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APPLICATIONS OF MODERN CONTROL THEORY TO THE CONTROL OF INCOMPLETELY SPECIFIED CHEMICAL PROCESSES

by

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Abstract

APPLICATIONS OF MODERN CONTROL THEORY TO THE CONTROL OF INCOMPLETELY SPECIFIED CHEMICAL PROCESSES

by

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Six criteria are offered for the evaluation of the performance of process controllers. Classical techniques for the design of P.I.D. controllers and modern control theoretic techniques for controller design are reviewed. The P.I.D. controller and various optimum controllers are compared by examining their performance when controlling a simple first-order process with time-delay. It is found that the overall performance of the P.I.D. controller is quite acceptable in a practical sense whereas the "optimum" designs have significant shortcomings. A discussion of these results is included together with some indications for necessary future work.

The effects of imprecise knowledge of system parameters on the reconstruction error of linear observers and on the stability of a class of linear regulators is also examined. An upper bound on the reconstruction error linear unforced systems is obtained. It is shown that in the regulator problem consideration of parameter uncertainty leads to the inclusion of step disturbances. An upper bound on allowed parameter variations that guarantees stability of a class of closed-loop regulators is obtained. It is also shown that by properly choosing parameters of a low-order observer, the output of this low-order model can be made to follow closely the output of a higher-order dynamic systems. The effect of modeling errors on randomly-peturbed systems is also examined.

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FOREWORD

Aaron Kestenbaum was killed in action during the October 1973 Middle East War. This dissertation is an edited version of a draft that he completed in September 1973.

Introduction

The work of Aaron Kestenbaum presented here has focused on the setting up of meaningful design criteria for process controllers. Although many mathematical optimization procedures have been proposed in recent years for process controller design, these techniques usually focus on only one or two aspects of the desired system performance. In Section I.2 below, six criteria are presented for evaluating the performance of competing process controller designs. These criteria focus on the ability of a controller to maintain a given set point, perform rapid and smooth set-point changes, yield stable performance despite system parameter variations, avoid excessive control effort, minimize the effect of random disturbances, and be insensitive to assumptions regarding the structure of the system to be controlled.

Section I.3 contains an evaluation of the classical P.I.D. controller in terms of the six design criteria, and Section I.4 contains a summary of some modern optimization procedure which have been applied to process controller design. In Section I.5 a comparison is made between the performance of a specific plant, characterized by a transfer function with a single time constant and transport delay, using a P.I.D. controller and using controllers designed via an optimization procedures with a quadratic performance criterion. Nyquist, Bode, and transient response plots are presented for the feedback system employing the P.I.D. controller and optimum controllers both when the parameters of the plant have their nominal values and when the parameters of the plant differ from the nominal values upon which the controller designs are based.

It was shown that although an optimum controller may be designed to achieve superior performance to a P.I.D. controller with respect to one of the six performance criteria, the P.I.D. controller actually achieves a successful compromise among all six requirements. This clearly points out the inadequacy of recent mathematical approaches to process control theory and establishes a firm foundation for a more practical approach to these problems.

In Section II an examination of the sensitivity of linear state reconstructors and regulators is given. It was shown that if bounds on parameter uncertainty are known, then the asymptotic stability of the resulting closed-loop regulator can be guaranteed. This result is an important step in the development of modern techniques for process controller design.

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SECTION I

DESIGN CONCEPTS FOR PROCESS CONTROL

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Abstract

Six criteria are offered for the evaluation of the performance of process controllers. Classical techniques for the design of P.I.D. controllers and modern control theoretic techniques for controller design are reviewed. The P.I.D. controller and various optimum controllers are compared by examining their performance when controlling a simple first-order process with time-delay. It is found that the overall performance of the P.I.D. controller is quite acceptable in a practical sense whereas the "optimum" designs have significant shortcomings. A discussion of these results is included together with some indications for necessary future work.

1. Introduction

In recent years there has been a growing literature on the applications of optimal control techniques in the process industries [1-8]. However, it is generally felt that these methods have not made the expected impact on the industrial practice of process control, a problem which was the subject of panel discussions at several recent AICH. E. meetings [9, 10].

Often the hypothesis is raised that the lack of applications is due to the lack of sophistication of the practitioner and might be the result of the normal time lag between the conception of modern control system design techniques in the academic community and their industrial application. Some are less generous and claim that the problems dealt with in the academic world are too far removed from reality to lead to useful results. One of the authors (R.S.) has been involved in industry for many years and has often tried to apply some of these optimal control techniques with rather limited success and reluctantly has come to the conclusion that in the present state of the art it is quite difficult to apply optimal control theory to an industrial problem. It is not that the techniques are not useful. In fact they often contribute significantly to our understanding of the problem. It is rather that they are often not in a state where their application is straight-forward enough to allow their use in a reasonable amount of time without extensive study and research, and where straight forward application might even lead to serious troubles.

For several years we have tried to look at basic problems in applying optimal control theory in a systematic way to systems characteristic of those found in an industrial environment, and in this paper we summarize some of our results.

The principal general conclusions that we have come to are that currently there are inherent difficulties in the application of most optimal control techniques, that the state of the art is not yet suitable for wide applications and that the time has come to rethink and re-evaluate our whole approach to the problem. Problems associated with the application of modern control theory to controller design for complex processes have been considered by Foss [10]. However, significant problems are revealed in considering even simple precesses. Hence in our treatment we quantitatively evaluate competing controller designs through application to a first-order process with delay.

Since optimal control is a very wide subject, we have no pretense that our report below relates to all of it. Neither is it our intent to provide a comprehensive process control literature review. Rather we have chosen examples to illustrate some difficulties in applications of control theory to the process industry. As the main illustration we use the rather simple and perhaps trivial problem, the continuous feedback control of a simple linear process shown in Fig. 1 which is stable, overdamped, has a single measured variable y (t), and is controlled by the manipulation of a single input variable u (t). For this process, the dynamic process is characterized by a transfer function $G_p(s) = e^{-s\theta}/(1+\tau s)$, and actuator and sensor dynamics are neglected.

In this paper we do not deal with computer control but with the problem of designing an analog controller whose inputs are the measured output y (t) and the set-point setting x_p and whose output is the control signal u (t). We admit that most industrial problems of real interest are more complex, are often characterized by processes with multiple inputs and outputs and often by processes which are neither overdamped nor linear. Furthermore, many simple control problems can be solved satisfactorily by simple PID controllers. However, the fact that the problem is rather simple and has satisfactory solutions is an advantage in testing out various competing control system design methods especially if they are mathematically complex.

An important feature of a good control system design algorithm is that it provides the practicing engineer with a framework within which to cast his problem and provides a systematic design procedure which can be applied to a larger number of similar problems. If, however, an algorithm gives unsatisfactory results, we have to understand why, and put proper safeguards into the design algorithm. Hence, in section 2 below we state our interpretation of the principal desirable characteristics that should be achieved by a process controller. In section 3 we describe and compare various design methods for the classic PID controller in terms of their achieving these goals.. In section 4 we describe some approaches to the design of process controllers using frequency-domain and time-domain optimization procedures. Section 5 contains an evaluation of three competing process controller designs for a simplified problem. Our conclusions are summarized in Section 6.

2. Design Criteria for Process Control

Before discussing in detail the advantages or disadvantages of specific controllers, let us restate the aims and goals of process control and the specifications that such a controller must fulfill. We will list them first without any intention of ranking them. Again we are referring here to a simple continuous feedback controller, as discussed in the optimal control literature [11] which may be part of a more complex system but which can be designed separately.

It is important to remember that in process control we seldom totally rely on such controllers, and that they normally are part of a more complex scheme which is managed by an operator who achieves the desired control of the total process by adjusting the set-points of individual controllers. Interestingly, contrary to quality control in mass production by machine tools, there has been little systematic research on the way an operator should or does control a complex unit. But we still have a pretty good idea on what demands are put on the controller. In the following an attempt is made to summarize these into 6 criteria.

1). Ability to Maintain the Controlled Variable at a given Set-Point

The first demand seems rather trivial, as this is the most obvious goal of process control. But this most essential requirement of process control is often the most difficult to fulfill as it creates mathematical difficulties for most of the optimization algorithms proposed thus far in the process control literature [8] and we therefore, would like to define it rather precisely.

The fact that the process of Fig. 1 can be controlled by a controller measuring a single variable y (t) and manipulating another u does not mean that the process has a single input. In fact it may have several feed-streams as well as other uncontrolled inputs such as environmental temperatures, moisture, etc. When the operator makes a set-point change he may also simultaneously change the set-points of other units which may change some of the input to the process under control. The value of the controller is that it is able to maintain the controlled variable x at a given set-point x_p and compensate for all the other unknown and changing inputs to the plant. Obviously, in a real system it can only accomplish this for a given range of inputs, and when one of the inputs exceeds that range the operator must step in. But within its operating range this controller must be able to handle the process to be controlled, without any specific knowledge about the value of the different inputs.

Therefore, when changing a set-point an operator has only very imperfect knowledge as to what the proper steady-state control should be and the controller must be able to estimate this input, which in conventional control is achieved by the integral control mode.

2). Set-Point Changes Should Be Fast and Smooth

As the overall system may be slow and complex, it is important for the operator to be able to perform individual set-point changes as fast as possible. However, minimum time response often leads to large excursions in the system transient response, and smooth response (or low overshoot) has a significant advantage. The operator normally does not know what the proper set-point is. If there is severe overshoot he will normally try to counteract and thereby often aggravate the problem. While this problem of minimum-time response has attracted a great deal of attention in the optimal control literature [11] , and while it is a very important one, fast time response cannot be treated in isolation and is only one part of a process controller specification.

3). Asymptotic Stability and Satisfactory Performance for a Wide Range of Frequencies

The total system (not necessarily the controller) should obviously be asymptotically stable to be suitable for operator control. This asymptotic stability should be achieved even though the process parameters may change within a reasonable range of system parameter values.

Furthermore, the closed-loop transfer function frequency response should not have peaks indicating strong amplification of certain input signals. This means that the maximum amplification in the transfer function from disturbance input to process output should be low. This should be true for disturbances such as w_d in Fig. 1 which are filtered through the entire plant as well as measurement noise, disturbances which appear in an "unfiltered" form such as $\dot{w}(t)$ in Fig. 1. Most of the inputs to a plant are filtered through the plant and, though the transfer function from each disturbance to the output may be different, they all may have the same denominator. Some disturbances, however, may be created in the process itself or may be entering the process in a later stage. The first type are normally the more important as in most cases they determine the steady-state level of the manipulated inputs as well as of the uncontrolled state variables.

4). <u>The Controller Should Be Designable With A Minimum of Information With Respect To The Nature Of the Input and the Structure of the System</u>

In many cases in process control we have a rather imprecise knowledge of the nature of the disturbances and their variation with time. If the payoff is sufficient, we can try to obtain that knowledge. But even if we can obtain this knowledge we would still like the controller to protect us against contingencies, sudden changes in its inputs, etc.

The second requirement of obtaining a controller design despite lack of knowledge of the system structure is more stringent. In most situations the process to be controlled is only inaccurately known, or is so complex that we try to get away with using a design based upon a simple, approximate model of the actual process. We have to be careful that the control action achieved in theory is not strongly dependent on that part of our model which is inaccurate. A short example will illustrate this. Consider, for example, a distributed-parameter system (as for example, a heat exchanger) which features both mixing process and transport delays. For mixing studies we might successfully model it as a series of three stirred tanks. However, if we design an optimal controller for three stirred tanks we might obtain a controller which combines derivative action with a very high gain. While this would function well in three stirred tanks, it will lead to instability in the real system due to the finite time-lags involved. It is this need for structural insensitivity which is the hardest one to evaluate in practical applications of optimal control and will be discussed in detail later.

5). <u>The Controller Should Be Insensitive To Change In System Parameters</u>

In a real control situation the parameters of the system are not accurately known and in addition often change with time, and the controller must be able to handle reasonable changes, with a sufficient stability margin. The reason for this need is twofold. First, the throughput through process equipment changes due to varying overall needs of the plant. That means in a process with a time lag the controller must be able to perform while the actual time constants of the system change and these changes are in no way negligible (a factor of two is a reasonable range for many processes). The second reason is that as mentioned above, linear system equations are often a linearization around a steady-state and when the steady-state set-point is changed these linearized system parameters may change significantly.

6). Excessive Control Actions Should Be Avoided

There are two main reasons for limiting the control effort. The first is mathematical. When dealing with a linear problem we neglect one important nonlinearity, the finite limits on the magnitude of allowed control signals. To avoid errors we must put reasonable limits on magnitude of control or we have to take this nonlinearity into account in our design. Strong control action might also be costly and there is an economic reason for limiting control action as, for example, in space flight. In process control reducing the control actions is seldom of economic significance as there are very few cases when a minimization of control effort can really be justified on cost considerations alone.

If one looks at any of the optimum control strategies as to the way they fulfill these requirements it becomes obvious that in the present state of the art it is impossible to incorporate all of them simultaneously into an algorithm. All existing algorithms are written for one or two of the above points and the rest must be tested for in a rather pragmatic way. After several such tests in practical situations it soon becomes evident that some of these requirements are contradictory, such as 2 and 4, and that a compromise is needed. And here we come to a basic problem in the application of optimal design methods for problems which basically demand a compromise solution. It is not clear a priori that a method that searches for an optimum for one specification or maybe for two together (2 and 6 are quite accessible together) leads even to the proper specification of the structure of a controller which will give a sensible compromise for all of them. One of the advantages of the PID controller is that we have for its design, methods which will almost always lead to a reasonable, workable compromise. Hence before we can find wide general applications to optimal control, we have to work out similar methods which either guarantee a sensible compromise or tell the designer more clearly the type of problems for which the method will lead to a good overall controller.

To illustrate the problems involved we will discuss in more detail the properties of PID controllers and of controllers designed using the framework of optimization procedures.

3. The P.I.D. Controller

Several conventional design methods have been proposed in the literature for the P.I.D. controller [12, 13] and some as the Cohen and Coon method, deal specifically with the case which is the base for our comparison, an overdamped system with delay. It is interesting to note that all of these design techniques lead to controllers which are almost identical [8], despite the fact that they are based on different design procedures. We will indicate below one possible reason for this similarity in controller structures.

First consider the Ziegler-Nichols method [13] which is based principally on stability considerations. The method can briefly be described as follows: First the maximum allowable gain, K_c, for proportional control only is found (experimentally or by caluclation). This gain $K_{c\ max}$ is one that will cause the system to be on its stability limit and any larger gain (or certain changes in the parameters of the system) will cause the overall system to become unstable. In order to prevent this from happening, the actual gain used is reduced to insure a sufficient gain margin (~ 2). In order to satisfy requirement No. 1 above a limited integral action is added. This leads to a P-I controller of the form

$$U(s) = -K_c\left[Y(s) + \frac{1}{\tau_I s} Y(s)\right] \qquad (3\text{-}1)$$

Since the integral mode has a tendency to decrease the stability safety margin the setting of K_c is decreased a little more. In order to speed up the control of the process a small derivative action is added resulting in the control law

$$U(s) = -K_c\left[1 + \frac{1}{\tau_I s} + \tau_D s\right] Y(s) \tag{3-2}$$

This has a tendency to increase the safety margin and therefore a slightly larger gain K_c can be used. The reason only a small derivative action is used, is because high values of τ_D would tend to lead to saturation of the controller or the valve if high frequency content disturbances are present near the input to the controller.

In Fig. 2 we give the response of the system to a unit-step disturbance in the load w (t) and in Fig. 3 we show the response to a unit-step change in set-point x_p. The process parameters are $\theta = .5$, $\tau = 1$. The plots are given for both the ideal P.I.D. controller and for a practival P.I.D. controller. The main difference between the two is that the practical P.I.D. controller has a high and low frequency filter in addition to the terms given by (3-2). The transfer function is

$$G_c(s) = \frac{U(s)}{Y(s)} = \frac{K_c\left(1 + \tau_D s + \frac{1}{\tau_I s}\right)}{\frac{\tau_D}{\gamma} s + 1 + \frac{1}{\alpha \tau_I s}} \tag{3-3}$$

where

$$\gamma >> \tau_D \,, \quad \alpha >> \frac{1}{\tau_I}$$

In general, the effect of the low frequency filter is very small and for all practical applications we may neglect it. G_c (s) then becomes

$$G_c(s) = \frac{K_c\left(1 + \tau_D s + \frac{1}{\tau_I s}\right)}{\left(\frac{\tau_D}{\gamma} s + 1\right)}$$

The controller parameters used to generate Figs. 2 and 3 are: $\tau_D = .17$, $\tau_I = 1$, $K_c = 2$, $\gamma = 8$. From the step response we note that the performance of the actual P.I.D. controller is slightly worse than that of the ideal P.I.D. controller. However it eliminates one of the main problems of derivative control action, which is the high control effort due to disturbances with appreciable high frequency content. For completeness we give also the Nyquist plot and the frequency response of that system controlled by a P.I.D. controller. (Fig. 4, 5, 6, 7).

The Ziegler-Nichols settings on a P.I.D. controller give a reasonable compromise among all of the requirements we defined in the previous section. In addition to the ability to maintain a desired set point (requirement 1) it has a good step response (requirement 2) and reasonably low amplification (requirements 3 and 6); it is quite insensitive to parameter changes and not very sensitive to the structure of the plant (requirement 5). This will be demonstrated by the simulation results presented below.

One of the frequently encountered process changes in chemical processes is an increase or reduction of through-put. This causes a change in the residence time of the plant, thus changing the characteristic times in the plant's model. However, the ratio between these characteristic times do not change. This change should be followed by an appropriate change in the controller settings since these settings depend on the individual characteristic times (τ_D and τ_I). Obviously we would take care of extreme changes by changing the controller parameter settings, but the controller should not be too sensitive to such changes even at fixed settings. In Fig. 6 and 7 we give the frequency response of the output y to input disturbances w_d and output disturbances w when no change in throughput has occurred (curve c) and when a change in throughput has occurred such that a 20% error in the previously assumed time characteristics have resulted (curve d). We observe that the difference between the two curves is not significant and the controller has a very reasonable frequency response. It is also insensitive to the exact form of the plant model assumed since the only important parameters of the model are the maximum allowable gain K_c*max and the critical frequency ω_{cr}** (τ_D and τ_I will depend on ω_{cr}). In any case, approximate knowledge of those features ($K_{c\ max}$ and ω_{cr}) is a minimum requirement for any controller design.

* $K_{c\ max}$ is the maximum gain that can be used (and still maintain stability) when only proportional control is used.

** ω_{cr} is the frequency of oscillation of the system when only proportional control of gain $K_{c\ max}$ is used.

An approximate model is therefore sufficient. Let us for example consider a cascade of three identical stirred tanks, for which $K_{c\ max} = 8$ when only proportional controller is used. If we would really believe that the three stirred tank model accurately represents the process to be controlled we could by appropriately designing a controller (for example using a strong derivative control with a large proportional gain) actually achieve nearly perfect control.

In this case a double derivative controller $K_c\,(\tau_{DD}\, s^2 + \tau_D\, s + 1)$ will allow for infinitely large K_c where the system is perfectly stable and can have an excellent step response or frequency response. However this may be a fiction. A real system has a finite delay in it and a high gain K_c would cause that system to be unstable. Thus if we want to stabilize the system by derivative action we need much more detailed information about the system itself to guarantee the stability.

Cohen and Coon [12] derive a method for finding P.I.D. controller settings, which results in settings very close to that of Ziegler-Nichols. They consider a system which consists of a single stirred tank and a delay which is subject to a step disturbance. The problem is to find the settings of the controller that gives a "best" compromise among certain requirements that one would be interested in. Since it is not easy to incorporate all of the requirement in a straight forward way they find the setting that gives the "best" response with regard to one or two requirements and then modify those settings in order to satisfy some of the other requirements. For example using a P.I.D. controller one requirement is to minimize the area under the response curve. This results in a controller which is on the verge of its stability limit. Although the step response will be a good one this is obviously not a useful controller. By stability arguments this controller is modified and a version similar to that of Ziegler-Nichols is obtained. A second minor modification is performed to satisfy a requirement for faster response.

The trouble with the Cohen and Coons procedure is, that it is difficult to generalize it even though it is a very ingenious one. In the end in both the Ziegler-Nichols and the Cohen and Coon approaches success depends on finding a proper compromise between the conflicting performance requirements. A proper guess of where to start can be made by quite different arguments. The procedure used by Cohen and Coon has one big advantage, as it shows how close the step response of the actual controlled system is to the best possible step response.* Cohen and Coon Achieve their structure insensitivity in an interesting way. Of all over-damped systems the single stirred tank with delay represents the most difficult case to control. As the real system to be controlled is different, the "optimum response" is not necessarily optimum, but actually is a compromise design.

There is a third way devised by the authors whose value for this case is merely didactic as it leads to similar results. In other cases this point of view might be useful since it may lead to a more direct design procedure. We try to combine the first three requirements mentioned in section 2, in terms of requirements on the frequency response of the system. Denoting the magnitude of the amplitude ratio of the frequency response of the system by $G(\omega)$ we may impose conditions that will satisfy the actual physical requirements. The first requirement dictates that $G(0) = 0$. To insure fast response $G(\omega)$ should stay as low as possible at low frequencies. This can be specified by $\left.\frac{dG(\omega)}{d\omega}\right|_{\omega \to 0} \leq \epsilon$.

Requirement 3 can be expressed quantitatively by limiting the maximum value of $G(\omega)$. However, the last two specifications may be contradicting indicating the need for a compromise. If we specify $\left.\frac{dG(\omega)}{d\omega}\right|_{\omega \to 0} = \epsilon$ then the settings that minimize $G_{max}(\omega)$ (where $G_{max}(\omega) = \max_{\omega} G(\omega)$) might be considered to be the best, or when $G_{max}(\omega)$ is specified one might try to minimize $\left.\frac{dG(\omega)}{d\omega}\right|_{\omega \to 0}$.

Hence what is needed is not a controller that is optimim in a mathematical sense but a "mini-max" controller.

For example, when a system consisting of three stirred tanks is considered (the one stirred tank and delay is mathematically more tedious) with a P.I. controller it is found that all mini-max settings lie approximately on a line in the $K_c - K_c/\tau_I$ plane as indicated by the solid line in Fig. 8. Note that both the Ziegler-Nichols and the Cohen and Coon controller settings are close to the mini-max line. It is also interesting to note that there is a minimum controller parameter setting S_{min} on the line below which smaller values of K_c have very little effect on $G_{max}(\omega)$. Similarly there is an upper limiting parameter setting

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S_{max} which is determined by the need to insure a reasonable safety margin with respect to closed-loop system stability. Clearly this mini-max procedure does not take into account stability in a direct way. Stability is taken into consideration indirectly by limiting the maximum allowable value of G (ω).

The success of the conventional three mode controller has puzzled many workers. However, in this case the difference between the maximum allowable value of K_c to insure stability and its reasonable minimum value is rather small. This might explain why many different design procedures result in similar controller settings for the process under consideration.

4. Optimization Techniques

Optimization procedures that have been offered for the design of process controllers usually involve a two-step design procedure: (1) the mathematical minimization of a scalar functional performance criterion

$$V[\underline{x}(t), \underline{u}(t)] \tag{4.1}$$

where $\underline{x}$ denotes the process dynamic state vector, $\underline{u}$ denotes the control vector which is to be determined, and where the control interval may be finite or infinite; (2) the physical realization of a feedback control law

$$\underline{u} = \underline{\phi}(\underline{x}, t) \tag{4.2}$$

to effect the minimization of (4.1).

The optimization procedures that have received the greatest attention for process plant regulation are those that involve a quadratic performance criterion. The problems are stated either in a deterministic formulation where a controller is sought that minimizes an integral involving quadratic forms in the state and control variables for a given initial deviation in state, or in a stochastic formulation where the controller is sought to minimize the expected value of an integral of a weighted sum of quadratic forms in the state and control variables, conditioned on knowledge of the random processes perturbing the process to be controlled and on the available measurements.

4.1 Deterministic Formulation

One class of deterministic optimum control problems can be stated as follows: given the linear system of first-order differential equations

$$\dot{\underline{x}} = A\underline{x} + Bu(t), \quad \underline{x}(0) = \underline{x}_0 \tag{4.3}$$

where $\underline{x}$ is an n-vector and u is a scalar input, find the feedback control law

$$u = f(\underline{x}(t)) \tag{4.4}$$

that will minimize the quadratic performance criterion,

$$J = \frac{1}{2}\int_0^\infty (x'Qx + u'Ru)\,dt \tag{4.5}$$

where $(\quad)'$ denotes the transpose of a matrix or vector.
Under the conditions that (a) the pair (A, B) is completely controllable [14] (b) matrices Q and R are positive semi-definite and positive definite, respectively, the optimum control law for this problem is a linear feedback control law

$$u = -K\underline{x} \tag{4.6}$$

where

$$K = R^{-1}B'M \tag{4.7}$$

and M is the positive-definite solution of the following nonlinear algebraic equation:

$$MA + A'M - MBR^{-1}B'M + Q = 0 \tag{4.8}$$

When conditions (a) and (b) above are satisfied the closed-loop system that results from use of control law (4.6) is guaranteed to be asymptotically stable. Note, however, that control law (4.6) requires measurement of all process state variables. This may be costly or physically impossible in certain practical applications.

It is also important to realize that care must be taken in properly translating a physical process control problem into the above mathematical framework in order to insure that the resulting controller will yield a feedback control system that meets the performance requirements mentioned in section 2 above. For example, when a change in process steady-state operation is desired one standard procedure for employing the state-feedback formulation is as follows:

assume that the process is described by the non-linear differential equation

$$\dot{z} = f(z, v) \tag{4.9}$$

and is operating in a steady-state $z = x_{s1}$ with a corresponding manipulated input $v = u_{s1}$,

$$0 = f(x_{s1}, u_{s1}) \tag{4.10}$$

Now suppose it is desired to drive the system to a new steady-state x_{s2} where the manipulated input required to maintain this new steady-state is u_{s2}. Let us suppose for the moment that u_{s2} is known. Then

$$0 = f(x_{s2}, u_{s2}) \tag{4.11}$$

If we define x and u as deviations,

$$\begin{aligned} x &= z - x_{s2} \\ u &= v - u_{s2} \end{aligned} \tag{4.12}$$

and expand the right hand side of (4.11) in a Taylor series about x_{s2}, u_{s2}, retaining terms up to first-order, we obtain

$$\dot{x} = Ax + Bu \tag{4.13}$$

Now, the objective is to drive the system (4.13) from $x(0) = x_{s1} - x_{s2}$ to the origin $x = 0$. A performance criterion of the form (4.5) can be used to penalize deviations from the origin and excessive amounts of control effort as measured by the quadratic forms in the integrand of (4.5).

However, an important characteristic of control problems for chemical processes is that the required control u_s to maintain a steady-state x_{s2} will not generally be known so that the parameters of the A and B matrices in (4.13) will not be known precisely. Hence, a designer will use a nominal linearized model which we will denote by

$$\dot{x}_c = A_c x_c + B_c u_c(t) \tag{4.14}$$

Now, this parameter uncertainty in the linear state model, will be shown to indicate that step disturbances should be included in a proper formulation of the process control problem. Suppose a desired steady-state operating point is $x_d = d$, then:

$$\dot{d} = 0 = Ad + Bu_d \tag{4.15}$$

where d is a constant vector and u_d is the steady-state control action, that would be calculated from (4.15) if A and B were known. It is assumed that a solution for u_d does exist. A more detailed discussion is given in [15] .

The control u (t) depends on the performance criteria.* Since the system parameters are not known accurately the following model for the process to be controlled is used:

$$\dot{x}_c = A_c x_c + B_c u_c \tag{4.16}$$

where A_c is an nxn matrix and B_c is an nx1 vector.

The steady-state control action that will be applied to the actual process (4.3) will result from the solution of

$$0 = A_c d + B_c u_{cd} \tag{4.17}$$

Substitution of $u(t) = u_{cd} + u_1(t)$ into (4.3) , gives:

$$\dot{x} = Ax + Bu_{cd} + Bu_1(t) \tag{4.18}$$

Define $e(t) = x(t) - d$ and substract (4.15) from (4.18)

$$\dot{e} = Ae + B(u_{cd} - u_d) + Bu_1(t) \tag{4.19}$$

where $u_{cd} - u_d$ is an unknown constant which plays the role of a step disturbance. The goal of the controller design now is to find a control law $u_1(t)$ that will drive e (t) to zero and will optimize some performance criterion.

In early attempts to apply optimum control theory [8] to chemical processes, parameter uncertainties and external disturbances were often ignored and linear models of the form (4.14) were employed. When these parameter uncertainties or, equivalently, step disturbances are ignored in the formulation of the first-order state equation for the plant of Fig. 1 it is found [8] that the controller transfer function is

* The control can be written as $u = u_d + u_1(t)$ where $u_1(t)$ depends on the performance criterion.

$$G_c(s) = \frac{K e^{-\theta_c} (\tau_c s + 1)}{(\tau_c s + 1) + K(1 - e^{-\theta_c(1+s)})} \tag{4.20}$$

where θ_c is the nominal value of the plant transport delay and τ_c is the nominal plant time constant. K is the gain of the controller and depends on the weight given to the control effort in (4.5). When no weight is given to control effort, the transfer function of the feedback controller becomes

$$G_c(s) = \frac{e^{-\theta_c} (\tau_c s + 1)}{1 - e^{-\theta_c(1+s)}} \tag{4.21}$$

Section 5 contains an evaluation of this controller with reference to the design criteria of section 2.

If the designer desires to account for parameter uncertainties or external step disturbances a class of compensators, called observer-based controllers, after the work of Luenberger and others [16, 17] might be employed. These controllers act on the available measurements to provide estimates of unmeasured state variables and unmeasurable disturbances.

The derivation of this controller is based on [15] and [16] . In the first-order test system the constant disturbance resulting from applying an inaccurate steady-state control action is modeled and augmented to the system equations. The constant disturbance is then approximately reconstructed using a Luenberger observer. Following the work of C.D. Johnson [15] , constant disturbances can be counter-acted by the control law, which is designed to be a sum of two terms, one that acts to cancel disturbances, and the other which acts to satisfy the design requirements as if the constant disturbance did not exist. Although the resulting controller will no longer be strictly optimum for performance index (4.5), this kind of procedure will result in an integral control action, thus eliminating off-sets of some states.

For a single manipulated-input, single measured-output system the construction of the observer eliminates the need for taking derivatives in order to reconstruct the state vector. Indeed for a second order system the compensator achieved by the above method resembles a P.I.D. controller where the actual system "derivative" action is an approximation to a pure derivative, thus tending to eliminate frequent saturation of the controller due to the effect of measurement noise.

When applying this procedure to the base comparison system the resulting feedback controller becomes:

$$G_c(s) = \frac{[c_1(s-f) + c_2](s+1)}{s^2 - (f-1-k)s + se^{-\theta s}[-fe^{-\theta}(f-1) + f(1-k)] - f(1+k)[1-e^{-\theta s}]} \quad (4.21a)$$

where

$$c_1 = ke^{-\theta} - f[1 + k(1-e^{-\theta})] \qquad f = \text{observer pole}, < 0$$

$$c_2 = -(1+f)[1 + k(1-e^{-\theta})] \qquad k = \text{controller gain}$$

We see that this controller has a pole at zero which indicates that integral control action will be applied in order to eliminate offsets.

In this design the value k is to be determined on the basis of the relative weights given to the state and control effort in the performance index and the value of f is arbitrary but negative. However, in order for the observer to accurately reconstruct the state of the system, the value of f should be larger (one order of magnitude or more) than the value of $1+k$, since only then will the eigen-value of the observer be much more negative than the eigen-value of the system it observes, thus causing the observation error to rapidly approach zero.

The effect of modeling errors on the observer has been investigated by two of the authors [18] and it is shown that caution must be taken when designing an observer, both in regard to measurement noise amplification and in regard to sensitivity with respect to parameter variations.

4.2 Stochastic Formulation

The above approach of considering a linear process model and a quadratic performance criterion can be extended to provide a design procedure which explicitly accounts for stochastic disturbances to the process dynamics and measurement noise.

a) First consider the case where a white noise random process perturbs the process dynamics,

$$\dot{x} = Ax + bu + w(t) \tag{4.22}$$

where w (t) is white Gaussian noise with zero mean, E $\{w(t)\} = 0$, and with covariance matrix

$$E\{w(t)w'(t_1)\} = W\delta(t-t_1)$$

The performance criterion to be minimized is

$$J = E\left\{\frac{1}{2}\int_0^{\infty}(x'Qx + ru^2)\,dt\right\} \tag{4.23}$$

It can be shown that the control law that minimize (4.23) is (4.6), the same control law that minimizes (4.5) for the deterministic control problem. The optimum value of (4.23), of course, differs from the optimum value of (4.5) because of the presence of the disturbance w (t) in (4.22).

b) Next assume that the process to be controlled is given by (4.22) but that the measurements available to controller are perturbed by random noise and are given by

$$y = Cx + v(t) , \tag{4.24}$$

where the measurement vector y is of dimension $m \leq n$ and v (t) is zero-mean white Gaussian noise with

$$E\{v(t)v'(t_1)\} = V\delta(t-t_1)$$

The performance criterion is (4.23) but now the expectation is conditioned on the available measurements.

The solution to this problem is expressed by the Separation Therem [11] : the optimum controller comprises a Kalman-Bucy filter which provides a minimum-variance estimate $\hat{x}$ (t) of the process state and a control law,

$$u = -\frac{1}{r}b'M\hat{x}(t) \tag{4.25}$$

that has the same form as the deterministic optimum control law (4.6). The optimum estimate $\hat{x}(t)$ is generated by a dynamic system,

$$\dot{\hat{x}} = A\hat{x} + b\,u(t) + S(t)\left[y - C\hat{x}\right], \quad \hat{x}(0) = 0 \tag{4.26}$$

where time-varying gain matrix $S(t)$ is given by

$$S = P(t)\,C'V^{-1} \tag{4.27}$$

and P (t) is the positive-definite solution of the matrix Raccati equation,

$$\dot{P} = AP + PA' + W - PC'V^{-1}CP \tag{4.28}$$

If it is assumed that the measurements available to the controller have been observed over the entire interval $-\infty \leq \tau \leq t$, where t denotes the current time, then the controller has the time-invariant structure specified by (4.25) and (4.26) with a constant gain matrix S given by (4.27) where the matrix P is the solution to the nonlinear algebraic equation

$$0 = AP + PA' + W - PC'V^{-1}CP \tag{4.29}$$

Extension of this separation theorem to systems with time-delays [22] yields a controller identical in form to (4.20).

One may observe the resemblance between the gain matrix S of the Kalman filter equ. (4-26) and the gain matrix G of the Luenberger observer.

While the Kalman filter gain (S) is calculated on the basis of the knowledge of the measurement noise covariance matrix V the observer gain is arbitrary as long as it satisfies the requirement that the observer error dynamics be asympototically stable [17] .

Earlier work [3] on the control of processes subject to random disturbances was based on techniques developed by Wiener [19, 20] for the filtering and prediction of stationary random processes. Using this approach the perfromance criterion to be minimized is given by

$$J = E\{y^2(t)\} + \lambda E\{u^2(t)\} \tag{4.30}$$

where λ is a positive number denoting the relative weight of control effort. Following the development described in [3], it is assumed that the output disturbance w has zero mean, that the spectral density $\Phi_{ww}(s)$ is known, and that the plant transfer function $G_p(s)$ is also known. Minimization of (4.29) is performed on an equivalent open-loop system, Fig. 9, consisting of the plant and a cascaded compensator, K (s), from which the feedback controller is then constructed. After K (s) is found via spectral factorization, the feedback controller G_c (s) is given by

$$G_c(s) = -\frac{K(s)}{1 - K(s)G_p(s)} \tag{4.31}$$

It is important to note that the stability of the cascaded compensator, K (s), is guaranteed by the procedure, but not that of the feedback controller derived from it [1, 2, 3]. While K (s) has all its poles in theleft hand plane it is not guaranteed that G_c (s) will have all its poles in the left hand plane [3]. The problem of the stability of the controller has been treated by Bankoff [1] and Cegla [3]. The former solves the problem by constraining the stucture of the cascaded compensator, but does not consider the stability of the controller itself. The latter stabilizes the controller by imposing a larger weight on the control effort (λ), which consequently changes the parameters of the controller and thus effects its stability. A method of how to choose a suitable weight λ, that meets some constraints on the variance of the control effort is described in [1].

When applied to the plant that is the base for the comparison of controllers to be discussed in section 5, we use in the design

$$G_p(s) = \frac{e^{-\theta_c s}}{1 + \tau_c s}$$

and find the controller transfer function that minimizes (4.30) to be identical with (4.20) and, of course to be identical with the asymptotic form of the controller that employs a Kalman filter to provide an estimate of the process state for the time-delayed system with no measurement noise.

In section 5 below we examine the performance of a number of the above controllers designed using the framework of optimum control theory and compare the resulting performance with that of P.I.D. controllers.

5. Controller Evaluation

In this section we evaluate the performance of the controllers described above under what may be considered "real" conditions. As noted earlier the degree of uncertainty in the description of most chemical processes is usually much higher than for systems encountered in aero-space or electrical industry for which most modern optimal control techniques have been developed and successfully applied. Thus, the performance of the controllers will be examined for a situation in which the actual parameters of the process to be controlled are different than the nominal parameters used in the controller design. Since in many real processes the nature of the disturbances are not really known (without excessive detailed research) a comparison will be made of the magnitude of the frequency response of the appropriate transfer functions for each closed-loop system.

In the process industry two kinds of errors may be encountered. First is the error in the estimation of the individual parameters of the system and the second is the error that is encountered because of load changes. These load changes, for example, not only introduce an input disturbance but also cause an uncertainty in the parameters of the system. This parameter uncertainty is equivalent to a change in the steady-state operation of the system. While the set-point itself does not necessarily change, the controller should be able to handle an uncertainty in the true steady-state input.

The first kind of error, that caused by measurement inaccuracies, may not be very large, but the second kind may some times be of 100%, since it may correspond to increasing or decreasing the throughput by a factor of 2.

Specifically, we examine the case where the plant has a transfer function given by:

$$G_p(s) = \frac{K_p e^{-\theta s}}{\tau s + 1} \tag{5-1}$$

The first kind of error is such that $\theta_c \neq \theta$, $\tau_c \neq \tau$ and $K_{p_c} \neq K_p$ where the subscript c denotes the estimated value of the appropriate parameter. The second kind of error may be represented by:

$$\frac{\theta_c}{\tau_c} = \frac{\theta}{\tau} \quad but \quad \theta_c \neq \theta, \ \tau_c \neq \tau$$

Thus while the ratio between the two time characteristics of the system remains constant the actual value of these parameters differ from the estimated ones. For the sake of convenience and simplicity we normalize the gain and the times such that the transfer function G_p becomes

$$G_p(s) = \frac{e^{-\theta_c s}}{1+s} \tag{5-2}$$

where s is the new dimensionless Laplace transform variable and θ_c is the dimensionless estimated delay of the system, and actually is the ratio between the estimated delay and the estimated time constant of the system.

We now proceed with the controller evaluation. For the purpose of comparison we choose the plant that has a ratio between the delay and the time constant of one half, hence θ_c in (5-2) is equal to 0.5.

P.I.D. Controller

The settings of the P.I.D. controller were taken approximately according to Ziegler-Nichols settings with a small shift towards the Cohen and Cohen settings. Hence, the particular settings chosen were

$\tau_D = .1$ [illegible] , [illegible] , [illegible] , $K = 2.0$

The Nyquist plot of the process with an actual P.I.D. controller is given in Fig. 5, curve a from which we see that the degree of stability is such that the gain margin is 2 and the phase margin is 50^0. These are satisfactory margins. We note that the magnitude of $G_c(s)\,G_p(s)$ is always decreasing with increasing frequency, until it approaches zero. This is actually the difference between the real P.I.D. controller and the ideal one. There is almost no difference between the two in the low frequency region; the difference is evident in the high frequency region. When the ideal P.I.D. controller would be used the magnitude of $G_c(s)\,G_p(s)$ is also decreasing but approaches a positive constant value $(K_c \tau_d / \tau)$.

The frequency response of the overall system for input and output disturbances are given in Fig. 6 (curve C) and in Fig. 7 (curve C), respectively. We observe that while the P.I.D. controller yields an improvement over the uncontrolled system's response in the low frequency region, it obviously is worse in the mid-frequency regions, and approaches the uncontrolled system's response in the high-frequency region. This is to be expected, since it is easier to control disturbances of low-frequency content than those of high-frequency content. The frequency response of the control action for output disturbance is plotted in Fig. 10 (curve C). We observe that the magnitude of the frequency response increases (on the average) with increasing frequency until it reaches a constant mean value of ($K_c \gamma$). This is in contrast to the response of the system with an ideal P.I.D. controller where the magnitude increases indefinitely with increasing frequency. Thus the high frequency filter imposed on the ideal P.I.D. controller serves to limit the increase of the magnitude of the frequency response. Although the control effort for a purely white noise output disturbance is infinite, this controller is acceptable since any real disturbance has a finite cutoff frequency and since any output disturbance is usually filtered through a transfer function G_d (s). Hence, for practical applications an infinte control effort for output white-noise disturbance does not really concern the process designer.

We stress the point that the acceptable performance of the actual P.I.D. controller for white noise disturbance is achieved by the high frequency filter. The designer uses the idea that not too much can be done to control the high frequency content disturbance and that the controller should be employed to control the response to the low frequency content disturbances. Since it is the high frequency region that gives rise to the infinte control effort, introduction of the high-frequency filter does not affect the operation of the controller in the low-frequency region too much, but overcomes the problem of infinte control effort.

We now proceed to examine the sensitivity of this controller. Consider first the first kind of error, namely $\tau_c = \tau$ but $\theta_c \neq \theta$. In this case, defining $\frac{\theta}{\theta_c} = 1+ \beta$ it is easy to find that the maximum allowable error β is ≈ 0.775 and no limit exists on negative β (except for practical reasons $\beta > -1$). We repeat the Nyquist plots of the system for an existing error of $\beta = 0.2$ and $\beta = -0.1$. and the result can be seen in Fig. 5 curves B and C respectively. The frequency response has also been plotted for $\beta = 0.2$ and the results are given in Figs. 6 and 7, curves denoted by D. We observe that although the performance has deteriorated a bit, the main features of the curves remain the same.

We consider now the second kind of error. The ratio $\frac{\theta}{\tau} = \frac{\theta_c}{\tau_c}$ but $\theta = \theta_c(1+\alpha)$ and $\tau = \tau_c(1+\alpha)$ where $\tau_c = 1$ and α is the relative error. We find that the limits on the allowable error α is such that the system is stable for $-0.65 < \alpha \leq 1.5$. This range for α is rather large and, hence, the controlled system can tolerate large load changes.

We see that the P.I.D. controller with Ziegler-Nichols settings performs quite well. It has a reasonably fast step response (Fig.). It eliminates off-sets, it performs quite well for input and output disturbances over a wide range of frequencies; it has a very simple structure and not complicated setting procedures; it is not sensitive to parameter uncertainty and load changes. These are actually the requirements a process control system has to satisfy and the P.I.D. controller seems to combine these requirements by some reasonable and acceptable compromise..

Deterministic, quadratic performance minimization

The controller for the deterministic design is the same as that for input white noise in the Wiener-Hopf design and is given by (4-20).

a) Unweighted control effort

We first consider the controller when no weight is given to the control effort in the performance criteria. The controller is given by (4-21) and is repeated here:

$$G_c(s) = \frac{e^{\theta_c}(\tau_c s + 1)}{1 - e^{-\theta_c(1+s)}} \qquad (4.21)$$

The Nyquist plot of $G_p(s)\,G_c(s)$ for this control system is given in Fig. 11. We observe that the system has a satisfactory gain and phase margin. However, the fact that the plot consists of a closed curve raises the suspicion that a small error in estimation of the system's parameters may eliminate the cancellations of terms and cause the closed curve to become a spiral-like curve that may increase with increasing frequency, thus causing the system to become unstable. Indeed, when the Nyquist plot was obtained, for the cases where $\beta = 0.2$ and $\beta = -0.1$, this conjecture was found to be true, (Fig. 12). An analytic detailed proof that this system becomes unstable for an infinitesimal error in the delay term ($\theta \neq \theta_c$) has been obtained. When checked for the second kind of error with α =-0.1 it has also been found that the system is unstable. Hence, this controller is very sensitive to certain parameter inaccuracies. It is also the fact that this controller has an infinte control effort, and, unlike the case for the actual P.I.D. controller, the frequency response of the control effort to output white-noise disturbances $U(s)/W(s)$ will increase indefinitely with increasing frequency. However, when comparing the two effects, the most important consideration is the stability sensitivity problem. This is actually a feature of any unconstrained optimal controller we have investigated. All are extremely sensitive to small structural changes (such as a small delay not included in the model) or to small changes in the parameters, and while they are stable for design conditions, are really unstable controllers in any practical sense.

b) Weighted control effort

The two problems mentioned above can be solved by introducing a weight on the control effort in the performance index. While this is obviously a good way to reduce the control effort the question still remains as to whether it is a good way to reduce the sensitivity to parameter uncertainties, and especially where the infinite control effort is secondary in its importance to the stability-sensitivity. However, this is the standard way used in the literature and we will therefore evaluate the overall performance of such a constrained controller. It is evident that when the gain of the controller in (4-20) is zero, which is the case when no control is applied, the system is stable for any error β or α . Hence there exist a finite value of K that will tolerate some finite allowable error. The limits of the allowable errors β and α as a function of K are given in Fig. 13 and 14, respectively. We observe that if one wants the limits of the allowable errors to be at least as large as those of the P.I.D. controller the value of K is reduced very much. For example if α =-0.65 the value of K should not exceed 6.2. For the purpose of demonstration we choose two values of K and plot the frequency response of the system for both cases. The two values chosen were K=6.2 which is approximately equivalent in sensitivity to a P.I.D. controller, and a higher value of K=40.

Let us first examine K=40 which results in better performance at design conditions but allows only a much smaller error before it becomes unstable (β_{max} = 0.26) and α =-0.15). The frequency response of the system for input and output disturbances is given, together with the P.I.D. controller in Fig. 6 and Fig. 7. The corresponding control effort is shown in Fig. 10. We note immediately that despite the fact that the controller is much more sensitive to parameter changes, its performance is far worse than the P.I.D. Clearly at correct design conditions it has lower amplification in the mid-frequency range. But we achieved this by sacrificing the low frequency behavior and the offset is much too large. The controller therefore does not fulfill the main requirement of a process controller which was criterion one, the ability to control the system at a given set point. We should remind ourselves that even in a P.I.D. controller we could have made a different compromise and achieved a lower amplification in the mid-range by sacrificing low frequency behavior. The control effort in the low frequency range is lower for the constrained optimal controller, but it is far too low for any sensible process control. The least we want in this region is "perfect" compensation or $U(j\omega)/W_\lambda(j\omega) \rightarrow -1$, and any value less than 4 is a rather reasonable effort. The fact that the control effort of the constrained controller for small frequencies is less than unity is rather detrimental and it is really a result of our optimization procedure.

For design conditions the stability of the controller is excellent as can be seen from the Nyquist plot shown in Fig. 15. However, as shown in Fig. 16 for β =0.2 the controller is close to instability.

In Figs. 6, 7, and 10 we have also plotted the Bode design for controllers operating in a system where the parameters have slightly changed (β =0.2) and we note the poor overall performance of the constrained optimal controller, curves B, as compared with a P.I.D. controller at the same conditions, curves D. The P.I.D. controller due to its large stability limits is much more able to handle parameter changes.

The performance of the system using a constrained optimum controller with controller gain K=6.2 is displayed in Fig. 17. Curve (A) represents the frequency response for a system with no parameter error, α =0. Curve (C) represents the frequency response for a system with α =0 that uses the P.I.D. controller. Note that although the performance of the optimum controller might be considered to be an improvement in the mid and high-frequency region, this has been achieved at the expense of worsening the response in the low-frequency range. This is also evident from the larger offset that this optimum controller will have (.48) compared to (.40) the offset of the optimum controller with K=40. In regard to parameter sensitivity, the value of controller gain K=6.2 allows a maximum error α =-0.65. The value α =-0.2 was used in generating curve (B) which shows that the performance of the constrained optimum system using controller gain K=6.2 is less sensitive to the parameter variations than is the optimum controller with gain K=40. So although the optimum controller with lower gain is less sensitive to parameter variations, its offset is larger and its overall performance is quite poor.

In Figure 18 we give the step response of the unconstrained optimal controller, and the constrained optimal controller which has the same sensitivity to flowrate changes as the P.I.D. controller. We note that while the system using the unconstrained optimal controller reaches a steady output faster than does the system that uses the P.I.D., the "optimum system" does not go to the proper steady state, but has a considerable offset.

We come here to the somewhat surprising but inescapable conclusion that the standard P.I.D. controller is a far superior controller than the one we designed by a complex optimization procedure. We mean here better in the sense that is allowed a better compromise between the six criteria defined in section 2. It fulfills the main criteria of any good controller, to bring the system to the correct steady-state, regardless of the inputs. It is not too sensitive to structure and parameter changes and it does not amplify too badly. It also give a reasonable fast step response, whereas the constrained optimal controller does not fulfill the first criterion and is only marginally better on the last two.

Now if we just consider one of the criteria we can always get an optimal controller that is much better. In fact, for any single criterion that can be treated by optimal methods, the optimal controller is obviously going to be superior. One of the main applications of optimal control is that it gives us exact bounds as to what can be achieved for any single criterion.

The optimum controller whose performance is shown in Figs. 6, 7, 10, and 17 were obtained as a compromise between fast response and low control effort. However, the unconstrained controller did not really have an excessive control effort in the practical frequency range. The control effort of the constrained controller is far lower than desirable, and the reason it is that low, is that we constrained the control effort to improve the controller's stability in the face of parameter uncertainty. The only reason we chose control effort as a variable is that the resulting optimization problem was then mathematically easier to formulate. There are procedures for correctly formulating the problem, when the system parameters are themselves stochastic variables but they have not been applied to this case. [21] And even this procedure is not very useful since it only protects against changes in system parameters but offers no protection for another type of sensitivity, namely the fact that the real transfer function of the system is unknown and often different from the one used by the controller designer.

McGuire has treated the same example [2] by solving the Wiener Hopf equation with a value of ν corresponding to the delay time of the plant. The resulting controller is similar to that in equation (4.20) and has the same deficiencies.

Cegla [3] tried to solve the problem of bad low frequency response by including a low frequency noise in the formulation of the optimization problem. In his case the input to the system contains a Gaussian disturbance with a spectral density of the form

$$\Phi_{nn}(\omega) = \frac{2\nu_1}{\nu_1^2+\omega^2} + k\,\frac{2\nu_2}{\nu_2^2+\omega^2} \quad , \; k>0$$

By proper choice of ν_1 , and ν_2 , we can obtain a proper compromise between fast response and low offset. However, the main disadvantage of the controller in Figs. 6 and 17 remains. If no weight is given to the control effort, the controller is very sensitive to parameter variations, and if the control effort is constrained via a standard optimization procedure the control effort is too low for good control.

We can also obtain good low frequency control by introducing an observer for the inputs into the optimization procedure but again it is in general difficult to include sensitivity to parameter variations or structural sensitivity. In Figs. 19 and 20 we give the response of such a nonoptimal higher-order observer which has the structure indicated by equation (4.21a). The value of controller gain K was taken as K=6.2, the same value as used for the optimum controller. The pole of the observer was taken to be f=-5K. We note that for the conditions the controller is designed for it is significantly better than the P.I.D. If the parameters change the behavior of the controller is also acceptable. Although the Bode diagram has a peak in the higher frequency range, this is not that important, as in most systems inputs with such high frequencies will be filtered before they enter. This controller while having the same stability margins for parameter variation is however more sensitive to parameter variation than the P.I.D. controller. With further design effort or by trial and error this might be improved but an investigation of such a controller design is outside the scope of this paper. However, this is again a question of judgements. We want in no way to imply that such a controller is optimal. In fact it is perfectly possible that better overall performance would be achieved by a controller using a higher-order observer of a different stucture.

For the case discussed it is even questionable if it is sufficiently better than a P.I.D. to make the effort worthwhile. What is important is to realize that any improved overall performance is not achieved by straightforward mathematical optimization, but by systematic trial and error. Modern computer design techniques make such trial and error methods more feasible. Optimal methods are useful for such trial and error methods as they reveal what is achievable for each one of the first four criteria and furthermore they suggest controller structures for the trial and error procedure.

6. Discussion and Conclusion

In the last section we presented a comparison between a P.I.D. controller and different optimum control schemes. We purposely chose an extremely simple case in which the P.I.D. controller gives rather satisfactory performance. As mentioned earlier none of the optimum controllers has an overall performance better than that of the classical controller and most are definitely inferior. The optimum control techniques described above have been very successful in applications to aerospace technology and one has to ask oneself why it is so difficult to translate these techniques to process control applications. A possible answer might be that the goals of process control are much more difficult to formulate in a mathematically accessible way. In process control applications we search for completely different compromises, and those criteria which are easiest to formulate for the application of the algorithms of optimum control are not necessarily the criteria which are most important to the control of processes. In the classical aerospace problems control effort is one of the prime parameters to be minimized, whereas its importance in process control is minimal. Furthermore, in aerospace control problems we normally have a much better description of the system and the control actions are usually large as compared to the disturbance encountered.

The main fault of all the optimum schemes that were examined above was their sensitivity to the exact structure of the designed system. We would expect that a proper optimum design algorithm should use the available information to the largest extent possible, and it is quite difficult to build into the algorithm the fact that is should not really "believe" our process model.

Although it is possible to include parameter uncertainties in the formulation of the optimum control problem, it is the structural sensitivity which is more difficult to include and which is so important. In most cases we design controllers for processes which are not only very imperfectly known but are also so complex that any manageable process model is only a very crude description of the process.

If one includes a penalty on the control effort in the optimum control problem formulation then this structural and parameter sensitivity is strongly reduced. However, this is not necessarily a reasonable way to deal with sensitivity since the control effort of the unconstrained controller in the range of practical frequencies is small. By including a bound on control effort, one thereby obtains an optimum controller in the sense that it is the best controller for this control effort, but since the control effort is far too small this is not a useful result.

Optimum control methods have one important application. They give an exact limit as to what can be achieved for any given criterion. Any practical controller will be a compromise between the different criteria, and here a knowledge of what could be achieved is an excellent guideline as to how good a compromise the real controller is.

For the example given above it is quite evident that a classical P.I.D. controller is a good compromise. In more complex cases there will be much more room for improvement, but at present the best we can do is to use systematic trial and error methods. In conclusion it should be pointed out that the above discussion dealt mainly with continuous controllers. Sampled data controllers with infrequent sampling involve other problems which will be discussed in a future paper.

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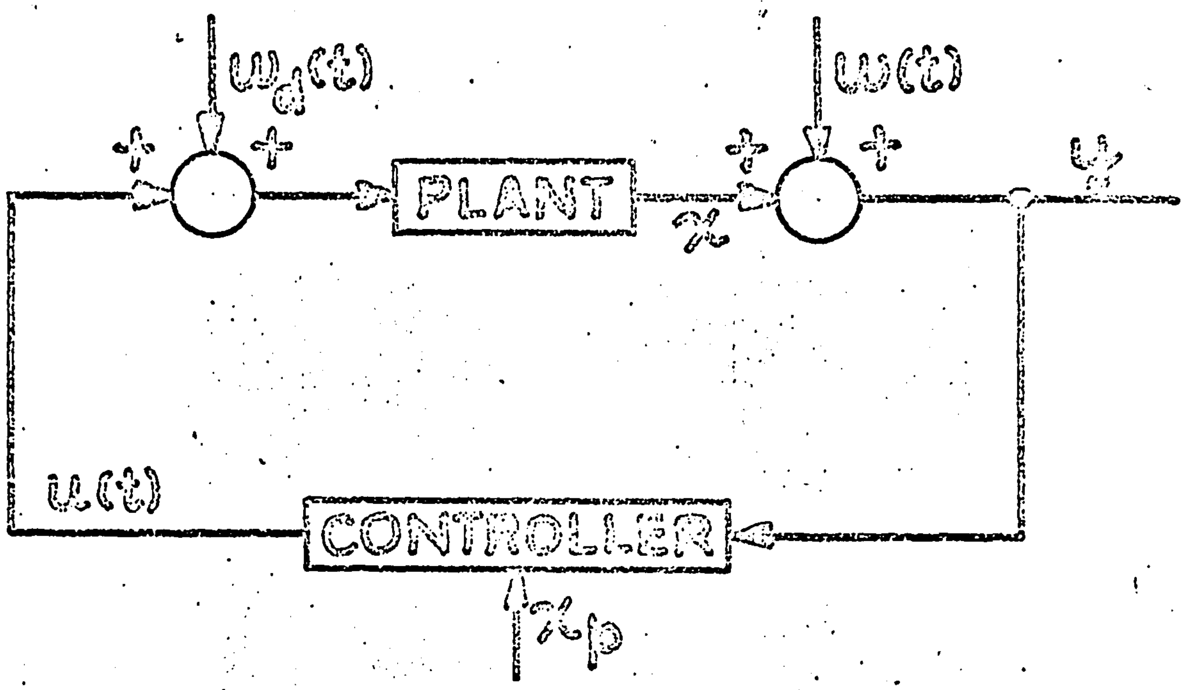

Figure 1. Process Control Configuration

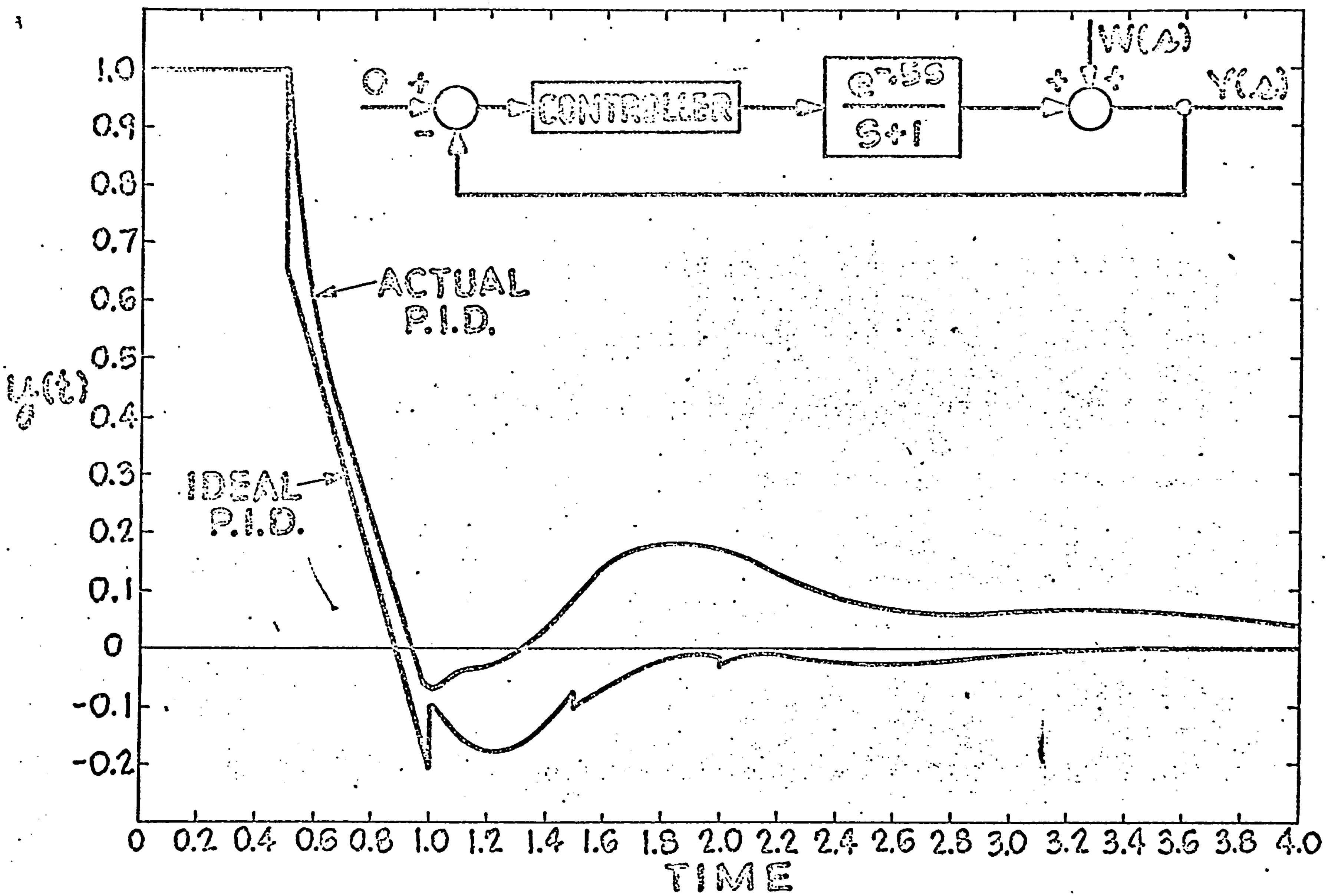

Figure 2. Time Response of PID Controller to Output Unit Step Disturbance

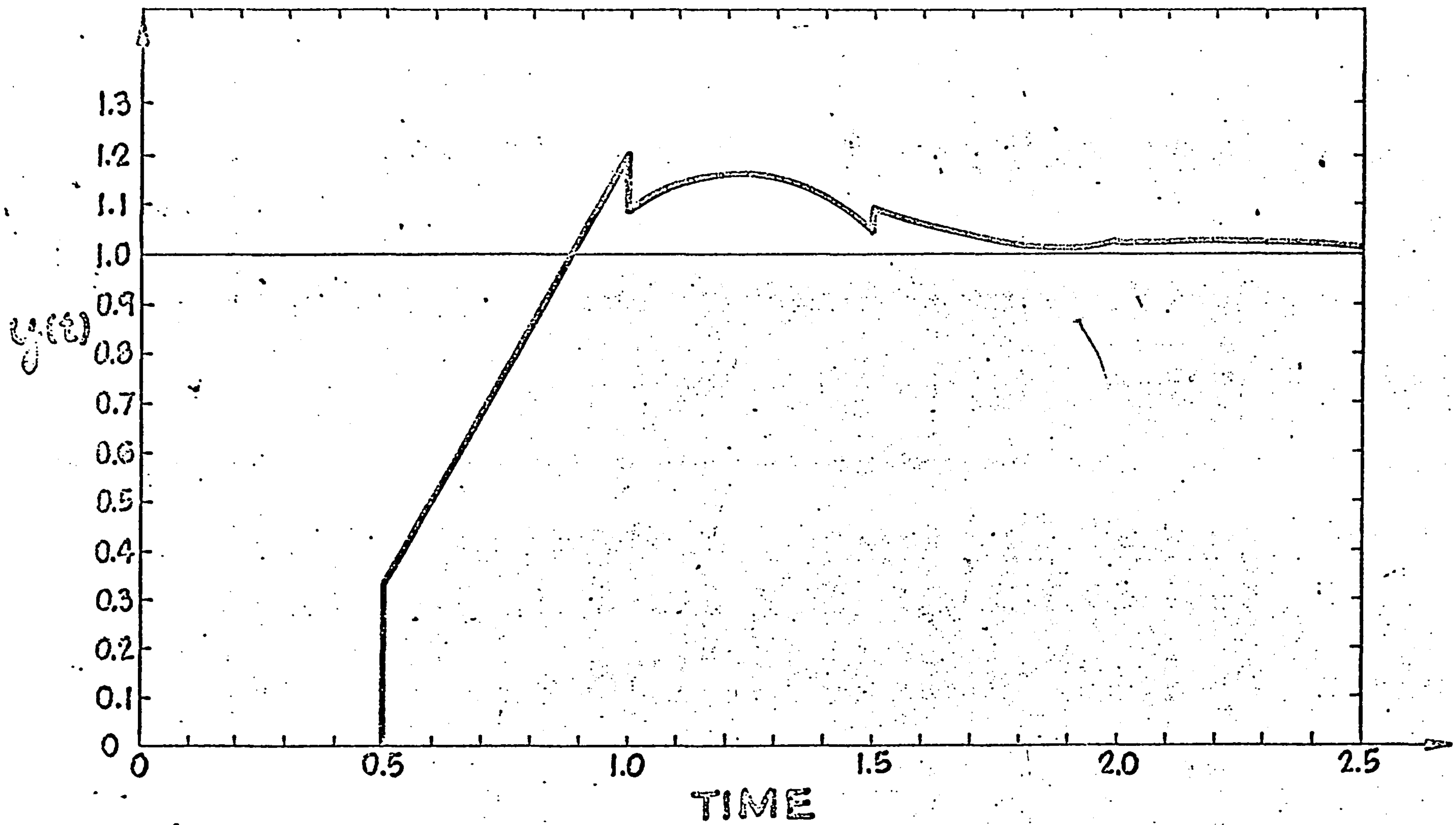

Figure 3. Time Response of Ideal PID Controller to Set Point Change

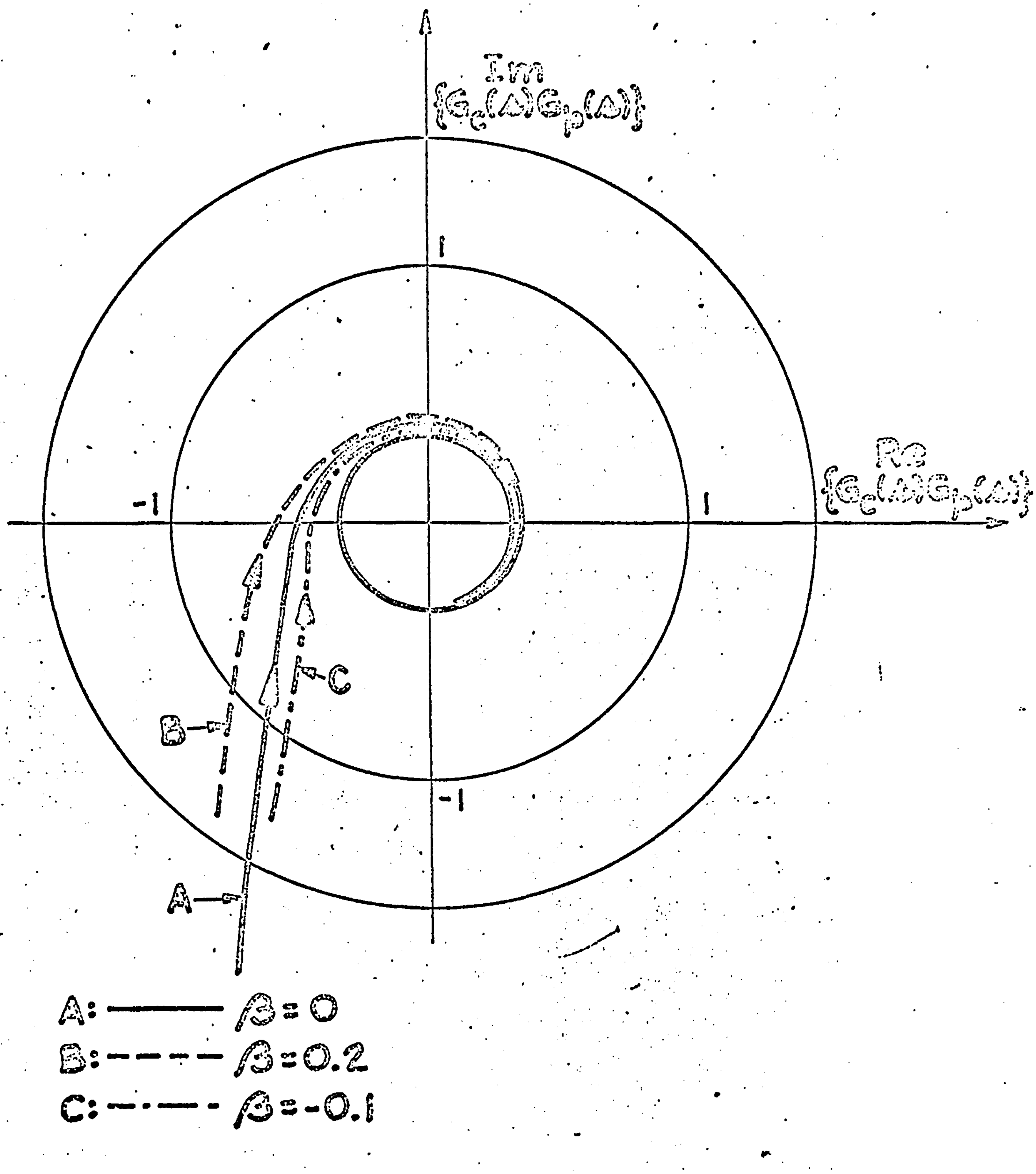

Figure 4. The Effect of Parameter Changes on Nyquist Plot of System Using Ideal PID Controller

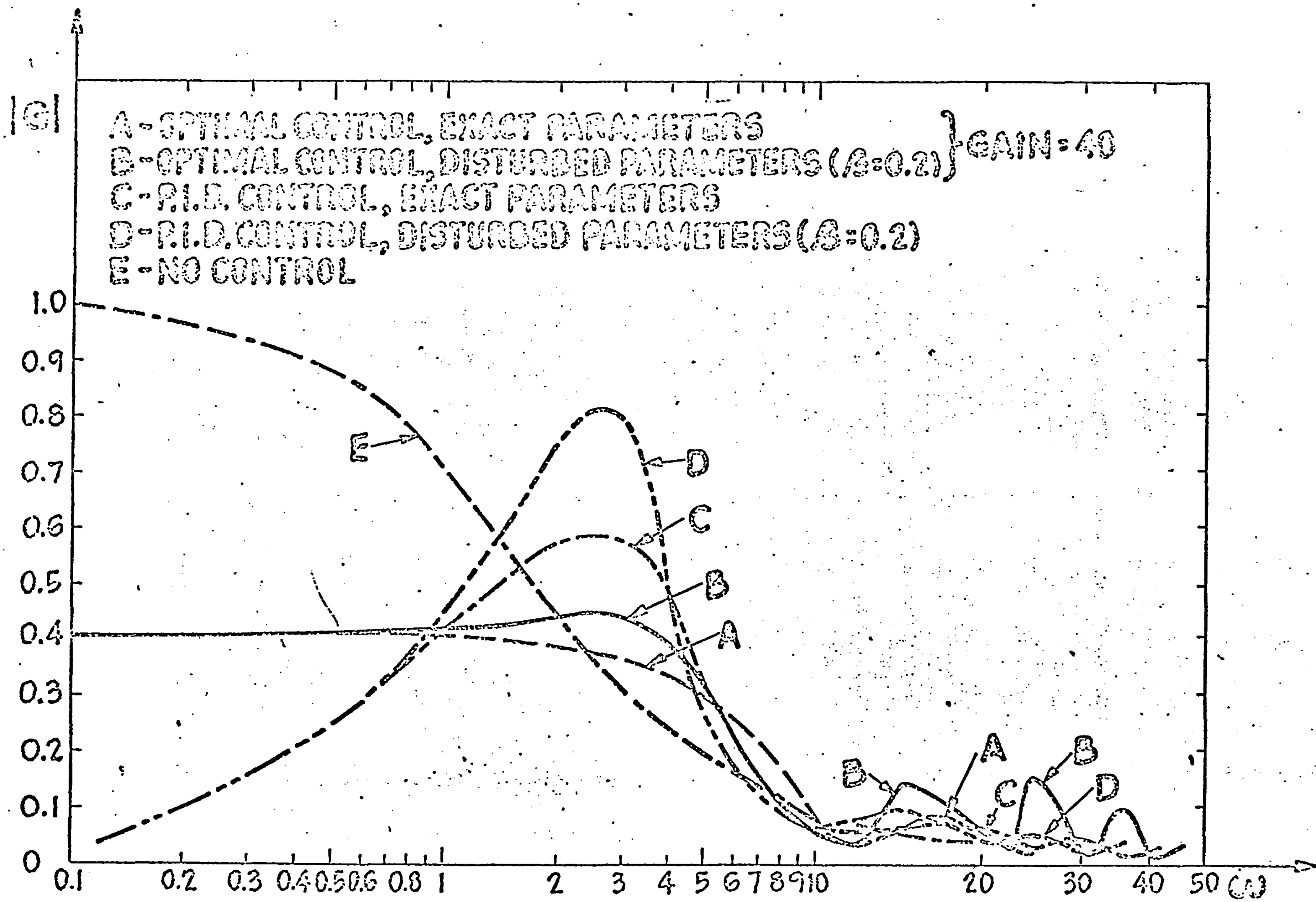

Figure 6. Magnitude of Transfer Function from Input Disturbance to Output Response for Closed Loop System

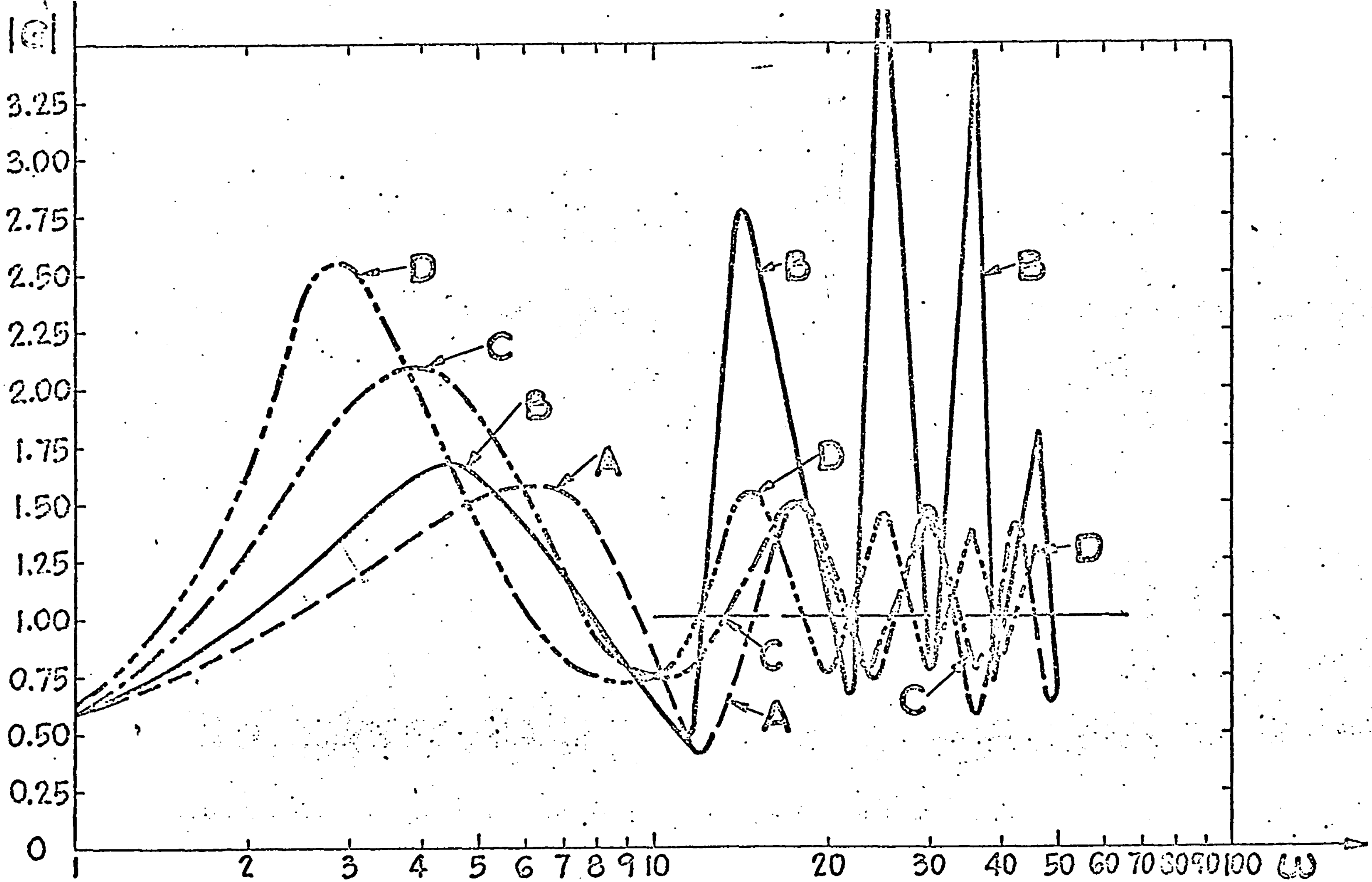

Figure 7. Magnitude of Transfer Function from Output Disturbance to Output Response

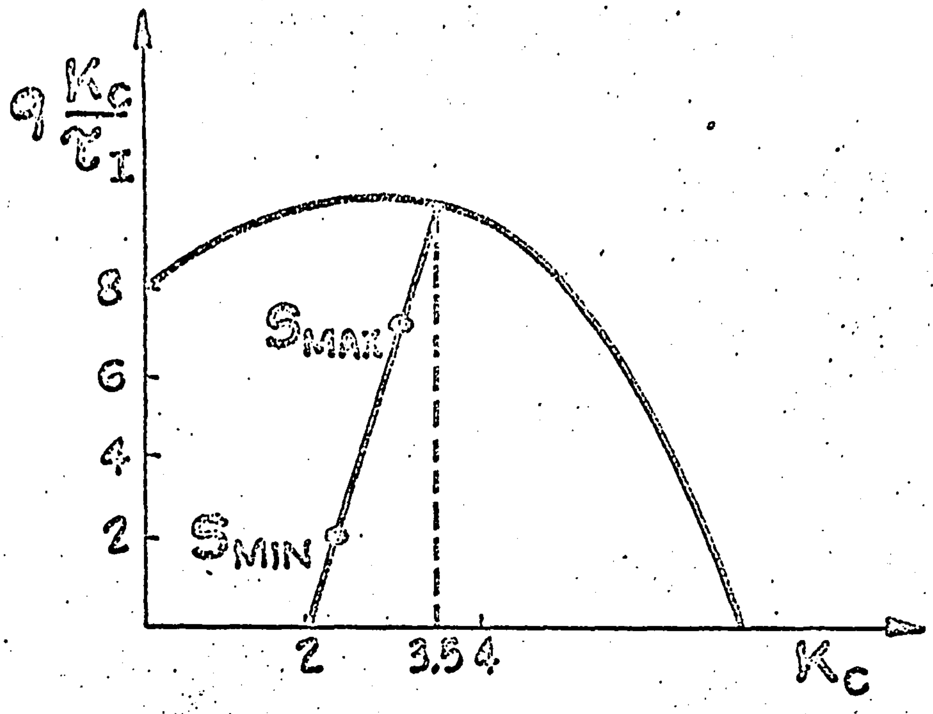

Figure 8. Parameter Plane for PI Controller

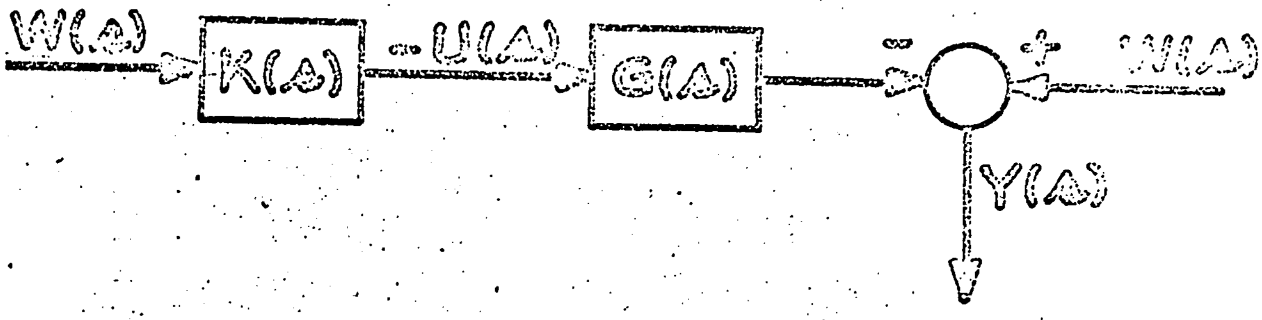

Figure 9. Open Loop Configuration for Wiener Design

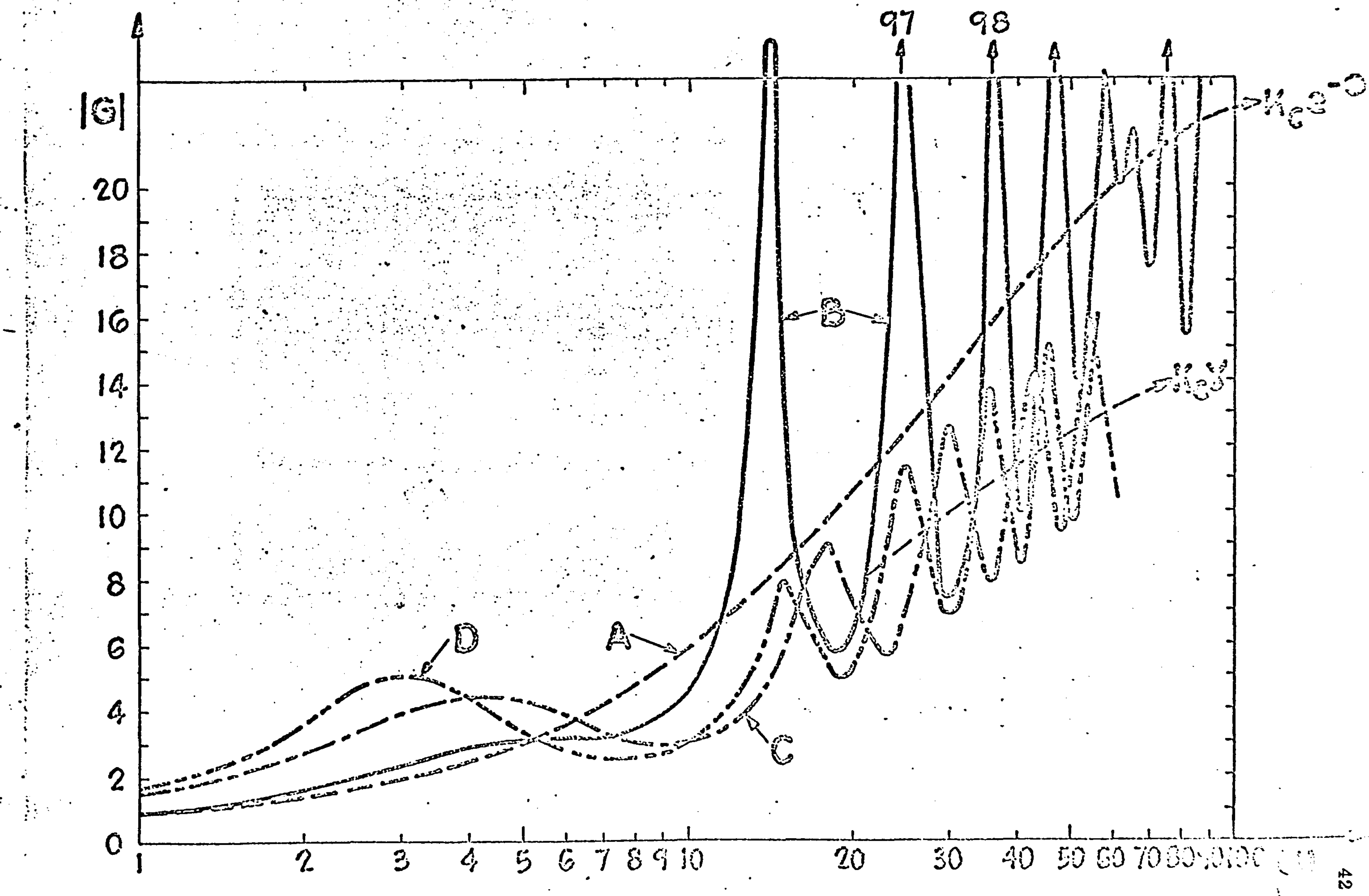

Figure 10. Magnitude of Transfer Function from Output Disturbance to Controller Output

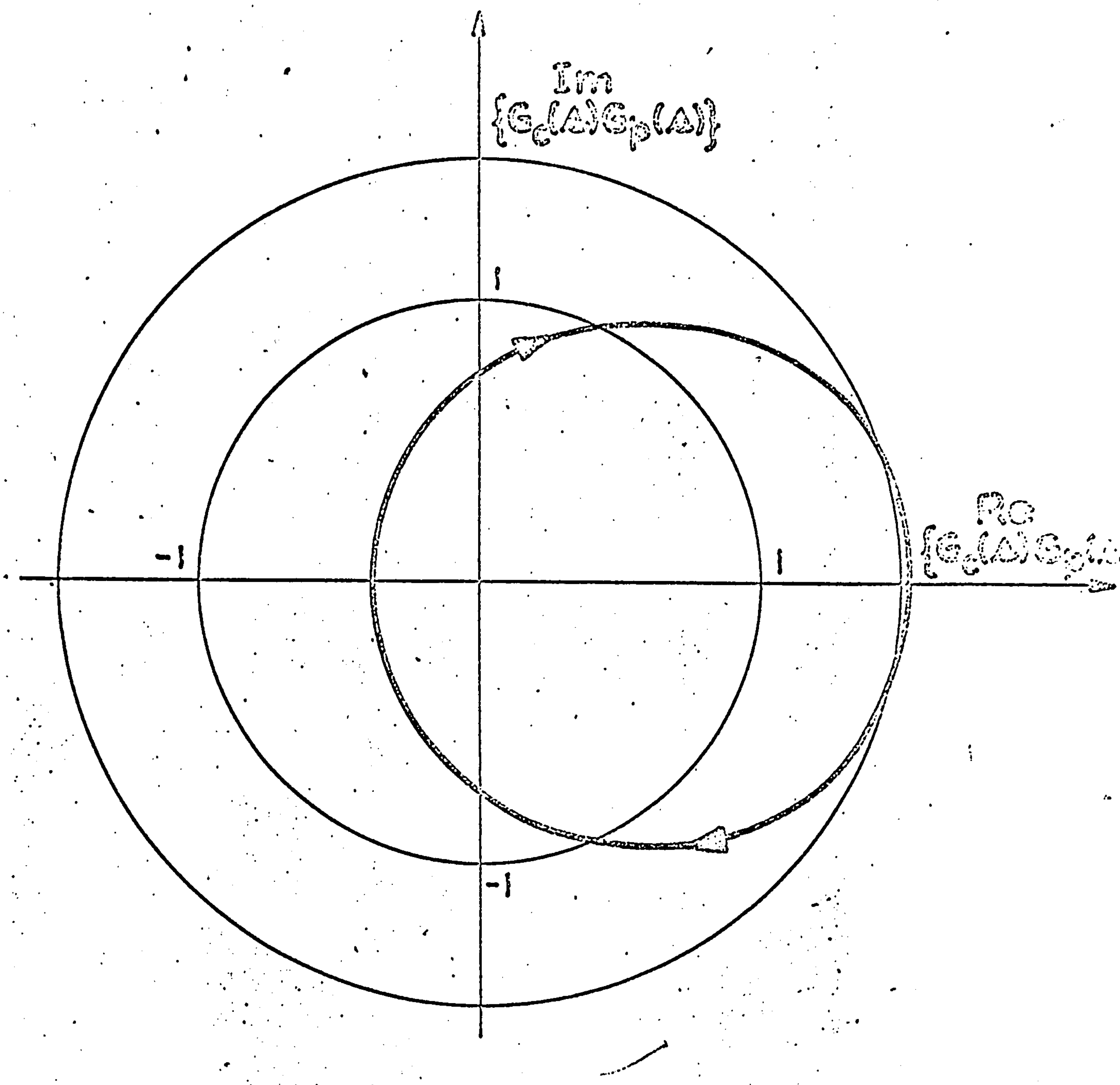

Figure 11. Nyquist Plot of Systems Using Optimal Controller

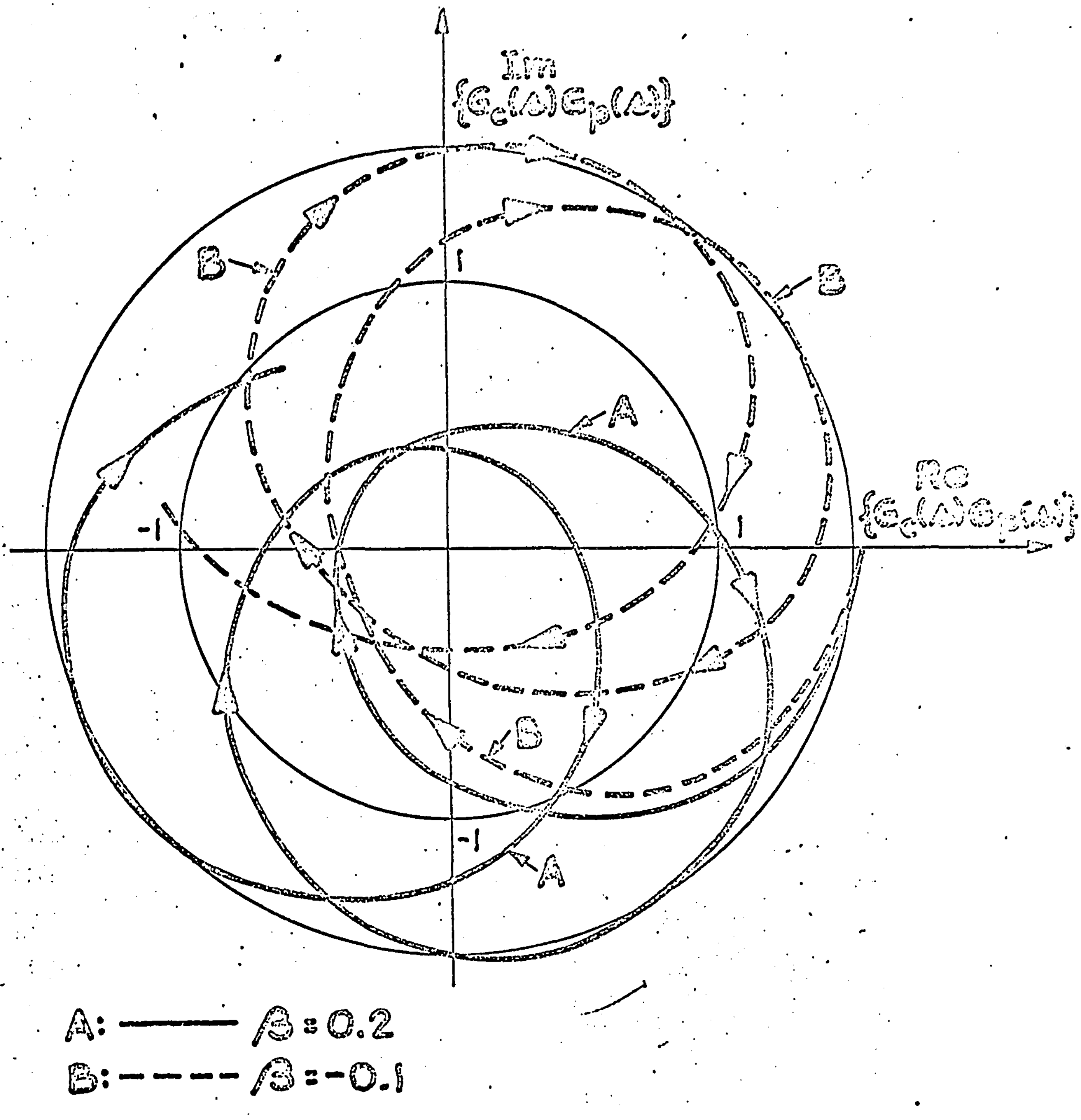

Figure 12. Effect of Parameter Variations on Nyquist Plot of Systems Using Unconstrained Optimal Controller

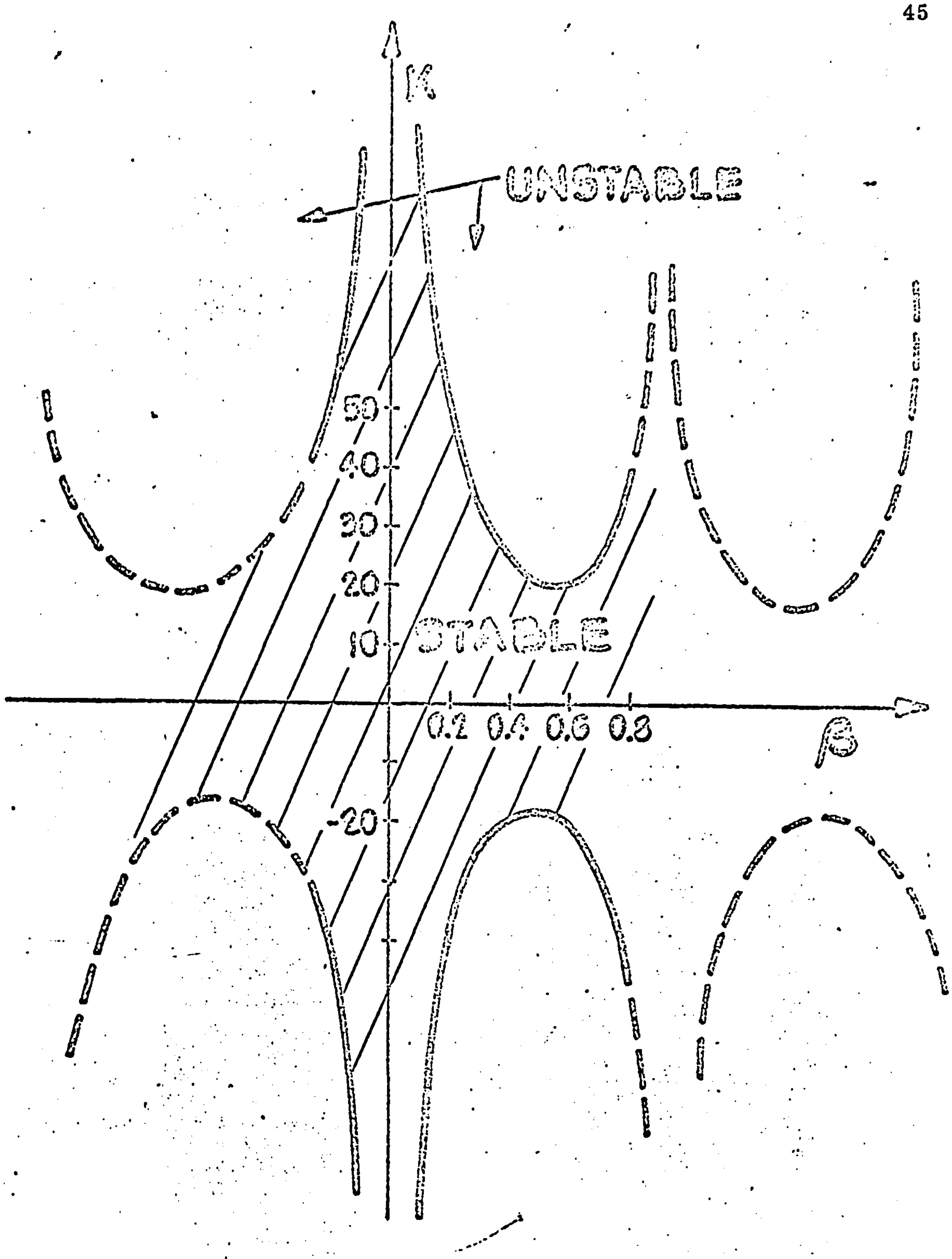

Figure 13. Stability Limits of Optimal Controller as a Function of Estimation Error

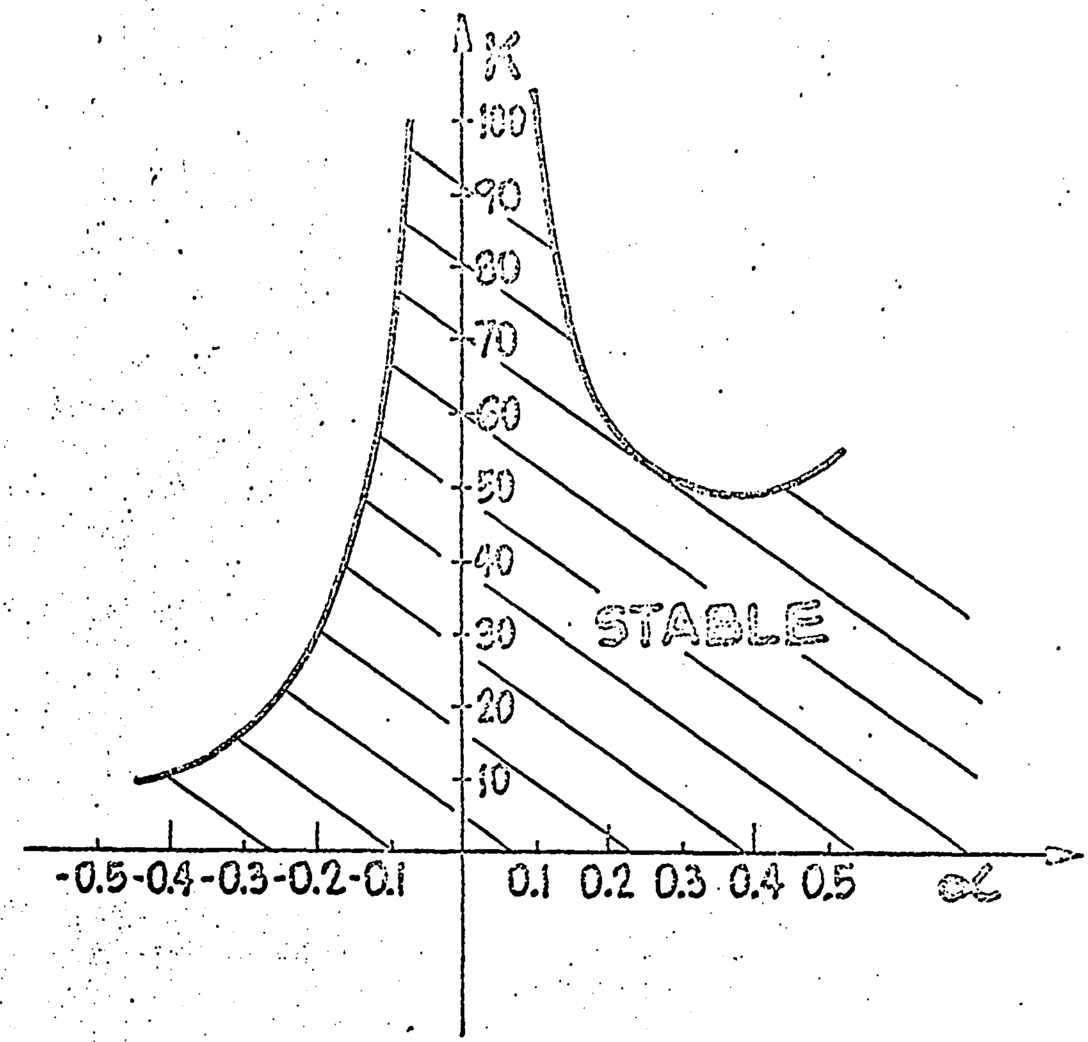

Figure 14. Stability Limits of Optimal Controller as a Function of Load Change

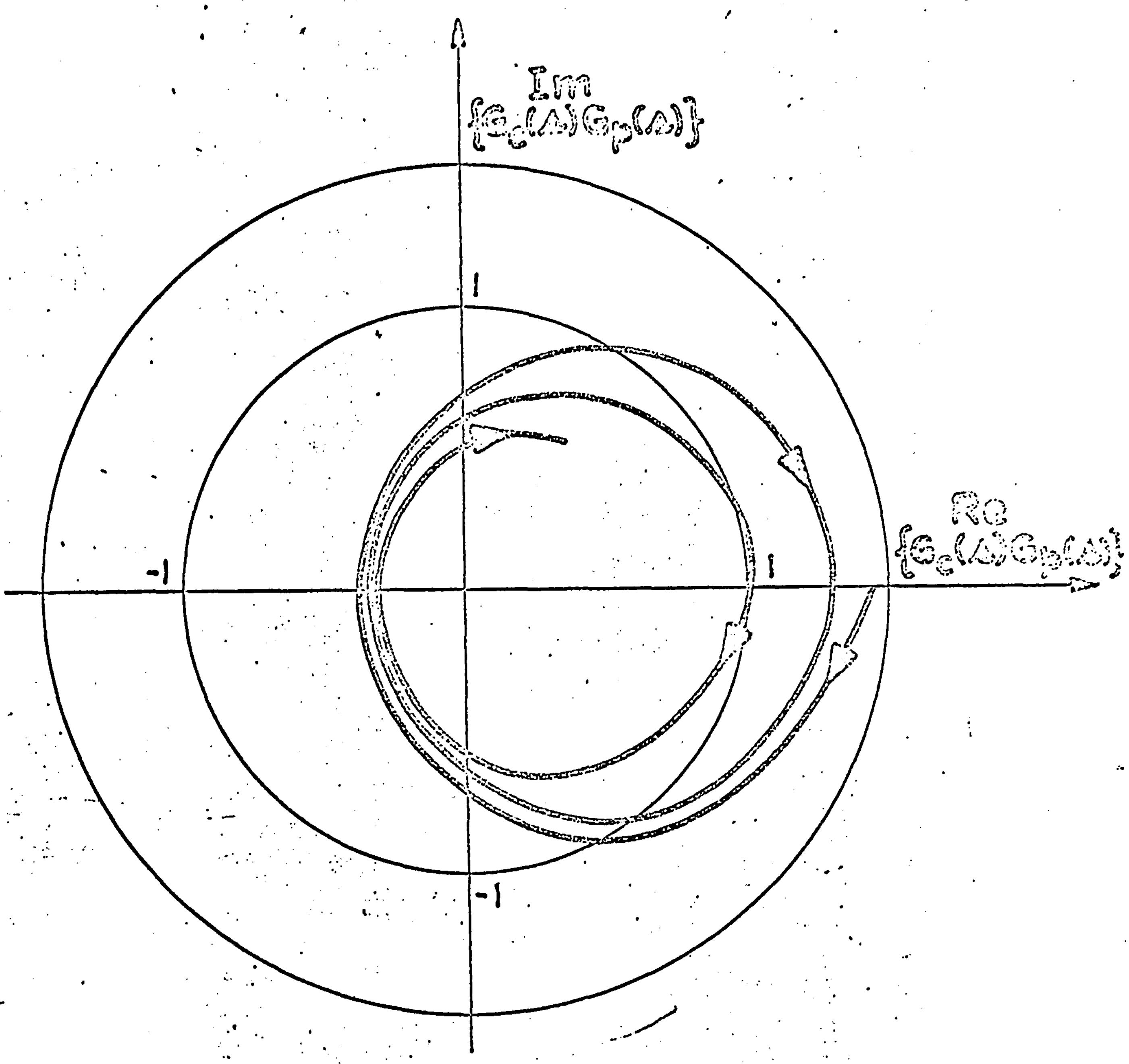

Figure 15. Nyquist Plot of System Using Constrained Optimal Controller

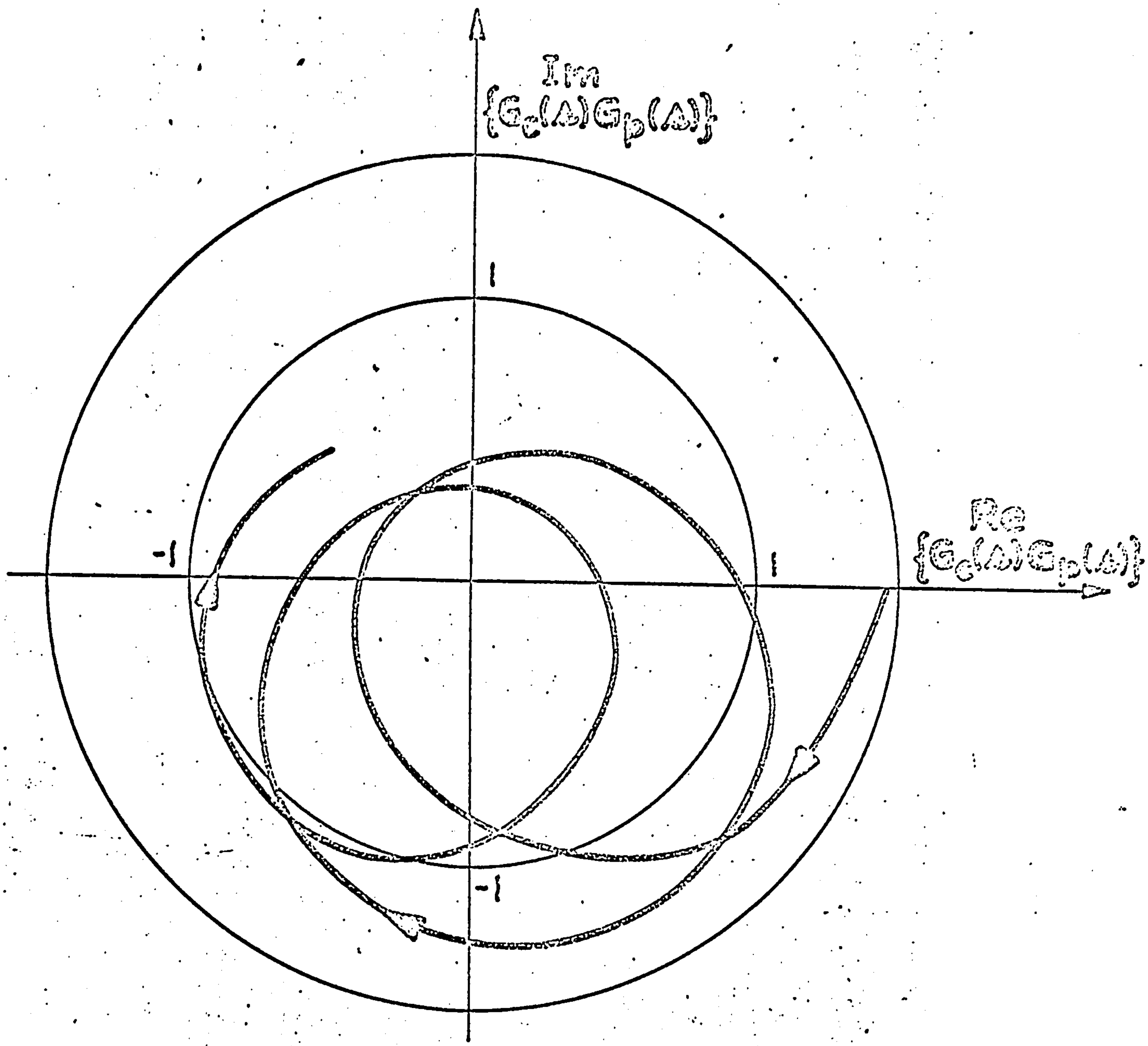

Figure 16. Nyquist Plot of System with Parameter Variation ($\beta = 0.2$), Using Constrained Controller ($K = 40$)

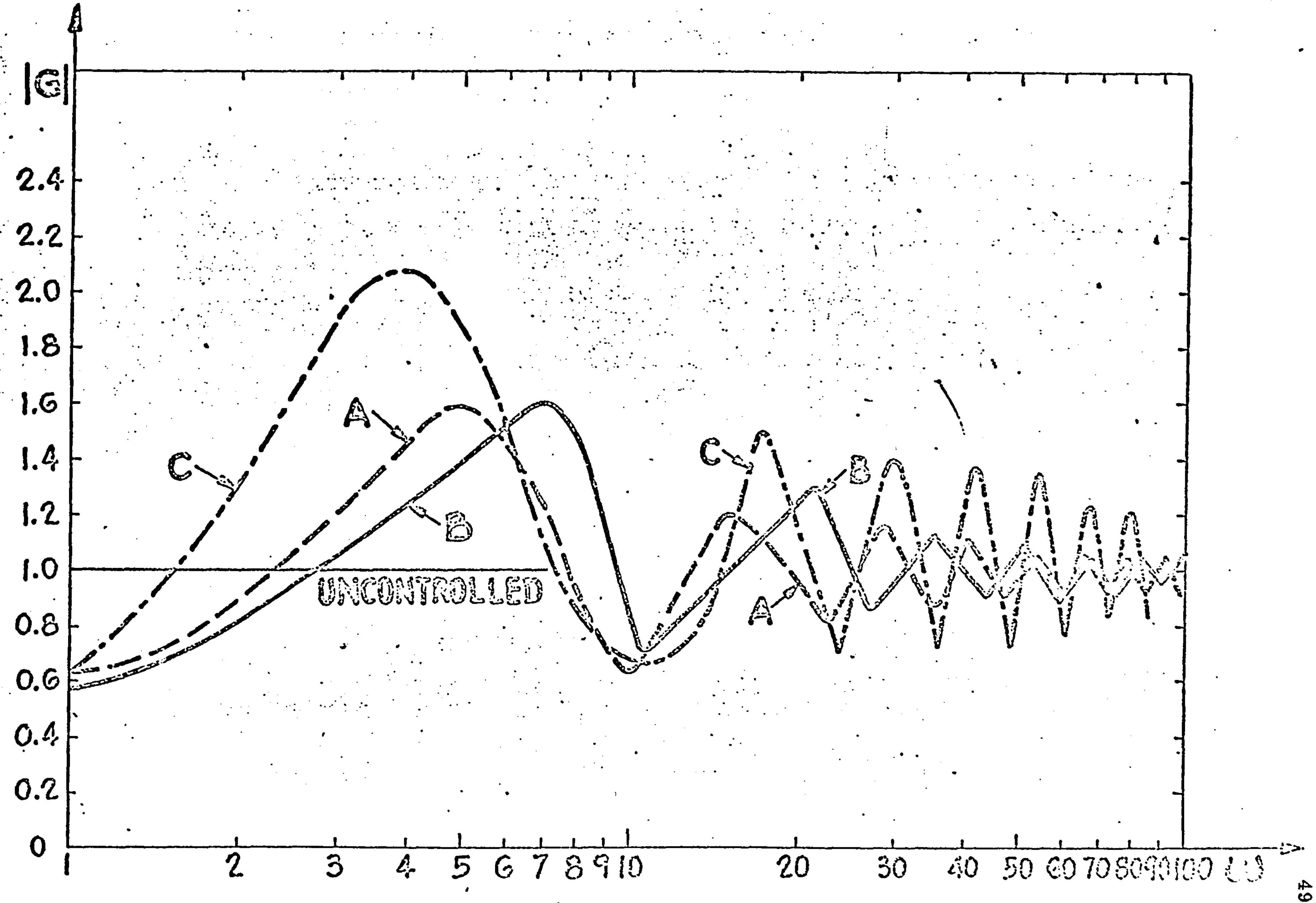

Figure 17. Magnitude of Frequency Response from Output Disturbance to Process Output for Systems Using Optimal Controller (Gain = 6.2)

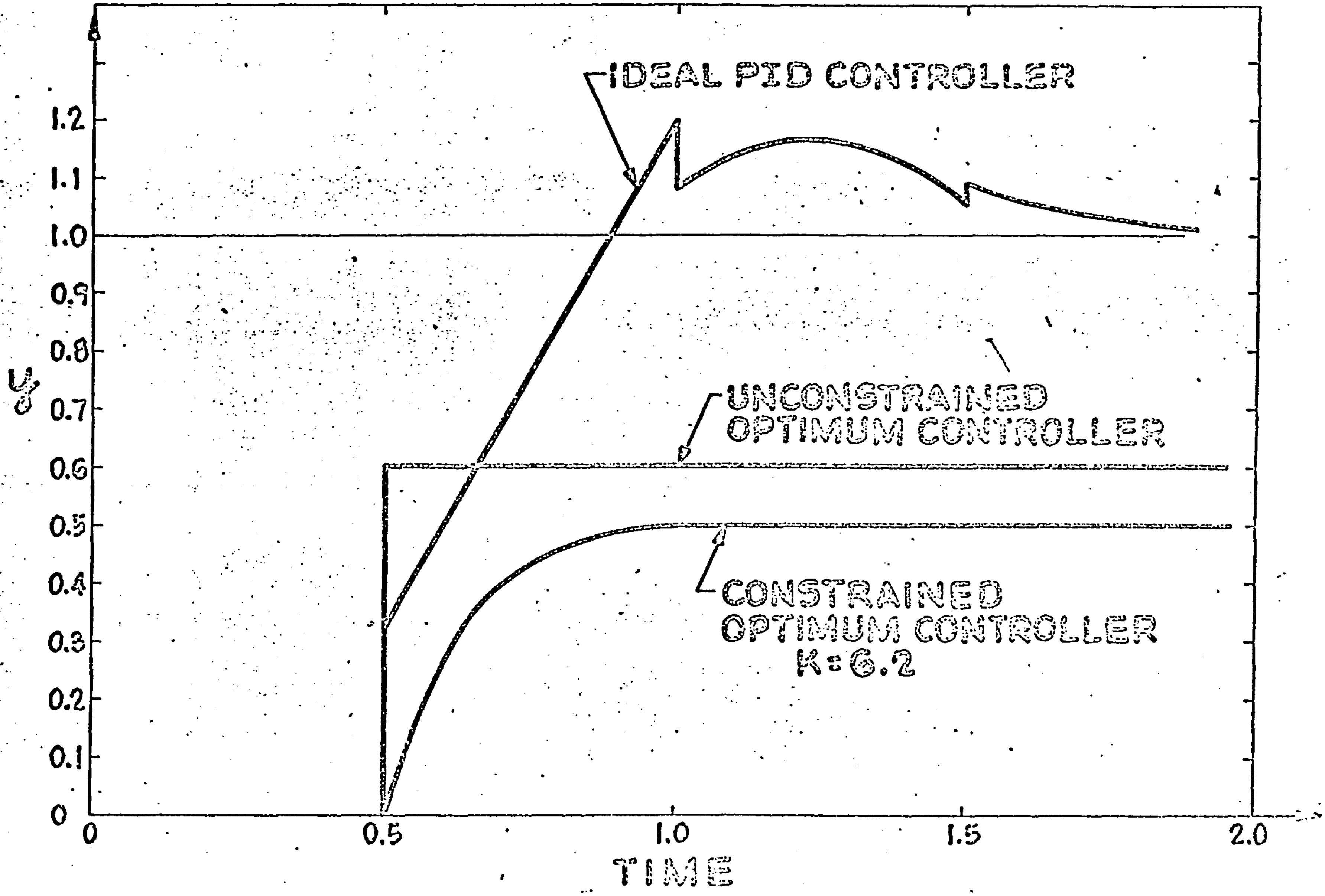

Figure 18. Resonse of Closed Loop System to Stop Change in Set Point

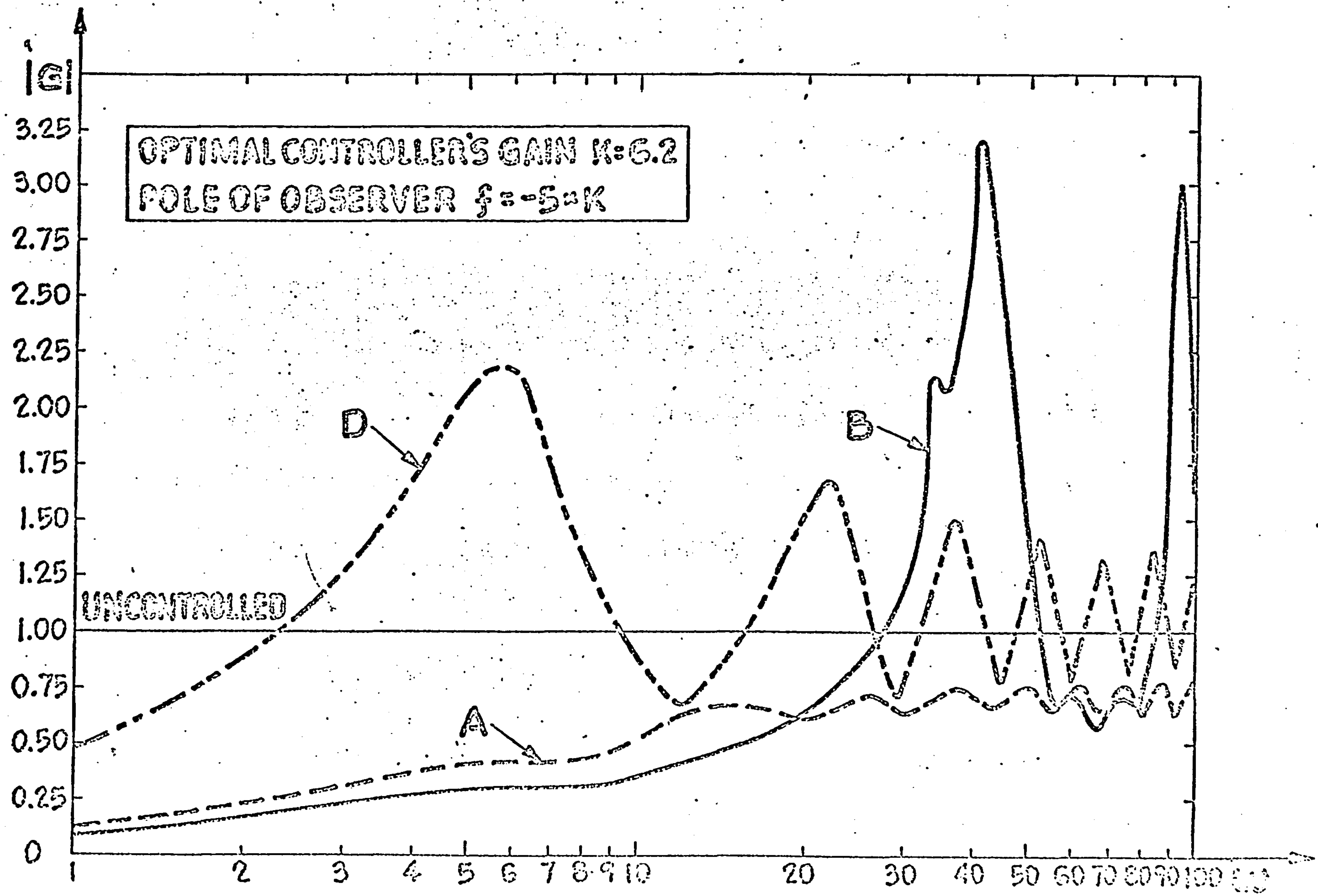

Figure 19. Magnitude of Frequency Response from Output Disturbance to Process Output for System Using Observer Controller

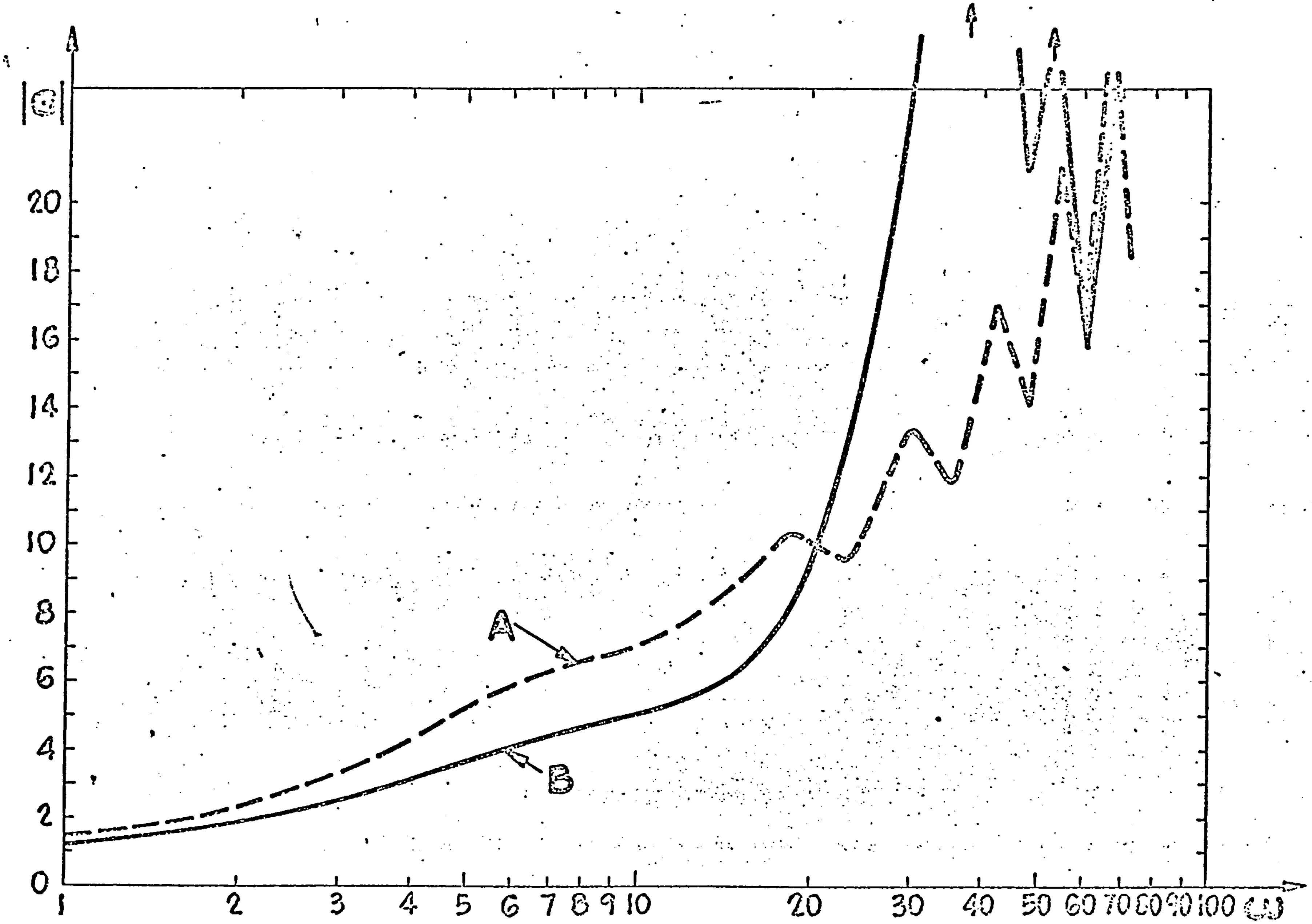

Figure 20. Magnitude of Frequency Response from Output Disturbance to Controller Output for Observer Controller.

SECTION II

THE EFFECT OF MODELING ERRORS ON LINEAR STATE RECONSTRUCTORS AND REGULATORS

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ABSTRACT

The effect of imprecise knowledge of system parameters on the reconstruction error of linear observers and on the stability of a class of linear regulators is examined. An upper bound on the reconstruction error for linear unforced systems is obtained. It is shown that in the regulator problem consideration of parameter uncertainty leads to the inclusion of step disturbances. An upper bound on allowed parameter variations that guarantees stability of a class of closed-loop regulators is obtained. It is also shown that by properly choosing parameters of a low-order observer, the output of this low-order model can be made to follow closely the output of a higher-order dynamic systems. The effect of modeling errors on randomly-peturbed systems is also examined.

1. Introduction

The design of linear controllers for linear plants where only a limited number of state measurements are availalbe has been the concern of a number of research workers in recent years [3]. Techniques using linear state reconstructors have been developed for designing linear feedback control laws that result in asymptotically stable closed-loop systems. Methods have also been developed [4] for controlling disturbed plants, where the disturbances can be modeled as the output of linear dynamic systems.*

These techniques are based on having precise knowledge of the parameters of the plant to be controlled, the measurement sub-system, and the disturbance model. It is the objective of this paper to examine the effect of imprecise knowledge of these parameters on the performance of linear state reconstructors and regulators.

Two types of modeling errors will be studied. First it will be assumed that the dynamic order of the plant to be controlled is known and only parameter uncertainty is of significance. In section 2 an upper bound on the observation error is obtained for linear unforced systems of the form

$$\dot{x} = A x(t) \tag{1}$$

where A is an nxn matrix, with measurements

$$y(t) = C x(t) \tag{2}$$

where C is an mxn matrix, and where there is uncertainty in the parameter of A and C. In section 3 the regulator problem is considered

$$\dot{x} = A x(t) + B u(t) \tag{3}$$

*A number of papers related to the use of observers and linear regulators may be found in the IEEE Transactions of Automatic Control, Vol. AC-16, No. 6, December 1971.

with measurements (2) where B is an nxr matrix and where the designer does not have precise knowledge of the matrices A, B, and C. It is shown that uncertainty in system parameters leads to consideration of step disturbances. An upper bound on some linear combination of the uncertainties in A, B, and C that guarantees the stability of a closed-loop regulator is found.

In section 4 it is assumed that there may be uncertainty in the dynamic order of the process to be controlled. It is shown that if a low order model is used to approximate the actual process then by properly choosing a linear observer the output of the observer can be made to approximate a desired process output response despite uncertainty in the order of the actual process.

Model uncertainties described thus far do not include the consideration of additive noise that might corrupt the measurements (2). The effect of these perturbations is examined in section 5.

2. Observing an Unforced System with Uncertain Parameters.

It has been shown in [1], [2] and [3] that if the system (1), (2), is completely observable, then the nth-order observer of the form

$$\dot{\hat{x}} = F\hat{x} + G\,y(t) \qquad \text{where} \qquad F = A - GC$$

a stable matrix, will yield an observation error, $e(t) = x(t) - \hat{x}(t)$ which will approach zero at a rate determined by the eigen-values of F. It was also shown that those eigen values can be arbitrarily fixed, by selecting G. This result holds for forced and unforced systems alike. Furthermore, the rate at which the observation error is reduced to zero is independent of the behavior of the original state x(t), i. e. x (t) may be decaying, unstable, or of constant magnitude.

Unfortunately, this is not the case when the system's parameters are not known exactly. In this section the effect of two different modes which may arise in an unforced system, namely, unstable, or bounded responses, on the observation error, will be discussed. It will be shown that if the system parameters are not known precisely, then:

1) Under certain circumstances an unstable system is impossible to observe.

2) For a system with zero-real part eigen values, a steady-state error will always exist. However, the magnitude of that error, can, to some degree, be controlled by a suitable choice of the matrix G.

3) For a stable system the error will approach zero, but at a rate which is not faster than the original system. However, the magnitude of the error can, to some degree, be controlled.

Consider the system (1), (2) where it is assumed that the system is described by

$$\dot{x}_c = A_c x_c(t) \qquad (4)$$
$$y_c(t) = C_c x_c(t)$$

where A_c is an nxn matrix and C_c is an mxn matrix. An nth order observer is built to estimate the state $x(t)$,

$$\dot{\hat{x}}_c = F_c \hat{x}_c + G y(t) \qquad (5)$$
$$= A_c \hat{x}_c + G(y - C_c \hat{x}_c)$$

(If C_c is of rank m, one could construct an observer of order (n-m) and in that case an analysis similar to what follow would hold).

Subtraction of (5) from (1) and the addition and subtraction of appropriate terms to the result yields

$$\dot{e} = F_c e + (F - F_c) x(t)$$
$$= F_c e + (\Delta F) x(t) \qquad (6)$$

where $F_c = A_c - G C_c$ and $\Delta F = (A - A_c) - G(C - C_c)$.

The eigen values of the augmented system (1), (6) are the eigen- values of A and F_c. Hence, depending on the structure of ΔF, e(t) might grow without bound if A is unstable.

Now, consider system (1) with a bounded response x(t). Solving for e (t) gives

$$e(t) = \exp\left[F_c(t-t_0)\right] e(t_0) + \int_{t_0}^{t} \exp\left[F_c(t-s)\right] \Delta F\, x(s)\, ds$$

Assume that bounds on x (t), ΔF and $\exp(F_c\,(t-t_0))$ exist, such that*

$$\|\Delta F\| \le \|\Delta F\|_{max}$$
$$\|x(t)\| \le \|X\|_{max} \qquad (8)$$
$$\|\exp[F_c(t-t_0)]\| \le c_1 e^{-c_2(t-t_0)}\,{}^{**}, \quad c_1, c_2 > 0$$

Taking the norm of (7) yields

$$\|e(t)\| \le \|\exp[F_c(t-t_0)]\| \cdot \|e(t_0)\| + \int_{t_0}^{t} \|\exp(F_c(t-s))\|\,\|\Delta F\|\,\|x(s)\|\,ds \qquad (9)$$

Substitution of the bounds (8) gives

$$\|e(t)\| \le c_1 e^{-c_2(t-t_0)}\|e(t_0)\| + \|\Delta F\|_{max}\|X\|_{max}\int_{t_0}^{t} c_1 e^{-c_2(t-s)}\,ds \qquad (10)$$

Integrating and rearranging results in

$$\|e(t)\| \le c_1 e^{-c_2(t-t_0)}\|e(t_0)\| + \frac{c_1}{c_2}\|\Delta F\|_{max}\|X\|_{max}\left(1 - e^{-c_2(t-t_0)}\right) \qquad (11)$$

---

* A short introduction to definitions, properties, and the calculation of bounds on vector and matrix norms may be found in the Appendix to [5].

**A bound of this form is guaranteed if the eigenvalues of F_c all have negative real part [5].

To obtain a small bound on the norm of e (t), two parameters must be adjusted (by choice of G): 1) c_2 should be large to yield fast decay, and 2) c_1/c_2 should be small to yield a small steady-state error bound. When $\Delta F = 0$, c_2 can be chosen to be very large and then the error will decay quickly towards zero. However, when $\Delta F \neq 0$, the elements of F_c have to be chosen with care, to guarantee a low bound on $\|e(t)\|$. Sometimes a tradeoff between conditions (1) and (2) may be necessary because of the steady-state error resulting from the second term on the right hand side of (11) and because c_1 and c_2 may not be independent for a particular matrix F_c. The last remarks will be illustrated by the following example:

Consider the 2nd order completely observable system:

$$\begin{aligned} \dot{x}_c &= \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix} x_c(t) \\ y_c &= [1, 0]\, x_c(t), \quad t_0 = 0 \end{aligned} \tag{12}$$

Let $F_c = A_c - GC_c$ where $G = [g_1, g_2]'$. Then the eigen values of F_c are given by

$$\begin{aligned} a &= -f_1 + \sqrt{f_1^2 - f_2} \\ b &= -f_1 - \sqrt{f_1^2 - f_2} \end{aligned} \tag{13}$$

where

$$\begin{aligned} 2f_1 &= 2 + g_1 > 0 \\ f_2 &= g_2 > 0 \end{aligned}$$

Two cases should be investigated

a) Two different real roots

b) Two equal real roots

The case where the roots are complex ie qualitatively similar to case a, since the oscillation do not effect the rate of decay.

Case a: Two different real roots.

It is seen that a is the slower mode. If the observer is designed such that the error will decay quickly towards zero, f_1 and f_2 (g_1 and g_2) could be chosen such that f_1 will be very large and f_2 could accordingly be chosen to make the discriminant (in (13)) very small. Note that

$$\exp[F_c t] = \frac{1}{a-b}\left[\begin{array}{c|c} ae^{at}-be^{bt} & e^{at}-e^{bt} \\ \hline -f_2\left(e^{at}-e^{bt}\right) & (2f_1+a)e^{at} - (2f_1+b)e^{bt} \end{array}\right] \tag{14}$$

It is easily verified that under the aobve conditions a bound on the elements of exp (F_c t) is f_2 exp (at) /(a-b) and hence $c_1/c_2 = -f_2'$ ((a-b) a).
However under the above conditions

$c_1/c_2 \approx f_1/\sqrt{f_1^2-f_2}$ which is very large. Hence, if c_2 is made large by the above procedure, this might result in an undesirably large steady-state observation error. Hence a trade-off is necessary in this case if both decay of error and steady-state error are to be considered in the observer design.

Case b: Two equal real roots.

$$a = b = -f_1 \,, \quad f_2 = f_1^2$$

Here to obtain fast decay of the observation error, f_1 could be chosen very large.. Note that

$$\exp[F_c t] = \left[\begin{array}{c|c} (1-f_1 t)e^{-f_1 t} & t e^{-f_1 t} \\ \hline -f_2 t e^{-f_1 t} & (1+f_1 t)e^{-f_1 t} \end{array}\right] \tag{15}$$

For the above choice of f_1 and f_2 a bound on the elements of $\exp(F_c t)$ is:

$$(1+f_2 t)e^{-f_1 t} = (1+f_1^2 t)e^{-f_1 t}$$

When a similar analysis to the one in equations (7)-(11) is performed, where the bound on the norm of $\exp(F_c t)$ is now of the form $(c_0 + c_1 t)\exp(-c_2 t)$, it is found that the norm of $e(t)$ satisfies:

$$\|e(t)\| \le \left\{(c_0 + c_1 t)\exp(-c_2 t)\right\} \cdot \|e(t_0)\| + \|\Delta F\|_{max}\|X\|_{max}\left\{\frac{c_0}{c_2}\left(1-e^{-c_2 t}\right)+\frac{c_1}{c_2^2}\left(1-(1+c_2 t)e^{-c_2 t}\right)\right\} \quad (16)$$

In the above example $c_0 = 1$ $c_1 = f_1^2$ $c_2 = f_1$.

Hence $(c_0/c_2) = 1/f_1$, $(c_1/c_2^2) = 1$, and an upper bound on the steady-state error is found to be

$$\|\Delta F\|_{max}\|X\|_{max}\left(\frac{1}{f_1} + 1\right)$$

The larger f_1 the smaller the steady-state bound. In this example it is seen that the elements of F_c should be chosen such that F_c will have equal eigen values, since only then will it be possible to satisfy both conditions of fast decay of the error and small steady-state error bound. In this latter case no trade-off between the two conditions is necessary. The above example is included to illustrate problems involving the choice of observer gain parameters. The problem of finding a "best" choice of observer gain for general higher-order systems is currently unsolved. Further issues to be considered in designing observers and observer-based regulators are examined in the following sections.

3. Regulating a Process with Uncertain Parameters

In this section it will be assumed that, based on approximate knowledge of plant parameters a linear control law is designed to stabilize the system (3) and to optimize some performance criterion. The effect of imprecise knowledge of the system's parameters will be examined.

3.1 Inclusion of Step Disturbances

Before we begin the main argument, it will be shown that imprecise knowledge of system parameters leads to consideration of "step disturbances", which must be observed, and counter-acted by the control action. The linear time-invariant system which is to regulated is governed by (3), (2). Let the desired steady-state be $x_d = d$ then:

$$\dot{d} = 0 = Ad + Bu_d \tag{17}$$

where d is a constant vector and u_d is the steady-state control action, that would be calculated from (16) if A and B were known. It is assumed that a solution for u_d does exist. A more detailed discussion is given in [4].

The control u(t) is then given by

$$u(t) = u_d + u_1(t)$$

where $u_1(t)$ depends on the performance criteria. Since the system parameters are not known accurately the following model for the process to be controlled is used:

$$\begin{aligned} \dot{x}_c &= A_c x_c(t) + B_c u_c(t) \\ y_c(t) &= C_c x_c(t) \end{aligned} \tag{18}$$

where A_c is an nxn matrix, B_c is an nxr matrix, and C_c an mxn matrix.

The steady-state control action that will be applied to the actual process (3) will result from the solution of

$$0 = A_c d + B_c u_{cd} \tag{19}$$

Substitution of $u(t) = u_{cd} + u_1(t)$ into (3), gives:

$$\dot{x}(t) = Ax(t) + Bu_{cd} + Bu_1(t) \tag{20}$$

Define $e(t) = x(t) - d$ and subtract (16) from (20) to give

$$\dot{e} = Ae + B(u_{cd} - u_d) + Bu_1(t) \tag{21}$$

where $u_{cd} - u_d$ is an unknown constant which plays the role of a step disturbance. The goal now is to find a control law $u_1(t)$ that will drive $e(t)$ to zero and will optimize some performance criterion.

3.2 The Effect of Parameter Uncertainties

Following the result of section 3.1 consider the following system:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{w} \end{bmatrix} &= \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) \\ y(t) &= [C, 0] \begin{bmatrix} x \\ w \end{bmatrix} \end{aligned} \tag{22}$$

where $x(t)$ is to be regulated at zero. Note, that (22) is not completely controllable. However, the system (22) must be completely observable (the pair $\left([C,0], \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}\right)$ must be observable.)

Because of imprecise knowledge of A, B, and C, the designer assumes that the system is described by

$$\begin{aligned} \dot{x}_c &= A_c x_c + B_c w + B_c u_c \\ \dot{w} &= 0 \\ y(t) &= C_c x_c \end{aligned} \tag{23}$$

Suppose the linear control law is given by

$$u_c(t) = -L_c \hat{x}(t) - \hat{w}(t) \tag{24}$$

where the 2nd term is due to the counteraction to the disturbance $w(t)$ [4]. $\hat{x}(t)$ and $\hat{w}(t)$ are estimates of x(t) and w(t), and are the state variables of an $(n+r)$-th order observer for the augmented system:

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{w}} \end{bmatrix} = F_c \begin{bmatrix} \hat{x} \\ \hat{w} \end{bmatrix} + G\, y(t) + B_c u_c \tag{25}$$

where

$$F_c = \begin{bmatrix} A_c & B_c \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} G_x \\ G_w \end{bmatrix} [C_c, 0]$$

and

$$G = \begin{bmatrix} G_x \\ G_w \end{bmatrix}$$

G is chosen as to make the eigen values of F_c large and negative (As indicated in section 2, if a lower order observer is used, an analysis similar to what follows would hold).

Define an observation error

$$e(t) = \begin{bmatrix} e_x(t) \\ e_w(t) \end{bmatrix} = \begin{bmatrix} x(t) - \hat{x}(t) \\ w(t) - \hat{w}(t) \end{bmatrix} \tag{26}$$

and substitute

$$u_c(t) = -L_c \left(x(t) - e_x(t) \right) - \left(w(t) - e_w(t) \right) \tag{27}$$

into (22) and (25) to give the closed-loop system:

$$\begin{bmatrix} \dot{x} \\ \dot{w} \\ \dot{e}_x \\ \dot{e}_w \end{bmatrix} = \left[\begin{array}{c|c|c|c} A - BL_c & 0 & BL_c & B \\ \hline 0 & 0 & 0 & 0 \\ \hline A - A_c - G_x(C - C_c) - (B - B_c)L_c & 0 & A_c - G_x C_c + (B - B_c)L_c & B \\ \hline -G_w(C - C_c) & 0 & -G_w C_c & 0 \end{array}\right] \begin{bmatrix} x(t) \\ w(t) \\ e_x(t) \\ e_w(t) \end{bmatrix} \tag{28}$$

Rearranging and addition and subtraction, of the same terms, to the right hand side of (28) yields

$$\begin{bmatrix} \dot{x} \\ \dot{e}_x \\ \dot{e}_w \end{bmatrix} = \left[\begin{array}{c|c|c} A_c - B_c L_c & B_c L_c & B_c \\ 0 & A_c - G_x C_c & B_c \\ 0 & -G_w C_c & 0 \end{array}\right] \begin{bmatrix} x(t) \\ e_x(t) \\ e_w(t) \end{bmatrix} \tag{29}$$

$$+ \left[\begin{array}{c|c|c} A - A_c - (B - B_c)L_c & (B - B_c)L_c & (B - B_c) \\ A - A_c - \{G_x(C - C_c) + (B - B_c)L_c\} & (B - B_c)L_c & (B - B_c) \\ -G_w(C - C_c) & 0 & 0 \end{array}\right] \begin{bmatrix} x(t) \\ e_x(t) \\ e_w(t) \end{bmatrix}$$

where the decoupled equation for w(t) was omitted.

Note, that when there is precise knowledge of the system's parameters, $e_w(t)$ is identically zero (since $w(t) \equiv 0$), the observation error $e_x(t)$ is decoupled from x(t), and the second line of (28) reduces to

$$\dot{e}_x = F_c\, e_x(t) = F e_x(t) \tag{30}$$

as given by Luenberger in [2].

Equation (29) can be written in a short form as

$$\dot{q} = A_0 q(t) + A_1 q(t) \tag{31}$$

where A_0 is a known matrix and A_1 is unknown and depends on some linear combination of the uncertainty in the process parameters. The relation between the bound on A_1 and the stability of the designed closed loop system is to be investigated next.

A_0 is a stable matrix, since its eigen-values are those of the designed (n+r) the order observer, and of the designed closed-loop control system. Since both were designed to have stable roots, so will A_0. Hence a Lyapunov function

$$V(q) = q' P q \tag{32}$$

for $\dot{q} = A_0 q(t)$ exists where P is a positive-definite, symmetric matrix resulting from the solution of

$$A_0' P + P A_0 = -Q \tag{33}$$

and (

$$A_0' P + P A_0 = -Q \tag{33}$$

of (31

and Q is a positive-definite matrix. Evaluating $\dot{V}(q)$ along trajectories of (31) results in

$$\dot{V}(q) = -q' Q q + 2 q' P A_1 q \tag{34}$$

Using Rayleigh's principle, $q'Qq \geq \lambda_{min}(Q)\, q'q$
where $\lambda_{min}(Q)$ denotes the smallest eigen-value of Q and taking the norm of (34), [5] , results in the inequality:

$$\dot{V}(q) \leq -\lambda_{min}(Q)\, q'q + 2\,\|q'P\| \cdot \|A_1 q\| \tag{35}$$

or

$$\dot{V}(q) \leq -\lambda_{min}(Q)\, q'q + 2\,\|P\| \cdot \|A_1\|\, q'q \tag{36}$$

If the following inequality holds, $\dot{V}(q)$ will be negative definite

$$\|P\| \cdot \|A_1\| < \lambda_{min}(Q)/2 \tag{37}$$

or

$$\|A_1\| < \lambda_{min}(Q)/2\,\|P\| \tag{38}$$

Since, [5] , $P = \int_0^\infty \exp(A_0' s)\, Q \exp(A_0 s)\, ds$ (39)

If $\|\exp(A_0 s)\|$ is bounded such that

$$\|\exp(A_0 s)\| \leq c_1 e^{-c_2 s} \tag{40}$$

then

$$\|P\| \leq \{c_1^2/2c_2\} \cdot \|Q\| \tag{41}$$

Note that A_0, and hence c_1 and c_2 are under the control of the designer. Substitution of (41) in (38) yields the bound on the norm of A_1, for the system (31) to be stable,

$$\|A_1\| < \{(c_2/c_1^2)/\|Q\|\} \cdot \lambda_{min}(Q) \tag{42}$$

where c_2/c_1^2 can be adjusted by picking A_0. Note that c_2 characterizes, in some way, the slowest mode of A_0. The larger c_2, the more stable is A_0, and hence, larger deviations A_1, can be tolerated.

As in most applications of Lyapunov's second method the bound obtained depends on the choice of the positive definite matrix Q. Once Q is selected (33) will yield a unique positive definite matrix P [5]. Finding that particular Q that results in the sharpest bound must be examined in each application.

The stability of (31) can also be examined directly as follows: write the solution to (31) as

$$z(t) = e^{A_0 t} z(0) + \int_0^t e^{A_0(t-\lambda)} A_1 z(\lambda) d\lambda \tag{43}$$

Then using (40) yields

$$\|z(t)\| \leq c_1 e^{-c_2 t} \|z(0)\| + \int_0^t c_1 e^{-c_2(t-\lambda)} \|A_1\| \|z(\lambda)\| d\lambda \tag{44}$$

Rearranging this inequality gives

$$e^{c_2 t} \|z(t)\| \leq c_1 \|z(0)\| + \int_0^t c_1 e^{c_2 \lambda} \|A_1\| \|z(\lambda)\| d\lambda \tag{45}$$

Applying the Gronwall-Bellman inequality [5] yields

$$\|z(t)\| \leq c_1 \|z(0)\| e^{-[c_2 - c_1 \|A_1\|]t} \tag{46}$$

Hence, if the deviation matrix A_1 is bounded such that

$$\frac{c_2}{c_1} > \|A_1\| \tag{47}$$

then the closed-loop regulator will still be asymptotically stable.

4. Modeling a Process Despite Uncertainty in Its Dynamic Order

In this section the problem of modeling a process of the form (1), (2), is considered, where the designer does not know n, the dynamic order of the process. A design objective is to determine a kth- order observer, ($k \geq m$) of the form

$$\dot{z} = F_c z(t) + G y(t)$$
$$y_c(t) = C_c z(t) \qquad (48)$$

such that the output of the observer $y_c(t)$ will be close in some sense to the output of y(t). Hence (48) will be (in this sense) the best kth-order model of the process whose dynamic order is unknown. Conditions on F, G, and C_c will be found for the case where y and y_c are scalars.

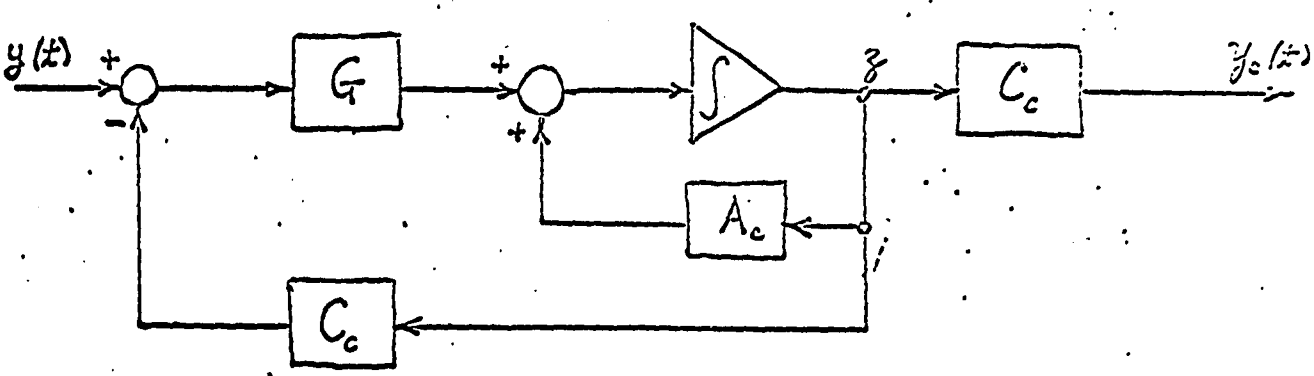

Figure 1. Structure of Low-Order Model

It will be assumed that

$$F_c = A_c - G\,C_c$$

where A_c is some kxk stable matrix in the canonical form and,

$$A_c = \begin{bmatrix} -a_{k-1} & 1 & 0 & \cdots & 0 \\ -a_{k-2} & 0 & 1 & \cdots & 0 \\ \vdots & & & & \vdots \\ & & & & 1 \\ -a_o & 0 & & \cdots & 0 \end{bmatrix}, \quad G = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_k \end{bmatrix}, \quad C_c = [1, 0, \ldots, 0] \qquad (49)$$

The kth-order observer has the structure shown in Figure 1. The transfer function of the model is

$$H(s) = Y_c(s)/Y(s) = C_c\,(sI - F_c)^{-1} G \qquad (50)$$

Let the cofactors of the ijth element in $(sI-F_c)$ be denoted by $|sI-F_c|_{ij}$

Then

$$H(s) = \frac{g_1|sI-F_c|_{11} + \cdots + g_k|sI-F_c|_{k1}}{(s+g_1+a_{k-1})|sI-F_c|_{11} + (g_2+a_{k-2})|sI-F_c|_{21} + \cdots + (g_k+a_o)|sI-F_c|_{k1}} \qquad (51)$$

From (51) it is seen that if

$$g_i >> a_{k-i}\ , \quad i = 1, \ldots, k \qquad (52)$$

$$g_1/g_j >> 1\ , \quad j = 2, \ldots, k$$

then

$$H(s) \approx \frac{g_1/s}{1+(g_1/s)}$$

and for large g_1, $H(s) \approx 1$. Hence, by choosing the parameters of the low-order model (48), one can insure that $y_c(t)$ will follow closely $y(t)$. Construction of a model of this kind might be useful in designing a control law for the original process based on controlling the output of the low-order model.

5. The Effect of Modeling Uncertainties for Perturbed Systems

The analysis above does not include the effect of process disturbances or measurement noise. Suppose the process of section 3.2 is perturbed by the additive noise $n_1(t)$,

$$\begin{aligned} \dot{x} &= Ax + Bw + Bu(t) + n_1(t) \\ \dot{w} &= 0 \end{aligned} \tag{53}$$

and that the measurements available to the observer are perturbed by the additive noise $n_2(t)$,

$$y = Cx + n_2(t) \tag{54}$$

In this case (31) becomes

$$\dot{g} = A_0 g(t) + A_1 g(t) + n(t) \tag{55}$$

where

$$n(t) = \begin{bmatrix} n_1(t) \\ n_1(t) - G_x n_2(t) \\ -G_w n_2(t) \end{bmatrix} \tag{56}$$

Bounds on $\|g(t)\|$ can be calculated assuming that the disturbances $n_1(t)$ and $n_2(t)$ are bounded.

Note that if $n_2(t) \equiv 0$ and if the system parameters are known precisely, then $e_w \equiv 0$, $G_w = 0$, and (55) reduces to

$$\dot{z} = \tilde{A}_0 z + \tilde{n}$$

where

$$\tilde{A}_0 = \left[\begin{array}{c|c} A_c - B_c L_c & B_c L_c \\ \hline 0 & A_c - G_x C_c \end{array}\right], \quad \tilde{n} = \begin{bmatrix} n_1(t) \\ n_1(t) \end{bmatrix}$$

One can now examine the effect of neglecting the additive noise $n_1(t)$ in the regulator design. If $n_1(t)$ is a zero-mean white noise process with

$$E\{n_1(t)\, n_1'(\tau)\} = N\, \delta(t-\tau)$$

where the elements of N are not known precisely, then

$$E\{\tilde{n}(t)\, \tilde{n}'(\tau)\} = \left[\begin{array}{c|c} N & N \\ \hline N & N \end{array}\right] \delta(t-\tau) = \tilde{N}\, \delta(t-\tau)$$

If P(t) is defined as

$$P(t) = E\{z(t)\, z'(t)\}$$

then

$$\dot{P} = \tilde{A}_0 P + \tilde{A}_0' P + \tilde{N}$$

Hence

$$P(t) = \Phi(t,t_0)\, P(t_0)\, \Phi'(t,t_0) + \int_{t_0}^{t} \Phi(t,\tau)\, \tilde{N}\, \Phi'(t,\tau)\, d\tau$$

where $\Phi(t,t_0)$ is the fundamental matrix for $\tilde{A}_0$. If

$$\|\Phi(t,t_0)\| \le c_1 e^{-c_2 t} \qquad c_1, c_2 > 0$$

then

$$\lim_{t\to\infty} \|P(t)\| \le \frac{c_1^2}{2c_2} \|\tilde{N}\|$$

It is seen that in the steady-state the norm of the covariance matrix has an upper bound that depends on the fundamental matrix determined by the observer and controller designs. Here, as in section 2 above, a trade-off between rapid transient response and small steady-state error may be necessary if a regulator with the above structure is to be designed to take into account random process disturbances.

Note also that if $n_1(t) \equiv 0$ then (55) reduces to

$$\dot{z} = A_0 z + A_1 z + \begin{bmatrix} 0 \\ -G_x n_2 \\ -G_w n_2 \end{bmatrix} \qquad (57)$$

Hence "large" gains G_x and G_w might enhance the effect of measurement noise. If it is known that $n_2(t)$ is bounded, one could attempt to pick the "best" L_c, G_x and G_w by minimizing a scalar performance criterion that would take into account the need for asymptotic stability of (57) with $n_2(t) = 0$, low sensitivity to parameter variations A_1, and suppression of measurement noise. This is a problem currently receiving attention.

6. Conclusion

In many realistic control problems the dynamic order and parameter values of the plant to be regulated are not known precisely. The use of observers and linear regulators which are based on precise knowledge of the system's order and parameters, was examined. Conditions on the allowed "amount of uncertainty" for unforced and forced systems were found, under which an observer and an overall closed-loop regulator will still be stable despite possible parameter variations. Application of these results to the control of imprecisely defined chemical processes is a subject of current research.

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VITA

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