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CHILDREN'S UNDERSTANDING OF NUMBER SYMBOLS IN FORMAL AND INFORMAL CONTEXTS

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CHILDREN'S UNDERSTANDING OF

NUMBER SYMBOLS IN FORMAL

AND INFORMAL CONTEXTS

by

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ABSTRACT

A focus of the present study is an examination of the relationship between cognitive development and social knowledge. This was investigated by exploring children's developing understanding of number symbols in the domain of school and their everyday lives. One guiding assumption of this research is that when children do poorly on mathematical tasks one should not rely on cognitive structural deficits or on an inability to understand mathematical operations as explanations of poor performance. Another assumption guiding this work is one taken from Vygotsky which views cognitive activity -- e.g. mathematical knowledge -- not as an isolated "in the head" mental activity but an activity which is social in its origins and how it is conducted. Children in this society, depending on their race, sex and class, have different social relationships to the activity of mathematics.

There was a total of fifty-four black low income children in this study, ranging from first through third grades. Three tasks were designed for the study. Two of them, the On and Off Paper Tasks, were

designed to explore children's understandings of the meaning of numbers as quantities and numbers as labels in the context of written and verbal tasks. They were also designed to explore if and when children were likely to confuse the two. The third task, the Conventions Task, was constructed to examine children's understandings of the grammar of mathematics tasks.

The hypotheses that the performance of on and off grade level children was a result of the inability of poor performers to distinguish between numbers as labels and numbers as quantities was not supported. However, there were developmental trends in the children's ability to make these differentiations and in the children's understanding of mathematical conventions.

Finally, the two areas of focus developing from the research project have to to with the development of formal and informal concepts and why the question of differential performance in young children is a major question in developmental psychology. In the area of the development of formal and informal number concepts evidence is provided to support Vygotsky's position that formal and informal concepts develop in reverse directions.

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INTRODUCTION

A major theme to be explored in this paper is the relationship between cognitive development and social knowledge. This theme will be investigated by examining children's developing understanding of number symbols in the domains of school and their everyday lives. One guiding assumption of the present research is that when children do poorly on mathematics tasks one should not rely on cognitive structural deficits or on an inability to understand mathematical operations as explanations of this performance. The question of interest here is: can poor performance on mathematical tasks be explained in ways that involve the extension of informal meanings of number symbols to formal mathematical learning in ways which can hinder performance.

Another assumption guiding this work is one which views cognitive activity -- e.g. mathematical knowledge -- not as an isolated "in-the-head" mental activity but as an activity which is social in its origins and in how it is conducted. Children in this society, depending on their race, sex, and class, have different social relationships to the activity of mathematics. Therefore, it is believed that this difference in how children's relationships are socially organized in relationship to mathematics also impacts upon their performance and is part of the so called "miscellaneous" (e.g., Walkerdine, 1979) information that is operating when they are doing mathematics.

It is clear from the findings of a pilot study and from observations of young children that children do participate in numerous interactions about number symbols in informal everyday encounters both with adults and peers. What is being argued here is that the knowledge that arises from these interactions must be acknowledged as possible sources of conflict or influence with school or formal number learning.

Although it is generally accepted that children learn something about numbers in their early preschool environment little attention is given to the possibility that some of the child's number information is at odds, in contradiction with, or just not useful to the formal mathematics environments encountered in school. It seems to be assumed that the child either receives little information from his home experiences (Kirk, Hunt & Volkmar, 1975) or that the number knowledge that he does receive can serve as prerequisite knowledge for the development of later mathematical concepts and skills.

The present study examines children's understandings of number symbols as labels versus their understandings of these same symbols as indicators of quantities. Children in first, second and third grades were also interviewed about their understanding of the conventions of mathematics tests. The potential conflicts among the different understandings of number symbols were explored.

The following is a discussion of several theoretical perspectives that have been useful to this research and a review of pertinent empirical research. Finally, the present research project will be introduced.

Contributing Theoretical Frameworks

Piaget's theory of mathematical development is of course reflective of his structuralist approach to cognitive development. Thus, for Piaget, any model of the young child's use and understanding of numbers will be qualitatively different from that of the mature adult. Even more importantly, one would not expect too much similarity between the number concepts of the preschooler and those of the second or third grader. These differences are of course tied to the older child's development of the ability to conserve.

Piaget (1941) sees the concept of conservation as an essential principle which forms the basis for a framework of numerical reasoning. The child in his attempts to construct a concept of number, must come to coordinate the operations of series and seriated classes. Thus in The Child's Conception of Number Piaget presents a series of experiments designed to investigate both the child's ability to conserve quantity as a prerequisite to acquiring notions of quantity and to investigate the stages involved in the development of an understanding of series and seriated classes (Piaget, 1941; Elkind, 1968).

Piaget (1941) has also demonstrated that the non-conserver does not use counting as a way to correct his perceptual misunderstandings about two equal arrays of objects. This, for Piaget, is evidence that the young child does not have a mature conception of quantity. Thus, the child's use of counting is at most an empty imitation of adult behavior (Elkind, 1968) and cannot be considered an effective symbol system until the child comes to understand the principles of number conservation (in Saxe, 1977).

Thus from a Piagetian perspective, what can one expect about the relationship between the preschooler's everyday mathematical knowledge and his formal school knowledge? Prior to the development of conservation, Piaget's preschooler is described as possessing some amorphous and ambiguous concept of quantity (Zimiles, 1968.) Piaget describes the number concepts of preschoolers as having hints of quantity in that they are tied to the notion of "more" but he sees this quantification as non-numerical. Although for Piaget, the number experiences of the preschooler's everyday environment would not be valueless, they would also not contribute directly to the knowledge that is necessary for formal school mathematics. For example, the counting activities of the preschooler would be of value because it is the child's "grouping, ordering, and counting action that, on the mental plane give rise to class relations and arithmetic operations" (Elkind, 1968). But these counting activities are not evidence of the child's ability to reason about numbers. Once the child achieves an understanding of invariance, his counting activities achieve the status of a symbolic system and his interpretation, understanding, and use of his practical knowledge about numbers becomes incorporated into the more sophisticated structures of the conserver.

Piaget does recognize that in the case of small sets children are able to make "intuitive" judgments about numbers. However, he does not give these intuitive judgments the status of number reasoning but describes them as perceptual, that is, linked to a primitive ability to perceive the "wholeness" of certain sets. Bubler (1975) is making a similar argument when he states: "When the child appears to have grasped the four and five of domino blocks, it is configurations and not real groups which are concerned." Klahr and Wallace (1973) suggest that small sets are processed by a subitizing mechanism and larger sets by a counting procedure. Thus, the child's perceptions according to Piaget and Bubler would not be based on any understanding of the relationships between a whole and its parts. The child is, therefore, incapable of reasoning numerically about activities performed even on sets of small sizes.

Piaget's model of children's development of mathematical knowledge has been described as being based on the "ideal world" of mathematics to the exclusion of real world knowledge (Walkerdine & Sinha, 1981; Glick (in press)). The practical, day to day activities of counting and labeling with numbers is of interest only to the extent that it lays the groundwork for the acquisition of logical mathematical principles. Moreover, for Piaget, these principles are the product of the developing genetic epistemology--an organization of knowing activated by the child's interaction with the environment. Thus, for Piaget, a developing mathematical understanding is a kind of learning that is not social in origin but biological. This theory then denies the impact of history, and the sensuous activity of real children in real times and real places developing their mathematical skills. This research takes issue with locating mathematical knowledge outside of real world activity and seeks to demonstrate how it is and mathematical activity is always a part of the real world.

Gelman (1980) suggests that the conservation task underestimates young children's understanding of numbers. It makes them appear much less capable than they actually are. Thus, an important aspect of her research is the investigation and identification of the capacities of preschoolers, (Gelman, 1978(a)) as opposed to a somewhat traditional trend to contrast their performance to a more competent model while highlighting their inadequacies. Gelman's work is, to a large degree, motivated by the question of what it is that preschoolers can do.

Gelman (1978(a)) and Gelman and Gallistel (1978) have investigated the child's ability to reason about numbers and their use of counting as a way to represent the numerical values of a display. They have demonstrated (Gelman and Gallistel, 1978) that much of the number activity in children ages two to five is rule governed regardless of the child's ability to verbalize the rules.

For Gelman and Gallistel (1978) counting plays a major role in the number experiences of preschool and primary school children. They have analyzed counting skill into five component processes. They are the one-to-one principle, the stable-order principle, the cardinal principle, the abstraction principle -- any collection of items may be collected together to be counted -- and the order-irrelevance principle which suggests that number words are arbitrary tags and any item in a set can receive any tag. Their work on preschoolers' grasp of these component skills has led to evidence that preschoolers do indeed reason numerically about these processes.

Gelman and Gallistel's (1978) model, however, is based on the notion that preschoolers need not always be skillful counters or demonstrate some exact replica of adult behavior in order to have some capacity. They instead make a distinction between the ability to reason about these principles and the mistakes that are made in performance. They have found that although children do not always use conventional words in a conventional order, close scrutiny of what it is that they do suggests the existence of rule governed principles. For example, the children in a pilot study conducted for this research who counted by saying 3, 4, 5, 6, 7 and indicated that they had 7 oranges, would be described by Gelman and Gallistel as being aware of the cardinal principle. Also the fact that a child in the same pilot applied one tag to one item (there were a total of 5 items) indicates an understanding of the one-to-one principle. There are a number of examples from the data of the pilot study to support Gelman's notions of underlying rule-governed principles in the young child's counting behavior.

Gelman (1980) and Gelman and Gallistel (1978) view the early arithmetic capacities of the preschooler as natural and universal, similar to the universal and innate grammatical rules that govern Chomsky's theory. What appears to be developing in the young child according to their model has little to do with underlying structural changes and more to do with improvement in accessing various existing skills. The argument for the universality of counting procedures can be tied to Zaslavsky (1973), Saxe (1979(a)) and Conant's (1954) work which explores the use of counting in other cultures.

Although Gelman (1980) and Gelman and Gallistel (1978) disagree with Piaget's argument that conservation precedes numerical reasoning, they do not dismiss the importance of conservation as a developmental milestone in the intellectual development of the child. They argue that although it is not clear exactly what the ability to conserve is related to, it is quite possible that conservation indexes the beginning development of higher order arithmetic, reasoning abilities.

Gelman's (1980) suggestion that conservation tasks underestimate young children's understanding of numbers is a very important one. Much of the research on early mathematical abilities has been involved in demonstrating the effects of number conservation on the child's ability to perform other mathematical skills. Numerous studies have suggested that conservers do better on various arithmetic tasks (Almy, Chittendam & Miller, 1966; Robinson, 1968; Steffe, 1967). However, assuming that conservation is necessary before the child can form an understanding of numbers can interfere with the identification of the various component skills that are necessary in the performance of early arithmetic operations. Some of these skills have been discussed in the work of Heller and Resnick (1979) and of course by Gelman and Gallistel (1978).

Work by Mpiangu and Gentile (1975) and by Ginsberg (1977a) has explored arithmetic abilities in young children who are non-conservers. Ginsburg and Russell (1979) reports that almost none of the children in his middle class or inner city preschool group succeeded at conservation in spite of their demonstrations of other mathematical skills. Also Saxe (1979(b)) reports evidence that quantitative counting strategies develop prior to the development of number conservation.

Gelman and Gallistel (1978) argue that the preschooler's understanding of small set sizes is not an outgrowth of non-rational thinking. They would argue instead that the young child is capable of numerical reasoning and that this capacity is best understood when analyzing their behavior in relationship to sets of small sizes. These small sets allow the child the opportunity to investigate linkages between their reasoning principles and reality and are a domain where the demands of the task are not so overwhelming that the child is unable to demonstrate his or her skills (Gelman and Gallistel, 1978). They present evidence to suggest that the number representations in the case of small sets are first obtained by counting rather than by subitizing and that it is through the constant application of these counting procedures that these small sets become almost immediately recognizable to young children.

Gelman & Gallistel's (1978) work raises issues about the importance of developing methodologies that are effective in identifying skills in preschoolers' development. How they choose to understand the development of those skills is somewhat problematic and once again raises the issue of mathematics skills as genetic intellect. Gelman (1980) characterizes these skills as natural "arithmetic abilities similar to how Chomsky discusses "natural" linguistic abilities." Gelman (1980) also cautions, however, against assuming that the existence of 'natural' arithmetic abilities downplays the importance of mathematical instruction. Although she argues that there are natural arithmetic abilities, she also argues that young children do not spontaneously develop an awareness of the rules that govern their behavior. Gelman rejects the notion that conservation is necessary for preschool rational number knowledge, identifies logical underpinnings in the preschooler's numbering activity and then evokes biological explanations for why it is that these skills exist. It is not clear what is gained by using a biological explanation in understanding these logical activities in preschoolers' interactions with numbers. What it helps to do, however, is to relegate mathematics once again to this plane of development that occurs in the genes rather than in the real world experiences of young children.

There is also a case to be made based on Gelman and Gallistel's (1978) findings that there exists in the preschooler's early number experiences some potential conflict between this information and that of formal school arithmetic. This is especially possible in those cases where the child learns to use idiosyncratic lists as counting tags. In addition, depending on the number of experiences encountered, children might also enter school as poor counters -- their performance skills may not necessarily reflect their competencies. However, overall, Gelman and Gallistel's preschoolers' everyday experiences should be of considerable value in the learning of formal school mathematics, especially if the teachers are aware of the performance-competence distinction and are able to identify the true source of children's problems.

Neither Piaget (1941) nor Gelman and Gallistel's (1978) theories speak directly to the possibility of children's everyday number learning as a source of potential confusion once the child has entered school. Results from a second pilot study (see page 38) suggest that this is very possible. Children's experiences with number symbols in the everyday environment can and at times do serve as sources of conflict if the child does not learn to make distinctions between number symbols as signalers of number operations versus number symbols as labels.

Ginsburg's work on children's mathematical errors (1977a) and his more recent work in the area of mathematical development on low income and learning disabled children (Ginsburg and Russell, 1979, 1980) is also of particular interest to this research. In his earlier work on children's errors Ginsburg demonstrated that children's errors are often the result of systematic rules and not simply random error. He offers two clear examples of how children's early preschool and informal experiences with numbers can potentially lead to confusion. One is of a child who although she understood that ten years of age was older than one year would identify ten as equal to one once it was written on paper because she had also learned that zero had no value. She could not represent her knowledge in this instance in number symbols. Another concerns a child who could add correctly when "altogether" was used instead of "plus". Ginsburg explains this as a failure to understand arithmetic operations unless they are framed in very specific language. He suggests that "plus" is a word used most frequently in school while "altogether" is a word for addition which was probably learned in informal interactions.

Ginsburg's work on black low income and on learning disabled children has been very useful for developing the theoretical perspective of the present work. In an introduction to his work on black low income children he points out that although assumptions abound concerning poor children's performance on mathematics tasks there has been very little actual research on the competencies and weaknesses of these children. The one study cited that dealt with this issue (Kirk, Hunt & Volkmar, 1975) was filled with methodological flaws. Ginsburg's and Russell's (1979, 1980) work demonstrates that black low income children are not deficient in their mathematical abilities and that their developmental progression on math tasks is "normal".

Ginsburg, in a discussion of his work on learning disabled children, concluded that many of the children labelled learning disabled were misidentified. He suggests that rather than focussing on the "insides of their heads" as a way to explain school math failure it is important to focus on the educational system, the methods of teaching and the methods of evaluating children's progress. He described children who were learning disabled not as bizarre thinkers but as children who often appeared to have difficulty remembering number facts, though the reasons for this difficulty are not clear. He stresses, however, their use of strategies in math tasks that were similar to those used by their "normal" peers. Finally he suggested a clinical interview method as a more precise way than typical arithmetic tasks for getting at what it is that children are both doing and are capable of doing.

Ginsburg's research raises important issues of interest to this work. It is clear that the circumstances often exist for there to be conflicts between informal and school number knowledge. His work and observations of black low income and of learning disabled children raises issues about how it is that children are socially organized in relationship to mathematics, which includes how they are organized to remember or not remember "number facts." The assessment that teachers make of children's ability or inability to learn math is an important determinant of what and how children are able to learn. Many teachers are predisposed to assume that the poor or black child is relatively inept at mathematics.

Out of all the theorists cited in this research Vygotsky's work has been the most valuable in developing the theoretical perspective of this research. There are several aspects of his theory that are particularly relevant here. One has to do with Vygotsky's understanding that the development of higher mental processes begins on a social plane. In contrast to other theorists (e.g. Piaget), Vygotsky argues that the skills and processes of thinking first occur on the interpsychological level and only later are transferred to the intrapsychological (individual) level.

> ...All higher mental functions are internalized social relationships....Their composition, genetic structure and means of action -- in a word, their whole nature is social. (Vygotsky, 1978, p. 164).

This understanding of mental processes locates these processes in the midst of the social activity of the species. Vygotsky emphasizes the conception of social as both historical and cultural. For Vygotsky "the activities of human beings, at all stages of development and organization, are social products and must be seen as historical developments, not merely as interpersonal developments" (Hood, In Press).

This understanding of the development of mental processes is in sharp contrast with theories which emphasize and see as predominant the biological origins of thinking activities -- e.g. mathematical abilities. One way to understand this aspect of Vygotsky's theory is that mental abilities are not the development of the organism's inherited biological characteristics which are then realized in certain social conditions, but that instead these mental abilities are themselves the "tools and results" developed in living and working in the world.

Krutetski's (1976) discussion of differing perspectives taken by Western and Soviet psychologists on the development of abilities helps to illustrate this point. Western psychologists (e.g. Thorndike and Anastasi) argue that individual differences in abilities are related to biological abilities and to constraints on the individual. Therefore individual differences serve as the basis for arguing class and race distinctions. For example, in American society, racial and class supremacy are not explained, as Vygotsky would insist, in relationship to socio-historical causes but instead on the basis of the innate and invariable potential of individuals.

Another aspect of Vygotsky's theory which has been useful here and which accounts for the development of a child's mental processes from the interpsychological to the intrapsychological plane is the zone of proximal development.

> It is the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers (Vygotsky, 1978).

This concept is of particular importance here because of its emphasis on processes rather than endpoints in development. The tendency to concentrate on standardized testing in the evaluation of mental capacity as a way to assess the presence of certain abilities or concepts in a child is characteristic of American psychology. What does not evolve from this focus is an understanding of the child's potential or of the processes involved in mental functioning. The interview format used in this study was constructed with an interest in understanding process.

Another aspect of Vygotsky's theory that has been important in formulating this research is his work on the development of spontaneous and scientific concepts. The focus of the present research is to examine the impact of number symbol concepts which develop informally and those concepts which develop in the context of school instruction. This research supports Vygotsky's observations that: the two concepts must differ in their development as well as their functioning and that these two variants of the process of concept formation must influence each other's evolution (Vygotsky, 1962).

Piaget (1969) raised similar issues in his discussions of the development of spontaneous and non-spontaneous concepts. He described spontaneous concepts as representing the child's ideas of reality which evolve mainly through his or her own independent mental efforts. Non-spontaneous concepts, on the other hand, evolved from the direct influence of adults and did not bear the same stamp of the child's mentality as did spontaneous ones. Therefore to best understand the special quality of child thought, the psychologist probes for the "uncontaminated", spontaneous thought processes.

Vygotsky (1962) insists that despite the different origins of Piaget's non-spontaneous concepts, they are by no means separate from or unorganized by the distinct character of the child's thought. Vygotsky claims that all concepts--spontaneous or "non-spontaneous"--are social in origin though to be sure internalized and assimilated to the child's conceptual framework.

Using a somewhat different taxonomy, Vygotsky makes the distinction between <u>spontaneous</u> vs. <u>scientific</u> concepts. He does so not to identify which kind is more the child's own, but to make distinct their different origin, function, and trajectory of development. Children learn spontaneous concepts in their everyday lives before entering formal training. Once in school, they learn more formal, scientific concepts which reorganize their thinking about both scientific and spontaneous ideas. Spontaneous concepts develop in a process where the child is focusing on the object to which the concept refers and not on the act of thought itself. On the other hand, Vygotsky suggests that the rudiments of systematization first enter the child's mind through his contact with scientific concepts. This systematization is only later transferred to everyday concepts. In this sequence of development, spontaneous concepts lay the groundwork for learning scientific ones, and the learning of scientific concepts raises spontaneous concepts to a new level of self-consciousness.

The relevance of Vygotsky's work to the present research has to do with the role that informal number symbol (spontaneous) concepts play in learning formal (scientific) number symbols in school. Given Vygotsky's claim as to the developmental unity and interplay among scientific and spontaneous concepts, how should informal number knowledge be organized to teach young children mathematics?

Piaget's characterization leads us to assume a mutually antagonistic relationship between informal and formal number symbol concepts. Hence, the role of academic instruction is to use the spontaneous concept as a means to a formal mathematical end. Vygotsky, on the other hand, suggests that there is a mutual interrelationship between the development of these concepts and that the two are complementary in the process of formal education. This distinction between Piaget's and Vygotsky's understandings of the development of spontaneous and non-spontaneous concepts also has implications for the issue raised earlier concerning the value and impact and development of informal thinking. It also raises issues about how it is that informal thinking and concepts are related to in academic settings. The results from this research will be examined in relation to Vygotsky's and Piaget's positions on the development of scientific and spontaneous concepts.

The claim of the present research is that the study of children's number knowledge is not the study of the development of formal mental structures in children's heads. Often, as with Piaget, formal mathematical learning is treated as the development of "knowledge" where knowledge results from the elaboration of internal mental structures. Rather, mathematical knowledge, like other mental abilities, has to be understood as a complicated social activity organized through the child's interactions in the world.

Walkerdine and Corran's (1979, 1981) distinction between metonymy and metaphor raises issues about the nature of formal mathematical learning. Metaphor is a means for making something comprehensible by comparing it to something else; metonomy refers to what is self-explanatory or self (and only self)-referential. Foucault (1972) describes bodies of knowledge such as mathematics as regulated systems of statements which he calls discourses. Walkerdine and Corran (1979, 1981) examine the way in which mathematical discourse differs from others. Mathematical discourse is described as metonymic when it contains statements that do not allow the individual comprehending them to make sense through reference to any other discourse. Mathematical discourse is a self-contained discourse much like logical syllogisms, whose resolutions are located in syllogistic statements, not in the real world.

The question then is to what degree is mathematical discourse engaged in by children in a metonymic form. Walkerdine and Corran (1979) contrast the differences between written and spoken math statements as a way to illustrate the metaphoric-metonymic difference. They describe the written statement 2+3 = 5 as metonymic having no metaphoric meaning. They demonstrate how in the inclusion of natural language, written or spoken, metaphoric content is introduced and the statement is thus transformed in various ways depending on the words used. The difference between reading this statement as "two plus three equals five" or "two plus three makes five" can lead to drastic differences in how this mathematical statement is read and acted upon. In other words, what I will argue is that for children there is no such thing as "pure" mathematics; children always understand mathematics not exclusively in a metonymic mode, i.e. without reference to anything "outside," but also metaphorically.

It is not clear that children ever operate on the formal rather than the social logic of the problem. One guess is that this is tied to the context in which the problem is presented. This is complicated by the assumption that the less contextual support present the more intellectual the task. But it will be argued here that it is not that contextual support is ever absent but that one is unaware of what is evoked (Walkerdine and Sinha, 1981).

In a discussion of semiotics Walkerdine and Corran (1979) discuss the limitations of Piaget's reliance on a mathematical model which neglects the social nature and context of tasks and which "relegate to the signifier the role of articulating operational structures which themselves arise outside of any relationship to any system of signs." Vygotsky's formulation of mental processes arising in the course of social interaction gives social significance to the signifier and allows for the imposition of context in formal mathematics. Thus one task of the young child in mathematical development is to learn when or when not to leave out the metaphoric (social logic) relations that are present in their lives. This has implications for how one structures the math learning of young children. It is not a "natural" process.

The social interaction contexts involving number symbols organize the child's understanding of these symbols very differently. The following will involve a characterization of the informal everyday nature of number learning environments and that of school or formal number learning.

Pertinent Literature on
Informal Number Learning

One distinction made here is between scientific and spontaneous number symbol learning. Walkerdine and Corran's (1979) discussion of metonymic-metaphor is also useful here. "Children incorporated in everyday practices through the production of signs learn to position and locate themselves through (metaphoric relations) which are historically and culturally loaded." Thus one of the tasks of formal schooling is to teach children to "exclude metaphor," to operate on metonymic relations alone. What Walkerdine and Corran identify as metonymic are those operations involving pure mathematics with no interference from extraneous, metaphorical, everyday number knowledge. It is not clear that numbers are ever related to in a purely metonymic sense.

Formal number learning is the systematic learning of number symbols and number concepts which generally occurs in school interactions and leads to performance on math tasks in "school-like" ways. Formal number learning can also include errors in performance (e.g. incorrect number facts). For example, understanding numbers as things you perform operations on can interfere with pre-formal uses of numbers (such as labeling). Formal math learning is thought to occur when the child comes to operate on metonymic operations alone. However, one of the issues raised here is the problem with treating school learning, the development of formal concepts, as metonymic, abstract, isolated and non-social.

Much of the number symbol learning that occurs outside of the school context takes place in social interactions that do not stress or highlight metonymic operations. Much of this learning provides the metaphoric relations that can enter into and sometimes impede progress in school math. Thus informal number learning describes information about number symbols which can influence performance on a task but which lies outside of the information necessary to math tasks. What is focused on in this paper are the possible sources of metaphoric influences, many of which arise in social interactions, that occur outside and inside of school environments.

What is it that children learn about number symbols prior to schooling? There are numerous reasons for numbers to be of concern to young children. They are interested in ages, in the channel numbers of their favorite television programs, they have discussions about bedtime and about the number of pieces of candy they are allowed to have. Observation demonstrates that children are aware of these number symbols that appear throughout their environments, and they even speak of these number symbols in ways that are appropriate to the various contexts. For example, when referring to the number symbols on scales children talk about pounds; or when referring to measurements made using tape measures they speak of inches. However, exactly what it is that children understand or know about these common number symbols is not clear.

Most of the research on children's early number learning is designed to determine what it is that children understand about operations prior to entering kindergarten. This research will be reviewed in the following section.

There have been very few studies that have addressed the issue of how home number learning influence, is the same as or different from learning about and using number symbols in school. Payne and Rathmell (1975) suggest that any planned mathematics program must take into account the knowledge already possessed by children. However they also focus on the knowledge that is compatible with school number knowledge. Robinson (1947) in a study investigating out-of-school use of arithmetic among pupils in first and second grade had parents record the experiences in which their children used numbers in the home. Robinson (1947) reported that activities centering around money and time accounted for more than half of the home usage of numbers. Robinson's (1947) findings however are possibly confounded in two ways. One is that it is not clear what instructions were given to parents and therefore what parents identified as number use. Another possible source of confounding has to do with the influence of school on the child's use of numbers at home since these subjects had already had one or more years of schooling.

A pilot was conducted in the spring of 1980 in an attempt to determine how and when preschoolers, ages 2-3 years, use numbers. A total of twelve mothers participated in this pilot. The mothers were instructed to keep daily notes of their children's use of numbers regardless of how bizarre, repetitive, incorrect or mundane the instances appeared. Each mother was given a sheet on which she was asked to record:

1) what was said
2) items involved
3) what child was doing and with whom
4) other information of interest

The pilot lasted for eight weeks and the sheets were collected weekly. The subjects were participants in a play group or were siblings of children in a New York City Headstart Program. Most of the subjects had not yet begun school. Mothers were also given a twenty-five dollar bonus at the completion of the study. Although twelve subjects is a very small sample, detailed data was collected on these children's use of numbers and it provided useful information that aided in the formulation of the present study.

The pilot showed that children talk about numbers, use number symbols and apply number operations across a wide variety of contexts. They count all types of household objects, food, and toys. There are also abundant opportunities for these children to observe number symbols in their environments.

The data was analyzed to determine where children see and acknowledge number symbols, to investigate their displayed understandings of number concepts and to explore the "scripts" or contexts in which the numbers are usually embedded. The overall interest was to begin to determine how the home is similar to or different from the school as an environment for learning about number symbols.

There are a number of places where young children experience number symbols in their preschool environments. There are the number symbols that are actually on many common household items (washing machines, radios, clocks, etc.). They also hear number symbols on records, on television and radio programs. They recognize number symbols that are written by planes in the sky and they find them in the shapes of pretzels, candle holders and toy train tracks. Much of this identification of number symbols as is much of their counting is spontaneous and applied in situations that are unlikely imitations of adult counting. For example, these children counted splashes in the bathtub, sides of a sheet of paper, drops of honey as it fell from a muffin and the spins on the dishwasher. Thus, there are many opportunities for children to experience and use number symbols in their home environments.

It should be noted here that among this group of preschoolers, except for the extensive counting, there were very few examples of activities that looked like school number exercises. Some of the children did write their numbers in imitation of an older sibling or because they had to "do their homework." Others expressed awareness of their ability to count, "I can count from 1 to 10"; or their inability to do so, "I cannot count 17, 18, 19 and 20." The latter example was taken from an instance where the child was trying to write his numbers. The only other school-like activity observed in the home was done by a child who attended preschool. She wanted to know what was seven in Spanish and how much 4 and 5 was equal to in Spanish. This is not to suggest that children do not learn "school-like" operations from their home experiences with number symbols. It is simply to suggest that the experiences that children do have with number symbols in their home do not mimic school arithmetic. And there is no evidence to suggest that they learn naturally the need to operate on metonymic operations. The results from the pilot data show that children encounter numbers at home in a wide variety of situations that could potentially lead to conflicts in their understanding and use of numbers. Although data from this pilot did not show actual metaphoric conflict, they did indicate the possibility of such conflict.

There are also many instances where children confuse the meanings of number symbols or where they use incorrect symbols to represent number sets.

- two, five. I have two sneakers
- mommy how old are you?
  (Thirty)
  I'm 3. I'm 3 too.
- how old are you?
  (I'm 35)
  I'm 3 too.
- (Are you going to church too?)
  No, I'm going to one church.
- (Someone on TV said she had worked somewhere for 3 years)
  Mommy I'm 3. She's not 3. She's too big to be 3. Me 3.
- (Using a ruler to measure his sister's leg. Points to number 6.)
  You're six Ainka. You're six years old.

These instances highlight some of the work that has to be done by the preschooler who is confronted by number symbols which vary in their meanings. He must learn the names of the symbols; he has to learn to recognize them; he has to learn their meanings; and he has to learn that the same number symbol does not always represent the same underlying concept.

The notion that number symbols should be introduced only after concepts are taught is, as indicated earlier, an important one in the literature on preschooler's mathematical experiences. Educators are suggesting that if the child's early experiences have caused him/her to attend to the numbers superficially at the level of language and counting, then there is a need for activities which help the child to associate meaningfully and conceptually the language with the appropriate groupings. For example, a young child in counting two toys may apply the appropriate labels but when directed to put two toys in his toybox, he may carry only the second toy (Gibb & Castaneda, 1975). Thus it is clear that the word two is being associated with a particular object but not being understood conceptually.

In analyzing the concepts that appear to be understood by this preschool group, it was rather clear that children can appear much more sophisticated in their number concepts than they really are. They read price tags: "This cost $17 Mommy." They read the scale: "I weigh 25 pounds." They count money: "I have fifty pennies." However when children are asked to demonstrate their understanding of concepts such as cost, weight, or value, in fact they are unable to do so.

The pilot data was analyzed to determine how often when using the number language and symbols children actually demonstrated through their behavior an understanding of a number concept. Eighty-one examples were found where children's understandings of concepts could be examined. The concept that the children consistently demonstrated an understanding of was two. Forty of the examples involved discussion of the number two. There were only seven instances where the children demonstrated an incorrect understanding of the concept. There were also eleven instances where understandings of number three were demonstrated and five where the number was used incorrectly. There were two correct demonstrations of the concept four, one of six, and four correct instances of number one. These findings help to support Gibb's and Castaneda's (1975) recommendation that teachers should start by teaching children about sets of two because the body provides so many model sets for two. It is also apparent from the many examples that the home environment also provides many experiences upon which to build knowledge about the concept "two" and its number symbol. It is also apparent that children's use of number language and symbols often exceeds their understanding of the underlying concepts.

What else can be learned about children's everyday preschool experiences with numbers? Some attention needs to be given to the contexts or "scripts" in which these number symbols appear.

A description of social scripts is presented by Shank and Abelson (1977) where they define a script as a "very basic kind of organizing structure in human memory." A script enables people to infer unstated propositions from events and statements, to interpret simple stories and to predict probability sequences of events. It may also be personal or social and, to the degree that it is social, it represents shared knowledge about culture, socially shared events. Nelson (1977, 1978) has adopted, to some degree, the script model as a way to describe and analyze the structure of children's knowledge of everyday events, to specify components and to compare knowledge about these events across ages and events. A method used by Nelson has been that of asking children through verbalizations or play to demonstrate acts or events involved in going to a birthday party or eating a meal or eating in a restaurant.

The script model seems a useful method for investigating children's knowledge and understanding of events and for the investigation of how this knowledge develops. There are a number of ways in which the model can be useful. Scripts are said to evolve from the child's everyday experiences. They are also said to be used by the child to make predictions about routines and procedures as a way to know what is expected next in a sequence of events. In addition, Nelson (1978) has discussed the possibility of script mismatch for children coming from different backgrounds or from a different set of experiences.

What is of particular interest here are the situations in which children learn about or encounter number symbols in their home environments. An investigation of those situations reveals that much of numbering or uses of number symbols is embedded in other scripts. Children's uses of number symbols in their everyday environment are embedded in playing football, weighing oneself, calling daddy, using the elevator, playing store, reading the "TV Guide," going shopping, dancing, riding the subway, playing game show, playing baseball, playing hide and seek, etc. Often, the pragmatic contexts or scripts of everyday living neatly identify the relationship of the number symbol and underlying score. Number symbols then appear as part of some other script where the child does not have to know or understand the underlying relationship. The contexts specify, as in the case of pounds on the scale or time on the clock, what to do with numbers -- at least how to talk about them -- and the child need not know what a pound is or what time actually means. Thus, much of the child's informal number symbol use and knowledge is context or script bound.

What kinds of knowledge or scripts do children have about number symbols as a result of their everyday activities? The two examples presented earlier from Ginsburg (1977b) are useful here. The examples suggest two things. One is that there are differences in the kinds of number language that preschoolers develop at home and in school. Another is that the information that is developed about the number symbols can be sometimes confusing, faulty and misleading. An example from the observations made by mothers is provided below:

> (Amani, using a ruler, holding it up against his sister as if to measure her leg. Points to number 6.)
>
> You're six Ainka. You're six years old.

This is a clear example of the type of confusion that is possible. This child is obviously unclear about what the numbers on the ruler represent. His sister is six years old and he does see a six on the ruler. He knows that number is relevant. However when confronted with a familiar number in a not so familiar situation he uses number in a habitual way. This confusion is likely to occur in those situations where children have not clearly come to understand the variety of meanings a number symbol can represent. These different meanings of the same number symbol can be thought of as situations where number has either quantitative or non-quantitative meaning. The number symbols encountered in everyday environments are sometimes to be added and subtracted in the same way that school numbers are. However there are also numerous examples of number symbols that serve only as labels or as ways of indicating position. One task of the preschoolers is to come to make this distinction between numbers as quantities -- school-like numbers -- and numbers as labels.

A second pilot study was done in July 1980 to begin to explore children's understanding of the meanings of number symbols which represent quantities versus those which are only identifiers or labels. Eleven children, kindergarten through third grade, were asked questions about the meanings of the number symbols that appear on buses, telephones, clocks, radios, televisions, elevators, math papers, football jerseys, etc. and the kinds of operations that were appropriate. For example, "Can you add a #2 bus plus a #1 bus" or in dialing a phone number can one dial 3 plus 3 for six.

There were clear developmental differences in the type of responses and explanations given. Third graders clearly stated that numbers representing nominal relationships could not be added:

- Can't turn to channel 2 plus channel 3. It won't work. You don't plus the numbers.

- You can't plus buses and you can't plus TV channels. You can't plus the numbers on them. You can say one bus plus another bus equals two buses.

Kindergarteners either had no idea why the numbers were used on objects or gave the following responses:

- Yes, you can add the #1 bus and the #2 bus and get a #3 bus (where is the #3 bus?) I don't know but you can.

- They put numbers on trucks so they can know which one is the biggest.

There was also a third type of response given by some first and second graders who either said that you could "plus" some nominal numbers but not others or if their initial responses were challenged would switch to a response which was that you could add nominal numbers. Often these children gave reasons like the following:

- Can add #1 plus #2 trucks to get #3 trucks but you then have to cut the #2 truck in half.

These children also made a distinction between "plus" numbers and those that fall in the nominal category. These plus numbers were identified as being "always" On Paper Task where the children were asked to "solve" arithmetic and non-quantitative problems (e.g. If you have a number 1 bus plus a number 2 bus, how many buses do you have?). Results from the pilot data show that some of the children -- third graders -- who were clear that nominal numbers presented in familiar contexts could not be added together, "added" these same numbers once they were presented as school-like numbers in an On Paper Task. These findings suggested that there was potential confusion between number symbol learning in the informal versus school environments and that some of the demands of school arithmetic learning can interfere with children's abilities to conceptualize problems.

School As A Number Learning Environment

It is important to make explicit that although formal math is encountered in school, all that children learn about numbers there does not contribute to formal knowledge. In the context of an interaction with teachers children learn many things about their relationship to number symbols which are not essential to formal mathematical learning but which can have considerable influence on how they do math tasks. It is important to look at literature that makes this point.

It is clear that children have conflicts at times between what is being taught and what is learned informally. What is also being argued here is that performance is impacted upon by how children are located socially within this society. Ingleby (1974) asserts that practically every act in relation to a child from birth and even before reflects constraints dictated by that child's place in the political system. He points out also how psychologists (and educators) are trained to treat the child's political location as an extraneous variable rather than as having profound impact on how they understand and are understood, how they do and are done to in all aspects of their lives including "mental" activities.

Anyon (1981) further elaborates this point in an article entitled "Social Class and School Knowledge" where she shows how the organization of knowing is guided and constrained by class differences that are maintained through the process of schooling. She shows through observations and interviews in school settings where parental income varies from $12,000 or less to $100,000 the social activities by which children come to understand themselves as knowers and how profoundly different those activities are arranged depending on social class. These differences are present in the psychological and educational activities in various forms.

For example, one area in the context of schooling where "extraneous" information has potential impact is in the interaction and understanding between teachers and black children who are described as poor math performers. Just as in the case of sex differences, researchers and educators can be hampered by assuming that what they are measuring "is an abstracted notion of individual progress rather than locating that performance in terms of concrete achievement in a particular place and time." McNamee, Katz & Bowman (1982) present data on the nature of the teacher-child interaction showing how attitudes and knowledge interact to compromise learning. They highlight the teacher's difficulty in separating cognitive characteristics from personality characteristics when responding to questions concerning why particular children performed poorly on math tasks. The teacher's comments almost always pointed to person centered rather than cognitive characteristics. This point supports the contention that mathematical evaluations are not descriptions of these abstract intellectual abilities.

Another example of a context within the school setting where children's social relationships to numbers is important and which involve "extraneous" information are those interactions involving children who are successful and non-successful students. How do teachers' and children's perceptions of good-bad performers operate on their interactions around number symbols. McDermott's (1977) investigation of slow and fast reading groups revealed that children in slow reading groups are subjected to a significantly higher number of interruptions during reading time which leads to tremendously less time on task. And that all people -- children and teachers -- participate in constructing the social setting where this is the norm. In his theory of "how the smart get smarter," Rowher (1980) explains that smart children become more so because their teachers leave them alone. That is, teachers pay more attention--interest with, intervene in the thought processes of--slower students. It is therefore quite likely that the information involved in interactions around number symbols is loaded with "extraneous" messages.

Another example of importance here has to do with differences in the type of demands placed on the child by school arithmetic. What new things must the child learn about these number symbols? What are the new ways in which he/she must come to use them? What is the "social" understanding of his/her potential for using school number symbols? What are some of the characteristics that are unique to school arithmetic?

One clear distinction between everyday and school math is that school math is abstracted out of meaning. It is steps removed from the real world and it is this abstractness that often makes it difficult for the young child to use his everyday number knowledge where appropriate in this domain. The number symbols that appear in the child's everyday experiences are often presented as part of a script of something else. In the context of school, the meanings of number symbols are not as well specified and it may not always be clear to the child what is expected of him. Much of what is encountered in the child's everyday number environment is orally coded and functionally engaged. Much of what is required in the school number environments has to do with written symbolism.

Donaldson (1978) discusses the abstractness of schooling by asserting that the major task for young school children is that of disembedded thinking. This concept suggests the same activities that underlie Walkerdine's notion of metonymic thinking. However, what Donaldson does not engage are the ways in which mathematics training and learning are very much a part of the real world. She discusses school failure as if it is due to the child's inability to do disembedded thinking without discussing the politics of this training in a society where valued people become the mathematicians. Donaldson (1978) ignores all of the potential metaphoric confusion.

Another difference between the everyday number environment and that of the school centers around the introduction of symbolism and the demands of orthography in school math. In explaining the differences between spoken and written speech, Vygotsky suggests that written speech -- even its minimal development -- requires a high level of abstraction. He states: The motives for writing are more abstract, more intellectualized and further removed from immediate needs. It is not difficult to see how this might also apply in the area of written and symbolic math. This point is further supported by Ginsburg in a discussion of children's use of tallies. According to Ginsburg (1977a), tally making, as other forms of written arithmetic, can be viewed as an extra task which requires too much attention and effort.

Another characteristic of school learning is that number symbols and number concepts are presented to children in a very systematic and organized way. This is very different from the spontaneous use of them by young children in their everyday environments. In their everyday experiences children clearly have many more opportunities to stumble upon number symbols and they have the option of taking them in or of walking away from the experience. As was mentioned earlier, Piaget (1969) makes a distinction between the young child's spontaneous concepts which evolve mainly from his own efforts as a result of his everyday experiences and those non-spontaneous concepts which are influenced by adults. A large part of how the adult influences the development of the non-spontaneous concepts is through school instruction. Instruction is a principal source of school children's concepts and is a powerful force in directing their evolution (Vygotsky, 1962).

Studies on sex differences in mathematical abilities raise critical issues about the presentation of number facts in social interaction and how young girls come to know themselves as number knowers. Benbow and Stanley (1980) gave SAT exams to junior high school boys and girls. They conclude that since the girls in their studies do not have a pattern of course taking that is different from boys, sex differences in achievement result from superior mathematical ability in boys. They assume that until junior high school all the students had received essentially "identical formal instruction in mathematics." Why they make this assumption is not clear. Walkerdine and Eynard suggest that researchers in investigations of sex differences in math are often hampered by the idea that they are measuring an abstracted notion of individual progress rather than locating that performance in terms of concrete achievement in a particular place and time. Thus what has been neglected is what goes on in the classrooms in which math is learned and taught. The work of Walkerdine and Eynard at least suggests that the nature of these interactions can influence differences in attitudes and performance on math tasks.

Other considerations in classroom observations is that mathematics learning is a cumulative process and that for the most part, schools in poor black neighborhoods are notoriously bad academic settings. Finally, a study by the Minority Engineering Education Effort (1978) reported that many of the high school math teachers in black high schools are teachers who were trained in other disciplines as opposed to teachers in white high schools who are mathematicians. What then is the nature of the interaction and the learning process when participated in by non-mathematicians? What also are the assumptions underlying the relationship between the subject of math and the child's social location when student and teachers know that math is being taught by an "art" teacher?

Obviously, this is a major way in which the school context differs from that of the everyday environment in relation to the development of ideas and knowledge of number symbols.

A number of studies have been done by Russian psychologists (Istomina, 1974) which focus on the developing consciousness of children in the performing of mental deeds. Studies of metacognition by Flavell (1976) and Brown (1978) address similar issues while focusing on the use of strategies by children as they come to "know" what it is that they must know and do in order to perform some cognitive task. Piaget (1941) discusses how the concepts of the young school child are marked primarily by a lack of conscious awareness of relationships. Thus, the young child is able to use certain concepts spontaneously and correctly but he does not know how to use them deliberately or to reflect on their use. Gelman's (1980) discussion of the "natural" arithmetic abilities that develop in young children without the support of schooling supports this argument. Although her young preschoolers do show evidence of rule governed counting behaviors, she does not dismiss the importance of school instruction. Instead she suggests that one role of school instruction is to aid in the development of deliberation or consciousness in the use of mathematical skills. The doing of arithmetic in the early grades helps to make conscious the processes that young children often glibly display in early number behavior but rarely understand.

There are obvious differences in the demands of the school and home environment in relationship to the development of knowledge about number symbols. The previous review of literature demonstrates that although some attention has been paid to the role of everyday experiences in the development of number concepts, almost nothing has been said about the potential influence of everyday concepts of number symbols on the child's early school arithmetic skills and concepts. The textbooks used in the earlier grades make little mention of the likelihood that the child does have some "knowledge" about these number symbols and that this "knowledge" might be very different from what he must come to know and how he must come to use these symbols in the school environment.

Present Project

The intent of this research is to investigate what it is that children know about the number symbols that they encounter as part of their everyday and school experiences and how the influence of the everyday knowledge of number symbols interacts with the development of school concepts. It is clear from pilot studies that children experience number symbols in their everyday environments both as quantities and in non-quantitative ways. As quantities they count with them, use them to indicate amounts, see them on money, etc. As non-quantities, these same number symbols appear as labels and/or indicators of positions on buses, football jerseys, elevators and as addresses. In school, the child is faced with the same symbols which 1) may or may not be used in ways which are most familiar to him; 2) are used with different demands affixed to them; and 3) he may have some information that does not fit into the school's use of number symbols. What is investigated here is how an understanding of number symbols in their different roles as indicators of quantitative and non-quantitative relationships develops and if this confusion can contribute to incorrect performance on school math tests.

Another source of interest here is in the child's understanding of the grammar and conventions of mathematics (Allardice, 1976). Many children who can correctly verbalize mathematical operations demonstrate a misunderstanding of the same operations when they are put on paper. Do these children understand the symbols for operations (+, -, ÷, x, =)? Do they understand the signs which indicate spacing between sets (.. ...)? Do they know where to place correct answers or on which side to add or subtract first? Finally, what happens to this understanding once these problems are put on paper? Although this knowledge is not directly related to an understanding of cognitive operations, it is an essential knowledge if children are to succeed in school mathematics.

The design of this research project was also greatly influenced by the work of the Human Development Laboratory at Rockefeller University which was headed by Dr. Michael Cole. An important activity of the laboratory was to explore issues about the methodological constraints and limitations of traditional psychology and to grapple with how research could be reorganized to bypass some of these problems. One exciting area of research in this laboratory was to examine the strengths of black, latino and poor children in settings where they were displayed (informal settings) as a way to understand the documented failure of some of these children in academic settings.

This emphasis on and interest in formal versus informal learning became a key concept in the formulation of the present research topic. The intent was to develop a research project that allowed for the expression of creativity of black low income children while at the same time locating the academic "failures" of these children somewhere other than in the children's cognitive structures--in their heads.

The study of mathematical development was selected because of the tremendous bias in this country that blacks, women and the poor are less equipped to do well in mathematics. The intent from the initial conception of this study was to challenge that theory that black low income children have cognitive deficits. The issue then became how and what was needed to do this research. It was clear that the tools did not already exist to study what was of interest, so based on contributions from the Human Development Laboratory and Vygotsky and Ginsburg, steps were taken to create those tools. It would be helpful to take a look at that process.

One point that Ginsburg (1977a) makes emphatically in his research on children's number learning is that there are enough opportunities in the lives and environments of all children for them to learn all kinds of things about numbers. Operating from this assumption the researcher designed a simple pilot study to determine what those opportunities were. Twelve mothers and their children were selected to participate in the collecting of information about the instances when children spontaneously identified or interacted in any way with numbers. The children in this pilot were pre-schoolers and as mentioned earlier provided many and rich examples of early informal number use.

Based on information from this pilot questions emerged specifically around the differences between numbers as labels versus numbers as quantities. This led to another pilot study parts of which were conducted at the University of San Diego in California and parts of which involved children in New York City. The children in this pilot were asked questions about numbers and the meaning of those numbers on a variety of items. In the midst of this pilot it also became clear that there might be some difference in whether the number was written on paper or on an item. The conception for the

On and Off Paper Tasks which will be described in detail in the next section, emerged here.

The idea for a Conventions Task developed when it became clear that not all children understood the symbols, spacing, slashes and rules that were necessary to do well on mathematical tasks. This led to visits to elementary school classrooms during math time to see if conventions were ever self-consciously taught. It became clear that the self-conscious teaching of these conventions was rare. This observation then led to a perusal of elementary standardized tests from which the Conventions Task was designed based on conventions that needed to be known in order to carry out mathematical operations.

The design for this study was both time consuming and tedious. It involved spending a lot of time with many children of all ages, not always being clear about what was being asked, what was an expected answer or what the answers meant. The actual project has involved the use of non-standardized tasks which raises issues about what it is that is being tested as well as what the children think they are being tested to do.

In spite of these issues, however, the design and conducting of this research was a valuable and useful experience.

The purpose of this research is as follows:

1) to determine if there is a developmental trend in how children come to differentiate between number symbols as labels versus number symbols as quantities.

2) to determine if the lack of differentiation appears most often in children who are performing below grade level in mathematics.

The hypotheses are that poor performance on a math test could be a result of:

1) the lack of differentiation among number symbol meanings,

2) the inclusion of "extraneous" information in the activity, and/or

2) a misunderstanding of the conventions of a task.

METHOD

The children used in this study were from black, low income families all of whom attended the same elementary school in Brooklyn. Three tasks were designed for this study. Two of them, the On and Off Paper Tasks, were aimed at determining the children's understandings of the different meanings of number symbols in the contexts of On and Off paper. Both the On and Off Paper Tasks explore how children understand the meaning of numbers as quantities and the meaning of numbers as qualities. The On Paper Task is presented in much the same way as a conventional school "test." In the Off Paper Task, children are asked similar questions, and shown similar items, but are required to respond to them verbally (See Appendices B and C for examples). The third was constructed to examine children's understandings of the grammar of mathematics tests.

<u>Experimental Tasks</u>: Conventions, On Paper; Off Paper.

<u>Conventions Task Subjects</u>: There were a total of 54 subjects who were given the Conventions Task. There were 18 first graders (x = 6.1; range 6-7 years),

17 second graders (x = 7.4; 7-9 years), and 19 third graders (x = 8.6; 8-10 years). There was a total of 28 males and 26 females. Thirty of these children were on or above grade level and 24 were below. Grade level was determined in two ways. First graders were assigned on or below grade level depending on their scores on a District-wide test given to all first grade children in January 1981. Second and third graders were designated as on or below grade level based on their scores on a City-wide math test given to them in April.

Materials. This task involved questions about the grammar of mathematics. All items were selected by examining standardized math tests used in first, second and third grades and asking questions about the symbols and structures of many of the math conventions that children need to know to solve math problems. For example, children were asked questions about the signs used to indicate arithmetic operations, about the use of slashes to indicate subtraction and about the use of spacing as a way to indicate two sets of numbers (see Appendix A).

Procedures: Subjects were tested individually for approximately 10 minutes by a black female experimenter. The experimental task was introduced in the following way:

> I need your help. I need to help some little kids learn about numbers but I'm not sure how to do this. What I need to do is to talk to some kids like you. Show them toys and ask them to tell me what they know. This is a tape recorder. I am going to tape what we say. Do you have any questions?

Children were then asked each question and their responses were both typed and written down. The presentation of the three tasks were counter-balanced. This task was presented at the same session as the On Paper Task.

Coding:

The coding system developed for the Conventions and the On Paper Task was similar. Subjects were assigned a score of 3, 2, or 1A, 1B, 1C. The following is a description of each of the scores.

Code 3 was given if the response was correct and based on an understanding of a difference between numbers as quantities and numbers as labels. To get a 3, subjects had to demonstrate an understanding of the fact that one cannot confuse the underlying meanings of number symbols. An example of that understanding is presented below.

> "No. They (the numbers on buses) are numbers that always stay the same. They don't need to be added or subtracted. You can add it but they're not equal to bus number 3. If you add the numbers they wouldn't be 3 buses. It would be 3."

Code 2 was given if the subject's response was correct based on an understanding of a difference between numbers as quantities and numbers as labels. However, the explanations are based on the functional real world use of the item.

> "I'll say equal. Cuz when I turn on T.V. and it be at channel 9 and I turn to channel 5 there be something on channel 5...and everytime you want to watch something on channel 5, there be something on channel 9. There's always something on channel 9."

> or

> "I don't know. You'd have to race them first."

Code 1A is given when a subject is unable to differentiate between numbers as labels and numbers as quantities. For example, the child confuses numbers used as labels with ideas of size or speed.

> "Yes. I think so...cuz channel 12 is bigger than channel 3. Cuz channel 9 is bigger than channel 5. (This answer was given in response to "Does channel 12 have more programs than channel 3?" etc.)

> "Yes. House number 7 is bigger than house number 3, number 5 or number 1."

Code 1B is given if a response is incorrect and the explanation offered is for a reason other than those indicated for Code 1a.

> "Number 1 because it would go faster. (Why?) I just think so."

Code 1C is given when explanations or answers are not provided.

On Paper Task

Subjects: The same children as in the Conventions Task served as subjects.

Materials: This task investigated the children's understandings of number symbol meaning across various contexts. This task had two types of problems, school arithmetic addition and subtraction problems which were grade appropriate and a set of non-quantitative problems. An example of a non-quantitative problem is: Mary's favorite channel is 3; Marvin's favorite channel is 6; which channel has the most programs? Why?

The presentation of the type of problems were counterbalanced with half of the subjects receiving the quantitative problems first and half receiving the non-quantitative problems first (see Appendix B).

Procedures: Subjects were tested individually. Each child was given a "test" and a pencil. The experimenter read along with each child and then allowed the child to write the answer. The instructions for this task were the same as those given for the Conventions Task. This task lasted approximately 15 minutes, but as with all tasks, subjects were allowed all the time needed to complete the task.

Coding: Subject's responses on the conventions task were given either a 3 for correct or 1 for incorrect.

Off Paper Tasks

Subjects. A total of 58 subjects participated in the Off Paper Task; 54 of these children were the same as those who had participated in the conventions and On Paper Tasks. There were 20 first graders (x = 6.1; range 6-7 years); 18 second graders (x = 7.3; range 6-9 years) and 20 third graders (x = 8.6; range 8-10 years). There were a total of 32 males and 26 females. Thirty-one of these subjects were on grade level and 27 were below grade level. The grade level designation was determined in the same way as in the Conventions Task.

Materials: The children were presented with a series of toy items: buses, telephones, television sets, houses with addresses, real dollar bills and real coins (see Appendix C).

Procedure: The children were then interviewed for approximately 20 minutes about the meaning of the number symbols on each of the items. For example, they were presented with two buses which were alike in every way except one had the symbol number 1 on it and the other number 2.

Bus Questions

1. What are these?

2. What are they used for?

3. If you are standing on a corner and two buses come along, how do you know which one to take?

4. Why do people put numbers on buses?

5. What would happen if there were no numbers on buses?

6. If these two buses were in a race, which do you think would win?

7. Is either of these buses bigger than the other? Is either smaller? Is bus number 2 bigger than bus number 1?

8. Can I say bus number 1 plus bus number 2 is equal to bus number 3? Why? Why not?

9. Are the numbers on buses numbers that you can add together?

10. What do you do with the numbers on buses?

Questions of this type were asked for each set of items. Questions were also asked that involved comparing the meaning of number symbols across the items.

The format for interviewing across each of the tasks used in this project was based on Vygotsky's zone of proximal development. That is, the goal of the interview was to assess, but not in a static way, what the child knew or understood. In other words, the test is to determine not where the child is "at" at any single moment, but whether and how far they can move. Children were asked questions several times, allowed to change their responses and were always asked to explain why it was that they responded as they did. All sessions were individually administered and tape recorded.

The Off Paper Tasks included questions about busses phones, televisions, addresses and dollars. The protocols were judged in each of these areas for indication of the subject's understanding or lack of concerning the meaning of the number symbols on buses, telephones, televisions, etc. The students were then assigned into group one if they understood the function of numbers of four or more of the items. They were assigned into group two, meaning that they confused the meanings of the number symbols, if they failed on two or more of the items.

Students also received scores for responses to questions about ordinality. There were four questions about ordinality and they received 4 if four were correct, 3 if three were correct, etc. Finally, students were scored on their responses to questions about money values. They received a 1 if all the responses were correct and a 2 if any were incorrect. The reliability coefficients obtained between the experimenter and one judge was as follows. For the Off Paper Task, the reliability coefficient was .94. For the On Paper Task, the range was .83 through .96 with an overage of .91. For the Conventions Task the reliability coefficient ranged from .94 to 1.0 with an average of .95. These coefficients are based on the protocols of 60 children--20 from each condition.

RESULTS

The results present an analysis of the relationship between three independent variables--developmental level (as indicated by grade); sex; and grade level (as measured by standardized test)--and eleven dependent variables. Following is a brief description of dependent variables two through eleven inclusive (the first dependent variable is self-explanatory).

2. Number Three Level Responses: correct response, explanation indicates understanding of difference between quantitative and qualitative numbers.

3. Number Two Level Responses: correct response, but explanation is "true" without being relevant.

4. One-A Level Responses: incorrect response; explanation demonstrates confusion between quantitative and qualitative numbers.

5. One-B Level Responses: incorrect response; explanation is ambiguous.

6. One-C Level Responses: incorrect response; no explanation.

7. Traditional Items: number of correct responses to arithmetic or traditional items on paper task.

8. <u>Non-Traditional Items</u>: number of correct responses to non-arithmetic or traditional items on paper task.

9. <u>Ordinal Responses</u>: number of correct responses to questions about ordinality on off paper task.

10. <u>Distinguish Numerical Value</u>: refers to a code assigned to child's ability to distinguish numerical value from labeling function on off paper task.

11. <u>Knowledge of Money Value</u>: refers to a code assigned to child's ability to understand the concept of monetary value.

Of the eleven variables, the first nine were interval scale variables since each consisted of the number of responses of a given type provided by the subject. The last two variables were dichotomies.

The results of the analysis of data will be presented in the following order: 1) the comparison of subjects in all grades (1, 2, or 3) with respect to each of the dependent measures; 2) the comparison of subjects on grade level to subjects off grade level with respect to each of the dependent measures; 3) the comparison of subjects on grade level to those off grade level for each of the three grades considered individually; 4) the comparison of male to female subjects on each of the dependent measures; and 5) the comparison of subjects' performance in grades 1, 2, 3 and on and off grade level on the On and Off Paper Task.

Comparison of Subjects in Three Grades

In comparing subjects from three different grades on the eleven variables noted above, the researcher employed either one-way analysis of variance with Scheffe contrasts, Kruskal-Wallis analysis of variance by ranks, or crosstabulation with Chi Square tests of association. The choice of statistic in each case was based upon the level of measurement of the variable and the degree to which the frequency distributions conformed to assumptions of normality and homogeneity of variance.

Table 1 presents the results of comparisons of subjects in the three different grade levels on the nine interval scale measures.

Insert Table 1 about here

The Table indicates whether a parametric one-way anova or a non-parametric one-way Kruskal-Wallis Test was employed in the comparison, and the Table indicates where significant results were obtained.

Note that subjects in the three grades differed with respect to number of correct responses on the

Conventions Task ($F\ (2,51) = 19.61$, $p \leq .001$). Post-hoc Scheffe contrasts indicated that subjects in grade 1 ($x = 8.50$) were significantly lower than subjects in either grade 2 ($x = 12.06$) or grade 3 which did not differ significantly from one another.

Subjects in the three grades also differed significantly on number of 1A level responses ($F\ (2,51) = 3:11$; $p \leq .050$). Here post-hoc contrasts indicated that the only significant comparison was between grade 1 ($\dot{x} = 4.89$) and the average of grades 2 and 3 ($x = 6.89$). The number of 1A level responses generated by first graders is lower than the number of such responses generated by second and third graders.

Conversely, with respect to the number of 1C level responses, the Kruskal-Wallis Test was significant ($(x^2) = 12.26$, $p \leq .002$). As a follow-up on this test, the researcher conducted three Mann-Whitney U tests, comparing grade 1 to grade 2, grade 1 to grade 3, and grade 2 to grade 3. These follow-up tests indicated that the only significant pairwise difference was between grade 1 and grade 3. It was concluded that first graders were more likely to give 1C level responses than third graders.

A significant difference among the grades also emerged for number of correct responses to traditional items in the On Paper Task (F = 2,51) = 9.29, $p \leq .001$). Post-hoc Scheffe contrasts indicated that third grade students (x = 2.89) achieved significantly greater numbers of such responses than did either first or second graders.

Finally, a significant Kruskal-Wallis test was obtained for the number of correct ordinal responses in the Off Paper Task ($x^2 = 28.00$, $p \leq .001$). Mann Whitney U tests performed as follow-ups indicated that first graders (x = 2.00) were significantly lower than either second graders (x = 3.22) or third graders (x = 3.60) on this measure.

Insert Table 2 about here

Table 2 presents the crosstabulation of ability to distinguish numerical significance by grade. Note that only 2 (10%) of the first graders could distinguish numerical significance, compared with 7 (22.2%) of the second graders and 11 (55%) of the third graders. The Chi-Square Test of association was significant ($x^2(2) = 10.41$, $p \leq .006$). It was concluded that children in higher grades were more likely to distinguish numerical significance.

Insert Table 3 about here

Table 3 presents the crosstabulation of knowledge of money value by grade (1, 2 or 3). Note that 4 (20%) of the first graders were correct on this item, compared to 7 (38.9%) of the second graders and 18 (90%) of the third graders. Here again the Chi-Square test of association was significant ($x^2(2) = 20.89$, $p \leq .001$). It was concluded that subjects from grade three were more likely to understand money value than subjects from lower grades.

In addition to the analyses reported above, the researcher also calculated Spearman correlations between chronological age and each of the dependent variables having at least an ordinal level of measurement. The results are presented in Table 4.

Insert Table 4 about here

Note that age is positively correlated with: 1) number of correct responses on the Conventions Task (rho = .46, $p \leq .001$); 2) number of three-level responses (rho = .28, $p \leq .021$); 3) number of non-traditional items correct (rho = .39, $p \leq .013$); and number correct in Off Paper Task (rho = .57, $p \leq .001$). Age was negatively related to Number 1B level responses (rho = -.25, $p \leq .037$) and number of 1C level responses (rho = -.40; $p \leq .001$).

Comparison of Subjects on Grade Level to Subjects Below Grade Level (Total Sample)

Subjects' grade levels were determined by the scores on the District-wide test given to first graders in the winter of 1981 and the scores from a City-wide math test given to the second and third graders in April, 1981. Subjects on grade level were compared to subjects below grade level on a total of eleven dependent variables as described above.

In comparing those on grade to those off grade levels with respect to their performance on these measures, one of three different types of statistical tests were employed: 1) the independent sample t-test; 2) the Mann-Whitney U test; or 3) the Chi-Square test of association. With respect to interval scale variables, the t-test for independent samples was employed if the data conformed to the assumptions of normality and homogeneity of variance. On a number of these variables, frequency distributions were found to be skewed, and for this reason the non-parametric Mann-Whitney U was employed as a substitute. In the case of the two dicotomous variables, the variable of interest was crosstabulated with on or below grade level, and the Chi-Square test of association was calculated.

The results of comparisons for the first nine variables are presented in Table 5.

Insert Table 5 about here

The table indicates whether the t-statistic or the Mann-Whitney U statistic was employed to compare the two groups, and it also indicates where significant differences were obtained. The results indicate that subjects performing on grade level demonstrate significantly greater numbers of correct responses to the Conventions Task (.t $(52) = 1.92$, $p \leq .030$) and significantly greater numbers of correct responses to the traditional items in the On Paper Task (t $(52) = 4.05$, $p \leq .001$). No significant differences were detected with respect to the remaining seven variables presented in this table. These areas of no significant differences included all of the non-traditional variables.

The crosstabulation of ability to distinguish numerical value by On versus Off grade level is presented in Table 6.

Insert Table 6 about here

Note that 12 (38.7%) of the 31 subjects who were on grade level could distinguish the numerical significance of numbers. In contrast, only 5 (18.5%) of the 27 subjects who were off grade level could distinguish numerical significance. However, this difference was not statistically significant given the sample size ($x^2(1) = 1.95$, $p \leq .160$).

The crosstabulation of recognition of money value by On versus Off grade levels for the total sample is presented in Table 7.

Insert Table 7 about here

Note that 20 (64.5%) of the 31 subjects who were on grade level correctly answered the item dealing with the value of money. In contrast, only 9 (33.3%) of the 27 subjects who were off grade level correctly responded to this question. Note that this difference was significant (x^2 (1) = 4.43, $p \leq .035$), indicating that those on grade level are more likely to understand money value than those off grade level.

Comparison of Subjects on Grade Level to Subjects Below Grade Level (For Each Grade Separately)

Subjects who were on grade level in first, second and third grades were compared to subjects off grade level in each of those grades on the eleven variables mentioned above. The paragraphs which follow present the results of these analyses for each of the three grade level groupings separately.

First Graders. Table 8 presents the results of independent sample t-tests or Mann-Whitney U-tests comparing first graders who are below grade level on the nine dependent measures which are of interval scale.

Insert Table 8 about here

Note that the on grade level group is significantly higher than the off grade level group on number correct in the Conventions Task ($t(16) = 2.36$, $p \leq .016$); the number correct among the traditional items ($t(16) = 1.94$, $p \leq .036$); and the number of ordinal items correct in the Off Paper Task ($u = 20.0$, $p \leq .011$). No significant differences emerged between first graders on and off grade level with respect to the six non-traditional measures presented in this Table.

Table 9 presents the crosstabulation for first graders of ability to distinguish numerical value of the number from the labeling value of the number by on grade level years off grade level.

Insert Table 9 about here

And Table 10 presents the crosstabulation of correct versus incorrect response to the money value item by on versus off grade level for the first graders.

Insert Table 10 about here

Note that in neither case was a significant difference found between those on and off grade level.

Second Graders. Table 11 presents the results of independent sample t-tests on Mann-Whitney U-tests comparing on and below grade level second graders on the nine interval scale dependent measures.

Insert Table 11 about here

Note that the only significant difference to emerge here was on number of two level responses. Second graders who were on grade level tended to rank higher than second graders who were below grade level in terms of the number of two level responses shown ($U = 22.0$, $p \leq .024$).

Table 12 presents the crosstabulation of ability to distinguish numerical value of numbers and the variable on versus off grade level for the sample of second graders.

Insert Table 12 about here

The Fisher exact probability test applied to this table indicates no difference between second graders on grade level and second graders off grade level in this ability

$(p \leq .515)$. Table 13 presents the crosstabulation of correct versus incorrect response to the money value item by the variable on versus below grade level for second graders.

Insert Table 13 about here

The Fisher exact test applied to this Table indicates that among second graders the proportion of on grade level subjects who were correct on this item (71.4%) was significantly $(p \leq .039)$ higher than the proportion of below grade level subjects who were correct on the item (18.2%).

Third Graders. Table 14 presents the results of independent sample t-tests or Mann-Whitney U-tests comparing third graders who are on grade level to third graders who are below grade level on the nine interval scale dependent measures.

Insert Table 14 about here

Note that only one significant finding emerged: third graders who were on grade level were significantly higher than those who were off grade level on the number of correct items from among the traditional items $(t(17) = 3.65, p \leq .001)$. No other significant differences emerged between third graders on and off grade level with respect to the variables presented in Table 14. Table 15 presents the crosstabulation for the third grade sample of ability to distinguish numerical value and on versus below grade level.

Insert Table 15 about here

Table 16 presents the crosstabulation for the third grade sample of correct versus incorrect response to the money value item by on versus below grade level.

Insert Table 16 about here

Note that in neither of these tables was a significant difference obtained between those on grade level and those below grade level.

Comparison of Male and Female Subjects on Dependent Measures

Table 17 presents the results of independent sample t-tests or Mann-Whitney U-tests comparing females to males on the nine interval scale dependent measures.

Insert Table 17 about here

Note that the females score significantly higher than the males on the number correct among the traditional items ($t(52) = 2.05$, $p \leq .023$). No significant differences emerged between males and females with respect to the eight other measures presented in this table.

Table 18 represents the crosstabulation of ability to distinguish numerical value of numbers and the variable of male versus female.

Insert Table 18 about here

The Fisher exact probability test applied to this table indicates no significant difference between male and female subjects on this variable.

Table 19 presents the crosstabulation of correct versus incorrect responses to the money value item by the variable of sex.

Insert Table 19 about here

Note that 17 (53.1%) of the 32 subjects who were male correctly answered the item dealing with the value of money. In contrast, only 12 (46.2%) of the 26 subjects who were female correctly responded to this question. Note that this difference was significant ($x^2(1) = 0.07$, $p \leq .792$), indicating that males were more likely to understand money values than females.

Comparison of Subjects in Three Grades Relative to On and Off Paper Performance

In order to compare the performance differential between On and Off Paper Tasks from grade to grade, it was necessary to take into account possible overall differences in task difficulty levels. Accordingly, new scores representing performance in On and Off Paper Tasks were standardized across all grades. Then differences were calculated between three pairs of standardized scores representing On and Off paper performance: 1) between the standardized score on Number of correct "three level" responses and the standardized score for the Off Paper Task, D1; 2) between the standardized score on number of correct "two level" responses and the standardized score on the Off Paper Task, D2, and 3) between the standardized score on the total number of correct responses (level three plus level two) and the standardized score on the Off Paper Task, D3. Each of these three differences was then broken down by grade, and a one-way analysis of variance was performed. The results of these analyses are presented in Table 20.

Insert Table 20 about here

A positive mean indicates relatively better performance in the On Paper Tasks, while a negative mean represents relatively better performance in the Off Paper Tasks. Note that in all three cases the F-ratio for the one-way Anova is significant, indicating that the difference in performance between On and Off paper tests is not the same from group to group. Post hoc Scheffe contrasts indicated in each case that first graders differed significantly from second and third graders in that their performance On paper relative to their performance Off paper was better.

The same three variables were employed to compare students on grade level to those off grade level with respect to relative performance On and Off paper. The results are presented in Table 21.

Insert Table 21 about here

Note that no significant differences were obtained.

DEVELOPMENT OF FORMAL (SCIENTIFIC) AND INFORMAL (SPONTANEOUS) NUMBER CONCEPTS

The main hypothesis for this study was that there would be conflict between numbers as labels and numbers as quantities and that the conflict would be expressed as a significant difference between children who are on and children who are off grade level in mathematical performance. This hypothesis, however, was not confirmed. There are therefore two ways in which one can respond to this finding. One would be to further search for other possible significant differences in the data that would explain the differences between on and off grade children. For example, one could do an item analysis reviewing on each task, each item to see if there are differences between on and off grade children.

Another way, which is the way being chosen here, is to consider the non-significant as a positive finding and to discuss this finding in light of Vygotsky's discussion of spontaneous and scientific concepts.

Both Piaget and Vygotsky have addressed the issue of the development of spontaneous and scientific concepts in young children. Piaget's (1941) work in this area was primarily focused on the development of spontaneous concepts as a way of demonstrating the peculiarities of child thought. Piaget also described these two forms of thinking as mutually anatogonistic involved in constant conflict at each successive developmental level until finally adult thought--scientific thought--wins out.

Vygotsky (1962) in his discussion of the development of these concepts disagrees with Piaget's formulation of them as mutually antagonistic and instead asserts that they are part of a single process, related to and constantly influencing the other. Piaget's claim that scientific thought wins out seems to be another way of discussing the human mind as ahistorical, with children's experiences playing no major role in the development of their mental capacities.

In an earlier discussion of the pilot studies for this research which explored the development of spontaneous concepts and the differences between every-day and school concepts, a number of differences between the two were outlined. However, Vygotsky elaborates further on differences in the development between the two that are key to understanding the results of this research. These differences have to do with the self conscious, systematic and non-familiar way in which scientific concepts are developed. He points out that the advantage of familiarity is on the side of everyday concepts.

The issue of familiarity is a misleading one. The way in which its effects were expected to operate in this research was that the influence of the familiarity of the everyday concepts would enhance the child's performance on the problems based on these concepts. However that was not the case. Children who were able to do quite well on the scientific concepts, the traditional problems taken from arithmetic tests, often responded incorrectly to problems about everyday concepts. Here are some examples.

(Third grade girl, 10 years, on grade level got eleven out of twelve correct on traditional problems.)

- "#1 is bigger because it looks like it would go faster."

- "Yes. Because on channel 12 it has plenty pictures and on channel 3 they have fewer pictures."

(Third grade, 8 years, on grade level, all twelve traditional problems correct.)

- "Yes. Bus #2 is bigger than bus #1. And you can add bus #2 and #1 to get bus #3. You can add the numbers on buses."

- "Yes. Channel 12 has twelve programs and there are only 3 programs on channel 3."

(Second grade, 8 years on grade level, 8 correct on traditional items.)

- "If I lived at 130 Gate Ave. and you switched my number to 132 and gave my neighbor 130, my address would still be 130."

- "Yes. Because channel 12 is a lot bigger than channel 3."

In discussing why it is that problems involving scientific concepts are solved correctly more often than similar problems involving everyday concepts, Vygotsky points out how no one would assume that a child knows less about bicycles, children or school than about class struggle. However, he suggests that problems involving life situations are harder because children lack the awareness of their everyday concepts and cannot operate with them at will as the task demands.

This point is also related to the methodological issues raised by Glick (1978) in a discussion of how familiarity cannot in all experimental situations be related to as an aide in the solution of problems. This is definitely an important area in child development that warrants more work. Many psychologists and educators who point out correctly the racism, classism and sexism of educational institutions and their tools, often suggest that this could be resolved by using teachers, texts and tools that were more culturally familiar. This is obviously a too simple answer to what is methodologically, socially and politically a very complex problem.

In discussing the relationship between spontaneous and scientific concepts, Vygotsky points out how concepts which originate in the classroom have a vastly different development from those emerging from the child's personal experience. "The mind faces different problems when assimilating concepts at school than when left to its own devices" (Vygotsky, 1962). The impact of the adult's help on scientific concepts, is that the adult's help enables the child to solve these problems much earlier than everyday problems. However, Vygotsky makes clear that this does not mean that scientific concepts are ahead of everyday ones. Instead he points out how mastering a higher level in the realm of scientific concepts also raises the level of spontaneous concepts.

The data that Vygotsky (1962) offers points out how spontaneous and scientific concepts develop in reverse direction.

> The child becomes conscious of his spontaneous concepts relatively late: the ability to define them in words, to separate them at will appears long after he has acquired the concepts. He has the concept...but is not conscious of his own act of thought. The development of a scientific concept, on the other hand, usually begins with its verbal definition and its use in non-spontaneous operations--with working on the concept itself. It starts its life in the child's mind at the level that his spontaneous concepts reach only later. (Vygotsky, 1962, p. 108).

Vygotsky's position is supported by the results of the present research. It can be seen clearly in the different ways children respond to questions about the two types of concepts. The following responses from first graders give evidence of the much more self-conscious use of scientific concepts.

- "You can't add them cuz there are no plusses on the phone."
- "People don't like to add numbers on phones cuz it's not their notebooks."
- "Can't add the numbers on TV sets 'cause it's not your notebook, too."

These comments are opposed to the extremely non-conscious discussions that occurred in relationship to numbers used in everyday contexts.

- "Yes. You can add number one and number two buses to get a number three bus because its just like the dollars you just can't see the number three bus."
- "Yes. You can dial two plus two and get four if you want to call your mother."

The distinction between conscious and non-conscious awareness of scientific and spontaneous concepts was demonstrated to different degrees across all age groups and also among the children who were on and off grade level in their arithmetic skills. There is ample evidence in support of Vygotsky's position. This is also another area where more research would be quite helpful in the study of concept development.

The finding of critical importance here is the displaying of quantity-label confusions by the older children across grade levels. Although there is a significant positive correlation between age and the number of correct responses (third and second graders get more responses correct), third and second graders also give, at a significant level, more responses which confuse the meaning of numbers as labels and numbers as quantities. On the other hand, first graders give, at a significantly higher rate, more 1B and 1C responses. The 1B responses are incorrect but the explanations for them are based on some reason other than this quantity--label confusion. The 1C responses are incorrect responses with no explanation offered although they were asked to give one.

The fact that second and third graders are more likely to give responses that confuse quantity--label meanings of numbers is an important one. The conflict can be seen not as evidence of the mutually antagonistic nature of these concepts but instead as a part of the process in the necessary reorganization of this development. Evidence of this confusion in older children is demonstrated in the following.

> (In response to: Are either of the buses bigger? (See Appendix C.))
>
> - "Number 2 because it has a higher number. Number two is bigger cause . . . Number one is smaller cause number two is the biggest number," (third grade, 9 years old).
>
> (Is bus number 1 plus bus number 2 equal to bus number 3?)

- "Yes. It means that you're adding it up. It's right there with one and two. But you can't see it cause you're adding one and two up together," (third grade, 9 years old).

Some examples of first grade 1B responses are below.

(Are either of these buses bigger than the other?)

- "No. Yeah number two. Because I can tell," (first grade, 6 years old).
- "The number one. 'Cause the number one can win," (first grade, 6 years old).
- "Yes, the number two is smaller than number one. 'Cause the back is longer than this back," (first grade, 6 years old).

As pointed out earlier, although first graders comments on scientific concepts reflect the self consciousness that Vygotsky referenced, what is also demonstrated by these results is the potential for confusion between the spontaneous and scientific in the older children. This confusion can be regarded as part of the process present in the reorganization of scientific and spontaneous concepts. It is further evidence of their interaction:

> The development of a spontaneous concept must have reached a certain level for the child to be able to absorb related scientific concept. For example, historical concepts can begin to develop only when the child's everyday concept of the past is sufficiently differentiated... Scientific concepts grow down through spontaneous concepts; spontaneous concepts grow upward through scientific concepts. (Vygotsky, 1962, p. 109).

Two sets of protocols were selected to further examine the 1A confusions in relationship to the development of spontaneous and scientific concepts. Both sets are from third grade girls. One, who is identified here as Lolita, is in the top third grade class and scored sixth grade, four months on a citywide math test that was given in the spring of 1981. The other child, identified as Debbie, is in the lowest third grade class and scored second grade, two months on the citywide math test.

The potential for confusion between number symbols as labels and number symbols as quantities is demonstrated, in different ways, in these two sets of protocols. Both of these children demonstrated the label-quantity confusion in responding to the non-traditional items on the On Paper Task. There were a total of eleven questions. Debbie gave nine 1A responses and Lolita, eleven. It is important to look more closely at the differences in their 1A responses.

Debbie's responses to the non-traditional questions on the On Paper Task are more typical than Lolita's of the response given by children who demonstrate the quantity-label confusion. Debbie simply related to these questions as if they represented quantities. She, like most of the children, demonstrated this confusion both on the On and Off Paper Tasks. So her response to "John turned to channel two and channel three, which channel did he get?" was five because two plus three makes five. Likewise, her response to "which bus gets the child home faster, eight or two" was eight because it goes faster, it's bigger than two. It was clear throughout the protocol that Debbie treated number symbols which indicated labels as if they were quantifiable.

Lolita's responses on the On Paper Task also suggested a confusion in the quantity-label meanings of number symbols. However, the difference for Lolita was that it seemed that the context of this paper and pencil task -- her schooling -- influenced the way she chose to perform on the task. Lolita responded to each of the non-traditional tasks as if they were addition problems. She actually went one step further in obtaining an answer for these items. For example, when asked which channel had the most programs, channel three or six, Lolita added up the two numbers and responded by saying that channel nine had the most programs. In response to "which bus gets the child home faster, number eight or two," Lolita responded that the number ten bus gets the child home faster. "You add up the numbers on the bus."

Debbie's and Lolita's 1A responses were different in that they represented confusions of different sources. Although both treated these labels as if they were quantifiable, Lolita responded in a more school-like way to the task. This difference in how Lolita and Debbie responded was consistent with how they related to other tasks. Both of them did well on the Conventions Task getting most of the items correct. However, they responded very differently to the Off Paper Task. Debbie was assigned to group two on the Off Paper Task where for four of the five items she demonstrated the qualitative-quantitative conflict. However, Lolita after an initial confusion, gave answers that revealed an understanding of that distinction.

In the initial questions about bus numbers on the Off Paper Task, Lolita responded to "Are the numbers on buses plus numbers?" in the following way:

> "Yes. It means that you're adding it up. It's right there with 1 and 2 but you can't see it cuz you're adding 1 and 2 up together. You can add it if you want to. If you add 1 and 2 you get 3."

Lolita's response to where's bus number 3 (It's right there with 1 and 2 but you can't see it) becomes clearer if we reflect upon formal number symbols. Children do in math on paper add up 1 and 2 to get 3 without being responsible for explaining or perhaps even understanding where 3 is. However, Lolita's responses about telephone, television, address and dollar numbers were correct. They indicate her understanding of the qualitative quantitative distinction. It's after she answers questions about telephones and televisions that she recognizes her contradiction about bus numbers as quantities.

- "No. You can't add up the numbers on TV because if you turn to channel 2 you'll get one show. If you turn to channel 3 you'll get the same show that was on channel 2 but you won't get the same show that was on channel 5."
- "2 plus 3 is 5 but that's only when you're supposed to be adding up in school and stuff like that. Can't do this on TV sets or phones."
- ("But I can do it on buses.")
- "No! No! No! I just realized that. Because if you add up 1 bus plus 2 bus you won't get number 3 bus. When you were just talking about the TV's I was just realizing that and I had to change my answer."

As a result of further probing on this verbal task, Lolita was able to demonstrate her understanding of the qualitative-quantitative distinction. Vygotsky's position on the zone of proximal development that allows for the possibility of examining the child's potential in interaction with adults guided the selection of this interview procedure for this study. Children in this study were interviewed individually and questioned closely about the nature and meanings of their responses. There were clearly developmental differences in the sophistication of these explanations. However, children of all ages provided an explanation. Their thinking was based on some logic. It was also in the midst of these explanations that some children discovered contradictions in their thought and attempted to grapple with them. Lolita's protocol is an example of this.

Although Lolita was able to demonstrate an understanding of this distinction in the Off Paper Task, she was clearly unable to do this in the On Paper Task where she received 1A's for all eleven non-traditional items. It is possible that the paper-pencil task made it harder to see the number symbols on houses, buses and phones as labels and not quantities. Unlike Debbie who demonstrated this confusion on both tasks, Lolita's confusion seemed to be impacted upon by the more obviously school-like task.

As was mentioned earlier, the ability to make this quantitative-qualitative distinction was not a significant source of difference between children on and below grade level. One of the "smartest" set of responses on the non-traditional tasks was given by a young male third grader who was in the lowest third grade class and who had scored third grade, two months on the citywide math test. Ernest, as he'll be referred to here, gave the following responses to the non-traditional questions on the On Paper Task.

- "He'll get channel 2 and channel 3 cuz you can't add TV sets. He ain't doing math."
- "They both is equal cuz no channel is more than the other."
- "Nobody. It ain't no bigger house."

Ernest's answers to the Off Paper responses were similar.

- "No, unless it's on a commercial or they be showing a cartoon and they be teaching you about numbers and they have a bus with a plus . . . but that's the only time they can do that."

Evidence that children recognize that number symbols in different contexts are to be responded to differently is present throughout the protocols even in cases where children are unable to make the distinction.

- "No, a plus number is something that you add. No, you don't add the numbers on TV. You're suppose to look at it and see if that's what you want," (third grade, 8 years old).
- "No. Cuz television is an electrical thing. You can't say plus five. It doesn't work like that. It be like channel two plus five equals seven. It won't work like that. You'll be turning to station to station. Three stations is what you'll be looking at," (third grade, 10 years old).

> - "No. Cuz my phone won't ring. Cuz the telephone won't work. You can't dial three plus three for six cuz that's gonna be wrong...Cuz it don't have no plusses on it." (first grade, 7 years old).

It is clear from the protocols that children's spontaneous and scientific concepts develop differently and that they operate on them in different ways. Perhaps what the 1A confusion allows is a glimpse of the process of the reorganization--the downward development of scientific and the upward development of spontaneous concepts.

The Conventions Task is also important to discuss in terms of spontaneous and scientific concepts. Although the conventions of mathematics are first introduced in school there is no reason to believe that the teaching of conventions is very conscious or systemmatic.

In the course of a mathematics test many demands are placed on children, some of which are clearly separated from the operations of addition, subtraction, etc. One such demand has to do with the learning of symbols, spacing and conventions that are extremely crucial to successful performance on a math test. The imparting of systematic knowledge to children involves teaching them about many things that they cannot directly see or experience. In the case of these conventions, they are present but it is not clear that adequate attention is given to them in teaching children mathematics, nor is it clear what it is that very young children understand about them. Hood, Fiess and Aron (1982) point out that one of the things we learn when we are learning the content of particular norms and conventions is that there are such "things" as norms and conventions.In the case of doing a written math test it is as important to know where answers go, how to procede on an exam, how to do the test, as it is to get the right answer. The Conventions Task was developed to determine how self-conscious young children are in the area.

Third and second graders differ significantly from first graders on their understandings of mathematical conventions. This was also one of the areas where there was a distinction between first grade subjects who are on and below grade level. First graders who were on grade level performed significantly better on the Conventions Task than did first graders who were below. This differentiation did not exist, however, among second and third graders who were on and below grade level.

One interesting point that will be helpful in illustrating the importance of conventions in math has to do with problems involving three digit addition. The problems:

$$8)\ \begin{array}{c}3\\2\\\underline{2}\end{array}\qquad 9)\ \begin{array}{c}4\\2\\\underline{2}\end{array}$$

were presented without operators and were designed to explore the child's understanding of the arbitrary use of conventions in mathematics. Children have to learn that these problems are to be added. The children's responses indicated that many children do not automatically add these problems. In fact, even in third grade almost one-half of the children were confused about this rule of addition.

The children provided very interesting explanations of what is to be done. One explanation, which was common for number eight was: "You add this cause you can't take away four (2+2) from three" but that in number nine, "You can subtract four (2+2) from four if you don't mind getting zero."

For each of the 13 problems there is a developmental trend. However, the results from third graders demonstrate that even third grade children are not clear about the relationship between the number of the problem and the problem itself. Nor do they demonstrate a clear understanding of the use of spacing to indicate two sets of numbers. It is apparent that young children while learning mathematical operations are at the same time struggling to know the conventions of the task.

It is also apparent both from observations of elementary school classrooms and from elementary school textbooks that very little formal time is spent teaching young children about mathematical conventions. However, there is a developmental trend in "knowing" mathematical conventions as demonstrated by children's responses on this task. Children who are often able to verbalize the correct response don't always have a self-conscious awareness of the critical role of the symbols spacings and rules involved in doing mathematics. It is important that educators begin to be more sensitive and self-conscious about development in this area.

DISCUSSION

The question this research project attempted to answer is why it is that some black children from low income homes perform below grade level on mathematical tasks. This research was guided by the commitment to not rely on cognitive structural deficits or on an inability to understand mathematical operations as explanations of children's poor performance. Instead, the hypotheses posited were that poor performance on a math test could be a result of 1) the lack of differentiation between number symbol meanings; 2) the inclusion of "extraneous" information in the mathematical activity; and/or 3) a misunderstanding of the conventions of a task.

The present results, however, do not support these hypotheses. Overall, the strongest findings had to do with developmental trends in the children's ability to differentiate between number meanings and in the children's understanding of mathematical conventions. There were also significant differences found between children on and off grade level in their performance on the Conventions Task, on traditional arithmetic items and in their ability to understand money values. However, as was indicated in the previous chapter the non-significant is being treated as a positive finding in that it provides valuable insight into the development of spontaneous and scientific concepts.

There are two areas of focus emerging from this research project. One has to do with the findings concerning the development of formal and informal number concepts and was discussed in the previous chapter based on Vygotsky's understanding of the development of spontaneous and non-spontaneous concepts. This was presented in the previous chapter. The other area of interest has to do with the question of differential performance in young children and the history of this question in the area of developmental psychology. The following discussion rather than having emerged from the findings of this study are being discussed as important considerations in future research in the area of children and mathematical development.

This question of differential performance in young children is a common one. It is indeed a dominant question in the history, the methodology of the social sciences in this country. There are many explanations in the psychological literature concerning why it is that the performance of black and low income children differs from that of white middle class children on cognitive tasks including mathematical tests. These theories vary from the genetic explanations of Jensen (1969), to the cultural deficit explanations offered by Hess and Shipman (1965), to the work by Baratz (1970) describing differential performance between whites and blacks as stemming from cultural differences.

Although the issue of differential performance is a key issue in Western psychology, what is rarely if ever engaged and what was not engaged at the outset of this research project is why the question of differential performance is a question that psychologists ask; what is the history of this question especially in relationship to blacks and mathematics; what is its social meaning and the assumptions underlying it.

One way to proceed is to examine further the positions of Piaget and Vygotsky on the development of mathematical abilities and the history of the question of differential performance in Western and Soviet psychology.

It is clear that Vygotsky's theory of human development is better known to psychologists in this country than it was ten years ago. However, it is not clear how well understood are the implications of Vygotsky's position or how seriously the theoretical and methodological differences between his work and that of Western psychologists have been taken. There is perhaps no better area in which to seriously engage these issues than in the area of mathematics.

One frequently referenced position held by Vygotsky is that all the higher functions develop first on the interpsychic and then later the intrapsychic levels. As Hood (In Press) points out in a paper entitled "Pragmatism and Dialectical Materialism in Language Development," this claim that higher psychological processes are internalized social functions "does not merely reverse the order of the traditional Western explanation of development. It leads to a qualitatively different methodology." What are the implications of this qualitatively different methodology for the development of mathematics?

In Western psychology, mathematical skills are related to (as are other cognitive activities) as if they develop on an intrapsychic plane. Quotes from Piaget's (1941) work on number development demonstrate this point:

> It now remains in order to discover the mechanisms that determine thought to investigate how the sensory motor schemata of assimilating intelligence are organized in operational systems on the plane of thought. (Piaget, 1941, p. vii)

(or)

> Our hypothesis is that the construction of number goes hand in hand with the development of logic and that a pre-numerical period

> corresponds to the pre-logical level. In our view, logical and arithmetical operations therefore constitute a single system that is psychologically natural, the second resulting from generalization and fusion of the first under the two complementary headings of inclusion of classes and seriation of relations quality being disregarded. (Piaget, 1941, p. viii)

The position that human thought develops on an intrapsychic plane has its roots philosophically in idealism and this has real implications for how it is that one studies, researches, knows and understands the development of mental activities.

Idealism, having its roots in Greek philosophy and Aristotle's insistence that the essence of man is rationality, is the dominant philosophical position in Western psychology. Piaget's work comes out of this tradition. Vygotsky, on the other hand, is a dialectical materialist; he is a Marxist. Any comparison of the vastly different perspectives of these two men on the development of mental activities (e.g. mathematical development) has to seriously grapple with these methodological and philosophical differences. This is precisely the position being expressed by Hood (In Press), that Vygotsky's claim of interpsychic to intrapsychic development of mental activities is not merely the reverse of the traditional Western explanation. It is of critical methodological importance.

Glick (In Press) further points out two important implications of the influence of idealism in Piaget's work. One has to do with the acultural and ahistorical characteristics of idealism.

> If the problem is to provide for the conditions of the possibility of transcendent forms of thought which surpass intuitions at every turn, the Piagetian model of inquiry is quite successful...[However] thought and action in culture is not free--it is constrained--precisely by those things that give us some tangible sense of culture and social life. Thus, in some important way, the transcendent notion of mind developed within the idealist tradition is fundamentally acultural.

To relate to the mind as acultural and ahistorical, as Piaget does, has serious implications for what type of activity one takes the learning of mental activities, i.e. mathematics, to be. It is as if these activities are developing in one's mind regardless of the child's experiences in the world. Piaget's insistence on the unfolding of genetic structures as an explanation of how it is that thought develops, his delegation of conservation as the "in the mind" activity that elevates pre-logic numbering to the logical plane and his reference (in the second quote on page 84) to logical and arithmetic operations as "psychologically natural" are all examples of relating to mathematical development outside of experience, outside of the social, outside of history. Thus, in the best idealist tradition, Piaget insists that history be understood in terms of the development of our mental capacities rather than our mental capacities being understood in terms of history. This points to an extremely important difference between Piaget and Vygotsky. For Vygotsky, a Marxist, mathematical activities are social as are all human activities. They are in the world. Therefore the mind, mental activities, mathematical development are historical; they do not float above or exist outside of history.

Glick's (In Press) other observation regarding Piaget's idealism has to do with Piaget's understanding of universality and interaction.

> Piaget has preserved the notion of organism-environment interactions as a central aspect of his account of development but has given it the sort of twist necessary to meet the idealist requirement. For Piaget, the environment that is interacted with is one that is constrained by universal laws (e.g. those of physics) which operate through all possible encounters.

Thus, the twist that Glick cites once again places development outside of history.

Piaget's position, upon careful scrutiny, is at base a position drawing heavily on genetic explanations of behavior. The environment referenced by Piaget leaves no room for or evidence of the environments in which real children grow, learn and create or are kept from doing so. There is, for example, no place in Piaget's conception of interaction to discuss how it is that mathematics in this society is an activity that black children, latino children, girl children and poor children are most often organized in relation to as failures. What is the history of that organization?

Piaget's interactionist explanation of human development is often related to as a corrective to the purely genetic explanations of development provided, for example, in Jensen's (1969) work. However, what must be understood is how Piaget's position is not inconsistent with innatist explanations. This point is of critical importance in the study of the cognitive activities of black, low income children. Indeed, one of the ways in which Piaget's theory was related to by black researchers in the early seventies and to some degree today, is as more neutral and less attacking than Jensen's claim of black genetic inferiority. In fact, what is being exposed here is how their explanations are similar.

The basis for the appeal of Piaget's work for many black and latino researchers lay in his claim that the cognitive operations and skills of his theory develop in all people; an appeal to the universality of the development of mental activities. So in spite of the studies that attempted to demonstrate how these structures would develop later in people of color, it seemed that Piaget's use of universality gave all children an equal chance. After all, the development of Piagetian structures had very little to do with what was going on in the world, only with what was going on in the children's heads.

This position of neutrality is very seductive especially to those black, latino and progressive researchers who were fighting Jensen's position of innate genetic inferiority. However, upon closer scrutiny it is clear that Piaget's position of cognitive neutrality is not very neutral at all. The problem with this neutrality, the acultural and ahistorical nature of Piaget's theory, is that the social, the historical does have a real impact on children's development. Racism, sexism, classism cannot be separated out from their learning of mathematics or from their learning of anything else. The point being made here repeatedly is that when black, girl and poor children learn that two plus two equals four they are also learning their location within that activity. All of that is being identified here as the activity of mathematics.

What would it mean to study the development of cognitive activities in the black poor children in this study, leaving out the very basis of how it is that they are organized into the activity of learning? Actually, it is clear what it means. If indeed mathematical development is exempt from the history in which these children find themselves, it leads psychologists to ask the kinds of questions that they ask: Why is it that poor children, black and latino children are not the same as white middle class children in their performance of cognitive tasks especially since all children have equal opportunities in particular in that they have ahistorical minds. Thus this leads to the question of differential performance which leads to social scientists choosing among a geneticist, environmental or interactionist explanation of behavior without ever engaging the history of that question or the historical location of mathematics and of children's performance.

The point being made here is that the question of differential performance is often addressed by over-focussing on the genetic contributions to intellectual development. This focus on "genetic intelligence" denies the impact of socio-historical determinants on the organization of the activity of learning and the role of racism, sexism, and classism in children's learning.

Several psychologists in this country have made contributions and raised critical issues about the methodological constraints and limitations of traditional psychology. Scribner's (1975) work on the role of the experiment in cross-cultural research, Glick's (1978) work on methodological concerns and the work of Cole, Hood and McDermott (1978) and Franklin and Fulani (1979) are part of a tradition out of which the present study developed. It is important to recognize, however, that although Vygotsky's methodological position allows for the engaging of similar issues, it cannot be simply incorporated into traditional psychology. One reason for this is Vygotsky's understanding of what it means for a psychology to be social and historical.

Vygotsky's understanding of the social and historical has implications for a qualitatively different methodology for the study of mathematical development. As Hood (In Press) points out:

> For Vygotsky, social does not mean interpersonal... For Vygotsky, the activities of human beings at all stages of development and organization are social products and must be seen as historical developments, not merely as interpersonal developments. Social does not reduce to interpersonal social activity is not merely social interaction.

Wertsch's (1979) work represents a real attempt to build off of Vygotsky's contributions in his study of communication and mother-child interactions. In his attempt to trace the transition from inter- to intra-psychological functioning, Wertsch, in the tradition of

Vygotsky, relates to the development of language neither as cognitive nor social but as a complex human activity embedded in a particular social relationship. Hood (In Press) however critiques Wertsch's work by pointing out how his work is ultimately "still bound to the perceptual, to what can be observed, what is dynamically changing. The cultural, historical conditions under which this mother-child interaction takes place is not addressed."

Luria (1976) further elaborates on Vygotsky's conception of the social.

> Following Marx, Vygotsky concluded that in order to understand the human mind, it is critical to understand the social origins of mind...Man is not only a product of his social environment he is an active agent in creating that environment. It is in the individual's relations with the external world that we had to seek the origin of higher forms of conscious behavior. We needed, as it were, to step outside the organism, to discover the sources of specifically human forms of psychological behavior. (Luria, 1976, p. 13)

What Vygotsky means by social is that it is part of the social activity that human beings do in the transforming of nature. This conception of social is foreign to traditional psychology. The social in traditional psychology is treated as separate from the mental activities of human beings.

This difference between the two approaches makes for a qualitative difference in how one studies mathematical development. Thus for Vygotsky mathematical skills are not merely cognitive, nor interactional, they must be understood from their social origins. For example, mathematical activity has to be viewed as a socially organized human activity whose history cannot be separated from the history of society. Vygotsky does not believe that some of the things that human beings do are social and others cognitive. He begins with a concept of human activity which includes all of what it is that humans do.

The development of a historical psychology is an important conception in Vygotsky's theoretical work. In many ways, it is antithetical to the understandings and conceptions of Piagetian theory. For one, it would make the study of mathematical development a study where mathematics and other cognitive activities are not related to as neutral or ahistorical. The study and learning of mathematics would be viewed as a product of socio-historical conditions. A quote from Vygotsky further develops this distinction.

> The separation of the intellectual side of our consciousness from its affective, volitional side is one of the fundamental defects of all traditional psychology. Because of it thinking is inevitably transformed into an autonomous flow of thoughts thinking themselves. It is separate from all the fullness of real life, from the living motives, interests and attractions of the thinking human. (Vygotsky, 1956, p. 53).

For Vygotsky mathematical development is not somewhere in a mind that exists outside of history. It is not somewhere in a laboratory that exists apart from the history of society. Mathematics is a social activity. It has to be studied and understood historically.

An extension of Vygotsky's work is represented in the methodological position of how what is known is constrained by how it is known. This point is made explicit in a discussion of the development of the language of causality by Hood, Feiss and Aron (1982) where they are hypothesizing about the kinds of things children learn through exchanges with adults:

> ...one of the things being taught is a particular way of making sense of the world, one that, of course conforms to the norms, customs, and ideology of our society. It is important to delineate substantively what these norms are. But that is not all. In our view the important fact to be recognized is not that language activities such as these teach particular norms, but that they teach that there are such things as norms. The very way we use language treats the content of it as somehow separate, independent and pre-existent. So, children learn through discourse about causality that there are things to be explained, as if such things existed independently of their being explained. (Hood, Feiss & Aaron, 1982, p. 281).

The point being made here is that of how the teacher also expresses to young children not only what to learn but what there is to learn. This point is an important contribution that has evolved from work done at the New

York Institute for Social Therapy and Research (Hood & Newman, 1979).

Vygotsky's commitment to the zone of proximal development and his emphasis on process as a way of understanding cognitive development is an example of the indistinguishability of what one knows from how one knows it. This is in contrast with the methodology of traditional psychology where children are tested (i.e. how it is known) and the results are treated as the answers (i.e. what is known) with no recognition of how it is that methodology interacts with what is known.

In the area of mathematics Ginsburg's study of children's errors in arithmetic helps to raise this issue. The relegation of capabilities to a test score leads to qualitatively different questions, responses and understandings of what it is that children know. Ginsburg (1977a) in his recognition that the errors on those tests obscured all kinds of relevant information about the process of development, contributed to an understanding of how what is known by social scientists is informed by how knowing is done.

It is clear that Vygotsky's understanding of the social and historical raises critical issues for how it is that mathematical development in children is studied and understood. The study of developmental process for Vygotsky is not a study of neutral processes. In addition, how they are studied and what is known is very much related to the socio-historical conditions. These conceptions are drastically different from traditional psychology and therefore as has been demonstrated lead to different methods, questions and understandings of human development.

CONCLUSION

The history of the present research project has been a long and exciting one. What began as a research task designed to demonstrate that black low income children too are smart, has evolved into a serious inquiry of why it is that this is and has always been a concern of Western psychology--that of demonstrating that some children have and some do not.

The contrasting of Piaget and Vygotsky's work; of their vastly different methodologies provided a context for raising this issue. One important contribution from Vygotsky is the idea of self-reflexivenss. (Note that the concept of self-reflexiveness has been a major one in philosophical and psychological writings, see Holzman and Newman (In Press) for a discussion of the history of the use of this term in psychology.) This act of self-reflexiveness, as Vygotsky points out, is the very act that sets man apart from animals. It is this activity that makes human beings special and that is the source of their creativity. The absence, for the most part, of this activity in Western psychology is an important issue to further explore. It is what contributes largely to why Western psychology is ahistorical.

It is clear, though it was not as self-conscious at the outset of this research project, that the intent of this project was to demonstrate that black low income children were smart. The providing of proof of the smartness of black, latino and poor children was a major activity in the Laboratory of Human Development at Rockefeller University. One way of doing so was at times to insist that school learning did not matter as much because it was clear that these children could learn in the streets. However, it was also clear that street and school learning carry very different implications for children in this society. Also what was never engaged as part of the activity of the research was the history of cognitive activity and black, low income children. Why did the laboratory exist? What made it necessary for these children to be studied separately?

Another issue raised in this research has to do with the separation of cognitive activities from history. The study of mathematical development in children is a source of numerous examples of how this is done. What is being emphasized in the present research is that mathematical development cannot be understood without understanding how it has been and is organized as a social activity. This position that all cognitive activities have their origins in the social world, in history, is a very controversial one. It is also one of the reasons why Vygotsky and Soviet Psychology cannot be simply integrated into Western methodology.

It is clear that understanding mathematical development as located not in people's heads but in history makes it necessary to concern oneself not only with how children learn or do not learn mathematics but also with why they do or do not learn. A statement that has been repeated throughout this work is that when black, poor and girl children learn two plus two is equal to four, they also are learning their relationship to that activity. All of this is the activity of mathematics.

Future research directions abound for the issues raised here. The work on spontaneous and scientific concepts has all kinds of implications for the teaching and examination of mathematical development. In exploring the development of children's self-reflexiveness in relationship to math concepts much can be learned about the process of that development.

Other issues raised have to do with the assumptions about what is and is not a school like task. Although the Off Paper Task was originally conceived as a non-school like task, it was not responded to by the children in that way. The institutional constraints of schools as they are presently organized and the impact of these constraints on learning are not removed by doing an artificial non-school like task in an academic setting. The children in this study were still "doing school."

Another issue worth mentioning is the work involved in developing the skills to work as an adult with children based on the concepts outlined in Vygotsky's zone of proximal development. This concern is related to the recognition that the biases of Western psychologists and educators leads to an understanding that it is only what one creates individually that provides any real evidence of what one knows. It was clear that the method of questioning and the raising of contradictions with these children provided a context for them to give more. It is also clear that much more work needs to be done to improve how this is done in recognition of the strong biases that exist.

Finally, progressive researchers and psychologists who are grappling daily with these issues recognize that if one wants to change the mathematical performance of black, low income children, one has to change much more than textbooks, curriculum teacher training and experimental techniques. What has to be recognized by psychologists and educators is the important methodological point that what it is that one knows cannot be separated from how it is that one knows. In a statement describing the issues involved in the creating of a Marxist therapy Hood and Newman (1979) share the following:

> In another sense, the engagement of reality relates to the kinds of practical questions that are considered theoretically basic. In social therapy, the crucial questions are neither existential nor epistemological; rather they are ontological. But they are ontological in a quite special sense, namely in an epistemological sense. In a word, they are methodological questions in the Marxist sense, which forbids separating what is known from how it is known. (Tools from results) . . . For Marxism, how to transform (reorganize) reality and how to understand it are inseparable--indeed, in a quite important sense, indistinguishable. (Hood & Newman, 1979, p. 18).

The point that is being emphasized here in the present research and the work by Vygotsky is that issues relating to poor math performance in children go far beyond the techniques of the classroom. How we know what we know, how the children know what they know is very much related to our methods of knowing. What must be grappled with is the tradition of separation, of isolation, that involves studying cognitive activities as if they are these ahistorical, idealistic structures that exist outside of human control. What must be further appreciated is the depth of this tradition in Western psychology and the issues that will be raised by viewing mathematical development as a social activity.

It is important if one's true concern is the improving of black, poor and girl children's mathematical performance to bring that activity back into history, to relocate it in the world.

Table 1

Comparison of Subjects in Grades 1, 2, and 3 on Dependent Measures

Measure	Grade 1			Grade 2			Grade 3			Anova or Kruskal-Wallis (x^2) test	P
	N	$\overline{X}$	SD	N	$\overline{X}$	SD	N	$\overline{X}$	SD		
Correct Responses on Conventions Task	18	8.50	3.59	17	12.06	2.08	19	13.73	1.73	F=19.61***	.000
Number Three Level Responses - On Paper	18	1.33	1.61	17	1.59	1.37	19	2.52	3.04	x^2=2.03	.362
Number Two Level Responses - On Paper	18	0.22	0.73	17	0.24	0.66	19	0.37	0.60	x^2=2.55	.280
Number One-A Level Responses - On Paper	18	4.89	2.95	17	6.82	1.98	19	6.95	3.21	F=3.11*	.050
Number One-B Level Responses - On Paper	18	2.17	1.98	17	1.23	1.09	19	0.84	0.89	x^2=4.87	.088
Number One-C Level Responses - On Paper	18	2.39	2.62	17	1.12	1.46	19	0.32	0.58	x^2=12.26**	.002
Number Correct: Traditional Items - On Paper	18	7.50	1.95	17	6.59	2.48	19	9.47	1.74	F=9.29***	.000
Total Number Non-Traditional Items Correct (Two Level Responses plus Three Level Responses)	18	1.56	1.92	17	1.82	1.74	19	2.89	3.14	x^2=2.64	.268
Number Ordinal Correct in Off-Paper Task	20	2.00	.079	18	3.22	0.73	20	3.60	0.60	x^2=28.00	.000

** $p < .01$
*** $p < .001$

Table 2

Crosstabulation of Ability to Distinguish
Numerical Value of Number from Labeling
Value of Number by Grade (1, 2, or 3)

	Grade 1		Grade 2		Grade 3	
	N	%	N	%	N	%
Can Distinguish	2	10.0	4	22.2	11	55.0
Cannot Distinguish	18	90.0	14	77.8	9	45.0
Total	20	100.0	18	100.0	20	100.0

$x^2(2) = 10.41$, $p \leq .006$

Table 3

Crosstabulation of Money Value Item by

Grade (1, 2, or 3)

	Grade 1		Grade 2		Grade 3	
	N	%	N	%	N	%
Correct	4	20.0	7	38.9	18	90.0
Wrong	16	80.0	11	61.1	2	10.0
Total	20	100.0	18	31.0	20	34.5

$x^2(2) = 20.89$, $p \leq .000$

Table 4

Spearman Correlations Between Age and the Dependent Measures

Variable Correlated With Age	Rho	n	p
Correct Responses on Conventions Task	.46***	54	.001
Number Three Level Responses - On Paper	.28*	54	.021
Number Two Level Responses - On Paper	.21	54	.067
Number One-A Level Responses - On Paper	.22	54	.055
Number One-B Level Responses - On Paper	.25*	54	.037
Number One-C Level Responses - On Paper	.40***	54	.001
Number Correct: Traditional Items - On Paper	.28	54	.021
Total Number Non-Traditional Items Correct (Two Level Responses plus Three Level Responses)	.30*	54	.013
Number Ordinal Correct in Off-Paper Task	.57***	58	.001

* $p < .05$
*** $p \leq .001$

Table 5

Comparison of Subjects
On and Off Grade Level
On Dependent Measures

Measure	On Level			Off Level			t-test or Mann-Whitney U-test	One-tailed
	N	$\overline{X}$	SD	N	$\overline{X}$	SD		P
Correct Responses on Conventions Task	30	12.33	3.38	24	10.50	3.18	t=1.92*	.030
Number Three Level Responses - On Paper	30	1.87	2.16	24	1.79	2.28	U=351.0	.435
Number Two Level Responses - On Paper	30	0.30	0.60	24	0.25	0.74	U=326.0	.191
Number One-A Level Responses - On Paper	30	6.50	3.06	24	5.88	2.69	t=0.79	.218
Number One-B Level Responses - On Paper	30	1.23	1.50	24	1.63	1.47	U=291.0	.106
Number One-C Level Responses - On Paper	30	1.10	1.97	24	1.46	1.89	U=315.0	.202
Number Correct: Traditional Items - On Paper	30	8.93	1.93	24	6.63	2.26	t=4.05***	.000
Total Number Non-Traditional Items Correct (Two Level Responses plus Three Level Responses)	30	2.17	2.43	24	2.04	2.42	U=348.5	.419
Number Ordinal Correct in Off-Paper Task	31	3.06	0.89	27	2.78	1.09	U=359.5	.168

* $p < .05$
*** $p \leq .001$

Table 6

Crosstabulation of Ability to Distinguish

Numerical Value of Number from Labeling

Value of Number by On and Off Grade Level

(Total Sample)

	On Grade Level		Off Grade Level	
	N	*%*	*N*	*%*
Can distinguish numerical value from labeling function	12	38.7	5	18.5
Cannot distinguish	19	61.3	22	81.5
Total	31	100.0	27	100.0

$x^2(1) = 1.95$, $p \leq .16$

Table 7

Crosstabulation of Money Value Item by

On or Off Grade Level

(Total Sample)

	On Grade Level		Off Grade Level	
	N	%	N	%
Correct	20	64.5	9	33.3
Wrong	11	35.5	18	66.7
Total	31	100.0	27	100.0

$x^2(1) = \underline{4.43}$, $p \leq .035$

Table 8

Comparison of First Graders On Grade Level to Those Off Grade Level On Dependent Measures

Variable	On Grade Level			Off Grade Level			t-test or Mann-Whitney U-test	One-tailed
	N	$\bar{X}$	SD	N	$\bar{X}$	SD		P
Number Correct on Conventions Task	12	9.75	3.65	6	6.00	1.79	t=2.36*	.016
Number Three Level Responses	12	1.58	1.83	6	0.83	0.98	U=29.5	.261
Number Two Level Responses	12	0.08	0.29	6	0.50	1.23	U=32.5	.274
Number One-A Level Responses	12	5.33	3.20	6	4.00	2.37	t=0.90	.191
Number One-B Level Responses	12	1.83	1.95	6	2.83	2.04	U=25.5	.156
Number One-C Level Responses	12	2.17	2.76	6	2.83	2.48	U=28.5	.287
Number Correct from Traditional Items	12	8.08	1.44	6	6.33	2.42	t=1.94	.036
Number Correct from Non-Traditional Items (Three Level plus Two Level Responses)	12	1.67	1.97	6	1.33	1.97	U=32.5	.365
Number Ordinal Correct in Off-Paper Task	12	2.33	0.78	8	1.50	0.54	U=20.0*	.011

* $p < .05$

Table 9

Crosstabulation of Ability to Distinguish Numerical Value of Number from Labeling Value of Number by On and Off Grade Level (First Graders)

	On Grade Level		Off Grade Level	
	N	%	N	%
Can distinguish numerical value from labeling function	2	16.7	0	0.0
Cannot distinguish	10	83.3	8	100.0
Total	12	100.0	8	100.0

Fisher exact probability = .347 (one-tailed)

Table 10

Crosstabulation of Money Value Item by

On or Off Grade Level

(First Graders)

	On Grade Level		Off Grade Level	
	N	%	N	%
Correct	4	33.3	0	0.0
Wrong	8	66.7	8	100.0
Total	12	100.0	8	100.0

Fisher exact probability = .10
(one-tailed)

Table 11

Comparison of Second Graders On Grade Level to Those Off Grade Level On Dependent Measures

Variable	On Grade Level			Off Grade Level			t-test or Mann-Whitney U-test	One-tailed
	N	$\bar{X}$	SD	N	$\bar{X}$	SD		P
Number Correct on Conventions Task	6	13.33	2.58	11	11.36	1.43	t=1.73	.064
Number Three Level Responses	6	1.67	1.51	11	1.55	1.37	U=32.0	.457
Number Two Level Responses	6	0.67	1.03	11	0.00	0.00	U=22.0*	.024
Number One-A Level Responses	6	6.67	2.42	11	6.91	1.81	t=-0.23	.409
Number One-B Level Responses	6	1.33	1.37	11	1.18	0.98	U=32.0	.459
Number One-C Level Responses	6	0.67	0.82	11	1.37	1.69	U=24.5	.182
Number Correct from Traditional Items	6	7.83	2.32	11	5.91	2.39	t=1.62	.067
Number Correct from Non-Traditional Items (Three Level plus Two Level Responses)	6	2.33	2.34	11	1.55	1.37	U=27.5	.279
Number Ordinal Correct in Off-Paper Task	7	3.43	0.79	11	3.09	0.70	U=28.0	.152

* $p < .05$

Table 12

Crosstabulation of Ability to Distinguish Numerical Value of Number from Labeling Value of Number by On and Off Grade Level (Second Grade)

	On Grade Level		Off Grade Level	
	N	%	N	%
Can distinguish numerical value from labeling value	2	28.6	2	18.2
Cannot distinguish	5	71.4	9	81.8
Total	7	100.0	11	100.0

Fisher exact probability = .515 (one-tailed)

Table 13

Crosstabulation of Money Value Item by

On or Off Grade Level

(Second Grade)

	On Grade Level		Off Grade Level	
	N	%	N	%
Correct	5	71.4	2	18.2
Wrong	2	28.6	9	81.8
Total	7	100.0	11	100.0

Fisher exact probability = .039
(one-tailed)

Table 14

Comparison of Third Graders On Grade Level to Those Off Grade Level On Dependent Measures

Variable	On Grade Level			Off Grade Level			t-test or Mann-Whitney U-test	One-tailed
	N	$\bar{X}$	SD	N	$\bar{X}$	SD		P
Number Correct on Conventions Task	12	14.17	1.53	7	13.00	1.92	t=1.46	.081
Number Three Level Responses	12	2.25	2.77	7	3.00	3.65	U=37.0	.329
Number Two Level Responses	12	0.33	0.49	7	0.43	0.79	U=42.0	.500
Number One-A Level Responses	12	7.58	3.00	7	5.86	3.49	t=1.14	.135
Number One-B Level Responses	12	0.58	0.67	7	1.29	1.11	U=26.0	.074
Number One-C Level Responses	12	0.25	0.45	7	0.42	0.79	U=39.0	.371
Number Correct from Traditional Items	12	10.33	1.30	7	8.00	1.41	t=3.65***	.001
Number Correct from Non-Traditional Items (Three Level plus Two Level Responses)	12	2.58	2.97	7	3.42	3.60	U=34.5	.259
Number Ordinal Correct in Off-Paper Task	12	3.58	0.52	8	3.63	0.74	U=42.5	.306

Table 15

Crosstabulation of Ability to Distinguish Numerical Value of Number from Labeling Value of Number by On and Off Grade Level (Third Grade)

	On Grade Level		Off Grade Level	
	N	%	N	%
Can distinguish numerical value from labeling value	8	66.7	3	37.5
Cannot distinguish	4	33.3	5	62.5
Total	12	100.0	8	100.0

Fisher exact probability = .205 (one-tailed)

Table 16

Crosstabulation of Money Value Item by

On or Off Grade Level

(Third Grade)

	On Grade Level		Off Grade Level	
	N	%	N	%
Correct	11	91.7	7	87.5
Wrong	1	8.3	1	12.5
Total	12	100.0	8	100.0

Fisher exact probability = .652
(one-tailed)

Table 17

Comparison of Female and Male Subjects On Dependent Measures

Measure	Females N	Females $\bar{X}$	Females SD	Males N	Males $\bar{X}$	Males SD	t-test or Mann-Whitney U-test	One-tailed P
Correct Responses on Conventions Task	26	11.69	3.96	28	11.25	2.78	t=0.47	.314
Number Three Level Responses - On Paper	26	1.81	2.19	28	1.86	2.24	U=353.0	.421
Number Two Level Responses - On Paper	26	0.38	0.75	28	0.18	0.54	U=308.0	.076
Number One-A Level Responses - On Paper	26	6.31	3.11	28	6.14	2.73	t=0.21	.163
Number One-B Level Responses - On Paper	26	1.62	1.70	28	1.21	1.26	U=325.0	.241
Number One-C Level Responses - On Paper	26	0.88	1.21	28	1.61	2.38	U=318.5	.201
Number Correct: Traditional Items - On Paper	26	8.58	2.56	28	7.29	2.02	t=2.05*	.023
Total Number Non-Traditional Items Correct (Two Level Responses plus Three Level Responses)	26	2.19	2.45	28	2.04	2.41	U=343.5	.357
Number Ordinal Correct in Off-Paper Task	26	2.81	1.06	32	3.03	0.93	U=370.0	.225

* $p < .05$

Table 18

Crosstabulation of Ability to Distinguish Numerical Value of Number from Labeling Value of Number by Male and Female Subjects

	Male		Female	
	N	%	N	%
Can distinguish numerical value from labeling value	11	34.4	6	23.1
Cannot distinguish	21	65.6	20	76.9
Total	32	100.0	26	100.0

$x^2(1) = 0.42$, $p = 0.52$

Table 19

Crosstabulation of Money Value

Item by Sex

	Females		Males	
	N	%	N	%
Correct	12	46.2	17	53.1
Wrong	14	53.8	15	46.9
Total	26	100.0	32	100.0

$x^2(1) = 0.07$, $p = .792$

Table 20

Mean Differences Between Standardized Scores for On and Off Paper Tasks by Grade

	Grade 1 (N=17)		Grade 2 (N=17)		Grade 3 (N=19)		F	P
	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD		
D1	0.70	0.933	-0.42	0.80	-0.34	1.33	6.01	.005
D2	0.93	1.39	-0.38	1.00	-0.52	1.43	6.61	.003
D3	0.71	1.01	-0.43	0.80	-0.33	1.36	5.72	.006

Table 21

Mean Differences Between Standardized Scores for On and Off Paper Tasks by On and Off Grade Level

Difference Score	Group					
	On Grade Level N=29		Off Grade Level N=24			
	$\bar{x}$	SD	$\bar{x}$	SD	t	p
D1	-0.16	1.11	0.12	1.22	-0.81	.39
D2	-0.09	1.16	0.09	1.73	-0.46	.65
D3	-0.14	1.12	0.11	1.28	-0.76	.45

Appendix A

Conventions Task

1 + ▱ = 2	What does ▱ mean in this problem? Why is it there?
3 + 2 = ▱	What does ▱ mean in this problem? Why is it there?
4. $\begin{array}{r} 5 \\ +2 \\ \hline \end{array}$	Why?
●●●● ●●	Why is there a space here? What does it mean?
●●●●●̸●̸	Why do these have a line drawn through them?
2 + 3 = $\begin{array}{r} 2 \\ +3 \\ \hline \end{array}$	Why?
21 or 12	Do these two 2's mean the same thing?
$\begin{array}{r} 3 \\ 2 \\ \underline{2} \end{array}$	What should you do to this problem? Should you add or subtract?

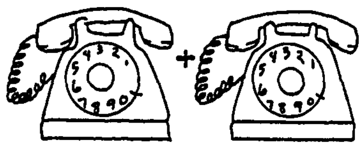
4 2 2	What should you do to this problem? Should you add or subtract?
11 +18	Add this.
+	What is this?
-	What is this?
x	What is this?
÷	What is this?
=	What is this?
	Where is problem #1 on this page?
	Where is the last problem on this page?

Appendix B

On Paper Task

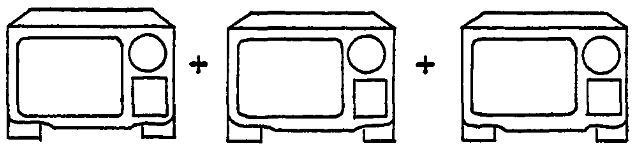
1.

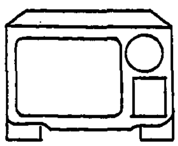
4					
9	25	20	24	17	10
+ 2	- 10	+ 80	- 3	+ 8	- 5

2. There are 2 telephones  upstairs in Cindy's house

and 3  downstairs.

How many telephones are in her house?

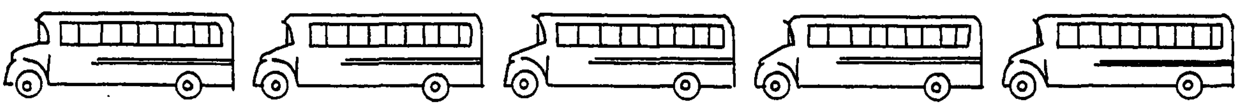
3. John has 3  television sets.

One  is in the shop.

How many does he have left in his house?

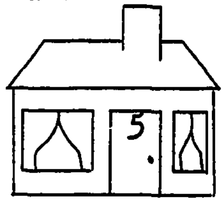
There were 6 busses in the parking lot.

5 were driven away.

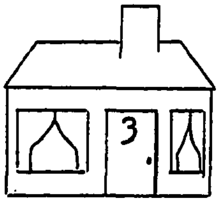


How many busses were left?

Name ____________________

5. Marvin lives at 

Fulton Street. He moves to

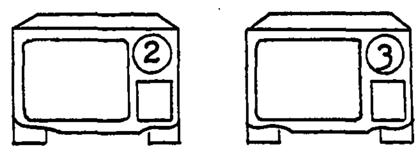


Gates Avenue. How many houses did Marvin live in?

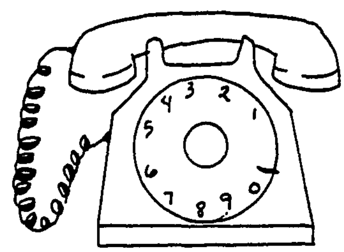
If you could have one of these which would you choose? Why?

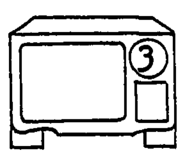
20¢ 11¢ 12¢ 3¢

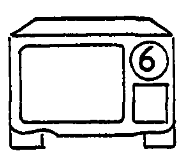
John turned to channel 2 and channel 3.
Which channel did he get? Why?



Marcy's phone number is 555-3133.
If I dial 5 and 5 and 5 and 3 and 1
and 3 and 3.
What number would I get?

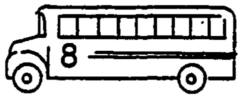


Mary's favorite channel is 3. 

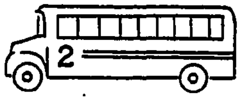
Marvin's favorite channel is 6. 

Which channel has the most programs? Why?

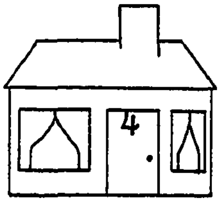
Name ______________________

Kareem comes home on the 

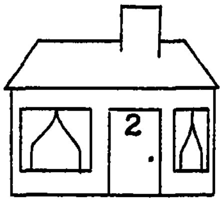
school bus.

Marvin comes home on the 

Which bus gets the child home fastest? Why?

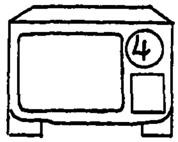
Mary lives at 

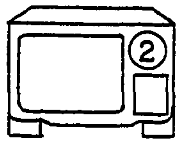
Grey Street. Marvin lives at



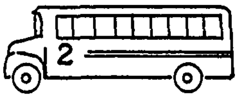
Grey Street. Who lives in the biggest house?

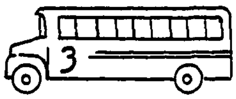
John turned to channel 4 + channel 2.
Which channel did he get? Why?



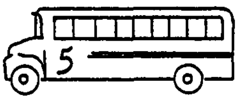
+


=

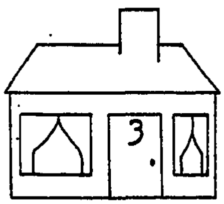
Here is bus 

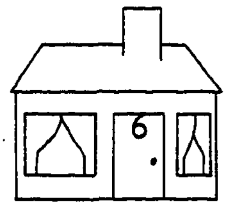
+


.

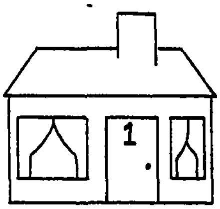
Are they equal to 

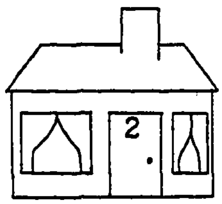
? Why? Why not?

Grey Street has a 

and 

house.

Fulton Street has a 

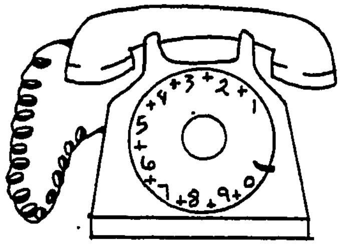
and 

house.

Which street has the most houses? Why? Why not?

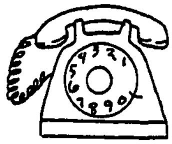
Name ______________________

John's phone number is 322-1212.
If I dial 3 + 2 + 2 + 1 + 2 + 1 + 2
on a phone.
What number would I get?

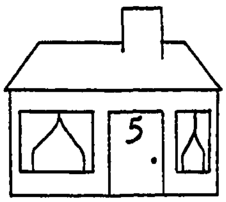


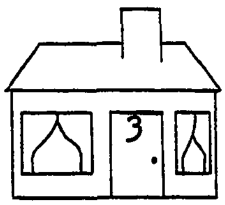
17 Mary's phone number is 222-1212.

Kim's phone number is 564-6666.

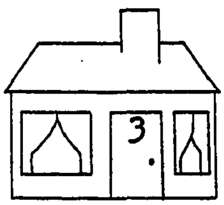


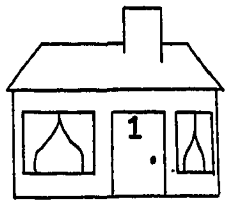
Who has the biggest number?
What does it mean to have the biggest number?

Gates Avenue has a 

\+ 

= 8.

Adams Street has a 

\+ 

= 4.

Which Street has the most houses?   Why?   Why not?

Appendix C

Off Paper Task

Task: Off Paper Questions

Busses

1. What are these and what are they used for?
2. How do you know which bus to get?
3. Why do people put numbers on busses?
4. What would happen if there were no numbers on busses?
5. If in a race, which of these busses would win?
6. a. Is either of these busses bigger?
   b. Is either of these busses smaller?
   c. Why? Why not?
7. a. Can I add bus number 1 to bus number 2 to get bus number 3?
   b. Why? Why not?
   c. Where is bus number 3?
8. a. Are the numbers on the bus plus numbers?
   b. Why? Why not?

Telephones

1. What is this and what are they used for?
2. How do you call someone?
3. How and what do they dial?
4. Why do people put numbers on phones?
5. Could everyone have the same number?

6. a. Do you have a phone number?

   b. What is it?

   c. If I want to call you, can I dial 2 (3+3) for 6?

   d. Why? Why not?

7. How much is 3 + 3 then?

8. Are the numbers on phones plus numbers?

9. a. What is the first number on your phone?

   b. What is the last number on your phone?

<u>Television</u>

1. What is this and what is it used for?

2. If I want to look at something else what should I do?

3. a. Why would I have to turn the channel?

   b. What is a channel?

   c. How can you tell one channel from another?

4. Why would anyone put numbers on a channel?

5. a. Does channel 12 have more shows than channel 3?

   b. Does channel 9 have more shows than channel 5?

   c. Why? Why not?

6. a. What is the first and last numbers on your TV set?

   b. If I want to see channel 5 what numbers must I past to get to it?

7. What numbers come after channel 7 on your TV set?

8. a. If I want to see channel 5 can I turn to channel 2 plus channel 3 to get channel 5?

   b. Why? Why not?

9. How much is 2 plus 3 then?
10. Are the numbers on the TV set plus numbers?

Addresses

1. What are these and what are they used for?
2. How can you tell one house from another?
3. Why would anyone put numbers on a house?
4. Can everyone have the same numbers on their houses?
5. a. Can I mix up the numbers like this?

   b. Which house do you now live in?
6. a. Is anyone of these houses biggest?

   b. Is anyone of these houses smallest?

   c. Is house number 7 bigger than house number 1?

   d. Is house number 3 smaller than house number 5?
7. Mr. Jones lives in the 3rd house on the street. Which is the 3rd house?

   1 3 5 7
8. My address is number 5 Grey Street. If you want to visit me, can you go to house number 1 plus house number 4?
9. How much is 4 plus 1 then?
10. Are the numbers on houses plus numbers?

Dollars / $5 $1 $1 $2 /

1. What are these and what are they used for?
2. How do people know which one to use?
3. Why do people put numbers on dollars?

4. Here is a grocery slip. The groceries cost $2.

   a. What should I give them?

   b. Could I also give them this?

5. a. Are any of these dollars more than the others?

   b. Are any of these dollars less than the others?

6. a. If some said you could have these three or this one, which would you choose?

   b. Is someone was to give you one of these, which would you want?

7. a. Is dollar number 2 more than dollar number 5?

   b. Is two one dollars bills more than number 5?

8. a. Are the numbers on dollars plus numbers?

   b. Can I say $1 plus $1 equals $2?

   c. Why? Why not?

9. a. Can you show me 20¢?

   b. Can you show me 25¢?

10. a. Show me 12¢?

    b. Show me 7¢?

11. Group together the items with numbers that go together.

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