

INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps. Each original is also photographed in one exposure and is included in reduced form at the back of the book.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

U·M·I

University Microfilms International
A Bell & Howell Information Company
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
313/761-4700 800/521-0600

Order Number 9207047

**Technical analysis, the stochastic properties of security prices,
and profits in trading**

Baskin, Brenda Leigh, Ph.D.

City University of New York, 1991

Copyright ©1991 by Baskin, Brenda Leigh. All rights reserved.

U·M·I
300 N. Zeeb Rd.
Ann Arbor, MI 48106

A

TECHNICAL ANALYSIS, THE STOCHASTIC PROPERTIES
OF SECURITY PRICES, AND PROFITS IN TRADING

by

BRENDA BASKIN

A dissertation submitted to the Graduate Faculty in Economics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

1991

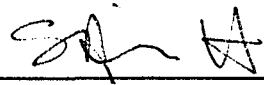
© 1991

Brenda Baskin

All Rights Reserved

This manuscript has been read and accepted for the Graduate Faculty in Economics in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

July 30, 1991
Date


Chair of Examining Committee

July 31, 1991
Date

Michael Grossman
Executive Officer

Dr. Salih N. Neftci

Dr. Michael Grossman

Dr. Ronald Anderson
Supervisory Committee

Abstract

**TECHNICAL ANALYSIS, THE STOCHASTIC PROPERTIES
OF SECURITIES PRICES, AND PROFITS IN TRADING**

by

Brenda Baskin

Advisor: Professor Salih Neftci

Speculative traders in financial markets earn their living from trading profits that arise from correct forecasts of the direction and size of movements in security prices. A popular method among traders for such predictions is **TECHNICAL ANALYSIS**, the study and investigation of movements in past security prices and market averages.

This paper reviews, formalizes, and tests methods of **TECHNICAL ANALYSIS**. After an evaluation of the well-known practiced methods, we construct a mathematical framework for analyzing the stochastic properties and information structures in a long time series of security price data. **TECHNICAL ANALYSIS** are said to be feasible only when they generate stopping rules.

Moving average methods, stochastic methods, ratio methods, divergence methods, and cycle methods are shown to be feasible **TECHNICAL ANALYSIS** techniques in the sense that they are based on past information only, and not on the

intuition or hunches of traders. Given the feasible methods, it is shown that these can have predictive value above the long autoregressive process prediction theory of Weiner-Kolmogorov only if the underlying security price process is nonlinear.

Empirical tests are suggested to determine the significance of the feasible TECHNICAL ANALYSIS methods, the linearity of various security price series, and ex post profits from trading rules from the set of feasible methods over varied time periods.

The paper provides a framework for evaluating all methods of TECHNICAL ANALYSIS, an important criterion for the feasibility of trading rules, and strong support for the predictive value of the feasible trading rules explored. Important inferences are drawn about the plausibility of the efficient markets hypothesis for securities markets.

Acknowledgements

Professors Grossman and Neftci have been a kind and encouraging influence throughout my time at the Graduate Center. My experience as a graduate student at CUNY has been exceptionally positive, and these two individuals are entirely responsible.

Writing this dissertation has actually been my most pleasurable and rewarding period as a graduate student. If this is what all the coursework, exams, seminars, and discussions with peers come down to, then my time has been well spent. Professor Neftci has provided very meaningful, always constructive suggestions towards the completion of this dissertation. I feel very fortunate to have had the opportunity to work with an economist so full of ideas; certainly his type is the exception rather than the rule in our field. Needless to say, only I am responsible for any remaining errors.

**TECHNICAL ANALYSIS, THE STOCHASTIC PROPERTIES
OF SECURITIES PRICES, AND PROFITS IN TRADING**

Table Of Contents

			Page
		Acknowledgements	vi
		List of Figures	ix
Chapter	1	Technical Analysis: A Systematic Art of Profit Making?	1
	1.1	Technical Analysts at Work	6
	1.2	The Relationship Between the Way Technical Analysts Behave and the Way Security Markets Work?	10
Chapter	2	Methods of Technical Analysis	15
	2.1	Tools and Methods of Technical Analysis	15
		2.1.1 Types of Charts	16
		2.1.2 Terms Used to Characterize Charts	20
		2.1.3 Pattern Recognition Methods	21
		2.1.4 Other Charting Methods - Gaps, Trendlines, and Moving Averages	27
		2.1.5 Numerical Indicators	30
		2.1.5.1 Momentum	30
		2.1.5.2 Timing of a Trend Reversal	32
		2.1.5.3 Sentiment, or Speculative Confidence	37
		2.1.5.4 Cycles	39
	2.2	Literature Review	40
		2.2.1 Mechanical Trading Rules	42
		2.2.2 Volume and Price	45
		2.2.3 Short Interest, Put-Call Ratios, Odd-Lot Trading, and Advance-Decline	47
		2.2.4 Relative Strength	50
		2.2.5 Patterns	50
		2.2.6 Conclusions From the Literature	51
Chapter	3	Technical Analysis Revealed: A Formalized Study of Trading Rules	54
	3.1	A Mathematical Mileu for Technical Analysis	55
	3.2	A Mathematical Reformulation of Technical Analysis Methods	75

	3.3	Technical Analysis Methods Put to a Feasibility Test	84
	3.4	Modeling Security Prices as Markov Processes	87
		3.4.1 Reasons for the Markov Specification	87
		3.4.2 Markov Models of asset Prices in the Literature	88
Chapter	4	The Predictive Power of Technical Analysis	92
Chapter	5	Summary and Conclusions	101
	5.1	Summary	101
	5.2	Suggested Empirical Tests	102
	5.3	Is the Efficient Markets Hypothesis a Shared Act of Faith?	103
		5.3.1 Forms of the Efficient Markets Hypothesis	103
		5.3.2 Efficient Markets and the Random Walk Hypothesis	105
		5.3.3 Empirical Evidence	105
		5.3.4 The Spurious Link With Technical Analysis	106
		5.3.5 Theoretical Argument for the Efficient Markets Hypothesis	108
		5.3.6 Lessons Learned	108
		Bibliography	110

List of Figures

	Page
1.1 Stock Market Index	3
1.2 Bond Market Index	4
1.3 Commodity Market Index	5
2.1 Line Chart	17
2.2 Bar Chart with Arithmetic Scale	19
2.3 Bar Chart with Logarithmic Scale	19
2.4 Head and Shoulders Pattern	23
2.5 Ascending Triangle	25
2.6 Flag Pattern	26

CHAPTER 1

TECHNICAL ANALYSIS: A SYSTEMATIC ART OF PROFIT MAKING?

Traders of financial securities seek to predict changes in security prices.

Many financial economists consider the task impossible. They believe that security markets are "efficient," in the sense that all available information is reflected in today's security price¹.

If markets are efficient, reasoning goes, then individuals using the same information as other individuals should not be able to earn trading profits, or the difference between the high price one sells at minus the low price they bought at, less any transactions costs and an adjustment made for the degree of risk undertaken in the trade. By the same token, if markets are not efficient, or today's security price does not reflect all information, then predicting security prices would be possible, and individuals with the best predictions would take part in a virtual money machine, earning extraordinary profits on mere buy and sell orders.

But speculative traders, in contrast with economists, make their living on the basis of actions undertaken on the belief that security prices are predictable. Consider the speculative trading environment of that group of individuals who believe that

¹ A formally strong way of restating this belief is to say that security prices follow a martingale. A definition for this term will be given in Chapter 4.

security prices are predictable. To do this we focus on the proverbial security price chart.

This chart reveals a winding path pulled first in one direction and then in another by apparently mysterious, unforeseen forces. The path of prices over time, or the price process, is rarely linear - the market appears to steer a meandering course between floors of support and ceilings of resistance. Occasionally, prices go relentlessly in one direction. Figures 1.1 to 1.3 show some actual examples from the stock, bond, and commodities markets.

Based on a purely visual inspection of these and other price charts, we can both ascribe characteristics to the price process and draw inferences about the implications for any model of security prices. The path of prices appears nonlinear; price movements resemble a curve, arc, or wave - not a line. The path is often trending; trades in one direction of a recent price change seem more profitable than trades against a trend. Most short term price fluctuations seem random; a model of the price process will have to dampen the insignificant movements in price before a trend can be used for analysis. Finally, prices appear to alternate between periods of up/down fluctuations and strong one directional movements; the most profitable time to initiate trades appears to be when prices emerge from a trend.

These observations as to how security prices actually evolve over time will allow us to delineate two aspects of the practice of technical analysis. First we will consider technical analysts at work, which will in turn lead to a discussion of how the way technical analysts behave relates to the way security markets work.



Figure 1.1: Dow-Jones Industrial Average,
January 1990-June 1991

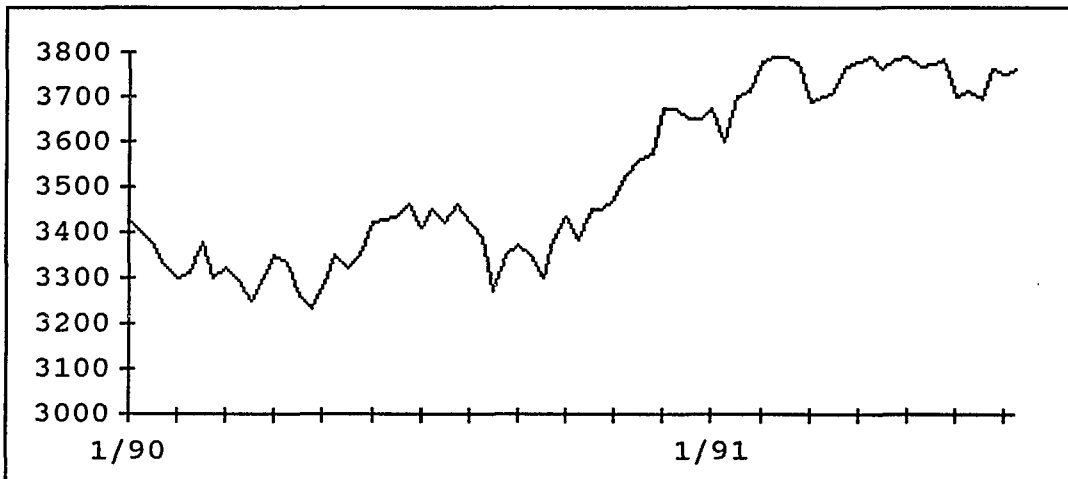


Figure 1.2: Lehman Brothers T-Bond Index,
January 1990-June 1991

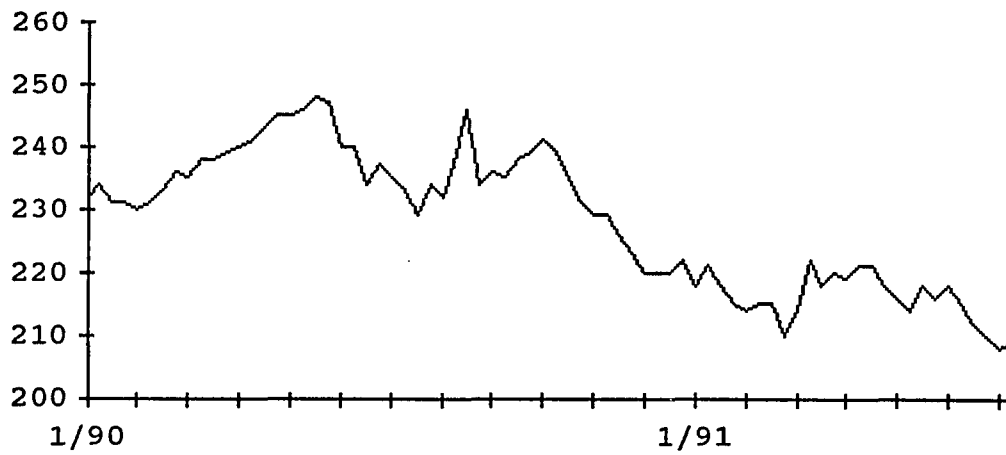


Figure 1.3: CRB Commodity Futures Index,
January 1990-June 1991

1.1 Technical Analysts at Work

Technical analysis is the study of historic asset prices, market averages, and other numeric measures that seek to describe "momentum", the timing of price "reversals", the "sentiment" of traders and investors, and the cyclic activity of prices. Technical analysts include a group of "chartists," who seek to detect subtle patterns that could recur in the future. The market technicians consider far more information in the many numeric indicators².

The goal of technical analysis, the oldest known form of market analysis³, is to separate meaningless price movements from profound ones with the hope of generating signals that allow the trader to issue profitable buy and sell orders. The techniques used in the practice of technical analysis yield only the signals that potentially provide a criterion for trades. Technical analysis can be applied to stocks,

² Market technicians believe that such real world event information is already reflected in the price of a security. They use measures of market sentiment to show how the behavior of a trader may have been influenced by such events. A trader correctly anticipating how events will affect prices, based on today's indicator of market sentiment, will predict the extent to which information of such events is being discounted by the market. Trades are undertaken to profit from any discrepancy in the way the market discounts and the trader's personal prediction of price, based on some notion of how investors are behaving. Some technicians believe such information is usually reflected in measures of sentiment. Some of the sentiment indicators will be discussed in Chapter 2.

³ As an early application of trend analysis, an important technical method described in the next chapter, the price of rice was plotted in Japan in the 17th Century. Technical analysis as we know it today has its roots in the late 19th Century work of Charles Dow. Early charts associated with Dow Theory were published in the early 20th Century in the Wall Street Journal. Dow was interested in "divergences" in the trend of major market averages. This work was done well before company financial information associated with fundamental analysis became publicly available.

bonds, options, futures, mutual funds, and commodities. The data used can be tic-by-tic, intraday, daily, weekly, monthly, or other.

What is so "technical" about the jobs of these technical analysts? "Technical" refers to the fact that the analysts study the action of the prices themselves, not the underlying securities traded in the market. Technicians speak of "breadth," "momentum," and "resistance levels" rather than earnings, management, and new products. The attraction of technical analysis is that the trader does not need to understand⁴ the economic forces that drive the market higher or lower, only that under similar circumstances in the past, the market has performed with some consistency⁵ Technical analysis seeks to determine the predominant trends in the market, the level of support (or lower bound) for price movements, the level of resistance (or upper bound), and the point of a breakout. The manner in which technical analysis is employed by traders making buy and sell orders in the market varies. In describing a technical analyst's view of the June 1991 U.S. equity market, the Wall street Journal recently reported:

Market technicians study a raft of statistical data to measure the market's internal condition and to forecast

⁴ Though most certainly do. Never is the information ignored.

⁵ This fact leads however to great inconsistency in the way in which technical analysts, looking at the same set of indicators, read their systems of indicators. They can reach very different conclusions about individual security price movements and movements of the market as a whole. In "Will the Bull Market Gore 3200? Technicians are Divided," by John R. Dorfman, The Wall Street Journal, (February 20, 1991): C1, seven market technicians are interviewed: three thought the market headed up, four thought the market going down or remaining stable. Another case of disagreement over the market signals, this time in the bond market, is "Whose Right? Bond Bulls or Technicians," by Constance Mitchell, The Wall Street Journal, (May 20, 1991): C1.

its direction. There isn't universal agreement about what all the charts and numbers mean, but most technical analysts feel upbeat.⁶

Yet each technical analyst has his or her own system of prediction, typically based on a distinctive set of technical analysis indicators drawn from the larger set of known market indicators⁷. An analysis of the "practice" of technical analysis is difficult, for all technical analysts appear to use a heady mix of objective (recognizable or quantifiable) indicators and subjective (intuitive) indicators. Some technicians follow the signals of their devised system mechanistically while others intuitively veer from the system's signal. This point is best illustrated with profiles of four technical analysts⁸. The first three analyze the equities market, while the last considers the commodities market.

Kenneth Spence, director of the technical analysis group at Salomon Brothers, considers one of the simplest ways to analyze the market is to look at the market as the percentage of stocks trading above their 200-day moving average. In describing why he believes the stock market will rise, he notes that "Somewhere on the order of 70% of stocks are above their long-term average, indicating that nearly three out of four stocks are under accumulation." He compares this with one year ago, when only forty-five percent of the stocks were above their 200-day moving average. At the

⁶ Douglas R. Sease, "Technicians Say the Market Shows Strong Signs of Life," The Wall Street Journal, (June 17, 1991): C1.

⁷ These indicators are the topic of Chapter 2.

⁸ These examples are reported in the February 21, 1991 and June 17, 1991 issues of the Wall Street Journal.

same time however, Spence acknowledged that the percentage of cash in mutual funds is relatively low. He considers this a sign of a declining market, as it indicates that there is little cash left to fuel any further increase in stock prices. Yet Spence uses this combination of technical indicators to predict that the market will continue to rise, especially in the "cyclical" stocks, representing the industrial sectors that will benefit from an economic recovery - autos, airlines, newspapers, and banks.

Analyzing the same market as Spence, Robert Ritter, a technical analyst at Ladenburg Thalmann & Co., also says that the market will go up, also based among other things on his count of the number of stocks increasing in price, or advancing, versus those that are declining. But like a contrarian, who believes in doing the opposite of the majority of investors and traders, he warns that the market will not rise too soon because the market is too "bullish." He cites polls revealing that a large number of traders are bullish. He sees this as an indicator of a declining market, as there is no longer a pool of traders waiting to jump into the market.

Considering the same market, but reading an opposite signal of market direction was Ricky Harrington, director of investment policy at Marion Bass Securities in Charlotte, N.C. He says that "those negatives (signals of a declining market) would certainly warrant caution. We've been in a trading range now for seventeen to eighteen weeks and there's a good chance that sometime in the third quarter we'll break down below a Dow of 2480."

The Wall Street Journal regularly surveys the top twelve commodity funds, or

professionally managed futures trading portfolios⁹. Commodity trading advisors buy and sell commodity futures and options contracts for individual investors, private investment pools, and commodity funds. These advisors fall into two broad groups: technicians and fundamentalists. More than ninety percent of all those trading in commodities markets today are technicians¹⁰. On average for 1990, the funds earned twenty percent returns. Most of the large gains for the year came from "trend followers," advisors who use mathematical techniques to determine when a rising or falling trend in price has started. "Dr. Druz, a 37-year-old emergency room physician who lives in Hawaii and commutes monthly to practice medicine in Alaska, had the best performing funds not only for 1990, but also for the three years through December 31." Druz claims to be a technician who always follows the dictate of his system. He purports to use a trend following strategy of buying and selling futures when he believes a price trend has started up or down. Not surprisingly, Druz did not reveal this strategy to the Wall Street Journal.

1.2 Is There a Relationship Between the Way Technical Analysts Behave and the Security Markets Work?

The framework used by technical analysts to predict security prices has little

⁹ "Dr. Druz Captures Commodity Crown," The Wall Street Journal, February 21, 1991, C1.

¹⁰ Stanley W. Angrist, "Commodity Advisors Show Their Mettle," Wall Street Journal, (November 15, 1990): C1. Note however on this point that statistics quoted in this article also indicate that seventy percent of all individual commodity traders come out as losers on an annual basis.

relationship with the classical view in economics about how prices evolve over time. Models of financial markets in the economic literature have traditionally assumed that security prices are linear. Based on a notion that the security market is "efficient," economists have modeled security prices as evolving linearly (as a martingale) over time. Today's security price is equal to yesterday's price plus some (positive or negative) amount that reflects the value of the information that arrives today. Hence, we have the idea of economists that prices evolve linearly over time. Additionally, in order to model the economic "long run," economists have assumed that prices will over time converge to a steady state if they are not upset by some exogenous force, like strikes, raw material shortages, changes in government policy, changes in taste, etc¹¹. The problem with this view, recognized decades ago by security traders, is that real-world security prices exhibit what appears to be nonlinear and cyclic behavior¹².

Among all security traders, there are two lines of thought about how security prices are determined: fundamentalists and technical analysts¹³. The fundamentalists believe that security prices reflect fundamental values in the economy. Security prices are determined by actual and expected developments concerning these values. Then, forecasting security values is a matter of forecasting fundamentals.

¹¹ For the statement of this belief, see E. Fama, "The Behavior of Stock Market Prices," *Journal of Business*, 38 (1965): 34-105.

¹² Evidence for this view will be discussed in Chapter 4.

¹³ Forms of market analysis include the random walk theory, modern portfolio theory, economic forecasts, fundamental earnings analysis, and technical methods. The majority of security traders tend to focus on the fundamental and technical methods.

The technical analyst takes little account of fundamental values because it is assumed that most investor expectations concerning value (and other information) are already reflected in prices. The perception is that financial markets attempt to anticipate the future of the underlying securities. Although market activity is thought to reflect day-to-day business developments, technical analysts believe it to be primarily concerned with future expectations. Hence, changes in financial market prices are thought to precede changes in investor perceptions, perhaps triggered by actual changes in the fundamentals or other factors affecting investor sentiment.

For many years, technical analysts have recognized that: one, certain (recurring) price patterns have predictive power, two, the extent of price movements can be calculated in advance, and three, that there are regular price cycles working in the market. The problem however in attempting to evaluate these claims is that few practitioners of technical analysis understand why in terms of formal statistical prediction theory their forecasting techniques work. The typical methods used by the technical analyst include pattern recognition, moving averages, and various indicators.

Technical analysts believe that asset prices move in long lasting trends, where any change in the direction of prices is caused by changes in investor sentiment, which cause shifts in the supply and demand for securities. Technical analysts seek to predict changes in the supply and demand for a security. If the reason for these changes is emotions, or animal spirits, prediction is difficult, but indicators of changes in belief can be devised (some of which are documented and discussed in the next chapter).

The reason to attempt to formalize and study technical analysis is its

widespread use, its attempt to recognize crowd psychology, and the fact that nonlinearity of security prices implies that standard (linear) econometric method offer little benefit. The purpose of this text is to evaluate the claims of technical analysts. Chapter 2 presents a tedious survey of the current popular (documented) methods employed by technical analysts, and ends with a survey of the extant economic literature regarding the effectiveness of these methods in issuing signals that lead to profitable buy and sell orders in security markets. In Chapter 3 a formal environment for evaluating the claims of technical analysts will be introduced, and broad categories of their methods, based on the review in Chapter 2, will be evaluated on the basis of their feasibility as predictive tools. Based on a theory of natural systems, we seek to establish a framework appropriate for a description of the dynamic process of security prices. The framework provides a unifying foundation for the traditional methods of technical analysis - like moving averages, trendlines, price patterns, and even cyclic waves. In addition, a criterion is developed to determine which techniques should in theory be meaningful for prediction. Contrary to popular thought amongst economist, the methods of technical analysis will be revealed as relatively easy to evaluate, even when acknowledging the breadth of the methods employed and the imprecise and inconsistent ways in which the trading rules are often applied. Several technical analysis methods prove to be feasible in the sense that the strategies generate trading rules based only on today's information set. Chapter 4 formally considers the predictive power of technical analysis techniques. Confirming the results of Neftci (1989), we show that these trading rules can only have predictive value above standard (linear) econometric techniques if the price process is nonlinear. Chapter 5

contains a summary and brief set of conclusions. Based on the theoretical results on prediction in securities markets, several suggestions are made for future empirical work. Finally, some implications of our work on the efficient markets hypothesis will be drawn.

CHAPTER 2

METHODS OF TECHNICAL ANALYSIS

The methods employed in technical analysis range from intuition to sophisticated econometrics. What follows is an overview of some of these techniques, with emphasis placed on actual practice, rather than on the similarities that might exist with existing econometric methods. In practice, technicians gather information: price, timing, volume, sentiment, trend, and so on. From this information, they devise systems to indicate price momentum, the timing of a change in the direction of prices, sentiment, and cycles. The categorization of techniques for discussion in this chapter is therefore practical, rather than methodological, as based on the type of information to be exploited from the data available.

The survey of technical analysis methods will consider first the language, or jargon, of technical analysts and then their methods, or devised techniques. The second section of this chapter is a survey of the economic literature about the effectiveness of various trading rules in yielding profitable trading strategies.

2.1 Tools and Methods of Technical Analysis

Traditionally, technical analysis has been concerned with the recognition of patterns. Technical analysts graph prices on several types of charts: line charts, bar

charts, point and figure charts, equivolume charts, and candlestick charts. There are literally hundreds of patterns that technical analysts try to spot; the most popular are head and shoulders, double tops/bottoms, triple tops/bottoms, broadening formations, diamonds, triangles, flags, pennants, wedges, rounding tops/bottoms, and gaps. In addition, technical analysts graph trendlines and moving averages. Various numerical indicators have aided technical analysts locate reversal points in a trend. In the next sections we discuss the types of charts used by technical analysts, the technician's jargon for describing changes in price and volume on the chart, and the popular patterns identified by some technicians.

2.1.1 Types of Charts

Line charts, bar charts, point and figure charts, equivolume charts, and candlestick charts are the most common chart types used in technical analysis. Line charts depict the path of a single variable over time. Examples of line charts include daily closing prices of stocks, weekly market volume, or monthly values for a trader sentiment indicator. Line charts are also used to depict moving averages and relative strength. Other variables are also illustrated on line charts. Figure 2.1 illustrates the put-call ratio for February 4 through April 30¹.

The bar chart is the most common technical chart tool. It shows the high, low

¹ The Put-Call Ratio (PCR) is discussed later in Chapter 2. This line chart shows the daily volume of put options divided by the daily volume of call options on the S&P 100 stock index. Some technicians believe a ratio above 1.3 indicates a bullish market and below 0.6 indicates a bearish market. The data for this chart was collected from issues of The Wall Street Journal.

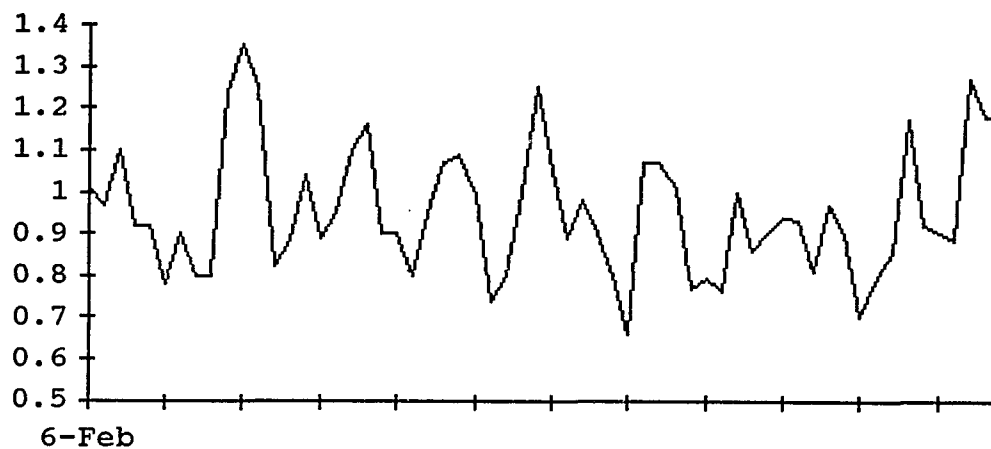


Figure 2.1: Line Chart of the Put-Call Ratio

and closing prices of a particular security over time. The increment of time for the bar chart might be weekly, monthly, or yearly. For both the line and bar charts, numbers may be arithmetic (for measuring price changes on an absolute basis) or logarithmic (for measuring price changes on a percentage basis). Figures 2.2 and 2.3 show the same bar chart with arithmetic and logarithmic scales for the same time period.

Another type of chart is the point and figure chart. The chart contains X's to reflect a price move up or down of one point. The active stocks will have charts with more Xs than the charts for inactive stocks. Each column in the point and figure chart reflects the direction of price. There are two types of point and figure charts: the one point and the three point reversal charts. The one point reversal chart illustrates movements of one point or more in each direction. On the three point reversal chart, each column of the graph illustrates movements of at least three points in one direction. One might use the three point reversal point and figure chart when one is interested in portraying data over a longer time period. We see that the point and figure charts plot price changes only when prices have changed by a prespecified amount. The charts give no indication of calendar time.

Equivolume charts are used to show the relationship between price changes and volume changes. A bar on the equivolume chart shows the daily closing price, while the width of the bar indicates the volume of trades on the same day.

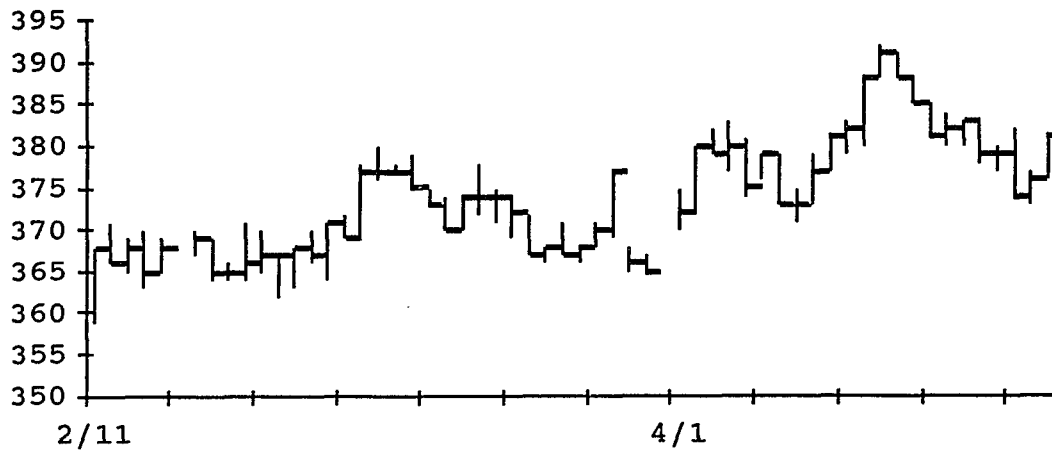


Figure 2.2: Bar Chart with Arithmetic Scale

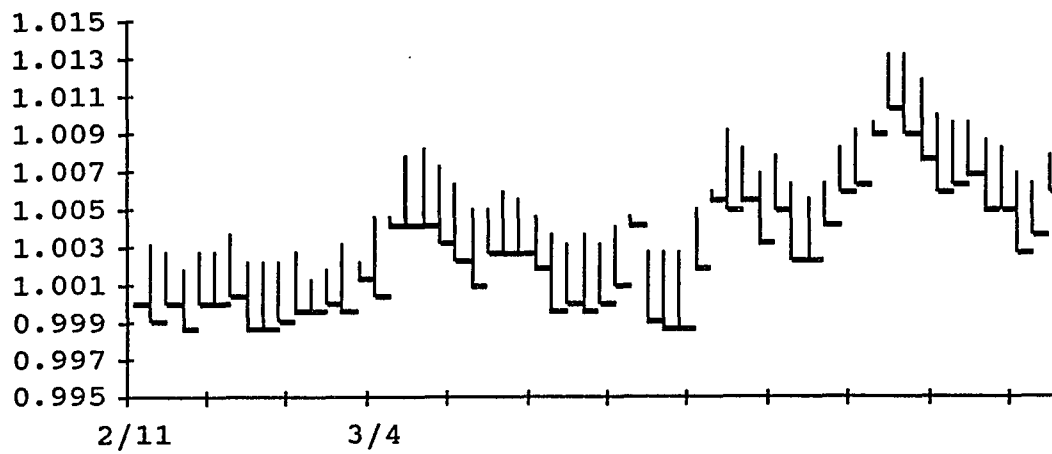


Figure 2.3: Bar Chart with Logarithmic Scale

The candlestick method² uses boxes to represent price movements for the day. The box length is determined by the opening and closing prices of the day, and the box's shade is determined by whether prices closed higher or lower than the previous day. If the close was higher than the open, the box is left hollow; the box is shaded if the close was lower than the open. A line beneath the box represents the day's high and low price.

2.1.2 Terms Used to Characterize Charts

Technical analysts speak of "support" and "resistance," terms that have to do with the supply and demand of assets. Traders in the market think of changes in supply and demand as due primarily to changes in the crowd psychology of the market, causing changes in the behavior of market participants. A level of support is a price, or price range, where there is demand, or investors willing to buy. Resistance is the price range where there is excess supply, or where investors are willing to sell³.

A reversal is the pattern of prices that marks a change in trend. Reversal patterns at market tops are sometimes known as distribution patterns (selling points), and those at market bottoms are sometimes called accumulation patterns (buying points).

² McMurray, Scott. "Japan's 'Candlesticks' Light Traders' Path." The Wall Street Journal (Nov. 8, 1990): C1 and Morris, Greg, "East Meets West: Candlepower Charting," Technical Analysis of Stocks and Commodities, (December 1990): p. 16.

³ For an example of the concept applied see Craig Torres, "Dow 'Resistance' is the Reason Why 3001 is Still Sci-Fi," The Wall Street Journal, (March 7, 1991): C1.

A breakout is a penetration out of pattern or a previous level of either support or resistance. A breakout at a level of resistance is a victory for buyers, as increasing demand is met with less resistance from supply.

The question then for the technical analyst is when a change will occur. We said earlier that technical analysts in general believe that the market follows long run trends that will continue until a reversal occurs. But reversals sometimes take time to occur. Hence the transition from an uptrending to a downtrending market, for instance, will be characterized by patterns reflecting a change in crowd psychology. Each of the terms above give us insight into the characteristics of trend; technical analysts will seek indicators to identify these characteristics. The use of these terms is illustrated in Figure X, depicting the Dow-Jones Industrial Average from 1986 to the beginning of 1991. In the next section we describe some patterns recognized by technical analysts.

2.1.3 Pattern Recognition Methods

A recent statement in the technical analysis literature follows:

Pattern recognition has been the topic of much technical literature...traders have been told that some fairly well-defined chart patterns indicate the market's position and the most likely outcome of the ensuing market activity, as indicated from a particular chart pattern.⁴

Some patterns indicate a continuation of market trend, others indicate a market reversal. Market reversals will be indicated by head and shoulders patterns, double

⁴ Robert Miner, "Form and Pattern as a Trading Tool," The Technical Analysis of Stocks and Commodities, (May 1991): 62.

tops, triple tops, broadening formations, diamonds, triangles, key reversal days, and gaps. Continuation patterns are flags, pennants, and wedges; they are short term price patterns developing at points along a trend. In anticipation of the discussion in the next chapter, market highs and lows will be referred to as local extrema, where a market high is called a local maxima and a market low is called a market minima.

The head and shoulders pattern occurs at market tops and bottoms, or just before the point of a market reversal⁵. The pattern consists of the head (major extrema) surrounded by two shoulders (lesser local extrema on each side of the major extrema). Figure 2.4 illustrates.

Volume changes over time during the formation of the pattern allow for an assessment of the validity of the pattern. In a valid head and shoulders pattern, we expect the heavy volume during the formation of the left shoulder and as the price reaches its overall maximum. Heavy volume occurs again at the pattern breakout. A line joining the minimum extrema is called the neckline. The head and shoulders top pattern illustrated above indicates a market downturn. A head and shoulders bottom indicates a market climb. The penetration of the neckline is the signal of the trend reversal.

The double top pattern is characterized by two maxima separated by a local minima. Typically the volume during the formation of the second extrema is less than during the formation of the first. The double top pattern will indicate a market downturn. The double bottom pattern is just the inverse of the double top; two

⁵ Shaw (1988), p. 341.

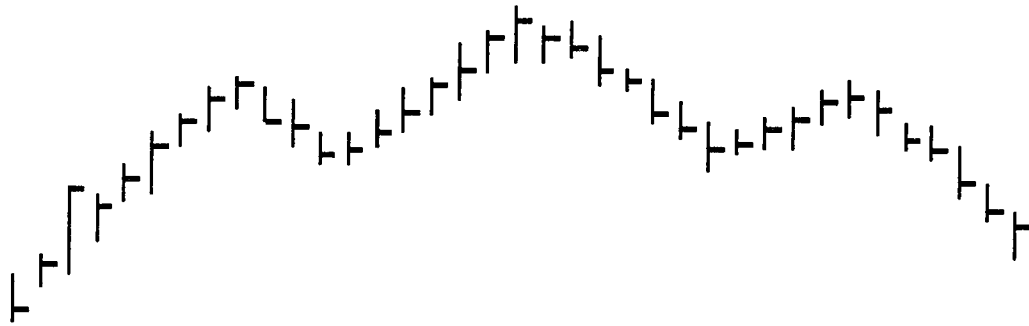


Figure 2.4: The Head and Shoulders Pattern

minima are separated by a local minima. The triple top and triple bottom patterns are similar; three extrema are separated and volume diminishes throughout.

The diamond pattern is characterized by a series of local minima and maxima that can be enclosed in a diamond shaped figure.

Triangles are thought to be the most common, but least reliable pattern. The triangles may be symmetrical, ascending, or descending. The symmetrical triangle pattern encloses decreasing price fluctuations in two converging lines with approximately the same slopes in absolute value. The pattern demonstrates a failure to advance during the period, and indicates a market that will fall⁶. The right angled triangle encloses the price fluctuations in converging lines where one of the lines is horizontal. Neither type of triangle pattern indicates the direction of the price process once it leaves the triangle. The ascending triangle is illustrated in Figure 2.5.

The flag pattern is the first of our continuation patterns, and is formed during a pause in a sharp or vertical price trend. During the formation of the pattern, the volume contracts. When the formation is completed, the price continues in the direction it was moving before the pattern. The pattern may take anywhere from five days to five weeks to form. The flag pattern is illustrated in Figure 2.6.

A pennant pattern is formed in the same way as the flag pattern except that the pause can be enclosed in converging lines tangent to the peaks and troughs. The wedge pattern is like the triangle except that the converging lines are either rising or falling.

⁶ Miner, p. 62.

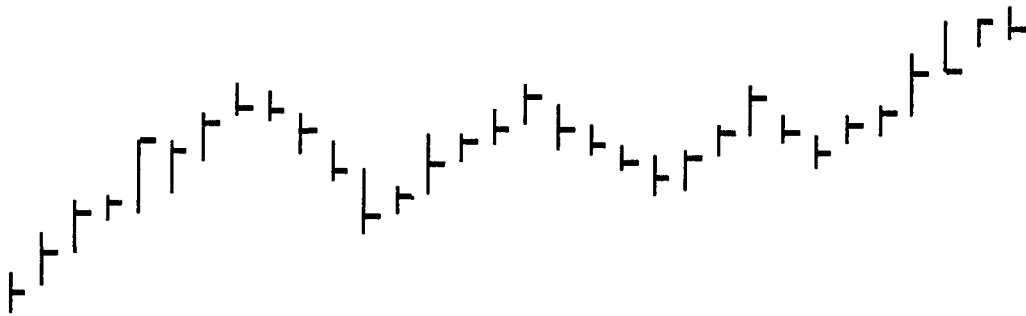


Figure 2.5: Ascending Triangle Pattern

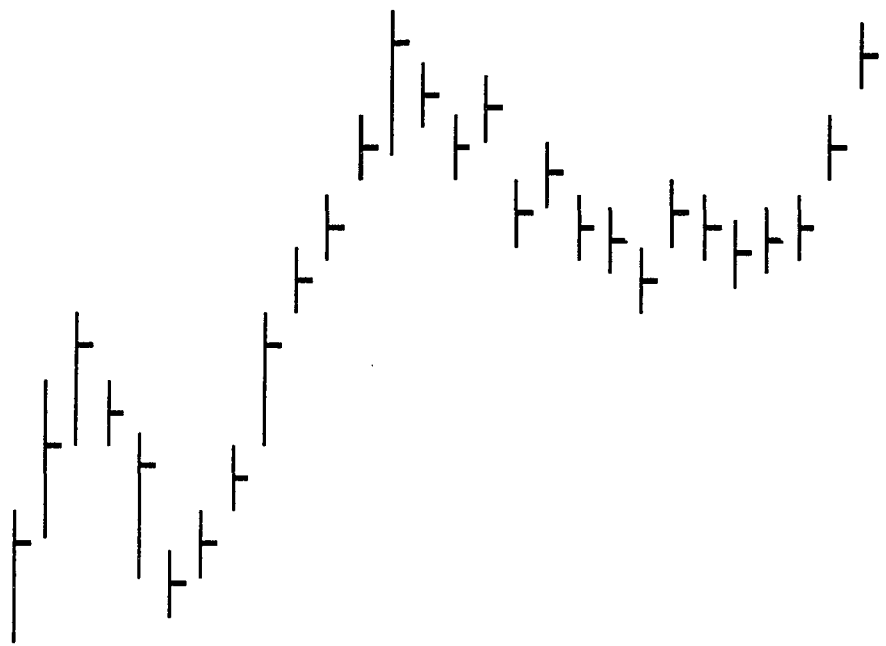


Figure 2.6: The Flag Pattern

A market reversal usually occurs in either a rounding top, rounding bottom, or a key reversal day. The key reversal day is accompanied by high volume. During the rounding top or rounding bottom, volume will rise and then fall. The key reversal day is a one day price pattern representing a short term trend reversal

Shaw (1980) claims that bar charts and point and figure charts are most likely to reveal either the head and shoulders, double top, or triple top patterns. Point and figure are said to most likely show all types of triangles, indicating price "consolidation." Shaw⁷ also provides the following caveat:

Technical students of the market will need to experiment for themselves with each of the two charting techniques (bar charts and point and figure charts) to discover which one they feel most comfortable with.

2.1.4 Other Charting Methods - Gaps, Trendlines, and Moving Averages

A gap is a trading leap over a range of prices to reach a new level, leaving a blank space in the price chart. They occur when the lowest price of a specific trading day is above the highest level of the previous trading period, or vice versa. If the spaces are wider than the "usual" one-eighth to one-half point difference in bids, this implies that significantly greater price movements are expected in the future⁸. A breakaway gap occurs when the price breaks out of a price pattern. High volume usually accompanies, or comes just after, the gap. A runaway gap occurs during a straight line advance or decline when the price moves rapidly. The runaway gap

⁷ Shaw (1980), p. 351.

⁸ Thom Hartle and Melanie F. Bowman, "Gaps," The Technical Analysis of Stocks and Commodities, (November 1990): 26.

indicates high emotions. The exhaustion gap usually occurs on the second or third runaway gap in a short time. It indicates that the trend is soon likely to reverse. Breakaway gaps occur at the start of a move, runaway gaps in the middle, and exhaustion gaps at the end. Gaps are thought to indicate a change in crowd psychology.

Trendlines may be either upward sloping, downward sloping, or horizontal. An example of a trendline is the neckline of the head and shoulders pattern. The overall significance of the trendline is determined by the number of tangencies with the price process and the length of the trendline. A trendline is characterized by at least three tangencies. A valid break in the trendline occurs if the price goes "far" beyond the trendline, and the break is accompanied by increasing volume, as when closing prices go, say, three percent above the trendline or when the price closes outside of the trendline for, say, two consecutive days. It is thought that the longer a trendline is in place without having been crossed by closing prices, the more important the trendline⁹. The angle of the trendline is also considered important; the steeper the line, the sooner the trendline is likely to end.

Trend channels occur when we can draw a two parallel trendlines joining two sets of extrema. The two trendlines indicate the lines of support and resistance. Reversals are signaled when either the price breaks out at the bottom of an increasing trend channel or if the price breaks out of the top of a decreasing trend channel.

Moving averages, probably the most popular tool in technical analysis, are

⁹ Melanie F. Bowman and Thom Hartle, "Trendlines," The Technical Analysis of Stocks and Commodities, (February 1991): 58.

used to reduce the fluctuations in the daily price movements, to filter data noise, and to identify the underlying trend. The average of the daily closing price indicates the general direction of market movement. There are three types of moving averages used by the technical analysts: simple, weighted, and exponential.

The crossing of two moving averages of different period some believe indicates a change in the direction of a price. A special type of moving average is the exponential moving average, which is more sensitive to price changes than simple moving averages, and is thus helpful in identifying trend reversals. In addition it remembers all past data, only assigns it less weight, and it is less sensitive to either the very low or the very high points. Some use the exponential moving average to ensure that the average falls quickly to better reflect large, sudden drops in price. The difficulty in using this technique is deciding the appropriate length of the moving average. If the moving average is too short, the moving average will be too sensitive, while a moving average that is too long will lag the price movement. A statistical criterion has been suggested for determining the appropriate length of a moving average.¹⁰ Some computer software programs allow the technician to optimize the length of the moving average to ensure the best prediction for each security or market average analyzed¹¹.

Because of the way in which moving averages are computed, they will always lag the security price movement. If the price falls below an uptrending moving

¹⁰ George Arrington, "Building a Variable Length Moving Average," The Technical Analysis of Stocks and Commodities, (June 1991): 18.

¹¹ Shaw (1980), p. 336.

average, this signals impending weakness in the market. An example of the use of moving averages follows: using weekly data, compare the price today with the average price in the last 40 weeks. If the current price rises above the moving average, buy the stock; but if the price is less than the moving average, sell the security short. If the current price then rises above the moving average, all short positions should be covered. If the price falls below the moving average, all long positions should be eliminated.

2.1.5 Numerical Indicators

1. Momentum

While trendlines and pricing patterns indicate changes in direction after they have already taken place, momentum measures the rate at which prices are rising or falling. Momentum indicators can signal the duration of a trend. Some technical analysts believe that prices tend to rise the fastest just before their peak; prices falling fastest may be near their bottom.

Relative strength¹² can be computed in several ways. The most common method is to divide the daily (or weekly) closing price of the stock (or group) by a market average or index, most often the S&O 500 Index¹³. It shows the strongest performers on the basis of price performance a sample of stocks. Typically an investor would wish to purchase the stocks with rising relative strength, and sell the

¹² Robert L. Hand, Jr., "Relative Strength Investing," The Technical Analysis of Stocks and Commodities, (May 1991): 42.

¹³ Shaw (1988), p. 337.

stock's whose relative strength is falling.

There are two types of Advance/Decline Momentum indicators: difference indicators and ratio indicators¹⁴. The most common type of difference indicator is the simple absolute value of the difference between the number of advancing stocks and the number of declining stocks.

Advance/decline ratios measure the ratio of advancing stocks to declining stocks, usually measured on a ten day percentage basis. At 1.25, the ratio indicates that the market may be overbought, and is likely to decline. A ratio of .75 indicates that the market may soon rise.

The ADX indicator¹⁵ is designed to rate the intensity of a security price move. To calculate the ADX, compare the current day's trading range with the previous day's range. The difference between today's high and yesterday's high is called directional movement (DM). DM is divided by the true range (TR), where TR is the largest of either the price difference between today's high and low, the price difference between yesterday's high and yesterday's close, or the price difference between today's low and yesterday's close. Then sum the last fourteen days for +DM, -DM, and TR. Calculate +DI(14) and -DI(14). The DM Index (DX) is

¹⁴ Fay H. Dworkin, "Defining Advance/Decline Indicators," The Technical Analysis of Stocks and Commodities, (July 1990): 66.

¹⁵ Thom Hartle, "ADX." The Technical Analysis of Stocks and Commodities. April 1991, p. 24.

$$DM\ Index(DX) = \frac{|+DI(14) - (-DI(14))|}{+DI(14) + (-DI(14))}$$

The ADX is the fourteen day moving average of the DX.

Another suggested indicator of momentum is ten day momentum¹⁶, or the momentum oscillator:

$$10\ Day\ Momentum = \frac{\sum_{n=1}^{10} (dailypricechange \times dailyvolume)}{10}$$

2. Timing of a Trend Reversal

Fibonacci number ratios are used to predict support and resistance levels.¹⁷

The most common numbers are 38% and 61.8% price changes as indicators of the market turning point. Once a trend exceed two-thirds of the previous move, some say to expect a reversal¹⁸.

The stochastic indicator attempts to predict the timing of trend reversals by looking at a closing price within a range of recent highs and lows. The close price is expressed as a percentage of a five to eighteen day range of highs and lows. If prices are rising, they should fall closer and closer to the high within the range of the recent

¹⁶ Darryl W. Maddox, "Calculating Momentum in a New Way," The Technical Analysis of Stocks and Commodities, (April 1991): 58.

¹⁷Slezak, George "How Fibonacci Can Forecast Stock Market Resistance Levels" Futures July 1989, pp 36-37.

¹⁸ John J. Kosar, "Support and Resistance Levels," The Technical Analysis of Stocks and Commodities, (January 1991): 62.

highs and lows. Internal weakness of the price trend is indicated if the close prices sag further away from the range high. The stochastics indicator is %K:

$$K = \frac{(\text{Today's close} - \text{low of range})}{(\text{High of range} - \text{low of range})} \times 100$$

However, the most commonly used version of this indicator is %D:

$$\%D = \frac{\sum (\text{today's close} - \text{low of range})}{\sum (\text{high of range} - \text{low of range})} \times 100$$

where the sums are over the latest three day period. However, this chosen daily length can vary across technicians. An overbought zone is when %D is 80 percent¹⁹ or more; the oversold zone is 20 percent or below. Some technicians use 70 and 30 percent.

Divergences between the Dow Industrials and the Dow Transports.

Divergences between %D and the price is also used to indicate a trend reversal.

Divergences are thought to indicate changes in market direction (called an advance/decline or A/D indicator). The oldest, going back to the genesis of Dow Theory, prescribes a change in market price movements, or trend reversal, when for instance the DJIA starts moving in a direction different from the movement of another major index, like the DJTA. A divergence is indicated in this case when $\text{sign}(\Delta \text{DJIA}) \neq \text{sign}(\Delta \text{DJTA})$. Another example is the Moving Average Convergence/Divergence (MACD) measure which looks at the crossing point of two exponentially smoothed moving averages. Another example is some form of the

¹⁹ Mike Takano, "Stochastic Oscillator," The Technical Analysis of Stocks and Commodities, April 1991, p. 24.

difference between the number of market advances less the number of market declines.

An indicator that tells whether the stocks that are increasing or the stocks that are decreasing are receiving more volume is the Arms Index, or Short Term Trade Index (TRIN).²⁰ The Arms Index is based on the idea that the volume of trades is closely linked with the sentiment prevailing in the market. If A is the ratio of advancing issues over declining issues, and B is the ratio of advancing volume over declining volume, then the Arms Index is the ratio of A to B. When the Arms Index is greater than one, this means that the volume of the average stock that fell is greater than the volume of the average stock that rose, or that the sellers dominate the market. When the Index is less than one then the buyers control the market. An increasing market is accompanied by low numbers for the Arms Index. Users of the Index warn that the direction and speed of changes in the Index values are more important than the absolute value of the Index. If the Index quickly moves toward higher numbers, regardless of the current reading, the indicator warns of a forthcoming downward move in the market. Rapid moves towards lower numbers indicates that the market will rise.

However, when we convert this index into a long run moving average, it can be used to indicate market conditions. A low moving average indicates an

²⁰ Peter Eliades, "Buying Opportunity: That's What the Technical Indicator Says," Barrons, September 21, 1987 and Richard A. Arms, Jr., "Using the ARMS Index in Intraday Applications," The Technical Analysis of Stocks and Commodities, (April 1991): 36 and Richard A. Arms, Jr., "Cross Your Arms," The Technical Analysis of Stocks and Commodities, (May 1991): 34 and "Arms on Arms," Interview With Richard A. Arms, Jr., Technical Analysis of Stocks and Commodities, (July 1991): 42.

overbought, or falling market, as market prices have risen too sharply too fast. The twenty-one and fifty-five day moving averages are recommended in constructing the 21/55 Crossover Arms indicator; if the twenty-one day line is below the fifty-five day line, this indicates a falling market where stocks should either not be bought or be short sold.

The Moving Average Convergence Divergence (MACD) Indicator²¹ graphs the difference between the 26 and 12 day exponential moving average and the 9 day exponential moving average of this difference and is used to signal overbought or oversold conditions. The typical constant used in the exponential moving average formula is between .07 and .15, from the formula $2/(n+1)$, where n is the moving average length. When prices change, the first line will cross the second. Indicating continuation or reversal of a trend, a buy signal is given when the first line crosses above the second. The aggregate supply of stocks trading is measured as the total dollar volume of stocks trading. Shrinking supply would drive up prices.

On-Balance Volume (OBV)²², the running sum of each day's volume, is used to show a positive or negative divergence from the price line in a particular market. A change in the direction, or breakout, in this index, will indicate a change in the

²¹ Alexander Elder, "How to Use MACD to Catch Price Trends Early," Futures (September 1986):68-69 and Thom Hartle, "MACD," The Technical Analysis of Stocks and Commodities (April 1991): 25.

²² Joseph Barics, "WAD Trades ABX," The Technical Analysis of Stocks and Commodities, (February 1990): 16 and Daniel E. Downing, "On Balance Volume and the Dow Jones Utility Index," The Technical Analysis of Stocks and Commodities, (March 1991): 61.

direction of prices. For example, if the index becomes positive, this may indicate a change in the demand for stocks that may lead to higher stock prices in the next six months. OBV is perhaps the best-known example of the cumulative sum methods in technical analysis.

The Tick Index²³ is a contrarian indicator calculated from intraday price tick readings. The Tick Index is the difference between the number of shares on an uptick (where the current price is higher than the previous price), and the number of shares traded on a downtick (where the current price is lower than the previous price). For example, if an exchange has 1000 issues trading on an uptick and 250 shares trading on a downtick, then the Tick Index is equal to $1000 - 250 = 750$. When the market is filled with buyers, the index will become very large. The contrarian would sell stocks when the Index becomes too large and buy stock when the Index is relatively small. Although the Index is based on data received with each market tick, the positions taken on the basis of the index will typically last from one to ten days.

The trend exhaustion index (TEI)²⁴ is thought to give an indication of a change in market direction. the TEI is calculated by dividing the number of new highs on the NYSE (H) by the number of advancing NYSE stocks (A) and plotting a ten-day exponential moving average of this value:

In an advancing market, the TEI shows a pattern of rising peaks, each peak higher

²³ Tim Ord, "Picking Tops and Bottoms With the Tick Index," The Technical Analysis of Stocks and Commodities, (June 1991): 45.

²⁴ Clifford L. Creel, "Trend Exhaustion Index," The Technical Analysis of Stocks and Commodities, (January 1991): 18-22.

$$TEI = TEI_{-1} + .18 \left(\frac{H}{A} - TEI_{-1} \right)$$

than the last. And a lower peak after a period of rising peaks in the TEI signals an end to a market advance.

Percentage retracements are considered indicators of a change in market direction²⁵. The most common retracement level used by technicians is to be fifty percent. It is thought that a correction in a trending market will retrace approximately fifty percent of the previous move before resuming the trend. Other common retracement percents are thirty-three and sixty-six.

3. Sentiment, or Speculative Confidence²⁶

The Put-Call Ratio (PCR) is calculated as the volume of puts divided by the volume of calls. The PCR is often calculated using daily or weekly volume figures for the S&P 100 Index Option (OEX) or the Chicago Board Options Exchange (CBOE) equity options. The ratio is considered important in market timing strategies, or indicator of the investing public's sentiment the market. Professionals use the measure as a contrarian indicator under the premise that option trading is dominated by an unsophisticated public, not by trading professionals. An increase in the volume of puts relative to the volume of calls indicates that the investing public is acting as if the market will decline. Contrarians believe this to indicate that the market will

²⁵ Kosar, p. 62.

²⁶ An example of sentiment applied to an analysis of the market is in Douglas R. Sease, "Consumer Euphoria Doesn't Guarantee a Stock Rally," The Wall Street Journal, (March 25, 1991): C1.

increase.

A call-put TRIN²⁷ is calculated as another sentiment indicator:

$$\text{Call-Put TRIN} = \frac{\frac{\text{callvolume}}{\text{callopeninterest}}}{\frac{\text{putvolume}}{\text{putopeninterest}}}$$

The NYSE Bullish Percent²⁸, based on the percentage of stocks on the NYSE that have point and figure charts indicating increasing prices, is a sentiment indicator:

$$\text{NYSE Bullish Percent} = \frac{\text{number of stocks giving a buy signal}}{2000 \text{ stocks on NYSE}}$$

where the "number of stocks giving a buy signal" equals the number of stocks currently above their moving average.

The Mendelson Sentiment Indicator²⁹, named for its originator, John Mendelson, seeks to determine the point in which investors have "lost control" (due to the fact that they are either overly anxious to get either in or out of the market). The indicator has three components: the rate of change in net volume, two, the rate of price change for the market's ten most active stocks, and the CBOE PCR. When the

²⁷ James P. Merrill, "Updating Option Ratios With Market Sentiment," The Technical Analysis of Stocks and Commodities, (February 1991): 58.

²⁸ Thomas J. Dorsey, "NYSE Bullish Percent as an S&P Indicator," The Technical Analysis of Stocks and Commodities, (July 1990): 58.

²⁹ The Wall Street Journal, March 12, 1991, p. C1.

combination of these indicators hits a prespecified extreme, Mendelson uses this indicator as a contrarian signal of when to buy or sell. Other well-known sentiment indicators include the ratio of insider buyers to insider sellers, the 30 day upside/downside volume, the count the number of stocks in the S&P 500 that are above their 30 week moving average, and the large block ratio, which measures the number of trades of 50,000 shares or more made on the upticks or the downticks.

4. Cycles

Many technical analysts believe that temporal cycles have an influence on the stock and commodities markets³⁰. Cycles fall into four major categories: seasonal cycles, long-term cycles, intermediate term cycles, and short-term cycles.³¹ A knowledge of cycles helps in the prediction of tops and bottoms. An example of a long-term cycle is the Kondratieff 54 year wave. Four year cycles are thought to exist because when we use the standard deviation of the market average as a risk parameter, risk averaged over a four year period becomes relatively smooth³². Nine year cycles mimic the tree ring cycles. Eleven year cycles mimic solar radiation

³⁰ James A. Arnold, "Four Year Cycles," The Technical Analysis of Stocks and Commodities, (September 1990): 76 and Lewis Carl Mokrash, "Looking at 10-Year Stock Price Patterns," The Technical Analysis of Stocks and Commodities, (April 1991): 26.

³¹Wilson, Jeff "Using Cycles to Observe and Respond to Markets." Futures June 1989, pp 20-21.

³² See Arnold, p. 76.

cycles. Unfortunately, statistical studies have found these cycles difficult to verify.³³

Elliott Wave Theory is a pattern recognition technique established by Ralph Nelson Elliott in 1939³⁴. Elliott believed that the stock market follows a rhythm or pattern of five waves up and three waves down to complete an eight wave cycle. The three waves down are considered a "correction" of the five waves up.

Listed above is a mere description of some technical analysis methods³⁵. The review above reflects more the documentation available than which techniques are applied most often. As illustrated in Chapter 1, the application of the methods to the prediction of security prices varies from trader to trader. A "correct" application of the methods is described by technical analysts to be more a learned art than a science, and styles will vary. For an application of these methods to the prediction of the degree and timing of price changes for individual stocks, industries or groups of stocks, and the market, see Shaw (1988), Pring (1986), Barrons (frequent issues, but at least once a month), The Wall Street Journal (often articles appearing Section 3, Money and Investing), and numerous other texts on technical analysis.

2.2 Literature Review

³³ See Mokrash, p. 26.

³⁴ "Timing the Bond Market With Elliott and Fibanacci," The Technical Analysis of Stocks and Commodities, (November 1990): 54.

³⁵ Based on some of these methods, many online software packages have been developed for market technicians. PC Magazine (April 15, 1986) reviews fifty technically oriented software packages. Reviews of the most recently developed computer packages for technical analysts appear monthly in Technical Analysis of Stocks and Commodities.

In the first half of this century, there was a general belief among both economists and securities traders that stock prices followed repeating patterns.³⁶ Study of these patterns was thought to provide the information required to predict future prices. But by the mid-1960s these ideas were overshadowed by the new idea of "efficient markets," which says that security markets are efficient in that today's security price reflects all relevant information. The efficient market economists think security prices change only to reflect new information; since new information arrives at random moments, the changes in security prices were considered to be random changes.³⁷ If markets then are efficient, it was considered futile to use past information to predict future prices. This means any mechanical trading rule applied to an individual security would not persistently outperform a simple buy and hold strategy.

Since the late 1960s, the finance literature has played out the battle between the efficient market types, who believe that price changes are random and hence unpredictable, and a group of "nonbelievers", which included the technical analysts, who claim that profitable trading strategies can be devised. The literature has developed in three major areas: price based rules sometimes called mechanical trading rules, trading rules based on price and volume, and trading rules based on other

³⁶ Evidence of this can be seen from the popularity of Dow theory, for example, in the various editions of Robert E. Edwards and John Magee, Technical analysis of Stock Trends (Springfield, Mass: John Magee, 1966) whose first edition appeared in 1948 and by the fact that approximately one-third of the book Stock Market Theory and Practice by R.W. Schabacker (New York: B.C. Forbes, 1930) was devoted to technical analysis.

³⁷ See Fama (1970) for a description of and evidence supporting this idea.

variables like short interest, odd-lot trading, and advance-decline statistics. The studies related to each area will be discussed in turn below. The discussion is followed by a brief critique.

2.2.1 Mechanical Trading Rules

Mechanical trading rules are security trading strategies based purely on information derived from past prices. Alexander (1961) considered the X% filter rule: if the security price moves X% above a previous low, then buy the security; if the security price falls X% below a previous high, then sell the security. Price changes less than X% in either direction are ignored. Compared with the returns that could be expected from a naive "buy and hold" strategy, he found that substantial profits could be earned with a 5% filter. The problem with his analysis was that the costs of commissions were not included in the simulation test of trading returns. Alexander (1964) reworked the earlier analysis, this time including the cost of trading commissions, and found that the filter rule trading profits were still superior, but reduced greatly from the earlier paper, to the naive "buy and hold" strategy.

Cootner (1964) developed a "reflecting barriers model," where security prices fluctuate between two barriers that develop due to the information differences that develop between two investor groups: professionals and non-professionals. The barriers shift due to changes in information over time. Using weekly data, he compared the price today with the average price in the last 40 weeks. If the current price rises above the moving average, buy the stock; but if the price is less than the moving average, sell the security short. If the current price then rises above the

moving average, all short positions should be covered. If the price falls below the moving average, all long positions should be eliminated. Using a five percent threshold before buy and sell orders are actually executed, he found that the moving average trading rule yielded greater profits than the "buy and hold" strategy. But these trading profits become insignificant once transactions costs are accounted for.

The first discussion of the application of the efficient markets hypothesis on the usefulness of technical analysis can be found in Fama and Blume (1966). They considered the thirty stocks in the Dow Jones Industrial Average using daily closing prices and used a filter expressed as a percentage change from the previous peak or trough. The rule employed was to buy the stock when the stock's price exceeded the lowest previous closing price from the day the position was opened by the size of the filter. These positions were held until the closing price dropped below the highest preceding closing price from the day the position was opened by the size of the filter. When the stock price fell from a previous high by an amount equal to the size of the filter, the strategy was to short sell the stock. Calculating returns for this strategy, and comparing these returns to the simple "buy and Hold" strategy, they concluded that a mechanical trading rule cannot outperform the market.

Levy (1967) took a portfolio approach to mechanical trading rules. In a "reverse variable ratio model" he considered portfolios composed of stocks and bonds, where the proportion invested in stocks increased or decreased depending on whether the market went up or down. After allowing for commissions, Levy found the portfolio results superior to a buy and hold policy. Van Horne, et al (1967) considered the moving averages of thirty stocks calculated for 100, 150, and 200 days

prior to each day's closing price. If the daily closing price exceeded the moving average of past prices by X% for two consecutive days, a buy order was placed. If the closing price was below the moving average by X% for two consecutive days, a sell order was issued. Decision rules were tested for the investor who took only a long position and both a long and short position. In the latter case, a short position is taken as soon as the long position was liquidated. Using five different thresholds and three moving averages with long and long and short positions, thirty variations were tested and none proved more profitable than the buy and hold strategy.

Van Horne and Parker (1968) repeated the previous study using weighted moving averages where more emphasis was placed on the more recent prices. Tests were run on 32 variations, and the profits generated were even less than the profits generated in the previous study. Seelenfreund, et al (1968) again repeated the study but this time used a quadratic predictive model employing adaptive exponential smoothing. After commissions, the buy and hold strategy on average yielded higher annual rates of return than the trading rule strategy. However, for some stocks, annual returns were higher when the trading rule was used than when the buy and hold strategy was employed. James (1968) used monthly data to show that unweighted and exponentially smoothed moving averages offer no benefits for investors.

Sweeney (1988) replicates the Fama and Blume (1966) tests and finds the opposite result: simple filter rules generate excess returns, even when risk and transactions costs are accounted for. The only difference in the tests is the period of data uses: Fama and Blume used data for the early 1960s, while Sweeney applied the

tests to data for 1970-1982.

Recent evidence tends to support the moving average rule. Anderson (1989) reconsiders the Cootner (1964) results and finds, like Cootner, that the moving average rule can be applied to yield trading gains superior to that of the buy and hold strategy. Neftci (1989) finds that the 150-day moving average is an significant indicator of stock prices. Brock, Lakonishok, and LeBaron (1991) use ninety years of daily data to consider both the moving average method and their version of the trend-crossing method (which unfortunately is incongruous with the way technical analysts use trendlines). They find that trading rules beat the simple buy and hold strategy when transactions costs are not taken into account.

2.2.2 Volume and Price

Several economists have considered the relationship between security price and volume. Granger and Morgenstern (1963) employed spectral analysis on both weekly and monthly data for stock indices and individual stocks. They find "...at least in the short run, and for normal day to day or week to week workings of the stock exchange the movements in the amount of stock sold are unconnected with the movements in price." Godfrey, Granger, and Morgenstern (1964) employed spectral analysis on daily data for several stocks. The only strong correlation they observed was between volume and the differences between the high and low price for the day. They found no strong correlations between the first difference in price and volume. Moore (1965) employed volume as one of the independent variables seeking to explain the negative serial dependence of prices for individual stocks. He found that volume had

little to do with the observed serial dependence.

Ying (1966) studied the relationship between daily data for the S&P 500 stock average and the volume of sales on the New York Stock Exchange. Employing both analysis of variance and spectral analysis, he finds significant relationships between volume and price. A fall in volume is normally associated with falling prices, and increasing volume or large volume indicates increases in price. He found that changes in volume tend to lead to movements in stock prices.

Granger and Morgenstern (1970) present evidence of the existence of a relation between price variability and volume using daily for common stocks. Epps (1975) and Smirlock and Starks (1985) show evidence of a positive correlation between the absolute value of price changes in the market and changes in transaction volume. Epps and Epps (1976) find theoretical and empirical support for the hypothesis that there exists a stochastic dependence between transaction volume and changes in the log of price from one transaction to the next. Rogalski (1978) shows that knowledge of both prices and transactions volume information may be more valuable in predicting future stock movements than prices alone. Like Ying, Epps and Epps, and Rogalski in the equity market, Cornell (1981) finds a positive relationship between price variability (measured by squared price changes) and trading volume in the futures market. Similarly, Tauchen and Pitts (1983) find a relationship between the variability of daily price changes and the volume of trading. Karpoff (1987) develops a model to explain why volume on price upticks is greater than volume on price downticks based on the idea that institutional rules raise the cost of selling short.

Jain and Joh (1989) apply the Granger-Sims causality test to find the intraday relationships between hourly common stock index returns and transactions volume. They observed returns causing volume. In fact, trading volume was found to be positively correlated with returns lagged up to four hours.

2.2.3 Short Interest, Put-Call Ratio, Odd-Lot Trading, Block Trading, and Advance-Divide

Another group of economists have considered the relationships between price and such variables as short interest, odd-lot trading, and advance-divide statistics. Short interest is the volume of short sales, while odd-lot trading indicates the investment activity of the small investor, as it measures the volume of trades that occurred in blocks of less than 100 shares. The technicians using these indicators are most often called "contrarians." The majority of investors will buy a stock whose price they believe to be rising and short sell stock when they believe that the price is falling. But the contrarians act in just the opposite manner, assuming if everyone has bought (sold), there is no one left in the market to bid prices any higher (lower). However, Biggs (1966) found no consistent long run relationships between short interest and price.

Seneca (1967), using monthly data from January 1946 to July 1965, used regression analysis where the S&P 500 stock market average was the dependent variable. Corporate dividends and short interest were the explanatory variables. He found no long run indication that larger short interest indicates higher stock prices. Major (1968) employed regression and simulation techniques, but failed to find any

significant relationships between short interest and stock prices. He found that most short sellers suffer substantial losses.

The Put-Call Ratio (PCR) is calculated as the volume of puts divided by the volume of calls. Billingsley and Chance (1988) use a correlation analysis to find that OEX and CBOE options data do not always give comparable signals, as indicated by the low correlation between the PCRs for both exchanges. Taking long and short positions depending on the PCR signal, they found that the PCR is a good market forecasting tool and can be used to gauge market direction.

Odd lot sales and purchases are often studied as an indication of what the small investor is doing. Kewley and Stevenson (1967) found no significant advantage achieved by using buy and sell signals developed from the ratio of odd-lot sales to odd-lot purchases. In a later test, Kewley and Stevenson (1969) found it is possible to make valid buy recommendations based on a moving average odd-lot sale to purchase ratio. Kewley and Stevenson found however that the procedure does not yield valid sell signals. Kaish (1969) performs the same tests using data on individual stocks rather than aggregate data, and finds similar results.

A round lot of shares on a stock market can range from 100 to several thousand shares. Economists argue that the number of shares traded should have no effect on a security price because all individuals are assumed to possess the same information. Kraus and Stoll (1972) considered the effect on price of block trades of ten thousand shares or more on the NYSE from July 1, 1968 to September 30, 1969. By the close of the day of the trade, prices had adjusted back to an equilibrium level. Using intraday price data, they showed that the block trades affect price. Based on

the same time period, but looking only at "large" price changes, Dann, Mayers, and Raab (1977) show that abnormal trading profits net of commissions and New York State transfer taxes cannot be earned using publicly available information. Only trades executed within five minutes of the block trade could earn abnormal returns.

Several economists have considered the predictive value of Advance Decline statistics. Theil and Leenders (1965) and Fama (1965) use information theory to examine the percentage of stocks that are increasing, decreasing, or remaining unchanged. Theil and Leenders conclude there is considerable positive dependence in successive values of the proportions of stocks advancing, declining and remaining unchanged of the Amsterdam Stock Exchange. Fama finds much weaker evidence of the same for the New York Stock Exchange. Kakon and Pennypacker (1968) use correlation analysis to determine if the Advance-Decline line is a leading indicator of general stock market peaks. Using a weekly, rather than a daily, index, they find that the Advance-Decline line coincides with peaks in the S&P 425 stock average. Dryden (1969) employs the theory of Markov processes, and concludes that there is some evidence that dependence exists among successive daily price changes for securities in the United Kingdom.

Solt and Statman (1988) consider the usefulness of the bearish sentiment index, a private investor newsletter published by Investors Intelligence, calculated as the ratio of the number of investment advisors who are bearish to the total number of advisors who are either bearish or bullish. The contrarians use this index to buy when investment advisors are too bearish and to sell when the advisors are seen to be too bullish. Using monthly data from January 1963 to September 1985, they show in

a regression analysis that the index is useless as an indicator of forthcoming stock price changes: The number of correct forecasts was offset by the number of incorrect forecasts.

2.2.4 Relative Strength

Levy (1967b) early found that "stock prices follow discernable patterns which have predictive significance, and the theory of random walk has been refuted." Jensen and Bennington (1970) concluded that the relative strength trading rule outperformed the buy and hold only when transactions costs are not taken into account. Bohan (1981) and Brush (1986) consider the usefulness of relative strength indicators, and both find considerable price persistence as measured by these indicators. Pruitt and White (1988) test the multi-component CRISMA (cumulative volume, relative strength, moving average) technical trading system and find that the system outperforms the market over time, even after adjusting for trade timing, risk, and two-way transactions costs.

2.2.5 Patterns

Roberts (1959) showed that the "head and shoulders" pattern of stock prices can be generated from a table of random numbers. He described other techniques that could be employed to distinguish real changes from random ones.

Levy (1971) considered thirty-two configurations of the five-point pattern characterized by either two highs and three lows or three highs and two lows. He showed that his test included several variations of channels, wedges, diamonds,

symmetrical triangles, head and shoulders, reverse head and shoulders, triple tops, and triple bottoms. He sought to test the belief among chartists that the appearance of certain patterns followed by a "breakout" gives a profitable buy and sell signal by testing the predictive significance of the thirty-two patterns. After taking trading costs into account, he found that none of the thirty-two patterns showed any evidence of profitable forecasting ability either when the market was increasing or when the market was decreasing.

2.2.5 Conclusions From the Literature

The evidence on mechanical trading rules is mixed, depending on the type of data used, the time period studied, whether the rule is based on moving averages or filter rules, and the particular company being tested. The more recent studies of Sweeney (1988) and Anderson (1989) indicate that filter rules and moving average rules, respectively, yield higher returns than the simple buy and hold strategy.

Based on the evidence presented on the relationship between price and volume, price movements today may not be independent of volume history. The evidence on short interest and odd-lot sales indicates that these variables have no relationship with price changes. However, the PCR seems a useful predictor of price.

The results of Theil and Leenders (1965) suggest that past data does provide some information about the future proportions of stocks that will advance, decline, or remain unchanged. Dryden's (1969) results suggest that the advance-decline statistics may provide some information that is not already available. Relative strength yielded profitable buy and sell strategies in each of the its three studies.

Overall, the evidence on technical analysis is mixed, though several indicators - filter rules, moving averages, volume, PCRs, Advance-Decline lines, and relative strength, have shown themselves to be useful in the ability to predict prices. But Roberts (1959) and Levy (1971) find that price patterns have no predictive significance. However, the problem with both studies is that neither makes mention of volume. Chartists tend to rely heavily on volume to confirm pattern formations.

There are several problems in using these studies to draw conclusions about the potential usefulness of technical analysis. First, most technical analysts use a group of indicators. The studies performed to date, with the exception of Pruitt and White's (1988) study of the CRISMA trading system, have considered trades based only on the signal of one indicator. In reality, traders may have one or two core indicators, but they will not be used for trading purposes until they are "confirmed" by any number of other indicators. In addition, both the core and confirming indicators may change depending, for instance, on the current direction of the market and/or breadth of activity in the market. Other factors that may cause technical analysts to change their system include a change in the rate of change of prices in the market, a change in the percent of institutional investment funds already invested in stocks, and change in the rate at which the Fed is creating money in the economy. Some analysts believe that any excess money in the market will always flow into the stock market.

Technical analysis is performed on market averages, individual stock prices, options prices, bond prices, and mutual funds. For each of these security markets, data on volume, sentiment, odd-lot trades, etc. are used in addition to the data on

price. However, the formal empirical studies of the success of trading rules have only considered almost exclusively the equity market in their studies, and most have been confined to the use of price and volume information. In addition, there is little in the literature to suggest the trading effects of various institutional constraints to trading in the many securities markets.

Each of these criticisms will not be answered in the small empirical analysis that follows in Chapter 5. However, the analysis is done with these criticisms in mind.

CHAPTER 3
TECHNICAL ANALYSIS REVEALED: A FORMALIZED
STUDY OF TRADING RULES

The purpose of this chapter is to investigate the mathematical properties of the types of technical analysis methods outlined in the previous chapter. This will entail three things. First, we will create the descriptive mathematical environment appropriate for a formalization of the techniques outlined in the previous chapter. Second, we will mimic the buy and sell decision rules given by the technical analysis methods with formal mathematical rules. Third, we will devise a formal theoretical test to determine if the mathematical rules will have predictive power, in the realm of linear time series prediction theory. Finally, the mathematical framework will be defended with an eye towards the results attained.

Technical analysis is concerned with two practical issues: one, issuing buy and sell signals when the series reaches a new extreme, or inflection point, and two, discerning recognizable, repeating patterns in the price series. Note that standard linear statistical methods for Gaussian processes speak of patterns found only in the first and second moments of a price series. This is basically any pattern that can be mimicked by a quadratic (rolling, smooth shaped) function. Any "non-smooth" event could never be foreseen by such models.

Both of the issues that give rise to technical analysis are nonlinear problems. To issue buy and sell orders, an analyst must, at random moments, issue signals to either reverse previous trading actions or initiate new positions. A trader following linear rules can only do this with a considerable lag, as will be discussed in the following chapter. The technical analysis method of pattern recognition involves pinpointing the extreme point of a sequence, or maxima and minima. Clearly, predicting the point of the maximum or minimum of a sequence is again a nonlinear prediction problem.

The survey of the methods of technical analysis methods in the previous chapter has led us to two general categories of technique: charts and numerical. The purpose of chart techniques is to spot an ordered series of extreme points. For example, the double top pattern is said to occur when the chartist sees two maximum points and three minimum points, where the maximum points separate the three minimum points. Numerical techniques are those technical analysis methods that can be restated with a formal mathematical rule. For example, the moving average rule says to buy stocks when the price X_t exceeds a declining moving average Z_t , or $X_t > Z_t$. Numerical techniques include trend crossing methods, moving average methods, divergence methods, percentage range methods, cumulative sum methods, ratio methods, and cycle methods. Our first task will be to create the mathematical environment appropriate for the mathematical reformulation of the methods that follows in Section 3.2.

3.1 A Mathematical Milieu for Technical Analysis

To use mathematics to describe a natural or experimental phenomenon, the phenomenon must be described by a mathematical model. The mathematical model must at a minimum describe the details of the phenomenon of which the experimenter is concerned. Security prices are the phenomenon, and will be modeled as Markov processes. The procedure we follow to construct the mathematical model is to establish events on a probability space, define random variables, establish the criterion for a stochastic process, define the Markov property, describe persistent states of nature as those that will be returned to, and finally describe how stopping times are generated. An optimal stopping time will be defined, and an example is used to provide the criterion for establishing whether the stopping time will be finite. The extrema of a series are defined. With this mathematical framework in place, we will be ready to reformulate (in Section 3.2) the methods of technical analysis in the mathematical model set out in this section. The model follows loosely from Billingsly (1986). Most of the probability statements are well-known, but are included for completeness and continuity of the argument that leads to the results of Section 3.3.

Our model should first describe the set of possible outcomes, often called the sample space of the experiment. The sample space will be denoted by the symbol Ω , which will contain the set of points ω . In probability, a subset A of Ω is an event, and an element ω of Ω is a sample point. If the sample space is finite or countable infinite, we call it discrete.

Let Ω be an arbitrary nonempty space. A class \mathfrak{S} of subsets of Ω is called an algebra if it contains Ω itself and is closed under the formation of complements and finite unions:

- (i) $\Omega \in \mathfrak{S}$
- (ii) $A \in \mathfrak{S}$ implies $A^c \in \mathfrak{S}$
- (iii) $A, B \in \mathfrak{S}$ implies $A \cup B \in \mathfrak{S}$

A class \mathfrak{S} of subsets of Ω is a σ -algebra if it is an algebra and if it is closed under the formation of countable unions:

- (iv) $A_1, A_2, \dots \in \mathfrak{S}$ implies $A_1 \cup A_2 \cup \dots \in \mathfrak{S}$

A set in a given class \mathfrak{S} is said to be measurable in \mathfrak{S} . The largest σ -algebra in Ω consists of all the subsets of Ω ; the smallest consists only of the empty set \emptyset and \mathfrak{S} itself.

A set function is a real-valued function defined on some class of subsets of Ω .

A set function P on an algebra \mathfrak{S} is a probability measure if it satisfies:

- (i) $0 \leq P(A) \leq 1$ for $A \in \mathfrak{S}$
- (ii) $P(\emptyset) = 0, P(\Omega) = 1$
- (iii) if A_1, A_2, \dots is a disjoint sequence of \mathfrak{S} -sets and if $\bigcup_{k=1}^{\infty} A_k \in \mathfrak{S}$, then

countable additivity holds, or

$$P\left(\bigcup_{k=1}^{\infty} A_k\right) = \sum_{k=1}^{\infty} P(A_k)$$

If \mathfrak{S} is a σ -algebra in Ω and P is a probability measure on \mathfrak{S} , the triple $(\Omega, \mathfrak{S}, P)$ is called a probability measure space, or simply a probability space. A support of P is any \mathfrak{S} set A for which $P(A)=1$.

If $(\Omega, \mathfrak{S}, P)$ is a probability space, then an event is any subset of Ω . If $A =$

$\{\omega_1, \omega_2, \dots, \omega_k\}$, then the probability of A is defined as $P(A) = \sum_{i=1}^k P(\omega_i)$. Then

$P(\emptyset) = 0$ and $P(\Omega) = 1$. The information contained in the probability space $(\Omega, \mathfrak{S}, P)$ is contained in \mathfrak{S} . The σ -algebras \mathfrak{S}_t have the property that they are nested, or $\mathfrak{S}_t \subset \mathfrak{S}_{t+1} \subset \dots$. Intuitively, \mathfrak{S}_t can be thought of as the information known at t . Recall that experiments occur in Ω , and outcomes are the mappings from Ω to \mathfrak{R}^1 . Then the real number is used to represent the experimental outcome. Usually, the goal of a model is to determine the properties of these outcomes.

We will say that the function that maps a sample space into the real numbers is called a random variable. Let $(\Omega, \mathfrak{S}, P)$ be an arbitrary probability space and let X be a real valued function on Ω ; X is a simple random variable if it has a finite range, or assumes only finitely many values, and if $\{\omega: X(\omega) = x\} \in \mathfrak{S}$ for each real x . Then the probabilities $P[\omega: X(\omega) = x]$ are defined. Generally, argument ω is omitted and X stands for $X(\omega)$.

There are two general purposes for defining a random variable on a sample space: (i) each outcome is renamed to be a real number, and (ii) some of the useless information (in the context of the experiment) contained in the outcome is lost in the mapping from \mathfrak{S} into \mathfrak{R}^1 . This simplification allows us to focus on the properties of the outcome.

Let S be defined as the range of X , or the subset of real numbers. We need to

know the probability of each of these numbers assigned to the random variables occurring. Since the original sample space Ω has a probability measure, we can provide the space S with a measure. If the probability measure on S is denoted by P^* , then define a probability measure on $S = \{s_1, s_2, \dots, s_n\}$ as $P^*(s_i) = P[\omega: X(\omega) = s_i]$. This measure on S is called the distribution of X . A new probability space (S, P^*) has been constructed from (Ω, P) using the random variable X .

A stochastic process is a family of random variables defined on the sample space Ω . Let S be a countable or finite set. Suppose that to each pair i and j in S there is assigned a nonnegative number p_{ij} and these numbers satisfy the constraint

$$\sum_{j \in S} p_{ij} = 1, \quad i \in S. \quad \text{The set of distinct values assumed by the stochastic process is}$$

called the state space, corresponding to S . The stochastic process will be viewed as a function of two variables, $X_i(\omega) = X(t, \omega)$. For fixed t , the function is a random variable. For fixed ω , the real valued function of t is called the sample path. A finite discrete time process will be denoted $\{X_n\}$.

Let $(\Omega, \mathfrak{F}, P)$ be a probability space and let A, B be two subsets of Ω . The probability of both A and B occurring is denoted $P(A \cap B)$. The conditional probability that B occurs given that A has occurred is defined to be

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

where $P(A) \neq 0$. Once a stochastic process has been defined, restriction on the joint and conditional distributions of the process allow us to define special types of

stochastic processes, like a Markov Process. A definition of the Markov property and transition matrices will allow us, using conditional probabilities, to calculate the joint probability of several events.

A process is called "nonhereditary" if the future is determined by the last observed state only - the present. The mathematical concept of nonheredity in a stochastic context is Markov dependence.

A stochastic process $\{X_k\}$, $k=1,2,\dots$ with state space $S = \{1,2,3,\dots\}$ is said to satisfy this Markov property if for any n and all states i_1, i_2, \dots, i_n it is true that

$$\begin{aligned} P[X_n=j | X_{n-1}=i_{n-1}, X_{n-2}=i_{n-2}, \dots, X_1=i_1] &= P[X_n=j | X_{n-1}=i_{n-1}] \\ &= p_{ij}^{(n-1)} \end{aligned}$$

for every n . If $X_{n-1} = i$, then we mean the process is in state i at time $n-1$. But we might want to know where the process will be at n . This is unknown, but can be represented as the probability distribution over the states in S , conditional on where the process was at time $n-1$, or $P[X_n=j | X_{n-1}=i]$. The movement of the process among the states of S is determined by the conditional probabilities, $P[X_n=j | X_{n-1}=i]$. These probabilities are written $p_{ij}^{(n-1,n)}$. In a special case, these probabilities will depend

on the time when the transition from state i to j takes place.

Markov theory is concerned with the transition of a system from one state to another. In the case of a sequence of observations on security prices, the states of a system may be thought of as the set of all possible prices that might be observed for a given security. A more convenient way to characterize our set of states may be in

terms of price changes or price change classes (those that increase, decrease, or remain unchanged).

Many models of economic events are stochastic ones in the sense that we assume that the outcomes from one period to the next are generated by some underlying probability process. Markov processes are an example of a class of stochastic processes where the probability of a particular outcome in any period depends on the outcomes during the previous period, all prior history irrelevant. If the probability structure is invariant over time, or stationary, then we can use historical data to estimate these probabilities.

Our Markov chain is considered to be stationary if the probability of going from one state to another is independent of the time at which each step is being made, or

$$P[X_n=j | X_{n-1}=i] = P[X_{n+k}=j | X_{n+k-1}=i].$$

If the process is stationary, the transition probabilities do not vary with n .

Example: (Type of sequence that is stationary) If X_n represents the price change of a series between times $n-1$ and n , then the sequence is stationary if the probability of any particular price change remains constant throughout the duration of the price series.

Example: (To show that even serially dependent price processes can be modeled as Markov processes): Suppose that one thought that the probability of today's price depended on the probability of yesterday's price. In other words, the system carried momentum. To consider the events independent (necessary for modeling the price series as a Markov process), we could redefine states and probabilities. $X_n(\omega)$ is

today's price. Consider the events $[X_n=2|X_{n-1}=2]$, $[X_n=2|X_{n-1}=1, X_{n-2}=2]$, and $[X_n=2|X_{n-1}=1, X_{n-2}=1]$. These events are: one, that today's price is 2, given that yesterday's price was 2; two, the price today is 2, given that it was 1 yesterday and 2 the day before; and three, today's price is two, given that it was 1 for the last two days. The Markov property fails if there is some relation between these events. In the second and third cases above, price today is conditioned on prices over the previous two days. If price today depended on prices six days ago, then these events could be rewritten and new probabilities assigned to show this dependence. Our state space will have increased, but once again events will be considered independent.

The elements of S are thought of as the possible states of the system, X_n representing the state at time n . The sequence or process X_0, X_1, X_2, \dots then represents the history of the system, and evolves according to p_{ij} . The conditional distribution of the next state X_{n+1} given the present state X_n must not depend further on the past X_0, \dots, X_{n-1} .

When the process is stationary we can write $p_{ij}^{(n-1,n)} = p_{ij}$. Note that p_{ij} is a conditional probability as it represents the probability that the process is in state i and goes to state j next given the probability that the process was in state i . The conditional probabilities are called the transition probabilities. A Markov chain is called regular if all the transition probabilities are nonnegative.

If $\{X_t\}$ denotes a discrete time stationary Markov chain with finite state space $S = \{1, 2, \dots\}$, then there are n^2 (number of probabilities times the number of states)

transition probabilities, to be recorded in the transition matrix P . Row i denotes the probabilities, and column j the states. Note that for each row, the entries sum to one:

$$p_{i1} + p_{i2} + \dots + p_{in} = P[X_k=1 | X_{k-1}=i] + P[X_k=2 | X_{k-1}=i] + \dots + P[X_k=n | X_{k-1}=i]$$

Example: (Unrestricted random walk) Let S consist of all integers $i=0, \pm 1, \pm 2, \dots$ and take $p_{i,i+1}=p$ and $p_{i,i-1} = q = 1-p$. The series represents a random walk without barriers, with the process free to move anywhere in the space of integers. The walk is symmetric if $p = q$.

Example: (Security prices represented as a three state Markov process)

Consider a three state Markov process for security prices: prices can increase, decrease, or remain unchanged. the transition probabilities describing the movement of security prices from $t-1$ to t are described by a 3×3 matrix, where the rows of the matrix sum to one. If we assume that the transition probabilities are stationary, then we can define the limiting matrix of transition probabilities. If P is the matrix of transition probabilities, then P^n approaches a limiting matrix T . The probability of being in any particular state is, in the long run, independent of the current state occupied by the system.

Traders are interested in the prices that are likely to occur. These prices are the states of nature that the price process is most likely to return. A subset C of the state space S is called closed if $p_{ik} = 0$ for all $i \in C$ and $k \notin C$. If a closed set consists of a single state, then the state is called an absorbing state. For Markov chains, closure refers to the impossibility of escape. So a subset C of S is closed if once the chain enters C it cannot leave C .

Markov chains which have at least one state which once entered cannot be left

(i.e., the state's transition probability is one) and where from every state an absorbing state could be reached display some interesting properties. If there exists precisely one absorbing state, then one can determine the probability that the process will terminate in a particular absorbing state, the length of time until the process is absorbed, and the distribution of time spent in the nonabsorbing state(s).

Example: (Absorbing states) When the stochastic process being modeled is security prices, there are several states that could be considered absorbing. A price of zero is absorbing for all many securities like stocks and bonds as it indicates bankruptcy of the underlying company. another absorbing state from the viewpoint of many investors is the prespecified price at which a buy or sell order will be executed. Also, when security prices are denoted as percentage changes (or the logarithm of today's price divided by yesterday's price), the filter rule strategy, where buy and sell orders are placed if prices rise X percent from a previous low or fall X percent from a previous high, makes some states absorbing from the investor's perspective.

A Markov chain is called irreducible if there exists no nonempty closed set other than S itself. If S has a proper closed subset, it is called reducible.

Example: A price series is reducible because once the series goes to zero, then the security does not exist. A price equal to zero is an absorbing state. If the possibility of bankruptcy were denied, then the set of positive prices would be the irreducible set.

Example: A series of price changes is irreducible as there is no closed subset of price changes. No price change state is absorbing. We could consider the set of positive stock prices to be irreducible if we assume that the price cannot go to zero. Once the

price process hits zero, it is absorbed (i.e., bankruptcy).

Even when we have no absorbing state (i.e., the Markov chain is irreducible), the conclusions we drew above about absorbing states can be applied to determine the time spent on average before reaching a particular state for the first time. Say nonabsorbing state k in the transition matrix is really absorbing. Replace the computed probability for that state p_{kk} with one (and make the other elements in the row equal zero). Then the average time until absorption into this state can be computed. These times are commonly interpreted as the mean first passage time from state k to the other states.

Two states i and j are said to intercommunicate if for some $n \geq 0$, $p_{ij}^{(n)} > 0$

and for some $m \geq 0$, $p_{ji}^{(m)} > 0$.

This says that it is possible for the chain to go from i to j in n steps and it is possible to go from j to i in m steps, where it may be that $n \neq m$. If all states intercommunicate, the chain is irreducible. When a chain is irreducible, states may be returned to with some periodicity. The period can be used to describe the motion from state to state. The period of state j tells the number of times we may return to state j .

Let

$$f_{ij}^{(n)} = P[X_{n+k}=j | X_{n+j-1} \neq j, X_{n+k-2} \neq j, \dots, X_{k+1} \neq j, X_k \neq j]$$

be the probability of a first visit to j at any time n for a system that starts at i . If $i =$

j we refer to $f_{ij}^{(n)}$ as the probability that the first return to state i occurs at time n .

By definition, $f_{ij}^{(0)} = f_{ii}^{(0)}$. Let

$$f_{ij} = P\left(\bigcup_{n=1}^{\infty} [X_n = j]\right) = \sum_{n=1}^{\infty} f_{ij}^{(n)}$$

be the probability of ever visiting state j from state i . If $i = j$, let $f_{ii} = \sum_{n=1}^{\infty} f_{ii}^{(n)}$

denote the probability of ultimately returning to state i .

A state is persistent if the system starting at j is certain to return to j , or $f_{jj} =$

1. The state is transient in the opposite case $f_{jj} < 1$.

If i.o. means infinitely often, and $i = j$, then

$$P[X_n = 1 \text{ i.o.}] = 0 \text{ (if } f_{jj} < 1) \text{ or } 1 \text{ (if } f_{jj} = 1).$$

Or, in terms of the $p_{ij}^{(n)}$'s, transience is equivalent to $P[X_n = j \text{ i.o.}] = 0$ and

$$\sum_n p_{jj}^{(n)} < \infty \text{ and persistence is equivalent to } P[X_n = j \text{ i.o.}] = 1 \text{ and } \sum_n p_{jj}^{(n)} = \infty.$$

The economic meaning of persistence for prices is that the price will eventually occur again. However, the trader is still uncertain as to when a particular price will occur.

It is possible for a system starting at i to reach j ($f_{ij} > 0$) if and only if

$p_{ij}^{(n)} > 0$ for some n . If this is true for all i and j , the Markov chain is irreducible.

In the case where state j is persistent, the numbers $f_{jj}^{(n)}$ form a probability

distribution on the times of first return. Then $f_{jj}^{(n)} \geq 0$ for all n and $\sum_{n=1}^{\infty} f_{jj}^{(n)} = 1$.

In this case, we can consider the expected time until first return. A persistent state with an infinite expected return time is called null persistent. If the expected return time is finite, the state is called positive persistent. As the motion of the particle becomes more complicated, expected return times become more difficult to calculate.

Example: Positive prices can only be positive persistent. Suppose not. If prices were null persistent, then there would be positive prices that the series could never hit, as the return time is infinite. This is clearly not the case for security prices.

When a chain is irreducible, any state can be used to classify all states. If the chain is not irreducible, the first step is to identify the various irreducible closed subsets of S . One way to find a closed subset is to start with a persistent state, j , and generate an irreducible closed subset. Here, all states are persistent, and they have the same period. One must only know how to find the persistent state. When the state space is finite, we can be sure that all persistent states are positive persistent.

If the Markov chain is irreducible, then one of the following alternatives

holds: either all states are transient,

$$P(\cup[X_n=j \text{ i.o.}])=0 \quad \forall i,j, \text{ also } \sum_n p_{ij}^{(n)} < \infty \quad \forall i,j$$

or all states are persistent

$$P(\cap[X_n=j \text{ i.o.}])=1 \quad \forall i, \text{ also } \sum_n p_{ij}^{(n)} = \infty \quad \forall i,j$$

The irreducible chain itself can accordingly be called persistent or transient. In the persistent case, the system visits every state infinitely often. In the transient case, it visits each state only finitely often, hence visiting each finite set only finitely often, and so may be said to go to infinity.

Due to the importance in what is to follow of the notion of stationarity, we wish to relate the ideas above to the existence of a stationary distribution of states for our stochastic process. We can relate the existence of a recurrent set of states with the existence of a stationary distribution of states using the following result. For Markov Chains with countably many states, there exists a unique stationary distribution if and only if the set of states contains precisely one positive recurrent class of intercommunicating states¹.

Consider a Markov chain with both persistent and transient states. Suppose that the persistent states form two closed irreducible subsets, C_1 and C_2 . When a state space is finite, the chain leaves the transient state with probability one. It will eventually be absorbed into either of the subsets C_1 or C_2 . Then the questions are

¹ A.N. Shiryaev, Probability, (New York: Springer-Verlag, 1984), p. 543.

where it will go and the expected waiting time until absorption into either of the subsets. If the chain is ergodic then there is only one closed subset, and only the second question is relevant. Then the space of positive prices is the smallest closed subset and we are only interested in when the price process will be absorbed into some specified boundary.

In tracking the path of our sequence of random variables, we will be interested in either the absorption of the random variable into a set of persistent states or the first time the variable hits a prespecified point in the state space.

Let $(\Omega, \mathfrak{F}, P)$ be a probability space and $\{\mathfrak{F}_n, n \geq 1\}$ an increasing sequence of sub- σ -algebras of \mathfrak{F} , or $\mathfrak{F}_1 \subset \mathfrak{F}_2 \subset \dots \subset \mathfrak{F}_n$. A measurable function $\tau = \tau(\omega)$ taking values $1, 2, \dots, \infty$ is called a stopping time relative to $\{\mathfrak{F}_n\}$ if $\{\tau = j\} \in \mathfrak{F}_j, j = 1, 2, \dots$. A stopping time is completely determined by the sets $\{\tau = n\}, 1 \leq n < \infty$. A stopping time is said to be finite if $P(\tau = \infty) = 0$ and defective if $P(\tau = \infty) > 0$. A finite stopping time is called a stopping rule. This idea can be rephrased; consider a function τ on Ω for which $\tau(\omega)$ is a nonnegative integer for each ω . Let $\mathfrak{F}_n = \sigma(X_0, X_1, \dots, X_n)$; τ is a stopping time if $\{\omega: \tau(\omega) = n\} \in \mathfrak{F}_n$ for $n=0, 1, \dots$

This says that the first time the process X_t hits the boundary $(-c, c)$ is a stopping time. We contrast this with the notion from renewal theory, a field in engineering, that deals with the last time a process $\{X_t\}$ hits the boundary $(-c, c)$.

If f is a real function on the state space, then $f(X_0), f(X_1), \dots$ are simple random variables. An observer could follow the successive states X_0, X_1, \dots of the system. He stops at time τ , when the state is X_τ (or $X_{\tau(\omega)}(\omega)$) and receives a reward or payoff $f(X_\tau)$. The definition of a stopping time prevents prevision on the part of

the observer. This is a kind of game, the stopping time is the strategy, and the problem is to find the strategy that maximizes the expected payoff $E[f(X_\tau)]$.

The stopping time problem is to choose τ so as to maximize simultaneously expected payoffs $E_i[f(X_\tau)]$ for all initial states i . If X lies in the range of f , which is finite, and if τ is everywhere finite, then $[\omega : f(X_{\tau(\omega)}) = x] = \bigcup_{n=0}^{\infty} [\omega : \tau(\omega) = n, f(X_n(\omega)) = x]$ lies in \mathfrak{S} , and so $f(X_\tau)$ is a simple random variable.

The game with payoff function f has at i the value $v(i) = \sup E_i[f(X_\tau)]$, the supremum extending over all Markov times τ . It will turn out that the supremum at X_i is achieved: there will always be an optimal stopping time. It will also be true that there is an optimal τ that works for all initial states i . The problem is to calculate $v(i)$ and find the best τ .

The notion of a stopping time derives from gambling. Its definition is simply the mathematical formulation of the fact that, since an honest gambler cannot peer into the future, her decision to stop gambling at any time n must be based solely on the outcomes X_1, X_2, \dots, X_n up to that time, and not on the subsequent outcomes, X_j , $j \geq n$. By analogy, the securities trader seeks to maximize her payoff by issuing buy and sell orders at the time τ when value $v(i)$ of a particular trading strategy is maximized.

If the chain is irreducible, the system must pass through every state, and the best strategy is obviously to wait until the system enters a state for which f is maximal. This describes an optimal τ , and $v(i) = \max f$, for all i . The more difficult cases to analyze is when some states are transient and others are absorbing (or persistent with $p_{ii} = 1$). The trader may then find that the asset price goes to zero

or to infinity. In the case where the price goes to infinity, the stopping time may be infinite. But infinite stopping times are useless to the trader because the buy and sell signal could never be issued. Below we show that trend crossing methods in technical analysis generate such infinite stopping times.

The relevant properties of stopping times could be given as a series of theorems, or simply be characterized by some examples. We will use the technique of examples.

Example: (Independent random variables) In a sequence of Bernoulli trials with parameter $p = 1/2$, i.e. $S_n = \sum_{i=1}^n X_i$ where $\{X_n\}$ are i.i.d. random variables with $P(X_i = \pm 1) = 1/2$, let

$$\tau = \tau_1 = \inf\{n \geq 1 : S_n = 0\}$$

and
$$\tau_{j+1} = \inf\{n \geq 1 : S_{j+n} = 0\}$$

Then τ_j are the return times to the origin.

Example: (First passage time) If $\tau_j = \inf\{n \geq 1 : S_n = j\}$, then τ_j is the first passage time through the barrier at j . Again, when c is an upper horizontal boundary, then the first passage time across this boundary is $\tau_c = \inf\{n \geq 1 : S_n > c\}$, $c > 0$.

Now we will define the optimal stopping time. Consider the trader who makes a series of market trades with outcomes X_n , $n \geq 1$, where X_n , $n \geq 1$, are i.i.d. random variables and who has the option of stopping at any time $n \geq 1$ with a fortune $Y_n = \max(X_n - cn)$, where c is some positive cost per trade. Since the trader is not clairvoyant, her choice of a rule for stopping trade is a stopping time. In other words, her decision to stop trading at a specific time n must be based only on

X_1, \dots, X_n (and not on $X_j, j \geq n$). There exists an optimal stopping rule that maximizes EY , over the class of all finite stopping times.

In more general cases of $\{X_t\}$, we may not know whether τ is finite. But when the process is stationary, we can consider the time that the process hits its boundary. In particular, we are interested in whether or not the time until the boundary is hit is finite. Above, we illustrated a first passage time of a price process. We can think of this happening often through time, and hence the stopping time can be defined as any time the price goes up through the prespecified boundary. Clearly, when X_t is ergodic, then $P(X_n \in A \text{ i.o.}) = 1$; hence the process is recurrent and the stopping times are well-defined with probability one. This will be the case for asset prices, as we showed that the set of positive prices is positive persistent.

But when the process is not recurrent, then the transient process needs a further process to make the stopping times well-defined. We will note now though that the smaller the subset of recurrent states in a process, the longer the time it will take for the process to hit the boundary, one of the states in the recurrent subsets.

The following example provides a criterion for determining whether a stopping time will be finite.

Example: (Brieman, 1968, p. 123) Suppose A is a recurrent subset of \mathfrak{S} . If X_0, X_1, \dots is a stationary process such that $P(X_n \in A \text{ at least once}) = 1$ then the $\tau_i, i = 1, 2, \dots$ are finite with probability one. On our sample space $\Omega = \{\omega: X_0 \in A\}$ the τ_i form a stationary sequence under the probability measure P and Hence, the smaller A , the lower the probability of X_0 being in A at the start of the process and the longer the wait until the stopping time τ_1 . This also means that in

$$E(\tau_1 | X_0 \in A) = \frac{1}{P(X_0 \in A)}$$

order for the waiting time to be finite, the process must start in the recurrent state A . So once we are in the recurrent set A , the times that we hit the prespecified boundary are finite. Then the random variable stopping times for another stationary process.

As stated above, all positive asset prices belong to an irreducible set of recurrent states. Hence, satisfaction of the conditions established in the Brieman example establish for traders that a stopping time will be finite.

Example: (Average waiting times) When the process $\{X_t\}$ is ergodic, then we can specify the average time until the boundary is hit as

$$\frac{\tau_1 + \tau_2 + \dots + \tau_n}{n} \rightarrow \frac{1}{P(X_0 \in A)}$$

with probability one. Again, the higher the probability that X_0 starts at a recurrent state, the lower the average waiting time (Breiman, p. 124). For traders, this means that if they can "intuitively" reduce the state space for the asset price process and they trade according to some well-defined stopping rule, then they will have a relatively short wait until the next buy or sell signal can be issued.

Counterexample: Suppose that the process $\{X_n\}$ is not stationary, or the path depends on time. When the process is not stationary, then $E(Z_t) \neq 0$ and $\lim_{n \rightarrow \infty} \dots$. Clearly

the process only stops at infinity. This is no help to anyone trying to issue buy and sell orders.

Before concluding our mathematical excursion, several additional definitions must be given. These terms will be important for our description of pattern recognition methods in technical analysis in Section 3.3. In the context of locating sequence extrema, we are often interested in the times the sequence exceeds a certain level. When the sequence is continuous, these levels are called upcrossings. Let X_t be any stationary process, not necessarily Markov. Let $M_n = \max(X_1, X_2, \dots, X_n)$ be the maximum of the first n random variables. Then $m_n = \min(X_1, X_2, \dots, X_n) = -\max(-X_1, -X_2, \dots, -X_n)$. Define u as some boundary, where u may be some function of X_t , $u(X_t)$. For any real u , introduce a class G_u of functions f that are continuous on the positive real line, and not identically equal to u in any subinterval. Then the sample paths of our stationary process X_t are, with probability one, members of G_u (Leadbetter, 146).

Definition: A strict upcrossing of u occurs at point $t_0 > 0$ if for some $\epsilon > 0$, $f(t) \leq u$ in the interval $(t_0 - \epsilon, t_0)$ and $f(t) \geq u$ in $(t_0, t_0 + \epsilon)$ (Leadbetter, p. 147).

Definition: An upcrossing (or ϵ -crossing) for a function $f \in G_u$ occurs for u at t_0 if for some $\epsilon > 0$ and all $\eta > 0$, $f(t) \leq u$ for all t in $(t_0 - \epsilon, t_0)$ and $f(t) > u$ for some t (hence infinitely many) in $(t_0, t_0 + \epsilon)$ (Leadbetter, p. 147).

If $f(t_1) < u < f(t_2)$ with $t_1 < t_2$ write $t_0 = \sup\{t > t_1; f(s) \leq u\}$ for all $t_1 \leq s \leq t_2$. Then $t_1 < t_0 < t_2$ and t_0 is an upcrossing point of u by f . Then downcrossings (strict or not) may be defined by making the changes above.

Let u -values be points t_0 where $f(t_0) = u$, which are not crossings, like points where f is tangential to u or point t_0 such that $f(t) - u$ is both positive and negative in every right and left neighborhood of t_0 (Leadbetter, 147). Let N_u denote the number

of upcrossings of the level u by X_t in the bounded interval I , where $N_u(t) = N_u((0,t])$.

When the technical analyst seeks a local maxima, and the sequence reaches some level u , we say that we have a crossing. The crossings need not be \mathfrak{F}_t measurable. When the analyst is seeking the first time our (Markov) process crosses a prespecified boundary c , we say they seek a stopping time. When u is a prespecified real number, we can consider stopping times to be a special case of a first crossing.

3.2 A Mathematical Reformulation of Technical Analysis Methods

With the framework in place, our goal is to formalize the categories of technical analysis. First, some general definitions will be given, and then the technical analysis methods will be characterized in the framework above. We find that several of the well-known technical analysis techniques will meet our criterion for potentially having predictive powers. Only broad categories of the practiced methods in technical analysis, encompassing the majority of the documented methods, will be formalized.

As indicated by the definition of a stopping rule given above, technical analysis methods that generate buy or sell orders must be conditioned on past information known today in order to be stopping rules. If the method incorporates future information, then it cannot be a stopping rule, as it would then be anticipating the future. This information is based on trader "hunches" or "intuition" that is unique to the trader, and not part of the common knowledge of today's information set.

As the example above illustrated, technical analysis methods relying on pattern

recognition are frightfully easy ex-post. However, applied to a price series today, the technique clearly relies on prevision of a future price pattern. This cannot be a stopping rule. But a survey of the literature as well as direct interviews with "well-known" technical traders reveals that graphical methods are the most common tool of all the traders, with some traders basing transactions only on this technique. The technique is not predictive, is based only on the idea that historical patterns repeat themselves, and yet is the most common tool among Wall Street traders. Clearly, the contradictions these issues raise with standard economic theory merit academic attention.

We can characterize those technical analysis methods that will generate stopping rules and those that will not. Clearly, methods that require recognition of an extreme point today are not stopping rules, as the event is not measurable today based on the information available today. To recognize today's price as a local maximum, you must know that tomorrow's price is lower. Yet there are technical analysis methods based on the recent achievement of a maximum that is known today. These methods we will formalize as stopping rules. The methods that do provide stopping rules will clearly be feasible and predictive methods.

From the example on first entry times, we know that the first time a series hits the barrier $(-c, c)$ is a stopping rule. Our analysis will seek to redefine "buy" and "sell" orders as first passage times across some well-defined prespecified boundary, where the boundary is some fixed number or known function of historical data; in other words, a boundary "knowable" based on today's information set.

Example: We start with the simplest case of a technical analysis method that is a

stopping rule. Suppose you "know" as an informed investor that there is no reason why the value of your share of stock should fall below \$z. Thus you tell your broker to buy more shares as soon as the price goes below \$z, for surely the value of the share will rise shortly thereafter.

Example: Here is an example that is clearly not a stopping rule. You request that your broker sell all your shares of ABC stock when the share price reaches its annual maximum. Unfortunately, your broker is not equipped with a crystal ball.

More complicated technical analysis trading methods will be formalized below. We assume that traders seek to prescribe finite stopping rules. If not, then any resources used in analysis could never be rewarding as the buy or sell signal would never be issued. Although infinite stopping times are imaginable, we have defined them to be unfeasible for the reason above.

Pattern Recognition Methods: Traditionally, a large group of technical analysts have been referred to as "chartists," for their fondness of making and interpreting stock charts. They study historical prices, sometimes accompanied with volume information, for clues to the direction of future price changes. As outlined in the previous chapter, they construct line charts, bar charts, equivolume charts, and candlestick charts. More complex charts include point and figure charts and charts based on Elliot Wave theory. On these charts, they search for patterns: head and shoulders, triangles, pennants, flags, channels, rectangles, double tops, triple tops, wedge formations, and diamonds.

Chartists assume implicitly that patterns tend to repeat themselves. Their trade is based on the belief that a price series will eventually reach a new extreme point;

they wish to know when. We saw above that pattern recognition methods are based on the underlying belief that prices are nonlinear, as pattern recognition will involve the determination of the extrema, or threshold, of a time series. Formal statistical techniques for analyzing nonlinear problems are still new, although formal algorithms have been developed for estimation (examples are in Tong, 1983 or 1990). Chartists have taken an informal approach to the determination of such extrema. Theirs is a visual identification technique, based only on a "hunch" that the price series will indeed reach the extreme point "predicted" by the apparent formation of a recognized pattern. Experienced chartists seek to distinguish the pattern as it forms and base trades on their "hunch" about when the expected extrema will occur.

The patterns recognized by traders are learned from experience, and only known for the frequency with which they repeat themselves. A particular market environment may lend itself to a greater likelihood of a pattern appearing. However, patterns are recognized primarily from an ex-post price chart. Experienced chartists seek to distinguish the pattern as it forms and base trades on their "hunch" about when the expected extrema will occur. Clearly, the currently practiced informal recognition of patterns is an empirical, rather than a statistical technique.

Defining these patterns mathematically requires that we specify a path for our price process that involves at least one local minimum or maximum. Our discussion of local minimum and maximum follows Leadbetter (1984). Technical analysis proves a useful application of the techniques described there and the Leadbetter framework provides some basis for that used here. The head and shoulders and broadening formations patterns are formalized as examples.

Formally, the head and shoulders pattern consists of two local maxima enveloping the overall series maxima. Let M_1 and M_2 represent the two local maximas and M the overall maxima (m_1 and m_2 will denote the local minima). The pattern would resemble:

$$X_t < X_{t+1} < \dots < X_{t+i} < M_1 > X_{t+i+1} > X_{t+i+2} > \dots > X_{t+j} > m_1 < X_{t+j+1} < \dots < X_{t+k-1} < M > X_{t+k+1} > \dots > X_{t+n-1} > m_2 < X_{t+n+1} < \dots < X_{t+n+1} < M_2 > X_{t+n+1} > \dots$$

The broadening formations pattern occurs when three upcrossings connected by two downcrossings are such that the extrema may be connected with a straight line. The downcrossings are connected by a horizontal line and the upcrossings by an upward sloping line. Formally the pattern can be written:

$$X_t < X_{t+1} < \dots < X_{t+i} < f(t_1^H) > X_{t+i+1} > \dots > X_{t+j} > f(t_2^L) < X_{t+j+1} < \dots < X_{t+k-1} < f(t_3^H) > X_{t+k+1} > \dots > X_{t+n-1} > f(t_4^L) < X_{t+n+1} < \dots < X_{t+n+1} < f(t_5^H) > X_{t+n+1} > \dots > f(t_6) > \dots$$

where t_i^H are the u-levels tangent to the upward sloping top boundary, t_i^L are the u-levels tangent to the horizontal lower boundary, and $f(t_6)$ represents the time that the process pierces the lower boundary. The lines joining the extrema will diverge.

When the series leaves the linear boundaries described above, it is expected to go down. The opposite shaped broadening formation can be formed by reversing the signs and changing the boundary assumptions. A broadening formation between a horizontal top boundary and a downward sloping line indicates that the price series is headed to.

Filter Rule Methods: Other than the simple example provided above, filter rules are perhaps the simplest applications of technical analysis. A typical filter rule will

specify a buy order if the stock price rises 10% above a recent low and sell if the stock price falls 10% from its recent high. These percentages vary of course with preferences. Mathematically, this rule can be restated: X_t represents the price process and let H and L , respectively, represent the recent price maximum and minimums. Fix X_H at the most recent price high and X_L at the most recent price low. Then we sell if X_t falls below $.9X_H$ and we buy if X_t exceeds $1.1X_L$. We will show in a proposition below that filter rules are stopping times that are finite if the price series is monotonic (increasing or decreasing).

Moving Average Methods: As discussed in the previous chapter, the most common types of moving averages applied to technical analysis are simple and exponential ones. We consider the simple moving averages here, as both types are analogous in the context of this study. They differ only on the relative weights they give to past observations. In typical applications, buy and sell orders are generated when the moving average crosses either another moving average, a market index average of prices, or a particular price series. Mathematically, the n -day moving average can be stated:

$$Z_t = \sum_{i=1}^n \frac{X_i}{n}, \quad \text{for } (X_1, X_2, \dots, X_n)$$

where n is the number of day's prices in the moving average. The buy signal is issued if $X_t > Z_t$, while the sell signal is issued if $X_t < Z_t$. In the next section, we will prove that moving average methods generate stopping rules.

Trend Crossing Methods: Trendlines for the technical analyst are very different from those one might statistically estimate. Trend crossing methods fit lines tangent to the

extreme points in a series. Whether the tangent line is above or below the series depends on the direction of the series. When the market is rising, technical analysts say they have a market of ascending bottoms, where the troughs in price movements are tangent to an upward sloping line. A trendline is said to be more significant the greater both the length of the line and the number of tangencies with the price process. The trend is said to be broken or slowed when the process breaks the trendline. Trendlines with greater slope are the most likely to be broken. A trader seeks to buy or sell at the point of a trend reversal, when the previous trendline is pierced and a new trend line established.

Formally, we seek a series of u-levels and connect the tangency points with a line. Suppose that M_1 , M_2 , m_1 , and m_2 are two local maxima and minima, respectively, where $M_2 > M_1$ and $m_1 < m_2$:

$$X_t < X_{t+1} < \dots < X_{t+i-1} < M_1 > X_{t+i+1} > \dots > X_{t+j-1} > m_1 < X_{t+j-1} < \dots < X_{t+k-1} < M_2 > X_{t+k+1} > \dots > X_{t+l-1} > m_2 < X_{t+l+1} < \dots$$

where m_1 and m_2 are u-levels, or $f(t_1^+)$ and $f(t_2^+)$. For this upward sloping line we can write the equation for the line tangent to the minimas:

$$T(t) = a + \left(\frac{m_2 - m_1}{t_2 - t_1} \right) t$$

where a is some constant determined by the time when the trendline starts. A simple example of a sell signal is the time when the price process breaks through below this upward trend.

Stochastic (Or, Percentage Range) Methods: These methods are based on the observation that as a price declines, the daily closes tend to be located nearer to the

extreme low of the daily low. Or, as a price is increasing, the daily close price tends to be closer and closer to the high price of the preceding days. Stochastic methods produce what is called oscillating lines, %K and %D, where

$$\%K = 100 \left(\frac{\text{current close} - \text{lowest low}_r}{\text{highest high}_r - \text{lowest low}_r} \right)$$

where r is the number of days in the range considered (usually 5), and

$$\%D = 100 \left(\frac{\sum_{n=1}^3 (\text{current close} - \text{lowest low}_r)}{\sum_{n=1}^3 (\text{highest high}_r - \text{lowest low}_r)} \right)$$

%D is just a three day average of %K. Buy and sell signals occur when %K crosses %D, provided both lines are in an overbought or oversold zone. The overbought zone is typically 80% and above; the oversold zone is 20% and below. We will show that stochastic method generate stopping rules that are finite.

Divergence Methods: Divergences are thought to indicate changes in market direction (called an advance/decline or A/D indicator). The oldest, going back to the genesis of Dow Theory, prescribes a change in market price movements, or trend reversal, when for instance the DJIA starts moving in a direction different from the movement of another major index, like the DJTA. A divergence is indicated in this case when $\text{sign}(\Delta \text{DJIA}) \neq \text{sign}(\Delta \text{DJTA})$. Another example is the Moving Average Convergence/Divergence (MACD) measure which looks at the crossing point of two exponentially smoothed moving averages. Another example is some form of the difference between the number of market advances less the number of market declines.

Cumulative Sum Methods: As described in the previous chapter, there are several cumulative sum methods; on balance volume, used to measure the intensity of investor emotion, is perhaps the best known. Once computed, these indicators either look for divergences with the price line, or "large" changes in direction. For example, if there is high cumulative volume on days when the price is increasing, a bullish market is implied.

Ratio Methods: Like the divergence methods, the ratio indicators attempt to indicate a change in market direction. The simplest example is simply the ratio of the number of advancing stocks divided by the number of declining stocks. Another more complicated version is the market breadth indicator, or

$$\text{Market Breadth} = \left(\frac{|adv - dec|}{adv + dec + unch} \right)$$

where *adv* is the number of advancing stocks, *dec* is the number of declining stocks, and *unch* is the number of unchanged stocks. Other ratios discussed in the previous chapter include the relative strength measures, random walk indicator, TRIN, and put/call ratios. The different ratios will produce different numbers, but all prescribe critical regions for the ratio where the trader is signaled to buy or sell. In the next section, we will show that ratio methods do provide stopping rules.

Cycle Methods: Many technical analysts believe that stock prices follow cycles. To see the cycles, they give ways to remove trend: first differencing the data, use regression residuals, or use MACD numbers, one technique that allows you to take the difference between two moving averages. The timing of the cycle will determine the market highs and lows.

3.3 Technical Analysis Methods Put to a Feasibility Test

All predictive technical analysis methods can be formulated mathematically as stopping rules. These rules we will call signals. All other technical analysis methods we will call indicators, or simply precursors to a possible upcoming stopping rule.

Proposition: All technical analysis rules will provide useful buy and sell signals if and only if they define stopping rules.

proof: If an optimal stopping rule has been defined, then it is based only on information up to the current period. Then the signals issued provide a feasible basis for trading activity. A useful buy and sell signal is provided only when it is based on historic data and locates the point at which a series reaches a prespecified threshold. ■

The indicators, or non-stopping time rules, "anticipate" the future and hence only rely on data to repeat certain patterns recognized in previous time periods.

The technical analysis methods that define stopping time rules we will call feasible in a market where prices are observed as Markov processes.

Definition: A feasible technical analysis trading method is one that defines a stopping rule, and hence is based on the information set \mathfrak{S}_t , or historical data.

Infeasible technical analysis methods are those that force traders to anticipate the future. Whether or not a technical analysis method is feasible will depend on whether the method generates a stopping time. The next relevant question is whether these stopping times are finite. If so, then based on the framework of section II, we called these stopping rules. The following propositions serve to distinguish the technical analysis methods that are stopping times and those that are not; if they are stopping times, the propositions describe the cases when finite stopping rules are

generated.

Proposition: Pattern recognition methods never generate stopping times.

proof: The chart type used in this proof is the "head and shoulders" type formalized above. For the extrema M_1 , M , and M_2 to be \mathfrak{S}_t measurable, or recognizable from past data, they must be known by the trader when they occur. In other words, the trader must "know" that X_{t+i+1} , X_{t+k+1} , and X_{t+n+1} will be lower than M_1 , M , and M_2 , respectively. Such foresight is impossible.

The deduction of this proof can be applied to all other patterns. The statement of the proposition follows. ■

Note that the chart seeks to locate extrema, with no prescribed boundary for the price process. A stopping rule has not even been defined!

Proposition: Filter rules are stopping times that are finite if the series is not monotonic.

proof: As detailed above, filter rules prescribe that trades be made when the series moves some percentage away from its last nearest extrema. By the Breiman example, the rule provides finite stopping times for the price process that is not strictly monotonic (increasing or decreasing). In the case of strict monotonicity, the stopping time is \mathfrak{S}_t measurable, but infeasible since the prescribed trading rule could never be reached.

In the next proof above we have assumed that the price process is never so explosive that the price series never crosses its moving average. In other words, over a certain time period, there are enough high and low observations on X_t that Z_t is in this high-low range. We will call this our good behavior assumption about prices.

Proposition: If prices adhere to our behavioral assumption, then moving average methods generate (finite) stopping rules.

proof: Define Z_t as our moving average process. Breiman (1968, p. 105) shows that if the process $\{X_t\}$ is stationary then the function defined on $\{X_t\}$ defines another stationary process $\{Z_t\}$. If stationary, then $E(Z_t) = 0$ and $0 < P(Z_t \geq 0) < 1$. The moving average sell signal occurs if $Z_t > X_t$ and the buy signal occurs if $Z_t < X_t$. As defined, $Z_t = f(X_t)$. By the Breiman example above, if $\{X_t\}$ is stationary and $P(X_t \in f(X_t) \text{ at least once}) = 1$, then $\tau_i, i=1,2,\dots$ is finite. We use a simple example that $X_t \in f(X_t)$ at least once and the result is established. Let X_1, X_2, X_3, X_4 all equal 0, then $Z_t = f(X_t) = 0, t = 1, 2, 3, 4$ and $\{Z_t\}$ as defined above. ■

Proposition: Trend crossing methods do not generate stopping times.

proof: In order to formulate the trendline tangent to any two (or more) extrema, we must know today the u -levels $f(t_1)$ and $f(t_2)$. Today, such foresight is impossible. Hence the u -levels are not \mathfrak{F}_t measurable, the trendline cannot be formulated, and it is therefore impossible to know when the price process crosses a trendline that does not exist. ■

Proposition: Stochastic methods, divergence methods, cumulative sum methods, and ratio methods generate (finite) stopping rules.

proof: As described above, the calculation of the averages, sums, and ratios according to these methods is based on historical data. The trading signal in each case is a prescribed rule (either a real number independent of the price process, a time of intersection between two calculated series, or the time when the sign of the price change for two moving averages is inconsistent), known at the outset. Clearly these

rules are \mathcal{F}_t measurable, and stopping times are generated. Stochastic and divergence methods generate finite stopping rules if the series is not monotonic. Cumulative sum and ratio methods may not generate finite stopping rules. ■

Under the condition that the price series is monotonic, stochastic and divergence methods provide feasible trading criteria. Stochastic, divergence, and cumulative sum methods may prove infeasible, depending on the movement of the price series and the size of the critical region determining the decision to trade.

Proposition: Cycle methods do generate stopping times.

proof: Cyclic methods require the trader to identify peaks and troughs in the series of past prices. These points are \mathcal{F}_t measurable. Traders will use these past extrema in the cycle to predict the next cyclic extrema. As described above, the timing of the next extrema will depend on some numeric method, like ratios of Fibonacci numbers. ■

3.4 Modeling Security Prices as Markov Processes

3.4.1 Reasons for the Markov Specification

Markov theory is relevant to the analysis of security prices in two ways. First, the theory allows us to make probabilistic statements about future security price levels, and hence is an important alternative to traditional regression forecasting techniques. second, the theory allows us to extend the random walk hypothesis for series of independent random variables to a discussion of a price process where the random variables are dependent.

Modeling security prices as a markov process allows us to concentrate on the

temporal behavior of prices themselves without reference to their dependence on other variables such as earnings, dividends, etc. Our purpose has not been to explain security prices or their movements in terms of a set of independent variables.

Giving security prices the Markov property implies that past prices are independent of future prices, but that the current price is shown to depend on the past. Intuitively this fits the widely held notion that security prices today depend only on past information plus any new information that arrives today. Security prices are modeled this way for what the framework gives us: the notion of recurrent states, the notion of a probability of any state occurring, etc. In the model, this leads logically to the notion of a stopping time, which can be easily demonstrated graphically for a Markov process. In addition, security prices modeled as a Markov process allows for the possibility that price processes are serially dependent, and hence predictable. Also, the developed notion that the distribution of probabilities over the states is ergodic allows us to infer that our state space of positive prices is recurrent. Hence a stopping time will exist. Most importantly however, the Markov specification seems the correct one given what we observe about prices. Developed as such, we were able to use the model to establish a feasibility criterion for the use of technical analysis methods that will allow us in the next chapter to draw conclusions about the predictive power of the methods.

3.4.2 Markov Models of Asset Prices in the Literature

Modeling asset prices, and in particular security prices, as Markov processes is not without precedent. Niederhoffer and Osborne (1966) use a seven state Markov

chain to show nonrandom behavior in the preference of traders for placing buy and sell orders at (in order of descending order) integers, halves, quarters, and odd eighths. Dryden (1969) analyzes stock prices in the United Kingdom in a Markov process framework. Using a quadratic programming approach to calculate transition probabilities, he finds that the matrix of transition probabilities is stationary, providing strong evidence of a tendency of share prices to repeat themselves. He sees the framework as an important one for providing decision rules for portfolio managers.

Fielitz and Bhargava (1973) and Bhargava (1975) use the Markov chain specification of changes in the natural logs of stock prices to investigate the dependency of price movements for individual securities and for the market as a whole. They show that share prices display short term memory in that price behavior is dependent on immediately preceding daily price changes. But they conclude the dependency has nonstationary transition probabilities, and hence the steady state transition matrix cannot be found. however, ninety percent of the chains tested were stationary

Ryan (1973) investigates the Markovian characteristics of weekly data for individual stocks on the london exchange. he finds that transition probabilities for weekly price changes are stationary, calculates the steady state distribution, the mean first passage times, the distribution of the mean first passage times, and the variance of the first passage times. he concludes that the Markov framework for security prices provides an important tool for determining trading rules.

Samuelson (1988) uses the markov chain framework to explore some

implications of mean regressing stock returns. Lo and MacKinlay (1988) use a Markov chain model and find positive serial correlation in weekly returns on the NYSE.

Turner, Startz, and Nelson (1989) devised a model of the stock market where excess returns are drawn from a mixture of two normal densities. In the model, the market is assumed to switch between two states, where the state in each period determines which of the two normal distributions are used to generate the returns for the period. The two states are characterized by the variances of their densities, as high variance states or low variance states. the state itself is assumed to be generated by a first order Markov process. The model is used to explore the relation between the time dependent variance of stock returns and the risk premium in the stock market.

Like Turner, Startz, and Nelson (1989) where the focus is on changes in price volatility and hence risk, Cecchetti, Lam, and Mark (1990) consider a two state Markov process, but focus on the properties of mean real returns. McQueen and Thorley (1991) use a two state Markov chain to test the random walk hypothesis using weekly return data for 1947-1987. They reject the random walk hypothesis, which would restrict their probabilities to .50/.50, as they find nonrandom behavior in that low (high) returns tend to follow runs of high (low) returns.

Markov chains have been used to model other asset markets. Gregory and Sampson (1987) use a Markov model to describe properties of forecast errors in the forward foreign exchange market. Schwert (1988) uses a Markov model of nominal returns to show that the variance of asset prices is heterogeneous and predictable.

Hamilton models the long term trend in GNP as a Markov process.

CHAPTER 4

THE PREDICTIVE POWER OF TECHNICAL ANALYSIS

This chapter will examine prediction techniques for a time series of security prices. We will seek to determine the conditions under which technical analysis methods will provide predictions superior to those given by standard linear econometric techniques. Linear techniques for prediction are well-known; Ordinary Least Squares (OLS) and Box-Jenkins are two examples. It is a well-known result in econometrics that when a process is linear, these techniques will provide the best estimates of the process in a mean squared error sense¹.

The framework provided here for statements about prediction theory will follow closely the analysis of Tong (Chapter 1, 1983 and 1990). Our notion of stationarity will be refined, the difference between linear and nonlinear systems will be explored, best predictors for linear systems will be reviewed and examples given, and a conclusion about the predictive abilities of the technical analysis techniques will be inferred. The empirical testing of the specifications will be left for the following chapter. Also in the

¹ OLS provides linear estimators in the sense that its unbiased estimator of the dependent variable is a linear combination of the independent variables. The Gauss-Markov theorem says that the least squares estimators have minimum variance in the class of linear unbiased estimators, or estimators are best linear unbiased. This minimum variance property of least squares estimators is the primary reason for the widespread use of OLS. See Johnston (1984) or Dhrymes (1978) for a discussion.

next chapter, the linearity of our time series will be inspected over various time periods.

In the previous chapter, we defined a stationary process. Here we refine that definition a bit.

Definition: A stochastic process $\{X_t\}$ is called strict sense stationary (sss) if its statistical properties are invariant to a shift in the origin.

Then processes $\{X_t\}$ and $\{X_{t+c}\}$ have the same statistics for any c . If the random sequence $X = (X_1, X_2, \dots)$ is sss then for Borel set A (a set on which the probability will always be defined) and $n \geq 1$, $P\{(X_1, X_2, \dots) \in A\} = P\{(X_{n+1}, X_{n+2}, \dots) \in A\}$ ². Then, $E(X_t^2) < \infty$ and $E(X_n)$ is independent of n , or $E(X_t) = E(X_n)$ and $\text{cov}(X_{n+m}, X_n) = E[X_{n+m} - E(X_{n+m})][X_n - E(X_n)]$ depends only on m so $\text{cov}(X_{n+m}, X_n) = \text{cov}(X_{1+m}, X_1)$.³

Definition: A stochastic process $\{X_t\}$ is called wide sense stationary (wss) if its mean is constant, $E(X_t) = \eta$, and its autocorrelation function depends only on $T = (t_2 - t_1)$, or $\rho_T = E[X_{t+T}X_t]$.

Recall that the Bernoulli random walk discussed in the previous chapter applies to a sequence of independent random variables. Two examples of dependent random variables are Markov chains and martingales. As discussed in the previous chapter, a Markov process is known as a memoryless system. To be more explicit, we can say that a system is memoryless if its output can be given by $y_t = g(X_t)$, where $g(X)$ is a function of X . Then at any given time $t=t_i$, the output y_{t_i} depends only on X_{t_i} and not on any past or future values of X_t . Suppose that the input to a memoryless system is an sss

² Shiriyayev, A.N. Probability (New York: Springer Verlag, 1984), 387.

³ Shiriyayev, 387.

process X_t . Then the resulting output y_t is also sss.

Say that the notation $y_t = L(X_t)$ indicates that y_t is the output of a linear system with input X_t . Then

$$L(a_1X_{1t} + a_2X_{2t}) = a_1L(X_{1t}) + a_2L(X_{2t})$$

for any a_1 , a_2 , X_{1t} , and X_{2t} .

Theorem: For any linear system⁴,

$$E[L(X_t)] = L(E[X_t])$$

Linear Gaussian processes are an example of such linear systems. We can construct such a process by putting conditions on the following autoregressive (AR) model:

$$X_t = a_0 + \sum_{j=1}^k a_j X_{t-j} + \epsilon_t$$

where the a_j 's are real constants, k is a finite positive integer referred to as the order of the AR model, and the ϵ_t 's are zero mean uncorrelated random variables called white noise with a common variance σ^2_ϵ . A slight variation of this model is when the ϵ_t is replaced by a weighted average of ϵ_t 's. When the ϵ of our AR process is replaced by a weighted average of $(\epsilon_t, \epsilon_{t-1}, \dots, \epsilon_{t-l})$ then we are left the following autoregressive moving average (ARMA) process:

$$X_t = a_0 + \sum_{j=1}^k a_j X_{t-j} + \sum_{j=0}^l b_j \epsilon_{t-j}$$

The b_j 's are real constants and l is a finite positive integer. Both the AR and the ARMA

⁴ Papoulis, 220

model are referred to as linear Gaussian processes when $\{e_t\}$ is a sequence of identically distributed independent random variables, each distributed normally, $N(0, \sigma^2)$ and when the roots of the equations have a modulus of less than one. Under these assumptions, the $\{X_t\}$ is stationary and jointly Gaussian, and we have a linear Gaussian model. The advantage of the linear Gaussian model is that the theory is well-known and well-developed, computational time is limited, and the model has proven an effective forecasting tool. Another advantage of this specification is that a well-defined linear Gaussian model for $\{X_t\}$ is completely specified by its mean and autocovariances. For these models, estimation procedures providing minimum mean squared prediction error are well-known.

Weiner (1949) and Kolmogorov (1941) developed the theory for constructing an optimal linear predictor for a linear stochastic process when the spectrum, or covariance function, of the stationary process is known. They considered a weakly stationary process $\{X_t\}$, where $E(X_t) = 0$. Assume we observed X_t, \dots, X_{t+m} , $m > 0$ and we wish to estimate X_{t+m+v} , $v > 0$ by a linear combination of the observed stochastic variables that is best in the sense of achieving the smallest mean squared prediction error. In other words, we wish to predict v steps ahead. The criterion of minimizing mean squared error was chosen because of the tradeoff between the asymptotic behavior of bias and the variance of estimates. If the estimate is modified to improve the asymptotic behavior of bias, the variance will increase, and vice-versa. Mean squared error is a function of both bias and variance, and hence allows us to control the tradeoff between the two variables. For our linear Gaussian ARMA process, the Weiner-Kolmogorov prediction theory will provide optimal predictions based on the criterion of minimizing this mean squared error.

However, there are many times in which the ARMA representation is incorrect as a time series model. When ϵ_t has zero variance, X_t will be constant. Due to its symmetric joint distribution, the stationary Gaussian ARMA models may not be ideal for asymmetric data. In addition, due to the normality of ARMA models, they are not appropriate for data exhibiting large and sudden thrusts of movement at irregular time intervals. Only if the probability of such large price changes is negligible may the ARMA specification be correct. Finally, if the data is not time irreversible, in the sense defined below, ARMA is not an appropriate specification.

Definition: A stationary time series $\{X_t\}$ is time irreversible if for every positive integer n and (t_1, t_2, \dots, t_n) , the vectors $(X_{t_1}, X_{t_2}, \dots, X_{t_n})$ and $(X_{t_1}, X_{t_2}, \dots, X_{t_n})$ have the same joint distributions.

Example: Both linear models and stationary ARMA Gaussian models are time irreversible, the latter because the covariance functions of the Gaussian distributions are symmetrical.

Tong (1990)⁵ shows that stationary Gaussian ARMA models can be characterized as a Markov process. This makes a ARMA model representation the correct choice if the amount of information about the future as obtained by past and present data is finite.

The problem with the ARMA representation is however when these assumptions are not met. For a series of security prices, these conditions are clearly not met. The Markov representation still represents the same nonheredity in our system, but it is no longer equivalent to the stationary Gaussian ARMA process. Hence, all the advantages

⁵ H. Tong, Non-linear Time Series, (New York: Oxford University Press, 1990), p. 188.

of the ARMA representation listed previously are irrelevant.

Proposition: If our time series X_t is linear, then the best prediction method is a linear time series technique.

proof: Linear prediction techniques, based on Weiner-Kolmogorov prediction theory are known to minimize mean squared prediction error⁶. ■

Several examples of linear processes are given below. The best prediction methods to apply to such series, in the sense of minimizing mean squared prediction error, are the Weiner-Kolmogorov methods alluded to above.

Example: If security prices are Gaussian, linear prediction techniques provide the best forecasts. However, in the real world, there is much evidence contradicting the application of this normality assumption to security prices⁷.

Definition: A stochastic sequence $X = \{X_n, \mathfrak{F}_n\}$ is a martingale⁸ if for all $n \geq 0$,

$$E|X_n| < \infty \text{ and } E(X_n | \mathfrak{F}_n) = X_n$$

Example⁹: If $\{\epsilon_n; n \geq 0\}$ is a sequence of independent random variables with $E(\epsilon_n) = 0$ and $X_n = \epsilon_0 + \dots + \epsilon_n$ where $\mathfrak{F}_n = \sigma\{\omega; \epsilon_0, \dots, \epsilon_n\}$, the stochastic sequence $X = \{X_n, \mathfrak{F}_n\}$ is a martingale.

Martingales are linear processes. Hence, any stopping times τ_i calculated from

⁶ This fact was simultaneously proved by Weiner and Kolmogorov in 1940-41, but published in 1949 and 1941, respectively. The Weiner-Kolmogorov prediction theory will be discussed in more detail in the next chapter.

⁷ See Akgiray (1989) for a discussion of this evidence.

⁸ A.N. Shiriyayev, Probability, (New York: Springer-Verlag, 1984), p. 446.

⁹ Shiriyayev, p. 446.

past X_i 's following a martingale process cannot be useful in prediction.

Example: Martingales have a linear representation. If a process can be represented as a martingale, then technical analysis methods could never be valuable in prediction. This result follows from the proposition above, as the Weiner-Kolmogorov prediction method is known to be the best when the underlying process is linear.

What are the implications of these results for a study of technical analysis? The technical analysis methods can be useful for prediction only if the process is nonlinear.

Proposition: Technical analysis methods may lead to predictions superior to those provided by linear estimation techniques only if the security prices have a nonlinear representation.

proof: We know that linear estimation procedures yield the best estimates in a mean squared error sense. When a price process is nonlinear, the application of linear estimation techniques will not yield the best predictions. Therefore, for technical analysis procedures to have predictive value beyond that of linear estimation procedures, it must be true that the price process is nonlinear. ■

As mentioned above, Markov processes have an ARMA representation under restricted conditions. As mentioned above, the ARMA process is linear, and prediction methods are well-known and fast. But linear techniques will not provide the best estimates of the process if one of the restrictive conditions fails, like when the white noise term is non-Gaussian. Furthermore, linear techniques will clearly not provide the best estimates when we abandon the linearity assumption altogether. The economics literature provides us with evidence that security prices may be nonlinear.

Neftci (1984) shows that there is no reason to expect that prices will be either

linear or independent. Hinich and Patterson (1985) present evidence of nonlinearity in daily stock returns. Poterba and Summers (1986) show that stock market returns are not independent. Akgiray (1989) finds evidence that daily stock return series display "significant levels of second order dependence, and ...cannot be modeled as linear white noise processes." The daily series do not appear normally distributed, and the nonlinear dependence seems due to changing variances (as suggested earlier by Epps and Epps (1976) and Tauchen and Pitts (1983)).

The evidence against linearity leads to the question of the best way to represent nonlinear time series. Tong¹⁰ (1990) gives the conditions under which the nonlinear stationary time series $\{X_t\}$ can be given a polynomial representation.

If traders have chosen not to apply linear estimation techniques in their formulation of profitable trading signals, then without formally knowing or specifying the price process as such, traders observed and accounted for this nonlinearity with their own set of techniques. Nonlinear estimation techniques are relatively new in the statistics literature and have been applied (albeit sparsely) to security price data only very recently. Separately, technical analysts developed and continue to develop their own set of techniques that yield favorable predictions when the underlying price data is nonlinear. Indeed, the majority of the methods of technical analysis must over time yield profitable

¹⁰ Tong (1990), p. 202. His discussion is based on a 1960 theorem by Nisio, who shows that every stationary time series may be approximated by a two-sided polynomial. When the degree of the polynomial is equal to one, our time series is linear. When the degree of the polynomial is two or more, $\{X_t\}$ is called a nonlinear time series.

trades, or technical analysts would not exist¹¹. However, from the perspective of both the economist and the statistician, these techniques are atheoretical and, for the most part, derived from no unifying framework.

To analyze the success of technical analysis then, we can only accept the techniques as practiced, and then apply historical data to determine the significance of these indicators over time in leading to profitable trades. Techniques for performing the empirical tests suggested above are briefly discussed in the next chapter.

¹¹ The arc sine law (See Feller (1971)) discussed in Dale and Workman (1980) notwithstanding. They use the law to explain why technical analysts exist. The result from probability theory says that one player in a "fair game" played over time will with a ten percent probability be in the lead for over 300 rounds of the game. They conclude that trading rules applied by technical analysts to price movements will result in long periods of cumulative success, but equally long periods of cumulative failure.

CHAPTER 5

SUMMARY AND CONCLUSIONS

In this chapter we summarize the findings of the text, discuss the appropriate empirical tests of these conclusions, and draw inferences about the plausibility of the efficient markets hypothesis in light of our results. The paper is unique in the sense that it has drawn its sources from the fields of economics, finance, probability, and statistical prediction theory to make statements about the practice of technical analysis. Despite theory to the contrary, this chapter shows that security prices are predictable, and that trading signals may be devised that lead to profitable trades.

5.1 Summary and Conclusions

In Chapter 2 we considered some indicators used by technical analysts and the extant economic literature on the usefulness of trading rules based on these indicators. We found that the practiced numerical methods of technical analysis fall into one of several categories: momentum, timing of trend reversals, sentiment, and cyclic.

In the review of literature, we found that the more recent studies of filter rules and moving averages using longer time series and improved empirical methodology show the methods to be useful in prediction. Studies indicate a correlation between volume and price, and that changes in volume tend to cause changes in price. Although short

interest and odd-lot sales appear uncorrelated with price, the put-call ratio seems a useful predictor of price. The advance-decline statistics and relative strength measure proved useful as a measure of price.

In Chapter 3 we devised a stochastic process framework for analyzing the dynamics of security prices. Our Markov specification for security prices allowed us to formalize the notion that security prices tend to reflect both past information and any new information received today. Certain restrictions in this framework allowed us to define a stopping time for Markov processes and demonstrate under what conditions a finite stopping rule will exist.

In this framework, we are able to describe which of the methods of technical analysis are feasible in the sense that the method is based only on past and present information, not on the intuition or hunches of traders. Moving average methods, stochastic methods, ratio methods, divergence methods, and cyclic methods are shown to be feasible technical analysis methods.

Given this set of feasible methods, we show in Chapter 4 that they can only have predictive value beyond the long autoregressive process prediction theory of Weiner-Kolmogorov if the underlying security price is nonlinear.

5.2 Suggestions For Empirical Study

Based on the theoretical results of Chapter 4 on the prediction of securities prices, several suggestions can be made for future empirical work. In particular, our work suggests three types of empirical tests; the Neftci (1989) test for the significance of technical analysis methods in prediction, simple tests for the linearity of various security

prices over different time periods, and simulations of investor returns generated with technical strategies versus simple buy and sell investment strategies. The linearity test is important in light of our proposition above that technical analysis methods may be useful in forecasting prices than the Weiner-Kolmogorov prediction technique only if the time series is nonlinear. Correspondence of the results on the significance of the technical analysis indicators with the technical analysis methods yield excess trading profits in the simulation study will link the Neftci (1989) test results with the previous work in the literature on technical analysis. The results in the simulation studies will also provide evidence on the efficiency of securities markets. Positive trading profits, after correcting for transactions costs and risk, would provide evidence against the efficient markets hypothesis.

5.3 Is the Efficient Markets Hypothesis a Shared Act of Faith?

Market efficiency describes whether investor expectations provide unbiased estimates of the "fair" value of securities. Fairly valued securities implies that investors will earn a return on holding the securities which appropriately reflects the security's risk. In the efficient market, no securities will be over or under priced. Then all forms of security analysis would be useless. Yet those that argue that markets are efficient say that there will exist some well-informed trader who will arbitrage away any profits that at any time result from any mispricing of securities.

5.3.1 Forms of the Efficient Markets Hypothesis

Fama (1970) defined market efficiency in terms of a probabilistic "fair game" where security prices fully reflect the available information. He distinguished between

three forms of market efficiency. The weak form efficient market hypothesis says that today's security prices fully reflect all information contained in historical security prices. This implies that investors cannot earn excess returns by developing trading rules based on historical price or return information. In particular, the methods of technical analysis which rely on charts of past prices or moving average and filter rule methods cannot be useful in the prediction of security prices. Under the hypothesis, profitable use of these trading rules is impossible because current prices already reflect all information in past price patterns.

The semistrong form of the efficient markets hypothesis says that security prices fully reflect all publicly available information. Then investors could not earn excess returns based on trading rules based on earnings reports, dividend announcements, annual reports, and newspaper articles. In particular, under this hypothesis, technical analysts should not be able to devise profitable trading strategies based on volume information, published sentiment indicators, put-call ratios, relative strength measures, etc. This type of information should be quickly reflected in prices so that investors cannot consistently earn abnormal returns when acting on such public information. Traditional tests of the semistrong form of the efficient markets hypothesis include how quickly security market "news," like accounting earnings reports or block trading is reflected in security prices.

The strong form efficient markets hypothesis says that security prices fully reflect all information, both public and private. Traditional empirical tests of the strong form efficient markets hypothesis are tests of insider trading, specialist trading, and the performance of professional money managers.

5.3.2 Efficient Markets and the Random Walk Hypothesis

If the efficient markets hypothesis is correct, past price changes contain no useful information about future price changes. With two additional assumptions, this theoretical notion can be translated into an empirical statement. If the expected return of holding stock is constant and the volatility of stock prices does not change during the time period examined, the efficient markets hypothesis says that observed changes in security prices should be uncorrelated and the past price changes should not exhibit long sequences of successive changes that are greater or less than the median change for the sample. Once these conditions are met, we can say that our time series of price changes or returns follow a random walk. Statistically, security prices will follow a random walk only if the price changes are independent and identically distributed over time. This means that the mean price change must be constant over time, and the distribution of price changes about the mean must be constant.

5.3.3 Empirical Evidence

The idea that securities markets are efficient forms the basis for much research in financial economics. Much empirical literature, reviewed in Fama (1970) supports the hypothesis. Following Fama, academic studies have defined the information set associated with weak form efficiency to be only the past price (or rate of return) series. These studies have found that price changes are independent over time and are thus useless to investors and traders as a basis for prediction of future price changes. Jensen (1978) calls the efficient markets hypothesis the best established empirical fact in economics. This testing of the weak form efficient markets hypothesis has been equated

with technical analysis. Based on these empirical tests of the weak form efficient markets hypothesis in the literature, academics have traditionally considered technical analysis to be useless.

5.3.4 The Spurious Link with Technical Analysis

The major problem with accepting the ideas relating acceptance of the efficient markets hypothesis with rejection of technical analysis is the lack of a one-to-one correspondence between the weak form tests and the practiced methods of technical analysis. Technical analysis is broader than the use of past price changes alone. The analysis of past price history is only a part of the practice of technical analysis¹.

Most of the tests of the weak form efficient markets hypothesis are not direct tests of specific forms of technical analysis. Our literature review in Chapter 2 makes this point abundantly clear. In addition, many technical analysis schemes have not been tested. It therefore seems premature and incorrect to judge the validity of technical analysis based on the limited evidence to date.

There have been two empirical tests applied to technical analysis. The indirect tests have yielded correlation studies between either price changes or returns and their supposed indicators. With respect to the relationships between these indirect tests and the usefulness of technical analysis it is difficult to say whether low correlations imply that no, or only small, profits are possible if the indicators are used in devising trading

¹ See the discussion of mechanical trading rules and pattern recognition methods in Chapter 2. The review of some of the popular methods in technical analysis reveals that the practice of technical analysis makes use of far more information than past prices alone.

rules. Similarly, would high correlations indicate the possibility that high abnormal trading profits could be earned? Direct tests of technical analysis have been simulation studies of trader returns over various time periods when risk and transactions costs are taken into account. Because of the difficulty in interpreting the results of the indirect tests, direct tests provide the best test of the historical success of technical analysis.

Finally, because technical analysis methods include those based on past prices, as well as numerous other methods, a distinction must be made in testing methods of technical analysis between whether one is testing the weak form or the strong form efficient markets hypothesis. Simulation tests of the charting methods and mechanical trading rules would provide evidence on the weak form efficient markets hypothesis, as these methods only make use of past price data. Simulation studies of all other technical analysis methods would provide evidence on the validity of the semi-strong form of the efficient markets hypothesis.

The central message of the huge literature on market efficiency is the supreme difficulty in earning abnormal returns when making use of only publicly available information. The assumption of investor rationality assumes that all investors have access to all publicly available information, understand all its implications, and correctly use the information to make investment decisions. Summers (1986) questions whether this implication of the efficient markets hypothesis that market prices represent rational assessments of fundamental values. He questions the power of common tests of market efficiency where inefficiencies result from large and persistent deviations of price from a rational market fundamental.

5.3.5 Theoretical Argument for the Efficient Markets Hypothesis

One theoretical argument for market efficiency is that unless securities are priced efficiently, there will be opportunities for excess returns. Noting the opportunities for abnormal returns, speculative traders will arbitrage away these inefficiencies in how securities are priced. The problem with this view, as noted by Summers (1982) is just how speculators identify these opportunities. He gives several reasons why speculators are unlikely to eliminate miscalculations of fundamentals.

Merton (1985) points out that strong evidence in favor of the efficient markets hypothesis is the repeated finding that professional money managers do not consistently outperform the market². On the whole, money management results are mixed. But there does exist convincing evidence that individuals can outperform the market, as described in the recent bestseller Market Wizards by Jack D. Schwager. The profiles of successful traders in the stock, bond, and commodities markets suggest that some hidden models with market forecasting ability do indeed exist.

5.3.6 Lessons Learned

An assessment of the validity of technical analysis means that simulation studies must be performed that accurately measure trading profits in strategies that mimic the actual practice of technical analysis. In addition, designers of these simulation tests must remember that sophisticated technicians may use a combination technical factors in

² In the "Money and Investing" section of the Wall Street Journal reports monthly results on the "dart Throwers verses professional money managers. Reading the reported results over a long period of time certainly gives mixed results. But the series only reports on traders in the stock market.

varying weights depending on the market situation.

Simulations should correct for risk and transactions costs, to ensure that our results enable us to draw conclusions about the efficient markets hypothesis. A simulation study showing that positive trading profits are earned after correcting for risk and transactions cost would provide strong evidence against the efficient markets hypothesis. A rejection of the efficient markets hypothesis implies acceptance of the alternative hypothesis that security prices are predictable. Driven then by a profit motive, the technical analyst seeks to develop a profitable trading strategy and administer that money machine that guarantees profits from speculative trading.

In theory then, for markets that provide more than a zero sum game (as in futures markets) all traders with the best long run trading signals could earn profits in a growing market. In a shrinking market, some traders would continue to earn profits, but losses accrued by other traders would be equal to the amount of market retraction plus the profits earned by the few.

Bibliography

- Akgiray, V. "Conditional Heteroscedasity in Times Series of Stock Returns: Evidence and Forecasts." Journal of Business 62 (1989): 55-80.
- Alexander, S.S. "Price Movements in Speculative Markets: Trends or Random Walks." Industrial Management Review II (May 1961): 7-26.
- Alexander, S.S. "Price Movements in Speculative Markets: Trends or Random Walks, No. 2." Industrial Management Review V (Spring 1964): 25-46.
- Anderson, S.C. "Evidence on the Reflecting Barriers Model: New Opportunities For Technical Analysis?" Financial Analysts Journal (May-June 1989): 67-71.
- Berkowitz, S.A., D.E. Logue, and E.A. Noser. "The Total Costs of Transactions on the NYSE." Amos Tuck Business School, Dartmouth College (1986).
- Biggs, B.M. "The Short Interest - A False Proverb." Financial Analysts Journal XXII (July-August 1966): 111-116.
- Billingsley, Patrick. Probability and Measure. New York: Wiley Press, 1986.
- Billingsley, R.S. and D.M. Chance. "Put-Call Ratios and Market Timing Effectiveness." Journal of Portfolio Management (Fall 1988): 25-28.
- Breiman, Leo. Probability. New York: Addison-Wesley, 1968.
- Brock, W.J., J. Lakonishok, and B. LeBaron. "Simple Trading Rules and The Stochastic Properties of Stock Returns." (1991) Social Science Research Institute, University of Wisconsin, Madison, Working Paper 9022.
- Brown, S. and J. Warner. "Using Daily Stock Returns: the Case of Event studies." Journal of Financial Economics. (March 1985), 3-32.
- Cecchetti, S.G., P. Lam, and N.C. Mark. "Mean Reversion in Equilibrium Asset Prices." The American Economic Review. 80 (1990): 398-418.
- Cootner, P.H. "Stock Prices: Random vs. Systematic Changes." Industrial Management Review III (Spring 1962): 24-45.
- Cornell, B. "The Relationship Between Volume and Price Volatity in Futures Markets." Journal of Futures Markets 1 (1981): 303-316.
- Dale, C. and R. Workman. "The Arc Sine Law and the Treasury Bill Futures Market." Financial Analysts Journal. (November-December 1980): 71-74.

- Dann, L., D. Mayers, and R. Raab. "Trading Rules, Large Blocks and the Speed of Adjustment." Journal of Financial Economics. (January 1977): 3-22.
- Dryden, M.M. "Share Price Movements: A Markovian Approach." Journal of Finance. XXIV (March 1969): 49-60.
- Edward, R.D. and J. Magee. Technical Analysis of Stock Trends. Springfield, Mass.: John Magee, 1966.
- Epps, T. "Security Price Changes and Transaction Volumes: Theory and Evidence." American Economic Review. (September 1975): 586-597.
- Epps, T.W. and M.L. Epps. "The Stochastic Dependence of Security Price Changes and Transactions Volumes: Implications for the Mixture of Distributions Hypothesis." Econometrica. Vol. 44, No. 2 (March 1976): 305-321.
- Fama, E.F. "Tomorrow on the New York Stock Exchange." Journal of Business. XXXVIII (July 1965): 285-299.
- Fama, E.F. "Efficient Capital Markets: A Review of Theory and Empirical Work." Journal of Finance. 25 (May 1970): 383-417.
- Fama, E.F. and M.E. Blume. "Filter Rules and Stock-Market Trading." XXXIX Journal of Business. 39 (January 1966): 226-241.
- Feller, W. "Fluctuations in Coin Tossing and Random Walks." in Chapter 3, An Introduction to Probability Theory and its Applications. (Volume I) New York: John Wiley & Sons, Inc, 1968: 67-97 or (Volume II) New York: John Wiley & Sons, Inc, 1971: 417-423.
- Fielitz, B.D. "On the Stationarity of Transition Probability Matrices of common Stocks." Journal of financial and Quantitative Analysis. 10 (1975) 327-339.
- Fielitz, B.D. and T.N. Bhargava. "The Behavior of Stock Price Relatives: A Markovian Analysis." Operations Research. 21 (1973): 1183-1199.
- Godfrey, M.D., C.W.J. Granger, and O. Morgenstern. "The Random Walk Hypothesis of Stock Market Behavior." Kyklos. XVII (1964): 1-30.
- Granger, C.W.J. "Forecasting Stock Market Prices - Lessons For Forecasters." Paper presented at the July 1991 meeting of the International Institute of Forecasters, New York.
- Granger, C.W.J. and O. Morgenstern. "Spectral Analysis of New York Stock Market Prices." Kyklos. XVI (1963): 16.

- Granger, C.W.J. and O. Morgenstern. Predictability of Stock Market Prices.
Lexington, Mass.: D.C. Heath & Co., 1970.
- Gregory, A.W. and M. Sampson. "Testing the Independence of Forecast Errors in the Forward Foreign Exchange Market Using Markov Chains: A Cross-Country Comparison." International Journal of Forecasting. 3 (1987): 97-113.
- Hamilton, J.D. "A New Approach to the Economic Analysis of nonstationary Time Series and the Business Cycle." Econometrica. 57 (1989): 357-384.
- Hinich, M. and D. Patterson. "Evidence of Nonlinearity in Daily Stock Returns." Journal of Business and Economic Statistics. 3 (1985): 69-77.
- Jain, P.C. and G. Joh. "The Dependence Between Hourly Prices and Trading Volume." Journal of Financial and Quantitative Analysis. (September 1988): 269-283.
- James, F.E., Jr. "Monthly Moving Averages - An Effective Investment Tool?" Journal of Financial and Quantitative Analysis. III (September 1968): 315-326.
- Jensen, M. "Symposium on Some Anomalous Evidence Regarding Market Efficiency." Journal of Financial Economics. 6 (June/September 1978): 93-330.
- Jensen, M. and G. Bennington. "Random Walks and Technical Theories: Some Additional Evidences." Journal of Finance. 25 (1970): 460-482.
- Kaish, Stanley. "Odd-Lotter Trading of High and Low Quality Stocks." Financial Analysts Journal. XXV (March-April 1969): 88-91.
- Karpoff, J.M. "The Relation Between Price Changes and Trading Volume: A Survey." Journal of Financial and Quantitative Analysis. 22 (March 1987): 109-126.
- Kewley, T.J. and R.A. Stevenson. "The Odd-Lot Theory as Revealed by Purchase and Sales Statistics for Individual Stocks." Financial Analysts Journal. XXIII (September-October 1967): 103-106.
- Kewley, T.J. and R.A. Stevenson. "The Odd-Lot Theory for Individual Stocks: A Reply." Financial Analysts Journal. XXV (January-February 1969): 99-104.
- Kraus, A. and H. Stoll. "Price Impacts of Block Trading on the New York Stock Exchange." Journal of Finance. (June 1972): 569-588.
- Leadbetter, M.R.; Lindgren, Georg; and Rootzén, Holger. Extremes and Related

Properties of Random Sequences and Processes. New York: Springer-Verlag, 1983.

Levy, R.A. (a) "Random Walks: Reality or Myth." Financial Analysts Journal. XXIII (November-December 1967): 69-77.

Levy, R.A. (b) "Relative Strength as a Criterion for Investment Selection." Journal of Finance. 22 (1967): 595-610.

Levy, R.A. "The Predictive Significance of Five-Point Chart Patterns." Journal of Business. 71 (July 1971): 316-323.

Lo, A. and A. MacKinlay. "Stock Prices Do not Follow random Walks: Evidence from a Simple Specification Test." The Review of Financial Studies. 1 (1988): 41-66.

Mayor, T.H. "Short Trading Activities and the Price of Equities: Some Simulation and Regression Results." Journal of Financial and Quantitative Analysis. III (September 1968): 283-298.

McQueen, G. and S. Thorley. "Are Stock returns Predictable? A Test using markov Chains." Journal of Finance. XLVI (March 1991): 239-263.

Merton, R.C. "On the Current State of the Stock Market Rationality Hypothesis." Sloan School of management Workin Paper No. 1717-85. (October 1985).

Moore, A.B. "Some Characteristics of Changes in Common Stock Prices." reprinted in Cootner, The Random Character of Stock Market Prices, 162-188.

Neftci, S.N. "Are Economic Time Series Asymmetric Over the Business Cycle?" Journal of Political Economy. 92 (1985): 307-328.

Neftci, S.N. "Naive Trading Rules in Financial Markets and Weiner-Kolmogorov Prediction Theory: A Study of "Technical Analysis". Journal of Business. Fall 1991.

Niederhoffer, V. and M.F.M. Osborne. "Market making and Reversal on the Stock Exchange." Journal of the American Statistical Association. 61 (1966): 897-916.

Phillips, S.M. and C.W. Smith, Jr. "Trading Costs for Listed Options: The Implications for Market Efficiency." Journal of Financial Economics, 8 (June 1980), 179-201.

Poterba, J. and L. Summers. "The Persistence of Volatility and Stock Market

- Fluctuations." American Economic Review. 76 (1986): 1142-1151.
- Pring, Martin J. Technical Analysis Explained. New York: McGraw-Hill, 1985.
- Pruitt, S.W. and R.E. White. "The CRISMA Trading System: Who Says Technical Analysis Can't Beat the Market?" Journal of Portfolio Management. (Spring 1988): 55-58.
- Roberts, H.V. "Stock-Market 'Patterns' and Financial Analysis: Methodological Suggestions." Journal of Finance. 24 (1959): 1-10.
- Rogalski, R. "The Dependence of Prices and Volume." Review of Economics and Statistics. (May 1978): 268-274.
- Ryan, T.M. "Security Prices as Markov Processes." Journal of Financial and Quantitative Analysis. 8 (January 1973): 17-36.
- Samuelson, P. "Longrun Risk Tolerance When Equity Returns are Mean regressing: Pseudoparadoxes and Vindication of Business Man's Risk." presented at the Colloquium for James Tobin at Yale University. (1988).
- Schabacker, R.W. Stock Market Theory and Practice. New York: B.C. Forbes, 1930.
- Schwager, J.D. Market Wizards: Interviews with Top Traders. New York: Harper & Row, 1990.
- Schwert, G.W. "Business cycles, Financial Crises, and stock Volatility." Unpublished manuscript. University of Rochester, Rochester, N.Y.
- Seelenfreund, A., G.C.C. Parker, and J.C. Van Horne. "Stock Price Behavior and Trading." Journal of Financial and Quantitative Analysis. III (September 1968): 263-281.
- Seneca, J.J. "Short Interest: Bullish or Bearish?" Journal of Finance. XXII (March 1967): 67-70.
- Shaw, Alan R. "Market Timing and Technical Analysis." Financial Analysts Handbook. ed. Sumner Levine. New York: Standard and Poors, 1988, 312-372.
- Shiryayev, A.N. Probability. New York: Springer-Verlag, 1984.
- Smirlock, M. and L. Starks. "A Further Examination of Stock Price Changes and Transaction Volume." Journal of Financial Research. (Fall 1985): 217-226.

- Solt, M.E. and M. Statman. "How Useful is the Sentiment Index?" Financial Analysts Journal. (September-October 1988): 45-55.
- Summers, L.H. "Do We Really Know Financial Markets Are Efficient?" NBER Workin Paper No. 836. (January 1982).
- Summers, L.H. "Does the Stock Market Rationally Reflect Fundamental Values?" Journal of Finance. XLI (July 1986): 591-602.
- Sweeney, R.A. "Some Filter Rule Tests: Methods and Results." Journal of Financial and Quantitative Analysis. vol. 23, no 3 (September 1988): 285-300.
- Tauchen, G.E. and M. Pitts. "The Price Variability-Volume Relationship in Speculative Markets." Econometrica. Vol. 51, No. 2 (March 1983): 485-505.
- The Technical Analysis of Stocks and Commodities. Seattle, Wa: Technical Analysis of Stocks and Commodities, various issues.
- Theil, H. and C.T. Leenders. "Tomorrow on the Amsterdam Stock Exchange." Journal of Business. XXXVIII (July 1965): 277-284.
- Tong, H. Threshold Models in Non-linear Time Series Analysis. New York: Springer-Verlag, 1983.
- Tong, H. Non-Linear Time Series. New York: Oxford University Press, 1990.
- Turner, C.M., R. Startz, and C.R. Nelson. "A Markov Model of Heteroscedasticity, Risk, and Learning in the Stock Market." Journal of financial Economics. 25 (1989): 3-22.
- Van Horne, J.C. and G.G.C. Parker. "The Random Walk Theory: An Empirical Test." Financial Analysts Journal. XXIII (November-December 1967): 87-92.
- Van Horne, J.C. and G.C.C. Parker. "Technical Trading Rules: A Comment." Financial Analysts Journal. XXIV (July-August 1968): 128-132.
- The Wall Street Journal. New York: Dow Irwin Company, various issues.
- Ying, C.C. "Stock Market Prices and Volume of Sales." Econometrica. XXXIV (July 1966): 676-685.
- Zakon, A.J. and J.C. Pennypacker. "An Analysis of the Advance-Dcline Line as a Stock Market Indicator." Journal of Financial and Quantitative Analysis. III (September 1968): 299-314.