900 in New York State 384,760 boys between the ages of 10 and 15. 14.4 per cent of them were breadwinners. The total number of girls between those same ages was 384,901 of whom 9.5 per cent were breadwinners. How many boys and girls in this State were breadwinners in 1900?" (p. 136)

The treatment given to the 'child labor problem' in this text differs from that which will be given to it by McMurry and Benson in their *Social Arithmetic* (1927)

> When Tom was 11 years old, he averaged $1.50 a week in earnings.... In some of our factories children as young as 8 years old have been found sewing buttons on clothing. Perhaps you can find a book about 'child labor' in an encyclopedia. Find out all you can about it and explain to the class why child labor is a bad thing.

Of course, in none of these cases we know how these problems

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[45]E.g. Stevens, Butler, and Stevens, *A Practical Arithmetic* (1909), Calfee, *Rural Arithmetic* (1913), Burkett and Swartzel, *Farm Arithmetic to Be Used with any Text-Book of Arithmetic or Without* (1914). It is worth mentioning also Nelson L. Roray's *Industrial Arithmetic for Girls* (1917), subtitled "An Elementary Text in Home Economics." Its preface states that "while most of the problems are applied problems, a few have been inserted to impress a mathematical principle or to cause the pupils to do a little mental gymnastics" (p. v). Another popular genre was that of 'household arithmetics,' designed for women to rationalize the economy of the home (e.g. Ball and West, *Household Arithmetics*, 1920).

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were read by those to whom they are addressed.

In the context of massive migration to the cities, which created urban unemployment, housing shortages, and other forms of 'social unrest,' authors of 'rural' or 'farm arithmetics' took upon themselves the task of inspiring in their young readers a love for the life and work in the country.[46] A compelling example is *Rural Arithmetic: A Course in Arithmetic Intended to Start Children to Thinking and Figuring on Home and Its Improvement*, written by John Calfee--a Professor of Mathematics at a Normal School in Kentucky--and published, in 1913, also by Ginn and Company.

Calfee laments that most "old-time arithmetics" were written for students to pass examinations, rather than for the "child who was never to enter college." No provision was made for those "who must toil and do the world's work by the sweat of the brow." The sad result was that "the soil has been abused and worn out, much of the timber wasted, and many once fertile farms abandoned." By means of "real and practical" farm problems, his *Rural Arithmetic* was designed

> to develop in the children a broad view of the unbounded opportunities which the farm offers for *the accumulation of wealth and happiness* (emphasis added).

It is worth referring here to Carson's *Essays in Mathematical Education*, written in the same year as Calfee's text. Carson elaborates on the distinction between

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[46]For a critique of the idealized representation of rural life in British readers, see Ahier 1988.

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formalism--"an opening discourse, which allows for the play of the imagination, and transports students to a world of abstract thought"--and utilitarianism--"a closed discourse which limits the play of the imagination, and keeps students in the world of the concrete." He recommends:

> Our children must learn to think, and to think about the world as it now is and the manner of its evolution. Some few there may be who can with profit to us all devote themselves to one or other side of this world of thought and action, but the mass of men must be fitted to play their part between the two (p. 35).

As Carson would see it, arithmetic texts (and curricula) designed with the sole aim of adjusting children to their present, or projected, social and geographical location, fall short of the overall educative value of mathematics, that is, to introduce all students to the realm of abstract thought.

The *Walsh and Suzzallo Arithmetics* series, published in 1915 by D. C. Heath Company, was written by John Walsh and Henry Suzzallo, an Associate Superintendent of New York City Schools and a Professor of Philosophy of Education at Teachers College. The authors at giving to the child "all the arithmetical power necessary to cope with the situations of common everyday life." After vaguely noting that a "social survey of the usual responsibilities of men has determined omissions and additions," they declare a thorough application of both "a standard of social efficiency" and one of "pedagogical economy." Yet, they add a suggestive

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statement:

> No attempt has been made to project the pupil into *unfamiliar situations*. His *experience* is already greater than his *power of mathematical interpretation*. The teacher of primary arithmetic has accomplished enough when she has taught the child *to solve his own problems* (emphasis added).

This disclaimer anticipates the paramount theme of the 1920s and beyond: problem-solving. Experts working in this booming area will confront, again and again, an apparently unsolvable mystery: despite the multiple attempts at changing the face of word problems from obscure riddles and puzzles to interesting and engaging stories, and despite the proliferation of problem-solving methods and techniques, most teachers continue to have trouble teaching them, and most students encounter the same difficulties and produce the same errors when solving them. If the 'problem of representation' has been intrinsic to problem solving in classrooms and textbooks, as the decades go by, the emphasis will shift back and forth, from representation understood as a mental, pictorial, linguistic, and social question, without ever 'solving' it.

Before entering the 1920s, a few more textbooks from the late 1910s will be presented. Their introductory sections illustrate the difficulties faced by authors as they tried to accommodate to the demands of arithmetic, the new scientific findings of educational research, everyday life applications, the needs and proclivities of the child, and the interests of business was becoming increasingly

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problematic.

As for business interests, unknowingly, some textbook authors will unmask the hidden function of capitalist schools, i.e. the contribution of formal education to the reproduction of a disciplined and skilled labor force (Bowles and Gintis 1979). For example, in their *Wheeler's Graded Arithmetics*, dated 1919, Frederick Somerville and Ellis Graff observed that

> The keen business man does not expect young boys and girls to have a knowledge of business practice; but he has the right to expect that they shall be habitually careful, systematic, and accurate.

This statement makes sense in a context of widespread calls for textbooks to respect the notations and procedures which are commonly employed in the business world. As the authors continued,

> While it is true that the methods of all business and manufacturing firms develop certain details of form that are not found in school textbooks, it is also true that these details are merely variations which extend beyond the fundamentals that the school must thoroughly teach.

With this in mind, Somerville and Graff express their committment to helping develop the qualities above enumerated, "in order to render the best service for both the pupil and the future employer." In pursuing this aim, they will avoid "certain tendencies and theories which, while attractive as experiments, have not yet established themselves as a part of practical arithmetic."

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A common rhetorical strategy at the time was to argue that the textbook interpellates the 'normal,' 'common,' or 'average' child. Let us consider William Boylan and Floyd R. Smith's *City Arithmetic* (Four Year, First Half), published in 1916 by Charles E. Merrill and Company. The authors--a District Superintendent of New York City Public Schools, and the Principal of Public School 167, in Brooklyn--begin by pointing to a "noticeable weakness in elementary education," namely "the pupil's inability to use a textbook correctly." As they promise, *The City Arithmetic* will "remove this weakness" by using a language which children can comprehend and providing easy-to-grasp explanations. And, as for the problems, these are "within the experience of the average child."

What is meant an average child? And what was known about his or her experience? These questions were becoming central, as the nation and its schools continued to suffer the impact of heavy immigration flows. Yet, if the scientific treatment of whatever concerns the child--i.e. child study--had to operate, by necessity, with a notion of an 'average child,' this conflicted with a central tenet of the dominant educational discourse, namely, the need to respect and cherish 'individual differences.' In fact, in his introduction to Thorndike's *Individuality*, written in 1911, Henry Suzzallo notes as a sign of progress to have escaped

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> from the tyranny of traditional notions such as what constitutes an average child under average conditions, and to [have] reach[ed] the belief--general vague though it be--that schools must be respecters of individuality.

Although textbook authors did not usually participate in these debates,[47] in their choices of 'problem material,' they were forced to face the 'average child vs. each-child-as-a-unique-personality' dilemma. In their accommodations to these and other demands, some textbook authors unmasked the dubious assumptions underneath notions such as universal, normal, common, and average. *Applied Arithmetics: The Three Essentials* (Book I), written by N. J. Lennes and Frances Jenkins, and published in 1919 by J. B. Lippincott Company, speaks suggestively to this unmasking. Although they do employ expressions such as "nearly all normal children" and "the general child," they also bring to the fore the problem of the social context of teaching and learning.

Lennes and Jenkins--a Professor of Mathematics at the University of Montana and an Assistant Professor of Elementary Education at the University of Cincinnati--complain that too often

> use has been made of material that actually comes from within the experience of a comparatively small number of children. Thus, manual training

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[47]In fact, with the passing of the decades and up to the end of the period under study, textbook authors will be less and less involved in such endeavors, as most of them will be recruited not from the ranks of the mathematics education community, but from those of school administration.

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> may furnish most excellent resources for number work, *provided* the child has had experience with just that kind of manual training; otherwise it is very bad material.

Instead, in their textbooks, "the elementary facts of Arithmetic are derived from situations which are *within the range of experience of nearly all normal children*" (emphasis added). As they see it, whenever arithmetic is connected with other activities "which in themselves are of compelling interest to the child," the result is "*a source of joy and life* when the Arithmetic elements alone would lead to *sadness and forced labor*." Unfortunately, they maintain, many textbooks

> have been based on work done in some one very excellent school or system of schools, where children have been *subjected to special experiences*, such as construction work (emphasis added).

In their view, placing such books "in the hands of children not subjected to these particular experiences," could lead to "disastrous results."[48]

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[48]After restating that the "applications of arithmetic should likewise be made to things of *general* child interest," Lennes and Jenkins expressed what comes closest to recognizing the ultimate irrelevance of the arithmetic textbook, namely that "[t]he most effective application of Arithmetic can be made only when considerable local material is brought it. Such material is most effective when brought in by the child himself." A similar statement appears in D. W. Werremeyer's preface to *Arithmetic by Practice* (1913): the problems "are made as practical as possible, considering that the conditions in different localities are different, and that what is practical in one community may not be practical in another community." And, in *A Course of Study in Arithmetics with Answers to the Everyday Arithmetics* (1915) by Franklin Hoyt and Harriet Peet: "No textbook can hope to supply all the problems that a class could solve, nor would it be a wise thing for a book to attempt it. Much of the work should grow out of a child's immediate environment and current interests."

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What were these "special experiences"? In 1918, William Kilpatrick released "The Project Method," a monograph which soon became one of the highest-selling education texts of all times. Together with his other writings, this essay was used as an instruction manual for experimentation in progressive schooling. In this text, Kilpatrick proposed that, since education was to be made akin to life rather than as a preparation for it, school activities had to be organized not around formal disciplines, but as "projects" or "purposeful acts," that is, as "the typical units of worthy life in democratic society."

Historians have found that most of the schools which housed experiments in progressive education and the project method were attended by white, middle class, English-speaking children (Cuban 1993, Zilversmit 1993). In these environments, textbooks were hardly used and, instead, problems emerged from the classroom, the school, and the community. But the project method had an impact beyond these experimental schools. A watered-down, distorted version of this approach soon entered most schools, in the form of textbook-generated projects. As Lennes and Jenks seem to suggest, to the extent that experience in 'construction work' was already part of students' experience and within their field of interests, meaningful learning of arithmetic occurred, as they engaged in solving 'construction work problems' (e.g. measurement, ratio, fractions, etc). If this

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was not the case, this approach may have led to "disastrous results."

In the area of problem solving, by the end of the 1910s a consensus emerged that arithmetic problems had to be "interesting and instructive." Yet, new questions arose: What needs to be known about the students in order to make word problems *really meaningful* for them? How do educators know whether a given situation will be *interesting enough* for the children? And how may one anticipate whether or not a given word problem would introduce *unnecessary difficulties*, either in the vocabulary used, or in the cultural milieu depicted therein?

## 3.6 The Mystery of Problem Solving

The 1920s was a period of economic and social prosperity. Educational systems continued to expand, especially at the high school and college levels.[49] While there was an overall movement towards a more 'practical' training for the majority of high school students,[50] at the elementary level, the progressive ideas of Dewey and

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[49]Claiming to base the problems in their textbook series on reliable, authoritative sources, in *Social Arithmetic* (Book Two), dated 1927, McMurry and Benson inform their young readers that in 1870 "4,077,00 children attended our public schools." In 1900 this figure raised to 10,633,000 pupils, and by 1920 "the number in daily attendance had increased to 16,150,000 pupils" (pp. 193-4). We will discuss, later in this section, the peculiar ways in which 'social arithmetics' fictionalize this school subject.

[50]Less students would study Latin and algebra, and more would attend courses in home economics, typewriting, and other vocational subjects.

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Kilpatrick were gaining increasing acceptance. The decade was inaugurated with the adoption by the Progressive Education Association of its seven cardinal principles: "freedom to develop naturally," "interest, the motive of all work," "the teacher as guide, not as taskmaster," "scientific study of pupil development," "greater attention to all that affects the child's physical environment," "cooperation between school and home to meet the needs of child-life," and "the progressive school as a leader in educational movements" (Progressive Education Association 1920).

The year 1920 saw the founding of the National Council of Teachers of Mathematics, an event which is usually to the need for responding to the prevailing attacks on the place of mathematics in the curriculum. This period corresponds also to the peak of the so-called scientific movement. This movement was influencing school curricula through new types of textbooks in 'skill subjects.' In particular, arithmetic textbooks

> were offered to schools on the argument that the skills operations included in them had been shown by investigation to represent the practical everyday needs of grown-ups and children (Rugg 1926).

As the decades go by, textbook authors will be more willing to respond to the tendencies in vogue, particularly in the area of problem solving. The fact that questions such as 'what arithmetic problems are most worth?' and 'how does one

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diagnose and prevent errors in problem-solving?' will now occupy most of the space both in specialized journals and textbook prefaces and introductions,[51] justifies an in-depth discussion of the main debates in this newly-emerging field.

During the period from the 1920s to the mid 1930s, daily classroom routines and experimental studies in arithmetic learning were guided by Thorndike's motto "Whatever exists at all, exists in some amount."[52] Relying on the newly-born psychology of education, researchers in mathematics education put at work a variety of research tools in studies of students' problem solving abilities. These efforts spanned from the tabulation of types of errors (Osburn 1922, 1925), to experimental inquiries into the relative difficulty of different kinds of problems: problems about easily or not-easily visualizable objects (Hydle and Clapp 1926), conventional and imaginative story problems (Wheat 1929), and problems depicting familiar vs. those depicting unfamiliar situations (Brownell 1931).

A year after the end of the First World War, a shortage of arithmetic problems was declared. "Campaigns for food

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[51]Most space in specialized journals and textbook prefaces will be dedicated to discussing how to best provide for practice (or 'drill') in computation. As we move into the mid 1930s and 1940s, with the displacement of connectionism by Gestalt theory and the "meaningful method"--which, among other changes, reflects on a de-emphasizing of isolated computation and a concern for the application of arithmetic to quantitative situation--the area of 'problem solving' will increasingly occupy more space.

[52]For an application of this motto to educational measurement, see William McCall 1923.

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conservation and for general thrift" demanded "new reading material and new problems for classes in arithmetic." As the 19th Yearbook of the NSSE further explained, "periods of social stress or rapid reorganization" make apparent "the conservative character of American textbook-controlled instruction" (NSSE 1920). Despite their efforts to introduce engaging and up-to-date material, textbook authors could not escape the blame for the "monotony," "abstractness," and "lack of contact with practical life" of most of their word problems (ibid.)

The same Yearbook (1920) welcomed the forming of a "cooperative movement" for gathering problems material relevant for each locality. "How much oil will I burn in a month if I use 1/2 gal. a week?", and "When eggs are 64c a dozen, what are 31 dozen eggs worth?" were among the problems collected. Aside from updated prices to account for post-WWI inflation, these 'community-based' problems were not much different from those found in most textbooks. The following remarks by Margaret Campbell, a textbook author of the mid 1920s, speak rather suggestively about this problematic.

In the foreword to her *Workaday Arithmetic: Practical Lessons in Arithmetic for the Employed Youth and the Junior High School arranged for Individual Work* (1924), Campbell distinguishes two stages in the history of arithmetic textbook-making: first, the old formal arithmetic, when

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problems--including the 'practical' ones--were selected only as ocassions for drill in the manipulation of numbers and formulas; and second, the modern, "realistic" arithmetic, when traditional "catch" problems were replaced with problems more in tune with "the real affairs of life." However, these new problems were still stated "*not in the language of the trade but in the schoolmaster's, and solved accordingly*" (p.xiv, emphasis added). "After all," she complaints,

> there is but little difference between calculations for building *imaginary walls* after the manner of the older texts and those for setting *imaginary ems of type* or making *imaginary board feet of lumber* into bungalows! (emphasis added).

Campbell's comment highlights one of the central dilemmas of math education, namely, that the very fact that an arithmetic problem is a *school problem* seems to invite, from most students, a mechanical approach. Either there is no escape from the imaginary nature, artificiality, or pseudorealism of arithmetic problems, or the way out is to be found, not in the problem *per se* but in the context in which it is read, interpreted, and solved.[53]

While communities documented their own problems, textbook authors improved their practice material, and

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[53]To be fair, Campbell (1924) is rather more optimistic about the possibility of getting out of this dilemma. The progress in textbook making towards a more realistic treatment of problems indicates that "the shortcomings" of the previous stages may eventually be overcome.

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students in progressive schools engaged in their own projects,[54] the persistence and proliferation of children's errors in the solution of (old and new, dry and fun) word problems was increasingly perceived as a "tragedy" (Myers 1925). After analyzing "thirty thousand errors made by six thousand children in eighteen counties and one large city," Osburn (1922) concluded that the greatest cause of error in problem-solving (30 percent) was "total failure to comprehend the problem." As he continued, with visible indignation, "[w]hen children of normal inteligence

> show a total failure to comprehend a problem, one suspects that the problem is not well stated, that sufficient information is not well stated, or that it is too difficult for children of the degree of maturity to whom it is presented. Certainly the school cannot expect proper reasoning responses when children are presented with situations which they totally fail to comprehend (cited in Buswell, 1927).[55]

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[54]For example, see the collection of 285 'projects' for grades four, five, and six, in the 20th Yearbook of the NSSE, Part I, dated in 1921. This list includes: "Learning thrift by keeping accounts," "Building and furnishing a cardboard house," and "Playground arithmetic." See, also, Hotchkiss 1924, Whipple 1921, Hanna 1935.

[55]Commenting on this and other "curriculum problems in arithmetic," Guy Buswell put the blame back on textbooks: "Textbooks should tell children how to solve problems, not by giving them a formal statement summarizing the thinking experiences of adults but by pointing out to them the many ways in which problems may be solved by children, and critically evaluating these various ways. If children can be shown that certain methods of reasoning are open to erroneous results and if they can be shown why, there is reason to believe that they will tend to avoid these ways and search for more productive methods of reasoning. at the present time, children are left very much to their own devices in the solution of problems" (p. 87). As Buswell saw it, if at stake in arithmetic textbook-making was the point of view of the child, the emphasis had to be put, less on children's whims and interests--about which we hardly know--and more on their thinking processes. Buswell (1927) emphasized the need for "individual diagnosis of the actual thinking carried out by children." Until such a survey is

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Researchers also explored whether the language used in arithmetic problems affected students' ability to solve them. And they found that to be the case. Fortunately, in 1921 Thorndike published the *Teacher's Word Book*, a text which included a list of 10,000 words specifying the vocabularies appropriate for children of different ages. Eager for clear and concise recommendations as to "the elimination of unnecessary linguistic difficulties," most authors would check their vocabularies against Thorndike's word book.

While those textbook authors preoccupied with children's *mastery* of number combinations and their real life applications,[56] could maintain the fiction of linguistic transparency by relying upon approved word lists, those concerned primarily with offering 'socially significant' arithmetic problems, had more difficulties sustaining 'readability' claims. Combining a criteria of intelligibility with one of social significance often proved a difficult enterprise. How could one depict the complexities of social life with the few hundred words supposedly familiar to the intended child-reader?

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available, he continued, "it will be difficult to supply suitable instructional material for problem-solving (pp. 85-6).

[56]Terms such as 'mastery' and 'number 'facts' were widely used in textbooks circulating during the 1930s up to the mid-1950s. As I will later discuss, the aim of arithmetic teaching would soon be defined as the mastery of number facts and their application to real life situations.

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*Thorndike Arithmetics* (Book Three), dated 1924, and published by Rand McNally and Company and the *Social Arithmetics* series (Books One, Two, and Three) (1926 and 1927) by Frank McMurry and George Benson, and released by the MacMillan Company, are the best representatives of the two opposite logics at work in 1920s textbook-making, i.e. social efficiency and social significance.

Thorndike's textbooks express his theory of learning, one which, between roughly 1920 and 1937, dominated almost uncontested the field of math education. Thorndike's connectionism--a modern version of 19th century British associationist psychology--reflects the view that thinking and learning result from joining things together. Rather than endowing the human mind with an extraphysical existence, Thorndike conceives of it as the sum of connections between life situations and a person's responses to those situations. For Thorndike, to think is to have a number of impressions--'mental bonds'--of the elements of a situation.[57] In opposition to mental discipline theorists, Thorndike conceives of reason not as a "mythical faculty," but as "the cooperation, organization, and management of habits."

Within his model, all learned connections are labelled

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[57]Many of Thorndike's critics read this as a version of Pavlov' approach, despite the fact that Thorndike himself defined 'situation' and 'response' more broadly than as stimulus and response, respectively (cf. Joncich 1962).

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as 'habits,' and education is defined as the process of forming socially desirable habits. Thus, and here I quote at length, the "psychology of the elementary school subjects

> is concerned with the connections whereby a child is able to respond to the sight of printed words by thoughts of their meanings, to the thought of "six and eight" by thinking "fourteen," to certain sorts of stories, poems, songs, and pictures by appreciation thereof, to certain situations by act of skill, to certain others by acts of courtesy and justice, and so on and on through the series of situations and responses which are provided by the systematic training of the school subjects and the less systematic tranining of school life during their study (Thorndike 1922, p. xi)

As many critics pointed out, such a narrow view of education translated, in practice, into the production of "automatons, individuals whose behavior is narrowed, nonthinking, and drill-induced" (Joncich 1962, p. 14).

"Nothing is included merely for mental gymnastics," claims Thorndike in the preface to his 1924 textbook, in reference to the still somewhat influential, mental faculty doctrine. "Instead, training is obtained through content that is of intrinsic value." As he further explains, the material is intended as a practice "not for the verbally described problems of examination papers, but for the actual problems of life." As we see, Thorndike's realism denies the inherently didactical, hence artificial, character of arithmetic problems.

In Kilpatrick's pedagogical model, authenticity claims are elaborated on the grounds that projects bring real life into the classroom, with all its complexity and

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interconnectedness. While Kilpatrick's project method declares a moratorium on the use of textbooks, in Thorndike's pedagogy these artifacts play a major mediating role. In his view, a textbook's authenticity resides in the fact that the arithmetic problems included therein are "just like those which commonly appear in daily life." A good textbook is one in which "problems whose answers must be known to frame the problems or whose conditions are fantastic are rigorously excluded." Thus, for example, as the first edition of *Thorndike Arithmetic* appeared in 1917, the author felt the need to add a short new preface in which he stated that

> Since it is now practically certain that prices will not return to their pre-war level for many years, if ever, the problem material has been altered to fit present price conditions.

Within the discursive boundaries of Thorndike's psychology, it is neither discipline nor interest, but systematic and varied practice (drill) and the positive reinforcement of correct mental bonds, which link arithmetic and the child. And, since this perspective assumes the absence of transfer in learning (Thorndike and Woodworth 1901), each number combination and each arithmetical process is to be fixed separately in the child's mind.[58] As his

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[58]Thus, provision was to me made for systematic practice in all number facts separately, even in the case of related pairs (e.g. 3+4=7 and 4+3=7). For, within Thorndike's paradigm, there was no certainty that when children acquired the first bit of knowledge, they were be able to infer the second one. It goes without saying that this no-transfer approach was highly space- and time-consuming for textbooks and teachers, respectively.

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critics pointed out, this approach amounts to a *de facto* dissolution of the structure of arithmetic into a multiplicity of disconnected and apparently meaningless 'number facts.'

In the manner of social efficientists, Thorndike maintains that the aim of teaching arithmetic is "that pupils shall approximate 100 percent efficiency with thinking of which they are capable." To achieve this goal, it is crucial to eliminate

> the irrelevant linguistic difficulties, the unrealizable pretenses at deductive reasoning, and the unorganized computations which have burdened courses in arithmetic.

In *New Methods in Arithmetic* (1921), Thorndike commented that in the "older" methods "the teacher could set any problem that was a problem regardless of whether it would ever occur as a real problem in real life." Instead, the "newer" methods will devote attention to

> the arithmetic that a person actually borrowing or lending money will really need to know, and to the general significance of interest for thrift and business credit operations (p. 3).

In reference to those who would unnecessarily complicate arithmetic with extraneous elements, he asked "Why risk losing interest in the problem and its solution when Tom, Dick, Mary, a boy, and a girl are just as good arithmetically as Horace Mann?" (ibid. p. 15)[59] When the

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[59]For example, in the preface to the *Alexander-Dewey Arithmetic*, Intermediate Book, from 1922, Georgia Alexander and John Dewey, express their desire to introduce subject matter "of contemporary

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subject is thus presented, "the game of thought [is] spoiled by the intrusion of linguistic difficulties" (ibid.).

The table of contents in Thorndike's textbook reveals that the author 'fictionalizes' arithmetic based on the principle of social utility. Sections are entitled "Owning, Buying, and Selling," "The Arithmetic of Public Business," "The Arithmetic of the Home," or "The Arithmetic of Science and Industry."

In line with Thorndike's school-life correspondence principle, curriculum-makers and textbook authors scrutinized the possible practical uses of each arithmetical concept and process. Big numbers, long division, seldomly used fractions, the decimal metric system, certain cases of percentage, and place value systems other than base ten,[60]

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interest," so that their textbook would bring "into the otherwise isolated schoolroom the great world where mathematics is found in every basic activity." With this mind, they include ready-to-be-dramatized (that is, already scripted) story problems such as the one about Mr. and Ms Henderson discussing with their children, at the breakfast table, the advantages of buying a farm house in Meadowbrook; as well as word problems such as "George Washington died in 1799 at the age of 67. In what year was he born? How old was he in 1776?"

[60]Consider the arguments during the late 1940s and early 1950s, for and against teaching the decimal metric system in American schools. Whereas some appealed to the fact that this measurement system is not used in daily life, others argued in favour of incorporating this content area as a way of enlarging students' understanding of mathematics as well as giving them a fairer chance to compete with their counterparts in foreign nations (cf. NCTM 20th Yearbook, 1948: *The Metric System of Weights and Measures*) The inclusion of place value systems other than our decimal, or base ten system--e.g. base 2--became a highly controversial matter during the 1960s new math. It is worth noting that an argument for the inclusion of non-base 10 systems was given, much earlier, by Buckingham in his contribution to the 1930 Report of the Committee on Arithmetic (NSSE, *29th Yearbook*).

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became suspicious in that they failed the 'social utility' test. Yet, since some of these topics were seen as an indispensable part of the arithmetic curriculum, to protect them from the eliminationist impulses, an appeal had to be made to criteria other than utility.[61]

Criteria other than utility permeate the *Social Arithmetic* series by McMurry and Benson.[62] In the preface to their Book Three, published in 1926, we are told that

> in this book, as well as in the other two books of the series, the data used for problems, instead of being supposed facts, are actual facts from reliable and authoritative sources.[63]

The necessity of this claim is made clear given the nature of the problems in this textbook, one of which the preface thus anticipates:

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[61]In *Thorndike Arithmetics-Handbook for Teachers* (1927), after asking "What does a child think about more willingly than buying and selling, planning for childish pleasures, his play, his activities in the school and in the home?" (pp. 7-8), Ida Baker announces that, in her Book One, addition has been connected with playing story (p. 9) and she comments that "even though there are not many immediate uses for it in the fourth grade, long division is included here in the learning material."

[62]Together with his brother Charles, Frank McMurry had been a crucial figure in the Herbartian movement. The National Herbart Society--which also included John Dewey--grew up in overt opposition to the traditional humanist approach to the curriculum. Whereas for humanists such as Harris elementary school was to focus on the 'five windows of the soul;' for the Herbartians, a particular subject--for example, Robinson Crusoe--could be used as a focal point for all subjects, in order to achieve a more unified curriculum.

[63]In *Social Arithmetic*, Book One (1927), in a section entitled "Sources of Data and Other Valuable Help," McMurry and Benson argue that "unless the statement that a good egg-laying hen averages 180 eggs per year is authoritative, there si no reliance to be placed in the answers to the series of problems about the number of hens required to keep a family in eggs" (p. vi).

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> The prices of food and clothing have so increased in recent years that the cost of living is said to have doubled. But the laborer who used to receive $1.50 or $2 per day is now receiving $4. Can he not live just as comfortably now as formerly? What, then, is meant by increase in cost of living? And are workers better or worse off now than they used to be?

For Thorndike, prices were 'a problem' only in so far as their fluctuation forced him to update them in the new edition of his text. If "whatever exists at all, exists in some amount," the outside world may be brought into a textbook, only to the extent that it can be turned into an arithmetic problem.[64] An entirely different logic permeates McMurry and Benson's *Social Arithmetic* texts. In this series, aside from introducing the various topics in arithmetic, each "quantitative subject matter"--e.g. "The Story of Berries," and "The Grocer's Profits and Some of his Troubles" (Book One, 1927), "How City Boys and Girls Earn Money" (Book Two, 1927), or "The Value of a Good Education" (Book Three, 1926)--represents a fragment of the larger social world.[65] For example, the compelling arithmetic

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[64]As suggested by Georg Simmel (1900), "By and large, one may characterize the intellectual functions that are used at present in coping with the world and in regulating both individual and social relations as *calculative* functions. Their cognitive ideal is to conceive of the world as a huge arithmetical problem, to conceive events and the qualitative distinction of things as a system of numbers" (pp. 443-4).

[65]In a similar vein, the function of arithmetic textbooks as mediating texts between the classroom and its 'outside,' is suggestively described by Georgia Alexander and John Dewey: "The subject matter is of contemporary interest which brings into the otherwise isolated schoolroom the great world where mathematics is found in every basic activity" (Preface to *Alexander-Dewey Arithmetic*, Intermediate Book, 1921).
Interestingly enough, in the title page, the authors include the

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problem/social problem about higher prices and lower living standards, is followed by an invitation which is as compelling as the problem itself:

> It is plain from these examples that arithmetic contains questions that are fully as interesting and important as those in history or geography. But in arithmetic the answers are not usually given to you as they are in other studies. On the contrary, you have to work to get them. You are given problems to solve, and as you solve them the answers that you find supply the answers to the big questions. That may seem an odd way for one to study arithmetic, but it is the way in which answers to such questions were originally found.[66]

The discussion so far suggests that McMurry and Benson's story problems are as pregnant with social significance as Thorndike's word problems are straightforward and clearly stated. Yet, when confronted with these two kinds of arithmetic problems, schoolchildren continue to produce an almost unlimited amount and variety of errors.

In light of this situation, the need was becoming apparent for assessing whether the situations depicted in

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following epigraph by M. Laisant: "The problem is always the same: to interest the pupil, to induce research, to give him the notion continually, the illusion, if you please, that he is discovering for himself that which is being taught to him."

[66]One may compare this with Karl Marx's discussion of the 'social problem of surplus value, presented also in the manner of an 'arithmetic' problem (Chapter One: "The Commodity," Volume One of *Capital*). It is only a critical attitude towards an apparently inocuous arithmetical relationship which allowed Marx to uncover the mystery of the commodity form.
In *How We Think* (1910), John Dewey, emphasizes the puzzle-like character of authentic problems in that these induce human beings to an attitude of critical reflection. In other words, worthwhile problems are not mere exercises or puzzles, to be solved by recipe-like procedures, but are truly 'problematic.' See, also, Lave 1992, 1993.

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arithmetic problems did in fact interpellate their intended child-readers. It was assumed at the time that for a problem to be meaningful, the situation depicted in it had to fall within the 'visualization power' of the child. Yet, if a focus on vocabulary proved too limited a view for addressing failure in problem-solving, the 'visualization' approach had its shortcomings as well.

Hydle and Clapp (1926) found that the percentage of students' errors in the solution of a problem about two boys who mowed a lawn for $.90 an hour was almost double compared to the percentage of errors in the solution of another problem--with the same numbers and logical structure--but involving the selling of rabbits. In the authors' opinion, "selling rabbits is more easily visualized than devoting time to mow a lawn." Yet, Clapp and Hydle found also that children had less trouble with problems about pounds of meat, than with those involving grapefruits--despite the obvious fact that grapefruits are more visualizable, or imaginable, than *pounds* of meat. Since this difficulty was found in the case of younger rather than older children, Clapp and Hydle could easily attribute this anomaly to the youngster's unfamiliarity with this fruit.

The puzzling issue of 'easy-to-imagine rabbits' and 'hard-to-picture grapefruits' suggests an alternative approach. Perhaps children were more in tune with the imaginary scenario of conducting their own rabbit-selling

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business, than with the familiar and quite realistic situation of mowing someone else's lawn.[67] And, in so far as grapefruits were concerned--as some critics of Hydle and Clapp's Clapp interpretation suggested--the errors could be traced to the fact that unlike apples, these items were uncommon among the lower classes. Beyond vocabulary and visualization, the issue of 'interesting and instructive' problems seemed to touch a more complex problematic involving work, mathematics, and the imagination.

Since textbook authors could no longer limit their problem solving material to dry scenarios in which apples were cut, and portions were shared among two, three, or more children, they eventually had to grapple with grapefruits. A typical way out of the 'apples and grapefruits' dilemma was to create the illusion of authenticity self-referentially, that is, by presenting the problems as if these had been brought by the children themselves,[68] claiming that the problems had been "taken from the daily conversation of children"[69] or, more commonly, insisting that the problems

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[67]In Walkerdine's discussion of the shopping game (1988), the discusses the two choices available for children: playing the shopping game as an excuse for learning how to subtract, or experiencing the game as the self-fulfilling fantasy of becoming rich, and in the course of doing so, failing to subtract correctly.

[68]For example, "The children in the Jackson Fourth Grade brought problems to school about their vacation," (from *Practical Arithmetics*, Fourth Year, by Engelhardt, Smith, and Kline, published in 1929).

[69]E.g. *The Modern Life Arithmetics* (Book Four), by Fowles and Goff, dated 1929.

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had been experimentally tried with actual children in real classrooms.

The standard textbook preface for the decades to follow is best illustrated by Clark, Otis and Hatton's introductory note to their *Modern School Arithmetics* (1929), in which "seven cardinal principles" for textbook-making are enumerated:

> ... child experience and interests; guidance in problem solving; fined graded steps in presentation, abundance of practice material; diagnosis and remedial practice; measurement of progress; and experimental use before publication.

Similarly, in *Foundation and Practice of Arithmetic*, also published in 1929 and authored by Arthur Gorton, Leslie Lynch, and Burt Seelye,[70] it is stated that since

> the purpose of arithmetic is chiefly utilitarian, long explanations have been omitted. Emphasis has been placed on the HOW, rather than on the WHY.

As the authors continue, with the aim of "fixing the fundamental data of number in the pupils' minds," extensive practice on "the 100 addition and the 100 subtraction combinations, the 95 multiplication and 95 division combinations" is provided, together with problems depicting "situations which are to be found in real life."

In the decades to come, mathematics educators, educational psychologists, and other noospherians will

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[70]As for the authors' affiliation, Gorton and Seelye are Principals of Public Schools in New York City, and Lynch is a mathematics teacher at a NYC junior high school. The book is published by Simmons and Peckham Company.

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provide authors with clear and concise recommendations on what should be included in their books. But, despite so many recipes for handling the books, solving the problems, and checking one's work, the mystery of problem solving remained unsolved. And, during the "bewildered thirties," the whats, the hows and the whys of arithmetic teaching became, once again, the topic of fierce debates; and, one more time, textbook authors provided unexpected and quite suggestive answers.

The three quotes below anticipate the discussion to follow:

> We do not expect pupils to become good spellers simply through incidental spelling connected with reading and writing; we do not expect pupils to become good penmen simply from incidental writing connected with other subjects. Should we, then, expect proficiency in adding and subtracting simply from the incidental practice pupils get in solving problems?[71]

> A group of fourth grade pupils spent three forty-minute arithmetic periods in learning how to use thermometers in measuring temperature. The number and variety of thermometers used, the confusion incident to the handling of the material, the inability of all pupils to read the same thermometer at the same time and thus understand instructions, and the lack of a clear cut understanding on the part of the teacher as to the important facts and meanings that should have been developed resulted in a situation where 'a joyous time was had by all' but where little or no arithmetical experience was gained.[72]

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[71]Bennett, Conger, and Conger, preface to their *Drill Book in Fundamental Operations with Integers* (1924).

[72]Drushel, Noonan, and Withers, *Arithmetical Essentials: A Teaching Manual* (Part One, Third and Fourth Grades), 1926, pp. 34-5.

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> Waste of time must be guarded against. A game in which number is merely incidental is expensive of time. It gives too much play with too little number.[73]

## 3.7 Incidentalism and Other Discontents

The crisis-ridden decade of the 1930s opened with an exhaustive Report by the NSSE Committee on Arithmetic.[74] In the introduction, the Committee announced that

> the treatment of arithmetic in this Yearbook is as progressive as modern thinking, courses of study, textbook construction, and scientific experimentation give it a right to be, and much more progressive than much of present classroom practice (pp.2-3).

Given the pervasiveness of incidentalism, the collective author of this document chose a moderate line. While stating their preference for "a liberal school in contrast to the lock-step, teacher-driven school of the '90s" (p.3) the Committee considered the excesses indulged in the name of child-centeredness: the child ought to be the center of interest, yet, "not a part of him, but the whole child, and this includes not only his present but also his future." As they saw it, if in the old school

> there was an overconfidence in drill; today we are in danger of allowing felt needs to degenerate into whims (often requiring less effort)

Their moderate line was clearly stated:

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[73]Bruce Watson, *Simplified Fourth Frade Arithmetic* (1924).

[74]Published in 1930, the 29th Yearbook of the NSSE included a first part in which "some aspects of modern thought in arithmetic" were discussed, and a second part which reviewed recent research findings in arithmetic. Included among the members of the Committee were Brownewll, Buswell, Buckingham, and Knight.

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> not on the one hand, running to seed, as in the case of the older school, on dead tasks for the tasks' sake; and not, on the other hand, declaring a Roman holiday, and becoming concerned only with the factor of interest (pp.6-7).

In matters of method, the Committee adopted Thorndike's psychological theory "viewing skills and habits as fabrics of connections."

During this period, psychologists were busy gathering data about 'the child.' Studies were conducted of children's mental, physical and emotional development; their rates of learning; and their changing attitudes, interests, and needs. 'Readiness' and 'development stages' became generally accepted as key factors in the learning process. For example, it was judged unwise to introduce a topic before the learner had reached a mental age high enough to profit from it.[75]

In the meantime, humanists like William Reeve (1936) and Charles Judd (1936) continued their struggle against instrumentalism, insisting that arithmetic was a way of organizing experience and developing meaning, rather than a 'tool subject.' In a decade of severe economic and social problems such as the 1930s, arithmetic--especially high school mathematics, due to the expansion of this level of

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[75]In fact, in 1930, Washburne contributed his "Grade Placement of Arithmetic Topics: A Committee of Seven' Investigation," to the *29th Yearbook* of the NSSE (1930). Although his study was criticized at the time as careless and unscientific, it had widespread influence on the placement of arithmetic topics in classrooms and textbooks.

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schooling--was forced to defend its place as a legitimate school subject.[76] As Reeve himself acknowledged (1936), the problem of

> weighing the values of the subjects of the school becomes acute during depression. With bewildered taxpayers demanding economy and the elimination of the nonessentials all school material should be given a fair hearing with all the evidence considered.

At the same time, in reaction to the over-emphasis on drilling in computation of Thorndike's connectionism, the NCTM endorsed, in 1935, the 'meaningful method.' Heavily influenced by Gestalt and 'field' theories of learning, this new approach to arithmetic teaching emphasized insight, meaning, and understanding.[77] In the words of William Brownell, to whom this method is attributed, and whom we will meet as a textbook author, arithmetic is best defined as "a closely-knit system of understandable ideas, principles, and process." Within this framework, drilling is seen as a subordinated practice aimed at "securing retention when an idea or process is already understood."

In 1934, the New York City became the site of the largest demonstration of progressive educational practices in the nation--i.e. the Activity Program--a six-year long

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[76]See the *New York Times* editorial from May 2, 1932, an argument against the teaching of mathematics; and the response by Reeve ("Attacks on Mathematics and How to Meet Them" 1936).

[77]Notions such as these were far from well-defined, a fact which soon became a central concern among curriculum, tests, and text-makers. See, for example, efforts to operationalize understanding ("The Measurement of Understanding," NSSE, *45th Yearbook*, 1946), and to clarify "the meaning of meaning" (NCTM, *21st Yearbook*, 1953).

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experiment involving 75,000 students, 69 schools, and 2,200 teachers (Cuban 1993). Yet, in 1941, a study of 24 classes in traditional and activity program schools showed that the time spent on formal subjects was the same in both (Cuban 1993). It appear that, in these classrooms, a compromise between computation skills by means of drilling and meaningful learning via projects had been arrived at, whereby an hour was spent daily on 'drills and skills' in reading, writing, and arithmetic. Thus, while the agenda of progressivism redefined the didactic contract towards giving a more active role to students in the learning process, the perceived incompatibility between the logic of arithmetic and students' spontaneous interests, translated *de facto* in a special clause whereby basic numeracy skills were secured in the traditional, teacher-centered fashion.

Starting in the early 1930s, the 'mathematical ennui' pervading schools--e.g. mechanical teaching, lack of meaning, and deficiencies in basic skills--reached the textbook industry:

> The emphasis on rules and calculation would no longer suffice a society whose technological complexities would increase in a geometric progression.

"We had to have meaning, not just the what but the why," continued George Russell, the mathematics editor at Scott and Foresman (Black 1967). In their search for meaning, and their efforts to make their texts more palatable to children, satisfy the demands of business, and comply with

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the prescriptions of one or another of the theories of learning in vogue, textbook authors didactically transposed arithmetic in such a way that, by the 1940s and mid-1950s, this school subject became almost unrecognizable as a structured field of knowledge.

As we open Book Two of *Mastery Arithmetic*--a text written in 1934, by Bodley, Gibson, Hayes, and Watson--we read about the careful selection of vocabulary, the attention to the child's surroundings, and the clarity, exactitude, and completeness of the directions given throughout the text. In line with current views on how children learn arithmetic, plenty of opportunities to "sense numbers," "see relations," and "visualize situations" are provided. In the authors' opinion, the road towards mastery is rather straightforward: after numbers are sensed, relations are seen, and situations are visualized,

> the procedure to be followed.... is obvious. The solving becomes then merely the adaptation of a means to an end.

Nevertheless, certain provisions must be taken, such as using "words as well as figures for the expression of numbers," a redundancy which is necessary as a safeguard

> against the habit, quite common among children, of combining thoughtlessly the numbers of a problem, without having read the problem.

Throughout *Mastery Arithmetic*, the focus is on the solving of problems--"You know that you are studying arithmetic to learn to solve problems." And, since a "good

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problem always gives the facts you need to use in its solution," it is a matter of deciding "how to use the given information--and what processes to use to obtain the required result." Yet, this procedure is not left to chance. Bodley, Gibson, Hayes, and Watson offer their readers a four-step procedure: "Read the whole problem to find what is asked, and what is given that can be used to find the answer," "Decide what operation is required," "Estimate the answer," and "Solve the problem." As it was common in an 'era of instructional recipes' (Devault and Kriewall 1969), new formulas are provided for obtaining children's understanding of arithmetical processes, securing their mastery of number facts, and preventing errors in the solution of problems.

While *Mastery Arithmetics* instructs students in the art of problem solving, *Iroquois New Standard Arithmetic*, written in 1938 by Degroat and Young, presents arithmetic "so simply and clearly that the child easily *understands* basic principles, method, and applications" (emphasis added). In their view, a concern for the child translates into the fact that

> The wealth of activities and appealing problems, built around normal child interests in the home, school, and community, gives further *incentive for mastering* the subject.

And, in so far as linguistic difficulties are concerned, to write for the child means that the language

> is simple, direct, clear, concise, and fitted to

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> the child's stage of development. Therefore, *errors in reasoning and computation, due to language difficulties, will not occur.*

Yet, the ability to decipher a word problem proved to be a necessary, but insufficient condition for arriving at the correct answer. As the authors of *The New Trend Arithmetics* put it,

> Proficiency in solving verbal problems is largely the outgrowth of meaningful experience that establishes associations of kinds of problem situations and appropriate arithmetical processes.... Cultivating skills in problem solving requires *intensive practice, cumulative practice*, and mixed practice at intervals, just as in the mechanics of manipulation of numbers (Gillet, Durell, and Durell, 1936, emphasis added).

Increasing anxiety about children's errors in problem solving, even after so many experiences and so much practice, was leading to a more sophisticated technique, i.e. problem solving heuristics.[78] For example, an initial pause was recommended during which, when 'confronted' by a problem, problem-solvers would ask themselves a series of questions as part of the 'method of attack.' As ennumerated in DeGroat and Young's 1938 *Iroquois New Standard Arithmetics*,

> Answer these questions about each problem below before you begin work: 1. What does the problem tell? 2. What does the problem ask? 3. What must I do with the numbers to get the right answer.

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[78]We will have to wait two more decades for the definite masterpiece on this subject: George Polya's *How to Solve It* (1957). And the perspective on 'reflective thinking,' offered by Dewey in *How We Think* (1910) was not so easily translatable into recipes, despite the various efforts in that direction.

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From now on, authors will devote their introductions, less to articulating the principles guiding the construction of their text, and more to providing specific instructions for the child on how to handle the textbook,[79] and how to solve its problems.

In *Social Utility Arithmetics* (1937), Strayer and Upton argue that as in "developing "the ability to solve one-step problems," the pupil is taught

> the more important language expressions in arithmetic, these phrases often serving as cues that tell the pupil whether he is to add, subtract, multiply, or divide to get the right answer.

However, as it corresponds to an era of 'meaningful arithmetics,' the emphasis on the mechanics of cues, is accompanied by an attention to worthwhile content. As the authors promise, their book

> contains the most carefully prepared collection of projects to be found in any textbook in arithmetic. These projects are simple, wholesome, and natural. They relate to things that *interest* children; they are alive and joyous.

At last, everything seems under control: problems have been made meaningful and are clearly written; multiple steps are provided; and batteries of tests are included for the diagnosis, evaluation, and self testing of computation and problem solving skills. Textbook writers seem to have proved that the question "Must arithmetic be dull in order to be

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[79]"Do not write on this book," "Use a folding paper for writing down your answers," or "Measure your own progress by means of the battery of tests included at the end of the text."

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acceptable?"[80] can be answered with a "No."

Nevertheless, life in the classrooms where these textbooks circulated was not substantially different from earlier in the century. A sad picture--almost as sad as Rice's--emerges from the commissioned report on New York State schools published in 1939.[81] In this study, Brueckner, also one of our textbook writers, and his collaborators reveal that in many arithmetic lessons they observed,

> the pupils merely copied long lists of examples from the blackboard and worked them at their seats, a deadly routine procedure. Fewer than ten percent of the teachers observed made any attempt to relate the arithmetic being studied to local situations (cited in Zilversmit 1993, p.29).

If in the old teacher-centered or recitation model there were too many rules and not enough child; the alternative, childcentered, project-based model could easily degenerated into too much play and too little number. As Reeve warned in 1937,

> We cannot make it purely 'child's play' because to do so would kill the science in it. That is why playing store, bank, and the like, if carried to the extreme, may be fatal.

## 3.8 Banality and Fatality in Arithmetic Teaching

In a trend which may be traced back to the mid-1930s 'meaningful method,' during the 1940s, theorists began to

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[80] *Buckingham-Osburn Searchlight Arithmetics* (Book I), p.ix.

[81] *The Changing Elementary School, The Regent's Inquiry into the Character and Cost of Public Education in the State of New York.*

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consider the possibility that the *structure* of arithmetic in itself could provide a fertile ground for organizing its teaching. In sympathy with the German-born Gestalt psychology, these theorists proposed a departure from the practice of building arithmetic programs according to considerations which were not directly mathematical. However, such timely concerns for structure, meaning, insight, and understanding, could not be made to fully operate within the trends which, almost uncontested, still dominated the curriculum field--i.e. the social efficientist emphasis on vocationalism and the 'life adjustment' movement.[82]

The year 1943 saw the release of *Vocational Education in the Years Ahead*, a study commissioned by the US Office of Education. Its authors recommended that 20% of the high-school student population be given college-preparation courses, another 20% be prepared for skilled occupations, and the remaining 60% be offered "the life adjustment

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[82]Although many of Piaget's books appeared in the United States before the 1940s--e.g. *The Language and Thought of the Child* 1932, *The Child's Conception of the World* 1929, *The Child's Conception of Physical Causality* 1930, and *Judgement and Reasoning in the Child* 1928--his genetic-epistemological approach will not become influential on American mathematics education until the late 1950s and early 1960s. (In his literature review on problem solving written for the 41st Yearbook of the NSSE, Brownell discusses the contribution of Piaget to this area of study as well as the most common criticisms of his work in the United States). Then, an elective affinity will be established, between the structures of cognition and those of mathematics. On the basis of this pressumed affinity, it would be possible for students to apprehend mathematics with no recourse to extra-mathematical devices. Whether or not meaning and structure did permeate the teaching of elementary mathematics, will be discussed in the next chapter.

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training they need and to which they are entitled as American citizens" (cited in Kliebard, 1986). During this period, the emphasis on children's adjustment to their present and projected life conditions was tied to a celebration of American culture and its scientific and technological progress.

*Real Life Arithmetic* (Grade Four) was first published by Charles Scribner's Sons in 1938 and in a new edition, in 1943. Its four authors--E. A. Bond (Head of the Mathematics Department at Western Washington College of Education), Rollo Reynolds (Principal of Horace Mann School, where?), Carl N. Shuster (Head of the Mathematics Department of the New Jersey State Teachers College, Trenton, Roscoe West (President of New Jersey State Teachers College in Trenton)--announce that "the subject of arithmetic is of ever-increasing social significance," and that it holds an important place "in the life of every person from the child playing hopscotch to the engineer building a bridge." In implicit reference to David E. Smith's 1928 thought-experiment ("Let us imagine that arithmetics disappears from the face of the earth..."[83]), they continue,

> As regularly as the hands move around the clock, we are confronted by number. The clock, the thermometer, the telephone, are constant reminders of the necessity for it. In a world without number we would have no radio, no electric lights, no oil

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[83]NCTM 3rd Yearbook. This thought-experiment was further exploited by Buckingham, in his critical contribution to the 29th Yearbook by the NSSE (1930).

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burners, not even books.

Along the same lines, *Arithmetic for Young America,* Grade Four (1944), celebrates American democracy in its cover, picturing a children's parade with banners inviting people to vote for President. The authors--John Clark, Ruth Baldwin, and Caroline Hatton Clark--do not find it necessary to introduce their text.

In the same year, Leo Brueckner, Foster Grossnickle, and Fred Bedford--all of them professors (either of mathematics or of education)--published Book Three of their *Mathematics We Use* series (John C. Winston Company). The first page invites us to admire "'the world's largest office structure'--the massive Pentagon Building, Washington, D.C., as seen from the air." The authors assert that arithmetics and other branches of mathematics "mirror social change." Since this metaphor could misleadingly suggest that mathematical truths are affected by social transformations, *Mathematics We Use* assures its child-readers that "This does not mean that 2+2 no longer equal 4." What has radically changed is "the setting." That is,

> the setting in which 2+2 is presented must be indicative of the age in which the pupil will live and in which he will make his living. High school youth of today and tomorrow will be exposed to many kinds of problems resulting from the navigation of aircraft.

In *Arithmetic for You*, written a year later by Clifford Woody, Frederick Breed and James Overman (Lyons and Carnahan Publishers), we also find references to arithmetic and

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social change. As the title indicates, this book is made for the child--"The curriculum of the school has one pole anchored in the nature of the child." Its other pole "is anchored in the nature of society" and, as society changes, education, "as part and parcel of our general social structure," must change accordingly. As the authors acknowledge, although

> arithmetic is one of the most conservative fields of study and therefore one of the most resistant to change, nevertheless the extent of changes in the subject is impressive.

After referring to the additions and ommisions occurring during previous decades, the authors emphasize that the battleground is now "the field of *instructional methodology*." As they further comment, "this also has been the battleground of liberal and conservative leaders speaking for education in general during the bewildered thirties."[84]

*Mathematics We Use* and *Arithmetic for You* do a fair job in convincing their readers of the pervasiveness of the changes underway, and of the ways in which arithmetics is

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[84]The following questions, quoted at length from the preface to *Arithmetic for You*, illustrate the dilemmas faced by textbook-makers in those times: "Is the problem approach the basic method of instruction? Are difficult computational processes properly analyzed and gradually developed? Is the pupil pushed into new areas of learning without making amends for vacation loss? Are unnecessary language difficulties in vocabulary and sentence structure carefully weeded out? Does the pupil encounter the primary number combinations with a frequency proportional to their experimentally determined difficulty? Is the emphasis on problem solving commensurate with the importance of quantitative thinking in everyday life? Are the modern techniques of diagnoses and remedial treatment made integral parts of the instruction throughout the books? Is there content appeal?"

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made attuned with its times. Although 2+2 will continue to equal 4, and this will be applied to an increasing array of social settings, during the post-war period, the need will become apparent for changing more than the setting of problems.

### 3.9 The Death of the Arithmetic Textbook

The post-war era was a period of economic growth, and a strengthening of the role of the US as a worldwide hegemonic power. In the early 1950s, educational systems around the world began a process of unprecedented expansion. In many places, student enrollments more than doubled, expenditures on education rose at an even faster rate, and education emerged as the largest local industry (Coombs 1968).

In the US, school systems began to face a severe crisis--a combination of alarming drop out rates with an increasing dissatisfaction with the 'life adjustment' approach to curriculum-making (Kliebard 1986). Widespread concerns emerged about the anti-intellectual tendencies in American culture and about what was perceived as one of its most pervasive effects: schools had been diverted from their central function, i.e. the development of the intellect.

This period was characterized by the creation of the NSF (1950), an agency of the executive branch of government dedicated to the support of mathematics and other sciences. There was also an increase in the use of educational

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technologies and a whole array of 'multi-sensory teaching aids' (NCTM 1945). In line with the activity movement and project method, a 'laboratory' approach to arithmetic teaching was recommended. In 1948, Foster Grossnickle, whom we recently met as a textbook author, suggested that classrooms needed to become laboratories where pupils would manipulate things to discover arithmetical principles (1948). From this time on, textbooks will become only one among the many instructional devices such as 'real experiences, manipulative materials, pictorial materials, and symbolic materials' (Grossnickle *et al.* 1951).

Yet, a certain impatience accompanied these innovations. Laboratory approaches and discovery methods had their pros and cons. As Buswell himself expressed it in 1945,

> ... nothing is to be gained by using pupil's time to try to discover a fact which can be readily be found by reference to the appropriate source in a library (Buswell 1945)

While acknowledging the value of direct experience, Buswell was aware of the limitations of classroom time. Thus, it was part of the task of teaching to decide

> when a child can afford discovery and when he must economize time by making use of the organized knowledge which is available, and has already been formulated for teaching purposes.

An awareness was also emerging that, despite the efforts, by textbooks and by teachers, to provide 'methods of attack,' 'tips, 'hints,' and 'cue words,' and despite the

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variety of settings, situations, and contexts in which problems were presented, children insisted in making the same errors when solving them.

In 1951, the NSSE devoted the second part of its 50th Yearbook to "The Teaching of Arithmetic." As one of the authors made clear, the three-fold aim of arithmetic teaching was to be "computational skill," "mathematical understandings," and "sensitivity to number in social situations and the habit of using number effectively in such situations" (Horn p. 7). The same contributor warned "the development of important abilities cannot be left safely to incidental learning" (p. 18).

This may sound as more of the same. Yet, by the end of World War II, textbook publishers began to realize that more of the same would not do. Scott and Foresman and Co. started to develop "a dramatic new series of textbooks" (Black 1967). What was so radical about these new texts? One of their first-grade books consisted entirely of pictures and numbers. The team of authors working for this publisher was concerned also with teaching the teachers. Thus, for example, whereas the second-grade textbook contained only 216 pages, the teacher's manual that explained it totalled 748 pages (Black, op. cit).

Among the series released by Scott and Foreman Co. is *Study Arithmetics*, of which we will quickly survey its Book Five, dated 1948. With three authors--F. B. Knight, J. W.

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Studebaker and G. M. Ruch--and two illustrators, this text does not include a preface, such is the hurry to get "Ready for New Work." The first section, subtitled "Flying and Figuring," offers a picture of an airplane, with workers preparing for the take off, and a group of children and a dog watching the scene. As the authors explain, very much within the life adjustment logic, yet with some qualification,

> Getting an airplane ready for a flight takes very careful work and checking by many people... Everytime the pilot reads an instrument or a map he is using arithmetic... You may not plan to be an airplane worker when you grow up. But no matter what kind of work you do, you will need to know how to use numbers.

Once a child has decided that it is worth to 'study arithmetic'--whether or not he or she is planning to get into aviation--only part of the issue is taken care of. The remaining part is, obviously, how to go about solving arithmetic problems. *Study Arithmetics* takes care of this need by recommending that when solving a problem

> first read it carefully. Then answer these questions before you do any figuring. 1. What is the question in the problem? 2. Should I add, subtract, multiply, or divide? 3. What numbers should I use to find the answer?

After more than half a century of teachers "trying to find the best way to give their pupils an understanding of the processes of arithmetic;" so many decades of "Committee reports, national conferences, and yearbook studies" recommending additions, omissions, and postponements; and so

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much innovation in activity programs, project methods, and laboratory work, meaninglessness continued to pervade classrooms and textbooks.

With these words, Catherine Stern introduces *Experimenting With Numbers*, a manual for teachers using her *Structural Arithmetics*, published by Houghton Mifflin Company in 1950. What is truly innovative about this text is the emphasis on structure. As Stern presents it, hers is

> a uniquely integrated method, centered around concrete teaching devices which, for the first time in arithmetic instruction, have all the properties of abstract numbers.

When children "play *Structural Arithmetic* games with colorful blocks and cubes that fit into attractive boxes," they begin "their adventures in arithmetic with full enjoyment. Yet,

> since each block represents a number, play with these devices gives the children fundamental insight into the structure of the number system.

The era of embodiments has arrived! Rather than clothing concepts and processes in attractive costumes--an effort which frequently translated into unreadable formulations, hard-to-visualize scenarios, and unfamiliar settings--children would be presented with the naked truth of arithmetical structures. Since these abstract entities had to be made material, the formula for keeping their transparency--or minimizing the 'noise'--was to 'embody' these in concrete materials homologous to the abstract entities they represented. Stern assures her teacher-readers

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that through

> a carefully developed sequence of demonstrations, experiments, and games, number relationships are presented so clearly and so unforgettably that the child himself can easily reconstruct them--first with the materials, later in his mind.

And she goes as far as to promise that "the mastery of arithmetic can be secured regardless of whether the teacher has had advanced training in mathematics, or only a general background in arithmetic." Furthermore, this approach may operate "in public as well as in private schools, and with or without elaborately equipped classrooms"... and, perhaps more importantly, "whether she [the teacher] has many, or few pupils to teach--whether her children are bright, average, or dull" (p.6). As Stern concludes, with her method,

> the children's number concepts grow out of their own experiments in childlike games with blocks and cubes. Since the concrete materials they use are true to the very nature of numbers, the acquired concepts are mathematically sound (p. 105).[85]

But the times were not ripe for structural approaches to arithmetic teaching. Notwithstanding the obvious appeal of this didactical novelty, until the end of the period under study, most arithmetics textbooks will insist on the same recipes and rules.

An example of this inertia is *Making Mathematics Work,*

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[85]As she further comments, "Unlike so many early childhood impressions of the world, these first number concepts are basic, and therefore they will never be set aside as unreasonable, unnecessary, or untrue" (ibid.).

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published in the same year, and by the same publishing house as *Structural Arithmetic*. For lack of a preface, the authors--G. D. Nelson and H. E. Grime--introduce their text in the first chapter, entitled "Analyzing Problems." In a manner which resonates with Carson's question, they ask,

> Whenever an intelligent person begins the study of a new subject, he quite naturally asks: 'What will I learn from this subject? What good will it do to me?'

But their answer is quite unlike that of our old-time humanist. For Nelson and Grime, the course they present

> will be worth while because in it you will learn how to solve the kinds of problems that arise in actual day-to-day living. Whether you work in an office, a factory, or a store, on a farm or at home, situations will arise in which a knowledge of budgeting, installment buying, insurance, and banking will be of value to you...

Yet, solving everyday life problems is "more than a matter of luck;" specific skills are required, such as being able to

> decide what you are looking for, what fact you have with which to work, and what computations you must make. You should be able to estimate your answer, so that you will have a general idea of what to expect.

Their approach is, once again, metacognitive. The idea is not so much to provide answers as to decide on the appropriate course of action: "Do not work out the answers to these problems. Instead select all the correct statements

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from the list that follows each problem."[86]

As we reach the mid 1950s, this text within a text--the preface--will become less and less convincing, until it will disappear altogether.[87] Since the use of arithmetic textbooks is increasingly becoming unthinkable without a teacher's manual, it is in these parallel texts that the whats, hows, and whys of arithmetic teaching will be now expressed.

It is worth reviewing *Teaching Arithmetic We Need*, a handbook for teachers published in 1955 and co-authored by Guy Buswell, William Brownell (from the University of California, Berkeley), and Irene Sauble (from the Detroit Public Schools).[88] The authors begin by arguing that, when taught properly and, thus, when properly learned

> arithmetic makes a positive contribution to the personality and to wholesome, happy adjustment to life situations (p.17).

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[86]And this command is followed by a long list of questions: Can you tell what are you to find?... Do you know what a problem tells you?... Can you tell what facts you need to solve a problem?... Can you tell what facts are not important? ...Do you know what arithmetic processes to use? ...Can you tell what to do to find the answer? ....How good are you at estimating answers?...

[87]*Scribner Arithmetic* (Madden, Gager and Beatty, 1955), includes only a note about the accompanying teacher's guides; *Arithmetic in My World* (Stokes, Adams, and Unkel, 1958), begins with a short paragraph, expressing the authors' belief that "the learning and teaching of arithmetic is based on child growth," and that, therefore, in their text, "children's problems" are organized "into units of work composed of activities which are in reality areas of development." In *Understanding Numbers* (1952), Brueckner, Merton, and Grossnickle do without any introductory statement; the same is the case with *Arithmetic We Need* (1955), by Buswell, Brownell and Sauble, and Henderson' and Pingry's *Using Mathematics* (1955).

[88]This handbook was published by Ginn and Company.

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*Teaching Arithmetic We Need* is forwarded as unique text: "You have never seen an arithmetic manual like this one." It is not "a book of recipes," that is, while it offers practical suggestions, it also provides "the reasons for them." As Buswell, Brownell and Sauble remind their teacher-readers, twenty years ago there "was no need for a manual of this kind," for in those days

> arithmetic was generally viewed as a tool subject or as a drill subject, and the teaching process was not very complicated. The children's books carried the burden of instruction. These books told the children what to do, showed them how to do it, and gave them plenty of abstract examples as practice to develop their skills.

But a radical change has taken place: "Arithmetic is no longer regarded as a textbook subject." Instead, it is something which pervades the whole curriculum, as

> something that influences thought and activity throughout the day, both in school and out of school.

Moreover, arithmetic has become bigger than anything a textbook can hold. School arithmetic "is too extensive a subject to be put within the covers of a book, no matter how good the book is."

As to the disappearance of prefaces, they confess that

> as authors we cannot talk to you in the confines of the pupil's book, but in the Manual we have the chance to tell you about the arithmetic program you will teach and to suggest ways of teaching it (p.5).

Thus, one reason for the disappearance is that all relevant

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pedagogical material is now given in the accompanying manual.[89]

The almost obligatory use of a teacher's manual may be interpreted as a gesture which further contributes to the fiction of symmetry in the teacher/student relationship. Both are now treated as students and, thus, both are to access the 'world of number' through their own text. This equating of teachers and students is implicitly made possible by the elimination of the 'knowledge' term in the didactic triangle: i.e. arithmetic. Instead of arithmetic, there is, on the one hand, 'textbook arithmetic'--written *for* the child--and, on the other hand, a set of instructions *to* the teacher for the appropriate transmission of this school subject.[90]

Yet, something more dramatic may be at stake in the sudden disappearance of prefaces and introductions, something which anticipates and sheds light on the imminent disappearance of arithmetic textbooks. As we discussed throughout this chapter, aside from operating as a marketing device, the preface introduced its readers to a didactically transposed object--i.e. school arithmetic--and justified

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[89]It may also be argued that, since most textbook series are now written by two or more people, there is no longer a single author eager to add a few preliminary words to his or her text.

[90]This may be seen as a particular instance of a wider phenomenon, typical of advanced capitalism, of the standarization of the process of teaching and the deskilling of teachers (e.g. Apple 1986).

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this transposition. Textbook prefaces are statements addressing the 'public domain' (Bernstein 1996). In these introductory notes, authors speak to the public about the relationship between arithmetic as a field of knowledge--the 'esoteric domain' (Bernstein)--and school arithmetic as they have fictionalized it in the pages that follow. Everything worth saying is now said in private to the teacher. The debate is no longer held at the frontiers between school and its outside. Instead, talk about the whats, the hows, and the whys of arithmetic teaching is displaced to the inside of school, becoming engulfed, so to speak, in the pedagogical realm.

Are we thus to conclude that the arithmetic text now stands for itself, or that school arithmetic finally become an unproblematic subject? Quite to the contrary. As these books succeeded in creating their own fiction--i.e. 'textbook arithmetic'--that is, a fictionalized version of arithmetic which became totally estranged from the discipline which gave it birth, the preface has lost its purpose as a mediating text.

From the early 1880s to the mid 1950s, this peculiar didactical artifact, the arithmetic textbook, suffered a whole metamorphosis. Through successive adaptations to the demands of the social order and the needs of the child, the arithmetic text has been transformed, from a concise and cryptic scientific treatise, to an instruction manual or

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recipe book which became quite foreign to the science of numbers, "killing the science in it."[91]

I close this story with *Making Arithmetic Clear*, published in 1957 by the Macmillian Company, and co-authored by Dale Carpenter and Dorothy Pepper. Eager to jump into "The World of Numbers," as the series is called, the authors dispense with any introduction. As we look into the table of contents, we discover that the entire book is "about numbers"--how to understand numbers, how to multiply integers, and how to divide fractions.

And, while American schoolchildren would suffer through sections such as "Using Numbers," and "Using Your Arithmetics," by 1957, events surrounding this text will suddenly make apparent that, in the way in which it has been didactically transposed, arithmetic--with its four fundamental operations, and its problem-solving recipes--has become a useless subject.

Under the new math regime, a new didactical transposition of mathematical knowledge will be effected, one which, far from solving all the problems, will further add to this complex problematic.

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[91]As Smith and Reeve feared it. Or, in the words of Dutch mathematics educator Jan de Lange (cited in Keitel 1987), these texts may bring mathematics for all, but what they bring is not mathematics at all.

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# CHAPTER FOUR: THE NEW MATH

> Once we have agreed that our aims are not entirely practical, the whole question of what mathematics to teach children is immediately thrown wide open. If mathematics-learning is to be regarded honestly as a cultural activity in the way we regard the appreciation of literature, art, and music, then the present syllabuses immediately loose their sanctity. Any syllabus would be suitable provided that with it most children could appreciate mathematics as a beautiful structure, irrespective of possible practical uses (Dienes 1963).
>
> Any subject can be taught effectively in some intellectually honest form to any child at any stage of development (Bruner 1960).

On October 4th, 1957, the Soviet Union launched the Sputnik I satellite. This event initiated that rapid chain of worldwide reactions which has gone under the name of 'modern mathematics' or 'new math.' As was generally thought, if the Soviets took this step before the Americans and the rest of the 'free world,' this was not only due to their advances in science and mathematics, but also because their schoolchildren encountered algebraic structures much earlier than their counterparts elsewhere.[1]

Assuming a simplistic causal model which turns schools into weapons for economic, political and scientifico-technological dominance, the teaching of mathematics was seen as the continuation of the (cold) war by other means.[2]

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[1]For example, in the Soviet Union, children are usually taught to solve word problems by expressing the conditions of the problem in equation form, rather than by the misleading 'key word' method (Freudenthal 1974). For a comparison of reforms in Soviet and American mathematics education, see Keitel 1982.

[2]This logic has already been employed, at the beginning of the century, with respect to Prussia; and is employed, in the present 'new world order' context, vis a vis Japan and the European Community.

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Thus from the late 1950s to the early 1970s, the modernization--i.e. algebraization--of school mathematics became a priority in most First and Third World school systems.[3]

In the United States, one immediate effect of Sputnik was the 1958 National Defense Education Act which, calculating that the security of the Nation required "the fullest development of the mental resources of its young men and women," allocated seventy million dollars to upgrade curricula and design new textbooks and other materials for the teaching of foreign languages, sciences, and mathematics.[4] "In this climate of turmoil, debate, and public apprehension" (Wooton, 1965, p.9), the American Mathematical Society formed, the same year, the School Mathematics Study Group (SMSG), with an initial National Science Foundation (NSF) grant of $100,000, and under the direction of Edward Begle, from Yale University.

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[3]The new math was not just an American phenomenon. Radical departures from traditional math curricula took place in most First and Third world countries. In 1960, the OEEC held in France the Royamount Conference (OEEC 1961). Also, there was the creation of the Centre Papy, the work of Gattegno in Belgium, and that of Mle. Picard and others in France. The First Inter-American Conference on Mathematics Education took place in 1961, in Bogota, Colombia (Fehr 1961), and the following year saw the creation of the International Study Group in Mathematics Learning (ISGML) which participated actively in reform efforts. For a critical consideration of innovations in math curricula in US and in Europe, see Howson, Keitel, and Kilpatrick 1981).

[4]Most of the funding was channeled through the National Science Foundation. Recall that the support for education in mathematics and sciences has been central to the NSF's mission ever since its creation, in 1950. Private funding sources were also available, such as the Carnegie Corporation of New York.

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Together with the 1958 Act and the pioneering work of the SMSG, a third event is generally identified as a foundational gesture of the new math reform: the 1959 Woods Hole Conference, a meeting which brought together more than thirty scientists, scholars, and educators to discuss how to improve science education in primary and secondary schools. Four years later, about the same number of mathematicians and natural scientists assembled at the Cambridge Conference on School Mathematics with the purpose of articulating an even more radical program for mathematics education 'for the year 1990' (Cambridge Conference on School Mathematics 1963).

Thus the new math was born, inaugurating an era which is generally remembered as being 'all about sets,' but which in fact was far more than that.

## 4.1 Sputnik and its Aftermath

The four events described so far make apparent two 'macro'-level features distinguishing the new math refom from previous interventions: the involvement of the federal government through NSF-funded projects operating in many locations across the nation,[5] and the direct participation of mathematicians in the definition of the aims of

[5]"As was the case for several of the foundations' research programs, Sputnik, that orbiting symbol of the Soviet's scientific and military supremacy, marked a decisive turning point in NSF's education programs" (Kleinman and Solovey 1995, p. 128).

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mathematics education as well as in the design of teaching materials. It is suggestive that, rather than the US Office of Education, the leading actor of this reform was the NSF, that is "the institutional embodiment of the federal concern with intellectual excellence" (Jackson 1983).

The Sputnik episode disclosed to the public a crisis which had been acknowledged already in academic circles. What had started, in the early 1950s, as a slow process of transformation in contents and pedagogy by a group of mathematics educators participating in the University of Illinois Committee on School Mathematics (UICSM),[6] was suddenly accelerated by the Sputnik scare. Yet, while recognizing the satellite effect on the new math 'revolution,'[7] many observers consider that the opening salvo for the reform was the change in high school mathematics courses initiated, in 1955, by the College Entrance Examinations Board (CEEB).

The CEEB is an insittution in charge of administering achievement tests in the standard high school subjects; the results of these tests are used by colleges and universities

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[6]Under the direction of Max Beberman, and with funding from the Carnegie Corporation of New York, the UICSM had been undertaking the preparation of math programs for grades 9 to 12, with strong emphasis on concepts and understanding. The committee experimented with teaching methods which provided more room for the active involvement of students in the learning process (Fehr 1964).

[7]NCTM, *The Revolution in School Mathematics: A Challenge to Administrators and Teachers*, A Report with the proceedings of the 1960 Regional Orientation Conferences in Mathematics, 1961. This document expresses the support of mathematics teachers to the changes underway.

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to select students for admission. The work of the CEEB is considered an unprecedented event in the history of math education in that it was "the first truly national group to concern itself with the high school mathematics curriculum" and it "was composed of both classroom teachers and university mathematicians working together on equal footing."[8] As the Board's Executive Director lamented,

> [i]f you dug up an old seventeenth-century don he could walk right into any classroom and start teaching math. Nobody would notice the difference, since the content of the courses hadn't changed in the last 300 years.[9]

The sad fact was that school mathematics--from kindergarten to 12th grade--had become aged, obsolete, filled with banalized topics like the four fundamental operations and tedious lists of word problems, and with no traces of the latest and dearest accomplishments of this booming scientific discipline.[10] Put differently, the gap between the daily routines of teachers and students in math classrooms and the discoveries of those scholars working at

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[8]Edward Begle, in his presentation to the First Inter-America Conference on Mathematical Education, Bogota, Colombia, December 1961, p. 138.

[9]Thirty-five years later, a similar point would be made by the Mathematical Sciences Education Board: "The mathematical content of today's school curriculum is about 500 years old. The core of this curriculum--arithmetic, geometry, and elementary algebra--differs in only superficial ways from the curriculum followed by tutors during the Renaissance" (MSEB 1990, p.4).

[10]These include topics in pure mathematics such as abstract algebra, topology, general theories of integration, functional analysis, and non-Euclidean geometries.

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the frontiers of the discipline had became unbearably wide.... and such a gap the American nation could ill afford.

Inscribed in the logic of human capital,[11] the general complaint was that outdated syllabi and inappropriate methods had resulted in the underutilization of children's intellectual powers. The failure "to develop at all efficiently, or at all adequately, the latent mathematics talents and interests of the average child," was defined by Marshall Stone, at an international encounter organized in 1959 by the Organization for European Economic Co-operation,[12] as an "extremely urgent pedagogical problem." Yet, as he reminded his audience,

> From a purely technical point of view as well, we are under a continuing obligation to all our mathematics students to eliminate dead, useless, outmoded or unimportant parts of mathematics, however hallowed by tradition, from the studies we expect them to pursue with zeal and diligence (OEEC, 1961, p. 17).

In this way, a new version of the uneasiness with 'educational waste' came to be articulated. Yet, whereas the

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[11]'Human capital' had become a dominant approach to educational matters after the end of the Second World War. The clearest exposition of this logic can be found in Theodore Schultz's 1960 Presidential Address to the American Economic Association ("Investment in Human Capital"). Put succintly, the argument is made that investment in human capital, particularly through the allocation of human and material resources for education, far from simply increasing the productivity of individuals, lays also the technical base for the skilled labor force required by the nation's economic growth.

[12]Dr. Stone, from the University of Chicago, presided the event--which was referred to as the Royamount Seminar--and he was also the Chairman of the International Commission on Mathematical Instruction of the International Union of Mathematicians.

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eliminationist curriculum politics of the 1910s was guided by a logic of social efficiency and economic utility, the new politics of elimination, was inscribed in a logic of mathematical and technological reason. However, as we will discuss below, this time it was not only a matter of eliminating dated topics and adding new ones but, instead, of radically reorganizing the teaching of mathematics on the basis of the underlying structures of the discipline.

A widespread fear was felt about the nation's scientific and technological backwardness, one of the indicators of which was the projected shortage of mathematicians and other 'hard' scientists, in the face of technological developments, particularly in the area of digital computers. In Stone's words,

> the teaching of mathematics is coming to be more and more the true foundation of the technological society which it is the destiny of our times to create. We are literaly compelled by this destiny to reform our mathematical instruction so as to adapt and strengthen it for its utilitarian role in carrying the ever heavier burden of the scientific and technological superstructure which rests upon it (OEEC, ibid. p. 18).

In more or less concerted fashion, forces emerging from political, economic, and technological fields, cleared the ground for the most radical reform ever attempted in US mathematics education. These forces provided support for a project which promised both to improve the quality of mathematics learning at all levels of the school system, and to channel more college-bound students into mathematics and

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science-related courses and careers. The result was a government-sponsored, top-down intervention at the level of changes in curriculum, textbooks, and other teaching materials.

## 4.2 Anti-Didactical Inversions

At the 'micro'-level of the classroom, the radical nature of the new math resided in its simultaneous departure from pedagogy-as-usual and math-as-usual. In matters of content, new math reformers put an unprecedented stress on *structures*, an emphasis which was undoubtedly linked to the structuralist mood of the 1960s. According to Jerome Bruner--the Harvard psychologist who took upon itself the task of presenting the collaborative work done at the Woods Hole Conference and one of the main architects of the new math reform--knowledge is "a model we construct to give meaning and structure to regularities in experience" (1962, p. 120). As he continued, this leads to two educational principles: first, "the structure of knowledge--its connectedness and the derivations that make one idea follow from another--is the proper emphasis of education" (p. 120) and, second, "the unity of knowledge is to be found within knowledge itself, if the knowledge is worth mastering" (p. 121).

In matters of method, the focus was on *discovery*, an approach which found its justification in the newly emerging constructivist theory of learning. These two defining

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features: structure--i.e. to know is to structure--and discovery--i.e. structures are not taught, but constructed--found their theoretical and experimental justification in the studies on the development of intelligence in young children conducted in Geneva since the 1940s by Jean Piaget. As Piaget saw it, 'discovery' or 'active' methods

> give broad scope to the spontaneous research of the child or adolescent and require that every new truth to be learned be rediscovered or at least reconstructed by the student (1948, p. 15).

Piaget's definition of mathematics learning as the discovery of structures, rather than as the memorization of number facts and mechanical acquisition of computation skills, carried sufficient rhetorical force to meet the expectations of the various sectors concerned with the well-being of math education in the US--whether these expectations were expressed in terms of efficiency, meaning, rigour, or honesty.

It was maintained that students could efficiently reconstruct any mathematical fact, concept, or procedure in case of forgetting it, if these were presented to them in a structured fashion; and even more so if the students had been given the opportunity to discover these facts, concepts, or procedures, instead of merely having been told them. It was argued also that it is always more compelling for children to 'freely' explore problems to discover the underlying patterns and structures of mathematics, in that this modality of learning responds to their 'natural'

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curiousity towards their surrounding environment. In addition, it was regarded as self-evident that an emphasis on structures would allow for a more rigorous, orderly, and honest presentation of mathematical knowledge. Finally, if one of the objectives fueling the reform was to motivate more youngsters to become mathematicians, a discovery approach would introduce them to the practice of doing mathematics, as this is experienced by mathematicians. Thus, the reformers' paramount goal became the introduction of mathematics as such in the elementary school, that is, "complete with its sense of adventure, its unsolved questions, and its groping toward the future."[13]

This goal required a new didactical transposition of mathematical knowledge. As we recall from Chapter Two, Chevallard (1985) defines 'didactical transposition' as the series of transformations (or, better said, deformations) whereby scientific knowledge is made into knowledge-to-be-taught. Yet, the transposition proposed proved a rather peculiar one, in that the intention was, for the first time, to present to young learners mathematics *in vivo*, that is, stripped of all didactical camouflages. By misrecognizing the inevitability of the didactical transposition, a central fiction was made to operate: i.e. the belief in the continuity between school mathematics and mathematics as a

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[13]Francis Keppel--the United States Commissioner of Education--in his foreword to the Report by the Cambridge Conference (Cambridge Conference on School Mathematics, 1963, p. vii).

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scientific discipline. Or, in Bruner's words,

> ... the assumption that there is a continuity between what the scholar does on the forefront of his discipline and what a child does in approaching it for the first time (1960, p. 28).

Concretely speaking, the new math amounted to a radical change in the contents of mathematical lessons. Almost overnight, everything mathematical was expressed in the language of *sets*. As recommended in 1961 by Mle. Croquet, at the First Inter-American Conference on Mathematics Education,

> No teaching will be possible if we do not clear out our domain; without the processes which bring with them an economy in thinking, we overwhelm our children under a mass of inheritances of the past. Instruction based on the historical method, for example, has become unthinkable.... We should accostum our students to think in terms of sets and operations as early as possible. It will be necessary to teach *the simple, universal, and precise language of sets* (Fehr 1962, pp. 76-7, emphasis added).[14]

At the same time, number systems of base two, five, or twelve began to be treated on equal footing than the commonly used decimal or base ten system. Although these were only two among the many innovations introduced during the new math period, many of the arguments against the

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[14]"There are concepts in mathematics having a unifying strength which goes beyond the scope of traditional mathematics. It might be said that those concepts extend mathematics by establishing patterns previously not envisaged.
An example of such a concept is a *set* (together with other pertinent concepts such as *element, subset, intersection, union, Cartesian product* and *relation*). If we regard grammatical notions such as *noun* and *adjective* as useful in developing clear thinking, we should attach even more importance to the following ideas about sets" (Servais and Varga 1971, pp-17-8).

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reform centered around them. Thus, it seems worth considering them in some detail.

What is a set? As described in *Key Topics in Mathematics for the Primary Teacher* (1962),

> [i]n mathematics, no attempt is made to define the term 'set,' for it is impossible to define every idea in terms of simpler ones; it is necessary to start with certain undefined terms, and the idea of set is so intuitive and basic that is a convenient starting point (p.9).

While logicians may feel at ease with indefinible and undefined terms, teachers and students seem to deserve something in the form of an explanation. As an aid for "the intuitive recognition of 'set,'" the text proposes we think of set "as synonymous with such words as 'collection,' 'group,' and 'aggreggate' (ibid. p.9). That is, a set can be understood as "any collection or group of things or ideas which is described precisely enough for us to tell whether or not a given object belongs to it" (ibid. p.9).

In a similar vein, but with a less didactical tone, as it may suit a text addressed to the general public, *A Guide to Modern Mathematics* (1964) makes creative use of the metaphor of the city, with its streets, its inhabitants, and its language, to introduce to puzzled readers the new (set) approach to the old science of numbers. The preface explains that mathematics

> is something like a very large city. At its center is a compact area that serves as a common meeting ground for everyone. From this center fan out hundreds of streets.... There are a few things that should be understood first, in order to

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> appreciate the general character of the city. *Sets* are the city's most important products, and almost all citizens are engaged in making them. Conversation revolves almost exclusively around the manufacturing, use, and nature of sets. A set is a collection of numbers, things, or ideas. When someone wishes to indicate that he has pulled several (perhaps unrelated) things together to form a set, he names or pictures them within braces, {}, or puts a border around the pictures or names.

As our *Guide* describes it, sets are the main currency in the general economy of the new math city. Everything--objects, proper names, numbers, points, and figures--is treated as either belonging or not belonging to a set. This logical procedure covers the whole spectrum, from the universal set, to that hard to grasp, yet rather handy entity, the empty set, which, by definition, is the set which contains no elements.[15]

Although the preface does not anticipate it, the "border around the pictures or names"--a potato-shaped curve encompassing diverse objects--is a 'Venn diagram.' Intendedly not drawn like a perfect circle, lest children may take it for a geometrical figure, yet always in some circular shape to convey the sense of encompassing,

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[15]In *Math in the Streets*--a videotape prepared by the British Broadcasting Corporation, and based on the work of Analucia Schliemann, a psychologist who studied the 'street' mathematics of child vendors in the North of Brazil--a group of children repeat with their teacher, in a parrot-like fashion, this tautological definition of an empty set. These children who, as skillful mental calculators in the streets, are able to grapple with different prices, change, and conversions back and forth between old and new currency, are unable to understand the new math as it is presented to them by their schoolteacher. See also Nunes, Schliemann, and Carraher 1993).

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including, or holding together, Venn diagrams soon became the most reified of all didactical objects. A superficial look into many of the written materials from the new math period justifies the name given to this reform by French mathematics educator Stella Baruk (1973): *mathematiques aux pommes de terre*.

Far from a new notational nuance, sets were forwarded as very useful artefacts. By means of them, the four arithmetical operations could be redefined, thus achieving a higher status than that of mere algorithms. The activities of counting and calculating--which are so central to all preschoolers' numeracy experiences--or the chanting the multiplication tables--which belong to the cultural capital of those (mostly middle class) children whose parents are eager to initiate them into the world of numbers--were now seen as remainders of an old, superseded era. In 1961, at the annual meeting of the NCTM in Chicago, a member of the SMSG proudly announced: "We no longer teach multiplication as the addition of equal sized groups, but as the Cartesian product of two sets" (p.417).[16] As explained by David Rappaport,

> A Cartesian product of two sets is the pairing of the elements of one set with the elements of another set. Thus, if a girl has 8 blouses and 6 skirts, she could have 48 outfits by pairing one blouse with one skirt.... This is considered an improvement over the traditional concept of multiplication in which the child learned that 8 x

[16]Cited by David Rappaport (in Vigilante 1969).

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> 6 was found by adding 8 sixes" (in Vigilante, 1969, p. 419).

At the same time, the embracing of non-decimal, place value numeration systems was so widespread that during the 1960s and early 1970s it was difficult to find an 'elementary mathematics' textbook series which would not include this topic. As the argument went, by going 'outside' of our familiar base-ten system, students would be forced to consider the underlying principles of all numeration systems, and this would result in deeper understandings. And, after recognizing the base-ten system as one particular case of a more abstract and general structure, children would be free to 'apply' it to practical matters (e.g. counting and calculating in real-life situations).

These two curricular innovations generated heated controversies. For example, Rappaport strongly objected to the use of the set approach for teaching the fundamental operations of arithmetic:

> Would it not be better to have the child learn the multiplication facts as the sum of equal sized groups and then apply this knowledge to the concept of Cartesian cross-products?" (in Vigilante 1969, p. 420).

As he continued, introducing "highly sophisticated" mathematical concepts to young children takes away from them the meaningful understanding of basic arithmetic concepts which they need not only in their everyday, practical experiences, but also in their "preparation for further

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learning in other areas" (ibid.).[17]

In the meanwhile, while some researchers questioned the extent to which children were capable of 'really' understanding these new concepts and structures, and teachers demanded from experts--and shared with each other--practical suggestions for introducing these rather difficult new topics, experts insisted on the risk that the true mathematical meaning of these topics could be seriously deformed. If these two topics generated such widespread uneasiness this was, in part, because through them the teaching of elementary school math dared, for the first time, to start from the strange, the abstract, and the unpractical, instead of from the familiar, the concrete, and the useful.

Various rhetorical strategies were employed in order to make the strange familiar, all of them aimed at conveying the message that we 'use' sets all the time. For example, a textbook author would tell his young readers:

> We deal constantly with sets and subsets. When we send clothes to the cleaners, for example, we are sending a subset of the set of clothes we own"

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[17]As he continued, although "the current trend may help a few to become abstract mathematicians, the large majority of elementary school children, will, I think, be confused by many elements of the program as it is now being developed" (1969, p. 420). Similarly, Cruikshank and Arnold evaluated non-decimal instruction programs. While they found no 'harmful effects,' their five studies suggest that the study of non-decimal systems does not convincingly support spending a 'disproportionate' amount fo time on them (1969).

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(Osborn *et al.*, 1968, p.19).[18]

Conversely, the familiar needed to be made strange, as in the many activities designed to help students come to terms with the fact that our base 10 number system belongs to the large family of all possible bases.[19]

These pedagogical inversions constituted a challenge to the two trends which had dominated the field of curriculum-making until the end of WWII: developmentalism and social efficientism. Advocates of these approaches would feel entitled to ask: 'Are young children *ready* to learn the abstract notions and the complex structure of set theory and non-base ten number systems?' And, 'what is the *use* of these topics for the normal child, the future worker, or the average citizen?' Appearing in teachers' journals, school corridors, and experts' conventions, these and similar

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[18]As an instance of their proposed 'pedagogy of situations'--i.e. a pedagogy which "provides for continued intellectual involvement of children in learning processes"--Kaufman and Haag (1977) describe five and six year-old students' play-like (guided) enactment of the logic of sets. (Kaufman and Haag were centrally involved in the Comprehensive School Mathematics curriculum development program (CSMP).
In a similar vein, Zina Steinberg--a first grade teacher working in a school in Manhattan (NYC)--describes her use of the language of sets (1971): "when it was time to get coats before lunch, I asked the members of the 'set of children wearing red' to get their coats first, then the members of the 'set of children whose names begin like MILK.' This became a very normal and routine part of the day..." (p.105).

[19]Thus, for example, teachers made use of money (quarters, nickels and pennies) for introducing base 5 ('making change' between base 10 and base 5); scales with four weights--1 oz., 2 oz., 4 oz., and 8 oz.--or the English system of liquid measure (Tucker 1973), for introducing base 2; and pizzas cut in six slices each for working in a base 6 system (e.g. Steinen 1971). On the use of 'classroom lights' for teaching the binary system to fourth graders, see Morgenstern 1973).

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questions indicate that the suspension of the principles of readiness and usefulness--a double break from the prevalent tendency towards the psychologization and vocationalization of school mathematics--would not be tolerated so easily.

In fact, the suspension of the principle of 'usefulness,' that is, the idea that the selection of contents for the mathematics curriculum was to be guided primarily by questions of social significance or practical value, was seen as a must by most new math reformers (Dienes 1963). As Bruner put it,

> [i]f set theory--now often the introductory section in newer curriculums on mathematics--had to be justified in terms of its relation to immediate experience and social life, it would not be worth teaching. Yet set theory lays a foundation for the understanding of order and number that could never be achieved with the social arithmetic of interest rates and bales of hay at so much per bale (1962, p. 121).

The principle of readiness had to be suspended as well, as a requisite for a more 'honest' presentation of the subject matter. As it is usually defined, 'readiness' refers to the notion that for any concept or theory to be learned, its didactical presentation ought to respect the stages in the development of children's cognitive abilities. Translated into teaching practices, readiness results in a carefully defined set of topics for each grade (i.e. 'scope and sequence'). Bruner offered a critique of the notion of

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readiness in the context of his 'honesty' argument[20]:

> [a]ny subject can be taught effectively in some intellectually honest form to any child at any stage of development (1960, p.33)

This oftquoted statement conveys, perhaps better than anything else, the new math's utopian impulse.

In sum, cold-war paranoia and the resulting drive for excellence, awareness of the negative effects of past curricular reforms, and impatience with constraints such as the classification of students in grades and the use of pacing calendars, helped shape the belief that, as long as it was done 'honestly,' no limits could be put on how much mathematics young children could be made to learn. Moreover, the support for the inclusion of new topics such as sets and non base-10 number systems into school mathematics was based not only on their mathematical and cognitive value, but also on evidence from experimental studies in which these topics had been taught to young children.[21]

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[20]"The 'curriculum revolution,' so-called, has made it plain even after only a decade that the idea of 'readiness' is a mischievous half-truth. It is a half-truth largely because it turns out that one *teaches* readiness or provids opportunities for its nurturance, one does not simply wait for it.... Readiness for Euclidean geomerty can be gained by teaching intuitive geometry or by giving children the opportunity to build increasingly elaborate constructions with polygons" (Bruner 1965, pp. 35-6).

[21]For example, it was proved that five and six year old children could learn to recognize the notational pattern of the numbers 4 and 5 in the binary number system in one experimental session, and that first-grade children could be taught the concepts of identity and equivalence of sets.

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## 4.3 The Fiction of Embodiments

The suspension of readiness and usefulness gave permission to didacticians and teachers for effecting the desired anti- didactical transposition. This was to be accomplished even at the risk of alienating parents and the general public, challenging teachers, confusing children, and leaving 'slow' learners behind.

Although it was generally acknowledged that the sudden upgrading of mathematics curricula could present some difficulties to students, it was hoped also that, thanks to the changes underway, the learning of mathematics would become, for most children, a source of fun and excitement, and a gift to their will to know. As reported by the ISGML (1966), experiments around the world have demonstrated "that children are capable of learning more mathematics with greater enjoyment than ever before" (p. 2).

The mode of instruction which accompanied the proposed curricular changes was based on children's play-like activities with attractive materials and their engagement in the solution of challenging problems. The expectation was that students would 'talk science' rather than talk about it.[22] Yet, the new math was not so much about talking math,

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[22]As Bruner put it, in reference to the teaching of physics, "It seemed natural that emphasis should shift to teaching basic principles, underlying axioms, pervasive themes, that one should 'talk physics' with students rather than 'talk about' it to them" (Preface to the 1977 edition of *The Process of Education*, 1960, p.ix).

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as it was about 'doing' it, that is, about manipulating physical, pictorial, and symbolic 'embodiments' of mathematical structures and processes.

Following Piaget, it was proposed that if children were placed in suitable environments, with access to appropriate materials, and allowed enough time, sooner or later they would discover the structures of mathematics. At work here was another fiction: the belief in the transparency of concrete materials or 'the fiction of embodiment.' This was coupled with the belief in the power of these materials to facilitate students' access to abstract structures without the teacher's didactical mediation. Dutch mathematics educator Hans Freudenthal, one of the earliest and most severe critics of the new math reform, has referred to this move as an 'anti-didactical inversion' (1973). That is, within the fiction of embodiments, it was deemed possible for the structures of mathematics to enter, in a clean and direct fashion, the children's cognitive structures.

To explore this fiction, we must take a closer look at the didactics of mathematics developed by Zoltan Dienes who, together with Bruner, was one of the key figures of the reform. Atuned to the politics of international cooperation in math education and in education in general which characterized the 1960s scenario, Dienes had conducted experimental studies in mathematics didactics in such diverse geographical locations as Boston, Sherbroke,

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Adelaide, and New Guinea.

Following one of the main axioms of the Piagetian paradigm, namely, that children acquire logico-mathematical structures by internalizing their actions with concrete materials in a process of reflexive abstraction, Dienes' didactics centers around children's involvement in play- and game-like activities with concrete objects or manipulatives.[23] Within his 'psychodynamic' model, learning is seen as resulting from the experience of categorizing, a process which involves cycles of increasing abstraction. As Dienes explains, each cycle consists of three stages: a preliminary, rather unstructured 'manipulative play' stage, a structured intermediate constructive stage, and a more sophisticated, rule-bound play stage. Once children have been led, through increasingly controlled manipulations, to grasp the structures embodied in the concrete materials, they can utilize pictures, graphs, maps, and mathematical symbols to represent their findings.

Since "everybody knows that mathematics is an abstract subject" (Dienes, 1971, p. 337), an urgent didactical task

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[23]The most widely-used manipulatives were the Cuisinaire rods, the Attribute Blocks, and the Multibase Arithmetic Blocks (MBA) or Dienes blocks. The Cuisinaire rods, developed by C. Gattegno in Belgium, are a set of 291 wooden rods varying in color and length, which are especially used to demonstrate number relationships. Since the rods are not affixed with numerals, they can be used to explore logic as well as quantitative relationships. The Dienes blocks are designed for the exploration of number systems of different bases, and the Attribute blocks are indicated for the teaching of logic and set theory.

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was the design of appropriate material apparati to faciliate the passage from the concrete to the abstract. Central to this approach was the assumption that, to the extent that learners manipulated this apparatus, they were actually operating with and within the given mathematical structures (.i.e. Brousseau's 'Dienes effect').

The structured materials favoured by this model were defined as free of distractors, in that, as opposed to beads, sticks, or coins, they had no use in daily life. As Dienes explained it in a report commissioned by UNESCO to the ISGML,

> [a]rtificial practical situations involving apparatus... can provide a much simpler and more relevant kind of embodiment of mathematics than would situations arising in the child's natural environment (1966).

Moreover, these materials were said to embody mathematical structures without being tied to symbolic notational systems. Thanks to their being beyond language as well as to their "simplicity and universal character," these materials could be used in all countries around the world.[24]

What made these materials so popular among math educators and classroom teachers during the 1960s and afterwards is, in part, that by allowing the child "freedom

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[24]This allowed for a whole parafernalia of didactical materials to be manufactured and to circulate in American classrooms and abroad. An analysis of the political economy of manipulatives would be a study in itself. It will include, for example, the cycle from a small school in Thuin (Belgium) where, around the 1940s, George Cuisinaire designed his rods, through the creation of the Cuisinaire Company of America, to the use of Cuisinaire rods in small African towns and in Latin American villages.

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to choose a process of abstraction and to develop confidence on his (sic) ability to discover" (Vigilante 1969, p. 305), their use was compatible with a child-centered classroom.[25] As "an attempt to engineer ways in which play- and game-like materials can be used in classrooms as the basis for mathematical experience" (Westbury 1973), Dienes' approach not only provided for the passage from the concrete to the abstract, but it also lent support to a classroom organization where groups of children were assigned to work at different 'stations' or 'corners,' depending on their interests and/or cognitive ability.

As Ian Westbury puts it (1973), since this shift of pedagogical model amounted to a transfer of control from the teacher to the materials, the effectiveness of this process necessitated that these be "self-instructing and, as far as possible, self-managing" (ibid.). Thus, the new mathematics classroom required a radical redefinition of the teacher's role. As Dienes himself prescribed it,

> children should be allowed to make their own constructions from these manipulations; a construction forced by the teacher is likely to result in parrot-like repetition of responses and is not likely to be genuine at all (1963, p. 157-

[25]It was also claimed that experience with rods provided "for visual and tactile manipulations necessary to reinforce number awareness among children who cannot deal with abstract work" (Vigilante, 1969). Yet, it was also noted that while developing a strong understanding of the structure of the number system, the abstract and artificial nature of these materials made them lacking in social application. Thus, it was deemed "desirable for teachers to continue to emphasize social applications" (Vigilante 1969).

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8).[26]

It was recommended that, in order to allow learners to construct knowledge by themselves, teachers not engage in frontal teaching but, instead, operate as facilitators, guides, or "organizers of the flow of communication" (Dienes and Golding 1971). That is, for manipulative play 'to do its job,' teachers had to refrain from what historically has been their job, i.e. teaching. Moreover, it was recommended that the material embodiments of mathematics be "designed and invented by experts who are aware of the concepts which must be embodied in a concrete way," rather than by teachers, who are "not usually capable of gathering or designing aids of this nature once the concepts involved reach the higher levels" (Dienes and Golding, 1971, p. 54). In line with Brousseau (1986, 1988), we may see in all these gestures an implicit attempt at establishing an 'anti-didactic contract' for the learning of mathematics.

The displacement of teachers from their traditional role was consistent with the dominant constructivist and child-centered approach to teaching and learning. Yet, this

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[26]Another crucial notion in Dienes' didactics is that of 'perceptual variability.' As he explains it, "to abstract a mathematical structure effectively, one must meet it in a number of different situations to perceive its purely structural properties" (p. 158). He refers to this as the principle of 'multiple embodiment,' i.e. the prescription that every concept ought to be presented in as many different ways as possible. Finally, there is the 'mathematical variability' principle, which proposes that since every mathematical concept involves essential variables, these ought to be varied in order to achieve the full generality of the mathematical concept (p. 158).

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displacement should also be understood as a response to what had been perceived, from the start, as a main obstacle to the reform's implementation, i.e. teachers' lack of preparation in modern mathematics. As a matter of fact, as they discussed the design of a mathematics curriculum for the year 1990, participants in the Cambridge Conference acknowledged that

> [i]t is not only that most teachers will be incapable of teaching most of the mathematics set forth in the curricula proposed here; most teachers would be hard put to comprehend it. No brief period of training will suffice. Even the first grade curriculum embodies notions with which the average teacher is totally unfamiliar (Cambridge Conference on School Mathematics, 1963, p. viii).

Thus, teachers were put in the uncomfortable position of mediating in a process which casted them only as secondary actors and yet one which they could easily damage due to their limitations.[27]

The use of self-instructing and self-managing technologies of learning--including not only manipulatives but also computarized programmes and other 'teaching machines'[28]--which was presented as a help for teachers' daily practices was a subtle attempt at getting them 'out of

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[27]The Committee on the Undergraduate Program in Mathematics (CUPM), part of the Mathematical Association of America took the leading role in establising recommendations for new content in the preparation of pre-service and in-service teachers (CUPM 1960).

[28]Cf. the computed-assisted instruction (CAI) programs for drill-and-practice and problem-solving developed, since the mid-1960s, at Stanford University by Patrick Suppes and his collaborators (Suppes *et al.* 1969).

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the way.' Beyond the seductive rhetoric of 'freeing' teachers from the labor-intensive and routine aspects of their job, both in the new math's didactics and in the approach to learning which was contemporary to (and overlapping) with it--i. e. programmed instruction[29]--the displacement of teachers by manipulatives, computers, videos, and other technological cornucopia solved, at least in principle, the problem of providing for the costly investment and long-term endeavor of in-service and pre-service preparation of teachers.

Dienes' "object-game oriented approach" was only one among many didactical models available during the new math period.[30] Competing in popularity was the "symbol-game oriented approach" followed by the University of Illinois Committee on School Mathematics (UICSM), the SMSG, and the Madison Project.[31] In this formalist model, symbols rather than concrete materials are used as playthings.[32] Although

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[29]Under the new math umbrella there were projects such as the one initiated in 1963 by Patrick Suppes at Stanford University, which centered around the use of programmed instruction for the teaching of mathematics (Suppes and Morningstar 1972).

[30]These are usually classified under six categories: basic set, arithmetically-oriented, geometrically-oriented, symbol-game, science-oriented, and object-game oriented approach (ISGML 1966). See, also, DeVault and Kriewall 1969 and Sherman 1972.

[31]On the use of the 'discovery' approach to math learning by the Madison Project, see Davis 1966.

[32]The first kind of symbol introduced is usually the open sentence--e.g. (♯ x 3) + 2 = * --where place-holders are represented by different sorts of frames. As critics soon pointed out, these are nothing different than the old equations, now called 'sentences' to stress the analogy with spoken language. Discussing his teaching experiment with "some low-IQ culturally deprived children

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these and other projects differed in their underlying assumptions, explicit aims, and technical features, their inclusion under the new math umbrella is justified in that they all 'made the abstract concrete' by means other than the 'clothing of mathematics' which was characteristic of the 'old' arithmetic instruction. Or, to use one of the favorite metaphors of that time, a systematic effort was made toward the reduction of 'noise'--that is, the elimination of any visual, linguistic, or cultural element which would interfere with the pure and transparent structures of mathematical thought.

Much more could be said about the assumptions and prescriptions of the projects at work during the new math period. Yet, for our purposes at hand, we must now consider to what extent--and with what effects--these theoretical models, practical recommendations, and material parafernalia translated into 'real' classroom practices. For, if one

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in the seventh grade," Robert Davis--from the Madison Project--admits that "with a middle-class teacher and a middle-class curriculum," students are lead to 'discover' that the set of numbers which make this sentence true constitute a pattern for linear equations. Advocates of this approach assume that, through playing with symbols, the properties of that which is symbolized are learned. Thus, rather than introducing symbols at the end of the process of abstraction--as in Dienes' didactics--symbols are introduced from the start, as a strategy for cutting through the irrelevant 'noise' during the process. Dienes suggests that the amount of noise may be minimal but, for that very reason, the probability that children would be motivated by the task may be low as well--"[f]ree play generates more noise than restricted play, but it also generates more zest, although there is also a measure of excitement in the restrictions and rules imposed, especially if they are naturally imposed or self-imposed" (p. 160-1).

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agrees with Basil Bernstein that the classroom is "the graveyard of the researcher's intentions" (Atkinson *et al.* 1995, p. 137), this was perhaps never more true than during the new math era.

## 4.4 The Revolution that Never Happened

From the mid to the late 1970s, the opposition to the new math began to increase. The most severe critic was Morris Kline, a professor of mathematics at the New York Courant. Kline considered that the new math's curriculum renewal was, at best, only appropriate for those few students who would become mathematicians. As described by Bruce Vogeli--a math professor at Teachers College--Kline's widely read *Why Johnny can't Add: The Failure of the New Math* (1973) "shook the confidence

> of parents, teachers, and administrators in the 'new math' movement, and they in turn have shaken the resolve of publishers to collaborate further with 'new math' reformers. The result has been a return to basic skills. The slogan 'back to the basics' is alone responsible for the commercial success of several new, reactionary, and thoroughly mediocre mathematics series that have appeared in recent years (1976, p.18).[33]

The NSF sponsored three studies of the impact of the recent reforms: a survey of teaching practices (Weiss 1978), a collection of case studies of classroom practices

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[33]Along the same lines as Kline's argument against the new math, there is the monograph by math educator Leo Macarow entitled *New Math or New Myth?* (1981).

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across the nation (Stake and Easley 1978), and a review of the literature on the status of precollegiate science, mathematics, and social studies from 1955 to 1977 (Suydam and Osborne 1977). In the latter, the authors concluded that "[a]s reflected in print, the content of school mathematics curricula changed... But the inclusion of 'new math' content in the elementary school may be illusory" (p. 48). Together, the three studies identified

> a consistent pattern of great differences between the apparent reality of mathematics education on most schools and the recommendations or practices of many prominent teachers, supervisors, and professional organizations (Fey 1979, p. 14).

A similar conclusion was reached in by Price, Kelley, and Kelley (1977), on the basis of a survey questionnaire they administered to more than 1200 second- and fifth-grade teachers across the country. Their conclusion was that, in the years between 1952 and 1977, things had changed far less than had been supposed. The average classroom continued to operate like a self-contained entity, with a 43-minute long math period, half of which was devoted to written work, and with heavy reliance on a single textbook. Further, the modality of transmission was generally whole-class instruction. And, as for the content of lessons, teachers were mostly teaching what they had been taught in school. As Price, Kelley and Kelley concluded, almost none of the concepts, methods, or big ideas of modern mathematics appeared in the average classroom. As cynically described by

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Cruickshank and de Flandre,

> Previously we chanted the 'times tables' and played fast-moving games of 'flash card.' Today, after nearly fifteen years of revolution in elementary school mathematics, we chant the associative property and ask, 'What plus what equals what plus what?' (1971, p. 443)

For some critics, the revolution in school mathematics which had begun in the early 1950s was actually *interrupted* by Sputnik. That is, 'the problem' was not the new math, but the pace at which the changes had been deployed. As Edwin Hirschi put it (1977),

> [i]f we had moved in a more cautious manner by a massive retraining of mathematics teachers at the inservice and college level I believe we could have warded off most of our present troubles.

Any visit to a math classroom, he continued, may reveal "a mishmash of old and new mathematics handled over by a discomfited teacher" who was never given the opportunity or encouragement to learn and understand the 'new math.'"[34] Thus,

> confused students carry their frustrations into the home where bewildered parents, presently somewhat relieved of their fear of Russian dominance, are now ready to abandon the program that they had accepted in desperation but never really understood (Hirshi, ibid. p. 224).

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[34]A similar point is made by Bruce Vogeli (1976), a professor of mathematics education at Teachers College: "Before rejecting the premise that stress on unifying concepts enhances mathematical learning, we must ask if the premise was, in fact, observed by elementary teachers. I believe it was not. Of the literally hundreds of elementary classrooms I visited in recent years, I can count on the fingers of one hand those in which the spirit of the new mathematics truly prevailed. Of course, the 'new math' forces must accept much of the blame for this failure: they were unable to get the message across to classroom teachers--through texts or guides, propaganda or coercion" (p.19).

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In their examination of changes in curriculum development in the US and abroad--in the light of the multiple social forces which affected those innovations--Howson, Keitel, and Kilpatrick (1981) conclude that "too much was attempted in too short a time with insufficient thought concerning aims and possible effects" (p. 238). True some of the new math projects carried the label of 'experiments.' However, as these authors remind us,

> an educational system is not like a chemical laboratory where a failed experiment means merely that certain chemicals and apparatus have been expanded. A failed experiment in education can mean deprived pupils and dispired teachers, and parents who look askance at further proposals for change (ibid.).

As they raised the question "Will there be a New 'New Math'?," Robert Hayden and William Rudolph (1984) depicted a similar scenario:

> [t]eachers and parents soon became threatened by children who seemed to know more than they did, or the expectation that they themselves should accomplish such prodigious feats. Finally, when the children who seemed so sophisticated about sets and the commutative law did not show a corresponding improvement in the kind of practical mathematics that balances checkbooks or launches Sputniks, the public, and many mathematics educators, began to rebel against 'new math' in general, and 'modern math' in particular (1984, p.?).

Thus, while some critics doubted that the desired changes had happened at all, others identified the multiple dystopian effects of this intervention in the larger social field, including the alienation of parents and the general public from the ongoing reform efforts.

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To be fair, the potential misuses of the new math's innovations had been anticipated by their own creators and supporters. For example, Dienes himself acknowledged that manipulatives could add their own 'noise'--i. e. children could learn the materials rather than acquire the structures (Dienes and Golding, 1971, p. 56).[35] As he proposed, this could be avoided with the 'multiple embodiments' principle, that is, by presenting every concept in as many different ways as possible, so as to allow for 'perceptual variability.' Yet, in actual classroom practices, the application of this as well as other principles met with a complex array of constraints, including limitations of time and of the teacher's understanding of mathematics, large class sizes, and the lack of appropriate material resources.

As for sets, let us offer the case of Herbert Vaughan (1970)-- a member of the UICSM--who retells the story, told to him by a mathematics supervisor, of a teacher who told her students: "Open your desks and take out your elements." As soon as children took out some colored counters, the teacher said: "Now take out your sets." And when children took out some loops of differently colored yarns, she continued "Now put your green elements in your yellow set."

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[35]Dienes claimed that children are able of forming instantaneously an *isomorphism* between the concrete materials they manipulate and the abstract structures of mathematics. Based on an empirical study of children's acquisition of the concept of 'group,' Lamon and Scott (1970) argue that fourth and sixth graders are not able to grasp such concept beyond the level of one-to-one correspondences between concrete objects.

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As we may have expected, Vaughan reacts strongly to such a misuse of the language of sets. A set "is not a physical object--it cannot be seen, it does not occupy space, nor can it be carried from one place in a classroom to another." And, he further explains,

> [w]hat most people think of when they speak of a set of dishes is very far removed in sense from what a mathematician means when he speaks of the set whose members are the dishes in question.... A set is not a heap (p. 55).[36]

As it materialized into classroom practices, the sudden redefinition of math learning as the discovery of structures produced a series of tensions and complications. For, if efficiency was at stake, a discovery approach often proved to be far from the best-suited didactical route. In fact, it was noted that, while capable of generating more excitement and desire to learn than traditional, teacher-led, frontal, or 'didactical' approaches, the discovery method was a time-consuming classroom technique, which did not always lead to

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[36]Vaughan blames also the authors of some of the new textbook series, in that their definition of sets as collections invites the misunderstanding that sets are simply heaps, or bunches of things. Moreover, Vaughan advises to teachers that, despite what their text may say, "one does not enclose the members of the set between braces." And he recommends: "Please don't 'construct' a set of pupils by having them stand in a line and having two others hold up cardboard braces at the ends. Instead, name the sets by listing the pupils.... While you are completely justified in pushing around members of sets--say, counters or popsicle sticks--in order to call attention to the sets you are talking about, neither you nor anyone else can move a set. Only physical objects can be moved, and no set is a physical object" (p.58)
This may be seen as an instance of what Seeger and Steinbring refer to as the 'mystification' or 're-mythologization' of mathematics: abstract operations and structures are naturalized or made concrete, by equating them with processes and objects belonging to children's environment (in Ernest 1994).

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the intended outcome (i.e. learning a pattern, a law, a property, or an algorithm). Thus, many teachers opted for a modified version (i.e. 'guided discovery'), a modality which, in practice, tended to degenerate into pseudo-discovery. That is, students and teachers jointly created the illusion of a mathematical discovery when, in fact, the teacher 'told' students what they were supposed to discover.

The reproduction of this illusion in so many 'reformed' classrooms signals one of the many ways in which the teaching of mathematics may be at odds with the child-centered, progressivist, and constructivist prescriptions which, together, constitute the modern pedagogical paradigm. If the teacher tells too much, the students do not learn. In Brousseau's terms, when the teacher falls into the temptation to tell, the didactic contract breaks down in ways which are not conducive to meaningful learning (1986).[37] If, instead, the teacher does not tell--either because she does not know what to tell, or because she decides not to intervene--the didactic contract breaks down as well, as the teacher has withdrawn from her responsibility of leading students towards learning.

The best, middle-of-the-road strategy may be 'guided discovery.' Yet, in order to guide children's discoveries effectively, teachers must have a strong understanding of

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[37]To tell or not to tell continues to be one of the main impasses of this pedagogical paradigm (Brousseau 1986, Lampert 1990, NCTM 1991, Ball 1993, Smith 1996).

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what is to be discovered, as well as the appropriate didactical tools and models to performence such guidance. In other words,

> [i]f the student is to be brought to the frontiers of knowledge, the teacher must know the whereabouts of those frontiers. If the student is to be encouraged to grope, the teacher must at least be able to suggest which of his roads are likely to be blind alleys (Keppel, 1963, p.viii).

Or, switching from a travel to a military metaphor, and in the words of old master Edward Thorndike, we may think of the teacher

> as a general who protects his (sic) army against... dangers, extricates them from this or that trap, and provides them with the best weapons and ammunition.... So the teacher's work includes measures to avoid misunderstandings and false steps, the diagnosis and cure of difficulties, and the selection or invention of just the best means for learning each topic (1921, p. 147).

The discovery approach has been shown to put an extra burden on 'classroom management.' Faced with overcrowded classrooms, given the 'natural' limitations of didactical time, and further constrained by their spontaneous philosophy of mathematics and their limited knowledge of this subject matter, many teachers, as good pragmatists, found ways of accommodating the reformers' prescriptions, by introducing the new topics with old-fashioned teaching methods. To this day, researchers continue to discuss whether this inconsistency in teachers' practices ought to be attributed to deficiencies in their conceptions of mathematics, teaching, and learning, or whether it ought to

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be interpreted as a realistic response to the multiple constraints affecting their daily work as well as the inconsistencies in the reformers' didactical recommendations.[38]

It soon became apparent to many observers that an emphasis on structures did not always make mathematics more meaningful for children. In a conference presentation entitled "Modern Math, Does it Exist?," French mathematician Rene Thom argued that, in their eagerness to find a once-and-for-all solution for all pedagogical troubles, modern math reformers were seduced by the double fiction that, if children were given direct access to mathematical structures, they would learn more effectively, and that if their implicit processes--classification, ordering, and so on--were made explicit, they will develop more powerful abstractions. In his view, far from furthering children's understanding, this 'making explicit the implicit' often resulted in a rather unproductive emphasis on mathematical rigour for its own sake. Thom saw this as a indicative of the naive conception, held by some reformers, of the place which rigour and formalism occupy in the production of mathematical knowledge. In fact,

> [o]ne accedes to absolute rigour only by

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[38]This debate, which I introduced in Chapter One, centers around the question of how to account for 'the persistance of recitation,' has occupied significant space in education journals since the 1960s. See, for example, Hoetker and Ahlbrand 1969; Westbury 1973; Stodolsky *et al.* 1981, and Sosniak *et al.* 1991.

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> eliminating meaning; absolute rigour is only possible in, and by, such destitution of meaning. But if one must choose between rigour and meaning, I shall unhesitatingly choose the latter. It is this choice one has always made in mathematics, where one works almost always in a semi-formalised situation, with a metalanguage which is ordinary speech, not formalized. And the whole profession is happy with this bastard situation and does not ask for anything better (Thom 1973, pp. 202-3).

The question of meaning brings us to the work of French math educator Stella Baruk (1973, 1977, 1985). In line with the critique of the structuralist approach to the teaching of mathematics, and constructing her argument with suggestive events collected as she helped children who 'fail' in school mathematics, Baruk describes the new math--and school mathematics in general--as a didactics without a subject (1985). To the question 'How did children manage to find their way in the new math city?'[39], Baruk would respond: by becoming 'automaths.' An automath is "a user-distributor of signs, messages, codes, and systems deprived of meaning; user-distributer in spite of itself, of mathematical language" (1985). In other words, an automath is a subject who follows rules blindly, with no concern as to whether or not what s/he does makes any sense. After all, as Baruk comments, in the mind of the average child, school mathematics is not supposed to make sense.

There is plenty of evidence that, during the new math era, structures were usually acquired by rote-learning. As

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[39]As it was described in the *Guide to Modern Mathematics*.

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described in 1976 by British mathematics educator Richard Skemp, in most classrooms there was more 'instrumental' than 'relational' understanding; more "rules without reason" than "knowing both what to do and why." As he continued (and here I quote at length),

> This is predictable from the difficulty of accommodating (restructuring) our existing schemas. To the extent that this is so, the innovations have probably done more harm than good, by introducing a mis-match between the teacher and the aims implicit in the content. For the purpose of introducing ideas such as sets, mappings, and variables is the help which, rightly used, they can give to relational understanding. If pupils are still being taught instrumentally, then a 'traditional' syllabus will probably benefit them more. They will at least acquire proficiency in a number of mathematical techniques which will be of use to them in other subjects, and whose lack has recently been the subject of complaints by teachers of science, employers, and others (p. 11).

Skemp's statement prefigures the restructuring of the math education field which occurred during the 1970s 'back to the basics,' a backlash movement which lasted about ten years.

The early 1970s witnessed a diminishing of the cold war paranoia and a severe worldwide crisis of capitalist states and economies. This phenomenon precipitated a fiscal crisis of the state which, in turn, resulted in the shrinking of resources for experimentation and innovation in education--a politics which further accentuated the "world educational crisis" already declared in 1967 (Coombs 1967). Accelerated by the nation-wide falling of scores in standardized mathematics tests, particularly in the area of basic

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computation skills (NACOME 1975; Carpenter *et al.* 1975, 1981; Austin and Garber 1982), this scenario brought about a general disillusionment with the new math reform. This was especially manifested in an increasing impatience in the face of the inefficiency of discovery methods, as well as in a widespread skepticism as to the value of insights into the underlying structure of mathematics for the majority of schoolchildren.

For example, Rappaport explained (1976) that elementary school pupils may be classified into three different groups: "the innovative and creative people who are really the great minds in society" (about 10 to 15% of the population), the highly capable people who are "capable of understanding and appreciating the work of the creative people" (about 15 to 25%), and those who compose the rest of society, that is to say, "the ones who make society function." As he continued,

> [t]he 60 percent of the children who are in the third group do not need to learn *mathematics*, certainly not as an abstract, structured system. These children do need to learn the basic arithmetic skills in order to function effectively in society (p. 344).[40]

As in Thorndike's era, the theme of 'basic arithmetic skills' became the central preoccupation of math educators,

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[40]A similar view is expressed by Gerald Brown--a professor of Education at California State College--in an article suggestively entitled "What happened to elementary school arithmetic?" (1971). In Brown's view, the reformers "did not bring us a substitute for the multiplication talbes, for the addition facts, or for the need to see how mathematics is applied. They didn't--nor are they able to--tell us how to handle slow learners or what mathematics is most appropriate for them (p. 174).

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school administrators, and other noospherians. And, during this backlash period, a renewed emphasis was put on drilling and practice in computational algorithms and problem-solving, and there was a return to practical or applied courses.

Within the US math education community, the new math has been dismissed, almost unanimously, as a failed attempt at imposing a centralized, quick-fix, top down reform, which made no provision for preparing teachers to effectively implement the desired changes. As a consequence, teachers responded either by distorting the true spirit of the proposed innovations, or by rejecting them altogether and continuing to teach the old basic skills.

I suggest that, notwithstanding all of the above, as a project, the new math constituted an unprecedented step towards a more effective teaching of mathematics in the elementary school. It did so by ignoring the old-time prescriptions that school mathematics be practical, useful, and socially significant and that it be atuned with the evolving needs and interests of the child. For the first time, the possibility was opened of staying within the world of number, without the pressure to establishing immediate connections with the 'real' world and to camouflaging mathematics in child-like costumes.[41]

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[41]However, as Freudenthal's disciples powerfully argued, if the new math was big on the vertical axis of mathematization, it definitely undervalued the horizontal axis (Treffers 1991).

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If the new math project did not succeed, or if was shamefully deformed, this may be attributed to the limitations of Piagetian constructivism as much as to the sudden shift in the overall political scenario from cold war paranoia to the crisis of the welfare state. One may also argue, following Keitel (1982), that since the new math's revolution in school mathematics was never given a real chance to happen, rather than designing new programs, the task at hand is to continue the slow process of reform where it has been interrupted.

But, instead, US noospherians opted to begin anew. Around the late 1970s, it started to become apparent to most sectors concerned with the well-being of math education in the US that there was no 'good old math' to recycle or, rather, that the 'old math' was no longer good enough for the new times. Much talk about securing basic skills by means of drill and practice--albeit modernized thanks to the newly available technological cornucopia--soon proved insufficient. To move forward, a new change in teaching methods was decreed and, in the 1980s, the learning of mathematics was once again radically redefined.

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# CHAPTER FIVE: THE NEW NEW MATH

> We are at risk of becoming a divided nation in which knowledge of mathematics supports a productive, technologically powerful elite while a dependent, semiliterate majority, disproportionately Hispanic and Black, find economic and political power beyond its reach. Unless corrected, innumeracy and illiteracy will drive America apart (Mathematical Sciences Education Board 1989).

> When students make public conjectures and reason with others about mathematics, ideas and knowledge are developed collaboratively, revealing mathematics as constructed by human beings in an intellectual community... Like a piece of music, the classroom discourse has themes that pull together to create a whole that has meaning (National Council of Teachers of Mathematics 1989).

The 1980s and 1990s brought hard times for the United States economy and its public schools. Jobs are scarce and students are not learning as they did in the past or, at best, what they learn is irrelevant for their future occupations. It is claimed that schools are out of sync with the emerging technological and information society. As we enter the 21st century, society turns post-industrial but schools are lagging behind. This is perceived as a severe crisis, not only due to the centrality of education in the nation's imaginary, but also because knowledge has become the new means of production.[1] A radical transformation is thus needed both in those sites where knowledge is produced-

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[1]Peter Drucker forecasts that by the end of the 20th century, 'knowledge workers' will make up more than a third of the nation's workforce. As knowledge becomes the key national resource, he argues, the quality of schools will be a critical factor in international economic competition (1994). For a discussion of the new status of knowledge in information societies, see Lyotard 1979).

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-i.e. laboratories and universities[2]--an in those where it is distributed: the schools.

This multilevel crisis is said to affect the entire educational system.[3] Yet, what is being most vociferously lamented is the chain of disfunctionalities which affect, from kindergarten to 12th grade and beyond, the learning of mathematics. For as it was shown by the Sputnik crisis and has not been challenged since then, mathematics, "the invisible culture of our age" (Mathematical Sciences Education Board, 1989), has been endowed with the highest exchange value among all fields of knowledge.

To make matters worse, the demographic trends projected for the next decades anticipate a need to reacommodate the school system similar to that which occurred at the turn of the century. Since minorities are becoming majorities, their school failure is tantamount to the failure of the nation as

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[2]The 1980s brought a fundamental change in the function of science. Whereas in the 1960s the power of science was linked to its potential contribution to social reform, in the 1980s, science is primarily measured in social efficiency terms (Popkewitz *et al.* 1986). Cf. also Aronowitz and DiFazio 1994.

[3]Between 1963 and 1977, the average scores on the Scholastic Aptitude Test (SAT) declined a total of 81 points, on a 200-800 scale. The Third National Assessment of Educational Progress indicates a decline in overall scores during the period 1972-1980 (NAEP 1983, Carpenter *et al.* 1983). For 1994, NAEP indicates that two fifths of 4th graders, including 2 out of 3 Blacks and Hispanic youngsters, can hardly read at all; among high school students, only 36% were proficient in this basic skill; and almost half of the entering freshmen in the California state-university system required remedial instruction in reading or mathematics or in both (US Department of Education 1994). Also, the 1992 National Adult Literacy Survey indicates that about 40 to 44 million of the 191 million adults (21-23%) demonstrated skills in the lowest level of prose, document, and quantitative proficiencies.

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a whole.[4] When applied to mathematics, this argument carries an extra weigth, given that racial and ethnic minorities--and women--are underrepresented in occupations and careers requiring skills and credentials in higher level mathematics, as well as in post-graduate teaching and research positions in mathematics, science, and technology.

## 5.1 The Need for Reform

Although the account just presented is only a sketch of a complex picture, it seems sufficient as the background for asserting that the current new new math reform engages a multitude of voices which harmonize in a way that is unprecedented since the late 1950s. This will become apparent as I review a series of documents released, from the early 1980s to the mid 1990s, by various constituencies that have a stake in the well-being of mathematics education in the United States: the federal government, the corporations and the private business sector, the scientific community, and the teachers of mathematics.

The federal government is preoccupied with 'excellence,' 'higher-order' skills, and 'world class'

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[4]Johnston and Parker emphasize the disastrous consequences of failure in mathematics for the nation's industrial needs (1987). Corporations spend as much every each year in on-the-job mathematics training courses for their workers, as is spent on school math education. Moreover, due to the expected growth in the minority composition of the workforce--i.e. the number of white males in the work force is decreasing--the failure in math of these groups has serious implications for the nation's economy. See, also Oaxaca and Reynolds 1990).

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standards. The argument is made that, due to the demographic trends delineated above, a concern for *excellence* ought to be coupled with one for *equality*. Government officials insist on tougher certification requirements for teachers, and call for a more appropriate assessment system to make states, districts, schools, teachers, and students accountable for meeting the new standards. It is proposed that, while curriculum and performance standards for each subject area are best formulated at the national level, their implementation is to be secured by local authorities, with the financial support of individual states, corporations, and the private sector.[5]

Representatives of corporations and the private sector argue that, due to the increasing computarization of the workplace, the teaching of mathematics ought to move beyond its traditional emphasis on *mechanical* skills, towards the development of *analytical* abilities. Pronouncements about the demands which the new workplace imposses on school mathematics assume an homogeneous labor market, in need of

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[5]Since 1991, NSF has signed cooperative agreements with 26 states to undertake reform initiatives in science, mathematics, and technology. This effort is known as the Statewide Systemic Initiatives Program (SSI). SSI projects intervene in research and development on new curriculum, training and professional development of teachers, production of educational resources, and design of new assessment instruments (CPRE 1995; NSF 1995; NYSSI 1995, 1996).

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highly skilled problem solvers or symbolic analysts.[6] For these workplaces, 'speed and accuracy in the performance of arithmetical operations' may, in fact, have become an obsolete skill.

Yet, these environments--computarized, highly paid, stable, full-time, and unionized, where people work in groups, and enjoy a relative autonomy and control over their daily assignments--constitute only the upper end of the job market, to be filled mostly by American White males (Reich 1991; Aronowitz and DiFazio 1994). Due to the local and global restructuring of the era of flexible capital, the lower end of the job market is increasingly made up of routine, less-than-full-time, temporary, non-unionized, semi or unskilled jobs, where people work individually and have little control over their daily tasks.[7] These jobs, which are filled mostly by women and ethnic and racial minorities, continue to demand disciplined laborers equipped with basic computation skills (even when these skills are irrelevant for the performance of the jobs in question). Either worried

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[6]Reich claims that the ratio of symbolic analysts affects the US political and economic strength. He defines symbolic analysts as those specialists, mostly white, male, and college educated, who "simplify reality into abstract images that can be rearranged, juggled, experimented with, communicated to other specialists, and then, eventually, transformed back into reality" (1991, p. 178).

[7]The largest proportion of new jobs are relatively unskilled and millions of skilled workers are jobless (Levin and Rumberger 1983). For every 1 computer programming position, 9 new cashier jobs are produced; and there are 1.5 times as many janitors than lawyers, accountants, investment bankers, stockbrokers, and computer programmers combined (cited in Berliner 1996).

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about the dollars spent in on-the-job basic skills courses, or eager for highly skilled problem solvers, corporations and private sector leaders have acknowledged the need for reform in mathematics education.[8]

Teachers have recognized the need for radical change in school mathematics. Reform is badly needed but, this time, they ought to be active participants in it. For a lesson learned during the 1960s new math is that no reform will be effectively implemented, unless teachers are involved. Elementary school teachers are interested in solving a long list of (old and new) problems, including the students' lack of motivation to learn math; their students (and their own) fear of mathematics;[9] the mechanical way in which most children go about solving word problems;[10] the deficient quality of textbooks and other resources; and severe

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[8]Since the early 1980s, Ford, Exxon, IBM, Apple, Kodak and other corporations have been providing financial support for various innovations, including in- and pre-service teacher training programs, the development of new technologies for math teaching, and a variety of university-based research projects which focus on different aspects of mathematics teaching and learning.

[9]This pathological reaction to mathematics is referred to as 'mathophobia' (Tobias 1980) or 'math anxiety' (Resek and Rupley 1980). That these notions have become part of the teachers' imaginary, may be gathered by leafing through *Arithmetic Teacher*. Consider the following titles: "Some thoughts for teachers on mathematics anxiety" (Brush 1981), "On the transmission of mathematics anxiety" (Bulmahn, and Young 1982), "A study of math anxiety/math avoidance in preservice elementary teachers" (Kelly and Tomhave 1985).

[10]Results of the problem solving part of the Third NAEP study are taken as evidence of the lack of sense in US math classrooms. An oft-quoted example is that nearly 30% of the children tested gave the answer of '31 remainder 12' to the problem: "An Army bus holds 36 soldiers. If 1,128 soldiers are being bused to their training site, how many buses are needed?" Fewer than one in four gave the correct answer to this problem (Carpenter *et al.* 1983).

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limitations in their own knowledge of mathematics. To this list another must be added, which is intrinsic to the overall conditions affecting their job: isolation; time constraints; work overload; limited opportunities for professional development; bureaucratic control through textbooks, pacing calendars, and unit tests; and the constant pressure of district, city, and statewide assessments.

Despite emphasis on one or another aspect of 'the problem,' since the late 1980s, everybody agrees that school mathematics ought to be reformed, and there seems to be a consensus on the direction that such changes ought to take. The focus is to shift away from teaching low-level, computation skills, towards developing high-level, problem solving abilities (NCTM 1980, 1989, 1991). 'Mathematical power' will be defined as the ability to solve problems; communicate and reason about conjectures, strategies, and solutions; and understand the connections within mathematics, and between mathematics and daily life, science, technology, language arts, and other human and social sciences (NCTM 1991). The dry and dull word problems found in most textbooks will be replaced with relevant and meaningful story problems, full-fledged inquiries, worthwhile investigations, and student-generated projects

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(NCTM 1991).[11] Computers and calculators will be used in all classrooms, and computational skills will be acquired, not by means of rote learning, daily worksheets, and drill and practice, but incidentally, that is, in the course of problem solving (NCTM 1991). Finally, there is a move away from an exclusive reliance on multiple choice tests, towards gathering multiple sources of evidence of students' understanding through 'portfolios' and 'performance' assessments (Stenmark 1991; Wiggins 1991; NCTM 1995; Lesh and Lamon 1992; Romberg 1992, 1995).

The trends just outlined constitute the 'new new math.' As we recall from Chapter One, the expression 'new new math' is meant to emphasize the idea of newness as well as to suggest a parallel with the 1960s new math reform.

Yet, unexpected dissonances are emerging in the midst of this harmonious chorus. Disagreements, tensions, concerns, and skeptical attitudes may be found, for example, at the two ends of the spectrum of social groups which have an interest in the field of math education, i.e. on the one hand, among the experts in mathematics education or 'noospherians' (Chevallard 1990), whose perspectives on the subject occupy thousands of pages in more than 250

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[11]As expressed by Thomas Romberg--professor of mathematics education at the University of Madison (Wisconsin) and Director of the NCTM Assessment Standards Working Group--students "are not going to live in a textbook. They are going to live in a world awash with numbers, where problem-solving [is] essential to everyday life" (cited in West 1995, p.5).

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specialized journals in the United States and abroad; and, on the other hand, among the parents, whose hopes and worries about their children's math performances reach classroom teachers every other 'open school night' and enter the public domain through the mass media and opinion polls.

Although math education experts agree on the need for reform, they arrive to this realization from different perspectives on mathematics, teaching and learning. Since the middle of the century, the field of mathematics education faces a dilemma between two antagonistic schools in the psychology of learning: cognitivism (including 'information-processing' and 'task analytical' approaches) (Gagne 1977, Resnick and Ford 1981, Schoenfeld 1987), and Piagetian constructivism (including, more recently, 'radical' and 'social' constructivism) (Cobb 1987, 1990; Cobb, Yackel, and Wood 1992; Steffe and Gale 1995; Cobb and Bauersfeld 1995.[12] Linked to these two theories of learning are two opposing views on the aims of math education, i.e. the transmission of useful skills *vs*. the development of autonomous and reasoning individuals (Keitel 1982).[13] Yet,

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[12]The twin dominance of cognitivism and Piagetian constructivism(s) is contested by alternative paradigms such as 'cognitive apprenticeship' (Schoenfeld 1987; Lampert 1986, 1990; Brown, Collins and Duguid 1989); 'situated cognition' (Lave and Wenger 1991), 'realistic mathematics instruction' (Streefland 1991, Treffers 1991, Gravemeijer 1994), and activity theory/sociohistorical approach (Wertsch 1991; Saxe *et al.* in Daniels 1993; Saljo and Wyndhamn, in Chaiklin and Lave 1993).

[13]Keitel made this statement in the context of her comparison of US and Soviet math education reforms. In the Soviet Union, "student learning is seen as based on objective *activity*, but the mental process interlacing

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lurking underneath, albeit in updated versions, is the behaviorist approach to teaching and learning which has dominated the field for most of the century (Keitel 1992).

Noospherians discuss themes such as what level of mathematical literacy is needed for citizens to manage their daily affairs and become full participants in a democratic society;[14] what math skills are required by the restructured workplace and thus should be taught in the nation's schools; what portions of school math have been made obsolete by modern technologies and, therefore, ought to be removed from the curriculum; how may computers be utilized to improve the teaching of mathematics; and whether or the emphasis on conventional computation algorithms and the use of hand-held calculators have harmful effects on the development of

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this activity is not conceived, as it is in Piaget, as hermetic, but as dependent on and largely influenced by instruction."

[14]The math education community notes that the information revolution demands "higher standards of quantitative literacy for effective participation in democratic society" and that the new jobs "demand analytical rather than merely mechanical skills" (Mathematical Sciences Education Board 1990). From a more critical perspective, the argument is made that, as mathematics becomes 'implicit' or 'invisible' in almost all aspects of social life--i.e. the 'mathematisation' of society--students are less and less involved in doing 'explicit mathematics,' that is, in acquiring the tools for 'mathematizing' reality (Keitel "Implicit mathematical models in social practice and explicit mathematics teaching by applications" 1993). Put differently, the "reducing demands on particular mathematical skills of many citizens, within societies in which mathematics itself is playing an ever increasing role, is a paradox. One result is that there is an employed elite on whom greater, and changing, mathematical demands are placed and who, as a result, are coming to enjoy increasing power" (Howson and Wilson 1986, p.3).

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'number sense.'[15]

Survey results and research findings indicate that 'the public,' a category which in educational matters is generally constructed as 'the parents,'[16] is concerned about the failure of public schools--particularly the deficient quality of students' math performances--and would support any intervention which promises to address these seemingly intractable problems.[17] Yet, it is noted also that parents

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[15]Piagetian math educator Constance Kamii considers the 'harmful' effects of conventional algorithms vs. the value of algorithms *invented* or *constructed* by children (1985). In her view, due to the imposition of the former, children give up "their own numerical thinking." Instead, Ball (1990), Lampert (1990), and other math educators consider the dilemmas faced by teachers as they mediate between children's idiosyncratic inventions and the conventions of mathematics as an historically constituted knowledge field.
As for calculators, it is lamented that although research studies have not found empirical support for the claim that calculators rot the mind--e.g. Sydam and Dessart, 1980--these studies have not yet reached the public (cited in Black and Atkin 1996). Yet, Howson and Wilson argue that, although technology has redefined 'useful' mathematical knowledge, making obsolete conventional procedures such as as long division, "the concept of division retains its importance, and even the algorithm itself may still be 'useful' in a propaedeutical sense to the minority of students who may later wish to factorise polynomials" (1986, p.3).
An alternative to, both, the mechanical approach to conventional algorithms of traditional arithmetic teaching and the flight from algorithms favored by radical constructivists, is the 'guided reinvention' approach developed by Dutch math educators. In this pedagogical model, teachers play a central didactical role in children's *reconstruction* of the long division algorithm in the course of solving realistic problems (Gravemeijer 1994).

[16]'The public' and 'the parents' are two distinct categories, not to be conflated. When discussing the increase in negative public opinion about schools, it must accounted for the fact that 'the public' is increasingly made up of people without children and who, in general, tend to have more negative opinions towards public schools (Tyack and Cuban, 1995).

[17]According to a Gallup poll, in 1981, on the average, citizens rated public schools as 'C' institutions, as compared to an average rating of 'B' in 1974 (cited in Tyack and Cuban 1995, p. 13). Yet, poll after poll shows that although the public is dissatisfied with the educational system, people are pleased with their own schools--they may in general support the idea of reform, but we reluctant to embrace radical experiments in their children's

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have conservative views on mathematics and how to teach it: they want their children to learn the way they themselves have been taught.

While most people agree that in our modern technological society, basic skills are no longer sufficient, they are not ready to substitute these with unspecified higher order abilities.[18] Some parents consider that the new new math is too fuzzy.[19] They resist to the idea that there could be "more than one way to count

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school (Rothman 1995). Research also indicates that, as opposed to in other industrialized nations, parents in the US tend be satisfied with their children's math performances (Stevenson, Lee, and Stigler 1986). NCTM officials worry about the 'apathy' of the American public vis a vis students' performances in mathematics (Meserve and Suydam, in Morris and Arora 1992).

[18]Acording to Public Agenda--a non partisan research and education organization--many Americans believe that the public school is not meeting its most elementary goal: ensuring children's mastery of basic, essential skills such as the ability to read and write English and do simple arithmetic by hand, along with a 'common knowledge' understanding of science, history, and geography (Johnson and Immerwahr 1994-5).

[19]In "The New 'New Math'?" (*Education Week*, May 10 1995), Peter West discusses parents' opposition to State System Initiative (SSI) projects in Montana and Palo Alto (CA). In the latter, the reformed math curriculum has been challenged by a parents' group called HOLD (Honest, Open and Logical Debate on Mathematics Reform. As West reports, the education adviser to the State's Governor referred to the reform as a 'fuzzy math' which dismisses basic skills. West voices also the concerns of the Johnsons, whose children attend a 'reformed' high school in Kalispell (Montana). As they see it, due to the use of calculators, "children aren't developing the mental discipline that would come from, say, rote memorization of the multiplication tables." The owner of a computer assembly business, Tom Johnson values highly his own as well as his employees's "ability to perform mental calculations without electronic aids." There has been also parents' opposition in Ames (Iowa), where "Maths in Context"--a reform initiative influenced by the Freudenthal Institute (Netherlands)--has been blamed for recent decreases in students' scores on standardized math tests. As expressed in the *Ames Daily Tribune* (Editorial, February 22, 1995), the controversy boils down to "Do children learn the basic facts of math better by encountering then in the context of solving problems or simply by memorizing the multiplication tables and the like?"

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peanuts,"[20] and that student-generated algorithms may be given the same value that is given to conventional algorithms. Although ready to admit that, in their daily affairs, they seldom use school-taught arithmetical procedures, they believe these are worth learning.[21] Also, while valuing calculators, many parents object to the indiscriminate use of these artifacts in the classroom; for, children may employ them as a black box, without understanding the operations performed with them. Finally, some parents feel uneasy with the new assessments administered together with (or instead of) multiple-choice tests. In particular, for many lower class, non-English speaking, or semiliterate parents, test scores and exam grades such as 'at or above the 50% percentile,' 'B,' or 'passed' seem more objective and easier to understand than the more descriptive evaluations now proposed.[22]

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[20]This is how Peter West entitles a vignette illustrating the new approach to assessment (*Education Week*, May 31, 1995, p. 5). This expression refers to a 1st grade classroom episode in which children proposed different solution strategies for the problem "The African elephant ate 37 peanuts. The Indian elephant ate 43 peanuts. How many fewer peanuts did the African elephant eat than the Indian elephant?" (NCTM 1995, pp. 32-34).

[21]Cf. #19.

[22]Rothman 1995. Bernstein (1975) explores the relationship between social class and modalities of evaluative discourse. Whereas visible pedagogies evaluate students with reference to a group, generally by means of standardized tests, invisible pedagogies evaluate the 'whole child,' by means of 'dossiers' which include performance assessments, portfolios, and journals. This modality of evaluation is continuous with the modalities of socialization and regulation of the middle classes.

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All these worries, objections, controversies, and dilemmas may be seen as reminders of the constraints which limited the effects of past reforms and which may threaten the changes currently underway. In line with my overall thesis, I suggest that these expressions of uneasiness with the new modalities of teaching and evaluating mathematics bring to the fore the tension between an effective teaching of mathematics and the child-centered, progressivist, and constructivist paradigm. But, let us not anticipate the plot.

## 5.2 Risk, Excellence, and Educational Goals

In 1983, the National Commission on Excellence in Education released *A Nation at Risk: The Imperative for Education Reform*. This text provided the framework for contemporary debates on education. Despite the shifts in political power underlying the move from Reagan to Clinton, and the ideological modulations around the Cold War and its aftermath, this viewpoint on educational matters remained essentially unchanged up to the present.

As these oft-quoted statements illustrate, *A Nation at Risk* is shaped by cold war rhetoric:

> ... the educational foundations of our society are presently being eroded by *a rising tide of mediocrity that threatens our very future as a Nation and a people*. What was unimaginable a generation ago has begun to occur--*others are matching and surpassing our educational attainments*....

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> [I]f an unfriendly foreign power had attempted to impose on America the mediocre educational performance that exists today, we might well have viewed it as *an act of war*. As it stands, we have allowed this to happen to ourselves. We have even squandered the gains in student achievement made in the wake of the Sputnik challenge.... We have, in effect, been committing *an act of unthinking, unilateral educational disarmament* (p. 5, emphasis added).

*A Nation at Risk* blames education for the difficulties faced by the US in its competition with Japan and West Germany. As the causal link is defined, the 'mediocre' quality of American schools results in lower productivity rates and in the decline of the nation's scientific and technological progress.[23]

The Commission reported also that not enough 'academically able' students were being attracted to the teaching career; that teachers had low salaries and little influence in decision making; and that their preparation focused too much on methods, at the expense of the subjects to be taught. The report did not only diagnose these social illnesses. It also made a series of recommendations for its

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[23]As described by Westbury (1984), the 1980s economic recession resulted in an increasing awareness "of the lack of international competitiveness of many American basic industries." Thus, the 'solution' to this problem is said to be "the emergence of a 'new industrial order' based on high technology with all this implies for manpower training and new kinds of collaboration between universities, community colleges (technical schools) and industry." In particular, it is argued that, due to the lack of skilled workers, employers are 'forced' to look abroad for skilled workers. Yet, as many critics have argued, in the present stage of flexible capital the massive relocations of capital may be due more to the comparative advantages of investing in areas with weaker unionization, lower wages and taxes, and less strict environmental protection laws, than to the deficiencies in the skills of prospective employees. In any case, the idea that the underpreparation of the domestic labor market is the main cause of job flight continues to enjoy popular appeal.

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cure, including the need for curriculum and assessment reform,[24] and changes in the preparation of teachers. In addition, it called citizens to make schools accountable for bringing about the desired changes, and communities and the private sector to provide the necessary financial support.

With *A Nation at Risk*, the drive for excellence skyrocketed to the first place among the goals of public education. Yet, whereas in the past excellence had been linked to increased education budgets,[25] the 1980s and 1990s school reform is to be achieved at the lowest possible cost. As Republican President Reagan said, in line with his agenda of cutbacks in federally funded welfare programs, "If money alone were the answer, the problem would have long ago been solved." Instead, it is proposed that the best strategy is to enlist the aid of the states, the private sector, and the corporate world.

The Reaganite education revolution could not do without strong moral exhortations:

> Our children need to do *more work* and *better work*, and that includes *homework*. We must also require that our students learn *the basics.... I'm not talking about establishing order only in our*

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[24]The Commission recommended "more rigorous and measurable standards and higher expectations for student performance," and standardized achievement tests to be administered at major transition points form one level of schooling to another, as part of a nationwide, but not federal, system of state and localized system of assessments.

[25]Recall the generous funding of compensatory education programs during the 1960s, and the spectacular rise in federal spending for research and development projects in mathematics and sciences which made the new math reform possible.

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> *classrooms and hallways, but in our students' hearts and minds*. We're training our children for life in a democracy, so we must provide not only classroom discipline, but we must teach self-discipline as well. If it's sometimes difficult to assert *rightful adult authority*, we must ask: 'Who should correct the child's arithmetic? His math teacher? Or years later, his boss? Who should teach the child *respect for rules*? His principal? Or some day, law officers?" (in Marshner 1984, pp. 4, 5, emphasis added).

The 'basics' are linked to discipline, a respect for authority and the following of rules. In its move away from mathematics as rule-following and from a reliance on the teacher's authority, the new new math may be bound to disappoint the republicans' crusade for disciplined classrooms, hallways, hearts, and minds.

Also in 1983, the Task Force on Education and Economic Growth--a commission appointed during the Reagan years, and composed of government, business, labor and education leaders--released *Action for Excellence: A Comprehensive Plan to Improve our Nation's Schools*. The Task Force offered its own set of recommendations: curriculum renewal, periodic testing in all subject areas, and clear standards and goals for educational outcomes. After stating that technological change and global competition put pressure on public schools to equip students with skills that go beyond 'the basics,' the authors of *Action for Excellence* presented "a matching of basic skills with today's spectrum of jobs."

The Task Force defines 'basic skills' as the minimum skills necessary for a person's economic survival. Their

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first category includes 'unskilled' jobs, i.e. jobs that can be performed adequately with less than 'basic skills'--e.g. simple hauling or janitorial work. Next are 'basic' jobs, which require today's basic skills--e.g. clerical work in non-computerized stores. 'Learning-to-learn' jobs are those requiring basic skills and the capacity to acquire new ones, as in most factory and service-industry jobs. Finally, 'professional' jobs are occupations such as scientific programmers and analysts, and middle-to-upper corporate managers which, aside from adaptability, require more sophisticated intellectual skills. In the authors' view, it is with respect to 'learning-to-learn' jobs that schools face the greatest need for improvement. For, these occupations demand analytical and problem solving skills, which are those that national assessments find lacking among American students.[26]

Just as *A Nation at Risk*, *Action for Excellence* alerted the public on the deficiencies affecting the educational system. The 72% increase in enrollment in the remedial math

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[26]The Second and the Third NAEP studies indicate that students have difficulties in solving 'non-routine,' 'multi-step' word problems (Carpenter *et al.* 1980, 1983; NAEP 1983). McKnight *et al.* (1987) report on the low ranking of 8th grade American students in the second IEA math study (at the end of the 1981-2 school year). This panorama remained unchanged throughout the 1980s and early 1990s. For example, commenting on results from the Fourth NAEP study, Kouba *et al.* (1988) note that problem solving "continues to be an area of central concern" (see, also Dossey *et al.* 1988). According to the 1992 NAEP survey, problem solving abilities continue to be particularly lacking among African American and other minority children, as well as among children from low-income families (Dossey *et al.* 1993).

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courses of four-year colleges between 1975 and 1980--in a period when college attendance increased only by 7%--testifies to the inadequacy of high school mathematics and science courses. Moreover, 50% of high school graduates take no math or science beyond 10th grade; and there has been an overall decline in students' performance on higher order skills such as inference, analysis, interpretation, and problem solving. In addition, the country faces a shortage of math teachers; and there is neither a way to reward exceptional teaching through 'merit pay' nor--due to the tenure system--a mechanism for taking bad teachers out of the classroom.

Around the end of 1983, another document was released--*Educating Americans for the 21st Century*--which expressed the views of the scientific community. Prepared by the National Science Board Commission on Precollege Education in Mathematics, Science and Technology, *Educating Americans* focuses on the crisis in mathematics, science, and technology education. Its subtitle is worth quoting at length:

> A plan of action for improving mathematics, science, and technology education for all American elementary and secondary students *so that their achievement is the best in the world by 1995* (emphasis added).

The executive summary begins with the expected complaint:

> The Nation that dramatically and boldly led the world into the age of technology is failing to provide its children with the intellectual tools needed for the 21st century.

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Given this scenario, which threatens to make of the United States "an industrial dinosaur," the Board highlights the central role of the federal government in financing and maintaining "a national mechanism for measuring student achievement and participation," which will allow national, state, and local authorities to evaluate educational progress.

*Educating Americans* recommends that elementary school children have at least 60 minutes of daily instruction in mathematics. Yet, unfortunately, many teachers "are not qualified to teach mathematics and science for even 30 minutes a day." The Board acknowledges that teachers work long hours (46 hours per week in average), that the purchasing power of their salaries has decreased compared to that of 1970-1, and that, paradoxically, due to the breaking down of race and gender discrimination barriers, the teaching profession is loosing those women and minorities "who earlier had only teaching careers open to them [and] now are often being hired by business, or enter other professions" (p.28).[27] Aside from proposing extra compensation for the teaching of mathematics, science, and technology, the report outlines a plan to 'retrain' the 1.17 million elementary and 200,000 secondary school teachers in these fields.

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[27]Yet, as compared to the early 1980s, in the early 1990s, the pool of school teachers is older, female, and ethnically diverse (US Department of Education 1994).

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The Board notes that students' performances in mathematics indicate "that they are learning to be *technicians* but not *problem solvers*" (p.42, emphasis added). Although we "must return to the basics,

> the 'basics' of the 21st century are not only reading, writing and arithmetic. They include *communication and higher problem solving skills, and scientific and technological literacy*--the *thinking tools* that allow us to understand the technological world around us (p.v, emphasis added).

*Educating Americans* stresses the need for "early 'hand-on' experience, *disciplined and rigorous study*, a substantial amount of *time-on-task* and *homework* at all levels" (emphasis added). A crucial component in this proposal is the use of information technologies, especially computers: all children should learn about computers (i.e. computer literacy), with computers (i.e. computers as learning tools; e.g. drill and practice software), and through computers (e.g. artificial environments).[28]

The mathematics education field gave its own response to the calls for reform. In 1983, two meetings were held: one sponsored by the Conference Board of the Mathematical Sciences--with NSF funds--and the other co-sponsored by the NCTM and the University of Wisconsin--with funds from the US

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[28]The Board acknowledges that the use of new technology may accentuate the gap between "economically advantaged and disadvantaged students." The latter "may have only limited access to computers, and then only in school for drill and practice. Equal access for all segments of the student population to the best technology available is clearly an important issue" (p.53).

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Department of Education. Both conferences indicated the need for a new curriculum standards and frameworks for kindergarten through grade 14th mathematics. In 1986, the NCTM created the Commission on Standards for School Mathematics, and assigned it the drafting of new standards. Revised on the basis of 2,000 suggestions from business leaders, parents, and mathematicians, this draft resulted in the *Curriculum and Evaluation Standards for School Mathematics*, dated 1989.

In the area of curriculum renewal, the year 1985 saw the birth of *Project 2061*, a nationwide, long term, top down, and scientist-driven plan to revise the contents of science teaching.[29] With an initial funding of 8 million dollars from the Carnegie Corporation, and under the direction of John Rutherford, from the American Association for the Advancement of Science,[30] *Project 2061* follows the 1960s approach to reform. That is, unlike the NCTM Standards and other ongoing projects in the US and elsewhere,[31] where teachers participate in all stages of reform, *Project 2061*

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[29]In Project 2061, 'science' is broadly defined to include physical and biological sciences, earth sciences, mathematics, technology and social science (Black and Atkin 1996). These efforts resulted in a path breaking document (American Association for the Advancement of Science 1989).

[30]"a highly-respected figure in science education world wide and "a veteran of the serious and famous attempts in the 1960s to change science education in the United States" (Black and Atkin 1996, p. 180).

[31]For a review of 23 ongoing school science, mathematics and technology reform projects in 14 OECD countries, see Black and Atkin 1996.

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began with the gathering of what scientists at the forefront of each discipline considered as knowledge worth transmitting to the younger generations.[32] After the new frameworks were drafted, teachers and other educators were invited to help transpose these into knowledge to be taught.

In the meanwhile, responding to the pressure for improving teachers education, two reports appeared in 1986: one by the Holmes Group,[33] entitled *Tomorrow's Teachers,* and the other by the Carnegie Forum on Education and the Economy, with the title *A Nation Prepared: Teachers for the 21st Century*. Both reports call for strengthening teachers' liberal arts background, improving their subject matter preparation, and rewarding excellence by taking advantage of differences in knowledge, skill, and motivation among teachers (i.e. merit pay).

As the Holmes Group sees it, educational reform is best effected when teachers implement the pedagogical innovations developed by university-based researchers. In *Tomorrow's Schools: Principles for the Design of Professional Schools* (1990), the Holmes Group describes teacher education, teachers' work, and children's learning with a language which is permeated with concepts from cognitive psychology

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[32]Project 2061 argues that, if students are to leave school knowing and understanding science, this necessitates less emphasis on memorizing lists of science facts and, instead, a focus on the development of scientific 'habits of mind' (AAAS 1989).

[33]The Holmes Group brings together a group of deans of schools of education at major universities.

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(e.g. 'mental schemes,' and teaching as 'scaffolding'), and the emphasis throughout is on problem solving abilities and metacognitive processes.[34]

The problem of 'incompetent' teachers might be solved with massive retraining, tougher certification requirements and merit pay, or by teaming teachers with education researchers. An alternative strategy, which has the obvious appeal of a quick fix, is the use of machines. If teachers are not good, we may take advantage of the newly available technological cornucopia to compensate for their deficiencies, or to simply bypass them.[35] The following document, released in 1988 by the US Congress Office of Technology Assessment (OTA), illustrates the eternal return of the "faith in electronic pedagogy" (Tyack and Cuban, 1995, p. 111).

*Technology and the American Economic Transition* restates the generally agreed upon viewpoint that schools must accommodate to the present and future demands of our information and technology era. The new skills are described

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[34]On the emergence of cognitive psychology as a dominant discourse of reform due to its rhetorical quality of interrelating multiple political strategies, see Labaree 1992. The Holmes Group recently relased a third document: *Tomorrow's Schools of Education* (1995). For a critique of the Holmes Group's proposals for reform in teacher education, see Labaree and Pallas 1996.

[35]Consider, for example, Lindquist's invention of the scoring machine in the early 1950s. Apart from its value as a time- and cost-saving device--the scoring machine could read answer sheets, produce raw scores, and translate these into standard scores in seconds--this artifact was seen as enhancing "the seeming objectivity of test scores by removing teacher judgement from the picture (Rothman 1995, p. 38).

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in a similar way as in *Action for Excellence*: skills in working in teams, capacity for self-education, and ability to deal with ambiguity and cope with 'too much or too little information.' Yet, the OTA report considers also the role of technology in fostering educational reform. With a triumphant tone which contrasts with most early 1980s reports, the authors announce that the US educational system

> is on the brink of a major change. This results both from a fundamental change in the demand for educational services and from the fact that *new technology makes it possible to consider real improvements in the productivity of both teaching and learning* (emphasis added).

Technoscience is both the rationale for educational change, and the means for effecting it. As the OTA sees it, "more capital equipment and expansion of teaching beyond traditional school settings will lead to much greater differentiation in an occupation now called 'teachers.'" The OTA's vision of the impact of technology on education is worth quoting at length:

> Artificial intelligence techniques could help *diagnose defects* in what a student understands, while less exotic software could help with needed *drill and practice*. Simulations that use images, sounds, and text can lend realism, and can *remove the barriers of abstraction that so often impose themselves between formal education and the practical mastery of a subject* (emphasis added).

While ordinary software programs may alleviate tedious aspects of the teaching job such as the detection of errors and the provision of drill and practice, more exotic artificial learning environments may take over the function,

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traditionally performed by textbooks, of connecting the otherwise isolated classroom to the outside world.[36] That computers bring into the classroom only a virtual version of reality should not matter, since

> the working life of many graduates will increasingly be spent viewing reality through computer screens--whether they are operating nuclear plants or analyzing a commercial insurance policy--the distinction between simulation and reality can be extremely small.

Computers do create the illusion of transparency. Consider LOGO, a program developed in the mid 1970s by Seymour Papert and his colleagues at MIT. This artificial learning environment is inhabited by a small 'tortoise' which moves left, right, up, and down the screen, in response to children's instructions. LOGO's *Mathland* is a utopian microworld where each child is its computer's teacher (Papert 1980, 1993; Hoyles and Sutherland 1989, Nesher 1989, Noss 1990). Yet, computers are "noisy little critters." In 'real' classrooms, students must cut through the artificial noises generated by these artifacts, in order to interpret the didactical signs embedded therein (Chevallard 1990).[37] While the promise to reduce the

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[36]This is usually described as a distinction between teaching machines (computer assisted instruction or 'CAI') and 'learning machines' (microcomputer artificial environments).

[37]See, also Jahnke (1983). Similarly, Howson and Wilson (1986) are skeptical about recent attempts at replacing the new math's 'macrostructures' with computer microworlds. As they see it, many problems of traditional math teaching resulted from similar efforts at creating microworlds--e.g. 'the microworld of formal arithmetic' (p. 54).

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didactic triangle to a student-machine dyad returns again and again, classroom practices remind us of the futility of attempts at bypassing the teacher's didactical mediation.

Also, since the technological revolution in the classroom usually takes the form of a crash, top down intervention, teachers--who, in general, are 'computer illiterate'--tend to appropriate from the new machinery only that which enhances their daily practices. Thus, computers are underutilized-- e.g. used only for drill and practice. Given that this phenomenon is more likely to occur in classrooms attended by lower class and minority children, far from providing universal access to the wonders of the information era, high-tech reforms may create a technological underclass in America's public schools.[38] The OTA does acknowledge that "each opportunity for progress" carries with it

> opportunities for serious missteps. Even change that clearly leads to overall economic growth can have very *uneven effects*.... Change can weaken the bargaining power of some groups while strengthening that of others. Change can result in *a growing gap between those fortunate enough to have the talents, education, and connections needed to seize emerging opportunities and those forced into narrowly defined, heavily monitored, temporary positions*. This latter group could be forced to bear the costs of uncertainty (emphasis added).

As the document anticipates, in times of accelerated change, the dislocation of those groups who lack the necessary

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[38]As noted by Pillar (1992), cited in Tyack and Cuban, 1995.

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"talents, education, and connections," may be the inevitable price to pay.

During the 1980s, commissions, task forces, and boards continued to release crisis reports and policy papers which brought more bad news to an already concerned public. Finally, in April of 1991, a bipartisan agreement between representatives of the states and the Bush administration resulted in *America 2000: An Education Strategy*. The six goals specified in this path-breaking document included that all children will start school ready to learn; schools will be safe, disciplined, and alcohol- and drug-free; high school graduation rates will increase to at least 90%; "every school in America will ensure that all students learn to use their minds well, so that they may be prepared for responsible citizenship, further learning, and productive employment in our Nation's modern economy;" the nation will ensure a good quality teacher education and professional development; and "by the year 2000, United States students will be first in the world in science and mathematics achievement."[39]

The goal for math education is the only one phrased in competitive terms. In the post Cold War era, the political

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[39]Corporation leaders were asked which of the goals they would support the most. Most of them chosed 'to make math education the best in the world' (Teltsch 1991). Also in 1991, a representative sample of parents were asked if they thought that these goals would be accomplished. While 4% believed that "every school will be free of drugs and violence," and 14% thought that "every American Adult will be literate," 19% believed that American students would become first in the world in math and (Tyack and Cuban 1995).

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and economic well-being of the United States is still linked to the standing of its students in international mathematics tests. In that respect, the 1980s offered a bleak scenario. In 1981-2, 8th grade American students scored third from the bottom in mathematical achievement among fourteen countries (surpassing only students in Sweden and New Zealand). In the same period, American 12th graders scored last in algebra and calculus, in a list of twelve countries and provinces.

Commenting on these embarrasing results, observers note the problematic nature of these comparisons.[40] Those countries which are ahead in the math race--Japan, the Netherlands, France, Belgium, and Hungary--have a centralized educational system; a national curriculum; a longer school year; a culture which values mathematics, teaching, and education; heterogeneous grouping; a well-trained cadre of mathematics teachers; higher teachers' salaries; a more teacher-centered or 'didactic' pedagogy; a more demanding curriculum; or all of the above.[41] One may

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[40]The samples of test-takers are often incomparable--e.g. comparing the average score of more than three-fourths of the age-group in the US with the average of the top 9% students in West Germany, the top 13% in the Netherlands, and the top 45% in Sweden" (Rotberg 1990, cited in Tyack and Cuban 1995).

[41]The interpretation of these results continues to be a matter of heated debate. For example, Westbury (1992, 1993) argues that the source of the higher math performances of Japanese students is not ineffective teaching or social factors, but the curriculum to which students are exposed. He asserts also that the 'problem' is only at the 'bottom half' of the educational ability spectrum. Stedman (1994) shows that an underachieving curriculum cannot be the only explanatory factor, and that the 'problem' is not only the bottom, but also the top half of the student population. It appears that, taken in isolation, none of these factors accounts fully for the students' math performances in each of these high-

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anticipate that, unless changes are effected in these fundamental structures, no significant improvement in the mathematical performances of American students would result.[42]

During the Clinton administration, a new version of *America 2000* was released: *National Education Goal Report: Building a Nation of Learners* (1994). Restating the goals defined in 1991, this Report added two more: all teachers will have access to professional development, and every school will promote partnerships to increase parental involvement in educational matters. The *National Education Goal Report* called for all Americans, including public officials, educators, parents, business and community leaders, and students to participate in the reform endeavors.[43]

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ranking countries. Yet, it is likely that the absence of all of these factors which characterizes the US math education system--i.e. decentralization, low social social value given to teaching and to mathematics, deficiencies in teachers' preparation in mathematics and its didactics, homogeneous grouping, a less demanding curriculum, and a shorter school year--may account for the low math performances of its students.

[42]The National Education Goals Panel indicates that American students continue to be outperformed by students in Korea, Switzerland, and Taiwan in all areas of the international math assessment conducted in 1995, and by students in France and Hungary in four out of five areas (1995, p.36).

[43]Assuming that 'informed decision making' is central to any reform process, the Goals Panel created a Community Action Toolkit, which included a handbook with "the steps required to collect reliable data so that informed decision-making can take place at the local level" (p.16).

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## 5.3 Skills, Standards, Opportunities, and Assessments

As the story is told, due to its emphasis on structures, the 1960s new math reform contributed to de-skilling a whole generation of schoolchildren. But the 1970s 'back to the basics' proved also lacking as a solution to the problem of skills, schools, and mathematics.[44] And, by the early 1980s, it was claimed that the computerization of the workplace and of almost all aspects of social life urged a redefinition of the math skills to be taught in the nations' schools.

In 1988, an inquiry group funded by the Ford Foundation, and led by a 12-member panel representing the business, academic, civil rights, and labor communities, published *Toward a More Perfect Union: Basic Skills, Poor Families, and Our Economic Future*. In the panelists' view, underneath the three most severe problems affecting the nation--inadequate schools, teenage pregnancy, and economic competitiveness[45]--lies a common theme: lack of basic

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[44]"In the 1970s the concern of the public was directed towards problems evidenced almost exclusively in test scores. Schools have responded to this concern in a variety of ways, but a clear-cut and carefully reasoned sense of direction that looks towards the future has been lacking" (NCTM 1980).

[45]As the panelists see it, the quality of education, especially at the pre-college level, determines the ability of the labor force to adapt to changes resulting from new productivity-enhancing technologies. The paper cites three documents from the early 1980s: *A Nation at Risk*, a report by the National Research Council's Panel on Adolescent Pregnancy and Childbearing, and the Committee on Economic Development's *Strategy for US Industrial Competitiveness* (1984). Twelve years after *A Nation at Risk*, the Fall Issue of *Daedalus* considers the question "Are there certain new conditions--one forebears to call them 'crises'--that the National Commission on Excellence [...] failed to anticipate?" As

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skills, defined as "the ability to read, write, communicate, and compute."

As suggested in the paper's subtitle, the importance of skills goes beyond pure instrumentalism. Those with "better basic skills

> do better in school, at work, and in other areas of their lives. They are more likely to perform well in school, obtain a high school diploma, go on to complete college, work more hours, earn higher wages, be more productive workers, and avoid bearing children out of wedlock. Conversely, *those who are deficient in basic skills are more likely to be school drop outs, teenage parents, jobless, welfare dependent, and involved in crime....* In short, *basic skills bear a distinct relation to the future well-being of workers, families, firms, and the country itself* (pp.1-2, emphasis added).

Why is it, ask the panelists, that some students acquire skills while others don't? With sophisticated sociological sensitivity, yet one which stops short of a critical analysis by implicitly blaming it all on the 'culture of poverty,' the report attributes disparities in skills-acquisition to the fact that "disadvantaged children are limited by the *language* and *problem solving skills* they learn from their often poorly educated parents" (emphasis added). And, since language development is a predictor of cognitive ability,

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the preface authors argue, the end of the Cold War has removed one of the major incentives to education reform. The nation's attention is now turned towards its domestic agenda and, in this context, the debate about schooling is now framed by social problems such as unemployment, teenage pregnancy, drug abuse, abortion, crime, and divorce (Daedalus 1995).

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> the intergenerational transmission of low basic skills promises to perpetuate the social problems described above.

For the Ford Foundation, inner city teachers may contribute to the alleviation of urban poverty by enhancing the relationship between education and work (Popkewitz and Myrdal 1993). In 1986 the Foundation began funding the *Urban Mathematics Collaborative Project* (UMC). Operating in eleven urban centers throughout the nation, this project aims at 'empowering' teachers to change their classroom practices. To address one of the main obstacles for reform--i.e. teachers' isolation--since the early 1990s, the UMC Project has organized workshops and meetings for teachers to discuss the implementation of the new NCTM standards.

Case studies conducted at the UMC sites indicate that most inner city teachers still view mathematics as a science of right and wrong answers. Also, despite so much emphasis on higher order abilities, the tension between drill instruction and conceptual learning remains intact in the mind of most of these teachers (Popkewitz and Myrdal 1993). As math education experts see it, these as well as other assumptions about mathematics, teaching, and learning are operating as obstacles for an effective reform.[46]

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[46]Thus, math educators involved in teacher education stress the need for changes not only in teachers' subject matter preparation, but also in their epistemological assumptions about mathematics. These include, the work of Deborah Ball, at the *National Center for Research on Teacher Learning* (Ball 1988, 1990, 1990a); Catherine Fosnot, Martin Dolk, and Willem Uittenbogaard at the *Mathematics in the City* in-service teacher training program (CUNY, City College); Paul Cobb and his collaborators at the *Purdue Problem-*

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It all seems to come down to skills, skills, and more skills. Despite *Action for Excellence, Towards a More Perfect Union,* and similar reports, it looked as if the new beyond-the-basic skills had not been clearly defined, and this underspecification--or the mixed signals being sent--made it difficult for schools to perform their job. In 1990, the newly formed Secretary's Commission on Achieving Necessary Skills (SCANS) took on the threefold aim of defining the new skills, specifying required levels of proficiency, and recommending mechanisms for the assessment of these skills.

The Commission's efforts materialized in *A SCANS Report for America 2000: What Work Requires of Schools* (1991) and *Learning a Living: A Blueprint for High Performance* (1992). The *SCANS Report* relies on an image of the workplace which, as the authors admit, exist in only ten percent of today's organizations. In these places, work is problem-oriented and flexible, and is carried out by teams operating under decentralized control in an atmosphere of labor-management cooperation. "Once upon a time, a strong back,

> the willingness to work, and a high school diploma were [...] all that was needed to make a start in America. They are no longer. A well-developed

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*Centered Mathematics Project* (Cobb, Yackel, and Wood 1992); the *Summermath for Teachers Program at Mount Holyoke College* (Simon and Schifter 1991); and the work of Carpenter, Fennema, and others in the *Cognitively Guided Instruction* program (CGI), at the University of Wisconsin, Wisconsin Research and Development Center (Carpenter *et al.* 1988, Fennema *et al.* 1996).

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> mind, a passion to learn, and the ability to put knowledge to work are the new keys to the future of our young people, the success of our business, and the economic well-being of our nation (p.1).

The skills identified by SCANS include personal qualities such as sociability, self-management, integrity, ethics; interpersonal skills; and subject matter skills in reading, writing, mathematics, time management, and applications of technology.

In particular, the Commission recommends that schools deliver to employers a job-seeker who performs *basic computations*; uses basic numerical concepts such as whole numbers and percentages in practical situations; makes reasonable estimates of arithmetic results *without a calculator*; and uses tables, graphs diagrams, and charts to obtain and convey quantitative information. In line with recent findings from cognitive science on the effectiveness of learning 'in context,' SCANS suggests that basic and problem-solving skills be taught simultaneously.[47] For example,

> Learning how to compute percentages in the context of a realistic budget problem will be much more profitable than if taught in the abstract or with artificial word problems (p.22).

*Learning a Living* acknowledged the vast disparities in

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[47]"Placing learning objectives within real environments is better than insisting that students first learn in the abstract what they will then be expected to apply" (p.19).

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educational opportunity in the nation's schools.[48] Linking the drive for excellence with a concern for equity, the second SCANS report stated that every person "must have the opportunity to reach the higher level of skills and competencies the Commission found to be associated with high-wage jobs." A member of SCANS, well-known cognitive psychologist Lauren Resnick, put it bluntly:

> Expectations are not the same for all, the opportunities for learning are not equivalent. Almost everywhere, *we expect less of children from poor and minority families; we therefore ask less of them, and, above all, we offer them less....* The adoption of minimum competency testing in forty-seven states resulted in *some modest gains in basic skills achievement for low-income minority students, but little educational progress in other respects* (pp. 59, 60, emphasis added).

Her statement highlights a central theme in the dynamics of reform. When the upgrading of standards through more demanding courses, certification requirements, and examinations, is not accompanied with equal opportunities for all students to achieve them, this may exacerbate the existing unequalities in the distribution of academic capital. That is, children from disadvantaged social groups may find it harder to respond to the new demands and thus be denied competency certifications (Pullin 1994).

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[48]It is worth mentioning the publication, a year earlier, of Jonathan Kozol's *Savage Inequalities*. Kozol describes the decrepit condition of school buildings and the deficient quality of teaching in such cities as St. Louis, Chicago, Camden, and New York City.

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In the case of mathematics, critics have argued that special attention must be paid to those who have been historically 'underserved' and, as a result, seem destined to fill the 'underachievement' ranks, namely, lower class children, members of racial and ethnic minorities, and girls.[49] If the universally cherished *excellence plus equality* goal is to be reached, a delicate balance must be kept between expectations and opportunities. For, the lowering of standards and expectations for some students--e.g. by substituting regular math courses with 'business' or 'practical mathematics'--may lead to a situation where 'mathematics for all' becomes 'not mathematics at all.'[50]

In 1992, the National Council on Education Standards and Testing (NCEST),[51] released *Raising Standards for American Education*. Sensitive to a distinctive traits of the

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[49]The NCTM's Research Advisory Committee (1989) raises its concerns for the differential impact of the new standards on students from "underserved and underrepresented" segments of the school population. See, also Frankenstein 1989; and Secada, Fennema, and Adajian 1995.

[50]"Mathematics for all is not mathematics at all" (Keitel 1987). Oakes (1985) compares the expectations for students' learning which are held by high school teachers working in high-track classes *vs.* those of teachers working in low-track classes. The former include "love and respect for math," "to reason logically," "to think critically," "to convert problems into numerical order," to "determine the best approach to problem solving," and to have a "better feeling for their own abilities." On the contrary, the latter include "respect for each other," "respect my position," "punctuality, self-discipline and honesty," "good work habits," "how to fill out insurance forms. Income tax returns," and "practical math skills for everyday living" (pp. 79-83).

[51]Created in 1991, NCEST was a 32-member council asigned the task of examining the "desirability and feasibility" of national standards and assessments.

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US educational system--i.e. its decentralized structure[52]--NCEST asserted that its aim was not a national examination, but a system of assessments of students and districts.[53] Concerned with the need for all students, including those with 'special needs' and LEP students, to receive opportunities to learn, the Council introduced the notion of 'school delivery standards,' i.e. standards for the instructional opportunities that schools need to provide for enabling students to reach the learning standards set for them.

In an effort to help define these standards through "a community of schools using innovative approaches to assessment, curriculum, and instruction," the *New Standards Project* began its work in the early 1990s. As a partnership of seventeen states and six school districts, and under the direction of Lauren Resnick,[54] this project is jointly led by the Learning Research and Development Center at the University of Pittsburgh, and the National Center on

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[52]As described by Cohen (1995), the US school 'system' is, "in some critical respects, a non-system, a congeries of more than 100,000 schools situated in 15,000 independent local governments, governed by fifty state governments and hundreds of intermediate and special district governments in between, as well as by several federal agencies and influenced by countless of private organizations" (p.13).

[53]On the controversies surrounding NCEST, and the opposition to the idea of national assessments, see Rothman 1995.

[54]Simmons and Resnick 1993, p.15. The New Standards Project originated as a direct response to *America's Choice: High Skills or Low Wages!* (National Center on Education and the Economy 1990), a document calling for ambitious standards for students performance and new assessments to measure them.

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Education and the Economy.[55]

The *New Standards Project* has been developing a system of assessments which responds to six principles. First, the new assessment instruments are linked to 'content' standards; second, the same high standards are expected from all students; third, valued outcomes are assessed directly, by means of 'performance' examinations and 'portfolios,' and based on the principle that test items ought to be samples of the kind of work students are doing in their classrooms; fourth, the new assessment system operates with 'performance' standards which are publicly and widely known; fifth, the standards are to be internalized by all teachers and students; and sixth, the Project is committed to linking assessment reform with all other elements of systemic change (Resnick *et al.* 1992; Simmons and Resnick 1993).

The *New Standards Project* proposes a radical redefinition of the 'assessment contract' as a strategy for changing the didactic contract.[56] Taking advantage of the well-known 'what you test is what you get' phenomenon, Resnick and her collaborators are designing a set of

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[55]The Project gathered teachers from all centers to discuss the possibility of setting common performance standards. A series of tasks were developed and tried in a pilot test with fourth, fifth, and eight graders in 1992 and 1993. In 1994, the Project administered a full-scale math exam to a sample of students in each of the participating states.

[56]In line with Brousseau's didactic contract, the assessment contract is understood here as the more or less implicit set of rights and obligations binding students and teachers in 'test' situations (van den Heuvel-Panhuizen 1996).

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examinations based on 'world class' standards which, in their view, will pressure schools to teach to those standards (Resnick *et al.* 1992). Yet, it is also well-known that when the didactic and the assessment contracts are out of synch, one tends to overwrite the other. This effect tends to occur in classrooms attended by lower class and minority children, where the pressure to perform in standardized tests is higher than elsewhere.[57] In these schools, teachers lack the autonomy they need to temporarily suspend the old assessment practices, while they work towards establishing a new didactic contract.

Perverse effects occur when teachers 'teach to the test,' by providing rules, recipes, and hints to prepare students for multiple-choice questions.[58] Or, conversely, when faced with performance examinations which consist of open-ended story problems and the solicitation to explain how the answers were obtained, teachers provide instruction in problem solving strategies,[59] and train students to deliver the procedural narratives elicited by the 'show your

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[57]For example, in a 1992 study of the impact of mandated testing on curriculum and instruction in math and science, teachers and administrators working with minority students reported spending significatively more classroom time training children in the specific skills and contents included in the tests, than did teachers and administrators working with non-minority children (cited in Resnick *et al.* 1992). See, also Harvard Educational Review 1994).

[58]Thus, the rupture of the didactic contract towards the 'algorithmization' of teaching (Brousseau 1986, Chevallard 1985).

[59]Brousseau (1986) refers to this phenomenon as 'metacognitive sliding.'

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work' command.[60] As a pragmatic response to the constraints affecting their classrooms, teachers and students develop 'survival strategies' which *de facto* distort the new modalities of learning, teaching, and assessment.

Thus, change can be effective and occur with a minimum of unintended effects when it happens simultaneously on all fronts. Luckily, in the 1980s and 1990s, 'systemic reform' is the buzzword for educational change. As in the Sputnik era, the rationale for reform is presented as a national security problem. The argument is made that the US cannot afford to waste its human resources, expectations are too low, and opportunities are not equal for all. It is lamented that the 'nation's report card' shows too many failures--more conspicuously in mathematics--and that this handicaps its hegemonic position in the new world order. Given this scenario, rather than effecting piecemeal changes, the whole system ought to be 'restructured.' And the first step in this direction has been the establishment of national curriculum and assessment standards for all school disciplines.[61]

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[60]A report by Mary Lee Smith on the Arizona Student Assessment Program performance exams showed that some teachers tried to 'disintegrate' the exams, that is, to break an integrated unit into pieces and teach the parts (cited by *Fairtest Examiner*, Spring 1994, p.9).

[61]With differing degrees of consensus, this has been accomplished. Standards documents have been released for most school subjects, from the rather uncontroversial NCTM Standards, to the standards for science and for history which created heated, and still unresolved debates.

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Yet, side by side with calls for a centralized approach to reform, there are signs of an increasing decentralization and commodification of public education, as neo-liberals demand that state bureaucracies withdraw from educational matters. Viewing education as a tax burden rather than as a social right, and guided by the diagnosis that the 'public schools crisis' originates from an educational system which operates like a 'failed monopoly,' leaders of the corporations and the private sector propose the privatization of education through 'vouchers' and 'school choice.' As they see it, the best strategy for improving the quality of educational 'services' and 'outputs' is to create a system where schools compete with each another in attracting 'costumers.'[62] Decentralization also finds its allies among progressive educators who, since the early 1960s, have been struggling against the bureaucratic control of schools. In their view, effective, lasting, and meaningful reform occurs only when local communities--administrators, teachers, parents, and students--are

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[62]Kearns and Doyle (1989) propose the following reform plan: choice (public schools should compete with one another); restructuring (schools should reorganize as magnet schools, be open year-round, and be run by teachers and principals); professionalism (teachers must take more control, setting their own curricula and raising standards to elevate their status to one equal to other professions); standards (academic standards must be raised and children held strictly accountable to them--just as it is the teacher's job to teach, it is the student's job to learn); values (children should understand the great documents of American citizenship and the ethical, moral, and religious underpinnings of their creation); and federal responsibility (the federal government's role in education should continue to be limited). See, also Chubb and Moe 1990.

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directly involved in designing and implementing the changes.[63]

The co-existance of these two forces (i.e. centralization *vs.* decentralization) accounts for the impasses suffered by past reform plans and may anticipate the faith of present interventions. When attempts are made to impose a centralized, top down reform, teachers respond, as they did during the new math period, either by modifying the new content to the point of turning it unrecognizable to the experts who designed it,[64] or by rejecting it altogether and continuing to teach the old basic skills. But, when standards and curriculum frameworks are meant only as timid suggestions from which teachers and administrators are to pick and choose, this often acts as an obstacle for significant and lasting classroom change.[65]

What is then to be done? David Tyack and Larry Cuban (1995) suggest that curriculum reforms ought to be designed with the expectation that teachers will 'hybridize' them. A central question would thus be: to what extent teachers are prepared to effect the inevitable didactic hybridizations,

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[63]Bastian *et al.* 1985.

[64]Recall, for example, the banalization of set notation and the overutilization of Venn diagrams during the new math era.

[65]This is accentuated by the uneven distribution of resources (school budgets and other forms of funding). While the proposed reforms are highly demanding in terms of resources (e.g. access to new technology such as computers and modems), not all schools and districts have equal access to the resources necessary to effect the desired transformations.

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while remaining faithful to the reformers' original intent? Unfortunately, in the systemic reform(s) currently underway, teacher preparation continues to be low in the list of priorities (Cohen 1995). Aside from a few exceptions (e.g. Vermont, Kentucky), states have been more busy upgrading standards and renewing assessments, than investing in the long-term process of preparing their teachers in mathematics and its didactics. As Cohen notes (1995), many factors account for this situation: first, spending money in teacher education is politically unattractive; second, the agencies concerned with the K-12 schooling system and those involved in teacher education are disconnected; and third, in most states, there are not enough providers of professional development in mathematics.[66]

In line with my general hypothesis, I suggest that to this list of political, economic, and bureaucratic factors, we must add the tendency, installed at the center of the threefold modern pedagogical paradigm dominant in the United States, to underestimate the didactic role played by teachers in introducing children to the world of literate (and numerate) culture. As I will illustrate below, with its

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[66]Thus, while new frameworks replace the underachieving math curriculum, new resources (textbooks and 'replacement units') supplement the old math textbook, and new assessment instruments complement--or, in some cases, replace--the old multiple choice tests, teachers continue to be ill-prepared to make sense of the up-graded standards, use productively the innovative materials, and help their students perform in the reformed math examinations (Cohen 1995). For the case of the California math reform, see Ball and Cohen 1994.

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underlying child-centered pedagogy, its progressivist view of curriculum, and its constructivist theory of learning, the new new math reform may perpetuate the fiction of an anti-didactical contract for mathematics learning.

## 5.4 The New Didactical Transposition

All that has been said so far about the crisis in math education, the need for reform, and the excellence + equality goal converge in *Everybody Counts: A Report to the Nation on the Future of Mathematics Education*, a concise document prepared, in 1989, by the Mathematical Sciences Education Board (MSEB), a division of the National Research Council.

*Everybody Counts* asserts that, in the coming decades, the United States must revitalize its entire system of math education. As the Board sees it, change is now possible thanks to the emerging consensus on the aims of math teaching and the means to achieve them. *Everybody Counts* represents the views of

> 70 leading Americans, among them classroom teachers; college and faculty administrators; research mathematicians and statisticians; scientists and engineers, mathematics supervisors; school principals; school superintendents; chief state school officers; school board members; members of state and local governments: and leaders of parent groups, business and industry.

And change is now urgently needed, not only because mathematics is the foundation of technoscience and technoscience has 'mathematized' society, but also due to

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the unequal access to mathematical knowledge and credentials among students from different social and economic backgrounds.

*Everybody Counts* assigns to schools the task of preparing "the more highly skilled work force that the nation needs." In the 21st century, work "will be less manual but more mental; less mechanical but more electronic; less routine but more verbal; and less static but more varied" (p.11). Rather than focusing on calculation, school mathematics must adapt to a world economy which requires 'mentally fit' workers, that is,

> workers who are prepared to absorb new ideas, to adapt to change, to cope with ambiguity, to perceive patterns, and to solve unconventional problems.

Yet, the focus is not only on workplace demands. Mathematics should be taught in order for students "to learn practical skills for their daily lives, to understand quantitative aspects of public policy, to develop problem solving skills, and to prepare for careers." Furthermore, if education is about "learning to think" (Resnick 1987), this is especially the case for mathematics,

> an area in which even young children can solve a problem and have confidence that the solution is correct--not because the teacher says it is, but because its inner logic is so clear (p. 3).

Throughout its pages, *Everybody Counts* describes the uniqueness of mathematics as a knowledge field and the special place it occupies as a school subject. Mathematics,

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"the invisible culture of our age," is not only "the key to career and job opportunities" but it has become a central component of functional literacy in the modern world. Without it, "one cannot fully comprehend modern writing such as that which appears in the daily newspapers." As "the language of science and technology" and "the science of pattern and order," the central role of mathematics in education "is a consequence of its universal applicability."

School mathematics is "especially sensitive to deficiencies in the effectiveness of the educational system." While it is "the worst curricular villain in driving students to failure in school," when properly taught, it offers "special opportunities for children to learn the power of thought as distinct from the power of authority." Although studying mathematics "can help develop critical habits of mind--to distinguish evidence from anecdote, to recognize nonsense, to understand chance, and to value proof," due to the way in which it is taught in schools and represented in society, "children begin to view mathematics as a rigid system of externally dictated rules governed by standards of accuracy, speed and memory."

"Early school experiences suggest that all mathematics problems have a single correct answer." Thus, "the public--to its great risk--tends to believe uncritically any expert who employs mathematical arguments." Also, since most parents believe that performing well in math requires an

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innate ability, they "often accept--and sometimes even expect--their children's poor performance." Finally, while "among the many subjects taught in school, mathematics is probably the most universal, depending least on a student's background and culture," it continues to be "primarily part of upper- and middle-class male culture." Thus, although mathematics "should transcend the cultural diversity of our nation," at the present time, its knowledge (or lack thereof) threatens to divide the nation along economical and racial lines.

In sum, since school mathematics has distorted the true nature of mathematics, it is necessary to undo the existing didactical perversions. The MSEB invites all interested parts, especially the teachers,[67] to participate in the desired transformations. A new transposition of mathematical knowledge, together with a redefinition of the didactic and assessment contracts will allow the equalizing, instrumental, and critical potential of mathematics to materialize into classroom practices. The new new math is forwarded as a truly utopian project.

That same year, the NCTM responded to the call for everybody to count with the *Curriculum and Evaluation Standards for School Mathematics*, the first attempt by a

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[67]"... whatever any of us do as individuals or through organizations to improve education, we see our role as supporting the efforts of the central person who can bring about meaningful and lasting change: the teacher" (Preface).

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teachers' organization to specify nationwide curriculum standards. The endorsement of fifteen professional groups testifies to the unprecedented consensus built around it.[68] The document, written to 'make sense' of school math for the common reader, was seen by the press as signaling a national awareness of the importance of mathematics education.

Organizing its recommendations in four headings--i.e. problem solving, communication, reasoning, and connections--the *Standards* indicate that students must learn to value mathematics and develop confidence in their own ability; become mathematical problem solvers;[69] learn to communicate mathematically by means of symbols, tables, charts, and graphs; learn to reason mathematically--i.e. making conjectures, gathering evidence, and building arguments in support of their views; and understand the connections within mathematics, between mathematics and everyday life, and between mathematics and other fields of knowledge.

For kindergarten through grade four, the NCTM declares a moratorium on rote practice of computation algorithms, memorization of rules, worksheets, and 'teaching by

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[68]These included the Mathematical Association of America, the Mathematical Sciences Education Board, the National Council of Supervisors of Mathematics, the American Mathematical Society, the Association of Women in Mathematics, and the Operations Research Society of America.

[69]This ability requires working on problems "that may take hours, days, and even weeks to solve."

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telling.'[70] Instead, the *Standards* encourage the use of manipulatives, calculators, and computers; a focus on problem-solving; cooperative work; discussing, questioning, and justifying mathematical procedures and conjectures; and the integration of mathematics with other school subjects. The importance of talking and writing mathematics is highlighted. Verbal interactions help students construct knowledge, understand others' ideas, and clarify their own thinking. And, through writing--e.g. describing how they solved a problem or expressing their feelings about math -- students reflect on their practices and develop deeper understandings.

Math learning has been equated to communicating and reasoning about mathematical problems. As the NCTM explains, "information is the new capital and the new material, and communication is the new means of production" (p.3). In placing problem solving at the center of the math classroom, the *Standards* follows its own recommendation for the 1980s.[71] But problem solving has now been made even more central. Since performance in problem solving "will measure the effectiveness of our personal and national possession of mathematical competence" (p.2), it "should permeate the entire program and provide the context in which concepts and

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[70]'Telling' refers to the central teaching action of demonstrating the proper sequence of steps in mathematical procedures (Stodolsky 1985).

[71]"Problem solving must be the focus of school mathematics" (NCTM 1980).

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skills can be learned" (p.23).

'Word' or 'story' problems have become the paramount didactical objects. It is around these singular micro-narratives that the new teaching and assessment practices will operate. For this to happen, students need to experience 'genuine' math problems, that is, situations which are "sufficiently simple to be manageable but sufficiently complex to provide for diversity in approach" (p.11). For children in the early grades, the *Standards* suggest that,

> most problem situations will arise from school and other everyday experiences. When mathematics evolves *naturally* from problem situations that have meaning to children and are related to their environment, it becomes relevant and helps children link their knowledge to many kinds of situations (p.23, emphasis added).

Thus, an end may be put to children's disaffection with school mathematics by substituting the dry and dull word problems of traditional arithmetic lessons, with story problems emerging from situations which are familiar to children.

Michael Apple (1992) describes the *Standards* as a slogan system: its recommendations are sufficiently vague "so that powerful groups or individuals who would otherwise disagree can fit under the umbrella" and, yet, specific enough to provide guidance to practicioners; and the vision outlined has "the ability to charm." In its descriptive

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rather than prescriptive tone,[72] the NCTM document does not pose a threat to local autonomy. Nevertheless, its authors articulate a plan for mathematical-literacy-for-all-and-how-to-get-it with enough appeal to seduce a wide range of social groups. As Apple sees it, the *Standards* interpellate

> *economic modernizers and conservatives* who want to transform mathematics into economically useful knowledge and want a technically prepared and flexible workforce; *process-oriented and constructivist educators and researchers* who want more dynamic and interactive styles of curriculum and teaching; *democratically inclined educators* who seek to make mathematics somewhat more based on community needs and who question the patterns of differential achievement that now exist; *mathematicians* who are concerned about the state of mathematics as a field; *advocates of a national common culture and common curriculum*--most, not all, of whom are cultural conservatives worried about the decline in standards and the loss of a vision of the 'western tradition;' and, finally, many *teachers* who are laboring under difficult and uncertain conditions in the schools (p. 414, emphasis added).

Wherein exactly resides the *Standards'* ability to charm? In addressing this question, I draw a parallel with the 1960s reform.[73]

As we recall, the new math was mainly about 'doing' mathematics. Underlying this approach was the belief that it was possible to design transparent or 'noiseless' materials embodying the formal structures of mathematics. It was assumed that, as children manipulated these *concrete*

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[72]The *Standards* are presented as 'design solution,' not a set of measures for 'quality control' (NCTM 1989, p. 251).

[73]As does Apple (1992) when he compares the *Standards* with Bruner's *Process of Education*.

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materials, they would discover the *abstract* structures embedded therein. The conception of math learning as *the acquisition of structures* charmed scientists who wanted 'real' mathematics in the schools, as well as process-oriented educators who advocated *a discovery approach*.[74] In this pedagogical model, the role of social interaction was minimal: each individual child was engaged in 'constructive play' with pre-designed materials, the teacher acted as a guide, and the other children were ocassional playmates and fellow 'proto-mathematicians' discovering mathematical structures.[75] In the wake of Sputnik, the new math appeared as a quick fix solution to the seemingly intractable twin problem of 'incompetent' teachers and an 'underachieving' curriculum.[76]

While in the 1980s and 1990s students are also expected to do hands-on mathematics--with computers, calculators, and

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[74]As we recall, the determinant force fueling the new math was the perceived external threat of the Soviets. This allowed new math reformers enough leeway to suspend the two principles which have historically guided curriculum building in the US--i.e. readiness and usefulness--which correspond, respectively, to the two dominant interest groups in the field: developmentalists and social efficentists.

[75]Skovmose (Restivo *et al.* 1993) critically discusses the 'monological' epistemology underlying this approach. To this he opposes a dialogical model, based on the the conception of knowledge acquisition as a process which happens in conversation with others.

[76]This was done not only through manipulatives, but also by using instructional films, teaching machines, and other cognitive technology. As Bruner self-critically reflects, during the 1960s, scientists were "convinced that the trouble with schools was the shoddy stuff they taught. The core was to narrow the gap between knowledge locked up in the university library or the scholar's mind and the fare being taught in schools. If, as in physics, an untrained teacher stood between the knowledge and the student, then bypass the teacher" (Bruner 1983, pp.179-80)

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manipulatives--an unprecedented emphasis is put on 'talking mathematics.' At work in here is the fiction of the classroom as a public sphere. In this 'microcosm of mathematical culture,' each individual student is expected to *construct* his or her 'own' mathematics as he or she participates, in *a community of discourse* in the context of which alternative procedures, conjectures, and strategies for *the solution of math problems* are presented and discussed (Schoenfeld 1988, Lampert 1990, Ball 1990, Cobb *et al.* 1992, Borasi 1992, Cobb and Bauersfeld 1995).

With its view of learners as constructors of mathematical knowledge, the mathematics classroom as a community of discourse, and mathematics learning as the activity of problem solving, this model seduces radical and social constructivist educators alike, cognitive scientists, and all those sectors concerned with the upgrading of basic skills in the direction of problem solving abilities. Nervertheless, the new new math reform does not promise to dispense with teachers.[77] Quite to the contrary, reformers acknowledge that the desired transformations are highly demanding for teachers in terms of time, motivation (willingness to 'take risks'), and mathematical knowledge.

Another feature which distinguishes the current reform

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[77]With the exception perhaps of those pedagogical models centered exclusively around artificial learning environments or 'microworlds.'

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from its 1960s predecesor and accounts for its ability to charm, is the emphasis on *connections*. In tune with the structuralist mood of the 1960s, new math reformers understood 'connections' as internal to the architecture of mathematics. The whole story of mathematics was told through the metanarrative of sets.[78] Nowadays, aside from stressing the underlying structures of the discipline--i.e. connections *within* mathematics--reformers emphasize the relationship *between* mathematics and everyday life, the links *among* mathematics, science and technology, and the integration of mathematics *with* other school subjects. With postmodern incredulity towards metanarratives (Lyotard 1979), and in line with the prevalent efficientist and vocationalist tendencies in curriculum-building, mathematics is valued "as a tool for problem solving, communication and reasoning."[79]

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[78]"Curriculum reform [...] was about letting kids see through the flawed instances of nature to the pure, unflawed idea behind it: the deep structure. The strategy was to find means that would help the learner get through the surface clutter to the simpler, more beautiful underlying forms" (Bruner 1983, p.181). New math reformers "hinged on the idea that the display of mathematics as a formal system, unified by the idea of *sets*, would allow pupils to see mathematics as a coherent whole." As Richard Noss continues (1994), it was expected that this would "offer a greater degree of involvement and understanding, and result in a more substantial fraction of pupils studying mathematics successfully at post-school levels (p. 41).

[79]Meserve and Suydam (in Morris and Arora 1992, p. 12). For a discussion of 'connections' in school math, see the 1995 NCTM Yearbook. Comparative studies of math textbooks indicate that whereas problem solving occupied a central place in the 'old math' textbooks of the 1950s, the number of word problems decreased during the 1960s and 1970s new math and 'aftermath,' and began to increase again in the early 1980s' 'new old math' (Nibbelink *et al.* 1987).

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The emphasis on problem-solving suggests that cognitive science has become the dominant paradigm in math education.[80] That cognitive scientists--together with constructivist educators--have taken the leading role in the reform was, to an extent, a predictable shift. The 'spectacular failure' of the new math was almost universally attributed to its elitist aims (seducing students into taking college math courses) and its structuralist approach (its inattention to the child's agency), all of which supposedly resulted from the exclusive involvement of pedagogically-blind mathematicians in the design of new curricula.

As Chevallard describes it (1985), the new math's transposition created a sharp discontinuity between school math and everyday math and, in the same stroke, made school math continuous with disciplinary mathematics. With its emphasis on connections and in a curriculum integrated via problem solving, the current reform makes school math continuous with out of school math, as well as with all other fields of knowledge. Will this move increase the distance between school math and disciplinary math and thus led eventually to delegitimizing the former in the eyes of

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[80]For example, for Garofalo and Lester (1985) problem solving is a process involving the highest faculties--visualization, association, abstraction, comprehension, manipulation, reasoning, analysis, synthesis, and generalization. As they see it, each of these faculties need to be 'managed' and all of them need to be 'coordinated.'

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mathematicians? This is hard to anticipate and, to an extent, depends on the highly debated issue as to whether mathematics may, in fact, be defined as (or reduced to) problem solving.[81]

Behind the *Standards'* insistence on problem solving as a way of stressing the meaningfulness of school mathematics--which, as the story goes, is always hidden underneath mountains of senseless symbols--lays a fundamental misrecognition of the didactical transposition of mathematical knowledge. What is being denied is the fact that the alchemy whereby school math is constituted as an 'imaginary subject' and an 'esoteric knowledge domain,' inevitably distances it not only from disciplinary mathematics--in itself also an esoteric domain--but also from all manifestations of mathematics in the 'public domain.'[82] As the chapters to follow will illustrate, children's systematic errors, unpredictable confusions, and unexpected misunderstandings, bring to our attention the negative effects of these misrecognitions.

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[81]This matter is linked to the perennial divide between those who protect the distinction between pure and applied mathematics, and those who argue that all mathematics is applied mathematics. Even among cognitive scientists, there is disagreement on the issue of problem solving. For example, Schroder and Lester express their concern that problem solving is being made into a math topic. As they see it, math teaching should not be *about* nor *for* problem solving, but *via* problem solving (1989). See, also, Hiebert *et al* (1996).

[82]E.g. street mathematics, ethnomathematics, informal mathematics, or children's protomathematics. On 'imaginary subjects and the esoteric/public domain, see Bernstein (1990, 1996) and Dowling (in Harris 1991).

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During the new math era, *mathematical structures* were embedded in concrete materials with the expectation that, through accomodating and assimilating them, children's *cognitive structures* would be transformed. These materials were designed in a top-down fashion, that is, with no attention to how children actually learn. As Freudenthal noted (1983), underlying this 'didactical inversion' was the belief that mathematical structures could affect cognitive structures in a direct manner, that is, by-passing the teacher's role in providing *didactical structures* (models, tools, and strategies).[83]

I suggest that a didactical inversion of a different sort may occur, as the recommendations of present-day reformers translate into classroom practices. In this case, portions of reality are imported into the classroom via 'non-routine' and 'open-ended' story problems, and the teacher is expected to play a proactive role in helping children construct mathematical concepts as they all go about solving them. Yet, due to the retreat from didactics which is consistent with the dominant child-centered and constructivist paradigm, most teacher education programs do not provide their candidates with a strong preparation in the didactics of mathematics. Instead, these programs consist mainly of mathematics and general pedagogy

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[83]A similar argument is made by Vygotsky (1978) in his critique of Piaget. The latter underestimates the effects of instruction in the development of children's everyday and scientific concepts.

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courses.[84] Thus, the indispensable didactical models for *mathematizing* everyday life and other kinds of situations are not acquired by teachers and, therefore, do not reach the students.[85]

Apart from the prescription not to 'tell'--that is, refrain from being 'too didactic'[86]--teachers are left mostly on their own in making crucial didactical decisions in their daily practices. At best, the thrust is placed on the classroom-as-a-community-of-practice, where children, *qua* mathematicians, will construct mathematical concepts in the course of negotiating meanings, conjectures, and strategies,[87] or batteries of problem solving strategies are

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[84]Buchman 1982. This problem goes back at least to the turn of the century (e.g. on the preparation of teachers in Prussia, cf. Young 1900). Consider, for example, the list of traits which, in the mind of 3rd and 4th grade students at River East (an alternative, child-centered, progressive elementary school in East Harlem, New York City) constitutes 'an excellent teacher': participates; gets along with students; is fair; listens to the students; makes sure students are working; cares about others; respects everyone's feelings; involves everyone; and helps people work out their problems. Note that nothing is said about the teacher's subject matter knowledge and his/her ability to communicate it to students.

[85]Together with problem solving contexts, didactical models--e.g. diagrams, drawings, manipulatives, etc.--play a crucial mediating role between informal and formal mathematics. Cf. Lampert (1986), on models for multiplication; Ball (1990), on models for the teaching of fractions; Gravemeijer (1994), on the use of the open numberline model; Beishuizen (1993), on the use of base 10 blocks *vs.* the 'hundredsquare' model for teaching place value; or Whitenack (1995), and Cobb *et al* (in press), on the use of the 'arithmetic rack' for teaching addition and subtraction up to 20. More general discussions of problems of representation in math education may be found in Janvier 1987, Biehler *et al.* 1994, and Pimm 1995.

[86]It is symptomatic of the flight from didactics which results from the child-centered and constructivist schooling paradigm that, in the Anglo-saxon world, the term 'didactics' is either not used at all, or is used in peyorative form.

[87]Cobb *et al.* 1995, Cobb and Bauersfeld 1995, Yackel and Cobb 1996.

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suggested, with the teacher acting as the expert problem solver and the students as novice apprentices.[88]

With its fiction of continuity, the new new math also appeals to anthropologists, radical pedagogues, and other 'democratically inclined educators.' As these critics rightfully argue, mathematics is not culturally neutral and neither is school mathematics. Nothing conveys this more strongly than the content of the word problems confronting children all over the world through textbooks, workbooks, and tests. In an implicit critique of the damaging effects of textbooks on children's representation of mathematics, the *Standards* authors acknowledge that too often students believe that

> *mathematics is an academic exercise that occurs only in schools, whereas solving problems outside of schools is different.* Many believe that it is not mathematics to explore the meaning of one-third by sharing a pitcher of milk equally among three people; to count on a clock face how long until it is time to go to a friend's house; or to figure 100 4 by thinking, "Four quarters make a dollar, so it's 25", or "It's 100 divided by 2 and 2 again" (p.35)[89]

This effort at recognizing the mathematical character of mundane activities is a "desperate attempt" by math

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[88]Schoenfeld 1987, 1988.

[89]Compare this with Dienes' remark: "Once we have agreed that our aims are not entirely practical, the whole question of what mathematics to teach children is immediately thrown wide open. If mathematics-learning is to be regarded honestly as a cultural activity in the way we regard the appreciation of literature, art, and music, then the present syllabuses immediately loose their sanctity. Any syllabus would be suitable provided that with it most children could appreciate mathematics as a beautiful structure, irrespective of possible practical uses (1963, p. 155).

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education experts to prove "that mathematics is *not foreign* to the child's everyday experience" (Chevallard 1990, p.6).[90] We may see this as one more resolution of the perennial math *vs*. the child dichotomy. Yet, something radically new has taken place: children's *premathematics* (Freudenthal 1971) have been legitimized and integrated into *ethnomathematics*. Within this newly emerging field, noospherians also include the informal mathematical procedures and notations invented by practicioners in different activities and occupations.[91] *Multicultural mathematics*, a more overtly political strand of ethnomathematics, is particularly well received by the most radical reformers in the US and elsewhere in that it questions the Eurocentric character of official school math curricula, that is, the belief that mathematics can be equated with Western mathematics.[92]

The recognition of the omnipresence of mathematics

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[90]For a hilarious critique of the fiction of continuity gesture, as if seen through the eyes of a child, see Scieszka + Smith 1995. As the story goes, when the teacher tells her class that they can think of almost anything as a math problem, one student acquires a math anxiety which becomes a real curse.

[91]Scribner 1975; Lave, Murtaugh, and de la Rocha 1984 (on 'grocery shopping arithmetic'; Carraher *et al*. 1985; Saxe 1988 (on the mathematics of children who work as candy sellers in Brazil); Scribner and Stevens 1989 (on the mathematical methods used by workers in different occupations); Nunes, Schliemann and Carraher 1993 (a review of studies on street mathematics); Masingila 1994 (on the mathematical practices of carpet layers).

[92]Bishop 1988 (the cultural imperialism of Western mathematics); Joseph 1991 (on the non-Western roots of Western mathematics; D'Ambrosio in Harris, 1991 (on ethnomathematics); Nelson *et al*. 1993 (on multicultural mathematics).

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humanizes the discipline by presenting it as the product of human activity. Moreover, opening the doors of math education to cultural differences and the social contexts of learning is undoubtedly a positive development, particularly in a field which for centuries has been defined as beyond the social. Yet, interpreting everything as mathematical may contribute to further mystifying mathematics, by turning esoteric what, in fact, are mundane, everyday activities (Chevallard 1990).[93] And, as for the efforts to *reconstruct* the multiple and often scattered manifestations of non-Western mathematics and transform them into 'culturally-sensitive' mathematics, this gift is often received with suspicion by the children themselves, as well as by their parents who, with practical wisdom, prefer that schools leave to their own communities the preservation of endogenous numeracy practices and, instead, teach the 'real' mathematics (Chevallard 1990).[94]

Researchers have shown that everyday numeracy practices are embedded in specific contexts, and that their incorporation into school mathematics entails a cycle of decontextualization and recontextualization which is far from unproblematic (Bernstein 1975, 1990, 1996; Walkerdine 1988; Lave 1988; Solomon 1989). Although sometimes children

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[93]See also Dowling (in Harris 1991).

[94]Lisa Delpit (1995) makes a similar point about the teaching of Standard English to African-American students.

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do get trapped in the camouflages of school mathematics, they may also offer an implicit critique to the fiction of continuity. For most of them, school math and street math are two different activities, especially when it comes to money.[95]

Will the new new math ever become classroom reality so that everybody can count? After celebrating the broad consensus built around the reform, the *Standards* authors consider the many obstacles to be faced. These include "the strongly held beliefs, expectations, and attitudes of all people in education about specific aspects of the reform;" teachers who believe in computational skills; administrators used to charting group scores on standardized tests; and

> parents who expect students to do mathematics homework on paper at a desk rather than by gathering real data to solve a problem (pp. 254-5).

Other barriers relate to the political framework within which schools operate. As the NCTM acknowledges, some of the standards may be "fully implemented

> only by changing directives about the selection of texts, mandated testing, and so forth, in consultation with professionals in mathematics and mathematics education (p. 255).

The final obstacle is cost. Excellence costs money and, in

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[95]In school we use money, in real life we use coupons, explains to me Hillary (4th grade); Eliza, also from the 4th grade, notes that in school we don't do the tax; and his classmate Byron reminds us that in school we use only numbers, but out there we use real money.

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the current scenario of limited federal funding, it is likely that the already scarce resources will become even scarcer. As the document warns, money "must be found and used judiciously."[96]

One way to use resources judiciously is to produce guidelines, workbooks, and other resources, to help teachers change their antiquated, textbook-based practices. With different degrees of centralization, teacher-proof models have been adopted in those countries which produce the largest number of high-ranking students in international mathematics competitions.[97] Yet, a move in this direction would be a threat to the dominant decentralized approach to education.[98] Moreover, an endeavor which acknowledges a constructivist theory to learning, ought to leave room for

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[96]As we discussed so far, the present political conjuncture is dominated by a reactionary and conservative right. Education is seen more as a tax drain than as a public right; there is a decrease in funding for urban, public education, local school districts are facing economic crisis. "Because of the distressingly unequal results of school finance policies at the national, state, and local levels, students who are poor, minority, or who live in inner cities or rural areas are not sharing in the benefits (Kozol 1991)." (Cited in Apple 1992).

[97]In the Netherlands, the shift from the traditional approach to realistic mathematics instruction took place in 10 years (from 1980 to 1990). This was due, on the one hand, to the fact that the new math never florished in Dutch schools and, on the other hand, to the development and dissemination of a new 'realistic' textbook series (Streefland 1991).

[98]In California which, together with Texas, is a giant market for the textbook industry, new new math reformers have been able to produce and distribute new texts and 'replacement units.' As Cohen notes (1995), more than 90% of elementary teachers report that they use at least one of the newly released math curriculum materials. Yet, he also remarks that, due to the lack of professional development opportunities, most teachers are ill-prepared to use these innovative materials more extensively.

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teachers to 'construct' their own reform.

## 5.5 Towards New Didactic and Assessment Contracts

Two years after the publication of the *Curriculum and Evaluation Standards*, the NCTM released the *Professional Standards for Teaching Mathematics*. This document consists of a collection of classroom 'vignettes' organized around four principles--i.e. discourse, environment, task, and analysis--and meant to help teachers move "toward excellence in teaching mathematics." We may read these fragments of exemplary lessons as evidence that a new didactic contract can, in fact, be established by teachers and students in their math clasrooms.

The *Professional Standards* propose that math teaching should aim at the development of 'mathematical power,' that is

> the ability to explore, conjecture, and *reason* logically; to *solve* non routine problems; to *communicate* about and through mathematics; and to *connect* ideas within mathematics and between mathematics and other intellectual activity (p.1, emphasis added).[99]

Yet, the NCTM laments that the daily routines of most present-day classrooms are not conducive to these ambitious goals. In these math lessons, answers are given for the previous day's assignments, and the difficult problems are

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[99]As the document continues, "Mathematical power also involves the development of personal self-confidence and a disposition to seek, evaluate, and use quantitative and spatial information in solving problems and in making decisions" (p.1).

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worked out at the chalkboard by the teacher or the students. Next follows a brief explanation of new material, and assignments are given for the next day. The rest of the time, students work at their desks on 'do now' tasks, while the teacher walks around answering questions.

How do we move away from this teacher-centered model?[100] Rather than as an assembly line for the production of rule-followers and rote memorizers, the classroom ought to function like a 'community of mathematical *discourse*.'[101] This requires that the teacher learns the art of 'teaching without telling,' a modality of instruction which consists of asking probing questions to help students construct their own knowledge. The teacher's mastery of these rhetorical skills will keep classroom conversations from degrading into a cacophony of disonant voices, continuing endlessly without

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[100]Five major shifts are recommended: 1) toward classrooms as mathematical communities--away from classrooms as simply a collection of individuals; 2) toward logic and mathematical evidence as verification--away from the teacher as the sole authority for right answers; 3) toward mathematics reasoning--away from merely memorizing procedures; 4) toward conjecturing, inventing, and problem solving--away from an emphasis on mechanistic answer-finding; and 5) toward connecting mathematics, its ideas, and its applications--away from treating mathematics as a body of isolated concepts and procedures (NCTM 1991, p.3).

[101]The NCTM defines 'classroom discourse' as "the ways of representing, thinking, talking, and agreeing and disagreeing that teachers and students use to engage in those tasks. The discourse embeds fundamental values about knowledge and authority. Its nature is reflected in what makes an answer right and what count as legitimate math activity, argument, and thinking. Teachers, through the ways they orchestrate discourse, convey messages about whose knowledge and ways of knowing are valued, who is considered able to contribute, and who has status in the group" (1991, p.20).

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resolution, or deteriorating into an 'elicitation' or 'funnel pattern of interaction' (Bauersfeld 1988).

In the funnel pattern of interaction, a mode of pedagogical communication which is typical of child-centered classrooms, teacher and students engage in a sequence of questions and answers as a result of which evidence of students' understanding is elicited.[102] Eager to obtain this evidence--without which she may feel uneasy about moving on with the lesson--the teacher will help those students who are 'stuck' by *filtering* their incorrect answers and *directing* them subtly towards the correct responses. In their effort to please the teacher by producing the right answers and using the appropriate words, the students will read her mind and, eventually, produce the desired outcome. Rather than an authentic expression of students' thinking, the teacher obtains a response which has been forced by her very questioning and, thus, has only the appearance of understanding. As Brousseau notes (1986), this is one of the most common ways in which the didactic contract breaks down (i.e. the 'Topaz effect'). Since, in this modality of pedagogical communication, students and teachers follow hidden regulations of which they are rarely aware, what results is a series of

context- and problem-specific routines and skills

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[102]The FPI replaces the 'initiation-response-evaluation' pattern (IRE) which is typical of traditional, teacher-centered classrooms (Mehan 1979).

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> rather than insight, self-confidence, flexible strategies, and autonomy. *The mathematical logic of an ideal teaching-learning process thus becomes replaced by the social logic of this type of interactions* (Bauersfeld 1988, pp. 37-8).

If the logic of mathematics is to unfold and, in this way, realize its potential for developing a reasoning and autonomous child, pedagogical questioning has to become less-authoritative and more open-ended. To this effect, the *Professional Standards* suggest a menu of probing questions:

> What do others think about what J... said? Do you agree? Do you disagree? Does anyone have the same answer but a different way to explain it? Would you ask the rest of the class that question? Do you understand what they are saying? Can you convince the rest of us that that makes sense? Why do you think that? How did you reach that conclusion? Can you make a model to show that? Does that always work? Is that true in all cases? Can you think of a counterexample? How could you prove that? What assumptions are you making?

If carefully practiced, this new art of questioning will prevent funnel-like communications and other pathological ruptures of the didactic contract.

Signing a new contract requires, on the part of students and teachers, a negotiated redefinition of their respective new roles in the didactic game.[103] If like "a piece of music, the classroom discourse has themes that pull together to create a whole that has meaning" (p.35), the

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[103]Given my theoretical framework (Brousseau, Chevallard, Bourdieu, Wittgenstein) I employ the language of 'games' and 'performances,' even as I discuss texts such as the *Professional Standards*, which operates throughout with a 'musical' metaphor. This will inevitably result in two overlapping imageries for describing the math classroom.

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teacher, as the main 'orchestrator,' must find ways of pulling the themes together. For the flip side of the funnel pattern of interaction is a classroom conversation gone astray. Given that "many more ideas will come up

> than are fruitful to pursue at the moment, teachers must *filter and direct* the students' explorations by picking up on some points and by leaving others behind. Doing this prevents student activity and talk from becoming too diffuse and unfocused.... Decisions about when to let students struggle to make sense of an idea or a problem without direct teacher input, when to ask leading questions, and when to tell students something directly are crucial to *orchestrating productive math discourse* in the classroom (p.36).

How do teachers manage to take such important decisions 'on the spot'? The NCTM acknowledges that this depends on teachers' understanding both of their students and of mathematics.[104]

For this new contract to operate, students must be made to realize that whatever the teacher says is no longer true simply because she says so. Only after the teacher's authority has been displaced, students may be ready to take on the responsibility for their own learning. This includes their ability (and willingness) "to verify, revise, and discard claims on the basis of mathematical evidence," using tools such as "drawings, diagrams, invented symbols, and

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[104]As discussed by Lampert (1990, 1990a) and Ball (1993), the child-centered and constructivist pedagogical model confronts teachers with a series of dilemmas for which, as they see it, no general solutions are available. These include the need to honor children's inventions while making sure they are introduced to the conventions of mathematics, representing mathematical contents in such a way that the integrity of mathematics is respected, and creating and fostering a classroom community.

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analogies." In line with constructivism, it will be acceptable for students to use invented procedures and notations. Moreover, the *Professional Standards* recommend that teachers "introduce conventional notations only when doing so can further the work of the discourse at hand" (p.52).[105]

Obstacles to the smooth and peaceful unravelling of mathematical discourse are anticipated. The NCTM emphasizes that students should learn to justify their claims without becoming hostile or defensive, and that they must "give one another time to think, without bursting in, frantically waving hands, or showing impatience" (p.58). This requires a learning *environment* in which there is

> a genuine respect for other's ideas, a valuing of reason and sense-making, pacing and timing that allow students to puzzle and to think, and the forging of a social and intellectual community (p.57).

A further obstacle for the whole-class performace of the mathematical symphony is that

> some students, particularly those who have been successful on more traditional math classrooms, may be resistant to talking, writing, and reasoning together about mathematics (p.35)

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[105]Since teachers are to welcome children's invented algorithms, a key didactical question is when--if ever--should they introduce conventional algorithms. There are three perspectives on this matter: 1) traditional instruction (rote learning); 2) radical constructivism: algorithms are harmful; thus eliminate (Kamii); 3) social constructivism, activity theory, cognitive apprenticeship, and realistic mathematics instruction (the teacher must establish meaningful links between children's inventions and mathematical procedures and notations (Lampert 1990a, Ball 1990a, Gravemeijer 1994).

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This may require, on the teacher's part, a subtle imposition of the obligation to paraphrase, by questions such as "Who would like to repeat, using your own words, what J... just explained?" Aware that the will to understand and explain can hardly be imposed,[106] teachers may easily turn the former solicitation into "If you don't pay attention, you will not be able to paraphrase J...'s explanation"--a clever displacement of discipline for the sake of a softer modality of pedagogical transmission.

The *Professional Standards* recommend that students and teachers pursue 'worthwhile mathematical *tasks*.' As opposed to *bare* problems (e.g. 9 - 4= ), which are meant only to secure basic computation skills, or *routine, one-step* word problems (e.g. There are 9 birds on a tree. Four fly away. How many are left?), which meant as applications of those skills, worthwhile mathematical tasks are *non-routine, multi-step* story problems and investigations which provide opportunities for students to develop skill in computation while they engage in their solution (e.g. "How many handshakes will occur if everyone of the 15 guests shakes hands with each of the others?" NCTM 1989).

Yet, for a task to be worthwhile, more than this is required. As we recall from our seventy-five year history of

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[106]Some students simply choose not to understand and, instead, prefer to follow rules; others refuse to explain or may lack the words to describe their mental processes (Lampert 1990, Walkerdine 1990). On the limits of spoken and written language to express thoughts and to serve as a vehicle for understanding, see Vygotsky 1978.

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arithmetic textbooks, it has become part of the noospherians' *doxa* that for a problem to be worth solving, it must engage the child. Math problems must be *interesting*. Yet, since classrooms are becoming increasingly multicultural, the criteria of interest ought to be tied with a concern for cultural relevance. Math problems must be *interesting for all children*. This may prove, once again, an impossible demand. As expressed by two math educators from Kansas, with so much stress on cultural differences

> it is no wonder that a farm boy in Kansas would have trouble identifying with word problems developed by someone who didn't know a Hereford from a shorthorn or a drill from a cultivator (Muller and Kurtz 1981, p.13).

The best math problems may be those invented by teachers themselves, with a perspective on the students' learning processes, with an eye on their school as well as out-of-school experiences and, especially, with attention to the mathematics embedded therein.[107]

Unfortunately, researchers have found that, when teachers invent math problems, their main concern is to make them *fun*. This denounces teachers' belief that mathematics is intrinsically *boring* and thus ought to be camouflaged in

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[107]Lampert (1985, 1990) considers the need for teachers to perceive mathematics through the mind of the learner while perceiving the mind of the learner through mathematics. Hiebert *et al.* (1996) argue that what matters is that problems become problematic for the students--a fact which depends not on the problems themselves but on the classroom settings in which these problems are presented--and that they contain enough 'mathematical residue' (i.e. what remains in terms of cognitive gain after the activity of problem solving is over).

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child-like costumes (Ball 1990, 1990b). Even when teachers are made to realize the importance of focusing on the mathematical concepts and strategies which may be elicited by the problems they create, most of them lack the didactical training to design such artifacts.

In my examination of the introductory notes in old arithmetic textbooks, I addressed the unresolved dilemma faced by authors of balancing a concern for meaningfulness and cultural significance with an attention to linguistic transparency (i.e. 'readability'). In their efforts at depicting other cultures and other ways of life, present-day designers of culturally sensitive and socially relevant math problems tend to rely more heavily on language. Interestingly enough, this shift towards more wordy word problems is generally seen as a strategical move towards turning math lessons into ocassions for reading and writing (e.g. reading and writing 'across the curriculum').

I anticipate that this 'linguistic turn' in school mathematics may have consequences for Limited English Proficiency (LEP) students as well as for students who have difficulties in reading. For these students, the new, long and detailed, problem formulations may appear as 'noisy little narratives,' which they must filter to read the didactical signs embedded therein. In other words, they must cut through a mountain of unknown and unpronounciable words to grasp the structure of the problem and choose the

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appropriate model for its solution.[108]

Aware of the possible limiting effects of the criteria of relevance, the NCTM argues that, while sensitivity "to the diversity of students' backgrounds and experiences is crucial in selecting worthwhile tasks," not always

> should a concern for 'interest' limit the teacher to tasks that relate to the familiar everyday worlds of the students. What is crucial is for teachers to select tasks that provide them with *windows to students' thinking* (p.27).[109]

Hence, mathematical tasks ought to be *interesting for the teacher*. That is, these tasks must give her the opportunity to access and assess students' thinking and diagnose their level of mathematical understanding.

The continuous gathering and *analysis* of data on children's mathematical performances--including their skills, abilities, preconceptions, prejudices, dispositions, attitudes, feelings, and beliefs--is a central tenet of the

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[108]An alternative approach, developed in the Netherlands as part of the didactics of 'arithmetic in a second language,' is the use of drawings for representing the contexts to be mathematized. Although this strategy does not 'solve' the problem of representation, it does minimize the verbal component in problem solving (van den Heuvel-Panhuizen 1996).

[109]As the authors continue, although "instruction should incorporate real world contexts and children's experiences and, when possible, should use children's language, viewpoints and culture," it must be noted that "culture-sensitive does not mean a focus on the traditional arts, foods, and folklore of a group. Instead, culture-sensitive means sensitivity to "relatively subtle aspects of interactional ettiquete [that] are likely to go unrecognized by non-minority teachers" (p. 146).

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new new math reform.[110] Since the early 1980s, a relatively ample consensus has emerged on the need for a new assessment system which will gather 'multiple sources of evidence' on students' mathematical performances.[111] As the argument goes, constructivist teachers and students constructing mathematics cannot be subjected to exams which have been designed for behaviorist classrooms, teachers who tell, and students who repeat and memorize.[112] In line with the *New Standards Project*, old-fashioned exams are now being complemented or replaced with performance assessments which, via open-ended and 'authentic' problems,[113] and the solicitation that students show their work, are proposed as a more sensitive measurement of students' mathematical power.

As a result of this new development, in daily classroom practices, the didactic contract appears to have been taken over by the assessment contract. The NCTM recommends that

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[110]As the NCTM proposes it (1991), the ongoing monitoring of classroom life--i.e. 'analysis'--is aimed at assessing "how well the tasks, discourse, and environment foster the development of every student's mathematical literacy and power."

[111]On assessment reform as a requisite for durable transformation in the teaching of mathematics (Resnick *et al.* 1992, Romberg 1995, NCTM 1995).

[112]"The test theory that dominates educational measurement today is the application of twentieth century statistics to nineteenth century psychology" (Mislevy, cited in Romberg 1995).

[113]'Authentic' assessment tasks are those tasks whose attributes make them worthwhile activities beyond their use as evaluation tasks (Romberg 1995). The authenticity of a test item is often defined in terms of its alignment with the new curriculum standards (Lesh and Lamon 1992).

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teachers not be satisfied with short and straightforward answers and, instead, elicit from children detailed, and exhaustive accounts of their solution procedures.[114] With singular rhetorical force, reformers announce that the traditional distinction between instruction and evaluation no longer holds (NCTM 1989, 1991, 1993, 1995). This does not necessarily make of every math lesson an examination machine. For just as teaching is to be done without telling, assessment is to happen without testing.[115] In the new new math classroom, teachers do the asking and students the telling; and the more the student tells, the more informed is the teacher to decide what to do next. In a turn towards a more invisible invisible pedagogy, the most strictly bounded and feared aspect of the schooling process--i.e. the

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[114]E.g. if a student says "Six plus seven is thirteen," the teacher must ask "How did you figure that out?" At first, the students may remain silent, as if puzzled by the obviousness of the question; later, they may say "I just knew it," or "I did it in my head," or "I counted with my fingers. After many episodes of this sort, children may learn to articulate something like "I put the seven in my head, and then I did eight, nine, ten...". In a self-evaluating gesture which teachers will certainly welcome, children will eventually deliver an explanation such as "I started with the bigger number, and then added on." (It is a well-known fact that, universally, children would do these kinds of sums by starting with the bigger number and then add on).

[115]Wiggins (1991) calls for an 'assessment bill of rights.' This would include "worthwhile (engaging, educative, and 'authentic') intellectual problems that are validated against worthy 'real world' intellectual problems, roles, and situations" (p.28). He notes that whereas a test is a device for measuring behavior, assessment is "a comprehensive, multifaceted analysis of performance" and, thus, it "must be judgment-based and personal" (p.13). Similarly, other experts argue for the possibility of examining students and monitoring their progress "without relying on a test (or other non-productive ordeals), "without reducing it to a letter grade of a single-number score, without assigning value to students' current states of knowing, and without comparing students with one another along a simplistic 'goo-bad' scale" (Lesh and Lamon 1992).

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examination--goes underground.[116]

Although instruction and evaluation appear to merge, these two discourses correspond to two different modalities of the student-teacher relationship--i.e. the didactic and the assessment contracts. On the one hand, the students--soon to become members of the community of mathematically literate people (Lave and Wenger 1991)--and the teacher--the most expert knower in the classroom[117]--come to a more or less implicit agreement that the former are to participate in the construction (reconstruction or reinvention) of mathematical knowledge, and that the latter is to intervene, whenever she judges appropriate, to help students arrive at this knowledge. Due to constraints such as limitations of classroom time, the teacher's epistemology, and the pressure from standardized testing, this didactic contract breaks down in pathological ways. As we discussed, a series of perverse effects may result including algorithmization (emphasis on rules), metacognitive sliding (instruction in

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[116]This new modality of continuous monitoring, is concomitant with two fictions: the entry of the child's subjectivity into the process of math learning--e.g. the use of math journals--and the belief in the possibility of accessing the child's mind while in operation. These technologies become part of the more general tendency, which is inherent in invisible pedagogies, towards a more subtle regulation of students' bodies, practices and discourses. "It is the examination which, by combining hierarchical surveillance and normalizing judgement, assures the great disciplinary functions of ditribution and classification, maximum extraction of forces and time, continuous and genetic accumulation, optimum combination of aptitudes and, thereby, the fabrication of cellular, organic, genetic, and combinatory individuality" (Foucault 1979, p.192).

[117]And, thus, the one who has "the potential to demonstrate the nature of expertise to those who seek to acquire it" (Lampert 1990, p. 41).

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problem solving strategies), the funnel pattern of interaction (or Topaz effect), and the production of 'automaths' (Baruk 1985).

On the other hand, in so far as it materializes in high-stakes, on-demand, standardized, multiple-choice, time-restricted, and machine-scored examinations, the assessment contract consists of an implicit agreement among test-designers, who are interested in finding out how much students know;[118] teachers, who are to administer these exams by abiding to a set of instructions, including that they restrain, under all circumstances, from helping the students; and students, who will recall number facts, problem solving strategies, test-taking tips, and other survival skills to answer the exam items.

For the didactic and the assessment contracts to truly merge, it is not sufficient that instructional discourse turns evaluative. Assessment itself must become didactic. As Marja van den Heuvel Panhuizen describes it (1996), didactic assessments are administered and evaluated directly by classroom teachers, with the exclusive aim of gathering information to improve their teaching. Thus, in written tests, rather than eliciting clean, step-by-step reconstructions of their solution procedures via 'show work'

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[118]While tests may be seen as "technologies for knowing what young people know" (Rothman 1995, p.149), their structure as well as the conditions of their implementation suggest that most tests are actually designed to find out what students do *not* know.

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commands, the teacher makes children understand the importance of sharing with her their scribblings on 'scrap paper' (van den Heuvel Panhuizen 1996). While it certainly does produce more authentic evidence of children's mathematical understanding, this modality of assessment is more time-consuming for teachers and, perhaps more importantly, it demands from them a more robust understanding of children's thinking processes, mathematics, and its didactics.[119]

Other reasons, external to the didactic system, make it unlikely that the shift towards didactic assessment will occur in the near future.[120] Due to the overall political trend towards more transparent mechanisms and instruments for making schools 'accountable,' to the fact that standardized, multiple choice tests are cheaper to administer and score than extended-response, performance assessments (Rothman 1995), and to the lack of consensus among educational measurement experts as to whether performance assessments may be made to comply with criteria

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[119]Another modality of assessment has been proposed--i.e. 'dynamic assessment'--which, relying on Vygotsky's 'zone of proximal development' (ZPD), allows for teachers to help students during the test (Minick, in Lidz 1987). Dynamic assessment is perhaps the more radical reformulation of the assessment contract.

[120]Bernstein (1996) discusses the increasing dominance of the 'official recontextualizing field' over 'pedagogical recontextualizing field.' See also Broadfoot's analysis of the social and political dimensions of new developments in assessment (1996).

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such as validity, reliability, and generalizability,[121] the newly designed system of performance assessments will co-exist with the old system of standardized testing.

In the meanwhile, in real math classrooms, teaching and testing will continue to function as two strictly bounded instances of pedagogical practice, and as such will be lived by teachers and students. As the chapters to follow will illustrate, this is especially the case in inner city, lower class, minority segreggated classrooms, where testing is more demanding and there is more at stake.

## 5.6 On Skills and Other Dilemmas

As we discussed, the early 1980s saw the emergence, among the various social groups which have a stake in the well-being of mathematics education in the US, of an unprecedented consensus around the need for reform. Some groups have stressed the need for the United States to recuperate its hegemonic position in the world economy; others are concerned with maintaining a competitive edge in technoscientific development and making reality the hope that American students will become first in the world in mathematics and science; and others are motivated by their long-standing dream of designing and securing a good-quality

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[121]Moreover, Messick (1994) warns about the possible unintended social consequences of performance assessment, and Linn (1994) notes that the heavier reliance on language may make open-ended tasks more difficult for LEP students.

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mathematics education for all students. All these sectors have agreed that the mathematical skills for today are unlike those for yesterday; that these are best described as 'higher order,' 'analytical,' and 'problem solving' abilities; that schools are failing to teach them, especially the schools attended by lower class and minority children; and that, due to all of the above, it is time to reform.

And reform is, indeed happening. Ongoing since the late 1980s, the new new math has intervened in all four dimensions of the system of mathematics education. A new didactical transposition has been effected which suspends the teaching of computation skills 'in isolation' and, instead, makes of problem-solving the paramount activity of the math classroom. Cognitive technologies are being upgraded to such an extent that a math lesson without manipulatives, hand-held calculators, computer-assisted instructional programs, or artificially-designed learning environments will son become unthinkable. A new didactic contract has been recommended, which turns classrooms into communities of discourse where mathematics is constructed in the course of the negotiation of meanings, computation algorithms, notations, and solutions to worthwhile mathematical tasks. In particular, the textbook-generated word problems typical of old arithmetic teaching are now being replaced with 'story' problems, student-generated

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inquiries, and other mathematical investigations. And, finally, a new assessment contract has been designed which replaces the obsolete, unreliable, and repressive examination machine with an assessment process whereby students' mathematical performances are continuously monitored.

Yet, sooner or later, depending on the relative impatience of funding agencies and education policy-makers, the effects of the new new math will be assessed. These evaluations will be based on whether teachers across the nation are delivering, to parents, employers, and college professors, a generation of students endowed with the ability to solve mathematical problems and communicate effectively to others about them. If the results of these evaluations cannot be anticipated, this is not only due to the relative unpredictability of test results, but also because, underneath the apparent consensus, there are incompatible expectations on the skills that schoolchildren ought to learn in their math lessons.

Republicans and conservatives endow the new basic math skills with the power to help the United States recover its competitive edge, but they also link them to their crusade for tougher discipline, more homework, and respect for traditional family values. Whereas liberals and democrats seem content with replacing basic skills with those problem solving abilities which will allow every citizen to make

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sense of the quantitative dimensions of social life, social reformists and radical educators insist on the need to expand the notion of basic skills to include critical skills, that is, the ability to decode and demystify the quantitative reifications of the social world.

Depending on which segment of the job market they consider, corporations and the private sector understand by new basic skills either an updated version of the old mechanical habit to perform arithmetical calculations with 'speed and accuracy,' or the new analytical ability to solve problems collaboratively and 'cope with too much or too little information.' For technological utopians, the new basic skills are those closely connected to computer literacy. Although scientists conceive of new basic skills as higher order thinking tools and welcome the use of calculators and computers, they also worry that, in the move 'beyond the basics,' number sense may be discarded as an aim of mathematics teaching. There is, at last, the 19th century mental faculty doctrine made common sense, for example, in parents' defense of the disciplinary value and 'mental tonic' effect of having children memorize the multiplication tables.

As we consider these and other divergent expectations about beyond-the-basics math skills, I suggest we place the definition of the aims of math education as the development of problem solving skills, in a social, political, and

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economic atmosphere in which the State no longer fulfills the responsibility of caring for the basic problems of its citizens--i.e. health, housing, education, and so on. Wither the Welfare State, everyone is expected to become a problem solver.

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# CHAPTER SIX: CLASSROOM PERFORMANCES

> Welcome to the wonderful world of problem exploration!... *Let's Think About It* is a booklet with eighteen nonroutine problems for you to solve. We suggest that each week your teacher should read a problem to the class. Then you should reread the problem, think about the problem, organize your thoughts, and attack the problem. You have the *entire page* to work on, and you can use more paper if you need it. Explain your thinking. while the answer is important, even more important is *your thought process*--how you think, plan, organize, and work on the problem. When you have finished, check your answer and ask yourself, "Does it make sense?"...Enjoy! (Nagler and Gallin 1992)

The main argument in this thesis is that due to the conception of mathematics as a formalized language, a value-free science, and a discourse without a subject, as well as due to the central place that mathematics has been made to occupy in the educational system, its effective teaching in the elementary school is in tension with the three tenets of the modern schooling paradigm (child-centeredness, progressivism, and constructivism).[1] When brought to the extreme, these tenets result in a series of prescriptions for teachers which have damaging effects for the learning of mathematics[2]: reductionism (omit whatever will not engage the child); anti-didacticism (restrain from explicit

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[1]Within the boundaries of this paradigm, the child has been placed at the center of pedagogical practices, turning the teacher into a provider of rich learning environments (child-centeredness). Further, a relationship of continuity has been posited between school and society, whereby the former is to mirror the latter (progressivism). Finally, it has been decreed that for meaningful and lasting learning to occur, children are to be active constructors of knowledge, rather than their passive receptors (constructivism).

[2]In line with realistic mathematics education, I understand by learning mathematics the process whereby students engage in the *mathematization* of realistic or 'realizable' contexts and, eventually, in the analysis of their own mathematical activity, i.e. *mathematizing mathematics* (Freudenthal 1973, 1991; Gravemeijer 1994).

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teaching); and incidentalism (address a topic only when it proves necessary for accomplishing the project at hand).

I argued that the modern paradigm rests on three highly problematic assumptions: that the power/knowledge relationship between students and teachers may be made symmetrical; that pedagogical action is a productive practice whereby students are lead to construct their own knowledge; and that 'the child' as constituted by pedagogical discourse--i.e. 'the student' --may be equated with actual children.

Instead, I maintained throughout this dissertation that the teacher is the one supposed-to-know and the one supposed-to-anticipate what the students will know; thus, their power/knowledge relation is always necessarily asymmetrical; that teaching mathematics consists in introducing students to a pre-constructed body of knowledge, even when this process takes the form of guided reinvention; and that the categories of 'child' and 'student' are best kept separate--a distinction which may allow, for instance, to consider in a different light perennial dichotomies such as the child vs. mathematics.

I proposed that the normal functioning of the system of elementary school math education demands that the tension between the effective teaching of mathematics and the modern pedagogical paradigm be misrecognized. While in real classroom practices, an effective teaching of mathematics

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may require the suspension of some of the assumptions underlying this paradigm, ways will be found to maintain the coherence of the overall ideological framework. What resuls is a series of math fictions, which I defined as a set of discursive practices which maintain the illusion that mathematics can, in fact, be taught and learned within the constraints of the dominant paradigm.[3]

In Chapter Three, I have traced the fiction of continuity of math and out of school math and that of the transparency of the mathematical text in the prefaces and introductions of a sample of arithmetic textbooks in circulation during the period from 1882 to 1957. In Chapters Four and Five, I analyzed the seductive power and considered the effects of the fictions framing, on the one hand, the 1960s' new math and, on the other hand, the new new math reform underway since the early 1980s. This chapter and the one that follows explore the effects of the new new math's fictions, as these materialize in the daily classroom practices at a bilingual, elementary school in El Barrio (East Harlem), New York City.

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[3]These fictions include: the autonomy of the didactical system vis a vis its surrounding environment; the continuity of school- and out-of-school mathematics; a student/teacher relationship defined as a dyad (with exclusion of the third term, i.e. knowledge); the classroom modelled as a microcosm of mathematical culture; the transparency of mathematical texts and the neutrality of visual and other forms of representation; the accessibility to the student's mind while in operation; and the entry of the student's subjectivity into the process of learning and assessing mathematics.

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PS XYZ[4] is a minority-segregated school located in a low-income geographical area of the city. Low income, working-class, and non-white students attending this and similar schools, have historically scored below average level on traditional, standardized assessments. Labeled 'at risk,' classified as Limited English Proficiency (LEP) and placed in bilingual classrooms, special education programs, 'resource rooms,' 'math labs,' or all of the above, these students are caught in an examination machine which, denying them access to high quality learning further contributes to perpetuating their social and economic marginality.[5]

It may be argued that far from representing a pure instanciation of the modern schooling paradigm, PS XYZ--like most inner city, minority-segregated public schools--is a site where this paradigm appears, if at all, in its most distorted form. One may also object that since this school is not a 'demonstration site' of the reform, whatever happens behind its classroom doors ought to be seen, not as a perverse effect of the ongoing transformation in school

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[4]For reasons of confidenciality, the school's name has been disguised as PS XYZ. The first names of my main informants--the 4th and 5th grade students attending this school--have not been changed. As for the teachers and other school personnel, I identify them with initials.

[5]As we discussed in the previous chapter, ability level tracking and standardized testing entrap these students in a vicious cycle of low scores and low-level learning opportunities. They are taught in ways that mimic the rote learning and passive format of multiple-choice tests; moreover, the worse they do in those tests, the more time they spend rehearsing for further test performances, in detriment of meaningful learning practices. Cf. Oakes 1985, Cole and Griffin 1987).

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mathematics, but as a shameful remainder of an old regime about to be superseded.

As for the first objection, I suggest that in classrooms attended by lower class and minority children precisely *because* severe constraints on the didactic contract force the tenets of child-centeredness, progressivism, and constructivism to appear in their most distorted form, this tension will be stronger and will produce the most damaging effects. For, in the case of children from the dominant classes, this tension may be mitigated by the fact that they arrive to school already endowed with the prerequisites for operating in childcentered, progressive, and constructivist environments.

In what concerns the second objection, the impact of the reform is clearly making itself apparent at PS XYZ. In this school, teachers have been made aware of the NCTM *Standards*--e.g, in schoolwide staff development sessions and districtwide 'math liaison' meetings[6]--and, with varying degrees of enthusiasm and commitment, are aligning their practices to these recommendations. Also, since 1993, the New York City Board of Education introduced two new

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[6]The math program at District Four aims at preparing "our children for the twenty-first century by providing an enriched experience based curriculum.... In keeping with the National Council of Teachers of Mathematics *Standards* (1989), the District Four Math Program's goals for students reflects the importance of mathematical literacy. The goals are that students: learn to value mathematics as a tool for problem solving; become confident in their ability to do mathematics; become more adept at communicating mathematically; learn to reason mathematically; and become life long learners" (Community School District Four, August 1993).

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examinations--the CAT-5 and the PAM[7]--which reflect the current trends in assessment. It is thus fair to say that my fieldwork at PS XYZ coincided with a transitional period in which the didactic and the assessment contracts were being redefined along the lines of the new new math reform. Ultimately, whatever can be said about math teaching and learning in this school contributes to further understanding the dystopian effects which result from the overlap of old and new pedagogical regimes.

The math education research community has recognized the lack of studies of math learning among poor and minority children. The NCTM's Research Advisory Committee notes that

> It is unclear how current conceptions of mathematics learning, curriculum intervention, and teacher development can be effective in improving the mathematics education of underserved and underrepresented groups without considering the unique traits and characteristics of learners who constitute these groups, or without including members of these groups in the research on which current conceptions are based (1989, p. 373).

If we add to this the fact that, as the projected demographic trends for the year 2000 indicate, schools like PS XYZ will become typical throughout the United States, all this should more than justify my choice of school setting.[8]

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[7]California Achievement Test, Fifth Version and Performance Assessment in Mathematics.

[8]By the year 2000, one in every three American students will be minority.... Of those under 18, the proportion of minorities is already nearing 40 percent.... Already, the 10 largest school distrcits in the United States are 70 percent Black and Hispanic. Because of high birth rates and regular immigration, the Hispanic population in the United States grows at five times the national average.... More than one child in four comes from a family living

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Despite its utopian rhetoric, best encapsulated in the 'Everybody Counts' motto, the new new math reform may have dystopian effects. As I discussed in the previous chapter, this may be particularly the case in locations attended by low income, non-white, non-English speaking students, where old practices of instruction and testing are being affected by new standards and assessments. In these settings, the didactic contract is already fragile, affected as it is by high student-teacher ratios, tracking by ability level, the pressure of traditional on-demand testing, districtwide pacing calendars, limited material resources, and deficiencies in teachers' knowledge of mathematics and its didactics. Yet, this contract may be made even weaker when it is overwritten, almost overnight, by new didactic and assessment contracts which have neither been explicitly formulated nor negotiated by the two main players in the didactic game: the students and the teachers. It is reasonable to expect that this overlap may result in further ruptures of the didactic contract in ways which are not conducive to meaningful learning.

## 6.1 New York City, El Barrio, PS XYZ

El Barrio (East Harlem) is an inner-city neighborhood

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that lives in poverty; nearly one in five comes from a home in which English is not spoken..." (Mathematical Sciences Education Board 1989, pp. 18-9). See, also Johnston and Parker 1987; Oaxaca and Reynolds 1990.

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in which most of the 120,000 residents have a low to medium per capita income (around $4,000) and a predominantly Latino origin, largely Puerto Rican. Corresponding to Geographical District Eleven and to Community School District Four, its 180 blocks extend from 96th Street to 125th Street, and from Fifth Avenue to the East River.

East Harlem is considered one of the poorest urban areas in the United States. It has the eighth highest rate of welfare recipients of the twenty-six poverty areas of New York City. One in seven adults in East Harlem is out of work. Twenty-seven percent of the households are headed by single parents. Young mothers face scarce employment and educational opportunities, and day care waiting lists. Nearly one third of East Harlem teenagers are unemployed. The New York City Bureau of Police Statistics reports that the number of violent crimes in East Harlem's 'mean streets' is about twice that of the city average.

Like other communities across the United States, East Harlem is experiencing socio-economic and cultural shifts associated with urban disinvestment, changes in local industries, and influxes of immigrants.[9] In recent years, a significant number of Mexicans have begun to populate the

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[9]Before the 1920s, El Barrio's population was mostly from Italian origin. Then Puerto Ricans began to arrive. Racial tensions and violent gang wars affected the area in the 1950s and 1960s. In the mid-1960s Italians began to abandon El Barrio in search for better living conditions. In the meanwhile, new immigrants began to arrive, from Puerto Rico, Dominican Republic, and more recently, from Mexico.

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area. Due to their illegal status, these families do not even qualify for social security assistance. A sign of their presence is that at Jefferson Park, while in the past one would only see people playing football or baseball, today there is plenty of soccer-playing.

Around twenty years ago, at the center of El Barrio's social and economic life was 'La Marqueta.' Although this market has been in steady decline since then, in the last few years it has experienced a slow revival. Except for a few *bodegas*, most store owners are from ethnic groups other than Blacks and Puerto Ricans (e.g. Asians, Dominicans, Yemenite Arabs and, most recently, Mexicans). The drug trade is a major force in the local underground economy. This business is conducted by young children selling on the streets to support their families and by dealers working in understocked grocery stores.

*Housing problems* have affected East Harlem as early as in the 1960s, when landlords found that they could no longer run their buildings with sufficient profit. Many buildings were abandoned, torched for the insurance, or destroyed by their tenants. As a twin effect of this housing nightmare, the neighborhood's landscape is made up of garbage-filled vacant lots and overcrowded homes. Extended families live in the same building, sometimes two or three in one apartment, and six or more people sharing one room. Except for drug dealers and their clients, El Barrio streets are empty after

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school and on weekends. Most children spend their leisure time in the hallways, watching Spanish TV and playing Nintendo.

*Garbage collection* is a severe problem and, linked with it, is the overwhelming presence of rats and cockroaches. These living conditions are far from unrelated to parental involvement in students' school performances. As one of the mothers shared with me, she wishes she could keep her daughter's notebooks from the previous years. *"Hay que botar todito, por las ratas."* (We must throw everything away, because of the mice). Responding in writing to "What are the main problems in today's world?", fifth grader Anabel complained:

> *Lo que pasa en este mundo es que uno tira basura en las calles siempre. Si el mundo se limpiara se biera (sic) mas mejor que ahora."* (What happens in this world is that folks are always throwing garbage in the streets. If the world would clean itself up, it would look much nicer).

Similarly, for fourth grader Lakeysha,

> We have too much *litring* (littering). It would look better if people will learn not to *litre* (litter) and *batful* (beautiful) *fowrle* (flowers) will *gowun* and *gowun* (grown).

There aren't many trees in El Barrio. Wherever there is one, it so attracts the children's attention that they include it in their drawings.[10] But there are lots of flowers, especially in community gardens and in the Puerto Rican

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[10]As I commented on the tree in their drawing, they both referred with nostalgia to their countries of origin (Dominican Republic and Puerto Rico) as places full of trees.

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*casitas*. Lakeysha shared with me that one day, as she and her mother walked past an abandoned lot, her mother remarked: "No matter what, flowers always grow."

Community School District Four serves more than fourteen thousand students. Of these, 60 percent are Latinos, 35 percent are Black, 4 percent are white, and 1 percent Asian. Almost 80 percent of the students are eligible for free or reduced lunch programs. Ten percent of the students in the district are classified as LEP. These children come from homes where English is not the first language, and they have scored below the 20th percentile on a test of English language ability. The drop out rate for Blacks and Latino students has reached overwhelming proportions, and only one third of the total district population has reached a high school level of education.

Unique to District Four is its politics of 'open zoning,' that is, the presence of a variety of alternative schools for students and parents to choose from, each organized around a particular theme or vocation. Functioning since the early 1970s, some these schools have the best programs in New York City and nationwide. Autonomous, in that teacher/directors run them independently of by-the-book principals, racially integrated, and small, these schools attract not only students who are bright, attend school often, and are well-behaved, but also *problem students* from other schools and districts. Students from other

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neighborhoods crowd the 'schools of choice,' sometimes leaving waiting lists for East Harlem residents.[11] Yet, alternative schools do not draw the largest number of the District's students. Most schools in the district are 'minority group isolated.' PS XYZ is one of such schools.

Once described as 'a gem of a school in the devil's playground,' PS XYZ was founded in 1961. In 1977 it was made bilingual. Its bilingual program "is intended to strengthen English language abilities while providing instruction in areas such as mathematics, science, and social studies in Spanish and to continue the development of Spanish language abilities."[12] Although listed as a 'school of choice' due to its bilingual program, PS XYZ is not unlike most other inner city schools. Its student and teacher attendance are high (90% and 97%, respectively), but its mobility rate is high as well (about 50%)--admissions and discharges occur almost every school day. The totality of its population (493) qualifies for student lunch.

Ninety three percent of the students have a Latino background, and 6 percent are African American. Sixty five percent of them are classified as LEP.[13] Sixty percent of the kindergartners speak no English when they enter school

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[11]On alternative schools in District Four, see Fliegel and Macguire 1993, and Meir 1995.

[12]Extracted from a letter written by the Assistant Principal to the parents whose children qualify for the bilingual program.

[13]With the exception of two teachers, the staff at PS XYZ is bilingual.

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and, each year, at least ten immigrant children enter each grade directly from non-English speaking countries. In the last few years, newly arrived Mexican families began to send their children to PS XYZ. Today they constitute about 17 percent of the school population. *I Hate English*, a proposal seeking funds for a bilingual project, describes rather vividly the *language problem* at PS XYZ:

> English is not the language of East [...] Street in El Barrio. It is not spoken on the street, in shops, or in living rooms. No one watches CNN nightly news. Business, entertainment, and family life take place in Spanish. It is difficult to promote English at Public School [...] in Manhattan. It is no longer the language of the playground or the lunchroom. It is sometimes the language of the classroom by force of the teachers' will.

Many children live in shelters or in the 'projects,' or they come from foster care or single-parent homes, where they are given household responsibilities such as shopping, cleaning, cooking, and baby-sitting.[14] As described in the same proposal, "most of our students do not have *problems* themselves, but are growing up in *problematic* situations."

"PS [...] is 100% computerized," proudly states a pamphlet which announces a "computer hardware summer loan for teachers." The school has acquired various sorts of computer devices, and students from grades two to six attend

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[14]From Michelle's journal (4th grade): i hate rules because my mother send me to clean the house. i don't like it because i have to wash the dishes and mope and suipe and fix my room and my family's and i hat it because i when i wash the dishes it max my hands grisi and it is hard for me that's why i don't like it.

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every other month the 'computer lab.'[15] There is also an English as a Second Language (ESL) computer lab where "our newest immigrants learn English by creating multi-media stories and books, and perfecting their New York accents by doing their own voice tracks" (*I Hate English*).

The school also offers a 'math lab.' Euphemistically called a 'lab,' this is a pull-out program which provides remedial instruction for students scoring low on standardized state and citywide mathematics tests. As defined by the District's Office of Curriculum and Professional Development,

> At each target site, a teacher and a paraprofessional team provide corrective mathematics instruction emphasizing error patterns, hands-on experiences, problem exploration, test taking skills, and mental math (Community School District Four 1993, p.25).

Miss H., the school's math specialist, explained to me that candidates for the math lab are chosen either on the basis of teachers' judgement or in terms of their low test scores. "How low?," I asked. "Since the room is small, the placement is done from the bottom up until the seats are occupied. As we lose kids--those who no longer need the lab--we replace them with new ones."

There is also a special project (SPROUT), which exposes

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[15]There are 64 computers in the school, that is, one for every seven students. While for the case of math, computers are used mostly for drill and practice, every child in grades four to six uses Hyper Studio to write three editions of their memoirs: a bound book, an interactive disk copy with sound and occasional animation, and a video of the disk copy.

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parents to the curriculum by involving them in parallel activities. About twenty-five to thirty mothers participate regularly in it. Project SPROUT is one aspect of a larger effort to include parents in their children's schooling process. As described in a proposal seeking funds for a writing projects,

> We work with parents as we work with their children: keeping them abreast of curriculum and methods in regular meetings and in weekly hands-on curriculum workshops, inviting them to observe classes and meet with teachers on a regular basis, and sending home books to be read aloud each night.[16] The office staff, all of whom are bilingual, help parents negotiate the forms and procedures that are so much a part of American life.

Notwithstanding the violence in the neighborhood, the schoolyard stays open during lunch time, and the students play there without major disruptions. Yet, during a significant part of my fieldwork, the schoolyard was closed due to renovation work. As the Principal told me at the beginning of the school year, the playground

> will be closed for a few months; two more months. They say they'll make it into a fabulous playground. We'll see. In the meantime, children have to play outside in the street. Street traffic will be closed and parents and staff will take care of security.

As a result, to the regular noises coming from classrooms

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[16]As part of the school's regular routine mothers, parents, or guardians are to fill in a daily chart indicating what have they read to their children (or with them) and for how long. Although this regulation is generally obeyed, this approach does not preempt the possibility of subversion. For example, Byron (4th grade, high ability class) confesses that his mother who reads him "all that boring staff about Abraham Lincoln to make me fall asleep."

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and corridors, significantly more was added by the bulldozers fixing the schoolyard.

*Discipline problems* abound. 'Detention' plays a major role in the school's daily routine and in the children's minds. If you misbehave, this may translate either to a short visit to 'the office'--"where you will be asked to stay still, waiting for an audience, on the long wooden seat near the Principal's desk"[17]--or to a lunch plus break period in 'detention room'--"where you will sit quietly under the surveillance of Mr. H., the computer teacher."[18]

School staff define their practices as child-centered and emphasize the thematic and project-based character of the school's curricula:

> The nature of our school mandates student-centered curricula. All teachers know they must start with the child's language and knowledge base; we never retain a child or refer him to special education because his information does not match our curriculum. The early childhood curriculum has always been thematic, and we find that our special projects for older children have become interdisciplinary as well.[19]

As described by a group of visitors from the Principals Institute at Bank Street College, in a letter addressed to

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[17]The following note was posted over the clock by the Principal around the end of May: "Please do not send children to the office. The number of children being sent far exceeds our capacity to accommodate them. Please, work out a buddy system with a colleague to exchange students when they become difficult. If you have an emergency, send a neutral child with a note to get help from the office."

[18]According to 4th grader Rafael (high ability class), Mr. H. makes more money than the other teachers, "'cos he has to teach computers and has to scream to kids all the time."

[19]From the proposal seeking funds for a writing project.

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the Principal, PS XYZ is "a childcentered school;" it is "warm and alive," has a "terrific" library, its "cleanliness is impressive," and there is "a lot of trust."[20]

How do students at PS XYZ perform in the two major subject areas, i.e. reading and math? From the 621 New York City public elementary schools, PS XYZ ranks roughly above the 510th place. Thirty-three percent of the students score above grade level in reading, and 45 percent score above grade level in mathematics, as measured by citywide and statewide tests. At PS XYZ, math scores are higher and are improving at a faster rate than reading scores.[21] While there have been many efforts at improving students' reading and writing performances,[22] mathematics occupies the central place. (This may explain Miss L.'s supportive attitude towards my research.)

The Principal conducts in person the monthly math challenges held in the school's auditorium. An effective control device for the quality of students' performances in mental math, this activity is nonetheless very much enjoyed

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[20]Cohort 7, Principals Institute (Bank Street), dated November 5, 1993.

[21]These data were compiled from: the Office of Institutional Development, Community School District 4: Community District Statistics (City of New York, Department of City Planning); New York City Public Schools: Division of Strategic Planning/Office of Educational Data Services; New York City Board of Education; and from verbal information provided by the school Principal.

[22]For example, since the mid-1980s, the school has worked with the Teacher's College Writing project to train teachers in whole language and writing process.

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by the children.[23] While generally satisfied with the level of these public recitations, Miss L. worries about the effect of the heavily verbally-oriented new tests (PAM and CAT-5) on her students' achievement in mathematics.[24]

On the basis of their reading scores, students are placed either in the low or in the high ability track. As a result, the low ability class has a higher proportion of ESL, mostly Spanish-speaking students. As Miss H. explained to me,

> high and low is not really a difference in curriculum or in pace. The difference is the teacher. Good teachers get the top classes.

As we enter the world of classroom performances at PS XYZ,

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[23]In these events, grades compete with one another, and the winner gets a 'diploma' at the end of the month and the year. Children come up to the stage in groups of four; they are to stand up straight, look neat, and respond as fast as they can. No cheers are allowed, but a moderate celebration is permitted at the end.

[24]Thus, Miss L. has written many letters regarding this matter. "... there is a new trend in math testing, one which purports to stress higher order thinking but really stresses verbal ability at the expense of numerical skills. I believe that this trend is dangerous to all students and to society at large, and that it will have disastrous effects on the academic achievement of LEP students.... Please note that, on these tests, a mathematically correct answer will receive less credit than a cogent essay. Also, please, read the sample question in the *Times* article. The old question may be stilted, but it is straightforward and has practical applications to carpentry. The new, enlightened question is not only idiotic; it shows a clear cultural bias towards the mythical American family and a profound ignorance of the eating habits of cats." [excerpted from a letter she sent to the Bilingual Education Division of the NYC Board of Education (11/23/92)].
Miss L. is referring to a question which appeared in the pilot version of the PAM test. The problem--presented by Board of Education officials as a "multistep problem and a real life question, that is, one in which they [the students] can connect to their real life experience" (see #26 below)--tells about a fat cat who eats 1/2 a can of food each day and a thin cat who eats 1/3 of a can. Given that the price of cat food is 3 cans for $1.00, the question is "How much did Diana spend for a 60 day supply of cat food?"

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we will encounter closely the following six teachers:

| Name | Grade Level | Ability Track | Years of Experience |
|---|---|---|---|
| Miss R. | 4th | high | 6 |
| Miss O. | 4th | low | 15 |
| Miss C. | 5th | high | 17 |
| Miss P. | 5th | low | 2 |
| Miss H. | math lab | high/low | 21 |
| Miss S. | aide | high/low | student teacher |

The average class size at PS XYZ is approximately 33 students. In the fourth grade, Miss R.'s class has 37 students and Miss O.'s has 34. In the fifth grade, Miss C.'s class has 33 students and Miss P.'s has 32.

As they move throughout the system, students at PS XYZ face a complicated examination machine, composed of 'unit' or 'chapter' tests and district, city, and statewide examinations. In April 1993, New York City students in grades second to eighth took for the first time a new, norm-referenced multiple-choice test: the California Achievement Test, fifth edition (CAT-5).[25] The CAT-5 "reflects the guidelines promulgated in the *Curriculum and Evaluation Standards* (NYC Board of Education, April 1993, p.2). What is new about this test is that none of its items test computational skills in isolation. Instead,

> the CAT-5 has greater stress on logical thinking and solving problems that involve relationships among several concepts and application of multi-step procedures in sequential order (ibid. p. 3).

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[25]The CAT is one of the most popular standardized achievement battery in the United States. It is published by CTB/McGraw Hill of Monterey, California.

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That computation is not tested in isolation makes of the CAT-5 a more verbally loaded test than the one it replaced.[26] Yet, it is argued that the price is worth paying since this allows for higher doses of realism and relevance. As a Board of Education official explained to parents a month before the CAT-5 was given for the first time, at a conference entitled "What's New About the New Math?,"[27] all computations

> are embedded in the solution of problems, and these problems are real life kinds of problems, that is, kinds of situations that students run into in their daily lives.... If you want to have a good test, you have to engage the students. They have to be interested in the material in the test. If they are not interested, they may not perform up to their capacity. The items in this test are interesting for kids.

The Performance Assessment in Mathematics (PAM), also referred to as the "Show or Explain How You Got Your Answer Test" (NYC Board of Education, pamphlet, 11/93), was also administered for the first time in April 1993, but only to students in the fifth and the seventh grades. (A Practice Version of the PAM was given in January). The PAM has an open-ended format, which allows students to "create their own solutions rather than choose one from the answers given to them" (NYC Board of Education, *PAM: Parent Brochure*,

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[26]The CAT-5 replaces the Metropolitan Achievement Test, 6th version. An emphasis on problem solving does not need to make tests more verbally-oriented. Consider, for example, the items designed by Dutch didacticians for 'arithmetic as a second language.' In these items, productive use is made of pictures and diagrams to represent robust problem contexts (van den Heuvel-Panhuizen 1996).

[27]This conference took place on March 31, 93, at the McGraw-Hill Building.

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p.2). In line with the NCTM *Standards,* the aim is to assess "children's ability to reason logically about mathematical concepts, and to communicate their knowledge clearly" (ibid. p.3). To that effect, the test includes the command: "Show or explain the steps you took to solve this problem. Label your work. Be as complete as you can." The 0-6 scoring rubric for the PAM identifies three levels of competency in terms of the sophistication of strategies, and the clearness and completeness of explanations. The levels are as follows:

> High--the student has a clear grasp of what needs to be done, does it well, and provides a good explanation of how the solution was arrived at. There is no doubt that the student is competent (6 and 5). Medium--the student shows knowledge of an appropriate strategy, but makes a major error, or several minor ones, in logic, computation, labeling, or notation. The explanation may be unclear or incomplete (4 and 3). Low--the student chooses an inappropriate strategy and has only superficial knowledge of the concepts and skills required to solve the problem. Explanations are absent or incomplete. The student may try to rationalize the strategy chosen, but cannot apply it appropriately (2, 1, and 0).[28]

As opposed to multiple-choice tests, which are machine-scored, the PAM is graded by teachers; thus, they have to receive proper training. As the District's math coordinator explained to them at a PAM training session,

> Even if you did it mentally, you have to show your work. *If you just do the math, it is not enough.* If you do it in your head, you can do it on the paper... It's all training. *The more you do 'how did you do it?' the more they learn to explain....* This is the difference: in the CAT there is a

[28]Teachers are given also a specific scoring rubric for each of the problems which appear in the PAM test.

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> right or wrong answer; in the PAM you need to explain. *We need to see each step.*

Thousands of teachers throughout the district (and the city) spent a considerable amount of classroom time training students to 'show their steps.'[29] In the words of one teacher,

> we do it once a week, like religion, and we work in groups. It's collaborative, that way it's more than math. It's a social thing.

As we recall, the *Standards* recommend that the classroom ought to function like a 'community of discourse' where students and teachers 'communicate mathematically.' The *New York City Scope and Sequence* suggests questions such as

> What do you think? Why do you think that? Can you explain another way to solve it? Can you convince us of that? Who has a different way?

Along the same lines, PAM training manuals propose that teachers motivate students to show work by means of solicitations such as

> Write your answer as if you were explaining it to a student who was absent and does not understand the question (NYC Board of Education, *Teacher's Guide for the PAM*, September 3, 1993, p.5).

While practice for the PAM may be more easily integrated into a dynamics of group work than preparation for multiple choice tests, given that assessments are administered to individual students, sooner or later, the classroom-as-community fiction breaks down.

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[29]It did not take long for the preparation for performance assessment to become standardized. See, for example, Osmus (1997), *Math Strategies: Problem Solving and Peformance Assessment*.

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The pressure for training children to explain has entered every classroom at PS XYZ, affecting how answers, solutions, and explanations are handled in whole class discussions, individual and group problem solving activities, homework checking and, most conspicuously, preparation for the PAM. This new modality of evaluation, which demands that students explain their thinking *retrospectively* by providing written accounts of their strategies,[30] generates a new modality of instruction whereby students are to explain *introspectively* their thinking in oral classroom performances.

In the course of my fieldwork, I observed and audiotaped math lessons in Miss R.'s fourth grade (high ability) class, three times a week, for approximately one and a half years. I visited, though less frequently, Miss O.'s fourth grade (low ability) class. To observe how teachers prepare their students for the new tests, I also visited Miss P.'s and Miss C.'s classrooms during the weeks prior to these events. I complemented these observations with weekly visits to the 'math lab.'

After the first semester of fieldwork, I identified twelve fourth grade students in the high ability class (six

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[30]"Think of your answer as a road map (using numerical, symbolic, diagrammatic, and verbal explanations) that guides the reader to the answer," recommends NYC Board of Education officials that teachers suggest to their students in order to help them "write quality answers" (NYC Board of Education, *Teachers' Guide for the PAM*, September 1993, p.5).

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boys and six girls) for a more in-depth study: Odette, Ana, and Gianny, from the Dominican Republic; Jonathan and Wilfred, from Puerto Rico; Eric, Eliza, Rafael and Josaphat, from Mexico City; and Felicia, Lakeysha, and Byron, from New York City. As described by Miss R., their teacher,

> Odette is the perfect student--far better than his brother Oscar, who is in fifth grade. Wilfred is an excellent student; he is extremely smart. Felicia is also very good.
> Rafael is good, especially in math; but, he has some trouble expressing himself, and needs to read more often; Jonathan is also a good student, but does not always pay attention; Josaphat and Gianny are good also. Same with Eric, Eliza, and Ana. Byron can seduce you very easily, but he just has no sense of right and wrong; he is a good thinker, especially in math, but he cannot get it on paper; his handwriting and spelling are terrible; he is absent quite often, does not pay attention in class, and sometimes does not even do the homework; Lakeysha is another case, smart but a very difficult child; always distracted, does not focus, and lacks basic skills.[31]

With these twelve children, I conducted informal interviews, focus groups, and sessions on problem solving.[32] In addition, I observed them and talked to them during lunch time and in the schoolyard. These observations and conversations proved fruitful for collecting data on their general attitude towards the world of grown-ups, school, learning, tests, and mathematics.

Presently, there is an increasing emphasis on the need

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[31]It was my intention to choose students whose school performances are average or above average, according both to the teacher's judgement and to their math and reading scores.

[32]See questionnaires in the Appendix.

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for parents, especially mothers, to participate in their children's schooling experience.[33] These efforts materialize in written communications to them regarding homework and night-time reading habits, explanations of the new tests' scoring systems, and invitations to 'family math' workshops. Interested in how these demands are perceived, I visited the homes of these twelve children, and interviewed their parents (in most cases, the mothers).

I attended schoolwide staff development sessions and monthly districtwide 'math liaison' meetings, and participated in two citywide conferences dedicated to the discussion of the new assessments. Finally, I had occasional conversations with math coordinators both at the district level and at the New York City Board of Education.

In the manner of the NCTM *Professional Standards*, the sections below are made up of a series of vignettes. Each of them chosen as representative of a typical math classroom performance, these vignettes are meant to illustrate the dystopian effects of the new new math reform at PS XYZ.

## 6.2 Tricks

> BZ: What is a word problem? Suppose you have to describe what a word problem is to someone who comes from another planet. How would you explain to this creature what a word problem is?

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[33]This is in line with the invisible pedagogy model, which relies on the central role of the mother as "an agent of cultural reproduction" (Bernstein 1975).

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José: They have a lot of words to trick you.

Immunity against traps is an enormously useful capacity, but should it be tested along with arithmetic? (Freudenthal 1978, p. 86)

Every school year at PS XYZ, the preparation of students for standardized testing in mathematics occupies a significant portion of classroom time. As students may see it, many agents participate in this endeavor: from the teacher who tells you what you need to know, through your mother who helps you with the homework and may even come to an occasional family math workshop, to Miss L., who inspects the entire school's mental math performances, on each and every one of the monthly challenges held at the auditorium. Yet, sooner or later children are also made to realize that, aside from those concerned with their learning, there are some fellows trying to trick them. Usually referred to as 'they,' these agents have available a whole menu of tricks, such as formulating problems upside down, offering false clues, using difficult words, listing misleading answers to choose from, or providing too much or too little information.

A few weeks after I began my fieldwork, Anabel became my first informant. She was a ten year old girl who had arrived three years earlier from Mexico City together with her mother, father, two brothers, and two sisters. Apart

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from observing her math classes,[34] and talking to her at lunch time and in the playground, I visited her home and met her parents. One evening, Anabel was working on an assignment which consisted of a series of multiplication and division word problems 'about the Incas.' I noticed that she was proceeding at a relatively fast speed, considering her less than average reading skills in English.[35] I also observed that, except for a few computation errors, Anabel was correctly solving all the problems. In order to check the girl's reaction to the 'culturally sensitive' nature of these materials, I asked her to read one problem aloud and to tell me what it was that she supposed to do. The problem read as follows

> The Incas never discovered the wheel. Instead, they transported things on the back of animals called llamas. Suppose the Inca emperor had 117 gold bricks, and each llama could carry 9 bricks. How many llamas would the emperor need to carry his load? (Fennel *et al*, 1987, p. 52)

Visibly annoyed with the idea of redoing[36] but willing to please me, she began to read the problem. What follows is a segment of our conversation.

BZ: Could you read this one aloud?

---

[34]During the first semester of my 1 1/2 year long fieldwork, Anabel attended the 4th grade, high ability class (Miss R.). The following year, she was a student in the low ability 5th grade class (Miss P).

[35]Anabel scored in the 14th percentile (1993 DRP Reading test).

[36]As she had recently explained to me, 'redos'--that is, when the teacher, for one or another reason, asks children to redo their homework--are the worst part of school life.

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Anabel: "The incas never discovered the... the whe..." What is this, miss?

BZ: Wheel. It means *rueda*.

Anabel: The wheel. "Instead, they trans... transported things on the back of animals called... llamas." Llamas?

BZ: Yes, llamas, you know, the animals...

Anabel: Oh, yeah! "Suppose the Inca empe..." What is emperor?

BZ: In Spanish we say *emperador*.

Anabel: "The Inca empe... emperor had 117 gold bricks."
*Que son brickes*? (What are bricks?)

BZ: Gold bricks are *lingotes de oro*.

Anabel: Oh, I see. "And each llama could carry 9 bricks. How many... It's easy, miss! I have to divide!

BZ: Wait, wait! Let's finish reading it.

Anabel: How many lla... llmas would the empe.. emperor need to carry his load? Load?

BZ: Load. It means *carga*.

Anabel: I have to divide 117 divided into 9.

BZ: How do you know that?

Anabel: Cos it says "each" and then it says "how many"!

A month after the 'Incas episode,' Anabel shared with me that, the past weekend, she and Maria (one of her sisters) had helped their mother prepare food for sale at a community *fiesta* in Jefferson Park. Pleased with the opportunity to further explore her problem-solving skills, I asked Anabel to invent a 'division problem' using this context. In this way, I would also test her skill as a

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'problem-maker.' To my surprise, Anabel constructed a rather meaningful and realistic 'multi-step' problem.

She told me that her mother had spent $50 in the ingredients ("meat, onions, *tortillas*, and all that"), and that thirty plates were prepared and sold ("at three dollars and fifty cents each"). Anabel wrote on her notebook: "10 *pequenos* (small), 10 *grandes* (big), and 10 *grandotes* (very big)." She then wrote "How much money did we make *altogether*?", and quickly figured out that it was $55. As I looked in her notebook, I noticed that she had performed--columnwise, with all necessary 'carrying' and borrowing,' and without missing any '.00'--the following computations: 3.50 x 30 = $105.00, and 105.00 - 50.00 = $55.00.

BZ: Are you done?

Anabel: No, miss. Now we have to find out *como lo vamos a repartir entre las tres*. (how are we going to share it among the three of us)

BZ: Will you divide $55 in three equal parts?

Anabel: No, that wouldn't be fair, 'cos my mother did all the cooking.

She scribbled, vertically, 12.50 + 12.50 + 12.50 = 37.50

Anabel: Mm... *no, no da*. (No, that doesn't work)

Then she wrote, also columnwise 30 + 25 = 55.

Anabel (*visibly satisfied*): That's it! Twenty five dollars for my mom, 20 dollars for me, and 10 for Maria.[37]

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[37]The 'sharing' portion of this problem was a 'math fiction.' In reality, Anabel's mother kept the 55 dollars but, as the girl explained to me, she would buy new clothes for the two girls.

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As a vertical sum, she wrote: 25 + 20 + 10 = 55. Amazed by the sense of number she displayed as she calculated a fair way to share the food sale revenues among three people of different ages, (after having deduced the costs of production!) I stretched the realism of her math problem, by asking "Did all the plates cost the same?"

> Anabel (*with a condescending tone*): No, miss! *I just wanted to make it easier for you*. So, I pretended it was $3.50. (my emphasis)
>
> BZ: Do you remember the prices?
>
> Anabel: Yes, it was $2.00, $3.50, and $5.00.

Realizing that my informant was more eager to play *ahorcado* (hang-man) than to continue responding to my problem-solving inquisition, I formulated my last, almost irresistible question.

> BZ: Wait a minute. I see a lot of additions, subtractions, and multiplications in here. Are you sure this was a division problem?

Anabel looked at me, then at her scrap notes, and laughed. My question remained unanswered.

Several months later, I gave the 'Incas problem' to my twelve informants, all of them in the 4th grade, high ability class. In all except for two cases--i.e. Gianny, who took her time to learn how to pronounce the words on the problem and expressed some curiosity as to the Inca transportation system, and Ana, who subtracted 9 from 117--the children responded just like Anabel did. That is, they quickly screened the problem in search for the 'clue' (or

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'key word') and performed, either correctly or with 'minor' errors, the requested division.

These scenarios seem to suggest that 'culturally sensitive' materials such as Inca transportation systems, Maya calendars, Mexican tortillas, or African-American astronauts, are often seen by students as noise, that is, as obstacles to overcome in order to find out 'what *they* are asking me to do.' This may occur even when the problems contain elements which may be close to the students' personal experience or that of their ancesters. As I will later illustrate, the use of the 'key word' method for solving problems is a common 'official' practice at PS XYZ. Yet, in the case of students with limited verbal skills in English like Anabel, the decision to search for clues without attempting to decipher the problems' narrative content, may be interpreted as a clever 'survival' strategy in the face of a math curriculum which is increasingly becoming verbally-loaded.

For most children, the realization of the 'trickiness' of word problems is akin to a fall from innocence experience, as a result of which they develop a *paranoid* attitude towards math problems in general. This attitude is best illustrated by José's definition "They have a lot of words to trick you." A Spanish-speaking fifth grader who arrived three years ago from Mexico City, José believes that, although "math can be fun cos it's about numbers," one

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must beware of tricks. In his view, long, unknown, and 'hard-to-pronounce' words cloud what otherwise would be an enjoyable activity. And he is not the only one who sees things that way.

Today José is an average student. But, when he was in fourth grade and was still struggling with his English, he 'fell behind' in reading and in math. To help him catch up with his classmates, it was recommended that he receive remedial instruction by attending the resource room[38] and the math lab. As he recalls,

> they send you to the small room and you do problems with Miss H. and Miss S.... You use the workbook and they give you cubes, and chips and money, not real money, plastic coins, to help you figure it out.

The following scenario, from a fourth grade math lab session, illustrates an experience of the sort that José probably went through. The event, which took place at the beginning of March, that is, less than two months before the citywide tests, involves eleven students, the math lab teacher, and her aid. Students and teachers discuss the following question

> When 729 is subtracted from 1,534 the difference is...
>
> A: 2,263
> B: 1,215
> C: 815
> D: 805

In keeping with the standard classification, I will refer to

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[38]The resource room is a pull-out program for students who score low in reading tests (or who are referred by their teachers).

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this is a 'bare' problem, that is, a problem 'without context.' Yet, in the light of my overall framework, it is not without significance to emphasize that there is not such thing as a problem 'without context.' For, even 'bare' problems occur in a context, namely, the school context and, more specifically, the math classroom (Bernstein 1975, Walkerdine 1988, Lave 1993).

As for the format, this is obviously a multiple-choice problem. Aside from the correct answer (D), the menu of options includes A, obtained by adding the two numbers; B, which results from subtracting the numbers in the order in which they appear (729 - 1,534) and without 'borrowing;' and C, which is obtained by subtracting 729 from 1,534, but with a borrowing 'bug.'[39] As it is typical of multiple-choice items, this menu is intended to test whether students will proceed automatically to subtract the numbers as these appear in the problem, or whether they will start with the larger number. An item designed to detect automaths.

Miss H. copies the problem on the board and asks, pointing to 'A': "Does *he* know the rule?"

All: No!

Miss H. *(pointing to B)*: Does he know the rule?

---

[39]"One of the insights of the cognitive science approach to learning theory is that many of the mathematics errors students make are systematic. These errors are bugs, like bugs in computer programs, not slips" (Maurer, in Schoenfeld 1987, p. 165). "Thousands of grade school students have been tested, in the United States and overseas, and about 40% of the students in grades 3 through 6 have consistent subtraction bugs" (Brown and Burton, 1978, cited by Maurer, in Schoenfeld 1987, p. 169.

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All: No!

Miss H.: Do you know why he did this?

Silence. If a deception scenario is built up, this is only for the teacher, not for the students. Through her words, the first imaginary character has entered the scene: the student who does not know 'the rule.' Interestingly enough, the test-designer assumes that this student *does* know the rules, albeit the wrong ones[40]--e.g. subtract the numbers in the order in which they appear, even if the second number is bigger than the first (option B),[41] or most math problems are 'about adding' (option A).

"Do you know why he did this?" is a strange question for these students, as demonstrated by their silence. To answer it, they have to put themselves in the mind of the test-designer, and see that the option B (1,215) results from the erroneous computation of 729 - 1,534. Realizing that this analysis of "error patterns in subtraction" is

---

[40]Consider, for example, the oft-quoted case of Benny, who constructed and then systematically applied 'his own' rule for converting fractions into decimals. Given 2/3, he would do 2 + 3 = 5, and then put the decimal point, thus obtaining 0.5. Apparently, it did not trouble Benny that, by his method, 3/2 would result also in 0.5 (Erlwanger 1973; discussed by Noddings, in Davis, Maher and Noddings 1990).

[41]See Maurer (Schoenfeld 1987) for a discussion of students' bugs in multidigit subtraction. As Maurer describes it, students' 'smaller from larger' bug "is a natural generalization from their previous experience with singledigit subtraction problems. Faced, for example, with problems such as 'What is the difference between 7 and 3?... Between 4 and 9?,' students apply the rule "subtract the smaller from the larger."

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beyond the students' reach,[42] the teacher reformulates the question. This time, she brings it closer to home by replacing the imaginary student-who-does-not-know-the-rules with

> Do you know why most of us, you children... Why? Which number do they say first?

The second imaginary character enters the scene, 'they': those who made up the problem. The students have no trouble responding.

> All: Seven hundred and twenty nine.
>
> Miss H.: And then what do they say?
>
> All: One thousand five hundred and thirty four.

Miss. H. writes on the board:

```
    729
-  1,534
   ------
```

And she continues: "Do you think they do that to confuse you?" The chorus-like negative reply is immediate. Puzzled by the students' naivete, Miss H. looks at her aid, and then looks at the students in a way that makes them change their answer to "Yes!"

> Diana: It's a trick!
>
> Miss H.: They do that to *trick* you! And when you see this, because Miss H. and Miss S. have been telling you forever, when you see this, you should

---

[42]This is similar to the labeling of students' errors as confusions, e.g. "You confuse A with B." As if one can conjugate the verb to confuse in present tense. As soon as one is able to say: "I confuse A and B," that moment one is no longer confused (Baruk 1985). Far from a logical nuance, this is a central problem in the constitution of the didactical reason.

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say to yourself, wait a second now... I can't do this!

Miss H. continues, while quickly drawing nine sticks:

If I had seven hundred and twenty nine sticks... How can I subtract one thousand from there? Can you do that? It's like if you have two fingers, how can you take away five?

The students laugh at her suggestion. Monique, pointing to the computation on the board says "I know why he did it like that!"

Miss H.: I just told you that, because they wrote down the number seven hundred and twenty nine *first*. Look, Harry, keep it simple, don't confuse yourself, Harry, please. The only reason he did it was because look which number came first. That's why he put it on first, because it came on top.

Harry: Oh!, they trick you!

Miss H.: Yes, they trick you. And on tests especially. That's why you have to be smarter. So, let's just quickly change this because we know it's wrong.

A similar event took place a few weeks later. Miss H., Miss S., and a group of fifth graders were discussing a word problem 'with extraneous data.'

Miss H.: Do we need to know how many people live in Kansas to solve this problem?

All *(chorus)*: No!

Miss H.: No, we don't. They're asking us how long would it take for Peter to get there, aren't they?

All *(chorus)*: Yes!

Miss H.: So, they are giving us too much information. Do you think they do that to confuse you?

All *(chorus)*: No!

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Miss H.: Yes, they do that to trick you...

As we see, the teacher makes explicit her role as mediator between agents devising tricky problems and students solving them. While playing this role as best as they can, teachers are caught between two contradictory binds: a didactic contract according to which they are to work so that their students will *get to know*, and an assessment contract which materializes in a set of instruments designed by agents located far from the classroom, with the purpose of determining what the students *do not know*.

## 6.3 Clues and Hints

Teacher: What are some of the words that let us know what operation we're gonna use? What if it says 'what is the sum'?
All: Add!
Teacher: We're gonna add. What if I say to you 'how many of each'?
All: Miss! Miss! Multiply!
Teacher: Multiply?
Student: Divide.
Teacher: You're gonna divide. What if I say 'how many in all'?
All: Multiply.
Teacher: And if I say 'how many altogether'?
All: Add!
Teacher: And 'the difference'?
All: Subtract!

Children also learn that, together with those mischevious characters devising tricks, there may be some good souls eager to help them. But they understand that, since telling them 'how to do it' defeats the purpose, this help is usually given indirectly, for example, by means of

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clues or hints. If an awareness of the possible presence of tricks generates in children a paranoid attitude, attention to clues and hints invites them to read math problems like a *detective* trying to make sense of a difficult and mysterious case. One must be very careful, read the problem many times,[43] and examine each word, for it is likely that something in the text may 'tell you what to do.'

One day, I asked students in the two fifth grade classes "How do you solve word problems?" These are some of their responses:

> Read the problem find clues solve it (Jason). What I do first is look for clues and then do what they say and that how I solve problems (Alex). Look at the words they tell you in the paragf [paragraph] then look if they're asking you to add, take away, multiply, or divide. Then solve the problem (Diana). You solve a word problem by looking for the clues and the information (Mary). By looking for word clues like each, times, into (Brenda). I look for key words (David). First you find the clues in the problem then you find the question, then you answer it (Christopher).[44]

---

[43]As I was told by Anabel days after the PAM test, this approach was recommended by Miss P., her teacher. Anabel: "Yo a cinco y media le reste los veinte minutos." BZ: "Y que fue lo que pensastes?" Anabel: "No pense." BZ: "Algo tenes que haber pensado." Anabel: "Primero lei la pregunta tres veces." BZ: "Y por que tres veces?" Anabel: "La maestra dijo que si no lo sabemos... que lo leamos y lo leamos y lo leamos hasta que aprendamos... entonces yo lo lei tres veces."

[44]As for the question "Give me an example of an easy word problem. Why is it easy?", the numbers where lower than I expected: in the same high ability class, only seven (7) out of twenty seven (27) children responded that an easy word problem is one where the clues tell you what to do (e.g. "because she or he tells you that together means add so right away you know," or "because you have too much clues," or "it says left everyone should know that left means that they subtracted"). The same questions were asked to the ten fourth graders in Miss R.'s class who were my main informants. This was done in the course of informal one-to-one conversations and focus groups. See Appendix # for a sample of the questionnaire.

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To my question "Give me an example of an easy word problem. Why is it easy?", Annatalia, a fifth grade student in the high ability class, responded

> Bill has 10 toy cars he gave away 6 how many cars does he have left? I think this word problem is easy because at the end it says left. *Everyone should know that left means subtract.*[45]

One of the most efficient tricks for entrapping blind rule-followers is, of course, the use of misleading key words (Nesher and Teubal 1975; Nesher 1980; Lester and Garofalo 1982). Interestingly enough, although experiences in which 'all,' 'left,' or 'each' do *not* indicate addition, subtraction, or division, may make students and teachers aware that this method is not fool proof, they continue to rely on it.

An antidote against misleading key words, or the trap of too little or too much information is the following self-regulatory procedure: understand (ask yourself what you know and what you need to find out); try (come up with an idea for finding out what is being asked); and look back (check to see if your answer makes sense).[46] Cognitive scientists maintain that this procedure works best when combined, in

---

[45]Mimicking the 'word problem genre,' when students are asked to make up their own problem, they include key words such as 'in all' and 'altogether' (cf. Anabel's 'division' problem; see, also Chapter Seven, Section 7.2).

[46]Heath, *Math Connections*.

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the manner of Polya's heuristic,[47] with a menu of 'problem solving strategies.'[48] It is assumed that, if students practice these strategies, they will develop and perfect their problem solving ability. Hence the proliferation of problem solving strategies in textbooks, workbooks, and lessons.

Two pathological ruptures of the didactic contract result from this approach: metacognitive and metamathematical sliding (Brousseau 1986). These terms refer to direct instruction in problem solving strategies, and discussion of the relative merits of one or another solution strategy for a given problem.[49] Metacognitive and metamathematical sliding may take the place of what cognitive scientists as well as children[50] identify as one

---

[47]Since its publication in 1945, Polya's *How to Solve It* has become the *sine qua non* reference in cognitive scientists' and math educators' discussions of problem solving.

[48]The menu of strategies may include: make a list, use objects/act out, make a drawing/use a model, make a table, make a graph, guess and check, work backward, use a pattern, use logic, and solve a simpler problem. A defense of metacognitive approaches to mathematics teaching can be found in Lester and Garofalo 1982; Schoenfeld 1985; and Silver 1985; Garofalo and Lester 1985.

[49]With the term *glissement metacognitif*, Brousseau (1986) refers to a situation whereby when an activity of teaching has failed, the teacher may be led to turn that activity into one where his or her own explanations and heuristic means are taken as objects of study in place of mathematical objects (p.43). *Glissement metamathematique* refers to turning the activity of solving a mathematical problem into a debate about the logic employed for its solution (p. 63).

[50]When asked to respond to the question: "What do you need in order to solve word problems? (You can choose more than one option): A: to have good memory, B: to be good in reading and vocabulary, C: to know how to write well, D: to know your times tables, E: To know how to draw pictures and make charts, and F: to have a lot of practice in solving word problems," sixteen (16) out of twenty one (21) fifth grade students from the high ability class chose option F. As for

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of the main traits of 'expert' problem solvers: practice at solving problems.[51]

Together with practice, attention to the context is considered crucial for good problem solving. Cognitive scientists have shown that rather than trying general strategies--as 'novices' do-- 'expert' problem solvers tend to use context- or content-specific strategies (Newell and Simon 1972; Hinsley, Hayes, and Simon 1977; Simon and Simon 1978; Larkin *et al.* 1980). Considering this matter from a radically different theoretical framework, Freudenthal discussed a problem which involved the calculation of the total rent for a piece of land (1981). As he noted, the students who remained in the context while performing their calculations arrived at the correct answer, while those who ignored it, and treated the problem as a bare one, did not.[52]

Situating oneself 'in the context' may serve the cause of good problem solving, only when the problems are truly realistic, and when the criteria for judging the

---

the rest of the options: A: 3, B: 8, C: 1, D: 3, and E: 0.

[51]In tune with the new NCTM recommendations, practice in problem solving was far from lacking in the lessons I observed--from the solitary activity of solving endless lists of textbook problems, to small-group and whole-class engagement in solving the 'problem of the day.'

[52]Similarly, in a problem such as "1296 supporters are taking buses to go to a soccer game. If each bus can carry 38 passengers, how many buses are needed?" staying 'in the context' may prevent students from giving unacceptable answers such as '34 remainder 4' (Gravemeijer 1994).

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appropriateness of the solutions also take the context into account. In line with Dutch didacticians, *realistic* is understood here not as real or realized but as *realizable* or *imaginable* (Freudenthal 1973, 1981, 1991; Gravemeijer 1994; Treffers 1987; van den Heuvel-Panhuizen 1996).[53] On the contrary, when the problem is camouflaged in a pseudorealistic context,[54] it is generally by *ignoring the context* that students may successfully arrive to the problem's solution.[55]

Consider these two multiple choice word problems which appeared in the Second NAEP Study (Carpenter *et al.* 1980, 1980a):

> Problem 1. One rabbit eats 2 pounds of food a week. There are 52 weeks in a year. How much food will 5 rabbits eat in a week?

---

[53]In Dutch, 'zich realiseren' also means 'to realize,' in the sense of 'to picture or imagine something concretely' (van den Heuvel Panhuizen 1996). My use of the expression 'truly realistic' does not intend to open up a philosophical debate on the problem of realism. My main concern is, of course, didactical. In other words, what matters is not whether a problem is realistic or not, but whether it rings true in the ears of the children to whom such problem is intended (a fact which, of course, can only be anticipated to a certain extent).

[54]On the classification of mathematics problems in terms of the role of the context, see Jan de Lange, in Romberg 1995. Most word problems are, in fact, dressed up bare mathematical texts--they are "rather unappealing dressed up problems in which the context is merely window dressing for the mathematics put there. One context can be exchanged for another without substantially altering the problem" (Treffers and Goffree, cited in van den Heuvel-Panhuizen 1996).

[55]In fact, most 'dressed up' problems have easy mathematical solutions only when the constraints of life are ignored. In the oft-quoted problem "There were 10 birds in a tree; 2 were killed by a gunman; how many were left?" the answer 8 is obviously unrealistic in that all living birds would have flown away (Gravemeijer 1990). Of course no child is expected to treat this problem as something other than a puzzle.

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Problem 2.

MENU

| | | | |
|---|---|---|---|
| Hamburger | .85 | Milk | .20 |
| Hot dog | .70 | Soft drink | .15 |
| Grilled cheese sandwich | .55 | Milk shake | .45 |
| French fries | .40 | Ice cream | .40 |

Sue had a hot dog, french fries, and milk. How much did she spend?

In Problem 1, about 50 percent of 9-year old children chosed incorrect answers such as 52, 104, or 520 pounds; that is, they got trapped in the 'too much information' trick. In Problem 2, almost 60 percent of children of the same age chosed the correct answer--i.e. $1.30--and those who did not, opted for an answer which was close to the correct one (i.e. $1.50). Far from concluding that the 'extraneous information'--52 weeks in a year and the prices of the items not bought by Sue--had a stronger negative effect in the 'rabbits problem' than in the 'menu problem,' one may argue that in truly realistic scenarios such as the latter, there isn't such a thing as 'extraneous information' to begin with.[56]

Far from using the 'truth effects' of word problems, that is, the different ways in which these narratives interpellate the subjects-supposed-to-solve-them, for an *a posteriori* classification of problems according to their doses of realism, these phenomena suggest we interpret those

[56]As Carpenter *et al.* suggest (1980a), "these results say more about how students approach school problems than about their ability to sort out data in a real world problem" (p.11).

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instances in which students arrive at a correct solution of a problem by ignoring its context, as the students' immanent critique of the claim of realism of the problem in question. For, as Lave suggests, the "real trick" does not consist in

> finding a correspondence between everyday problems and school problems, but [in] *making word problems truly problematic for children in school*--that is, part of a practice for which the children are practicioners. Given lively imaginations, it does not matter whether the problems conform to life experience, but it is important that they engage the imagination, that they become really problematic (1992, p. 88).

The reasons which may lead some children either to ignore or to consider the context pertain to an ongoing debate involving issues of gender, social class, and cultural and linguistic background. Children may decide to ignore the context when

> a) they experience or anticipate difficulties reading and understanding the words in the problem (ESL students and students with reading difficulties),[57]
>
> b) rather than taking at face value the implicit instruction to contextualize ('Let's pretend that this is a real life scenario'), they see this instruction as being overwritten by the instruction to take any school task as a decontextualized task (boys and middle-class children),[58]

---

[57]For example, Jose's awareness of tricks, or Anabel's skipping of supposedly culturally relevant themes and her focus on key words.

[58]Bernstein 1977, Holland 1981, Walkerdine 1988 (on the 'shopping game'), Cooper 1992. Aware of this phenomenon, some teachers may decide to give 'extra credit' to children's decision not to take the context into account. Take the case of an African American teacher who would score 'above standard' the performance of a student who, invited to choose what to 'buy' from a list of items, included a pencil, instead of 'spending' all his money in candy. Similarly, when asked to share a square-shaped birthday cake among 8 guests, middle class

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> c) they read word problems in terms of their underlying mathematical structure, rather than their 'cover stories' (high ability students),[59]
>
> d) after many years of experience in traditional math classrooms, they learn to treat mathematics as something devoid of meaning; therefore, they do not expect that the context ought to be taken into account (automaths).[60]

The children in my study belong to at least one of these categories.[61] It should thus not come as a surprise that there were instances when, faced with a word problem, some children chosed to bracket the context and others did not. But, before discussing these instances, an apparently trivial case of 'misunderstanding,' will set up the broader framework within which all these classroom vignettes should be located.

The episode took place at the math lab, around the month of May. About ten fourth grade boys and girls were sitting around two small tables, working with Miss H. and Miss S. Among them were Lakeysha and Atiya. Miss S. has just

---

second graders saw through the 'cutting the cake' context the solicitation to 'divide a square in eight equal pieces. Instead, lowed class second graders took the instruction literally, and made uneven cuts (for children and for adults), or left some of the cake uncut "for the neighbours next door," or "for the next day."

[59]Hinsley, Hayes and Simon 1977; Simon and Simon 1980; Larkin *et al.* 1980; Lester 1980; Silver 1985. To an extent, this may be seen as a circular argument. For, 'high ability in math' is usually understood as the ability to think in the abstract, that is, to decontextualize.

[60]Cf. 'l'age du capitaine' effect in Baruk 1985. See, also, Schoenfeld 1985; Davis 1989; Saljo and Wyndhamn 1993.

[61]There are girls and boys; most of them are Spanish-speaking; some are 'high ability' and others are 'low ability' students; they all have a low socio-economic background, and have been exposed to at least three years of traditional math instruction.

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given out workbooks to the students. One of the problems read as follows:

What number belongs in the circle?

| START | divide | add | subtract | multiply | END |
|---|---|---|---|---|---|
| (.?.) | by 14 | 39 | 18 | by 6 | 216 |

HINT: Work Backwards

> Miss S *(looking at Atiya's notebook, where she was struggling with 216 divided by 6)* : Atiya, they tell you to work backwards. I will give you another hint. What I'm giving you is 'work backwards,' but don't do exactly what they're telling you to do. No, if you do exactly what they're telling you to do, you're not gonna get the right answer. So, if you don't do that, what else can you do? Try the opposite. That's the only word I will say. Try the opposite of what they say to do.
>
> Atiya: Miss, I got it!
>
> Lakeysha *(whose attempt at multiplying 216 by 6 was interrupted by the teacher's suggestion)*: I don't get it, miss!
>
> Miss H.: If you take a minute and give yourself a break... just to think about it, you might get it. You're not gonna get it in less than one second. You need some time.... Lakeysha, why are you writing backwards? Upside down rather?
>
> Lakeysha *(appearing naive)*: Miss, you said...

As we recall, the assumption has been made throughout this work that, in the classroom, the discourse of instruction is always embedded in a discourse of regulation.[62] I suggest that the juxtaposition, in Lakeysha's mind, of the 'work backwards' hint and her

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[62]As we recall from Chapter Two, while *instructional discourse* creates specialized skills and their relationship to one another, *regulative discourse* creates order, relations, and identity (Bernstein 1990, 1996).

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teacher's advice "Try the opposite of what they say to do," exposes the inherent tension between these two discourses. Lakeysha's response brings this tension to the surface, much like the French boy named Florent who, in his willingness to comply with the neatness rule cherished by his teacher, would not write down the zeros when approaching the margin of his notebook and, instead, would cut off the numbers or start all over on a clean page (Baruk 1977).

We may endlessly speculate as to whether the responses by Anabel, Lakeysha, or Florent result from fatal misunderstandings or constitute solitary acts of resistance. Are these students being tricked or are they the *tricksters*? In either case, as Baruk reminds us (1985), these responses are only possible in a context where things are not supposed to make sense.[63]

## 6.4 Thirty Two Circles and Two Steps

> I'd rather you draw thirty two circles and separate them and get the right answer than not understand what you're doing! (Miss. S.)

The following episode also happened at the math lab. One would have predicted that the problem

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[63]Davis (1989) tells the story of a child who, when invited to share five balloons with his partner, took the Salomonic decision of breaking the fifth balloon in half. "Was this boy really thinking about solving the actual problem (i.e. effectively sharing the five balloons) or was he trying to accommodate himself to the peculiar tribal culture of the American classroom?" (Davis 1989, p. 144)

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> In a class of 32 students, 15 have brown hair and 10 have black hair. How many students have hair of other colors?"

would not have generated much debate. That it did may be linked to the interplay between two opposite forces. The first relates to the problem itself and to the context in which it was presented. This is a dressed up subtraction problem which, as suggested by its location in the 'Two-Step Problems' section *had* to be solved in two steps.[64] The second belongs to the broader political and pedagogical context framing this event. As we discussed, the NCTM *Professional Standards* recommend that teachers encourage students to 'communicate mathematically' their conjectures, reasoning, procedures by using "drawings, diagrams, invented symbols, and analogies;" and that they convey to them the message that there is always more than one way to solve any math problem.

As soon as Miss S. uttered "hair of other colors?," Raymond screamed "It's fifty seven!" A typical automath response--the numbers in the problem are combined in a mechanical way. Assuming that either Raymond did not understand what 'they' were asking, the teacher clarified what was at stake:

> Raymond, that's not what you're really trying to find out. They didn't ask you how many students

---

[64]The Second, Third, and Fourth NAEP studies identify as one of the weaknesses in students' math achievement their performance in 'non-routine,' 'two-step' and 'multi-step' word problems (Carpenter *et al.* 1980; Lindquist *et al.* 1983, Kouba *et al.* 1988).

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> altogether, or how many students have black and brown hair. They ask you how many students have the other color hair, maybe red, or purple... or green. They didn't ask for the black and brown students that have the hair... that color hair. They wanna know how many students have green hair, I don't know, another color hair.

Raymond added 15 and 10 and put his pencil down. Either motivated by her diagnosis that the student had trouble 'visualizing' the problem, or by her belief in the usefulness of the 'make a picture' strategy--or perhaps responding to so much insistence on the need for students to show their work--Miss S. invited Raymond to represent the problem in pictorial form.[65]

> That's good, that's the first part, but what's the second part? Think about it, read it again, read it again. You can do this with a picture.

In this case, pictures--e.g. circles representing the children's heads--may serve less as a model than as a concession to the need to explain. In particular, drawing would not help if one starts with the parts. That is, by making fifteen circles and then adding ten more, the circles needed to complete the whole (32) will result from adding on from 25 to 32--i.e. seven more circles--a task for which the picture is hardly a support. Miss S. helped Raymond use the

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[65]As Bauersfeld suggests (1992) in order to develop in students a set of 'clear images' of mathematical operations, teachers "introduce special material and pictures which serve as embodiments for the operations. The use of these materials and the sanctioned interpretations of the related pictures are exercised, if not drilled, like a ritual--fixed in an unequivocal relation between object and activity. If a teacher then assumes that a student is having difficulties, he or she points hopefully at the material embodiment in order ot elicit the related sanctioned interpretation."

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circles, coloring fifteen in brown and ten in black, and moved to another table.

> Miss S.: Ok, you guys, is table two working? Oh, you know what's the problem? You're doing the one step only. And that's not what they want. I told you at the beginning. *These are two-step problems! When you do one thing, one operation or one step, you should know that there's another one missing,* because I gave you the hint that these are multi-step, two-step problems!

'Step' is not a mathematical but a *paramathematical* notion, that is, one which is mobilized by the didactic contract and thus may need to be made explicit (Chevallard 1985).[66] Making this and other paramathematical notions explicit--e.g. the over-used and yet, underspecified notion of 'pattern,' or what counts as a *different*, a *sophisticated*, or an *efficient* solution--is part of a process which social constructivist math educators identify as "the negotiation of sociomathematical norms" (Cobb, Yackel, and Wood 1992; Lo and Wheatley 1994; Cobb and Baursfeld 1995; Yackel and Wood 1996). Yet, for teachers to play a 'proactive' role in the negotiation of these notions, they themselves ought have a clear grasp of them, a fact which requires more than mathematical knowledge and pedagogical skill, namely, training in the didactics of mathematics.

---

[66]As opposed to *mathematical* notions such as 'addition,' 'circle,' or 'derivation,' *paramathematical* notions such as 'strategy,' 'pattern,' 'label,' and 'step,' are not the object of explicit teaching and, thus, the ability to 'choose the appropriate strategy,' 'find a pattern,' 'label the work,' and 'include all the steps' are excluded from direct assessment (Chevallard 1985).

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Most children believe that you have steps only when you do it 'the long way,' not when you do 'it in your head.' Furthermore, they may think: 'If I include many steps, would that convince my teacher that I *really* understood how to solve the problem? Is a step the same thing as an operation? And if I do it all in my head, where are my steps?' As Kayser (4th grade, high ability track) explained to me, "Steps is mm... when it's hard and you have to show work. When it's easy you just do it in your head."

Children's mathematical, protomathematical (and paramathematical) notions are best elicited through 'free' or 'own productions' (Streefland 1991). Nowadays, a popular format is the invitation to 'make up your own problem,' referred to, by cognitive scientists, as 'problem-formulating' or 'problem-posing' (Kilpatrick 1987; Brown and Walter 1993; Silver and Cai 1996). Consider, for example, my conversation with Byron:

> BZ: Byron, do you know what a 'two-step word problem' is?
>
> Byron: Yes, it's when...
>
> BZ: No, no, don't tell me. I just want to know if you know what it means. Now, let's say I write this down, $3.75. Could you come up with a two-step word problem so that its solution will give you this number?
>
> *(Byron writes down a problem in his notebook.)*
>
> BZ: Could you read it to me?
>
> Byron *(reads)*: Tommy bought a hip hop tape that costed $1.00 and a pepperoni pizza that costed

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$2.75. How much money did he spend?[67]

BZ: Ok, now...how do you solve this problem?

Byron: It's easy, miss! You just have to add. See...

He has written:

```
+  1.00
   2.75
   ----
   3.75
```

BZ: So, how many steps?

Byron: Two steps.

BZ *(pointing to his calculation)*: Show me the steps. I see only one here.

Byron: Miss, you don't understand! First step you go to the record store to buy the tape, and second step you go to the pizza place. You couldn't buy the tape at the pizza place!

Back to Mis S.'s lesson. The most suggestive exchange occurred when the teacher, anticipating that her question "Could we draw thirty two circles?" would be interpreted rhetorically, was taken by surprise by a chorus-like negative answer. After all, as part of their experience with time-restricted tests, students have learned that they cannot rely on having enough time to draw as many sticks as they may need in order to find a sum. Also, the message has been repeatedly conveyed to them that 'doing it with pictures' is not the most sophisticated way to do it. Moreover, since teachers themselves often distinguish

---

[67]Byron had written: "Tomy brot a hip hop tape that cotsed $1.00 and a peprony pizza cost 2.75. How many did he spet=3.75."

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between 'doing it with pictures'[68] and 'doing it mathematically,' most students believe that heads, pictures, fingers, and cubes are one thing, and mathematics is another.

As a typical instance of the elicitation pattern of interaction (EPI), Miss S. managed to obtain the desired answer. Yet, puzzled by the students' initial response, she continued,

> What you mean no! Of course we can! We could draw thirty two circles! Why not? *Who told you you can't do thirty two circles?* I rather sometimes... *I rather you draw thirty two circles and separate them and get the right answer than not understand what you're doing!*

Her comment shows a desperate effort to undo the students' belief that drawing thirty two circles would not be an appropriate strategy. As we discussed, central to the new didactic contract is that students ought to open up the 'window' of their mathematical thinking. It is assumed that, when asked to show their work, students will expose the trajectory--as awkward as this may be--which led them from the problem to the solution, or from the question to the answer. As part of their wisdom of practice, teachers know that they must first undo children's preconceptions and prejudices about different ways of doing mathematics.[69]

---

[68]Or 'in your head,' or 'with cubes, chips, or sticks.'

[69]This applies not only to the making of drawings, but also to fingers, that forbidden counting crutch. If in traditional math instruction children would hide their finger-counting under the desk, nowadays, with so much insistence on showing work, students are slowly

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After a short period in which they all worked out the problem using the drawings on the board, the teacher insisted on the advantage of pictures,

> If you were just real quick, if you were to draw thirty two students, you take away twenty five, twenty six, twenty seven, twenty eight, twenty nine, thirty, thirty one, thirty two. And you would've circled fifteen and then circled ten, and end up with seven. *Without even having to do any adding, you could have drawn your picture*. That's what I'm trying to tell you... Sometimes, I don't care about no people with hair and pretty eyes. I don't want that! I just want the thought down on paper. You didn't even have to do any adding or subtracting. Sometimes it's hard to do it. So, if you can think about and *see* it, is also good.

It would be up to the students to assess in which context it may be convenient to draw pictures. Such decisions are part of a process of apprenticeship at playing, simultaneously, two different language-games: filling in bubbles in time-restricted, multiple-choice tests and performing in untimed, open-ended assessments.

Invited to offer "another way of doing it," Grace showed proudly her strategy: "I did 15 plus 10 in my head. And then I did only one step: take away. And I didn't use pictures." Miss S. corrected her use of 'step': "You mean you used one *operation*, subtraction. But you did it in how many *steps*?" Insisting on the indispensable role of pictures, the teacher re-interpreted Grace's procedure in terms of her own drawing.

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beginning to realize that to the question "How did you do it?" it is acceptable to respond "I counted with my fingers."

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"Can we bring a child to solve a problem by coercing him (sic) to produce a drawing?" (Janvier *et al.* 1987) When students are forced to make drawings for the solution of math problems, far from materializing their "thought down on paper," the representations they produce are "simple contextual correspondences" which neither clarify, for the students, the relations contained in those problems nor provide, for the teachers, access to students' thinking processes.

As Dutch didacticians propose, the teacher's role is not simply to encourage children to make pictures, but to facilitate their *mathematization* of contexts by means of *didactical models* (e.g. arithmetic racks, number lines, hundred boards, etc.). For this to happen, teachers themselves must be skillful users of these models, and they ought to have an understanding of children's mathematical thinking (Ball 1988; Carpenter *et al.* 1988a). Most American teachers are hardly ready for this 'didactical engineering' (Artigue 1989, 1994). In line with my general thesis, I suggested that preparing teachers for these explicit didactical interventions and authorizing them to effect such interventions may necessitate, among other social, political, and economic requisites, the suspension of some of the tenets the constructivist and child-centered pedagogical paradigm.

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## 6.5 Cubes and Coins

Since the perception among many math educators is that one reason why some children fail in mathematics is their difficulty operating 'in the abstract,' remedial instruction tends to emphasize hands-on experiences and the use of manipulatives.

The role of manipulatives has been a topic of heated controversy at least since the beginning of the century, and especially since the 1960s new math reform.[70] Although these debates are far from closed, a consensus has emerged that what is at stake is not whether or not to use manipulatives, but teachers' ability to decide which materials are most appropriate for a given problem or investigation, and how are these to be 'manipulated' by the students.[71] In making such decisions, factors extrinsic to mathematics and its didactics may enter into play, including the limitations of classroom time and the effect of the use of such materials on 'classroom management.' In many cases, teachers choose either not to use manipulatives at all, or to make available different kinds of them, so that children may choose the ones they 'feel most comfortable with.'

---

[70]As we recall, one of the central fictions operating during this period was the belief that students could be helped to bridge the gap between the concrete and the abstract by recourse to artificially designed, structured materials--e.g. Dienes blocks or Gattegno's arithmetic rods--which would embody the abstract relations to be learned, so that by manipulating these devices, students would be operating with those abstract concepts and relations.

[71]Fennema 1972; Suydam 1977, 1984; Resnick and Ford 1981; Resnick and Osmanson 1987; Cobb 1987; Ball 1992; Beizhuizen 1993.

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When students solve math problems with a combination of mental computations, written calculations, and concrete materials (or pictures), possibilities are open for discrepancies in the outcomes obtained by means of each of these procedures. Different kinds of errors may result: the miscalculation errors which are typical of mental math, 'bugs' in written algorithms, or a degree of inaccuracy which is tied to the handling of concrete materials (e.g. in measurement). To these must be added yet another source of error: students' ability to follow instructions.[72]

While 'math experiments' create the illusion that results can be obtained by manipulating objects, the sense is conveyed to students that solving problems with materials is somehow not the same as solving them 'mathematically.' Moreover, if results obtained mathematically differ from those obtained 'experimentally,' students may be put in the dilemma of deciding which results are more reliable. Of course, as part of the framing of these experiments, it is assumed, first, that students will realize that *if* they have followed the instructions correctly, the outcome will be as intended; and, second, that 'if something goes wrong,' they

---

[72]As Garfinkel has taught us (1967), inherent in the interpretation of instructions and rules are issues of indexicality, incompleteness, and ambiguity. As suggestively put by Amerine and Bilmes, in the context of their discussion of a science experiment, "for children, the translation from instructions to performance is particularly hazardous, engendering diverse, unforeseen, and quaint difficulties. The result is not that children do not learn, but that they learn something rather different from what the 'experiment' is designed to teach them" (in Lynch and Woolgar 1990, p. 324).

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will know that the results obtained 'mathematically' are, by definition, more reliable than those obtained experimentally.

An interesting math experiment happened at the math lab in December. Seven 4th grade students were sitting around two tables in the small classroom.[73] The first ten minutes of the lesson consisted in "a quick review of our fractions," during which students and teacher revisited together the meaning of 'one half,' 'one third,' and 'one sixth.' After discussing how to write each of these unit fractions, Miss H. drew a big circle on the board, and asked for pictorial representations of each of these three fractions.[74]

> Miss S.: Good. Now, if you have a circle and you break it in halves, or in thirds, or in sixths... Which fraction would be the biggest? Anthony.
>
> Anthony/All: Six.
>
> Yanique: No, one half is bigger cos...
>
> Anthony: Oh! that's what you meant. I thought you said what number is bigger.
>
> Miss S. *(while she cut the circle in two halves)*: So, this is the biggest fraction and this is the

---

[73]Since the teacher's aide was absent the day this event took place, the extra burden on classroom management in an activity involving the use of manipulatives ought to be taken into account as we make sense of the event. It is well-known than, when it is not closely monitored, the use of concrete materials may result in 'deviant solution behaviors.'

[74]As expected, the teacher used the 'pizza model' as a representational context for fractions. The didactical models teachers choose to introduce a given topic may have a limiting effect on students' understanding of that topic (Adda 1976; Janvier 1981, 1987; Mason 1989; Ball 1990).

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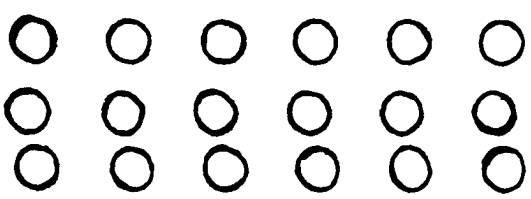
> smallest. *(She drew another circle and divided it into six parts)*. Very Good. Open your workbook on page eight.

The problem included a written formulation, a series of probing questions, a picture showing three rows each of six empty circles, a hint, and a list of four instructions:

> Hollie has 18 coins. One sixth of the coins are quarters, one third of the coins are dimes, and one half of the coins are nickels. What is the value of Hollie's coins?
> Questions:
> a) What is 1/6 of 6 coins?
> b) How can you find 1/6 of 18 coins?
> c) Is 1/3 more or less than 1/6?
> Is 1/3 more or less than 1/2?
> d) Do you think Hollie has more quarters, dimes, or nickels?
> e) What did you do to find out the value of Hollie's coins?
> Apply the strategy.
> Use the picture to help you solve the problem.
> Show that one sixth of the coins are quarters.
> This means that you should mark out one out of every six coins in the picture. Then show which coins are dimes and nickels.



Since the teacher decided to make manipulatives available, this problem solving activity now involved a complex chain of signifiers made up of:

> wooden cubes (18 units),
> fractions (1/2, 1/3, and 1/6),
> fake coins (9 nickels, 6 dimes, and 3 quarters),
> drawings (18 empty circles), and
> amounts of money (45c, 60c, and 75c).

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Miss S. included, as part of the whole-class discussion, the calculation of the number of nickels (1/2 of 18), dimes (1/3 of 18), and quarters (1/6 of 18) that Hollie had. Since the students had no difficulties obtaining the numbers 9, 6, and 3, she invited them to move to the 'experimental' phase of the activity.

> Now, we are also working with money, with coins. Instead of giving you coins, I'll give you cubes... to substitute for the coins... because we don't know what kinds of coins there are...This is what you're going to do: take eighteen of these. *(She points to a box full of cubes)*. It will help you figure it out.

Miss S. placed a handful of wooden cubes on each table. The intention was that the children would first figure out 1/2, 1/3, and 1/6 of eighteen with the cubes; second, they would trade the cubes for coins (9 nickels, 6 dimes, and 3 quarters); third, they would place the 18 coins on top of the 18 empty circles; and finally they would calculate 'the value of Hollie's coins.'

> Miss H. *(to Anthony)*: No, don't build them up! Count them! Make sure you have eighteen. If you don't have eighteen to start with, you'll figure it out wrong... Take the eighteen of them, what could half of eighteen be? Half of eighteen is what? Use your cubes.
>
> Anthony makes a group of nine cubes.
>
> Miss S.: Are they correct? Did you get nine? Michael, do you agree?
>
> Michael: Yes, cos nine plus nine is eighteen.
>
> Miss H.: So, what can we add to this information? The information that they give us is that half of all of them is nickels. How many nickels are there?

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All: Nine.

Miss H.: There are nine nickels. How did we figure this out *mathematically*?

Atiya: We divided.

Miss H.: Into how many groups?

Atiya: Half.

Miss S.: Two. How do we write that in a division sentence, Atiya?

Atiya: Nine divided by... no two...

Miss S.: Start with eighteen.

A: Eighteen divided by two is nine.

Miss S.: Good. Now we have to find out how many dimes. You can do it either mathematically or with the cubes... So, show me your paper or show me with the cubes.

The distinction between doing it 'with the cubes' and doing it 'mathematically' was made clear by the teacher herself, and the checking of one result against the other was also legitimated. Students worked individually. After figuring out the answer without the cubes, Lakeysha was amusing herself building a wall.

Lakeysha wrote: "There are six dimes in one third."

Miss S. (*looking at Lakeysha's notebook*): Very good. She did six groups of three for the quarters. Atiya, how would you do it mathematically?

Atiya: Miss, I finished, look. I did it math... mathematically.

Miss S. *(looking at Atiya's work)*: Very good. Take your information with you.

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Atiya got up from her seat with her 'information' in order to trade her cubes for the coins. In the meantime, Raymond was busy fitting his coins into the empty circles. "Miss, they don't fit!" "It doesn't matter, just put them on top. Just make sure you have eighteen," replied Miss S. But Raymond insisted, looking at his six quarters, "I think I got too many quarters." In his notebook, he had written

45¢
60¢
75¢

He has gotten the correct answer 'mathematically' but, when he traded the cubes for coins, he ended up with more quarters than he was supposed to. He thought that his answer was wrong and started to erase it.

> Miss S.: No! no! That's correct... *Don't you see?* You got the wrong amount of quarters!

Raymond looked confused at the coins and at his answer on the page, while the teacher addressed the rest of the class:

> Miss S.: Now, count your money and put down the amount on your paper. The question is "What is the value of Hollie's coins?"
>
> Lakeysha *(counting in a soft voice, almost to herself)*: 5, 10, 15, 20, 25, 30, 35, 40, 45 and 60 more is 105, and three quarters is 75 more... mm. A dollar eighty.

Judging from what she wrote in her notebook, i.e.

9 nickels
6 dimes
3 quarters
---
$1.80

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Lakeysha had figured out the amount for each of the coins in her head. For, $1.80 cannot result from simply adding 9 nickels, 6 dimes, and 3 quarters, unless one expresses it as 9 x 0.5 + 6 x 0.10 + 3 x 0.25 = $1.80.

As soon as Lakeysha finished her counting, Atiya screamed: "I also got 1.80!" She had written "Hollie had 18 coins. She added it and it was $1.80", and next to it, the same column addition as Lakeysha's. Close to the margin, she had neatly written:

20 40 60
70 80 90 100 105
130 155 180

As this list indicates, Atiya first counted by twenties the six dimes (up to 60), then counted the nine nickels as four groups of two and one more (up to 105), and to this she added the value of the three quarters (up to 180).

In the meantime, Raymond was still struggling with his cubes and coins, and Michael--who also got the correct answer on his notebook--ended up somehow with four quarters.

> Miss S. *(To Michael)*: But, you weren't supposed to have four quarters. Atiya, can you help him. *(Looks at her watch)*. It's fifteen minutes to twelve. Get ready to go. Yanique, collect the coins, please. Forget the money, we have to go.

One may wonder what was the purpose of all this manipulation. Was intended as an aid for finding fractional parts of a whole? This had already been accomplished during the whole class discussion. That during the 'experimental' segment of the lesson students were to trade the cubes for

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coins and then figure out the value of the eighteen coins, suggests that the goal was to learn about money. Two different ways of calculating the value of Hollie's coins were proposed by Lakeysha and Atiya, none of whom used the cubes, coins, and drawings. Yet, since most of the time was spent trading cubes for coins and fitting them into circles, the opportunity was missed for discussing these mental strategies.

In this episode, students operated in a fictional world of 'real money,' wooden cubes, plastic coins, circles, and the symbolic representation of unit fractions. The overlapping of two different practices--calculating with numbers and 'manipulating' cubes, coins, and drawings--occurred in such a way that, in the face of discrepancies, it was not obvious for some students which procedure had to be taken as trustworthy.

Raymond was about to erase his correct calculation on the basis of his 'experimental' result. Luckily, Miss S. was there to warn him. "Don't you see?" After all, it was just an experiment.

## 6.6 Good Reasons To Share a Pizza

> What sane person would ever want to solve a problem or work on an exercise in arithmetic for any purpose other than to obtain the correct answer? (Osburn 1924, p. 124).

The following episode, which occurred around the end of

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February, involved Miss O. and her fourth grade students in the low ability class. At some point during the lesson, they 'attacked' the following problem:

> There are 15 pieces in a large pizza. The pizza will be shared equally among 5 people. How many pieces can each person have?

The problem had a multiple choice format, with five choices (A: 5, B: 2, C: 4, D: 1, and E: none of these). If there was any 'trick,' it was perhaps that the correct answer--i.e. 15 divided by 5, that is, 3--was 'none of these.'

At stake in the ensuing classroom conversation was neither the (mis)understanding of a hint, nor the drawing of pizzas or people, nor the use of concrete materials. Instead, the classroom debate was framed by the teacher's expectation that her students would provide sufficient reasons for their choice of division as the operation for solving the problem.

This 'pizza problem' may be solved simply by dividing 15 by 5. Other solutions are possible as well: successive addition of fives, successive subtraction of fives from fifteen, or drawing the fifteen slices in a fraction pie and coloring groups of five. A comparison of these strategies could have led, for example, to establishing a link between division as *grouping* (i.e. how many groups of five can one make with fifteen objects?), and division as *dealing* (i.e. if we deal fifteen objects among five people, how many objects will each person get?) Yet, rather than around the

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'hows' of obtaining the solution, the conversation evolved around the 'whys' of dividing.

> Miss O. *(reading)*: "There are 15 pieces in a large pizza" ... Boy!, this must be a very big pizza...! "The pizza will be shared *equally* among 5 people. How many pieces can each person have?" First of all, say to yourselves: What is it that they're asking? And you say it again to yourself: "There are 15 pieces in a large pizza. The pizza will be shared equally among 5 people. How many pieces can each person have?"... And then they give you answers 5, 2, 4, 1, none of these. And you can have all the answers in the world, but if you don't know what you are going to do.... doesn't help.
>
> Dana: Divide.
>
> Miss O.: Why divide?
>
> Dana: Cos, cos five can't go in the one, so you take the fifteen, divided by five three.
>
> Miss O.: No, no, no I wanna know *why* do you have to divide.

Dana had turned the teacher's 'why' question into a 'how' question--a reasonable move, given the constant pressure towards procedural explanations. As soon as her explanation was rejected, Dana offered another one, this time focusing not on the mechanics of division but on the logic of numbers:

> Cos if you multiply five times fifteen, it will give us a bigger number and then...

Reformulating Dana's suggestion in terms of the problem's context, Miss O. pushed for a more convincing explanation:

> It will give us more pieces in the pizza, right. And they are telling you that there are fifteen pieces. They're already telling you how many pieces there are! And then they are telling you what? What else are they telling you?

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She then interrupted John's explanation: "If you add, you get more pieces, and if you subtract...," with this monologue:

> See, I don't understand, I'm a little confused. You're on the right track. But *why* we have to divide, I'm not so sure why we have to divide, *I'm not so sure why you're choosing those choices*. If I add I get this, if I multiply I get the other. Are you talking about because the numbers that they gave you are smaller? Ok, that's interesting... He is using what we call *common sense*... They give me five, they give me two, they give me four, and they give me one. If I multiply, I may get a big number, if I add I may get a large number; if I subtract I'm gonna get what? Thirteen, twelve, no I may get ten... I guess I must divide, but still that's not a good answer. It's logical, you use *logic* which is... is great, you're using logic... It's if I do this, or I do this other, I get this or that...doesn't make sense... but I wanna know the *reason*. If you chose division, why? What are the *clues*? You have to look for clues. Not because some of the answers look like this or like that... but why?
>
> Ana: Cos there are five people.
>
> Miss O.: And so?
>
> Jose: Because you have to share the pizza.
>
> Miss O.: The fifteen pieces... Because, he said, they use the words you have to *share*, *equally*. It's like you have a little party at your house, and there is a pizza, and you have five partners, there's five people at home, and each of them will get then what?
>
> All: Five/Three.
>
> Miss O.: Five?.... no, not five, three slices. Why?

Although the students' responses were "on the right track," Miss O. was pushing them to identify the-clues-indicating-division. Precisely at the point when logic and common sense were discarded as the appropriate criteria, Ana

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and Jose resisted the teacher's invitation to rely on clues, and appealed instead to the context as a justification for the need to divide. The teacher's reaction to these two interventions made explicit, once again, her agenda: the use of key words. But, unknowingly, she brought the students back to the specific situation depicted in the problem. Ana, who had previously given a context-related explanation, replied, rather circularly, "Cos if you divide by five you get three." Miss O. ended the discussion with:

> Right, fifteen divided by five is three. Right. It says the word "they share equally." When you're sharing... you are dividing. Because if I have something and I have to share it with you, what am I doing?
>
> All: You're dividing.

In their attempts at second-guessing their teacher, students moved back and forth between 'logic,' 'common sense,' big and small numbers, pizzas, slices, and people.

Aside from an instance of the EPI at work, this episode brings to the surface that the new clauses added almost overnight to the didactic contract--explain your thinking/show your work/include all the steps/justify your answer--may be interpreted in unexpected and unpredictable ways by the different actors playing the didactical game. If pushed too far to explain what perhaps for them 'goes without saying,' children may end up confessing, in the manner of Osburn, their true reasons: "I divided because I wanted to get the right answer."

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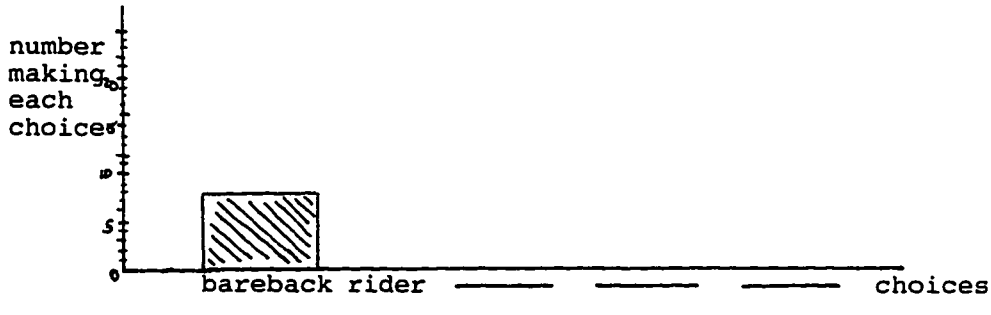
## 6.7 Circus Performers

In this episode, we meet Miss R. and her students in the fourth grade, 'high ability' track. The lesson took place at the end of November. As the teacher specified in her planning book, the topic was 'making bar graphs.' At stake was a workbook problem entitled "Organizing Data in a Bar Graph," which had been assigned as homework.[75] What follows is the complete transcript of the lesson.

> Complete the table and the bar graph. Use the data in the table to complete the bar graph. Then use the bar graph to answer each question.
>
> TABLE
> Circus Job Choices of Fourth-Graders at Carver school
>
> | Job Choice | Tally | Total |
> |---|---|---|
> | Bareback Rider | 𝍸 III | |
> | Clown | 𝍸 𝍸 𝍸 I | |
> | Lion Tamer | 𝍸 𝍸 𝍸 III | |
> | Trapeze Artist | 𝍸 𝍸 II | |
>
> BAR GRAPH




After the students had completed the tallies and total, Miss R. called upon Marcos to "read the first question at the bottom."

> Marcos *(reading)*: "How did the table help you to choose a tit, titl...

---

[75] *Practice and Assessment Skill Pad*, Heath Connections, Level 4, p.33.

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Miss R.: Title.

Marcos *(reading)*: ... title for the bar graph?

Miss R.: Title!... What title can we give to this bar graph? Marcos, what title did you give, what title would you give to this graph?

Marcos speaks softly.

Miss R.: Marcos, make believe you're in the playground. There is not a sound in here and we can't hear you!

Marcos: How much volunteers...

Miss R.: How much volunteers? Ok, we're talking about a title. What title would you give to this bar graph? Felicia.

Felicia: I would call it "Circus Performers."

Miss R.: Circus Performers? Another title. Why would you call it Circus Performers?

Felicia: Because all of them... because if you go to the circus you'll see all of them.

Miss R.: Alright, and *that's what we are talking about*. Another title. Jonathan?

Jonathan: I would call it "Circus Attraction."

Miss R.: Circus what?

Jonathan: Circus Attraction.

Miss R.: Circus attraction. Ok, *more specific*. Lakeysha

Lakeysha: I'll call it.... mm... Lion tamers, clowns, bareback riders, and trapeze artists.

Miss R.: You would call it *each individual name*? *No, we want a title for the whole thing, like a summary*... When we did this bar graph--*(points to a bar graph posted at the front of the classroom)*--I called that "Birthday Graph 4-323". Why did I call it that? Because it's a graph showing the birthdays for class 4-323... Gianny, what did you call it?

Gianny: Circus for the children, no, I mean, jobs

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for the circus, jobs for circus.

Miss R.: Jobs for circus? Ok, one more, Richard.

Richard: The Circus at the Carver school.

Miss R.: Well, ok, but that's not a good title for this graph... Anyway. Let's go to question number two.

As is typical of the EPI, when confronted with an open-ended task--'Give a title to a graph and justify your choice'--students must figure out what the teacher has in mind. Most of them took this as an invitation to draw upon their imagination to produce the most original, suggestive, and fun title. As the previous week she had been to the *Big Apple Circus*, Felicia did not wait to respond.

Miss R. tried hard to make students realize that, unlike movie titles or book titles, it is okey for bar graph titles to be boring. The teacher's request for something more "specific" than "Circus Performers" or "Circus Attraction," led Lakeysha to propose a title in which she enumerated all the available choices for the circus performance. Since to "call it each individual name" was obviously not a way to go either, the teacher defined the title as a "summary," and recalled an example from a previous lesson.

To produce a good title-as-summary, students had to discriminate what was mathematically relevant from what not. In other words, they had to see the need to include in the title some reference to the two variables whose relationship

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the bar graph represented (choices and number of children making those choices). The teacher's reaction to Gianny's and Richard's suggestions is quite revealing in that respect. Although theirs were concise and dry formulas, the main point--i.e. choices--was still missing. After many attempts at eliciting the right title, and visibly annoyed by the students' inability to respond properly to the task, the teacher moved on to the next question.

One may wonder what the overwhelming presence--in corridors, bulletin boards, and notebooks--of double entry-tables, pie charts, bar graphs, pictographs and other representations of data may represent for the students. This episode suggests that, far from seeing it as a simple and efficient way of handling quantitative information, students understand this activity as one which consists of moving data from one to another form of representation. Undoubtedly, it is a fun activity to go around collecting data on students' favorite ice-cream flavors, sneakers, and the like. Yet, the reasons why one may want to deliver that information in one or another form remains for most children a mystery.

Incidentally, students may have learned from this lesson that the more they resist the temptation to enter the *fictional* world of circus performances, the more they will be prepared to play their role in the *real* world of classroom performances.

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# CHAPTER SEVEN: THE PERFORMANCE OF ASSESSMENT

> Is it the same to solve problems in school as to solve problems in real life? Hillary: No, 'cos in school we do it with money, but outside we use coupons. Eliza: No, it's not the same because in school we don't do the tax.

As may be gathered leafing through textbooks, workbooks, curriculum guides, and pacing calendars, money is a conspicuous topic in elementary school mathematics.[1] Money may appear either as a topic of study ('money sense') or as a context for mathematizing daily experience. Through the activities of shopping, saving, working, and playing store and other games, children acquire a significant amount of knowledge about money and the social relations it encapsulates. It has long been recognized that this fund of knowledge may be relied upon for the teaching of mathematics.[2]

Yet, whereas outside of school, children are likely to refer to the names of coins rather than to their numerical

---

[1]Over the decades, many reasons have been given for the centrality of money in school math: teaching children about coins and bills, saving, investing, and tax-paying prepares them as future consumers (social efficientism); money has a high motivational power for young children (child-study, developmentalism); and learning about money develops the quantitative thinking which is fundamental for shaping a responsible citizenry (social meliorism).

[2]Cf. Ginsburg on the informal mathematical knowledge of pre-schoolers (1983), Resnick on the gap between school mathematics and everyday life (1987), Nunes, Schliemann, and Carraher (1993) on the differences in the treatment of money in the 'street' and in school. Also, money is used as a context for understanding negative numbers (Ball 1990), or for exploring the properties of multiplication (Lampert 1986). In the Dutch realistic math instruction, 'money sense' is often used as a road to 'number sense,' for example, in mental math strategies.

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value,[3] in school, the approach to money is additive: students are taught first to group pennies, then pennies and nickels, then pennies, nickels, and dimes.[4] Thus, school does not capitalize on children's understanding of the relationships between coins and the dollar bill. Also, in school, monetary value is treated in the same way as other 'measurement' concepts such as length, area, or volume. Thus, money relations--value, price, wage, etc.--are treated outside of the social relations from which they emerge--e.g. the value of a commodity and the labor power that produced it--and independently of the social relations regulating them--e.g. prices and market fluctuations.[5]

Depending on their socio-economic and cultural background, young children may or may not have learned about scarcity, they may or may not have heard of the term 'real estate,' and they may or may not have seen a one thousand

---

[3]Researchers have studied children's experiences with money, such as going to the store carrying a few quarters or a one dollar bill--more than they need-and bringing back change. As Brenner describes it, based on her study of Hawaian preschoolers, the naming of money "precedes more quantitative concepts. Children talk about the name of a coin rather than its numerical value. Thus they learn money as an ordinal system in which coins are organized by their relative value..." (cited in Lave 1992).

[4]See, for example, the story told in a fourth grade math textbook, at the beginning of the chapter on place value: "Each member of the Krieger family has a penny bank. Every few months, they get together to count their pennies. They put stacks of 50 pennies into paper wrappers. A full wrapper is called a *roll* of pennies. When they finish, the family cash in their pennies at the bank. They use the money to buy something for the whole family (...) Today the Kriegers wrapped a large pile of pennies. How many rolls of pennies did they wrap? How much money is that? (A picture showing a stack of twenty five rolls of pennies)" *Heath Mathematics Connections* 1992.

[5]Keitel 1978, Lave 1986, Walkerdine 1988.

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dollar bill or a booklet of WIC coupons. Yet, across these differences, all children know about penny banks, and are familiar with the names and values of the coins and bills in circulation. Even children from recently migrated families develop early on a strong practical understanding of money.

While a considerable amount of transfer takes place from out of school into school and vice versa--or from street mathematics to school mathematics (Nunes, Schliemann, and Carraher 1993)--these processes are far from unproblematic.[6] Dealing with money in daily life is an improvised practice, usually performed orally and rather mechanically; it is tied to a real context (e.g. shopping) and is guided by a pragmatic motive (e.g. paying the right amount, verifying the change, or comparing prices to decide on the best buy).[7] On the contrary, in school, monetary transactions are approached algorithmically and generally in written form; they are tied to an evoked context (i.e. word problems involving money or the shopping game) and are guided by a didactic motive (e.g. learning to subtract, relating the decimal and the monetary systems, producing coin combinations, or minimizing the number of coins and

---

[6]Although "non-mathematical practices can be used as the foil for, and therefore be transformed into, mathematics practices," this process entails "a shift on the relations of signification involved in the transition from one discursive practice into another" (Walkerdine 1988, p. 94). See also Lave 1993.

[7]The logic of shopping and that of mathematics do at times conflict, as in the choices made by supermarket shoppers regarding brands, sizes, etc. (Lave, Murtaugh, and de la Rocha 1984; Lave 1986).

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bills in a situation of giving/receiving change).

Inscribed simultaneously in two different social logics, the solving of money problems in school brings to the surface the inherent tension between two different contexts and two different discursive practices. As Hillary's coupons and Eliza's taxes suggest, and as the following conversation indicates as well, children are aware of these distinctions.

> BZ: I have a question for you. Some people think that solving problems in school is like solving problems in real life. What do you think?
>
> Byron: No way! Not the same 'cos you have the real stuff, that's just words *(while he points to a word problem in his notebook)*. Out there you got the real stuff... real money. If you're trying to count something, you can't do it as well as you can do it in a book, 'cos they make you count money these days. That's the nineties.
>
> BZ: So, it's easier to do it in school or in real life?
>
> Byron: In school.
>
> BZ: Why is that?
>
> Byron: 'Cos they teach you more than outside... *they tell you what to do.*

Assuming that most young children are skillful practicioners of street mathematics, a relevant question is how do they read and solve the money word problems they encounter in school? As the following episode illustrates, through their confusions, errors, and misunderstandings, students bring to the surface, often unknowingly, the tension between these two logics. At the same time, the teacher's reaction to their often unexpected responses

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exposes the extent to which, as soon as it enters the classroom, even the most realistic money problem is transposed into a school problem, in that it becomes subjected to the regulative discourse operating therein.[8] Ultimately, as Byron put it, solving money problems in school is easier not only because you use "just words," but also because "they tell you what to do."

## 7.1. Ninety-Eight Dollar Bills

The following event took place at the beginning of November, in the fourth grade, high ability class. Miss R. and her 37 students were working on a textbook problem which requested different combinations of coins and bills for ninety-eight dollars and five pennies ($98.05).[9] What follows is the full transcript of the event.

> Miss R.: Number nine, could you read it, William?
>
> William *(reading)*: What bills and coins could you use to get the amount in exercise three? Ninety-eight dollars and five pennies.
>
> Miss R.: Ok, Atiya?
>
> Atiya: Five pennies and one, and nine...
>
> Miss R.: Tell me the dollar bills, what dollar bills did you use?
>
> Atiya: I used a ninety eight dollar bill.

---

[8] In Goffman's terms (1974), the framing of a task, that is, the overall context in which it is embedded, determines to a significant extent the kinds of cognitive actions which participants will pursue.

[9] *Heath Mathematics Connections*, Book Four, p. 27.

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Miss R.: No, no! *There is not such thing as a ninety-eight dollar bill!* You can use fives, ones, or a ten dollar bill.

Hector: Twenty.

Byron: Uh, uh! Miss, miss!

Ana: Tens!

Miss R.: Ok, let's hear Wilfredo.

Wilfredo: Four twenty dollars bills...

Miss R.: Ok, four twenty dollar bills.

The teacher writes this on the board.

Miss R.: How much is that?

Wilfredo: Eighty... and one ten dollar bill ...

Miss R.: One ten dollar bill.

Wilfredo: That would be ninety...

Miss R.: Ninety.

Wilfredo: Eight one dollar bill...

Miss R.: That's eight, and five pennies that would be up here. Ninety-eight dollars and five cents. We could have done it differently.

Josaphat: You could do one five dollar bill. That's five dollars and then you use three dollars.

Miss R.: Right, instead of the eight singles, you could use a five dollar bill. Who thought of another way? What other bills and coins could you use to make up this combination?

All: Miss, miss!

Miss R.: Let me hear from Jonathan.

Jonathan: *(inaudible)*

Miss R.: I can't hear him. I know you're trying to help him, but people are being rude... Jonathan?

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Jonathan: Two forty dollar bills...

Miss R.: *Is there a forty dollar bill? When you go to the store, is there such a thing as a forty dollar bill?*

Jonathan: No.

All *(laughs)*: No!!!

Miss R.: Let's make the bills. What's the smallest bill?

The teacher began writing down on the board the value of the bills, as the students named them.

All: One!

Miss R.: Then?

All: Five!

Felicia: Two!

Miss R.: Two?

Felicia: *There's a two dollar bill!* I've got one at home.

Miss R.: There are only a few printed two dollar bills. After five?

All: Ten!

Miss R.: After ten?

All: Twenty!

Miss R.: After twenty?

All: Fifty!

Miss R.: And after fifty?

All: A hundred!

Miss R.: Ok, those are dollar bills...

Hillary: A thousand...

Miss R. *(impatiently)*: Yes, that would be next.

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Ana: Ten thousand!

Miss R.: Correct, and then fifty thousand.

Byron: And then one hundred thousand!

Miss R.: Yes. Now, based on what we know, give me another combination for ninety-eight dollars and five cents. William?

William: Sixteen five dollar bills... and eight... no... one, eight one dollar bill.

Richard *(mimicking the teacher)*: There's not such thing as a sixteen dollar bill!

Miss R.: No, Richard. He's talking about sixteen five dollar bills. William, the objective is to use...

William *(lifting one finger for each of the numbers he says)*: five, ten, fifteen, twenty, twenty-five, thirty, thirty-five, forty, forty-five, fifty, fifty-five, sixty, sixty-five, seventy, seventy-five, and eighty. So is sixteen times.... and then eight one dollar bills.

Miss R.: That will only be eighty eight dollars... you have to get to *ninety* eight. But William, *we're trying to use the biggest bills that we can*... One more, let me hear from Byron.

Byron: One fifty dollar bill, plus two twenty dollar bills,...

Miss R. started copying Byron's solution on the board.

Miss R.: Good! How much are two twenty dollar bills?

All: Forty.

Miss R.: Can we let Byron finish?

Anticipating that Byron would give her the combination which minimized the number of coins, the teacher wrote down '1 $5.'

Byron: And eight one dollar bills...

Miss R. *(erasing '1 $5' and replacing it with 8 $1)*: Ok, *you're choosing to do it like that.*

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Byron: And a nickel.

Miss R. *(smiling, visibly pleased that Byron did not say five pennies)*: A nickel. *There are many ways to do it. I just wanted you to get the idea that there are several ways of doing it.* Let's move on to the next one.

The correct solution of this problem required that the students apply their (common) sense knowledge of bills and coins. The fact that Atiya and Jonathan used non-existent bills, pushed Miss R. to interrupt the activity, and propose a new task aimed at checking the students' 'prior knowledge.' Aside from Felicia's desire to include (her) two-dollar bill in the collectively constructed list--a gesture which the teacher quickly, but politely, dismissed with "There are only a few printed..."--the new task did not present major difficulties. In fact, the children became so excited, that she could barely stop them from naming bigger and bigger bills. Satisfied after having established a commonly shared knowledge about bills, Miss R. re-oriented the students to the problem with "Now, based on what we know...."

In the course of the activity, the teacher added a constraint: that the combination for $98.05 had to involve the *least* amount of bills and coins (eight pieces). With this constraint, *the problem has only one correct solution*:

| | |
|---|---|
| 1 | $50 bill |
| 2 | $20 bills |
| 1 | $5 bill |
| 3 | $1 bills |
| 1 | nickel |
| ---- | --------- |

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8 pieces $98.05

Since the original formulation did not ask for the minimal combination, Miss R. made it clear with: "But William, we are trying to use the biggest bills that we can." Yet, some students kept producing combinations which did not minimize the number of bills and coins, and others went half through the production of the minimal solution (e.g. Wilfred, Byron). Eventually, Miss R. accepted Byron's almost correct solution--1 $50 bill, 2 $20 bills, 8 $1, and a nickel; a total of 12 pieces--and commented that she just "wanted you to get the idea that there are several ways of doing it."

I suggest that in this activity, the students found it difficult to operate, simultaneously, within constraints which brought them back and forth from the street, to the textbook, to the classroom. These included

1) real-world constraints (use any of the existing bills and coins, including rare pieces such as $2 bills, $1 coins, and so on),

2) the implicit, later on made explicit 'classroom' constraint (use only the most commonly used currency),

3) the constraints of the textbook's problem ("What bills and coins could you use...?", that is, produce different combinations of coins and bills), and

4) the constraint added by the teacher ("We are trying to use the biggest bills as we can," i.e. obtain the combination which minimizes the amount of pieces).

These difficulties may be attributed to their missing the point of the activity in the first place. Both the

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students' knowledge about bills and coins, and their ability to find mathematically correct combinations for $98.05, suggest that 'the problem' was neither mathematics nor money, but their struggle to position themselves vis a vis what they may have perceived as a meaningless task.

Towards the end of the lesson, Miss R. anticipated that Byron would use a five dollar bill. But Byron chose eight singles. Although visibly disappointed, Miss R. accepted his solution with "Oh, you're choosing to do it like that." Resisting the temptation to turn her verbal exchange with Byron into an EPI, but eager to "move on to the next one," Miss R. left open for 'multiple choices' what minutes earlier she herself had defined as a problem with only one correct answer.

Although money is a relevant context and an ever-present reality in children's lives, most of them approach the solving of money word problems as one more routine and meaningless task. Compare the request to obtain the combination with the least number of coins for $98.05, with a similar, yet more dilemmatic scenario invented by Mabel, a fifth grade teacher attending an in-service seminar at City College:

> Have you ever noticed the jingle sound when Mr. McDonald walks? No wonder since, in his pocket, he has 8 quarters, 10 dimes, 10 nickels, and 10 pennies! He buys milk for $1.83. Can we help him get rid of as many coins as possible (the most coins). Is that the only solution?

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## 7.2 Make up Your Own Problem

Perhaps as a result of years of experience in traditional math classrooms, most students respond like *automaths*, even when faced with the invitation to make up their own math problems (Baruk, 1985). In these 'free productions,' students are most likely to produce narratives which have what they perceive are the surface characteristics of word problems,[10] than they are to follow the (implicit) demand that the invented scenarios make sense. In other words, mimicking the problems they are routinely given, students will *reproduce* dressed-up bare computations, pseudorealistic contexts, or nonsensical scenarios, rather than *construct* truly problematic situations (Lave 1992).

For example, in response to my invitation to invent a two-step word problem with $3.75 as the answer, fifth grader

---

[10]For Anabel a word problem is "a thing that you have to think and resolve it" (Anabel). For her classmates, word problems are "about math," "they have a lot of words," " they come with big or small numbers," "they have math signals," "are like plus, take away, multiply, divide, or fractions." A word problem "is like a sum or something you subtract and you get your answer," "a question about math," "a paragraph with a question," "something to find out." Word problems have "clues," "key words," "words that are hard to pronounce." Or, as Jose warned us, "they have a lot of words to trick you." (Responses of fifth graders, from the low ability class to *What is a word problem? How would you explain what a word problem is to somebody who came form another planet and never went to school?*). Most word problems are about "math," "apples," "money," "people," "pizzas;" they are "about putting things together," "like how many altogether or how many do you have left," "about fractions and divisions," "like giving somebody something, "about how do you know how to explain your answer," "explaining how you got it." Or, as Nancy--Anabel's best friend--explains, "it's like when the teacher writes down something on the board and you have to read it and do the math." (Responses of the same students to *What are most word problems about?*)

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Eric, from the high ability class, wrote

> La senora Ramona compra una camisa que cuesta 5.00 y ella paga 8.75. cuanto le dan de cambio? 3.75 (Ramona bought a shirt that cost 5.00 and she paid 8.75. How much change did she get?

Given a previous conversation with Byron,[11] I decided to let go of the fact that 'Ramona's problem' could be solved in one step. Instead, I wondered aloud on the reason why she would give $8.75 in payment for a $5.00 item. As if embarassed by the nonsensical situation he created, Eric scribbled quickly a subtraction on his notebook, and changed the numbers. The shirt's price was now $6.25 and Ramona payed with a $10 bill.

When they do not act like automaths, children play their own tricks back to us, as illustrated by the responses of four of Eric's classmates to the same task. Particularly concerned with delivering *two-step* problems, Dawn, Kelly, Michael, and Alex decided that bare sums would do:

Dawn:

```
+ $1.75    + $1.75
  $0.00      $2.00
------     -----
 $1.75      $3.75
```

Kelly:

```
+  1.39
   2.36
   -----
   3.75
-  0
   ----
   3.75
```

Michael

```
25¢
x17
-----
$3.75
```

3.75 ÷ 1 = 3.75

Alex:

```
2.00
+ 1.75
----
```

[11]Chapter Six, Section 6.4.

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```
$3.75
   x1
----
$3.75
```

We may not be able to assess whether Eric, Dawn, Kelly, Alex, and Michael--or any of their classmates--are unable or unwilling to invent a *reasonable two*-step *word* problem with $3.75 as the answer. Yet, what these performances suggest is not only that these students have learned the properties of zero in addition and subtraction, and of 1 in multiplication and division, but that they are skillful players of the make-up-your-own-problem-language game.

Invitations to invent 'difficult' or 'fun' problems may give us indirect access to how word and story problems interpellate the subjects supposed to solve them (Ellerton 1986). The following narratives, made up by 5th graders, from the high ability class, almost speak for themselves.[12]

> *Question: Give me an example of a <u>difficult</u> word problem. Why is is difficult?*
>
> With <u>all</u> the math signals in it (Taurice, 30).
>
> Lorena had twelve coins and she wants to spend it so she lend 10 coins to Molly and she stayed with two coins so don't understand can't think (Yerina, 30).
>
> Framme had three quarts of soda he bought two more quarts of soda. Will it become a litter? It's hard cause I don't know whats quarts (Alexis, 52).
>
> Like saying John had 23 bottoms that he gave two away then if he squares the number it's going to

[12]I have listed the problems in ascending order, according to the students' scores on the CAT-5, given in 1994. The two questions are part of a written questionnaire on word problems (see Appendix).

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take a while to guess (Michael, 52).

There were 72 frogs in a pond the 9 frogs in a group left how many groups of frogs were left? There are 8 groups of frogs (Maria, 66)

If a boy had 2 seconds to answer this question would he make it? 22 x 66 + 33 - 42 - 2 + 7 + 31 + 6 = (Dianna, 69).

Max folded a piece of paper into 12 equal parts he used 1/3 and cut off 5 pieces how many were left. It does not tell me exactly what to do (David, 69).

Rosa had 14 roses Carolina had the triple that Rosa had. rosa *desided* to buy the same amount that Carolina had. how many roses will rosa have after she buys the same amount Carolina has? It is hard because you have to do two steps (Lorena, 87).

You did not have enough information so now you have to look for clues (Mary, 94).

*Question: Give me an example of an <u>fun</u> word problem. Why is it easy?*

Jorge has 6 Supernintendo games he gave 3 to Alex how many did he have left? It's fun because talk about games and that's what kid's are into this year (Alex, 41).

If Jason likes football, soccer, and basketball. what do you like? It is fun because you get to pick you own sports (Christopher, 48).

Change gold to mold
gold
---
---
---
mold
It's fun because I like these kinds of problems (David, 69).

If Chris likes chocolate cookies, oreos and dorito. Which one would you pick? It's fun because you get to decide (Jason, 82).

I don't know one (Kelly, 85).

Because it has pictures and exciting questions

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(Mary, 94).

> Anna ran 25 miles Yessica ran 37 miles and Mary ran 39 miles. Draw on a graph how each person ran. People like to draw (Brenda, 96).

Sometimes children do get carried away by the fantasy of telling their own story problem. We may thus discover that for some of them a 'story problem' may be a story without a problem, that is, one where there is no question, or where the question is accompanied by its answer. For example:

> David went to a computer store to buy things for his computer. He saw a printer. The sale price is $272.95 and a disk for $19.50. A quiet (?) for $259.50. The person said plus tax at $13.00. Will you buy it? Ok, I will take it. Have a nice day, sir. (Carlos, 5th grade, low ability class)

Finally, we may witness children produce, almost as a revenge, non-sense scenarios such as

> There was 9 bags in a tree. 4 fall down. How many are left? 9 - 4 = 5 4 - 9 = 5 5 - 9 = 4 5 - 4 = 9. (Maria)

> I put 4 fingers up from my hand. I put 11 more fingers up. How many fingers are up? 15 (Joseph).[13]

Note how Maria put more effort in reproducing, albeit mistakenly, a common theme in arithmetic teaching--i.e. family facts: 4+5=9, 5+4=9, 9-5=4, and 9-4=5--than in creating a meaningful scenario. Finally, Joseph's treatment

---

[13]I have given children a series of cards to choose from, and asked them to make up a story problem using what was written on that card. Maria chose '9 bags' and Joseph chose '4 fingers.' Sometimes this activity tells us more than we may be prepared to handle--as Diana's problem 'about apples': "I like apples. My mother does not give me any apples. How many apples my mother does not give me?"

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of fingers may be read as a quite revealing testimony of the difficulties entailed by finger-counting when numbers surpass ten.

## 7.3 Funny Jobs

When children are invited to read a math problem *as if it were a realistic scenario,* whether or not they accept this invitation may affect their chances of solving it. If they accept it, this will bring into play the relationship between the context of the problem and their knowledge about that context. But, in all cases, the overall context in which the practice of problem solving is embedded--i.e. the classroom--demands, albeit implicitly, that students handle math problems *as school problems*, that is, as puzzles, rather than dilemmas.[14]

Together with knowledge about currency and prices, children bring to school their own ideas about jobs--e.g. how many days a week people usually work; how much money a person earns in a week, or in a month; and the amount and frequency of wage raises. The following event, which happened in Miss R.'s class by the end of October, developed around the following 'problem of the day'[15]:

---

[14]Lave 1992. In other words, 'doing school' entails the adoption of a theoretical rather than a pragmatic attitude (Luria 1976). See, also, Scribner (1985) on arithmetic problems as a specialized genre.

[15]Researchers found that a pedagogical technique employed by teachers in Japan and in China to provide more "coherence across the events that constitute a mathematics class," is the use of only one, two, or at most three problems for an entire 40 minute class (Stigler and Perry

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> Suppose you had 50c on Monday night. You took a job for a week that doubled your money each day. For example Tuesday night you had $1.00. How much money did you have the next Monday night?[16]

As Miss R. later explained to me, this problem would give students the opportunity "to practice doubling, organize data in a double-entry chart, and apply this to money."

The problem is obviously unrealistic. Wage increases do not occur on a daily basis, nor is it likely that these will be of 100 percent. To operate within the constraints of this problem, students had to bracket their knowledge about jobs and wages. Furthermore, since the problem is ambiguous, unless a decision is collectively made about the length of the week, it is likely that some students may exclude the weekend.[17]

After posting the problem on the board, Miss R. underlined 'doubled,' and gave the students ten minutes to work individually. A conversation evolved afterwards, during which she wrote down in a double entry chart the students'

---

1988). The NCTM standards recommends this practice for American math classrooms.
In general, problems 'of the day' do not come from math textbooks. These problems are made available to teachers in school-based staff development or in district-wide math liaison meetings. Less frequently, teachers find them in specialized journals (e.g. *Arithmetic Teacher*) or, even less frequently, they themselves make them up.

[16]This problem was included in a booklet prepared by the district's math curriculum office.

[17]Consider, for example, the following problem included in the *Treviso Arithmetic* (1478): 'If 17 men build 2 houses in 9 days, how many days will it take 20 men to build 5 houses?' Roger Saljo (1991) notes that the correct solution of this problem necessitates the suspension of common sense. Arguments "rooted in an external reality, in which people do not work for 25 h. a day can--and have to--be temporarily disregarded; if they are not, the problem becomes difficult to handle" (p. 263). See, also, Saljo and Wyndhamn 1988, 1993.

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suggestions and her own corrections.

> Miss R.: Could someone give me some ideas on how are we going to solve this? Who can tell me what's the first thing we have to do? Gianny.
>
> Gianny: You have to put Monday fifty cents, then you have to add fifty cents for each day, Tuesday, then Wednesday...
>
> Miss R *(while she writes down what Gianny dictates to her)*: One moment, for Tuesday what you did was 50c plus 50c equals one dollar?
>
> All: One dollar.
>
> Gianny: Then Wednesday it's fifty cents plus fifty cents plus fifty cents...
>
> Miss R.: One moment. The problem, there's a word that I underlined in the problem. What is it?

The teacher's comment indicates that she had diagnosed Gianny's error as resulting from a careless reading of the problem. When she pointed the attention to the highlighted word, the chorus-like response was immediate: "Double."

> Miss R.: Double what?
>
> All: The money.
>
> Miss R.: Double each day. Double fifty, two fifties is...
>
> All: A dollar!

It looked as if the students would be able to follow the doubling rule step by step towards the completion of the week. But, to Miss R.'s question "What am I doing for Wednesday?, Gianny responded: "Adding fifty cents." Luckily, there was Eric, who corrected her with "Double one is two."

> Miss R.: Two dollars. Why Eric?
>
> Eric: Cos it says double.

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All: Oh!

Gianny: Oh, you have to double the dollar?

All: Oh!

Such a generalized surprise could have been interpreted as a warning sign. Since Miss R. had just emphasized the word 'double,' they knew *that they were supposed to double*. Moreover, after almost two months of daily drilling in mental math, it was reasonable to expect that they would know *how to double*. What was the source of this surprise? No time to consider this question. Once the 'keep doubling' rule was made explicit, it was necessary to move on.

Miss R.: Right. Then the next day I have one dollar plus one dollar.

Wilfred: Two dollars.

Miss R.: Two dollars. How about Thursday?

Eric: Four dollars.

Miss R.: How did you get that?

All: Miss, miss!

Miss R. *(ignoring the hands up and the screams)*: Every day... the following day you double it, and so on... I don't want any sound effects. I'll wait.

Noticing an increase in the level of noise, the teacher walked towards the light switch. Since this classroom management technique had long been established as a routine practice, the noise diminished before she turned off the lights.

Miss R.: Tina, what do we do next?

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Tina: Miss, I don't know...

Miss R.: If you were listening to what we are doing on each step you would know what comes next.

Tina *(pointing to Lakeysha)*: Miss, she was talking to me.

Miss R.: Then you ignore her. You don't blame that on her. If you don't pass the test what are you gonna say, Lakeysha was talking to me, that's why I didn't pass the test? Luis.

Not-knowing-what-to-do-next may result either from lack of attention, or from difficulties interpreting the 'and so on' clause (or from both). Since the former appeared to be the case, Tina was reprimanded, and the teacher moved on. After indicating that each 'step' would lead naturally to the following one, Miss R. was searching for attentive rule-followers.

Luis: Six dollars?

Miss R.: How did you get six dollars?

Luis: Because you've gotta double four dollars.

Luis' error--four *plus* two instead of four *times* two--is a typical one. In a math challenge, this would have gotten him in trouble with his classmates. In the matter concerning us, it does not inform us about his interpretation of the problem.

Miss R.: Double four is eight.

Wilfred: Eight dollars.

After Miss R. and Wilfred jointly corrected Luis' error, it appeared as if now students were on the right track.

Jorge: Two times eight is sixteen. Monday is

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sixteen dollars.

For Jorge, whose father works five days a week parking cars, it made sense to jump from Friday to Monday. Yet, in her eagerness to move ahead, the teacher missed the opportunity to bring to the surface the mismatch between the problem's constraints and the students' expectations.

> Miss R.: Wait a minute. How many days do we have to go to?
>
> All: Until Monday!
>
> Miss R.: To Monday night. Saturday, now Tina, what do we have to do?
>
> Tina: You have to add four.
>
> Miss R. *(impatiently)*: Not four!
>
> All: Eight.
>
> Lakeysha: You have to do eight plus eight.
>
> Miss R.: Anthony.
>
> Anthony: You gotta double the eight two times, and that equals sixteen dollars.

Visibly satisfied with Anthony's response, Miss R. wrote it down and, while adding 'Sunday' to the chart, called upon Yanique: "Sunday, Yanique?" Silence. The teacher continued:

> I don't understand, we've been doing it together since Monday. It is Sunday, you should know what I'm doing, Jackie.

Laughs. Perhaps too concerned with students' inability to follow the rule, Miss R. did not notice what was funny in her statement. Before Jackie had a chance to respond, Yanique was ready to give her answer: "You put sixteen plus sixteen."

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Miss R.: How much is that?

Richard: Thirty two dollars.

Eric: You have to do it one more time.

Although the students appeared to be moving in the right direction, Miss R. probably doubted whether they had really understood what had been accomplished thus far, or whether they were following her indications blindly. She thus pointed their attention again to the problem's formulation.[18]

---

[18]That some students had difficulties with the vocabulary was demonstrated a week later, during a conversation with Lakeysha.
L *(after reading silently the problem)*: Monday night you took a job for a week that doubles your money. So Tuesday is another dollar, because you have 50 cents. Wednesday is another dollar, and like that, and then you add at the end.
BZ: Why did you add at the end?
L: You have to add it up and see how much you get in one week.
BZ: What did you do with all this money? Did you save it or spend it?
L: I spent it.
BZ: So, if you spent it, how come you're adding at the end?
L: I mean, I'm saving it, cos before she has 50 cents, but she didn't spend the 50 cents, she kept the 50 cents. So that tells you right there that if you don't spend it, she should get the right amount!
BZ: Ok, and what about this word here: 'doubled'?
L: It's like this: you get 50 cents for a job, and the next day you get 50 more cents, because they double your money.
BZ: So, how much do you get the next day?
L: Fifty more cents because they doubled your money.
BZ: And the next day?
L: You have a dollar.
BZ: You don't double it again?
L: No, they already doubled the fifty cents.
BZ: So, you don't have to double the dollar?
L: Oh! Yeah! Double one dollar is two dollars, I remember.
BZ: And then?
L: Three.
BZ: Double two is three?
L: No, four!
BZ: And the next day?
L: Eight and then sixteen, and then...
BZ: Remember how the teacher did it?
L: Yes, like she kept doubling it.
BZ: Show me.
Lakeysha writes:

| Day | Money |
|---|---|
| M | 1.00 |
| W | 2.00 |

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Miss R.: Hillary, why are we going up to this Monday?

Hillary: I know, is that... it says at the beginning suppose you have fifty cents on Monday night, and then it goes... at the end it says, how much money did you have the next Monday.

All: Oh!

Miss R.: Very good, so it's not one week later, it's next Monday night, that means she works...

Hillary: All days.

Miss R.: One week.

Atiya: *It's a very tricky one!*

Math lab students like Atiya are constantly made aware of the possible presence of tricks in word problems. Atiya may have found it tricky that the problem did not say that you had to include the weekend.[19]

As we recall, the new new math has declared a moratorium on tricky problems. Thus, if a 'problem of the day' feels tricky to the students, this may originate from the fact that, as opposed to the puzzle-like character of the 'old' word problems, these problems, like most problems in real life, are non-routine, open-ended, and dilemma-like

---

T 4.00
F 8.00
S 16.00
S 32.00
BZ: What happened on Tuesday?
L *(pointing to the problem)*: It says 'except Tuesday.'
BZ: Let's see. Read it again.
L *(reading it again)*: Oh! 'For example'!

[19]Her solution read as follows: Monday 50c, Tuesday $1.00, Wednesday $1.50, Thursday $2.00, Friday $2.50, Monday $3.00. Answer: $3.00.

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situations, which are meant to function as multivocal texts;[20] that is, these texts are susceptible of multiple interpretations, solution strategies, and answers. However, these problems *become tricks* when, in the context of a given classroom and under the constraints which may operate on this context, they are treated as closed texts.

The teacher moved on.

Miss R.: What do we do here? Gianny.

Gianny: You add thirty two plus thirty two equals sixty four and that's the answer.

Miss R.: Excellent.

Hillary: It was hard!

Miss R.: Not hard, but you have to think about it. How many of you got close to this?

Miss R.: When I asked the question I expected to see your hands. I don't expect to hear: 'oh, I did it, I did this, I did that.' No. I just wanna see hands, that's all. Would anyone like to explain to me how they try to do it? Jahmar.

Jahmar: I thought I had to do fifty plus fifty plus fifty plus fifty plus fifty plus fifty...

Miss R.: Ok, what part were you missing there?

Jahmar: To double it.

Miss R.: Did anyone try another way to do this? Michael.

Michael: I put Monday fifty cents, Tuesday one dollar, Wednesday a dollar fifty, Thursday two dollars like that, and then I added.

---

[20]Lotman (1988) discusses the "functional dualism of texts" in the cultural system: whereas *univocal* texts fuction as unambiguous texts and lend themselves to only one interpretation (e.g. artificial languages), *multivocal* texts function as ambiguous texts and may be interpreted in many ways (e.g. poetry). See, also Wersch 1991.

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Miss R.: Ok, you added because it says each time, no?
Byron, what were you doing?

Byron: I was doing the same as you. Monday fifty cents, Tuesday one dollar, Wednesday two dollars, Thursday four dollars, Friday eight dollars, Saturday sixteen dollars, Sunday thirty two and Monday sixty four dollars.

Miss R.: So you did it like this?

Byron: Yes.

Miss R. *(barely able to hide her surprise)*: You did this on your own, before we went over it?

All: No!

Richard: Miss, he got eight fifty.

Byron: I don't got eight fifty.

Miss R.: All right, we are going to do mental math and then we are going to gym.

I propose that children's difficulties in solving this problem had more to do with their assumptions about jobs, wages, and increases, than with the arithmetical operations they were asked to perform. As in similar episodes, children were subjected to a double (and often dilemmatic) solicitation: make believe that problem solving in school is like problem solving in real life, but accept the constraints of the problem, even if these conflict with your expectations.

As I walked around the room looking into students' notebooks, I noticed that none of them had obtained $64. The solutions produced fell into the following eleven categories:

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I) 50, 1.00, 1.50, 2.00, 2.50, 3.00. Answer: $3.00
(Atiya)

II) 50, 50, 50, 50, 50, 50, 50. Answer: $3.50
(6 students, including Gianny)

III) 50, 1.00, 1.50, 2.00, 2.50. 3.00, 3.50, 4.00. A: $4.00
(4 students, including Rafael and Josaphat)

IV) 50, 1.00, 2.00, 3.00, 4.00, 5.00, 6.00, 7.00. A: $7.00
(4 students)

V) 50, 1.00, 1.00, 1.00, 1.00, 1.00, 1,00, 1.00, 1,00. A: $8.50
(3 students, including Byron)

VI) 50, 1.00, 2.00, 4.00, 6.00, 8.00, 10.00. A.: $10.00
(3 students)

VII) 50 x 7 = 3.50, 10.00 + 3.50 = 13.50. A.: $13.50
(Hillary)

VIII) 50, 1.00, 1.50, 2.00, 2.50, 3.00, 3,50. A.: $14.00
(3 students, including Lakeysha)

IX) 50, 1.00, 2.00, 4.00, 8.00. 16.00. A.: $16.00
(4 students, including Jorge and Jonathan)

X) 50, 1.00, 2.00, 4.00, 8.00, 16.00, 32.00. A.: $32.50
(5 students, including Wilfred, Odette, Eric, and Felicia)

XI) Blank (2 students)

More than 60 percent of the students obtained an answer which was equal to or less than $10.00 (21 out of 34 students), and only 38 percent obtained an answer which was higher than $10.00 (13 out of 34 students). This distribution, which is skewed towards small numbers, indicates that most students positioned themselves vis a vis this problem as if it were a real life scenario and, therefore, the situation depicted acted as an obstacle.

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This assertion may be further supported with the responses given a week later by the same group of students to 'My Great-Grandfather,' a problem with the same structure, but embedded in a context which, although removed from their experience, was certainly more imaginable.

> If my great-grandfather had put $10 in a savings bank in 1900, and if the amount of money doubled every eight years, how much money would he have in 1980?[21]

Only 14 out of 37 students attempted to solve this problem. Of these, only Felicia and Byron arrived at the correct answer. Felicia's solution read as follows: "10, 20, 40, 80, 160, 320, 640, 1280, 2560, 5120, 10,240. Ans: $10,240."[22] Byron wrote (column-wise) "1,280 + 1,280 = 2,560, 2,560 + 2,560 = 5,120, 5,120 + 5,120 = $10,240."

None of the students seemed troubled by the fact that, as they performed the successive doubling, the money kept growing and growing. For example, Wilfred did (vertically) "10 x 80=800, 800 x 4= 6,400, 6,400 + 6,400= 12,800;" Agustin wrote (also column-wise): "80, 160, 220, 440, 880, 980, 1020, 2040, 4080. Answer: 4080;" and Hillary put "1,000, 10,000, 100,000, 1,000,000, 10,000,000..."

---

[21]The problem (Dale Seymour 1980) included a picture of an old-looking, bearded many carrying a cart full of money bags. As for the setting of this problem solving activity, I had 'borrowed' Miss R'.s class for a 15-minute period. Students were given the problem on a written sheet, and where told to work individually and include their scrap work on that sheet.

[22]Felicia's first attempt got her from 10 to 657130. She had gone well past the ten eight-year periods. As she realized it, she did it again, and obtained $10,240.

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(unfinished). It appeared perfectly imaginable for these students that, when left for eighty years in a bank, $10.00 would soon turn into a fortune!

Of course, in the 'Week's Job' problem, the obstacle could have been surmounted as part of the whole-class discussion. Yet, for that to happen, the teacher herself would have to have been aware of the, so to speak, problematic nature of this problem.[23] As the transcript indicates, in her eagerness to get through the activity, Miss R. did not properly identify the reasons behind the students' errors. Instead, she attributed them to a deficient knowledge of 'math facts,' misunderstanding of the problem's formulation, difficulties in implementing problem-solving strategies, lack of attention to the task-at-hand, or some mysterious confusion.

It may be argued that the problem was too problematic, that the teacher did know how to handle it, or that the children did not engage in the activity as skillful problem solvers. In line with my overall argument, I suggest instead that the conditions under which this problem was read, interpreted, discussed, and solved, that is, the didactic contract operating in this classroom did not allow it to become truly problematic.

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[23]She was not, as I was able to verify days later, when I shared with her my impressions about the students' solutions.

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7.4 Short Cuts

BZ: Why do you have to solve so many word problems in school?

Rafael: So that when you go to the store, and if you buy like ten books, if you don't know how to count money like that, you cannot given them money.

BZ: And what about people who didn't go to school?

Rafael: I don't know.

BZ: They can still go to the store and buy stuff, can't they?

Wilfred: They stay dumb.

Byron: They don't stay dumb. They gonna have to count.

Wilfred: They count with their fingers.

Byron: If you have pennies, they gonna have to count all day, like one, two, three.... The lady behind them is gonna go: C'mon man! It's better to know, so you can do it fast.

BZ: When you go to the store, do you use what you learn in school?

Wilfred: Yeah. I count it for my mother, because sometimes she has a little bit of trouble. But I say, mom, it's eighty four cents. *Like if it's two ninety five and she paid four dollars and she wants to know how much she's gonna get back before the guy gives it to her, so I do it faster, cos I go all the way up to three, three dollars and three dollars all the way up to four dollars, is one dollar and five cents (writes down $1.05)*

BZ: Is that the way you do it in school? Because I've seen you all do it like this:

```
4.00
2.95
----
1.05
```

Wilfred: Yeah! *They* do that. I do it here *(points to his head)*.

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Byron: I do it the fast way.

BZ: And in school, how do you do it?

Wilfred: The slow way. You have to write down everything.

Byron: Cos you gotta show the steps.

As they become mathematically literate, children learn that there are 'short' and 'long' ways of performing arithmetical operations and solving word problems. Children also learn that, in school, these two ways of doing are not equally legitimate and that, for reasons which they may not fully grasp, the long way tends to be higher in the hierarchy. Eventually, they come to realize that the commonsensical rule 'If there is a short way and a long way to do something, choose the former,' does not work inside the classroom.[24]

As a genre, word problems have the effect of suspending common sense, even when they are situated in familiar, real life contexts. Like Eric's problem about Ramona's shirt, Wilfred's 'shopping with my mother' scenario, presents a relationship between the price and the amount paid which can hardly occur in real life.[25] Wilfred's solution to his own problem must be read as symptomatic. In his very attempt at

---

[24]School mathematics and common sense have long been divorced (Baruk 1985, Schoenfeld 1985, Davis 1989). That this has been recognized by the international mathematics education community as a problem worth discussing may be demonstrated by the topic chosen for the 47th CIEAEM Conference, held in Berlin in 1995: "Mathematics Education and Common Sense" (Keitel *et al.* 1996).

[25]Paying $8.75 for a shirt that costs $5.00 is as nonsensical as paying with $4.00 something that costs $2.95.

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illustrating the relevance of school math for everyday life, he exposes the contradiction between these two practices. Although the school-taught subtraction algorithm is slower than the 'adding up' mental strategy he uses outside--2.95 up to 3.00 is 5 cents, and one more dollar to get to 4.00; so it's $1.05--everybody knows that those who don't go to school 'stay dumb' and are doomed to 'count with their fingers.'

Whether or not they are aware of this contradiction, most students learn to cope with it. Although short cuts allow you to proceed in a simpler and more efficient fashion--and may even be more fun!--it is only by 'doing it the long way' that you can show all your 'steps.' The discourses of regulation and evaluation dominate over both common sense and mathematics.[26]

As I discussed in Chapter Five, new new math reformers recommend the constant monitoring of students' math performances. This translates into the merging of the instructional and the evaluative discourses. Thus, while in daily math lessons, teachers are to assess their students'

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[26]Yet, most of the beauty and elegance of mathematical procedures, notations, and solutions derives from the efficient use of curtailing methods or 'short-cuts.' Aware of the need for teachers and students to establish explicit agreements as to what may count as an 'acceptable,' a 'different,' an 'efficient,' and a 'sophisticated' solution, social constructivists propose, as a *sine qua non* step in the establishment of a new didactic contract, the joint negotiation of these 'sociomathematical norms,' together with the classroom's social norms (Cobb, Yackel, and Wood 1992; Lo and Wheatley 1994; Cobb and Bauersfeld 1995; McClain *et al.* 1995; Yackel and Cobb 1996).

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knowledge with questions such as 'How did you do it?,' in written tasks, students are asked to explain the step-by-step procedures which led them from questions to answers. These oral and written accounts are valued as 'windows' to students' reasoning processes and problem-solving abilities.[27] More specifically, these narratives are read as evidence as to whether students *really* understand mathematics or they blindly follow its rules.

As we also recall, in the New York City school system, this new assessment contract materialized in two examinations: the CAT-5 and the PAM. The latter consists of three open-ended problems to be solved in forty five minutes, all of which include the command "Show or explain the steps you took to solve this problem. Label your work. Be as complete as you can." Time-restricted, multiple-choice tests such as the CAT encourage 'short' ways of doing math--e.g. using the 'key word method,' guessing, rounding, employing short cuts, and discarding unlikely choices. Instead, the PAM invites 'long' ways of doing math such as listing and explaining each step, labeling all the work, and making drawings, charts, and graphs.

To prepare students for these two performances, which take place during the same week in April, teachers instruct students both in the long and the short ways of doing math.

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[27]This includes, not only performance assessments, but also more unstructured ways of collecting evidence of students' understanding such as 'journals,' 'portfolios,' and 'daily logs' (NCTM 1993).

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In schools with low-scoring students such as PS XYZ, the test preparation period may start as early as in November. The expectation is that students will accommodate to the constraints and demands of each examination. They will realize that there is no point in showing work in the CAT, since no one will read the yellowish scrap paper they may have used to figure out whether the answer was A, B, C, D, or none of these. And they will know that if they do not show their work on the PAM, they may get only 2 out of 6 points, even if they arrived at the correct answer.

That these two rehearsals occur simultaneously may create confusion as to which modality of work they are to choose for each occasion. For, once the common sense rule has been discredited--i.e. the short way is not always the best way--students are left with no obvious criteria according to which one is preferable to the other. In this, as well as in other school matters, the safest move is always to 'follow directions.' The following two events--from the fourth grade, high ability class-- illustrate some of the difficulties students face as they respond to the 'show your work' solicitation.

It was the beginning of March. Miss R. and her thirty seven students were reviewing long division. As part of the daily routine, they were checking the homework from the previous day. Assigned was a workbook page entitled "Two-Digit Quotients," with fifteen division problems (e.g. 128

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divided by 9, 44 divided by 4, and 487 divided by 9).[28]

> Miss R.: Workbook page eighty two. We're only going to go over the odd numbers. Livia, number one, Michelle, number three, Marcos, number five. Have your workbook out.
>
> Kayser: Miss, I showed work!
>
> Raymond: Miss, my mother helped me!
>
> Marcos: I was absent.
>
> Odette: I was absent, but I did it anyway cos I called Gianny...
>
> Miss R. *(while she walks around the room, looking at the children's notebooks)*: This is long division. *I can't do it in my head, so you can't possibly do it...* Zero, if you didn't do it the long way! You need to show your work... If you did not show the work, how did you do it? I'm really curious.
>
> Richard: My mother helped me.
>
> Eliza: I showed the work, but I lost the paper.
>
> Miss R.: Not a reason.
>
> Tina: I did it in my head.
>
> Miss R.: That's not a way to do it.
>
> Lakeysha: My mother, she told me to keep the paper, but then I lost it.
>
> Miss R.: *You cannot do long division in your head.* All of you who got a zero will do it tomorrow showing work... We spent about five class periods doing it. I taught you the short way and the estimating way... It's your part to study it.

As she saw it, the teacher's responsibility was to show students the 'short' way as well as the 'estimating' way of dividing. Apart from studying it, students' responsibility

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[28]*Practice and Assessment Skill Pad*, Level 4, D. C. Heath.

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was, as always, to follow directions.[29] If the teacher says you must show work, you must do so, even if it was not *your* work--your mother helped you--or if there was no work--you did it in your head--or if you lost your paper--you did not think that your work was worth showing, anyway.

It must be noted that Miss R. did not trust that the students could have done these divisions 'in their head,' for as she put it, she herself could not do it. The pedagogical reason is permeated by the logic of suspicion: rather than indicative of students' mental calculation strategies, the absence of written traces means calculators, parental help, or both.

This event suggests that since the new assessment contract has not been understood, negotiated, and agreed upon by the teacher and the students, the 'show work' solicitation has a distorting effect on the didactic contract. That students are being subjected to a rule the reasons behind which they do not see, results in a series of

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[29]For some of Miss R.'s students, 'following directions,' showing work,' or 'looking over' are not transparent instructions. Consider the following exchange, which had occurred a few weeks earlier:
William: Miss, what does 'write directions' mean?
Miss R.: Directions means that in every different part they'll say write the amount of money or write the pattern... write the amount using the dollar sign and the decimal point, I want you to write each part... Put your heads down. Write the question means, if for example the first one is $18 and 2 quarters, I want you to write this part of it, I want you to write what is it that you have to do. Then, next to it, write the amount of money.
William: What is 'look over'?
Miss R.: Look it over. If you see that you've done it wrong, you do it over, if you see that you've done it right, you look it over. For each part you have to do something else. William, I'm talking to you! Write the money or write the pattern, I want you to write that. No, that's the directions, the questions are...

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perverse effects: rather than showing work, they may show that they do not know what 'show work' means; even if they know what show work means, they may resist showing it; or they may show, not the work that they have actually done, but an *ex post facto*, reconstructed, and idealized version, which includes only what they believe is worth showing. Sooner or later, most children end up accepting that one must show work to show that one knows how to show work, that is to say, that one knows how to 'follow directions.'

One would expect that after many experiences of this sort, Miss R.'s students would have learned what show work means as well as when and how to do it. Yet, two months later, this aspect of the contract has to be renegotiated. In this episode, the teacher send almost her entire class to 'redo' the homework, because they had not done it in the way she specified.

> Miss R. *(walking around the room)*: Your homework should be on your desk. Let's go over it, please. Very nice... You didn't show your work... That's an incomplete. Another one that does not show work! That's not showing work. I need to see how you did it. Excuse me! Luis. Josaphat, let me see yours. Another one that does not show work. Out of thirty seven students, out of thirty seven students, two handed me the homework the way I wanted it to be done, two! I'm gonna explain it. Jackie, put your pencil down.
>
> Tina: What does it mean to show work?
>
> Byron *(almost crying)*: *You told me to show work but you didn't tell me how!*
>
> Miss R.: Oh, Byron was the only one, poor Byron, I told the rest of the class, and I did not tell Byron!

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Byron *(inaudible)*: ...

Miss R.: Excuse me, excuse me Byron, I didn't hear you.

Byron: You told us show work but you didn't say how.

> Miss R.: Ok, I told you, I pointed it on the board, how to show work. I did about ten problems yesterday... You have your pen in your hand. Put it down, cause I'm gonna explain it one more time. Math textbook, page twenty three, numbers fourteen to twenty five.[30] I see papers with forty three thousand, forty five thousand... I don't know what page, I don't know what number, I don't know if *you* did it, I don't know *how* you did it. This is not acceptable homework. As a matter of fact, all of you, except for Ana and Odette will be redoing the homework for me. I'm gonna show you again how we round, what it means, because to some of you... the short cut, doesn't make sense here, because you don't know what the short cut means.

The homework consisted of twelve multi-digit numbers to be rounded. Most of the students did the twelve exercises correctly. Yet, Miss R. realized that they had made happy use of the 'short cut' they had learned the previous week:

> Underline the proper place value; then look at the number to the right of it. If this number is five or bigger than five, you round up, and if it's smaller, you round down.

Is it that the students did not understand her request or that they did not bother following it? Recall Tina's question "What does it mean to show work?" and Byron's complaint "You told us show work but you didn't say how!" In any case, it was necessary to re-do.

As a matter of fact, what Miss R. expected the students to show was to apply another rule, one which she had given

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[30]*Heath Mathematics Connections*, Book Four 1992.

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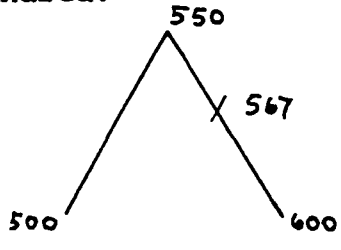
to them the same day she introduced the short cut procedure. What follows is a fragment of that lesson:

> Miss R. *(while she draws)*: Draw a little triangular shape like this, or a pyramid. Now, pretend this is a car going up and down the hill. On both sides of this shape, write down the two rounded numbers in between which lies the number that they gave you. For example, if you are rounding, let's say, five hundred and sixty seven to the nearest hundred, you should put...
>
> All: Five hundred and six hundred.
>
> Miss R.: Very good. Then write down the number which would be *exactly* in the middle of the two rounded numbers. In this case it would be what?
>
> All: Five hundred and fifty.
>
> Miss R.: Yes, you put five hundred and fifty in the middle, in the peak of the hill. And then see on which side of the slope would be the number you're supposed to round. This will tell you whether you should round up or down. Five hundred and fifty seven is on the right side, so you round up, and you get...
>
> All: Six hundred.




When students bring their homework 'redone,' Miss R. would assess their ability to draw twelve hills with three numbers on each, and locate the numbers to be rounded in one or another side of each hill. This written material would not necessarily be a more compelling evidence of their understanding of rounding, than twelve successive applications of the 'short cut' rule. Yet, since this

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procedure was not explicitly taught as a rule, it could be taken as evidence of a deeper understanding.

The lesson continued with a discussion of three more rounding examples. As they rounded the numbers, some students were mutilating them. For example, when rounding 382,693 to the nearest hundred with the short cut method, Hillary turned it into 3,827. And Atiya, who was using the pyramid method to round 382,693 to the nearest thousand, truncated it to 3,000. Miss R. corrected these errors with:

> The six, rounding up becomes a seven. Hillary, everything else becomes a zero. If I'm rounding to the nearest hundredth, my answer must end in hundred. Atiya, everything else stays the same. People, you don't drop the beginning of the number, because it won't be the same number!

Tina was now called upon to check Atiya's work.

Miss R.: Tina, which number is in the thousandth place?

Tina: The two.

Miss R.: What is the six telling you to do, Tina?

Tina: Round up.

Miss R.: So, what does it become?

Tina: It stays the same.

Miss R. *(rather impatiently)*: If you're rounding up, does it stay the same? Rounding up and staying the same is not the same thing. Tina, the two becomes a three. So, what's your answer?

Silence.

Miss R.: Laura, help her out.

Laura: Three hundred and eighty two.

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All: Three hundred and eighty two!

Another mutilation.

Laura: Three hundred and eight three... and zero zero zero.

Visibly satisfied, Miss R. wrote down '383,000' on the board and added "Very good. If you want it to the nearest thousand, I want it to end in thousand. Raise your hand if you understand this concept now." Most hands were up.

Such horrible mutilations could have been avoided by situating these bare rounding exercises in a meaningful context--e.g. money, population, or miles. In the context of these scenarios, it is unlikely that the numbers will be mutilated.[31] Also, the use of pyramids or hills gives the mistaken impression that numbers go *up* to 5, 50, 55, 500, 550, etc. and then go *down*. The placing in a triangular shape of "the two rounded numbers between which lies the number that they gave you," invites students to leave out the first digits of the number in question. If, instead, the numbers are located on an 'empty number line,' it is unlikely that this would happen.[32]

---

[31]For example, no one would round $382,693 into $3,827.

[32]The empty number line model for teaching of elementary school math has been proposed in 1990 by Dutch didacticians Treffers and de Moor (Streefland 1991, Gravemeijer 1994). The term 'empty' (or 'open') alludes to the fact that, in this model, children write down only the numbers they need for their calculations. Classroom research has shown that, while the full number line evokes primitive counting strategies, the empty number line invites curtailing strategies (Gravemeijer, in Streefland 1991). Yet, research has also shown that, perhaps guided by preconceptions about mathematical exactitude, many children (and teachers!) are unwilling to use this model in a flexible manner, e.g. writing the numbers 'somewhere' on the line (Streefland 1991, Gravemeijer 1994a).

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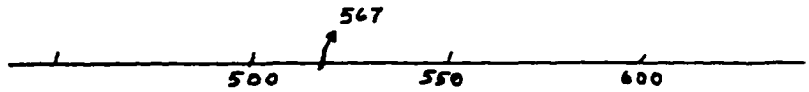


Towards the end of the lesson, Miss R. said: "Does everyone follow? This is what rounding means," to which Richard, impatiently, responded with his own question: "Do we have to write all that?" And Miss R. answered:

> The whole thing. The two thousands, the midway point, where the number falls in relation to the pyramid, and which place value is it closest to.

I suggest that this 'redo' experience confirms the sense that many students have that homework assignments and 're-dos' are a daily punishment rather than a productive aspect of their school experience.[33] Although they may wonder about the point of learning a short cut which they are not allowed to use, students end up accepting this demand. Ultimately, the reason why, in certain cases, it is best to do things 'the fast way,' while in other cases speed and simplicity are to be sacrificed, remains for them a mystery.

---

In an implicit critique of the laissez-faire approach to math teaching which is typical of American child-centered constructivism, Gravemeijer (1994a) notes that the didactical models used by teachers in their lessons ought to have 'level-raising qualities': "It was argued that a new model should leave students the freedom to develop their own solution procedures. However, this argument is not sufficient; there has to be room for improvement. Or better, employing the model should foster the development of more sophisticated strategies" (p. 461).

[33]Responding to my question "What is it that you don't like about school?", Lakeysha and Anabel coincided that what they don't like about school is that they "always have to do everything over and over again and again."

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Perhaps this is a mystery only to researchers. When invited to elaborate on the matter, children provide rather reasonable reasons for the current pressure towards procedural explanations. Consider, for example, the responses fourth and fifth grade students[34] to my questioning on 'show your work.'

> BZ: Why do you think that some math problems come with a phrase at the end that says 'show your work'?
>
> Josaphat (4th grade, HA): Si nada mas pone la respuesta ella va a decir que uno la adivino (If you only put the answer, *she may think that you guessed it*).
>
> BZ: Y si uno no sabe como explicar lo que hizo? (And if you don't know how to explain what you did?)
>
> Josaphat: Then *you get in trouble!*

---

> BZ: What does it mean to 'show work'?
>
> Eliza (4th grade, HA): Show work is.. if there is a problem that is difficult, then you have to show if you can do it... and *they give you extra credit.*
>
> Gianny (4th grade, HA): You have to work it out *cos you may copy it from someone else*, then the teacher knows.

---

> BZ: That business of 'show work' that your teacher is always talking about. Why do you have to show work?
>
> Wilfred (4th grade, HA): So the teacher can think you're smart.

---

[34] 'HA' refers to 'high ability class,' and 'LA' indicates 'low ability class.

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Rafael (4th grade, HA): They wanna make sure you did it all by yourself.

Byron (4th grade, HA): So she can see what you did. If she don't know what you did you can't get nowhere.

BZ: Why do they ask you to show your work?

Marcos (5th grade, LA): *They wanna make sure you didn't copy it from your neighbor.*

Anabel (5th grade, LA): Como *la maestra no nos comprende mucho*, entonces tenemos que escribir primero, segundo, tercero, que fue lo que hicimos. (*Since the teacher does not understand us*, we have to write down what was it that we did, like first, second, and third).

BZ: Y la maestra dice que hay que hacer eso para que? (And does the teacher tell you why you have to show your work?)

Anabel: Para... *porque asi va a estar en el test.* (So that, cos *that's how it's gonna be in the test.)*

## 7.5 Thirty Three Ways of Unlearning Division

In the following event, which took place in the middle of April, Miss C. prepared her thirty three students for the upcoming PAM test. As the lesson began, the problem of the day was posted on the board:

The 'All Year Store' celebrates its anniversary. It has been open for 9497 days. Which anniversary does it celebrate?

Miss C.: We're starting a few minutes late, so we won't do mental math today. I'll give you a problem. This is *individual* work. You do it on your own.... Then, if we have time, we will do another one in groups, ok? You have to copy the problem. Please remember that this is *in-di-vi-dual*... There will be more, if we can get this one

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done, I'll give you more. Just to make sure that you can read it, Lorena, can you read it please?

Lorena *(reading)*: The 'All Year Store' celebrates its ann... anni...

Miss C.: Anniversary.

Lorena: anniversary. It has been open for 9497 days. Which an... ann... anni...Which anniversary does it celebrate?

Miss C.: I want to see your work....and I want you to explain what you do, nothing new. And this is *in-di-vi-dual*... There is some information that we need to have that is not on the problem. I'm not gonna say anything more, cos I want you to do this on your own. *We are pretending that it is the twenty second, and we are having the PAM test.*

Individual work is not part of the routine in Miss C.'s lessons. In line with the NCTM *Standards*, a didactic contract has been established in this class whereby the teacher proposes a problem, students work in pairs or in small groups and outline their solutions on chart-paper, and those groups chosen by the teacher come to the front to share their work. During the ensuing whole-class discussion, students and teacher consider the merits of the different strategies proposed. Since this lesson is intended as a simulacrum of the PAM test, the assessment contract overwrites the didactic contract: the work must be done individually. Yet, that the teacher would help students with probing questions, as well as the ensuing collective debate, distinguish this situation from a test scenario.

The solution of this problem requires an understanding of 'anniversary.' As Miss C. shared with me after class,

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> I chose this one because I thought, anniversary, I thought they will know it in Spanish is *aniversario*. But some of them didn't.

The 'missing' piece of information--the number of days in a year--is what makes the 'All Year Store' an 'open-ended' problem. But this is a dressed up (or pseudorealistic) problem, in that it is unlikely that, in real life, one would know how many days a store was open, without knowing how many years that is.

This problem can be solved directly by dividing the number of days the store has been opened (9497) by the number of days in a year (365), obtaining an answer of 26, that is, the 26th anniversary. The 'remainder' (7 or one week) ought to be ignored--except perhaps for noticing that the celebration must have taken place a week before the store's 'birthday.' What children make of the 'remainder' in a division problem such as this one is usually read as a symptom of their attitude towards the context depicted therein. A response such as '26 remainder 7' suggests that the context has not being taken too seriously.[35]

The students worked for fifteen minutes. Noticing that Annatalia did not to know how to start, Miss C. asked her "What's the information that's missing?" Jorge's immediate reaction: "Wait! Don't say it!" betrays that he is

---

[35]Consider the oft-quoted problems about how many buses are necessary to transport a given number of people, or how many tables are needed to seat a certain number of guests. In these contexts, there cannot be remainder, unless one is willing to leave some people behind and others without seat (Streefland 1991, Gravemeijer 1994).

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approaching the problem as if it were a puzzle.

> Miss C.: I won't say it. You can go on, I'm just trying to help those who want some help. I'm gonna stop. Just for a second. Take it easy! If you don't need this, keep working, pay no attention to me. There is information missing. What is the information that's missing? David.
>
> David: How many days are in a year.
>
> Miss C.: Did that help anybody?
>
> Miss C.: I'm just asking little questions to see what it triggers. Also, don't forget what you're supposed to do with your work. You have to explain.

The teacher kept walking around, looking at children's work.

Five minutes later:

> Miss C.: I know that other people have it, but I cannot have everybody put it on the chart. *(To Daniel, who was showing her his work in progress)*: *Ok, there's nothing wrong with trial and error... especially on the PAM, on the CAT you can't do too much of that.* Ok, now I want you to take five minutes to... mm... talk about it in groups, to share what you've done, what answers you've got, and see if you can come up with new ideas.

Students were now allowed to work in small groups. Five minutes later, the teacher raised the issue of vocabulary.

> Miss C.: Alright, three more minutes. Let's stop one more time. Just a few things that might help you, if you're still stuck. How many of you got stuck because you did not know what 'anniversary' means?
>
> A few hands were up.
>
> Miss C.: I mean, some of us don't know that word... And I can explain it now, on the test I can't. What is an anniversary?
>
> Liz: Something that gets celebrated every year to remind you of what happened on a certain date.
>
> Miss C.: It is a celebration that happens once a

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year, once a year, you needed to know that.

Liz: It's like a birthday.

Miss C.: Yes, exactly, did you hear what Liz just said? Liz, could you explain it to Jessica?

Liz: It's like a birthday!

Miss C.: It's once a year... a birthday is an anniversary of the day we were born. That's right. And you needed to know what an anniversary is... *and you also needed to know... how many days are there in a year.*

All: Yeah!

Miss C: Now I'm going to give you another three minutes and then we're going to give a long explanation on the board.

As I walked around, I saw that each student was doing something different. Peter had just finished the column addition 31 + 28 + 31 + 30 + 31 + 30 + 31 + 31 + 30 + 31 + 30 + 31 = 365. As he explained to me, he would "double 365 until 9497," but "it will take me too long." Louis was "doing trial and error" starting with 365 x 3 and 365 x 4. Daniel started with 365 x 15, 365 x 16, and then jumped to 365 x 30 = 10,950--"It's gotta be less than thirty years," he said. Brenda began with 365 x 23, then 365 x 24, then 365 x 25, and then added 365 more, and got 26. Rafael was trying to divide 9497 by 30: "I'll figure out how many months." "And then?," I asked him. "Then I'll find out how many years that is." "How?" He responded: "I don't know." Elizabeth was attempting 9497 divided by 400. "Why 400?" I asked. "I rounded it up, to make it easier."

A few students had shown their work in written form.

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Taurice's answer read as follows:

> 9497 % 365 = 26
> First step: I multiplied 365 into 9497 and it equalled 26.
> Second step: Then I multiplied 365 times 6 and my answer was 2190.
> Third step: Then my answer was 26.
> by Taurice Ortiz

Vidmarie did 365 x 25, then 365 x 26 = 9490, 9490 + 7 = 9497. Yet, she wrote this description:

> I did trial and error... If I multiply 365 x 27 it will give me too high, so I added one more week that is 7 days. So, it's 26 years and one more week.

Lorena explained:

> 9497 % 345 = 27
> 1) First I divided but when I got my answer I knew that is not the way to solve this problem. 2) Then I found out that a anniversary is 365 days.

In Michael's notebook I read: "12 months x 30= 360," and Alex's page was blank. On the margin of his notebook, Oscar wrote "9497 % 365 = 26, and next to it "Answer: 26 years." A LEP student who had come from the Dominican Republic three years ago, Oscar loves math but, as he told me, he *hates* to show work.

These written accounts are rather revealing. Although Taurice managed to list three steps, his second step was part of the multiplication algorithm, and his third step was the answer. A mad proliferation of steps or a literal interpretation of the 'show your work' command? Vidmarie's narrative is a rather unclear and inaccurate version of her actual procedure. Also, while she solved the problem rather

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skillfully, her answer: "It's 26 years and one more week," suggests a mechanical approach to the problem. In an amusingly self-reflective gesture, Lorena shared her back and fourths in solving this problem due to her ignorance of the 'missing' information. Her explanation is somewhat unclear; yet, I noticed, looking in her notebook, that she had crossed the division of 9497 by 345, which gave her "27 R182," probably realizing then that "it was not the way to solve this problem."

While Lizmarie was busy performing successive subtractions of 365 from 9497, Mary had chosen to do successive additions of 365. (Neither of them went past four iterations). While trying himself to divide 9497 by 360, David told Mary: "That's too long, Mary, too long!," and to Jessica, who had already done 365 x 20, then 365 x 21, and 365 x 22, and seemed like she was about to give up: "Skip two, go to 24. It's above 24!"

Assuming that the division of 9497 by 365 was within the level of this fifth grade, high ability class, I asked Miss C. why only a few students had attempted it. She replied:

> They all know that if they do the division they get the answer right away. But they are trying to find their own way of solving it, so they can come up to the board and show their work to the class. But they also know that the goal is to find the right answer.. I mean, I don't know, I think they know...

"I have to say, though," she continued, "that most of them

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are afraid of division. They try to avoid it as much as they can."[36]

How did Miss C.' students 'avoid' division? The spectrum of their strategies included:

I) successive addition of 365 up to 9497 (e.g. Mary),

II) successive subtraction of 365 from 9497 (e.g. Lizmarie),

III) trial and error multiplication, starting with a one-digit number (e.g. 3, 4, 5) (e.g. Louis),

IV) addition of twelve months to figure out the 'missing' piece of information and then successive doubling of 365 (e.g. Peter),

V) trial and error multiplication, starting with a double digit number (e.g. 15 or a number in the 20s) (e.g. Daniel, Jessica, Brenda).

Other strategies (combining long division with rounding):

VI) dividing first 9497 by 30 (days in a month) and then "I will find out how many years that is" (Rafael), and

VII) first rounding 365 to 400, then dividing 9497 by 400 (Elizabeth).

---

[36]It is a well-known fact among teachers and math education researchers worldwide that long division--together with fractions and other topics--ranks high in the list of dreaded school math topics. In the last decades, it has become increasingly apparent that the source of such fear resides in the 'irrational' ways in which these topics are presented to schoolchildren.
'Mainstream' math educators argue for the need to teach the long division algorithm in a more rational manner, that is, by actually explaining the reasons behind each step. Others propose that teachers provide suitable contexts (or problematic situations) together with the necessary guidance to help children 'reinvent' long division. Still others argue for the need to eliminate conventional algorithms from the school math curriculum, and suggest that teachers let children invent their own ways of dividing, not only because, in this way, their number sense will not be damaged, but also because long division has long been made obsolete by the universal availability of hand-held calculators.

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What follows is the remainder of the lesson:

> Miss C.: Brenda and Oscar, get your posters and come to the board. All right, I want everyone's attention. Oscar is going to come up and explain what he did. *His chart may not be mm... complete, but whatever is not on the chart, you'll explain it*, right Oscar? Everyone else should be sitting. You should be very quiet, so that we can listen to what he did. Listen carefully and watch.
>
> Oscar: *(inaudible)*
>
> Miss C.: Oscar, you ought to speak louder. Some children are still confused, and you have a great explanation there.

On his poster, Oscar had written:

```
9497 ÷  365 = 26
       26
     9497    365        26 anniversary
```

> Miss C.: Don't read it. Just tell me why did you decide to divide.
>
> Oscar: Cos you have to find out how many years are there in 9497 days.
>
> Miss C.: Good. So you knew what anniversary meant... Did everybody see what he did?
>
> All *(chorus)*: Yes!
>
> Miss C.: Thank you. Brenda, could you come up, cos you did it differently. *There are many different methods, you can use any method that you like*. Some are quicker than others. This was a fast way, and it really had only one step (emphasis added).

Brenda came to the board. On her poster, she had written:

```
 365      365       9125
  x24     x25     +  365
----     ----     -----
8760     9125      9490
```

> "First I tried trial and error. I tried 365 x 24 to see which is the closest to 9497. I did this because it says how many days were there. I multiplied 365 because that's how many days were in a year. Then I tried 365 x 25, that was close.

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Then I added another 365... My answer is the 26 anniversary."

Brenda: I first tried trial and error. I multiplied 365 by 23.

Miss C.: Why did you do that?

Brenda: Cos that's how many days are in a year... So I multiplied and it gave me 8395. Then I tried 365 times 24 and it was 8760. It was still smaller. Then I tried 25...

Miss C.: And what did you do at the end?

Brenda: I wanted to make sure that there wasn't another year, so I added 9125 and 365, and there was another year.

Miss C.: Did everybody understand that?

All: Yes!

Miss C. *(looking at her watch)*: So, we don't have time for another one. This afternoon, if we have time, we will do another one.

Although students' strategies spanned various levels of mathematical sophistication, perhaps in her anxiety about their performances in the upcoming tests, Miss C. focused only on those strategies which fitted the modalities demanded by these tests. With her choice of Oscar's long division--a rather bare explanation--and Brenda's 'trial and error' approach--a detailed written account[37]--Miss C. covered, in one stroke, the CAT's and the PAM's modalities of math performance. If the first may lead some children to mechanically reproduce the division algorithm, the second may invite them to 'unlearn' division, to have more work to

[37]Miss C. later confessed to me that she wished to have had more time to go over some of the strategies proposed by the other students.

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show.[38]

Making the link explicit between Oscar's and Brenda's solutions would have opened the possibility for students and teacher to jointly 'reinvent' the long division algorithm.[39] This link could have been established as follows:

| Brenda | | Brenda | | Oscar | |
|---|---|---|---|---|---|
| 365 | | 365 | | 26 | |
| x24 | | x20 | 20 | 365⟌9497 | |
| ---- | | ---- | | 7300 | 20 |
| 1460 | | 7300 | | ---- | |
| 730 | | 730 | | 2197 | |
| ---- | | ---- | | 2190 | 6 |
| 8760 | 24 | 8030 | 2 | ---- | ---- |
| + 365 | | 730 | | 7 | 26 |
| ---- | | ---- | | | |
| 9125 | 25 | 8760 | 2 | | |
| + 365 | | 730 | | | |
| ---- | | ---- | | | |
| 9490 | 26 | 9490 | 2 | | |
| | | | ---- | | |
| | | | 26 | | |

Two constraints militated against this accomplishment: it was necessary to move on rehearsing for the tests and, despite her strong background in mathematics and well-known

---

[38]As she explained to me later, students were doing "CAT training in the morning, and PAM practice in the afternoon." As an outstanding teacher, she tries to anticipate and prevent her students' confusions, errors, and misunderstandings. I imagine that she fears two tragic scenarios. If the students do CAT thinking on the PAM, they will not 'get a six,' even if they arrive at the correct answer. Perhaps even worse, if they approach the CAT in a PAM-like manner, they may not even get half through the test, having, as they do, less than one minute per question. Also, since the CAT is graded by scanners, no one will decipher their scrap notes. An unfortunate paradox of the examination machine, in that this scrap paper may contain more revealing evidence of students' thinking than the *ex post facto*, idealized procedural accounts elicited by the 'show your work' command.

[39]The notion of 'guided reinvention' is central to the realistic approach to mathematics education developed at the Freudenthal Institute in the Netherlands. For a lesson on long division, see Streelfand 1991.

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pedagogical skill,[40] Miss C. lacked the in-depth knowledge of long division which would have been necessary for her to provide the appropriate didactical guidance for the students to reinvent it.

## 7.6 A Pizza Party

Two days later, Miss C. and her students were engaged with another problem of the day:

> For my party, my mother bought 6 pizzas. They were cut into eighths. Twelve of my friends ate 3 slices each. I ate 2. How many slices are left?

After working individually for ten minutes, many students indicated to her that they were ready to share.

> I know that you're racing to get it on the chart. I know that, but this is not a race, and I cannot get to all of you. If you think you've finished and you don't need anything, you know what you should be doing... No more charts, we don't have the time.

Jorge was called to the board first. On his chart paper, he had made six circles divided into eight parts each, and shaded all, except for ten parts. No written explanation was included. He demonstrated how he shaded "the

---

[40]Miss C. was recently given the '1996 American Hero in Education' award (*Reader's Digest*). One day that I met Miss C. in the subway after a staff development meeting, she commented, with visible indignation: "Some of my colleagues say sometimes: 'I taught such and such, but my students did not learn it.' I think that's wrong. If the students did not learn it, it means you did not teach it."
This resonates with Henry Poincare's assertion: "What is a satisfactory definition? For the philosopher or the scholar, a definition is satisfactory if it applies to those things and only those things that are being defined; this is what logic demands. But in teaching, this will not do: a definition is satisfactory only if students understand it."

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slices that my twelve friends ate," counting aloud by 3s up to 36; he then showed how he shaded "the two slices that I ate" from the sixth circle. "There are ten leftovers. Two over here, four, six, eight, and ten."

Annatalia was called next. She read aloud what was on her poster:

> "Answer: 10 slices left. What I did is that I drew pictures I drew 6 pizzas then I divided them into 8. Then I multiplied and it gave me 48. Then I did 12 times 3. Then I added 2 more that I ate. And then I took away 38 from 48 and it gave me ten."

Since the rest of the students claimed to have obtained the same answer and most of them said they had drawn the six pizzas, the teacher decided to move on to another problem.

At first sight, the 'Pizza Party' problem did not offer much food for thought. The question "How many *slices* are left?" did not invite a discussion around different ways of obtaining the leftover slices--e.g. as a whole pizza and two more slices (Annatalia's solution), or as ten loose slices, two from five of the six pizzas (Jorge' solution). However,

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'in real life,' either the six pizzas have been presented at the same time--each pizza cut into eight parts--and the children took their slices from any of them; or the pizzas have been brought one after another and, after the thirteen children took their share, there was a whole pizza and two more slices left.[41]

The children's drawings looked very much alike: six circles--or squares--divided into eight parts each; the thirty eight 'eaten' parts are either numbered or shaded; and the ten leftovers appear as one whole pizza and two more slices (as in Annatalia's drawing). One students drew seven circles, another one drew eight, Taurice chose to make squares, and a few other students drew the six circles but left them blank. One may infer that, in general, the drawings were not used as a model for solving the problem.[42]

---

[41]Of course, these two scenarios could be linked to whether the pizzas were all of the same kind or not; that is, whether or not it would have matter to take a slice from one or another pizza, and so on...

[42]As I walked around the room, looking at children's work, I found that all of them had drawn the circles. These are some of their explanations.
Rafael had written, in column form: 6x8=48, 48-36=12 12-2=10. Steps: 1) First I multiplied 6 times 8 because it said my mother bought 6 pizzas and cut them into 8 pieces. 2) Then I took away 36 because it says my friends eat 3 slices and there were 12 friends. 3) and then I take away two cos it says that I ate two. And it gave me ten."
Lorena wrote, also in column form, "6x8=48, 12x3=36, 36+2=38, 48-38=10. Answer: 10 slices left."
Taurice drew six squares, each divided into eight parts, and wrote: "First I made 6 pizzas and I cut them into eighths and...then I did 18 take away 2 and it gave me 16."
Lilian made a drawing of seven circles divided into eighths, and her computations were exactly like Lorena's.
Mary did the same calculations as Rafael and she wrote:
"I drew 6 pizzas and I colored the ones that we ate.... *And then to make sure that it was right I did it by numbers because you don't know.*"

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After all, the problem seemed too easy to begin with. Yet, a comparison of Annatalia's and Jorge's drawings could have led--with a trong intervention by the teacher in the form of probing questions--into a discussion about equivalent fractions.

Annatalia had colored the first four circles, she then colored all except for two slices on the fifth circle, and left the sixth circle blank. This is 8 + 2 = 10 slices which, in fractional form, may be written as 1 2/8 (that is, 'one whole pizza and two slices'). Instead, Jorge's '10 slices' appeared as two blank slices in each of the first five pizzas, that is, 2 + 2 + 2 + 2 + 2 = 10 or 5 x 2 = 10 slices which, in fractional form, is equivalent to 5 x 2/8, that is, 10/8.

Starting from Jorge's solution--i.e. 10 slices, or ten eighths--10/8 may be expressed as 5/4; 5/4, as a mixed number, is 1 1/4; 1 1/4 may be written as 1 2/8; thus, we arrive at Annatalia's solution--i.e. one whole pizza and two slices. (And vice versa, if we start from Annatalia's). In the course of investigating the two pictured solutions, the connections between 1 + 2/8, 1 2/8, 8/8 + 2/8, 5 x 2/8, 10/8, 5/4, 1 1/4, and 1 + 1/4 could have been easily visualized. Thus, what is usually taught as mere symbol manipulation,[43] could have been connected to the pizza-

[43]E.g. convert fractions into mixed numbers and viceversa; add, multiply, and reduce fractions, and so on.

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sharing context.

What would have been necessary for these fruitful discussions to happen? This investigation would have taken more than twenty minutes.[44] The problem was meaningful enough in that it makes sense to consider what remained of the six pizzas after the party was over; and it allowed for solutions at different levels of sophistication--e.g. counting one by one, repeated addition of twelve groups of three and one more group of two, a combination of multiplication and addition, or the use of fractions. Of course, nothing in the problem's formulation, except the fact that it was about pizzas, suggested fractions.

In line with my general argument, I suggest that if this potentially open-ended problem was *de facto* treated as a closed text, the reasons must be sought in the broader context in which the problem was interpreted and solved. At work in this event was a chain of interlocking contexts: the 'pizza party' problem, a simulacrum for the PAM, this fifth-grade classroom, PS XYZ, and the larger social context embedding all these contexts. I propose that the students'--and teacher's--pragmatic decision not to take the context too seriously reveals that, despite the claims of multivocality, open-endedness, authenticity, realism, and relevance, the problems of these days function just like

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[44]The NCTM acknowledges that developing the students' mathematical power requires working on problems which "may take hours, days, even weeks to solve" (1989).

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those univocal, closed, pre-fabricated, pseudo-realistic, and ultimately irrelevant problems typical of the traditional math instruction which the current reform aims at superseding. This suggestion ought to be read not so much as a call for truly multivocal, open-ended, authentic, realistic, and relevant texts, than as a suggestion for further exploring the multiple constraints which affect the reception of math texts in such a way as to force their closure.

Let us step off the elaborate stage of the elementary school classroom, let us leave behind the exciting activity of problem exploration, and let us reflect on our play. For, in the modern school, everything is play, activity, discovery, construction, fun. There is no more work! When we embarked, we were told that at the end of the journey we needed to consider whether our answer made sense. But, as we finish without a solution, we cannot check the answer. Can we still make sense of the classroom performances just presented?

Some of these classroom vignettes were clearly about rules. For example, we saw that it was not by following explicitly taught problem solving strategies but, instead, by obeying implicit rules, that students in general--and particularly those who have difficulties in reading and writing in English--developed the survival tactics they needed to attack the word problems they are confronted with

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in their daily math lessons. Some of these tactics were successful: you got the right answer, you convinced the teacher that you *really* understood, you passed the test, and you got a new bike. Other tactics were not: you got distracted by extraneous information, you wasted your time, you failed the test, and your Nintendo got confiscated until summer.

These scenes also highlight the dystopian effects which result when culture--in the form of everyday life themes, multicultural stories, and realistic problem situations--is introduced into math classrooms, textbooks, and workbooks. Madness seems to emerge in the course of the students' struggle to sift through the noises which, under the new new math regime, cloud the supposedly transparent discourse of mathematics.

A third, more optimistic reading of our classroom scenarios would consider the symptomatic nature of children's errors, confusions, and misunderstandings. As we have been constantly reminded, math is a very tricky thing. To confront it, most students become automaths, and a few exceptions become tricksters themselves. They lie, they copy from a neighbor, they save their play time--or they optimize their leisure time--by skipping the content of word problems and going right to the cues; and they all find clever ways of wasting as many minutes of class time as they possibly can. Consider the case of Lakeysha, who combined in an

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unexpected way the hints given to her by the workbook and by her math lab teacher--i.e. 'work backwards' and 'try the opposite.' Was she being tricked or was she the trickster? In either case, her reaction is only possible in a context in which things are not supposed to make sense. By bringing out the nonsensical in the teaching machine, the students' view from below may be seen as an immanent critique of the pretensions of realism, suspect gestures of inclusion, and illusions of control which characterize cutting-edge mathematical pedagogies.

At the risk of stating the obvious, I will suggest what this dissertation is not. It is not intended as a rejection of the new new math project. Neither does it aim at denouncing multicultural math education or at proposing the replacement of pseudo-narratives with genuine, true-to-life math texts. Moreover, the intention is not to propose the elimination of all math narratives in favor of a totally transparent, noiseless mathematical discourse. For, at stake is not so much the text of mathematics as the ways in which this text has been made to operate within old and new pedagogical discourses and practices. In other words, this thesis is about the seductive power of math fictions and their effect on the students as well as on the other players in the didactic game.

Although Byron does not always manage to get to school--even though he lives right around the corner--he does take

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his classroom work seriously. He is always aware that 'doing school' does not teach you how to 'do life.' Yet, he also knows that, for better or for worse, in school he will learn what he couldn't learn outside. Byron has a clear sense of the times he is living in. Although he hates rules, to some extent their existence must be reassuring for someone who is only nine-years old: in school things are easier because "they tell you what to do." Yet, sometimes all Byron wants to do is play outside, but he is not allowed. "Why?" I asked him one day. "Too dangerous, they could kill you."

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# CHAPTER EIGHT: MATH FICTIONS

> ... truly democratic education is education which sets itself the unconditional goal of enabling *the greatest possible number of individuals to appropriate, in the shortest possible time, as completely and as perfectly as possible, the greatest possible number of the abilities which constitute school culture at a given moment...* (Bourdieu and Passeron 1964).

> Rules are empty in themselves, violent, and unfinalized; they are impersonal and can be bent to any purpose. The success of history belongs to those who are capable of seizing these rules, to replace those who had used them, to disguise themselves so as to pervert them, invert their meaning, redirect them against those who had initially imposed them; controlling this complex mechanism, they will make it function so as to overcome the rulers through their own rules (Foucault 1977).

Since the end of World War II, critical theorists, post-structuralist and deconstructivist philosophers, feminist critics, and practitioners in the strong programme of the sociology of knowledge, have contributed new ways of thinking about mathematics where power, ideology, language, discourse, desire, gender, subjectivity, culture, and social interest occupy a central place. Yet, despite these powerful critiques, the dominant conception of mathematics in Western culture is still that of a formal discipline, an abstract science, a de-contextualized field of knowledge, a transparent or noiseless language, a value-free field of inquiry, and a subjectless and monological discourse.

As for the place of mathematics within the educational system, given the accumulative character of this discipline, a deficient acquisition of its basic concepts and skills in the elementary grades results, for many students, in poor

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performances in job-entrance exams or in high-stakes academic assessments. Much has been done in the last decades to neutralize or counteract the gate-keeping function of mathematics, as well as to provide all students with equal opportunities for acquiring this highly valued form of academic capital. Yet, despite these efforts, for women, lower class, Black, Latino, and other minority students, mathematics continues to operate as an almost inescapable educational filter.

The main argument of this thesis is that, in so far as the two conditions above are maintained, an effective teaching of elementary school mathematics will be in tension with the three tenets which, slowly emerging one after another since the turn of the century, have constituted the modern pedagogical paradigm: child-centeredness, progressivism, and constructivism.

Within the boundaries of this paradigm, the child has been placed at the center of the classroom, turning the teacher into a guide or a provider of rich learning environments; a relation of continuity has been postulated between school and society so that the classroom has become a micro version of the surrounding world; and, finally, it has been proposed that for meaningful and lasting learning to occur, students must be made to participate in the process of learning as active constructors of knowledge.

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*When pushed to the limit* or, better said, in its most deformed or distorted version, the modern pedagogical paradigm results in a peculiar set of recommendations to classroom teachers: a) omit whatever does engage the student/child (i.e. reductionism); b) teach something when and if it proves useful for accomplishing the project or activity at hand (i.e. incidentalism); and c) restrain from explicit teaching so that to students may construct knowledge by themselves (i.e. anti-didacticism).

If these prescriptions may be said to conflict with an effective teaching of mathematics in all classrooms, this is particularly the case in those inner city public schools which are attended by poor and minority students. At least three reasons can be given to support this assertion.

1) In these schools, teaching and learning is subject to a chain of severe constraints which includes high teacher-student ratios; tracking by ability level; teachers' deficient preparation in mathematics and its didactics; and the constant pressure from statewide and citywide tests as well as districtwide unit tests and pacing calendars.

2) As opposed to their white and middle-class counterparts, poor and minority students do not arrive to school endowed with the prerequisites to operate in child-centered, progressive and constructivist environments.

3) Anti-didacticism, reductionism, and incidentalism are especially damaging to poor and minority students in that,

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for most of them, school is their only opportunity for acquiring the literacy and numeracy skills which, despite vociferous claims to the contrary, are still indispensable tools for successful performance in academic as well as in job markets.

I suggested that the modern pedagogical paradigm rests on three highly problematic assumptions: the belief that the power/knowledge relationship between teachers and students may be made symmetrical; the conception of the process of learning as a productive practice in which students construct 'their own' knowledge; and the conflation of 'the child' as constituted by the pedagogical gaze, that is, 'the learning subject,' with actual children.

Throughout this dissertation I assumed, instead, that the relationship between teachers and students is always asymmetrical, in that the teacher is the one supposed to know and supposed to anticipate what the students will know; that, to a significant extent, teaching mathematics consists of introducing students to a pre-constructed body of knowledge, even when this process takes the form of a guided reinvention; and that a distinction must be kept between the child as the subject of pedagogical discourses and actual children.

For teachers and students working in real classrooms, the tension between an efficient teaching of mathematics and the modern pedagogical paradigm translates into very

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difficult dilemmas. Included in these are a series of dichotomies which are recurrent themes in the elementary school: mathematics *vs.* the child, the computational *vs.* the social value of mathematics, and students' inventions *vs.* the conventions of mathematics.

For example, with their conception of teaching as the facilitation of rich environments for the construction of mathematical knowledge, their unprecedented concern with honoring students' spontaneous inventions, and their trust in the possibility of symmetrical classroom communications, constructivist math educators are often at pains in developing a coherent program for transmitting--to more than thirty students and given the limitations of didactic time and of teachers' understanding of mathematics and its didactics--a cultural artifact made up of sophisticated structures and models, efficient algorithmic procedures, and universally shared conventions and notations.

However, the normal functioning of the system of mathematics teaching requires that this as well as other tensions and dilemmas be misrecognized. The result of this misrecognition is a set of math fictions which help maintain the illusion that mathematics can, in fact, be taught and learned within the boundaries of the dominant paradigm. These fictions include the belief that:

1) the didactic system of math teaching is autonomous vis a vis its surrounding environment;

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2) school mathematics may be made continuous with out-of-school mathematics (e.g. with the mathematics of the mathematicians, or with 'street' mathematics, 'multicultural' mathematics, or 'ethnomathematics');
3) the student/teacher relationship may be made symmetrical (by implicitly underplaying the element which accounts for the difference between them, that is, subject matter knowledge);
4) written texts, concrete materials, visual imagery, and other embodiments of mathematical structures and processes may be made transparent;
5) the classroom may be made to operate as 'microcosms of mathematical culture;'
6) it is possible to access the mind of the students while in operation thus obtaining an accurate picture of their mathematical understanding; and
7) room can be made for the learner's subjectivity in the processes of teaching and assessing mathematics.

Far from developing in unison, the emergence of these fictions is shaped by the specific set of pedagogical discourses which dominates the field of math education in each historical conjuncture. In particular, each period of reform may be characterized by the emphasis placed on one or another of these fictions. For example, many projects at work during the 1960s' new math revolution relied upon the embodiments fiction, that is, on the hope that if young

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children were made to play with especially designed materials, sooner or later, they would discover the mathematical structures embodied therein. Today, some advocates of the new new math reform sustain the fiction of a classroom modelled as a microcosm of mathematical culture, that is, the expectation that students will acquire 'mathematical power,' as they participate--with their teacher and classmates--in a 'community of discourse' in the context of which alternative conjectures and strategies for the solution of math problems are shared and negotiated.

I have traced the fiction of continuity of mathematics and out-of-school mathematics as it materialized in the prefaces of a sample of arithmetic textbooks in circulation from 1882 to 1957. This period spans from the time when the provision of free textbooks was made mandatory in most school districts, to the Sputnik crisis, when the obsolescence of American *arithmetic* textbooks was declared and, as a result, these texts were suddenly upgraded into *mathematics* textbooks.

Then I explored the singular rhetorical force with which the fictions operating during the 1960s' revolution in school mathematics and those invoked by the reform underway since the late 1980s, interpellate the various social constituencies which have a stake in the well-being of math education in the United States.

Finally, I observed and analyzed the rather complex and

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often unanticipated ways in which the fictions sustaining the current reform affect the realities of classroom life in the fourth- and fifth-grade classes at PS XYZ--a public, bilingual, elementary school in El Barrio (East Harlem), New York City.

This choice was motivated by my belief that telling the story from three angles--e.g. a historical inquiry into old-time arithmetic books, a discourse analysis of the reforms' official documents, and an ethnographic exploration of daily classroom performances--would keep me at a distance both from a fatalist conception of math teaching as a system affected by inescapable social and economic determinations, and from a banal, rather voluntaristic view of elementary math education as a field operating outside of all external or macro level constraints.

Once my 'topic of study' was demarcated, I considered the menu of available theoretical tools. This process entailed an exhaustive review of the literature on education and social reproduction; theories of cognition and learning; and almost everything that has ever been written, in the United States and elsewhere, in the field of elementary mathematics education. After conducting a series of thought experiments and writing more than a dozen 'early drafts,' in which the hermeneutic and explanatory power of some of these tools was put to trial for the subject at hand, I opted for a 'theoretical framework' which relies heavily on French and

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British social and educational theory.

My point of departure was Bourdieu/Passeron's theory of education as an institutional instance of social reproduction, including the notions of habitus and cultural capital; the view of pedagogic action as the systematic inculcation of a cultural arbitrary by means of symbolic violence; and the belief in the inherently asymmetric nature of pedagogical communications (1977).

I borrowed from Bernstein his model of visible and invisible pedagogies and his theory of restricted and elaborated codes (1971, 1973, 1975). From his later work (1990, 1996), I made use of his analysis of the embeddedness of instructional discourses in evaluative discourses, and his description of the complex dynamics of contextualization, decontextualization, and recontextualization which is inherent in the transmission of knowledge in school settings.

Why Bourdieu/Passeron and Bernstein? Anticipating that as the months--and years!--would go by, my research and writing would become increasingly specialized, as a disciplined soon-to-become sociologist, my intention was to stay grounded in the strongest and most critical tradition in the sociology of education. Of course, this choice entailed the risk of channeling my thinking along structuralist and reproductivist lines. But the price was worth paying, in that this choice allowed me to maintain a

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sense of the 'whole picture,' while getting momentarily lost in the programmatic statement of one or another textbook author, the grandiose rhetorical claims made by each generation of reformers, and the microscopic details of classroom life at PS XYZ. In other words, while exploring the discursive practices surrounding the production of mathematically literate subjects, I tried to remain faithful to the premise that every instance in the functioning of the education machine ought to be located in and understood as part of the broader social field.

In order to move a step further into real (math) classroom practices, I made use of a theoretical arsenal developed in the last ten years by researchers in the French *didactique des mathematiques*--in particular, the work of Guy Brousseau (1986, 1988) and Yves Chevallard (1985, 1990). More specifically, Brousseau's didactic contract theory and Chevallard's concept of didactical transposition proved the most useful theoretical tools for my undertaking.

In the United States, the contributions of Brousseau and Chevallard to a critical analysis of the tensions and dilemmas of teaching and learning elementary mathematics continue to be underestimated. There is no better proof of this disregard than the fact that their major works have not been translated into English. Thus, I saw my dissertation as an opportunity to assess the relevance of these foreign-born models for a field which, historically, has been limited by

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its provincialism and which, judging from recent borrowings from German and Dutch didactics (e.g. Cobb and Bauersfeld 1995; Romberg 1995), seems ready to widen its horizons.

As one of my major preoccupations was to explore the ways in which the teaching and assessing of elementary school mathematics is implicated in the production of reasoning subjects, the investigations on 'girls and maths' conducted by British developmental psychologist Valerie Walkerdine and her collaborators (Walden and Walkerdine 1982; Walkerdine 1988, 1990) proved a fruitful perspective. As the discussion in Chapter One made it apparent, the ultimate (philosophical) inspiration of my dissertation is Arendt's essay on "The Crisis in Education" (1954). However, my central thesis--i.e. that there is an inherent tension between an effective teaching of mathematics and the modern pedagogical paradigm--was also motivated by Walkerdine's (Foucauldian) critique of child-centeredness, as well as her skeptical approach to a pervading and, at the same time, always elusive distinction in the field of math education, namely, that between rule following and real understanding.

But these texts alone could not have sufficed to make sense of the contradictions, ambiguities, and impasses which affect the teaching and learning of mathematics in elementary school classrooms. What at first was little more than a theoretically supported and politically motivated hypothesis--namely, that an effective teaching of

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mathematics in the early school years necessitates the bracketing of at least one of the prescriptions of the modern pedagogical paradigm--was further supported by my acquaintance with another foreign-born set of discourses and practices.

I am referring here to the work developed in the Netherlands since the early 1970s by Hans Freudenthal and his disciples in the 'didactical phenomenological' approach, later to be called the 'realistic' approach to math education (Freudenthal 1971, 1973, 1983; Treffers 1987; Gravemeijer *et al.* 1990; Streefland 1991; Gravemeijer 1994; and van den Heuvel Panhuizen 1996). For in education, more than in other fields, except perhaps in healing and governing (in Freud's words, the other two 'impossible professions'), one is given permission to describe what 'does not work' if one is able to offer--at least tentatively or as a utopian vision--a model which does work.

Put bluntly, the Dutch didactics of mathematics works. And, interestingly enough, it does so by taking seriously that learning mathematics consists of the (essentially human) activity of mathematizing reality and, eventually, mathematizing mathematics. This activity is one which no child can realize by him or herself. Instead, it requires a) the assumption that mathematics is best learned when embedded in realizable or imaginable contexts;
a) a classroom full of children whose 'cognitive abilities'

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span at different levels;
b) a set of instructional goals and curriculum materials carefully designed in cycles of developmental research;
c) freedom from the pressure of standardized testing; and
d) a teacher who is equipped with the necessary didactical tools and models for guiding students in their collective reinvention of mathematics.

Towards the end of my seventy-five-year long excursion into the microworld of arithmetic textbooks, I considered the disappearance of prefaces and introductions. This disappearance--which occurred around the late 1940s--is, of course, linked to the increasing popularity of the teacher's manual. If needed, commentaries by the authors will now appear--together with pages and pages of didactical recommendations--in these accompanying recipe-books. In line with my general hypothesis, I suggested that the almost obligatory use of the teacher's manual may be seen as yet one more subtle way in which the teacher/student relationship is made into a symmetrical one. Both actors in the didactic game are treated as students, each invited to access the 'world of number' through their own text. On the one hand, there is school arithmetic, written *for* the student and, on the other hand, there are instructions *to* the teacher for the appropriate delivery of this subject matter.

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Many of the arithmetic textbooks in circulation before the 1940s also had an accompanying teacher's manual and, yet, this fact did not prevent authors from making public, in the textbook itself, their views on the whats, the hows, and the whys of arithmetic teaching. These included statements about the disciplinary and/or instrumental value of arithmetic; a rationale for the kinds of word problems contained therein; a presentation of the structure of the text; and instructions regarding the handling and use of the book.

It could be argued that, in so far as the function of the preface was to introduce the general public to a didactically transposed object--i.e. school arithmetic--as well as to justify this transposition, its disappearance should indicate that arithmetic has become, at last, an unproblematic subject. On the contrary, I argued that, if talk about arithmetic became engulfed in the pedagogical field, this gesture is best read as symptomatic of the ultimate didactical deformation of arithmetic.

In their efforts to accommodate their texts to the prescriptions of child-centeredness and progressivism, these authors finally succeeded in manufacturing their own fiction--i.e. an 'applied' and child-centered textbook arithmetic--that is, a body of knowledge which was totally estranged from the discipline from which it originated. In this context, the preface lost its function as a mediating

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text.

The disappearance of the preface anticipated the imminent death of the arithmetic textbook. While giving birth to the new math's revolution, the 1957 Sputnik crisis justified the channeling of material and intellectual resources which allowed for the old arithmetic textbook to re-incarnate as a textbook in 'elementary mathematics,' written in the language of sets.

The utopian desire of most new math reformers was to do away with all forms of didactical transposition. At last the structure of mathematics would be presented in an honest fashion, that is, stripped from any kind of didactical deformation. For that to happen, it was necessary to suspend the principle of immediate utility and social significance, as well as to put a moratorium on an exclusive concern with the immediate interests of the child. The Sputnik paranoia made it possible for these two radical gestures to occur.

From the early elementary school grades up to the college level, new topics such as set theory, non-metric geometry, and non-base ten number systems were suddenly introduced. It was argued that if children were placed in suitable environments with access to the appropriate materials, sooner or later, they would discover the mathematical structures embedded in these artifacts. Thus, classrooms were stacked with Cuisinaire rods, Dienes blocks, and other paraphernalia. It was recommended that for

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students to discover the concepts, rules, and laws of mathematics by themselves, teachers had to restrain from frontal teaching and assume, instead, the role of guides or 'organizers of the flow of communication.'

Central to the fiction of embodiments was not only the belief in the transparency of manipulatives, but also the anti-didactical assumption that these materials would facilitate students' access to abstract structures without the teacher's mediation. This displacement of the teacher's traditional role was consistent with the dominant child-centered and constructivist pedagogical paradigm. At the same time, this displacement was an opportunistic response to what had been perceived, from the early days of the reform, as a main obstacle to the desired transformations: teachers' lack of preparation in modern mathematics.

Faced with overcrowded classrooms and constrained by their limited knowledge of mathematics and its didactics, many teachers accommodated the reformers' prescriptions by introducing the new topics in old ways. Since the prescribed discovery method proved rather time-consuming and did not always lead to the expected outcomes, many teachers opted for guided discovery. This modality of classroom practice degenerated easily into pseudo-discovery, that is, teacher and students would jointly create the illusion of a mathematical discovery when, in fact, the former had told the latter what was there to be discovered.

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The reproduction of this illusion in so many 'reformed' classrooms indicates one of the ways in which the teaching of mathematics is at odds with the modern pedagogical paradigm. Since, according to this model, students are deprived from fruitful learning experiences either when the teacher tells too much or when she does not tell at all, the best strategy may have been, in fact, guided discovery. But to effectively guide students' discoveries, teachers themselves needed to understand what was there to be discovered, and they needed to be equipped with the appropriate didactical tools for performing such a guidance.

The 'failure' of the new math forced experts to reconsider their epistemological and didactical assumptions. The 1970s' back to the basics movement was short-lived, and the 1980s was decreed by the NCTM as the era of problem solving. By the late 1980s, the emerging new new math project promised to deliver all that both old and new models of math teaching had left out of the picture: the proactive role of the teacher, the classroom as a community, the requisite that math tasks be worthwhile projects, and the rhetorical dimensions of mathematics.

Informed by Piagetian constructivism, 1960s' reformers saw math learning as the series of transformations in the student's cognitive structures which resulted as he or she engaged--just like a mathematician--in manipulative play (or constructive work). Guided by less individualist versions of

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of constructivism, present-day reformers see math learning as the transformations in students' cognitive structures which may result from their participation in a community of discourse where mathematical conjectures, procedures, notations, and solutions are communicated and negotiated.

Despite this healthy shift from the individual to the social, the conflation of two distinct phenomena remains intact. The reproductive process whereby school mathematics is taught and learned in classroom contexts is still equated with the productive process whereby new knowledge is generated and validated within the scientific community. In their attempts at accommodating the teaching of mathematics to Piaget's theory of learning--i.e. a theory which underplayed the effect of instruction on cognitive development--new math and new new math reformers alike are doomed to underestimate the didactic role played by teachers in introducing children to the world of numerate culture and the mastery of mathematical reason.

The 1960s' new math has been described as a failed attempt at imposing a centralized, quick-fix, top down reform, with no provision to prepare teachers for implementing the desired changes. Teachers responded either by distorting the true spirit of the proposed innovations, or by rejecting them altogether and continuing to teach the old basic skills. Nevertheless, the new math project may be seen as a crucial step forward in the direction of a more

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effective teaching of mathematics. It did so by daring to ignore the prescriptions that school math be practical, useful, and socially significant, and that it be attuned with the immediate interests of the child. For the first time, it was permitted to remain within the world of number, without the need to establish connections with the 'real' world and camouflage mathematics in child-like costumes.

If the new math project did not succeed--or, if it was shamefully deformed when translated into practice--we may attribute this failure to the limitations of Piagetian constructivism, the sudden shift in the political scenario from cold war paranoia to the crisis of the Welfare State, or both. One may also propose, with Howson, Keitel, and Kilpatrick (1981), that since the new math's revolution was never given a real chance to happen, rather than designing newer programs, the task at hand is to continue the work of reform where it has been interrupted.

Yet, by the late 1980s, math education experts in the United States decided, instead, to start anew. The current reform scenario is characterized by new nationwide curriculum and new assessment standards which are meant to be implemented in the context of a highly decentralized educational system. Delivered in the form of classroom vignettes, the new recommendations by the NCTM have been designed with the expectation that, as teacher put them into practice, they will hybridize them (NCTM 1991).

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We may wonder whether teachers are, this time, better prepared for effecting these necessary didactic hybridizations while honoring the reformers' original intent. This question brings us back to the issue of didactics. Unfortunately, since the early 1990s, more emphasis has been placed in upgrading standards and renewing assessments, than in investing in the long-term, technically complex, and economically demanding enterprise of training and re-training teachers in mathematics and its didactics.

Moreover, as the recommendations of present-day reformers translate into classroom practices, a peculiar anti-didactical perversion may begin to occur. Portions of everyday reality are imported into the classroom in the form of ill-structured, non-routine, multi-step, and open-ended problems. Teachers are asked to play a proactive role in helping students construct mathematical concepts as they engage in the solution of these problems. Yet, since the dominant child-centered and constructivist paradigm has resulted in a de-emphasis of mathematics and its didactics in pre-service preparation of teachers, the didactical models for *mathematizing* these contexts are not acquired by teachers and, therefore, may never reach the students. Thus, with its child-centered pedagogy, its progressivist view of curriculum, and its constructivist theory of learning, the reform currently underway risks perpetuating the fiction of an anti-didactical contract for the learning of mathematics.

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By redefining the didactic contract in the direction of a classroom modelled as a microcosm of mathematical culture, the new new math reform aims at moving beyond the elicitation pattern of interaction. That is, in reformed classrooms, teachers' interrogations are no longer pseudoquestions but genuine requests for explanations. As for the students, they are asked to put more effort in understanding each other's mathematical procedures and explaining to one another their conjectures and solutions, than in figuring out the right answers.

In the course of my fieldwork at PS XYZ, I gathered that the conditions are not given for this modality of pedagogical communication to operate. In these classroom sites, the constraints affecting the didactic contract are such that teachers are usually impatient to get through each day's lesson and prepare children for the tests. There is no time to hear what the children have to say. And, even when there is time, the teachers' insufficient preparation in mathematics and its didactics hinders their ability to make sense of students' often idiosyncratic questions, procedures, and explanations, as well as to anticipate and prevent their confusions, errors, and misunderstandings. Thus, rather than jointly *producing* a microcosm of mathematical culture, students second-guess their teachers, and both play their part in *reproducing* a set of more or less scripted classroom performances.

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Since the early 1980s, problem-solving has become a central feature in the math classroom. It is suggested that, rather than working through long lists of problems, only one or two are to be tackled during one lesson, to allow for more in-depth discussions. Since many of these texts implicitly suggest "let's pretend this is a real life scenario," a tension often results between this invitation and the implicit solicitation that math problems be handled as school problems, that is, as puzzles, rather than dilemmas.

My classroom observations and informal conversations with 4th and 5th graders at PS XYZ led me to conclude that, in most cases, the devolution of the problem does not occur. Instead of approaching word problems out of an intrinsic desire to solve them, children adopt vis à vis these artifacts, four alternative subject-positions: the automath, the detective, the paranoic, and the trickster. Far from pathologizing these responses, I interpreted them as pragmatic adaptations as well as immanent critiques of the fact that, under the existing conditions, even the most open-ended math problems are treated as univocal texts.

The new new math reform confronts students with solicitations such as "Pretend that school is like real life," "See your culture included in this text," "Share your thinking with your teacher and classmates," and "Show me that you really understand it." Due to the constraints

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affecting schools like PS XYZ, most students soon learn that to play the new language game of school mathematics--one which translates, *de facto*, into more talking, more reading, more doing, and more writing--the implicit rule consists in ignoring each and all of these solicitations.

In the classrooms I observed, an already fragile didactic contract is overwritten, almost overnight, by an assessment contract which has neither been made explicit nor negotiated by the two main players in the didactic game. Aside from precipitating the various pathological breakdowns of the didactic contract described so far, this results in a rupture of the assessment contract itself. That is, rather than providing access to the inner workings of their minds, students respond to the 'show your work' solicitation either by ignoring, resisting, or misinterpreting it. The step-by-step narratives produced as they reflect *ex post facto* on their problem-solving trajectories, are a window to the dirtiest secret of the didactic contract now in place: the algorithm. We may see in this innocent gesture the students' revenge on their teachers who, after all, are testing them in something that they have not taught. As Byron put it, "Miss, you told as to show work but did not say how!"

Instead of using the 'show work' solicitation, which is often interpreted as a compulsion to draw, describe, label, and explain, Dutch mathematics didacticians capture traces of students' thinking by asking them to share their scrap

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notes with the teacher. This modality of assessment necessitates teachers with enough training to decipher the students' often obscure and idiosyncratic inscriptions. This is a more demanding and time-consuming task than that of checking step-by-step procedures and, of course, it demands from the teacher's part, a stronger preparation in the didactics of mathematics.

The responses to story problems offered by these 4th and 5th graders reveal their awareness of the fictional status of the claim that school mathematics may be made continuous with out-of-school mathematics. It is as if, through the claims of open-endedness and realism, students see the misrecognized truth of the didactical transposition. As the episodes in Chapters Six and Seven have illustrated, the topic of money is one in which this fiction appears with more strength and yet is most skillfully demystified. We may interpret this as students' rebellion against the deficient instruction which, due to their location in the social map, they seem destined to receive.

After so much time, energy, ink, and paper spent in learning about money, solving long lists of pseudo-realistic, puzzle-like word problems, and deciphering unnecessarily wordy problems, these children continue to be deprived from accessing that tool which is a central task of the school to provide: the ability to mathematize reality and to mathematize mathematics.

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This dissertation proposed a reformulation of the old-time, forever recurrent, and universally worrisome 'problem' of widespread failure in school mathematics and what-may-be-done-about-it. Far from offering one more solution to this problem, I attempted to demonstrate that, given the epistemological status and social function of mathematics, its effective teaching in the elementary school is in tension with the tenets which define the modern pedagogical paradigm. Moreover, I argued that this tension appears with more strength and produces its most damaging effects in school sites attended by poor and minority students.

From a different angle, this dissertation offers a sociological analysis of the two most recent reforms in North American school mathematics. Situating each reform in its broader political scenario and exploring the seductive power of the constellation of math fictions invoked by them, I located these two interventions within an overall shift towards more subtle modalities of social regulation.

Ultimately, *Math Fictions* may be read as an exploration of the often unintended and unanticipated ways in which the inter-generational transmission of a depersonalized discourse, a decontextualized field of knowledge, an extremely elaborated linguistic code, and an overvalued form of academic capital--i.e. elementary school mathematics--contributes to the reproduction of social and economic inequalities.

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# APPENDIX

Name: Grade:
Born in: Years in the US:
Language you speak at home:

## A QUESTIONNAIRE ABOUT WORD PROBLEMS

1. What is a word problem? Suppose you have to describe what a word problem is to someone who comes from another planet. How would you explain to this creature what a word problem is?

2. Suppose you have two columns, one called fiction (a story that is not real) and the other called non-fiction (a story that is a true story). Where would you put word problems? Why?

3. Give me an example of

a) an easy word problem. Why is it easy?

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b) a difficult word problem. What makes it difficult?

c) a fun word problem. Why is it fun?

4. Who do you think makes up the word problems that they give you in school?

5. Why do you think it is important to learn how to solve word problems?

6. How do you solve word problems?

7. What are most word problems about?

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8. What do they mean when they give you a word problem and they ask you to show your work? Why do you think they want you to do that?

9. What do you need in order to solve word problems? (You can choose more than one option)

( ) A. to have good memory
( ) B. to be good in reading and vocabulary
( ) C. to know how to write well
( ) D. to know your times tables
( ) E. to know how to draw and make graphs
( ) F. to have a lot of practice in solving word problems

10. Some people say that solving problems in school is just like solving problems in your daily life (for example: shopping, cooking, paying bills or tips, etc.). What do you think of that?

11. Do you think that word problems do always have a solution? What about problems in real life?

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12. Is it true that word problems always have only one answer? What about problems in real life?

13. Write a two-step word problem so that its answer would be $3.75.

14. Solve this problem. Please make sure you include all your scrap work on this page.

> Suppose you had 50c on Monday night. You took a job for a week that doubled your money each day. For example, Tuesday night you had $1.00. How much money did you have the next Monday night?

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# LIST OF ABBREVIATIONS

## Associations

| | |
|---|---|
| American Education Research Association | AERA |
| National Council of Teachers of Mathematics | NCTM |
| National Education Association | NEA |
| National Society for the Study of Education | NSSE |

## Journals

| | |
|---|---|
| Arithmetic Teacher | AT |
| Educational Leadership | EL |
| Educational Researcher | ER |
| Educational Studies in Mathematics | ESM |
| Harvard Educational Review | HER |
| Journal for Research in Mathematics Education | JRME |
| Journal of Curriculum Studies | JCS |
| Teachers College Record | TCR |

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