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EFFECTS OF HEAVY PARTICLES AT LOW ENERGIES

by

ALEKSANDAR ŠOKORAC

A dissertation submitted to the Graduate Faculty in Physics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

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Abstract

EFFECTS OF HEAVY PARTICLES AT LOW ENERGIES

by

Aleksandar Šokorac

Adviser: Professor Rabindra Nath Mohapatra

In this thesis we discuss the low-energy effects of heavy particles in context of the quantum field theories with local gauge invariance and in general. We present the evaluation of the one-loop heavy-particle contribution to a low-energy relation valid in the SU(5) grand unified theory of Georgi and Glashow. The calculation shows that heavy particles decouple (i.e. have negligible effect) in agreement with the decoupling theorem conjectured by Appelquist and Carazzone but proved only in a simple gauge model with one mass scale. The decoupling theorem is confronted with the recent counterexamples in literature and their difference with grand unified theories is stressed. We propose the more precise version of the decoupling theorem valid in a general case. As a byproduct of our calculation we obtain the heavy-particle contribution to $\sin^2\theta$ in the SU(5) theory which together with the existing light-particle calculation agrees with the pioneering result of Georgi, Quinn and Weinberg obtained by the renormalization-group method and assuming the validity of the decoupling theorem.

Acknowledgments

The work described in this thesis is contained in papers done in colaboration with my friend Goran Senjanović who also suggested the problem. I enjoyed the colaboration and learned a great deal from it. I wish to express my sincere thanks for his invaluable help and continuous encouragement especialy when it was needed most. Also I am greatly indebted to my adviser Professor Rabindra Nath Mohapatra for numerous, illuminating discussions, encouragement and support throughout my graduate work.

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I. Introduction

During the past decade the standard Weinberg-Salam [1] unified gauge model based on the spontaneously broken $SU(2)_L xU(1)$ [2] group and Quantum Chromodynamics [3] (QCD) based on the unbroken SU(3) gauge group established themselves as the most promising candidates for a theory of electro-weak and strong interactions respectively. It was a natural step further to unify [4] all these interactions within a simple gauge group G which will contain SU(3)xSU(2)xU(1) as a subgroup. In these, so called grand unified theories, we have one free gauge coupling constant instead of three and quarks and leptons are put into the same multiplets so that many parameters which would be otherwise unrelated in the SU(3)xS(2)xU(1) model are now related or fixed (finite fixed $\sin^2\Theta$,. various relations between quark and lepton masses, explanation of charge quantization etc.) The price we have to pay is the introduction of new interactions that have not yet been observed in Nature. Namely, quarks and leptons being in the same multiplets can convert one into others via new kind of gauge mesons - "lepto-quarks" now necessarily present in the theory (in addition to the "ordinary" gluons, W, Z mesons and photon) thus violating the experimentaly established baryon and lepton conservation laws. In order to suppress the unobserved interactions these exotic gauge mesons have to get the enormously large masses through the spontaneous symmetry breaking of G $\to$ SU(3)xSU(2)xU(1). Namely, the present lower bounds on proton lifetime $\tau_p \gtrsim 10^{30}$ years imply the existence of $m_{H} \sim 10^{14}$-10^{16} GeV mass scale. The ordinary W, Z, gauge mesons get their masses $m_L \sim 10^2$ GeV at the second stage of symmetry breaking SU(3)xSU(2)xU(1) $\to$ SU(3)xU(1) (with gluons and photon remaining massless.

The problem immidiately arises as to whether these superheavy particles manifest themselves at ordinary energies so they might spoil the well known low-energy phenomenology. Working within an unbroken non-Abelian gauge theory with massive (heavy) fermions Appelquist and Carazzone [5] considered the effects of heavy particles (fermions) in processes involving light particles (in this case the massless gauge mesons) at low energies. Using the formal power-counting arguments they were able to establish the following fact for the 1PI (one-particle-irreducible) graphs with external mesons of momentum k which is much smaller than mass m_H of heavy fermions in the internal lines. If the subgraph with heavy fermions is convergent it will be suppressed by factor k/m_H relative to the graph with the same external lines but without internal heavy fermions. On the other hand, if the subgraph with heavy fermions is primitively divergent by adjusting the accompaning counterterm so that the normalization mass is of order of external momenta k the effects of heavy particles will be absorbed into the coupling constant and the field strength. They conjectured that the so called decoupling theorem applies to any renormalizable theory with different mass scales. The decoupling theorem asserts that in a theory with heavy (m_H) and light (m_L) sector in which the light sector is renormalizable itself the heavy sector will have the vanishing effect $O(m_L^2/m_H^2)$ on low-energy ($E \sim m_L$) processes of light particles appart from the possible dependance of coupling constant on heavy mass (m_H).

The theorem has immidiately been applied [6] to a unified gauge theory in order to explain the widely different strengths of strong, weak and electromagnetic interactions at low energies. The renormalization effects responsible for that disparity were calculated under assumption that the effective coupling constants of the "observed" $SU(3)\times SU(2)\times U(1)$ group obey their own renormalization-group equation, i.e. that the superheavy particles which are the only communication between the above groups effectively decouple. In such a way the grand unification mass scale (at which all three coupling constants become equal to the symmetric limit) was easily found to be consistent with $m_H \sim 10^{14}$-10^{16} GeV obtained from proton-decay estimates. Also $\sin^2\Theta$ was found to be 0.2 at low energies consistent with experiment.

Another question raised in context of the grand unified theories was validity of large gauge hierarchies $m_H \gg m_L$. Namely, it was argued [7] that the radiative corrections may destroy the gauge hierarchy established at the tree level. Subsequent investigation [8] showed that higher order corrections in perturbation theory do not spoil the desired hierarchy when certain conditions are met. Finally, Weinberg [8] has shown using the effective field theory based on the decoupling theorem and the renormalization-group arguments that a full theory can have a large gauge hierarchy. Therefore we see that the decoupling theorem plays an important role in understanding a possible unification of elementary-particle forces.

Recently it was pointed out by the different authors [9] simultaneously that in some cases in the spontaneously broken gauge theories the heavy particles could manifest themselves at low energies contrary to the decoupling theorem. Namely, the important feature of these theories is that the gauge invariance (or sometimes a discrete symmetry) may eliminate some of the tree level divergences so we do not have for every higher order divergence the corresponding counterterm to remove the infinity. These divergences must now cancel between the different graphs because the theory is renormalizable. In this case the contribution of heavy particles can not be absorbed into a counterterm as argued by Appelquist and Carazzone and we will have a finite physical effect which grows with heavy-particle mass.

All these examples [9] against the decoupling theorem were not, admitedly, given in context of grand unified theories where, as we stressed above, it's validity is of the great importance. In view of that, we have undertaken [10] an explicit calculation involving superheavy particles in a grand unified theory. In this work we present the outcome of our analysis. We will work within the SU(5) theory of Georgi and Glashow [11] which is one of the first proposed realistic grand unified theories. It is a minimal theory that contains SU(3)xSU(2)xU(1) as a subtheory, it needs relatively simple Higgs sector and serves as a prototype of more general and more complicated theories. We have evaluated one-loop corrections due to heavy particles to the expression:

$$\Delta \equiv \frac{m_W^2 - m_Z^2 \cos^2\theta}{m_W^2} \tag{1.1}$$

which vanishes in their model at the tree level in the limit of infinitely heavy particles:

$$\Delta_0 \equiv \frac{m_W^{o2} - m_Z^{o2} \cos^2\theta_0}{m_W^{o2}} \propto \frac{m_L^2}{m_H^2} \tag{1.2}$$

(L,H stand for light and heavy particles respectively and index o denotes the bare values). The expression Δ_0 in (1.2) was already used in literature [12] in context of pure Weinberg-Salam model (where it vanishes exactly) to calculate heavy-particle effects. Namely, due to the high level of divergence of W and Z propagators (quadratic) used to calculate m_W^2 and m_Z^2 in Δ (1.1) the dominant effects linear in m_H^2/m_L^2 are expected (and obtained in literature [12]) because there is no counterterm coming from (1.2) to absorb it. Let us only mention that $\cos^2\Theta$ calculated from dimensionless wave function renormalization constants ($\sin^2\Theta = e^2/g^2$) can give only $\ln(m_H^2/m_L^2)$ terms.

Let us emphasize at this point the difference between a) the theories with examples against the decoupling theorem which happen to be with one basic mass scale v_0 and b) the grand unified theories with two mass scales $v \gg v_0$. In the case a):

$$m_H \sim b v_0 \quad , \quad m_L \sim c v_0 \tag{1.3}$$

while in the case b):

$$m_H \sim a v + b v_0 \quad , \quad m_L \sim c v_0 \tag{1.4}$$

where a,b,c are some coupling constants of the theory.

In the case a) the hierarchy $m_H \gg m_L$ is achieved by $b \gg c$ so the eventual heavy-particle effect in Δ (1.1) will be of the form:

$$\Delta \propto \frac{m_H^2}{m_L^2} \sim B \tag{1.5}$$

where B is some constant dependent on $b/c \gg 1$.

In the case b) the hierarchy $m_H \gg m_L$ is achieved by $v \gg v_0$ and the heavy particles will eventually give the effect in Δ (1.1) of the form:

$$\Delta \propto \frac{m_H^2}{m_L^2} \sim A \frac{v^2}{v_0^2} + B \tag{1.6}$$

or they decouple if:

$$A = B = 0 \tag{1.7}$$

Now we expect $A=0$ by the more general principle of U(1) gauge invariance. Namely, after the first stage of symmetry breaking ($v\neq 0$, $v_0=0$) when the SU(3)xSU(2)xU(1) gauge invariance remains, W and Z mesons are massless and they can not get mass radiatively from v^2 contributions of heavy particles in (1.4). They will remain massless as a consequence of the ordinary U(1) gauge invariance. Only after the second stage of symmetry breaking ($v_0\neq 0$) when W and Z pick up the tree level mass they can get mass radiatively corrected from the v_0^2 contributions of heavy particles in (1.4). Therefore the validity of the decoupling theorem alone depends on the B term in (1.6). We have calculated both A [13] and B terms in (1.6) and obtained that the heavy particles do decouple at low energies, i.e. (1.7) holds.

We find that there is a good reason not to be suprised at the decoupling of superheavy particles: it is the $SU(2)_L xU(1)$ gauge invariance of the light-particle sector at the stage when heavy particles get their masses. In grand unified theories (case b) above) that happens after the first stage of symmetry breaking ($v\neq 0$, $v_0=0$). In the previously found examples [9] of the low-energy manifestation of heavy particles (case a) above) the gauge invariance of the light-particle sector gets broken at the stage when heavy particles get their masses because there is only one basic mass scale v_0. In such cases the heavy particles will not necessarily decouple. In Chapter IV we discuss this in detail by analysing most of the found examples of heavy-particle effects at low energies. We also draw the parallel with work described in this thesis by putting the special emphasis on the gauge invariance of light sector of the theory. We do not offer any general proofs but we believe that our remarks do have a general nature since they seem to be applicable in all the known cases.

As a byproduct of our calculation we obtain the heavy-particle contribution to $\sin^2\Theta$ in the SU(5) theory which is needed together with already found [14] light-particle contribution in the Weinberg-Salam model to make $\sin^2\Theta$ finite at low energies. Therefore our work also serves the purpose of demonstrating explicitely how the renormalization works in a complicated gauge theory. The net finite result for $\sin^2\Theta$ agrees exactly with pioneering result of Georgi et al. [6] who used the renormalization-group method and the decoupling theorem. Therefore this result can be viewed as an additional confirmation that the heavy particles decouple at low energies.

The rest of the work is organized in the following manner:

Chapter II is used to review some of the relevant properties of the SU(5) model of Georgi and Glashow and to establish the notation. We followed closely the work of Buras et al. [15] but we also included the full physical spectrum of the theory completed by us and which is needed for our computation.

In Chapter III which presents the main body of our work we describe our computation. We also discuss some of the aspects of the renormalization program in the t'Hooft's gauge (using n-dimensional regularization) which we employ in the paper.

Chapter IV is devoted to the analysis of the previously found examples of the low-energy manifestation of heavy particles. The more precise version of the decoupling theorem is offered.

We conclude with Chapter V in which we summarize the results of the work.

In the Appendix we give some properties of the SU(5) group and the Feynman rules for the SU(5) theory of Georgi and Glashow which are needed for our calculation. Many of the details of the straightforward but tedious calculation needed in Chapter III are given.

II. SU(5) Grand Unified Theory of Georgi and Glashow

Let us now consider some of the properties of the SU(5) model [11]. We will follow closely the work of Buras et al. [15]. Included is also, for the first time the full physical spectrum of the theory which is needed for our computation.

There are 24 generators $I_a = 1/2\,\lambda_a$ (a=1,2,...24) (see Appendix A) of the SU(5) group which satisfy the following commutation relations

$$[I_a, I_b] = i f_{abc} I_c \quad (a,b,c = 1,\dots 24) \tag{2.1}$$

where f_{abc}'s are totally antisymmetric in a,b,c indices (see Appendix A). We normalize the generators I_a so that:

$$\mathrm{Tr}\, I_a I_b = \frac{1}{2}\delta_{ab} \tag{2.2}$$

The SU(5) group with the bare coupling constant g_u^0 contains the following subgroups: colour $SU(3)_c$ group of strong interactions (with $I_1,\dots I_8$) generators and the bare coupling constant g_3^0), SU(2) (with $I_{21}, I_{22}, I_{23} \equiv T_1, T_2, T_3$ generators and the bare coupling constant g_2^0) and U(1) groups (with $I_{24} \equiv T_0$ generator and the bare coupling constant g_1^0) of the weak and electromagnetic interactions. The invariance under SU(5) group implies that:

$$g_1^0 = g_2^0 = g_3^0 = g_u^0 \tag{2.3}$$

If the charge operator Q of the theory is given by:

$$Q = T_3 - K T_0 \tag{2.4}$$

the bare charge coupling constant e_0 is determined by:

$$\frac{1}{e_0^2} = \frac{1}{g_2^{0\,2}} + \frac{K^2}{g_1^{0\,2}} \tag{2.5}$$

so we can indentify the usual SU(2) and U(1) coupling constants:

$$g_0 \equiv g_2^0 \qquad ; \qquad g_0' \equiv \frac{g_1^0}{K} \tag{2.6}$$

and the bare mixing angle Θ_0 is defined by:

$$\sin^2\Theta_0 \equiv \frac{e_0^2}{g_0^2} = \frac{g_0'^2}{g_0^2 + g_0'^2} = \frac{1}{1+K^2} \tag{2.7}$$

i.e. is a fixed constant for the group. In any representation of SU(5) we have from (2.2) and (2.4):

$$\mathrm{Tr}\, Q^2 = (1+K^2)\, \mathrm{Tr}\, T_3^2 \tag{2.8}$$

so that using (2.7) and (2.8):

$$\sin^2\Theta_0 = \frac{\mathrm{Tr}\, T_3^2}{\mathrm{Tr}\, Q^2} = \frac{\sum_n T_{3n}^2}{\sum_n Q_n^2} \tag{2.9}$$

where the summation is over all members of any given representation of the SU(5) group.

The pure gauge meson Lagrangian of the theory is:

$$\mathcal{L}_{GM} = -\frac{1}{2}\, \mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} \tag{2.10}$$

where:

$$F_{\mu\nu} = \frac{i}{g_0}\, [D_\mu, D_\nu] \tag{2.11}$$

$$D_\mu = \partial_\mu - i g_0 A_\mu \tag{2.12}$$

and

$$A_\mu = \sum_{a=1}^{24} I_a A_\mu^a \tag{2.13}$$

i.e. adjoint $\underline{24}$ representation of the SU(5) group. We can write explicitely in a matrix form:

$$\sqrt{2}\,A_\mu = \begin{pmatrix} A_{8\mu} + \sqrt{\frac{2}{15}}\,A^0_\mu\, I & \underset{\sim}{X}_\mu & \underset{\sim}{Y}_\mu \\ \underset{\sim}{X}^\dagger_\mu & \frac{1}{\sqrt{2}} A^3_\mu - \sqrt{\frac{3}{10}}\,A^0_\mu & W^+_\mu \\ \underset{\sim}{Y}^\dagger_\mu & W^-_\mu & -\frac{1}{\sqrt{2}} A^3_\mu - \sqrt{\frac{3}{10}}\,A^0_\mu \end{pmatrix} \tag{2.14}$$

where A_8 is 3x3 gluon matrix with SU(3) colour indices, i.e. it is formed of a=1,2,...8 terms in (2.13) (which is not involved in our calculation), I is 3x3 unity matrix and:

$$\begin{aligned} \underset{\sim}{X}^\dagger_\mu &\equiv \left(X^+_{1\mu},\ X^+_{2\mu},\ X^+_{3\mu} \right) \\ \underset{\sim}{Y}^\dagger_\mu &\equiv \left(Y^+_{1\mu},\ Y^+_{2\mu},\ Y^+_{3\mu} \right) \end{aligned} \tag{2.15}$$

where:

$$\begin{aligned} X^\pm_i &\equiv \frac{1}{\sqrt{2}} \left(A^{7+2i} \mp i\, A^{8+2i} \right) \\ Y^\pm_i &\equiv \frac{1}{\sqrt{2}} \left(A^{13+2i} \mp i\, A^{14+2i} \right) \end{aligned} \qquad (i = 1,2,3) \tag{2.16}$$

are the gauge mesons with $Q=\pm\frac{4}{3}$, $T_3=\pm\frac{1}{2}$ and $Q=\pm\frac{1}{3}$, $T_3=\mp\frac{1}{2}$ respectively and

$$W^\pm \equiv \frac{1}{\sqrt{2}} \left(A^{21} \mp i\, A^{22} \right) \tag{2.17}$$

are the gauge mesons with $Q=\pm 1$, $T_3=\pm 1$, while:

$$A_3 \equiv A^{23} \quad ; \quad A_0 \equiv A^{24} \tag{2.18}$$

are the gauge mesons with $Q=0$, $T_3=0$.

Now we can find $\sin^2\theta_0$ using in (2.9) the gauge mesons (2.16), (2.17) and (2.18) of the adjoint representation (gluons do not contribute because they have $Q=T_3=0$):

$$\sin^2\theta_0 = \frac{2\cdot 3\cdot\left(\frac{1}{2}\right)^2 + 2\cdot 3\left(\frac{1}{2}\right)^2 + 2\cdot 1^2}{2\cdot 3\left(\frac{4}{3}\right)^2 + 2\cdot 3\left(\frac{1}{3}\right)^2 + 2\cdot 1^2} = \frac{3}{8} \tag{2.19}$$

The SU(5) group is spontaneously broken in two stages: first superstrongly down to SU(3)xSU(2)xU(1) by the adjoint 24 representation Ω and then weakly down to SU(3)xU(1) by the spinorial 5 representation ϕ (as in Weinberg-Salam model).

At the first stage, the most general Higgs potential (imposing a discrete $\Omega \to -\Omega$ symmetry for simplicity) is:

$$V(\Omega) = -\frac{1}{2}\mu^2 \mathrm{Tr}\,\Omega^2 + \frac{1}{4} a \left(\mathrm{Tr}\,\Omega^2\right)^2 + \frac{1}{2} b\, \mathrm{Tr}\,\Omega^4 \tag{2.20}$$

where:

$$\sqrt{2}\,\Omega = \sum_{a=1}^{24} I_a \Omega^a \tag{2.21}$$

or in a matrix form:

$$\Omega = \begin{pmatrix} \Omega_8 + \sqrt{\frac{2}{15}}\,\Omega_0 I & \underset{\sim}{\Omega}_X & \underset{\sim}{\Omega}_Y \\ \underset{\sim}{\Omega}_Y^{\dagger} & \frac{1}{\sqrt{2}}\Omega_3 - \sqrt{\frac{3}{10}}\,\Omega_0 & \Omega^+ \\ \underset{\sim}{\Omega}_X^{\dagger} & \Omega^- & -\frac{1}{\sqrt{2}}\Omega_3 - \sqrt{\frac{3}{10}}\,\Omega_0 \end{pmatrix} \tag{2.22}$$

where Ω_8 is 3x3 matrix formed of a=1,2,...8 terms in (2.21) (not involved in our calculation) and simmilary as for gauge mesons:

$$\begin{aligned} \underset{\sim}{\Omega}_X^{\dagger} &\equiv \left(\Omega_{X_1}^+,\ \Omega_{X_2}^+,\ \Omega_{X_3}^+\right) \\ \underset{\sim}{\Omega}_Y^{\dagger} &= \left(\Omega_{Y_1}^+,\ \Omega_{Y_2}^+,\ \Omega_{Y_3}^+\right) \end{aligned} \tag{2.23}$$

where:

$$\Omega^{\pm}_{X_i} \equiv \frac{1}{\sqrt{2}}\left(\Omega^{7+2i} \mp i\,\Omega^{8+2i}\right) \quad \left(Q = \pm\frac{4}{3}\ , T_3 = \pm\frac{1}{2}\right)$$

$$\Omega^{\pm}_{Y_i} \equiv \frac{1}{\sqrt{2}}\left(\Omega^{13+2i} \mp i\,\Omega^{14+2i}\right) \quad \left(Q = \pm\frac{1}{3}\ , T_3 = \mp\frac{1}{2}\right)$$

$$\Omega^{\pm} \equiv \frac{1}{\sqrt{2}}\left(\Omega^{21} \mp i\,\Omega^{22}\right) \quad \left(Q = \pm 1\ , T_3 = \pm 1\right) \tag{2.24}$$

$$\Omega_3 \equiv \Omega^{23}\ ; \quad \Omega_0 \equiv \Omega^{24} \quad \left(Q = 0\ , T_3 = 0\right)$$

The vacuum expectation value which gives the desired symmetry breaking SU(5) $\rightarrow$ SU(3)xSU(2)xU(1) is:

$$\langle\Omega\rangle = \begin{pmatrix} V & & & & \\ & V & & 0 & \\ & & V & & \\ & & & -\frac{3}{2}V & \\ & 0 & & & -\frac{3}{2}V \end{pmatrix} \tag{2.25}$$

where

$$\mu^2 = (15a + 7b)\frac{V^2}{2} \tag{2.26}$$

and the condition for minimum is $b > 0$, $a > (-7/15)b$.

At this stage the exotic gauge mesons X_i and Y_i get superlarge masses:

$$m_X^2 = m_Y^2 = \frac{25}{8} g_0^2 V^2 \tag{2.27}$$

as well as $\Omega^{\pm}$, Ω_3, Ω_0 Higgs scalars:

$$\begin{aligned} m^2_{\Omega^\pm} &= m^2_{\Omega_3} = 10\,b\,V^2 \\ m^2_{\Omega_0} &= (15a + 7b)V^2 = 2\mu^2 \end{aligned} \tag{2.28}$$

while the rest of particles remain massless.

If we now "switch on" the weak breaking ϕ we add to the Higgs potential

$$V(\phi) = -\frac{1}{2}\nu\,\phi^{+}\phi + \frac{1}{4}\lambda\,(\phi^{+}\phi)^2 \tag{2.29}$$

term, where

$$\phi = \begin{pmatrix} \phi_1^- \\ \phi_2^- \\ \phi_3^- \\ \phi^+ \\ \frac{1}{\sqrt{2}}(\phi_1^0 + i\phi_2^0) \end{pmatrix} \qquad (2.30)$$

and where $\phi_i^{\pm}$ have $Q=\pm\frac{1}{3}$, $T_3=0$, $\phi^{\pm}$ have $Q=\pm 1$, $T_3=\frac{1}{2}$ and $\phi_{1,2}^0$ have $Q=0$, $T_3=-\frac{1}{2}$ and the vacuum expectation value giving the SU(3)xSU(2)xU(1) $\longrightarrow$ SU(3)xU(1) breaking is:

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{v_0}{\sqrt{2}} \end{pmatrix} \qquad (2.31)$$

where

$$2\nu^2 = \lambda v_0^2 \qquad (2.32)$$

and

$$v_0 \ll V \qquad (2.33)$$

Now we must allow the couplings between Ω and ϕ so that the full Higgs potential includes the

$$V'(\Omega, \phi) = \alpha(\phi^+\phi)\, \mathrm{Tr}\,\Omega^2 + \beta\, \phi^+\Omega^2\phi \qquad (2.34)$$

piece respecting the gauge and the discrete $\Omega \to -\Omega$ symmetries which in return induces the change in the vacuum expectation value (2.25):

$$\langle \Omega \rangle \to \langle \Omega \rangle = \begin{pmatrix} v & & & & \\ & v & & & \\ & & v & & \\ & & & -\frac{v}{2}(3+\epsilon) & \\ & & & & -\frac{v}{2}(3-\epsilon) \end{pmatrix} \qquad (2.35)$$

where

$$\epsilon = \frac{3\beta}{20\theta}\frac{v_0^2}{v^2} + O\left[\left(\frac{v_0^2}{v^2}\right)^2\right]$$

$$\mu^2 = (15a + 7\theta)\frac{v^2}{2} + \left(\alpha + \frac{3}{10}\beta\right)v_0^2 + O\left(v_0^2\frac{v_0^2}{v^2}\right) \qquad (2.36)$$

$$\nu^2 = \frac{1}{2}\lambda v_0^2 + \left(15\alpha + \frac{9}{2}\beta - 3\epsilon\beta\right) + O\left(v_0^2\frac{v_0^2}{v^2}\right)$$

compared to (2.26) and (2.32).

Now the ordinary gauge mesons $W^{\pm}$ and

$$Z \equiv A_3\cos\theta_0 - A_0\sin\theta_0 \qquad (2.37)$$

get masses:

$$m_W^2 = \frac{1}{4}g_0^2 v_0^2 + O\left(v_0^2\frac{v_0^2}{v^2}\right)$$
$$m_Z^2 = \frac{1}{4}\frac{g_0^2 v_0^2}{\cos^2\theta_0} \qquad (2.38)$$

while

$$A \equiv A_3\sin\theta_0 + A_0\cos\theta_0 \qquad (2.39)$$

is photon.

From (2.38) we see easily that the relation (1.2) holds:

$$\Delta_0 \equiv \frac{m_W^{0\,2} - m_Z^{0\,2}\cos^2\theta_0}{m_W^{0\,2}} = O\left(\frac{v_0^2}{v^2}\right) \qquad (2.40)$$

where we explicitely put o index for the bare quantities in (2.38).

Also the exotic X_i and Y_i gauge mesons get the v_0 contribution to the masses so that (2.27) becomes:

$$m_X^2 = \frac{25}{8} g_0^2 v^2 (1 + \delta_1 x^2) + O(m_W^2 x^2)$$
$$m_Y^2 = \frac{25}{8} g_0^2 v^2 [1 + (1-\delta_1) x^2] + O(m_W^2 x^2) \qquad (2.41)$$

where

$$x \equiv \frac{\sqrt{2}\, v_0}{5v} + O\left(\frac{v_0^3}{v^3}\right) \qquad (2.42)$$

and

$$\delta_1 \equiv \frac{3\beta}{4e} \qquad (2.43)$$

From (2.38) and (2.41) we see that:

$$x = \frac{m_W}{m_X} \qquad (2.44)$$

and we will need from (2.41):

$$m_Y^2 - m_X^2 = m_W^2 (1 - 2\delta_1) + O(m_W^2 x^2) \qquad (2.45)$$

In the Higgs sector the mass matrix will consist of the following terms:

1) $$\Omega_{X_i}^{+} \; 0 \; \Omega_{X_i}^{-} \qquad (2.46)$$

(where 0 means zero and the summation over i=1,2,3 is understood)

2) $$(\Omega_{Y_i}^{+}, \phi_i^{+}) \begin{bmatrix} M x^2 & M x \\ M x & M(1 - 3\delta_1 x^2) \end{bmatrix} \begin{bmatrix} \Omega_{Y_i}^{-} \\ \phi_i^{-} \end{bmatrix} + O(m_W^2 x^2) \qquad (2.47)$$

where

$$M = -\frac{5}{3} e \delta_1 v^2 \qquad (2.48)$$

3) $$(\Omega^{+}, \phi^{+}) \begin{bmatrix} M_1\left(1+\frac{\delta_1}{3}x^2\right) & -M_1\delta_1 x \\ -M_1\delta_1 x & M_1\delta_1^2 x^2 \end{bmatrix} \begin{bmatrix} \Omega^{-} \\ \phi^{-} \end{bmatrix} + O(m_W^2 x^2) \tag{2.49}$$

where

$$M_1 \equiv 10\beta v^2 \tag{2.50}$$

4) $$(\Omega_3, \Omega_0, \phi_1^0) \frac{1}{2} \begin{bmatrix} M_1\left(1+\frac{\delta_1}{3}x^2\right) & 0 & M_1\delta_1 x \\ 0 & M_2 & M_2\delta_2 x \\ M_1\delta_1 x & M_2\delta_2 x & M_0 \end{bmatrix} \begin{bmatrix} \Omega_3 \\ \Omega_0 \\ \phi_1^0 \end{bmatrix} + O(m_W^2 x^2) \tag{2.51}$$

where

$$\begin{aligned} M_2 &\equiv (15\alpha + 7\beta) v^2 \\ M_0 &\equiv \frac{1}{2}\lambda v_0^2 \\ \delta_2 &\equiv \frac{\sqrt{15}}{2}\,\frac{3\beta + 10\alpha}{15\alpha + 7\beta} \end{aligned} \tag{2.52}$$

and finally

5) $$\frac{1}{2}\phi_2^0 \; 0 \; \phi_2^0 \tag{2.53}$$

From (2.46) and (2.47) we get the following Goldstone bosons (necessarily present in the t'Hooft's gauge which we employ):

$$\begin{aligned} G^{\pm}_{X_i} &= \Omega^{\pm}_{X_i} \\ G^{\pm}_{Y_i} &= \left(1 - \frac{x^2}{2}\right)\Omega^{\pm}_{Y_i} - x\phi_i^{\pm} + O(x^3) \end{aligned} \tag{2.54}$$

with charges $Q=\pm\frac{4}{3}$ and $Q=\pm\frac{1}{3}$ respectively while the orthogonal combination:

$$H^{\pm}_{Y_i} = \left(1 - \frac{x^2}{2}\right)\Phi^{\pm}_i + x\,\Omega_{Y_i}^{\pm} + O(x^3) \tag{2.58}$$

with charge $Q = \pm\frac{1}{3}$ has the mass:

$$m^2_{H_Y} = M\left[1 + (1 - 3\delta_1)x^2\right] + O(m_W^2 x^2) \tag{2.56}$$

From (2.49) the Goldstone boson is

$$G^{\pm} = \left(1 - \frac{\delta_1^2}{2}x^2\right)\Phi^{\pm} + \delta_1 x\,\Omega^{\pm} + O(x^3) \tag{2.57}$$

with charge $Q = \pm 1$ while the orthogonal combination:

$$H^{\pm} = \left(1 - \frac{\delta_1^2}{2}x^2\right)\Omega^{\pm} - \delta_1 x\,\Phi^{\pm} + O(x^3) \tag{2.58}$$

with charge $Q = \pm 1$ has the mass:

$$m^2_{H^{\pm}} = M_1\left[1 + \frac{1}{3}\delta_1(1 + 3\delta_1)x^2\right] + O(m_W^2 x^2) \tag{2.59}$$

From (2.51) we get the following neutral (Q=0) physical eigenstates:

$$H_3 = \left(1 - \frac{\delta_1^2}{2}x^2\right)\Omega_3 + O(x^2)\,\Omega_0 + \delta_1 x\,\Phi^0_1 + O(x^3)$$

$$H_0 = \left(1 - \frac{\delta_2^2}{2}x^2\right)\Omega_0 + O(x^2)\,\Omega_3 + \delta_2 x\,\Phi^0_1 + O(x^3)$$

$$\eta = \left[1 - \frac{\delta_1^2 + \delta_2^2}{2}x^2\right]\Phi^0_1 - \delta_1 x\,\Omega_3 - \delta_2 x\,\Omega_0 + O(x^3) \tag{2.60}$$

with masses:

$$m_{H_3}^2 = M_1 \left[1 + \frac{1}{3}\delta_1 (1+3\delta_1) x^2 \right] + O(m_W^2 x^2)$$

$$m_{H_0}^2 = M_2 \left(1 + \delta_2^2 x^2 \right) + O(m_W^2 x^2) \tag{2.61}$$

$$m_{\eta}^2 = M_0 - M_1 \delta_1^2 x^2 - M_2 \delta_2^2 x^2 + O(m_W^2 x^2)$$

while from (2.53)

$$G^0 = \phi_2^0 \tag{2.62}$$

is the neutral (Q=0) Goldstone boson.

We will also need the couplings of the light fermions with the exotic superheavy gauge mesons. The fermions are accomodated [11] in the spinorial $\underline{5}$ representation and the antisymmetric $\underline{10}$ representation (obtained from $\underline{5}\times\underline{5}=\underline{15}+\underline{10}$) of SU(5):

$$\underline{5} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ e^+ \\ \nu_e^c \end{pmatrix}_R ; \quad \underline{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{pmatrix}_L \tag{2.63}$$

where u_i, d_i are up and down quarks with i=1,2,3 colour indices, e^+, ν_e^c are positron and its neutrino, superscript c denotes charge conjugation, subscript L,R denotes left- and right- handed components of the fermion fields:

$$\psi_{L,R} \equiv \frac{1 \mp \gamma_5}{2} \psi \tag{2.64}$$

and we neglected the Cabbibo mixing for simplicity.

The relevant couplings are [15]:

$$\mathcal{L}_F = \frac{g_0}{\sqrt{2}} X_{i\mu}^{-} \left(\epsilon_{ijk} \bar{u}^c_{kL} \gamma^\mu u_{jL} + \bar{d}_i \gamma^\mu e^+ \right) + h.c$$

$$+ \frac{g_0}{\sqrt{2}} Y_{i\mu}^{-} \left(\epsilon_{ijk} \bar{u}^c_{kL} \gamma^\mu d_{jL} - \bar{u}_{iL} \gamma^\mu e^+_L + \bar{d}_{iR} \gamma^\mu \nu^c_R \right) + h.c \; . \tag{2.65}$$

where i,j,k=1,2,3 are colour indices.

III. Calculation and Comments

Let us now turn to the calculation itself. As we promised in the Introduction we will consider the one-loop corrections due to the heavy particles in SU(5) to the expression:

$$\Delta \equiv \frac{m_W^2 - m_Z^2 \cos^2\theta}{m_W^2} \tag{3.1}$$

which vanishes at the tree level in the limit of infinitely heavy particles:

$$\Delta_o \equiv \frac{m_W^{o\,2} - m_Z^{o\,2}\cos^2\theta}{m_W^{o\,2}} = O\left(\frac{m_L^2}{m_H^2}\right) \tag{3.2}$$

where L,H stand for light and heavy particles respectively and index o denotes the bare quantities) so it is calculable in that limit. (Note that Δ_o in (3.2) is exactly equal to zero in pure Weinberg-Salam model).

The connection between renormalized and unrenormalized (bare) quantities is:

$$\begin{aligned} m_W^2 &= m_W^{o2} + \delta m_W^2 \\ m_Z^2 &= m_Z^{o2} + \delta m_Z^2 \\ \cos^2\theta &= \cos^2\theta_o + \delta\cos^2\theta \end{aligned} \tag{3.3}$$

We have by substituting (3.3) into (3.1) and using (3.2):

$$\Delta = \Delta_o + \frac{\delta m_W^2}{m_W^2} - \frac{\delta m_Z^2}{m_Z^2} - \frac{\delta\cos^2\theta}{\cos^2\theta_o} \tag{3.4}$$

to the one-loop level. We will further neglect Δ_o and all quantities of order $O(m_L^2/m_H^2)$ since they mean the decoupling by definition.

The squared-mass shift δm^2 are determined from the polarization tensors (see (C.4) in the Appendix).

We define $\sin^2\Theta$ as:

$$\sin^2\Theta = \frac{e^2(0)}{g^2_{e\nu W}(m_W)} \tag{3.5}$$

where e(0) (called e afterwards) is the renormalized (physical) electromagnetic coupling constant (electric charge) evaluated at subtraction point $\mu^2=0$ ($\alpha(0) \equiv e^2(0)/4\pi = 1/137$) and $g_{e\nu W}(m_W)$ (called g afterwards) is the renormalized weak coupling constant between neutrino, electron and W meson evaluated at $\mu^2=m_W^2$. Notice that $g(m_W)$, the way we define it, can be determined from β or μ decay once m_W is known precisely [16]. In the renormalizable R gauge of t'Hooft, which we employ, the renormalized charge squared e^2 is given by:

$$e^2 = e_0^2 \frac{Z_2^{(e)^2} Z_3^{(A)}}{Z^2_{1eeA}} \tag{3.6}$$

where e_0 is the bare charge and the renormalization constants Z_3,Z_2 and Z_1 are defined in the Appendix C by (C.5),(C.73) and (C.85) respectively. Notice that in R gauge $Z_1 \neq Z_2$, i.e. the Ward identity does not hold [17] (although it holds for the parity-violating pieces [18]: $Z_1^5=Z_2^5$ defined in the Appendix C by (C.85) and (C.73) respectively; see (C.78) and (C.90)) and the A-Z mixing is to be included in calculation of Z_{1eeA} [17]. Using the definitions (C.5),(C.73) and (C.86) we can write (3.6) in the form:

$$e^2 = e_0^2 + \delta e^2 \tag{3.7}$$

where:

$$\delta e^2 = e_0^2 \left(-2 Z_{1eeA} + 2 Z_2^{(e)} + Z_3^{(A)} \right) \tag{3.8}$$

The renormalized coupling constant squared g^2 is given by:

$$g^2 = g_0^2 \frac{Z_{2L,R}^{(e)} Z_{2L,R}^{(\nu)} Z_3^{(W)}}{Z_{1e\nu W}^2} \tag{3.9}$$

where g_0 is the bare coupling constant and the renormalization constants Z_3, Z_2 and Z_1 are defined in the Appendix C by (C.5),(C.73) and (C.87) respectively. Using the definitions (C.5),(C.73) and (C.88) we can write (3.9) in the form:

$$g^2 = g_0^2 + \delta g^2 \tag{3.10}$$

where:

$$\delta g^2 = g_0^2 \left(-2 Z_{1e\nu W} + Z_{2L,R}^{(e)} + Z_{2L,R}^{(\nu)} + Z_3^{(W)} \right) \tag{3.11}$$

Now using (3.5),(3.6) (3.7),(3.9) and (3.10) we can express $\sin^2\Theta$ in the form:

$$\sin^2\Theta = \sin^2\Theta_0 + \delta \sin^2\Theta \tag{3.12}$$

where

$$\delta \sin^2\Theta = \sin^2\Theta_0 \left(\frac{\delta e^2}{e_0^2} - \frac{\delta g^2}{g_0^2} \right) \tag{3.13}$$

or using (3.8) and (3.11):

$$\delta \sin^2\Theta = \sin^2\Theta_0 \Big[2 \left(Z_{1e\nu W} - Z_{1eeA} \right) + \left(2 Z_2^{(e)} - Z_{2L,R}^{(e)} - Z_{2L,R}^{(\nu)} \right) + \left(Z_3^{(A)} - Z_3^{(W)} \right) \tag{3.14}$$

Finally using $\delta\cos^2\Theta = -\delta\sin^2\Theta$ we get from (3.14):

$$\frac{\delta \cos^2\theta}{\cos^2\theta_0} = -tg^2\theta_0 \left[2\left(Z_{1eOW} - Z_{1ee\Lambda}\right) + \left(2Z_2^{(e)} - Z_{2L,R}^{(e)} - Z_{2L,R}^{(\nu)}\right) + \left(Z_3^{(\Lambda)} - Z_3^{(W)}\right) \right] \tag{3.15}$$

Before we start with the results of our calculation let us make few remarks. The tree level relation:

$$m_W^{o2} - m_Z^{o2}\cos^2\theta_0 = O\left(m_L^2 \frac{m_L^2}{m_H^2}\right) \tag{3.16}$$

in (3.2) is the consequence of the SU(2) symmetry breaking by particular Higgs mechanism. The heavy particles that could correct this relation are in the SU(2) multiplets (with equal masses) before the weak symmetry breaking. Suppose that they, after the SU(2) symmetry breaking is "switched on", "approximately" remain in the SU(2) multiplets (i.e. their masses can be set equal up to the terms we neglect in our approximation). Then such particles "don't know" that the SU(2) symmetry is broken and therefore we should expect them not to correct Δ_0 in (3.2), i.e. their contribution to Δ in (3.4) should cancel.

Consider, for example, the q^2- dependent terms of the polarization tensors $\Pi_{\mu\nu}^{(W,Z,\Lambda)}(q)$ which have the dimension of mass squared. Since we put $q^2=m_M^2$ $(M=W,Z,\Lambda) \sim m_L^2$ the masses of the heavy particles mentioned above can be set equal (corrections are of order $O[m_L^2(m_L^2/m_H^2)]$) Therefore the contributions of these terms to $\delta m_{W,Z}^2$ and $Z_3^{(W,\Lambda)}$ in Δ should cancel. In order to check that explicitely let us write Δ in (3.4) in the form:

$$\Delta = \left(\frac{\delta m_W^2}{m_W^2} - \frac{\delta m_Z^2}{m_Z^2}\right)_0 + \left(\frac{\delta m_W^2}{m_W^2} - \frac{\delta m_Z^2}{m_Z^2}\right)_1 - \left[\left(\frac{\delta\cos^2\theta}{\cos^2\theta_0}\right)_0 - \left(\frac{\delta\cos^2\theta}{\cos^2\theta_0}\right)_1\right] \tag{3.17}$$

where subscripts o and 1, as explained in the Appendix C by (C.55), designate the terms calculated from $B_0^{(M)}$ and $B_1^{(M)}$, i.e. $q^2=0$ and $q^2\neq 0$ terms of $B^{(M)}(q^2)$ respectively. The quantities $\delta m_M^2/m_M^2\big|_{o,1}$ are already defined by (C.63) and (C.64). Using (3.15) we define:

$$\left.\frac{\delta\cos^2\theta}{\cos^2\theta_0}\right|_0 = -tg^2\theta_0\left[2\left(z_{1e\nu W}-z_{1ee A}\right)+\left(2z_Z^{(e)}-z_{Z L,R}^{(e)}-z_{Z L,R}^{(\nu)}\right)\right] \tag{3.18}$$

and

$$\left.\frac{\delta\cos^2\theta}{\cos^2\theta_0}\right|_1 = -tg^2\theta_0\left(z_3^{(A)}-z_3^{(W)}\right) \tag{3.19}$$

since z_1, z_2 constants in (3.18) are calculated for $q^2=0$ as discussed in C.III. and z_3 constants in (3.19) are calculated from $B_1^{(M)}$ (see (C.69)), i.e. $q^2\neq 0$ terms.

Now as we discussed above the "q^2-dependent" terms should cancel in Δ (3.17), i.e.:

$$\left(\frac{\delta m_W^2}{m_W^2}-\frac{\delta m_Z^2}{m_Z^2}\right)_1-\left(\frac{\delta\cos^2\theta}{\cos^2\theta_0}\right)_1=0 \tag{3.20}$$

Realy, from (C.66) and (C.68) we have:

$$\left(\frac{\delta m_W^2}{m_W^2}-\frac{\delta m_Z^2}{m_Z^2}\right)_1=\frac{g_0^2}{16\pi^2}\left(-\frac{1}{8}A_X+\frac{3}{8}A_{11}-\frac{1}{40}A_{11Y}-\frac{1}{4}\right) \tag{3.21}$$

while from (C.70),(C.71) and (3.19):

$$\left(\frac{\delta\cos^2\theta}{\cos^2\theta_0}\right)_1=\frac{g_0^2}{16\pi^2}\left(-\frac{1}{8}A_X+\frac{3}{8}A_{11}-\frac{1}{40}A_{11Y}-\frac{1}{4}\right) \tag{3.22}$$

where we used $tg^2\Theta_0=3/5$. From (3.21) and (3.22) we see that (3.20) is satisfied as we expected.

Let us calculate the rest of terms in (3.17). From (C.65) and (C.67) we have:

$$\left(\frac{\delta m_W^2}{m_W^2} - \frac{\delta m_Z^2}{m_Z^2}\right)_0 = -\frac{g_0^2}{16\pi^2}\frac{1}{6}A_X +$$

$$+\frac{g_0^2}{16\pi^2}\left\{\frac{3}{2m_X^2}\left[I_1(Y,H_Y) - I_1(X,H_Y)\right] +\right.$$

$$+ 6\left[I_2(Y,Y) - I_2(X,Y) + I_2(X,H_Y) - I_2(Y,H_Y)\right] \qquad (3.23)$$

$$+\frac{\delta_1^2}{4m_X^2}\left[2I_1(H^+,W) - I_1(H^+,\eta) - I_1(H^+,Z) + I_1(H_3,Z) - I_1(H_3,W)\right]$$

$$\left. + \frac{\delta_2^2}{4m_X^2}\left[I_1(H_0,Z) - I_1(H_0,W)\right]\right\}$$

and from (3.18),(C.77),(C.79),(C.84),(C.89) and (C.91) we have:

$$\left(\frac{\delta\cos^2\theta}{\cos^2\theta_0}\right)_0 = -\frac{g_0^2}{16\pi^2}\frac{1}{6}A_X \qquad (3.24)$$

We see that (3.24) cancels the first term of (3.23) in Δ (3.17). This can be explained simmilary as the cancelation of q^2-dependent terms (3.20). Namely, z_1 and z_2 constants in (3.18) are dimensionless and as discussed in the Appendix C.III and C.IV we put $m_X \simeq m_Y$ so the particles X,Y "don't know" that the SU(2) symmetry is broken. The corresponding contribution in (3.23) is the first term which comes from δm_Z^2 in (C.67) and is calculated from diagrams in Fig.2 with heavy particles Y, C_Y, G_Y of the same mass $m_Y \simeq m_X$ contributing to $B^{(Z)}(q^2)$. Again, the diagrams with one mass only "don't know" about SU(2) symmetry breaking and above cancelation is expected.

So finally we have from (3.17),(3.20),(3.23) and (3.24):

$$\Delta = \frac{g_0^2}{16\pi^2}\left\{ \frac{3}{2m_X^2}\left[I_1(Y,H_Y) - I_1(X,H_Y)\right] + \right.$$

$$+ 6\left[I_2(Y,Y) - I_2(X,Y) + I_2(X,H_Y) - I_2(Y,H_Y)\right] + \qquad (3.25)$$

$$+ \frac{\delta_1^2}{4m_X^2}\left[2I_1(H^+,W) - I_1(H^+,\gamma) - I_1(H^+,Z) + I_1(H_3,Z) - I_1(H_3,W)\right]$$

$$+ \frac{\delta_2^2}{4m_X^2}\left[I_1(H_0,Z) - I_1(H_0,W)\right]$$

The quantities I_1/m_X^2 (where I_1 is defined by (C.27)) of the first row and the I_2 quantities (defined by (C.38)) of the second row in (3.25) are dimensionless. Since $m_Y^2 - m_X^2 \sim m_W^2$ (see (2.45)) they cancel in the limit $m_W \to 0$ so for a finite m_W the correction is of order $O(m_W^2/m_X^2)$ which we neglect in our approximation.

The simmilar argument holds for the third and fourth rows in (3.25) where I_1 terms cancel in the limit $m_W \to 0$ and for finite m_W the corrections are again negligible of order $O(m_W^2/m_X^2)$.

Therefore we have for the one-loop heavy-particle contribution to   :

$$\Delta_H \equiv \left(\frac{m_W^2 - m_Z^2\cos^2\theta}{m_W^2}\right)_H = O\left(\frac{m_L^2}{m_H^2}\right) \qquad (3.26)$$

In conclusion, we have shown that the superheavy particles in the SU(5) theory do not manifest themselves at low energies in this particular relation which we believe is a more general result. We will discuss our findings in detail in the subsequent section.

As a byproduct of our calculation we can get the contribution of the heavy particles to $\sin^2\theta$ in the SU(2) theory. Namely, from the definition (3.14) and using (3.18),(3.19),(3.22) and (3.24) we get:

$$\left(\delta \sin^2\theta\right)_H = \frac{3}{8}\,\frac{g_0^2}{16\pi^2}\left(\frac{35}{72}A_X - \frac{5}{8}A_H + \frac{1}{24}A_{HY} + \frac{5}{12}\right) \tag{3.27}$$

where we put explicitely $\sin^2\Theta_0 = 3/8$. Using the definitions of A quantities (C.51) in (3.27) we have finally:

$$\left(\delta \sin^2\theta\right)_H = -\frac{3}{8}\,\frac{g_0^2}{16\pi^2}\left\{\frac{109}{12}\left(\frac{1}{n-4} + \frac{1}{2}\gamma - \ln 2\sqrt{\pi}\right) + \right.$$

$$\left. + \frac{105}{12}\ln m_X + \frac{5}{12}\ln m_H - \frac{1}{12}\ln m_{HY} + \frac{5}{12}\right\} \tag{3.28}$$

The coefficient of the infinite term is equal and opposite in sign to the same coefficient for the light particles [14] one-loop contribution to $\sin^2\Theta$ as it should be (i.e. the total one-loop contribution is finite since $\sin^2\Theta$ is calculable in the SU(5) theory). The noteworthy feature of this result is that the value for $\sin^2\Theta$ obtained in Ref.14 and here is exactly equal to the pioneering result of Georgi et al. [6] who used the renormalization-group methods and decoupling theorem. Therefore this result can be viewed as an additional confirmation that the heavy particles decouple at low energies.

IV. Decoupling Theorem and Heavy Particles at Low Energies

In the previous section we have seen how the superheavy particles in grand unified theories decouple at low energies. The reason, we believe [19], is the gauge invariance $SU(2)_L \times U(1)$ of the standard model which remains unbroken at the energies of order of the superheavy particles masses. Here we discuss in detail how the lack of the gauge invariance leads to the physical low-energy effects due to the heavy particles which would apparently invalidate the decoupling theorem.

The first discussion of the heavy-particle manifestation at low energies is, we believe, due to Veltman [12]. He has calculated the one-loop contribution of the heavy fermions to the relation $\Delta = (m_W^2 - m_Z^2 \cos^2\Theta)/m_W^2$ (the same relation that constitutes the main body of this work) and found in a manner simmilar to ours the effects proportional to heavy fermion masses. Let us discuss, briefly, his results. If we have two fermions in a doublet representation of the standard model, then one can show that they lead to the following expression for Δ (see formula (C.27) in this work):

$$\Delta = \frac{G_F}{4\sqrt{2}\pi^2}\left[\frac{m_1^2 m_2^2}{m_1^2 - m_2^2}\ln\frac{m_1^2}{m_2^2} - \frac{1}{2}(m_1^2 + m_2^2)\right] \propto I_1(1,2) \qquad (4.1)$$

The above expression is actually quite general. Namely, its form holds true for any two particles in a doublet, in particular if such particles are Higgs scalars as shown by Toussaint [9] and independently by us [9].

Obviously if we take a) $m_1 \ll m_2$ in (4.1) we get:

a) $$\Delta \simeq - \frac{G_F}{8\sqrt{2}\pi^2} m_2^2 \qquad (4.2)$$

i.e. the heavy fermions do not decouple at low energies since Δ increases with their masses. Now if the gauge model is of the sequential Weinberg-Salam type then without this doublet the theory would be renormalizable. Does it mean that the result (4.2) violates the decoupling theorem of Appelquist and Carazzone [5]? The answer would be yes if we interpret the decoupling theorem as originally stated: if the light subtheory is renormalizable then the effects of heavy particles vanish in the limit of their increasing mass. However, the above statement is obviously not completely precise since (4.2) violates it. The point is the following: if the heavy fermions are superheavy, i.e. their mass is many orders of magnitude bigger than m_W, then we expect them to get the mass at the level when the low energy gauge group is still unbroken. In such case b) $\Delta m \equiv m_1 - m_2 \ll m_i$ $(i=1,2)$. It is easy to see that (4.1) then reduces to the following expression (see (C.32)):

b)
$$\Delta \simeq - \frac{G_F}{24\sqrt{2}\pi^2} \frac{(\Delta m)^2}{m_1^2} \tag{4.3}$$

which means decoupling. What happened then in the case a)? Well, in this case the mass of a heavy fermion must be of order m_W since otherwise $SU(2)_L xU(1)$ would be too strongly broken. In other words, when heavy fermion gets mass, $SU(2)_L xU(1)$ gets also broken - there is no light gauge invariance preserved which would make the effects small as in (4.3).

Alternatively in the case a) we could consider particle with mass m_2 to be "heavy" and the other one with mass m_1 to belong to the light sector. Then, of course, the light subsector is not renormalizable by itself and so the decoupling theorem is not applicable anyway so we have the heavy-particle effects.

Let us illustrate the above point in more detail. Let us concentrate on a four-quark Weinberg-Salam model. We will consider up and down quarks as light and charm and strange quarks as heavy. The theory with only up and down quarks is, of course, renormalizable. A particularly interesting example of the heavy-particle effect was devised by Collins et al. [9] who found that the axial isoscalar current $\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d$ (which is forbidden at the tree level) gets a contribution of the form:

$$J_\mu^5 \propto \alpha_s^2 \ln \frac{m_c}{m_s} \tag{4.4}$$

which indicates the strong dependance of the heavy-quark masses (α_s is the strong coupling constant). Again in this example $SU(2)_L xU(1)$ is broken at the level of m_c and m_s or otherwise we would have $m_c - m_s \ll m_c$ which we know is not true (or if it is, hypothetically, we would expand $\ln(m_c/m_s) = \ln(1+\Delta m/m_s) \simeq \Delta m/m_s$ and get decoupling simmilary to (4.3)). What happens is the following: now we have the hierarchy of masses: $m_c \gg m_s \gg m_d \sim m_u$. But obviously then there is an intermediate stage where we can ignore m_s compared to m_c. At this stage m_s would belong to a light sector and as we know , the light subtheory would not be renormalizable. That prompts the following necessary requirement for the decoupling of heavy particles: <u>at each stage of symmetry breaking the light subtheory has to be renormalizable</u>. In other words, whenever we isolate the heavy sector of the theory, the gauge invariance of the remaining light sector should not be broken. Then in any weak multiplet $\Delta m \ll m$ and so the heavy particles decouple as explained above. This is exactly what happened in the case of superheavy particles in SU(5) as discussed in Chapter III.

It is obvious that the requirement of the light unbroken gauge invariance is necessary. We were not able to show that it is also sufficient, although the previous examples and our new calculation indicate so. The problem stems from the fact that in such cases $\Delta m_H \sim m_L$ (L,H stand for light ang heavy particles respectively) where Δm_H is the mass diference of heavy particles in some weak isomultiplet. We do not see why the expressions which are of order m_L do not pick up any contribution of order Δm_H but rather negligible contributions of order $(\Delta m_H)^2/m_H$. Since the decoupling theorem was proved rigorously only in the simple case of massive fermions coupled to a non-Abelian massless gauge field in which case such a problem will not appear since $m_L=0$ to all orders in perturbation theory, we believe that a more general proof should be offered which would guarantee the decoupling of heavy particles when two or more mass scales exist as in the case of the spontaneously-broken gauge theories.

Let us comment on few other examples of heavy-particle effects discussed in literature. One is the decay of Higgs boson into two gluons discussed by Wilczek [20]. The process is possible at the one-loop level through the quark loop. He observed that the process is independent of the internal quark mass m_q. Namely, $1/m_q$ from the internal propagation gets canceled by the Yukawa coupling which in the standard model is gm_q/m_W. Again we have the contradiction with the decoupling theorem. However the explanation is the one given above: the Yukawa coupling is proportional to m_q only if the heavy quark gets the mass from the same Higgs scalar that breaks $SU(2)_L xU(1)$ symmetry. In this case, there is again no unbroken light gauge invariance and the decoupling theorem does not apply. If on the other hand the quark is superheavy, i.e. $m_q \gg m_W$ so that W is still massless when the quark gets the mass then $1/m_q$ will not get killed and such superheavy quarks then decouple at low energies.

Simmilar example is provided by the calculation [21] of the axion mass which is proportional to the number of flavors, so that the heavy quarks contribute as mush as light ones. Again the explanation for the decoupling is exactly the same as in the above example (it has nothing to do with the axial anomaly as is commonly thought).

Finally, we would like to emphasize that the examples discussed above are genuine of heavy particles which can not be absorbed in the renormalization of the coupling constants of the theory. It is maybe worth noticing that the effects of heavy particles to the coupling-constant renormalization may sometimes be physical the best example being provided by $\sin^2\Theta$ in grand unified theories. The same will be true whenever some couplings are fixed due to the symmetry of the theory.

V. Summary

Let us now summarize our results. Our main achievement is the explicit computation of superheavy-particle effects on a low-energy relation which is directly experimentally testable. As was expected, but not proved before, they decouple, giving us practically vanishing contribution at the ordinary energies much below their thresholds. This result is somewhat negative since their low energy-effects will be only welcomed. Namely, the main problem in the unified gauge theories stems from the fact that the superheavy particles which the theories predict are not observable due to their decoupling. The possible exception seems to be the proton decay [22], which although extremely slow, appears to be in the reach of experimental verification [23]. We would definitely prefer a situation in which the superheavy particles give also nonegligible contributions in the usual weak or strong interactions which would enable us to test the grand unified theories more precisely.

Admitedly , we have checked only a particular relation. A more general result would be desirable since the decoupling theorem was proved only in the case of one mass scale with unbroken gauge symmetry. As we discussed at length in the previous section the governing principle seems to be the gauge invariance of the light subsector of the theory. When it is preserved at the energies of the heavy-particle masses then the heavy particles decouple whereas in the opposite case they do not decouple. That explains why the superheavy particles in the grand unified theories decouple: at the unification scale we can ignore the breaking of the low-energy gauge group $SU(3)_c \times SU(2)_L \times U(1)$. Therefore the superheavy-particle mass differences in the weak isomultiplets are extremely small ($\sim m_W$) compared to the masses itself which in turn is responsible for their decoupling.

The particular relation Δ which we analyzed was chosen for its high level of divergence of individual contributions. That makes the effect of superheavy particles rather likely so we believe that our results have more general nature and that they confirm the decoupling of superheavy particles in any grand unified theory (the SU(5) theory was chosen for its simplicity). The last statement still needs a proof.

Our result probably did not come as a suprise to the reader but we believe that the more formal arguments [5] are not completely justified in a complicated gauge theory with two widely different mass scales. If anything, our calculation should serve the purpose of demonstrating explicitely the renormalizability of such a theory. Also, the computation of the heavy-particle contribution to $\sin^2\Theta$ completes the previous program of calculating $\sin^2\Theta$ in perturbation theory. The result is completely in agreement with the pioneering work of Georgi, Quinn and Weinberg [6] who used the renormalization-group arguments and assumed the decoupling of the superheavy particles in the renormalization-group equations. Our result can be viewed as an useful check and confirmation of such a program.

APPENDIX

A. SU(5) Lie Algebra

$$\left[\tfrac{1}{2}\lambda_a , \tfrac{1}{2}\lambda_b\right] = i f_{abc}\,\tfrac{1}{2}\lambda_c \qquad \text{(A.1)}$$

$$(a, b, c = 1, 2, \ldots 24)$$

$$\lambda_1 = \begin{bmatrix} 0&1&0&0&0\\ 1&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{bmatrix} \quad \lambda_2 = \begin{bmatrix} 0&-i&0&0&0\\ i&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{bmatrix} \quad \lambda_3 = \begin{bmatrix} 1&0&0&0&0\\ 0&-1&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{bmatrix}$$

$$\lambda_4 = \begin{bmatrix} 0&0&1&0&0\\ 0&0&0&0&0\\ 1&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{bmatrix} \quad \lambda_5 = \begin{bmatrix} 0&0&-i&0&0\\ 0&0&0&0&0\\ i&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{bmatrix}$$

$$\lambda_6 = \begin{bmatrix} 0&0&0&0&0\\ 0&0&1&0&0\\ 0&1&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{bmatrix} \quad \lambda_7 = \begin{bmatrix} 0&0&0&0&0\\ 0&0&-i&0&0\\ 0&i&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0 \end{bmatrix} \qquad \text{(A.2)}$$

$$\lambda_8 = \frac{1}{\sqrt{3}}\begin{bmatrix} 1&0&0&0&0\\ 0&1&0&0&0\\ 0&0&-2&0&0\\ 0&0&0&0&0 \end{bmatrix}$$

$$\lambda_9 = \begin{bmatrix} 0&0&0&1&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 1&0&0&0&0\\ 0&0&0&0&0 \end{bmatrix} \quad \lambda_{10} = \begin{bmatrix} 0&0&0&i&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ -i&0&0&0&0\\ 0&0&0&0&0 \end{bmatrix}$$

$$\lambda_{11} = \begin{bmatrix} 0&0&0&0&0\\ 0&0&0&1&0\\ 0&0&0&0&0\\ 0&1&0&0&0\\ 0&0&0&0&0 \end{bmatrix} \quad \lambda_{12} = \begin{bmatrix} 0&0&0&0&0\\ 0&0&0&i&0\\ 0&0&0&0&0\\ 0&-i&0&0&0\\ 0&0&0&0&0 \end{bmatrix}$$

$$\lambda_{13} = \begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&1&0\\ 0&0&1&0&0\\ 0&0&0&0&0 \end{pmatrix} \qquad \lambda_{14} = \begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&i&0\\ 0&0&-i&0&0\\ 0&0&0&0&0 \end{pmatrix}$$

$$\lambda_{15} = \begin{pmatrix} 0&0&0&0&1\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 1&0&0&0&0 \end{pmatrix} \qquad \lambda_{16} = \begin{pmatrix} 0&0&0&0&i\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ -i&0&0&0&0 \end{pmatrix}$$

$$\lambda_{17} = \begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&1\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&1&0&0&0 \end{pmatrix} \qquad \lambda_{18} = \begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&i\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&-i&0&0&0 \end{pmatrix} \qquad (A.2)$$

$$\lambda_{19} = \begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&1\\ 0&0&0&0&0\\ 0&0&1&0&0 \end{pmatrix} \qquad \lambda_{20} = \begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&i\\ 0&0&0&0&0\\ 0&0&-i&0&0 \end{pmatrix}$$

$$\lambda_{21} = \begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&1\\ 0&0&0&1&0 \end{pmatrix} \qquad \lambda_{22} = \begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&-i\\ 0&0&0&i&0 \end{pmatrix}$$

$$\lambda_{23} = \begin{pmatrix} 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&0&0\\ 0&0&0&1&0\\ 0&0&0&0&-1 \end{pmatrix}$$

$$\lambda_{24} = \frac{s}{c} \begin{pmatrix} 2/3&0&0&0&0\\ 0&2/3&0&0&0\\ 0&0&2/3&0&0\\ 0&0&0&-1&0\\ 0&0&0&0&-1 \end{pmatrix}$$

where:

$$s \equiv \sin\Theta_0 = \sqrt{\frac{3}{8}} \quad ; \quad c \equiv \cos\Theta_0 = \sqrt{\frac{5}{8}} \tag{A.3}$$

The generators of the SU(5) group:

$$I_a = \frac{1}{2}\lambda_a \qquad (a = 1, 2, \ldots 24) \tag{A.4}$$

The third component of isospin is:

$$T_3 = I_{23} = \frac{1}{2}\lambda_{23} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & -1/2 \end{pmatrix} \tag{A.5}$$

The hypercharge is:

$$\frac{Y}{2} = -\frac{c}{s} I_{24} = -\frac{1}{2}\frac{c}{s}\lambda_{24} = \begin{pmatrix} -1/3 & 0 & 0 & 0 & 0 \\ 0 & -1/3 & 0 & 0 & 0 \\ 0 & 0 & -1/3 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 1/2 \end{pmatrix} \tag{A.6}$$

The charge is:

$$Q = T_3 + \frac{Y}{2} = \begin{pmatrix} -1/3 & 0 & 0 & 0 & 0 \\ 0 & -1/3 & 0 & 0 & 0 \\ 0 & 0 & -1/3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{A.7}$$

Structure constants f_{abc} (a,b,c=1,2,...24) different from zero:

$$f_{123}=f_{(21)(22)(23)}=1$$

$$f_{147}=-f_{156}=f_{19(12)}=-f_{1(10)(11)}=f_{1(15)(18)}=-f_{1(16)(17)}=f_{246}=f_{257}=$$
$$=f_{29(11)}=f_{2(10)(12)}=f_{2(15)(17)}=f_{2(16)(18)}=f_{345}=f_{39(10)}=f_{3(15)(16)}=$$
$$=-f_{367}=-f_{3(11)(12)}=-f_{3(17)(18)}=f_{49(14)}=-f_{4(10)(13)}=f_{4(15)(20)}=$$
$$=-f_{4(16)(19)}=f_{59(13)}=f_{5(10)(14)}=f_{5(15)(19)}=f_{5(16)(20)}=f_{6(11)(14)}=$$
$$=-f_{6(12)(13)}=f_{6(17)(20)}=-f_{6(18)(19)}=f_{7(11)(13)}=f_{7(12)(14)}=f_{7(17)(19)}=$$
$$=f_{7(18)(20)}=f_{9(15)(22)}=-f_{9(16)(21)}=-f_{9(10)(23)}=f_{(10)(15)(21)}=$$
$$=f_{(10)(16)(22)}=-f_{(11)(12)(23)}=f_{(11)(17(22)}=-f_{(11)(18(21)}=f_{(12)(18)(22)}=$$
$$=f_{(12)(17)(21)}=-f_{(13)(14)(23)}=f_{(13)(19)(22)}=-f_{(13)(20)(21)}=$$
$$=f_{(14)(20)(22)}=f_{(14)(19)(21)}=f_{(15)(16)(23)}=f_{(17)(18)(23)}=$$
$$=f_{(19)(20)(23)}=1/2$$

$$f_{458}=f_{678}=\sqrt{3}/2 \tag{A.8}$$

$$f_{89(10)}=f_{8(11)(12)}=f_{8(15)(16)}=f_{8(17)(18)}=1/2\sqrt{3}$$

$$f_{8(13)(14)}=f_{8(19)(20)}=-1/\sqrt{3}$$

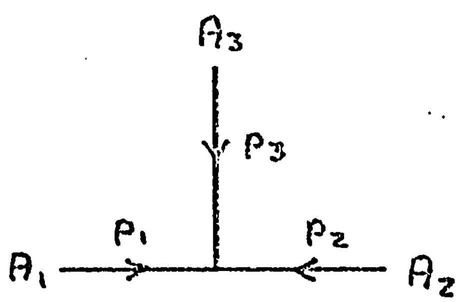
$$f_{9(10)(11)}=f_{(11)(12)(24)}=f_{(13)(14)(24)}=f_{(15)(16)(24)}=f_{(17)(18)(24)}=$$
$$=f_{(19)(20)(24)}=(1/2)(c/s)$$

where $c/s = \sqrt{5/3}$ from (A.3)

B. Feynman Rules

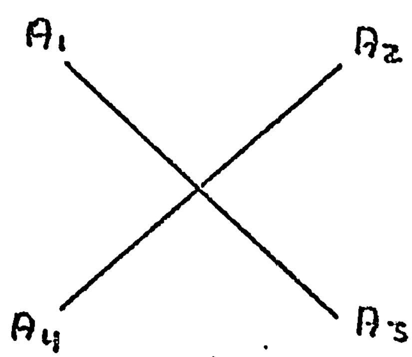
Notation:

1. Colour indices: i=1,2,3 ; ϵ_{ijk} - totally antisymmetric tensor = ±1.
2. Lorentz indices: $\alpha,\beta,\gamma,\ldots\mu,\nu,\ldots=0,1,2,3$; $g_{\mu\nu} \sim \begin{pmatrix} 1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$
3. $s \equiv \sin\Theta_0 = \sqrt{3/8}$, $c \equiv \cos\Theta_0 = \sqrt{5/8}$.
4. $x \equiv m_W/m_X$.
5. $\Gamma_{\alpha\beta\gamma}(p_1,p_2,p_3) \equiv (p_1-p_2)_\gamma\, g_{\alpha\beta} + (p_2-p_3)_\alpha\, g_{\beta\gamma} + (p_3-p_1)_\beta\, g_{\alpha\gamma}$.
6. $\Delta_{\alpha\beta\gamma\delta} \equiv g_{\alpha\beta}\, g_{\gamma\delta} - 1/2(g_{\alpha\gamma}\, g_{\beta\delta} + g_{\alpha\delta}\, g_{\beta\gamma})$.
7. $\delta_1 = 3\beta/4b$; $\delta_2 = (\sqrt{15}/2)(3\beta + 10\alpha/15a+7b)$.
8. $L = (1/2)(1-\gamma_5)$, $R = (1/2)(1+\gamma_5)$.
9. Trilinear couplings:



$\equiv \quad -ig_0 a_1 a_2 a_2$

10. Quartic couplings:



$\equiv \quad -2ig_0^2 b_1 b_2 b_3$

I. Trilinear Couplings

Table I.1

Heavy gauge mesons $X^{\pm}_{i}$ $(Q=\pm\frac{4}{3})$ and $Y^{\pm}_{i}$ $(Q=\pm\frac{1}{3})$ (and corresponding ghosts)

$a_1 = \pm\,\Gamma_{\alpha\beta\mu}(p_1, p_2, p_3)$ $\quad (a_1 = p_{1\mu})$				
A_1	A_2	A_3	a_2	a_3
$X^{\pm}_{i\alpha}$ (C_{X_i})	$Y^{\mp}_{i\beta}$ (C_{Y_i})	$W^{\mp}_{\mu}$	$\frac{1}{\sqrt{2}}$	1
$X^{\pm}_{i\alpha}$ (C_{X_i})	$X^{\mp}_{i\beta}$ (C_{X_i})	Z_{μ}	$\frac{1}{c}$	0
$Y^{\pm}_{i\alpha}$ (C_{Y_i})	$Y^{\mp}_{i\beta}$ (C_{Y_i})			$-c^2$
$X^{\pm}_{i\alpha}$ (C_{X_i})	$X^{\mp}_{i\beta}$ (C_{X_i})	A_{μ}	s	$\frac{4}{3}$
$Y^{\pm}_{i\alpha}$ (C_{Y_i})	$Y^{\mp}_{i\beta}$ (C_{Y_i})			$\frac{1}{3}$

Table I.2

"Heavy" Goldstone bosons $G^{\pm}_{X_i}(Q=\pm\frac{4}{3})$, $G^{\pm}_{Y_i}(Q=\pm\frac{1}{3})$ and heavy Higgs scalars $H^{\pm}_{Y_i}(Q=\pm\frac{1}{3})$.

$a_1 = \pm(p_2-p_1)_\mu$				
A_1	A_2	A_3	a_2	a_3
$G^{\pm}_{X_i}$	$G^{\mp}_{Y_i}$	$W^{\mp}_\mu$	$\frac{1}{\sqrt{2}}$	$1-\frac{1}{2}x^2$
$G^{\pm}_{X_i}$	$H^{\mp}_{Y_i}$			x
$G^{\pm}_{X_i}$	$G^{\mp}_{X_i}$	Z_μ	$\frac{1}{c}$	0
$G^{\pm}_{Y_i}$	$G^{\mp}_{Y_i}$			$-c^2+\frac{1}{2}x^2$
$H^{\pm}_{Y_i}$	$H^{\mp}_{Y_i}$			$-\frac{1}{3}s^2-\frac{1}{2}x^2$
$G^{\pm}_{Y_i}$	$H^{\mp}_{Y_i}$			$-\frac{1}{2}x$
$G^{\pm}_{X_i}$	$G^{\mp}_{X_i}$	A_μ	s	$\frac{4}{3}$
$G^{\pm}_{Y_i}$	$G^{\mp}_{Y_i}$			$\frac{1}{3}$
$H^{\pm}_{Y_i}$	$H^{\mp}_{Y_i}$			$\frac{1}{3}$

Table I.3

Heavy gauge mesons X_i and Y_i mixed with "heavy" Goldstone bosons G_{X_i}, G_{Y_i} and heavy Higgs scalars H_{Y_i}.

$a_1 = -g_{\mu\nu}$				
A_1	A_2	A_3	a_2	a_3
$G_{X_i}^{\pm}$	$Y_{i\mu}^{\mp}$	$W_\nu^{\mp}$	$\frac{1}{\sqrt{2}}$	$m_X(1-2\delta_1 x^2)$
$G_{Y_i}^{\mp}$	$X_{i\mu}^{\pm}$			$m_Y[1+2x^2(\delta_1-1)]$
$H_{Y_i}^{\mp}$	$X_{i\mu}^{\pm}$			$2m_X X$
$G_{X_i}^{\pm}$	$X_{i\mu}^{\mp}$	Z_ν	$\frac{1}{c}$	0
$G_{Y_i}^{\pm}$	$Y_{i\mu}^{\mp}$			$m_Y(-c^2+x^2)$
$H_{Y_i}^{\pm}$	$Y_{i\mu}^{\mp}$			$m_X X$
$G_{X_i}^{\pm}$	$X_{i\mu}^{\mp}$	A_ν	s	$\frac{4}{3} m_X$
$G_{Y_i}^{\pm}$	$Y_{i\mu}^{\mp}$			$\frac{1}{3} m_Y$

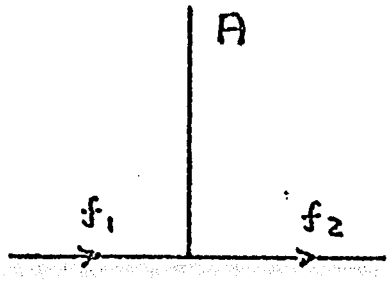
Table I.4

"Light" Goldstone bosons $G^{\pm}(Q=\pm 1)$, $G^0(Q=0)$, heavy $H^{\pm}(Q=\pm 1)$, $H_3(Q=0)$, $H_0(Q=0)$ and light $\eta\,(Q=0)$ Higgs scalars.

$a_1 = \pm(p_2 - p_1)_\mu$				
A_1	A_2	A_3	a_2	a_3
$G^{\pm}$	η	$W_\mu^{\mp}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}\left[1 + x^2\left(\delta_1^2 - \frac{1}{2}\delta_2^2\right)\right]$
$G^{\pm}$	H_3			$-\frac{1}{\sqrt{2}}\delta_1 x$
$G^{\pm}$	H_0			$\frac{1}{\sqrt{2}}\delta_2 x$
$G^{\pm}$	G_0			$\mp\frac{i}{\sqrt{2}}\left(1 - \frac{1}{2}\delta_1^2 x^2\right)$
$H^{\pm}$	η			$\frac{1}{\sqrt{2}}\delta_1 x$
$H^{\pm}$	H_3			$-\sqrt{2}\left(1 - \frac{1}{2}\delta_1^2 x^2\right)$
$H^{\pm}$	H_0			0
$H^{\pm}$	G_0			$\pm\frac{i}{\sqrt{2}}\delta_1 x$
$G^{\pm}$	$G^{\mp}$	Z_ν	$\frac{1}{c}$	$-\frac{1}{2} + c^2 + \frac{1}{2}\delta_1^2 x^2$
$H^{\pm}$	$H^{\mp}$			$c^2 - \frac{1}{2}\delta_1^2 x^2$
$G^{\pm}$	$H^{\mp}$			$\frac{1}{2}\delta_1 x$
η	G_0			$\pm\frac{i}{2}\left[1 - \frac{1}{2}x^2\left(\delta_1^2 + \delta_2^2\right)\right]$
H_3	G_0			$\pm\frac{i}{2}\delta_1 x$
H_0	G_0			$\pm\frac{i}{2}\delta_2 x$
$G^{\pm}$	$G^{\mp}$	A_ν	s	1
$H^{\pm}$	$H^{\mp}$			1

Table 1.5

Fermions with heavy gauge mesons X_i and Y_i.



$$\equiv -ig_0 a_1 a_2 a_3$$

$a_1 = -\gamma_\mu$				
f_1	f_2	A	a_2	a_3
u_j	u_k^c	$X_{i\mu}^-$	$\frac{1}{\sqrt{2}}$	$\epsilon_{ijk} L$
e^+	d_i			1
d_j	u_k^c	$Y_{i\mu}^-$	$\frac{1}{\sqrt{2}}$	$\epsilon_{ijk} L$
e^+	u_i			$-L$
ν^c	d_i			R

II. Quartic Couplings

Table II.1

Heavy gauge mesons X_i and Y_i.

A_1	A_2	A_3	A_4	B_2	B_3
$B_1 = (\Delta_{\alpha\beta\gamma\delta} + \Delta_{\alpha\delta\beta\gamma})$					
$X^{\pm}_{i\alpha}$	$X^{\mp}_{i\beta}$	$W^{\pm}_{\gamma}$	$W^{\mp}_{\delta}$	$\left(\frac{1}{\sqrt{2}}\right)^2$	1
$B_1 = (\Delta_{\alpha\beta\gamma\delta} + \Delta_{\alpha\gamma\beta\delta})$					
$Y^{\pm}_{i\alpha}$	$Y^{\mp}_{i\beta}$	$W^{\pm}_{\gamma}$	$W^{\mp}_{\delta}$	$\left(\frac{1}{\sqrt{2}}\right)^2$	1
$B_1 = \Delta_{\alpha\beta\gamma\delta}$					
$X^{\pm}_{i\alpha}$	$X^{\mp}_{i\beta}$	Z_{γ}	Z_{δ}	$\left(\frac{1}{c}\right)^2$	0
$Y^{\pm}_{i\alpha}$	$Y^{\mp}_{i\beta}$				$(-c^2)^2$
$X^{\pm}_{i\alpha}$	$X^{\mp}_{i\beta}$	A_{γ}	A_{δ}	s^2	$\left(\frac{4}{3}\right)^2$
$Y^{\pm}_{i\alpha}$	$Y^{\mp}_{i\beta}$				$\left(\frac{1}{3}\right)^2$
$X^{\pm}_{i\alpha}$	$X^{\mp}_{i\beta}$	A_{γ}	Z_{δ}	$\frac{s}{c}$	0
$Y^{\pm}_{i\alpha}$	$Y^{\mp}_{i\beta}$				$-\frac{1}{3}c^2$

Table II.2

"Heavy" Goldstone bosons G_{X_i}, G_{Y_i} and Higgs scalars H_{Y_i}.

A_1 A_2	A_3 A_4	B_2	B_3
		$B_1 = -g_{\mu\nu}$	
$G^{\pm}_{X_i}$ $G^{\mp}_{X_i}$	$W^{\mp}_{\nu}$ $W^{\pm}_{\mu}$	$\left(\frac{1}{\sqrt{2}}\right)^2$	$\frac{1}{2}$
$G^{\pm}_{Y_i}$ $G^{\mp}_{Y_i}$			$\frac{1}{2}(1-x^2)$
$H^{\pm}_{Y_i}$ $H^{\mp}_{Y_i}$			$\frac{1}{2}x^2$
$G^{\pm}_{X_i}$ $G^{\mp}_{X_i}$	Z_{ν} Z_{μ}	$\left(\frac{1}{c}\right)^2$	0
$G^{\pm}_{Y_i}$ $G^{\mp}_{Y_i}$			$c^2 - s^2x^2$
$H^{\pm}_{Y_i}$ $H^{\mp}_{Y_i}$			$\left(\frac{1}{3}s^2\right)^2 + s^2x^2$
$G^{\pm}_{X_i}$ $G^{\mp}_{X_i}$	A_{ν} A_{μ}	s^2	$\left(\frac{4}{3}\right)^2$
$G^{\pm}_{Y_i}$ $G^{\mp}_{Y_i}$			$\left(\frac{1}{3}\right)^2$
$H^{\pm}_{Y_i}$ $H^{\mp}_{Y_i}$			$\left(\frac{1}{3}\right)^2$
$G^{\pm}_{X_i}$ $G^{\mp}_{X_i}$	Z_{ν} A_{μ}	$\frac{s}{c}$	0
$G^{\pm}_{Y_i}$ $G^{\mp}_{Y_i}$			$-\frac{1}{3}c^2$
$H^{\pm}_{Y_i}$ $H^{\mp}_{Y_i}$			$-\frac{1}{9}s^2$

Table II.3

"Light" Goldstone bosons $G^{\pm}$, G^0, heavy $H^{\pm}$, H_3, H_0 and light η Higgs scalars.

$b_1 = -g_{\mu\nu}$					
A_1	A_2	A_3	A_4	b_2	b_3
$G^{\pm}$	$G^{\mp}$	$W^{\mp}_{\nu}$	$W^{\pm}_{\mu}$	$\left(\frac{1}{\sqrt{2}}\right)^2$	$\frac{1}{2}(1+\delta_1^2 x^2)$
$H^{\pm}$	$H^{\mp}$				$1-\frac{1}{2}\delta_1^2 x^2$
η	η				$\frac{1}{2}\left[1+x^2(3\delta_1^2-\delta_2^2)\right]$
H_3	H_3				$2(1-\frac{3}{4}\delta_1^2 x^2)$
H_0	H_0				$\frac{1}{2}\delta_2^2 x^2$
G_0	G_0				$\frac{1}{2}$
$G^{\pm}$	$G^{\mp}$	A_{ν}	A_{μ}	s^2	1
$H^{\pm}$	$H^{\mp}$				1
$G^{\pm}$	$G^{\mp}$	Z_{ν}	Z_{μ}	$\left(\frac{1}{c}\right)^2$	$(-\frac{1}{2}+c^2)^2+s^2\delta_1^2 x^2$
$H^{\pm}$	$H^{\mp}$				$c^2-s^2\delta_1^2 x^2$
η	η				$\frac{1}{4}\left[1-x^2(\delta_1^2+\delta_2^2)\right]$
H_3	H_3				$\frac{1}{4}\delta_1^2 x^2$
H_0	H_0				$\frac{1}{4}\delta_2^2 x^2$
G_0	G_0				$\frac{1}{4}$
$G^{\pm}$	$G^{\mp}$	Z_{ν}	A_{μ}	$\frac{s}{c}$	$-\frac{1}{2}+c^2+\frac{1}{2}\delta_1^2 x^2$
$H^{\pm}$	$H^{\mp}$				$c^2-\frac{1}{2}\delta_1^2 x^2$

C. Calculation of Self-Energy Functions and Vertex Functions

C.I. Introduction

Consider a theory with two widely different mass scales $m_L \ll m_H$ (L stands for light and H stands for heavy): for example the SU(5) theory with $m_L \sim v_0$ and $m_H \sim v$ where $v_0 \ll v$. We will be calculating the dimensionless quantities A for which we have the expansion of type:

$$A \sim a_0 + a_1 \frac{m_L^2}{m_H^2} + \dots \tag{C.1}$$

and the quantities B with the dimension of mass have the expansion of type:

$$B \sim m_H^2 \left[b_0 + b_1 \frac{m_L^2}{m_H^2} + b_2 \left(\frac{m_L^2}{m_H^2}\right)^2 + \dots \right] \tag{C.2}$$

We will calculate only a_0, b_0 and b_1 terms of the above quantities since their presence or absence means coupling or decoupling of heavy particles in the sense of the Appelquist and Carazzone's theorem. In order to avoid the proliferation of symbols $O[m_L^2(m_L^2/m_H^2)]$ and $O(m_L^2/m_H^2)$ which mean decoupling by definition, its presence should be understood in the subsequent calculations.

C.II. Vector-Meson Polarization Tensor

The polarization tensor $\Pi_{\mu\nu}^{(M)}(q)$ of the vector meson M is defined as (-i) times the sum of all proper Feynman diagrams with two external vector-meson M legs only and where these legs are amputated. The general structure of the polarization tensor is:

$$\Pi^{(M)}_{\mu\nu}(q) = B^{(M)}(q^2) g_{\mu\nu} + C^{(M)}(q^2) q_\mu q_\nu \tag{C.3}$$

The squared-mass shift δm_M^2 and the vector meson wave function renormalization constant $Z_3^{(M)}$ are then given by:

$$\delta m_M^2 = \mathrm{Re}\, B^{(M)}(q^2) \Big|_{q^2 = m_M^2} \tag{C.4}$$

$$Z_3^{(M)} \equiv 1 + \dot{Z}_3^{(M)} = 1 + \frac{d}{dq^2} \mathrm{Re}\, B^{(M)}(q^2) \Big|_{q^2 = m_M^2} \tag{C.5}$$

We shall present in some detail the evaluation of the one-loop heavy-particle contribution in the SU(5) theory to the $\Pi^{(M)}_{\mu\nu}(q)$ where M mesons will be W,Z mesons and photon A. Our calculations are performed in the t'Hooft's R gauge using n-dimensional regularization.

Let us consider first some typical integrals which appear in calculation of a general $\Pi^{(M)}_{\mu\nu}(q)$ in a general gauge theory. For that purpose let us introduce two arbitrary particles 1 and 2 with masses m_1 and m_2 contributing in a closed loop.

1). <u>The trilinear-coupling diagram with gauge mesons</u> 1 and 2 in a loop (for example Fig.1a with 1=X and 2=Y) gives up to a specific factor the integral:

$$J^{(1)}_{\mu\nu}(1,2;q^2) = \int \frac{d^nK}{(2\pi)^n} \frac{\Gamma_{\alpha\beta\mu}(K,-K-q,q)\,\Gamma^{\beta\alpha}{}_{\nu}(K+q,-K,-q)}{(K^2-m_1^2)\left[(K+q)^2-m_2^2\right]} \tag{C.6}$$

Following the usual sequence of steps: using Feynman parametrization formula:

$$\frac{1}{ab} = \int_0^1 dx \frac{1}{[ax + b(1-x)]^2} \tag{C.7}$$

shifting the integration variable k=Q-qx, performing some algebraic manipulations (respecting the rules of the n-dimensional algebra) and the symmetric integration $Q_\mu Q_\nu \to (Q^2/n) g_{\mu\nu}$ we end up with:

$$J^{(1)}_{\mu\nu}(1,2;q^2) = \int_0^1 dx \int \frac{d^4Q}{(2\pi)^4} \frac{1}{[Q^2 - C_{12}(x;q^2)]^2} \Big\{ 6\left(1-\frac{1}{n}\right) Q^2 g_{\mu\nu} +$$

$$+ (q^2 g_{\mu\nu} - q_\mu q_\nu)[(x-2)^2 + (x+1)^2] + q_\mu q_\nu (n-1)(2x-1)^2 \Big\} \qquad \text{(C.8)}$$

where

$$C_{12}(x;q^2) \equiv m_1^2 + (m_2^2 - m_1^2)x + q^2 x(x-1) \qquad \text{(C.9)}$$

2). The quartic-coupling diagrams with gauge mesons 1 and 2 in a loop (for example Fig.1b with 1=X and 2=Y) give respectively up to a specific factor:

$$J^{(2)}_{\mu\nu}(1) = \int \frac{d^4K}{(2\pi)^4} \frac{-(n-1)g_{\mu\nu}}{K^2 - m_1^2} = \int \frac{d^4K}{(2\pi)^4} \frac{[-(n-1)g_{\mu\nu}][(K+q)^2 - m_2^2]}{(K^2-m_1^2)[(K+q)^2 - m_2^2]} =$$

$$= \int_0^1 dx \int \frac{d^4Q}{(2\pi)^4} \frac{1}{[Q^2 - C_{12}(x;q^2)]^2} \Big\{ -(n-1)Q^2 g_{\mu\nu} - (n-1) q^2 g_{\mu\nu} (x-1)^2 + $$
$$+ (n-1)\, m_2^2 g_{\mu\nu} \Big\} \qquad \text{(C.10)}$$

and

$$J^{(2)}_{\mu\nu}(2) = \int \frac{d^4K}{(2\pi)^4} \frac{-(n-1)g_{\mu\nu}}{K^2 - m_2^2} = \int \frac{d^4K}{(2\pi)^4} \frac{[-(n-1)g_{\mu\nu}](K^2 - m_1^2)}{(K^2-m_1^2)[(K+q)^2 - m_2^2]} =$$

$$= \int_0^1 dx \int \frac{d^4Q}{(2\pi)^4} \frac{1}{[Q^2 - C_{12}(x;q^2)]^2} \Big\{ -(n-1)Q^2 g_{\mu\nu} - (n-1) q^2 g_{\mu\nu} x^2 + $$
$$+ (n-1)\, m_1^2 g_{\mu\nu} \Big\} \qquad \text{(C.11)}$$

3). The contribution of ghosts C_1 and C_2 up to a specific factor (for example Fig.1c with $1=C_X$ and $2=C_Y$):

$$J^{(3)}_{\mu\nu}(1,2;q^2) = \int \frac{d^4K}{(2\pi)^4} \frac{-2(K+q)_\mu K_\nu}{(K^2-m_1^2)[(K+q)^2 - m_2^2]} =$$

$$= \int_0^1 dx \int \frac{d^4Q}{(2\pi)^4} \frac{1}{[Q^2 - C_{12}(x;q^2)]^2} \Big\{ -\frac{2}{n} Q^2 g_{\mu\nu} + 2 q_\mu q_\nu x(1-x) \Big\} \qquad \text{(C.12)}$$

4). The graphs that mix Higgs scalars H_1 and gauge meson 2 and Higgs scalar H_2 and gauge meson 1 (for example Fig.1d,e) give typically

$$J^{(4)}_{\mu\nu}(H_1, 2; \varkappa, q^2) = \int \frac{d^4K}{(2\pi)^4} \frac{-\varkappa g_{\mu\nu}}{(K^2 - m_{H_1}^2)[(K+q)^2 - m_2^2]} =$$
$$= \int_0^1 dx \int \frac{d^4Q}{(2\pi)^4} \frac{1}{[Q^2 - C_{H_1 2}(x; q^2)]^2} \{-\varkappa g_{\mu\nu}\} \tag{C.13}$$

and

$$J^{(4)}_{\mu\nu}(1, H_2; \tilde{\varkappa}, q^2) = \int \frac{d^4K}{(2\pi)^4} \frac{-\tilde{\varkappa} g_{\mu\nu}}{(K^2 - m_1^2)[(K+q)^2 - m_{H_2}^2]} =$$
$$= \int_0^1 dx \int \frac{d^4Q}{(2\pi)^4} \frac{1}{[Q^2 - C_{1H_2}(x; q^2)]^2} \{-\tilde{\varkappa} g_{\mu\nu}\} \tag{C.14}$$

respectively where m_{H_1} and m_{H_2} are masses of H_1 and H_2 and $\varkappa$, $\tilde{\varkappa}$ are some parameters with dimension of mass squared.

5). <u>The trilinear-coupling diagrams with Higgs scalars</u> H_1 and H_2 (for example Fig.1f) give typically up to a factor:

$$J^{(5)}_{\mu\nu}(H_1, H_2; q^2) = \int \frac{d^4K}{(2\pi)^4} \frac{(2K+q)_\mu (2K+q)_\nu}{(K^2 - m_{H_1}^2)[(K+q)^2 - m_{H_2}^2]} =$$
$$= \int_0^1 dx \int \frac{d^4Q}{(2\pi)^4} \frac{1}{[Q^2 - C_{H_1 H_2}(x; q^2)]^2} \left\{ \frac{4}{n} Q^2 g_{\mu\nu} + q_\mu q_\nu (2x-1)^2 \right\} \tag{C.15}$$

6). <u>The quartic-coupling diagrams with Higgs scalars</u> H_1 and H_2 (for example Fig.1g) give typically up to a factor:

$$J^{(6)}_{\mu\nu}(H_1) = \int \frac{d^4K}{(2\pi)^4} \frac{-g_{\mu\nu}}{K^2 - m_{H_1}^2} = \int \frac{d^4K}{(2\pi)^4} \frac{-g_{\mu\nu}[(K+q)^2 - m_{H_2}^2]}{(K^2 - m_{H_1}^2)[(K+q)^2 - m_{H_2}^2]} =$$
$$= \int_0^1 dx \int \frac{d^4Q}{(2\pi)^4} \frac{1}{[Q^2 - C_{H_1 H_2}(x; q^2)]^2} \{-Q^2 g_{\mu\nu} - q^2 g_{\mu\nu}(x-1)^2 + m_{H_2}^2 g_{\mu\nu}\} \tag{C.16}$$

and

$$J^{(6)}_{\mu\nu}(H_2) = \int \frac{d^4K}{(2\pi)^4} \frac{-g_{\mu\nu}}{K^2 - m_{H_2}^2} = \int \frac{d^4K}{(2\pi)^4} \frac{-g_{\mu\nu}(K^2 - m_{H_1}^2)}{(K^2 - m_{H_1}^2)[(K+q) - m_{H_2}^2]} =$$
$$= \int_0^1 dx \int \frac{d^4Q}{(2\pi)^4} \frac{1}{[Q^2 - C_{H_1 H_2}(x; q^2)]^2} \{-Q^2 g_{\mu\nu} - q^2 g_{\mu\nu} x^2 + m_{H_1}^2 g_{\mu\nu}\} \tag{C.17}$$

Let us consider the several combinations of the above integrals which we will need for the subsequent calculations:

i) First consider the contribution of gauge mesons (C.8),(C.10), (C.11), ghosts (C.12) and Goldstone bosons mixed with gauge mesons (C.13),(C.14) (for example Fig.1a,b,c,d,e) up to specific factors:

$$\sum_{i=1}^{4} J^{(i)}_{\mu\nu}(1,2;q^2) \equiv J^{(1)}_{\mu\nu}(1,2;q^2) + J^{(2)}_{\mu\nu}(1) + J^{(2)}_{\mu\nu}(2) + \\ + J^{(3)}_{\mu\nu}(1,2;q^2) + J^{(4)}_{\mu\nu}(1,2;m_1^2,q^2) + J^{(4)}_{\mu\nu}(1,2;m_2^2,q^2) \tag{C.18}$$

where we remind the reader that in $J^{(4)}_{\mu\nu}$ integrals Goldstone bosons in t'Hooft's gauge have masses of corresponding gauge mesons, i.e. $m_{H_1} = m_1$, $m_{H_2} = m_2$ and the coupling masses are $\mathcal{H} = m_1^2$, $\tilde{\mathcal{H}} = m_2^2$.

Now we have from (C.8),(C.10),(C.11),(C.12),(C.13),(C.14) and (C.18):

$$\sum_{i=1}^{4} J^{(i)}_{\mu\nu}(1,2;q^2) = \int_0^1 dx \int \frac{d^4Q}{(2\pi)^n} \frac{1}{[Q^2 - C_{12}(x;q^2)]^2} \Big\{ (n-2)\, g_{\mu\nu} \times \\ \left[\left(\frac{4}{n}-2\right)Q^2 + m_1^2 + m_2^2\right] + (q^2 g_{\mu\nu} - q_\mu q_\nu)\left[(x-2)^2 + (x+1)^2\right] - \\ - (n-1)\, q^2 g_{\mu\nu}\left[(x-1)^2 + x^2\right] + q_\mu q_\nu \left[(n-1)(2x-1)^2 + 2x(1-x)\right]\Big\} \tag{C.19}$$

Since in the calculations we will put $q^2 = m_M^2 \sim m_L^2$ we will expand in q^2 and keep only zeroth and first order terms in q^2 due to the discussion in the Introduction of the Appendix C. Using the expansion:

$$\frac{1}{[Q^2 - C_{12}(x;q^2)]^2} \simeq \frac{1}{[Q^2 - C_{12}(x)]^2} + \frac{2q^2 x(x-1)}{[Q^2 - C_{12}(x)]^2} \tag{C.20}$$

where $C_{12}(x;q^2)$ is defined by (C.9) and we put:

$$C_{12}(x) \equiv C_{12}(x;0) = m_1^2 + (m_2^2 - m_1^2)x \tag{C.21}$$

we obtain:

$$\sum_{i=1}^{4} J_{\mu\nu}^{(i)}(1,2;q^2) = \int_0^1 dx \int \frac{d^4Q}{(2\pi)^4} \left\{ \frac{(n-2)\, g_{\mu\nu}\left[\left(\frac{4}{n}-2\right)Q^2 + m_1^2 + m_2^2\right]}{\left[Q^2 - C_{12}(x)\right]^2} + \right.$$

$$+ \frac{(q^2 g_{\mu\nu} - q_\mu q_\nu)\left[(x-2)^2 + (x+1)^2\right] - (n-1)\, q^2 g_{\mu\nu}\left[(x-1)^2 + x^2\right]}{\left[Q^2 - C_{12}(x)\right]^2} +$$

$$+ \frac{q_\mu q_\nu\left[(n-1)(2x-1)^2 + 2x(1-x)\right]}{\left[Q^2 - C_{12}(x)\right]^2} + \frac{2q^2 x(x-1)\left[-\frac{2}{n}(n-2)^2 Q^2\right] g_{\mu\nu}}{\left[Q^2 - C_{12}(x)\right]^3} +$$

$$\left. + \frac{2q^2 x(x-1)(n-2)\, g_{\mu\nu}(m_1^2 + m_2^2)}{\left[Q^2 - C_{12}(x)\right]^3} \right\} \tag{C.22}$$

The zeroth-order term in q^2 gives:

$$\sum_{i=1}^{4} J_{\mu\nu}^{(i)}(1,2;q^2)\Big|_{q^2=0} = -(n-2)\, g_{\mu\nu} \frac{i}{16\pi^2} I_1(1,2) \tag{C.23}$$

where we defined:

$$I_1(1,2) = 16\pi^2 i \int_0^1 dx \int \frac{d^4Q}{(2\pi)^4} \frac{\left(\frac{4}{n}-2\right)Q^2 + m_1^2 + m_2^2}{\left[Q^2 - C_{12}(x)\right]^2} \tag{C.24}$$

The integral $I_1(1,2)$ is evaluated via the known formula [24]:

$$\int \frac{d^4Q}{(2\pi)^4} \frac{(Q^2)^r}{(Q^2 - C)^m} = \frac{i(-1)^{r-m}}{(16\pi^2)^{\frac{n}{4}}} C^{r-m+\frac{n}{2}} \frac{\Gamma\left(r+\frac{n}{2}\right)\Gamma\left(m-r-\frac{n}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\Gamma(m)} \tag{C.25}$$

and using the expansion of $\Gamma(2-n/2)$ around $n=4$:

$$\Gamma\left(2-\frac{n}{2}\right) = -\frac{2}{n-4} - \gamma + O(n-4) \tag{C.26}$$

where γ is Euler's constant we get the finite result:

$$I_1(1,2) = \frac{m_1^2 m_2^2}{m_1^2 - m_2^2} \ln \frac{m_1^2}{m_2^2} - \frac{1}{2}(m_1^2 + m_2^2) \tag{C.27}$$

so finally we can put n=4 in (C.23):

$$\sum_{i=1}^{4} J_{\mu\nu}^{(i)}(1,2;q^2)\Big|_{q^2=0} = -\frac{i}{16\pi^2} g_{\mu\nu}\, 2 I_1(1,2) \qquad \text{(C.28)}$$

We will encounter two possible cases for the relation between masses m_1 and m_2 which will also simplify the calculation of the integrals:

$$\text{a)} \quad m_1^2 - m_2^2 \sim m_i^2 \qquad (i=1,2) \qquad \text{(C.29)}$$

in which case it will turn out from the Feynman rules that we can neglect q^2-dependent terms in our approximation so that (C.28) becomes:

$$\sum_{i=1}^{4} J_{\mu\nu}^{(i)}(1,2;q^2) = -\frac{i}{16\pi^2} g_{\mu\nu}\, 2 I_1(1,2) \qquad \text{(C.30)}$$

$$\text{b)} \quad m_1^2 - m_2^2 \ll m_i^2 \qquad (i=1,2) \qquad \text{(C.31)}$$

i.e. $m_1^2 - m_2^2 \sim m_L^2$ and $m_i^2 \sim m_H^2$ (i=1,2) in the notation of the Introduction C.I. In this case we get from (C.27):

$$I_1(1,2) \simeq -\frac{1}{6}\frac{(m_1^2-m_2^2)^2}{m^2} \simeq 0 \qquad \text{(C.32)}$$

(where $m_1 \simeq m_2 \simeq m$) in our approximation so we drop such terms from the forthcoming expressions. Now the only contribution will come from q^2-dependant terms of (C.22) and we get using (C.25) and (C.26):

$$\sum_{i=1}^{4} J_{\mu\nu}^{(i)}(1,2;q^2) = -\frac{i}{16\pi^2}\, 2(q^2 g_{\mu\nu} - q_\mu q_\nu)\Big\{\frac{10}{3}\Big(\frac{1}{n-4} + \frac{1}{2}\gamma - \ln 2\sqrt{\pi} + \ln m\Big) - \frac{1}{3}\Big\} \qquad \text{(C.33)}$$

ii) We will need the contribution of graphs that mix Higgs scalars with masses m_{H_1} and m_{H_2} with gauge mesons with masses m_1 and m_2 (for example Fig.1d,e) and where $\varkappa = m^2 \sim m_L^2$ and $\tilde{\varkappa} = -\varkappa$. We have from (C.13) and (C.14):

$$J^{(4)}_{\mu\nu}(H_1,2;\tilde{m}^2,q^2) + J^{(4)}_{\mu\nu}(1,H_2;-\tilde{m}^2,q^2) =$$

$$= -\tilde{m}^2 g_{\mu\nu}\int_0^1 dx \int \frac{d^4Q}{(2\pi)^4}\left\{\frac{1}{[Q^2 - C_{H_12}(x;q^2)]^2} - \frac{1}{[Q^2 - C_{1H_2}(x;q^2)]^2}\right\} \tag{C.34}$$

which neglecting q^2- terms in view of $\tilde{m}^2 \sim m_L^2$ becomes in our approximation:

$$-\tilde{m}^2 g_{\mu\nu}\int_0^1 dx \int \frac{d^4Q}{(2\pi)^4}\left\{\frac{1}{[Q^2 - C_{H_12}(x)]^2} - \frac{1}{[Q^2 - C_{1H_2}(x)]^2}\right\} \tag{C.35}$$

Finally using (C.25) and (C.26):

$$J^{(4)}_{\mu\nu}(H_1,2;\tilde{m}^2,q^2) - J^{(4)}_{\mu\nu}(1,H_2;\tilde{m}^2,q^2) =$$
$$= \frac{i}{16\pi^2}\tilde{m}^2 g_{\mu\nu}\left[I_2(H_1,2) - I_2(1,H_2)\right] \tag{C.36}$$

where

$$I_2(1,2) \equiv \int_0^1 \ln C_{12}(x)\,dx \tag{C.37}$$

and $C_{12}(x)$ is defined by (C.21). We get:

$$I_2(1,2) = \frac{m_1^2 \ln m_1^2 - m_2^2 \ln m_2^2}{m_1^2 - m_2^2} - 1 \tag{C.38}$$

In the case b) $m_1^2 - m_2^2 \ll m_i^2$ $(i=1,2)$ $I_2(1,2)$ becomes:

$$I_2(1,2) \simeq \ln m^2 \tag{C.39}$$

(where $m_1 \simeq m_2 \simeq m$) in our approximation.

iii) Consider finally the pure contribution of Higgs scalars with masses m_1 and m_2 (for example Fig.1f,g):

$$\sum_{i=5}^{6} J^{(i)}_{\mu\nu}(1,2;q^2) \equiv J^{(5)}_{\mu\nu}(1,2;q^2) + J^{(6)}_{\mu\nu}(1) + J^{(6)}_{\mu\nu}(2) \tag{C.40}$$

We have from (C.15),(C.16) and (C.17):

$$\sum_{i=5}^{6} J_{\mu\nu}^{(i)}(1,2;q^2) = \int_0^1 dx \int \frac{d^n Q}{(2\pi)^n} \frac{1}{[Q^2 - C_{12}(x;q^2)]^2} \times \tag{C.41}$$

$$\times \left\{\left[\left(\frac{4}{n}-2\right)Q^2 + m_1^2 + m_2^2\right] g_{\mu\nu} + q_\mu q_\nu (2x-1)^2 - q^2 g_{\mu\nu}\left[(x-1)^2 + x^2\right]\right\}$$

The simmilar considerations as in the case i) give:

$$\sum_{i=5}^{6} J_{\mu\nu}^{(i)}(1,2;q^2)\Big|_{q^2=0} = -\frac{i}{16\pi^2} g_{\mu\nu} I_1(1,2) \tag{C.42}$$

and again we have cases a) $m_1^2 - m_2^2 \sim m_i^2$ (i=1,2) when (C.42) becomes:

$$\sum_{i=5}^{6} J_{\mu\nu}^{(i)}(1,2;q^2) = -\frac{i}{16\pi^2} g_{\mu\nu} I_1(1,2) \tag{C.43}$$

and b) $m_1^2 - m_2^2 \ll m_i^2$ (i=1,2) when (C.41) becomes:

$$\sum_{i=5}^{6} J_{\mu\nu}^{(i)}(1,2;q^2) = -\frac{i}{16\pi^2} 2(q^2 g_{\mu\nu} - q_\mu q_\nu)\left\{-\frac{1}{3}\left(\frac{1}{n-4} + \frac{1}{2}\gamma - \right.\right.$$
$$\left.\left. - \ln 2\sqrt{\pi} + \ln m\right\} \tag{C.44}$$

Now we will present in some detail the evaluation of the one-loop heavy-particle contribution in the SU(5) theory to $\Pi_{\mu\nu}^{(W)}(q)$:

i) The contribution of gauge mesons X_i, Y_i (Fig.1a,b), ghosts C_{X_i}, C_{Y_i} (Fig.1c) and Goldstone bosons that mix with gauge mesons (Fig.1d,e) is:

$$-\frac{3}{2} g_0^2 \left\{\sum_{i=1}^{4} J_{\mu\nu}^{(i)}(X,Y;q^2) - 4 J_{\mu\nu}^{(4)}(X,Y;m_W^2,q^2) + \right.$$
$$\left. + 4\delta_1 J_{\mu\nu}^{(4)}(X,Y;m_W^2,q^2) - 4\delta_1 J_{\mu\nu}^{(4)}(X,Y;m_W^2,q^2)\right. \tag{C.45}$$

where we put 1=X and 2=Y in definitions (C.8) - (C.14) and (C.18).

ii) The graph that mixes physical Higgs scalar H_{Y_i} with gauge meson X_i (Fig.1e) gives:

$$-\frac{3}{2} g_0^2 \, 4 J_{\mu\nu}^{(4)}(X, H_Y; m_W^2, q^2) \tag{C.46}$$

Since in (C.45) we have case b) (C.31) because $m_Y^2 - m_X^2 \sim m_W^2 \ll m_X^2, m_Y^2$ see (2.45)) we can sum up (C.45) and (C.46) using (C.33) and (C.36) to get the contribution of heavy gauge mesons, ghosts, Goldstone bosons and Higgs scalars mixed with gauge mesons to $\Pi_{\mu\nu}^{(W)}(q)$:

$$-\frac{g_0^2}{16\pi^2} 3 \left\{ (q^2 g_{\mu\nu} - q_\mu q_\nu) \left[\frac{10}{3} \left(\frac{1}{n-4} + \frac{1}{2}\gamma - \ln 2\sqrt{\pi} + \ln m_X \right) - \right.\right.$$

$$\left.\left. - \frac{1}{3} \right] + 2 m_W^2 g_{\mu\nu} \left[I_2(X,Y) - I_2(X, H_Y) \right] \right\} \tag{C.47}$$

iii) The contribution of Goldstone bosons and Higgs scalars with charges $Q = \pm\frac{4}{3}, \pm\frac{1}{3}$ only (which belong naturally to the X-Y system) to $\Pi_{\mu\nu}^{(W)}(q)$ is (Fig.1f,g):

$$-\frac{3}{2} g_0^2 \left\{ \left(1 - \frac{m_W^2}{m_X^2}\right) \sum_{i=5}^{6} J_{\mu\nu}^{(i)}(X,Y;q^2) + \frac{m_W^2}{m_X^2} \sum_{i=5}^{6} J_{\mu\nu}^{(i)}(X, H_Y; q^2) \right\} \tag{C.48}$$

where we put $H_1 = X$, $H_2 = Y$ for Goldstone bosons and $H_1 = X$, $H_2 = H_Y$ for Goldstone bosons mixed with Higgs scalars in (C.15) and (C.16) (remember that masses of G_X, G_Y Goldstone bosons are equal to the masses of corresponding X,Y gauge mesons). First term in (C.48) satisfies the case b) (C.31) and second term in (C.48) satisfies the case a) (C.29) so we have using (C.43) and (C.44) the contribution of the heavy Higgs scalars and Goldstone bosons with charges $Q = \pm\frac{4}{3}, \pm\frac{1}{3}$ only, to $\Pi_{\mu\nu}^{(W)}(q)$

$$-\frac{g_0^2}{16\pi^2} 3 \left\{ (q^2 g_{\mu\nu} - q_\mu q_\nu) \left[-\frac{1}{3} \left(\frac{1}{n-4} + \frac{1}{2}\gamma - \ln 2\sqrt{\pi} + \ln m_X \right) \right.\right.$$

$$\left. + \frac{1}{2} \frac{m_W^2}{m_X^2} I_1(X, H_Y) g_{\mu\nu} \right\} \tag{C.49}$$

The simmilar but more tedious calculation gives the contribution of heavy Higgs scalars alone and mixed with light Higgs scalars and Goldstone bosons with charges $Q = \pm 1, 0$ only, to $\Pi_{\mu\nu}^{(W)}(q)$ (Fig.1f,g):

$$-\frac{g_0^2}{16\pi^2}\Big\{-\frac{2}{3}(q^2 g_{\mu\nu}-q_\mu q_\nu)\Big(\frac{1}{n-4}+\frac{1}{2}\gamma-\ln 2\sqrt{\pi}+\ln m_{H+}\Big)+$$

$$+\frac{\delta_1^2}{4}\frac{m_W^2}{m_X^2}\Big[I_1(H_3,W)+I_1(H^+,\gamma)+I_1(H^+,Z)\Big]g_{\mu\nu} \quad (C.50)$$

$$+\frac{\delta_2^2}{4}\frac{m_W^2}{m_X^2}I_1(H_0,W)\,g_{\mu\nu}\Big\}$$

At this point let us introduce the notation for the infinite quantities which will frequently appear in the subsequent expressions:

$$A_X \equiv -18\Big(\frac{1}{n-4}+\frac{1}{2}\gamma-\ln 2\sqrt{\pi}+\ln m_X\Big)$$

$$A_H \equiv \frac{2}{3}\Big(\frac{1}{n-4}+\frac{1}{2}\gamma-\ln 2\sqrt{\pi}+\ln m_{H+}\Big) \quad (C.51)$$

$$A_{HY} \equiv 2\Big(\frac{1}{n-4}+\frac{1}{2}\gamma-\ln 2\sqrt{\pi}+\ln m_{HY}\Big)$$

where the numbers in front of parentheses are characteristical for the particles given by indices.

So finally we have from (C.47),(C.49) and (C.50) the heavy-particle contribution to $\Pi_{\mu\nu}^{(W)}(q)$ in the SU(5) theory:

$$\Pi_{\mu\nu}^{(W)}(q)\Big|_H=\frac{g_0^2}{16\pi^2}(q^2 g_{\mu\nu}-q_\mu q_\nu)\Big(\frac{1}{2}A_X+A_H+1\Big)-$$

$$-\frac{g_0^2}{16\pi^2}m_W^2\Big\{\frac{3}{2m_X^2}I_1(X,H_Y)+6\big[I_2(X,Y)-I_2(X,H_Y)\big]+ \quad (C.52)$$

$$+\frac{\delta_1^2}{4m_X^2}\big[I_1(H_3,W)+I_1(H^+,\gamma)+I_1(H^+,Z)\big]+\frac{\delta_2^2}{4m_X^2}I_1(H_0,W)\Big\}$$

where subscript H denotes the heavy-particle contribution.

We will only list the results of our calculation for Z meson (Fig.2):

$$\Pi_{\mu\nu}^{(Z)}(q)\Big|_H=\frac{g_0^2}{16\pi^2}(q^2 g_{\mu\nu}-q_\mu q_\nu)\Big[\frac{5}{8}(A_X+A_H)+\frac{1}{40}A_{HY}+\frac{5}{4}\Big]+$$

$$+\frac{g_0^2}{16\pi^2}m_Z^2 g_{\mu\nu}\frac{1}{6}A_X-\frac{g_0^2}{16\pi^2}m_Z^2 g_{\mu\nu}\Big\{\frac{3}{2m_X^2}I_1(Y,H_Y)+6\big[I_2(Y,Y)$$

$$-I_2(Y,H_Y)\big]+\frac{\delta_1^2}{4m_X^2}\big[2I_1(H^+,W)+I_1(H_3,Z)\big]+\frac{\delta_2^2}{4m_X^2}I_1(H_0,Z)\Big\} \quad (C.53)$$

and photon A (Fig.3):

$$\Pi^{(A)}_{\mu\nu}(q)\Big|_H = \frac{g_0^2}{16\pi^2}(q^2 g_{\mu\nu} - q_\mu q_\nu)\left(\frac{17}{24}A_X + \frac{3}{8}A_H + \frac{1}{24}A_{HY} + \frac{17}{12}\right) \tag{C.54}$$

We will need only $B^{(M)}(q^2)$-terms defined by (C.3) from $\Pi^{(M)}_{\mu\nu}(q)$ in order to calculate δm_M^2 and $Z_3^{(M)}$ according to (C.4) and (C.5). For the convenience of calculation we will write $B^{(M)}(q^2)$ in the form (neglecting $O[(q^2)^2]$ terms in our approximation):

$$B^{(M)}(q^2) \simeq B^{(M)}(0) + \frac{d}{dq^2}B^{(M)}(q^2)\Big|_{q^2=0}\cdot q^2 \equiv \tag{C.55}$$

$$\equiv B_0^{(M)} + B_1^{(M)} q^2$$

and present all the subsequent results in such a form: divide $q^2=0$ terms and $q^2\neq 0$ terms.

We have from (C.52),(C.3) and (C.55):

$$B_0^{(W)} = -\frac{g_0^2}{16\pi^2} m_W^2 \left\{ \frac{3}{2m_X^2} I_1(X,H_Y) + 6\left[I_2(X,Y) - I_2(X,H_Y)\right] + \right. \tag{C.56}$$

$$\left. + \frac{\delta_1^2}{4m_X^2}\left[I_1(H_3,W) + I_1(H^+,\eta) + I_1(H^+,Z)\right] + \frac{\delta_2^2}{4m_X^2} I_1(H_0,W)\right\}$$

and

$$B_1^{(W)} = \frac{g_0^2}{16\pi^2}\left(\frac{1}{2}A_X + A_H + 1\right) \tag{C.57}$$

From (C.53),(C.3) and (C.55) we have:

$$B_0^{(Z)} = \frac{g_0^2}{16\pi^2} m_Z^2 \frac{1}{6} A_X - \frac{g_0^2}{16\pi^2} m_Z^2 \left\{ \frac{3}{2m_X^2} I_1(Y,H_Y) + 6\left[I_2(Y,Y) - \right.\right. \tag{C.58}$$

$$\left.\left. - I_2(Y,H_Y)\right] + \frac{\delta_1^2}{4m_X^2}\left[2I_1(H^+,W) + I_1(H_3,Z)\right] + \frac{\delta_2^2}{4m_X^2} I_1(H_0,Z)\right\}$$

and

$$B_1^{(Z)} = \frac{g_0^2}{16\pi^2}\left[\frac{5}{8}(A_X + A_H) + \frac{1}{40}A_{HY} + \frac{5}{4}\right] \tag{C.59}$$

From (C.54),(C.3) and (C.55) we have:

$$B_0^{(A)} = 0 \tag{C.60}$$

and

$$B_1^{(A)} = \frac{g_0^2}{16\pi^2}\left(\frac{17}{24}A_X + \frac{3}{8}A_H + \frac{1}{24}A_{HY} + \frac{17}{12}\right) \tag{C.61}$$

Now we can calculate $\delta m_M^2/m_M^2$-quantities defined by (C.4):

$$\frac{\delta m_H^2}{m_H^2} \equiv \left.\frac{\delta m_H^2}{m_H^2}\right|_0 + \left.\frac{\delta m_H^2}{m_H^2}\right|_1 \tag{C.62}$$

where in view of (C.4) and (C.55):

$$\left.\frac{\delta m_H^2}{m_H^2}\right|_0 = \frac{\mathrm{Re}\, B_0^{(H)}}{m_H^2} \tag{C.63}$$

$$\left.\frac{\delta m_H^2}{m_H^2}\right|_1 = \mathrm{Re}\, B_1^{(H)} \tag{C.64}$$

We have from (C.56) and (C.63):

$$\left.\frac{\delta m_W^2}{m_W^2}\right|_0 = -\frac{g_0^2}{16\pi^2}\left\{\frac{3}{2m_X^2}I_1(X,H_Y) + 6\left[I_2(X,Y) - I_2(X,H_Y)\right] + \right.$$
$$\left. + \frac{\delta_1^2}{4m_X^2}\left[I_1(H_3,W) + I_1(H^+,\eta) + I_1(H^+,Z)\right] + \frac{\delta_2^2}{4m_X^2}I_1(H_0,W)\right\} \tag{C.65}$$

and from (C.57) and (C.64):

$$\left.\frac{\delta m_W^2}{m_W^2}\right|_1 = \frac{g_0^2}{16\pi^2}\left(\frac{1}{2}A_X + A_H + 1\right) \tag{C.66}$$

Also, we have from (C.58) and (C.63):

$$\left.\frac{\delta m_Z^2}{m_Z^2}\right|_0 = \frac{g_0^2}{16\pi^2}\frac{1}{6}A_X - \frac{g_0^2}{16\pi^2}\left\{\frac{3}{2m_X^2}I_1(Y,H_Y) + 6\left[I_2(Y,Y) - \right.\right.$$
$$\left.\left. - I_2(Y,H_Y)\right] + \frac{\delta_1^2}{4m_X^2}\left[2I_1(H^+,W) + I_1(H_3,Z)\right] + \frac{\delta_2^2}{4m_X^2}I_1(H_0,Z)\right\} \tag{C.67}$$

and (C.59) and (C.64) we have:

$$\left.\frac{\delta m_Z^2}{m_Z^2}\right|_1 = \frac{g_0^2}{16\pi^2}\left[\frac{5}{8}(A_X + A_H) + \frac{1}{40}A_{HY} + \frac{5}{4}\right] \tag{C.68}$$

We will finally need $Z_3^{(M)}$-quantities calculated from $B_1^{(M)}$ using the definitions (C.5) and (C.55):

$$Z_3^{(M)} = B_1^{(M)} \tag{C.69}$$

We have from (C.57) and (C.69):

$$Z_3^{(W)} = \frac{g_0^2}{16\pi^2}\left(\frac{1}{2}A_X + A_H + 1\right) \tag{C.70}$$

and from (C.61) and (C.69) we have:

$$Z_3^{(A)} = \frac{g_0^2}{16\pi^2}\left(\frac{17}{24}A_X + \frac{3}{8}A_H + \frac{1}{24}A_{HY} + \frac{17}{12}\right) \tag{C.71}$$

C.III. Fermion Self-Energy Functions

The self-energy function $\Sigma^{(f)}(\hat{q})$ of the fermion f (where $\hat{q} \equiv q_\mu\gamma^\mu$) is defined as i times the sum of all amputated proper Feynman diagrams with two external-fermion f legs. The general structure of the fermion self-energy function in a theory with parity-violating interaction is [24]:

$$\begin{aligned}\Sigma^{(f)}(\hat{q}) = \Sigma^{(f)}(m_f) + B^{(f)}(\hat{q}-m_f) + C^{(f)}(\hat{q}-m_f)^2 + \ldots \\ - A^{(f)}\hat{q}\gamma_5 - E^{(f)}\hat{q}\gamma_5(q^2-m_f^2) + \ldots\end{aligned} \tag{C.72}$$

The left-handed (right-handed) fermion f wave function renormalization constant is defined by:

$$\begin{aligned}Z_{2L,R}^{(f)} &\equiv (1 + z_{2L,R}^{(f)}) = (1 + B^{(f)} \pm A^{(f)}) = (1 + B^{(f)})(1 \pm A^{(f)}) \equiv \\ &\equiv Z_2^{(f)} Z_2^{(f)5} \equiv (1 + z_2^{(f)})(1 + z_2^{(f)5})\end{aligned} \tag{C.73}$$

i.e. explicitely:

$$\begin{aligned}z_2^{(f)} &= B^{(f)} \quad ; \quad z_2^{(f)5} = \pm A^{(f)} \\ z_{2L,R}^{(f)} &= B^{(f)} \pm A^{(f)} = z_2^{(f)} + z_2^{(f)5}\end{aligned} \tag{C.74}$$

The only contribution to $B^{(f)}$ and $A^{(f)}$ from heavy particles consistent with our approximation comes from heavy gauge mesons X and Y (the couplings of the light fermions which we only consider to the heavy Higgs scalars are supressed by factor m_L/m_H). Since $B^{(f)}$ and $A^{(f)}$ are dimensionless quantities of type A (C.1) where we calculate only a_0 term in (C.1) we can put $m_X \simeq m_Y$ (remember that $m_Y^2 - m_X^2 \sim m_W^2 \sim 0$), fermions to be massless and $q^2=0$.

Now it is easy to find $B^{(e^-,e^+)}$ and $A^{(e^-,e^+)}$ constants for electron (positron) (Fig.4a):

$$B^{(e^-,e^+)} = \frac{g_0^2}{16\pi^2}\left(-\frac{1}{4}A_X + \frac{9}{8}\right) \tag{C.75}$$

$$A^{(e^-,e^+)} = \pm\frac{g_0^2}{16\pi^2}\left(\frac{1}{12}A_X - \frac{3}{8}\right) \tag{C.76}$$

so using the definitions (C.74):

$$Z_2^{(e)} = \frac{g_0^2}{16\pi^2}\left(-\frac{1}{4}A_X + \frac{9}{8}\right) \tag{C.77}$$

$$Z_2^{(e)5} = \frac{g_0^2}{16\pi^2}\left(\frac{1}{12}A_X - \frac{3}{8}\right) \tag{C.78}$$

$$Z_{2L,R}^{(e)} = \frac{g_0^2}{16\pi^2}\left(-\frac{1}{6}A_X + \frac{3}{4}\right) \tag{C.79}$$

which now hold for both electron and positron (e).

Also we find $B^{(\nu,\nu^c)}$ and $A^{(\nu,\nu^c)}$ constants for neutrino (antineutrino) (Fig.4b):

$$B^{(\nu,\nu^c)} = \frac{g_0^2}{16\pi^2}\left(-\frac{1}{12}A_X + \frac{3}{8}\right) \tag{C.80}$$

$$A^{(\nu,\nu^c)} = \frac{g_0^2}{16\pi^2}\left(\mp\frac{1}{12}A_X + \frac{3}{8}\right) \tag{C.81}$$

so using the definitions (C.74):

$$z_Z^{(\nu)} = \frac{g_0^2}{16\pi^2}\left(-\frac{1}{12}Ax + \frac{3}{8}\right) \tag{C.82}$$

$$z_Z^{(\nu)5} = \frac{g_0^2}{16\pi^2}\left(-\frac{1}{12}Ax + \frac{3}{8}\right) \tag{C.83}$$

$$z_{ZL,R}^{(\nu)} = \frac{g_0^2}{16\pi^2}\left(-\frac{1}{6}Ax + \frac{3}{4}\right) \tag{C.84}$$

which now hold for both neutrino and antineutrino ($\bar{\nu}$).

C.IV. Vertex Functions

Consider the proper vertex Feynman diagram $V_{eeA}(q_1,q_2)$ with electrons $e^-(q_1),e^-(q_2)$ (positrons $e^+(q_1),e^+(q_2)$) and photon $A(\epsilon_\mu)$ as external lines. Then the vertex-renormalization constants Z_{1eeA} and Z_{1eeA}^5 are defined by:

$$V_{eeA}(q_1,q_2)\Big|_{\hat{q}_1=\hat{q}_2=m_e\ ;\ q_1\to q_2} = \tag{C.85}$$
$$= ie_0\epsilon^\mu\bar{e}(q_1)\gamma_\mu\left[Z_{1eeA}^{-1} \mp (Z_{1eeA}^5)^{-1}\gamma_5 - 1\right]e(q_1)$$

Again we will define:

$$\begin{aligned} Z_{1eeA} &\equiv 1 + z_{1eeA} \\ Z_{1eeA}^5 &\equiv 1 + z_{1eeA}^5 \end{aligned} \tag{C.86}$$

For the proper vertex Feynman diagram $V_{e\nu W}(q_1,q_2)$ with electron (positron) $e_{L,R}(q_1)$, neutrino (antineutrino) $\nu_{L,R}(q_2)$ and $W(\epsilon_\mu)$ meson as external lines the vertex-renormalization constant $Z_{1e\nu W}$ is defined by:

$$V_{e\nu W}(q_1,q_2)\Big|_{\hat q_1 = mc,\ \hat q_2 = 0;\ q_1 \to q_2} = \tag{C.87}$$

$$= i\,\frac{g_0}{\sqrt{2}}\,\epsilon^{\mu}\,\bar e(q_1)\gamma_\mu\left(Z^{-1}_{1e\nu W}-1\right)\frac{1\mp\gamma_5}{2}\,\nu(q_1)$$

and we define also:

$$Z_{1e\nu W} \equiv 1 + z_{1e\nu W} \tag{C.88}$$

Since the vertex-renormalization constants Z_1 are dimensionless the same approximation as for self-energy constants B and A applies for their calculation.

Again it is not difficult to get from (Fig.5):

$$z_{1eeA} = \frac{g_0^2}{16\pi^2}\left(-\frac{13}{18}Ax + \frac{9}{8}\right) \tag{C.89}$$

$$z^5_{1eeA} = \frac{g_0^2}{16\pi^2}\left(\frac{1}{12}Ax - \frac{3}{8}\right) \tag{C.90}$$

and from (Fig.6):

$$z_{1e\nu W} = \frac{g_0^2}{16\pi^2}\left(-\frac{1}{2}Ax + \frac{3}{4}\right) \tag{C.91}$$

References and Footnotes:

1. S. Weinberg, Phys.Rev.Lett. 19, 1264 (1967);
A. Salam, in "Elementary Particle Theory: Relativistic Groups and Analiticity", p.367, ed. N. Svartholm (Almqvist and Wiksell, Stockholm, 1968);
S. L. Glashow, J. Iliopoulos and L. Maiani, Phys.Rev. D2, 1285 (1970).

2. S. L. Glashow, Nucl.Phys. 22, 579 (1961).

3. For a review of QCD see:
W. J. Marciano and H. Pagels, Phys.Rep. 36C, 137 (1978).

4. For the pioneering work see:
J. C. Pati and A. Salam, Phys.Rev. D10, 275 (1974);
H. Georgi and S. L. Glashow, Phys.Rev.Lett. 32, 438 (1974).

5. T. Appelquist and J. Carazzone, Phys.Rev. D11, 2856 (1975).

6. H. Georgi, H. R. Quinn and S. Weinberg, Phys.Rev.Lett. 33, 451 (1974).

7. E. Gildener, Phys.Rev. Phys.Rev. D14, 1667 (1976);
See also:
K. T. Mahanthappa and D. G. Unger, Phys.Lett. 78B, 604 (1978).

8. A. J. Buras, J. Ellis, M. K. Gaillard and D. V. Nanopoulos, Nucl. Phys. B135, 66 (1978);
R. N. Mohapatra and G. Senjanović, Hadronic Journal 1, 903 (1978);
I. Bars and M. Serdaroglu, Yale preprint (1978);
O. K. Kalashnikov and V. V. Klimov, Phys.Lett. 80B, 75 (1978);
S. Weinberg, Phys.Lett. 82B, 387 (1979);
K. T. Mahanthappa, M. A. Sher and D. G. Unger, Univ. of Colorado preprint (1979);
For a review see:
I. Bars, Invited talk at Orbis Scientiae (Coral Gables) (1979).

9. J. C. Collins, F. Wilczek and A. Zee, Phys.Rev. D18, 242 (1978);
D. Toussaint, Phys.Rev. D18, 1626 (1978);
G. Senjanović and A. Šokorac, Phys.Rev. D18, 2708 (1978);
See also recent papers:
L. H. Chan and T. Hagiwara, Phys.Rev. D20, 1698 (1979);
L. H. Chan, T. Hagiwara and B. Ovrut, Phys.Rev. D20, 1982 (1979);
T. Hagiwara and B. Ovrut, Phys.Rev. D20, 1988 (1979).

10. G. Senjanović and A. Šokorac, Nucl.Phys. to be published.

11. H. Georgi and S. L. Glashow, Ref.4;
For a detailed analysis see:
A. J. Buras et al. Ref.8.

12. W. J. Marciano, Nucl.Phys. B84, 132 (1975);
M. Veltman, Acta Physica Polonica B8, 475 (1977); Nucl.Phys. B123, 89 (1977);
M. Chanowitz, M. Furman and I. Hinchliffe, Phys.Lett. 78B, 285 (1979); Nucl.Phys. B153, 402 (1979);
D. Toussaint, Ref.9.

13. After the paper in Ref.10 has been typed we became aware of the work by F. T. Hadjioannou, A. B. Lahanas and C. E. Vayonakis, Phys.Lett. 84B, 427 (1979) which has some overlap with our work. These authors have verified that A=0 for the light-gauge-meson polarization tensors which is not enough to show the decoupling as we discussed in the Introduction.

14. W. J. Marciano, Phys.Rev. D20, 274 (1979);
For the original calculation in the standard model see:
C. G. Bollini, J. J. Giambiagi and A. Sirlin, Nuovo Cimento 16A, 423 (1973).

15. A. J. Buras et al. Ref.8.

16. A word of caution: this $\sin^2\theta$ is slightly different from the one which is probed by the neutral currents (see Marciano, Ref.14).

17. D. A. Ross and J. C. Taylor, Nucl.Phys. B51, 125 (1973).

18. M. Baće and N. D. Hari Dass, Annals of Physics 94, 349 (1975).

19. This explanation was suggested to us by F. Wilczek. We thank him for a very useful discussion on that point.

20. F. Wilczek, Phys.Rev.Lett. 40, 279 (1978).

21. The work of M. Peskin and S. Weinberg as reported by S. Weinberg, Phys.Rev.Lett. 40, 223 (1978).

22. Proton decay was first suggested in context of a grand unified theory by Pati and Salam (see Ref.4 and papers quoted therein).

23. M. Goldhaber et al., "Proposal for nucleon decay detector", Brookhaven / Irwin / Michigan (1979).
The detailed calculations on the estimate of proton lifetime in the SU(5) theory have been done by:
A. J. Buras et al. Ref.8;
D. A. Ross, Nucl.Phys. B140, 1 (1978);
C. Jarlskog and F. J. Yndurain, Nucl.Phys. B149, 29 (1979);
W. J. Marciano, Ref.14;
T. J. Goldman and D. A. Ross, Phys.Lett. 84B, 208 (1979); Caltech preprint CALT-68-759 (1980);
M. E. Machacek, Harvard preprint HUTP 79/A021;
J. Donoghue, MIT preprint CTP 824 (1979);
N. P. Chang, A. Das and J. Perez-Mercader, CCNY-HEP-79/24,25 and CCNY-HEP-80/4.

24. C. G. Bollini et al. Ref.14.

Figure Captions:

Fig.1 The one-loop heavy-particle contribution to the W-meson polarization tensor $\Pi^{(W)}_{\mu\nu}(q)$ (i=1,2,3):

(a),(b) The pure contribution of heavy gauge mesons $X_i^{\pm}$ ($Q=\pm 4/3$) and $Y_i^{\pm}$ ($Q=\pm 1/3$);

(c) The contribution of ghosts C_{X_i} and C_{Y_i};

(d),(e) The graphs that mix "heavy" $G^{\pm}_{X_i}$ ($Q=\pm 4/3$), $G^{\pm}_{Y_i}$ ($Q=\pm 1/3$) Goldstone bosons and the heavy Higgs scalar $H^{\pm}_{Y_i}$ ($Q=\pm 1/3$) with $X_i^{\pm}$ and $Y_i^{\pm}$ gauge mesons.

(f),(g) The set of diagrams with Higgs particles only. G stands for $G^{\pm}_{X_i}$, $G^{\pm}_{Y_i}$ and the "light" $G^{\pm}$ ($Q=\pm 1$), G_0($Q=0$) Goldstone bosons, H stands for $H^{\pm}_{Y_i}$, the heavy H ($Q=\pm 1$), H_3($Q=0$) and H_0($Q=0$) Higgs scalars and η for the light Higgs scalar. The set consists of all the diagrams consistent with charge conservation and with at least one heavy Higgs particle.

Fig.2 The one-loop heavy-particle contribution to the Z-meson polarization tensor $\Pi^{(Z)}_{\mu\nu}(q)$.(For the explanation of the particle content see Fig.1.)

Fig.3 The one-loop heavy-particle contribution to the photon A and A-Z mixing polarization tensors $\Pi^{(A)}_{\mu\nu}(q)$ and $\Pi^{(A-Z)}_{\mu\nu}(q)$. (For the explanation of the particle content see Fig.1, except note that G and H particles do not mix when coupled to photon.)

Fig.4 The one-loop contribution of heavy gauge mesons X_i and Y_i to the (a) electron (positron) and (b) neutrino (antineutrino) self-energy functions. u and d stand for up and down quarks.

Fig.5 The one-loop contribution of heavy particles to the V_{eeA} vertex. The blob on the A-Z line stands for all such graphs depicted on Fig.3. (For the explanation of the particle content see Fig.1 and Fig.4)

Fig.6 The one-loop contribution of heavy gauge mesons X_i and Y_i to the $V_{e\nu W}$ vertex.

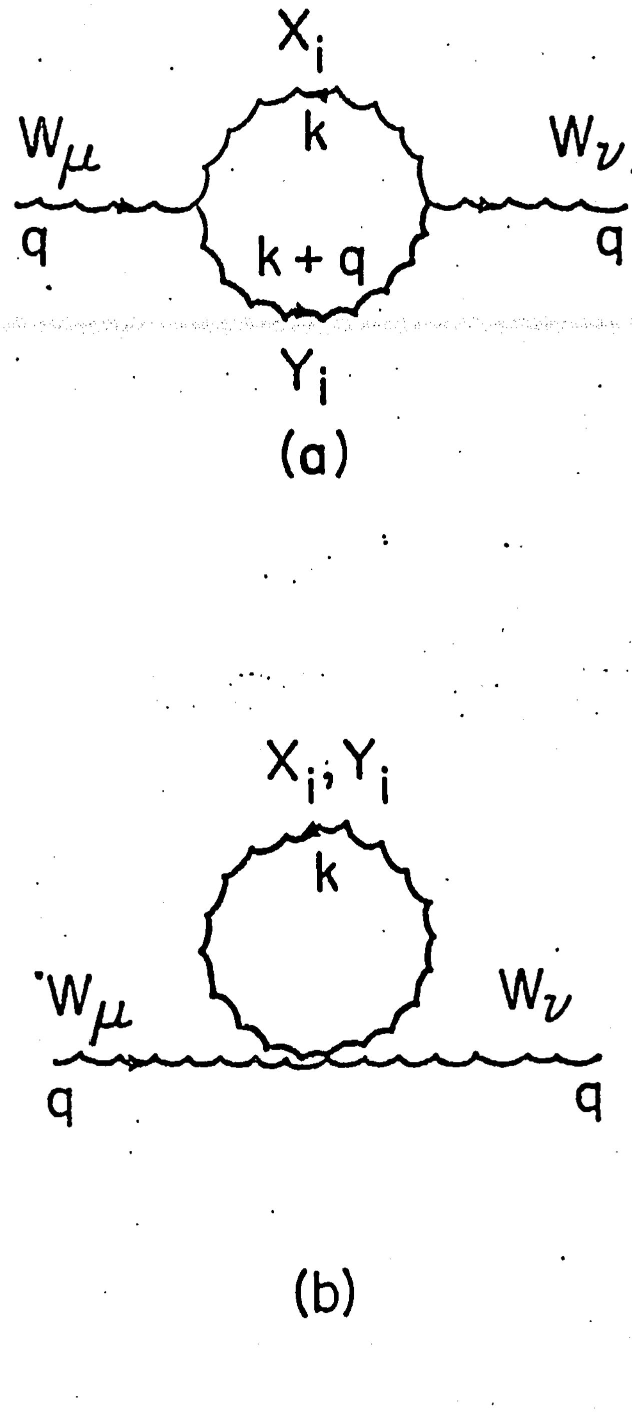
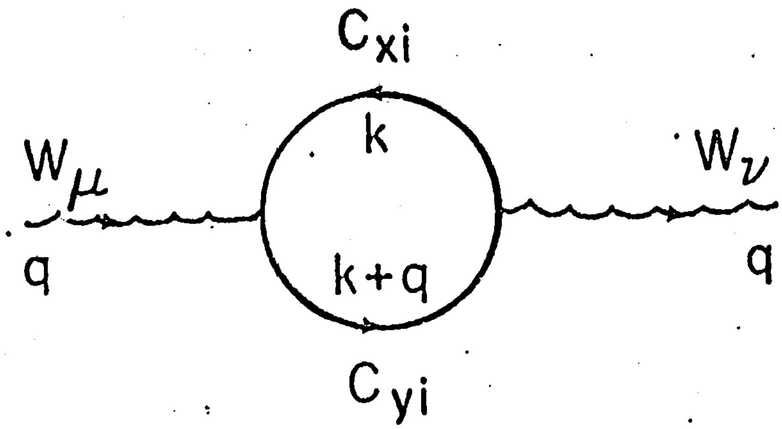
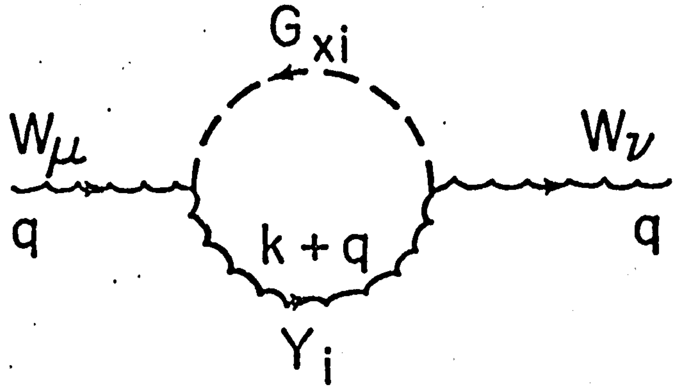


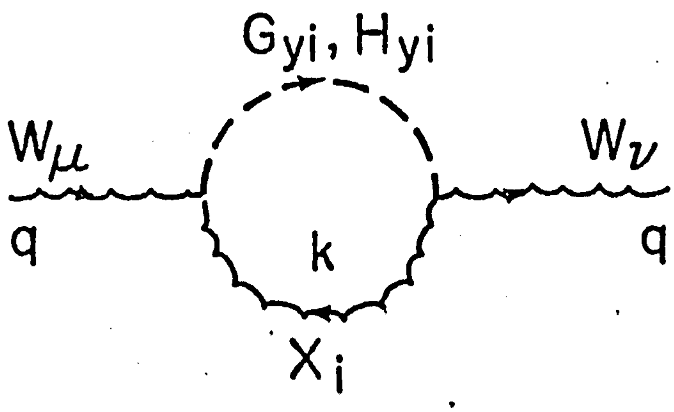
FIG. 1



(c)

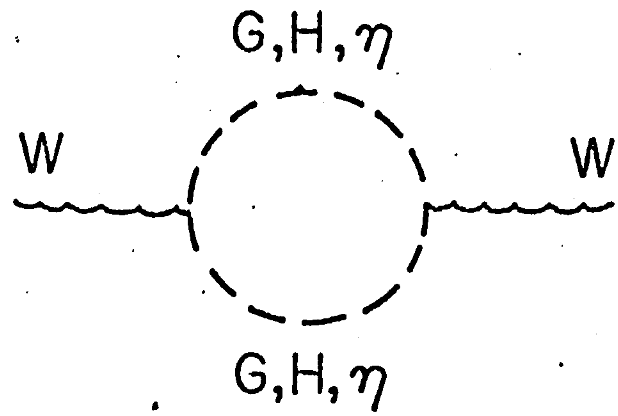


(d)

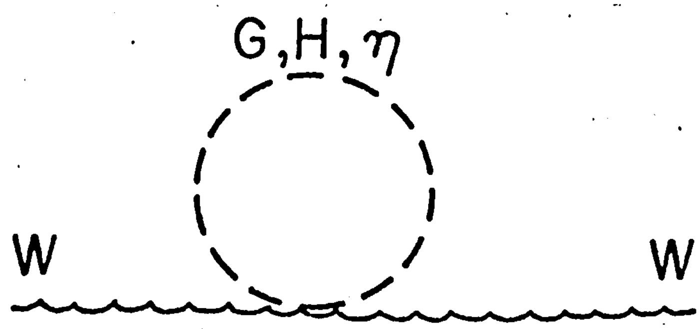


(e)

FIG. 1

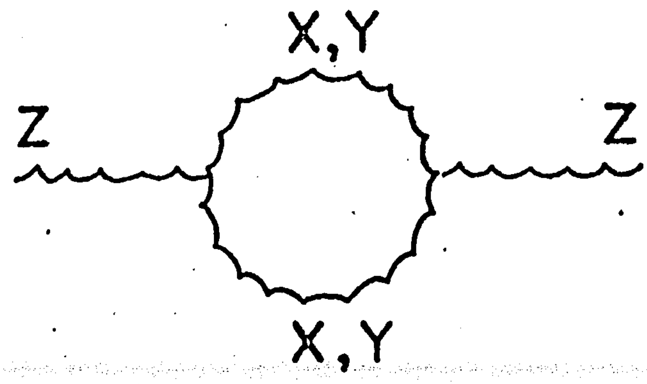


(f)



(g)

FIG. 1



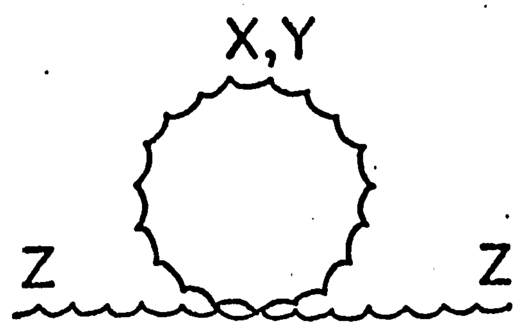
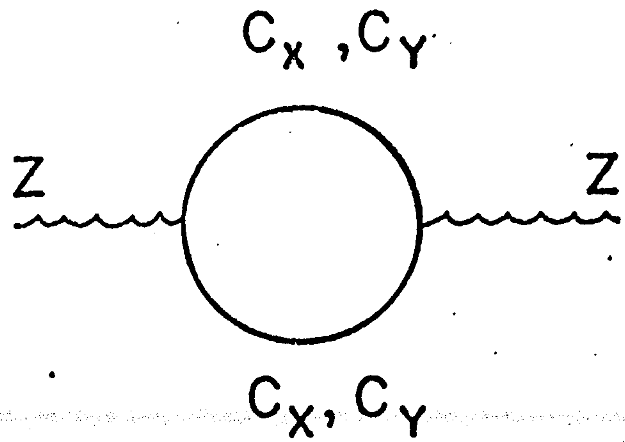


FIG. 2



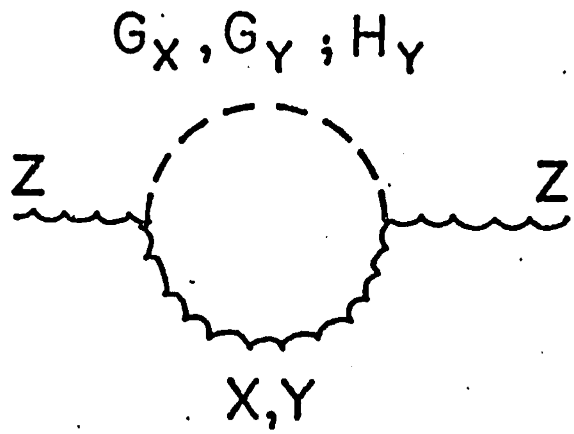


FIG. 2

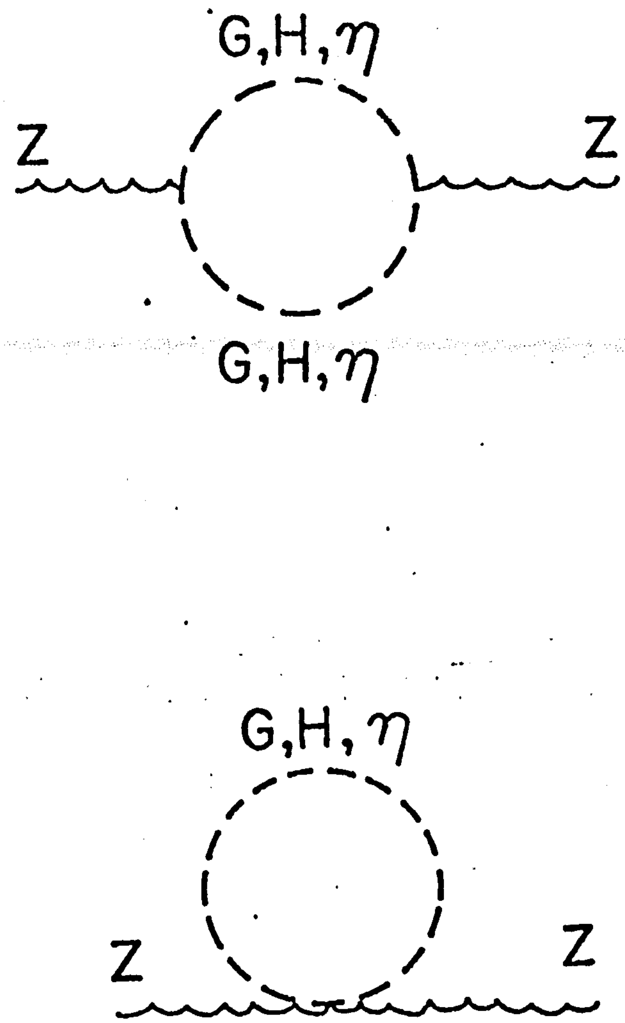


FIG. 2

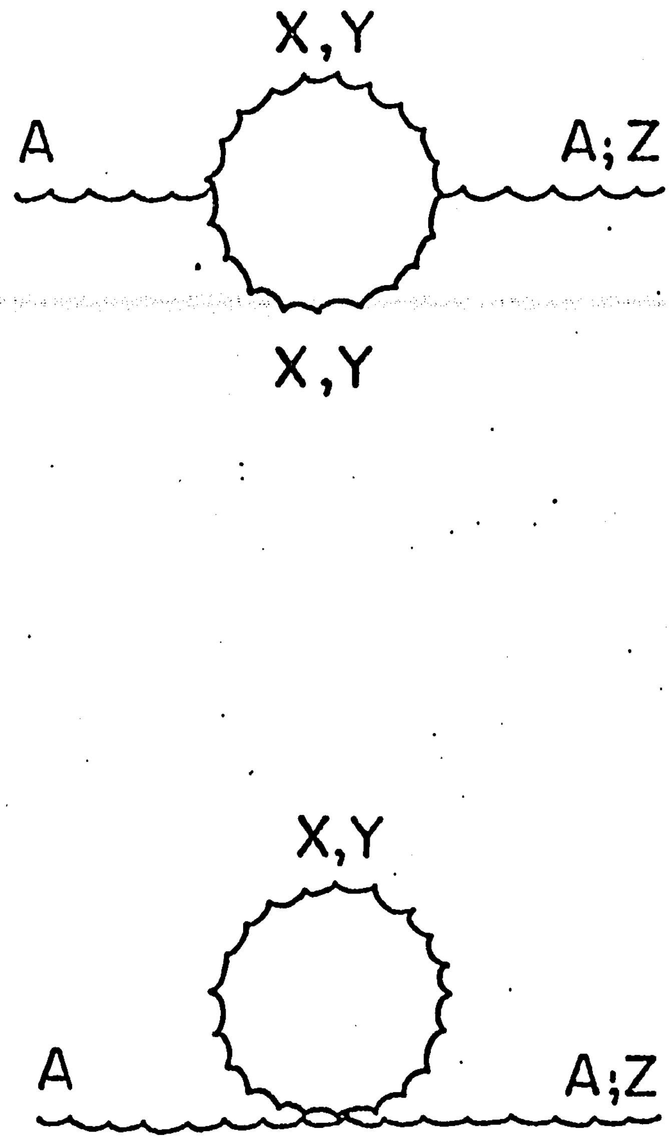


FIG. 3

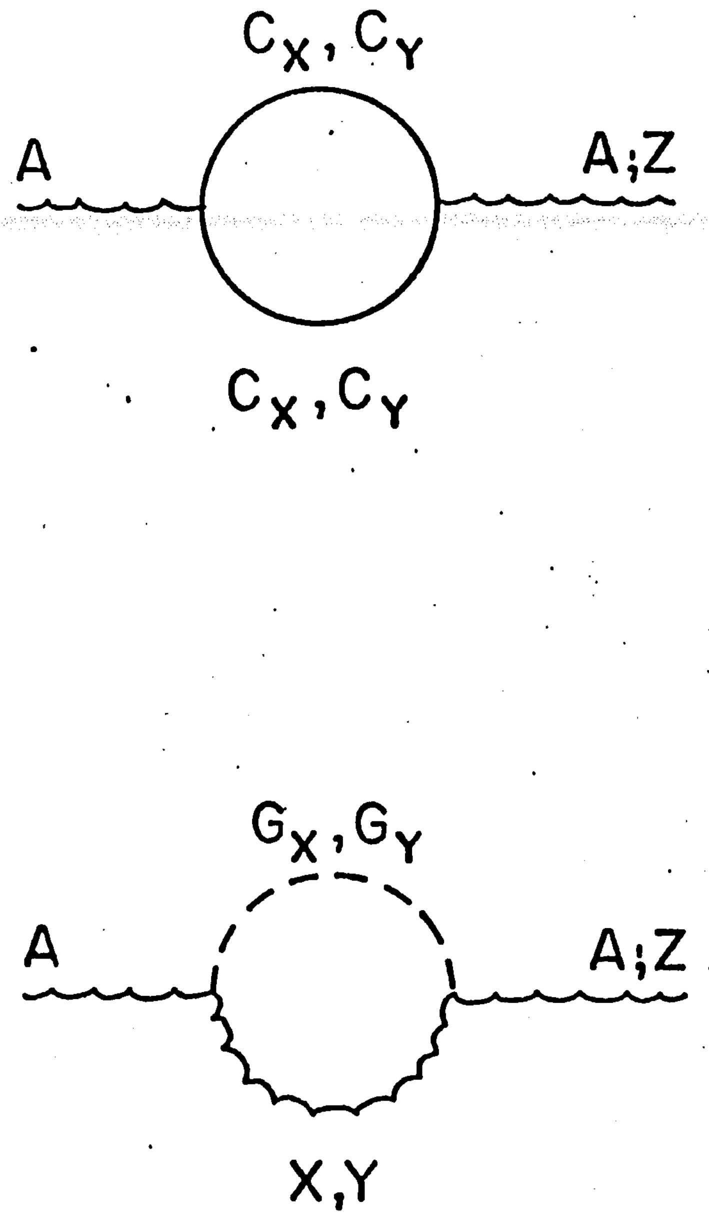


FIG. 3

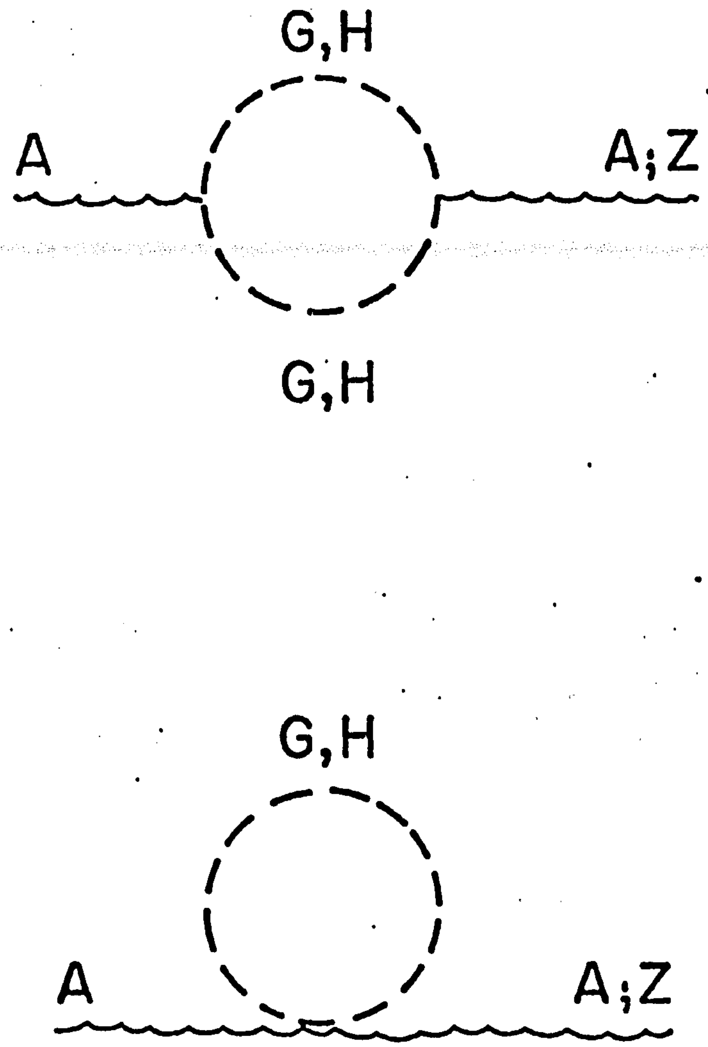


FIG. 3

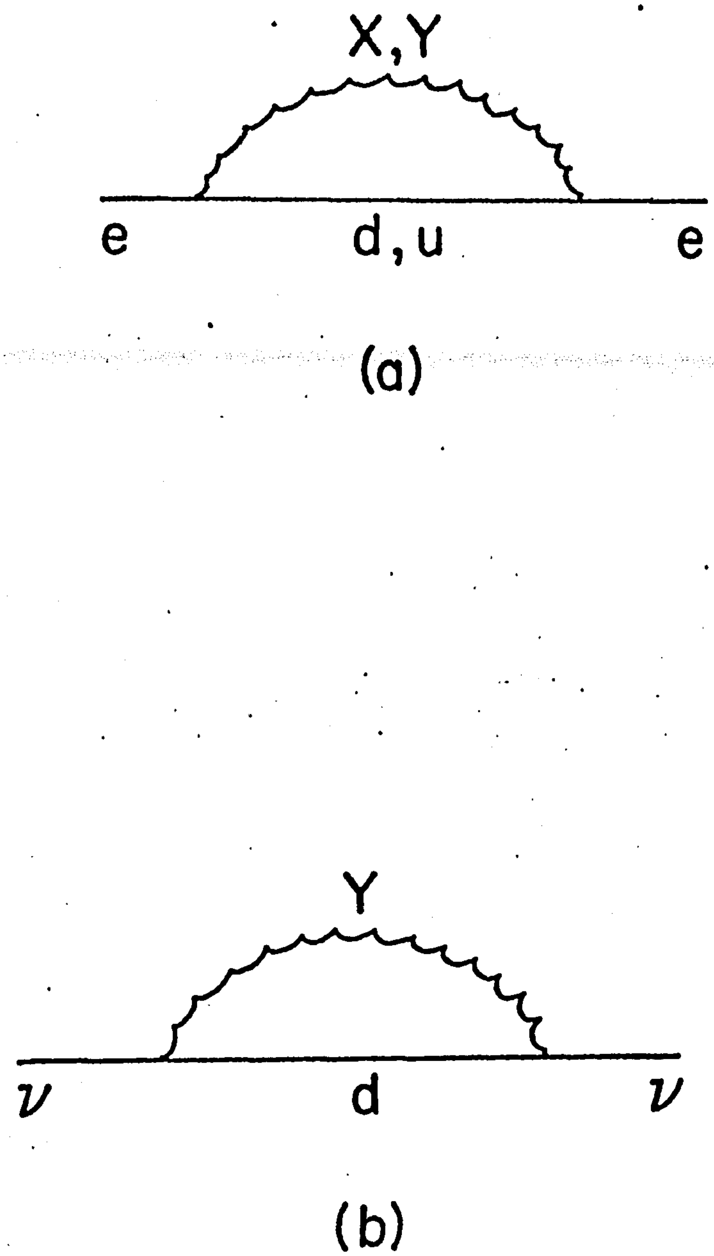


FIG. 4

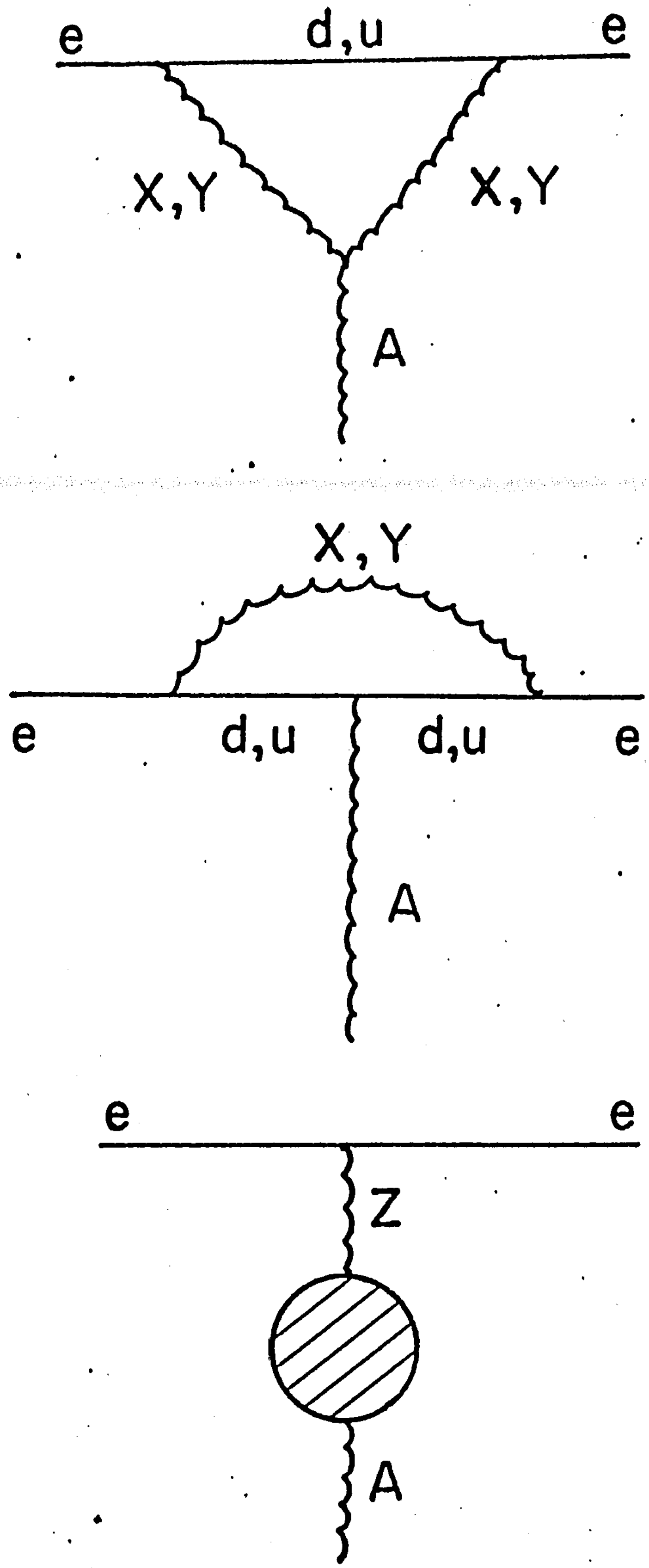


FIG.5

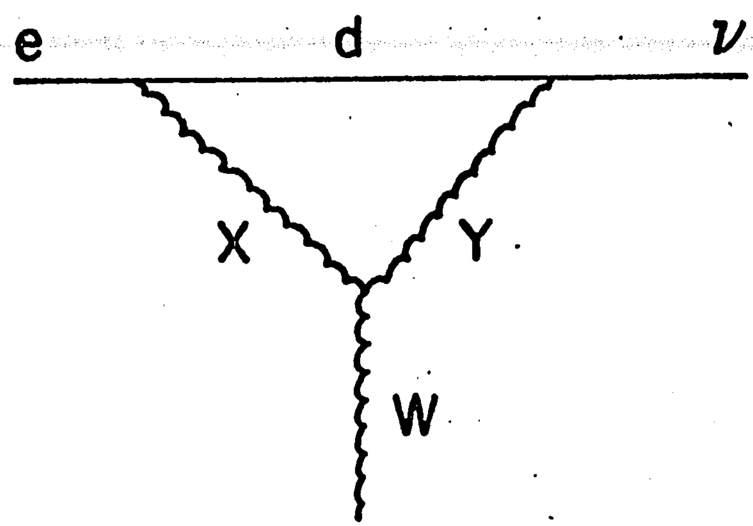


FIG. 6