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WEIGHTING OF VARIABLES IN MARKET SEGMENTATION

by

XAVIER JOSEPH NAMPIAPARAMPIL

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August 17, 1972
date

Edward N. Wolf
Chairman of Examining Committee

August 19, 1976
date

Executive Officer

Prof. Edward Wolf, Chairman

Prof. Ronald Gatty

Prof. Mathew Goldstein

Mr. Charles Jacobson

Supervisory Committee

The City University of New York

Abstract

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by

Xavier Joseph Nampiaparampil

Adviser: Professor Edward Wolf

The bi-multivariate relationship between buying behavior and personal characteristics is central to the whole concept of market segmentation. Weighting schemes were proposed in the present study to incorporate this relationship explicitly in the analysis. And it was hypothesized that segmentation procedures where the predictor variables, namely, the personal characteristics, were weighted relative to the criterion set, namely, the buying behavior, would do a better job of segmenting a market than procedures where the variables were not differentially weighted.

Three types of variables were considered in the study. These were (1) canonical variates, (2) orthogonal factors, and (3) original variables. Each type of variable was weighted three different ways: (1) variables were all given an equal weight of unity, (2) variables were weighted based on correlations with criterion set variables, and (3) variables were weighted based on the concept of

redundancy for canonical variates and factor scores, and weighted based on correlations with orthogonal factors of the criterion set for the original variables.

The study design involved simulation of segmentation with generated data, and a real application of segmentation to survey data. Simulation was done using an analysis sample and a validation sample. Each sample consisted of observations from two different populations. Six types of data were used for the simulation: multivariate normal distributions with little and large separation between populations, multivariate binomial distributions with little, moderate and large separation between populations, and multivariate multinomial distributions with moderate separation. Cluster analysis was performed on the analysis sample several times. Each time a different set of weights for predictor set variables or their linear combinations was used. Cluster analysis was performed on the validation sample too, but using the criterion set variables. Observations in the validation sample were then classified several times using discriminant functions derived from the cluster solutions obtained for the analysis sample. After 100 trials for each data condition, results were examined for segmentation effectiveness. Measures of effectiveness used were: (1) total sum of squared error variance for criterion set variables of the analysis sample, (2) number of misclassifications in the validation sample based on membership as determined by criterion set variables, and (3) total sum of squared error variance for criterion set variables of the validation sample.

Results showed that weighting increased segmentation effectiveness for the three variate classes when the data were from populations with no large separation, and when the data were not highly skewed. Usefulness of weighting decreased with separation between populations for all the variate classes. With skewness, usefulness decreased very greatly for the factors, substiantially for the original variables, and only moderately for the canonical variates.

Application of weighting schemes to survey data involved factors and canonical variates. Segmentation was done a number of times using as bases different sets of personal characteristics. Results showed that conclusions drawn from experimentation with generated data held good for survey data too. Thus, for the survey data which was characterized by extreme skewness, weighting was found useful in the case of canonical variates but not in the case of factors. And canonical variates weighted relative to the criterion set produced the best solution in all cases. Measure of segmentation efficiency, derived from consideration of squared error variance of the criterion set, was found useful in evaluating alternative bases of segmentation. Transformations to reduce skewness of distributions increased the efficiency of procedures in segmentation.

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CHAPTER I

INTRODUCTION

In the past and still at present, there have been developed several different statistical procedures to segment markets. Though these procedures are widely used for segmenting markets, it seems that proper attention is not often given to the selection and weighting of variables used in the analysis. One of the common objectives in segmentation is to explain buying behavior of consumers through personal characteristics. Selection and weighting of personal characteristics should therefore be made based on their relative importance in explaining buying behavior. As buying behavior is a complex of many different factors, the procedure used for weighting the personal characteristics should be such that it can account for the multidimensional nature of buying behavior.

The concept of redundancy developed by Stewart and Love (1968) and Miller (1969) in the field of Psychology is of interest here. It is a concept developed to account for relationship between bi-multivariate measurement sets. The focus of the work of Stewart and Love, and Miller was on summary description of data. In the present study, it is proposed that the concept be used for prediction of criterion.

Specifically, it is suggested that the concept be utilized for weighting the personal characteristics of questionnaire respondents such that the weights thus given reflect the importance of the different characteristics in explaining the complex phenomenon of buying behavior. As things stand, the interrelationships between buying behavior and personal characteristics are not usually treated appropriately in market segmentation. Thus, in segmentation studies using factor analysis, cluster analysis, or multidimensional scaling techniques, there is no traditional way to assign proper weights to personal characteristics as buying behavior is not explicitly considered in the analysis along with personal characteristics. And in studies where buying behavior is explicitly considered, the multidimensional nature of buying behavior is not accounted for, as in studies using regression analysis, AID, MCA, and two-group discriminant analysis. Or the linear combinations of the personal characteristics are not properly weighted to reflect their importance in explaining buying behavior, as in studies using multiple-group discriminant analysis and canonical correlation analysis. The application of the concept of redundancy in segmentation will make it possible for the analyst to consider buying behavior explicitly in all studies, and also to assign weights to the personal characteristics such that the weights reflect the importance of the different characteristics in explaining buying behavior. Thus, it is hoped that the present study will contribute to improvement in current market segmentation practices. In this introductory chapter, it seems proper to define market segmentation and to discuss the role of statistical procedures in segmenting markets. Accordingly, these topics will be discussed here.

Definition of Market Segmentation

To a marketer, the market is all the actual or potential consumers for a product or service. Theoretically, to the extent that individual needs and desires are unique, each consumer might be treated as a separate market and could be selected as a target to be reached with a distinct marketing mix - the four P's of product, place, promotion and price. This might conceptually be the case when the size of the total market is small. But when the size of the market increases, it becomes necessary to group consumers because of (1) diseconomies of scale present in tailoring of specialized programs, (2) information constraints like lack of information about the response characteristics of consumers reached by promotional media, (3) institutional constraints on the flexibility of using information about response characteristics of consumers, and (4) cost of research required to implement the individualized programs. To what level can aggregation of consumers be carried forward and still satisfy the needs and desires of individual consumers with the limited resources the marketer commands? This is the problem of market segmentation.

The concept of market segmentation thus involves grouping of consumers. Groups formed should be of sufficient size to warrant separate marketing approaches. And grouping should meet two important criteria:

1. Members in a group should have similar buying behavior, that is, the members should be homogeneous with respect to their elasticities for the various marketing mix variables. This is the primary requirement of market segmentation. If this is not met, it would not be possible to create a distinct mix to satisfy the needs of members of a group. Grouping here is dictated by diseconomy of scale, and the difficulty in estimating response functions at the disaggregate level.

2. Groups formed should be such that they account for information and institutional constraints making it possible for the marketer to focus marketing effort on specified groups. In other words, grouping is to be done based on consumer characteristics which are related in some way to the marketing mix. Marketing mix is nothing but a set of marketing tool variables and the marketing tool variables are to some degree under the control of the marketer. There are many consumer characteristics like demographics, benefits, etc., which are related to marketing tool variables.

Criterion 1 is concerned with grouping of consumers based on buying behavior which is not under the control of the marketer, whereas criterion 2 is concerned with grouping of consumers based on characteristics which are related to marketing tool variables under the control of the marketer.[1] Are there consumer characteristics which are related to buying behavior just as there are characteristics which are related to marketing tool variables? In other words, is there any relationship between personal characteristics and buying behavior? If the answer is in the affirmative, segmentation studies may be directed towards finding those characteristics which are related to both buying behavior and marketing tool variables. Theories of consumer behavior developed by Howard and Sheth (1969) and Nicosia (1966) suggest that (1) relationships exist between a consumer's personal characteristics and his/her purchase decision process, and (2) individuals with similar purchase decision process are likely to exhibit overall similarity in buying behavior. Consequently, as Lessig and Tollefson (1971) point out "... one would expect a relationship between personal characteristics and buying behavior type" (p. 480). Thus, segmentation study may be aimed at finding personal characteristics which are

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[1]It is possible to modify the response functions of consumers. But it is quite difficult and only rarely does marketing effort focus primarily on modifying response functions.

related to both marketing tool variables and buying behavior variables.

There are many personal characteristics which are related to marketing tool variables, as noted before. Information is to be gathered on those characteristics which are related to marketing tool variables and which the marketer thinks are also related to buying behavior. These personal characteristics, chosen for their relationship to marketing tool variables, are to be related to actual buying behavior to make sure that the marketer is not led astray by his/her own hunches. Hence, information is to be gathered on relevant buying behavior characteristics, too. To gather accurate information on buying behavior is to obtain all the information needed for estimating the functional form and parameter values of marketing response functions for individual consumers of a market. Such information would be quite expensive and difficult to collect. The alternative is to choose surrogates of buying behavior which are related to the response functions of the consumers. The commonly used surrogates are total consumption, private brand proneness, average price paid, innovativeness, store patronage, deal proneness, attitudes, and purchase intentions. Information on buying behavior and personal characteristics is usually obtained from a full-scale questionnaire survey. Before launching the survey, a pilot study may be necessary to decide on the proper buying behavior and personal characteristics measures which are to be used in the survey.

Once market data are available, the marketer may begin segmentation analysis. Since only those personal characteristics believed to be related to marketing tool variables are included in the survey, segmentation study may now be confined to finding the relationship between these characteristics and buying behavior. If the marketer can find characteristics which are related to buying behavior, grouping based on these characteristics will accomplish market segmentation satisfying criteria 1 and 2. The different statistical procedures are the means the marketer employs to accomplish his/her objectives in this important phase of market segmentation.

Role of Statistical Procedures in Market Segmentation

Regression Analysis, AID, MCA, Discriminant Analysis, Canonical Correlation Analysis, Factor Analysis, Cluster Analysis, and Multidimensional Scaling Techniques are the major statistical procedures used in market segmentation. The role of all these procedures is forming groups of consumers satisfying the two criteria described earlier. To study how this role is performed by the different statistical procedures, it would be useful to examine them in the light of one of the classification schemes suggested by E.J. Wilson (1974), namely, criterion-related techniques versus descriptive techniques. Regression analysis, AID, MCA, discriminant analysis and canonical correlation analysis are criterion-related techniques. Analysis using these procedures yields results useful in determining predictor set variables which are in some way closely related to the criterion set. Factor analysis and cluster analysis are descriptive. Analysis using these procedures yields results which provide an interpretive description of the subject matter. In the case of multi-dimensional scaling procedures, some are criterion-related while the others are descriptive. Thus, MDPREF which locates subjects in relation to their preferences for different stimuli is criterion-related whereas MDSCAL which portrays the stimuli in multidimensional space to reflect the proximity contained in the similarity/dissimilarity matrix is descriptive. That is for techniques *per se*. In the context of market segmentation, the above classification needs modification in the case of multidimensional scaling techniques. This is so because multidimensional scaling techniques are not directly used for segmenting markets. What multidimensional scaling techniques accomplishes for the marketer is to locate stimuli and/or subjects in a multidimensional metric space of lowest possible dimensionality to account for the subject's perceptions and/or preferences. To segment a market, the researcher has to apply some other statistical procedures. Thus, from the marketer's point of view, multidimensional scaling summarizes data in a quite useful form. In other words, multidimensional scaling techniques are descriptive in the context of market segmentation. The role of statistical procedures may now be discussed under the above classification scheme.

Criterion-Related Procedures

When criterion-related procedures are applied in market segmentation, the criterion set is represented by buying behavior and the predictor set by personal characteristics. All the criterion-related techniques help in finding personal characteristics which are correlates of buying behavior.

Regression analysis, AID, and MCA have only one variable in the criterion set. This variable should be at least interval-scaled or a dichotomy. Hence buying behavior is to be represented by a unidimensional attribute in analyses using these procedures. In regression analysis, all interactions and nonlinearities in the predictor set are to be specified a priori along with other candidate set of variables. Use of dummy variables, of course, obviates the necessity of specifying nonlinearities a priori. And this is what MCA does, which is equivalent to regression analysis using dummy variables. Here only the interactions are to be specified a priori. In AID, one need not concern oneself with a priori specification of interactions or nonlinearities. The multiple classification of variables will take care of nonlinearities. As for interactions, the AID analysis is expected to detect them and account for them.

Analysis using regression analysis and MCA provides coefficients for personal characteristics to predict buying behavior and also to indicate their importance in prediction. The analyst may, therefore, form groups of consumers of similar buying behavior by cross classifying the consumers against the characteristics found significant in the analysis. AID analysis directly provides groups of consumers with similar buying behavior. However, since all the splits except the first one are based on a subsample of observations, the results may not be quite reliable. Hence, it would be preferable to make sequential use of regression analysis or MCA using the relevant personal characteristics and the interactions detected in the AID analysis. The results of regression analysis and MCA may then be used for forming market segments as described before.

Discriminant analysis may be viewed as a technique with a nominal-scale dependant variable. Depending on whether the dependant variable is a dichotomy or a multichotomy, buying behavior may be represented by a unidimensional or multidimensional measure. Nonlinearities and interactions are to be specified a priori as in the case of regression analysis. Market segmentation using discriminant analysis is carried out as follows. First, groups of consumers with similar buying behavior are formed. Groups may be formed subjectively by forming an a priori classification system or by using a statistical procedure like cluster analysis. Discriminant analysis is then applied. The values of the different discriminant criterion show the importance of the corresponding discriminant functions and the coefficients of the personal characteristics in the discriminant functions, the importance of the characteristics.

In canonical correlation analysis, the criterion set consists of two or more variables. Thus it is possible to represent buying behavior in canonical correlation analysis by a multidimensional measure. For segmenting a market, the canonical variates are first extracted. Canonical scores of the predictor set are then used to form groups of consumers with similar buying behavior. Grouping may be done either by clustering the subjects based on their canonical variate scores, or by cross classifying them against these scores.

Descriptive Procedures

As was noted before, there is no criterion set in these procedures. These procedures are performing only a summarizing function. It is left to the marketer to see that these summarizations are meaningful from the market segmentation aspect. The marketer does this by the proper selection of variables, that is, by choosing personal characteristics which are related to buying behavior.

Factor analysis, both R and Q type, begins with personal characteristics data. In R factor analysis, components are extracted from a variable by variable correlation matrix. For each component, variables which have high loadings on it are assigned to a group. Now, for the different groupings of the variables, simple sum scores are computed for each individual. These scores are used to classify consumers in to different groups. In Q factor analysis, segmentation is more direct. Components are extracted from a person-by-person correlation matrix. For each component, members who have high loadings on it are assigned to a group.

In cluster analysis, consumers are grouped based on their degree of likeness. Degree of likeness may be represented by distance measures, covariance or correlation measures, or matching type measures. These measures are derived from personal characteristics related to buying behavior. There are a large number of clustering techniques to group subjects.

In techniques using multidimensional scaling, respondent's perceptions of different products and/or preferences for them are used to represent products and/or respondents in the same geometric space or different spaces. The input data may be metric or nonmetric. The algorithms will produce ratio-scaled output. It is possible to do analysis either at the aggregate level or at the disaggregate level for perception data. In the case of preference data, the analysis can be either internal or external; in internal analysis, only preferences of respondents are taken in to account whereas in external analysis, both perceptions and preferences are taken in to account. Market segmentation may be accomplished by using output from multidimensional scaling analysis with criterion-related techniques or with a combination of descriptive and criterion-related techniques. In the first case, perception and preference measures given by multidimensional scaling are used either as dependent variables or as independent variables. These measures are then related to personal characteristics like demographics, psychographics, etc., or to buying behavior variables like product usage, brand loyalty, etc. In the second case, output from multidimensional scaling is used to form groups of consumers with homogeneous perceptions and/or preferences by some clustering technique. The resulting groups are then related to buying behavior or personal characteristics by a criterion-related technique.

CHAPTER II

LITERATURE SURVEY

The concept of market segmentation as a marketing strategy was developed in the late 1940s. At the end of World War II, a large number of industrial firms in the United States which were producing military goods shifted their production to consumer goods. The great recovery following depression of the 1930s, coupled with the imposed economic austerity of the war time period, resulted in high discretionary income for people at the end of war. Under the conditions of overabundance of consumer products and high level of discretionary income, the concept of market segmentation originated of necessity.

Developments in market segmentation have been along two different but related lines - selecting appropriate bases for segmentation and finding suitable statistical procedures for creating the segments. Since the present study is mainly concerned with statistical procedures, only a brief review of selection of bases for segmentation will be given here. As for statistical procedures, applications of each of them in segmentation will be treated separately. Segmentation is an approach, as made clear elsewhere, to accomplish certain objectives in marketing of products and services. Thus segmentation studies may involve application of different statistical procedures in the same study. In such cases, the study will be described under that procedure which mainly served the function of segmentation. As research related to development of redundancy has taken place outside the field of Marketing, progress of research in that area will be surveyed in a separate section.

Bases for Market Segmentation

According to McCann's (1974) definition, "A segmentation base refers to the particular characteristic of consumers that is used to assign these consumers to segments" (p. 402). The role of segmentation bases and the criteria to be used in their evaluation were briefly noted when market segmentation was defined in the introductory chapter. A detailed description of their role, and the criteria for evaluating them can be found in Frank, Massy and Wind (1972). There are many personal characteristics which can be used as bases to segment a market.

Geographic variables like region, population density, climate, etc., have been the earliest bases to be used in segmentation. These variables were in use even before segmentation was developed as a systematic approach to achieve certain marketing objectives. The late 1940s saw the introduction of demographic and socioeconomic variables as bases for segmentation. Usually only one demographic or socioeconomic variable was used. Thus, market was segmented on the basis of race (Steele, 1947), sex (Alexander, 1947), age (Gilbert, 1948), and income (White, 1948). Soon there appeared segmentation based on volume (Alexander, 1948). For some reason, volume segmentation was not followed up for more than a decade. During this interval, more and more demographic and socioeconomic variables were introduced, and analyses were done using the variables simultaneously when suspected of intercorrelations and/or interactions. Also, composites of demographic and socioeconomic variables, developed by sociologists in the Social Sciences began to appear in the segmentation field. Thus, Barton (1955) introduced the composite index, stage-of-life cycle developed by Loomis (1936) from demographics, and Martineau (1958b) the composite index, social class developed by Gordon (1949) from socioeconomic variables.

In the mid-fifties, purchase patterns were introduced as bases for segmentation. Based on orientation towards stores and purchasing habits, Stone (1954) classified shoppers into four groups - the economic shopper, the personalized shopper, the ethical shopper, and the apathetic shopper. Cunningham (1955, 1956) used brand loyalty and Martineau (1958a) store loyalty for segmenting markets. In all these studies, groups were first formed based on purchase patterns. Afterwards the groups were examined to see how they differed on demographic and socioeconomic characteristics. Though consumers could be classified into distinctive groups based on purchase patterns, these groups were not quite distinct with regard to demographic and socioeconomic characteristics.

What was true of purchase patterns was true of buying behavior in general; for many product categories, buying behavior could not be accounted for sufficiently by demographic and socioeconomic variables whether used singly, simultaneously or as composites. Research interest, therefore, focussed on finding new segment bases. Evans (1959) and Koponen (1960) used personality inventories to segment markets. In 1962, Rogers (1962) proposed using participation in the adoption and diffusion process. In an important paper published in 1964, Yankelovich (1964) listed a set of suitable bases for market segmentation. In his paper, Yankelovich stated, "Markets should be scrutinized for important differences in buyer attitudes, motivations, values, usage patterns, aesthetic preferences, or degree of susceptibility" (p. 89). In 1965, Kassarjian (1965) introduced in Marketing the concepts of "inner directed" individuals and "other directed" individuals developed earlier by Riesman (1950) in the Social Sciences.

The period covering 1950s and early 1960s was characterized by indiscriminate use in marketing research of all types of concepts and techniques developed in clinical psychology. It was the period of raging controversy over the relevance of motivation research to marketing. When Ditcher (1964) argued for motivation research, Politz (1956) attacked it saying that motivation research lacked a scientific basis. The net result of the controversy, so far as segmentation research was considered, was the availability of a large set of variables called psychographics which can be used as segmentation bases in quantitative analysis. As Wells (1975) notes, there is no common definition for psychographics that meet with general approval. Depending on the researcher's objectives and taste, psychographics may cover activities, interests, opinions, needs, values, attitudes, and personality traits. There is a large number of studies which used psychographics as base for segmentation. Examples are Bernay (1971, 1973), Wells and Tigert (1971), Ziff (1971), Peterson (1972), and Plummer (1974).

If psychographics can be meaningful bases for segmentation, so should product usage be, argued Alpert and Gatty (1969). From the marketer's point of view, psychographics reflect the overall manner in which people live and spend time and money. The products people consume also reflect the same thing. They are the two sides of the same coin. On that reasoning, Alpert and Gatty (1969) introduced a new base for segmentation, namely, "behavioral life styles" by product usage.

Volume segmentation which was introduced earlier by Alexander (1948) came into vogue in the 1960s under the name, "'Heavy Half' Theory". Credit for its popularity should go to Twedt (1964a, 1964b, 1970, 1972). Twedt showed that for many consumer products, if consumers are ranked according to their usage of the product class and divided into two classes at the median, the group of heavy users thus obtained would account for a substantial share of total product consumption. Volume segmentation was, however, attacked from a conceptual point of view by Haley (1968) and from a statistical point of view by Frank (1968a), According to Haley (1968), although the heavy half are the most valuable consumers, " not all heavy consumers are available to the same brand because they are not all seeking the same kind of benefits from a product " (p. 31), and hence, all the heavy users cannot be expected to respond similarly to changes in the marketing mix. According to Frank (1968a), "Heavy Half" theory concentrates only on the heavy half, leaving the light half which is not that light when one considers their share of total consumption and the market share of individual brands in many product categories.

Marketing factor is a segmentation base which was introduced in the 1960s. In this form of segmentation, consumers are grouped based on their responses to different marketing factors like price and price deals, product quality, retail advertising, and so on. As Frank (1967a, pp. 27-28) notes:

> If a manufacturer knew that one identifiable group of his customers were more responsive to changes in advertising expenditures than others, he might find it advantageous to increase the amount of advertising aimed at them. The same sort of tailoring might also be appropriate if it was found that customers reacted differently to changes in pricing, packaging, product quality, etc.

An example of the application of marketing factor as segmentation base is the study reported by Frank and Massy (1965). A problem with marketing factor segmentation may be that it is difficult to identify the different segments by socioeconomic or personality characteristics.

A new base for segmentation, namely, "benefits" was introduced by Haley (1968). According to Haley, " . . . the benefits which people are seeking in consuming a given product are the basic reasons for the existance of true markets" (p. 31), and he called his approach "benefit segmentation". Since benefits are a subset of attitudes, benefit segmentation may be viewed as a special case of attitude segmentation. Benefit segmentation has some drawbacks when applied to certain product categories. It is based on the presumption that consumers can rate all the benefits they seek from a product. As Knobler (1976) notes, " . . . consumers cannot articulate what they truly want, or else they merely play back what they had been taught-- taught by advertising" (p. 20). Another drawback of benefit segmentation is that it may be quite ineffective for product categories with an overwhelmingly dominant benefit or for categories where all the possible benefits are sought.

Stefflre (1968, 1971) has introduced a base for segmentation where situation-specific variables are used for segmenting markets. Situational segmentations are based on the assumption that people are basically the same but have different needs when in different situations. The procedure to be followed in situational segmentation is outlined in Tauber (1974).

One of the segmentation bases introduced in the early '70s is the "product space", characterized by consumer's perceptions and preferences. Product-space segmentation is based on the work of psychometricians like Shepard (1962a, 1962b) and Carroll (Carroll, 1972; Carroll and Chang, 1972) who developed techniques to represent subjects and/or stimuli in multidimensional geometric space. Applications to date of product space approach to market segmentation include Johnson (1971), Green and Wind (1973), and Ritchie (1974).

Another segmentation base introduced in the '70s is utilities for different product attributes. Using conjoint measurement, information is obtained on individual consumer's utilities for different attributes. Consumers with similar utilities are grouped together to form different segments. Application of segmentation based on utilities may be found in Green and Wind (1973). A major problem with this type of segmentation is data collection for conjoint measurement.

Still another base introduced in '70s is problems with products (Burgi, 1976, Knobler, 1976). In problem-oriented segmentation, consumers are grouped based on the importance they give to different problems encountered with existing products. Problem-oriented segmentation may be useful in some situations when benefit segmentation is ineffective. A major disadvantage of problem-oriented segmentation, as noted by Burgi (1976), is that segmentation is difficult in many cases as there are not many problems encountered by consumers in a large number of product categories. Besides, it is not applicable to entirely new type of products as problem-oriented segmentation is limited to existing product classes by its very definition.

All the variables described above are currently in use as segmentation bases. Choice of the base is dependent on the nature of the product, characteristics of the market, and statistical procedures used for analysis. In many situations, it has been found that addition of certain segmentation bases like psychographics to demographic and socioeconomic variables increases the proportion of explained variance in buying behavior variables. In some of the studies, information provided by psychographics are quite independent of that provided by demographic and socio-economic variables. Thus it may be necessary to combine different bases to properly segment certain markets. The availability of computers and applications programs has made it possible for the analyst to use as many variables as he/she thinks relevant to a particular problem. And a good many studies in fact involve a large number of variables often in 100s for segmentation. By an initial application of factor analysis, these variables are reduced to a fewer number of independent factors.

It has been found that situation specific-variables like attitude towards the product or product class are more useful in explaining buying behavior than general bases like personality variables. Advances in the field of multidimensional scaling have contributed to increased use of situation-specific bases of perceptions and preferences. The major stumbling block to wider use of perceptions and preferences seems to be the large amount of data that are to be collected for the application of multidimensional scaling techniques.

Most of the current research is directed towards refining these bases. Thus, personality inventories like Gordon Personal Profile (Gordon, 1963), Jackson's Personality Research Form (Jackson, 1967), etc., are used in recent segmentation studies (Kernan, 1968, Sparks and Tucker, 1971, Worthing et al., 1972) replacing Edward's Personal Preference Schedule which was invariably used in earlier studies (Evans, 1959, Koponen, 1960). Traits measured by Gordon's Personal Profile and Jackson's Personality Research Form are found more relevant to consumption situations than those measured by Edward's Personal Preference Schedule. Progress of related work currently done and suitable areas for future research in refining segmentation bases are described in Frank, Massy and Wind (1972) and Frank and Massy (1975).

Applications of Statistical Procedures

Application of statistical procedures for segmenting markets began in 1940s. Initially, cross-classification was the only procedure used. Some buying behavior characteristic like product usage formed the dependent variable. Independent variables consisted of demographic and socioeconomic characteristics. Cross-classification was done using the dependent variable against one independent variable at a time. Later, to account for intercorrelations and interactions among the independent variables, analysis was done simultaneously cross-classifying the respondents against different variables. However, with increase in the number of variables used for analysis, cross-classification proved unsuitable because of the large number of cells required for studying the effect of these variables. The marketer, therefore, turned to more advanced statistical procedures which are described below.

Regression Analysis

The method of least squares which is one of the methods used in estimating parameters in regression analysis was first published by Adrien Legendre (1805). However, its first treatment along the lines now familiar was not done until 1821. Gauss (1821) showed that method of least squares gives estimators with minimum variance among unbiased linear estimators. The term "regression" was first used by Galton in the context of describing the relationships of traits of parents to the traits of offspring. About 1890, Karl Pearson, stimulated by the work of Galton, began to investigate bivariate distributions which led to regression analysis. Fisher (1925) clarified and extended ideas on the subject of estimation and emphasized the importance of the method of maximum likelihood as providing a very general technique for obtaining "best" estimators. Fisher was also instrumental in emphasizing the importance of rendering the predictor variables as independent as possible. Other major theoretical contributions to the development of regression analysis came from Laplace (1812), Chebyshev (1859), Markov (1898), Wald (1940), Neyman (1944), Kolmogorov (1946), Rao (1946), and David (1951). Suits (1957) showed how regression analysis can be done using dummy variables when the independent variables are only nominal-scaled.

There are a number of studies where regression analysis was used to segment markets. One of the early applications of regression analysis in market segmentation was by Ferber (1958). Ferber applied regression analysis to account for variations in retail sales between cities for different categories of products. Koponen (1960), and Frank, Massy and Lodhal (1969) applied regression analysis to find relationship between purchase rates of products and socio-economic and personality variables. In media research, regression analysis was used to relate viewer's preference for program types to personal characteristics by Gensch and Ranganathan (1974) and Villani (1975). Among other studies using regression analysis to segment markets are those by Frank and Boyd (1965), Frank and Massy (1965), Fry (1967), Myers (1967), Frank, Massy and Boyd (1967), and Bass, Tigert and Lonsdale (1968). The study by Bass, Tigert and Lonsdale is described below.

The data for the study was collected from a mail questionnaire survey. Questionnaires were sent to 8000 households in the Milwaukee area, selected on a probability basis. The sample return was more than 80 percent. Information was obtained on (a) the quantities purchased for ten grocery products like beer, catsup, etc., and (b) demographic and socioeconomic variables such as age of male head, number of children under 18 years, family income, education of head, occupation of head, and television viewing yesterday by head. For each product, a separate regression analysis was performed. The dependent variable was the purchase rates of grocery products. The independent variables were demographic and socioeconomic variables. To avoid assumptions of linearity and continuity, dummy variables were used for the set of independent variables. Regression was done, first, using all the variables and then deleting in a stepwise manner non-significant ones using F ratio. The six socioeconomic and demographic variables were found significant for all products at 0.10 level. Consumers were classified based on these variables. Mean purchase rates for different segments were computed from the final stage regression estimates by summing the regression coefficients included in the segment description and then summing over variables not included. There was wide range of variation between segments. However, R^2 values were low.

Interpretation of the low value of R^2 in regression analysis with specific reference to market segmentation has created sort of a controversy in the field. The low R^2 values obtained in several studies using demographic and socio-economic variables led Frank (1968b) to conclude that "Based on the research reported in the preceding sections, for the most part socioeconomic characteristics are not particularly effective bases for segmentation either in terms of their association with household differences in average purchase rate or in response to promotion " (p. 53). But according to Bass, Tigert and Lonsdale (1968), a high value of R^2 is not essential for carrying out a successful segmentation strategy. This is how they stated their position:

> The evidence from the regression and discriminant studies clearly indicates that it is impossible to predict usage rates very well on an individual basis with socioeconomic variables. For purposes of market segmentation, however, it is sufficient that the variables yield large differences in the mean purchase rates. . . Even though the criterion-group variance is great, the fact that the choice is between groups not persons permits segmentation by group means (p. 265).

The accepted view now seems to be that group means are sufficient for determining segments to be chosen for marketing cultivation and that a low R^2 only indicates that it may be possible to separate the group means further by adding more personal characteristics (Frank, Massy and Wind, 1972, pp.229-232; Bass, 1975, pp. 1-36).

Remarks on the use of regression analysis. Regression analysis is the most widely used statistical procedure in market segmentation. Its properties are well known and tests of significance are available. However, it has certain limitations. To use regression analysis, the researcher has to specify the regression equation. In this respect, it is different from AID where interactions, etc., are not to be specified a priori. Another limitation is that (linear) regression analysis assumes linearity. Hence it may be necessary to apply transformations to the dependent and independent variables for proper representation of the relationship. However, in the case of independent variables, it is possible to use dummy variables to avoid the assumption of linearity as well as continuity. Still another limitation of regression analysis is that only a univariate measure of buying behavior can be represented by the dependent variable.

AID and MCA

AID and MCA originated at the Institute of Social Research, University of Michigan, USA. Sonquist and Morgan (1964) developed AID, and Andrews, Morgan, and Sonquist (1967) developed MCA. In 1973, Sonquist, Baker and Morgan (1973) proposed a modified version of AID, called AID3. For both AID (and AID3) and MCA, the dependent variable must be measured on an interval scale, or be dichotomous and should not be badly skewed. In another version of AID, called THAID, proposed by Messenger and Mandell (1972) and Morgan and Messenger (1973), the dependent variable is nominal-scaled. For dichotomous dependent variable, both AID and THAID are applicable. However, the results may be different because the split criteria in THAID and the criterion in AID may not rank order predictors identically.

There are a few applications of AID and MCA in market segmentation. Assael, Kofron and Burgi (1967) applied AID to study readership as a function of ad characteristics. Peters (1970) used AID to segment car markets. Assael (1970) applied AID to relate product and brand usage to demographic and attitudinal variables. Carmen (1970) performed AID analysis and Newman and Werbel (1973) MCA to find correlates of brand loyalty.

Combined use of AID and MCA was made in two studies reported by Newman and Staelin. One of these studies (Newman and Staelin, 1972) was for segmenting buyers of automobiles and major household appliances based on amount of information seeking while the other study (Newman and Staelin, 1971) was for segmenting buyers based on buyer decision time. The latter study is described here.

The data for the study were collected from 652 households which had bought cars or major household appliances during a two year period preceding the interview. Respondents were adults randomly selected from a probability sample of 1300 households in the United States. Duration of the decision process - the time between the first thought of buying and purchase - was the dependent variable. There were 36 independent variables representing household characteristics, conditions of purchase, and activities in the decision process. It was difficult to quantify a priori the functional form of relationships between the dependent variable and the independent variables. AID technique was therefore applied to analyze the data. AID formed groups of households with different average decision times and also pointed to interactions of variables. The information obtained in AID was used to specify the structure for MCA analysis. Analysis helped to identify segments with markedly different average decision times.

Doyle and Fenwick (1975) report a work where combined use of AID and regression analysis was made to study the Sunday reading habits of adults in U.K. and its relationship to personal characteristics. Information regarding media usage, attitudes and socioeconomic characteristics was obtained by personal interview from a representative sample of over 2000 adults. Data on attitudes were factor analyzed prior to the application of AID. An initial AID run was made with low parameter values to provide maximum information on the characteristics of the data and to eliminate variables lacking a useful causal interpretation. The dependent variable was the respondent's estimate of time spent reading the major popular Sunday papers. After the initial AID analysis, the respondents were randomly assigned to two equal groups to form an analysis and a validation sample. An AID analysis was then performed on the analysis sample to determine the variables to be included in the regression analysis. In addition to individual variables, the regression equation contained six interaction terms detected by AID analysis and two other interaction terms which were not shown by AID but were suspected to be present in the data. Of the eight interaction terms, only four were found significant in the regression analysis. All four were from the set of six interactions discovered in AID analysis. Regression analysis had an R^2 of 0.52.

To test the predictive power of the model obtained from the above analysis, validation sample was utilized. AID was performed and the tree obtained from the validation sample was compared with the tree obtained from analysis sample. The new tree looked very different from the earlier analysis. As replication was not possible by AID, predictive power of the model was tested by using analysis equation to generate readership levels for the validation sample. Explained variation was now only 25 percent which was much lower than the earlier value and cannot be attributed to chance.

Robustness of the interactions were tested by fitting the original equation structure to the validation sample and performing regression analysis. The coefficients were very different from the previous analysis, although the differences were not statistically significant. Besides, the two "test" interactions now turned out to be significant. R^2 was 0.40. The sequential use of regression analysis and the utilization of validation sample were thus quite useful in avoiding the pitfalls of AID and in determining the relationship between reading habits and personal characteristics.

Remarks on the use of AID and MCA. AID is well suited for application in situations when (1) it is difficult for the analyst to specify a priori the relationship between dependent and independent variables, (2) linearity and additivity assumptions are not warranted, and (3) a criterion-related technique is required for the analysis.

The limitations of AID are listed below:

(1) The optimal split at each level except the first one is based on the predictor's contribution to reducing error variance in the subgroup rather than the total sample.

(2) For splitting, AID selects the variable which yields the highest value for the ratio between sum of squares/total sum of squares. All subsequent splits are contingent upon the subgroups formed by the first split. It is possible a second variable was almost as discriminating as the first. If the sample were split by the second variable, the results

could be totally different. (3) AID may make spurious partitions. This may occur when (a) there is high skewness in the dependent variable, (b) the sample size is small, and (c) the amount of random variation actually present in the data is large. (4) There is no statistical basis for the stopping rules. (5) AID may not detect interactions because of data problems as reported by Doyle and Fenwick (1975). (6) Finally, there is only one variable in the criterion set, and hence, a multivariate measure of buying behavior cannot be used in the analysis.

MCA is equivalent to regression analysis in which the independent variables are categorized and used as a set of dummy variables. Compared to dummy variable regression analysis, input to MCA is simpler and the results are easier to interpret. A major disadvantage is that any departures from linearity including those caused by random fluctuations from category to category of a variable may be attributed to the effect of that particular variable on the dependent variable.

The proper use of AID is in exploratory phase of investigation. If a study is final, sequential use of MCA or regression analysis to AID is to be recommended.

Discriminant Analysis

Discriminant analysis originated with the work of Fisher in 1936. Fisher (1936) proposed the discriminant criterion in connection with his two-group discriminant function and gave as an example the assigning of iris plants to one of two species on the basis of the lengths and widths of the sepals and petals. Other major theoretical contributions have come from Bartlett (1947), Brown (1947), Rao (1948b, 1952, 1965) and Anderson (1958).

Starting with Evans' (1959) classic study of Ford versus Chevrolet buyers in 1959, discriminant analysis has been one of the widely used techniques in market segmentation. Evans attempted to discriminate new Ford and Chevrolet buyers based on personality needs, socioeconomic variables, and a combination of both. But no significant differences were found between the groups. Based on intention to buy rather than purchase, Ito (1967) discriminated loyal and switching Ford and Chevrolet buyers using nine attitude scales.

Burger and Scott (1972) used discriminant analysis to find the characteristics of private brand users. The data consisted of responses from 136 buyers of jams and jellies (consumer goods) and 111 buyers of irons (durable good). Data were collected on demographics, attitudes, interests, opinions and brand purchase. Depending on the responses, the subjects were classified as private brand buyer, manufacturer's brand buyer, or mixed buyer or non buyer of the product. Those in the third category were excluded from further analysis. Factor analysis of the variables measuring attitudes, interests and opinions yielded three factors: price attitude, advertising attitude and careful shopping. Discriminant analysis was done separately for consumer and durable items. For both items, socioeconomic variables, advertising attitude, and careful shopping were not important in differentiating the groups. But price conscious attitude and brand loyalty made significant differentiation between the groups in the case of both consumer items and durable item.

Wiseman (1972) reports a study where discriminant analysis was applied to find whether status consciousness and economy-minded measures could differentiate between buyers of "new season" cars and "new left-over" cars. A research design was used in which buyers of new left-over and new season cars were each subdivided into three groups- buyers of "full size low price", "full size high price", and "intermediate" cars. This was done to reduce heterogeneity present in the "new left-over" and "new season" market segments. 35 buyers were chosen from each of the six groupings. The sampling procedure used was to randomly select dealers from a particular geographic area and then individuals within each of these dealerships. Data were collected through personal interviews for four sets of variables: (1) purchase and usage related, (2) demographic and socioeconomic, (3) personality, and (4) attitudes toward styling in automobiles. Separate discriminant analyses were done for the three subgroups of new season and new left-over buyers. The results showed that new left-over purchasers were more economy minded and less status conscious than new season buyers. The economy-minded measures achieved greater success in discriminating the groups than the status conscious measures.

Shuchman and Riesz (1975) applied discriminant analysis to determine the distinctive characteristics of consumers who adopted Crest tooth paste following its endorsement by American Dental Association. The data for the study were from the MRCA panel of household purchase records and included information on purchasing, demographic, and socioeconomic variables for 3028 households which had not purchased Crest prior to the endorsement. Among the households, 837 converted to Crest and remained "loyal" to it in the post-endorsement period and the rest did not convert to it during a period of about 32 months following the endorsement for which data were available. Discriminant analysis showed that the two groups differed with respect to socioeconomic, demographic, and purchasing variables. To find out how well these variables could predict the two groups, split sample prediction-validation technique was used. For this stage of analysis, equal sized samples of 837 converters and 837 nonconverters chosen randomly from among the 2191 nonconverting households were used. Four runs were made to determine classification effectiveness. The average correct percentage of classifications for the analysis samples was 64.9 while the average for validation samples was 60.9, both figures better than the chance 50 percent.

Other studies where discriminant analysis was used to find the distinctive characteristics of adopters and innovators include King (1965), Frank and Massy (1964), Frank, Massy and Morrison (1965), Pessemier, Burger and Tigert (1967), Robertson and Kennedy (1968), and Darden and Reynolds (1974).

Claycamp (1965) used discriminant analysis to compare the characteristics of owners of thrift deposits in commercial banks and savings and loan Associations. Etzel (1975) applied discriminant analysis to find how three types of credit card holders, namely, installment users, convenience users and inactives differ on general credit attitudes, attitudes toward bank credit cards, and demographics.

Among other studies using discriminant analysis in market segmentation are those by Massy (1965), Brody and Cunningham (1968) and Anderson and Cunningham (1972). Massy applied discriminant analysis to evaluate the similarities and dissimilarities among the audiences of five FM radio stations. Brody and Cunningham used discriminant analysis to study the relationship between personality variables and consumer decision process. Anderson and Cunningham used discriminant analysis to study the characteristics that discriminate between individuals with favorable attitude towards foreign products and those with unfavorable attitude.

Remarks on the use of discriminant analysis. Discriminant analysis is the statistical procedure to be used for finding the relationship between independent variables and a nominal-scale dependent variable. It enables the analyst to determine the distinguishing characteristics of different groups. There is no assumption of linear relationship between the independent variables and the dependent variable. Hence, in situations where there are nonlinear relationships and the analyst has no idea about their nature and extent, an interval- or ratio-scaled dependent variable may be deliberately downgraded for using the procedure of discriminant analysis.

There are a few limitations to discriminant analysis. The dependent variable has to be nominal-scaled. Hence, for using the procedure it is necessary to throw away useful information if the dependent variable is interval- or ratio-scaled. As in the case of regression analysis, the analyst has to specify interaction terms, etc., a priori. In two-group discriminant analysis, a multivariate measure of buying behavior cannot be represented by the dependent variable. In multiple discriminant analysis, a multivariate measure of buying behavior can be represented but the analyst has to devise an a priori classification system for using the procedure.

Canonical Correlation Analysis

Canonical correlation analysis originated with the work of Hotelling (1935, 1936). Other contributions to the development of canonical correlation analysis came from Bartlett (1941, 1947), Mariott (1952), Rao (1952, 1965), Roy (1957), Anderson (1958), Lawley (1959), Meredith (1964), and Gower (1966).

Though application of canonical correlation analysis began in Psychology soon after its introduction by Hotelling, the first reported application in market segmentation, or for that matter in the whole field of Marketing did not appear until 1966. In 1966, Green, Halbert and Robinson (1966) published a study to illustrate the use of the technique in segmentation of markets. Sample consisted of 36 students and the two sets of variables were risk taking behavior and personality characteristics.

There are a number of studies where canonical correlation analysis was applied to relate product usage to personal characteristics: Darden and Reynolds (1971), Sparks and Tucker (1971), Worthing, Venkatesan and Smith (1972), Alpert (1972), Frank (1972) and Frank and Strain (1972). The study reported by Frank and Strain (1972) is described below.

The data for the study were collected from a panel of households maintained by Homarket, Inc. A sample of 800 households kept continuous records of their purchases for

a product category over a period of 25 weeks. Analysis was done based on 499 individuals who completed the questionnaires seeking information on attitude toward the product category under investigation, general life style and personal characteristics. Dependent variables were measures of total units of purchases for each of two product types. The independent variables were 26 factors extracted from the attitudes, interests and opinions battery of 151 items by factor analysis, and a set of demographic and socioeconomic measures. Canonical correlation analysis yielded two significant variates. Segments were then delineated as follows:
(1) Respondents were cross classified based on the magnitude of canonical variate scores of individuals for the independent variable set. Division of each variate into low, medium and high yielded nine "microsegments". (2) For each microsegment, the standardized mean was computed for each variable in the analysis. To facilitate interpretation, means for variables which had been previously excluded from analysis because of collinearity were also computed.
(3) Macrosegments were then created by combining microsegments judgementally. For the segments thus created, percentage contribution of unit volume for the two products by selected brand was computed. There were clear differences among segments in purchase behavior.

Lessig and Tollefson (1971) used canonical correlation analysis to relate group membership in one of several behavioral clusters to personal characteristics. Prior to the canonical correlation analysis, behavioral clusters were formed by applying a clustering technique to group consumers based on buying behavior characteristics. For the canonical correlation analysis only those personal characteristics which were found to be related to buying behavior by means of separate chi-square tests were included.

Perry and Hamm (1969) and Sweeney, Mathews and Wilson (1972) applied canonical correlation analysis to relate risk taking behavior to personal characteristics. Segmentation studies by Kernan (1968), Baumgarten and Ring (1972), Alpert (1972) and Lessig (1972) have reported the use of canonical correlation analysis in such applications as (a) relating personality variables to use of alternative choice criterion under uncertainity, (b) relating media readership to personal characteristics of audiences, (c) finding the relationship between personality and determinants of product choice, and (d) relating store loyalty to store image. Other applications of canonical correlation analysis in market segmentation include studies by Alpert and Peterson (1972), Gensch and Turner (1972), and Neilson and Stanton (1972).

Remarks on the use of canonical correlation analysis: Canonical correlation analysis is an invaluable tool for studying the overall relationships existing between two sets of variables. A multivariate measure of buying behavior can be used for analysis in this procedure.

There are a number of limitations to canonical correlation analysis as noted by Lambert and Durand (1975): (1) The canonical correlations are deficient in indicating the variance shared by the sets. (2) The canonical coefficients used in computing the canonical variate scores are extremely sensitive to sampling variations. (3) There are problems of interpretation if an attempt is made to relate subsets of the criterion variables to predictor variables. Such problems arise because there are no canonical counterparts to partial correlation coefficients.

Factor Analysis

The earliest work on factor analysis may be traced to Karl Pearson and to the beginning of this century. In 1901, Pearson (1901) published a paper on lines and planes of closest fit to systems of points in space. This work formed the basis for what is now called principal-axes method of factoring. In 1904, Spearman (1904) published his paper, "General Intellegence Objectively Determined and Measured", which was the beginning of his developing the Two-Factor theory. As time went on, more and more researchers began to concentrate on studying human intellegence and various types of tests became available to make an assessment of intellegence. Soon it became clear that Spearman's was an oversimplification of the facts, and multiple factor concepts were developed. L.L. Thurstone (1935, 1947), Thomson (1939), and Burt (1940) were in the forefront of this movement. In 1940, Lawley (1940) published an important paper which dealt with the problem of factor estimation.

Q factor analysis was first introduced by Burt (1937). Burt (1940) along with Thomson (1939) and Stephenson (1953) were instrumental in the development of its philosophy and methodology.

Principal components analysis was developed by Hotelling (1933). Bartlett (1950, 1954) has developed procedures for testing the equality of the remaining latent roots of a matrix after the first k have been extracted. This procedure is sometimes used in making a decision regarding the number of factors to be extracted in a study.

In the area of rotational criteria, Thurstone (1947) introduced the important concept of simple structure. Since then, several researchers have proposed different analytical methods for determining simple structure. Their list include Carroll (1953), Newhaus and Wrigley (1954), Saunders (1953), and Kaiser (1958). Contributions in the area of rotation to a specified structure have come from Mosier (1939), Saunders (1960), Ahmavaara (1954, 1970), Horst (1956), Hurley and Cattell (1962), Cliff (1962, 1966), Joreskog (1966), Shonemann (1966), Shonemann and Carroll (1970), Gruvaeus (1970) and Lingoes and Shonemann (1974).

Other important contributions to the development of factor analysis came from Guttman (1953, 1955, 1958), Rao (1955), Anderson and Rubin (1956), Harris (1962), Tucker (1964, 1966) and Harman (1967).

Application of factor analysis to segment markets began in the late '50s. Among the early applications were studies by Pilgrim and Kamen (1959), Stoetzel (1960), Gatty and Allais (1961), and Gatty and Heim (1961). Pilgrim and Kamen applied R factor analysis to find patterns of food preferences among the army personnel and were able to relate them to personal characteristics like age, region, time spent in the army, etc. Stoetzel used R factor analysis to classify French consumers of liquor based on their preferences.

A study done by Alpert and Gatty (1969) which used R factor analysis is worth describing here. The purpose of the study was (1) to determine the behavioral life styles of men from usage patterns of products and services, and (2) to find the differences in behavioral life styles and usage patterns between heavy and light users of two brands of a product (beer). The data were drawn from 1965 Brand Rating Index data base. Frequency of usage of 80 products and services for all of the 5424 male respondents were first collected. The 80 variables included heavy drinking of brand Y beer, light drinking of brand Y beer, heavy drinking of brand W beer, light drinking of brand W beer, and drinking of beer. 25 factors were drawn from the 80x80 correlation matrix by principal component analysis. The loading of products on factors showed that there were certain patterns of product usage. To find the life style and usage patterns of the heavy and light users of the two brands of beers, the following steps were taken. (1) Scores were found for all individuals on the 25 factors extracted. (2) From product usage rates, subjects were identified as beer drinker and non-beer drinker, heavy user of brand Y and light user of brand Y, and heavy user of brand W and light user of brand W. (3) For each of the groups identified in step (2), average factor score was computed as a deviation from the total male segment for the 25 factors and a table was constructed to display the results. (4) Another table was constructed, where factors were arranged in rank order of differences from non-consumers of beer (measured in factor loadings) for each of the five groups - all beer consumers, heavy consumers of brand Y, heavy consumers of brand W, light consumers of brand Y, light consumers of brand W. The study showed that heavy beer drinkers were distinctly different from light beer drinkers. There was clear distinction between heavy drinkers of the two brands. For light drinkers too, there were differences between the brands though they were less pronounced. The study also threw light on the general life styles of men.

There are several studies in media research where R factor analysis was applied for segmentation purpose. Banks (1967), Swanson (1967) and Wells (1969) used factor analysis to derive program groupings. They started with a person by program data matrix where one row of the matrix showed program preferences or viewing behavior of a respondent for the period under study. Program by program correlation matrix was factor analyzed and loadings were used to assign programs to different groups. Ehrenberg (1968) questioned the findings of such studies and suggested that the viewing patterns which emerged were primarily influenced by the day and hour of the programs, programs that precede and follow, the strength of the individual television channels and the quality of the competition. Frank, Becknell and Clokey (1972) used preferences data for studying program types and removed the influences of (1) personal characteristics of the individuals - socioeconomic, demographic, set ownership, and usage pattern, and (2) scheduling characteristics of program - network, day of week, time of day and lead-in, by regression analysis. The residuals were then factor analyzed. The results supported the view that there were distinct program types. Rao (1975) replicated the above study using behavior data instead of preference data and came to the conclusion that television types are largely determined by prior knowledge variables like individual characteristics and program scheduling variables.

In some of the television media studies, attempts were made to find relationship between television viewing behavior and personal characteristics and/or product usage.

Villani (1975) found no significant relationship between demographics and psychographics, and program groupings obtained from factor analysis. The results of this study contradicted findings of two previous studies reported by Tigert (1972) and Gensch and Ranganathan (1974), where relationships could be found between program groupings and personal characteristics. In addition, relationships were found between product usage and program groupings as well, in these two studies. The study by Gensch and Ranganathan is reported below.

The data for the study were collected from Brand Rating Index data base. Information was obtained on the following: (a) exposure scores of individuals for 81 programs as measured by the number of exposures viewed out of the last four, for each of the shows, (b) demographics, and (c) usage of ten products, six of which were related to female grooming and the rest general. Analysis consisted of several phases and for each phase, a new subsample was drawn from a total sample of size over 16000. Different phases of analysis are described below.

1. Factor analysis was performed using the raw viewing scores for 1000 individuals. Groupings of programs were obtained which were dominated by scheduling influences.

2. A random sample of 100 individuals were chosen and this time, factor analysis was performed after removing the scheduling influences by factor analysis. Fifteen

factors which had eigenvalues of more than 2.0 were retained. These factors accounted for 65.3 percent of the variance in the refined data.

3. To check the stability of factors obtained in the previous analysis, the procedure was repeated for a sample of size 150. Fifteen factors accounted for 59.3 percent.

4. The factor solutions obtained in (2) and (3) were studied using a similarity index. It was found that the number of meaningful dimensions was less than ten. Programs which appeared to have a common dimension and had positive correlations among them were grouped yielding eight program configurations.

5. A sample of 775 individuals were drawn and separate regression analysis was performed with usage for each product as dependent variable and the number of exposures for the eight groupings as independent variables. Though the values for R^2 were low, a number of configurations were found significant.

6. Step (5) was repeated with another sample of 775 to validate the results. The results were somewhat similar and many configurations which were found significant in the analysis sample for different products were found significant against the same products in the validation sample too.

7. Two more regression analyses were performed for

each product using product usage as the dependent variable. The independent variables were demographics in one case, and demographics and number of exposures for the eight configurations in the other case. For seven out of the ten products, R^2 obtained when show type and demographics were used in the analysis was equal to the sum of R^2 obtained when these variables were used separately in the regression. Thus, in these cases, information on program viewing is independent of demographic data. An advertiser of products in this group should treat information on program types as independent and additive to the information on demographics.

8. For products in which demographics were not independent of viewing behavior, individuals were first segmented based on demographics found to be related to product usage. Regression analysis was then performed for the set of individuals who ranked high in usage of the product. Analysis thus helped to identify the show type which was most suitable for advertising to this particular segment.

There are a few studies where Q factor analysis was used to segment markets. The first application of Q factor analysis in market segmentation was due to Haley (1968). Haley used Q factor analysis to segment tooth paste market based on benefits consumers sought in the product.

Morrison and Sherman (1972) used Q factor analysis to group people responding differently to sex in advertising. Cunningham and Crissy (1972) applied Q factor analysis to see whether the compact car market could be segmented based on motivational and attitudinal measures. The details of the study on the compact car market are given below.

The sample for the study was drawn from individuals who had bought new Mavericks, Toyotas, and Renaults from February, 1970 through May, 1970 in Ingham county, Michigan. 61 Maverick and 59 foreign car owners were included in the study. Measurements were taken on socioeconomic and demographic variables and the following motivational and attitudinal variables: status concern, conservatism, attitude toward big business, and dogmatism. Preliminary analysis of the data showed that significant difference existed between Maverick car owners and foreign car owners on individual variables. Q factor analysis was used to find the differences between the groups in motivational and attitudinal measures on the whole. As the program used for the analysis had a capacity to handle only 99 variables, 49 Maverick and 50 foreign car owners were randomly selected from the original sample. The person by person correlation analysis was factor analyzed with varimax rotation to yield 2, 3, and 4 factors. The two-factor solution accounted for 43 percent of the total variance, and the factors differentiated between the Maverick and foreign car owners. The three-factor solution accounted for 47 percent of total variance and the first and third factors differentiated between the groups. The four-factor solution accounted for 52 percent of total variance and the first and the third factors differentiated between the groups. The study showed that the US compact car market could be segmented into two markets - American and foreign.

Remarks on the use of factor analysis. The major use of factor analysis has been in summarization of data. Variation in a mass of data can be described through factor analysis. In many cases, a few factors may account for most of the variation in data, and consequently management, analysis, and understanding of the data are facilitated. There are several other uses of factor analysis as Rummel (1970) has noted. These are listed below.

1. Factor analysis is often used for transforming data to a form suitable for use in other techniques. An example would be the factor analysis of correlated predictor variables to derive orthogonal factors for later use in regression.

2. Factor analysis may be used to determine the separate patterns of interrelationships among a set of variables.

3. The basic structure of a domain may be studied by means of factor analysis. An example would be determining the primary dimensions of variation in group characteristics and behavior.

4. Factor analysis may be used in developing an empirical typology. Factor analysis has thus been used for grouping closely related variables into descriptive categories, and

objects with similar profile values into types.

5. Factor analysis is often used for developing scales on which different individuals or groups may be rated. "Each factor", as Rummel (1970) notes, " defines a group of interrelated characteristics - a functional unity that can be used as a scale " (p. 30). Factor scores of objects show their rating on that scale.

6. Factor analysis may be used for hypothesis-testing. If there is reason to believe that a particular buying behavior is associated with distinct personal characteristics, factor analysis may be done to test the empirical existance of this association.

7. Factor analysis may be used to map phenomenological terrains like personality, cognitive meaning, etc.

8. Finally, factor analysis is useful as a tool in exploratory studies in new domains of interest.

With all its wide range applicability, factor analysis has a number of limitations. Some of them are noted below.

1. Factor analysis is done on a matrix derived from the original data. Use of derived data is proper only when original data is ratio- or interval-scaled.

2. (Linear) factor analysis assumes that redundancy of observed variable is attributable to their linear dependence on a smaller set of variables. The assumption of linearity may not be warranted in many cases when factor analysis is used for segmentation. This is not a serious limitation in that the analyst can use transformations to meet

the requirement of linearity. Or the analyst can use nonlinear factor analysis. Nonlinear factor analysis models are, however, quite complicated and their applications are quite limited.

3. Statistical tests in factor analysis are not of much practical use. Green and Tull (1975) have given two reasons for this limitation :

> First, distribution theory in factor analysis is quite complex, and few of the significance tests are available in easily applied form. Second, with a large number of objects relative to the number of variables, the number of statistically significant factors tend to equal the number of variables if one uses, say, principal-component analysis. In such cases components that account for a very small portion of total variability might still turn out to be statistically significant, even though their practical significance from an interpretive standpoint might be nil (p. 555).

4. Factor analysis is a descriptive procedure, and hence the relationship with a criterion variable cannot be considered explicitly in the analysis.

5. When factor analysis is used to form groups of members, there may remain a large undefined group. This may occur in R factor analysis when members have moderate factor scores on a number of factors, or in Q factor analysis when members have moderate loadings on a number of factors but no high loading on any single factor. There may be similarity among these members, but these are not considered in R or Q factor analysis. This is in contrast to cluster analysis where no large undefined group is left.

6. Finally, there are the problems of validity and reliability. This is how Sheth and Armstrong (1969) state the problems:

> Factor analysis summarizes in a few factors much of the variance in a data matrix but there is no way to support the arguement that the derived factors are valid

> or reliable. This is particularly important because on the same set of data one can obtain a variety of factors depending upon the options chosen. It is then possible to derive "meaningful" factors even from data which are randomly generated (p. 142)

Between R and Q factor analyses, though Q factor analysis readily yields groups of consumers, the number of applications of Q factor analysis in segmentation is much less than that of R factor analysis. The difficulty in interpreting the results, the problem caused by bipolar factors, and the loadings of members on several factors seem to be inhibiting its popular use in segmentation. The requirement of large computer memory to analyze person by person matrix for a large sample is no longer a problem; the transformations suggested by Johnson (1970)may be applied to the matrix in such cases.

Cluster Analysis

The first work in cluster analysis may be a paper by Zubin (1938), in which he proposed a technique for grouping "likeminded" individuals. Development of cluster analysis was quite slow in the beginning. Rapid developments began in '60s and most of them had to do with the work of numerical taxonomists. Major contributions came from Tryon(1939,1958a,1958b, 1970), Rao(1948a), Mcquitty(1954,1967), Cronback and Gleser (1953), Sokal and Sneath(1963), Ward(1963), Ball and Hall(1967), Johnson(1967), Friedman and Rubin(1967), and Gower(1967), to mention just a few. There is no well-developed statistical model for variability in clustering. The fundamental paper in this area is Rao's (1948a).

Relative merits of different measures of likeness - distance measures, covariance and correlation measures, and matching type measures - are discussed in Anderberg (1973), Bijnen (1973), Fleiss and Zubin (1969), Green and Carmone (1970), and Sokal and Sneath (1963). Green and Rao (1969) showed that different types of likeness measures produce different types of clusters. Description of the various clustering routines can be found in Anderberg (1973), Bijnen (1973), Green and Carmone (1970), and Hartigan (1975). Mathematical treatment of different aspects of clustering can be found in Cole (1969) and Jardine (1971).

Application of cluster analysis to market segmentation began to appear in the literature only in the '60s. Joyce and Channon (1966) published an article in 1966 in which they reported a number of studies where cluster analysis was used to segment markets. One of these studies was in the area of press readership and was aimed at grouping people based on their readership pattern. The basic data consisted of statements from forty women about "regular readership" of 46 items. Likeness measure used was a matching type coefficient. Linkage type of cluster analysis was used. Analysis yielded six groups with different readership patterns.

One of the other studies reported by Joyce and Channon (1966) in the same article was on segmentation of women based on their behavior pattern as shown by the order of acquisition of eight household durables. The sample consisted of the same people described in the previous study. The data were the order of acquisition of eight household durables which all these people already had. Likeness measure used was Spearman's rank correlation coefficient. Linkage type of cluster analysis was used here too. Thirty respondents could be grouped to seven clearly identifiable segments. The remaining ten respondents had low correlations with all the other respondents.

Greeno, Sommers and Kernan (1973) report a study where cluster analysis was used to relate personality to implicit behavior patterns. Q-sorts of 38 products by 190 housewives were the data used for cluster analysis. Grouping was done using Ward's (1963) clustering algorithm. There were six groups of housewives. Members of these groups differed on the emphasis they placed on five implicit behavior patterns. For these six groups, means of various personality and socioeconomic variables were quite different indicating the relationship between implicit behavior patterns and personal characteristics.

Another application of cluster analysis can be found in Sexton (1974). The purpose of Sexton's study was to estimate market response functions. For accurately estimating response functions for a market, it is necessary, first, to estimate market response functions for each segment separately. Cluster analysis was used to form different segments. Panel data on the purchases over a two and a half year period of all brands of regular coffee, instant coffee, and tea by 569 families were used for the study. Mahalanobis D^2 statistic computed from thirteen purchase habit variables, was the likeness measure. Clustering was done to yield 3, 4, 5, and 6 groups. From the results of cluster analysis, a four group structure was chosen judgementally for further analysis.

Peterson (1972) used cluster analysis to relate media exposure patterns to personal characteristics. The basic data for the study were responses from 196 adults to questions on personality, attitudinal, demographic, and media exposure items. Distances between respondents were computed from exposure responses to different media. Clustering routine developed by Ward (1963) yielded five groups. For these five groups, discriminant analysis was done using, first, demographic and then personality variables. There were significant group differences in both cases. Another discriminant analysis was performed to discover whether there is any significant difference among groups in the use of media as sources of primary information. Sources of information were obtained from responses of subjects indicating the medium from which they received most of their product information. Discrimination of the groups in this regard was found to be marginal.

Among other studies using cluster analysis for market segmentation are Kernan (1968), Kamer (1970), Lessig and Tollefson (1971), and Ritchie (1974).

Remarks on the use of cluster analysis. Cluster analysis is the most flexible technique for grouping of objects. There is flexibility in the choice of likeness measures and techniques. In contrast to factor analysis, cluster analysis does not leave a large undefined group when it is used to form groups of members.

However, there are some limitations to cluster analysis. The major limitations are the following.

1. Cluster analysis is a descriptive procedure and, hence, criterion variables cannot be considered explicitly in the analysis.

2. Solutions obtained by using different types of likeness measures and techniques vary from solution to solution.

3. Cluster analysis generally provide only locally optimal solutions.

4. Relationships among all members are not considered simultaneously in cluster analysis. In analysis where groups are formed by aggregation, attention is given only to two members which are most similar. Since groups formed depend to a great extent on the characteristics of the members of the nuclei, groups from cluster analysis are not stable. This is not the case with factor analysis.

5. There is no way to test the statistical significance of the results.

Multidimensional Scaling

The birth of multidimensional scaling may be traced to 1938 when Young and Householder (1938) proved a set of theorems on distances between points. Fully metric multidimensional scaling methods were developed based on these theorems and major contributions came from Richardson (1938), Klinberg (1941), and Torgerson (1958). Coombs (1950) and Bennet and Hays (1960) contributed to the development of fully nonmetric multidimensional scaling. Development of multidimensional scaling with ordinal input and metric output began in 1962 with the publication by R. N. Shepard (1962a, 1962b) of an innovative paper, in two issues of Psychometrika. Shepard's contributions have been significant since then; Shepard (1963, 1964, 1966); Shepard and Carroll (1966, 1972); Shepard and Kruskal (1964). Other contributions in the areas of models, programs, and applications came from Carroll (1968, 1969a, 1969b, 1971, 1972); Carroll and Chang (1964a, 1964b, 1969, 1970a, 1970b, 1970c, 1972); Carroll and Shepard (1966); Carroll and Wish (1971); Kruskal (1964a, 1964b, 1965); Kruskal and Carmone (1969); Kruskal and Carroll (1969); Lingoes (1965, 1966, 1967); Wish (1967, 1969); Wish and Carroll (1971); Wish, Deutsch, and Biener (1970, 1972); McGee (1968a, 1968b); Tucker and Messick (1963) Young (1968); Young and Pennell (1967); Young and Torgerson (1967); Guttman (1968); and Horan (1969). The INDSCAL (Carroll and Chang, 1969a, 1970a, 1970b, 1970c) and the PREFMAP (Carroll and Chang, 1972) models were a major breakthrough in the field of scaling. Credit for making multidimensional scaling techniques popular in Marketing should go to Green along with Carmone, Rao and Wind; Green and Carmone (1970); Green and Rao (1970, 1971a, 1971b); and Green and Wind (1973). They have made a number of imaginative applications of scaling techniques in different areas of marketing.

Application of multidimensional scaling techniques in market segmentation began in late '60s. Of the various studies conducted, only a few are reported in the literature. Johnson (1971) has reported a pilot study where multi-dimensional scaling techniques were applied to segment the beer market. The data were collected from members of Market Facts' Consumer Mail Panel. Approximately 500 male beer drinkers rated eight brands of beer on 35 attributes. Discriminant functions derived from multiple discriminant analysis were used to construct spatial model of the product category. Products were located in a geometric space by using their average scores on each dimension. Respondents were also represented in the same space by finding each person's "ideal point". The study does not give details as to (a) which method was used to locate the ideal points, (b) whether respondents were clustered to form segments, and (c) whether any attempt was made to relate consumer's preference to personal characteristics or to buying behavior.

Ritchie (1974) used multidimensional scaling techniques to segment respondents based on their individual differences in perception of twelve leisure activities. There were two samples: one primary and the other secondary. The secondary sample consisted of twenty women chosen for convenience. They were used to study variability in rank order measurement when subjects made similarity judgements of the twelve leisure activities in repeated trials. The primary sample consisted of 225 subjects drawn from a consumer panel of 700 females in London, Ontario. Six categories of data were collected from members of the primary sample: (1) similarities/dissimilarities data for the twelve stimuli, (2) criteria of similarities and their relative importance, (3) rank order of preference for stimuli, (4) ratings of stimuli on twelve bipolar scales, (5) interests and participation levels for the activities, and (6) individual values as measured by Rokeach value scales in terms of their relative importance in their personal lives. The first three items were collected from a personal interview and the remaining items by means of a questionnaire left with the respondents. Experimentation with secondary sample had indicated that individual perceptions tended to stabilize after a brief "familiarization period". Hence, similarities/dissimilarities data for primary sample were collected by asking the subjects, first, to sort the activities into unspecified number of categories based on similarity and then to rank order all possible pairs of activities. Data from 200 members were used in the analysis after leaving those who did not provide complete information and those who were judged by the interviewer to be careless or confused.

The 200 subjects were divided into two equal groups, an analysis sample and a validation sample. To determine the dimensionality required to conduct the analysis, two different approaches were employed:

a. Dissimilarity matrices for members of sample A were analyzed by means of INDSCAL for one to five dimensions. Incremental variance accounted for by each additional dimension showed that either two or four dimensions should be used in the remainder of the analysis.

b. INDSCAL does not provide a direct measure for the dimensionality underlying the input data. To provide an outside measure, namely, Kruskal's Stress Index, the average dissimilarities matrices for samples A and B were used as input to TORSCA and solutions were obtained for one to six dimensions. Kruskal's Stress Index showed that three dimensions were sufficient to provide a good fit.

Since INDSCAL and TORSCA did not provide identical solutions to the problem of finding the proper dimensionality to conduct the analysis, consideration was given to another factor. It was decided to find out the increase in the explanation of an individual's data with increase in dimension of INDSCAL solution. An examination of the four-dimensional solution showed that variance accounted for in the case of certain individuals by the fourth dimension was as high as 33 percent. Thirty five percent of sample A was found to weight dimension four at a level of 0.30 or greater. Hence, it was decided to choose a four-dimensional solution for further analysis.

To identify the four dimensions, Carroll and Chang's (1969b) PROFIT routine was first run using as input group average of similarity data ratings of stimuli on the twelve bipolar scales. A four dimensional configuration explained much of the variation in the bipolar scales. Now, identification of the dimensions was made taking the following into account: (a) percentage of variation of the bipolar scales explained by the solution obtained in the PROFIT analysis, (b) the order of importance of different criteria in the similarity judgement for sample A, and (c) the degree of importance of different dimensions in the INDSCAL solution in accounting for the total variance. The results were validated using sample B.

The subjects in sample A and B were now separately clustered using Johnson's (1967) hierarchical clustering routine to form perceptual segments. A distance measure derived from the four-dimensional INDSCAL solution was used to represent likeness of members. To determine the exact number of clusters which "best" described the data, the following criteria were used: (1) the average distance within the cluster, (2) the cluster diameter, that is, the maximum distance between any two members of the cluster, (3) the ratio of the between-cluster to within-cluster average distance, and (4) the ability to obtain replicating clusters across samples A and B. Based on these criteria, a five cluster solution was selected. The weighting of the four dimensions by the clusters was used to define the segments. Every cluster was classified as having placed a high, medium, or low emphasis on each of the four dimensions. The clusters were then defined by the emphasis pattern.

A one-way analysis of variance was used to determine if the five groups differed with respect to (a) mean interest and participation levels of the twelve leisure activities, and (b) mean scores on each of the eighteen Rokeach values. While some of the variables examined provided significant F values across both samples A and B, the majority did not. Finally, an attempt was made to predict an individual's INDSCAL weights by multiple correlation using demographics, interest and participation levels for the activities, and scores on Rokeach values. The results were not satisfactory.

Green and Wind (1973) report three studies where multidimensional scaling techniques were used to relate individual differences in perception/preference to personal characteristics. In all three studies, samples were chosen for convenience and the sizes were small. Differences in perception/preference could not be related to personal characteristics in any of the studies. The authors have this to say about these studies: " . . . our choice of convenience samples has probably led to greater within-sample homogeneity than would have been obtained under conditions involving more representative sample " (p. 228).

Remarks on the use of multidimensional scaling procedures. Multidimensional scaling procedures have certain advantages over traditional statistical procedures. One advantage is in the area of input data. In generating data in multidimensional scaling, the respondents need make only simple comparisons and simple choices, and are to provide only rank-ordered data. Ratio-scaled input will be generated by the various multidimensional scaling procedures.

Another advantage of multidimensional scaling is its wide range applicability. This advantage can be better appreciated if multidimensional scaling procedures are compared with factor analysis. Both may be viewed as procedures to determine the smallest set of dimensions that will adequately represent data. To determine the dimensions, factor analysis begins with a matrix derived from the original data necessitating collection of interval- or ratio-scaled input data. Multidimensional scaling does not require any derived data thus obviating the need for higher level data. Factor analyis assumes that redundancy of observed variables is attributable to their linear dependence on a smaller set of underlying variables. With multidimensional scaling, there is no assumption about the nature of dependence. So, when the underlying structure is severely nonlinear, only multidimensional scaling methods can properly represent data. Thus, multidimensional scaling is applicable in situations where traditional procedures cannot be applied.

However, there are some serious limitations to multidimensional scaling. The major limitations are the following:

1. Output from multidimensional scaling is difficult to interpret.

2. There are no statistical tests to determine the significance of dimensions.

3. Though the major advantage of multidimensional scaling is in the area of input data, collection of data does create certain problems. It is true that multidimensional scaling requires only rank ordered data, but the number of times the ranking is to be done increases rapidly with increase in the number of stimuli. The long time required to complete an interview affects a study several ways: (a) many respondents may be unwilling to take part in the survey, making the final sample used for analysis unrepresentative of the population being studied. (b) fatigue on the part of interviewee may produce incorrect responses, and (c) longer interview may cost more in terms of money for data collection.

4. Finally there is the problem of indeterminateness of solution. Not only are there instances of indeterminateness of solution in multidimensional scaling but nothing much

is known about its relationship to number of stimuli, noise level, type of transformation, etc. Green and Carmone (1970) have cited two instances where the problem has occured: " . . . some nonmetric programs will yield meaningless solutions, with zero stress, under situations where the points can be arranged in two clusters such that all between-cluster dissimilarities exceed within-cluster dissimilarities. Similarly, nonmetric joint space programs can break down if, . . . all individuals prefer a particular stimulus to all other stimuli " (p. 127).

Association Between Multivariate Measurement Sets

Investigation of the relationship between two sets of variables originated with Hotelling (1935, 1936). In two papers, he proposed the idea of canonical correlation analysis. Initially, it was assumed that only the first and largest canonical variate was of importance. But, in 1948, Bartlett (1948) showed that each successive canonical variate makes a distinctive contribution to the relationship between the two sets of variables.

The major problem with canonical correlation analysis has been that the canonical variates may have little correlations with the original variables. This fact was noted by Hotelling (1935) himself in his original paper on the most predictable criterion: " . . . It is entirely possible that the most predictable criterion . . . will have little correlation with many of the original criteria, especially if they are numerous " (p. 142). This fact was also noted by psychologists like Thomson (1940) and Peel (1948) who used canonical correlation analysis to investigate bi-multivariate relationships. Burt (1948) suggested several approaches for studying multivariate relationships and raised questions such as " . . . whether the two general factors underlying two distinct sets of assessments formed in fact essentially one and the same general factor " (p. 95), and " What if any, is the relation between the factors obtained by ordinary factor analysis and the factors which we may suppose to be responsible for these canonical correlations? " (p. 99).

In the mid-fifties, researchers in the area of bi-multivariate relationship could have proceeded along the line of canonical correlation analysis opened by Hotelling or along the line of general factors suggested by Burt. The fact that canonical variates may have little correlations with the original variables seemed to present a dead end. So researchers proceeded along the line of general factors. Of course, there were other reasons too. Tucker (1958) derived interbattery factors based on correlation of tests in one battery with the tests in the other battery. Factors thus derived are common to both batteries. Gibson (1960) pointed out that Tucker's method does not provide orthogonal matrices, that is, matrices whose elements are interpretable as correlations between tests and uncorrelated factors. Gibson then proposed a modification to yield proper orthogonal matrices. Kristof (1967) pointed out deficiencies inherent in the models of Tucker and Gibson. Kristof showed that there was a serious drawback in the interbattery model of Tucker: the emergence of " negative residual communalities and residual correlations exceeding unity . . . after the extraction of interbattery factors " (p. 200). And Gibson's procedure, according to Kristof, " is devised to yield orthogonal interbattery factors if some underlying hardly verifiable assumptions are met " (p. 200). Kristof's solution was to derive two sets of factors - (1) those orthogonal factors that are shared by the batteries and account for the observed interbattery correlations, and (2) those orthogonal factors that are exclusive to one battery or the other. The method involves an orthogonal transformation applied to a preliminary orthogonalization of the combined intra-battery and interbattery correlation matrix. The preliminary factoring employs a communalities solution, substituting communality estimates for the diagonal of the combined correlation matrix. Any estimate of interbattery relationship based on Kristof's interbattery factors would thus necessarily be interpretable only as a proportional measure of the common variance in one battery explained by another battery. Hence, Kristof's model was not suitable to be used as a basis for deriving a comprehensive measure of relationship between two multivariate sets.

Because of the problem inherent in the interbattery models of Tucker, Gibson, and Kristof, research interest focussed again on the canonical correlation analysis model of Hotelling. Proceeding from the fundamentals set forth by Hotelling at the beginning, Stewart and Love (1968) and Miller (1969) working independently developed the concept of redundancy to account for the relationship between two sets of variables. Redundancy in a set of variables is the proportion of variance in the set that is predictable from another set. Stewart and Love, and Miller treated canonical correlation analysis as the equivalent of two principal component analyses on each of the two sets of variables with orthogonal rotations to meet certain criteria. Viewed this way, the sum of squared correlations (loadings) of a canonical variate (factor) with the individual variables within a set may be defined as the variance extracted by that variate from the set of variables. That is, by denoting one set of variables, say, $Y_1, Y_2, \ldots, Y_m$ the criterion set Y and the other set, say, $X_1, X_2, \ldots, X_n$ the predictor set X,

Variance of Y extracted by the jth canonical variate CY_j of the criterion set is given by $\sum_{i=1}^{m} L_{ji}^2$

where L_{ji} is correlation (loading) between Y_i and CY_j.

$$\text{The proportion of variance of Y extracted by } CY_j = \frac{\sum_{i=1}^{m} L_{ji}^2}{m}$$

Therefore, the proportion of variance of Y predictable from the jth canonical variate, CX_j, of the predictor set $= \frac{\lambda_j \cdot \sum_{i=1}^{m} L_{ji}^2}{m}$

where λ_j is the squared jth canonical correlation coefficient.

The proportion of variance of Y predictable from the other set, that is, the redundancy, $R_{Y/X}$ $= \frac{\sum_{j=1}^{k} \lambda_j \cdot \sum_{i=1}^{m} L_{ji}^2}{m}$

where k is the number of canonical variates that may be extracted from the set of variables.

There is a major drawback in using canonical correlation analysis to study bi-multivariate measurement sets, which is due to the restriction that the canonical variates may be maximally correlated. The externally imposed canonical restriction may in some cases contravene to alter intrinsic structural relationships. Besides, the same restriction may lead to derivation of factors which are difficult to interpret. Miller (1969) extended the scope of the concept of redundancy by showing that it can be applied to any set of orthogonal factors, thus freeing the analyst from the restriction of canonical correlation analysis.

Applications to date of the concept of redundancy are quite few. Studies where the concept was applied include Stewart and Love (1968), Miller (1969), Miller and Farr (1971), Wood (1972), and Alpert and Peterson (1972). In the study reported by Stewart and Love (1968), three scores obtained in Pintner General Ability Test and ten scores in the Metropolitan Achievement Test by 230 junior high school students formed one set of variables. The seventh- and eighth-year course grades in English, arithmetic, social studies and science formed the other set. The first two canonical correlations were 0.90 and 0.66. Application of the concept of redundancy showed that 92 percent of the redundant variance of student grades and 93 percent of redundant variance of the thirteen test scores were associated with the first canonical variate. The study showed how important each of the canonical variates were in accounting for variance in the two sets.

Alpert and Peterson (1972) used the concept of redundancy to study the relationship of demographics to the consumption of cold cereal and beer and the frequency of eating out and attending movies. The concept was useful in determining the contribution of the different canonical variates of the predictor set in explaining the variation of the criterion set.

Remarks on using the concept of redundancy to improve prediction of the criterion set: There does not appear in the literature any application where the concept was used for differentially weighting the canonical variates of the predictor set to improve prediction of the criterion set. In the field of Marketing, even the question of weighting of variables does not seem to have attracted much attention. There are exceptions, however.

In an article reviewing a cluster analysis procedure of Green, Frank and Robinson (1967) using distance measures, Morrison (1967) suggested weighting of variables to get meaningful results. Taking as an example selection of test markets for a product involving baby diapers, Morrison wanted more weight to be given to the proportion of households with infants than to newspaper circulation, if the marketer considers the infant variable more important. The author suggested assigning subjective weights to the different variables. Morrison defended his position this way:

> . . . we are inclined to give up "objectivity" . . . for the sake of "relevancy" or "managerial meaningfulness". Since the importance of the variables vis-a-vis the particular problem area is relevant, any relevant distance measure must explicitly take this importance of the variables into account (p. B-779).

In a study reported by Frank and Strain (1972) and described earlier in this chapter, the authors commented on the use of canonical correlation analysis:

> In segmenting groups of consumers, we made use of information on the association between personal characteristics and purchasing behavior. The resulting classification procedure plausibly weighted the original variables in terms of the relative importance of their relationship to purchasing behavior (p. 390).

This was the same weighting scheme suggested by Burt (1948):

> We are not always at liberty to weight our criteria . . . If, on the other hand, we have a priori reasons for believing that the criteria

> do not, as they stand, yield the best possible criteria . . . we may feel justified in giving differential weights to the criteria (p. 98).

In Burt's opinion, the best approach was the "most predictable criterion" developed by Hotelling (1935, 1936). Only the first canonical variate was considered a relevant measure of interbattery relationship in that early period and that might be why Burt suggested using a single variate.

It is true that the canonical variates are weighted combinations of original variables derived to satisfy certain conditions regarding the relationship between the two sets of variables. But, when one is using more than one canonical variate in segmentation as is usually the case, what about weighting the weighted combinations to account for their relative importance? The present study is mainly concerned with this question, and the rationale for weighting the linear combinations is discussed in the following chapter.

CHAPTER III

HYPOTHESES

The present study is concerned with weighting of personal characteristics to better predict buying behavior. All the statistical procedures assign some weights to the personal characteristics. Thus, in regression analysis, the regression coefficients denote the weights assigned to the characteristics. In factor analysis, it is the factor score coefficients. If personal characteristics are used as such in the analysis, say, in a study using cluster analysis, it means that all the chosen characteristics are given an equal weight of unity. If some characteristics are not used in the analysis, it means that these are assigned a weight of zero. So all the personal characteristics receive some weight in segmentation studies. The question is whether the weights the different personal characteristics receive are the proper weights in the context of market segmentation. In this chapter, an attempt is made to answer this question. And some hypotheses are formulated in the belief that a weighting procedure different from the current practices would be better for certain segmentation problems. However, before taking up the question of weighting of the variables, it may be necessary to develop certain criteria for evaluating the effectiveness of different statistical procedures in accomplishing market segmentation objectives. These criteria may be used to test the different hypotheses to be formulated in this chapter. Developing these criteria before statement of the hypotheses may help presenting the latter in a more lucid manner.

Criteria for Evaluating Effectiveness of Statistical Procedures in Market Segmentation

The objectives of market segmentation were set forth earlier in the introductory chapter. The concern here is how to objectively evaluate statistical procedures used in market segmentation. For an answer, one may study current practices in the segmentation field. In studies where segments are formed based on personal characteristics, the analyst examines the various segments to see whether buying behavior characteristics vary across different segments. If there is no variation across the different segments, segmentation is considered not effective. If there are considerable differences, segmentation is considered quite effective. In studies using a large number of personal characteristics, it is not uncommon for the analyst to try different bases of segmentation until significant differences in buying behavior are found across the different segments. Thus, when two different procedures are applied to the same set of data, the procedure which gives rise to greater differences in buying behavior among the various segments may be considered more effective. Now, if the same set of data is used for segmentation, the total sum of squares for all variables is constant, and the greater the difference in buying behavior among the various segments, the smaller the sum of squared error variance. Hence, the squared error variance summed over all buying behavior variables and all segments may be used as a measure of effectiveness of segmentation. Since the sum of squared error variance gets smaller and smaller with increase in the number of groups formed, it is necessary to keep the number of segments the same for objectively evaluating different statistical procedures.

There may be cases where the analyst has prior information of the class memberships of the observations. An example would be a study designed to compare the different procedures, where the sample of observations is generated from populations with known parameter values. In such cases it is possible to find the number of misclassifications when the different procedures are used for segmentation. It may be worthwhile examining the usefulness of this measure in evaluating the procedures. In general, a procedure which results in a lower number of misclassifications would be considered superior to other procedures. However, in the context of market segmentation, a procedure which results in a lower number of misclassifications need not be superior. This is so because market segmentation is not concerned with the assigning of observations into different groups based on their class membership. There may be a large number of characteristics, both buying behavior and personal, which distinguish the different populations. The primary objective of market segmentation is to form groups of consumers with similar buying behavior. Thus, the focus is on buying behavior characteristics and not on personal characteristics. A procedure which gives more importance to buying behavior than other procedures need not necessarily produce lower number of misclassifications.

There is also another reason why number of misclassifications may not be very important in market segmentation. This is the requirement of substantiality - the segments formed should be large enough to warrant separate marketing approaches. Thus, if the observations came from a large number of different populations, the requirement of substantiality might call for the creation of a smaller number of market segments resulting, of course, in an increase in the number of misclassifications.

In the discussion so far, the term "misclassification" referred to the one based on membership in the original populations. A meaningful measure of misclassification can be developed if group membership of observations is determined by criterion set variables. For this purpose, a cluster analysis may be performed using criterion set variables, namely, buying behavior characteristics. The clusters formed are buying behavior clusters in that members in a group are similar to each other with regard to buying behavior. The clustering of observations using criterion set variables in a two-group case is equivalent to the first-level split in the Monitored Automatic Interaction Detection, MAID-M (Gillo and Shelly, 1974). Hence, if a procedure weighting the predictor set variables differently were to obtain groups identical to the buying behavior clusters, the procedure would be combining the best of both worlds of inductive and deductive techniques with none of their drawbacks. The number of misclassifications thus defined based on buying behavior characteristics may, therefore be, used as a criterion for evaluating the effectiveness of different statistical procedures.

Up to now, the discussion has been confined to finding ways of evaluating the statistical procedures from results in an analysis sample. Results of any study which utilizes information in the sample to improve the results may be good only for the data on which the analysis was performed. If generalizations are to be made from the particular set of data, it is necessary to validate the results in a new sample. Validation may be done as follows:

To begin with, there should be an analysis sample and a validation sample. Clusters are formed from the analysis sample using different statistical procedures. For each of these procedures, discriminant functions are derived from the clusters. These discriminant functions are used to classify observations in the validation sample into different groups. Sum of squared error variance of the criterion set variables may then be computed to evaluate the different statistical procedures.

For a two-group case, evaluation of the statistical procedures may be done also using the measure of number of misclassifications based on membership determined by criterion set variables. To apply this measure, sets of two-group clusters are formed from the analysis sample using different procedures. A two-group cluster solution is obtained for the validation sample using the criterion set variables. Observations in the validation sample are then classified using discriminant functions derived from the two-cluster solutions of the analysis sample.

Use of discriminant analysis and classification procedure in this way is a bit different from their applications in the research field. When these procedures are used for validation purpose, observations from the same set of groups are generally used for both the procedures. This is not so in the present case. And the objective of using these procedures here is also different. It is not to predict membership as such in the validation sample but to find out how well the discriminant functions can discriminate the two groups obtained for the validation sample. The importance of the test lies in the fact that the two-group solution is the best for the validation sample. To the extent a discriminant function discriminates the two groups, the procedure yielding that discriminant function is effective in segmentation.

It is to be noted that groups in the validation sample do not have any fixed correspondence with groups in the analysis sample. As such, there is no way of obtaining the measure of number of "hits" from the results of classification analysis. But this is not a problem in the two-group case. Since one is interested only in the discriminatory power of the procedure, groups obtained in classification may be named group 1 or group 2 so as to reduce the number of misclassifications for the analysis. The most effective procedure is the one which gives the minimum number of misclassifications as defined here.

For the case of more than two groups, the relationship between discriminatory power of the procedure and the number of misclassifications as it is determined for the two-group case may not be direct. Thus, the measure of number of misclassifications may not be applicable to the case of more than two groups for validation purpose.

Hypothesis Based on Criterion-Related Procedures

As was noted before, regression analysis, AID, MCA, discriminant analysis, and canonical correlation analysis are the major criterion-related procedures used in market segmentation. Only a unidimensional measure of buying behavior can be used in regression analysis, AID, MCA, and two-group discriminant analysis. A multidimensional measure of behavior is possible in multiple-group discriminant analysis and canonical correlation analysis. Thus, it would be useful to make a subclassification of criterion-related techniques in market segmentation, namely, unidimensional criterion-related techniques and multidimensional criterion-related techniques.

In the case of unidimensional criterion-related techniques, weighting of predictor set variables does not pose a problem. In studies using regression analysis, MCA, or two-group discriminant analysis, the coefficients obtained in the analysis for the different personal characteristics may be considered the proper weights for predicting behavior. In the case of AID, it would be necessary to make a sequential use of MCA or regression analysis to determine the proper weights to be assigned to the characteristics. In all these cases, one is not considering how important each of these variables, taken individually, is in predicting buying behavior; rather, one is considering how important each variable is, given a set of variables. It is this aspect of weighting, which is the subject matter of the present study.

From the discussion above, it is seen that in studies using unidimensional criterion-related techniques, weighting of personal characteristics to predict buying behavior is a simple matter. But how useful is analysis using unidimensional measure of buying behavior? The answer is not much, in many cases. For a segmentation study to be meaningful, it is often necessary to represent buying behavior by a multidimensional measure. This position has theoretical justification and empirical support. As was noted before and pointed out by Lessig and Tollefson (1971), theories of consumer behavior point to a " relationship between personal characteristics and buying behavior type, rather than some single facet of behavior; and in determining this relationship a multivariate measure of behavior should be used " (p. 480). This is how Frank and Massy (1975, pp. 157-158) put the matter:

> Much of the behavior we are interested in studying is a complex of many factors, that is, it is multidimensional in character. We often sidestep this complexity by picking some unidimensional attribute which we assume is an indicator of the more complex phenomena we seek to understand. By far, the most common way of entering a set of purchase variables . . . is to analyze the relationship between each of them and the set of independent variables separately. . . .
>
> For many purposes, however, these measures may be too limited a measure of purchasing behavior in that they fail to come close to fully describing a rather complex phenomenon. . . .
>
> . . . In our opinion what is needed is to extend the analysis of purchasing behavior so that the specification of the dependent variable permits the simultaneous analysis of the complete set of whatever purchase measures are appropriate in the context of the study.

Empirical support for the position taken comes from Kernan (1968). Kernan studied the relationship between personality and use of alternative choice criteria under conditions of uncertainity. No relationship was found between personality variables and univariate measures of the usage of choice criteria. But significant relationship could be found between a multivariate measure of the usage of choice criteria and personality variables.

The above discussion points to the desirability of using procedures which allow taking into account the multidimensional nature of buying behavior. Obviously, multiple-group discriminant analysis and canonical correlation analysis are such type of procedures. In fact, both these procedures can account for a multidimensional measure of buying behavior explicitly in the analysis. There is, however, a difference in the way these two techniques specify the criterion set. To use multiple-group discriminant analysis, the analyst has to use an a priori classification system taking into account the multidimensional nature of buying behavior. Canonical correlation analysis has no such limitation. Frank and Massy (1975) have described the advantages and disadvantages of a priori classification. The only major advantage is that the researcher is less likely to be led astray by spurious classifications arrived at from noise in the data. But an a priori classification system has serious disadvantages in market segmentation. For example, there is no well-developed theory of consumer behavior to guide the researcher in the proper classification of consumers. In addition, when the number of buying characteristics like brands and product types is great as is usually the case, it becomes impossible for the analyst to consider properly all the relevant classifications before reaching a decision. Thus one is led to look for a technique which does not call for an a priori classification of observations. The choice is, obviously, canonical correlation analysis. This does not mean that canonical correlation analysis is superior to multiple-group discriminant analysis for segmenting markets. Multiple-group discriminant analysis is in fact much better than canonical correlation analysis to deal with data problems like weak scale of measurement for the criterion set, nonlinear relationship between the criterion and predictor sets, etc. However, since multiple discriminat analysis may be considered a special case of canonical correlation analysis and since it has the drawback of requiring an a priori classification for buying behavior, the remainder of this section will be focused on canonical correlation analysis to the exclusion of discriminant analysis.

Several studies report the use of canonical correlation analysis for segmenting markets: Green, Halbert and Robinson (1966), Kernan (1968), Alpert (1972), Alpert and Peterson (1972), Baumgarten and Ring (1972), Darden and Reynolds (1971), Lessig and Tollefson (1971), Lessig (1972), Sparks and Tucker (1971), Worthing, Venkatesan and Smith (1972), Frank (1972), Frank and Strain (1972), Gensch and Turner (1972), Neilson and Stanton (1972), Sweeny, Mathews and Wilson (1972). In none of these, is there any indication that the different canonical variates were treated differently in segmenting markets. Alpert and Peterson (1972) emphasized the importance of redundancy in interpreting the results, but they did not use the concept in forming segments.

Canonical variates are linear combinations of original variables derived for maximum correlation between these combinations. As pointed earlier, Hotelling (1935) himself had remarked that " . . . it is possible that the most predictable criterion will have little correlation with the original criteria" (p. 142). Thus, when canonical variates of the predictor set are used for segmentation, one is using a set of variates whose only qualification is that they are maximally correlated with the corresponding pair of the canonical variates of the criterion set, which may or may not have any significant correlations with the criterion set in which the marketer is really interested.

But there is a redeeming feature. As Miller (1969) notes, " . . . canonical analysis does preserve as an intrinsic function of its method, all components of variance relevant to inter-battery dependency" (pp. 132-133).

Thus, when the marketer uses canonical variates for segmentation, he/she is using a set of variates which account fully for the interrelationship between the criterion and predictor sets. What the marketer should do is to weight the canonical variates of the predictor set based on the proportion of criterion set variance explained by each of these variates. It is hoped that a procedure which assigns weights as suggested above will be more effective in segmentation. Accordingly, the following hypothesis is formulated:

Hypothesis 1. In market segmentation studies using canonical correlation analysis, weighting of predictor set variates based on redundancy will result in more effective market segmentation.

Hypotheses Based on Descriptive Procedures

Descriptive procedures are, as Heller (1973) aptly notes, "completely blind" (p. 24). These procedures will form groups on whatever variables put into the analysis. It is left for the marketer to be certain that only the relevant variables are used in the analysis. The variables used in these procedures are personal characteristics. As was noted earlier, the primary objective of market segmentation may be to form groups such that members in a group have similar buying behavior. Thus, though the techniques are not criterion related, the marketer has to make them "criterion related" by choosing variables related to the criterion set. Researchers using descriptive procedures have employed different methods to relate data in the analysis to some outside criterion. Since factor analysis is the most commonly used descriptive procedure in market segmentation, the discussion below will be centered on this procedure.

Some researchers in other areas have performed factor analysis by including criterion set variables as well in the analysis. Only those factors on which at least one criterion variable has a loading above a minimum cut-off level are retained for further analysis. One problem with this method is that it might be necessary to retain a large number of factors. For example, if the cut-off level is set at 0.5, a variable can have high loading on four factors even in the orthogonal case.

To overcome the problem of too many factors, one may use what Hurley and Cattell (1962) call the procrustean transformation. Gruvaeus (1970) suggested a procrustean transformation procedure making it possible for the analyst to rotate factors such that any variable can have specified loadings with the factors. In other words, the analyst is free to specify elements of the factor structure. Thus, it is possible for the analyst to rotate the factors in such a way that the criterion variables will have high loadings on the factors. Here the number of factors to be retained will depend on the number of criterion variables. If each criterion variable is specified to have a loading of 1.0 with a separate factor, then the analysis will yield as many factors as there are variables in the criterion set. If the number of variables in the criterion set is not large, the analysis may be satisfactory as far as the number of factors to be retained in the study is considered. However, the method has some drawbacks. One drawback is that of oblique factors. Another, and a serious one in the context of market segmentation, is that it limits the freedom of the marketer in rotating the factors in the light of the predictor set variables. In segmentation studies, it is imperative that the factors provide meaningful interpretation so that the marketer can create proper marketing mixes. Thus, the marketer should not be constrained to rotate the factors in directions determined by the criterion set variables.

The method to be suggested will account for problems the above methods have. It will at the same time weight the factors in such a way that each of them will get importance based on their relationship to the criterion set. The method involves one more factor analysis - principal component analysis of the criterion set, namely, the buying behavior characteristics. There is a restriction that factors derived from both the analyses, namely, factor analyses of the buying behavior characteristics and the personal characteristics, be orthogonal. Since the factors used in most segmentation studies are orthogonal, restriction to orthogonality may not be considered a limitation. Once factors are derived, factors of the predictor set should be weighted based on the proportion of variation of criterion set explained by each of the factors. It is expected that factors thus weighted will do a better job of segmenting markets than the one with no differential weighting. Accordingly, the following hypothesis is formulated:

Hypothesis 2. In segmentation studies using orthogonal factors, weighting of predictor set variables based on redundancy will result in more effective market segmentation.

There are several instances where segmentation is done using predictor set variables, namely, personal characteristics as such in the analysis. An example is grouping consumers based on personal characteristics by a clustering algorithm. Since personal characteristics are generally correlated, there is no way to determine how important each of these variables is in explaining the total variation of the criterion set variables. Under these conditions, the best thing to do, it seems, is to weight each of the personal characteristics in proportion to the square root of the sum of simple correlations squared between characteristics and each orthogonal factor of buying behavior set. Unlike the two cases discussed earlier, there is strictly speaking, no theoretical justification to weight the variables based on correlations. Besides, there is the problem of suppressor effect - a predictor with zero or low correlation with the criterion contributing markedly through its correlation with other predictors to variance accounted for in the criterion variable. However, as McNemar (1969) notes, the finding of variables which will qualify as suppressants is hard to realize in practice. This, along with the fact that a characteristic highly correlated with buying behavior will always get a greater weight than a characteristic with low correlations with the buying behavior set leads one to hope that any problem created by weighting will be more than offset by advantages resulting from it. Accordingly, a third hypothesis is formulated:

Hypothesis 3. In segmentation studies using personal characteristics as such in the analysis, it would be better to weight them based on their correlations with orthogonal factors of buying behavior characteristics for segmentation to be more effective.

Ancillary Hypotheses

All the three hypotheses formulated earlier are based on correlations of the predictor set with orthogonal factors on the criterion set. It would be interesting to see how the results would differ if the predictor set variates are weighted based on their correlations with the original criterion set variables rather than factors of the criterion set. Weighting of the predictors in this way is expected to give better results than if they are not differentially weighted. Accordingly, the following ancillary hypotheses are formulated:

Hypothesis 1a. In market segmentation studies using canonical correlation analysis, weighting of predictor set variates based on correlations with criterion set variables will result in more effective market segmentation.

Hypothesis 2a. In segmentation studies using factor analysis, weighting of orthogonal factors of the predictor set based on correlations with criterion set variables will result in more effective market segmentation.

Hypothesis 3a. In segmentation studies using personal characteristics as such in the analysis, it would be better to weight them based on their correlations with buying behavior characteristics for segmentation to be more effective.

CHAPTER IV

METHODOLOGY

The basic methodology adopted for the study was simulation. Simulation was done on a sample of observations generated from two different populations. Each observation had measurements on two sets of variables. One set was to represent the criterion set, namely, buying behavior and the other set the predictor set, namely, the personal characteristics. The sample of observations was grouped several times. Each time a different set of weights for predictor set variables or their linear combinations was used. Groups obtained under different weighting procedures were examined to determine the effectiveness of each procedure in accomplishing marketing objectives. Results were then validated on a new sample of observations. Experimentation was done for a sufficiently large number of trials so that statistical inference could be drawn from the results of the study.

Since the complete description of the methodology is lengthy, it may be useful to give an outline first. Simulation was done using observations generated from two different populations. To account for different types of data that are found in market segmentation, six different types of data were used in the study.

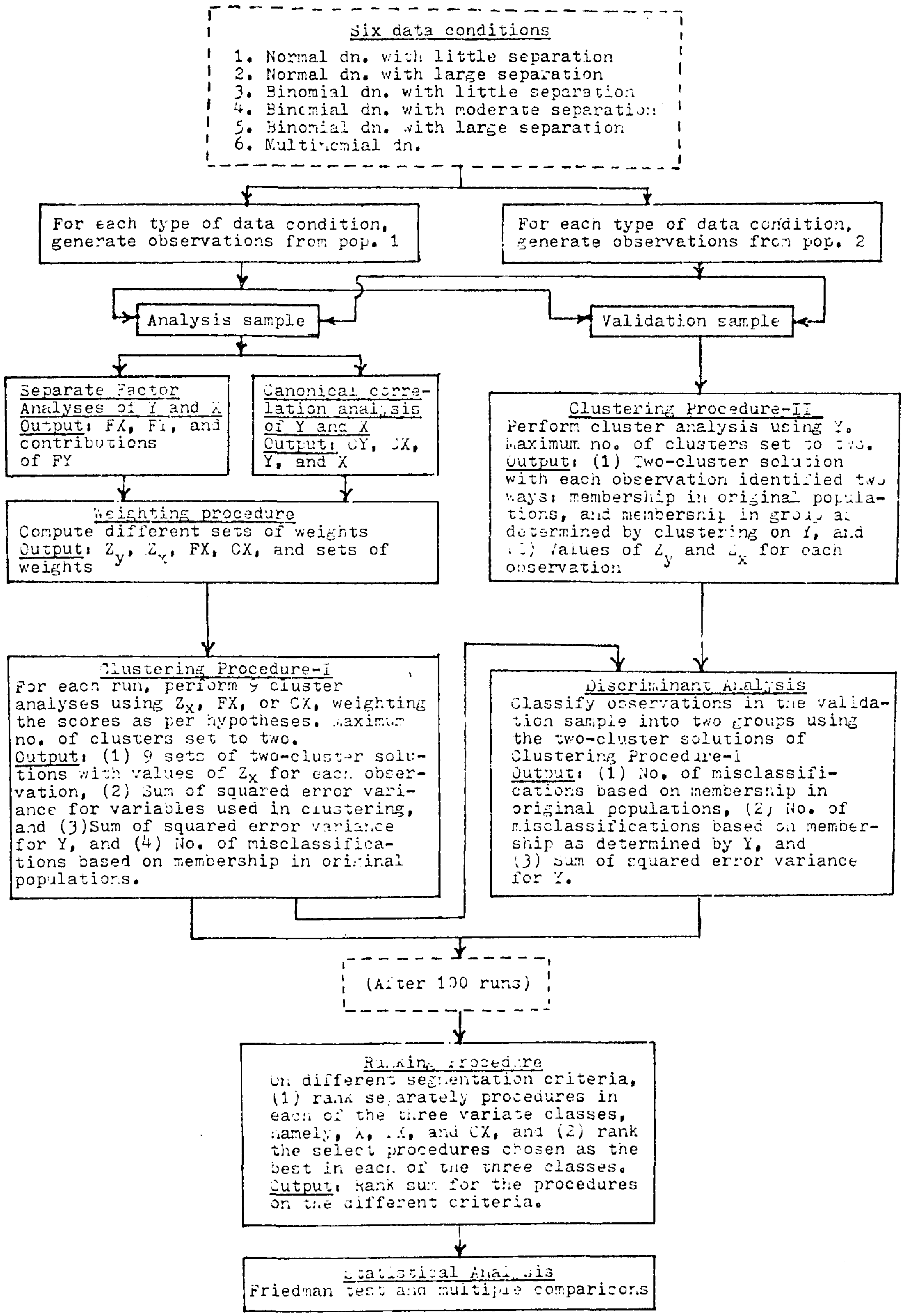
1. Observations from multivariate normal distributions with means of populations close to each other.
2. Observations from multivariate normal distributions with means of populations farther apart.
3. Observations from multivariate binomial distributions with means of populations close to each other.
4. Observations from multivariate binomial distributions with means of populations moderately apart.
5. Observations from multivariate binomial distributions with means of populations farther apart.
6. Observations from multivariate multinomial distributions.

For each case, experimentation was done for 100 trials. The schematic diagram of flow of analysis (see Exhibit 1) shows the sequence of procedures followed for simulation. The first step in simulation was generating observations. Observations generated from two different populations were split into two groups - an analysis sample and a validation sample. Both the samples were of equal size and each consisted of an equal number of observations from the two different populations.

The following operations were performed on the analysis sample. First, canonical correlation analysis was applied. Afterwards, two factor analyses were performed using only the predictor set variables in one case, and only the criterion set variables in the other. Output from

EXHIBIT 1

SCHEMATIC DIAGRAM OF
FLOW OF ANALYSIS



the canonical correlation analysis and the factor analyses were used to compute different sets of weights to be assigned to the predictor set variables and their linear combinations. Three different cluster analyses were then performed assigning predetermined sets of weights to each set of predictor variables, factor scores, and canonical variate scores. In all, there were nine cluster analyses of the analysis sample per run. Each cluster analysis was to yield two clusters, and output included among various measures, the sum of squared error variance for the criterion set variables and number of misclassifications.

Observations in the validation sample were clustered into two groups using measurements on the criterion set variables. This was to determine group membership of each observation based on the criterion set. The same set of observations were then classified nine times, each time using a different discriminant function derived from the two clusters obtained for the analysis sample under each of the weighting schemes. Output of discriminant analysis were three types of measures: (1) number of misclassifications based on membership in the original population, (2) number of misclassifications based on membership determined by criterion set variables, and (3) sum of squared error variance for the criterion set variables.

After 100 trials for each of the six data conditions, ranking procedure was applied to those measures which were indicative of segmentation effectiveness. Results from the ranking procedure were used to draw conclusions regarding the various hypotheses.

Experimental Procedure

The various steps in the experimental procedure are described below.

Generating Observations

Observations were generated from multivariate normal distribution, multivariate binomial distribution, and multivariate multinomial distribution. Thus, there are three different distributions to be considered here.

Multivariate Normal Distribution

Twenty observations each were drawn from two populations having a multivariate normal distribution. The number of twenty was chosen after considering two conflicting factors. The larger the number of observations the better would be the approximation of the distribution to multivariate normal. However, a larger sample would cost more in terms of computer time for the different steps in the analysis. A number of twenty observations from a population was seen to satisfy the condition of multivariate normal distribution. Hence, it was decided to choose twenty observations from a population. Each observation had measures on six variables. The first three, say, Y_1, Y_2, and Y_3 represented buying behavior , and the remaining three say, X_1, X_2, and X_3 the personal characteristics.

Experimentation was done for two different cases.

Population Means Closer to Each Other

In this case, the distributions were assigned the following values for their means:

	Y_1	Y_2	Y_3	X_1	X_2	X_3
M_1	$+.2s_{y_1}$	$+.2s_{y_2}$	$+.2s_{y_3}$	$+.2s_{x_1}$	$+.2s_{x_2}$	$+.2s_{x_3}$
M_2	$-.2s_{y_1}$	$-.2s_{y_2}$	$-.2s_{y_3}$	$-.2s_{x_1}$	$-.2s_{x_2}$	$-.2s_{x_3}$

Here M_i stands for the mean of population i and s for population standard deviation.

Population Means Farther Apart

In this case, the distributions were assigned the following values for their means:

	Y_1	Y_2	Y_3	X_1	X_2	X_3
M_1	$+1.5s_{y_1}$	$+1.5s_{y_2}$	$+1.5s_{y_3}$	$+1.5s_{x_1}$	$+1.5s_{x_2}$	$+1.5s_{x_3}$
M_2	$-1.5s_{y_1}$	$-1.5s_{y_2}$	$-1.5s_{y_3}$	$-1.5s_{x_1}$	$-1.5s_{x_2}$	$-1.5s_{x_3}$

In both the above cases, each of the two population distributions had the same correlation matrix and all the variables had the same standard deviation of unity. Correlation matrix assigned for the distributions is shown in Table 1. Elements of the correlation matrix were so chosen that the criterion set had higher correlations with X_1 than with either X_2 or X_3, and in the same way, the criterion set had higher correlations with X_2 than with X_3. The exact values chosen for the different elements were arbitrary.

TABLE 1

CORRELATION MATRIX FOR NORMAL AND BINOMIAL DISTRIBUTIONS

Variables	Y_1	Y_2	Y_3	X_1	X_2	X_3
Y_1	1.00	0.71	0.50	0.76	0.50	0.10
Y_2	0.71	1.00	0.50	0.71	0.30	0.10
Y_3	0.50	0.50	1.00	0.76	0.35	0.15
X_1	0.76	0.71	0.76	1.00	0.50	0.20
X_2	0.50	0.30	0.35	0.50	1.00	0.65
X_3	0.10	0.10	0.15	0.20	0.65	1.00

Description of the algorithm used for generating the distribution is given in Appendix B-I.

Multivariate Binomial Distribution

As was the case with the multivariate normal distribution, there were three criterion set variables and three predictor set variables. One hundred observations each were drawn from two different populations.

Experimentation was done for three different cases. Marginal probabilities assigned to the different variables for these cases were as follows:

Population Means Closer to Each Other

	Y_1	Y_2	Y_3	X_1	X_2	X_3
Pop. 1	.525	.525	.525	.525	.525	.525
Pop. 2	.475	.475	.475	.475	.475	.475

Population Means Moderately Apart

	Y_1	Y_2	Y_3	X_1	X_2	X_3
Pop. 1	.600	.600	.600	.600	.600	.600
Pop. 2	.400	.400	.400	.400	.400	.400

Population Means Farther Apart

	Y_1	Y_2	Y_3	X_1	X_2	X_3
Pop. 1	.900	.900	.900	.900	.900	.900
Pop. 2	.100	.100	.100	.100	.100	.100

The same correlation matrix (see Table 1) which was used for generating observations for multivariate normal distribution was used for all the above cases.

Description of the algorithm is given in Appendix B-II.

Multivariate Multinomial Distribution

Here too, there were three criterion set variables and three predictor set variables. One hundred observations each from two different populations were generated. The observations were generated under the assumption of independence of variables.

Every observation had one of five values, namely, 1, 2, 3, 4, or 5 on each of the six variables. Marginal probabilities for the variables to receive different values are given in Table 2.

TABLE 2

MARGINAL PROBABILITIES FOR MULTINOMIAL DISTRIBUTION

Marginal Probability	Variables					
	Y_1	Y_2	Y_3	X_1	X_2	X_3
	Population 1					
p_1	.100	.100	.200	.100	.100	.200
p_2	.100	.100	.200	.100	.100	.200
p_3	.100	.100	.200	.100	.100	.200
p_4	.100	.100	.200	.100	.100	.200
p_5	.600	.600	.200	.600	.600	.200
	Population 2					
p_1	.400	.400	.100	.400	.400	.100
p_2	.150	.150	.100	.150	.150	.100
p_3	.150	.150	.100	.150	.150	.100
p_4	.150	.150	.100	.150	.150	.100
p_5	.150	.150	.600	.150	.150	.600

Here, p_i is the marginal probability for a variable to receive the value i.

Canonical Correlation Analysis

The program used for analysis was BMD09M, one of the programs in the Biomedical Computer Program package of University of California at Los Angeles (Dixon, 1973). Input for the analysis was the sample of observations generated in the previous procedure. Analysis was done using Pearson's product moment correlation matrix. For binomial and multinomial data, Pearson's r is not satisfactory. However, its use was predicated on its widespread use in segmentation studies. All the variates were drawn in the analysis so that they might account for 100 percent variance. Output from the analysis consisted of (1) original values of the variables Y_1, Y_2, Y_3, X_1, X_2, and X_3 for the sample of observations and (2) the canonical variate scores CY_1, CY_2, CY_3, CX_1, CX_2, and CX_3 for the same set of observations.

Factor Analysis

The program used for analysis was FACTAN, one of the programs in the OSIRIS III package of the Institute of Social Research, University of Michigan, Ann Arbor, Michigan (Institute of Social Research, 1973). Analysis was done separately for the criterion set and the predictor set. Input was the set of observations from samples generated. Pearson's product moment correlation matrix was used in all cases. Varimax rotation was performed as it is a common practice in market segmentation. All the factors were drawn in to account for 100 percent variance. Output from the analyses were (1) factor scores FY_1, FY_2, and FY_3 for the criterion set, (2) contributions of each of the above three factors, and (3) factor scores FX_1, FX_2, and FX_3 for the predictor set.

Weighting Procedure

In this stage, different sets of weights were computed to be used in cluster analysis. Input data for the weighting procedure were the output from the previous two procedures, namely, canonical correlation analysis and factor analyses. The first step in weighting was computation of the correlation matrix for the three sets of variables, namely, original values, canonical variates, and the factors. In all, there were 18 variables. Different sets of weights were computed as follows.

Weights for Canonical Variate Scores of the Predictor Set

There were two sets of weights to be computed, one based on relationship with canonical variate scores of the criterion set and the other based on relationship with the original criterion set variables.

Weights Based on Relationship with Canonical Variates of the Criterion Set

Since the canonical variates CY and CX are orthogonal, the concept of redundancy can be used to compute the proportion of variance of the criterion set explained by each of canonical variate of the predictor set. Proportion of the variance of the criterion set explained by the jth canonical variate CX_j may be computed as follows:

Since $r_{CY_i CX_j} = 0$, if $i \neq j$,

$$\text{the variance of the criterion set explained by } CX_j = r^2_{CY_j CX_j} \sum_{i=1}^{n} r^2_{CY_j Y_i}$$

where n is the number of variables in the criterion set.

$$\text{Therefore, the proportion of the variance of the criterion set explained by } CX_j = \frac{r^2_{CY_j CX_j} \sum_{i=1}^{n} r^2_{CY_j Y_i}}{\sum_{j=1}^{m} r^2_{CY_j CX_j} \sum_{i=1}^{n} r^2_{CY_j Y_i}}$$

where m is the number of canonical variate pairs.

$$\text{The weight to be assigned to } CX_j = \left(m. \frac{r^2_{CY_j CX_j} \sum_{i=1}^{n} r^2_{CY_j Y_i}}{\sum_{j=1}^{m} r^2_{CY_j CX_j} \sum_{i=1}^{n} r^2_{CY_j Y_i}} \right)^{1/2}$$

Scale factor m is used in the above computation to keep the total variance the same under different weighting procedures.

Weights Based on Relationship with Original Variables of the Criterion Set

Since the variables in the criterion set are correlated, weighting based on criterion set would involve repetitive incorporation of identical information.

The variance in the criterion set explained by CX_j $= \sum_{i=1}^{n} r^2_{CX_jY_i}$

Based on the reasoning followed earlier, the weight to be assigned to CX_j

$$= \left(m. \frac{\sum_{i=1}^{n} r^2_{CX_jY_i}}{\sum_{j=1}^{m}\sum_{i=1}^{n} r^2_{CX_jY_i}} \right)^{1/2}$$

Weights for Factors of the Predictor Set

As before, there were two sets of weights to be computed here, one based on relationship with factors of the criterion set, and the other based on the relationship with the original variables of the criterion set.

Weights Based on the Relationship with Factors of the Criterion Set

Variance explained by the jth factor of the predictor set $= \sum_{i=1}^{n} L_i \cdot r^2_{FY_iFX_j}$

where L_i is the contribution of the ith factor of the criterion set, and n the number of factors in the same set.

Weight to be assigned to the jth factor

$$= \left(m. \frac{\sum_{i=1}^{n} L_i \cdot r^2_{FY_iFX_j}}{\sum_{j=1}^{m}\sum_{i=1}^{n} L_i \cdot r^2_{FY_iFX_j}} \right)^{1/2}$$

where m is the number of factors in the predictor set.

Weights Based on Relationship
with Original Variables of
the Criterion Set

Weight to be assigned to the jth factor of the predictor set $= \left(m. \frac{\sum_{i=1}^{n} r^2_{FX_jY_i}}{\sum_{j=1}^{m}\sum_{i=1}^{n} r^2_{FX_jY_i}} \right)^{1/2}$

where n is the number of variables in the criterion set and m the number of factors in the predictor set.

Weights for Original Variables of the Predictor Set

As before, there were two sets of weights to be computed here, one based on the relationship with factors of the criterion set, and the other based on the relationship with the original variables of the criterion set.

Weights Based on the Relationship
with Factors of the
Criterion Set

Weight to be assigned to the jth variable of the predictor set $= \left(m. \frac{\sum_{i=1}^{n} Li.r^2_{FY_iX_j}}{\sum_{j=1}^{m}\sum_{i=1}^{n} Li.r^2_{FY_iX_j}} \right)^{1/2}$

where n is the number of factors for the criterion set and m the number of variables in the predictor set

Weights Based on the Relationship
with Original Variables of
the Criterion Set

Weight to be assigned to the jth variable of the predictor set $= \left(m. \frac{\sum_{i=1}^{n} r^2_{Y_iX_j}}{\sum_{j=1}^{m}\sum_{i=1}^{n} r^2_{Y_iX_j}} \right)^{1/2}$

where n is the number of variables in the criterion set and m the number of variables in the predictor set.

Clustering Procedure - I

Clustering procedure - I was used for grouping of observations in the analysis sample. Singleton-Kautz algorithm formed the core of the procedure. The objective of the Singleton-Kautz is minimization of within-group variance. The algorithm does not guarantee a global minimum. However, for the present study, it was not considered to be a serious drawback. The purpose of the study was to compare different systems of weightings, and there was no reason to suspect that the algorithm was biased in favor of any particular set of weighting of variables. The application of Singleton-Kautz algorithm for the present study was predicated on its widespread use in market segmentation. In a study designed to simulate market segmentation, application of an algorithm widely used in market segmentation was preferable to others.

Clustering of observations in Singleton-Kautz algorithm is done using the values of these observations on a set of variables and the results of sum of squared error variance and means for different clusters are given in terms of the same set of variables, namely, predictor set variables. But the present study required sum of squared error variance for the criterion set variables and number of misclassifications. Hence, Singleton-Kautz algorithm was modified to give results on sum of squared error variance for the criterion set variables and the number of misclassifications. Modification of the algorithm was also made to meet two other requirements: One requirement was that clustering be done under different weighting schemes; the other was that output include for each clustering solution information regarding predictor variables for observations in each of the two clusters so that this information might be used later to derive discriminant functions for classifying observations in the validation sample.

Input for cluster analysis were (1) standardized scores for the criterion set and the predictor set variables, (2) factor scores for the criterion set and the predictor set, (3) canonical variate scores for the predictor set, and (4) the different sets of weights to be applied to the above scores.

On each analysis sample, nine different cluster analyses were performed using variables and weights as noted below.

1. Original variables X_j were standardized and each variable was given equal weight of unity.
2. Same variables as in (1), but weighted based on their correlations with the criterion set variables Y_i.
3. Same variables as in (1), but weighted based on

their correlations with factors FY_i of the criterion set.

4. Factor scores FX_j of the predictor set, each given an equal weight of unity.
5. FX_j weighted based on their correlations with the criterion set variables Y_i.
6. FX_j weighted according to the proportion of variance of the criterion set explained by it.
7. Canonical variate scores CX_j of the predictor set, each given an equal weight of unity.
8. CX_j weighted based on their correlations with criterion set variables.
9. CX_j weighted according to the proportion of the variance of the criterion set explained by it.

Option of no standardization of variables was chosen for analysis so that the weighting of variables could be effected in clustering. In all cases, the number of clusters to be formed was set to two, which was the same as the number of different populations from which the sample was drawn.

Output included among other measures (1) the sum of squared error variance for the criterion set variables, (2) the number of misclassifications based on membership in the original populations, and (3) the values of predictor set variables for observations in each of the two clusters.

Description of the algorithm is given in Appendix B-III.

Clustering Procedure - II

Clustering procedure II was used to group observations in the validation sample. As was made clear in the earlier chapter, number of misclassifications based on membership in the original populations may not be quite useful in evaluating the effectiveness of different statistical procedures in accomplishing market segmentation objectives. A meaningful measure is the number of misclassifications based on membership as determined by criterion set variables. It was to obtain this measure that clustering procedure - II was applied.

Clustering procedure - II was basically the same as clustering procedure - I. Observations were clustered using criterion set variables. The number of clusters to be formed was set to two. Output from the analysis consisted of values of criterion set and predictor set variables for observations in each of the two clusters.

Discriminant Analysis

Discriminant analysis was performed to test the validity of results obtained in clustering procedure - I.

Input to the program were (1) nine sets of two-group clusters obtained for the analysis sample, and (2) the two groups of observations obtained for the validation sample. For each of the nine sets of the clusters of the analysis sample, a discriminant function was derived. This discriminant function was used to classify members of the validation sample into two groups. Information used for classification was values of predictor set variables for members of the sample.

Output of the program were (1) sum of squared error variance for the criterion set variables of the validation sample summed over variables and groups, (2) number of misclassifications of the validation sample based on membership in the original populations, and (3) number of misclassifications based on membership as determined by criterion set variables.

Ranking Procedure

Procedures in each of the three classes, namely, those based on original variables, factors, and canonical variates were ranked separately. Ranking was done using values obtained for the procedures on different measures of segmentation effectiveness. Another ranking was performed using the best procedure from each of the three classes.

Method of Analysis

For the six different cases considered in the present study, each run in the experiment produced the following measures for the nine different segmentation procedures: (1) the total sum of squared error variance for criterion set of the analysis sample, (2) the number of misclassifications for analysis sample, (3) the total sum of squared error variance for the criterion set of validation sample, (4) the number of misclassifications for validation sample based on membership as determined by criterion set variables, and (5) the number of misclassifications based on membership in the original populations.

The above measures were used to test the different hypotheses formulated for the study. As these measures did not warrant the assumption of any particular distribution to make use of standard parametric tests, non-parametric tests were used. It was for applying these non-parametric tests that the above measures were converted to rank-ordered data. Tests were applied separately for the three groups of procedures - those using canonical variates, factors, and original variables. S' statistic (Hollander and Wolfe, 1973) associated with Friedman rank sums was used for testing overall difference among the methods in each of the three different groups. The number of trials was sufficiently large to make a Chi-square approximation to the S' statistic. S' was computed using the following formula:

$$S' = \frac{12 \sum_{j=1}^{k} (R_j - nR_{..})^2}{nk(k+1) - 1/(k-1) \sum_{i=1}^{n} \left(\sum_{j=1}^{g} t_{i,j}^3 \right) - k} ,$$

where R_j = sum of ranks for method j summed over n trials,

k = number of methods,

$R_{..} = (k+1)/2$

g_i = number of tied groups in trial i

and $t_{i,j}$ = size of the jth tied group in block i, and untied values in a trial are counted as ties of size 1.

If S' was found significant, multiple comparisons among the methods were made. The methods u and v are considered significantly different at an experimentwise error rate, α

$$\text{if } \left| R_u - R_v \right| \geq q(\alpha, k, \infty) \left(\frac{n(k)(k+1)}{12} \right)^{1/2}$$

where $q(\alpha, k, \infty)$ is the upper percentile point of the range of k independent N(0,1) variables. The formula is taken from Hollander and Wolfe (1973, p. 151).

Of the five criteria, two had to do with results in the analysis sample, and the remaining three with results in the validation sample. If the criteria involving the analysis sample did not differentiate the methods being studied, there was no need to validate the results. If one method was found superior in the analysis sample, its superiority had to be validated in a new sample. Thus, for a method to be judged better than another one, it had to fare better both in the analysis sample and in the validation sample.

CHAPTER V

ANALYSIS OF DATA

Analysis of data was first done separately for each of the three classes of procedures, namely, those based on canonical variates, factor scores, and original variables. Best procedure from each of the three classes was then chosen, and final analysis was done using these select procedures.

For analysis of data, information was available on five measures: (1) number of misclassifications for analysis sample based on membership in the original populations, (2) total sum of squared error variance for analysis sample computed for criterion set variables, (3) number of misclassifications for validation sample based on membership in the original populations, (4) number of misclassifications for validation sample based on membership as determined by criterion set variables, and (5) total sum of squared error variance for validation sample computed for criterion set variables. Of these five measures, two were misclassifications based on membership in the original populations. As made clear in an earlier chapter, such measures are of no relevance to segmentation of a market. Hence these two measures were not used directly for analyzing the data.

These measures were, however, useful in giving an idea of the composition of the two groups formed in clustering and discriminant analysis procedures. Values of the two measures for different data types are given in Tables 15-26 in appendix A.

Analysis was performed using mainly the remaining three measures. For brevity and clarity of expression, two short forms are used in this chapter. The short forms are (1) "number of misclassifications" for "number of misclassifications based on membership as determined by criterion set variables", and (2) "sum of squared error variance" for "total sum of squared error variance computed for criterion set variables".

Application of canonical correlation analysis and factor analysis involved use of Pearson's r and so did the computation of different sets of weights. Hence, an understanding of r is important in analyzing the results. Accordingly, the assumptions underlying the use of r are detailed first. Analysis of the results for the different procedures is taken up afterwards.

Assumptions in the Use of Pearson's r

Pearson (1896) introduced the coefficient r to denote the relationship between two variables. Computation of r does not require any assumption beyond applicability of the axioms of ordinary algebra. But the r thus computed has only limited use as generalization from the data is not possible. To generalize from the data, it is necessary to introduce certain assumptions regarding the data. With the introduction of these assumptions, r may be put in the realm of statistical models and the deductive property of the formal model may be utilized to draw statistical inferences.

Some of the assumptions that may be made to increase the deductive power of r are the following:

1. There is a linear relationship between the two variables X and Y.

2. Observations are from the same populations.

3. X and Y have symmetrical distributions

4. Y is related to X in such a way that (a) Y may be expressed as $Y = A + BX + E$, where A and B are constants and E is a random error term, (b) the X's may be considered fixed, that is, they are chosen prior to making experimental observations and do not vary from sample to sample, and (c) the E's are normally and independently distributed with means equal to zero and equal variances.

5. X and Y have a bivariate normal distribution.

When assumption (1) is met, r will be a more meaningful measure of the relationship between the two variables than when the assumption is not met. For example, if the relationship is nonlinear, r will underestimate the relationship between the variables.

When assumption (2) is not met, that is, when the observations are from different populations, the computed value of r does not represent the true relationship between the two variables. Correlations are now affected by the differences in means of the populations.

When assumption (3) is not met, that is, when distributions are skewed, r may not be a meaningful measure of the relationship between two variables. Since r describes data in terms of deviations from the mean, an addition of a single deviant case of sufficient magnitude can make the direction of deviation from the mean the same for all but the new case, thus changing the directions of many observations. Skewness can therefore, affect correlations between variables tremendously. Skewness also affects the range of r. The range of r will not be -1.00 to +1.00, but will be restricted. Use of r with a limited range other than ± 1.00 can introduce distortions into the analysis (Rummel, 1970, p. 275). Carroll (1961) has suggested a method for finding out the limiting values of r for skewed distributions.

A special case of skewness occurs when the data are dichotomous with unequal frequencies. If there are extreme frequencies the range of r will be severely restricted. To overcome this problem, Cureton (1959) suggested using ϕ/ϕ_{max}, and Carroll (1961) the tetrachoric r. But both the methods involve assumptions regarding the underlying distribution thus restricting their applicability. Extreme frequencies in dichotomous data create other problems too. With extreme frequencies, there is a wide fluctuation in the value of r with minor changes in the cell values. Also, differences between values obtained for correlation measures such as Pearson's r and tetrachoric r becomes greater with increase in the extremeness of frequencies. Thus, there is no proper correlation measure for dichotomies with extreme frequencies.

When assumption (4) is met, the analyst can draw all the inferences that are applicable to the linear regression model. Thus, interpretive power of r is greatly increased on satisfying this assumption.

When assumption (5) is met, r will provide a complete description of the joint distribution from information of marginal distributions. As in the case of linear regression model, bivariate normal model has also wide interpretive power. A major useful property of bivariate normal distribution is that the relationship between the two variables is linear (Kendall and Stewart, 1958, vol 1, p. 383). Linearity in the bivariate relationships is a fundamental assumption in factor analysis and canonical correlation analysis. The place of assumptions in the use of r is discussed in Nefzger and Drasgow (1957), Furfey (1958), LaForge (1958), Milholland (1958), and Binder (1959).

Because of the desirable properties of bivariate normal distribution, it is common in studies using various statistical procedures to transform data so that the variables have normal distributions. Though normal marginal distributions do not ensure a bivariate normal distribution, they increase the likelihood. It is possible to transform data to approximate normal distribution in many cases. But dichotomous data are not suitable for such transformation.

Analysis of Procedures Using Canonical Variates

Results were first examined for overall difference among the procedures. If difference was found, multiple comparisons were made.

Tests for Overall Difference

Values of different statistical measures for each of the 100 runs are given in Tables 27-44 in Appendix A. Rank sums, value of S' statistic, and significance level for results obtained are presented in Table 3.

As seen in table 3, on all three criteria, there was an overall difference among the methods for all types of data conditions at a significance level of less than 0.001.

Multiple Comparisons

Multiple comparisons were made for all the six data types. Results of the comparisons are presented in Table 4.

TABLE 3

TESTS FOR OVERALL DIFFERENCE AMONG METHODS USING CANONICAL VARIATES (Number of trials = 100)

Criterion	Type of Data[a]	Rank Sums			Value of S'	Level of Significance ≤
		CX not Differentially Weighted	CX Weighted with respect to Y	CX Weighted with respect to CY		
Squared error variance for analysis sample	A	265.00	167.50	167.50	83.12	0.001
	B	285.00	158.50	156.00	149.23	0.001
	C	300.00	150.00	150.00	200.00	0.001
	D	299.00	150.50	150.50	198.00	0.001
	E	258.00	102.00	240.00	153.35	0.001
	F	299.00	150.50	150.50	182.94	0.001
Number of misclassifications for validation sample	A	141.50	229.50	229.00	69.37	0.001
	B	122.00	238.00	240.00	143.75	0.001
	C	172.00	214.00	214.00	15.68	0.001
	D	150.00	225.00	225.00	57.02	0.001
	E	150.50	288.50	161.00	159.93	0.001
	F	168.00	215.50	216.50	30.88	0.001
Squared error variance for validation sample	A	270.50	162.50	167.50	96.96	0.001
	B	278.50	162.00	159.50	141.71	0.001
	C	288.00	156.00	156.00	154.88	0.001
	D	273.00	163.50	163.50	107.66	0.001
	E	234.50	113.50	252.50	141.62	0.001
	F	265.00	165.50	169.50	123.21	0.001

[a]A: Normal Distribution, Means Closer to Each Other; B: Normal Distribution, Means Farther Apart; C: Binomial Distribution, Means Closer to Each Other; D: Binomial Distribution, Means Moderately Apart; E: Binomial Distribution, Means Farther Apart; F: Multinomial Distribution.

TABLE 4

ABSOLUTE DIFFERENCES IN RANK SUMS ON SEGMENTATION CRITERIA FOR METHODS USING CANONICAL VARIATES (Number of trials = 100)

Criteria[a]	Methods	Difference in Rank Sums: CX Weighted with respect to Y	Difference in Rank Sums: CX Weighted with respect to CY
Normal Distribution with Means Closer to Each Other			
A	CX not differentially weighted	97.5**	97.5**
	CX weighted with respect to Y		0.0
B	CX not differentially weighted	88.0**	87.5**
	CX weighted with respect to Y		0.5
C	CX not differentially weighted	108.0**	103.5**
	CX weighted with respect to Y		4.5
Normal Distribution with Means Farther Apart			
A	CX not differentially weighted	127.0**	129.5**
	CX weighted with respect to Y		2.5
B	CX not differentially weighted	116.0**	118.0**
	CX weighted with respect to Y		2.0
C	CX not differentially weighted	116.5**	119.0**
	CX weighted with respect to Y		2.5
Binomial Distribution with Means Closer to Each Other			
A	CX not differentially weighted	150.0**	150.0**
	CX weighted with respect to Y		0.0
B	CX not differentially weighted	42.0*	42.0*
	CX weighted with respect to Y		0.0
C	CX not differentially weighted	132.0**	132.0**
	CX weighted with respect to Y		0.0

[a]A: Squared error variance for analysis sample; B: Number of misclassifications for validation sample; C: Squared error variance for validation sample.

*$p < .01$

**$p < .0001$

TABLE 4—*Continued*

Criteria[a]	Methods	Difference in Rank Sums: CX Weighted with respect to Y	Difference in Rank Sums: CX Weighted with respect to CY
Binomial Distribution with Means Moderately Apart			
A	CX not differentially weighted	148.5**	148.5**
	CX weighted with respect to Y		0.0
B	CX not differentially weighted	75.0**	75.0**
	CX weighted with respect to Y		0.0
C	CX not differentially weighted	109.5**	109.5**
	CX weighted with respect to Y		0.0
Binomial Distribution with Means Farther Apart			
A	CX not differentially weighted	156.0**	18.0
	CX weighted with respect to Y		138.0**
B	CX not differentially weighted	138.0**	10.5
	CX weighted with respect to Y		127.5**
C	CX not differentially weighted	121.0**	17.5
	CX weighted with respect to Y		138.5**
Multinomial Distribution			
A	CX not differentially weighted	148.5**	148.5**
	CX weighted with respect to Y		0.0
B	CX not differentially weighted	47.5*	48.5*
	CX weighted with respect to Y		1.0
C	CX not differentially weighted	99.5**	95.5**
	CX weighted with respect to Y		4.0

CX Weighted Relative to CY versus CX Not Differentially Weighted

Except for the case of multivariate binomial distribution with means of populations farther apart, the weighting schemes were different at a significance level of less than 0.01. In the case of multivariate binomial distribution, the difference between the two weighting schemes decreased with increase in the distance between the two population means. Finally, when the means were farther apart, the difference was not significant to reject the null hypothesis of no difference between the two weighting schemes. In all cases where a difference between the two schemes was found, the one weighted relative to CY produced better results from a segmentation point of view than the one involving no differential weighting. Thus, except for the case of multivariate binomial distribution with means farther apart, the hypothesis was supported that canonical variates weighted based on redundancy would create better segments than variates not differentially weighted.

CX Weighted Relative to Y versus CX Not Differentially Weighted

In all cases, the differences were large enough to reject the null hypothesis of no difference between the two weighting schemes at a significance level of less than 0.01. The differences were in a direction pointing to the superiority of the differential weighting scheme over the other one.

Thus, the hypothesis was supported that canonical variates weighted relative to criterion set variables would create better segments than canonical variates not differentially weighted.

CX Weighted Relative to CY versus CX Weighted Relative to Y

There was no difference between the weighting schemes for all data types except one. This particular case involved multivariate binomial distribution with means of populations farther apart. In this case, the differences were large enough on all the three criteria to reject the null hypothesis of no difference. The direction of differences indicated that from a segmentation point of view, CX weighted relative to Y was better than CX weighted relative to CY.

Discussion

The performance of the weighting schemes in the case of binomial distribution with means farther apart needs an explanation. CX weighted relative to CY failed to show any improvement over CX not differentially weighted. The data had the following characteristics: (1) data were from two different populations, and (2) data were dichotomies with extreme skewness. These characteristics indicated that two of the assumptions in the use of r were not satisfied. Violation of the first of these two assumptions, that is, the data should be from the same population, might not have any serious effect on the results. This was so because weighting was found useful in the case of multivariate normal distribution even when the means of populations were farther apart. Extreme skewness of the data was thus seen to be the reason for obtaining results different from expected.

Effect of skewness was quite serious in the application of canonical correlation analysis. When the populations were farther apart, grouping of members in a population to a separate cluster should not have been difficult. Such a grouping would have satisfied all the three different criteria. However, clustering based on CX failed in discriminating the two groups. In none of the 100 runs did CX have the number of misclassifications below ninety when misclassifications were determined for analysis sample based on membership in the original populations (see table 23 in appendix A). It should be noted that number of misclassifications as defined for the present study could not exceed 100. Hence results obtained with CX were not much different from a random assignment of the observations into two groups. Thus, when data were skewed, canonical variates lost their "orientation" completely. However, relating CX to Y resulted in the proper re-orientation of CX. So one may infer that canonical variates maintained the distinctive characteristics of the data in some form. Transformation based on weighting relative to Y resulted in the proper re-orientation of data, and better discrimination of observations from the two different populations.

Analysis of Procedures Using Factor Scores

As in the case of canonical variates, analysis was done first for overall difference among the procedures. If difference was found, multiple comparisons were made.

Tests for Overall Difference

Values of different statistical measures for each of the 100 runs are given in tables 27-44 in appendix A. Rank sums, value of S' statistic, and significance level for results obtained are presented in Table 5.

As seen in table 5, there were differences among the procedures on all the three criteria for the following four data types: (1) multivariate normal distribution with means of populations closer to each other, (2) multivariate normal distribution with means farther apart, (3) multivariate binomial distribution with means closer to each other, and (4) multivariate binomial distribution with means moderately apart. The level of significance was less than 0.002 on all the three criteria.

For multivariate binomial data with means farther apart, S' had low values showing that there was not much difference among the procedures.

For multivariate multinomial data, the sum of squared error variance for the analysis sample showed differences

TABLE 5

TESTS FOR OVERALL DIFFERENCE AMONG METHODS
USING FACTORS
(Number of trials = 100)

Criterion	Type of Data[a]	Rank Sums			Value of S'	Level of significance <
		FX not Differentially Weighted	FX Weighted with respect to Y	FX Weighted with respect to FY		
Squared error variance for analysis sample	A	276.00	159.00	165.00	105.56	0.001
	B	245.00	185.50	169.50	37.80	0.001
	C	298.00	149.50	152.50	194.08	0.001
	D	286.00	155.50	158.50	162.62	0.001
	E	197.00	199.50	203.50	0.49	—
	F	246.50	176.50	177.00	53.11	0.001
Number of misclassifications for validation sample	A	134.50	235.00	230.50	88.30	0.001
	B	165.00	213.00	222.00	24.15	0.001
	C	175.00	212.50	212.50	12.50	0.003
	D	157.00	223.00	220.00	41.16	0.001
	E	193.50	199.00	207.50	2.97	—
	F	199.00	200.50	200.50	1.00	—
Squared error variance for validation sample	A	277.50	158.50	164.00	110.73	0.001
	B	243.00	183.00	174.00	35.40	0.001
	C	292.00	152.50	155.50	171.05	0.001
	D	271.00	164.50	164.50	110.79	0.001
	E	196.50	202.50	201.00	0.46	—
	F	201.00	199.50	199.50	1.00	—

[a]A: Normal Distribution, Means Closer to Each Other; B: Normal Distribution, Means Farther Apart; C: Binomial Distribution, Means Closer to Each Other; D: Binomial Distribution, Means Moderately Apart; E: Binomial Distribution, Means Farther Apart; F: Multinomial Distribution.

among the procedures at a significance level of less than 0.0005. However, there was practically no difference among the methods when judged against sum of squared error variance for the validation sample, or number of misclassifications in the validation sample. Since the results were not validated, it was concluded that there was no difference among the procedures for the case of multivariate multinomial distribution.

Multiple Comparisons

Multiple comparisons of the weighting procedures were made for the four data conditions where overall differences were found. Results of the comparisons are given in Table 6.

FX Weighted Relative to FY versus FX Not Differentially Weighted

There was a difference between the two weighting schemes on all three criteria for the four different data conditions. The significance level was less than 0.025. The direction of difference indicated that differential weighting improved results from a segmentation point of view. Thus, the hypothesis was supported that for multivariate normal distribution, and multivariate binomial distribution with means not farther apart, FX weighted based on redundancy would create better segments than FX not differentially weighted.

TABLE 6

ABSOLUTE DIFFERENCES IN RANK SUMS ON SEGMENTATION CRITERIA FOR METHODS USING FACTOR SCORES (Number of trials = 100)

Criteria[a]	Methods	Difference in Rank Sums	
		FX Weighted with respect to Y	FX Weighted with respect to FY
Normal Distribution with Means Closer to Each Other			
A	FX not differentially weighted	117.0***	111.0***
	FX weighted with respect to Y		6.0
B	FX not differentially weighted	100.5***	96.0***
	FX weighted with respect to Y		4.5
C	FX not differentially weighted	119.0***	113.5***
	FX weighted with respect to Y		5.5
Normal Distribution with Means Farther Apart			
A	FX not differentially weighted	59.5***	75.5***
	FX weighted with respect to Y		16.0
B	FX not differentially weighted	48.0*	57.0**
	FX weighted with respect to Y		9.0
C	FX not differentially weighted	60.0***	69.0***
	FX weighted with respect to Y		9.0
Binomial Distribution with Means Closer to Each Other			
A	FX not differentially weighted	148.5***	145.5***
	FX weighted with respect to Y		3.0
B	FX not differentially weighted	37.5*	37.5*
	FX weighted with respect to Y		0.0
C	FX not differentially weighted	139.5***	136.5***
	FX weighted with respect to Y		3.0

[a]A: Squared error variance for analysis sample; B: Number of misclassifications for validation sample; C: Squared error variance for validation sample.

*$p < 0.025$

**$p < 0.005$

***$p < 0.0001$

TABLE 6—Continued

Criteria[a]	Methods	Difference in Rank Sums	
		FX Weighted with respect to Y	FX Weighted with respect to FY
Binomial Distribution with Means Moderately Apart			
A	FX not differentially weighted	130.5***	127.5***
	FX weighted with respect to Y		3.0
B	FX not differentially weighted	66.0***	63.0***
	FX weighted with respect to Y		3.0
C	FX not differentially weighted	106.5***	106.5***
	FX weighted with respect to Y		0.0

FX Weighted Relative to Y versus
FX Not Differentially Weighted

The results were similar to those obtained for the case of FX weighted relative to FY versus FX not differentially weighted. Thus, the hypothesis was supported that for multivariate normal distribution, and multivariate binomial distribution with means not farther apart, FX weighted relative to Y will create better segments than FX not differentially weighted.

FX Weighted Relative to FY versus
FX Weighted Relative to Y

The two procedures were not different for any data condition.

Discussion

Weighting was found useful for four data conditions and not useful for two data conditions. The characteristics of the data were studied to find out why weighting schemes did not increase the effectiveness of segmentation in the two cases. For normal distribution, when the distance between the means of the populations increased, the improvement due to weighting decreased. The same held true for binomial case when the two criteria based on sum of squares were considered. For criteria based on misclassifications, the results were a bit different. However, there seemed to be justification to conclude that when distance between means of populations increased, the effectiveness of weighting decreased.

When populations were farther apart, it should have been a simple matter to group the observations in a population to a separate cluster. And this would have satisfied the three different criteria. However, an examination of actual values obtained for misclassifications based on membership in the original populations showed that FX did not discriminate the groups properly in the case of binomial distribution with means farther apart (see table 23 in appendix A). As seen in table 23, there was not a single run with number of misclassifications less than ninety. This was not much different from a random assignment of the observations into two groups. The case of normal distribution with means farther apart was different (see table 17 in appendix A). There were, in fact, three cases with 100 percent correct classification. Hence, differences in means of populations alone might not have reduced the effectiveness of FX to such a low level as in the case of binomial distribution with means farther apart. When the fact was considered that binomial distribution in this case was also characterized by extreme skewness, the reason for the reduction in effectiveness of FX and the weighting schemes became clear. The reason was this: distance between populations affected FX and weighting procedures to some extent, but skewness of distribution had much more effect on both of them.

The results obtained for multivariate multinomial distribution could also be explained. In this case, the data were characterized by (1) moderate distance between means of populations, and (2) moderate skewness. Together, these two characteristics made the weighting schemes ineffective.

Analysis of Procedures Using Original Variables

As in the previous two cases, analysis was done first for overall difference among the procedures. Multiple comparisons were then made for those cases where overall differences were found significant.

Tests for Overall Difference

Values of different statistical measures for each of the 100 runs are given in tables 27-44 in Appendix A. Rank sums, value of S' statistic, and significance level for results obtained are presented in Table 7.

As table 7 shows there were differences among the procedures on all the three criteria for the following three data types: (1) multivariate normal distribution with means of populations closer to each other, (2) multivariate binomial distribution with means closer to each other, and (3) multivariate binomial distribution with means moderately apart.

There was no difference among the weighting procedures on all the three criteria when the data were from multivariate binomial distribution with means farther apart.

TABLE 7

TESTS FOR OVERALL DIFFERENCE AMONG METHODS USING ORIGINAL VARIABLES (Number of trials = 100)

Criterion	Type of Data[a]	Rank Sums			Value of S'	Level of significance <
		X not Differentially Weighted	X Weighted with respect to Y	X Weighted with respect to FY		
Squared error variance for analysis sample	A	297.00	143.50	159.50	166.57	0.001
	B	224.00	184.50	191.50	35.20	0.001
	C	282.50	158.00	159.50	137.52	0.001
	D	290.00	155.00	155.00	162.00	0.001
	E	200.00	201.50	198.50	3.00	—
	F	185.00	204.00	211.00	4.51	—
Number of misclassifications for validation sample	A	129.50	243.50	227.00	94.89	0.001
	B	199.50	201.50	199.00	0.35	—
	C	170.50	215.50	214.00	18.94	0.001
	D	153.00	223.50	223.50	47.51	0.001
	E	199.00	200.50	200.50	1.00	—
	F	211.50	195.50	193.00	4.83	—
Squared error variance for validation sample	A	292.00	147.50	160.50	151.70	0.001
	B	200.50	198.50	201.00	0.39	—
	C	288.50	156.50	155.00	170.28	0.001
	D	283.00	158.50	158.50	148.15	0.001
	E	201.00	199.50	199.50	1.00	—
	F	154.50	223.00	222.50	71.80	0.001

[a]A: Normal Distribution, Means Closer to Each Other; B: Normal Distribution, Means Farther Apart; C: Binomial Distribution, Means Closer to Each Other; D: Binomial Distribution, Means Moderately Apart; E: Binomial Distribution, Means Farther Apart; F: Multinomial Distribution.

In the case of multivariate normal distribution with means farther apart, there was difference among the procedures when the criterion was the sum of squares for the analysis sample. But on the other two criteria based on validation sample, the procedures did not differ. Since difference among the weighting schemes was not validated, it was considered that there was no difference among the procedures for this data type.

In the case of multivariate multinomial data, there was no difference among the procedures on the criteria of sum of squares for the analysis sample. On the criteria of misclassifications for the validation sample too, no difference could be found among the procedures. However, the procedures were found to differ when sum of squares for validation sample was used as a criterion. Since there was no difference among the procedures to begin with in the analysis sample, no importance was attached to the difference found in the validation sample for sum of squares.

Multiple Comparisons

Multiple comparisons of the weighting procedures were made for the three data conditions where overall differences were found. Results of comparisons are given in Table 8.

TABLE 8

ABSOLUTE DIFFERENCES IN RANK SUMS ON SEGMENTATION CRITERIA FOR METHODS USING ORIGINAL VARIABLES
(Number of trials = 100)

Criteria[a]	Methods	Difference in Rank Sums: X Weighted with respect to Y	Difference in Rank Sums: X Weighted with respect to FY
Normal Distribution with Means Closer to Each Other			
A	X not differentially weighted	153.5***	137.5***
	X weighted with respect to Y		16.0
B	X not differentially weighted	114.0***	97.5***
	X weighted with respect to Y		16.5
C	X not differentially weighted	144.5***	131.5***
	X weighted with respect to Y		13.0
Binomial Distribution with Means Closer to Each Other			
A	X not differentially weighted	124.5***	123.0***
	X weighted with respect to Y		1.5
B	X not differentially weighted	45.0**	43.5*
	X weighted with respect to Y		1.5
C	X not differentially weighted	132.0***	133.5***
	X weighted with respect to Y		1.5
Binomial Distribution with Means Moderately Apart			
A	X not differentially weighted	135.0***	135.0***
	X weighted with respect to Y		0.0
B	X not differentially weighted	73.5***	73.5***
	X weighted with respect to Y		0.0
C	X not differentially weighted	124.5***	124.5***
	X weighted with respect to Y		0.0

[a]A: Squared error variance for analysis sample; B: Number of misclassifications for validation sample; C: Squared error variance for validation sample.

*p $<$ 0.01

**p $<$ 0.005

***p $<$ 0.0001

Original Variables Weighted Relative to FY versus Original Variables Not Differentially Weighted

In all three cases there was difference between the procedures on all three criteria. The direction of the differences indicated that weighting improved the results from a segmentation point of view. Thus, for the cases of multivariate normal and multivariate binomial data types with means not farther apart, the hypothesis was supported that weighting relative to FY increased the effectiveness of the original predictor variables in segmentation of markets.

Original Variables Weighted Relative to Y versus Original Variables Not Differentially Weighted

The results were almost similar to the above cases where weighting relative to FY was considered against no differential weighting. Thus, the hypothesis was supported that for the case of multivariate normal and multivariate binomial data with means not farther apart, original variables weighted relative to Y created better segments than original variables not differentially weighted.

Original Variables Weighted Relative to FY versus Original Variables Weighted Relative to Y

There was no difference between the procedures for the three data types.

Discussion

Weighting was useful for the case of multivariate normal distribution and multivariate binomial distribution when the means of populations were not farther apart. For the above distributions, weighting was not found effective when the means of populations were farther apart. In the case of multivariate multinomial distributions too, weighting was not effective. The ineffectiveness of weighting for the three different data types needed an explanation.

When the means of populations were farther apart, original variables were able to group observations from a population to a separate cluster with only a few misclassifications (see tables 17 and 23 in appendix A). Groups thus formed were effective from a segmentation point of view on all the three criteria. Since original variables did the job of properly grouping the observations, there was nothing to be gained by weighting. Besides, the effect of r decreased when the means of populations were farther apart. These two factors were responsible for the ineffectiveness of differential weighting in the case of multivariate normal distribution with means farther apart.

In the case of binomial data, skewness also could have played a part in reducing the effectiveness of weighting. Consequently, no difference could be found among the procedures when the data were from multivariate binomial distribution with means farther apart.

Multivariate multinomial data used in the study were characterized by moderate distance between the means of populations and moderate skewness. So, to a certain extent, the same factors were operative here as those in the case of multivariate binomial data with means farther apart. And the effect was also the same; weighting did not serve any useful purpose.

Analysis of Select Procedures

In this stage of the analysis, the procedure which was found to be the best in each of the three variate classes was chosen, and a comparative study of the select procedures was done. Since there was no procedure which was superior across all data types in any of the three classes, it was only proper to choose the best procedure separately for each of the six different data types. The procedures were chosen based on the sum of squared error variance for the analysis sample. If two procedures tied on this criteria, consideration was given to sum of squared error variance for the validation sample. Procedures selected from each variate class for the six data types are given in Table 9. The procedures were studied first for overall difference. If difference was found, multiple comparisons were made.

TABLE 9

SELECT PROCEDURES FROM VARIATE CLASSES
FOR DIFFERENT DATA TYPES

Type of Data[a]	Variate Class		
	Original Variables	Factor Scores	Canonical Variates
A	X weighted with respect to Y	FX weighted with respect to Y	CX weighted with respect to Y
B	"	FX weighted with respect to FY	CX weighted with respect to CY
C	"	FX weighted with respect to Y	"
D	"	"	"
E	X weighted with respect to FY	FX not differentially weighted	CX weighted with respect to Y
F	X not differentially weighted	FX weighted with respect to Y	"

[a]A: Normal Distribution, Means Closer to Each Other;
B: Normal Distribution, Means Farther Apart;
C: Binomial Distribution, Means Closer to Each Other;
D: Binomial Distribution, Means Moderately Apart;
E: Binomial Distribution, Means Farther Apart;
F: Multinomial Distribution.

Tests for Overall Difference

Values of different statistical measures for each of the 100 runs can be found in tables 27-44 in appendix A. Rank sums, value of S' statistic, and significance level for results obtained are presented in Table 10.

As seen in table 10, on the criteria of sum of squares for analysis sample, there was an overall difference among the procedures for the case of normal distribution with means closer to each other. But the difference was not significant on the two criteria based on validation sample. Hence it was concluded that there was no difference among the procedures when the data were from normal distributions with means of populations closer to each other. Overall difference was not significant also in two other cases where the data were from multivariate binomial distributions with means not farther apart.

There were differences among the procedures for the three remaining three data types. Significance level for the three cases on all the three criteria was less than 0.001.

Multiple Comparisons

Multiple comparisons of the procedures were made for the three data types where overall differences were found. Results of the comparisons are given in Table 11.

TABLE 10

TESTS FOR OVERALL DIFFERENCE AMONG SELECT PROCEDURES (Number of trials = 100)

Criterion	Type of Data[a]	Rank Sums			Value of S'	Level of Significance <
		Procedure Using X	Procedure Using FX	Procedure Using CX		
Squared error variance for analysis sample	A	185.0	217.0	198.0	5.20	0.100
	B	155.5	299.0	145.5	157.35	0.001
	C	199.5	201.0	199.5	2.00	—
	D	196.0	209.5	194.5	18.20	0.001
	E	177.0	294.5	128.5	165.59	0.001
	F	210.0	273.0	117.0	123.18	0.001
Number of misclassifications for validation sample	A	194.5	200.5	205.0	0.67	—
	B	169.0	271.5	159.5	102.85	0.001
	C	199.0	200.5	200.5	0.50	—
	D	198.0	203.0	199.0	1.40	—
	E	161.5	286.0	152.5	131.77	0.001
	F	202.0	223.5	174.5	17.24	0.001
Squared error variance for validation sample	A	201.0	195.0	204.0	0.46	—
	B	169.5	275.0	155.5	103.15	0.001
	C	198.0	199.5	202.5	3.50	—
	D	195.0	205.0	200.0	5.00	0.100
	E	167.0	285.0	148.0	127.38	0.001
	F	183.5	258.0	158.5	76.01	0.001

[a]A: Normal Distribution, Means Closer to Each Other; B: Normal Distribution, Means Farther Apart; C: Binomial Distribution, Means Closer to Each Other; D: Binomial Distribution, Means Moderately Apart; E: Binomial Distribution, Means Farther Apart; F: Multinomial Distribution.

TABLE 11

ABSOLUTE DIFFERENCES IN RANK SUMS ON SEGMENTATION CRITERIA FOR SELECT PROCEDURES
(Number of trials = 100)

Criteria[a]	Methods	Difference in Rank Sums	
		Procedure using FX	Procedure using CX
Normal Distribution with Means Farther Apart			
A	Procedure using X	143.5***	10.0
	Procedure using FX		153.5***
B	Procedure using X	102.5***	9.5
	Procedure using FX		112.5***
C	Procedure using X	105.5***	14.0
	Procedure using FX		119.5***
Binomial Distribution with Means Farther Apart			
A	Procedure using X	117.5***	48.5**
	Procedure using FX		166.0***
B	Procedure using X	124.5***	9.0
	Procedure using FX		133.5***
C	Procedure using X	118.0***	19.0
	Procedure using FX		137.0***
Multinomial Distribution			
A	Procedure using X	63.0***	93.0***
	Procedure using FX		156.0***
B	Procedure using X	21.5	27.5*
	Procedure using FX		49.0**
C	Procedure using X	74.5**	25.0*
	Procedure using FX		99.5***

[a]A: Squared error variance for analysis sample; B: Number of misclassifications for validation sample; C: Squared error variance for validation sample.

*$p < 0.1$

**$p < 0.005$

***$p < 0.001$

Procedure Based on Original Variables versus Procedure Based on Factor Scores

There was significant difference between the procedures in the case of the three data types - multivariate normal distribution with means of populations farther apart, multivariate binomial distribution with means farther apart, and multivariate multinomial distribution. For these data types, the significance level was less than 0.001 for sums of squares and nearly 0.2 for misclassification. The direction of the differences indicated that procedures based on original variables would be better for segmentation than procedures based on factor scores.

Procedure Based on Original Variables versus Procedure Based on Canonical Variates

Considering,first,the data type of multivariate normal distribution with means farther apart, values obtained for the procedure based on canonical variates were better than the other on all the three segmentation criteria. However, the differences were not large enough to reject the null hypothesis of no difference between the procedures. For the data type of multivariate binomial distribution with means farther apart, there was difference between the procedures at a significance level of less than 0.005 on the criterion of sum of squares for the analysis sample, and nearly 0.2 on the criterion of sum of squares for the validation sample. For the data type of multivariate multinomial distribution, there was difference between the procedures on all the three criteria. The significance level was less than 0.001 on the criterion of sum of squares for the analysis sample and nearly 0.1 on the two criteria involving the validation sample. The direction of the differences indicated that, from a segmentation point of view, procedures based on canonical variates were superior to procedures based on original variables.

Procedure Based on Factor Scores versus Procedure Based on Canonical Variates

There was difference between the procedures on all the three criteria in the case of the three data types - multivariate normal distribution with means farther apart, multivariate binomial distribution with means farther apart, and multivariate multinomial distribution. The level of significance was less than 0.001 in all cases. The direction of the differences indicated that procedures based on canonical variates were better for segmentation than procedures based on factor scores.

Discussion

Analysis showed that there was no difference among the select procedures when the means of populations were not farther apart or when the distributions were not highly skewed.

These were the very same situations when the different assumptions in the use of factor analysis and canonical correlational analysis as well as in the computation of weights were satisfied. As found in the separate analyses of the procedures, weighting was most effective for data types of this description. Thus, when data types were such as to satisfy the various assumptions in the use of the techniques and the computation of weights, the procedures based on differential weighting of the variates were all quite effective, and there was no difference among the select procedures chosen from the three variate classes.

When the means were farther apart or when the distributions were highly skewed, there appeared overall differences among the procedures. These were the situations when some of the the assumptions in the application of factor analysis or canonical correlation analysis and in the computation of weights were violated. Three of the data types used in the study had these characteristics, and in each of these cases, procedure based on factor scores fared badly.

The poor performance of factor-analytic procedures relative to others needed an explanation. When the means of populations were farther apart, grouping of observations in a population to a separate cluster based on original variables was a simple matter. Clusters thus formed by original variables could be expected to satisfy to a great extent the different segmentation criteria. For data types considered in the present study, skewness resulted from the distance between means of populations. So, grouping based on original variables could be effective in grouping observations from skewed distributions too, at least for data types used in the study. But this was not the case with factor-analytic procedures. When the means were farther apart, or when the distributions were skewed, factor analysis failed in maintaining the distinct structure of data as made clear earlier when procedures based on factor scores were studied separately. Thus, for data types where overall differences among the procedures were found, factor-analytic procedures had to fare badly relative to procedures based on original variables.

Performance of procedures based on canonical variates was in stark contrast to that of procedures based on factor scores. Though both procedures used orthogonal factors, canonical correlation analysis, unlike factor analysis, maintained the distinct structure of data in some form even when the distributions were highly skewed or when the means of populations were farther apart. That was why factor-analytic procedures could not perform as well as procedures based on canonical variates.

The wide range of applicability of procedures based on canonical variates was well demonstrated in cases where the data were from (a) multivariate binomial distribution with means farther apart, and (b) multivariate multinomial distribution. In both cases, the data were skewed, and there was at least moderate separation between the populations; therefore, procedures based on original variables could be expected to do well in grouping the observations properly. Still, procedures based on canonical variates outdid those based on original variables. And the differences were significant on all three criteria.

The values obtained for the procedure based on canonical variates were better, also, in the case of multivariate normal distribution with means farther apart. However, the differences were not large enough to reach a conclusion regarding the superiority of this procedure over the one using original variables.

CHAPTER VI

ILLUSTRATIVE APPLICATION

Experimentation with generated data showed the effect of weighting of variables on segmentation. It would then only be natural to apply the different weighting schemes to actual consumer data. The present chapter is the result of endeavour in this regard.

Description of Data

The data for the study were obtained from Haley Overholser & Associates, Inc. The data were collected for devising a marketing strategy for a particular brand of shave cream. Information on buying behavior and personal characteristics was obtained from a national probability sample of well over 1000 respondents. Most of the information was collected through personal interview and the rest by means of a questionnaire left with the respondents.

The present study used only a small portion of the vast amount of information collected during the survey. Three types of information regarding the respondents were used in the study: (1) ratings of four different brands of shave cream by the respondents, (2) importance of different benefits sought from the product by the respondents, and (3) psychographics.

Relevant portions of the questionnaire are given in Appendix C.

Procedure and Design of Study

There were 802 respondents who answered all the questions considered in the present study.[1] These respondents were randomly assigned to two equal groups of size 401. Two separate independent samples were used in the study to get some feel as to whether results obtained for a sample were specific to that particular sample. Since the major purpose of the study was to determine the effect of weighting on segmentation, it was decided to segment consumers in a number of ways using different bases and different weighting schemes. Bases used for segmentation were benefits and psychographics. Procedure followed in segmenting the market using these bases are described below.

Segmentation Based on Benefits

The importance respondents gave to different product benefits was used as personal characteristics set for segmentation purpose. Ratings of four different brands formed the buying behavior set.

A study of correlation coefficients showed that the ratings of different brands were almost independent. However, principal components analysis was performed on both samples

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[1]Information regarding a few respondents were lost in tape mishandling.

to obtain truly orthogonal variates. The program used for components analysis was FACTAN, one of the programs in the OSIRIS III Statistical Package (Institute of Social Research, 1973). Separate factor analyses of benefits were performed on the samples, and in each case all the factors were drawn in. After examining the values of characteristic roots, and the loadings of variables on the factors, it was decided to draw in four factors. These four factors accounted for 53.59 percent variance in sample 1 and 51.74 percent variance in sample 2. Varimax rotations were used to improve interpretability of factors.

Canonical correlation analysis was done using the factors obtained for ratings and benefits. The program used was BMD09M of BMD computer program package (Dixon, 1973). The factors, and canonical variates of benefits sets were weighted with respect to original criterion set variables, namely, ratings, and their orthogonal factors. Cluster analysis was then performed using factors and canonical variates of the benefits under different weighting schemes. Cluster analysis was set to yield two-cluster and four-cluster solutions. Cluster analysis was also performed using factors of ratings to obtain a measure of segmentation effectiveness for the different procedures.

The data of ratings and benefits had extreme skewness for a number of variables. Since Pearson's correlations were not suitable for such data, various transformations were applied to reduce skewness. Segmentation was done as before. Use of highly skewed data with and without transformations in segmentation might be expected to throw some light on the usefulness of distributional transformations in segmentation studies using factor analysis and canonical correlation analysis. To save computer time, analysis was confined to two-cluster solutions.

Segmentation Based on Psychographics

Psychographics were used to segment the market in two different ways. In one case, ratings of brands formed the buying behavior set, and in the other, attitudes towards different product attributes, that is, benefits.

Segmentation procedures followed for the case which used ratings of brands for buying behavior set were quite similar to those used earlier in the case of benefits. Four factors were drawn in from both samples, and clustering was done using factors and canonical variates of psychographics under different weighting schemes.

For the case where benefits formed the buying behavior set, segmentation procedures used were somewhat different. First, a correlation analysis was performed using the 34 benefit variables and the 40 psychographic variables. Those psychographic variables which were not significantly related to benefits were not used for further analysis. Ten variables were thus eliminated. The remaining thirty variables were factor-analyzed and ten factors were drawn in both samples. These factors explained 54.67 percent variance in sample 1, and 54.7 percent variance in sample 2.

Number of factors drawn in for benefits was increased from four to six. Analysis with four factors had showed that three factors were common to both samples and the remaining factor was specific to each of the two samples. The 'specific' factor accounted for 4.27 percent of total variance in sample 1, and 5.37 percent of total variance in sample 2. The reason for increasing the number of factors drawn in was to account for the problem of these 'specific' factors explaining a significant portion of variance in the samples.

Canonical analysis was performed using factors of benefits for one variable set, and factors of psychographics for the other set. The first six canonical variate pairs were retained for further analysis. The remaining steps in segmentation were exactly the same as in the case of segmentation using benefits.

Results

Results of segmentation based on benefits and psychographics are presented below.

Segmentation Based on Benefits

Squared error variance summed over clusters and the four rating variables for different segmentation procedures are given in Table 12.

To analyze the results, it was thought better first to derive a measure of efficiency for segmentation procedures. Such a measure was derived as follows:

Since the primary requirement of segmentation is grouping of members with similar buying behavior, clustering based on buying behavior variables could be expected to give the best solution as far as this primary requirement was considered. And the squared error variance of buying behavior obtained for clustering based on buying behavior variables would be the minimum squared error variance for a given number of clusters. The higher the squared error variance of buying behavior variables obtained by the application of a segmentation procedure the less efficient the procedure would be in segmenting a market. Hence, efficiency of a segmentation procedure, η, may be defined as

$$\eta = 100 - (A - B)/B \times 100$$

where,

A = Squared error variance of criterion set variables obtained for a segmentation procedure, and

B = Squared error variance of criterion set variables when criterion set variables are used for clustering.

Of course, A and B should be obtained for the same number of clusters or groups, and the measure would be applicable only in situations when $(A - B) \leq B$.

TABLE 12

SQUARED ERROR VARIANCE OF RATINGS OF BRANDS FOR PROCEDURES USING BENEFITS AS BASES OF SEGMENTATION

Sample Number	Number of Clusters	Variates Used for Clustering[a]						
		FY	FX1	FX2	FX3	CX1	CX2	CX3
		No Data Transformation Used						
1	2	1235.55	1594.41 (71.0)	1569.68 (73.0)	1593.62 (71.0)	1594.41 (71.0)	1557.08 (74.0)	1557.08 (74.0)
	4	896.84	1557.50 (26.3)	1567.25 (25.2)	1559.65 (26.1)	1557.50 (26.3)	1555.10 (26.6)	1553.68 (26.8)
2	2	1266.88	1587.93 (74.7)	1578.60 (75.4)	1578.60 (75.4)	1587.93 (74.7)	1573.00 (75.8)	1573.00 (75.8)
	4	875.08	1580.96 (19.3)	1564.17 (21.3)	1564.17 (21.3)	1580.96 (19.3)	1561.94 (21.5)	1561.94 (21.5)
		Data Transformations Used						
1	2	1284.93	1590.44 (76.2)	1583.59 (76.8)	1582.83 (76.8)	1590.44 (76.2)	1577.82 (77.2)	1577.82 (77.2)
2	2	1292.36	1585.08 (77.3)	1586.80 (77.2)	1586.80 (77.2)	1585.08 (77.3)	1578.92 (77.8)	1578.92 (77.8)

NOTE: Figures in parantheses denote efficiencies of the procedures.

[a]FY: Factors of Ratings of Brands; FX1: Factors of Benefits Not Differentially Weighted; FX2: Factors of Benefits Weighted Relative to Ratings of Brands; FX3: Factors of Benefits Weighted Relative to FY; CX1: Canonical Variates of Benefits Set Not Differentially Weighted; CX2: Canonical Variates of Benefits Set Weighted Relative to Ratings of Brands; CX3: Canonical Variates of Benefits Set Weighted Relative to Canonical Variates of Ratings Set.

Efficiencies of segmentation procedures for the different cases are presented in Table 12 below the values for squared error variance.

Weighting of canonical variates was seen to increase efficiency in all six cases. And the procedure in which the canonical variates were weighted based on the concept of redundancy was the most efficient in each of the six cases. Weighting of factor scores did not always increase efficiency. In fact, efficiency was reduced in two of the six cases.

Though the efficiencies were sufficiently high for the two-cluster solutions, they were very low for the four-cluster solutions.

Transformations used for reducing skewness did serve a useful purpose; they increased the efficiency of segmentation procedures.

Segmentation Based on Psychographics

Results for segmentation based on psychographics are presented in Table 13 and 14. Weighting was found useful in all eight cases involving canonical variates. And canonical variates weighted based on the concept of redundancy produced the most efficient segmentation in each of these eight cases. As in the case of benefits, here too, weighting of factors produced mixed results. When efficiencies were computed taking ratings of brands for buying behavior, values obtained for four-cluster solutions

TABLE 13

SQUARED ERROR VARIANCE OF RATINGS OF BRANDS FOR PROCEDURES USING PSYCHOGRAPHICS AS BASES OF SEGMENTATION

Sample Number	Number of Clusters	Variates Used for Clustering[a]						
		FY	FX1	FX2	FX3	CX1	CX2	CX3
1	2	1235.55	1596.69 (70.8)	1597.09 (70.7)	1597.13 (70.7)	1596.23 (70.8)	1592.61 (71.1)	1593.73 (71.0)
	4	896.84	1591.62 (22.5)	1588.36 (22.9)	1581.79 (23.6)	1592.00 (22.5)	1677.68 (24.1)	1578.58 (24.0)
2	2	1266.88	1596.09 (74.0)	1593.55 (74.2)	1594.87 (74.1)	1596.09 (74.0)	1592.04 (74.3)	1592.04 (74.3)
	4	875.08	1589.67 (18.3)	1590.31 (18.3)	1591.97 (18.1)	1588.85 (18.4)	1585.50 (18.8)	1585.51 (18.8)

NOTE: Figures in parantheses denote efficiencies of the procedures.

[a]FY: Factors of Ratings of Brands; FX1: Factors of Psychographics Not Differentially Weighted; FX2: Factors of Psychographics Weighted Relative to Ratings of Brands; FX3: Factors of Psychographics Weighted Relative to FY; CX1: Canonical Variates of Psychographics Set Not Differentially Weighted; CX2: Canonical Variates of Psychographics Set Weighted Relative to Ratings of Brands; CX3: Canonical Variates of Psychographics Set Weighted Relative to Canonical Variates of Ratings Set.

TABLE 14

SQUARED ERROR VARIANCE OF BENEFITS FOR PROCEDURES USING PSYCHOGRAPHICS AS BASES OF SEGMENTATION

Sample Number	No. of Clusters	Variates Used for Clustering[a]				
		FY	FX1	FX2	CX1	CX2
1	2	3724.57	3989.76 (92.9)	3970.76 (93.4)	3981.19 (93.1)	3937.36 (94.3)
	4	3320.74	3931.66 (81.6)	3951.79 (81.0)	3921.76 (81.9)	3881.04 (83.1)
2	2	3720.12	3979.32 (93.0)	3977.72 (93.1)	3976.69 (93.1)	3945.18 (94.0)
	4	3332.97	3955.82 (81.3)	3945.71 (81.6)	3922.38 (82.3)	3878.55 (83.6)

NOTE: Figures in parantheses denote efficiencies of the procedures.

[a]FY: Factors of Benefits; FX1: Factors of Psychographics Not Differentially Weighted; FX2: Factors of Psychographics Weighted Relative to FY; CX1: Canonical Variates of Psychographics Set Not Differentially Weighted; CX2: Canonical Variates of Psychographics Set Weighted Relative to Canonical Variates of Benefits Set.

were low (see Table 13). But when benefits were considered surrogates of buying behavior, efficiencies were high in all cases (see Table 14).

Discussion

Several conclusions may be drawn from the results of the study. First and foremost, the findings demonstrated that the conclusions drawn earlier from experimentation with generated data held good for consumer data too. Experimentation with generated data had shown that (i) usefulness of weighting decreased with increase in skewness of variables, and (ii) for highly skewed data, weighting was not useful in the case of factors but still useful in the case of canonical variates. Data for the consumer study were highly skewed. Hence one might expect increase in segmentation effectiveness with weighting for canonical variates but not for factors. And the results were no different from what were anticipated. There were fourteen different segmentation runs using consumer data. In five of these runs, weighting of factors did not improve efficiency. But in the case of canonical variates, weighting improved efficiency in all the fourteen runs. Hence weighting may be confidently recommended for use in situations when canonical correlation analysis is applied for segmentation purpose.

Another conclusion that may be drawn from results of the study has to do with selection of bases for segmentation. There were segmentation runs with the same set of variables, namely, ratings of brands used as surrogates of buying behavior, but with different sets of variables for personal characteristics, namely, benefits and psychographics. For such runs, efficiencies obtained for benefits were slightly higher than those for psychographics. The measure of efficiency as defined for the present study can thus be used in the selection of bases for segmentation.

Efficiency of segmentation as defined in the study may also be used in determining whether segmentation can be accomplished with a given set of personal characteristics. In a number of cases, four-cluster solutions were obtained with a very low value for efficiency (see Tables 12 and 13). The low values indicated that bases used for segmentation had very little relationship with surrogates of buying behavior used in the analysis. Thus, the results showed that ratings of brands as a whole had very little relationship with benefits or psychographics. If ratings of brands truly represented buying behavior, grouping based on benefits or psychographics would not accomplish segmentation objectives for this particular product class.

Segmentation runs using benefits for buying behavior and psychographics for personal characteristics were seen to have high values for efficiency. Benefits used in the study were just attitudes towards different product attributes.

If <u>these attitudes were proper surrogates of buying behavior</u>, segmentation based on psychographics would accomplish the primary objective of market segmentation. Efficiencies computed for the different procedures are meaningful to the extent the criterion set variables represent actual buying behavior.

CHAPTER VII

SUMMARY AND SUGGESTIONS FOR FUTURE RESEARCH

This chapter represents the culmination of work done for the present study. In this chapter are presented a summary of the whole project, implications of the results, limitations of the study, and suggestions for future research.

Summary

A survey of literature of the application of various multivariate statistical procedures in market segmentation showed that there was no traditional way of differentiating the predictor set variables in terms of their relationship to criterion set variables. Since the bi-multivariate relationship between buying behavior and personal characteristics is central to the whole concept of market segmentation, it was thought essential to incorporate this information explicitly in the analysis. The concept of redundancy developed by Stewart and Love (1968), and Miller (1969) to account for the relationship between bi-multivariate measurement sets should therefore be relevant to market segmentation, and the present study originated from this feeling. It was hypothesized that canonical variates of the predictor set weighted based on the concept of redundancy would create better segments than canonical variates not differentially weighted. The scope of the study was later enlarged to encompass different types of variates and different sets of weights.

Three types of variates were used in this study. These were (1) canonical variates, (2) factor scores, and (3) original variables. Each type of variates was weighted three different ways: (1) variates were all given an equal weight of unity, (2) variates were weighted relative to criterion set variables, and (3) variates were weighted based on the concept of redundancy for canonical variates and factor scores, and weighted relative to factor scores of the criterion set for the original variables.

The study design used was simulation. Each type of variates was assigned different sets of weights and the segmentation process was simulated in its essentials. To account for different types of data conditions found in segmentation studies, six types of data were used in the simulation: (1) multivariate normal distribution with means of populations closer to each other, (2) multivariate normal distribution with means farther apart, (3) multivariate binomial distribution with means closer to each other, (4) multivariate binomial distribution with means moderately apart, (5) multivariate binomial distribution with means farther apart, and (6) multivariate multinomial distribution. A validation sample was used so that generalizations of the results could be made. To compare the different weighting schemes, it was necessary to develop certain criteria for judging the effectiveness of different procedures in segmenting a market. Three different criteria were developed for the purpose. These were (1) total sum of squared error variance for criterion set variables of the analysis sample, (2) number of misclassifications in the validation sample based on membership as determined by criterion set variables, and (3) total sum of squared error variance for criterion set variables of the validation sample. In order to draw statistical inference from the study, experimentation was done for a sufficient number (= 100) of trials.

Experimentation with canonical variates showed that for all data conditions, canonical variates of the predictor set weighted relative to criterion set variables were more effective in segmentation than variates not differentially weighted. The same variates when weighted relative to canonical variates of the criterion set were found more effective than variates not differentially weighted for all data types except one. The exception was in the case of multivariate binomial distribution with means of populations farther apart. This case thus involved dichotomies with extreme skewness. It was, therefore, concluded that weighting of canonical variates of the predictor set relative to canonical variates of the criterion set was not quite useful in segmentation when the data involved dichotomies with extreme skewness.

Experimentation with factor scores showed that effectiveness of weighting depended on data conditions. Thus, for data types with symmetrical distributions and with no large separation between populations, weighting improved segmentation effectiveness. There was a decrease in effectiveness with skewness of distribution and separation between populations. In the case of multivariate multinomial distribution characterized by moderate to large skewness, weighting was found not useful. These results were different from those obtained in the case of canonical variates when weighting was applied. In cases where weighting was found useful for factor scores, no difference could be found between weighting schemes based on factors of the criterion set, and the criterion variables.

Weighting of original variables was useful when there was no large separation between populations. Weighting was not effective in the case of large separation. Original variables could then form groups satisfying different segmentation criteria. There was thus little left for the differential weighting schemes to accomplish by way of improving segmentation effectiveness. In cases where weighting was found useful, no difference could be detected between procedures weighted with respect to criterion set variables and criterion set factors.

Analysis of select procedures showed that there was no difference among the procedures when there was no large separation between populations or when the distributions were not highly skewed. In fact, these were the very same situations when weighting was found most effective for all the variate classes. When the means were farther apart, or when the distributions were highly skewed, there appeared overall difference among the select procedures. And in all these cases, factor-analytic procedures fared badly relative to procedures based on original variables or canonical variates. Between the latter two sets, the superiority of procedures based on canonical variates was clearly seen in the cases of data drawn from (1) multivariate multinomial distribution with moderate separation between populations, and (2) multivariate binomial distribution with large separation. Though values obtained for multivariate normal distribution with large separation were better for procedures based on canonical variates, the differences were not large enough to establish the superiority of these procedures over those based on original variables.

Analysis of consumer data showed that conclusions drawn from experimentation with generated data held good for survey data too. The data were highly skewed for a number of variables used in the analysis. Still, weighting was found useful in the case of canonical variates. And canonical variates weighted based on the concept of redundancy yielded the best solution in all cases. The measure of efficiency of segmentation derived from consideration of squared error variance of criterion set variables was found useful in evaluating alternative bases of segmentation. When ratings of brands were used for buying behavior set, segmentation based on benefits were seen to be more efficient than segmentation based on psychographics. However, the values of efficiency were generally low indicating that benefits or psychographics would not accomplish segmentation objectives for the product class if ratings of brands truly represented buying behavior. Transformations of data for reducing skewness were found to increase efficiency of the procedures.

Implications of the Results

Three classes of predictor variables, namely, canonical variates, factor scores, and original variables were included in the study. Each class of variables was studied for three different weighting schemes under six data conditions. Results showed that each class of variables with a particular set of weighting has its own place in market segmentation. The different classes of variables are considered separately in discussing the implications of the results to market segmentation.

Predictor set variables are used as such in segmentation when they are somewhat independent and when their number is small. If there is reason to believe that the means of predictor set variables for different populations are not farther apart, it would be better to weight them relative to the criterion set. If the means are farther apart, weighting is not of much use. Here, the original variables should be able to form groups satisfying the different segmentation criteria. It is only when the means are not farther apart that the original variables fail to accomplish proper segmentation. And it is at this critical situation that weighting offers its maximum advantage. Thus, together with weighting, original variables can be useful in segmentation of markets whether the means of populations are farther apart or not.

Factor scores are used in segmentation when the predictor set contains a large number of correlated variables. It is better to weight the factor scores if the means of the populations are thought to be not farther apart. When the means of populations are farther apart, or when the distributions are badly skewed, results obtained by using factor scores are poor whether the scores are weighted or not. Hence, if factor analysis is to be used in the study, the analyst should give attention to the selection of variables. Variables with distributions approaching normality are always to be preferred. However, it is not always possible to find variables which approximate normal distributions. For variables similarly measured, one may therefore choose those which are continuous and symmetrical. In the case of dichotomies, those with extreme skewness should be avoided. Conversion of higher level measures to ordinal scale may sometimes be necessary to reduce skewness. Various transformations should also be considered.

When canonical variates are used in segmentation, weighting the canonical variates based on criterion set variables will always produce better results. Except for data characterized by extreme skewness, weighting based on canonical variates of the criterion set is preferable to the case when the variates are not differentially weighted. Hence, when weighting is to be done based on canonical variates of the criterion set, care should be taken in the selection of variables. For variables similarly selected, those with extreme skewness are to be avoided.

As used in the study, methods based on factor scores and canonical variates were criterion related. And both involved weighting of orthogonal factors. The question then naturally arises as to the superiority of one method over the other. The present study was mainly concerned with weighting of variables in each of the three separate classes, namely, original variables, factor scores, and canonical variates.As such it was not aimed at the selection of the best procedure across the different classes. However, the study did point to certain situations when canonical correlation analysis would be preferable to factor analysis. But before discussing the implications of the study in this regard, one may study the instances where canonical correlation analysis or factor analysis is to be preferred for segmentation from other considerations.

When one set of variables is quite small compared to the other, there is a major advantage if canonical correlation analysis is used for segmentation. In this case, there will only be a few canonical variate pairs since the maximum number of canonical variates is limited by the number of variables in the smaller of the two sets. These variates will account for all the information in the data relevant to segmentation of a market. This is so because, as Miller (1969) puts it, " . . . canonical analysis does preserve as an intrinsic function of its method, all components of variance relevant to inter-battery dependency " (pp. 132-133). Thus, the use of canonical correlation analysis may enable the analyst to work with fewer variates with no loss of information regarding the bi-multivariate relationships.

Use of canonical correlation analysis may be called for from theoretical considerations, too. There may be cases where variation in the criterion set may be due to distinct components in the set. And these components may be related to corresponding ones in the predictor set. Since canonical correlation analysis extracts mutually orthogonal pairs of maximally correlated components, application of canonical correlation analysis should result in drawing of canonical variates each relating a distinct component in the criterion set to a corresponding component in the predictor set. The variates thus formed should be more meaningful and interpretable than factors obtained in a factor analysis. For an example, one may consider a criterion set consisting of usage rates for different brands of a product class, and a predictor set consisting of personal characteristics. Variation in the buying behavior set may be made up of components of variation common to some combination of product types and those specific to each product type. If there is reason to believe that each type of variation in buying behavior is related to a distinct type of variation in personal characterisitics, canonical correlation analysis is needed to draw in the different pairs of components. The canonical variate pairs will show how distinct types of buying behavior are associated with distinct personal characteristics.

There are certain situations when factor analysis is preferable to canonical correlation analysis. Canonical correlation analysis is subject to the restriction that the different canonical variate pairs be maximally correlated. This externally imposed restriction may contravene to change intrinsic structural relationships. Factor analysis is free from such restriction. Besides, factors are extracted exclusively from each set of variables, and orthogonal rotations may be performed independently on each set of factors for maximum interpretability. Thus, factor analysis may be preferred in many cases from consideration of the interpretability of results.

If the choice of canonical correlation analysis or factor analysis is not dictated by the above considerations, findings of the study may be useful as a guide in selecting one of the procedures for analysis. The study showed that when analyzing data with means of populations not farther apart and distributions not highly skewed, there was no difference among the procedures with or without weighting. Weighting improved segmentation effectiveness for both procedures. Hence, one may use either of the procedures for segmentation purpose.

It is when the populations are farther apart, or when the distributions highly skewed that the findings of the study have more relevance in the selection of canonical correlation analysis or factor analysis. The results showed that when the data were of the above description, factor-analytic procedures fared very badly. Hence, if there is reason to suspect that the data comprise observations from widely different populations or that the variables are badly skewed, the choice is obviously canonical correlation analysis. And weighting is to be done based on original variables of the criterion set.

The major implication of the results of the study has to do with the practical usefulness of weighting in market segmentation. Weighting was found most useful in situations when the traditional methods were least effective in segmenting a market. These were the situations when the data were characterized by little or moderate separation between populations. Thus, the weighting procedures used in the study can be of help to segmentation analyst when the traditional methods fail in properly segmenting a market. An important characteristic of the weighting schemes, so far as practical usefulness is considered, is that they do not entail anything more by way of data collection. Buying behavior characteristics are routinely collected in segmentation studies. This information on buying behavior is used to check whether segmentation arrived at is effective or not. The present study showed that information on buying behavior could be used not only for checking whether segmentation was effective but also for making segmentation more effective. And it is here that the present study is expected to make its major contribution.

Limitations

Main limitations of the present study were to do with restrictiveness of the models used to generate data. Some restrictive characteristics of the models which limited generalizations of the results are the following:

1. Data were drawn from only two populations.

2. Number of variables characterizing the populations was small.

3. All the variables were assigned equal variances,and the variance-covariance matrices were the same for the two populations.

4. The study did not involve any mixed models; observations from both populations had the same type of probability distribution for each of the six data conditions.

5. Populations studied for the different data conditions were were located symmetrically with respect to the axes; if the mean of one population were + k standard deviations from the axes, the mean of the other population was - k standard deviations from these axes.

6. The only model which best approximated data conditions in actual segmentaion studies was multivariate multinomial distribution. Here, the observations were

generated under the assumption of independence of variables.

There was also a limitation in the analysis of data. Same set of data were used in testing the various hypotheses and in comparing the select procedures.

Suggestions for Future Research

Weighting of variables is quite a potential area of research in market segmentation. Research work may be undertaken by removing the various limitations imposed on the present study. There is also scope in extending the study on the relationship of measurement sets from bi-multivariate to tri-multivariate.

Most of the limitations of the present study were deliberately introduced so as to determine the effect of weighting on segmentation in an unconfused manner. And they did serve their purpose. However, they also led to narrowing possible generalizations from the findings of the study. A fruitful avenue for further research is along the line which has none or only some of the limitations of the present study. Thus, studies may be designed using samples with observations drawn from several populations. The populations may have measurements on many variables. The variables may have different probability distributions. Variance-covariance matrix may be the same among populations in some cases and different in others. Cluster analysis may be performed several times, each time setting a different value for the number of clusters to be formed. Sample sizes may be varied as well the number of variables included in the analysis. Studies of this type may be useful in determining how effectiveness of weighting is affected by factors like (i) increase in the number of populations, (ii) addition or dropping of variables, (iii) unequal variances and covariances, (iv) number of segments formed, (v) mixed distributions, and (vi) sample size.

The present study used only two sets of variables. One set was to represent buying behavior and the other personal characteristics. However, there is one more set of variables, namely, marketing tool variables which are relevant to segmentation. In many segmentation studies, marketing tool variables are not explicitly considered in the analysis. In such studies, only those personal characteristics which are under the control of the marketer are included in the analysis. Judicious selection of personal charateristics in this way may account for the marketing tool variables in many cases. However, there are situations when marketing tool variables are to be explicitly considered to accomplish segmentation objectives. An example would be the selection of media to reach consumers of a certain product class. Here, the analyst should take explicitly into account the interrelationships among the three sets of variables - buying behavior, personal characteristics, and media variables. In this context, it would be interesting and worthwhile extending the concept of redundancy to cover the association among the tri-multivariate measurement sets. Horst's (1961) model for relations among more than two sets of variables, or Tucker's (1964, 1966) three-mode factor analysis model, or Carroll's (1969a, 1972) INDSCAL model may be used as a starting point for such a project.

APPENDIX A

TABLES

TABLE 15

MULTIVARIATE NORMAL DISTRIBUTIONS WITH MEANS CLOSER TO EACH OTHER: NUMBER OF MISCLASSIFICATIONS BASED ON MEMBERSHIP IN ORIGINAL POPULATIONS (ANALYSIS SAMPLE)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	20	14	14	15	15	15	15	13	13
2	17	18	18	17	17	17	17	18	18
3	15	18	18	17	18	18	17	18	18
4	13	15	14	13	17	17	13	16	16
5	20	19	19	18	19	19	18	20	20
6	13	10	10	19	13	13	19	11	11
7	20	13	14	18	15	15	18	14	14
8	19	16	16	16	16	16	16	18	18
9	19	19	17	19	19	19	19	19	17
10	14	14	14	16	19	19	16	16	16
11	19	15	20	20	16	18	20	15	18
12	13	17	13	16	17	18	18	19	19
13	10	10	10	15	12	12	15	10	10
14	13	12	12	16	14	14	16	12	12
15	12	15	15	19	19	19	19	17	17
16	16	18	18	18	18	18	18	19	19
17	17	16	16	20	16	16	20	15	16
18	19	19	19	18	20	19	18	20	20
19	17	20	20	16	18	17	16	20	20
20	20	19	20	14	20	20	14	20	20
21	20	20	20	18	19	19	18	19	19
22	14	14	14	14	14	14	14	12	12
23	18	18	19	14	15	15	14	15	15
24	16	16	17	18	18	19	18	16	16
25	19	20	17	19	20	20	19	19	19
26	17	20	20	13	19	19	13	19	19

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 15-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	10	11	11	10	11	11	10	10	9
28	14	16	16	20	17	17	20	18	18
29	10	13	13	19	16	16	19	14	14
30	16	15	17	16	16	16	16	16	16
31	17	19	19	16	16	16	16	16	17
32	20	19	19	16	19	19	18	17	20
33	20	19	18	20	20	17	20	18	18
34	18	16	16	20	16	16	20	16	16
35	14	10	11	12	11	16	12	10	10
36	14	16	16	18	18	18	18	16	16
37	19	13	13	13	14	14	13	13	13
38	14	17	17	19	18	17	19	17	17
39	13	13	13	17	15	15	17	15	15
40	15	14	14	16	15	15	16	13	15
41	17	19	19	16	16	16	16	18	19
42	20	19	19	18	19	19	16	14	14
43	20	19	18	20	20	17	20	19	19
44	18	16	16	20	16	16	15	8	8
45	14	10	11	12	11	16	17	12	12
46	14	16	16	18	18	18	18	14	14
47	19	13	13	13	14	14	19	12	14
48	14	17	17	19	18	17	18	17	16
49	13	13	13	17	15	15	19	18	18
50	15	14	14	16	15	15	18	17	17
51	19	16	19	13	17	17	13	19	19
52	17	17	17	18	16	17	18	17	17
53	14	17	17	17	19	19	17	18	18
54	16	14	14	19	19	19	19	20	20
55	19	14	16	17	12	13	17	13	13
56	12	18	18	19	16	16	19	18	18
57	18	16	16	20	17	17	20	17	17
58	14	13	14	11	12	12	11	11	11
59	15	16	15	17	16	16	17	17	17
60	19	17	17	15	18	18	15	17	17
61	16	16	14	15	15	15	18	18	18
62	15	15	14	12	14	14	18	15	15
63	17	17	17	20	18	18	10	4	4
64	14	17	17	19	16	16	16	9	9

TABLE 15-Continued

Trial No.	Weighting Schemes[a] X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	19	14	14	16	18	18	18	18	18
66	14	18	17	14	16	16	8	8	8
67	17	17	17	14	17	17	20	9	9
68	18	17	17	16	20	20	16	5	5
69	16	15	14	17	11	11	18	17	17
70	14	19	19	14	18	18	13	13	13
71	9	17	14	15	16	15	15	15	16
72	18	19	19	18	19	19	18	16	15
73	20	19	19	17	20	20	17	18	18
74	18	16	20	20	19	19	20	16	16
75	20	20	19	18	17	17	18	19	17
76	19	17	16	17	17	17	17	20	20
77	15	12	12	18	11	11	18	12	12
78	14	19	19	13	20	20	13	20	20
79	19	20	20	19	19	19	19	20	20
80	16	14	18	13	15	15	13	18	17
81	19	15	15	14	16	15	14	17	17
82	15	14	16	20	15	15	20	16	16
83	19	14	16	18	14	14	18	14	14
84	10	12	9	14	14	13	14	11	11
85	14	14	14	17	16	16	17	12	12
86	20	20	20	20	18	19	20	17	17
87	11	13	13	18	14	13	18	12	12
88	17	16	16	16	18	18	16	15	16
89	13	12	11	19	13	13	19	12	12
90	18	16	18	20	20	20	20	19	19
91	13	16	16	15	17	17	15	15	15
92	19	19	19	20	15	15	20	19	19
93	17	15	14	15	15	15	16	15	15
94	18	16	16	20	17	17	20	19	19
95	12	13	13	14	13	14	14	14	13
96	17	18	19	19	18	20	19	19	19
97	12	12	12	15	14	14	15	13	13
98	12	16	15	15	17	17	15	15	16
99	13	14	14	13	12	12	13	15	16
100	20	19	17	17	17	17	17	17	17

TABLE 16

MULTIVARIATE NORMAL DISTRIBUTIONS WITH MEANS CLOSER TO EACH OTHER: NUMBER OF MISCLASSIFICATIONS BASED ON MEMBERSHIP IN ORIGINAL POPULATIONS (VALIDATION SAMPLE)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	16	19	19	16	18	18	16	18	18
2	20	17	17	18	19	19	18	17	17
3	19	15	15	17	14	14	17	15	15
4	14	16	14	16	14	14	16	17	17
5	19	16	19	19	16	18	18	16	18
6	18	20	17	17	18	19	19	18	17
7	17	19	15	15	17	14	14	17	15
8	15	14	16	14	16	14	14	16	17
9	17	19	16	19	19	18	18	18	16
10	18	18	20	17	17	19	19	19	18
11	18	13	16	18	17	18	18	13	19
12	15	16	15	16	16	15	16	15	15
13	17	19	19	15	15	15	15	18	19
14	18	14	14	20	13	13	20	15	15
15	13	18	13	16	18	18	18	18	13
16	19	15	16	15	16	15	15	16	15
17	15	17	19	19	15	15	15	15	18
18	19	18	14	14	20	13	13	20	15
19	15	13	18	13	16	17	18	18	18
20	13	19	15	16	15	16	15	15	16
21	19	19	19	18	19	19	18	19	19
22	14	16	16	16	17	17	16	14	14
23	15	19	19	15	19	19	15	19	18
24	11	12	13	13	16	16	13	12	13
25	14	19	19	19	18	19	19	18	19
26	19	14	16	16	16	17	17	16	14

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 16-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	14	18	19	19	15	19	19	15	19
28	18	11	12	13	13	16	16	13	12
29	13	14	19	19	19	19	19	19	18
30	19	19	14	16	16	17	17	17	16
31	16	15	15	20	17	17	20	20	20
32	17	19	19	19	19	20	19	19	16
33	15	12	13	19	15	16	19	12	12
34	19	15	15	19	19	17	19	15	15
35	16	16	15	15	20	17	17	20	20
36	20	17	19	19	19	20	20	19	19
37	16	15	12	13	19	16	16	19	12
38	12	19	15	15	19	17	17	19	15
39	15	16	16	15	15	17	17	17	20
40	20	20	17	19	19	19	20	20	19
41	16	18	18	18	20	20	17	18	18
42	18	16	16	18	18	18	18	20	20
43	15	16	16	20	16	17	19	15	15
44	17	16	16	18	19	19	17	19	19
45	15	16	18	18	18	20	20	17	18
46	18	18	16	16	18	18	18	18	20
47	20	15	16	16	20	17	17	19	15
48	15	17	16	16	18	19	19	17	19
49	19	15	16	18	18	20	20	20	17
50	18	18	18	16	16	18	18	18	18
51	17	12	12	20	14	14	20	12	12
52	14	15	15	19	19	20	19	17	17
53	16	19	19	16	18	18	16	18	18
54	19	17	17	17	18	18	17	16	16
55	17	17	12	12	20	14	14	20	12
56	12	14	15	15	19	20	20	19	17
57	17	16	19	19	16	18	18	16	18
58	18	19	17	17	17	18	18	17	16
59	16	17	17	12	12	14	14	14	20
60	12	12	14	15	15	19	20	20	19
61	19	19	20	18	19	19	19	19	19
62	13	17	16	16	15	15	18	17	17
63	15	12	12	15	12	12	16	15	15
64	12	11	11	14	14	12	14	10	10

TABLE 16-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	17	19	19	20	18	19	19	19	19
66	19	13	17	16	16	15	15	18	17
67	17	15	12	12	15	12	12	16	15
68	15	12	11	11	14	12	12	14	10
69	10	17	19	19	20	19	19	19	19
70	19	19	13	17	16	15	15	15	18
71	18	15	15	19	11	11	19	10	11
72	17	17	17	17	14	14	17	18	20
73	18	16	16	19	15	15	19	16	16
74	13	16	14	16	18	18	16	16	16
75	17	18	15	15	19	11	11	19	10
76	11	17	17	17	17	14	14	17	18
77	20	18	16	16	19	15	15	19	16
78	16	13	16	14	16	18	18	16	16
79	16	17	18	15	15	11	11	11	19
80	10	11	17	17	17	14	14	14	17
81	18	15	14	14	16	16	14	13	13
82	9	10	9	13	13	13	13	13	13
83	18	20	18	18	20	20	18	20	20
84	12	14	15	19	15	16	19	20	15
85	15	18	15	14	14	16	16	14	13
86	13	9	10	9	13	13	13	13	13
87	13	18	20	18	18	20	20	18	20
88	20	12	14	15	19	16	16	19	20
89	15	15	18	15	14	16	16	16	14
90	13	13	9	10	9	13	13	13	13
91	17	19	19	20	20	20	20	19	19
92	20	18	18	19	20	20	19	18	18
93	19	17	16	15	14	14	14	17	17
94	20	19	19	20	18	18	20	16	16
95	19	17	19	19	20	20	20	20	19
96	19	20	18	18	19	20	20	19	18
97	18	19	17	16	15	14	14	14	17
98	17	20	19	19	20	18	18	20	16
99	16	19	17	19	19	20	20	20	20
100	19	19	20	18	18	20	20	20	19

TABLE 17

MULTIVARIATE NORMAL DISTRIBUTIONS WITH MEANS FARTHER APART: NUMBER OF MISCLASSIFICATIONS BASED ON MEMBERSHIP IN ORIGINAL POPULATIONS (ANALYSIS SAMPLE)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	1	1	1	13	9	9	13	5	5
2	1	1	1	19	9	9	19	2	2
3	4	3	3	4	8	8	4	4	4
4	2	2	2	15	10	9	15	3	4
5	3	3	3	8	5	5	8	4	4
6	1	1	1	18	10	10	18	3	3
7	0	0	0	16	10	11	16	2	2
8	0	0	0	14	7	2	14	1	1
9	0	0	0	9	12	6	9	2	2
10	1	1	1	18	10	10	18	2	1
11	1	3	3	11	13	7	11	4	4
12	1	1	1	18	19	19	18	1	1
13	0	0	0	16	11	11	16	2	2
14	6	4	4	20	8	4	20	2	2
15	1	1	1	14	7	7	14	1	1
16	4	4	4	19	7	7	19	3	3
17	2	2	2	5	8	8	5	3	5
18	2	1	1	17	9	9	17	2	2
19	1	1	1	20	11	11	20	1	1
20	2	3	3	1	12	12	1	5	5
21	0	1	0	11	10	7	10	2	2
22	3	3	3	18	3	19	11	3	3
23	0	0	0	16	7	11	14	0	0
24	1	1	1	20	8	4	1	0	0
25	0	0	0	14	7	7	10	1	1
26	2	1	2	19	8	7	15	3	3

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 17-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	2	1	2	5	8	8	14	0	0
28	1	2	1	17	9	7	7	2	2
29	0	0	0	20	17	11	0	1	1
30	1	1	1	1	12	12	1	0	0
31	1	1	1	16	10	11	16	0	0
32	1	1	1	17	10	10	17	1	1
33	1	2	1	4	9	9	4	2	2
34	0	0	0	0	9	9	0	0	0
35	4	2	2	4	10	12	4	2	2
36	2	2	2	2	6	6	2	3	3
37	1	0	0	8	11	4	8	0	0
38	0	0	0	0	10	8	0	0	0
39	0	0	0	14	12	11	14	1	1
40	0	0	0	16	13	13	16	4	1
41	0	0	0	16	9	9	16	1	1
42	1	1	1	1	9	8	1	2	2
43	2	2	2	13	11	15	13	5	5
44	3	4	4	13	13	13	13	4	4
45	2	2	2	2	10	7	2	3	3
46	2	2	2	18	16	16	18	2	2
47	2	1	2	18	9	9	18	2	2
48	2	2	2	3	7	7	3	2	2
49	0	0	0	15	9	11	15	2	2
50	0	1	1	19	6	6	19	1	1
51	0	0	0	10	9	12	10	0	0
52	2	2	2	7	11	12	7	3	3
53	2	0	0	12	7	6	12	4	3
54	0	1	1	8	10	10	8	2	2
55	1	1	1	14	10	10	14	1	1
56	0	0	0	5	5	5	5	1	1
57	0	0	0	18	7	7	18	1	1
58	3	4	4	12	13	14	12	5	5
59	2	2	2	1	7	7	1	4	4
60	1	1	1	19	12	9	19	1	1
61	5	1	1	10	6	6	10	3	2
62	1	1	1	18	11	11	18	3	3
63	1	1	1	14	7	6	14	2	2
64	1	1	1	12	10	10	12	3	3

TABLE 17-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	1	1	1	9	9	9	9	2	2
66	1	0	0	14	6	6	14	0	0
67	1	1	1	18	9	9	18	3	3
68	2	2	2	10	11	12	10	4	4
69	1	1	1	14	5	4	14	1	1
70	0	0	0	19	10	15	19	0	0
71	0	0	0	12	8	8	12	3	3
72	3	2	2	20	6	6	20	3	3
73	2	1	1	15	6	6	15	4	3
74	0	0	0	14	9	13	14	1	0
75	0	1	1	17	9	9	17	2	2
76	1	2	2	3	8	8	3	3	3
77	1	1	1	15	6	2	15	0	0
78	0	0	0	15	8	8	15	0	0
79	2	2	2	1	7	7	1	2	2
80	0	0	0	17	16	16	17	1	1
81	0	0	0	14	9	8	14	2	2
82	0	0	0	0	9	9	0	0	0
83	1	2	2	15	9	9	15	2	2
84	0	0	0	10	8	7	10	2	2
85	0	0	0	14	4	2	14	1	1
86	5	3	4	5	7	5	5	1	1
87	0	0	0	6	9	9	6	4	0
88	1	3	3	19	10	7	19	2	2
89	1	0	0	12	9	9	12	3	3
90	2	2	2	13	5	5	13	2	2
91	0	0	0	18	4	4	18	1	1
92	2	1	1	19	8	8	19	1	1
93	0	1	1	2	8	9	2	2	2
94	1	1	1	1	12	12	1	2	2
95	0	0	0	18	4	1	18	0	0
96	0	0	0	15	12	14	15	4	2
97	1	0	0	16	11	11	16	1	1
98	1	1	1	20	8	8	20	1	1
99	0	0	0	17	8	7	17	1	1
100	0	0	0	14	10	7	14	1	1

TABLE 18

MULTIVARIATE NORMAL DISTRIBUTIONS WITH MEANS FARTHER APART: NUMBER OF MISCLASSIFICATIONS BASED ON MEMBERSHIP IN ORIGINAL POPULATIONS (VALIDATION SAMPLE)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	0	0	0	2	2	2	2	1	1
2	4	2	2	19	16	16	19	2	2
3	4	5	5	4	11	11	4	4	4
4	1	1	1	13	13	13	13	1	2
5	2	0	0	0	2	2	2	2	1
6	1	4	2	2	19	16	16	19	2
7	2	4	5	5	4	11	11	4	4
8	4	1	1	1	13	13	13	13	1
9	2	2	0	0	0	2	2	2	2
10	1	1	4	2	2	16	16	16	19
11	0	0	0	9	6	4	9	0	0
12	0	0	0	13	15	15	13	0	0
13	1	1	1	14	8	8	14	0	0
14	0	0	0	13	6	2	13	1	1
15	2	0	0	0	9	4	4	9	0
16	0	0	0	0	13	15	15	13	0
17	0	1	1	1	14	8	8	14	0
18	0	0	0	0	13	2	2	13	1
19	1	2	0	0	0	6	4	4	9
20	0	0	0	0	0	15	15	15	13
21	0	0	0	2	0	0	4	0	0
22	1	1	1	2	0	2	14	0	0
23	2	2	2	5	3	3	10	2	2
24	1	1	1	20	0	2	1	0	0
25	1	0	0	0	2	0	0	4	0
26	0	1	1	1	2	2	2	14	0

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 18-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	0	2	2	2	5	3	3	10	2
28	2	1	1	1	20	2	2	1	0
29	0	1	0	0	0	0	0	0	4
30	0	0	1	1	1	0	2	2	14
31	4	4	4	17	12	13	17	4	4
32	0	0	0	18	11	11	18	0	0
33	1	0	1	0	1	1	0	0	0
34	0	0	0	0	2	2	0	0	0
35	1	4	4	4	17	13	13	17	4
36	4	0	0	0	18	11	11	18	0
37	0	1	0	1	0	1	1	0	0
38	0	0	0	0	0	2	2	0	0
39	0	1	4	4	4	12	13	13	17
40	4	4	0	0	0	11	11	11	18
41	0	0	0	19	6	6	19	1	1
42	1	1	1	2	2	2	2	2	2
43	2	2	2	12	9	14	12	1	1
44	1	1	1	7	7	7	7	1	1
45	3	0	0	0	19	6	6	19	1
46	1	1	1	1	2	2	2	2	2
47	2	2	2	2	12	14	14	12	1
48	1	1	1	1	7	7	7	7	1
49	1	3	0	0	0	6	6	6	19
50	1	1	1	1	1	2	2	2	2
51	0	0	0	7	6	11	7	0	0
52	0	0	0	7	11	11	7	1	1
53	3	3	3	8	6	5	8	5	4
54	1	3	3	6	7	7	6	1	1
55	4	0	0	0	7	11	11	7	0
56	0	0	0	0	7	11	11	7	1
57	1	3	3	3	8	5	5	8	5
58	4	1	3	3	6	7	7	6	1
59	1	4	0	0	0	6	11	11	7
60	0	0	0	0	0	11	11	11	7
61	3	3	3	7	10	10	7	5	5
62	0	0	0	16	5	5	16	1	1
63	1	1	1	15	2	2	15	1	1
64	0	0	0	6	7	7	6	0	0

TABLE 18-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	2	3	3	3	7	10	10	7	5
66	5	0	0	0	16	5	5	16	1
67	1	1	1	1	15	2	2	15	1
68	1	0	0	0	6	7	7	6	0
69	0	2	3	3	3	10	10	10	7
70	5	5	0	0	0	5	5	5	16
71	1	1	1	5	4	4	5	0	0
72	1	2	2	16	13	16	16	5	5
73	3	2	2	13	1	1	13	2	2
74	2	2	2	14	8	17	14	2	2
75	1	1	1	1	5	4	4	5	0
76	0	1	2	2	16	16	16	16	5
77	5	3	2	2	13	1	1	13	2
78	2	2	2	2	14	17	17	14	2
79	2	1	1	1	1	4	4	4	5
80	0	0	1	2	2	13	16	16	16
81	2	2	2	9	9	8	9	3	3
82	0	0	0	0	5	5	0	0	0
83	2	2	2	13	4	4	13	2	2
84	1	1	1	8	8	4	8	1	1
85	1	2	2	2	9	8	8	9	3
86	3	0	0	0	0	5	5	0	0
87	0	2	2	2	13	4	4	13	2
88	2	1	1	1	8	8	4	8	1
89	1	1	2	2	2	9	8	8	9
90	3	3	0	0	0	5	5	5	0
91	3	3	3	16	5	5	16	3	3
92	0	0	0	14	14	14	14	0	0
93	1	2	2	3	9	9	3	2	2
94	1	1	1	2	9	9	2	2	2
95	0	3	3	3	16	5	5	16	3
96	3	0	0	0	14	14	14	14	0
97	0	1	2	2	3	9	9	3	2
98	2	1	1	1	2	9	9	2	2
99	2	0	3	3	3	5	5	5	16
100	3	3	0	0	0	14	14	14	14

TABLE 19

MULTIVARIATE BINOMIAL DISTRIBUTIONS WITH MEANS CLOSER TO EACH OTHER: NUMBER OF MISCLASSIFICATIONS BASED ON MEMBERSHIP IN ORIGINAL POPULATIONS (ANALYSIS SAMPLE)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	98	96	96	99	96	96	99	96	96
2	98	92	92	98	92	92	98	92	92
3	96	100	100	96	100	100	96	100	100
4	90	93	90	90	93	93	90	93	93
5	86	93	93	97	93	93	97	93	93
6	97	93	93	100	93	93	100	93	93
7	98	95	95	98	95	95	98	95	95
8	91	91	91	99	91	91	99	91	91
9	97	100	100	97	100	100	97	100	100
10	99	99	99	100	99	99	100	99	99
11	94	97	97	97	97	97	97	97	97
12	99	95	95	93	95	95	93	95	95
13	87	86	86	99	86	86	99	86	86
14	97	99	99	97	99	99	97	99	99
15	93	82	82	89	82	82	89	82	82
16	96	98	98	96	98	98	96	98	98
17	100	97	97	99	97	97	99	97	97
18	98	97	97	96	97	97	96	97	97
19	97	96	96	93	96	96	93	96	96
20	87	90	90	99	99	99	99	90	90
21	95	97	97	97	97	97	97	97	97
22	95	87	87	92	87	87	92	87	87
23	93	84	84	97	84	84	97	84	84
24	93	89	89	95	89	89	95	89	89
25	98	88	88	95	88	88	95	88	88
26	100	98	98	96	98	98	96	98	98

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 19-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	94	94	94	98	94	94	98	94	94
28	88	91	91	99	91	91	99	91	91
29	96	96	96	100	96	96	100	96	96
30	97	95	95	98	95	95	98	95	95
31	99	95	95	99	95	95	99	95	95
32	89	93	93	97	93	93	97	93	93
33	92	89	89	96	89	89	96	89	89
34	94	93	93	99	93	93	99	93	93
35	90	95	95	93	95	95	93	95	95
36	80	89	89	100	89	89	100	89	89
37	86	81	81	95	81	81	95	81	81
38	99	95	95	97	95	95	97	95	95
39	90	81	81	98	81	81	98	81	81
40	99	99	99	97	99	99	97	99	99
41	99	96	96	97	96	96	97	96	96
42	95	96	96	99	96	96	99	96	96
43	97	97	97	99	97	99	99	97	97
44	93	87	87	100	87	87	100	87	87
45	88	77	77	97	77	77	97	77	77
46	96	98	98	98	98	98	98	98	98
47	91	96	96	99	96	96	99	96	96
48	95	97	97	99	97	97	99	97	97
49	89	85	85	99	85	85	99	85	85
50	99	97	97	98	97	97	98	97	97
51	98	90	90	96	90	90	96	90	90
52	90	88	88	99	88	88	99	88	88
53	98	98	98	97	98	98	97	98	98
54	94	96	96	100	96	96	100	96	96
55	81	82	82	98	82	82	98	82	82
56	97	94	94	92	94	94	92	94	94
57	97	87	87	95	87	87	95	87	87
58	98	98	98	100	98	98	100	98	98
59	100	98	98	98	98	98	98	98	98
60	91	89	89	100	89	89	100	89	89
61	86	81	81	93	81	81	93	81	81
62	95	96	96	91	96	96	91	96	96
63	96	97	97	95	97	97	95	97	97
64	98	[illegible]	98	95	98	98	95	98	98

TABLE 19-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	96	98	98	98	98	98	98	98	98
66	95	100	100	97	100	100	97	100	100
67	96	91	91	99	91	91	99	91	91
68	89	87	87	95	87	87	95	87	87
69	89	87	87	98	87	87	98	87	87
70	90	86	86	97	86	86	97	86	86
71	96	99	99	99	99	99	99	99	99
72	78	80	80	82	80	80	82	80	80
73	100	97	97	97	97	97	97	97	97
74	89	91	91	94	91	91	94	91	91
75	92	100	100	93	100	100	93	100	100
76	93	99	99	84	99	99	84	99	99
77	97	89	89	98	89	89	98	89	89
78	97	96	96	100	96	96	100	96	96
79	96	96	96	96	96	96	96	96	96
80	92	94	94	98	94	94	98	94	94
81	86	80	80	94	80	80	94	80	80
82	85	82	82	99	82	82	99	82	82
83	98	93	93	99	93	93	99	93	93
84	87	90	90	96	90	90	96	90	90
85	97	90	90	91	90	90	91	90	90
86	87	92	92	91	92	92	91	92	92
87	98	94	94	89	94	94	89	94	94
88	98	94	94	99	94	94	99	94	94
89	89	96	96	94	96	96	94	96	96
90	88	92	92	95	92	92	95	92	92
91	97	97	97	97	97	97	97	97	97
92	92	94	94	98	94	94	98	94	94
93	100	98	98	91	98	98	91	98	98
94	98	94	94	97	94	94	97	94	94
95	87	89	89	97	89	89	97	89	89
96	83	85	85	98	85	85	98	85	85
97	95	99	99	98	99	99	98	99	99
98	96	91	91	95	91	91	95	91	91
99	84	81	81	94	81	81	94	81	81
100	85	88	88	96	88	88	96	88	88

TABLE 20

MULTIVARIATE BINOMIAL DISTRIBUTIONS WITH MEANS CLOSER TO EACH OTHER: NUMBER OF MISCLASSIFICATIONS BASED ON MEMBERSHIP IN ORIGINAL POPULATIONS (VALIDATION SAMPLE)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	95	99	99	93	99	99	93	99	99
2	94	96	96	98	96	96	98	96	96
3	99	96	96	95	96	96	95	96	96
4	95	100	95	95	100	100	95	100	100
5	98	88	88	87	88	88	87	88	88
6	98	92	92	94	92	92	94	92	92
7	88	86	86	98	86	86	98	86	86
8	96	97	97	97	97	97	97	97	97
9	87	87	87	92	87	87	92	87	87
10	100	89	89	93	89	89	93	89	89
11	93	99	99	97	99	99	97	99	99
12	99	98	98	99	98	98	99	98	98
13	99	99	99	97	99	99	97	99	99
14	91	91	91	98	91	91	98	91	91
15	83	92	92	99	92	92	99	92	92
16	98	94	94	98	94	94	98	100	100
17	100	98	98	98	98	98	98	98	98
18	85	95	95	92	95	95	92	95	95
19	94	79	79	93	79	79	93	79	79
20	92	87	87	98	98	98	98	87	87
21	100	92	92	90	92	92	90	92	92
22	92	97	97	99	97	97	99	97	97
23	82	85	85	99	85	85	99	85	85
24	89	88	88	97	88	88	97	88	88
25	90	99	99	92	99	99	92	99	99
26	94	96	96	89	96	96	89	96	96

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted; CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 20-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	89	89	89	97	89	89	97	89	89
28	89	85	85	94	85	85	94	85	85
29	96	96	96	99	96	96	99	96	96
30	90	93	93	98	93	93	98	93	93
31	87	87	87	96	87	87	96	87	87
32	96	100	100	96	100	100	96	100	100
33	97	92	92	100	92	92	100	92	92
34	98	92	92	96	92	92	96	92	92
35	86	99	99	99	99	99	99	99	99
36	86	89	89	96	89	89	96	89	89
37	99	100	100	100	100	100	100	100	100
38	100	100	100	95	100	100	95	100	100
39	97	99	99	90	99	99	90	99	99
40	92	88	88	92	88	88	92	88	88
41	93	95	95	99	95	95	99	95	95
42	89	86	86	97	86	86	97	86	86
43	86	95	95	100	95	100	100	95	95
44	74	81	81	89	81	81	89	81	81
45	97	97	97	98	97	97	98	97	97
46	91	99	99	92	99	92	92	99	99
47	93	98	98	98	98	98	98	98	98
48	99	93	93	94	93	93	94	93	93
49	96	93	93	98	93	93	98	93	93
50	92	97	97	93	97	97	93	97	97
51	93	93	93	94	93	93	94	93	93
52	94	87	87	93	87	87	93	87	87
53	86	86	86	100	86	86	100	86	86
54	94	94	94	98	94	94	98	94	94
55	92	85	85	99	85	85	99	85	85
56	96	95	95	99	95	95	99	95	95
57	100	89	89	96	89	89	96	89	89
58	97	91	91	98	91	91	98	91	91
59	96	96	96	99	96	96	99	96	96
60	98	98	98	94	98	98	94	98	98
61	99	96	96	95	96	96	95	96	96
62	95	95	95	93	95	95	93	95	95
63	93	95	95	98	95	95	98	95	95
64	95	99	99	100	99	99	100	99	99

TABLE 20-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	82	82	82	100	82	82	100	82	82
66	94	95	95	96	95	95	96	95	95
67	99	90	90	96	90	90	96	90	90
68	99	96	96	95	96	96	95	96	96
69	90	93	93	97	93	93	97	93	93
70	100	92	92	98	92	92	98	92	92
71	89	84	84	95	84	84	95	84	84
72	88	92	92	86	92	92	86	92	92
73	97	92	92	88	92	92	88	92	92
74	95	96	96	98	96	96	98	96	96
75	96	99	99	97	99	99	97	99	99
76	79	80	80	94	80	80	94	80	80
77	94	100	100	96	100	100	96	100	100
78	92	96	96	92	96	96	92	96	96
79	98	92	92	98	92	92	98	100	100
80	93	92	92	94	92	92	94	92	92
81	83	82	82	92	82	82	92	82	82
82	95	96	96	97	96	96	97	96	96
83	86	89	89	97	89	89	97	89	89
84	85	86	86	91	86	86	91	86	86
85	80	90	90	100	90	90	100	90	90
86	100	99	99	98	99	99	98	99	99
87	91	90	90	99	90	90	99	90	90
88	100	97	97	97	97	97	97	97	97
89	88	98	98	98	98	98	98	98	98
90	95	99	99	99	99	99	99	99	99
91	98	96	96	96	96	96	96	96	96
92	99	95	95	100	95	95	100	95	95
93	99	99	99	95	99	99	95	99	99
94	100	98	98	92	98	98	92	100	100
95	99	99	99	98	99	99	98	99	99
96	99	97	97	91	97	97	91	97	97
97	94	92	92	94	92	92	94	92	92
98	94	96	96	100	96	96	100	96	96
99	88	87	87	95	87	87	95	87	87
100	89	89	89	94	89	89	94	89	89

TABLE 21

MULTIVARIATE BINOMIAL DISTRIBUTIONS WITH MEANS MODERATELY APART: NUMBER OF MISCLASSIFICATIONS BASED ON MEMBERSHIP IN ORIGINAL POPULATIONS (ANALYSIS SAMPLE)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	91	69	69	90	69	69	90	69	69
2	86	81	81	96	81	81	96	81	81
3	72	79	79	93	79	79	93	79	79
4	74	73	73	96	73	73	96	73	73
5	58	70	70	95	70	70	95	70	70
6	75	81	81	95	81	81	95	81	81
7	82	77	77	98	77	77	98	77	77
8	74	73	73	99	73	73	99	73	73
9	84	83	83	88	83	83	88	83	83
10	77	80	80	96	80	80	96	80	80
11	78	79	79	97	79	79	97	79	79
12	86	90	90	90	90	90	90	90	90
13	69	73	73	93	73	73	93	73	73
14	84	74	74	96	74	74	90	74	74
15	74	69	69	94	69	94	94	69	69
16	79	82	82	94	82	82	94	82	82
17	84	81	81	81	81	81	81	81	81
18	85	87	87	98	98	98	98	87	87
19	99	73	73	97	73	73	97	73	73
20	74	73	73	88	73	73	88	73	73
21	72	73	73	99	73	99	99	73	73
22	76	76	76	99	76	76	99	76	76
23	76	73	73	93	73	73	93	73	73
24	76	70	70	97	70	70	97	70	70
25	77	81	81	93	81	81	93	81	81
26	98	88	88	94	99	99	94	88	88

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 21-Continued

Trial No.	Weighting Schemes[a] X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	81	66	66	100	66	66	100	66	66
28	100	73	73	100	73	73	100	73	73
29	87	83	83	90	83	83	90	83	83
30	85	85	85	91	85	85	91	85	85
31	91	76	76	93	76	76	93	76	76
32	79	65	65	86	65	65	86	65	65
33	77	75	75	97	75	75	97	75	75
34	75	71	71	95	91	91	95	71	71
35	89	77	77	94	94	94	94	77	77
36	66	67	67	98	98	98	98	67	67
37	75	76	76	99	76	76	99	76	76
38	73	70	70	99	70	70	99	70	70
39	69	71	71	92	71	71	92	71	71
40	73	79	79	94	79	79	94	79	79
41	99	82	82	98	82	82	98	82	82
42	97	74	74	97	74	74	97	74	74
43	69	82	82	96	82	82	96	82	82
44	66	67	67	98	67	67	98	67	67
45	70	62	62	100	62	62	100	62	62
46	72	77	77	99	77	77	99	77	77
47	76	76	76	96	76	76	96	76	76
48	79	80	80	99	80	80	99	80	80
49	67	67	67	99	67	67	99	67	67
50	79	79	79	99	79	79	99	79	79
51	78	66	66	94	66	66	94	66	66
52	66	68	68	95	68	68	95	68	68
53	72	76	76	94	94	94	94	76	76
54	78	88	88	96	88	88	96	88	88
55	69	71	71	97	71	71	97	71	71
56	100	74	74	100	74	74	100	74	74
57	99	75	75	97	75	75	97	75	75
58	77	79	79	99	79	79	99	79	79
59	78	86	86	93	86	86	93	86	86
60	69	73	73	96	73	73	96	73	73
61	71	71	71	91	71	71	91	71	71
62	85	77	77	84	77	77	84	77	77
63	78	76	76	95	76	76	95	76	76
64	73	80	80	99	80	80	99	80	80

TABLE 21-Continued

Trial No.	Weighting Schemes[a] X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	78	78	78	97	78	78	97	78	78
66	69	80	80	98	80	80	98	80	80
67	72	84	84	91	84	84	91	84	84
68	84	89	89	96	89	89	96	89	89
69	93	69	69	93	69	69	93	69	69
70	71	69	69	95	69	69	95	69	69
71	78	77	77	95	77	77	95	77	77
72	69	65	65	99	65	65	99	65	65
73	77	73	73	95	73	73	95	73	73
74	75	75	75	80	75	75	80	75	75
75	71	74	74	98	74	74	98	74	74
76	86	82	82	96	82	82	96	82	82
77	74	81	81	92	81	81	92	81	81
78	72	77	77	99	99	99	99	77	77
79	98	99	99	99	99	99	99	78	78
80	81	74	74	95	74	74	95	74	74
81	63	61	61	96	61	61	96	61	61
82	69	60	60	93	60	60	93	60	60
83	68	80	80	100	80	80	100	80	80
84	86	93	93	99	93	93	99	93	93
85	95	78	78	95	95	95	95	78	78
86	73	77	77	96	77	77	96	77	77
87	70	77	77	93	77	77	93	77	77
88	68	72	72	97	97	97	97	72	72
89	74	81	81	95	81	81	95	81	81
90	82	78	78	100	78	78	100	78	78
91	69	76	76	95	76	76	95	76	76
92	97	83	83	94	83	83	94	83	83
93	77	77	77	98	77	77	98	77	77
94	75	69	69	94	69	69	94	69	69
95	67	66	66	90	66	66	90	66	66
96	71	66	66	92	66	66	92	66	66
97	85	77	77	95	77	77	95	77	77
98	77	73	73	100	73	73	100	73	73
99	67	61	61	95	61	61	95	61	61
100	72	69	69	96	69	69	96	69	69

TABLE 22

MULTIVARIATE BINOMIAL DISTRIBUTIONS WITH MEANS MODERATELY APART: NUMBER OF MISCLASSIFICATIONS BASED ON MEMBERSHIP IN ORIGINAL POPULATIONS (VALIDATION SAMPLE)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	100	100	100	95	100	100	95	100	100
2	75	68	68	89	68	68	89	68	68
3	81	86	86	99	86	86	99	86	86
4	72	77	77	100	77	77	100	77	77
5	73	76	76	96	76	76	96	76	76
6	68	78	78	90	78	78	90	78	78
7	59	61	61	98	61	61	98	61	61
8	72	66	66	97	66	66	97	66	66
9	72	79	79	92	79	79	92	79	79
10	72	72	72	100	72	72	100	72	72
11	77	87	87	94	87	87	94	87	87
12	78	78	78	98	78	78	98	78	78
13	72	75	75	99	75	75	99	75	75
14	61	72	72	89	72	72	92	72	72
15	68	77	77	95	77	95	95	77	77
16	68	100	100	98	100	100	98	100	100
17	85	90	90	90	90	90	90	90	90
18	68	78	78	90	90	90	90	78	78
19	95	65	65	97	65	65	97	65	65
20	79	71	71	98	71	71	98	71	71
21	73	100	100	90	100	90	90	100	100
22	75	81	81	96	81	81	96	81	81
23	70	70	70	96	70	70	96	70	70
24	72	75	75	97	75	75	97	75	75
25	83	77	77	98	77	77	98	77	77
26	100	100	100	87	96	96	87	100	100

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 22-Continued

Trial No.	Weighting Schemes[a] X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	84	82	82	99	82	82	99	82	82
28	89	100	100	89	100	100	89	100	100
29	74	73	73	99	73	73	99	73	73
30	69	71	71	98	71	71	98	71	71
31	99	78	78	97	78	78	97	78	78
32	73	73	73	94	73	73	94	73	73
33	78	76	76	98	76	76	98	76	76
34	79	77	77	96	97	97	96	77	77
35	100	82	82	98	98	98	98	82	82
36	100	75	75	95	95	95	95	75	75
37	74	70	70	96	70	70	96	70	70
38	100	100	100	93	100	100	93	100	100
39	82	76	76	95	76	76	95	76	76
40	72	67	67	91	67	67	91	67	67
41	100	100	100	98	100	100	98	100	100
42	92	73	73	89	73	73	89	100	100
43	67	82	82	89	82	82	89	82	82
44	66	61	61	93	61	61	93	61	61
45	74	69	69	97	69	69	97	69	69
46	100	73	73	100	73	73	100	73	73
47	64	72	72	98	72	72	98	72	72
48	77	81	81	97	81	81	97	81	81
49	79	73	73	98	73	73	98	73	73
50	66	77	77	100	77	77	100	77	77
51	100	88	88	98	88	88	98	88	88
52	100	77	77	98	100	100	98	77	77
53	69	68	68	98	98	98	98	68	68
54	60	76	76	100	76	76	100	76	76
55	64	64	64	90	64	64	90	64	64
56	91	75	75	91	75	75	91	75	75
57	90	100	100	90	100	100	90	100	100
58	72	75	75	98	75	75	98	75	75
59	76	77	77	98	77	77	98	77	77
60	81	81	81	100	81	81	100	81	81
61	79	85	85	92	85	85	92	85	85
62	83	81	81	80	81	81	80	81	81
63	100	81	81	97	81	81	97	81	81
64	69	72	72	97	72	72	97	72	72

TABLE 22-Continued

Trial No.	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
	Weighting Schemes[a]								
65	64	63	63	91	63	63	91	63	63
66	70	77	77	96	77	77	96	77	77
67	68	76	76	97	76	76	97	76	76
68	82	83	83	95	83	83	95	83	83
69	93	78	78	93	78	78	93	78	78
70	100	100	100	98	100	100	98	100	100
71	76	81	81	98	81	81	98	81	81
72	72	83	83	86	83	83	86	83	83
73	83	81	81	98	81	81	98	81	81
74	69	75	75	67	75	75	67	75	75
75	72	75	75	94	100	100	94	75	75
76	63	64	64	93	64	64	93	64	64
77	64	70	70	99	70	70	99	70	70
78	70	73	73	91	91	91	91	73	73
79	98	98	98	98	98	98	98	100	100
80	69	73	73	87	73	73	87	73	73
81	70	68	68	95	68	68	95	68	68
82	81	80	80	98	80	80	98	80	80
83	66	71	71	95	71	71	95	71	71
84	79	69	69	99	69	69	99	69	69
85	99	100	100	95	95	95	95	100	100
86	86	82	82	97	82	82	97	82	82
87	74	78	78	99	78	78	99	78	78
88	69	73	73	94	94	94	94	100	100
89	100	77	77	98	77	77	98	77	77
90	77	81	81	96	81	81	96	81	81
91	82	82	82	97	82	82	97	82	82
92	96	77	77	92	77	77	92	77	77
93	81	78	78	94	78	78	94	78	78
94	83	79	79	99	79	79	99	79	79
95	75	75	75	96	75	75	96	75	75
96	87	71	71	98	71	71	98	71	71
97	75	72	72	93	72	72	93	72	72
98	74	70	70	96	70	70	96	70	70
99	75	73	73	98	73	73	98	73	73
100	74	77	77	96	77	77	96	77	77

TABLE 23

MULTIVARIATE BINOMIAL DISTRIBUTIONS WITH MEANS FARTHER APART: NUMBER OF MISCLASSIFICATIONS BASED ON MEMBERSHIP IN ORIGINAL POPULATIONS (ANALYSIS SAMPLE)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	9	9	9	99	94	99	99	7	99
2	6	6	6	95	95	95	95	10	10
3	3	3	3	95	95	95	95	3	99
4	4	4	4	95	95	95	95	4	99
5	5	5	5	98	98	98	98	5	98
6	11	11	11	100	100	100	100	11	100
7	6	6	6	99	99	99	99	6	99
8	3	3	3	98	96	98	98	3	94
9	7	7	7	97	97	97	97	7	7
10	5	5	5	93	93	97	93	5	97
11	8	8	8	100	98	98	100	8	95
12	7	7	7	94	96	97	97	7	100
13	2	2	2	99	98	98	99	2	95
14	8	8	8	98	98	98	98	8	98
15	3	3	3	97	97	97	97	4	100
16	4	4	4	100	100	100	100	4	100
17	5	5	5	92	96	92	92	13	87
18	5	5	5	96	96	96	96	5	97
19	5	5	5	94	99	99	94	11	100
20	5	5	5	94	97	97	94	8	96
21	7	7	7	98	93	93	98	15	92
22	3	3	3	98	98	98	98	9	100
23	6	6	6	96	96	98	96	18	98
24	3	3	3	99	93	93	99	10	100
25	5	5	5	94	94	94	94	17	94
26	6	6	6	98	97	98	98	13	98

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 23-Continued

Trial No.	Weighting Schemes[a] X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	4	4	4	99	99	99	99	8	98
28	100	100	100	99	99	98	99	6	98
29	3	3	3	91	91	98	91	3	100
30	7	7	7	99	99	99	99	7	99
31	99	99	99	99	99	99	99	5	99
32	96	96	6	96	96	96	96	6	96
33	4	4	4	95	95	95	95	4	93
34	8	8	8	91	99	98	91	14	14
35	99	99	99	99	99	99	99	4	99
36	2	2	2	98	98	98	98	4	98
37	8	8	8	95	97	95	95	13	95
38	4	4	4	96	96	96	96	4	97
39	1	1	1	100	100	100	100	1	99
40	3	3	3	98	98	99	98	10	99
41	96	96	96	96	100	100	96	8	96
42	2	2	2	99	99	99	99	7	98
43	4	4	4	99	98	98	99	7	97
44	98	98	98	98	100	98	98	10	98
45	5	5	5	98	96	96	98	5	96
46	4	4	4	93	97	93	93	4	86
47	98	98	98	100	98	100	100	14	100
48	5	5	5	100	89	100	100	5	100
49	4	4	4	98	99	99	98	4	99
50	4	4	4	97	97	97	97	4	97
51	2	2	2	98	99	98	98	2	100
52	5	5	5	99	99	99	99	5	93
53	5	5	5	98	94	94	98	12	99
54	4	4	4	93	93	93	93	9	97
55	2	2	2	96	96	96	96	2	96
56	9	9	9	94	94	94	94	9	9
57	2	2	2	100	98	100	100	7	99
58	9	9	9	99	95	99	99	9	91
59	4	4	4	98	95	95	98	8	97
60	4	4	4	100	100	100	100	6	97
61	6	6	6	98	98	98	98	10	99
62	8	8	8	90	90	90	90	17	100
63	4	4	4	99	99	97	99	4	97
64	98	98	98	98	98	98	98	5	96

TABLE 23-Continued

Trial No.	Weighting Schemes[a] X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	3	3	3	99	99	99	99	5	97
66	3	3	3	93	93	93	93	3	98
67	2	2	2	99	99	98	99	2	98
68	3	3	3	100	100	100	100	3	99
69	2	2	2	99	99	99	99	2	99
70	3	3	3	99	99	97	99	6	99
71	3	3	3	100	100	97	100	3	97
72	4	4	4	100	100	100	100	6	94
73	4	4	4	93	97	89	93	4	89
74	11	11	11	95	95	98	95	12	98
75	96	96	96	95	95	95	95	3	93
76	95	100	95	95	95	95	95	5	95
77	6	6	6	98	98	98	98	6	99
78	4	4	4	99	99	99	99	4	93
79	3	3	3	96	95	95	96	7	95
80	5	5	5	98	98	91	98	5	99
81	6	6	6	98	97	98	98	10	99
82	2	2	2	98	98	98	98	5	98
83	5	5	5	93	93	98	93	15	97
84	3	3	3	97	97	100	97	3	100
85	10	10	10	96	96	96	96	10	100
86	6	6	6	97	97	97	97	11	98
87	4	4	4	97	97	97	99	4	97
88	4	4	4	98	98	99	98	4	97
89	6	6	6	99	99	99	99	6	99
90	3	3	3	96	97	100	96	3	96
91	3	3	3	100	100	100	100	3	100
92	6	6	6	93	93	93	93	6	98
93	4	4	4	92	92	92	92	11	100
94	5	5	5	96	96	96	96	5	96
95	3	3	3	99	96	96	99	5	99
96	4	4	4	99	99	99	99	4	98
97	3	3	3	94	98	94	94	14	94
98	4	4	4	99	99	99	99	6	95
99	7	7	7	98	93	93	98	10	98
100	4	4	4	97	97	97	97	11	99

TABLE 24

MULTIVARIATE BINOMIAL DISTRIBUTIONS WITH MEANS FARTHER APART: NUMBER OF MISCLASSIFICATIONS BASED ON MEMBERSHIP IN ORIGINAL POPULATIONS (VALIDATION SAMPLE)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	2	2	2	99	94	99	99	4	99
2	7	7	7	99	99	99	99	5	5
3	10	10	10	98	98	98	98	10	100
4	5	5	5	92	92	92	92	5	97
5	6	6	6	98	98	98	98	6	97
6	3	3	3	99	99	99	99	3	99
7	2	2	2	98	98	98	98	2	100
8	2	2	2	99	99	99	99	2	99
9	6	6	6	100	99	100	100	6	6
10	3	3	3	99	99	99	99	3	99
11	7	7	7	98	96	96	98	7	100
12	7	7	7	95	95	95	92	100	100
13	1	1	1	99	96	96	99	1	95
14	5	5	5	100	100	100	100	5	100
15	4	4	4	96	96	96	96	6	99
16	5	5	5	99	99	99	99	5	96
17	9	9	9	90	93	90	90	13	87
18	11	11	11	99	99	99	99	11	100
19	8	8	8	98	99	99	98	8	100
20	2	2	2	98	96	96	98	4	96
21	6	6	6	96	94	94	96	7	90
22	6	6	6	100	100	100	100	6	100
23	4	4	4	98	98	100	98	1	100
24	3	3	3	100	94	94	100	11	97
25	4	4	4	94	94	94	94	4	94
26	7	7	7	95	98	95	95	8	95

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 24-Continued

Trial No.	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
	Weighting Schemes[a]								
27	7	7	7	99	97	99	99	3	100
28	100	100	100	96	96	100	96	5	100
29	7	7	7	97	97	100	97	7	100
30	4	4	4	99	99	99	99	4	99
31	100	100	100	93	100	93	93	4	100
32	98	98	8	98	98	98	98	8	98
33	5	5	5	94	94	94	94	5	94
34	2	2	2	95	96	95	95	8	8
35	100	100	100	98	100	100	98	100	98
36	4	4	4	99	99	99	99	5	99
37	7	7	7	97	100	97	97	9	99
38	4	4	4	97	97	97	97	4	100
39	4	4	4	95	99	99	95	4	99
40	3	3	3	94	94	100	94	9	100
41	98	98	98	91	98	99	91	100	98
42	3	3	3	99	99	99	99	8	97
43	8	8	8	97	95	95	97	10	97
44	100	100	100	97	98	100	97	100	100
45	8	8	8	99	98	98	99	14	98
46	6	6	6	93	94	93	93	6	96
47	93	93	93	92	93	92	92	5	92
48	4	4	4	100	95	100	100	4	100
49	5	5	5	100	98	98	100	5	98
50	7	7	7	99	99	99	99	7	93
51	100	100	100	96	98	96	96	100	99
52	6	6	6	98	98	98	98	6	97
53	4	4	4	100	94	94	100	2	100
54	5	5	5	90	90	90	90	7	92
55	9	9	9	100	100	100	100	9	100
56	4	4	4	95	95	95	95	4	4
57	2	2	2	99	98	99	99	10	100
58	6	6	6	98	100	98	98	6	95
59	6	6	6	99	95	95	99	7	96
60	7	7	7	99	99	99	99	7	95
61	10	10	10	100	100	100	100	10	100
62	5	5	5	95	95	95	95	6	100
63	11	11	11	99	99	98	99	11	98
64	95	95	95	95	95	95	95	8	100

TABLE 24-Continued

Trial No.	Weighting Schemes[a] X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	5	5	5	97	100	100	97	5	100
66	4	4	4	92	92	92	92	4	100
67	6	6	6	100	100	98	100	6	98
68	9	9	9	100	100	100	100	100	100
69	5	5	5	99	99	99	99	5	100
70	5	5	5	100	95	91	100	15	100
71	8	8	8	97	97	97	97	8	97
72	6	6	6	94	94	94	94	14	93
73	7	7	7	98	96	98	98	7	98
74	11	11	11	93	93	98	93	15	98
75	96	96	96	96	96	96	96	5	99
76	100	97	100	96	96	96	96	4	100
77	4	4	4	99	99	99	99	4	100
78	5	5	5	100	100	100	100	5	98
79	5	5	5	94	95	95	94	3	95
80	2	2	2	95	95	93	95	2	100
81	5	5	5	100	97	100	100	5	100
82	7	7	7	99	99	99	99	7	99
83	9	9	9	96	96	100	96	10	95
84	5	5	5	99	99	100	99	5	100
85	7	7	7	95	95	95	95	7	96
86	9	9	9	100	100	100	100	8	99
87	8	6	8	100	96	96	96	8	96
88	2	2	2	95	95	100	95	2	100
89	3	3	3	99	99	99	99	3	98
90	9	9	9	99	94	93	99	9	99
91	6	6	6	99	99	99	99	6	100
92	9	9	9	98	98	98	98	9	100
93	12	12	12	98	98	98	98	12	100
94	5	5	5	100	100	100	100	5	100
95	8	8	8	99	100	100	99	6	99
96	5	5	5	99	99	99	99	5	100
97	8	8	8	98	100	98	98	9	98
98	6	6	6	98	98	98	98	5	95
99	5	5	5	99	99	99	99	7	99
100	6	6	6	100	100	100	100	6	97

TABLE 25

MULTIVARIATE MULTINOMIAL DISTRIBUTIONS: NUMBER OF MISCLASSIFICATIONS BASED ON MEMBERSHIP IN ORIGINAL POPULATIONS (ANALYSIS SAMPLE)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	46	52	52	71	58	58	71	43	43
2	44	44	45	72	98	72	72	37	37
3	50	52	52	54	58	58	54	38	38
4	43	42	41	61	60	60	61	38	38
5	55	54	55	72	70	70	72	43	41
6	74	70	66	75	70	70	75	49	49
7	42	43	44	69	65	65	69	34	34
8	50	62	62	80	69	69	80	49	49
9	58	57	57	85	71	71	85	40	40
10	47	50	50	91	63	63	91	46	46
11	63	57	58	63	63	63	63	46	46
12	40	44	44	80	60	60	80	42	42
13	61	57	56	67	67	67	67	47	47
14	61	59	59	71	62	62	71	42	44
15	50	52	50	61	81	66	61	39	39
16	51	51	51	75	67	67	75	48	48
17	50	56	56	80	59	59	80	47	47
18	59	57	57	76	70	70	76	49	49
19	63	63	63	78	71	71	78	42	42
20	47	58	58	71	71	71	71	46	46
21	51	60	72	72	72	72	72	52	52
22	38	46	44	75	55	55	75	39	39
23	49	46	49	71	72	72	71	42	42
24	45	50	50	68	87	61	68	46	46
25	62	56	56	72	66	66	72	44	44
26	45	52	52	66	65	65	66	36	36

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 25-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	42	54	54	65	62	62	65	44	44
28	51	54	54	64	60	56	64	42	44
29	55	50	50	50	59	98	50	46	46
30	49	47	47	65	61	72	65	40	40
31	40	45	45	58	72	72	58	38	38
32	62	56	56	84	73	73	84	45	45
33	67	51	50	80	63	63	80	56	56
34	38	44	44	72	52	52	72	38	38
35	40	47	47	81	44	44	81	39	39
36	54	51	46	79	62	62	79	40	40
37	49	51	53	53	56	57	53	40	40
38	36	57	57	69	68	68	69	41	41
39	53	63	63	66	62	62	66	55	55
40	47	48	47	49	58	58	49	43	43
41	38	55	46	64	81	81	64	35	35
42	61	50	50	66	64	64	66	42	42
43	44	52	52	86	57	57	86	43	48
44	43	52	52	59	61	61	59	42	42
45	50	60	60	71	62	62	71	43	43
46	39	44	43	74	62	62	74	37	37
47	42	50	51	89	60	60	89	45	45
48	44	52	54	84	57	57	84	50	50
49	54	48	48	71	55	55	71	44	44
50	43	56	56	83	59	64	83	44	44
51	71	50	50	72	51	51	72	46	46
52	68	54	54	83	63	63	83	50	50
53	48	49	50	66	61	67	66	44	44
54	55	63	63	68	72	72	68	46	46
55	39	49	49	70	54	54	70	36	36
56	71	53	70	86	61	61	86	45	45
57	50	58	58	84	71	71	84	44	44
58	36	54	54	54	61	61	54	36	36
59	49	42	50	67	53	53	67	40	40
60	61	61	61	71	71	71	71	32	32
61	76	50	50	90	59	59	90	47	47
62	43	50	50	65	55	55	65	35	35
63	75	60	60	59	59	59	59	49	49
64	46	59	59	60	69	69	60	47	47

TABLE 25-Continued

Trial No.	Weighting Schemes[a] X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	68	59	59	63	62	62	63	57	59
66	38	37	37	52	65	65	54	35	33
67	57	47	47	87	59	59	87	42	42
68	69	68	68	76	71	71	76	56	56
69	52	43	42	65	64	64	65	35	35
70	40	53	53	88	65	65	88	39	39
71	47	58	54	69	69	69	69	47	46
72	57	47	47	88	57	57	88	51	51
73	51	63	63	66	62	62	66	42	42
74	32	34	26	75	75	73	75	27	27
75	58	62	62	67	65	65	67	46	46
76	47	57	57	66	72	72	66	44	44
77	53	54	54	84	63	63	84	43	43
78	60	63	63	77	67	67	77	44	44
79	52	56	55	58	69	69	58	44	44
80	49	54	54	79	62	62	79	47	47
81	48	57	57	83	65	65	83	46	47
82	54	54	54	67	68	68	67	40	42
83	53	63	63	78	74	74	78	57	57
84	51	46	50	64	56	56	64	38	38
85	46	65	65	93	71	71	93	47	47
86	36	41	41	76	60	60	76	37	37
87	75	49	49	81	60	60	81	42	42
88	54	54	54	54	63	63	54	49	49
89	45	53	46	68	63	67	68	44	44
90	47	53	53	63	62	62	63	38	38
91	40	49	49	89	65	65	89	42	44
92	48	52	52	64	63	63	64	42	42
93	44	52	52	57	60	60	57	49	49
94	48	51	51	84	62	61	84	41	41
95	59	62	56	76	75	74	76	48	48
96	68	59	59	72	59	59	72	48	50
97	45	55	55	72	57	57	72	49	49
98	45	43	53	71	75	75	71	53	53
99	51	62	64	77	78	78	77	47	47
100	73	49	49	88	57	57	88	44	44

TABLE 26

MULTIVARIATE MULTINOMIAL DISTRIBUTIONS: NUMBER OF MISCLASSIFICATIONS BASED ON MEMBERSHIP IN ORIGINAL POPULATIONS (VALIDATION SAMPLE)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	100	100	100	100	100	100	100	100	100
2	79	79	78	100	100	100	100	57	57
3	100	100	100	100	100	100	100	90	90
4	81	100	100	100	100	100	100	100	100
5	72	100	100	100	100	100	100	67	62
6	100	100	100	100	100	100	100	82	82
7	100	100	100	100	100	100	100	87	87
8	63	100	100	100	100	100	100	100	100
9	81	100	100	100	100	100	100	100	100
10	100	100	100	100	100	100	100	100	100
11	100	100	100	100	100	100	100	66	66
12	86	100	100	100	100	100	100	68	68
13	69	100	100	100	100	100	100	100	100
14	76	100	100	100	100	100	100	94	100
15	83	76	83	100	100	100	100	61	61
16	66	100	100	100	100	100	100	100	100
17	62	100	100	100	100	100	100	56	56
18	88	100	100	100	100	100	100	65	65
19	100	100	100	100	100	100	100	90	90
20	70	100	100	100	100	100	100	78	78
21	63	100	100	100	100	100	100	58	58
22	75	100	100	100	100	100	100	100	100
23	90	100	100	100	100	100	100	95	95
24	90	100	100	100	85	100	100	100	100
25	100	100	100	100	100	100	100	92	100
26	100	100	100	100	100	100	100	93	93

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 26-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	66	100	100	100	100	100	100	100	100
28	100	100	100	100	100	100	100	100	100
29	80	100	100	100	100	90	100	89	89
30	86	100	100	100	100	100	100	57	68
31	91	100	100	100	100	100	100	91	91
32	100	100	100	100	100	100	100	93	93
33	100	100	100	100	100	100	100	100	100
34	80	100	100	100	100	100	100	100	100
35	74	100	100	100	100	100	100	91	91
36	91	100	94	100	100	100	100	77	77
37	100	82	100	100	100	100	100	68	68
38	83	100	100	100	100	100	100	100	100
39	100	100	100	100	100	100	100	100	100
40	100	100	100	100	100	100	100	74	74
41	92	100	100	100	100	100	100	85	85
42	100	100	100	100	100	100	100	100	100
43	100	100	100	100	100	100	100	89	85
44	91	100	100	100	100	100	100	76	76
45	100	100	100	100	100	100	100	85	85
46	62	100	100	100	100	100	100	70	70
47	100	100	100	100	100	100	100	95	95
48	74	100	100	100	100	100	100	100	100
49	100	100	100	100	100	100	100	100	100
50	92	100	100	100	100	100	100	88	88
51	100	100	100	100	100	100	100	90	90
52	100	100	100	100	100	100	100	88	82
53	88	100	100	100	100	100	100	88	88
54	60	100	100	100	100	100	100	74	74
55	71	100	100	100	100	100	100	92	92
56	100	100	100	100	100	100	100	76	76
57	51	100	100	100	100	100	100	100	100
58	69	100	100	100	100	100	100	86	86
59	100	100	100	100	100	100	100	84	84
60	100	100	100	100	100	100	100	48	48
61	100	100	100	100	100	100	100	100	100
62	100	100	100	100	100	100	100	74	74
63	100	100	100	100	100	100	100	100	100
64	100	100	100	100	100	100	100	100	100

TABLE 26-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	100	100	100	100	100	100	100	100	100
66	47	58	58	100	100	100	100	66	68
67	100	100	100	100	100	100	100	80	80
68	100	100	100	100	100	100	100	100	100
69	100	79	78	100	100	100	100	56	56
70	93	100	100	100	100	100	100	74	74
71	81	100	100	100	100	100	100	93	91
72	78	100	100	100	100	100	100	74	74
73	75	100	100	100	100	100	100	92	92
74	75	84	65	100	100	100	100	67	67
75	100	100	100	100	100	100	100	50	50
76	81	100	100	100	100	100	100	55	55
77	100	100	100	100	100	100	100	100	100
78	100	100	100	100	100	100	100	74	74
79	87	100	100	100	100	100	100	75	75
80	100	100	100	100	100	100	100	89	89
81	89	100	100	100	100	100	100	71	72
82	100	100	100	100	100	100	100	86	82
83	82	79	79	100	100	100	100	76	76
84	81	100	100	100	100	100	100	79	79
85	100	100	100	100	100	100	100	100	100
86	72	100	100	100	100	100	100	100	100
87	100	100	100	100	100	100	100	88	88
88	100	100	100	100	100	100	100	100	100
89	100	100	100	100	100	100	100	84	84
90	100	100	100	100	100	100	100	100	100
91	100	100	100	100	100	100	100	100	100
92	100	100	100	100	100	100	100	84	84
93	79	100	100	100	100	100	100	100	100
94	50	100	100	100	100	100	100	100	100
95	75	100	89	100	100	100	100	79	79
96	100	100	100	100	100	100	100	65	73
97	47	100	100	100	100	100	100	100	100
98	86	77	100	100	100	100	100	54	54
99	81	100	100	100	100	100	100	71	71
100	100	100	100	100	100	100	100	100	100

TABLE 27

MULTIVARIATE NORMAL DISTRIBUTIONS WITH MEANS CLOSER TO EACH OTHER: SUM OF SQUARED ERROR VARIANCE OF THE CRITERION SET VARIABLES OBTAINED FOR ANALYSIS SAMPLE
(Results Rounded-off to Nearest Integer)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	94	70	70	107	81	81	107	73	73
2	93	70	70	77	70	70	77	63	63
3	108	58	58	111	62	62	111	58	58
4	99	67	73	110	72	72	110	73	73
5	89	76	76	63	61	61	63	60	60
6	75	68	63	104	73	73	104	63	63
7	84	60	56	115	60	60	115	59	59
8	114	72	72	110	72	72	110	84	84
9	101	85	87	116	85	92	116	93	87
10	85	66	66	112	75	75	112	65	65
11	100	88	77	85	76	82	85	88	77
12	79	67	69	114	77	79	114	69	69
13	75	63	63	108	86	86	108	61	63
14	85	73	73	68	82	82	68	77	77
15	94	76	76	114	82	82	114	91	91
16	87	65	66	108	72	72	108	64	64
17	84	66	68	116	84	84	116	73	75
18	83	76	76	91	76	76	91	76	76
19	83	61	61	109	73	65	109	61	61
20	94	78	76	101	76	76	101	76	76
21	80	80	80	74	92	92	74	88	88
22	113	72	72	102	81	81	102	68	68
23	81	70	72	101	69	69	101	71	71
24	91	71	75	90	73	99	90	71	71
25	100	100	103	100	87	87	100	80	80
26	100	89	89	114	85	85	114	88	88

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 27-Continued

Trial No.	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
	Weighting Schemes[a]								
27	79	78	78	86	78	78	86	78	78
28	101	72	80	111	76	76	111	73	73
29	91	83	83	97	95	95	97	73	73
30	97	67	77	68	75	75	68	66	66
31	94	86	86	111	62	62	111	65	57
32	114	80	80	115	85	87	115	80	86
33	88	73	71	100	75	83	100	68	68
34	78	63	63	103	66	67	103	67	67
35	100	80	90	105	73	86	105	78	78
36	92	84	84	105	85	85	105	84	84
37	106	69	69	61	62	62	61	69	69
38	93	92	92	112	79	83	112	83	83
39	80	77	77	86	75	75	86	77	77
40	81	73	73	93	79	79	93	72	77
41	94	86	86	111	62	62	116	115	116
42	114	80	80	115	85	87	110	115	115
43	88	73	71	100	75	83	116	113	113
44	78	63	63	103	66	67	116	114	114
45	100	80	90	105	73	86	114	105	105
46	92	84	84	105	85	85	116	114	114
47	106	69	69	61	62	62	115	113	114
48	93	92	92	112	79	83	114	116	116
49	80	77	77	86	75	75	115	115	115
50	81	73	73	93	79	79	116	116	116
51	116	80	84	87	80	80	87	84	84
52	97	78	78	108	73	72	108	75	75
53	101	84	84	103	76	76	103	76	76
54	87	76	76	94	89	89	94	83	83
55	94	79	89	100	87	85	100	81	81
56	114	61	57	114	61	61	114	61	61
57	107	85	85	115	81	81	115	80	80
58	85	81	82	81	80	80	81	81	81
59	78	72	78	112	66	66	112	72	72
60	113	73	73	115	75	75	115	73	73
61	107	83	87	92	84	84	115	115	115
62	88	71	71	116	67	67	114	116	116
63	112	86	86	112	87	87	107	111	111
64	98	72	72	112	69	70	115	116	116

TABLE 27-Continued

Trial No.	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
	Weighting Schemes[a]								
65	88	69	69	75	82	82	100	100	100
66	97	77	89	110	82	82	109	109	109
67	104	82	82	108	76	76	116	114	114
68	75	74	74	90	77	77	114	113	113
69	100	79	81	103	76	76	116	116	116
70	90	88	88	90	88	86	115	115	115
71	110	77	82	114	79	81	114	75	72
72	96	69	69	115	73	73	115	64	71
73	115	69	69	105	74	74	105	63	63
74	99	88	87	108	89	89	108	88	88
75	94	77	78	95	71	71	95	74	71
76	96	75	73	81	75	75	81	67	67
77	82	81	81	107	81	80	107	81	81
78	106	103	103	108	84	84	108	84	84
79	95	78	78	108	79	79	108	78	78
80	102	77	81	96	82	82	96	81	76
81	116	79	75	78	67	70	78	76	76
82	92	82	84	115	81	81	115	78	78
83	86	72	66	104	75	75	104	72	72
84	94	87	88	86	91	89	86	85	87
85	90	79	81	83	86	76	83	75	75
86	86	78	78	86	73	73	86	70	70
87	108	69	69	112	64	60	112	65	65
88	79	65	60	60	72	70	60	67	65
89	89	50	54	111	53	53	111	50	50
90	97	97	97	100	95	95	100	84	84
91	100	80	80	82	85	85	82	85	85
92	99	69	69	106	80	80	106	69	69
93	95	73	74	69	69	69	70	73	73
94	102	74	74	105	79	79	105	73	73
95	101	95	95	92	104	103	92	96	94
96	100	80	83	91	81	86	91	81	81
97	107	58	58	94	63	63	94	58	58
98	106	72	73	75	74	74	75	70	72
99	76	63	66	87	70	70	87	64	66
100	105	85	82	84	82	82	84	81	81

TABLE 28

MULTIVARIATE NORMAL DISTRIBUTIONS WITH MEANS CLOSER TO EACH OTHER: NUMBER OF MISCLASSIFICATIONS BASED ON MEMBERSHIP AS DETERMINED BY CRITERION SET VARIABLES (VALIDATION SAMPLE)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	14	7	7	14	14	14	14	12	12
2	11	10	10	17	14	14	17	12	12
3	19	7	7	19	6	6	19	7	7
4	6	6	6	10	10	10	10	7	7
5	14	20	20	15	16	16	15	18	18
6	9	7	7	15	8	8	15	6	6
7	9	4	4	15	3	3	15	3	3
8	20	11	11	16	11	11	16	14	14
9	14	7	8	15	8	6	15	4	7
10	18	14	14	19	10	10	19	9	9
11	15	6	9	7	6	9	7	6	8
12	20	19	18	7	17	16	7	18	18
13	12	8	8	18	12	12	18	9	8
14	8	2	2	8	9	9	8	3	3
15	6	2	2	13	8	8	13	7	7
16	17	12	11	19	8	8	19	10	10
17	7	5	5	18	7	7	18	8	9
18	10	8	8	10	7	6	10	6	6
19	10	7	7	16	6	7	16	7	7
20	5	3	2	15	2	2	15	2	2
21	13	9	9	10	11	11	10	7	7
22	17	7	7	15	4	4	15	9	9
23	11	10	10	16	4	4	16	2	5
24	20	19	20	16	11	7	16	19	20
25	12	10	13	12	8	8	12	10	10
26	12	9	9	15	7	7	15	11	11

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 28-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	11	12	12	11	9	9	11	10	12
28	14	9	4	18	5	5	18	4	5
29	17	7	7	13	11	11	13	9	9
30	11	7	9	6	9	9	6	7	7
31	17	18	18	19	12	12	19	15	9
32	19	9	9	15	11	12	15	7	10
33	9	2	3	13	3	10	13	2	2
34	15	13	13	15	17	17	15	11	11
35	13	7	9	19	10	11	19	8	8
36	19	9	9	12	12	12	12	9	9
37	10	14	14	14	15	15	14	14	14
38	10	9	9	12	8	11	12	9	9
39	7	7	7	12	7	7	12	8	8
40	9	9	9	14	10	10	14	9	8
41	7	5	5	19	17	17	18	17	17
42	11	5	5	9	3	3	9	11	11
43	8	9	9	9	7	8	14	8	8
44	11	10	10	16	11	11	15	15	15
45	17	14	12	17	17	10	20	12	12
46	11	12	12	13	9	9	17	7	7
47	8	11	11	10	10	10	20	15	16
48	13	12	12	13	7	11	11	17	13
49	8	7	7	7	7	7	15	20	20
50	14	12	12	17	16	16	15	12	12
51	20	7	7	17	7	7	17	7	7
52	13	8	8	18	10	9	18	8	8
53	13	8	8	11	7	7	11	7	7
54	10	4	4	16	9	9	16	5	5
55	15	6	11	15	10	12	15	6	6
56	12	6	8	17	9	9	17	6	4
57	17	7	7	19	7	7	19	8	8
58	8	7	7	8	8	8	8	8	8
59	11	9	11	16	10	10	16	8	8
60	10	9	9	12	9	9	12	9	9
61	9	7	6	8	7	7	17	17	17
62	13	5	6	16	5	5	4	17	17
63	18	13	13	16	13	13	11	12	12
64	13	10	10	13	7	9	11	11	11

TABLE 28-Continued

Trial No.	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
	Weighting Schemes[a]								
65	12	8	8	9	13	13	14	14	14
66	13	7	9	16	4	4	15	15	15
67	15	5	5	19	11	11	9	20	20
68	13	9	9	13	6	6	20	10	10
69	18	11	10	19	5	5	4	12	12
70	11	9	9	11	7	10	17	17	17
71	11	10	10	16	6	6	16	7	6
72	8	4	4	20	11	11	20	3	3
73	12	8	8	11	11	11	11	10	10
74	11	6	8	12	6	6	12	6	4
75	14	15	15	17	13	13	17	13	13
76	12	3	4	5	3	3	5	4	4
77	8	4	4	20	4	4	20	4	4
78	14	17	17	14	9	9	14	9	9
79	9	6	6	15	12	12	15	6	6
80	13	5	5	6	6	6	6	5	5
81	15	8	9	9	15	15	9	10	10
82	14	11	10	20	6	6	20	14	14
83	20	16	16	18	16	16	18	16	16
84	17	11	14	16	8	9	16	17	14
85	16	10	12	10	8	6	10	12	12
86	14	10	10	14	8	7	14	7	7
87	13	8	8	20	6	8	20	8	8
88	9	4	6	4	4	4	4	3	4
89	10	8	8	12	7	7	12	8	6
90	15	11	15	15	5	5	15	9	9
91	13	7	7	8	8	8	8	5	5
92	12	6	6	15	6	6	15	6	6
93	10	2	3	8	3	3	7	2	2
94	10	9	9	18	8	8	18	6	6
95	10	12	12	16	10	13	16	12	11
96	12	15	13	15	20	18	15	20	20
97	13	7	7	13	6	6	13	8	8
98	12	5	7	7	6	6	7	6	5
99	10	4	7	7	8	8	7	5	5
100	14	17	18	18	17	17	18	19	19

TABLE 29

MULTIVARIATE NORMAL DISTRIBUTIONS WITH MEANS CLOSER TO EACH OTHER: SUM OF SQUARED ERROR VARIANCE OF THE CRITERION SET VARIABLES OBTAINED FOR VALIDATION SAMPLE
(Results Rounded-off to Nearest Integer)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	169	116	116	168	123	123	168	119	119
2	106	101	101	94	91	91	94	94	94
3	116	71	71	117	70	70	117	71	71
4	87	72	76	103	83	83	103	74	74
5	113	86	86	96	88	88	96	85	85
6	106	94	94	143	106	106	143	89	89
7	99	74	74	112	68	68	112	68	68
8	85	64	64	74	64	64	74	72	72
9	88	63	54	91	64	54	91	59	53
10	113	101	101	155	119	119	155	99	99
11	117	92	93	81	83	82	81	92	94
12	110	102	107	140	102	102	140	101	101
13	106	80	80	119	88	88	119	80	80
14	79	64	64	89	86	86	89	66	66
15	75	61	61	98	78	78	98	81	81
16	103	81	79	109	78	78	109	83	83
17	78	63	63	91	66	66	91	66	67
18	97	80	80	107	76	76	107	74	74
19	71	63	63	89	65	62	89	63	63
20	78	65	62	125	62	62	125	62	62
21	85	75	75	73	80	80	73	63	63
22	133	89	89	138	87	87	138	89	89
23	109	104	104	122	82	82	122	75	84
24	155	123	116	142	109	123	142	123	116
25	112	105	114	112	81	81	112	89	89
26	67	58	58	82	60	60	82	61	61

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 29-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	82	82	82	85	75	75	85	76	82
28	140	92	78	169	87	87	169	77	87
29	82	72	72	72	73	73	72	67	67
30	104	84	95	82	95	95	82	81	81
31	101	101	101	121	89	89	121	91	89
32	112	74	74	101	77	85	101	68	80
33	100	68	70	115	76	93	115	68	68
34	105	103	103	113	99	96	113	95	95
35	99	68	73	123	74	89	123	69	69
36	102	94	94	99	94	94	99	94	94
37	105	95	95	94	97	97	94	95	95
38	101	93	93	102	91	102	102	93	93
39	89	81	81	98	78	78	98	84	84
40	105	104	104	135	114	114	135	104	99
41	97	82	82	115	89	89	102	119	119
42	158	98	98	131	84	84	119	131	131
43	72	65	71	87	65	70	105	72	72
44	106	103	103	120	92	92	125	120	120
45	95	85	85	103	84	94	102	91	91
46	84	84	84	114	80	80	115	82	82
47	86	86	86	85	88	88	117	100	104
48	155	133	133	173	140	166	147	214	173
49	95	90	90	88	92	92	119	127	127
50	79	65	65	75	68	68	87	78	78
51	128	90	83	118	88	88	118	83	83
52	84	70	70	93	77	73	93	67	67
53	95	91	91	100	89	89	100	89	89
54	89	67	67	114	84	84	114	73	73
55	89	79	84	106	87	96	106	79	79
56	92	61	73	98	67	67	98	61	58
57	73	50	50	77	50	50	77	51	51
58	104	98	100	104	104	104	104	104	104
59	101	99	101	122	89	89	122	84	84
60	95	80	80	111	80	80	111	80	80
61	107	92	92	101	92	92	156	156	156
62	91	69	69	102	64	64	69	105	105
63	122	94	94	125	94	94	91	87	87
64	91	73	73	94	77	76	96	96	96

TABLE 29-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	95	71	71	79	90	90	73	73	73
66	106	88	89	121	77	77	111	111	111
67	108	79	79	124	98	98	113	121	121
68	116	98	98	107	75	75	132	106	106
69	113	89	89	123	80	80	76	108	108
70	115	104	104	115	93	97	132	132	132
71	74	61	61	83	59	59	83	60	59
72	77	54	54	102	70	70	102	50	54
73	112	82	82	111	87	87	111	89	89
74	110	79	81	124	79	79	124	79	73
75	95	77	77	130	87	87	130	76	87
76	113	78	77	66	78	78	86	81	81
77	64	56	56	100	54	56	100	56	56
78	111	114	114	107	83	83	107	83	83
79	72	70	70	97	74	74	97	70	70
80	108	78	77	88	74	74	88	77	77
81	127	80	79	79	95	89	79	76	76
82	84	70	72	109	69	69	109	77	77
83	92	79	79	94	78	78	94	79	79
84	73	65	64	68	63	62	68	63	64
85	115	85	100	91	79	72	91	87	87
86	133	120	120	133	97	94	133	99	99
87	76	61	61	87	52	55	87	56	56
88	106	84	93	84	76	84	84	79	84
89	89	74	76	121	78	78	121	74	70
90	148	122	148	140	94	94	140	102	102
91	69	77	77	76	76	76	76	67	67
92	94	75	75	113	73	73	113	75	75
93	105	72	73	87	72	72	83	72	72
94	90	77	77	96	76	76	96	69	69
95	84	79	79	105	79	82	105	79	82
96	115	104	106	105	106	108	105	110	110
97	105	84	84	92	83	83	92	85	85
98	134	81	92	92	89	89	92	85	81
99	62	62	72	74	71	71	74	66	68
100	80	76	74	81	74	74	81	77	77

TABLE 30

MULTIVARIATE NORMAL DISTRIBUTIONS WITH MEANS FARTHER APART: SUM OF SQUARED ERROR VARIANCE OF THE CRITERION SET VARIABLES OBTAINED FOR ANALYSIS SAMPLE
(Results Rounded-off to Nearest Integer)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	50	50	50	70	55	55	70	46	46
2	40	28	28	110	63	63	110	27	27
3	44	35	35	34	48	48	34	34	34
4	33	33	33	112	64	55	112	33	34
5	37	37	37	94	49	49	94	39	39
6	33	33	33	112	59	59	112	34	34
7	37	37	37	116	72	75	116	39	39
8	22	22	22	113	63	33	113	26	26
9	35	35	35	86	75	48	86	31	31
10	30	30	30	116	60	60	116	33	30
11	36	37	37	73	77	47	73	35	35
12	29	29	29	94	94	94	94	29	29
13	39	39	39	90	71	71	90	37	37
14	42	38	38	115	59	37	115	27	27
15	27	27	27	113	54	54	113	27	27
16	43	43	43	111	50	50	111	36	36
17	33	33	33	32	53	53	32	35	32
18	36	29	29	111	61	61	111	30	30
19	31	31	31	111	71	71	111	31	31
20	38	39	39	35	67	67	35	40	40
21	36	37	36	107	86	91	77	36	36
22	35	35	35	115	53	115	107	33	33
23	24	24	24	111	70	99	96	24	24
24	32	32	32	116	75	60	32	28	28
25	20	20	20	102	72	72	97	20	20
26	42	35	42	113	85	77	111	35	35

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 30-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	40	35	40	59	79	79	100	29	29
28	32	32	32	103	85	77	55	29	29
29	31	31	31	116	114	97	31	35	35
30	29	29	29	40	98	98	29	26	26
31	27	27	27	106	70	75	106	23	23
32	29	29	29	116	70	70	116	29	29
33	44	43	44	46	60	60	46	38	38
34	34	34	34	34	61	61	34	34	34
35	47	38	38	47	67	79	47	37	37
36	34	34	34	34	47	47	34	32	32
37	35	30	30	73	87	45	73	30	30
38	27	27	27	27	71	65	27	27	27
39	33	33	33	90	82	76	90	33	33
40	34	34	34	116	75	75	116	38	37
41	30	30	30	116	61	61	116	32	32
42	31	31	31	31	62	54	31	33	33
43	35	35	35	109	58	90	109	36	36
44	40	37	37	72	72	72	72	37	37
45	36	36	36	36	71	54	36	33	33
46	37	37	37	115	89	89	115	37	37
47	43	35	43	115	57	57	115	35	35
48	30	30	30	32	50	50	32	30	30
49	32	32	32	116	57	71	116	29	29
50	35	26	26	112	53	53	112	26	26
51	35	35	35	90	72	88	90	35	35
52	44	44	44	76	78	83	76	43	43
53	46	32	32	108	41	35	108	28	29
54	32	35	35	70	72	72	70	35	35
55	31	31	31	104	70	70	104	31	31
56	31	31	31	36	36	36	36	29	29
57	27	27	27	108	57	57	108	30	30
58	48	42	42	91	83	86	91	44	44
59	31	31	31	30	53	53	30	35	35
60	35	35	35	112	77	59	112	35	35
61	55	33	33	92	50	50	92	34	37
62	38	38	38	114	67	67	114	38	38
63	33	33	33	93	58	52	93	33	33
64	32	32	32	88	61	61	88	33	33

TABLE 30-Continued

Trial No.	Weighting Schemes[a] X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	33	33	33	70	70	70	70	32	32
66	36	29	29	93	53	53	93	29	29
67	39	39	39	106	63	63	106	37	37
68	34	34	34	73	78	76	73	33	33
69	31	31	31	108	44	39	108	31	31
70	32	32	32	112	80	83	112	32	32
71	37	37	37	76	51	51	76	35	35
72	38	33	33	116	45	39	116	37	37
73	52	41	41	100	42	42	100	41	36
74	29	29	29	98	66	95	98	26	29
75	41	31	31	116	58	58	116	33	33
76	39	39	39	39	53	53	39	39	39
77	27	27	27	115	52	32	115	26	26
78	27	27	27	116	59	59	116	27	27
79	40	36	36	33	49	49	33	31	31
80	40	40	40	115	98	98	115	43	43
81	33	33	33	115	60	57	115	37	37
82	31	31	31	31	59	59	31	31	31
83	35	31	31	115	61	61	115	31	31
84	29	29	29	73	63	54	73	32	32
85	24	24	24	90	42	32	90	25	25
86	55	40	47	55	65	54	55	35	35
87	29	29	29	45	53	53	45	39	29
88	34	33	33	112	66	50	112	30	30
89	40	29	29	110	55	55	110	29	29
90	43	43	43	94	52	52	94	41	41
91	24	24	24	115	36	36	115	27	27
92	43	37	37	114	60	60	114	30	30
93	33	27	27	31	43	48	31	27	27
94	39	39	39	39	69	69	39	38	38
95	23	23	23	113	49	24	113	23	23
96	40	40	40	88	73	81	88	49	43
97	40	36	36	86	64	64	86	34	34
98	36	36	36	115	60	60	115	36	36
99	25	25	25	106	62	53	106	28	28
100	29	29	29	89	59	50	89	33	33

TABLE 31

MULTIVARIATE NORMAL DISTRIBUTIONS WITH MEANS FARTHER APART: NUMBER OF MISCLASSIFICATIONS BASED ON MEMBERSHIP AS DETERMINED BY CRITERION SET VARIABLES (VALIDATION SAMPLE)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	1	1	1	1	1	1	1	2	2
2	4	2	2	17	14	14	17	2	2
3	3	2	2	1	8	8	1	1	1
4	1	1	1	15	11	11	15	1	2
5	1	1	1	7	3	3	7	1	1
6	3	3	3	19	7	7	19	3	2
7	1	1	1	18	6	9	18	1	1
8	2	2	2	19	8	2	19	2	2
9	0	0	0	11	13	5	11	1	1
10	0	0	0	11	5	5	11	1	0
11	2	2	2	7	6	2	7	2	2
12	0	0	0	13	15	15	13	0	0
13	3	3	3	12	6	6	12	2	2
14	1	1	1	14	5	3	14	2	2
15	4	4	4	18	5	5	18	4	4
16	1	1	1	15	5	5	15	3	3
17	3	3	3	3	3	3	3	3	3
18	3	1	1	17	10	10	17	3	3
19	2	2	2	16	14	14	16	2	2
20	2	2	2	2	10	10	2	2	2
21	2	2	2	4	2	2	2	2	2
22	1	1	1	2	0	2	14	0	0
23	2	2	2	7	3	3	12	2	2
24	2	2	2	19	1	3	2	1	1
25	1	1	1	5	1	1	13	1	1
26	0	1	0	1	2	3	15	1	1

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 31-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	0	0	0	2	2	2	13	0	0
28	1	1	1	10	2	3	3	2	2
29	1	1	1	20	14	3	1	1	1
30	1	1	1	1	3	3	0	0	0
31	3	3	3	20	9	10	20	3	3
32	2	2	2	20	9	9	20	2	2
33	2	1	2	1	2	2	1	1	1
34	1	1	1	1	1	1	1	1	1
35	1	1	1	1	9	14	1	1	1
36	2	2	2	2	6	6	2	2	2
37	1	1	1	4	7	2	4	1	1
38	4	4	4	4	9	5	4	4	4
39	1	1	1	14	11	12	14	1	1
40	4	4	4	15	6	6	15	4	4
41	0	0	0	19	6	6	19	1	1
42	1	1	1	0	2	0	0	0	0
43	2	2	2	12	9	14	12	1	1
44	2	2	2	6	6	6	6	2	2
45	2	2	2	2	8	2	2	1	1
46	0	0	0	14	15	15	14	0	0
47	3	2	3	19	6	6	19	1	1
48	0	0	0	0	4	4	0	0	0
49	2	2	2	18	3	8	18	3	3
50	1	1	1	11	6	6	11	1	1
51	0	0	0	7	6	11	7	0	0
52	0	0	0	7	11	11	7	1	1
53	4	4	4	9	5	4	9	4	3
54	3	5	5	4	3	3	4	3	3
55	3	3	3	10	9	9	10	3	3
56	5	5	5	3	3	3	3	5	5
57	2	2	2	14	3	3	14	2	2
58	2	2	2	13	15	15	13	2	2
59	2	2	2	2	4	4	2	2	2
60	1	1	1	18	14	9	18	1	1
61	2	4	4	8	9	9	8	4	6
62	1	1	1	17	4	4	17	0	0
63	1	1	1	13	0	0	13	1	1
64	1	1	1	5	6	6	5	1	1

TABLE 31-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	3	3	3	10	10	10	10	3	3
66	3	3	3	12	3	3	12	3	3
67	3	3	3	16	3	3	16	2	2
68	0	0	0	8	12	11	8	1	1
69	3	3	3	13	8	5	13	5	5
70	2	2	2	15	5	6	15	2	2
71	3	3	3	5	2	2	5	2	2
72	3	4	4	18	9	12	18	5	5
73	3	4	4	11	1	1	11	2	2
74	2	2	2	14	8	17	14	2	2
75	4	3	3	17	10	10	17	3	3
76	3	3	3	2	5	5	2	2	2
77	2	2	2	11	5	2	11	1	1
78	1	1	1	13	7	7	13	1	1
79	3	3	3	3	7	7	3	3	3
80	1	1	1	20	14	14	20	1	1
81	6	6	6	11	5	4	11	5	5
82	0	0	0	0	5	5	0	0	0
83	3	3	3	14	3	3	14	3	3
84	1	1	1	8	8	4	8	1	1
85	2	2	2	12	3	2	12	3	2
86	5	0	2	3	8	4	3	1	1
87	0	0	0	4	7	7	4	1	0
88	1	1	1	9	6	3	9	1	1
89	0	0	0	14	6	6	14	1	1
90	2	2	2	11	6	6	11	2	2
91	3	3	3	16	5	5	16	3	3
92	2	2	2	14	12	12	14	2	2
93	0	1	1	2	8	8	2	1	1
94	3	3	3	2	7	7	2	2	2
95	1	1	1	18	2	1	18	1	1
96	5	5	5	13	9	9	13	4	4
97	2	2	2	17	7	7	17	1	2
98	3	3	3	20	6	6	20	3	3
99	2	2	2	18	11	10	18	1	1
100	3	3	3	6	8	0	6	1	1

TABLE 32

MULTIVARIATE NORMAL DISTRIBUTIONS WITH MEANS FARTHER APART: SUM OF SQUARED ERROR VARIANCE OF THE CRITERION SET VARIABLES OBTAINED FOR VALIDATION SAMPLE
(Results Rounded-off to Nearest Integer)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	120	120	120	127	127	127	127	129	129
2	171	130	130	310	259	259	310	130	130
3	129	122	122	115	189	189	115	115	115
4	122	122	122	388	259	259	388	122	124
5	112	112	112	201	125	125	201	112	112
6	121	121	121	426	230	230	426	121	114
7	136	136	136	421	172	262	421	136	136
8	102	102	102	341	220	102	341	102	102
9	97	97	97	257	227	128	257	107	107
10	84	84	84	312	118	118	312	95	84
11	103	103	103	217	205	130	217	103	103
12	110	110	110	306	261	261	306	110	110
13	153	153	153	466	274	274	466	134	134
14	128	128	128	413	151	137	413	136	136
15	97	97	97	305	159	159	305	97	97
16	125	125	125	335	143	143	335	130	130
17	150	150	150	117	117	117	117	117	117
18	159	116	116	358	234	234	358	133	133
19	118	118	118	331	256	256	331	118	118
20	105	105	105	105	206	206	105	105	105
21	126	126	126	157	126	126	150	126	126
22	123	123	123	209	112	125	430	112	112
23	132	132	132	238	130	169	210	132	132
24	129	129	129	363	109	155	129	109	109
25	122	122	122	147	113	113	373	122	122
26	79	89	79	89	102	131	383	89	89

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 32-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	134	134	134	149	149	149	368	134	134
28	132	132	132	378	154	183	183	154	154
29	146	146	146	479	420	188	146	146	146
30	87	87	87	87	122	122	86	86	86
31	133	133	133	367	225	243	367	133	133
32	121	121	121	376	229	229	376	121	121
33	109	102	109	102	119	119	102	102	102
34	88	88	88	88	121	121	88	88	88
35	110	110	110	110	202	287	110	110	110
36	113	113	113	113	147	147	113	113	113
37	122	122	122	178	234	122	178	122	122
38	130	130	130	130	286	185	130	130	130
39	102	102	102	309	259	248	309	102	102
40	92	92	92	294	165	165	294	92	92
41	122	122	122	432	195	195	432	131	131
42	98	98	98	94	118	94	94	94	94
43	93	93	93	275	201	265	275	86	86
44	117	117	117	162	162	162	162	117	117
45	130	130	130	130	223	130	130	115	115
46	90	90	90	350	320	320	350	90	90
47	111	99	111	354	162	162	354	98	98
48	91	91	91	91	143	143	91	91	91
49	117	117	117	360	151	204	360	129	129
50	89	89	89	263	147	147	263	89	89
51	77	77	77	238	161	254	238	77	77
52	83	83	83	221	272	272	221	121	121
53	144	144	144	295	147	131	295	131	126
54	116	162	162	165	146	146	165	116	116
55	127	127	127	279	185	185	279	127	127
56	141	141	141	129	129	129	129	141	141
57	99	99	99	311	129	129	311	99	99
58	188	188	188	332	349	349	332	179	179
59	109	109	109	109	124	124	109	109	109
60	100	100	100	383	323	239	383	100	100
61	152	159	159	314	248	248	314	166	185
62	100	100	100	389	149	149	389	92	92
63	98	98	98	325	97	97	325	98	98
64	108	108	108	180	206	206	180	108	108

TABLE 32-Continued

Trial No.	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
	Weighting Schemes[a]								
65	124	124	124	255	255	255	255	124	124
66	120	120	120	290	129	129	290	120	120
67	121	121	121	369	118	118	369	106	106
68	124	124	124	229	266	246	229	145	145
69	125	125	125	333	202	161	333	141	141
70	126	126	126	406	158	195	406	126	126
71	113	113	113	153	91	91	153	89	89
72	100	106	106	275	158	183	275	110	110
73	128	170	170	320	107	107	320	125	125
74	127	127	127	401	242	410	401	127	127
75	149	125	125	416	235	235	416	125	125
76	150	150	150	140	194	194	140	140	140
77	133	133	133	312	155	123	312	114	114
78	124	124	124	377	201	201	377	124	124
79	108	108	108	108	167	167	108	108	108
80	113	113	113	395	294	294	395	113	113
81	141	141	141	325	174	148	325	143	143
82	78	78	78	78	154	154	78	78	78
83	107	107	107	291	133	133	291	107	107
84	87	87	87	171	166	115	171	87	87
85	110	110	110	327	113	110	327	114	107
86	228	131	168	169	248	185	169	143	143
87	89	89	89	132	166	166	132	99	89
88	121	121	121	324	202	168	324	121	121
89	106	106	106	421	164	164	421	108	108
90	158	158	158	362	186	186	362	157	157
91	109	109	109	334	136	136	334	109	109
92	119	119	119	342	270	270	342	119	119
93	111	115	115	129	170	170	129	115	115
94	106	108	108	104	180	180	104	104	104
95	119	119	119	387	139	119	387	119	119
96	139	139	139	312	244	259	312	128	128
97	122	122	122	370	169	169	370	113	122
98	135	135	135	481	190	190	481	135	135
99	105	105	105	324	221	217	324	98	98
100	116	116	116	195	218	88	195	91	91

TABLE 33

MULTIVARIATE BINOMIAL DISTRIBUTIONS WITH MEANS CLOSER TO EACH OTHER: SUM OF SQUARED ERROR VARIANCE OF THE CRITERION SET VARIABLES OBTAINED FOR ANALYSIS SAMPLE
(Results Rounded-off to Nearest Integer)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	573	264	264	530	264	264	530	264	264
2	412	325	325	569	325	325	569	325	325
3	319	333	333	578	333	333	578	333	333
4	568	342	568	568	342	342	568	342	342
5	486	374	374	571	374	374	571	374	374
6	549	340	340	580	340	340	580	340	340
7	308	344	344	585	344	344	585	344	344
8	358	306	306	557	306	306	557	306	306
9	415	336	336	563	336	336	563	336	336
10	372	349	349	587	349	349	587	349	349
11	372	284	284	563	284	284	563	284	284
12	403	368	368	579	368	368	579	368	368
13	332	324	324	566	324	324	566	324	324
14	336	338	338	561	338	338	561	338	338
15	397	395	395	593	395	395	593	395	395
16	571	337	337	571	337	337	571	337	337
17	397	357	357	568	357	357	568	357	357
18	522	292	292	549	292	292	549	292	292
19	474	371	371	581	371	371	581	371	371
20	373	333	333	590	590	590	590	333	333
21	565	294	294	561	294	294	561	294	294
22	363	347	347	588	347	347	588	347	347
23	468	340	340	570	340	340	570	340	340
24	447	378	378	554	378	378	554	378	378
25	553	315	315	554	315	315	554	315	315
26	367	286	286	595	286	286	595	286	286

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted; CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 33-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	556	377	377	592	377	377	592	377	377
28	376	395	395	583	395	395	583	395	395
29	323	323	323	584	323	323	584	323	323
30	421	378	378	580	378	378	580	378	378
31	453	353	353	561	353	353	561	353	353
32	413	341	341	547	341	341	547	341	341
33	379	352	352	579	352	352	579	352	352
34	448	298	298	523	298	298	523	298	298
35	549	366	366	578	366	366	578	366	366
36	382	313	313	584	313	313	584	313	313
37	427	344	344	577	344	344	577	344	344
38	528	297	297	561	297	297	561	297	297
39	410	395	395	583	395	395	583	395	395
40	479	335	335	548	335	335	548	335	335
41	541	333	333	579	333	333	579	333	333
42	435	366	366	575	366	366	575	366	366
43	489	367	367	586	367	586	586	367	367
44	368	282	282	552	282	282	552	282	282
45	429	316	316	575	316	316	575	316	316
46	396	365	365	561	365	561	561	365	365
47	495	357	357	571	357	357	571	357	357
48	374	355	355	586	355	355	586	355	355
49	434	321	321	579	321	321	579	321	321
50	343	356	356	469	356	356	469	356	356
51	562	337	337	534	337	337	534	337	337
52	410	329	329	573	329	329	573	329	329
53	328	313	313	564	313	313	564	313	313
54	398	369	369	566	369	369	566	369	369
55	445	367	367	568	367	367	568	367	367
56	542	347	347	579	347	347	579	347	347
57	568	356	356	578	356	356	578	356	356
58	374	328	328	578	328	328	578	328	328
59	436	351	351	574	351	351	574	351	351
60	341	367	367	585	367	367	585	367	367
61	415	320	320	570	320	320	570	320	320
62	431	362	362	561	362	362	561	362	362
63	399	341	341	560	341	341	560	341	341
64	530	309	309	563	309	309	563	309	309

TABLE 33-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	413	346	346	580	346	346	580	346	346
66	492	343	343	585	343	343	585	343	343
67	421	317	317	587	317	317	587	317	317
68	381	403	403	585	403	403	585	403	403
69	410	353	353	571	353	353	571	353	353
70	569	351	351	573	351	351	573	351	351
71	413	335	335	581	335	335	581	335	335
72	428	324	324	453	324	324	453	324	324
73	423	338	338	558	338	338	558	338	338
74	569	375	375	570	375	375	570	375	375
75	398	287	287	534	287	287	534	287	287
76	416	364	364	579	364	364	579	364	364
77	552	330	330	557	330	330	557	330	330
78	461	331	331	593	331	331	593	331	331
79	575	359	359	575	359	359	575	359	359
80	405	353	353	589	353	353	589	353	353
81	480	322	322	569	322	322	569	322	322
82	433	375	375	579	375	375	579	375	375
83	439	368	368	588	368	368	588	368	368
84	362	362	362	464	362	362	464	362	362
85	425	361	361	568	361	361	568	361	361
86	566	257	257	456	257	257	456	257	257
87	351	303	303	572	303	303	572	303	303
88	596	362	362	595	362	362	595	362	362
89	475	354	354	567	354	354	567	354	354
90	369	370	370	582	370	370	582	370	370
91	471	299	299	553	299	299	553	299	299
92	406	360	360	580	360	360	580	360	360
93	343	339	339	583	339	339	583	339	339
94	566	305	305	539	305	305	539	305	305
95	312	304	304	576	304	304	576	304	304
96	423	333	333	571	333	333	571	333	333
97	388	358	358	579	358	358	579	358	358
98	432	369	369	571	369	369	571	369	369
99	531	321	321	596	321	321	596	321	321
100	382	378	378	580	378	378	580	378	378

TABLE 34

MULTIVARIATE BINOMIAL DISTRIBUTIONS WITH MEANS CLOSER TO EACH OTHER: NUMBER OF MISCLASSIFICATIONS BASED ON MEMBERSHIP AS DETERMINED BY CRITERION SET VARIABLES (VALIDATION SAMPLE)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	36	90	90	46	90	90	46	90	90
2	46	22	22	86	22	22	86	22	22
3	29	18	18	81	18	18	81	18	18
4	96	83	96	96	83	83	96	83	83
5	85	83	83	38	83	83	38	83	83
6	77	15	15	73	15	15	73	15	15
7	71	89	89	49	89	89	49	89	89
8	74	15	15	75	15	15	75	15	15
9	41	21	21	74	21	21	74	21	21
10	39	14	14	76	14	14	76	14	14
11	36	16	16	76	16	16	76	16	16
12	45	22	22	93	22	22	93	22	22
13	31	15	15	89	15	15	89	15	15
14	18	18	18	73	18	18	73	18	18
15	80	99	99	48	99	99	48	99	99
16	93	15	15	93	15	15	93	87	87
17	87	77	77	53	77	77	53	77	77
18	68	96	96	49	96	96	49	96	96
19	49	20	20	94	20	20	94	20	20
20	76	17	17	60	60	60	60	17	17
21	81	15	15	75	15	15	75	15	15
22	49	22	22	72	22	22	72	22	22
23	51	26	26	96	26	26	96	26	26
24	47	86	86	45	86	86	45	86	86
25	44	13	13	98	13	13	98	13	13
26	86	100	100	43	100	100	43	100	100

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 34-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	66	18	18	88	18	18	88	18	18
28	40	26	26	85	26	26	85	26	26
29	83	83	83	56	83	83	56	83	83
30	74	99	99	50	99	99	50	99	99
31	56	100	100	41	100	100	41	100	100
32	37	17	17	79	17	17	79	17	17
33	27	14	14	66	14	14	66	14	14
34	43	23	23	71	23	23	71	23	23
35	77	16	16	90	16	16	90	16	16
36	79	98	98	35	98	98	35	98	98
37	68	97	97	23	97	97	23	97	97
38	82	82	82	65	82	82	65	82	82
39	53	19	19	98	19	19	98	19	19
40	90	100	100	90	100	100	90	100	100
41	33	95	95	47	95	95	47	95	95
42	32	19	19	98	19	19	98	19	19
43	59	96	96	41	96	41	41	96	96
44	66	99	99	45	99	99	45	99	99
45	51	95	95	46	95	95	46	95	95
46	40	16	16	95	16	95	95	16	16
47	65	12	12	100	12	12	100	12	12
48	57	19	19	92	19	19	92	19	19
49	49	12	12	75	12	12	75	12	12
50	33	14	14	70	14	14	70	14	14
51	68	14	14	93	14	14	93	14	14
52	39	14	14	90	14	14	90	14	14
53	93	93	93	51	93	93	51	93	93
54	19	19	19	71	19	19	71	19	19
55	35	16	16	76	16	16	76	16	16
56	52	99	99	41	99	99	41	99	99
57	75	18	18	61	18	18	61	18	18
58	40	16	16	69	16	16	69	16	16
59	43	17	17	82	17	17	82	17	17
60	33	23	23	79	23	23	79	23	23
61	36	15	15	90	15	15	90	15	15
62	85	85	85	53	85	85	53	85	85
63	41	17	17	96	17	17	98	17	17
64	43	89	89	38	89	89	38	89	89

TABLE 34-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	37	19	19	97	19	19	97	19	19
66	52	19	19	74	19	19	74	19	19
67	69	94	94	46	94	94	46	94	94
68	27	16	16	95	16	16	95	16	16
69	63	96	96	54	96	96	54	96	96
70	83	21	21	77	21	21	77	21	21
71	43	22	22	99	22	22	99	22	22
72	81	21	21	73	21	21	73	21	21
73	76	93	93	51	93	93	51	93	93
74	62	15	15	83	15	15	83	15	15
75	67	90	90	34	90	90	34	90	90
76	30	15	15	81	15	15	81	15	15
77	76	78	78	100	78	78	100	78	78
78	54	94	94	46	94	94	46	94	94
79	36	90	90	36	90	90	36	14	14
80	39	16	16	98	16	16	98	16	16
81	82	89	89	53	89	89	53	89	89
82	41	20	20	75	20	20	75	20	20
83	32	11	11	85	11	11	85	11	11
84	31	16	16	59	16	16	59	16	16
85	34	16	16	98	16	16	98	16	16
86	61	92	92	63	92	92	63	92	92
87	32	15	15	90	15	15	90	15	15
88	17	98	98	36	98	98	36	98	98
89	60	90	90	30	90	90	30	90	90
90	23	13	13	67	13	13	67	13	13
91	57	85	85	49	85	85	49	85	85
92	51	19	19	76	19	19	76	19	19
93	40	20	20	86	20	20	86	20	20
94	78	16	16	98	16	16	98	78	78
95	88	88	88	49	88	88	49	88	88
96	48	98	98	38	98	98	38	98	98
97	40	14	14	68	14	14	68	14	14
98	64	94	94	42	94	94	42	94	94
99	51	14	14	100	14	14	100	14	14
100	17	17	17	78	17	17	78	17	17

TABLE 35

MULTIVARIATE BINOMIAL DISTRIBUTIONS WITH MEANS CLOSER TO EACH OTHER: SUM OF SQUARED ERROR VARIANCE OF THE CRITERION SET VARIABLES OBTAINED FOR VALIDATION SAMPLE
(Results Rounded-off to Nearest Integer)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	144	93	93	137	93	93	137	93	93
2	119	90	90	139	90	90	139	90	90
3	104	88	88	142	88	88	142	88	88
4	143	148	143	143	148	148	143	148	148
5	132	86	86	132	86	86	132	86	86
6	142	81	81	138	81	81	138	81	81
7	106	96	96	145	96	96	145	96	96
8	140	83	83	141	83	83	141	83	83
9	112	91	91	142	91	91	142	91	91
10	115	79	79	140	79	79	140	79	79
11	110	82	82	143	82	82	143	82	82
12	119	94	94	140	94	94	140	94	94
13	105	82	82	135	82	82	135	82	82
14	90	90	90	141	90	90	141	90	90
15	106	90	90	139	90	90	139	90	90
16	141	80	80	141	80	80	141	148	148
17	104	84	84	145	84	84	145	84	84
18	145	79	79	135	79	79	135	79	79
19	127	87	87	140	87	87	140	87	87
20	135	84	84	136	136	136	136	84	84
21	148	83	83	146	83	83	146	83	83
22	125	94	94	141	94	94	141	94	94
23	130	104	104	142	104	104	142	104	104
24	131	84	84	141	84	84	141	84	84
25	125	83	83	144	83	83	144	83	83
26	91	79	79	147	79	79	147	79	79

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 35-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	140	87	87	145	87	87	145	87	87
28	103	89	89	146	89	89	146	89	89
29	87	87	87	146	87	87	146	87	87
30	105	93	93	140	93	93	140	93	93
31	127	80	80	141	80	80	141	80	80
32	114	88	88	137	88	88	137	88	88
33	100	82	82	141	82	82	141	82	82
34	115	94	94	141	94	94	141	94	94
35	144	80	80	137	80	80	137	80	80
36	100	85	85	149	85	85	149	85	85
37	113	82	82	149	82	82	149	82	82
38	147	147	147	134	147	147	134	147	147
39	125	85	85	147	85	85	147	85	85
40	136	83	83	136	83	83	136	83	83
41	134	82	82	134	82	82	134	82	82
42	105	91	91	145	91	91	145	91	91
43	123	80	80	139	80	139	139	80	80
44	127	86	86	141	86	86	141	86	86
45	137	89	89	141	89	89	141	89	89
46	113	83	83	140	83	140	140	83	83
47	134	80	80	140	80	80	140	80	80
48	131	90	90	144	90	90	144	90	90
49	130	76	76	133	76	76	133	76	76
50	109	87	87	131	87	87	131	87	87
51	139	88	88	138	88	88	138	88	88
52	111	76	76	139	76	76	139	76	76
53	91	91	91	143	91	91	143	91	91
54	87	87	87	142	87	87	142	87	87
55	108	83	83	144	83	83	144	83	83
56	131	88	88	144	88	88	144	88	88
57	146	79	79	133	79	79	133	79	79
58	109	82	82	139	82	82	139	82	82
59	118	88	88	143	88	88	143	88	88
60	107	92	92	143	92	92	143	92	92
61	117	88	88	140	88	88	140	88	88
62	83	83	83	139	83	83	139	83	83
63	112	83	83	138	83	83	138	83	83
64	134	73	73	137	73	73	137	73	73

TABLE 35-Continued

Trial No.	Weighting Schemes[a] X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	113	89	89	142	89	89	142	89	89
66	129	85	85	141	85	85	141	85	85
67	112	88	88	146	88	88	146	88	88
68	100	83	83	139	83	83	139	83	83
69	114	100	100	146	100	100	146	100	100
70	148	94	94	143	94	94	143	94	94
71	115	93	93	142	93	93	142	93	93
72	138	88	88	134	88	88	134	88	88
73	104	86	86	135	86	86	135	86	86
74	135	80	80	146	80	80	146	80	80
75	111	78	78	134	78	78	134	78	78
76	105	87	87	138	87	87	138	87	87
77	139	147	147	141	147	147	141	147	147
78	134	91	91	141	91	91	141	91	91
79	140	87	87	140	87	87	140	148	148
80	113	84	84	146	84	84	146	84	84
81	116	89	89	144	89	89	144	89	89
82	114	87	87	141	87	87	141	87	87
83	104	76	76	140	76	76	140	76	76
84	94	78	78	122	78	78	122	78	78
85	115	91	91	147	91	91	147	91	91
86	132	79	79	115	79	79	115	79	79
87	97	72	72	147	72	72	147	72	72
88	148	85	85	144	85	85	144	85	85
89	124	98	98	139	98	98	139	98	98
90	97	81	81	140	81	81	140	81	81
91	116	74	74	139	74	74	139	74	74
92	119	89	89	140	89	89	140	89	89
93	104	78	78	147	78	78	147	78	78
94	148	85	85	141	85	85	141	148	148
95	90	90	90	146	90	90	146	90	90
96	134	77	77	144	77	77	144	77	77
97	116	84	84	134	84	84	134	84	84
98	117	79	79	143	79	79	143	79	79
99	126	83	83	146	83	83	146	83	83
100	79	79	79	142	79	79	142	79	79

TABLE 36

MULTIVARIATE BINOMIAL DISTRIBUTIONS WITH MEANS MODERATELY APART: SUM OF SQUARED ERROR VARIANCE OF THE CRITERION SET VARIABLES OBTAINED FOR ANALYSIS SAMPLE
(Results Rounded-off to Nearest Integer)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	546	313	313	588	313	313	588	313	313
2	483	256	256	547	256	256	547	256	256
3	480	308	306	588	308	306	588	308	308
4	353	321	321	581	321	321	581	321	321
5	403	327	327	553	327	327	553	327	327
6	404	287	287	571	287	287	571	287	287
7	363	359	359	579	359	359	579	359	359
8	438	298	298	554	298	298	554	298	298
9	339	309	309	596	309	309	596	309	309
10	361	311	311	555	311	311	555	311	311
11	433	270	270	574	270	270	574	270	270
12	420	341	341	591	341	341	591	341	341
13	348	317	317	578	317	317	578	317	317
14	415	354	354	585	354	354	572	354	354
15	414	329	329	588	329	588	588	329	329
16	534	341	341	571	341	341	571	341	341
17	363	340	340	340	340	340	340	340	340
18	419	380	380	586	586	586	586	360	380
19	575	324	324	530	324	324	530	324	324
20	417	308	308	574	308	308	574	308	308
21	461	308	308	574	306	574	574	308	308
22	318	322	322	583	322	322	583	322	322
23	345	325	325	568	325	325	568	325	325
24	397	348	348	575	348	348	575	348	348
25	415	318	318	585	318	318	585	318	318
26	540	337	337	569	570	570	569	337	337

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 36-Continued

Trial No.	Weighting Schemes[a] X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	464	379	379	563	379	379	563	379	379
28	547	314	314	547	314	314	547	314	314
29	415	314	314	564	314	314	564	314	314
30	421	385	385	578	385	385	578	385	385
31	537	336	336	574	336	336	574	336	336
32	398	355	355	582	355	355	582	355	355
33	401	325	325	557	325	325	557	325	325
34	386	387	387	583	591	591	583	387	387
35	513	386	386	595	595	595	595	386	386
36	540	330	330	595	595	595	595	330	330
37	325	316	316	575	316	316	575	316	316
38	423	345	345	575	345	345	575	345	345
39	418	291	291	571	291	291	571	291	291
40	431	359	359	570	359	359	570	359	359
41	545	292	292	572	292	292	572	292	292
42	574	378	378	572	378	378	572	378	378
43	465	351	351	570	351	351	570	351	351
44	424	333	333	595	333	333	595	333	333
45	468	357	357	566	357	357	566	357	357
46	554	334	334	575	334	334	575	334	334
47	358	344	344	580	344	344	580	344	344
48	313	272	272	565	272	272	565	272	272
49	427	347	347	595	347	347	595	347	347
50	482	362	362	582	362	362	582	362	362
51	539	349	349	583	349	349	583	349	349
52	467	295	295	574	295	295	574	295	295
53	402	378	378	593	593	593	593	378	378
54	465	334	334	562	334	334	562	334	334
55	363	347	347	573	347	347	573	347	347
56	561	300	300	561	300	300	561	300	300
57	583	322	322	568	322	322	568	322	322
58	454	354	354	590	354	354	590	354	354
59	500	309	309	576	309	309	576	309	309
60	353	326	326	522	326	326	522	326	326
61	364	322	322	583	322	322	583	322	322
62	345	380	380	465	380	380	465	380	380
63	546	308	308	572	308	308	572	308	308
64	403	298	298	565	298	298	565	298	298

TABLE 36-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	364	338	338	584	338	338	584	338	338
66	426	321	321	585	321	321	585	321	321
67	461	346	346	567	346	346	567	346	346
68	406	359	359	579	359	359	579	359	359
69	583	313	313	583	313	313	583	313	313
70	385	301	301	555	301	301	555	301	301
71	379	310	310	580	310	310	580	310	310
72	397	323	323	573	323	323	573	323	323
73	405	317	317	570	317	317	570	317	317
74	388	355	355	471	355	355	471	355	355
75	399	321	321	578	321	321	578	321	321
76	392	264	264	545	264	264	545	264	264
77	332	305	305	581	305	305	581	305	305
78	327	325	325	588	588	588	588	325	325
79	573	579	579	579	579	579	579	311	311
80	453	316	316	564	316	316	564	316	316
81	382	318	318	579	318	318	579	318	318
82	445	363	363	575	363	363	575	363	363
83	449	317	317	580	317	317	580	317	317
84	462	394	394	581	394	394	581	394	394
85	505	321	321	545	545	545	545	321	321
86	408	329	329	569	329	329	569	329	329
87	387	280	280	557	280	280	557	280	280
88	418	365	365	583	583	583	583	365	365
89	561	334	334	590	334	334	590	334	334
90	508	309	309	560	309	309	560	309	309
91	449	335	335	570	335	335	570	335	335
92	540	332	332	582	332	332	582	332	332
93	436	310	310	542	310	310	542	310	310
94	440	305	305	570	305	305	570	305	305
95	404	304	304	572	304	304	572	304	304
96	428	337	337	575	337	337	575	337	337
97	575	289	289	563	289	289	563	289	289
98	415	303	303	582	303	303	582	303	303
99	393	270	270	564	270	270	564	270	270
100	324	331	331	586	331	331	586	331	331

TABLE 37

MULTIVARIATE BINOMIAL DISTRIBUTIONS WITH MEANS MODERATELY APART: NUMBER OF MISCLASSIFICATIONS BASED ON MEMBERSHIP AS DETERMINED BY CRITERION SET VARIABLES (VALIDATION SAMPLE)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	18	18	18	41	18	18	41	18	18
2	57	22	22	81	22	22	81	22	22
3	50	21	21	84	21	21	84	21	21
4	40	15	15	72	15	15	72	15	15
5	49	94	94	32	94	94	32	94	94
6	38	18	18	88	18	18	88	18	18
7	26	18	18	81	18	18	81	18	18
8	40	26	26	97	26	26	97	26	26
9	26	11	11	72	11	11	72	11	11
10	24	14	14	66	14	14	66	14	14
11	43	11	11	90	11	11	90	11	11
12	65	9	9	93	9	9	93	9	9
13	65	20	20	100	20	20	100	20	20
14	40	17	17	98	17	17	69	17	17
15	66	21	21	81	21	81	81	21	21
16	54	88	88	80	88	88	80	88	88
17	26	15	15	15	15	15	15	15	15
18	65	87	87	43	43	43	43	87	87
19	76	16	16	78	16	16	78	16	16
20	68	22	22	97	22	22	97	22	22
21	50	81	81	87	81	87	87	81	81
22	32	16	16	79	16	16	79	16	16
23	19	19	19	69	19	19	69	19	19
24	68	91	91	37	91	91	37	91	91
25	42	20	20	77	20	20	77	20	20
26	95	95	95	88	91	91	88	95	95

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 37-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	52	20	20	81	20	20	81	20	20
28	94	83	83	94	83	83	94	83	83
29	48	19	19	83	19	19	83	19	19
30	41	19	19	94	19	19	94	19	19
31	49	94	94	43	94	94	43	94	94
32	41	21	21	90	21	21	90	21	21
33	43	17	17	93	17	17	93	17	17
34	66	20	20	95	80	80	95	20	20
35	86	14	14	78	78	78	78	14	14
36	86	13	13	73	73	73	73	13	13
37	81	99	99	45	99	99	45	99	99
38	85	85	65	74	85	85	74	85	85
39	68	22	22	89	22	22	89	22	22
40	40	17	17	75	17	17	75	17	17
41	86	86	86	74	86	86	74	86	86
42	94	23	23	93	23	23	93	82	82
43	46	17	17	86	17	17	86	17	17
44	68	25	25	79	25	25	79	25	25
45	87	98	98	34	98	98	34	98	98
46	67	26	26	85	26	26	85	26	26
47	33	17	17	69	17	17	69	17	17
48	38	10	10	72	10	10	72	10	10
49	49	85	85	50	85	85	50	85	85
50	46	13	13	70	13	13	70	13	13
51	80	16	16	66	16	16	66	16	16
52	88	13	13	74	88	88	74	13	13
53	62	97	97	33	33	33	33	97	97
54	73	17	17	69	17	17	69	17	17
55	35	15	15	75	15	15	75	15	15
56	50	90	90	50	90	90	50	90	90
57	92	78	78	98	78	78	98	78	78
58	67	12	12	91	12	12	91	12	12
59	62	17	17	84	17	17	84	17	17
60	26	26	26	69	26	26	69	26	26
61	71	19	19	70	19	19	70	19	19
62	73	87	87	56	87	87	56	87	87
63	61	20	20	76	20	20	76	20	20
64	38	19	19	92	19	19	92	19	19

TABLE 37-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	31	20	20	98	20	20	98	20	20
66	58	13	13	68	13	13	68	13	13
67	52	92	92	37	92	92	37	92	92
68	69	22	22	80	22	22	80	22	22
69	55	86	86	55	86	86	55	86	86
70	91	91	91	91	91	91	91	91	91
71	31	14	14	99	14	14	99	14	14
72	69	16	16	87	16	16	87	16	16
73	76	98	98	43	98	98	43	98	98
74	28	20	20	62	20	20	62	20	20
75	57	24	24	95	99	99	95	24	24
76	48	15	15	74	15	15	74	15	15
77	40	16	16	79	16	16	79	16	16
78	87	98	98	40	40	40	40	98	98
79	29	43	43	43	43	43	43	15	15
80	61	25	25	83	25	25	83	25	25
81	55	91	91	36	91	91	36	91	91
82	51	26	26	74	26	26	74	26	26
83	63	94	94	38	94	94	38	94	94
84	67	21	21	91	21	21	91	21	21
85	89	82	82	89	89	89	89	82	82
86	76	96	96	37	96	96	37	96	96
87	22	14	14	83	14	14	83	14	14
88	57	21	21	92	92	92	92	88	88
89	97	20	20	87	20	20	87	20	20
90	48	14	14	89	14	14	89	14	14
91	54	96	96	39	96	96	39	96	96
92	69	24	24	79	24	24	79	24	24
93	64	17	17	81	17	17	81	17	17
94	54	18	18	72	18	18	72	18	18
95	66	22	22	79	22	22	79	22	22
96	56	92	92	37	92	92	37	92	92
97	68	13	13	98	13	13	98	13	13
98	64	86	86	36	86	86	36	86	86
99	43	21	21	94	21	21	94	21	21
100	32	13	13	80	13	13	80	13	13

TABLE 38

MULTIVARIATE BINOMIAL DISTRIBUTIONS WITH MEANS MODERATELY APART: SUM OF SQUARED ERROR VARIANCE OF THE CRITERION SET VARIABLES OBTAINED FOR VALIDATION SAMPLE
(Results Rounded-off to Nearest Integer)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	149	149	149	132	149	149	132	149	149
2	126	91	91	146	91	91	146	91	91
3	121	86	86	138	86	86	138	86	86
4	108	83	83	142	83	83	142	83	83
5	127	78	78	140	78	78	140	78	78
6	103	83	83	144	83	83	144	83	83
7	99	88	88	143	88	88	143	88	88
8	116	99	99	145	99	99	145	99	99
9	94	76	76	142	76	76	142	76	76
10	87	73	73	137	73	73	137	73	73
11	120	77	77	141	77	77	141	77	77
12	127	69	69	144	69	69	144	69	69
13	129	91	91	146	91	91	146	91	91
14	106	85	85	143	85	85	140	85	85
15	129	91	91	145	91	145	145	91	91
16	125	148	148	145	148	148	145	148	148
17	91	77	77	77	77	77	77	77	77
18	108	87	87	146	146	146	146	87	87
19	143	75	75	130	75	75	130	75	75
20	127	92	92	144	92	92	144	92	92
21	121	148	148	137	148	137	137	148	148
22	107	90	90	146	90	90	146	90	90
23	89	89	89	139	89	89	139	89	89
24	107	73	73	133	73	73	133	73	73
25	114	90	90	145	90	90	145	90	90
26	149	149	149	146	145	145	146	149	149

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 38-Continued

Trial No.	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
	Weighting Schemes[a]								
27	123	88	88	139	88	88	139	88	88
28	141	146	146	141	146	146	141	146	146
29	120	88	88	138	88	88	138	88	88
30	115	92	92	142	92	92	142	92	92
31	136	74	74	141	74	74	141	74	74
32	115	91	91	146	91	91	146	91	91
33	110	82	82	139	82	82	139	82	82
34	132	92	92	147	146	146	147	92	92
35	148	72	72	143	143	143	143	72	72
36	149	76	76	142	142	142	142	76	76
37	107	94	94	146	94	94	146	94	94
38	148	148	148	143	148	148	143	148	148
39	132	91	91	144	91	91	144	91	91
40	111	79	79	141	79	79	141	79	79
41	148	148	148	144	148	148	144	148	148
42	144	92	92	139	92	92	139	147	147
43	118	84	84	145	84	84	145	84	84
44	132	89	89	144	89	89	144	89	89
45	122	81	81	139	81	81	139	81	81
46	149	97	97	148	97	97	148	97	97
47	96	79	79	140	79	79	140	79	79
48	108	73	73	139	73	73	139	73	73
49	127	91	91	140	91	91	140	91	91
50	117	79	79	142	79	79	142	79	79
51	148	87	87	141	87	87	141	87	87
52	148	74	74	141	148	148	141	74	74
53	120	67	67	142	142	142	142	67	67
54	124	80	80	140	80	80	140	80	80
55	99	74	74	142	74	74	142	74	74
56	141	89	89	141	89	89	141	89	89
57	144	146	146	143	146	146	143	146	146
58	125	73	73	142	73	73	142	73	73
59	136	87	87	138	87	87	138	87	87
60	104	104	104	142	104	104	142	104	104
61	132	86	86	140	86	86	140	86	86
62	93	89	89	113	89	89	113	89	89
63	147	82	82	144	82	82	144	82	82
64	111	85	85	142	85	85	142	85	85

TABLE 38-Continued

Trial No.	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
	Weighting Schemes[a]								
65	98	86	86	146	86	86	146	86	86
66	122	76	76	139	76	76	139	76	76
67	129	91	91	143	91	91	143	91	91
68	127	90	90	145	90	90	145	90	90
69	147	87	87	147	87	87	147	87	87
70	147	147	147	139	147	147	139	147	147
71	99	80	80	146	80	80	146	80	80
72	129	82	82	133	82	82	133	82	82
73	105	98	98	146	98	98	146	98	98
74	93	86	86	126	86	86	126	86	86
75	126	96	96	148	149	149	148	96	96
76	113	74	74	139	74	74	139	74	74
77	104	81	81	144	81	81	144	81	81
78	103	85	85	146	146	146	146	85	85
79	140	140	140	140	140	140	140	149	149
80	129	98	98	145	98	98	145	98	98
81	126	90	90	148	90	90	148	90	90
82	119	96	96	142	96	96	142	96	96
83	123	79	79	148	79	79	148	79	79
84	128	78	78	143	78	78	143	78	78
85	138	148	148	145	145	145	145	148	148
86	104	85	85	146	85	85	146	85	85
87	83	69	69	142	69	69	142	69	69
88	128	93	93	141	141	141	141	148	148
89	149	95	95	144	95	95	144	95	95
90	124	74	74	136	74	74	136	74	74
91	115	80	80	146	80	80	146	80	80
92	133	90	90	144	90	90	144	90	90
93	127	76	76	142	76	76	142	76	76
94	120	90	90	143	90	90	143	90	90
95	128	88	88	143	88	88	143	88	88
96	122	75	75	145	75	75	145	75	75
97	135	80	80	147	80	80	147	80	80
98	106	85	85	143	85	85	143	85	85
99	110	86	86	141	86	86	141	86	86
100	96	72	72	145	72	72	145	72	72

TABLE 39

MULTIVARIATE BINOMIAL DISTRIBUTIONS WITH MEANS FARTHER APART: SUM OF SQUARED ERROR VARIANCE OF THE CRITERION SET VARIABLES OBTAINED FOR ANALYSIS SAMPLE
(Results Rounded-off to Nearest Integer)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	175	175	175	591	585	591	591	140	591
2	170	170	170	587	587	587	587	142	142
3	146	146	146	593	593	593	593	146	595
4	100	100	100	585	585	585	585	100	594
5	134	134	134	591	591	591	591	134	591
6	200	200	200	594	594	594	594	200	594
7	116	116	116	593	593	593	593	116	596
8	176	176	176	592	588	595	592	176	587
9	214	214	214	595	592	595	595	214	214
10	221	221	221	590	590	596	590	221	596
11	210	210	210	595	595	595	595	210	595
12	185	185	185	588	594	592	590	142	594
13	153	153	153	594	591	591	594	153	592
14	191	191	191	592	592	592	592	191	592
15	78	78	78	595	595	595	595	76	592
16	166	166	166	592	592	592	592	166	589
17	202	202	202	587	592	587	587	184	582
18	157	157	157	591	591	591	591	157	595
19	167	167	167	592	591	596	592	161	594
20	146	146	146	590	594	594	590	122	596
21	136	136	136	595	586	586	595	115	588
22	177	177	177	593	593	593	593	154	592
23	198	198	198	588	588	595	588	204	595
24	152	152	152	593	594	594	593	130	589
25	160	160	160	588	588	588	588	119	588
26	184	184	184	591	594	591	591	166	591

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 39-Continued

Trial No.	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
	Weighting Schemes[a]								
27	109	109	109	593	596	593	593	101	590
28	594	594	594	592	592	587	592	123	587
29	160	160	160	580	580	593	580	160	592
30	146	146	146	586	586	591	586	146	591
31	592	592	592	590	592	590	590	143	592
32	596	596	115	595	595	595	595	115	595
33	143	143	143	589	589	589	589	143	584
34	144	144	144	588	587	592	586	123	123
35	595	595	595	594	595	595	594	166	594
36	126	126	126	591	591	591	591	120	591
37	182	182	182	588	590	588	588	151	584
38	105	105	105	594	594	594	594	105	593
39	112	112	112	594	593	593	594	112	593
40	145	145	145	590	590	592	590	115	594
41	587	587	587	587	595	588	587	186	587
42	123	123	123	596	596	596	596	101	591
43	158	158	158	586	593	593	586	134	587
44	590	590	590	595	590	590	595	90	590
45	116	116	116	590	590	590	590	108	590
46	166	166	166	585	591	585	585	166	580
47	588	588	588	593	588	593	593	176	593
48	193	193	193	596	588	594	596	193	594
49	152	152	152	593	596	596	593	152	596
50	176	176	176	595	595	595	595	176	593
51	86	86	86	595	591	595	595	86	588
52	183	183	183	596	596	596	596	183	585
53	163	163	163	593	583	583	593	126	596
54	191	191	191	588	588	588	588	184	587
55	115	115	115	592	592	592	592	115	592
56	130	130	130	588	588	588	588	130	130
57	109	109	109	595	587	595	595	98	594
58	189	189	189	593	590	593	593	189	591
59	156	156	156	592	588	588	592	135	591
60	154	154	154	591	591	591	591	140	590
61	115	115	115	590	590	590	590	116	592
62	150	150	150	581	581	581	581	158	593
63	138	138	138	593	593	590	593	138	590
64	593	593	593	593	593	593	593	125	583

TABLE 39-Continued

Trial No.	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
	Weighting Schemes[a]								
65	169	169	169	593	594	594	593	152	592
66	200	200	200	592	592	592	592	200	593
67	133	133	133	596	596	590	596	133	590
68	185	185	185	596	596	596	596	182	595
69	162	162	162	594	594	594	594	162	592
70	150	150	150	593	593	593	593	147	593
71	154	154	154	592	592	591	592	154	591
72	133	133	133	587	587	587	587	121	586
73	177	177	177	583	591	579	583	177	579
74	136	136	136	590	590	590	590	133	590
75	594	594	594	594	594	594	594	110	579
76	583	594	583	583	583	583	583	156	583
77	157	157	157	588	588	588	588	157	593
78	187	187	187	595	595	595	595	187	591
79	178	178	178	590	589	589	590	167	589
80	179	179	179	590	590	583	590	179	594
81	101	101	101	592	593	592	592	94	591
82	86	86	86	592	592	592	592	100	592
83	190	190	190	583	583	593	583	169	587
84	132	132	132	594	594	597	594	132	597
85	154	154	154	590	590	590	590	154	594
86	162	162	162	591	591	591	591	139	589
87	118	118	118	593	593	593	593	118	593
88	172	172	172	585	585	596	585	172	590
89	222	222	222	590	590	590	590	222	593
90	135	135	135	593	591	591	593	135	588
91	123	123	123	589	589	589	589	123	595
92	168	168	168	588	588	588	588	168	586
93	177	177	177	584	584	584	584	178	593
94	184	184	184	587	587	587	587	184	587
95	129	129	129	595	589	589	595	115	595
96	97	97	97	595	595	595	595	97	595
97	206	206	206	589	594	589	589	158	589
98	133	133	133	593	593	593	593	126	589
99	160	160	160	591	588	588	591	143	591
100	162	162	162	592	592	592	592	155	595

TABLE 40

MULTIVARIATE BINOMIAL DISTRIBUTIONS WITH MEANS FARTHER APART: NUMBER OF MISCLASSIFICATIONS BASED ON MEMBERSHIP AS DETERMINED BY CRITERION SET VARIABLES (VALIDATION SAMPLE)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	8	8	8	91	92	91	91	12	91
2	14	14	14	92	92	92	92	14	14
3	17	17	17	97	97	97	97	17	95
4	15	15	15	92	92	92	92	15	89
5	14	14	14	96	96	96	96	14	95
6	11	11	11	91	91	91	91	11	91
7	5	5	5	97	97	97	97	5	97
8	7	7	7	100	100	100	100	7	100
9	15	15	15	91	96	91	91	15	15
10	18	18	18	86	86	86	86	18	86
11	14	14	14	95	93	93	95	14	93
12	16	16	16	92	92	92	93	93	93
13	15	15	15	93	92	92	93	15	95
14	18	18	18	87	87	87	87	18	87
15	17	17	17	91	91	91	91	9	88
16	14	14	14	92	92	92	92	14	93
17	19	19	19	100	93	100	100	19	99
18	8	8	8	98	98	98	98	8	99
19	13	13	13	91	94	94	91	13	95
20	14	14	14	90	92	92	90	10	92
21	16	16	16	94	88	88	94	13	92
22	10	10	10	96	96	96	96	10	96
23	13	13	13	93	93	91	93	10	91
24	9	9	9	96	94	94	96	11	97
25	19	19	19	89	89	89	89	17	89
26	20	20	20	96	89	96	96	17	96

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 40-Continued

Trial No.	Weighting Schemes[a] X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	14	14	14	94	94	94	94	12	93
28	89	89	89	91	91	87	91	16	87
29	10	10	10	100	100	99	100	10	99
30	11	11	11	90	90	92	90	11	92
31	93	93	93	90	93	90	90	11	93
32	93	93	15	93	93	93	93	15	93
33	20	20	20	93	93	93	93	20	93
34	16	16	16	87	88	87	87	16	16
35	91	91	91	89	91	91	89	91	89
36	16	16	16	93	93	93	93	17	93
37	11	11	11	99	92	99	99	7	97
38	18	18	18	91	91	91	91	18	92
39	9	9	9	94	94	94	94	9	94
40	9	9	9	100	100	98	100	13	98
41	92	92	92	91	92	93	91	90	92
42	11	11	11	95	95	95	95	12	91
43	8	8	8	97	97	97	97	14	99
44	92	92	92	95	92	92	95	94	92
45	96	96	96	7	4	4	7	92	4
46	14	14	14	97	98	97	97	14	98
47	93	93	93	92	93	92	92	19	92
48	14	14	14	94	91	94	94	14	94
49	11	11	11	96	94	94	96	11	94
50	19	19	19	93	93	93	93	19	91
51	5	5	5	5	7	5	5	5	6
52	18	18	18	88	88	88	88	18	91
53	9	9	9	95	93	93	95	9	95
54	13	13	13	92	92	92	92	15	94
55	14	14	14	97	97	97	97	14	97
56	9	9	9	92	92	92	92	9	9
57	15	15	15	88	91	88	88	15	89
58	18	18	18	90	94	90	90	18	91
59	20	20	20	89	89	89	89	17	88
60	12	12	12	98	98	98	98	12	96
61	95	95	95	5	5	5	5	95	5
62	11	11	11	93	93	93	93	12	96
63	12	12	12	96	96	97	96	12	97
64	95	95	95	95	95	95	95	22	98

TABLE 40-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	12	12	12	94	93	93	94	14	93
66	15	15	15	89	89	89	89	15	91
67	14	14	14	86	86	94	86	14	94
68	20	20	20	95	95	95	95	95	95
69	9	9	9	97	97	97	97	9	98
70	18	18	18	87	90	90	87	14	87
71	12	12	12	97	97	95	97	12	95
72	14	14	14	94	94	94	94	16	87
73	14	14	14	93	93	93	93	14	93
74	17	17	17	95	95	94	95	13	94
75	97	97	97	93	93	93	93	12	100
76	93	90	93	91	91	91	91	11	93
77	13	13	13	92	92	92	92	13	89
78	13	13	13	98	98	98	98	13	98
79	12	12	12	89	96	96	89	14	96
80	17	17	17	90	90	90	90	17	87
81	8	8	8	97	96	97	97	8	93
82	17	17	17	93	93	93	93	17	93
83	12	12	12	99	99	99	99	11	96
84	14	14	14	90	90	93	90	14	93
85	11	11	11	95	95	95	95	11	94
86	18	18	18	97	97	97	97	19	96
87	18	18	18	92	88	88	98	18	88
88	14	14	14	95	95	90	95	14	88
89	10	10	10	94	94	94	94	10	93
90	22	22	22	94	89	92	94	22	94
91	19	19	19	94	94	94	94	19	87
92	24	24	24	85	85	85	85	24	87
93	18	18	18	88	88	88	88	18	92
94	12	12	12	97	97	97	97	12	97
95	10	10	10	99	98	98	99	14	99
96	12	12	12	94	94	94	94	12	95
97	9	9	9	99	99	99	99	8	99
98	15	15	15	95	95	95	95	16	94
99	17	17	17	97	95	95	97	13	97
100	20	20	20	90	90	90	90	20	93

TABLE 41

MULTIVARIATE BINOMIAL DISTRIBUTIONS WITH MEANS FARTHER APART: SUM OF SQUARED ERROR VARIANCE OF THE CRITERION SET VARIABLES OBTAINED FOR VALIDATION SAMPLE
(Results Rounded-off to Nearest Integer)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	22	22	22	147	145	147	147	25	147
2	34	34	34	147	147	147	147	32	32
3	43	43	43	148	148	148	148	43	149
4	37	37	37	145	145	145	145	37	147
5	32	32	32	147	147	147	147	32	148
6	31	31	31	147	147	147	147	31	147
7	15	15	15	148	148	148	148	15	149
8	21	21	21	149	149	149	149	21	149
9	34	34	34	148	147	148	148	34	34
10	38	38	38	146	146	146	146	38	146
11	38	38	38	146	147	147	146	38	147
12	43	43	43	145	146	145	145	148	147
13	37	37	37	148	148	148	148	37	147
14	39	39	39	147	147	147	147	39	147
15	38	38	38	146	146	146	146	25	146
16	34	34	34	148	148	148	148	34	148
17	46	46	46	146	148	146	146	48	145
18	33	33	33	146	148	148	148	33	149
19	35	35	35	147	148	147	147	35	149
20	27	27	27	147	148	148	147	23	148
21	40	40	40	146	146	146	146	35	145
22	29	29	29	149	149	149	149	27	149
23	30	30	30	147	147	149	147	22	149
24	27	27	27	149	146	146	149	28	147
25	38	38	38	146	146	146	146	35	146
26	43	43	43	146	148	146	146	39	146

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 41-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	33	33	33	148	148	148	148	29	149
28	148	148	148	147	147	147	147	34	147
29	30	30	30	148	148	149	148	30	149
30	31	31	31	147	147	147	147	31	147
31	147	147	147	145	147	145	145	31	148
32	148	148	42	148	148	148	148	42	148
33	44	44	44	146	146	146	146	44	146
34	38	38	38	146	146	145	146	38	38
35	147	147	147	146	147	147	146	147	146
36	40	40	40	148	148	148	148	42	148
37	30	30	30	147	148	147	147	22	149
38	42	42	42	148	148	148	148	42	147
39	24	24	24	146	148	148	146	24	148
40	29	29	29	147	147	149	147	32	149
41	148	148	148	146	147	14[illegible]	146	148	148
42	32	32	32	147	147	147	147	31	146
43	27	27	27	147	146	146	147	36	148
44	149	149	149	147	148	149	147	149	149
45	31	31	31	148	148	148	148	32	148
46	35	35	35	146	147	146	146	35	147
47	146	146	146	145	146	145	145	44	145
48	36	36	36	149	146	149	149	36	149
49	31	31	31	149	148	148	149	31	148
50	48	48	48	147	147	147	147	48	147
51	149	149	149	146	148	146	146	149	148
52	43	43	43	146	146	146	146	43	146
53	23	23	23	149	146	146	149	22	149
54	36	36	36	144	144	144	144	37	145
55	37	37	37	147	147	147	147	37	147
56	28	28	28	147	147	147	147	28	28
57	36	36	36	147	146	147	147	35	147
58	43	43	43	146	147	146	146	43	147
59	43	43	43	147	146	146	147	37	146
60	35	35	35	148	148	148	148	31	148
61	44	44	44	149	149	149	149	44	149
62	32	32	32	148	148	148	148	33	149
63	38	38	38	147	147	147	147	38	147
64	148	148	148	148	148	148	148	48	148

TABLE 41-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	28	28	28	149	149	149	149	30	149
66	37	37	37	145	145	145	145	37	148
67	33	33	33	147	147	148	147	33	148
68	48	48	48	149	149	149	149	149	149
69	28	28	28	148	148	148	148	28	149
70	38	38	38	147	146	145	147	37	147
71	35	35	35	147	147	147	147	35	147
72	42	42	42	145	145	145	145	41	143
73	34	34	34	147	147	147	147	34	147
74	42	42	42	146	146	146	146	35	146
75	148	148	148	147	147	147	147	33	148
76	148	148	148	146	146	146	146	29	148
77	31	31	31	147	147	147	147	31	147
78	36	36	36	149	149	149	149	36	148
79	32	32	32	146	146	146	146	37	146
80	38	38	38	145	145	144	145	38	147
81	22	22	22	149	147	149	149	22	148
82	41	41	41	146	146	146	146	40	146
83	37	37	37	148	148	[illegible]	148	30	145
84	36	36	36	147	147	146	147	36	148
85	33	33	33	148	148	148	148	33	148
86	46	46	46	149	149	149	149	47	149
87	41	41	41	147	146	146	146	41	146
88	35	35	35	146	146	148	146	35	147
89	24	24	24	148	148	148	148	24	148
90	55	55	55	147	146	144	147	55	146
91	43	43	43	145	145	145	145	43	147
92	50	50	50	146	146	146	146	50	148
93	42	42	42	147	147	147	147	42	147
94	31	31	31	147	147	147	147	31	147
95	31	31	31	148	149	149	148	30	148
96	36	36	36	148	148	148	148	36	149
97	27	27	27	148	149	148	148	25	148
98	39	39	39	147	147	147	147	39	146
99	39	39	39	148	148	148	148	30	148
100	47	47	47	148	148	148	148	47	147

TABLE 42

MULTIVARIATE MULTINOMIAL DISTRIBUTIONS: SUM OF SQUARED ERROR VARIANCE OF THE CRITERION SET VARIABLES OBTAINED FOR ANALYSIS SAMPLE
(Results Rounded-off to Nearest Integer)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	556	566	566	585	571	571	585	553	553
2	563	563	566	581	595	581	581	560	560
3	567	567	567	563	566	566	563	550	550
4	555	559	556	578	576	576	578	555	555
5	569	572	574	581	588	588	581	568	562
6	583	577	570	585	582	583	585	566	566
7	568	567	568	579	586	586	579	558	558
8	575	583	563	593	580	580	593	570	570
9	561	561	561	591	578	578	591	546	546
10	582	585	585	595	585	585	595	583	583
11	574	561	565	574	574	574	574	565	565
12	555	561	561	579	574	574	579	558	558
13	588	581	580	589	589	589	589	571	571
14	582	570	570	588	583	583	588	566	561
15	562	561	562	573	587	575	573	539	539
16	583	570	570	589	580	580	589	570	570
17	562	565	565	594	576	576	594	556	556
18	576	570	570	582	583	583	582	566	566
19	575	575	575	587	575	575	587	547	547
20	558	564	564	586	586	586	586	548	548
21	575	576	579	579	579	579	579	574	574
22	546	567	560	592	576	576	592	547	547
23	561	567	571	585	589	589	585	552	552
24	554	564	564	587	594	580	587	554	551
25	581	556	556	592	567	567	592	544	539
26	563	565	565	579	582	582	579	548	548

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 42-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	542	559	559	576	567	567	576	549	549
28	558	564	564	575	571	568	575	543	546
29	574	569	569	567	576	593	567	557	557
30	574	576	576	587	586	586	587	570	568
31	543	552	552	566	582	582	566	538	538
32	573	571	571	594	578	578	594	557	557
33	585	578	577	594	581	581	594	579	579
34	538	557	557	587	568	568	587	539	539
35	543	553	553	588	571	571	588	541	541
36	576	570	570	589	579	579	589	559	559
37	570	566	575	569	575	572	569	548	548
38	551	556	556	585	561	561	585	536	536
39	583	588	588	590	581	581	590	581	581
40	562	559	561	560	574	574	560	551	551
41	537	561	550	569	585	585	569	536	536
42	577	563	563	584	573	573	584	549	549
43	565	577	577	588	577	577	588	563	568
44	554	556	556	567	568	568	567	549	549
45	554	563	563	588	570	570	588	548	548
46	550	549	548	585	562	562	585	544	544
47	548	559	558	592	577	577	592	556	556
48	551	557	561	588	566	566	588	554	554
49	584	574	574	591	580	580	591	568	568
50	560	576	576	589	577	577	589	560	560
51	587	570	570	589	573	573	589	575	575
52	575	554	554	589	569	569	589	549	554
53	553	555	556	581	572	581	581	546	546
54	567	572	572	583	577	577	583	551	551
55	543	557	557	584	567	567	584	536	536
56	585	569	582	593	578	578	593	569	569
57	566	571	571	596	577	577	596	555	555
58	546	567	567	564	569	569	564	540	540
59	564	567	565	563	573	573	563	563	563
60	572	572	572	571	571	571	571	548	548
61	595	570	570	596	568	568	596	561	561
62	562	561	561	580	570	570	580	563	563
63	587	580	580	580	580	580	580	569	569
64	558	563	563	574	570	570	574	557	557

TABLE 42-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	582	567	567	574	569	569	574	573	576
66	563	566	566	579	567	567	579	556	552
67	576	550	550	591	567	567	591	542	542
68	589	587	587	590	589	589	590	573	573
69	577	569	570	584	586	586	584	576	576
70	562	578	578	589	585	585	589	553	553
71	545	562	559	562	571	571	562	548	547
72	568	565	565	592	575	575	592	562	562
73	570	571	571	584	574	574	584	551	551
74	575	572	565	589	589	590	589	566	566
75	575	569	569	586	579	579	586	555	555
76	567	576	576	582	583	583	582	574	574
77	570	567	567	594	575	575	594	571	571
78	582	584	584	590	583	583	590	568	568
79	568	575	573	580	586	586	580	562	562
80	563	558	558	577	555	555	577	548	548
81	576	564	564	594	574	574	594	550	547
82	551	551	551	580	576	576	580	540	544
83	548	570	570	588	584	584	588	555	555
84	572	560	567	580	571	571	580	551	551
85	569	578	578	593	580	580	593	560	560
86	537	553	553	585	573	573	585	552	552
87	583	570	570	588	581	581	588	558	558
88	557	560	560	560	561	561	560	555	555
89	571	572	568	582	579	583	582	567	567
90	570	576	576	581	581	581	581	567	567
91	542	564	564	595	583	583	595	543	548
92	565	569	569	581	581	581	581	556	556
93	574	558	558	585	560	560	585	555	555
94	571	571	571	594	584	585	594	560	560
95	569	578	571	581	581	580	581	558	558
96	585	577	577	583	571	571	583	564	568
97	562	565	565	587	577	577	587	556	556
98	579	575	581	587	588	588	587	579	579
99	569	568	580	589	588	588	589	563	563
100	589	551	551	596	571	571	596	551	551

TABLE 43

MULTIVARIATE MULTINOMIAL DISTRIBUTIONS: NUMBER OF MISCLASSIFICATIONS BASED ON MEMBERSHIP AS DETERMINED BY CRITERION SET VARIABLES (VALIDATION SAMPLE)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	96	96	96	96	96	96	96	96	96
2	97	97	96	92	92	92	92	79	79
3	97	97	97	97	97	97	97	89	89
4	65	96	96	96	96	96	96	96	96
5	96	86	86	86	86	86	86	71	68
6	92	92	92	92	92	92	92	94	94
7	92	92	92	92	92	92	92	83	83
8	92	81	81	81	81	81	81	81	81
9	94	85	85	85	85	85	85	85	85
10	78	78	78	78	78	78	78	78	78
11	79	79	79	79	79	79	79	97	97
12	73	71	71	71	71	71	71	87	87
13	83	94	94	94	94	94	94	94	94
14	85	93	93	93	93	93	93	97	93
15	87	84	87	96	96	96	96	75	75
16	75	83	83	83	83	83	83	83	83
17	70	82	82	82	82	82	82	64	64
18	100	96	96	96	96	96	96	97	97
19	80	80	80	80	80	80	80	76	76
20	74	84	84	84	84	84	84	82	82
21	91	100	100	100	100	100	100	86	86
22	74	81	81	81	81	81	81	81	81
23	76	72	72	72	72	72	72	75	75
24	85	89	89	89	84	89	89	89	89
25	75	75	75	75	75	75	75	67	75
26	99	99	99	99	99	99	99	96	96

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 43-Continued

Trial No.	Weighting Schemes[a] X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
27	85	99	99	99	99	99	99	99	99
28	77	77	77	77	77	77	77	77	77
29	93	95	95	95	95	89	95	92	92
30	93	85	85	85	85	85	85	72	75
31	96	97	97	97	97	97	97	96	96
32	76	76	76	76	76	76	76	77	77
33	77	77	77	77	77	77	77	77	77
34	85	87	87	87	87	87	87	87	87
35	79	85	85	85	85	85	85	80	80
36	83	84	80	84	84	84	84	77	77
37	93	91	93	93	93	93	93	79	79
38	95	100	100	100	100	100	100	100	100
39	92	92	92	92	92	92	92	92	92
40	96	96	96	96	96	96	96	80	80
41	91	91	91	91	91	91	91	88	88
42	99	99	99	99	99	99	99	99	99
43	99	99	99	99	99	99	99	88	84
44	84	85	85	85	85	85	85	83	83
45	89	89	89	89	89	89	89	86	86
46	88	94	94	94	94	94	94	94	94
47	73	73	73	73	73	73	73	68	68
48	81	85	85	85	85	85	85	85	85
49	84	84	84	84	84	84	84	84	84
50	93	87	87	87	87	87	87	97	97
51	91	91	91	91	91	91	91	87	87
52	90	90	90	90	90	90	90	86	82
53	97	95	95	95	95	95	95	97	97
54	85	83	83	83	83	83	83	71	71
55	83	92	92	92	92	92	92	98	98
56	87	87	87	87	87	87	87	83	83
57	73	98	98	98	98	98	98	98	98
58	82	97	97	97	97	97	97	95	95
59	89	89	89	89	89	89	89	87	87
60	79	79	79	79	79	79	79	69	69
61	92	92	92	92	92	92	92	92	92
62	93	93	93	93	93	93	93	85	85
63	83	83	83	83	83	83	83	83	83
64	82	82	82	82	82	82	82	82	82

TABLE 43-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	93	93	93	93	93	93	93	93	93
66	67	74	74	100	100	100	100	80	80
67	93	93	93	93	93	93	93	87	87
68	78	78	78	78	78	78	78	78	78
69	92	83	84	92	92	92	92	80	80
70	96	89	89	89	89	89	89	91	91
71	79	80	80	80	80	80	80	83	81
72	87	79	79	79	79	79	79	83	83
73	86	91	91	91	91	91	91	89	89
74	88	87	84	99	99	99	99	84	84
75	94	94	94	94	94	94	94	80	80
76	86	91	91	91	91	91	91	84	84
77	89	89	89	89	89	89	89	89	89
78	86	86	86	86	86	86	86	76	76
79	74	79	79	79	79	79	79	68	68
80	98	98	98	98	98	98	98	91	91
81	85	88	88	88	88	88	88	81	82
82	95	95	95	95	95	95	95	93	93
83	76	71	71	74	74	74	74	78	78
84	94	79	79	79	79	79	79	74	74
85	86	86	86	86	86	86	86	86	86
86	77	99	99	99	99	99	99	99	99
87	100	100	100	100	100	100	100	92	92
88	87	87	87	87	87	87	87	87	87
89	81	81	81	81	81	81	81	79	79
90	94	94	94	94	94	94	94	94	94
91	86	86	86	86	86	86	86	86	86
92	97	97	97	97	97	97	97	91	91
93	95	86	86	86	86	86	86	86	86
94	72	92	92	92	92	92	92	92	92
95	83	74	75	74	74	74	74	79	79
96	91	91	91	91	91	91	91	72	74
97	71	78	78	78	78	78	78	78	78
98	63	62	71	71	71	71	71	71	71
99	74	75	75	75	75	75	75	98	98
100	100	100	100	100	100	100	100	100	100

TABLE 44

MULTIVARIATE MULTINOMIAL DISTRIBUTIONS: SUM OF SQUARED ERROR VARIANCE OF THE CRITERION SET VARIABLES OBTAINED FOR VALIDATION SAMPLE
(Results Rounded-off to Nearest Integer)

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
1	1540	1540	1540	1540	1540	1540	1540	1540	1540
2	1510	1510	1506	1547	1547	1547	1547	1498	1498
3	1464	1464	1464	1464	1464	1464	1464	1453	1453
4	1484	1511	1511	1511	1511	1511	1511	1511	1511
5	1438	1468	1468	1468	1468	1468	1468	1384	1384
6	1520	1520	1520	1520	1520	1520	1520	1500	1500
7	1507	1507	1507	1507	1507	1507	1507	1491	1491
8	1333	1413	1413	1413	1413	1413	1413	1413	1413
9	1458	1500	1500	1500	1500	1500	1500	1500	1500
10	1493	1493	1493	1493	1493	1493	1493	1493	1493
11	1571	1571	1571	1571	1571	1571	1571	1491	1491
12	1557	1564	1564	1564	1564	1564	1564	1539	1539
13	1459	1508	1508	1508	1508	1508	1508	1508	1508
14	1545	1582	1582	1582	1582	1582	1582	1573	1582
15	1539	1527	1539	1567	1567	1567	1567	1497	1497
16	1565	1615	1615	1615	1615	1615	1615	1615	1615
17	1503	1548	1548	1548	1548	1548	1548	1477	1477
18	1500	1511	1511	1511	1511	1511	1511	1493	1493
19	1519	1519	1519	1519	1519	1519	1519	1483	1483
20	1436	1457	1457	1457	1457	1457	1457	1444	1444
21	1546	1560	1560	1560	1560	1560	1560	1534	1534
22	1464	1492	1492	1492	1492	1492	1492	1492	1492
23	1500	1510	1510	1510	1510	1510	1510	1503	1503
24	1499	1509	1509	1509	1492	1509	1509	1509	1509
25	1518	1518	1518	1518	1518	1518	1518	1472	1518
26	1544	1544	1544	1544	1544	1544	1544	1530	1530

[a] X1: X Not Differentially Weighted; X2: X Weighted Relative to Y; X3: X Weighted Relative to FY; FX1: FX Not Differentially Weighted; FX2: FX Weighted Relative to Y; FX3: FX Weighted Relative to FY; CX1: CX Not Differentially Weighted;CX2: CX Weighted Relative to Y; CX3: CX Weighted Relative to CY

TABLE 44-Continued

Trial No.	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
	Weighting Schemes[a]								
27	1425	1542	1542	1542	1542	1542	1542	1542	1542
28	1511	1511	1511	1511	1511	1511	1511	1511	1511
29	1516	1532	1532	1532	1532	1519	1532	1517	1517
30	1485	1495	1495	1495	1495	1495	1495	1417	1443
31	1488	1522	1522	1522	1522	1522	1522	1488	1488
32	1594	1594	1594	1594	1594	1594	1594	1590	1590
33	1463	1463	1463	1463	1463	1463	1463	1463	1463
34	1400	1436	1436	1436	1436	1436	1436	1436	1436
35	1586	1626	1626	1626	1626	1626	1626	1612	1612
36	1530	1536	1526	1536	1536	1536	1536	1504	1504
37	1523	1508	1523	1523	1523	1523	1523	1487	1487
38	1538	1562	1562	1562	1562	1562	1562	1562	1562
39	1540	1540	1540	1540	1540	1540	1540	1540	1540
40	1476	1476	1476	1476	1476	1476	1476	1428	1428
41	1522	1528	1528	1528	1528	1528	1528	1517	1517
42	1568	1568	1568	1568	1568	1568	1568	1568	1568
43	1629	1629	1629	1629	1629	1629	1629	1574	1543
44	1440	1449	1449	1449	1449	1449	1449	1425	1425
45	1566	1566	1566	1566	1566	1566	1566	1539	1539
46	1427	1462	1462	1462	1462	1462	1462	1436	1436
47	1580	1580	1580	1580	1580	1580	1580	1551	1551
48	1506	1544	1544	1544	1544	1544	1544	1544	1544
49	1519	1519	1519	1519	1519	1519	1519	1519	1519
50	1527	1546	1546	1546	1546	1546	1546	1507	1507
51	1512	1512	1512	1512	1512	1512	1512	1494	1494
52	1580	1580	1580	1580	1580	1580	1580	1555	1537
53	1517	1520	1520	1520	1520	1520	1520	1517	1517
54	1427	1493	1493	1493	1493	1493	1493	1456	1456
55	1426	1504	1504	1504	1504	1504	1504	1476	1476
56	1542	1542	1542	1542	1542	1542	1542	1506	1506
57	1440	1559	1559	1559	1559	1559	1559	1559	1559
58	1432	1497	1497	1497	1497	1497	1497	1479	1479
59	1513	1513	1513	1513	1513	1513	1513	1505	1505
60	1530	1530	1530	1530	1530	1530	1530	1458	1458
61	1509	1509	1509	1509	1509	1509	1509	1509	1509
62	1482	1482	1482	1482	1482	1482	1482	1459	1459
63	1443	1443	1443	1443	1443	1443	1443	1443	1443
64	1592	1592	1592	1592	1592	1592	1592	1592	1592

TABLE 44-Continued

Trial No.	Weighting Schemes[a]								
	X1	X2	X3	FX1	FX2	FX3	CX1	CX2	CX3
65	1550	1550	1550	1550	1550	1550	1550	1550	1550
66	1447	1468	1468	1516	1516	1516	1516	1453	1456
67	1540	1540	1540	1540	1540	1540	1540	1527	1527
68	1496	1496	1496	1496	1496	1496	1496	1496	1496
69	1544	1521	1525	1544	1544	1544	1544	1502	1502
70	1503	1524	1524	1524	1524	1524	1524	1472	1472
71	1446	1472	1472	1472	1472	1472	1472	1455	1450
72	1415	1430	1430	1430	1430	1430	1430	1384	1384
73	1509	1559	1559	1559	1559	1559	1559	1555	1555
74	1500	1519	1509	1574	1574	1574	1574	1501	1501
75	1478	1478	1478	1478	1478	1478	1478	1425	1425
76	1572	1593	1593	1593	1593	1593	1593	1533	1533
77	1626	1626	1626	1626	1626	1626	1626	1626	1626
78	1542	1542	1542	1542	1542	1542	1542	1518	1518
79	1428	1444	1444	1444	1444	1444	1444	1394	1394
80	1599	1599	1599	1599	1599	1599	1599	1582	1582
81	1554	1559	1559	1559	1559	1559	1559	1537	1540
82	1541	1541	1541	1541	1541	1541	1541	1497	1504
83	1512	1509	1509	1531	1531	1531	1531	1523	1523
84	1563	1571	1571	1571	1571	1571	1571	1542	1542
85	1519	1519	1519	1519	1519	1519	1519	1519	1519
86	1471	1551	1551	1551	1551	1551	1551	1551	1551
87	1480	1480	1480	1480	1480	1480	1480	1434	1434
88	1470	1470	1470	1470	1470	1470	1470	1470	1470
89	1503	1503	1503	1503	1503	1503	1503	1481	1481
90	1537	1537	1537	1537	1537	1537	1537	1537	1537
91	1491	1491	1491	1491	1491	1491	1491	1491	1491
92	1522	1522	1522	1522	1522	1522	1522	1491	1491
93	1519	1541	1541	1541	1541	1541	1541	1541	1541
94	1434	1502	1502	1502	1502	1502	1502	1502	1502
95	1527	1554	1550	1554	1554	1554	1554	1541	1541
96	1597	1597	1597	1597	1597	1597	1597	1528	1537
97	1356	1452	1452	1452	1452	1452	1452	1452	1452
98	1547	1548	1602	1602	1602	1602	1602	1547	1547
99	1505	1515	1515	1515	1515	1515	1515	1419	1419
100	1528	1528	1528	1528	1528	1528	1528	1528	1528

APPENDIX B

DESCRIPTION OF ALGORITHMS

I. MULTIVARIATE NORMAL RANDOM GENERATOR

The program to generate random multivariate normal data points is based on methods described in Hammersley and Handscomb (1964).

Given specified values of μ and Σ, the parameters of a p-variate normal population, n random independent observations, x_k, (k=1,2,...,n) are generated in the following manner:

(1) A sequence of random uniform observations y_i is generated on the interval (0,1) using the following recursive algorithm:

$$w_i = (aw_{i-1} + c)\text{modulo } m$$

where

m is a large power of 2 ($16777216 \equiv 2^{24}$)

a = (1)modulo 4

c is an odd arbitrary integer (99191)

i = 1,2,..., p+2 if p is even.

= 1,2,..., p+3 if p is odd.

Set $y_i = w_i/m$, then $0 \leq y_i \leq 1$.

y_1 and y_2 are discarded. The initial value w_0 is supplied by the user.

(2) Pairs of normal pseudo random-observations z_i are calculated as follows:

$$z_i = (-2 \ln y_{i+2})^{1/2} \cos 2\pi y_{i+3}$$

$$z_{i+1} = (-2 \ln y_{i+2})^{1/2} \sin 2\pi y_{i+3}, \quad i=1,3,\ldots,p$$

if p is odd, z_{p+1} is discarded.

(3) Form the px1 vector $z=(z_1,z_2,\ldots,z_p)$

(4) Factor Σ into $T'T$ where T is a triangular pxp matrix. Then $x=T'z+\mu$.

(5) $x_k = x$

(6) Repeat steps (1) through (5) for k=1 to k=n.

II. MULTIVARIATE BINOMIAL RANDOM GENERATOR

The algorithm follows methods suggested by Bahadur (1961).

Let X denote the set of all points $x = (x_1, x_2, \ldots, x_n)$ with each $x_i = 0$ or 1, and let p(x) be a given probability distribution on X. Since there are 2^n points in X, 2^n-1 independent parameters can determine the probability distribution. These parameters are defined below.

For each i = 1, . . ., n, let $\alpha_i = E(x_i)$

$$0 < \alpha_i < 1, \quad i = 1, \ldots, n$$

Now using standardized variables,

$$z_i = \frac{x_i - \alpha_i}{\sqrt{\alpha_i (1 - \alpha_i)}},$$

define second-order, third-order, . . ., nth-order correlations as follows:

$$r_{ij} = E(z_i z_j), \qquad i < j;$$

$$r_{ijk} = E(z_i z_j z_k), \qquad i < j < k;$$

. . . .

. . . .

$$r_{12\ldots n} = E(z_1 z_2 \ldots z_n).$$

Probability distribution p(x) is completely determined by the 2^n-1 parameters defined above.

Bahadur (1961, pp. 159-160) has shown that

$$p(x) = p_1(x) \cdot f(x) ,$$

where $p_1(x)$ is the joint distribution of the x_i's when the x_i's are independent and have the same marginal distributions as under the given distribution p(x), and

$$f(x) = 1 + \sum_{i<j} r_{ij} z_i z_j + \cdots + r_{12\ldots n} z_1 z_2 \cdots z_n.$$

An approximation to p(x) is obtained by omitting correlations above a certain order. Second-order approximation to p(x) used in the present study is given by

$$p_2(x) = p_1(x) \; (1 + \sum_{i<j} r_{ij} \; z_i z_j)$$

Once the probabilities are computed, generating the observations is a simple matter. The cumulative distribution can be used for the purpose.

As Bahadur (1961) notes, the "approximation" may in certain cases give negative values for some points, thus, violating one of the fundamental axioms of probability. Such problems will not arise if the correlation parameters chosen are small in absolute value.

III. CLUSTERING PROCEDURE-I

Clustering Procedure-I is basically the Singleton-Kautz algorithm (Singleton, 1969). The algorithm is designed to form groups such that within group variance is at a minimum. It works in the following way:

1. All subjects are assigned to a single group.

2. The subject farthest from the group mean is assigned to a second group.

3. All subjects are sequentially tested to determine if an assignment to the second group will minimize the sum of squared error variance.

4. When it is no longer possible to reduce the sum of squared error variance by reassigning any single subject to a different group, the number of groups is increased by one and the process repeated. The maximum number of groups that can be formed is set by the analyst.

5. When the number of groups formed reaches the limit set by the analyst, the cycle is reversed and the number of groups is reduced by combining the two groups which will minimally increase the sum of squared error variance. If the sum of squared error variance found at this stage of the cycle is smaller than that found in any previous stage, the clustering now obtained is substituted for the previous one.

6. The increasing and decreasing of the number of groups is continued until it is not possible to reduce the

sum of squared error variance. At this stage, the process terminates.

Some modifications of Singleton-Kautz algorithm are made in Clustering Procedure-I. Singleton-Kautz algorithm uses only one set of variables whereas Clustering Procedure-I uses two sets of variables - criterion set as well as predictor set variables. Clustering is done with the predictor set variables. For the clusters thus formed, the procedure gives, in addition to output in Singleton-Kautz algorithm, the sum of squared error variance of the criterion set summed over variables and clusters.

Total sum of squared error variance of the criterion set is given by

$$\sum_{i=1}^{p} \sum_{j=1}^{m} \sum_{k=1}^{n_i} (y_{ijk} - y_{ij})^2$$

where y_{ijk} denotes the value of the criterion variable j for member k in cluster i, n_i the number of subjects in cluster i, m the number of criterion set variables, and p the number of clusters.

Besides, the output includes measurement on predictor set variables for each observation in the final clusters. For a two-cluster solution, the procedure also gives the number of misclassifications in clustering for data drawn from known populations. Another modification to Singleton-Kautz algorithm is that it is possible to weight the variables used for clustering.

APPENDIX C

QUESTIONNAIRE

Reproduced below are extracts of the questionnaire relevant to the present study. Brand names have been masked by the letters A, B, C, and D. Also omitted are rating board, and the photographs of the products that were used to help respondent identify the brands in the original study.

A. Here are some statements about aerosol shaving creams. Above each page of statements are some phrases that describe how important these qualities are to you in an aerosol shaving cream.

Any of these qualities could be extremely important, very important, somewhat important, slightly important, or unimportant.

Read each of these statements and place an "X" next to that statement and under the phrase that best describes how important you feel that quality is in an aerosol shaving cream. Do the same for all of the statements listed.

	Extremely Important	Very Important	Somewhat Important	Slightly Important	Unimportant
Lather comes out hot from the can	( )	( )	( )	( )	( )
Has a nice fragrance	( )	( )	( )	( )	( )
Makes face feel good while shaving	( )	( )	( )	( )	( )
Has a rich lather	( )	( )	( )	( )	( )
Easy to spread on face	( )	( )	( )	( )	( )
Is medicated	( )	( )	( )	( )	( )
Makes you feel as if you had a shave at a barbershop	( )	( )	( )	( )	( )

A. (Cont'd.)

	Extremely Important	Very Important	Somewhat Important	Slightly Important	Unimportant
Lather is deep soaking	( )	( )	( )	( )	( )
Made by a modern company	( )	( )	( )	( )	( )
Stays wet while shaving	( )	( )	( )	( )	( )
Economical	( )	( )	( )	( )	( )
Comes in several fragrances	( )	( )	( )	( )	( )
Good for sensitive skin	( )	( )	( )	( )	( )
Made by a well-known company	( )	( )	( )	( )	( )
Reduces razor drag	( )	( )	( )	( )	( )
Contains a skin conditioner	( )	( )	( )	( )	( )
Contains special ingredients for skin blemishes	( )	( )	( )	( )	( )
Good for tough beards	( )	( )	( )	( )	( )
Contains an after shave ingredient	( )	( )	( )	( )	( )
Has a longer lasting fragrance	( )	( )	( )	( )	( )
Doesn't irritate blemished skin	( )	( )	( )	( )	( )
Gives a soothing shave	( )	( )	( )	( )	( )
Gives face a tingling feeling	( )	( )	( )	( )	( )
Good for light beards	( )	( )	( )	( )	( )

A. (Cont'd.)

	Extremely Important	Very Important	Somewhat Important	Slightly Important	Unimportant
Can lasts a long time	( )	( )	( )	( )	( )
Good for men who shave more than once a day	( )	( )	( )	( )	( )
Helps prevent nicks and cuts	( )	( )	( )	( )	( )
Softens beard	( )	( )	( )	( )	( )
Has a firm lather	( )	( )	( )	( )	( )
Prevents irritation	( )	( )	( )	( )	( )
Moisturizes beard	( )	( )	( )	( )	( )
Gives a close shave	( )	( )	( )	( )	( )
Makes face feel good after shaving	( )	( )	( )	( )	( )
Stays moist while shaving	( )	( )	( )	( )	( )

B. Now we would like you to do something different. We'd like to have your opinions about some brands of shaving cream. There are no right or wrong answers. If you have never tried some of these brands of shaving cream, we would still like to have your opinions about them in terms of how you think they would be.

(LAY OUT RATING BOARD). Here is a board with some phrases on it which describe how good or bad you feel something is. It could be "very good", "good", "neither good nor bad", "poor", or "very poor".

(HAND RESPONDENT PHOTO DECK # 2). Here are four cards. Each card has a picture of a brand of shaving cream on it. Please place each card on the phrase on the board that best describes how you feel about that brand. (AFTER RESPONDENT HAS PLACED ALL FOUR CARDS ON THE BOARD, PICK THEM UP ONE AT A TIME AND RECORD RESPONSES BELOW. LEAVE RATING BOARD IN FRONT OF RESPONDENT.)

	Brand	Very Good	Good	Neither Good Nor Bad	Poor	Very Poor
( )	A	( )	( )	( )	( )	( )
( )	B	( )	( )	( )	( )	( )
( )	C	( )	( )	( )	( )	( )
( )	D	( )	( )	( )	( )	( )

C. The final section consists of some statements. Read each statement and decide whether you agree or disagree with it as it applies to you.

If you "agree completely" with a statement you would place an "X" next to that statement and under "agree completely".

If you "agree somewhat" with a statement you would place an "X" next to that statement and under "agree somewhat".

If you "disagree somewhat" with a statement you would place an "X" next to that statement and under "disagree somewhat".

If you "disagree completely" with a statement you would place an "X" next to that statement and under "disagree completely".

Remember to give YOUR OWN OPINION OF YOURSELF. Do not leave any statement blank if you can avoid it.

	Agree Completely	Agree Somewhat	Disagree Somewhat	Disagree Completely
I am too busy to spend a lot of time fussing over myself	( )	( )	( )	( )
I get annoyed when people tell me that I need a shave	( )	( )	( )	( )
I like to do things on the spur of the moment rather than planning ahead	( )	( )	( )	( )
I think that wearing conservative clothing is the mark of a gentleman	( )	( )	( )	( )

C. (Cont'd.)

	Agree Completely	Agree Somewhat	Disagree Somewhat	Disagree Completely
Shaving is a bother and a nuisance	( )	( )	( )	( )
I trim my sideburns every time I shave	( )	( )	( )	( )
I am not easily angered	( )	( )	( )	( )
I get a kick out of the skill with which I shave	( )	( )	( )	( )
I am perfectly happy when I am all by myself	( )	( )	( )	( )
I enjoy shaving only when I have plenty of time	( )	( )	( )	( )
I sweat very easily even on cool days	( )	( )	( )	( )
I believe that a husband should be "king of the castle"	( )	( )	( )	( )
I easily become impatient	( )	( )	( )	( )
I like to go out drinking with the boys once in a while	( )	( )	( )	( )
I like to wear old clothes on weekends	( )	( )	( )	( )
If I have a choice I would rather not shave on weekends	( )	( )	( )	( )
My eyesight is not as good as it used to be	( )	( )	( )	( )
I use hair tonic	( )	( )	( )	( )
I am bothered by forgetting where I put things	( )	( )	( )	( )

C. (Cont'd.)

	Agree Completely	Agree Somewhat	Disagree Somewhat	Disagree Completely
I shave only because I have to	( )	( )	( )	( )
I get annoyed when my hair is out of place	( )	( )	( )	( )
I like to wear bright colors	( )	( )	( )	( )
It upsets me more than most people to see animals suffer	( )	( )	( )	( )
I think it's important to live up to the ideas and standards which the majority of people believe in	( )	( )	( )	( )
I like poetry	( )	( )	( )	( )
I seldom worry about my health	( )	( )	( )	( )
I get annoyed when I miss part of my face shaving	( )	( )	( )	( )
I like games where I can show my ability	( )	( )	( )	( )
I have few friends, but they are all rather close	( )	( )	( )	( )
I like my wife/girl friend to pick out my clothes	( )	( )	( )	( )
I become upset when I cut myself shaving	( )	( )	( )	( )
I spend more time getting dressed than most men	( )	( )	( )	( )

C. (Cont'd.)

	Agree Completely	Agree Somewhat	Disagree Somewhat	Disagree Completely
When I get home from work I usually change into casual clothes	( )	( )	( )	( )
I am never unsure of myself	( )	( )	( )	( )
I don't like losing	( )	( )	( )	( )
It takes me a while to feel at ease with new people	( )	( )	( )	( )
When I go out I worry about whether or not I locked the door	( )	( )	( )	( )
I shave very carefully	( )	( )	( )	( )
I am always clean-shaven even on weekends	( )	( )	( )	( )
I like to have women depend on me	( )	( )	( )	( )

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