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**TERM STRUCTURE OF INTEREST RATES  
AND BOND VALUATION : AN APPLICATION TO BRADY BONDS**

**by**

**Ricardo Rivera**

**A dissertation submitted to the Graduate Faculty in Economics in partial fulfillment  
of the requirements for the degree of Doctor of Philosophy, The City University of  
New York.**

**1996**

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**Abstract**

**TERM STRUCTURE OF INTEREST RATES AND BOND VALUATION: AN  
APPLICATION TO BRADY BONDS**

by

**Ricardo Rivera**

Adviser: Professor Tom Thurston

This thesis presents a general method of estimating the term structure of interest rates and to value the default risk premium of emerging market countries' debt consistent with Brady bond prices and the U.S. term structure of interest rates. The purpose is to obtain an empirical measure of the default risk premium that permits one to perform a relative value analysis among different securities and across countries and to find potential arbitrage opportunities. In addition, the term structure of interest rates of a emerging market country is used to price derivatives and perform price sensitivity analysis of Brady securities. The model of the term structure of interest rates of the emerging market country is based on a binomial tree. This tree represents the random evolution of future short rates, given some inputs and an assumption about the stochastic process for the U.S short interest rate. The inputs of the model are the yields on zero-coupon and coupon-bearing bonds for various maturities, the yield volatilities for the same bonds, and the current Brady bond prices. Several authors have implemented models that derive the term structure of interest rates from the behavior of the riskless interest rate. While these studies have focused on the riskless term structure of the U.S. Treasury, our tree includes a constant spread value, the default risk premium, which is a spread over the return on Treasury securities. This spread is interpreted as an empirical measure of the default risk premium. Summarizing, the model of the term structure of interest rates implemented in this study is a practical tool to estimate the default risk premium, the option values, and duration and convexity measures for an emerging market country.

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## INTRODUCTION

This study presents a general method of estimating the term structure of interest rates and to value the default risk premium of emerging market countries' debt based on the price of Brady bonds and consistent with the U.S. term structure of interest rates. The purpose is to obtain an empirical measure of the default risk premium that permits one to perform a relative value analysis among different securities and across countries and to find potential arbitrage opportunities. The term structure of interest rates of a emerging market country is also useful in pricing derivatives. The model can be used to find arbitrage opportunities on relative mispricing -that is, on a divergence between a derivative's price and the price of a theoretical portfolio that duplicates the payoffs of that derivative. In addition, the term structure derived from the model is employed to perform price sensitivity analysis of Brady securities -in other words, how bond prices change when interest rates change.

We take the perspective of a US-based investor who is interesting in pricing these bonds and their derivatives in the secondary market. Debt securities created from renegotiated bank loans through the Brady Plan are mostly dollar denominated, consequently US-base investors bear no foreign exchange risk although default risk exists.

A Brady bond can be viewed as a package of cash flows, with each cash flow seen as a zero-coupon instrument and maturing on the date it will be received. Each cash flow is discounted at its corresponding rate to obtain the present value or current market bond price. This procedure is equivalent to discounting the cash flows at a series one-period

future interest rates. In this paper the *term structure of interest rates* of the *emerging market country* is modeled by a *binomial tree*. This tree represents the random evolution of future short rates, given some inputs and an assumption about the stochastic process for the future U.S. Treasury (U.S.T) short interest rate. The inputs of the model are the yields on zero-coupon and coupon-bearing U.S.Treasury bonds for various maturities and the yield volatilities for the same bonds.

We need first to define several concepts that will be used throughout the paper. For example, the relationship between the yields and maturity for these bonds is called *the yield curve*, while the relationship between the yield volatilities and different maturities is called *the volatility curve*<sup>1</sup>. These curves together form the U.S *term structure*. On the other hand, *the on-the-run yield curve* is the relationship between the yield-to-maturity and maturity for bonds of similar quality trading at par. The most recently auctioned Treasury issues for each maturity are referred as on-the-run or current coupon issues. Since any bond can be viewed as a package of zero-coupon instruments, one needs to know the yield on a zero-coupon Treasury with the same maturity to determine the value of each zero-coupon instrument. This yield is called the *spot rate*. *The zero-coupon yield curve* is a curve showing the relationship between spot rates (i.e., zero-coupon yields) and maturity. *A theoretical spot-rate curve* is constructed from the yield curve based on the observed yields of Treasury bills and Treasury coupon securities. The procedure to create

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<sup>1</sup> In Section IV there is a more detailed explanation of the assumptions and the method of estimation of the interest rate volatility.

a theoretical spot-rate curve in this way is called *bootstrapping*<sup>2</sup>. The n-year spot interest rate is the interest rate on an investment that is made for a period of time starting today and lasting for n years. Thus, the 3-year spot rate is the rate of an investment lasting 3 years, the 5-year spot rate is the rate of interest on an investment lasting 5 years, and so on. All the interest and the principal is repaid to the investor at the end of year n. Finally, *forward interest rates* are the rates of interest implied by current spot rates for periods of time in the future.

Several papers<sup>3</sup> have proposed to construct a tree of possible future short rates by using the current U.S.T term structure (as defined previously). While most of these authors have derived pricing models for *default free bonds* or *options on default free bonds*, our model extends these studies to the case of a country whose term structure of interest rates includes a *default risk premium*. The model's measure of default risk premium over the risk-free rate of Treasury securities is similar to the notion of *Option-Adjusted Spread* (OAS). This methodology was initially developed for evaluating the impact of the embedded options in Mortgage backed Securities and callable corporate bonds. We obtain first the binomial short rate tree for a U.S. Treasury coupon bond traded at par (on-the run yield curve), with a cash flow pattern identical to that of the particular Brady bond that is analyzed. Then we estimate the constant value over this binomial tree

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<sup>2</sup> The basic principle of the bootstrapping method is that the value of the Treasury coupon security should be equal to the value of the package of zero-coupon Treasury securities that duplicates the bond's cash flow.

<sup>3</sup> See, for example, Fischer Black, E Derman, and W Toy (1990).

that makes the theoretical (model) present value of the bond equal to its current market price. This constant value, or default risk premium, is interpreted as the return over the U.S.T short rate tree. By assumption, the binomial tree of future short rates corresponding to the U.S.T bond used as benchmark, has a zero default risk (that is, riskless term structure of interest rates). The default risk premium is represented graphically by a parallel shift of the entire U.S.T tree by an amount equal to the constant spread value. Given the U.S Treasury term structure and the resulting tree of future short rates for the emerging market country, the model is used to value options on Brady bonds. Investors also are interested in the sensitivity of a bond's price to changes in interest rates. The *effective duration* and *Effective convexity* quantify price sensitivity to changes in interest rates. In terms of our binomial interest rate tree, price response to changing interest rates is found by shifting the tree up and down by a few basis points.

To summarize, the main contribution of this paper is to present a consistent valuation model, that is, the term structure of interest rates of a emerging market country, the default risk premium, the value of the option, and the duration and convexity measures are simultaneously computed, given Brady bond market prices and the current U.S. term structure.

The first purpose of this paper, as noted above, is to find a measure of default risk premium for an emerging market country. The motivation for this is the following. Two yield measures are commonly cited by market participants to measure the potential return from investing in a Brady bond, yield to maturity (YTM) and stripped yields (SY). *Yield to maturity* is the discount rate that equates the present value of future cash flows of a Brady

bond to its market price. *Stripped yields* measure the yield to maturity on the uncollateralized or risky portion of a Brady bond. Spreads relative to Treasuries are obtained by comparing a Brady bond's YTM and SY to the yield of a Treasury with a similar maturity. For example, for a semiannual Brady bond whose next coupon payment will be received six months from now, the yield to maturity and the stripped yield are computed by solving the following relationships for  $y$  and  $s$ :

$$P = \frac{c}{(1+y)^1} + \frac{c}{(1+y)^2} + \frac{c}{(1+y)^3} + \dots + \frac{c}{(1+y)^n} + \frac{pv}{(1+y)^n}$$

and

$$P = \frac{c}{(1+s)^1} + \frac{c}{(1+s)^2} + \frac{c}{(1+s)^3} + \dots + \frac{c}{(1+s)^n} + \frac{pv}{(1+r)^n}$$

where

$P$  = market price (\$);

$c$  = semiannual coupon payment (\$);

$y$  = one-half the yield to maturity;

$s$  = one-half the stripped yield;

$r$  = one-half the U.S.T zero-coupon (n-year spot) yield;

$n$  = number of periods (number of years x 2);

$pv$  = principal value (\$).

For a semiannual-pay bond, doubling the interest rates or discount rates  $y$  and  $s$  gives the annual yield to maturity and stripped yields. For any Brady bond, its yield-to maturity and stripped yield equal the risk-free rate (US Treasury) plus a risk premium. The latter compensates the investor for assuming credit risk. The formulas become

$$P = \frac{c}{(1 + r_f + d)^1} + \frac{c}{(1 + r_f + d)^2} + \dots + \frac{c}{(1 + r_f + d)^n} + \frac{pv}{(1 + r_f + d)^n}$$

and

$$P = \frac{c}{(1 + r_f + d)^1} + \frac{c}{(1 + r_f + d')^2} + \dots + \frac{c}{(1 + r_f + d')^n} + \frac{pv}{(1 + r)^n}$$

where  $r_f$  is the risk-free rate, while  $d$  and  $d'$  are the corresponding risk premiums.

There are several problems in this traditional analysis. First, in calculating the yields, the same discount rate is used to calculate the present values of cash flows received at different points in time. Therefore, it ignores the term structure of interest rates and its implication that the market assigns different discount rates to cash flows of different maturities (though this is a problem of any yield to maturity). We need to find a sequence  $y_1, y_2, \dots, y_n$  of yield to maturities where each  $y_i$  is made up of a US risk-free rate plus a constant spread,  $d$ , which gives a measure of the default risk premium over the US Treasury rates. It would be possible to use U.S.T zero yields for each cash flow and just find a constant spread that added to these rates gives a value equal to the current Brady price. However, the effect of interest-rate volatility on securities with interest-rate-

contingent cash flows is particularly relevant but is ignored in the traditional yield analysis. In this study interest-rate volatility is explicitly included in the estimation of the term structure of the interest rates, in other words, our binomial short rate tree depends on the volatility of the interest rates. When a Brady bond is callable, its cash flows are uncertain. The volatility of interest rates affects the possibility of a option to be exercised, so we must consider that volatility to compute the binomial short rate tree. Since Brady bonds are callable at any time, one must consider the volatility of interest rates as their volatility will affect the possibility of the call option being exercised. The *volatility-dependent short rate tree* is used to discount the cash flows of any bond in order to find the bond value.

The finding of a country's volatility-dependent short rate tree is also related to the second purpose of this study, that is, the valuation of a bond option. Since the market of derivatives for Brady bonds became very active in the last years, the model can be a useful tool to value these options. Section IV shows how to value Brady bond options using the country's binomial tree of short rates. We find first the future prices of a Brady bond at various nodes on the tree using an arbitrage argument. For example, we construct a portfolio of  $n$  zero-coupon bonds with the same annual payoffs as a Brady bond with  $n$ -years to maturity. So the portfolio and the brady bond have the same value. We then find the call value at expiration along the price tree as the difference between the bond's price (without accrued interest) and the strike price. Knowing the call values of an European option at expiration, we compute the present value of the call option working backward along the tree until the root of the tree is reached. Given the resulting call value tree, we

estimate the change of option prices to changes in the price of the underlying bond. This measure is called *the hedge ratio* (or *delta*).

To construct the emerging market country's term structure of interest rates, the model employs several inputs : (i) the current Brady bond prices, (ii) the observed U.S.T term structure, which includes zero-coupon and coupon yield curves, as well as their estimated volatilities, (iii) the assumed process for the U.S.T short-term interest rate, and (iv) the volatility of interest rates. There are two broad approaches to constructing models of the term structure. The first approach, here referred to as *the equilibrium model* of the term structure of interest rate, assumes a stochastic process for the short-term rate in a risk-neutral world and deduces bond and option prices. Section III presents a more detailed explanation of this type of model. The second approach, here referred to as *the arbitrage-free model* or *no arbitrage model*, takes the current term structure as given and develops an arbitrage-free yield curve model so that it is consistent with the market prices. In this paper we use this second approach to model the term structure of interest rates. The arbitrage-free model requires the term structure of spot rates as an input. In other words, all bonds are assumed to be priced correctly. To ensure that the future short rates are arbitrage-free, the model's parameter must be chosen so that theoretical prices for zeros and coupon-bearing bonds match market prices. *Arbitrage-free future rates* means that an investor is indifferent between a sequence of two (for example) six-month investments at these future rates and a one-year investment at the one-year spot rate. The idea of arbitrage-free rates is closely related to the idea of consistency. To say that the theoretical term structure of interest rates is *consistent* with the UST term structure,

means that one estimates future short rates in a way to ensure that theoretical (model) prices for U.S.T zeros and U.S.T coupon bonds match market prices. For example, given the current six-month and one-year spot rates and an assumption of how the six-month rate evolves over the future, one can compute the future (six months from now) six-month rate. Section IV describes in detail how the entire term structure is computed.

As noted above, our arbitrage-free model is implemented by a binomial tree where the short-term interest rate (i.e., the one-year interest rate) are lognormally distributed. It is assumed that the U.S.T short-term rates follow a *geometric Brownian motion process* where the drift or trend in interest movements depends on the level of the rates and the short-term rate volatility varies over time (*time-varying volatility* of the interest rates). We construct a binomial tree to approximate the stochastic process of the short interest rate by assuming that over a short period of time, the change in interest rate can only take two possible values, each with probability of 1/2. For example, a two-year zero-coupon bond has a known price at the end of the second year, independent of what short rate prevails. Its possible prices after one year can be obtained by discounting the expected two-year price by the possible short rates one year out. The entire binomial short rate tree is obtained by an iteration procedure called *the Newton-Raphson method*. Section II presents an example of how to construct a binomial tree for zero-coupon bonds for a sample term structure. Section IV extends Section II in two ways. First, we estimate the binomial tree of short rates for U.S.T zero-coupon and coupon bonds given the *actual* term structure. Second, we obtain the country's term structure by computing the constant default risk premium.

Note here that two different short rate trees are constructed: one is a binomial tree for zeros and the other is a one for U.S.T coupon-bearing bonds (based on on-the-run yield curve). This distinction stems from the fact that the principal of Brady bonds are fully secured through U.S.T zero-coupon bonds, but coupon payments are uncollateralized<sup>4</sup>. The first tree is a representation of the U.S.T term structure of zero-coupon bonds. The volatility of the logarithm of the short rate is a time-varying volatility of yields on zero-coupon bonds. This tree is used to value the collateralized part (principal) of Brady bonds. The second tree is a representation of the U.S.T term structure of coupon-bearing bonds and used to value the uncollateralized part (coupon payments) of Brady bonds. It is also constructed with the assumption of a time-varying volatility of the short rate. In particular, it is assumed that the volatility of the n-year spot yield is equal to the natural logarithm of the ratio of the short rates. The volatility of the n-year spot yield is estimated by using a measure of historical volatility based on the observed yield-to-maturity of U.S.T zeros and coupon bonds in the last thirty trading days.

The paper consists of four sections. The first section illustrates the general procedure to construct an arbitrage-free binomial model for a sample term structure. The second section describes the historical background and the institutional aspects of the Brady Plan. The third section gives a literature review of existing models of the term structure of interest rates. In particular, this section explains equilibrium models of the interest rates. Several stochastic differential equations for interest rates are presented. The

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<sup>4</sup> In some cases twelve or eighteen months of interest are guaranteed on a rolling basis through an escrow account. In this paper we assume that all coupon payments are fully uncollateralized.

term structure of interest rates is derived from the behavior of the short-term interest rates, and from a set of assumptions about investor's aversion to risk on default free bonds. This term structure is a partial differential equation for bond prices that must be satisfy in equilibrium. We present an example of finite difference method to value a security by solving numerically the differential equation that the bond price satisfies. Section four is the major contribution of this study. It presents a binomial tree of the future short term rate in an arbitrage-free model for an emerging market country. This section is divided into five steps: (1) Finding a short rate tree, (2) Computing the present value of coupon payments on a Brady bond, (3) Finding the term structure and the default risk premium of the country, (4) Pricing a callable bond, the value of the option, and duration and convexity measures, and (5) Pricing floating coupon and amortizing bonds. The country's term structure is developed in step one to step three. Step four illustrates how to value options on Bradies, while step five is just an extension of the model to the case of floating coupon and amortizing Brady bonds. The steps are presented below.

*Step One* : at each point in time a complete (for each maturity) yield curve is extracted by fitting a mathematical curve to the yield points from different maturities on U.S.T zeros and U.S.T coupon-bearing bonds. This is performed by a Piecewise Regression Model whose regression function is a cubic spline function. In addition, complete volatility curves are estimated from time series data. We compute first a binomial tree of future short rates consistent with (i) the current U.S. Treasury yield curve for zeros, (ii) its yield volatility, and (iii) the assumed stochastic processes for the short-term interest rate. For example, a two-year zero-coupon bond has a known price at the end of

the second year, independent of what short rate prevails. Its price after one year is obtained by discounting the expected two-year price by the possible short rates one year out. The price today is then determined by discounting the one-year price (in a binomial tree, the average of the two possible one-year prices) by the current short rate. Using the Newton-Raphson method for each time step, the entire binomial short rate tree is computed.

*Step Two* : given the short rate tree obtained in step one, we compute a complete *binomial price tree* for the principal of a Brady bond. The value of the principal at the root of the binomial price tree is the present value of the Brady bond's principal that will be received at maturity. The present value of the principal is then compared to the current market price of the Brady bond and the difference between these two values is computed. This value is *the theoretical present value or the market's assessment of the future uncollateralized coupon payments* on a Brady bond. Investors take into consideration this value to estimate the second binomial tree of future short rates.

*Step Three* : the country's term structure of interest rates (including its default risk premium) is estimated. The inputs are: (a) the stream of future coupon payments on a Brady bond, (b) the on-the-run yield curve for a theoretical bond with a cash flow identical to the Brady bond that is analyzed, (c) the U.S.T yield volatility curve, and (d) the theoretical present value (or market's assessment) of future coupon payments that was found in step two. The country's term structure is derived by (1) finding the short rate tree consistent with (a, b, and c), and by (2) computing the constant spread that added to the short rate tree makes the model present value of future coupon payments equal to the

theoretical present value of future coupon payments (d). This measure of default risk over the risk-free rate of Treasury securities is called *default risk premium*. It is interpreted as the constant spread that makes the theoretical present value of the security equals to its market price. The default risk premium is interpreted as the extra return over the U.S. short rate tree. The default risk premium is computed simultaneously with the term structure of interest rates implied in Brady bond prices.

*Step Four* : the binomial short rate tree found in step three is used to value options on Brady securities. We compute callable and noncallable bonds as well as an entire option value tree. Knowing these options values at each node, their possible values before expiration are calculated by the same discounting procedure used to find the term structure. In addition, we obtain measures of delta hedging, effective duration and effective convexity for Brady bonds.

*Step Five* : it shows how the model can be extended to the cases of floating coupon and amortizing Brady bonds.

The last part of the paper presents the conclusions and outlines some ways in which the basic interest rate tree can serve as a practical tool.

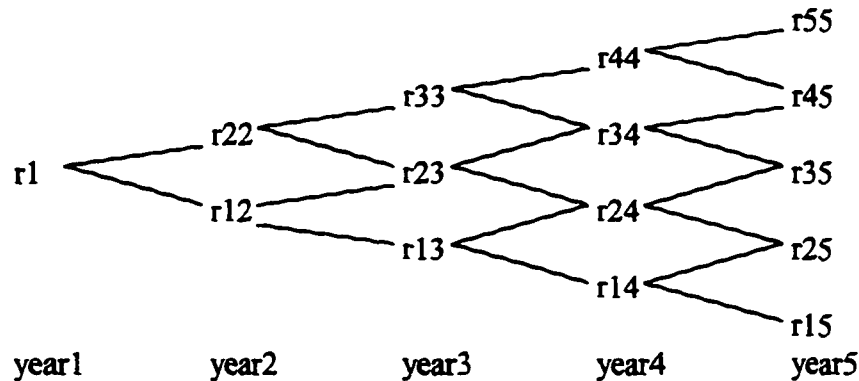
## SECTION I

### A SIMPLE BINOMIAL INTEREST RATE TREE

This section presents a simple binomial model to illustrate the technique of arbitrage-free pricing. Section IV shows how to place the framework in a realistic setting with respect to U.S.T yield curves and with respect to the other inputs of the model. Figure A shows an example of a binomial interest tree. Columns in the tree represent dates, while rows are different possible outcomes or states of the world in a particular date.

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**Figure A** A Binomial Interest Rate Tree



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We use the following notation to describe the binomial interest rate tree. Let :

$y_n$  = n-year zero yield

$\sigma_j$  = yield volatility at time j.

$r_{ij}$  = one-year rate j years forward in state i.

The tree is a discrete representation of a lognormal distribution over time of future one-year rates with time-varying volatility. In this tree, the nodes are spaced one year apart in time. We assume that the one-year interest rate can take on two possible values one year

from today and these two future rates are equally likely. Point  $r_1$  is the root of the tree and the current one-year rate. From this node, the one-year may move to  $r_{12}$  or  $r_{22}$ , which are the rates that correspond to the second year. It is important to note that these rates are not necessarily higher or lower than the one-year rate in the preceding year.

To obtain the short rates for different states  $i$  at each time step  $j$  in the tree, we have to solve a system of equations given by the equations (1) to (3). We obtain today's prices from future prices by using

$$(1) \quad B_{ij} = [1/2 ( B_{i,j+1} + B_{i+1,j+1} ) ] / ( 1 + r_{ij} ),$$

Equation (1) says that today's price of a zero-coupon bond is equal to the expected price of  $B$  one year from now,  $1/2 ( B_{i,j+1} + B_{i+1,j+1} )$ , discounted by today's short rate (one-year rate). We start with the zero's face value at maturity and find the price at each earlier node by discounting future prices using the valuation formula (1) and the short rate at each node. We work back to the root of the tree and find the price today. For example, a one-year zero with face value of \$100 and a short rate,  $r_1 = 5\%$ . Then  $B_{12} = B_{22} = \$100$ , and  $B_1 = \$100/(1+0.05) = \$95.24$ . In addition, today's annual yield,  $y$ , of the  $n$ -year zero in terms of its price,  $B$ , is given by the  $y$  that satisfies:

$$(2) \quad B_{ij} = \frac{100}{(1 + y)^n}$$

We want to find the short rates (the complete binomial interest tree in Figure A) that assure that the model's term structure matches today's market term structure (Table I gives a simple term structure in columns 1, 2 and 3).

**Table I** A sample Term Structure

Maturity (years)	Yield (%)	Yield Volatility (%)	Zero Price (dollars)	Forward Rate (%)	Avg.Short Rates (%)
1	5.00	15	95.24	5	5.00
2	6.00	18	89.00	7	7.02
3	7.00	17	81.63	9	9.18
4	7.50	16	74.88	9	9.36
5	8.00	15	68.06	10	10.67

We want now to find the short rates,  $r_{12}$  and  $r_{22}$ , one year from now by looking at the yield (6%) and volatility (18%) for a two-year zero in Table I. We assume that the volatility of the  $n$ -year spot yield is equal to the natural logarithm of the ratio of the short rates<sup>1</sup>:

$$(3) \quad \sigma_t = 0.5 [ \ln ( r^u / r^d ) ]$$

where  $r^u$  and  $r^d$  are up and down states of the short rate, respectively. This equation implies a relationship between the short rates :

$$\sigma_2 = \ln ( r_{22} / r_{12} ) / 2$$

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<sup>1</sup> This assumption is taken from Black, Derman, and Toy (1990).

or

$$r_{22} = r_{12} \exp(2 \sigma_2)$$

where the volatility of the two-year yield is the natural logarithm of the ratio of the short rates one year from now<sup>2</sup>. Looking at Table I we know today the short rate for the first year is 5 percent and its estimated volatility is 15 percent. We want to find the two short rates one year from now consistent with the yield and volatility rates (6.0% and 18.0%, respectively) for a two-year zero with maturity value of \$100 and today's bond price (\$89.00). These unknown future short rates are  $r_{12}$  and  $r_{22}$ , where  $i$  denotes state of the world (i.e., 2 refers to the higher of the two rates and 1 the lower one) and  $j$  denotes years (i.e., year 2). We obtain the short rates,  $r_{12}$  and  $r_{22}$ , by an iterative process as follows :

- (i) In the first trial we select an initial arbitrary value for  $r_{12}$ , for example  $r_{12} = 5.77\%$ .
- (ii) By using equation (3), calculate the corresponding value for  $r_{22}$ ,  $r_{22} = 8.27\%$  ( $= 5.77\% \exp(0.36)$ ).
- (iii) By using equation (1), compute the bond's value in the second year by discounted its maturity value (\$100.00) for each of the two interest rates,

$$\begin{aligned} B_{12} &= 1/2 (100 + 100) / (1 + r_{12}) \\ &= 100 / 1.0827 = \$92.36 \end{aligned}$$

and

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<sup>2</sup> In Section IV we discuss more in detail about volatility and the procedure to calculate it.

$$B_{22} = 1/2 (100 + 100) / (1 + r_{22})$$

$$= 100 / 1.0577 = \$94.54$$

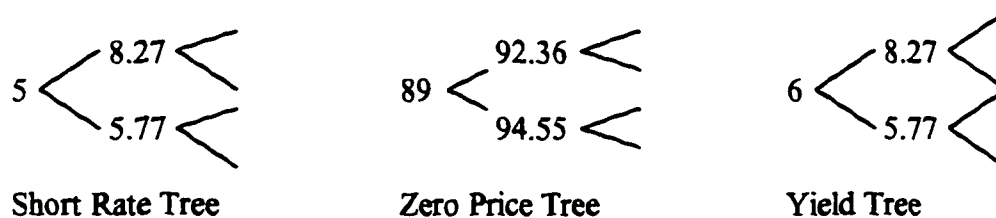
(iv) Determine the price of a zero today as the expected price one period in the future discounted to today by the short rate,

$$B_1 = 1/2 ( 92.36 + 94.54 ) / ( 1 + r_1 )$$

$$= 93.45 / 1.05 = \$89.00$$

(v) Compare the value obtained in (iv) with the target value of \$89.00. If the two values are the same, then  $r_{12}$  and  $r_{22}$  are the ones we seek. If the value found in (v) is not equal to the target value of \$89.00, our assumed value ( $r_{22}=5.77\%$ ) is not consistent with today's term structure. In this case, we have to repeat (i)-(v) with different values for  $r_{12}$  and  $r_{22}$ . Figure B shows how to find the one-year short rates (one year rates, one year from now) using a two-year zero.

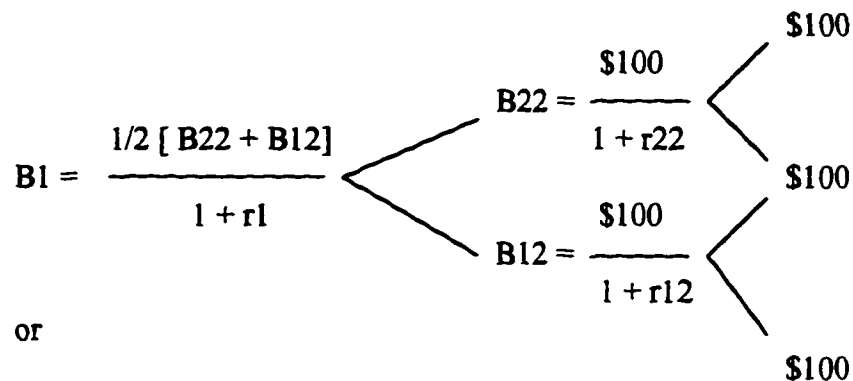
**Figure B** Finding the one-year short rates using a two-year zero



Given the short rates (5.77% and 8.27%), we use the valuation equation to get zero's prices. For example, \$92.36 ( $=\$100/1.0827$ ) and \$94.55 ( $=\$100/1.0577$ ). Today's price is

given by equation (1) by discounting the expected one year-out price by today's short rate:  $(1/2)(\$92.36 + \$94.55) / (1.05) = \$89.00$ . The short rates (5.77% and 8.27%) also satisfy equation (2) and the sample yield volatility. Thus,  $\$100/(1.06)^2 = \$89$ , and  $\sigma_2 = \ln(8.27/5.77) / 2 = 18\%$ .

The same five steps must be applied to find the short rates for years 3, 4 and 5. In Section IV the complete short rate tree is computed by the Newton-Raphson method. The Newton-Raphson procedure to find a short rates  $r_{12}$  and  $r_{22}$  using a two-year zero is summarized in the price tree presented below. Given  $B_1 = \$89.00$ ,  $\sigma_2 = 18\%$ ,  $r_1 = 5\%$ , and  $r_{22} = r_{12}(e^{2\sigma})$ , we obtain the values of  $r_{12} = 5.77\%$  and  $r_{22} = 8.27\%$ :



$$B_1 = \frac{1/2 [ (\$100/(1 + r_{22}) + \$100/(1 + r_{12})) ]}{1 + r_1}$$

or

$$\$89 = \frac{1/2 [ (\$100/(1 + r_{12} e^{2(0.18)}) + \$100/(1 + r_{12})) ]}{1 + 0.05}$$

Figure C shows the complete binomial interest rate tree given the assumed term structure of zero-coupon bonds in Table I. The last two columns in Table I show the implied forward rates and the average of short rates given the term structure of Figure C. For example, the average of short-rates in year 2 is 7.02% ( $= 8.27\% + 5.77\% / 2$ ).

**Figure C A Binomial Interest Rate Tree using the sample Term Structure in Table I.**

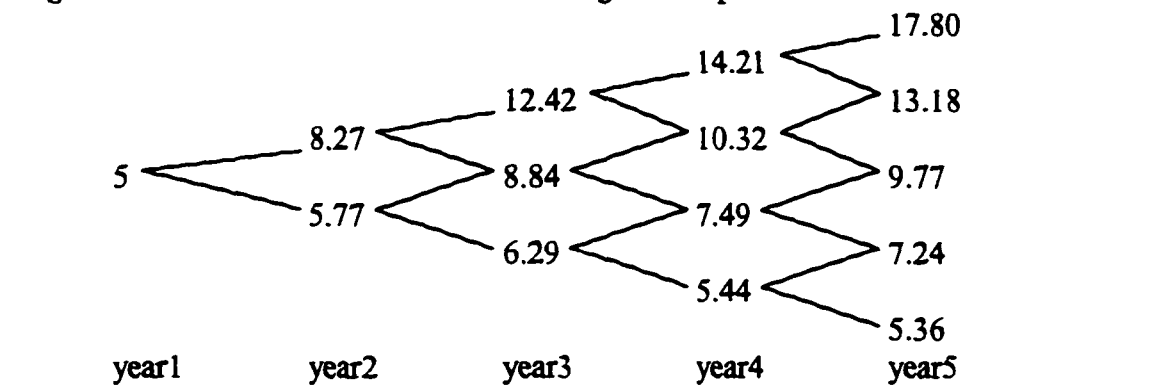
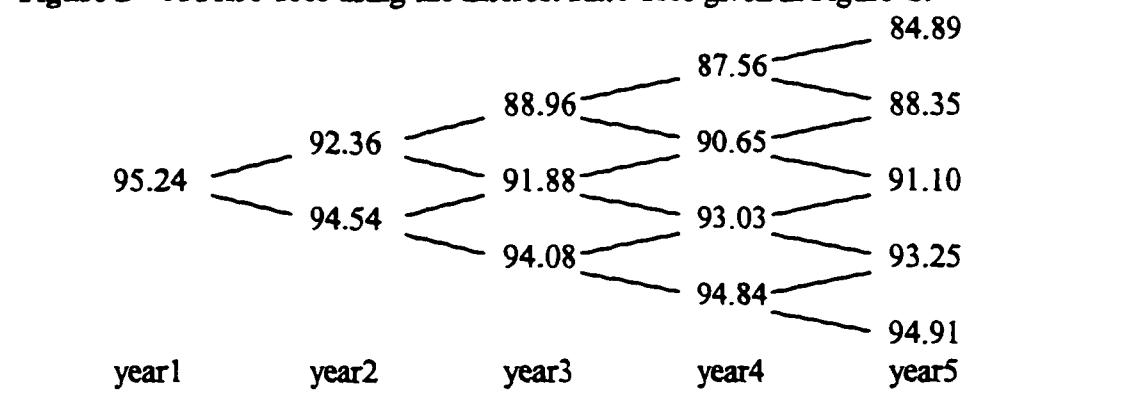


Figure D shows the corresponding values of zeros determined by using equation (1) and the short rate at each node. For example, an interest rate  $r_{55}$  of 17.8% produces a bond value equal to \$84.89 ( $= 100 / 1.178$ ) in year 5.

**Figure D A Price Tree using the Interest Rate Tree given in Figure C.**



**Section IV extends the simple model in different ways. First, it finds the short rate tree for the current U.S.T yields of zero and coupon-bearing bonds. Second, it computes the country's term structure of interest rates estimating the Option Adjusted Spread.**

## **SECTION II**

### **THE BRADY PLAN**

In March 1989, U.S. Treasury Secretary Brady proposed a new approach to resolving the country debt problem and restoring the creditworthiness of restructuring countries. From the debt crisis in mid-1982, financial packages for restructuring countries had emphasized new lending to give countries time to grow out of their debt-servicing difficulties. However, few countries returned to normal debt servicing and financing was becoming harder to arrange. In 1989 Mexico became the first country to implement the Brady Plan. For the first time since 1982 commercial banks agreed to a substantial reduction in their claims on a country.

The Brady Plan proposed a shift in emphasis toward permanent relief through market-based debt and debt service reduction. Instead of providing new money, banks would voluntarily reduce their claims on the debtor countries in return for credit enhancements on their remaining exposure, such as collateral accounts to guarantee the principal and/or interest in a bond exchange, or cash payments in the context of buybacks. The new approach contained the growth in debt. Fundamental results of the new approach have also been the recovery in the secondary market of the value of creditor claims, the enhancement of the claims' liquidity, and the expansion of income opportunities in the secondary market in terms of trade of restructured bank claims and the underwriting of securities flows to restructuring countries.

In implementing the Brady approach, countries and bank steering committees negotiated packages that offered two to six "menus" of debt and debt service reduction options. The restructurings usually securitized the claims, that is, converted the form of the claims from loans to bonds, and lengthened the repayment periods. Additionally, much of the remaining exposure was converted from floating to fixed rate obligations. The options were : (1) par exchange, (2) discount exchange, (3) buybacks, (4) debt conversion/new money, and (4) temporary interest reduction exchange. Buybacks and new money options were omitted in some cases. Discount and par exchanges were the most popular options. Creditors swapped existing loans for new bonds with a lower principal amount (discount exchange) or with the same principal but submarket, fixed interest rates (par exchanges). For example, the negotiated settlement between Mexico (the leading case) and its creditors signed in early 1990 involved : (i) 42 percent of the debt was swapped against a bond whose face value was written down by 35%; (ii) 46 percent of the debt was swapped against a bond with a reduced nominal interest rate; and (iii) 12 percent of the debt capitalized part of the interest due. On the other hand, the non-Mexican creditors opted for par bond 46.7 percent, discount bond 38.2 percent, and new money and others 15.1 percent. On the discount and par bonds, principal was fully secured through the escrowing of zero-coupon bonds that, at maturity, will exactly match the principal due. In addition, eighteen months of interest was guaranteed on a rolling basis through an escrow account ("rolling guarantee"). The escrow accounts, which amounted to \$7 billion, were funded by World Bank and IMF contributions, Mexico's own reserves, and a loan from the government of Japan. Interest accruing on the interest support

account is paid out to Mexico. The debt operations entailed large up-front cash outlays for buybacks, collateral purchases for the bond exchanges, and down payments on arrears. Around three-fifths of the overall financing came from official sources, although in every case the debtor also made a significant contribution.

In general, the range of options included varied from package to package, reflecting the circumstances of particular cases. The richer menu of options made negotiations more complex. As a result of these and other negotiations factors, implementing the debt packages has been time consuming. Still, by end-June 1995, 21 countries had completed deals that restructured commercial bank debts with a face value of \$170 billion, obtaining roughly \$76 billion in debt reduction in present value terms<sup>1</sup>. Table 1 presents the bank menu choices in debt-restructuring packages. Allocations to the options in the various debt packages have differed, reflecting in some cases explicit limits and the view of the holders of the debt regarding the expected future values of the debt instruments issued in exchange for the old bank claims.

The official strategy encourages the maintenance of debt conversion schemes. For most countries, however, these debt conversions placed a smaller role to the Brady packages in reducing countries' debt service burdens. The principal exceptions to this rule have been Chile and Argentina, which account for about three-fifths of the debt converted under official schemes since 1989. For example, in the case of Argentina, debt conversions were an integral part of the country's overall debt reduction strategy. From 1990 onward

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<sup>1</sup> International Monetary Fund. "Commercial Bank Debt Restructuring", Private Market Financing for Developing Countries, World Economic and Financial Surveys, November 1995.

these conversions consisted entirely of exchanges of debt for equity in privatized firms.

The reduction in bank claims through such operations exceeded that achieved through the debt package and more than offset the \$8 billion accumulation of bank debt from 1988-92 stemming from interest arrears. For Chile, debt-equity conversions constituted the primary means of reducing debt owed to banks. For the period 1989-95, debt conversions reached \$11.3 billion, \$1.7 billion, \$5.3 billion and \$3 billion for Argentina, Brazil, Chile and Mexico, respectively.

**Table 1. Bank Menu Choices in Debt-Restructuring Packages**  
(In percent of total eligible bank debt)

Country	Debt Reduction		Debt-Service Reduction		New Money	Other
	Buyback	Discount Exchange	Principal Par Exchange	Collat Par Exchange		
Argentina	-	34	66	-	-	-
Bolivia	46	35	19	-	-	-
Brazil	-	35	32	5	6	22
Bulgaria	13	60	-	27	-	-
Costa Rica	63	-	-	37	-	-
Dom. Republic	35	65	-	-	-	-
Ecuador	-	58	42	-	-	-
Jordan	-	33	67	-	-	-
Mexico	-	43	47	-	11	-
Nigeria	62	-	38	-	-	-
Philippines (1989)	100	-	-	-	-	-
Philippines (1992)	28	-	42	17	13	-
Poland	25	54	18	-	4	-
Uruguay	39	-	33	-	28	-
Venezuela	7	9	38	15	31	-
Total <sup>1</sup>	8	34	39	5	9	5

<sup>1</sup> International Monetary Fund. "Commercial Bank Debt Restructuring", Private Market Financing for Developing Countries, World Economic and Financial Surveys, November 1995.

<sup>1</sup> Weighted Average.

Although the Brady operations canceled a significant amount of claims, they did not provide countries with more cash to finance growth than did the previous approach.

The Plan on average tended to leave net transfers unchanged. Net cash flow impacts have varied, however, among countries. For Argentina and Brazil, for example, restoring normal relations with creditors required significant increase in debt service payments.

The Brady Plan departed from the previous new money approach by extending the time horizon of contractual relief. The central goal was to reduce countries' debt service outflows to manageable levels on a permanent basis. To achieve this, countries prepaid a portion of their debt service obligations at a discount and locked in the interest rates on a significant portion of the remainder. The coupon profiles on par bonds mimicked the slope of the U.S. yield curve at the time of the agreement. For example, in the cases of Argentina and Brazil, par exchange agreements provided for interest rates that rose gradually from 4 to 6 percent over a six-year period. Table 1 in Appendix B describes the principal terms on Argentina dollar and peso denominated bonds.

**Table 2. International Bond Issues by selected Latam countries**  
(In billions of U.S. dollars)

Country	1990	1991	1992	1993	1994	1995 <sup>1</sup>
Argentina	0.02	0.8	1.6	6.3	5.3	1.6
Brazil	-	1.8	3.7	6.5	4.0	3.5
Chile	-	0.2	0.1	0.3	0.2	-
Colombia	-	-	-	0.6	1.0	0.4
Mexico	2.5	3.7	6.3	11.3	7.0	1.1
Venezuela	0.3	0.6	1.0	3.4	-	-

Sources: Euromoney database, Financial Times, and International Financial Review.

<sup>1</sup> Correspond to the first semester of the year.

As perceptions of improving economic performance and rising payments capacity led to lower yield spreads on the restructured long-term Brady debt, yield spreads on new

unenanced issues decreased, the volume rose significantly, and the composition shifted to unenhanced issues<sup>2</sup>. Table 2 illustrates market reentry of selected Latin American (Latam) countries for the period 1990-mid 95, while Chart 1 in Appendix A shows the performance of return indexes on fixed and floating coupon Brady bonds.<sup>3</sup>

Bond issuance in international capital markets has become a major vehicle for developing countries' external financing, although this activity has fluctuated considerably. For example, in the wake of a tightening of monetary policy in the United States in February 1994, the pace of bond issuance slowed markedly and the weighted-average spread tended to increase after that date. Also, the Mexican devaluation prompted a more severe and prolonged reversal in investor confidence, manifested in the sharp decline in the number and value of new issues. Secondary market yield spreads for Eurobonds issued by Latin American borrowers widened and total returns on these bonds fell dramatically (Chart 1). After the crunch of December 1994, Latin American countries have gradually returned to international bond markets. April 1996 marks a new stage in Brady Bond debt for Latam countries. Mexico launched an offer to exchange its U.S. dollar denominated collateralized Floating Rate Bonds due 2019 (Discount Bonds) and collateralized Fixed

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<sup>2</sup> Unenhanced bonds are new issues that do not carry equity conversion rights and are not backed by collateral or receivable accounts.

<sup>3</sup> The Emerging Bond Index (EMBI) tracks total returns for traded external debt instrument in the emerging markets. The instruments include external-currency-denominated Brady bonds. The EMBI is concentrated in instruments from the three major Latin American countries (Argentina, Brazil, and Mexico).

**Rate Bonds due 2019 (Par Bonds) for a new 30-year U.S. dollar denominated Global Bond. The \$2 billion issue of the new Global Bond will reduce the outstanding \$23 billion in Mexico Brady debt by an estimated of \$3-3.5 billion once the discounted exchange price of Bradys is factored in. Table 1 in Appendix B presents some selected indicators of Latam Brady bonds prior to the Mexican bond exchange.**

### **SECTION III**

#### **LITERATURE REVIEW- TERM STRUCTURE MODELS**

Generally speaking, there are two broad approaches to constructing models of the term structure to value a bond option. The first approach, here referred to as the equilibrium model of the term structure of interest rate, assumes a stochastic process for the short-term rate in a risk-neutral world and deduces bond and option prices. Although these models do not explicitly show how to make the models consistent with the observed market prices of zero coupon bonds, in practice the parameters of the model are chosen so that it reflects market data as closely as possible. These models also can be differentiated by the number of state variables used to describe the evolution of the term structure of interest rates over time. Vacisek (1977), Dothan (1978), Richard (1978), Courtadon (1982) have derived pricing models for default free bonds or options on default free bonds which depend on one state variable, the short term rate of interest. Authors like Brennan and Schwartz (1979), (1982) and Schaefer and Schwartz (1984), have used a two state variable model to value default free securities as well as Brennan and Schwartz (1983), and Campbell and Schwartz (1986), who have used this approach to value options on default free bonds.

The second approach takes the current term structure as given and develops an arbitrage-free yield curve model so that it is consistent with market prices. This qualifier can also be applied to the models of the first approach, once a methodology is

implemented to force these models to respect the observed zero coupon yield curve. The arbitrage free debt option pricing literature includes papers by Cox, Ingersoll, and Ross (1985), Ho and Lee (1986), Jamshidian (1989), Black, Derman and Toy (1990), Black and Karasinski (1991), Hull and White (1990a,1990b), Heath, Jarrow and Morton (1992), and Hull and White (1993).

### **Equilibrium Models of The Term Structure of Interest Rates<sup>4</sup>**

Equilibrium models assume that the term structure of interest rates can be derived from the behavior of the instantaneous rate of interest rate, also referred to as the short rate, and from a set of assumptions regarding investor's aversion to risk on default free bonds. These models use a particular stochastic processes to represent the short interest rate and obtain a bond value formula that is function of the short rate and of time.

#### **Assumptions**

It is assumed that investors buy and sell pure discount bonds. A discount bond maturing a time  $T$  is a security that promises to pay one dollar at time  $T$  and nothing at any other time. Let denote the price of this bond at time  $t$ ,  $t \leq T$ , as  $B(t,T)$ . The maturity condition is

$$(1) \quad B(T,T) = 1$$

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<sup>4</sup>The presentation of the model in this paper is based on personal contribution as well as on my interpretation of the existing literature. This section presents a model based on one state variable.

The yield to maturity is the internal rate of return at time  $t$  on a bond with maturity date  $T$ ,

$$(2) \quad R(t, T) = - (1/T) \ln B(t, T)$$

Equation (2) is the term structure of interest rates or the yield curve. For fixed  $t$ , the term structure is an increasing or decreasing function of  $T$  depending on the equilibrium values of bond prices. For any given maturity  $T$ , the yield is positive, zero or negative depending on whether  $B(t, T)$  is less, equal or greater than one, respectively. The instantaneous spot rate of interest is the yield on the currently maturing bond,

$$(3) \quad r(t) = R(t, 0)$$

The instantaneous spot rate of interest is the rate of return investors can earn over the next very short interval of time.

$$(4) \quad B(t, T) = B[t, T, r(t)],$$

Equation (4) says that the bond price depends on the short-run interest rate,  $r$ , as the only state variable.

Several equilibrium models assume that the risk-free interest rate follows a *diffusion processes*, that is, the underlying variable is a continuous function of time and follows a *Markov process*<sup>5</sup>. A general expression of this is given by

$$(5) \quad r(t) = a[r(t), t] dt + b[r(t), t] dz,$$

where the parameters  $a$  and  $b$  are functions of the value of the underlying variables,  $r$ , and time,  $t$ . The generalized Wiener process given in equation (5) has an expected drift rate,  $a[.]$ , the volatility rate,  $b[.]$ , which may change over time, and  $z$  is a variable which follows a Wiener process. The price  $B(t, T)$  of a pure discount bond maturing at time  $T$ , is determined by the forecast, at time  $t$ , of the future interest rates over the term of the bond. The path of the interest rates over an interval  $(t, T)$ ,  $t \leq T$ , depends on the current value  $r(t)$ , given its values prior to time  $t$ , plus a term which represents noise or variability to the path followed by  $r$ .

#### Stochastic Differential Equations for the Interest Rate

A general formula for the short-term riskless interest rate,  $r$ , is

$$(6) \quad dr = (a + b r) dt + \sigma r^a dz,$$

or

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<sup>5</sup> For a complete explanation of diffusion processes, see Arnold (1974), Karlin and Taylor (1975), and Ross (1983).

$$(7) \quad dr = b ( a/b + r ) dt + \sigma r^\alpha dz,$$

Expression (6) is a stochastic differential equation where  $a$ ,  $b$  and  $\alpha$  are positive constants,  $\sigma$  is the volatility of the proportional change in the interest rates ( $dr/r$ ), and  $dz$  is a Wiener process with  $E [dz]=0$  and  $E[dz^2]=dt$ . Equation (7) implies that the conditional mean and variance of changes in the short -term rate depend on the level of  $r$ . Let assume  $\alpha=1$ , the instantaneous expected drift rate (i.e., average drift per unit of time) in  $r$  is  $( a + b r )$ , and its instantaneous variance rate (i.e., variance per unit of time) is  $\sigma^2 r^2$ . Both the expected drift rate and variance rate are functions of the value of the underlying variable,  $r$ . The assumption here is that the variance of the percentage change of the interest rate, ( $dr/r$ ), in a short period of time,  $dt$ , is constant and independent of the level of interest rates. Equation (6) says that the change in the interest rate from  $t$  to  $t+dt$  is normally distributed with mean  $( a + b r ) dt$  and variance  $\sigma^2 r^2 dt$ . In a very short interval of time,  $dt$ , the expected increase in  $r$  is  $( a + b r ) dt$ . Thus, both the expected instantaneous drift and the magnitude of the unanticipated change in the interest rate are proportional to the current level of that rate. Equation (7) can also be interpreted as describing a *mean reverting processes* since the non-random component of the change in the short rate,  $b ( a/b + r ) dt$ , always tends to bring the short rate back to its long run level. In effect,  $a/b$  is the long-run average level of the riskless interest rate, that is, the interest rate is pulled toward this level  $a/b$  at rate  $b$ . When  $r$  is above  $a/b$ , mean reversion causes a negative drift; when  $r$  is below  $a/b$ , mean reversion causes a positive drift in the interest rate. Equation (7) represents the

unrestricted model of short-term interest rate, and alternative models can be obtained by imposing parameter restrictions.

### The Interest Rate Models

The stochastic differential equation given in (6) defines a broad class of interest processes which can be obtained by placing the appropriate restrictions on the parameters  $a$ ,  $b$ ,  $\sigma$ , and  $\alpha$ . Table 3 summarizes different specifications of the short-term riskless rate that have appeared in the literature. Model 1, used in Merton (1973), specifies a Brownian motion with drift for the stochastic processes for the riskless rate. Model 2 is the Ornstein-Uhlenbeck process used by Vasicek (1977) in deriving an equilibrium model of discount bond prices. This model was also used by Jamshidian (1989) and Gibson and Schwartz (1990) to value futures and bond options. In these two models the conditional volatility of changes the riskless rate is constant.

**Table 3. The Interest Rate Models**

Author	Model Specification	Parameter Restrictions
1. Merton	$dr = a dt + \sigma dz$	$b=0, \alpha=0$
2. Vasicek	$dr = (a + b r) dt + \sigma dz$	$\alpha=0$
3. CIR SR	$dr = (a + b r) dt + \sigma r^{1/2} dz$	$\alpha=1/2$
4. Dothan	$dr = \sigma r dz$	$a=0, b=0, \alpha=1$
5. Black-Scholes	$dr = b r dt + \sigma r dz$	$a=0, \alpha=1$
6. Brennan-Schwartz	$dr = (a + b r) dt + \sigma r dz$	$\alpha=1$
7. CIR VR	$dr = \sigma r^{3/2} dz$	$a=0, b=0, \alpha=3/2$
8. Cox-Ross	$dr = b r dt + \sigma r^\alpha dz$	$a=0$

Model 3 is the squared root (SR) process of Cox, Ingersoll, and Ross (CIR) (1985) single factor general equilibrium term structure model. This model has been also used in valuation models for interest-rate-sensitive contingent claims. For example, the mortgage-

backed security valuation model of Dunn and McConnell (1981), the discount bond option model of CIR (March 1985b), the futures option pricing models of Ramaswamy and Sundaresan (1986), the swap pricing model of Sundaresan (1989), and the yield option valuation model of Longstaff (1990). The CIR SR model assumes that the conditional volatility of changes in  $r$  is proportional to  $r$ . Model 4 is used by Dothan (1978) in valuing discount bonds and also by Brennan and Schwartz (1977) in developing numerical models of savings, retractable, and callable bonds. Model 5 is the geometric Brownian motion process of Black and Scholes (1973) and also used in Marsh and Rosenfeld (1983). Model 6 is used in Brennan and Schwartz (1980) for convertible bond prices and in Courtadon (1982) for discount bond option prices. Models 4 to 6 imply that the conditional volatility of changes in the riskless rate is proportional to  $r^2$ . Model 7 is applied by CIR (1980) in their study of variable rate (VR) securities and also by Constantinides and Ingersoll (1984) to value taxable bonds. Lastly, Model 8 is the constant elasticity of variance (CEV) process found in Cox (1975), and Cox and Ross (1976).

*Stochastic Differential Equations for bond prices*

Applying *Ito's lemma* to equations (5) and (6), the instantaneous rate of return on the bond in a risk-neutral world is

$$(8) \quad dB/B = \mu(t, T, r, \sigma) dt + \delta(t, T, r, \sigma) dz,$$

where

$$(9) \quad \mu(t, T, r) = (1/B) [ b r (\partial B / \partial r) + \partial B / \partial t + (1/2) \sigma^2 r^{2\alpha} (\partial^2 B / \partial r^2) ]$$

$$(10) \quad \delta(t, T, r) = (1/B) (\partial B / \partial r) \sigma r^\alpha$$

$$(11) \quad \delta(t, T, r) = (r/B) (\partial B / \partial r) \sigma \quad \text{with } \alpha = 1,$$

Equation (8) represents the instantaneous return on the riskless discount bond. The behavior of the instantaneous rate of return on bonds is explained by its expected rate of return,  $\mu(\cdot)$ , and by the  $\delta dz$  term, where  $\delta$  is the instantaneous standard deviation of return. According to Equation (9), the required expected return on bonds depends on the level of interest rates, the variance of percentage change of interest rates, parameters  $a$  and  $b$ , and the  $\partial B / \partial t$  term. Given  $\partial B / \partial r < 0$ ,  $b < 0$ , and  $\partial^2 B / \partial r^2 > 0$ , higher levels of interest rates implies a greater expected rate of return. If  $\partial^2 B / \partial r^2 > 0$ , the greater the variance rate of proportional change of interest rates,  $\sigma^2$ , the greater the expected return required on bonds.  $\mu(\cdot)$  also depends on  $\partial B / \partial t$ . Assuming  $\partial B / \partial t > 0$ , this term may be interpreted as the scientific amortization of the pure discount bond, that is, the value of the bond marches toward par value (\$1) at the final maturity date. Equation (8) also expresses the return on bond as a function of the noise term,  $\delta dz$ . At the same time, equation (11) shows that the local standard deviation of  $dB/B$ ,  $\delta$ , can be regarded as a function of the elasticity interest rate of bond prices and  $\sigma$ , while the stochastic increment  $dz$  is the same Wiener process as in equation (6). Thus, the instantaneous changes in  $r$  and  $B$  are perfectly correlated.

### *The Market Price of Instantaneous Interest Rate Risk*

Following Merton (1971) and (1973), consider an investor forming a three-discount bond portfolio by issuing an amount  $W_1$  of a bond with maturity date  $T_1$ , and using the proceeds to finance an amount  $W_2$  of a bond maturing at time  $T_2$ , and amount  $W_3$  of a riskless bond with return,  $r$ , equal to the risk-free interest rate. The holder of this portfolio is short  $W_1$  and long  $W_2$  and  $W_3$ . The condition of zero total investment is  $W = W_1 + W_2 + W_3 = 0$ . If  $dW$  is the instantaneous dollar return to this portfolio,  $B_1$ ,  $B_2$  and  $B_3$  are the prices of bond 1, bond 2 and bond 3 respectively, by application of Ito' formula from equation (8), we have

$$(12) \quad dW = W_1 (dB_1/B_1) + W_2 (dB_2/B_2) + W_3 (dB_3/B_3) \\ = \{ W_1 [\mu(t, T_1, r) - r] + W_2 [\mu(t, T_2, r) - r] \} dt + \\ + [ W_1 \delta(t, T_1, r) + W_2 \delta(t, T_2, r)] dz,$$

assuming  $dz_1 = dz_2 = dz$ , and substituting  $W_3 = -(W_1 + W_2)$ . Suppose we choose a portfolio  $W_1 = W_1^*$  and  $W_2 = W_2^*$ , such that the coefficients of  $dz$  is always zero, then the return on that portfolio would be nonstochastic. Since the portfolio requires zero total investment, the expected return on this portfolio is zero under conditions of no arbitrage opportunities. Thus,

$$(13) \quad W_1^* \delta(t, T_1, r) + W_2^* \delta(t, T_2, r) = 0$$

$$(14) \quad W_1^* [\mu(t, T_1, r) - r] + W_2^* [\mu(t, T_2, r) - r] = 0,$$

The *zero risk condition* (13) and the *no arbitrage condition* (14) form a system of two linear homogeneous equations in the two portfolio proportions. A nontrivial solution to equation. (13) and (14) exists if only if

$$(15) \quad [\mu (t, T_1, r) - r] / \delta (t, T_1, r) = [\mu (t, T_2, r) - r] / \delta (t, T_2, r),$$

Equation (15) shows that the ratio  $[\mu (t, T, r) - r] / \delta (t, T, r)$  is independent of maturity dates. Let  $\lambda (t, r)$  denotes the common value of such ratio,

$$(16) \quad \lambda (t, r) = [\mu (t, T, r) - r] / \delta (t, T, r) \quad \text{with } T \geq t,$$

$\lambda (t, r)$  is *the market price of risk of the interest rate* and it is a measure of the trade-off between risk and return for every security depends on  $r$ . From equation (15),

$$(17) \quad \mu - r = \lambda \delta = \lambda ( \partial B / \partial r ) ( r / B ) \sigma,$$

$$(17a) \quad \mu = r + \lambda \delta$$

Equation (17) is an equilibrium relationship which constrains the relative risk premia on bonds of different maturities. It shows the instantaneous risk premium on a discount bond of any maturity as the product of the market price risk of the interest rate and the noise in the rate of return of the bond. It is also expressed as a function of the elasticity interest rate of bond prices,  $( r / B ) ( \partial B / \partial r )$  and the volatility of the proportional change in

interest rates. Since equation (17) depends on  $\lambda$ , the utility function of market participants is relevant in the determination of the risk premium. The market price of short-term interest rate risk can be estimated by a non-linear procedure<sup>6</sup>. Equation (17a) indicates that the instantaneous expected rate of return for any asset can be written as the instantaneous risk free return plus a risk premium.

### Partial Differential Equation for Bond Prices

Expressions for  $\mu (\cdot)$  and  $\delta (\cdot)$  are substituted into equation. (17) to obtain :

$$\begin{aligned} & (1/B) [ \partial B / \partial t + (a + b r) ( \partial B / \partial r ) + (1/2) \sigma^2 r^2 ( \partial^2 B / \partial r^2 ) ] - r = \\ & = \lambda (1/B) ( \partial B / \partial r ) \sigma r , \end{aligned}$$

$$\begin{aligned} (18) \quad & \partial B / \partial t + (a + b r) ( \partial B / \partial r ) + (1/2) \sigma^2 r^2 ( \partial^2 B / \partial r^2 ) \\ & - \lambda \sigma r ( \partial B / \partial r ) - r B = 0 \end{aligned}$$

$$(19) \quad \partial B / \partial t + [(a + b r) - \lambda \sigma r] ( \partial B / \partial r ) + (1/2) \sigma^2 r^2 ( \partial^2 B / \partial r^2 ) - r B = 0$$

Equation (19) is the term structure equation which is a *partial differential equation* for  $B$  (  $t, T, r$  ). It says that the value of a pure discount bond is obtained as a function of the short-run interest rate and the time to maturity, subject to the boundary conditions :

$$\begin{aligned} (20) \quad & \lim_{r \rightarrow \infty} B ( t, T, r ) = 0 \quad T \geq 0, \\ & \quad \quad \quad r \rightarrow \infty \end{aligned}$$

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<sup>6</sup> See, for example, Brennan and Schwartz (1979a).

$$(21) \quad B(t, 0, r) = 1$$

$$(22) \quad B(t, 0, r) = \text{Max} [0, B(t, T, r) - E]$$

Equation (20) implies that the price of the bond tends to zero for very high interest rates at any time prior to maturity. Equation (21) says that the value of the bond at maturity is exactly one dollar. In equation (22),  $B(t, T, r)$  is the value of the underlying bond at time  $t$  and  $E$  is the exercise price of the option. The coefficients of the partial differential equation depend upon two functions,  $(a + b r)$  and  $\sigma^2 r^2$ ; and the market price of short-term interest rate risk. Equation (19) is solved with  $B(t, 0, r) = 1$ , and the resulting values of  $B(t, T, r)$  will be the discount factors which give the present value of \$1 bond at maturity date. The model bond values are calculated by applying these discount factors to the future cash flows on any straight default-free bond. Using equation (9), and dividing equation (18) by  $B$ , we have

$$(22) \quad (1/B) \left[ \frac{\partial B}{\partial t} + (a + b r) \left( \frac{\partial B}{\partial r} \right) + (1/2) \sigma^2 r^2 \left( \frac{\partial^2 B}{\partial r^2} \right) \right] =$$

$$r + (1/B) \lambda \sigma r \left( \frac{\partial B}{\partial r} \right)$$

Thus, the expected rate of the bond is  $r + (1/B) \lambda \sigma r \left( \frac{\partial B}{\partial r} \right)$ , or  $r + \lambda \sigma (r/B) \left( \frac{\partial B}{\partial r} \right)$ . The instantaneous return premium on a bond is proportional to its interest elasticity. As Cox, Ingersoll and Ross (1985b) pointed out,  $\lambda r$  is the covariance of changes in the interest rate with percentage changes in optimally invested wealth. Given

$\partial B / \partial r < 0$ , a negative covariance ( $\lambda < 0$ ) implies a positive premium. The interest elasticity of the bond's price,  $(r/B) (\partial B / \partial r)$ , can be seen as similar to the concept of interest-rate duration (Duri), that is, the effect of a change in  $r$  on  $B$ . Analogously,  $\sigma (r/ B) (\partial B / \partial r)$  is a measure of interest-rate risk.

In general, the valuation equation cannot be solved analytically and one must use a numerical solution technique. The next section presents an example of *finite difference method* to value a derivative security by solving numerically the differential equation that the bond's price must satisfy.

#### Application of the Explicit Finite Difference Method to Bond Valuation

Given our earlier assumptions, the value at time  $t$  of a pure discount bond maturing at time  $T$  satisfies the following system of equations:

$$(25) \quad \partial B / \partial t + (b r - \lambda \sigma r) (\partial B / \partial r) + (1/2) \sigma^2 r^2 (\partial^2 B / \partial r^2) = r B$$

subject to :

$$(26) \quad B (t, 0, r) = 1, \quad (\text{for default-free bonds}).$$

To apply the explicit difference method, we have to derive finite difference approximations for the derivatives of the bond price with respect to the interest rate and time, and substitute these approximations in the original valuation equation (equation.(25)). Given the boundary condition that the bond price at maturity date  $T$  is equal to  $1$ , we can use the partial differential equation to solve for the value of the bond at

time  $T - k$  and proceed backwards by solving for the bond price every period of time of length  $k$  back to time  $t$ .

Assume that the time period  $T-t$  is divided in  $M$  intervals of length  $k$  such that  $\Delta t = k = (T - t) / M$ . A finite  $N$  number of interest rates are chosen and suppose  $r_{\max}$  is the highest interest rate of this  $N$  sample. Define  $\Delta r = r_{\max} / N$ . A grid of a total of  $N$  times  $M$  points is constructed over which the bond takes value and we compute the bond prices for each of these  $N$  times  $M$  combinations of interest rate and time. The  $(i, j)$  point on the grid is the point that corresponds to time  $i \Delta t$  and interest rate  $j \Delta r$ .  $B_{ij}$  will denote the value of the bond at the  $(i, j)$  point. The method starts from the maturity date,  $T$ , and works backwards in time by solving for the value of the bond at time  $T-k, T-2k, \dots, t$  ( $t = T - Mk$ ). To compute a finite difference approximation of the partial derivatives in equation (25), we use Taylor series for  $B(r + h, t)$ ,  $B(r - h, t)$ ,  $B(r, t + k)$ , and  $B(r, t - k)$  about  $(r, t)$ . In finite difference form the approximations to the partial derivatives are :

$$(27) \quad (\partial^2 B / \partial r^2)_{ij} = (B_{i+1,j+1} - 2 B_{i+1,j} + B_{i+1,j-1}) / h^2 + O(h^2),$$

$$(28) \quad (\partial B / \partial r)_{ij} = (B_{i+1,j+1} - B_{i+1,j-1}) / 2 h + O(h^2),$$

$$(29) \quad (\partial B / \partial t)_{ij} = (B_{i+1,j} - B_{i,j}) / k + O(|k|),$$

The asymptotic  $O$  notation represents the truncation errors of these approximations. We can now substitute equation (27), (28), and (29) into equation (25). The partial differential equation is replaced at every time step by a system of  $N-1$  difference equations,

$$(B_{i+1,j} - B_{i,j}) / k + (a + b r - \lambda \sigma r) (B_{i+1,j+1} - B_{i+1,j-1}) / 2 h +$$

$$+ (1/2) \sigma^2 r^2 (B_{i+1,j+1} - 2B_{i+1,j} + B_{i+1,j-1}) / h^2 - r B_{i,j} = 0$$

with  $1 \leq j \leq N - 1$ ,

$$(30) \quad B_{i,j} = (1/1 + rk) [P_1 B_{i+1,j-1} + P_2 B_{i+1,j} + P_3 B_{i+1,j+1}]$$

with  $1 \leq j \leq N - 1$ ,

where

$$P_1 = [(k/2h^2) \sigma^2 r^2 - (a + br - \lambda \sigma r)(k/2h)]$$

$$P_2 = [1 - (k/h^2) \sigma^2 r^2]$$

$$P_3 = [(a + br - \lambda \sigma r)(k/2h) + (k/2h^2) \sigma^2 r^2],$$

where  $P_1 + P_2 + P_3 = 1$ . Equation (30) represents a system of  $N - 1$  equations which gives the value of the bond at time  $i$  as a function of the value of the bond at time  $i + 1$ .  $B_{i,j}$  is the difference approximation of  $B(r, t)$  at the point  $t = t_0 + ik$ , and  $r = r_{\min} + jh$ . If  $i = 0$ , then  $t = t_0$ . If  $j = 0$ , then  $r = r_{\min}$ . We can express the system of equations. (30) in matrix form :

$$(31) \quad B_i = (1/1 + rk) P B_{i+1},$$

where  $B_j$  is a  $(N - 1) \times 1$  column vector of values  $B_{i,1}, B_{i,2}, \dots, B_{i,N-1}$ .  $P$  denotes a tridiagonal matrix of coefficients  $P_1, P_2$ , and  $P_3$ .  $B_{i+1}$  is a  $(N - 1) \times 1$  column vector of values  $B_{i+1,1}, B_{i+1,2}, \dots, B_{i+1,N-1}$ . According to equations. (30) and (31), the value of the contingent claim at time  $i$  is given by its expected value at  $i+1$  discounted at the riskless

rate,  $r$  (CIR, 1985a, Lemma 4). As Brennan and Schwartz pointed out, the stability of the explicit solution requires that the coefficients of equation (30) be nonnegative.

### Conclusions about Equilibrium Models

The strength of these models is the fact that the value of the underlying bond and the value of the option are determined in a consistent manner from a set of assumptions determining the term structure of interest rates. However, a crucial drawback of this type of model is that the valuation equation depends on the market price of risk, which is an unknown function and determined by the investor utility function. Moreover, the assumption of a specific functional form for the market price of risk can lead to internal inconsistencies of the model. A sufficient condition to maintain internal consistency and to prevent arbitrage opportunities with respect to today's spot curve is to assume that the function  $\lambda(r,t)$  is a function of time  $\lambda(t)$ . The existing estimation procedure of the market price of risk based on a sequential matching of the price of zero coupon securities through backward induction is cumbersome and requires a great amount of computer time. Still, an equilibrium model with internal consistency can lead a situation in which the term structure observed in the market is not an equilibrium of the model. More important, the term structure of interest rates of an emerging market country is determined by its default risk. Recall that most of these equilibrium models deal with default free bonds. While their main concern is the market price of risk of the interest rate, these studies do not explicitly deal with default risk. The model proposed in Section IV solves the option value by using a binomial approach, rather than solving the corresponding partial differential equation and

finding numerically the market price of risk. On the other hand, Cox, Ingersoll, and Ross present a closed form methodology to fit their model to the observed zero coupon curve. In our model, the market's price of risk must be reflected in the shape of the yield curves and thereby affects bond prices. Equilibrium models do not take bond prices as given whereas arbitrage-free models do. Arbitrage-free model has the advantage that uses the full information of the current term structure of interest rates. Unlike equilibrium models, the market's price of risk is not a problem since it must be reflected in the current term structure. The shape of the yield curve determines present values of cash flows and the price of the bond. The next section presents an arbitrage-free model of the short rate based on a binomial tree.

## SECTION IV

### A BINOMIAL ARBITRAGE-FREE PRICING MODEL FOR BRADY BONDS

The purpose of this section is to present a binomial arbitrage-free model to estimate the default risk premium and the term structure of interest rates implied by the market prices of Brady bonds and the current U.S.T yield and volatility curves. Once the implied emerging market country's term structure of interest rates for US-based investors is computed, we find the value of a bond option as well as different measures of sensitivity such as option hedge ratios, effective duration and effective convexity.

We are interested in finding a tree of future short rates similar to the one in Figure C (Section I), but now using *the current U.S. term structure* rather than a sample one. We want to find the one-year rate beginning today, the one-year rate beginning one year from now, the one-year rate beginning two years from now, the one-year rate beginning three years from now, and so on. Thus, the binomial tree that describes the country's term structure is a tree of future short rates (one-year rates). These future one-year rates are called *arbitrage-free future rates*. These future rates are arbitrage-free because an investor would be indifferent between a sequence of two (for example) one year investments at these rates and a two-year zero investment at the two-year spot rate. For example, given the sample term structure of Table I and Figure C in Section I, the investor would be indifferent between a two-year investment in a zero-coupon bond yielding 6 percent a year and a sequence of two one year investment at 5

percent in the first year and an expected future short rate of 7.02 percent ( $=1/2 (8.27\% + 5.77\%)$ ) in the second year. This example illustrates how the tree of future short rates is *consistent* with the current structure of yield to maturity and their estimated volatilities, and why these future short rates are called *arbitrage-free*.

The *inputs* of the model are : (1) yield curves, which give for each maturity the current yield on U.S. Treasury zero-coupon bonds and coupon-bearing bonds; (2) volatility curves, which give for each maturity the current estimated yield volatility on U.S Treasury zero-coupon bonds and coupon-bearing bonds, (3) current market Brady prices, (4) Swap rate spreads<sup>1</sup>. The *outputs* are : (1) the U.S.T short interest rate trees, (2) the country's term structure of interest rates, which includes the U.S.T short rate trees and the default risk premium or Option Adjusted Spread (OAS), (3) future prices for Brady bonds, (4) option prices, and (5) delta hedging, effective duration and convexity measures.

Two different short rate trees are constructed, one is a binomial tree for zeros and the other is a one for U.S.T coupon-bearing bonds. This distinction stems from the fact that the principal of Brady bonds are fully secured through U.S.T zero-coupon bonds, but coupon payments are uncollateralized<sup>2</sup>. The first tree is a representation of the U.S.T term structure of zero-coupon bonds. The volatility of the log of the short rate is a time-varying volatility of yields on zero-coupon bonds. This tree is used to value the collateralized part (principal) of

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<sup>1</sup> Coupon payments on floating Brady bonds depend on the 6-month LIBOR plus an spread (i.e., 13/16%). We use the swap rate spreads to estimate the 6-month LIBOR.

<sup>2</sup> In some cases twelve or eighteen months of interest are guaranteed on a rolling basis through an escrow account. In this paper we assume that all coupon payments are fully uncollateralized.

Brady bonds. The second tree is a representation of the U.S.T term structure of coupon-bearing bonds and used to value the uncollateralized part (coupon payments) of Brady bonds. It is also constructed with the assumption of a time-varying volatility of the short rate. Given these two short rate trees and current Brady bond prices, we construct a binomial tree of future short rates for an emerging market country. This short rate tree, as the ones mentioned above, is consistent with the current U.S.T term structure (the observed yield curves as well as their estimated volatilities). However, our tree includes a constant spread, the Option Adjusted Spread (OAS), which is an incremental return or risk premium over Treasury securities. This spread is interpreted as an empirical measure of the default risk premium.

The model is developed in five steps. Step One : Finding a U.S.T Short Rate Tree for Zeros, Step Two: Finding the Market Value of Future Coupon Payments on Brady Bonds, Step Three: Finding the Country's Term Structure of Interest Rates, Step Four: Pricing Callable Bonds and Options on Brady bonds<sup>3</sup>, and Step Five: Pricing Floating and Amortizing Brady bonds<sup>4</sup>.

## **1. The Model**

### **1.1. Assumptions**

This section presents a *one-factor model* of interest rates. It is assumed that *short rates are lognormally distributed*, all security prices are perfectly correlated and there is no taxes or

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<sup>3</sup> Most Bradies are callable at any time at par.

<sup>4</sup> Bradies are bullet and amortizing bonds. A bullet bond is a bond for which there are no amortizing payments.

other transaction costs. Because one factor (the short rate) governs all security prices, the returns on all securities are perfectly correlated. For example, knowing the short term return on a 5-year bond is sufficient to determine the short term return on a 10-year bond. In general, rates of different maturities are positively, but not perfectly correlated. We want to find a tree of future short rates consistent with the current U.S.T term structure. The term structure is made up of the yield curve and the volatility curve. The former is an array of yields on zero-coupon and coupon Treasury Bonds for various maturities, and the second is an array of estimated yield volatilities for the same bonds. We assume throughout this section that U.S.T bond is a benchmark for a *risk-free* security.

The model assumes that the new short rate is equal to the old one multiplied by a random shock,

$$(1) \quad r_{t+1} = r_t e^{(m_t dt + \sigma_t dz)},$$

where  $e$  is the base of the natural logarithm 2.71828....The stochastic processes of *the change in the logarithm of the short rate* is :

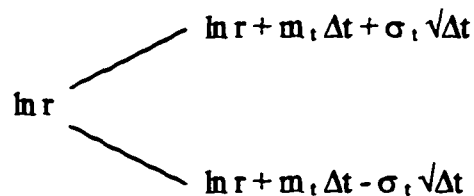
$$(2) \quad d \ln r = m_t dt + \sigma_t dz,$$

where  $m_t$  and  $\sigma_t$  are, respectively, the annual drift and the *proportional volatility* of the short rate. The subscript  $t$  indicates that both variables are time dependent, that is, they change from one time step to another. The variable  $z$  follows a Wiener process, where  $dz$  has a mean zero and variance  $dt$ . The proportional volatility equals the *basis point volatility* divided by the interest rate. Alternatively, basis point volatility is equal to the proportional volatility times the interest rate. For example, if  $\sigma = 15\%$  or 0.15 and the current interest rate is 5% or 0.05, then the basis point volatility equals 75 basis points. The proportional volatility is estimated using historical time series data and represents an input of the model.

The assumption of lognormal distribution of short rates eliminates the possibility of negative interest rates. Unlike the normal distribution, the lognormal distribution is skewed to the left. To make the diffusion process discrete, we assume that over a short period of time  $dt$ , the change in logarithm of the short rate can only take two possible values, each with probability  $1/2$ . Thus, the discrete processes of the short rate take the following form,

$$\begin{array}{l}
 \begin{array}{c} 1/2 \\ \nearrow \\ r_1 \end{array} \begin{array}{l} r_{22} = r_1 e^{(m_t \Delta t + \sigma_t \sqrt{\Delta t})} \\ \\ \searrow \\ 1/2 \end{array} \begin{array}{l} \\ \\ r_{12} = r_1 e^{(m_t \Delta t - \sigma_t \sqrt{\Delta t})} \end{array}
 \end{array}$$

where the short rate can take either the up-state ( $r_{22}$ ) or the down-state ( $r_{12}$ ). If the original short rate,  $r$ , is positive, then all subsequent rates are also positive. Taking the natural logarithm of the short rates along the tree, we obtain the following tree :



### 1.2 The Volatility of the Interest Rate

The construction of the short rate tree depends on the assumption about the volatility of the log of the short rate. The logarithm of the new short rate is equal to the logarithm of the old rate plus  $m_r \Delta t$  plus or minus  $\sigma_r \sqrt{\Delta t}$ . Since  $m_r \Delta t$  is added to the logarithm of the short rate in both the up- and down- states, it is called drift or trend of the logarithm of the short rate. Since  $\Delta t$  (the time step) is measured here in years,  $m_r$  is an annual drift ( $\Delta t = 1$ ). Since  $\sigma_r \sqrt{\Delta t}$  is added in the up-state but subtracted in the down state, it is interpreted as the random deviation of the short rate from its trend.  $\sigma_r \sqrt{\Delta t}$  is the volatility of the logarithm of the short rate and  $\sigma_r$  is the annual volatility of the same rate (with  $\Delta t = 1$ ).

As was stated above, the *proportional volatility* of the n-year spot yield is equal to the natural logarithm of the ratio of the short rates <sup>5</sup>:

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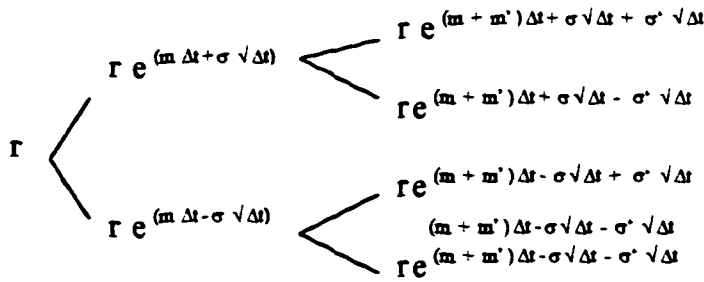
<sup>5</sup> This assumption is taken from Black, Derman, and Toy (1990).

$$(3) \quad \sigma_j = 0.5 [ \ln ( r^u / r^d ) ]$$

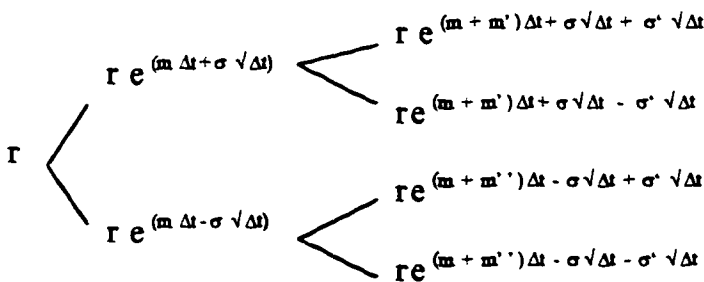
where  $r^u$  and  $r^d$  are up and down states of the short rate, respectively. To prove this, suppose a lognormal rate tree with up-state,  $r^u$ , a down-state,  $r^d$ , and the random short rate at the next time step,  $r$ . The proportional volatility of  $r$  equals *the standard deviation* of the logarithm of  $r$ . Thus,

$$\begin{aligned} E[\ln r] &= 1/2 [ \ln r^u + \ln r^d ] \\ V[\ln r] &= 1/2 \{ \ln r^u - E[\ln r] \}^2 + 1/2 \{ \ln r^d - E[\ln r] \}^2 \\ &= 1/2 \{ 1/2 [ \ln r^u - \ln r^d ] \}^2 + 1/2 \{ 1/2 [ - \ln r^u + \ln r^d ] \}^2 \\ &= [ 1/2 \ln r^u - 1/2 \ln r^d ]^2 \\ &= 0.25 [ \ln r^u - \ln r^d ]^2 \\ &= 0.25 [ \ln ( r^u / r^d ) ]^2 \\ \sigma &= \sqrt{V[\ln r]} = 0.5 [ \ln ( r^u / r^d ) ] \end{aligned}$$

Under this assumption,  $\sigma$ 's can be now empirically estimated by using actual spot yields and then applied them to determine short rates at each time step. Since the model allows the volatility of the short rate (also called *local volatility*) to change at each time step, it may lead to a *non-recombining tree*:



A non-recombining tree will be almost impossible to implement. For example, after N annual periods there will be  $2^N$  possible values for the one-year rate. So, a 30-year Brady bond will, in the rightmost column, have over 1 billion nodes. In a recombining tree, the up-down and down-up states are forced to have the same rate value. This tree recombines only if  $\sigma = \sigma'$ , which is not possible since by definition the volatility of the interest rates change at each time step. To address this problem, we allow the drift from the up-state to differ from the drift from the down-state:



where  $m'$  is the drift from the up-state and  $m''$  is the drift from the down-state. This tree will recombine if the up-down and down-up states are equal, that is

$$r e^{(m + m')\Delta t + \sigma\sqrt{\Delta t} - \sigma'\sqrt{\Delta t}} = r e^{(m + m'')\Delta t - \sigma\sqrt{\Delta t} + \sigma'\sqrt{\Delta t}}$$

or

$$(m'' - m')\sqrt{\Delta t} = 2(\sigma - \sigma')$$

Thus, time-varying local volatilities implicitly impose a relationship between the up-state drift and the down-state drift.

### 1.3. A Valuation Equation

The bond price changes are binomial in a short period of time  $\Delta t$ . Let  $B$  represent the current price of the bond. Over the next interval of time, the price can either move up to a value  $uB$  or down to  $dB$ , where  $u > 1$  and  $d < 1$ .  $p$  is the probability of an up movement and  $(1-p)$  the probability of a down movement. A conventional binary tree with probabilities of  $1/2$  is adopted here. At each node  $(i, j)$ , the estimated value of the bond is the expected value of the sum of the expected price plus the coupon payment  $c$ , discounted at the short rate  $r_{ij}$ . Let  $B_{ij}$  be the value of the coupon bearing bond in state  $i$  at time step  $j$  when the interest rate is  $r_{ij}$ , thus

$$(4) \quad B_{ij} = \exp(-r_{ij}\Delta t) [p B_{i,j+1} + (1-p) B_{i+1,j+1} + c]$$

Equation (4) defines a recursive relationship between the value of the bond at time  $t + \Delta t$  and its value at time  $t$ . Given the value of the bond on its maturity date and the short-rate term structure, we can solve for the value of the bond at time  $T - \Delta t$  and proceed backwards by discounting future expected prices using the valuation equation (4) and the short rate at that node. We repeat this procedure until the root of the tree and find the price today. Alternatively, the valuation equation can be written as

$$(5) \quad B_{ij} = [1/2 ( B_{i,j+1} + B_{i+1,j+1} ) + c_{j+1} ] / ( 1 + r_{ij} ),$$

which says that the price of a coupon bond today is the expected price one period in the future plus the coupon payment in the next period ( $c_{j+1}$ ) discounted to today using the short rate. Note that the cash flow at any node ( $CF_{ij}$ ) is equal to the value of the bond at this node plus the corresponding coupon payment, that is, the numerator in equation (5). Thus,

$$(6) \quad CF_{ij} = 1/2 ( B_{i,j+1} + B_{i+1,j+1} ) + c_{j+1}$$

For amortizing bonds, we need to add the future amortizing payments ( $a_j$ ) to the cash flows:

$$(7) \quad CF_{ij} = 1/2 ( PVC_{i,j+1} + PVC_{i+1,j+1} ) + c_{j+1} + a_{j+1}$$

and

$$(8) \quad B_{ij} = [1/2 ( B_{i,j+1} + B_{i+1,j+1} ) + c_{j+1} + a_{j+1} ] / ( 1 + r_{ij} ),$$

where the subscript  $i$  was eliminated from coupon and amortization payments because there is only one of them at each step size. For zero-coupon bonds:

$$(9) \quad B_{ij} = [1/2 ( B_{i,j+1} + B_{i+1,j+1} ) ] / ( 1 + r_{ij} ),$$

where its *expected price* one year from now is  $1/2 ( B_{i,j+1} + B_{i+1,j+1} )$  and its *expected return* is  $1/2 ( B_{i,j+1} + B_{i+1,j+1} )/B$  which is equal to  $\exp( r_{ij} \Delta t )$  or  $( 1 + r_{ij} )$ . To compute the present value of future coupon payments ( $PVC_{ij}$ ) in step three, we apply the following valuation formula :

$$(10) \quad PVC_{ij} = [ 1/2 (PVC_{ij+1} + PVC_{i+1,j+1} ) + c_{j+1} ] / ( 1 + r_{ij} ),$$

## 2. Model Implementation

The model is implemented in five steps. The country's term structure is developed in step one to step three. Step four illustrates how to value options on Bradies, while step five is just an extension of the model to the case of floating coupon and amortizing Brady bonds. To illustrate the general valuation methodology, we arbitrarily take the Argentine PAR bond to

find the term structure of interest rates of the emerging market country. Chart 1 in Appendix exhibits the main characteristics of Argentine Brady bonds.

### **2.1. Step One: Finding a Short Rate Tree**

We want to find the short rates that will be consistent with (i) current n-year spot interest rates (zero yields), (ii) their estimated yield volatilities and (iii) zero bond prices for different maturities. We use the same procedure described in Section I, but employing actual U.S. Treasury yields. We reproduce Table I in Section I using the actual interest rates on U.S. Treasury zeros for different maturities on May 31, 1993. This specific date was chosen because coupon payments on Argentine Par are payable semiannually on May 31 and November 30. The interest rate data were collected from REUTERS, BLOOMBERG services, and The Wall Street Journal.

A complete yield and volatility curves are extracted by fitting a mathematical curve to the yield and volatility points from different maturities on zero-coupon and coupon-bearing bonds. To obtain an smooth yield curve, we assume a functional form for this curve and then estimate the parameters of that function. We employ a *Piecewise Regression Model* where the regression function is a cubic spline function. This means that the yield curve is represented by three *cubic polynomials* where the first cubic describes the yield function for maturities between 0 and 10 years, the second for maturities between 10 and 20 years, and the third for maturities between 20 and 30 years. The regression functional form is

$$(11) \quad Y_t = g_1(t) I [1, t_0](t) + g_2(t) I [t_0, t_1](t) + g_3(t) I [t_0, T](t) + e$$

where  $I(\cdot)$  are indicator functions that take the value one if the argument is in the states interval and zero otherwise. The  $g_i(t)$ ,  $i = 1, 2, 3$ , are cubic polynomials of the form

$$g_i(t) = a_i + b_i t + c_i t^2 + d_i t^3$$

Points  $t_0$  and  $t_1$ , where the two cubic function meet, are called “knot points”. Since we need an smooth yield curve, the functions  $g_i(t)$  satisfy the following smoothness restrictions :

$$g_1(t_0) = g_2(t_0), g_2(t_1) = g_3(t_1),$$

$$g'_1(t_0) = g'_2(t_0), g'_2(t_1) = g'_3(t_1),$$

$$g''_1(t_0) = g''_2(t_0), g''_2(t_1) = g''_3(t_1),$$

which represent a set of linear equality restrictions on the coefficients of the cubic polynomials.

The *restricted least squares* regression model (RLS) is used to estimate the unknown coefficients. Table II in Appendix reports in its first and second column the complete estimated yield curve for zeros corresponding to May 31, 1993, obtained by using equation (11), while Table III shows the estimated par yield curve for coupon-bearing bonds.

Historical interest rate volatilities in the same table are computed from time series data. For purposes of estimation, the volatility is the *standard deviation* of the daily change in the natural logarithm of the yield, expressed as an annual rate. Assuming we have daily yield observations from the last  $n + 1$  trading days, estimating yield volatilities from historical data implies the following steps :

- (1) Take the natural logarithms of the yields.
- (2) Calculate the changes in the logarithms. There will be  $n$  changes. We denote them as  $y_t$ .
- (3) Compute the mean,  $\bar{y}$ , of the  $y_t$  observations.
- (4) Compute the  $n$  deviations from the mean ( $y_t - \bar{y}$ ). Square these deviations and sum them up.

$$s = \sum_{t=1}^n (y_t - \bar{y})^2$$

- (5) The estimate of the daily variance is  $V^2(y_t) = s / (n - 1)$ . Annualize the volatility by multiplying  $V^2$  by the number of trading days in a year (about 260) to get  $\sigma^2$  for each maturity.
- (6) The volatility  $\sigma$  is the square root of  $\sigma^2$ .

Note that these six steps should be done for each maturity. Then we obtain a complete yield volatility curve by fitting a mathematical curve to these estimated volatility points in the same way as it was done for the yield points. The results are reported in columns three of Table II and Table III. These figures indicate that the volatility rate of closer maturities is greater than

those of distant ones <sup>6</sup>. Zero prices represent the present value of zero-coupon bond of value \$100 at certain maturity date. These present values are given by  $B = 100 / (1 + y_i)^n$ , where  $y_i$  is the interest rate that make the present value of the future cash flow (\$100) equals to the current market price.

Tables II and Table III also report forward interest rates, which are the rates of interest implied by the current spot rates for periods of time in the future. The formula for the *implied forward rate* is :

$${}_n f_t = [ (1 + y_{n+t})^{n+t} / (1 + y_n)^n ]^{1/t} - 1$$

where  ${}_n f_t$  is the forward rate  $n$  periods from now for  $t$  periods, and  $y_n$  is a  $n$ -year spot rate.

Given the U.S.T term structure of Table II in Appendix, we construct a short rate tree for zeros by using the same technique illustrate in Section I. Figure D in Appendix displays in the full tree of short rates at one-year intervals that matches the term structure of Table II. For instance, short rate values of 5.08 and 4.14 per cent in year 2 produce a two-year yield of 3.98 per cent and volatility of 10.21 percent which are consistent with the same values in Table II. In similar manner, short rate values of 6.33, 5.2 and 4.27 per cent in year 3 match a three-year yield of 4.4 per cent and volatility rate of 9.84 percent as shown in Table II. These short rates

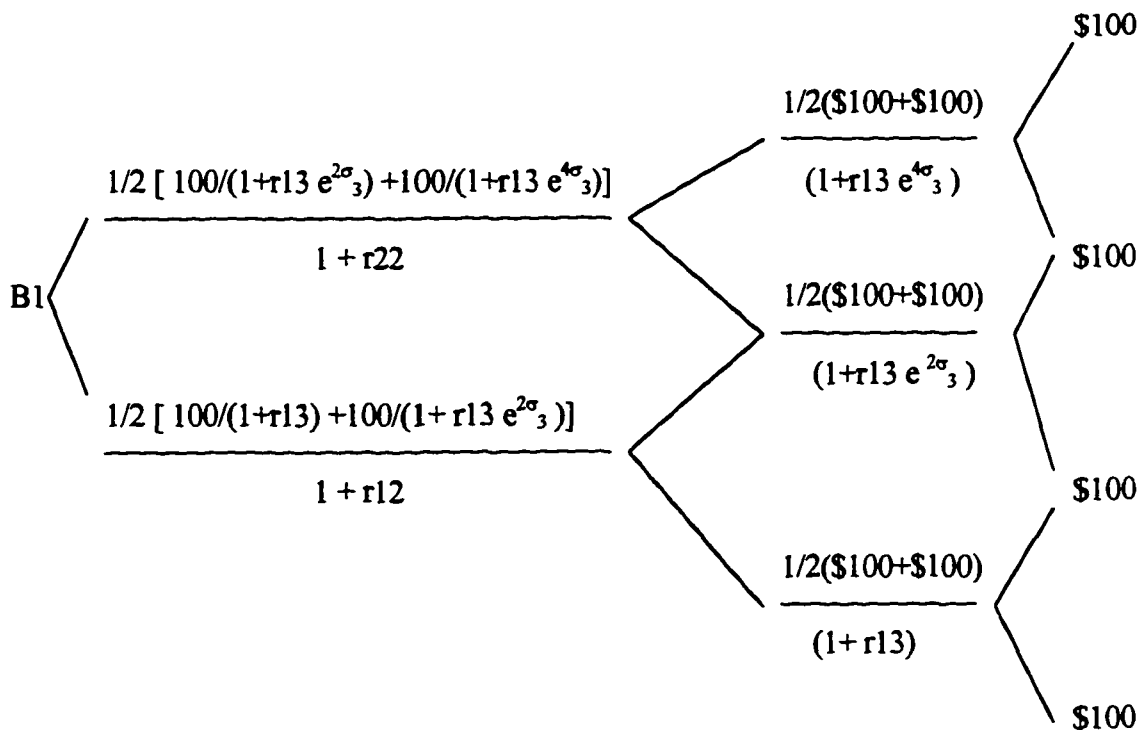
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<sup>6</sup> One explanation for this phenomenon is as follow. Long-term rates are a complex average of short-term rates and forward rates. Since forward rates are less volatile than short-term rates, long-term rates will also be less volatile than short-term rates. Short-term rates are determined by current economic conditions which can change rapidly, while expectation about the short-term rate in 10 years does not change much from day to day.

are computed by employing the *Newton-Raphson method* at each time step. Starting from an initial value  $r_{i,j} = r_{i,j}^*$ , it solves an equation of the form  $f(r_{i,j}) = 0$ , producing successive estimates of the solution by using the formula

$$r_{t+1} = r_t - f(r_t) / f'(r_t)$$

until  $f(r_t^*) = 0$  and then  $r_{t+1}^* = r_t^*$ , which is the solution of the equation system. For example, suppose we want to find the short rates in year 3 ( $r_{31}$ ,  $r_{32}$ , and  $r_{33}$ ) using a 3-year zero. We know that  $r_1 = 3.36\%$ ,  $r_{12} = 4.14\%$ ,  $r_{22} = 5.08\%$ ,  $y_3 = 4.4\%$ ,  $\sigma_3 = 9.84\%$ , and today's zero price is  $\$87.88 (= \$100 / (1.044)^3)$ . Then the price tree is



then,

$$\$87.88 = \frac{\frac{1/4 [100/(1+r_{13} e^{2\sigma_3}) + 100/(1+r_{13} e^{4\sigma_3})]}{1 + r_{22}} + \frac{1/4 [100/(1+r_{13}) + 100/(1+r_{13} e^{2\sigma_3})]}{1 + r_{12}}}{1 + r_1}$$

where the only unknown variable is  $r_{13}$ , which is computed by the Newton-Raphson iterative procedure. We use the same procedure for each time step. The last cell of each column in this tree shows *the average short rate or expected return* for the corresponding period of time. The last column of Table II shows the arithmetic average of those short rates found in Figure D. Note that forward interest rates are lower than the average short rates.

## 2.2. Step Two: Finding the Market Value of the Future Coupon Payments on Bradies

The value of a bond at each node is determined by discounting the average of the two possible bond's prices by the current short interest rates found in step one. Brady bonds are priced by breaking them down into their collateralized principal and uncollateralized coupon payments. Since the principal of the bond is secured by a U.S. Treasury zero-coupon bond, its present value is calculated by using the U.S. term structure for zero-coupon bonds given in Figure D. This present value is compared to the current market price of the bond and the difference between the two values is calculated. This difference may be interpreted as *the market's assessment of the present value of future coupon payments on Bradies*.

The key idea here is that the valuation model discounts the collateralized principal by the short rate term structure on U.S. Treasury zero-coupon bonds and the uncollateralized coupon payments by the short rate term structure on U.S. Treasury coupon-bearing bonds. The logic behind this idea is that the risky part of Brady bonds is given by the uncollateralized coupon payments. Therefore, the implicit interest rates that investors use to discount these default risk coupon payments made up the term structure of an emerging market country. Thus, the country's term structure of interest rates is the one implied by the market value of future coupon payments in Brady bonds. As we said before, any bond may be seen as a package of cash flows, with each cash flow representing a zero-coupon instrument maturing on the date it will be received. We break Argentine Brady bond down into its constituent coupon and principal payments. We use the U.S. term structure obtained in step one to price the principal payment (\$100). Then we compare the present value of this principal with its current market price. The difference between these two values may be interpreted as the market's assessment of the present value of coupon payments on a Brady bond, assuming the short rate tree of Figure D represents the actual U.S term structure employed for investors to value default risk-free zero-coupon securities. Figure E in Appendix exhibits the resulting price tree for a zeros. It shows today's price (\$13.4) of the principal payment that will be received after 30 years, computed by using the assumed term structure in Figure D and the valuation equation for zeros. Since the current par bond's price in the market is \$48.93 (as of 05.31.93), then the market's assessment of the present value of coupon payments is \$35.50 ( $=\$48.93 - \$13.4$ ).

### **2.3. Step Tree: Finding the Country Term Structure of Interest Rates**

To find the implied Argentine term structure of interest rates in Brady bond prices, we introduce the notion of option-adjusted spread (OAS) which is defined as the spread to U.S. short-term interest rates that equals the theoretical price of a bond to its market price. Given the future cash flows of coupon payments and the arbitrage-free U.S. term structure on coupon-bearing bonds, we use the valuation equation to determine the present value of coupon payments. We compare this value to the market's assessment of the present value of coupon payments which was found in step two. Then we compute the constant spread that added to the arbitrage-free short rates on coupon-bearing bonds, makes the theoretical present value of coupon payments equal to their current market's assessment. Therefore, the Argentine term structure implied in Brady bond prices is derived by a parallel shift of the entire U.S. short-rate tree by an amount equal to the OAS. The latter compensates investors for assuming credit risk and can be interpreted as an average extra return for investing in this particular security rather than in a default risk-free U.S. Treasury security. The option-adjusted analysis is a valuation methodology that introduces interest-rate volatility and provides a measure of risk premium over the risk-free rate of Treasury securities, which is called the option-adjusted spread (OAS). This methodology was initially used to evaluate the impact of the embedded options in mortgage-backed securities and callable corporate bonds. To compute the Argentine term

structure implied by the Argentine PAR bond, we have to construct a binomial short rate tree consistent with a theoretical U.S.T term structure for coupon-bearing bonds (on-the-run yield curve) with cash flows identical to the ones of the PAR bond. Suppose the theoretical term structure for the PAR bond is that of Table 4.

Table 4 A sample Term Structure (On-the-Run yield Curve)

Maturity	Coupon Rate	YTM	Value
1 year	4%	4%	100
2 year	4% and 5%	4%	100

We want, for instance, to construct a two-year interest rate tree that values a two-year bond with 4% coupon the first year and 4.25% the second year at 100.

Figure K Finding the future short-term rate for year one using two-year, 4% and 4.25%, On-the-Run Issue ( $\sigma_2=10\%$ ).

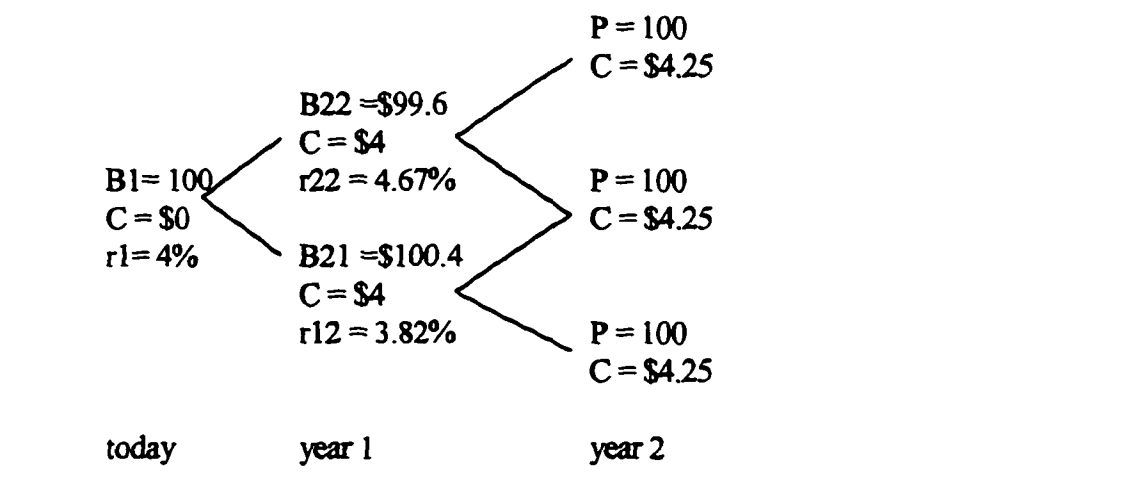


Figure K shows a binomial interest rate tree that gives the cash flow at each node. The root rate for the tree is simply the current one-year rate, 4%. This rate was chosen because it will value a one-year bond with a 4% coupon at 100. We need to find the two possible one-year rates one year from today, consistent with the assumed volatility (=10%), that result in a value of 100 for a bond with 4% coupon the first year and 4.25% the second year. Given the sample term structure of Table 4, we find the short rates one year from today as follows:

(1) Select a value for  $r_{12}$ . In the first trial, we arbitrary select a value of 3.82%. The corresponding value for the future short rate if rates rise is 4.67% ( $= r_{22} = 3.82\% e^{2(0.10)}$ ). The bond's value two years from now is \$104.25 ( $= \$100 + \$4.25$ ). The bond's present values one year from now are \$99.6 ( $= B_{22} = \$104.25 / 1.0467$ ), and \$100.4 ( $= B_{21} = \$104.25 / 1.0382$ ). To obtain cash flows one year from now, add the coupon to both  $B_{12}$  and  $B_{22}$  and use equation (4) to compute prices of coupon-bearing bond. Thus,  $B_1 = 0.5(\$99.6 + \$100.4) + \$4 / 1.04 = \$100$ . Since this value is equal to the *target value* of \$100,  $r_{12} = 3.82\%$  and  $r_{22} = 4.67\%$  are the arbitrage-free short rates. If, however, the value found is not equal to the target value of \$100, our initial value  $r_{12}$  ( $= 3.82\%$ ) is not consistent with the assumed term structure. In this case, repeat the procedure with a different value for  $r_{12}$ . The same steps must be followed to find the one-year rates along the tree. Figure F in Appendix shows the entire interest rate tree using the current term structure in Table III and the procedure described here.

(2) by applying the binomial short rate tree of Figure F and the valuation equation (9), compute the present value of future coupon payments. Note here that the stream of future cash flows is only represented by coupon payments but excludes the face value of the bond.

(3) find the constant spread that added to the short rates in Figure F, makes the theoretical (model) value of the coupon payments equal to the market's assessment of the future coupon payments (Recall from step two that the market's assessment of the present value of coupon payments is \$35.50). According to our definition, the risk premium over the riskless short rate is the value that makes the present value of the security equal to the market price of the security. Since the default risk-free rate has been defined as the return received by investing in U.S. Treasury bonds and assuming they are efficiently priced, the U.S. Treasury yield curve should have a default risk premium of zero. In this case, the target value is \$35.50 and the default risk premium over the entire U.S. Treasury yield curve is 0.0943685, or 9.44% (944 basis points). Traditional measures of yield to maturity spread and stripped spread are 3.89% and 5.84%, respectively. Figure G shows the implied term structure on Argentine Par bond calculated according to this method. The last value of each column represents the average value of short rates at each time step. The constant value (i.e., 0.0943685) is the default risk premium that added to the current U.S. Treasury short rate tree, makes the model's present value of future cash flows equals to the current market's assessment of these cash flows. The default risk premium may be viewed as an average spread over the entire U.S. Treasury term. It represents a parallel shift of the entire yield curve by an amount equal to its value. A

particular value of default risk premium does not provide sufficient information to determine whether a bond is rich or cheap unless its value is compared with those of other securities. The purpose is then to keep track of these estimates of default risk to make comparison for different Brady bonds combined with time series data of default risk premiums for these securities to determine if there have been any recent changes in their spread values. Figure H shows the present value of coupon payment cash flows along the tree calculated using the assumed binomial short rate tree of Figure G.

Tables IV and V in Appendix display alternative measures of risk premiums. Columns (2) and (3) of Table IV are the same as columns (2) and (6) of Table III. We compare these interest rates with the average short rates of Figure G (column (4) of Table IV), and we find that the risk premiums over the U.S. Treasury bond yields for year 2, for example, are 10.39% ( $= 14.36\% - 3.97\%$ ) and 9.63% ( $= 14.36\% - 4.73\%$ ), respectively. In Table V, risk premiums over the U.S. Treasury strips yields for year 2 are 10.38% ( $= 14.36\% - 3.98\%$ ) and 9.75% ( $= 14.36\% - 4.61\%$ ), respectively. We also can determine theoretical future bond's prices along the tree by using the values given in Figures E and H. For example, today's bond price is \$48.93 ( $= B1 = \$13.43 + \$35.50$ ). One year from now theoretical prices will be \$46.21 ( $= B22 = \$12.29 + \$33.92$ ), and \$51.89 ( $= B12 = \$15.47 + \$36.42$ ). In other words, Figure I is the sum of Figure E and Figure H. This method of breaking down the bond's value into its coupon and principal payments allow us to see how they evolve until maturity.

#### **2.4. Step Four: Pricing Callable Bonds and Options on Brady Bonds**

The binomial interest tree is used to value call options included in Brady bonds. We find the values of both a noncallable bond and a callable bond and the difference between the two values is the value of the call option. In addition, an entire call value tree is also computed by using the future prices of Brady bonds at various points in time. Knowing the values of a call at each node, their possible values before expiration are calculated by the same discounting procedure used to price the bond. We also obtain measures of delta hedging, effective duration and effective convexity for the same bonds.

Brady bonds are callable at any time at par value, we can use the binomial interest rate tree to value these callable bonds. The procedure is the same as in the case of the option-free bond but now we change the bond's price at the node where the call option can be exercised by the issuer. Bond's values in the price tree greater than \$100 should be replaced by the call price at the corresponding node. For instance, at node (17,16) in Figure I, a bond value of \$102.49 is replaced with \$100. We do the same thing for the next nodes (17,17), (17,18), (18,17), (18,18), (18,19) and so on. Note that the price tree shown in Figure I is a price tree for a noncallable PAR bond. To determine the value of a callable bond we follow the next steps :

(1) Change the bond's value at the node in which the call price is lower than the price of the bond (the \$100 call is in the money).

(2) Recalculate the binomial price tree by using the term structure of short rates of Figure G. Figure H shows the value at each node of the new price tree. In this particular case,

the difference in bond prices is very small (compare Figures G and H) due to the call option is in the money just in year 16 on.

(3) Determine the call option value as the difference between the value of the option-free bond (a noncallable bond) and the value of the callable bond. In this example, the value of the noncallable bond is \$48.93 and the value of the callable bond is \$48.92, so that the value of the call option is \$0.01.

One important aspect should be noted here : if two bonds have identical characteristics but they differ in the fact that one of them is callable and the other is not, then the former bond must be valued at lower price than the latter one. Since we use the same term structure to find the value of both bonds, the default risk premiums are the same for the callable and noncallable bonds. According to this interpretation, the distinction between these two bonds is reflected only in price differences and not in their corresponding term structure. Currently, there is an active secondary market for options on Brady bonds. The values of *call options* can be found by the same discounting procedure used to price the bond. As an example, suppose we want to value a two-year *European-style call option* on the Par bond with strike price \$100, at node (17,17) in Figure I. One year from now in nodes (17,18) and (18,18) the bond values are \$100.47 and 103.71, respectively. The bond in two years may have a price of \$99.59, \$102.74 or \$105.69. In year 19 the \$100 call is in the money if the bond's price is \$102.74 or \$105.69, otherwise the call is out the money and its value is zero. The value of the call at each node is defined as the difference between the value of the bond and its strike price. Thus the call value

is \$2.74 if the bond is trading at \$102.74 at expiration and \$5.69 if the bond is trading at \$105.69. The call is out of the money when the bond is trading at \$99.59. Given these call values two years ahead (in year 19), we compute today's call values (at node (17,17)) by applying a formula similar to the valuation equation (8) but replacing the bond's values for the values of the options at each node. In Figure G the corresponding short interest rates are 12.04 per cent at node (17,18), 11.77 per cent at node (18,18), and 11.89 per cent at node (17,17). Therefore, the call values are

$$1/2 (0.00 + 2.74) / 1.1204 = 1.22 \quad (\text{at node (17,18)}),$$

and

$$1/2 (2.74 + 5.69) / 1.1177 = 3.77 \quad (\text{at node (18,18)}),$$

and

$$1/2 (1.22 + 3.77) / 1.1189 = 2.23 \quad (\text{at node (17,17)})$$

**Figure L** A Two-year European-Style Call Price Tree (in dollars).

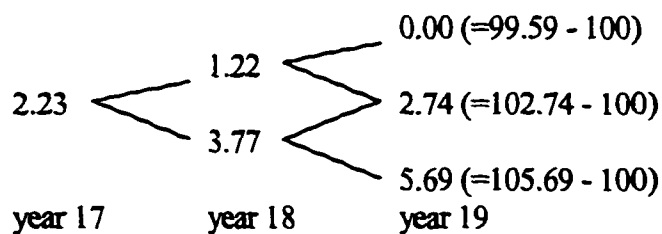


Figure L shows the two-year European option on Argentine PAR bond. We must to find the future call prices at each node on the tree and discount all of them using the same term

structure of Figure G. Today's call price is the sum of the present values of each call price (the present value of the call option portfolio).

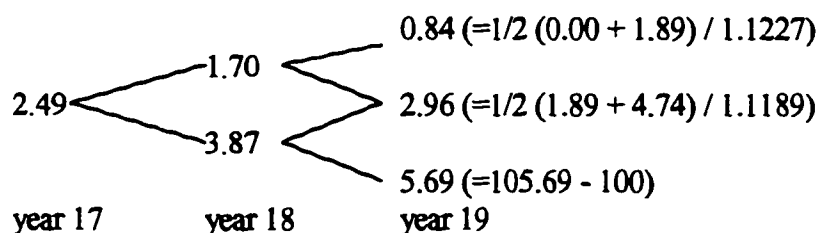
*American-style options* can be valued with a similar procedure along the bond's price tree. Since an American option may be exercised at any time, we have to compare at any node its value if held with its value if exercise and select then the greater of these two values. Its value if exercised is the difference between the bond price and the strike price. Its value if held is the present value of the option values in the next time step. Suppose we want to determine the value of the American option in node (1, 19), that is, state 1 in year 19 in Figure M which corresponds to a node (19,19) in Figure I. If the option is exercised, its value is \$5.69 ( $=\$105.69 - \$100$ ). Its value if held is

$$1/2 (\$4.74 + \$7.40) / 1.1172 = \$5.43$$

where \$4.74 ( $=\$104.74 - \$100$ ) and \$7.40 ( $=\$107.40 - \$100$ ) are the option values one year in the future.

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**Figure M** A Two-year American-Style Call Price Tree (in dollars).



The short rate 11.72% is given in node (19,19) of Figure G. Since the option value if exercised (\$5.69) is greater than if held (\$5.43), \$5.69 is the option value that corresponds at that node.

For states 2 and 3 in year 19, the option values of \$2.96 and \$0.84 if held are greater than \$2.74 and \$0 if exercise as it is shown in Figure M. Today's American-style option value (= \$2.49) is greater than the corresponding European-style option value (= \$2.23).

The binomial tree is also useful to obtain measures of *Delta Hedging*, *Effective Duration* and *Effective Convexity*. Delta hedging ratio is a measure of option value responsiveness to changes in a bond's price. The delta of a call option on bond is defined as the rate of change of its price with respect to the bond's price. Thus, the hedge ratio is :

$$(12) \quad \Delta = \Delta c / \Delta B = c_{i,j+1} - c_{i+1,j+1} / B_{i,j+1} - B_{i+1,j+1}$$

where  $c_{i,j+1}$  and  $c_{i+1,j+1}$  are the call prices one year from now and  $B_{i,j+1}$  and  $B_{i+1,j+1}$  are the bond's prices  $i$  and  $i+1$  in the  $j$  year. Knowing the call values in Figure L and the Par prices in Figure I, we compute

$$\begin{aligned} \Delta &= (1.22 - 3.77) / (100.47 - 103.71) \\ &= 0.787 \end{aligned}$$

This means that a small change in bond's price will yield to a change in the call option price by about 79 per cent of that amount. Investors are also interested in the sensitivity of a bond's

value to changes in interest rates. Effective duration is a measure of the percentage change in price for a small change in interest rate (e.g.; 25 basis points). The bond's price change in response to a small change in interest rate is represented in this model by shifting the entire binomial interest rate tree up and down by 25 basis points. Therefore, effective duration assumes that short rate changes affect future cash flows and take this into account. To estimate the effective duration, we assume that the default risk premium remains constant. We increase the short rate by 25 basis points and we compute the new bond value ( $B^+$ ). Decreasing the short rates by the same basis points we determine the bond value ( $B^-$ ). An approximate effective duration (ED) formula is given by

$$(13) \quad ED = (1 / 2 B_0) (\Delta B / \Delta r)$$

or

$$(14) \quad \Delta B / B_0 = 2 ED \Delta r$$

where  $\Delta B = B^- - B^+$ . Equation (14) measures the average percentage price change per one basis point change in the short rate. Note that the formula does not provide a good estimate of the percentage price change for large rate changes. Since interest rates are expressed per year, the effective duration measure is also expressed in years. Given the new bond prices, we recalculate the new option prices. The same formula then can be used to find the duration of an option. For example, consider the short rate tree in Figure G where the initial interest rate is

12.80 per cent and its corresponding bond's price ( $B_0$ ) is \$48.93. Suppose we shift now the yield curve up 25 basis points, refitting the tree and revaluing the bond gives a value for  $B^+$  of \$46.95. Similarly, shifting the curve down by the same amount gives a value for  $B^-$  of \$51.03.

Thus

$$\begin{aligned} ED &= (1/2 (48.93)) (4.08 / 0.0025) \\ &= 16.67688 \end{aligned}$$

or

$$\Delta B / B_0 = 0.0834, \text{ or } 8.34\%$$

A yield change of 25 basis point (0.0025 or 0.25%) implies an approximate percent price change of 8.34%. The dollar price change due to duration for a 25-basis-point change in yield is \$4.08. For large interest movements, a better approximation for bond price changes is obtained using both duration and convexity. Convexity measures the rate of change of duration as yield change. Effective convexity (EC) of any bond can be approximated by

$$(15) \quad EC = (B^+ + B^- - 2 B_0) / 2 B_0 \Delta r^2$$

## 2.5. Step Five: Pricing Floating and Amortizing bonds

Cash flows in *floating* coupon Brady bonds (i.e., discount bonds) depend on 6-month London Interbank Offer Rate (LIBOR). We assume that the LIBOR follows an stochastic process similar to that of the U.S. Treasury short rate. Thus, we use the binomial model as a discrete representation of the continuous-time processes followed by the LIBOR. Once the U.S. Treasury short rate tree has been estimated, we obtain an estimate of the LIBOR by adding a *swap spread* to the short rate at each node in the tree. Coupon payments on floaters are then equal to the estimated LIBOR plus a constant *spread* (i.e., 13/16%). Cash flows for the bond along each interest rate path in the tree are computed. We calculate the present value for those cash flows along each possible interest rate path by discounting them by the U.S. short rate tree estimated in step three. The value of these present values over all possible interest rate path at the root of the tree is the theoretical value of the bond corresponding to a default free risk security. Finally, we compare this theoretical coupon payments' value with its market price and we determine the option adjusted spread for those floating coupon securities.

To value *amortizing* Brady bonds (see Chart 2 in Appendix), the procedure requires little extra effort. Rather than only one principal at maturity as in the case of bullet bonds, we have now a bond's principal that is thought as a package of cash flows, with each amortization payment viewed as a zero-coupon bond maturing on the date it will be received. The present value of each amortization payment is computed by discounted it for the short rate tree estimated in step three and using equation (7) above.

## **SECTION V**

### **SUMMARY AND CONCLUSIONS**

This study has proposed a valuation methodology to estimate the term structure of interest rates of emerging market countries. Our goal was to obtain an empirical measure of the default risk premium that permits one to perform a relative value analysis among different Brady bonds and across countries and to find potential arbitrage opportunities. Additionally, the term structure of interest rates of an emerging market country was also used to price derivatives and to perform price sensitivity analysis of Brady securities.

The model of the term structure of interest rates was based on a binomial tree. This tree represents the random evolution of future short rates, given some inputs and an assumption about the stochastic process for the U.S. short interest rate. The inputs of the model are the yields on zero-coupon and coupon-bearing U.S. Treasury bonds for various maturities, the yield volatilities for the same bonds, and the current Brady bond prices.

Several authors have implemented models that derive the term structure of interest rates from the behavior of the riskless interest rate. They have used the current U.S.T. yield curves and their estimated volatilities to construct a tree of possible future short rates. Given the resulting tree of short rates, the models were employed to value bond options. In similar fashion, we have constructed a binomial tree of future short rates to model the term structure of interest rates for an emerging market country. This short rate tree, as the ones mentioned above, is consistent with the current U.S.T. term structure (the observed yield curves as well as their estimated volatilities). However, while these studies have

focused on the riskless (meaning that market participants do not perceive these securities of having any default risk) term structure of the U.S. Treasury, our tree includes a constant spread, the default risk premium, which is a spread over the return on Treasury securities. In this study this spread is interpreted as an empirical measure of the default risk premium. It is obtained by finding the constant value over the riskless U.S.T binomial short rate tree that makes the theoretical present value of a Brady bond equal to its market price. Graphically, the default risk premium is given by a parallel shift of the entire U.S.T short rate tree by an amount equal to the constant spread value. It is important to note that an default risk premium value by itself does not provide sufficient information to determine whether a bond is rich or cheap. The default risk premium on a Brady bond must be compared with those of another securities. Also, we can compare time series data of strip spreads with that of default risk premium on the same Brady bond, and see if one series behaves differently from the other. A different pattern can be interpreted as a sign that potential arbitrage opportunities exist.

A second difference with previous studies is that our model is made up of two short rate trees. The first one is a binomial tree for zeros and the other is a one for U.S.T coupon-bearing bonds. This distinction stems from the fact that the principal of Brady bonds are fully secured through U.S.T zero-coupon bonds, but coupon payments on these bonds are uncollateralized. The first tree is a representation of the U.S.T term structure of zero-coupon bonds and is used to value the collateralized part (principal) of Brady bonds. The second tree is a representation of the U.S.T term structure of coupon-bearing bonds (on-the-run yield curve) and used to value the uncollateralized part (coupon payments) of

Brady bonds. The logic behind this idea is that the risky part of any Brady bond is given by the uncollateralized coupon payments. Therefore, the implicit interest rates that investors use to discount these default risk coupon payments make up the term structure of an emerging market country.

A second purpose of finding a term structure for an emerging market country was to use the resulting tree of future short rates to value options on Brady bonds. By using an arbitrage argument, we found the future prices of a Brady bond at various nodes on the price tree. For example, we constructed a portfolio of  $n$  zero-coupon bonds with the same annual payoffs as a Brady bond with  $n$ -years to maturity. The portfolio and the Brady bond have the same value. Then we found the call value at expiration along the tree as the difference between the bond's price (excluding accrued interest) and the strike price. Given the values of the call of an European-style option at expiration, we worked back through the tree until the root of the tree is reached. The value of the call at the root of the tree is today's value. Put option values can be derived in a similar manner.

### **Improving the model**

To transform the basic interest rate tree into a practical tool requires several refinements. In effect, the short rate tree constructed in Section IV assumed one-year time steps, Brady bonds paid annual coupons, and options could only be exercised once a year. In addition, we built a one-factor model of interest rates, and tax considerations did not enter in our estimations of the term structure of interest rates.

To get more accurate solutions for option values, we need a tree with finely spaced steps between today and the option expiration. In practice it is almost impossible to build- for instance, a tree with one-day steps and a 30-year horizon- so that coupon payments and option exercise fall exactly on a node. To solve this, we can use two trees to value an option on Brady bonds. One tree with enough steps to value the bond from its maturity back to today, and the second one with enough steps to value the option accurately from its expiration until today. We find the Brady prices on the first tree from maturity to today by following the same procedure explained in Section IV. Then we interpolate these Brady prices onto the second tree. Maturity, expiration and coupon dates that fall between nodes are interpolated to the nearest node in time.

The other refinement is to take account of tax rates. The standard approach is to discount after-tax cash flows at after-tax rates. We can implement two different approaches. First, we can multiply the entire on-the-run yield curve by  $(1 - \text{tax rate})$ . Using the resulting after-tax yield curve, one can determine a new after-tax short rate tree which can then be used to discount after-tax cash flows. Alternatively, one can construct an after-tax binomial interest rate tree by simply multiplying each short rate on a pretax tree by  $(1 - \text{tax rate})$ . If one constructs an interest rate tree based on an after-tax yield curve, the implied underlying pretax interest rate process is different from that obtained directly from the pretax yield curve.

Our binomial tree was based on a one-factor model of interest rates. It is possible to build a more complex model with more than one factor to describe shifts in the yield curve. A two factor model, for example, can improve the results but is much harder to

work with than a single-factor model. It also takes much more computer time. Still, it is useful to work with a single-factor model but assuming different stochastic processes for the short-rate. For example, it would be possible to assume a process for the short-term interest rate with mean reversion and local volatility that vary deterministically through time.

## **APPENDIX**

**Chart 1. Selected characteristics of Argentine Brady Bonds & Sovereign Loans.**

**DISCOUNT BONDS**

**issuer** Republic of Argentina  
**size of issue** Approx. US\$4.3 billion  
**interest rate** LIBOR + 0.8125%, payable semi-annually  
**issue date** March 31, 1993  
**amortization** 30-year bullet  
**final maturity** March 31, 2023  
**remarks** Principal secured by a 30 year U.S. Treasury zero-coupon bond and 12 months of interest collateral.

**PAR BONDS**

**issuer** Republic of Argentina  
**size of issue** Approx. US\$12.7 billion  
**interest rate** year 1: 4.00%, year 2: 4.25%, year 3: 5.00%, year 4: 5.25%, year 5: 5.5%, year 6: 5.75%, years 7-30: 6.00%; payable semi-annually.  
**issue date** March 31, 1993  
**amortization** 30-year bullet  
**final maturity** March 31, 2023  
**remarks** Eligible for local privatization program. Principal secured by a 30 year U.S. Treasury zero-coupon bond and 12 months of interest collateral

**FLOATING RATE BONDS (FRB)**

**issuer** Republic of Argentina  
**size of issue** Approx. US\$7.1 billion  
**interest rate** Six-month LIBOR + 0.8125, payable semi-annually  
**issue date** March 31, 1993  
**amortization** 12-year bonds repayable in 19 semi-annual installments after 3 years grace; installments 1-7: 1.0% of principal, installment 8: 5.0% of principal; installments 9-19: 8.0% of principal.  
**final maturity** March 31, 2005  
**remarks** issued in exchange for past due interest arrears.

**BANCO DE LA NACION ARGENTINA FRN**

**issuer** Banco de la Nacion Argentina  
**guarantor** Republic of Argentina  
**size of issue** US\$195 million  
**interest rate** LIBOR + 0.75%, payable semi-annually  
**issue date** November 19, 1987  
**amortization** 8 equal semi-annual installments starting April 15, 1994  
**maturity** October 15, 1997

**NEW MONEY BONDS**

**issuer** Republic of Argentina  
**size of issue** US\$500 million  
**interest rate** LIBOR + 0.875%, payable semi-annually  
**issue date** August 1, 1987

amortization maturity 15 equal semi-annual installments starting October 25, 1992  
October 25, 1999

**BONEX**

issuer Central Bank of Argentina  
guarantor Republic of Argentina  
size of issue Total Approx. US\$6.5 billion (Series 1984: US\$1 billion, Series 1987: US\$1 billion; Series 1989: US\$4.5 billion)  
interest rate LIBOR flat payable semi-annually  
issue date Series 1984: December 20, 1984; Series 1987: September, 1987; Series 1989: December 28, 1989.  
amortization 8 equal annual installments starting 3 years from date of issue  
final maturity 10 years from date of issue of all series  
remarks prices include accrued interest and are quoted on residual values. Callable at any time at the residual value plus accrued interest.

**BOTE 1 & 2 (Bonos del Tesoro)**

issuer Central bank of Argentina  
guarantor Republic of Argentina  
size of issue Approx. US\$1.9 billion (BOTE 1: US\$1.3 billion; BOTE 2: US\$600 million)  
interest rate LIBOR flat payable quarterly  
issue date Series 1: March 31, 1991; Series 2: June 1, 1992.  
amortization 16 quarterly installments of 6% and one final of 4%  
final maturity 5 years and 2 months from date of issue  
remarks Prices include accrued interest and are quoted on residual values. Callable at par plus accrued interest.

**BOTE 5 & 10 (Bonos del Tesoro)**

issuer Central Bank of Argentina  
guarantor Republic of Argentina  
size of issue Approx. US\$800 million (Series 5: US\$300 million; Series 10: US\$500 million)  
interest rate LIBOR flat payable quarterly  
issue date Series 5: April 1, 1991; Series 10: April 1, 1990  
amortization Series 5: 14 quarterly installments of 6.6% and one final of 7.6% starting on October 1, 1991; Series 10: 29 quarterly installments of 3.3% and one final of 4.3% starting on January 1, 1993  
final maturity Series 5: 5 years; Series 10: 10 years  
remarks Prices include interest and are quoted on residual values. Callable in the issue's entirety or partially at par plus accrued interest

**BOCON Pre1 & Pre3 (Bonos de Consolidacion De Deudas Previsionales)**

issuer Central Bank of Argentina  
guarantor Republic of Argentina  
size of issue Pre1: Ps.1.3 billion; Pre3: Ps.800 million  
interest rate Monthly interest calculated by the Central Bank at a rate which represents the average effective monthly interest rate on savings account deposits. Interest is compounded monthly and capitalized for six years.  
issue date Pre1: April 1, 1991; Pre3: Sep.1, 1992.

amortization 47 equal monthly installments of 2.08% and one final installment of 2.24% starting May 1, 1997 (Pre1) and Oct. 1, 1998 (Pre3)  
 final maturity Pre1: April 1, 2001; Pre3: Sep. 1, 2002  
 remarks Prices include accrued interest and are quoted on residual values. Callable any time at the residual value plus accrued interest. Eligible for debt equity conversions.

**BOCON Pre2 & Pre4 (Bonos De Consolidacion De Deudas Provisionales)**

issuer Central Bank of Argentina  
 guarantor Republic of Argentina  
 size of issue Pre2: US\$2.0 billion; Pre4: US\$1.9 billion  
 interest rate One month LIBOR, interest is compounded monthly and capitalized for the first six years. Payment will commence with principal amortization installments  
 issue date Pre2: April 1, 1991; Pre4: Sep. 1, 1992  
 amortization 47 equal monthly installments of 2.08% and one final installment of 2.24% starting May 1, 1997 (Pre2) and Oct. 1, 1998 (Pre4)  
 final maturity Pre2: April 1, 2001; Pre4: Sep. 1, 2002  
 remarks Prices include accrued interest and are quoted on residual values. Callable any time at the residual value plus accrued interest. Eligible for debt equity conversions.

**BIC 5 (Bonos De Inversion y Crecimiento)**

issuer Central bank of Argentina  
 guarantor Republic of Argentina  
 size of issue Ps. 1.3 billion  
 interest rate Monthly interest calculated by the Central Bank at a rate which represents the average effective monthly interest rate on time deposits and saving accounts plus 0.9% per month.  
 issue date May 1, 1991  
 amortization 117 equal monthly installments of 0.85% beginning Aug. 1, 1991 and final installment of 0.55% at maturity  
 final maturity May 1, 2000  
 remarks Prices include accrued interest and are quoted on residual values. Callable any time at the residual value plus accrued interest.

**BOCON Pro I & Pro II (Bonos de Consolidacion)**

issuer Republic of Argentina  
 size of issue Pro I: Ps. 1.7 billion; Pro II: US\$0.6 billion  
 interest rate Pro I: One month LIBOR, payable monthly. Pro II: monthly interest rate on deposit in common savings account. Both the BOCON's monthly coupons are capitalized for the first 6 years.  
 issue date April 1, 1991 for both bonds  
 amortization 120 monthly installments with 6 year grace; the first 119 installments are 0.84% and one final of 0.04%  
 final maturity April 1, 2007 for both bonds  
 remarks Physical bonds have been partially released as of January 26, 1993. Eligible for debt to equity conversions and payment of tax related debt.

**CEDULAS HIPOTECARIAS (Banco de la Nacion Series 1)**

<b>issuer</b>	<b>Banco de la Nacion Argentina</b>
<b>guarantor</b>	<b>Republic of Argentina</b>
<b>size of issue</b>	<b>US\$300 million</b>
<b>interest rate</b>	<b>Fixed for the first two years at 7.9%; LIBOR + 2.9% thereafter, payable annually</b>
<b>issue date</b>	<b>September 1, 1993.</b>
<b>amortization</b>	<b>6 annual consecutive installments commencing on September 1, 1995; installments 1-4: 16%; installments 5-6: 18% of principal.</b>
<b>final maturity</b>	<b>September 1, 2000</b>
<b>remarks</b>	<b>Callable at any time at par plus accrued interest.</b>

**Chart 1. Brady Bonds Return Indexes**

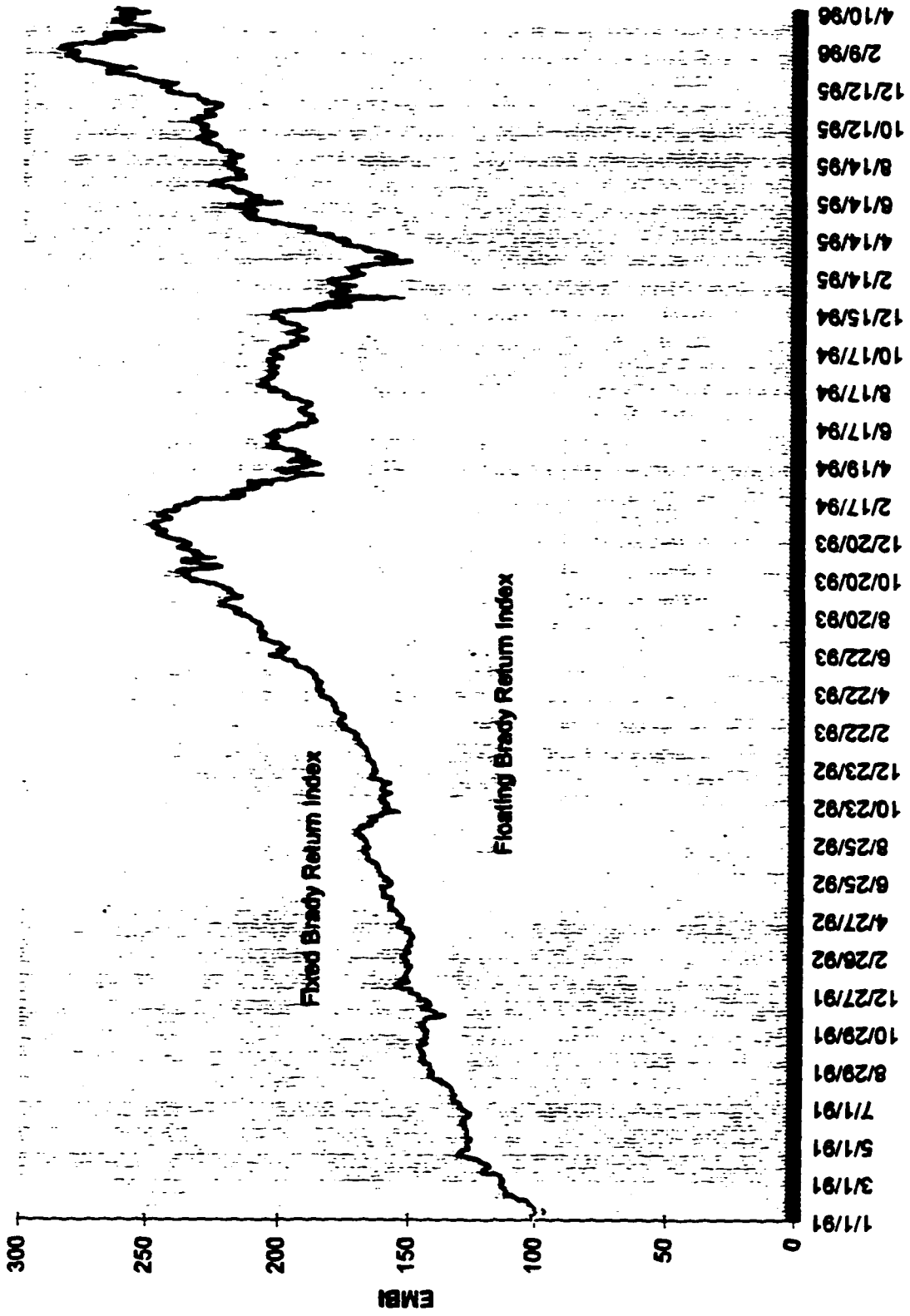


Table 1. EMERGING MARKET SOVEREIGN DEBT (4/18/88)

	Offer Price	Curr. Yield	YTM	Stkps Yield	YTM Spread	Stkps Spread	Avg. Life	Collat. Prio	Collat. Int.	Curr. Cpn.	Spd. LEARN	Final Maturity	Amount \$ miln	Rating Moys
<b>Argentina</b>														
BONEX 89*	48.30	8.00	9.00	9.00	308.00	308.00	2.18	-	-	8.88	-	12/28/89	3.40	B1
FRB*	77.12	8.27	13.38	13.38	703.00	703.00	8.89	-	-	8.37	81.25	3/30/05	8.50	B1
Discount	88.00	9.51	12.25	15.38	851.00	853.00	28.88	18.88	8.44	8.88	81.25	3/31/23	4.14	B1
Par	84.87	8.57	11.20	15.00	448.00	828.00	28.88	15.88	4.89	5.25	-	3/31/23	12.50	B1
<b>Brazil</b>														
NMB 1988*	88.37	7.00	8.18	8.18	325.00	325.00	1.97	-	-	8.75	81.25	10/15/89	0.87	B1
IDU*	91.50	8.97	10.71	10.71	485.00	485.00	2.93	-	-	8.38	81.25	1/1/01	8.84	B1
EI*	75.88	8.88	13.16	13.16	677.00	677.00	8.88	-	-	8.81	81.25	4/15/08	4.30	B1
FLIRB*	57.00	7.89	14.81	15.01	830.00	850.00	9.97	-	4.17	4.50	-	4/15/08	1.70	B1
NMB 1992*	88.75	8.88	14.01	14.01	754.00	754.00	8.88	-	-	8.88	87.50	4/15/09	2.10	B1
DCB*	84.82	10.84	14.38	14.38	785.00	785.00	11.88	-	-	8.88	87.50	4/15/12	8.50	B1
Exit*	88.80	10.08	14.13	14.13	782.00	782.00	10.14	-	-	8.00	-	8/15/13	1.00	B1
C-Bond*	81.00	7.38	14.18	14.18	784.00	784.00	12.88	-	-	4.50	-	4/15/14	7.70	B1
Discount	87.50	10.09	12.80	15.88	584.00	850.00	27.88	14.87	8.88	8.81	81.25	4/15/24	10.80	B1
Par	82.82	8.50	11.43	15.21	487.00	845.00	27.88	18.82	-	5.00	-	4/15/24	12.18	B1
<b>Colombia</b>														
IEB	72.50	8.52	14.20	14.20	787.00	787.00	6.51	-	-	8.50	81.25	12/21/04	0.19	NR
PDH	44.75	8.88	17.87	17.87	1128.00	1128.00	14.88	-	-	3.00	81.25	2/15/15	2.42	NR
Discount	57.25	10.58	14.48	18.84	788.00	1208.00	28.85	14.17	8.42	8.08	81.25	2/28/25	1.44	NR
Par	38.88	8.81	12.31	18.57	554.00	1180.00	28.85	14.17	3.00	3.25	-	2/28/25	1.81	NR
<b>Mexico</b>														
Aztec	88.37	7.80	8.45	12.78	281.00	628.00	11.88	42.87	-	7.81	182.50	3/31/08	2.88	NR
Discount	80.80	8.40	10.75	13.07	403.00	637.00	23.88	18.75	8.81	8.77	81.25	12/31/18	13.80	Ba2
Par	88.50	8.40	8.88	12.88	325.00	688.00	23.88	18.75	8.12	8.25	-	12/31/18	17.88	Ba2
<b>Philippines</b>														
FRN	88.00	7.28	10.24	10.24	418.00	418.00	3.04	-	-	8.75	100.00	8/1/02	0.41	NR
<b>Venezuela</b>														
FLIRB*	84.82	8.97	17.74	17.74	1138.00	1138.00	8.41	-	-	8.38	-	3/31/07	3.02	Ba2
DCB*	84.25	10.21	17.81	17.81	1123.00	1123.00	8.85	-	-	8.58	87.50	12/18/07	5.35	Ba2
Discount	82.50	10.18	13.58	18.97	888.00	1228.00	23.88	18.23	8.82	8.37	81.25	3/31/20	1.23	Ba2
Par	88.25	11.88	12.14	17.45	543.00	1075.00	23.88	18.23	8.11	8.75	-	3/31/20	7.45	Ba2

\* Amortizing bonds.

Sources: Merrill Lynch, Chase Securities, Inc.

Table II U.S.T Term Structure for Zeros (as of 5/31/93)

Maturity (years)	Spot Rates (%)	Yield Volatility (%)	Zero Price (dollars)	Forward Rate (%)	Avg. Short Rate (%)
1	3.36	9.11	95.75	3.36	3.36
2	3.98	10.21	92.49	4.60	4.61
3	4.40	9.84	87.88	5.25	5.27
4	4.82	9.11	82.84	6.09	6.16
5	5.20	8.58	77.61	6.73	6.86
6	5.44	7.94	72.77	6.65	6.83
7	5.66	7.43	68.02	6.99	7.23
8	5.80	7.20	63.70	6.79	7.09
9	5.93	7.02	59.54	6.98	7.41
10	6.04	6.88	55.63	7.04	7.54
11	6.13	6.62	51.97	7.03	7.63
12	6.21	6.40	48.53	7.09	7.79
13	6.28	6.21	45.30	7.12	7.92
14	6.35	6.04	42.24	7.26	8.15
15	6.41	5.89	39.38	7.25	8.26
16	6.47	5.75	36.67	7.37	8.53
17	6.52	5.63	34.17	7.32	8.53
18	6.57	5.52	31.81	7.42	8.78
19	6.61	5.43	29.64	7.33	8.80
20	6.65	5.34	27.59	7.41	9.07
21	6.69	5.26	25.67	7.49	9.26
22	6.72	5.19	23.91	7.35	9.26
23	6.75	5.12	22.26	7.41	9.46
24	6.78	5.06	20.71	7.47	9.67
25	6.81	5.00	19.26	7.53	9.95
26	6.83	4.95	17.95	7.33	9.76
27	6.86	4.90	16.67	7.64	10.51
28	6.88	4.86	15.52	7.42	10.00
29	6.90	4.82	14.44	7.46	11.00
30	6.92	4.78	13.43	7.50	10.81

Table III U.S.T Term Structure for Coupon Bonds (as of 5/31/93)

<b>Maturity (years)</b>	<b>Yield to Maturity (%)</b>	<b>Yield Volatility (%)</b>	<b>Treasury Bond Price (\$)</b>	<b>Forward Rate (%)</b>	<b>Avg. Short Rate (%)</b>
1	4.00	10.25	100.00	4.00	3.36
2	4.00	10.08	100.00	4.00	4.73
3	4.00	9.94	100.00	4.00	4.82
4	5.00	9.02	100.00	8.06	5.06
5	5.00	8.64	100.00	5.00	5.33
6	5.00	7.83	100.00	5.00	5.59
7	5.00	7.26	100.00	5.00	5.86
8	5.00	7.34	100.00	5.00	5.92
9	5.00	6.99	100.00	5.00	5.99
10	5.00	6.75	100.00	5.00	6.05
11	5.00	6.56	100.00	5.00	6.11
12	5.00	6.62	100.00	5.00	6.22
13	5.00	6.29	100.00	5.00	6.28
14	5.00	5.96	100.00	5.00	6.32
15	5.00	5.92	100.00	5.00	6.40
16	5.00	5.79	100.00	5.00	6.52
17	5.00	5.60	100.00	5.00	6.55
18	6.00	5.59	100.00	24.53	6.70
19	6.00	5.40	100.00	6.00	6.73
20	6.00	5.29	100.00	6.00	6.80
21	6.00	5.20	100.00	6.00	6.92
22	6.00	5.15	100.00	6.00	7.04
23	6.00	5.11	100.00	6.00	7.13
24	6.00	5.00	100.00	6.00	7.23
25	6.00	5.04	100.00	6.00	7.37
26	6.00	4.97	100.00	6.00	7.53
27	6.00	4.98	100.00	6.00	7.67
28	6.00	4.83	100.00	6.00	8.00
29	6.00	4.82	100.00	6.00	8.00
30	6.00	4.60	100.00	6.00	8.07

Table III U.S.T Term Structure for Coupon Bonds (as of 5/31/93)

Maturity (years)	Yield (%)	Volatility (%)	Treasury Bond Price (\$)	Forward Rate (%)	Avg. Short Rate (%)
1	3.38	10.25	100.00	3.38	3.38
2	3.97	10.08	100.00	4.58	4.73
3	4.37	9.94	100.00	5.17	4.82
4	4.80	9.02	100.00	6.10	5.08
5	5.19	8.64	100.00	6.76	5.33
6	5.41	7.83	100.00	6.52	5.59
7	5.62	7.26	100.00	6.89	5.86
8	5.79	7.34	100.00	6.99	5.92
9	5.90	6.99	100.00	6.78	5.99
10	5.98	6.75	100.00	6.70	6.05
11	6.09	6.56	100.00	7.20	6.11
12	6.17	6.62	100.00	7.05	6.22
13	6.22	6.29	100.00	6.82	6.26
14	6.31	5.96	100.00	7.49	6.32
15	6.38	5.92	100.00	7.36	6.40
16	6.43	5.79	100.00	7.18	6.52
17	6.49	5.60	100.00	7.45	6.55
18	6.53	5.59	100.00	7.21	6.70
19	6.58	5.40	100.00	7.48	6.73
20	6.60	5.29	100.00	6.98	6.80
21	6.65	5.20	100.00	7.65	6.92
22	6.69	5.15	100.00	7.53	7.04
23	6.70	5.11	100.00	6.92	7.13
24	6.74	5.00	100.00	7.66	7.23
25	6.77	5.04	100.00	7.49	7.37
26	6.79	4.97	100.00	7.29	7.53
27	6.83	4.98	100.00	7.88	7.67
28	6.84	4.83	100.00	7.11	8.00
29	6.86	4.82	100.00	7.42	8.00
30	6.88	4.80	100.00	7.46	8.07

Table IV Interest Rates and Risk Premiums on Argentina PAR Bonds (5/31/93)

Maturity (years)	U.S.T Bond Yield (%)	Avg. Short Rate (%)	PAR bond Short Rate (%)	Risk Premium I (%)	Risk Premium II (%)
1	3.36	3.36	12.80	9.44	9.44
2	3.97	4.73	14.36	10.39	9.63
3	4.37	4.82	14.46	10.09	9.64
4	4.80	5.06	14.75	9.95	9.69
5	5.19	5.33	15.04	9.85	9.71
6	5.41	5.59	15.33	9.92	9.74
7	5.62	5.86	15.64	10.02	9.78
8	5.79	5.92	15.72	9.93	9.80
9	5.90	5.99	15.78	9.88	9.79
10	5.98	6.05	15.84	9.86	9.79
11	6.09	6.11	15.93	9.84	9.82
12	6.17	6.22	16.04	9.87	9.82
13	6.22	6.28	16.09	9.87	9.81
14	6.31	6.32	16.14	9.83	9.82
15	6.38	6.40	16.25	9.87	9.85
16	6.43	6.52	16.33	9.90	9.81
17	6.49	6.55	16.40	9.91	9.85
18	6.53	6.70	16.53	10.00	9.83
19	6.58	6.73	15.82	9.24	9.09
20	6.60	6.80	15.91	9.31	9.11
21	6.65	6.92	15.99	9.34	9.07
22	6.69	7.04	16.90	10.21	9.86
23	6.70	7.13	16.25	9.55	9.12
24	6.74	7.23	16.34	9.60	9.11
25	6.77	7.37	17.30	10.53	9.93
26	6.79	7.53	17.42	10.63	9.89
27	6.83	7.67	16.82	9.99	9.15
28	6.84	8.00	16.89	10.05	8.89
29	6.86	8.00	17.82	10.96	9.82
30	6.88	8.07	18.00	11.12	9.93

**Table V Interest Rates and Risk Premiums on Argentina Par Bonds (5/31/93)**

<b>Maturity (years)</b>	<b>Spot Rates (%)</b>	<b>Avg. Short Rate (%)</b>	<b>PAR bond Short Rate (%)</b>	<b>Risk Premium I (%)</b>	<b>Risk Premium II (%)</b>
1	3.36	3.36	12.80	9.44	9.44
2	3.98	4.61	14.36	10.38	9.75
3	4.40	5.27	14.46	10.06	9.19
4	4.82	6.16	14.75	9.93	8.59
5	5.20	6.86	15.04	9.84	8.18
6	5.44	6.83	15.33	9.89	8.50
7	5.66	7.23	15.64	9.98	8.41
8	5.80	7.09	15.72	9.92	8.63
9	5.93	7.41	15.78	9.85	8.37
10	6.04	7.54	15.84	9.80	8.30
11	6.13	7.63	15.93	9.80	8.30
12	6.21	7.79	16.04	9.83	8.25
13	6.28	7.92	16.09	9.81	8.17
14	6.35	8.15	16.14	9.79	7.99
15	6.41	8.26	16.25	9.84	7.99
16	6.47	8.53	16.33	9.86	7.80
17	6.52	8.53	16.40	9.88	7.87
18	6.57	8.78	16.53	9.96	7.75
19	6.61	8.80	15.82	9.21	7.02
20	6.65	9.07	15.91	9.26	6.84
21	6.69	9.26	15.99	9.30	6.73
22	6.72	9.26	16.90	10.18	7.64
23	6.75	9.46	16.25	9.50	6.79
24	6.78	9.67	16.34	9.56	6.67
25	6.81	9.95	17.30	10.49	7.35
26	6.83	9.76	17.42	10.59	7.66
27	6.86	10.51	16.82	9.96	6.31
28	6.88	10.00	16.89	10.01	6.89
29	6.90	11.00	17.82	10.92	6.82
30	6.92	10.81	18.00	11.08	7.19

**Figure D A Binomial Interest Rate Tree using the data in Table A**

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12	Year 13	Year 14	Year 15	Year 16
3.36%	5.08%	6.33%	7.93%	9.40%	9.80%	10.80%	11.12%	12.18%	12.97%	13.57%	14.31%	15.00%	15.92%	16.80%	17.82%
	4.14%	5.20%	6.61%	7.91%	8.36%	9.31%	9.63%	10.58%	11.30%	11.89%	12.59%	13.25%	14.11%	14.75%	15.71%
	4.61%	4.27%	5.51%	6.67%	7.13%	8.03%	8.34%	9.16%	9.65%	10.41%	11.06%	11.70%	12.50%	13.11%	14.00%
		5.27%	4.59%	5.82%	6.09%	6.92%	7.22%	7.99%	8.59%	9.12%	9.75%	10.34%	11.06%	11.66%	12.48%
			6.16%	4.73%	5.19%	5.96%	6.25%	6.94%	7.48%	7.99%	8.57%	9.13%	9.82%	10.36%	11.13%
				6.86%	4.43%	5.14%	5.42%	6.03%	6.52%	7.00%	7.54%	8.08%	8.70%	9.21%	9.92%
					6.83%	4.43%	4.68%	5.24%	5.68%	6.13%	6.64%	7.12%	7.71%	8.19%	8.84%
						7.23%	4.08%	4.58%	4.95%	5.37%	5.84%	6.29%	6.83%	7.28%	7.88%
							7.08%	3.98%	4.31%	4.70%	5.14%	5.55%	6.06%	6.47%	7.02%
								7.41%	3.76%	4.12%	4.52%	4.91%	5.37%	5.75%	6.20%
									7.54%	3.61%	3.96%	4.33%	4.76%	5.11%	5.58%
										7.63%	3.50%	3.83%	4.21%	4.54%	4.97%
											7.79%	3.38%	3.74%	4.04%	4.43%
												7.92%	3.31%	3.59%	3.95%
													8.15%	3.19%	3.52%
														8.26%	3.14%
															8.53%

Year 17	Year 18	Year 19	Year 20	Year 21	Year 22	Year 23	Year 24	Year 25	Year 26	Year 27	Year 28	Year 29	Year 30
16.12%	16.14%	19.70%	20.65%	21.61%	22.36%	23.41%	24.51%	25.76%	25.90%	26.50%	26.56%	30.03%	31.35%
16.19%	17.14%	17.66%	18.73%	19.63%	20.17%	21.13%	22.15%	23.34%	23.46%	25.64%	25.91%	27.27%	28.50%
14.46%	15.35%	15.66%	16.94%	17.07%	18.16%	19.07%	20.02%	21.12%	21.25%	23.43%	23.51%	24.77%	25.90%
12.92%	13.74%	14.23%	15.13%	15.91%	16.39%	17.21%	18.09%	19.11%	19.25%	21.24%	21.33%	22.46%	23.54%
11.55%	12.31%	12.76%	13.60%	14.32%	14.77%	15.54%	16.35%	17.29%	17.43%	19.26%	19.36%	20.42%	21.39%
10.32%	11.02%	11.45%	12.22%	12.89%	13.32%	14.03%	14.77%	15.64%	15.76%	17.46%	17.57%	18.55%	19.44%
9.22%	9.87%	10.27%	10.96%	11.60%	12.00%	12.66%	13.35%	14.16%	14.30%	15.83%	15.94%	16.84%	17.67%
8.24%	8.84%	9.21%	9.87%	10.44%	10.82%	11.43%	12.07%	12.81%	12.95%	14.35%	14.46%	15.29%	16.06%
7.36%	7.91%	8.27%	8.87%	9.40%	9.75%	10.32%	10.91%	11.59%	11.73%	13.01%	13.12%	13.89%	14.59%
6.56%	7.06%	7.41%	7.97%	8.46%	8.79%	9.31%	9.86%	10.49%	10.63%	11.80%	11.91%	12.61%	13.26%
5.86%	6.35%	6.65%	7.16%	7.62%	7.92%	8.41%	8.91%	9.49%	9.62%	10.70%	10.80%	11.45%	12.05%
5.25%	5.66%	5.97%	6.44%	6.86%	7.14%	7.59%	8.05%	8.56%	8.72%	9.70%	9.80%	10.40%	10.95%
4.69%	5.09%	5.35%	5.79%	6.17%	6.44%	6.85%	7.28%	7.77%	7.90%	8.76%	8.90%	9.45%	9.96%
4.16%	4.56%	4.80%	5.20%	5.56%	5.80%	6.16%	6.56%	7.03%	7.15%	7.97%	8.07%	8.58%	9.05%
3.75%	4.08%	4.31%	4.67%	5.00%	5.23%	5.58%	5.94%	6.36%	6.46%	7.23%	7.32%	7.76%	8.22%
3.35%	3.65%	3.86%	4.20%	4.50%	4.72%	5.04%	5.37%	5.76%	5.87%	6.55%	6.65%	7.07%	7.47%
2.99%	3.27%	3.47%	3.77%	4.05%	4.25%	4.55%	4.85%	5.21%	5.31%	5.94%	6.03%	6.42%	6.79%
2.63%	2.93%	3.11%	3.39%	3.65%	3.83%	4.10%	4.39%	4.71%	4.81%	5.39%	5.47%	5.83%	6.17%
2.28%	2.58%	2.76%	3.05%	3.28%	3.45%	3.71%	3.96%	4.26%	4.36%	4.86%	4.96%	5.30%	5.61%
1.93%	2.18%	2.34%	2.61%	2.86%	3.02%	3.34%	3.58%	3.86%	3.95%	4.43%	4.50%	4.81%	5.10%
1.58%	1.78%	1.91%	2.14%	2.36%	2.53%	2.81%	3.02%	3.24%	3.24%	3.64%	3.71%	4.09%	4.37%
1.23%	1.38%	1.48%	1.68%	1.86%	2.01%	2.26%	2.43%	2.64%	2.64%	2.93%	3.07%	3.27%	3.46%
0.88%	0.98%	1.05%	1.18%	1.28%	1.41%	1.58%	1.73%	1.91%	1.91%	2.11%	2.11%	2.27%	2.46%
0.53%	0.58%	0.62%	0.71%	0.78%	0.86%	0.96%	1.07%	1.19%	1.19%	1.31%	1.31%	1.41%	1.51%
0.18%	0.21%	0.23%	0.28%	0.32%	0.37%	0.43%	0.49%	0.56%	0.56%	0.63%	0.63%	0.69%	0.74%
0.03%	0.04%	0.05%	0.06%	0.07%	0.08%	0.09%	0.10%	0.11%	0.11%	0.12%	0.12%	0.13%	0.14%
0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%

**Figure E A Zero Price Tree using the Interest Rate Tree In Figure D (Dollars)**

Today	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12	Year 13	Year 14	Year 15	Year 16	Year 17
13.43	12.29	11.41	10.69	10.16	9.79	9.47	9.25	9.06	8.96	8.97	9.02	9.16	9.36	9.71	10.15	10.74	11.47
	15.47	14.43	13.57	12.91	12.44	12.03	11.74	11.49	11.36	11.31	11.34	11.47	11.69	12.04	12.50	13.14	13.91
		17.79	16.79	16.01	15.43	14.93	14.57	14.25	14.06	13.97	13.97	14.06	14.29	14.64	15.12	15.79	16.61
			20.32	19.42	18.73	18.13	17.69	17.31	17.06	16.92	16.86	16.96	17.16	17.51	18.00	18.69	19.54
				23.08	22.29	21.60	21.09	20.63	20.32	20.13	20.05	20.09	20.27	20.61	21.10	21.80	22.67
					26.06	25.29	24.70	24.18	23.81	23.56	23.43	23.44	23.59	23.91	24.39	25.08	25.97
						29.13	28.48	27.89	27.46	27.16	26.96	26.95	27.07	27.37	27.83	28.53	29.40
							32.37	31.73	31.25	30.69	30.67	30.60	30.66	30.95	31.39	32.06	32.91
								35.64	35.11	34.70	34.43	34.32	34.36	34.60	35.01	35.66	36.49
									36.99	36.54	36.23	36.06	36.09	36.29	36.67	37.28	38.06
										42.37	42.03	41.84	41.81	41.98	42.32	42.89	43.85
											45.76	45.56	45.49	45.62	45.92	46.46	47.16
												49.20	49.11	49.20	49.46	49.96	50.83
													52.62	52.68	52.91	53.36	53.98
														56.05	56.24	56.64	57.21
															59.43	59.79	60.32
																62.60	63.26
																	66.09

year18	year19	year20	year21	year22	year23	year24	year25	year26	year27	year28	year29	year30
12.40	13.55	15.03	16.91	19.24	22.23	26.12	31.27	37.78	47.05	59.20	76.13	100.00
14.92	16.15	17.72	19.71	22.15	25.25	29.24	34.45	40.95	50.05	61.76	77.82	100.00
17.68	18.97	20.62	22.69	25.22	28.40	32.45	37.68	44.11	53.01	64.27	79.43	100.00
20.65	21.99	23.70	25.84	28.42	31.65	35.73	40.93	47.26	55.89	68.07	80.95	100.00
23.81	25.18	26.93	29.11	31.72	34.97	39.03	44.17	50.36	58.70	68.97	82.38	100.00
27.12	28.51	30.28	32.47	35.09	38.33	42.35	47.39	53.39	61.41	71.16	83.72	100.00
30.54	31.93	33.70	35.89	38.49	41.70	45.64	50.55	56.35	64.02	73.24	84.99	100.00
34.05	35.43	37.18	39.34	41.90	45.04	48.89	53.65	59.21	66.52	75.21	86.16	100.00
37.60	38.95	40.67	42.78	45.29	48.34	52.07	56.65	61.97	68.91	77.07	87.27	100.00
41.15	42.46	44.14	46.19	48.62	51.58	55.16	59.55	64.62	71.17	78.62	86.29	100.00
44.68	45.94	47.56	49.55	51.89	54.73	58.16	62.34	67.14	73.32	80.47	89.24	100.00
48.16	49.36	50.91	52.82	55.06	57.77	61.04	65.01	69.54	75.34	82.01	90.13	100.00
51.55	52.70	54.17	55.99	58.12	60.70	63.80	67.55	71.82	77.24	83.44	90.95	100.00
54.85	55.93	57.32	59.04	61.07	63.51	66.43	69.86	73.96	79.02	84.78	91.70	100.00
58.02	59.04	60.35	61.97	63.88	66.18	68.93	72.24	75.98	80.69	86.02	92.40	100.00
61.07	62.01	63.24	64.76	66.55	68.71	71.29	74.36	77.67	82.25	87.16	93.05	100.00
63.97	64.84	65.99	67.41	69.09	71.10	73.51	76.40	79.63	83.69	88.25	93.64	100.00
66.72	67.53	68.59	69.92	71.47	73.35	75.59	78.26	81.28	85.04	89.23	94.19	100.00
69.32	70.08	71.05	72.27	73.72	75.46	77.54	80.03	82.81	86.28	90.14	94.69	100.00
	72.45	73.35	74.49	75.82	77.44	79.36	81.67	84.23	87.43	90.98	95.15	100.00
		75.51	76.55	77.79	79.28	81.06	83.19	85.55	88.50	91.76	95.57	100.00
			78.48	79.62	80.99	82.63	84.59	86.77	89.48	92.47	95.98	100.00
				81.32	82.58	84.09	85.89	87.69	90.38	93.12	96.31	100.00
					84.05	85.44	87.09	88.93	91.21	93.72	96.64	100.00
						86.68	88.20	89.89	91.97	94.27	96.94	100.00
							89.22	90.70	92.66	94.77	97.21	100.00
								91.57	93.32	95.24	97.46	100.00
									93.91	95.66	97.68	100.00
										96.04	97.89	100.00
											98.08	100.00

**Figure F A Binomial Interest Rate Tree using the data in Table III**

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12	Year 13	Year 14	Year 15	Year 16
3.30%	5.42%	6.05%	6.82%	7.89%	8.41%	9.20%	9.93%	10.40%	10.92%	11.40%	12.35%	12.80%	13.00%	13.75%	14.30%
	4.43%	4.98%	5.68%	6.47%	7.19%	7.96%	8.58%	9.05%	9.54%	10.08%	10.82%	11.19%	11.54%	12.21%	12.74%
	4.92%	4.09%	4.75%	5.44%	6.15%	6.88%	7.40%	7.87%	8.34%	8.84%	9.48%	9.87%	10.24%	10.85%	11.35%
		5.02%	3.97%	4.58%	5.28%	5.95%	6.39%	6.84%	7.28%	7.75%	8.30%	8.70%	9.09%	9.64%	10.11%
			5.31%	3.85%	4.50%	5.15%	5.52%	5.95%	6.38%	6.80%	7.27%	7.67%	8.07%	8.56%	9.00%
				5.60%	3.85%	4.45%	4.77%	5.17%	5.58%	5.98%	6.37%	6.77%	7.18%	7.60%	8.02%
					5.88%	3.85%	4.12%	4.50%	4.88%	5.23%	5.58%	5.97%	6.38%	6.76%	7.14%
						6.21%	3.55%	3.91%	4.24%	4.59%	4.88%	5.28%	5.64%	6.00%	6.38%
							6.28%	3.40%	3.71%	4.02%	4.28%	4.64%	5.01%	5.33%	5.68%
								6.34%	3.24%	3.53%	3.75%	4.09%	4.45%	4.74%	5.04%
									6.41%	3.10%	3.29%	3.61%	3.95%	4.21%	4.49%
										6.49%	2.88%	3.18%	3.50%	3.74%	4.00%
											6.60%	2.81%	3.11%	3.32%	3.58%
												6.65%	2.78%	2.95%	3.17%
													6.71%	2.82%	2.83%
														6.82%	6.89%

Year 17	Year 18	Year 19	Year 20	Year 21	Year 22	Year 23	Year 24	Year 25	Year 26	Year 27	Year 28	Year 29	Year 30	
14.75%	15.59%	15.97%	16.55%	17.13%	17.93%	18.74%	19.29%	20.50%	21.24%	22.39%	22.72%	23.79%	24.91%	
13.18%	13.94%	14.34%	14.89%	15.44%	16.16%	16.92%	17.45%	18.53%	19.23%	20.26%	20.63%	21.60%	22.63%	
11.79%	12.46%	12.87%	13.39%	13.91%	14.59%	15.27%	15.79%	16.75%	17.41%	18.34%	18.73%	19.62%	20.56%	
10.54%	11.15%	11.55%	12.05%	12.54%	13.17%	13.76%	14.26%	15.15%	15.76%	16.60%	17.01%	17.81%	18.67%	
9.42%	9.97%	10.37%	10.84%	11.30%	11.88%	12.45%	12.93%	13.70%	14.27%	15.03%	15.44%	16.18%	16.98%	
8.42%	8.91%	9.31%	9.75%	10.16%	10.72%	11.24%	11.70%	12.36%	12.92%	13.60%	14.02%	14.69%	15.41%	
7.53%	7.97%	8.35%	8.77%	9.18%	9.67%	10.15%	10.59%	11.19%	11.70%	12.31%	12.73%	13.34%	14.00%	
6.73%	7.13%	7.50%	7.89%	8.27%	8.72%	9.16%	9.58%	10.12%	10.59%	11.15%	11.56%	12.11%	12.72%	
6.02%	6.37%	6.73%	7.10%	7.45%	7.87%	8.27%	8.67%	9.15%	9.59%	10.09%	10.49%	11.00%	11.56%	
5.36%	5.70%	6.04%	6.39%	6.72%	7.10%	7.47%	7.84%	8.27%	8.69%	9.13%	9.53%	9.99%	10.50%	
4.81%	5.10%	5.42%	5.75%	6.05%	6.40%	6.74%	7.10%	7.46%	7.86%	8.27%	8.65%	9.07%	9.54%	
4.30%	4.56%	4.87%	5.17%	5.46%	5.78%	6.09%	6.42%	6.76%	7.12%	7.46%	7.85%	8.24%	8.68%	
3.85%	4.07%	4.37%	4.65%	4.92%	5.21%	5.50%	5.81%	6.11%	6.44%	6.77%	7.13%	7.46%	7.87%	
3.44%	3.64%	3.92%	4.18%	4.43%	4.70%	4.96%	5.26%	5.53%	5.83%	6.13%	6.47%	6.79%	7.15%	
3.07%	3.26%	3.52%	3.76%	3.99%	4.24%	4.46%	4.76%	5.00%	5.28%	5.55%	5.86%	6.17%	6.50%	
2.75%	2.91%	3.10%	3.39%	3.60%	3.83%	4.05%	4.30%	4.52%	4.79%	5.02%	5.34%	5.60%	5.90%	
2.46%	2.61%	2.84%	3.05%	3.24%	3.45%	3.69%	3.89%	4.09%	4.33%	4.55%	4.84%	5.09%	5.36%	
0.97%	2.33%	2.55%	2.74%	2.92%	3.11%	3.30%	3.52%	3.69%	3.92%	4.12%	4.40%	4.62%	4.87%	
	7.06%	2.28%	2.46%	2.63%	2.81%	2.98%	3.19%	3.34%	3.55%	3.73%	3.99%	4.20%	4.42%	
		7.18%	2.22%	2.37%	2.53%	2.69%	2.86%	3.02%	3.21%	3.37%	3.63%	3.81%	4.02%	
			7.25%	2.14%	2.29%	2.43%	2.61%	2.73%	2.91%	3.05%	3.29%	3.46%	3.65%	
				7.33%	2.06%	2.19%	2.36%	2.47%	2.63%	2.76%	2.99%	3.14%	3.32%	
					7.46%	1.98%	2.14%	2.23%	2.36%	2.50%	2.71%	2.85%	3.01%	
						7.56%	1.93%	2.02%	2.16%	2.27%	2.40%	2.59%	2.74%	
							7.69%	1.82%	1.95%	2.05%	2.24%	2.35%	2.49%	
								7.86%	1.77%	1.86%	2.03%	2.14%	2.26%	
									7.96%	1.68%	1.84%	1.94%	2.05%	
										1.61%	1.67%	1.76%	1.88%	
											1.67%	1.60%	1.69%	
												1.60%	1.68%	
													1.54%	
														6.56%

**Figure G A Binomial Interest Rate Tree for Argentina PAR bond**

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12	Year 13	Year 14	Year 15	Year 16
12.80%	14.85%	15.48%	16.28%	17.12%	17.85%	18.64%	19.37%	19.84%	20.36%	20.93%	21.76%	22.13%	22.44%	23.16%	23.74%
	13.86%	14.39%	15.13%	15.90%	16.63%	17.39%	18.01%	18.48%	18.88%	19.52%	20.25%	20.63%	20.97%	21.65%	22.18%
	14.36%	13.50%	14.19%	14.88%	15.59%	16.32%	16.84%	17.30%	17.77%	18.26%	18.91%	19.31%	19.66%	20.26%	20.76%
		14.46%	13.41%	14.01%	14.70%	15.39%	15.83%	16.26%	16.72%	17.16%	17.74%	18.14%	18.53%	19.07%	19.54%
			14.75%	13.29%	13.93%	14.58%	14.98%	15.38%	15.80%	16.24%	16.71%	17.11%	17.51%	18.00%	18.44%
				15.04%	13.26%	13.89%	14.20%	14.61%	15.00%	15.40%	15.81%	16.20%	16.60%	17.04%	17.45%
					15.33%	13.26%	13.55%	13.93%	14.29%	14.67%	15.02%	15.40%	15.79%	16.19%	16.58%
						15.64%	12.99%	13.35%	13.68%	14.02%	14.32%	14.70%	15.06%	15.44%	15.80%
							15.72%	12.84%	13.15%	13.48%	13.72%	14.08%	14.45%	14.77%	15.10%
								15.78%	12.68%	12.97%	13.19%	13.53%	13.88%	14.17%	14.46%
									15.84%	12.53%	12.72%	13.04%	13.36%	13.64%	13.93%
										15.93%	12.31%	12.62%	12.94%	13.17%	13.44%
											16.04%	12.24%	12.55%	12.76%	13.00%
												16.09%	12.20%	12.39%	12.61%
													16.14%	12.06%	12.26%
														16.25%	11.95%
															16.33%

Year 17	Year 18	Year 19	Year 20	Year 21	Year 22	Year 23	Year 24	Year 25	Year 26	Year 27	Year 28	Year 29	Year 30
24.16%	25.02%	25.41%	25.98%	26.57%	27.37%	28.18%	28.73%	29.93%	30.88%	31.92%	32.18%	33.23%	34.34%
22.62%	23.38%	23.77%	24.32%	24.87%	25.62%	26.36%	26.66%	27.97%	28.67%	29.70%	30.07%	31.04%	32.08%
21.22%	21.90%	22.31%	22.83%	23.35%	24.03%	24.71%	25.23%	26.19%	26.85%	27.78%	28.17%	29.05%	29.99%
19.97%	20.58%	20.99%	21.49%	21.98%	22.60%	23.23%	23.73%	24.58%	25.20%	26.04%	26.44%	27.25%	28.11%
18.86%	19.40%	19.81%	20.28%	20.74%	21.31%	21.89%	22.37%	23.13%	23.71%	24.47%	24.88%	25.61%	26.40%
17.66%	18.35%	18.74%	19.19%	19.62%	20.15%	20.68%	21.14%	21.82%	22.30%	23.04%	23.46%	24.13%	24.85%
16.97%	17.41%	17.79%	18.21%	18.61%	19.10%	19.59%	20.02%	20.63%	21.14%	21.75%	22.16%	22.76%	23.44%
16.17%	16.56%	16.94%	17.33%	17.71%	18.16%	18.60%	19.02%	19.56%	20.03%	20.58%	20.99%	21.55%	22.18%
15.46%	15.81%	16.17%	16.54%	16.89%	17.30%	17.71%	18.10%	18.56%	19.03%	19.53%	19.93%	20.44%	20.98%
14.82%	15.14%	15.48%	15.82%	16.15%	16.53%	16.91%	17.28%	17.71%	18.12%	18.57%	18.96%	19.43%	19.93%
14.25%	14.53%	14.86%	15.18%	15.49%	15.84%	16.18%	16.53%	16.92%	17.30%	17.70%	18.09%	18.51%	18.97%
13.74%	13.99%	14.31%	14.61%	14.89%	15.21%	15.53%	15.86%	16.20%	16.55%	16.92%	17.29%	17.68%	18.10%
13.26%	13.51%	13.81%	14.09%	14.35%	14.65%	14.93%	15.25%	15.55%	15.86%	16.21%	16.57%	16.92%	17.31%
12.86%	13.08%	13.36%	13.62%	13.87%	14.14%	14.40%	14.69%	14.99%	15.27%	15.57%	15.91%	16.23%	16.59%
12.51%	12.70%	12.96%	13.20%	13.43%	13.68%	13.92%	14.19%	14.43%	14.72%	14.99%	15.31%	15.61%	15.93%
12.19%	12.35%	12.60%	12.82%	13.04%	13.28%	13.48%	13.74%	13.98%	14.22%	14.46%	14.77%	15.04%	15.34%
11.89%	12.04%	12.27%	12.48%	12.68%	12.89%	13.09%	13.33%	13.52%	13.77%	13.99%	14.26%	14.52%	14.80%
16.40%	11.77%	11.98%	12.18%	12.38%	12.55%	12.73%	12.96%	13.13%	13.36%	13.55%	13.84%	14.08%	14.31%
	16.53%	11.72%	11.90%	12.07%	12.25%	12.41%	12.63%	12.78%	12.99%	13.18%	13.43%	13.63%	13.86%
	15.82%	11.65%	11.85%	11.81%	11.97%	12.12%	12.32%	12.46%	12.65%	12.81%	13.06%	13.25%	13.48%
		15.91%	11.59%	11.56%	11.72%	11.86%	12.05%	12.17%	12.35%	12.49%	12.73%	12.90%	13.09%
			15.99%	11.50%	11.50%	11.63%	11.80%	11.90%	12.07%	12.20%	12.43%	12.58%	12.75%
				16.90%	11.42%	11.42%	11.57%	11.67%	11.82%	11.94%	12.15%	12.29%	12.45%
					16.90%	11.25%	11.37%	11.45%	11.60%	11.70%	11.90%	12.03%	12.17%
						16.25%	16.34%	11.26%	11.39%	11.49%	11.67%	11.79%	11.92%
							16.34%	17.30%	11.21%	11.29%	11.47%	11.57%	11.70%
								17.30%	17.42%	11.12%	11.26%	11.36%	11.49%
									16.82%	16.82%	11.11%	11.20%	11.30%
										16.89%	16.89%	11.04%	11.13%
											17.82%	17.82%	16.00%

**Figure H A Coupon Price Tree using the Interest Rate Tree in Figure G (Dollars)**

Today	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12	Year 13	Year 14	Year 15	Year 16	Year 17
35.50	35.02	34.94	34.34	33.66	32.97	32.16	31.22	30.36	29.51	28.67	27.84	27.12	26.37	25.56	24.76	24.01	23.16
	37.07	37.00	36.37	35.67	34.92	34.06	33.08	32.17	31.26	30.37	29.49	28.70	27.88	27.01	26.18	25.33	24.44
		36.92	36.28	37.57	36.77	35.88	34.88	33.92	32.97	32.03	31.10	30.24	29.36	28.44	27.54	26.63	25.68
			40.07	39.35	38.54	37.63	36.60	35.59	34.60	33.62	32.66	31.73	30.79	29.83	28.87	27.90	26.89
				41.02	40.20	39.27	38.23	37.19	36.17	35.16	34.15	33.17	32.18	31.17	30.16	29.13	28.08
					41.75	40.82	39.77	38.71	37.66	36.62	35.58	34.55	33.51	32.46	31.40	30.31	29.18
						42.27	41.21	40.13	39.07	38.00	36.93	35.86	34.78	33.69	32.58	31.44	30.26
							42.56	41.47	40.36	39.30	38.21	37.10	35.98	34.86	33.71	32.52	31.29
								42.70	41.62	40.52	39.41	38.27	37.12	35.96	34.77	33.54	32.27
									42.76	41.65	40.53	39.37	38.19	37.01	35.78	34.51	33.19
										42.71	41.57	40.39	39.19	37.98	36.72	35.41	34.05
											42.54	41.34	40.12	38.89	37.60	36.26	34.86
												42.22	40.99	39.74	38.43	37.05	35.62
													41.79	40.52	39.19	37.79	36.33
														41.25	39.90	38.47	36.98
															40.55	39.10	37.59
																39.69	38.15
																	38.00

	year18	year19	year20	year21	year22	year23	year24	year25	year26	year27	year28	year29	year30
	22.39	21.52	20.59	18.56	16.51	17.33	15.97	14.47	12.69	10.58	7.88	4.47	6.00
	23.57	22.63	21.63	20.54	19.38	18.11	16.65	15.04	13.14	10.89	8.07	4.54	6.00
	24.74	23.72	22.65	21.46	20.23	18.66	17.31	15.56	13.56	11.19	8.25	4.62	6.00
	25.87	24.79	23.64	22.39	21.05	19.59	17.94	16.10	13.96	11.47	8.42	4.66	6.00
	26.97	25.82	24.59	23.27	21.85	20.28	18.54	16.60	14.35	11.73	8.58	4.75	6.00
	28.03	26.81	25.51	24.11	22.61	20.96	19.12	17.07	14.71	11.99	8.73	4.81	6.00
	29.05	27.76	26.39	24.92	23.34	21.60	19.67	17.52	15.06	12.22	8.87	4.86	6.00
	30.01	28.67	27.23	25.69	24.03	22.21	20.19	17.94	15.38	12.44	9.00	4.91	6.00
	30.93	29.53	28.03	26.42	24.69	22.79	20.68	18.34	15.69	12.65	9.12	4.96	6.00
	31.80	30.34	28.79	27.11	25.30	23.33	21.14	18.71	15.97	12.85	9.23	5.00	6.00
	32.62	31.11	29.50	27.76	25.89	23.84	21.58	19.06	16.24	13.03	9.33	5.04	6.00
	33.39	31.83	30.16	28.37	26.43	24.31	21.96	19.39	16.48	13.19	9.43	5.06	6.00
	34.10	32.50	30.78	28.94	26.94	24.76	22.38	19.89	16.71	13.35	9.52	5.11	6.00
	34.77	33.12	31.37	29.47	27.41	25.17	22.71	19.97	16.92	13.49	9.60	5.15	6.00
	35.39	33.71	31.90	29.96	27.86	25.55	23.03	20.23	17.12	13.63	9.68	5.16	6.00
	35.97	34.25	32.41	30.42	28.26	25.91	23.33	20.48	17.30	13.75	9.75	5.20	6.00
	36.50	34.75	32.87	30.84	28.64	26.24	23.61	20.70	17.47	13.86	9.81	5.23	6.00
	36.99	35.21	33.30	31.23	28.99	26.54	23.87	20.90	17.63	13.97	9.87	5.25	6.00
	37.44	35.63	33.69	31.59	29.31	26.82	24.10	21.09	17.77	14.06	9.93	5.27	6.00
		36.02	34.05	31.92	29.61	27.06	24.32	21.27	17.90	14.15	9.98	5.29	6.00
			34.39	32.23	29.86	27.32	24.52	21.43	18.02	14.23	10.02	5.31	6.00
				32.51	30.13	27.53	24.70	21.57	18.13	14.30	10.08	5.32	6.00
					30.36	27.73	24.87	21.71	18.23	14.37	10.10	5.34	6.00
						27.92	25.02	21.83	18.32	14.43	10.14	5.35	6.00
							25.16	21.94	18.40	14.49	10.17	5.36	6.00
								22.04	18.46	14.54	10.20	5.37	6.00
									18.55	14.59	10.22	5.38	6.00
										14.63	10.25	5.39	6.00
											10.27	5.40	6.00
												5.41	6.00
													6.00

**Figure 1 A Price Tree for Argentina PAR bonds (Dollars)**

Today	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12	Year 13	Year 14	Year 15	Year 16	Year 17
48.93	47.31	46.35	45.03	43.64	42.77	41.63	40.47	39.43	38.49	37.64	36.87	36.28	35.75	35.27	34.94	34.75	34.65
	52.54	51.43	49.93	48.59	47.36	46.09	44.83	43.66	42.62	41.68	40.84	40.17	39.57	39.05	38.68	38.47	38.36
		56.71	55.07	53.58	52.21	50.82	49.45	48.17	47.03	46.00	45.07	44.31	43.64	43.08	42.67	42.42	42.29
			60.38	58.77	57.27	55.78	54.29	52.90	51.67	50.55	49.54	48.69	47.95	47.33	46.87	46.59	46.43
				64.10	62.49	60.88	59.32	57.83	56.50	55.29	54.20	53.26	52.45	51.78	51.26	50.93	50.73
					67.81	66.11	64.47	62.88	61.47	60.17	59.01	57.99	57.10	56.37	55.79	55.40	55.15
						71.40	69.69	68.03	66.53	65.18	63.92	62.81	61.85	61.08	60.41	59.97	59.68
							74.93	73.20	71.63	70.19	68.86	67.70	66.66	65.60	65.09	64.58	64.20
								78.34	76.72	75.22	73.84	72.59	71.49	70.56	69.78	69.20	68.75
									81.75	80.19	78.76	77.45	76.28	75.30	74.44	73.79	73.26
										85.08	83.60	82.23	81.00	79.96	79.04	78.31	77.70
											88.32	86.90	85.62	84.51	83.53	82.72	82.04
												91.42	90.10	88.94	87.89	87.01	86.25
													94.41	93.21	92.10	91.15	90.31
														97.29	96.13	95.11	94.19
															99.98	98.90	97.90
																102.49	101.42
																	104.75



**Figure J A Price Free for Argentina PAR bond (Dollars), callable at any time at 100.**

Today	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12	Year 13	Year 14	Year 15	Year 16	Year 17
48.92	47.31	46.35	45.03	43.84	42.77	41.63	40.47	39.43	38.49	37.64	36.87	36.28	35.75	35.27	34.84	34.45	34.05
	52.52	51.42	49.93	48.58	47.36	46.09	44.83	43.66	42.62	41.68	40.84	40.17	39.57	39.05	38.68	38.47	38.36
	58.88	58.08	55.05	53.57	52.20	50.81	49.45	48.17	47.03	46.00	45.07	44.31	43.64	43.08	42.67	42.42	42.29
		60.34	60.34	58.74	57.25	55.75	54.29	52.90	51.67	50.55	49.54	48.69	47.95	47.33	46.87	46.59	46.43
			64.04	64.04	62.44	60.85	59.30	57.82	56.49	55.29	54.20	53.26	52.45	51.76	51.26	50.93	50.73
				67.72	66.05	64.43	62.86	61.45	60.17	59.01	57.99	57.09	57.10	56.37	55.79	55.40	55.15
					71.27	71.27	69.60	67.97	66.49	65.14	63.91	62.81	61.85	61.06	60.41	59.97	59.66
						74.75	73.07	71.54	70.13	68.85	67.76	66.85	66.05	65.30	64.69	64.20	64.20
							78.10	76.54	75.09	73.76	72.54	71.46	70.55	69.78	69.20	68.75	68.75
								81.43	79.95	78.58	77.32	76.20	75.25	74.42	73.78	73.28	73.28
									84.68	83.28	81.97	80.81	79.83	78.96	78.28	77.89	77.89
										87.78	86.47	85.27	84.25	83.34	82.60	81.97	81.97
											90.72	89.58	88.49	87.52	86.74	86.06	86.06
												93.50	92.54	91.53	90.64	89.93	89.93
													96.06	95.32	94.45	93.50	93.50
														98.28	97.68	97.22	97.22
															100.00	100.00	100.00
																	100.00



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