8103917

CHAKRABARTI, JAYPROKAS

FLAVOR UNIFICATION

City University of New York PH.D. 1980

University
Microfilms
International 300 N. Zeeb Road, Ann Arbor, MI 48106

FLAVOR UNIFICATION

by

JAYPROKAS CHAKRABARTI

A dissertation submitted to the Graduate Faculty in Physics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York

1980

This manuscript has been read and accepted for the Graduate Faculty in Physics in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

June 24, 1980
date

Rabindra N. Mohapatra
Chairman of Examining Committee

June 24, 1980
date

Frank Masters
Executive Officer

Edward P. Tryon

Martin A. Kramer

John Smith

Bunji Sakita

Supervisory Committee

The City University of New York

Acknowledgements

The author is indebted to Professor Rabindra Nath Mohapatra for suggesting the problem as also for the numerous help he rendered during the course of the work. The work on the grand unified theories were done in collaboration with M. Popovic and R. N. Mohapatra. The author is grateful to Professor B. Sakita and Professor N. P. Chang for numerous discussions. Several important conversations with Dr. G. Branco, Dr. A. Das, A. Guha, J. Perez-Mercader and A. Sokorac are acknowledged. Special thanks are due to M. Popovic who developed several new ideas that are contained in this work. Finally, the author thanks Mrs. Shirley Burton for typing of the manuscript.

Abstract

FLAVOR UNIFICATION

by

Jayprokas Chakrabarti

Adviser: Professor Rabindra Nath Mohapatra

The present unified field theories-namely, (i) the $SU(2)_L(X)U(1)$ theory of the weak and electromagnetic interactions and (ii) the SU(5) grand unified theory - treat fermions in terms of families. This is because these are incomplete theories. We attempt flavor unification by introducing a "horizontal" gauge group. At the level of the weak and electromagnetic interactions we conjecture that the b-quark could decay dominantly via the horizontal interactions. Bounds are derived on the lifetime of the b-quark and $B_o - \bar{B}_o$ mixing, assuming this to be true. The multilepton signature for such a scenerio in the decay of upsilon is evaluated. It is shown that the multilepton events get related to the branching ratios for the decay of the τ-lepton.

Next we investigate the problem of incorporating the flavor generations within the framework of a grand unified SU(N) gauge theory. The requirement of no triangle anomaly and an asymptotically free gauge coupling constant appears to limit the number of "light" fermion generations to four. Specializing

to an SU(8) model, we exhibit a Higg's mechanism that splits the fermions into heavy and light sets, the later containing only three generations. We argue that the mass of the heavy generations is in the Tev. region.

We discuss the radiative effects on the Weinberg angle in such theories. Further using one loop renormalization group equations we obtain the scale at which the flavor-grand unification occurs. We argue that logical completeness requires that we consider SU(10) and SO(15) gauge groups as serious candidate theories.

TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

The theory of beta decay as proposed by Fermi involves the hamiltonian interaction part goes as:[1]

$$H_W = \sum_i G_i \bar{\psi}_p O_i \psi_n \bar{\psi}_e O_i \psi_{\nu_e} \qquad (1)$$

where

O_i = 1 i.e., scalar interaction

$= \gamma_\mu$ vector interaction

$= \gamma_\mu \gamma_5$ axial vector interaction

$= \sigma_{\mu\nu}$ tensor interaction

$= \gamma_5$ pseudo scalar interaction

However, the symmetries or the lack of symmetries of the weak interactions rule out several of the above possibilities. Isospin, hypercharge, parity, are all broken through the weak interactions. The observed parity violation of the weak interactions led Marshak and Sudarshan to propose that the interaction lagrangian be written as

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \bar{\psi}_p \gamma_\mu (1+\gamma_5) \psi_n \bar{\psi}_e \gamma_\mu (1+\gamma_5) \psi_{\nu_e} \qquad (2)$$

Aside from the CP violation such tree interaction term account for the usual weak interactions. The coupling constant, which carries a dimension is experimentally $G_F = 1.435 \times 10^{-49}\ erg \cdot cm^3$ (3)

or $$\frac{G_F}{\hbar c} = 1.165 \times 10^{-5}\ (Gev)^{-2}$$

Similar interaction lagrangian can be written at the level of quarks. For example, in the SU(3) nomenclature the weak interaction hamiltonian is written as:

$$H = -\frac{G_F}{\sqrt{2}} J_\mu J_\mu^\dagger \qquad (4)$$

where $J_u = l_u + h_u$; l_u denotes the leptonic current given as, $l_u = \bar{\psi}_e \gamma_\mu (1+\gamma_5) \psi_{\nu_e} + \bar{\psi}_\mu \gamma_\mu (1+\gamma_5) \psi_{\nu_\mu} + ..$ and the quark-hadron current is

$$h_\mu = (V_\mu^{1+i2} + A_\mu^{1+i2}) \cos\theta + (V_\mu^{4+i5} + A_\mu^{4+i5}) \sin\theta \qquad (5)$$

$$V_\mu^i = \frac{i}{2} \bar{q} \gamma_\mu \lambda_i q \quad , \quad A_\mu^i = \frac{i}{2} \bar{q} \gamma_\mu \gamma_5 \lambda_i q$$

First part of the hadronic current conserve strangeness. The second part, that changes strangeness is Cabibbo suppressed.

There are two parts about these interactions that need to be stressed. The first, that they lead to violations of unitarity and second that these interactions are non-renormalizable.

Unitarity[2]

Consider, for example, the process $\bar{\nu}_\mu \mu^- \to e^- \bar{\nu}_e$ The total cross-section, is given as:

$$\sigma = \frac{\pi}{K^2} \sum_\ell (2\ell+1)(2 - f_\ell - f_\ell^*) \tag{6}$$

where the f_e's are the partial wave amplitudes which are bounded in magnitude as $|f_\ell| \leq 1$. However, the born cross-section for the above process with the interaction hamiltonian given in is computed as follow:

$$\sigma = \frac{1}{flux} \frac{|M|^2}{T} \tag{7}$$

where $M = \frac{G_F}{\sqrt{2}} \bar{\psi} \gamma_\mu (1+\gamma_5) \psi . \bar{\psi} \gamma_\mu (1+\gamma_5) \psi$ in the centre of mass frame the

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2}{4\pi^2} . E_{cm}^2 \tag{8}$$

Keeping only the s-wave term in the partial wave analysis we have

$$\sigma = \frac{G_F^2}{\pi} . E_{cm}^2 \tag{9}$$

$$\sigma \lesssim \frac{2\pi}{E^2} \tag{10}$$

$$\frac{G_F^2}{\pi} E_{crit}^2 \approx \frac{2\pi}{E_{crit}^2} \tag{11}$$

$$E_{crit}^2 \approx \frac{2\pi^2}{G_F^2} \tag{12}$$

This leads to an unitarity bound in the range of around 300 Gev.

Attempts to improve the high-energy behavior led to the postulate of intermediate vector bosons. The lagrangian is now replaced as

$$\mathcal{L} = \frac{g}{\sqrt{2}} \left\{ J_\mu^+ W_\mu + J_\mu W_\mu^+ \right\} \tag{13}$$

where g unlike before is a dimensionless coupling constant and J as before is the sum of the leptonic and the hadronic currents. However the high energy behavior improves somewhat; it is still critical as indicated in the following analysis.

The propagator for the W boson is given as

$$\Delta_{\mu\nu} = \frac{1}{i} \frac{\delta_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2}}{q^2 - M_W^2} \tag{14}$$

The process $\nu_\mu \mu^- \to \nu_e e^-$ can be computed. The answer is

$$\sigma = \frac{g^4 \cdot E^2}{(m_W^2 - q^2)^2} \tag{15}$$

In the centre of mass frame, the scattering angle, θ, and energy E are related to the q^2 as

$$q^2 \simeq -(1-\cos\theta)E^2 \qquad \text{for E large.}$$

Therefore, the high energy behavior of the theory is improved for this process. However, the process $e^+e^- \rightarrow W^+W^-$, $\nu\bar{\nu} \rightarrow W^+W^-$ are still badly behaved. The first renormalisable model of the weak interaction were constructed in 1967-8 by Weinberg and Salam.[3] The theory has an exact local guage symmetry of $SU(2)_L \times U(1)$ which is broken by the ground state leaving only the U(1) of electromagnetism as the symmetry realized in the physical states. The fermionic representation go as follows:

$$Q_{iL} \equiv \begin{pmatrix} u^0 \\ d^0 \end{pmatrix}_L ; \begin{pmatrix} c^0 \\ s^0 \end{pmatrix}_L ; \begin{pmatrix} t^0 \\ b^0 \end{pmatrix}_L ; \cdots \tag{16}$$

the leptons belong to:

$$\begin{pmatrix} \nu_1 \\ e \end{pmatrix}_L ; \begin{pmatrix} \nu_2 \\ \mu \end{pmatrix}_L ; \begin{pmatrix} \nu_3 \\ \tau \end{pmatrix}_L \tag{17}$$

The right handed objects are taken as singlets as:

$$
\begin{aligned}
Q_{iR}^{2/3} &\equiv u_R\,,\ c_R\,,\ t_R \\
Q_{iR}^{-1/3} &\equiv d_R\,,\ s_R\,,\ b_R \\
& e_R\,,\ \mu_R\,,\ \tau_R
\end{aligned}
\tag{18}
$$

Aside from the fermions the theory contains Gauge Bosons, Higg's bosons and the Ghosts. The Higg's bosons in the standard version is a doublet transforming as:

$$
\begin{pmatrix} \phi \\ \phi^{0} \end{pmatrix} \qquad \left(\tfrac{1}{2}, 1\right)
\tag{19}
$$

The lagrangian for the gauge bosons and the fermions is given as

$$
\mathcal{L} = -\bar{Q}_{iL}\gamma_{\mu}\left(\partial_{\mu} - \frac{ig}{2}\vec{\tau}.\vec{W} + \frac{ig'}{2}.I\,B_{\mu}\right) Q_{iL}
\tag{20}
$$

$$
\vec{\tau}.\vec{W} = \begin{pmatrix} W_{\mu}^{3} & W_{\mu}^{1} - iW_{\mu}^{2} \\ W_{\mu}^{1} + iW_{\mu}^{2} & -W_{\mu}^{3} \end{pmatrix} = \begin{pmatrix} W_{\mu}^{3} & \sqrt{2}\,W_{\mu}^{+} \\ \sqrt{2}\,W_{\mu}^{-} & -W_{\mu}^{3} \end{pmatrix}
$$

The interaction lagrangian for one generation of leptons is:

$$
\begin{aligned}
\mathcal{L} = {} & \frac{i}{2}\bar{\nu}_{eL}\gamma_{\mu}\nu_{eL}\left(gW_{\mu}^{3} - g'B_{\mu}\right) + \frac{i}{2}\left(\bar{\nu}_{e_L}\gamma_{\mu}\bar{e}_L W_{\mu}^{+} + \bar{e}_L\gamma_{\mu}\nu_{e_L}W_{\mu}^{-}\right) \\
& - \frac{i}{2}\bar{e}_L\gamma_{\mu}e_L\left(gW_{\mu}^{3} + g'B_{\mu}\right) - ig'\bar{e}_R\gamma_{\mu}e_R B_{\mu}.
\end{aligned}
\tag{21}
$$

The lagrangian for the higg's bosons read

$$\mathcal{L} = -(D_\mu \phi)^\dagger (D_\mu \phi) - V(\phi)$$

$$= -\left(\partial_\mu \phi^\dagger + \frac{ig}{2}\phi^\dagger \vec{\tau}\cdot\vec{W}_\mu + \frac{ig'}{2}\phi^\dagger B_\mu\right)\left(\partial_\mu \phi - \frac{ig}{2}\vec{\tau}\cdot\vec{W}_\mu \phi - \frac{ig'}{2}B_\mu \phi\right) - V(\phi) \quad (22)$$

where,

$$V(\phi) = -\frac{\mu^2}{2}(\phi^\dagger\phi) + \frac{\lambda}{4}(\phi^\dagger\phi)^2 \quad (23)$$

The minimum of the potential corresponds to

$$\left(\frac{\partial V}{\partial \phi}\right)_{\phi = \langle\phi\rangle} = 0 \quad (24)$$

this gives

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ \frac{a}{\sqrt{2}} \end{pmatrix} \quad (25)$$

This gives the following mass terms for the gauge bosons

$$\mathcal{L}_m = -\frac{1}{4}g^2 a^2 W_\mu^+ W_\mu^- - \frac{a^2}{8}(g' B_\mu - g W_\mu^3)^2 \quad (26)$$

The neutral gauge bosons, that are mass eigenstates, are

$$Z_\mu = (g' B_\mu - g W_\mu^3)/(g^2 + g'^2)^{\frac{1}{2}} \quad (27)$$

$$A_\mu = (g B_\mu + g' W_\mu^3)/(g^2 + g'^2)^{\frac{1}{2}} \quad (28)$$

The mass eigenvalues are

$$M^2_{W+} = \frac{g^2 a^2}{4} \quad (29)$$

$$M^2_Z = \frac{(g^2 + g'^2)a^2}{4} \quad (30)$$

Define $$g'/(g^2+g'^2)^{\frac{1}{2}} = \cos\theta_W \tag{31}$$

$$g/(g^2+g'^2)^{\frac{1}{2}} = \sin\theta_W \tag{32}$$

In terms of this angle we get

$$Z_\mu = \cos\theta_W B_\mu - \sin\theta_W W_\mu^3 \tag{33}$$

$$A_\mu = \sin\theta_W B_\mu + \cos\theta_W W_\mu^3 \tag{34}$$

We obtain from the interaction lagrangian the following identifications:

$$e = \sqrt{g^2+g'^2} \cdot \sin\theta_W \cdot \cos\theta_W \tag{35}$$

$$g' = e/\sin\theta_W \tag{36}$$

$$g = e/\cos\theta_W \tag{37}$$

The μ-decay into $e\text{-}\bar{v}_e v_u$ gives

$$\frac{G_F}{\sqrt{2}} = \frac{e^2\cos\theta_W}{8m_W^2} \tag{38}$$

$$m_W^2 = \frac{\sqrt{2}\,e^2}{8\,G_F\cdot\cos^2\theta_W} \tag{39}$$

Similarly,

$$m_Z^2 = \frac{e^2 a^2}{4\cos^2\theta_W \sin^2\theta_W} \tag{40}$$

These are estimated as

$$m_W \simeq 80\ \text{Gev.} \tag{41}$$

$$m_Z \simeq 90\ \text{Gev.} \tag{42}$$

Hadronic Part:

The transformation properties of the hadronic matter is listed in eqn.16 The feature of hadronic matter that is of interest is the mass matrix. The mass terms for the fermions arise due to the Yukawa Interactions:

$$\mathcal{L}_Y = h_{ij}\,\bar{Q}_{iL}\,\phi\,Q_{jR}^{-\frac{1}{3}} + H_{ij}\,\bar{Q}_{iL}\,\tilde{\phi}\,Q_{jR}^{2/3} \tag{43}$$

where i and j are the generational index. When ϕ is replaced by $\langle\phi\rangle$ we get the fermion masses. In such a scheme the identically charged fermions mix arbitrarily amongst themselves. These in general lead to several important consequences:

(i) The physical fermions display cabibbo like mixings

(ii) The possibility arises for CP violation in some of the processes resulting from complex phases appearing in the mass-matrix.

(iii) The neutral currents conserve flavor quantum number because the mixing matrix is unitary and that all the fermions of a given charge have identical transformation with respect to the gauge group.

(iv) In the leptonic sector since the neutrino is massless, the mixing of fermions do not have any physical significance.

There are several features of this theory that need mentioning:

(i) The theory is renormalisable;

(ii) The quarks and leptons come in generations.

CHAPTER II

HORIZONTAL GAUGE SYMMETRY

"Let those facts be causes
with whom coordinated other facts
arise.
Non-causes will they be,
So far the other facts have not
arisen".

(Treatise on the Middle Doctrine)

Motivations to go beyond the standard theory:

There are several motivations to go beyond the standard model. The standard model leaves out the strong interactions. According to present indications, the strong interactions are described by the SU(3) "color" gauge theory.[4] Therefore, the group of physical interest is at least $SU(2)_L \otimes U(1) \otimes SU(3)$.

The standard model, further is wedded to the concept of generations. If the quarks and the leptons are all fundamental and elementary objects it is difficult to understand that they come in families. We shall address ourselves mainly to this issue.[5]

The standard model treats the left-handed and the right-handed fermions on different footing. Thus the lagrangian is not invariant under discrete symmetries like parity, charge, conjugation, etc. It is appealing to think that, like other symmetries in nature, these too are softly broken.[6]

We shall focus almost exclusively to the second issue though we shall have several occasions to go into the other points as well. There are several physical motivations for raising the family issue. The standard $SU(2)_L(X)U(1)$ model with three families of quarks and leptons at the level of the lagrangian has at least seventeen undetermined parameters. It is therefore natural to think of $SU(2)_L(X)U(1)$ as coming from a larger symmetry where the parameters are constrained. Physically, such larger symmetries could give rise to mass relations amongst quarks and leptons, determine the mixing angle and fix the value of Weinberg angle.

It was also noted that in the standard framework in the limit that the quarks are massless there is no distinction between particles of identical charge. Thus the up and the charm quark are identical if they are massless. Thus in this limit there is an extra symmetry in the system at least in the gauge interactions.

Horizontal Gauges - Historical Notes

There have been several attempts to utilize this symmetry in order to constrain the free parameters in the lagrangian. There are three early attempts. In order to get the cabibbo angle A. Zee introduced an electric charge carrying SU(2) triplet of gauge bosons.[7] An attempt was made to calculate the electron-muon mass ratio by introducing neutral SU(2) gauge triplet in addition to the standard model. However, these two models do not utilize the family symmetry as it is being presently done. The first attempt in the direction of family symmetry appears in the work of R. N. Mohapatra, J. C. Pati and L. Wolfenstein[8] where they use such a symmetry to understand the superweak nature of CP violation. The family symmetry in the way it is used today was introduced by S. M. Barr and A. Zee in the context of calculable electron-muon mass ratio.[9]

The more recent attempt in this direction have been made by F. Wilczek and A. Zee, C. L. Ong, K. C. Wali, Koca and A. Davidson as also by J. Chakrabarti.

We shall review the model of F. Wilczek and A. Zee for two reasons:

(i) The model has the feature that all the quark mixing angles are computable.

(ii) The model yields somewhat unusual results when attempt is made towards unification with the strong interactions.

The Model of Wilczek and Zee[10]:

The gauge group is

$$SU(2)_L (X) SU(2)_H (X) U(1)$$

The fermions transform as follows:

$$\left[\begin{pmatrix} u^0 \\ d^0 \end{pmatrix}_L \quad \begin{pmatrix} c_0 \\ s_0 \end{pmatrix}_L \quad \begin{pmatrix} t_0 \\ b_0 \end{pmatrix}_L \right] \tag{44}$$

$$\left[u_R^0 \quad c_R^0 \quad t_R^0 \right] \tag{45}$$

$$\left[d_R^0 \quad s_R^0 \quad b_R^0 \right] \tag{46}$$

If the following Higg's bosons ϕ_α^{ij} and η_α^i (where i.j are the horizontal indices and α the vertical index) are chosen and if there vacuum expectation values are

$$\langle \phi_{\alpha=+1}^{ij} \rangle \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{47}$$

and $\langle \eta^i_{\alpha=+1} \rangle \propto \begin{pmatrix} 1 \\ 0 \\ \epsilon \end{pmatrix}$ where $\epsilon << 1$

(48)

then, the charge 2/3 quark mass matrix is

$$M^{2/3} = \begin{pmatrix} 0 & +c & 0 \\ -c & -a & +b \\ 0 & -b & +a \end{pmatrix} \tag{49}$$

Similar, mass-matrix with a, b, c, replace by a', b', c', prevail for the charge 1/3 sector. The Cabibbo angle is computed as

$$\theta_c \approx \sqrt{\frac{m_d}{m_s}} \tag{50}$$

The other angle - the mixing between the first and the third generation is given by the angle

$$\frac{(m_d m_s)^{\frac{1}{2}}}{m_b} - \frac{(m_u m_c)^{\frac{1}{2}}}{m_t} \tag{51}$$

This theory has the important feature that it predicts that the bottom quark decays preferentially into the up quark. Further, the lifetime of $b\bar{b}$ is shorter than that predicted by the standard model. The unobservability of flavor changing neutral currents is administered by making the horizontal gauge bosons heavy. While the calculability of all the fermion mixing angles make the theory attractive there are two points about this theory that need to be emphasized. First of all - we shall discuss this point more explicitly later - this theory faces difficulties when attempt is made to grandunify it. Second, the idea that all the horizontal gauge bosons are heavy make such a theory physically unattractive.

Attempts at realizing the possibility that the horizontal gauge bosons need not be heavy encounter the usual difficulty that flavor-changing neutral currents between the known fermions (more precisely, only between the first two generations) has to be considerably suppressed compared to known interactions. Their strength is at best as $G_F\alpha^2$.

However, a theory that solves the above mentioned difficulties is easy to construct.[11] To do that let's consider the gauge group:

$$SU(2)_L(X)SU(2)_H(X)U(1)$$

where $SU(2)_H$ mediates interactions in the horizontal direction. The unrotated hadronic constituents then transform as follows:

$$Q_{1L} = \left\{ \begin{pmatrix} u^0 \\ d^0 \end{pmatrix}_L \begin{pmatrix} t^0 \\ b^0 \end{pmatrix}_L \right\} \qquad \qquad (52)$$

$$\left(\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{3}\right)$$

$$Q_{2L} = \left\{ \begin{pmatrix} c^0 \\ s^0 \end{pmatrix}_L \begin{pmatrix} h^0 \\ g^0 \end{pmatrix}_L \right\} \qquad \qquad (53)$$

$$q^{2/3}_R \equiv \{ u^0_R \quad t^0_R \} \qquad \left(0, \tfrac{1}{2}, \tfrac{4}{3}\right) \qquad (54)$$

$$\equiv \{ c^0_R \quad h^0_R \}$$

$$q_R^{-\frac{1}{3}} \equiv \{d_R^0 \quad b_R^0\} \qquad \left(0, \tfrac{1}{2}, -\tfrac{2}{3}\right) \tag{55}$$
$$\equiv \{s_R^0 \quad g_R^0\}$$

Amongst the leptons we assume that the horizontal interaction mediates between the electron and the tau multiplet as also between muon and M multiplet. The higg's content of the theory is as follows:

$$\chi_1 = \begin{bmatrix} \phi_1^+ & \phi_2^+ & \phi_3^+ \\ \phi_1^0 & \phi_2^0 & \phi_3^0 \end{bmatrix} \tag{56}$$

$$\left(\tfrac{1}{2}, 1, 1\right)$$

$$\chi_2 = \begin{bmatrix} \phi_4^+ & \phi_5^+ & \phi_6^+ \\ \phi_4^0 & \phi_5^0 & \phi_6^0 \end{bmatrix} \tag{57}$$

We impose the following discrete symmetry

$$D: \quad Q_{iL} \to Q_{iL} \; ; \; q_R^{\frac{2}{3}} \to -q_R^{2/3} \; , \; q_R^{\frac{1}{3}} \to q_R^{\frac{1}{3}}$$
$$L_{iL} \to L_{iL} \; , \; \ell_{iR} \to -\ell_{iR}$$
$$\chi_1 \to -\chi_1 \; , \; \chi_2 \to \chi_2$$

We assume that the discrete symmetry is softly broken in the lagrangian by terms of the type $X_1^+ X_2$. The vacuum expectation value that minimizes the effective potential is given as

$$\begin{bmatrix} 0 & 0 & 0 \\ V_1 & V_2 & 0 \end{bmatrix} \qquad (58)$$

$$\begin{bmatrix} 0 & 0 & 0 \\ V_4 & V_5 & 0 \end{bmatrix} \qquad (59)$$

The invariant Yukawa interactions are:

$$
\begin{aligned}
\mathcal{L}_Y = {} & \frac{a}{\sqrt{2}}\left[-\sqrt{\tfrac{2}{3}}\,\bar{\psi}_{1L}\phi_4 b_R + \cdots\right] \\
& + \frac{a'}{\sqrt{2}}\left[-\sqrt{\tfrac{2}{3}}\,\bar{\psi}_{2L}\phi_4 g_R + \cdots\right] \\
& + \frac{b}{\sqrt{2}}\left[-\sqrt{\tfrac{2}{3}}\,\bar{\psi}_{1L}\phi_4 g_R + \cdots\right] \\
& + \frac{b'}{\sqrt{2}}\left[-\sqrt{\tfrac{2}{3}}\,\bar{\psi}_{2L}\phi_4 b_R + \cdots\right] \\
& + \frac{A}{\sqrt{2}}\left[-\sqrt{\tfrac{2}{3}}\,\bar{\psi}_{1L}\tilde{\phi}_1 t_R + \cdots\right] \\
& + \frac{A'}{\sqrt{2}}\left[-\sqrt{\tfrac{2}{3}}\,\bar{\psi}_{2L}\tilde{\phi}_1 h_R + \cdots\right] \\
& + \frac{B}{\sqrt{2}}\left[-\sqrt{\tfrac{2}{3}}\,\bar{\psi}_{1L}\tilde{\phi}_1 h_R + \cdots\right] \\
& + \frac{B'}{\sqrt{2}}\left[-\sqrt{\tfrac{2}{3}}\,\bar{\psi}_{2L}\tilde{\phi}_1 t_R + \cdots\right]
\end{aligned}
\tag{60}
$$

Similar Yukawa couplings arise in the leptonic part of the theory. Note that owing to the discrete symmetry D, the leptons couple only to X_1. This is crucial for the determination of the mass of the t quark.

The mass matrices are

$$
\begin{array}{c}
\begin{array}{cccc} \quad u \quad & \quad c \quad & \quad t \quad & \quad h \quad \end{array} \\
\frac{1}{\sqrt{6}}\begin{bmatrix}
v_2\begin{pmatrix} A & B' \\ B & A' \end{pmatrix} & 0 \\
-\sqrt{2}\,v_1\begin{pmatrix} A & B' \\ B & A' \end{pmatrix} & -v_2\begin{pmatrix} A & B' \\ B & A' \end{pmatrix}
\end{bmatrix}
\end{array}
\tag{61}
$$

The mass matrix for the leptons is

$$\frac{1}{\sqrt{6}}\begin{array}{c} \begin{array}{cccc} e & \mu & \tau & M \end{array} \\ \begin{bmatrix} V_2\begin{pmatrix} A_t & B_t' \\ B_t & A_t' \end{pmatrix} & 0 \\ -\sqrt{2}V_1\begin{pmatrix} A_t & B_t' \\ B_t & A_t' \end{pmatrix} & -V_2\begin{pmatrix} A_t & B_t' \\ B_t & A_t' \end{pmatrix} \end{bmatrix} \end{array} \tag{62}$$

where A_t, A_t', B_t, and B_t' are the Yukawa couplings in the lepton sector. Note also that since the neutrinos are massless the mixing of the lepton does not have any impact as far as the vertical interactions are concerned.

For d, b, s, g we get

$$\frac{1}{\sqrt{6}}\begin{bmatrix} a\begin{pmatrix} V_5 & 0 \\ -\sqrt{2}V_4 & -V_5 \end{pmatrix} & b'\begin{pmatrix} V_5 & 0 \\ -\sqrt{2}V_4 & -V_5 \end{pmatrix} \\ b\begin{pmatrix} V_5 & 0 \\ -\sqrt{2}V_4 & -V_5 \end{pmatrix} & a'\begin{pmatrix} V_5 & 0 \\ -\sqrt{2}V_4 & -V_5 \end{pmatrix} \end{bmatrix} \tag{63}$$

It is known that the above matrices can be diagonalized by the following biorthogonal transformations:

$$U_L^{(i)} M_i U_R^{(i)T} = D_i \equiv \text{diagonal matrix.} \tag{64}$$

$$\begin{pmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{pmatrix}\begin{pmatrix} A & B' \\ B & A' \end{pmatrix} U_R^{1T} = D_1 \tag{65}$$

$$\begin{pmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{pmatrix}\begin{pmatrix} V_2 & 0 \\ -\sqrt{2}V_1 & -V_2 \end{pmatrix} U_R^{2T} = D_2 \tag{66}$$

$$\begin{pmatrix} \cos\theta_3 & \sin\theta_3 \\ -\sin\theta_3 & \cos\theta_3 \end{pmatrix} \begin{pmatrix} a & b' \\ b & a' \end{pmatrix} U_R^{3T} = D_3 \tag{67}$$

$$\begin{pmatrix} \cos\theta_4 & \sin\theta_4 \\ -\sin\theta_4 & \cos\theta_4 \end{pmatrix} \begin{pmatrix} V_5 & 0 \\ -\sqrt{2}V_4 & -V_5 \end{pmatrix} U_R^{4T} = D_4 \tag{68}$$

Then, so far as the vertical interactions are concerned, we get the following effective flavor states:

$$\begin{Bmatrix} u \\ \eta[d(\theta_c)] + \varsigma[b(\theta_c)] \end{Bmatrix}_L, \begin{Bmatrix} c \\ \eta[s(\theta_c)] + \varsigma[g(\theta_c)] \end{Bmatrix}_L$$

$$\begin{Bmatrix} t \\ \eta[b(\theta_c)] - \varsigma[d(\theta_c)] \end{Bmatrix}_L, \begin{Bmatrix} h \\ \eta[g(\theta_c)] - \varsigma[s(\theta_c)] \end{Bmatrix}_L \tag{69}$$

where $\theta_c = \theta_3 - \theta_1, \quad \phi = \theta_4 - \theta_2, \eta = \cos\phi, \varsigma = \sin\phi$

$$d[\theta_c] = d\cos\theta_c + s\sin\theta_c, \; s[\theta_c] = -d\sin\theta_c + s\cos\theta_c \tag{70}$$

$$b[\theta_c] = b\cos\theta_c + g\sin\theta_c, \; g[\theta_c] = -b\sin\theta_c + g\cos\theta_c$$

The angle ϕ is computable in terms of masses of the quarks and the relationship is

$$\phi = \left(\frac{m_d}{m_b + m_d}\right)^{\frac{1}{2}} - \left(\frac{m_u}{m_t + m_u}\right)^{\frac{1}{2}} \tag{71}$$

Therefore,

$$\phi = \left(\frac{m_d}{m_b}\right)^{\frac{1}{2}} \tag{72}$$

Comparing with Eqs. (7) and (13) we find that Eq. (31) is a good estimate for ς .

From the mass matrices for the fermions we get the following mass relations:

$$m_u/m_c = m_t/m_h \tag{73}$$

$$m_d/m_b = m_s/m_g \tag{74}$$

$$m_e/m_\tau = m_\mu/m_M \tag{75}$$

$$\frac{m_e}{m_\tau} = \frac{m_u}{m_t} \qquad (76)$$

As a result of the above relations, masses of all the upcoming fermions are determined in the theory. In particular, the mass of the t quark, which is of considerable interest at the present time, is predicted to lie between 14 to 27 GeV.

Experimental Signatures: Implication for the b Quark

In the event that the above is realized the b quark decays dominantly into the up quark (through verticle interaction) and down quark (through horizontal interaction). This is one of the important predictions of the theory and differs considerably from the present standard (KM) view. The multilepton events that follow the production of Υ (Rev. 10) can distinguish between these theoretical alternatives. There are two other features that are worth noting. The first is the mixing between the B^o and $\bar{B}^o$. Horizontal interactions connect b to d and s to g. They do not connect b to g or d to s. As a result the mixing between B^o and $\bar{B}^o$ may begin to differ from the standard view. The second distinguish feature is that owing to the presence of horizontal interactions, the lifetime of the b quark become shorter than that predicted by the KM model. These features clearly distinguish this case from all other currently held views about the b quark.

Flavor Changing Neutral Currents -

The horizontal gauge bosons mediate flavor changing neutral currents. However the pattern of fermion mixing ensure the following:

(i) There is no direct neutral current connecting up with charm.

(ii) There is no direct neutral current connecting down with strange.

Thus, these processes are naturally suppressed.

However,

(i) There is a direct neutral coupling between the bottom and the down quark

(ii) Similar direct neutral coupling exists between the strange and the g-quark.

Experimentally no bounds exist on these processes. Therefore, physically the horizontal gauge bosons need not necessarily be heavy. However, in future if these processes remain unobserved (or largely suppressed) it would be necessary to make the horizontal gauge bosons heavier than the usual W and Z bosons.

However, the leptonic sector can be used tc provide bound on the masses of the horizontal gauge bosons. The

The present data on the decay of the τ-lepton into the muon and the electron are as follows:

$$\tau \to \mu \bar{\nu} \nu \tag{77}$$
$$\tau \to e \bar{\nu} \nu$$

In the horizontal gauge sector there are two unknowns. First, the gauge coupling constant and second, the masses of the horizontal gauge bosons. We shall assume in the spirit of unification that the gauge coupling constant in the horizontal sector and the usual vertical sector are roughly equal. This leaves us with one unknown.

The experimental data on -decay may be interpreted to mean that the lepton decays into the electron at most 25% more than lepton into the muon. Thus, we get:

$$\frac{m_{W_H}^4}{m_W^4} \geqslant 4 \tag{78}$$

where M_{W_H} denotes the masses of the horizontal gauge bosons and M_W is the mass of the usual W boson mediating beta decay.

Therefore,

$$m_{W_H}^4 \geqslant 4 m_W^4 \tag{79}$$

$$m_{W_H} \geqslant 100 \text{ Gev.} \tag{80}$$

A somewhat better bound is obtained by noting that the decay of lepton that comes about through the horizontal gauge boson is not pure V-A. It depends on the details of mixing of the right-handed leptons. For the mass-matrixes, that we have obtained, the mixing in the left and right sector are not equal. In the absence of any experimental numbers on lepton mixings we will pretend that these mixings, if any, are small. Since the lepton decays are known to be of the V-A type we get: $M_H \gtrsim 4M_W$ (81)

<u>The b-Quark:</u>

Owing to flavor mixing the b-quark decays, the standard prediction, with Kobayashi and Maskawa mixing matrix, is somewhat arbitary. However, it may be interpreted to mean that the b-quark decays, in the hadronic sector, dominantly into the charm quark. Experimentally, it is known that the b-quark does decay ($\tau_b < 5\times10^{-8}$ sec.). However, the decay mode of the b-quark remains unverified.

In the pattern of flavor mixing that we are discussing the b-quark once again decays. The decay modes however contrast with the standard prediction. The vertical interactions make b-quark decay dominantly into the up quark. The horizontal interactions make b-quark decay into the down quark. Since the horizontal interactions are not necessarily suppressed (for reasons that we have indicated) the two rates could be comparable.

Since we have a bound on the mass of the horizontal gauge bosons, it is possible to put a bound on the lifetime of the b-quark. Further the decay modes are experimentally distinguisable from the standard scheme. Also, there is more mixing between B^o and $\bar{B}_o$ mesons.

<u>b-Quark Lifetime</u>:

Assuming that the dominant modes of decay of the b-quark are as predicted by this theory we get the following channels for the decay of the b-quark.

$$
\begin{aligned}
b \rightarrow\ & u, c\ (e\bar{\nu}_e) \\
& u, c\ (\mu\bar{\nu}_\mu) \\
& u, c\ (\tau\bar{\nu}_\tau) \\
& u, c\ (d\bar{u}) + u, c\ (s\bar{u}) \\
& u, c\ (d\bar{c}) + u, c\ (s, \bar{c}) \\
& d\ (\nu_\tau\bar{\nu}_e) \\
& d\ (\tau^- e^+) \\
& d\ (\nu_M\bar{\nu}_\mu)
\end{aligned}
\tag{82}
$$

The decay of b into d quark is the dominant decay mode for the b-quark. All other decay modes are suppressed by a factor of $\xi^2 \approx 10^{-6}$. Thus the bound on the lifetime of the b-quark is easily computable.

We assume in the analysis below that the mass of the horizontal gauge bosons is the four times the mass of the usual W-bosons. Thus, the decay rate of the b-quark is given as

$$\gamma_0 = \gamma_x \left(\frac{m_b}{m_\mu}\right)^5 \frac{1}{16} \left[1 + f\left(\frac{m_\tau}{m_b}\right) + 1\right] \tag{83}$$

where
$$\gamma_x = \frac{G_F^2 \, m_\mu^5}{192 \pi^3} \tag{84}$$

and
$$f(y) = (1-y^4)(1-8y^2+y^4) - 12y^4 \ln y^2 \tag{85}$$

This gives us a lifetime of the bottom quark of

$$\tau_b \gtrsim 10^{-14} \text{ sec.} \tag{86}$$

This compares with the prediction of the standard model of

$$\tau_b \gtrsim 10^{-14} \text{ sec.}$$

$B_o - \bar{B}_o$ Mixing

Though the lifetime of the b-quark does not differ radically from the standard model, the picture of B_o-$\bar{B}_o$ mixing is entirely different. In the standard model, the mixing comes about by the diagrams in figure 1. Note that the mixing takes place at the one loop level. However in our case the B_o-$\bar{B}_o$ mixing appears in the tree level as shown in the figure 2.

In order to evaluate the B_o-$\bar{B}_o$ mixing it is necessary to write down the effective lagrangian. Here B stands for the bottomness quantum number. From the tree graph mediating $\Delta B = 2$ process the effective lagrangian is

$$\mathcal{L}_{\Delta B=2} = \frac{g^2}{8M_H^2}\eta^4\cos^4\theta_c\left[\bar{b}\gamma_\mu(\alpha+\beta\gamma_5)d\right]\left[\bar{b}\gamma_\mu(\alpha+\beta\gamma_5)d\right] \tag{87}$$

where α and β depend~~s~~ on the magnitudes of the left and the right-handed mixings of the fermions. From the mass-maxtix displayed in eqn 63 it is clear that the left-handed and the right-handed mixings are not identical. Further the mixings are not computable as far as the right handed sector is concerned.

Let us evaluate the upper bound on the matrix element between B_o and $\bar{B}_o$. In order to evaluate the upper bound let us insert the vacuum state between two $\Delta B = 1$ currents. Therefore,

$$\langle \bar{B}^0 | [\bar{b}\gamma_\mu(\alpha+\beta\gamma_5)d]^2 | B_0\rangle$$

$$\simeq 4\langle \bar{B}_0 | \bar{b}\gamma_\mu(\alpha+\beta\gamma_5)d|0\rangle\langle 0|\bar{b}\gamma_\mu(\alpha+\beta\gamma_5)d|B_0\rangle \tag{88}$$

where the factor of 4 appears because of various Fierz ordering of the four fermion operator. Since,

$$\langle 0|\bar{b}\gamma_\mu d|B^0\rangle = 0 \tag{89}$$

because of parity, therefore, the upper bound on $M_B=M_{B_L}-M_{B_S}$ is obtained by setting $\beta=1$. Thus,

$$\Delta m_B \leq \frac{4g^2}{8m_H^2}\eta^4\cos^4\theta_c f_B^2 \cdot m_B \tag{90}$$

$$\Delta m_B \leq \frac{G_F}{4\sqrt{2}}\eta^4\cos^4\theta_c f_B^2 m_B \tag{91}$$

$$\Delta m_B \lesssim \frac{G_F f_B^2 m_B}{4\sqrt{2}} \tag{92}$$

$$\lesssim 0.3\times 10^{-5}\ \text{Gev.} \tag{93}$$

Where we assume that f_B = 0.5 Gev. and M_B = 5 Gev.

The decay width of B^o is estimated as

$$\Gamma_{B^o} = \frac{G^2 m_b^5}{192\pi^3} \cdot \frac{1}{16} \tag{94}$$

Therefore the ratio $\frac{\Delta m_B}{\Gamma_{B^o}}$ is given

by

$$\frac{\Delta m_B}{\Gamma_{B^o}} \simeq 10^4 \tag{95}$$

This compares with the value of 10^{-1} given in KM theory.

More on the b-Quark:

So far as the b-quark is concerned the horizontal interactions make one further contribution. If we assume that the horizontal mode of decay as the dominant mode (as we have done so far), then it is clear that the b-decay is not V- A type. It is a admixture of vector and axial vector currents the precise mixture depending on the relative magnitudes of the left and the right hand mixings.

Multilepton Events:

So far we have assumed that the dominant mode of decay of the b-quark is through the horizontal interactions. We shall continue to assume the above in the following discussions.

What is the experimental signature of the above scenerio? The multilepton events that follow from the decay of the b-quark after the production of upsilon may yield indirect support for the horizontal interactions.[12] We assume in the subsequent discussions that the decay of t and $\bar{b}$ are uncorrelated. If α, β and γ are defined as probabilities of the b-quark going into an e-, e+ and e^+e^- respectively, then,

$$\sigma_0 = \frac{\sigma(b\bar{b} \to no\ e^{\pm})}{\sigma(b\bar{b})} = (1-\alpha-\beta-\gamma)^2 \tag{96}$$

$$\sigma_1 = \frac{\sigma(b\bar{b} \to e^+) + \sigma(b\bar{b} \to e^-)}{\sigma(b\bar{b})} = 2(\alpha+\beta)(1-\alpha-\beta-\gamma) \tag{97}$$

$$\sigma_{+-} = \frac{\sigma(b\bar{b} \to e^+e^-)}{\sigma(b\bar{b})} = \alpha^2+\beta^2+2\gamma(1-\alpha-\beta-\gamma) \tag{98}$$

$$\sigma_{ss} = \frac{[\sigma(b\bar{b} \to e^+e^+) + \sigma(b\bar{b}) \to e^-e^-)]}{\sigma(b\bar{b})} = 2\alpha\beta \tag{99}$$

and

$$\sigma_3 = \frac{[\sigma(b\bar{b} \to e^+e^+e^-) + \sigma(b\bar{b} \to e^+e^-e^-)]}{\sigma(b\bar{b})} = 2(\alpha+\beta)\gamma \tag{100}$$

$$\sigma_4 = \frac{\sigma(b\bar{b} \to e^+e^+e^-e^-)}{\sigma(b\bar{b})} \simeq \gamma^2 \tag{101}$$

In this model

$$\beta \approx \frac{\Gamma[b \to d\tau e^+]}{\Gamma[b \to all]} \tag{102}$$

$$\gamma \approx \frac{\Gamma[b \to d\tau e^+]}{\Gamma[b \to all]} \cdot \frac{\Gamma[\tau \to e^-]}{\Gamma[\tau \to all]} \tag{103}$$

Therefore, we get,

$$\sigma_3 \approx 2\beta\gamma = 2\left\{\frac{\Gamma[b\to d\tau e]}{\Gamma[b\to all]}\right\}^2 \frac{\Gamma[\tau\to e]}{\Gamma[\tau\to all]} \tag{104}$$

$$\sigma_4 \approx \left[\frac{\Gamma[b\to d\tau e]}{\Gamma[b\to all]}\right]^2 \left[\frac{\Gamma[\tau\to e]}{\Gamma[\tau\to all]}\right]^2 \tag{105}$$

$$\frac{\sigma_4}{\sigma_3} = \frac{1}{2}\,\frac{\Gamma[\tau\to e]}{\Gamma[\tau\to all]} \tag{106}$$

This relates to the multilepton events in the decay of upislon to the branching ratio of the lepton going into electron. This, therefore, serves as an experimental test of the picture of the decay of the b-quark that we have presented.

An Alternative Scenerio: Calculable Cabibbo Angle:[13]

An alternative horizontal interaction scenerio is obtained by regrouping quarks and leptons.

In this scheme, due to regrouping, the possibility of the horizontal gauge bosons being of the same order of mass as w is destroyed. However the cabibbo angle becomes calculable. Further, we show how CP violation arises naturally in these horizontally extended modes.

As for the question of CP violation we show that CP violation may be more widespread than the present theoretical ideas admit. In fact, they may be present in almost all forms of weak interactions. However, roughly speaking, the CP violating component is proportional to the deviation from universality. Thus the CP-violating component is not so well preceived. They of course yield the observed CP-violationg mixing between the K^o and $\bar{K}^o$mesons.

As a by-product of this analysis we recover a definite prediction about the decay mode of the bottom quark. The dominant decay of the bottom is into the up quark. Further, we get relations connecting the masses of the fermions. We make prediction about the masses of some of the upcoming fermions. These fermions will be accessible in the next generation of machines.

- Horizontal extension.

The gauge group is $SU_{2,L}$ (X) $SU_{2,H}$ (X) U_1. The hadronic-constituent trasform is

$$Q_{1L} \equiv \boxed{\psi_{1L} = \begin{pmatrix} u^o \\ d^o \end{pmatrix}_L , \ \psi_{2L} = \begin{pmatrix} c^o \\ s^o \end{pmatrix}_L} , \ Q_{2L} \equiv \boxed{\psi_{3L} = \begin{pmatrix} t^o \\ b^o \end{pmatrix}_L \ \ \psi_{4L} = \begin{pmatrix} h^o \\ g^o \end{pmatrix}_L} \tag{107}$$

$$q_R^{2/3} \equiv \boxed{u_R^o \ \ c_R^o} , \ \boxed{t_R^o \ \ h_R^o} \tag{108}$$

$$q_R^{1/3} \equiv \boxed{d_R^o \ \ s_R^o} , \ \boxed{b_R^o \cdot g_R^o} \tag{109}$$

The leptons transform as

$$L_{1L} \equiv \boxed{\begin{pmatrix} \nu_1 \\ e \end{pmatrix}_L \ \ \begin{pmatrix} \nu_2 \\ \mu \end{pmatrix}_L} , \ L_{2L} \equiv \boxed{\begin{pmatrix} \nu_3 \\ \tau \end{pmatrix}_L \ \ \begin{pmatrix} \nu_4 \\ M \end{pmatrix}_L} \tag{110}$$

$$L_{1R} \equiv \boxed{e_R \quad \mu_R}\,, \quad L_{2R} \equiv \boxed{\tau_R \quad M_R} \tag{111}$$

Note that all the netrinoes are taken to be massless.

Higg's bosons of the theory are X_1 and X_2 both transforming as $(\frac{1}{2}, 1, 1)$, where

$$X_1 \equiv \boxed{\phi_1 \quad \phi_2 \quad \phi_3} \qquad X_2 = \boxed{\phi_4 \quad \phi_5 \quad \phi_6} \tag{112}$$

We add the following discrete symmetry:

$$(\mathrm{P}) \qquad \begin{array}{lll} Q_{iL} \to Q_{iL} & q_R^{2/3} \to -q_R^{2/3} & q_R^{1/3} \to q_R^{1/3} \\ L_{iL} \to L_{iL} & L_{iR} \to -L_{iR} & X_1 \to -X_1 \,, \; X_2 \to X_2 \end{array}$$

We assume that the above discrete symmetry is broken softly by terms like $\bar{X}_1^{\dagger} X_2$.

Vacuum expectation values that are consistent with the minimum of the potential are

$$\langle X_1 \rangle = \boxed{V_1 \quad V_2 \quad 0} \,, \quad \langle X_2 \rangle = \boxed{V_4 \quad V_5 \quad 0} \tag{113}$$

The Yukawa couplings are:

$$\begin{aligned} \mathcal{L}_Y = {} & \frac{a}{\sqrt{2}}\left[-\sqrt{\tfrac{2}{3}}\,\bar{\Psi}_{1L}\phi_4 s_R + \bar{\Psi}_{1L}\phi_5 d_R \cdot \sqrt{\tfrac{1}{3}} + \cdots\right] + \frac{a'}{\sqrt{2}}\left[-\sqrt{\tfrac{2}{3}}\,\bar{\Psi}_{3L}\phi_4 g_R + \sqrt{\tfrac{1}{3}}\,\bar{\Psi}_{3L}\phi_5 b_R + \cdots\right] \\ & + \frac{b}{\sqrt{2}}\left[-\sqrt{\tfrac{2}{3}}\,\bar{\Psi}_{1L}\phi_4 g_R + \cdots\right] + \frac{b'}{\sqrt{2}}\left[-\sqrt{\tfrac{2}{3}}\,\bar{\Psi}_{3L}\phi_4 s_R + \cdots\right] \\ & + \frac{A}{\sqrt{2}}\left[-\sqrt{\tfrac{2}{3}}\,\bar{\Psi}_{1L}\tilde{\phi}_1 c_R + \cdots\right] + \frac{A'}{\sqrt{2}}\left[-\sqrt{\tfrac{2}{3}}\,\bar{\Psi}_{3L}\tilde{\phi}_1 h_R + \cdots\right] \\ & + \frac{B}{\sqrt{2}}\left[-\sqrt{\tfrac{2}{3}}\,\bar{\Psi}_{1L}\tilde{\phi}_1 h_R + \cdots\right] + \frac{B'}{\sqrt{2}}\left[-\sqrt{\tfrac{2}{3}}\,\bar{\Psi}_{3L}\tilde{\phi}_1 c_R + \cdots\right] \end{aligned} \tag{114}$$

where we assume that a, a', b, b', A, A', B and B' are all complex. The mass matrices are:

$$
\frac{1}{\sqrt{6}}
\begin{array}{c}
\begin{array}{cccc} \quad u & \quad c & \qquad t & \quad h \end{array} \\
\left[\begin{array}{cc}
A\begin{pmatrix} V_2 & 0 \\ -\sqrt{2}V_1 & -V_2 \end{pmatrix} & B\begin{pmatrix} V_2 & 0 \\ -\sqrt{2}V_1 & -V_2 \end{pmatrix} \\
B\begin{pmatrix} V_2 & 0 \\ -\sqrt{2}V_1 & -V_2 \end{pmatrix} & A'\begin{pmatrix} V_2 & 0 \\ -\sqrt{2}V_1 & -V_2 \end{pmatrix}
\end{array}\right]
\end{array}
\tag{115}
$$

$$
\frac{1}{\sqrt{6}}
\begin{array}{c}
\begin{array}{cccc} \quad d & \quad s & \qquad b & \quad g \end{array} \\
\left[\begin{array}{cc}
a\begin{pmatrix} V_5 & 0 \\ -\sqrt{2}V_4 & -V_5 \end{pmatrix} & b'\begin{pmatrix} V_5 & 0 \\ -\sqrt{2}V_4 & -V_5 \end{pmatrix} \\
b\begin{pmatrix} V_5 & 0 \\ -\sqrt{2}V_4 & -V_5 \end{pmatrix} & a'\begin{pmatrix} V_5 & 0 \\ -\sqrt{2}V_4 & -V_5 \end{pmatrix}
\end{array}\right]
\end{array}
\tag{116}
$$

Similar Yukawa couplings arise amongst the leptons. The mass matrix for the leptons is

$$
\frac{1}{\sqrt{6}}
\left[\begin{array}{cc}
A_l\begin{pmatrix} V_2 & 0 \\ -\sqrt{2}V_1 & -V_2 \end{pmatrix} & B_l'\begin{pmatrix} V_2 & 0 \\ -\sqrt{2}V_1 & -V_2 \end{pmatrix} \\
B_l\begin{pmatrix} V_2 & 0 \\ -\sqrt{2}V_1 & -V_2 \end{pmatrix} & A_l'\begin{pmatrix} V_2 & 0 \\ -\sqrt{2}V_1 & -V_2 \end{pmatrix}
\end{array}\right]
\tag{117}
$$

Note that since the neutrinos have been taken to be massless, mixing amongst the leptons does not have any physical significance as far as the vertical interactions are concerned.

We know that the mass matrices are diagonalized by biunitary transformation of the type

$$U_L^{i\prime} M_i U_R^{i\dagger} = D_i \equiv \text{Diagonal} \tag{118}$$

Let

$$\exp[i\phi_1]\begin{pmatrix} \cos\theta_1 e^{i\alpha_1} & e^{i\beta_1}\sin\theta_1 \\ -e^{-i\beta_1}\sin\theta_1 & e^{-i\alpha_1}\cos\theta_1 \end{pmatrix}\begin{pmatrix} A & B' \\ B & A' \end{pmatrix} U_R^{1\dagger} = D_1 \tag{119}$$

$$\begin{pmatrix} \cos\theta_2 & \sin\theta_2 \\ -\sin\theta_2 & \cos\theta_2 \end{pmatrix}\begin{pmatrix} V_2 & 0 \\ -\sqrt{2}V_1 & -V_2 \end{pmatrix} U_R^{2\dagger} = D_2 \tag{120}$$

and

$$e^{i\phi_3}\begin{pmatrix} e^{i\alpha_3}\cos\theta_3 & e^{i\beta_3}\sin\theta_3 \\ -e^{-i\beta_3}\sin\theta_3 & e^{-i\alpha_3}\cos\theta_3 \end{pmatrix}\begin{pmatrix} a & b' \\ b & a' \end{pmatrix} U_R^{3\dagger} = D_3 \tag{121}$$

$$\begin{pmatrix} \cos\theta_4 & \sin\theta_4 \\ -\sin\theta_4 & \cos\theta_4 \end{pmatrix}\begin{pmatrix} V_5 & 0 \\ -\sqrt{2}V_4 & -V_5 \end{pmatrix} U_R^{4\dagger} = D_4 \tag{122}$$

Then the flavor eigenstrates are

(a) $$\begin{bmatrix} e^{i\phi_1}\{e^{i\alpha_1}\cos\theta_1\, u(\theta_2) + e^{i\beta_1}\sin\theta_1\, t(\theta_2)\} \\ e^{i\phi_3}\{e^{i\alpha_3}\cos\theta_3\, d(\theta_4) + e^{i\beta_3}\sin\theta_3\, b(\theta_4)\} \end{bmatrix}$$

(b) $$\begin{bmatrix} e^{i\phi_1}\{e^{i\alpha_1}\cos\theta_1\, c(\theta_2) + e^{i\beta_1}\sin\theta_1\, h(\theta_2)\} \\ e^{i\phi_3}\{e^{i\alpha_3}\cos\theta_3\, s(\theta_4) + e^{i\beta_3}\sin\theta_3\, g(\theta_4)\} \end{bmatrix}$$

(c) $$\begin{bmatrix} e^{i\phi_1}\{-e^{-i\beta_1}\sin\theta_1\, u(\theta_2) + e^{-i\alpha_1}\cos\theta_1\, t(\theta_2)\} \\ e^{i\phi_3}\{-e^{-i\beta_3}\sin\theta_3\, d(\theta_4) + e^{-i\alpha_3}\cos\theta_3\, b(\theta_4)\} \end{bmatrix}$$

and

$$\begin{bmatrix} e^{i\phi_1}\{-e^{-i\beta_1}\sin\theta_1\, c(\theta_2) + e^{-i\alpha_1}\cos\theta_1\, h(\theta_2)\} \\ e^{i\phi_3}\{-e^{-i\beta_3}\sin\theta_3\, s(\theta_4) + e^{-i\alpha_3}\cos\theta_3\, g(\theta_4)\} \end{bmatrix}$$

(d)

$$\sin^2\theta_2 \approx \frac{m_u}{m_c}\,, \quad \sin^2\theta_4 \approx \frac{m_d}{m_s} \tag{123}$$

where $u(\theta) = u\cos\theta + c$ and $c(\theta) = -u\sin\theta + c\cos\theta$ etc.

The angles θ_2 and θ_4 are calculated as

$$\theta_2^2 \approx \frac{m_u}{m_c} \quad , \quad \theta_4^2 \approx \frac{m_d}{m_s}$$

Note that θ_4 is roughly the Cabibbo angle. We get also the following Mass relationship:

$$\frac{m_u}{m_e} = \frac{m_t}{m_h} \tag{124}$$

$$\frac{m_d}{m_b} = \frac{m_s}{m_g} \tag{125}$$

$$\frac{m_e}{m_\mu} = \frac{m_\tau}{m_M} \tag{126}$$

$$\frac{m_e}{m_\mu} = \frac{m_u}{m_c} \tag{127}$$

CP violation:

We eliminate the redundant phases by the following procedure. We define the physical fields with a subscript p. Thus,

$$u_p(\theta_2) = e^{i(\alpha_1+\phi_1)} u(\theta_2)\,, \quad t_p(\theta_2) = e^{i(\beta_1+\phi_1)} t(\theta_2)\,, \quad c_p(\theta_2) = e^{i(\alpha_1+\phi_1)} c(\theta_2)$$

$$h_p(\theta_2) = e^{i(\beta_1+\phi_1)} h(\theta_2)$$

$$d_p(\theta_4) = e^{i(\alpha_3+\phi_3)} d(\theta_4)\,, \quad s_p(\theta_4) = e^{i(\alpha_3+\phi_3)} s(\theta_4)\,, \quad b_p(\theta_4) = e^{i(\beta_3+\phi_3)} b(\theta_4) \tag{128}$$

$$g(\theta_4) = e^{i(\beta_3+\phi_3)} g(\theta_4)$$

Then the physical flavour states are

(a) $$\begin{bmatrix} \cos\theta_1\, u_p(\theta_2) + \sin\theta_1\, t_p(\theta_2) \\ \cos\theta_3\, d_p(\theta_4) + \sin\theta_3\, b_p(\theta_4) \end{bmatrix}$$

(b) $$\begin{bmatrix} \cos\theta_1\, c_p(\theta_2) + \sin\theta_1\, h_p(\theta_2) \\ \cos\theta_3\, s_p(\theta_4) + \sin\theta_3\, g_p(\theta_4) \end{bmatrix}$$

$$
\text{(c)} \quad \begin{bmatrix} e^{-i(\alpha_1+\beta_1)}\{-\sin\theta_1\, u_p(\theta_2) + \cos\theta_1\, t_p(\theta_2)\} \\ e^{-i(\alpha_3+\beta_3)}\{-\sin\theta_3\, d_p(\theta_4) + \cos\theta_3\, b_p(\theta_4)\} \end{bmatrix}
$$

and

$$
\text{(d)} \quad \begin{bmatrix} e^{-i(\alpha_1+\beta_1)}\{-\sin\theta_1\, c(\theta_2) + \cos\theta_1\, h(\theta_2)\} \\ e^{-i(\alpha_3+\beta_3)}\{-\sin\theta_3\, s(\theta_4) + \cos\theta_3\, g(\theta_4)\} \end{bmatrix}
$$

Note that the horizontal interaction mediate between (a_p) and (b_p) and also between (c_p) and (d_p). Thus horizontal interactions do not carry any phases. Therefore, they conserve CP.

In the vertical direction we have the freedom to redefine that W_v^t. We define W_v^t (physical) such that in the multiplies (a_p) and (b_p) there is no CP-violating interactions. Therefore, we have CP violation in the (c_p) and (d_p) multiplets. Note also that the CP-violating amplitude has only one undertermined phase angle.

We have, therefore, established the existence of CP violation. The magnitude depends on the values of angles O_1 and O_3. In the K^o-$\bar{K}^o$ system the CP violating mixing comes about through the diagrams in Figure 3.

Experimental signatures - the decay of the bottom quark: The decay of the bottom quark may provide some small evidence in favour of the above-mentioned model. The model predicts that the decay of the bottom quark takes place dominantly into the up-quark. Such a situation is distinguishable from one where the bottom goes into charm by counting the number of leptons that are produced in the decay of the upsilon. We would like to add that there are a few theories where a similar scenario prevails with the respect to the bottom quark.

The masses of the horizontal gauge bosons are, presumably, larger than the ones mediating the usual weak interactions. This is necessitated by the fact that they mediate unobserved flavor-changing neutral processes.

One other observation that may have some bearing on experiments by the next generation of machines is the mass of the g-quark. It is predicted to lie between 80 to 100 Gev.

Discrete Summetries:

It has been proposed by several authors that flavor unification be achieved by imposition of discrete symmetries amongst the fermion families.[14] Such a choice is interesting because it does not necessitate an enlargement of the gauge group. Further, in many such models one obtains relations amongst the quark masses and mixing angles.

Amongst many such examples, the groups very widely used are permutation group of order n - ie, S_n. Other non-abelian point groups, like C_{3v} , have also been used.

As an example consider the work of S. Pakavasa and H. Sugawara[15], where they use the group $SU(2)_L$ (X) U(1) (X) S_3(X) R. The quark multiplets are defined as

$$\left\{ \begin{pmatrix} u \\ d \end{pmatrix}_L , \begin{pmatrix} c \\ s \end{pmatrix}_L \right\} \tag{129}$$

$$\{u_R\} , \{c_R\} \tag{130}$$

$$\{d_R \ . \ s_R\} \tag{131}$$

Higg's fields are

$$\{\phi_0\} \tag{132}$$

$$\{\phi_1 , \phi_2\} \tag{133}$$

The reflection symmetry, R is defined as

$$\phi_0 \rightarrow -\phi_0 \tag{134}$$

Analysis of the Yukawa interactions and the quark mass-matrices yield:

$$\tan\theta_c = \frac{m_d}{m_s} \tag{135}$$

There are several such schemes that lead to mass relations and calculable mixing angles. However, the class of discrete symmetries that do this is quite large. Further, the discrete symmetries, though they reduce the number of free parameters in the theory, do not have any observable consequences.

CHAPTER III
FLAVOR-GRAND-UNIFICATION

Grand Unification:

The standard $SU(2)_L$ (X) U(1) gauge theory describes only the weak and electromagnetic interactions. The strong interactions are described by SU(3) color gauge theory. In the spirit of unification they can be combined into an SU(5) theory describing all the three interactions. Such a theory was proposed by Georgi and Glashow. The content of such a theory is given as follows:[16]

Gauge Group SU(5)

Fermions belong in the representations 5^* and 10 as follows:

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ e \\ \nu \end{pmatrix}_L \tag{136}$$

and

$$\begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ & 0 & u_1^c & -u_2 & -d_2 \\ & & 0 & -u_3 & -d_3 \\ & & & 0 & -e^c \\ & & & & 0 \end{pmatrix}_L \tag{137}$$

The basic representations are repeated once for each generation. Thus to accommodate the known fermions three 5_L^* and three 10_L's are required.

In the simplest version of SU(5) the higg's bosons are as follows:

(i) ϕ belonging to the representation 5 of SU(5)

(ii) χ belonging to the 24 dimensional representation of SU(5).

The gauge bosons connect the d quarks to the electron. Thus in these theories the possibility for the decay of proton arises. This constrains the possible choice of vacuum expectation values as follows:

$$\langle\phi\rangle = O(10^3 \text{GeV}) \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \qquad (138)$$

The pattern of symmetry breaking is as follows:

$$\langle \chi_1 \rangle = O(10^{15}\,\mathrm{Gev.}) \begin{bmatrix} 1 & & & & \\ & 1 & & 0 & \\ & & 1 & & \\ & 0 & & -\frac{3}{2} & \\ & & & & -\frac{3}{2} \end{bmatrix} \tag{139}$$

Aside from proton decay the theory predicts the following:

$\sin^2\theta_W \approx .20$ present energies.

$M_e = M_d$; $M_\mu = M_s$; $M_\tau = M_b$ at energies of the order of 10^{15} Gev.

However, at the level of grand unification the generation issue resurfaces. The idea that the fundamental fermions come in families has no physical justification. At the level of SU(5) the issue remains unsolved.

FERMION GENERATIONS IN GRAND UNIFIED THEORIES

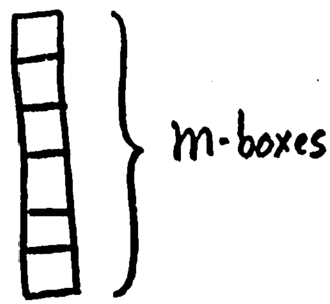
At the level of SU(5) the problem of generation persists. To explain the present spectrum of quarks and leptons one is forced to take three 5^*'s and 10 representations. There are three different approaches to this problem.

The Minimal Extension -[17]

The minimal extension of SU(5) to solve the generation problem was proposed by J. Chakrabarti, M. Popovic and

The constraint on the fermionic content of this scheme come from several considerations. The first of these is the idea that all fermions belong to 1, 3 and 3^* of color. Of course, no fermions that belong to any other representation of color is "known".

The above-mentioned criterion imposes severe restrictions on the possible representation of interest. It says that only totally antisymmetric representations are of physical interest. Totally autisymmetric representation of SU(N) is denoted by young tableaux as:



where $(m \leq N,)$

Such representation will be represented as (N, m). The dimension of the representation is $\binom{N}{m}$. Before proceeding further it is convenient to decompose (N,m) into its SU(5) content. The decomposition is as follows:

$$(N,m) = \binom{N-5}{m}\underline{1} + \binom{N-5}{m-1}\underline{5} + \binom{N-5}{m-2}\underline{10} + \binom{N-5}{m-3}\underline{10}^* + \binom{N-5}{m-4}\underline{5}^* + \binom{N-5}{m-5}\underline{1} \tag{140}$$

where 1, 5, 10, 10*, 5^* are all the antisymmetric representations of the SU(5) gauge group.

Following the above procedure let us decompose the well known representations 5^* and 10 of SU(5) into its color content. The representation 5^* of SU(5) in our notation is (5, 4). The decomposition into color, therefore, is

$$[5,4] = \binom{5-3}{4-2}\underline{3}^* + \binom{5-3}{4-3}\underline{1} \tag{141}$$

$$= \underline{3}^* + 2\,(\underline{1}) \tag{142}$$

Similarly, the representation 10 is denoted in our notation by (5, 2) and its color content is

$$[5,2] = \binom{5-3}{2}\underline{1} + \binom{5-3}{1}\underline{3} + \binom{5-3}{0}\underline{3}^* \tag{143}$$

$$= \underline{1} + 2(\underline{3}) + \underline{3}^* \tag{144}$$

Therefore, in 5^* and 10, number of times the representation 3 of color appears is two. The same is true for the representation 3^* of color. Thus, in the simple SU(5) of Georgi-Glashow color is real. As we shall see that this feature of grand unification presists when the families are tied together.

One further remark is in order. The model of Wilczek and Zee gets naturally excluded under this constraint. It is easy to verify that attempt at grand unifying their model would necessarily require fermions belonging to representations other that 1, 3 and 3^* of color.

The second constraint is as follows. Several representations of the type (N, m) are to be taken. However, this set be such that no irreducible representation be repeated. We shall denote the set of interest by $\{N,m\}$ It is convenient for future reference to set up the decomposition of $\{N,m\}$ with respect to the SU(5) subgroup.

$$\{N,m\} = n(1)\,\underline{1} + n(5)\,\underline{5} + n(10)\,\underline{10} + n(10^*)\,\underline{10}^* + n(5^*)\,\underline{5}^* \qquad (145)$$

where n(5), for example, denotes the number of times the representation 5 of SU(5) appears in the set

The thrid constraint on the fermionic content comes from renormalizability of the theory. In order for the theory to be free of divergent VVA triangle graphs, the fermionic content must be chosen in a special way. The problem has been investigated by S. Okubo[18] and the result as applying to the totally antisymmetric representations is as follows:

The VVA triangle anomaly A(n,m) is given by

$$A(N,m) = -A(N, n-m) = \frac{(N-2m)\ (n-3)!}{(N-m-1)!\ (m-1)!} \tag{146}$$

Thus in order for the anomaly to cancel, we must require that the anomaly associated with the set (N, m) must be zero. Thus,

$$\sum_{\{m\}} \frac{(N-2m)(N-3)!}{(N-m-1)!\,(m-1)!} = 0 \tag{147}$$

It is convenient at this point to discuss the anomaly cancellation at the level of SU(5) grand unified theory. Applying the formula, the anomaly associated with the representation 5^* of SU(5) is -1, while the anomaly associated with 10 of SU(5) is +1. Therefore the net anomaly is zero. Let's for future reference list this result

$$A(5, 4) = -A(5, 2) \tag{148}$$

similarily, $$A(5, 1) = -A(5, 3) \tag{149}$$

The fourth observation relates to the question of identification of the known quarks. In any scheme that relates to family issue several new fermions are introduced. Thus a process of identifiying the known fermions - also called "light" fermions - become necessary. In the process of putting the generations together it becomes necessary to go to a group like SU(N) with N 5. The embedding of SU(5) in SU(N) may be exemplified by breaking SU(N) into SU(5) X SU(N-5) x U(1). The group SU(N-5) is called the horizontal group.

The identification scheme for the known - or "light" fermions - is as follows. We demand that the "light fermions" transform as the simplest representation (i.e., the vector representation) of the horizontal SU(N-5) group. As we shall see that this provides the minimal extension of the SU(5) theory.

As a consequence of the above mentioned identification it is easy to verify (as we will) that SU(5+n) group gives n generations of "light fermions".
The above choice of "guidelines" lead to the following set of group and representations.

(1) SU(7) : $(7, 1)_L + (7, 3)_L + (7, 5)_L$ (150)

(2) SU(8) : $(8, 3)_L + (8, 6)_L + (8, 7)_L$ (151)

(3) SU(9) : $(9, 2)_L + (9, 5)_L$ (151)

(4) SU (10): $(10, 3)_L + (10, 5)_L + (10, 6)_L$ (153)

and so on.

The decomposition of the above representations with respect to SU(5) representations are as follows:

SU(7):

$$(7, 1) = 2(1) + 5 \tag{154}$$

$$(7, 3) = 5 + 2(10) + 10^* \tag{155}$$

$$(7, 5) = 10^* + 2(5^*) + 1 \tag{156}$$

We note that only two "light" generations exist in the SU(7) model.

SU(8)

$$(8, 3) = 1 + 3^*(5) + 3(10) + 10^* \tag{157}$$

$$(8, 6) = 3(1) + 3^*(5^*) + 10^* \tag{158}$$

$$(8, 7) = 3^*(1) + 5^* \tag{159}$$

Note that these contain only three "light" generations.

SU(9):

$$(9, 2) = 6(1) + 4(5) + 10 \tag{160}$$

$$(9, 5) = 5 + 4(10) + 6(10^*) + 4(5^*) + 1 \tag{161}$$

Note four generations.

SU(10):

$$(10,\ 3) = 10(1) + 10(5) + 5(10) + 10^{*} \tag{162}$$

$$(10,\ 6) = 5 + 5(10) + 10(10^{*}) + 10(5^{*}) + 5(1) \tag{163}$$

$$(10,\ 5) = 1(1) + 5^{*}(5) + 10^{*}(10) + 10\,(10^{*}) + 5(5^{*}) + 1(1) \tag{164}$$

Note five generations.

Partial Flavor Unification and SU(7):

It is interesting to ponder over partially unified models before flavor unification is achieved completely. It turns out that they contain the model that we have discussed at the level of horizontally extended standard $SU(2)_L$ (X)(1) model.

SU(7) serves as a good example of this phenomenon of paritial flavor unification. In order for this to happen we relax the criteria that the representations cannot be repeated in the set $\{7, m\}$. Allowing for repeatition, we get the following set of interest.

$$\{7, m\} \equiv 2(7,1)_L + 2(7,3)_L + 2(7,5)_L \tag{165}$$

Owing to the decomposition given previously, we note that this set contain four light generations. If we assume that (7, 3) - which 2(10) of SU(5) - to contain the e, μ, d, τ, t s, b in one and μ, c, s, M, h, g in the other then we have made a partially flavor unified model that just embeds the model we discussed.

However, the model of Wilczek and Zee remains outside the periphery of this scheme of flavor unification because they require fermions with exotic color. We have allowed only for fermions to transform as 1, 3, 3^* of color.

Flavor Unification and SU(8):

Flavor unification is achieved in SU(8). It is the minimal group that does the job and the representation of interest as indicated is (8, 3) + (8, 6) + (8, 7). The total number of helicity states is

$$\binom{8}{3} + \binom{8}{6} + \binom{8}{7} = 92 \qquad (166)$$

Note that aside from the 3 "light" generations, it has two "heavy" generations that have V + A weak interactions. Note also that one of these heavy generations does not come with its charge 2/3 quark.

Fermion Mass Matrix:

In order for the identification of "light" and "heavy" fermions to go through it is necessary to display their mass matrices.

Do the mass-matrices for the fermions respect such identification?

There are two types of mass-matrices that will respect the above identification. The first one is complete decoupling where the mass matrix would look like:

$$\begin{array}{cc} \text{Light Sector} & \text{Heavy Sector} \\ \multicolumn{2}{c}{\begin{bmatrix} X & 0 \\ 0 & Y \end{bmatrix}} \end{array} \tag{167}$$

However, in practice such a mass matrix is difficult to realize. The other mass-matrix that will respect the identification mentioned before

$$\begin{array}{cc} \text{Light Sector} & \text{Heavy Sector} \\ \multicolumn{2}{c}{\begin{bmatrix} X & \epsilon \\ \epsilon & Y \end{bmatrix}} \end{array} \tag{168}$$

$\epsilon < X, Y$

In the SU(8) theory we show that it is possible to realize this mass matrix.

To do that we introduce the following Higg's multiplet in the theory:

$$\phi^{\alpha\beta},\ \phi^{\alpha\beta\gamma\delta},\ \phi^{\alpha}_{\beta\gamma},\ \phi^{\alpha\pi\eta}_{\beta\gamma\lambda\rho\sigma}$$

The "heavy" fermions acquire their masses and mixing from the following gauge invariant couplings:

(i) $$\psi^{T\alpha\beta\gamma\lambda\sigma} C^{-1} \psi_{\eta\pi\rho}\, \phi^{\pi\eta\rho}_{\alpha\beta\gamma\lambda\sigma} \tag{169}$$

(ii) $$\psi^{T\alpha} C^{-1} \psi_{\alpha\beta\gamma}\, \phi^{\beta\gamma} \tag{170}$$

(iii) $$\psi^{T\alpha\beta} C^{-1} \psi_{\alpha\beta\sigma\lambda\mu\nu}\, \phi^{\sigma\lambda\mu\nu} \tag{171}$$

(IV) $$\psi^{T\alpha\beta} C^{-1} \psi_{\mu\alpha\nu}\, \phi^{\mu\nu}_{\beta} \tag{172}$$

Where ψ's with superscripts are obtained by applying the totally antisymmetric ϵ-symbol for the SU(8) group. The desired pattern of heavy particle masses ensue on setting

$$\begin{gathered} \langle \phi^{ab5}_{abABe} \rangle \neq 0,\ \langle \phi^{AB} \rangle \neq 0,\ \langle \phi^{5ABC} \rangle \neq 0 \\ \langle \phi^{AB}_{5} \rangle \neq 0,\ \langle \phi^{abc}_{abc5A} \rangle \neq 0 \end{gathered} \tag{173}$$

where a, b, c stand for SU(5) indices, A, B, ... stand for the horizontal SU(3) indices and α, β, γ, stand for SU(8) indices.

The masses of the "light" sector is obtained by introducing $\phi^{\alpha}_{\beta\gamma\lambda}$ with $\langle \phi^{A}_{5BC} \rangle \neq 0$ and $\phi^{\alpha\beta}_{\gamma}$, $\langle \phi^{5A}_{B} \rangle \neq 0$. The latter contributes to the mass matrix for the down quarks whereas the former contributes to the up-quark masses.

Aside from the couplings that we have displayed the rest of the gauge invariant yukawa couplings are taken to come with smaller coupling constrants. This ensures that the resulting mass matrix is as given in eqn.168.

SU(9):

SU(9) gives flavor unification with four generations. The novel feature of this model is that all the light fermions are contained in one single irreducible representation of SU(9). Thus (9, 5) contains $4(5^*)$ and 4(10) of SU(5) gauge theory.

This feature of SU(9) may provide ways of relating the masses of the charge 2/3 quarks and leptons. In particular, it may give the mass of the top quark.

As in SU(8) theory the heavy quarks "decouple" from the light quarks in the mass-matrix.

"Heavy" Fermions:

A question that naturally arises in the context of flavor unification is what are the masses of the unknown quarks that are introduced?

Physical constraint on the masses of the "heavy" quarks appear from the asymptotic freedom of the color SU(3) group. Experimental indications favor free parton like picture of deep inelastic collisions. It is conjectured (through not experimentally verified) that the strong interaction is asymptopically free - i.e., the coupling constrant decrease with increase of energy.

In order for this feature of the theory to be true it is required that the number of flavors not exceed 16. In the process of flavor unification several new fermions are introduced. However in order for the color subgroup to be asymptotically free we may follow three alternative paths:

(i) Introduce less than 16 flavors. An example is SU(8) theory.

(ii) If more than 16 flavors make the "unwanted" fermions superheavy. In such a scheme these fermions will decouple from the low-energy phenomenology. The color subgroup will remain asymptotically free up to superheavy energies ($\approx 10^{15}$ Gev.). Example of such a theory could be SU(9) theory.

(iii) In case the scenerio (ii) appears (as in SU(9) it is in principle possible to claim that the experimental indications of asymptotic freedom is an epiphenomenon and that the strong interaction gauge coupling may tend to become asymptotically strong as the threshold for these new fermions are reached. The overall gauge group though is still asymptotically free. An example of such a theory is SU(9). It may not require "superheavy" fermions.

There is a fourth possibility which we shall discuss subsequently.

<u>Asymptotic Freedom in SU(N) Gauge Theory</u>

If we restrict our discussions to gauge bosons and fermions only and demand that the overall representation be complex we arrive at the following set of interest

$$SU(5): \quad (5,\ 1) + (5,\ 3) \tag{174}$$

$$SU(7): \quad (7,\ 1) + (7,\ 3) + (7,\ 5) \tag{175}$$

$$SU(8): \quad (8,\ 3) + (8,\ 6) + (8,\ 7) \tag{176}$$

$$SU(9): \quad (9,\ 2) + (9,\ 5) \tag{177}$$

$$\text{or } (9,\ 1) + (9,\ 3) + (9,\ 5) + (9,\ 7) \tag{178}$$

$$SU(10): \quad (10,\ 3) + (10,\ 6) \tag{179}$$

If however we demand that the "light" fermions belong to the simplest (ie, the vector) repn of the horizontal group, then SU(10) is ruled out. The number of "light" generations seem bounded by four.

Symmetry Breaking:

Since SU(5) is the minimal group encompassing the weak, electromagnetic and the strong interactions therefore the breaking of SU(8) must realize SU(5) at some stage. Recently it has been shown by Ruegg et al that it is possible to achieve the above mentioned breaking if we introduce two higg's bosons one in the fundamental and another in the adjoint representation of SU(8). We assume the scale at which SU(8) breaks is higher than 10^{15} Gev. The exact estimate of this scale will be arrived at later. We assume that it breaks into SU(5) (X)$SU(3)_H$. Subsequently, the breaking pattern of SU(5) will follow the scenerio of the usual Georgi-Glashow SU(5). The horizontal symmetry will break completely down at around 10^2 Tev. Detailed analysis of the scales and coupling constants are discussed subsequently.

Alternative Viewpoints - The "Dynamical" Approach: -[19]

Two alternative viewpoints which assume various degrees of dynamical mechanisms exist in the literature.

Approach I: This approach, due to Georgi, preceeded the approach that we have discussed so far. The assumptions run parallel to ours except in the identification of "light" fermions.

For this purpose the generation number, g. is defined. From the eqns. 148. it follows that cancellation of triangle anomalies require

$$n(10) - n(10^*) = n(5^*) - n(5) \tag{180}$$

where $n(x)$ represents the number of times the representations χ of SU(5) occurs in the set $\{N, m\}$ of SU(N).

If there is a "dynamical mechanism" that make $n(10^*)$ combine with n(10) to give heavy masses keeping only $n(10) - n(10^*)$ massless (similarly for the n(5)'s and $n(5^*)$), then, the above eqn. makes for an interesting definition of the generation number g

$$g = n(10) - n(10^*) = n(5^*) - n(5) \tag{181}$$

In such a scheme, the minimum group that has three generations is SU(11) and the set N, m is

$$\{N, m\} = (11, 4) + (11, 8) + (11, 9) + (11, 10) \tag{182}$$

Note that though a generation number has been defined the light fermions remain unidentified. The identification is to come about "dynamically".

The theory is also asymptotically strong and the author takes the view that the unwanted fermions are superheavy (10^{15} Gev.) . in order for color subgroup to retain asymptotic freedom up to 10^{15} GeV.

Approach II:[20] This approach "identifies" the "light" fermions in the same way as Georgi. However, the restriction on repetition of irreducible representation in the set N, m is lifted as follows:

The decomposition of $\{N, m\}$ into its irreducible components is written as:

$$\{N, m\} = \sum_{\{m\}} C_m \, (N, m) \qquad (183)$$

Repeatition of (N, m) is allowed such that C_ms do not all have a common factor. Model with three generations is obtained in SU(7). The representation is:

$$\{N, m\} = 2\,(7, 2) + (7, 3) + 8\,(7, 6) \qquad (184)$$

Physical Consequences of Flavor Unification

There are several observable consequencies of flavor unification. In order to be specific, let us concentrate on the two models that we have used extensively. These are the $SU(2)_L(X)SU(2)_H(X)U(1)$ model and the SU(8) flavor - grand unified model.

The first obvious signature of flavor unification is the possible existence of the flavor-changing neutral processes. We have shown in the context of $SU(2)_L(X)$ $SU(2)_H(X)U(1)$ that such processes could exist with appreciable rate between the top and the up family without running afoul of the present experimental bounds on the first two families. Such a scenerio may be further confirmed by decay modes of the bottom quark, the B_o-$\bar{B}_o$ mixing and the multilepton events that follow the production of upsilon. These we have already discussed.

In the context of flavor, grand unified theory namely SU(8) theory the physical signatures are as follows. These signatures are given in decreasing order of importance

(1) The fourth generation has V + A weak interactions. This happens irrespective of the presence or the absence of the top quark.

The model with the top quark has already been presented. It is easy to observe that fourth generation onward. The structure of weak interactions become V + A.

The model without the top quark is obtained by reversing our argument about identification of the light fermions. If the objects that transform as vector of the horizontal SU(3) is identified as "heavy" (instead of "light" as before) then we obtain a model without the top quark. However, even in this scenario the fourth generation has V + A interactions.

We note parenthetically that V - A interaction of the fourth generation require going to SU(9) theory that we have discussed.

(2) <u>Neutrino Oscillations and Neutrino Decays</u>:
Let's display the SU(8) representation and its SU(5) decomposition for the above mentioned purpose.

$$(8,\ 3)_L = 1 + 3^*(5) + 3(10) + 10^* \tag{185}$$

$$(8,\ 6)_L = 3(1) + 3^*(5^*) + 10^* \tag{186}$$

$$(8,\ 7)_L = 3(1) + 5^* \tag{187}$$

Note the presence of extra "neutrinoes" that are singlets of SU(5). The higg's boson's that we have introduced mixes these SU(5) singlets with the neutrinoes that are contained in $3^*(5^*)$ or $3^*(5)$ and 5^* representations.

This therefore can lead to the phenomenon of neutrino oscillation. Further such mixing can give rise to the neutrino decay[22]

$$\nu_1 \rightarrow \nu_2 + \gamma \tag{188}$$

by the diagram shown in figure 4.

Radiative Effects:

The renormalisation of θ_W: -[23]

To be specific we shall confine ourselves to the SU(8) gauge theory. In order to understand the effect on the renormalized value of θ_W calculate θ_W in the SU(8) model.

The angle θ_W in our theory is given as:

$$\sin^2\theta_W = \frac{\sum_i T_{3Li}^2}{\sum_i Q_i^2} \tag{189}$$

where i goes over all fermions in an irreducible representation. Using this we get,

$\sin^2\theta_W = 3/8$ in our example

We shall like to emphasize the pattern of symmetry breaking at this point. We have assumed that the group SU(8) breaks down as follows:

$$\begin{array}{l} SU(8) \\ \big\downarrow \; \langle\phi_\alpha\rangle, \langle\phi^\alpha_\beta\rangle \text{ at } >10^{15}\,\text{Gev.} \\ SU(5)\times SU(3)^H \times U(1) \\ \big\downarrow \; 10^{15}\,\text{Gev.} \\ SU(3)^C \otimes SU(2) \otimes U(1) \otimes SU(3)^H \otimes U(1) \end{array}$$

$$
\begin{array}{c}
SU(3)^c \otimes SU(2) \otimes U(1) \otimes SU(3)^H \otimes U(1) \\
\Big\downarrow \; 10^2 \text{ Tev.} \\
SU(3)^c \otimes SU(2) \times U(1) \\
\Big\downarrow \; 300 \text{ Gev.} \\
SU(3)^c \otimes U(1)^{em.}
\end{array}
$$

It is important to note two points at this stage:

(i) That SU(8) breaks into SU(5)(X)SU(3)(X)U(1) at a scale above 10^{15} Gev.

(ii) The breakdown of SU(5) follows the conventional pattern. The intermediate scale that we have introduced (10^2 Tev) breaks the horizontal SU(3) group. Therefore, the renormalization of the SU(2) and U(1) coupling constants are unaffected by introduction of this scale.

To proceed further, let us use the "decoupling theorem" e la Appelquist, Carrazone. It says that when "light" particles are in the external lines and their momenta are small, then any superheavy particle may be omitted from internal lines provided their effect has been taken account of in the renormalization of coupling constant.

With the above provision we may now write down the renormalization group equations for the subgroups. We, of course, assume that the low energy subgroup is $SU(3)^C(X)SU(2)(X)U(1)$. Let's denote the coupling constants by g_1, g_2, g_3 respectively.

Thus,

$$\frac{dg_1}{dt} = \beta(g_1) \tag{190}$$

$$\frac{dg_2}{dt} = \beta(g_2) \tag{191}$$

$$\frac{dg_3}{dt} = \beta(g_3) \tag{192}$$

where, $t = \ln u$.

Using the for $SU(3)^c$, $SU(2)$ and $U(1)$ we get

(193)

$$\frac{dg_1}{dt} = -\frac{g_1^3}{16\pi^2}\left[11 - \frac{2}{3}\sum_m n(m)\binom{3-2}{m-1}\right] = -\frac{g_1^3}{16\pi^2}\cdot X_1 \tag{194}$$

$$\frac{dg_2}{dt} = -\frac{g_2^3}{16\pi^2}\left[\frac{22}{3} - \frac{4}{3}n_f\right] = -\frac{g_2^3}{16\pi^2}\cdot X_2 \tag{195}$$

$$\frac{dg_3}{dt} = \frac{g_3^3}{16\pi^2}\cdot\frac{4}{3}n_f = \frac{g_3^3}{16\pi^2}\cdot X_3 \tag{196}$$

The relationship between the unifying coupling constant (ie., the SU(5) coupling constant), with g_1, g_2 and g_3 can be obtained by integrating the above equations from g down to $g_i(\mu)$, where u is the low energy scale

$$\int_{g_i(\mu)}^{g_0} \frac{dg_i}{g_i^3} = -\frac{1}{16\pi^2}\ln\frac{M}{\mu}\cdot X_i \tag{197}$$

Integrating, we get

$$\frac{1}{g_{i0}^2} - \frac{1}{g_i(\mu)^2} = \frac{1}{8\pi^2} \cdot X_i \ln \frac{M}{\mu} \tag{198}$$

Thus,

$$\frac{1}{g_i(\mu)^2} = \frac{1}{g_{i0}^2} - \frac{X_i}{8\pi^2} \cdot \ln \frac{M}{\mu} \tag{199}$$

In our case

$$X_1 = \frac{11}{3}, \quad X_2 = 0, \quad X_3 = \frac{22}{3}$$

We note that

$$\frac{1}{e^2} = \frac{1}{g_2^2} + \frac{5}{3}\frac{1}{g_1^2} \tag{200}$$

Therefore

$$\sin^2\theta_W = \frac{e^2(\mu)}{g_3^2(\mu)} = \sin^2\theta_0 \left[1 - \frac{e^2(\mu)}{8\pi^2} \cdot \frac{22}{3} \cdot \cot^2\theta_0 \cdot \ln \frac{M}{\mu}\right] \tag{201}$$

This yields the value of 0.21 at low energies.

The value of the coupling constant at 10^{15} Gev. is obtained by using the eqns

$$\sin^2\theta_W = \frac{e^2(\mu)}{g_2^2(\mu)} \tag{202}$$

and using the renormalization group equation for $g_2(\mu)$ get the value at $g_2^2(10^{15}\,\text{Gev})$. Using $\sin^2\theta_W = .22$ and $\frac{e^2}{4\pi} = \frac{1}{137}$ at low energies we get

$$g_2^2(\mu) = \frac{e^2(\mu)}{\sin^2\theta_W} \tag{203}$$

$$= \frac{4\pi}{137} \cdot \frac{1}{0.22} \tag{204}$$

$$g_2^2(\mu) = .43 \tag{205}$$

Using the renormalization group eqn for g_2 we get the value of SU(5) coupling constants g_5 at 10^{15} Gev is $g_5^2 \approx .43$.

<u>The Trajectory of g_4^2 (μ): Horizontal Coupling Constants</u>:

In order to obtain the actual trajectory of g_4 (μ) we shall make the following observations:

(i) If finite renormalization effects are disregarded, the trajectory of g_4^2 (μ) will be as if the horizontal symmetry is unbroken.

(ii) The effective strength of horizontal interactions is bounded by $G_F \alpha^2$ at low energies.

The second observation gives:

$$\frac{g_2^2 \alpha^2}{8 M_W^2} \approx \frac{g_4^2}{8 M_H^2} \tag{206}$$

$$g_4^2 \approx \frac{g_2^2 . \alpha^2 . M_H^2}{M_W^2} \tag{207}$$

Assuming $M_H \approx 10$ Tev., $M_W \approx 100$ Gev., $\alpha \approx 10^{-2}$, $g_2^2 \approx 4$, we get

$$g_4^2 \approx .4 \tag{208}$$

at low energies.

Disregarding finite renormalization, have the following renormalization group eqn for g_4.

$$\frac{dg_4}{dt} = -\frac{g_4^3}{48\pi^2}\left[33 - 30\right] \tag{209}$$

$$\frac{dg_4}{dt} = -\frac{g_4^3}{16\pi^2} \tag{210}$$

Integrating this eqn with g_4^2 0.4 at low energies gives the trajectory of the horizontal coupling constant.

<u>Unification Scale</u>:

Next we like to find the scale at which g_4 given by eqn 210 "merge" with the SU(5) gauge group which is good at 10^{15} Gev.

The trajectory of g_4 is fixed by the above equation along with the low energy value of g_4=00.4 at ordinary energies. In order to estimate the scale at which this merges with the SU(5) coupling constant let's investigate the scale behaviour of SU(5) coupling constant, g_5.

The scale variation of g_5 is given by the eqn.

$$\frac{dg_5}{dt} = -\frac{g_5^3}{48\pi^2}\left[55 - 2\sum n(m)\binom{5-2}{m-1}\right] \tag{211}$$

$$\frac{dg_5}{dt} = -\frac{g_5^3}{48\pi^2}\cdot\left[55 - 42\right] \tag{212}$$

$$= -\frac{g_5^3}{48\pi^2}\,13 \tag{213}$$

$$\frac{dg_5}{dt} = -\frac{13 g_5^3}{48\pi^2} \tag{214}$$

The value of g_5 at 10^{15} Gev. has been obtained earlier. It is $g_5^2 \approx .4$. Integrating the renormalization group eqn for SU(5) gauge group we get

$$\frac{1}{g_5^2(\mu)} = \frac{1}{g_0^2} - \frac{1}{8\pi^2}\cdot 4.\ln\frac{M'}{\mu} \tag{215}$$

$$\frac{1}{g_{5,0}^2} = \frac{1}{g_5^2(\mu)} + \frac{1}{8\pi^2} 4.\ln\frac{M'}{\mu} \tag{216}$$

The horizontal gauge coupling constant g_4 eqn can be similarly integrated. We get that

$$\frac{1}{g_{4,0}^2} = \frac{1}{g_4^2(\mu')} + \frac{1}{8\pi^2}\cdot\ln\frac{M'}{\mu}, \tag{217}$$

Equating the above two equations we get

$$\frac{1}{g_5^2(\mu')} + \frac{1}{8\pi^2}\cdot 4\ln\frac{M'}{\mu'} = \frac{1}{g_4^2(\mu)} + \frac{1}{8\pi^2}\cdot\ln\frac{M'}{\mu} \tag{218}$$

for $\mu' = 10^{15}\,\mathrm{Gev},\ g_5^2 = 0.4$ and for $\mu \approx 1\,\mathrm{Gev},\ g_4^2 \approx .4$

Thus, we get,

$$\frac{1}{0.4} + \frac{1}{8\pi^2}\cdot 4.\ln\frac{M'}{10^{15}\,\mathrm{Gev}} = \frac{1}{0.4} + \frac{1}{8\pi^2}\cdot\ln M' \tag{219}$$

which gives $M' \simeq 10^{20}$ Gev. Thus, the unification of horizontal interaction with SU(5) gauge interaction occurs at 10^{20} Gev.

We note parenthetically that since 10^{19} is the planck mass, therefore, the flavor unification appears to take place almost at the gravitational scale.

CHAPTER IV

Fermion Irreducibility:

A logical extension of what we have discussed so far is to put all the fermions in one irreducible representation.

At the level of SU(8) or SU(9), we have noticed that the representation set of interest is such as to contain no repeatitions. However, the fermionic set is still reducible.

There are at least two ways to get irreducible fermionic content. These are in SU(10) or SO(15). These are the minimal groups that allow for fermion irreducibility.

SU(10) and Fermion Irreducibility:

The irreducible representation of interest is (10, 5). The decomposition of (10, 5) into its SU(5) content is as follows:

$$(10,\ 5) = 1(1) + 5^{*}\ (5) + 10^{*}(10) + 10(10^{*}) + 5(5^{*}) + 1(1) \tag{220}$$

Note that the representation is real and therefore contain no triangle anomalies.

Embedding of SU(8) in SU(10): - This group has 99 gauge bosons. We now want to show that the representation (10, 5) embeds the representations (8, 3) (+) (8, 6) (+) (8, 7). In order to do that let's decompose (10, 5) according to the following breaking of SU(10) into SU(5) of Georgi and Glashow.

$$\begin{array}{c} SU(10) \\ \Big\downarrow \; > 10^{15}\text{Gev} \\ SU(8) \times SU(2) \times U(1) \end{array}$$

$$SU(8) \times SU(2) \times U(1)$$

$$\Big\downarrow \; > 10^{15} \text{ Gev.}$$

$$SU(5) \otimes SU(3) \otimes SU(2) \otimes U(1)$$

We now display the $SU(5) \otimes SU(3) \otimes SU(2)$ content of $(10, 5)$.

The decomposition of 10, 5 according to the above subroup is as follows:

$$\begin{aligned}(10, 5) = {} & 1(1) + (5, 3^*, 1) + (5, 1, 2^*) \\ & + (10, 3^*, 2) + (10, 3, 1) \\ & + (10, 1, 1) + (10^*, 3, 2) \\ & + (10, 3^*, 1) + (10, 1, 1) + (5^*, 3, 1) \\ & + (5^*, 1, 2) + 1(1)\end{aligned} \tag{221}$$

Note that this contains 3 "light" generations as in SU(8).

There are two remarks to be made about SU(10) gauge theory. The first is that the gauge coupling constant with the representation 10, 5 is asymptotically free. The second point is that the repn is real. This may be problematic. However, we believe that the reality of the representation does not preclude it from being physically interesting.

SO(15): This group which has 105 gauge fields also provides an irreducible basis for the fermions. The representation of interest is the spinorial representation of SO(15). This has a dimension of $2^7=128$.

Special Orthogonal Groups:[24, 25]

It is generally easier to work with special unitary groups. Therefore, it is interesting to work with special orthogonal groups in the basis of special unitary groups. Such a basis was set up by Mohapatra and Sakita.

This is easily constructed. If a_i $(i=1, \ldots N)$ and its hermitean conjugate satisfy

$$\{a_i, a_j^\dagger\} = \delta_{ij} \qquad (222)$$
$$\{a_i, a_j\} = 0$$

then, the generators, X^i_j, of U(N) group, which satisfy:

$$[X^i_j, X^k_l] = \delta^k_l X^i_l - \delta^i_l X^k_j \qquad (223)$$

are constructed of a's as

$$X^i_j = a^\dagger_i a_j \qquad (224)$$

The O(2N) basis is obtained by defining $\Gamma_\mu \left(\mu = 1, \ldots 2N\right)$,

$$\Gamma_{2j-1} = -i\left(X_j - X_j^{+}\right) \tag{225}$$

$$\Gamma_{2j} = \left(X_j + X_j^{+}\right) \tag{226}$$

Thus,

$$\left\{\Gamma_\mu , \Gamma_\nu\right\} = 2\,\delta_{\mu\nu} \tag{227}$$

The generators are obtained as

$$\Sigma_{\mu\nu} = \frac{1}{2i}\left[\Gamma_\mu , \Gamma_\nu\right] \tag{228}$$

For SO(2N+1) the element is to be included in the above algebra, where Γ_0 is defined as

$$\Gamma_0 = i^N \Gamma_1 \ldots \Gamma_{2N} \tag{229}$$

The spinorial representation of SO(2N+1) has the dimension 2^N. A specific example of how this spinorial representation embeds SU(5) representations will be shown for SO(15).

Yukawa Couplings:

The Yukawa couplings are given as

$$\tilde{\psi} B C^{-1} \Gamma_\mu \psi \phi_\mu , \tilde{\psi} B C^{-1} \Gamma_\mu \Gamma_\nu \Gamma_\alpha \psi \phi_{\mu\nu\alpha} \cdots$$

where B satisfies

$$B^{-1} \tilde{\Gamma}_\mu B = + \Gamma_\mu \tag{230}$$

The solution of the B in terms of Γ_μ is given as

$$B = \prod_{\mu = \text{odd}} \Gamma_\mu \qquad \text{for } N = \text{even} \tag{231}$$

$$B = \prod_{\mu = \text{even}} \Gamma_\mu \qquad \text{for } N = \text{odd} \tag{232}$$

Embedding SU(5) in SO(15)

The SO(15) spinorial basis has the dimension 2^7. It is constructed as follows:

SO(15) Spinor Basis	Dimension	SU(5) Decomposition
$\|0\rangle$	1	$\underline{1}$
$a_i^+ \|0\rangle$	7	$\underline{5} + 2(\underline{1})$
$\frac{1}{2!} a_i^+ a_j^+ \|0\rangle$	21	$\underline{10} + 2(\underline{5}) + \underline{1}$
$\frac{1}{3!} a_i^+ a_j^+ a_k^+ \|0\rangle$	35	$\underline{10}^* + 2(\underline{10}) + \underline{5}$
$\frac{1}{4!} a_i^+ a_j^+ a_k^+ a_l^+ \|0\rangle$	35	$2(\underline{10}^*) + (\underline{5}^*) + \underline{10}$
$\frac{1}{5!} a_i^+ a_j^+ a_k^+ a_l^+ a_m^+ \|0\rangle$	21	$\underline{10}^* + 2(\underline{5}^*) + \underline{1}$
$\frac{1}{6!} a_i^+ a_j^+ a_k^+ a_l^+ a_m^+ a_n^+ \|0\rangle$	7	$\underline{5}^* + 2(\underline{1})$
$\frac{1}{7!} a_i^+ a_j^+ a_k^+ a_l^+ a_m^+ a_n^+ a_p^+ \|0\rangle$	1	$\underline{1}$

Note that on this basis, we have $4(5^*)$'s and $4(10)$'s in all. Thus, this representation contains four "light" generations. It is interesting to note that the SO groups have even number of families only. Further, we find that this repn of SU(15) gives asymptotically free gauge coupling.

CONCLUSIONS

The present standard theories - namely (i)$SU(2)_L(X)U(1)$ theory of the weak and electromagnetic fields and (ii) the SU(5) grand unified theory - are both incomplete. This arises because of repetitions of identical representations for the fermions.

It has been suggested by several authors that a step towards making the above theories logically complete would involve introducing horizontal gauge symmetry. Physically, these symmetries would give rise to the possibility of flavor-changing neutral processes.

In the framework of the weak and electromagnetic interactions it is known that the flavor changing neutral current do certainly exist. This is exemplified by several systems such as K^o-$\bar{K}^o$, D^o- $\bar{D}^o$ and perhaps B^o - $\bar{B}^o$. However, it is also known that these processes take place at most with strength $G_F\alpha$, if we have only four quarks (ie, u, d, s, and c). This compares with strength G_F for the usual weak and electromagnetic interactions.

It is known now that there are more than four quarks. In particular the fifth quark, b, has been "found". It has also been conjectured that the charge 2/3 counterpart of the b-quark is there.

As these quarks are "discovered" the question naturally arises about flavor changing neutral processes. The standard $SU(2)_L$ (S)U(1) scenerio predicts that as far as these currents are concerned they are suppressed as before.

However, such a conclusion is not inevitable. In fact, we show that there are theories where in spite of suppression of FCNC between u, c and d, and s,the FCNCs between b to d or t to u are not suppressed. These are theories precisely with horizontal gauge symmetries.

In such a model we discuss in detail about the picture of the b-quark. The lifetime of the b-quark is shown to be bounded by $\tau_b > 10^{-14}$ sec.. Because of horizontal interactions the b-decay need not be pure V-A. Further because of neutral current connecting b to d we show that the mixing between B^0 and $\bar{B}^0$ may exceed present expectations.

We deal then with experimental signatures of such a scenario. In particular, we show that if we focus on the multilepton events that follow the production of upsilon, then the ratio σ_4 to σ_3 (defined in the text) is related to the branching ratio for the decay of the -lepton into electron. The relationship is:

$$\frac{G_4}{G_3} = \frac{1}{2} \frac{\Gamma[\tau \to e \ldots]}{\Gamma[\tau \to all]} \tag{233}$$

In another version of such a model we display how CP violating currents arise. The $\Delta S = 2$, CP violating amplitude is hown to be proportional to deviations from universality of the weak interactions and, therefore, suppressed. Coming then, to the question of grand unification we embed the above models in a theory based on SU(7). However, owing to repetitions of the basic fermionic set in SU(7) we find that the theory is still incomplete.

The complete flavor unification arises at the level of SU(8). In such flavor unified schemes the horizontal guage bosons that mediate FCNC are perhaps all suppressed. The strength G_F is used to define the masses of the corresponding guage bosons.

We discuss how to arrive at plausible fermion mass-matrixes in these theories. We also note as an aside that the SU(8) theory is good irrespective of whether a top quark exists or not.

Experimental signature of flavor unification are two fold. First we expect that beyond the third or fourth generations the weak interactions become V + A as against V - A of the "light" fermions. Further these theories quite generally lead to neutrino oscillations and neutrino decay.

Aside from these peculiarities that appear at the tree lagrangian, we discuss some of the radiative effects. The one loop renormalization group equations are used to arrive at the value of the Weinberg angle θ_W at low energies. We note that this value seems insensitive to the number of fermion generations.

We use these renormalization group equations to discuss the scale at which flavor grand unification occur. While the "conventional" SU(5) unification appears at the 10^{15} Gev. scale, the flavor-grand unification appears at 10^{19-20} Gev. We note in passing that at such a scale the gravitational effects perhaps are important.

The story of flavor unification is still not complete. A complete flavor unification requires all fermions to belong to one irreducible multiplet of the gauge group. To that effect, we discover that SU(10) is the minimal group that is capable of achieving the above objective. We show that the representation

(10, 5) embeds the representation of SU(8) in one single irreducible multiplet.

In spite of the fact that SU(10) embeds SU(8) multiplet, the representation (10, 5) is plagued with difficulties. The most important difficulty is the fact that the representation is real and it is difficult to arrive at V - A nature of the weak interaction for the fermion mass eigenstates.

It may be more profitable to embed SU(8) in the SO(15) gauge group. We show how this is achieved. It is interesting to note that SO(15) allows an embedding of left-right symmetry.

REFERENCES

1. R. E. Marshak, Riazuddin and C. P. Ryan, "Theory of Weak Interactions in Particle Physics", John Wiley (1969).

2. J. C. Taylor, "Gauge Theories of Weak Interaction", Cambridge University Press (1975).

3. S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967)
A. Salam, in Elementary Particle Theory: Relativistic Groups and Analyticity (Nobel Symposium No. 8), Ed. N. Svartholm (Almquist and Wiksell, Stockholm, 1968) p. 367.

4. O. W. Greenberg and C. A. Nelson, Phys. Rep. 32C, 71 (1977)

5. H. Georgi, Nuc. Phys. B156, 126 (1979)

6. J. C. Pati and A. Salam, Phys. Rev. D10, 275 (1975)
R. N. Mohapatra and J. C. Pati, Phys. Rev. D11, 566 (1975)
G. Senjanovic and R. N. Mohapatra, Phys. Rev. D12, 1502 (1975)
H. Fritzsch and P. Minkowski, Nucl. Phys. B103, 61 (1976)

7. A. Zee, Phys. Rev. D9, 1772 (1974)

8. R. N. Mohapatra, J. C. Pati and L. Wolfenstein, Phys. Rev. D11, 3319 (1975)

9. S. M. Barr and A. Zee, Phys. Rev. D17, 1854 (1978)

10. F. Wilczek and A. Zee, Phys.Rev. Lett. 42, 421 (1979)
A. Davidson, M. Koca and K. C. Wali, ibid 43, 92 (1979)
C. L. Ong, Phys.Rev. D19, 2738 (1979)
J. Chakrabarti, Phys. Rev. D20, 2411 (1979)

11. See J. Chakrabarti in Ref. 10.

12. C. Quigg and J. Rosner, Berkeley Report No. LBL-7961, (1978)
G. Branco and R. N. Mohapatra, Phys. Rev. D18, 42446 (1978)

13. J. Chakrabarti, Nuovo Cimento, 55, 1(1980)

14. A. de Rujula, H. Georgi and S. L. Glashow, Ann of Phys. (N.Y.) 109, 258 (1977);
F. Wilczek and A. Zee, Phys. Lett. 70B, 436 (1977);
S. Weinberg, Transactions of the New York Academy of Sciences, Vol. 38 (1977);
H. Fritzsch, Phys. Lett. 73B, 317 (1978);
R. N. Mohapatra and G. Senjanovic, Phys. Lett. 73B, 176 (1978);
G. Branco, Phys. Lett. 76B, 605 (1978);
T. Hagiwara, T. Kitazoe, G. Mainland and K. Tanaka, Phys. Lett. 76B, 602 (1978);
M. deCrombruggie, Phys. Lett. 80B, 365 (1979);
H. Harari, H. Haut and J. Weyers, Phys. Lett. 78B, 459 (1978);
G. Segre, A. Weldon, J. Weyers, Phys. Lett. 83B, 351 (1979);
R. Barbieri, R. Gaatto and F.Strocci, Phys. Lett. 74B, 344 (1978);
For a review, see R. Gatto, Invited Talk at the Einstein Symposium at Jerusalem (1979).

15. S. Pakvasa and H. Sugawara, Phys. Lett. 73B, 61 (1978)

16. H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974)

17. J. Chakrabarti, M. Popovic and R. N. Mohapatra, City College, Prepring CCNY-HEP-79/12 (1979). To appear in Phys. Rev. D.

18. S. Okubo, Phys. Rev. D16, 3528 (1979).
J. Banks and H. Georgi, Phys. Rev. D14, 1159 (1976)

19. H. Georgi, See Ref. 5.

20. P. Frampton and S. Nandi, Phys. Rev. Lett. 43, 1460 (1979).

21. F. Buccella, H. Ruegg and C. A. Savoy, University of Geneve preprint UGVA-DPT 1979/11-221.

22. A. de Rujula, Proceedings of the First Workshop on Grand Unification. (University of New Hampshire, 1980). To be published

23. H. Georgi, H. Quinn and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974).

24. H. Fritzsch and P. Minkowski, Ann of Phys. (N.Y.) 93 192 (1975)

25. M. Chanowitz, J. Ellis and M. K. Gaillard, Nuc. Phys. B129, 506 (1977)
N. P. Chang and J. P. Mercader, Phys. Rev. D18, 4721 (1978)
R. N. Mohapatra and B. Sakita, Phys. Rev. D21, 1062 (1980)

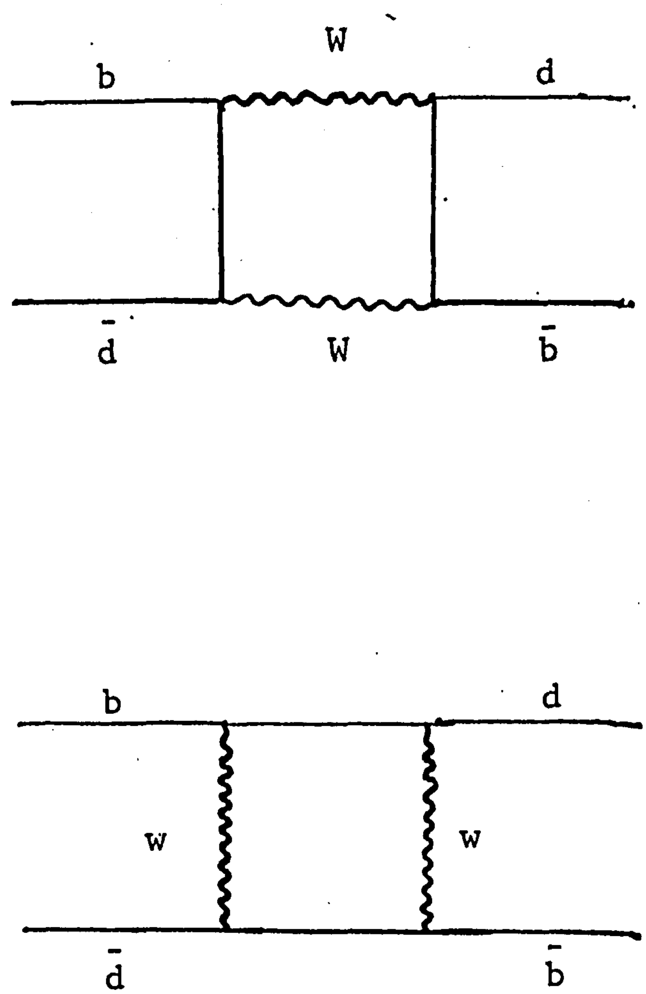


Figure 1

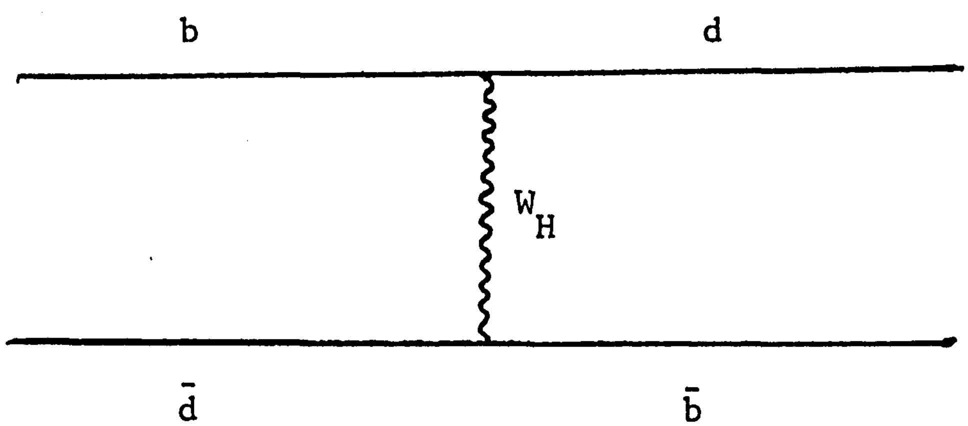


Figure 2

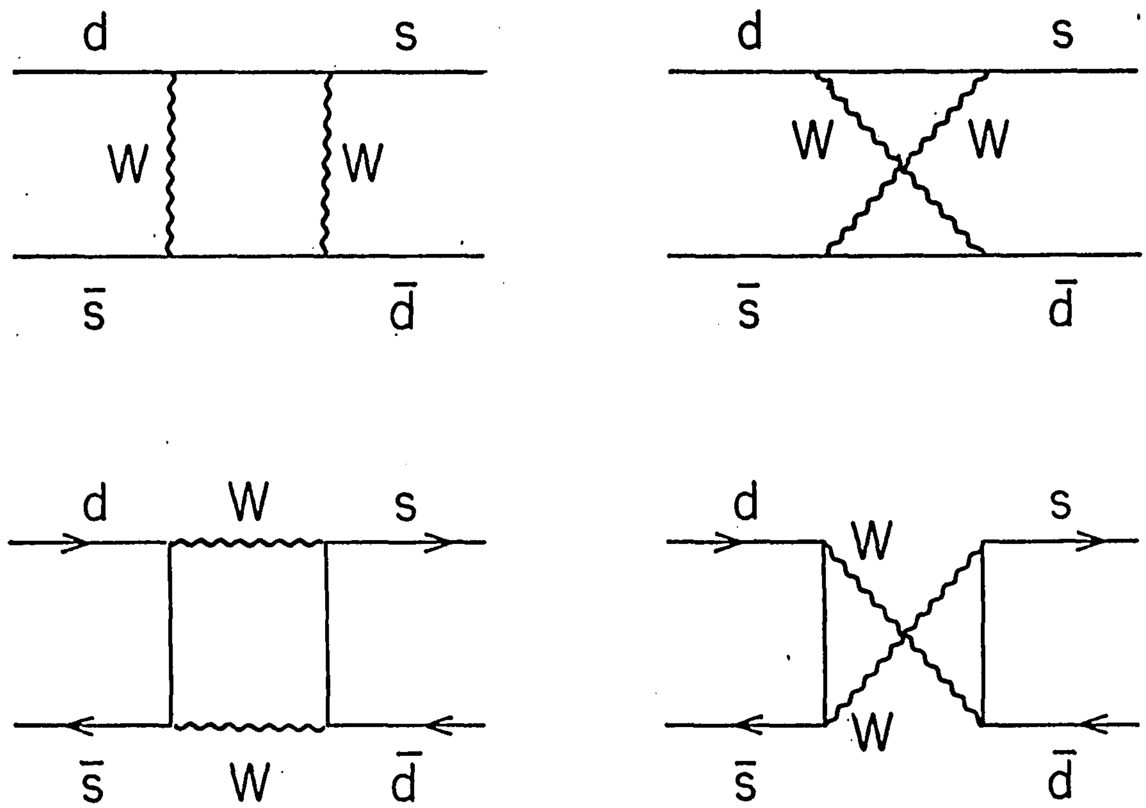


Figure 3

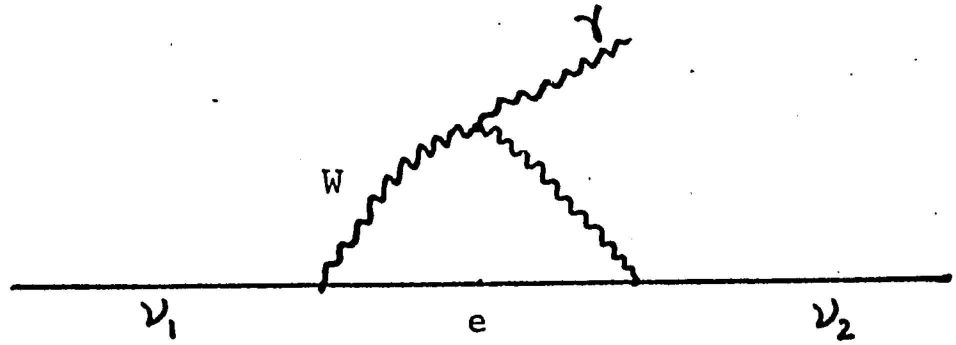


Figure 4