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# INSTRUCTIONAL ALGORITHMS DERIVED FROM MATHEMATICAL LEARNING MODELS :

## AN APPLICATION IN COMPUTER ASSISTED INSTRUCTION OF PAIRED-ASSOCIATED ITEMS

by

ALICE CHIANG

A dissertation submitted to the Graduate Faculty in Education in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

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Abstract

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Adviser: Professor Carl Helm

The design of an instructional system for optimal item selection was investigated using a decision theoretic approach. Three different adaptive strategies were derived based on the General Forgetting Theory model for paired-associate learning. All three strategies analyze student response histories to select items for learning, but differ in their assumptions and their complexities. The homogeneous parameter (HOP) strategy assumes equal item difficulty and subject ability in making its selection. The heterogeneous parameter (HEP) strategy considers individual differences among items and subjects. The latency heterogeneous parameter (LHEP) strategy augments the HEP strategy by incorporating state and subject dependent latency information.

Two experiments were carried out to evaluate and to compare the efficacy of these instructional algorithms. The content of the experiments involved the learning of Chinese vocabulary using a computer controlled system. The results of the first experiment showed that learning was improved significantly by the use of these adaptive strategies, when compared with a standard strategy which presents items an equal number of times in a random cyclical way. The results of the second experiment suggested that the three strategies were equally effective and that the computationally simpler HOP strategy would be optimum for the instructional situations of the types considered.

The problem of practical application of these strategies was investigated. The implementation of the instructional strategies derived from mathematical models involve complex algorithms for 1) determining the current state of knowledge of the student, 2) estimating the parameters of the learning model and 3) calculating the gains of possible instructional actions. For use in practical situations, several simpler variable based procedures that approximate the selection behavior of the parameter dependent adaptive strategies were formulated. Extensions of adaptive instructional algorithms are suggested for future investigation.

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# CHAPTER I

## A DECISION THEORETIC APPROACH TO THE PROBLEM OF OPTIMAL ITEM SELECTION FOR COMPUTER INSTRUCTIONAL SYSTEMS

The problem being investigated in this study is a common one which confronts teachers regularly. Consider the situation where a student is to learn a large number of items in a limited amount of time. How should the items be selected so that each student learns as much as possible in that time period, and can an adequate formalization of such selection strategies be developed? Teachers usually arrive at the solution to this item selection problem heuristically based on their accumulated experiences with the students and the subject matter. This approach, depending heavily on teacher intuition and on trial and error, is inefficient and too often does not guarantee the desired outcomes. If one believes that there are elements common to effective instructional decision making, then a systematic study of selection strategies is needed and worthwhile.

The formalization of instructional strategies is also motivated by the recent utilization of the computer as a teaching device. The past decade of experimentation with computer-assisted instruction (CAI) has emphasized the design of hardware and software for man-machine communications and the development of curriculum materials appropriate for the new technology. Currently, many CAI systems are designed to retrieve and present subject matter to the student according to a pre-specified curriculum. The promise of the computer as a teaching machine lies in its potential to process information about students and subject matter material such that it can adapt instruction to individual students and maximize the learning outcomes. The purpose of this study is to derive and to investigate instructional strategies for item selection which optimize and individualize instruction.

A formal solution to the problem of optimal item selection is offered by the tools of decision theory. The problems of optimal instructional strategies can be viewed as a subset of a set of optimization problems that have been studied in such diverse areas as physics, electrical engineering, economics and operations research within the decision theoretic analysis framework. By this approach, instruction is viewed as a multi-stage decion process. At each stage, instructional decisions are made according to decision algorithms that maximize the desired learning outcomes. A variety of optimization problems in instrution can be considered, such as optimal block size, optimal order of presentation, optimal study time.

The derivation of an optimal algorithm requires that the instructional problem be stated in a form amenable to decision theoretic analysis. Atkinson (1972) has identified four elements that need to be specified in the instructional situation for the derivation of an optimal instructional strategy (see Figure 1). The first requirement is a model of the learning process which describes the possible states of nature of the learner and the transformation of the states resulting from instructional actions. Secondly, there need to be specifications of instructional objectives. Next, a measure of the costs and payoffs of the achievement of instructional objectives must be assigned to possible instructional actions. Lastly, admissible instructional actions need to be defined.

Figure 1

Elements that need to be specified for the derivation of optimal instructional algorithms using a decision theoretic analysis approach.

1. Model of the learning process in terms of states of learning and transformation of states.

2. Instructional goals.

3. Measures of costs or payoffs of instructional actions for instructional goals.

4. Instructional actions.

When a description of the instructional situation is specified in terms of these four components, it is possible to derive optimal or near optimal instructional algorithms according to the different assumptions made in the learning model. To evaluate the efficacy of these strategies, experimental studies need to be designed and conducted comparing the learning outcomes due to the implementation ot these strategies.

The present study investigates the problem of optimal item selection from a decision theoretic analysis framework. The instructional situation is one in which a student is to learn a set of N items during S sessions, where $n < N$ items are presented for studying at each session. The items are assumed to be heterogeneous and independent in the sense that learning items i does not effect learning item j for all $i \neq j$. This independence assumption is realistic in many list learning and vocabulary learning situations. The goal is to maximize the student's performance on the achievement test given after all S sessions have been administered. The instructional task is to specify a strategy that selects n different items out of a set of N items to be learned (i.e. $n_i \neq n_j$; $i,j = 1,\ldots n$; $n < N$) for every session according to the subject's past responses on all items such that the number of items learned in S sessions is maximized. A mathematical model of paired-associate learning is used to represent the learning process. From this model, measures of instructional gains are specified. Chapter III discusses in detail this model and the measures of payoffs of the instructional actions.

In the present study, three instructional algorithms are derived from the mathematical model used. These stragtegies are experimentally tested for the computer-assisted instruction of Chinese vocabulary. This investigation is carried through two experimental cycles, where modificafications are made in the second experiment based on the results of the first experiment. By following a systematic cyclical procedure of theory formation and experimental validation, we can develope more effective instructional models for optimizing instruction.

It is important to distinguish between a theory of learning and a theory of instruction. A theory of learning constitutes a description of learning processes, whereas a theory of instruction prescribes the pedagogical actions which can effectively produce the desired learning outcomes, i.e. the aquisition of new information (Bruner, Hilgard ed.,1964). Furthermore, a theory of learning is confirmed or rejected to the degree that it can predict data on learning; a theory of instruction is evaluated according to its usefulness in improving the learning outcomes of instruction. Building a theory of instruction then is an educational engineering process whereas developing a theory of learning is a process of unravelling the nature of human cognitive processes. Although an understanding of learning leads to the identification of more effective instructional strategies, a detailed and complete description of the ways information is organized, processed, retrieved and augmented in the human mind is not a pre-requisite for investigations of instructional models or theories. In other words, the development of instructional theories need not wait for a complete theory of learning to be developed, just as diagnosis and treatment in the medical sciences need not wait for studies in the biological sciences to be completed. Investigations in instructional strategies can proceed for specific instructional situations for which a learning model exists that adequately describes the learning behaviors.

The specific goal of the present study is to contribute to the decision making capabilities of an instructional system that would individualize and optimize instruction for the learning of independent items. But a more general goal is to add to the theory of instruction.

# CHAPTER II

# HISTORY: A REVIEW OF STUDIES OF OPTIMIZATION STRATEGIES FOR INSTRUCTION

## Conceptual Framework of Optimal Decision Mechanism Introduced by Smallwood

The idea of incorporating an optimal decision mechanism into a computer instructional system was first pursued by Smallwood (1962). He explored the design of an adaptive system which is characterized by 1) the ability of the system to optimally sequence a student through an instructional program dependent upon his individual history and 2) the ability of the system to improve its decision making as it gains more information about all students. In order to design an adaptive system, a decision logic needed to be specified to utilize the available past histories in some meaningful way to guide the students toward some a priori goal of instruction. Smallwood introduced the concept of a general branching network as representing a student's possible paths through an instructional program. Each node in this tree (Figure 2) represents either an instructional decision made by the system or a response made by the student. The past response

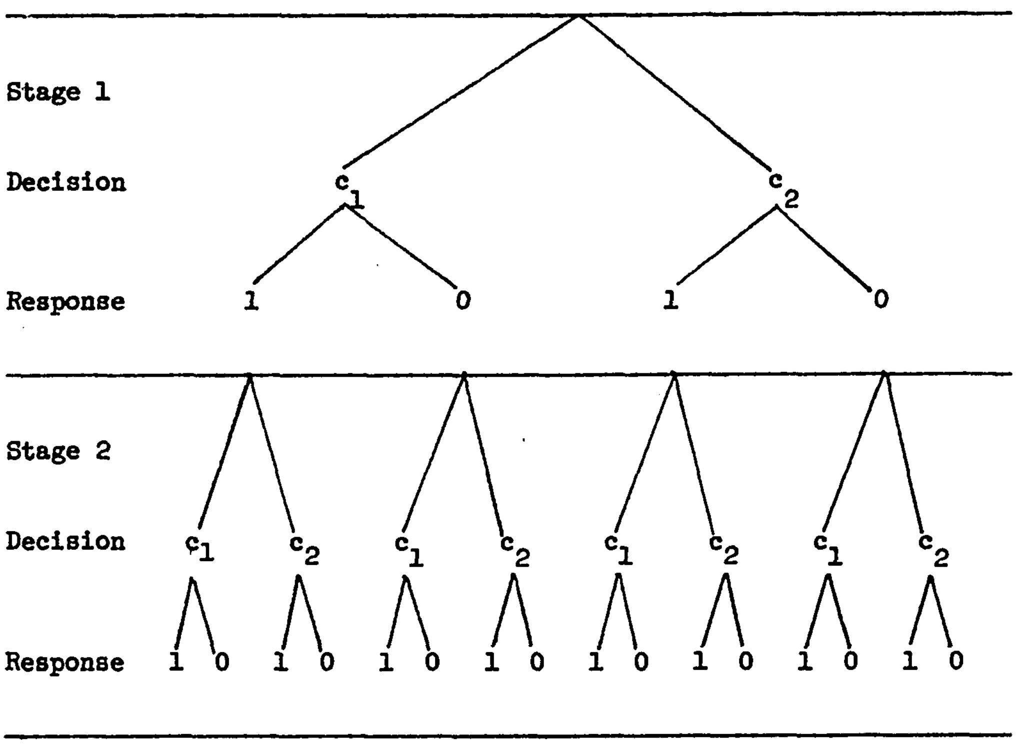



Figure 2

A Two Stage Decision Response Tree for Two Items ($c_1$,$c_2$) and Two Response Alternatives Denoted as 1 (incorrect) and 0 (correct).

history of a student can be mapped out on such a network and all possible future decision-response alternatives can be projected. Associated with each instructional alternative is a value (utility function) of selecting that alternative. Using this schema and dynamic programming techniques, Smallwood derived a backward recursive equation which calculated the expected total utility of all instructional paths, given a past response history. The optimal path was the one which yielded the maximum expected total utility. In other words, the optimal path was the one which has the greatest probability of leading to the desired final outcomes with least cost incurred.

The solution required predetermined quantitative representations of costs and payoffs of each instructional alternative and of the goal of instruction, as well as a learning model with associated parameters. The possible paths of an instructional program become phenomenally large as instructional alternatives and instructional stages increase. The computation involved would be impractical even for a high speed computer. For this reason and for the fact that no learning model existed for the general solution considered by Smallwood, no attempts were made to implement this decision algorithm in a CAI system. Nevertheless, this theoretical solution provided an useful conceptual framework with which to view the adaptive decision making process in a computer aided teaching system.

Instructional Strategies Derived from Mathematical Models of Paired-associate Learning

The subsequent studies in optimizing instruction were limited to paired-associate learning because a large body of research on mathematical learning models existed in this area. Also, the independence of items in this learning situation simplified the instructional problem since the decision mechanisms could be studied without considering the inherent structure of the subject matter. These simpler problems are in themselves interesting as they exemplify learning that occurs in some educational situations, (e.g. vocabulary learning). More important, experience gained from the solution to simpler problems would provide a knowledge base for the solution of more complex instructional problems where materials are hierarchically arranged.

Suppes' Application of Linear Learning Model. A first attempt to derive an instructional strategy from a mathematical learning model was made in a study by Suppes (1964). He considered the optimal allocation of vocabulary items in a limited number of trials. Suppes applied a linear learning model to this situation. In a linear model, the probability of a wrong response $q_{n+1}$ on trial n + 1 is a linear function of the response probability on the previous trial:

$$q_{n+1} = \alpha q_n \; .$$

In other words, a sampled stimulus becomes conditioned with some fixed probability $\alpha$; learning is incremental. Assuming that all items are equally difficult, Suppes analytically showed that if learning occurs faster than forgetting, cycling through the whole list in any order is superior to breaking up the list into sub-lists. This procedure of randomly cycling through the whole list has come to be known as the standard procedure or the randomized cyclical procedure.

<u>The All-or-none Model Studied by Karush and Dear</u>. A slightly more complicated strategy for optimally allocating items was derived by Karush and Dear (1966) from the extensively studied all-or-none model. In this model, an item is either in the learned state or the unlearned state. A correct response is always made from the learned state and can occur by quessing from the unlearned state. A transition from the unlearned state occurs with a fixed probablity c on any reinforced trial. Thus, the model can be re-presented by a matrix and a vector :

$$\begin{array}{cc} & \text{trial } n+1 \\ \text{trial } n & \begin{array}{cc} L & U \end{array} \\ \begin{array}{c} L \\ U \end{array} & \begin{bmatrix} 1 & 0 \\ c & 1-c \end{bmatrix} \end{array} \qquad \begin{array}{cc} & P(\text{correct response}) \\ \begin{array}{c} L \\ U \end{array} & \begin{bmatrix} 1 \\ g \end{bmatrix} \end{array} .$$

According to this model, once an item has been learned, it will not be forgotten; therefore, there is no need to present it for further study.

<u>The Principle of Maximizing the Immediate Payoff</u>. Assuming that all items are homogeneous, Karush and Dear proved that the optimal presentation strategy for this learning model is the one which "chooses in each trial that item for which the current probablity of being in the learned state is the least" (Karush and Dear, 1966, p. 19). This strategy is computationally simple because it only involves incrementing a counter of the item by one when a correct response is made and setting the counter to zero for a wrong response. The procedure selects for presentation the items with the lowest counters. This strategy is also conceptually simple because the allocation decision is equivalent to selecting items that have the maximal expected probability of being learned on the next trial. All future events can be ignored. Karush and Dear in effect showed that for this learning model, the single trial "locally optimal" strategy is equivalent to the "globally optimal" strategy which looks to the terminal states in minimizing the total expected losses or maximizing total expected gains. The principle of only planning one step ahead is sometimes also called the principle of maximizing the immediate payoff (MIP).

The MIP principle reduces the computational burden of the selection strategy, since computations are carried out only one stage ahead. Policies derived under this restriction are generally suboptimal as they do not consider the terminal utility of the strategy. Under which conditions is the MIP strategy also optimal? Matheson (1964) proved that for the case of the linear model and the all-or-none model the MIP principle is optimal if the items are homogeneous. With heterogeneous items, he showed that small additional gains can be made if more steps are considered by the optimization strategy. In simulation studies, Calfee (1966) demonstrated that the MIP principle is near optimal for heterogeneous parameters, and that for N-state Markov models with homogeneous parameters, the principle of selecting items with the largest immediate gain is optimal, under certain conditions pertaining to the gain function.

## Experimental Studies Comparing the Strategies Derived from Mathematical Models

Dear, Silberman, Estavan and Atkinson's Study. Two experimental studies comparing the relative efficiency of the all-or-none procedure with the standard procedure were conducted by Dear, Siberman, Estavan and Atkinson (1967). The learning task involved associating two-digit integers with one of four push button switches, over a large number of trials during one session. Here, the items were selected such that they would be homogeneous as required by the Karush and Dear theorem. The results of the first experiment demonstrated slightly better but not significantly different performance on the items selected by the all-or-none procedure. The second study differed in that study trials terminated when a certain learning criterion was reached. The results showed learning to be significantly better under the standard procedure. These results led to the conclusion that the all-or-none model was not an adequate description of the learning processes, as it did not account for the forgetting process.

Lorton's Study with Computer Assisted Spelling Instruction. Contrary to the above experimental results, an experiment conducted by Lorton (1969) in computer-assisted spelling instruction with elementary school children showed that the all-or-none strategy was significantly more effective than the standard procedure. The apparent discrepency between these results and those of the Dear, Siberman, Estavan and Atkinson (1967) study was attributed to the different experimental conditions. In the Lorton study, learning occurred over 24 sessions. It was concluded that the all-or-none model provided an accurate description of learning when items were well spaced, but was not adequate under conditions where items were massed into one session. An important conclusion from these two experiments was that different learning models are suitable for different instructional situations. The items used in Lorton's study were probably heterogeneous. Because the all-or-none strategy is sensitive to the subject's past responses, it can detect differences in item difficulty from the subject history and thus would present the difficult items more often than the easy items. This may also explain why the all-or-none strategy was more effective.

Application of the Random-trial-increment Model to the Instruction of Swahili Vocabulary by Laubsch. Laubsch (1969) investigated the principle of maximal immediate payoff using the random-trial-increment (RTI) model. This model is a combination of the linear model and the all-or-none model in that on any trial an item may become partially learned with a fixed probability. For this model, the probability of an error response q on trial n+1 is

$$q_{n+1} = \begin{cases} q_n & \text{with probability } 1-c \\ Aq_n & \text{with probability } c \end{cases},$$

where A is the proportion of the yet unlearned part and c is the probability that an item can be partially learned on a reinforced trial. For the two earlier models discussed, the derived optimal strategies were relatively simple as the strategies did not depend on the values of the model parameters. Furthermore, a student's response protocol could be condensed to a simple sufficient history (Atkinson and Paulson, 1970, p, 15), consisting of one index per item without losing relevant information for the presentation decision. For the RTI model, it was not possible to state a simple algorithm for the optimal presentation strategy. Laubsch derived an algorithm for selecting items with maximal expected immediate gain for the RTI-model as an approximation of the optimal selection strategy.

Laubsch's study differed from earlier studies more significantly in that, in previous studies, items and subjects were assumed to be homogeneous, whereas in Laubsch's study, parameters were allowed to vary with subjects and items : heterogenous parameters were estimated from response histories. As more data were accumulated about items and subjects, more reliable estimates were derived. This selection strategy is senstive to individual differences in subjects and items, and since it can improve its selection based on the accumulation of data about students and items, it possesses adaptive characteristics as defined by Smallwood.

Laubsch conducted an experiment comparing the three selection strategies; 1) the standard strategy, 2) the all-or-none strategy, and 3) the adaptive strategy (AOP) based on the RTI model. The experimental situation involved the learning of Swahili vocabulary words throughout 20 sessions. Results showed small but significant gains in learning due to the AOP strategy. Two reasons were given for the small gains. First, there were extensive intervening test sessions where additional learning occurred and the models did not account for learning on these test sessions. Secondly, the RTI model was an inadequate description of the learning processes because it did not include a time-dependent forgetting process.

Atkinson's Application of the Three State Markov Learning Model to Instruction of German Vocabulary. At the same time, studies in paired-associate learning models (Atkinson and Crothers, 1964; Rumelhart, 1967) had shown that a three-state Markov model which distinguishes between long-term and short-term retention and allows for forgetting between successive presentation of the same stimulus item is a good representation of this learning situation (a detailed description of this learning model follows in the theory of this study). Using this model, Atkinson (1972) derived a strategy that used heterogenous item parameter estimates in calculating the maximal expected immediate payoff. He experimentally tested this strategy against three others; 1) the standard strategy, 2) a learner controlled strategy and 3) a homogeneous parameter MIP strategy. The experimental situation consisted of learning a large set of German vocabulary items, during many trials over a two hour session. Results on a test administered one week later showed large significant gains due to the heterogeneous parameter MIP strategy.

## Summary

In summary, studies in the optimization of instructional strategies have advanced in several ways. First, the representation of the learning process has evolved from simple to more complex learning models that are more adequate descriptions of the learning process. Secondly, strategies have been developed that are more sensitive to differences among subjects and materials. Thirdly, the instructional situations being considered have gradually become more realistic, from the learning of nonsense syllables in one session to the learning of vocabulary items over many sessions.

In this evolving chain of theoretical experimental advances in the formalizing of optimal instructional strategies derived from mathematical learning models, the present study was designed to investigate the effectiveness of instructional algorithms derived from a learning model that better describes the learning of independent items spread throughout many sessions. A modified three state Markov model, which includes a time-dependent forgetting process and which accounts for learning during both study trialssand test trials, is used. Algorithms are derived for selecting items according to the MIP principle, using both homogeneous and heterogeneous parameters. In addition, latency information is incorporated into the decision structure to further improve the effectiveness of the selection strategy. The situation used to test the model involves the learning, in a CAI enviroment, of a set of 252 Chinese-English words during twelve study sessions spread out over five weeks.

# CHAPTER III

## THE THEORETIC COMPONENTS:

## THE LEARNING MODEL AND THE INSTRUCTIONAL ALGORITHM

Within the decision theoretic analysis framework of instructional problems, the instructional process can be conceptualized as the mechanism for changing the internal state of information of the student. The goal of instruction is to change the student's state of knowledge from the unlearned state to the learned state. How does an instructional system know the internal state of knowledge of the student? How does it measure and predict the changes in the learning states due to instructional actions? What instructional actions should be taken to bring about the desired changes in the learning states?

Because a subject's state of information is not directly observable, we need to make some inferences about the student's state from his observable responses. Having a learning model which describes the student in terms of his possible learning states and the transformation of the states resulting from instructional action, an equation can be derived that utilizes the student response history information to estimate his current state of knowledge. The parameters of the learning model need to be known and can be estimated from the response history data. From the known parameter values and the current state probability vector, it is possible to calculate the transformation of the current states due to instructional actions and to compute the instructional gain in terms of changes in states. An instructional strategy can then be specified which selects the instructional materials according to the instructional goal, the possible instructional actions and the instructional gain.

The design of the instructional process from a decision theoretic analysis approach can be thought of as having two parts (see Figure 3): 1) a description of the learning process and 2) the derivation of an instructional algorithm from the learning model. The implementation of such algorithms requires: 1) a method of calculating the gains of instructional actions, 2) an equation for determining the current state of knowledge of the student and 3) a way of estimating the parameters of the learning model. In this section, we will discuss the learning model and the instructional algorithm used in the proposed solution to the problem of optimal item selection.

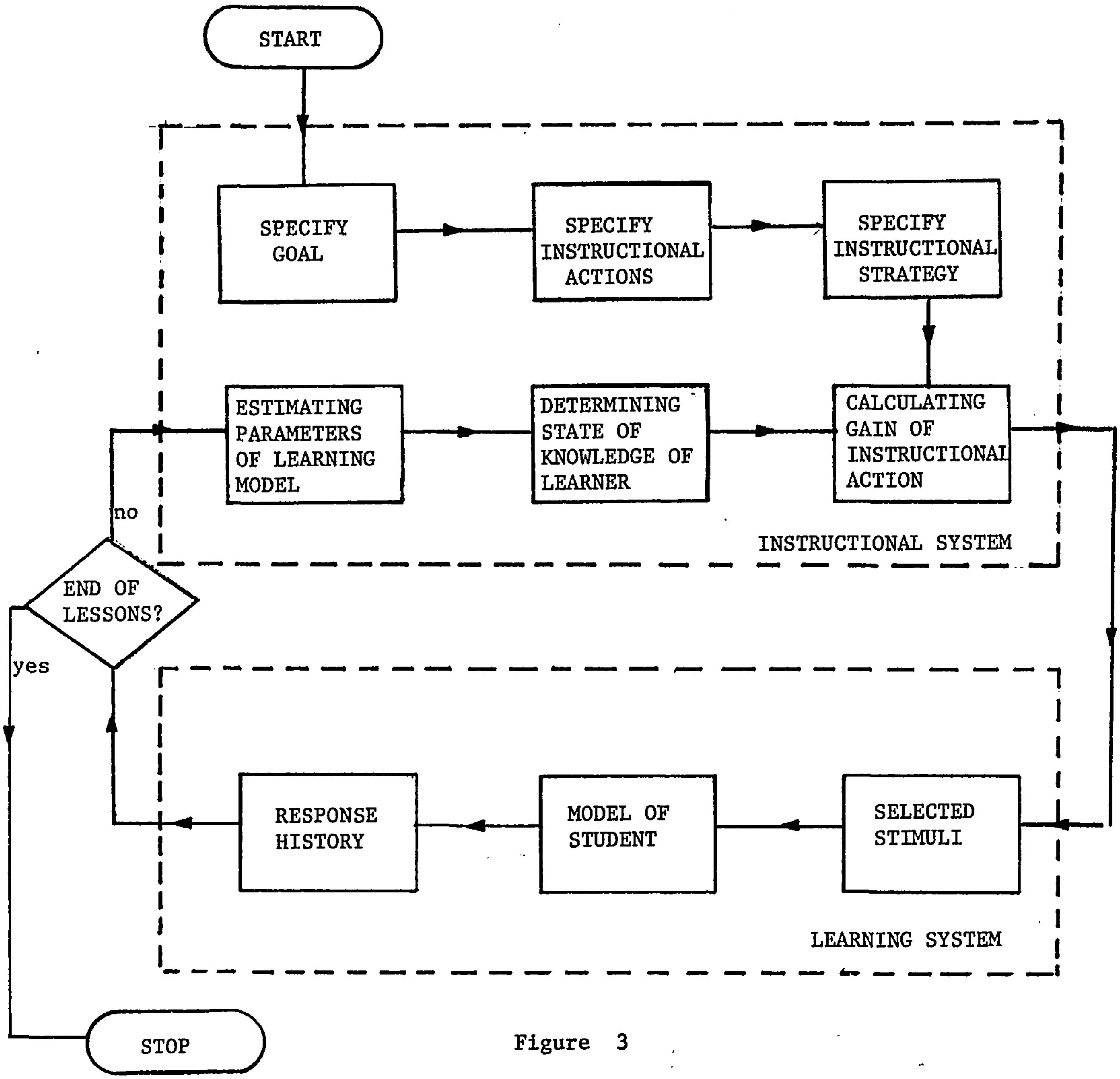



Figure 3

Model of Instructional Decision Process.

## The General Forgetting Theory as a Model of the Learning Process

The learning task under consideration is that of learning a set of N independent paired-associate items, as it occurs in vocabulary and code learning situations. This aspect of verbal learning has received a great deal of attention in experimental psychology (Kintsch,1970) and a detailed knowledge about mathematical models of paired-associate learning has evolved (Luce,Bush,Galanter,1965; Restle and Greeno, 1970). The general forgetting theory (GFT),investigated by Rumelhart (1970), describes both the learning and forgetting processes that occur in paired-associate learning and has been shown to be an accurate representation of this learning situation. In this study, we adopt the GFT to represent the learner and introduce minor modifications to it to suit the learning situations under consideration.

The GFT models assume that a subject can be in three or four states of knowledge with regard to a paired-associate item: 1) a learned state, L, in which the item is permanently stored and retrievable; 2) a transitory or intermediate memory state, T, in which the correct response is coded and retrievable, but from which forgetting may occur, i.e. storage is not permanent; 3) a forgotten state from which the subject cannot retrieve the association except by guessing; and 4) an unlearned state, U, in which each item is assumed to be initially, allowing retrieval only by random guessing. It is possible to lump the forgotten state and the unlearned state in most cases, reducing the model to a 3-state Markov model.

The Response Function. According to these models, a correct response is always given if the item is in State L or State T, but in State U, a correct response is emitted only by guessing. The following R matrix represents the probability of a student's response, given his internal state:

$$
\begin{array}{c} \\ \text{State} \\ \\ R = \end{array}
\begin{array}{c} \text{Response} \\ \begin{array}{ccc} & \text{Correct} & \text{Wrong} \end{array} \\
\begin{array}{c} L \\ T \\ U \end{array}
\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ g & 1-g \end{bmatrix}
\end{array}
$$

In State U, the subject responds by guessing randomly from a set of r response alternatives; hence, the probability of guessing correctly, g, is equal to 1/r. We define the response function

$$r(k, x_i) = P(x_i \mid S_i = k)$$

as the probability that a subject makes a response x on trial i given that the item was in State k, where x = 0 for correct response and x = 1 for incorrect response, and S = 0 for State U, 1 for State T, and 2 for State L.

Transition Between States. According to the GFT models, the state of knowledge is changed by two processes : learning and forgetting. The transitions between states are not deterministic; in other words, the transitions occur with certain probability. When an item is presented on a trial, learning may occur. Thus, an item in the unlearned state may enter the intermediate state or the permanent state of remain unlearned; an item in the intermediate state may become permanently learned or remain in the same state; a learned item would remain in the learned state. Between successive presentations for an item, forgetting may occur. Forgetting only effects items in the intermediate state, and would cause it to fall into the unlearned state. The changes of the states for an item can be represented by transition matrices (see below).

When an item is presented for a reinforced trial, that is, when an item is presented and the correct answer is given after a subject emits a response so that he can study the item, the state of knowledge changes according to the following transition matrix.

$$
M_r = \begin{array}{c} \\ L \\ T \\ U \end{array}
\begin{array}{c} \begin{array}{ccc} L & T & U \end{array} \\
\begin{bmatrix} 1 & 0 & 0 \\ b\gamma & 1-b\gamma & 0 \\ a\gamma & \gamma(1-a) & 1-\gamma \end{bmatrix} \end{array}
$$

The rows of the matrix represent the state of the item at the start of the trial and the columns represent its state at the end of the trial. Parameter $\gamma$ is the attention parameter and represents the probability that the subject attends to the presentation of the item and thus changes its state. The GFT with the introduction of the attention parameter, $\gamma$, constitutes a modified GFT which was successfully applied by Rumelhart to paired-associate learning experiments. Parameter a is the probability that an item moves to State L from State U given that the subject pays attention on that trial; that is, the probability of making a permanent correct association when such an association did not exist. Parameter b is the probability that the item moves to State L given that it had occupied State T and the subject attended to the presentation; in other words, the probability that an item enters permanent from intermediate memory. The matrix shows that when an item is presented for study, it can remain in the same state or move to a higher state. This notion is intuitively correct, as one's state of knowledge for an item should not decrease with the presentation of that item for study.

Between successive presentations of an item, forgetting may occur for an item in the transitory state, due to decay over time or to the interference of intervening items. In most paired-associate learning experiments, learning occurs over successive trials during a single session. However, in the instructional situation under consideration, instruction occurs over several sessions separated by several days. Therefore, we introduce to the representation of the forgetting processes of GFT a time factor d, which represents the number of days between presentation trials for an item. In other words, forgetting is measured in terms of the number of days between successive presentation. The forgetting process is represented by the following matrix:

$$M_f = \begin{array}{c} \\ L \\ T \\ U \end{array} \begin{array}{c} \begin{array}{ccc} L & T & U \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & \theta^d & (1-\theta^d) \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

where θ is the probability of not forgetting in one day while in the transitory state. The rows of the matrix represent the state of the item after a trial and the columns represent its state before the next trial.

Combining the forgetting and learning processes, the complete transition matrix from the beginning of a study trial to the beginning of the next trial is:

$$M_r \times M_f = \begin{array}{c} \\ L \\ T \\ U \end{array} \begin{array}{c} \begin{array}{ccc} L & T & U \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ b\gamma & (1-b\gamma)\theta^d & (1-b\gamma)(1-\theta^d) \\ a\gamma & (1-a)\gamma\theta^d & \gamma(1-a)(1-\theta^d) + (1-\gamma) \end{bmatrix} \end{array}$$

Another difference between the learning situations which the GFT models represent and the instructional situation under investigation is that in the present study the instructional sessions include both reinforced trials and unreinforced (test) trials. It is known that some learning occurs also during test trials. Therefore, we propose to add to the GFT the following transition matrix to account for learning on an unreinforced trial:

$$M_u = \begin{array}{c} \\ L \\ T \\ U \end{array} \begin{array}{c} \begin{array}{ccc} L & T & U \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ c\gamma & 1-c\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

where parameter c is the probability of establishing a permanent code when an item is in the transitory state and the subject attends the presentation. The rows of the matrix represent the state of the item before the test trial and the columns represent its state after the trial. The matrix indicates that learning on an unreinforced trial only occurs from the transitory state; that is, on a test trial, only items in the transitory state are effected It is reasonable to postulate no learning from the unlearned state on a test trial, since the subject does not have the correct association and is not given clues in regard to retrieving the correct response. In the transitory state, however, the subject has enough information available to retrieve the correct response. Even though no confirmation and no study interval is given to the subject, some encoding is assumed to occur as a result of rehearsal.

To summarize, a modified GFT is used for representing the learning and forgetting process under investigation. When an item is presented for a reinforced study trial, transition matrix $M_r$ represents the possible changes in states; when presented for an unreinforced or test trial, transition matrix $M_u$ is applied; and in between any two trials, transition matrix $M_f$ is applied to describe the changes in states.

The State Transition Function. We define

$$t(k,\ell,u_i)$$

as the probability that an item makes a transition to State $\ell$, given that it was in State K and given the conditions u on trial i, where $u_i$ is a tuple $\{m_{i-1}, d_i\}$ that indicates whether trial i-1 is an unreinforced trial (m = 0) or a reinforced trial (m = 1) and how many days have past since trial i-1 ($d_i$ = 1, 2...). In other words,

$$t(k, \ell, u_i) = P(\text{state prior to trial } i = \ell \mid \text{state prior to trial } i\text{-}1 = k \text{ and } u_i)$$

for k and $\ell$: 0 = State U, 1 = State T and 2 = State L.

Restrictions on the Parameter Space $\Omega$

The GFT models described thus far has a parameter space $\Omega$ consisting of five elements (a, b, c, $\theta$, $\gamma$). The GFT subsumes a variety of Markov models and under specific restrictions of the parameter space, it reduces to the long-short models investigated by Atkinson and Crothers (1964), Bernbach's forgetting model (1965), the trial-dependent forgetting model (Calfee and Atkinson, 1965), Greeno's coding theory (1966) and Bjork's all-or-none forgetting model (1966). If data or computing time is scarce, it would be useful to impose restrictions on the parameter space in order to avoid fitting noise or running costly estimation. Six alternative, meaningful ways of reducing the number of parameters for learning from four to two are proposed:

1) $a = b$, $\gamma = 1$. This is the LS-2 model (Atkinson and Crothers, 1964) which assumes (a) that the probability of making a permanent association is the same in the transitory state or the unlearned state, and (b) that some encoding is always attempted.

2) $b = c$, $\gamma = 1$. In this submodel, the transitory state is conceived as being a state where the correct association has been formed and needs only additional practice, regardless of reinforcement, to be permanently stored. In other words, in the transitory state, the subject learns as much on a reinforced trial as on an unreinforced trial. Some encoding is also assumed to be attempted on all study trials.

3) $a = 0$, $\gamma = 1$. This model is supported by Bernbach's experiment (1965). It assumes that in order to get to the learned state an item must first pass through the transitory state. Bernbach suggested the notion of a single memory store which holds associations with degrees of strength rather than separate short-term and long-term memories as suggested by Atkinson and Crothers (1964). Encoding is also assumed to be attempted on all study trials.

4) $a = b = c$. Rumelhart (1969) found that in his experiment, parameter $\gamma$ played the most important role in predicting learning and that the restriction $a = b$ had a much smaller effect. The restrictions assume that the subject (a) learns

as much on a test-trial as on a study-trial, if the item is in the transitory state, and (b) has the same chance of establishing a permanent association from the transitory state or the unlearned state on a study-trial.

5) $a = 0$, $b = c$. This model is a compromise between Rumelhart's model and Bernbach's model (models 3 and 4).

6) $b = c = 0$. This model conforms to Greeno's (1966) coding theory, which assumes that parameter $a > b = 0$; that is, no learning occurs from the transitory state. Greeno interprets State T as one in which a bad code has been established and it has to be forgotten before a new coding attempt is made. In a subsequent experiment, Bjork (1966) supported Greeno's notion that $b = 0$.

These submodels differ mainly 1) in their interpretation of the transitory state 2) in the values given to the probabilities of entering the learned state from the other states, and 3) in the assumption that some encoding always occurs when an item is presented. The different models suggest different values and relationships for the learning parameters ($a > b = 0$; $b > a = 0$; $a = b = c$; $a = b > c$; $b = c$; $\gamma = 1$; $\gamma = 0$).

The present study adopts the GFT for describing the verbal learning under investigation. However, to simplify the GFT model, restrictions are made on the parameter space $\Omega$ , thus reducing the number of parameters. We do not have a priori knowledge of which of the above six submodels of the GFT most accurately describes the instructional situation of the study, as the instructional situation differs significantly from the conditions of the cited paired-associate learning experiments. We propose to use the LS-2 model (model 1) for the first experiment, as a similar model, LS-3, has been shown by Fisherman, Keller and Atkinson (Atkinson and Wilson, 1969) to be an adequate description of vocabulary learning in a CAI environment. From the data of experiment one, we will apply the other models and determine which model provides the best fit. Subsequently, this model will be used for the second experiment. The adaptivity of an instructional system through model selection was first mentioned by Smallwood (1962). This idea is incorporated into our experimental procedure, with the aim that experimental studies on optimal instructional strategies would also lead to improving the descriptive power of learning models for real instructional situations (see Figure 4).

## Instructional Strategy: the MIP Selection Strategy Derived from GFT Models

The instructional goal of the present study is to select

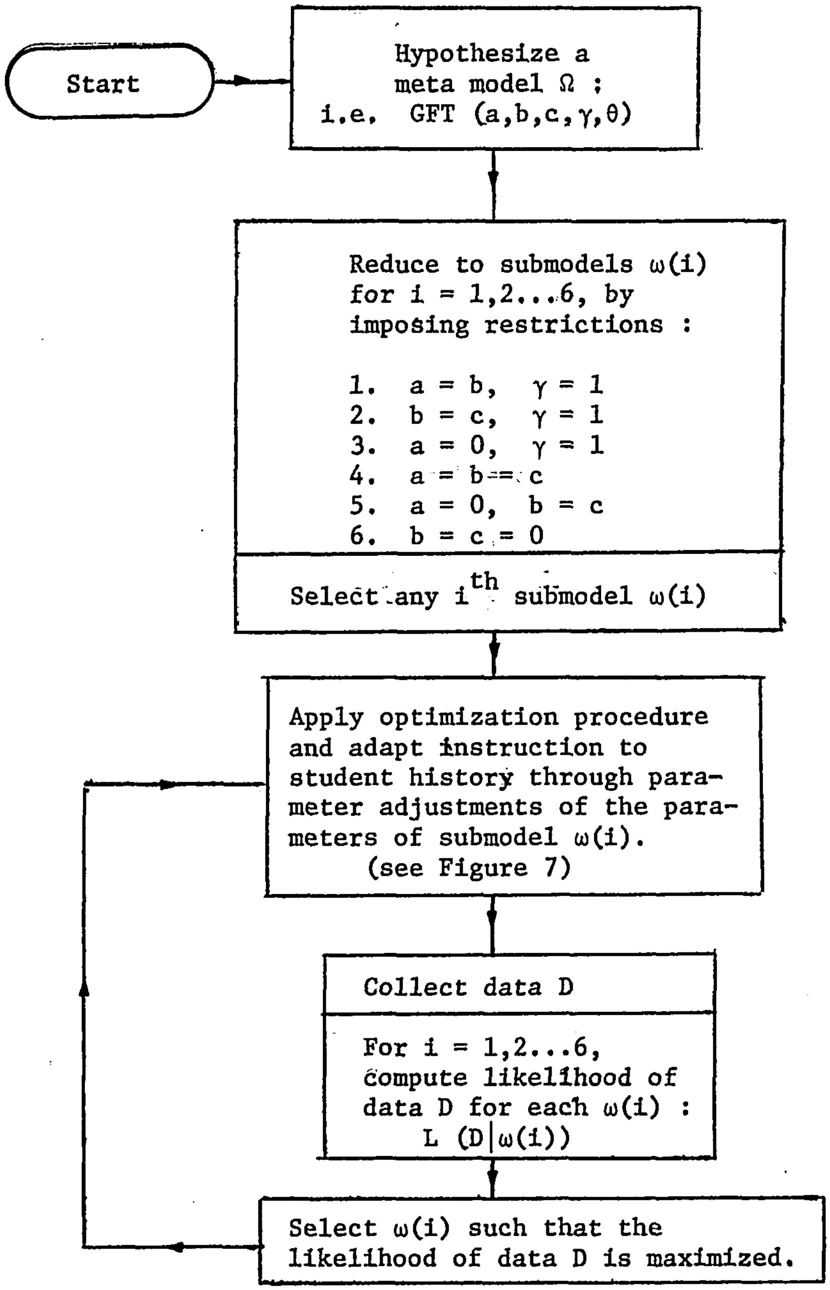



Figure 4

Adaptivity of Instructional System Through Model Selection items for reinforced trials such that after T sessions, the number of items in learned State L would be maximized. The restriction is that only n different items, $n < N$ total items, can be presented for study at each session. The general problem of finding an optimal strategy was studied by Smallwood (1962) in terms of dynamic programming. The main idea is to consider all possible item-response sequences for all the remaining trials and to select the items that have the highest probability of being in the learned state at the termination of all instructional sessions. The optimal policy which looks N stages ahead to compute the terminal utility becomes extremely complex computationally, particularly if the number of items is large and the remaining stages are many. The necessary computations are too time consuming even for a large computer; therefore, it is not practically feasible to implement this N-stage policy.

For this study, a simpler strategy was used which selects items that have the highest probability of entering State L on the next study trial. This one stage ahead policy is known as the maximum immediate payoff (MIP)strategy. For the linear model and the all-or-none model, Matheson (1964) proved that the MIP strategy is equal to the optimal strategy. For N-state Markov models, the MIP strategy is generally suboptimal. However, Monte Carlo studies with N-state Markov models show that the MIP strategy is a good approximation of the optimal policy (Groen and Atkinson, 1966;

Calfee, 1970; Laubsch, 1970). Also, from computations and formal manipulations for various Markov models using heterogeneous items, Matheson's (1964) showed that the gains of additional stages are small compared to the gain of the first stage.

Let us define $\Delta_i$ as the immediate expected gain in presenting an item for study on trial i. Then by the MIP principle, the n items with the largest $\Delta_i$ are selected for studying at trial i. From the GFT model, we can derive an algorithm for computing $\Delta_i$ for each item, for each subject. The derivation is given in Appendix A. Suffice it to mention here that we used Bayes theorem, characteristics of Markov processes and quantitative manipulations to obtain

$$\Delta_i = a\gamma\ \pi_i(0) + b\gamma\ \pi_i(1) \qquad (1)$$

where $\pi_i(0)$ is the probability of being in State U before trial i

$\pi_i(1)$ is the probability of being in State T before trial i

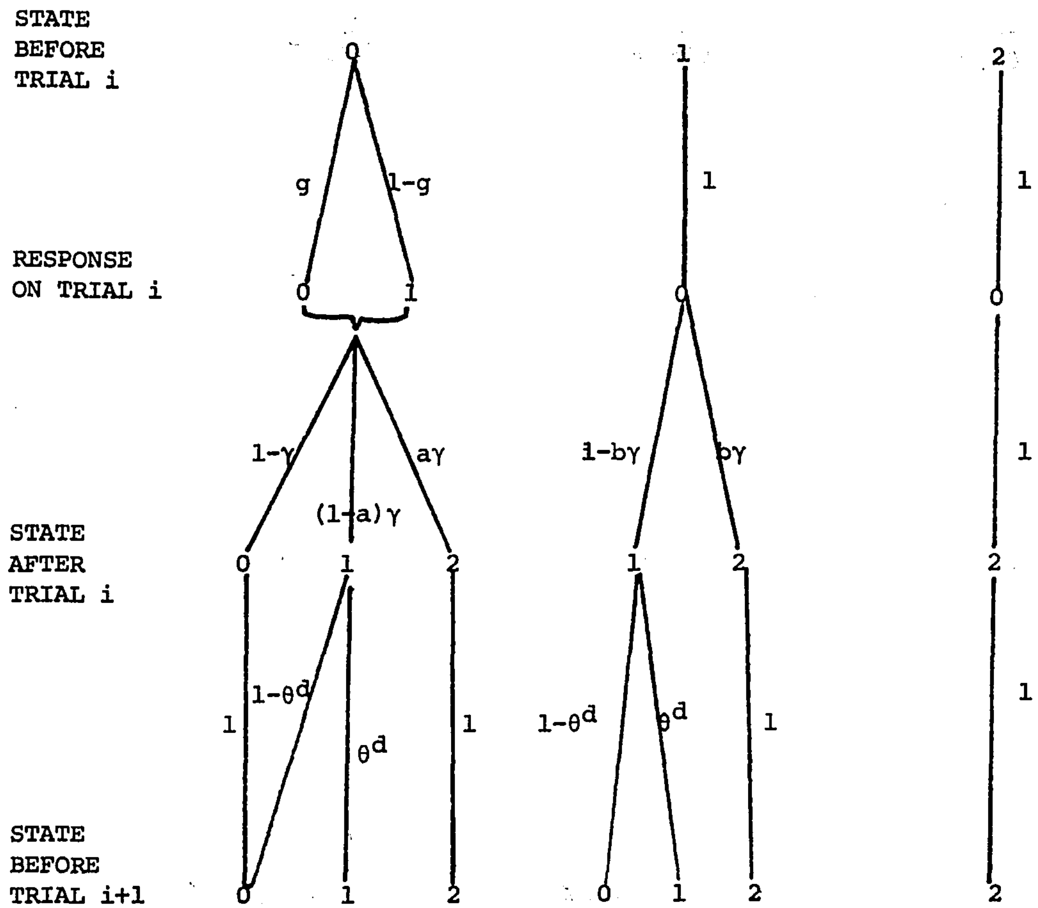
and $a\gamma$ and $b\gamma$ are parameters from the transition matrix.

In other words, the expected immediate gain is equal to the probability that the item is in the unlearned state times the probability that it would enter the learned state in the next study trial plus the probability that the item is in the transitory state times the probability that it would enter the learned state on the next study trial. This equation depends on the parameters a, b, $\gamma$ as well as the estimates of the current state of knowledge $\pi_i(K)$.

Determining the State of Knowledge of the Learner

Since instructional gains are measured in terms of changes in state of knowledge, a method is needed for determining the student's state of knowledge. These states are not directly observable but we can make inferences about these states from a subject's observable responses and give probability estimates to these states. From the GFT model, an algorithm can be derived for estimating the learner's state of knowledge that depends on his response history.

The GFT model with response and transition probabilities can be represented by a tree diagram where the nodes of the tree represent a subject's state or his responses, and the branches of the tree represent the probability of making the response or transition. Figure 5 is a diagram which indicates all the possible paths which an item can take between the onset of two trials. The likelihood of any one of these paths is equal to the product of the probabilities of taking each branch of the path. To calculate the probability of being in a certain state before trial i+1 we can sum up the probabilities of all the different paths which can lead to this state from the states of the previous trial, trial i given that we know what response was made on trial i. To deter-




Response : 0 = correct, 1 = wrong
Trial i = reinforced trial

**Figure 5**

**Tree Representation of GFT Model**

mine the state probability vector at trial i, we need to know the state probabilities of trial i-1 and response on trial i-1 and so on, working backwards until we come to the state of the initial trial, which is known, as all items are assumed to be initially in the unlearned state. Using this logic, a recursive equation can be derived for determining the state of an item at any trial, which depends on the state probability vector of the previous trial, the state transition function and the response probability function. The formal derivation is given in Appendix B. Here, we will state the equation.

We define $\pi_{i+1}(\ell)$ as the probability of an item being in State $\ell$ before trial i+1, for $\ell$ = 0, 1, or 2. Then the algorithm for determining $\pi_{i+1}(\ell)$ is:

$$\pi_{i+1}(\ell) = \frac{\sum_{k=0}^{2} \pi_i(k)\ t(k,\ell,u_i)\ r(k,x_i)}{\sum_{k=0}^{2} \pi_i(k)\ r(k,x_i)} \qquad (2)$$

We know that initially $\pi_1(0) = 1$, $\pi_1(1) = 0$, $\pi_1(2) = 0$. Note that for the computation of equation 2, the values of $t(k, \ell, u_i)$ which are the transition probabilities of the learning model need to be known. If these transition values are known, the previous state probability vector constitutes a sufficient history for updating the current state probability vector, and all the previous response history can be discarded. The burden of keeping large amounts of data can therefore be drastically reduced when parameter values are known.

Estimating Parameters of the GFT Model

The parameter values (a, b, c, $\gamma$, $\theta$) from the GFT models are not known and need to be estimated from available response data. In other words, from a set of response data (D), we need to estimate a set of parameter values (P). An expression is needed for the likelihood function P(D|P), the probability of obtaining the data (D) given the parameters (P) and to choose a set of parameters that maximizes the likelihood function. Thus, the estimation involves two parts: an algorithm to calculate the likelihood of a response sequence and a method of finding the parameters that maximize that likelihood.

In the present study, the available response history for each item for each subject consists of a response sequence with three arguments. Let $y_i$ be the triplet $(m_{i-1}, d_i, x_i)$ associated with each response, where

$m_i$ = type of presentation on trial i; 0 = unreinforced, 1 = reinforced

$d_i$ = number of days between trial i - 1 and trial i

$x_i$ = response on trial i; 0 = correct, 1 = incorrect .

Let $(y^i)$ be the entire sequence $(y_1, y_2, \dots y_i)$. We want to find the parameter values such that the probability of the entire sequence $P(y_1, y_2 \dots y_i)$ or $P(y^i)$ is maximized.

The entire response sequence can be mapped out on a branching tree, where each node represents one of two responses, or one of the two presentation modes or one of three states for i trials. The probability of any response sequence occuring is the sum of the probabilities of all the branches on which this sequence can occur. Using this schema we can list all the branches generated by a response sequence $y^i$ and obtain the equation of computing the probability of a response sequence $P(y^i)$ as a function of the state transition function and the response probability function. That is,

$$P(y^i) = \sum_{j_o=0}^{2} \{\pi_0(j_0) \sum_{j_1=0}^{2} \{t(j_0,j_1,u_1)\ r(j_1,x_1)$$

$$\sum_{j_2=0}^{2} \{t(j_1,j_2,u_2)\ r(j_2,x_2)\dots$$

$$\sum_{j_{i-1}=0}^{2} \{t(j_{i-2},j_{i-1},u_{i-1})\ r(j_{i-1}, x_{i-1})$$

$$\sum_{j_i=0}^{2} \{t(j_{i-1},j_i,u_i)\ r(j_i,x_i)\}\dots\}\}\}$$

Laubsch (1971) has specified a backward recursive algorithm for computing this likelihood function. Let us define,

(1) $A(k=1,j_k) = 1$, for $i = 0,....k$

(2) $A(i,j_{i-1}) = \sum_{j_i=0}^{2} t(j_{i-1},j_i,u_i)\ r(j_i,x_i)\ A(i+1,j_i)$ for $i>0$

Then,

$$P(y_1,y_2...y_k) = P(y^k) = A(1,0)$$

Using this recursive algorithm, the likelihood of any response sequence can be calculated, given the parameter values of the learning model. The parameter values that provide the best fit for the data are the parameters which maximize the likelihood of the response sequences.

Heterogeneous vs. Homogeneous Parameters

One of two assumptions about the model parameters can be made: (1) all items and subjects are homogeneous, i.e., the same parameter values exist for all subjects and all items, or (2) items vary in difficulty and subjects differ in ability, i.e., there are different parameter values for different subjects and items.

Making this second assumption of heterogeneous parameters, ideally we would like to know values of parameters for each particular subject-item sequence; thus, P parameters for each of the N items and S subjects totalling P x N x S parameters. In order to reduce the number of parameters and to obtain reliable estimates, we assume that the parameters can be partitioned into a subject and an item effect. The item component, characterizing its difficulty, can be estimated from the data of all the students for that item, while the subject component, characterizing its difficulty, can be estimated from the subject's performance on all items. These component parameters can then be combined to form composite parameters for each item-subject sequence. In this way, the number of parameters to be estimated can be reduced to P x ( N+S ).

A quasi-additive partitioning method formulated by Laubsch (1969, p.29) to describe the functional relationship between the component parameters and the composite parameter is used. Let $\sigma_i$ and $p_j$ denote item and subject components and $A_{ij}$ denote the composite parameter, then

$$A_{ij} = \begin{cases} \sigma_i + (1-\sigma_i)\, p_j & \text{if } p_j \geq 0 \\ \sigma_i + \sigma_i p_j & \text{if } p_j < 0 \end{cases}$$

The underlying intuitive notion in this relationship is that "a given subject possesses some underlying trait that will consistently cause an increment of the item parameter proportionally to their complement to 1 or decrease that parameter directly proportional to its value" (Laubsch, 1969, p.29). In other words, the item parameter represents the average difficulty of the item for all subjects. This item would be easier to learn for a subject with high ability than for a subject with low ability. Therefore, we increase or decrease the item parameter according to the subject's ability, to form the composite parameter, which represents how difficult a particular item is for a particular subject.

An Approximation Method of Maximizing the Likelihood Function

Now that a method of calculating the likelihood of a response sequence has been specified, a method for obtaining the parameters that maximize that likelihood function is needed. An iterative "up and down" walk method (Laubsch, 1969, Appendix B) is used for estimating the parameters that maximize the likelihood function and thus, best fit the data. By systematically varying the parameter being estimated by a constant value c and fixing all other parameters from the model, the likelihood estimates of the response sequences are computed. The varying parameter yielding the maximum likelihood is retained. The amount of response data being analyzed at one time is a subset of the complete history. The search for the parameter value that yields the maximum likelihood estimate is repeated until all response sequences have

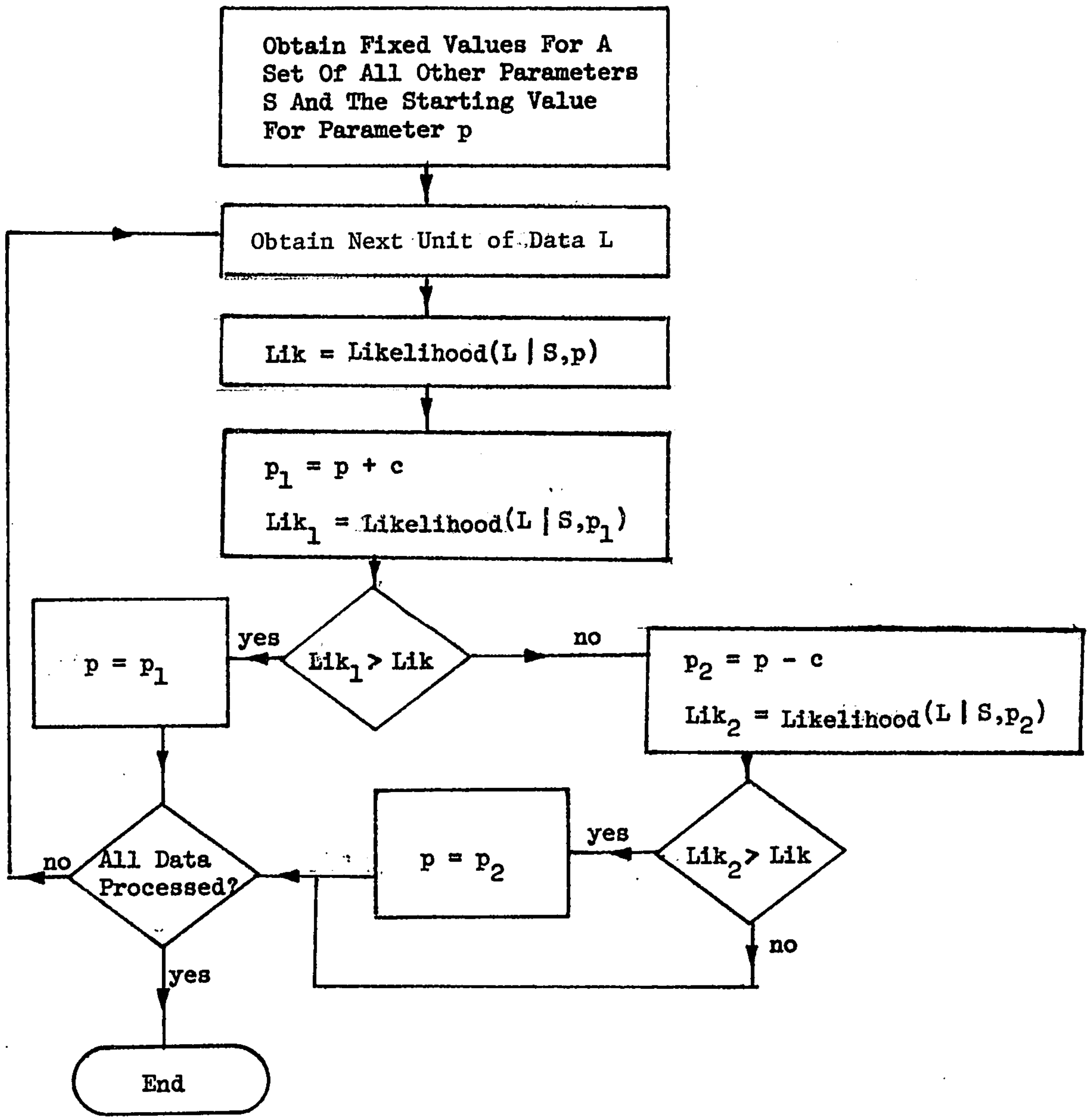



Figure 6

The Up and Down Walk for Estimating a Particular Parameter p by 1) Computing the Likelihood of an Unit of Data L for a Set of Fixed Parameters S and for Varied Values of p ( p varied by c ) and 2) Retaining the Value of p that Maximizes the Likelihood of L.

been analyzed for each item and each subject (See Figure 6).

In the estimation of homogeneous parameters, response sequences for all subjects and for all items are used in estimating the five parameters of the GFT model. Thus, the search for homogeneous parameters requires passing through the complete response history for all subjects and items five times, each time holding four parameters constant and varying one parameter by a constant value c. The parameter values that yield the maximum likelihood of the response data are retained. In the estimation of heterogeneous parameters, all response sequences pertaining to a particular item are used to estimate that item's parameters and all response sequences pertaining to a particular subject are used for estimating that subject's parameters. Thus each item component parameter is estimated from S subject response sequences for that item and each subject component parameter is estimated from N item response sequences for that subject.

## Homogeneous Parameter (HOP) Strategy and Heterogeneous Parameter (HEP) Strategy

From the GFT models, we have derived an instructional algorithm that selects items which have the largest expected immediate gain. This algorithm is an approximation to the optimal strategy as it looks only one step ahead to compute the immediate gains of an instructional action. The instructional gain is defined in terms of changes in the state of knowledge. In order to calculate this gain, we have specified a method of determining the state of the learner and a method of estimating the state transition probabilities from the student response history. Making the assumption that the same transition functions exist for all subjects and items in calculating the maximal immediate gain, we obtain the homogeneous parameter (HOP) stategy. The alternative assumption that different parameter values exist for different items and subjects leads to a heterogeneous parameter (HEP) strategy that uses heterogeneous parameters to compute the immediate gain. Both the HOP and HEP strategies are adaptive to the extent that they select items according to the histories of individual students and the parameter values change with the accumulation of data from all students. However, the HEP stategy is sensitive to individual differences in subject ability and item difficulty in estimating the parameters of the learning model. That is, both strategies base their selection behavior on the response protocol of the subjects, but the HEP strategy utilizes additional information relating to subject and item characteristics in making its selection. Figure 7 summarizes the various procedures in the implementation of these adaptive strategies.

Now that we have derived theoretically these near optimal adaptive instructional strategies, based on the GFT learning model, their efficacy need to be tested experimentally.

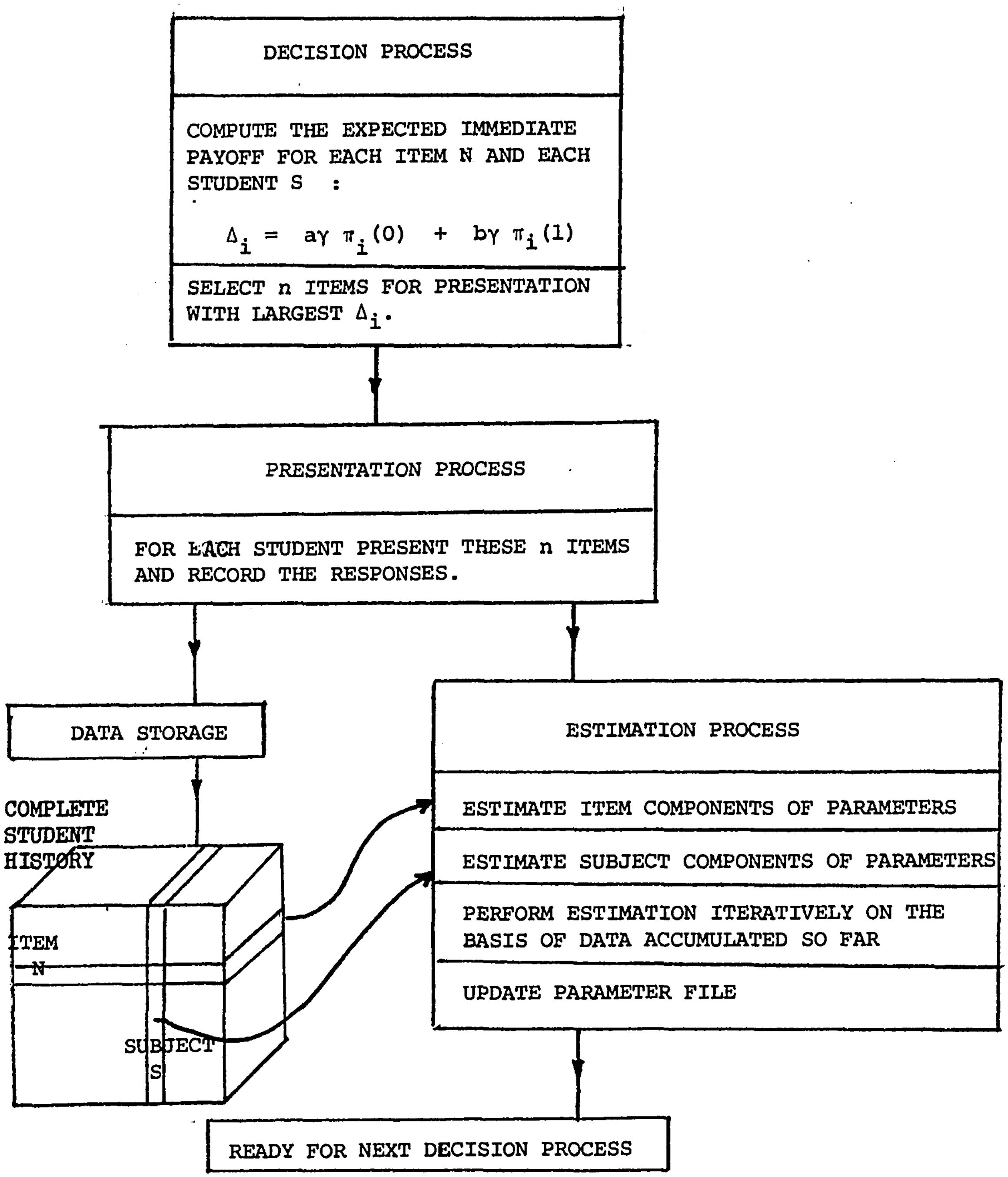



**Figure 7**

**Procedure for an Adaptive Teaching Strategy**

# CHAPTER IV

# THE DESIGN AND IMPLEMENTATION OF COMPUTER CONTROLLED EXPERIMENTS

The purpose of these experiments was primarily to test the efficacy of the instructional algorithms derived from the mathematical models and secondly to evaluate the adequacy of the proposed learning models. Two experiments were carried out at the Graduate School of the City University of New York. The first experiment was designed to compare the effectiveness of HEP and HOP adaptive strategies with a standard procedure used as a control. The second experiment then tested both the effect of incorporating parameter-heterogeneity and of adding latency information to the adaptive strategies on the basis of the best submodel from experiment 1. The experimental methods, materials, subject population, apparatus and procedures were the same for both experiments.

## Design

The experimental design used in comparing the three instructional strategies was a single factor design with repeated measures. All subjects were presented with items selected by all strategies. Subjects and items were assigned to strategies in a balanced design. Each S took the same number of items under each strategy and each item occurred equally often under each strategy. An achievement test consisting of all items was administered after all instructional sessions had been administered and was used to compare Ss performance on items selected by different strategies.

Subjects

In each experiment, twelve different paid volunteer students from the Graduate School of The City University of New York served as subjects. All subjects professed no prior knowledge of the Chinese language and had no prior course work in Chinese. The low performance of all subjects on the first test and study session also indicates that the subjects were not familiar with the Chinese characters innitially.

Materials

The learning materials consisted of 252 frequently used Chinese characters, typically taught in a first year course in the Chinese language. The characters were sampled from all parts of speech and ranged in number of strokes from 2 to 9. Figure 8 gives

Figure 8 : Sample of Chinese Characters

| | | |
|---|---|---|
| 2 strokes | 人 | Man |
| 3 strokes | 也 | Also |
| 4 strokes | 今 | Today |
| 5 strokes | 冬 | Winter |
| 6 strokes | 行 | Virtue |
| 7 strokes | 休 | To Rest |
| 8 strokes | 忠 | Faithful |
| 9 strokes | 屋 | House |

a sample of the characters used. The reasons for choosing Chinese vocabulary for instruction were because foreign language vocabulary learning is a realistic and typical example of paired-associate learning that occurs in the classroom and because the dissimilarities of Chinese-to-English and to other foreign languages minimized the transfer effects due to the knowledge of other languages. In addition, the current interest in learning the Chinese langauge and the visual characteristics of the Chinese word, which could be conveniently displayed on a CRT unit and make the learning more interesting than learning from a teletype, made it easier to obtain and to retain subjects for the experiment. The S's task was to associate the visual representation of a Chinese character with its English equivalent from a set of five alternative English words. The English word list consisted of the 252 equivalent words and 252 other commonly used English words which served as distractores.

## Apparatus

The experiments were conducted on an IMLAC Graphics Display* unit connected to a PDP-8I* computer running under the standard time sharing system TSS-8. The IMLAC displayed stimulus material,

* The PDP-8I computer is a 16K general purpose mini-computer, manufactured by the Digital Corporation, Maynard, Mass.. The IMLAC PDS-1 Graphics Display unit is a CRT display unit with a 8K mini-computer, manufactured by IMLAC Corp. of Needham, Mass..

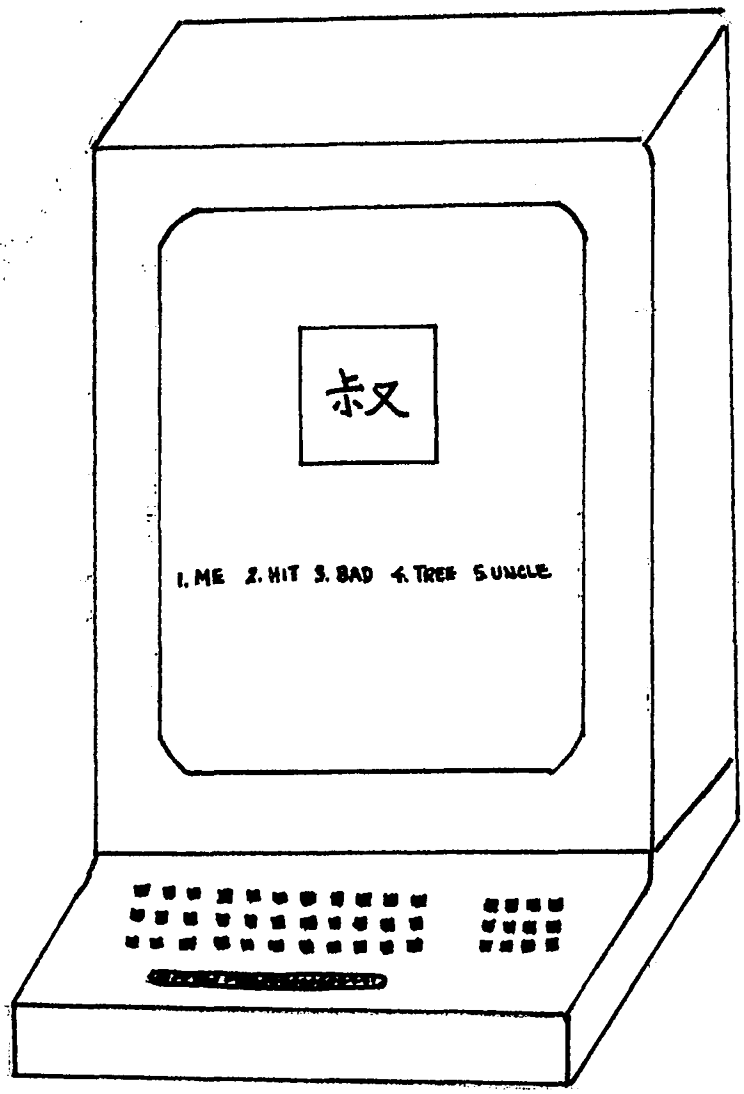



Figure 9

Picture of Imlac Screen Display

received keyboard responses and recorded response latency. The Chinese characters were graphically drawn and stored in the IMLAC computer and were referenced through a table of their starting lo-- cations. The Chinese characters were displayed one at a time in a three inch square box in the middle of the IMLAC creen. Beneath this box, the English word set was displayed ( see Figure 9 ). All other functions of the instructional system were performed by programs running on the PDP-8. These functions included on-line interaction with the Ss through IMLAC, in presenting and evaluating items, collecting and analyzing response data from every session, updating student history, estimating parameters and state probabilities, and preparing lessons for the next sessions. For a more detailed description of the computer instructional system, refer to Appendix C.

Experimental Procedure

The Ss were run one at a time, each taking 14 sessions with no more than one session on the same day. Session one, approximately one hour, was a study session during which all 252 Chinese-English vocabulary pairs were randomly presented in intervals of 15 seconds. Sessions 2-13, approximately a half-hour each, were "test and study" sessions which consisted of both study trials and test trials. Session 14, approximately one hour, was a test session, taken three days after session 13, presenting all items randomly for testing. See Table 1. The entire experiment was carried out over five weeks.

Table 1

Session Sequence

| Session Number | Type | Number of Items |
|---|---|---|
| 0 | Study Only | 252 |
| 1-12 | Test and Study | 63-126 |
| 13 | Test Only | 252 |

In both test trials and study trials a set of five numbered English words appeared on the screen, followed by the display of the Chinese character. This set of words consisted of the English equivalent word and four distractor words that were randomly selected, two from the English word set corresponding to the Chinese words and two from the set of English distractors. The position of the words in this answer set is random. S was required to choose and type the number of the English word which he thought corresponded to the Chinese character. In a study trial, the system responded to S by displaying "correct" if the answer were correct and "wrong" if the answer were wrong. The correct answer was displayed following an error response so that S could study the item again. If a subject did not respond within 10 seconds, he was timed-out and treated as if he had made a wrong response. In a test trial, no feedback is given. The purpose of the test trials were to allow the system to obtain more information about the student's state of knowledge on each item and to detect differences in performance among items belonging to different strategies at each session.

Items for test and study sessions were allocated in the following way (See Table 2). 21 items were selected by each of the 3 strategies for study, yielding 63 study items for every session. In addition, 63 test items were randomly selected; 21 items from each of the 3 conditions. If an item were selected for both a study and a test trial, it would be presented only once as a study item, but counted as a test item as well on the subject's total scores for the session. Thus no item appeared more than once during any session.

Table 2

Allocation Of Types Of Items

For Test And Study Session

| CONDITION / PRESENTATION MODE | STRATEGY ONE | STRATEGY TWO | STRATEGY THREE | TOTAL |
|---|---|---|---|---|
| TEST AND STUDY | 21 | 21 | 21 | 63 |
| TEST | 0-21 | 0-21 | 0-21 | 0-63 |
| TOTAL | 21-42 | 21-42 | 21-42 | 63-126 |

Data Collection

On a given session for every item and subject, two response variables were measured by the computer instructional system: 1) correct or incorrect, where time-out was considered as incorrect and 2) response latency, measured as the time interval between the onset of the Chinese character display and the typing of a response by the subject. Time-outs were considered to have the maximum time allowance of 10 seconds. At the end of each day, records of each subject's performance for that day were printed and also updated onto the complete student history file.

# CHAPTER V

## EXPERIMENT ONE: RESULTS AND CONCLUSIONS

### Experimental Hypotheses

Experiment One was designed to test the effectiveness of the adaptive instructional strategies derived from the GFT models against a standard strategy, used as a control. The 3 strategies compared were:

(1) A Standard Strategy which randomly cycles through the list of 84 items, presenting each item once in sessions 2-5, 6-9, 10-13; thus presenting all items an equal number of times in a random cyclical way.

(2) A Homogeneous Parameter Strategy (HOP) which selects items according to MIP principle based on the LS-2 version of the GFT models, assuming average parameters.

(3) A Heterogeneous Parameter Strategy (HEP) which is similar to HOP, but uses additional information about items and subjects in its selection procedure by using heterogeneous parameter estimates.

The experimental hypotheses tested were:

(1) A higher proportion of items would be learned using the adaptive strategies (HOP and HEP) rather than the Standard strategy.

(2) A higher proportion of items would be learned using heterogeneous rather than homogeneous parameter estimates.

## Results

Analysis of Test Data. The curve for performance on test trials and study trials are plotted in Figure 10. The abscissa refers to the pooled response data for all students for four consecutive sessions; the reason for combining the items from four consecutive sessions is that every four session cycle constitutes a complete cycle of item presentation by the Standard strategy. The data from the final test (Session 14) is plotted as cycle 4. Both adaptive strategies yielded a consistently higher learning effect than the standard procedure for test trials during all sessions. In order to test the hypotheses of no difference between strategies, an analysis of variance with repeated measures on one factor was performed on the final test scores. The results are summarized in Table 3.

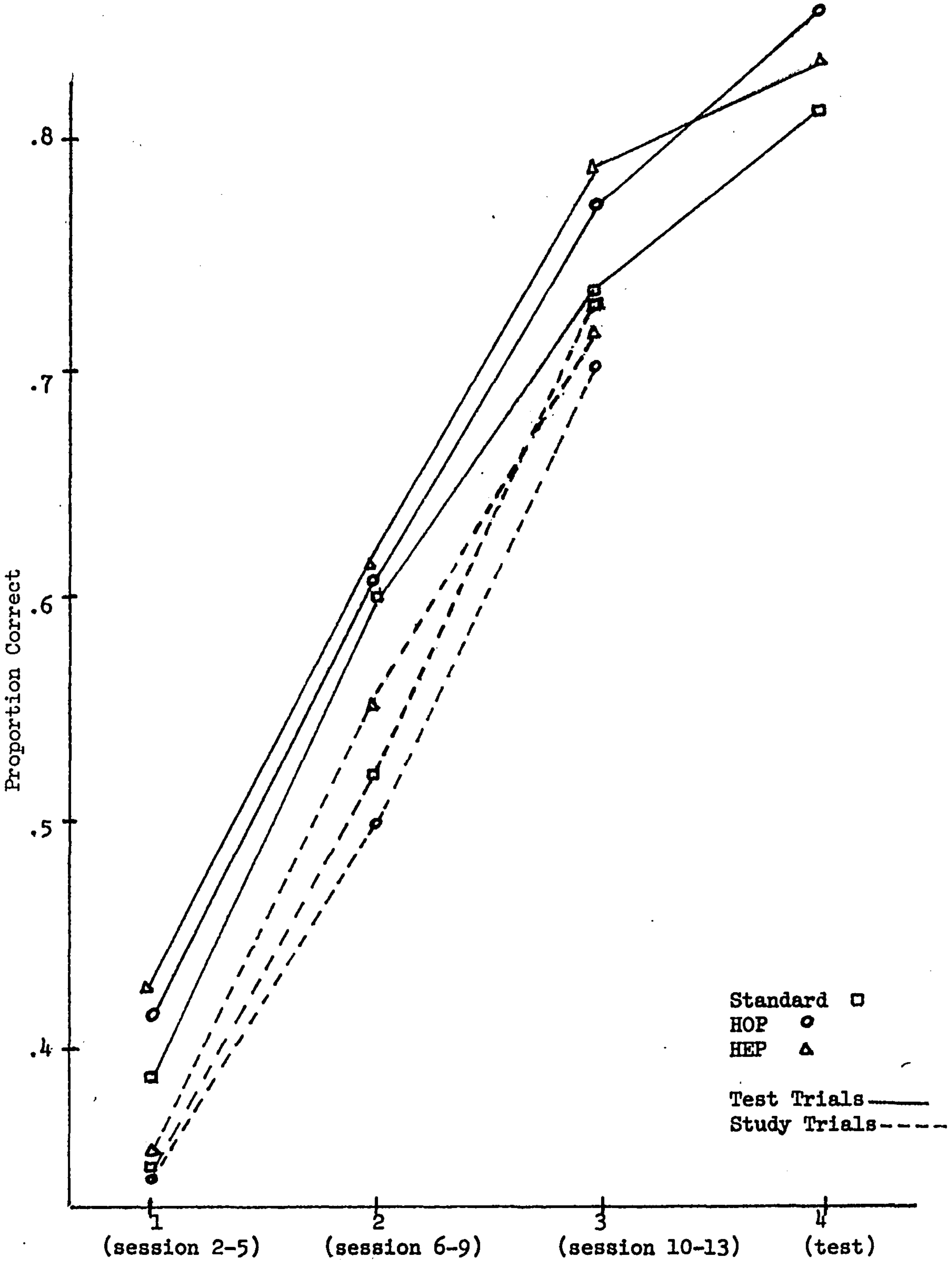



Figure 10

Proportion of Correct Responses for Test Trials and Study Trials
Experiment One

Table 3

Analysis of Variance

Items Wrong on Final Test

Experiment I

| Source of variation | SS | df | Ms | F |
|---|---|---|---|---|
| Between subjects | 2307.56 | 11 | | |
| Within subjects | 269.33 | 24 | | |
| Presentation strategy | 97.39 | 2 | 48.70 | 6.23* |
| Residual | 171.94 | 22 | 7.82 | |

The hypothesis of no effect due to the presentation strategy can be rejected at $\alpha < .01$ ($F_{.99}$ (2,22) = 5.72). The Newman-Keuls test of differences between pairs of scores showed that the difference between HOP-strategy and Standard strategy is significant at $\alpha <$ .01; the difference between HEP-strategy and Standard strategy is significant at $\alpha < .05$; no significant difference existed between the HOP- and HEP- strategies. The experiment demonstrated significant improvements in learning, as measured by performance on final test, due to the implementation of both adaptive strategies. However, no differences were found between the HEP- and HOP- strategies.

A further analysis (Table 4) on the total test items correct on all study sessions also showed significant differences between the adaptive strategies and the Standard strategy, at $\alpha < .05$ ($F_{.95}$ $(2,22) = 3.48$). Contrary to the final test and in accordance with our expectation the HEP-strategy performed slightly better than the HOP-strategy, but the differences were not significant.

Table 4

Analysis of Variance

Test Items Correct on Test and Study Sessions

| Source of variation | SS | df | Ms | F |
|---|---|---|---|---|
| Between sessions | 61284 | 11 | 5571 | |
| Within sessions | 1608 | 24 | 67 | |
| Presentation strategy | 492 | 2 | 246 | 4.84* |
| Residual | 1116 | 22 | 50.72 | |

Analysis of Study Data. From Figure 10 it can be observed that performance on study items was consistently lower than on test items. This result is expected since adaptive presentation strategies tend to select the less learned items for study. There were no significant differences among presentation strategies on study trials. However, it can be observed that the HOP-strategy tended to detect the least well known items better than the other strategies and it presented those more frequently.

Latency for correct responses on study trials is also plotted by strategy (Figure 11) and shows differences among strategies in the expected direction. If one assumes that the items in the unlearned state need a longer response time than the items in the learned or transitory state, then the observed longer latency under adaptive strategies shows that the adaptive strategies were better at selecting unlearned items for studying.

<u>Response Latency Data</u> Average response latencies for correct and wrong responses on all trials are plotted by sessions (Figure 12) to see whether this data could yield useful information concerning the state of the learner. We assumed that a wrong response indicated that the item was in the unlearned state, whereas a correct response could be elicited when an item was in the learned state or the transitory state, or when the subject guessed correctly in the unlearned state. The marked differences in the latency curves suggest that latency information is indicative of the state of the learner and may be valuable in obtaining more accurate estimates of the learning state.

<u>Evaluation of Parameters</u>. The LS-2 version of the GFT models assumes that 1) the probability of entering the learned state is

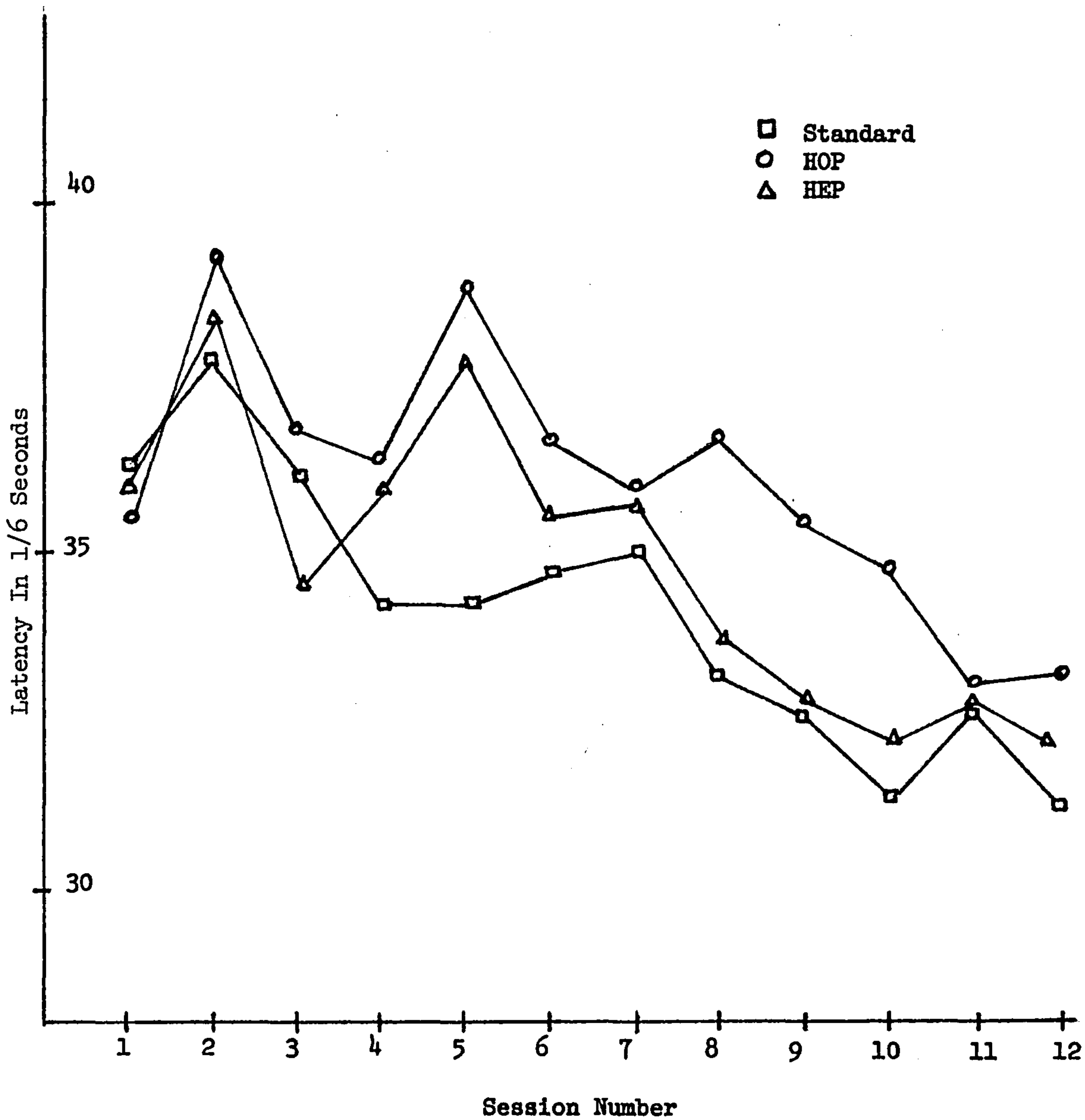



Figure 11

Latency for Correct Responses for Study Trials
Experiment One

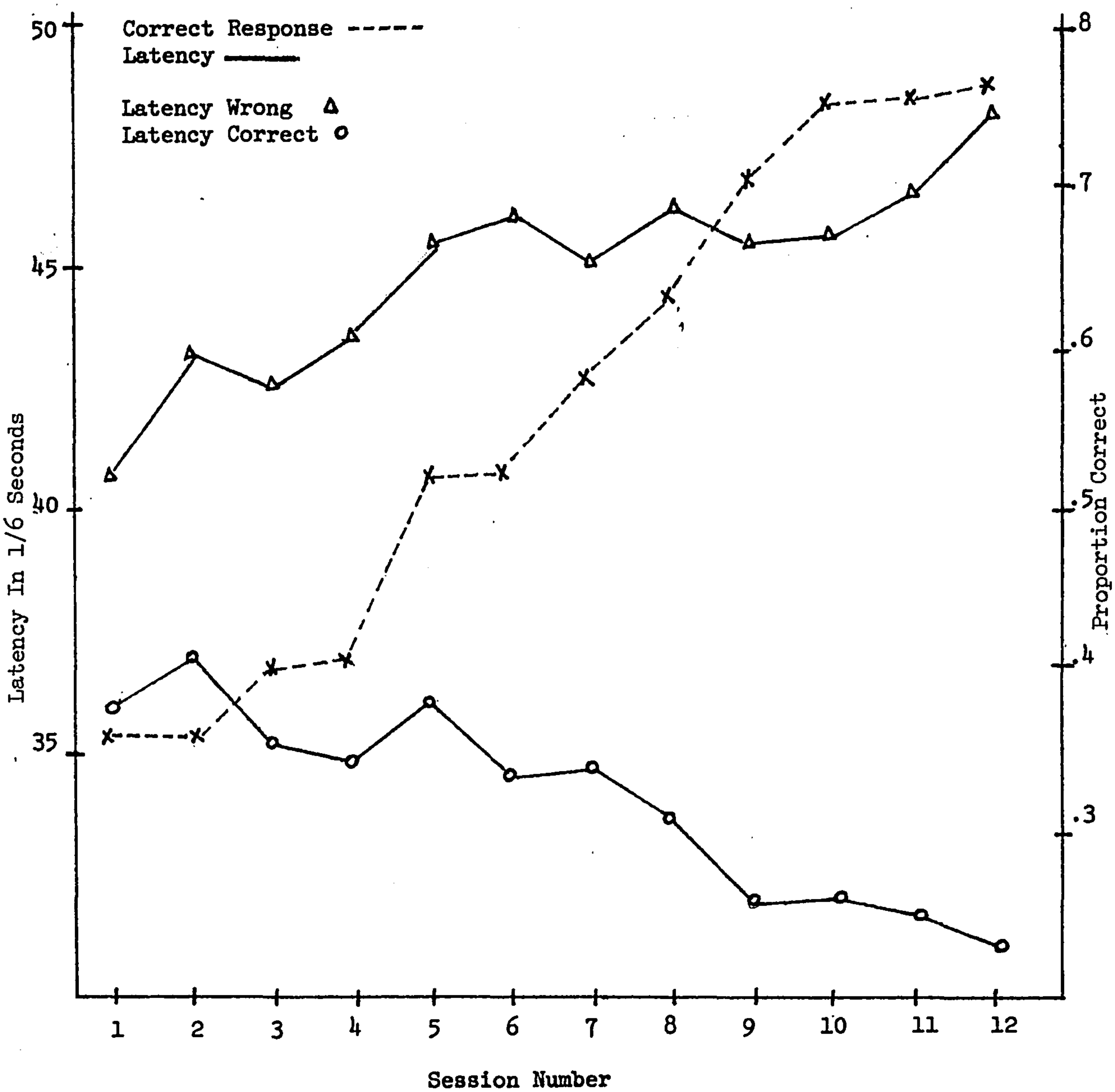



Figure 12

Correct Response Probability and Response Latency as a Function of Session Number Experiment One the same whether an item is in the transitory state or the unlearned state, i.e. parameters a = b and 2) some encoding always occurs when an item is presented, i.e. $\gamma = 1$. Therefore, we only need to estimate 3 parameters: a, c, $\Theta$.

Subject Parameters. In order to demonstrate that the individual subject parameters a, c, $\Theta$ reflect the observed individual differences between subjects, their correlations with the S's total number of correct responses on the final test were computed. These results are shown in Table 5.

Table 5

Correlation of S's Total Number Correct on Final Test with Subject Parameters.

| | a | c | $\theta$ | a+c+$\theta$ |
|---|---|---|---|---|
| S's final test score | .34 | .83 | .70 | .76 |

The high correlation between S's performance and the sum of the parameters indicate that subject parameters predicted final test performance reasonably well. However, the low correlation between parameter a and subject performance indicates that this parameter is not very useful in predicting subject's performance. We would expect parameter a to be a good predictor of subject's performance as parameter a represents the probability of learning on a reinforced trial. The low correlation of parameter a indicates that unrealistic assumptions were made in applying the LS-2 model to the instructional situation.

Item Parameters. The mean and SD for parameter a were .17 and .15 and for parameter c were .59 and .34 . The large SDs indicate the heterogeneity of the items. It was expected that parameter a would be larger than parameter c, since the probability of entering the learned state on a trial where the correct answer is presented for study should be greater than on a trial where the subject is not given a study interval. This result points to the unrealistic assumption of the LS-2 model, which assumed that the probability of entering the learned state is the same in the transitory state as in the unlearned state. The small value of parameter a and the much greater value of parameter c indicate that separate learning parameters need to be introduced to account for learning from each state on a reinforced trial.

## Best Fitting Model

From the data of experiment one, the six submodels of the GFT were compared on the basis of the maximum likelihood fit of the models for the data. The results are shown in Table 6.

Table 6

Results of Model Tests for Experiment One

| Models and Restrictions | Parameters a | b | c | γ | θ | Log Likelihood | Ranking |
|---|---|---|---|---|---|---|---|
| 1: a=b, γ=1 | .19 | .19 | .53 | 1 | .57 | -10153.15 | 5 |
| 2: b=c, γ=1 | .14 | .56 | .56 | 1 | .60 | -9968.83 | 1 |
| 3: a=0, γ=1 | 0 | .45 | .60 | 1 | .69 | -10158.13 | 6 |
| 4: a=b=c | .37 | .37 | .37 | .60 | .77 | -10138.38 | 4 |
| 5: a=0, b=c | 0 | .72 | .72 | .62 | .79 | -10134.74 | 2 |
| 6: b=c=0 | .45 | 0 | 0 | .57 | .79 | -10136.85 | 3 |

The inadequacies of the LS-2 model, model (1), for the instructional situation became apparent. The best fitting model, model (2), introduced separate parameters for learning from the transitory state and the unlearned state on a study trial and assumed that in the transitory state, learning does not differ greatly for a reinforced or an unreinforced trial. The high values for parameter b makes Greeno's contention that learning only occurs in State U (b=0) very tenuous in this context. The next best fitting models (models 4,5,6) were those for which the attention parameter γ was not equal to 1. This result supports

Rumelhart's (1967) findings that the attention parameter affects the fit of the model greatly. The attention parameter can be interpreted as an immediate memory parameter, i.e. the probability that some encoding of the stimulus item was made such that the correct response could be elicited immediately. The introduction of this immediate memory parameter increases the value of parameter $\theta$, the probability of not forgetting in 1 day, as part of the short term forgetting can be interpreted as failure to attend in state U, i.e. $1-\gamma$.

## Conclusions

The results of experiment one confirmed the experimental hypothesis that a higher proportion of items would be learned using the adaptive strategies (HOP and HEP) rather than the Standard strategy. These results indicate that these instructional algorithms that approximate the optimal strategy derived from a learning theory that accounts for forgetting are more effective than the Standard strategy, which is an optimal strategy derived from a simple learning model that does not account for forgetting. This result is especially encouraging because the system was started with no apriori knowledge about the parameters. A further improvement can be inferred for the case where the system could begin with known parameters.

The small difference noted between the homogeneous and heterogeneous parameter strategies did not concur with our expectations, as it was hypothesized that the additional information concerning subjects and item would increase the effectiveness of the adaptive strategy. This small difference may be due in part to the delayed effect of improved parameter estimates. In other words, because the item and subject parameter were not known prior to the experiment and were estimated as instruction proceded, the HEP strategy did not have reliable and stable parameter estimates until late in the experiment. Hence, the advantages of utilizing heterogeneous parameter estimates were not realized during most of the experiment.

The inadequacies of the LS-2 model also became apparent. A model that fits the learning data better would improve the effectiveness of the adaptive strategies. The latency data suggest that student's response time constitutes valuable information for determining his state of knowledge. Incorporating the assumption of state dependent latencies in the learning model should make the state determination more reliable, hence, lead to a better optimization strategy.

# CHAPTER VI

# EXPERIMENT TWO: MODIFICATIONS, RESULTS AND CONCLUSIONS

Experiment two was designed to further explore whether the MIP instructional algorithm derived from the GFT model can be improved by incorporating additional information, such as heterogeneous parameters and state-dependent response latencies, into its decision structure. **Apriori** item parameter estimates and a GFT model (submodel 2) that describes the learning process better, obtained from the data of experiment one, are used for the second experiment.

## Addition of Response Latency for LHEP Strategy

Experimental studies in paired-associate learning have suggested that response latency may be a useful index of the associative strength of learning (Judd and Glaser, 1969). However, this measure has not been used systematically for instructional decisions mainly because latency is not easily measured in most instructional situations. In computer assisted instruction, response latency measures are easily available. Judd and Glaser (1969) suggest that response latency may serve as a supplement to frequency measures in relating student's responses to optimal presentation schemes in computer-based instruction. Including the subject's response on a trial as well as his response latency should lead to a better prediction of student's state of learning and therefore to a more effective instructional strategy.

The question is how can response latency information be incorporated into the instructional decision mechanism. How can response latency be included in the models of the learning and instructional processes? From experiment one and from previous studies on latency (Judd and Glaser,1969; Suppes,1966), two characteristics of response latency seem evident. First, there is a wide variability in response latency between Ss. Secondly, latency for correct and wrong responses are significantly different. Making the two assumptions that latency on any item is dependent only upon the subject and the state of learning for that item, we can relate latency data to the GFT models in the following way.

Let us define $\lambda(z_i,k)$ as the probability of a response latency $z$ on trial $i$ given that the item is in state $k$. The following table represents the relationship between a student's learning state and his response and latency response probabilities:

| State | Response $x_i = 0$ | $x_i = 1$ | Latency $z_i$ |
|---|---|---|---|
| L = 2 | 1 | 0 | $\lambda$ (z,2) |
| T = 1 | 1 | 0 | $\lambda$ (z,1) |
| U = 0 | g | 1-g | $\lambda$ (z,0) |

In order to estimate $\lambda$ (z,k) for k = 0, 1, 2, we make the assumption that response latency is normally distributed for each state with a different $\mu_k$ for each state but all with the same variance. We need to estimate four latency parameters ($\mu_0$, $\mu_1$, $\mu_2$, $\sigma$) for each subject from the latency data. To obtain $\mu_0$ and $\sigma$, we can obtain the mean and variance of response latencies for each subject's error responses, since all error responses imply that the subject is in the unlearned state. With $\mu_0$, $\sigma$, and all other parameters in the model fixed, we can then use the iterative up and down walk search method for obtaining the maximum likelihood estimates on the response sequences for parameters $\mu_1$ and $\mu_2$.

Adding latency information to the response history, the complete response history for a student now becomes a sequence with four arguments; that is, $y_i = (m_{i-1}, d_i, x_i, z_i)$. Introducing the latency probability $\lambda$ (z,k) to equation (3) for computing the likelihood of a response sequence, we have the equation:

$$P(y^i) = \sum_{j_o=o}^{2} \{\pi_o(j_o) \sum_{j_1=o}^{2} t(j_o,j_1,u_1)\ r(j_1,x_1)\ \lambda(z_1,j_1)$$

$$\sum_{j_2=o}^{2} \{t(j_1,j_2,u_2)\ r(j_2,x_2)\ \lambda(x_2,j_2) \ldots$$

$$\sum_{j_i=0}^{2} \{t(j_i-1,j_i,u_i)\ r(j_i,x_i)\ \lambda(z_i,j_i)\}\ldots\}\} \qquad (5)$$

To compute $P(y^i)$, we note that the above equation can be simplified to a backward recursive algorithm as shown earlier on page 41. A computer program is written to do this computation.

To derive a MIP strategy that uses the latency data, we need to modify the algorithm for determining the state of knowledge of the learner to include latency information. This is done by adding the term $\lambda(z_i,k)$ to equation (2). Thus we obtain:

$$\pi_{i+1}(j) = \frac{\sum_{k=u}^{2} \pi_i(k)\ t(k,\ell,u_i)\ r(k,x_i)\ \lambda(z_i,k)}{\sum_{k=0}^{2} \pi_i(k)\ r(k,x_i)\ \lambda(z_i,k)} \qquad (6)$$

Equations 5 and 6, then specify how state-dependent response latencies can be incorporated into the adaptive instructional algorithms derived from GFT models. Using heterogeneous parameters and this latency information in computing the maximal immediate payoff, we have the LHEP strategy.

## Experimental Hypotheses

In the second experiment, we compared the effectiveness of the three adaptive strategies, HOP, HEP, and LHEP. We wanted to see whether the adaptive strategy can be improved by using more refined techniques such as heterogeneous parameters and latency information. The hypotheses tested were:

(1) Heterogeneous parameter strategies (HEP and LHEP) would be more effective than homogeneous parameter strategy.

(2) The strategy that uses response latency information (LHEP) would be more effective than the strategies (HOP and HEP) that do not.

For all strategies, the best fitting submodel (model 2) was used to describe the learning situation and the system began with apriori item parameters obtained from experiment one. The same experimental procedures and methods employed in experiment one were applied to the second experiment, except that study session Ø was omitted and sessions 1 and 2 consisted of all study trials. The reason for this change was that the 1-hour study session Ø was felt to be too long by the subjects, and thus was broken down to two separate study sessions.

## Results

### Analysis of Test Data

Test Scores. Performance data, consisting of proportion correct and latencies for correct responses on test trials, are used to compare the effectiveness of the different strategies. The curves for the proportion correct on test trials and for the correct response latencies are plotted in Figures 13 and 14. Figure 13 indicates little differences in the learning curves under each strategy for test trials. In order to test the hypothesis of no difference among procedures, an analysis of variance with repeated measures on one factor was performed on the results of the final test and on total test items correct on study sessions. The results are summarized in Tables 7 and 8.

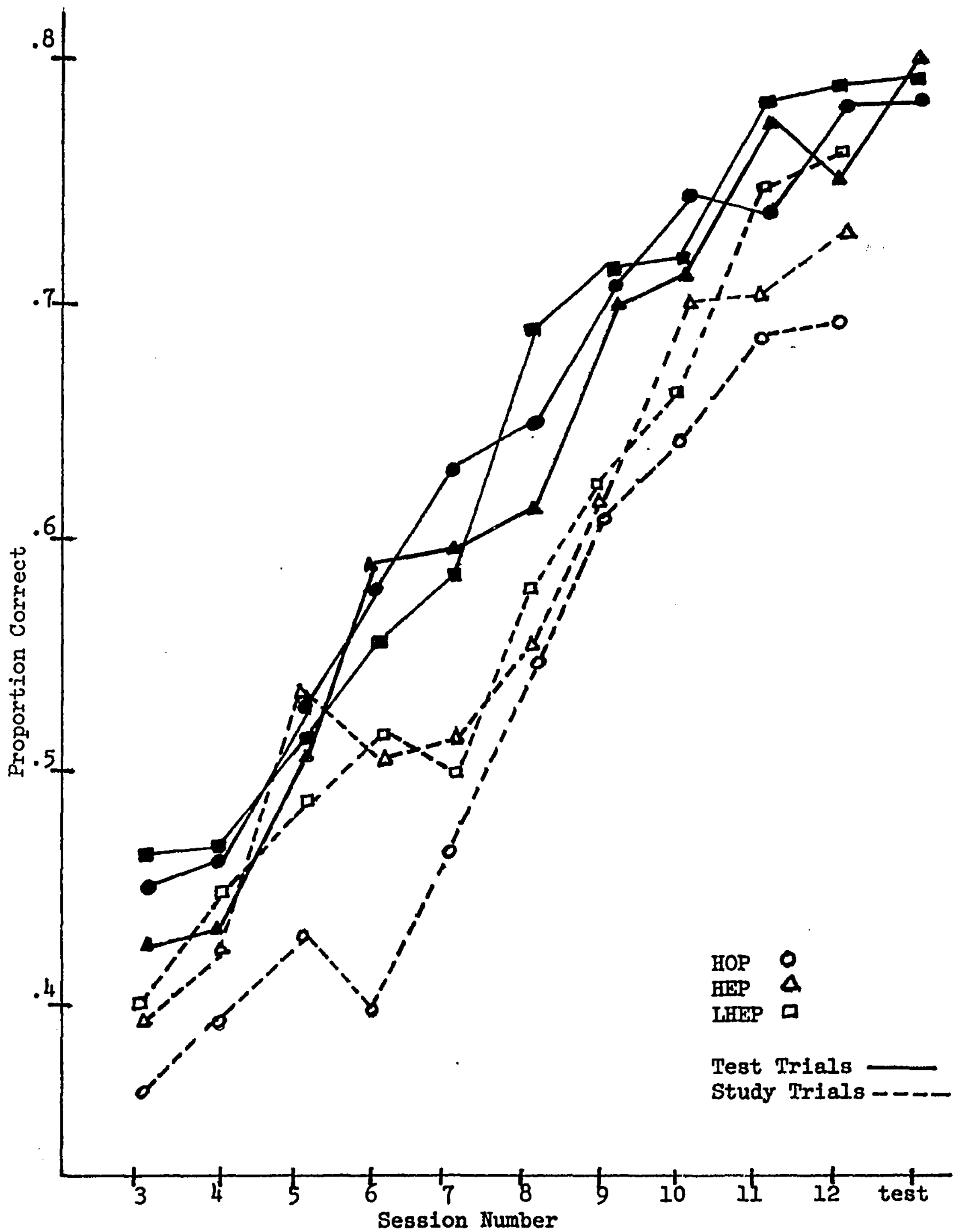



Figure 13

Proportion of Correct Responses for Test Trials and Study Trials
Experiment Two

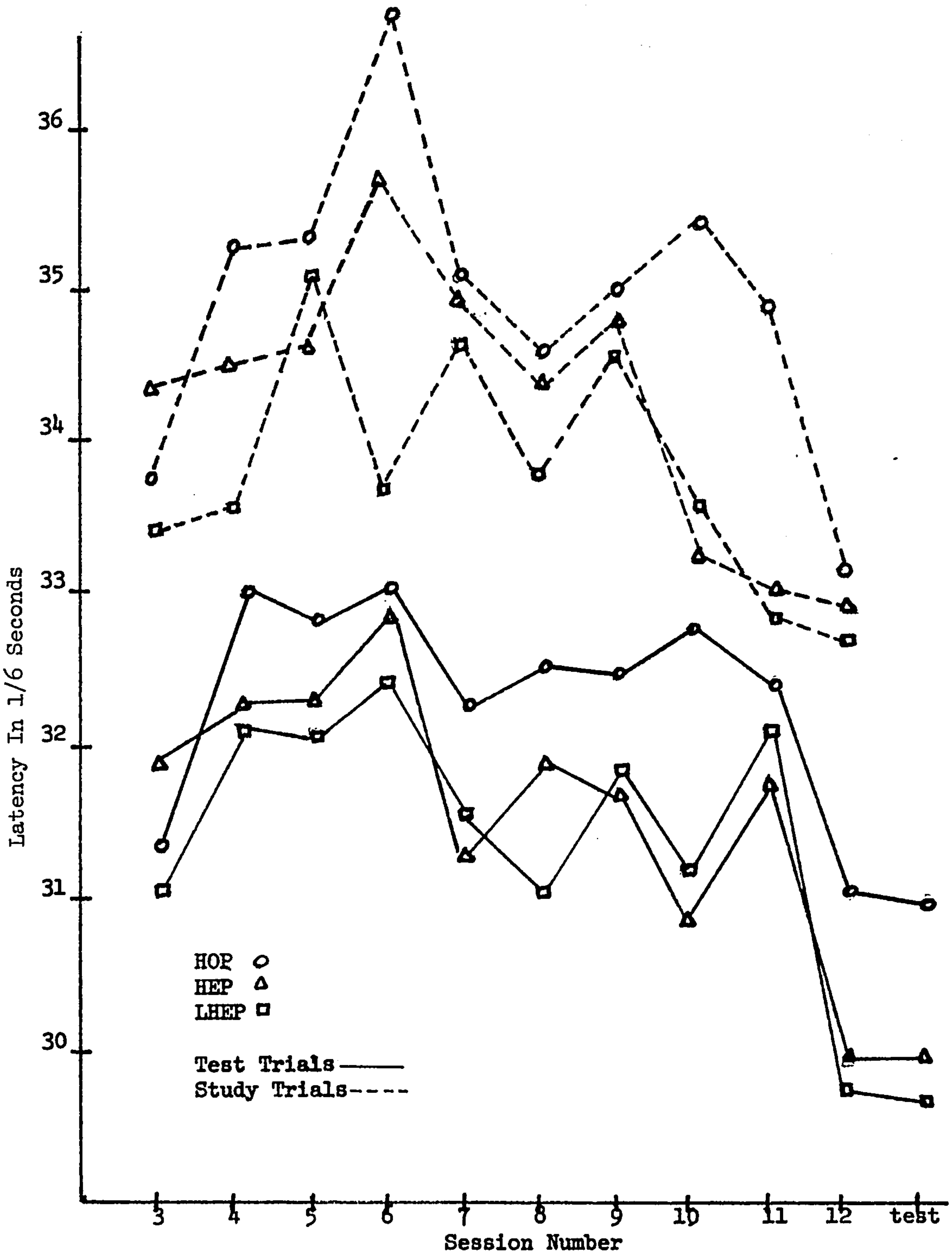



Figure 14

Latency for Correct Responses for Study Trials and for Test Trials
Experiment Two

Table 7

Analysis of Variance

Items Wrong on Final Test

Experiment Two

| Source of Variation | SS | df | MS | F |
|---|---|---|---|---|
| Between subjects | 3282.56 | 11 | | |
| Within subjects | 236.33 | 24 | | |
| Presentation strategy | 20.22 | 2 | 10.11 | 1.03 |
| Residual | 216.11 | 22 | 9.82 | |

Table 8

Analysis of Variance

Total Test Items Correct for Sessions 3-12

Experiment Two

| Source of Variation | SS | df | MS | F |
|---|---|---|---|---|
| Between sessions | 151264.97 | 9 | | |
| Within sessions | 583.34 | 20 | | |
| Presentation strategy | 57.06 | 2 | 28.53 | 1.00 |
| Residual | 515.34 | 18 | 28.63 | |

No significant differences among strategies were found on the final test ($F_{.95}(2,22) = 3.44$) or on the total test items correct on study sessions ($F_{.95}(2,18) = 3.55$).

Response Latency Correct. Figure 14 shows that latency correct for test items was consistently higher for the HOP stategy. An analysis of variance on the latencies correct for the final test (Table 9) leads to a rejection of the hypothesis of no difference due to presentation procedure at $\alpha < .05$ ($F_{.95}(2,22) = 3.44$). Contrasts reveal that response time under HEP and LHEP strategies are both significantly shorter than under HOP strategy at $\alpha < .05$. The difference between LHEP and HEP strategies are not significant.

Table 9

Analysis of Variance

Latencies Correct on Final Test

Experiment Two

| Source of Variation | SS | df | MS | F |
|---|---|---|---|---|
| Between subjects | 576.91 | 11 | | |
| Within subjects | 41.33 | 24 | 5.54 | 4.01* |
| Presentation strategy | 11.07 | 2 | 1.38 | |
| residual | 30.26 | 22 | | |

Similarly, significant differences ($\alpha < .01$, $F_{.99}(2,18) = 6.01$) are found among strategies for average latencies correct on test items during the test and study sessions (Table 10). A contrast shows that the differences between HOP and HEP is significant at $\alpha < .05$, and between HOP and LHEP is significant at $\alpha < .005$. No significant differences exist between the heterogeneous parameter strategies although response time is shorter for the LHEP strategy.

Table 10

Analysis of Variance
Latencies for Correct Responses on Test Items
During Sessions 3-12

| Source of Variation | SS | df | MS | F |
|---|---|---|---|---|
| Between sessions | 23.42 | 9 | | |
| Within sessions | 12.61 | 20 | | |
| Presentation strategy | 6.4 | 2 | 3.2 | 9.41* |
| Residual | 6.2 | 18 | .34 | |

The results of the test data for proportion of correct responses do not support the experimental hypotheses of differences among the effectiveness of selection strategies under consideration. These results are in agreement with the results of experiment one; that is, adaptive strategies using heterogeneous parameters for item difficulty and subject ability do not result in improvement of learning for the instructional situation under investigation. The improved learning model and improved item parameter estimates obtained from experiment one did not contribute to a more effective heterogeneous parameter strategy. Furthermore, the addition of latency information to the selection strategy did not result in improvement of the adaptive strategy.

However, the results of the latency data on test items do support the experimental hypotheses. Subjects responded significantly faster to items selected by the heterogeneous strategies, indicating that they learned these items better, if response latency is considered a measure of how well an item is learned. But the actual differences in response time is small. For the final test, the average difference between HOP and heterogeneous parameter strategies is about 1/6 second. Practically speaking, a gain in response time of 1/6 second is not impressive.

In conclusion, subjects did not learn significantly more items under any strategy, but the items selected by the heterogeneous parameter strategies were learned a little better.

## Analysis of Study Data

The adaptive strategies were designed such that they would select the items for study that are not yet learned but most likely to enter the learned state upon the next presentation. According to this MIP principle, performance on the test trials should be better than performance on the study trials, as the items selected for studying are those that subjects have not formed permanent associations for, whereas the test items are randomly selected from the entire list of items. Comparisons of average latencies for correct responses and of proportion correct on test trials and on study trials by sessions can be seen from Figures 13 and 14. Both figures indicate that performance on the test trials is consistently better than performance on the study trials, showing that the adaptive strategies were indeed selecting items that the subjects had not yet learned. Tests for differences between means for correlated samples indicate that for both the proportion of correct responses and for the response latencies correct, the difference between test trials and study trials is significant at $\alpha < .001$.

Comparison of Strategies for Study Trials

From Figure 13 it can be observed that the proportion correct on study trials was consistently lower for items selected by the HOP strategy than those selected by the heterogeneous parameter strategies. An analysis of variance of the total items correct on study trials by strategy (Table 11) indicates significant differences among strategies at $\alpha < .01$

($F_{.99}(2.18) = 6.01$). Contrasts reveal that the differences between HOP and HEP strategies and between HOP and LHEP strategies are both significant at $\alpha < .01$. No significant differences between HEP and LHEP strategies exists, although performance is slightly better for the LHEP items. These results indicate that the HOP strategy is presenting more of the less learned items than the heterogeneous parameter strategies.

Table 11

Analysis of Variance
Total Study Items Correct During Sessions 3-12
Experiment Two

| Source of Variation | SS | df | MS | F |
|---|---|---|---|---|
| Between sessions | 22288.8 | 9 | | |
| Within sessions | 24.8 | 20 | | |
| Presentation strategy | 1189.4 | 2 | 594.7 | 8.71* |
| Residual | 1228.4 | 18 | | |

This result is also supported by the latency data in Figure 14. The response times for HOP items were consistently longer than they were for the heterogeneous parameter items. However, an analysis of variance of correct response latencies for study trials (Table 12) shows no significant differences among strategies ($F_{.95}(2,18) = 3.55$).

Table 12

Analysis of Variance

Latencies Correct for Study Trials During Sessions 3-12

Experiment Two

| Source of Variation | SS | df | MS | F |
|---|---|---|---|---|
| Between sessions | 21 | 9 | | |
| Within sessions | 22.2 | 20 | | |
| Presentation strategy | 5 | 2 | 2.5 | 2.6 |
| Residual | 17.2 | 18 | .96 | |

Both the latency data and the proportion correct data for study trials support the findings of experiment one (Figures 10 and 11); i.e., the homogeneous parameter strategy selects items for study that are less well learned than the heterogeneous parameter strategies. However, the actual differences in the number of correct responses averages to be 1 item for each subject for each session, which practically speaking is not large.

Evaluation of Parameters

Item Parameters. The GFT model used in experiment two had the two restrictions parameters $b = c$ and parameter $\gamma = 1$. The mean and SD for parameter a were .23 and .19, and for parameter b were .37 and .24. A frequency plot of the values of parameters a and b (figure 15) also shows the wide range and large variability of these parameters, demonstrating that the items used were indeed heterogeneous. Correlation between parameters a and b was .25.

Subject Parameters. Subjects performance on final test was correlated with subject parameters for the six submodels of the GFT. The results are shown in Table 13. The total number wrong on the final test was used as a measure of subject performance; therefore, the correlations have negative values. The high correlation

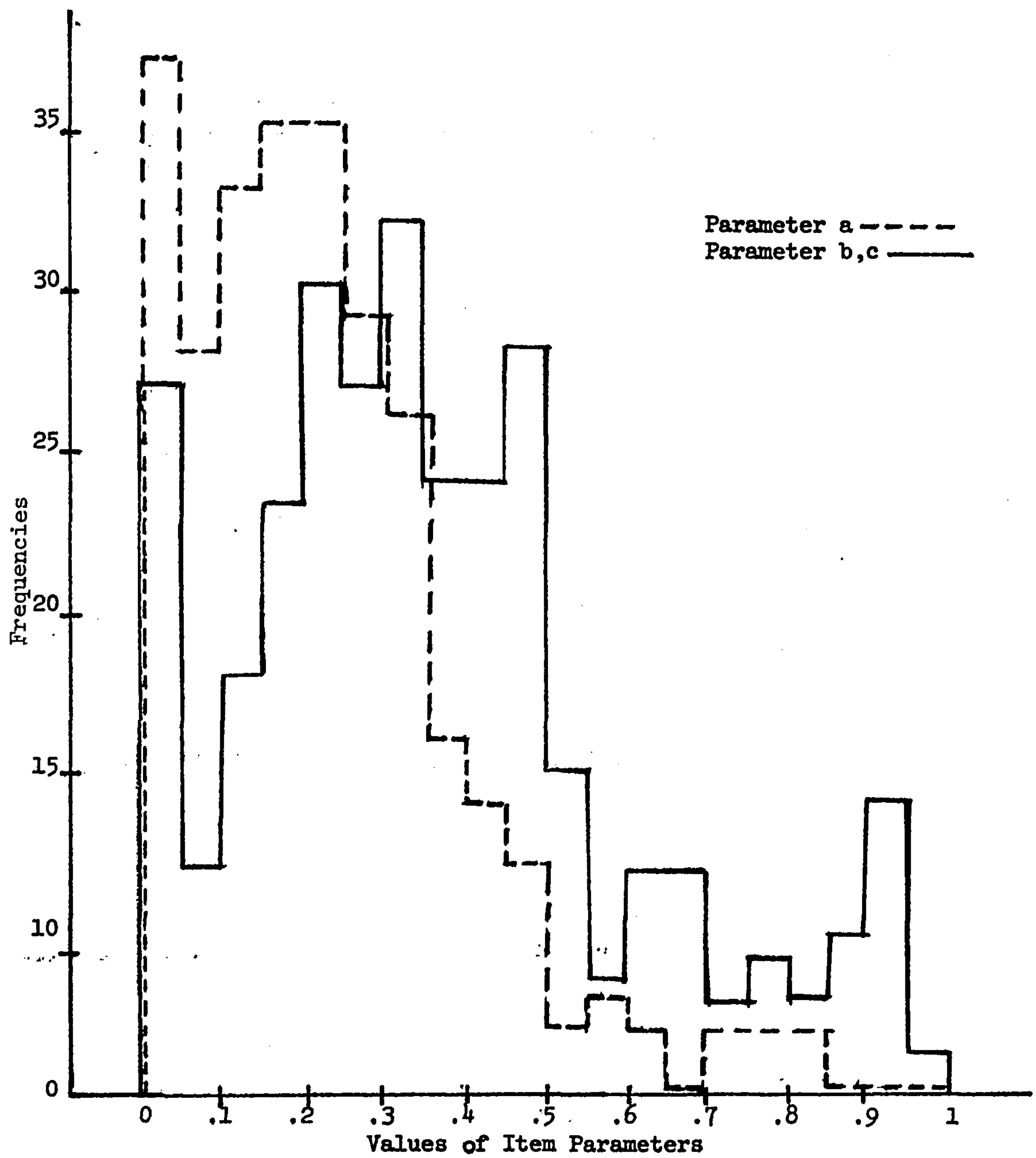



Figure 15

Frequency Distributions of Item Parameter Values
in Intervals of .05
Experiment Two

between subject parameters and subject performance indicate that the subject parameters predict the subject's performance well. Of the six models, the 3 models (models 4, 5, 6) whose parameters correlated higher with subject performance have the common characteristic that parameter $\gamma \neq 1$; that is, the subject does not always attend the presentation. The parameters of model 2, the model used in the experiment, did not correlate as highly as the other model parameters. This lower correlation implies that model 2 may not be the best description of the learning processes for this experiment.

Table 13

Correlation of S's Total Number Wrong on Final Test with the Sum of Subject Parameters for Different Models

| | Sum of Subject Parameters | | | | | |
|---|---|---|---|---|---|---|
| | Model | | | | | |
| | 1 | 2 | 3 | 4 | 5 | 6 |
| S's final test score: number wrong | -.61 | -.64 | -.59 | -.87 | -.77 | -.92 |

Latency Parameters

The average response latency for correct and wrong responses for all subjects is plotted by sessions in Figure 16. Again, there were marked differences in the response time for correct and wrong answers, indicating that response latency is a good indicator of the state of the learner. The curves also show a wider range and larger variance for response latencies for the wrong responses. The mean and SD for incorrect responses by sessions were 7.1 seconds and .4 seconds, and for correct responses were 5.5 seconds and .15 seconds. In the experiment, we assumed that latency was dependent upon the subject and state, while the variance in response time would be equal for all states for each subject; i.e., variance was dependent only on subject, and not the state. This assumption is unrealistic, as the data indicates that the variance for incorrect responses is much greater than for correct responses.

The latency parameters from experiment two are shown in Table 14. These parameters support the notion that the response time varies for different subjects according to the different states that they are in. However the $\sigma$ obtained from the incorrect responses and used for all states is large. The difference between $\mu_2$ and $\mu_0$ is almost always greater than 1 SD; however, the differences between $\mu_0$ and $\mu_1$ and between $\mu_1$

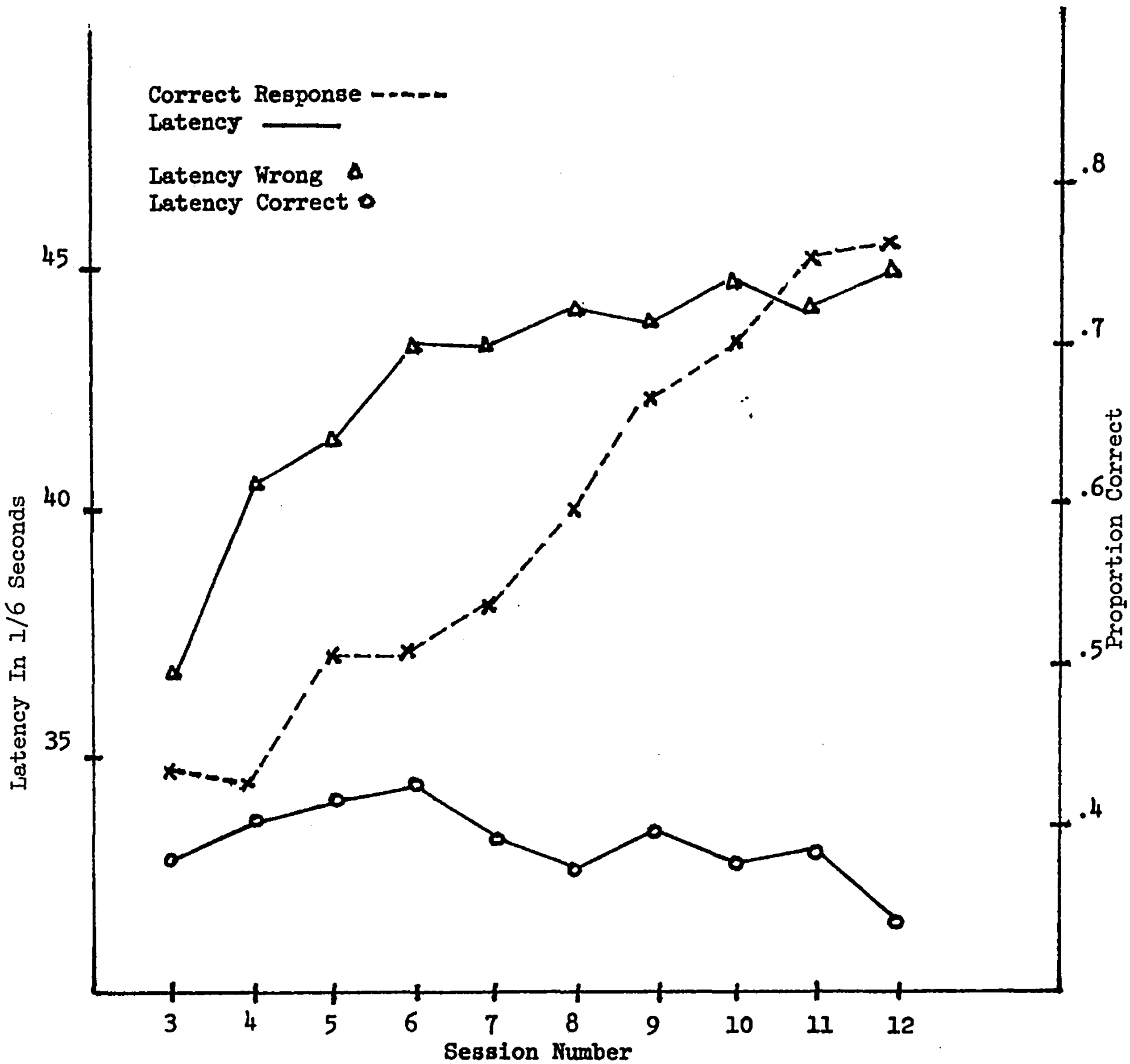



Figure 16

Proportion of Correct Responses and Response Latency
as a Function of Session Number
Experiment Two

Table 14

Latency Parameters from Experiment Two

| Subject | $\mu_0$ | $\mu_1$ | $\gamma_2$ | $\sigma$ |
|---|---|---|---|---|
| 1 | 6.3 seconds | 4.8 seconds | 3.5 seconds | 1.8 seconds |
| 2 | 7.2 " | 5.8 " | 4.3 " | 1.8 " |
| 3 | 6.1 " | 4.5 " | 3.3 " | 1.8 " |
| 4 | 6.5 " | 5.9 " | 4.2 " | 2.0 " |
| 5 | 6.4 " | 5.0 " | 3.5 " | 2.0 " |
| 6 | 6.7 " | 6.0 " | 4.8 " | 2.0 " |
| 7 | 7.4 " | 5.0 " | 4.5 " | 2.1 " |
| 8 | 7.5 " | 5.2 " | 4.2 " | 1.8 " |
| 9 | 5.7 " | 4.3 " | 3.8 " | 1.9 " |
| 10 | 7.0 " | 6.5 " | 5.0 " | 2.0 " |
| 11 | 5.2 " | 4.8 " | 3.8 " | 1.8 " |
| 12 | 6.9 " | 6.0 " | 4.8 " | 2.0 " |

and $\mu_2$ are not greater than 1 SD. It is probably erroneous to assume the same variance for latencies for different states. Obtaining separate $\sigma_o$ , $\sigma_1$ , $\sigma_2$ would improve the usefulness of the latency data for state determination. We conclude that in the present experiment the large variance in the state latency parameters reduced the sensitivity of latency strategy to detect differences in state from the latency information. A transformation of the means and variances of the latencies to eliminate the effects of the correlation that necessarily exists might serve to improve the effectiveness of the latency measures.

## Best Fitting Models

Table 15

Results of Model Tests for Experiment Two

| Model and Restrictions | Parameters | | | | | Log Likelihood | Ranking |
|---|---|---|---|---|---|---|---|
| | a | b | c | γ | θ | | |
| 1 : a=b, γ=1 | .28 | .28 | .37 | 1 | .59 | -12531.44 | 5 |
| 2 : b=c, γ=1 | .25 | .34 | .34 | 1 | .60 | -12455.31 | 4 |
| 3 : a=0, γ=1 | 0 | .54 | .43 | 1 | .73 | -12755.29 | 6 |
| 4 : a=b=c | .40 | .40 | .40 | .53 | .77 | -12328.85 | 1 |
| 5 : a=0, b=c | 0 | .52 | .52 | .53 | .88 | -12430.69 | 3 |
| 6 : b=c=0 | .47 | 0 | 0 | .54 | .73 | -12347.85 | 2 |

The six submodels were again compared on the basis of the maximum likelihood fit of the models for the data of experiment two. The results are shown in Table 15. These results indicate certain characteristics about the learning processes. That is, some encoding does not always take place upon the presentation of an item. Secondly, neither parameter a nor parameter b are equal to zero: i.e. some learning takes place whether from the transitory or the unlearned state. Thirdly, parameters b = c; that is, in the transitory state the complete association has been formed and needs only additional practice (regardless of reinforcement) to be stored permanently. The GFT model with those characteristics would be the best description of the learning process investigated in this study.

In general, the differences in the likelihood fit of the models for the data of both experiment I and II are not large, indicating not a great deal of differences in the submodels. The main difference in the ranking of the models for experiment I and II is that in the second experiment, the models where the attention parameter $\gamma$ is not equal to 1 fit the data better. Otherwise, the ranking of the models does not differ greatly. This difference can be attributed to the fact that subjects in the first experiment attended the lessons better than the subjects in the second experiment.

Discussion

Experiment I failed to demonstrate that the heterogeneous strategy was more effective than the homogeneous parameter strategy for the instructional situation under investigation. It was concluded that an improved learning model and apriori parameter estimates may lead to improvements in the heterogeneous parameter strategy. Such modifications were made to the second experiment. However, the results of experiment II failed also to confirm the experimental hypothesis that the efficiency of the adaptive strategy using homogeneous parameters could be further improved by using heterogeneous parameters and latency information. The failure of further improvements in instructional efficiency by the more refined techniques, such as mathematical models of learning with heterogeneous parameters and state dependent response latency, indicates that the computationally simpler strategy, the homogeneous parameter strategy, would suffice for the instructional situation under consideration. However, the question still remains, why did heterogeneous parameter strategies not result in improved learning outcomes?

Previous research on using heterogeneous parameters for optimization strategies has been scarce; both theoretical and experimental ground work is lacking. Intuitively, we believe that a strategy that is sensitive to item difficulty and subject ability should make the strategy more effective. But the effectiveness of a strategy is dependent on many other factors including: 1) the adequacy of the learning model used; 2) the principle used for selecting the items; 3) the instructional material and instructional situation. We need to ask under what conditions does using heterogeneous parameters improve the effectiveness of the optimization strategy or how does using heterogeneous parameters interact with the above 3 factors.

## The Optimality of the MIP Principal for GFT Models with Heterogeneous Parameters

Studies which have demonstrated the optimality of the MIP principal have mostly dealt with simpler learning models and assumed homogeneous parameters. The question is how optimal is this one-step ahead maximization principal when heterogeneous parameters are used with a learning model that includes forgetting. Matheson (1964) showed by computational examples that small additional gains can be made by using heterogeneous parameters for the one-element model, if a j-step (j > 1) maximization procedure were used. Calfee's simulation study (1970) also supports the notion that there is little gain in using heterogeneous parameters when a MIP strategy is used for the one element model. Perhaps heterogeneous parameter strategies require more than looking at the immediate gain in order to be more effective.

In the present study, we used the one step maximization procedure which defines the instructional gain as the probability of an item entering the learned state on the next presentation trial. For the simple 2-state all-or-none model, this gain is computed as the probability that an item is in the unlearned state times the probability of moving to the learned state. For the 3-state GFT model with forgetting, the immediate gain is computed as the probability that an item would enter the learned state from either the unlearned state or the transitory state on the next presentation trial, that is, the gain $\Delta_{i+1}$ for presenting an item for study on trial i + 1 is :

$$\Delta_{i+1} = a\gamma\ \pi_i(0) - b\gamma\ \pi_i(1)$$

where $\pi_i(0)$ and $\pi_i(1)$ are the probabilities of being in the unlearned state and the transitory state on trial i, and parameter a and parameter b are transition probabilities, and parameter $\gamma$ is the attention parameter. This method of computing the gain ignores the learning that occurs in the transition from the unlearned state to the transitory state; that is, we are discounting the additional learning gain $\pi_i(0)\ \gamma(1-a)$ , where $\gamma(1-a)$ is the probability of moving from the unlearned state to the transitory state.

To see the differences in the gains due to a policy which includes this additional gain, we computed and compared the gains by the maximal immediate payoff principle (MIP) and by the MIP principle which includes the state 0 to state 1 transition gains ($MIP^+$) for items of easy, average and hard difficulty levels and for various state distributions. The results are shown in Table 16. From this table we observe that the gains computed by the MIP and $MIP^+$ principles are vastly different. The differences are especially large for the difficult items that are less well learned; that is, when parameter a is small (thus (1-a) is large) and when $\pi(0)$ is large. The MIP principle, by discounting the learning gain $\pi(0)\gamma(1-a)$ is ignoring a great deal of the potential of entering the learned state. As a result, the selection behavior of the MIP and the $MIP^+$ policies would be quite different. The MIP principle would present the easier items that are better learned before the harder items that are less well learned, whereas the $MIP^+$ principle would select harder items for studying. Thus, we conclude that the MIP principle may not be a good selection principle to use for GFT models with heterogeneous parameters, because it ignores too much of the transitory learning gains, especially for difficult items. The MIP principle does not have the same effect on the GFT models with the homogeneous parameters, as the amount of transitory learning discounted is the same for all items, such that a MIP or a $MIP^+$ principle would not

Table 16

Comparison of Instructional Gains Computed by the MIP Principle and by the $MIP^+$ Principle, Given the State of Item and Difficulty of the Item

| State of Item | | | Gain by MIP $\Delta$ | | | Gain by $MIP^+$ $\Delta^+$ | | |
|---|---|---|---|---|---|---|---|---|
| (0) | (1) | (2) | Easy | Average | Hard | Easy | Average | Hard |
| 80 | 15 | 5 | .83 | .26 | .08 | .91 | .85 | .85 |
| 60 | 30 | 10 | .76 | .26 | .12 | .82 | .70 | .70 |
| 40 | 40 | 20 | .65 | .24 | .15 | .69 | .54 | .54 |
| 20 | 50 | 30 | .54 | .22 | .18 | .56 | .37 | .38 |
| 10 | 30 | 60 | .31 | .13 | .11 | .32 | .20 | .20 |
| .05 | .05 | .90 | .08 | .03 | .02 | .09 | .07 | .07 |

| Parameter values for | a | 1-a | b | $\gamma$ |
|---|---|---|---|---|
| Easy item | .91 | .09 | .72 | 1 |
| Average item | .26 | .74 | .34 | 1 |
| Hard item | .03 | .97 | .35 | 1 |

result in very different selection behaviors for homogeneous items.

The experimental results support the fact that the HEP strategies selected easier items for studying than the HOP strategy did. Thus, performance on study trials was lower for HOP items than for HEP and LHEP items. It would be worth while to investigate experimentally whether the addition of the state 0 to state 1 transition gains to the gains computed by the MIP principle would make a practical difference in the learning outcome for the GFT models with heterogeneous parameters.

Considerations of the Instructional Situation

Atkinson (1972) carried out the only other experimental study comparing the effectiveness of an adaptive strategy which used heterogeneous item parameters with one that used average and equal parameters. Atkinson used a similar 3-state learning model and the same immediate payoff principal for the selection strategies as the present study. However, contrary to the findings of this study, Atkinson found that using heterogeneous parameters for an adaptive strategy resulted in a 25% increase in learning over the strategy that used homogeneous parameters. Why are there conflicting results between the two studies which basically used the same theory for describing the learning processes and same selection procedure?

A comparison of Atkinson's experiment with the present one indicates major differences in the instructional situations for which these strategies were investigated. The two studies were similar only in that both involved the learning of foreign language vocabulary: German and Chinese. In Atkinson's study, the learning occurred over a two hour study session which consisted of 336 study trials. A test of all items was administered 7-8 days later to measure the amount of learning. The entire list of items to be learned consisted of 84 German words, which were broken down into 7 lists of 12 words each. Each of the 7 lists was displayed 48 times in a round-robin way. A trial consisted of displaying one list on the screen and presenting one item from the list for study. Thus a selection strategy was selecting 1 item from 12, making the selection ratio 1/12.

We observe that some of the major differences between the two experimental situations are:

1. Learning occurred over 1 long session vs. 12 shorter sessions spread throughout 5 weeks.
2. Testing occurred 7-8 days after the study trials were completed vs. test trials were interspersed with study trials throughout the sessions.
3. More selection stages with fewer items selected at each stage and a lower selection ratio vs. fewer selection stages with more items selected at each

stage and a higher selection ratio. The selection ratio refers to the proportion of the number of items chosen to the number of items selected from at each selection stage. In Atkinson's study, the selection ration is 1/12; in our study, the selection ratio is 21/84 or 1/4.

4. The average number of presentations for an item is higher in Atkinson's study.
5. The number of items to be learned by each subject was much greater in our study.

The two factors in the instructional situation of the present study that most likely negated the differences that may have existed among the instructional strategies were the frequent test trials and the high selection ratio. The learning models showed that given that an item is partially learned, i.e. in the transitory state, the subject learns as much on reinforced as on unreinforced trials. Since almost half the trials in the experiment for all strategies were test trials, a great deal of learning for all strategies occurred because of the testing. the too frequent test trials may have cancelled out differences in learning due to different selection strategies. The purpose of the test trials were 1) to obtain more data for the estimation of parameters and 2) to observe how the strategies differ in learning outcomes at various stages of the instructional process. This frequent testing would not be necessary for real instructional situations. In future studies, it would be advisable to eliminate or to reduce the test trials during the learning stages.

Secondly, the selection ratio may have been too high to be sensitive to a difference in the learning gain detected by the use of heterogeneous parameters. In other words, basically all 3 strategies (HOP, HEP,LHEP), are alike in that they adapt to individual students by utilizing subject history information for computing the instructional gains. Even though item difficulty and subject ability are reflected in the subject history, the heterogeneous parameters offer additional refinement on accuracy in computing the instructional gain. However, when the selection ratio is high (i.e., selecting 21 items from 84), the additional differences in gains detected by a heterogeneous parameter strategy may not make any difference in the actual items that were selected. In effect then, even though the gains computed by different strategies may have been different, the 21 items with the largest gain may be the same items for all strategies. All strategies may select the same items for studying or the overlap in the items selected may be so large that there would be no differences among the strategies in the learning outcome. We observe that in Atkinson's study, the selection ratio was much lower: 1/12, such that the chances that different strategies would select the same items are much lower. This may explain why the heterogeneous parameter strategy performed much better in Atkinson's study than in the present study. It would be worthwhile to investigate systematically the amount of overlap in the items chosen by the homogeneous and heterogeneous parameter strategies for different selection ratios.

The obvious but important conclusion obtained from this analysis of the instructional situation is that the comparative benefits of various instructional strategies are dependent upon the instructional situation for which these strategies are tested. In designing the instructional decision mechanism for an instructional system, one must consider the instructional situation in order to assess the benefits derived from using different strategies. A list of these instructional variables include :

N : the number of items to be learned

S : the total number of study sessions

T : the total number of study trials

A : the average number of presentations of an item for study by the Standard procedure : T/N

I : the number of selection stages

n : the number of items selected at every stage by one strategy

B : the block of items from which n items are selected at each stage

R : the selection ratio : n/B

In our investigation of optimal selection strategies, we have concentrated on the design of decision mechanisms that utilize response history information in maximizing the learning outcomes. Future studies need also to examine how the instructional situational variables influence the effectiveness of selection strategies.

## Conclusions

Experiment two showed no significant differences in the learning outcomes among the HOP, HEP and LHEP stategies. These results imply that the simpler adaptive strategy using homogeneous parameters is as effective as the more complicated adaptive strategies which utilize heterogeneous parameters and latency information for the instructional situation under consideration. Two aspects of the instructional situation account for the lack of differences among the strategies : 1) the too frequent test trials increased learning under all strategies and cancelled out differences in learning due to different strategies, and 2) the selection ratio was too high to be sensitive to a difference in the learning gain detected by different strategies such that the overlap of the actual items that were chosen was very large.

A more basic reason for why the heterogeneous parameter strategies were not more effective is that the one step ahead immediate payoff principle is not a good approximation to the optimal strategy when heterogeneous parameters are used. The MIP principle discounts the learning gain from the unlearned state to the transitory state; thus, it tends to select the easier items for studying when heterogeneous parameters are used. The addition of latency information did not result in the improvement of the adaptive strategy because the large variance in the latency parameters used for all states reduced the sensitivity of the latency strategy to detect differences in the states from the latency information.

Although the results of the second experiment did not confirm our initial hypotheses, these results are not discouraging as they point to some improvements and directions for future studies dealing with adaptive instructional algorithms derived from mathematical learning models. First, we recommend that experimental studies comparing the effectiveness of instructional strategies eliminate or greatly reduce the number of test trials throughout the study sessions so that the test trials would not cancel out the varying benefits in learning due to the selection strategies. Secondly, there needs to be a more systematic study of how different selection ratios effect the amount of overlap in the items chosen by various strategies. From such investigations, we can determine the best selection ratio; i.e. one which allows the different instructional gains computed by different instructional strategies to be represented by the differences in the actual items that are chosen. Next, we hypothesize that the MIP principle that computes the instructional gain by including the additional gain from the unlearned state to the transitory state would result in a more effective heterogeneous parameter strategy. Finally, we suggest that the state dependent latency parameters could be used more effectively for estimating the state of the learner if separate variances were obtained for each state. In other words, we need to examine more carefully how to effectively incorporate heterogeneous parameters and latency parameters into an adaptive strategy.

# CHAPTER VII

## SIMPLIFICATION OF PARAMETER DEPENDENT ALGORITHMS TO VARIABLE BASED PROCEDURES FOR PRACTICAL APPLICATIONS

The research on instructional algorithms derived from mathematical learning models is directed towards obtaining prescriptive rules for the individualization and optimization of instruction based on theories of learning. As instructional designers, we are concerned with both the theoretical soundness and the practical feasibility of these algorithms. The value of this research lies in its applicability to real instructional situations. Can these algorithms be easily implemented for an operational computer instructional system? What are the computational and storage requirements, the programming complexities of these strategies?

Previous research on instructional strategies has shown that computationally simple strategies can be derived from simple learning models, under the assumption of homogeneity of learning parameters. For the linear model, the optimal strategy is equivalent to presenting all items the same number of times by cycling through the entire list in a random order. For the all-or-none model, the optimal strategy is equivalent to presenting the items with the lowest current number of successive correct responses. Note that for the implementation of these strategies, a student's entire response history can be reduced to one index per item; that is, a counter of the number of successive correct responses or a counter of the number of presentations. However, these learning models have been shown to be inadequate for describing the paired-associate learning process, and their inadequacies imply that the instructional strategies derived from them would also be inadequate.

The GFT model used in this study incorporates a component to account for the forgetting process between trials, which the simpler models do not do and has been shown to be a good representation of learning in paired-associate learning studies. From this model, an algorithm which is an approximation of an optimal strategy has been derived using the maximum immediate payoff principle. The present study and Atkinson's study (1972) have both demonstrated experimentally that the MIP algorithm derived from the GFT model is significantly better than the standard strategy derived from the linear model. Thus, by introducing a forgetting factor into the learning model, we have obtained a more effective instructional strategy.

Implementation of the MIP Algorithm Derived from the GFT Model

The MIP algorithm derived from the GFT model is computationally more complex than the strategies derived from simpler models. This is because the algorithm is dependent on the values of the learning and forgetting parameters. If these parameter values are not known prior to instruction, they must be estimated and updated from the response histories as instruction proceeds. This estimation process requires both high computational resources and large storage devices as well as a great deal of computational time. In the present study, we did not have a set of apriori student and item parameters; these parameters were re-estimated after every session as new student data were acquired. Because these parameter values were not stabilized, we also had to update the student's state probability vector using the entire response history with updated parameters. Only in experimental studies is this constant updating of parameters tolerable. For operating CAI systems, we would require that the parameters be obtained prior to instruction and remain fixed throughout instruction.

The number of parameters needed depends on the assumptions made concerning the parameters. For the homogeneous parameter strategy which assumes equal parameters for all subjects and items, only a set of five parameters $P = (a,b,c,\gamma,\theta)$ averaged over all subjects and all items needs to be estimated. For the heterogeneous parameter strategy, parameter values must be estimated for each subject S and for each item N, increasing the number of parameters by a factor of $P \times (N + S)$. If a state dependent latency strategy is used, then an additional set of latency parameters for each subject needs to be estimated. Thus, the implementation of the HEP and LHEP strategies is further complicated by the large number of parameters that need to be estimated and stored. The failure of the second experiment to demonstrate further improvements in instructional efficiency by the more refined techniques, such as the use of heterogeneous parameters and latency parameters, indicates that the computationally simpler homogeneous parameter strategy would suffice for the instructional situation such as the one considered in this study.

If the average parameter values are known, little storage space and minor computations are necessary to implement the HOP strategy. The entire student response history consists of a record for each trial that an item has been presented, his response on that trial, the response time and the intervals between trials. A sufficient history summarizes the information of a complete history and contains all the information necessary to implement the instructional strategy. For the HOP strategy with known parameters, the response history can be reduced to a sufficient history consisting of the current state probability vector and a counter of the number of days since the last presentation for each item. As instruction proceeds, the current state can be updated using the recursive state determination algorithm (equation 2). The instructional gain is then computed from the state probability vector and the values of the learning parameters. The items with the largest expected gain are selected for studying.

To recapitulate, for the implementation of the HOP strategy, the space requirements are 1) a set of parameter space P for parameters $a, b, c, \gamma, \theta$, 2) a state probability vector consisting of three elements for each item and each subject and 3) a counter of the number of days since the last presentation for each item and subject.

The computational procedures for the HOP strategy are :

1. After each session, for each item, update the state probability vector using the algorithm

$$\pi_{i+1}(\ell) = \frac{\sum_{k=0}^{2} \pi_i(k)\, \xi(k,\ell,d_{i+1})\, r(k,x_i)}{\sum_{k=0}^{2} \pi_i(k)\, r(k,x_i)}$$

for $\ell = 0,1,2$ .

2. Compute $\Delta$ for each item using the equation

$$\Delta_{i+1} = a\gamma\, \pi_i(0) + b\gamma\, \pi_i(1) \ .$$

3. Select the n items with the largest $\Delta_{i+1}$.

Eventhough this instructional algorithm can be easily implemented by a computer, it is nevertheless conceptually quite complex and computationally more involved than the simpler variable dependent strategies. This strategy is conceptually more difficult because it deals with values that are not directly observable and which need to be stated in terms of probability estimates. In order to understand this algorithm, one needs to be familiar with probability theory, estimation techniques and algebraic computations. Curriculum designers or teachers are often not equipped with this knowledge to be able to incorporate these more sophisticated control mechanisms into an instructional system. It would be desirable to simplify this parameter dependent algorithm to a parameter independent or variable based procedure which approximates the selection behavior of the parameter dependent strategy. In other words, can we translate the MIP instructional algorithm derived from the GFT model to an instructional procedure that makes its decision directly based on some response variables?

## A Variable Based Procedure that Approximates the HOP Strategy

To formulate a variable dependent procedure that approximates the HOP strategy, we explored how response history variables influence the selection behavior of the HOP strategy in order to determine which variables are important. These variables include such things as the number of trials an item has been presented, the intervals between trials, the number and sequence of correct responses. Using the GFT model with the average parameters obtained from the first experiment ( $a=.14$, $b=.54$, $\gamma=.9$, $\theta=.64$ ) we examined the selection behavior of the homogeneous parameter strategy by computing the instructional gain for various response history sequences; that is, histories with various combinations of correct and wrong responses and with various intervals between trials. From these computational examples, the following observations were developed.

## For an Incorrect Response

When an incorrect response has been made, the GFT model assumes that the subject is in the unlearned state no matter what his previous response history was. The resulting state probability distribution after a wrong response then does not depend on the prior state probability distribution, but rather only on the parameter values and on the number of days since the trial of the wrong response. In other words, if the response on trial i were incorrect and the interval between trial i and trial i+1 were $d_{i+1}$ days, then the state probability distribution before trial i+1 would be equal to the transition probability from the unlearned state to the other states for $d_{i+1}$ days.

This can be shown analytically from the state determination equation :

$$\pi_{i+1}(\ell) = \frac{\sum_{k=0}^{2} \pi_i(k)\ t(k,\ell,d_{i+1})\ r(k,x_i)}{\sum_{k=0}^{2} \pi_i(k)\ r(k,x_i)} \qquad \text{for } \ell = 0,1,2 .$$

Expanding this equation for $\ell = 0$ ( that is, the probability of being in the unlearned state before trial i+1) and letting $x_i = 1$ ( that is, an incorrect response was made on trial i, we obtain

$$\pi_{i+1}(0) = \frac{\pi_i(0)\ t(0,0,d_{i+1})\ r(0,1) + \pi_i(1)\ t(1,0,d_{i+1})\ r(1,1) + \pi_i(2)\ t(2,0,d_{i+1})\ r(2,1)}{\pi_i(0)\ r(0,1) + \pi_i(1)\ r(1,1) + \pi_i(2)\ r(2,1)} .$$

Since the probability of making a wrong response in the transitory state and the learned state is zero ( that is, $r(1,1) = 0$ and $r(2,1) = 0$ ), the last two terms in the numerator and the denominator drop out, leaving

$$\pi_{i+1}(0) = \frac{\pi_i(0)\ t(0,0,d_{i+1})\ r(0,1)}{\pi_i(0)\ r(0,1)} \quad .$$

By simple cancellation, this equation becomes

$$\pi_{i+1}(0) = t(0,0,d_{i+1}) \quad .$$

Similarly, it can be shown that

$$\pi_{i+1}(1) = t(0,1,d_{i+1})$$

and

$$\pi_{i+1}(2) = t(0,2,d_{i+1}) \quad .$$

This means that in determining the state of the learner from his response history, we can ignore all past histories up to the trial of the last error response. All previous histories up to the last error response can be reduced to two variables : c, the current count of the number of correct responses which would be equal to zero and d, the number of days since the trial of the last error.

The state probability distribution $\pi_{i+1}(k)$ and the expected instructional gain $\Delta_{i+1}$ were then computed for histories where an incorrect response was made on trial i, with various $d_{i+1}$ intervals between trial i and trial i+1. From this table (Table 17) it can be observed that for all items whose last response was incorrect, the items with the smallest number of days since the error trial would have the largest gain and would be selected for presentation by the HOP strategy.

Table 17

State Probability Distribution Before Trial i+1
Given an Incorrect Response on Trial i

| Number of Successive Corrects | Days Between Trials $d_{i+1}$ | State Probability Before Trial i+1 $\pi_{i+1}(0)$ | $\pi_{i+1}(1)$ | $\pi_{i+1}(2)$ | Instructional Gain $\Delta$ |
|---|---|---|---|---|---|
| 0 | 1 | .38 | .49 | .13 | .30 |
| 0 | 2 | .55 | .32 | .13 | .25 |
| 0 | 3 | .67 | .20 | .13 | .19 |
| 0 | 4 | .74 | .13 | .13 | .17 |
| 0 | 5 | .78 | .09 | .13 | .15 |
| 0 | 6 | .82 | .05 | .13 | .14 |
| 0 | 7 | .83 | .04 | .13 | .13 |

It can also be shown from Table 17 that as the number of days since the last error trial increases, the probability of being in the unlearned state also increases, while the probability of being in the transitory state decreases and the probability of being in the learned state remains unchanged. This is to be expected. The variable d determines the amount of forgetting between trials. Since the learning model assumes no forgetting from the learned state, then the probability of being in that state would remain constant between trials and would not be effected by the interval between trials. On the other hand, the model defines the transitory state as a state where forgetting occurs. Thus, as the number of days between trials increases, the probability of forgetting increases and therefore, the probability of being in the transitory state decreases while the probability of being in the unlearned state increases.

For a History with One Correct Response

The above discussion has shown that 1) all histories up to the trial of the last error can be summarized by the two variables, the count of successive corrects and the number of days since the last error trial, and 2) given that the current count of successive corrects is zero, the HOP strategy would select the items with the smallest number of days since that last error trial. What would the selection behavior be for a history with one correct response since the trial of the last error?

The state probability distribution and the expected gain resulting from histories with a current count of one successive correct response, for different number of days between the last error trial and the subsequent correct response trial and for different number of days between the correct response trial and the current trial, were then computed. In other words, histories were examined where the response on trial i-1 was incorrect and on trial i was correct, for various $d_i$ and $d_{i+1}$ days between trials. Table 18 shows these results.

How do the intervals between trials effect the selection behavior? Table 18 indicates that $d_i$ does not result in much differences in the instructional gain, whereas the variable $d_{i+1}$ does. The number of days between trials is to measure the amount of forgetting that has occurred. For all previous trials, the forgetting that has occurred is also accounted for by whether a correct response was made on subsequent trials, so that for practical purposes, the history of days between all previous trials can be ignored. However, the number of days since the last trial is needed to determine how much forgetting has occurred since the last trial, which would in turn effect the current state probability distribution and the instructional gain. It can be concluded that the intervals between previous trials is not an important variable in determining the selection behavior the adaptive strategies and can be ignored, but that the number of days since the last trial is an important variable and should not be ignored.

Table 18

State Probability Distribution Before Trial i+1
Given 1 Successive Correct Response and
Various $d_i$ and $d_{i+1}$

| Number of Successive Corrects | Days Between Trials $d_i$ | $d_{i+1}$ | State Probability Before Trial i+1 $\pi_{i+1}(0)$ | $\pi_{i+1}(1)$ | $\pi_{i+1}(2)$ | Instructional Gain Δ |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | .17 | .29 | .54 | .17 |
| 1 | 2 | 1 | .18 | .28 | .54 | .16 |
| 1 | 3 | 1 | .19 | .27 | .54 | .16 |
| 1 | 4 | 1 | .19 | .27 | .54 | .16 |
| 1 | 1 | 2 | .28 | .18 | .54 | .13 |
| 1 | 2 | 2 | .28 | .18 | .54 | .13 |
| 1 | 3 | 2 | .28 | .18 | .54 | .13 |
| 1 | 4 | 2 | .29 | .17 | .54 | .13 |
| 1 | 1 | 3 | .33 | .13 | .54 | .11 |
| 1 | 2 | 3 | .34 | .12 | .54 | .10 |
| 1 | 3 | 3 | .35 | .11 | .54 | .10 |
| 1 | 4 | 3 | .36 | .10 | .54 | .10 |
| 1 | 1 | 4 | .37 | .09 | .54 | .09 |
| 1 | 2 | 4 | .38 | .07 | .54 | .09 |
| 1 | 3 | 4 | .39 | .07 | .54 | .09 |
| 1 | 4 | 4 | .39 | .07 | .54 | .09 |

How important a variable is the current count of successive corrects? A comparison of Tables 17 and 18 shows large differences in the instructional gain for histories with zero successive correct and with one successive correct, indicating that this is indeed an important variable in determining the instructional gain. Let us explore further the effects of this variable.

The Count of Successive Correct Responses

The MIP principle selects items for study that are not in the learned state but which have the largest probability of entering the learned state. We deduce that the smaller the probability of being in the learned state, the larger the instructional gain would be, if homogeneous parameters are used. What variable determines the probability that an item is in the learned state? The probability of being in the learned state increases every time a correct response is made such that as the count of successive corrects increases, the probability of being in the learned state increases and the instructional gain decreases.

This relationship can be shown mathematically. The equation for determining the probability of being in the learned state is

$$\pi_{i+1}(2) = \frac{a\gamma\,\pi_i(0)\ r(0,x_i) + b\gamma\,\pi_i(1)\ r(1,x_i) + \pi_i(2)\ r(2,x_i)}{\pi_i(0)\ r(0,x_i) + \pi_i(1)\ r(1,x_i) + \pi_i(2)\ r(2,x_i)}\ .$$

Since the parameters for the learning model are known, their values can be entered for the above equation where $a\gamma = .13$ and $b\gamma = .50$. Thus,

$$\pi_{i+1}(2) = \frac{.13\ \pi_i(0)\ r(0,x_i) + .50\ \pi_i(1)\ r(1,x_i) + \pi_i(2)\ r(2,x_i)}{\pi_i(0)\ r(0,x_i) + \pi_i(1)\ r(1,x_i) + \pi_i(2)\ r(2,x_i)} .$$

The above shows the value of $\pi_{i+1}(2)$ is determined by $x_i$, whether the response on trial i was correct or not, and by the state probability distribution for trial i ( $\pi_i(k)$ for k = 0,1,2 ). It has been shown above that for an incorrect response, the prior state distribution can be ignored and the value of $\pi_{i+1}(2) = .13$ for the parameter values assumed. For a correct response, we note that $r(0,0) = .2$, $r(1,0) = 1$, and $r(2,0) = 1$. Thus, for a correct response

$$\pi_{i+1}(2) = \frac{\pi_i(0)\ (.13)\ (.2) + \pi_i(1)\ (.50) + \pi_i(2)}{\pi_i(0)\ (.2) + \pi_i(1) + \pi_i(2)} .$$

The above equation shows that the value which makes the largest contribution to $\pi_{i+1}(2)$ is $\pi_i(2)$ and the probability of being in state 2 increases every time a correct response is made. We conclude that the variable , the current count of successive correct responses, is the key variable that determines the state of the learner and in turn effects most the selection behavior of the HOP strategy.

A Variable Based Procedure

The preceding discussion indicates that the two variables which influence the estimation of the state of the learner most are the current count of successive correct responses and the number of days since the last trial. The instructional gain was then computed for histories with different counters of successive correct responses and different number of days since the last trial. Table 19 shows these results. The histories resulting in the largest instructional gains are selected for studying.

Table 19

Table of Instructional Gains Computed for Histories with Different Counters of Successive Correct Responses and Different Number of Days Since the Last Trial, Given Homogeneous Parameter Values $a = .14$, $b = .54$, $\gamma = .9$, $\theta = .64$

| Number of Successive Corrects | Days Since the Last Trial | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 0 | .30 | .25 | .19 | .17 | .15 | .14 | .13 |
| 1 | .17 | .13 | .11 | .10 | .08 | .06 | .06 |
| 2 | .07 | .05 | .05 | .04 | .03 | .03 | .03 |
| 3 | .02 | .02 | .015 | .01 | .01 | .01 | .01 |

From this table, a simple procedure can be reduced for the HOP strategy whereby :

1. The items with lowest count of successive correct responses are selected for studying.

2. If the items have equal counts, then the items for which the number of days since the last presentation is the smallest are selected for studying.

3. If the number of days since the last presentation is too long, then an item which has a higher count of successive corrects and fewer days since the last presentation would be selected. For the parameters considered in Table 16, an interval of 4-5 days since the last presentation would result in selecting items with higher counts.

The above procedure is conceptually and computationally simple to implement and reduces the history space required from 4 indices per item to 2 indices per item. Any curriculum designer or teacher can easily understand this selection procedure and can easily incorporate it into their instructional program. However, this procedure is only an approximation to the MIP algorithm derived from the GFT model,assuming homogeneous parameters. It would be worthwhile to experimentally compare this variable dependent procedure with the HOP strategy, to see whether the learning outcomes would indeed be similar for these two strategies. If so, then the variable based procedure would certainly be a more useful procedure than the parameter dependent algorithms because of its simplicity.

It can be observed that the variable based procedure for the HOP strategy does not differ greatly from the optimal strategy for the two state all-or-none model assuming homogeneous parameters. For this simpler model, the optimal strategy is equivalent to presenting the items with the lowest count of the number of successive correct responses. By incorporating the forgetting process into the learning model, we have refined the instructional procedure by taking into account the additional variable, the number of days since the last presentation. Otherwise, the two procedures are basically similar in considering the count of successive correct responses for its instructional action.

Item Difficulty Variable for a Heterogeneous Parameter Strategy

Average parameter values have been assumed above in computing the instructional gains for different histories. It is also possible to compute instructional gains for these histories assuming that items vary in difficulty. Introducing an item difficulty variable for calculating the instructional gain, a variable based procedure that approximates the HEP strategy can be obtained.

To examine the selection behavior of the HEP strategy, items of three levels of difficulty were chosen and the instructional gains were computed for histories with different counts of successive correct responses, assuming a two day interval since the last trial. For an easy item, parameter values of a = .64,and b = .97 were used; for an average item, parameter values of a = .14 and b = .54; and for the difficult item, parameter values of a = .06 and b = .10.

Table 20

Instructional Gains for Different Parameter Values, Given Histories with Different Counts of Successive Correct Responses and Two Days Since the Last Trial

| Number of Successive Corrects | Easy Item | Average Item | Hard Item |
|---|---|---|---|
| 0 | .28 | .25 | .06 |
| 1 | .05 | .135 | .045 |
| 2 | .005 | .07 | .03 |
| 3 | .0 | .02 | .02 |

It is not as simple to reduce a selection strategy that uses varying item parameters to a simple variable procedure. Table 20 shows that for an item with no successive correct response, the easier items are selected for presentation before the average items and the latter are selected before the hard items. But for items with one successive correct response, the average items would be selected before the easier items which would be selected before the hard items. Further more, an average item with one successive correct response would be selected before a harder item with no correct responses. It would not be difficult however to keep a table of instructional gains for items of a few levels of difficulty in the instructional program. With such a table, the system can look up the instructional gain for various response histories with various item difficulty. Items would then be sorted according to their gain values. The items with the largest gains would be selected for studying. This table driven procedure would be one method of implementing a selection strategy that accounts for item difficulty.

In the discussion of Chapter VI, one reason given for why the heterogeneous parameter strategy was not more effective in our experiments was that a one step ahead MIP principle was used in computing the expected instructional gain. This principle ignores the gain in learning from the unlearned state to the transitory state. For learning models with heterogeneous parameters, the net result in ignoring this gain would be that easier items tended to be selected for studying. We concluded that a MIP principle which included the State U to State T transition gains ($MIP^+$) in computing the instructional gain would be a better selection strategy when heterogeneous parameters were used, but would not make any difference in the case where homogeneous parameters were used. To illustrate this point, instructional gains were computed for histories with different counts of successive correct responses for both homogeneous parameters and heterogeneous parameters using the $MIP^+$ principle. The results are shown in Tables 21 and 22.

Table 21

Table of Instructional Gains Computed for Histories with Different Counters of Successive Correct Responses and Different Number of Days Since the Last Trial Using the MIP$^+$ Principle with Homogeneous Parameter Values

| Number of Successive Corrects | Days Since the Last Trial | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 0 | .49 | .43 | .33 | .27 | .22 | .18 | .15 |
| 1 | .26 | .22 | .18 | .15 | .12 | .08 | .08 |
| 2 | .10 | .08 | .08 | .06 | .05 | .04 | .04 |
| 3 | .03 | .03 | .027 | .02 | .02 | .01 | .01 |

A comparison of Table 19 and Table 21 shows that, although the values of the instructional gains are different when the MIP and the MIP$^+$ principles are used, the resulting variable based procedure that selects items with the largest instructional gain are the same. For the homogeneous parameter strategies, it doesn't make any difference to the selection process if the MIP or the MIP$^+$ principles are used.

However, a comparison of the instructional gains in Table 20 and Table 22 for heterogeneous parameters, shows that the two resulting instructional policies are quite different. Using the MIP$^+$ principle, more of the difficult items would be selected before the easier items. We believe that using the MIP$^+$ principle would result in an improved selection strategy in the case of the heterogeneous parameters because the potential gain of entering the learned state as well as the actual gain are both accounted for by the $MIP^+$ principle.

Table 22

Instructional Gains for Different Parameter Values, Given Histories with Different Counts of Successive Correct Responses and Two Days Since Last Trial,Using $MIP^+$ Principle

| Number of Successive Corrects | Easy Item | Average Item | Hard Item |
|---|---|---|---|
| 0 | .32 | .40 | .27 |
| 1 | .05 | .225 | .22 |
| 2 | .005 | .11 | .17 |
| 3 | .0 | .03 | .11 |

Another suggestion for accounting for item difficulty in the selection strategy would be to measure the difficulty of an item by obtaining the average number of trials before the last error for each item. In other words, a count T can be obtained of the average number of presentations needed for an item to be learned. As instruction proceeds, we can keep a count P of the number of times an item has been presented and compute the ratio P/T. We can then see how this ratio interacts with the response history to influence the selection strategy. Perhaps items with smaller P/T ratios should be selected before items with larger P/T ratios. This would be another area of further investigation.

## Summary

In the above discussions, we have delineated some areas that we think are worthwhile for the future research of optimal instructional strategies. The direction of this research is towards obtaining variable based procedures that approximate the theoretically sound instructional algorithms derived from mathematical models and which are conceptually and computationally simpler to implement. The method suggested for this investigation is to examine the relationship between certain response variables with the selection behavior of the instructional algorithms. From such investigations, the parameter dependent algorithms can be translated to procedures that are dependent on certain key variables. These procedures then need to be experimentally tested to verify their efficacy. If the variable based procedures prove to be as effective as the parameter dependent procedures, then the variable based procedures would be more useful for operational computer instructional systems.

# CHAPTER VIII

## CONCLUDING REMARKS

The present study represents one approach to the solution of the optimal item selection problem. Using decision theoretic analysis, instructional algorithms have been derived from the GFT mathematical models of paired-associate learning. The usefulness of the decision theoretic approach to the solution of instructional problems is that it provides a framework for systematically and formally investigating the task of specifying instructional rules for optimizing learning outcomes. These rules direct and control instruction towards specified goals through quantitative measures of payoffs of instruction that are based on a theory of learning. They adapt instruction to the individual's response history in maximizing the learning outcome.

From our experiments testing the efficacy of these algorithms for computerized teaching of Chinese vocabulary, it was concluded that the adaptive instructional algorithms do improve the learning outcomes. Two directions for future research are suggested from the results of our investigation. 1) Investigations need to be directed towards more effective ways of incorporating into the instructional decision structure additional information concerning subjects and items, such as item difficulty, subject ability and latency data. 2) For practical applications, these algorithms need to be converted to variable based procedures which reduce computational complexity while maintaining adaptivity and sensitivity to individual subjects.

The present study focuses on one aspect of the design of the control system; that is, how the system can adapt instruction to the individual student based on response history. In the multidimensional task of the design of instructional decision mechanisms, other factors in addition to subject history need to be considered. These variables can be grouped into three catagories : 1) learner variables, 2) situational variables and 3) content variables. Learner variables include response histories such as those investigated in this study, as well as learner ability, cognitive style, anxiety state and previous knowledge. Situational variables include the amount of time given for instruction, the terminal criterion, the selection ratio etc. (see discussion in Chapter VI). Content variables include the value of learning a specific item and the hierachical structure of the material to be learned.

Our study was restricted to the study of independently learned paired-associate items. Learning models that consider the structural relationship between items in deriving adaptive strategies should be the subject of future research. Hopefully, future studies in psychology will provide mathematical and process models for more complex learning behaviors such as concept formation and problem solving. This would be a pre-requisite to the study of instructional algorithms for a broader class of instructional situations where independence of items is not assumed.

One other major factor in the total systems design which we have not considered is the cost/benefit structure of various control systems. This factor must be considered in the implementation of instructional decision mechanisms for real instructional systems.

Difficulties in carrying out research on optimal instructional algorithms became apparent in the course of this research. The implementation of the optimization strategies in CAI systems involves considerable programming efforts, especially with limited software and hardware resources. This type of research would be greatly facilitated if software support for computer instructional systems and large computational facilities were readily available. Ideally, this research should be carried out at institutions where there is on-going development of computer assisted instructional programs. In this way, experiments testing the efficacy of various instructional strategies could be carried out in less artificial circumstances than was the case in this study. Furthermore, instructional materials need not be specially developed for the experiments; they could come from the regular classroom curriculum materials. Modifications made to the theory could be easily tested out on subsequent student populations. In this way, the cycles of theory formation and experimental validation would be greatly facilitated.

Studies of optimal selection strategies are directed towards the accumulation of a whole body of knowledge related to the optimization of learning. The problem of optimal item selection represents one sub-problem in an array of optimal learning problems. It is through such systematic investigations of specific sub-problems that a general theory of instruction will emerge.

## Appendix A: Derivation of a MIP Selection Algorithm From GFT Models

The MIP one stage optimization strategy selects items that have the highest probability of entering the learned state on the next study trial. Let us define $\Delta_i$ as the probability of an item not being in the learned state before trial i and entering the learned state after study trial i. From the GFT models, we can derive an algorithm for computing $\Delta_i$ for each item, for each subject.

The available response data for each item for each subject consist of response sequences with three arguments, that is

$x_i$ = response on trial i where 0 = correct, 1 = incorrect ,

$m_i$ = type of presentation on trial i where 0 = unreinforced, 1 = reinforced ,

$d_i$ = the number of days between trial i-1 and trial i.

We define $y_i$ as the triplet $(m_{i-1}, d_i, x_i)$ and represent the entire response sequence $(y_1, y_2, \ldots y_i)$ as $(y^i)$. Next we define

$$\pi_i(j) = P(\text{state before trial } i = j \mid y^{i-1}, u_i) \text{ for } j = 0, 1, 2$$

where $u_i = (m_{i-1}, d_i)$.

Now we can define

$$\Delta_i = \pi_{i+1}(2) - \pi_i(2) \qquad \text{(A1)}$$

for $u_{i+1} = (0,0)$.

Introducing the previous State K to the first term of equation (A1) we obtain,

$$\pi_{i+1}(2) = \sum_{k=0}^{2} P(S_{i+1}=2,\ S_i=k|y^i,u_{i+1}). \qquad \text{(A1a)}$$

Using Bayes rule, we can write

$$\pi_{i+1}(2) = \sum_{k=0}^{2} P(S_{i+1}=2|S_{i=k},y^i,u_{i+1}) \quad P(S_i=k|y^i,u_{i+1}). \qquad \text{(A1b)}$$

Since the transition probability does not depend upon the previous history but rather only on the previous state, and since the state at trial i does not depend on the response on that trial, we can rewrite (A1b) as

$$\pi_{i+1}(2) = \sum_{k=0}^{2} P(S_{i+1}=2|S_i=k,u_{i+1}) \cdot P(S_i=k|y^{i-1},u_i). \qquad \text{(A1c)}$$

We recall by definition

$$\pi_i(k) = P(S_i=k|y^{i-1},u_i)$$

and $t(k,\ell,u_i) = P(S_i=\ell|S_{i-1}=k,u_i)$.

We can now write (A1c) as

$$\pi_{i+1}(2) = \sum_{k=0}^{2} t(k,2,u_{i+1}) \quad \pi_i(k). \tag{A1d}$$

Substituting this term in equation (A1), we obtain

$$\Delta_i = \sum_{k=0}^{2} t(k,2,u_{i+1}) \, \pi_i(k) - \pi_i(2). \tag{A2}$$

Since $t(2,2,u_{i+1}) = 1$, equation (A2) can be written as

$$\Delta_i = \sum_{k=0}^{1} t(k,2,u_{i+1}) \quad \pi_i(k) + \pi_i(2) - \pi_i(2)$$

$$\Delta_i = \sum_{k=0}^{1} t(k,2,u_{i+1}) \quad \pi_i(k)$$

$$\Delta_i = t(0,2,u_{i+1}) \quad \pi_i(0) + t(1,2,u_{i+1}) \quad \pi_i(1) \quad . \tag{A2a}$$

Since $u_{i+1} = (0,0)$; that is $m_i = 0$ as trial i is a reinforced trial, and we're interested in the state immediately after transition, before forgetting occurs, so $d_{i+1} = 0$, we have the following values for the transition function:

$t(0,2,u_{i+1}) = a\gamma$

$t(1,2,u_{i+1}) = b\gamma$ .

Substituting these values for equation (A2a), we obtain

$$\Delta_i = a\gamma\ \pi_i(0) + b\gamma\ \pi_i(1)\ . \tag{A2b}$$

Using this equation (2b), we can calculate the expected immediate gain for each item.

Appendix B: Derivation of State Probability Algorithm

Since a subject's state of knowledge is only partially observable, we need some way of making inferences about the student's state from his observed response: we need an algorithm for determining the state of the learner from his response history. The available response data for each item for each subject consist of response sequences with three arguments, that is

$x_i$ = response on trial i, where 0 = correct, 1 = incorrect,

$m_i$ = type of presentation on trial i, where 0 = unreinforced, 1 = reinforced ,

$d_{i+1}$ = the number of days between trial i and trial i+1 .

We define $u_i$ as the tuple $(m_{i-1},d_i)$ and represent the entire response history sequence until trial i, excluding the response on trial i, as

$$h(i) = \{u_1;\ u_2,x_1;u_3,x_2;\ldots\ldots;\ u_{i-1},x_{i-2};\ u_i,x_{i-1}\} .$$

We define the state of the learner prior to trial i, given response history up until trial i as

$$\pi_i(\ell) = P(S_i=\ell\,|\,h(i)) \qquad \text{for } \ell=0,1,2, \tag{B1}$$

which can be alternatively written as

$$P(S_i=\ell|h(i-1),u_i,x_{i-1}). \tag{B1a}$$

Since $P(A|B,C) = P(AB|C)/P(B|C)$, we can write (B1a) as

$$\frac{P(S_i=\ell,x_{i-1}|h(i-1)u_i)}{P(x_{i-1}|h(i-1)u_i).} \tag{B2}$$

Introducing the previous state $S_{i-1}$ to both the numerator and denominator, we obtain:

$$\frac{\sum_{k=0}^{2} P(S_i=\ell,S_{i-1}=k,x_{i-1}|h(i-1),u_i)}{\sum_{k=0}^{2} P(S_{i-1}=k,x_{i-1}|h(i-1),u_i).} \tag{B3}$$

Using the rule $P(A,B,C|D) = P(A|B,C,D)\ P(B|C\ D)\ \ P(C|D)$ for the numerator of (B3) and the rule $P(A,B|C) = P(A|B\ C)\ \ P(B|C)$ for the denominator of (B3), we obtain

$$\frac{\sum_{k=0}^{2} \{P(S_i=\ell|S_{i-1}=k,x_{i-1},h(i-1),u_i)\ P(x_{i-1}|S_{i-1}=k,h(i-1),u_i)\ P(S_{i-1}=k|h(i-1),u_i)\}}{\sum_{k=0}^{2} \{P(x_{i-1}|S_{i-1}=k,h(i-1),u_i)\ \ P(S_{i-1}=k|h(i-1),u_i)\}.} \tag{B4}$$

By the definition of the models, the current state depends only on the previous state, not on the total previous history and the response on a trial depends only on the state of that trial, thus (B4) can be written as

$$\frac{\sum_{k=0}^{2} \{P(S_i=\ell|S_{i-1}=k,u_i) \quad P(x_{i-1}|S_{i-1}=k) \quad P(S_{i-1}=k|h(i-1)\}}{\sum_{k=0}^{2} \{P(x_{i-1}|S_{i-1}=k) \quad P(S_{i-1}=k|h(i-1))} \quad \text{(B5)}$$

Recalling our definition of the state transition function as

$$t(k,\ell,u_i) = P(S_i=\ell|S_{i-1}=k,u_i),$$

and the response probability function

$$r(k,x_i) = P(x_i|S_i=k),$$

we can substitute these functions for the terms in equation B5 and obtain

$$\frac{\sum_{k=0}^{2} \{t(k,\ell,u_i) \quad r(k,x_i) \quad P(S_{i-1}=k|h(i-1)\}}{\sum_{k=0}^{2} \{r(k,x_i) \quad P(S_{i-1}=k|h(i-1)\}.} \quad \text{(B6)}$$

Since $P(S_{i-1}=k|h(i-1) = \pi_i(k)$, we obtain the recursive equation

$$\pi_{i+1}(\ell) = \frac{\sum_{k=0}^{2} t(k,\ell,u_i)\ r(k,x_i)\ \pi_i(k)}{\sum_{k=0}^{2} \pi_i(k)\ r(k,x_i)} \quad \text{(B7)}$$

Since an item is initially assumed to be in the unlearned state, we know that

$$\pi_0(0) = 1$$

$$\pi_0(1) = 0$$

$$\pi_0(2) = 0.$$

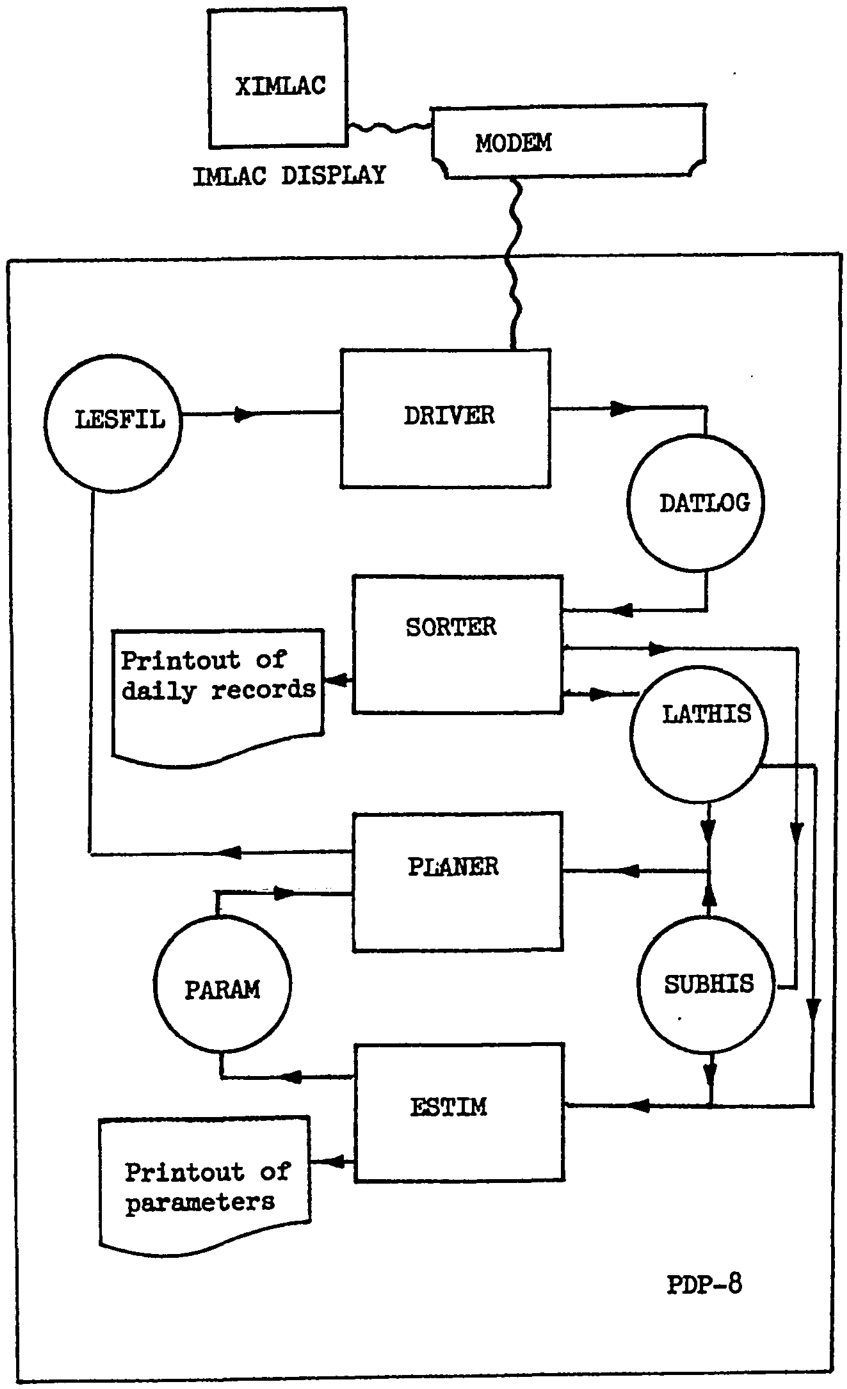
# APPENDIX C : SYSTEM SPECIFICATION

## General System Information

| | | |
|---|---|---|
| Machine : | PDP-8<br>IMLAC | |
| Operating System : | PDP-8 TSS-8 | |
| Language : | PDP-8 Assembly language<br>IMLAC Assembly language | |
| System Programs : | XIMLAC : | Interacts with subjects by displaying items and receiving answers. |
| | DRIVER : | Presents,evaluates and records items for subject's daily session. |
| | SORTER : | Updates subject histories with daily records. |
| | PLANER : | Plans lessons for subjects according to selection strategies. |
| | ESTIM : | Estimates item and subject parameters. |
| System Files : | DATLOG : | Daily records of student responses. |
| | SUBHIS : | Accumulated response histories for all subjects. |
| | LATHIS : | Accumulated latency response records for all subjects. |
| | LESFIL : | Planned lessons for subject's next session. |
| | PARAM : | Subject and Item Parameters. |
| | WORDS : | Chinese characters stored in Imlac. |
| System Output : | Printout analysis of subject's daily sessions.<br>Printout of parameter estimates. | |

## System Flow : Interface Of Programs And Files





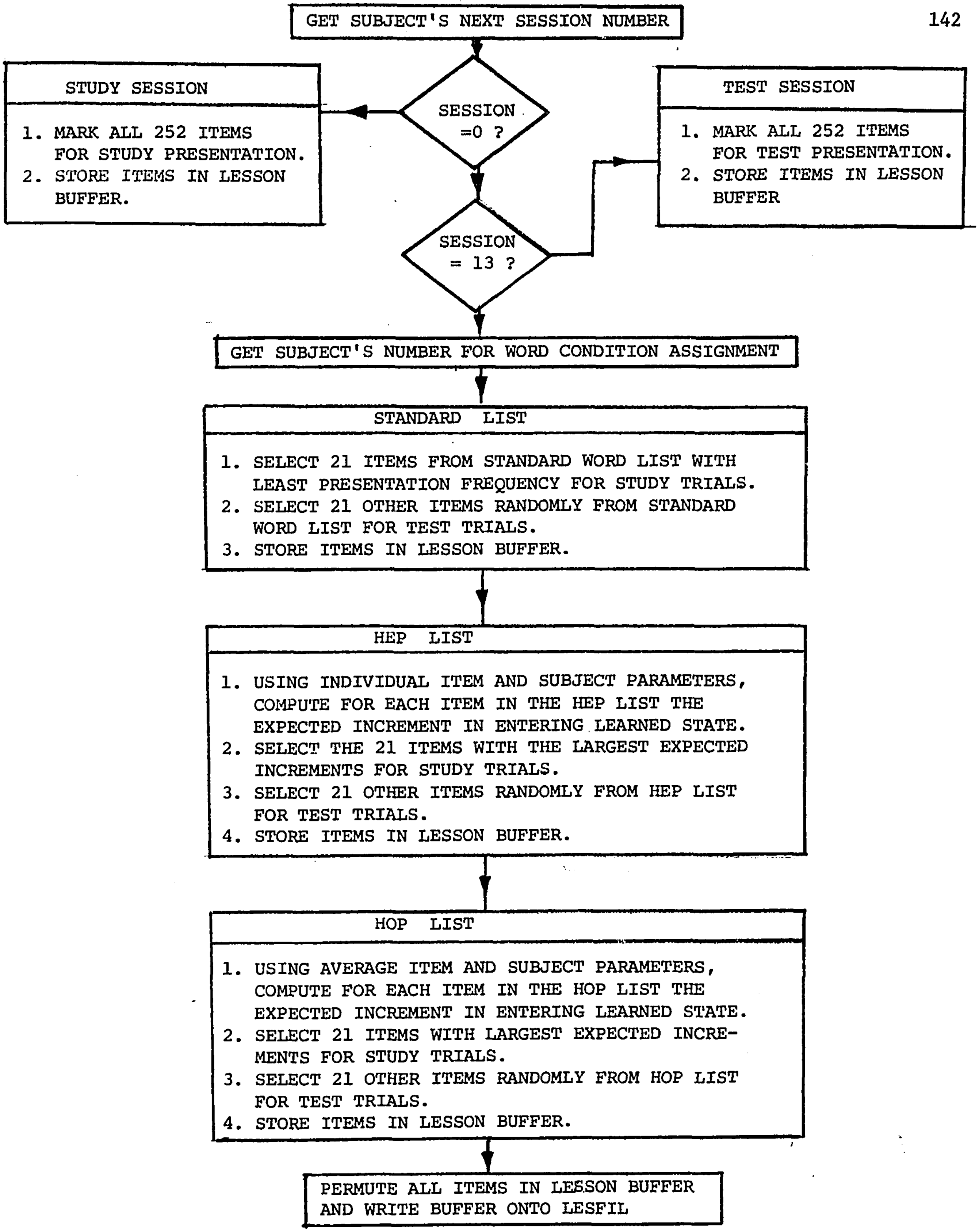



DIAGRAM OF PROGRAM 'PLANER' FOR PLANNING LESSON

Appendix D: Glossary of Key Terms Used in Study

Adaptive strategy - A strategy that selects items according to the histories of individual students and for which the parameter values change with the accumulation of data from all students.

All-or-none procedure, or drop-out procedure - The procedure of selecting items for presentation that have the lowest current count of successive correct responses. This is the optimal procedure for the two state all-or-none model, assuming homogeneous parameters.

Complete history - a record of the entire past performance of the subject.

General Forgetting Theory (GFT) - A three or four state Markov model which describes the learning and forgetting processes as they occur in paired-associate learning situations.

Homogeneous Parameter Strategy (HOP) - An adaptive strategy derived from the GFT model using the MIP principle and assuming homogeneous parameters.

Heterogeneous Parameter Strategy (HEP) - An adaptive strategy derived from the GFT model using the MIP principle and assuming heterogeneous item and subject parameters.

LHEP - A heterogeneous parameter strategy which incorporates state and subject dependent latency information in making its selections.

Linear model - A learning model with the characteristic that the probability of a correct response on any trial is a linear function of the response probability on the previous trial.

Maximum Immediate Payoff (MIP) principle - The principle of selecting items which yield the largest expected gain on the next trial.

Optimal strategy - A strategy which chooses the instructional path that maximizes the expected total utility for a given history.

Random Trial Increment model (RTI) - This model is a combination of the linear model and the all-or-none model in that on any trial an item may be come partially learned with a fixed probability.

Reinforced trial - A trial where the subject is given feedback in his response, and the correct answer is presented for studying.

Standard procedure or randomized cyclical procedure - The procedure of randomly cycling through a whole list. This procedure is the optimal instructional procedure based on a linear model, assuming homogeneous parameters.

Sufficient history - Summarizes the information of a complete history and contains all the information necessary to implement an instructional strategy.

Two state all-or-none model - A two state learning model which assumes an item is either in the learned or unlearned state, and a transition from the unlearned state to the learned state occurs with a fixed probability.

Unreinforced trial - A test trial, where the subject is not given feedback to his response.

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