A

Essays in Asset Pricing

by

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Abstract

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This dissertation consists of two distinct models. The first model offers a detailed study of the interaction between durability and habit formation and their impact on asset pricing. The second model is a brief inquiry into the effects of power risk aversion in a dynamic asset pricing setting.

In the first model, we study a representative agent pure exchange economy with two goods, one durable and one perishable. The representative agent's preferences are habit forming and defined over perishable good consumption and the service flow from the durables, which is produced according to a linear technology. Thus his past consumption patterns affect his current utility directly through the service flow from the durable good and indirectly through his habit persistence level on both goods. We find that the two goods economy leads to more realistic levels of habit formation in justifying the high premium observed in the U.S. data. Durability also helps significantly to reduce the volatility of the risk-free rate. Hence the combination of durability and habit formation in a two - goods model is a, more consistent with U.S. data, representation of reality.

In the second model, we consider a pure exchange economy with a Power Risk Aversion (PRA) utility. Shorter simulated consumption paths and low relative risk aversion lead to a reasonable equity risk premium and improved the correlation strucutre of returns. These results may, for example, imply that countries with different levels of GDP or different histories of consumption growth may have different equity premium structures.

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Chapter 1

Does Durability Help Asset Pricing with Habit Formation to Conform to U.S. Data?

1.1 Introduction

Mehra and Prescott (1985) introduced the equity premium puzzle. They demonstrated that the observed difference between the returns from risky equity and risk-free bonds is too high to be justified by a reasonable degree of risk aversion in a time - and state - separable utility framework. Interest in the study of the role of habit persistence and durability in resolving the equity premium puzzle comes from their implication of time-non-separable felicity from consumption, and also from their more realistic representation of an economy. With durability, past purchases continue to provide service flows and generate felicity in the current period which makes the current purchase less desirable. With habit formation, higher past consumptions build up higher appetites for current consumption. To achieve the same level of satisfaction, higher current consumption is required. In other words, durability captures the notion that consumption is substitutable over time and habit formation implies that consumption is complementary over time.

With habit formation, a consumer's level of current consumption (or service flow) is compared against his "habit", i.e. the trend of his past consumption (or service flows). Higher consumption in the current period will produce a higher level of habit during the next period, which will prospectively lead to a decrease in future utility and a complementary increase in future marginal utility (Constantinides,1990, Cochrane and Campbell, 1999 and Abel, 1990). Therefore, a consumer may show dissatisfaction when faced with a small decline in consumption after many years of relatively favorable consumption growth and even when the current consumption may be, say, the second best in history. On the other hand, the effects of durability of the service flow from consumption are the opposite. An increase in current consumption will increase future utility and decrease future marginal utility (Hindy and Huang, 1993). After a period of high consumption, a consumer would prefer to reduce consumption in a nearby period since he is still receiving the felicity from the past period's consumption.

While the habit formation model successfully solves the risk-free rate puzzle, it has limited power in explaining the equity premium since the implied relative risk aversion (RRA) is counterfactually high. However, both Dunn and Singleton (1986) and Giannikos (2004) find that incorporating durability only with linear service flow is not sufficient to resolve the equity premium puzzle. Using a broader definition of service flow, Detemple and Giannikos (1996) argue for the benefits of including durability in a continuous time framework. It follows that a model combining both durability and habit formation could possibly provide a better approach towards resolving the equity premium puzzle and related questions.

The opposing effects of past consumption on the Euler equation make it interesting to see which effect dominates the consumption decision. By studying an economy with one durable good in which the consumer develops a habit of consumption over time, Ferson and Constantinides (1991) present evidence that the habit formation effect dominates the durability effect. Heaton (1993, 1995) finds that, after adjusting for time averaging data and seasonality, durability alone helps to reconcile actual U.S.

market data with the representative agent model while habit persistence alone does not. Furthermore, Heaton (1993 ,1995) also finds that given durability, habit formation helps to improve the explanatory power of the model. Adding the interaction between durability and habit formation, Hindy, Huang and Zhu (1997) find that when durability dominates, an agent invests a higher portion of his wealth in risky assets (such as stocks) and when habit formation dominates, the agent becomes more risk averse and invests less in the risky assets. These results from Hindy, Huang and Zhu (1997) hint that a two-goods economy which nests durability and habit formation may better explain the behavior of security returns.

The high correlation between non-durable and durable expenditures in the U.S. economy suggests that the utility of consumption of both goods is not separable. Dunn and Singleton (1986) and Detemple and Giannikos (1996) document that introducing the interaction between durable good and non-durable good helps to improve the fit of the model with durability.

In this paper, we consider an economy with two goods, one perishable and one durable, where the representative agent has habit forming preferences defined by the consumption of the perishable good and by the service flow derived from the stock of durable goods. The service flow from the durable good is produced according to a linear technology. Thus the agent's past consumption patterns affect his current utility directly through the service flow from the durable good and indirectly through his habit persistence level on both goods. We consider Cobb-Douglas utility for the felicity derived from the non-durable and the durable, which allows for an interaction between the consumption of two goods, and a Constant Relative Risk Aversion (CRRA) outer utility function.

We confirm that in an economy with two goods, habit formation and durability have opposite effects on the equity risk premium and the volatilities of return on securities. Habit persistence causes the equity premium and volatilities of return to increase, while durability causes a decrease in the equity risk premium and volatilities. In a model combining both durability and habit formation with two goods, we find it much easier to reconcile the representative agent model with the stylized facts observed in the U.S. stock market. The equity premium increases dramatically due to the increases in local risk aversion from habit formation. Durability on the other hand helps to significantly reduce the volatility of the risk-free rate. The introduction of the two-good economy leads to a more realistic necessary level of habit persistence, which in turn produces the observed equity premium as noted by Mehra and Prescott (1985). In addition to matching the first two moments of the risk premium, we are able to produce co-movements between securities returns and the growth rates of outputs which are closer to the observed patterns in the U.S. economy.

We also document an interesting impact on the interaction between the durable and non-durable goods consumption on the returns of a risky portfolio. When the surplus service flow over the habit level of the durable good is relatively safe, the agent is willing to invest more on the risky portfolio as his habit persistence level for the non-durable good increases. As the surplus service flow becomes more risky, the agent will invest less on the risky portfolio as his consumption of the non-durable good becomes more risky. Higher demand for the risky portfolio lowers its return and the opposite holds for lower demand for the risky portfolio.

The remainder of this paper is organized as follows. Section 1.2 presents an overview of the optimization problem encountered by the representative agent, and establishes the equilibrium relation between aggregate consumption and asset prices. Significantly, exact linear solutions are found for all the relative prices of securities and the durable good. Section 1.3 presents details of the model calibration and a cogent discussion of the numerical results. Section 1.4 presents a summary and conclusion of our findings.

1.2 The Model

There are two firms in the economy, one producing the durable good and the other producing the non-durable good. The stock market consists of two shares of stock, one issued by the perishable good producing firm and the other issued by the durable good producing firm. Both shares of stock are assumed to be perfectly divisible. The infinitely-lived representative agent in this economy chooses his consumption plan by maximizing his expected lifetime utility, which is derived from the difference between consumptions and the habit persistence levels:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t u \left[\left(c_t - h_{c,t} \right), \left(ds_t - h_{ds,t} \right) \right] \tag{1.1}$$

subject to his lifetime budget constraint:

$$c_t + q_t f_t + p_{1,t} z_{1,t+1} + p_{2,t} z_{2,t+1} \leq (p_{1,t} + x_t) z_{1,t} + (p_{2,t} + y_t q_t) z_{2,t} \tag{1.2}$$

$$c_t \geq 0, \qquad f_t \geq 0, \qquad 0 \leq z_{1,t+1} \leq 1, \qquad 0 \leq z_{2,t+1} \leq 1$$

where $E_0[.]$ is the expectations operator based on the information available at time 0, $u(.,.)$ is utility for every period and is strictly increasing and strictly concave in both arguments, β is the rate of time-preference, c_t is per capita non-durable consumption at time t, ds_t is the service flow from the durable stock at time t, q_t is the relative price of the durable good at time t, (the non-durable good serves as the numeraire,) $z_{1,t}$ and $z_{2,t}$ denote the number of shares that the agent holds of the firms producing the non-durable and the durable good, respectively, at the beginning of period t, $p_{1,t}$ and $p_{2,t}$ are the relative prices of those two securities, x_t and y_t are the outputs (or more simply, the dividends) from these two firms, $h_{c,t}$ and $h_{ds,t}$ are the habit persistence level parameters for consumption of the perishable good and service flow from the durable good respectively, and finally f_t represents current purchases of the durable good.

The technology that transforms the durable good purchases into service flow is assumed to be linear and of the form:

$$ds_t = (1 - \Omega)[f_t + \Omega f_{t-1}] \tag{1.3}$$

where $0 \leq \Omega \leq 1$ proxies the depreciation factor of the durable good. The linear technology implies that a consumer's tolerance for erratic consumption is higher since his consumption now is relatively smooth as a result of the fact that his utility is determined by the weighted average of his current and past consumption streams.

In the simplest possible case, the habit persistence levels of non-durable consumption and service flow from the durable good are proportionally related to the respective consumptions in the previous period.

$$h_{c,t} = \xi c_{t-1}, \qquad h_{ds,t} = \eta ds_{t-1} \tag{1.4}$$

where $0 \leq \xi \leq 1$ and $0 \leq \eta \leq 1$.

The necessary and sufficient first order conditions of the aforementioned optimization problem for the representative agent are as follows:

$$(u_t^c - \beta\xi E_t[u_{t+1}^c])p_{1,t} = E_t[(\beta u_{t+1}^c - \beta^2\xi u_{t+2}^c)(p_{1,t+1} + x_{t+1})] \tag{1.5a}$$

$$(u_t^c - \beta\xi E_t[u_{t+1}^c])p_{2,t} = E_t[(\beta u_{t+1}^c - \beta^2\xi u_{t+2}^c)(p_{2,t+1} + y_{t+1}q_{t+1})] \tag{1.5b}$$

$$(u_t^c - \beta\xi E_t[u_{t+1}^c])q_t = (1-\Omega)u_t^{ds} + E_t[(1-\Omega)(\Omega-\eta)\beta u_{t+1}^{ds} - \beta^2\eta\Omega(1-\Omega)u_{t+2}^{ds}] \tag{1.5c}$$

where u_t^c and u_t^{ds} denote marginal utility at time t with respect to consumption of the perishable good and service flow from the durable good, respectively. We detail the derivation of the first order conditions for the optimization problem in Appendix A.

The intuition for these conditions is as follows. If $\xi = \eta = 0$, the utility level is determined by the absolute consumption of the non-durable good and the service flow of the durable good in the current period. The equations in (1.5) are thereby reduced to the first order conditions in the standard one durable good and one perishable good economy as presented in Giannikos (2004). Equations (1.5a) and (1.5b) demonstrate that the loss in today's utility related to the purchase of an additional share of the non-durable or durable firm (LHS) should equal the expected discounted gain in the future utility stemming from holding an additional share of one of those securities(RHS). Equation (1.5c) defines the trade-off between the present value of utility derived from service flows received from the durable good and the disutility from foregone non-durable good consumption now. Note that demand for the perishable and the durable depend on each other when utility is not additively separable. When $\xi > 0$ and $\eta > 0$, the impact of additional purchases of securities on the utility are felt beyond the current period. On the one hand, lower consumption today of the non-durable good reduces today's utility, while on the other hand, it reduces tomorrow's habit stock and hence increases tomorrow's marginal utility. For the durable good, a reduction in today's consumption will decrease service flows tomorrow and therefore reduce the utility both today and tomorrow. At the same time, the habit stock of service flows from the durable good is lower and marginal utility derived from service flows from the durable good is higher.

The LHS of equations (1.5a) and (1.5b) represent the net loss in marginal utility from additional unit purchases of the security of the non-durable (durable) firm. The RHS of (1.5a) (and (1.5b)) represents the discounted future net gain in the marginal utility from holding a share of the non-durable (durable) firm. Equation (1.5c) posits the following. One additional unit of durable good consumption will lower the utility from perishable good consumption this period and increase the marginal utility obtained from perishable good consumption in the next period. The net loss in the utility from the perishable consumption (LHS) should equal the discounted net gains from future utilities derived from durable good consumption (RHS). The future net gains in the utilities are related to (i) delayed service flows from the current period of durable good purchasing, (ii) an increased level of future habit level for the service flow from the durable good. The equations in (1.5) show that the prices of and demand for the securities and goods are linked directly to the future service flows from current consumption and the future habit persistence level from current consumption. Making the felicity function indirectly non-time-separable via habit formation and durability may generate a different covariation between the security returns and consumptions, and possibly solve the equity premium puzzle.

At equilibrium, the representative agent owns all the shares and is entitled to their total output:

$$z_{1,t} = z_{2,t} = 1, \qquad c_t = x_t, \qquad f_t = y_t \ . \tag{1.6}$$

To further study the implication of the first order conditions, we proceed by specifying the functional form for the representative agent's utility. We use the Cobb-Douglas utility for the felicity derived from the non-durable consumption and the service flow from the durable good, and a CRRA outer utility function:

$$u(c_t, ds_t, h_{C,t}, h_{ds,t}) = \frac{1}{1-\nu}\left[(c_t - h_{c,t})^{\delta}(ds_t - h_{ds,t})^{1-\delta}\right]^{1-\nu}. \tag{1.7}$$

Following Giannikos (2004), we assume that there are two states of the economy for the non-durable good and two states of the economy for the durable good. The exogenous joint process in the growth of the consumption of the durable good and the non-durable good is defined by a four-state stationary Markov process and is governed by the following transition matrix:

$$
M = \begin{array}{c} (\lambda_1,\gamma_1) \\ (\lambda_1,\gamma_2) \\ (\lambda_2,\gamma_1) \\ (\lambda_2,\gamma_2) \end{array}
\overset{\begin{array}{cccc} (\lambda_1,\gamma_1) & (\lambda_1,\gamma_2) & (\lambda_2,\gamma_1) & (\lambda_2,\gamma_2) \end{array}}{\begin{bmatrix} \phi+\frac{\Delta}{2} & \pi-\Delta & \sigma & H+\frac{\Delta}{2} \\ \pi & \phi & H & \sigma \\ \sigma & H & \phi & \pi \\ H+\frac{\Delta}{2} & \sigma & \pi-\Delta & \phi+\frac{\Delta}{2} \end{bmatrix}} \qquad (1.8)
$$

where λ_1 and λ_2 are growth rates in the good state and the bad state, respectively, for the non-durable good; and γ_1 and γ_2 are the growth rates for the durable good in the good state and bad state, respectively. Therefore, dividends of the durable good and non-durable good producing firms at time $t+1$ are described by

$$
x_{t+1} = \tilde{\lambda}_{t+1}\, x_t, \qquad y_{t+1} = \tilde{\gamma}_{t+1}\, y_t \qquad (1.9)
$$

where $\tilde{\lambda}_{t+1}$ and $\tilde{\gamma}_{t+1}$are growth random variables defined by equation (1.8) by the Markov Process.

We are interested in finding the prices of the financial securities (equity in each firm) and the price of the durable good, all relative to the numeraire (the non-durable good). We find linear solutions to the system of first order conditions (1.5). Details of the derivation are in Appendix B.

$$p_1(x_t, y_t; x_{t-1}, y_{t-1}; y_{t-2}) = W(\lambda_t, \gamma_t, \lambda_{t-1})x_t \tag{1.10a}$$

$$p_2(x_t, y_t; x_{t-1}, y_{t-1}; y_{t-2}) = Z(\lambda_t, \gamma_t, \lambda_{t-1})x_t \tag{1.10b}$$

$$q(x_t, y_t; x_{t-1}, y_{t-1}; y_{t-2}) = G(\lambda_t, \gamma_t, \lambda_{t-1})\frac{x_t}{y_t} \tag{1.10c}$$

The relative prices of both risky securities, in this economy, are multiples of consumption of the non-durable good while the relative price of the durable good is a multiple of the ratio of the non-durable consumption to the durable consumption. All the multipliers are determined by the state in the joint growth process.

The return of the security of the non-durable good producing firm from period t to $t+1$ is

$$r^1_{t,t+1} = \frac{p_{1,t+1} + x_{t+1}}{p_{1,t}} - 1. \tag{1.11}$$

The return of the security of the durable good producing firm in the same period as above is

$$r^2_{t,t+1} = \frac{p_{2,t+1} + y_{t+1}\, q_{t+1}}{p_{2,t}} - 1. \tag{1.12}$$

As a consequence, the return on the market portfolio (i.e. holding all equities in both shares) is

$$1 + r^M_{t,t+1} = \frac{p_{1,t+1} + x_{t+1} + p_{2,t+1} + y_{t+1}\, q_{t+1}}{p_{1,t} + p_{2,t}}. \tag{1.13}$$

A risk-free security, which guarantees one unit of the perishable good in the next period, can be priced following the same fashion. The net marginal utility loss in period t from postponing units of the non-durable consumption which equal to the price of the risk-free security is the same as the net marginal utility gain in period $t+1$ from one extra unit of consumption of the perishable good associated with the ownership of the risk-free security. The necessary and sufficient first order condition for the risk-free security is

$$(u_t^c - E[\beta\xi u_{t+1}^c])p_{f,t} = E[\beta u_{t+1}^c - \beta^2\xi u_{t+2}^c]. \tag{1.14}$$

Accordingly, the price of the risk-free security and the one period risk-free rate are

$$p_{f,t} = E(\frac{\beta u_{t+1}^c - \beta^2\xi u_{t+2}^c}{u_t^c - \beta\xi u_{t+1}^c}) \tag{1.15}$$

and

$$r_{t,t+1}^f = \frac{1}{p_{f,t}} - 1, \tag{1.16}$$

respectively.

1.3 Calibration and Numerical Results

The transition matrix M in (1.8), the growth rate for the non-durable, λ_1 and λ_2, and the growth rate in the durable good, γ_1 and γ_2, are chosen to match the first two moments and the autocorrelations of the joint process of the growth rates of the output of the two goods. A detailed explanation is included in Appendix C. We have $\lambda_1 = 1.036$, $\lambda_2 = 1.014, \gamma_1 = 1.133, \gamma_2 = 0.991$ and the transition matrix for the U.S. economy:

$$M = \begin{array}{c} \\ (\lambda_1,\gamma_1) \\ (\lambda_1,\gamma_2) \\ (\lambda_2,\gamma_1) \\ (\lambda_2,\gamma_2) \end{array} \begin{array}{c} \begin{array}{cccc} (\lambda_1,\gamma_1) & (\lambda_1,\gamma_2) & (\lambda_2,\gamma_1) & (\lambda_2,\gamma_2) \end{array} \\ \begin{bmatrix} 0.5182 & 0.0750 & 0.0070 & 0.3997 \\ 0.7750 & 0.1682 & 0.0497 & 0.0070 \\ 0.0070 & 0.0497 & 0.1682 & 0.7750 \\ 0.3997 & 0.0070 & 0.0750 & 0.5182 \end{bmatrix} \end{array} \tag{1.17}$$

We construct artificial consumption paths for both the durable good and the non-durable good for 100,000 periods using the parameters specified above. When $\xi = \eta = \Omega = 0$, the model reduces to a two-perishable-good model in the absence of habit formation, where preferences are time separable. This is the two goods case corresponding to the Mehra and Prescott (1985) framework. When $\xi = \eta = 0$ and $\Omega > 0$, the model is the general case of problem (1) in Giannikos (2004), which captures the durability in one of the consumption goods. When $\xi > 0, \eta > 0$ and $\Omega = 0$, the model applies to an economy with two non-durable goods with habit formation.

Table 1.1: Summary Return Statistics: Comparison Among Submodels
All Returns and standard deviations are in percentage.

(1)	U.S. Data (2)	Two Perishable Goods $\xi = \eta = 0$ $\Omega = 0$ (3)	One Perishable Good and One Durable Good $\xi = \eta = 0$ $\Omega = 0.2$ (4)	Two Perishable Goods With Habit Formation $\xi = \eta = 0.5$ $\Omega = 0$ (5)	One Perishable Good and One Durable Good With Habit Formation $\xi = \eta = 0.5$ $\Omega = 0.1$ (6)
E(Re)	6.98	17.28	17.32	16.31	15.22
S.D.(Re)	16.54	3.78	3.94	32.95	25.16
E(Rf)	0.8	17.21	17.4	7.75	9.08
S.D.(Rf)	5.67	2.99	3.22	23.97	17.94
E(RP)	6.18	0.07	−0.08	8.56	6.13
S.D.(RP)	16.67	2.28	2.26	21.26	16.85

Table 1.1 summarizes the first two moments for returns on the risky portfolio, the risk-free security and the market risk premium (R_p) under various parameter configurations. It shows that when the utility is time-separable, the two-good economy is not volatile enough to warrant a high premium. The durability not only smooths the exogenous volatility of the durable through the service flow but also provides a useful hedging tool for the durable good consumption. Sometimes the durability feature can make the risky security very desirable and leads to a negative premium, as is the case in the fourth column. With habit formation, the representative agent

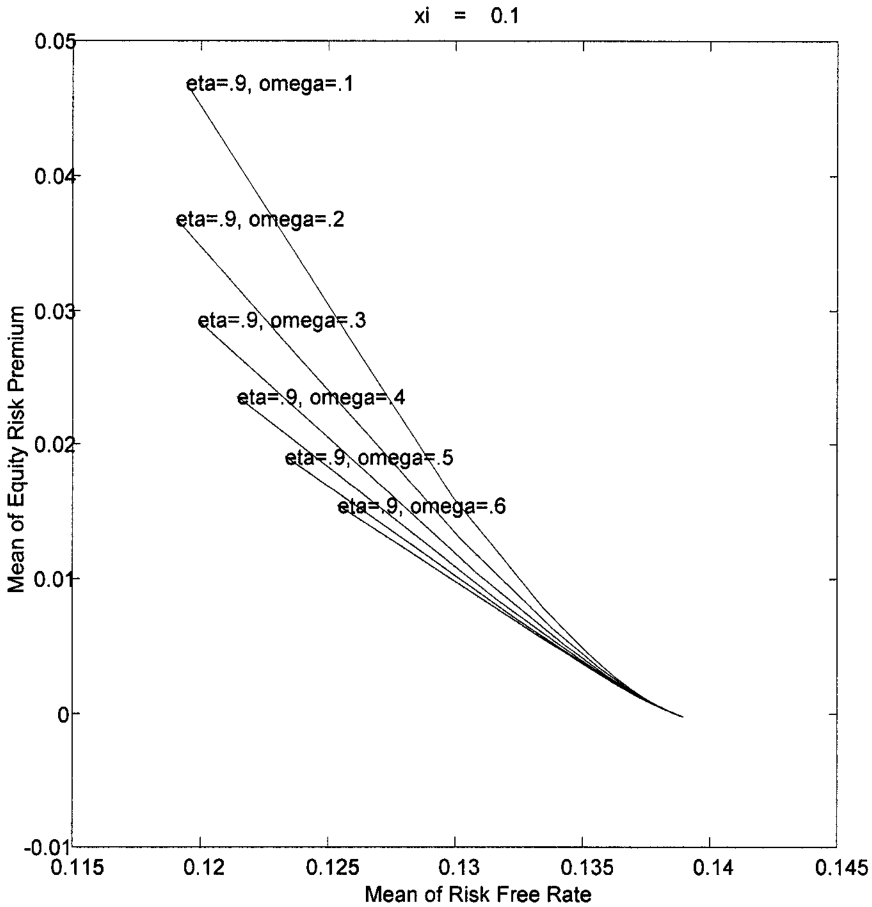
is only concerned about his surplus consumption above the habit which is much more volatile than his absolute consumption. The induced higher effective risk aversion increases the risk premium. In the fifth column, we find that the model can easily explain the risk premium with relative low habit persistence levels of 0.5. However, the standard deviations of both risky returns and the risk-free rate are too high as compared to the U.S. data. Finally, the case that incorporates habit formation and durability (Column 6) helps to reduce the volatilities.

We try different combinations of habit persistence levels and durability and report the dynamics of the average risk-free returns, returns on the market portfolio and equity risk premia in Figure 1.1, Figure 1.2 and Figure 1.3. The standard deviation of returns on all securities are reported in Figure 1.4 and Figure 1.5. To avoid inifinite marginal utility, we restrict the habit persistence levels in the consumptions of both goods to be less or equal to 0.9.

Examining all the subfigures in Figure 1.1 and Figure 1.2, we find that higher durability decreases the equity risk premium. The durability feature makes the representative agent's effective risk aversion smaller by smoothing the service flows and results in lower risky returns and a higher risk-free rate.

The dynamics of the returns on risky and risk-free securities are worth noting. They result from the interaction between the durable good and non-durable good in the agent's utility function. While the risk-free rate is always decreasing in the habit persistence levels of either good (Figure 1.1, Figure 1.2), the dynamics of the return on the risky portfolio are mixed when habit persistence levels change (Figure 1.3). The interaction between the durable good consumption and the perishable good consumption causes the returns on the risky portfolio to increase in ξ when the agent is highly risk averse in his durable good consumption and to decrease otherwise. The dynamics of the returns of the risky portfolio are not only the result from the interaction between the durable good consumption and the non-durable good consumption, but also the result from the contradicting effects from the habit formation of the durable good (for destabilizing the service flow) and the durability (for stabilizing the service flow). The returns on the risky portfolio are decreasing when the stabilizing effect is dominating and increasing when the destabilizing effect is dominating.

Figure 1.1: Dynamics of the Risk-free Rate and the Equity Premium - The Impact of Durability and Habit Persistence Level of the Durable Good
For all the subfigures in this section, relative risk aversion, $\nu = 3$; preference parameter $\delta = 0.75$. Omega is the durability of the durable good, Ω. Eta is the habit persistence level of the durable good, η. Xi is the habit persistence level of the perishable good, ξ.



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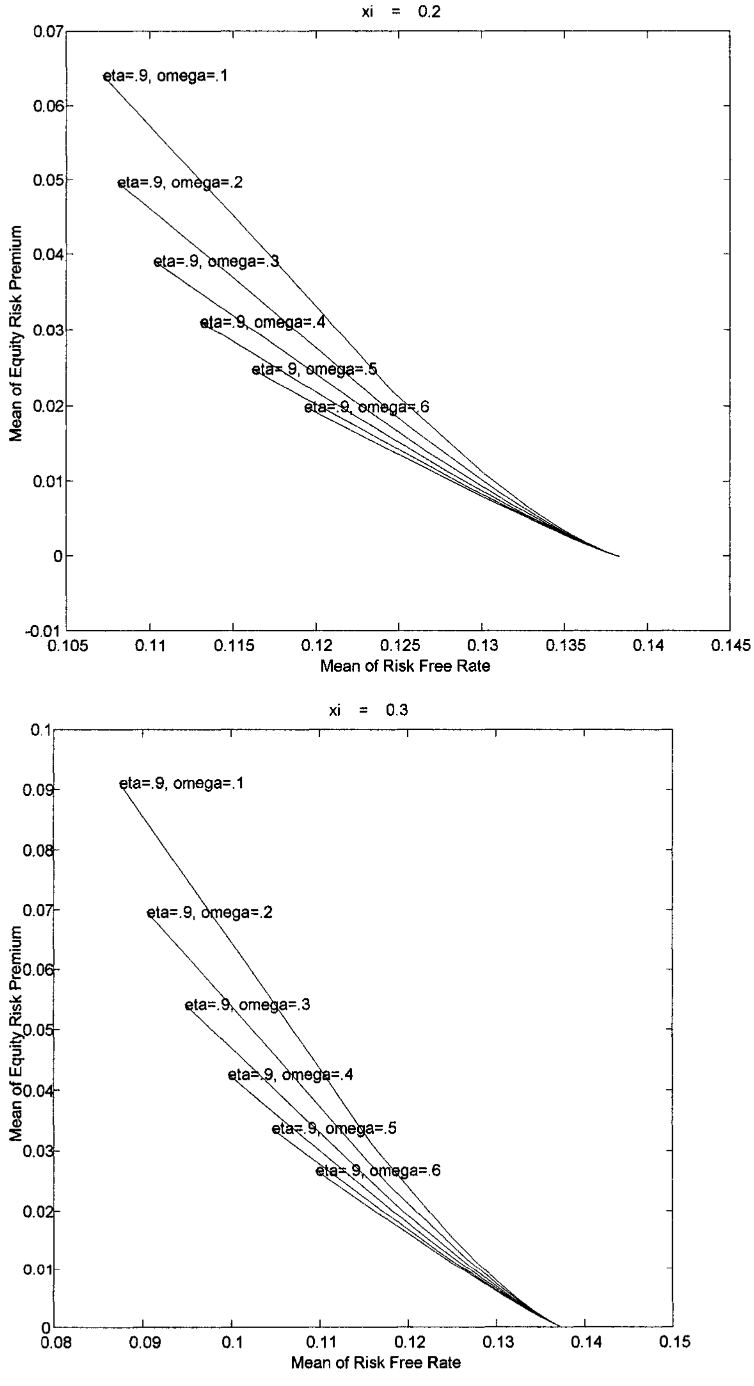

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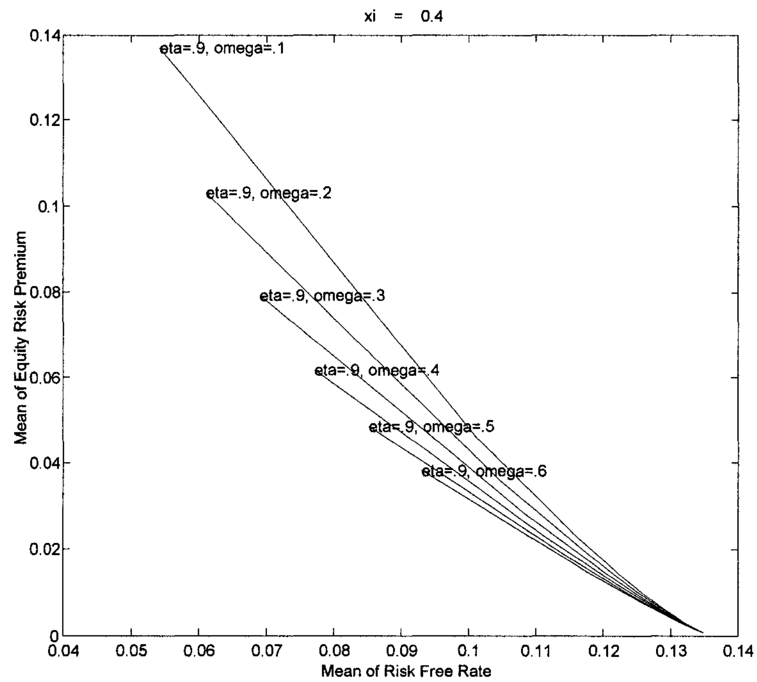

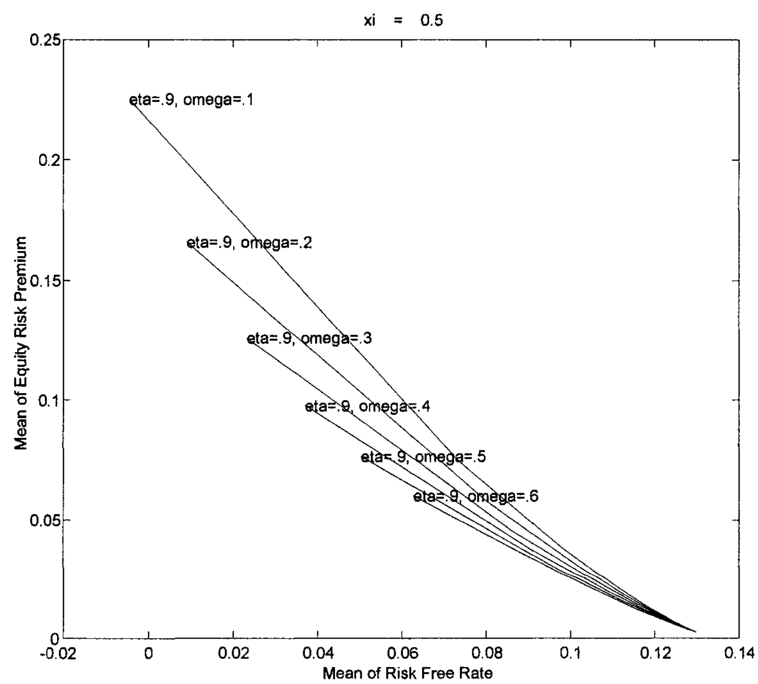

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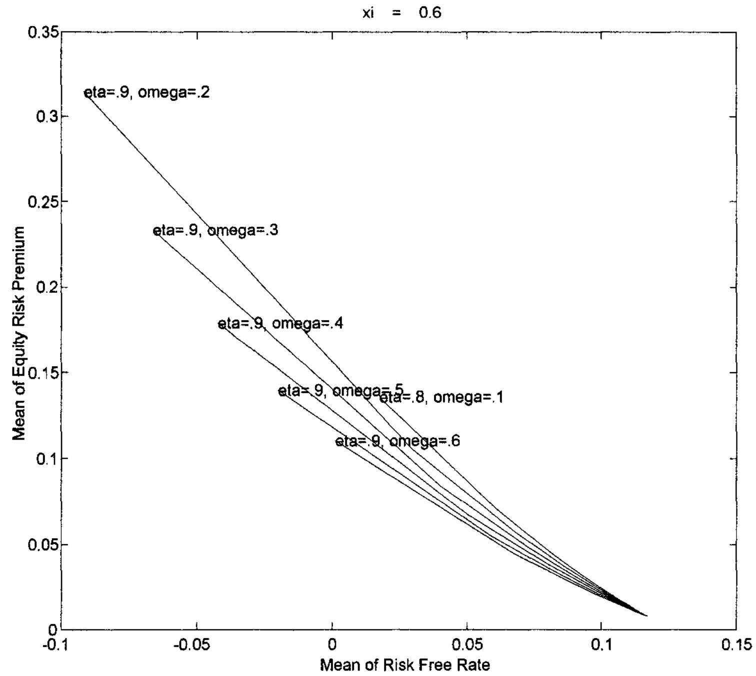


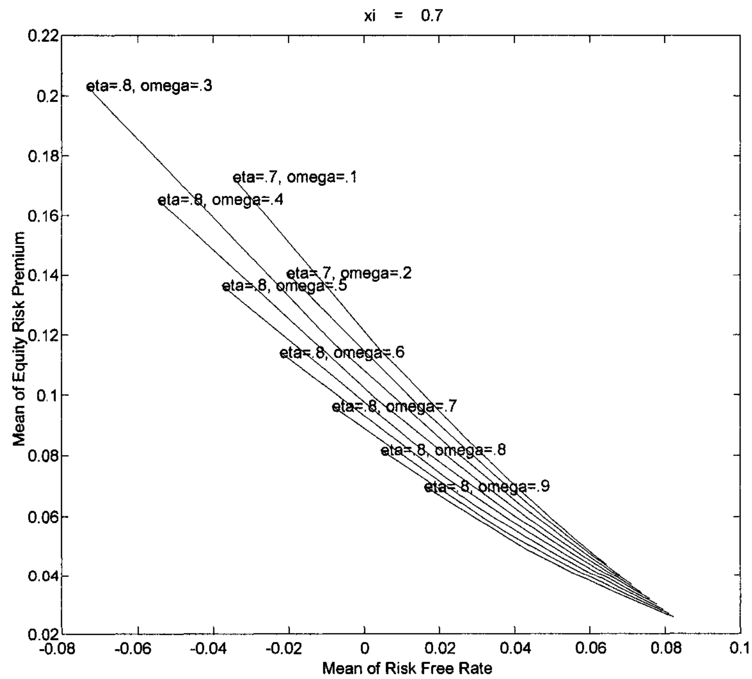
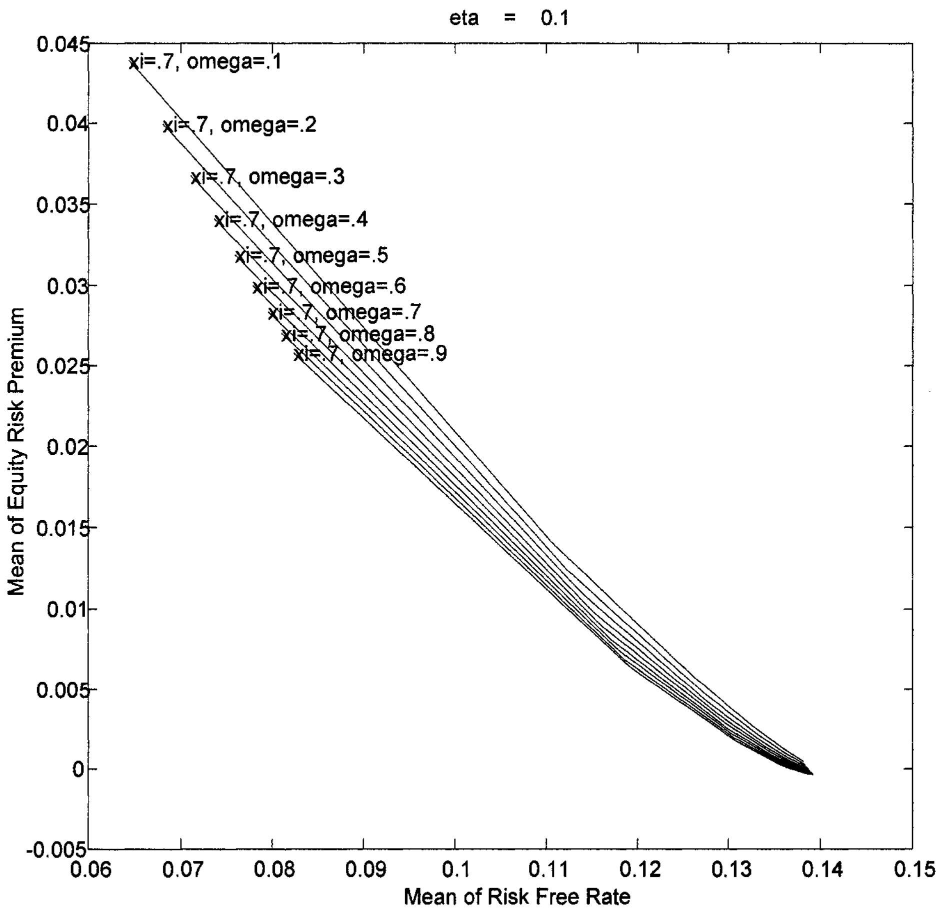


Figure 1.2: Dynamics of the Risk-free Rate and the Equity Premium - The Impact of Durability and Habit Persistence Level of the Non-durable Good
For all the subfigures in this section, relative risk aversion, $\nu = 3$; preference parameter $\delta = 0.75$. Omega is the durability of the durable good, Ω. Eta is the habit persistence level of the durable good, η. Xi is the habit persistence level of the perishable good, ξ.



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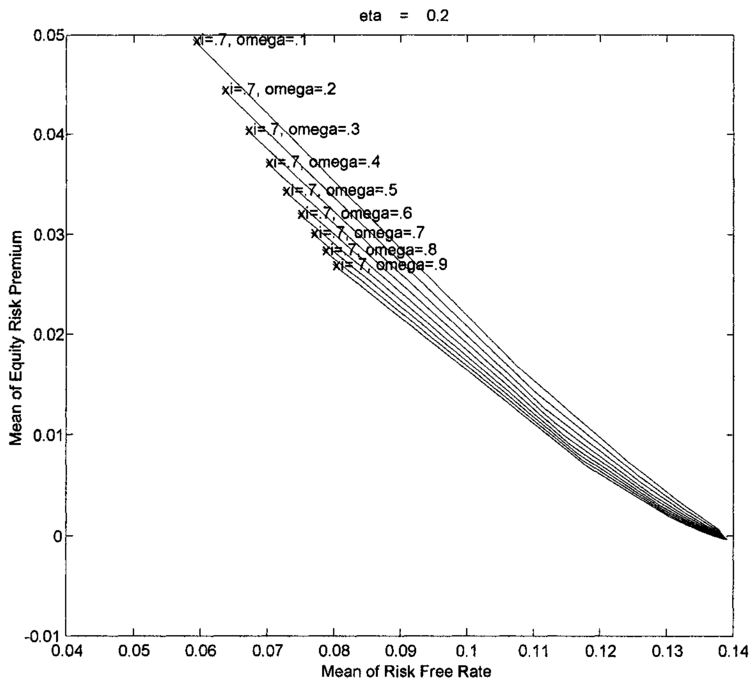

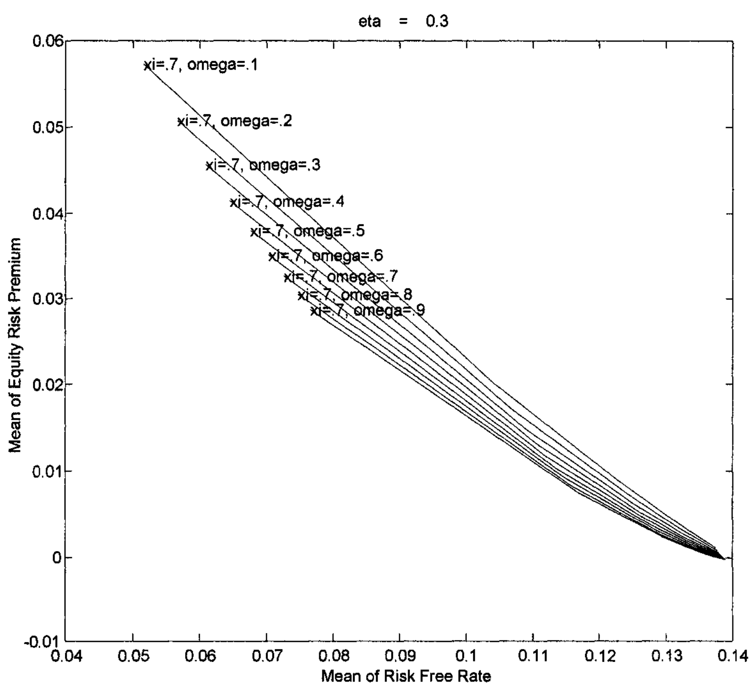

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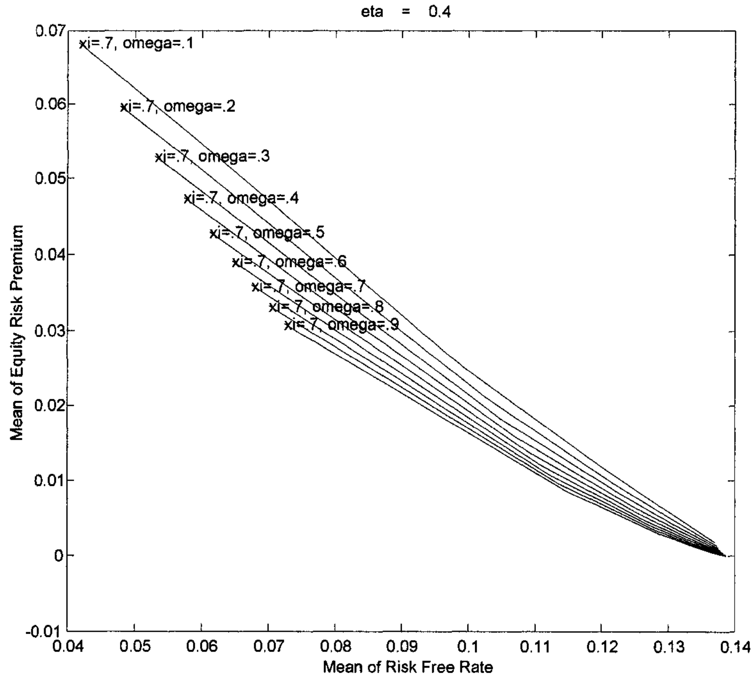

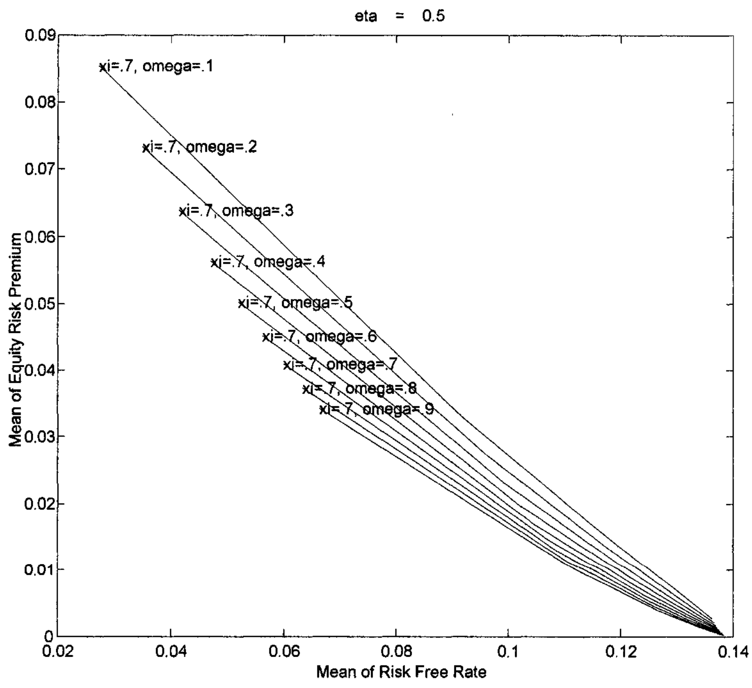

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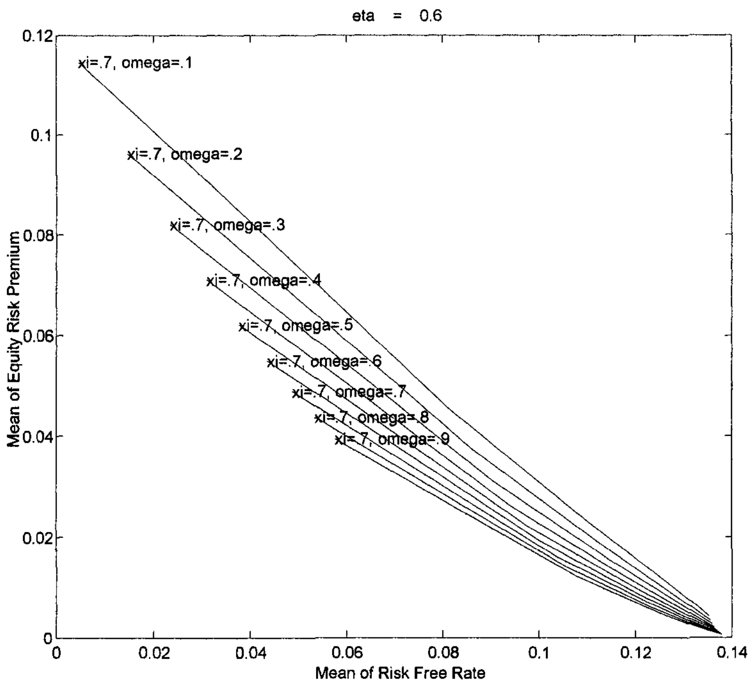

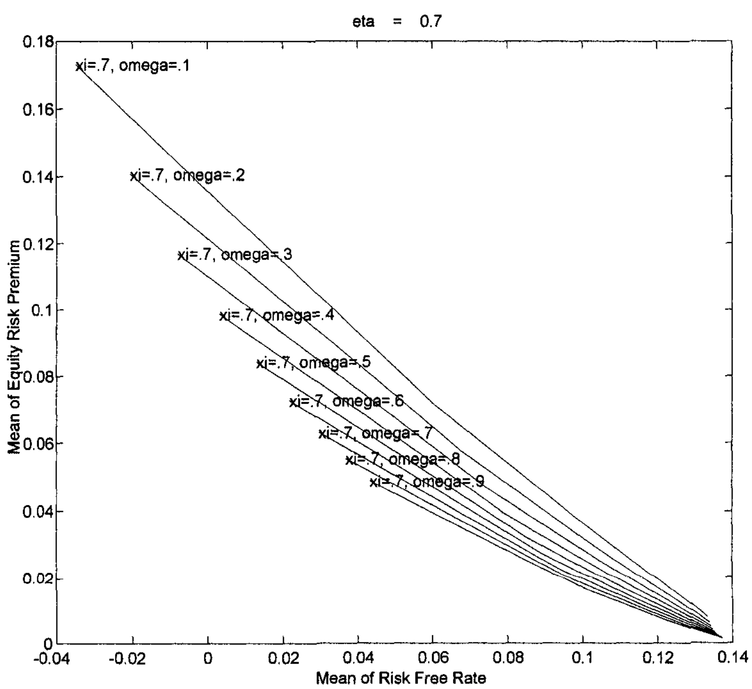

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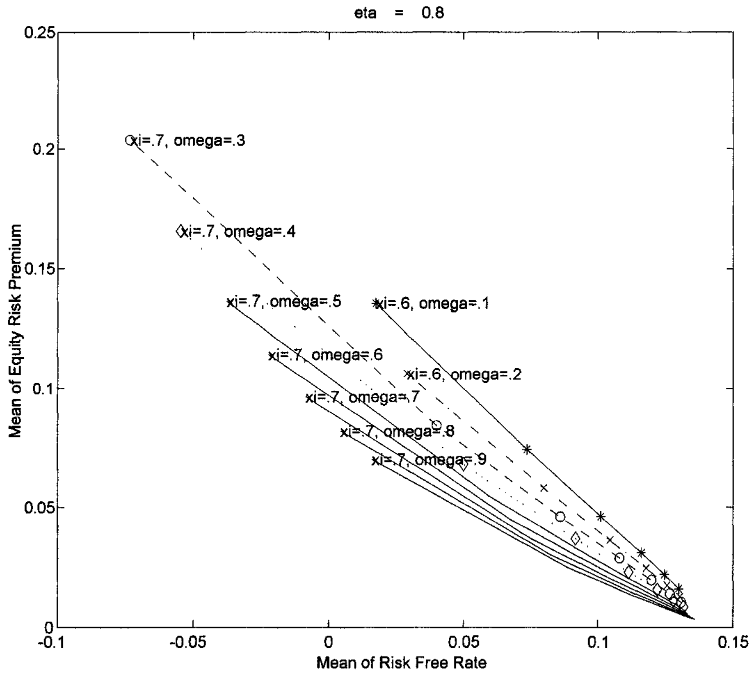


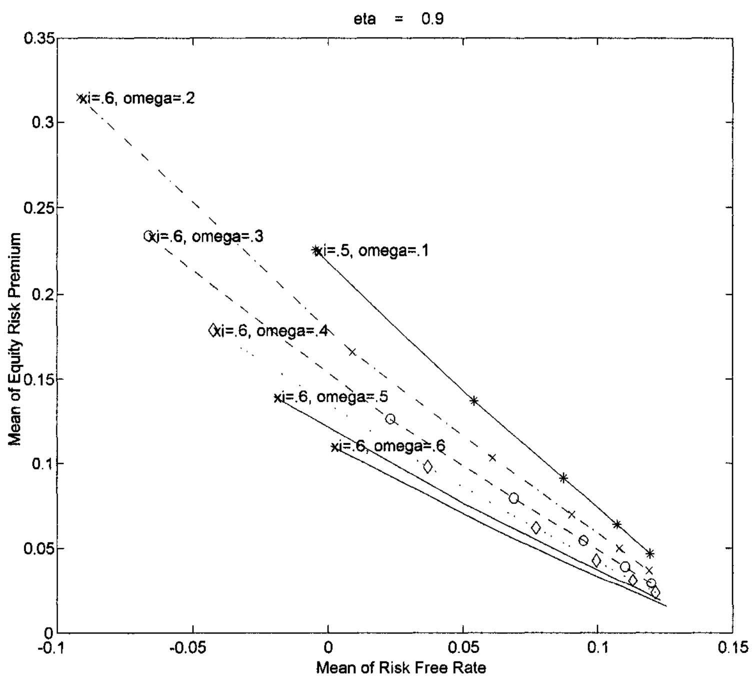
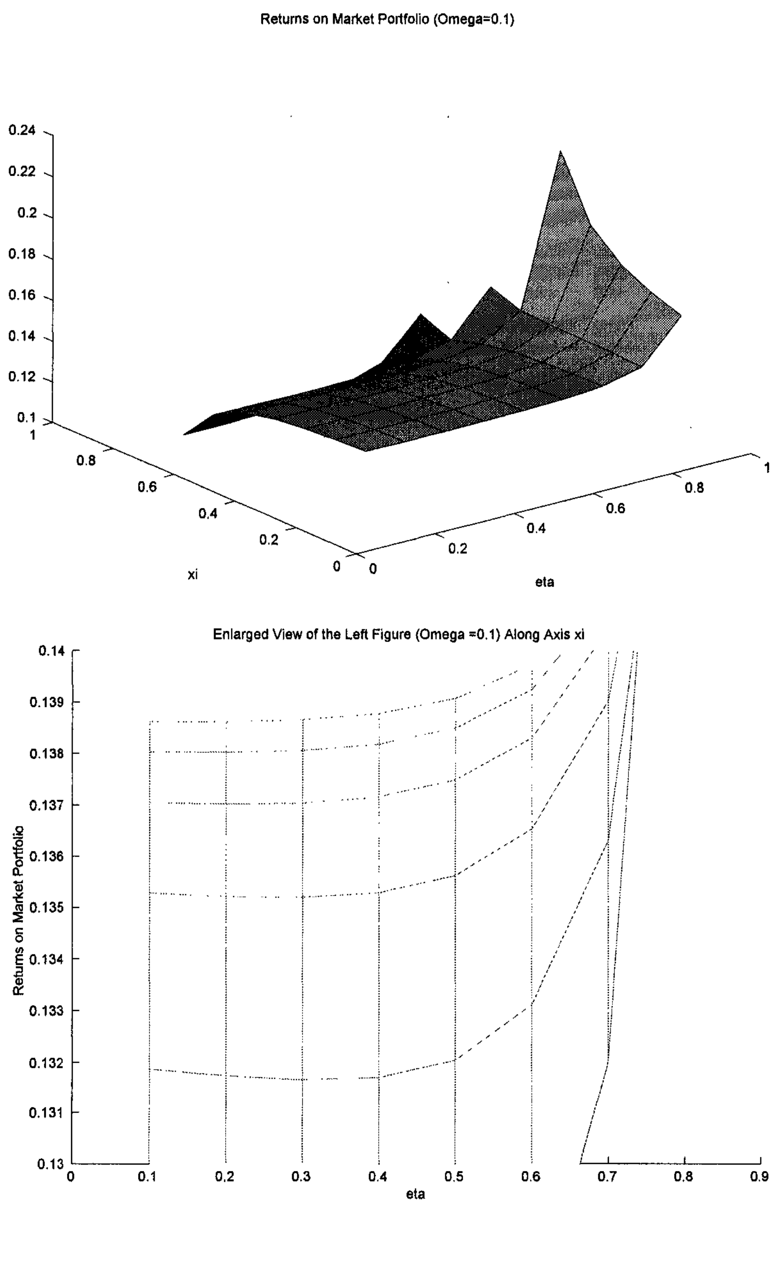


Figure 1.3: Dynamics of the Returns on the Market Portfolio - The Impact of Durability and Habit Formation

For all the subfigures in this section, relative risk aversion, $\nu = 3$; preference parameter $\delta = 0.75$. Omega is the durability of the durable good, Ω. Eta is the habit persistence level of the durable good, η. Xi is the habit persistence level of the perishable good, ξ.



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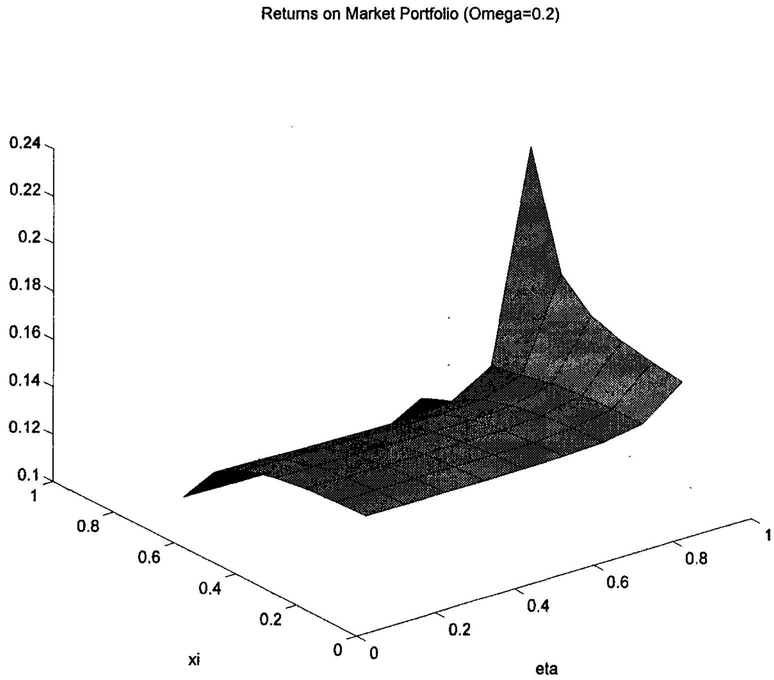

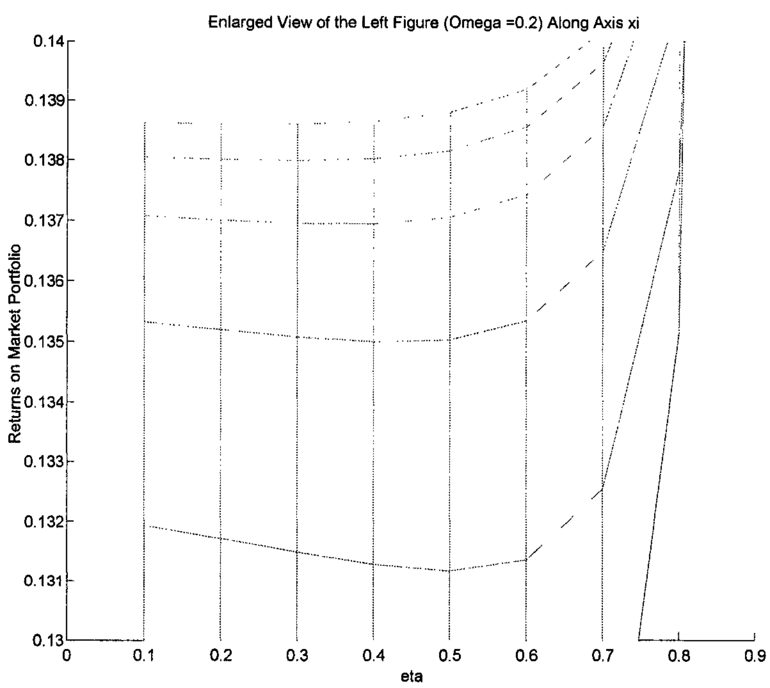

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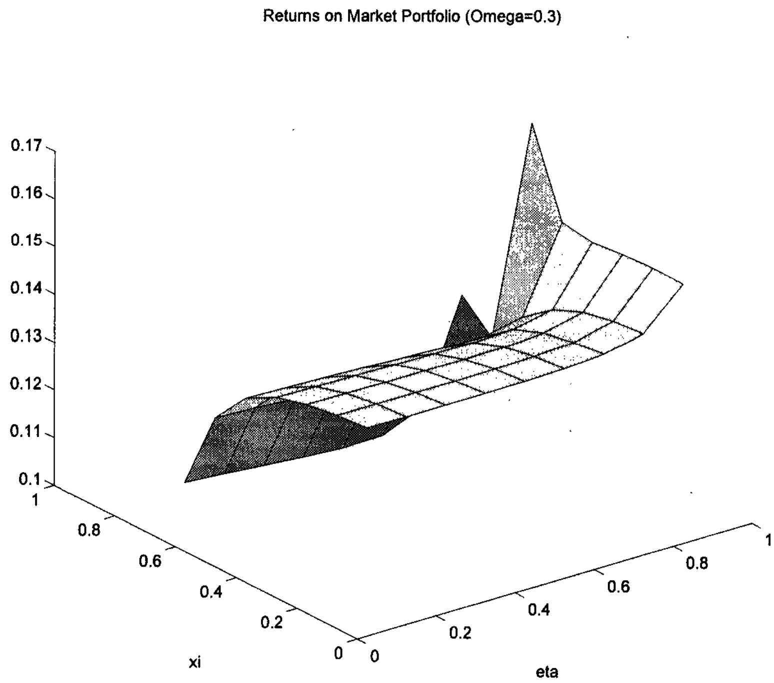

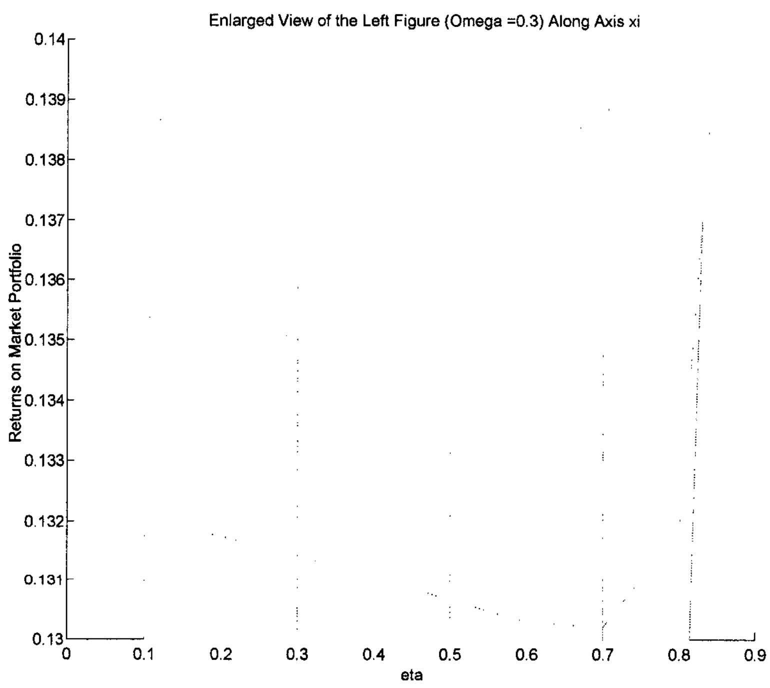

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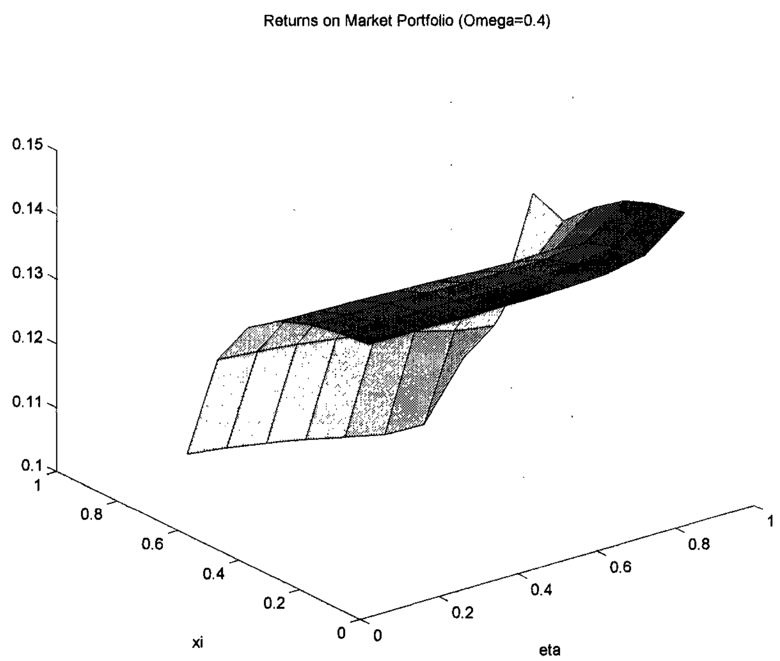

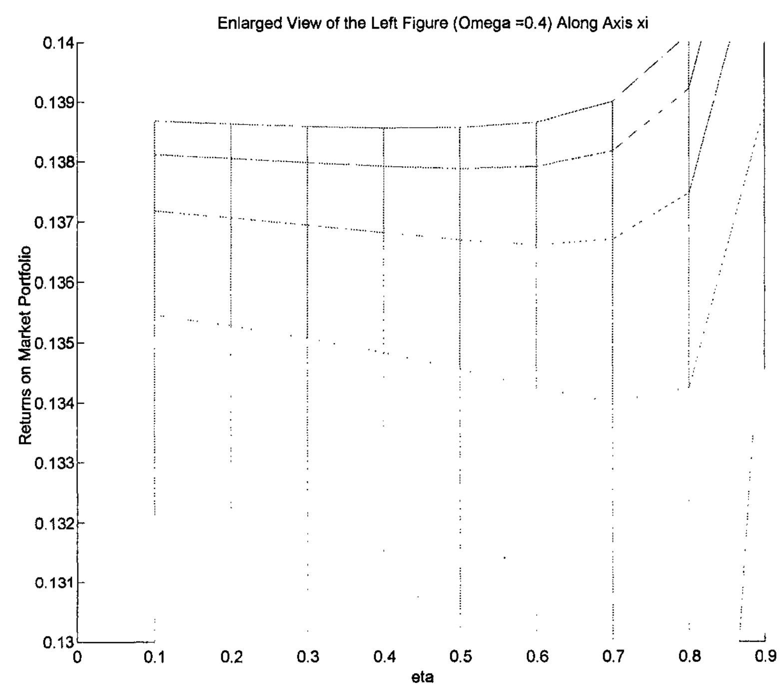

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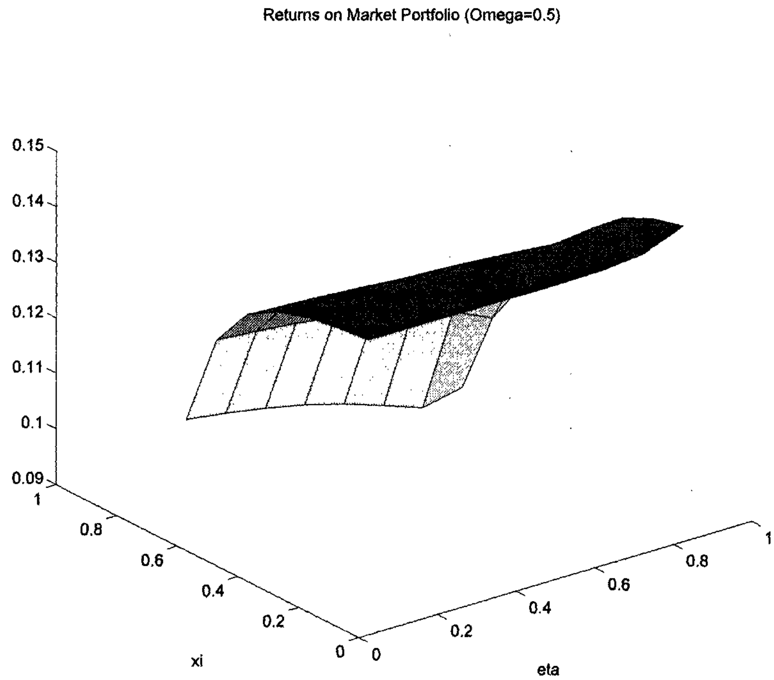

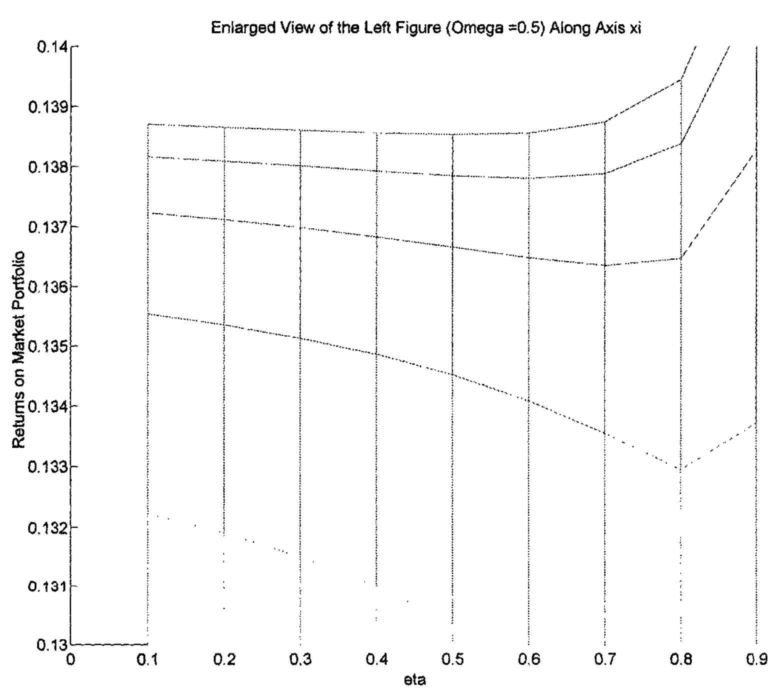

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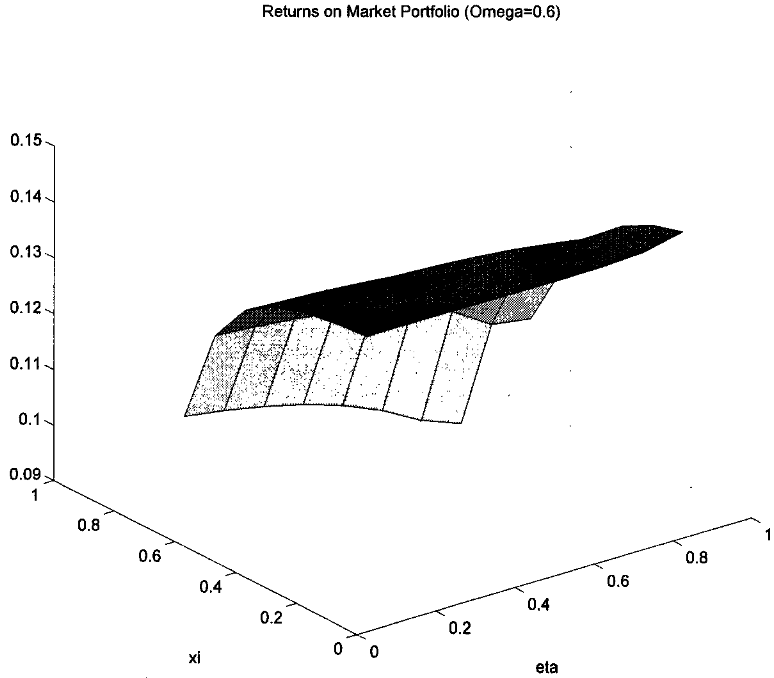

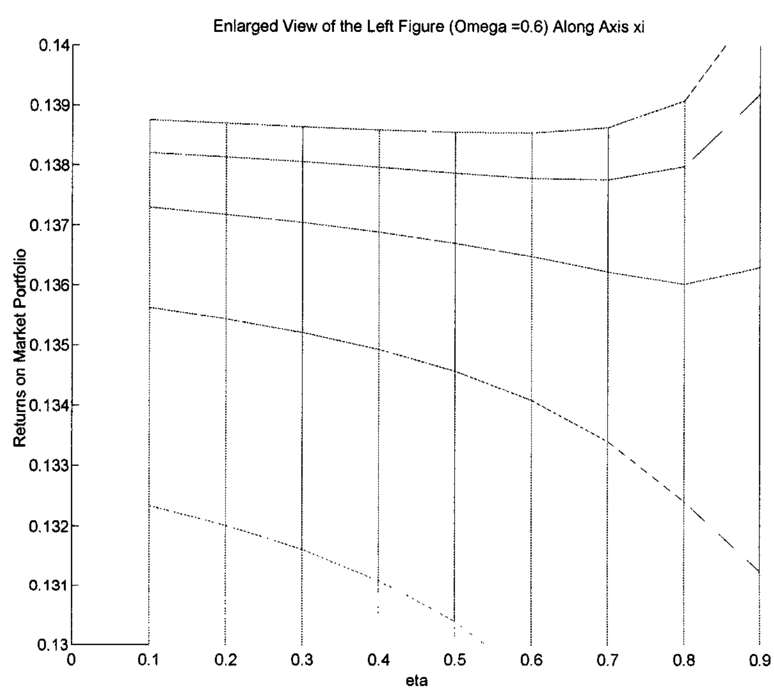

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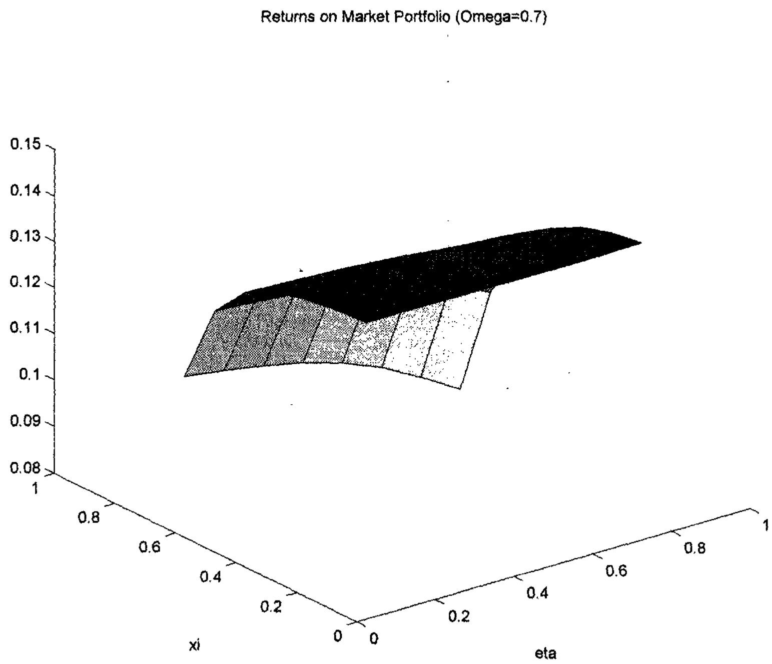

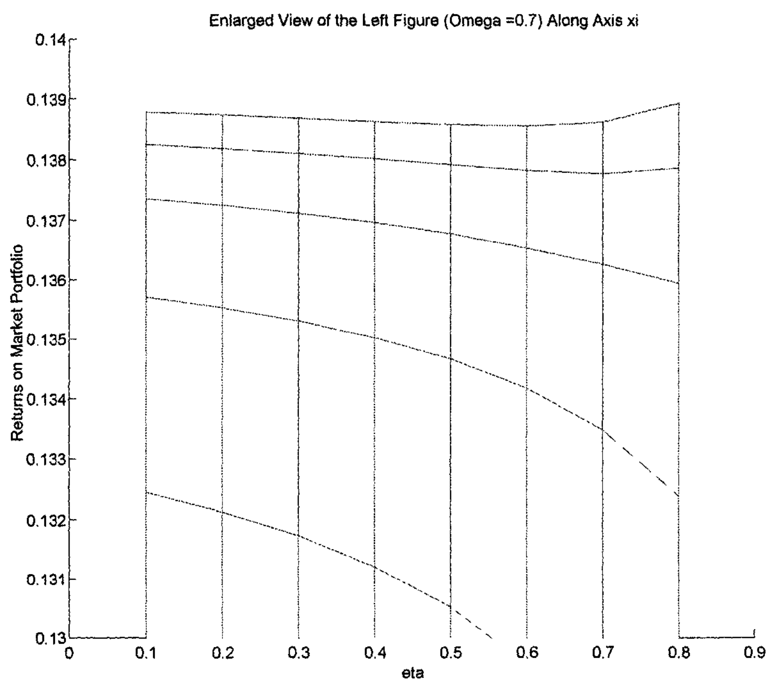

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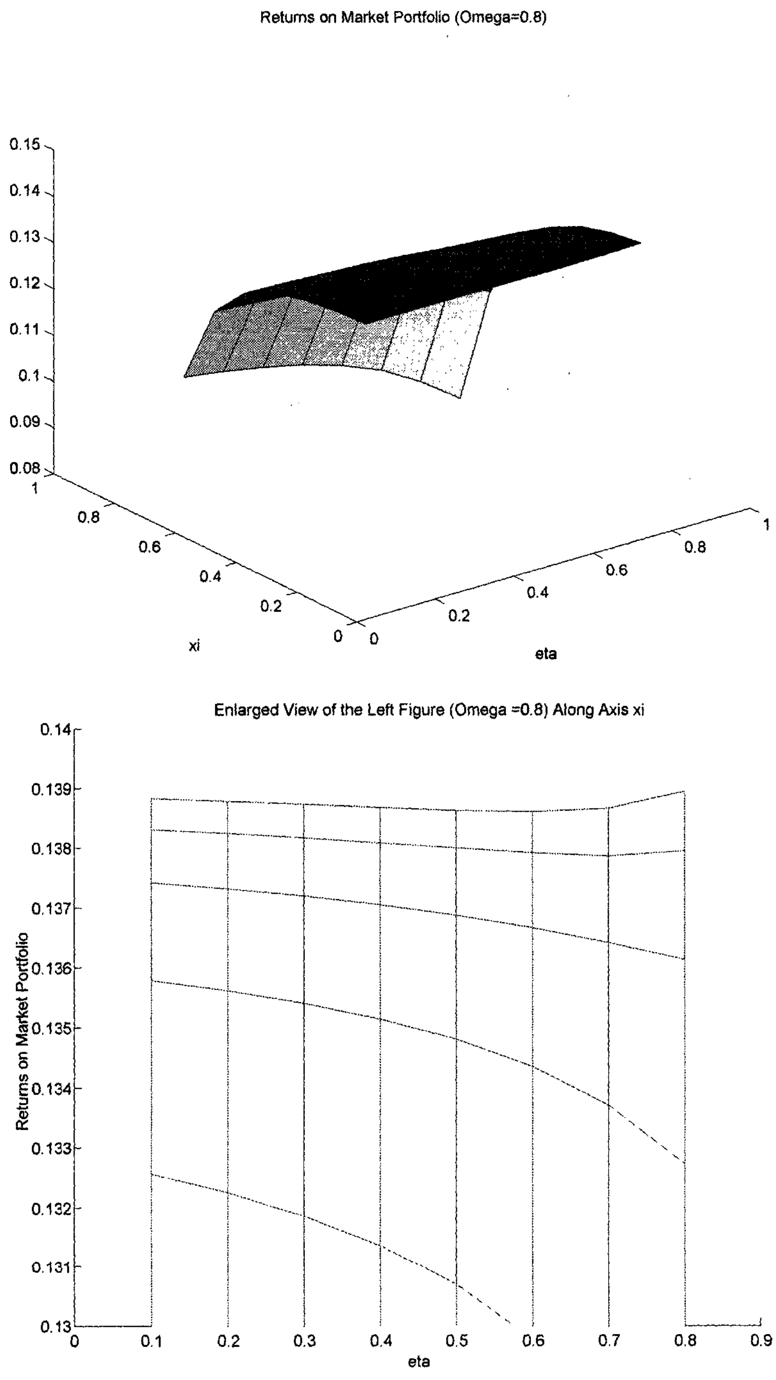

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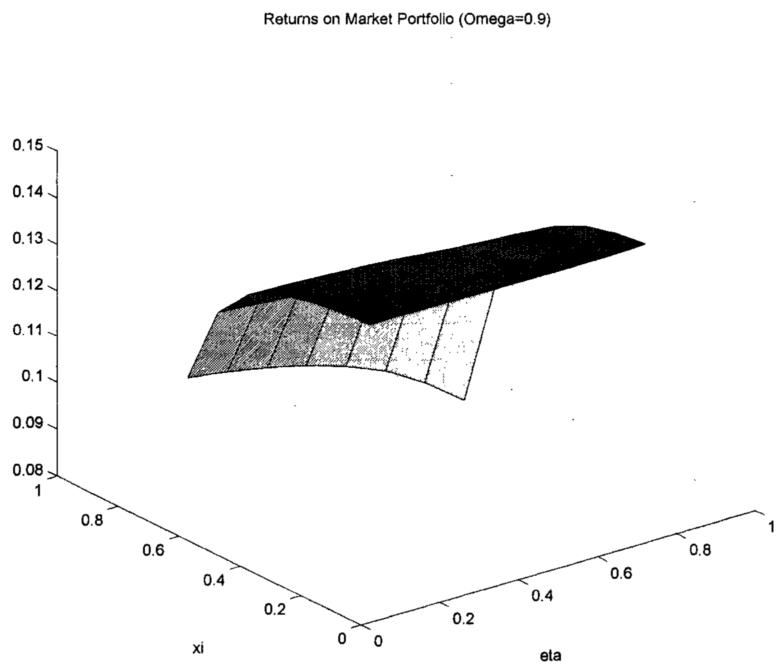
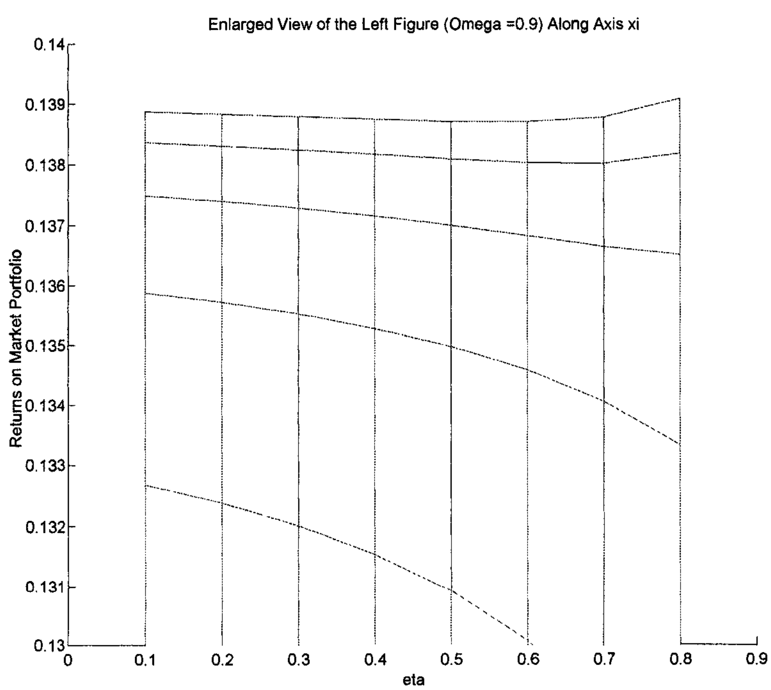




The reason for the decreases in the risk-free rate when habit persistence level parameters for both goods increase is obvious. As ξ or η increases, the agent is more reluctant to face the volatility in the surplus consumptions, the risk-free security that ensures the agent with the non-durable the next period is the safest tool to reduce the downside risk in the surplus consumption of the non-durable good. Higher demand generates lower returns.

When η is extremely high ($\eta = 0.8$ and $\eta = 0.9$) and Ω is low ($\Omega < 0.5$), the agent is highly averse to the volatility in the surplus service flow (above the habit persistence level) from the durable good. As ξ increases, his increased aversion to the volatility in the perishable good consumption will force the agent to avoid additional risky positions and demand more of the risk-free security. The lower demand for the risky portfolio results in higher returns. When η is not high enough ($\eta < 0.8$) or the durable good is more durable ($\Omega \geq 0.5$), the surplus service flow from the durable good is less volatile and there is more room for the risk tolerance for the surplus consumption of the perishable good. When the only unit of the perishable consumption in the next period guaranteed by the risk-free security is not enough to satisfy the growing appetite for the perishable good (as ξ increases), the demand for the risky portfolio increases and drives its return down. This effect is more profound as the durability of the durable good increases. This observation confirms the interaction between the durable good and non-durable good in the economy – when the durable good is relatively safe for the agent, it provides comfort to the agent in non-durable consumption.

How do we explain the relation between η and the returns on the market portfolio? When η increases, there are two conflicting effects. On the one hand, the agent becomes more risk averse to the future surplus consumption of the durable good. The risky portfolio becomes very unattractive and he demands a higher return to hold it. On the other hand, the durability of the durable good makes the service flow less risky since it is the weighted average of two periods' purchases of the durable good. This effect will reduce the return of the risky portfolio. When the habit effect dominates, we observe that the return on the risky portfolio increases as η increases. The opposite will happen when the durability effect dominates. Examining from Figure 1.3, we find that the decreases in the risky returns when the relatively low η

increases are very small.[1] With $\Omega < 0.5$, it is more obvious that the risky returns increase as the relatively high η increases. From the enlarged view of 3D plots alone Axis xi (ξ) in Figure (1.3), we can discern the 'U' shape pattern of risky returns when η increases. The decreasing pattern of the risky returns lasts longer as durability increases or as the agent's utility derives more from the service flows from the durable good[2] and / or when the position of the non-durable consumption is more risky. When durability makes the service flow from the durable good more smooth with $\Omega \geq 0.5$, with riskier position in consumption of the non-durable good, the agent benefits more from the decreased volatility in consumption and demand less return as η increases.

As shown in Figure 1.4 and Figure 1.5, volatilities of the returns in all the securities increase as the habit persistence levels increase, and decrease as the durability increases.

We are quite satisfied with the mean and standard deviation of equity risk premium generated from the two-good model with durability and habit formation. It would be interesting to see whether this model is also a better representation of the

[1] Some of the returns on the market portfolio only increase by 1/100,000 when habit persistence level of durable consumption increases by 10 percentage points.

[2] Not reported in this paper.

Figure 1.4: Volatility of the Returns on the Market Portfolio

For all the subfigures in this section, relative risk aversion, $\nu = 3$; preference parameter $\delta = 0.75$. Omega is the durability of the durable good, Ω. Eta is the habit persistence level of the durable good, η. Xi is the habit persistence level of the perishable good, ξ.

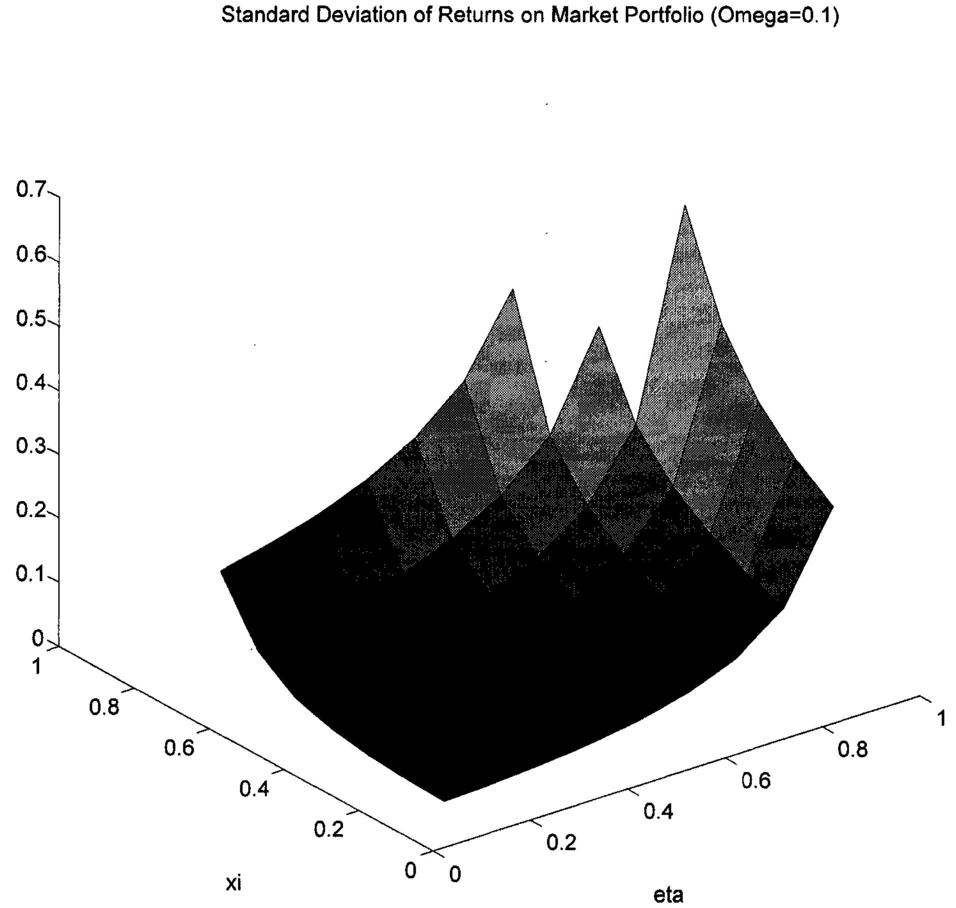
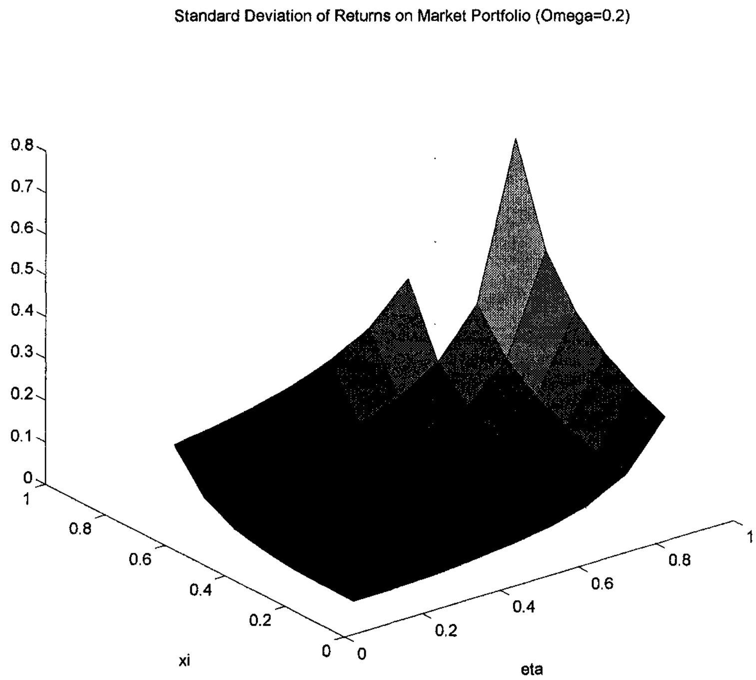
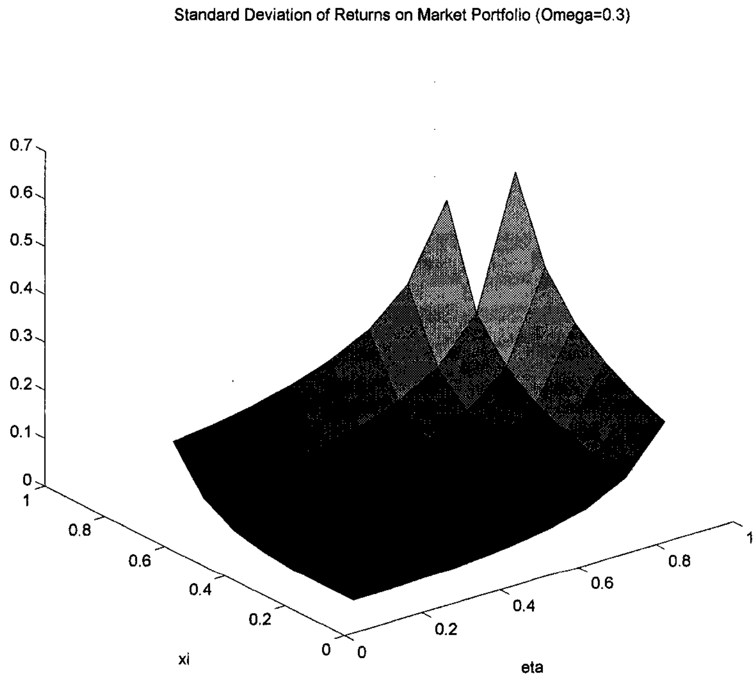


Figure 1.4: continued





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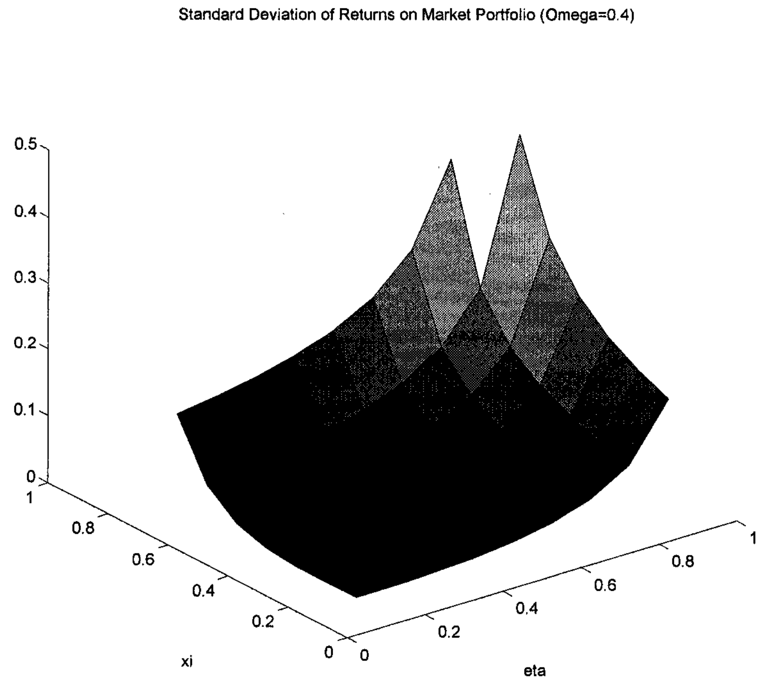

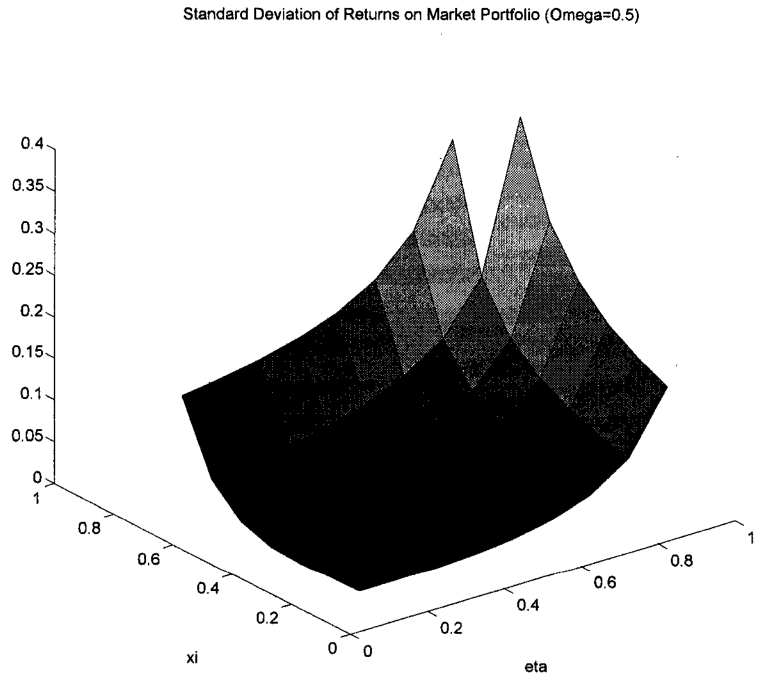

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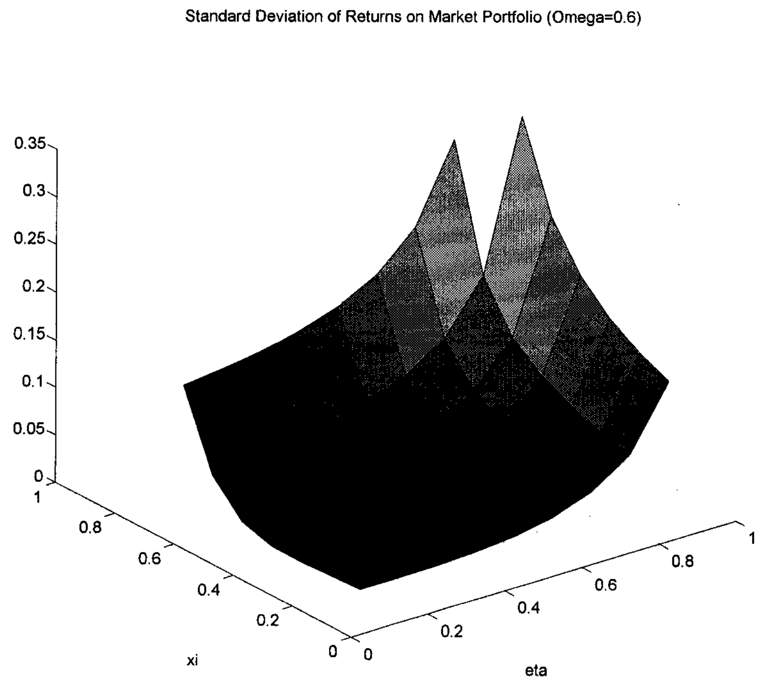


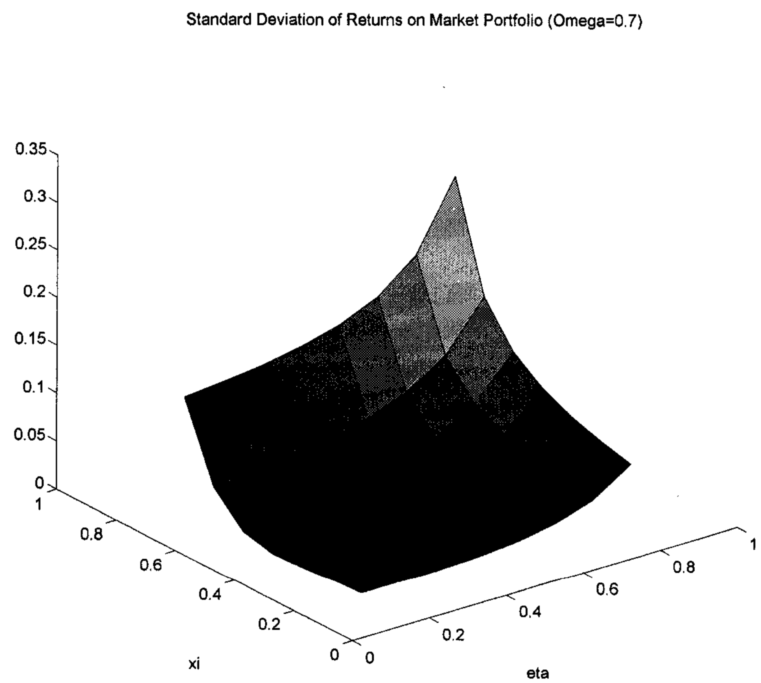
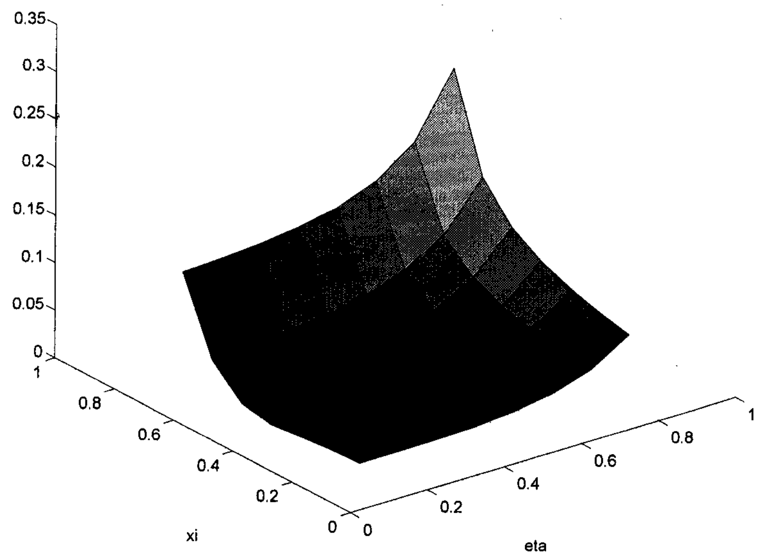


Figure 1.4: continued

Standard Deviation of Returns on Market Portfolio (Omega=0.8)



Standard Deviation of Returns on Market Portfolio (Omega=0.9)

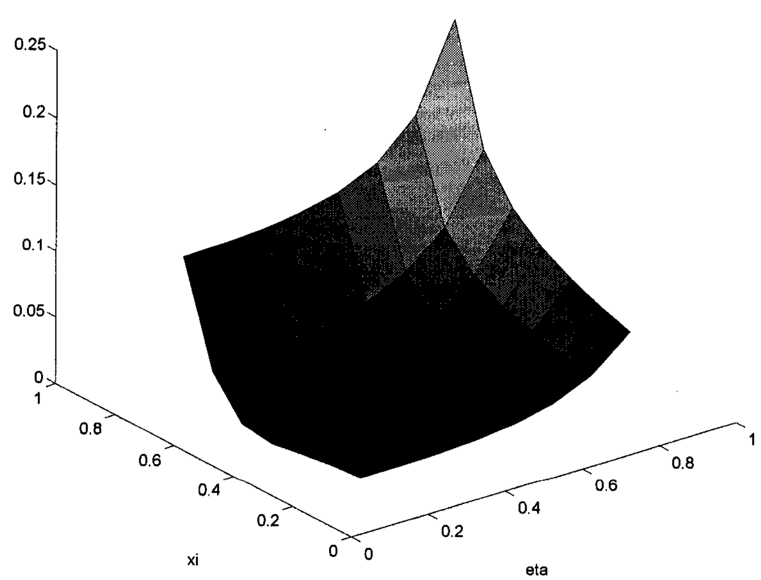
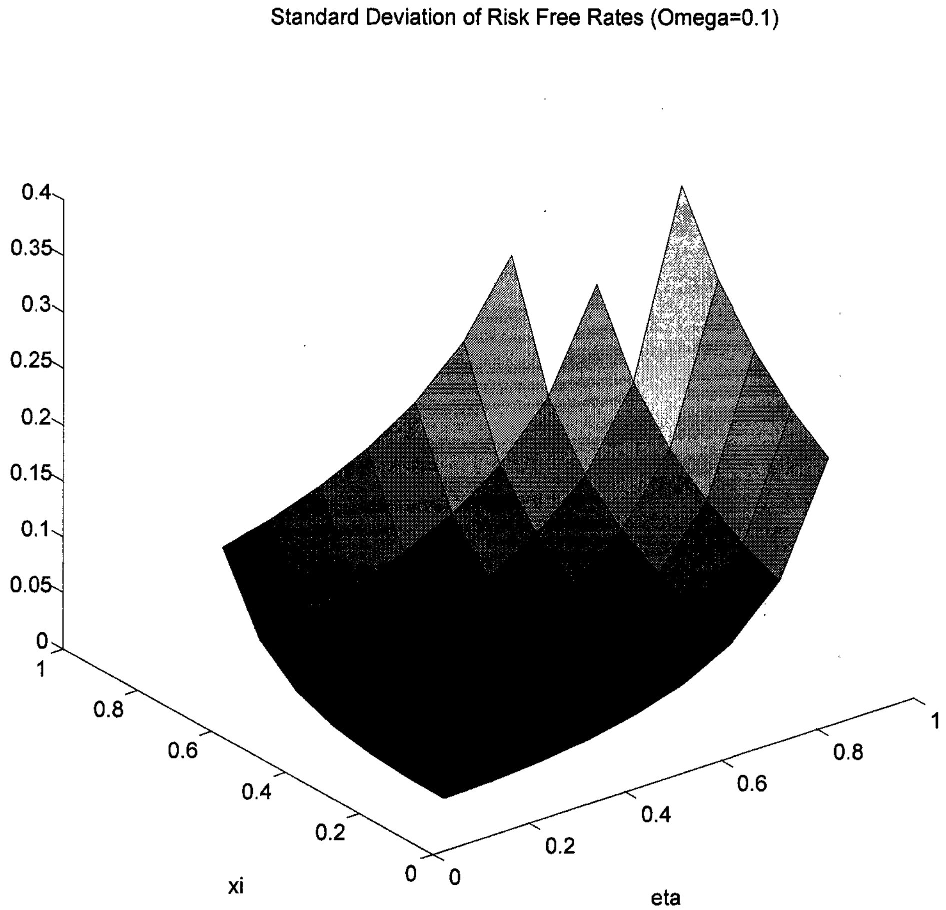


Figure 1.5: Volatility of the Risk-free Rates

For all the subfigures in this section, relative risk aversion, $\nu = 3$; preference parameter $\delta = 0.75$. Omega is the durability of Durable good, Ω. Eta is the habit persistence level of durable good, η. Xi is the habit persistence level of perishable good, ξ.



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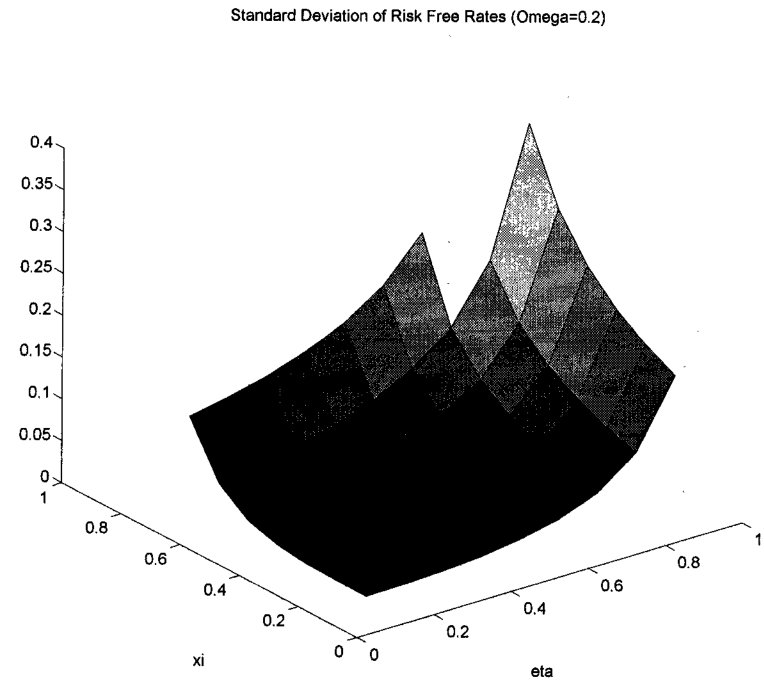


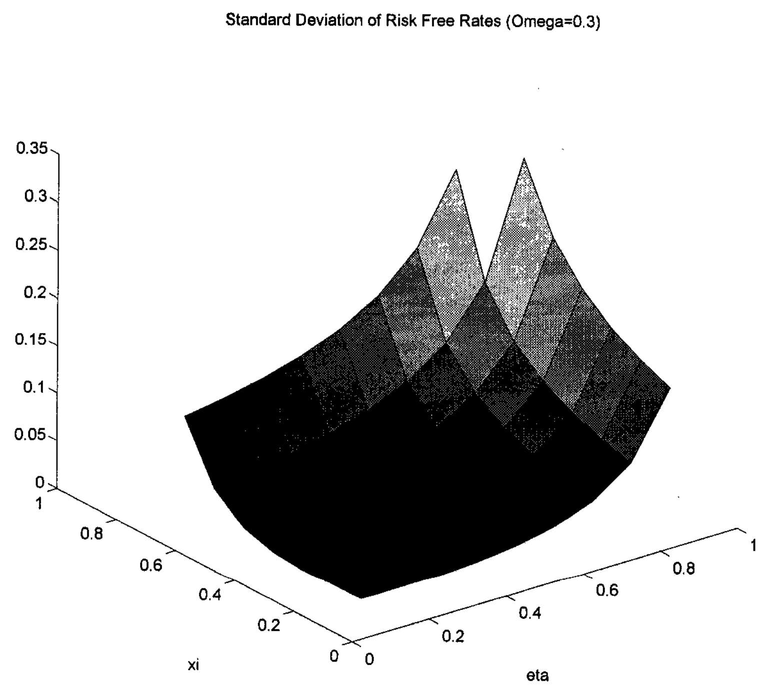
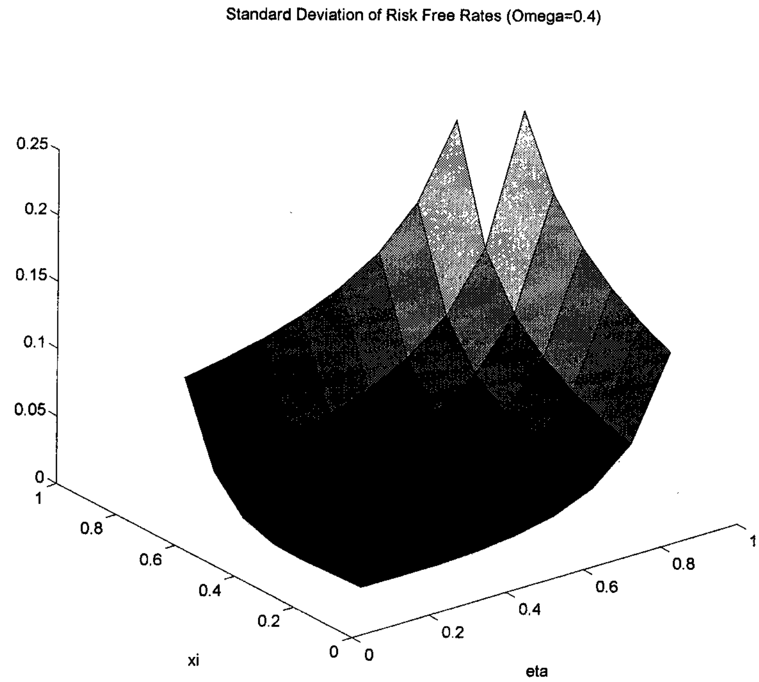
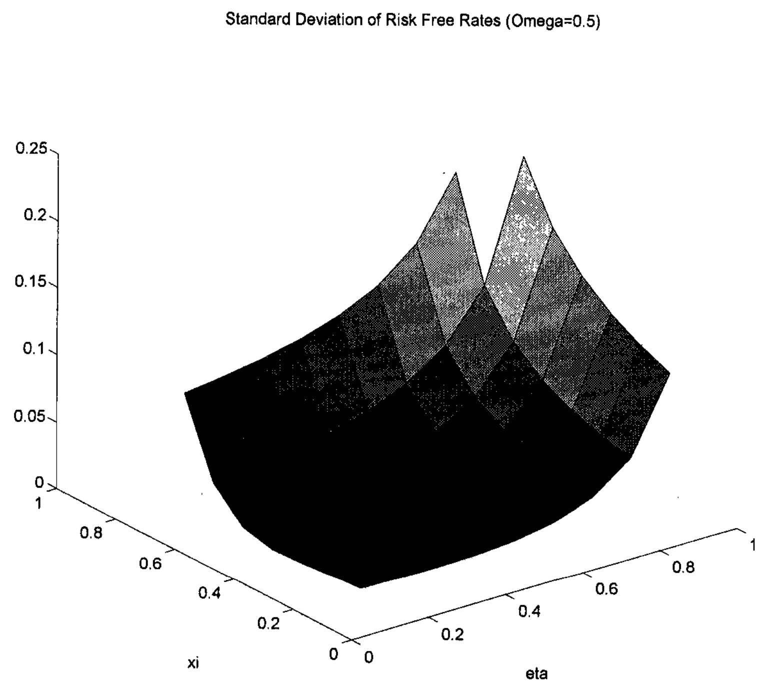


Figure 1.5: continued





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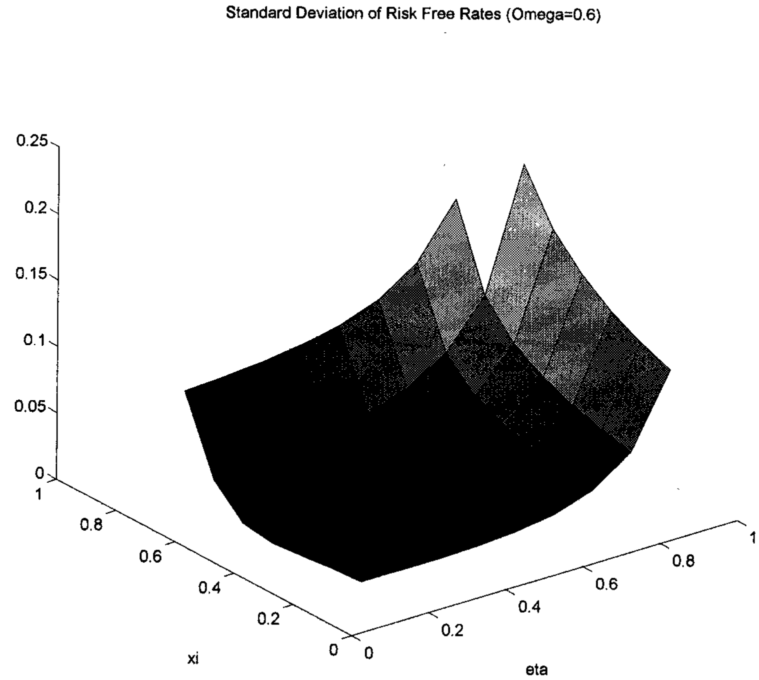


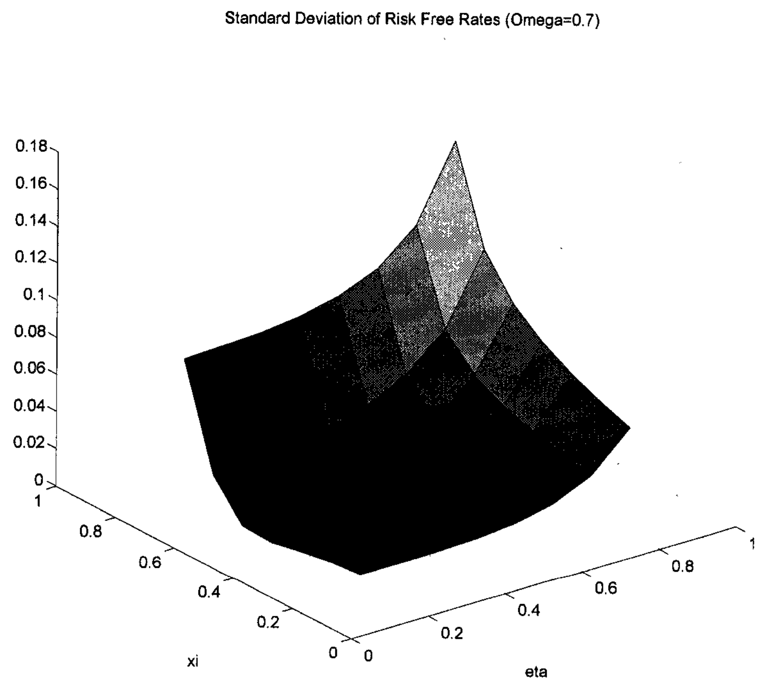
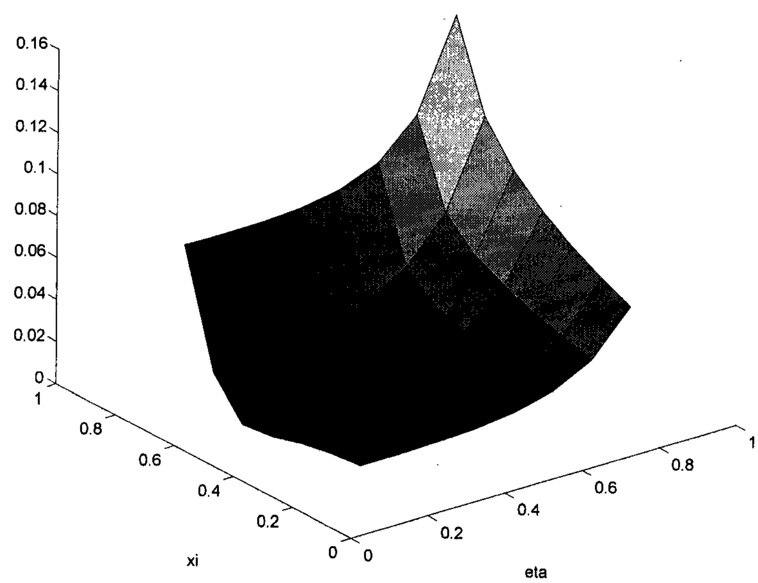
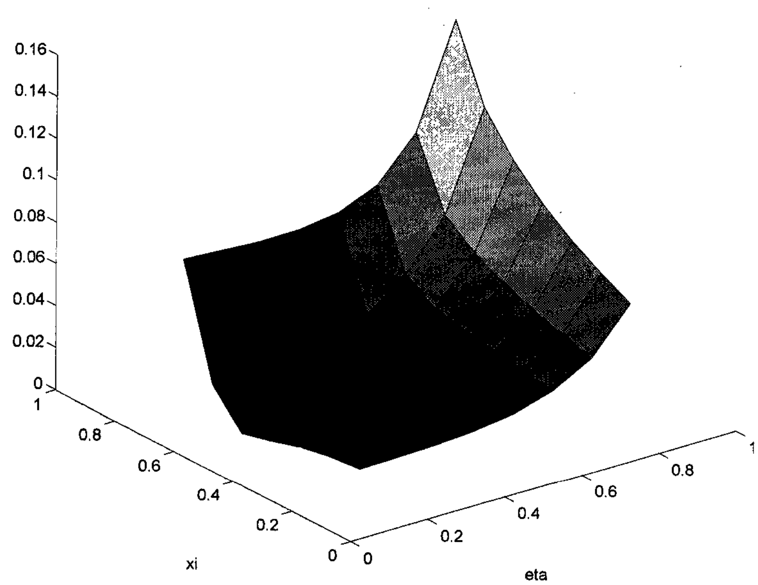


Figure 1.5: continued

Standard Deviation of Risk Free Rates (Omega=0.8)



Standard Deviation of Risk Free Rates (Omega=0.9)



co-movement between returns and co-movement between returns and growth rates.

Figure 1.6 and Figure 1.7 present the autocorrelation of the returns on the market portfolio and the real risk-free rate. The autocorrelation of returns on the market portfolio, $\rho(R_{m,t-1}, R_{m,t})$ is negative in all cases. It confirms that higher return this period will be followed by lower return next period. The autocorrelation of the real risk-free rate, $\rho(R_{f,t-1}, R_{f,t})$, is positive in most cases. The weak negative $\rho(R_{f,t-1}, R_{f,t})$ is mostly associated with cases with $\Omega = 0.1$, with risky non-durable consumption and safe durable consumption when $0.1 < \Omega < 0.5$ or with safe non-durable consumption and risky durable consumption when $\Omega \geq 0.7$. Although cases with high Ω and high habit persistence levels generate $\rho(R_{f,t-1}, R_{f,t})$ to be around 0.4, which is much lower than 0.87 from the U.S. historical data,[3] they are able to generate very weak negative $\rho(R_{m,t-1}, R_{m,t})$ that matches with the U.S. economy.[4] Cases with high Ω and high habit persistence levels generate lower $\rho(R_{f,t-1}, R_{f,t})$ around 0.3 and can reproduce $\rho(R_{m,t-1}, R_{m,t})$ in the U.S. economy.

Figure 1.8 demonstrate high correlation between the returns on the market port-

[3] From Danthine, Donaldson, Giannikos and Guiruis (2004).

[4] Danthine, Donaldson, Giannikos and Guiruis (2004) record an autocorrelation of -0.03 for return on market portfolio.

Figure 1.6: First Order Autocorrelation of the Returns on the Market Portfolio
For all the subfigures in this section, relative risk aversion, $\nu = 3$; preference parameter $\delta = 0.75$. Omega is the durability of the durable good, Ω. Eta is the habit persistence level of the durable good, η. Xi is the habit persistence level of the perishable good, ξ.

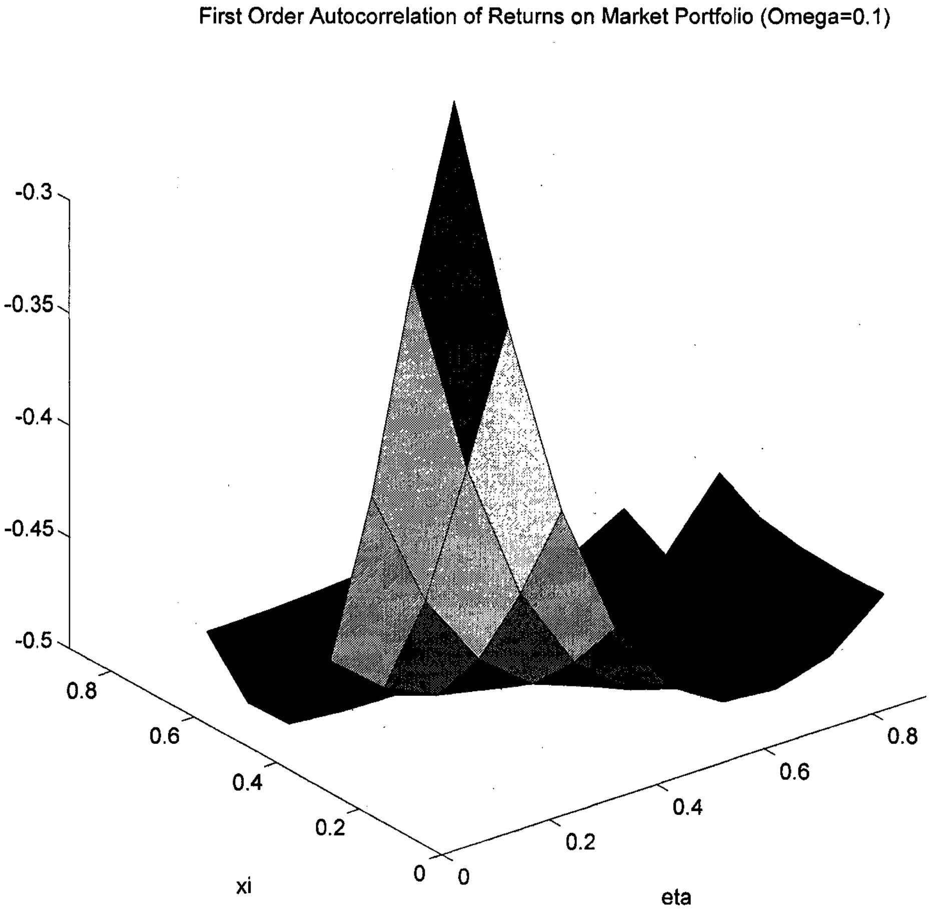
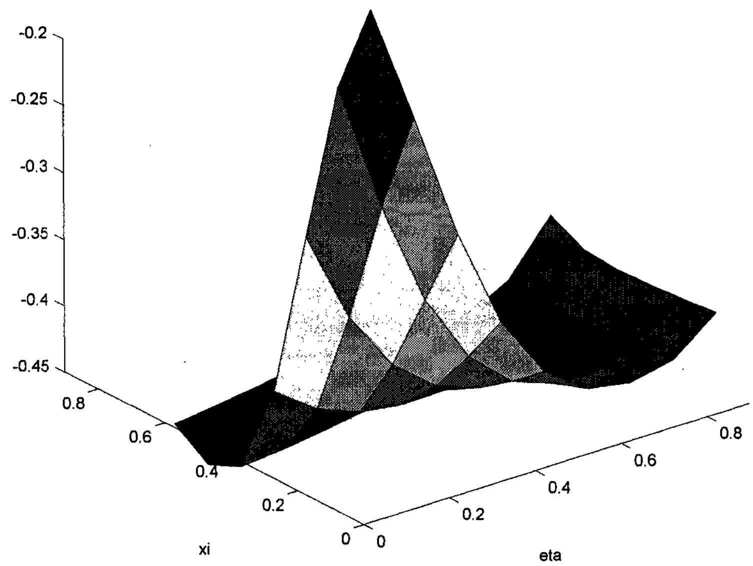


Figure 1.6: continued

First Order Autocorrelation of Returns on Market Portfolio (Omega=0.2)



First Order Autocorrelation of Returns on Market Portfolio (Omega=0.3)

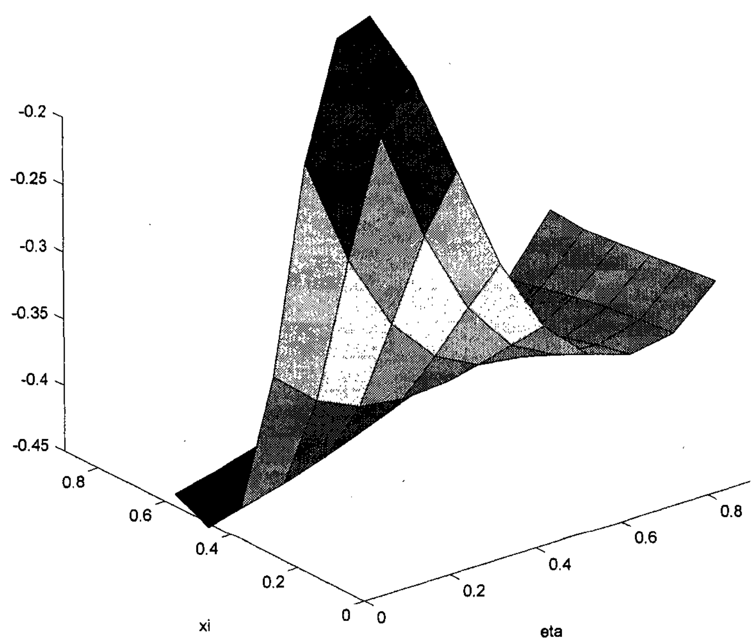
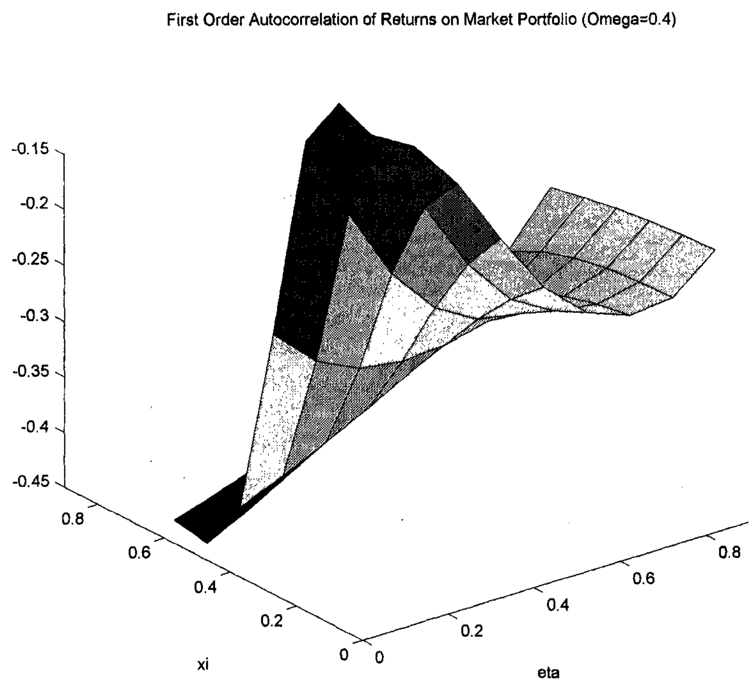


Figure 1.6: continued



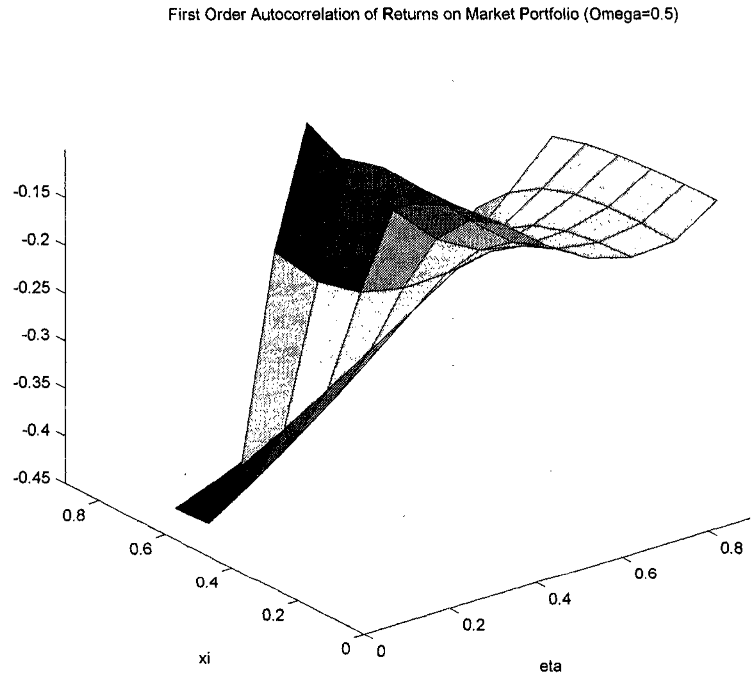
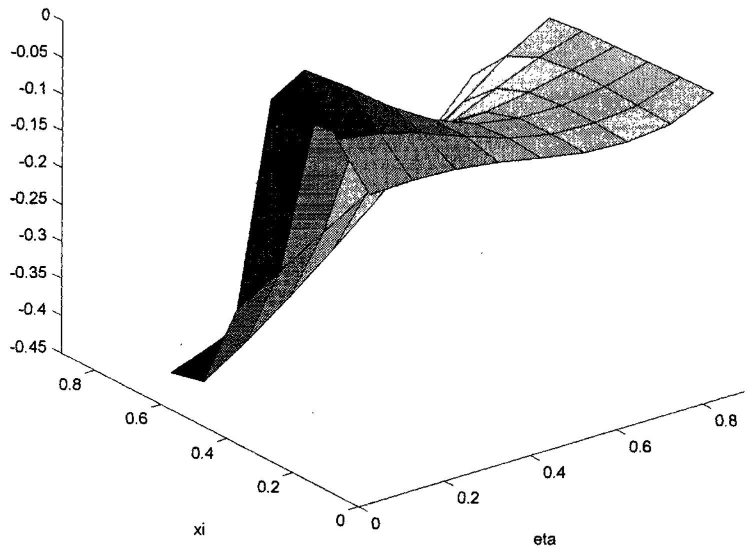


Figure 1.6: continued

First Order Autocorrelation of Returns on Market Portfolio (Omega=0.6)



First Order Autocorrelation of Returns on Market Portfolio (Omega=0.7)

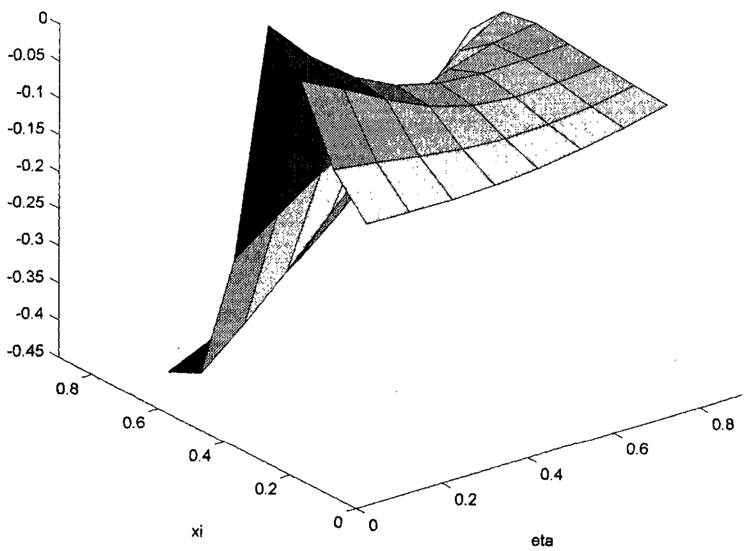
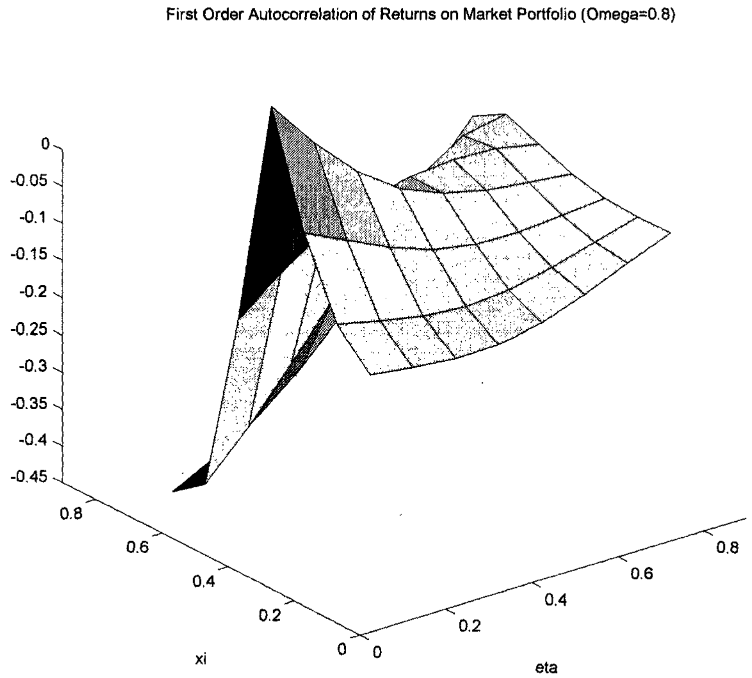


Figure 1.6: continued



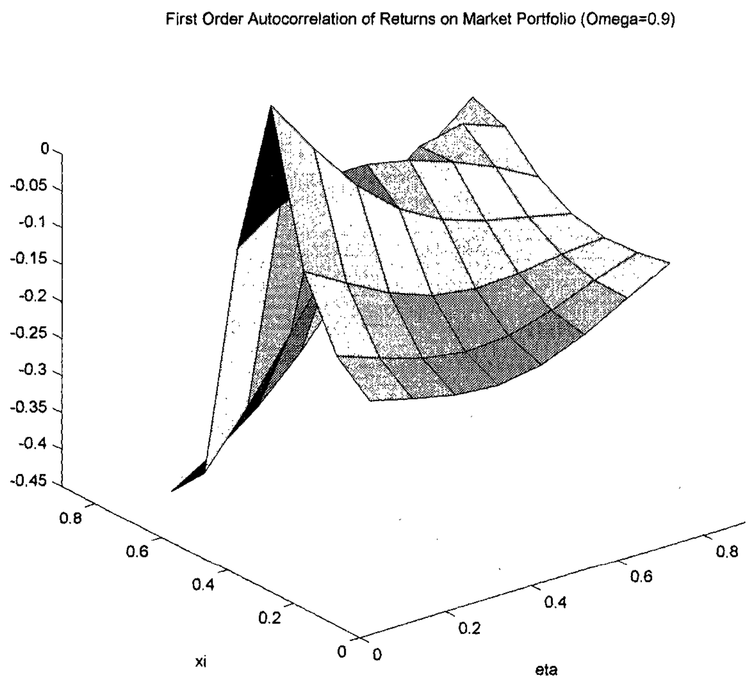


Figure 1.7: First Order Autocorrelation of the Real Risk-free Rates
For all the subfigures in this section, relative risk aversion, $\nu = 3$; preference parameter $\delta = 0.75$. Omega is the durability of Durable good, Ω. Eta is the habit persistence level of durable good, η. Xi is the habit persistence level of perishable good, ξ.

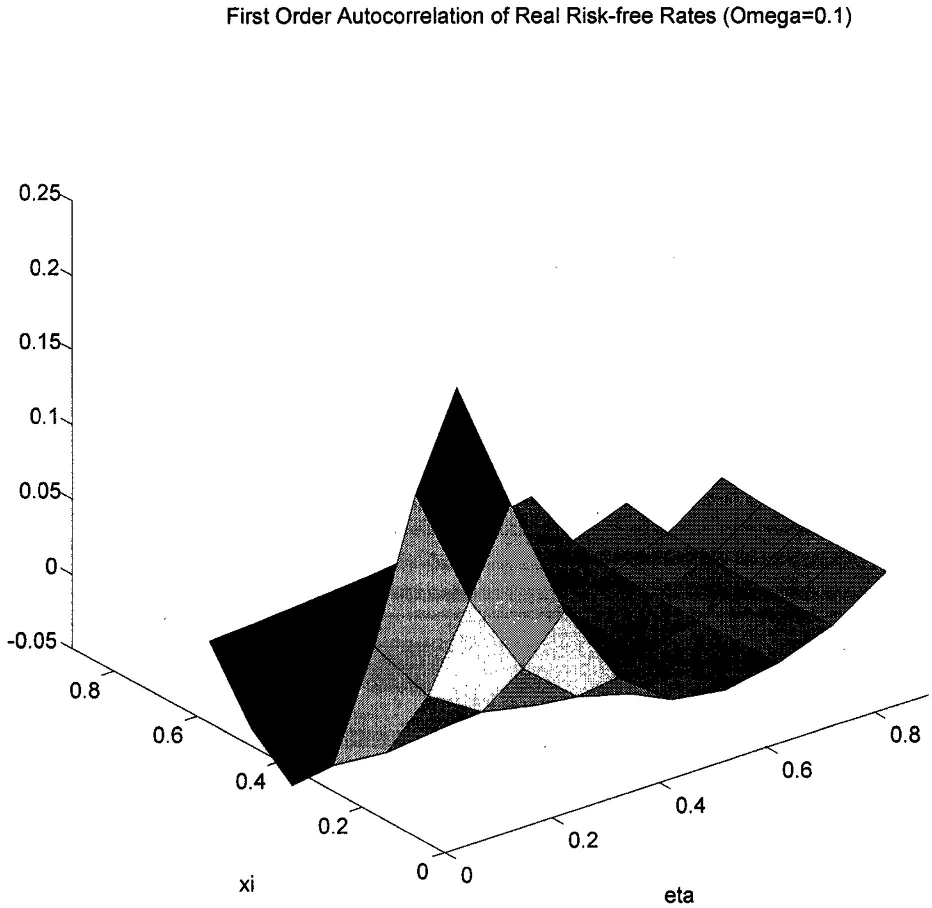
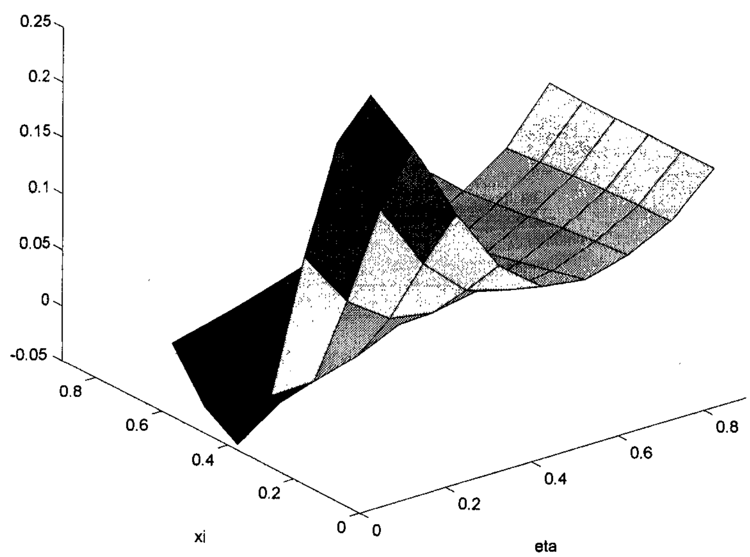


Figure 1.7: continued

First Order Autocorrelation of Real Risk-free Rates (Omega=0.2)



First Order Autocorrelation of Real Risk-free Rates (Omega=0.3)

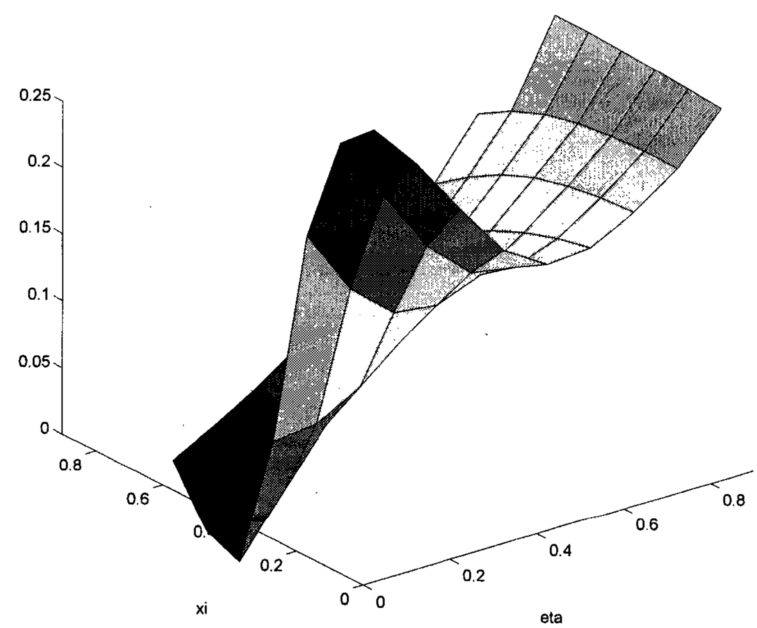
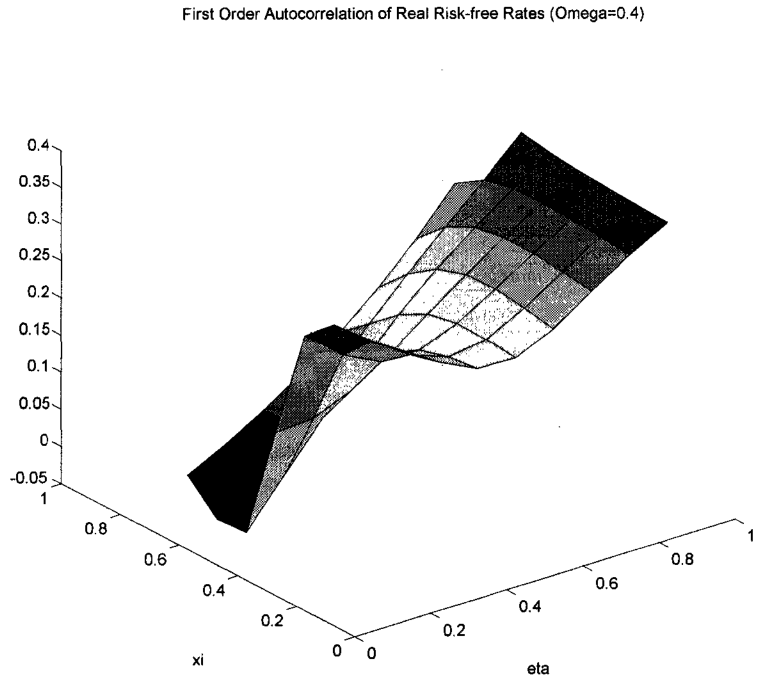


Figure 1.7: continued



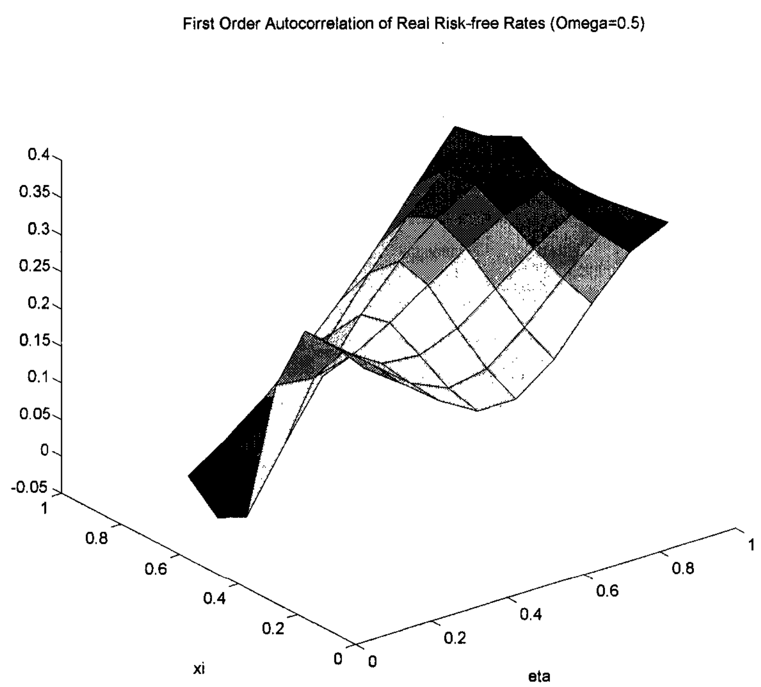
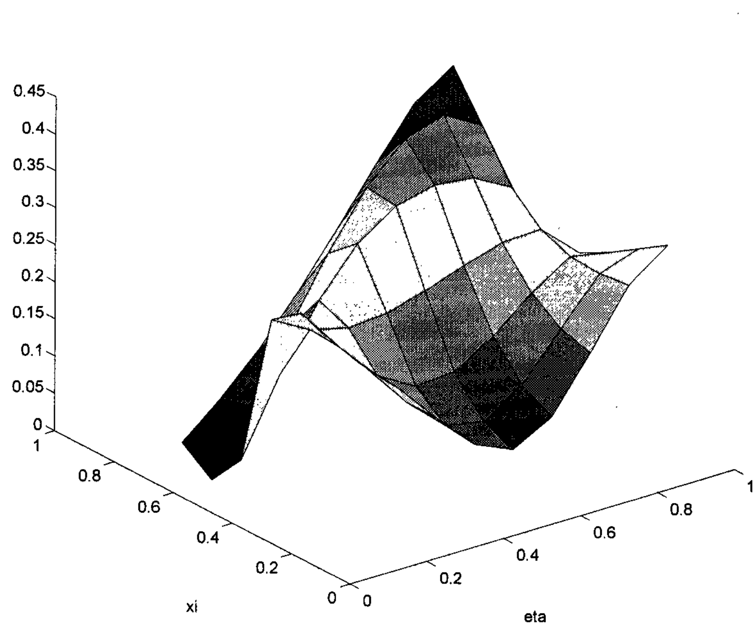


Figure 1.7: continued

First Order Autocorrelation of Real Risk-free Rates (Omega=0.6)



First Order Autocorrelation of Real Risk-free Rates (Omega=0.7)

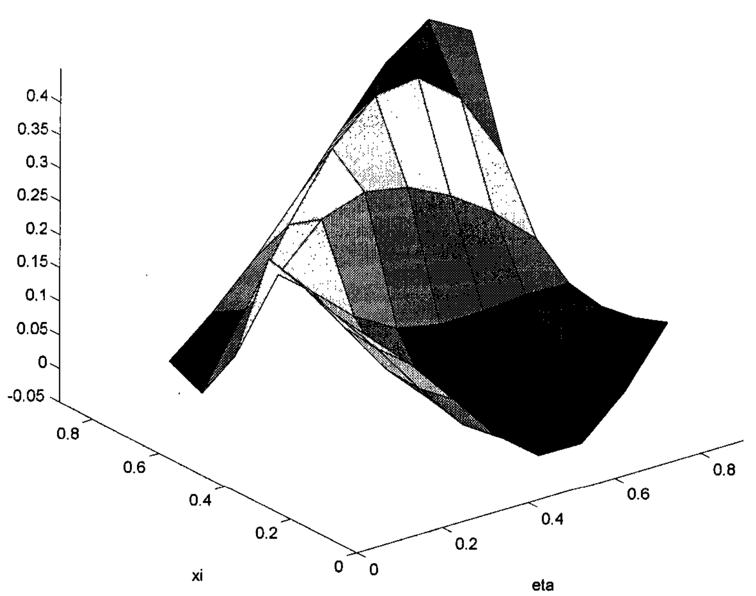
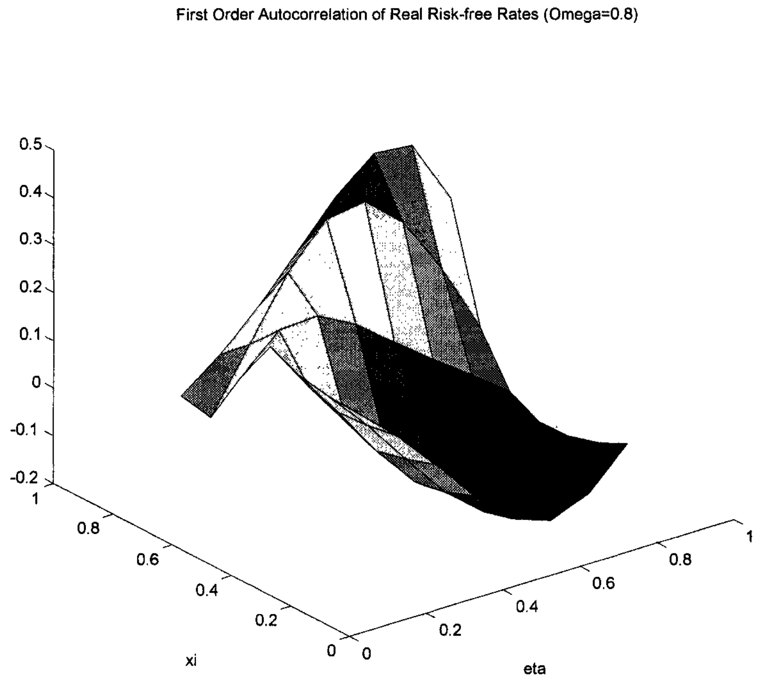
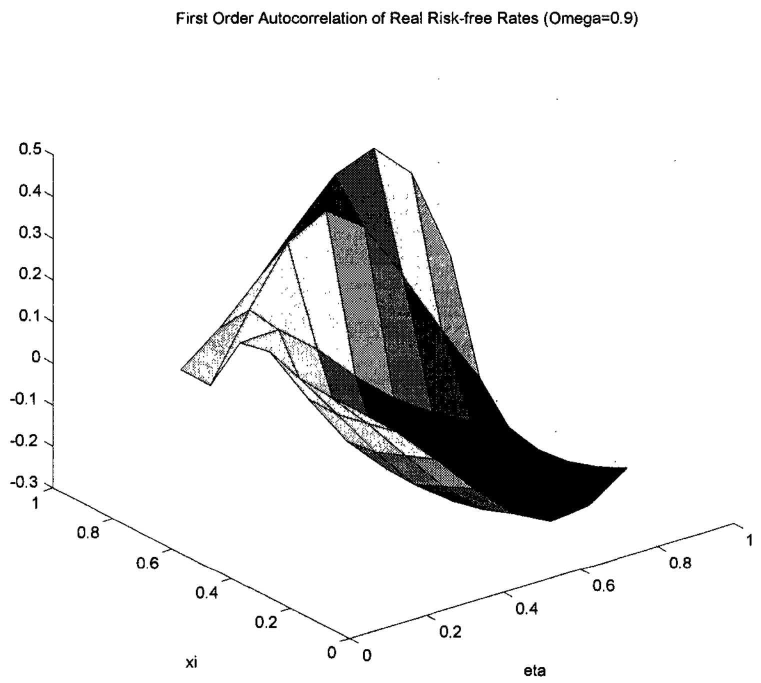


Figure 1.7: continued





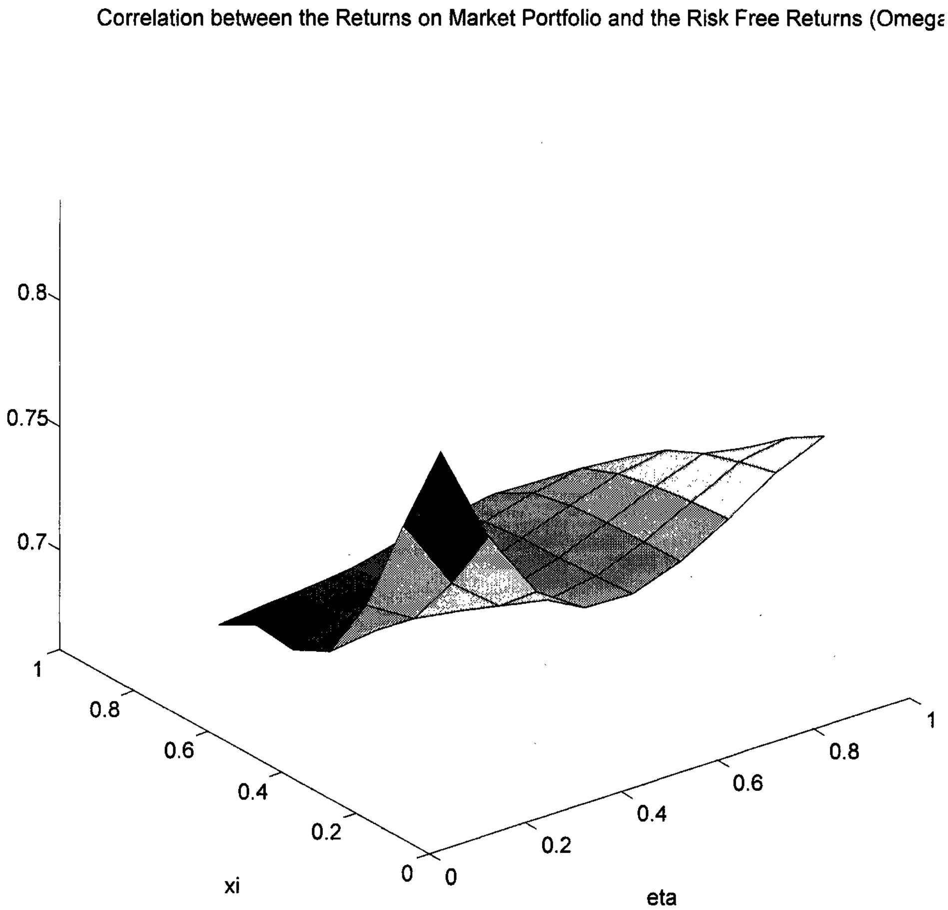
folio and the real risk-free rate, $\rho(R_{m,}R_f)$. In all the cases simulated, the correlation are too high as compared to the very weak negative correlation of -0.09 observed in the U.S. market.[5]

The correlations between the returns on securities and the growth rates of the non-durable good, λ, are reported in Figure 1.9 and Figure 1.10. The patten of these correlations are similar to those of the correlations between the returns and the growth rates of the durable good (Figure 1.11 and Figure 1.12).

[5]From Danthine, Donaldson, Giannikos and Guiruis (2004)

Figure 1.8: Correlation between the Returns on the Market Portfolio and the Risk-free Rate

For all the subfigures in this section, relative risk aversion, $\nu = 3$; preference parameter $\delta = 0.75$. Omega is the durability of the durable good, Ω. Eta is the habit persistence level of the durable good, η. Xi is the habit persistence level of the perishable good, ξ. Rm is returns on market portfolio and Rf is risk-free rates.



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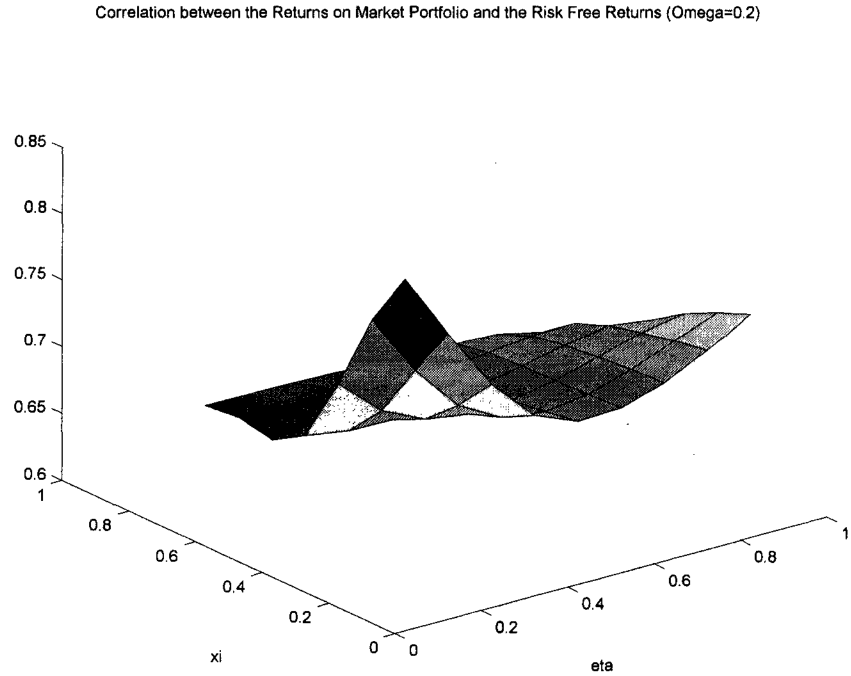

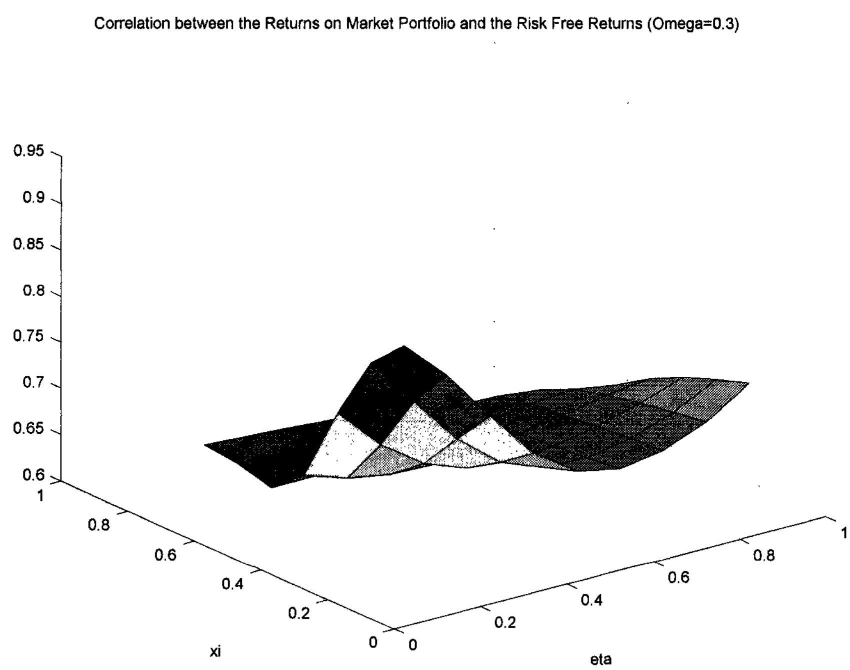

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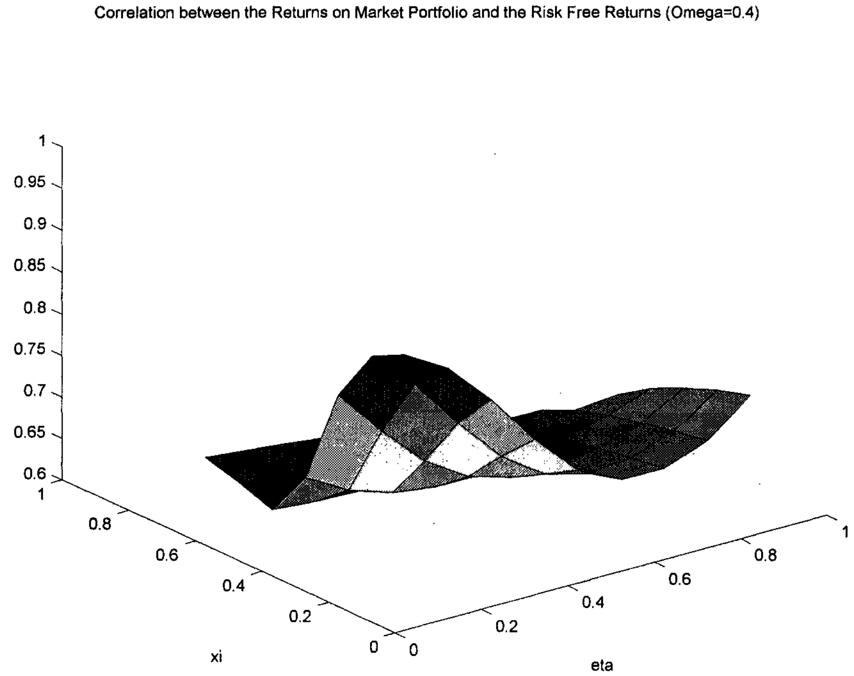


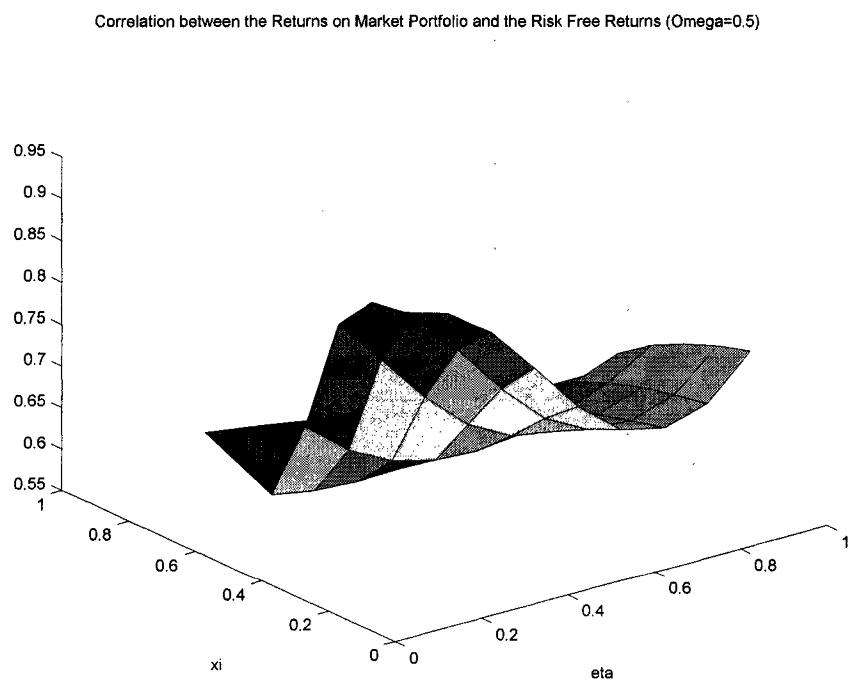
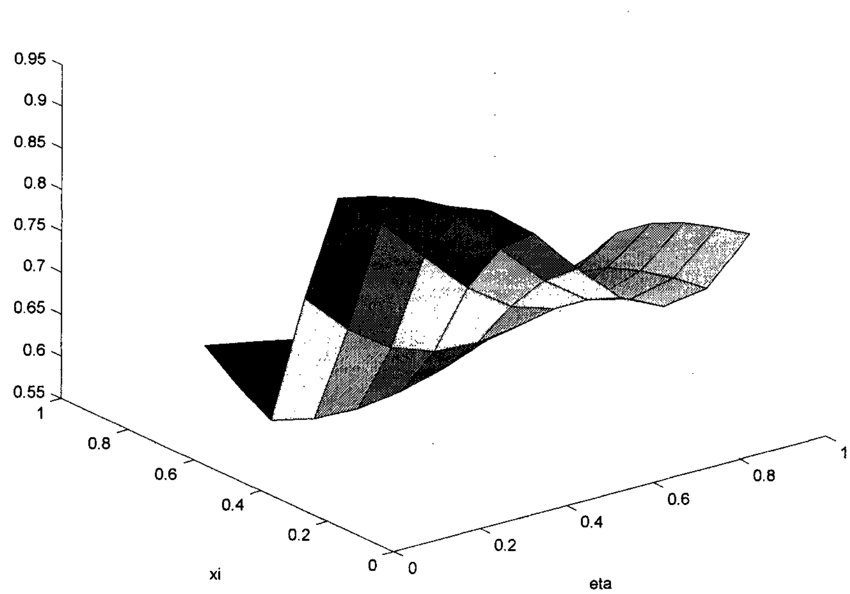


Figure 1.8: continued

Correlation between the Returns on Market Portfolio and the Risk Free Returns (Omega=0.6)



Correlation between the Returns on Market Portfolio and the Risk Free Returns (Omega=0.7)

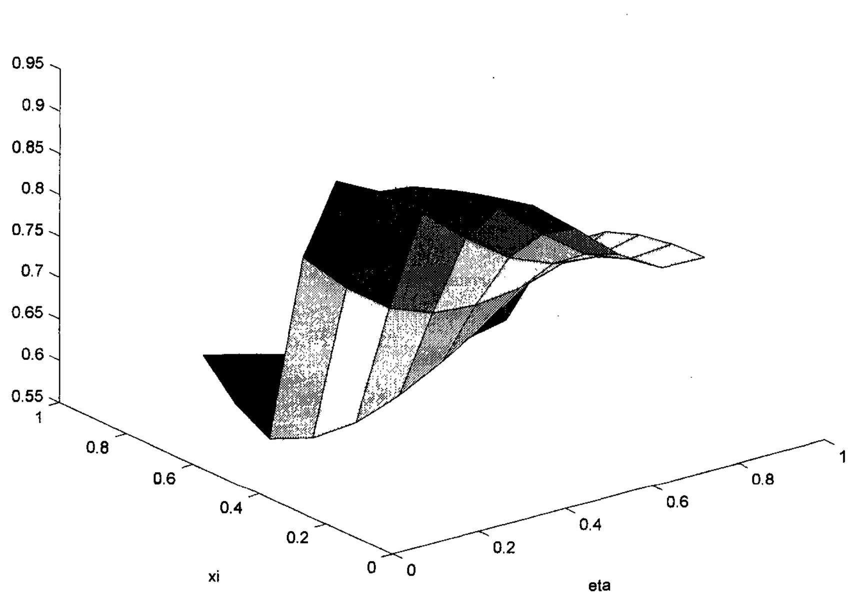
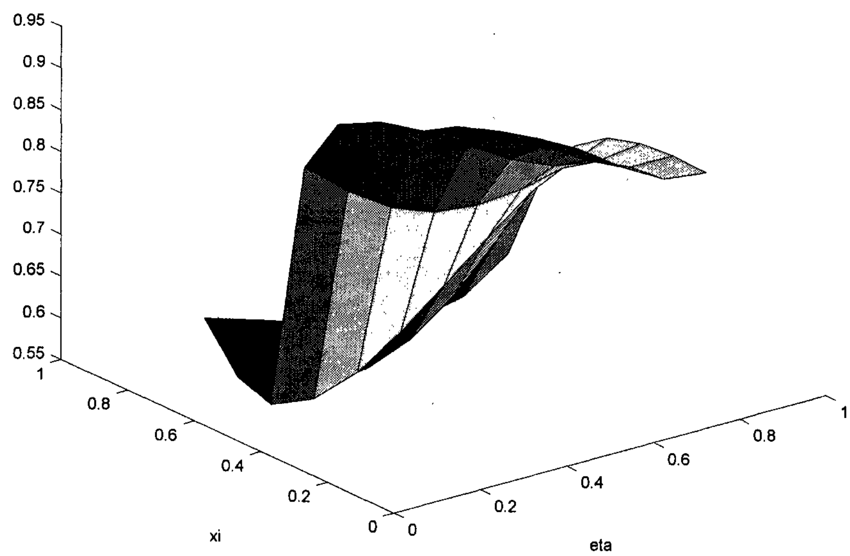


Figure 1.8: continued

Correlation between the Returns on Market Portfolio and the Risk Free Returns (Omega=0.8)



Correlation between the Returns on Market Portfolio and the Risk Free Returns (Omega=0.9)

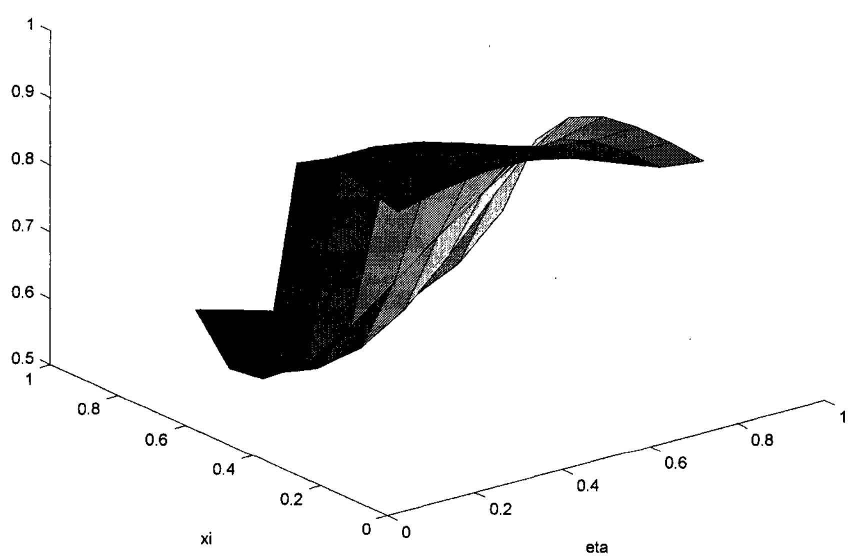


Figure 1.9: Correlation between the Returns on the Market Portfolio and the Growth Rate of the Non-durable Good Output
For all the subfigures in this section, relative risk aversion, $\nu = 3$; preference parameter $\delta = 0.75$. Omega is the durability of the durable good, Ω. Eta is the habit persistence level of the durable good, η. Xi is the habit persistence level of the perishable good, ξ. Rm is the return on market portfolio, Rf is the risk-free rate and lambda is the growth rate of the non-durable good output, λ.

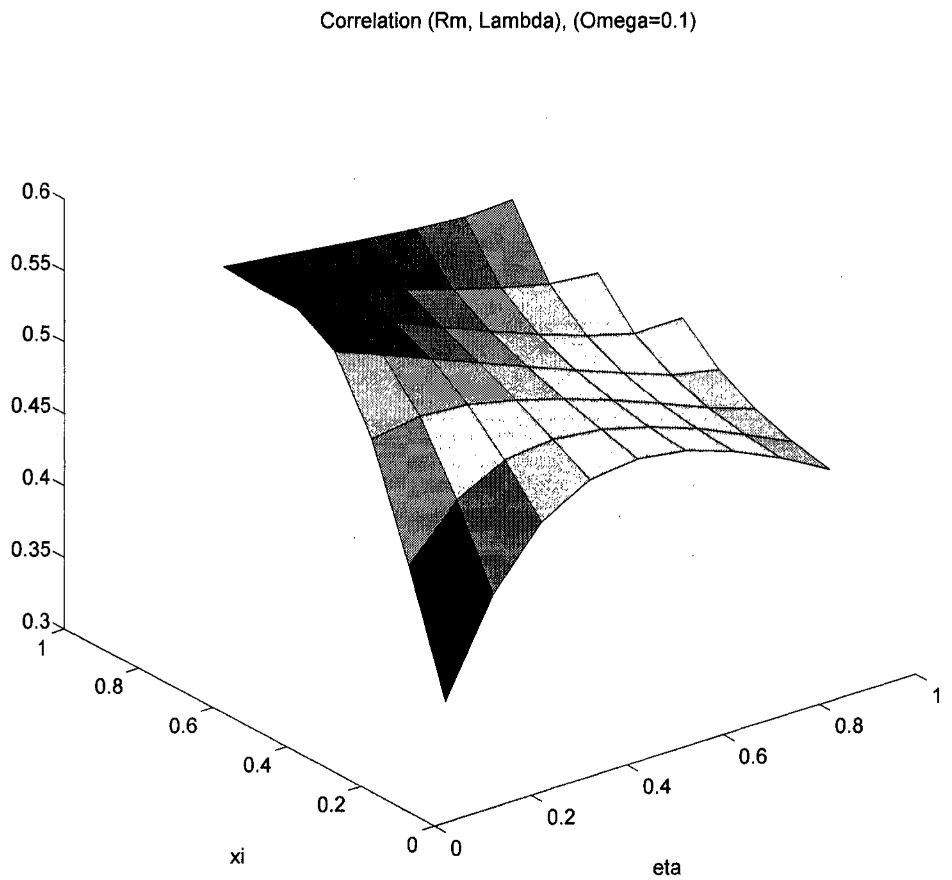
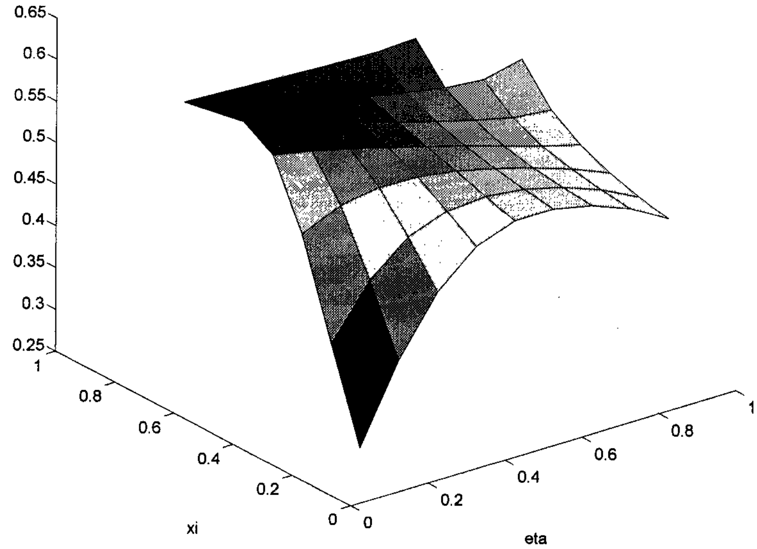


Figure 1.9: continued

Correlation (Rm, Lambda), (Omega=0.2)



Correlation (Rm, Lambda), (Omega=0.3)

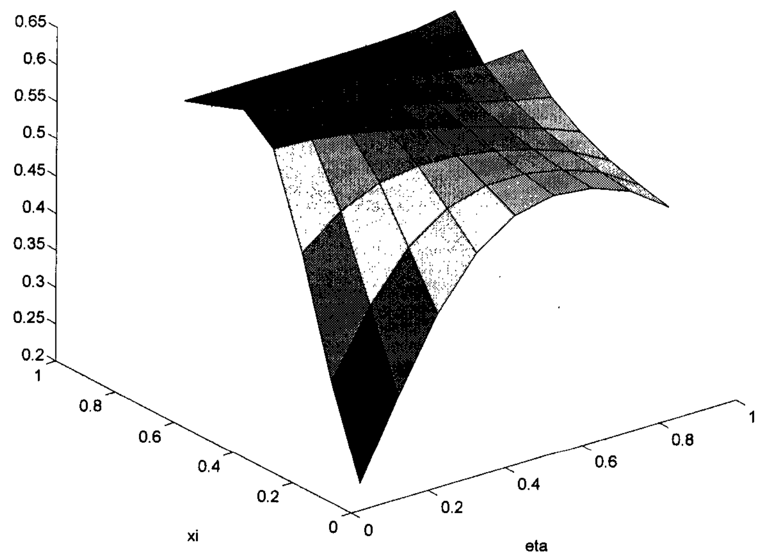
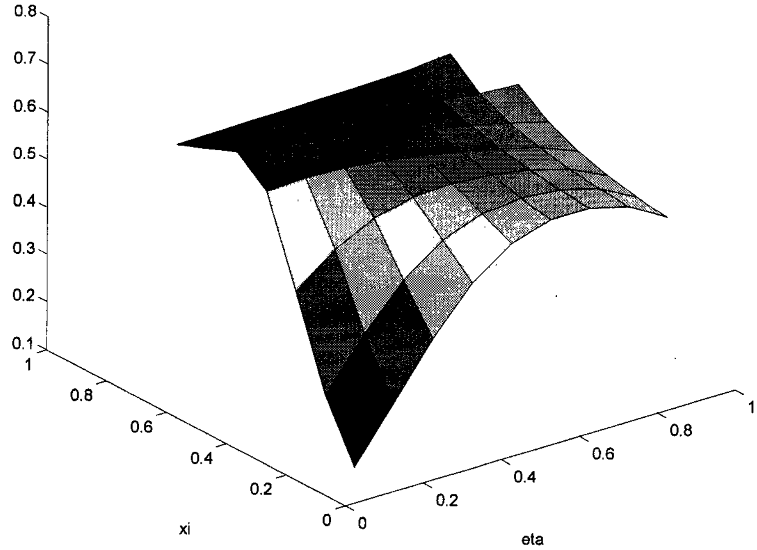


Figure 1.9: continued

Correlation (Rm, Lambda), (Omega=0.4)



Correlation (Rm, Lambda), (Omega=0.5)

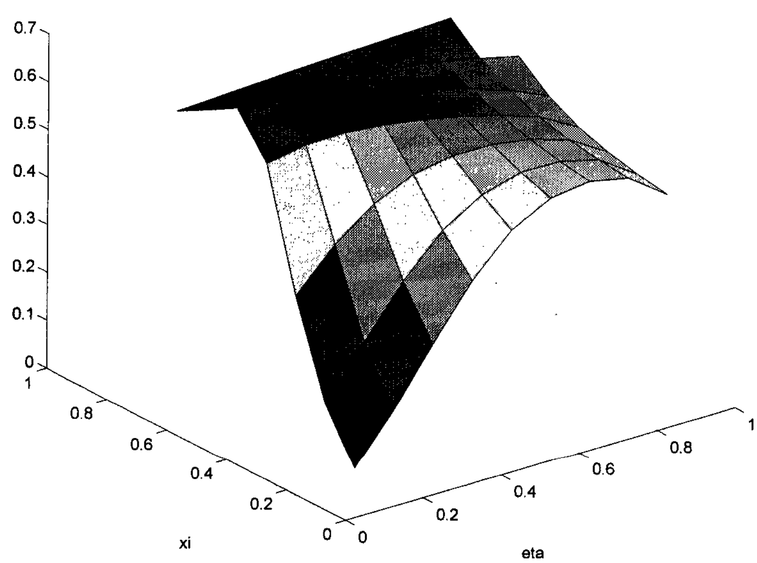
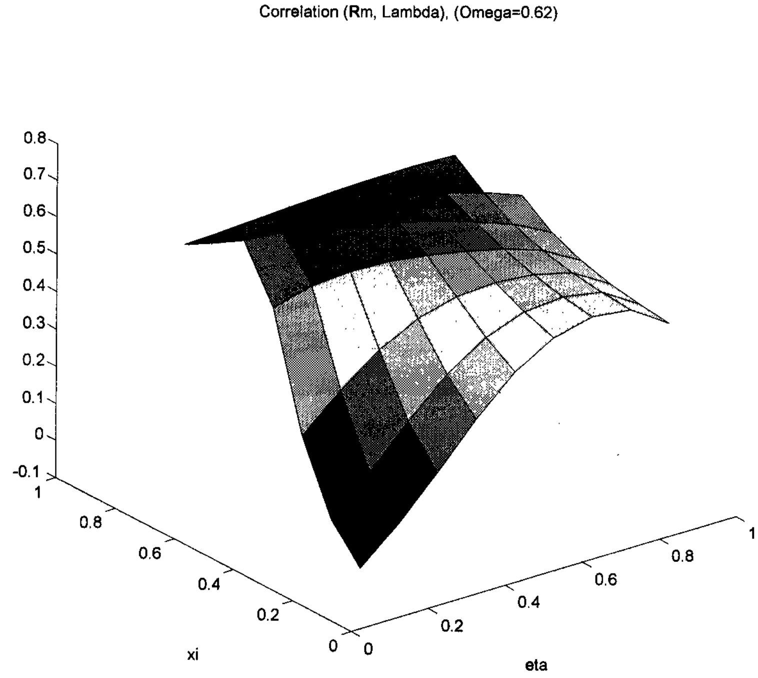
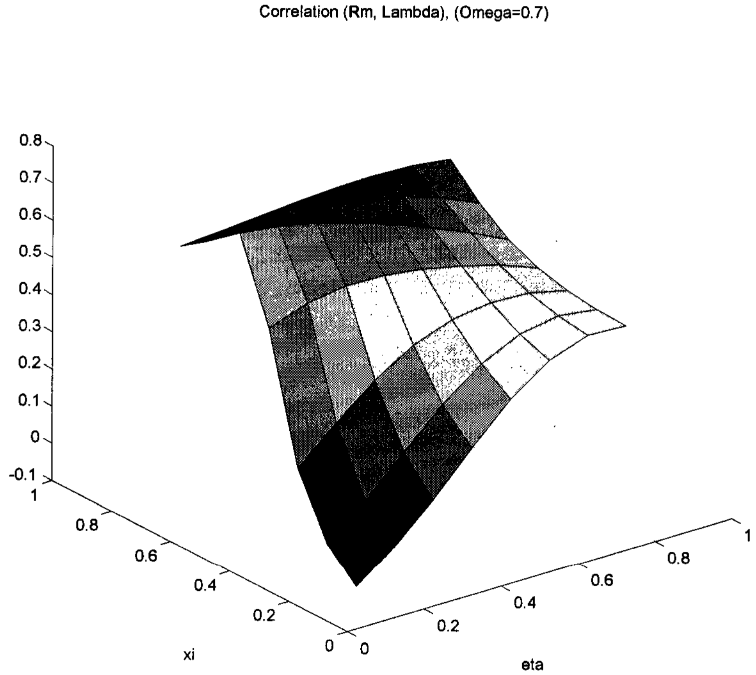


Figure 1.9: continued





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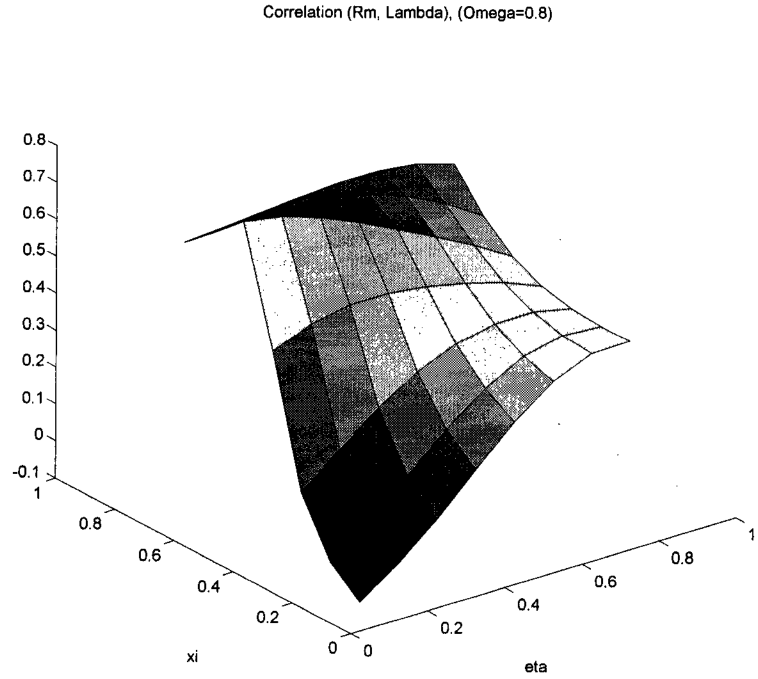


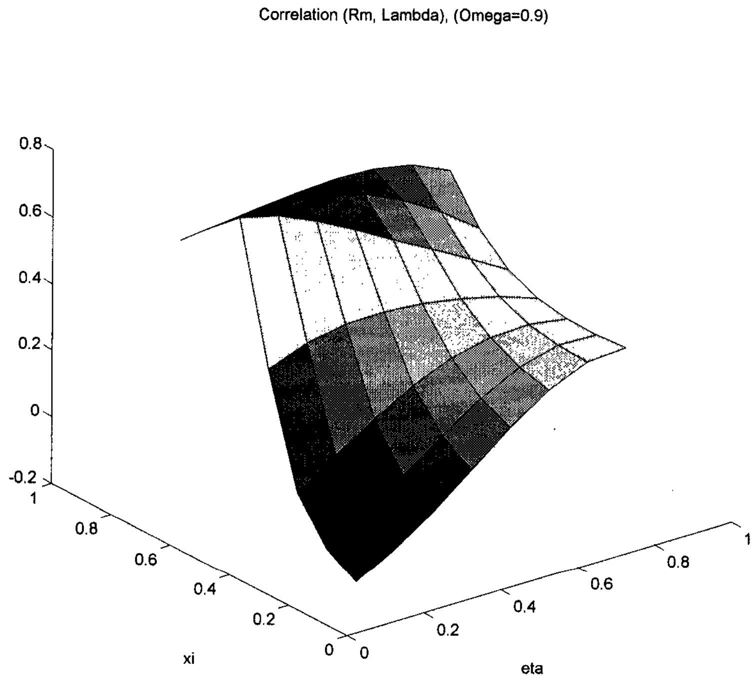


Figure 1.10: Correlation between the Risk-free Rate and the Growth Rate of the Non-durable Good Output

For all the subfigures in this section, relative risk aversion, $\nu = 3$; preference parameter $\delta = 0.75$. Omega is the durability of the durable good, Ω. Eta is the habit persistence level of the durable good, η. Xi is the habit persistence level of the perishable good, ξ. Rm is the return on market portfolio, Rf is the risk-free rate and and lambda is the growth rate of the non-durable good output, λ.

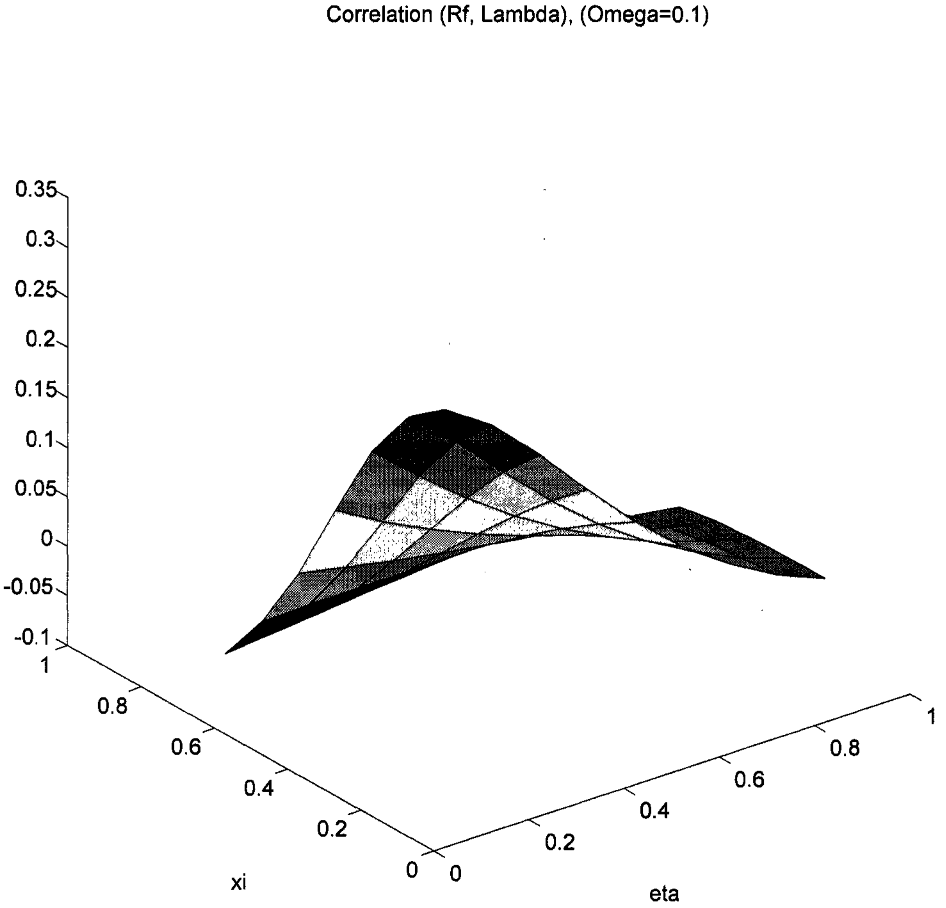
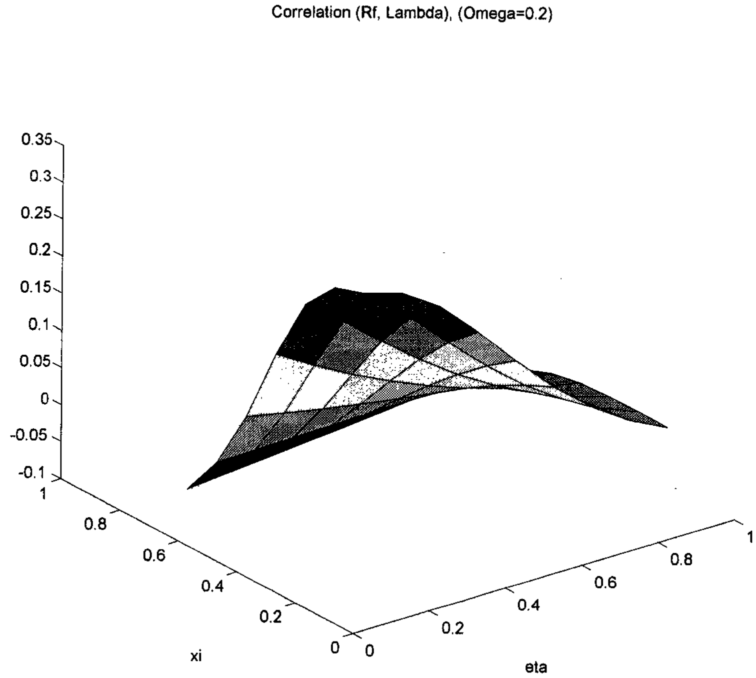
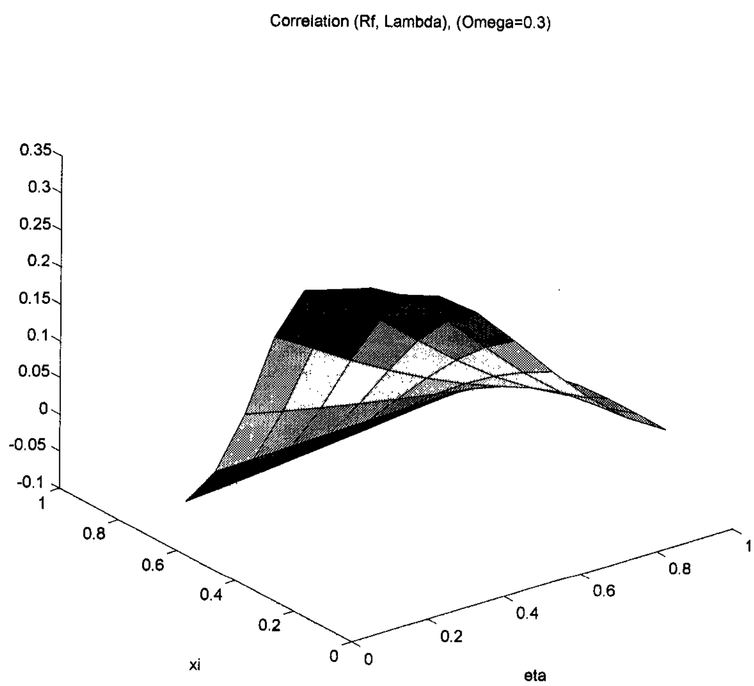


Figure 1.10: continued





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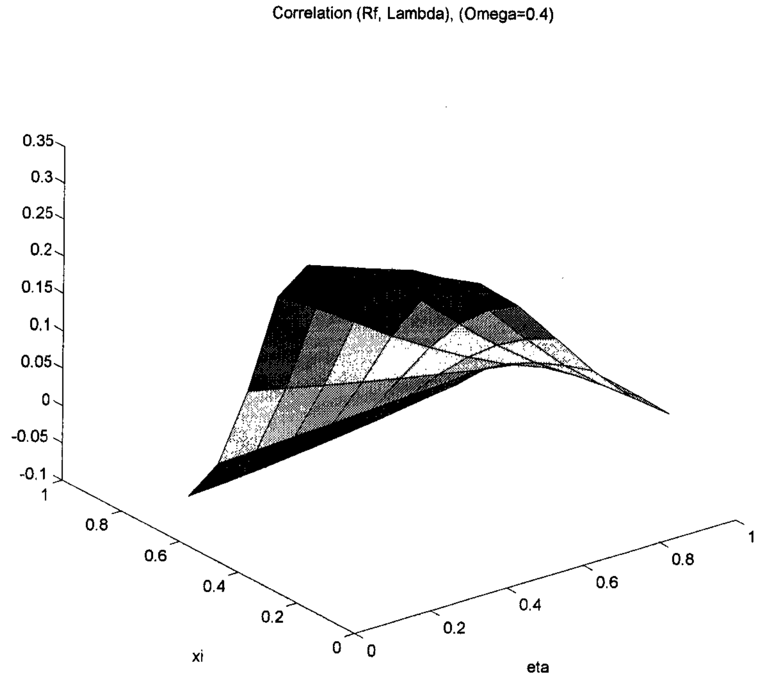


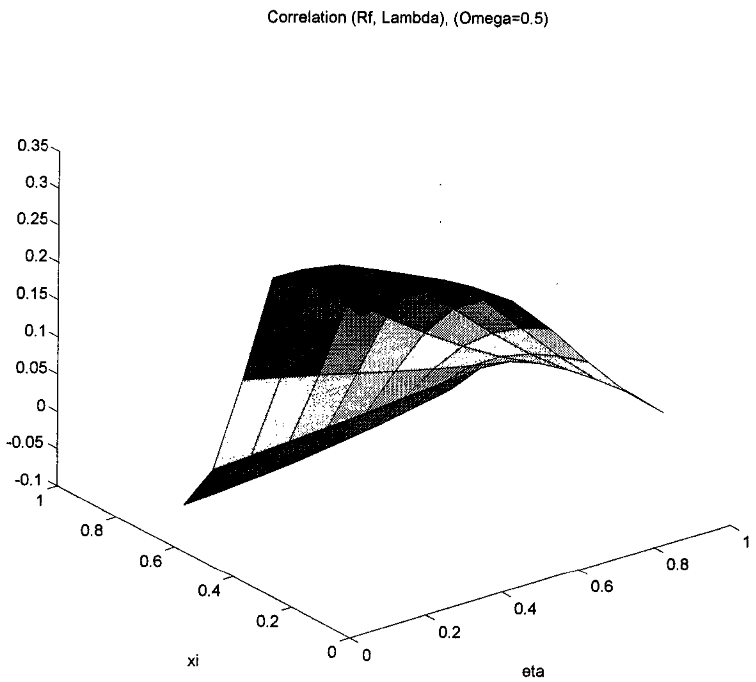
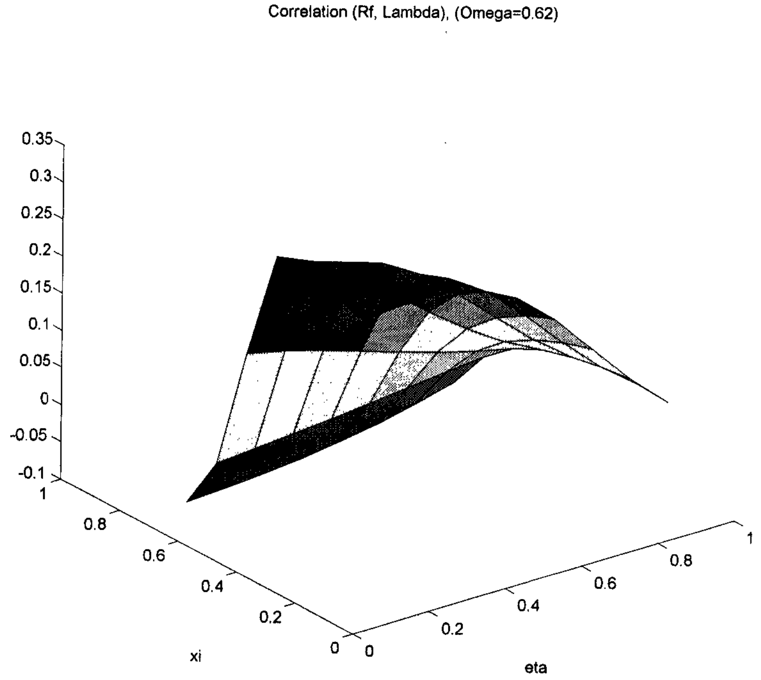


Figure 1.10: continued



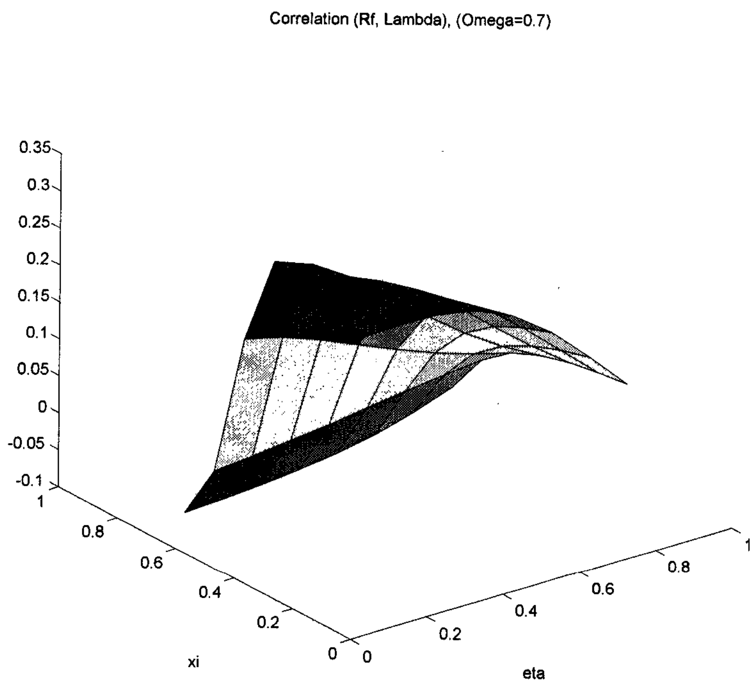
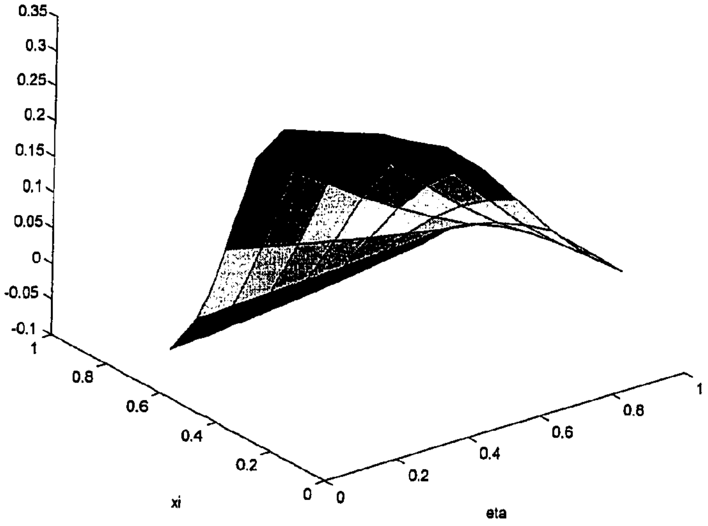
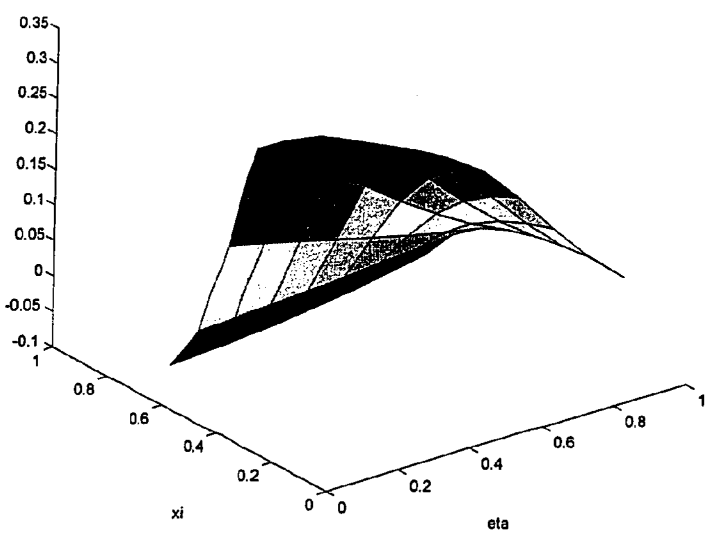


Figure 1.10: continued

Correlation (Rf, Lambda), (Omega=0.4)



Correlation (Rf, Lambda), (Omega=0.5)



With very low Ω and habit persistence levels, $\rho(R_{m,}\lambda)$ is around 0.3, which is observed in the U.S. market. As Ω increases, the necessary habit persistence levels to reproduce $\rho(R_{m,}\lambda)$ of 0.3 increase. The highest $\rho(R_{m,}\lambda)$ simulated from the model with high ξ and η is around 0.8. Although it is much higher than 0.3, it is lower than the correlation of 0.92 which is reported in Campbell and Cochrane (1999). With relatively high ξ ($\xi > 0.5$), the weakly negative $\rho(R_f, \lambda)$ produced by the model is consistent with the observation in Chapman (1997).

Examining all the simulation with various different paramenters, we find results from two cases matches the U.S. economy fairly well. They are reported in Table 1.2. In order to match the returns of all the securities, the necessary habit persistence levels of consumption of both goods increase to 0.7 and 0.8. Although the implied relative risk aversion is very high if we wanted to match the moments of the returns of individual securities, this model is able to reproduce the co-movements of risk-free rate and the growth rate of the ouput in the U.S. economy.

Figure 1.11: Correlation between the Returns on the Market Portfolio and the Growth Rate of the Durable Good Output

For all the subfigures in this section, relative risk aversion, $\nu = 3$; preference parameter $\delta = 0.75$. Omega is the durability of the durable good, Ω. Eta is the habit persistence level of the durable good, η. Xi is the habit persistence level of the perishable good, ξ. Rm is the return on market portfolio, Rf is the risk-free rate and gamma is the growth rate of the durable good output, γ.

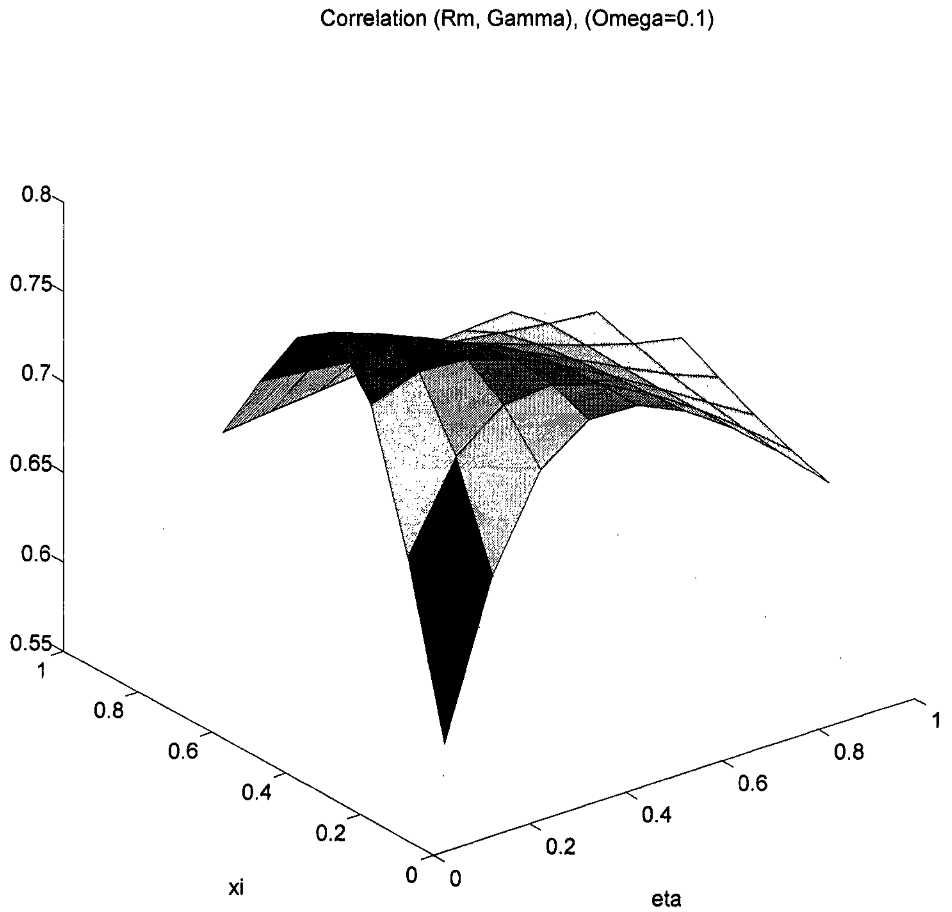
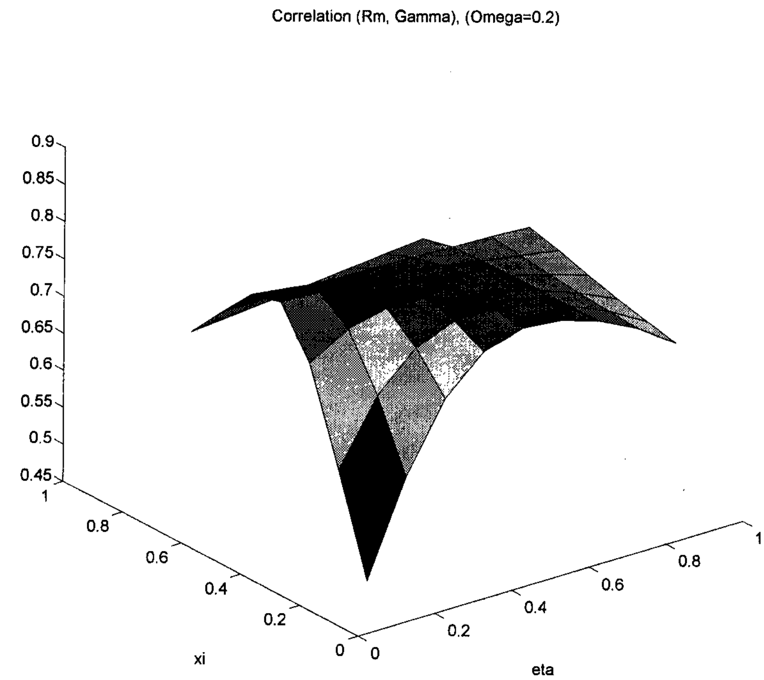


Figure 1.11: continued



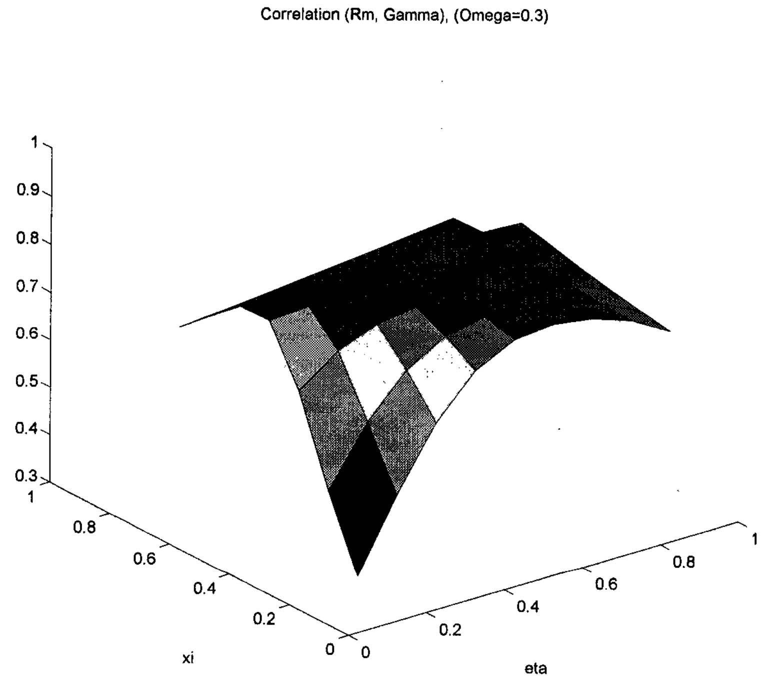
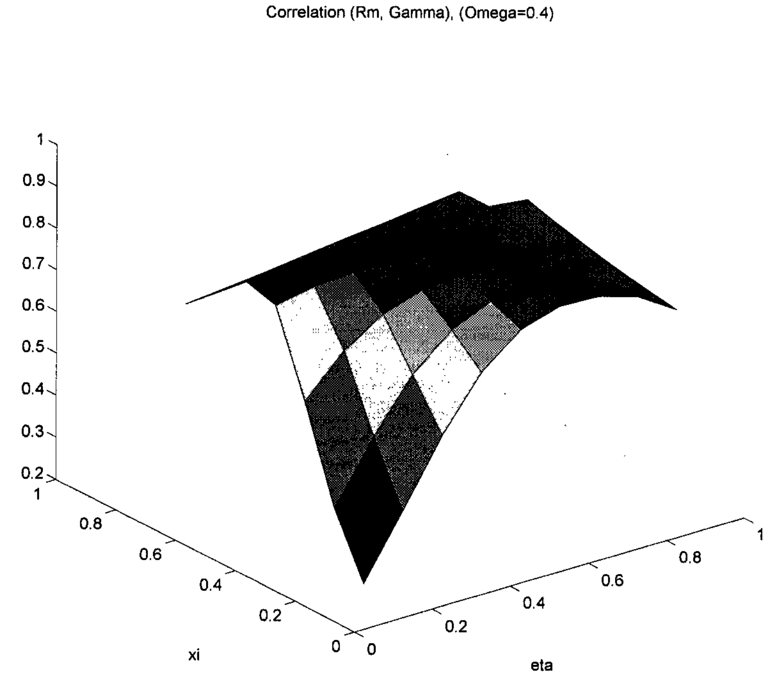


Figure 1.11: continued



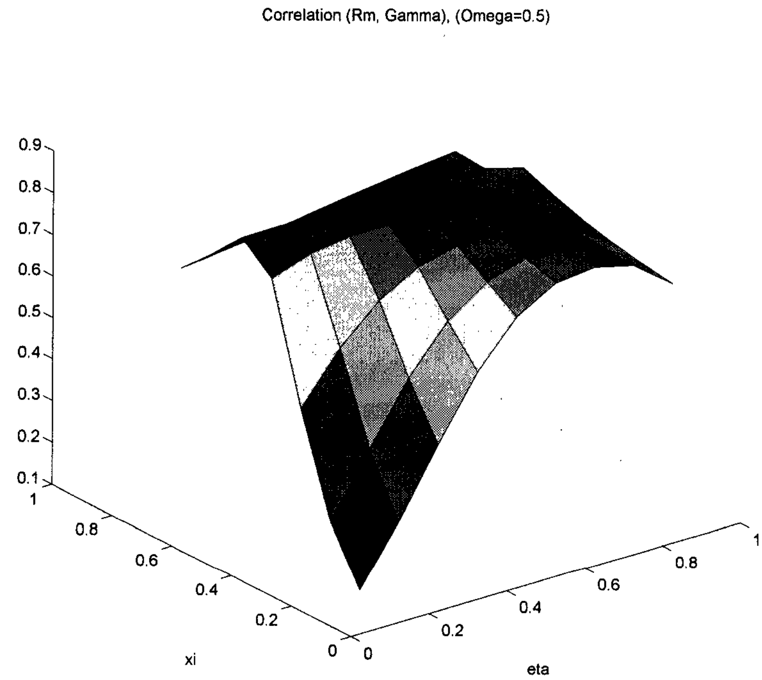
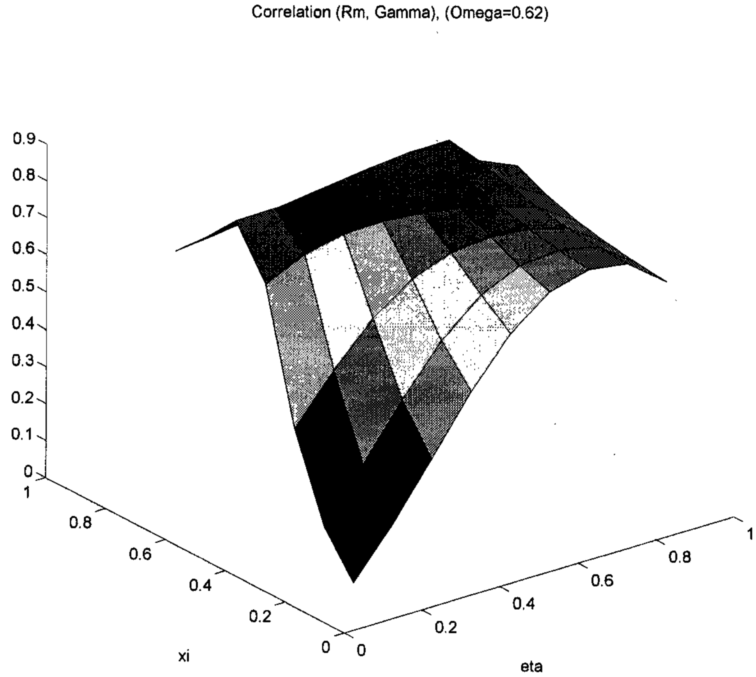


Figure 1.11: continued



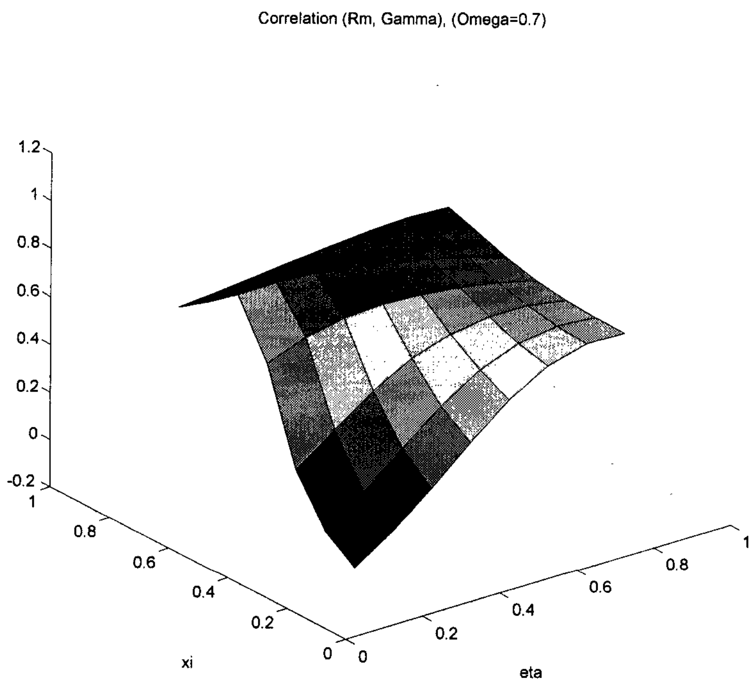
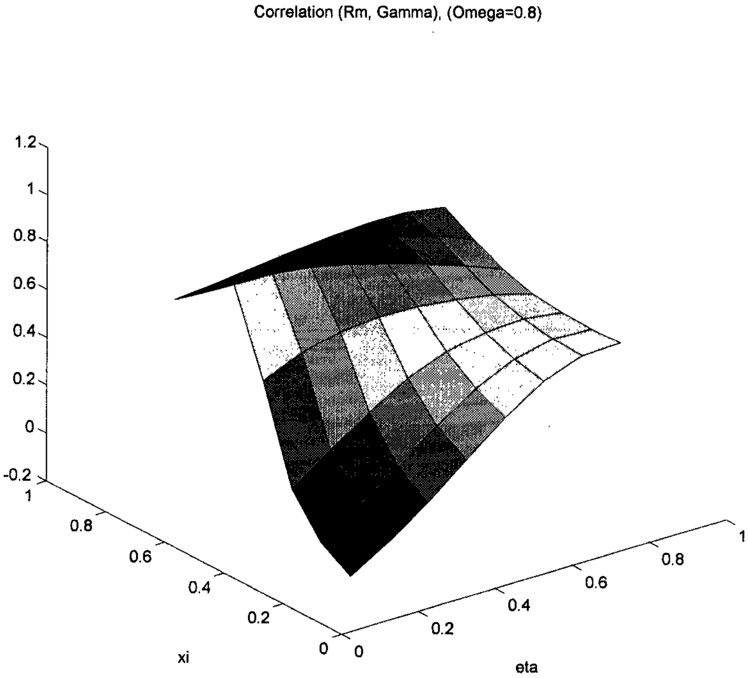


Figure 1.11: continued



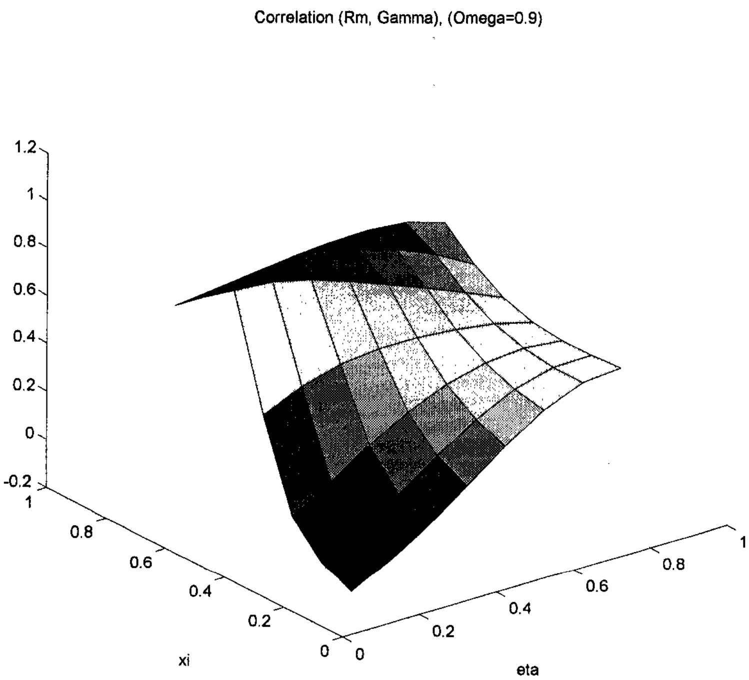


Figure 1.12: Correlation between the Risk-free Rate and the Growth Rate of the Durable Good Output

For all the subfigures in this section, relative risk aversion, $\nu = 3$; preference parameter $\delta = 0.75$. Omega is the durability of the durable good, Ω. Eta is the habit persistence level of the durable good, η. Xi is the habit persistence level of the perishable good, ξ. Rm is the return on market portfolio, Rf is the risk-free rates and gamma is the growth rate of the durable good output, γ.

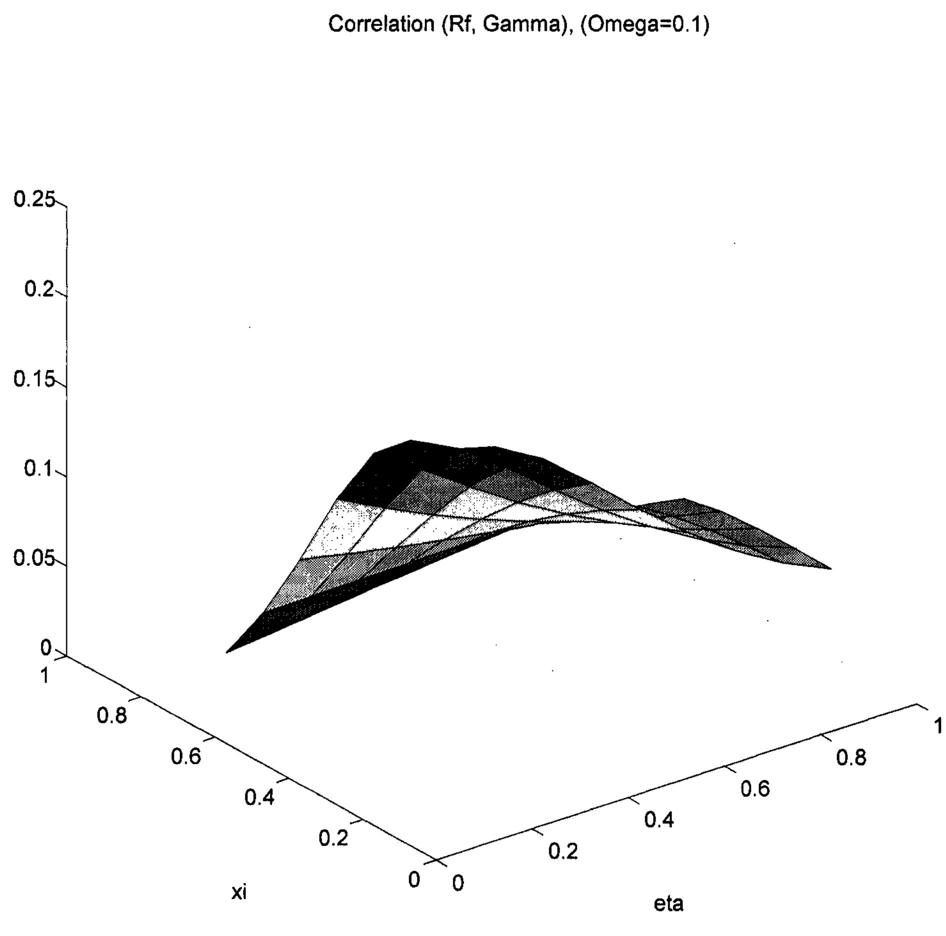
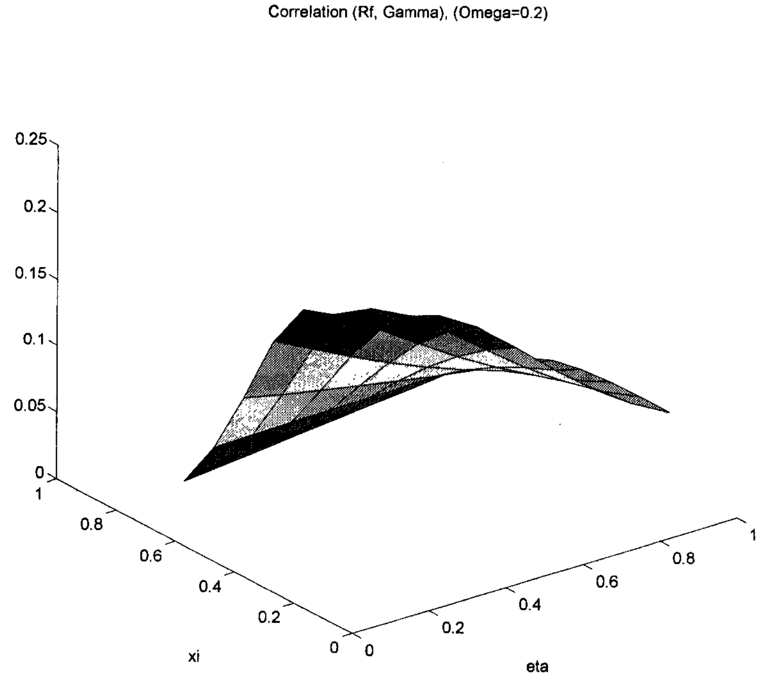


Figure 1.12: continued



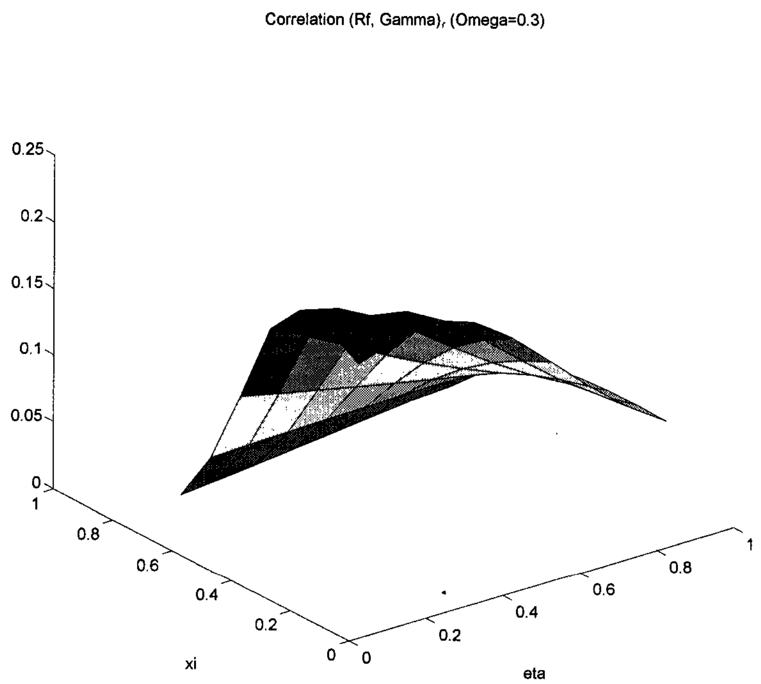
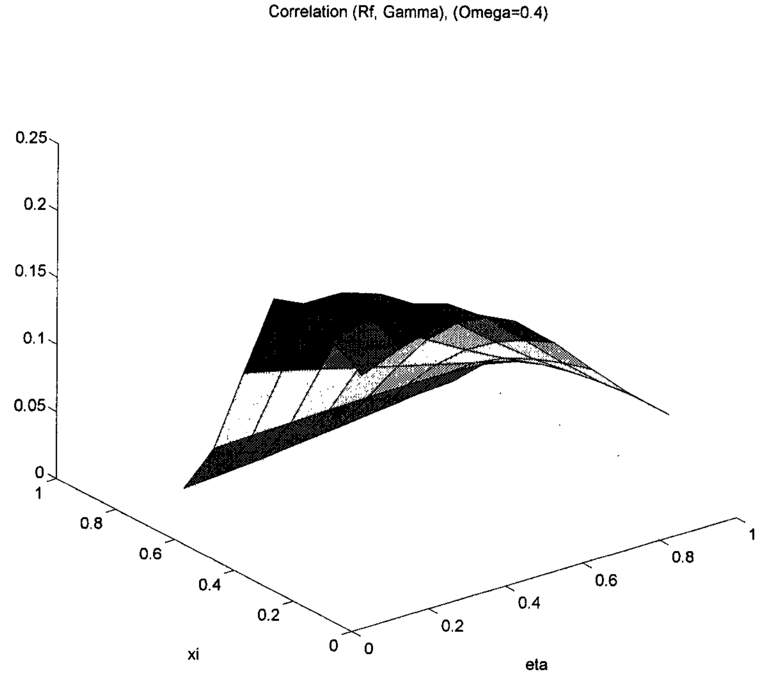


Figure 1.12: continued



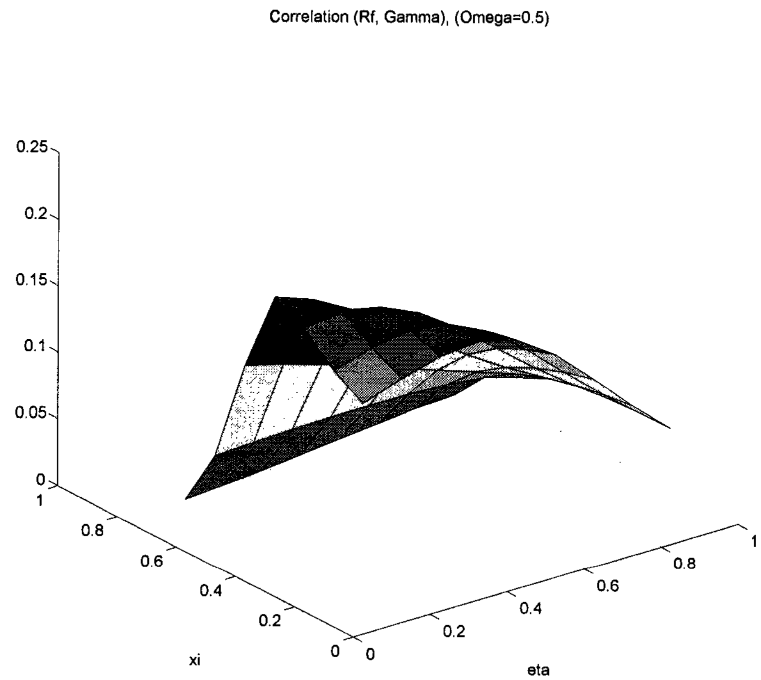
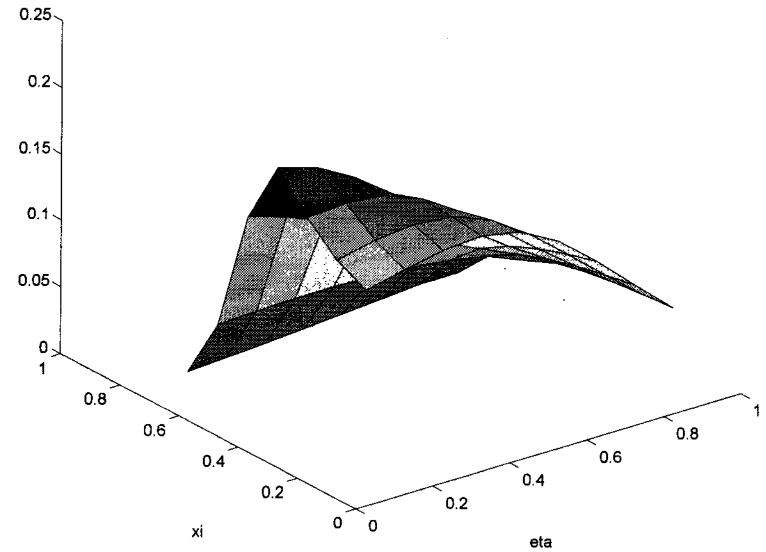


Figure 1.12: continued

Correlation (Rf, Gamma), (Omega=0.62)



Correlation (Rf, Gamma), (Omega=0.7)

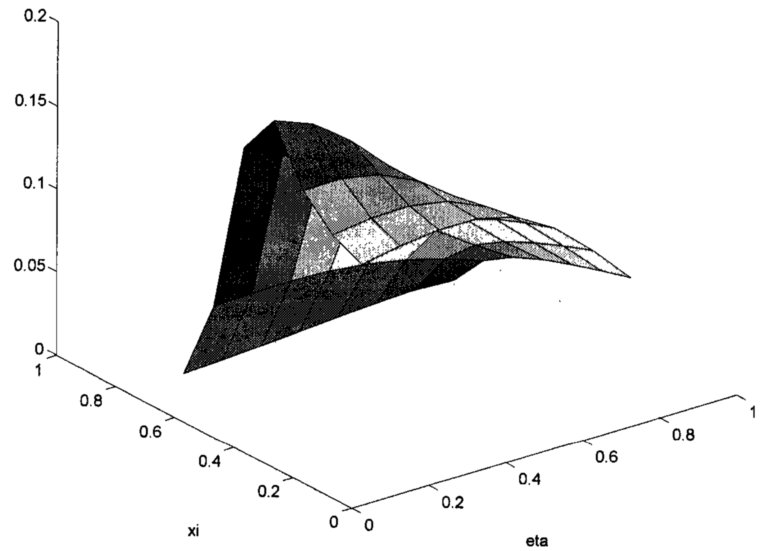
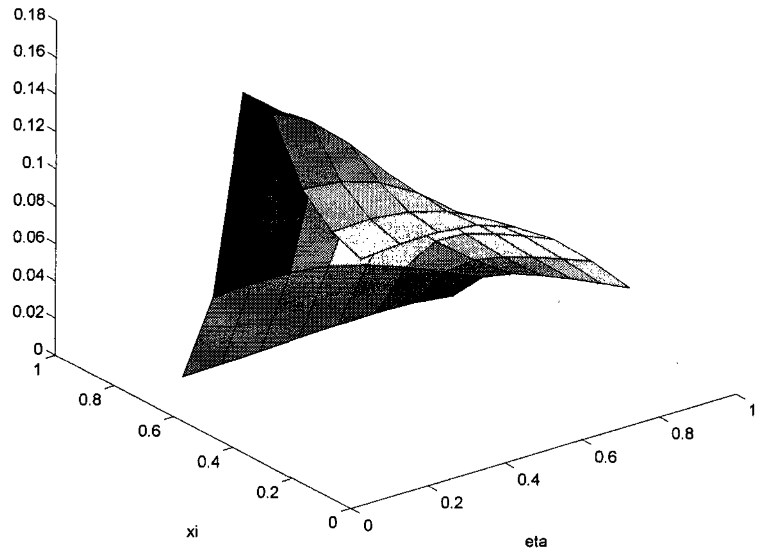


Figure 1.12: continued

Correlation (Rf, Gamma), (Omega=0.8)



Correlation (Rf, Gamma), (Omega=0.9)

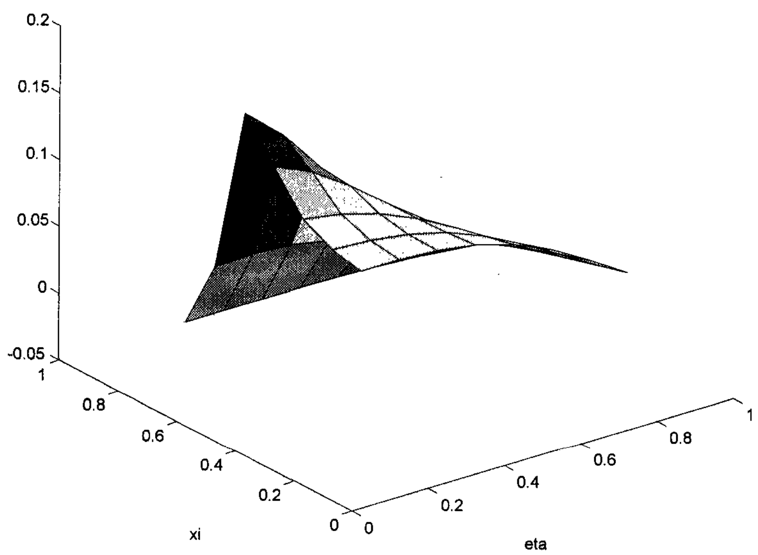


Table 1.2: Summary Return Statistics: Maching Cases
All Returns and standard deviations are in percentage.

	U.S. Data	$\upsilon = 2$ $\delta = 0.25$ $\Omega = 0.8$ $\xi = 0.7$ $\eta = 0.8$	$\upsilon = 2$ $\delta = 0.25$ $\Omega = 0.5$ $\xi = 0.7$ $\eta = 0.7$
$E(Re)$	6.98	7.06	7.26
$S.D.(Re)$	16.54	28.57	21.82
$E(Rf)$	0.8	0.5	0.52
$S.D.(Rf)$	5.67	22.62	13.02
$E(RP)$	6.18	6.56	6.74
$S.D.(RP)$	16.67	16.92	17.22
$corr(r^e_t, r^e_{t-1})$	-0.03	-0.062	-0.15
$corr(r^f_t, r^f_{t-1})$	0.87	-0.2558	0.3837
$corr(r^e_t, r^f_t)$	-0.09	0.8059	0.6148
$corr(r^e, \lambda)$	$corr(r^e$,growth in total output)	0.4317	0.5952
$corr(r^e, \gamma)$	≈ 0.30	0.5618	0.8011
$corr(r^f, \lambda)$	$corr(r^f$,growth in total output)	0.0423	0.0315
$corr(r^f, \gamma)$	[-0.03, 0.01]	0.0063	0.0585

1.4 Conclusion

We study an exchange economy with one non-durable good and one durable good, where the representative develops a taste for consumption through time via habit formation and durability. We find that habit formation helps to increase the equity premium through the dramatic increase in the effective relative risk aversion. Incorporating durability helps to reduce the conterfactually high volatility in the risk-free real interest rate. We also show that this model can better represent the co-movement between the securities returns and growth rates of outputs.

Chapter 2

A Note on Asset Pricing with Preferences Represented by A Power Risk Aversion Utility

2.1 Introduction

The equity premium puzzle was first introduced by Mehra and Prescott (1985). Over the period of 1889 - 1978, the unconditional mean of (ex-post) real return on the US T-bill is 0.8% while the unconditional mean of real returns (dividend inclusive ex-post returns) on the S & P 500 index is 6.98%. The unconditional mean of the risk premium of 6% is much larger than that predicted by the representative agent model with Constant Relative Risk Aversion (CRRA) utility using reasonable parameter of risk aversion.

The utility function adopted in Mehra and Prescott (1985) is the additive power utility function, in which the preference is time- and state-separable. Over the past 20 years, the popular solutions for the equity premium puzzle are to replace the time-and state-separable additive utility function with state-non-separable utility function (Epstein and Zin, 1989, 1991 and Weil, 1989) or with time-non-separable utility function exhibiting habit formation (Campbell and Cochrane, 1999, Constantinides,1990) or durability (Detemple and Giannikos,1996). Attempts to explain the equity premium puzzle with the use of time- and state- separable additive utility functions has not been successful. By incorporating the possibility of a third rare but catastrophic state in the standard Mehra and Prescott (1985) setup, Rietz (1988) finds that a coefficient of relative risk aversion of 10 is big enough to reconcile the representative agent asset pricing model with the observed risk premium in the U.S. market and that the coefficient of the relative risk aversion decreases as the probability of the catastrophic state increases. However, Mehra and Prescott (1988) argue that the disaster scenario and the implied movement in the real interest rate are not observed in the recorded U.S. data.[1]

Another feature of the power utility function is that the representative agent has

[1] The probability of the catastrophic state (with a 25% declined in consumption) assumed in Rietz (1988) is 1%. The model implies that there is a negative relation between the real interest rate and the probability of the extreme event. However, we do not observe the real interest rate increasing in the periods after World War II, when the perceived probability of a depression was decreasing over time.

CRRA, with which his optimal decision will be invariant of the scale of his endowments. It is a very strong restriction on the risk preference structure. Pratt (1964) suggests that the relative risk aversion should be first decreasing then increasing as wealth increases. Under the habit formation model, when investors have their consumption just above the habit persistence level, their local relative risk aversion can be very high (Constantinides, 1990). As their endowment (and the surplus consumption over the habit persistence level) grows in the future, their relative risk aversion must be decreasing. Most empirical studies reject constant absolute risk aversion (CARA) in favor of the decreasing absolute risk aversion (DARA) (Chavas and Holt, 1990, Pope and Just, 1991). Studies on investors' relative risk attitudes, however, produce mixed results. Cohn, Lewellen, Lease and Schlarbaum (1975) find strong decreasing relative risk aversion (DRRA) when studying the effect of the investors' wealth on the portfolio allocation. Friend and Brume (1975) claim that CRRA can describe investors' demand for risk assets well. Morin and Suarez (1983) support the DRRA after controlling for the life-cycle effect. Using international consumption data, Xie (2000) finds increasing relative risk aversion (IRRA). It seems fair to say that there is no consensus on the pattern of relative risk aversion.

To relax the CRRA assumption, we use the power risk aversion utility introduced by Xie (2000), which includes all the possible cases of the risk preference structure other than Increasing Absolute Risk Aversion, depending on the range of the parameters. Xie (2000) suspects that the bigger admissible range for the risk aversion may improve the performance of the Consumption Asset Pricing Model. Since the equity risk premium is a result of the uncertainty about the investors' future consumption, it may be more comprehensive if we can study the dynamic of the overall distribution of the excess return.

With long simulated consumption paths, we find that the results from the Power Risk Aversion utility are very close to those from the CRRA utility function. This is not surprising since the CRRA utility function is a special case of Power Utility function over the long run. With shorter simulated consumption paths and low long-run relative risk aversion, we are able to replicate the equity risk premium in the U.S. economy with a small long-run relative risk aversion of 1.5. The correlation structures of the returns on securities are also closer to the dynamics of the U.S.

data. These results may, for example, imply that countries with different GDP levels or consumption growth histories may have different equity premium structures.

The rest of the chapter is organized as as follows. Section 2.2 describes the economy and the Power Risk Aversion utility function and sets up the appropriate ranges for parameters. Section 2.3 discusses the simulation results. Sections 2.4 concludes.

2.2 Model

2.2.1 Basic Structure

In the Lucas' (1978) tree-type pure exchange economy, the long-lived representative agent maximizes his expected utility over a random consumption path by solving the following problem subject to his life-time budget constraint:

$$\max : E[\sum_{t=0}^{\infty} \beta^t U(c_t)], \qquad 0 < \beta < 1 \tag{2.1}$$

$$subj. : c_t + p_t z_{t+1} \leq z_t(y_t + p_t) \qquad 0 \leq z_t \leq 1 \tag{2.2}$$

where $E_0[.]$ is expectation based on the information available at time 0, $u(.)$ is period utility with $u'(.) > 0$ and $u''(.) < 0$, β is the rate of time-preference at which the utility in the future is discounted to the present; a low β means that the agent is highly impatient; c_t is the per capita consumption at time t, z_t is the agent's security holdings at the beginning of the period t, p_t is relative price of the security as the consumption good is served as numeraire and y_t is the output in the economy.

To capture the non-stationary in per capita consumption series, following Mehra and Prescott (1985), we assume that the growth rate of the endowment follows a Markov process. There are two states of the economy according to the growth rate, the good state and the bad state. The growth rates and a symmetric matrix of transition probabilities are chosen to match the per capita consumption statistics of the U.S. in the period between 1889 and 1978. The details for the growth rates and the transition matrix can be found in the next sub-section. At equilibrium, aggregate consumption equals the aggregate output in the economy, i.e. $c_t = y_t$.

The necessary and sufficient first order condition is

$$U'(y_t)p_t^e = \beta \int U'(y_{t+1})(p_{t+1}^e + y_{t+1})d(x_{t+1;}x_t) \tag{2.3}$$

where x_{t+1} is the growth rate of output from time t to time $t+1$;

$$y_{t+1} = y_t x_{t+1}. \tag{2.4}$$

This pricing relation says that, at equilibrium, the loss in the marginal utility this period by deferring consumption and buying one unit of security at p_t should be equal to the expected gain in the discounted marginal utility of consuming additional $(p_{t+1} + y_{t+1})$ units of goods next period. This pricing mechanism can also be applied to the risk-free security that guarantees one unit of numeraire goods next period. The necessary and sufficient first order condition for risk-free security is

$$U'(y_t)p_t^f = \beta \int U'(y_{t+1})d(x_{t+1;}x_t). \tag{2.5}$$

The price of risky security and risk-free security are

$$p_t^e = E_t[\sum_{j=1}^{\infty} \beta^j \frac{U'(c_{t+j})}{U'(c_t)} c_{t+j}] \tag{2.6}$$

$$p_t^f = E_t[\beta \frac{U'(c_{t+1})}{U'(c_t)}] \tag{2.7}$$

and the one-period return on the risky security and risk-free security are

$$R_t^e = \frac{p_{t+1} + c_{t+1}}{p_t} \tag{2.8}$$

$$R_t^f = \frac{1}{p_t^f} \quad . \tag{2.9}$$

The equity risk premium is

$$R_t^e - R_t^f. \tag{2.10}$$

Knowing the pricing mechanism, we can now proceed by specifying the functional form for the reprehensive agent's utility. Since there is no a priori risk aversion structure for the representative agent, we consider the Power Risk Aversion (PRA)

Utility proposed by Xie (2000):

$$u(c) = \frac{1}{\gamma}[1 - \exp(-\gamma(\frac{c^{1-\sigma}-1}{1-\sigma}))], \qquad \sigma \geq 0, \qquad \gamma \geq 0. \tag{2.11}$$

$$\gamma = 0 : u(c) \equiv \lim_{\gamma \to 0} \frac{1}{\gamma}[1 - \exp(-\gamma(\frac{c^{1-\sigma}-1}{1-\sigma}))] = \frac{c^{1-\sigma}-1}{1-\sigma}, \sigma \geq 0, \tag{2.12}$$

$$\sigma = 1 : u(c) \equiv \lim_{\sigma \to 1} \frac{1}{\gamma}[1 - \exp(-\gamma(\frac{c^{1-\sigma}-1}{1-\sigma}))] = \frac{1-c^{(-\gamma)}}{\gamma}, \gamma \geq 0. \tag{2.13}$$

This utility function is more flexible and allows for changes for both relative risk aversion and absolute risk aversion. The relative risk aversion (RRA) and the absolute risk aversion (ARA) defined by the PRA utility are as follows:

$$RRA = \sigma + \gamma c^{1-\sigma} \tag{2.14}$$

$$ARA = \sigma c^{-1} + \gamma c^{-\sigma}. \tag{2.15}$$

Both risk aversion measures are function of consumption. With $\sigma \neq 0$, absolute risk aversion is decreasing with the levels of consumption. The decreasing absolute risk aversion is consistent with both intuition and observation of rational behavior. With $\sigma < 1$, the relative risk aversion is increasing with consumption. With $\sigma > 1$, the relative risk aversion is decreasing with consumption. In the long run, as the consumption is approaching infinity an investor will have his relative risk aversion approaching σ. CRRA and CARA are specials cases of the PRA. When $\gamma = 0$ or $\sigma = 1$, the representative agent has CRRA of σ or $1+\gamma$, respectively; when $\sigma = 0$, the the representative agent has a CARA of γ. Since we don't have empirical evidence to assume the structure of the risk preference, adopting the PRA utility avoids the danger of taking special results as general solutions.

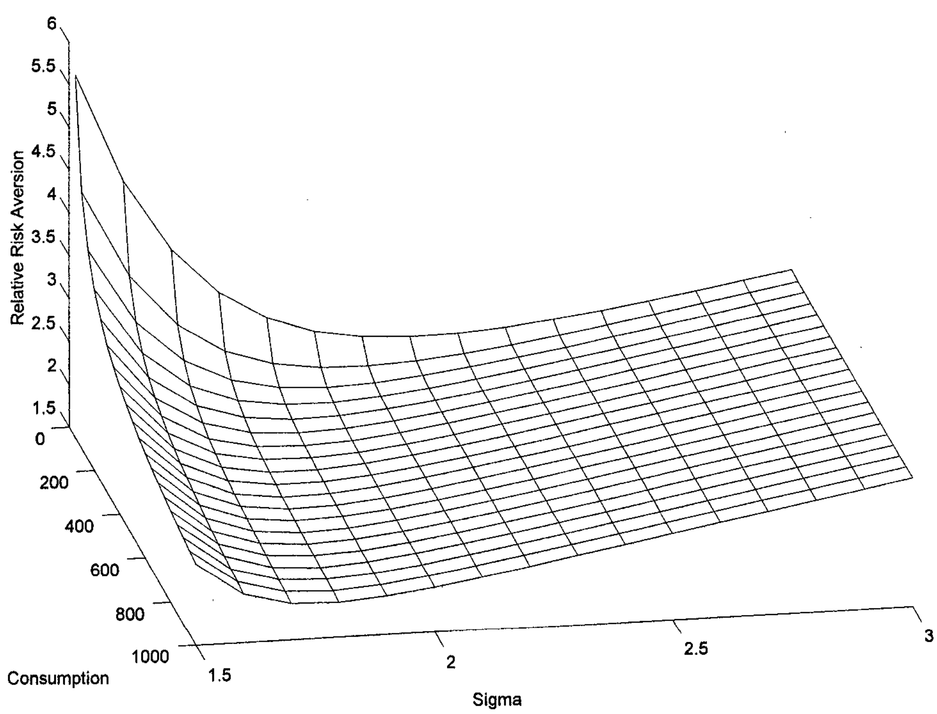
Although both γ and σ increase the curvature of the PRA utility, they affect the measure of the risk aversion differently. While higher γ always makes the investor more risk averse both absolutely and relatively, higher σ can increase or decrease the risk aversion according to the consumption level. Figure 2.1 shows the dynamics of the RRA when $\gamma = 30$. The comparative statics for relative risk aversion (RRA) with respect to σ are:

$$\begin{aligned} \frac{\partial RRA}{\partial \sigma} &\geq 0 \qquad \text{if } C^{1-\sigma} \leq \frac{1}{\gamma} \\ \frac{\partial RRA}{\partial \sigma} &< 0 \qquad \text{if } C^{1-\sigma} > \frac{1}{\gamma}. \end{aligned} \tag{2.16}$$

Due to the non-monotonic relation between the RRA and long-run relative risk aversion, σ, we maybe able to increase the short-run relative risk aversion and decrease the long-run relative risk aversion simultaneously. Consequently, we may be able to reconcile the PRA utility with the recorded U.S. economy.

With PRA utility, we could not find the exact solutions for the securities' prices since the prices are no longer homogeneous of degree one with the consumption level. To solve the problem numerically, we need to choose reasonable ranges for σ and γ.

Figure 2.1: The Dynamics of Relative Risk Aversion



2.2.2 Appropriate parameters

The appropriate range for the parameters (σ and γ) should yield the desired properties for the price of the risk-free security.

Following the setup in Mehra and Prescott (1985), we define x_1 and x_2 to be the growth rates in the good state and bad state, respectively. More specifically,

$$x_1 = 1 + \mu + \delta \tag{2.17}$$

$$x_2 = 1 + \mu - \delta \tag{2.18}$$

where μ and δ are mean and standard deviation of the growth rate of the output in the representative economy.

We also assume the transition probability for the Markov Chain to be the following:

$$M = \begin{bmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{bmatrix} = \begin{bmatrix} \pi & 1-\pi \\ 1-\pi & \pi \end{bmatrix}. \tag{2.19}$$

The probability of staying in the same state in the next period is π and the probability of switching to the other state in the next period is $1 - \pi$. To match the mean, standard deviation and the first order autocorrelation of the per capita consumption in the U.S. economy from 1889 to 1978, Mehra and Prescott (1985) choose $\mu = 0.018$, $\delta = 0.036$ and $\pi = 0.43$.

The conditional prices for the risk-free security are:

$$p^f(y_t, x_1) = \beta[\pi x_1^{-\sigma} \exp(\frac{-\gamma}{1-\sigma} y_t^{1-\sigma}(x_1^{1-\sigma} - 1) + (1-\pi) x_2^{-\sigma} \exp(\frac{-\gamma}{1-\sigma} y_t^{1-\sigma}(x_2^{1-\sigma} - 1)] \tag{2.20}$$

$$p^f(y_t, x_2) = \beta[(1-\pi) x_1^{-\sigma} \exp(\frac{-\gamma}{1-\sigma} y_t^{1-\sigma}(x_1^{1-\sigma} - 1) + \pi x_2^{-\sigma} \exp(\frac{-\gamma}{1-\sigma} y_t^{1-\sigma}(x_2^{1-\sigma} - 1)] \tag{2.21}$$

where $p^f(y_t, x_i)$ is the price of the risk-free security in state i at time t.

As output grows without bound ($y_t \rightarrow \infty$), the return on the risk-free security will behave differently according to σ.

1. If $\sigma < 1$, the conditional price of the risk-free security in both states will explode and result in a conditional return of –1 in both states. Furthermore, the portion of wealth that is invested in the risk-free security will also explode, ($\frac{p^f(y_t, x_i)}{y_t} \longrightarrow \infty$). Therefore, $\sigma < 1$ with increasing relative risk aversion is not

appropriate.

2. If $\sigma > 1$, the conditional prices of the risk-free security are

$$\lim_{y_t \to \infty} p^f(y_t, x_1) = \beta(\pi x_1^{-\sigma} + (1-\pi)x_2^{-\sigma})$$

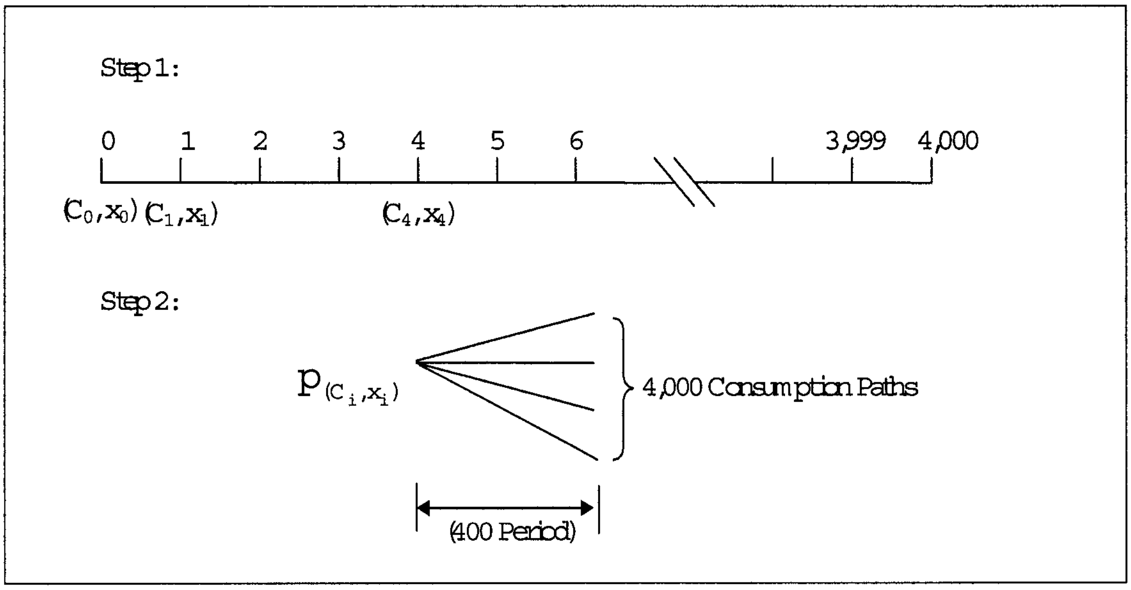
$$\lim_{y_t \to \infty} p^f(y_t, x_2) = \beta((1-\pi)x_1^{-\sigma} + \pi x_2^{\sigma}).$$

Note that the converged prices are exactly the same as the exact solution from the CRRA preference. The risk-free return in the good state is $\frac{1}{\beta(\pi x_1^{-\sigma}+(1-\pi)x_2^{-\sigma})}$ and is $\frac{1}{\beta((1-\pi)x_1^{-\sigma}+\pi x_2^{-\sigma})}$ in the bad state. This risk-free rate is a function of the time preference and the expected growth rate next period. The more impatient the investor is, the lower the risk-free rate. The higher the expected growth rate in the next period, the higher the risk-free rate, because a consumption smoothing agent is more reluctant to save. The proportion of the wealth invested in the risk-free security approaches zero as the output goes to infinity. Therefore, $\sigma > 1$ are good candidates to test the equity premium.

Since the relative risk aversion is approaching σ in the long run (equation (2.14)), we consider $1 \leq \sigma \leq 3$ being reasonable range to account for investor's risk attitude. Any γ will be acceptable since over the long run it will be washed out in the risk aversion measures by huge consumption. High γ can generate a high risk-aversion measure (both relative and absolute) when the consumption level is relatively low. Higher σ, however, does not always increase the relative risk aversion. When $c > \gamma^{\frac{1}{\sigma-1}}$, higher σ will lower the investor's relative risk aversion.

2.3 Numerical Results

Figure 2.2: Simulation Procedure (Step 1 - Step 2)



2.3.1 Simulation Procedures

We adopt the following simulation procedures as in figure 2.2.

1. Simulate a single consumption path for 4,000 periods according to the Markovian growth rate specified in the previous section with $y_0 = 1$.

2. Choose $\beta = 0.96$ as the annual subjective discount rate and calculate the equilibrium prices for both risky and risk-free securities given the consumption growth and level simulated in each period in Step 1.[2]

3. Find the returns for both securities and the equity risk premium in each period according to equation (2.8) and equation (2.9).

4. Calculate the average risky and riskless returns for the simulated path.

5. Repeat Step 1 to Step 4 for 4,000 times.

[2]In order to find the equilibrium price for a pair of consumption levels and (growth) states, we need to simulate a separate set of 4,000 consumption paths each with 400 periods. All the 4,000 consumption paths start with the aforementioned pairs. Equation (2.6) and equation (2.7) will give us the prices for the risky security and the risk-free security, respectively, at the beginning of every single path. The average prices for the 4,000 paths are calculated. These average prices are not sensitive to the increase in the length of each simulation and in the number of the simulations. We interpret these average prices as the equilibrium prices for the pair of consumption levels and (growth) states specified at the beginning of each of the 4,000 simulations.

6. Find the average statistics for the returns in the risky and riskless securities.

2.3.2 Simulation Results

With $\sigma = 3$, the simulation with PRA Utility shows that at and above the consumption (dividend) level of $377 per year, the expected prices for both securities are exactly the same as those from CRRA utility. If the simulation lasts for long enough, all the statistics with PRA Utility will converge to the standard Mehra and Prescott (1985) case with a relative risk aversion coefficient of 3. The result is not surprising since the long-run relative risk aversion from the PRA utility is 3. With a simulation path up to the different consumption levels, we get different correlation statistics in the returns on both risky and risk-free securities.

The results of the simulation with up to 4,000 periods are reported in Table 2.1. Panel A provides the statistics on historical securities price in U.S. economy from 1897-1998. Corresponding statistics with standard CRRA utility with CRRA = 3 (Mehra and Prescott,1985) are presented in Panel B. Panel C summarizes the results with Power Risk Aversion utility, with long-run CRRA = 3. In addition to the standard statistics, we report the the auto-correlation between the risky and risk-free returns and the comovement between them.

Table 2.1: Summary of Financial Statistics: Comparison of Results from Mehra and Prescott (1985) and Power Risk Aversion Utility
All returns and standard deviations are in percentage.

	A U.S. Data	B Mehra-Prescott(1985) $\sigma = 3$; $\gamma = 0$	C Power Risk Aversion Utility $\sigma = 3$; $\gamma = 10$
$E(R^e)$	6.98	9.58	9.67
$S.D.(R^e)$	16.54	5.00	5.20
$E(R^f)$	0.8	9.10	9.07
$S.D.(R^f)$	5.67	1.57	1.74
$E(R^P)$	6.18	0.48	0.59
$S.D.(R^P)$	16.67	4.73	4.84
$corr(r^e_t, r^e_{t-1})$	-0.03	-0.33	-0.2928
$corr(r^f_t, r^f_{t-1})$	0.87	-0.15	-0.0202
$corr(r^e_t, r^f_t)$	-0.09	0.33	0.3682

With 4, 000 simulation periods, a PRA utility with long-run CRRA of 3 produces an unconditional expected equity premium of 0.59%, which is still much lower than what we observed in the market. The returns on both securities in the U.S. market are too volatile as compared to those predictions from the CRRA and PRA models. Both models predict negative first-order autocorrelations in returns on both equity and the risk-free security. This suggests that the price changes in both securities tend to be reversed. The negative auto-correlation in the risk-free rate is not consistent with the data in the U.S. economy.

With CRRA utility[3], return on the risk-free security is determined by the growth rate and its probability structure. With the probability structure assumed, a lower growth rate implies higher expected growth next period. High expected growth rate in the next period would deter a risk-averse agent from saving and the risk-free security becomes less desirable. Since the growth rate is less likely to stay in the same state for consecutive periods, we should see that high returns tend to be followed by low returns for the risk-free security. Although the sign of the first-order autocorrelation in equity returns matches the data from the U.S. economy, the mean reversion prediction on the risk-free security is not observed in the market.

The results from the simulations for different consumption levels are reported in Table 2.2. We choose $\sigma = 3$ and $\gamma = 10$. With shorter simulations, the returns from both securities and their volatilities with PRA utility are higher than those with CRRA utility. The equity premia from the PRA model are slightly higher than those from the CRRA model mainly due to the fact that PRA utility with $\gamma = 10$ produces

[3]The PRA utility will converge to CRRA in the long run.

higher relative risk aversion when the consumption levels are relatively low.

Table 2.2: Summary of Financial Statistics: Comparison of Results from Different Simulation Lengths ($\sigma = 3$ and $\gamma = 10$)
All the returns and standard deviations are in percentage.

	A U.S. Data	B 100 periods	C 200 periods	D 300 periods	E 400 periods
$E(R^e)$	6.98	13.70	11.73	11.02	10.66
$S.D.(R^e)$	16.54	9.34	7.79	7.06	6.64
$E(R^f)$	0.8	11.90	10.52	10.01	9.76
$S.D.(R^f)$	5.67	0.04	3.37	2.99	2.74
$E(R^P)$	6.18	1.80	1.22	1.01	0.09
$S.D.(R^P)$	16.67	7.89	6.58	6.03	5.74
$corr(r^e_t, r^e_{t-1})$	-0.03	-0.1981	-0.1522	-0.1591	-0.1732
$corr(r^f_t, r^f_{t-1})$	0.87	0.1620	0.2660	0.2680	0.2520
$corr(r^e_t, r^f_t)$	-0.09	0.5464	0.5474	0.5302	0.5122

With relatively lower levels of consumption, PRA utility produces positive autocorrelation in return on the risk-free security, which conforms to the U.S. data qualitatively. When the consumption is low, the agent's relative risk aversion is $\sigma + \gamma c^{1-\sigma}$ and the price of the risk- free securities is not only determined by the growth rate, but is also determined by the level of the endowment. Since a high growth rate (good state) is more likely to be followed by a low growth rate (bad state) in the probability structure assumed in this model, the high level of consumption is more likely to drop in the next period. As a result, the agent's relative risk aversion increases. In the low growth period following the high growth period, there are two effects on the agent's saving decision. On the one hand, the representative agent becomes more risk adverse and increases his precautionary saving. On the other hand, the higher expected growth rate in the next period makes the agent decrease his precautionary saving. In the higher growth period, the agent's saving decision is exactly the opposite. The positive first-order autocorrelation in the returns on the risk-free security we observed may suggest that different decision factors dominate in the precautionary savings at different growth states.

Keeping the long-run relative risk aversion constant and raising the short-run relative risk aversion by increasing the parameter γ to 30 is not enough to match our model with the U.S. economy. Results in Table 2.3 show little improvement on the risk premium with shorter simulations.

After trying various combinations of γ and σ, we find that a Power Risk Aversion Utility with $\gamma = 30$ and $\sigma = 1.5$, is able to produce an equity risk premium that is very close to the recorded U.S. economy premium when we simulate the consumption path for only 100 periods or 200 periods. The summary of the return statistics

Table 2.3: Financial Statistics from Shorter Simulations ($\sigma = 3$; $\gamma = 30$)
All returns and standard deviations are in percentage.

	A U.S. Data	B 100 Periods	C 200 Periods	D 300 Periods	E 400 Periods
$E(R^e)$	6.98	16.41	13.27	12.04	11.43
$S.D.(R^e)$	16.54	12.67	10.28	9.05	8.31
$E(R^f)$	0.8	12.97	11.19	10.46	10.09
$S.D.(R^f)$	5.67	6.51	5.11	4.39	3.94
$E(R^P)$	6.18	3.44	2.08	1.58	1.33
$S.D.(R^P)$	16.67	11.87	9.27	8.09	7.42
$corr(r^e_{t,} r^e_{t-1})$	-0.03	-0.2709	-0.1545	-0.1314	-0.1318
$corr(r^f_{t,} r^f_{t-1})$	0.87	0.0306	0.1462	0.1812	0.1896
$corr(r^e_{t,} r^f_t)$	-0.09	0.3768	0.4345	0.4479	0.4485

on the risky and risk-free securities with different simulation lengths are reported in Table 2.4. With longer simulations, the equity risk premium and its volatility will eventually drop to the levels that are consistent with the results from a power utility with CRRA of 1.5. Although we find decreases in both the equity risk premium and its volatility, the autocorrelations in the risky and risk-free returns are closer to those observed in the U.S. economy. The Power Risk Aversion Utility is not able to generate a cross-correlation between the returns from the risky and risk-free securities that conforms to the recorded cross-correlation from the U.S. economy.

The dramatic increases in the risk premium with shorter simulations in Table 2.4

Table 2.4: Financial Statistics from Shorter Simulations ($\sigma = 1.5$; $\gamma = 30$)
All returns and standard deviations are in percentage

	A U.S. Data	B 100 Periods	C 200 Periods	D 300 Periods	E 400 Periods
$E(R^e)$	6.98	21.99	20.40	17.89	15.83
$S.D.(R^e)$	16.54	13.47	12.37	11.52	10.95
$E(R^f)$	0.8	14.46	15.01	13.87	12.65
$S.D.(R^f)$	5.67	10.13	8.19	7.10	6.56
$E(R^P)$	6.18	7.53	5.38	4.02	3.18
$S.D.(R^P)$	16.67	10.51	9.98	9.18	8.50
$corr(r^e_t, r^e_{t-1})$	-0.03	-0.5042	-0.4423	-0.2899	-0.1514
$corr(r^f_t, r^f_{t-1})$	0.87	-0.0339	-0.0374	0.0209	0.1225
$corr(r^e_t, r^f_t)$	-0.09	0.6347	0.5932	0.6026	0.6310

result from the increase in the short-term risk aversion which is not only associated with the increase in parameter γ,but also related to the decrease in the long-run relative risk aversion, σ. The different financial statistics presented in Table 2.2, Table 2.3 and Table 2.4 with different simulations lengths may imply that countries with different GDP levels or different histories of consumption growth may have different equity premium structures.

2.4 Conclusion

This paper is an exploration of the effects of Power Risk Aversion (PRA) Utility on the structure of asset pricing. PRA utility introduces DRRA, IRRA and DARA. Furthermore, CRRA and CARA are included as special cases. Since there is no consensus on the structure of the risk preference, adopting PRA utility avoids the danger of taking special results as general solutions. We find the PRA utility with DRRA and CRRA is appropriate for asset pricing. By incorporating DRRA, we are able to replicate the return statistics in the U.S. economy using simulation data of 100 periods or 200 periods. We also find return correlation structures that are closer to those observed in U.S. data with DRRA.

Appendix A

Equilibrium Pricing Functions

The Lagrangian for the representative agent's maximization problem is

$$L = E\sum_{i=0}^{\infty}\{\beta^t u(c_t, ds_t, h_{ct}, h_{ds,t}) - k_t[c_t + q_t f_t + p_{1,t}z_{1,t+1} + p_{2,t}z_{2,t+1}$$
$$-(p_{1,t} + x_t)z_{1,t} - (p_{2,t} + y_t q_t)z_{2,t})]\} \tag{A.1}$$

where k_t is the Lagrangian Multiplier for the representative agent's lifetime budget constrain.

The first order conditions are

$$\frac{\partial L}{\partial c_t} = \beta^t u_t^c + \beta^{t+1} u_{t+1}^h - k_t = 0 \tag{A.2a}$$

$$\frac{\partial L}{\partial z_{1,t}} = k_t(p_{1,t} + x_t) - k_{t-1}p_{1,t-1} = 0 \tag{A.2b}$$

$$\frac{\partial L}{\partial z_{2,t}} = k_t(p_{2,t} + y_t q_t) - k_{t-1}p_{2,t-1} = 0 \tag{A.2c}$$

$$\begin{aligned}\frac{\partial L}{\partial f_t} &= \beta^t u_t^{ds}\frac{\partial(ds_t)}{\partial f_t} + \beta^{t+1}u_{t+1}^{h_{ds}}\frac{\partial h_{ds,t+1}}{\partial(ds_t)}\frac{\partial(ds_t)}{\partial f_t} + \beta^{t+1}u_{t+1}^{ds}\frac{\partial(ds_{t+1})}{\partial f_t} \\ &\quad + \beta^{t+2}u_{t+2}^{h_{ds}}\frac{\partial h_{ds,t+2}}{\partial(ds_{t+1})}\frac{\partial(ds_{t+1})}{\partial f_t} - k_t q_t = 0.\end{aligned} \tag{A.2d}$$

Solving k_t from equation (A.2a), and replacing it in the rest of the equations in the system, with the second order conditions

$$\left(\frac{\partial L}{\partial^2 c_t} \leq 0,\ \frac{\partial L}{\partial^2 f_t} \leq 0,\ \frac{\partial L}{\partial^2 z_{1,t}} = 0,\ \frac{\partial L}{\partial^2 z_{2,t}} = 0\right). \tag{A.3}$$

we have the necessary and sufficient first order condition in (1.5a), (1.5b) and (1.5c).

Appendix B

EQUILIBRIUM PRICES

The necessary and sufficient first order conditions are:

$$(u_t^c - \beta\xi E[u_{t+1}^c])p_{1,t} = E[(\beta u_{t+1}^c - \beta^2\xi u_{t+2}^c)(p_{1,t+1} + x_{t+1})] \quad \text{(B.1)}$$

$$(u_t^c - \beta\xi E[u_{t+1}^c])p_{2,t} = E[(\beta u_{t+1}^c - \beta^2\xi u_{t+2}^c)(p_{2,t+1} + y_{t+1}q_{t+1})]$$

$$(u_t^c - \beta\xi E[u_{t+1}^c])q_t = (1-\Omega)u_t^{ds} + E[(1-\Omega)(\Omega-\eta)\beta u_{t+1}^{ds} - \beta^2\eta\Omega(1-\Omega)u_{t+2}^{ds}].$$

With the utility function specified in equation (1.7), the marginal utility at time t with respect to the perishable good consumption and service flow from the durable good are:

$$u_t^c = \delta(c_t - \xi c_{t-1})^{\delta(1-\nu)-1}(ds_t - \eta ds_{t-1})^{(1-\delta)(1-\nu)} \quad \text{(B.2a)}$$

$$u_t^{ds} = (1-\delta)(c_t - \xi c_{t-1})^{\delta(1-\nu)}(ds_t - \eta ds_{t-1})^{(1-\delta)(1-\nu)-1}. \quad \text{(B.2b)}$$

At equilibrium, market clears:

$$z_{1,t} = z_{2,t} = 1, \qquad c_t = x_t, \qquad f_t = y_t \ . \tag{B.3}$$

Let's define λ and γ as growth rates of the outputs in the non-durable good and durable good known at the beginning of the period t. N apostrophes next to the growth rate represent the corresponding growth rates to be know n periods ahead. λ_{-i} and γ_{-i} are growth rates revealed at the beginning of the period $t-i$.

Substituting (B.2) into (1.5a), with simple algebra rearrangement, we have

$$p_{1,t} = E_t[\frac{\beta(\lambda' - \xi)^{\delta(1-\nu)-1} a_2^{(1-\nu)(1-\delta)} - \beta^2 \xi(\lambda'\lambda'' - \xi\lambda')^{\delta(1-\nu)-1} a_3^{(1-\nu)(1-\delta)}}{(1 - \frac{\xi}{\lambda})^{\delta(1-\nu)-1} a_1^{(1-\nu)(1-\delta)} - \beta\xi(\lambda' - \xi)^{\delta(1-\nu)-1} a_2^{(1-\nu)(1-\delta)}}(p_{1,t+1} + \lambda' x_t)] \tag{B.4}$$

where

$$\begin{aligned} a_1 &= 1+\frac{\Omega}{\gamma}-\eta(\frac{1}{\gamma}+\frac{\Omega}{\gamma\gamma_{-1}}) \\ a_2 &= \gamma'+\Omega-\eta(1+\frac{\Omega}{\gamma}) \\ a_3 &= \gamma''\gamma'+\Omega\gamma'-\eta(\gamma'+\Omega). \end{aligned}$$

When the joint process of the growth rates of the durable good and non-durable good follows markov chain process, the (relative) stock price for the non-durable good producing firm is a linear function of its output. The coefficient of the linear relation is depending on the growth rates of both goods in the past two periods.

$$p_{1,t} = w(\lambda,\gamma,\gamma_{-1})x_t. \tag{B.5}$$

Substitute (B.2) into (1.5c) and rearrange it,

$$\begin{aligned} q_{1,t} &= \frac{x_t}{y_t}\frac{1-\delta}{\delta} * E_t[\frac{(1-\frac{\xi}{\lambda})^{\delta(1-\nu)}a_1^{(1-\nu)(1-\delta)-1}+\beta(\lambda'-\xi)^{\delta(1-\nu)}a_2^{(1-\nu)(1-\delta)-1}(\Omega-\eta)}{(1-\frac{\xi}{\lambda})^{\delta(1-\nu)-1}a_1^{(1-\nu)(1-\delta)}-\beta\xi(\lambda'-\xi)^{\delta(1-\nu)-1}a_2^{(1-\nu)(1-\delta)}} \\ &\quad -\frac{\beta^2\Omega\eta(\lambda'\lambda''-\xi\lambda')^{\delta(1-\nu)}a_3^{(1-\nu)(1-\delta)-1}}{(1-\frac{\xi}{\lambda})^{\delta(1-\nu)-1}a_1^{(1-\nu)(1-\delta)}-\beta\xi(\lambda'-\xi)^{\delta(1-\nu)-1}a_2^{(1-\nu)(1-\delta)}}]. \end{aligned} \tag{B.6}$$

With the joint markov process of the growth rates of both goods, the (relative) price for the durable good is a linear function of the ratio of outputs in the durable good to those of the non-durable good

$$q_t = G(\lambda, \gamma, \gamma_{-1})\frac{x_t}{y_t}. \tag{B.7}$$

For (B.2) and (1.5b), we have

$$p_{2,t} = E_t[\frac{\beta(\lambda' - \xi)^{\delta(1-\nu)-1} a_2^{(1-\nu)(1-\delta)} - \beta^2 \xi(\lambda'\lambda'' - \xi\lambda')^{\delta(1-\nu)-1} a_3^{(1-\nu)(1-\delta)}}{(1 - \frac{\xi}{\lambda})^{\delta(1-\nu)-1} a_1^{(1-\nu)(1-\delta)} - \beta\xi(\lambda' - \xi)^{\delta(1-\nu)-1} a_2^{(1-\nu)(1-\delta)}}(p_{2,t+1} + y_{t+1}q_{t+1})]. \tag{B.8}$$

Substituting (B.7) into (B.8), we have

$$\begin{aligned} p_{2,t} &= E_t[\frac{\beta(\lambda' - \xi)^{\delta(1-\nu)-1} a_2^{(1-\nu)(1-\delta)} - \beta^2 \xi(\lambda'\lambda'' - \xi\lambda')^{\delta(1-\nu)-1} a_3^{(1-\nu)(1-\delta)}}{(1 - \frac{\xi}{\lambda})^{\delta(1-\nu)-1} a_1^{(1-\nu)(1-\delta)} - \beta\xi(\lambda' - \xi)^{\delta(1-\nu)-1} a_2^{(1-\nu)(1-\delta)}} \\ &\quad *(p_{2,t+1} + G(\lambda, \gamma, \gamma_{-1})\lambda' x_t]. \end{aligned} \tag{B.9}$$

Again, with the mokovian property of the joint process of the growth rates, we find that the (relative) stock price of the durable good producing firm is a multiple of the non-durable good's output. The multiplier is depending on the realized growth rates of both goods in the past two periods.

$$p_{2,t} = Z(\lambda, \gamma, \gamma_{-1})x_t. \qquad \text{(B.10)}$$

Appendix C

Transition Matrix, M

We use the history pattern of growth rates of the durable and non-durable goods output (Table C.1) summarized by Giannikos (2004) to estimate the transition matrix, M specified in equation (1.8) and 4 growth rates.

Table C.1: US Statistics

Statistics	**Value for U.S. Data**
$E(\lambda)$	1.025
$S.D.(\lambda)$	0.011
$E(\gamma)$	1.062
$S.D.(\gamma)$	0.071
$corr(\lambda_t, \lambda_{t-1})$	0.253
$corr(\gamma_t, \gamma_{t-1})$	−0.016
$corr(\lambda_t, \gamma_t)$	0.81

Following Giannikos (2004), if the above statistics are defined by markov chain process with the transition matrix, there are seven relations between the observed statistics and the nine unknown parameters (four growth rate parameters and five probability parameters in the transition matrix, M). It is also obvious from the specification of M (equation (1.8)) that $\sigma + H + \phi + \pi = 1$. In the system of eight equations with nine unknowns, we treat Δ as a free parameter. Only those Δs that produce positive probabilities will be considered. Luckily, we find that Δ has to be extremely close to 0.7, if not equals to 0.7, to get positive solution for all other probability parameters in M.

Bibliography

[1] A. B. Abel, Asset prices under habit formation and catching up with the joneses, The American Economics Review 80 (1990) 38–42.

[2] J. Y. Campbell, J. Cochrane, By force of habit: A consumption-based explanation of aggregate stock market behavior, Journal of Political Economy 107 (1999) 205–251.

[3] D. A. Chapman, The cyclical properties of consumption growth and the real term structure, Journal of Monetary Economics 39 (2) (1997) 145–172.

[4] J.-P. Chavas, M. T. Holt, Acreage decisions under risk: The case of corn and soybeans, American Journal of Agriculture Economics 72 (1990) 529–538.

[5] R. A. Cohn, W. G. Lewellen, R. C. Lease, G. G. Schlarbaum, Individual investor risk aversion and investment portfolio composition, Journal of Finance 30 (1975) 605–620.

[6] G. M. Constantinides, Habit formation: A resolution of the equity premium

puzzle, Journal of Political Economy 98 (3) (1990) 519–543.

[7] C. G. Jean-Pierre Danthine, John B. Donaldson, H. Guirguis, On the consequences of state dependent preferences for the pricing of financial assets, Finance Research Letters.

[8] J. B. Detemple, C. I. Giannikos, Asset and commodity prices with multi-attribute durable goods, Journal of Economic Dynamics and Control 20 (1996) 1451–1504.

[9] K. B. Dunn, K. J. Singleton, Modeling the term structure of interest rates under non-seperable utility and durability of goods, Journal of Financial Economics 17 (1986) 27–55.

[10] L. G. Epstein, S. E. Zin, Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework, Econometrica 57 (1989) 937–969.

[11] L. G. Epstein, S. E. Zin, Substitution, risk aversion, and the temporal behavior of consumptin and asset returns: An empirical analysis, Journal of Political

Economy 99 (1991) 263–286.

[12] W. Ferson, G. M. Constantinides, Habit persistence and durability in aggregate consumption: Empirical tests, Journal of Financial Economics 29 (1991) 199–240.

[13] I. Friend, M. E. Blume, The demand for risky assets, The American Economics Review 64 (1975) 900–921.

[14] C. I. Giannikos, Asset pricing in the presence of durable goods, Journal of Economic Dynamics and Control (Forthcoming).

[15] J. Heaton, The interaction between time-nonseparable preferences and time aggregation, Econometrica 61 (1993) 353–385.

[16] J. Heaton, An empirical investigation of asset pricing with temporally dependent preference specifications, Econometrica 63 (1995) 681–718.

[17] A. Hindy, C.-F. Huang, Optimal consumption and portfolio rules with durability and local substitution, Econometrica 61 (1) (1993) 85–121.

[18] A. Hindy, C.-F. Huang, S. H. Zhu, Optimal consumption and portfolio rules wih

durability and habit formation, Journal of Economic Dynamics and Control 21 (1997) 525–550.

[19] R. E. J. Lucas, Asset prices in an exchange economy, Econometrica 66 (1978) 1429–1445.

[20] R. Mehra, E. Prescott, The equity premium: A puzzle, Journal of Monetary Economics 15 (1985) 145–161.

[21] R. Mehra, E. Prescott, The equity premium: A solution?, Journal of Monetary Economics 22 (1988) 133–136.

[22] R.-A. Morin, A. F. Suarez, Risk aversion revisited, Journal of Finance 38 (4) (1983) 1201–1216.

[23] R. D. Pope, R. E. Just, Empirical estimation and use of risk preferences: An appraisal of estimaion methods that use actual economics decisions, American Journal of Agriculture Economics 73 (1991) 743–748.

[24] J. W. Pratt, Risk aversionin the small and in the large, Econometrica 32 (194) 122–136.

[25] D. Xie, Power risk aversion utility functions, Anals of Economics and Finance 1 (2000) 265–282.