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Chapter I

INTRODUCTION

Between 1939 and 1960, the relative wage rate of elementary, high school and college graduates has remained nearly constant, as can be seen in Table 1.[1,2]

TABLE 1

RATIO OF MEAN ANNUAL EARNINGS OF MALES, TWENTY-FIVE YEARS AND OVER, FOR THE EDUCATIONAL FACTORS: 1939, 1949 and 1959

	1939	1949	1959
High school/College	.637	.612	.663
Elementary/College	.456	.458	.510
High school/Elementary	1.398	1.338	1.300

Source: Z. Griliches, "Education in Production Functions and Growth Accounting," Education, Income and Human Capital, ed. W.L. Hansen (New York, 1970), pp. 71-115 (Table 2).

However, over approximately the same period of time (1940-1960), the percentage of college graduates 25 years and over in the U.S. increased from 4.6% to 7.7%, while the percentage of high school graduates 25 years and over increased from 24.1% to 40.1%.[3] The rise in the supply of college and high school graduates over time relative to elementary school graduates, and the failure of the wage rate of these factors to decline relative to that of elementary school graduates suggests a paradox concerning the role of education in the production process.[4]

Three possible explanations can be offered for reconciling this paradox.[5,6] There are two plausible explanations on the demand side which could have offset the increase in the relative supply of high school and college graduates, and one on the supply side. Here are the three possible explanations:

(1) Those industries which are the most rapidly expanding are also the most educational-intensive industries. Thus, as the output of such industries increases, the relative share of the more-educated inputs increases relative to the share of less-educated inputs in that industry;

(2) The change in the composition of production may have increased the demand for more-educated labor by increasing the productivity of this factor relative to

less-educated labor. That is, the changing composition of production may have had a non-neutral effect on relative educational labor productivities. This could have occurred in two ways:

(a) in the production process, the pattern of factor substitution may be such that non-labor inputs are relatively enhancing to more-educated than less-educated labor. Thus, as the quality and quantity of non-labor inputs increases, the productivity of more-educated labor will increase relative to less-educated labor; or

(b) technological change may be such that it increases the productivity of more-educated labor, relative to labor inputs with less education.

It may be difficult to distinguish empirically between the alternative hypotheses. For example, if the rise in the quality of non-labor inputs is considered as embodied technical change, then it is difficult to distinguish empirically between (a) and (b) above. For this reason, (a) and (b) will sometimes be referred to as "non-neutrality of technological change." Also, those industries which are most rapidly expanding may also have a rapid rate of technical change. If technical change is non-neutral, then it may be difficult to "partial" out the effects of (1) and (2).

(3) Production of human capital may be such that over time, there has been an increase in the rate of learning per unit of resource allocated.

The only empirical evidence at the present time on the changing composition of output hypothesis is provided by Griliches.[7] He finds that the correlation between the percentage change in the employment of males between 1950 and 1960 and the logarithm of mean school years completed by industry in 1960 is .33.[8]

As for (2), it is surprising that although economists realize that an understanding of patterns of factor substitution can help explain some apparent economic paradoxes and open up a pregnant range of new perspectives for economic research, little empirical research has been done on this question.[9,10] I expect that the lack of extensive research on this question is due to a crucial constraint acting upon the potential investigator: the lack of knowledge of a production function which permits estimation of patterns of factor substitution without restricting such patterns a priori. In Chapter 3, I discuss this problem. Here, I summarize three previous empirical studies that are aimed at investigating patterns of factor substitution.

All three studies, as well as my own, operate under a fundamental constraint. The fundamental relationship between household consumption and the supply curve of different levels of labor are eliminated from the model. The wage determining operation in the labor market are thus removed from the economic processes under consideration. The most important ceterius paribus assumption, therefore, is contained in the phrase "given a set of wage rates."

Using no a priori production function and a two-dimensional breakdown of labor, "schooled" and "unschooled", Griliches examines patterns of factor substitution in U.S. manufacturing in 1954 and 1963.[11] He finds that "schooled" labor is more complementary with physical capital than this latter factor is with "unschooled" labor.

Bowles, using aggregate inter-country data, three educational inputs (0-7, 8-11 and 12+ years of school completed) and a two-level aggregated production function, finds that the elasticity of substitution between any pair of educational inputs is sufficiently high so that the three educational inputs can easily be substituted for each other in the production process.[12] He, therefore, concludes that all educational inputs can be aggregated into a single educational measure, since little information is lost about the educational services used in the production process. Vis a vis the production process, each factor is identical, except that the "quantity" of input of each educational factor in the production process is not the same.[13]

Welch, like Bowles, uses three educational classes: college graduates, high school graduates and persons with 1-4 years of schooling. Welch treats persons in intermediate educational levels as linear combinations of persons in the above three educational classes. The empirical analysis is concerned with the factors determining the productivity of schooling in agriculture in the U.S. in 1959.[14] Welch finds that much of the "leverage" associated with college-educated labor can be attributed to changing technology. That is, ceterius paribus, the productivity of college-educated labor increases relative to that of non-college-educated labor as a result of technological change. Thus, as technological change continues in agriculture, one would expect that there will be an increase in college-educated labor employed in agriculture relative to non-college-educated labor. However, he also finds that non-labor inputs have a negative effect on the relative productivity of college- to non-college-educated labor.

(3) above deals with the quality of schooling over time. A knowledge of educational production functions is necessary. An educational production function measures the relationship between school and student inputs and a measure of output. The difficulties involved in estimating

an educational production function are discussed by Bowles.[15]

This study examines the first two hypotheses above with emphasis on the second hypothesis. With respect to the second hypothesis, the empirical analysis is concerned with U.S. manufacturing. I do not empirically investigate the third hypothesis in this study. In Chapter 2, I explore the effect of the changing composition of output and the non-neutrality of technological change on the demand for educational factors between 1940 and 1960, and also present some preliminary evidence for U.S. manufacturing. In Chapter 4, I examine patterns of factor substitution for U.S. manufacturing in 1958. Chapter 3 is a discussion of production functions and models which can be used to estimate these patterns.

Footnotes to Chapter 1

1 For occupations which reflect educational levels, the constancy of the relative wages between 1949 and 1959 also holds. The ratio of wage and salary income in 1959 to 1949 for each major occupational class is:

Occupation	Earnings 1959/ Earnings 1949
All	1.721
Prof. & Technical	1.698
Managers (excluding farm)	1.666
Farm managers	1.532
Clerical workers	1.652
Sales workers	1.694
Craftsmen & foremen	1.695
Operative workers	1.671
Service workers	1.630
Laborers	1.697

(Based on males who worked 50-52 weeks.
Source: 1950 Census of Population PC(4), 1B, p.199.
1960 Census of Population PC(2), 7A, p.356.)

2 There is an extensive literature on the changing skill differential over time between "skilled" and "unskilled" labor. However, in this literature, skilled workers are usually considered some sort of manual craftsmen, such as carpenters, millwrights, electricians, etc. Unskilled workers are laborers, sweepers, watchmen, etc. In this literature, it is observed that the wage of unskilled labor is rising, relative to that of skilled labor. See:

M.W. Reder, "Wage Differentials: Theory and Measurement," Aspects of Labor Economics (Princeton, 1962), pp. 257-311.

and

R. Perlman, "Forces Widening Occupational Wage Differentials," Review of Economics and Statistics, XL (1958), 107-115.

3 U.S. Bureau of the Census, Abstract of the United States: 1966 (Washington: 1966), Table 154.

4 It is also possible for the relative demand for the two more-educated labor inputs to increase less than the demand for the lower-educated labor input and still observe no change in the relative wage rate, even under the observed supply shifts. Such a result would depend upon the elasticity of the initial supply and demand curves. The possibility is more likely the (a) greater the elasticity of demand for more-educated labor; (b) the less elastic the demand for less-educated labor; (c) the more inelastic the supply of more-educated labor; and (d) the more elastic the supply of less-educated labor.

5 These three explanations are offered by Welch in trying to explain why the private internal rate of return to college and high school graduates has not fallen, even though there has been a relatively large increase in the supply of these factors.

F. Welch, "Education in Production," Journal of Political Economy, LXXVII (1970), 35-59.

The constancy of the relative wage rate of educational factors is consistent with an increase in the private internal rate of return because it implies that if the wage rates are increasing, then the absolute wage differential will increase. An increase in the absolute wage differential will increase the private internal rate of return.

6 Institutional factors, such as the effect of minimum wage laws, migration laws and trade union pressure, are omitted. All three of these factors, I believe, would act to increase the wage rate of elementary school graduates, relative to the two other educational classes under consideration.

7 Z. Griliches, "Education in Production Functions and Growth Accounting," Education, Income and Human Capital, ed. W.L. Hansen (New York, 1970), pp. 71-115.

8 His result is statistically significant. The correlation for females is not statistically different from zero.

9 For example, an understanding of factor substitution can help explain the well-known "Leontief Paradox" of international trade.

10 I eliminate from this review studies which introduce an "educational measure" into the production function, and instead, consider those studies in which labor of differ-

ent educational levels is included in the production function.

For example, Hildebrand and Liu introduce an educational measure into the exponent of the Cobb-Douglas production function to test if education affects labor's share in production. The hypothesis that education affects all parameters of the production function is tested by Brown and Conrad.

G.H. Hildebrand and T.C. Liu, Manufacturing Production Function in the U.S., 1957 (Cornell University Studies in Industrial and Labor Relations, 1965).

M. Brown and A. Conrad, "The Influence of Research and Education on C.E.S. Production Relations," The Theory and Empirical Analysis of Production, ed. M. Brown (New York, 1967), pp. 341-371.

11 Griliches uses an approximation to the input demand equations as his model to make inferences about patterns of factor substitution. The model is discussed in

Z. Griliches, "Education in Production Functions and Growth Accounting."

The empirical test of the hypothesis that "schooled" labor is more complementary with physical capital than this latter factor is with "unschooled" labor is in

Z. Griliches, "Capital-Skill Complementarity," Review of Economics and Statistics, LI (1969), 465-468.

In this paper, Griliches defines skilled or "schooled" labor in two ways: (i) all workers who are not laborers, operators or service workers, and (ii) using the educational distribution.

12 S. Bowles, "Aggregation of Labor Inputs in the Economics of Growth and Planning," Journal of Political Economy, LVIII (1970), 68-81.

13 For example, suppose there are three educational labor classes - X_1, X_2 and X_3. Assume that all three factors are perfect substitutes for each other; that is, the elasticity of substitution between any pair of factors is infinite. Also assume that X_3 is three times as productive as X_1, but only twice as productive as X_2, and that this relationship is independent of the quantity of physical capital employed; that is,

$$X_3 = 1/3\ X_1$$

and

$$X_3 = 1/2\ X_2$$

at all levels of physical capital employed.

Then, from the point of view of the production function, all three factors are identical, except that each factor represents a different number of units of input.

Now suppose there are two industries, A and B, such that the educational labor distribution is:

Number in Educational Class

Industry	X_1	X_2	X_3
A	6	8	3
B	3	12	1

Then, under the assumptions made above, the educational distribution for each industry can be described by an aggregate "educational measure." In terms of X_3, industry A employes 9 units, while B employes 8 units. (Or in terms of X_1, industry A employes 27 units while industry B employes 24 units.)

Bowles' results suggest that aggregation into an aggregate "educational measure" is valid. However, because of the lack of data on physical capital, Bowles could not test if the marginal productivity relationships are independent of the level of physical capital employed. The condition that a subset of factors can be aggregated if, and only if, the marginal rate of factor substitution between the subset of factors is independent of the remaining factors, is given by the Leontief separability theorem.

W. Leontief, "A Note on the Inter-relationship of Subsets of Independent Variables of a Continuous Function with Continuous First Derivatives," Bulletin of American Mathematical Society, (1947), 343-350.

and

________, "Introduction to a Theory of Internal Structure of Relationships," Econometrica (1947), 361-372.

14 Welch, F., "Education in Production."

Welch's findings do not support the conclusion of Bowles; that is, that the elasticity of substitution between all pairs of educational labor inputs is sufficiently high so that aggregation into an aggregate "educational measure" is valid.

Welch finds that production can be viewed as a two-stage or nested C.E.S. production process. First, the two lower educational classes combine via the C.E.S. production function to produce what he calls "conventional labor." Then "conventional labor," college-educated labor and physical capital combine to produce output. On the other hand, Bowles' findings would suggest a two-stage or nested production process that differs from that suggested by Welch. First, all three educational classes combine, via the fixed proportion model, to produce "labor," and then "labor" would combine with physical capital to produce output.

15 S. Bowles, "Towards an Educational Production Function," Education, Income and Human Capital, pp. 11-61.

Chapter 2

THE CHANGE IN DEMAND FOR EDUCATIONAL INPUTS

Three explanations are offered for the failure of the relative wage rate of college and high school graduates to elementary school graduates to fall between 1939 and 1958, even though over the same time span, the relative supply of these factors of production has increased. In this Chapter, I consider the effect of changes in the composition of output and non-neutrality of technological change as possible factors in offsetting the relative rise in the supply of college- and high school-educated labor.

I first discuss the observed growth and distribution of output by sector in the U.S. between 1939 and 1960. After this, I discuss the observed growth in the demand for each factor of production over the same period, along with the observed relative factor ratios, and attribute the observed change in factor demand for each educational labor input to: (1) change in demand due to technological change; (2) change in demand due to constant growth of output in all sectors of the economy; that is, the percentage composition of output is unchanged; and (3) change in demand due to different growth rates of output between sectors of the economy.

Finally, I look at 17 two-digit SIC manufacturing industries and examine the relationship between both the change in the capital-average labor ratio and real output between 1948-50 and 1961-63 with the 1960 educational factor ratios.

(2.1) Composition and Growth of Output in the U.S. Economy: 1939 - 60

Between 1939 and 1960, real output in 1954 dollars in the U.S. increased from $184 billion to $440.1 billion. In 1950, real output in 1954 dollars was $323.1 billion. The percentage distribution of real output in the three years 1939, 1950 and 1960 for eight sectors of the U.S. economy are given in Table 1. As can be seen in Table 1, there has not been a drastic shift in the relative composition of output between 1939 and 1960. Both the agriculture and government sector showed a continual decrease over the 21-year period in the percentage contribution to total output. Manufacturing's share increased

Table 1

PERCENTAGE DISTRIBUTION OF REAL OUTPUT (IN 1954 DOLLARS) IN THE U.S. ECONOMY BY EIGHT MAJOR SECTORS: 1939, 1950 and 1960

(percent)

Sector	1939	1950	1960
Agriculture, forestry and fisheries	7.78	6.25	5.16
Mining	2.23	2.73	2.45
Construction	3.16	4.62	4.27
Manufacturing	25.05	28.66	28.52
Wholesale and retail trade	16.97	18.97	17.72
Transportation, communications and public utilities	10.39	8.91	10.22
Other services	22.66	20.95	22.63
Government and government enterprises	11.75	8.91	9.03

Source: See the appendix to this Chapter.

between 1939 and 1950, but was approximately the same in 1960 as it was in 1950. The trade sector increased between 1939 and 1960, but the share in total output in 1960 fell from its 1950 level. The share of "other services" remained the same between 1939 and 1960, but fell in 1950. The annual compounded growth rate of total output between 1939 and 1950 was 5.3 percent, as compared to 3.1 percent between 1950 and 1960. Sector growth rates are shown in Table 2.

(2.2) Observed Demand for Educational Labor Inputs

No data on the educational distribution is available for the three years 1939, 1950 and 1960 by sector. However, data is available on the occupational distribution by sector for 1940, 1950 and 1960 from the Census of Population. I, therefore, use the occupational distribution as a proxy for the educational distribution.

The decision I made was to combine the nine occupational classes in the following way: college-graduated labor - professionals and managers (excluding farm managers); high school-educated labor - craftsmen, foremen, farm managers, clerical and sales workers; and elementary school-educated labor - laborers, operators and service workers. The decision is based on both the median educational attainment and wage and salary income of males in both 1949 and 1959. Table 3 provides the

Table 2

ANNUAL GROWTH OF REAL OUTPUT (IN 1954 DOLLARS) IN THE U.S. ECONOMY FOR EIGHT MAJOR SECTORS FOR SELECTED PERIODS

(percent)

Sector	1939-50	1939-60	1950-60
ALL	5.3	4.2	3.1
Agriculture, forestry and fisheries	3.2	2.2	1.1
Mining	7.2	4.7	2.0
Construction	8.9	5.8	2.3
Manufacturing	7.0	5.0	3.1
Wholesale and retail trade	6.4	4.5	2.5
Transportation, communications and public utilities	3.8	4.2	4.6
Other services	4.5	4.2	3.9
Government and government enterprises	2.6	2.9	3.2

Table 3

EDUCATIONAL ATTAINMENT AND WAGE AND SALARY INCOME BY MAJOR OCCUPATION: 1949 AND 1959 (MALES)

Occupation	Median School Years[1] 1949	1959	Median Wage and Salary Income[2] 1949	1959
ALL	9.7	11.1	$3110	$5354
Prof. & Technical	16.0+	16.3	4030	6841
Managers (non-farm)	12.2	12.5	4327	7208
Farm managers	NA	11.5	2572	3936
Clerical workers	12.2	12.3	3136	5181
Sales workers	12.2	12.3	3270	5541
Craftsmen & foremen	9.5	10.5	3378	5727
Operative workers	8.9	9.6	2924	4886
Service workers	8.8	9.7	2425	3953
Laborers	8.2	8.7	2366	4016

[1] 1950 Census of Population PC(4), 1B, p. 107.
1960 Census of Population PC(2), 7A, p. 116.

[2] Based on males who worked 50-52 weeks in the respective years.
1950 Census of Population PC(4), 1B, p. 199.
1960 Census of Population PC(2), 7A, p. 356.

relevant data. Non-farm managers are included in the college-educated class because their median wage and salary income was above that of professionals in both 1949 and 1959, even though median educational attainment was just above twelve years. Craftsmen and foremen are included in the high school-educated class because of their wage and salary income in both years.

The educational distribution was imputed from the male-female occupational distribution by using relative wage rates. The computed distribution by sector for each of the three years is given in the appendix to this Chapter. The observations are treated as the demand for each educational factor. However, what is really being observed are equilibrium points which are a result of both the shifts in supply and demand for each factor. In order to interpret the observations as changes in the demand for each factor for all sectors, it is necessary to assume that each sector acts as a perfect competitor in the factor market, so that the supply curve each sector faces for each type of labor is perfectly elastic at the prevailing relative wage rate. Since the relative wage rate of the educational factors has remained nearly constant, the shifts observed are then due to changes in demand.

Table 4 shows the growth of demand by sector for each educational input. The annually compounded growth in demand for college-educated labor was 3.2 percent

Table 4

ANNUAL GROWTH IN DEMAND FOR EDUCATIONAL FACTOR INPUTS BY SECTOR
(percent)

Sector	College	High School	Elementary School
All			
1940-50	3.2%	2.2%	1.0%
1940-60	2.0	.7	.2
1950-60	.7	- .9	- .8
Agriculture, forestry and fisheries			
1940-50	13.2	-2.0	-2.5
1940-60	7.5	-2.3	-3.0
1950-60	2.2	-5.0	-3.0
Mining			
1940-50	3.4	2.7	-1.0
1940-60	2.8	1.1	-3.1
1950-60	2.2	- .5	-3.0
Construction			
1940-50	8.5	3.1	4.3
1940-60	5.7	2.8	2.6
1950-60	3.0	.8	.9
Manufacturing			
1940-50	5.8	3.2	2.8
1940-60	5.3	2.7	1.8
1950-60	4.7	2.4	.9
Wholesale and retail trade			
1940-50	1.7	3.9	3.0
1940-60	.1	.7	1.2
1950-60	-1.5	-2.0	-1.6
Transportation, communications and public utilities			
1940-50	3.9	3.4	2.2
1940-60	3.1	1.9	.8
1950-60	2.2	.4	-1.5

Table 4 (concluded)

ANNUAL GROWTH IN DEMAND FOR EDUCATIONAL
FACTOR INPUTS BY SECTOR
(percent)

Sector	College	High School	Elementary School
Other services			
1940-50	3.0%	3.2%	-2.4%
1940-60	1.0	.2	-3.0
1950-60	-1.0	-2.1	-3.1
Government and government enterprises			
1940-50	2.9	6.2	.1
1940-60	3.1	4.2	.8
1950-60	3.0	2.2	1.7

between 1940 and 1950, while only 2.2 and 1.0 percent for high school- and elementary school-educated labor over the same period. Growth of total output over approximately the same period of time was 5.3 percent. Thus, the demand for each factor grew less than the total output. Similarly, over the 1940-1960 and 1950-1960 periods, the growth in demand for each educational factor was less than the growth in total output. However, in all periods, the growth in the demand for college-educated labor was greater than the demand for the two less-educated factors, and always positive. In the 1950-1960 period, there was a decrease in the demand for both high school- and elementary school-educated labor.

The observed relative factor ratios for the total economy are shown in Table 5 for each of the three years. Notice that between 1940 and 1960, more college-educated labor is being employed, relative to the two less-educated labor inputs. More high school-educated labor was used, relative to elementary school-educated labor in 1950 and 1960, as compared to 1940.

Table 5

OBSERVED EDUCATIONAL FACTOR RATIOS
IN THE ECONOMY: 1940, 1950 AND 1960

	1940	1950	1960
High school/College	2.60	2.36	2.05
Elementary/College	2.52	2.02	1.79
High school/Elementary	1.07	1.17	1.14

Why has there been an increase in the demand for more-educated labor, relative to less-educated labor, even though relative wage rates have remained constant between educational factors? Such changes in total demand may reflect: (1) increases in the output of the economy as a whole; (2) different sector growth rates of output; and (3) technological change.

Technological change may come about as a result of changes in the price of capital relative to the price of the educational factors. The price of capital relative to the average price of labor fell in the private domestic economy by about 23% between 1948-50 and 1961-63.[1] For each sector, the change in the relative price of capital to average labor for the same period is given in Table 6.

Table 6

CHANGE IN THE RELATIVE PRICE OF CAPITAL TO AVERAGE LABOR: 1961-63

(percent change with 1948-50 = 100)

Sector	Change in relative price of capital to average labor
Farms	88.7%
Mining	73.5
Construction	49.9
Manufacturing	67.1
Transportation	75.2
Communications	118.2
Electric and gas, etc.	99.6
Wholesale and retail trade	47.1

Source: J.W. Kendrick, "Industry Changes in Nonlabor Costs," Table 5.

Since the educational factor wage ratios were constant and the price of capital fell relative to that of average labor, this implies that the price of capital fell relative to the price of all educational inputs. Some reasons which can be offered for the decrease are: (1) liberalization of depreciation laws; (2) increase in the minimum wage; and (3) trade union pressure.

Thus, the growth in demand can be attributed to the first three reasons stated above. Tables 7-9 show my estimates of the decomposition of the change in demand for each educational factor for the three periods 1940-50, 1940-60 and 1950-60.

These estimates are computed under two alternative situations. First, when the base period is used as the point of reference, and second, when the terminal year is used as the point of reference. Technology is defined as the quantity of educational input employed per unit of real output. When the reference point is the base year, the technology of that year is used; while,the technology of the terminal year is used when that year is the reference point. The procedure for obtaining the estimates on Tables 7-9 are given in the appendix to this Chapter.

From Tables 7-9, it can be seen that changes due to technological change, when either the initial year or base year is used as the reference point, shows that such changes have resulted in a decrease in the demand for

Table 7

DECOMPOSITION OF THE CHANGE IN OBSERVED DEMAND FOR EACH EDUCATIONAL INPUT: 1940-50

Reference Year	Total Change	Due to Technological Change	Due to Constant Growth in All Sectors	Due to Changing Composition of Output
College				
1940	2 322 806	- 2 551 438	4 654 824	219 420
1950	2 322 806	- 1 358 038	3 629 505	51 339
High School				
1940	3 976 921	- 8 027 755	12 120 644	-115 968
1950	3 976 921	- 4 504 798	8 719 522	-237 803
Elementary				
1940	1 616 785	-10 667 902	11 712 234	572 453
1950	1 616 785	- 5 962 723	7 210 378	369 130

Table 8

DECOMPOSITION OF THE CHANGE IN OBSERVED DEMAND
FOR EACH EDUCATIONAL INPUT: 1940-60

Reference Year	Total Change	Due to Technological Change	Due to Constant Growth in All Sectors	Due to Changing Composition of Output
College				
1940	2 920 573	- 6 051 721	8 582 139	390 155
1960	2 920 573	- 2 464 272	5 151 874	232 971
High School				
1940	2 560 484	-16 981 629	22 346 936	-2 804 823
1960	2 560 484	- 8 114 155	11 052 034	- 377 395
Elementary				
1940	758 042	-20 915 731	21 593 947	79 826
1960	758 042	- 8 833 902	9 293 371	299 573

Table 9

DECOMPOSITION OF THE CHANGE IN OBSERVED DEMAND

FOR EACH EDUCATIONAL INPUT: 1950-60

Reference Year	Total Change	Due to Technological Change	Due to Constant Growth in All Sectors	Due to Changing Composition of Output
College				
1950	597 767	-2 643 442	3 072 705	168 504
1960	597 767	-1 875 161	2 393 897	79 031
High School				
1950	-1 416 437	-7 697 670	7 251 133	-969 900
1960	-1 416 437	-5 922 668	5 107 128	-600 897
Elementary				
1950	- 858 743	-6 739 266	6 200 943	-320 420
1960	- 858 743	-4 978 723	4 398 646	-278 666

each educational factor. However, each educational factor was not affected to the same extent. Table 10 shows what the factor ratios would have been in 1940 had the technology of 1950 and 1960 prevailed. Under both technological states, the ratio of non-college- to college-educated labor would have been less than the ratio that was observed in 1940 with that state of technology. Thus, college-educated labor employed would have increased,, relative to the two less-educated inputs, even though technological change decreased the demand for all factors. Using the 1960 technology, the decrease in all factors was larger than with the 1950 technology. Again, however, the impact of technological change decreased the number of high school- and elementary school-educated labor employed, relative to college-educated labor, as can be seen by comparing the 1940 factor ratios, if the technology of 1950 or 1960 existed in 1940. Table 10 also shows that had the technology in 1960 existed in 1950, the demand for college-educated labor would have decreased less than that of the two less-educated inputs.

Tables 7-9 show that sector differences in the rate of growth of real output has resulted in an increase in the demand for college-educated labor. That is, the changing composition of output increased the demand for that factor. On the other hand, between 1940-1950 and 1940-1960, elementary school-educated labor also increased,

Table 10

ESTIMATED EDUCATIONAL FACTOR RATIOS IN 1940 AND 1950
HOLDING TECHNOLOGY CONSTANT

	Observed ratio in 1940	Ratio in 1940 with 1950 Technology	Ratio in 1940 with 1960 Technology	Ratio in 1950 with 1960 Technology
High School/College	2.60	2.40	2.14	2.13
Elementary/College	2.52	1.99	1.80	1.84
High School/Elementary	1.07	1.21	1.19	1.16

Table 11

ESTIMATED EDUCATIONAL FACTOR RATIOS IN 1950 AND 1960
DUE TO THE CHANGING COMPOSITION OF OUTPUT

	Ratio in 1950 with 1940 Technology	Ratio in 1960 with 1940 Technology	Ratio in 1960 with 1950 Technology
High School/College	2.54	2.35	2.24
Elementary/College	2.52	2.45	1.96
High School/Elementary	1.01	0.96	1.14

due to the changing composition of output. There was, however, a decrease in the demand for elementary school-educated labor between 1950 and 1960. In all the periods considered, the demand for high school-educated labor decreased, due to the changing composition of output. Table 11 shows what the factor ratios would have been if the technology of 1940 prevailed and the output of each sector in 1950 and 1960 was unchanged. The estimated factor ratios for 1950 and 1960 are due to the change in the composition of output. In 1950, the ratio of elementary- to college-educated labor would have been unchanged from its 1940 level. Thus, between 1940 and 1950, the increase in demand for both of these factors, due to the changing composition of output, left the ratio unchanged. On the other hand, between 1940 and 1960, the demand for college-educated labor increased relative to elementary school-educated labor, so that the ratio fell. The remaining ratios simply reflect the fact that the demand for college-educated labor increased, due to the changing composition of output relative to the two less-educated labor inputs.

(2.3) A Preliminary View of the Demand for Education in U.S. Manufacturing

Of primary concern of this study is the use of educational factors in U.S. manufacturing. Here, I take a preliminary look at the first two hypotheses stated in Chapter 1 by examining the relationship between the change in selected variables between the late 40's and early 60's and the educational distribution in 1960.

Table 12 shows the educational factor ratios in 1960 for 17 two-digit SIC manufacturing industries. For the same industries, Table 13 shows the change in the following four variables between the periods 1948-50 and 1961-63, using the former period as the base year: (1) relative capital input; (2) relative price of capital; (3) relative capital share; and (4) real output.

The change in the relative use of capital is significantly related to the change in the average price of capital. The Spearman rank correlation is -.665.[2] That is, the greater the decrease in the price of capital relative to the average labor compensation, the greater the quantity of capital used per unit of labor.

For those industries in which the capital per unit of average labor increased the most, the greater were the number of high school- and elementary school-educated labor employed relative to college-educated labor in 1960.[3] This observation lends some preliminary support to the hypothesis that increases in the quantity of capital utilized appears to be associated with those industries in which more non-college-educated labor is employed relative to college-educated labor.[4] On the

Table 12

EDUCATIONAL FACTOR RATIOS IN 1960 FOR
17 TWO-DIGIT SIC MANUFACTURING INDUSTRIES

Industry (SIC Code)	High School / College	Elementary / College	Elementary / High School
Food and kindred prod. (20)	5.21	3.89	.75
Tobacco (21)	5.81	6.46	1.11
Textile mill prod. (22)	7.44	9.21	1.24
Apparel (23)	8.19	7.72	.94
Furniture and fixtures (25)	6.49	6.48	1.00
Paper and allied prod. (26)	3.10	4.72	.66
Printing and publishing (27)	3.25	1.41	.43
Chemical and allied prod. (28)	1.89	1.00	.53
Rubber and misc. plastics (30)	4.20	2.51	.60
Leather (31)	9.96	10.12	1.02

Table 12 (Concluded)

EDUCATIONAL FACTOR RATIOS IN 1960 FOR
17 TWO-DIGIT SIC MANUFACTURING INDUSTRIES

Industry (SIC Code)	High School / College	Elementary / College	Elementary / High School
Stone, clay and glass prod. (32)	4.60	3.97	.86
Primary metal ind. (33)	5.01	4.08	.81
Fabricated metal prod. (34)	3.60	2.16	.60
Machinery, exc. elec. (35)	3.90	2.04	.52
Electrical machinery (36)	2.83	1.11	.39
Transportation equipment, exc. motor vehicles (37-371)	2.70	1.23	.46
Motor vehicles and equip. (371)	5.24	3.44	.66

NOTE: The 1960 educational distribution is taken from the 1960 Census of Population. The distribution includes eight classifications for both sexes. The distribution is combined into three groups: elementary, high school and college male graduates by using the relative average hourly wage with the average hourly wage of male graduates as the group representative. The average hourly wage in 1960 is obtained from:

V.R. Fuchs, Differentials in Hourly Earnings by Region and City Size, 1959, Occasional Paper 101, National Bureau of Economic Research (1967).

Table 13

RELATIVE CAPITAL INPUT, RELATIVE PRICE OF CAPITAL, CAPITAL SHARE OF INCOME AND RELATIVE CHANGE IN REAL PRODUCT FOR 17 TWO-DIGIT SIC MANUFACTURING INDUSTRIES, 1961-63

(index number 1948-50 = 100)

Industry (SIC Code)	Relative capital input	Relative price of capital	Capital share of income	Relative change in real product
Food and kindred prod. (20)	123.6	72.2	89.2	85.0
Tobacco (21)	120.0	99.4	119.3	94.4
Textile mill prod. (22)	149.3	52.2	77.9	75.9
Apparel (23)	122.6	80.2	98.3	85.1
Furniture and fixtures (25)	112.2	80.9	91.3	81.3
Paper and allied prod. (26)	129.5	64.1	83.0	104.8
Printing and publishing (27)	119.5	85.6	99.9	91.4
Chemical and allied prod. (28)	119.4	76.3	91.1	149.3
Rubber and misc. plastics (30)	117.3	104.9	123.0	110.0
Leather (31)	147.0	61.3	90.1	60.9

Table 13 (Concluded)

RELATIVE CAPITAL INPUT, RELATIVE PRICE OF CAPITAL, CAPITAL SHARE OF INCOME AND RELATIVE CHANGE IN REAL PRODUCT FOR 17 TWO-DIGIT SIC MANUFACTURING INDUSTRIES, 1961-63

(index number 1948-50 = 100)

Industry (SIC Code)	Relative capital input	Relative price of capital	Capital share of income	Relative change in real product
Stone, clay, and glass prod. (32)	150.8	67.2	101.3	92.6
Primary metal ind. (33)	143.4	59.5	85.3	66.5
Fabricated metal prod. (34)	142.5	56.1	79.9	99.2
Machinery, exc. electrical (35)	131.7	69.8	91.9	92.4
Electrical machinery (36)	108.4	69.9	75.8	161.9
Transportation equipment exc. motor vehicles (37-371)	82.7	125.5	103.8	194.7
Motor vehicles and equip. (371)	147.7	64.6	95.4	104.6

NOTE: Source: J.W. Kendrick, "Industry Changes in Non-labor Costs," The Industrial Composition of Income and Product, ed. J.W. Kendrick (New York, 1968), pp. 151-176.

other hand, there is no statistically significant relationship between the change in capital's relative share and the educational factor ratios in 1960.[5]

As for the relationship between the growth of output between the two periods under consideration and the educational factor ratios in 1960, the faster-growing industries, as measured by real output, are those industries in which more college-educated labor is employed relative to non-college-educated labor.[6] This appears to support the changing composition of output hypothesis for U.S. manufacturing.

As stated in Chapter 1, it may be difficult to separate the effect of the changing composition of output and the non-neutrality of technology, if the two variables are significantly related. That is, if the most rapidly growing industries are also those industries which have a rapidly changing technology, then distinguishing between the two hypotheses is made difficult. However, from the data in Table 13, there is no significant statistical relationship between the growth of real output and the change in the relative share of capital employed relative to labor.[7]

(2.4) Summary

For the U.S. economy between 1940 and 1960, the changing demand for each educational factor is decomposed into changes due to : (1) changes in output when the composition of output is held constant; (2) changes in output when the composition of output changes; and (3) technological change.

The results imply that technological change resulted in a decrease in all educational factors of production. This finding does not lend support to the hypothesis that changing technology increased the demand for more-educated labor, and hence, prevented the relative wage rate of such factors from falling relative to that of less-educated labor.

It was also found, however, that technological change had a greater effect on decreasing the demand for less-educated labor inputs relative to college-educated labor.

On the other hand, the findings support the hypothesis that the changing composition of output has increased the relative demand for college-educated labor. For all the time periods considered, the demand for college labor increased while the demand for high school-educated labor decreased. Between 1940 and 1950, the demand for elementary school labor increased at the same rate as college-educated labor; however, between 1950 and 1960, the demand for this factor decreased as a result of the changing composition of output.

Focusing on U.S. manufacturing, data on changes in the capital-average labor ratio between 1948-50 and

1961-63, and the educational factor ratios in 1960, support the hypothesis that the non-neutrality of capital may increase the demand for non-college-educated labor relative to more college-educated. This finding appears to be contrary to what I observe for the total U.S. economy.

The data for U.S. manufacturing, however, supports the hypothesis that the changing composition of output may have increased the relative demand for college labor. This hypothesis is also supported for the total economy.

In Chapter 4, I concentrate on the question of the non-neutrality in production in U.S. manufacturing by examining patterns of factor substitution between physical capital and each of the educational labor inputs. Before such patterns are estimated, I first discuss alternative definitions of the elasticity of substitution and discuss possible production functions for estimating these patterns.

Footnotes to Chapter 2

1 See: J.W. Kendrick, "Industry Changes in Nonlabor Costs," The Industrial Composition of Income and Product, ed. J.W. Kendrick (New York, 1968), pp. 150-184.

2 The standard error is .25 and the t-value is -6.66. At the 10 percent level of significance, the computed t-value is significant. All of the statements made in this section are based on the Spearman rank correlation.

3 The Spearman rank correlation is .488 between the change in the relative quantity of capital employed and the ratio of high school- to college-educated labor employed; while the rank correlation is .528 for the ratio of elementary- to college-educated labor. At the 10 percent level of significance, both rank correlations are statistically significant.

4 Between the change in relative capital employed and the ratio of elementary to high school graduates employed, the rank correlation is .553 and statistically significant. This implies that industries in which capital per unit of average labor is growing the fastest, more elementary school graduates are employed relative to high school graduates.

5 The Spearman rank correlation between the change in the relative share of capital and the educational factor

ratios in 1960 are .029 for the ratio of high school- to college-educated labor, -.100 for the ratio of elementary- to college-educated labor and -.024 for the ratio of elementary- to high school-educated labor. All the rank correlations are not significantly different from zero.

6 The Spearman rank correlations are respectively -.882, -.728 and -.647 between the change in real output and the following educational ratios in 1960 - high school/ college, elementary school/ college and elementary school/ high school. All correlations are statistically significant at the 10 percent level.

7 Between the change in the relative share of capital and the growth of real output, the Spearman rank correlation is .223 and -.393 between the change in the relative quantity of capital employed and the growth of real output. Both rank correlations are not statistically different from zero at the 10 percent level.

Chapter 3

ESTIMATING PATTERNS OF FACTOR SUBSTITUTION

Economists interested in estimating patterns of substitution in production processes which employ more than two factor inputs encounter two significant problems at the very outset of their work. First, in a multi-input production process with more than two factor inputs, there is no "standard" or "traditional" definition of the elasticity of substitition (ES) between a pair of factor inputs as there is in a two factor input world. Instead, there are several alternative definitions of the ES which incorporate different ceterius paribus conditions.

Once a definition of the ES is accepted, the second problem is to select a production function whose derived equations permit the estimation of the desired measure, yet does not restrict patterns of substitution a priori.

Here, I discuss alternative definitions of the ES in a four factor input world. The discussion can be generalized into any number of factor inputs. After each measure is discussed, I present a production function which holds each measure constant, and I discuss the restrictions placed upon the patterns of substitution for each production function. Finally, I discuss an alternative procedure for analyzing patterns of substitution. This procedure does not assume an a priori production function. Instead, it utilizes an approximation to the demand equation for each factor input, in order to make inferences about patterns of substitution, and is later used to make such inferences.

(3.1) Elasticity of Substitution with Four Factors of Production: One-Factor-One-Price and Two-Factor-Two-Price Measures

The ES, according to Hicks, is a measure of the ease with which one factor can be substituted for another along a given isoquant. With only two factors in the production process, no problems arise for the definitions of this measure. However, once we admit more than two factors, no unique definition of the ES exists.

The most commonly used definition is the Allen partial ES.[1] The interpretation of the Allen partial ES is that it is the elasticity of factor i with respect to total cost, for a change in the price of factor j while holding output and all other factor prices constant. For this reason, the Allen partial ES is known as a one-factor-one-price measure of the ES. The Allen partial ES can also be interpreted as the cross demand elasticity between factors i and j, normalized by the relative change in total cost. This definition reduces to the standard two-factor ES measure when there are only two factors.

Another approach is to apply the standard measure of the two factor input ES to each pair of factors, while holding all other factor inputs constant. This definition is known as the direct partial ES.[2] In the Allen partial ES definition, all other levels of factor inputs were not held constant. Since the direct partial ES measure involves the change in two factor inputs with respect to a change in their relative prices, the direct partial ES is known as a two-factor-two-price measure of the ES.

The direct partial ES is also subject to simple economic interpretation. It tells us how the relative factor shares of a pair of factors in total cost change when we change their relative prices, holding all other factor levels and output constant.

If we combine elements of both the Allen partial ES and direct partial ES, we obtain the shadow partial ES. In this case, we apply the standard two factor input ES measure to each pair of factors, permitting the level of the remaining input factors to vary, but holding the other factor prices and the total cost fixed.[3] The shadow partial ES is, thus, also known as a two-factor-two-price ES measure.

Denoting factor inputs by X_i and its respective price by W_i, output by Q and total variable cost by C, we can summarize the above measures of the ES and the ceterius paribus relationship involved in each measure as:

One-factor-one-price measure:

Allen partial -

$$\left.\frac{\partial \log X_i}{\partial \log W_j}\right| \begin{array}{ll} Q = \bar{Q} & \\ W_k = \bar{W}_k & \text{for all } k \neq j. \end{array}$$

Two-factor-two-price measure:

Direct partial -

$$\left.\frac{\partial \log\left(\frac{X_i}{X_j}\right)}{\partial \log\left(\frac{W_i}{W_j}\right)}\right| \begin{array}{ll} Q = \bar{Q} & \\ W_k = \bar{W}_k & \text{for all } k \neq i \text{ or } j. \\ X_k = \bar{X}_k & \text{for all } k \neq i \text{ or } j. \end{array}$$

Shadow partial -

$$\left.\frac{\partial \log\left(\frac{X_i}{X_j}\right)}{\partial \log\left(\frac{W_i}{W_j}\right)}\right| \begin{array}{ll} Q = \bar{Q} & \\ C = \bar{C} & \\ W_k = \bar{W}_k & \text{for all } k \neq i \text{ or } j. \end{array}$$

(3.2) Production Functions with Constant One-Factor-One Price ES and Two-Factor-Two-Price ES

In attempting to analyze patterns of substitution in a production process with two or more factors of production, one seeks a production function which has derived functional forms which include the relevant parameters and that can be estimated by means of econometric techniques available. This was generally done by imposing the constancy of either the one-factor-one-price or two-factor-two-price ES measure. In the case of a production process with only two factors, a linear homogeneous production function, with the constancy of both measures imposed, is given by the well-known SMAC-CES production function:[4]

$$Q = (A_1 X_1^a + A_2 X_2^a)^{1/a}$$

where the ES equals 1/(1-a) and is the same for all measures of the ES.

Using the Allen partial ES, Uzawa has generalized the CES production function to multi-factor inputs and has shown that if such elasticities are to remain constant, no further generalizations are possible.[5] The natural extension of the CES to four factors is given by

$$(1) \quad Q = \left[\sum_{i=1}^{4} A_i X_i^a\right]^{1/a}$$

For this case, all six pairs of Allen partial ES are constant and equal to the common value 1/(1-a).[6] Slight flexibility is added if we partition the factors into different groups. In such a case, Uzawa has shown that (i) the Allen partial ES is unity for all pairs of factors in different groups; (ii) the Allen partial ES for any pair of factors in the same group is constant and equal to a common value; and (iii) the common group ES value does not have to be equal across groups.

Working with the two-factor-two-price ES measure - that is, the direct partial ES and shadow partial ES - McFadden has also shown restrictions on the generalized CES. In the case of the natural extension of the CES, given by (1), the restriction is identical to that imposed by the Allen partial ES - that is, all six possible pairs of ES are constant and equal to the common value 1/(1-a). However, the situation is more restricted when factors are partitioned than in the case using the Allen partial ES. For the direct partial ES, McFadden has shown that when factors are partitioned into groups: (i) the ES between factors in the same group is unity; (ii) the ES between factors in different groups is constant and equal to some common value; and (iii) this common value is the same between all factors in different groups. Thus, when using the partitioned version of the CES, if we use the direct partial or shadow partial ES, only two values can be taken for the ES - unity and some common value.

Hence, production functions with constant one-factor-one-price and two-factor-two-price ES measure are too restrictive for studying theoretical patterns of substitution between more than two factors of production. Another limitation is also present. If complementarity between factors does exist - that is, the ES is negative - such a case is not permitted in the CES framework by the restrictions imposed upon the parameters by cost minimization.

(3.3) Elasticity of Substitution with Four Factors of Production: Two-Factor-One-Price Case

Working within the Allen partial ES framework, one can generate measures of the ES by asking what happens to the ratio of the ES between two factors, i and j, when the price of a third factor changes while all other factor prices and output are held constant. Or one can ask what happens to the relative quantity of factors of i and j employed when the price of factor k changes, with all other factor prices and output held constant.[7] This is why this ES is called a two-factor-one-price measure.

In the former case, the two-factor-one-price elasticity is the ratio of the Allen partial ES between factors i and k and factors j and k - that is, $\sigma_{ik}^{A}/\sigma_{jk}^{A}$. The latter case results in just the difference of the Allen partial ES - that is, $\sigma_{ik}^{A}-\sigma_{jk}^{A}$.

The advantage of the two-factor-one-price measure over the other measures discussed is that the former measure yields production functions with constancy of such measures that are less restrictive and useful in studying patterns of substitution than production functions with constancy of the latter measure. It is to such functions that I now turn.[8]

(3.4) Production Functions with Constant Two-Factor-One-Price ES Measures

(3.4.1) The Constant Ratio of ES Case (CRES)

Production functions with constant ratios of ES have been used in studies as early as 1963. Such functions have been called the "Mukerji" function and written as:[9]

$$(3) \qquad Q = \left[\sum_{i=1}^{4} A_i \quad X_i^{a_i}\right]^{1/a}$$

The problem with (3) is that it is not homogeneous nor homothetic, and hence, the ES will vary with the factor bundle used and with the level of output. Another property of (3), as Hanoch has shown, is that for given factor prices, the expansion path may be curved in an undesirable way.[10,11]

Hanoch, however, has shown that (3) can be written as a linear homogeneous function implicitly as:[12]

$$(4) \qquad \phi \quad (Q, \tilde{X}) = \sum_{i=1}^{4} A_i \quad \left[\frac{X_i}{Q}\right]^{a_i} - 1 = 0$$

($\tilde{X}$ is a vector of inputs)

The derived equation under competitive cost minimization are obtained from the Lagrangian:

$$(5) \qquad L = \sum_{i=1}^{4} W_i \; X_i \; - \lambda \left[\sum_{i=1}^{4} A_i \left[\frac{X_i}{Q} \right]^{a_i} - 1 \right]$$

Denoting logarithms by the minor letter of the respective variables, the first-order conditions for competitive cost minimization are:[13]

$$x_i = b_i \; (\bar{\lambda} - w_i - a_i \; q + c_i) \quad i = 1,2,3,4$$

where $b_i = 1/(1 - a_i)$; $\log|\lambda| = \bar{\lambda}$ [14] and; $\log|A_i a_i| = c_i$.

The Allen partial ES for (4) can be shown to be[15]

$$\sigma_{ij}^{A} = \frac{b_i b_j}{\sum_K b_K s_K} \qquad \text{if } i = j$$

and

$$\sigma_{ii}^{A} = \frac{b_i^2}{\sum_K b_K s_K} - \frac{b_i}{s_i}$$

where s_k is the share of the kth factor in total cost; i.e.

$$s_k = \frac{W_k X_k}{\sum_i W_i X_i}$$

Thus, we observe that σ_{ij}^{A} has a common factor $1/(\sum_K b_K s_K)$ for all i,j $(i \neq j)$.[16] If we are now interested in comparing the Allen partial ES between, say, X_i and X_j, with the ES between X_j and X_m, it is necessary only to compare the magnitudes of b_j and b_m. That is, to test if

$$\sigma_{im}^{A} \gtrless \sigma_{ij}^{A}$$

we are equivalently testing whether

$$\frac{b_i \; b_m}{\sum_k a_k \; b_k} \gtrless \frac{b_i \; b_j}{\sum_k a_k \; b_k}$$

but since Hanoch shows that the denominator must be positive, the test reduces to if

$$b_m \gtrless b_j$$

(4) has many well-known production functions as special cases. For example, if all a_i are equal, we have the generalized CES given by (1). If all a_i equal unity (or equivalently $b_i = -\infty$) then we have the case of perfect substitution. If all a_i equal $-\infty$ (or $b_i = 0$), then we have the Leontief fixed-proportion production function; while, if all a_i equal zero (or $b_i = 1$), we have the Cobb-Douglas Case.

Although the CRESH production function does permit more flexibility than the previous multi-factor input functions discussed in this Chapter, it does have one fault: estimates of the elasticity of substitution vary depending upon the procedure used to estimate the function. Suggested estimation procedures are discussed in the Appendix to this Study. An illustration of the variation of the elasticity of substitution for alternative estimating procedures is also given in the Appendix. For this reason, The CRESH function is not used in the study.

(3.4.2) The Constant Difference of ES Case (CDE)

Attempts have been made to estimate parameters of the production function by using the duality relationship between production functions and cost functions. Nerlove, for example, in his study of the electricity supply, used this approach.[17] By using the duality relationship, economists can obtain two production functions for the price of one, both satisfying the same requirements but with different properties.[18] Hanoch has shown that the unique dual of the CRESH production function is a cost function with constant differences of Allen partial ES.[19]

(3.5) A Model Without An A Priori Production Function

It is possible to make inferences about patterns of factor substitution without assuming an a priori production function. Since the ES is a second-order parameter, the demand equation for each factor provides a starting point. Here, I use a model suggested by Griliches.[20] He suggests that if inputs are defined per unit of output and constant returns to scale are assumed, then we can approximate the demand function for inputs by:[21,22]

$$(6) \quad \log \frac{X_i}{Q} = a_i + \sum_{j=1}^{4} e_{ij} \log W_j \qquad i = 1,\ldots,4$$

where e_{ij} is the partial demand elasticity between factors i and j and a_i is a constant.

For the Allen partial we know that $e_{ij} = S_j \sigma_{ij}$ and can, therefore, write (6) as

$$\text{(7)} \quad \log \frac{X_i}{Q} = a_i + \sum_{j=1}^{4} s_j \sigma_{ij} \log W_j \qquad i = 1, \ldots, 4$$

Subtracting the demand equation for input k from the demand equation for input i, we have

$$\text{(8)} \quad \log \frac{X_i}{X_k} = (a_i - a_k) + \sum_{j=1}^{4} (\sigma_{ij} - \sigma_{kj}) \, s_j \log W_j$$

The sign of the coefficient of $s_j \log W_j$ then tells us if

$$\sigma_{ij} - \sigma_{kj} \gtrless 0$$

It is model (8) that I use in the next Chapter to make inferences about patterns of substitution.

Footnotes to Chapter 3

1 The Allen partial ES was first developed in:

R.G.D. Allen and J.R. Hicks, "A Reconsideration of the Theory of Value, II," Economica (1934), 196-219.

In this article, the authors use the terminology "elasticity of complementarity," (pp. 202-206). Allen developed the concept further in:

R.G.D. Allen, Mathematical Analysis for Economists (New York, 1938), pp. 503-505. For any linear homogeneous production function

$$Q = F(\tilde{X})$$

where Q is output

and $\tilde{X}$ is the input vector with elements X_i $(i = 1,2,3,4)$, the Allen partial ES between any pair of factors X_i and X_j is given by

$$\sigma_{ij}^{A} = \frac{Q}{X_i \, X_j} \frac{\bar{F}_{ij}}{\bar{F}}$$

where

$\bar{F}$ is the determinant of the bordered Hessian

$$\begin{vmatrix} 0 & F_1 & F_2 & F_3 & F_4 \\ F_1 & F_{11} & F_{12} & F_{13} & F_{14} \\ F_2 & F_{21} & F_{22} & F_{23} & F_{24} \\ F_3 & F_{31} & F_{32} & F_{33} & F_{34} \\ F_4 & F_{41} & F_{42} & F_{43} & F_{44} \end{vmatrix}$$

$\bar{F}_{ij}$ is the cofactor of F_{ij} and F_i and F_{ij} denote the first and second partial derivatives.

[2] This measure was first used in:

Allen and Hicks, pp. 211-214. The terminology used was the "elasticity of substitution between Y and Z in the YZ indifference direction." The concept was further applied to production theory in:

D. McFadden, "Further Results on C.E.S. Production Function," Review of Economic Studies, XXX (1963), 73-83.

The direct partial ES is defined by

$$\frac{(X_i\ F_i + X_j\ F_j)\ F_i\ \ F_j}{-\ F_{ii}\ F_j^2 + 2\ F_{ij}\ \ F_i\ F_j - F_{jj}\ F_i^2}$$

This definition holds regardless of any assumption about the returns to scale parameter. When F is homogeneous of any degree, the measure depends on factor proportions.

[3] The Shadow partial ES was first defined and applied to production theory in:

D. McFadden, pp. 76-77.

[4] K.J. Arrow, H.B. Chenery, B.S. Minhas and R.M. Solow, "Capital-Labor Substitution and Economic Efficiency." Review of Economics and Statistics, XLIII (1961), 225-250.

[5] H. Uzawa, "Production Functions with Constant Elasticity of Substitution," Review of Economic Studies, XXIX (1962), 291-299.

[6] The three special cases of the CES exist when a equals either unity, zero or infinity. In such instances, we have a production function with perfect substitution between factors , the Cobb-Douglas and Leontief production functions respectively.

7 The latter measure was first suggested in:

Y. Mundlak, "Elasticities of Substitution and the Theory of Derived Demand," Review of Economic Studies, XXXIV (1968), 225-236.

8 Another advantage of the two-factor-one-price measure over the two-factor-two-price measure is that the latter measure depends upon the relative magnitude of the prices which are being changed, and restrictions must be placed on such changes. Since such restrictions can be arbitrary, the result will be different measures for the two-factor-two-price measure. What one is really doing in the one-price measure is changing relative factor prices by just changing one factor price; thus, no restrictions need be placed. (See Mundlak, pp. 229-230).

9 V. Mukerji, "Generalized SMAC Function with Constant Ratios of Elasticities of Substitution," Review of Economic Studies, XXX (1963), 233-236.

10 G. Hanoch, "'CRESH' Production Functions," Discussion Paper No. 84; Harvard Institute for Economic Research, (Cambridge, Mass.: August, 1969), Appendix A.
Here it is shown that if any $a_i > 1$, then all other X_j $(j \neq i)$ will be inferior. That is, with all factor prices given as output increases, the quantity of factor X_j $(j \neq i)$ will decrease.

11 Gorman has shown that if the ratio of the Allen partial ES were to remain constant, no further generalization could be made.

W.M. Gorman, "Production Functions in Which the Elasticities of Substitution Stand in Fixed Proportion to Each Other," Review of Economic Studies, XXXII (1965), 217-224.

12 G. Hanoch, "'CRESH' Production Functions," I refer to (4) as the CRESH function as Hanoch does. I have restricted myself to the linear homogeneous case. The general CRESH production function is implicitly given by:

$$(4')\quad \phi\ (Q,\tilde{X}) = \sum_{i=1}^{4} A_i \left[\frac{X_i}{h(Q)}\right]^{a_i} - 1 \quad 0$$

where h(Q) is a linear function of the input vector. If $h(Q) = Q^{1/u}$, then (4') is homogeneous of degree u. When u = 1, we have the constant returns to scale case.

13 G. Hanoch, "'CRESH' Production Functions," p. 4.

14 Note that $\bar{\lambda}$ is a function of factor prices and output.

15 G. Hanoch, "'CRESH' Production Functions," pp. 4-5.

16 If we applied the direct partial ES, we would obtain

$$\sigma_{ij}^{D} = \frac{b_i\ b_j}{(s_i\ b_i + s_j\ b_j)\ /(s_i + s_j\ )}$$

Thus, the σ_{ij}^{D} do not vary proportionately with some common factor, and constant ratios do not hold. (SEE: G. Hanoch, "'CRESH' Production Functions," p. 12.)

[17] M. Nerlove, "Returns to Scale in Electricity Supply," Measurement in Economics: Studies in Mathematical Economics and Econometrics in Memory of Yehuda Grunfeld, ed. C. Christ (Stanford, 1963).

[18] For a discussion of duality and the conditions for duality, SEE:

G. Hanoch, "Generation of New Production Functions Through Duality," Discussion No. 118, Harvard Institute for Economic Research, (Cambridge, Mass.; April, 1970).

[19] G. Hanoch, "Production with Constant Two-Factors-One-Price Elasticities of Substitution," Discussion Paper No. 117, Harvard Institute for Economic Research, (Cambridge, Mass.; April, 1970).

[20] Z. Griliches, "Education in Production Functions and Growth Accounting."

[21] Here is the proof of equation (6):

The demand for any factor, X_i, is a function of all factor prices, W_j ($j = 1,\ldots,n$) and the level of output, Q. That is,

$$X_i = f(W_1,\ldots,W_n,Q)$$

Taking the total differential of the above function, we have

$$dX_i = \sum_{j=1}^{n} \frac{\partial X_i}{\partial W_j} dW_j + \frac{\partial X_i}{\partial Q} dQ$$

Dividing both sides by X_i, we have

$$\frac{dX_i}{X_i} = \sum_{j=1}^{n} \frac{\partial X_i}{\partial W_j} \frac{dW_j}{X_i} + \frac{\partial X_i}{\partial Q} \cdot \frac{dQ}{X_i}$$

which can also be expressed as

$$\frac{dx_1}{x_1} = \sum_{j=1}^{n} \frac{\partial x_1}{\partial w_j} \frac{w_j}{x_1} \frac{dw_j}{w_j} + \frac{\partial x_1}{\partial Q} \frac{Q}{x_1} \frac{dQ}{Q}$$

but

$$\frac{\partial x_1}{\partial w_j} \frac{w_j}{x_1} = \text{the cross demand elasticity}$$
$$= e_{1j}$$

and

$$\frac{\partial x_1}{\partial Q} \frac{Q}{x_1} = \text{the percentage change in } x_1 \text{ employed when output increases by 1 percent.}$$

When the production function is linearly homogeneous, this last value is 1. Substitution into the equation of the total differential, we have:

$$\frac{dx_1}{x_1} = \sum_{j=1}^{n} e_{1j} \frac{dw_j}{w_j} + \frac{dQ}{Q}$$

Holding the e_{ij}'s constant and integrating each side of the above equation, we have

$$\text{Ln } x_1 = \sum_{j=1}^{n} e_{1j} \text{ Ln } w_j + \text{Ln } Q + a_1$$

or

$$\text{Ln } x_1 - \text{Ln } Q = \sum_{j=1}^{n} e_{1j} \text{ Ln } w_j + a_1$$

where a_1 is a constant of integration.

22 When we attempt to estimate the ES, we are theoretically interested in movements along an isoquant. However, in cross sectional (or time series) data, we are generally given information in which production takes place at different levels of output. In such a case, we may not be measuring the theoretical ES. In order to avoid the above situation, the assumption that the production function is homogeneous or homothetic is substituted for the obviously unwarranted assumption that all firms in the industry are producing at the same level of output.

This can be exemplified geometrically. Let us assume that we know the world perfectly and that there are two factor inputs, X_1 and X_2, needed to produce an output Q measured in physical units. Suppose we have two observations on two firms in the industry producing Q. (See Diagram 1). Both firms are producing at output level $\bar{Q}$ and are in equilibrium at e_o and e_1 respectively, given the relative factor prices $\left(\frac{W_2}{W_1}\right)_o$ and $\left(\frac{W_2}{W_1}\right)_1$ they face. Then the ES we are interested in measuring is:

$$\sigma_{12} = \frac{\left(\frac{X_2}{X_1}\right)_o - \left(\frac{X_2}{X_1}\right)_1}{\left[\frac{\left(\frac{X_2}{X_1}\right)_o + \left(\frac{X_2}{X_1}\right)_1}{2}\right]} \div \frac{\left(\frac{W_2}{W_1}\right)_o - \left(\frac{W_2}{W_1}\right)_1}{\left[\frac{\left(\frac{W_2}{W_1}\right)_o + \left(\frac{W_2}{W_1}\right)_1}{2}\right]}$$

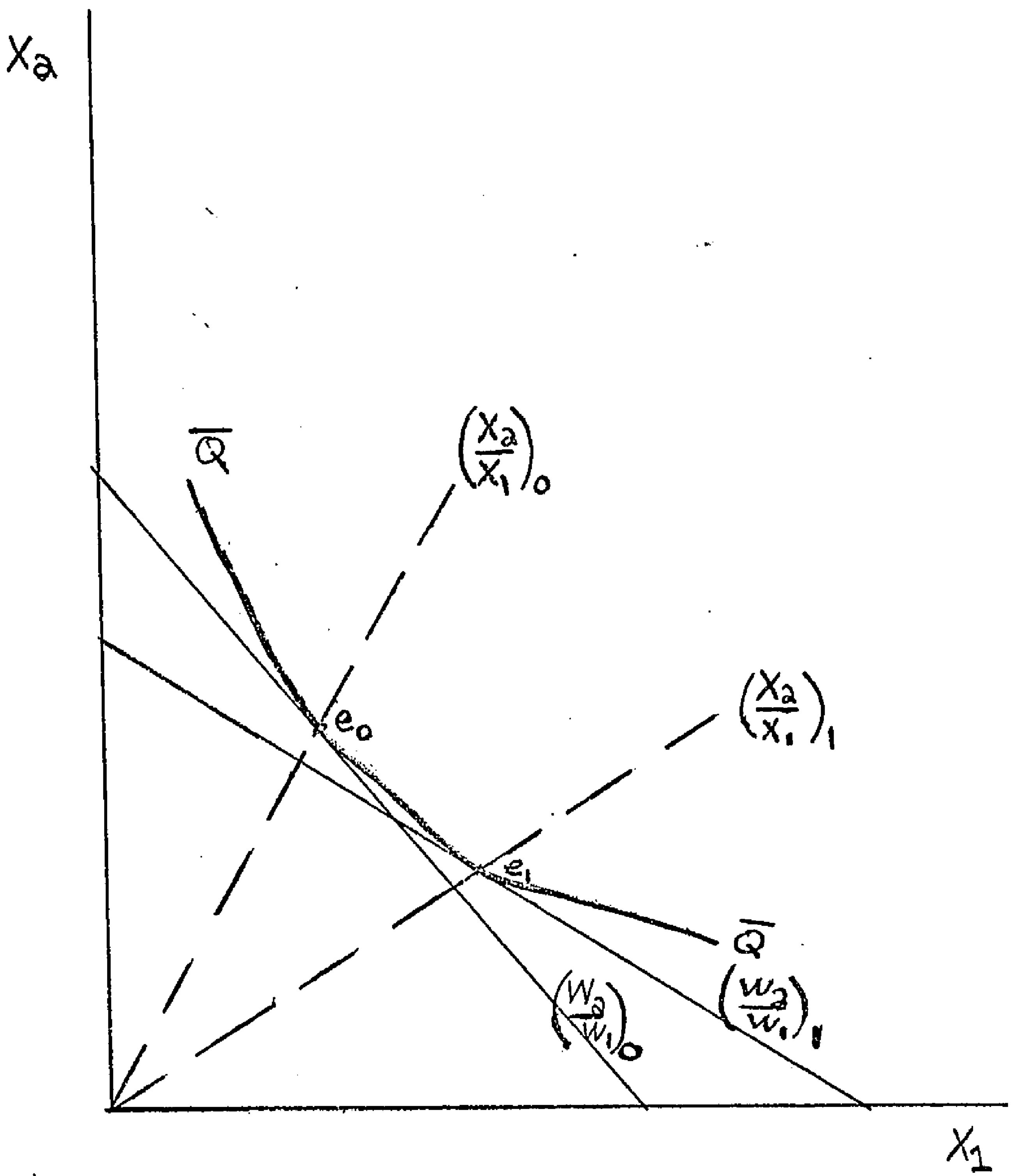
We would expect that as we move from e_o to e_1 - that is, as the wage of X_1 falls relative to that of X_2 - the ratio of $\frac{X_2}{X_1}$ will decrease. Now what would happen if one of the firms were producing at a higher level of output, say $\overline{Q}'$, and in addition, the production function is not homogeneous. Then, as will be illustrated, we will not only not be measuring the ES, but the estimate obtained may have a sign which is not expected a priori. To see this, assume the conditions as specified by Diagram 2. Then we would observe that the "measured" ES implies that as the wage rate of X_1 falls relative to that of X_2, the quantity of X_1 will decrease relative to X_2. However, this is not the "true" measure we are interested in.

If the production function is assumed to be homogeneous (or homothetic), we can then observe what happens from Diagram 3. We know from the property of homogeneous production functions that if one firm is producing at $\overline{Q}'$, the factor ratio at $\overline{Q}'$ will be the same as if equilibrium had occurred at $\overline{Q}$, with the given relative wage rate of $\left(\frac{W_2}{W_1}\right)_1$. Thus, the homogeneity assumption is important in empirical work because it permits the investigator to measure the ES when all firms are not producing at the same level of output.

Let us now drop the assumption that data on output is given in physical units, and now assume data is given in dollar value. Assume that the two firms sell their output at different prices. The homogeneity of the production function then assures us that the measurement of the ES will not be effected by errors in measurement of output, due to different selling prices of the two firms. For example, using Diagram 3, if both firms produce the same number of physical units, but one firm sells its output at p dollars per unit, while the other firm sells the output at $p+c$ dollars per unit ($c>o$), then the two observed output levels in dollar value would be $\bar{Q}$ and $\bar{Q}'$. But the homogeneity assumption assures us that our measurement of the ES will not be effected, as we discussed before.

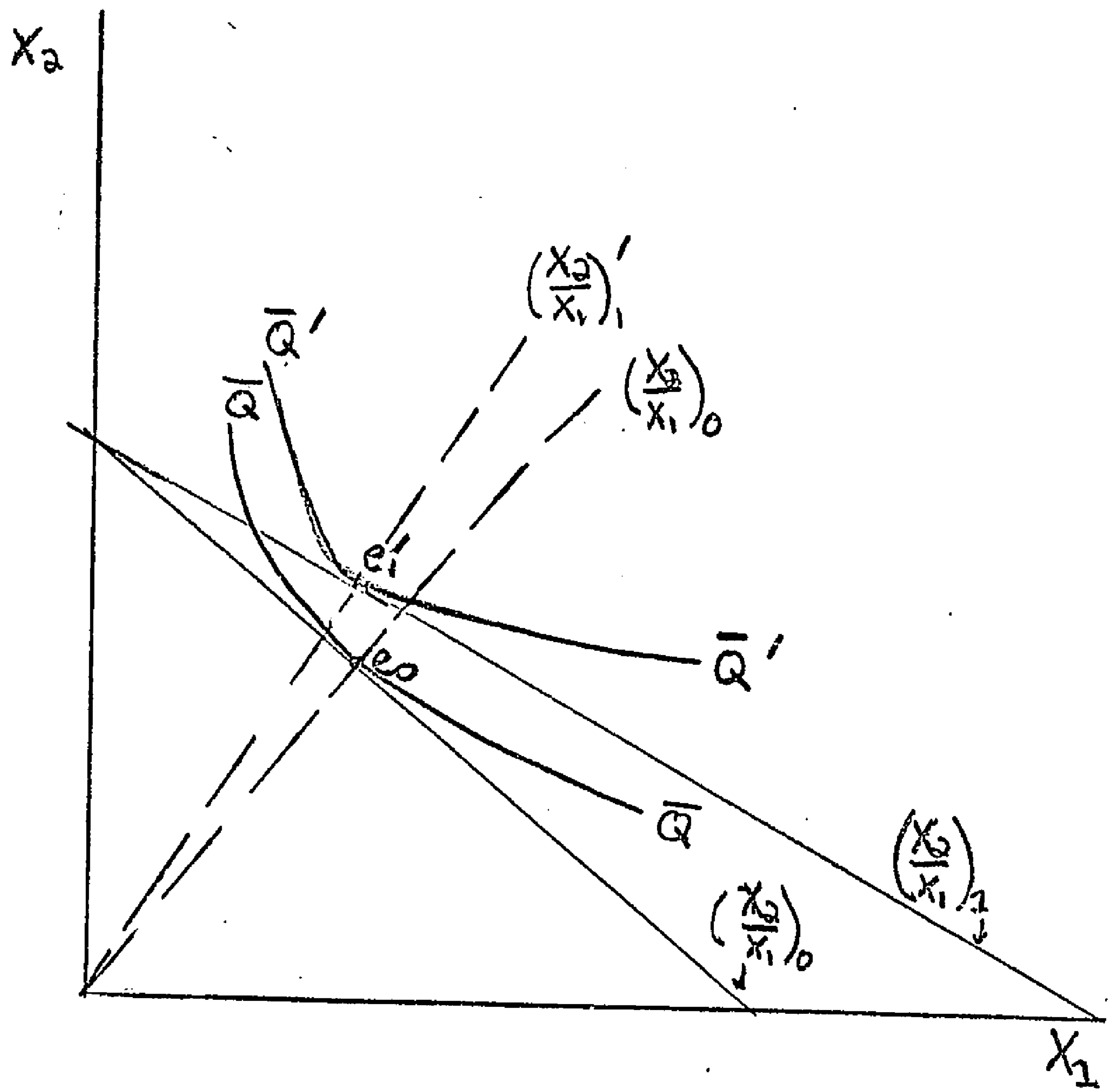
Thus, the homogeneity assumption is an important crutch in empirical work attempting to measure the ES.

Diagram 1



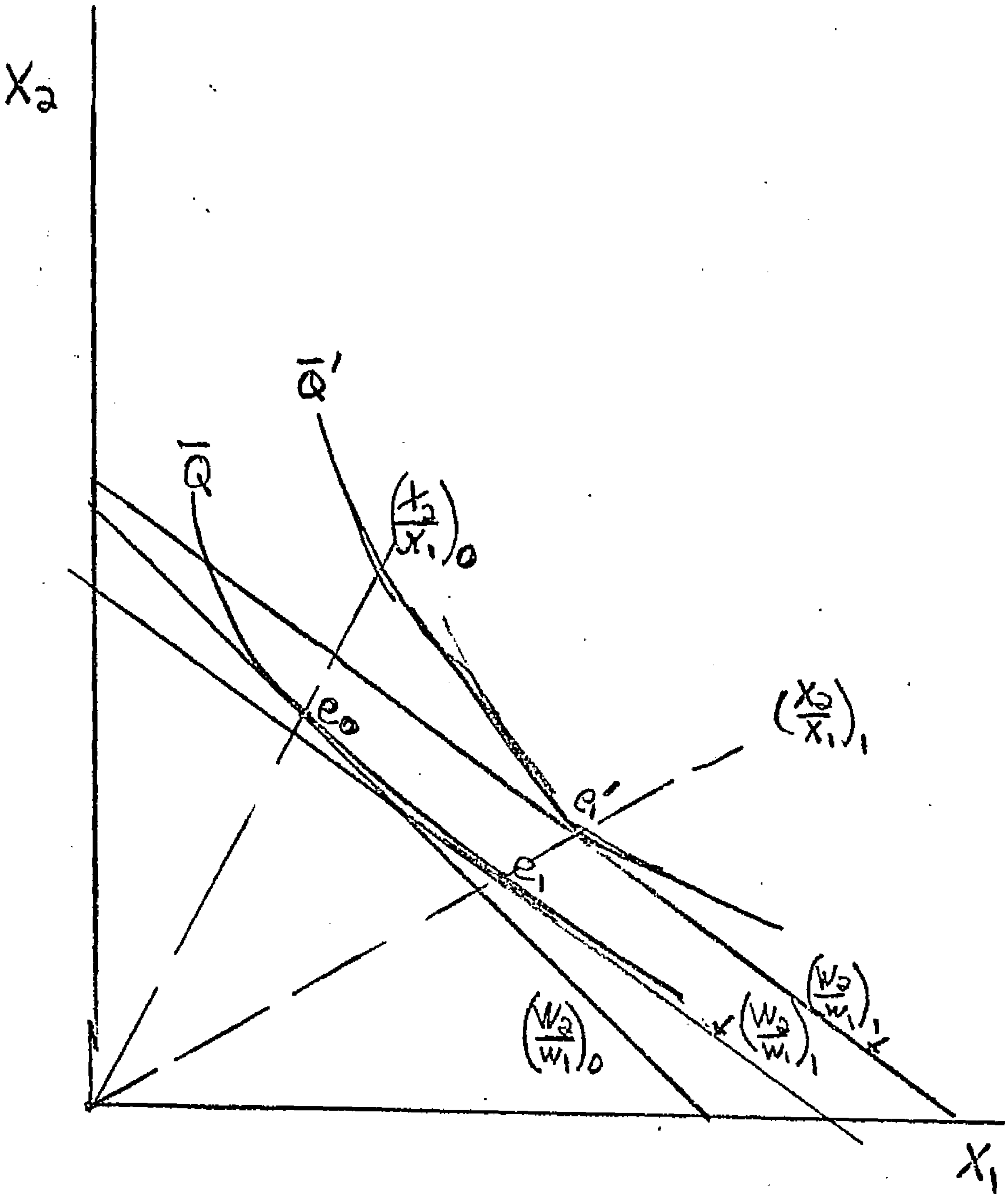
X_2
$\overline{Q}$
$\left(\frac{X_2}{X_1}\right)_0$
e_0
$\left(\frac{X_2}{X_1}\right)_1$
e_1
$\overline{Q}$
$\left(\frac{W_2}{W_1}\right)_0$
$\left(\frac{W_2}{W_1}\right)_1$
X_1

Diagram 2



X_2
$\left(\frac{X_2}{X_1}\right)_1'$
$\bar{Q}'$
$\bar{Q}$
$\left(\frac{X_2}{X_1}\right)_0$
e_1'
$\bar{Q}'$
e_0
$\bar{Q}$
$\left(\frac{X_2}{X_1}\right)_1$
$\left(\frac{X_2}{X_1}\right)_0$
X_1

Diagram 3



X_2
$\bar{Q}'$
$\bar{Q}$
$\left(\frac{X_2}{X_1}\right)_0$
e_0
$\left(\frac{X_2}{X_1}\right)_1$
e_1'
e_1
$\left(\frac{W_2}{W_1}\right)_1$
$\left(\frac{W_2}{W_1}\right)_1$
$\left(\frac{W_2}{W_1}\right)_0$
X_1

Chapter 4

PATTERNS OF FACTOR SUBSTITUTION IN U.S. MANUFACTURING INDUSTRIES: 1958

In the previous chapter, the problems encountered in estimating patterns of factor substitution were discussed, and a model for analyzing such patterns was developed. Here, I use this model to make inferences about patterns of substitution from cross-sectional data for 11 two-digit SIC industries in 1958, using the state as the unit of observation. Further comments on the model are first presented. I then describe the data set along with the necessary assumptions. At the end of the Chapter, I present the results and a summary of these results.

(4.1) The Model

The model developed in the previous chapter is

$$\log \frac{X_i}{X_k} = (a_i - a_k) + \sum_{j=1}^{n} (\sigma_{ij} - \sigma_{kj})\ S_j \log W_j$$

where

X_j = factor input j ;

W_j = price of factor input j ;

S_j = share of factor input j ;

σ_{ij} = Allen - partial ES between factors i and j ;

and a_i and a_k = constants.

In this study, I deal with four factor inputs: three educational levels and physical capital. These inputs I denote by the following subscripts:

Subscript	Descriptions
1	elementary school-educated labor (0-8 years)
2	high school-educated labor (9-12 years)
3	college-educated labor (13-16+ years)
4	physical capital

To make inferences about patterns of substitution between the three educational inputs themselves and between each educational input and physical capital, it is only necessary to use the logarithm of the ratio of the educational inputs. Data on physical capital is only needed to estimate the cost of capital, as I explain later on.

Although the model above can be used to make inferences about patterns of factor substitution for a given industry, it is also desirable to compare patterns of substitution between industries. To test for these differences using (1), it is only necessary to include the appropriate dummy variables. The model with four factors which will be estimated is:

$$(1) \quad \log \frac{X_i}{X_k} = (a_i - a_k) + \sum_{j=1}^{4} (\sigma_{ij} - \sigma_{kj})_R \, s_j \log W_j + \sum_{m=1}^{M} \sum_{j=1}^{4} (\sigma_{ij} - \sigma_{kj})_m \, \delta_m \, s_j \log W_j$$

where

$(\sigma_{ij} - \sigma_{kj})_R$ = the coefficient of the reference industry;

$(\sigma_{ij} - \sigma_{kj})_m$ = differential impact of industry m from the reference coefficient $(m = 1,\ldots,M)$;

and δ_m = 1 if the observation is in industry m
= 0 otherwise.

Then for industry m, $\sigma_{ij} - \sigma_{kj}$ is

$$(\sigma_{ij} - \sigma_{kj})_R + (\sigma_{ij} - \sigma_{kj})_m \quad .$$

(4.2) Data and Assumptions

The classical constraint acting upon all empirical work is the data with which the investigator has to work. For the present study, the data constraint restricts this investigator to work with cross-sectional intraindustry manufacturing data, with the state as the unit of observation. The problem is, therefore, to combine the micro-production functions of firms in the same industry, using capital of different vintages. Therefore, for each state, we are observing for all those firms in the industry of that state an average of productivities and an average of input combinations.[1] When we combine the information of the average productivities, we obtain the industry production function which is then interpreted as an average of average productivities. On the other hand, the production functions we are interested in for production theory is the relationship between best-practice factor ratios and relative factor input prices.[2] The extent to which we can interpret the estimated relationship as the theoretical relationship between input and output depends on how great is the divergence between average practice and best practice factor ratios.

Finally, before I explain the data, I would like to turn to the question of the homogeneity of the production function. The assumption which will be made is that the production function is homogemeous of degree one - that is, there are constant returns to scale. The theoretical reason is that if we assume that the data pepresents long-run equilibrium, which is production at minimum average cost, the production function must have constant returns to scale at points corresponding to minimum average cost.

(4.2.1) Industries Used in Study

The data set consists of observations on two-digit SIC industries in 1958, with the state as the unit of observation. Due to limitations on either capital from the Census of Manufactures (CM) or labor information from the Census of Population (CP), in which the educational inputs are derived, I can only analyze 11 of the 20 SIC industries given by the CM. The 11 two-digit industries I use, along with the number of observations, are shown in Table 1.

(4.2.2) Measure of Output (Q)

A measure of output is needed in order to compute factor shares and the cost of capital. Value-added is used as the measure of output, rather than gross output which requires data on raw materials. There are two alternative assumptions which can be made for using value-added in lieu of gross output, the first being the more restrictive:

(i) the ratio of raw materials to gross output is constant for all firms in the same industry, or

(ii) the production function is separable in terms of raw material input and all other inputs.[3,4]

Data on value-added by state and industry is taken from the <u>1958 Census of Manufactures</u>.[5]

Table 1

INDUSTRIES USED

Industry	SIC Code	No. of Observations
Food and kindred products	20	40
Textile mill products	22	21
Apparel and related products	23	24
Printing and publishing	27	17
Chemicals and allied products	28	31
Rubber and plastic products nec.	30	15
Primary metal industries	33	28
Fabricated metal products	34	33
Machinery, except electrical	35	30
Electrical machinery	36	25
Transportation equipment	37	27
Total		291

(4.2.3) Educational Inputs (X_1, X_2 and X_3)

No data is available which gives the educational distribution by state and industry, per se; however, it is possible to construct the educational distribution from the detailed occupational distribution, and the relationship between the occupation and educational distribution on the national level.

Let

C_{ij} = the number of males in occupation i and educational class j. $i = 1,...,55$

$j = 1,...,8$

This data is available on the national level from the 1960 Census of Population.[6]

Then, let

$$C_i = \sum_{j=1}^{8} C_{ij}$$ = the total number of persons in occupation i

and

$$P_{ij} = \frac{C_{ij}}{C_i}$$ = percentage of males in occupation i and in educational class j.

It was then assumed that the educational distribution by occupation is independent of the industry. This assumption was tested and the results are shown in Table 2 for eleven two-digit SIC manufacturing industries on the national level, for which the "true" educational

Table 2

COMPARISON OF "TRUE" AND ESTIMATED RELATIVE FREQUENCY
EDUCATIONAL DISTRIBUTION BY INDUSTRY

Industry		YEARS OF SCHOOL COMPLETED 0 - 4	5 - 7	8	9 - 11	12	13-15	16	16+
Food and kindred products	TRUE	.060	.139	.185	.250	.252	.074	.032	.010
	EST.	.057	.131	.168	.243	.246	.087	.050	.019
Textile mill products	TRUE	.118	.263	.158	.222	.161	.041	.029	.007
	EST.	.102	.216	.164	.223	.190	.058	.033	.013
Apparel and related products	TRUE	.084	.142	.185	.239	.216	.084	.038	.013
	EST.	.068	.132	.165	.226	.236	.094	.059	.021
Printing and publishing	TRUE	.013	.085	.160	.282	.269	.105	.062	.024
	EST.	.019	.062	.105	.209	.309	.155	.109	.032
Chemicals and allied products	TRUE	.036	.103	.125	.187	.263	.112	.111	.062
	EST.	.040	.111	.144	.218	.282	.111	.065	.028
Rubber and plastic products	TRUE	.036	.119	.169	.245	.271	.082	.056	.022
	EST.	.042	.128	.174	.242	.265	.082	.046	.022
Primary metal products	TRUE	.064	.158	.188	.242	.241	.060	.035	.012
	EST	.064	.152	.186	.243	.241	.067	.032	.014
Fabricated metal products	TRUE	.037	.116	.168	.242	.266	.096	.052	.023
	EST.	.040	.119	.169	.241	.265	.090	.052	.023
Machinery, except electrical	TRUE	.026	.101	.169	.240	.300	.096	.050	.018
	EST.	.032	.109	.169	.242	.286	.091	.050	.021
Electrical machinery	TRUE	.018	.073	.121	.204	.312	.133	.092	.048
	EST.	.027	.088	.135	.213	.292	.122	.083	.041
Transportation equipment	TRUE	.030	.109	.162	.242	.280	.098	.053	.024
	EST.	.037	.117	.170	.244	.275	.088	.046	.022

distribution is known from the Census of Population. The estimated distribution is obtained from the procedure above. The results suggest that one can approximate the educational distribution fairly well from the detailed occupational distribution.

For any given industry and state, we have an occupational distribution.[7]

Let

d_i = number of males in occupation i for the given state and industry.

Then $d_i \cdot P_{ij}$ tells us the number of males in occupation i and in educational class j for a given industry and state.

If we sum over all occupations - that is, over all i-we obtain the number of males in educational class j, call this e_j, which is

$$e_j = \sum_{i=1}^{55} d_i \cdot P_{ij}$$

To obtain the total number of persons in the industry and state, we sum over all educational classes

$$e_T = \sum_{j=1}^{8} e_j$$

Finally, we obtain the relative frequency educational distribution for the state and industry as

$$f_j = e_j / e_T \quad .$$

Given the relative frequency distribution f_1, f_2 and f_3, the wage rates and total payroll (TP) for each state and industry from the 1958 Census of Manufactures,[8] the "adjusted" total labor (L) is obtained. We know that

$$TP = L\,(W_1 f_1 + W_2 f_2 + W_3 f_3)$$

and can easily solve for L for each industry and state. Once L is known, we find each X_i by

$$X_i = L \quad f_i \qquad i = 1, 2, 3$$

and these X_i are the inputs to be used.

(4.2.4) The Wage Rates (W_1, W_2, W_3) and Factor Shares (S_1, S_2, S_3)

There is no data given directly on the wage rate by state, industry and educational level. Welch, however, has derived estimates for the average wage rate of a 45-54 year old white male by educational class for each state. A description of how the data was computed is given in two of his works.[9]

The computed factor shares are then obtained by

$$S_i = \frac{X_i W_i}{Q} \qquad i = 1, 2, 3$$

for the three educational labor inputs and the residual is physical capital's share. Table 3 gives the mean share for each factor by industry.

Table 3

MEAN FACTOR SHARES

Industry	S_1	S_2	S_3	S_4
Food and kindred products	.20	.16	.07	.57
Textile mill products	.35	.19	.06	.40
Apparel and related products	.29	.22	.10	.39
Printing and publishing	.12	.27	.19	.42
Chemicals and allied products	.13	.11	.08	.68
Rubber and plastic products nec.	.24	.20	.08	.48
Primary metal industries	.25	.21	.07	.47
Fabricated metal products	.25	.22	.10	.43
Machinery, except electrical	.25	.22	.10	.43
Electrical machinery	.19	.18	.13	.50
Transportation equipment	.31	.21	.09	.39

Subscripts denote: 1 = elementary school-educated labor;
2 = high school-educated labor;
3 = college-educated labor; and
4 = physical capital.

(4.2.5) Cost of Capital (W_4)

To estimate the cost of capital, I use the same measure as Griliches.[10] This measure gives the gross rate of return on capital and is computed for each state and industry for which data is available by

$$W_4 = \frac{(\text{Value-added} - \text{Total Payrolls})}{\text{Gross Book Value of fixed assets in 1957}}$$

The denominator of W_4 is obtained from the 1958 Census of Manufactures.[11]

(4.3) The Results

The estimates of model (1) for the three equations whose dependent variables are logarithms of X_1/X_2, X_1/X_3 and X_2/X_3 are given in Table 4.[12] The food industry (20) is the reference or "left-out" dummy variable. In order to see the impact of the dummy variable on the sign of the parameter, when the reference coefficient and dummy coefficient are of opposite sign, an F-test is used to see if the sum of the coefficients is equal to zero. For example, for the food industry (20), $(\sigma_{12} - \sigma_{32})_R$ is negative and significantly different from zero, while the dummy coefficient for the textile industry (22) is positive and significantly different from zero. An F-test is used to test if the sum of the two coefficients is zero. Such a test shows that the sum is significantly different from zero in this case. Since the value of the dummy coefficient is greater than the value of the reference coefficient, the

Table 4

REGRESSION ESTIMATES OF EQUATION 1

Coefficients[1,2]

Industry	$(\sigma_{13}-\sigma_{23})_R$	$(\sigma_{12}-\sigma_{32})_R$	$(\sigma_{21}-\sigma_{31})_R$	$(\sigma_{14}-\sigma_{24})_R$	$(\sigma_{14}-\sigma_{34})_R$	$(\sigma_{24}-\sigma_{34})_R$
Food (20)	.182* (.158)	-.302 (.162)	.086 (.039)	.009* (.056)	.067* (.106)	.058* (.078)
	$(\sigma_{13}-\sigma_{23})_m$	$(\sigma_{12}-\sigma_{32})_m$	$(\sigma_{21}-\sigma_{31})_m$	$(\sigma_{14}-\sigma_{24})_m$	$(\sigma_{14}-\sigma_{34})_m$	$(\sigma_{24}-\sigma_{34})_m$
Textiles (22)	-.110* (.170)	.385 (.179)	-.074* (.059)	-.135* (.136)	-.391 (.196)	-.275 (.141)
Apparel (23)	-.223* (.162)	.355 (.163)	-.132 (.044)	-.077* (.075)	-.255 (.112)	-.166 (.079)
Printing and publishing (27)	-.590 (.178)	-.043* (.235)	.089* (.138)	-.399 (.109)	-.605 (.205)	-.206* (.151)
Chemicals (28)	-.297 (.141)	-.364* (.250)	-.120* (.091)	-.105* (.070)	-.348 (.131)	-.241 (.097)
Rubber (30)	-.108* (.184)	.447 (.210)	-.112* (.067)	-.108* (.091)	-.037* (.172)	.071* (.126)
Primary metal (33)	-.191* (.189)	.178* (.214)	-.037* (.061)	-.053* (.076)	-.333 (.143)	-.279 (.105)
Fab. metal (34)	-.159* (.165)	.397 (.186)	-.075* (.070)	-.097* (.081)	-.169* (.152)	-.070* (.112)
Machinery, except electrical (35)	-.191* (.170)	.252* (.175)	-.173 (.081)	-.137 (.068)	-.353 (.129)	-.216 (.095)
Electrical machinery (36)	-.184* (.161)	.059* (.199)	-.201 (.081)	-.077* (.062)	-.152* (.117)	-.074* (.086)
Transportation equipment (37)	-.093* (.164)	.407 (.168)	-.081* (.113)	-.014* (.071)	-.006* (.135)	.007* (.099)

Footnotes to Table 4

(1) Subscripts denote: 1 = elementary school-educated labor;

2 = high school-educated labor;

3 = college-educated labor; and

4 = physical capital.

(2) The food industry (20) is the "left-out" or reference variable.

The bracketed value below the estimate of the coefficient is the standard deviation of the estimate.

The coefficients above are for equation 1. The coefficients $\sigma_{13} - \sigma_{23}$ and $\sigma_{14} - \sigma_{24}$ have the logarithm of X_1/X_2 as their dependent variable. The R^2 value is .98. The coefficients $\sigma_{12} - \sigma_{32}$ and $\sigma_{14} - \sigma_{34}$ have the logarithm of X_1/X_3 as their dependent variable. The R^2 value is .98. The coefficients $\sigma_{21} - \sigma_{31}$ and $\sigma_{24} - \sigma_{34}$ have the logarithm of X_2/X_3 as their dependent variable. The R^2 value is .97.

* Not significantly different from zero at the 10% level of significance. Two tail test used.

sign is taken as positive. The signs of the coefficients, after testing if the sum of the reference and dummy coefficient is significantly different from zero, is given in Tables 5 and 6.

Table 5 provides the results for patterns of substitution for the educational labor inputs with respect to each other; while Table 6 provides the results for patterns of substitution between the educational labor inputs and physical capital.

(4.3.1) Analysis of Patterns of Substitution between the Educational Labor Inputs

From Table 5, four patterns of substitution emerge. For two of the eleven industries, the results are inconsistent with respect to statements about patterns of substitution.[13] The four patterns of substitution are given in Table 7, along with the industries that conform to that pattern.

Pattern A, which four industries follow, implies that the two lower-educated labor inputs are more easily substituted for each other than they are for college-educated labor. The degree to which the two lower-educated educational inputs can be substituted for college-educated labor is about the same. That is, both lower-educated inputs are equally well suited to substitute for college-educated labor.

Table 5

SIGNS OF COEFFICIENTS BETWEEN EDUCATIONAL LABOR INPUTS BY INDUSTRY[1]

Industry	Coefficient[2]		
	$\sigma_{13}-\sigma_{23}$	$\sigma_{12}-\sigma_{32}$	$\sigma_{21}-\sigma_{31}$
Food and kindred product	0	-	+
Textile mill products	0	+	+
Apparel and related products	0	0	0
Printing and publishing	-	-	+
Chemicals and allied products	-	-	+
Rubber and plastic products nec.	0	+	+
Primary metal industries	0	-	+
Fabricated metal products	0	+	+
Machinery, except electrical	0	-	-
Electrical machinery	0	-	-
Transportation equipment	0	+	+

(1) These results come from combining the dummy and reference coefficient after testing if the sum is significantly different from zero.

(2) Subscripts denote: 1 = elementary school-educated labor; 2 = high school-educated labor; and 3 = college-educated labor.

Table 6

SIGNS OF COEFFICIENTS BETWEEN EDUCATIONAL LABOR INPUTS AND PHYSICAL CAPITAL BY INDUSTRY[1]

	Coefficient[2]		
Industry	$\sigma_{14} - \sigma_{24}$	$\sigma_{14} - \sigma_{34}$	$\sigma_{24} - \sigma_{34}$
Food and kindred products	0	0	0
Textile mill products	0	-	-
Apparel and related products	0	-	-
Printing and publishing	-	-	0
Chemicals and allied products	0	-	-
Rubber and plastic products nec.	0	0	0
Primary metal industries	0	-	-
Fabricated metal products	0	0	0
Machinery, except electrical	-	-	-
Electrical machinery	0	0	0
Transportation equipment	0	0	0

(1) See footnote 1 of Table 3.

(2) Subscripts denote: 1 = elementary school-educated labor; 2 = high school-educated labor; 3 = college-educated labor; and 4 = physical capital.

Table 7

SUMMARY OF PATTERNS OF SUBSTITUTION

BETWEEN THE EDUCATIONAL LABOR INPUTS

Pattern	Industry	Description
A	textile, rubber, fab. metal, transportation	$\sigma_{13} = \sigma_{23}$; $\sigma_{12} > \sigma_{23}$; $\sigma_{12} > \sigma_{13}$
B	Apparel	$\sigma_{12} = \sigma_{13} = \sigma_{23}$
C	printing, chemicals	$\sigma_{13} < \sigma_{23}$; $\sigma_{12} < \sigma_{23}$; $\sigma_{12} > \sigma_{13}$
D	machinery, electrical machinery	$\sigma_{13} = \sigma_{23}$; $\sigma_{12} < \sigma_{23}$; $\sigma_{12} < \sigma_{13}$
-	food, primary metal	Inconsistent

(Subscripts denote: 1 = elementary school-educated labor;
2 = high school-educated labor;
3 = college-educated labor; and
4 = physical capital.)

Only one industry, apparel (23), follows Pattern B, which implies that all educational labor inputs are equally well-suited to substitute for each other.

The implications of Pattern C are that the two higher-educated labor inputs are more easily substituted for each other than they are for elementary school-educated labor. However, between the two higher-educated labor inputs, high school-educated labor is more easily substituted for elementary school-educated labor.

Pattern D implies that college-educated labor is more easily substituted for the two lower-educated labor inputs than these inputs can be substituted for each other; while both lower-educated labor inputs are both equally well-suited to substitute for college-educated labor.

(4.3.2) Analysis of Patterns of Substitution Between the Educational Labor Inputs and Physical Capital

The major finding for the four patterns of substitution observed in Table 6 is that college-educated labor is <u>at least</u> as good a substitute for physical capital as the two lower-educated labor inputs, if not more of a substitute. The four patterns observed are shown in Table 8.

Table 8

SUMMARY OF PATTERNS OF SUBSTITUTION BETWEEN THE EDUCATIONAL LABOR INPUTS AND PHYSICAL CAPITAL

Pattern	Industries	Description
E	food, rubber, fab. metal, elect. machinery, transportation	$\sigma_{14} = \sigma_{24} = \sigma_{34}$
F	textile, apparel, chemicals, primary metal	$\sigma_{14} = \sigma_{24}$; $\sigma_{14} < \sigma_{34}$; $\sigma_{24} < \sigma_{34}$
G	printing	$\sigma_{14} < \sigma_{24}$; $\sigma_{14} < \sigma_{34}$; $\sigma_{24} = \sigma_{34}$
H	machinery, exc. electrical	$\sigma_{14} < \sigma_{24}$; $\sigma_{14} < \sigma_{34}$; $\sigma_{24} < \sigma_{34}$

(Subscripts denote: 1 = elementary school-educated labor;
2 = high school-educated labor;
3 = college-educated labor; and
4 = physical capital.)

Five of the eleven industries follow Pattern E, which implies that all educational inputs are equally well-suited to substitute for physical capital. On the other hand, Pattern F, to which four of the industries conform, implies that college-educated labor is more easily substituted for physical capital than the two lower-educated labor inputs can be substituted for physical capital. However, the two lower-educated labor inputs are equally well-suited to substitute for physical capital.

Only the printing and publishing industry (27) follows Pattern G. This pattern implies that high school- and college-educated labor are equally well-suited to substitute for physical capital; yet, both can be substituted more easily for physical capital than elementary school labor can be substituted for physical capital.

Pattern H. implies that the more educated the labor input, the more easily it can be substituted for physical capital. The machinery (except electrical) industry (35), conforms to Pattern H.

I next tried to determine the similarities of those industries which had the same patterns in Table 8. Unfortunately, I could not discover any characteristic of the production process which would result in the similarity of the patterns of factor substitution.

(4.4) Summary

For the eleven industries considered, four patterns of substitution between the educational labor inputs and four patterns between the educational labor inputs and physical capital are observed. As can be seen from Table 9, this implies seven different patterns of factor substitution. Only three industries, rubber (30), fabricated metal (34) and transportation (37), have the same overall pattern.

The major finding is that at this level of aggregation of both industry and unit of observation, there is no evidence that college-educated labor is more complementary to physical capital than lower-educated labor inputs.

Table 9

SUMMARY OF PATTERNS OF FACTOR SUBSTITUTION

Industry	Patterns: Between Educational Inputs	Patterns: Between Educational Inputs and Physical Capital
Food and kindred products	Inconsistent	E
Textile mill products	A	F
Apparel and related products	B	F
Printing and publishing	C	G
Chemicals and allied products	C	F
Rubber and plastic products nec.	A	E
Primary metal industries	Inconsistent	F
Fabricated metal products	A	E
Machinery, except electrical	D	H
Electrical machinery	D	E
Transportation equipment	A	E

Footnotes to Chapter 4

[1] Klein, in the CES context, has shown that if we want to move from the micro-production function to the aggregate estimated production function, the input and output variable used should be the geometric means of the variables. I will be using census data from which I can obtain the totals for each variable, and given data on the number of firms, I can calculate the arithmetic average; I cannot, however, get the individual values of each variable in order to get the geometric means. There is no way out of this situation unless one gets the individual observations used by the Census to obtain the published aggregates. This is not possible, however, under disclosure laws.

SEE: L.R. Klein, "Macroeconomics and the Theory of Rational Behavior," Econometrica (1946), 93-108.

[2] For an attempt to estimate best-practice production functions via linear programming,

SEE: D. Aigner, and S. Chu, "On Estimating the Industry Production Function," American Economic Review, LVIII (1968), 826-839.

[3] For the proof (given for non-labor inputs), SEE Appendix (p.240) of F. Welch, "Labor-Market Discrimination," Journal of Political Economy, LXXV (1967), 225-240.

4 This is suggested by

C. Sims in "Theoretical Basis for a Double-Deflated Index of Real Value-Added," Review of Economics and Statistics, LVI (1969), 470-471.

5 1958 Census of Manufactures, Vol. II, Table 3 for each state.

6 1960 Census of Population, PC (2) 7A, Table 9. 55 occupational classes are used.

7 1960 Census of Population, State books, Table 125. Since there is no data available by education and age for a given industry within a given state, no adjustment could be made for age.

8 1958 Census of Manufactures, Vol. II, Table 3 for each state.

9 The data which I use for the wage of the educational inputs I obtained from Professor Welch. The data was derived from the 1960 Census of Population data and gives the average wage rate of a 45-54 year old white male by educational class for each state. A description of how the data was obtained is given in two of his works:

F. Welch, "Linear Synthesis of Skill Distributions," Journal of Human Resources, (1969 Summer), 317n.

__________, "Education in Production," Journal of Political Economy, LXXVIII (1970), Appendix 55-58.

10 Z. Griliches, "Capital-Skill Complementarity," Review of Economics and Statistics, LI (1969), 465-468.

11 <u>1958 Census of Manufactures</u>, Vol. 1, Supplementary Table 1.

12 All the observations were first pooled to test for an "aggregate" estimate of patterns of substitution. The estimates of the parameters of the "aggregate" function are (the standard deviation of the estimates are given below the estimate and "*" denotes that the estimate is not significantly different from zero at the 10 percent level when a two tail test is used):

$$\sigma_{13} - \sigma_{23} = \underset{(.017)}{-.109} \qquad \sigma_{14} - \sigma_{24} = \underset{(.017)}{-.022^*} \qquad R^2 = .95$$

$$\sigma_{12} - \sigma_{32} = \underset{(.025)}{.047} \qquad \sigma_{14} - \sigma_{34} = \underset{(.026)}{-.041^*} \qquad R^2 = .94$$

$$\sigma_{21} - \sigma_{31} = \underset{(.012)}{.015^*} \qquad \sigma_{24} - \sigma_{34} = \underset{(.022)}{-.018^*} \qquad R^2 = .90$$

These estimates imply the following pattern about substitution between the educational inputs:

$$\sigma_{13} < \sigma_{23}$$

$$\sigma_{12} > \sigma_{32}$$

and $\sigma_{21} = \sigma_{31}$.

This pattern is inconsistent, since the first two statements do not imply the third.

Between physical capital and the educational inputs, the ppattern implied is

$$\sigma_{14} = \sigma_{24} = \sigma_{34}$$

Next, it is necessary to test if an "aggregate" function is appropriate - that is, if the dummy coefficients are all equal to zero. The F-test used to test if all $(\sigma_{ij} - \sigma_{kj})_m$ are equal to zero with no stipulation on $(\sigma_{ij} - \sigma_{kj})_R$ is given in

F.A. Graybill, An Introduction to Linear Statistical Models Vol. 1 (New York: 1961), pp. 130-140.

The computed F-values for each equation under the null hypothesis that all $(\sigma_{ij} - \sigma_{kj})_m$ are equal to zero are

dependent variable	computed F-value
$\log \frac{X_1}{X_2}$	11.10
$\log \frac{X_1}{X_3}$	8.80
$\log \frac{X_2}{X_3}$	13.20

The F-test under consideration requires that the computed F-value be less than or equal to an F with 242 and 44 degrees of freedom, in order to accept the hypothesis that all $(\sigma_{ij} - \sigma_{kj})_m$ equal zero, or equivalently, that an "aggregate" function can be used to express all patterns. All of the computed F-values are greater than an F(242, 44) at the 5 percent level. Thus, the "aggregate" estimate is rejected and the dummy variables for industries are used.

13 For both industries 20 and 33, the results are inconsistent, since they imply that

$$\sigma_{13} = \sigma_{23}$$

$$\sigma_{12} < \sigma_{32}$$

and $$\sigma_{21} > \sigma_{31}$$

which is impossible, since the first two statements imply that $\sigma_{21} < \sigma_{31}$, which is not implied by the last statement.

Chapter 5

CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

In Chapter 2, the impact of the changing composition of output and the non-neutrality of technological change on the demand for three educational inputs - college-, high school- and elementary school-educated labor - for the U.S. economy between 1940 and 1960 are examined. Using data on eight major sectors of the economy, the findings support the hypothesis that the changing composition of output increased the demand for college-educated labor relative to the two less-educated labor inputs. The findings do not, however, support the non-neutrality of technical change hypothesis. The demand for all educational inputs would have decreased because of technological change. Decreases in factor demand did not affect all factors to the same degree. The demand for college-educated labor fell less than the demand for the two less-educated inputs.

Data on the educational ratios in 1960 and the change in the capital to average labor ratio between 1948-50 and 1960-63 for 17 two-digit SIC manufacturing industries implies that those industries in which capital intensity increased the most are those industries in which there is less college-educated labor used relative to non-college labor. This does not support the non-neutrality hypothesis. However, the changing composition of output hypothesis is supported for U.S. manufacturing.

In Chapter 4, patterns of factor substitution are examined for 11 U.S. manufacturing industries in 1958. Four patterns of substitution between the educational labor inputs and physical capital are observed. Although the patterns do differ by industry, one general statement can be made: college-educated labor is as good a substitute for physical capital as the two lower-educated labor inputs are. This is quite contrary to what some economists have come to believe - that is, that college-educated labor is the most complementary educational input to physical capital.[1]

As for future research, I believe future studies on the role of education in the production function must wait for some concrete evidence on the technology of the learning industry - that is, about educational production functions. What is of primary interest is the factors which determine economic productivity and how such factors are produced. What is assumed in this study, as well as in all other studies, is that the number of years of education completed determines economic productivity.

If education does affect economic productivity, then changes in the technology of the learning industry or educational production function over time is important. A decrease in the cost of learning per unit of resource allocated has already been cited as one of the possible explanations of the paradox considered in the study.

But what is really known about educational production functions? All studies estimate such functions by relating inputs, such as characteristics of teachers and the environment of students, to output measured by IQ's.[2] What is under question here is how such inputs are related to economic productivity. Thus, the crucial question is the correlation between IQ's which measure the ability to succeed in school and economic productivity. The link between inputs and output measured by IQ's, I believe, falls far short of the mark set. Rather than look at IQ's, what should be looked at is how inputs of teacher characteristics and the environment of students relates to aspirations of students. Aspirations measure motivation, and it is this measure that is more closely related to economic productivity than IQ's, which simply measures the ability to succeed in school.[3]

Once factors affecting the educational production function are discerned, questions about how changes in such factors over time have affected learning per unit of input can be analyzed.

Footnotes to Chapter 5

[1] Recall the controversy in the 60's concerning structural unemployment due to automation.

[2] SEE Bowles, "Towards An Educational Production Function," for references of previous attempts at estimating educational production functions. One recent study not included in Bowles' paper, which uses data on individuals over time, is

E. Hanushek, "Teacher Characteristics and Gains in Student Achievement Estimation Using Micro-data," American Economic Review, LXI (1971), 280-288.

[3] For example, consider two twins who went through school together from the first grade until the completion of their freshman year in college, and terminated their education at this point. Suppose one of the twins had a substantially higher IQ than the other. Then the question is why didn't the twin with the higher IQ complete college or one can ask how did the twin with the lower IQ get as far as the other? The link is obviously the motivation factor.

Appendix to Chapter 2

(A.2.1) Data on Real Output

Data on real output in 1954 dollars in 1950 and 1960 is obtained for each sector from

M.L. Marimont, "GNP by Major Industries," *Survey of Current Business* (October, 1962), pp. 6-19.

The data is shown in Table A-1. The total differs from that given by Marimont by the two components "Rest of the World" and "Residual".

Data for both total output in the economy and agricultural output in 1939 in 1954 dollars is taken from V.R. Fuchs, *The Service Economy* (New York; 1968).

The output for each sector is then obtained by applying the non-agricultural output to the percentage distribution of non-agricultural output in 1939, given in

E.F. Denison, "Industrial Composition of National Income," *Survey of Current Business* (December, 1948), pp. 11-17.

(A.2.2) Educational Factor Inputs

As mentioned in Chapter 2, the occupational distribution is used to estimate the educational distribution. Since relative wage rates between occupations have not changed significantly, the 1960 occupational relative wage rates between occupations and sex are used to aggregate into the three educational classes. The wage rates are the median annual wage for each occupation by sex for persons who worked 50-52 weeks. Hourly wage rates would have been preferred, but were not available. The occupation with the highest wage within each educational class is taken as the group representative.

Table A-2 gives the imputed distribution by sector for the years 1940, 1950 and 1960. The basic data on the distribution by occupation is taken from the Census of Population subject reports in each year.

(A.2.3) Methodology for Estimating the Decomposition of the Demand for Each Educational Factor

In Chapter 2, the decomposition of the demand for each educational factor is given in Tables 7-9. The methodology for obtaining these estimates will be explained here. Decomposing the change in observed demand for each educational input between any two periods depends upon the year used as the reference point. That is, if the initial year is used as the reference point, the analysis proceeds in one way; while, if the terminal year is used, the analysis proceeds in another way.

(A.2.3.1) Analysis When the Initial Year Is Used As the Reference Point

The effect of technological change on the demand for each educational factor is measured as a residual when the initial year is used as the reference point. That is, first the effect of the growth of output, when all sectors grow at the same rate, is estimated. Then the change in demand, due to differences in the growth of output by sector, is estimated. The difference between the observed change and the estimated change in demand due to these two variables is then attributed to technological change.

To state the procedure mathematically, let X_{1mo} and X_{1m1} denote the quantity of educational factor 1 employed in sector m in the initial year (subscript o) and terminal year (subscript 1) respectively. Similarly, let Q_{mo} and Q_{m1} denote real output in sector m for the initial and terminal years respectively. Then total demand for any educational factor 1, which is observed in each year, is:

$$(1) \qquad X_{1o} = \sum_{m=1}^{M} \left[\frac{X_{1mo}}{Q_{mo}} \right] Q_{mo}$$

and

$$(2) \qquad X_{1_1} = \sum_{m=1}^{M} \left[\frac{X_{1m_1}}{Q_{m1}} \right] Q_{m_1}$$

What I want to explain is the change in demand between these two years - that is, $X_{1_1} - X_{1o}$.

Holding the technology of the initial year constant - that is, using the initial year as the reference point - the demand for each educational factor would have been λX_{1o}, had the output of each sector changed by λ. Thus,

$$(3) \qquad \lambda X_{1o} - X_{1o}$$

is the change in demand if the growth of output of all sectors had been the same. However, if the output of all sectors grew at different rates - that is, each sector changed by λ_m - then demand would have been

$$(4) \qquad X_{11g} = \sum_{m=1}^{M} \left[\frac{X_{1mo}}{Q_{mo}} \right] \lambda_m \; Q_{mo}$$

But $\lambda_m \; Q_{mo}$ is just the output of sector m in the terminal year - that is, Q_{m_1}.

Thus,

$$(5) \qquad X_{11g} = \sum_{m=1}^{M} \left[\frac{X_{1mo}}{Q_{mo}} \right] Q_{m_1}$$

The change in demand due to sector differences in the rate of growth of output is, then

$$(6) \qquad X_{11g} - \lambda X_{1o}$$

The difference between what demand would have been with sector differences in the rate of growth of output, and what actually is observed in the terminal year, is attributed to technological change. That is, this change in demand is measured by

$$(7) \qquad X_{1_1} - X_{1_1g}$$

The sum of these changes (3), (6) and (7) is then the change in total demand for each educational factor between any two periods:

$$X_{1_1} - X_{1o} \equiv (\lambda X_{1o} - X_{1o}) + (X_{1_1g} - \lambda X_{1o}) + (X_{1_1} - X_{1_1g})$$

(A.2.3.2) Analysis When the Terminal Year Is Used As the Reference Point

When the terminal year is used as the reference point, the technology of that year is held constant. Thus, technological change is measured directly by multiplying the input per unit of output in the terminal year for each sector by the output in the initial year. This tells what the demand for factor i would have been in the terminal year, had technology changed to that of the terminal year but had output remained unchanged - that is,

$$X_{i_{1T}} = \sum_{m=1}^{M} \left[\frac{X_{im_1}}{Q_{m1}} \right] Q_{mo}$$

The change in demand due to technological change is then measured by

$$(8) \qquad X_{i_{1T}} - X_{io}$$

If the growth of output of each sector has been the same and output increased by λ, then with the terminal year technology, demand for factor i would have been $\lambda X_{i_{1T}}$. Thus, the change due to the same growth of output in all sectors is

$$(9) \qquad \lambda X_{i_{1T}} - X_{i_{1T}}$$

The residual between the observed change and the change due to technological change and equal growth of output of all sectors is then attributed to differences in the growth of output of all sectors - that is, the changing composition of output. This residual is measured by

$$(10) \qquad x_{i_1} - \lambda x_{i_{1T}}$$

Total change in observed demand is then simply the sum of all changes (8), (9) and (10) - that is,

$$x_{i_1} - x_{i_0} \equiv (x_{i_{1T}} - x_{i_0}) + (\lambda x_{i_{1T}} - x_{i_{1T}}) + (x_{i_1} - \lambda x_{i_{1T}})$$

Table A-1

REAL OUTPUT IN THE U.S. IN 1954 DOLLARS FOR EIGHT MAJOR SECTORS: 1939, 1950 AND 1960

(In billions of dollars)

Sector	1939	1950	1960
All	184.0	323.1	440.1
Agriculture, forestry and fisheries	14.3	20.2	22.7
Mining	4.1	8.8	10.8
Construction	5.8	14.9	18.8
Manufacturing	46.1	92.6	125.5
Wholesale and retail trade	31.2	61.3	78.0
Transportation, communications and public utilities	19.1	28.8	45.0
Other services	41.7	67.7	99.6
Government and government enterprises	21.7	28.8	39.7

Table A-2

DEMAND FOR EDUCATIONAL INPUTS BY SECTOR

FOR 1940, 1950 AND 1960

Sector	College	High School	Elementary School
All			
1940	6 165 330	16 053 834	15 512 893
1950	8 488 136	20 030 755	17 129 678
1960	9 085 903	18 614 318	16 270 935
Agriculture, forestry and fisheries			
1940	17 486	5 126 643	2 620 433
1950	60 215	4 278 019	2 051 767
1960	74 753	2 639 728	1 481 794
Mining			
1940	47 887	147 572	707 304
1950	67 102	194 722	649 293
1960	83 264	185 521	360 578
Construction			
1940	177 341	1 251 274	525 465
1950	400 351	2 018 702	800 678
1960	539 280	2 181 820	878 385
Manufacturing			
1940	746 056	3 227 551	5 251 339
1950	1 305 123	4 363 507	6 902 608
1960	2 077 117	5 539 152	7 494 825
Wholesale and retail trade			
1940	2 039 532	2 578 111	1 740 357
1950	2 416 780	3 782 858	2 340 674
1960	2 065 082	2 998 713	2 202 942

Table A-2 (Concluded)

DEMAND FOR EDUCATIONAL INPUTS BY SECTOR

FOR 1940, 1950 AND 1960

Sector	College	High School	Elementary School
Transportation, communications and public utilities			
1940	293 755	1 207 733	1 423 881
1950	430 898	1 692 959	1 768 771
1960	539 523	1 768 905	1 654 526
Other services			
1940	2 478 032	1 941 001	2 726 276
1950	3 323 136	2 648 954	2 096 212
1960	3 304 704	1 999 626	1 580 793
Government and government enterprises			
1940	365 241	573 949	513 838
1950	484 531	1 051 034	519 675
1960	672 180	1 300 853	617 092

Appendix to the Study

THE "CRESH" PRODUCTION FUNCTION

In this study, the CRESH production function was introduced. Unlike the CES production function, the properties, parameter restrictions and econometric techniques for estimating the CRESH function are not documented in the economic literature at the time of this writing. The only discussion of the CRESH function is given by Hanoch in an economic research paper for Harvard University.[1] A revision of this paper will appear in Econometrica at some future time. All statements made here will refer to the research paper.

Here, I discuss the properties, parameter restrictions and econometric techniques possible for estimating the parameters of CRESH. I only provide some proofs omitted by Hanoch; all other results I state and the reader is referred to the corresponding pages of Hanoch's work. In discussing the econometric techniques and problems which arise in estimating the production function under consideration, data on an industry I discussed in Chapter 4 is used.

(A.1) Validity of the CRESH Production Function

A production function shows the relationship between inputs and output at a given point in time. For any bundle of inputs ($X_1, X_2, \ldots, X_n$), the production function states the maximum output, Q, possible. The production function is taken as a single-valued function. That is, for a given bundle of inputs, there is a unique quantity of output produced. In traditional production theory, the production function is expressed in explicit form - that is:

$$Q = f(X_1, X_2, \ldots, X_n) \quad \text{for all } X_i \geq 0.$$

Given the production function, economists require certain assumptions in order to obtain optimal conditions under profit maximization or competitive cost minimization conditions. The assumption is that the production function has continuous first and second order partial derivatives.

The CRESH production function is not given in explicit form. Rather, the function is expressed in implicit form by

$$(1) \qquad F(Q, X_1, \ldots, X_n) = \sum_{i=1}^{n} A_i \left[\frac{X_i}{Q}\right]^{a_i} - 1 = 0$$

for all $X_i \geq 0$ with at least one $X_j > 0$ and $Q > 0$.

What I demonstrate here is that the CRESH function is a valid production function in the sense that it provides a unique output for any non-negative bundle of inputs, and also has continuous first and second order partial derivatives. The proof relies on the Implicit Function Theorem.[2]

From this theorem we know that if the CRESH production function given in its implicit form by

(1) has continuous first order partial derivatives in all arguments (all X_i and Q), and further, $F_Q \neq 0$, then the following is true: (i) the explicit form implied by (1) is single-valued; (ii) the first order partial derivatives of the explicit function is continuous; and (iii) the first order partial derivative of the explicit function is given by

(2) $$f_i = -\frac{F_i}{F_Q}$$

where f_i is the first order partial derivative with respect to X_i; that is, $\frac{\partial Q}{\partial X_i}$ and F_i and F_Q are the first order partial derivatives of (1) with respect to X_i and Q.

The first step then is to show that (1) has continuous first order partial derivatives. The first order partial derivative of (1) with respect to any X_i is

(3) $$F_i = A_i \, a_i \, X_i^{ai - 1} \; Q^{ai}$$

F_i is then continuous for all $Q > 0$ and $X_i \geq 0$, since $X_i^{ai - 1}$ and Q^{ai} are continuous functions.[3]

The first order partial derivative of (1) with respect to output, Q, is

(4) $$F_Q = -\sum_{i=1}^{n} A_i \, a_i \, X_i^{ai} \; Q^{-(ai + 1)}$$

F_Q is then continuous for all $Q > 0$ and

$X_i \geq 0$, since X_i^{ai} and $Q^{-(ai+1)}$ are continuous functions.[4] Furthermore, $F_Q \neq 0$ for $Q \neq 0$, $X_i \geq 0$ with at least one $X_j > 0$.

Then, by the Implicit Function Theorem, (1) is, therefore, a single-valued function with continuous first order partial derivatives. Next, it is only necessary to show that (1) has continuous second order partial derivatives.

Since (1) satisfies the Implicit Function Theorem, we know that the first order partial derivative can be found from (2) by

$$(5) \qquad f_i = -\frac{F_i}{F_Q} \qquad \text{for all } X_i.$$

Substituting F_i from (3) and F_Q from (4) into (5), and taking the partial derivative of f_i with respect to X_i, we obtain, after algebraic manipulation

$$(6) \qquad f_{ii} = -\frac{F_Q A_i\, ai(ai-1) X_i^{ai-2} - A_i^2 ai^3 X_i^{2(ai-1)} Q^{-(ai+1)}}{F_Q^2}$$

$$+ \frac{f_i A_i\, ai X_i^{ai-1} \sum_i A_i\, ai(ai+1) X_i^{ai} Q^{-(ai+2)}}{F_Q^2}$$

Since $F_Q \neq 0$ and is continuous and f_i is continuous by the Implicit Function Theorem, f_{ii} is also continuous for $X_i \geq 0$, $Q > 0$ and at least one $X_j > 0$, since it is a composite of continuous functions.

Similarly, for $i \neq j$,

$$f_{ij} = \frac{- A_i A_j a_i a_j X_i^{ai-1} X_j^{aj-1} Q^{-(aj+1)}}{F_Q^2}$$

$$+ \frac{-f_j A_i a_i X_i^{ai-1} \sum A_i a_i (ai+1) X_i^{ai} Q^{-(ai+2)}}{F_Q^2}$$

is continuous, since f_j is continuous by the Implicit Function Theorem and $F_Q \neq 0$ and continuous. Thus, the CRESH production function given by (1) has continuous second order partial derivatives with respect to all inputs.

(A.2) Parameter Restrictions

The derived conditions under competitive cost minimization are obtained from the Lagrangian:

$$L = \sum_{i=1}^{n} W_i X_i - \lambda \left[\sum_{i=1}^{n} A_i \left[\frac{X_i}{Q}\right]^{a_i} - 1 \right].$$

Denoting logarithms by the minor letter of the respective variables, the first order conditions for competitive cost minimization are

(7) $\quad x_i = b_i \, (\bar{\lambda} - w_i - a_i q + c_i) \quad i = 1, \ldots, n$

where $b_i = \frac{1}{(1 - a_i)}$; $\log |\lambda| = \bar{\lambda}$ and

$$\log |A_i a_i| = c_i$$

Hanoch shows that the Allen partial ES are[5]

$$\sigma_{ij} = \frac{b_i b_j}{\sum_k b_k s_k} \qquad i \neq j$$

and

$$\sigma_{ii} \quad \frac{b_i^2}{\sum_k b_k s_k} - \frac{b_i}{s_i}$$

where s_k is the share of factor k.

Hanoch analyzes the parameter restrictions on the b_i's under competitive cost minimization. The restrictionson the b_i parameters for the function to meet the second order conditions for competitive cost minimization (quasi- concavity of the production function), Hanoch shows results in two cases.[6]

(i) all $b_i > 0$, whereby all pairwise ES will be positive; that is, all factors are substitutes or (ii) one $b_k < 0$ and all other $b_i \quad (i \neq k) > 0$. This latter case is possible if the following condition holds:

$$(8) \qquad -s_k b_k > \sum_{i \neq k} s_i b_i > 0.$$

In case (ii), factor k is a substitute for all other factor inputs, while the other factor inputs themselves will form a group of complements.[7]

Thus, in no case may there be more than one negative value for b_i, and if one negative value for b_i appears, the condition given by (8) must hold.

(A.3) A General View of the Econometric Problems of the CRESH Production Function

As stated in Chapter 3, the first order conditions for competitive cost minimization are:[8]

$$(9) \qquad X_i = b_i\,(\bar{\lambda} - w_i - a_i\,q + c_i) \qquad i = 1,\ldots,4$$

where $\bar{\lambda}$ is some function of all w_i and output. Let us now introduce the stochastic error term u_i for each equation which is assumed to have zero mean and constant variance:

$$(10) \qquad X_i = b_i\,(\bar{\lambda} - w_i - a_i\,q + c_i) + u_i \qquad i = 1,\ldots,4.$$

We shall also assume that each u_i is distributed independently of the wage rate - that is, we shall assume that the factor prices are determined exogenously.

In order to estimate the parameters, it is necessary to solve system (10) for $\bar{\lambda}$. This can be done by solving for $\bar{\lambda}$ in terms of one of the X_i, and, therefore, reducing the system to three equations which include only observed variables.

Let us assume a value for i, say i = 1, and solve (10) for $\bar{\lambda}$. We then obtain

$$(11) \qquad \bar{\lambda} = \frac{x_1}{b_1} + w_1 + a_1\,q - c_1 - \frac{u_1}{b_1}$$

Substituting $\bar{\lambda}$ into (2), we obtain for all $i = 2,3,4$

$$(12) \qquad x_i = \frac{1}{b_i} x_1 + \left(1 - \frac{b_i}{b_1}\right) q + b_i (w_1 - w_i) + (c_i - c_1) + \left(u_i - \frac{b_i}{b_1} u_1\right) .$$

We cannot, however, apply ordinary least squares to each equation of system (12) because it will be observed that although the composite stochastic error term of each equation may be independent of $(w_1 - w_i)$ by our assumptions, it is not independent of x_1. This can be seen by observing that in system (10), x_i depends upon u_i. Hence, x_1 is not independent of the composite stochastic error term. The correlation between the explanatory variable and the stochastic error term will result in estimates being biased and inconsistent.[9] The bias is due to the simultaneous presence of x_1, which is assumed to be endogenous to the system, on the right hand side of each equation.

Notice, also, that the composite stochastic error term in each equation are correlated with each other.[10] As will be explained later on, the correlation between the error terms in each equation could be used to obtain more efficient estimates of the parameters in the system.[11]

Another difficulty in estimating the system (12) is that restrictions must be placed on the parameters. We can see this by rewriting system (12) as

$$(13) \qquad x_1 = \alpha_1 \; x_1 + \alpha_2 \, q + \alpha_3 \, (w_1 - w_1) + \alpha_0 + v_1$$

where $\alpha_1 = b_1 / b_1$; $\alpha_2 = (1 - b_1/b_1)$; $\alpha_3 = b_1$ and ;

$$v_1 = u_1 - b_1 \, u_1$$

$$\alpha_0 = b_1 \, (c_1 - c_1)$$

then the restrictions necessary for each equation to assure that the estimates of b_1 and b_1 will be "consistent" are:[12]

$$\alpha_1 \alpha_3 - \alpha_2 = 0 \quad \text{and} \quad \alpha_1 + \alpha_2 = 1 \, .$$

Notice that the first restriction is <u>not</u> linear, and thus, increases the complexity of estimating the parameters.[13] In addition, restrictions must be set so that the estimate of b_1 is "consistent" between equations. That is, we will obtain an estimate of b_1 for each equation, and therefore, must restrict these estimates to be equal.

In the above exposition, X_1 was assumed to be the numeraire. However, we must determine the criteria for selecting the input factor which is to be used as the numeraire.

Thus, there are many econometric problems in estimating the desired parameters. What is necessary is a scheme which handles all these problems. It is to the development of such a scheme that I will now explain.

(A.3.1) A Scheme to Estimate the Parameters of CRESH

We have seen that there are four main problems in estimating the b_i's from system (12): (i) the simultaneous equation error bias caused by x_1 being an independent variable in each equation; (ii) correlation between the composite stochastic error term in each equation of the system; (iii) the nonlinear and linear restrictions for each equation and the linear restriction between equations; and (iv) the determination of what input factor to use as the numeraire.

To begin this scheme, we can first rewrite system (12) as

$$(14) \qquad (x_i - q) = \frac{b_i}{b_1}(x_1 - q) + b_i(w_1 - w_i) + (c_i - c_1) + v_i \qquad i = 2,3,4$$

By casting the system in this form, we have eliminated one linear restriction from each equation.[14]

From system (14), we would like to somehow remove the simultaneous equation error bias.[15] This can be done by using the method of instrumental variables and choosing as the instruments each $(w_1 - w_i)$ of the system. This method is identical to the estimates obtained by applying two-stage least squares to each equation, since in two-stage least squares, all the exogenous variables in the system are used to estimate $\widehat{(x_1 - q)}$.

The use of instrumental variables now affords us the opportunity to choose the factor input to be used as the numeraire. The gain in efficiency by using the instrumental variable technique depends upon the goodness of fit between the input per unit of output of the factor used as the numeraire and the relative wage rate of the non-numeraire factors to the numeraire factor. Hence, we have a criteria for selecting the numeraire.[16]

We can combine the problem of nonlinear restrictions and correlation between the error terms in each equation by rewriting system (14) as

$$(15) \qquad ({}^{x}i - q) = {}^{b}i \left[({}^{w}i - {}^{w}1) + \theta \widehat{({}^{x}1 - q)} \right] + ({}^{c}1 - {}^{c}i) + {}^{v}i \qquad i = 2,3,4$$

where $\theta = 1/{}^{b}1$. (We are still assuming for expositional purposes that ${}^{x}1$ is the numeraire.)

This suggests an iterative estimation procedure to find $\hat{\theta}$ or $(\widehat{1/{}^{b}1})$. However, we still have the restriction that $\hat{\theta}$ must be the same in all three equations. This, therefore, suggests that all equations be estimated simultaneously. Although the error terms will be correlated between equations, it is not necessary to use the variance-covariance of the error terms to estimate the system simultaneously by the "seemingly unrelated regressions" technique suggested by Hanoch.[17] The reason is that there will be a gain in efficiency, the greater the correlation between the error terms in each equation, but little gain if the correlation between the independent variables in each equation is high.[18] In the case of system (15), the independent variable in each equation is highly correlated. Thus, the variance-covariance structure was not utilized when estimating the parameters of the system.

A suggested procedure after deciding upon the factor to be used as numeraire is:

(i) estimate the unconstrained system (14) by using instrumental variables and then estimating each equation separately. Then, test if the restrictions within and between equations are met;

(ii) if the restrictions are not met, then estimate system (15) which imposes the within equation restriction. For each equation of (15), estimate that σ , which minimizes the residual sum of squares for that equation through an iterative technique.[19] There will then be three estimates of σ for each equation. A test is then necessary to ascertain whether these estimates are statistically equal.

(iii) if the σ estimates are not equal between equations, the final step is to estimate σ by constraining it to be equal between equations. The estimating criteria is to choose that σ which minimizes the generalized residual sum of squares of the system.

(A.3.2) Estimation by Approximate Equation[20]

Another approach is to estimate $\bar{\lambda}$, which is a nonlinear function of output and factor prices, by a first order Taylor expansion around some set of factor prices. Hanoch has shown that if we estimate $\bar{\lambda}$ by such an expansion and substitute the expansion into system (9), we obtain

$$(16) \qquad x_i = \text{constant} - b_i\,(w_i - \bar{w}) - (b_{ii} + 1)\,q \qquad i = 1, \ldots, 4$$

where $\bar{w} = \sum_{i=1}^{4} s_i b_i w_i$, that is, the average wage rate of all factors for a given industry using $s_i b_i$ as weights.

We can rewrite system (16) as

$$(17) \qquad (x_i - q) = \text{constant} + b_i\,(\bar{w} - w_i - q) + e_i \qquad i = 1, \ldots, 4.$$

where we have added the stochastic error term which is assumed to fulfill the assumptions of the general linear model.

The method suggested to estimate (16) or (17) is (i) first estimate either system by using $\bar{w} = \sum_{i=1}^{4} s_i w_i$ disregarding the b_i; (ii) then given the estimated $\hat{b}_i$ obtained in (i), re-estimate $\bar{w}$ by $\bar{w}' = \sum_{i=1}^{4} \hat{b}_i s_i w_i$; and (iii) continue to estimate new values of $\hat{b}_i$ and recompute $\bar{w}'$, testing if the new $\hat{b}_i$ reduce the generalized residuals of the system significantly or alternatively if the $\hat{b}_i$ tend to stabilize.

The advantage of estimating system (17) over system (16) is that the former system has the restrictions already imposed. An advantage of using approximate equations over the procedure suggested in the previous section is that the estimates do not depend upon what factor is used as the numeraire, since no numeraire is required with the approximate equations procedure.

(A.3.3) Estimating the Parameters under Profit Maximization

The mathematical interpretation of λ is that it is the Langrangian multiplier. However, λ has an important economic meaning. It is the total derivative of cost with respect to output, where output is measured in physical terms.[21] Under the assumption of profit maximization by entrepreneurs, the marginal cost of an additional one dollar of output is one dollar. Thus, under the profit maximation assumption, λ is equal to one dollar. Substituting $\lambda = 1$ into system (9), the system, after rearranging terms, becomes:

$$(18) \qquad x_i = -b_i w_i - (b_i - 1)\, q + d_i + v_i \qquad i = 1, \ldots, 4$$

where $\quad d_i = b_i + b_i c_i$

and v_i = stochastic error term which is assumed to meet the conditions of the general linear model.

Rewriting (18) to incorporate the within equation restriction, we have

(19) $$x_i - q = b_i (- w_i - q) + d_i + v_i \qquad i = 1,\ldots,4.$$

Notice that the systems (16) and (17) differ from systems (18) and (19) only by the inclusion of $\bar{w}$ in the former systems.

(A.4) Estimates of the Parameters of CRESH

Rather than report all the results for each of the 11 two-digit SIC industries discussed in Chapter 4, I report only the results of industry 34 - fabricated metal industry. All industries follow the same pattern - that is, the estimates vary by the estimation procedure I employ. In all cases, the restrictions on the estimates were not satisfied. Thus, systems with restrictions imposed are used - that is, only systems (15), (17) and (19) are reported.

The estimates of the b_i's using each educational input as numeraire and the iterative procedure suggested in (A.3.1) are shown in Table A-1. The estimates are given for both systems with and without instrumental variables. The reader can observe the wide variation in the estimate for each b_i, depending upon the numeraire used.

Table A-2 and A-3 shows the estimates of the b_i's, using approximate equations and the profit maximization assumption respectively. With the approximate equations estimate, the system converged after the first iteration. The estimates of the parameters are almost the same in value in the two procedures for the reason explained at the end of (A.3.3). However, these estimates do differ from those in Table A-1.

The computed elasticity of substitution, using the mean factor share for this industry and the various estimates of the b_i's under each estimating procedure, is shown in Table A-4. Because of the wide range of estimates for the elasticity of substitution, the CRESH production function was not used in this study.

Table A-1

ESTIMATES OF THE PARAMETERS OF "CRESH" BY THE ITERATIVE TECHNIQUE USING EACH EDUCATIONAL INPUT AS NUMERAIRE: FABRICATED METAL INDUSTRY, 1958[1]

Numeraire	b_1	b_2	b_3	b_4	$\theta^{(2)}$	$R^{2(3)}$	$F^{(4)}$
Without Instruments							
Elementary school graduates	5.650	.369 (.217)	.312 (.177)	.338 (.198)	.179 (.041)	.168	5.15
High school graduates	.724 (.088)	8.450	.775 (.334)	.878 (.504)	.118 (.033)	.165	5.06
College graduates	.017 (.002)	.015 (.006)	.∞	.012 (.006)	.000	.131	4.01
With Instruments							
Elementary school graduates	6.050	.547 (.217)	.200* (.444)	1.079 (.477)	.165 (.052)	.122	3.74
High school graduates	.472 (.148)	7.650	.367* (.535)	1.677 (.685)	.131 (.041)	.143	4.39
College graduates	.023 (.007)	.010* (.014)	.∞	.034* (.024)	.000	.071	2.17

Footnotes to Table A-1:

(1) The model estimated is the three equations of system (15):

$$(x_1 - q) = b_1 \left[(w_N - w_1) + \theta \widehat{(x_N - q)} \right] + (c_N - c_1) + v_1$$

where $\theta = \frac{1}{b_N}$

with the subscript N denoting the factor used as the numeraire. $\widehat{(x_N - q)}$ denotes the estimate of $(x_N - q)$ when this variable is regressed against all exogenous variables in the system. When instruments are used, $\widehat{(x_N - q)}$ is used. However, when instrumental variables are not used, $(x_N - q)$ is used.

(2) It was not possible to obtain the standard deviation of b_N, however, an asymptotic unbiased estimate for the standard deviation of θ was computed by:

(i) calculating the generalized residual sum of squares for θ which minimizes this quantity for the system (RSSo); and

(ii) calculating the generalized residual sum of square when θ is constrained to equal zero (RSS_1).

We then have the following F distribution:

$$\hat{F}_{1,n-7} = \frac{RSS_1 - RSS_o}{RSS_o} \times (3n - 7)$$

Since the square root of an F-distribution gives us a Student-t with n-7 degrees of freedom, we can then calculate the standard deviation of θ as:

$$\text{Std. Dev. } \theta = \frac{\hat{\theta}}{\sqrt{\hat{F}_{1,3n-7}}}$$

(3) R^2 is the ratio of the generalized explained sum of squares to the generalized total sum of squares of the complete system.

(4) The F-value is found by

$$R^2 \cdot \left(\frac{3n - 7}{7} \right)$$

* Not significantly different from zero at the 10% level. Based on 33 observations. Two tail test used. With the F-test, the 5% level is used.

Table A-2

ESTIMATES OF THE PARAMETERS OF THE "CRESH" PRODUCTION FUNCTION BY APPROXIMATE EQUATIONS: FABRICATED METAL INDUSTRY, 1958 (1)

Factor input	b_i	R^2
Elementary school graduates	.026 (.013)	.111
High school graduates	.032 (.017)	.102
College graduates	.039* (.029)	.056
Physical capital	-.034* (.053)	.014

Footnotes to Table A-2:

(1) The model estimated is the four equations of system (17);

$$(x_i - q) = \text{constant} + b_i \;(\bar{w} - w_i - q) + e_i$$

where $\bar{w} = \sum_{i=1}^{4} s_i\, b_i\, w_i$

The system is estimated simultaneously and converged on the first estimate of the b_i's.

* Not significantly different from zero at the 10% level. Based on 33 observations. Two tail test used.

Table A-3

ESTIMATES OF THE PARAMETERS OF THE "CRESH" PRODUCTION FUNCTION UNDER THE PROFIT MAXIMIZATION ASSUMPTION: FABRICATED METAL INDUSTRY, 1958[1]

Factor input	b_i	R^2
Elementary school graduates	.023 (.012)	.094
High school graduates	.028 (.016)	.086
College graduates	.033* (.028)	.042
Physical capital	.037* (.052)	.016

Footnotes to Table A-3:

(1) The model estimated is the four equations of system (19):

$$(x_i - q) = b_i(-w_i - q) + d_i + v_i$$

* Not significantly different from zero at the 10% level. Based on 33 observations. Two tail test used.

Table A-4

ESTIMATES OF THE ELASTICITY OF SUBSTITUTION FROM ALTERNATIVE ESTIMATION TECHNIQUES OF THE CRESH FUNCTION: FABRICATED METAL INDUSTRY, 1958[1]

Estimation Technique	σ_{12}	σ_{13}	σ_{14}	σ_{23}	σ_{24}	σ_{34}
Iterative procedure: Numeraire used without instruments						
Elementary school graduates	1.232	.846	1.129	.068	.074	.062
High School graduates	2.452	.219	.255	2.625	2.974	.273
College graduates[2]	.000	.170	.000	.150	.000	.120
With instruments						
Elementary school graduates	1.589	.000	3.113	.000	.281	.000
High school graduates	1.432	.000	.314	.000	5.100	.000
College graduates[2]	.000	.230	.000	.000	.000	.000
Approximate equations	.061	.000	.000	.000	.000	.000
Under profit maximization	.054	.000	.000	.000	.000	.000

Footnotes to Table A-4:

(1) The elasticities are computed by

$$\sigma_{ij} = \frac{b_i b_j}{\sum_k b_k s_k}$$

The s_k used for each factor is the mean factor share. For the industry under consideration, the mean for each factor is:

Factor input	Subscript	Mean s_k
Elementary school graduates	1	.25
High school graduates	2	.22
College graduates	3	.10
Physical capital	4	.43

(2) Estimates taken at the limit since $b_3 = \infty$.

Footnotes to the

Appendix to the Study

1 G. Hanoch, "'CRESH' Production Functions," Discussion Paper No. 84; Harvard Institute for Economic Research (Cambridge, Mass.; August, 1969).

2 SEE: A.E. Taylor, Advanced Calculus (Mass.; 1955), pp. 241-244.

3 The product of continuous functions is a continuous function.

4 See footnote 3.

5 Hanoch, pp. 4-5.

6 Hanoch, pp. 6-8.

7 Hanoch also shows that the b_i's can also be equal to zero (p. 9). In this case, the special case of the CRESH is the Leontief fixed proportion model.

8 Here, I return to the four factor case.

9 J. Johnston, Econometric Methods (New York; 1963), p. 233.

10 To see this, let:

$$v_i = u_i - \frac{b_i}{b_1} u_1 \quad \text{and} \quad v_j = u_j - \frac{b_j}{b_1} u_1 \quad ;$$

then, the covariance of the stochastic error term between equation i and j given by

$$E\left[v_i v_j\right] = E\left[\left(u_i - \frac{b_i}{b_1}u_1\right)\left(u_j - \frac{b_j}{b_1}u_1\right)\right]$$

since $E(v_i) = 0 = E(v_j)$.

Expanding the above expression, we find:

$$E\left[v_i v_j\right] = E(u_i u_j) - \frac{b_j}{b_1}E(u_1 u_i) - \frac{b_i}{b_1}E(u_j u_1) + \frac{b_i b_j}{b_1^2} E(u_1^2) \neq 0$$

Thus, even if $E(u_i u_j) = E(u_j u_1) = E(u_i u_1) = 0$, the covariance of the error term of the two equations will not be zero, since $\frac{b_i b_j}{b_1^2} \cdot E(u_1^2) \neq 0$,

11 By more efficient I mean estimation procedures which result in smaller variances for the estimates of the parameters b_i.

12 Here, by "consistent" I mean that we don't obtain different estimates for a given parameter.

13 Hanoch, p. 11, states incorrectly that such a restriction is linear.

14 Now the restriction for each equation of the system

$$(x_i - q) = \alpha_1 (x_1 - q) + \alpha_2 (w_1 - w_i) + \alpha_0 + v_i \qquad i = 2,3,4$$

where $\alpha_1 = b_i / b_1$; $\alpha_2 = b_i$; and $\alpha_0 = c_i - c_1$

is only $b_1 \alpha_1 - \alpha_2 = 0$. However, this restriction is still nonlinear and the restrictions must still be placed between equations so that b_1 is "consistent."

15 That is, obtain estimates which will be asymptotically unbiased.

16 Although we seek as the numeraire the factor j such that the regression

$$\widehat{(x_j - q)} = \sum_{k \neq j} \hat{\gamma}_k \, (w_j - w_k)$$

gives the largest R^2 there is another criteria to be considered. If only one $\hat{\gamma}_k$ is in the estimated relationship - that is, say

$$\widehat{(x_j - q)} = \hat{\gamma}_k \, (w_j - w_k) \qquad \text{for some } k,$$

then when $\widehat{(x_j - q)}$ is substituted for $(x_j - q)$ in the equation which includes $(w_j - w_k)$, the case of perfect multicollinearity will result and the estimation procedure breaks down.

17 The technique Hanoch suggests is developed in

A. Zellner, "An Efficient Method of Estimating SSeemingly Unrelated Regressions and Tests for Aggregation Bias," Journal of American Statistical Association, (1962), 348-368.

This approach relies on Aitken's generalized least squares to estimate the parameters simultaneously.

18 To see this, let us illustrate the two equation case given by:

$$y_i = \alpha_i z_i + u_i \qquad i = 1,2$$

where the variables are assumed to be measured in deviations from their respective means.

Let the variance-covariance matrix be given by:

$$S = \begin{bmatrix} s_{11} & s_{12} \\ s_{12} & s_{22} \end{bmatrix} = \begin{bmatrix} s_{11} & r\sqrt{s_{11} \; s_{22}} \\ r\sqrt{s_{11} \; s_{22}} & s_{22} \end{bmatrix}$$

where r is the correlation coefficient between the disturbances of the two equation; that is

$$r = \sqrt{s_{12}^2 \; / \; s_{11} \; s_{22}} \; .$$

The inverse of S is then

$$S^{-1} = \frac{1}{(1 - r^2)} \begin{bmatrix} 1/ s_{11} & - r/\sqrt{s_{11} \; s_{22}} \\ - r/\sqrt{s_{11} \; s_{22}} & 1/s_{22} \end{bmatrix}$$

the variance-covariance of the parameters is then given by

$$\frac{1}{(1 - r^2)} \begin{bmatrix} \Sigma z_1^2/s_{11} & \dfrac{- r(\Sigma z_1 z_2)}{\sqrt{s_{11} \; s_{22}}} \\ - \dfrac{r(\Sigma z_1 z_2)}{\sqrt{s_{11} \; s_{22}}} & \Sigma z_2^2/s_{22} \end{bmatrix}^{-1}$$

then the variance of α_1 can be shown to be

$$\operatorname{var}(\alpha_1) = \frac{s_{11}}{\Sigma z_1^2} \left\{ 1 + r^2 \; (1 - r^2) \left[1 - (\Sigma z_1^2)^{-1} (\Sigma z_2^2)^{-1} (\Sigma z_1 \; z_2) \right] \right\}^{-1}$$

but

$$\frac{\sum z_1 \; z_2}{\sum z_1^2 \sum z_2^2} = \text{say } \rho^2$$ which is the square of

the correlation between the independent variables in the two equations. Thus, we can write

$$\text{var } (\alpha_1) = \frac{s_{11}}{\sum z_1^2} \left[\frac{1}{1 + r^2 (1 - r^2)^{-1} (1 - \rho^2)} \right]$$

The estimate of the variance of α_1 under ordinary least squares is given by

$$\text{var } (\alpha_1)_{OLS} = \frac{s_{11}}{\sum z_1^2}$$

Thus, we see, if $r = 0$ and/or $\rho = 0$, we get the ordinary least squares estimates. However, given some value of $\rho \neq 0$, we observe that if the correlation between the disturbances, r, increases, the estimate of the variance from the generalized simultaneous estimation of the parameter decreases, relative to the ordinary least squares estimate. On the other hand, as the correlation ρ between the independent variables in the two equations increases, given some $r \neq 0$, then the smaller is the gain in efficiency over the ordinary least squares estimate.

[19] The technique of iterating in search of a global minimum of the generalized residual sum of squares is, in fact, a maximum likelihood estimation procedure which is similar to the procedure described in

C.E.P. Box and D.R. Cox, "An Analysis of Transformation," Journal of the Royal Statistical Society, Series B, (1963), pp. 211-252.

[20] This approach is first suggested by Hanoch, pp. 15-16.

[21] To see this, given a production function

$$Q = f(X_1, \ldots, X_n)$$

the Lagrangian equation for competitive cost minimization is

$$L = \sum_{i=1}^{n} X_i W_i - \lambda \left[Q - f(X_1, \ldots, X_n) \right]$$

The first order conditions are

$$\frac{\partial L}{\partial X_i} = W_i - \lambda f_i = 0 \qquad i = 1, \ldots, n.$$

Under profit maximization, the equation to be maximized is

$$\pi = P\,Q - \sum_{i=1}^{n} W_i X_i$$

where π = profits

and P = price of output.

The first order condition for profit maximization is

$$\frac{\partial \pi}{\partial X_i} = P f_i - W_i = 0$$

since $f_1 = \partial Q / \partial x_1$.

Thus, $P f_1 = W_1$ under profit maximization. But under profit maximization, output price is equal to marginal cost. Thus, we can write

$$\text{marginal cost} \times f_1 = W_1 .$$

Looking back at the first order condition for competitive cost minimization, we have

$$\lambda f_1 = W_1 .$$

Thus, λ is equal to marginal cost.

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