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CONSIDERATIONS IN DESIGN OF NONLINEAR CONTROLLERS FOR IDENTIFICATION AND OBSERVATION OF PARTIALLY UNKNOWN PLANTS

by

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A dissertation submitted to the Graduate Faculty in Engineering in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

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Abstract

CONSIDERATIONS IN DESIGN OF NONLINEAR CONTROLLERS FOR IDENTIFICATION AND OBSERVATION OF PARTIALLY UNKNOWN PLANTS

by

Walter K. Feldman

Adviser: Professor Ralph Mekel

This dissertation deals with design problems that occur in the application of a class of nonlinear and time-varying controllers whose function is to identify unknown linear plants from knowledge of their input and output time functions. The system configuration studied is one in which the identification controller continuously adjusts parameters of a model of the unknown plant until an error vector between the plant and model is driven to zero. This controller has learning capability in the sense that changes in the plant parameters will lead to corresponding changes in the model parameters through the operation of the identification controller.

The identification controller is designed using a formulated Liapunov function that is a function of the plant-model error state vector and the perturbational variations of the plant parameters about the model parameters. A design technique is presented that allows the computation of controller weighting parameters which insure the global asymptotic stability of the composite plant, model, and controller configuration and the identification of the plant. Explicit criteria are obtained which guarantee this behavior for a specified range of plant parameter values about the controller starting values. In addition, an index of performance is formulated that allows the designer to control the transient performance of the system. This control is achieved through the relationship of the identification controller parameters to the index of performance selected.

A key problem that develops in the type of system under consideration is the question of whether or not the convergence to zero of the model-plant output vector insures the convergence of the model-plant parameters. To answer this question, a matrix singularity criteria is derived that can easily be applied to the available plant input and output time quantities.

The identification controller under consideration requires the complete plant state vector to be available. In practice only the scalar plant output is directly measurable. Since differentiation of the plant output is not a practical solution to obtaining this state vector, the addition of an observer to estimate this quantity is made. As a conventional observer would require the plant to be known for its design, a modified observer which has its own observer controller dynamics is designed. The observer controller is designed, and provides estimates to the modified observer, in a manner similar to the way the identification controller updates the model.

Simulations of the plant model and controller system are presented in an illustrative example for a plant with unknown numerator and denominator parameters.

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Finally, I am indebted to my wife Helene for her understanding and patience under rather trying conditions.

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CHAPTER 1

INTRODUCTION

1.1 Motivation And Objective

One of the major broad areas of research over the last fifteen years in the modern control theory area has been the study of adaptive control systems, the term "adaptive" first appearing in the books by Tsien[17] and Bernner and Drenick[18] in the mid 1950's. Two broad sub-categories of adaptive control theory that will be discussed in this dissertation are model-reference adaptive techniques and learning-model identification, both of which are closely related but show significant differences upon detailed examination. Although as brought out in the next chapter, the literature in these fields dates back to 1958, many technical problems which may be called design considerations remain. As in many allied engineering fields these areas requiring further investigation become apparent as soon as the designer attempts to apply the theoretical principles to a particular problem and finds that the solution is not as straightforward as the literature implies.

The emphasis in this dissertation will be on the identification problem. This important area of investigation has received less detailed examination from the adaptive control system point of view than the model-reference problem. Specifically, major differences arise when techniques that can be used to implement the model-reference system are applied without modification to the identification problem.

While the basic differences between these problems can be seen from the block diagrams in Chapter 2, the procedure for designing the identification controller for a specific example requires a thorough understanding of the material contained in Chapters 4 and 5.

The class of identification controllers that are derived in Chapters 4 and 5 have numerous advantages that suggest their use for the identification of a wide variety of unknown plants. These controllers are easily implemented in real time with the use of basic electronic devices such as amplifiers, integrators, and multipliers. Given stored plant data, off line schemes are easily mechanized with the use of digital or analog computers to solve the required differential equations.

As referenced in Chapter 2, there are numerous other approaches to the identification problem that yield algorithms quite different from the learning model ones. Situations may arise in any particular application that indicate that an alternate to the learning model approach is the best choice. An example of this may be the use of a statistical averaging or smoothing technique for the situation when the measured data is highly contaminated by noise. However, it is felt that the application of identification algorithms has not been frequent enough to allow one particular class to emerge as generally superior over all others. Thus the emphasis in this dissertation will be on extending the usefulness of one class of identification controllers through clarification and solution of a number of design problems and the detailed examination of an illustrative example.

The class of identification controllers considered in this dissertation, as well as numerous other identification techniques found in the literature, require the availability of the plant state vector for implementation of the identification technique. This requirement leads directly into a second major area of research, first developed by Luenberger[3] in 1966, that of observer design for state vector estimation. A detailed examination of the observer design, which does not require n-1 differentiations of the nth order plants scalar output, indicates however that the plant must be known to design the observer correctly. This problem, that of needing the plant state vector to identify the unknown plant while requiring knowledge of the plant to estimate its state vector, led to the coupled identification controller and modified observer configuration formulated in Chapter 6 and applied to a design problem in Chapter 8.

The resulting observer part of the dual configuration has dynamics that can be considered dual to the identification controller dynamics. In fact, the total objective of the identification portion of the system can be viewed as providing a means of improving the observer's plant state vector estimates. These observer outputs might be required for a subsidiary control loop performing additional tasks.

A brief discussion of the reason for the identification modeling effort and the basic design approach is the subject matter of Section 1.2, after which Section 1.3 summarizes the major design accomplishments in this dissertation.

1.2 Reasons For Modeling And Design Approaches

From the viewpoint of the control system designer it is almost always mandatory that the input-output characteristics of a plant (the fixed portion of a total control system) be known and expressed mathematically. This requirement is based upon the desireability to be able to predict future performance of a plant for long time operation based upon an observation of the plant over a relatively short period of time and for one of a possibly infinite number of input time functions. This process of obtaining a mathematical model for a plant is called identification and in many cases is the first step in a complete control system built around a fixed plant. For the plants under consideration in this dissertation, the required model is a differential equation expressing the output of the plant as a function of its input and initial conditions.

As shown in the block diagrams of Chapter 2, the learning-model approach to identification first introduced by Margolis [19] in 1959 uses sensitivity functions for design and has the goal that the identification be performed without the insertion of externally generated test signals. This specification, which separates classes of identification techniques is highly worthwhile from a practical viewpoint, since the designer is looking for a model that is valid only under normal operating conditions. In addition, the learning-model approach relaxes the restriction that the exact form of the unknown plant differential equation be known since a lower-order model can easily approximate a higher-order process.

The identification controllers introduced in this text have the additional advantage that they can be implemented in real time as well as off-line. Real time identification becomes a requirement of the control engineer when the unknown plant dynamics are changing with time. (In this case, the identification must be resolved over a time period that is short compared with the time over which the plant's parameters undergo a significant change.)+ In these applications, the "identified" plant parameters usually are sent to another portion of the total control system to: a) implement a feedback control law, resulting in an external signal being applied to the plant, or b) evaluate system performance based upon the estimated plant parameters and some a priori derived performance equations.

The learning model approach in this dissertation has an additional advantage in that it models a continuous plant using a differential equation rather than a difference equation. Obtaining a differential equation model‡ for a continuous dynamical system is a subtle but possibly very important requirement since a difference equation only models the samples at a fixed particular sampling time. Thus, one may find after application of other identification techniques that a different model is obtained for different data sets with different sampling

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+It is assumed that the plant parameters can be treated as time invariant during the time interval over which the identification is resolved. This assumption is believed to be applicable in most practical cases.

‡The order of the differential equation model is set equal to what is believed to be the order of the differential equation describing the unknown plant. This a priori assumption is based upon experimentation performed on the unknown plant.

intervals, making the model "plant input" dependent and thus effectively limiting its usefulness.

An important consideration that makes this type of identification controller particularly attractive is the simplicity of the resulting plant, controller, and model configuration. The model dynamics are easily generated in an analog-computer type configuration while the identification controller requires basically simple additions and multiplications of the plant and model input and output quantities. In addition, use of Liapunov functions can provide useful stability and performance criteria usually difficult or impossible to obtain with other techniques.

In Chapter 4 the identification controller is derived directly from a quadratic Liapunov function which is a function of the plant-model error and deviations of the plant and model parameters. This technique has the following advantages:

- It leads to the design procedure of Chapter 5 that yields identification controller parameters which insure the global asymptotic stability of the composite system. Other techniques resulting in similar equations yield either equations which can be shown to be stable only in the small (usually due to the case of incomplete error state variable usage in the controller) or do not yield explicit techniques on how to obtain the controller parameters to guarantee stability. Although there are remaining controller gain parameters that still must be selected experimentally, the condition that they be positive insures stability in all cases.

- The use of the complete error state vector in the controller leads directly in Chapter 5 to a meaningful and useful index of performance. This index of performance is the integral of the negative derivative of the Liapunov function used to derive the dynamics of the identification controller. The integrand has the property of being a quadratic form in the error state vector. Terms selected by specifying relative weighting of the elements in the integrand allow the designer to control the transient response in a systematic way, rather than by trial and error. The lack of this option in designs using other techniques than Liapunov functions with useful performance indices is clearly evidenced in the literature.[20]
- The removal of a restriction of the Γ matrices+ in Chapter 5 result in a multitude of controllers open for future study beyond this dissertation.
- Sufficient conditions guaranteeing that driving the plant-model error vector to zero will also drive the model parameters to their correct values are obtained by a a detailed examination of the equilibrium conditions of the composite system composed of the plant, model, and controller. These conditions allow, from the examination

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+The elements of these matrices enter into the identification controller as design parameters.

of the singularity of an indicated computable matrix, the the determination of whether or not the indicated model parameters are correct when the error state vector is null. A test of this kind is a necessity in applications when the designer has no control over the plant input.

1.3 Organization Of The Dissertation

Chapter 2 contains a review of the literature on model-reference and identification systems together will a clear distinction made between the model-reference control techniques and the learning model identification systems. Included are block diagram configurations, the specification of the fixed and time-varying quantities, and the designed controller inputs and outputs.

After the presentation in Chapter 3 of basic definitions and theorems in the area of Liapunov stability theory, Chapter 4 derives the basic dynamical plant-model error equations that are used to design the identification controller.

Chapter 4 relates the plant's transfer function to its matrix differential equation in phase variable form and exhibits the fact that derivatives of the plant's input must be computed if the plant has finite zeros. This fact is not made clear in the literature (see Hsia and Vimolanich[20] for example) since these relationships are not detailed and the examples have usually all pole transfer functions for the unknown plant.

Chapter 5 develops design techniques to select controller parameters that insure stability and minimize an index of performance that controls the system's identification time.

In Chapter 6, the use of an observer is introduced as a state variable estimator when the plant state vector is either difficult to measure or unavailable.

In Chapter 7, an illustrative example with a plant having two unknown poles and two unknown zeros is carried out according to the procedures indicated in Chapters 4 and 5 for the non-observer case. Computer simulation results are included for various conditions of interest and appropriate performance conolusions are made.

In Chapter 8, the composite system of identification controller, model, and plant state vector observer is discussed. The example in Chapter 8 has a fixed known plant added to make the original plants state vector not directly available. The identification controller dynamics are the same as in Chapter 7, with the exception that a modified error vector resulting from the availability of only an observer estimated plant state vector is used. The observer dynamics are designed using a Liapunov function in a dual manner to the identification controller dynamics.

The results of a computer simulation of the total system, with both the identification controller and observer designed separately, are present in Section 8.5.

My indebtness to the work of many scholars in this area is indicated throughout this dissertation and in the Bibliography.

CHAPTER 2

HISTORICAL BACKGROUND

2.1 Introduction

In this Chapter the basic block diagram configurations for model-reference and learning model identification schemes are presented.

Section 2.2 discusses early developments, where the differences in both goals and implementation become evident. In addition, the specific techniques used for solving the model-reference and learning model identification problems are mentioned with particular emphasis placed on how the adjusting mechanism is derived. Section 2.3 concludes with a discussion of the recent developments in the literature. For a general background in the area of adaptive control systems, the text by Mishkin and Braun [24] is recommended.

2.2 Early Developments

The most general model-reference configuration is shown in Fig. 2-1 and appears in the work of Osburn [17], Whitaker [26], Yarmon, and Kezev [27], Rang [21] and Grayson [22, 23]. In Fig. 2-1, the object of the controller is to drive the output of the plant, w(t), to follow the output of the model, y(t), by generating a controller plant input u(t) from the reference input r(t) and the model output y(t).

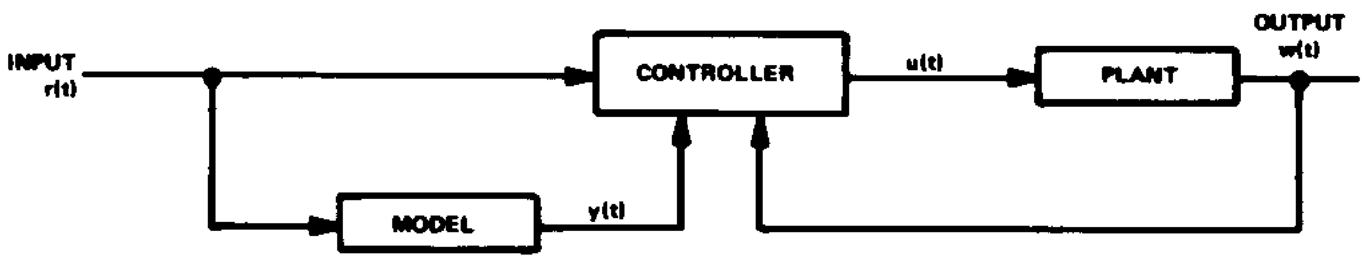

Figure 2-1. General Model-Referance Control System

One early scheme that appeared in 1961 was that of Donalson [25] who used what may be called a "parameter adjustment" technique, the configuration of which is shown in Fig. 2-2. Donalson's scheme, based upon using the gradient of the plant-model output error function, derived an adjusting mechanism which varied the parameters in the plant's prefilter, feedforward compensation and feedback compensation.

Rang [21] applied Liapunov's direct(second) method to the configuration in Fig. 2-1 to derive a plant input signal u(t) such that the error between the plant and model outputs went asymptotically to zero. Although his approach had many of the learning model identification characteristics, the configuration of Fig. 2-1 requires the plant to be operated with a derived controller signal u(t) rather than with the actual plant input r(t). This condition is of course not acceptable for general identification schemes, whose purpose is not to influence the plant's operation but to obtain a model of the plant as discussed in Chapter 1.

Before discussing the early references on identification techniques, the work of Grayson [22, 23] that has the configuration shown in Fig. 2-3 should be mentioned. The resulting system in Fig. 2-3 contains a nonlinear controller with relays. This controller has adaptive properties and is designed using a Liapunov function. However, there were also severe limitations on when the signal u(t) could be generated. In addition the plant input is now u(t) which is not the "normal" plant reference input r(t), although if the controller is properly designed w(t) should follow y(t).

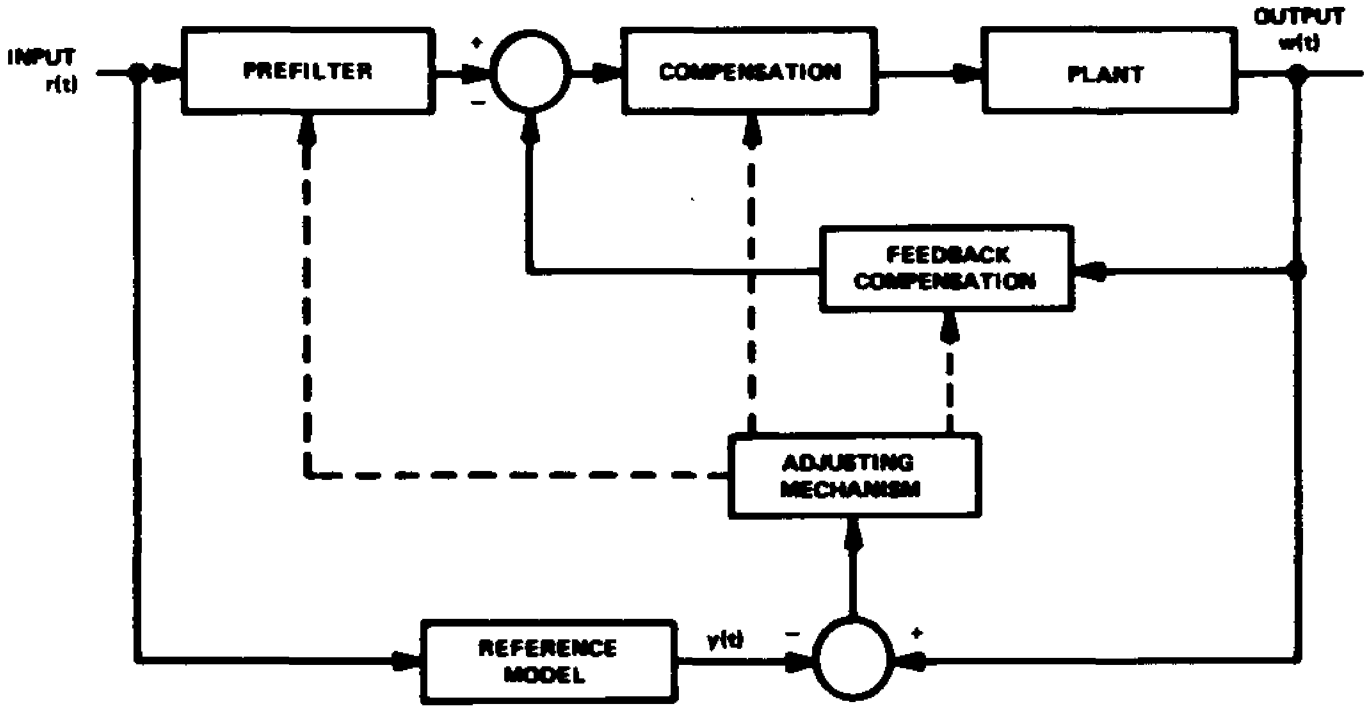

Figure 2-2. Parameter Adjustment Model-Reference System

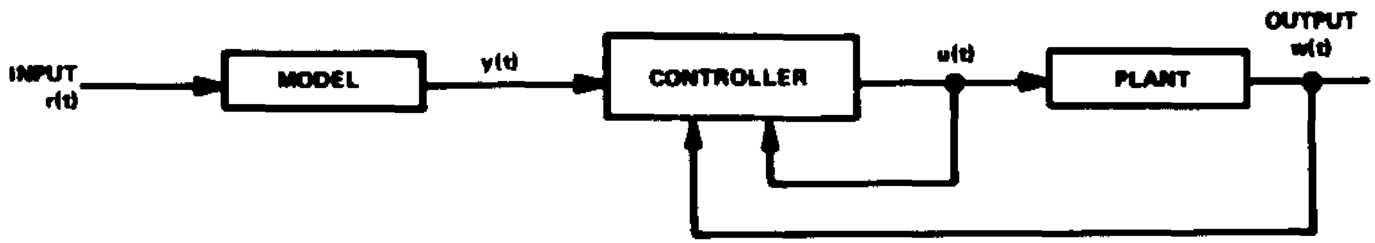

Figure 2-3. Additional Model-Reference Configuration

The general identification problem consists in determining a complete description of the relations between the input and output state of an unknown plant under the condition that the identification must

a) be made in the presence of normal operation signals,

b) must not disturb the normal operation of control and

c) must be made relatively faster with respect to the rate of variation of the plant parameters if the plant is not time invariant.

Early research in the field for linear plants can be found in the works by Lee [29], Anderson et al [30], Levin [32] and Lindenlaub [31]. The identification techniques in these studies are statistical in nature, using least squares theory, crosscorrelation analysis, and matched filter theory.

The work of Kalman [34] in this area employed samples of the input and output as well as a difference equation model of the unknown plant. An error criterion is presented and minimized by the selection of model coefficients at each sampling interval.

The learning model approach of Fig. 2-4 was first introduced by Margolis and Leondes [19] for discrete transfer functions; "tuning" of the model being obtained by a steepest descent procedure. Although this approach is easily extended conceptually to continuous transfer functions, the lack of a stability guarantee in the large for the initial model parameters and initial error conditions hampered the usefulness of this approach at the time.

It should be noted that the "models" in the model-reference schemes are of course fixed before the system's operation and do not contain time-varying parameters. However, the parameters in the learning

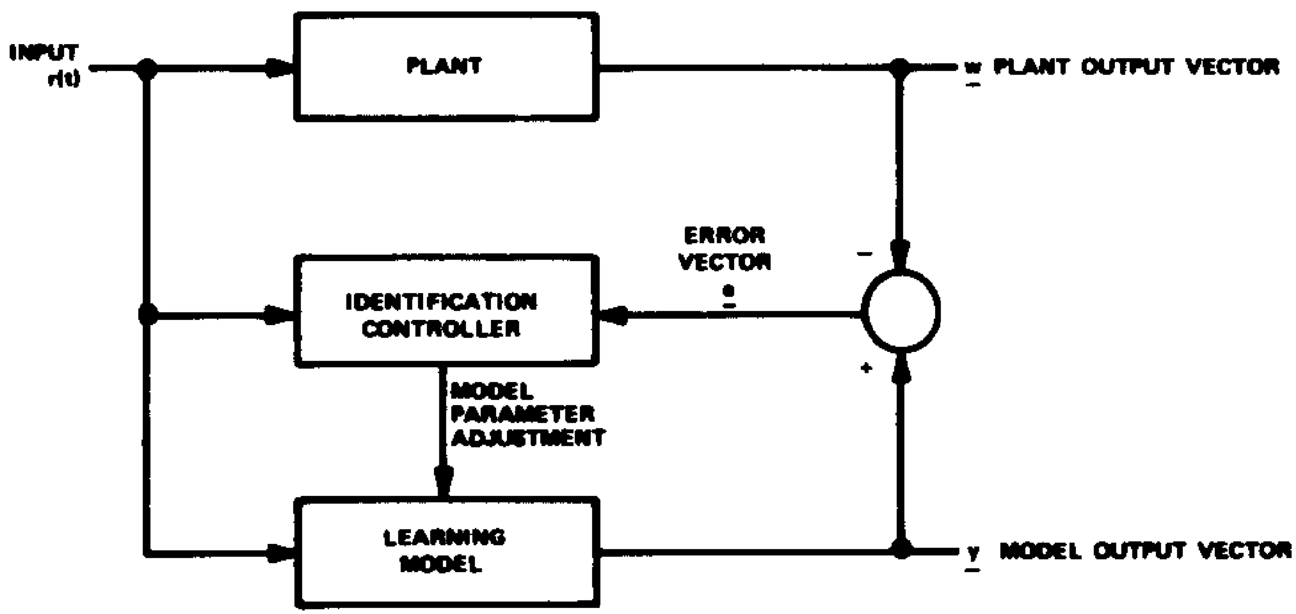

Figure 2-4. The Learning Model Identification Configuration

model identification configuration in Fig. 2-4 do contain time-varying parameters, and it is this difference between the function and implementation of the "model" that leads to considerable differences in the controller designs.

2.3 Recent Developments

The first design attempt employing Liapunov functions for the model-reference system that resulted in a controller similar to those generated by the procedure in Chapter 5 (for the identification problem) can be found in Parks [37]. In a simplified system of the framework shown in Fig. 2-2, Parks used his Liapunov function to adjust one time varying gain to force the plant's output to track the reference model output with zero error. Further improvements to Parks system was made by Phillipson [42], who suggested that his design can be improved with the addition of subsidiary feedback loops. Windsor and Roy [41] further generalized Parks work on model-reference system design.

Similar design contributions to those just mentioned appear in the paper by Dressler [38], in which a pertubational technique operating on an incremental error criterion is used to generate the controller, rather than a Liapunov function. Hsia and Vimolvanich [20] apply Dressler's work to the identification problem. However, for their resulting controller, which uses only a scalar model-plant error, (rather than the complete error state vector that is used in Chapter 5) there is great difficulty in proving global stability and/or selecting the controller design parameters efficiently without extensive a priori experimentation. Without control over the complete error state vector, selection of too large "gain" parameter values will easily over correct in an unstable manner.

A recent series of publications that discuss making an improvement in the performance of model-reference control systems, designed similar to the technique presented in Chapter 5, include Shahein et al [45], and comments to Shahein paper by Pazdera and Spence [58], Hang and Parks [59], and Mukerjee [60]. The conclusions drawn are that if the misalignments between the model and plant parameters were available, Shahein's technique would improve the system's performance. However, these misalignments are not available, as were pointed out by the papers commenting on Shahein et al, and thus the improved procedure is of academic interest only.

Luders and Narendra [57] derive conditions for the existence of quadratic Liapunov functions for vector differential equations that appear to generalize some of the previous work in the field of model-reference systems. However, they do not deal with any design considerations or mention the identification problem.

Butz [56] uses a Liapunov function to design a bang-bang controller to "learn" an unstable plant and to provide a stabilizing controller. The controller is generated with a system configuration, similar to that of Fig. 2-1, except that the time derivative of the Liapunov function actually provides the desired model for the model reference system.

Other plant identification techniques which point out different approaches to the problem include the Laplace Transform technique of Smith [36], the Volterra function approach of Roy and Sherman [39], and the stochastic approximation techniques of Holmes [43] and Saridus and Lobbia [44].

This dissertation bases the design of nonlinear controllers for identification and observation of partially unknown plants on Liapunov's direct method. The material on Liapunov's direct method is presented in the next chapter.

CHAPTER 3

THE DIRECT METHOD OF LIAPUNOV+

3.1 Introduction

In 1892 Liapunov [55] introduced a direct method for studying the stability of the equilibrium solution of nonlinear systems. The application of this technique to the stability analysis and design of nonlinear systems presupposes that the dynamical system under consideration can be expressed by a matrix differential equations of the form

$$\dot{\underline{x}} = \underline{f}(\underline{x}) \qquad (3.1\text{-}1)$$

where $\underline{x}$ is a column matrix of n state variables, $\dot{\underline{x}}$ is a column matrix of the time derivatives of the state variables and $\underline{f}(\underline{x})$ is a column vector of n nonlinear functions of the state vector $\underline{x}$ and, in general, the independent time variable t.

A constant solution $\underline{x}_0$ is said to be an equilibrium point [9] if for Eq. (3.1-1)

$$\underline{0} = \underline{f}(\underline{x}_0) \qquad (3.1\text{-}2)$$

If $\underline{x}_0$ is the only constant solution in a neighborhood of $\underline{x}_0$, then x_0 is called on isolated equilibrium. The application of stability theory in this dissertation is primarily concerned with motions about the equilibrium point of differential equations of the form in Eq. (3.1-1). For convenience, equations such as (3.1-1) are rewritten to have

$$\underline{x}_0 = \underline{0} \qquad (3.1\text{-}3)$$

+The material contained in this chapter is presented to make this dissertation self-contained and for the benefit of those readers unfamiliar with the use of Liapunov's direct method.

as an isolated equilibrium point, i.e.,

$$\underline{0} = f(\underline{0}) \qquad (3.1\text{-}4)$$

Further conditions on $\underline{f}$ in Eq. (3.1-1) may be required for use with various Liapunov functions, continuity and differentiability being two such conditions.

The basic definitions and theorems of Liapunov will now be presented+. An attempt is made to present the material in such a way that the reader, with a knowlegde of state-variable techniques, will understand the physical implications involved in the statements of the theorems. No proofs of these theorems will be given, since they have been adequately proved in the literature [9]. Before the introduction of basic definitions and Liapunov's theorems, the concepts of definiteness and closure will be presented.

3.2 Concepts of Definiteness and Closure

This section gives an exposition of the concepts of definiteness and closure which are utilized in the statements of Liapunov's theorems. Let the differential equation describing the system have the form as given by Eq. (3-1.1), with the conditions given by Eqs. (3.1.2) and (3.1.3). A scalar function V $(\underline{x})$, which is defined in a certain region of the origin, is considered. It is assumed that V $(\underline{x})$ is continuous, has continuous partial derivatives and becomes zero when $\underline{x} = 0$. The following definitions follow Hahn [9]

Definition 3.1 Positive (Negative) Definite

A function V $(\underline{x})$ is called positive (negative) definite in a neighborhood of the origin of a real state space

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+See footnote on page 21.

$$|\underline{x}| \leq h \qquad h \text{ arbitrary small positive number} \qquad (3.2\text{-}2)$$

if for all $\underline{x}$ in this neighborhood

$$V(\underline{x}) > 0 \qquad [V(\underline{x}) < 0] \qquad \text{for all } \underline{x} \neq 0 \qquad (3.2\text{-}3)$$

Note that $|\underline{x}|^2 = x_1^2 + \ldots + x_n^2$

<u>Definition 3.2</u> <u>Positive (Negative) Semidefinite</u>

$V(\underline{x})$ is said to be positive (negative) semidefinite in a neighborhood of the origin defined in Eq. (3.2-2) if

$$V(\underline{x}) \geq 0 \qquad [V(\underline{x}) \leq 0] \qquad \text{for all } \underline{x} \qquad (3.2\text{-}4)$$

<u>Definition 3.3</u> <u>Indefinite</u>

The scalar function $V(\underline{x})$ is said to be indefinite in a neighborhood defined by Eq. (3.2-2) if it assumes both positive and negative values in this neighborhood.

For h arbitrarily small in Definitions 3.1 and 3.2, $V(\underline{x})$ will be definite in an arbitrarily small region about the origin. The larger the h, the greater the region of definiteness for $V(\underline{x})$. If h is infinite, $V(\underline{x})$ is definite in the whole space.

An important concept that becomes inherent in the Liapunov stability theorems is that of closure. Following Letov [10], if $V(\underline{x})$ is a positive-definite function then the equation

$$V(\underline{x}) = C \qquad C > 0 \qquad (3.2\text{-}5)$$

represents a family of closed surfaces about $\underline{x} = 0$, and as $C \to 0$, the surface becomes the origin. For stability conditions in the large it will be necessary to insure that $V(\underline{x})$ satisfying Eq. (3.2-5) be closed for sufficiently large C, and in particular for $C = \infty$. We have the following theorem from Letov [10].

Theorem 3.1

If V ($\underline{x}$) is positive definite for all $\underline{x}$ and if

$$\lim_{|\underline{x}| \to \infty} V(\underline{x}) = \infty \qquad (3.2\text{-}6)$$

then the closure of the surface or surfaces defined in Eq. (3.2-5) is assured for all positive values of C.

3.3 Definitions of Stability

The question of stability is concerned with the solutions of Eq. (3.1-1) when the system is disturbed from its equilibrium condition $\underline{x} = 0$.

For linear autonomous systems, the concept of stability is quite simple.

Assuming that the system is completely controllable and observable, this system is said to be stable if its output response to every bounded input remains bounded. [11]

For the linear autonomous system shown in Eq. (3.3-1),

$$\dot{\underline{x}} = A\,\underline{x} \qquad (3.3\text{-}1)$$

where A is a constant matrix, a necessary and sufficient condition for stability is that the eigenvalues of A have negative real parts.

This stability condition for linear systems is not adequate or even meaningful when one deals with nonlinear systems. Therefore, new stability definitions must be developed. In fact Ingwerson [12] defines twenty different types of stability, Kalman and Bertram [13] define eight types and Antosiewicz defines nine types of stability.

Stability, asymptotic stability, and global asymptotic stability are considered in this dissertation.

The following definitions follow Hahn [9]. It is assumed that the system is expressed as in Eq. (3.1-1) and the equilibrium state being investigated is located at the origin. Let S (R) be a spherical region of radius R > 0 around the origin, where S (R) consists of point $\underline{x}$

$$|\underline{x}| < R \tag{3.3-2}$$

Also let S (r) be a spherical region of radius r > 0,

$$|\underline{x}| < r \tag{3.3-3}$$

where $|\underline{x}|^2 = x_1^2 + x_2^2 + . . . + x_n^2$, the Euclideen norm of the vector $\underline{x}$.

Definition 3.4 Stability

The origin is stable for Eq. (3.1-1) if corresponding to each S (R) there is an S (r) such that if $\underline{x}$ (t) is a solution whose initial position $\underline{x}_0 = \underline{x}$ (0) lies in S (r) then $\underline{x}$ (t) lies in S (R) for all time after.

Definition 3.5 Asymptotic Stability and Global Asymptotic Stability

If the origin is stable and additionally, if every solution starting in S (r) approaches the origin as $t \rightarrow \infty$, then the system is said to asymptotically stable.

If this condition holds for all points in the state space, then the equilibrium is said to be asymptotically stable in the whole, which La Salle calls global asymptotic stability (complete stability) [15].

Definition 3.6 Unstable (Completely)

The equilibrium is said to be unstable (completely) if given an initial condition x_0, there exists an $E > 0$ such that when

$$0 < \left| x_0 \right| < E \tag{3.3-4}$$

then for $t \geq 0$ and finite, each motion $\underline{x}$ (t) reaches the sphere

$$\left| \underline{x} \right| = E \tag{3.3-4a}$$

3.4 Liapunov Stability Theorems

Let V ($\underline{x}$) denote a scalar function of $\underline{x}$, as previously mentioned, and $\dot{V}$ ($\underline{x}$) denote its time derivative. Using Eq. (3.1-1) one obtains

$$\dot{V} = \left(\frac{\partial V}{\partial \underline{x}}\right)^T \dot{\underline{x}} = \left(\frac{\partial V}{\partial \underline{x}}\right)^T \underline{f}\ (\underline{x}) \tag{3.4-1}$$

Note that V ($\underline{x}$) does not depend explicitly on time.

Furthermore, let V ($\underline{x}$) be a positive definite function with the following properties

a) $V(\underline{0}) = 0$

b) V ($\underline{x}$) is continuous together with its first partial derivatives in an open region Ω about the origin

c) Outside the origin and in Ω, V ($\underline{x}$) > 0

Then one has from Hahn [9] the following two theorems:

Theorem 3.2 Stability

The origin is stable if $\dot{V}(\underline{x}) \leq 0$ in Ω. V ($\underline{x}$) is called a Liapunov function for the system in Eq. (3.1-1) if this condition holds.

Theorem 3.3 Asymptotic Stability

The equilibrium (origin) is asymptotically stable if $\dot{V}$ ($\underline{x}$) is negative definite, or likewise, $-\dot{V}$ ($\underline{x}$) is positive definite.

From LaSalle [15] and LaSalle and Lefschetz [16] we have a useful theorem on complete stability.

Theorem 3.4 Complete Stability (Global Asymptotic Stability)

If $V(\underline{x})$ is continuous with continuous first partial derivatives, and such that $V(0) = 0$ and

a) $V(\underline{x}) > 0$ for all $\underline{x} \neq 0$

b) $V(\underline{x}) \to \infty$ as $|\underline{x}| \to \infty$

c) $\dot{V}(\underline{x}) \leq 0$ for all $\underline{x}$ (at least negative semidefinite)

d) $\dot{V}(\underline{x})$ not identically zero along a solution of the system other than the origin, then the system in Eq. (3.1-1) is completely stable (globally stable).

3.5 Liapunov Functions-Brief Review

In using the second method of Liapunov as a design tool to guarantee the stability of a resulting nonlinear system, two important considerations should be kept in mind. The first is that the conditions obtained from a particular V function are sufficient but not in general necessary. The second is that the Liapunov function used to analyze or design a system is not unique. Thus, although the initial choice of a suitable V function is in general still an art rather than a science, a considerable amount of research has been done in the area, especially for systems governed by ordinary differential equations. Specifically, the books by Hahn [9], LaSalle and Lefschetz [16] and the works by Antosiewicz [14] and Kalman and Bertram [13] contain detailed summaries and extensive bibliographies of the research efforts up to the beginning of the 1960's. The texts by Lefschetz [46] and Aizerman and Gantmacher [47] consider the nonlinear regulator problem in extensive detail while the survey article by Brockett [51] summarizes and places in perspective recent contributions in stability theory and includes a bibliography of over 100 references.

As an illustration of specific techniques for finding a suitable Liapunov function for a system of nonlinear differential equation, three classes of Liapunov functions are discussed in this section.

The first class of Liapunov functions that find widespread use, particularly for design of nonlinear controllers for linear plants, is the V functions discussed by Bass and Weber [40], where $V = V(\underline{x})$ is of the form

$$V \triangleq \sum_{r=1}^{N} \phi_{2r} (\underline{x}) \tag{3.5-1}$$

where $\underline{x}$ is an nx1 state vector and for r = 1

$$\phi_2 = \underline{x}^T B \underline{x} , \tag{3.5-2}$$

matrix B being positive definite.

Note that ϕ_2 is a positive definite quadratic form and each $\phi_{2r} = \phi_{2r} (\underline{x})$ is a positive semidefinite homogeneous multinomial form of degree 2r, (r = 2, 3, 4, ...). The V function in Eq. (3.5-1) is usually applied to a plant whose differential equation is of the form

$$\dot{\underline{x}} = A\underline{x} + \underline{\xi} \tag{3.5-3}$$

where A is an arbitrary stability matrix and $\underline{\xi}$ is a control vector to be specified in terms of some operation on $V(\underline{x})$. The basis for the success of this technique rests upon solving the Liapunov matrix equation which is discussed in Section 3.6.

A second useful method for generating a class of V functions is the variable gradient method of Schultz and Gibson [50]. In this method an arbitrary column vector ∇V whose ceoeffcients are allowed to be functions of the state variables is assumed, ∇V signifying the gradient of the V function. From ∇V, $\dot{V}$ is computed since

$$\dot{V} = \underline{\nabla V}^T \dot{\underline{x}} \tag{3.5-4}$$

or

$$\dot{V} = \frac{\partial V}{\partial x_1} \dot{x}_1 + \frac{\partial V}{\partial x_2} \dot{x}_2 + \dots + \frac{\partial V}{\partial x_n} \dot{x}_n \tag{3.5-5}$$

where the system equations, $\dot{x}_i$ $(i = 1, 2, \dots, n)$ are given as functions of the x_i $(i = 1, \dots, n)$ state variables. After constraining $\dot{V}$ to be at least semidefinite, $(n-1)\,n/2$ curl equations are used to determine the remaining coefficients of ∇V. These curl equations are in vector form and given by

$$\nabla \times \nabla V = 0 \tag{3.5-6}$$

Eq. (3.5-6) allows V to be uniquely obtained by the line integration

$$V = \int_0^{(x_1, x_2, \dots, x_n)} \nabla V \cdot d\underline{x} \tag{3.5-7}$$

Finally, $\dot{V}$ is recomputed from V to check its semidefiniteness or definiteness and V is checked for closure. For further details, the reader is referred to the paper by Schultz and Gibson [50], which contains illustrative examples.

The third technique to be discussed is that of Mekel [48, 49]. This technique is especially useful for high order systems containing multiple nonlinearities. The differential equation describing the autonomous nonlinear system is expressed in matrix form as

$$\dot{\underline{x}} = A_{On}(\underline{x}) \cdot \underline{x} \tag{3.5-8}$$

where $\underline{x}$ is a column state vector and $A_{On}(\underline{x})$ is a square matrix that characterizes the system. Note that subscripts O and n denote the order of the system and the number of nonlinearities in the system respectively.

The Liapunov function is constructed as

$$V = \underline{x}^T \cdot M_{On}(\underline{x}) \cdot \underline{x} \tag{3.5-9}$$

where $M_{On}(\underline{x})$ is a symmetric matrix directly related to the system matrix $A_{On}(\underline{x})$. The time derivative of V, denoted by $\dot{V}$, is formulated as

$$\begin{aligned} \dot{V} &= \underline{x}^T (A_{On}^T M_{On} + M_{On} A_{On} + \dot{M}_{On}) \underline{x} \\ &= \underline{x}^T \cdot D_{On}(\underline{x}) \cdot \underline{x} \end{aligned} \tag{3.5-10}$$

To comply with Liapunov's criterion for $\dot{V}$, the elements of the $D_{On}(\underline{x})$ matrix are constrained to satisfy the conditions

$$d_{ii} \leq 0 \tag{3.5-11}$$

and

$$d_{ij} + d_{ji} = 0 \tag{3.5-11a}$$

where d_{ij} are the elements of the $D_{On}(\underline{x})$ matrix. Note that i and j denote the row and column respectively. These conditions permit the evaluation of the elements in the $D_{On}(\underline{x})$ and the $M_{On}(\underline{x})$ matrices.

Using these evaluated M_{On} ($\underline{x}$) and D_{On} ($\underline{x}$) matrices, V and its derivative $\dot{V}$ in Eqs. (3.5-9) and (3.5-10) can be evaluated. The formulation of the M_{On} ($\underline{x}$) matrix for seventeen types of nonlinear systems with multiple nonlinearities is given in the paper by Mekel [49].

Chapter 3 has discussed Liapunov stability theory and presented classes of Liapunov functions. In Chapter 4, a candidate Liapunov function is used as a design tool to generate a nonlinear identification controller for an unknown plant. The next chapter also discusses the basic design approach.

CHAPTER 4

SYSTEM CONFIGURATION AND BASIC CONTROLLER DESIGN

4.1 Introduction

This chapter presents the plant, model, and identification controller configuration under study. The differential equations describing the error dynamics between plant and model outputs are derived in terms of plant-model parameter misalignments. A Liapunov function formulated for use with these equations leads directly to a non-linear set of controller equations that drive the parameters in the model to the values which identify the unknown part of the given plant.

4.2 Plant and Model Equations

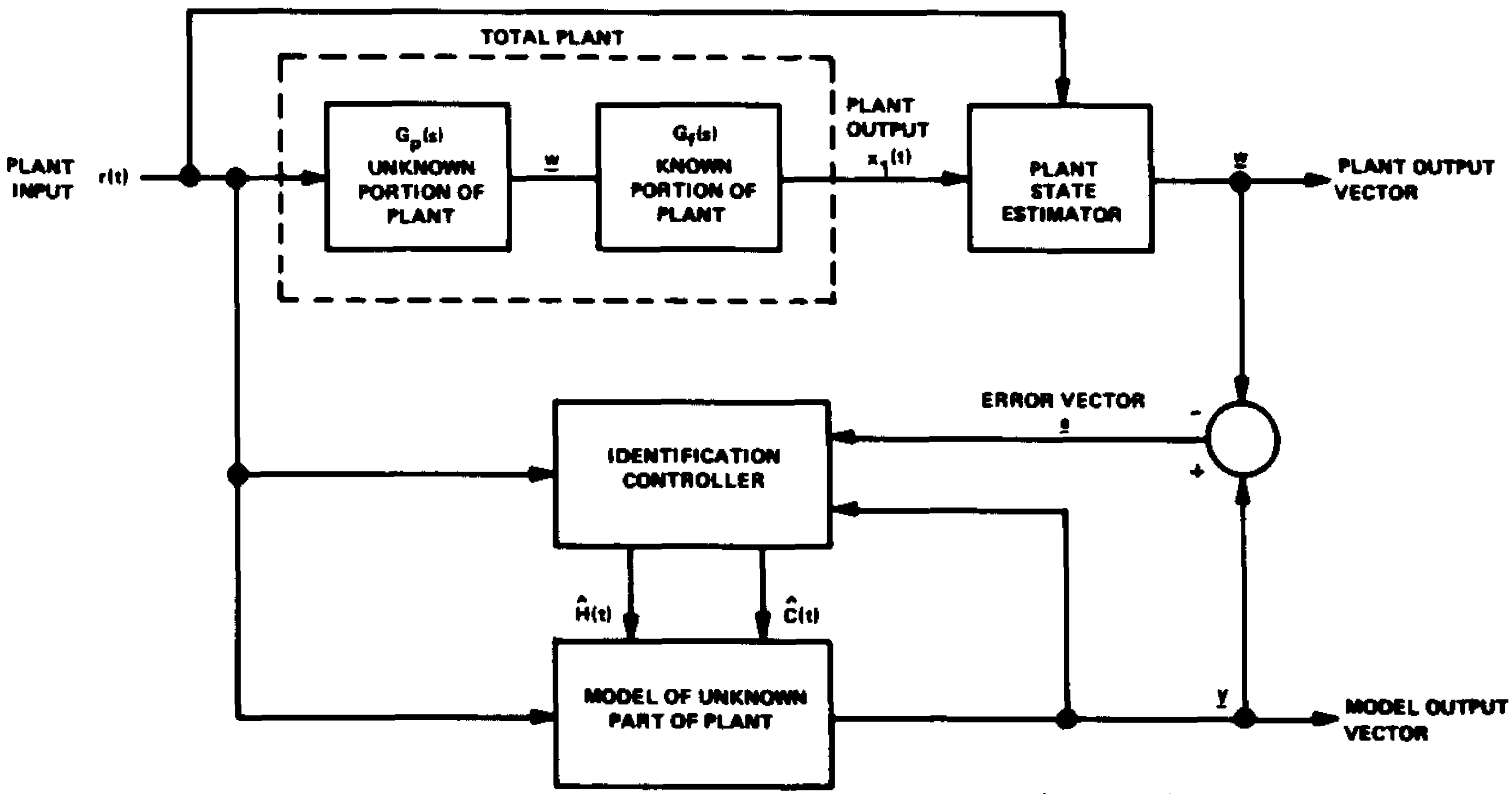
The basic plant, model, and controller configuration under study is shown in Figure 4-1 in block diagram form. The plant, with an input scalar time function $r(t)$ and output scalar time function $x_1(t)$, is assumed to be composed of two parts in tandem. The first part, denoted by the transfer function $Gp(s)$ in Figure 4-1, represents the unknown portion of the plant. This unknown plant is assumed to have its input-output behavior represented by a rational function $Gp(s)$ having n poles and n-1 zeros, i.e.,

$$Gp(s) = \frac{W(s)}{R(s)} = \frac{b_{n-1} s^{n-1} + b_{n-2} s^{n-2} + \ldots + b_o}{s^n + a_{n-1} s^{n-1} + \ldots + a_o} \qquad (4.2\text{-}1)$$

where $W(s)$ and $R(s)$ represent the Laplace transforms of the unknown plant output and input time functions respectively.

In Figure 4-1, a known transfer function $G_f(s)$ is used to indicate the transformation from the unknown plant output w(t) to a measurable plant output $x_1(t)$. This representation includes the situation when Gp(s) is an unknown plant and $G_f(s)$ a known plant; the output of Gp(s) is however not available for measurement. In this case only the unknown portion of the plant need be modelled. The system shown in Figure 4-1 also includes the case where $G_f(s)$ represents a known portion of an unknown plant and Gp(s) represents the unknown portion which is to be identified. In this case, w(t) would be a mathematical (fictional) time quantity, not a physical state variable [1].

In this chapter, as well as in Chapters 5 and 7, $G_f(s)$ is set to unity, resulting in a fully unknown plant Gp(s) describing the dynamics between the scalar output $x_1(t)$ and the scalar input r(t). In addition, the unknown plant's state vector $\underline{w}$ is assumed to be available without error. In Chapter 6 and 8, the restriction that $\underline{w}(t)$ be directly available is removed, and a modified Luenbarger observer is used to estimate $\underline{w}$ from $x_1(t)$. In addition, a known, non-unity transfer function $G_f(s)$ is assumed. In the case where $G_f(s)$ is unity, the observer still yields an estimate of $\underline{w}$ from $x_1(t)$, which would be only one component of $\underline{w}$. It should be mentioned here that the unknown plant is assumed to be both controllable and observable[2], which is a necessary and sufficient condition for the transfer function Gp(s) to be an accurate representation of the plant. It is obvious that having only the plant input-output quantities available for identifying the plant requires that the input excite all the plant's dynamics and that this behavior is evidenced at the plant output.



$$G_p(s) = \frac{b_{n-1} S^{n-1} + b_{n-2} S^{n-2} + \ldots + b_o}{S^n + a_{n-1} S^{n-1} + \ldots + a_o}$$

Figure 4-1. Plant, Model, and Controller Block Diagram Configuration

In Eq. (4.2-1), the a_i and b_j coefficients represent the unknown plant parameters. In practice many of these parameters may be known so that considerably fewer than the 2n parameters indicated in Eq. (4.2-1) need be identified. Even if all are unknown, one may find relationships among the a_i's and b_j's which would simplify the identification task. For the design to follow, all 2n plant parameters are assumed to be unknown and need be identified.

The unknown plant Gp(s) in Eq. (4.2-1) may be represented by the matrix differential equation

$$\dot{\underline{w}} = H\underline{w} + C\underline{r} \qquad (4.2\text{-}2)$$

where $\underline{w}$, $\dot{\underline{w}}$, and $\underline{r}$ are column vectors as shown

$$\underline{w} = \begin{bmatrix} w_1 \\ w_2 \\ \cdot \\ \cdot \\ \cdot \\ w_n \end{bmatrix} \qquad \dot{\underline{w}} = \begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \cdot \\ \cdot \\ \cdot \\ \dot{w}_n \end{bmatrix} \qquad \underline{r} = \begin{bmatrix} r_1 \\ r_2 \\ \cdot \\ \cdot \\ \cdot \\ r_n \end{bmatrix} \qquad (4.2\text{-}2a)$$

Referring to Figure 4-1, let $x_1^{(n)}$ denote the nth time derivative of $x_1(t)$, (the plant output) and $r^{(n)}$ denote nth time derivative of r(t), (the plant input). Then for Eq. (4.2-2), with $\underline{w}$, $\dot{\underline{w}}$, and $\underline{r}$ as given and $G_f(s) = 1$, $w_1 \equiv x_1(t)$, $w_2 \equiv x_1^{(1)}, \ldots, w_n \equiv x_1^{(n-1)}$, $r_1 \equiv r(t)$, $r_2 \equiv r^{(1)}, \ldots,$ and $r_n \equiv r^{(n-1)}$. Note that the superscript in parenthesis denotes the order of the time derivative.

Matrices H and C in Eq. (4.2-2) are square matrices containing the coefficients of Gp(s). These matrices have the form as shown:

$$H = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-2} & -a_{n-1} \end{bmatrix} \qquad (4.2\text{-}3)$$

and

$$C = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ b_0 & b_1 & b_2 & \cdots & b_{n-2} & b_{n-1} \end{bmatrix} \qquad (4.2\text{-}3a)$$

Matrix H is assumed to be stable, i.e., its eigenvalues have negative real parts.

In view of Eq. (4.2-2) one may consider the model shown in Figure 4-1 to have input-output dynamics described by the differential equation of the form

$$\dot{\underline{y}} = \hat{H}(t)\,\underline{y} + \hat{C}(t)\,\underline{r} \qquad (4.2\text{-}4)$$

where $\underline{r}$ is as previously described in Eq. (4.2-2) and $\underline{y}$ is the column vector shown

$$\underline{y} = \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix} \qquad (4.2\text{-}5)$$

where $y(t) = y_1$, $\dot{y}(t) = y_2$, $y^{(2)}(t) = y_3$, . . . , $y^{(n-1)}(t) = y_n$. Note that y(t) denotes the model scalar output and the superscript denotes the order of the time derivative. Matrices $\hat{H}(t)$ and $\hat{C}(t)$ in Eq. (4.2-4) have the same for as matrices H and C given by Eqs. (4.2-3) and (4.2-3a), except that $\hat{H}(t)$ and $\hat{C}(t)$ contain the time-varying parameters $\hat{a}_o(t)$, . . . , $\hat{a}_{n-1}(t)$, $b_o(t)$, . . . , $\hat{b}_{n-1}(t)$ as shown[+]

$$\hat{H}(t) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -\hat{a}_o & -\hat{a}_1 & -\hat{a}_2 & \cdots & -\hat{a}_{n-2} & -\hat{a}_{n-1} \end{bmatrix} \qquad (4.2\text{-}6)$$

$$\hat{C}(t) = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \hat{b}_o & \hat{b}_1 & \hat{b}_2 & \cdots & \hat{b}_{n-2} & \hat{b}_{n-1} \end{bmatrix} \qquad (4.2\text{-}6a)$$

It should be noted that since the model is generated by the system designer, all the state variables of the output vector $\underline{y}$ are available. The vector $\underline{y}$ is shown at the output of the model box in Figure 4-1.

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[+]In order to simplify notation, letter t in parenthesis will be omitted in future notations.

As indicated in Figure 4-1, the $\hat{H}(t)$ and $\hat{C}(t)$ matrices are obtained from the outputs of the box labelled "identification controller". This scheme, as discussed in Chapter 2, is the "learning model" identification system configuration. Note that the controller inputs consist of vectors $\underline{r}$, $\underline{y}$, and $\underline{e}$, where

$$\underline{e} = \underline{y} - \underline{w} \tag{4.2-7}$$

As may be inferred from the discussion in Chapter 2, the basic problem under consideration is to design the controller to provide outputs $\hat{H}(t)$ and $\hat{C}(t)$ that drive $\underline{e}(t)$ to 0, while at the same time driving $\hat{H}(t) \rightarrow H$ and $\hat{C}(t) \rightarrow C$, thus identifying the unknown plant parameters.

To obtain the desired goal as stated above, it becomes necessary to study in detail the dynamics of the $\underline{e}(t)$ vector defined in Eq. (4.2-7). After this is done in Section 4.3, the identification controller, specifying the time variation of the parameters of $\hat{H}(t)$ and $\hat{C}(t)$, is designed in Section 4.4.

4.3 Error Equations

Differentiating with respect to time the error vector given by equation (4.2-7), one obtains

$$\dot{\underline{e}} = \dot{\underline{y}} - \dot{\underline{w}} \tag{4.3-1}$$

Substituting Eqs. (4.2-2) and (4.2-4) into Eq. (4.3-1) yields

$$\dot{\underline{e}} = \hat{H}\underline{y} + \hat{C}\underline{r} - (H\underline{w} + Cr) \tag{4.3-2}$$

Collecting terms gives

$$\dot{\underline{e}} = \hat{H}\underline{y} - H\underline{w} + (\hat{C} - C)\,\underline{r} \tag{4.3-3}$$

Replacing $\underline{w}$ by $(\underline{y} - \underline{e})$ yields

$$\dot{\underline{e}} = \hat{H}\underline{y} - H(\underline{y} - \underline{e}) + (\hat{C} - C)\,\underline{r} \qquad (4.3\text{-}4)$$

Rearranging terms gives

$$\dot{\underline{e}} = H\underline{e} + (\hat{H} - H)\,\underline{y} + (\hat{C} - C)\,\underline{r} \qquad (4.3\text{-}5)$$

In order to simplify the derivation it is convenient to define matrices H' and C' as follows

$$H' = \hat{H} - H \qquad (4.3\text{-}6)$$

$$C' = \hat{C} - C \qquad (4.3\text{-}6a)$$

An examination on H' and C' shows that they represent "mis-alignments" of the model parameters from the unknown plant parameters. Before the use of these perturbational matrices becomes apparent, one may replace the term $\hat{H} - H$ and $\hat{C}$-C in Eq. (4.3-5) by H' and C' in Eqs. (4.3-6) and (4.3-6a). This yields

$$\dot{\underline{e}} = H\underline{e} + H'\underline{y} + C'\underline{r}. \qquad (4.3\text{-}7)$$

In view of Eqs. (4.2-3) and (4.2-6), H' in Eq. (4.3-6) can be decomposed as

$$H' = \underline{b}\,\underline{\mu}'^{T} \qquad (4.3\text{-}8)$$

where

$$\underline{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix} \qquad (4.3\text{-}8a)$$

and

$$\underline{u}' = \begin{bmatrix} a_o' \\ a_1' \\ \cdot \\ \cdot \\ \cdot \\ a_{n-2}' \\ a_{n-1}' \end{bmatrix} \qquad (4.3\text{-}8b)$$

Note that the elements of $\underline{u}'$ are given by

$$\begin{aligned} a_o' &= a_o - \hat{a}_o(t) \\ a_1' &= a_1 - \hat{a}_1(t) \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ a_{n-1}' &= a_{n-1} - \hat{a}_{n-1}(t). \end{aligned} \qquad (4.3\text{-}8c)$$

Note also that the superscript T in Eq. (4.3-8) denotes the transpose. Thus, the a_i', ($i = o, 1, \ldots, n-1$) represent the perturbational or misalignment parameters.

Similarly, using Eqs. (4.2-3a) and (4.2-6a) one may decompose matrix C' as follows

$$C' = -\underline{d}\,\underline{v}'^{T} \qquad (4\text{-}3\text{-}9)$$

where

$$\underline{d} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix} \qquad (4.3\text{-}9a)$$

and

$$\underline{v}' = \begin{bmatrix} b_o' \\ b_1' \\ \cdot \\ \cdot \\ \cdot \\ b_{n-1}' \end{bmatrix} \qquad (4.3\text{-}9b)$$

The elements of vector $\underline{v}'$ are

$$\begin{aligned} b_o' &= b_o - \hat{b}_o(t) \\ b_1' &= b_1 - \hat{b}_1(b) \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ b_{n-1}' &= b_{n-1} - \hat{b}_{n-1}(t) \end{aligned} \qquad (4.3\text{-}9c)$$

The physical meaning for the b_j', $(j = o, \ldots, n-1)$ set is the same as previously discussed for the a_i' parameters.

Using the decompositions given in Eqs. (4.3-8) and (4.3-9) for H' and C' respectively, Eq. (4.3-7) can be expressed as

$$\dot{\underline{e}} = H\,\underline{e} + \underline{b}\,\underline{u}'^{T}y - \underline{d}\,\underline{v}'^{T}\underline{r} \qquad (4.3\text{-}10)$$

Equation (4.3-10) above forms the basis for the controller design as discussed in the next section.

4.4 Basic Controller Design

Let us now assume for the moment that the time-varying model parameters could be varied in such a way that the resultant $\underline{e}(t)$, $\underline{u}'(t)$, and $\underline{v}'(t)$ time perturbations governed by the solution of Eq. (4.3-10) would be asymptotically stable about the equilibrium point $\underline{e} = \underline{u}'$ $\underline{v}' = o$. Then from Eqs. (4.3-8) and (4.3-9), one desires that,

$$H' \rightarrow o \qquad (4.4\text{-}1)$$

$$C' \rightarrow o$$

which from Eq. (4.3-6) results in

$$\hat{H} \rightarrow H$$

$$\hat{C} \rightarrow C \qquad (4.4\text{-}2)$$

These conditions would then identify the plant.

The essential idea behind the design procedure to follow is to use the Second Method of Liapunov discussed previously in Chapter 3 with a candidate Liapunov function selected in terms of the perturbational quantities e, $\underline{u}'$ and $\underline{v}'$. This selection is a natural one, since these quantities when driven to equilibrium force the plant-model output error to zero and the model parameters to the plant parameters. The resulting equations for $\underline{u}'$ and $\underline{v}'$, (which are time functions obtained from the time derivative of the Liapunov function) yield the identification controller design configuration.

Let the candidate Liapunov function be chosen as

$$V = \underline{e}^T P \underline{e} + \underline{u}'^T \Gamma_1 \underline{u}' + \underline{v}'^T \Gamma_2 \underline{v}' \qquad (4.4\text{-}3)$$

where P, Γ_1 and Γ_2 are constant symmetric matrices.

Differentiating Eq. (4.4-3) with respect to time, yields

$$\dot{V} = \dot{\underline{e}}^T P \underline{e} + \underline{e}^T P \dot{\underline{e}} + \dot{\underline{u}}'^T \Gamma_1 \underline{u}' + \underline{u}'^T \Gamma_1 \dot{\underline{u}}' + \dot{\underline{v}}'^T \Gamma_2 \underline{v}' + \underline{v}'^T \Gamma_2 \dot{\underline{v}}' \qquad (4.4\text{-}5)$$

One may define

$$\dot{\underline{u}}' = \boldsymbol{\Omega} \qquad (4.4\text{-}5a)$$

$$\dot{\underline{v}}' = \underline{\boldsymbol{\Phi}}$$

Substituting for $\dot{\underline{e}}$, $\dot{\underline{u}}'$, and $\dot{\underline{v}}'$ from the expressions given by Eqs. (4.3-10) and (4.4-5a), $\dot{V}$ becomes

$$\begin{aligned} \dot{V} = \; & (H\underline{e} + \underline{b}\, \underline{u}'^T \underline{y} - \underline{d}\, \underline{v}'^T \underline{r})^T P \underline{e} \\ + \; & \underline{e}^T P (H\underline{e} + \underline{b}\, \underline{u}'^T \underline{y} - \underline{d}\, \underline{v}'^T \underline{r}) \\ + \; & \boldsymbol{\Omega}^T \Gamma_1 \underline{u}' + \underline{u}' \Gamma_1 \boldsymbol{\Omega} \\ + \; & \underline{\boldsymbol{\Phi}}^T \Gamma_2 \underline{v}' + \underline{v}'^T \Gamma_2 \underline{\boldsymbol{\Phi}} \end{aligned} \qquad (4.4\text{-}6)$$

Since $\underline{b}^T P \underline{e}$ and $\underline{d}^T P \underline{e}$ are scalar quantities, one may rearrange $\dot{V}$ to yield

$$\dot{V} = \underline{e}^T [P H + H^T P] \underline{e} + 2 \underline{u}' [\boldsymbol{\Omega}^T \Gamma_1 + y^T (b^T P e)] + 2 \underline{v}' [\boldsymbol{\Phi}^T \Gamma_2 - \underline{r}^T (\underline{d}^T P \underline{e})] \qquad (4.4\text{-}7)$$

From Eq. (4.4-7) it can be seen that if one selects $\underline{\boldsymbol{\Omega}}$ and $\underline{\boldsymbol{\Phi}}$ such that

$$\boldsymbol{\Omega}^T \Gamma_1 = - \underline{y}^T (\underline{b}^T P \underline{e}) \qquad (4.4\text{-}8)$$

$$\boldsymbol{\Phi}^T \Gamma_2 = \underline{r}^T (\underline{d}^T P \underline{e}) \qquad (4.4\text{-}8a)$$

and assuming that Γ_1 and Γ_2 are invertible, then Eqs. (4.4-8) can be rearranged to give

$$\underline{\Omega} = \quad - \Gamma_1^{-1} \; \underline{y}(\underline{b}^T P \underline{e}) = \dot{\underline{u}}' \qquad (4.4\text{-}9)$$

$$\underline{\Phi} = \quad + \Gamma_2^{-1} \; \underline{r}(\underline{d}^T P \underline{e}) = \dot{\underline{v}}' \qquad (4.4\text{-}9a)$$

Defining

$$- Q = P H + H^T P \qquad (4.4\text{-}10)$$

one obtains from Eq. (4.4-7)

$$- \dot{V} = \underline{e}^T Q \underline{e} \qquad (4.4\text{-}11)$$

Note that the resulting $\dot{V}$ is negative semidefinite since it is a function of $\underline{e}$ alone.

Let us for the moment assume we can find a positive definite pair of matrices Q and P such that Eq. (4.4-10) is satisfied. (Note that if H were known and had eigenvalues with negative real parts, any positive definite Q would yield a positive definite P by using a theorem given in Reference [16].) Then if matrices Γ_1 and Γ_2 are selected to be also positive definite, V in Eq. (4.4-3) is seen to be positive definite in $\underline{e}$, $\underline{u}$, and $\underline{v}$. Selection of $\underline{\Omega}$ and $\underline{\Phi}$ as in Eqs. (4.4-9) yields $-\dot{V}$ to be positive definite in $\underline{e}$ and positive semi definite in $\underline{e}$, $\underline{u}$, and $\underline{v}$. Asymptotic stability then follows from the theorems in Chapter 3 after additional assumptions are added to the plant input to compensate for the fact that $-\dot{V}$ is only semi definite in $\underline{e}$, $\underline{u}$ and $\underline{v}$. This topic is discussed in Chapter 5, Section 5.4.

One must now relate the controller dynamics given by Eqs. (4.4-9) and (4.4-9a) to the design of the model parameter matrices $\hat{H}(t)$ and $\hat{C}(t)$. To do this one makes use of Eqs. (4.3-6) and (4.3-6a) from which it follows (since $\dot{H} = \dot{C} = o$) that

$$\dot{H}' = \dot{\hat{H}} \qquad (4.4\text{-}12)$$

$$\dot{C}' = \dot{\hat{C}} \qquad (4.4\text{-}12a)$$

Note that the plant parameters are assumed constant or slowly varying with respect to the time required for identification.

Differentiating Eqs. (4.3-8) and (4.3-9) gives

$$\dot{\hat{H}} = \underline{b}\, \dot{\underline{u}}'^{T} \qquad (4.4\text{-}13)$$

$$\dot{\hat{C}} = -\underline{d}\, \dot{\underline{v}}'^{T} \qquad (4.4\text{-}13a)$$

and from Eqs. (4.4-1a), (4.4-1b), (4.4-9) and (4.4-9a) one obtains

$$\dot{\hat{H}} = \underline{b} \left[-\Gamma_1^{-1} \ \underline{y}\, (\underline{b}^T P\, \underline{e}) \right] \qquad (4.4\text{-}14a)$$

$$\dot{\hat{C}} = -\underline{d} \left[+ \Gamma_2^{-1} \ \underline{x}\, (\underline{d}^T P\, \underline{e}) \right] \qquad (4.4\text{-}14b)$$

Equations (4.4-14a) and (4.4-14b) are integrated with respect to time to form the model matrices $\hat{H}$ and $\hat{C}$.

The plant, model, and identification controller developed in this chapter is summarized in Figure 4-2. The controller design as shown by Eqs. (4.4-14a) and (4.4-14b) is subject to the specification of Γ_1, Γ_2 and P. The controller initial conditions, $\hat{H}(o)$ and $\hat{C}(o)$ are selected as discussed in Chapter 5.

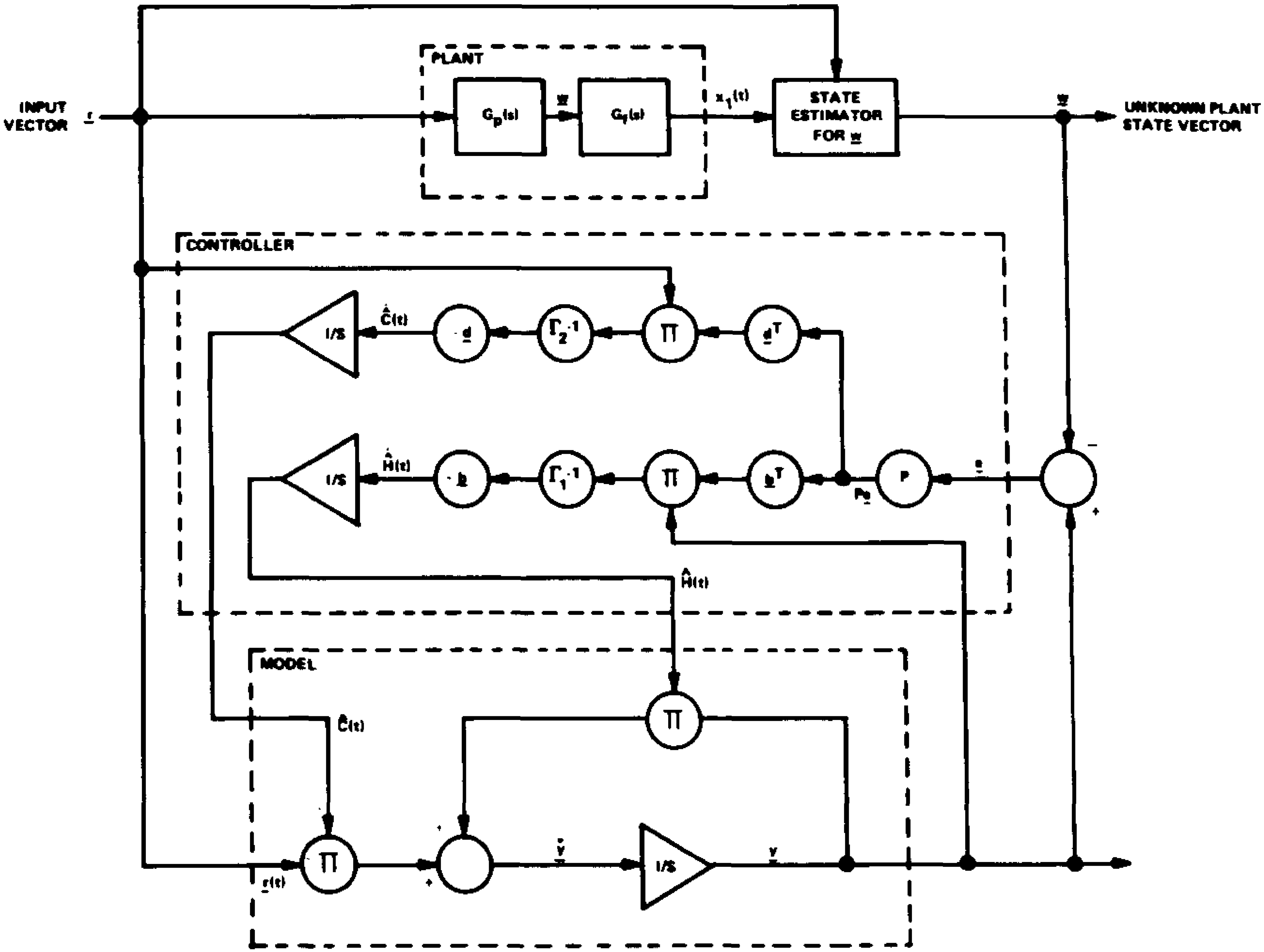

Figure 4-2. Plant, Model, and Controller Design Configuration

CHAPTER 5

SELECTION OF CONTROLLER PARAMETERS AND COMPLETION OF STABILITY ANALYSIS

5.1 Introduction

In this chapter, a technique is developed for designing the P and Γ parameter matrices needed to completely specify the identification controller. This technique is based upon a Liapunov function that establishes stability of the composite plant, model, and identification controller system, with an arbitrary set of initial conditions and for a computed range of unknown plant parameter values. It is shown that the use of the negative derivative of the Liapunov function as an index of performance allows the designer to control the shape of the transient error time function. To achieve a desired type of convergence, this index of performance can be minimized by selection of a design parameter matrix. The convergence of the plant-model error vector to its equilibrium state is examined and sufficient conditions are presented that imply convergence of the model parameters to the true plant parameters. These conditions are shown to be obtainable directly from the unknown plant output.

5.2 Selection of the P and Q Matrices

5.2.1 Selection of Q Matrix

In order to discuss the selection of the P matrix, one must first consider a Q matrix and the Liapunov function given by Eq. (4.4-3)

$$V = \underline{e}^T P \underline{e} + \underline{u}'^T \Gamma_1 \underline{u}' + \underline{v}' \Gamma_2 \underline{v}' \qquad (4.4\text{-}3)$$

It was shown in Section 4.4 that the selection of the identification controller dynamics indicated in Eqs. (4.4-14a) and (4.4-14b) led to the time derivative of V in Eq. (4.4-3) to be

$$-\dot{V} = \underline{e}^T Q \underline{e} \qquad (4.4\text{-}11)$$

where

$$-Q = P H + H^T P \qquad (4.4\text{-}10)$$

If the plant were known, that is, if the $[a_i]$ set of parameters in H (Eq. 4.2-3) had known numerical values, one could select Q to be any positive definite nxn matrix and solve for P. If H is a stability matrix, P would also be guaranteed to be positive definite. Thus if Γ_1 and Γ_2 were selected to be positive definite, V in Eq. (4.4-3) would be positive definite.+

However, for the identification problem, H is not known, and the problem of obtaining a positive definite P matrix requires the development of the design technique to follow.

In order to find matrix P, which is needed for the controller design, a matrix Q is selected. There are many ways one may select Q. In this development, matrix Q is selected to have the diagonal form

$$Q = \begin{bmatrix} q_{11} & & \bigcirc \\ & q_{22} & \\ & & \ddots & \\ \bigcirc & & & q_{nn} \end{bmatrix} \qquad (5.2\text{-}1)$$

+The solution of the Liapunov - Matrix Equation, Eq. (4.4-10), when H is a known constant matrix, and $Q > o$ is selected, is a topic of continued interest. Straightforward solutions for large n are readily available, as in Ziedon [8].

This selection has the effect of reducing the non zero number of elements of Q from n(n+1)/2 to n. .

A complete design procedure that results in a positive definite P matrix is presented in the remaining subsections of this section.

5.2.2 Selection of the Form of the $\tilde{Q}$ Matrix

A nominal matrix which has the same form as Q in Eq. (5.2-1) is selected and called $\tilde{Q}$, where

$$\tilde{Q} = \begin{bmatrix} \tilde{q}_{11} & & & \bigcirc \\ & \tilde{q}_{22} & & \\ & & \ddots & \\ \bigcirc & & & \tilde{q}_{nn} \end{bmatrix} \tag{5.2-2}$$

The elements of $\tilde{Q}$ are denoted by $\tilde{q}_{ii}$ $(i = 1, \ldots, n)$ and $\tilde{Q}$ is chosen positive definite, i.e.,

$$\tilde{q}_{ii} > 0 \quad \text{for} \quad i = 1, 2, \ldots, n \tag{5.2-2}$$

5.2.3 Selection of Nominal $\tilde{H}$ and $\tilde{C}$ Matrices

Nominal or best "a priori" assumed values for the unknown elements of the H matrix are selected. The selected $\{a_i\}$ parameters in Eq. (4.2-6) are denoted by $[\tilde{a}_i]$ and the H matrix corresponding to these parameters is denoted by $\tilde{H}$. These nominal plant parameter values can, for example, be obtained from an approximate knowledge of upper and lower bounds of the unknown plant parameters. One procedure would be to take nominal values corresponding to arithmetic or geometric means of the upper and lower bounds. Another source for their selection in a practical example might be previous research or experimentation conducted on the plant or similar plants under investigation. In almost all practical cases, the physics of the plant's operation limits the possible range of the plant parameter values. At present, all known identification algorithms require an initial set of plant parameter values. Those that do not employ a Liapunov function design approach cannot guarantee convergence for a stated variation of the true parameters about the initial set. Many identification algorithms lead to convergence which is guaranteed only for an arbitrarily small variation of the true plant values about the nominal or initial starting parameters.

The Liapunov function design technique presented in this Chapter however always yields a finite variational set of the true plant parameters about the initial starting set for which convergence is assured.

The range of this variational set however is expected to be smaller than the Routh Hurwitz stability criteria signify for a stable plant. This decrease in range is due to the fact that the Liapunov function design presented in this chapter yields conditions for the stability of the complete dynamical system shown in Figure 4-1. The Routh-Hurwitz criteria however only yields conditions for the input-output stability of Gp(s).

Before continuing the design procedure for P note that the nominal $\tilde{H}$ matrix is used as the initial condition matrix for $\hat{H}$ (at t = o). Thus let

$$\hat{H}(o) = \tilde{H} \qquad (5.2\text{-}3)$$

Similarly, a nominal matrix $\tilde{C}$ is used as the initial condition matrix for $\hat{C}(t)$ (at t = o). Thus let

$$\hat{C}(o) = \tilde{C} \qquad (5.2\text{-}3a)$$

Since this $\tilde{C}$ matrix is not involved in any stability criterion+, its selection can be quite arbitrary. Of course, the farther the a priori elements of $\tilde{H}$ and $\tilde{C}$ are from the true elements of the H and C matrices, the greater the time required for convergence.

5.2.4 Solution of Matrix Liapunov Equation

Substituting the selected $\tilde{Q}$ and $\tilde{H}$ matrices into Eq. (4.4-10) with Q and H replaced by $\tilde{Q}$ and $\tilde{H}$ respectively, one obtains

$$-\tilde{Q} = P\tilde{H} + \tilde{H}^T P \tag{5.2-4}$$

The matrix P which is the solution to Eq. (5.2-4) is guaranteed to be positive definite since $\tilde{Q}$ is positive definite and $\tilde{H}$ has eigenvalues with negative real parts.

At first glance it may appear that the complete symmetric P matrix with its n(n+1)/2 elements would be needed for computing $\dot{\hat{H}}(t)$ and $\dot{\hat{C}}(t)$ before these quantities are integrated. However, when Equations (4.4-14a) and (4.4-14b) are examined in detail with respect to Equations (4.3-8a) and (4.3-9a), it is seen that only n elements of the P matrix are needed for the controller. Specifically, if P has the form $P = \{p_{ij}\}$ as shown

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdot & \cdot & \cdot & p_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ p_{1n} & p_{2n} & \cdot & \cdot & \cdot & p_{nn} \end{bmatrix} \tag{5.2-5}$$

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+This will be illustrated in Sections 5.2-4 and 5.2-5. The selected $\tilde{H}$ matrix must have eigenvalues with negative real parts.

Then ($\underline{b}^T P \underline{e}$) and/or $\underline{d}^T P \underline{e}$ have the form

$$(\underline{b}^T P \underline{e}) = (\underline{d}^T P \underline{e}) = p_{1n} e_1 + p_{2n} e_2 + \cdot \cdot \cdot + p_{nn} e_n \qquad (5.2\text{-}6)$$

Thus $n(n-1)/2$ elements of P do not have to be obtained numerically since they are not required for the identification controller.[+] This important fact allows the modification presented in the next subsection to be implemented.

5.2-5 Constraints On the Liapunov Function And Its Derivative

At this point in the design, Eq. (5.2-4) is solved numerically for only the n elements of P needed for the controller, as indicated in Eq. (5.2-6). Then the remaining $n(n-1)/2$ P elements are selected such that the off-diagonal elements of Q in Eq. (4.4-10) are zero. Thus when the n numerical P controller parameters together with the $n(n-1)/2$ "non-controller" elements of P are substituted in Eq. (4.4-10), Q will have the diagonal form of Eq. (5.2-2).

This Q matrix has its elements as functions of the $\{a_i\}$ plant parameters and the numerical P element set. At this point the range of the $\{a_i\}$ unknown parameters that guarantees Q to be positive definite can be determined. This condition will insure that the quadratic form for $\dot{V}$ in Eq. (4.4-11) will be negative semi-definite. Additional conditions[‡] on the $\{a_i\}$ unknown plant parameters can insure P to be positive definite. If Γ_1 and Γ_2 are selected to

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[+] These elements of P are functions of the true plant matrix H and the design matrix $\tilde{Q}$.

[‡] The example in Section 7.2 illustrates how these additional conditions are obtained.

be positive definite, a positive definite P insures V to be positive definite. It should be repeated at this point that n(n-1)/2 elements of P are in terms of the $\{a_i\}$ unknown plant parameter set.

Thus, to guarantee stability of the equilibrium state, one has to find the intersection of two ranges of $\{a_i\}$ parameter sets such that $Q > 0$ and $P > 0$. This will guarantee $V > 0$ and $-\dot{V} > 0$ for this final set.

Before discussing the selection of the $\tilde{Q}$ matrix and the Γ_1 and Γ_2 matrices, it is appropriate to discuss the case when the true plant parameters do not lie in the region guaranteeing P and Q to be positive definite. For this case convergence of the model parameters to the true plant parameters may still occur since the conditions using Liapunov functions are sufficient but not necessary. Thus the true parameter region for asymptotic stability will lie between the determined region and the region obtained using the Routh-Hurwitz criteria.[+]

If convergence does not occur, a different set of $\tilde{H}$ parameters must be selected and the design procedure repeated. Since the parameter region which insures stability will be finite about the nominal $\tilde{H}$ values, repeated application of this procedure would cover all possible bounded sets of the $\{a_i\}$ true parameters of the H matrix.

5.3 Selection of Γ Matrices and $\tilde{Q}$ Matrix

Let V in Eq. (4.4-3)

$$V = \underline{e}^T P \underline{e} + \underline{u}'^T \Gamma_1 \underline{u}' + \underline{v}'^T \Gamma_2 \underline{v}' \qquad (4.4\text{-}3)$$

be made positive definite by choosing Γ_1 and Γ_2 to be positive definite matrices. In addition, let us restrict Γ_1 and Γ_2 to be

+ Only stable unknown plants are considered in this dissertation.

diagonal matrices with positive coefficients. Thus let

$$\Gamma_1 = \begin{bmatrix} \gamma_{11} & & & \bigcirc \\ & \gamma_{12} & & \\ & & \ddots & \\ \bigcirc & & & \gamma_{1n} \end{bmatrix} \tag{5.3-1}$$

$$\Gamma_2 = \begin{bmatrix} \gamma_{21} & & & \bigcirc \\ & \gamma_{22} & & \\ & & \ddots & \\ \bigcirc & & & \gamma_{2n} \end{bmatrix} \tag{5.3-1a}$$

where

$$\gamma_{11} > o, \quad \gamma_{12} > o, \ . \ . \ . \ , \quad \gamma_{1n} > o$$

$$\gamma_{21} > o, \quad \gamma_{22} > o, \ . \ . \ . \ , \quad \gamma_{2n} > o \tag{5.3-1b}$$

With Γ_1 and Γ_2 so defined, let an index of performance for the composite system be defined in terms of the error state vector $\underline{e}$, as

$$\text{I. P.} = \int_o^\infty (\underline{e}^T Q \, \underline{e}) \, d \, t \tag{5.3-2}$$

where Q was given previously in Eq. (5.2-1).+

Chapter 7 of Reference [1] shows that when V is a Liapùnov function and

$$\text{I. P.} = \int_o^\infty - \dot{V} \, d \, t \tag{5.3-2a}$$

then the index of performance equals V at $t = o$, i.e.,

$$\text{I.P.} \quad V \, (t = 0) \tag{5.3-2b}$$

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\+ The relationship of non-diagonal Q matrices in indices of performance such as Eq. (5.3-2) is discussed in Chapter 8 of Schulz and Melsa [1].

Thus for the case under consideration, using Eq. (4.4-3), one has

$$\text{I. P.} = \underline{e}^T(o)\, P\, \underline{e}(o) + \underline{u}'^T(o)\, \Gamma_1\, \underline{u}'(o) + \underline{v}'^T(o)\, \Gamma_2\, \underline{v}'(o) = \int_{\infty}^{\infty} \underline{e}^T Q\, \underline{e}\, dt \qquad (5.3\text{-}2c)$$

Since $\underline{e}(o)$, $\underline{u}'(o)$, $\underline{v}'(o)$, and P are fixed in Eq. (5.2-2c), the γ_{ij} elements of matrices Γ_1 and Γ_2 should be selected as small as possible if one wishes to minimize I.P.

While the I. P. in Eq. (5.3-2c) is the one actually to be minimized, the fact that Q is never known numerically forces the designer to select the nominal index of performance $\widetilde{\text{I.P.}}$

$$\widetilde{\text{I.P.}} = \int_0^{\infty} (\underline{e}^T \tilde{Q}\, \underline{e})\, dt \qquad (5.3\text{-}3)$$

Thus Eq. (5.3-3) indicates how the selection of the diagonal $\tilde{Q}$ is chosen. Given an $\tilde{H}$ matrix close to H, substitution of the P matrix obtained by the solution of Eq. (5.2-4) into Eq. (4.4-10) results in Q approximately equal to $\tilde{Q}$. This method of initiating the design procedure of Section 5.2 is justified under the assumption that Q does not differ substantially from $\tilde{Q}$.

However, as shown in the illustrative example in Chapter 7, the designer can actually compare Q with $\tilde{Q}$ by computing Q over the specified admissible $\{a_i\}$ parameter range. Thus whether this procedure will force $\underline{e}(t)$ to behave as the designer wishes will depend on the sensitivity of Q to the variation of H about $\tilde{H}$. For the example in Chapter 7 this sensitivity was relatively small and the selection of $\tilde{Q}$ forces $\underline{e}(t)$ to behave as predicted for small γ_{ij} parameters. Stability is guaranteed however in all cases for any selected $\tilde{Q}$, subject to the previous discussed true plant parameter variation constraints. To prove asymptotic stability for the designed composite system, the results of the following section are needed.

5.4 Sufficient Conditions for Asymptotic Stability

In order to prove asymptotic stability of the composite system consisting of the plant, model, and controller, one must prove that the condition $\underline{e}(t) = o$ implies that

$$\hat{H}(t) = H(t) \qquad (5.4\text{-}1)$$

and

$$\hat{C}(t) = C(t) \qquad 5.4\text{-}1a)$$

This indicates that the adjustment of the model parameters that forces the systems error vector to zero leads to final model parameters that equal the plant parameters.

For the positive definite Liapunov function V in Eq. (4.4-3), its negative derivative $-\dot{V}$ as seen from Eq. (4.4-11) is positive definite in the perturbational $\underline{e}$ state variables. Equation (4.4-11) also indicates that $-\dot{V}$ is only positive semidefinite in the $\underline{e}$, $\underline{u}'$, and $\underline{v}'$ state variables, since $-\dot{V}$ can be zero when $\underline{e} = o$ but $\underline{v}'$ and $\underline{u}'$ are not zero. However reference to theorem 3.4 (Chapter 3) indicates that this condition on $-\dot{V}$ and V is sufficient for the asymptotic stability of the equilibrium state $\underline{e} = \underline{u}' = \underline{v}' = o$ only if there are no other solutions for the dynamic equations that satisfy both $-\dot{V} = o$ in Eq. (4.4-11) and Eqs. (4.3-10) and (4.4-5a). Equations (4.3-10) and (4.4-5a) are rewritten as follows

$$\dot{e} = H\underline{e} + \underline{b}\,\underline{u}'^T\underline{y} - \underline{d}\,\underline{v}'^T\underline{r} \qquad (5.4\text{-}3)$$

$$\dot{\underline{u}}' = \underline{\Omega} \qquad (5.4\text{-}3a)$$

$$\dot{\underline{v}}' = \underline{\Phi} \qquad (5.4\text{-}3b)$$

From Eqs. (4.4-14a), (4.4-14b), (4.4-13) and (4.4-13a)

$$\underline{e} = \underline{o} \longrightarrow \dot{\underline{u}}' = \dot{\underline{v}}' = \underline{o} \quad \text{or}$$

$$\underline{u}' = \text{constant when } \underline{e} = o \qquad (5.4\text{-}4)$$

$$\underline{v}' = \text{constant when } \underline{e} = o \qquad (5.4\text{-}4a)$$

The question is then whether or not the constants in the above equations are zero. If they are zero, then

$$\underline{u}' = o \longrightarrow \hat{H} = H \qquad (5.4\text{-}5)$$

and

$$\underline{v}' = o \longrightarrow \hat{C} = C \qquad (5.4\text{-}5a)$$

(where $\longrightarrow$ means implies)

and thus the plant is identified by the model. It should be noted that this requirement has been mentioned in many of the references concerning the design of model reference control systems. For the identification problem in this dissertation there is no possible way known to make $-\dot{V}$ definite in $\underline{e}$, $\underline{v}'$ and $\underline{u}'$, since $\underline{u}'$ and $\underline{v}'$ are never measurable. An identification technique however was derived to enable the designer to insure that the values of $\hat{H}$ and $\hat{C}$ when $\underline{e} = o$ are the true values. This method also helps select a suitable r(t) to insure asymptotic stability of the composite system.[‡] In Chapter 7, a procedure is applied quantitatively

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†If $\underline{u}' = \text{constant} \neq o$ and $\underline{v}' = \text{constant} \neq o$, then the model parameters do not equal the plant parameters and the plant is not identified.

‡Situations may arise where this choice is a designer option.

to design a "suitable" input for the identification of the plant selected.

To determine necessary and sufficient conditions for $\left\{\begin{array}{l}\underline{e} = o \\ \underline{u}' = o \\ \underline{v}' = o\end{array}\right\}$ to be the only equilibrium state, let us set $\underline{e} = o$ into Equations (5.4-3),(5.4-3a), and (5.4-3b). In view of Eqs. (4.4-9) and (4.4-9a) one obtains

$$\underline{o} = o + \underline{b}\, \underline{u}'^T \underline{y} - \underline{d}\, \underline{v}'^T \underline{r} \qquad (5.4\text{-}6)$$

$$\dot{\underline{u}}' = o \qquad (5.4\text{-}6a)$$

$$\dot{\underline{v}}' = o \qquad (5.4\text{-}6b)$$

When $e = o$, then $\underline{y} = \underline{w}$ and $\underline{u}' = \underline{U}'$ (a constant), and $\underline{v}' = \underline{V}'$ (a constant). Thus we need only consider Eqs. (5.4-6), (5.4-6a), and (5.4-6b) which give

$$o = \underline{b}\, \underline{U}'^T \underline{w} - \underline{d}\, \underline{V}'^T \underline{r} \qquad (5.4\text{-}7)$$

In order to illustrate the relationships indicated by Eq. (5.4-7), one rewrites Eq. (5.4-7) with the time varying quantities as

$$o = \underline{b}\, \underline{U}'^T \underline{w}(t) - \underline{d}\, \underline{V}'^T \underline{r}(t) \qquad (5.4\text{-}7a)$$

Since, $\underline{b}$, $\underline{d}$, $\underline{u}'$ and $\underline{v}'$ are as shown in Eqs. (4.3-8a), (4.3-9a), (4.3-8b), and (4.3-9b), then Eq. (5.4-7a) reduces to

$$\begin{aligned} o = {} & (a_o - \bar{\hat{a}}_o)\, w_1(t) + (a_1 - \bar{\hat{a}}_1)\, w_2(t) + \dots + \\ & (a_{n-1} - \bar{\hat{a}}_{n-1})\, w_n \\ & - [(b_o - \bar{\hat{b}}_o)\, r_1(t) + \dots + (b_{n-1} - \bar{\hat{b}}_{n-1})\, r_n(t)] \end{aligned} \qquad (5.4\text{-}8)$$

where the $\bar{\hat{a}}_i$ represent $\hat{a}_i(t)$ when $\underline{e} = o$ and $\dot{\hat{a}}_i = o$, and the $\bar{\hat{b}}_j$ represent $\hat{b}_j(t)$ when $\underline{e} = o$ and $\dot{\hat{b}}_j = o$. Note that the dot denotes the time derivative.

Thus one has to show that

$$\begin{array}{ccc} a_o & = & \bar{\hat{a}}_o \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ a_{n-1} & = & \bar{\hat{a}}_{n-1} \end{array} \qquad (5.4\text{-}9)$$

and

$$\begin{array}{ccc} b_o & = & \bar{\hat{b}}_o \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ b_{n-1} & = & \bar{\hat{b}}_{n-} \end{array} \qquad (5.4\text{-}9a)$$

can be solutions of Eq. (5.4-8). (Or more generally, $\underline{u}' = o$ and $\underline{v}' = o$ solve Eq. (5.4-7a) uniquely).

From the theory of simultaneous equations,[+] one can develop conditions directly from Eq. (5.4-8) to guarantee this. Since Eq. (5.4-8) holds for all time when $\underline{e} = o$, it holds in particular for any 2n distinct time instances, $t_1, t_2, \ldots t_n$. Writing Eq. (5.4-8) at these time instances would yield 2n equations with the 2n unknowns $(a_o - \bar{\hat{a}}_o), \ldots, (a_{n-1} - \bar{\hat{a}}_{n-1}), (b_o - \bar{\hat{b}}_o), \ldots, (b_{n-1} - \bar{\hat{b}}_{n-1})$. The assumption that $\underline{y}(t)$ and $\underline{r}(t)$ are known quantities then forces Eqs. (5.4-9) and (5.4-9a) to be the unique solution to the 2n equations if and only if the matrix S in Eq. (5.4-10) is nonsingular.[+]

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[+]See Section 12 of Chapter 4 in Sokolnikoff and Redheffer [61].

$$S = \left[\begin{array}{cccc|ccc} w_1(t_1) & w_2(t_1) & \cdots & w_n(t_1) & -r_1(t_1) & \cdots & -r_n(t_1) \\ w_1(t_2) & w_2(t_2) & \cdots & w_n(t_2) & -r_1(t_2) & \cdots & -r_n(t_2) \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ w_1(t_n) & w_2(t_n) & \cdots & w_n(t_n) & -r_1(t_n) & \cdots & -r_n(t_n) \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ w_1(t_{2n}) & w_2(t_{2n}) & \cdots & w_n(t_{2n}) & -r_1(t_{2n}) & \cdots & -r_n(t_{2n}) \end{array}\right] \qquad (5.4\text{-}10)$$

The singularity of this 2n by 2n matrix can easily be determined since $\underline{w}(t)$ and $\underline{r}(t)$ are always available and known. This singularity condition is used in Chapter 7 to design a suitable plant input.

CHAPTER 6

THE OBSERVER AS A PLANT STATE ESTIMATOR

6.1 Introduction

As evidenced from the controller equations in Section 4.4, the complete state vector $\underline{w}$ of the unknown plant must be available in order to obtain the error vector $\underline{e}$. If $G_f(s)$ equals unity for the system shown in Fig. 4-1, then the plants scalar output $w(t)$ equals $x_1(t)$ and is directly measurable. The complete state vector $\underline{w}$ could be obtained by differentiating $w(t)$ (n-1) times when Gp(s) is an arbitrary n^{th} order unknown plant. In many practical cases, $G_f(s)$ is not unity but of m^{th} order, with Gp(s) of $(n-m)^{th}$ order, therefore only n-m differentiations would be required of $x_1(t)$. Differentiation, however, is an operation that control engineers have found to be impractical in most situations. The basic reason for not differentiating is that even low measurement noise levels can lead to significant errors in estimating first and/or higher order derivatives.

As an alternative to differentiation, state variable estimators called observers, or "Luenberger observers", have recently appeared in the control theory literature [3, 4, 5, 6, 7]. These observers for linear time invariant plants are themselves dynamic. The observer inputs are the plant's input and the available plant output(s). The observer output is an estimate of the unavailable plant state vector. For example, in Fig. 4-1, the observer inputs are $x_1(t)$ and $r(t)$ and its output is $\underline{\hat{w}}(t)$. Note that the dynamics of $G_f(s)$ have been assumed to be known. The fact that for conventional observer design Gp(s) must also be known creates the difficult problem discussed in Chapter 1. The use of a "modified" observer, as shown in Fig. 6-1, to generate the unknown plant's state vector is discussed qualitatively in this chapter and quantitatively illustrated in Chapter 8.

6.2 Design Philosophy

The use of the observer with its own dynamics immediately dissuades the design engineer from designing the complete configuration shown in Fig. 6-1 in one analytical operation. At present, the complexity of the resulting "modified" observer and identification controller dynamics led to the following simplified design procedure:

a) Design the identification controller structure discussed in Chapters 4 and 5 based upon the assumption that the needed plant state vector is available. Then substitute the observer state vector estimate in the designed control law in place of the assumed true state vector.

b) Design the observer using conventional design procedures, leaving the required unknown plant parameters as unknown observer design parameters.

Techniques that could possibly be used to design this modified observer are discussed in the next section.

6.3 Observer Configuration

Figure 6-1 suggests that two possible techniques can be formulated to design the modified observer. One technique is to use the best a priori values of the unknown plant parameters that are available.

The observer state vector estimate of the plant unknown states then will always be in error. While the identification might still be adequate, there are numerous cases where this technique probably would cause considerable error. One case for example is when the plant contains an unknown zero and the plant's input is unbounded over the identification time interval. The error in the observer's state estimate, designed using an a priori value for the plant zero, would then grow without bound.

Because of the obvious limitation of this technique it therefore seems logical to consider a second technique which can improve (update) in time the values of $\hat{H}$ and $\hat{C}$ fed into the modified observer[+], (see Fig. 6-1). This second technique proposes using a dynamic observer controller which will continuously provide values of H and C for use by the modified observer. As shown in Fig. 6-1, the inputs to this observer controller are the unknown plant's input vector $\underline{r}$, the modified observer's output vector $\underline{\tilde{W}}$, and the error vector $\underline{\tilde{e}}$. This observer controller is designed similar to the model identification controller. However, in the preliminary design phase the state vector $\underline{y}$ is assumed equal to the unknown state vector $\underline{w}$. When this observer controller is implemented, the model's state vector $\underline{y}$ is substituted for the unknown plant's state vector $\underline{w}$. If the model's state vector $\underline{y}$ is sufficiently close to $\underline{w}$, then the observer identification controller can improve its estimate $\underline{\tilde{w}}$ of $\underline{w}$. A better estimate of

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+ Matrices $\hat{H}$ and $\hat{C}$ are modified observer estimates of the unknown plant matrices H and C respectively.

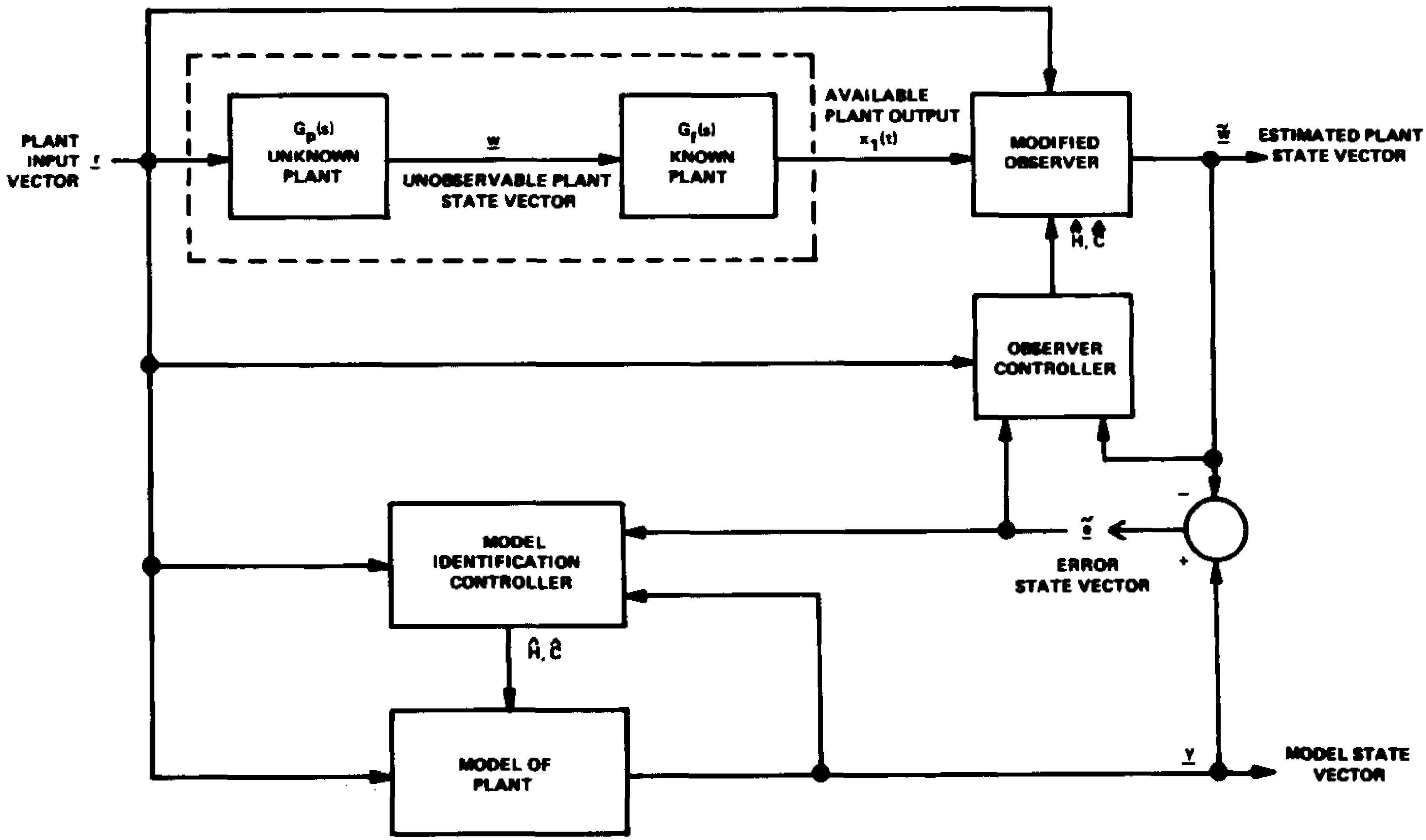

Figure 6-1. Observer - Identification Controller Configuration

$\tilde{\underline{w}}$ will then improve the error $\tilde{\underline{e}}$ which will then improve the model's estimate of the unknown plant. A better model of the unknown plant will further bring the model's state vector $\underline{y}$ closer to the plant's state vector $\underline{w}$.

As will be illustrated in detail in Chapter 8, before the modified observer design procedure can be completed, many important considerations must be resolved. One key consideration is the problem of how to select the design parameters of the observer that remain after the procedure of Appendix A is followed. [+] This problem is resolved by use of a Liapunov function to obtain constraints on these free parameters which guarantee that the observer is designed correctly. By a correctly designed observer it is meant that if the plant were known, the observed states would asymptotically converge to the true plant states.

The second key consideration is whether $\tilde{\underline{e}}$ in Fig. 6-1 going to zero implies that $\underline{e}$ (equal to $\underline{y}$ - $\underline{w}$) goes to zero and what possible constraints are required to force this condition. An intimately related third consideration is whether or not $\tilde{\underline{e}} = 0$ can be forced to be the only error portion of the equilibrium state for the total composite system in Fig. 6-1.

The above three considerations are fully discussed in detail in Chapter 8, Section 8.3, where an illustrative example is given and quantitative results discussed.

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[+]The W_{21} sub-matrix in Eq. (A-10) is not specified by the procedure given in Appendix A. The elements of this matrix are designated observer design parameters.

The fourth major design problem concerns the procedure at the point in the final design stage for the observer where the plant parameters must be known. For the illustrative example in Chapter 8, additional observer dynamics are added that estimate the required plant parameters. Both the design technique and the final observer dynamics appear dual to the identification controller design and dynamics. The results of computer simulations using the composite system are then presented.

An outline of the observer design procedure when the plant is known appears in the next section. This outline serves as an introduction to the design of the complete system shown in Fig. 6-1.

6.4 An Introduction To The Design Of An Observer

With reference to Fig. 6-1, let us for simplicity[+] consider the case where

$$G_f(s) = 1 \tag{6.4-1}$$

and

$$x_1(t) = w(t) = w_1(t) \tag{6.4-2}$$

Assuming that the plant's dynamics are represented by Eq. (4.2-2), one may add the additional transformation

$$\underline{x}_1 = \underline{D}\ \underline{w} \tag{6.4-3}$$

where $\underline{x}_1$ is a scalar and $\underline{D}$ is a 1 x n row vector defined as

$$\underline{D} = [\, 1 \mid 0 \mid 0 \ \ldots \ 0 \mid 0 \,] \tag{6.4-4}$$

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[+]The general procedure when $G_f(s) \neq 1$ is presented in Appendix A.

Thus, from Eqs. (6.4-3) and (6.4-4),

$$x_1(t) = w_1(t) \qquad (6.4\text{-}5)$$

From Appendix A, one observes that the estimated plant state vector $\tilde{\underline{w}}$ will be of the form

$$\tilde{\underline{w}} = W \begin{bmatrix} \underline{x}_1 \\ \underline{z} \end{bmatrix} \qquad (6.4\text{-}6)$$

where W is an nxn matrix and $\underline{z}$ an (n-1) vector. This $\underline{z}$ vector is the solution of the (n-1) dimension linear dynamic system

$$\dot{\underline{z}} = F\,\underline{z} + G\,\underline{x}_1 + R\,\underline{r} \qquad (6.4\text{-}7)$$

where F is an (n-1) by (n-1) matrix, G an (n-1) by 1 matrix and R an (n-1) by n matrix.

The design of the matrices W , F, R, and the matrix G + will complete the requirements for obtaining $\tilde{\underline{w}}$, since $\underline{z}(t)$ can be found from the solution of Eq. (6.4-7) and $\tilde{\underline{w}}$ from Eq. (6.4-6). Note that $x_1(t)$ is directly available.

If we assume the plant to be known, as discussed in Appendix A, the vector $\tilde{\underline{w}}(t)$ will be asymptotically convergent to $\underline{w}(t)$, i.e.,

$$\lim_{t \to \infty} \left[\underline{w}(t) - \tilde{\underline{w}}(t)\right] = 0 \qquad (6.4\text{-}8)$$

Note that the time required for convergence depends upon all the values of H, C, W, F, R, and G.

The above introduction to the design of an observer gives the reader a preview of the material presented in Chapter 8.

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+See Appendix A and Chapter 8 for details on how these quantities are designed.

CHAPTER 7

ILLUSTRATIVE CASE WITH PLANT STATES DIRECTLY AVAILABLE

7.1 Introduction

In this chapter the identification controller design indicated in Chapters 4 and 5 is applied to a plant with two unknown poles and one unknown zero. The design procedure is carried out to completion in Sections 7.2 and 7.3 for a nominal set of parameters using quadratic Liapunov functions. Simulation results are then presented in Section 7.4. In this Chapter the plant state vector is assumed available for use by the identification controller. In Chapter 8, an observer is designed which when used in a closed-loop fashion will provide the identification controller with estimates of the plant's state variables.

7.2 Controller Design Using The Quadratic Liapunov Function

The overall system diagram under consideration is that of Fig. 6-1, where the identification controller is designed on the assumption that the true unknown plant state vector $\underline{w}(t)$ is the output of the state estimator or observer.

The unknown plant transfer function Gp(s) is a second order system given by

$$Gp(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} = \frac{W(s)}{R(s)} \qquad (7.2\text{-}1)$$

Equation (7.2-1) may also be described by the differential equation

$$\ddot{w} + a_1 \dot{w} + a_0 w = b_0 r + b_1 \dot{r} \tag{7.2-2}$$

where w and r are the plant scalar output and input respectively. Letting $w = w_1$, $\dot{w}_1 = w_2$, $r = r_1$, and $\dot{r}_1 = r_2$, Eq. (7.2-2) can easily be expressed in phase variable form

$$\dot{\underline{w}} = H\underline{w} + C\underline{r} \tag{7.2-3}$$

where

$$\underline{w} = \begin{bmatrix} w1 \\ w_2 \end{bmatrix} \tag{7.2-4}$$

$$\underline{r} = \begin{bmatrix} r1 \\ r_2 \end{bmatrix} \tag{7.2-4a}$$

$$H = \begin{bmatrix} 0 & 1 \\ -a_0 & -a_1 \end{bmatrix} \tag{7.2-4b}$$

and

$$C = \begin{bmatrix} 0 & 1 \\ b_0 & b_1 \end{bmatrix} \tag{7.2-4c}$$

For the second order system under consideration, Eq. (7.2-3) may be expressed by two first order differential equations

$$\dot{w}_1 = w_2$$

$$\dot{w}_2 = -a_0 w_1 - a_1 w_2 + b_0 r_1 + b_1 r_2 \tag{7.2-5}$$

At this point one may consider a model for the unknown plant having input-output dynamics described the vector differential equation

$$\dot{\underline{y}} = \hat{H}(t)\ \underline{y} + \hat{C}(t)\ \underline{r} \tag{7.2-6}$$

where, as indicated by Eqs. (4.2-6), (4.2-6a), and (4.2-5),

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{7.2-7}$$

$$\hat{H} = \begin{bmatrix} 0 & 1 \\ -\hat{a}_0 & -\hat{a}_1 \end{bmatrix} \tag{7.2-7a}$$

and

$$\hat{C} = \begin{bmatrix} 0 & 0 \\ \hat{b}_0 & \hat{b}_1 \end{bmatrix} \tag{7.2-7b}$$

Note that $\underline{r}$ in Eq. (7.2-6) is the same vector as in Eq. (7.2-3). Also note that $\hat{a}_0$, $\hat{a}_1$, $\hat{b}_0$, and $\hat{b}_1$ are as defined in Chapter 4, Section 4.2.

In a manner similar to that for Eq. (7.2-3), one may express Eq. (7.2-6) by two first order differential equations

$$\dot{y}_1 = y_2$$

$$\dot{y}_2 = -\hat{a}_0\ y_1\ -\hat{a}_1\ y_2 + \hat{b}_0\ r_1 + \hat{b}_1\ r_2 \tag{7.2-8}$$

In view of Eqs. (4.2-7), (7.2-4), and (7.2-7), one has

$$\underline{e} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \tag{7.2-9}$$

$$e_1 = y_1 - w_1 \tag{7.2-9a}$$

$$e_2 = y_2 - w_2 \tag{7.2-9b}$$

Using Eqs. (7.2-4b) and (7.2-7a), one observes that H' defined by Eq. (4.3-6) is evaluated as

$$H' = \hat{H} - H = \begin{bmatrix} 0 & 0 \\ -(\hat{a}_0 - a_0) & -(\hat{a}_1 - a_1) \end{bmatrix} \qquad (7.2\text{-}10)$$

while C', defined by Eq. (4.3-6a), is obtained from Eqs. (7.2-4c) and (7.2-7b) as

$$C' = \hat{C} - C = \begin{bmatrix} 0 & 0 \\ (\hat{b}_0 - b_0) & (\hat{b}_1 - b_0) \end{bmatrix} \qquad (7.2\text{-}10a)$$

Using H, H', and C' in Eqs. (7.2-4b), (7.2-10), and (7.2-10a), Eq. (4.3-7) can be expressed as two first order differential equations

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= -a_0 \ e_1 \ -a_1 \ e_2 - (\hat{a}_0 - a_0)\, y_1 - (\hat{a}_1 - a_1)\, y_2 \\ &\quad + (\hat{b}_0 - b_0)\ r_1 + (\hat{b}_1 - b_1)\ r_2 \end{aligned} \qquad (7.2\text{-}11)$$

As indicated in Eqs. (4.3-8c) and (4.3-9c),

$$\begin{aligned} a_0' &= a_0 - \hat{a}_0(t) \\ a_1' &= a_1 - \hat{a}_1(t) \\ b_0' &= b_0 - \hat{b}_0(t) \\ b_1' &= b_1 - \hat{b}_1(t) \end{aligned} \qquad (7.2\text{-}12)$$

and $\underline{u}'$ and $\underline{v}'$ in Eqs. (4.3-8b) and (4.3-9b) become

$$\underline{u}' = \begin{bmatrix} a_0' \\ a_1' \end{bmatrix} \qquad (7.2\text{-}13)$$

$$\underline{v}' = \begin{bmatrix} b_0' \\ b_1' \end{bmatrix} \qquad (7.2\text{-}13a)$$

The $\underline{b}$ and $\underline{d}$ vectors corresponding to Eqs. (4.3-8a) and (4.3-9a) are

$$\underline{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad (7.2\text{-}14)$$

$$\underline{d} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Equations (4.3-8) and (4.3-9) can be expressed for this second order system as

$$H' = \begin{bmatrix} 0 & 0 \\ a_0' & a_1' \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} a_0' & a_1' \end{bmatrix} \qquad (7.2\text{-}15)$$

$$C' = \begin{bmatrix} 0 & 0 \\ -b_0' & -b_1' \end{bmatrix} = - \begin{bmatrix} 0 \\ +1 \end{bmatrix} \begin{bmatrix} b_0' & b_1' \end{bmatrix} \qquad (7.2\text{-}15a)$$

Note that Eq. (7.2-12) was used to express H' in Eq. (7.2-10) and C' in Eq. (7.2-10a) in the form given in Eqs. (7.2-15) and (7.2-15a).

Equation (4.3-10) is now completely defined in terms of the unknown plant matrix H, the model state vector output $\underline{y}$, the plant input vector $\underline{r}$, and the variational vectors $\underline{u}'$ and $\underline{v}'$. Equation (7.2-11) can now be written as two first order differential equations in terms of a_0', a_1', b_0', and b_1'

$$\dot{e}_1 = e_2$$

$$\dot{e}_2 = -a_0 \; e_1 \; -a_1 \; e_2 + a_0' \; y_1 + a_1' \; y_2 \; - b_0' \; r_1 - b_1' \, r_2 \qquad (7.2\text{-}16)$$

As presented in Section 4.4, the identification controller design is based upon the selection of a candidate Liapunov function. The V function presently under consideration is as given in Eq. (4.4-3), which is repeated here for convenience

$$V = \underline{e}^T P \underline{e} + \underline{u}'^T \Gamma_1 \underline{u}' + \underline{v}'^T \Gamma_2 \underline{v}' \qquad (4.4\text{-}3)$$

For the second order plant the matrices for P, Γ_1, and Γ_2 have the form

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \qquad (7.2\text{-}17)$$

$$\Gamma_1 = \begin{bmatrix} \gamma_{11} & 0 \\ 0 & \gamma_{12} \end{bmatrix} \qquad (7.2\text{-}17a)$$

$$\Gamma_2 = \begin{bmatrix} \gamma_{21} & 0 \\ 0 & \gamma_{22} \end{bmatrix} \qquad (7.2\text{-}17b)$$

Note that vectors $\underline{e}$, $\underline{u}'$, and $\underline{v}'$ are defined in Eqs. (7.2-9), (7.2-13), and (7.2-13a). Matrices Γ_1 and Γ_2, as discussed in Section 5.3, are selected as diagonal matrices with positive γ_{ij}'s. (See Eqs. (5.3-1), (5.3-1a), and (5.3-1b)).

Differentiating Eqs. (7.2-13) and (7.2-13a) leads to $\boldsymbol{\Omega}$ and $\boldsymbol{\Phi}$ as defined by Eq. (4.4-5a) to be

$$\boldsymbol{\Omega} = \dot{\underline{u}}' = \begin{bmatrix} \dot{a}_0' \\ \dot{a}_1' \end{bmatrix} \qquad (7.2\text{-}18)$$

$$\boldsymbol{\Phi} = \dot{\underline{v}}' = \begin{bmatrix} \dot{b}_0' \\ \dot{b}_1' \end{bmatrix} \qquad (7.2\text{-}18a)$$

Using matrix H given by Eq. (7.2-4b) and matrix P given by Eq. (7.2-17), one obtains in view of Eq. (4.4-10).

$$Q = \left[\begin{array}{c|c} 2\ a_0\ p_{12} & a_1\ p_{12} + a_0\ p_{22} - p_{11} \\ \hline a_1\ p_{12} + a_0\ p_{22} - p_{11} & 2(a_1\ p_{22} - p_{12}) \end{array}\right] \tag{7.2-19}$$

Combining Eqs. (4.4-5a), (4.4-9), (4.4-9a) with Eqs. (7.2-18) and (7.2-18a) yields

$$\begin{aligned} \dot{a}_0' &= -\frac{1}{\gamma_{11}}\ y_1\ (p_{12}\ e_1 + p_{22}\ e_2) \\ \dot{a}_1' &= -\frac{1}{\gamma_{12}}\ y_2\ (p_{12}\ e_1 + p_{22}\ e_2) \\ \dot{b}_0' &= \frac{1}{\gamma_{21}}\ \Gamma_1\ (p_{12}\ e_1 + p_{22}\ e_2) \\ \dot{b}_1' &= \frac{1}{\gamma_{22}}\ \Gamma_2\ (p_{12}\ e_1 + p_{22}\ e_2) \end{aligned} \tag{7.2-20}$$

Since a_0' , a_1', b_0', and b_1' are related to the model parameters $\hat{a}_0$, $\hat{a}_1$, $\hat{b}_0$, and $\hat{b}_1$ through Eq. (7.2-12), and since a_0, a_1, b_0, and b_1 are assumed to be constant,+ Eq. (7.2-20) yields the control laws for updating the parameters in the model's differential equation.

$$\begin{aligned} \dot{\hat{a}}_0 &= \frac{1}{\gamma_{11}}\ y_1\ (p_{12}\ e_1 + p_{22}\ e_2) \\ \dot{\hat{a}}_1 &= \frac{1}{\gamma_{12}}\ y_2\ (p_{12}\ e_1 + p_{22}\ e_2) \\ \dot{\hat{b}}_0 &= -\frac{1}{\gamma_{21}}\ r_1\ (p_{12}\ e_1 + p_{22}\ e_2) \\ \dot{\hat{b}}_1 &= -\frac{1}{\gamma_{22}}\ r_2\ (p_{12}\ e_1 + p_{22}\ e_2) \end{aligned} \tag{7.2-21}$$

+Note that a_0, a_{11} b_0, b_1 drop out when Eq. (7.2-12) is differentiated with respect to time and $\dot{a}_i' = -\dot{\hat{a}}_i$, $\dot{b}_j' = -\dot{\hat{b}}_j$.

Equation (7.2-21), together with Eqs. (7.2-5) and (7.2-8) specify the complete system behavior. Fig. 7-1 shows the complete system in an analog computer representation.

Based upon the design techniques presented in Section 5.4, it follows that if $\underline{r}(t)$ is a suitable [+] bounded input time function and the γ_{ij} are positive, the system is asymptotically stable for all initial error vectors $\underline{e}$ (t = 0). The range of the unknown plant parameters is subject to the constraints imposed by the design procedure presented in Section 5.2. This design procedure is now illustrated numerically in this section. The effect on system performance of changes in the controller design parameters is discussed in Section 7.4.

For this illustrative case study, the nominal or initial values for the model parameters are chosen to be:

$$\tilde{a}_0 = 100$$
$$\tilde{a}_1 = 10 \qquad (7.2\text{-}22)$$

It is assumed at this point that the plant behaves as a second order system. Therefore we assume a model of the same order.

The $\tilde{Q}$ matrix discussed in Section 5.2 is selected as

$$\tilde{Q} = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix} \qquad (7.2\text{-}23)$$

so that the index of performance $\widetilde{IP}$ given by Eq. (5.3-3) becomes

$$\widetilde{I.P.} = \int_0^{\infty} (10\ e_1^2 + e_2^2)\ dt \qquad (7.2\text{-}24)$$

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+ Suitable inputs, as discussed in Section 5.4, are those that force the model parameters to converge to the plant parameters when the system's error vector goes to zero.

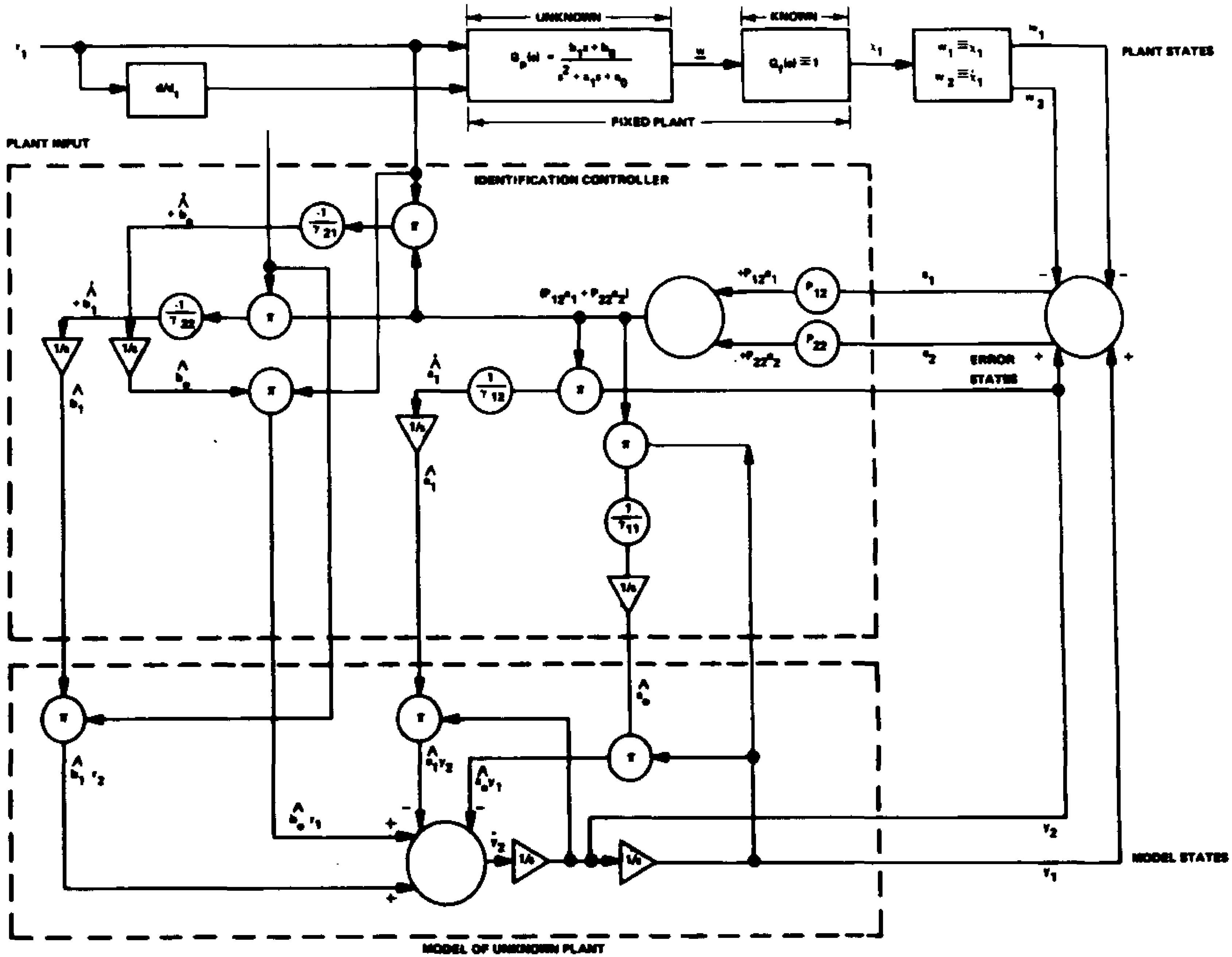

Figure 7-1. Plant, Model, and Controller Flow Diagram for Illustrative Case

Eq. (7.2-24) obviously weights $e_1(t)$ more heavily than its derivative $e_2(t)$. The effect of two different $\tilde{Q}$ matrices will be studied in Section 7.4 through system simulation.

Following the procedure indicated in Section 5.2, $\tilde{H}$ in view of Eqs. (5.2-3) and (7.2-22) has the form

$$\tilde{H} = \left[\begin{array}{c|c} 0 & 1 \\ \hline -100 & -10 \end{array}\right] \qquad (7.2\text{-}25)$$

Using $\tilde{Q}$ as given in Eq. (7.2-23), matrix P can be determined. Thus Eq. (7.2-19) with Q replaced by $\tilde{Q}$ becomes

$$\begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix} = \left[\begin{array}{c|c} 2\,p_{12}\quad \tilde{a}_0 & 0 \\ \hline 0 & 2\,(\tilde{a}_1\, p_{22} - p_{12}) \end{array}\right] \qquad (7.2\text{-}26)$$

If one lets

$$p_{11} = a_1 \; p_{12} + a_0 \; p_{22} \qquad (7.2\text{-}26a)$$

One notes from Section 5.2 that p_{11} is not needed numerically since it is not part of the controller, but will enter into the constraints for V to be positive definite.

Continuing the design procedure of Section 5.2, one has from Eqs. (7.2-26) and (7.2-22)

$$\begin{aligned} 10 &= 2 \; p_{12} \; (100) \\ 1 &= 2 \; (10 \; p_{22} - p_{12}) \end{aligned} \qquad (7.2\text{-}27)$$

or

$$\begin{aligned} p_{12} &= .05 \\ p_{22} &= .055 \end{aligned} \qquad (7.2\text{-}27a)$$

and from Eq. (7.2-26a)

$$p_{11} = a_1 \; (.05) + a_0 \; (.055) \qquad (7.2\text{-}27b)$$

Equations (7.2-27a) and (7.2-27b) are now used to compute Q in Eq. (7.2-19). This yields

$$Q = \begin{bmatrix} .1 & a_0 & 0 \\ 0 & .11 & a_1 - .1 \end{bmatrix} \qquad (7.2\text{-}28)$$

Thus for $-\dot{V} \geq 0$, one must have $Q > 0$. Therefore, one must have

$$\begin{aligned} a_0 &> 0 \\ a_1 &> 10/_{11} \end{aligned} \qquad (7.2\text{-}29)$$

Substituting from Eqs. (7.2-27a) and (7.2-27b) into Eq. (7.2-17) yields the P matrix

$$P = \left[\begin{array}{c|c} a_1\,(.05) + a_0\,(.055) & .05 \\ \hline .05 & .055 \end{array}\right] \qquad (7.2\text{-}30)$$

For $V > 0$, matrix P must be positive definite. Therefore, Eq. (7.2-30) must satisfy the following constraints

$$a_1\,(.05) + a_0\,(.055) > 0$$

$$\left[a_1\,(.05) + a_0(.055)\right] \; .055 - \;(.055)^2 > 0 \qquad (7.2\text{-}31)$$

The constraints in Eq. (7.2-31) when combined with those in Eq. (7.2-29) yield

$$\begin{aligned} a_0 &> 0 \\ a_1 &> 10/_{11} \end{aligned} \qquad (7.2\text{-}32)$$

Equation (7.2-32) implies both V and $-\dot{V}$ positive definite in $\underline{e}$. Thus for the initial parameter set selected by Eq. (7.2-22), convergence of the model parameters to the plant parameters is guaranteed so long as the true plant parameters satisfy Eq. (7.2-32). Criteria obtained using this type of procedure with Liapunov functions yield sufficient conditions for stability. The true range for stability will generally be larger than those obtained by the design procedure in Chapter 5.

7.3 Simulation Results

The complete system designed in Section 7.2 was simulated on an IBM 360 digital computer using the FORTRAN language and compiler. The details of the simulation performed are as follows:

1) The unknown portion of the plant has the transfer function given by Eq. (7.2-1) with a_0, a_1, and b_0 the unknown parameters. Parameter b_1 is known with value

$$b_1 = 1 \tag{7.3-1}$$

2) The plant's input time function $r_1(t)$ was selected as parabolic segments. This input satisfies the sufficient conditions given in Section 5.4 that are required to force the model parameters to the true plant parameters when the error vector $\underline{e}$ goes to zero. The plant input in view of Eqs. (7.2-4a) is chosen as

$$\left.\begin{aligned} r_1(t) &= 187 \ \frac{Zt^2}{2} \\ r_2(t) &= 187 \ Zt \end{aligned}\right\} \quad 0 \leq t \leq T_1 \tag{7.3-2}$$

and

$$\left.\begin{aligned} r_1(t) &= 187 \left(-\frac{Z}{2}(t - T_1)^2 + \frac{Z}{2} \ T_1{}^2 \right) \\ r_2(t) &= -187 \ Z(t - T_1) \end{aligned}\right\} T_1 \leq t \tag{7.3-3}$$

where

$$Z = 10.0 \qquad (7.3\text{-}4)$$

$$T_1 = 1.75$$

3) The model matrices $\hat{H}$ and $\hat{C}$ have the initial element values as given by Eq. (7.2-22). The initial value of $\hat{b}_0$ was selected as:

$$\hat{b}_0 = 3.0 \qquad (7.3\text{-}5)$$

4) The elements of the controller P matrix are as given in Eq. (7.2-27a). Using Eq. (7.2-30), V in Eq. (4.4-3) and $-\dot{V}$ in Eq. (4.4-11) are specified when the Γ_1 and Γ_2 matrices are selected, and values are assigned to a_0 and a_1, the true plant parameters.+

5) Select $a_0 = 121$ and $a_1 = 22$. Scalars V and $-\dot{V}$ are now

$$V = 7.755\ e_1^{\ 2} + .1\ e_1\ e_2 + .055\ e_2^{\ 2} + \frac{1}{\gamma_{11}}\ a_0'^{\,2} + \frac{1}{\gamma_{12}}\ a_1'^{\,2} + \frac{1}{\gamma_{21}}\ b_0'^{\,2}$$

$$-\dot{V} = 12.1\ e_1^{\ 2} + 2.32\ e_2^{\ 2} \qquad (7.3\text{-}6)$$

6) With the selection of γ_{11}, γ_{12}, and γ_{21} as

$$\frac{1}{\gamma_{11}} = 400$$

$$\frac{1}{\gamma_{12}} = 50$$

$$\frac{1}{\gamma_{21}} = 2.5 \qquad (7.3\text{-}7)$$

+ Since the true plant parameters are unknown, V and $\dot{V}$ are also unknown to the designer. For the simulation studies however, V and $\dot{V}$ are computed to illustrate the Liapunov theory design.

The model dynamics given in Eq. (7.2-21) are:

$$\dot{\hat{a}}_0 = 400 \; (.05 \; e_1 + .055 \; e_2) \; y_1$$

$$\dot{\hat{a}}_1 = 50 \; (.05 \; e_1 + .055 \; e_2) \; y_2$$

$$\dot{\hat{b}}_0 = -2.5 \; (.05 \; e_1 + .055 \; e_2) \; r_1 \qquad (7.3\text{-}8)$$

The parameters given by Eq. (7.3-7) were selected as a result of experimental simulation runs. It was found that a large $1/\gamma_{11}$ was needed to effectively move the $\hat{a}_0$ parameter while a small $1/\gamma_{21}$ would effectively move the $\hat{b}_0$ parameter. Any positive γ_{ij} values will of course keep $V > 0$ and force the identification eventually. However, the running time for the simulation program would become excessively large. The selection of r_1 given by Eqs. (7.3-2) and (7.3-3) was made basically to keep the input bounded over the time required for the identification to take place. The bounded input will avoid numerical integration errors that an unbounded parabolic input is likely to produce. The parameter T_1 in Eq. (7.3-4) was arbitrarily chosen. It should be noted that the response of the true plant due to initial conditions has a time constant of 1/11, which is smaller than than T_1.

7) With the following specification of the model and plant initial condition states at $t = 0$,

$$y_1(0) = 1.0$$

$$y_2(0) = 0.0$$

$$w_1(0) = 0.0$$

$$w_2(0) = 0.0 \qquad (7.3\text{-}9)$$

the system illustrated in Fig. 7-1 (with $G_f(s) \equiv 1$ and a perfect observer) is completely specified.

8) The results of this simulation were computed using a step size h = .0005 and a predictor-corrector numerical integration scheme. They can be illustrated best by plots of V, $\dot{V}$, $\hat{a}_0$, $\hat{a}_1$, and $\hat{b}_0$ versus time and a phase plane portrait of the error vector elements $e_1(t)$ and $e_2(t)$.

Figure 7-2 shows V and $\dot{V}$ for the time interval t = 0 to t = 3.41. The percentage decrease in V and $\dot{V}$ is considerable from their initial values, as it can be easily seen. These values are

$$\frac{V(3.41)}{V(0)} \times 100\% = \frac{.3617}{13.31} \times 100 = 2.72\%$$

$$\frac{+\dot{V}(3.41)}{\dot{V}(0)} \times 100\% = \frac{.04906}{12.19} \times 100 = 0.4\% \qquad (7.3\text{-}10)$$

Note that V(t) is of course positive and monotonically decreases while $-\dot{V}(t)$ is positive for all time. The maximum value of $-\dot{V}$ was approximately 3 1/2 times its initial value and occurred between t = .002 and t = .4.

The model parameter variations about their true values are shown in Fig. 7-3. As can be easily seen $\hat{a}_0$ moved monotonically from 100 to 112 compared to its true value of 121, while $\hat{a}_0$ moved from 10 to 24.3, passed its true value of 22. In comparison, $\hat{b}_0$ moved from 3.0 to .64, below its true value of 1.0 and then moved to 1.22, above its true value.

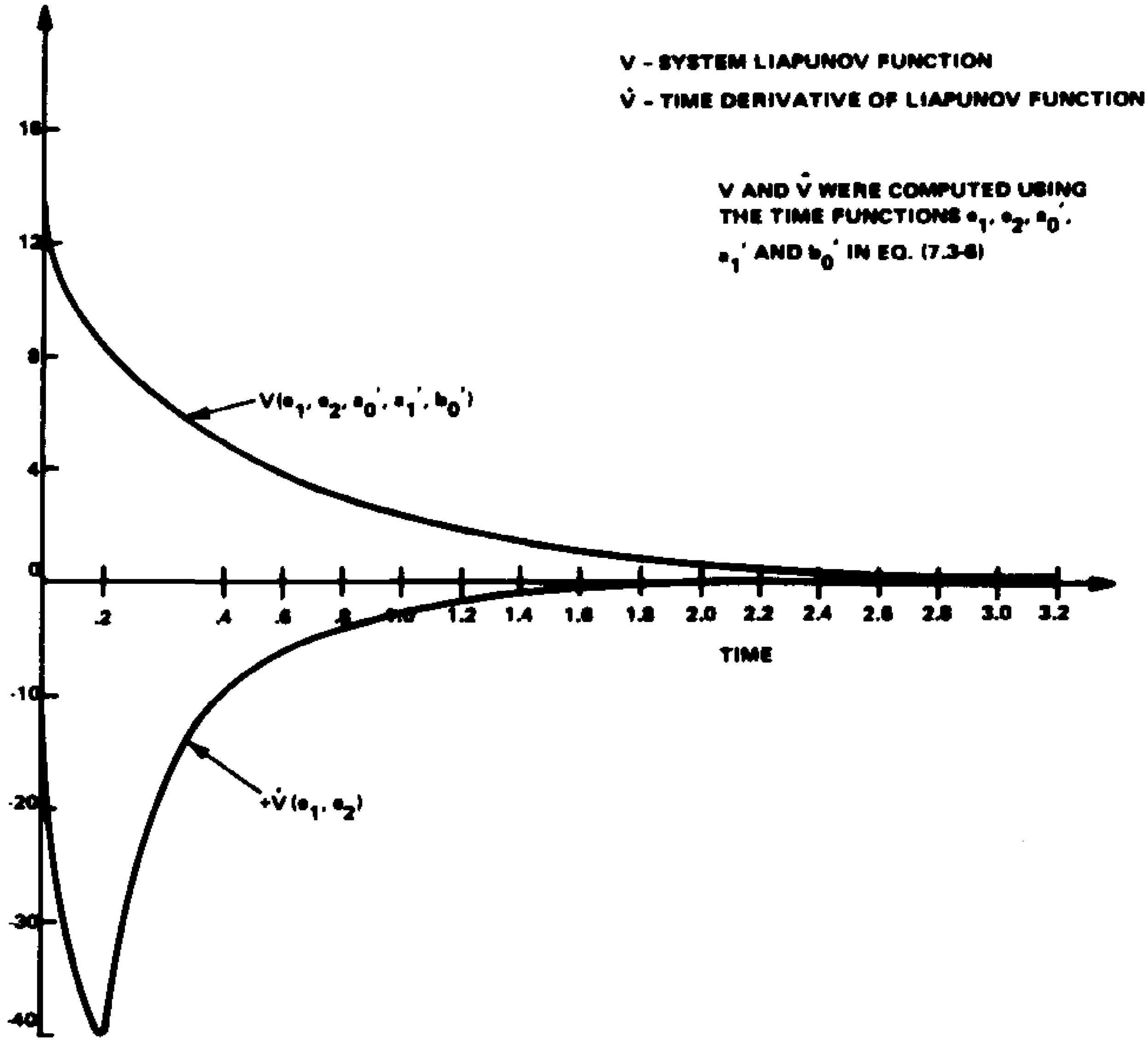

Figure 7-2. V and $\dot{V}$ Versus Time - Example 7-1

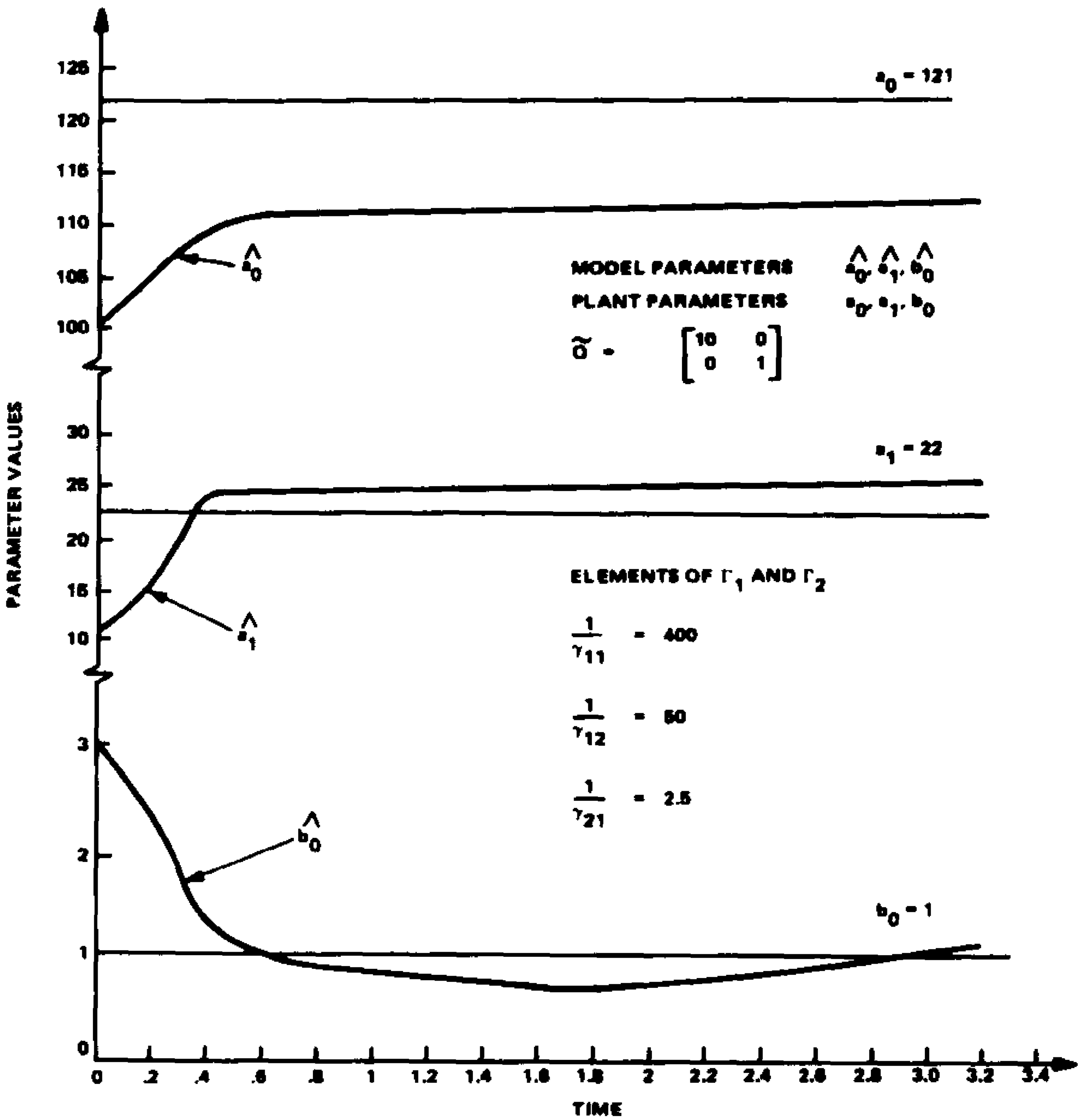

Figure 7-3. Model Parameter Time Variations - Example 7-1

Figure 7-4 presents a phase portrait of the error vector $\underline{e}$ versus time. It must be stressed that while this system contains five state variables, Fig. 7-4 is only a plot of the time variation of two of them. As seen from this figure, the system's behavior spirals into the origin about the line $e_2 = 0$. The terminal behavior as shown in Fig. 7-4 indicates that the system tends to track the line in state space where

$$p_{12} \; e_1 + p_{22} \; e_2 = 0 \tag{7.3-11}$$

or for the case under study

$$-.05 \; e_1 = .055 \; e_2 \tag{7.3-12}$$

Since $-\dot{V}$ is always greater than zero for e_1, e_2 (not both zero), the system of course cannot remain on this trajectory, but will vary about it until the equilibrium point, (the origin), is reached. Since $\dot{\hat{a}}_0$, $\dot{\hat{a}}_1$, and $\dot{\hat{b}}_0$ are small along this trajectory, as Eq. (7.3-8) indicates, the final identification is slow. As can be seen from Fig. 7-4, the error dynamics reached Eq. (7.3-11) at t = 1.0. The functions V and $\dot{V}$ were at this time considerably below their peak values as can be seen from Fig. 7-2. After t = 1.0, $\hat{a}_0$, $\hat{a}_1$, and $\hat{b}_0$ did not vary considerably in comparison to their previous behavior from t = 0 to t = 1.0. This fact is indicated by Fig. 7-3.

At this point it is instructive to compute Q in Eq. (7.2-28) for the true plant values, $a_0 = 121$ and $a_1 = 22$.

One has

$$Q = \left[\begin{array}{c|c} 12.1 & 0 \\ \hline 0 & 2.32 \end{array}\right]$$

which from Eq. (5.3-2) indicates that the true index of performance, I.P. is

$$\text{I.P.} = \int_0^{\infty} (12.1 \; e_1^2 + 2.32 \; e_2^2) \, dt$$

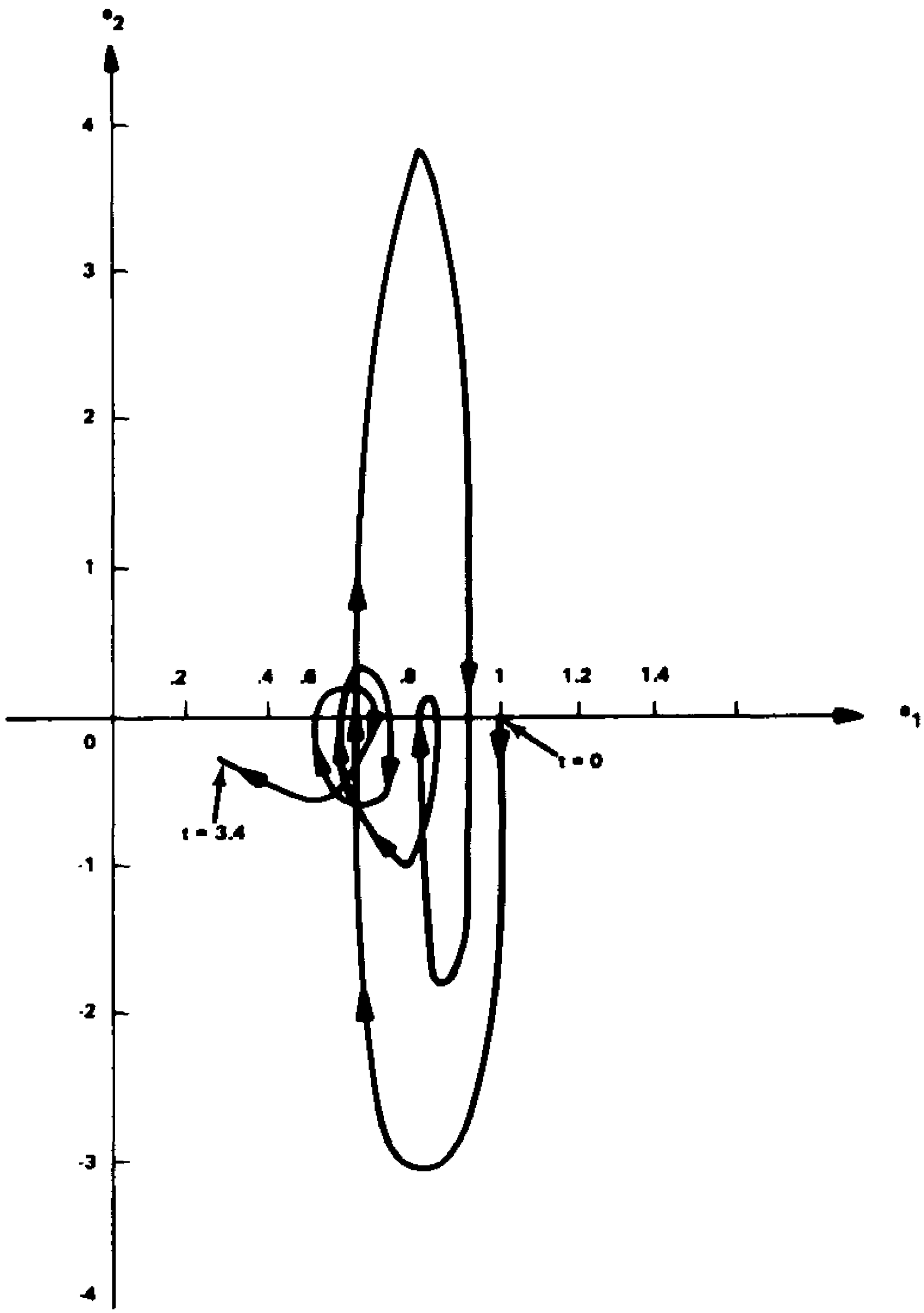

Figure 7-4. Phase Plane Error Portrait - Example 7-1

which is quite close (relatively speaking) to that of $\widetilde{I.P.}$ in Eq. (7.2-24). Since the parameters in the I.P. integrand for this type of performance index are never selected with precision, the difference between a 10/1 or a 12.1/2.32 ≈ 5.5/1 ratio is not very significant. In all cases however, convergence of the model parameters to those of the plant is assured.

As discussed in Chapter 5, changing $\tilde{Q}$ in Eq. (7.2-23) will change $\widetilde{I.P.}$ in Eq. (7.2-24) and will affect the behavior of the identification controller. To illustrate this effect, $\tilde{Q}$ for the example previously discussed was changed from Eq. (7.2-23) to that in Eq. (7.3-13)

$$\tilde{Q} = \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix} \tag{7.3-13}$$

and $\widetilde{I.P.}$ in Eq. (7.2-24) is changed to

$$\widetilde{I.P.} = \int_0^{\infty} (e_1^2 + 10\, e_2^2)\, dt \tag{7.3-14}$$

The result of this simulation can be seen in Figures 7-5, 7-6 and 7-7, where the step size in the numerical integration was $h = .00005$, one-tenth of the h used previously. The γ_{ij} parameters remained as those given in Eq. (7.3-7). The new elements of the P matrix become:

$$p_{12} = .005$$

$$p_{22} = .5005$$

$$p_{11} = a_1 (p_{12}) + a_0\, p_{22} \tag{7.3-15}$$

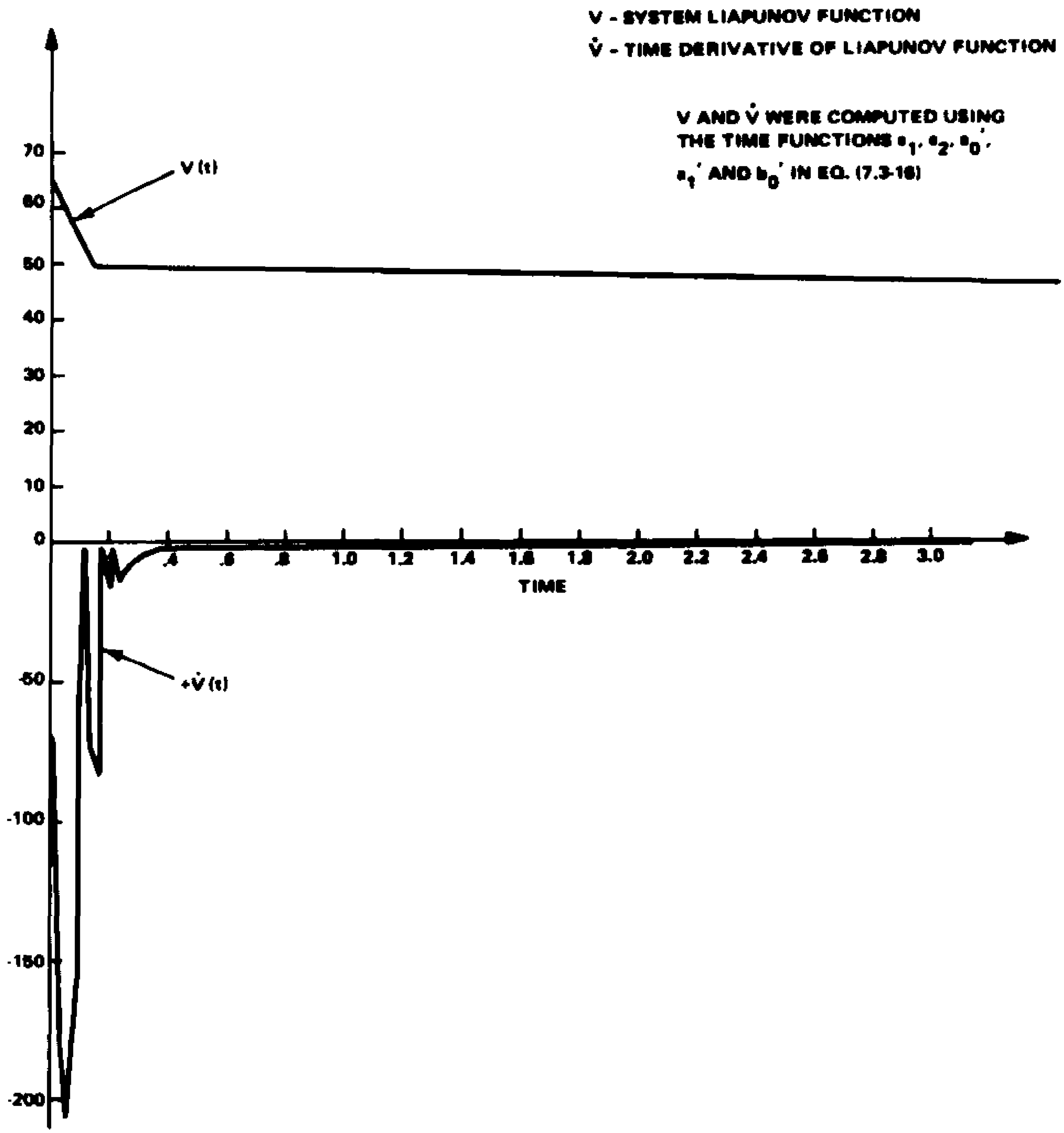

Figure 7-5. V and $\dot{V}$ Versus Time - Example 7-2

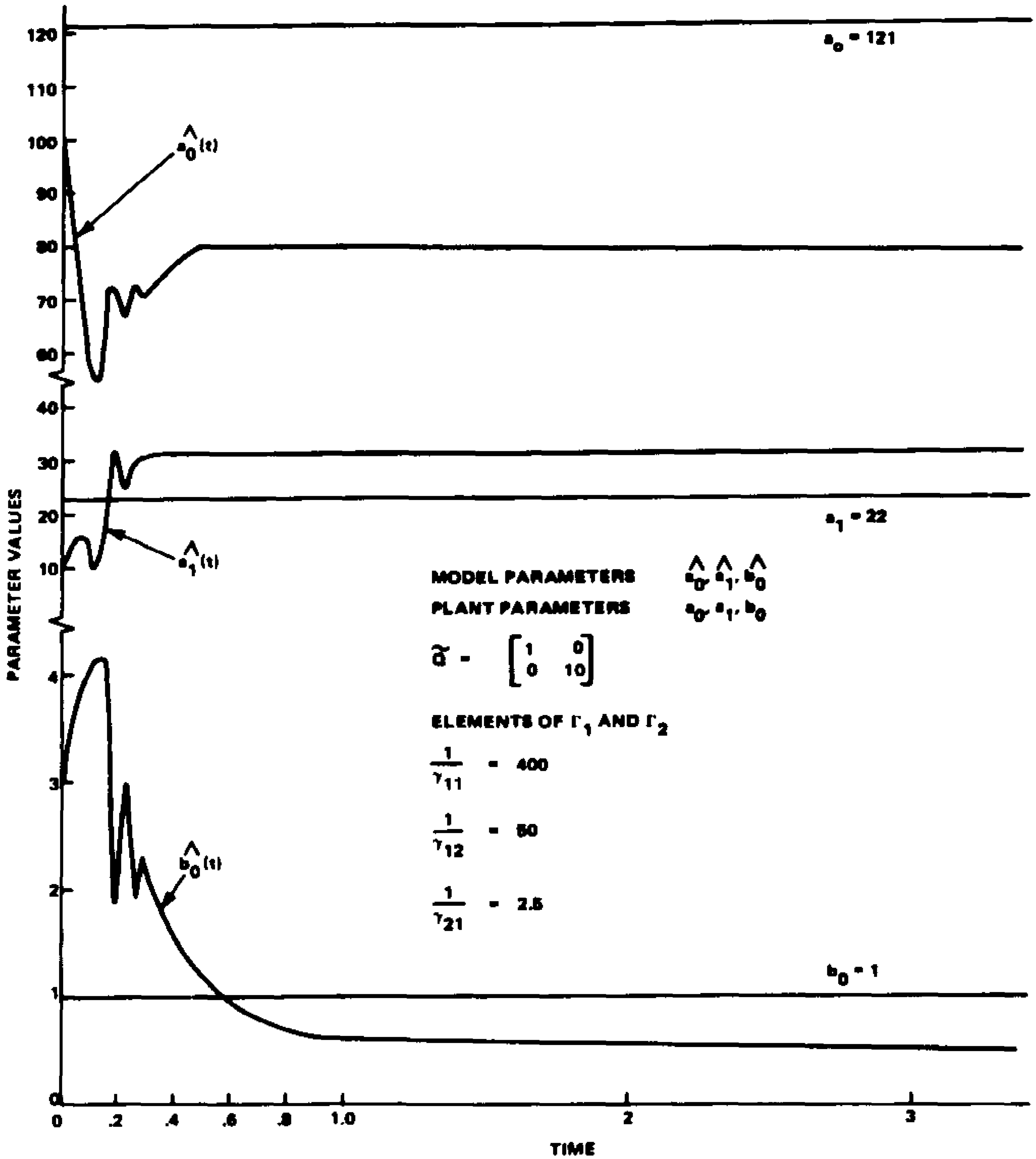

Figure 7-6. Model Parameter Time Variations - Example 7-2

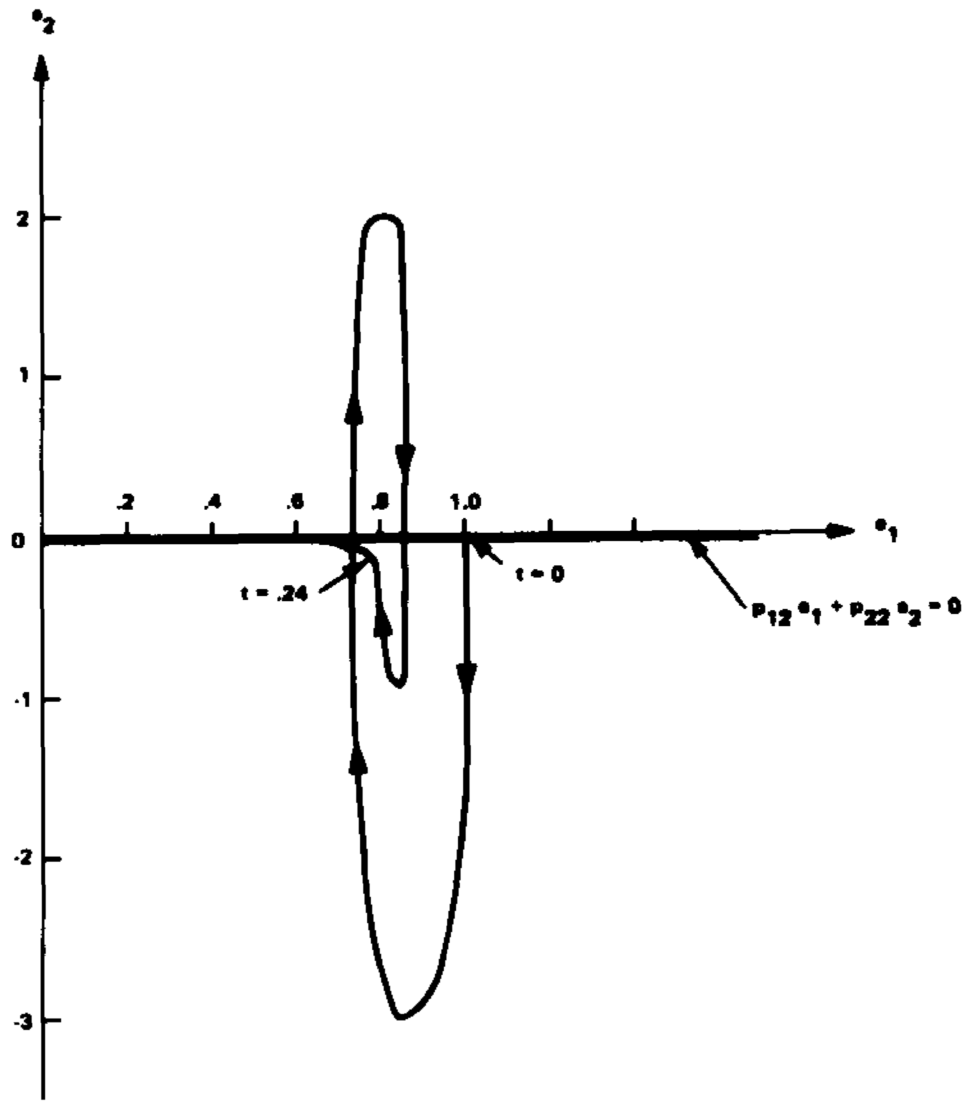

Figure 7-7. Phase Plane Error Portrait - Example 7-2

and V and $-\dot{V}$ can be found using Eqs. (4.4-11), (7.3-15) and (4.4-3) in a manner similar to that in the previous simulation. They are

$$V = 60.67 \; e_1^{\,2} + .01 \; e_1 \; e_2 + .5005 \; e_2^{\,2} + \frac{1}{\gamma_{11}} \; a_0'^{\,2} + \frac{1}{\gamma_{12}} \; a_1'^{\,2} + \frac{1}{\gamma_{21}} \; b_0'^{\,2}$$

$$-\dot{V} = 1.11 \; e_1^{\,2} + 22.012 \; e_2^{\,2} \qquad (7.3\text{-}16)$$

As seen from Figure 7-5, V(t) and $\dot{V}(t)$ were both monotonically decreasing after t = .34 at a very slow rate. After this time, identification by the controller effectively came to a halt and the parameters in the model varied little afterwards. After t = .34, $\dot{V}$ was reduced from its peak value by 99.5%, which from Eq. (7.3-16) would indicate that e_2, at the systems terminal behavior, would be small. This predictable behavior can be seen in Fig. 7-7, the phase error vector portrait. The error vector starts at $e_1 = 1.0$, $e_2 = 0.0$ at t = 0, spirals from t = 0.0 to approximately t = .30, and then heads toward the origin very slowly along the path $p_{12} \; e_1 + p_{22} \; e_2 = 0$.

As can be seen in Fig. 7-3 $\hat{a}_0(t)$ in the first study did not move significantly over the identification time interval. This result therefore indicated that better performance might be obtained if $1/\gamma_{11}$ in Eq. (7.3-7) were increased, thereby increasing a_0 in Eq. (7.3-8). Since in practice, the true value of a_0 would be unknown to the designer, Figure 7-4 would be used to gauge effective performance and decide on values for the γ_{ij} parameters.

The first case was rerun with $1/\gamma_{11}$ in Eq. (7.3-7) changed to $1/\gamma_{11} = 800$. All other information remained the same. Figure 7-8 shows the model's parameter variations from t = 0.0 to t = 1.75. As can be seen from this figure, the identification is excellent. Scalar time functions of V and $\dot{V}$ are shown in Fig. 7-9 and the phase portrait of e_1 and e_2 is shown in Fig. 7-10.

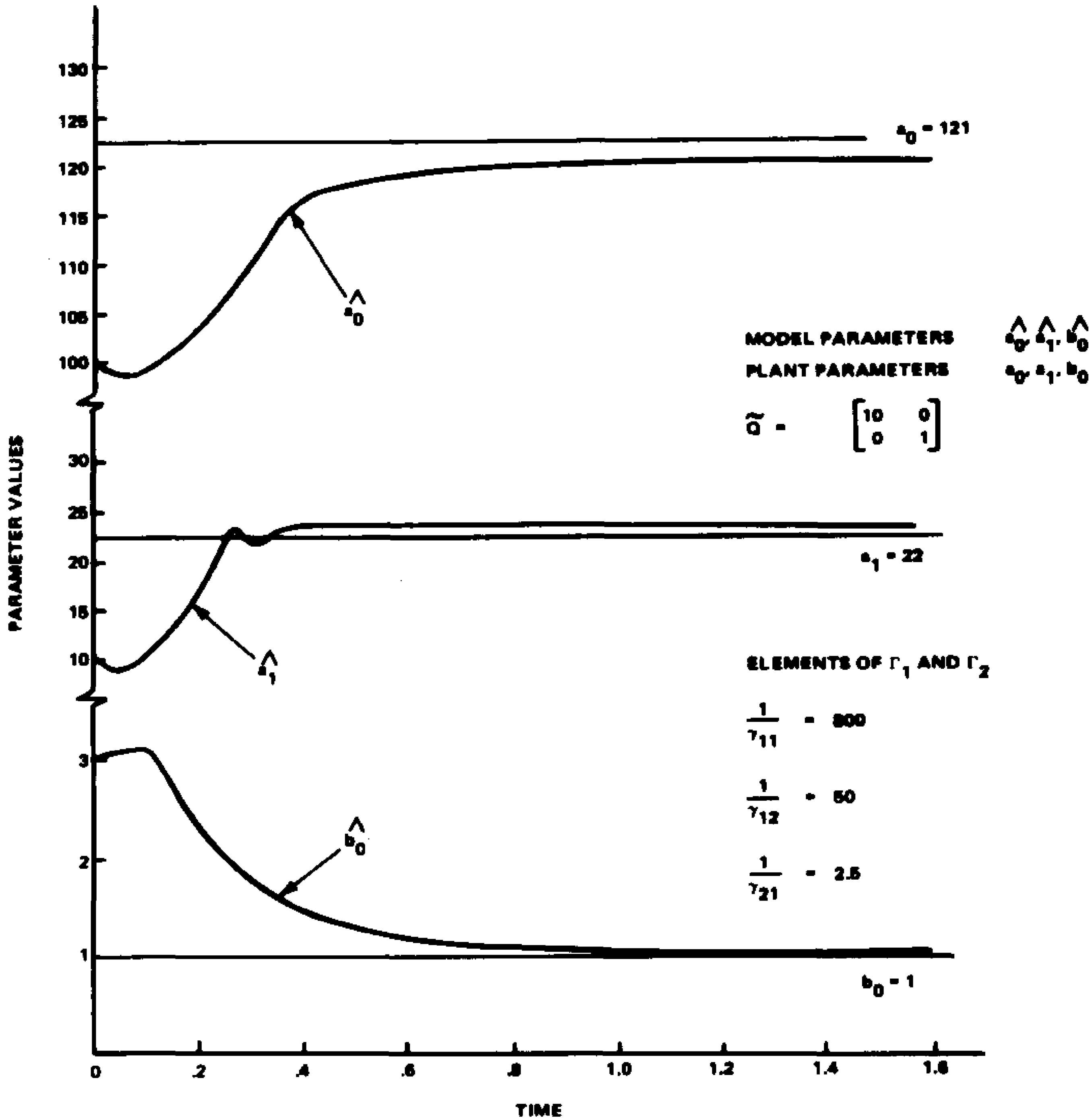

Figure 7-8. Model Parameter Time Variations - Example 7-3

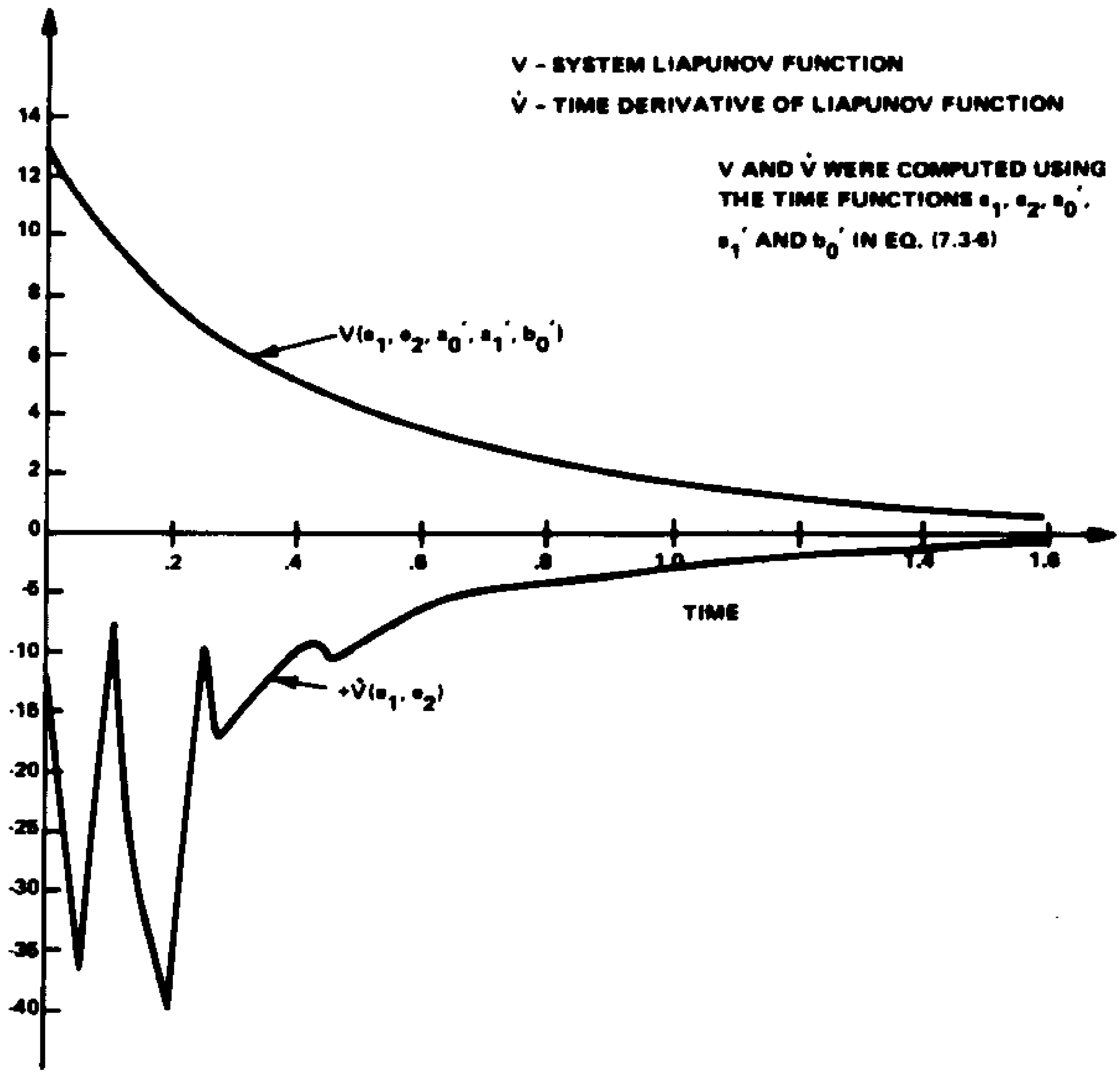

Figure 7-9. V and $\dot{V}$ Versus Time - Example 7-3

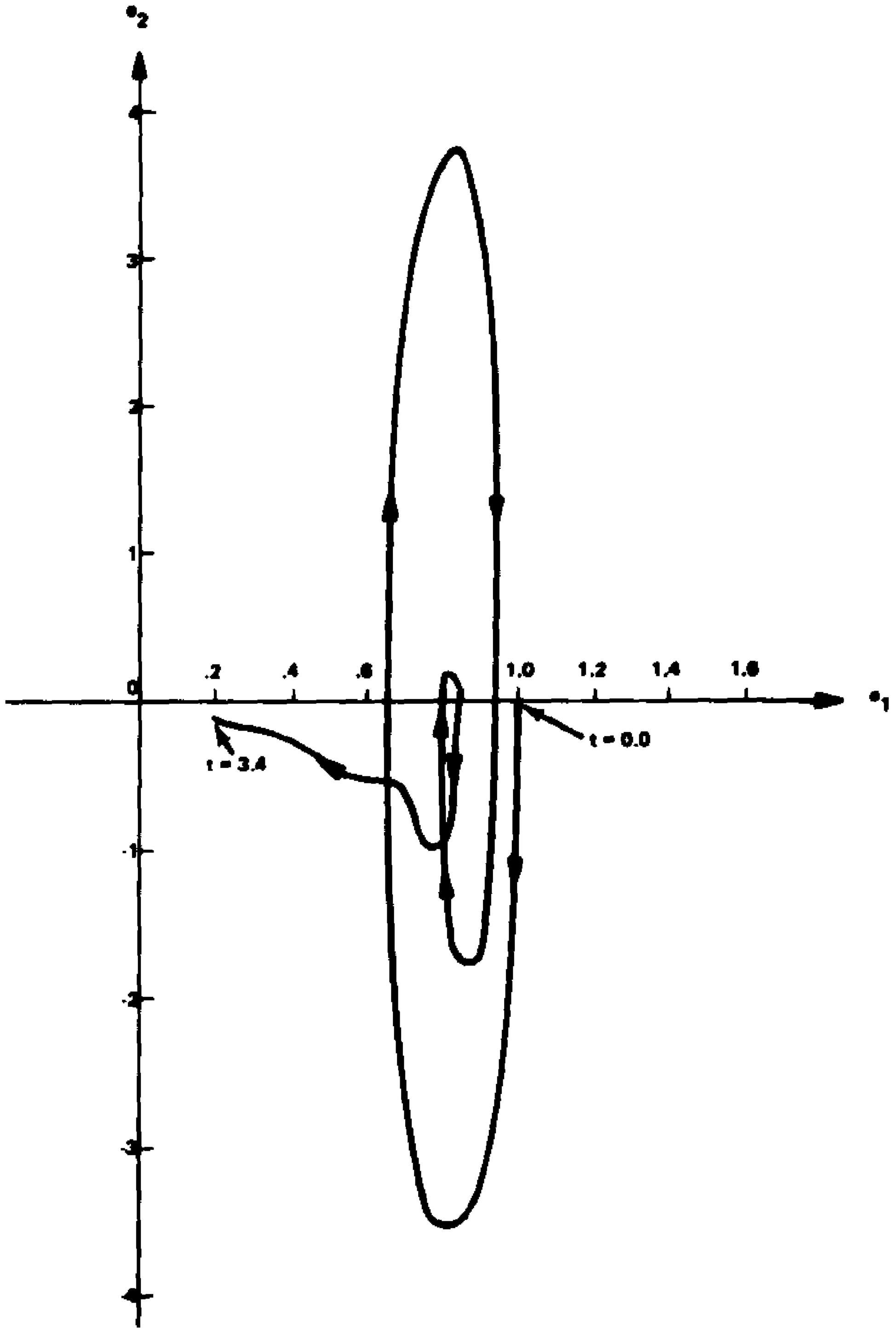

Figure 7-10. Phase Plane Error Portrait - Example 7-3

CHAPTER 8

ILLUSTRATIVE CASE WITH OBSERVER

8.1 Introduction

As previously discussed in Chapter 6, for the case when $G_r(s)$ in Fig. 6-1 is not unity, the identification controller is still designed under the assumption that the system error vector $\tilde{\underline{e}}$ equals the true error vector $\underline{e}$ in Eq. (4.2-7). In addition, the first step in the observers' design assumes that the matrices H and C of the unknown plant Gp(s) are known. However, the design philosophy adopted leaves the observer design in closed form, as a function of the elements of H and C. The complete closed-loop system in Fig. 6-1 is mechanized by providing the elements of matrices $\hat{H}$ and $\hat{C}$ to substitute for the elements of H and C respectively. Matrices $\hat{H}$ and $\hat{C}$ are generated by the observer controller in a closed-loop fashion as shown in Fig. 6-1.

In Section 8.2, the first observer design phase is carried out for the same illustrative case as in Chapter 7. Section 8.3 develops a Liapunov function technique for obtaining constraints required for the observer free parameters. These constraints insure that the observer output $\tilde{\underline{w}}$ will converge in time to the plant output vector $\underline{w}$, when Gp(s) is known. In Section 8.3 one finds further constraints which guarantee that when the system error vector $\tilde{\underline{e}}$ approaches zero, the true plant-model error vector $\underline{e}$ also approaches zero. This condition is essential for the total system in Fig. 6-1 to operate properly.

Section 8.4 completes the total system design by using a Liapunov function to design the observer controller in a manner similar to the design of the model identification controller. At this stage, the model's state vector $\underline{y}$ is assumed to be equal to the unknown plant's state vector $\underline{w}$. The final implemented observer controller uses the actual model vector $\underline{y}$.

Section 8.5 presents the results of computer simulations of the illustrative design case for selected observer parameter values.

8.2 Controller Design Using The Observer

For the illustrative case in this chapter, $G_f(s)$ in Fig. 6-1, the known portion of the plant, is selected as a first order stable system represented by the transfer function

$$G_f(s) = \frac{1}{s+\psi} \qquad \psi > 0 \tag{8.2-1}$$

The observer to be designed will provide estimates of the plant's two state variables w_1 and w_2 from $x_1(t)$, the output of $G_f(s)$.

As the observer design in this chapter is completely independent from the identification controller design of previous chapters, notation changes consistent with Appendix A will be made for convenience. Specifically, the plant's state vector $\underline{w}$ will now be denoted by $\underline{x}$, where $\underline{x}$ will also contain the state variables of $G_f(s)$. Let the $\underline{x}$ vector thus be defined as

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{8.2-2}$$

where x_1 is the output of $G_f(s)$, $x_2 \equiv w_1$, and $x_3 \equiv w_2$. Note that the state variables w_1 and w_2 are the elements of the previous plant vector $\underline{w}$.

Let the observer estimate of $\underline{x}$ be denoted by $\tilde{\underline{x}}$, where

$$\tilde{\underline{x}} = \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} \tag{8.2-3}$$

Note that $\tilde{x}_2$ is the observer estimate of x_2, $\tilde{x}_3$ is the observer estimate of x_3 and $\tilde{x}_1 = x_1$.

With the change of notation undertaken, the plant equations, for the illustrative example of Chapter 7, become from Eqs. (7.2-5) and (8.2-1)

$$\begin{aligned} \dot{x}_1 &= -\Psi x_1 + x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= -a_0\, x_2 - a_1\, x_3 + b_0\, r_1 + b_1\, r_2 \end{aligned} \tag{8.2-4}$$

where r_1, r_2, a_0, a_1, b_0, and b_1 are as previously described in Section 7.2. The first relationship in Eq. (8.2-4) was obtained from the differential equation represented by the transfer function $G_f(s)$ in Eq. (8.2-1).

Equation (8.2-4) can now be written as the matrix differential equation

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} \tag{8.2-5}$$

where

$$A = \begin{bmatrix} -\Psi & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -a_0 & -a_1 \end{bmatrix} \tag{8.2-6}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ b_0 & b_1 \end{bmatrix} \qquad (8.2\text{-}6a)$$

and

$$\underline{u} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \qquad (8.2\text{-}6b)$$

Since x_1, the output of $G_f(s)$ and the input to the observer, is denoted in Appendix A by the vector $\underline{y}$, one has the relationship

$$\underline{y} = C\,\underline{x} \qquad (8.2\text{-}7)$$

where

$$\underline{y} = y_1 \longrightarrow y_1 \equiv x_1 \qquad (8.2\text{-}7a)$$

and matrix C is given by

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \qquad (8.2\text{-}7b)$$

The observer estimate of $\underline{x}$, $\underline{\tilde{x}}$ is of the form given by Eq. (A-8) of Appendix A, repeated for convenience

$$\underline{\tilde{x}} = W \begin{bmatrix} y \\ \hline z \end{bmatrix} \qquad (A\text{-}8)$$

Vector $\underline{z}$ in Eq. (A-8) is a 2 x 1 vector of the form

$$\underline{z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \qquad (8.2\text{-}8)$$

where $\underline{z}$ is the solution of the vector differential equation in Eq. (A-9). For the case under consideration, F, G, and R in Eq. (A-9) are 2 x 2, 2 x 1, and 2 x 2 observer design matrices respectively.

The matrix W in Eq. (A-8) has the partitioned form shown by Eq. (A-10). For the case under consideration, W_{22} is from Eq. (A-11) the 2 x 2 identity matrix

$$W_{22} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (8.2\text{-}9)$$

Equation (A-3) partitions matrix C in Eq. (8.2-7b) such that C_1 is the 1 x 1 matrix

$$C_1 = [1] \quad (8.2\text{-}10)$$

and C_2 the 1 x 2 matrix

$$C_2 = [0 \quad 0] \quad (8.2\text{-}10a)$$

Using Eqs. (A-13), (8.2-10), and (8.2-10a), one has

$$W_{12} = [0 \;\vdots\; 0] \quad (8.2\text{-}11)$$

Note W_{12} is a 1 x 2 matrix

Let W_{21}, the 2 x 1 submatrix of W, in Eq. (A-10) be made up of elements K_1 and K_2 as shown

$$W_{21} = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} \quad (8.2\text{-}12)$$

Constants K_1 and K_2 are now observer design parameters. Eqs. (8.2-10), (8.2-10a), and (8.2-12) can now be combined to yield the 1 x 1 matrix W_{11} given by Eq. (A-14)

$$W_{11} = [1] \quad (8.2\text{-}13)$$

Equations (8.2-9), (8.2-11), (8.2-12), and (8.2-13) combine to yield W in Eq. (A-10) as

$$W = \begin{bmatrix} 1 & 0 & 0 \\ K_1 & 1 & 0 \\ K_2 & 0 & 1 \end{bmatrix} \tag{8.2-14}$$

Equation (A-8) can now be written as three scalar equations, using Eqs. (8.2-14), (8.2-8), and (8.2-7a).

$$\begin{aligned} \tilde{x}_1 &= y \\ \tilde{x}_2 &= K_1 \, y + z_1 \\ \tilde{x}_3 &= K_2 \, y + z_2 \end{aligned} \tag{8.2-15}$$

Thus $\tilde{x}_2$ and $\tilde{x}_3$, the observer estimates of x_2 and x_3 respectively, are linear combinations of y, z_1, and z_2.

To complete the first phase of the observers' design, matrix W in Eq. (8.2-14) is partitioned as indicated in Eq. (A-15), where

$$M = \begin{bmatrix} 1 \\ K_1 \\ K_2 \end{bmatrix} \tag{8.2-16}$$

and

$$N = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{8.2-16a}$$

Note that M is a 3 x 1 matrix and N a 3 x 2 matrix.

Matrix T in Eq. (A-16) can now be computed from W_{22}, W_{21}, C_1 and C_2 in Eqs. (8.2-9), (8.2-12), (8.2-10) and (8.2-10a) as

$$T = \begin{bmatrix} -K_1 & 1 & 0 \\ -K_2 & 0 & 1 \end{bmatrix} \qquad (8.2\text{-}17)$$

Note, T in Eq. (8.2-17) is a 2 x 3 matrix. Similarly, H in Eq. (A-17) is computed from Eqs. (8.2-6a) and (8.2-17) to be

$$R = \begin{bmatrix} 0 & 0 \\ b_0 & b_1 \end{bmatrix} \qquad (8.2\text{-}18)$$

Matrices F and G in Eqs. (A-18) and (A-19) are computed from Eqs. (8.2-17), (8.2-6), (8.2-16) and (8.2-16a) to be

$$F = \begin{bmatrix} -K_1 & 1 \\ -K_2 - a_0 & -a_1 \end{bmatrix} \qquad (8.2\text{-}19)$$

$$G = \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} \qquad (8.2\text{-}20)$$

where

$$g_1 \equiv -K_1 (-\psi + K_1) + K_2 \qquad (8.2\text{-}20a)$$

$$g_2 \equiv -K_2 (K_1 - \psi) - a_0 K_1 - a_1 K_2 \qquad (8.2\text{-}20b)$$

Note R, F, and G are 2 x 2, 2 x 2, and 2 x 1 matrices respectively.

The elevation of matrices F, G, and H now allows one to write the matrix differential equation for $\underline{z}$ in Eq. (A-9) as two explicit first order equations. Using Eqs. (8.2-18), (8.2-19), (8.2-20), (8.2-20a), (8.2-20b) in Eq. (A-9) one has

$$\dot{z}_1 = -K_1 z_1 + z_2 + g_1 y$$

$$\dot{z}_2 = (-K_2 - a_0) z_1 - a_1 z_2 + g_2 y + b_0 r_1 + b_1 r_2 \qquad (8.2\text{-}21)$$

The first phase of the observer's design is now completed. Section 8.3 presents constraints on the observer design parameters K_1 and K_2 that are necessary for the system to behave as desired. Section 8.4 presents the design of the observer controller that provides estimates of a_0, a_1, b_0, and b_1 for use in Eq. (8.2-21).

8.3 Selection of Observer Parameters

To obtain a correct seat of allowable values for the observer design K_1 and K_2, the technique proposed in Appendix A is adopted. This technique uses a Liapunov function to show that the variation $\delta\underline{x}$ of the observer state vector estimate about the true state vector is asymptotically stable for a parameter range specification on K_1 and K_2. It should be noted however that this procedure will guarantee that the observer is designed correctly only under the assumption that the unknown plant parameters are known. The overall stability of the composite system, including the observer controller, shown in Fig. 6-1 has not been proved theoretically.

For the case under consideration, the perturbation vector $\delta \underline{x}$ defined in Eq. (A-21) is given by

$$\delta \underline{x} = \begin{bmatrix} \delta x_2 \\ \delta x_3 \end{bmatrix} \tag{8.3-1}$$

where the elements of $\delta \underline{x}$ in Eq. (8.3-1) are given by

$$\delta x_2 = \tilde{x}_2 - x_2 \tag{8.3-2}$$

$$\delta x_3 = \tilde{x}_3 - x_3 \tag{8.3-3}$$

A candidate Liapunov function is now defined as

$$V = (\delta \underline{x})^T \; L \; (\delta \underline{x}) \tag{8.3-4}$$

where L is selected to have the form

$$L = \begin{bmatrix} l_{22} & l_{12} \\ l_{12} & l_{22} \end{bmatrix} \tag{8.3-5}$$

The time derivative of V in Eq. (8.3-4) is denoted by $\dot{V}$ and is given by

$$\dot{V} = 2\, l_{11}\, (\delta x_2)\, (\delta \dot{x}_2) + 2\, l_{12}\, \delta \dot{x}_2\, \delta x_3 + 2\, l_{12}\, \delta x_2\, \delta \dot{x}_3$$
$$+ 2\, l_{22}\, \delta x_3\, \delta \dot{x}_3 \tag{8.3-6}$$

From Eqs. (8.3-2) and (8.3-3), $\delta \dot{x}_2$ and $\delta \dot{x}_3$ are given by

$$\delta \dot{x}_2 = \dot{\tilde{x}}_2 - \dot{x}_2$$
$$\delta \dot{x}_3 = \dot{\tilde{x}}_3 - \dot{x}_3 \tag{8.3-7}$$

Differentiating the expressions for $\tilde{x}_2$ and $\tilde{x}_3$ in Eq. (8.2-15) and substituting them in Eq. (8.3-7) yields

$$\begin{aligned} \delta \dot{x}_2 &= K_1 \ \dot{y} + \dot{z}_1 - \dot{x}_2 \\ \delta \dot{x}_3 &= K_2 \ \dot{y} + \dot{z}_2 - \dot{x}_3 \end{aligned} \qquad (8.3\text{-}8)$$

Combining the expressions for $\dot{x}_2$ and $\dot{x}_3$ given by Eqs. (8.2-4), (8.3-8), and (8.2-7a) yields

$$\begin{aligned} \delta \dot{x}_2 &= K_1 (-\psi x_1 + x_2) + \dot{z}_1 - x_3 \\ \delta \dot{x}_3 &= K_2 (-\psi x_1 + x_2) + \dot{z}_2 + a_0 x_2 + a_1 x_3 \\ &\quad - b_0 r_1 - b_1 r_2 \end{aligned} \qquad (8.3\text{-}9)$$

Equation (8.3-9) can be simplified by using the expressions for $\dot{z}_1$ and $\dot{z}_2$ found in Eq. (8.2-21). Equation (8.3-9) thus can be written as

$$\begin{aligned} \delta \dot{x}_2 &= K_1 (-\psi x_1 + x_2) - K_1 z_1 + z_2 + g_1 y - x_3 \\ \delta \dot{x}_3 &= K_2 (-\psi x_1 + x_2) + \Big[(-K_2 - a_0) z_1 - a_1 z_2 + g_2 y \\ &\qquad + b_0 r_1 + b_1 r_2 \Big] \\ &\qquad + a_0 x_2 + a_1 x_3 - b_0 r_1 \\ &\qquad - b_1 r_2 \end{aligned} \qquad (8.3\text{-}10)$$

Substituting $\delta \dot{x}_2$ and $\delta \dot{x}_3$ into Eq. (8.3-6) and using the expressions for g_1 and g_2 in Eqs. (8.2-20a) and (8.2-20b) yields after some algebraic manipulation the following result

$$-\dot{V} = \begin{bmatrix} \delta x_2 & \vdots & \delta x_3 \end{bmatrix} \left[\begin{array}{c|c} 2(K_1\, l_{11} + a_0 l_{12} + K_2 l_{12}) & -l_{11} + l_{12} K_1 \\ & -l_{22}\,(-a_0 - K_2) \\ & -a_1\, l_{12} \\ \hline -l_{11} + l_{12}\, K_1 & \\ -l_{22}\,(-a_0 - K_2) & 2\,(a_1\, l_{22} - l_{12}) \\ +\, a_1\, l_{12} & \end{array}\right] \begin{bmatrix} \delta x_2 \\ \delta x_3 \end{bmatrix} \qquad (8.3\text{-}11)$$

Note that Eq. (8.3-11) is independent of ψ , the parameter of the fixed plant $G_f(s)$.

For simplification, l_{11}, l_{12}, and l_{22} in Eq. (8.3-5) are selected as

$$\begin{aligned} l_{12} &= 1 \\ l_{22} &= 1 \\ l_{11} &= K_1\, l_{12} - l_{22}\,(-a_0 - K_2) + a_1\, l_{12} \\ &= K_1 + a_0 + K_2 + a_1 \end{aligned} \qquad (8.3\text{-}12)$$

Using the expressions for l_{12}, l_{22}, and l_{11} in Eq. (8.3-11) results in

$$-\dot{V} = \begin{bmatrix} \delta x_2 & \delta x_3 \end{bmatrix} \left[\begin{array}{c|c} 2\, K_1\, l_{11} + 2(a_0 + K_2) & 0 \\ \hline 0 & 2(a_1 - 1) \end{array}\right] \begin{bmatrix} \delta x_2 \\ \delta x_3 \end{bmatrix} \qquad (8.3\text{-}13)$$

In addition, V in Eq. (8.3-4) now has the form

$$V = \begin{bmatrix} \delta\; 2 & \delta x_3 \end{bmatrix} \left[\begin{array}{c|c} l_{11} & 1 \\ \hline 1 & 1 \end{array}\right] \begin{bmatrix} \delta x_2 \\ \delta x_3 \end{bmatrix} \qquad (8.3\text{-}14)$$

where l_{11} is defined by Eq. (8.3-12).

Equations (8.3-13) and (8.3-14) indicate that for V and $-\dot{V}$ to be positive definite one must have

$$l_{11} - 1 > 0 \quad (8.3\text{-}15)$$

$$a_1 > 1 \quad (8.3\text{-}15a)$$

$$2 K_1 l_{11} + 2 a_0 + 2 K_2 > 0 \quad (8.3\text{-}15b)$$

The constraints in Eqs. (8.3-15) to (8.3-15b) can be simplified by substituting l_{11} from Eq. (8.3-12). This yields

$$a_0 + K_1 + K_2 + a_1 > 1 \quad (8.3\text{-}16)$$

$$a_1 > 1 \quad (8.3\text{-}16a)$$

$$K_1 \left[a_0 + K_2 + a_1 + K_1\right] + a_0 + K_2 > 0 \quad (8.3\text{-}16b)$$

Since $a_0 > 0$ for a stable plant Gp(s), then if one selects $K_1 \geq 0$ and $K_2 \geq 0$, only the condition in Eq. (8.3-16a) need be assumed for the observer to be properly designed. The fact that V and $-\dot{V}$ are positive definite indicates that V is a Liapunov function and that lim $\delta \underline{x} = 0$ as $t \rightarrow \infty$. This condition implies that $\tilde{x}_2 \rightarrow x_2$ and $\tilde{x}_3 \rightarrow x_3$, the desired behavior for the observer.

It is now convenient to denote the observer estimates of the true plant parameters with a double hat, i.e., $\hat{\hat{a}}_0$, $\hat{\hat{a}}_1$, $\hat{\hat{b}}_0$, and $\hat{\hat{b}}_1$. These are time varying quantities which will replace a_0, a_1, b_0, and b_1 respectively in the modified observer configuration in Fig. 6-1. Specifically, g_1, g_2, $\dot{z}_1$ and $\dot{z}_2$ in Eqs. (8.2-20a), (8.2-20b), (8.2-21) become

$$
\begin{aligned}
g_1 &= -K_1 (-\Psi + K_1) + K_2 \\
g_2 &= -K_2 (K_1 - \Psi) - \hat{a}_0 K_1 - \hat{a}_1 K_2 \\
\dot{z}_1 &= -K_1 z_1 + z_2 + g_1 x_1 \\
\dot{z}_2 &= (-K_2 - \hat{a}_0) z_1 - \hat{a}_1 z_2 + g_2 x_1 + \hat{b}_0 r_1 + \hat{b}_1 r_2
\end{aligned}
\qquad (8.3\text{-}17)
$$

Equation (8.3-17), together with Eq. (8.2-15) and $y_1 \equiv x_1$, form the modified observer's dynamics and provide estimates $\tilde{x}_2$ and $\tilde{x}_3$ of x_2 and x_3 respectively. The observer parameters $\hat{a}_0$, $\hat{a}_1$, $\hat{b}_0$, $\hat{b}_1$ can be thought of as belonging to an $\hat{H}$ and $\hat{C}$ set of matrices similar to the matrices $\hat{H}$ and $\hat{C}$ defined in Chapter 4. The observer parameters are generated by an observer controller which is designed in Section 8.4 of this chapter. They are presented in Eq. (8.3-18) at this point because it is important to consider whether further constraints must be placed on the observer constants K_1 and K_2 to force the equilibrium condition

$$\tilde{e}_1 = \tilde{e}_2 = 0 \text{ to imply } e_1 = e_2 = 0.$$

The differential equations whose solutions determine the observer estimates of the plant parameters are derived+ in Section 8.4 These equations are

$$
\begin{aligned}
\dot{\hat{a}}_0 &= -\frac{1}{\gamma'_{11}} \tilde{w}_1 (p'_{12} \tilde{e}_1 + p'_{22} \tilde{e}_2) \\
\dot{\hat{a}}_1 &= -\frac{1}{\gamma'_{12}} \tilde{w}_2 (p'_{12} \tilde{e}_1 + p'_{22} \tilde{e}_2)
\end{aligned}
$$

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+These equations are required at this point for the determination of sufficient conditions that force the system to have a unique equilibrium point at $\tilde{\mathbf{e}} = 0$, where $\tilde{\mathbf{e}} = \mathbf{y} - \tilde{\mathbf{w}}$.

$$\hat{\dot{b}}_0 = \frac{1}{\gamma_{21}'} r_1 (p_{12}' \tilde{e}_1 + p_{22}' \tilde{e}_2)$$

$$\hat{\dot{b}}_1 = \frac{1}{\gamma_{22}'} r_2 (p_{12}' \tilde{e}_1 + p_{22}' \tilde{e}_2) \qquad (8.3\text{-}18)$$

In Eq. (8.3-18), γ_{11}', γ_{12}', γ_{21}', and γ_{22}' are positive design constants, while p_{12}' and p_{22}' have restrictions similar to parameters p_{12} and p_{22} in Eq. (7.2-21). At this point the notation is changed to that of the previous chapters, with $\tilde{w}_1$ and $\tilde{w}_2$ now being the observer estimates of the unknown plant's state variables w_1 and w_2. The model's state variables are as in Chapters 4, 5 and 7, denoted by y_1 and y_2. Before examining the equilibrium condition of the equations given by Eqs. (8.3-18), (8.3-17), (7.2-21) (with $\tilde{e}_1$ and $\tilde{e}_2$ replacing e_1 and e_2 respectively) and (7.2-8), it should be noted that Eqs. (7.2-21) and (8.3-18) indicate that $\tilde{e}_1 = \tilde{e}_2 = 0$ is part of the equilibrium point. If $p_{12}' \neq p_{12}$ and $p_{22}' \neq p_{22}$, then this equilibrium point is unique. It is now assumed therefore that the preceeding inequality conditions hold.

Before proceeding, it is convenient to rewrite Eq. (8.2-15) to reflect the changed notation

$$\tilde{x}_1 \equiv x_1$$

$$\tilde{w}_1 = K_1 x_1 + z_1$$

$$\tilde{w}_2 = K_2 x_1 + z_2 \qquad (8.3\text{-}18a)$$

Note that x_1 is the output of $G_f(s)$, $\tilde{x}_2 \equiv \tilde{w}_1$, $x_3 \equiv \tilde{w}_2$, and $y \equiv x_1$.

Using the model Eq. (7.2-8) and the relationships in Eq. (8.3-18a), the derivatives of $\tilde{e}_1$ and $\tilde{e}_2$, which are defined as

$$\tilde{e}_1 = y_1 - \tilde{w}_1$$
$$\tilde{e}_2 = y_2 - \tilde{w}_2 \qquad (8.3\text{-}19)$$

can be evaluated. The result is

$$\dot{\tilde{e}}_1 = \dot{y}_1 - \dot{\tilde{w}}_1 = y_2 - K_1 \dot{x}_1 - \dot{z}_1$$
$$\dot{\tilde{e}}_2 = \dot{y}_2 - \dot{\tilde{w}}_2 = (-\hat{a}_0 y_1 - \hat{a}_1 y_2 + \hat{b}_0 r_1 + \hat{b}_1 r_2)$$
$$-(K_2 \dot{x}_1 + \dot{z}_2) \qquad (8.3\text{-}20)$$

If one substitutes the expressions for $\dot{z}_1$ and $\dot{z}_2$ from Eq. (8.3-17), then Eq. (8.3-20) can be simplified to be

$$\dot{\tilde{e}}_1 = y_2 - K_1 \dot{x}_1 - z_2 - g_1 x_1 + K_1 z_1$$
$$\dot{\tilde{e}}_2 = -\hat{a}_0 y_1 - \hat{a}_1 y_2 + \hat{b}_0 r_1 + \hat{b}_1 r_2$$
$$-K_2 \dot{x}_1 - (-K_2 - \hat{\hat{a}}_0) z_1 + \hat{\hat{a}}_1 z_2 - g_2 x_1 - \hat{\hat{b}}_0 r_1 - \hat{\hat{b}}_1 r_2$$
$$(8.3\text{-}21)$$

At equilibrium, $\tilde{e}_1 = \tilde{e}_2 = 0$ which implies $\dot{\tilde{e}}_1 = \dot{\tilde{e}}_2 = 0$. This condition applied to Eq. (8.3-21) yields

$$0 = y_2 - K_1 \dot{x}_1 - z_2 - g_1 x_1 + K_1 z_1 \qquad (8.3\text{-}22)$$
$$0 = -\hat{a}_0 y_1 - \hat{a}_1 y_2 + \hat{b}_0 r_1 + \hat{b}_1 r_2$$
$$-K_2 \dot{x}_1 + (K_2 + \hat{\hat{a}}_0) z_1 + \hat{\hat{a}}_1 z_2 - g_2 x_1$$
$$-\hat{\hat{b}}_0 r_1 - \hat{\hat{b}}_1 r_2 \qquad (8.3\text{-}23)$$

But the condition $\tilde{e}_1 = \tilde{e}_2 = 0$ implies from Eqs. (8.3-19) and (8.3-18a)

$$y_1 = \tilde{w}_1 = K_1 x_1 + z_1 \tag{8.3-24}$$

$$y_2 = \tilde{w}_2 = K_2 x_1 + z_2 \tag{8.3-25}$$

Rearranging Eqs. (8.3-24) and (8.3-25) results in

$$z_1 = y_1 - K_1 x_1$$

$$z_2 = y_2 - K_2 x_1 \tag{8.3-26}$$

Substituting Eq. (8.3-26) into Eq. (8.3-22) yields

$$0 = y_2 - K_1 \dot{x}_1 - y_2 + K_2 x_1 - g_1 x_1 + K_1 g_1 - K_1^2 x_1 \tag{8.3-27}$$

Substituting g_1 from Eq. (8.3-17) in Eq. (8.3-27) yields

$$0 = -K_1 x_1 + K_2 x_1 + K_1 y_1 - K_1^2 x_1 - x_1 \left[K_1 \psi - K_1^2 + K_2\right] \tag{8.3-28}$$

Equation (8.3-28) can now be simplified using Eq. (8.2-4) for x_1

$$0 = -K_1 \left[- \psi x_1 + w_1\right] + K_2 x_1 + K_1 y_1 - K_1^2 x_1 - K_1 x_1 \psi + K_1^2 x_1 - K_2 x_1 \tag{8.3-29}$$

Collecting terms gives

$$0 = -K_1 w_1 + K_1 y_1 \tag{8.3-29a}$$

Since $y_1 - w_1 = e_1$, then Eq. (8.3-29a) becomes

$$0 = K_1 e_1 \tag{8.3-30}$$

Thus if $\tilde{e}_1 \rightarrow 0$ and $K_1 > 0$, then e_1 will be forced to approach zero. If $e_1 = 0$, then $e_2 = \dot{e}_1$ also equals zero. Thus $y_1 \equiv w_1$ and $y_2 \equiv w_2$. Returning to the equilibrium equation, Eq. (8.3-23), the substitution for z_1 and z_2 results in

$$\begin{aligned} 0 = {} & (\hat{b}_0 - \hat{\hat{b}}_0)\, r_1 + (\hat{b}_1 - \hat{\hat{b}}_1)\, r_2 \\ & - \hat{a}_0\, y_1 - \hat{a}_1\, y_2 - K_2\, \dot{x}_1 + (K_2 + \hat{\hat{a}}_0)\,(y_1 - K_1\, x_1) \\ & + \hat{\hat{a}}_1\,(y_2 - K_2\, x_1) - g_2\, x_1 \end{aligned} \qquad (8.3\text{-}31)$$

Collecting terms and using g_2 from Eq. (8.3-17) in Eq. (8.3-31) results in

$$\begin{aligned} 0 = {} & r_1\,(\hat{b}_0 - \hat{\hat{b}}_0) + r_2\,(\hat{b}_1 - \hat{\hat{b}}_1) + y_1\,(\hat{\hat{a}}_0 - \hat{a}_0) + y_2\,(\hat{\hat{a}}_1 - \hat{a}_1) \\ & - K_2\, \dot{x}_1 + K_2\, y_1 - K_1\, K_2\, x_1 - \hat{\hat{a}}_0\, K_1\, x_1 - \hat{\hat{a}}_1\, K_2\, x_1 \\ & - \hat{\hat{a}}_1\, K_2\, x_1 - x_1 \left[- K_1\, K_2 + K_2 \psi - \hat{\hat{a}}_0\, K_1 - \hat{\hat{a}}_1\, K_2 \right] \end{aligned} \qquad (8.3\text{-}32)$$

Using Eq. (8.2-4) for $\dot{x}_1$ and cancelling like terms in Eq. (8.3-32) yields

$$\begin{aligned} 0 = {} & r_1\,(\hat{b}_0 - \hat{\hat{b}}_0) + r_2\,(\hat{b}_1 - \hat{\hat{b}}_1) + y_1\,(\hat{\hat{a}}_0 - \hat{a}_0) + y_2\,(\hat{\hat{a}}_1 - \hat{a}_1) \\ & - K_2 \left[- \psi\, x_1 + w_1 \right] + K_2\, y_1 - K_2\, \psi\, x_1 \end{aligned} \qquad (8.3\text{-}33)$$

With additional cancellation one has

$$0 = r_1 (\hat{b}_0 - \hat{\hat{b}}_0) + r_2 (\hat{b}_1 - \hat{\hat{b}}_1) + y_1 (\hat{\hat{a}}_0 - \hat{a}_0) + y_2 (\hat{\hat{a}}_1 - \hat{a}_1) + K_2 (y_1 - w_1) \qquad (8.3\text{-}34)$$

Using Eq. (7.2-9a), Eq. (8.3-34) can be expressed as

$$0 = r_1 (\hat{b}_0 - \hat{\hat{b}}_0) + r_2 (\hat{b}_1 - \hat{\hat{b}}_1) + (\hat{\hat{a}}_0 - \hat{a}_0)(e_1 + w_1) + (\hat{\hat{a}}_1 - \hat{a}_1)(e_2 + w_2) + K_2 e_1 \qquad (8.3\text{-}35)$$

Since Eq. (8.3-30) for $K_1 > 0$ implies $e_1 = 0$, then $e_2 = \dot{e}_1$ also equals zero, and Eq. (8.3-35) becomes

$$0 = r_1 (\hat{b}_0 - \hat{\hat{b}}_0) + r_2 (\hat{b}_1 - \hat{\hat{b}}_1) + (\hat{\hat{a}}_0 - \hat{a}_0) w_1 + (\hat{\hat{a}}_1 - \hat{a}_1) w_2 \qquad (8.3\text{-}36)$$

Furthermore, since $e_1 = e_2 = 0$ implies+ $\hat{a}_0 = a_0$, $\hat{b}_0 = b_0$, $\hat{a}_0 = a_0$, and $\hat{a}_1 = a_1$, then Eq. (8.3-36) becomes

$$0 = r_1 (b_0 - \hat{\hat{b}}_0) + r_2 (b_1 - \hat{\hat{b}}_1) + w_1 (\hat{\hat{a}}_0 - a_0) + (\hat{\hat{a}}_1 - a_1) w_2 \qquad (8.3\text{-}37)$$

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+Subject to the conditions established in Section 5.4.

Equation (8.3-37) indicates that $\hat{b}_0 = b_0$, $\hat{b}_1 = b_1$, $\hat{a}_0 = a_0$, and $\hat{a}_1 = a_1$ is indeed an equilibrium condition. However, from Eq. (8.3-37) alone other values of the observer parameters might be able to satisfy Eq. (8.3-37).

The condition $\tilde{e}_1 = \tilde{e}_2 = 0$ signifies that the observer can estimate the unavailable state vector w with zero error if the observer parameters $\hat{\hat{b}}_0$, $\hat{\hat{b}}_1$, $\hat{\hat{a}}_0$, $\hat{\hat{a}}_1$ happen to satisfy Eq. (8.3-37). In this case one would say that the observer is "perfect", as it functions with zero error.

8.4 Design of Observer Controller Dynamics

In this section the observer controller dynamics indicated previously in Eq. (8.3-18) are derived for the observer configuration indicated by Eq. (8.3-17). This derivation assumes that the model's state vector $\underline{y}$ in Fig. 6-1 is arbitrarily close to the unknown plant's state vector $\underline{w}$. If one defines $\tilde{\underline{e}}$ as

$$\tilde{\underline{e}} = \underline{w} - \tilde{\underline{w}} \tag{8.4-1}$$

then $\tilde{\underline{e}}$ can be used to generate a control law to adjust $\hat{a}_0$, $\hat{b}_0$, and $\hat{b}_1$ in the observer so that $\tilde{\underline{w}} \rightarrow \underline{w}$ and thus $\tilde{\underline{e}} \rightarrow 0$.

With $\tilde{\underline{e}}$ as in Eq. (8.4-1), let a candidate Liapunov function for the observer dynamics be

$$V = \tilde{\underline{e}}^T \; P^T \; \tilde{\underline{e}} + \gamma_{11}' a_0''^{\,2} + \gamma_{12}' a_1''^{\,2} + \gamma_{21}' b_0''^{\,2} + \gamma_{22}' b_1''^{\,2} \qquad (8.4\text{-}2)$$

where

$$P' = \begin{bmatrix} p_{11}' & p_{12}' \\ p_{12}' & p_{22}' \end{bmatrix} > 0$$

$$\tilde{\underline{e}} = \begin{bmatrix} \tilde{e}_1 \\ \tilde{e}_2 \end{bmatrix}$$

$$a_0'' = a_0 - \hat{a}_0$$

$$a_1'' = a_1 - \hat{a}_1$$

$$b_0'' = b_0 - \hat{b}_0$$

$$b_1'' = b_1 - \hat{b}_1$$

$$\gamma_{ij}' > 0 \quad i,j = 1, 2$$

and

$$\tilde{e}_1 = w_1 - \tilde{w}_1$$

$$\tilde{e}_2 = w_2 - \tilde{w}_2 \qquad (8.4\text{-}2a)$$

where the dynamics of $\underline{w}$ are given in Eqs. (7.2-3), (7.2-4).

The time derivative of V is computed as

$$\dot{V} = 2\, p_{11}' \, \tilde{e}_1 \, \dot{\tilde{e}}_1 + 2\, p_{12}' \, \dot{\tilde{e}}_1 \, \tilde{e}_2 + 2\, p_{12}' \, \tilde{e}_1 \, \dot{\tilde{e}}_2 + 2\, p_{22}' \, \tilde{e}_2 \, \dot{\tilde{e}}_2 + 2\, \gamma_{11}' a_0'' \, \dot{a}_0'' + 2\, \gamma_{12}' a_1'' \, \dot{a}_1'' + 2\, \gamma_{21}' b_0'' \, \dot{b}_0 + 2\, \gamma_{22} \, b_1'' \, \dot{b}_1'' \qquad (8.4\text{-}3)$$

From Appendix B, the time derivatives of $\tilde{e}_1$, $\tilde{e}_2$ are shown to be

$$\dot{\tilde{e}}_1 = -K_1 \tilde{e}_1 + \tilde{e}_2$$

$$\dot{\tilde{e}}_2 = -(a_0 + K_2)\tilde{e}_1 - a_1 \tilde{e}_2 - a_0'' \tilde{w}_1 - a_1'' \tilde{w}_2 + b_0'' r_1 + b_1'' r_2 \tag{8.4-4}$$

The expressions for $\dot{\tilde{e}}_1$ and $\dot{\tilde{e}}_2$ in Eq. (8.4-4) are now substituted in the expression for $\dot{V}$ in Eq. (8.4-3) to yield

$$-\dot{V} = [\tilde{e}_1 \; \tilde{e}_2] \left[\begin{array}{c|c} 2p_{11}' K_1 + 2p_{12}' (a_0 + K_2) & -p_{11}' + p_{12}' K_1 + p_{12}' a_1 + p_{22}' (a_0 + K_2) \\ \hline -p_{11}' + p_{12}' K_1 + p_{12}' a_1 + p_{22}' (a_0 + K_2) & -2p_{12}' + 2p_{22}' a_1 \end{array}\right] \begin{bmatrix} \tilde{e}_1 \\ \tilde{e}_2 \end{bmatrix}$$

$$+2a_0'' \left(-\gamma_{11}' \dot{a}_0'' + \tilde{w}_1 (p_{12}' \tilde{e}_1 + p_{22}' \tilde{e}_2)\right) \tag{8.4-5}$$

$$+2a_1'' \left(-\gamma_{12}' \dot{a}_1'' + \tilde{w}_2 (p_{12}' \tilde{e}_1 + p_{22}' \tilde{e}_2)\right)$$

$$+2b_0'' \left(-\gamma_{21}' \dot{b}_0'' - r_1 (p_{12}' \tilde{e}_1 + p_{22}' \tilde{e}_2)\right) + 2b_1'' \left(-\gamma_{22} \dot{b}_1'' - r_2 (p_{12}' \tilde{e}_1 + p_{22}' \tilde{e}_2)\right)$$

The expression for $-\dot{V}$ in Eq. (8.4-5) can be made positive definite by letting

$$-\dot{V} = \left[\tilde{e}_1 \; \tilde{e}_2\right] \; [Q'] \; \begin{bmatrix} \tilde{e}_1 \\ \tilde{e}_2 \end{bmatrix} \tag{8.4-6}$$

with the following constraints

$$\dot{a}_0'' = \frac{1}{\gamma_{11}'} \tilde{w}_1 (p_{12}' \tilde{e}_1 + p_{22}' \tilde{e}_2)$$

$$\dot{a}_1'' = \frac{1}{\gamma_{12}'} \tilde{w}_2 (p_{12}' \tilde{e}_1 + p_{22}' \tilde{e}_2)$$

$$\dot{b}_0'' = \frac{-1}{\gamma_{21}'} r_1 (p_{12}' \tilde{e}_1 + p_{22}' \tilde{e}_2)$$

$$\dot{b}_1'' = \frac{-1}{\gamma_{22}'} r_2 \left(p_{12}' \tilde{e}_1 + p_{22}' \tilde{e}_2\right) \qquad (8.4\text{-}7)$$

Since in Eq. (8.4-2a), a_0, a_1, and b_0 are constant,

$$\dot{a}_0'' = -\dot{\hat{a}}_0 \qquad \dot{a}_1'' = -\dot{\hat{a}}_1$$

$$\dot{b}_0'' = -\dot{\hat{b}}_0 \qquad \dot{b}_1'' = -\dot{\hat{b}}_1 \qquad (8.4\text{-}8)$$

Equation (8.3-18) follows immediately from Eqs. (8.4-7) and (8.4-8). Matrix Q' in Eq. (8.4-6) is selected positive definite in an identical manner to Q in Chapter 5. Matrix P' in Eq. (8.4-2) is also selected identically to matrix P in Eq. (4.4-3).

If the model output $\underline{y}$ is sufficiently close to $\underline{w}$, then the above observer controller will guarantee that $\tilde{\underline{e}} \rightarrow 0$ and $\tilde{\underline{w}} \rightarrow \underline{w}$. It should be noted that in the complete system mechanization, $\tilde{\underline{e}}$ will not be generated by Eq. (8.4-1), since vector $\underline{w}$ is not available, but will be generated by $\tilde{\underline{\varepsilon}}$ defined as

$$\tilde{\underline{\varepsilon}} = \underline{y} - \underline{w} \qquad (8.4\text{-}9)$$

8.5 Simulation Results

The complete system illustrated in Fig. 6-1 and designed in Sections 8.1 to 8.4 was simulated with a computer program similar to the one used in Section 7.3. The details of the simulations performed using the dynamic modified observer are as follows:

1) The unknown portion of the plant has the transfer function given by Eq. (7.2-1) with a_0, a_1, and b_0 the unknown parameters. Parameter b_1 is known with value $b_1 = 1$.

2) The known portion of the plant has the transfer function given by Eq. (8.2-1). Parameter ψ is known with value $\psi = 25$.

3) The plant's input is the same as was used in the simulations described in Section 7.3. The input $r_1(t)$, its derivative $r_2(t)$ and values of Z and T_1 are given by Eqs. (7.3-2), (7.3-3), and (7.3-4).

4) The identification controller was designed as given in Section 7.2, using initial values for a_0 and a_1 given by Eq. (7.2-22). The initial values were those used as initial conditions for the controller's dynamics. The p_{12} and p_{22} parameters were selected as $p_{12} = 1$, $p_{22} = .50$. The γ_{ij} parameters for the identification controller were selected as

$$\frac{1}{\gamma_{11}} = .20$$

$$\frac{1}{\gamma_{12}} = .10$$

$$\frac{1}{\gamma_{21}} = .00015 \qquad (8.5\text{-}1)$$

The controller dynamics are now given by Eq. (7.2-21) with e_1 and e_2 replaced by their respective observer estimates, $\tilde{e}_1$ and $\tilde{e}_2$. The identification controller dynamics thus become

$$\dot{\hat{a}}_0 = .20\ y_1\ (\tilde{e}_1 + .5\ \tilde{e}_2)$$
$$\dot{\hat{a}}_1 = .10\ y_2\ (\tilde{e}_1 + .5\ \tilde{e}_2)$$
$$\dot{\hat{b}}_0 = .00015\ r_1\ (\tilde{e}_1 + .5\ \tilde{e}_2) \qquad (8.5\text{-}2)$$

The initial conditions for Eq. (8.5-2) are

$$\hat{a}_0(0) = 100.0$$

$$\hat{a}_1(0) = 10.0$$

$$\hat{b}_0(0) = 2.0 \qquad (8.5\text{-}3)$$

5) The initial condition vectors $\underline{w}(0)$ and $\underline{y}(0)$ were selected to be zero.[+] Thus the initial error state $\underline{e}$ (0) = 0.

6) The modified observer has the dynamics indicated by Eq. (8.3-17). Initials conditions for z_1 and z_2 at t = 0 were selected as $z_1(0) = z_2(0) = 0$. The values of K_1 and K_2 used in the simulation are $K_1 = 21.0$ and $K_2 = 21.0$. These values satisfy constraints found in Sections 8.2 and 8.3 that insure that the observer is designed correctly and that $\tilde{\underline{e}} \to 0$ will force $\underline{e} \to 0$.

7) The observer controller's dynamics are those given by Eq. (8.3-18). The values of p_{12}' and p_{22}' were selected abritrarily in inverse ratio to p_{12}/p_{22}. Since

$$p_{12}/p_{22} = {}^{2.0}/_{1.0} = 2.0 \qquad (8.5\text{-}4)$$

p_{12}' was set equal to 1.0 and p_{22}' equal to .50 so that

$$p_{12}'/p_{22}' = {}^{1.0}/_{2.0} = .50 \qquad (8.5\text{-}4a)$$

The reason for the above choice of p_{12}' and p_{22}' was to keep the equilibrium point $\tilde{e}_1 = \tilde{e}_2 = 0$ well defined (unique), as discussed in Section 8.3.

Let us define α as

$$\alpha = \gamma'_{1j} / \gamma_{1j} \qquad (8.5\text{-}4b)$$

- - - - - -

[+]This condition corresponds to the plant being at equilibrium when its input is applied.

For α greater than unity, the observer controller's dynamics can be thought of as moving slower than those of the identification controller. For α less than unity, the opposite will be true. The performance of the composite system for $\alpha = 1$, $\alpha > 1$, and $\alpha < 1$ values were simulated.

Equation (8.3-18) can now be written as

$$
\begin{aligned}
\dot{\widehat{a}}_0 &= -\frac{.20}{\alpha}\,\tilde{w}_1\,(.5\,\tilde{e}_1 + \tilde{e}_2) \\
\dot{\widehat{a}}_1 &= -\frac{.10}{\alpha}\,\tilde{w}_2\,(.5\,\tilde{e}_1 + \tilde{e}_2) \\
\dot{\widehat{b}}_0 &= \frac{.00015}{\alpha}\,r_1\,(.5\,\tilde{e}_1 + \tilde{e}_2) \qquad (8.5\text{-}5)
\end{aligned}
$$

The initial conditions on $\widehat{a}_0$, $\widehat{a}_1$, and $\widehat{b}_0$ were the same as $\hat{a}_0$, $\hat{a}_1$, $\hat{b}_0$ respectively, given in Eq. (8.5-3).

8) The step size h used in the numerical integration of the systems differential equations is h = .0005. The procedure used was identical to the one used in Section 7.3.

9) The true plant parameters are $a_0 = 121$, $a_1 = 22$, and $b_0 = 1$. Thus Gp(s) in Eq. (7.2-1) has a double pole at s = -11.

Figure 8-1 shows the results of three simulation runs using α values of .2, 1, and 5. As can be seen from Fig. 8-1, the zero parameter $\hat{b}_0$ of the model moves rather quickly close to the plant value b_0 in the time interval t = 1 to t = 2. Among the three values of α, the identification for the case $\alpha = .2$ is clearly superior. This fact is true for the identification of a_1 and a_0, as can be seen from the temporal variation of $\hat{a}_1(t)$ and $\hat{a}_0(t)$ respectively. The identification of a_1 for the case $\alpha = .2$ can be considered successful,

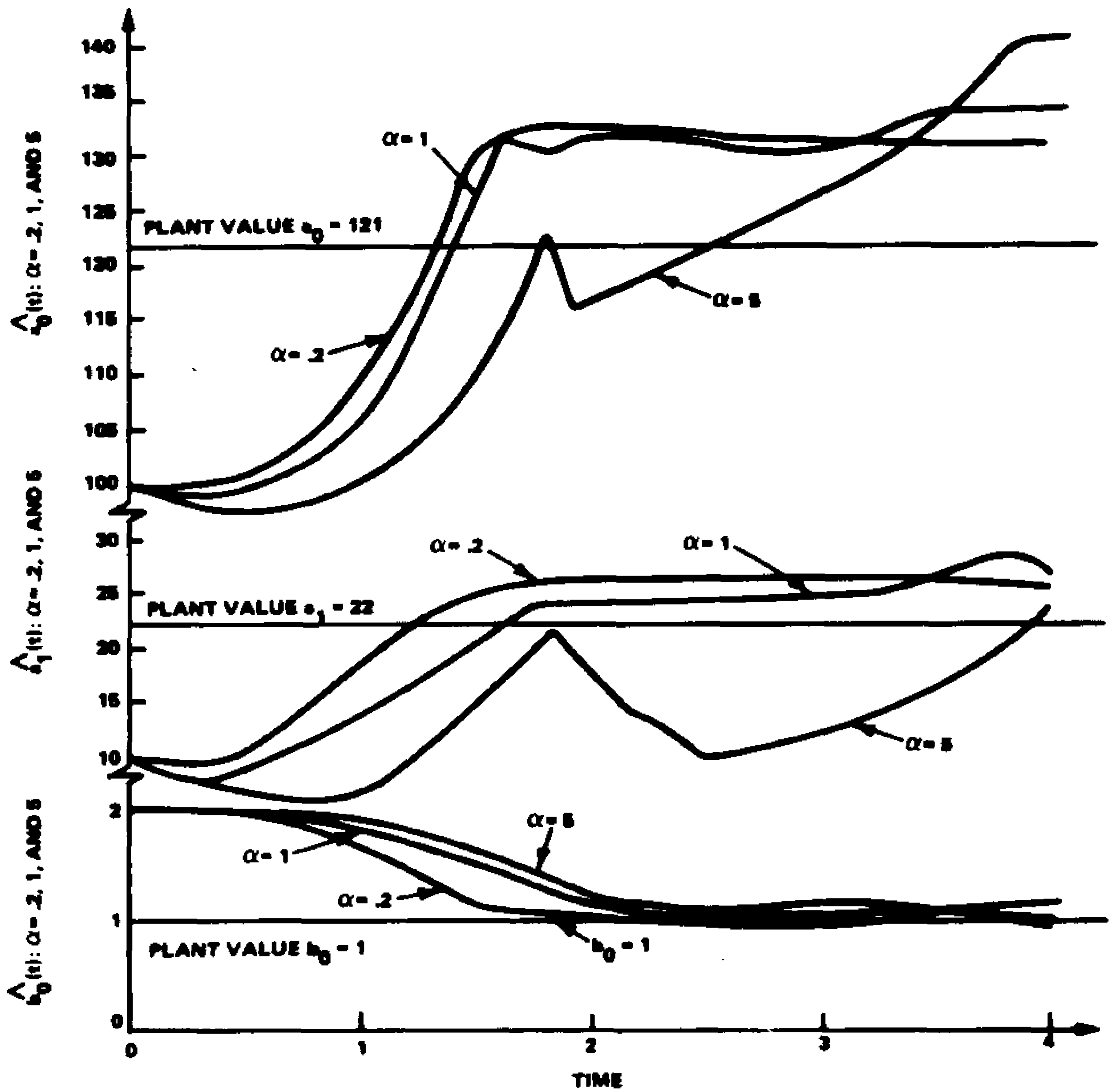

Figure 8-1. Model Parameter Dynamics With System Observer And Bounded Plant Input - Example 8-1

$\hat{a}_1(t)$ reaching a steady state value of approximately $\hat{a}_1(t) = 26$ for $t \geq 2$. The identification of a_0, while not as good as for b_0 and a_1, is good, with $\hat{a}_0(t)$ reaching a steady state value of approximately 130 for $t \geq 2$.

Figure 8-2 plots the state error variables $\tilde{e}_1$ and $\tilde{e}_2$ starting from the initial condition $\tilde{e}_1 = \tilde{e}_2 = 0$ for the $\alpha = .2$ case. The cyclic error portrait illustrated was evidenced in all the observer runs and is believed to indicate a good degree of stability for the composite system. Truncation error introduced by the numerical integration scheme used prevents however a definite answer to the question of whether $\tilde{e}_1$ and $\tilde{e}_2$ asymptotically go to zero as $t \rightarrow \infty$, or do they oscillate about the origin in a limit cycle mode. For the step size used, no instability was evidenced for the bounded plant input.

To further illustrate the advantage of using a learning modified observer configuration, the case of identification when the plant input is unbounded was simulated. This simulation was accomplished by letting T_1 in Eq. (7.3-4) equal infinity, so that $r_1(t)$ and $r_2(t)$ form the input state vector for all time. This input, given by Eq. (7.3-2), will lead to an observer estimate having a growing unbounded error if the observer is designed conventionally with an incorrect knowledge of the plant zero parameter b_0. This would surely cause the identification scheme designed in Chapters 4 and 5 to yield erroneous model parameter variations, further increasing the system's output error vector. The total system in such a case would quickly become unstable.

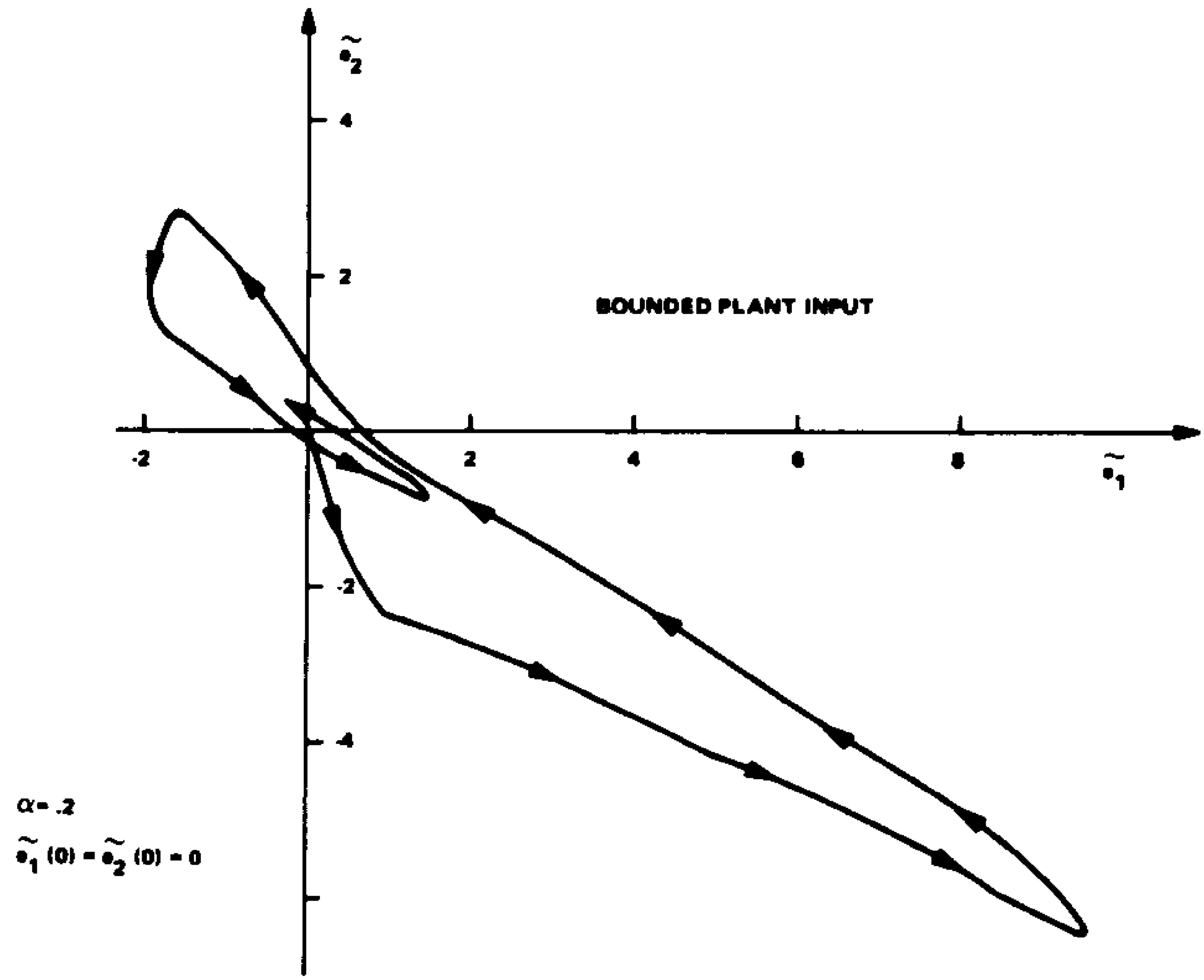

Figure 8-2. Observer-Model Error State Variable Portrait - Example 8-1

As indicated in Fig. 8-3 however, using the modified observer with its own observer dynamics results in successful identification, especially for the case $\alpha = 1$ and the $\hat{b}_0$ and $\hat{a}_1$ variations. The system eventually went unstable by t = 3 due to truncation error in the numerical integration scheme, which was expected since the input $r_1(t)$ is parabolic and builds up quite rapidly.

Figure 8-4 shows the output error $\tilde{e}_1$ and $\tilde{e}_2$ state variable portrait which is similar to that of Figure 8-2. This portrait indicated a stable system, possibly asymptotic, until the numerical integration errors made the system runaway for $t \geq 3$.

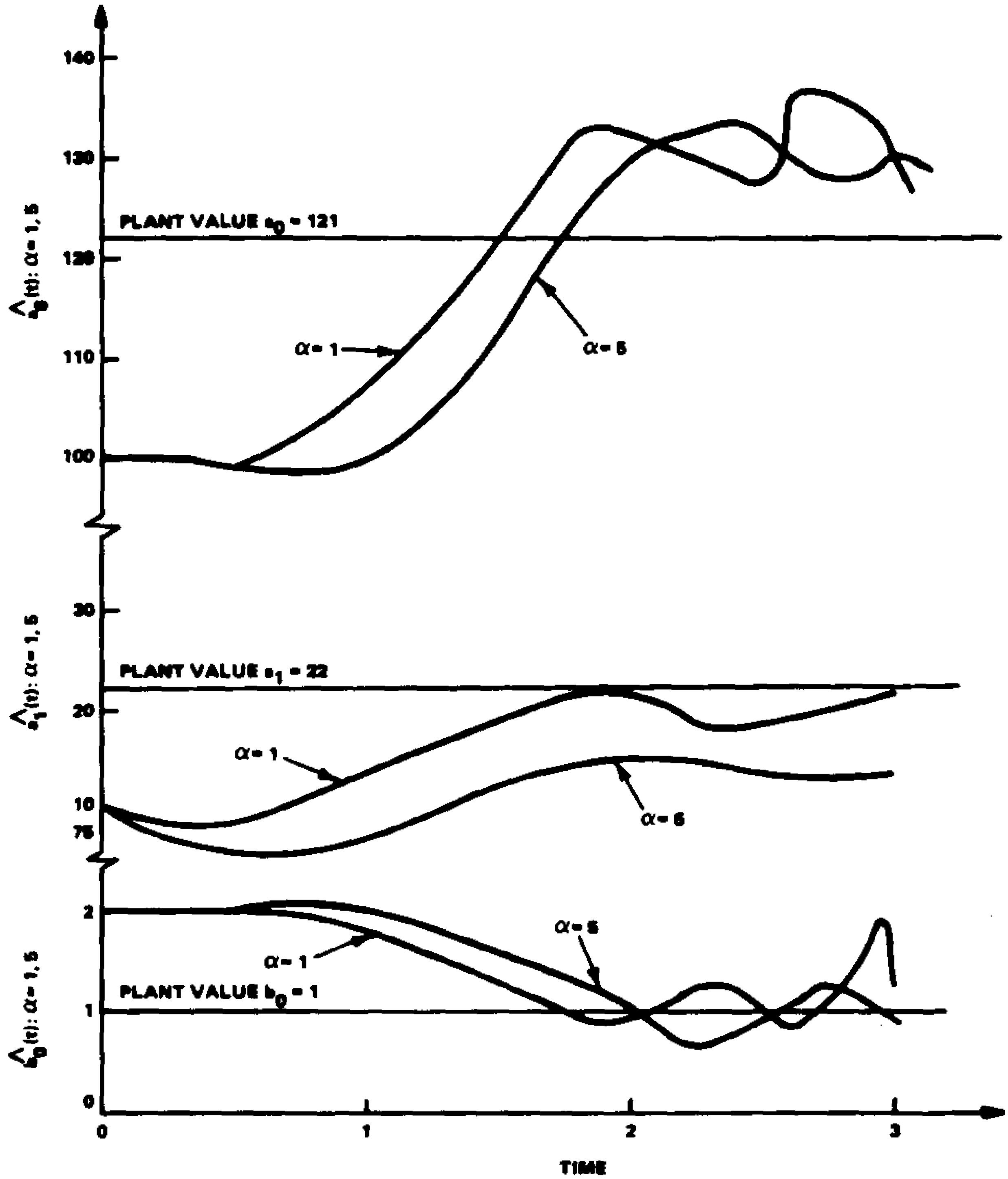

Figure 8-3. Model Parameter Dynamics With System Observer And Unbounded Plant Input - Example 8-2

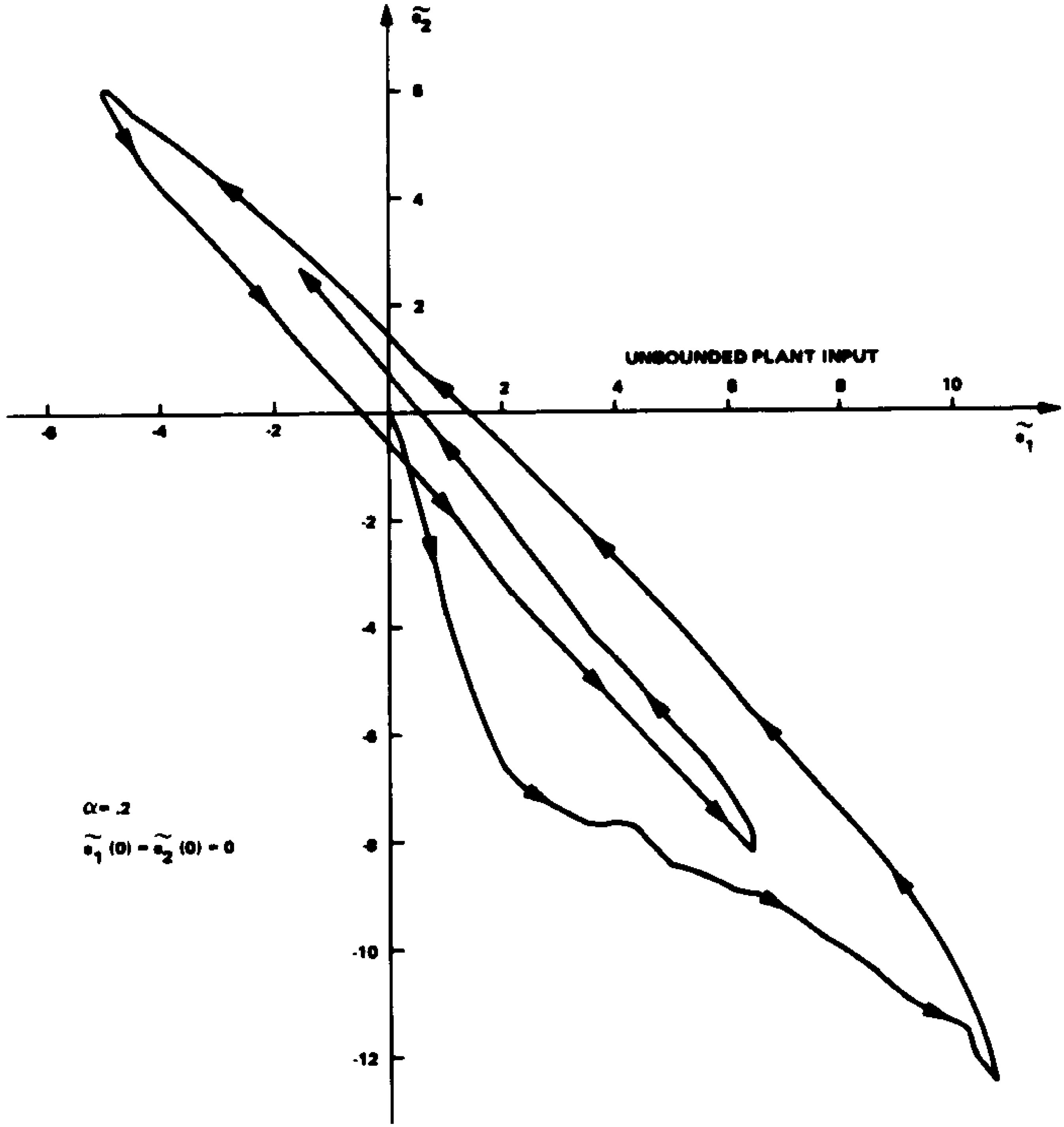

Figure 8-4. Observer-Model Error State Variable Portrait - Example 8-2

CHAPTER 9

SUMMARY AND CONCLUSIONS

9.1 Summary

This study concerned itself with the design and application of a class of identification controllers in what is known as the learning model identification configuration. The function of the identification controller in this configuation is to adjust the time-varying parameters of a model of the unknown plant so that they converge to their corresponding plant parameters. This function is by definition the identification task.

The class of plants considered were assumed linear, observable, controllable, and having fixed or slowly varying parameters. They have a single input time function which is the normal plant operating signal and is available for use by the identification controller. No portion of the plant input however is generated by the identification controller. This constraint that forces the identification to be done using only normal plant operating signals is highly desirable although not met by numerous other identification techniques. This constraint is desirable since signals generated by identification schemes if applied to the plant would likely disturb the normal operation of the plant. The identification controllers using the model parameters adjusting mechanisms operate in real-time and in no way disturb the plant.

In general, the plant can be either fully or partially unknown. The most general configuration considered has the plant input-output time functions separated by an unknown transfer function of known dimension followed in tandem by a known transfer function. The state vector of the unknown portion of the plant is assumed available, either directly or through a state variable estimator for use by the identification controller.

The basic design tool used to generate the class and dynamics of the identification controllers is Liapunov's Second or Direct method. While similar equations appear in what is known as the "model-reference control area," they often are not directly related to a Liapunov function. The importance of using a Liapunov function as a design tool is related to the fact that the identification controllers are inherently nonlinear, and that Liapunov theory indicates the performance of nonlinear systems without having to solve the system perturbational equations.

The relationship of the identification controller to the Liapunov function is described in Chapter 4. The design results obtained in Chapter 5 are applied to the illustrative example in Chapter 7. The results of the simulation are documented in Chapter 7. These results indicate the performance of the identification controllers and verify numerous theoretical conditions.

The fact that the identification controllers require the state output vectors to be available led to the second research activity of this study, namely, the design of an observer. In this study, the observer was designed to estimate the system state output vector.

However, the design of an observer requires the knowledge of the plant parameters which are unknown for the problem under consideration. In view of this situation, the design philosophy developed in Chapter 6 was formulated. This philosophy was used as a starting point for the design of a modified observer. The actual design of the modified observer is given in Chapter 8. In this chapter a fixed known plant was added between the unknown plant and the point where a measured plant scalar output is assumed available.

The class of identification controllers that were developed in this study are shown to be applicable to a wide variety of unknown plants. As shown in the controller implementation block diagrams for the illustrative examples, these controllers can easily be implemented in real-time with the use of basic analog computer electronics such as operational amplifiers, summers, and multipliers. In addition, the use of the identification technique for off-line identification, given stored plant data, becomes obvious from the study of the illustrative example.

The simplicity of the system mechanization together with the straightforward design procedure makes the identification technique particularly attractive from an economic point of view. Furthermore, the study derives simple criterion by which it can be determined whether or not identification has taken place. Once it is ascertained that the plant input satisfies the conditions given in Section 5.4, it was shown that reduction of the system error vector to zero assures that the identification has been accomplished. This condition was shown to be true for both the observer and non-observer cases.

9.2 Discussion of Results

In Chapter 4, the basic dynamical plant-model error equations that form the basis for all subsequent design are derived. These error equations contain the variations of the instantaneous model parameters about the true plant parameters. While these quantities are not directly available for design, their time derivatives are available and form the basis for the controller structure.

The design procedure for the coefficients of the identification controller developed in Chapter 5 yielded controller parameters which insured the global asymptotic stability of the composite system. The selection of the parameters of the P matrix that weigh the error state variables in the identification controller are determined by the use of an index of performance. This index of performance is related to the time derivative of the Liapunov function of the system. Terms selected by specifying relative weighting of the elements in the integrand allow the designer to control the system identification time.

A consideration studied in detail in Chapter 5 is the requirements necessary to force the model parameters to approach the plant parameters when the system error vector goes to zero. A test is derived in Chapter 5 from a study of the equilibrium conditions of the composite plant, model, and controller system.

In Chapter 6, the use of a modified observer that supplies the error state vector estimates to the identification controller is formulated. This modified observer receives estimates of the plant parameters from its own observer controller. The total observer and observer controller system is fully designed for an illustrative case in Chapter 8, using the design philosophy developed in Chapter 6. The observer controller dynamics were found to be dual to the identification controller dynamics with sign differences. In addition, constraints were developed on the free observer parameters to insure that when the composite system error vector $\underline{\tilde{z}}$ goes to zero, the true plant-model error vector $\underline{e}$ goes also to zero.

In Chapter 7, an identification controller was designed for an illustrative case with the plant state vector assumed to be available. The design procedure, developed in Chapter 5, was carried through in detail for two different specified performance indices and different controller gain parameters. The model parameter time variations were illustrated in Figures 7-3, 7-6, and 7-8, where the identified true plant parameter values are shown. In these illustrative examples, it is also shown how the identification time can be controlled by the choice of the performance index. The Liapunov function and its time derivative were computed as functions of time and their time behavior is shown in Figures 7-2, 7-5, and 7-9.

The dynamic identification of the model may also be observed via the error phase plane portraits. (See Figures 7-4, 7-7, and 7-10.) These plots show how the components of the model-plant error vector vary throughout the time over which the controller is identifying the plant.

Simulation results for the total system of unknown plant, model, identification controller, and modified observer (with its own observer controller) are given and discussed in Section 8.5.

Simulations were run for the two cases of bounded and unbounded plant inputs. For both cases, the convergence of the model to the plant parameters were illustrated. (See Figures 8-1 and 8-3.)

The system error state variable portraits were computed and their variation shown in Figures 8-2 and 8-4. These plots indicate the usefulness of the observer in estimating the unavailable portions of the plant state vector.

Important results were obtained from the study of the variation of the observer controller σ parameter. This study indicated that faster convergence of the model parameters to the plant parameters was obtained when the observer controller dynamics moved faster than the identification controller dynamics.

9.3 Conclusions and Recommendations

It is hoped that the contributions made in this dissertation will encourage the application of the described techniques in numerous practical applications. The design techniques developed here were shown to result in a stable plant, model, and identification controller composite system which can easily be implemented for a large class of unknown plants. An index of performance was developed that allows the designer to control the system identification time. The use of a modified observer to estimate the required plant state vector was shown to offer excellent possibilities for successful use.

The class of plants to be identified were restricted to those that are linear with slowly varying parameters. To extend the results of this dissertation to a wider class of plants, future research is suggested in the following areas:

- Extend the present design method to include classes of non-linear and/or time-varying plants.
- Investigate new forms of Liapunov functions for which different controller structures may be designed.

Furthermore, areas of future research that may result in improved total system performance are:

- A study of the stability of the composite observer and identification controller system in greater detail with the ultimate aim of obtaining a Liapunov function for the total system.
- For those situations where the choice of the plant input time function is a designer's option, investigation is suggested to determine which class yields optimum system performance.

APPENDIX A:

OBSERVER DESIGN FOR LINEAR TIME INVARIANT PLANTS +

The class of linear time invariant plants under consideration is characterized by the set of equations

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \qquad \text{(A-1)}$$

$$\underline{y} = C\underline{x} \qquad \text{(A-2)}$$

where A, B, and C are nxn, nxm, and rxn matrices respectively and x, u, and y are nxl, mxl and rxl vectors respectively. The C matrix is assumed to have row rank r and to be partitionable (with renumering of the state variables if necessary) as shown below

$$C = \left[C_1 \;\vdots\; C_2\right] \qquad \text{(A-3)}$$

where C_1 is an rxr matrix and C2 on rxp matrix, where p = n-r.

The problem under consideration is to design a state vector estimator, $\tilde{\underline{x}}$ of $\underline{x}$, from the measurement vector $\underline{y}$ and input vector $\underline{u}$, that converges to $\underline{x}$, i.e.,

$$\lim_{t \to \infty} (\underline{x} - \tilde{\underline{x}}) = 0 \qquad \text{(A-4)}$$

In the following design we shall assume that the plant described by Eqs. (A-1) and (A-2) is uniformly observable, i.e. $\{A, C\}$ is uniformly observable, which by definition is equivalent to saying that the rank of Q_0 is n, where

-----

+ The material in this Appendix is a simplification of Yuksel and Bongiorno [5].

$$Q_0 = \text{col} \left\{ Q_i \right\} \qquad \text{(A-5)}$$
$$i = 1, 2, \ldots, n$$

$$Q_1 = C \qquad \text{(A-6)}$$

$$Q_i = Q_{i-1} A + Q_{i-1} \qquad i = 2, 3, \ldots, n \qquad \text{(A-7)}$$

The design of an observer for the state vector x is now presented in "cook-book" form in terms of the plant A, B, and C matrices which are assumed known. However, a set of observer design parameters will be unspecified. These design parameters must be chosen so that Eq. (A-4) holds, i.e., $\underline{\tilde{x}}$ is asymptotically stable about x. One technique to do this, not presented in the references, is to use a Liapunov function. This technique is used in the illustrative example in Chapter 8 to select these observer parameters and at the same time to check the observer design.

Design Procedure:

1) Assume

$$\underline{\tilde{x}} = W \begin{bmatrix} \underline{y} \\ - \\ \underline{z} \end{bmatrix} \qquad \text{(A-8)}$$

where W is an nxm matrix and $\underline{z}$ is a p_{x1} vector (called on asymptotic state estimator vector) where p = n-r. The $\underline{z}$ vector will be the solution of the dynamic system below +

$$\dot{\underline{z}} = F\underline{z} + G\underline{y} + R\underline{u} \qquad \text{(A-9)}$$

The specification of the computational technique to find F, G, R, and W to compute $\underline{\tilde{x}}$ will complete the design procedure.

+ The observer is itself a linear time-invariant system, with state vector $\underline{z}(t)$.

2) Let W have the partitioned form

$$W = \left[\begin{array}{c|c} W_{11} & W_{12} \\ \hline W_{21} & W_{22} \end{array}\right] \qquad \text{(A-10)}$$

where W_{11}, W_{12}, W_{21}, and W_{22} are rxr, rxp, pxr and pxp matrices respectively. Since as discussed in Reference [5] , W_{22} can be chosen as any constant nonsingular matrix, let us select for simplicity

$$W_{22} = I_p \qquad \text{(A-11)}$$

where I_p signifies the pxp identity matrix.

From the partitioning of matrix C in Eq. (A-3), we compute W_{12} as

$$W_{12} = -C_1^{-1} \; C_2 \; W_{22} \qquad \text{(A-12)}$$

or from Eq. (A-11)

$$W_{12} = -C_1^{-1} \; C_2 \; I_p \qquad \text{(A-13)}$$

We also compute W_{11} as

$$W_{11} = C_1^{-1} \left[I_r - C_2 \; W_{21} \right] \qquad \text{(A-14)}$$

where I_r is the rxr identity matrix and W_{21} will be an observer design matrix, i.e., an as yet unspecified matrix of constants. With W_{21} an observer design matrix; and W_{11}, W_{12}, and W_{22} given in Eqs. (A-14), (A-12), and (A-11), W in Eqs (A-10) and (A-8) is specified.

3) Partition W as shown

$$W = \left[M \,|\, N \right] \qquad \text{(A-15)}$$

where M is an nxr matrix and N an nxp matrix (n = pxr).

4) Compute T as shown

$$T = W_{22}^{-1} \begin{bmatrix} -W_{21} & I_p \end{bmatrix} \begin{bmatrix} C_1 & C_2 \\ O_{p,p} & I_p \end{bmatrix} \qquad \text{(A-16)}$$

where $O_{p,r}$ is the pxr zero matrix, I_p is the pxp identity matrix, and W_{22}^{-1} from Eq. (A-11) is also I_p.

5) Compute R as shown from Eqs. (A-16) and (A-1).

$$R = TB \qquad \text{(A-17)}$$

6) Compute F as shown from Eqs. (A-16), (A-1) and (A-15).

$$F = TAN \qquad \text{(A-18)}$$

7) Compute G from Eqs. (A-16), (A-1) and (A-15).

$$G = TAM \qquad \text{(A-19)}$$

8) With the specification of W_{21}, the observer design is complete since $\underline{z}(t)$ can be generated from Eq. (A-9) (F, G, H and $\underline{u}(t)$ are all known) and $\tilde{\underline{x}}$ can be generated from Eq. (A-8) since $\underline{y}$ and $\underline{z}$ are now available.

A technique useful in specifying W_{21} so that Eq. (A-4) holds is to use a Liapunov function V

$$V = (\delta \underline{x})^T \; L \; (\delta \underline{x}) \qquad \text{(A-20)}$$

where

$$\delta \underline{x} = \tilde{\underline{x}} - \underline{x} \qquad \text{(A-21)}$$

and L is an nxr positive definite matrix. Thus if W_{21} can be selected such that $-\dot{V}$ is positive definite in $\delta \underline{x}$, the variation of the observer state about the true state, then $\tilde{\underline{x}} \to \underline{x}$ as $t \to \infty$. This technique will always yield an appropriate W_{21}, since its existence was proved in Reference [3], and the quadratic form in Eq. (A-20) is all that is required to obtain necessary and sufficient stability criteria for linear time invariant systems of ordinary differential equations. An example of how this technique is used can be found in detail in Sections 8.2 and 8.3.

APPENDIX B:

DERIVATION OF EQUATION (8.4-5)

The evaluation of Eq. (8.4-5) requires the substitution of derived expressions for $\dot{\tilde{e}}_1$ and $\dot{\tilde{e}}_2$ into the expression for $\dot{V}$ in Eq. (8.4-3). These expressions are now derived as follows:

Differentiating the first expression in Eq. (8.4-2a) with respect to time yields

$$\dot{\tilde{e}}_1 = \dot{w}_1 - \dot{\tilde{w}}_1 \tag{B-1}$$

Substituting Eqs. (7.2-5) and (8.3-18) into Eq. (B-1) results in

$$\dot{\tilde{e}}_1 = w_2 - (K_1 \dot{x}_1 + \dot{z}_1) \tag{B-2}$$

Using the expression for $\dot{x}_1$ in Eq. (8.2-17) and the expression for $\dot{z}_1$ in Eq. (8.3-18), Eq: (B-2) becomes

$$\dot{\tilde{e}}_1 = w_2 - K_1 (- \psi x_1 + w_1) - (-K_1 z_1 + z_2 + g_1 x_1) \tag{B-3}$$

which can be simplified to be

$$\dot{\tilde{e}}_1 = w_2 + \psi K_1 x_1 - K_1 w_1 + K_1 z_1 - z_2 - g_1 x_1 \tag{B-4}$$

Substituting g_1 given in Eq. (8.3-18) into Eq. (B-4) yields

$$\begin{aligned} \dot{\tilde{e}}_1 = {} & w_2 + \psi K_1 x_1 - K_1 w_1 + K_1 z_1 - z_2 - K_1 \psi x_1 \\ & + K_1^2 x_1 - K_2 x_1 \end{aligned} \tag{B-5}$$

which, after rearranging terms becomes

$$\dot{\tilde{e}}_1 = w_2 - K_1 w_1 + K_1 (z_1 + K_1 x_1) - (K_2 x_1 + z_2) \tag{B-6}$$

Using the expressions for $\tilde{w_1}$ and $\tilde{w_2}$ in Eq. (8.3-18) in Eq. (B-6) results in

$$\dot{\tilde{e_1}} = w_2 - K_1 w_1 + K_1 \tilde{w_1} - \tilde{w_2} \tag{B-7}$$

From Eqs. (8.4-2) definition of $\tilde{e_1}$ and $\tilde{e_2}$, Eq. (B-7) becomes

$$\dot{\tilde{e_1}} = -K_1 \tilde{e_1} + \tilde{e_2} \tag{B-8}$$

which yields the final expression for $\dot{\tilde{e_1}}$.

Differentiating $\tilde{e_2}$ defined in Eq. (8.4-2) results in

$$\dot{\tilde{e_2}} = \dot{w_2} - \dot{\tilde{w_2}} \tag{B-9}$$

Substituting Eq. (7.2-5) and Eq. (8.3-18) into Eq. (B-9) yields

$$\dot{\tilde{e_2}} = (-a_0 w_1 - a_1 w_2 + b_0 r_1 + b_1 r_2) - (K_1 \dot{x}_1 + \dot{z}_2) \tag{B-10}$$

Using the expression for $\dot{x}_1$ in Eq. (8.2-17) and the expression for $\dot{z}_2$ in Eq. (8.3-18), Eq. (B-10) becomes

$$\begin{aligned} \tilde{e_2} = & -a_0 w_1 - a_1 w_2 + b_0 r_1 + b_1 r_2 - K_2 (-x_1 \Psi + w_1) \\ & - (-K_2 z_1 - \hat{a}_0 z_1 - \hat{a}_1 z_2 + g_2 x_1 + \hat{b}_0 r_1 + \hat{b}_1 r_2) \end{aligned} \tag{B-11}$$

or

$$\begin{aligned} \tilde{e_2} = & -a_0 w_1 - a_1 w_2 + b_0 r_1 + b_1 r_2 + \Psi K_2 x_1 - K_2 w_1 \\ & + K_2 z_1 + \hat{a}_0 z_1 + \hat{a}_1 z_2 - g_2 x_1 - \hat{b}_0 r_1 - \hat{b}_1 r_2 \end{aligned} \tag{B-12}$$

Substituting the expression for g_2 given by Eq. (8.3-18) and using the definitions of the double primed variables in Eq. (8.4-2a), Eq. (B-12) becomes

$$\dot{\tilde{e}}_2 = -a_0 w_1 - a_1 w_2 + b_0 r_1 + b_1 r_2 + \psi K_2 x_1 - K_2 w_1 + K_2 z_1 + (a_0 - a_0'') z_1 + (a_1 - a_1'') z_2 - (-K_1 K_2 + \psi K_2 - \hat{a}_0 K_1 - \hat{a}_1 K_2) x_1 - (b_0 - b_0'') r_1 - (b_1 - b_1'') r_2 \quad \text{(B-13)}$$

With rearrangement and cancellation Eq. (B-13) can be written as

$$\dot{\tilde{e}}_2 = -a_0 w_1 - a_1 w_2 + \psi K_2 x_1 - K_2 w_1 + K_2 z_1 + a_0 z_1 + a_1 z_2 + K_1 K_2 x_1 - \psi K_2 x_1 + (a_0 - a_0'') K_1 x_1 + (a_1 - a_1'') K_2 x_1 + b_0'' r_1 + b_1'' r_2 - a_0'' z_1 - a_1'' z_2 \quad \text{(B-14)}$$

which can be further simplified to be

$$\dot{\tilde{e}}_2 = -a_0 w_1 - a_1 w_2 - K_2 w_1 + K_2 (z_1 + K_1 x_1) + a_0 (z_1 + K_1 x_1) + a_1 (Z_2 + K_2 x_1) - a_0'' (K_1 x_1 + z_1) - a_1'' (K_2 x_1 + z_2) + b_0'' r_1 + b_1'' r_2 \quad \text{(B-15)}$$

Substituting the expression for $\tilde{w}_1$ and $\tilde{w}_2$ given by Eq. (8.3-18) into Eq. (B-15) yields

$$\dot{\tilde{e}}_2 = -a_0 w_1 - a_1 w_2 - K_2 w_1 + K_2 \tilde{w}_1 + a_0 \tilde{w}_1 + a_1 \tilde{w}_2 - a_0'' \tilde{w}_1 - a_1'' \tilde{w}_2 + b_0'' r_1 + b_1'' r_2$$

which can be rearranged as

$$\dot{\tilde{e}}_2 = -(a_0 + K_2)(w_1 - \tilde{w}_1) - a_1 (w_2 - \tilde{w}_2) - a_0 \tilde{w}_1 - a_1 \tilde{w}_2 + b_0'' r_1 + b_1'' r_2 \quad \text{(B-16)}$$

Substitution of Eqs. (B-2) and (B-16) into $\dot{V}$ in Eq. (8.4-3) yields Eq. (8.4-5).

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