LONG MEMORY IN REAL EXCHANGE RATE CHANGES

by

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Abstract

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This study examines the time-series dynamics of real exchange rate changes in the fractionally integrated autoregressive moving average (ARFIMA) framework, which allows for long-memory in the data and is a generalization of the standard ARIMA model. Specifically, the differencing parameter of an ARFIMA model is not restricted to the integer domain and can assume real values. This generalization makes an ARFIMA model a parsimonious and flexible model to study long-memory, which induces persistence, and short-run dynamics simultaneously. Fractional integration is a more general way to describe dependence between observations some distance apart than the unit-root specification and provides an alternative perspective to examine the unit-root hypothesis. This empirical study provides supportive evidence of long-memory in real exchange rate changes, meaning that purchasing power parity may hold as a long-run relationship, based on a statistical procedure that is asymptotically robust to short-run memory and has a well defined distribution. In addition, when ARFIMA models are fitted to the

data the estimates of the AR and MA parameters imply that there is long-memory in all real exchange rate data. To better understand the long-memory characteristics of real exchange rates, estimates of the impulse response parameters are obtained based on the fitted models. Impulse-responses provide more convincing evidence of long-range dependence in the real exchange rate data and also give information regarding the pattern and speed with which shocks to purchasing power parity are propagated.

To my family, my wife Elena and my daughter Paula,
with gratitude for their patience, support and love.

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I. Introduction

One of the most important and debated issues in macroeconomics is that of whether or not the theory of purchasing power parity (PPP) holds either in the short-run or in the long-run. A key problem in the empirical literature is how to identify persistence or long-range dependence or long memory in real exchange rates, that summarizes the theory of PPP. In the past years, a series of studies has tried to solve the problem by analyzing the behavior of real exchange rates, and selecting statistical techniques that have low power in identifying the correlation of long period components. Based on the selection of a particular procedure these studies present evidence that PPP does not hold and that real exchange rates do not exhibit mean-reversion.

Long-run PPP is of fundamental importance in most models of exchange rate determination, for example the Dornbusch-Frankel and the Frenkel-Bilson models. These dynamic exchange rate models rely on PPP as a long-run equilibrium relationship for the exchange rate. The PPP relationship is generally viewed as the open-economy extension of the quantity theory and it simply states that, when measured in the same units, the monies of different countries should purchase the same basket of goods and services.

PPP as an exact relationship (strict parity) has been studied extensively and the conclusions of the research have shown that violations of this idea are common. Although the validity of PPP as a short-run relationship has failed, it may still be valid as a long-run relation. In other words, the hypothesis of long-run PPP, that is, a tendency for the real exchange rate to revert to its parity value, seems an attractive alternative for many economists.

Many empirical models of exchange rate determination report significant deviations from PPP in the short-run, but the validity of the long-run PPP remains controversial. For example Roll (1979), Diebold (1988), Adler and Lehman (1983), Mussa (1986), Hallio (1986) and Darby (1983) have failed to reject the random walk assumption or that real exchange rates followed martingale processes, which imply that economic shocks have a completely permanent effect on the level of the real exchange rates and that changes in real exchange rates are totally unpredictable. In short, these studies have shown that there is little or no evidence for nominal exchange rates and prices to adjust in such a way as to promote PPP. In effect, these models imply that short-run movements in real exchange rates are caused by short-run movements in nominal exchange rates and that offsetting movements in the price levels have no effect on the real exchange rate in the long-run.

In contrast, Abuaf and Jorian (1990) reported evidence of PPP reversion for the period 1900-1972 using multivariate unit-root tests. Kim (1990) also examined the same data set and also found evidence for long-run PPP using cointegration techniques. One important result of his analysis is that when consumer price indexes (CPIs) are used, little favorable evidence for long-run PPP can be found. On the basis of fractional differencing analysis Diebold, Husted and Rush (1991) found evidence of mean-reversion in the real exchange rate under the gold standard. For the recent period of floating exchange rate regimes, Huizinoa (1987) and Kaminsky (1988) found some evidence of mean reversion toward PPP using variance-ratio tests. Glen (1991) found evidence in favor of long-run PPP using long-horizon autocorrelations and variance-ratio tests and Cheung and Lai (1992-1993) also found evidence favorable to long-run PPP using cointegration and fractional cointegration techniques. Many empirical studies however, have failed to find cointegration between nominal exchange rates and relative price indices, implying that the two series tend to drift apart without bound, for example, Ballie and Sclover (1987), Corbae and Ouliaris (1988) and Taylor (1988).

The apparently mixed results reflects a major problem associated the testing of long-run PPP. A test for long-run PPP requires a proper modeling of the low-frequency dynamics of real exchange rates while allowing for significant deviations from the PPP equilibrium condition in the short-run. Empirical results can crucially depend on the power of statistical techniques employed to separate the low-frequency (long-period) components from the high-frequency (short-period) components.

The negative results obtained in previous empirical studies reflect the lower power of the tests used rather than evidence against PPP. The inappropriateness of the methodology used provided the result that long-run PPP does not hold. Thus, in order to examine long-run PPP is essential to use a statistical procedure that can identify a rich class of low-frequency dynamics. The methodology that is used in this empirical study is specifically designed to address long-run issues, that is, to examine the correlation between observations at long lags.

The idea that real exchange rates are non-stationary implies that the real exchange rate will take on any value within a given period of time. The real exchange rate is a relative price, it is the relative price of one country's goods and services in terms of another country's goods and services and as such, it is unlikely that it will take on any value within a given period of time.

This empirical study is concerned with applying recent developments in econometrics to examine long-range dependence in real exchange rates and to determine the validity of PPP as a long-run relationship.

The study employs data from the modern experience with floating exchange rates. Real exchange rates were generated by obtaining end-of-month nominal exchange rates for seven countries against the U.S. dollar, from the International Financial Statistics (IFS) data base. Deflating these by the Consumer Price Indexes (CPI), also obtained from the IFS produces the real exchange rates. The monthly time-series used cover the period 1974:1-1994:1 and the countries examined are: Canada, Germany, Italy, Japan, United Kingdom, France as the foreign countries and the United States as the home country.

In order to analyze and explain the behavior of the six real dollar exchange rates, section II briefly addresses the PPP relationship and discusses in some detail the approach used by Abuaf and Jorian, J. D. Glen and Cheung and Lai. Section III introduces fractional integration and dynamic models of real exchange rates. Section IV presents empirical results. It addresses the identification issue by estimating the fractionally differenced operator using the

Geweke and Porter-Hudak approach, which is one method of testing whether there is long memory in real exchange rates. Then, in order to capture the remaining short-run dynamics of the series, I estimate ARFIMA models, using similar approach used by Diebold, Husted and Rush (1991) and Cheung (1993). To analyze the effects of the impact of a unit innovation on the real exchange rate, I then estimate impulse responses. The last part of the empirical section offers tentative explanations of long memory behavior, and finally, section V summarizes the study.

II. PPP and Real Exchange Rates

The first part of this section describes some of the theory behind PPP.

1. Theoretical Review

The PPP theory was developed by Cassel (1919, 1922), and it states that currencies are valued for the goods they can buy, and in equilibrium a given basket of goods should cost the same in the domestic country as well as in foreign countries. Otherwise, international arbitrage should bring about adjustments in prices, nominal exchange rates or both to restore PPP.

Regarded as a theory, PPP is subject to important qualifications such as modifications to tariff policy and trade restrictions. When PPP is applied to the real world, the relation between nominal exchange rates and national price levels can be affected by many factors. It is also necessary to determine how the price level is calculated since not all prices of goods enganged in foreign trade are taken into account. There is the problem of tradable and nontradable goods, for example, which can weaken the relationship between exchange rates and price levels. The relationship can also be weakened by other real world complications such as transaction costs, exchange rate interventions, taxation and imperfect competition. In addition, price levels measurements carry with them the usual problems associated with aggregation and index construction. A change in the general price level can only be approximated by means of index numbers and the composition of the index number determines the proportionate effect on the exchange rate. The choice of an inappropriate price index can systematically affect judgments on the scope and direction of measurements in real exchange rates. The use of cost of living indexes (CPIs) include diverse commodities which have no influence on internationals trade. At the other extreme, wholesale price indexes (WPIs) ignore not only all forms of manufactured goods and products but the whole range of services and other invisible exports. It is generally argued that the price index used should attempt to measure the cost of production of tradable goods rather than actual prices which the forces of competition tend to equalize among countries.

In its absolute form PPP theory states that the nominal exchange rate is determined by the ratio of domestic and foreign prices P and P* respectively, so that

$$S_t = P_t / P_t^* \qquad (1.1)$$

the theory thus assumes the existence of a price index. The two indexes can be defined as

$$P = \prod_{i=1}^{N} (P_i)^{\alpha_i}$$

and

$$P^* = \prod_{i=1}^{N} (P_i^*)^{\alpha_i^*}$$

where it is assumed that the same N goods exist in both countries and α_i and α_i^* represent a system of weights with $\Sigma\,\alpha_i = \Sigma\,\alpha_i^* = 1$. These indexes are usually justified using consumer theory by assuming that the consumer devoted a constant fraction α_i of his or her budget to the ith good which is given independently of relative prices. Hence the level of welfare between consumers depends only on their purchasing power.

In the relative version of PPP it is no longer necessary for the exchange rate to be equal to the ratio of price indexes instantaneously but to simply remain at a constant ratio as

$$S_{t+1}/S_t = (P_{t+1}/P^*_{t+1})(P_t/P^*_t) \qquad (1.2)$$

Since the behavior of consumption in both countries is reflected in the composition of the indexes P_t and P^*_t, equation (1.2) can be rewritten in terms of the inflation rates π_t and π^*_t in each country. Since

$$\pi_t = (P_{t+1} - P_t)/P_t = (dP_t/dt)/P_t$$

$$(1.3)$$

$$\pi^*_t = (P^*_{t+1} - P^*_t)/P^*_t = (dP^*_t/dt)/P^*_t$$

and
$$(S_{t+1} - S_t)/S_t = (\pi_t - \pi^*_t)/(1 + \pi_t) \qquad (1.4)$$

and in the case where the foreign inflation rate π^*_t is low this can be further simplified to

$$(S_{t+1} - S_t)/S_t = \pi_t - \pi^*_t \qquad (1.5)$$

Thus the rate of change in prices is balanced by a corresponding change in the exchange rate.

Both the absolute and relative versions of PPP can be expressed in terms of the real exchange rate R_t as

$$R_t = S_t P_t^* / P_t \tag{1.6}$$

Thus, the real exchange rate if defined as the nominal exchange rate deflated by a ratio of foreign to domestic price levels.

Under long-run PPP, the long-run equilibrium real exchange rate is a time invariant constant which equals one for the absolute version of PPP and equals the previous year's value for the relative PPP. In contrast, short-run PPP is violated whenever the instantaneous real exchange rate does not equal its long-run equilibrium value. Taking logarithms,

$$r_t = s_t + p_t^* - p_t = \log(S_t P_t^* / P_t) \tag{1.7}$$

where r_t is the log real exchange rate, s_t is the log of the spot exchange rate, defined in local currency units per foreign currency units, p_t^* and p_t are the log of the foreign and domestic price levels respectively[1].

There are three ways in which PPP can be justified. The most traditional approach is the law of One Price, the second requires the invariance of relative

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[1] In accordance with the literature the log of real exchange rate is the object of analysis.

prices and the third is based on the Fisherian hypothesis concerning the real interest rate:

a. The law of one price can be explained by assuming that the behavior of consumers is the same in both countries, which implies that the system of weights will also be identical, so that

$$\alpha_i = \alpha_i^* \quad , \quad i = 1, 2, ..., N \qquad (1.8)$$

Thus, the real exchange rate can be written as

$$R_t = \Pi_{i=1}^{N} (S_t P_{it}^* / P_{it})^{\alpha} \qquad (1.9)$$

and is equal to one if the price of each of the N goods does not vary between one country and another, so that

$$P_{it} = P_{it}^* S_t \quad , \quad i = 1, 2, ..., N \qquad (1.10)$$

The last condition characterizes the law of one price in its absolute version. If only the relative version of this law applies then by definition

$$P_{i,t+1} S_{t+1} / P_{i,t+1} = P_{i,t}^* S_t / P_{i,t} \quad , \quad i = 1, 2, ..., N \qquad (1.11)$$

a condition which clearly implies that the real exchange rate is stable over time. Thus, when the price indexes in both countries are identical the law of one price justifies PPP.

b. If the composition of the indexes P and P^* differ, relative PPP can be verified providing that the relative version of the law of one price holds for all the N goods and that relative foods prices are not modified during the period.

c. The Fisher hypothesis states that real rates of interest m and m* are equalized between two countries through the following relationship between domestic and foreign nominal rates i and i* and price levels. Then

$$1 + m_t \equiv (1 + i_t)/(p_{t+1}/p_t) = (1 + i_t)/(p^*_{t+1}/p^*_t) \equiv 1 + m^*_t \qquad (1.12)$$

To move to PPP, it is sufficient to appeal to the uncovered interest rate parity condition, which states that, in equilibrium, the expected return abroad must equal the domestic rate of return, and can be wrtten as

$$(1 + i_t)S_t = (1 + i^*_t)S_{t+1} \qquad (1.13)$$

which indicates how the hypothesis of relative PPP is verified.

2. Analysis of related literature

This part of section II considers four empirical studies that find evidence suppotrive to long-run PPP or that real exchange rates exhibit mean reversion. Specifically, the study by Abuaf and Jorian (PPP in the long-run, 1990) is considered first. Second, Glen's model (Exchange Rates in the Short, Medium and Long Run, 1991) is examined. Third, Cheung's and Lai's (Long Run PPP During the Recent Float, 1992) study is examined and finally Cheung's and Lai's (Fractional Cointegration Analysis of PPP, 1993) extension of the previous study is considered. This part does not attempt to provide a complete theoretical review and it does not give full details of the various procedures. Rather, it discusses intuitively the main econometric procedures that are currently available and their relative strengths and weaknesses.

2.1. PPP in the long-run

Abuaf and Jorian (1990) find supportive evidence of PPP reversion for the 1973-1987 period, (1973-1987 monthly data for the recent flexible exchange rate period), using multivariate unit-root tests for ten industrial countries. The same analysis is also performed with annual data for the 1900-1972 period, which yields clearer evidence of mean reversion The annual analysis is performed because it may take a number of years for real exchange rate to revert to its parity value.

One of the main results of this analysis is that by using a system of univariate autoregressions the autoregressive coefficient is constrained to be same across countries, which leads to more precise parameter estimates than the usual country-by-country setting. Their results suggest that, though the real exchange rate is well captured by a first-order autoregressive process, the root of the process is slightly below unity, which implies that PPP holds in the long-run. The approach adopted by Abuaf and Jorion assumes that the real exchange rate is generated by a first-order autoregressive process,

$$e_{t+1} = \kappa_0 + \kappa_1 e_t + u_{1t+1} \qquad (2.1.1)$$

where e_t is the real exchange rate, κ_0 and κ_1 are constants and the error term is normally and independently distributed over time. The log of the long-term equilibrium real exchange rate $\bar{e}$ is defined as the unconditional expectation of the process in (2.1.1). Assuming that $|\kappa_1|\langle 1$, it can be written as

$$\bar{e} = \kappa_0/(1-\kappa_1) \qquad (2.1.2)$$

Long-run PPP does not hold if $|\kappa_1| \geq 1$ and if κ_0 and/or κ_1 are not time variant constants. Provided that long-run PPP holds, short-run PPP is violated whenever e_t does on equal its long-run value $\bar{e}$. If $\kappa_1 \langle 1$, however, shocks to the system are corrected at the rate of $(1-\kappa_1)$ per period.

The previous relationship is consistent with efficiently functioning capital markets. Under rational expectations and risk averse speculations in the foreign exchange market, movement in the real exchange rate can be described by

$$\Delta e_{t+1} = e_{t+1} - e_t = E_t[r_{t+1}] - E_t[r^*_{t+1}] - d_t + u_{2t+1} \tag{2.1.3}$$

where $E_t[r_{t+1}]$ is the expectation of the real interest rate from time t to t+1 based on the information available at time t, and d_t is the risk premium in the forward rate, defined as the difference between the forward rate and the expected future spot rate. Equation (2.1.3) indicates that the serial correlation properties of Δe depend on the time-series characteristics of expected real return differentials and of the risk premium. As a result, the martingale or random walk model for Δe breaks down when real interest differentials are mean-reverting, or when risk premia embodied in forward rates are time-varying.

Then, they proceed by estimating a model that has attracted a lot of attention in the literature and compare their results with the results obtained by the following relationship used by others,

$$\Delta e_{t+1} = b_0 + b_1 \Delta e_t + \ldots + b_\kappa \Delta e_{t-\kappa+1} + u_{t+1} \qquad (2.1.4)$$

where κ is the number of lags. This univariate representation is based on differences in the real exchange rate and can be used to test the validity of the long-run PPP by testing whether the real exchange rate has a unit root. The random walk hypothesis implies that the b_js coefficients should be zero for all j. All empirical work done based on equation (2.1.4) is unable to reject the hypothesis that the coefficients are different from zero, which implies that PPP does not hold in the long-run.

On the basis of the observation that regressions in levels are likely to yield more powerful tests (Dickey and Fuller 1979), they use a first-order autoregression in levels with $\kappa_1 \langle 1$ as an alternative hypothesis.

One drawback of this specification is that the usual test statistics do not apply. Fuller (1976) tabulates new tests under the null $\kappa_0 = 0$ and $\kappa_1 = 1$. The first test statistic is

$$\hat{\rho}_{\mu} = \tau\left(\hat{\kappa}_1 - 1\right) \qquad (2.1.5)$$

where κ_1 is the OLS estimate of κ_1 in (2.1.1) and the subscript μ indicates that the regression contains an intercept.

The second test statistic is (analogous to the t-statistics).

$$\hat{\tau}_{\mu} = \left(\hat{\kappa}_1 - 1\right)/\sigma\left(\hat{\kappa}\right) \qquad (2.1.6)$$

where $\sigma\left(\hat{\kappa}_1\right)$ is the OLS standard error of κ_1. Given a significance level, the critical value of $\hat{\tau}_{\mu}$ is lower than that of the t distribution, since κ_1 is downward bias in finite samples. Dickey and Fuller (1979-1981) indicate that the ρ_{μ} test is more powerful than the τ_{μ} test and also superior to likelihood ratio tests and Box-Pierce statistics. While these tests improve on the traditional test, they still have low power to differentiate between κ_1 equal to one and slightly less than one. Thus, it is essential to construct tests with increase power. This can be done by extending the Dickey and Fuller tests to the autoregression model in a multivariate setting.

The drawback of this method is that the distribution of these test statistics is unknown for a system of equations and therefore the empirical distributions have to be derived by simulation analysis. The simulation is conducted as follows: The data generation process is based on the autoregressive model with $\kappa_0 = 0$ and $\kappa_1 = 1$. The sample size is chosen to be T=180, which corresponds to the number of observation in the floating exchange rate period 1973-1987, and N=10 countries. In each experiment, N error terms u_t are generated jointly T times from a multivariate normal distribution with zero mean and a given covariance matrix. The covariance matrix for the simulation is taken from the data set.

This procedure allows to simulate e_t from the autoregressive model (2.1.1) and to compute a sample value of the statistics. Each experiment is then replicated 5000 times, which generates a sample distribution of the statistics for a known autoregression coefficient. The result of these Monte-Carlo experiments is the improvement of the above statistics.

Next, they investigate whether the previous simulations are sensitive to misspecification in the covariance matrix and they find that, changes in the structure of correlations have little effect on the tests. Then, to assess whether these results are sensitive to heteroscedasticity, the simulations are also run with the errors generated by an ARCH process. Specifically, an ARCH(1) process is fitted to the real exchange rate changes.

$$\Delta e_{t+1} | t \sim N(\mu, h_{t+1}), h_{t+1} = \alpha_0 + \alpha_1 (\Delta e_t - \mu)^2 \qquad (2.1.7)$$

where the conditional variance h_{t+1} is a linear function of the last squared innovation only. Since the ARCH effects is relatively small, they also investigate a process with an arbitrarily high value of α_1. In these simulations the error terms are taken to be conditionally distributed as

$$u_{t+1} | t \sim N(0, h_{t+1}), h_{t+1} = \alpha_0 + \alpha_1 u_t^2 \qquad (2.1.8)$$

Given the correlation coefficients, a new covariance matrix can be computed for each observation t. Again the tests are not sensitive to this alternative specification. As a result, such multivariate tests are likely to be much more informative than univariate tests. Next since, the AR(1) model restricts the dynamics of real exchange rates to only three possibilities, an explosive process, a random walk, or a monotonic adjustment to a constant value, they increase the number of lagged values to allow for a more general dynamic specification,

$$e_{t+1} = \kappa_0 + \kappa_1 e_t + \sum_{j=1}^{p} b_j (e_{t-j+1} - e_{t-j}) + u_{t+1} \qquad (2.1.9)$$

The result of this increased lagged valued model is the rejection of the random walk hypothesis.

In sum, this paper shows that long-run PPP may hold, and that nominal exchange rates are well approximated by a process with a unit-root, which indicates that price levels are the major reason for the long-term stability in real exchange rates.

2.2. Exchange rates in the short, medium and long run

Using long horizon autocorrelations and variance ratio tests, Glen (1991) finds significant deviations from random walk behavior in real exchange rates. The rejection can be attributed to mean reversion and confirms that PPP is a long-term phenomenon. Specifically, the PPP relationship that is tested is

$$p_t = \kappa s_t p_t^* \qquad (2.2.1)$$

where s_t is the nominal exchange rate at time t, p_t and p_t^* are the price levels in the home and foreign countries respectively, and κ is equal to one if PPP holds. The relative version of PPP allows κ to take on values different from one to allow for distortions κ is constant, if the distortions are constant and equation (2.2.1) will then hold in rate of change form.

An implication of the previous PPP relationship is that relative PPP must be expected to hold ex ante when κ is constant. That is,

$$E_t[d(t, t-1)] \equiv E_t[r_t - r_{t-i}] = 0 \qquad (2.2.1)$$

where r_t is the real exchange rate at time $t, d(t, t-i)$ is an innovation in r_t and $E_t[\cdot]$ is the time t expectation operator. Under ex ante PPP the real exchange rate will be a martingale. The implication of ex ante PPP is that acceptance of this hypothesis means that both absolute and relative PPP are rejected. Since there exist models that offer theoretical alternatives to the theory of ex ante PPP, that is, alternative theories of the real exchange rate which suggest that κ is not constant, Glen proceeds by estimating long-horizon autocorrelations and variance-ratio tests of real exchange rate movements in order to examine whether PPP holds as a long-run relationship, with the result that the signs of the estimated first order autocorrelations provide evidence in favor of long-term mean reversion in the real exchange rate.

To provide a stronger evidence supportive to long-term PPP, Glen then uses the variance-ratio test of long-term behavior. The test is constructed as follows : let x_t be a random variable which follows the diffusion process.

$$dx_t = \mu dt + \sigma dw(t).$$

If x_t is sampled at discrete intervals, the variance of its increments is linear in the observation interval. On the basis of this observation, he then uses the variance- ratio test, developed by Lo and Mackinlay (1988), of the random walk hypothesis. With homoscedastic increments the statistic

$$M_r(q) = [\sigma^2(q)/\sigma^2(1)] - 1 \qquad (2.2.3)$$

is asymptotically normally distributed and where

$$\sigma^2(q) = \tfrac{1}{m}\sum_{\kappa=q}^{nq}\left[x_\kappa - x_{\kappa-q} - q\mu\right]^2 \qquad (2.2.4)$$

and where $m \equiv q(nq-q+1)(1-q/nq), nq+1$ is the total number of observations. Inferences are drawn through the use of the statistic

$$Z1(q) \equiv M_r(q)\left[\tfrac{2(2q-1)(q-1)}{3q}\right]^{-\frac{1}{2}}\sqrt{nq} \qquad (2.2.5)$$

which asymptotically has a standard normal distribution.

In order to allow for a general form of heteroskedasticity in the variance of the increments of x_t, Lo and Mackinlay proposed using the same statistic, $M_{r(q)}$, but altered the estimate of its variance. The proposed statistic is

$$Z2(q) = M_r(q)/\sqrt{\theta} \qquad (2.2.6)$$

where $\theta(q) = \sum_{j=1}^{q-1}\left[\frac{2(q-j)}{q}\right]^2\delta(j)$

and where $$\delta(j) = \frac{\sum_{K=j+1}^{nq} (X_K - X_{K-1} - \mu)^2 \left(X_{K-j} - X_{K-j-1} - \mu\right)}{\sum_{K=1}^{nq} (X_K - X_{K-1} - \mu)^2}$$

There is more than one advantage of the variance ratio statistic over other tests of random walk behavior:

(a) The statistic is more powerful than other tests, especially for processes that exhibit slowly mean-reverting behavior,

(b) The statistic can be calculated so that it is consistent under general forms of heteroskedasticity. Thus any rejections that occur cannot be attributed solely to nonstationarity in the variance of the process, and

(c) By allowing for overlapping observations, the statistic permits evaluation of long-term serial correlation using a relatively small number of observations.

All these findings suggest that the variance ratio tests may provide additional evidence on long-run PPP behavior.

The results provide enough positive correlation in real exchange rate movements that the random walk behavior is rejected, that is, this paper finds significant deviations from random walk behavior in real exchange rates measured on both a monthly and an annual basis. The random walk hypothesis is rejected for most countries at lags greater than two years. Annual data provide more convincing evidence for mean reversion than the monthly data.

2.3 Long-run PPP during the recent floats

Cheung, Y and Lai K (1992) examine the relevance of long-run PPP during the post Bretton Woods period using cointegration techniques and find significant evidence favorable to long-run PPP. Five real exchange rates are examined between the U.S. as the home country and U.K., France, Germany, Switzerland and Canada as the foreign countries from January 1974 to December 1989. The theory of cointegration allows testing for long-run PPP while abstracting from the short-run dynamics and can account for non-stationarity in the time series of exchange rates and prices, that is, if PPP is true deviations from a linear combination of exchange rates and prices should be stationary and thus exchange rates and prices should be cointegrated.

Based on Taylor's (1988) and Taylor's and McMahm's (1988) observations that observed price series are imperfect proxies for the theoretical price variables, and that symmetry and proportionality restrictions under PPP, are not necessarily consistent with empirical data, they observe that the imposition of symmetry and proportionality can bias PPP tests on real exchange rates towards finding no mean reversion. The reason is that, a linear combination of nonstationarity series, except with the correct cointegration coefficients, is generally also nonstationary. Therefore, they proceed to test the validity of the two restrictions and the measurement error hypothesis because a violation of the two restrictions can be the source of not finding mean reversion toward PPP.

The empirical test of the long-run PPP hypothesis is motivated by the presence of measurements errors, thus, they test the following PPP relationship

$$s_t = e + \alpha_1 p_t - \alpha_2 p_t^* + u_t \qquad (2.3.1)$$

where e is some constant, s_t is the spot exchange rate, p_t and p_t^* are domestic and foreign price indexes respectively and u_t is an error term capturing deviations from PPP. All variables except u_t are expressed in logarithms. For PPP to hold in the long-run, u_t should be stationary. Symmetry between the domestic and foreign countries requires that $\alpha_1 = \alpha_2$ and the long-run proportionality between exchange rates and prices implies that $\alpha_1 = 1 = \alpha_2$.

Then, Cheung and Lai assume that long-run PPP holds for some theoretical price indexes, so that

$$s_t = h + g_t - g_t^* + d_t \qquad (2.3.2)$$

where h is some constant, d_t is a stationary process, g_t and g_t^* are, respectively theoretical domestic and foreign price indexes. Since theoretical price variables are not observable, they are proxied by same observed price indexes in empirical analysis. In order to capture the relationship between theoretical and observed price indexes they consider the following measurement equations,

$$p_t = a_1 + b_1 g_t + \varepsilon_{1t} \qquad (2.3.3)$$

$$p_t^* = a_2 + b_2 g_t^* + \varepsilon_{2t} \qquad (2.3.4)$$

where the parameters a_1, a_2, b_1 and b_2 capture systematic measurement errors, ε_{1t} and ε_{2t} are stationary stochastic terms, capturing non-systematic measurement errors. The stationarity in the stochastic terms means that the observed prices will not drift far apart from the theoretical prices which is a necessary requirement for the validity of long-run PPP. On the other hand, the parameters b_1 and b_2 can differ from one another due to differences between countries in the composition of goods and services and the weights used for index construction. The above two equations tell us that a one percent change in the observed price indexes can cause the theoretical price indexes to change by more than or less than one percent.

Combining equations (2.3.2), (2.3.3) and (2.3.4), yields equation (2.3.1).

Equation (2.3.1) can be considered as a PPP relationship with measurement errors in prices, since $\alpha_1 = 1/b_1$ or $\alpha_2 = 1/b_2$ or both can be different from one.

Expressing equation (2.3.1) in matrix form , we have

$$\alpha' X_t = e + U_t \tag{2.3.5}$$

where X_t is a vector series given by $(s_t, p_t, p_t^*)'$ and $\alpha = (1, -\alpha_1, \alpha_2)'$. When the series in X_t are integrated of order one, I(1), U_t is also I(1). But if there exists an α such that U_t is stationary, X_t is said to be cointegrated and $\alpha' X_t$ as implied by long-run PPP represents a long-run equilibrium relationship.

In order to test for cointegration, Cheung and Lai use the Johansen test which is based on the technique of reduced rank regression. This technique is constructed as follows : consider in general an nx1 time-series vector $X_t = (x_{1t}, x_{2t}, \cdots, x_{nt})'$. Regress ΔX_t on a constant and $\Delta X_{t-1}, \Delta X_{t-2}, \cdots, \Delta X_{t-\kappa}$, giving the residual u_{1t} . Then regress $X_{t-\kappa-1}$ on a constant and $\Delta X_{t-1}, \Delta X_{t-2}, \cdots, \Delta X_{t-\kappa}$ giving the residual u_{2t} . Define the product moment matrices of the residuals as $S_{ij} = T^{-1} \Sigma_{t=1}^{T} \hat{u}_{it} \hat{u}_{jt}$, $ij = 1, 2$. The likelihood ratio test statistic for the hypothesis of r cointegrating vector is

$$-2\ln Q_r = -T\Sigma_{j=r+1}^{n} \ln(1-\phi_j)$$

where $\phi_{r+1},\cdots,\phi_n$ are the $n-r$ smallest eigenvalues $S_{21}S_{11}^{-1}S_{12}$ with respect to S_{22} .

The ML estimate of the cointegrating vector α can be found as the eigenvector $V=(v_1,\cdots,v_r)$ which satisfies the normalization condition that $V'S_{22}V=I$.

Then to establish the low power of the standard residual-based test and to show that the Johansen test has a potentially significant power advantage over them, they proceed by using Monte Carlo simulation methods with the result that the Johansen test, which is based on full system estimation, can raise efficiency relative to single equation methods, that is, the Johansen test gives more accurate results than the augmented Dickey-Fuller test and Phillips-Perron test.

Next, nominal exchange rates and price indexes are checked for a unit-root using the two tests and the results suggested that the levels of exchange rates and prices are not stationary and that there is little evidence of cointegration between nominal exchange rates and prices. The Johansen test, $-2\ln Q_r$, is next performed and the results are totally different from those obtained from the residual-based tests. In all cases the series are cointegrated which is what the hypothesis of long-run PPP requires. To check whether the exchange rate is part of the cointegrating equation or not, a LR test is performed on the cointegrating vector and in all series except of the Canadian CPI the test indicates that exchange rate series should be included. These findings are in favor of long-run PPP.

To test the assumption that symmetry and proportionality restrictions do not necessarily hold empirically in the presence of measurements errors in prices, a chi-square test (Johansen 1991) is performed with the result that in nine out of ten real exchange rate series the restriction of either symmetry or proportionality is rejected statistically. This result implies that is better to work with a general unrestricted trivariate model in testing for cointegration and long-run PPP. Thus, when the symmetry or proportionality restriction is imposed, leading to a bivariate or univariate model, can bias the test towards finding no cointegration. As a result the findings of no cointegration can be interpreted as rejections of the imposed restrictions on the underlying equilibrium relationship rather than of the equilibrium condition itself. Moreover, symmetry and proportionality are restriction on the long-run coefficients only. Thus, when the trivariate model, (s,p,p*), is reduced to the bivariate model, (s,p-p*), or the univariate model, (S-p+p*), in addition to restrictions on the long-run coefficients, restrictions on the short-run coefficients must be imposed with the result that the restricted model ignores any possible interactions in the determination of prices and exchange rates that are possible in the unrestricted trivariate model. Then Johansen tests are used to illustrate the differences in test results among, trivariate, bivariate and univariate models. For the bivariate models Cheung and Lai find evidence of cointegration in six out of ten cases at the 5 percent level or better. These results are not as favorable to long-run PPP as those based on the trivariate models. The results for the univariate models are disappointing because in no case they could find supportive evidence for long-run PPP. On the other hand the results for the trivariate models are in favor to the long-run PPP hypothesis.

As a conclusion, they believe that, interpreting the results of tests for long-run PPP based only on the behavior of real exchange rates can be misleading. Nonstationarity in the real exchange rate can indicate a violation of the symmetry or proportionality conditions and not evidence against long-run PPP with measurement errors.

2.4. A Fractional Cointegration Analysis of PPP

In a more recent study Cheung and Lai (1993) examine the validity of long-run PPP using a fractional cointegration approach that integrates cointegration and fractional differencing. In that work, they study data for six countries for the period 1914-1989. The countries studied are the U.S. as the home country and U.K., France, Italy, Canada and Japan as foreign countries. The study finds favorable evidence to long-run PPP. Particularly, the PPP relationship that is tested is

$$sp_t = \alpha_0 + \alpha_1 p_t + e_t \qquad (2.4.1)$$

where α_0 is a constant, sp_t is the foreign price index converted to domestic currency units, p_t is the domestic price index and e_t is an error term capturing deviations from PPP. All variables are expressed in logarithms.

A necessary condition for PPP to hold in the long-run is that e_t is a mean-reverting process, that is, the effect of a shock to PPP will vanish in the long-run.

Fractional cointegration analysis implies a long-run equilibrium relationship, since fractionally integrated equilibrium errors are mean-reverting. In general, consider a pair of series x_{1t} and x_{2t} which are I(d) (integrated of order d). Let $X_t = (x_{1t}, x_{2t})'$. The linear combination $Z_t = \alpha x_t$ will also be I(d). If a vector α exists such that Z_t is I(d-b) with b>0, x_{1t} and x_{2t} are said to be cointegrated of order (d,b) and $\alpha x_t = 0$ represents an equilibrium constraint operating on the long-run components of x_t. The cointegrated system has an error correction representation of the form

$$H(L) = (1 - L)^D X_t = -T\left[\left(1 - (I - L)^B\right)\right](1 - L)^{d-b} Z_t + C(L)\varepsilon_t \quad (2.4.2)$$

where $H(L)$ is a matrix polynomial in the lag operator L with $H(O) = 1$, $C(L)$ is a lag polynomial with $C(1)$ finite and ε_t is a white-noise disturbance term.

Since the typical case considered in empirical work is the one in which b=d=1, cointegration requires that the equilibrium error, Z_t, is mean-reverting (even though x_{1t} and x_{2t} wander widely). To test whether the two variables are cointegrated they use the Engel-Granger testing procedure which consists of two steps: Regress x_{1t} on x_{2t} or x_{2t} on x_{1t} as the equilibrium or cointegrating regression, and then check if its residual is I(0) or not using a unit-root test and if the residual is found to be I(0), then the null hypothesis of no cointegration is rejected.

In general, however, Z_t does not have to be I(0) exactly, to be mean-reverting process. Fractional integrated processes have the property of mean-reversion. A fractionally integrated process is represented by

$$C(L)(1-L)^d Z_t = D(L)v_t \qquad (2.4.3)$$

where $C(L) = 1 - c_1 L - \cdots - c_p L^p$
$D(L) = 1 + D_1 L + \cdots + D_q L^q$

and all roots of the polynomials C(L) and D(L) lie outside of the unit circle and v_t is $iid(0, \sigma^2)$, and the fractionally differencing operator is defined by the binomial expansion (defined in section III) and can take on to noninteger values. To test for fractional integration in the equilibrium error they use the two-stage semiparametric procedure suggested by Geweke and Porter-Hudak.

The validity of PPP implies a long-run equilibrium relationship between sp_t and p_t . To test for this relationship they adopt the following test: they estimate equation (2.4.1) by a cointegrating regression and examine whether its least squares residual is I(d) in terms of their degree of integration with d<1. The least squares estimate of the cointegrating parameter is consistent and converges in probability which implies that the least squares estimate in the fractional cointegration framework is also consistent, but with different convergence rates according to the actual order of cointegration. The least squares estimation under fractional cointegration is as follows: If we assume that there are two time series, x_t and y_t, which are I(d) and are fractionally cointegrated of order (d,b) such that there exists a ς that

$$y_t = \zeta x_t + \varepsilon_t \tag{2.4.4}$$

where ε_t is I(d-b) with d>1/2 and $d \geq b > 0$. The least squares estimator of ζ is given by

$$\hat{\zeta} = \Sigma_{t=1}^{T} x_t y_t / \Sigma_{t=1}^{T} x_t^2 = \zeta + \Sigma_{t=1}^{T} x_t \varepsilon_t / \Sigma_{t=1}^{T} x_t^2 \tag{2.4.5}$$

The convergence rate of $\hat{\zeta}$ thus depends on those of $\Sigma_{t=1}^{T} x_t \varepsilon_t$ and $\Sigma_{t=1}^{T} x_t^2$. This can be examined in two possible situations. First by the Cauchy-Schwarz inequality (d-b>1/2)

$$\Sigma_{t=1}^{T} x_t^2 \Sigma_{t=1}^{T} \geq \left(\Sigma_{t=1}^{T} x_t \varepsilon_t \right)^2 \tag{2.4.6}$$

This implies that

$$\left(\Sigma_{t=1}^{T} x_t^2 / T^{2d} \right) \left(\Sigma_{t=1}^{T} \varepsilon_t^2 / T^{2(d-b)} \right) \geq \left(\Sigma_{t=1}^{T} x_t \varepsilon_t / T^{2d-b} \right) \tag{2.4.7}$$

Since it is known that

$$\Sigma_{t=1}^{T} x_t^2 = o(T^{2d}) \qquad \text{and} \qquad \Sigma_{t=1}^{T} \varepsilon_t^2 = o(T^{2(d-b)}) \tag{2.4.8}$$

Equation (2.4.8) implies that $\Sigma_{t=1}^{T} x_t\varepsilon_t = o(T^{\tau})$ with $\tau \leq 2d-b$, that is $\Sigma_{t=1}^{T} x_t^2/T^{2d}$ is bounded, and $\Sigma_{t=1}^{T} x_t\varepsilon_t/T^{2b-d+\delta}$ converges in probability to 0 for all $\delta \rangle 0$. It then follows from equation (2.4.6) that

$$T^{b-\delta}\left(\hat{\zeta} - \zeta\right) = \Sigma_{t=1}^{T} x_t\varepsilon_t/T^{2d-b+\delta} / \left(\Sigma_{t=1}^{T} x_t^2/T^{2d}\right) \tag{2.4.9}$$

converges in probability to 0 for all $\delta \rangle 0$. In the second case where $\frac{1}{2}\rangle d-b \geq 0$

$$\Sigma_{t=1}^{T} x_t^2 = o(T^{2d}) \qquad \text{and} \qquad \Sigma_{t=1}^{T} \varepsilon_t^2 = o(T) \tag{2.4.10}$$

since ε_t is $I(d-b)$, which is a stationary process for $d-b < \frac{1}{2}$. Applying the Cauchy-Scwarz inequality gives

$$\left(\Sigma_{t=1}^{T} x_t^2/T^{2d}\right)\left(\Sigma_{t=1}^{T} \varepsilon_t^2/T\right) \geq \left(\Sigma_{t=1}^{T} x_t\varepsilon_t/T^{d+\frac{1}{2}}\right) \tag{2.4.11}$$

This suggests that $\Sigma_{t=1}^{T} x_t\varepsilon_t = o(T^{\tau})$ with $\tau \leq d+\frac{1}{2}$ .A tighter band, however, can be obtained by observing that

$$\Sigma_{t=1}^{T} x_t\varepsilon_t/T^{2d-b} = \Sigma_{t=1}^{T} \left(x_t/T^{d-\frac{1}{2}}\right)\left(\varepsilon_t/T^{d-b-\frac{1}{2}}\right)/T \tag{2.4.12}$$

converges in distribution to some function of Brownian motions, following from the functional central limit theorem. This implies that $\sum_{t=1}^{T} x_t \varepsilon_t = o(T^{\tau})$ with $\tau \leq 2d - b$, so that the result in (2.4.10) still holds here.

III. Dynamic Real Exchange Rate Models and Fractional Integration

1. Long Memory Models and GPH Test

Integrated Autoregressive Moving Average (ARIMA) models and Unobserved Components (UC) models are the standard time-series models used to study the intertemporal dynamics of real exchange rates. Unrestricted ARIMA models are not correct specifications to study the long-run properties of real exchange rates because these models examine the short-run dynamics of time-series. Mean reversion or long memory in economic time-series depends crucially on correlations at long lags, which easily can be misspecified in simple ARIMA models. On the other hand, the UC model is a special case of the ARIMA model, that is, UC models are restricted ARIMA models. What we need, in order to examine long-range dependence is not a specialization but a generalization of the ARIMA model, a model that can capture the long-run behavior of a time-series. Thus, the behavior of real exchange rates would be modelled by using an extension of the standard ARIMA model which is called

Fractionally Integrated Autoregressive Moving Average (ARFIMA) model that allows modeling of long-period components.

ARFIMA models were introduced by Granger and Joyeux (1980) and Hoskings independently. The generalization makes an ARFIMA model a parsimonious and flexible model to study short-run dynamics and long-memory simultaneously.

Typically, standard unit-root tests are used to examine the presence of long-range dependence of real exchange rate time-series, that is, by testing for unit-roots in the autoregressive lag-operator polynomials. If a unit-root is found in an ARMA representation the series may be decomposed into the sum of a random walk component and a stationary component. The permanence of the random walk model implies long-range dependence.

McLeod and Hipel (1978) suggested that, a covariance stationary time series process exhibits long memory, if it satisfies the following condition:

$$\Sigma_{j=-n}^{n} |\rho(j)| \rightarrow \infty \qquad \text{as} \qquad n \rightarrow \infty,$$

where $\rho(j)$ is the autocorrelation at lag j. The implication of this infinite sum condition is that the correlation between observations some distance apart can be relatively high, that is, autocorrelations at long lags are not negligible. The slow decline in the autocorrelations (ACF) is the reflection of the long memory in the series.

To formalize, a weakly or covariance stationary process possesses a constant mean and variance and autocovariance function that is just a function of lag j and is independent of time. The Wold's decomposition theorem states that any stationary process, y_t, can be uniquely represented as the sum of two mutually uncorrelated processes ζ_t and η_t. These processes are respectively a purely indeterministic one-sided moving average (MA) process of a stationary uncorrelated sequence and the other a deterministic process. Thus ,

$$y_t = \zeta_t + \eta_t \qquad (1.1)$$

where $\zeta_t = \Sigma_{j=0}^{\infty} \psi_j \varepsilon_{t-j}$ and $\varepsilon_t \sim iid(0, \sigma^2), E(\varepsilon_t, \varepsilon_\tau) = 0, \tau \neq t$ .The validity of the Wold's decomposition also requires that $\Sigma_{j=0}^{\infty} \psi_j^2 < \infty$. Furthermore, the spectral distribution function y_t is a nondecreasing function that can be partitioned into the sum of three components,

$$F(\lambda) = F_1(\lambda) + F_2(\lambda) + F_3(\lambda) \qquad (1.2)$$

where $F_1(\lambda)$ is discontinuous (a step function) and is due to the presence of $\eta_t, F_2(\lambda)$ is absolutely continuous and corresponds to ζ_t and $F_3(\lambda)$ is a singular function that is insignificant and is ignored in most applications.

Since economic time-series are not generally considered to possess pure harmonic or deterministic components, they can be expressed in terms of ζ_t alone.

The mixed autoregressive moving average ARMA (p,q) process is

$$\Phi(L)r_t = \Theta(L)\varepsilon_t \qquad (1.3)$$

where $\Phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p, \Theta(L) = 1 - \theta_1 L - \cdots - \theta_q L^q$ and where L is the lag operator defined as $L^j r_t = r_{t-j}$ for integer values of j. For r_t to be stationary it is necessary that all the roots of $\Phi(L)$ lie outside of the unit circle. To avoid model multiplicity, it is required that all the roots of $\Theta(L)$ also lie outside the unit circle, the requirement for invertibility.

Empirical studies using unit-root tests have concluded that real exchange rates are well described by low-order ARMA processes with a single unit-root.

In addition to this result the above model is restrictive to the low-frequency dynamics, which justifies the use of the ARFIMA (p,d,q) model.

The ARFIMA (p,d,q) model is

$$\Phi(L)(1-L)^d r_t = \Theta(L)\varepsilon_t \quad , \quad \varepsilon_t \sim iid(0,\sigma^2) \qquad (1.4)$$

The differencing operator, d, of an ARFIMA model is not restricted to the integer domain and can assume real values. Fractional integration is a more general way to describe long-range dependence than the unit-root specification and provides an alternative perspective to examine the unit-root hypothesis. Operationally the fractional differencing operator is defined by the binomial series expansion

$$\Delta^d = (1-L)^d = \sum_{\kappa=0}^{\infty} \begin{bmatrix} d \\ \kappa \end{bmatrix} (-L)^\kappa$$

$$= \sum_{\kappa=0}^{\infty} \frac{\Gamma(\kappa-d)}{\Gamma(-d)\Gamma(\kappa+1)} L^\kappa$$

$$= 1 + \sum_{\kappa=1}^{\infty} \frac{\Gamma(\kappa-d)}{\Gamma(-d)\Gamma(\kappa+1)} L^\kappa$$

Since $\Gamma(x) = (x-1)\Gamma(x-1)$ for positive x ,

$$\Delta^d = 1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 + \cdots \tag{1.5}$$

where $\Gamma(\cdot)$ is the gamma or generalized factorial function.

For noninteger d, the filter $(1-L)^d$ is an infinite-order lag operator polynomial with slowly and monotonically declining coefficients. Hoskings (1981) showed that the autocorrelations of an ARFIMA process have a slower hyperbolic autocorrelation decay than ARMA processes. Specifically, ARIMA(p,d,q) autocorrelations decay exponentially and satisfy

$$\rho_r(\kappa) \sim b^{\kappa} \quad \text{as} \quad \kappa \to \infty \ ,$$

where b is a constant such that $|b| < 1$. In contrast, ARFIMA (p,d,q) processes satisfy

$$\rho_r(\kappa) \sim \kappa^{2d-1} \quad \text{as} \quad \kappa \to \infty \ ,$$

which means that autocorrelations decline monotonically and hyperbolically towards zero as the lag increases, and it also means that the correlation between observations some distance apart can be relatively high. The slow decline in the autoregressive weights in equation (1.5) reflects the long-memory property of the process.

Granger and Joyeux (1980) and Hosking (1981) also showed that if $|d| < \frac{1}{2}$, r_t is stationary and invertible,[2] with variance

$$Var(r_t) = \Gamma(1-2d)/[\Gamma(1-d)]^2 \qquad (1.6)$$

and autocorrelation function

$$\rho(\kappa) = \frac{\Gamma(1-d)\Gamma(\kappa+d)}{\Gamma(d)\Gamma(\kappa+1-d)} \quad , \quad \kappa = 1, 2, \cdots \qquad (1.7)$$

If $d \geq \frac{1}{2}$ the variance of r_t is infinite, that is, the autocorrelations do not have a finite sum, and the process is nonstationary.

The long-memory behavior of real exchange rates can also be seen in the frequency domain by observing the shape of its spectral density. A real exchange rate is said to display long-memory if its spectral density, $f_r(\lambda)$, is unbounded at angular frequency $\lambda = 0$ rather than bounded as for a stationary ARMA series, that is,

$$\lim_{\lambda \to 0} f_r(\lambda) = \infty \qquad (1.8)$$

For an ARFIMA (p,d,q) model, the spectral density behaves like λ^{-2d} as

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2 Covariance stationarity requires that d<1/2.

$\lambda \to 0$, so the ARFIMA model has the ability to capture a wide variety of low-frequency behavior with a single parameter, the differencing parameter d. In contrast, for an ARIMA (p,1,q) model the spectral density behaves like λ^{-2} as $\lambda \to 0$. The implication of these results is that we can get better and richer range of spectral behavior near the origin when the differencing operator d can assume noninteger values.

The memory property of the process depends crucially on the value of the differencing parameter d. When $0 < d < 1/2$, the ARFIMA model displays long-memory. Since, it is the parameter d that enables long-memory to be modeled , a model building strategy requires a method of estimating d independently of the rest of the model and the value chosen for it is obviously crucial in any empirical application. Typically this value is unknown and must therefore be estimated. A number of methods have been proposed as a two stage semiparametric procedure proposed by Geweke and Porter-Hudak (GPH, 1983), approximate frequency domain Maximum likelihood (ML) proposed by Fox and Taggu (1986), and exact time domain Maximm likelihood proposed by Sowell (1990). The approach proposed by GPH has been proved useful in many economic application and it is the one that is used in this empirical study. However, when the correct model specification is known, exact ML gives more accurate estimates than the GPH method, but correct model specification is usually unknown. Thus, following Diebold and Rudebusch (1989), first a consistent and asymptotically normal estimate of d will be obtained. Having obtained an estimate of $d, \hat{d}$, for each series the real exchange rate series can be transformed by the long-memory filter $(1-L)^{d}$. The transformed series is then modeled as an ARMA process to obtain consistent estimates of the remaining model parameters, Φ, Θ and σ^2. The first-stage estimate of d is based on the order of the spectral density function near the origin $(\lambda = 0)$. By writing

$$(1-L)^{d} r_t = (1-L)(1-L)^{d-1} r_t = (1-L)^{\hat{d}} y_t \qquad (1.9)$$

where $y_t = (1-L) r_t$ and $\hat{d} = d - 1$[3]. Then the ARFIMA model can be written as

$$(1-L)^{\hat{d}} y_t = \Phi^{-1}(L)\Theta(L)\varepsilon_t \equiv w_t \qquad (1.10)$$

where w_t is stationary process. The spectral density of the real exchange rate is

$$f_r(\lambda) = \; 1 - e^{-i\lambda} \; {}^{-2d} f_w(\lambda)$$

$$= 2^{-d}(1 - \cos\lambda)^{-d} f_w(\lambda)$$

$$= 4^{-d} \sin^{-2d}(\lambda/2) f_w(\lambda) \qquad (1.11)$$

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[3] Since d=d+1, a value of d=0 corresponds to a unit-root in real exchange rate.

where $f_w(\lambda)$ is the spectral density of the stationary process w_t. Taking logs gives

$$\log f_r(\lambda) = -d \log\left\{4\sin^2(\lambda/2)\right\} + \log f_w(\lambda) \qquad (1.12)$$

Adding the periodogram, $I(\lambda_j)$, defined by

$$I(\lambda_j) = (1/2\pi T)\left|\Sigma_{t=1}^{T} e^{it\lambda}\left(r_t - \bar{r}\right)\right|^2 \quad ,$$

to each side at the harmonic (or Fourier) frequencies $\lambda_j = 2\pi j/T, j = 0, 1, 2, \cdots, T-1$ and rearranging gives

$$\log I(\lambda_j) = \log\{f_w(0)\} - d\log\left\{4\sin^2(\lambda_j/2)\right\} + \log\{f_w(\lambda_j)/f_w(0)\} + \\ + \log\{I(\lambda_j)/f(\lambda_j)\} \qquad (1.13)$$

If attention is restricted to the lower frequencies, the penultimate term can be dropped as negligible,

$$\log I(\lambda_j) = \log\{f_w(0)\} - d\log\left\{4\sin^2(\lambda_j/2)\right\} + \log\{I(\lambda_J)/f(\lambda_j)\} \qquad (1.14)$$

This last result suggests treating the above equation as a simple linear reggression model in which the last term is the disturbance,

$$\log\{I(\lambda_j)\} = \beta_0 + \beta_1 \log\left\{4\sin^2(\lambda_j)/2\right\} + \eta_j \tag{1.15}$$

where β_0 is the constant $\log(f_w(0))$ and η_j is equal to $\log\{I(\lambda_j)/f(\lambda_j)\}$ are identically and independently distributed across the harmonic frequencies. Kunsch (1986) argues that frequencies around the origin need to be excluded to get a consistent estimator of d from regressing $\log I(\lambda_j)$ on $\log\left\{4\sin^2(\lambda_j/2)\right\}$. Therefore, if the number of low-frequency ordinates is a function of the sample size, that is, $\eta = g(T)$, then the negative of the OLS estimate of the slope coefficient gives a consistent and asymptotically normal estimate of d. With the proper choice of η, the asymptotic distribution of d depends on neither the ARMA component nor the distribution of the error term of the ARFIMA process. Brockwell and Davis (1987) and Shea (1989) recommended using $\eta = g(T) = T^{\alpha}$ and $\alpha = 0.5$. The variance of the estimate of b_1 is given by the usual OLS estimator and the theoretical asymptotic variance of the regression error $\eta_j, \pi/6$.

2. Estimating ARFIMA models

The frequency domain approach to the ML estimation is based on a Fourier transform that converts serial correlation into heteroskedasticity. Assuming normality the likelihood function of an ARFIMA model can be written as

$$L = (2\pi)^{-\frac{T}{2}}|\Sigma|^{-\frac{1}{2}}\exp\left(-\frac{x'\Sigma^{-1}x}{2}\right) \tag{2.1}$$

where Σ is the $T \times T$ variance-covariance matrix and is a function of $d, \Phi(L), \Theta(L)$ and σ^2. Taking logs gives,

$$\log L = -\tfrac{T}{2}\log 2\pi - \tfrac{1}{2}\sum_{j=0}^{T-1}\log 2\pi j - \pi \sum_{j=0}^{T-1}\frac{I(2\pi j/T)}{(2\pi j/T, \xi)} \tag{2.2}$$

where $I(\lambda_j)$ is the periodogram $I(\lambda_j) = (1/2\pi T)\left|\sum_{t=1}^{T} r_t e^{-i\lambda_j t}\right|^2$ and $f_r(\lambda_j)$ is the spectral density, defined at the Fourier frequencies $\lambda_j = 2\pi j/T, j = 0, 1, 2, \cdots, T-1$. For ARMA (p,q) models the second terms of equation (2.2) reduces to $[-T/2]\log\sigma^2$ and therefore, the maximization of $\log L$ with respect to σ^2 gives

$$\hat{\sigma}^2(\Phi, \Theta) = \frac{2\pi}{T}\sum_{j=1}^{T-1}\frac{I(\lambda_j)}{f_r(\lambda_j, \xi)} \tag{2.3}$$

Thus σ^2 may be concentrated out of the likelihood function, which led Fox and Taggu (1986) to suggest that maximization of the likelihood function is equivalent to minimization of

$$\sigma_T^2(\xi) = \sum_{j=1}^{T-1}\frac{I_r(2\pi j/T)}{f_r(2\pi j/T, \xi)} \tag{2.4}$$

with respect to the ARFIMA parameter vector $\xi = \{\phi_1, \phi_2, \cdots, \phi_p, \theta_1, \theta_2, \cdots, \theta_q\}$. $f(\lambda_j, \xi)$ is proportional to the spectral density of r at frequency λ ,

$$f_r(\lambda, \xi) = \left|1 - e^{-i\lambda}\right|^{-2d} \frac{\left|\Theta\left(e^{-i\lambda}\right)\right|^2}{\left|\Phi\left(e^{-i\lambda}\right)\right|^2} \tag{2.5}$$

and satisfies the normalization condition

$$\int_{-\pi}^{\pi} \log \{f_r(\lambda, \xi)\} d\lambda = 0 \tag{2.6}$$

The distribution of $\sqrt{T}\left(\hat{\xi} - \xi_0\right)$ can be approximated by

$$\sqrt{T}\left(\hat{\xi} - \xi_0\right) \to N\left(0, 2\sigma_T^2\left(\hat{\xi}\right)\right)\left[\frac{\partial^2 \sigma_T^2(\xi)}{\partial\xi\partial\xi'}\Big|_{\hat{\xi}}\right]^{-} \tag{2.7}$$

where ξ_0 is the true parameter vector. The general form of (2.7), was given by Fox and Taggu (1986). The frequency domain estimates reported are based on (2.4), (2.5) and (2.6).

3. Impulse - Response Functions

As mentioned earlier, unit-root tests are used to examine the existence of long-range dependence in the time-series, but a more interesting question is to measure the importance of the persistence in real exchange rate changes. One important measure of persistence of the estimated ARFIMA models used by

Campbell and Mankiw (1987) and by Watson (1986) is the cumulative impulse response function, that is, the sum of the coefficients of the moving-average lag-operator polynomial of the first-differenced series. Specifically, the ARFIMA model can be put into MA form

$$\begin{aligned}\Phi(L)(1-L)^d r_t &= \Theta(L)\varepsilon_t \\ (1-L)^d r_t &= \left(\Phi(L)^{-1}/\Theta(L)\right)\varepsilon_t \\ &= B(L)\varepsilon_t\end{aligned}$$

where $B(L) = \Theta(L)/\Phi(L)$

$$\begin{aligned}(1-L)^{d-1}(1-L)r_t &= B(L)\varepsilon_t \\ (1-L)r_t &= \left[B(L)/(1-L)^{d-1}\right]\varepsilon_t \\ (1-L)r_t &= A(L)\varepsilon_t\end{aligned}$$

where $A(L) = B(L)/(1-L)^{d-1}$,

$$\begin{aligned}(1-L)r_t &= (1 + a_1 L + a_2 L^2 + \cdots)\varepsilon_t \\ &= \textstyle\sum_{i=0}^{\infty} a_i \varepsilon_{t-i} \qquad \text{where } a_0 = 1.\end{aligned}$$

The MA parameters $a_i, i = 0, 1, 2, \cdots$, are called impulse responses and they show the impact on future real exchange rate changes to a unit-innovation. The impulse response function, $C(\kappa)$ is given by

$$C(\kappa) = \tfrac{\partial \Delta r_t}{\partial \varepsilon_{t-\kappa}} = 1 + a_1 + a_2 + \cdots + a_\kappa \qquad (3.1)$$

which measures the effect of a unit-shock on the k period ahead real exchange rate change. In the limit , we get the cumulative impulse responses, $C(\infty)$, which is the impact of a unit shock today on the level of real exchange rate in the future. For a stationary process , $C(\infty) = 0$, but for a random walk series, the effect is permanent. Parity reversion or long memory occurs when $d < 1$ and $C(\infty) = 0$. When $d > 1$ the $C(\infty) = \infty$ and therefore $C(\infty)$ diverges. $C(\infty)$ converges when d=1, the unit-root case, and $C(\infty)$ is finite and nonzero. In order to answer the question of how does a shock today affects the level of the real exchange rates in the short-run and in the long-run, the entire sequence of cumulative impulse responses $C = \{1, c_1, c_2, \cdots, c_\infty\}$ are examined and not the infinite cumulative impulse response because such an examination gives information regarding the pattern and speed with which shocks to PPP are propagated.

IV. Empirical Analysis

In this part of section IV, I examine whether there is evidence of fractional integration in real exchange rate data at montlhy frequencies. As a first step, I obtained estimates of the fractional intergation parameter, d, for each of the six series. Then, ML estimates of the selected ARFIMA models are obtained and finally, long-memory is examined in the series through the sequence of cumulative impulse responses.

1.Description of Results

The time-series properties of the six dollar real exchanges rates - US/Canada, US/Germany, US/Italy, US/Japan, US/United Kingdom, and US/France - are examined. Graphs of the log real exchange rate data and first differenced log data are presented in figure 1 and figure 3. Their corresponding spectra are presented in figure 4. Examination of these time-series graphs makes clear the need for a class of model enabling flexible parameterization of low-frequency dynamics. Note the rapid rise and subsequent fall in the value of the dollar over the period 1980-1987. Except for the U.S./Germany and U.S./Japan, this rise and fall characterized the behavior of most of the dollar real exchange rates.

These graphs also give some indication of the performance of PPP since the real exchange rates presented have experienced substantial fluctuations that appear to be systematically associated with movements in the corresponding nominal exchange rate. An illustration of this is that in mid - 1975 the dollar appreciated relative to the DM by nearly 10 percent and an associated increase can be seen in the real exchange rate. The depreciation of the dollar in the late 1977 and early 1978 is reflected in a declining real exchange rate.

From the graphs of the first differened log data, it is clear that real exchange rate changes appear to be random fluctuations around zero and have no obvious structural break during the sample period, that is, real exchange rate data seem to be difference stationary. All various spectra have peaks near the origin which implies that the corresponding components near $\lambda = 0$ are of greatest importance, that is, the spectra are dominated by long-period components, and that the high frequency components are of little importance. A peak in the spectrum indicates an important contribution to the variance from the components at frequencies in the corresponding interval since the spectrum may be interpreted as the decomposition of the variance of a process. Indeed, when the Dickey-Fuller, the augmented Dickey-Fuller tests and the corresponding Phillips-Perron test for a unit-root were conducted on individual real exchange rate series, these tests could not reject the unit-root hypothesis at a 5% level except for the US/Germany series (Appendix: TableA1, TableA2 and TableA3). The above results show little favorable evidence of long-run PPP. As noted by Diebold and Rudebusch (1991) and Sowell (1990), however, these standard unit-root tests have low power against fractional integration alternatives.

Thus, this empirical work proceeds by modeling the behavior of real exchange rates using ARFIMA models that provide a better and flexible approximation to the World's representation and low-frequency dynamics. In addition, the use of ARFIMA models allows us to discriminate between long memory from short memory behavior, that is, it enables the discrimination between slow parity reversion from nonreverting martingale behavior.

The potential value of a fractionally differenced model is indicated by the sample autocorrelation function (AFC) of the first differened series. Typically, the decision to difference the data prior to further modeling is based on an inspection of the sample ACF. A first difference is usually implied by a slow linear decay of the ACF. The sample ACF of each series indeed indicates the need for a first-difference, as each shows a clear linear decline. If a fractionally differenced model is appropriate the sample ACF of the first-differenced series displays a hyperbolic decay rather than the slow linear decay characteristic of the standard ARIMA model. The sample ACF's of the first - differences of the logs of the six real exchange rates all show a hyperbolic decay and slower than the usual decay associated with autoregressive processes (Appendix: Figure A). Hence a first-difference is used throughout the analysis.

The results of applying the GPH test on first-differenced log data are reported in Table1 along with asympotic standard errors (SE) and the associated p-values, for the no long memory null hypothesis $(d = 0)$. The asymptotic standard errors are computed using the known theoritical variance of η_j, $\pi^2/6$. The p-values give the probability, under the no long memory null hypothesis against the long-memory alternative $(d > 0)$, of obtaining the estimated d. In general, the marginal significance level or p-value of a test statistic is the probability that a value as large or larger would occur by chance. Thus, the p-value is small for extreme values. There is evidence of long-memory if the least squares estimate of d is significantly larger than zero. In addition to the $\hat{d}$ that is based on $\eta = T^{.5}$, which is commonly used to test for long memory, table 1 also reports $\hat{d}$ estimates based on $\eta = T^{.45}$ and $\eta = T^{.55}$, to check the sensitivity of the results to the choice of η. The $\hat{d}$ estimates corresponding to $\alpha = \cdot 55$ are smaller than the corresponding to $\alpha = \cdot 45$ or $\alpha = \cdot 5$. This result verifies GPH (1983) observation that the estimation result can be contaminated as more periodogram ordinates are included in the regression: " improper inclusion of medium-frequency ordinates will contaminate the estimate of d, while too small of a regression sample will lead to imprecise estimates ".

The GPH estimates of the differencing parameter, d, clearly suggest that there is long-memory in all six real exchange rate series. For each real exchange rate, the estimated value of d is significantly above zero and less than

TABLE 1.Results of the Geweke and Porter-Hudak (GPH) Test

Sample Period:1973:1-1994:1

Estimates of d

Data Series	0.45	0.50	0.55
US/Canada	0.27425	0.06101	-0.19496
	(0.141)	(0.113)	(0.107)
p-value	0.124	0.001	0.000
US/Germany	0.97746	0.63631*	0.32809***
	(0.117)	(0.110)	(0.107)
p-value	0.461	0.054	0.002
US/Italy	0.45460**	0.12458***	-0.21788***
	(0.119)	(0.118)	(0.117)
p-value	0.020	0.001	0.000
US/Japan	0.87052	0.42693**	0.05552***
	(0.129)	(0.146)	(0.160)
p-value	0.307	0.030	0.004
US/UK	0.62617**	0.45995***	-0.22030***
	(0.094)	(0.090)	(0.170)
p-value	0.047	0.004	0.001
US/France	0.52970	0.24016***	0.02304***
	(0.184)	(0.144)	(0.121)
p-value	0.107	0.008	0.001

Notes: Asymptotic standard errors appear in parentheses. "p-value" gives the p-value of the no long memory null hypothesis (d=0) against the long memory (d>0) alternative. *, **, and * indicate singnificance at 10%, 5%, and 2.5%.**

one except for the US/Canada series that is marginally significant. The d parameter estimates suggest that the memory of US/France, US/Japan, and the US/UK series is of comparable magnitude. The memory of the US/Germany, is the strongest among the six series, whereas the memory in the US/Canada series is the weakest. For the US/Canada, US/Germany, US/Italy, US/Japan, US/UK and US/France, the GPH test on the first-differenced series suggest a fraction integration parameter of 0.06, 0.64, 0.12, 0.43, 0.46, and 0.24 respectively. Thus the data are transformed by applying the filter $(1-L)^{0.94}$ to US/Canada, $(1-L)^{0.36}$ to US/Germany, $(1-L)^{0.88}$ to US/Italy, $(1-L)^{0.57}$ to US/Japan, $(1-L)^{0.54}$ to US/UK, and $(1-L)^{0.76}$ US/France series.

In sum, through the application of the GPH test, evidence favorable to long-memory is found in all six real exchange rate series. The unit-root hypothesis is rejected in favor of the long-memory alternative because the estimated ds are significantly larger than zero. By the construction of the GPH test, the empirical result is robust to both the short-run dynamics and the underlying distribution of the series. In other words, the asymptotic distribution of d depends on neither the order of the ARMA component nor the distribution of the error term of the ARFIMA process.[4] This result suggests that the evidence of a unit-root in real exchange rate data is not robust to fractional

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4 See Yagima 1989.

integration alternatives. These findings warrant the use of ARFIMA models to examine real exchange rate dynamics.

To capture the remaining short-run dynamics of real exchange rate date ARFIMA (p,d,q) models with p and q less than or equal to 3 are used, that is, 16 different model specifications are considered for each real exchange rate series. The parameters of each model specification are estimated by the frequency domain approximate ML method. Estimates are obtained by minimizing equation (2.4) using the Broyden-Fletcher-Foldfarb-Shanno (BFGS) algorithm which is a slight modification of the better known Davison-Fletcher-Powell (DFP) algorithm. The order of the ARFIMA model is initially determined by the Akaike information criterion (AIC) and then the Scwartz information (SIC) is used to choose the final ARFIMA specifications. Because the AIC and SIC have different optimality properties, the model selected by SIC is generally more parsimonious.

Maximum likelihood parameter estimates of the model selected are reported in table 2 along with their asymptotic standard errors. Overall, the estimation results are in accordance with the GPH rest results. The estimated ARFIMA model indicate that there is long-memory in real exchange rate changes, and the dynamics are more complicated than implied by the random-walk hypothesis. The estimates of the autoregressive and moving average parameters are also of interest. They generally imply that the persistence of memory implied by the model as a whole is moderately high. In particular, for those models for which the persistence appears to be short-memory (i.e. the estimated value of d is insignificantly different from zero), the autoregressive and moving average parameters imply relatively strong shock persistence. For instance, the real dollar-Canadian dollar exchange rate has the weakest memory of all series and the model selected by the SIC is an AR (1) in levels. The estimation of that model gives a parameter of 0.99902, which implies that the half-life of the shock that moves the real exchange rate away from its parity value is approximately 12 months.

Graphical analysis of the cumulative impulse response functions of the selected ARFIMA models is presented in figure 2. The imlulse responses are shown over a horizon of 60 months. An interesting result that emerges from these calculated impulse responses is that the effect of a unit-shock on real exchange rate changes are similar across the six real exchange rate series. It is evident from the impulse response graphs that the long-memory and the

TABLE 2. Parameter Estimates of the Selected ARFIMA Models

Countries	d	ϕ_1	ϕ_2	ϕ_3	θ_1	θ_2	θ_3
US/Canada	**0.06101** **(0.113)**	**0.99902** **(0.007)**	....	....	....	....	....
US/Germany	**0.63631** **(0.110)**	**0.98561** **(0.010)**	....	....	**0.07593** **(0.049)**	**-0.76594** **(0.051)**	....
US/Italy	**0.12458** **(0.118)**	**1.00088** **(0.001)**	....	....	....	....	....
US/Japan*	**0.42693** **(0.146)**	**0.99845** **(0.001)**	....	....	**0.00401** **(0.061)**	**-0.45110** **(0.063)**	**-0.19647** **(0.067)**
US/UK	**0.45995** **(0.090)**	**0.99447** **(0.008)**	....	....	**0.14624** **(0.059)**	**-0.58255** **(0.060)**	....
US/France*	**0.24016** **(0.144)**	**1.00038** **(0.002)**	....	....	**0.06719** **(0.068)**	**-0.31657** **(0.096)**	....

NOTES: Asymptotic standard errors appear in parentheses. * , indicates that there is agreement between the AIC $(2 \ln L - 2\kappa)$ **selected model and the SIC** $(2 \ln L - \kappa \ln T)$ **selected model.**

short-memory interact in such a way that the persistence, as measured by $C(\kappa)$, is usually moderate high and less than 1. This is also true for the US/Germany series, which has a relatively large estimate of d. The slow decay of $C(\kappa)$ indicates the long-memory effect (the eventual decay toward zero means parity reversion). The half life of a shock that moves away the real exchange rates from their parity value, averaged across the currencies, is 12 months.

Consider the response of the real US/Canada series to a unit-shock presented in figure 2.1. On impact real US/Canada series jumps by one unit (by 100%, because the log of the real exchange rate is used) and then it falls, roughly for 36 months until it reaches its long-run parity value. The half-life of the shock is approximately 10 months, after 24 months 15 percent of the shock is still present.

In the case of the real US/Italy exchange rate, presented in figure 2.3, the initial response to the one-unit shock is clearly 100 percent increase in the value of the real exchange rate and a subsequent decrease until it reaches its long-run value after 60 months. By 27 months out, the response has dropped to about 0.25 and after 48 months it is less than 0.1 and continues to decline. After 24 months 30 percent of the shock is present and it takes about 16 months for the half-life of the shock to die out.

Figure 2.4 shows the response of US/Japan real exchange rate to a unit-shock. The major difference between this series and all other real exchange time series is the initial shock magnification followed by shock dissipitation. On impact it increases by more than 100 percent, approximately by 110 percent, then it falls rapidly for 32 months until it reaches its long-run parity value. The maximum impulse response value occurs at less than 8 months and the rate of decline is approximately 5 percent per month.

An important result of this section is that the dynamic responses of all six real exchange time series follow a very similar pattern. The graphs of the impulse responses clearly show that there is long memory in all series, and that the typical half-life of the shock is approximately one year.

2. Sources of Long-Memory

There are different possible sources of long-memory in real exchange rates. For example, the long-memory behavior of real exchange rates can be related to the dynamic properties of other economic variables. The PPP hypothesis suggests that real exchange rate fluctuations are tied to the movements of nominal exchange rates and relative national price levels. Thus, by examining if there is long-memory in nominal exchange rates and national price levels one source of long-memory can be identified, which is due to the components of the real exchange rate.

Tables 3 and 4 report the tests for long-memory in the first differences of the six nominal exchange rates and relative national price series. The relative price indexes are all against the U.S. price index. The GPH test suggest that there is long-memory in both, nominal exchange rates and relative price indexes, and the evidence is quatitevely the same across different choices of n. The least squares estimates of the differencing parameters are greater than zero and less than one for both series. The PPP hypothesis suggests that these results of long-memory in nominal exchange rates and in relative national price level level changes reinforces the evidence of long-memory in real exchange rate changes.

Fundamentals may contribute to long-memory found in the real exchange rate data as well. Standard exchange rate determination models

TABLE 3. Tests for Long Memory in Nominal Exchange Rates

Sample Period: 1973:1-1994:

Estimates of d

Data Series			
CD	0.29337**	0.11820***	-0.16769***
	(0.165)	(0.121)	(0.119)
p-value	0.026	0.001	0.000
DM	0.67091*	0.34973***	-0.09559***
	(0.108)	(0.109)	(0.134)
p-value	0.072	0.004	0.001
IL	0.60105**	0.18649***	-0.15923***
	(0.088)	(0.119)	(0.116)
p-value	0.021	0.002	0.000
JY	0.84577	0.43226**	0.07901***
	(0.128)	(0.135)	(0.140)
p-value	0.545	0.024	0.002
BP	0.81891	0.57278**	-0.13647***
	(0.139)	(0.122)	(0.191)
p-value	0.273	0.045	0.003
FM	0.59318*	0.26381***	-0.06015***
	(0.151)	(0.134)	(0.114)
p-value	0.096	0.007	0.000

NOTES: Asymptotic standard errors appear in parentheses. The d estimates are obtained from GPH test using the first differences of the log nominal exchange rate series. "p-value" gives the p-value of the no long memory null hypothesis (d=0) against the long memory alternative. *, **, and * indicate significance at 10%, 5%, and 2.5% respectively.**

TABLE 4. Tests for Long Memory in Relative Price Indexes

Sample Perid: 1973:1-1994:1

Estimates of d

Data Series	0.45	0.50	0.55
Canada/US	0.30103*	0.13263***	-0.28202***
	(0.189)	(0.142)	(0.140)
p-value	0.043	0.004	0.000
Germany/US	0.15139**	0.19894***	0.18820***
	(0.141)	(0.100)	(0.076)
p-value	0.001	0.006	0.000
Italy/US	-0.06295***	0.02145***	-0.06123***
	(0.152)	(0.106)	(0.082)
p-value	0.003	0.000	0.000
Japan/US	-0.16198***	-0.31122***	-0.27530***
	(0.110)	(0.093)	(0.074)
p-value	0.000	0.000	0.000
UK/US	0.49797**	0.27774***	-0.13122***
	(0.099)	(0.086)	(0.107)
p-value	0.013	0.001	0.000
France/US	-0.24520***	-0.04746**	-0.16174***
	(0.265)	(0.187)	(0.147)
p-value	0.018	0.006	0.001

NOTES: Asymptotic standard errors appear in parentheses. The d estimates are obtained from GPH test using the first differences of the log CPI's series. "p-value" gives the p-value of the no long memory null hypothesis (d=0) against the long memory (d>0) alternative. *, **, and * indicate significance at 5%, 2.5%, and .5%, respectively.**

usually explain exchange rate dynamics by movements of relative money supplies, relative outputs, relative productivities and so forth. Recent empirical studies on the dynamic properties of the U.S. macroeconomic time-series, such as output consumption and money supply data, showed that these macroeconomic series may be fractionally integrated. (Diebold and Rudebusch 1989, 1991b, Haumrich 1989, Porter-Hudak 1990). The implication of these studies is that the long-memory in real exchange rate data may not be an isolated incident but it may be the product of fractional integration in other macroeconomic variables that determine the real exchange rate.

V. Summary and Conclusions

The time series properties of six major real exchange rate series are examined with the general ARFIMA model. The ARFIMA model, in which the differencing parameter d can assume nointeger values, provides a flexible and parsimonious way to model the low-frequency dynamics of deviation from PPP, and at the same time it provides a convenient framework to study both short and long-memory behavior. Accordingly, the ARFIMA model can identify a wide range of mean-reversion behavior which turns out to be important for a proper evaluation of long-run PPP. The ARFIMA model also suggests interesting long-memory alternatives to the unit-root hypothesis.

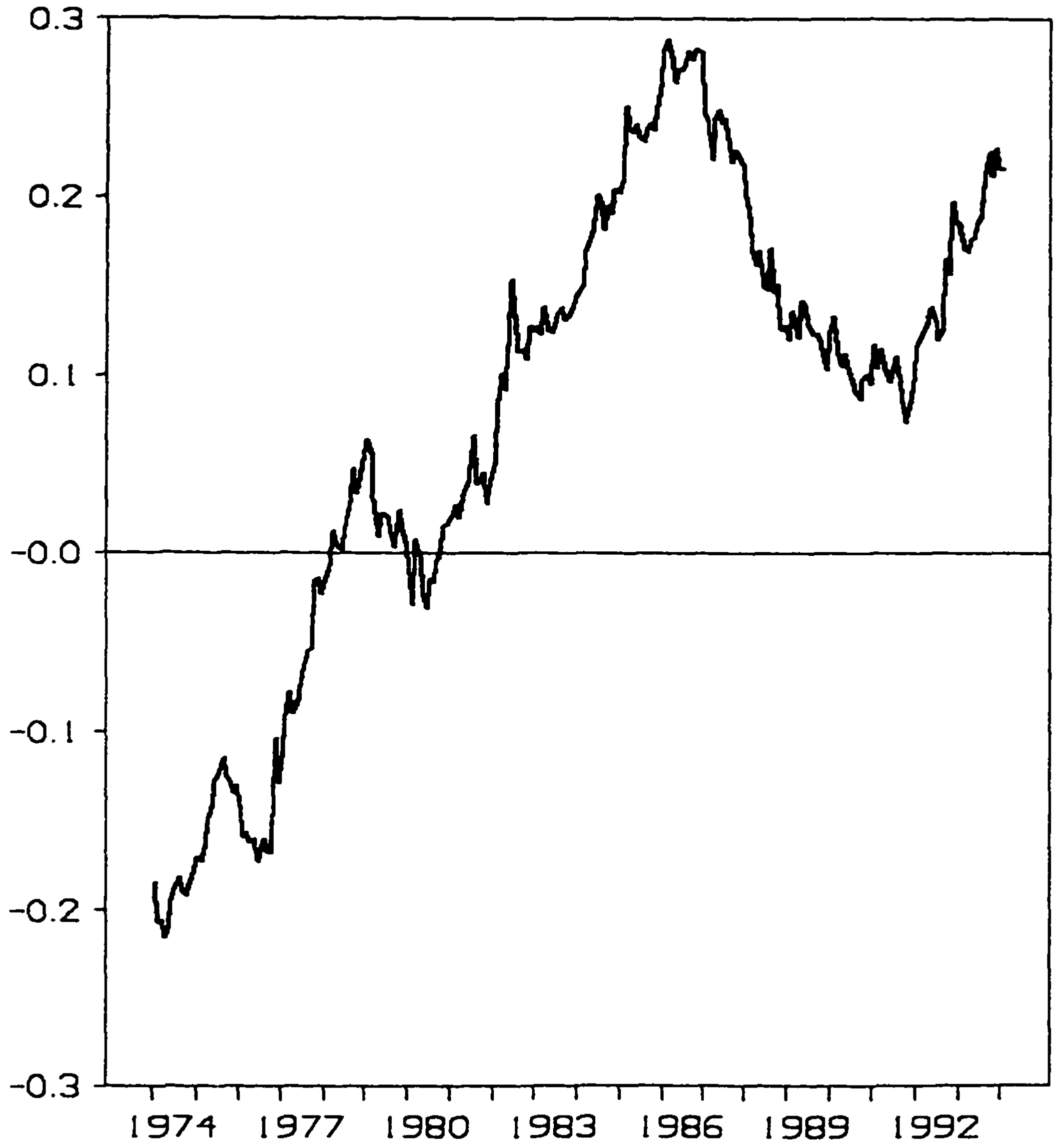
The empirical results show that there is evidence of long-memory in real exchange rate changes and that the evidence from the fractional ARIMA model is much more favorable to long-run PPP than from the standard ARIMA model using unit-root tests, that is , the test result is asymptotically robust to both the distribution of the innovation term and the short-run dynamics in the data. This implies that the empirical evidence of unit-roots in real exchange rate data is not robust to long-memory alternatives.

When ARFIMA models are fitted to the data, it is found that exchange rate dynamics are more complex than implied by a random walk. Analysis of impulse response functions indicates that the persistence in the real exchange rate changes is moderately high.

The findings of the long-memory property of real exchange rates for the different countries invite questions concerning the source of such property. Recent studies on the dynamic properties of macroeconomic time series indicate that the long-memory found in the real exchange rate data may not be an isolated incident. For example, Johansen and Juselius (1990b) suggested that deviations from PPP for the United Kingdom can be accounted for by interactions between exchange rates and interest rates. It is possible that the behavior of PPP deviations may in general reflect the statistical property of economic fundamentals such as the level of output, money supply, and interest rates. In this regard, the studies by Diebold and Rudenbusch (1989), Porter-Hudak (1990), Shea (1991) and Yin-Wong (1993) are particularly interesting. These studies individually report evidence of fractional integration in output, money supply, interest rates, and nominal exchange rates. I also found evidence of long-memory in monthly nominal exchange rate data and national consumer price indexes, which are closely related to real exchange rates under the PPP hypothesis. Of course, more systematic work is required to establish the relationship of the dynamics of real exchange rates and fundamentals and to determine the source of deviations from PPP.

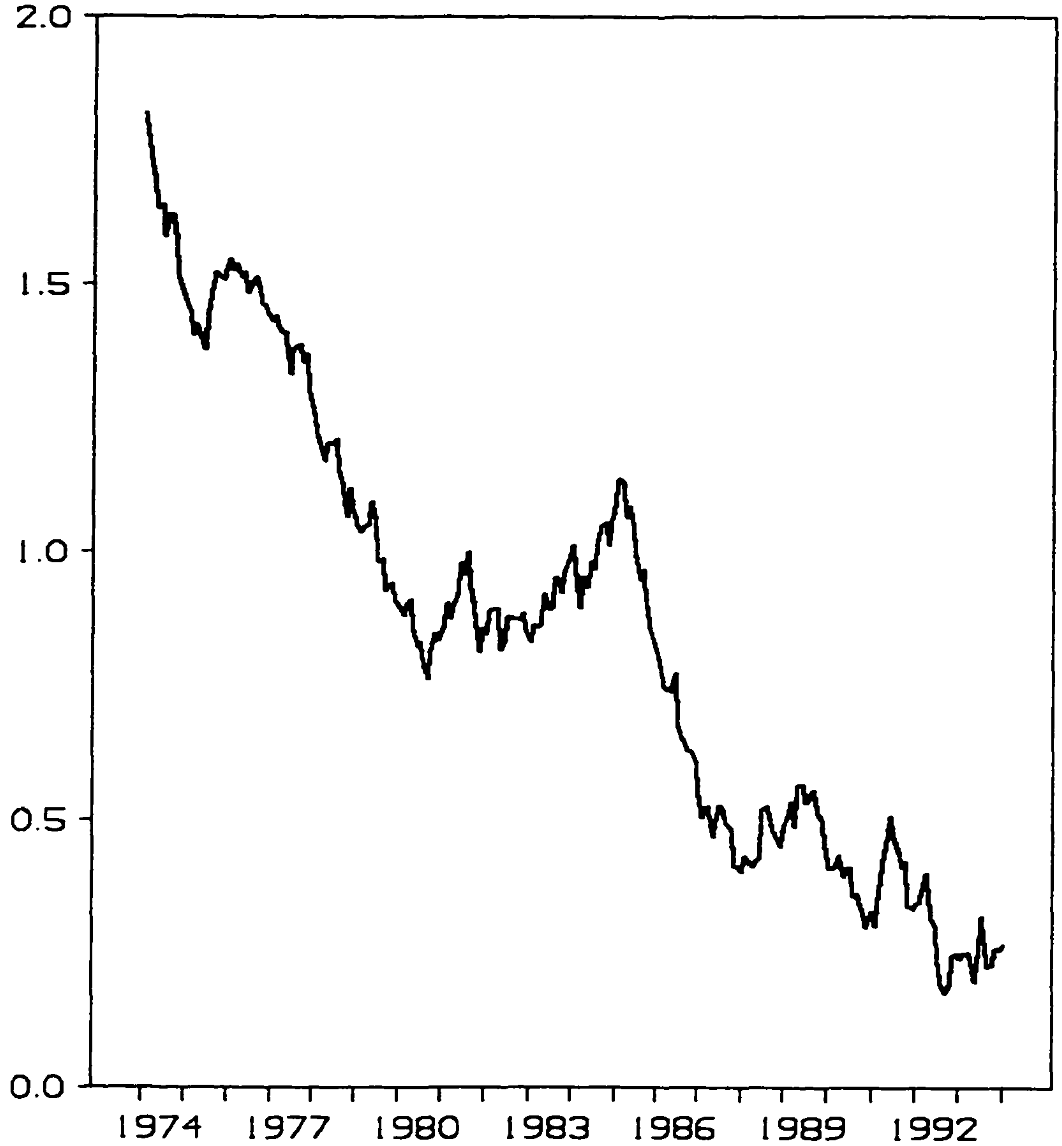
LOG REAL US/CANADA EXCHANGE RATE

FIGURE 1.1



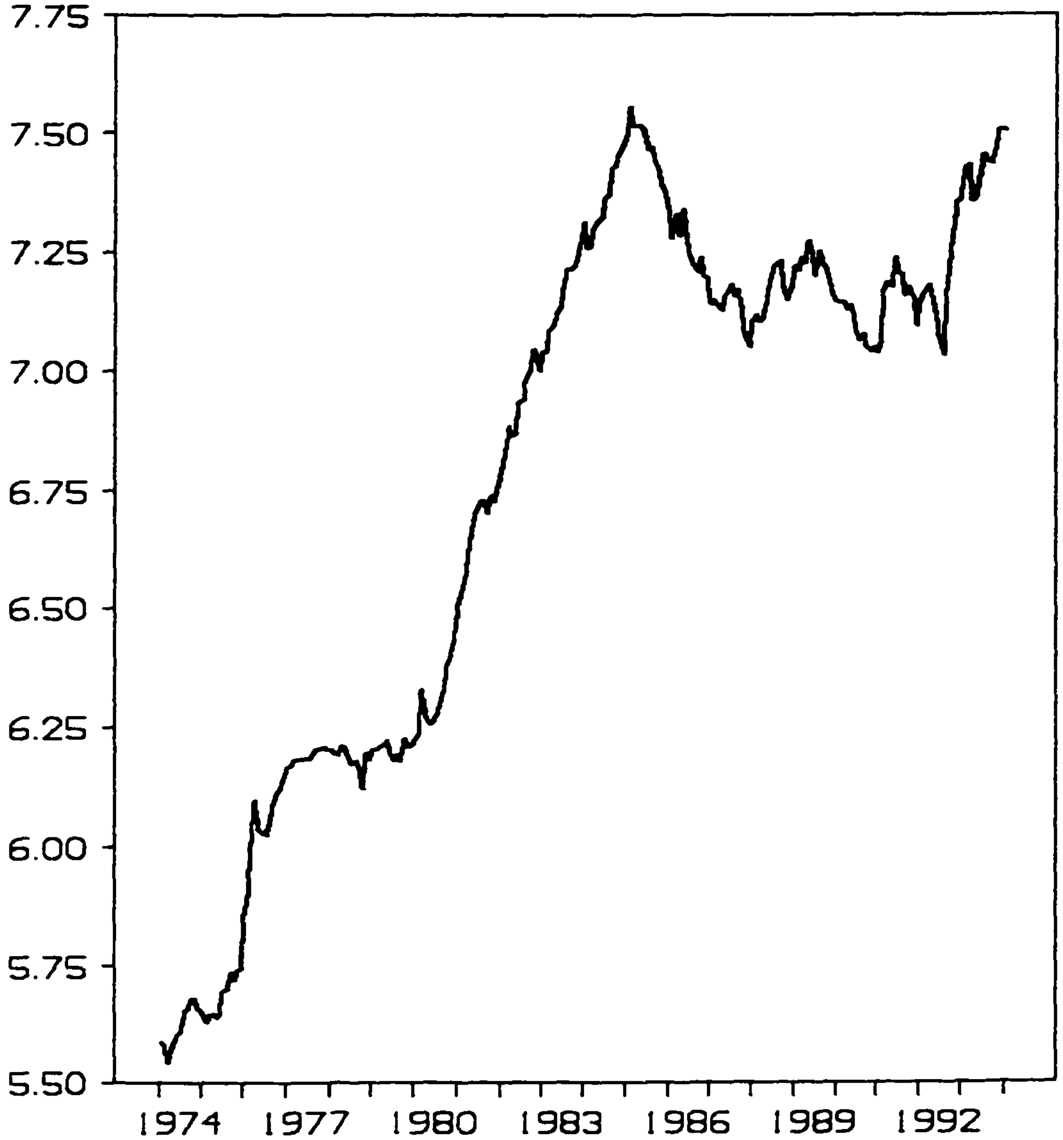
LOG REAL US/GERMANY EXCHANGE RATE

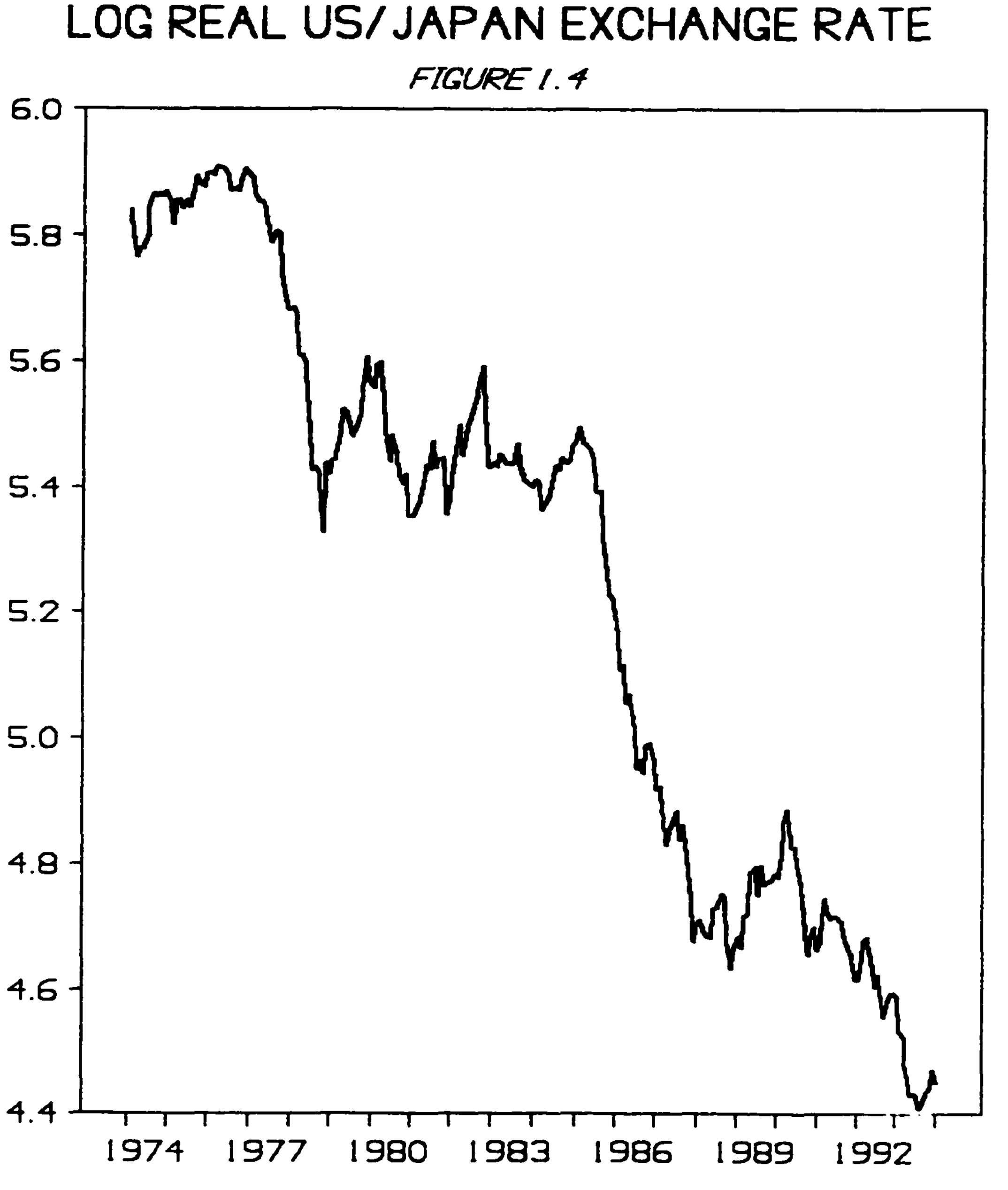
FIGURE 1.2

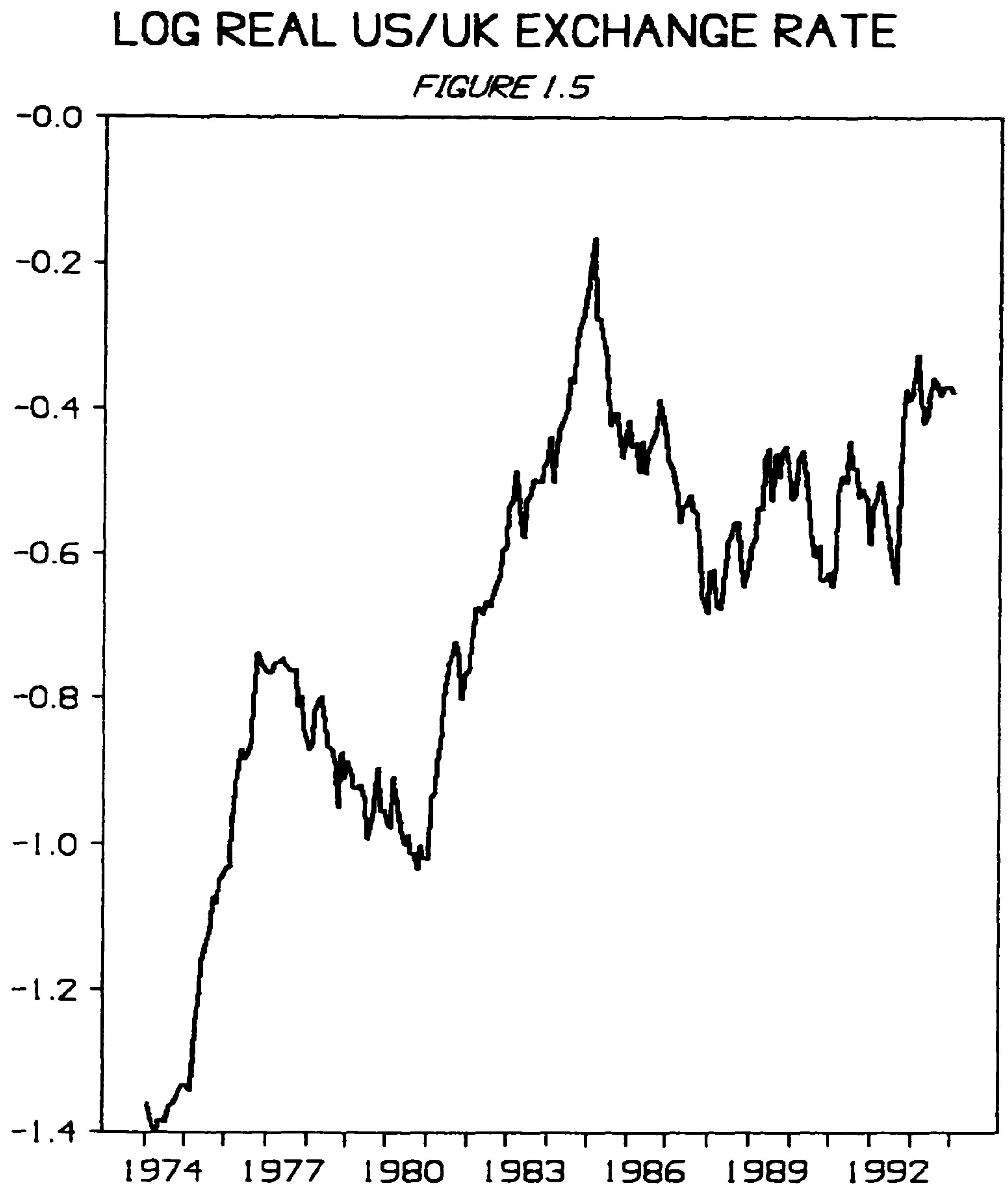


LOG REAL US/ITALY EXCHANGE RATE

FIGURE 1.3

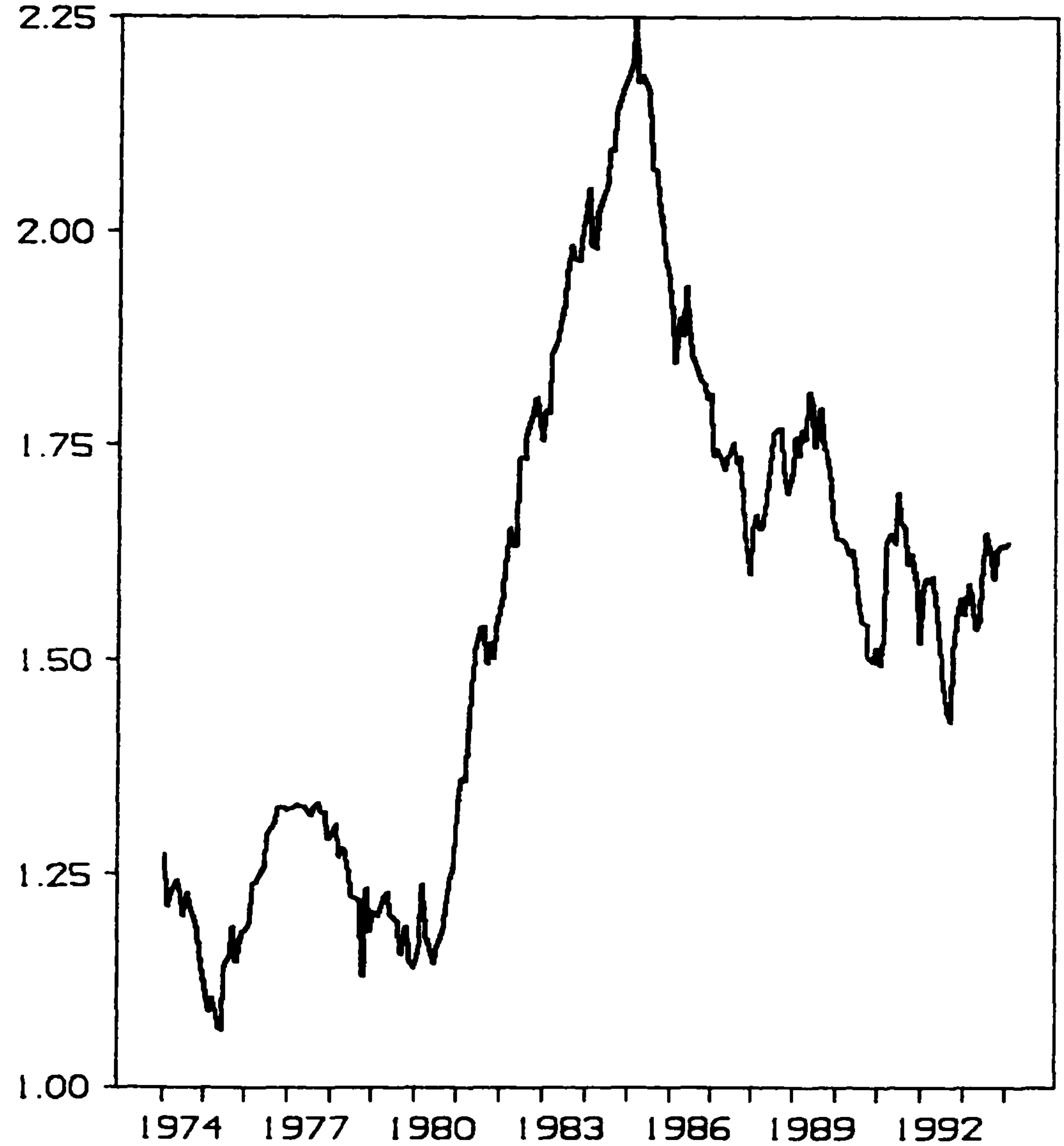



LOG REAL US/JAPAN EXCHANGE RATE
FIGURE 1.4
6.0
5.8
5.6
5.4
5.2
5.0
4.8
4.6
4.4
1974
1977
1980
1983
1986
1989
1992


LOG REAL US/UK EXCHANGE RATE
FIGURE 1.5
-0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
-1.4
1974
1977
1980
1983
1986
1989
1992

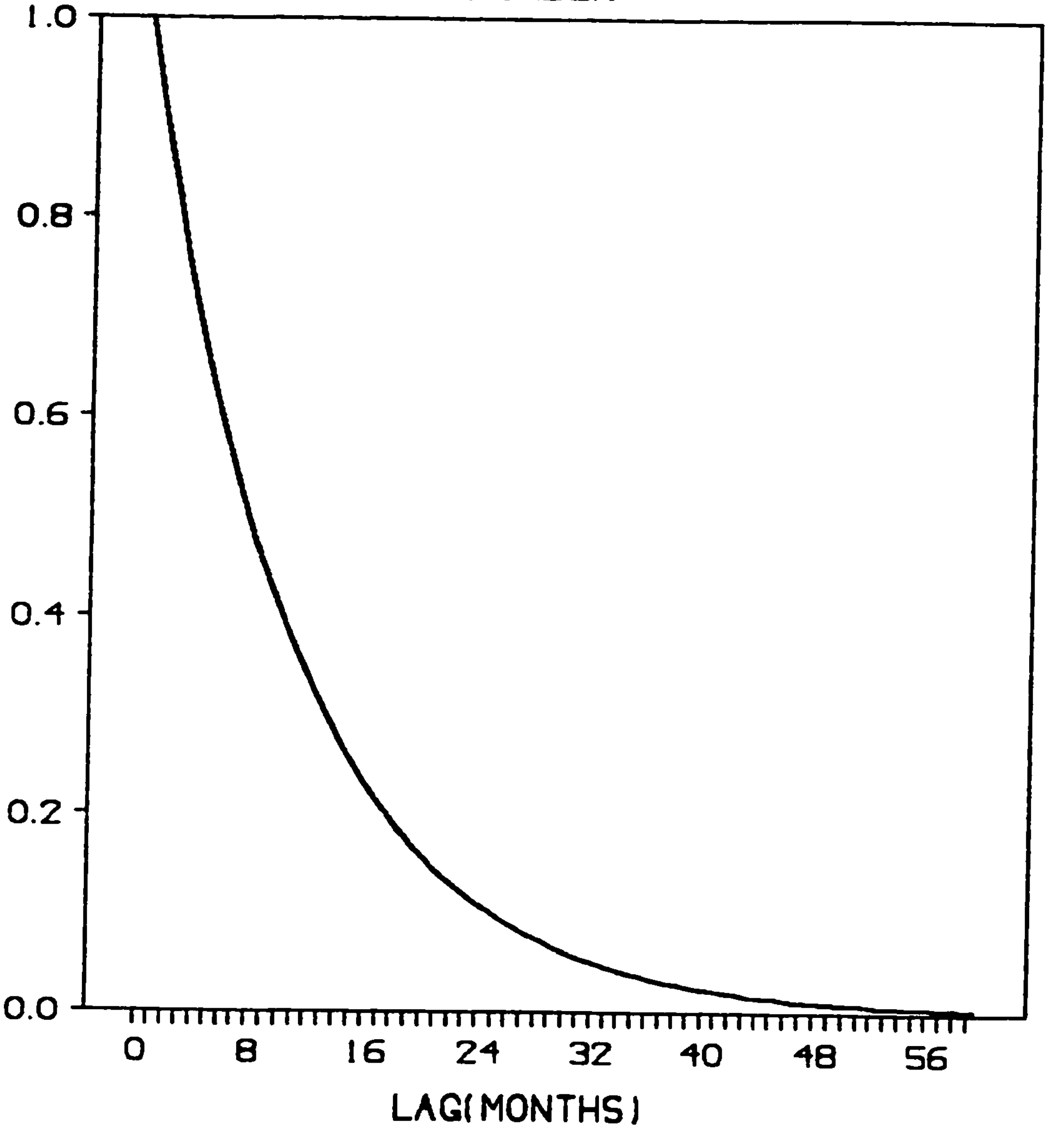
LOG REAL US/FRANCE EXCHANGE RATE

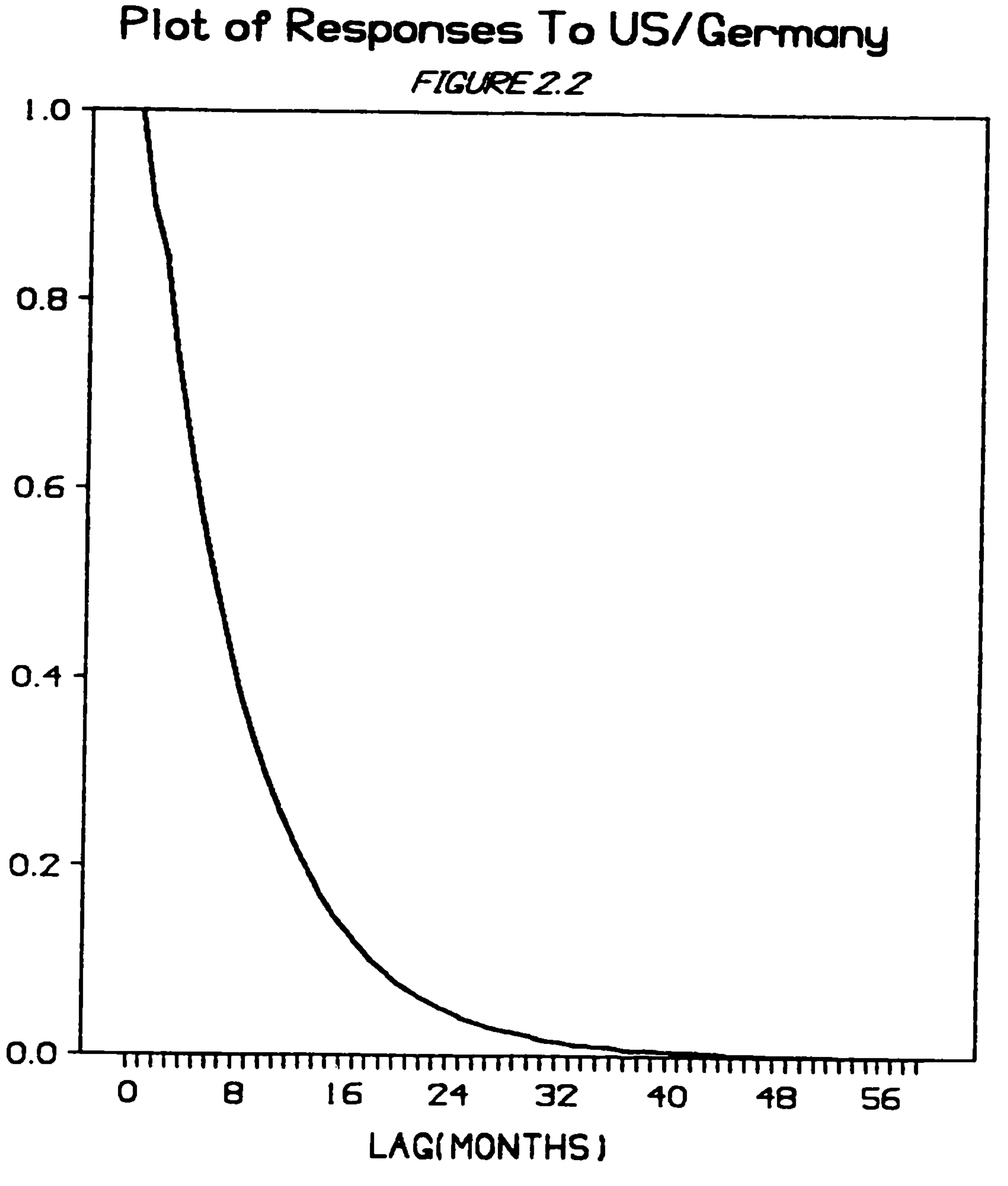
FIGURE 1.6



Plot of Responses To US/Canada

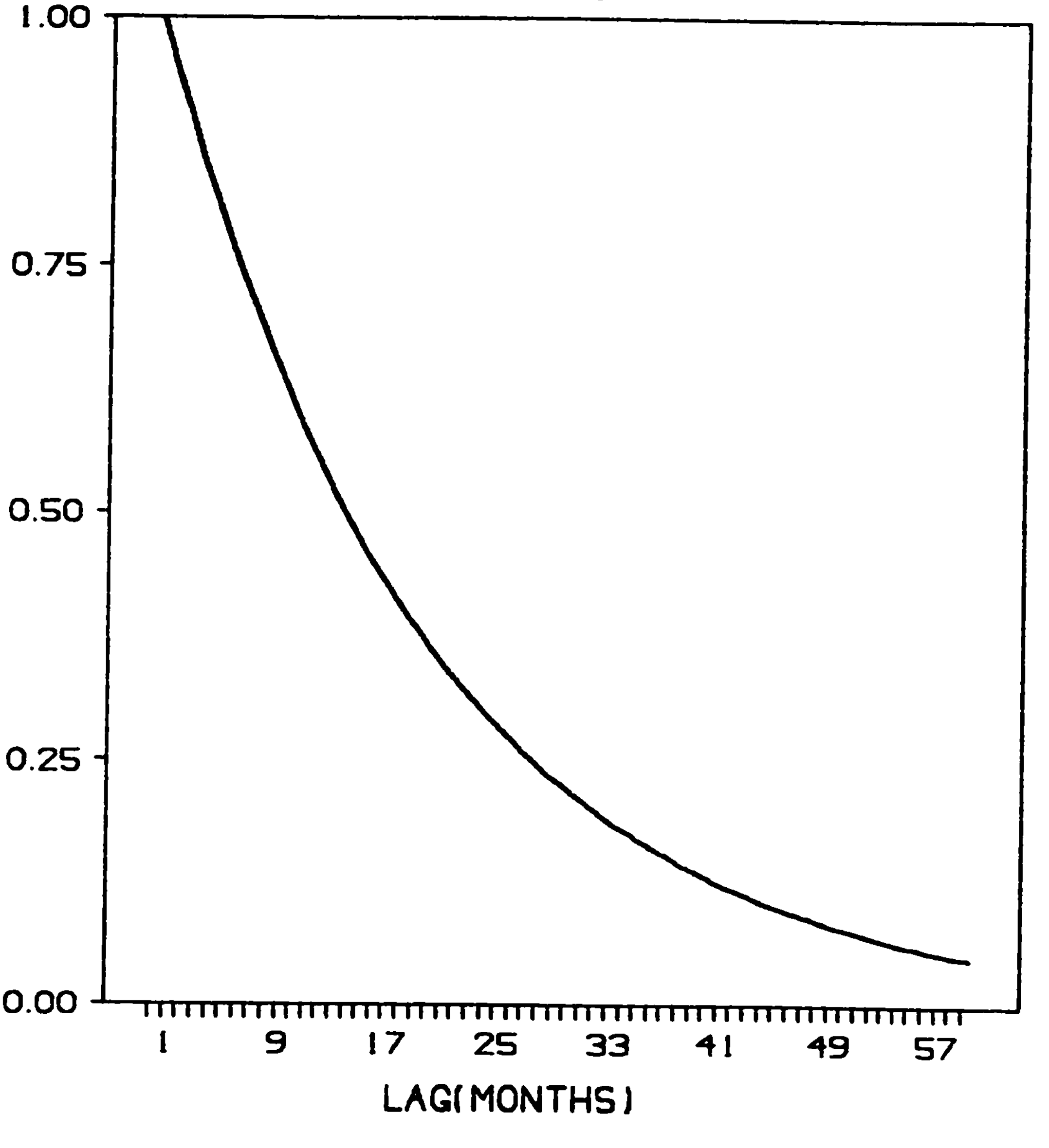
FIGURE 2.1

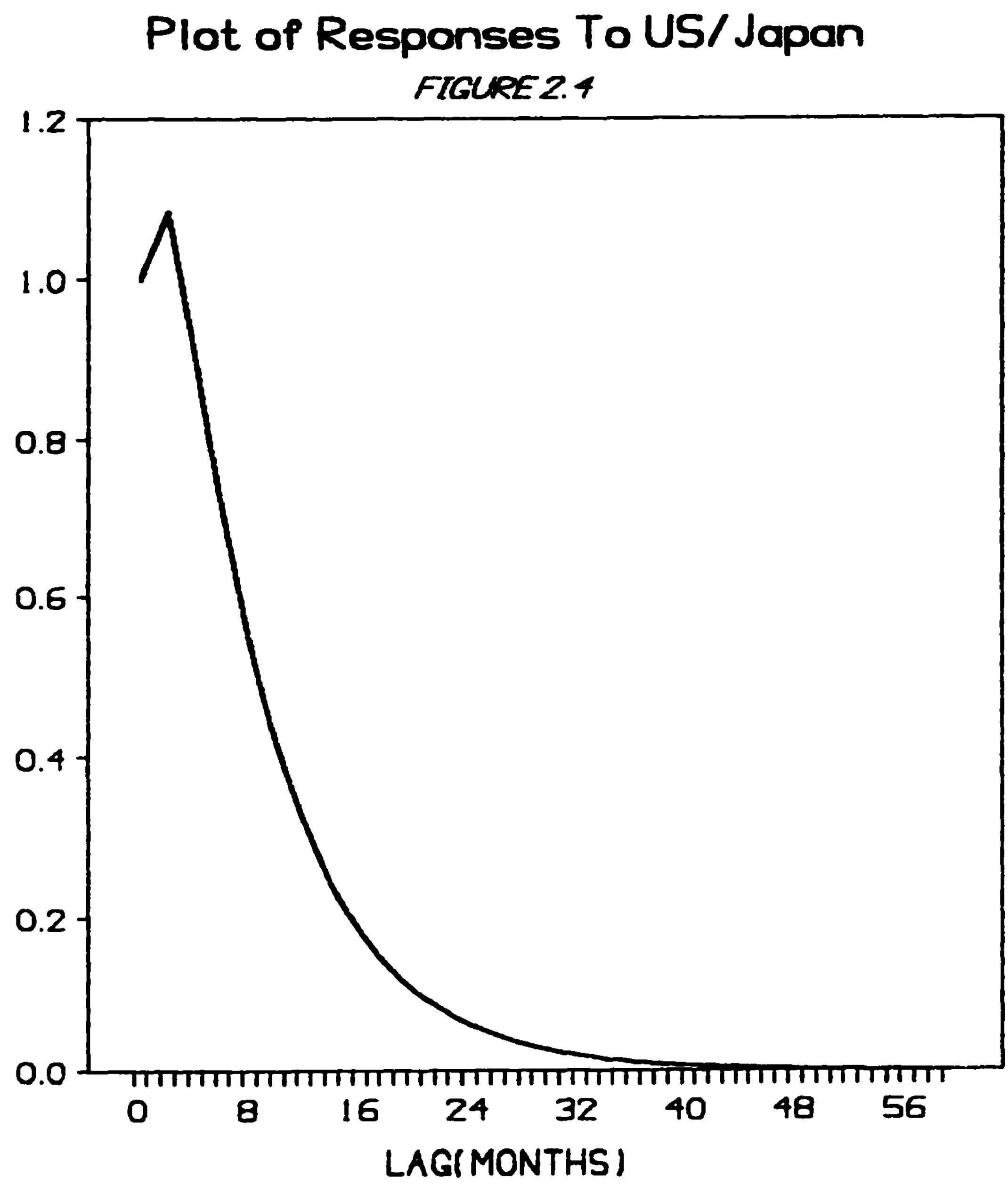



Plot of Responses To US/Germany
FIGURE 2.2
1.0
0.8
0.6
0.4
0.2
0.0
0
8
16
24
32
40
48
56
LAG(MONTHS)

Plot of Responses To US/Italy

FIGURE 2.3




Plot of Responses To US/Japan
FIGURE 2.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
8
16
24
32
40
48
56
LAG(MONTHS)

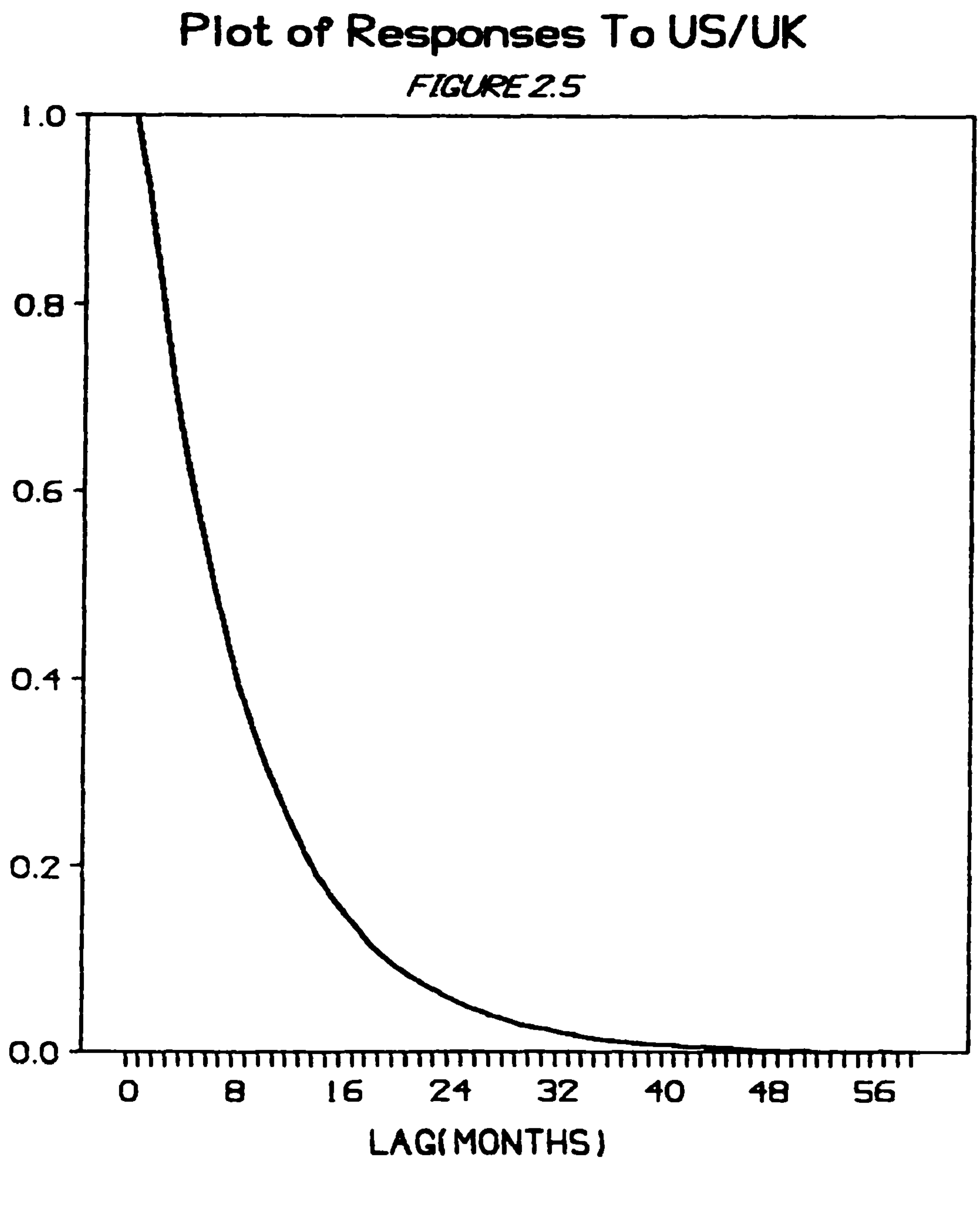
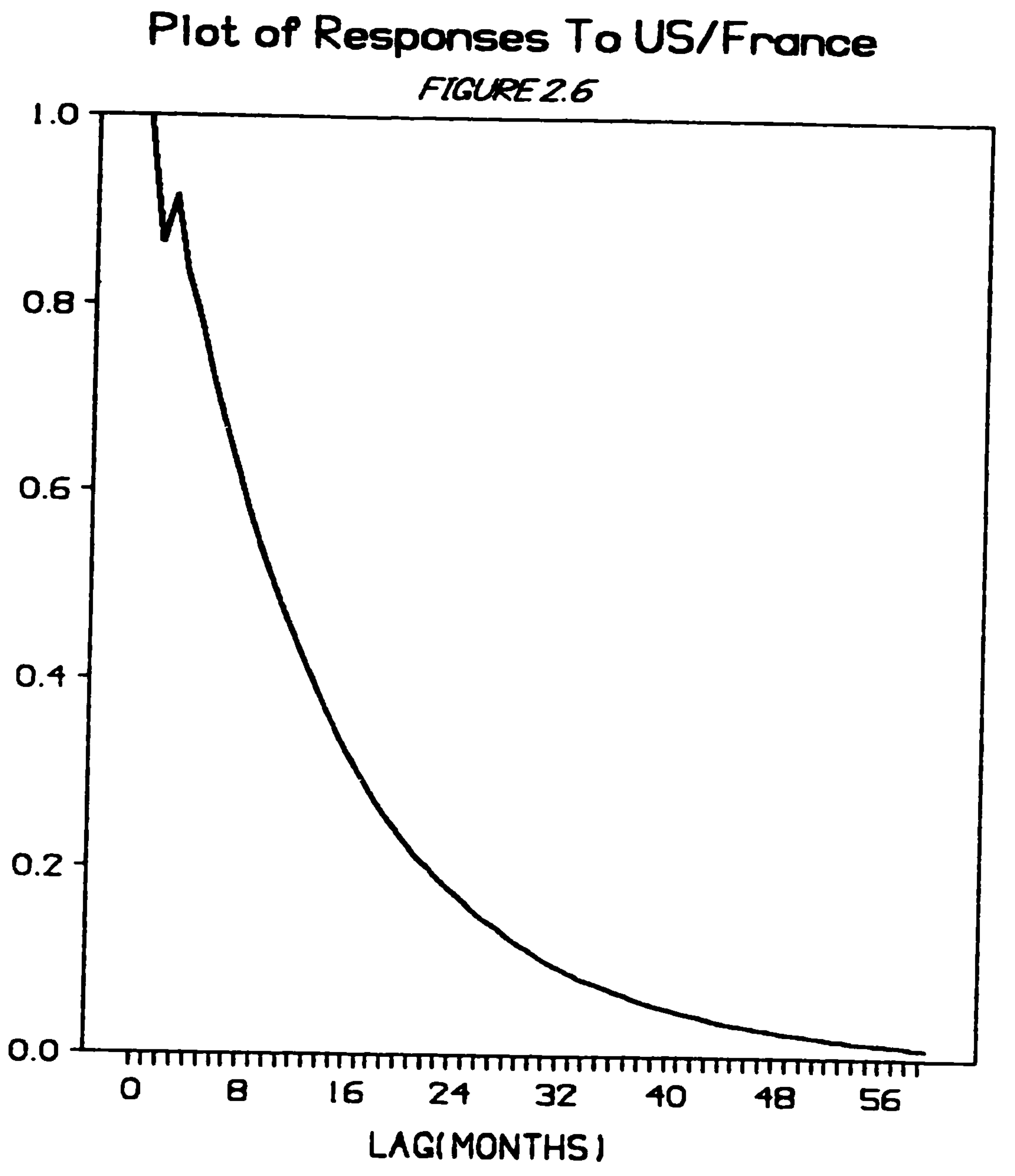
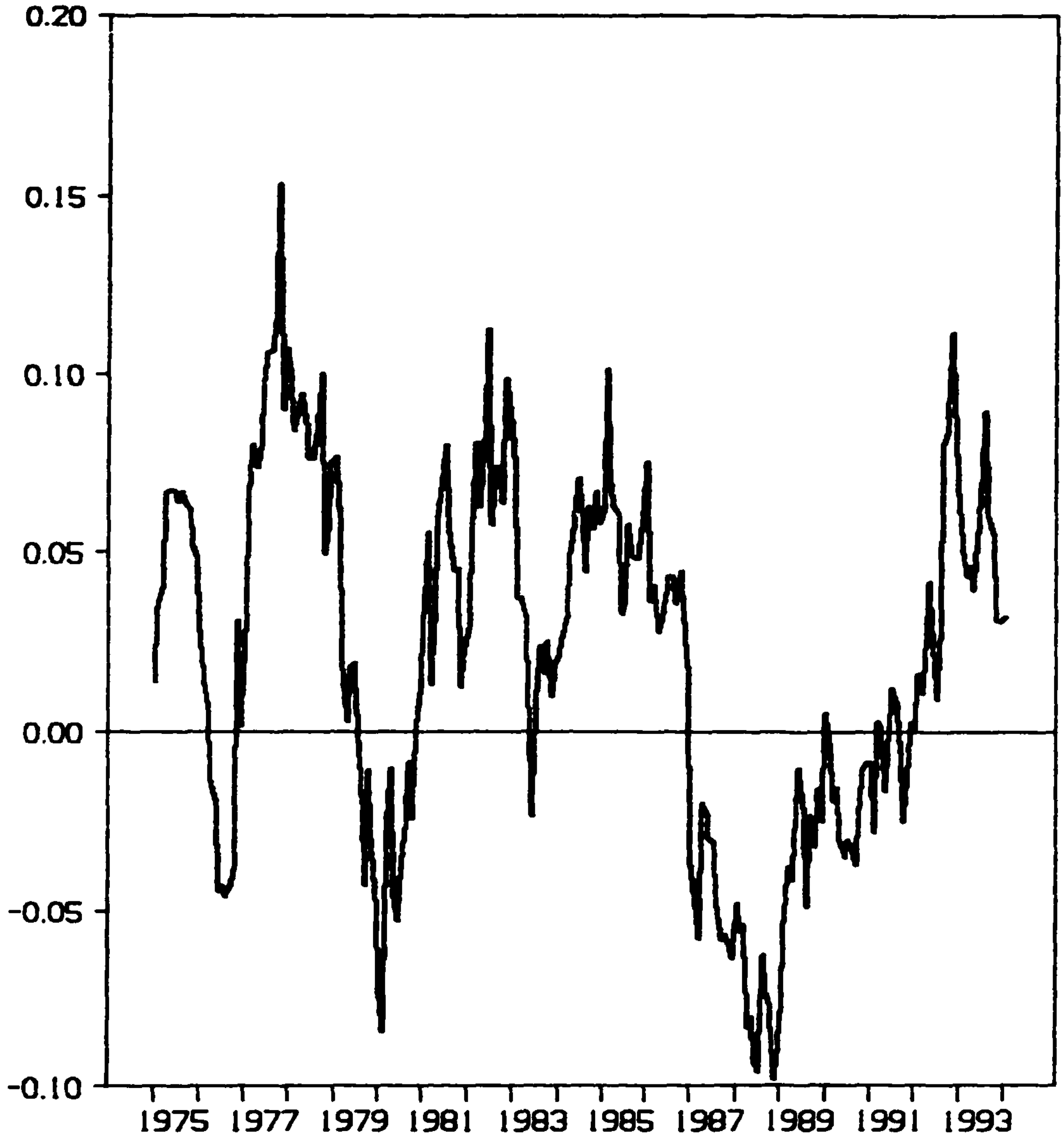

Plot of Responses To US/UK
1.0
0.8
0.6
0.4
0.2
0.0
0
8
16
24
32
40
48
56
LAG(MONTHS)

FIGURE 2.5


Plot of Responses To US/France
FIGURE 2.6
1.0
0.8
0.6
0.4
0.2
0.0
0
8
16
24
32
40
48
56
LAG(MONTHS)

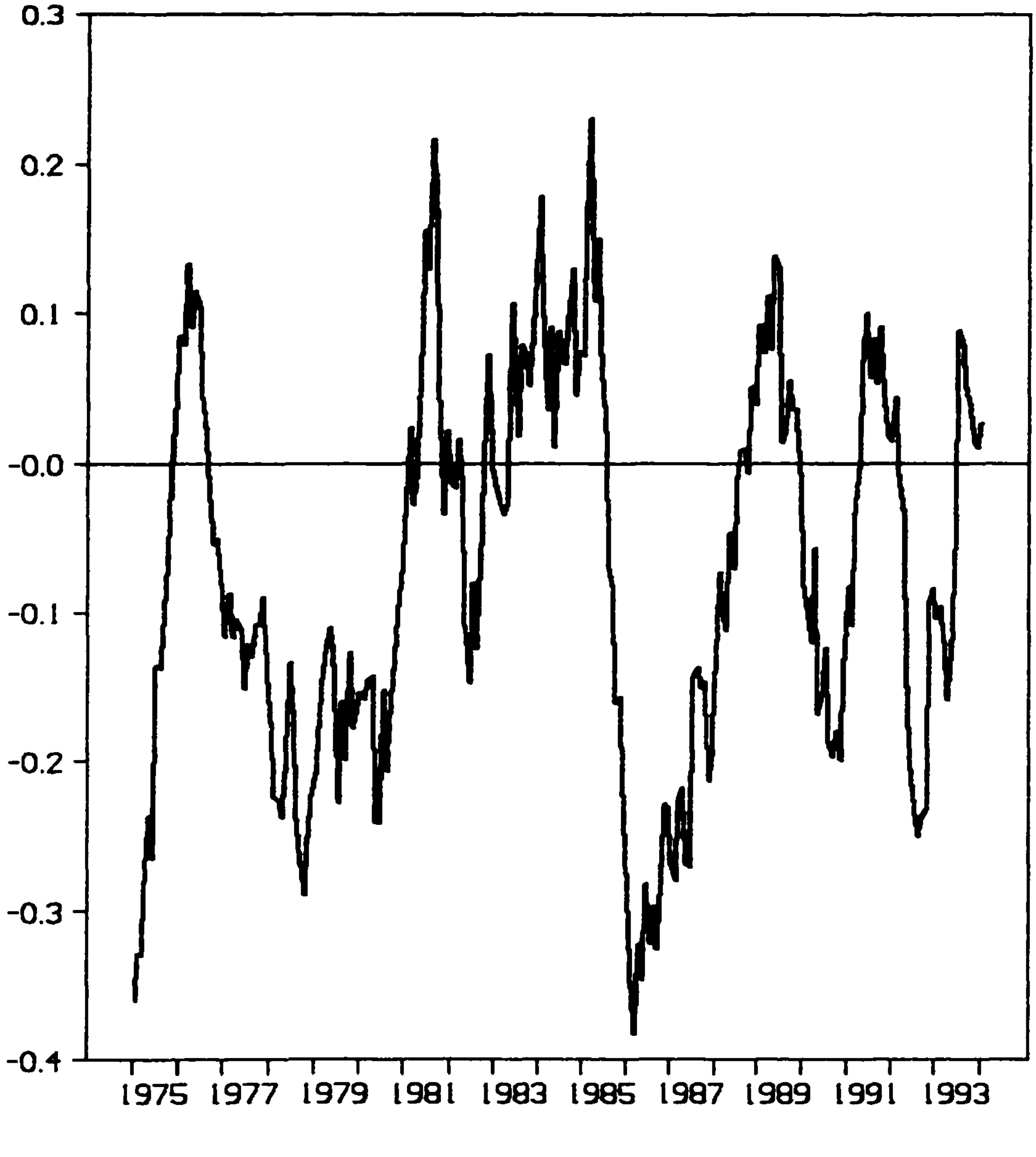
LOG REAL EXCHANGE RATE CHANGE US/CANADA

FIGURE 3.1



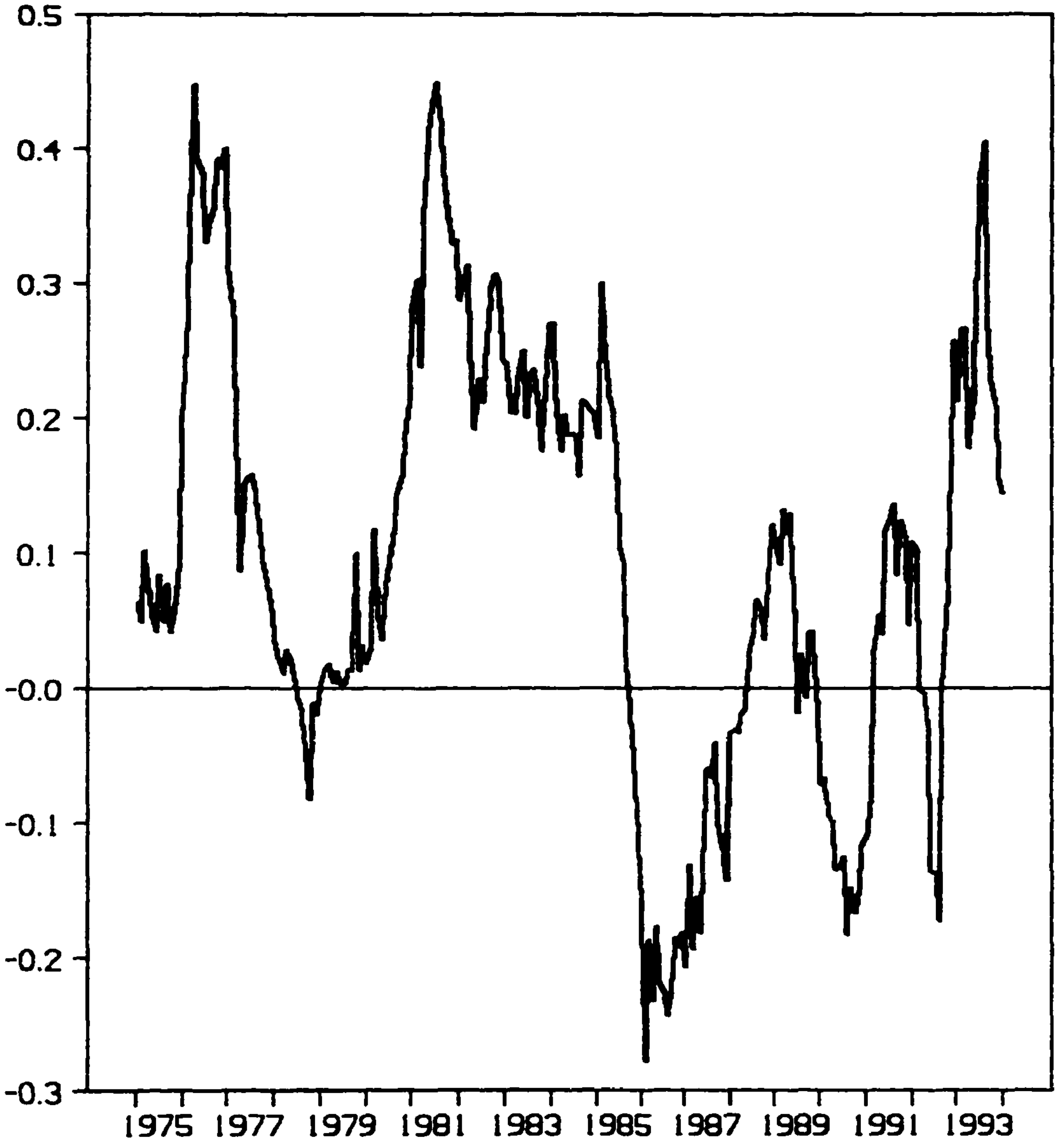
LOG REAL EXCHANGE RATE CHANGE US/GERMANY

FIGURE 3.2



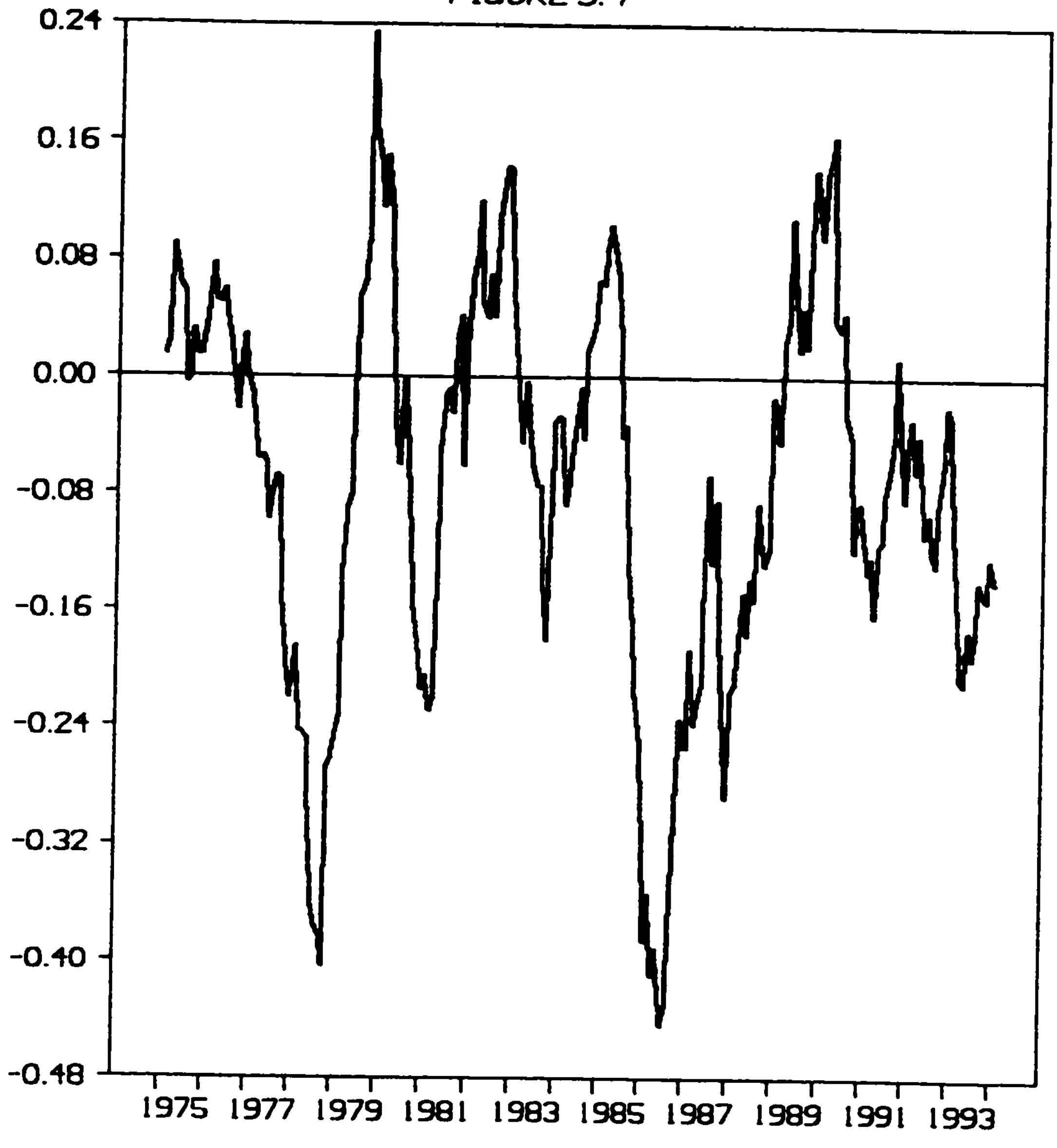
LOG REAL EXCHANGE RATE CHANGE US/ITALY

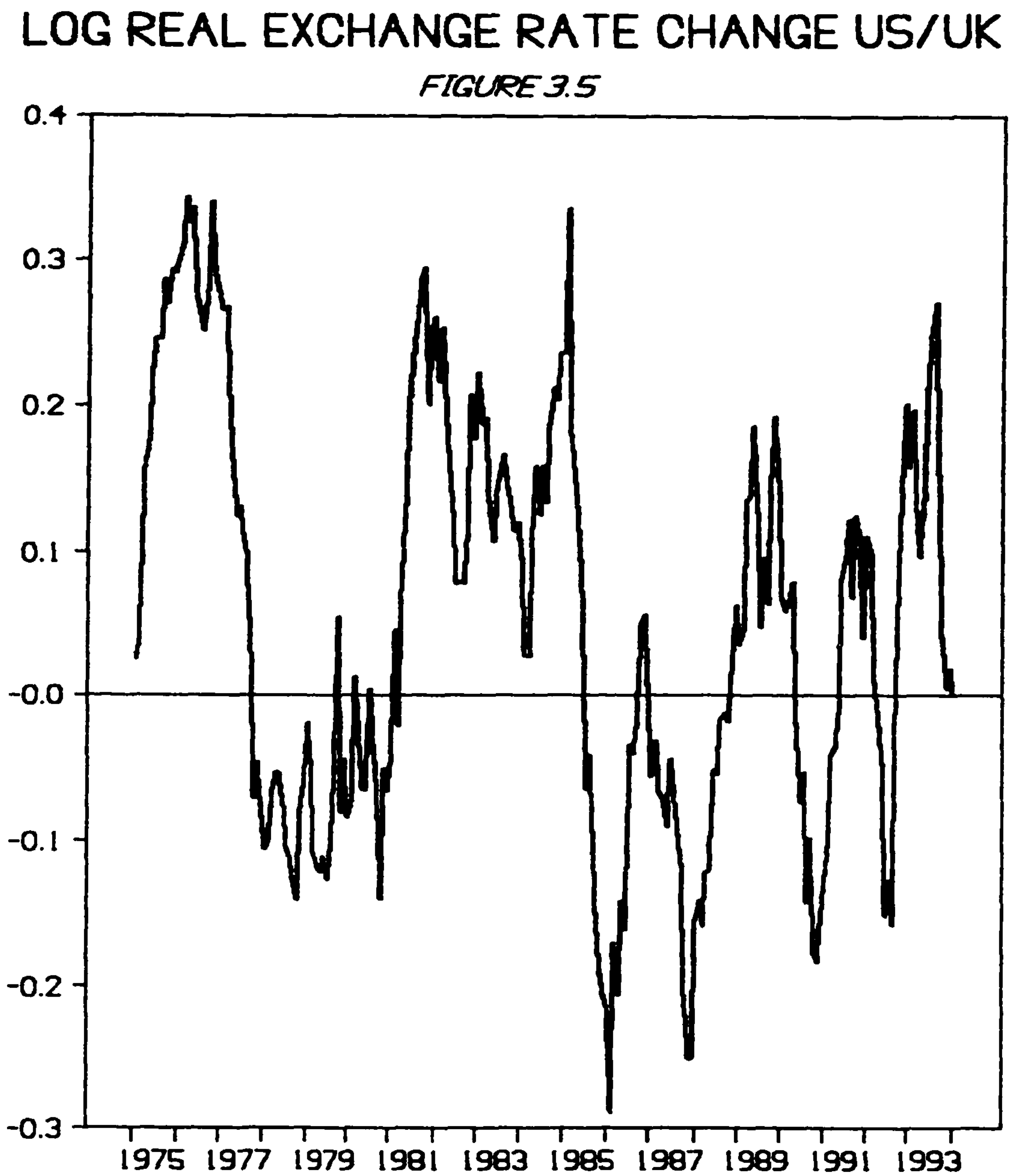
FIGURE 3.3



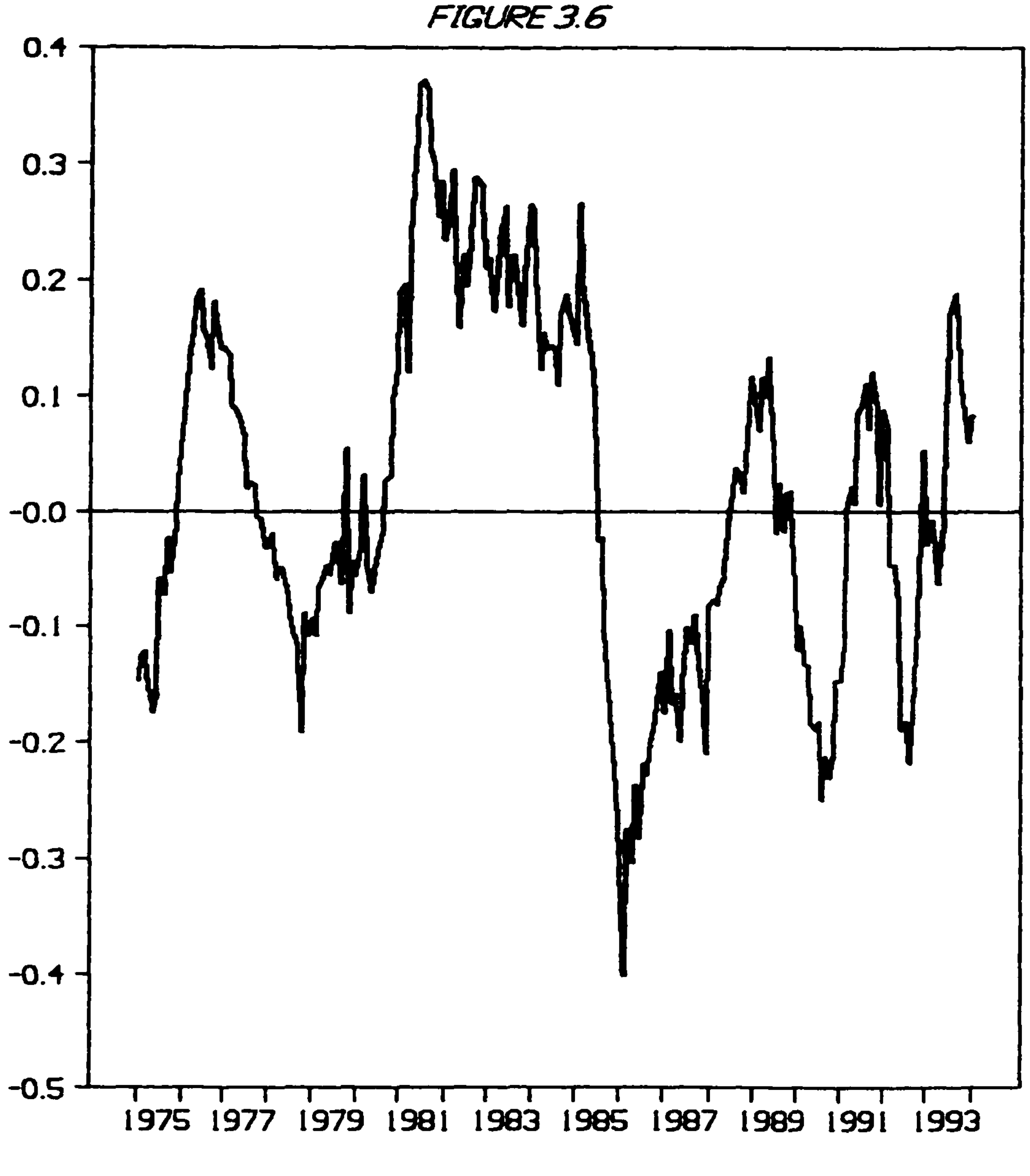
LOG REAL EXCHANGE RATE CHANGE US/JAPAN

FIGURE 3.4



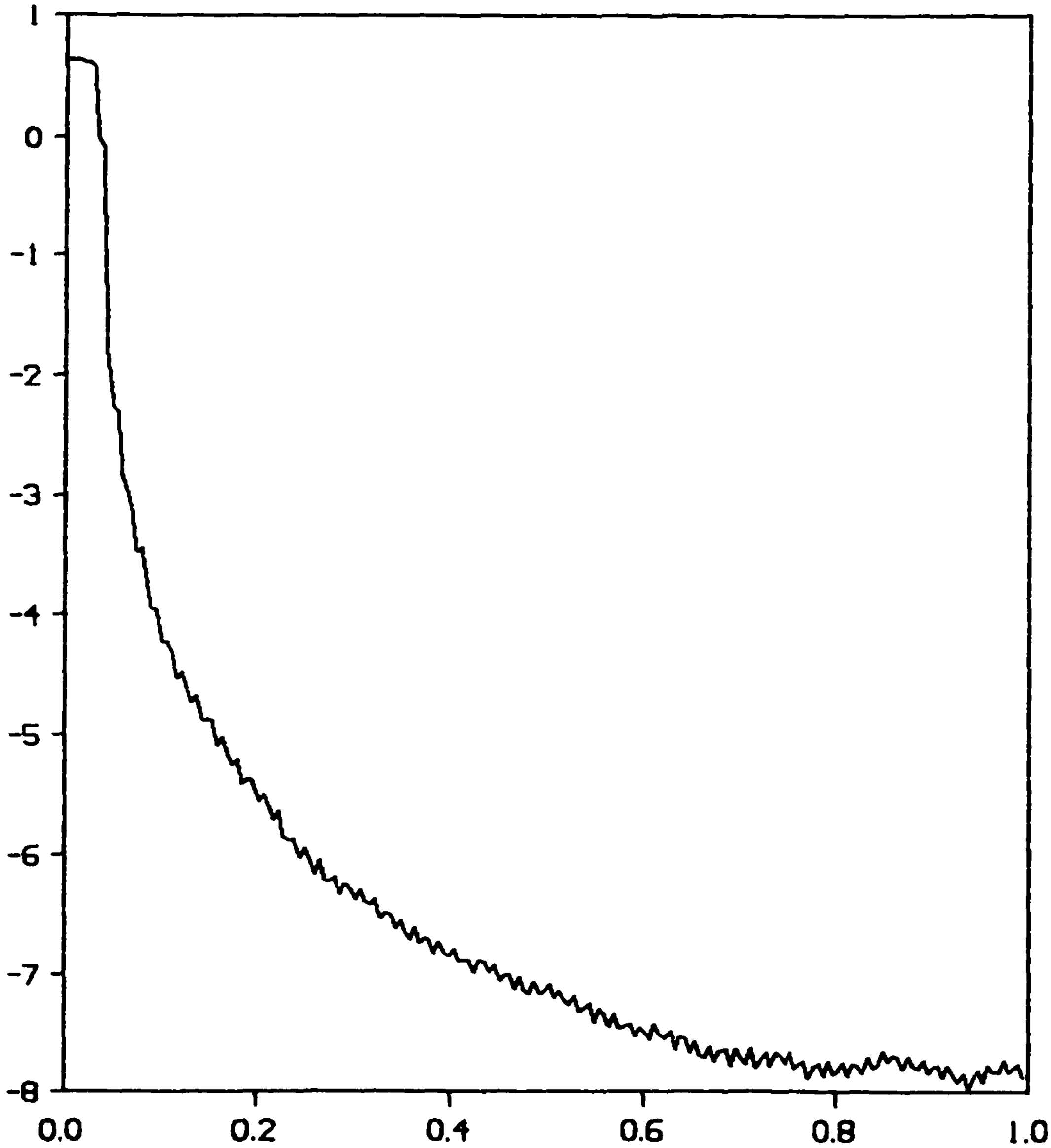

LOG REAL EXCHANGE RATE CHANGE US/UK
FIGURE 3.5
0.4
0.3
0.2
0.1
-0.0
-0.1
-0.2
-0.3
1975
1977
1979
1981
1983
1985
1987
1989
1991
1993

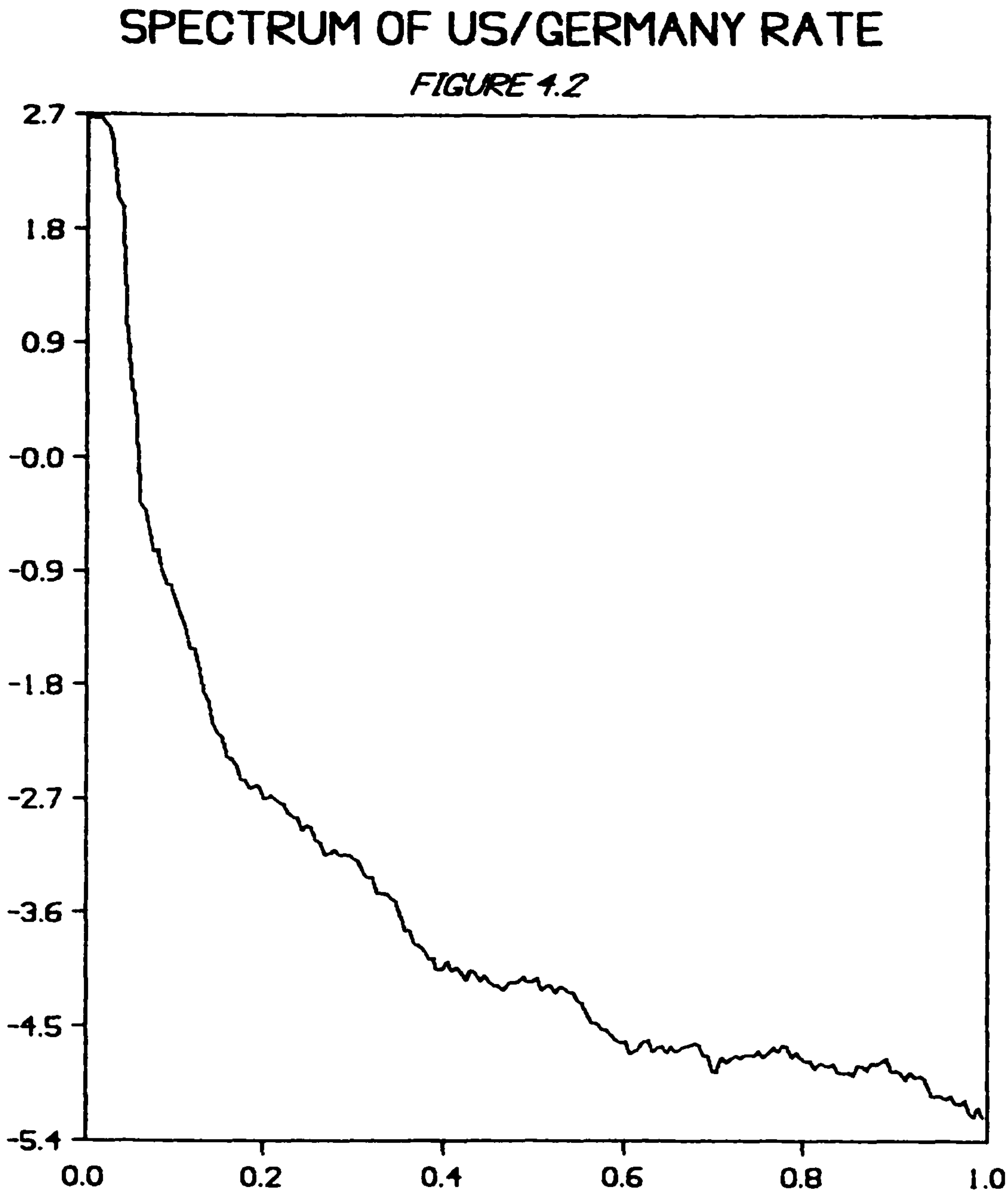
LOG REAL EXCHANGE RATE CHANGE US/FRANCE



SPECTRUM OF US/CANADA RATE

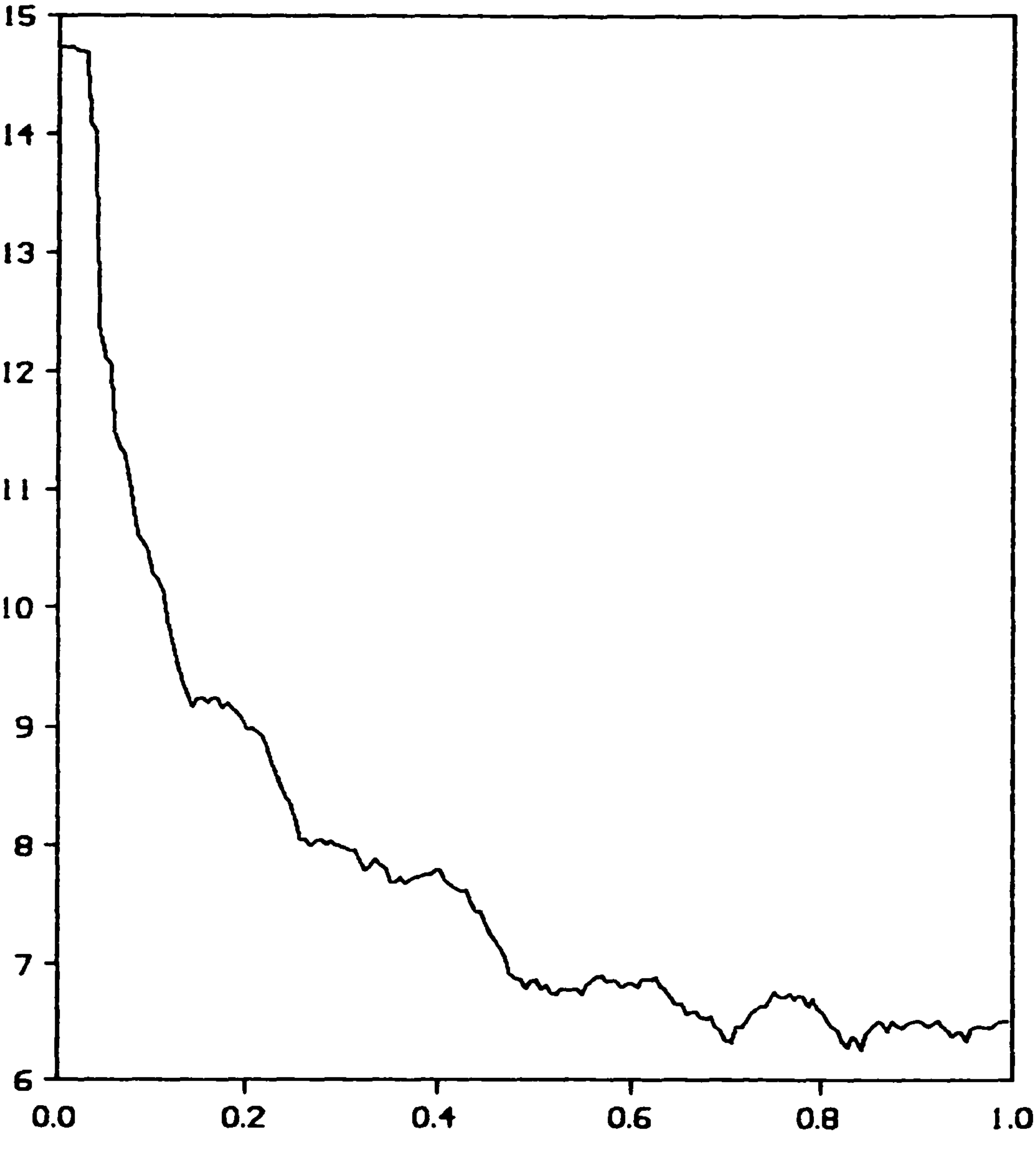
FIGURE 4.1




SPECTRUM OF US/GERMANY RATE
FIGURE 4.2
2.7
1.8
0.9
-0.0
-0.9
-1.8
-2.7
-3.6
-4.5
-5.4
0.0
0.2
0.4
0.6
0.8
1.0

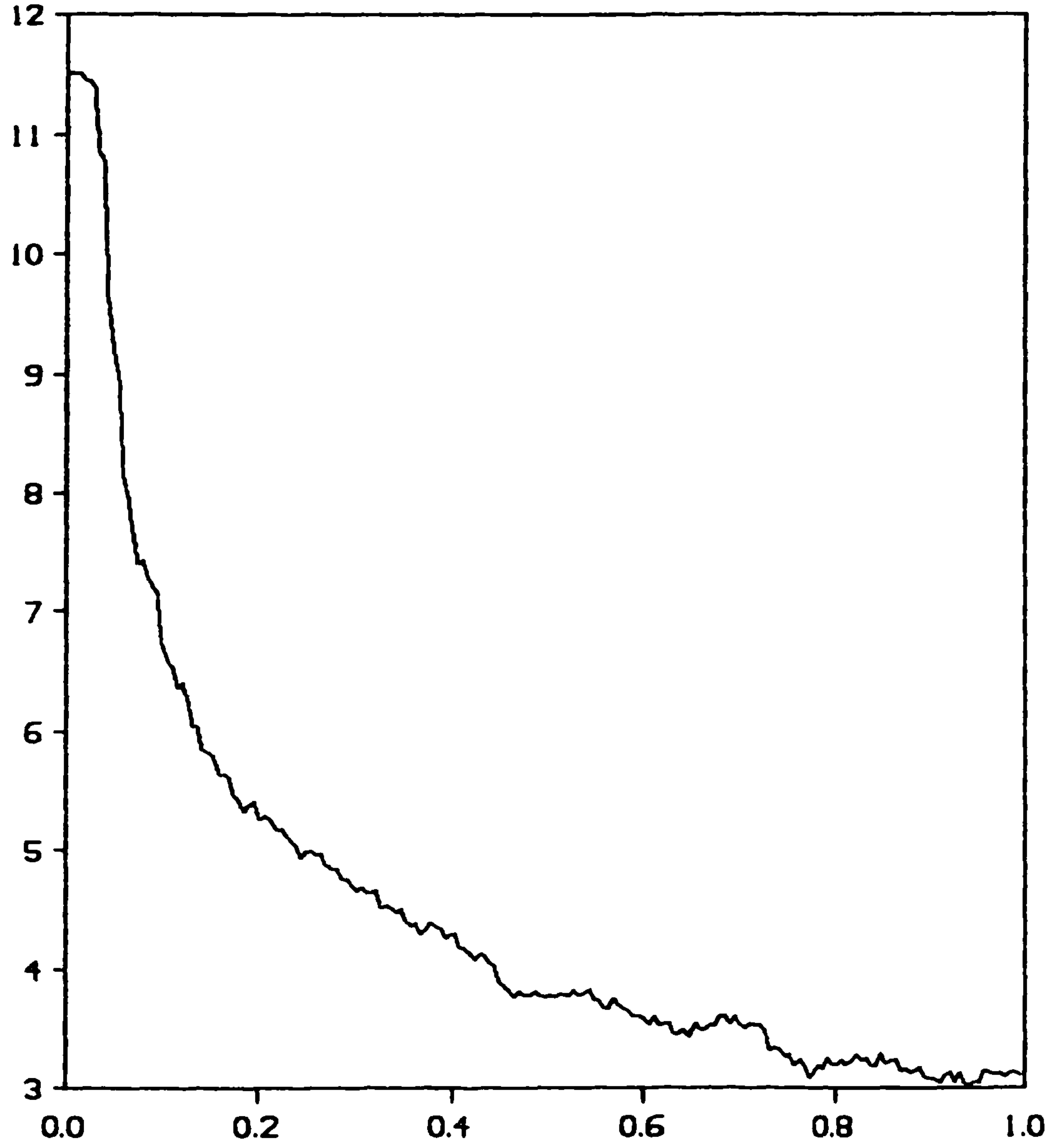
SPECTRUM OF US/ITALY RATE

FIGURE 4.3



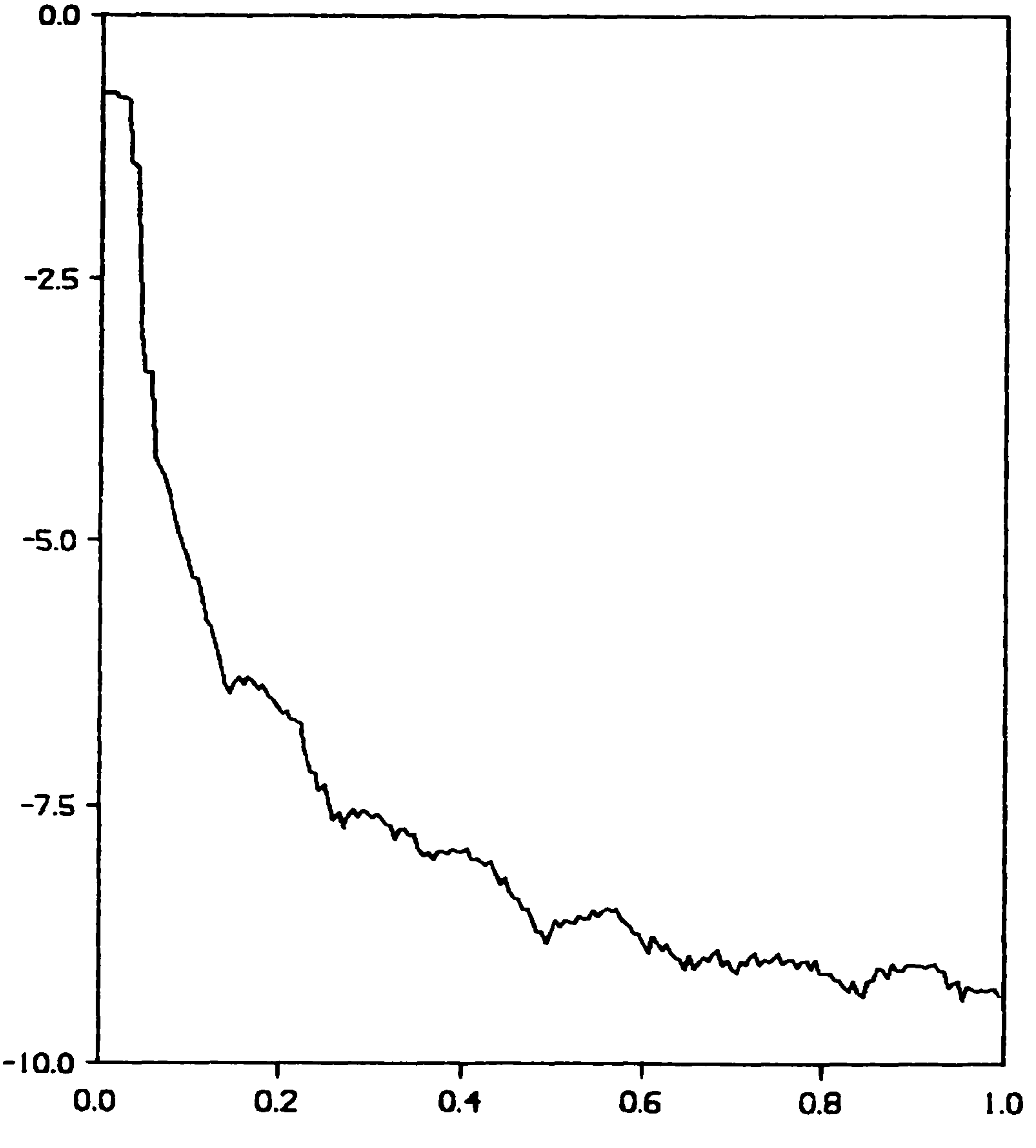
SPECTRUM OF US/JAPAN RATE

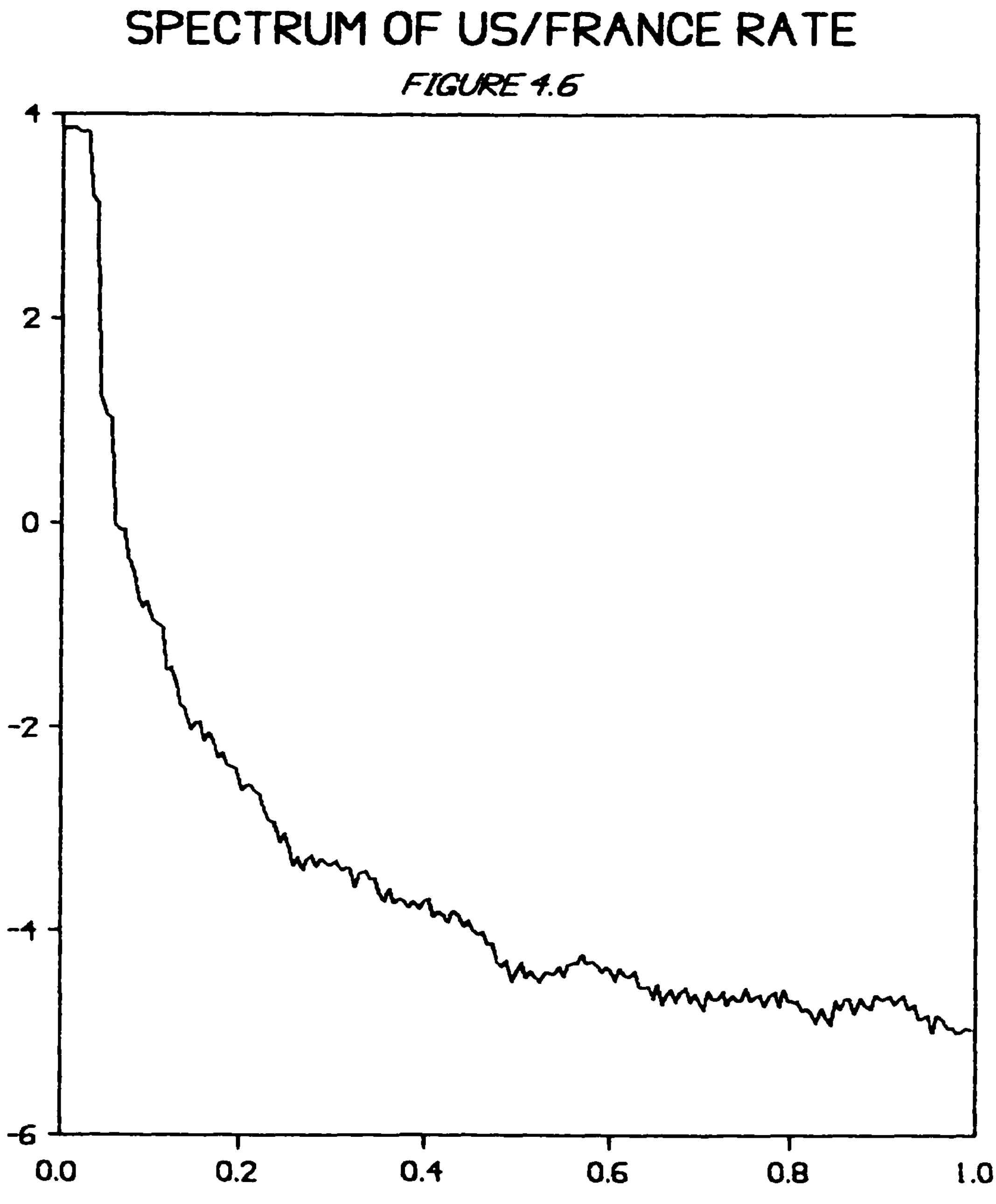
FIGURE 4.4



SPECTRUM OF US/UK RATE

FIGURE 4.5




SPECTRUM OF US/FRANCE RATE
FIGURE 4.6
4
2
0
-2
-4
-6
0.0
0.2
0.4
0.6
0.8
1.0

Appendix

TABLE A1. Unit-Root Tests Based on t Statistic (constant)

Data Series	Dickey-Fuller	Phillips-Perron
US/Canada	-1.50704	-1.51336
US/Germany	-4.13467	-4.15200
US/Italy	-0.49196	-0.49402
US/Japan	-0.75837	-0.76155
US/UK	-1.70250	-1.70963
US/France	-1.14161	-1.14639

NOTES: The truncation lag is equal to 0. The 5% critical value equals -2.88 and the 10% critical value equals -2.58.

Appendix

TABLE A2. Unit Root Based on t Statistics (constant with 2 lags)

Data Series	Aug. Dickey-Fuller	Phillips-Perron
US/Canada	-1.70124	-1.50852
US/Germany	-2.91583	-3.99167
US/Italy	-0.74404	-0.57552
US/Japan	-0.51136	-0.79501
US/UK	-1.88719	-1.75328
US/France	-1.33561	-1.19447

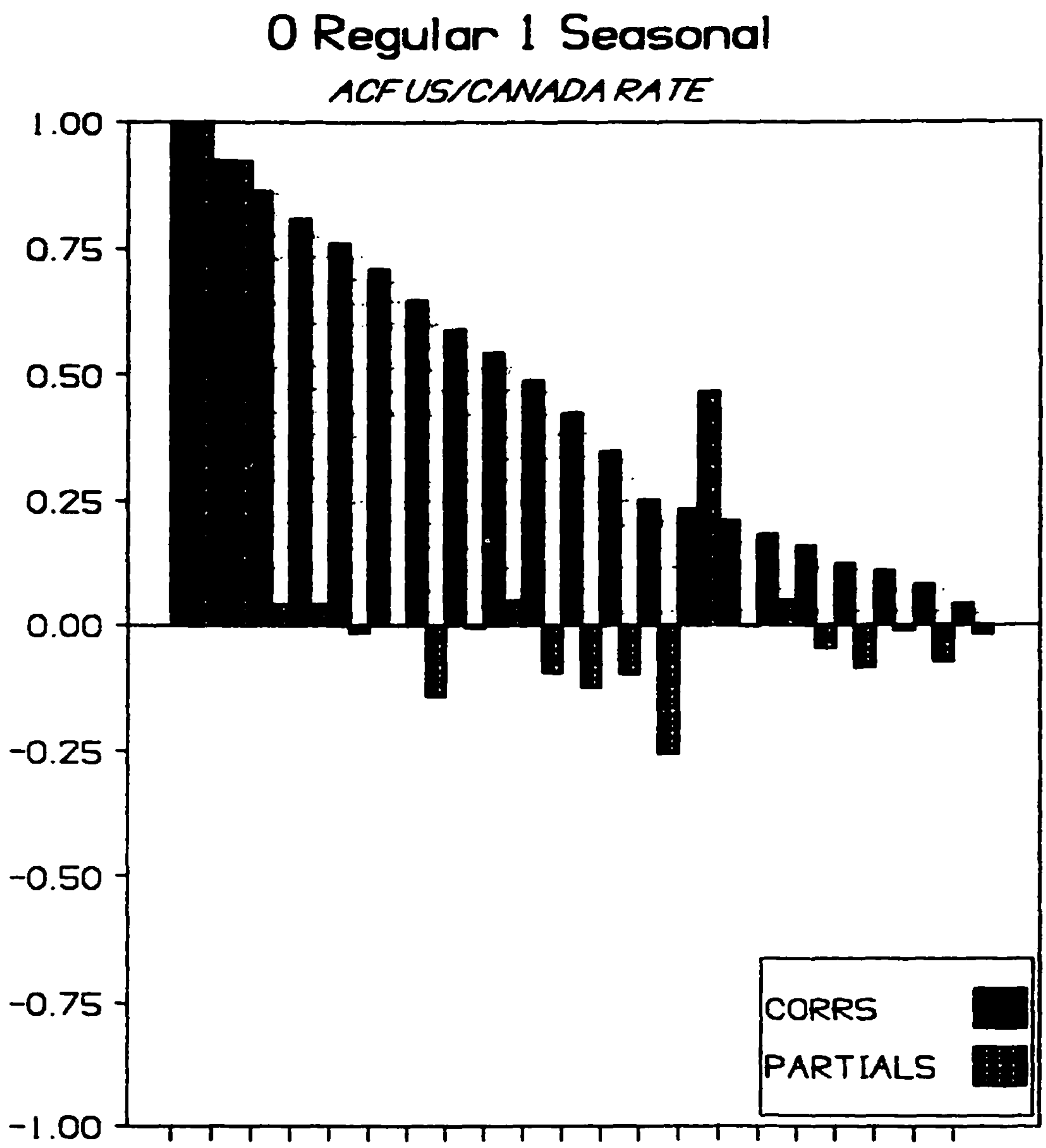
NOTES: The 5% and 10% critical values equal -2.88 and -2.58, respectively. The truncation lag is equal to 2.

Appendix

TABLE A3. Unit-Root Tests Based on t Statistic (with time trend)

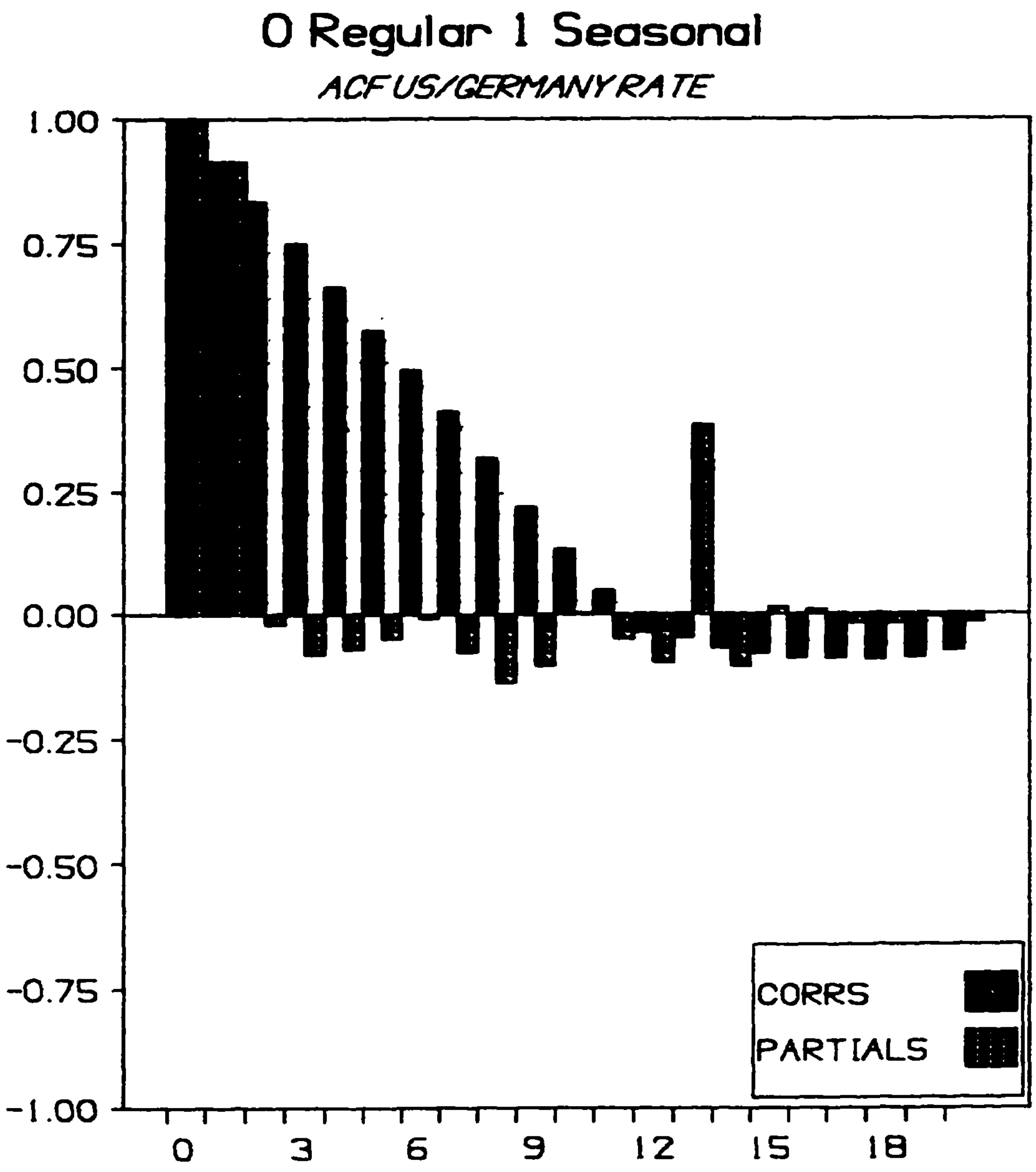
Data Series	Aug. Dickey-Fuller	Phillips-Perron
US/Canada	-1.40126	-1.35482
US/Germany	-3.00368	-3.93573
US/Italy	-1.45633	-1.29711
US/Japan	-2.27190	-2.09601
US/UK	-2.29842	-2.16125
US/France	-1.10271	-0.96053

NOTES: The 5% and 10% critical values equal -3.43 and -3.13 respectively. The truncation lag is equal to 2.



0 Regular 1 Seasonal
ACF US/CANADA RATE
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
0
3
6
9
12
15
18
CORRS
PARTIALS

FIGURE A1



0 Regular 1 Seasonal
ACF US/GERMANY RATE
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
0
3
6
9
12
15
18
CORRS
PARTIALS

FIGURE A2

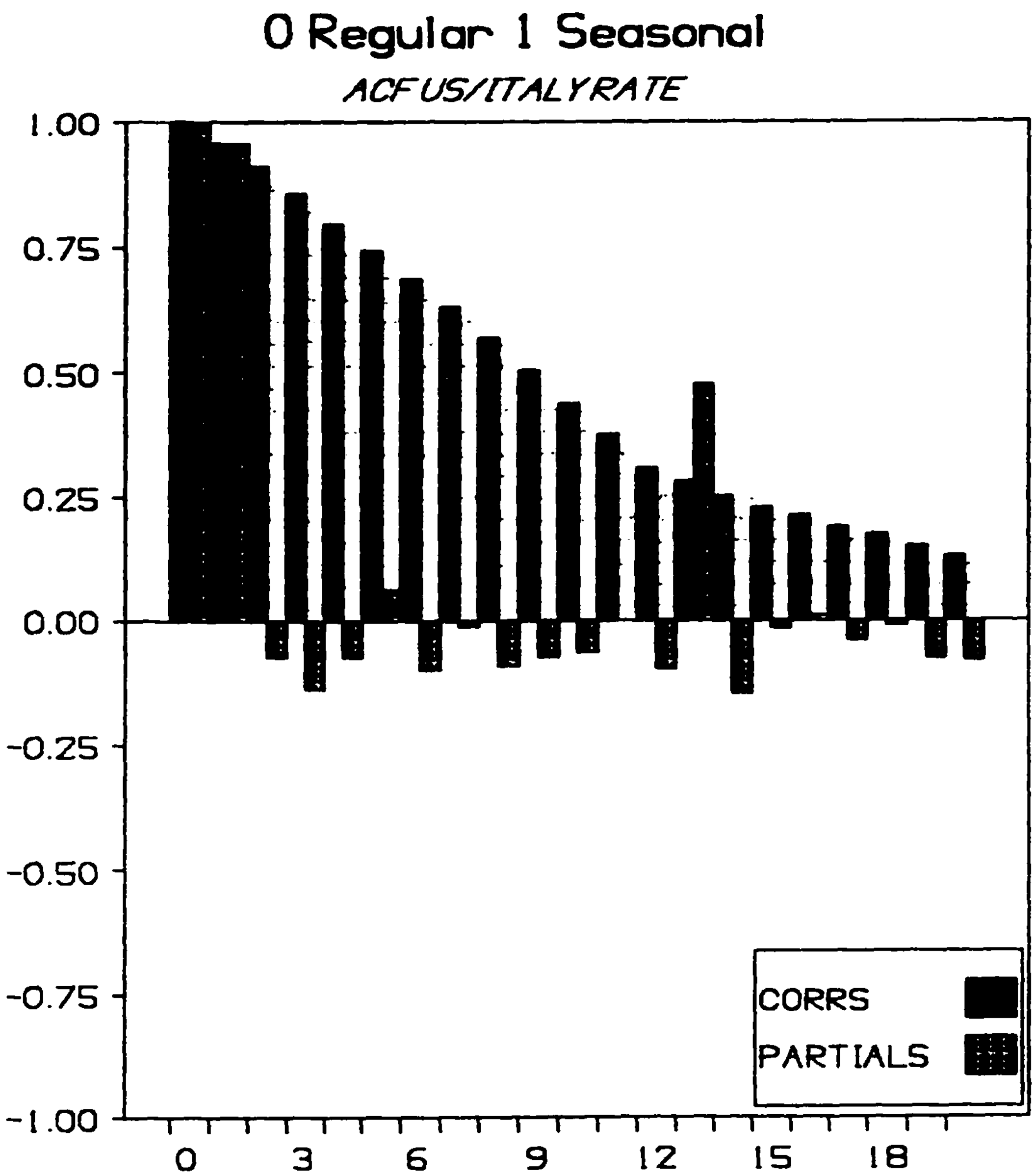
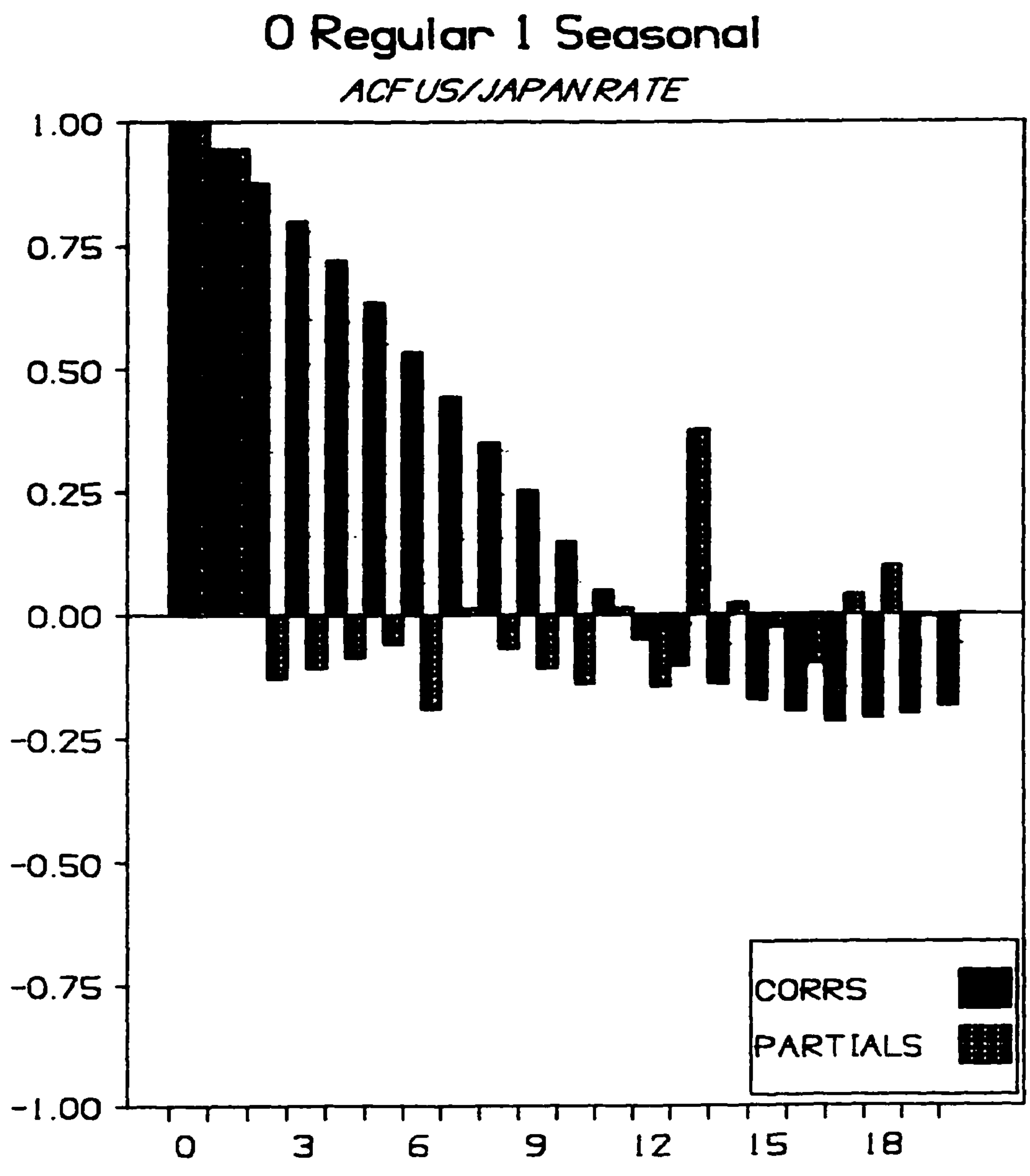

0 Regular 1 Seasonal
ACF US/ITALY RATE
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
CORRS
PARTIALS
0
3
6
9
12
15
18

FIGURE A3



0 Regular 1 Seasonal
ACF US/JAPAN RATE
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
0
3
6
9
12
15
18
CORRS
PARTIALS

FIGURE A4

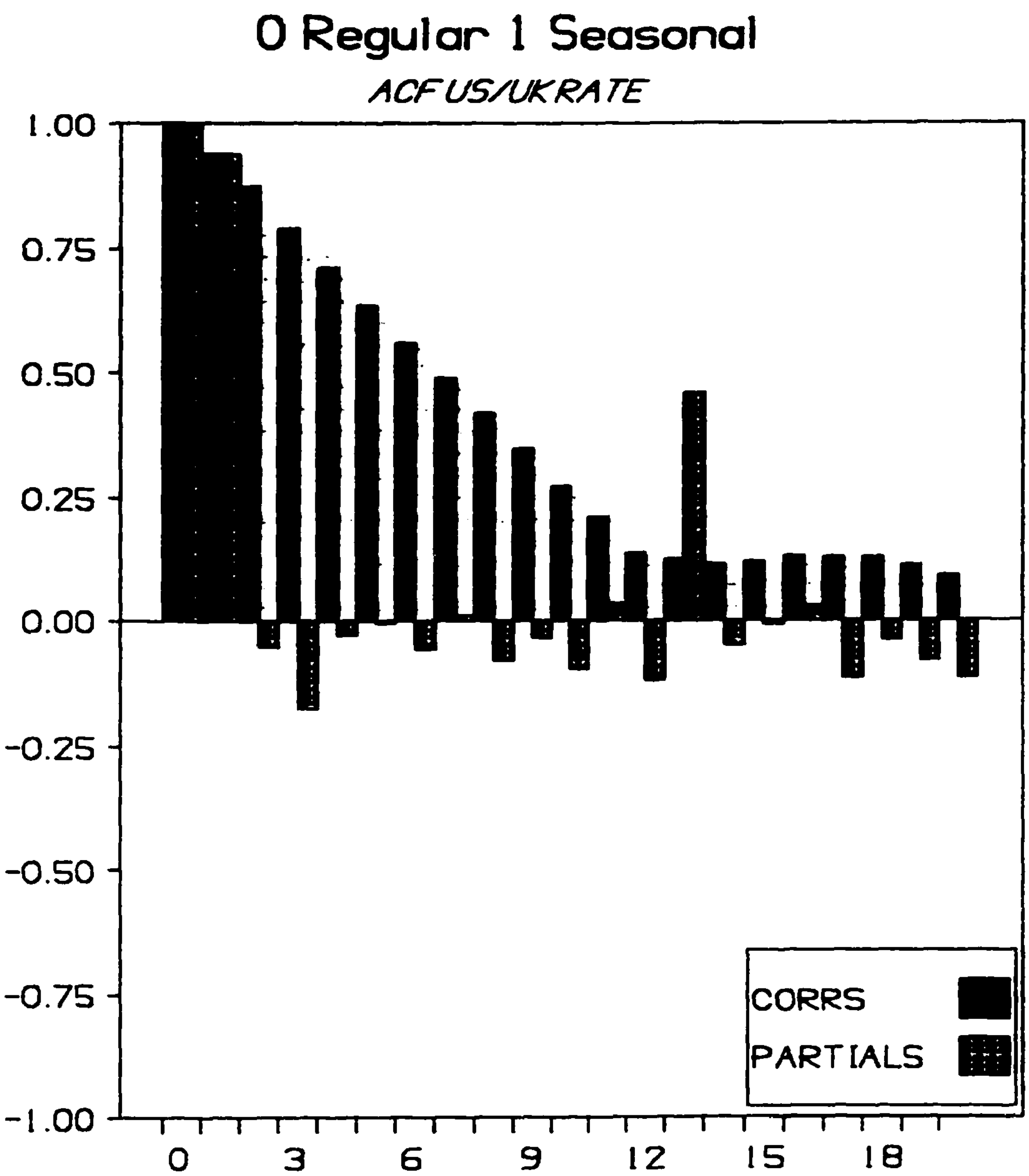

0 Regular 1 Seasonal
ACF US/UK RATE
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
0
3
6
9
12
15
18
CORRS
PARTIALS

FIGURE A5

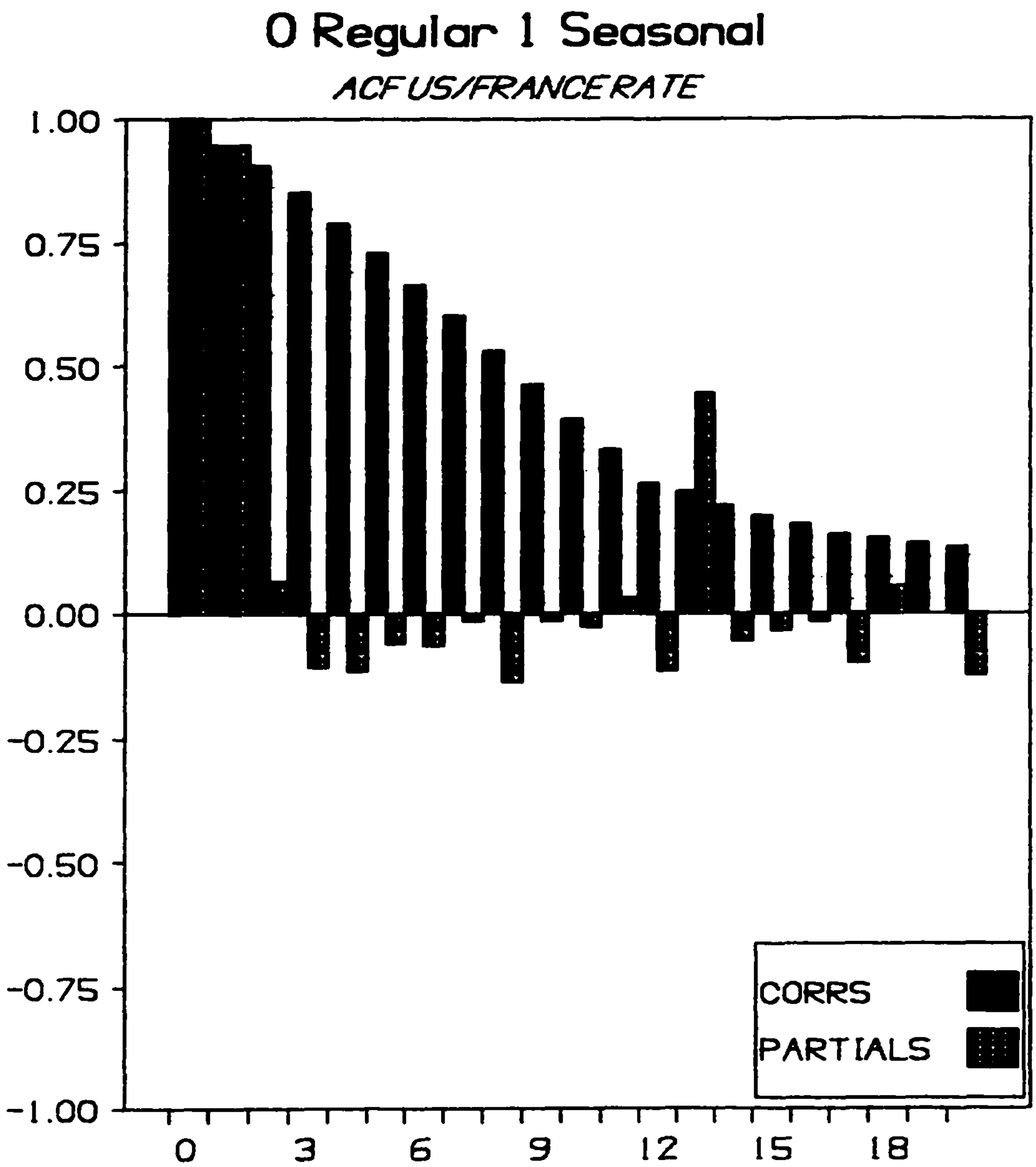

0 Regular 1 Seasonal
ACF US/FRANCE RATE
1.00
0.75
0.50
0.25
0.00
-0.25
-0.50
-0.75
-1.00
0
3
6
9
12
15
18
CORRS
PARTIALS

FIGURE A6

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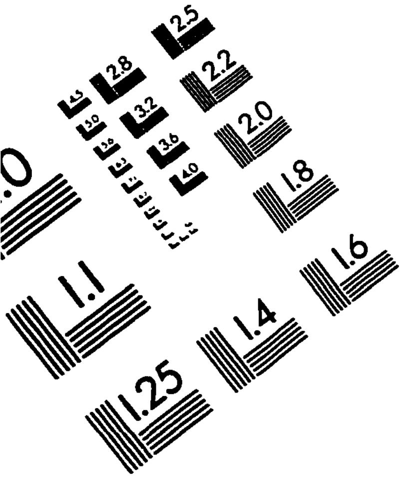
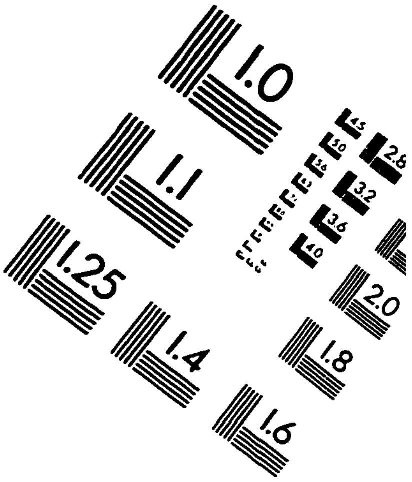
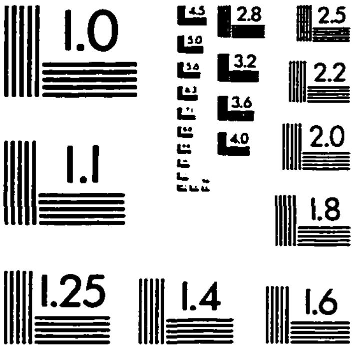
IMAGE EVALUATION TEST TARGET (QA-3)

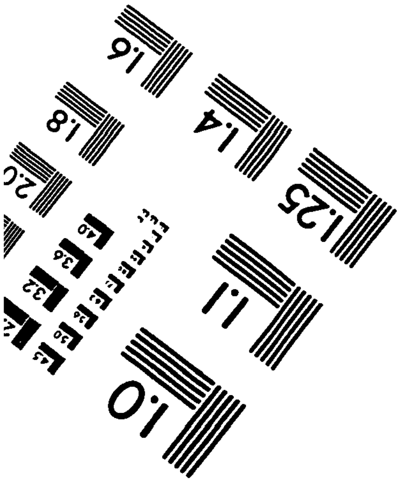


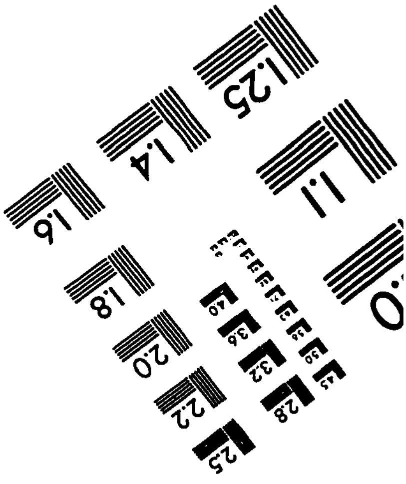




150mm

6″





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